Design References

LRFD AASHTO LRFD Bridge Design Specifications, 9th Edition

MAST 2 Lateral Stability of Long Prestressed Concrete Beams - Part 2, PCI Journal, Jan-Feb 1993

LIFTING FROM BED - VERTICAL CABLES

Concrete Properties

Concrete Compressive Strength, $f_{c.lift1}$ = 5.50 ksi

Unit Weight of Unreinforced Girder Concrete, w_c = 0.150 kcf

Correction Factor for Modulus of Elasticity, K_1 = 1.00

Concrete Density Modification Factor, λ = 1.00

Concrete Modulus of Elasticity, $E_{c.lift1}$: **LRFD Eq. 5.4.2.4-1**

$$E_{c.lift1} = 120000\, K_1\, w_c^2\, f_{c.lift1}^{0.33}$$

= 120000(1.00) (0.150 kcf)² (5.50 ksi) $^{0.33}$

= 4739 ksi

Modulus of Rupture, $f_{r.lift1}$: **LRFD 5.4.2.6**

$$f_{r.lift1} = -0.24\ \lambda \sqrt{f_{c.lift1}}$$

= -0.24(1.00)$\sqrt{5.50 \text{ ksi}}$

= -0.563 ksi

Prestress Force

Effective Prestress Force at Lifting, $P_{eff.lift1}$ = 1232 kips

CG of Strands at Midspan to Bottom of Girder, $y_{cgs.mid.lift1}$ = 5.000 in

Camber, $\Delta_{camb.lift1}$ = 2.920 in

Other Configuration Parameters

Lift Connection Locations from End of Girder, a_{lift1} = 9.00 ft

Rigid Extension Lift Connection above Top of Girder, y_{lift} = 0.000 in

Lift Connection Tolerance from Centerline of Girder, e_{conn} = 0.935 in

Other Loading Parameters

Lateral Wind Force, $w_{wind.lift1}$ = 0.000 klf

Vertical Wind Uplift Considered Negligible for Lifting

Lifting Impact Factor, IM_{lift1} = 0% = 0.00

Unit Weight of Girder, $w_{DC.girde}$ 0.876 klf

Effective Unit Weight of Girder, $w_{DC.girder.lift1}$:

$$w_{DC.girder.lift1} = w_{DC.girder}\ (1 + IM_{lift1})$$

= (0.876 klf)(1 + 0.00)

= 0.876 klf

Girder Eccentricities

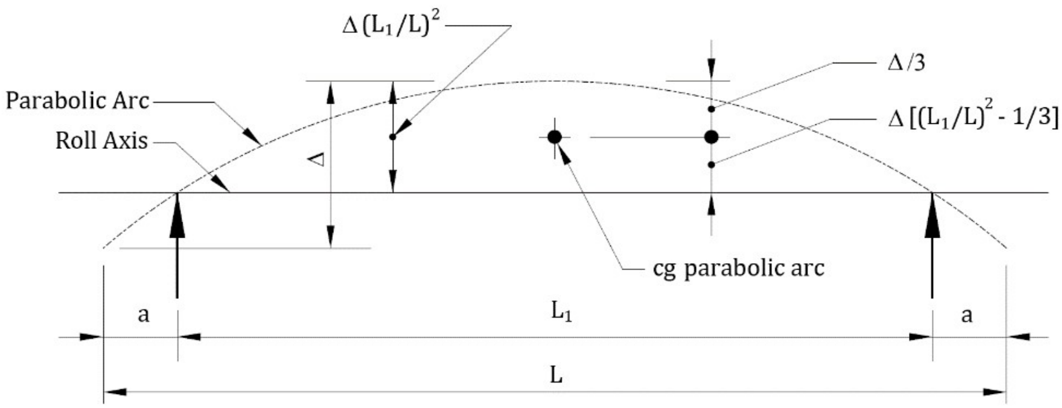
Total Lateral Deflection over Girder Length, $e_{i.total.lif}$ 0.850 in (Assumed Parabolic)

