

Characterization of Uncertainty
in
Measurements of Wind
from the
NSF/NCAR Gulfstream V Research Aircraft

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(and, we hope, others)

DRAFT 5/19/2015

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1 Introduction

1.1 Overview

Wind is the motion of the atmosphere relative to the Earth. Most research aircraft have the capability to measure wind, and these measurements have many uses in research projects using aircraft. They help define the flow and so provide context for other measurements, and they are often used to study fluxes of atmospheric constituents, turbulence, wave motions, cloud updrafts and downdrafts, convergence and divergence, and many other topics. They can provide important information transferred to models for data assimilation or for validation tests of model results.

The National Science Foundation (NSF) owns a Gulfstream V research aircraft that is operated by the Research Aviation Facility (RAF), Earth Observing Laboratory (EOL), National Center for Atmospheric Research (NCAR). In this report, this aircraft is referred to either as the NSF/NCAR GV or simply the GV. Its range and endurance makes it possible to measure wind over large distances and so to characterize mesoscale and even larger features in the atmosphere. However, its high speed (typically Mach 0.8, or about 240 m s^{-1} for flight near 40,000 ft) poses special problems for wind measurement. The flow distortion around the aircraft perturbs pressure measurements that are central to the measurement of wind, and the measurement of air temperature, needed in the calculation of wind, requires corrections of typically more than 20°C to account for dynamic heating of the sensors. Accurate measurement of wind thus is particularly challenging on this and other high-speed aircraft.

This report documents how measurements of wind are made from the GV and estimates the uncertainty associated with those measurements. These estimates apply to the system as it existed in 2014, in particular as it operated in the DEEPWAVE research project flown from New Zealand in June-July 2014. Key features of the instrumentation influencing the uncertainty in the measurements as characterized here are the presence of “OmniSTAR” GPS (Global Positioning System) measurements, providing measurements of the velocity of the aircraft relative to the earth, a calibration of airspeed provided by the Laser Air-Motion Sensor ([Cooper et al. \[2014\]](#)), and a newly developed all-weather wind sensor or “gust pod” employing a Rosemount 858 probe mounted under the wing of the GV. These complemented the standard wind-sensing system comprised of a Honeywell Laseref IV inertial reference unit, GPS measurements from Novatel and Garmin units, and a gust-sensing system based on pressure ports in the nose radome, on the fuselage, and at a pitot tube. The results obtained here do not necessarily apply to measurements from earlier projects when not all these components were available in their present form, but they should apply to measurements subsequent to 2014.

The intent in this report is to follow the conventions established by the International Committee on Weights and Measures and by the National Institute of Standards and Technology. Appendix A summarizes key aspects of those recommendations and how they are addressed in this report. This report also contains additional information resulting from various studies of the measurements that have tested the validity of the measurements or have been used for calibration.

The organization is as follows. Section 2 describes the components of the wind-measuring system in more detail, with specifications for the sensors involved and examples of the mea-

surements. Section 3 then describes calibration procedures and summarizes how measurement quality is monitored. Some empirical relationships used to deduce wind are also introduced in that section. The central discussion of uncertainty in wind measurements is contained in Sect. 4, where there is a tabulation of individual error sources along with many references to other parts of the document where studies have led to estimates of limits on those error sources. It is our hope that this section will provide a summary of the results that can either stand alone or provide a guide to the further information in this report.

Subsequent sections provide additional information and checks used to assess uncertainty. Section 5 describes in more detail how the key gust measurements have been calibrated, tested, and intercompared. Section 6 discusses the choice of measurement to represent the vertical motion of the aircraft and some aspects of the uncertainty in that measurement, the relative timing of acquisition of the measurements entering vertical wind, and a procedure for detecting the Schuler oscillation in the pitch measurement and applying a correction that reduces the uncertainty in that measurement that, uncorrected, accounts for the dominant uncertainty in vertical wind. Section 7 then uses drifting or Lagrangian circle maneuvers to establish limits on some critical components entering measurement of the horizontal wind, the true airspeed and offsets in heading and sideslip angle, which account for some of the bias limits used in Sect. 4, and it describes how the measurements of ground-speed components from the inertial units and GPS units have been combined to reduce uncertainty in the components of the horizontal wind. Appendix B presents some of the characteristics of turbulence measurements, including variance spectra and potential to measure fluxes, and notes some limitations of such measurements. Appendix C provides a discussion of reproducibility of this document, with links to the programs and data used to generate this report. At the end of the report there are a list of symbols, a list of acronyms and variable names from NCAR/EOL/RAF aircraft data files that are used in this report with definitions and page references, and an index.

1.2 The systems and equations

Three wind-sensing systems are available for use on the GV:

1. The “standard” wind sensing system that uses pressure ports on the radome combined with airspeed measured using a pitot tube and ground-speed measured by an inertial reference unit and a global positioning system (GPS) receiver.
2. A gust-pod system consisting of a Rosemount 858 airflow sensor mounted under the wing combined with an inertial system co-located with the airflow sensor and updated to GPS measurements via a Kalman filter.
3. A laser air-motion sensor (LAMS) described by [Spuler et al. \[2011\]](#) combined with its own IRU/GPS system. See also [Cooper et al. \[2014\]](#).

All three also depend on a measurement of temperature and, for the first two, humidity which is used to correct for the influence of moisture on the specific heats and gas constant of moist air. Those two also share dependence on the measurement of ambient pressure as delivered by

static sources on the fuselage. Although the other measurement components differ, for each of these the measurement of wind involves the vector sum of two components, the motion of the air relative to the aircraft and the motion of the aircraft relative to the Earth. The former is the “relative wind” and is measured as a three-component vector having magnitude equal to the “true airspeed” and angles relative to the aircraft reference frame characterized by the angle of attack and the sideslip angle. The angle of attack is considered positive if the relative wind is from below the aircraft, and the sideslip angle is considered positive if the relative wind is from the starboard side of the aircraft.¹ The relative wind defined in the coordinate system of the aircraft (conventionally with \hat{x}' forward, \hat{y}' in the starboard direction, and \hat{z}' obtained from the cross product $\hat{x}' \times \hat{y}'$ and so approximately downward but oriented to align with the aircraft reference frame) must be transformed to an Earth-based reference system (conventionally with \hat{x} toward east, \hat{y} toward north, and \hat{z} upward) so that the components can be combined to yield the Earth-relative wind. This transformation is a function of the attitude angles of the aircraft (heading ψ , pitch θ , and roll ϕ), measured by an inertial reference unit in all three cases discussed here. Once in an Earth-based reference system, the relative wind vector is added to the vector representing the aircraft motion relative to the Earth to obtain the wind. The sources of the various measurements entering this processing sequence vary among the three systems and will be discussed separately for each system.

1.2.1 The Relative Wind

In the standard aircraft coordinate system the three corresponding components of the relative wind \mathbf{v} (cf. [Lenschow \[1972\]](#) and [RAF Bulletin 23](#)) are:

$$\mathbf{v} = \begin{pmatrix} u_r \\ v_r \\ w_r \end{pmatrix} = \begin{pmatrix} V^* \\ V^* \tan \beta \\ V^* \tan \alpha \end{pmatrix} \quad (1)$$

where, if V is the true airspeed, $V^* = V / \sqrt{1 + \tan^2 \alpha + \tan^2 \beta}$ is the component of true airspeed along the aircraft longitudinal (\hat{x}') axis, α is the angle of attack and β the sideslip angle.² The sign convention is such that the relative wind is positive when *from* the direction of the axis for each component. The relative wind is therefore determined from measurements of true airspeed, angle of attack, and sideslip angle.

1.2.2 Transformation to an Earth reference frame

The orientations of the aircraft, the gust pod, and the LAMS are measured by IRUs located respectively in the fuselage and in the pod itself. Each independently measures heading, pitch, and roll, so the calculations of wind from the three systems can be fully independent except that,

¹There is potential confusion arising from the signs of yaw and sideslip. The terms have different meaning and opposite sign conventions. Yaw refers to the orientation of the aircraft about an axis perpendicular to the longitudinal and lateral axes (i.e., upward when level), and it increases as the nose moves to starboard. Sideslip refers to the direction of the relative wind, and it is positive if the relative wind is from the starboard side.

²The magnitude of \mathbf{v} is thus V as required.

because it is considered to have the smallest uncertainty, the true airspeed measured from the standard radome-based system is used also for the gust pod. In each case, the IRU measurements and GPS ground-speed components are used to transform the measurements to the reference frame of the Earth.

The required transformation is described by three rotation matrices, defined in RAF Bulletin 23 Eqs. 2.5 and 2.6:³

$$\begin{aligned}\mathbf{T}_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix} \\ \mathbf{T}_2 &= \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \\ \mathbf{T}_3 &= \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}\end{aligned}\tag{2}$$

where $\{\phi, \theta, \psi\}$ are {roll, pitch, heading}.

The transformation needs to be in the following order to conform to conventional definitions of the attitude angles:

1. Rotate by \mathbf{T}_1 using the roll angle ϕ (ROLL or CROLL_GP) to level the wings by a rotation about the \hat{x}' axis.
2. Rotate by \mathbf{T}_2 about the \hat{y}' axis using the pitch angle θ (PITCH or CPITCH_GP) to level the aircraft.
3. Rotate by \mathbf{T}_3 using the heading angle ψ (THDG or CTHDG_GP) to obtain components in a true-north reference frame. At this point, the relative-wind vector \mathbf{v}_r in an Earth-reference coordinate system is $\mathbf{v}_r = \mathbf{T}_3(\mathbf{T}_2(\mathbf{T}_1\mathbf{v}))$ where \mathbf{v} is given by (1).

The measured ground-speeds (with components VNS, VEW, VSPD) then can be added to the relative wind to get the true Earth-relative wind. In the “R” code associated with this document, the required transformations are coded to provide the described processing option, but the RAF “nimbus” routine “gust.c” provides the transformation as implemented in standard processing.

Each of the three wind-measuring systems provides its own measurement of the ground-speed components $\{VNS_x, VEW_x, VSPD_x\}$, where x denotes the system {standard, gust-pod, or LAMS}, normally labeled respectively {"C", "_GP", or "_LAMS"}. The final equations, defining the Earth-relative wind \mathbf{v}_E in terms of the three wind variables $\{WD_s, WS_s, WI_s\}$ where S

³An additional correction is applied to account for the effect of the rotation rate of the aircraft on the measurements. This correction is needed when the reference unit for motion relative to the Earth, the IRU, is separated from the measurement of relative wind. For the gust pod and single-beam LAMS, this is negligible because the IRU is co-located with the gust-measuring system. See the cited references for details.

denotes the measuring system and subscripts x or y indicate the respective east or north component of the wind, are:

$$\mathbf{v}_E = \mathbf{v}_r + \begin{pmatrix} -VNS_s \\ -VEW_s \\ VSPD_s \end{pmatrix} \quad (3)$$

$$WD_x = \arctan 2(v_{E,y}, v_{E,x}) \quad (4)$$

$$WS_x = \sqrt{(v_{E,x}^2 + v_{E,y}^2)} \quad (5)$$

$$WI_x = v_{E,z} \quad (6)$$

2 Components of the wind-sensing systems

2.1 The radome-based system

2.1.1 Overview

The primary measurement of wind on the GV is that based on measurement of true airspeed via a pitot tube, airflow angles via pressure differences measured on the nose radome, attitude angles measured by an inertial reference unit, and ground-speed components measured by the same inertial reference unit and also by a Global Positioning System receiver. A cursory description of this system was provided by [Cooper et al. \[2014\]](#). A more extensive description will be provided here. Table 1 provides a summary of the measurements used to determine the wind and the characteristics of the sensors used, and the [EOL instrument web pages](#) (cf. "State Parameters", "Wind") provide additional information on these measurements.

2.1.2 Attitude angles

Attitude angles (roll, pitch, and heading) are provided by a Honeywell IRU, with specifications as listed in Table 1. There are duplicate inertial systems, so a measure of uncertainty is how well they agree. For DEEPWAVE flight 16, the mean difference in pitch was 0.2° (reflecting a small difference in installed orientation relative to the aircraft axes) and the standard deviation in the difference between measurements was about 0.015° (for measurements with absolute value of the roll smaller than 3°). If the measurements are filtered to remove variations with period exceeding about 1000 s the slowly varying component of the difference has standard deviation of about 0.012° while the fast-varying component has standard deviation of 0.008° . This was characteristic of most flights, although there were two (8 and 20) that had slightly larger standard deviations. The project averages were 0.02° for the slow component and about 0.007° for the fast component. This is an indication that the IRU performs better than indicated by the specifications, and indeed additional evidence for this is provided in Section 6.4. As discussed later in Section 6.4, the error in pitch tends to precess with a period of about 84.4 min, so the slowly varying component tends to be dominated by this precession which, for periods short compared to 84.4 min, introduces a bias while the faster varying component has the character of a random error.

2.1.3 Ambient pressure

Ambient or “static” pressure is measured on the GV at pressure ports called static buttons that are located at positions on the fuselage where in normal flight the pressure is approximately the pressure present outside the airflow disturbance produced by the aircraft. Because there are residual effects of airflow that change with angle of attack and Mach number, corrections are applied to these measurements to obtain better representation of the true ambient pressure. These corrections are described in [this document on processing algorithms](#), Sect. 4.3, and in [Cooper et al. \[2014\]](#). The latter reference has additional information on the locations of the

Measurement (VARIABLE)	Instrument	Range, Char- acteristics	Standard Uncertainty	Comments
pitch, roll (PITCH, ROLL)	Honeywell Laseref IV HG2001 GD03	ring gyros, strap-down system	0.05°	mixed bias and random error
heading (THDG)	"	"	0.2°	"
ambient pressure (PSF)	Paroscientific 1000-15A-28 (absolute)	0–15 psi \simeq 0–1035 hPa	0.10 hPa	specs. assumed std. uncertainty
dynamic pressure (QCF, QCR)	Honeywell PPT0005- DXX2VB-5021	0–5 psi \simeq 0–345 hPa	0.34 hPa 0.68 hPa max.	"
pressure differences ({A,B}DIFR)	Honeywell PPT0001- DXX2VB-5021	\pm 1~psi \simeq \pm 68.95 hPa	0.07 hPa 0.14 hPa max.	the first is "typ.", average over the range
horizontal GV velocity components (VNS, VEW)	Laseref IV (see above)	strap-down ac- celerometers	2.1 m s^{-1}	0.1 m s^{-1} with slow updating to GPS
" (GGVNS, GGVEW)	Novatel OEMV differential GPS	correction via OmniSTAR XP	0.03 m s^{-1}	$<0.1 \text{ m s}^{-1}$ without OmniSTAR
vertical GV speed (VSPD)	Laserref IV (see above)	strap-down ac- celerometers	0.76 m s^{-1}	with baro-loop updating
" (GGVSPD)	Novatel GPS (see above)		0.03 m s^{-1} with OmniSTAR	0.1 m s^{-1} without OmniSTAR
temperature (ATX)	HARCO 100009-1	–80 to +40°C, anti-iced	0.3°	needed for true airspeed
dewpoint (DPX)	Buck Research 1011C	–70 to +30°C	<5°	for level flight

Table 1: Characteristics of measurements from the radome-based system that are used for the standard calculation of the wind. See the discussion of each measurement in the text of Sect. 2.1.

sensors and the system configuration. The transducer characteristics are listed in Table 1, and the transducer is temperature-compensated to maintain these characteristics in flight when the cabin-mounted transducer can encounter fluctuating temperature. It is a digital transducer with resolution of 0.001 hPa, equivalent to about 20-bit resolution, when sampling at 1 Hz. The characteristic response time of the sensor is 0.02 s and measurements are normally sampled at 50 Hz and filtered to 25 Hz. However, lines of length 2.3 m and inside diameter 0.21 inch (0.53 cm) connect the transducer to the static buttons, and there is an additional long line (5 m) extending to the low-pressure port on the differential sensor measuring dynamic pressure.. Some problems apparently arise from these lines that affect the high-frequency response, as discussed in Appendix B. More information is available in the EOL instrument pages; see [this link](#).

2.1.4 Dynamic pressure

The dynamic pressure is the pressure difference above ambient that develops if air is compressed and adiabatically brought to rest relative to the moving aircraft. The total pressure, the sum of ambient and dynamic pressure, is sensed using a pitot tube, a tube pointed in the direction of the relative airflow and designed to be relatively insensitive to small-angle changes in the direction of the relative airflow. Figure 1 shows the location of the research-system pitot tube on the GV as well as one of the avionics-system pitot tubes. The excess pressure q above the ambient sensed by a pitot tube or, approximately, by the center port on the radome is $q = 0.5\rho_a V^2$ where ρ_a is the density of air and V the airspeed, so this excess pressure can be used to determine the airspeed of the aircraft. On NSF/NCAR aircraft, the measurement of dynamic pressure is made using differential sensors connected between a static source and a total-pressure source from either a pitot tube (QCF) or the front port on the radome (QCR). The sensor used, with specifications listed in Table 1, has these additional characteristics: resolution is 0.0011% of full scale or 0.0076 hPa, which is better than 16-bit resolution; the maximum sampling rate is 120 Hz, response time 50 ms and response delay 21 ms or about one sample period at 50 Hz sample rate. The response time is affected further by the pressure lines between the pressure ports and the transducers, as discussed above. The transducer provides 50-Hz output that is then filtered digitally to 25 Hz or 1 Hz in processing.⁴

Because any errors affecting the measurement of static pressure also affect the difference between dynamic and static pressure, the same corrections that are applied to static pressure (for errors in the pressure delivered by the static ports) are also applied to the dynamic pressure. See the references in the preceding subsection for more information. Cooper et al. [2014] argue that the measurements of static and dynamic pressure, corrected for flow distortion or generation of a “static defect” at the static-pressure ports, each have standard uncertainty of 0.3 hPa and precision (for straight and level flight) of 0.1 hPa.

⁴The specifications indicate that the appropriate time lag to apply in processing would be 21 ms but there is additional delay introduced by the sample tubing. Most processing including preliminary processing for DEEPWAVE has not included a delay for QCF or QCR.

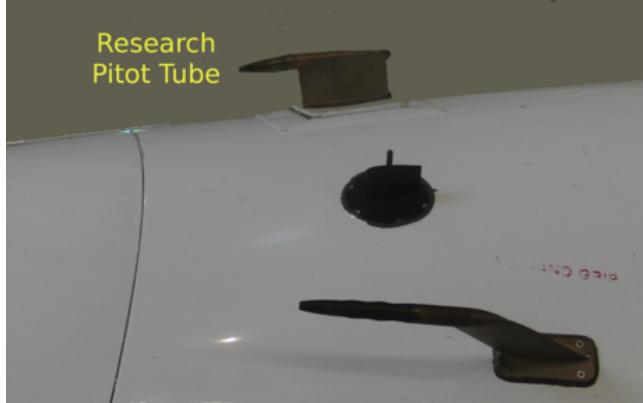


Figure 1: A pitot tube used for the measurement of dynamic pressure.

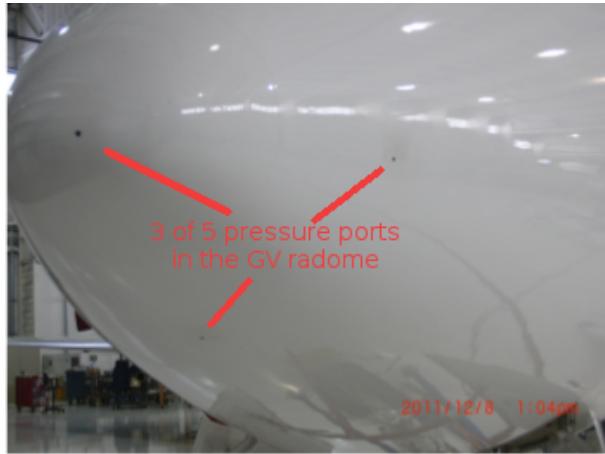


Figure 2: Photograph of the GV radome showing three of the five pressure ports on the radome used for measurement of components of the relative wind.

2.1.5 Airflow angles

The radome gust-sensing system consists of five pressure ports installed in a standard GV radome, as shown in Fig. 2.

The pressure ports are connected to differential pressure sensors, one between the top and bottom ports (variable ADIFR), one between the left and right ports (variable BDIFR), and one between the center port and the static source (variable QCR). The latter provides an alternative measurement of dynamic pressure but is not normally used. The measurements ADIFR and BDIFR are obtained from differential pressure transducers, with characteristics as shown in Table 1. The transducers have specified response times of 0.05 s and resolution 0.0015 hPa, with quoted stability of about 0.03 hPa per year. These measurements are used with procedures discussed in Section 5.1.2 to find the angle of attack and sideslip angle of the relative wind. Additional information is available at [this URL](#).

2.1.6 Components of aircraft velocity relative to the Earth

There are two sources of information regarding the ground-speed vector, an inertial reference unit (IRU) and a Global Positioning System (GPS).

