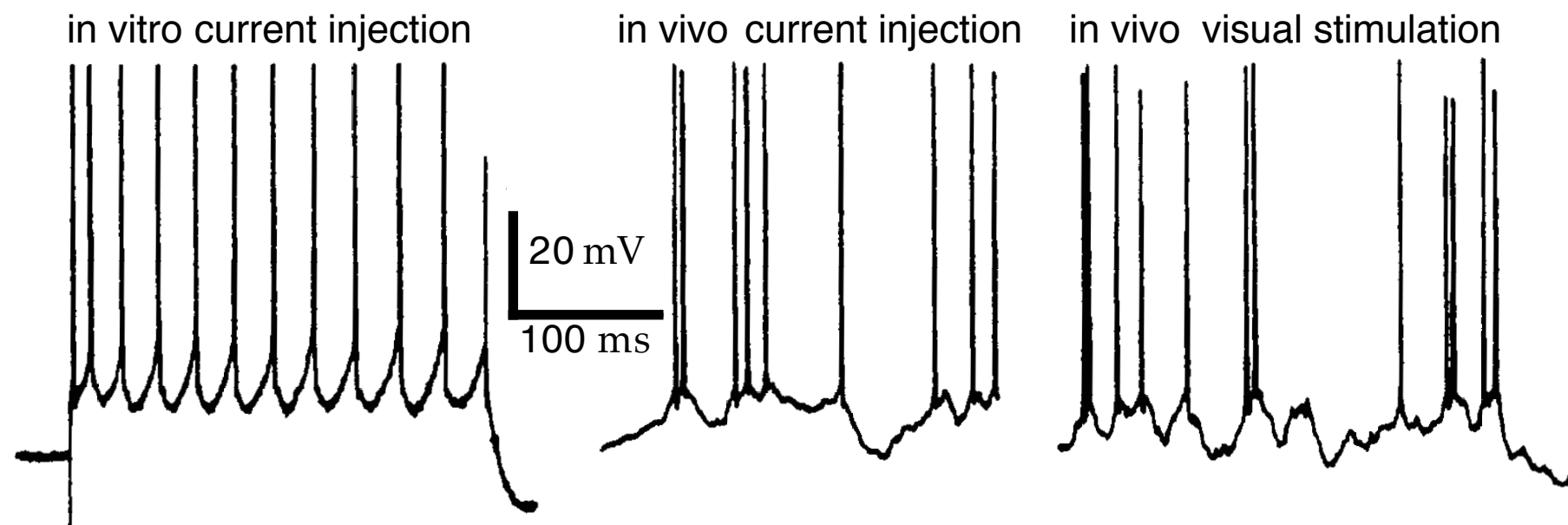
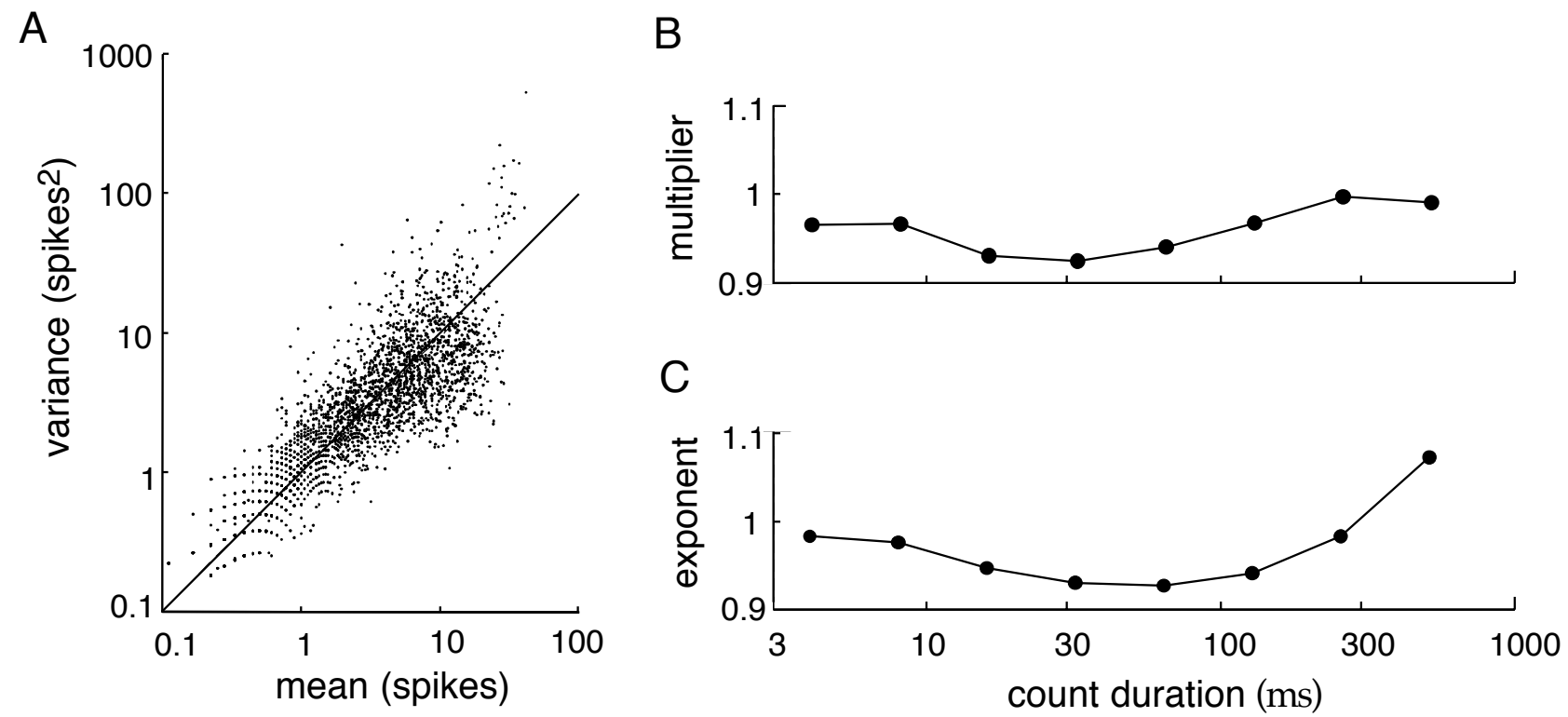