Center of Mass Eccentricity Reduction Factor, $offset_{lift1}$:

$$L_{1.lift1} = L_{girder} - 2\,a_{lift1}$$

= 136.00 ft - 2(9.00 ft)

= 118.00 ft



Offset to centroid of a parabolically deflected girder from roll axis (used horizontally and vertically)

based on MAST 2 Figure B1, with generic variables

$$offset_{lift1} = \left(\frac{L_{1.lift1}}{L_{girder}}\right)^2 - \frac{1}{3}$$

$$= \left(\frac{118.00\text{ ft}}{136.00\text{ ft}}\right)^2 - \frac{1}{3}$$

= 0.419

Center of Mass Eccentricity Due to Lateral Deflection, $e_{i.lift1}$:

$$e_{i.lift1} = e_{i.total.lift1}\, offset_{lift1}$$

= (0.850 in)(0.419)

= 0.357 in

Distance from the Center of Mass of the Cambered Girder below Roll Axis, $y_{r.lift1}$:

$$y_{r.lift1} = y_t - offset_{lift1}\,\Delta_{camb.lift1} + y_{lift}$$

= 35.400 in - 0.419(2.920 in) + 0.000 in

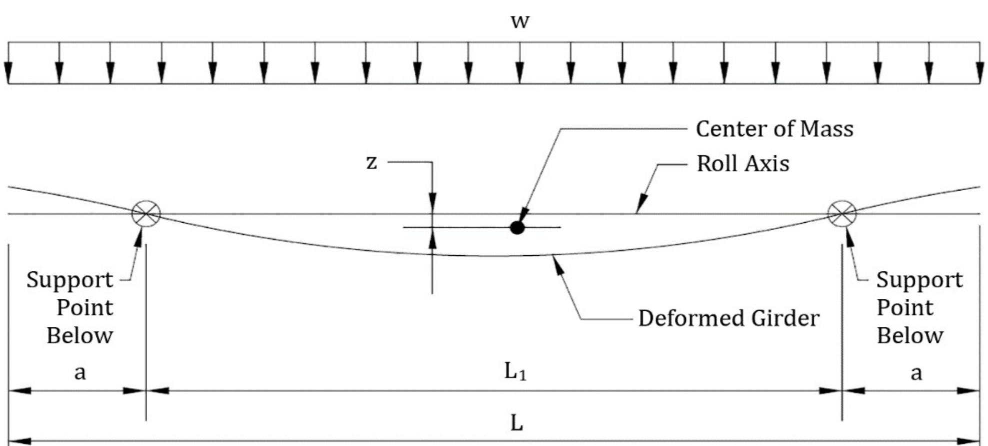
= 34.175 in

Center of Mass Eccentricity Due to Wind Deflection, $z_{wind.lift1}$:

$$z_{wind.lift1} = \frac{w_{wind.lift1}}{12\,E_{c.lift1}\,I_y\,L_{girder}}\left(\frac{L_{1.lift1}^5}{10} - a_{lift1}^2\,L_{1.lift1}^3 + 3\,a_{lift1}^4\,L_{1.lift1} + \frac{6}{5}\,a_{lift1}^5\right)$$

$$= \left(\frac{(0.000\text{ klf})(12\text{ in/ft})^3}{(12)(4{,}739\text{ ksi})(37{,}634\text{ in})^4\,(136.00\text{ ft})}\right)\left(\frac{(118.00\text{ ft})^5}{10} - (9.00\text{ ft})^2(118.00\text{ ft})^3 + 3(9.00\text{ ft})^4\,(118.00\text{ ft}) + \frac{6(9.00\text{ ft})^5}{5}\right)$$