1. The IRU: The inertial system on the GV is a Honeywell Laseref IV Model HG2001 GD03, with specifications listed in Table 1. There are three units on the aircraft, two of which are recorded via the ARINC data bus to standard data files. These are strap-down ring laser gyro micro inertial systems. The measurements of ground-speed components are affected by errors that arise from initial alignment errors or orientation errors resulting from gyro responses to acceleration and so often exhibit a characteristic Schuler oscillation with magnitude that can be as much as $1\text{--}3 \text{ m s}^{-1}$. This is the primary source of error in the measurements of wind, so for aircraft velocity components it is important to remove these errors by comparison to lower-uncertainty measurements provided by the GPS that are not subject to the Schuler oscillation. In addition, there are signal delays that are accounted for in post processing to align measurements with other recorded data, and there are some inherent filters in the IRU computer that affect the signals transmitted to the GV data system. The orientation of this unit was defined and checked by initial survey to coincide with the aircraft reference axes.
2. The GPS: The primary GPS unit is a Novatel OEMV differential GPS unit (L1/L2) with OmniSTAR XP satellite update for ionospheric and other corrections. As used on the GV, it reports ground-speed components at a rate of 5 Hz, although faster rates are possible and 10 Hz is in standard use after DEEPWAVE. The claimed standard uncertainty for position is 0.15 m for vertical position; the standard uncertainty in velocity is 0.03 m s^{-1} when OmniSTAR corrections are available and $<0.1 \text{ m s}^{-1}$ otherwise.

fix

2.1.7 Temperature

A measurement of temperature is needed to calculate the wind because the conversion from dynamic pressure to true airspeed involves the temperature, as documented in [this document on processing algorithms](#). The measurements of temperature were checked against expectations for height-vs-pressure changes from the hydrostatic equation by [Cooper et al. \[2014\]](#), with the result that the measurements were validated to an uncertainty of about 0.3°C . Documentation of the temperature uncertainty will be presented in more detail in a separate document.

2.1.8 Humidity (dew point)

The calculation of true airspeed from measured dynamic and static pressure involves the specific heats and gas constant for air, and this can depart from dry-air values when water vapor is present in significant amounts. This correction is usually insignificant for dew-point temperatures below about -20°C . The equations used are those in [the document on processing algorithms](#). The correction to true airspeed is approximately a factor of $(1+0.3q_h)$ where q_h is the dimensionless specific humidity, typically about 0.01 at 10°C dewpoint and 700 hPa pressure.

In this case the correction to airspeed, typically 150 m/s at this altitude, is about 0.45 m/s, so the correction is not negligible but is relatively insensitive to uncertainty in the measured humidity. The dewpoint measurements become more uncertain than listed here at the low end of this range, but the humidity correction is insignificant there. Those measurements are likely better than listed here for the upper range, in level flight, but lags and overshooting introduce errors when conditions are changing rapidly as in climbs or descents.

2.1.9 Examples of measurements

Typical measurements of the attitude angles are shown in Fig. 3. The large difference in pitch is a result of the gust pod being installed in a canister below the wing where it points downward by several degrees relative to the aircraft longitudinal axis. (The pods were designed this way to provide better approaching airflow for cloud-imaging probes and other sampling from the airstream.) There is also a significant difference in heading and in roll for similar reasons.

The measurements of pressures and the true airspeed calculated from these measurements are shown in Fig. 4 for the same period as in the preceding figure. Corrections have been applied to the pressure measurements according to the calibration determined from LAMS measurements, as described by [Cooper et al. \[2014\]](#); these corrections vary with flight conditions but normally are smaller than a few hPa so are not evident in these plots. They are nevertheless crucial to reducing the uncertainty in the true airspeed to about 0.3 m s^{-1} , as shown in that reference.

Figure 5 shows the measurements of differential pressure at the radome and the resulting angle of attack and sideslip angle calculated from those pressure differences. The calculation is described in Section 4 of this document. Fluctuations in sideslip angle are seldom more than a fraction of a degree, while there can be several-degree fluctuations in the angle of attack. The gradual decrease in angle of attack is a result of the change in fuel load on the aircraft, which requires a smaller angle of attack to keep the aircraft level as the weight becomes smaller.

The last set of components entering the measurement of wind consists of the measurements of the motion of the aircraft with respect to the Earth. These measurements must be combined with the measurement of relative wind to transform the measurements to an Earth-referenced measurement. Figure 6 shows the east and north components of the ground speed as measured by the IRU and GPS. They are close enough to lie almost on top of each other in this plot, but the red lines show the difference magnified by a factor of 100. They clearly show the Schuler oscillation that results from an IRU error in pitch, having magnitude of about $1\text{--}2 \text{ m s}^{-1}$. This error is discussed in the next section, and Section 7.2 discusses how the IRU measurements (having good short-term response but long-term drift) and the GPS measurements (having long-term accuracy but inferior short-term response) are combined in the measurement of wind. In addition to the Schuler oscillation, additional perturbations associated with turns result from the mixing of pitch, roll, and heading errors when the aircraft is banked.

Finally, Fig. 7 shows the resulting wind measurements for this flight. These measurements will be discussed extensively in the remainder of this report, and the uncertainty associated with them will be estimated in the closing section.

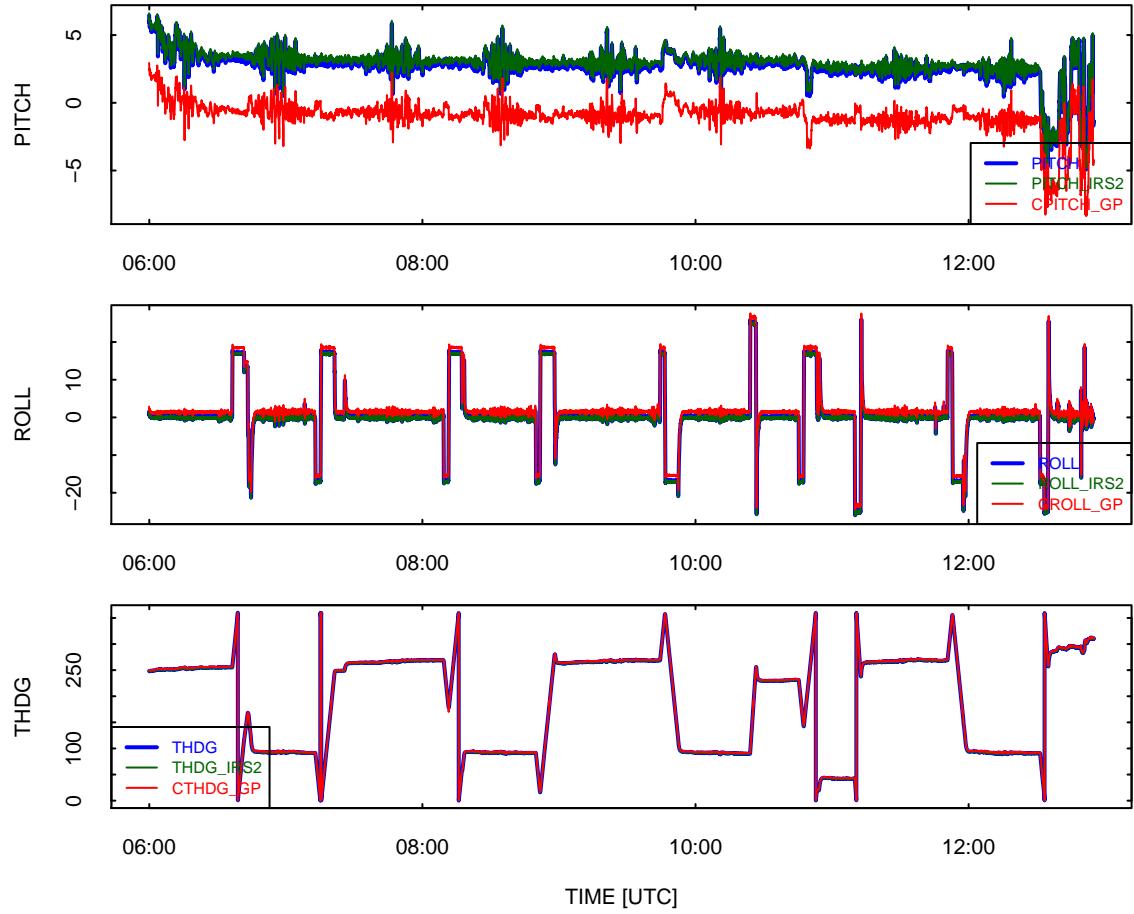


Figure 3: Attitude angles pitch, roll, and heading as measured by three independent systems inertial reference systems. The systems are: (1) the standard Honeywell IRU (PITCH, ROLL, THDG, blue lines); (2) a duplicate backup Honeywell IRU (PITCH_IRS2, ROLL_IRS2, THDG_IRS2, green lines), and the C-MIGITS IRU mounted in the gust pod (CPITCH_GP, CROLL_GP, CTHDG_GP, red lines). All units are degrees. Data from DEEPWAVE flight 16 (4 July 2014), 9:00:00 to 10:00:00.

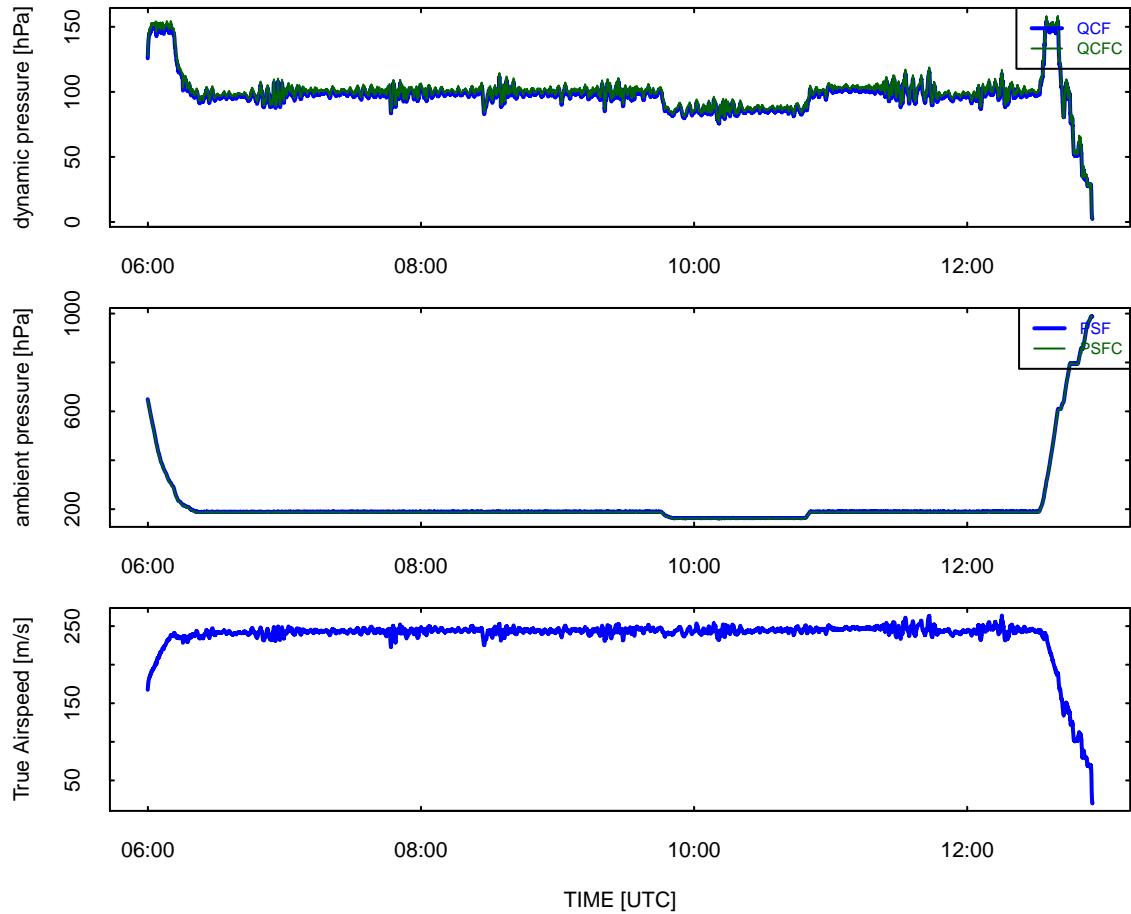


Figure 4: The measurements of dynamic pressure (QCF and, after correction QCFC), ambient pressure (PSF and corrected PSFC) and the resulting true airspeed TASX. Data from DEEP-WAVE flight 16.

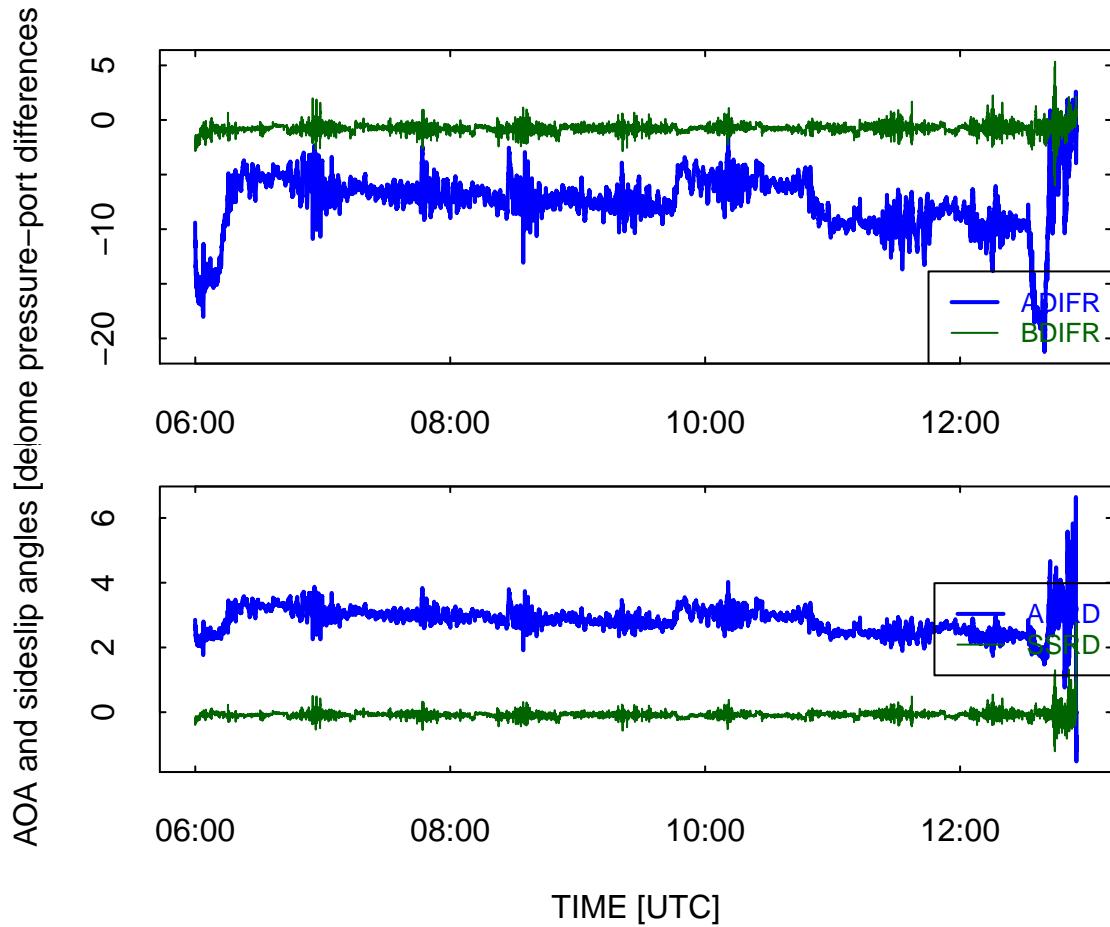


Figure 5: The pressure differences measured on the radome (ADIFR and BDIFR, respectively between the vertically separated ports and the horizontally separated ports) and the resulting airflow angles AKRD (angle of attack) and SSRD (sideslip angle). Data from DEEPWAVE flight 16.

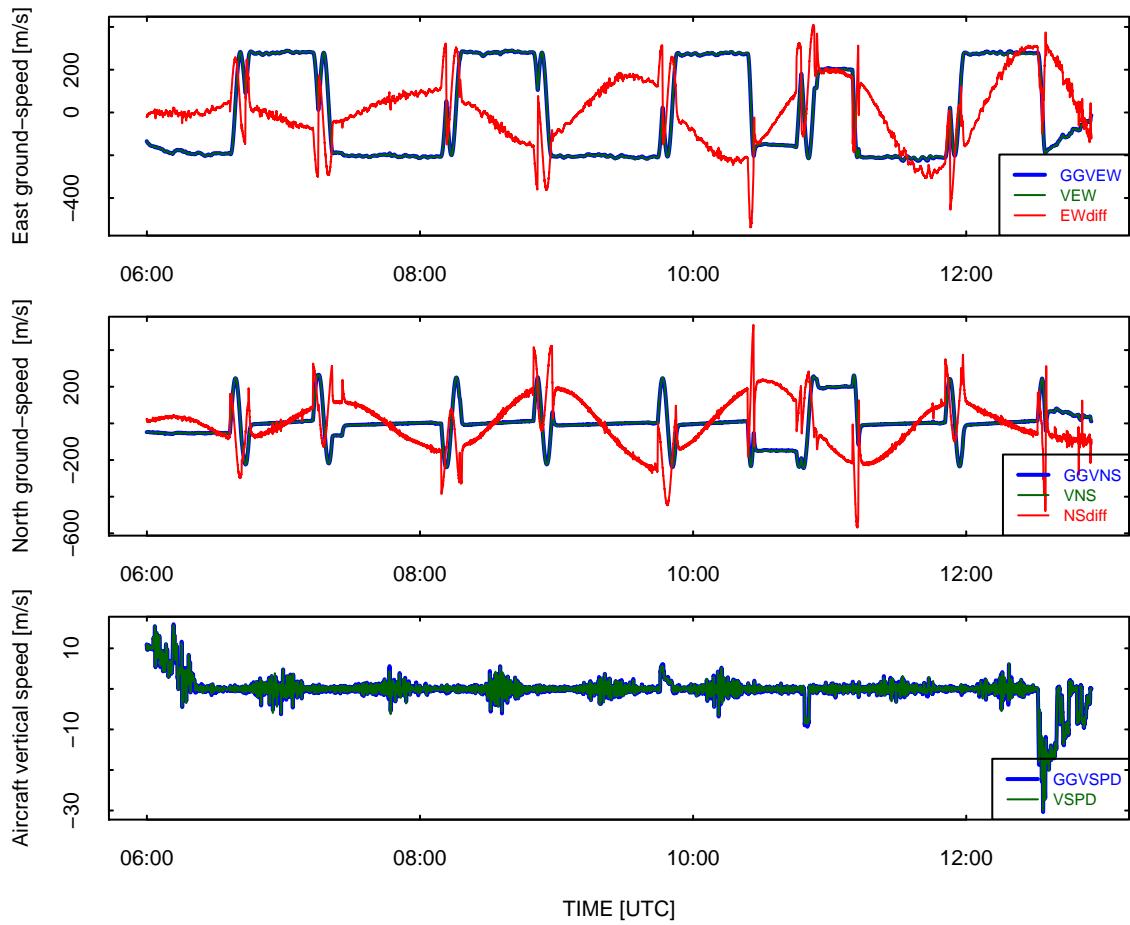


Figure 6: Top two panels: Ground-speed components as measured by the IRU and GPS, and (red lines) the difference between the two measurements multiplied by a factor of 100. Bottom panel: Aircraft vertical speed as measured by the IRU (with built-in damping to the pressure altitude) and by the GPS unit. Data from DEEPWAVE flight 16.