Spike-Train Statistics

Testing the Poisson Model



Testing the Poisson Model



Testing the Poisson Model

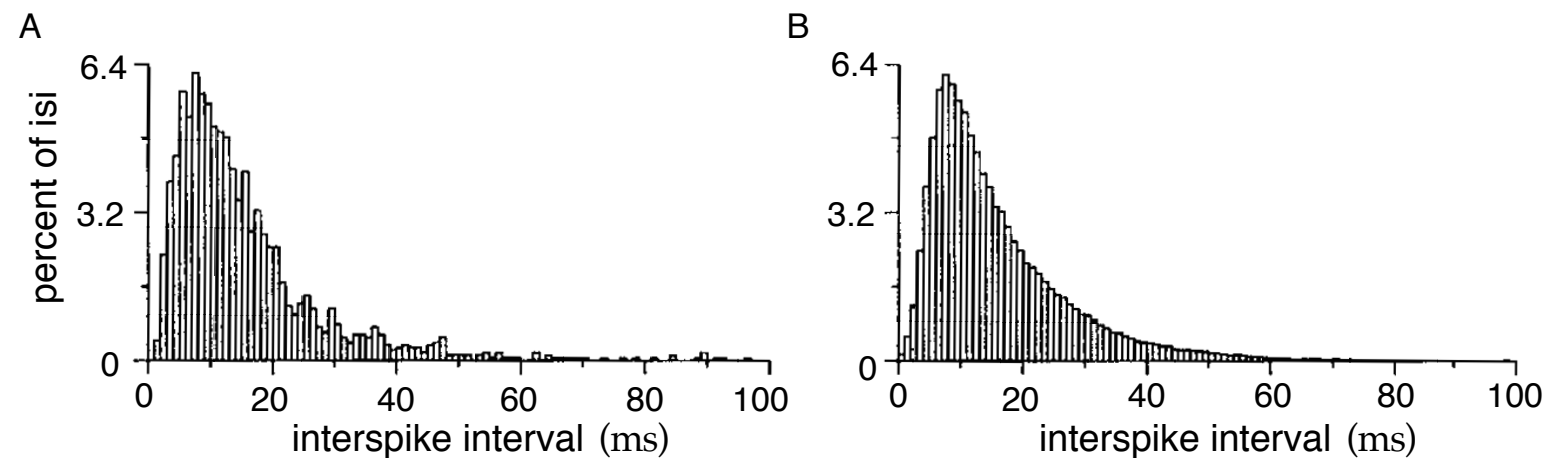
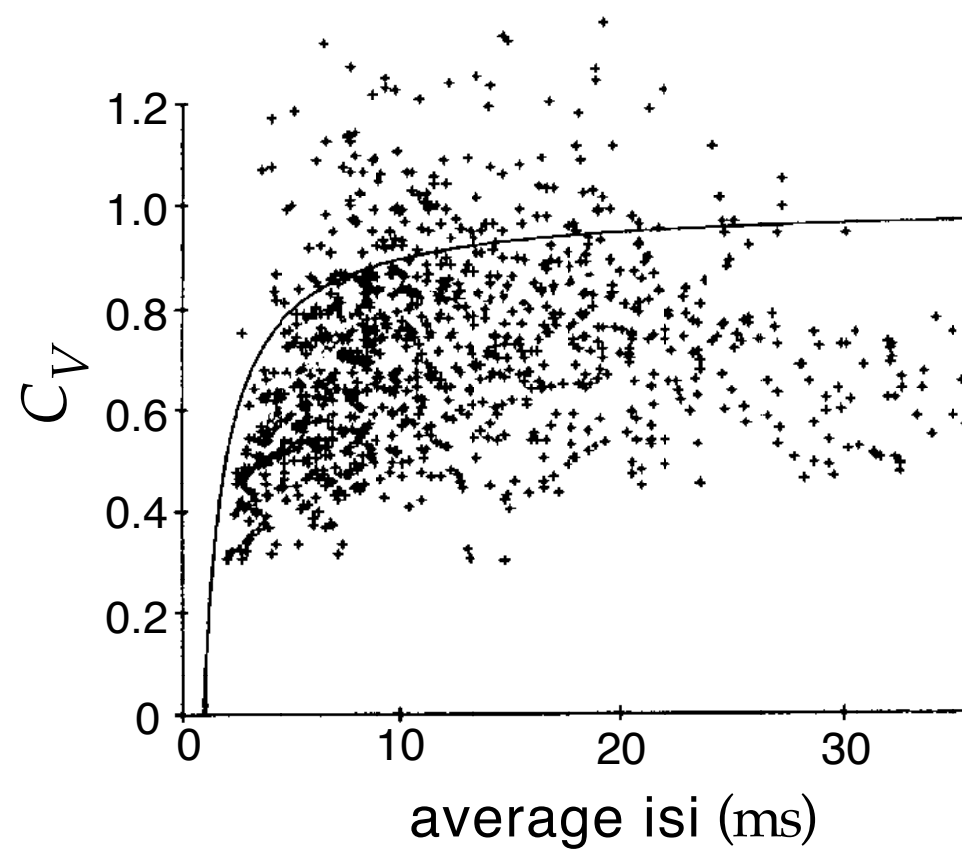


