

11/6/12 09 3:30 IR.

HW4

$$\begin{aligned} 1. P(\text{open} | u) &= P(\text{open} | u, \text{open}) \cdot P(\text{open}) \\ &\quad + P(\text{open} | u, \text{close}) \cdot P(\text{close}) \\ &= 1 \times 0.5 + 0.8 \times 0.5 \\ &= 0.9. \end{aligned}$$

$$2. \text{Bel}(\alpha_0) = 0.5. \quad \begin{matrix} \alpha_0 = \text{open} \\ \text{or} \\ \alpha_0 = \text{close} \end{matrix}$$

After  $z = \text{open}$   $u = \text{do-nothing}$

$$\overline{\text{Bel}}(\alpha_1) = \sum_{\alpha} P(\alpha | u_1, \alpha_0) \cdot \text{Bel}(\alpha_0).$$

$$\therefore \overline{\text{Bel}}(\alpha_1) = P(\alpha_1 | u_1 = \text{do-nothing}, \alpha_0 = \text{open}) \times \text{Bel}(\alpha_0 = \text{open}) + P(\alpha_1 | u_1 = \text{do-nothing}, \alpha_0 = \text{close}) \times \text{Bel}(\alpha_0 = \text{close}).$$

$$\text{Bel}(\alpha_1) = \eta \cdot \overline{\text{Bel}}(\alpha_1) \cdot P(z = \text{open} | \alpha_1).$$

$$\therefore \overline{\text{Bel}}(\alpha_1 = \text{open}) = 1 \times 0.5 + 0 \times 0.5 = 0.5$$

$$\overline{\text{Bel}}(\alpha_1 = \text{close}) = 0 \times 0.5 + 1 \times 0.5 = 0.5$$

$$\text{Bel}(\alpha_1 = \text{open}) = \eta \cdot \overline{\text{Bel}}(\alpha_1 = \text{open}) \cdot P(z = \text{open} | \alpha_1 = \text{open}) = 0.4\eta$$

$$\text{Bel}(\alpha_1 = \text{close}) = \eta \cdot \overline{\text{Bel}}(\alpha_1 = \text{close}) \cdot P(z = \text{open} | \alpha_1 = \text{close}) = 0.15\eta$$

$$\therefore \eta = (0.4 + 0.15)^{-1} \approx 1.8182$$

$$\text{Bel}(\alpha_1 = \text{open}) \approx 0.727$$

$$\text{Bel}(\alpha_1 = \text{close}) \approx 0.273$$

$$P(x_2) = \sum_{x_1} P(x_2 | u_2 = \text{push}, x_1) \cdot \bar{Bel}(x_1).$$

$$\begin{aligned} \therefore \bar{Bel}(x_2 = \text{open}) &= 1 \times \bar{Bel}(x_1 = \text{open}) + 0.7 \times \bar{Bel}(x_1 = \text{close}) \\ &= 0.9727 \end{aligned}$$

$$\begin{aligned} \bar{Bel}(x_2 = \text{close}) &= 0 \times \bar{Bel}(x_1 = \text{open}) + 0.1 \times \bar{Bel}(x_1 = \text{close}) \\ &= 0.0273. \end{aligned}$$

$$\begin{aligned} \therefore Bel(x_2 = \text{open}) &= \eta \cdot P(z_2 = \text{open} | x_2 = \text{open}) \cdot \bar{Bel}(x_2 = \text{open}) \\ &= 0.77816 \eta \end{aligned}$$

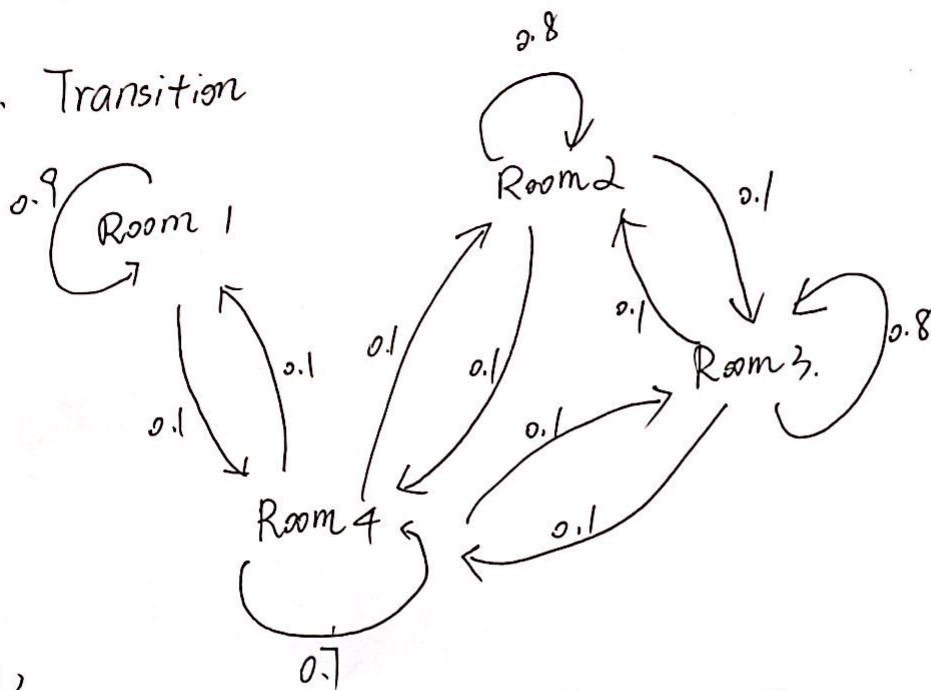
$$\begin{aligned} Bel(x_2 = \text{close}) &= \eta \cdot P(z_2 = \text{open} | x_2 = \text{close}) \cdot \bar{Bel}(x_2 = \text{close}) \\ &= 0.00819 \eta \end{aligned}$$

$$\eta = (0.00819 + 0.77816)^{-1} \approx 1.2717.$$

$$\therefore Bel(x_2 = \text{open}) \approx 0.990.$$

$$Bel(x_2 = \text{close}) \approx 0.010$$

### 3. Transition



(1)

∴ Markov model:

$$MK = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} \cdot \begin{bmatrix} R_1 & R_2 & R_3 & R_4 \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}$$

(2). Assume the robot is in Room 1 at first.

∴  $P_0 = [1, 0, 0, 0]$

$P_1 = P_0 \cdot MK$

∴  $P_n = P_{n-1} \cdot MK$

By using Matlab, I found that the static status was that

$P_{12} = [0.25 \ 0.25 \ 0.25 \ 0.25]$

which means the probability of the robot staying at each room is 0.25.

(3).  $P(\text{between 1 and 4} \mid \text{going through a door}) = \frac{P(1 \rightarrow 4) \cdot P(\text{at } R_1) + P(4 \rightarrow 1) \cdot P(\text{at } R_4)}{P(\text{going through a door})}$

$$= \frac{0.25 \times (0.1 + 0.1)}{0.25 \times (0.1 + 0.3 + 0.2 + 0.2)} = \frac{1}{4}$$