= 0.000 in



PLAN VIEW

Mid-Height of the Cambered Arc Below Roll Axis, $y_{w.lift1}$:

$$y_{w.lift1} = \frac{h_{girder}}{2} + y_{lift} - offset_{lift1}\,\Delta_{camber.lift1}$$

$$= \frac{72.000\text{ in}}{2} + 0.000\text{ in} - 0.419(2.920\text{ in})$$

= 34.775 in

Center of Mass Eccentricity Due to Girder Weight on Weak Axis, $z_{0.lift1}$:

$$z_{0.lift1} = \frac{w_{DC.girder.lift1}}{12\,E_{c.lift1}\,I_y\,L_{girder}}\left(\frac{L_{1.lift1}^5}{10} - a_{lift1}^2\,L_{1.lift1}^3 + 3\,a_{lift1}^4\,L_{1.lift1} + \frac{6}{5}\,a_{lift1}^5\right)$$

$$= \left(\frac{(0.876\text{ klf})(12\text{ in/ft})^3}{(12)(4{,}739\text{ ksi})(37{,}634\text{ in}^4)(136.00\text{ ft})}\right)\left(\frac{(118.00\text{ ft})^5}{10} - (9.00\text{ ft})^2(118.00\text{ ft})^3 + 3(9.00\text{ ft})^4\,(118.00\text{ ft}) + \frac{6(9.00\text{ ft})^5}{5}\right)$$

= 11.218 in

Eccentricity of Girder Dead Load to Equilibrate Wind Load, $e_{wind.lift1}$:

$$e_{wind.lift1} = \frac{w_{wind.lift1}\, y_{w.lift1}}{w_{DC.girder.lift1}}$$

$$= \frac{(0.000 \text{ klf})(34.775 \text{ in})}{0.876 \text{ klf}} = 0.000 \text{ in}$$

Check Stresses

Moment Due to Gravity Load, $M_{g.lift1}$ at $x = a_{harp}$ from girder end:

$$M_{g.lift1} = \frac{w_{DC.girder.lift1}\, L_{girder}}{2}\left(x - a_{lift1}\right) - \frac{w_{DC.girder.lift1}\, x^2}{2}$$

$$= \frac{(0.876 \text{ klf})(136.00 \text{ ft})}{2}(54.40 \text{ ft} - 9.00 \text{ ft}) - \frac{(0.876 \text{ klf})(54.40 \text{ ft})^2}{2}$$

$$= 1408 \text{ kip-ft} = 16898 \text{ kip-in}$$

Lateral Moment Due to Wind, $M_{wind.lift1}$ at $x = a_{harp}$ from girder end:

$$M_{wind.lift1} = \frac{w_{wind.lift1}\, L_{girder}}{2}\left(x - a_{lift1}\right) - \frac{w_{wind.lift1}\, x^2}{2}$$

$$= \frac{(0.000 \text{ klf})(136.00 \text{ ft})}{2}(54.40\text{ft} - 9.00 \text{ ft}) - \frac{(0.000 \text{ klf})(54.40 \text{ ft})^2}{2}$$

$$= 0.0 \text{ kip-ft} = 0 \text{ kip-in}$$

Base Concrete Stresses in Girder before Rotation and Wind

$$f_{t.lift1.base} = P_{eff.lift1}\left(\frac{1}{A_{girder}} - \frac{y_b - y_{cgs.mid.lift1}}{S_{x.t}}\right) + \frac{M_{g.lift1}}{S_{x.t}}$$

$$= 1{,}232 \text{ kips}\left(\frac{1}{767.0 \text{ in}^2} - \frac{36.600 \text{ in} - 5.000 \text{ in}}{15{,}421 \text{ in}^3}\right) + \frac{16{,}898 \text{ kip-in}}{15{,}421 \text{ in}^3}$$

$$= 0.177 \text{ ksi}$$

$$f_{b.lift1.base} = P_{eff.lift1}\left(\frac{1}{A_{girder}} + \frac{y_b - y_{cgs.mid.lift1}}{S_{x.b}}\right) - \frac{M_{g.lift1}}{S_{x.b}}$$

$$= 1{,}232 \text{ kips}\left(\frac{1}{767.0 \text{ in}^2} + \frac{36.600 \text{ in} - 5.000 \text{ in}}{14{,}915 \text{ in}^3}\right) - \frac{16{,}898 \text{ kip-in}}{14{,}915 \text{ in}^3}$$