Figure 1.15: (A) Interspike interval distribution from an MT neuron responding to a moving random dot image. The probability of interspike intervals falling into the different bins, expressed as a percentage, is plotted against interspike interval. B) Interspike interval histogram generated from a Poisson model with a stochastic refractory period. (Adapted from Bair et al., 1994.)

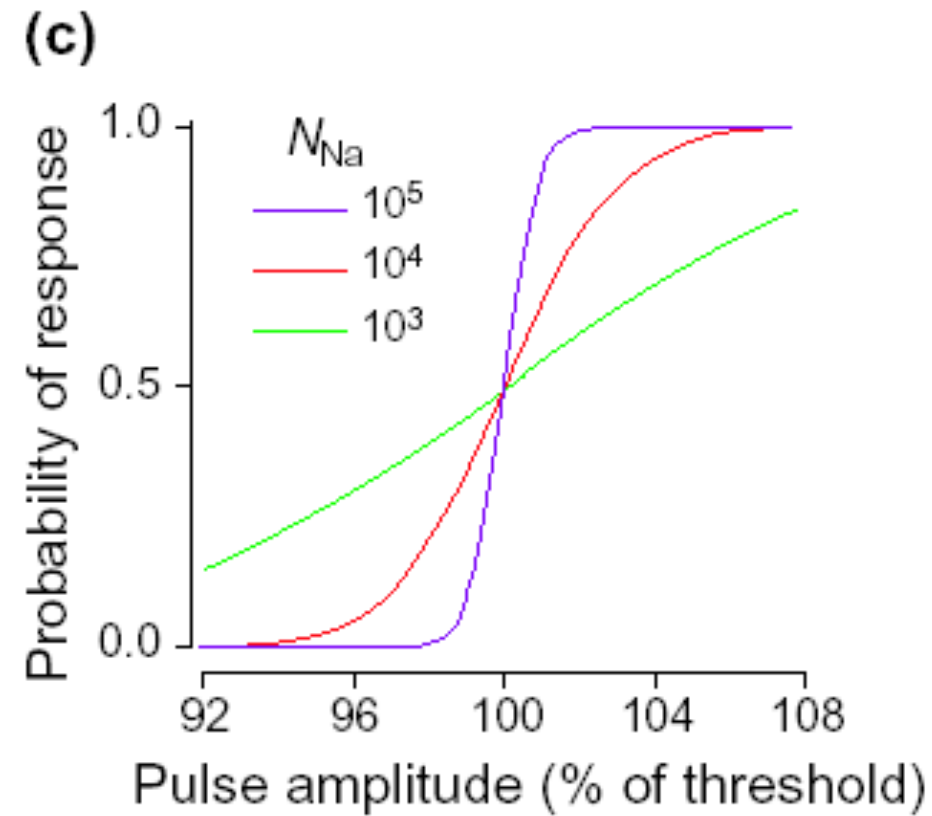
Testing the Poisson Model



Where does the stochasticity come from?