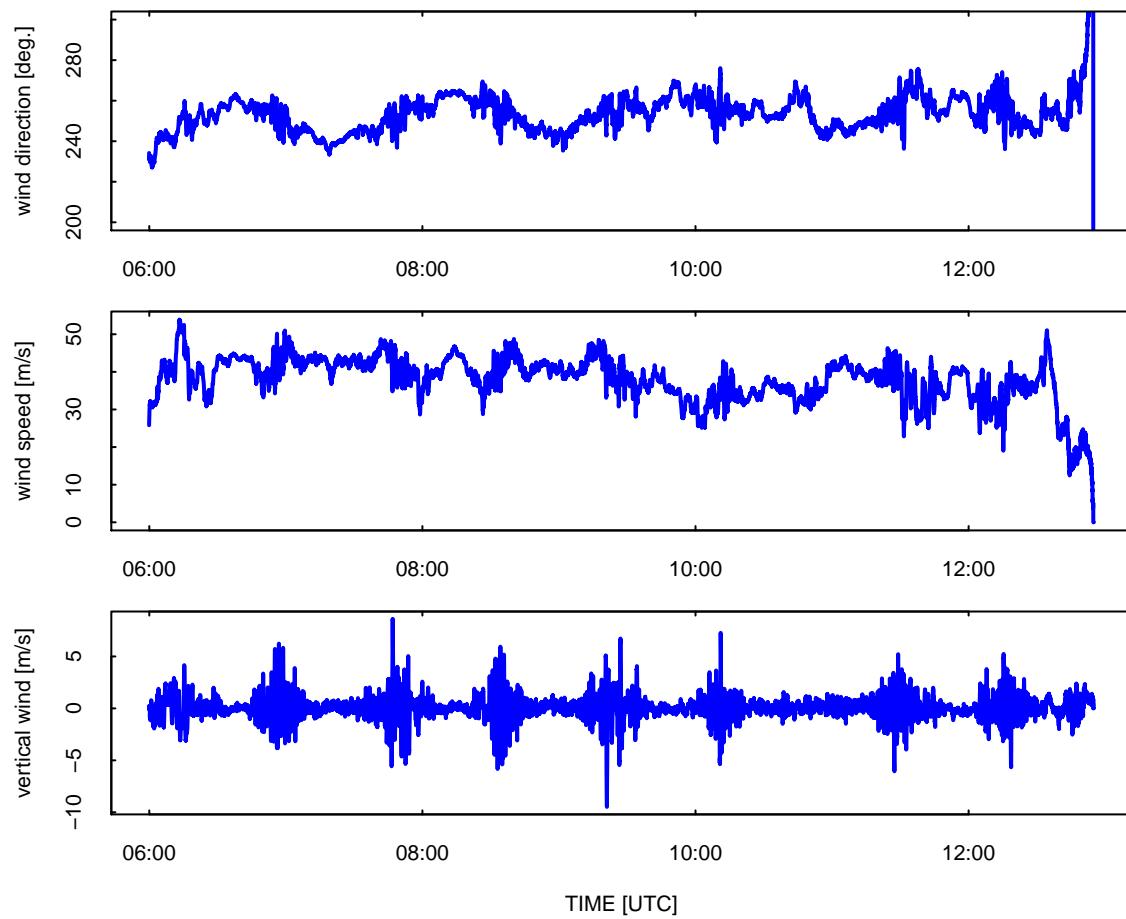


Figure 7: Wind measurements for DEEPWAVE research flight 16.

2.2 The gust-pod system

2.2.1 Overview

The all-weather gust pod was developed by Allen Schanot and is available for mounting under the wing of the GV, where it was installed during the 2014 project DEEPWAVE. It was still regarded as experimental for that project, but performed well as documented in this document, so it is now ready for deployment as a requestable instrument. The system has also been called the all-weather wind pod because the primary reason for its development was to provide a backup wind measurement for cases when the standard system malfunctions because its lines or apertures are blocked by ice. The gust pod fits into a standard PMS-style canister and uses a Rosemount 858 probe, but the location under the wing is one where there is substantial flow distortion in comparison to the free stream so an unconventional calibration is needed to use the measurements. The 858 probe is anti-iced by heaters and should be unaffected by icing or ice accumulation. Five ports are oriented forward, upward and downward 45°, and left and right 45° on the leading edge of the sensor, which has the shape of a hemisphere. There are also ports in a ring around the cylinder behind the hemisphere that provide a static source. The measurements are the pressure difference between the top and bottom ports (ADIF_GP), the pressure difference between the right and left ports (BDIF_GP), and the pressure difference between the forward port and the static ports (QC_GP). In addition, the pressure provided by the ring of static ports is recorded as PS_GP. The system incorporates a Systron Donner C-MIGITS III GPS/INS MEMS Tactical Grade navigation system (henceforth called the C-MIGITS IRU), which is mounted in the pod to be able to measure vibrations and wing-flex motions that will affect the measurements of wind. This unit provides measurements of attitude angles, ground-speed components, and accelerations and uses a GPS signal in a Kalman-filter feedback loop to reduce errors in the measurements. The relevant specifications are listed in Table 2.



Photograph of the Gust Pod (bottom left) and the ports on the Rosemount 858 probe (right).

2.2.2 Attitude angles

The C-MIGITS IRU unit provides measurements of the attitude angles, recorded as variables CROLL_GP, CPITCH_GP, and CTHDG_GP. The estimated standard uncertainty in measure-

Measurement (VARIABLE)	Instrument	Range, Char- acteristics	Standard Uncertainty	Comments
velocity components (CVNS_GP, CVEW_GP, CVSPD_GP)	C-MIGITS IRU	with GPS	0.1 m s^{-1}	horizontal and vertical
pitch, roll (CPITCH_GP, CROLL_GP)	C-MIGITS IRU	with GPS	$1 \text{ mrad} \simeq 0.06^\circ$	with Kalman filter
heading CTHDG_GP	C-MIGITS IRU	with GPS	1.5 mrad $\simeq 0.09^\circ$	valid when in motion ^a
pressure differences,858 ports (ADIFR_GP, BDIFR_GP)	Honeywell PPT0001- DXX2VB- S021	± 1 psi $\simeq \pm 68.95$ hPa	0.07 hPa 0.14 hPa max.	the first is “typ.”; same transducers as for radome
dynamic pressure (QC_GP)	Honeywell PPT0005- DXX2VB- S021	0–5 psi $\simeq 0$ –345 hPa	0.17 hPa 0.34 hPa max.	“
ambient pressure (PS_GP)	Paroscientific 6000-15A-28	0–15 psi $\simeq 0$ –1035 hPa	0.10 hPa	digital transducer

^aWithout occasional turns the heading error grows linearly at about $1\text{--}10^\circ/\text{hour}$

Table 2: Characteristics of measurements from the gust-pod that are used for the calculation of the wind. There is further discussion of each measurement in the text of Sect. 2.2.

ment of pitch, 1 mrad (cf. Table 2), is supported by comparison to the cabin-mounted inertial systems; the standard deviation in the difference between the two systems was about $0.1^\circ \simeq 1.7$ mrad for extensive multi-flight comparisons, while the expected difference for two systems each having standard uncertainties of 1 mrad would be 1.4 mrad. Some contribution would be expected from vibrations and wing flex that affect the gust pod, so these comparisons are good support for the approximate validity of the specifications. However, the feedback from the Kalman filter using GPS measurements can be ineffective in the absence of turns or maneuvers, so some of the specified uncertainties, esp. for heading, can be exceeded significantly in such cases, as noted in Table 2.

2.2.3 Ambient or static pressure

Ambient pressure (variable PS_GP) is measured by a digital transducer with low uncertainty, as listed in Table 2. However, the Rosemount 858 probe is located under the wing in a location where there is significant airflow distortion, so these measurements often differ from the measurements from the static buttons on the fuselage by 10–20 hPa and significant corrections are needed if these measurements are to be used for pressure measurements. With the gust-pod, the use of this measurement is as a reference for the differential measurement of dynamic pressure because the dynamic-pressure transducer is connected between the total-pressure port on the front of the 858 probe and the static ports. No calibration has been determined that would make this measurement useful as a measure of true ambient pressure, although that could be done by fitting to match the standard static pressure. In the absence of such a correction, PS_GP should not be considered an alternate measurement of ambient pressure. The use of this measurement to determine an alternate measure of true airspeed will be discussed in Section 5.3.

2.2.4 Dynamic pressure

The dynamic pressure QC_GP is measured by a differential pressure transducer, with specifications listed in Table 2. The measurement is the pressure difference between the forward-pointing port on the Rosemount 858 probe and the static ports on the side of that probe. Because the system is located under the wing in a region of disturbed airflow, the dynamic pressure requires unconventional correction to obtain the airspeed, as discussed in Sect. 5. This measurement is also used in the calculation of flow angles from the gust-pod pressure ports, as also discussed in that section.

2.2.5 Airflow angles

The difference between pressures at the top and bottom ports of the Rosemount 858 probe (ADIFR_GP) and the corresponding difference between right and left ports (BDIFR_GP) are also measured using differential transducers listed in Table 2. These are digital transducers that produce output with a fixed relationship to the pressure differences, and those digital outputs are recorded by the aircraft data system.

2.2.6 Components of aircraft velocity relative to the Earth

The C-MIGITS IRU, mounted with the gust pod, provides digital representations of the ground-speed components CVEW_GP and CVNS_GP and the vertical speed CVSPD_GP. (cf. Table 2). The unit incorporates a GPS receiver and uses GPS information as input to a Kalman filter for adjustment of these measurements and others discussed in this section.

2.2.7 Temperature and humidity

Air temperature and dew point are measured in the same way as discussed for the radome system in Sects. 2.1.7 and 2.1.8, and the same variables (ATX and DPX) are used.

2.2.8 Examples of measurements

A new calibration is developed in Section 5. On the basis of that calibration, it appears that the gust pod provides a useful alternative to the standard wind measurements. Plots and average values are presented in that section to support the validity of this measurement.

The following are some plots that show the results of this processing, in this case from DEEP-WAVE flight RF16 on 4 July 2014. Figure 8 shows a comparison of the vertical wind calculated from the gust pod (WI_GP) vs the conventional vertical wind (WIC). The standard deviation between the two measurements is 0.27 m/s.

A small segment of flight from a period with large-amplitude waves is shown in Fig. 10. The two measurements match quite well in regard to the structure of these waves and the amplitudes of the fluctuations. The measurements of horizontal wind speed are in similar agreement, but the wind direction for this period shows an offset for the gust pod measurement relative to the conventional measurement, varying from about 5° near the start of this period to about 1° near the end. This is a result of an apparent error in heading from the C-MIGITS IRU, a common feature to see near the start of flights but one that usually was made smaller during flight by GPS updating via the Kalman filter in that IRU.

The result of this error in wind direction and additional problems that occur with measured heading very close to southbound complicate the use of the measurements of horizontal wind from the gust pod and make them of lesser quality than the standard measurements. Fortunately, in DEEPWAVE and most projects, the horizontal-wind measurements are available from the radome-based system on all flights and are usually not compromised even when there is loss of the measurement of angle of attack from plugging of the lines in the radome, because the side-mounted ports for the measurement of sideslip seldom are plugged.

Figure 11 shows variance spectra from the two systems. There is a significant difference at frequencies above about 3 Hz, with the gust-pod distribution dropping faster and the standard wind WIC showing more variance. The high frequency spectrum from the gust pod may be more realistic; it is unusual to see high variance at these frequencies without a related generating source. The coherence between the radome and gust-pod measurements was above 0.9 for frequencies less than 1 Hz but then fell to around 0.2 at 10 Hz. This is an indication that

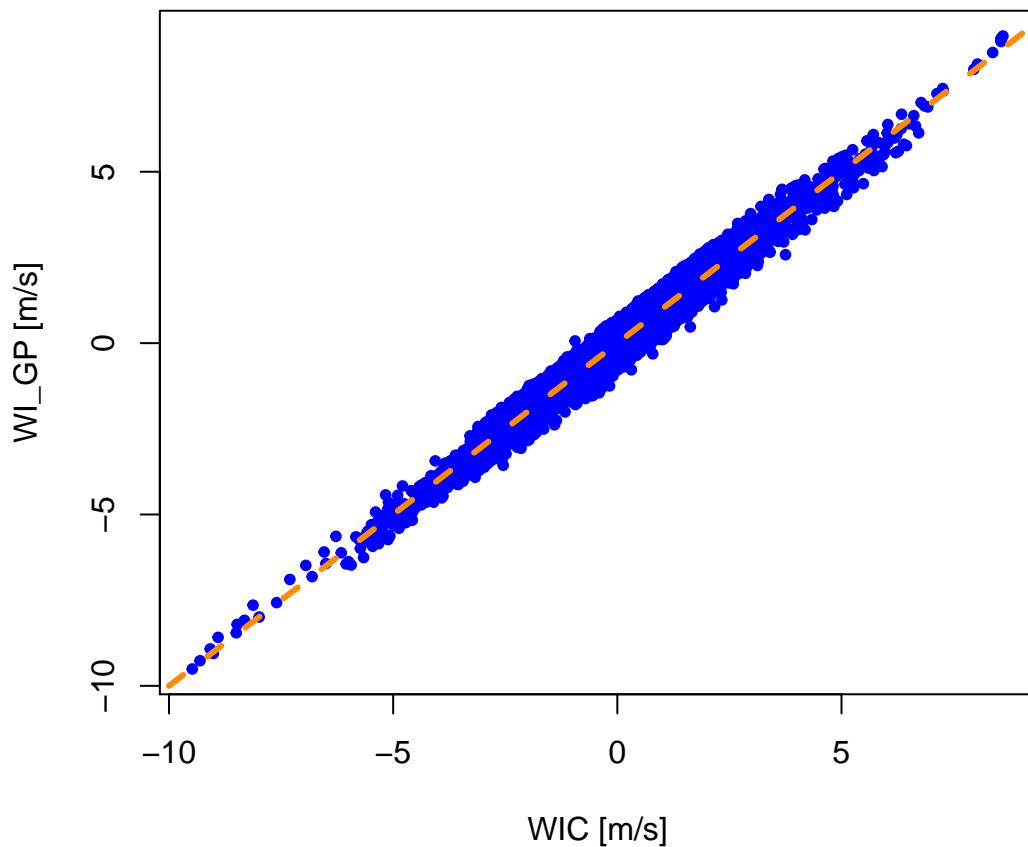


Figure 8: Comparison of vertical wind calculated from the gust pod (WI_GP) to the conventional measurement of vertical wind (WIC). The dashed orange line is a 1:1 reference line, and each blue dot represents a 1-s pair of corresponding measurements.

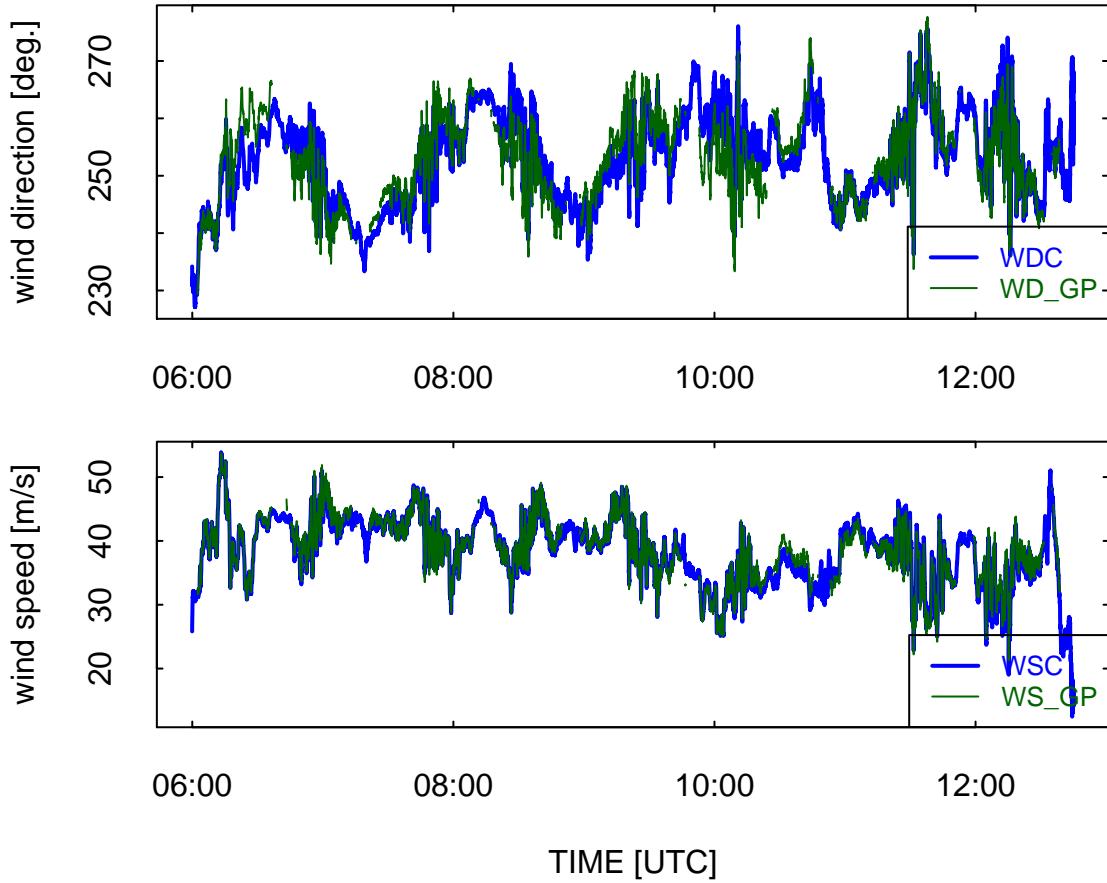


Figure 9: Comparison of horizontal wind direction (top panel) and horizontal wind speed (bottom panel) as determined from the gust pod and from the conventional radome-based system, for measurements where the true airspeed is greater than 130 m/s (to exclude periods of possible flap deployment). Only measurements considered valid are plotted for the gust pod; the restrictions where the gust-pod measurement is flagged as missing and therefore is not plotted here are: altitude (GGALT) > 5000 m, absolute value of roll (ROLL) < 5°. This causes exclusion of some measurements near the start and end of the flight and during turns.

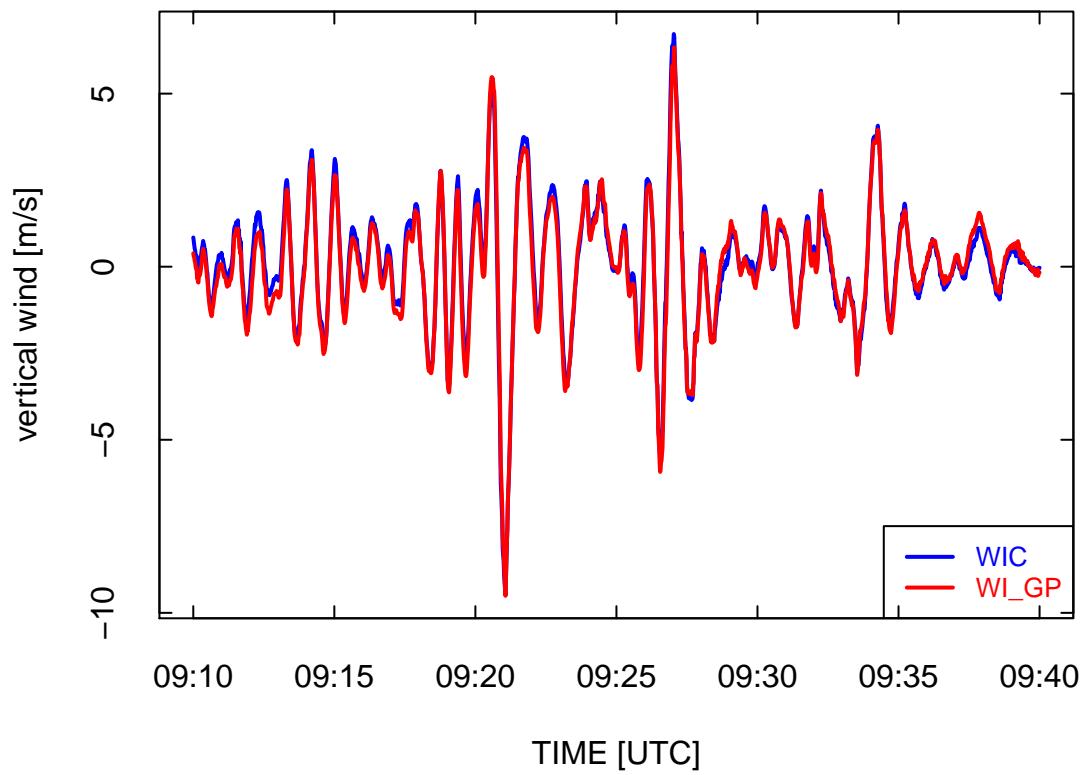


Figure 10: A 30-min segment from flight 16 of the DEEPWAVE project, showing the good agreement of the vertical wind measurements from the gust pod (WI_GP) and from the conventional wind-sensing system on the GV (WIC).

