CNeuron Homework1

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1 Optimal length of a dendritic arbor

1.1 题目

In the class, I have raised up a thought experiment about how to wire up a neural circuit of N neurons with all-to-all potential connectivity. Each neuron will use wire with diameter $d = 0.3\ \mu m$. I have considered several models.

- Point-to-point wires
- use axons that meander through the space to find the cell body of another neuron
- use both axons and dendrites and they are treated symmetrically
- dendrite would grow a spine with length s

Recall how I define potential connectivity between neurons, and recall how I make the scaling argument. Make a similar derivation for the last case, and check whether this argument makes sense. Show how the volume of the network depends on N, d and s, and how the total length of the dendrite L from a single neuron depends on these parameters.

1.2 解答

l 是一个神经元发出的线缆总长度，N 是神经元总数，R 是总的尺度（简单视为一个方体），d 是线缆的直径。设 $N = 10^5$，$d = 0.3um$.

以下各种情况中，根据方体的总体积，总有关系式

$$R^3 \doteq Nld^2$$

- Point-to-point wires

  如果所有的神经元彼此直接相连，则一个神经元要与方体中的各个神经元相连，因此有关系：

  $$l \doteq NR$$

  由此得

  $$R \doteq ND = 3cm.$$

- use axons that meander through the space to find the cell body of another neuron

  神经元之间并不一定直接相连，而是以直角的折线相连，一条线上串起一排神经元。设每两个神经元之间的跨度为 r，则有

  $$r \doteq \frac{R}{N^{1/3}}$$

  $$l \doteq Nr$$

  由此得

  $$R \doteq N^{5/6}d = 4.4mm$$

- use both axons and dendrites and they are treated symmetrically

  设一个神经元胞体发出的轴突和树突的空间占用率分别为 ρ_a，ρ_d，则有

  $$\rho_d = \rho_a = \frac{ld^2}{R^3}$$

  一个神经元的轴突必须与另一个神经元的树突相遇，即相遇的概率为 1，即

  $$\rho_a\rho_d\frac{R^3}{d^3} \doteq 1$$

  结合上式得

  $$R \doteq N^{2/3}d = 0.65mm$$

- dendrite would grow a spine with length s

  一个神经元的树突伸出小触手，考虑到它要接触所有的神经元发出的轴突，沿着树突与小触手构成的平面展开，其面积为

$$l2s \doteq Nd^2$$

结合

$$Nld^2 \doteq R^3$$

得

$$R^3 = \frac{N^2d^4}{s}$$

此即为总体积与 N, d, s 的关系式。

$$L \doteq \frac{Nd^2}{s}$$

2 Quantitative Analysis of Dendritic Morphology

2.1 题目

Attached you will find morphology data txt files (.swc) of one pyramidal dendrite, one Purkinjie dendrite, and one arbor from larval zebrafish. Use MATLAB or Python to write a simple program:

- Load the data file.
- Plot and visualize the neuronal 3D arbor shape.
- write a function to compute the mean path length form a dendritic segment to the soma
- Compute the spine reach zone area of the Purkinje dendrite, namely the gray area that can be reached by growing a spine on a dendrite (slide 44), assuming the spine length $s = 2\ \mu m$. And compare the total spine reach zone area with the arbor area, which may be defined