- Channel noise
- Presynaptic sources are noisy

Impact of channel noise on spike generation threshold



Neurons *in vitro* respond reliably to fluctuating stimulus

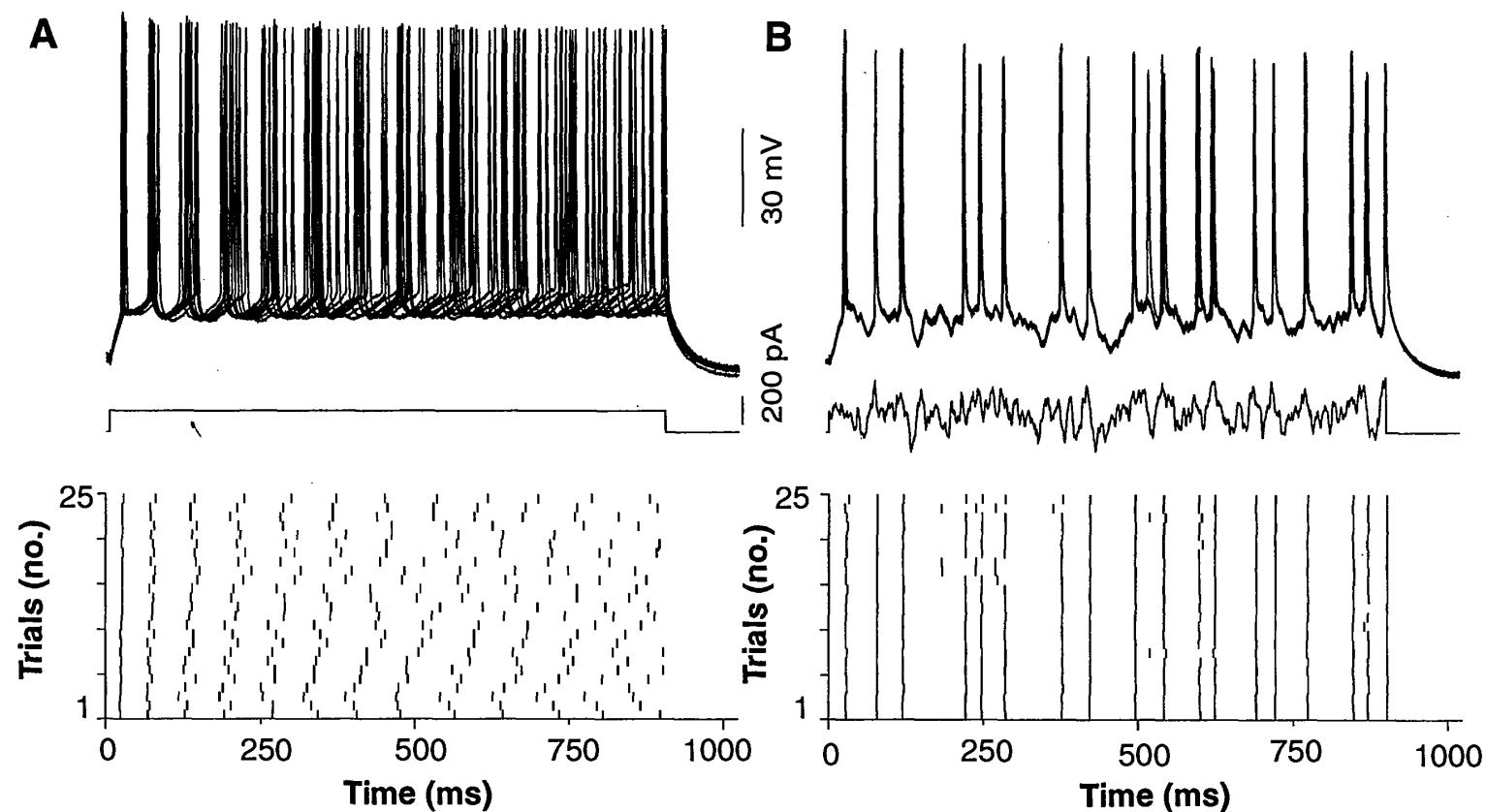
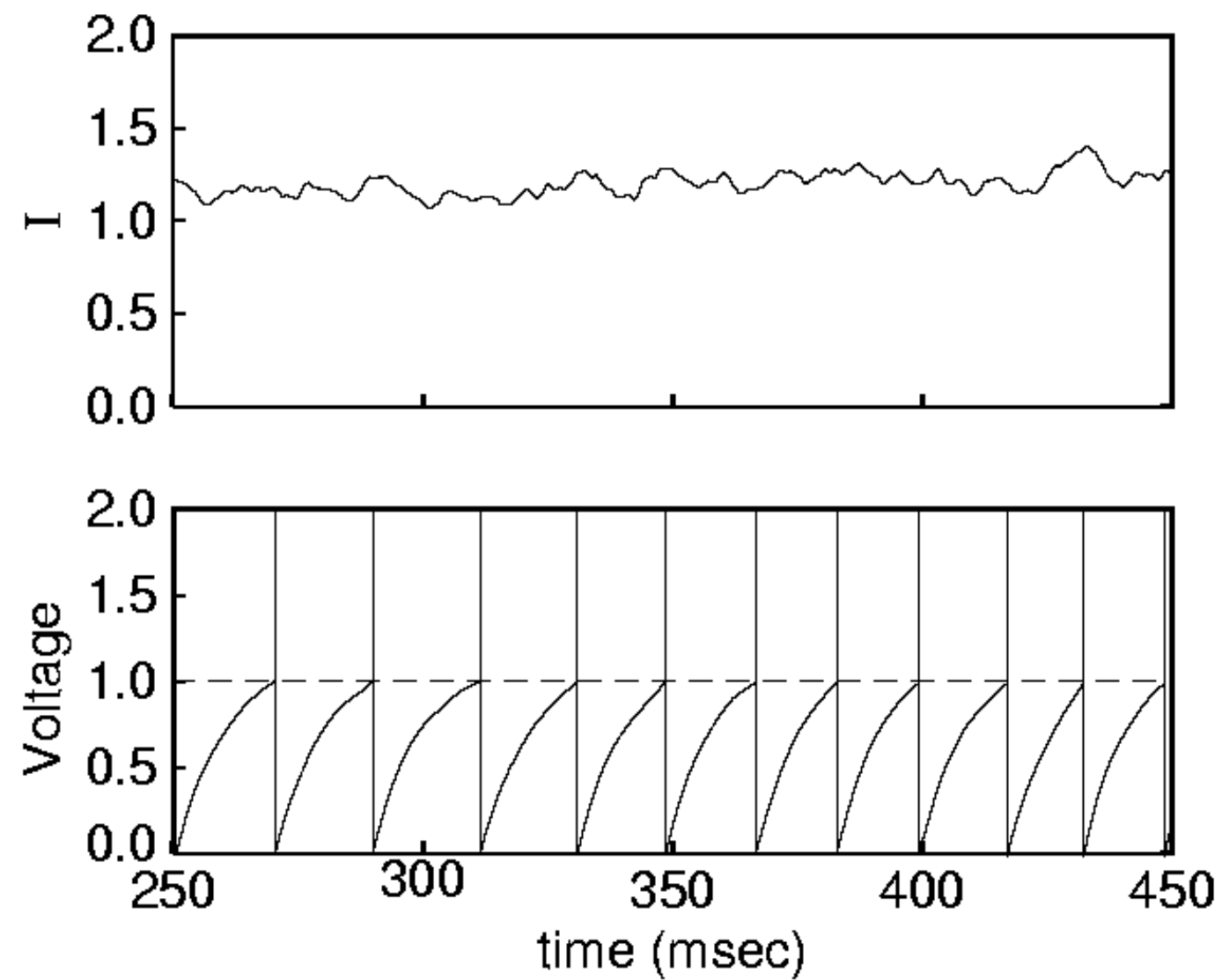


