

Hierarchy Ranking Method for Multimodal Multi-objective Problems with Local Pareto Fronts

Supplementary Material

Wenhua Li, Xingyi Yao, Tao Zhang, Rui Wang, and Ling Wang

Abstract—This is the supplementary material for Hierarchy Ranking Method for Multimodal Multi-objective Problems with Local Pareto Fronts. In this document, the detailed information of the proposed IDMP_e is given, including the images of PFs and PSs. In addition, the running data (IGDX and IGD) of all competitor algorithms is listed.

Index Terms—Multimodal multi-objective optimization, Indicator-based algorithms, Evolutionary computation, Diversity-preserving mechanisms

I. MULTIMODAL MULTI-OBJECTIVE IMBALANCED DISTANCE MINIMIZATION BENCHMARK PROBLEMS WITH LOCAL PARETO FRONTS

In this section, the proposed benchmark problems based on the IDMP test suit are further illustrated. Liu et.al [1] proposed the multimodal multi-objective imbalanced distance minimization benchmark problems (IDMPs). The main characteristic of IDMPs is that the computational resource of solving different PSs is different. The general objective functions of the two-objective IDMPs can be expressed as:

$$\begin{aligned} f_1(\mathbf{x}) &= \min(|x_1 + 0.6| + g_1(\mathbf{x}), |x_1 - 0.4| + g_2(\mathbf{x})) \\ f_2(\mathbf{x}) &= \min(|x_1 + 0.4| + g_1(\mathbf{x}), |x_1 - 0.6| + g_2(\mathbf{x})) \\ \text{s.t. } x_1 &\in [-1, 1], x_2 \in [-1, 1] \end{aligned} \quad (1)$$

where $g_1(\mathbf{x}) \geq 0$ and $g_2(\mathbf{x}) \geq 0$ are difficulty functions corresponding to the first and second equivalent PSs, respectively. Specifically, the authors proposed 4 types difficulty functions:

Type1:

$$\begin{aligned} g_1(\mathbf{x}) &= |x_2 + 0.5| \\ g_2(\mathbf{x}) &= \alpha |x_2 - 0.5| \\ \alpha &\geq 1 \end{aligned} \quad (2)$$

This work was supported by the National Science Fund for Outstanding Young Scholars (62122093), the National Natural Science Foundation of China (72071205), the Scientific Key Research Project of the National University of Defense Technology (ZZKY-ZX-11-04) and the Ji-Hua Laboratory Scientific Project (X210101UZ210).

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Type2:

$$\begin{aligned} g_1(\mathbf{x}) &= 100(x_2 + 0.5)^2 \\ g_2(\mathbf{x}) &= 100|x_2 - 0.5|^{2-\alpha} \\ \alpha &\in [0, 2) \end{aligned} \quad (3)$$

Type3:

$$\begin{aligned} g_1(\mathbf{x}) &= 100(x_2 + 0.5)^2 \\ g_2(\mathbf{x}) &= 100(x_2 - 0.5 + \alpha(x_1 - 0.5))^2 \\ \alpha &\in [0, 5] \end{aligned} \quad (4)$$

Type4:

$$\begin{aligned} g_1(\mathbf{x}) &= 100 \left((x_2 + 0.5)^2 - \cos(2\pi(x_2 + 0.5)) + 1 \right) \\ g_2(\mathbf{x}) &= 100 \left((x_2 - 0.5)^2 - \cos(2\pi\alpha(x_2 - 0.5)) + 1 \right) \\ \alpha &\text{ is a positive integer.} \end{aligned} \quad (5)$$

Specifically, the parameter α is introduced to determines the difficulty in obtaining the second PS. The larger value of α , the more difficult to obtain the second PS. It can be observed that when $g_1(\mathbf{x}) = 0$ and $g_2(\mathbf{x}) = 0$, the two equivalent PSs are $x_1 \in [-0.6, -0.4]$ and $x_2 \in [0.4, 0.6]$. In addition, the difficulty functions $g(\mathbf{x})$ can somehow control the location of the PSs. Take Type1 as an example, the first PS and the second PS can be expressed as $x_2 = -0.5$ and $x_2 = 0.5$, respectively.