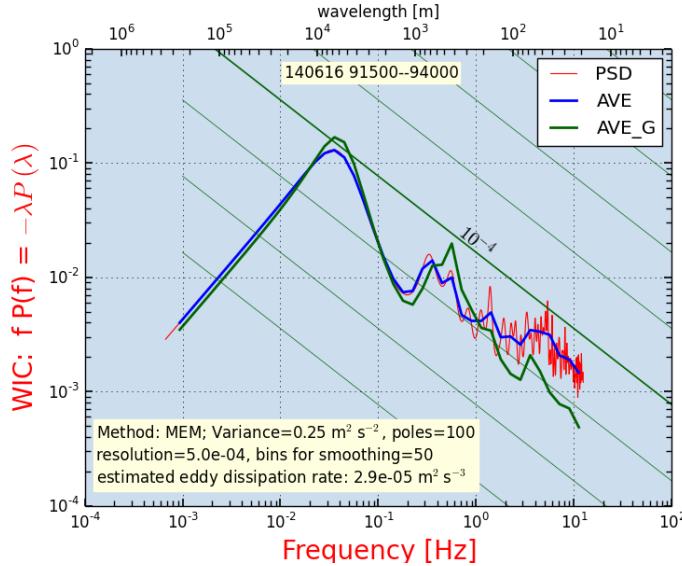


Figure 11: Variance spectra for WIC (red line labeled PSD, unsmoothed, and also shown smoothed as the blue line), compared to the smoothed spectrum for WIG from the gust pod (green line). Data from flight RF05, 9:15:00–9:40:00.

the two measurements are different in important ways at high frequency. This would not be the case if they were responding with different amplitudes; the signals must really be mostly incoherent at the highest frequency. The separation wing-to-fuselage is about 7 m lateral and 13 m longitudinal, so that doesn't seem enough to cause the low high-frequency coherence. The phase changes from in-phase at frequencies less than 1 Hz to 180° out-of-phase at 9 Hz, with WIG lagging, so this is consistent with the longitudinal offset. Shifting WIG relative to WIC also gave maximum coherence when WIG was shifted forward 1/25 s. Figure 12 shows an example of the good correspondence between gust-pod and radome measurements of vertical wind after application of such a shift to 25-Hz measurements.

2.2.9 Mixing of Attitude Angles for the Gust Pod

The attitude angles (pitch, roll, heading) as measured by the gust-pod IRU are defined relative to the orientation of the inertial unit in the gust pod, which is aligned a few degrees from the longitudinal axis of the aircraft. The canisters on the GV are oriented with axis pointing inward and downward relative to the aircraft longitudinal axis, in order to align with normal airflow at the pods. This is desirable for hydrometeor sampling and minimum drag, but it complicates the calculation of wind because roll introduced in turns, primarily a rotation about the aircraft longitudinal axis, will appear as a combination of attitude-angle changes in the gust pod. Errors arising from the initial alignment at the start of flights will also cause problems with the measured attitude angles, and it is likely that these will be more significant near the start of flights because the built-in Kalman filter uses GPS measurements to correct such errors in the course of the flight.

This problem with the reference frame for attitude angles has two consequences:

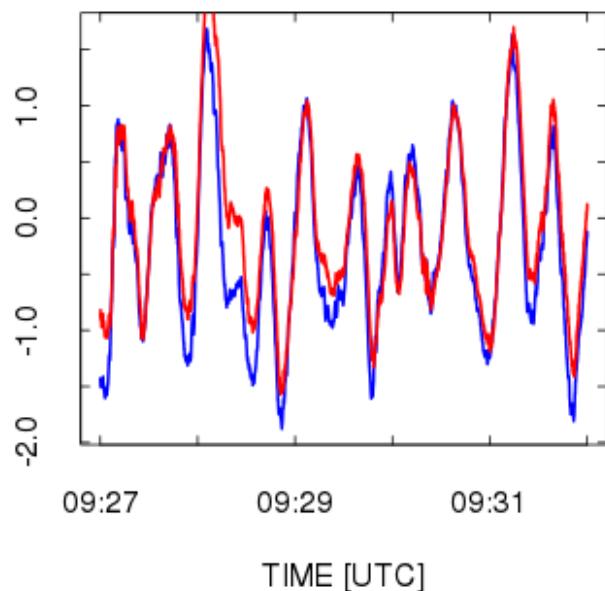


Figure 12: Comparison of WIG (red line) and WIC (blue line) after shifting WIG 1/25 s earlier to compensate for the longitudinal displacement of the sensor.

1. Measurements from the gust pod in turns have large errors in comparison to the errors in level flight, because for example relatively large errors in heading can contribute to errors in pitch. The measurements from the gust pod should probably be flagged as of poor quality whenever the roll exceeds some threshold like perhaps $\pm 5^\circ$. The measurements usually look reasonable in turns despite this worry, but the largest errors occur there.
2. There may be an offset introduced by the mixing of sideslip and angle of attack, arising from the difference in roll angle, and this will affect the reference or average value of the measured vertical wind. Some flights (e.g., DEEPWAVE flight 18) have a significant offset in vertical wind from the gust pod at the start of the flight, related to offsets in heading and roll, that gradually is corrected in the course of the flight via the C-MIGITSIII IRU Kalman filter.

Because the weight of the aircraft decreases during the flight, so does the angle of attack. When the wing flexes, the measured sideslip at the gust pod also varies with weight of the aircraft. This change in sideslip causes an offset in the mean lateral component of the wind as measured by the gust-pod system.

These effects suggest that the vertical wind measured by the gust pod may have an offset in some cases, particularly at the start of flights, and the measurement of sideslip can also have an offset that will contribute to the lateral component of the measured wind. These are weaknesses in the measurements from the gust pod that, at this stage, do not appear easily corrected. A future study implementing Schuler tuning in a post-processing step and correcting for the entwined-angle effects may be able to reduce these weaknesses, but that will require continued analysis not performed for this report. It may also be possible to develop special processing that corrects the measurement of attitude angles in turns, but that has not yet been developed or implemented.

2.3 The laser air-motion sensor

The laser air-motion sensor (LAMS) is still under development so results presented here will be more tentative than for the other systems. The characteristics and associated uncertainties in measured wind are discussed by Spuler et al. [2011] and Cooper et al. [2014]. Figure 13 shows the one-beam LAMS as installed on the GV, and Fig. 14 shows the configuration of the three or four-beam version.

The system consists of one, three or four fiber-based laser beams focused ahead of the aircraft and a collection system to detect the Doppler shift in light backscattered from aerosols. The transmitter optical components are mounted in an underwing pod, similar to that used for the gust pod, and like the gust pod the system incorporates a compact Systron Donner SDN500 GPS/INS MEMS Tactical Grade navigation system (henceforth called the SDN500 IRU) to measure the attitude angles and ground speed of the pod. Early measurements from this system have been used to determine corrections to the pressure measurements, and those serve an important role in reducing uncertainty in the wind measurements, as described in Cooper et al. [2014] and later in this report.

LAMS hardware supports up to four beams, with three pointing 35° off the forward direction and separated by 120° in the azimuthal direction. The fourth beam is directed forward from the

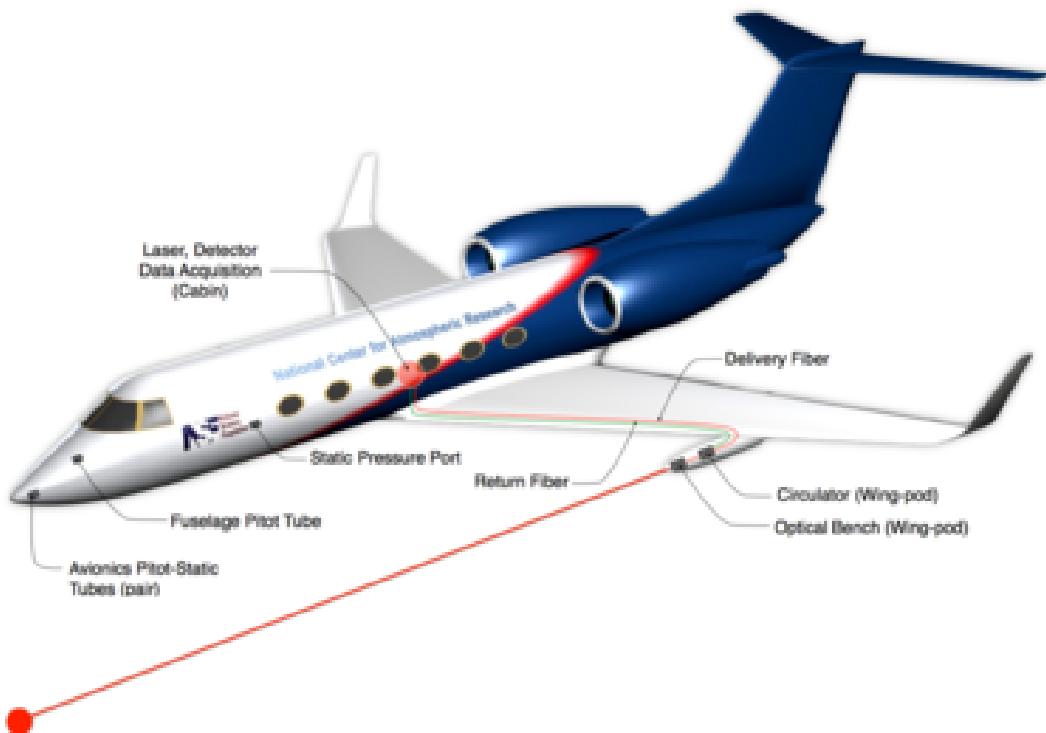


Figure 13: Diagram of the LAMS. Light generated by the laser in the cabin is transmitted by optical fibers to a wing pod, where it is transmitted in a beam that has a focal point well ahead of the aircraft (farther ahead than suggested by this not-to-scale diagram). The light backscattered from aerosol particles in the focal region is collected by the lens, and a circulator mixes a portion of the transmitted signal with the returned signal. The resulting signal, with interference patterns that measure the Doppler shift of the backscattered light, is returned via optical fiber to the cabin for digitization. Also illustrated in this figure are the approximate locations of the static pressure ports and the fuselage pitot tube used by the research data system to measure static and dynamic pressures. This figure appears in Applied Optics in the article by Spuler et al. [2011] and is used here with permission from the Optical Society of America.



Figure 14: The four-beam configuration of the LAMS optical head as installed in a GV under-wing pod.

LAMS unit. The LAMS channel designations changed after DEEPWAVE (summer 2014) when it was decided the instrument would benefit from having a lower noise channel for forward pointing measurements (the laser/detector on channel 4 had lower SNR than the other three channels). Table 3 summarizes the labels and pointing angles of each beam for both up to and after DEEPWAVE. Note the number designation refers to the laser, detector, and data processing channels. The letter designation is associated with the exit window on the LAMS head. The pointing angles reported here are ideal. The angle θ is measured relative to the forward direction of the LAMS pod (approximately the angle between the beam and \hat{x}') and the angle ϕ is the rotation angle about \hat{x}' relative to the \hat{z}' axis. This geometry is designed for GV installation where the LAMS pod is installed on the left wing, outboard pylon, outboard position. The pointing geometry relative to the aircraft may change based on the installation configuration on the C-130 (to be determined on ARISTO in fall 2015) where the LAMS optical fiber are routed to the aircraft right wing.

During IDEAS-4-GV, the LAMS beams were focused approximately 15 m from the LAMS exit ports on the pod. After this flight program, concerns were raised about the aircraft causing flow distortion in the LAMS sample volumes, and the beam focus was moved to 20 m from the exit port. In both cases the sample volume is approximately 2 m long.

2.3.1 Relative Beam Pointing

The relative pointing of the LAMS beams is defined by the orientation of each beam relative to the others. If the absolute pointing direction of one beam is known, the relative pointing may be used to determine the absolute pointing of all other beams. We define the relative pointing coordinates such that the z-axis is directed along the forward beam (D). The angle θ_L is the angle between the beam and the z-axis and the azimuthal angle ϕ_L is measured relative to the x-axis such that beam 1 (A) is positioned at $\phi_L = 0$.

180?

Beam Name After DEEPWAVE	Beam Name Before/During DEEPWAVE	θ_L	ϕ_L
Beam 1/A	Beam 1/A	35°	180°
Beam 4/B	Beam 2/B	35°	60°
Beam 3/C	Beam 3/C	35°	-60°
Beam 2/D	Beam 4/D	0°	-

Table 3: Designations and ideal orientations of LAMS beams for installation on the GV. θ_L is the angle from the longitudinal axis of the aircraft to the beam, and ϕ_L is the azimuthal angle of rotation about the longitudinal axis relative to the downward (aircraft-axis) direction, with positive values to starboard. In particular, beam 1 is upward at an angle of 35° from the centerline of the aircraft.

Two methods were used to determine the relative pointing angles. The first, performed prior to IDEAS-4-GV, used a Laser Survey, in which a commercial laser surveying company measured the positions of the LAMS head and the beam focal points.

The second method used a 30x beam expanded with focusing lens in a fixed position (referred to here as the receiver system) while LAMS was mounted in an astronomical telescope mount. A camera was placed at the focal point of the receiver system and the telescope mount was used to steer the LAMS beams into the receiver system such that each beam was centered on the CCD. This method had redundant pointing measurements, using both SDN500 data as well as the telescope mount angle read-outs.

Both methods were relatively accurate, with less than 0.2° difference in beam pointing. It should be noted that the fibers on the LAMS head may have moved slightly between the two measurement methods. The forward pointing beam was added after IDEAS-4-GV which required some disassembly of the LAMS head. So the relatively small difference in beam pointing may be partially attributable to that work.

Prior to IDEAS-4-GV in Sept.–Oct. 2013, a laser survey system was used to estimate the LAMS beam pointing angles of Beams 1, 2, and 3 (the forward pointing beam had not been added at that point). The LAMS head was positioned in the hanger so the bottom two beams were approximately parallel to the floor. Two IR card targets were placed at the focal points of those beams.

The laser survey system locked onto a corner cube reflector and register its position in space. The corner cube was first used to estimate the position of the LAMS exit ports by taking a series of points around their circumference. The tracker then registered the positions of the IR card targets by the same method. Once complete, the LAMS head was rotated by 120° and the process was repeated. This measurement was repeated for three different LAMS head rotations, where two beams were surveyed at a time.

After data was collected, the beam exit and target information were used to provide a best fit for the beam pointing.

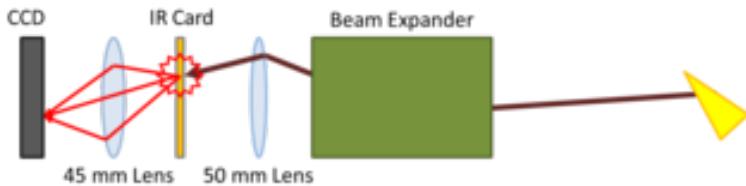


Figure 15: Diagram of the receiver system. The invisible infrared beam enters the beam expander which amplifies the angle of the beam relative to the telescope optic axis, and is then focused onto an IR card. The IR card fluoresces at a visible wavelength and that light is reimaged onto a CCD.

The benefits of this method are

- It measures the beam focal distance.

The weaknesses of this method are

- Only two beams can be surveyed at any one time.
- The measurement process can perturb the beam target.
- There is no reference to the SDN500 IRU.
- There is no independent redundant measurement and uncertainty is difficult to quantify.
- The survey can be expensive, requiring the hire of an outside vendor.

The results of the laser survey method were used to process IDEAS-4-GV data, but those angle calibrations have since been traded for the Telescope technique described below.

The relative positions of the LAMS system can be determined by defining a fixed reference vector and directing each beam onto it. The pointing of the LAMS module is recorded when each beam is directed on the reference. Without absolute knowledge of the reference vector, this only allows the relative beam geometry to be determined. Thus, we define a LAMS beam pointing coordinate frame such that the z-axis is along the forward beam.

The telescope beam measurement system was developed in house to provide position insensitive angular measurement. It uses a 30x beam expander consisting of an off axis parabola and convex secondary (exact type is unknown). The light from that beam expander is focused onto an IR card using a 50 mm lens, yielding a total focal length of 1.5 m. The IR card was then reimaged using a 45 mm lens onto a CCD to monitor the beam position (see Figure 15 and 16).

With the IR card at the back focus of the optical system, the beam image is position independent. Its location in this plane is dictated solely by the angle of the input beam. Even though each beam is slightly translated relative to its counterparts, we can still perform accurate angle measurements.



Figure 16: Photographs of the telescope angle measurement system in the RAF optics lab. View of the setup from behind the LAMS pod (top left), the receiver system (bottom left), LAMS mounted in the telescope mount (right).

The LAMS system is mounted in a precision telescope mount with accuracy on the order of $1''$ or about 0.0003° . The mount gives fine resolution in adjusting LAMS pointing into the receiver reference system and provides a digital readout of Alt/Az coordinates. These coordinates provide the first set of pointing data for the LAMS system. The second set of pointing data is provided by the SDN500 INS, which is attached and operating during our data collection. Thus, two measurements of the LAMS pointing are provided.

The telescope mount readout provides the quantities Alt, ϑ_T , and Az, φ_T . When a beam is directed into the reference telescope, it is accomplished through rotation operations:

$$\hat{r} = \mathbf{R}(\vartheta_T, \hat{x})\mathbf{R}(\varphi_T, \hat{y})\hat{u}, \quad (7)$$

where \hat{r} is the reference direction of the receiving telescope, \hat{u} is the beam pointing direction and $\mathbf{R}(\theta, \hat{v})$ is a rotation matrix of angle θ about the vector \hat{v} .

To determine the beam pointing angle, we invert the rotation operations (or perform the opposite rotations in the opposite order), which results in

$$\hat{u} = \mathbf{R}(-\varphi_T, \hat{y})\mathbf{R}(-\vartheta_T, \hat{x})\hat{r}. \quad (8)$$

To obtain a beam pointing vector, we first need to know the reference vector in some coordinate basis. We let Beam 4 (the forward pointing beam) define the z-axis in this coordinate basis and obtain \hat{r} by evaluating Eq. (7) for the recorded Alt/Az angles of Beam 4 (D) where $\hat{u} = [0 \ 0 \ 1]^T$. We then obtain all other beam vectors from their respective Alt/Az coordinates using Eq. (8).

The process for obtaining the beam pointing vectors from the SDN500 IRU is essentially identical to the process above, except that the SDN500 IRU provides roll, pitch and heading. It

Beam	Angle difference between SDN500 IRU and Telescope Mount
Beam 1 (A) (Upward)	0.024°
Beam 2 (B) (Down-In)	0.004°
Beam 3 (C) (Down-Out)	0.003°
Beam 4 (D) (Forward)	0.024°

should be noted, as will be addressed later, that the SDN500 IRU heading is not reliable when the device is stationary.

After the pointing angles of the beams are obtained from the telescope mount and SDN500 IRU, the pointing angles need to be compared. However, the beam vectors are recorded in different coordinate frames and SDN500 IRU heading data cannot be treated as reliable. A transformation matrix between these two frames and heading adjustments to SDN500 IRU are determined using minimization of errors so we can compare the beam pointing results. Table 2.3.1 shows the difference in angle between each of the four beams after optimizing the transformation between the two coordinate frames. The two beam pointing measurements give results that are quite close.

An assessment of the beam pointing accuracy was performed by translating the focused beam across the IR card and finding the spread of angles accepted by the optical system. The full angle field-of-view of the system was approximately $2'$ or about 0.02° . The beam spot position can be repeated to greater accuracy than this (we can see when it is well centered using the CCD), however this is probably a reasonable uncertainty figure because the IR card may not be located at the exact Fourier plane of the optical system.

The benefits of the telescope beam measurement method are:

- All four beams are measured without changes to the setup.
- The measurement is "hands off", so there is very little risk of perturbing the system during the measurement processes.
- The process provides redundant angle measurements (SDN500 IRU and the telescope mount).
- The entire LAMS pod is used and referenced directly to the same SDN500 INS used in flight.

Beam	θ_L Telescope	θ_L SDN500 IRU	θ_L Laser Survey	ϕ_L Telescope	ϕ_L SDN500 IRU	ϕ_L Laser Survey
Beam 1 (A) (Upward)	35.03°	35.08°	34.95°	— ^a	—	—
Beam 2 (B) (Down-In)	34.86°	34.85°	34.96°	119.74°	119.74°	120.01°
Beam 3 (C) (Down- Out)	34.86°	34.85°	35.01°	120.16°	120.16°	120.00°

^aBeam 1 is used as the basis for $\phi = 0$ in the relative pointing coordinate frame

The weaknesses of this method are:

- Beam focal positions are not measured.
- Setup and alignment of the system is time consuming (approximately 2 days).
- The procedure still does not provide an absolute pointing measurement because the telescope acceptance vector is not known.

The following uncertainty analysis treats unknown biases in pointing as random variables with standard deviations or variances. These figures represent uncertainty, not stochastic processes. The resultant variances provide bounds on the LAMS wind vector accuracy based strictly on the instrument characterization, independent of other factors such as signal fidelity.

