In the tables below we list the subresults

$$\sum_{|\lambda|=w} \left((-1)^{\sum_i \lambda_i} \cdot \sum_{S \in (\mathbb{P}^2 - \{\mathbf{P}\})(\lambda)} |\mathbf{C}_{\text{split}}(S)| \right)$$

for $0 \leq w \leq 5$ and

$$\sum_{|\lambda|=w} |\mathbf{C}_{\text{split}}(\mathbf{P}, \lambda)| \cdot \sigma_N(\lambda)$$

for $6 \leq w \leq 9$.

The columns marked q contain the results from our computations in the article. The colums marked $q = 2$ and $q = 3$ contain the results from our computer programs.

For the explicit cases where we have a singularity on an irreducible component we do not know whether that singularity is a split node, a non-split node, or a cusp. So there will be some question marks in the tables.
For split nodes we have

w	q	$q=2$	$q=3$
0	$\frac{q^{13}}{2(q-1)^2}$	4096	$\frac{1594323}{8}$
1	$-\frac{q^{12}+q^{11}}{2(q-1)^2}$	-3072	$-\frac{177147}{2}$
2	$\frac{q^{10}}{2(q-1)^2}$	512	$\frac{59049}{8}$
3	0	0	0
4	0	0	0
5	0	0	0
6	?	1	0
7	?	-5	-9
8	$\frac{1}{2}(2q^3 - 53q^2 + 222q - 252)$	-2	$-\frac{9}{2}$
9	$\frac{1}{2}(13q^2 - 49q + 56)$	5	13
sum	?	1535	$\frac{236195}{2}$

For non-split nodes we have

w	q	$q=2$	$q=3$
0	$\frac{q^{13}}{2(q^2-1)}$	$\frac{4096}{3}$	$\frac{1594323}{16}$
1	$-\frac{q^{11}}{2(q-1)}$	-1024	$-\frac{177147}{4}$
2	$\frac{q^{10}}{2(q^2-1)}$	$\frac{512}{3}$	$\frac{59049}{16}$
3	0	0	0
4	0	0	0
5	0	0	0
6	?	3	-4
7	?	1	5
8	$\frac{1}{2}(q^2-6)$	-1	$\frac{3}{2}$
9	$\frac{1}{2}q(q-1)$	1	3
sum	?	516	$\frac{118109}{2}$

For cusps we have

w	q	$q=2$	$q=3$
0	$\frac{q^{11}}{q-1}$	2048	$\frac{177147}{2}$
1	$-\frac{q^{10}+q^9}{q-1}$	-1536	-39366
2	$\frac{q^8}{q-1}$	256	$\frac{6561}{2}$
3	0	0	0
4	0	0	0
5	0	0	0
6	?	-1	-4
7	?	-1	0
8	0	0	0
9	0	0	0
sum	?	766	52484

For the sum of the split nodes, non-split nodes, and cusps we get

w	q when $\mathrm{char}(k)=2$	$q=2$	q when $\mathrm{char}(k)\neq 2$	$q=3$
0	$\frac{q^{11}(q^3+q^2-1)}{(q-1)^2(q+1)}$	$\frac{22528}{3}$	$\frac{q^{11}(q^3+q^2-1)}{(q-1)^2(q+1)}$	$\frac{6200145}{16}$
1	$-\frac{q^9(q^3+q^2-1)}{(q-1)^2}$	-5632	$-\frac{q^9(q^3+q^2-1)}{(q-1)^2}$	$-\frac{688905}{4}$
2	$\frac{q^8(q^3+q^2-1)}{(q-1)^2(q+1)}$	$\frac{2816}{3}$	$\frac{q^8(q^3+q^2-1)}{(q-1)^2(q+1)}$	$\frac{229635}{16}$
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	$-16q^2+70q-73$	3	$-16q^2+70q-74$	-8
7	$-q^3+35q^2-156q+175$	-5	$-q^3+35q^2-156q+176$	-4
8	$q^3-26q^2+111q-129$	-3	$q^3-26q^2+111q-129$	-3
9	$7q^2-25q+28$	6	$7q^2-25q+28$	16
sum	$q^{11}+q^{10}-q^8+1$	2817	$q^{11}+q^{10}-q^8+1$	229636