Based on the proposed IDMP test suit, in this paper, we propose the new IDMP_e test suit. The main characteristic of these problems is that, there exist more than one local PFs and PSs. Specifically, the distributions of the PSs and PFs for IDMP_e with two objectives are shown in Fig. 1. As we can see, for IDMPM2T1_e and IDMPM2T2_e, the global PF and local PS correspond to single global PS and single local PS, respectively. For IDMPM2T3_e, there are two different global PSs and one local PS. For IDMPM2T4_e, the global PF corresponds to two PSs. Moreover, there are two different local PFs and several local PSs. Notably, the difficulties in obtaining the different PSs are different, which means that even for the global PSs, the normal MMEAs are hard to obtain all of them.

In addition, based on IDMP test suit with three objectives, we proposed several new problems. The distribution of PFs and PSs is shown in Fig. 2.

II. RESULT OBTAINED BY COMPARED ALGORITHMS

TABLE I and TABLE II list the variance values of IGDX and IGD for the normal MMOPs; TABLE III and TABLE IV list the variance values of IGDX and IGD for the MMOPs.

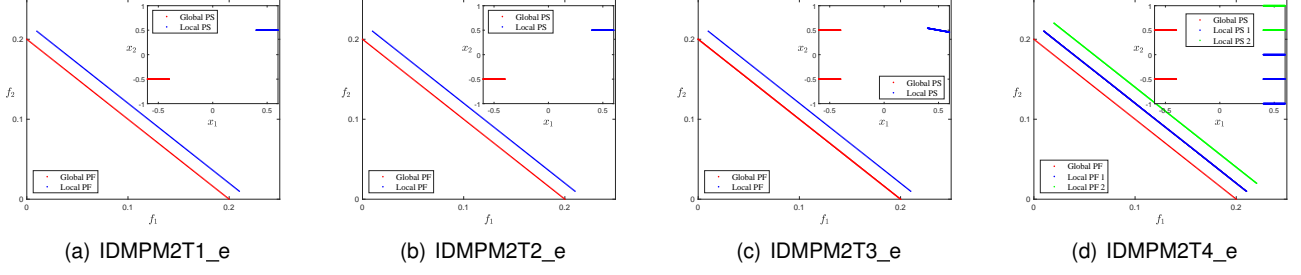


Fig. 1. The distribution of PFs and PSs of IDMP_e test suit with two objectives and two decision variables.

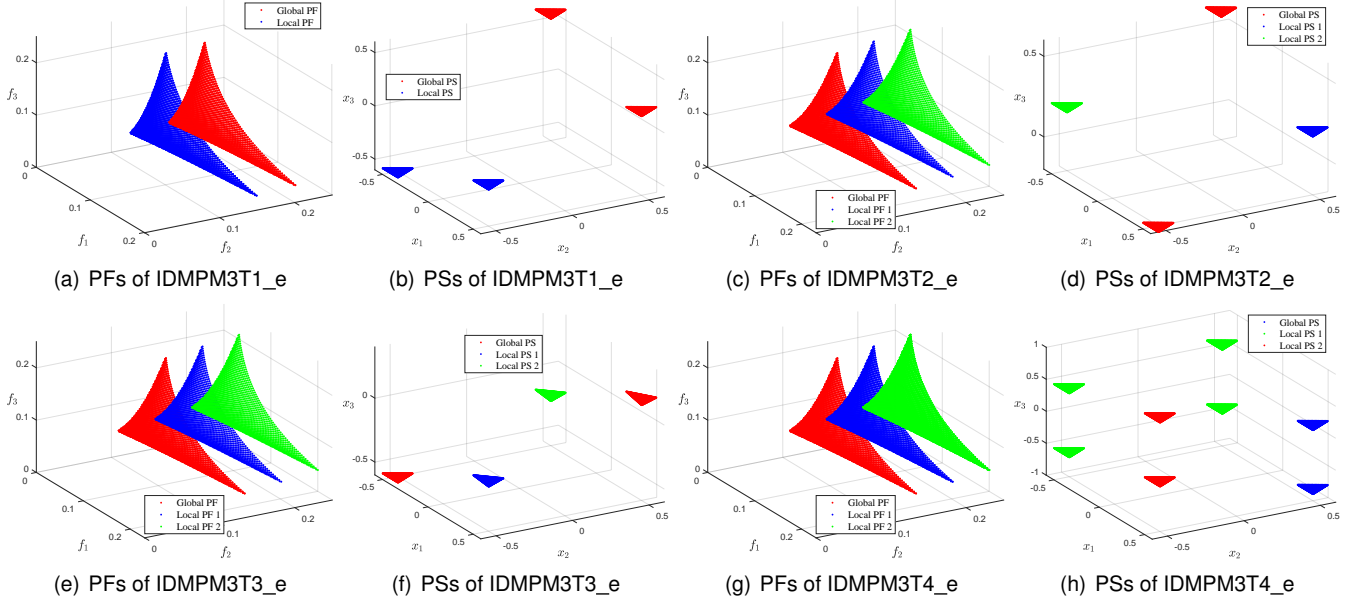


Fig. 2. The distribution of PFs and PSs of IDMP_e test suit with three objectives and two decision variables.