Fig. 1. Reliability of firing patterns of cortical neurons evoked by constant and fluctuating current. **(A)** In this example, a superthreshold dc current pulse (150 pA, 900 ms; middle) evoked trains of action potentials (approximately 14 Hz) in a regular-firing layer-5 neuron. Responses are shown superimposed (first 10 trials, top) and as a raster plot of spike times over spike times (25 consecutive trials, bottom). **(B)** The same cell as in (A) was again stimulated repeatedly, but this time with a fluctuating stimulus [Gaussian white noise, $\mu_s = 150$ pA, $\sigma_s = 100$ pA, $\tau_s = 3$ ms; see (14)].

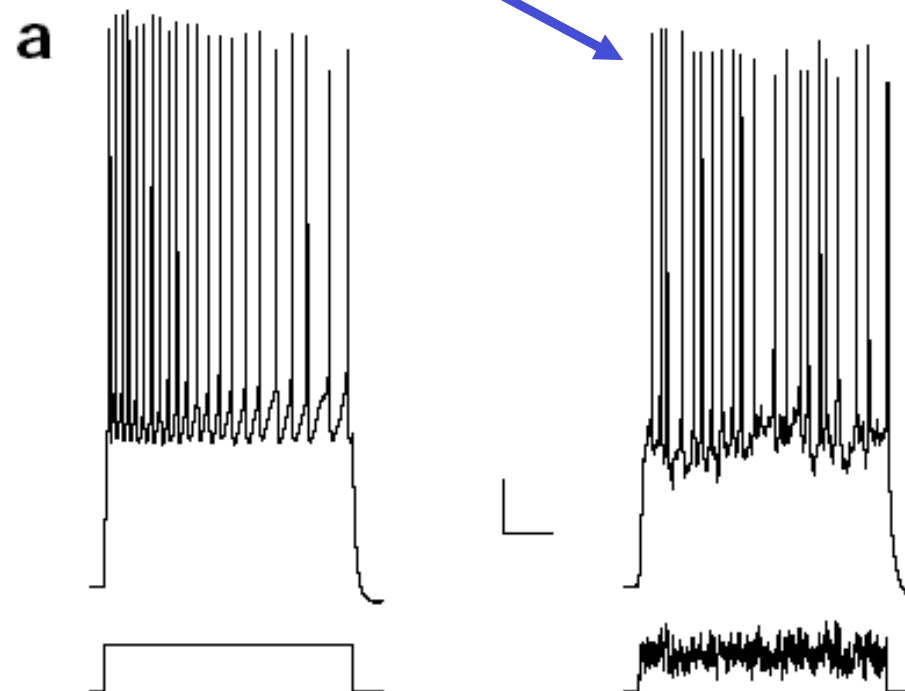
Where does the stochasticity come from?