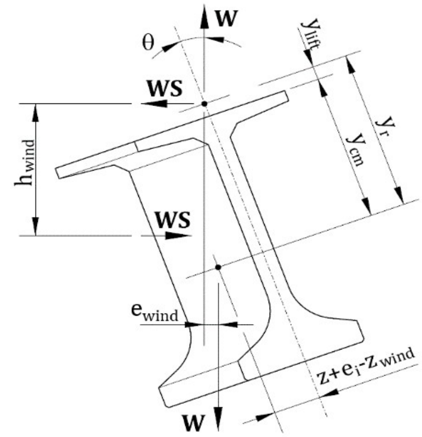
$$= 3.084 \text{ ksi}$$

Check Compressive and Tensile Stresses at Equilibrium Rotation - Wind Right

$$\theta_{eq.lift1.wr} = \frac{\left(e_{i.lift1} + e_{conn} - z_{wind.lift1} + e_{wind.lift1}\right)}{y_{r.lift1} - z_{0.lift1}}$$

$$= \frac{(\text{ 0.357 in + 0.935 in - 0.000 in + 0.000 in })}{\text{34.175 in - 11.218 in}}$$

$$= \quad 0.05626 \quad \text{rad}$$



Girder Free Body Diagram, Wind Right

Check of Bottom Flange Tips - Wind Right

$$f_{b.lift1.wr.left} = f_{b.lift1.base} - \frac{M_{g.lift1}\ \theta_{eq.lift1.wr}}{S_{y.b}} + \frac{M_{wind.lift1}}{S_{y.b}}$$

$$= \text{3.084 ksi} - \frac{\text{(16,898 kip-in)(0.05626 rad)}}{\text{2,895 in}^3} + \frac{\text{0 kip-in}}{\text{2,895 in}^3}$$

$$= \quad 2.755 \quad \text{ksi}$$

$$f_{b.lift1.wr.right} = f_{b.lift1.base} + \frac{M_{g.lift1}\ \theta_{eq.lift1.wr}}{S_{y.b}} - \frac{M_{wind.lift1}}{S_{y.b}}$$

$$= \text{3.084 ksi} + \frac{\text{(16,898 kip-in)(0.05626 rad)}}{\text{2,895 in}^3} - \frac{\text{0 kip-in}}{\text{2,895 in}^3}$$

$$= \quad 3.412 \quad \text{ksi}$$

$$f_{b.ck.lift1.wr} = \text{Max}(\,f_{b.lift1.wr.left}\,,\,f_{b.lift1.wr.right})$$

$$= \text{Max (2.755 ksi , 3.412 ksi)} = \quad 3.412 \quad \text{ksi}$$

Check of Top Flange Tips - Wind Right

$$f_{t.lift1.wr.left} = f_{t.lift1.base} - \frac{M_{g.lift1}\ \theta_{eq.lift1.wr}}{S_{y.t}} + \frac{M_{wind.lift1}}{S_{y.t}}$$

$$= \text{0.177 ksi} - \frac{\text{(16,898 kip-in)(0.05626 rad)}}{\text{1,792 in}^3} + \frac{\text{0 kip-in}}{\text{1,792 in}^3}$$

$$= \quad -0.353 \quad \text{ksi}$$

$$f_{t.lift1.wr.right} = f_{t.lift1.base} + \frac{M_{g.lift1}\ \theta_{eq.lift1.wr}}{S_{y.t}} - \frac{M_{wind.lift1}}{S_{y.t}}$$

