Description

1. Naive solution $O(n^3)$

"1234":
$\Downarrow$
"1 2 3 4"
"1 2 34"
"1 234"
"12 34"
"1234"
Five different ways

f[i][j]: str[i,j] be a unitary first one, then str[j+1,n] can be divide in how many ways that fit the problem description.

$$f[i][j] = \begin{cases} 0, & if(str[i] ==' 0') \\ 1, & if(j == n-1) \\ \sum_{k=j+j-i+1}^{n-1} f[j+1][k], & if(str[i,j] < str[j+1, j+j-i+1]) \\ \sum_{k=j+j-i+2}^{n-1} f[j+1][k], & if(str[i,j] >= str[j+1, j+j-i+1]) \end{cases}$$

$$Ans = \sum_{k=0}^{n-1} f[0][k]$$

	0	1	2	3
0				
1	×			
2	×	×		
3	×	×	×	

$\Downarrow$

	0	1	2	3
0				1
1	×			1
2	×	×		1
3	×	×	×	1

$$f[i][n-1] = 1$$

$$\Downarrow$$

	0	1	2	3
0			0	1
1	×		0	1
2	×	×	1	1
3	×	×	×	1

$$f[2][2] = 1, (str[2] < str[3])$$

$$f[1][2] = 0, (if str[1,2] < str[3,4] \ ?)$$

(note: there is no str[3,4])

$$f[0][2] = 0$$

(note: there is no str[3,5])

$$\Downarrow$$

	0	1	2	3
0		1	0	1
1	×	2	0	1
2	×	×	1	1
3	×	×	×	1

$$f[1][1] = f[2][2] + f[2][3] = 1 + 1 = 2, (str[1] < str[2])$$

$$f[0][1] = f[2][3] = 1, (str[0,1] < str[2,3])$$

$$\Downarrow$$

	0	1	2	3
0	3	1	0	1
1	×	2	0	1
2	×	×	1	1
3	×	×	×	1

$$f[0][0] = f[1][1] + f[1][2] + f[1][3] = 2 + 0 + 1 = 3, (str[0] < str[1])$$

$$\Downarrow$$

$$Ans = f[0][0] + f[0][1] + f[0][2] + f[0][3] = 5$$

2. Promotion 1. $O(n^2)$

Let $f[i][j] = \sum_{k=j}^{n-1} f[i][k]$, that is

$$f[i][j] = \begin{cases} 0, & if(str[i] ==' 0' \ \&\& \ j == n-1) \\ f[i][j+1], & if(str[i] ==' 0') \\ 1, & if(str[i]! =' 0' \ \&\& \ j == n-1) \\ f[i][j+1] + f[j+1][j+j-i+1], & if(str[i,j] < str[j+1, j+j-i+1]) \\ f[i][j+1] + f[j+1][j+j-i+2], & if(str[i,j] >= str[j+1, j+j-i+1]) \end{cases}$$

$$Ans = f[0][0]$$

	0	1	2	3
0				
1	×			
2	×	×		
3	×	×	×	

$$\Downarrow$$

	0	1	2	3
0				1
1	×			1
2	×	×		1
3	×	×	×	1

$$f[i][n-1] = 1$$

$$\Downarrow$$

	0	1	2	3
0			1	1
1	×		1	1
2	×	×	2	1
3	×	×	×	1

$$f[2][2] = f[2][3] + f[3][3] = 1 + 1 = 2, (str[2] < str[3])$$

$$f[1][2] = f[1][3] = 1, (if str[1, 2] < str[3, 4] \ ?)$$

(note: there is no str[3,4])

$$f[0][2] = f[0][3] = 1$$

(note: there is no str[3,5])

$$\Downarrow$$

	0	1	2	3
0		2	1	1
1	×	3	1	1
2	×	×	2	1
3	×	×	×	1

$$f[1][1] = f[1][2] + f[2][2] = 1 + 2 = 3, (str[1] < str[2])$$

$$f[0][1] = f[0][2] + f[2][3] = 1 + 1 = 2, (str[0, 1] < str[2, 3])$$

$$\Downarrow$$

	0	1	2	3
0	5	2	1	1
1	×	3	1	1
2	×	×	2	1
3	×	×	×	1

$$f[0][0] = f[0][1] + f[1][1] = 2 + 3 = 5, (str[0] < str[1])$$

$$\Downarrow$$

$$Ans = f[0][0] = 5$$

3. Promotion 2. Pre-Comparison(str[i,j], str[j+1,j+j-i+1])

Let s[i][j] denote the shift k of the first different char of the string.
That is,
str[i]==str[j], str[i+1]==str[j+1], ..., str[i+k-1]==str[j+k-1], str[i+k]!=str[j+k]

$$s[i][j] = \begin{cases} 0, & if(i == n \ || \ j == n) \\ 0, & if(str[i] != str[j]) \\ s[i+1][j+1] + 1, & if(str[i] == str[j]) \end{cases}$$