The accuracy to which we know the LAMS relative beam pointing directly impacts our estimates of airspeed and wind. A sequence of line-of-sight velocity measurements is given by the equation:

$$\vec{m} = \mathbf{U} \vec{v}_L, \quad (9)$$

where \vec{m} is the set of measurements (3 or 4 elements corresponding to the number of beams in use), \vec{v}_L is the air velocity vector in the LAMS beam coordinate basis and \mathbf{U} is the matrix describing each beam's pointing angle given by

$$\mathbf{U} = [\hat{u}_1 \quad \hat{u}_2 \quad \hat{u}_3 \quad \hat{u}_4]^T, \quad (10)$$

where \hat{u}_i is the i th beam pointing direction. The exact vector entries for \mathbf{U} are dependent on which beams are used for LAMS operation. For example, \hat{u}_4 would not be included for HCR-TEST (post-DEEPWAVE configuration) where the down-inboard beam was not used in that

three beam configuration. For this analysis the beam and air velocity vectors are defined in the LAMS relative coordinate frame. Though this is not inherently required, it allows the analysis presented in this section to flow directly into further uncertainty analysis described in Section 2.3.2.

The covariance matrix of a beam pointing vector is given by

$$\Sigma_{\hat{u}_i}^2 = \left(\frac{\partial \hat{u}_i}{\partial \phi} \right) \sigma_\phi^2 \left(\frac{\partial \hat{u}_i}{\partial \phi} \right)^T + \left(\frac{\partial \hat{u}_i}{\partial \theta} \right) \sigma_\theta^2 \left(\frac{\partial \hat{u}_i}{\partial \theta} \right)^T, \quad (11)$$

where σ_ϕ and σ_θ are the standard deviation of the beam pointing angles. Note that the uncertainty in these angles may be different for each beam, but this analysis will treat them as identical.

The variance in a measurement due to beam pointing uncertainty is given by

$$\sigma_{m_{ii}}^2 = \vec{v}_L^T \Sigma_{\hat{u}_i}^2 \vec{v}_L. \quad (12)$$

The uncertainty in the relative pointing angles are assumed to be independent for each beam (common uncertainty in the instrument pointing will be addressed in section 2.3.2). Thus the total measurement covariance matrix is diagonal and given by

$$\Sigma_{\vec{m}}^2 = \begin{bmatrix} \sigma_{m_{11}}^2 & 0 & 0 & 0 \\ 0 & \sigma_{m_{22}}^2 & 0 & 0 \\ 0 & 0 & \sigma_{m_{33}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{m_{44}}^2 \end{bmatrix}. \quad (13)$$

The total velocity uncertainty in the LAMS coordinate frame is ultimately bounded by its covariance matrix

$$\Sigma_{\vec{v}_L}^2 = \mathbf{U}_{inv} (\Sigma_{\vec{m}}^2 + \Sigma_f^2 + \mathbf{U} \Sigma_{\vec{v}}^2 \mathbf{U}^T) \mathbf{U}_{inv}, \quad (14)$$

where the velocity uncertainty resulting from beam pointing uncertainty is given by the covariance matrix $\Sigma_{\vec{m}}^2$ from Eq. (13), the uncertainty due to FFT Doppler peak estimation is described by the covariance matrix Σ_f^2 and is defined in section 2.3.3, the covariance matrix $\Sigma_{\vec{v}}^2$ is the result of velocity variability between the three or four beam sample volumes with \mathbf{U} being the beam projection matrix from Eq. (10) and \mathbf{U}_{inv} is the inversion matrix for finding the total velocity vector from the three or four line-of-sight measurements. In the case of the three beam LAMS, $\mathbf{U}_{inv} = \mathbf{U}^{-1}$. However, because the four beam system is overdefined, a pseudo-inverse may be used, or if the uncertainty in measurements are well known,

$$\mathbf{U}_{inv} = \left[\mathbf{U}^T (\Sigma_L^2)^{-1} \mathbf{U} \right]^{-1} \mathbf{U}^T (\Sigma_L^2)^{-1}, \quad (15)$$

where

$$\Sigma_L^2 = \Sigma_{\vec{m}}^2 + \Sigma_f^2 + \mathbf{U} \Sigma_{\vec{v}}^2 \mathbf{U}^T. \quad (16)$$

Assuming the angle uncertainty of all four beams is approximately 0.02° and an aircraft velocity of 200 m/s along the LAMS pointing direction, the total velocity uncertainty resulting from

beam pointing uncertainty is approximately 0.06 m/s in the horizontal and vertical directions with the four beam configuration, 0.09 m/s in the horizontal and vertical directions with the three beam configuration used in HCR-TEST. To first order approximation, the forward velocity is insensitive to small perturbations in beam pointing.

Add note that SDN500 IRU cannot provide reliable heading information without moving. Heading adjustments had to be added to the fit of the two measurements.

2.3.2 SDN500 IRU Absolute Beam Pointing

With the relative pointing angles of the beams, an air velocity vector can be retrieved reliably in the predefined LAMS coordinate frame. However, the exact transformation between the LAMS beams and the SDN500 IRU coordinate frame (defined by the unit's principal axes) is unknown. We typically assume that one SDN500 IRU axis is exactly aligned to the forward pointing beam and one is directed along the angle beam 1. However, it stands to reason that there will be some slight differences between the LAMS beam coordinate frame and the principal axes of the SDN500 IRU. At present, the only method we have for determining this transformation relies on making small angle adjustments based on flight maneuvers.

Let the transformation matrix between the LAMS relative coordinate frame and the SDN500 IRU principal axes be \mathbf{T}_L . The velocity from LAMS is converted to the SDN500 IRU coordinate frame using

$$\vec{v}_c = \mathbf{T}_L \vec{v}_L, \quad (17)$$

where \vec{v}_c is the air velocity vector in the SDN500 IRU coordinate frame and \vec{v}_L the air velocity vector in the LAMS relative beam coordinate frame. To propagate error in the transformation matrix we reframe the problem by vectorizing the matrix such that Eq. (17) becomes

$$\vec{v}_c = \mathbf{V}_L \vec{t}_L, \quad (18)$$

where the matrix \mathbf{T}_L has been converted to the vector \vec{t}_L given by

$$\vec{t}_L = [T_{11} \ T_{12} \ \cdots \ T_{33}]^T, \quad (19)$$

where T_{ij} is the element of \mathbf{T}_L from the i th row and j th column. The matrix \mathbf{V}_L is constructed from the LAMS coordinate frame velocity vector \vec{v}_L and is given by

$$\mathbf{V}_L = \begin{bmatrix} v_1 & v_2 & v_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_1 & v_2 & v_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v_1 & v_2 & v_3 \end{bmatrix}, \quad (20)$$

where v_i is the i th element of \vec{v}_L .

The covariance matrix of \vec{t}_L can be estimated from the uncertainties in the roll, pitch and yaw transformation angles denoted here as γ , β and α respectively using partial derivatives

$$\Sigma_{\vec{t}_L}^2 = \left(\frac{\partial \vec{t}_L}{\partial \gamma} \right) \sigma_\gamma^2 \left(\frac{\partial \vec{t}_L}{\partial \gamma} \right)^T + \left(\frac{\partial \vec{t}_L}{\partial \beta} \right) \sigma_\beta^2 \left(\frac{\partial \vec{t}_L}{\partial \beta} \right)^T + \left(\frac{\partial \vec{t}_L}{\partial \alpha} \right) \sigma_\alpha^2 \left(\frac{\partial \vec{t}_L}{\partial \alpha} \right)^T. \quad (21)$$

Thus the velocity covariance matrix in the SDN500 IRU coordinate frame is given by

$$\Sigma_{\vec{v}_c}^2 = \mathbf{V}_L \Sigma_{\vec{v}_L}^2 \mathbf{V}_L^T + \mathbf{T}_L \Sigma_{\vec{v}_L}^2 \mathbf{T}_L^T, \quad (22)$$

where $\Sigma_{\vec{v}_L}$ is obtained from Eq. (14).

It should be noted that a similar analysis can be performed for the transformation between SDN500 IRU and a Global or aircraft coordinate frame where uncertainties in SDN500 IRU roll, pitch and heading are better known.

ADD DETAILS ON DETERMINING THIS TRANSFORMATION MATRIX FROM FLIGHT DATA.

With the SDN500 IRU INS, the air velocity measurements can be transformed into a global coordinate frame. The transformation matrix is determined using \mathbf{T}_1 , \mathbf{T}_2 and \mathbf{T}_3 from the roll, pitch and yaw reported by SDN500 IRU. The analysis needed for this step is covered in Section 1.2.2.

2.3.3 Frequency Precision

The LAMS A/D samples each beam detection channel at 200 MHz and performs a 1024 point FFT. The frequency resolution of the FFT is thus given by the sample rate divided by the number of data points

$$\Delta f = \frac{f_s}{N_s} = 195\text{kHz}. \quad (23)$$

The Doppler shift measured on a particular beam is

$$f_D = \frac{2}{\lambda} \hat{u} \cdot \vec{v}, \quad (24)$$

where λ is the laser wavelength (1560 nm), \hat{u} is the beam direction and \vec{v} is the velocity vector of the air relative to the instrument. A factor of two is included because the Doppler shift is imposed twice, first when the beam is absorbed by the aerosol and second when it is re-emitted. Along the beam line-of-sight, each FFT bin corresponds to a velocity resolution of $\Delta v_{LOS} = 0.15\text{m/s}$. However, the peak in the detected Doppler spectrum can be determined with resolution of about 0.3 bin by using a fit to the derivative of the spectrum. This gives a line-of-sight velocity resolution of about $\Delta v_{LOS} = 0.05\text{m/s}$.

The resulting covariance matrix for each line-of-sight velocity measurement (only accounting for frequency accuracy) is diagonal with identically distributed variances

$$\Sigma_f^2 = \begin{bmatrix} \sigma_{v_{LOS}}^2 & 0 & 0 & 0 \\ 0 & \sigma_{v_{LOS}}^2 & 0 & 0 \\ 0 & 0 & \sigma_{v_{LOS}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{v_{LOS}}^2 \end{bmatrix}, \quad (25)$$

where, for our purposes, we assume $\sigma_{v_{LOS}} = \Delta v_{LOS}$.

The analysis presented here assumes variance in the sample rate to be small compared to other error sources.

Note that at GV speeds, the Doppler shift is expected to exceed the Nyquist frequency at the present sample rate. We use the true air speed measurements from the aircraft radome to determine which frequency fold contains the Doppler peak.

2.3.4 Flow Distortion

Initial operation of the three beam LAMS on IDEAS-4-GV revealed that the aircraft can influence flow fields in the LAMS sample volume. This issue became recognizable when data processing showed the down-inboard beam registered significantly slower velocities than the other two beams. An analysis using Gulfstream's computational fluid dynamics analysis confirmed that all three beams could be expected to observe some flow distortion. The expected flow distortion depends on the aircraft flight parameters. The down-inboard beam is expected to experience the largest effect, typically observing a line-of-sight flow effect between -0.5 and -2.0 m/s when the beam is focused at 20 m. The upward beam may see flow effects on the order of ± 0.5 m/s and the down-outboard may see flow effects between 0 and -0.5 m/s.

On the GV, LAMS no longer uses the down-inboard beam due to the substantial flow effects in its sample volume. The flow effects around the C-130 will be better determined after the ARISTO flight campaign in Fall 2015.

2.3.5 Uncertainty arising from separation of measurement volumes

In turbulent conditions the three sensitive volumes can be characterized by slightly different values of the wind vector $\mathbf{v} = \{u, v, w\}$. The single forward beam measures u directly while the 3-beam system must solve for u using the relative wind measurements at three locations displaced from each other. If there are variations in the wind vector at these three locations, that will introduce an error that can be significantly larger than the measurement errors for a single-beam-forward system.

If for simplicity it is assumed that the 3-beam system is aligned so that the longitudinal axis matches the u axis and the vertical axis matches the w axis, then the unique solution (for a 35° diverging-beam angle) for the true airspeed (u) is

$$u = \frac{(a_1 + a_2 + a_3)}{3 \cos(35^\circ)} \quad (26)$$

where a_i is the relative airspeed measured by the i th beam. If each beam measures relative airspeed in its direction of alignment to an uncertainty δ , then a one-beam system aligned along the airflow measures with uncertainty δ while a three-beam system measures to uncertainty $\sqrt{3}\delta/(3 \cos(35^\circ)) = 0.7\delta$, so if each beam is an independent measurement the 3-beam system measures TAS more accurately than a single-beam system. However, the unique solution for the wind vector obtained from the 3-beam system relies on the assumption that all three beams are viewing air that has the same wind vector \mathbf{v} . If there is variation in the wind vector at the three viewed locations, that variation is not necessarily just variation in u (that it might be desirable

to average) but can also result from other variations because the beams are not aligned along the u axis.

Because the uncertainty δ is less than 0.1 m/s, variations of this magnitude would introduce errors comparable to the measurement error. The spatial separation between any two sensitive volumes in the 3-beam system is about $1.5(\sqrt{2}L \sin(35^\circ)) \simeq 18$ m for focal distance $L=15$ m. The variance in the wind for points separated by 18 m can be estimated as follows:

$$(u')^2 = \int_{k_0}^{\infty} C\varepsilon^{2/3} k^{-5/3} dk = \alpha\varepsilon^{2/3} \frac{3}{2} k_0^{-2/3} \quad (27)$$

where $k_0 = 2\pi/\Delta$ with $\Delta = 18$ m. For modest eddy dissipation rates in the range $\varepsilon = 0.001$ to $0.01 \text{ m}^2\text{s}^{-3}$, and for $C = 1.5$ (FIND REFERENCE XXX), (27) results in estimates of the velocity variance of about $.02\text{--}0.1 \text{ m}^2\text{s}^{-2}$, or standard deviations of about 0.14 to 0.3 m/s. These fluctuations, entering (26), will cause errors in the estimate of u that are not negligible in comparison to the measurement errors in $\{a_1, a_2, a_3\}$. [FIX THIS: C or α ...]

XXX

3 Calibration procedures

3.1 Scope

The measurements from the components described in the preceding section are used to calculate the wind, but those calculations also involve several empirically determined relationships and adjustments arising either from calibrations or flight maneuvers. This short section outlines the procedures used for calibration, for determining the empirical relationships, and for quality control. It also provides links to sections of this report where the procedures are discussed in more detail.

3.2 Laboratory calibrations

The key pressure sensors are digital sensors having low uncertainty and good stability, so the laboratory calibrations of those sensors are used primarily to ensure performance within specifications rather than to make adjustments to the output.

The pressures on the GV for the measurements ADIFR, BDIFR, QCR, QCF, and PSF (cf. Table 1) are calibrated before each project, and a leak check is performed on the pressure lines. The calibrations are done at/on the GV by use of a portable pressure-calibration cart. All GV pressure transducers have serial outputs and quoted uncertainty limits similar to the calibration unit, so the calibration is treated as a check and external calibrations are not applied to these measurements.

The calibration cart consists of a Ruska Digital Control Unit/Pressure Gauge (model 7010), a vacuum pump and pressure pump, and a data-system interface. The various systems are sealed off for the leak check and then the calibrations are performed. (The static ports and radome ports are sealed off with tape. For the static-pressure calibration the calibrator is attached by a "T" at the transducer. For the radome ports the calibrator is attached to the drain lines and the ports are sealed with tape. For QCF the the pressure line is connected to the nose of the pitot itself.)

A set of calibration pressures are provided by the Ruska controller which has a resolution of 0.067 hPa, and the results are read out and recorded on the GV data system. For ADIFR and BDIFR the calibration pressures are +/- 50, 40, 25, 10, 0 hPa; for QCF and QCR, 0 to 200 hPa in increments of 25 hPa; and for PSF, 150, 200, 300, ..., 900, 1025 hPa.

The Ruska manual gives "precision" values for the controller that translate to about 0.02 hPa for the range used for ADIFR, BDIFR, QCF, and QCR and 0.2 hPa for the range used for PSF. All calibrations are recorded in an RAF calibration database, and any departures from standard calibrations are documented with comments in the attributes associated in the netCDF file with the measured variable.

The other basic variable entering the calculation of wind that is subject to calibration is temperature, which is needed for the calculation of airspeed. The temperature sensors are calibrated in a stirred bath to determine the appropriate relationship between resistance and temperature,

and then are calibrated in place on the aircraft to link that calibration to the properties of the analog-to-digital converters used to record the signals.

3.3 Quality-assurance checks

The output from the inertial sensors is monitored primarily by comparison to other similar units. The duplicate Honeywell inertial systems are compared on each flight to ensure that their performance remains consistent, and when present the gust-pod and LAMS inertial systems are also compared to each other and to the Honeywell systems. The alignment angles of the various inertial systems have been determined by survey and are not expected to change. Ground-speed measurements from the inertial systems are compared routinely to GPS measurements of ground speed, and the Schuler oscillations in ground-speed components are monitored as indicators of the quality of these measurements. The oscillating errors in ground speed normally do not exceed a few m/s and in the best cases are about 1 m/s.

Errors in the pitch measurement from the Honeywell inertial system will result in errors in ground-speed components, so the differences between inertial-system and GPS measurements of ground-speed components can be used as indicators of the quality of the measurement of pitch. This is discussed in detail in Sect. 6.4, where a procedure is also developed that can reduce the uncertainty in the measurement of pitch.

A calibration procedure for determining angle of attack and sideslip angle is developed in Sect. 5. To monitor the validity of the relationships determined there, the equations employed ((30) and (37)) are also used during projects for quality assurance to check that the sensitivity coefficients in use give angles consistent with these predictions.

Some special maneuvers are used frequently during projects to monitor the performance of the wind-measuring systems. Pitch maneuvers, in which variations in pitch lead to oscillating vertical motion of the aircraft, are used to check that this motion does not enter the vertical-wind measurement to any significant extent. (An example is discussed in Sect. 6.3; cf. Fig. 28.) Similar maneuvers with variations in forced sideslip are checked to see that no significant effects of the sideslip enter the final wind. Speed runs provide good checks on the angle-of-attack relationships, and reverse-heading maneuvers (where the aircraft flies over the same drifting track in opposite directions) are also used to check for offsets in measurements of airspeed and sideslip. Finally, circle maneuvers are used to check for errors in airspeed, sideslip offset, or heading offset, as discussed below (Sect. 3.6).

3.4 Finding the relative-wind angles

The angle of attack and sideslip angle are needed to determine the relative wind (cf. 1.2.1). For the radome-based and gust-pod systems, these angles are not measured directly but are determined from empirical relationships between the flow angles and pressure differences on the sensors. The procedures for determining these empirical relationships are discussed in Sect. 5.

In the case of angle of attack, the procedure for determining the needed empirical relationship relies on assuming that the vertical wind is negligible over the region covered by the flight

maneuver, which is normally a "speed run" where flight speed is changed gradually over the flight envelope of the aircraft. This assumption is checked by repeated calibrations, but the absence of an absolute standard for calibration leads to possible constant bias in the vertical wind that remains a key source of uncertainty. This is discussed in detail in Sect. 5.1.2 for the radome system and Sect. 5.1.3 for the gust-pod system.

For sideslip, the calibration uses the heading as a reference so the resulting empirical relationship relies on the heading being accurate. The fit procedure is discussed in Sect. 5.2, but an adjustment to that procedure is then determined from additional flight maneuvers that permit separation of the offsets in sideslip and heading. This is discussed in Sect. 3.6 below and in detail in Sect. 7.1.

The empirical relationships used to determine angle of attack and sideslip angle are summarized in Sect. 5.4.

3.5 True airspeed

The measurement of true airspeed for the radome-based system relies on the measurements of dynamic and ambient or static pressure and so will be affected by any error in the static pressure. This has been removed by calibration using the single-beam version of the LAMS, as discussed in detail by [Cooper et al. \[2014\]](#). The procedure is too involved to discuss in detail in this report, so that reference should be consulted for the determination of uncertainty in true airspeed from the conventional system on the GV. In that reference, an empirical fit to the corrections determined by the measurements from the LAMS was found, and that fit is now applied to all projects including those on which the LAMS is not present. The result is an uncertainty in true airspeed of about 0.1 m/s when the single-beam LAMS is available and about 0.3 m/s otherwise.

The gust-pod system also can be used to determine a measurement of true airspeed, but it requires an involved fit to obtain good results. This measurement is useful for two reasons. First if it may be desirable to obtain measurements of wind from the gust pod without reference to the usual radome-based system, for redundancy, backup, and quality control. Second, the gust pod provides high-frequency measurements that apparently are superior to those from the radome-based system. Determining true airspeed from the gust pod is discussed in Sect. 5.3, and there is a brief discussion of high-rate measurements and the usefulness of the gust pod for such measurements in Appendix B.

3.6 The circle maneuvers

Flying constant-roll-angle turns while drifting with the wind provides additional opportunities for checks and adjustment of the empirical relationships and the airspeed measurement. The drift of the aircraft in such a maneuver provides a good reference measurement of the wind that is independent of all wind-sensing components on the aircraft except the GPS. In addition, a plot of measured wind speed as a function of the angle between the heading of the aircraft and the wind direction is a very useful analysis tool because any error in true airspeed will

produce a difference from upwind to downwind flight while any error in the combination of heading and sideslip angle will produce a difference between the two portions of the circle that are crosswind. Furthermore, the sideslip angle should have opposite sign in right-turn circles vs. left-turn circles, so this makes it possible to separate a bias in the measurement of heading from a bias in the measurement of sideslip angle.