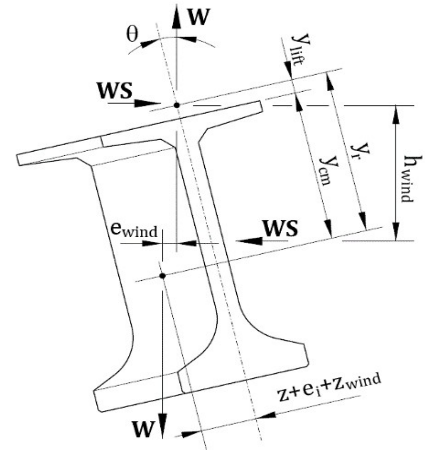
$$= \text{0.177 ksi} + \frac{\text{(16,898 kip-in)(0.05626 rad)}}{\text{1,792 in}^3} - \frac{\text{0 kip-in}}{\text{1,792 in}^3}$$

$$= \quad 0.708 \quad \text{ksi}$$

$$f_{t.ck.lift1.wr} = \text{Min}(\,f_{t.lift1.wr.left}\,,\,f_{t.lift1.wr.right})$$

$$= \text{Min (-0.353 ksi , 0.708 ksi)} = \quad -0.353 \quad \text{ksi}$$

Check Compressive and Tensile Stress esat Equilibrium Rotation - Wind Left



Girder Free Body Diagram, Wind Left

$$\theta_{eq.lift1.wl} = \frac{(e_{i.lift1} + e_{conn} + z_{wind.lift1} - e_{wind.lift1})}{y_{r.lift1} - z_{0.lift1}}$$

$$= \frac{(\ 0.357 \text{ in} + 0.935 \text{ in} + 0.000 \text{ in} - 0.000 \text{ in}\ )}{34.175 \text{ in} - 11.218 \text{ in}}$$

$$= 0.05626 \text{ rad}$$

Check of Bottom Flange Tips - Wind Left

$$f_{b.lift1.wl.left} = f_{b.lift1.base} - \frac{M_{g.lift1}\ \theta_{eq.lift1.wl}}{S_{y.b}} - \frac{M_{wind.lift1}}{S_{y.b}}$$

$$= 3.084 \text{ ksi} - \frac{(16{,}898 \text{ kip-in})(0.05626 \text{ rad})}{2{,}895 \text{ in}^3} - \frac{0 \text{ kip-in}}{2{,}895 \text{ in}^3}$$

$$= 2.755 \text{ ksi}$$

$$f_{b.lift1.wl.right} = f_{b.lift1.base} + \frac{M_{g.lift1}\ \theta_{eq.lift1.wl}}{S_{y.b}} + \frac{M_{wind.lift1}}{S_{y.b}}$$

$$= 3.084 \text{ ksi} + \frac{(16{,}898 \text{ kip-in})(0.05626 \text{ rad})}{2{,}895 \text{ in}^3} + \frac{0 \text{ kip-in}}{2{,}895 \text{ in}^3}$$

$$= 3.412 \text{ ksi}$$

$$f_{b.ck.lift1.wl} = \text{Max}(f_{b.lift1.wl.left}, f_{b.lift1.wl.right})$$

$$= \text{Max}\ (\ 2.755 \text{ ksi}, 3.412 \text{ ksi}\ ) = 3.412 \text{ ksi}$$

Check of Top Flange Tips - Wind Left

$$f_{t.lift1.wl.left} = f_{t.lift1.base} - \frac{M_{g.lift1}\ \theta_{eq.lift1.wl}}{S_{y.t}} - \frac{M_{wind.lift1}}{S_{y.t}}$$

$$= 0.177 \text{ ksi} - \frac{(16{,}898 \text{ kip-in})(0.05626 \text{ rad})}{1{,}792 \text{ in}^3} - \frac{0 \text{ kip-in}}{1{,}792 \text{ in}^3}$$

$$= -0.353 \text{ ksi}$$

$$f_{t.lift1.wl.right} = f_{t.lift1.base} + \frac{M_{g.lift1}\ \theta_{eq.lift1.wl}}{S_{y.t}} + \frac{M_{wind.lift1}}{S_{y.t}}$$

$$= 0.177 \text{ ksi} + \frac{(16{,}898 \text{ kip-in})(0.05626 \text{ rad})}{1{,}792 \text{ in}^3} + \frac{0 \text{ kip-in}}{1{,}792 \text{ in}^3}$$

$$= 0.708 \text{ ksi}$$

$$f_{t.ck.lift1.wl} = \text{Min}(f_{t.lift1.wl.left}, f_{t.lift1.wl.right})$$

$$= \text{Min}\ (\ -0.353 \text{ ksi}, 0.708 \text{ ksi}\ ) = -0.353 \text{ ksi}$$

