

$$P_n L(T, t) = P_n L_P(T, t) + P_n L_Q(T, t)$$

$$= E \left([C \cdot W_{tP} \Delta MV_P] - [C \cdot W_{tQ} \Delta MV_Q] \right)$$

(using the definition
of Δ in markets)

$$= E \left(\left[C \cdot W_{tP} \cdot \frac{P_T - P_t}{P_t} \right] - \left[C \cdot W_{tQ} \cdot \frac{Q_T - Q_t}{Q_t} \right] \right)$$

(using
 $\log\left(\frac{P_T}{P_t}\right) \approx \text{return}$)

$$\approx E \left(\left[C \cdot W_{tP} \cdot \log\left(\frac{P_T}{P_t}\right) \right] - \left[C \cdot W_{tQ} \cdot \log\left(\frac{Q_T}{Q_t}\right) \right] \right)$$

$$= E \left(C \left[\frac{1}{1+\beta} \cdot \log\left(\frac{P_T}{P_t}\right) - \frac{\beta}{1+\beta} \cdot \log\left(\frac{Q_T}{Q_t}\right) \right] \right)$$

$$= E \left(\frac{C}{1+\beta} \left[\log(P_T) - \log(P_t) - \beta \log(Q_T) - \beta \log(Q_t) \right] \right)$$

$$= E \left(\frac{C}{1+\beta} \left[(\log(P_T) - \beta \log(Q_T)) - (\log(P_t) - \beta \log(Q_t)) \right] \right)$$

(subbing in
values from
equation (1))

$$= E \left(\frac{C}{1+\beta} [\alpha + \varepsilon_T - \alpha - \varepsilon_t] \right)$$

$$= E \left(C \cdot W_{tP} (\varepsilon_T - \varepsilon_t) \right)$$