TABLE I

THE VARIANCE VALUES OF IGD_X RESULTS OF THE COMPARED ALGORITHMS ON MULTIMODAL MULTI-OBJECTIVE PROBLEMS WITH LOCAL PARETO FRONTS.

Problems	TriMOEA TAR	CPDEA	DN- NSGA-II	Omni_Opt	MO_Ring _PSO_SCD	MO_PSO _MM	MMOEA /DC	HREA
IDMPM2T1	5.62E-02	4.93E-08	1.06E-01	1.11E-01	2.05E-02	1.74E-05	1.38E-08	1.61E-04
IDMPM2T2	1.12E-01	1.11E-08	1.01E-01	8.36E-02	1.12E-05	6.96E-05	1.27E-08	1.60E-07
IDMPM2T3	6.90E-02	1.22E-06	8.70E-02	5.85E-02	2.73E-07	6.34E-07	2.92E-08	1.06E-05
IDMPM2T4	3.97E-02	8.28E-08	6.79E-02	5.78E-02	9.23E-02	4.96E-06	7.32E-02	5.67E-05
IDMPM3T1	7.15E-02	1.03E-07	4.19E-02	3.26E-02	2.43E-02	1.13E-02	1.23E-07	4.41E-08
IDMPM3T2	5.56E-02	2.82E-08	7.15E-02	6.74E-02	1.09E-02	1.17E-02	8.25E-08	1.67E-07
IDMPM3T3	9.51E-02	4.30E-06	5.08E-02	7.93E-02	2.81E-06	9.39E-06	2.44E-07	8.42E-06
IDMPM3T4	4.86E-02	7.68E-03	6.90E-02	3.67E-02	4.04E-02	2.96E-02	4.60E-04	2.27E-05
IDMPM4T1	8.99E-07	1.11E-01	7.11E-02	8.96E-02	8.48E-02	1.26E-01	3.89E-03	2.98E-03
IDMPM4T2	5.16E-02	1.22E-01	8.04E-02	6.24E-02	6.92E-02	4.25E-02	9.39E-07	1.20E-01
IDMPM4T3	8.44E-02	8.43E-02	9.45E-02	9.07E-02	1.04E-02	2.19E-02	6.23E-04	2.02E-01
IDMPM4T4	2.26E-02	1.30E-01	6.07E-02	5.21E-02	9.55E-02	5.71E-02	3.08E-03	8.46E-02
MMF1	7.88E-05	3.71E-07	1.69E-04	3.69E-04	4.25E-06	1.20E-06	1.48E-06	1.27E-06
MMF2	1.68E-03	5.42E-06	5.80E-03	5.12E-03	9.81E-05	2.24E-05	9.91E-05	4.44E-05
MMF3	7.97E-04	6.58E-06	1.46E-03	1.29E-03	1.03E-04	5.87E-05	2.54E-05	3.64E-05
MMF4	2.41E-02	3.12E-07	2.78E-04	4.54E-04	1.63E-06	8.43E-07	8.59E-06	2.48E-07
MMF5	6.55E-05	1.59E-06	3.18E-04	2.94E-04	1.22E-05	1.15E-05	1.03E-05	3.21E-06
MMF6	5.83E-05	1.93E-06	1.86E-04	2.17E-04	1.17E-05	5.45E-06	5.81E-06	3.68E-06
MMF7	2.25E-04	3.12E-07	9.12E-05	8.36E-05	1.28E-06	7.37E-07	1.14E-05	9.51E-07
MMF8	8.19E-03	2.68E-06	1.53E-02	1.11E-02	6.01E-05	1.06E-05	8.00E-05	1.20E-05

TABLE II

THE VARIANCE VALUES OF IGD RESULTS OF THE COMPARED ALGORITHMS ON MULTIMODAL MULTI-OBJECTIVE PROBLEMS WITH LOCAL PARETO FRONTS.