- Channel noise
- Presynaptic sources are noisy

Integrate and Fire Neuron with $K=1000$ uncorrelated Poisson synaptic inputs



Simulating synaptic inputs in-vitro



$CV=0.28$

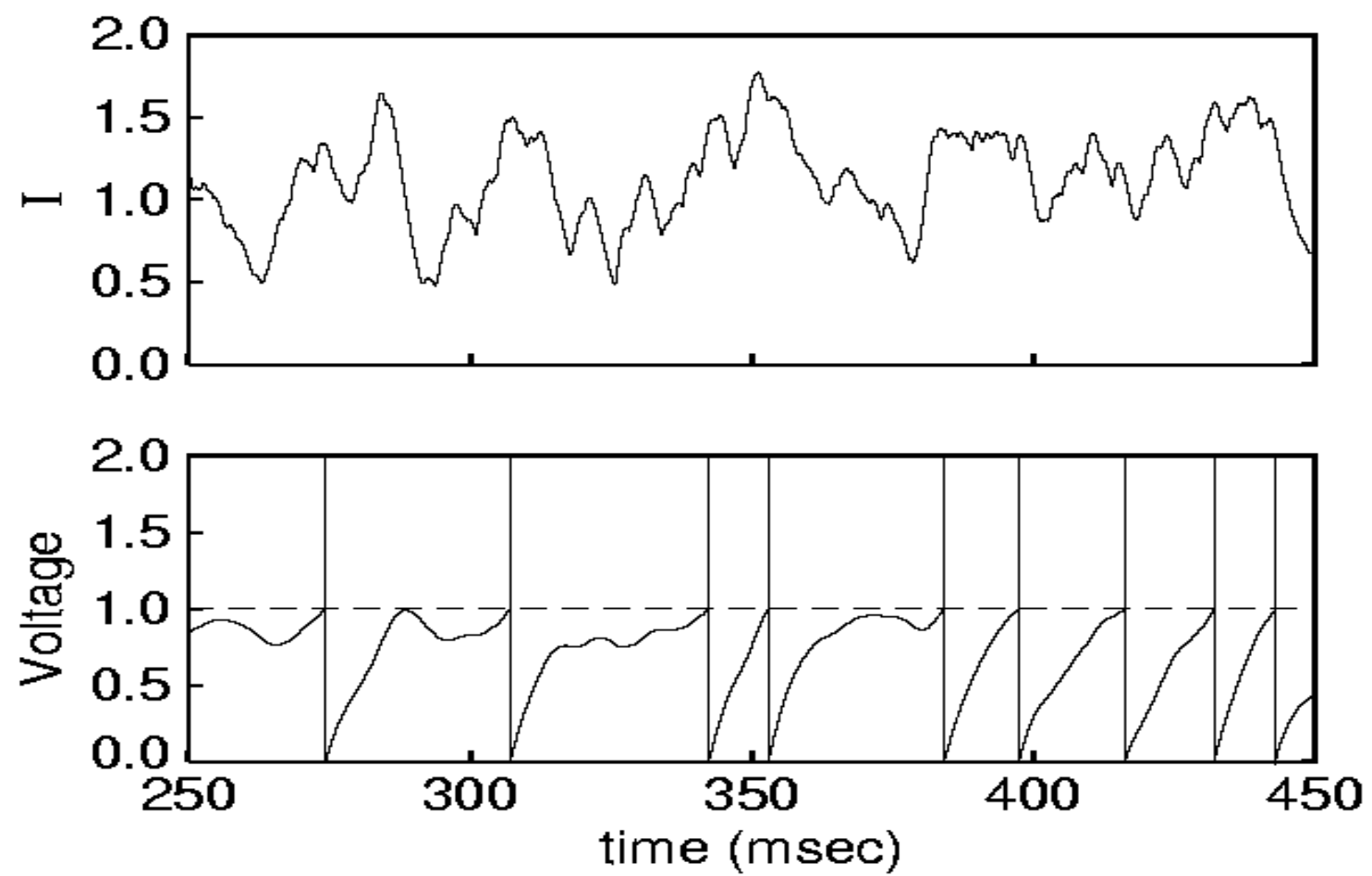