	Critical	Allowable	**LRFD 5.9.2.3.1a**		
Compressive Stresses	3.412 ksi	$0.65\, f_{c.lift1}$	= 0.65(5.50 ksi) =	3.58 ksi	OK
Tensile Stresses	-0.353 ksi	$f_{r.lift1}$ =	-0.563 ksi		OK

Check Factor of Safety Against Cracking

Analysis assumes $e_{i.lift1}$ is due to form misalignment and not eccentric prestressing.

Base Concrete Stresses in Girder with Wind (Left Top Tip)

$$f_{t.lift1.wr} = f_{t.lift1.base} + \frac{M_{wind.lift1}}{S_{y.t}}$$

$$= 0.177 \text{ ksi} + \frac{0 \text{ kip-in}}{1{,}792 \text{ in}^3}$$

$$= 0.177 \text{ ksi}$$

$$f_{t.lift1.wl} = f_{t.lift1.base} - \frac{M_{wind.lift1}}{S_{y.t}}$$

$$= 0.177 \text{ ksi} - \frac{0 \text{ kip-in}}{1{,}792 \text{ in}^3}$$

$$= 0.177 \text{ ksi}$$

Lateral Moment to Cause Cracking:

Wind Right:

$$M_{lat.lift1.wr} = \left(f_{t.lift1.wr} - f_{r.lift1}\right)S_{y.t}$$

= (0.177 ksi - (-0.563 ksi))(1,792 in³ /12 in/ft)

= 110.6 kip-ft

Wind Left:

$$M_{lat.lift1.wl} = \left(f_{t.lift1.wl} - f_{r.lift1}\right)S_{y.t}$$

= (0.177 ksi - (-0.563 ksi))(1,792 in³ /12 in/ft)

= 110.6 kip-ft

Tilt Angle at Cracking due to Lateral Deflection:

Wind Right:

$$\theta_{cr.lift1.wr} = \frac{M_{lat.lift1.wr}}{M_{g.lift1}}$$

$$= \frac{110.6 \text{ kip-ft}}{1{,}408 \text{ kip-ft}} = 0.07851 \text{ rad}$$

Wind Left:

$$\theta_{cr.lift1.wl} = \frac{M_{lat.lift1.wl}}{M_{g.lift1}}$$

$$= \frac{110.6 \text{ kip-ft}}{1{,}408 \text{ kip-ft}} = 0.07851 \text{ rad}$$

Factor of Safety Against Cracking:

Wind Right:

$$FS_{cr.lift1.wr} = \frac{y_{r.lift1}\,\theta_{cr.lift1.wr}}{z_{0.lift1}\,\theta_{cr.lift1.wr} + e_{conn} - z_{wind.lift1} + e_{wind.lift1} + e_{i.lift1}}$$

$$= \frac{(34.175 \text{ in})(0.07851 \text{ rad})}{(11.218 \text{ in})(0.07851 \text{ rad}) + 0.935 \text{ in} - 0.000 \text{ in} + 0.000 \text{ in} + 0.357 \text{ in}}$$

= 1.235

Wind Left:

$$FS_{cr.lift1.wl} = \frac{y_{r.lift1}\,\theta_{cr.lift1.wl}}{z_{0.lift1}\,\theta_{cr.lift1.wl} + e_{conn} + z_{wind.lift1} - e_{wind.lift1} + e_{i.lift1}}$$

$$= \frac{(34.175 \text{ in})(0.07851 \text{ rad})}{(11.218 \text{ in})(0.07851 \text{ rad}) + 0.935 \text{ in} + 0.000 \text{ in} - 0.000 \text{ in} + 0.357 \text{ in}}$$

