Derivation of θ'_{max} for $FS_{failure}$ of Seated Beam w/ Wind & Centrifugal force

$$FS_{cr} = \frac{K_\theta(\theta - \alpha)}{W[(z_0 + y_r)\theta + e_i + z_t) + M_{ot}}$$

when cracked $\bar{z}_0 = z_0(1 + 2.5\theta)$ [Mast, Part 2, Eq 23]

$$\frac{d}{d\theta} FS_{cr}(\theta'_{max}) = 0$$

take derivative, set equal to zero solve for θ'_{max}

$FS_{cr}(\theta)$ is of the form $\frac{u}{v}$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$$

$$u = K_\theta(\theta - \alpha)$$

$$v = W[(z_0(1 + 2.5\theta) + y_r)\theta + e_i + z_t] + M_{ot}$$

$$u' = K_\theta$$

$$v' = W[z_0(1 + 5\theta) + y_r]$$

for $\frac{u'v - v'u}{v^2} = 0$ $\qquad u'v - v'u = 0$

Substitute & simplify

$$2.5 z_0 W \theta^3 + W(y_r - 4z_0)\theta^2$$
$$+ \left[W(e_i + z_t + z_0(5\alpha - 1) - y_r) + M_{ot}\right]\theta + W(z_0 + y_r)\alpha = 0$$

Solve for θ

There can be up to 3 values for θ

θ_{max} = greatest value but not to exceed 0.4 rad.

0.4 rad limit comes from Mast Part 2

max tilt $\approx 23° = 0.4$ r

Project				Sheet No. of Sheets
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the pages that follow
are my actual calcs

$$FS_{cr} = \frac{K_\theta(\theta - \alpha)}{W\left[(z_o + y_r)\theta + e_i + z_w - z_{cc}\right] + M_{ot}}$$

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$$\frac{d}{dx}\frac{u}{v} = \frac{u'v - v'u}{v^2}$$

$$\bar{z}_o' = z_o(1+2.5\theta) \rightarrow$$ when cracked z_o becomes $\bar{z}_o$ (Mast Part 2, Eqn 23)

$$u = K_\theta(\theta - \alpha)$$

$$v = W\left[(z_o(1+2.5\theta) + y_r)\theta + e_i + z_w - z_{cf}\right] + M_{ot}$$

$$u' = K_\theta$$

$$v = W z_o \theta + W z_o 2.5\theta^2 + W y_r \theta + W e_i + W z_w - W z_{cf} + M_{ot}$$

$$v' = W z_o + 5 W z_o \theta + W y_r$$

$$v' = W\left[z_o(1+5\theta) + y_r\right]$$

$$\frac{\left[(K_\theta \theta)\left[W\left[(z_o(1+2.5\theta)+y_r)\theta + e_i + z_w - z_{cf}\right] + M_{ot}\right] - W\left[z_o(1+5\theta)+y_r\right]K_\theta(\theta-\alpha)\right]}{\left\{W\left[(z_o(1+2.5\theta)+y_r)\theta + e_i + z_w - z_{cf}\right] + M_{ot}\right\}^2} = 0$$

solve for θ

$$\cancel{K_\theta}\,\theta\left[W\left[(z_0(1+2.5\theta)+y_r)\theta+e_i+z_w-z_{cf}\right]+M_{ot}\right]$$

$$= W\left[z_0(1+5\theta)+y_r\right]\cancel{K_\theta}(\theta-\alpha)$$

$$\left\{W\left[\underbrace{(z_0(1+2.5\theta)+y_r)\theta}+e_i+z_w-z_{cf}\right]+M_{ot}\right\}\theta = W(\theta-\alpha)\left[z_0(1+5\theta)+y_r\right]$$

$$z_0\theta(1+2.5\theta)+y_r\theta$$

$$z_0\theta+2.5z_0\theta^2+y_r\theta$$

$$2.5z_0\theta^2+(z_0+y_r)\theta$$

$$2.5z_0W\theta^3+W(z_0+y_r)\theta^2+\left[(e_i+z_w-z_{cf})W+M_{ot}\right]\theta$$

$$= W(\theta-\alpha)(z_0+5z_0\theta+y_r)$$

$$= (W\theta-W\alpha)(z_0+5z_0\theta+y_r)$$

$$= Wz_0\theta+5Wz_0\theta^2+Wy_r\theta-Wz_0\alpha-5Wz_0\alpha\theta-Wy_r\alpha$$

$$= 5Wz_0\theta^2+(Wz_0+Wy_r-5Wz_0\alpha)\theta-W(z_0+y_r)\alpha$$

$$2.5z_oW\theta^3 + W\left[\overbrace{(z_o+y_r)-5z_o}^{y_r-4z_o}\right]\theta^2$$

$$+\left[W(e_i+z_w-z_{cf}-z_o-y_r+5z_o\alpha)+M_o+\right]\theta + W(z_o+y_r)\alpha = 0$$

$$5z_o\alpha - z_o$$

$$= z_o(5\alpha-1)$$