Fano factor=0.06

Two possible solutions

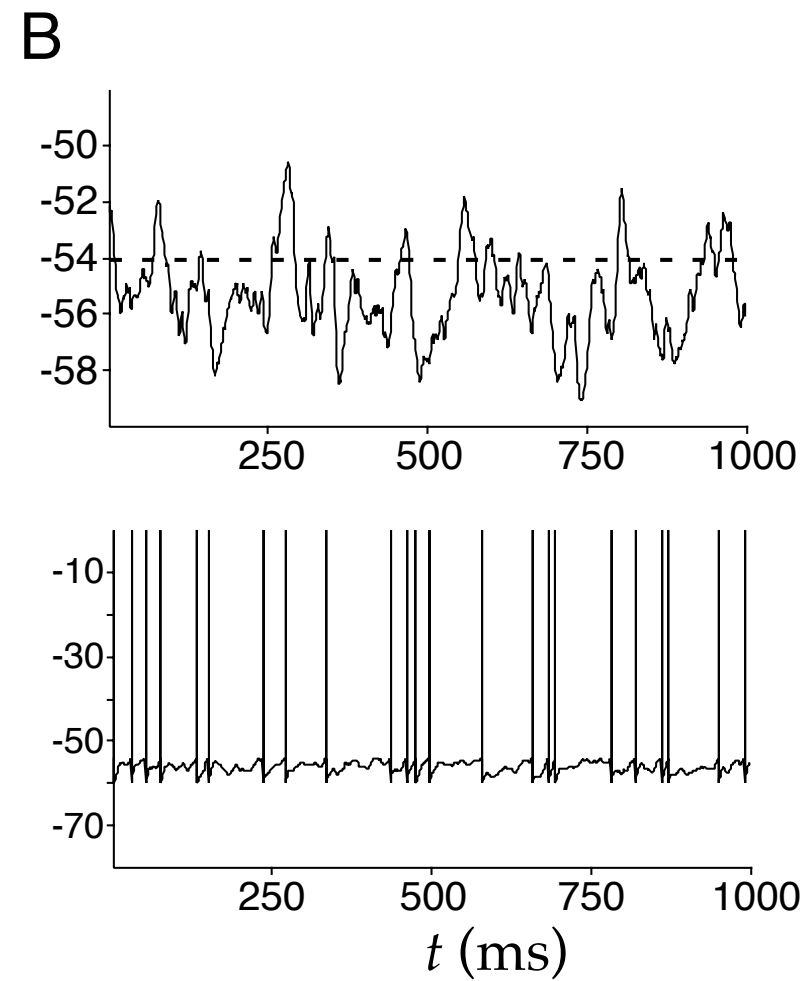
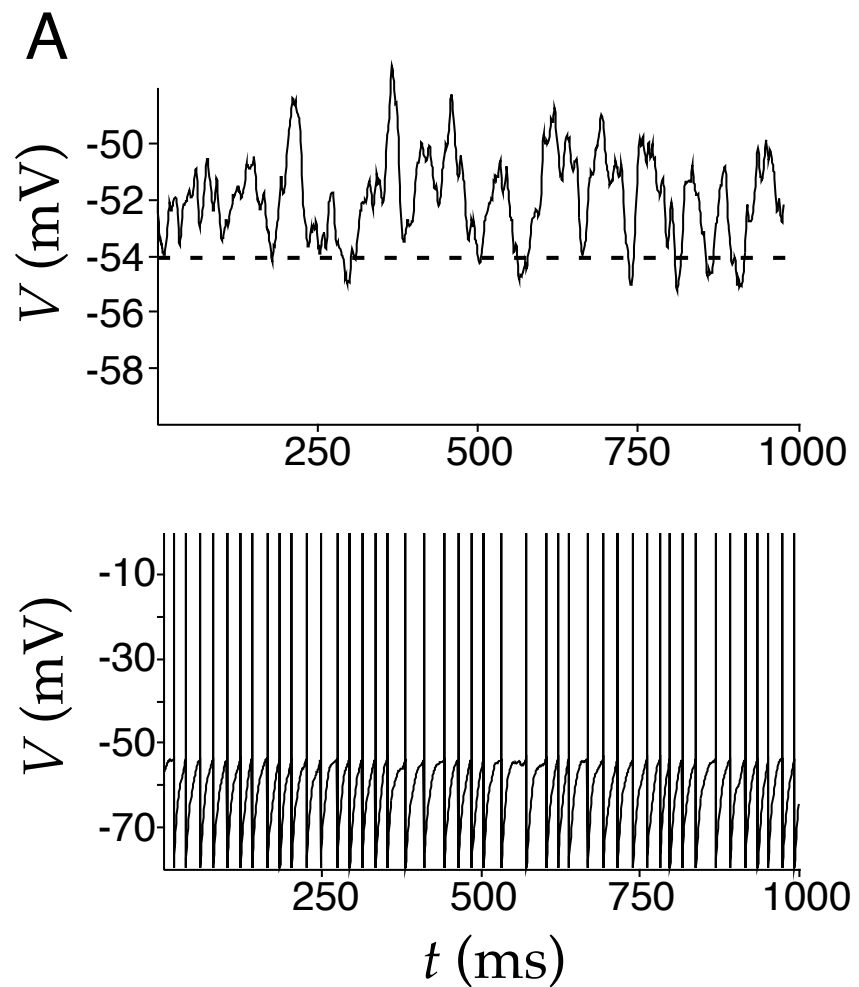
- Correlated Inputs
- Balanced Excitation and Inhibition to push the membrane potential near threshold