Problems	TriMOEA TAR	CPDEA	DN- NSGA-II	Omni_Opt	MO_Ring _PSO_SCD	MO_PSO _MM	MMOEA /DC	HREA
IDMPM2T1	6.40E-10	2.97E-08	7.17E-09	5.35E-09	4.29E-08	3.63E-07	3.67E-08	7.34E-09
IDMPM2T2	3.42E-10	1.10E-09	1.96E-09	2.54E-09	2.70E-08	5.23E-08	8.29E-09	8.42E-10
IDMPM2T3	2.32E-10	2.78E-09	1.83E-09	7.80E-10	2.20E-08	2.66E-07	9.57E-09	6.49E-10
IDMPM2T4	2.04E-10	3.01E-08	1.83E-06	1.24E-06	1.79E-07	2.38E-07	1.65E-08	5.64E-09
IDMPM3T1	2.91E-07	3.45E-08	7.49E-07	4.57E-07	1.28E-06	2.60E-06	2.41E-07	4.89E-08
IDMPM3T2	4.02E-07	2.73E-08	7.29E-06	1.09E-06	4.48E-07	1.25E-06	1.16E-07	2.12E-08
IDMPM3T3	8.67E-07	1.01E-08	1.07E-06	9.77E-07	8.59E-07	5.66E-07	2.09E-07	1.28E-08
IDMPM3T4	3.94E-07	1.07E-07	5.58E-05	1.51E-05	1.24E-05	4.07E-06	4.47E-07	2.98E-08
IDMPM4T1	3.31E-06	4.83E-07	7.68E-05	2.33E-05	8.61E-06	3.08E-05	3.40E-06	2.47E-06
IDMPM4T2	4.98E-06	2.59E-07	9.40E-05	6.13E-04	1.74E-05	2.35E-05	3.65E-06	1.51E-06
IDMPM4T3	6.86E-05	2.42E-07	3.23E-05	2.30E-05	2.34E-05	3.91E-05	2.39E-06	8.81E-07
IDMPM4T4	3.10E-06	2.31E-07	5.33E-03	3.24E-03	4.41E-04	1.42E-04	6.35E-06	1.86E-06
MMF1	6.49E-07	4.16E-09	1.74E-07	2.41E-07	3.33E-08	1.49E-06	2.31E-08	6.71E-08
MMF2	3.14E-03	7.28E-07	3.06E-04	3.34E-04	1.47E-05	3.42E-06	3.21E-07	1.94E-05
MMF3	2.58E-03	1.05E-06	1.43E-04	4.28E-04	1.30E-05	4.70E-06	3.17E-07	1.91E-05
MMF4	5.23E-03	2.28E-08	3.56E-08	5.19E-08	6.38E-08	4.53E-08	2.54E-07	3.91E-08
MMF5	1.07E-07	3.55E-09	1.28E-07	5.08E-08	2.48E-08	1.05E-08	1.68E-08	6.82E-08
MMF6	1.08E-06	4.92E-09	1.09E-07	3.51E-08	1.34E-08	8.51E-09	5.97E-08	5.55E-08
MMF7	8.40E-07	2.41E-09	6.47E-08	1.29E-07	8.35E-08	4.49E-09	3.60E-07	6.12E-08
MMF8	4.39E-05	6.54E-09	2.89E-07	6.45E-08	1.14E-07	4.36E-08	5.34E-08	6.82E-08

TABLE III

THE VARIANCE VALUES OF IGD_X RESULTS OF THE COMPARED ALGORITHMS ON MULTIMODAL MULTI-OBJECTIVE PROBLEMS WITH LOCAL PARETO FRONTS.

Problems	TriMOEA TAR	CPDEA	DN- NSGA-II	Omni_Opt	MO_Ring _PSO_SCD	MO_PSO _MM	MMOEA /DC	HREA
IDMPM2T1_e	2.25E-09	3.87E-08	9.68E-08	4.57E-08	1.24E-06	8.44E-07	2.91E-08	5.02E-09
IDMPM2T2_e	1.05E-08	7.72E-08	2.53E-07	9.91E-08	4.01E-07	3.44E-07	7.31E-09	7.63E-09
IDMPM2T3_e	2.75E-02	2.27E-07	3.51E-02	3.15E-02	1.95E-02	7.09E-03	1.38E-08	6.75E-09
IDMPM2T4_e	1.40E-02	8.45E-08	3.37E-02	4.27E-02	5.55E-02	8.28E-02	9.00E-03	3.92E-06
IDMPM3T1_e	3.23E-02	2.02E-06	3.07E-02	3.55E-02	7.02E-05	6.45E-03	2.49E-08	1.80E-08
IDMPM3T2_e	1.47E-03	5.12E-07	2.97E-02	3.02E-02	3.18E-02	2.73E-05	2.23E-08	1.14E-06
IDMPM3T3_e	5.89E-02	1.98E-05	5.41E-02	4.74E-02	2.41E-05	1.89E-05	1.13E-07	1.15E-05
IDMPM3T4_e	2.18E-02	1.61E-06	2.93E-02	1.81E-02	2.01E-02	2.21E-02	8.52E-04	3.32E-06
MMF10	2.13E-09	1.62E-09	1.18E-03	1.14E-03	1.30E-05	1.47E-04	2.35E-06	5.10E-07
MMF11	2.85E-09	8.59E-08	1.41E-07	5.50E-08	6.27E-04	5.43E-04	8.40E-08	2.24E-06
MMF12	1.03E-07	6.82E-08	3.09E-07	1.22E-04	1.91E-03	1.40E-03	2.32E-08	8.72E-09
MMF13	5.50E-05	4.99E-05	7.45E-05	1.22E-04	3.72E-04	1.38E-04	1.32E-04	4.53E-07
MMF15	5.06E-08	2.57E-04	6.44E-04	9.21E-04	3.14E-04	1.85E-04	3.88E-06	1.02E-06