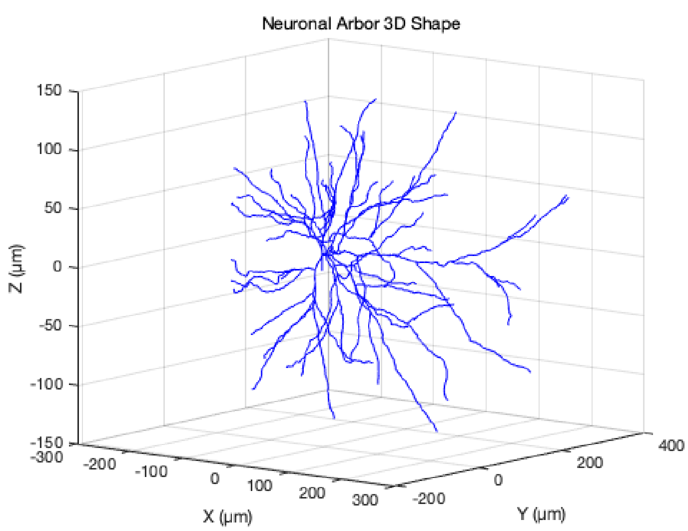
as the convex hull area that embed the 2D arbor. For 3D pyramidal dendrites, choose a random 2D projection of the 3D morphology. Perform the same analysis.

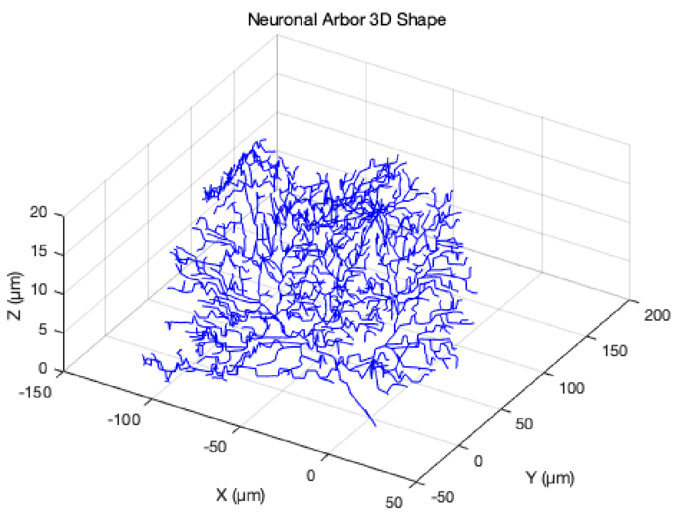
In the swc file, each column has the following meaning (from left to right): segment index, segment type (cell body =1, dendrite = 3), x coordinate (μm), y coordinate (μm), z coordinate (μm), segment diameter (μm), father segment index (root index = -1).

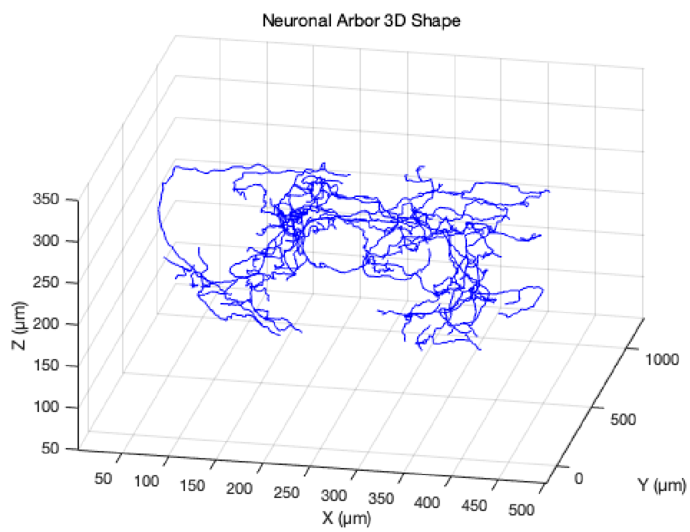
2.2 解答

程序见文件夹“Matlab”。

2.2.1 可视化结果







2.2.2 计算平均长度

对于 pyramidal dendrite, Purkinjie dendrite, and one arbor from larval zebrafish, 从树突节点到胞体的平均长度分别为 178.06 m, 144.06 m, 405.23 m.

