Course Code: HW

Your name

May 21, 2023

Question Title (i.e., "Griffiths XX.XX")

> This is a very simple layout that I used through the end of my Undergraduate studies. No real flairs and no reliance on importing a specific sty file. I would usually put relevant information from the question in this 'quote' block.

a) *This is a enumeration using lowercase letters.*

Question Title

This is an exerpt taken from one of my Intro to Quantum Mechanics courses (Introduction to Quantum Mechnaics by David J. Griffiths question 11.13 - *I do not remember if this was correct*). The purpose of this is to show how to format math in a quick and straitforward way; not so much to a new readers but within your own workflow.

A lot of the analysis for this one mirrors what we have already done in the first question. From that conversation we can copy that $\langle 100| z |210\rangle = \frac{2^8 a}{3^5\sqrt{2}}$. We also noticed that the equations were even in x and y for all of them except $|21 \pm 1\rangle$. This is the only matrix element that we still need to calculate.

$$\begin{aligned}
\langle 100| x |21 \pm 1\rangle &= \int \frac{1}{\sqrt{\pi a^3}} \frac{\mp 1}{\sqrt{64\pi a^3}} e^{-r/a} \frac{r}{a} e^{-r/2a} \sin(\theta) e^{\pm i\phi} x \, \mathrm{d}V \\
&= \frac{\mp 1}{8\pi a^4} \left[4! \left(\frac{2a}{3} \right)^5 \right] \frac{4\pi}{3} = \mp \frac{2^7}{3^5} a \,, \\
\langle 100| y |21 \pm 1\rangle &= \int \frac{1}{\sqrt{\pi a^3}} \frac{\mp 1}{\sqrt{64\pi a^3}} e^{-r/a} \frac{r}{a} e^{-r/2a} \sin(\theta) e^{\pm i\phi} y \, \mathrm{d}V \\
&= \frac{\mp 1}{8\pi a^4} \left[4! \left(\frac{2a}{3} \right)^5 \right] \frac{4\pi}{3} (\pm i) = -i \frac{2^7}{3^5} a \,.
\end{aligned}$$

Putting these all together give that

$$\begin{aligned}
&\langle 100| \mathbf{r} |200\rangle = 0 \,, & &\langle 100| \mathbf{r} |210\rangle = \frac{2^8 a}{3^5\sqrt{2}} \hat{z} & &\langle 100| \mathbf{r} |21 \pm 1\rangle = \frac{2^7}{3^5} a (\mp \hat{x} - i\hat{y}) \,, \\
&\langle 100| \mathbf{r} |200\rangle \to |\mathcal{P}|^2 = 0 & &\langle 100| \mathbf{r} |210\rangle \to |\mathcal{P}|^2 = (qa)^2 \frac{2^{15}}{3^{10}} & &\langle 100| \mathbf{r} |200\rangle \to |\mathcal{P}|^2 = (qa)^2 \frac{2^{15}}{3^{10}} \,.
\end{aligned}$$

We are given that $A = \frac{\omega^3 |\mathcal{P}|^2}{3\pi\varepsilon_0 \hbar c^3}$, and $\omega = \frac{E_2 - E_1}{\hbar} = -\frac{3}{4\hbar} E_1$. So, for all the states where $\mathcal{P} \neq 0$:

$$A = -\frac{3^3}{2^6} \frac{E_1^3}{\hbar^3} \frac{(ea)^2 2^{15}}{3^{10}} \frac{1}{3\pi\varepsilon_0 \hbar c^3} \to 6.27 \times 10^8 \mathrm{s}^{-1} \,,$$

$$\boxed{\tau = \frac{1}{A} \approx 1.60 \times 10^{-9} \mathrm{s}} \,. \qquad \checkmark$$

Obviously, the $|200\rangle$ state never has the opportunity to transition, so the lifetime is $\tau \to \infty$.