P3_Convolution_and_DCT

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1 P3 Convolution and DCT

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Convolution in the spatial domain is equivalent to multiplication in the frequency domain. This notebook attemps to show that convolving a 2D image with a kernel is the same as multiplying the transforms of the kernel and the image.

In [1]:
```
import numpy as np
import pandas as pd

import scipy
from scipy import ndimage
from scipy import fftpack

import cv2

import matplotlib.pyplot as plt
%matplotlib inline

from IPython.core.pylabtools import figsize

figsize(10, 8)

from sklearn.preprocessing import MinMaxScaler
```

2 Sharpen Filter

In [2]:
```
img = cv2.imread('img/ex.jpg')[:, :, 0]
img = cv2.resize(img, (222, 228))

sharpen_kernel = np.array([[-1, -1, -1],
                           [-1,  8, -1],
                           [-1, -1, -1]])

filtered = cv2.filter2D(img, -1, sharpen_kernel)
plt.imshow(img, cmap = 'gray'); plt.axis('off'); plt.title('Original Image'); plt.show
plt.imshow(filtered, cmap = 'gray'); plt.axis('off'); plt.title('Sharpened Image'); plt
```

Original Image

Sharpened Image

3 Discrete Cosine Transform and Inverse Cosine Transform

```
In [3]: def get_2D_dct(img):
            return fftpack.dct(fftpack.dct(img.T, norm = 'ortho').T, norm = 'ortho')

        def get_2D_idct(coefficients):
            return fftpack.idct(fftpack.idct(coefficients.T, norm = 'ortho').T, norm = 'ortho'

        def get_reconstructed_image(raw):
            img = raw.clip(0, 255)
            img = img.astype('uint8')
            return img
```

```
In [4]: kernel_transforms = get_2D_dct(sharpen_kernel)
        img_transforms = get_2D_dct(img)
```

4 Multiply the Kernel and Image Transforms

```
In [5]: start_height = 0
        stop_height = start_height + 3

        product = np.zeros(img_transforms.shape)

        for i in range(int(img.shape[0] / 3) - 1):
            start_width = 0
            stop_width = start_width + 3

            for j in range(int(img.shape[1] / 3)):

                patch = img_transforms[start_height:stop_height, start_width:stop_width]
                m = kernel_transforms * patch

                product[start_height:stop_height, start_width:stop_width] = m

                start_width = stop_width
                stop_width += 3

            start_height = stop_height
            stop_height += 3
```

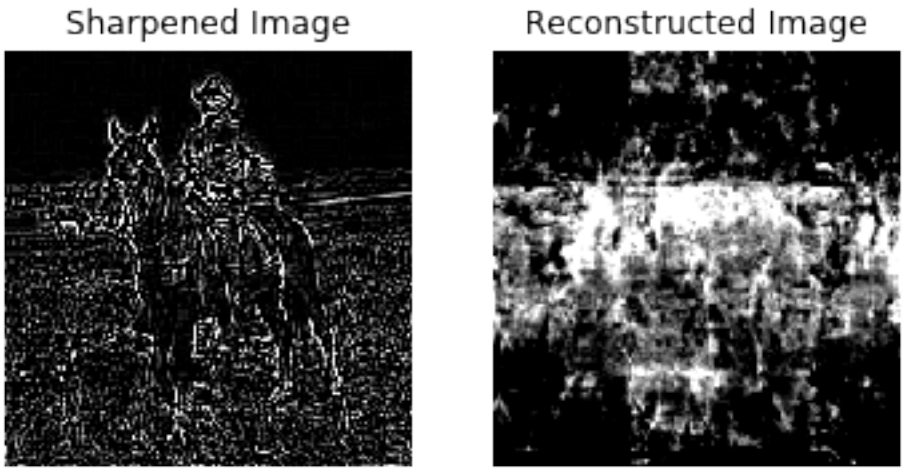
5 Visualize Reconstruction

```
In [6]: inv = get_2D_idct(product)
        inv_img = get_reconstructed_image(inv)

        plt.subplot(1, 2, 1)
```

```
plt.imshow(filtered, cmap = 'gray'); plt.axis('off'); plt.title('Sharpened Image');

plt.subplot(1, 2, 2)
plt.imshow(inv_img, cmap = 'gray');
plt.title('Reconstructed Image'); plt.axis('off')
plt.show();
```



This clearly did not work. Another attempt will first pad the kernel to be the same size as the image, take the transforms of the image and the kernel, multiply them together, apply the inverse transform, and visualize the results.

6 Pad Kernel before Transform

```
In [7]: pad = int((221 - 3)/2)
        padded_kernel = np.pad(sharpen_kernel, (pad,) , mode = 'constant')

        pt = get_2D_dct(padded_kernel)
        it = get_2D_dct(cv2.resize(img, (221, 221)))
```

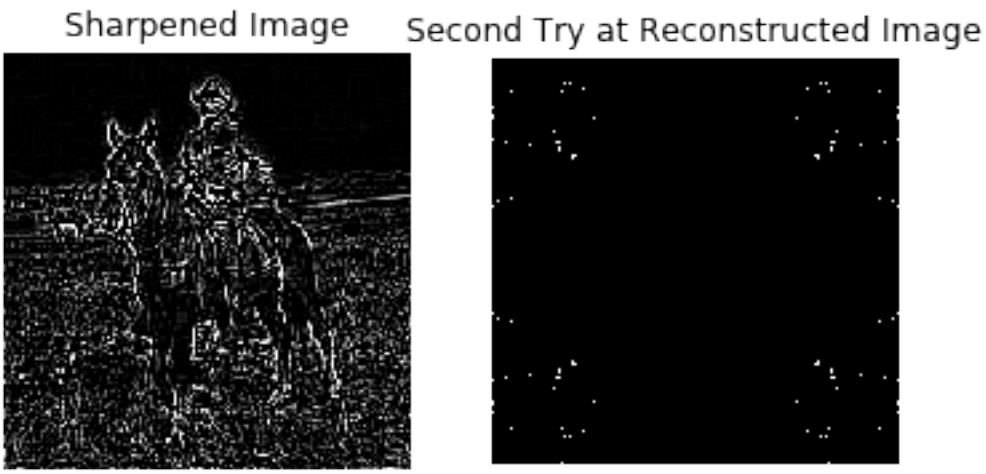
7 Multiply and Reverse Transformation

```
In [8]: m = it * pt
        inv = get_2D_idct(m)
        new_image = get_reconstructed_image(inv)
```

8 Visualize New Results

```
In [9]: plt.subplot(1, 2, 1)
        plt.imshow(filtered, cmap = 'gray'); plt.axis('off'); plt.title('Sharpened Image');
```

```
plt.subplot(1, 2, 2)
plt.imshow(new_image, cmap = 'gray');
plt.title('Second Try at Reconstructed Image'); plt.axis('off')
plt.show();
```



This attempt was no better (and in fact may have been worse). I tried several other approaches but never was able to figure out how to equate convolution in the frequency domain with multiplication in the spatial domain.

9 Conclusions

Unfortunately, I was never able to recover the image from the transformations. I think the problem is in the multiplication of the transformed kernel and the transformed image. My code tries to multiply the two transforms element by element, but this implementation does not appear to be correct. The transforms themselves are right, but it is the mutliplication of the two different-sized arrays with which I am having difficulty.