Large Fluctuations due to Correlated Inputs

Integrate and Fire Neuron with $K=1000$
Poisson inputs with $c=0.1$ correlations

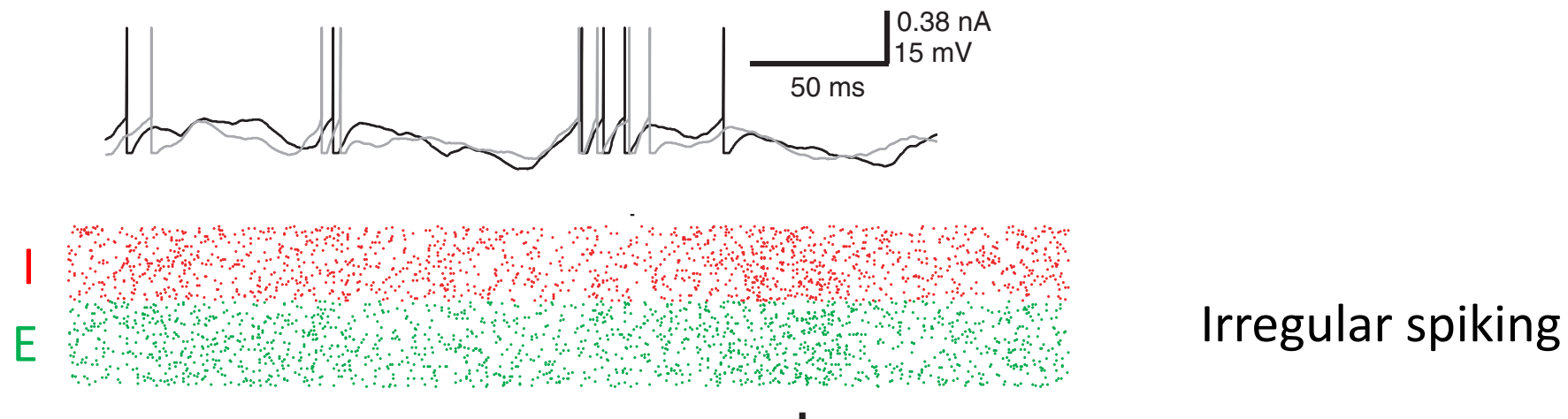


Membrane potential near threshold



The following slides are adapted from Professor
Yu Hu's lectures from HKUST

Irregular neural activity



Noise: stochastic opening of ion channels, spontaneous vesicle release,

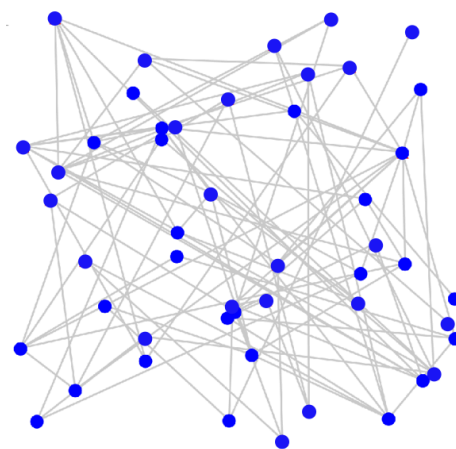
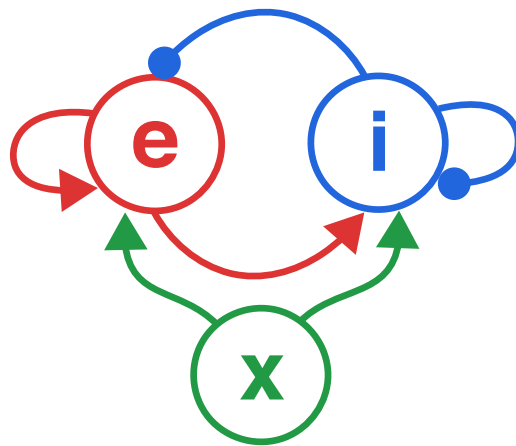
Example: input from Poisson neurons

$$I(t) = \frac{1}{N} \sum_{j=1, \dots, N} y_j(t), \quad y_j(t) \sim \text{Poisson}(r)$$

Puzzle: central limit theorem: in large N limit, the variance becomes negligible vs. the mean, $I(t)$ no longer fluctuates in time.

E-I balanced network

Local cortical circuits



Randomly connected to K neurons

$$W_{ij}^{ee} = 0 \text{ or } \frac{w_{ee}}{\sqrt{K}}$$

(1) Individual connections are strong
(vs. weak connections $\frac{w_{ee}}{K}$)

(2) K is large

$$\tau \frac{dI_i}{dt} = -I_i + \sum_j W_{ij}^{ee} y_j^e(t) - \sum_j W_{ij}^{ei} y_j^i(t) + \sum_j W_{ij}^x y_j^x(t)$$

$$\tau \frac{dI_i}{dt} = -I_i + \sum_j W_{ij}^{ee} y_j^e(t) - \sum_j W_{ij}^{ei} y_j^i(t) + \sum_j W_{ij}^{ex} y_j^x(t)$$

average over time:

$$\left\langle \sum_j^{N_e} W_{ij}^{ee} y_j^e(t) \right\rangle_t = \sum_j^{N_e} W_{ij}^{ee} r_j^e$$

average over different neurons j:

$$\left\langle \sum_j^{N_e} W_{ij}^{ee} r_j^e \right\rangle_j \approx \sum_j^{N_e} \langle W_{ij}^{ee} \rangle \langle r_j^e \rangle$$

$$\langle W_{ij}^{ee} \rangle = \frac{K}{N_e} \frac{w_{ee}}{\sqrt{K}} \qquad \langle r_j^e \rangle = r^e$$

$$\left\langle \sum_j^{N_e} W_{ij}^{ee} r_j^e \right\rangle \approx \sqrt{K} w_{ee} r^e$$

average input to an excitatory neuron:

$$\left\langle \sum_j W_{ij}^{ee} y_j^e(t) - \sum_j W_{ij}^{ei} y_j^i(t) + \sum_j W_{ij}^{ex} y_j^x(t) \right\rangle_{t,j} \approx \sqrt{K}(w_{ee}r^e - w_{ei}r^i + w_{ex}r^x)$$

average input to an inhibitory neuron:

$$\sqrt{K}(-w_{ii}r^i + w_{ie}r^e + w_{ix}r^x)$$

Since the input should be $O(1)$, we must have the E-I balanced condition

$$\mathbf{E} : w_{ee}r^e - w_{ei}r^i + w_{ex}r^x = 0$$

$$\mathbf{I} : w_{ie}r^e - w_{ii}r^i + w_{ix}r^x = 0$$

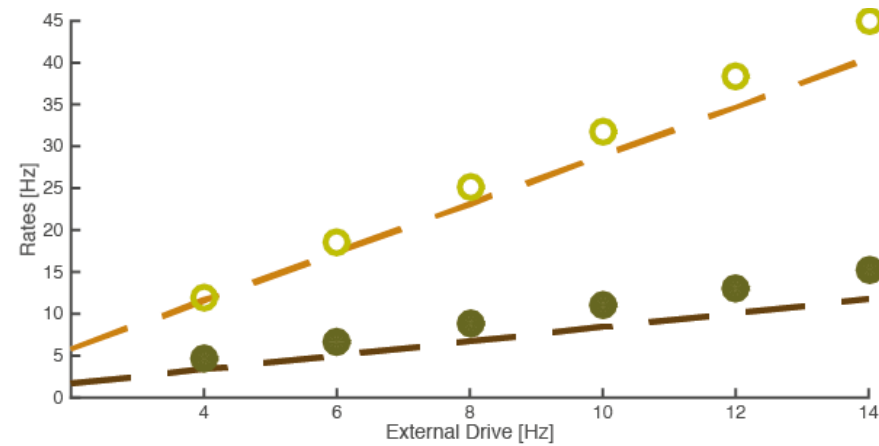
$$r^e \geq 0, r^i \geq 0, r^x \geq 0$$

$$E \quad w_{ee}r^e - w_{ei}r^i + w_{ex}r^x = 0$$

$$I \quad w_{ie}r^e - w_{ii}r^i + w_{ix}r^x = 0$$

- Solve for population firing rates $r = -W^{-1}w_x r_x$
- Predict linear population response to input!

E, filled and green
I, open and yellow



Vreeswijk and Sompolinsky 1996