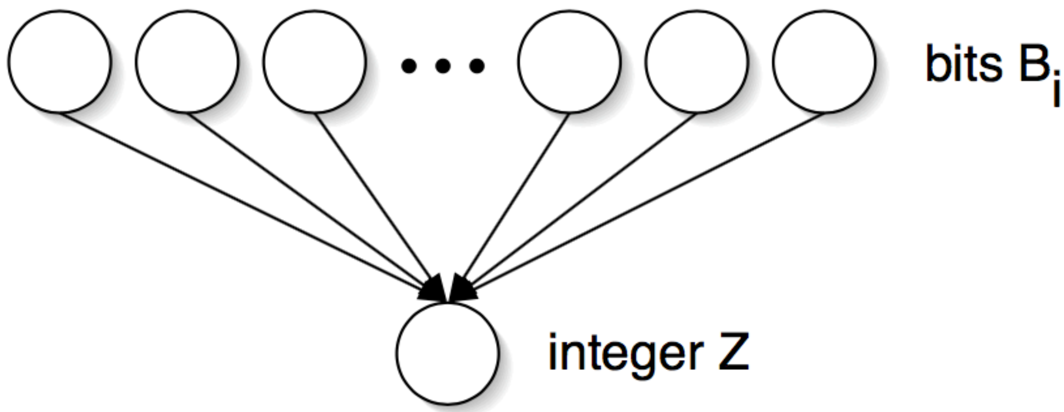
3.6 Stochastic simulation

Consider the belief network shown below, with n binary random variables $B_i \in \{0,1\}$ and an *integer* random variable Z. Let $f(B) = \sum_{i=1}^{n} 2^{i-1} B_i$ denote the nonnegative integer whose binary representation is given by $B_n B_{n-1} \ldots B_2 B_1$. Suppose that each bit has prior probability $P(B_i = 1) = \frac{1}{2}$, and that

$$P(Z|B_1, B_2, \ldots, B_n) = \left(\frac{1-\alpha}{1+\alpha}\right) \alpha^{|Z-f(B)|}$$

where $0 < \alpha < 1$ is a parameter measuring the amount of noise in the conversion from binary to decimal. (Larger values of α indicate greater levels of noise.)



(a) Show that the conditional distribution for binary to decimal conversion is normalized; namely, that $\sum_z P(Z = z|B_1, B_2, \ldots, B_n) = 1$, where the sum is over all integers $z \in [-\infty, +\infty]$.

(b) Consider a network with $n = 10$ bits and noise level $\alpha = 0.2$. Use the method of *likelihood weighting* to estimate the probability $P(B_i = 1|Z = 128)$ for $i \in \{2, 4, 6, 8, 10\}$.

(c) Plot your estimates in part (b) *as a function of the number of samples*. You should be confident from the plots that your estimates have converged to a good degree of precision (say, at least two significant digits).