TABLE IV

THE VARIANCE VALUES OF IGD RESULTS OF THE COMPARED ALGORITHMS ON MULTIMODAL MULTI-OBJECTIVE PROBLEMS WITH LOCAL PARETO FRONTS.

Problems	TriMOEA TAR	CPDEA	DN- NSGA-II	Omni_Opt	MO_Ring _PSO_SCD	MO_PSO _MM	MMOEA /DC	HREA
IDMPM2T1_e	6.31E-10	3.01E-09	1.46E-09	8.31E-10	2.26E-08	5.39E-08	6.04E-08	3.27E-08
IDMPM2T2_e	6.00E-11	1.33E-09	1.11E-09	4.03E-10	5.85E-09	4.89E-08	2.72E-09	1.06E-08
IDMPM2T3_e	6.76E-04	3.75E-10	1.73E-06	3.43E-06	6.11E-07	3.14E-07	3.46E-08	5.10E-09
IDMPM2T4_e	1.11E-05	5.31E-08	1.59E-05	2.23E-05	1.82E-06	1.04E-06	7.12E-07	5.02E-09
IDMPM3T1_e	1.66E-07	9.72E-08	4.56E-07	9.88E-07	4.37E-07	5.15E-07	1.45E-07	6.41E-08
IDMPM3T2_e	1.38E-07	5.99E-08	4.28E-06	7.69E-05	1.79E-06	4.84E-07	7.81E-08	1.05E-07
IDMPM3T3_e	8.00E-07	6.32E-08	9.05E-07	7.54E-07	5.44E-07	5.13E-07	2.90E-06	3.66E-06
IDMPM3T4_e	2.80E-07	4.62E-08	1.33E-05	9.67E-05	1.08E-05	6.04E-06	1.42E-07	1.91E-06
MMF10	1.22E-05	5.37E-08	1.54E-03	1.49E-03	4.66E-04	2.52E-04	6.18E-06	6.83E-05
MMF11	4.99E-05	5.67E-08	1.68E-06	7.01E-07	2.51E-05	3.39E-05	4.80E-07	2.10E-05
MMF12	4.61E-07	4.69E-08	7.00E-08	2.85E-05	2.03E-04	9.77E-05	1.62E-08	1.49E-07
MMF13	4.81E-05	5.22E-04	5.52E-05	7.79E-06	8.29E-04	5.90E-04	1.54E-05	6.44E-05
MMF15	2.70E-07	1.31E-05	8.47E-05	6.14E-05	1.36E-05	5.02E-06	1.06E-05	6.89E-06

Fig. 3 shows the final distribution of the obtained solution sets by HREA on IDMP_e test suits. In addition, the final distributions of solutions obtained by all competitor algorithms on IDMPM2T4 and MMF6 are presented in Fig. 4.

III. COMPARE WITH ALGORITHMS IN CEC 2019 COMPETITION

We also compare the performances of HREA and other algorithms in the IEEE CEC 2019 multimodal multiobjective optimization competitions. The average IGDX and IGD result can be seen from TABLE V and TABLE VI, respectively. It's worth mentioning that, the results of other algorithms except HREA come from the official website¹.

REFERENCES

- [1] Y. Liu, H. Ishibuchi, G. G. Yen, Y. Nojima, and N. Masuyama, "Handling imbalance between convergence and diversity in the decision space in evolutionary multi-modal multi-objective optimization," *IEEE Transactions on Evolutionary Computation*, 2019.