= 1.235

Critical Factor of Safety Against Cracking:

$$FS_{cr.lift1} = \min(FS_{cr.lift1.wr}, FS_{cr.lift1.wl})$$

= 1.235 ≥ 1.000 OK

Check Factor of Safety Against Failure - Wind Right

Rotation at Maximum Factor of Safety:

$$\theta_{max.ult.lift1.wr} = \sqrt{\frac{e_{i.lift1} + e_{conn} - z_{wind.lift1} + e_{wind.lift1}}{2.5\,z_{0.lift1}}}$$

$$= \sqrt{\frac{0.357 \text{ in} + 0.935 \text{ in} - 0.000 \text{ in} + 0.000 \text{ in}}{2.5(11.218 \text{ in})}}$$

$$= \sqrt{\frac{1.292 \text{ in}}{28.045 \text{ in}}}$$

Check for rotation of center of mass past vertical, defined with negative numerator.
If negative, wind left is critical case, set $FS_{ult.lift1.wr}$ to default value of 99.

= 0.21460 rad

Factor of Safety Against Failure:

$$FS_{ult.lift1.wr}(\theta) = \frac{y_{r.lift1}\,\theta}{(z_{0.lift1}\,\theta - z_{wind.lift1})(1 + 2.5\theta) + e_{wind.lift1} + e_{conn} + e_{i.lift1}}$$

$$= \frac{(34.175 \text{ in})(0.21460 \text{ rad})}{((11.218 \text{ in})(0.21460 \text{ rad}) - 0.000 \text{ in})(1 + 2.5(0.21460 \text{ rad})) + 0.000 \text{ in} + 0.935 \text{ in} + 0.357 \text{ in}}$$

= 1.470

Check Factor of Safety Against Failure - Wind Left

Rotation at Maximum Factor of Safety:

$$\theta_{max.ult.lift1.wl} = \sqrt{\frac{e_{i.lift1} + e_{conn} + z_{wind.lift1} - e_{wind.lift1}}{2.5\, z_{0.lift1}}}$$

$$= \sqrt{\frac{0.357 \text{ in} + 0.935 \text{ in} + 0.000 \text{ in} - 0.000 \text{ in}}{2.5(11.218 \text{ in})}}$$

$$= \sqrt{\frac{1.292 \text{ in}}{28.045 \text{ in}}}$$

Check for rotation of center of mass past vertical, defined with negative numerator.
If negative, wind right is critical case, set $FS_{ult.lift1.wl}$ to default value of 99.

= 0.21460 rad

Factor of Safety Against Failure:

$$FS_{ult.lift1.wl}(\theta) = \frac{y_{r.lift1}\, \theta}{(z_{0.lift1}\, \theta + z_{wind.lift1})(1 + 2.5\theta) - e_{wind.lift1} + e_{conn} + e_{i.lift1}}$$

$$= \frac{(34.175 \text{ in})(0.21460 \text{ rad})}{((11.218 \text{ in})(0.21460 \text{ rad}) + 0.000 \text{ in})(1 + 2.5(0.21460 \text{ rad})) - 0.000 \text{ in} + 0.935 \text{ in} + 0.357 \text{ in}}$$

= 1.470

Critical Factor of Safety Against Failure:

$$FS_{ult.crit.lift1} = min(FS_{ult.lift1.wr}, FS_{ult.lift1.wl})$$

= 1.470

Check Factor of Safety Against Failure for Critical Case

$$FS_{ult.lift1} = \max(FS_{ult.crit.lift1}, FS_{cr.lift1})$$

= 1.470 < 1.500 N.G.

Lateral Ultimate Moment Capacity Required, $M_{ult.y.lift1}$:

$$M_{ult.y.lift1} = \frac{1.5}{FS_{ult.lift1}} \left(M_{g.lift1} \max(\theta_{max.ult.lift1.wr}, \theta_{max.ult.lift1.wl})\right)$$

$$= \frac{1.5}{1.470}(1{,}408 \text{ kip-ft})(0.21460 \text{ rad})$$

= 308 kip-ft