The analysis of circle maneuvers is discussed in detail in Sect. 7.1, and there the offset in sideslip angle is adjusted. The circles provide good evidence in support of the low uncertainty claimed for the true airspeed measurement from the radome and for the quality of the measurements of heading and sideslip angle.

4 Elements of uncertainty

4.1 General structure

Here we follow a particular style for construction of an analysis of uncertainty (cf. Appendix A) by including these components:

1. A description of the measuring system. Section 2 of this report serves this function by providing extensive discussion of each component contributing to the measurement of wind, and it discusses what is known about specifications for uncertainty associated with those components. It also includes description of the algorithm leading to components of the measured wind, and it discusses the three independent systems available for measuring wind on the GV.
2. Tests and calibrations. Sections 3 and 5 provide key information on how calibration maneuvers are used to determine the sensitivity of some of the measurements to components of the wind. Section~5 on calibration is key to the uncertainty analysis because to a large extent many of the potential errors from sensors are removed by the calibrations in this section, so the calibration becomes the central factor affecting the final uncertainty. Other intermediate sections discuss some specific tests applied to the measurements in order to check or reduce the uncertainty limits associated with these measurements. This report goes somewhat beyond a conventional analysis of uncertainty in that there are new developments discussed here and some unconventional ways of checking the measurements.
3. Discussion of the elemental contributions to uncertainty. A standard tabulation of elemental sources of uncertainty is presented in the subsections 4.2 and 4.3 that follow.
4. Summary and comprehensive estimate of uncertainty. This follows at the end of this section and presents the key conclusions of this study.

Because three wind-measuring systems are characterized in this report, they will be discussed separately. However, the standard system is the radome-based system, so that will be treated in the most depth. The other two systems, the under-wing gust pod and the LAMS, are new systems and their characteristics are still being developed and explored, so their discussion necessarily will be incomplete until additional flight data are collected.

4.2 Elemental sources, vertical wind

Next, we tabulate the elemental sources of uncertainty in the measurement of vertical wind.

4.2.1 Radome-based system

Table 4 summarizes the results for the radome-based wind system. The following is a discussion of the individual elements in that table.

element	uncertainty source	bias	random	δ_w bias [m/s]	δ_w random [m/s]
1	ADIFR transducer	0.07 hPa	0.002 hPa	–	–
2	AKRD coefficients	0.01°	0.001°	0.04	0.004
3	BDIFR transducer	0.07 hPa	0.002 hPa	–	–
4	QCF transducer	0.34 hPa	0.01 hPa	<0.02	0.001
5	pitch	0.02°	0.007°	0.08	0.03
6	GV vertical velocity	0.03 m/s	<0.03 m/s	0.03	<0.03
7	PSF transducer	0.10 hPa	0.001 hPa	–	–
8	ATX	0.3°	0.1°C	–	–

Table 4: Elemental contributions to the uncertainty in measurement of vertical wind by the radome-based system. Entries ‘–’ indicate negligible contribution to uncertainty.

1. **ADIFR:** See Sect. 2.1.5 and Table 1. The uncertainty is assigned to bias because it is likely a calibration uncertainty and the resolution and stability are much smaller than this bias. However, a calibration bias in this measurement does not affect the final wind measurement because the procedure in Sect. 5.1.2 determines the angle of attack from flight data in a way that can be considered a calibration of the measurement of angle of attack, and a bias in ADIFR would be reflected in a change in sensitivity coefficients determined in that section that would compensate for that bias. Only random errors in ADIFR would propagate to the final measurement w , and such errors are thought to be negligible, so the propagated error for ADIFR is listed as negligible. The next item considers the determination of sensitivity coefficient for angle of attack and is the dominant contribution to uncertainty in w arising from the measurement of angle of attack.
2. **AKRD coefficients:** The calibration procedure of Sect. 5.1.2 effectively removes the effects of possible biases in ADIFR and QCF and instead replaces them with uncertainties arising from the coefficients $\{c_0, c_1\}$ in (34). The estimated bias and random error are those obtained and discussed in that section using the estimated uncertainties in the coefficients in (34). For propagation to vertical wind w , (28) indicates that the result is approximately $\delta w = V\delta\alpha$ where V is airspeed, with additional contributions from correlated errors involving V that are small in comparison to that listed. A typical value for V is about 240 m s^{-1} , leading to the listed elemental uncertainties in w arising from uncertainty in AKRD. It is important, though, that the calibration is dependent on the assumption that the mean vertical wind where the calibration data were collected be zero. This is discussed in Sect. 5.1.2. There is no independent way to check this except by comparing results from different regions as done in this report. That remains a major weakness in calibration and is the major contributor to uncertainty in angle of attack, but it introduces a bias arising from a possible calibration error. Fluctuations from the mean value are measured with uncertainty a factor of ten smaller than this bias. The assumed

mean value of the vertical wind leading to this bias was $\pm 0.03 \text{ m/s}$, which could well be an over-estimate for the large datasets used in Section 5.1.2.

3. **BDIFR:** The sideslip angle has negligible effect on the vertical wind as long as the roll angle is small, so for measurements made during straight-and-level flight this contribution to uncertainty in vertical wind is negligible.
4. **QCF:** The values listed are the characteristics of the transducer. Application of the calibration procedure based on comparison to the laser air-motion sensor (Cooper et al. [2014]) led to an alternate uncertainty estimate of 0.3 hPa . As in the case of ADIFR, the procedure to determine sensitivity coefficients removes any effect of bias in QCF by calibration in terms of the coefficients $\{c_0, c_1\}$ so the effect on bias in w is replaced by possible bias in those coefficients, as discussed for element 2. The effect of a random error in QCF of 0.01 hPa is, from (34), to introduce an uncertainty in angle of attack of about 0.0002° or a contribution to uncertainty in w of less than 0.001 m s^{-1} . QCF is also used to determine the true airspeed, which affects w , but the effect is negligible for the estimated uncertainty in QCF ($<0.2\%$ of the measured value of w , or 0.02 m s^{-1} for 10 m s^{-1} vertical wind).
5. **PITCH:** The estimates listed are those that apply without the pitch-correction procedure of Sect. 6.4. In that section, it was estimated that the standard error in pitch is 0.02° and that this is primarily in the form of a slowly varying error that, over measurement periods short compared to the Schuler oscillation period of 84.4 min, will appear as a bias. The correction procedure represented by Eq. 57 corrects for this error well enough to leave the residual bias negligible, so the bias entry in Table 4 can be eliminated by application of that algorithm. The partitioning between bias and random uncertainty depends on the interval considered, because Schuler precession will cause variation in this error with the Schuler-oscillation period of about 84 min. For periods long compared to this the error will have the character of a random-error component, so using 0.08 m/s would be appropriate for random uncertainty of such long-term measurements while the bias should be reduced substantially, perhaps to 0.02 m/s . For periods small compared to the Schuler period, the pitch error appears as a bias and there is a much smaller random error, evaluated in Sect. 2.1.2 to be about 0.007° in pitch or about 0.03 m/s in vertical wind. This is the usual case for measurements of interest, so the bias and random errors are partitioned as appropriate for this case in the table. The uncertainty in pitch is the leading contributor to the standard uncertainty in vertical wind and is also the leading contributor to the overall estimate of bias. The correction technique of Sect. 6.4 is not incorporated in routine processing so needs special calculation.
6. **Aircraft Vertical Velocity:** The measurement used for vertical motion of the aircraft is discussed in Sect. 6.2. The values listed here are those specified for measurements when “OmniSTAR” corrections are available; if not, the values should be increased to about 0.1 m s^{-1} and so will make a contributor to uncertainty in vertical wind that is comparable to the contributions from pitch and angle of attack. The error in aircraft vertical speed is likely a mixture of bias and random error, because the primary source is uncertainty in ionospheric corrections which will be persistent for important parts of flights but likely

to change at least from flight to flight. Because of the likely persistence of the error, it is assigned here primarily to bias.

7. **PSF:** The measured ambient pressure affects vertical wind only through the dependence of true airspeed T on PSF, as described in the document on [RAF processing algorithms](#), Section 4.7.1. Evaluation at typical values shows that the dependence of measured vertical wind on uncertainty in this variable is negligible. For example, the airspeed for $\text{PSF}=300 \text{ hPa}$, $\text{QCF}=80 \text{ hPa}$, and $\text{ATX}=-40^\circ\text{C}$ differs from that for $\text{PSF}=300.1$ by 0.03 m s^{-1} or about 0.01%, so this would also be the percentage change in vertical wind.
8. **ATX:** Temperature is needed to calculate the true airspeed, but as for ambient pressure the effect of uncertainty in temperature is very small. This was tested as for PSF by evaluating at representative points. A representative result was that the listed bias in temperature would lead to a bias in airspeed of about 0.05%, leading to a similar percentage change in the value of the vertical wind. This is negligible in comparison to other sources of uncertainty.

The result of adding the elemental sources of uncertainty in quadrature is a bias estimate of 0.10 m s^{-1} and a random-uncertainty estimate of 0.04 m s^{-1} , with pitch correction. Without the correction, the bias estimate increases to 0.35 m s^{-1} , so the pitch correction results in significant reduction in uncertainty and without that correction the uncertainty is dominated by the bias introduced by the measurement of pitch. In the corrected case, the dominant contributions are those from pitch and angle of attack, as was argued in the preliminary discussion in Section ???. The bias and random errors from pitch and from angle of attack, as listed here, arise from different sources so it is reasonable to combine them in quadrature to obtain composite estimates.

The APPLANIX IRU should provide another route to improvement in the measurement of pitch, because it offers significantly lower specified uncertainty. It achieves this through use of a full Kalman-filter correction to the measurements, which would remove the need for the pitch correction proposed in this document.

4.2.2 Gust-pod system

For the gust pod system, many of the uncertainties associated with measurement components are known less well than for the radome system, but some similar estimates can be made. This section will duplicate the structure of the radome-based system, but will be less definitive and more sketchy in some of the components while emphasizing the differences that apply to the gust pod.

Table 5 lists the elemental contributions to uncertainty in the measurement of vertical wind from the gust-pod system. The following is a discussion of the individual elements in that table.

1. **ADIF_GP:** See Sect. 2.1.5 and Table 1 and the discussion related to the radome. The same transducers are used for the pressure measurements on the gust pod, although the configuration of ports is different. The next item considers the determination of sensitivity

element	uncertainty source	bias	random	δ_w bias	δ_w random
				[m/s]	[m/s]
1	ADIF_GP transducer	0.07 hPa	0.002 hPa	–	–
2	AK_GP coefficients	0.01°	0.001°	0.04	0.004
3	BDIF_GP transducer	0.07 hPa	0.002 hPa	–	–
4	QCF/QC_GP transducer	0.34 hPa	0.01 hPa	0.02	0.001
5	pitch	0.04°	0.02°	0.17	0.08
6	GV vertical velocity	–	0.07 m/s	–	0.07
7	PS_GP transducer	0.10 hPa	0.001 hPa	–	–
8	ATX	0.3°	0.1°C	–	–

Table 5: Elemental contributions to the uncertainty in measurement of vertical wind using the gust pod. Entries ‘–’ indicate negligible contribution to uncertainty.

coefficient for angle of attack and is the dominant contribution to uncertainty in w arising from the measurement of angle of attack.

2. **AK_GP coefficients:** The calibration procedure of Sect. 5.1.3 effectively removes the effects of possible biases in ADIFR and QCF and instead replaces them with uncertainties arising from the fit coefficients $\{b_{0-3}\}$ in (35). It was necessary to use additional terms to obtain a good fit in this case, but the final result provided a very good representation of the data, as good as in the radome case. We have therefore used the same uncertainty estimate as for the radome, although with less justification and study. As for the radome, the dominant source of bias is again the uncertainty in vertical wind in the calibration region, which is the same for this data set as for that used to determine the radome sensitivity coefficients, so this estimate remains the same as for the radome. The uncertainties also propagate to the vertical wind in the same way. However, the restriction to low roll angle (less than 5° from vertical) is still more important in the case of the gust pod because the CMIGITS IRU used with the gust pod is not aligned with the aircraft longitudinal axis but rather is mounted in an under-wing pod that was designed to point into the airflow and therefore slightly inward relative to the longitudinal axis. That causes significant problems in turns because the IRU rotates in ways that mix the attitude angles.
3. **BDIF_GP:** The sideslip angle has negligible effect on the vertical wind as long as the roll angle is small, so for measurements made during straight-and-level flight this contribution to uncertainty in vertical wind is negligible.
4. **QCF and QC_GP:** Two measurements are listed because both are used in the calculation of vertical wind. QC_GP is used with ADIF_GP to determine the angle of attack, and the calibration described with item 2 effectively replaces uncertainty in this measurement with uncertainty in the sensitivity coefficients. However, true airspeed is determined using QCF because the conventional true airspeed is thought to be superior to the new value

determined solely from the gust pod (Section 5.3). Therefore, the effect of uncertainty in QCF on vertical wind is the same as that for the radome because the same calculated true airspeed is used for both.

5. **pitch (CPITCH_GP):** As was the case for the cabin-mounted inertial systems, there were two nearly identical inertial systems used in the wing pods, one for the gust pod and the other for the LAMS, so it is again possible to compare the measurements and obtain estimates of the random errors in their measurements. For both units (LAMS and gust pod), the inertial systems use GPS measurements with a Kalman filter to apply corrections, but they align independently and so have different errors and Schuler oscillations. There were many flights in DEEPWAVE where one of these was not operational: 1–4, 6–7, 15, 17, 19. For the other flights, the standard deviation in the difference in pitch between these two units was 0.06° , so this is a reasonable estimate of the random error that characterizes these measurements.⁵ As for the radome, a true bias in this measurement (e.g., from misalignment at installation) has been subsumed by the calibration of item 2 so does not enter in this item. However, the remaining error on most flights has a slowly varying component (consistent with the long time period of the Schuler oscillation) and so appears as a bias for any measurement made over a period short compared to the Schuler oscillation, so it appears appropriate to assign the observed standard deviation primarily to a bias because it will appear steady in normal applications that look at vertical wind over periods short compared to the Schuler oscillation. We have therefore partitioned the standard deviation into estimated components of 0.04° bias and 0.02° random error. The uncertainty in pitch is the leading contributor to the standard uncertainty in vertical wind and is also the leading contributor to the overall estimate of bias.
6. **Aircraft Vertical Velocity:** For the gust pod, the measurement of vertical motion of the aircraft must be that from the IRU mounted in the under-wing pod because the wing can flex and vibrate and the aircraft can roll in ways that cause that vertical motion to differ from that sensed in the cabin. Again, comparing the two units mounted in side-by-side wing pods provides the best indication of the random component of uncertainty in this measurement, because both units experience almost identical vertical motion. These two units measure project-mean vertical aircraft motions that differ by 0.04 m/s , with standard deviation in that difference of 0.07 m/s . It seems reasonable then to estimate the random component of uncertainty as 0.07 m/s , but the bias is more uncertain. Good flights usually produced mean vertical aircraft motion from takeoff to landing of less than 0.005 m/s , so it is reasonable to neglect the possible bias in this measurement (which is updated for stability in the IRU using pressure altitude as a reference).
7. **PSF:** The measured ambient pressure affects vertical wind only through the dependence of true airspeed on PSF, as described in the document on [RAF processing algorithms](#), Section 4.7.1. The effect is the same as for the radome, and is negligible; see the discussion above for the radome system.

⁵The standard deviation in the difference between two variables is actually $\sqrt{2} \times \delta$ where δ is the standard deviation in each variable, so a better estimate is 0.04; this correction has not been made throughout this document.

element	uncertainty source	bias	random	$\delta u_{\perp, \parallel}$ bias [m/s]	$\delta u_{\perp, \parallel}$ random [m/s]
1	BDIFR transducer	0.07 hPa	0.002 hPa	–	–
2	SSRD coefficients	0.03°	0.002°	(0.12, –)	(0.01, –)
3	ADIFR transducer	0.07 hPa	0.002 hPa	–	–
4	QCF transducer	0.34 hPa	0.01 hPa	(see item 9)	
5	heading	0.09°	0.04°	(0.38, –)	(0.17, –)
6	pitch	0.02°	0.02°	–	–
7	horiz. velocity of GV	0.03 m/s	<0.03 m/s	0.03	0.03
8	PSF transducer	0.10 hPa	0.001 hPa	–	–
9	ATX	0.3°	0.1°C	(–, 0.16)	(–, 0.05)
10	δq	0.2 hPa	0.1 hPa	(–, 0.3)	(–, 0.15)

Table 6: Elemental contributions to the uncertainty in measurement of horizontal wind by the radome-based system. Entries ‘–’ indicate negligible contribution to uncertainty. Entries with subscript \perp refer to the lateral component of the horizontal wind, and those with subscript \parallel indicating parallel refer to the longitudinal component (along the axis of the aircraft).

8. **ATX:** Temperature is needed to calculate airspeed, but as for ambient pressure the effect is negligible. See the radome discussion above.

The result of adding the elemental sources of uncertainty in quadrature is a bias estimate of 0.18 m s and a estimate of random uncertainty of 0.11 m s⁻¹. The dominant contribution in both cases is that from measured pitch, although the uncertainty in vertical motion of the aircraft also makes a significant contribution to the random component of uncertainty. It is important that these estimates only apply to cases where the roll is within 5° of level.

4.3 Elemental sources, horizontal wind

4.3.1 Radome-based system

Table 6 lists the elemental contributions to uncertainty in the measurement of horizontal wind from the radome-based system. The following itemization discusses each element.

1. **BDIFR:** The primary uncertainty in BDIFR is assigned to bias because it is likely a calibration uncertainty and the resolution and stability are much smaller than this bias. However, a calibration bias in this measurement does not affect the final wind measurement because the procedure in Sect. 5.1.2 determines the sideslip angle from flight data in a way that can be considered a calibration of the measurement of sideslip angle, and

a bias in BDIFR would be reflected in a change in sensitivity coefficients determined in that section that would compensate for that bias. Only random errors in BDIFR propagate to the final measurement of horizontal wind, and the effect of the listed random error is typically less than 0.0001 m/s in lateral wind, with even smaller contribution to the longitudinal wind. These contributions therefore are listed as negligible in the table. The next item considers the determination of sensitivity coefficient for sideslip angle and is the dominant contribution to uncertainty in horizontal wind arising from the measurement of sideslip angle.

2. **SSRD coefficients:** The calibration procedure of Sect. 5.1.2 effectively removes the effects of possible biases in BDIFR and QCF and instead replaces them with uncertainties arising from the coefficients $\{e_0, e_1\}$ in (38) and the ability of the selected formula to represent the calibration data. The uncertainty in the first coefficient, the main contributor to sideslip bias, is obtained from the standard deviation in the mean of results from the circle analysis, summarized in Sect. 7.1.6. For propagation to lateral horizontal wind, (36) indicates that the result is approximately $\delta u_{lateral} = V \delta \beta$ where V is airspeed, with additional contributions from correlated errors involving V that are small in comparison to that listed. A typical value for V is about 220 m s^{-1} , leading to the listed elemental uncertainties in horizontal wind arising from uncertainty in SSRD.
3. **ADIFR:** See Sect. 2.1.5 and Table 1. The angle of attack has negligible effect on the horizontal wind as long as the roll angle is small, so for measurements made during straight-and-level flight this contribution to uncertainty in horizontal wind is negligible.
4. **QCF:** The values listed are the characteristics of the transducer. Application of the calibration procedure based on comparison to the laser air-motion sensor (Cooper et al. [2014]) led to an estimated standard uncertainty of 0.1 m/s for steady flight conditions and 0.3 m/s for fluctuating conditions, so this is used for the table entry pertaining to the wind component in the direction of the longitudinal axis of the aircraft. For the lateral component, QCF affects the calculated sideslip angle SSRD, but as in the case of AKRD the procedure to determine sensitivity coefficients removes any effect of bias in QCF by calibration in terms of the coefficients $\{c_0, c_1\}$ so a potential bias in QCF does not enter the lateral component of the horizontal wind but instead is replaced by possible bias in those coefficients, as discussed for element 2. The effect of a random error in QCF of 0.01 hPa for a typical value of $\text{QCF} \approx 100 \text{ hPa}$ is, from (38), to introduce an uncertainty in sideslip angle of about 0.01% or, because typical values of sideslip angle are smaller in magnitude than 1° , an error propagated to horizontal wind smaller than 0.001 m/s. This contribution is therefore neglected.
5. **HEADING:** The random error in heading can be evaluated by comparing two duplicate IRUs, as was done for pitch. The two systems on the GV for DEEPWAVE differed in mean heading by about 0.45° , evidently a result of an alignment error on installation. However, the standard deviation of the difference between the two measurements was only 0.04° , a value that indicates the systems may perform better than the manufacturer's specification (0.2°) would indicate. The uncertainty in the bias evaluated from the circle-maneuver study of Sect. XXX is about 0.09° , so this will be used as the bias estimate

while 0.04° is considered the random component of uncertainty in heading.

