STA 532 Final

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5/1/23

	j	Y_j
1	1	118
2	2	74
3	3	44
4	4	24
5	5	29
6	6	22
7	7	20
8	8	19
9	9	20
10	10	15
11	11	12
12	12	14
13	13	6
14	14	12
15	15	6
16	16	9
17	17	9
18	18	6
19	19	10
20	20	10
21	21	11
22	22	5
23	23	3
24	24	3

5)

```r
# j
n = 501

# define MLE, for c(k,u)
mle = c(0.491, 4.48)

# define normalized Hessian from problem
normalized_H = matrix(c(305.82, -48.71, -48.71, 12.86), 2,2,byrow=T)/n

# define pi which we will differentiate
pi_func = expression((k / (u + k))^k)

pi_deriv_k = D(pi_func, 'k')
pi_deriv_u = D(pi_func, 'u')

k = mle[1]
u = mle[2]

deriv_vector = matrix(c(eval(pi_deriv_k), eval(pi_deriv_u)))

mle_asymptotic_variance = t(deriv_vector) %*% normalized_H %*% deriv_vector
pi_hat = eval(pi_func)

# print results
cat("Asymptotic variance of MLE :", mle_asymptotic_variance)
```

```
Asymptotic variance of MLE : 0.1228559
```

```r
cat("\n")
```

```r
cat("Asymptotic SD of MLE :", sqrt(mle_asymptotic_variance))
```

```
Asymptotic SD of MLE : 0.3505081
```

```r
cat("\n")
```

```
  cat("pi evaluated at MLE : ", pi_hat)
```

```
pi evaluated at MLE :  0.3208981
```

Numerically finding the confidence interval:

```
  moe = 2.24 * (1.601/sqrt(n))
  pi_hat - moe
```

```
[1] 0.1606767
```

```
  pi_hat + moe
```

```
[1] 0.4811195
```

```
  # checking 4.5

  f = expression(exp(lambda + (kappa/2)))
  l = D(f, 'lambda')
  kap = D(f, 'kappa')

  lambda = 4.93
  kappa = 0.07

  eval(f)
```

```
[1] 143.3086
```

```
  m = matrix(c(eval(l), eval(kap)))
  I = matrix(c(0.07, 0, 0, 2 * (0.07)^2),2,2,byrow=T)
  t(m) %*% I %*% m
```

```
         [,1]
[1,] 1487.93
```