¹<http://www5.zzu.edu.cn/cilab/info/1005/1300.htm>

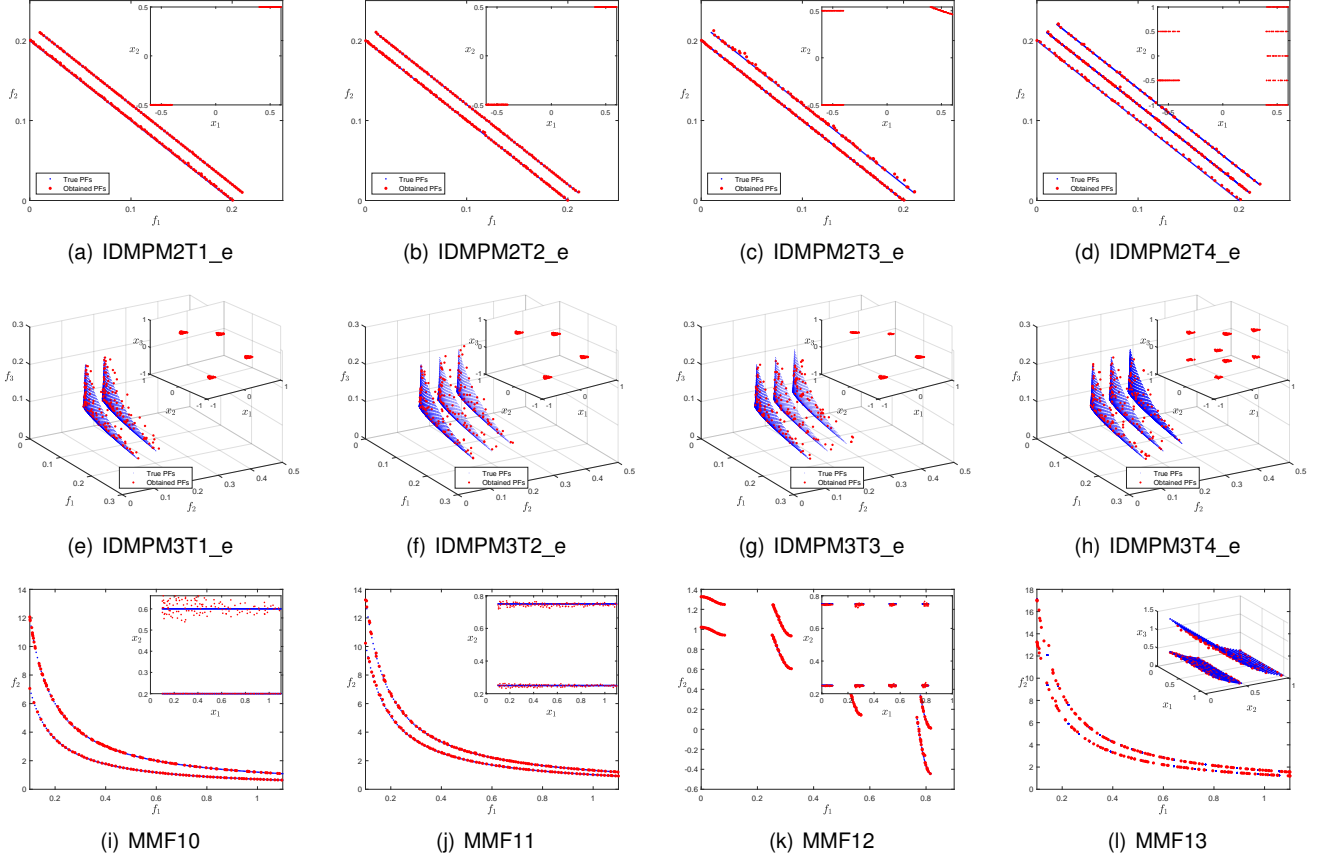


Fig. 3. The distribution of the obtained PFs and PSs by HREA.

TABLE V
THE AVERAGE VALUES OF IGD_X RESULTS OF THE COMPARED ALGORITHMS ON IEEE CEC 2019 MULTIMODAL COMPETITION TEST SUITES.

