Problem Set 3

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Integrate and Fire Model

The Integrate-and-fire model of neuron's firing consists of of the following equation for the membrane potential (in dimensionless units):

$$\begin{aligned} \tau \frac{dV}{dt} &= -V + I_e \\ V(t^-_{spike}) &= 1 \\ V(t^+_{spike}) &= 0 \end{aligned} \tag{1}$$

However, the model ignores two important biological observations. First, the action potential has a finite temporal width. Second, after firing an action potential, a neuron is less likely to fire an action potential in a short refractory period, contributed by the large persistent voltage-gated potassium current. To incorporate these ingredients into the model, Here we consider two different modifications.

We assume that after a spike the neuron's potential is strongly refractory, namely it is unable to respond to an external input for a period of time, τ_r where τ_r is of the order of a few milliseconds. Mathematically, this assumption can be written as,

$$V(t) = 0, t_{spike} < t < t_{spike} + \tau_r. \tag{2}$$

Please compute the $f - I_e$ curve (firing frequency vs applied current) of this neuron. Analyze its behavior for large I_e, and compare it to the behavior at large I of the normal I-F neuron (i.e., without refractoriness). *Hint*: Use Taylor expansion in $1/I_e$. Additionally, explore the effect of τ_r, by plotting the two curves (with and without refractoriness using the following parameters: $\tau = 20$ ms $\tau_r = 2$ ms.

Hodgkin and Huxley Model

The Hodgkin-Huxley model for generation of an action potential is constructed by a summation of leaky current, a delayed-rectified K^+ current, and a transient Na^+ current:

$$
\begin{aligned}
C_m \frac{dV}{dt} &= -\bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) - \bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) - \bar{g}_L (V - E_L) + I_e. \\
\frac{dn}{dt} &= \alpha_n (1-n) - \beta_n n. \\
\frac{dm}{dt} &= \alpha_m (1-m) - \beta_m m. \\
\frac{dh}{dt} &= \alpha_h (1-h) - \beta_h h.
\end{aligned}
\tag{3}
$$

(a) Please simulate the dynamic equations and check whether it could generate action potentials. Below I will provide detailed parameter values used in the Hodgkin and Huxley model.

$$
\alpha_n = \frac{0.01(V+55)}{1-\exp(-0.1(V+55))},\ \beta_n = 0.125\exp(-0.0125(V+65)),
$$

$$
\alpha_m = \frac{0.1(V+40)}{1-\exp(-0.1(V+40))},\ \beta_m = 4\exp(-0.0556(V+65)),
$$

$$
\alpha_h = 0.07\exp(-0.05(V+65)),\ \beta_h = \frac{1}{1+\exp(-0.1(V+35))}.
$$

These rates have dimensions ms^{-1}. The maximum conductances and reversal potentials used in the model are $\bar{g}_{\mathrm{K}} = 0.36$ mS/mm^2, $\bar{g}_{\mathrm{Na}} = 1.2$ mS/mm^2, $\bar{g}_{\mathrm{L}} = 0.003$ mS/mm^2, $E_{\mathrm{L}} = -54.387$ mV, $E_{\mathrm{K}} = -77$ mV, $E_{\mathrm{Na}} = 50$ mV, $C_m = 10$ nF/mm^2.

(b) Is there a threshold current above which the system generates periodic pulses? Explore the frequency of the pulses as a function of current, and discuss phenomenologically the similarity and difference between the H-H model and the integrate-and-fire model.