6. **PITCH:** The contribution to uncertainty from the measurement of pitch was discussed above in connection with measurement of the vertical wind. However, in the case of horizontal wind, for level flight with negligible roll an uncertainty in pitch makes negligible contribution to uncertainty in either component of the horizontal wind.
7. **Horizontal Velocity Components of the Aircraft:** The measurement of horizontal wind is the sum of the relative wind and the horizontal motion of the aircraft relative to the Earth, so uncertainty in this component enters directly into uncertainty in the measured wind components.
8. **PSF:** The measured ambient pressure affects horizontal wind only through the dependence of true airspeed on ambient pressure, as described in the document on [RAF processing algorithms](#), Section 4.7.1. Evaluation at typical values shows that the dependence of the measured lateral component of the horizontal wind on uncertainty in this variable is negligible. For example, TASF for PSF=300 hPa, QCF=80 hPa, and ATX= -40°C differs from that for PSF=300.1 by 0.03 m s^{-1} or about 0.01%, so this would also be the percentage change in the lateral component of the relative wind.
9. **ATX:** Temperature is needed to calculate airspeed. Other studies ([Cooper et al. \[2014\]](#)) indicate that the temperature uncertainty is about 0.3°C , and this error will propagate to uncertainty in both components of the horizontal wind. Typical values of Mach number for the DEEPWAVE project were 0.8, for which a temperature change of $+0.3^\circ\text{C}$ led to an increase in true airspeed of about 0.16 m/s. The temperature error is likely a bias, so this difference also should be treated as a bias. The result is that the longitudinal component of the horizontal wind has an elemental contribution from temperature of 0.16 m/s, while the lateral component (being small and having an error proportional to the error in airspeed of about $0.16/240$ or smaller than 0.1%) has negligible error from this source.
10. **PCOR:** The dynamic and static pressure measurements are corrected for the static defect at the pressure ports using the formulas developed in ([Cooper et al. \[2014\]](#)). The uncertainty in the determination of the correction was estimated in that source as less than 0.3 m/s. Here we use similar estimates of 0.2 (bias) and 0.1 hPa (random error), correlated such that the error in static pressure is the negative of the error in dynamic pressure. For DEEPWAVE research flights these errors propagate to 0.3 m/s bias and 0.15 m/s random uncertainty.

For the lateral component of the wind, adding the elemental contributions to uncertainty in quadrature leads to a net bias estimate of 0.4 m/s and a random uncertainty of 0.2 m/s. The measurement of heading makes a dominating contribution to each. For the longitudinal component, the corresponding results are 0.3 and 0.2 m/s. Here the dominant contribution arises from the corrections applied to dynamic pressure to address the measured static defect as determined from calibrations. Measurements of the longitudinal wind provided by the LAMS have uncertainty of only about 0.1 m/s, so when this instrument is available the uncertainty could be reduced; the tabulated uncertainty includes an estimate of how well the parameterized function used to correct pressure in the absence of LAMS actually represents those measurements

element	uncertainty source	bias	random	$\delta u_{\perp, \parallel}$ bias [m/s]	$\delta u_{\perp, \parallel}$ random [m/s]
1	BDIF_GP transducer	0.07 hPa	0.002 hPa	–	–
2	SS_GP coefficients	0.03°	0.3°	(0.12, –)	(1.25, –)
3	ADIF_GP transducer	0.07 hPa	0.002 hPa	–	–
4	QCF/QC_GP transducers	0.34 hPa	0.01 hPa	(see item 9)	
5	heading	0.17°	0.3°	(0.7, –)	(1.2, –)
6	pitch	0.02°	0.02°	–	–
7	horiz. velocity of GV	0.05 m/s	0.05 m/s	0.05	0.05
8	PSF transducer	0.10 hPa	0.001 hPa	–	–
9	ATX	0.3°	0.1°C	(–, 0.16)	(–, 0.05)
10	δq	0.2 hPa	0.1 hPa	(–, 0.3)	(–, 0.15)

Table 7: Elemental contributions to the uncertainty in measurement of horizontal wind from the gust pod.. Entries ‘–’ indicate negligible contribution to uncertainty. Entries with subscript \perp refer to the lateral component of the horizontal wind, and those with subscript indicating parallel refer to the longitudinal component (along the axis of the aircraft).

adequately. Some of the uncertainty entering this assessment arises because the LAMS and the radome gust-sensing system measure at locations displaced from each other and so may encounter slightly different wind conditions.

4.3.2 Gust-pod system

1. **BDIF_GP:** The primary uncertainty in BDIF_GP is assigned to bias because it is likely a calibration uncertainty and the resolution and stability are much smaller than this bias. However, a calibration bias in this measurement does not affect the final wind measurement because the procedure in Sect. 5.2.2 determines the sideslip angle from flight data in a way that can be considered a calibration of the measurement of sideslip angle, and a bias in BDIF_GP would be reflected in a change in sensitivity coefficients determined in that section that would compensate for that bias. Only random errors in BDIF_GP propagate to the final measurement of horizontal wind, and the effect of the listed random error is typically less than 0.0001 m/s in lateral wind, with even smaller contribution to the longitudinal wind. These contributions therefore are listed as negligible in the table. The next item considers the more uncertain determination of sensitivity coefficient for sideslip angle.
2. **SS_GP coefficients:** The calibration procedure of Sect. 5.2.2 effectively removes the effects of possible biases in BDIF_GP and QC_GP and instead replaces them with uncertainties arising from the coefficients $\{e_0, e_1\}$ in (38) and the ability of the selected

formula to represent the calibration data. For sideslip angle, the fit procedure used values of heading and ground speed components determined from the gust-pod IRU, but wind components determined from the radome system. This allowed better determination of sensitivity coefficients than would have been possible from "bootstrapping" gust-pod measurements using repeated iterations, because the wind measurements from the radome system have lower uncertainty than those from the gust-pod system. However, this means for example that the offset in sideslip angle or heading will be dependent on the values from the radome system. Adjustment using the circle maneuvers of Sect. 7.1.5 is not possible for the gust pod because the wind measurements are compromised at the high bank angles required for the circle maneuver, so other adjustment is necessary. Values of SS_GP are at least as uncertain as those from SSRD because the SS_GP calibration uses wind measurements determined from SSRD, so the values in Table 6. 0.03 and 0.002°, are lower limits for the uncertainty in calibration coefficients from the gust pod. However, the standard deviation between sideslip angle measured by the radome (SSRD) and that measured by the gust pod (SS_GP) is typically about 0.3°, an indication that the uncertainty in SS_GP may be much larger.⁶ This large standard deviation might arise partly from different turbulent components being measured at the radome and at the gust pod, but this seems unreasonably high for that explanation because the standard deviation corresponds to a standard deviation in the difference in lateral wind at the two locations of 1.25 m/s. Instead, it appears that there is some source of error affecting SS_GP and that the elemental uncertainty assigned to the random error arising from application of the SS_GP calibration must be increased to 0.3° until this discrepancy between SSRD and SS_GP can be resolved.

3. **ADIF_GP:** See Sect. 2.1.5 and Table 1. The angle of attack has negligible effect on the horizontal wind as long as the roll angle is small, so for measurements made during straight-and-level flight this contribution to uncertainty in horizontal wind is negligible.
4. **QCF and QC_GP:** The values listed are the characteristics of the transducers. Application of the calibration procedure based on comparison to the laser air-motion sensor (Cooper et al. [2014]) led to an estimated standard uncertainty of 0.1 m/s for steady flight conditions and 0.3 m/s for fluctuating conditions, so this is used for the table entry pertaining to the wind component in the direction of the longitudinal axis of the aircraft. For the lateral component, QC_GP affects the calculated sideslip angle SS_GP, but the procedure to determine sensitivity coefficients removes any effect of bias in QC_GP by calibration in terms of the coefficients $\{c_0, c_1\}$ so a potential bias in QC_GP does not enter the lateral component of the horizontal wind but instead is replaced by possible bias in those coefficients, as discussed for element 2. The effect of a random error in QCF of 0.01 hPa for a typical value of QCF \approx 100 hPa is, from (38), to introduce an uncertainty in sideslip angle of about 0.01% or, because typical values of sideslip angle are smaller in magnitude than 1°, an error propagated to horizontal wind smaller than 0.001 m/s. This contribution is therefore neglected.
5. **HEADING (CTHDG_GP):** The random error in heading can be evaluated by com-

⁶For comparison, the standard deviation in the difference between AKRD and AK_GP is only 0.09°.

paring the two similar IRUs for the gust pod and the LAMS, as was done for pitch. In DEEPWAVE, these two systems differed in mean heading by about 1.3° , evidently a result of being installed at different angles relative to the aircraft longitudinal axis. The standard deviation in the difference in heading measurements from the two systems, after excluding some additional flights (18, 22, 25) where there appeared to be problems with the measurement, was 0.3° , so the uncertainty associated with this measurement is much higher than that for the radome-based system. The mean difference between the two measurements of heading, averaged over flights, had a standard deviation of 0.17° , so this may be a reasonable estimate of bias, as entered into the table.⁷

6. **PITCH:** The contribution to uncertainty from the measurement of pitch was discussed above in connection with measurement of the vertical wind. However, in the case of horizontal wind, for level flight with negligible roll an uncertainty in pitch makes negligible contribution to uncertainty in either component of the horizontal wind.
7. **Horizontal Velocity Components of the Aircraft:** The measurement of horizontal wind is the sum of the relative wind and the horizontal motion of the aircraft relative to the Earth, so uncertainty in this component enters directly into uncertainty in the measured wind components. Comparison among the different measurements of velocity components of the aircraft (<{GGVEW, CVEW_GP, CVEW_LAMS} and {GGVNS, CVNS_GP, CVNS_LAMS}) indicate that, for DEEPWAVE flights with good IRU operation (flights 5, 8–14, 16, 20–21, 23–24, 26) the standard deviations among these measurements are consistent with an uncertainty of 0.05 m/s. This characterizes some combination of bias and random error, so to be conservative this value has been assigned to each in the table.
8. **PSF:** This measurement has the same effect on the wind measurement from the gust pod that it has on the measurement from the radome-based system because the same true airspeed measurement is used for both. See the discussion for the radome system that follows Table 6.
9. **ATX:** Temperature affects wind measured by the gust pod in the same way as that measured by the radome-based system. See the discussion for the radome system that follows Table 6.
10. **PCOR:** When the airspeed from the standard radome-based system is used for calculating the relative wind using the angle measurements from the gust-pod, the same correction to true airspeed is applied in this case as when calculating wind from the standard radome-based system. See the discussion for the radome system that follows Table 6. If instead

⁷Special processing, using a hybrid heading obtained by complementary filtering (cf. Sect. 7.2) with CTHDG_GP considered the "fast" signal and THDG the "slow" signal, with appropriate adjustment for the discontinuity at 360° and with exclusion of data during and for 1 min after turns, reduced the standard deviation of the difference in heading to less than 0.10° with a similar reduction in estimated bias. The resulting heading variable retains the high-frequency response of the gust-pod IRU, needed to address issues like wing flex or vibration of the pod, but used the higher-quality measurement of heading from the fuselage IRU for long-term updating. This can improve the measurement of horizontal wind from the gust pod significantly, at the expense of having the measurements use a reference measurement outside the instrument. If improvement in the measurement of horizontal wind from the gust pod becomes important, such processing can be used for special cases.

TAS_GP as parameterized by (39) and (40) in Sect. 5.3, no correction is applied because the fit was determined by fitting to TASF values that were already corrected using the pressure-correction equations.

For the gust-pod system, the uncertainties in the two components of the horizontal wind (lateral and longitudinal relative to the aircraft) are quite different. For the lateral component of the wind, adding the elemental contributions to uncertainty in quadrature leads to a net bias estimate of 0.7 m/s and a random uncertainty of 1.7 m/s. The measurement of heading makes a dominating contribution to each, and the values used for these estimates are the result of inter-comparisons between units and are much higher than the best specifications for the unit listed in Table 2, but as noted there the error can increase fairly significantly if not updated with frequent course changes. For the longitudinal component, the corresponding results are 0.3 and 0.2 m/s. Here the dominant contribution arises from the corrections applied to dynamic pressure to address the measured static defect as determined from calibrations, just as for the radome-based system, because the measurements of the longitudinal component of the wind are the same for both systems.

4.4 Summary

This subsection summarizes the net uncertainty in wind measurements as developed in the earlier parts of this section. See Table 8 for the key results.

4.4.1 The radome-based system.

The standard wind measuring system on the GV is called the radome-based system and results in the basic wind measurements WDC, WSC, and WIC representing the horizontal wind direction (degrees relative to true north), horizontal wind speed (m/s) and vertical wind speed (m/s). For this system, the estimated bias limit, random component of standard uncertainty, and combined standard uncertainty are listed in Table 8. The combined uncertainty is obtained by adding the estimate of bias and the estimate of random error in quadrature, but this characteristic can be questioned because the bias estimate does not have normal statistical characteristics. It is preferable to use the estimates of bias and of the random component of standard uncertainty separately when characterizing a measurement. As an approximation, it is reasonable to consider about 0.4 m/s as the uncertainty in each component of the measurement of horizontal wind and 0.1 m/s as the corresponding uncertainty in vertical wind. In any specific direction, the uncertainty in horizontal wind remains about 0.4 m/s, so this is also the uncertainty in measured wind speed. Translation to uncertainty in wind direction depends on the magnitude of the wind speed: If the wind speed is u and the uncertainty in the component of the wind transverse to the wind speed is δ_u , the uncertainty in wind direction $\delta\xi$ is about δ_u/u . For example, for $u = 20$ m/s and $\delta_u = 0.4$ m/s, $\delta\xi = 0.02$ rad. or about 1° .

There is some potential for improvement in the vertical wind if the pitch-correction algorithm of Section 6.4 is applied. The improvement in effect removes the bias contribution from pitch, so it reduces the estimated bias to 0.05 m/s and the standard uncertainty to about 0.06 m/s.

Table 8: Summary of uncertainty for measurements of wind from the GV. The two entries for bias for the horizontal wind are first the component lateral to the axis of the aircraft and second the component parallel to the axis of the aircraft.

Measurement	bias	random uncertainty	net uncertainty	notes
vertical wind, radome	0.1	0.04	0.1	lower with pitch correction ^a
horizontal wind components, radome	0.4 , 0.3	0.2	0.4	roll < 5° ^b
vertical wind, gust pod	0.18	0.11	0.2	roll < 5° ^c
horizontal wind components, gust pod	0.7, 0.3	1.7, 0.2	1.8, 0.4	best conditions ^d

^aWith application of the pitch-correction algorithm of Sect. 6.4, Eq. 57, the bias estimate is 0.05 m s^{-1} and the net uncertainty is 0.06 m/s .

^bExpect minor degradation in turns.

^cErrors may be much larger in turns.

^dSelected flights in DEEPWAVE; can be factor-of-2 more uncertain for worst flights. Must qualify heading measurement by comparison to another measurement to get the listed performance. Not valid in turns.

Further reduction would require independent evidence that the calibration maneuvers are flown where the average wind is smaller than the 0.05 m/s value assumed when obtaining these results, because this then is the dominant remaining uncertainty.

In the case of horizontal wind, the leading uncertainty is that associated with heading, which could be improved by implementation of a full Kalman filter to adjust the heading or by replacement of the IRU with a higher-quality system with inherent Kalman filtering. There are systems available with much lower specified uncertainty that could reduce the uncertainty in lateral wind significantly.

4.4.2 The gust-pod system

The vertical wind measured by the gust pod is surprisingly good, when it is considered that the measurements are made under the wing of the aircraft in a region of seriously distorted airflow. While of lesser quality than the measurements from the radome, the measurements based on the gust pod have estimated uncertainty only about twice that of the radome-based measurement. On the other hand, the measurements of horizontal wind from the gust pod have significantly greater uncertainty than those from the radome. The uncertainty approaches 2 m/s even in the selected best cases, and there are examples where the discrepancy between similar measurements of heading becomes much larger than the tabulated values and the associated uncertainty in horizontal wind becomes even larger. Vertical and horizontal winds are both problematic in turns and should not be used for roll angles exceeding about 5° in magnitude. The problem with measurements in turns arises because the gust-pod system is not aligned with the longitudinal axis of the aircraft so, in turns, the three attitude angles (pitch, roll, heading) become intermixed. It may be that the appropriate angle transformations can be found to handle this problem, but current processing leads to obvious errors in turns.

Some support for this value of uncertainty in vertical wind from the gust pod was provided by Fig. 8, where the two measurements of vertical wind were compared for all measurements from one flight. The standard error in the difference between the two measurements was 0.27 m/s, while the uncertainties in Table 8 would suggest an expected uncertainty in the difference of 0.22 m/s. While this is slightly lower than the measured difference, some of that difference can arise from real differences in vertical wind at the two locations on the aircraft and from timing differences, so the measured standard error is in reasonable agreement with the expectations from the uncertainty analysis.

Using measurements of horizontal wind from the gust pod is not recommended. That system was designed to provide a back-up measurement that could be anti-iced to remain operational in heavy cloud. In the case of vertical wind, it appears that the system fills this back-up role well. However, the horizontal wind from the gust pod is much inferior to that from the radome-based system and probably should be used only with much caution. That would involve checking that the measurement of heading from the gust-pod IRU provides measurements in reasonable agreement with other units, considering installation differences in orientation) and excluding turns. An additional restriction arises from the fit restrictions used to determine the coefficients in the equation representing angle of attack. Those restrictions were: true airspeed (TASF) greater than 130 m/s, absolute value of roll less than 5°, and altitude greater than 5000 m.

Outside these limits, extrapolation errors can lead to significant errors in the measurements from the gust pod.

The straightforward way to improve the measurements from the gust pod would be to improve the measurement of heading. It might be possible to calculate a surrogate heading from the fuselage IRU and the known installation offset of the gust-pod IRU, but this hasn't been investigated yet and would require continued study beyond that reported here.

4.4.3 Conclusion

The first three lines in Table 8 claim that wind can be measured with low uncertainty from a high-speed aircraft such as the NCAR/NSF GV. This is particularly challenging at high speed because the aircraft introduces flow distortions and pressure variations over and near the fuselage that affect many of the sensors used to measure wind. Calibration by comparison to a laser air-motion sensor has led to improvement in the measurement of horizontal wind and is the basis for achieving these tolerances. Calibration maneuvers, especially those involving flying circles, have provided evidence for the claimed limits to uncertainty and have refined some of the calibrations used to achieve these limits. The fourth line in the table indicates disappointing performance for the gust-pod measurement of the component of horizontal wind lateral to the aircraft, so in general this measurement should not be used for research without further improvement. However, the measurement⁴ of vertical wind from the gust pod provides a useful back-up to the conventional measurement.

5 Empirical coefficients

This section reviews the determination of empirical or "sensitivity" coefficients that provide parameterized measurements of the angles of the relative wind (angle of attack and sideslip angle) in terms of measured quantities like pressure differences between ports on the radome. These sensitivity coefficients are essential for measurement of the relative wind, as described in Sect. 1.2.1, Eq. (1). DEEPWAVE flight 15 on 3 July 2014 was devoted to calibration maneuvers, and measurements from that flight, combined with similar calibration maneuvers flown on flight 11 at 40,000 ft, are used in this section to determine sensitivity coefficients for measurements of angle of attack (AKRD and AK_GP) and of sideslip angle (SSRD and SS_GP). A larger data set, described below, is also used to study the representativeness and uncertainty of the resulting sensitivity coefficients.

5.1 Angle of Attack

5.1.1 Equations underlying the calibration

The first-order expression for the vertical wind w is

$$w = V \sin(\alpha - \theta) + w_p \quad (28)$$

where V is the true airspeed, α the angle of attack, θ the pitch, and w_p the vertical motion or rate-of-climb of the aircraft. The solution for the angle of attack is

$$\alpha = \theta + \arcsin \frac{w - w_p}{V} \quad (29)$$

If it is reasonable to assume for some period of flight that w is zero, or that it averages to zero, then

$$\alpha^* = \phi - \arcsin \frac{w_p}{V} \quad (30)$$

can be used as a reference angle of attack to which to fit a parameterized formula. This fit reference depends on measurements of pitch, rate-of-climb, and true airspeed. Even in the presence of waves, fitting functions of the radome measurements and other flight characteristics to this reference should average any real effects of vertical wind as long as the vertical wind over the flight segments used averages to zero.

The danger in this approach is that a particular data set may not have negligible average mean wind. For example, if a flight spent more time in the updraft regions in the ascending portion upwind of the island and less in the downdraft region downwind of the island, the mean measurement of vertical wind may not be negligible. The functions used for representation of angle of attack always include an offset term along with functions of measurements, so it may be

appropriate to adjust that offset if there is evidence that the mean vertical wind should not be zero. Other steps can be taken to check offset coefficient, as discussed in subsequent sections.