Problems	NMOHSA	PEN_MOBA	CEOA	MMO-ClusteringPSO	MM-NAEMO	MEMDMO	MMOEa-GD	IMDN_NSGAII	HREA
MMF1	4.27E-02	4.05E-02	4.50E-02	3.26E-02	4.87E-02	4.79E-02	4.22E-02	5.42E-02	3.36E-02
MMF2	2.21E-02	2.97E-02	2.11E-02	4.75E-02	1.18E-02	1.49E-02	1.88E-02	4.70E-02	2.35E-02
MMF3	1.99E-02	2.27E-02	3.50E-02	2.70E-02	1.39E-02	1.28E-02	1.22E-02	5.22E-02	4.10E-02
MMF4	2.46E-02	2.39E-02	2.52E-02	1.29E-02	3.12E-02	2.30E-02	2.12E-02	2.18E-02	1.83E-02
MMF5	8.02E-02	7.39E-02	7.85E-02	5.63E-02	8.71E-02	9.83E-02	7.48E-02	9.24E-02	5.84E-02
MMF6	7.04E-02	6.57E-02	6.78E-02	4.39E-02	7.42E-02	7.60E-02	6.38E-02	7.08E-02	5.37E-02
MMF7	2.15E-02	2.12E-02	2.76E-02	1.26E-02	2.28E-02	2.42E-02	2.44E-02	2.32E-02	1.95E-02
MMF8	7.30E-02	5.14E-02	7.13E-02	5.18E-02	3.53E-01	1.13E-01	5.42E-02	9.59E-02	4.06E-02
MMF10	1.60E-01	1.71E-01	9.48E-03	1.61E-01	1.23E-02	7.89E-02	6.53E-02	4.77E-02	7.41E-03
MMF11	2.32E-01	2.06E-01	7.42E-03	2.18E-01	4.18E-02	2.44E-01	2.31E-01	1.62E-01	7.46E-03
MMF12	1.76E-01	1.75E-01	3.19E-03	1.96E-01	5.02E-03	2.39E-01	1.84E-01	1.92E-01	2.76E-03
MMF13	2.30E-01	2.29E-01	9.99E-02	2.34E-01	1.88E-01	2.46E-01	2.39E-01	1.78E-01	4.85E-02
MMF15	1.42E-01	1.40E-01	5.78E-02	1.65E-01	5.28E-02	2.45E-01	2.56E-01	1.57E-01	5.24E-02
MMF15_a	1.61E-01	1.57E-01	9.29E-02	1.56E-01	8.65E-02	2.01E-01	1.97E-01	1.61E-01	5.95E-02