2.2.3 计算可达的面积

对于 Purkinje dendrite，计算结果为：
树突棘到达区面积：25829.42 m^2
凸包区面积：27698.03 m^2
比值为 0.93
对于 pyramidal dendrites，选择投影到 xy 平面，计算结果为：
树突棘到达区面积：53003.82 m^2
凸包区面积：250700.46 m^2
比值为 0.21

3 Optimal Orientation Map

3.1 题目

Consider neurons in primary visual cortex (V1), where each neuron has a preferred orientation tuning. In my slides, we use different colors to represent each neuron's orientation selectivity $\theta \in [-\pi/2, \pi/2]$, namely the activity of a neuron in response to a bar (or edge) is maximized at a given orientation. We ask for a given a connectivity rule what is the optimal spatial layout of the neurons that minimize the total wires. Let us consider a simplified case where neurons are uniformly distributed on a 2-dimensional sheet (or located on a 2-D lattice).

- Rule 1 Consider that each neuron will establish equal numbers of connections with neurons of all preferred orientations. Prove that the optimal arrangement is *Salt and Pepper*, namely neurons with different orientation selectivity are intermingled. In other words, in this layout, neurons of each preferred orientation are equally represented at every location.

- Rule 2 Consider a specific connectivity rule:

$$c(\Delta\theta) = c(0) \begin{cases} \sqrt{1 - \Delta\theta^2/\theta_{max}^2} & |\Delta\theta| < \theta_{max} \\ 0 & |\Delta\theta| > \theta_{max} \end{cases}$$

where $\Delta\theta$ is the orientation selectivity difference. Show that in this case, the optimal arrangement is *Icecube* such that the preferred orientation smoothly varies spatially, i.e., $\theta(\vec{r}) = k\vec{r}$, with an appropriate periodicity.

Hint: Think this problem from a geometric viewpoint. No complicated calculation is involved.

3.2 解答

参考：Alexei A. Koulakov, Dmitri B. Chklovskii. 2001. Orientation Preference Patterns in Mammalian Visual Cortex: A Wire Length Minimization Approach. Neuron, Vol.29, 519-527.

该文章于 Experimental Procedures 一节证明了：

（1）Salt and Pepper Is Optimal for the Uniform Connection Function

（2）Icecube Is Optimal for Semi-Elliptic Connection Functions

本作业参照了该文献的思路。

（1）如果要求每个神经元同相等数量的各种方向偏好的神经元连接，那么，Salt and Pepper 是最优的布局方式。

考虑任何一个神经元，以它为中心画一个元，恰好覆盖它需要连接的神经元的总数目，它与这圆内的所有神经元都连接将会使得总的连接长度最小化。相应的神经元布局必须是 Salt and Pepper。

（2）若要求按照半椭圆连接函数

$$c(\theta) = c(0)\sqrt{1 - \frac{\theta^2}{\theta_{max}^2}},\ |\theta| \le \theta_{max}$$

去连接有不同方向偏好的神经元，那么，Icecube 是最优的布局方式。

具体来说，就是按照

$$\theta(x) = x/h \tag{1}$$

周期性布局，其中 h 决定了周期。连接方式是让每一个神经元与它周边半径为 r 范围内的所有神经元相连接。

r 按如下方式确定：

$$\pi r^2 = \frac{1}{2}\pi c(0)\theta_{max}$$

（假设神经元以密度 1 分布在格点上），这样就恰好满足连接总数量的要求。

h 可由

$$\theta_{max} = \frac{r}{h}$$

确定。

下面验证这样的布局及连接方式满足要求：

考虑任何一个神经元，不妨设其方向偏好为 0，以它为中心，以 r 为半径的圆内，与方向偏好角为 θ 的神经元连接的数目应为

$$c(0)\sqrt{1 - \frac{\theta^2}{\theta_{max}^2}}$$

半径为 r 的圆中，方向偏好角为 θ 的神经元数目为

$$2\sqrt{r^2 - (h\theta)^2}h = 2r\sqrt{1 - \frac{\theta}{\theta_{max}}}\frac{r}{\theta_{max}} = c(0)\sqrt{1 - \frac{\theta^2}{\theta_{max}^2}}$$

因此恰好符合要求。