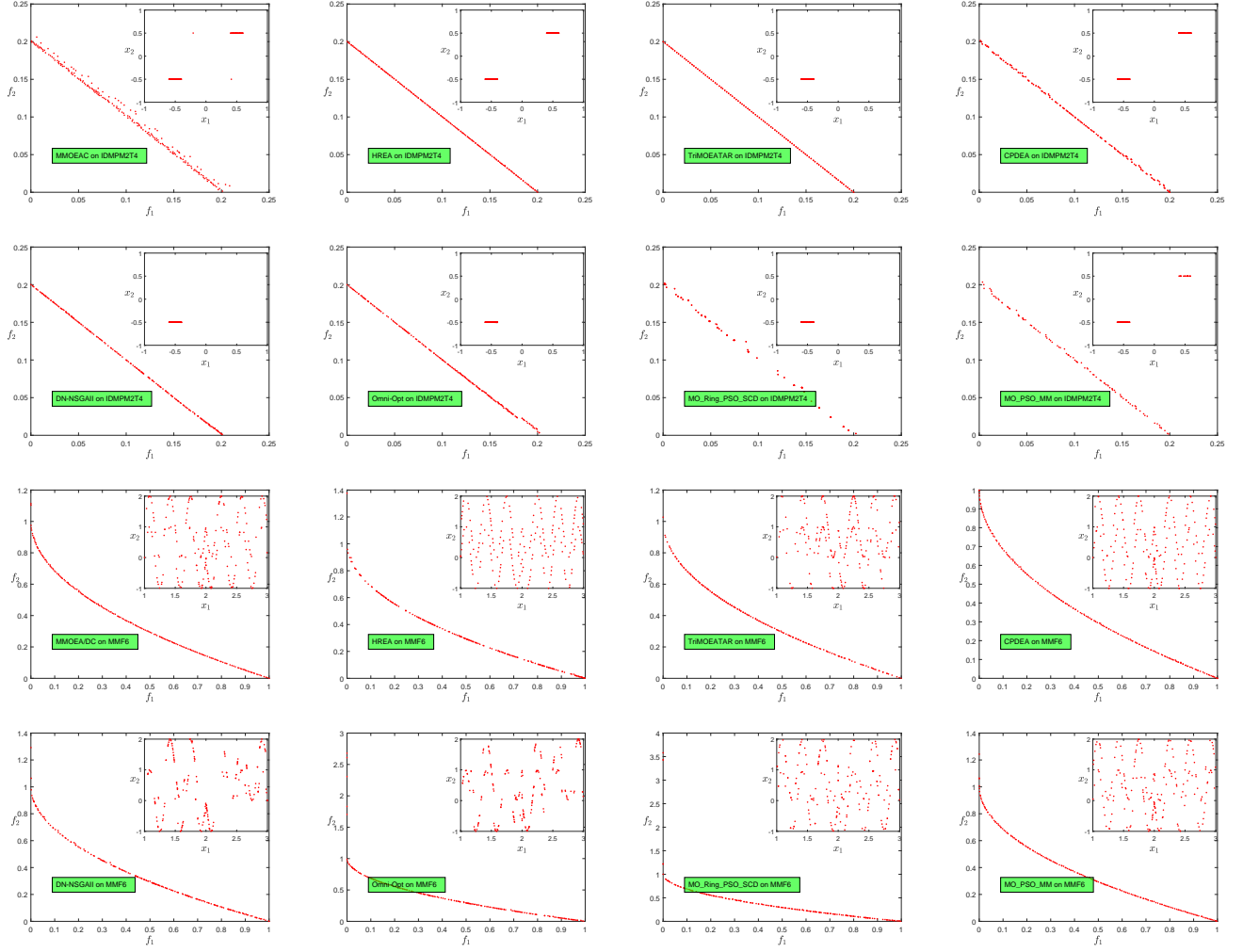


Fig. 4. The final distribution of the obtain PF and PSs on IDMPM2T4 and MMF6, where the PSs are shown in sub-figures in the upper right.

TABLE VI
THE AVERAGE VALUES OF IGD RESULTS OF THE COMPARED ALGORITHMS ON IEEE CEC 2019 MULTIMODAL COMPETITION TEST SUITES.

Problems	NMOHSA	PEN_MOBA	CEOA	MMO-ClusteringPSO	MM-NAEMO	MEMDMO	MMOEA-GD	IMDN_NSGAII	HREA
MMF1	2.74E-03	2.75E-03	3.44E-03	2.09E-03	4.03E-03	5.55E-03	3.02E-03	3.64E-03	3.44E-03
MMF2	1.12E-02	1.45E-02	9.74E-03	1.89E-02	8.28E-03	1.47E-02	1.11E-02	2.25E-02	5.82E-03
MMF3	9.86E-03	1.13E-02	9.69E-03	1.37E-02	8.49E-03	1.49E-02	9.31E-03	2.53E-02	5.86E-03
MMF4	2.66E-03	2.70E-03	2.62E-03	1.39E-03	3.42E-03	3.89E-03	2.56E-03	2.21E-03	2.45E-03
MMF5	2.72E-03	2.68E-03	3.61E-03	2.08E-03	3.68E-03	5.68E-03	3.10E-03	3.61E-03	3.56E-03
MMF6	2.59E-03	2.60E-03	3.42E-03	1.74E-03	3.58E-03	5.30E-03	2.85E-03	3.12E-03	3.46E-03
MMF7	2.72E-03	2.67E-03	3.51E-03	1.30E-03	3.47E-03	3.93E-03	2.75E-03	2.32E-03	3.54E-03
MMF8	3.07E-03	2.85E-03	2.71E-03	3.08E-03	3.68E-03	4.64E-03	3.81E-03	5.75E-03	2.43E-03
MMF10	1.52E-01	1.98E-01	1.77E-02	1.97E-01	6.59E-02	1.54E-01	9.89E-02	1.11E-01	2.50E-02
MMF11	8.38E-02	8.42E-02	2.08E-02	8.04E-02	9.31E-02	9.24E-02	9.24E-02	8.73E-02	2.79E-02
MMF12	5.80E-02	6.36E-02	3.95E-03	7.04E-02	1.46E-02	8.21E-02	5.79E-02	6.44E-02	6.48E-03
MMF13	8.72E-02	9.37E-02	2.57E-02	8.86E-02	1.17E-01	1.50E-01	1.32E-01	1.27E-01	1.74E-02
MMF15	1.67E-01	1.68E-01	1.06E-01	1.51E-01	1.11E-01	1.61E-01	1.74E-01	1.57E-01	1.23E-01
MMF15_a	1.70E-01	1.71E-01	1.38E-01	1.50E-01	1.26E-01	1.66E-01	1.70E-01	1.63E-01	1.30E-01