In the case of the standard radome-based system on the GV, the relevant variables are ϕ =PITCH, w_p =GGVSPD, and V =TASF. The system measures the pressure difference (ADIFR) between top and bottom ports on the radome, and this pressure is then normalized by some measure of dynamic pressure like QCF or QCFC. The former is preferable because the use of corrected QCFC requires the application of static-defect corrections that themselves depend on angle of attack, leading to circularity in the calculation. Other candidates QCR and QCRC are not chosen because they can be affected by icing or freezing of accumulated water in pressure lines even when ADIFR continues to function.

For the gust pod, the relevant variables are θ =CPITCH_GP, w_p =CVSPD_GP, and V =TASF. The gust-pod measurements differ from those measured relative to the fuselage; for example, the pitch of the gust pod is several degrees different from that of the fuselage because of the way in which the gust-pod IRU is installed. However, the true airspeed V in (30) is measured better by the fuselage system, so TASF will be used for V . The equation with the appropriate variables is then:

$$\alpha^* = \text{CPITCH_GP} - \arcsin \frac{\text{CVSPD_GP}}{\text{TASX}} \quad (31)$$

"Calibration" of the angle of attack (i.e., fitting to find the empirical relationship) then requires determining a function $\alpha_{fit}(\{m_i\}) = \alpha^*$ of measured quantities $\{m_i\}$ that approximates the values of α^* determined from (31). Possible terms m_i in that function may include pressure measurements from the radome (esp. ADIFR), dynamic pressure, and Mach number, as well as powers and products of these terms. For the Rosemount 858 sensor used with the gust pod, it is expected from theory that one element of $\{m_i\}$ will be ADIF_GP/QC_GP. Wind tunnel and theoretical studies predict how the pressure will vary on a hemispheric surface with changing angles, but those don't necessarily apply to the mounting location on the GV because there is considerable flow distortion at the under-wing location of the pod and that affects the pressure response to changes in flow angles. Therefore the above approach is used for that sensor also and determines a separate functional representation of the calibration data for the gust pod.

5.1.2 Application to the radome

The sensitivity coefficients for the radome are determined through the use of speed runs. In these maneuvers, the aircraft is slowed to a speed near the lower range of its operating range, then accelerated to near the upper limit, and then slowed again to normal cruise. If this is done while flying a level track, the angle of attack will vary through its normal range and the pitch will vary similarly. If there is no vertical wind or if a fluctuating vertical wind averages to zero, (30) provides a reference angle α^* that serves as reference for the parameterized fit.

There were three speed runs during DEEPWAVE flight 15, at the times 3:21–3:29, 4:15–4:23, and 5:01–5:11 UTC and at approximate altitudes of 3830 m, 5770 m, and 8800 m. On flight 11 of that project, there was a similar speed run flown from 10:30–10:40 UTC at an altitude of approximately 12,250 m. For the purpose of this first determination of sensitivity coefficients for angle of attack, only those periods were used. In addition, because some of the measurements at

minimum speed deviated from the otherwise simple fits, only measurements with true airspeed in excess of 130 m/s were used; this eliminated some of the slowest parts of the speed runs, but that is a flight speed not used in normal operation.

The sensitivity to the pressure difference between vertically separated ports is the most important part of the calibration of angle of attack. Secondary terms are sometimes needed to adjust the value to maintain a correct zero. Therefore, the fit was done in two stages. First, the four speed runs alone were used to determine the sensitivity to the pressure ratio, and then a larger dataset was used to incorporate a wider range of flight conditions to check that the fit determined from the speed runs remained representative of the larger data set.. The first fit was to the following simplified equation:

$$\alpha^* = c_0^* + c_1^* \frac{\Delta p_\alpha}{q} \quad (32)$$

where c_0^* and c_1^* are sensitivity coefficients determined by the fit. Once sensitivity coefficients are found, the angle of attack can then be calculated using α as produced by the right side of this same equation.

Figure 17 compares α obtained in this way to α^* for the data used in the fit. The fit results are tabulated in the following summary, which was produced by the “R” call at the top of the listing. AOAREFC is α^* with correction to pitch as in Section 6.4 and AQR= $\Delta p_\alpha/q$.

```
## lm(formula = AOAREFC ~ AQR, data = Data2)
## [1] "Coefficients:"
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.394    0.005672 774.6     0
## AQR         20.986   0.068937 304.4     0
## [1] "Residual standard error: 0.121, dof=1977"
## [1] "R-squared 0.979"
```

This fit gave coefficients c_0^* and c_1^* equal to 4.394° and 20.986° . The fit accounted for 97.9% of the variance and had a residual standard error of 0.12° , so it represents the speed-run data reasonably well.⁸ The entries "Std. Error" in the tabulated results show that the fit coefficients are constrained with small uncertainty limits. The more significant uncertainty, however, comes from the assumption that the vertical wind is zero for these periods of the speed runs. Therefore, additional studies will be used below for further evaluation of the uncertainty introduced by this assumption.

This calibration would be affected by a time difference between the IRU measurement of pitch and the data-system sampling of the pressures involved, especially that from the radome. The IRU outputs measurements with a time delay that can be variable and unknown but is expected to be <0.1 s, and normal processing uses an adjustment of 60 ms to compensate for this delay. To guard against this delay affecting the calibration, fits like that leading to the above formula and coefficients were repeated after shifting the measurement of pitch by various time intervals, both forward and backward. For shifts within about 0.5 s, the results did not change significantly,

⁸A residual error of this magnitude corresponds to the measured fluctuations in the vertical wind for this data set of about 0.3–0.4 m/s.

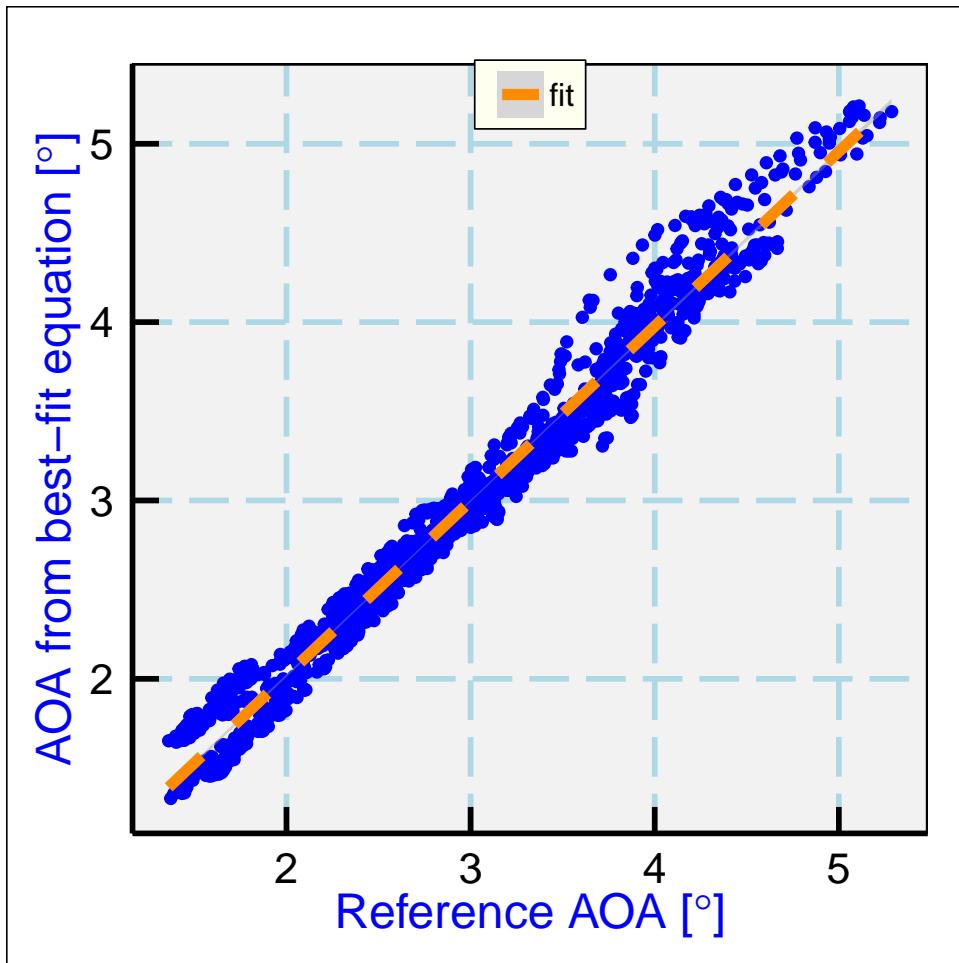


Figure 17: The angle of attack determined from the fit to pressure measurements from the radome, as a function of the reference angle provided by Equation (32), for the combination of data from all four speed runs as listed in the text.

although the fit with adjustment of pitch *backward* by about 0.5 s had the smallest standard error . Equivalently, a similar standard error was obtained if the measurement of the pressure ratio from the radome (δp_α) were moved forward in time by a similar amount. Neither of these shifts seems likely at this magnitude, and the change in standard error was only about 0.002° , so we interpret this as indicating insensitivity of the calibration to small shifts in time. That is likely a result of the calibration data being from speed runs where the flight speed is gradually increased and then decreased, so an effect of a lag partially cancels in such data segments.

The calibration would also be affected by pitch errors such as are discussed in Sect. 6.4. Therefore the correction procedure discussed in that section was applied to these data before finding the preceding fit.⁹ In comparison to the fit without this correction, the standard error was changed only negligibly and the two fit coefficients changed by -0.007 and -0.004, respectively, so this correction also had only minor effect on the fit, with changes comparable to or smaller than the standard errors in these coefficients. This insensitivity perhaps arises because the period of the calibration spanned a few hours and so included enough time for some averaging of the Schuler oscillation of errors in pitch.

Previous studies of the radome where low-altitude flight segments were included required an additional term in the calibration equation to match reference measurements (α^*) in the boundary layer over the ocean as well as at high flight levels. The standard calibration, determined from a large set of speed runs from past projects, is given in the RAF document on [Processing Algorithms](#):

$$\alpha = c'_0 + \frac{\Delta p_\alpha}{q} (c'_1 + c'_2 M) \quad (33)$$

with coefficients $\{c'\} = \{4.604^\circ, 18.67^\circ, 6.49^\circ\}$. The last term provides some adjustment dependent on Mach number M and corrects an offset often seen in vertical wind at low airspeed.

Figure 18 shows the angle of attack determined from this fit vs. that from the standard formula. The plotted shapes show the distribution in values for centered intervals in the predicted angle of attack, where the blue shapes and orange line are the results from the fit to the speed-run measurements and the green shapes are the distributions that would result from using the standard calibration. The standard calibration is close to that determined from the speed runs, but the standard deviation of the difference between predicted values and fit values increases from 0.12° for the speed-run fit to 0.15° for the standard fit. The difference is most evident for measurements at large angle of attack, where the standard coefficients produce increasingly higher values as the angle of attack increases.

Because the standard fit in use for the GV includes an additional term representing dependence on Mach number, the benefit of using such a fit for DEEPWAVE is worth considering. The primary reason for that fit has been that otherwise the measurements of vertical wind at low level tend to be biased, but most of the useful flight data from DEEPWAVE was at intermediate or high levels so this may not be a concern for this project.

⁹The correction is based on detection of the errors in ground-speed measurements and use of the Schuler relationship between pitch error and acceleration error to correct the pitch. This is discussed in detail in the referenced section.

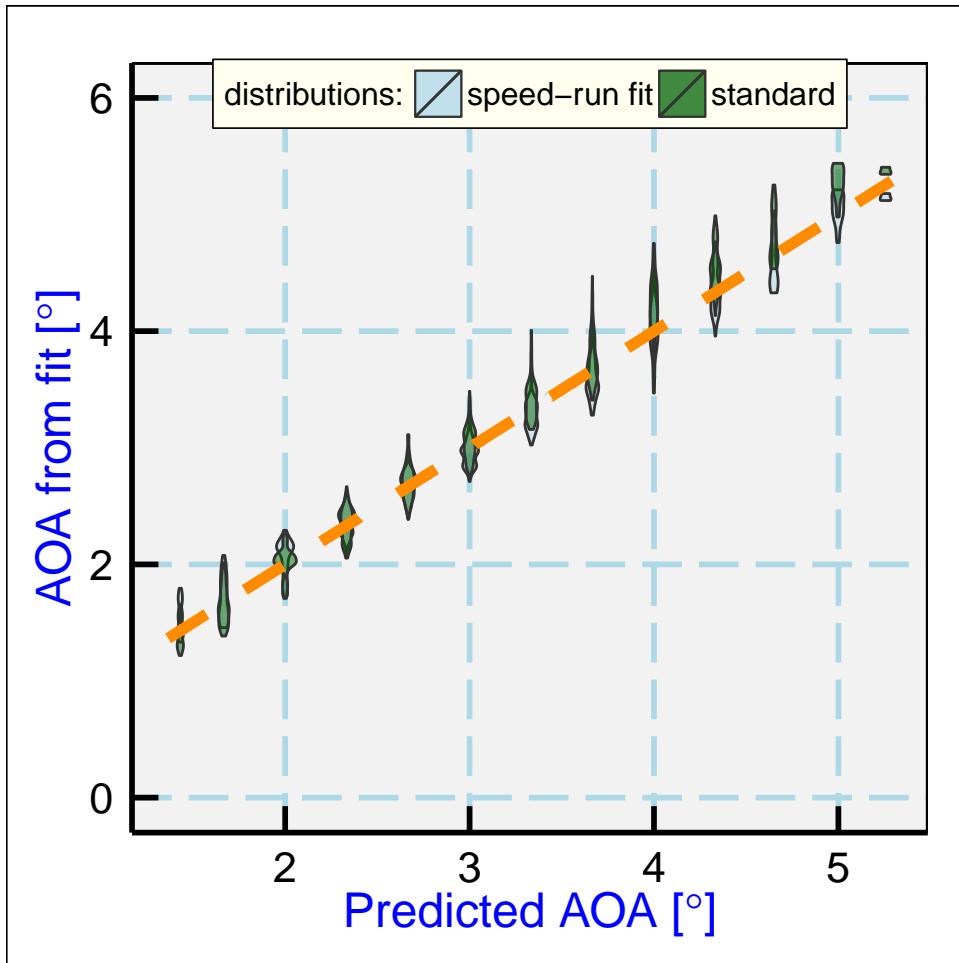


Figure 18: The angle of attack determined from the simple fit to the pressure ratio from the radome, as a function of the angle of attack predicted for zero vertical wind from the formula in the text (blue shapes), and the corresponding distribution that would result from using the standard calibration (green shapes). The shapes show the distributions for measurements in centered bins at (1/3)-degree increments in the predicted angle of attack. The dashed orange line is the best fit to the speed-run measurements.

```

## lm(formula = AOAREF ~ AQR + AQRM, data = Data2)
## [1] "Coefficients:"
##              Estimate Std. Error t value
## (Intercept)    4.387   0.005402 812.09
## AQR          17.924   0.216747  82.70
## AQRM         4.295   0.288585  14.88
##              Pr(>|t|)
## (Intercept) 0.000e+00
## AQR         0.000e+00
## AQRM        1.445e-47
## [1] "Residual standard error: 0.115, dof=1976"
## [1] "R-squared 0.981"

```

Fitting (33) to the speed-run data gave the results listed above, with a residual standard error that is reduced by $<0.006^\circ$ relative to the fit without the term dependent on Mach number, so for those measurements this small improvement does not seem to justify adding another term to the fit.

The next step was to expand the data set to include flights from a range of altitudes including up to FL450,¹⁰ to check if it is necessary to refine the fit to include additional terms like that in Eq. 33 to constrain the zero of the angle of attack while preserving the measured sensitivity as in (32). This expanded dataset included DEEPWAVE flight 14, with a FL450 leg from 11:30 to 12:20 UTC; flight 16, with a FL430 leg from 9:30 to 11:00 UTC; flight 3, all measurements at and above FL350 (to incorporate a long flight where fuel burn-off changed the angle of attack); and an expanded section from flight 11 (in addition to the speed runs) from 7:00 to 10:00 UTC, to include a long leg at FL400.

Various fit equations were explored involving terms including the Mach number, the air density, the measured pressures, and various products and powers of these terms including terms multiplied by the basic pressure ratio already included in (32). None of these produced enough improvement over the single-term fit to warrant their inclusion; the best improvement in the residual standard error was about 2%.¹¹

The two-coefficient fit to the expanded data set, using

$$\alpha = c_0 + c_1 \frac{\Delta p_\alpha}{q} , \quad (34)$$

gave best-fit coefficients $\{c_0, c_1\} = \{4.468^\circ, 21.481^\circ\}$ and a standard error of 0.12° for 17,715 measurements. The small increase in residual standard error in comparison to the speed-run measurements is expected because this expanded data set includes regions more likely to have

¹⁰FL450 denotes flight level 450 or flight at the pressure corresponding to 45,000 ft in the ICAO standard atmosphere.

¹¹Additional terms that did improve the fit significantly were those correlated with the terms in Eq. 30, esp. pitch, but including such terms is not consistent with finding a fit that would represent the angle of attack in conditions with non-zero vertical wind. For example, including "PITCH" as a term in the fit resulted in a coefficient for this term of about 0.4, and such a large correlation between resulting measurements of angle of attack and pitch would bias the response to a true vertical wind. As an extreme example, inclusion of a term based on the right side of Eq. 30 leads to a perfect fit, but use of such a fit would ensure that all measurements of vertical wind would be zero. For this reason, pitch and vertical aircraft motion were excluded from the candidate terms in the fit.

non-zero vertical wind, which would contribute to this residual error. Expanding this fit to include a third coefficient as in (33) resulted in negligible improvement (0.001°) in the residual standard error, and introduction of a set of eight possible dependencies while exploring for better fits only led to reduction in the standard error by 0.003° , so more complicated equations than (34) do not appear to be needed.¹² Another fit considered with this expanded data set was to set the slope parameter to the value obtained from the speed-run data, 20.986° , and then fit using the expanded data set to determine a value of the offset coefficient that minimized the mean vertical wind. That gave a value for the first coefficient of 4.43° and a standard error negligibly different from that for the two-coefficient fit, so that is another indication that the speed-run dataset and the expanded dataset give consistent results.

The recommended calibration for DEEPWAVE, determined with correction of the pitch errors as discussed in Sect. 6.4, is (34) with these values for the coefficients: $\{c_0, c_1\} = \{4.468^\circ, 21.481^\circ\}$. However, for other projects and general use the coefficients and standard fit (33) are better initial choices for processing.

Evaluation of all the points in the expanded DEEPWAVE dataset using three fits, that from the expanded dataset, from the speed runs alone, and using the "standard" fit discussed above, gave only very small differences in the results. In Fig. 19 the values of angle of attack obtained using the fit to data from the expanded dataset are compared to those obtained using the fit to the speed-run dataset. The angle of attack evaluated from the expanded-dataset coefficients resulted in values $0.036 \pm 0.007^\circ$ larger than those from the speed-run coefficients and $0.037 \pm 0.036^\circ$ larger than those from the standard coefficients, so using any of these formulas would give about the same results.

The standard errors in the coefficients for the expanded dataset are respectively 0.0035° and 0.046° , so the coefficients are tightly constrained by the fit. The correlation between error terms was very high and positive, but the second coefficient is applied to a term ($\delta p_\alpha/q$) that is typically negative with representative magnitude of about -0.0037° so the correlated uncertainties partially cancel, leaving a net uncertainty in angle of attack of 0.0007° (for 0.99 correlation between the uncertainty terms) as a result of uncertainty in the fit.

The expanded dataset provided results consistent with those from the speed runs alone but contained 17,721 measurements vs. only 2,019 for the speed-run dataset, so this larger set provides an opportunity to examine the consistency of results from subsets of the measurements. For this purpose, randomly selected but exclusive subsets of the data were selected repeatedly and the fit coefficients were determined from each of these subsets. In one example, the dataset was divided randomly into 100 exclusive subsets and the fit coefficients were calculated for each of those subsets, and then this process was repeated 50 times. The resulting standard deviations in the fit coefficients were 0.070° and 0.88° . With resampling, the independent samples entering these averages are fewer than 5000 but more than 100, suggesting that the means for the coefficients are known with uncertainties of between $0.007 - 0.001^\circ$ for the first coefficient and $0.09 - 0.012^\circ$ for the second. The estimates from the fit ($0.004^\circ, 0.053^\circ$) are midway in these

¹²This simpler representation applies to the normal research flight levels of the DEEPWAVE project, levels above about 10,000 ft. Because expanded representations were needed in other projects to represent low-level flight data, and low levels are not included in this expended data set, it may be necessary to revisit this calibration with more terms if lower-level flight segments are to be analyzed.

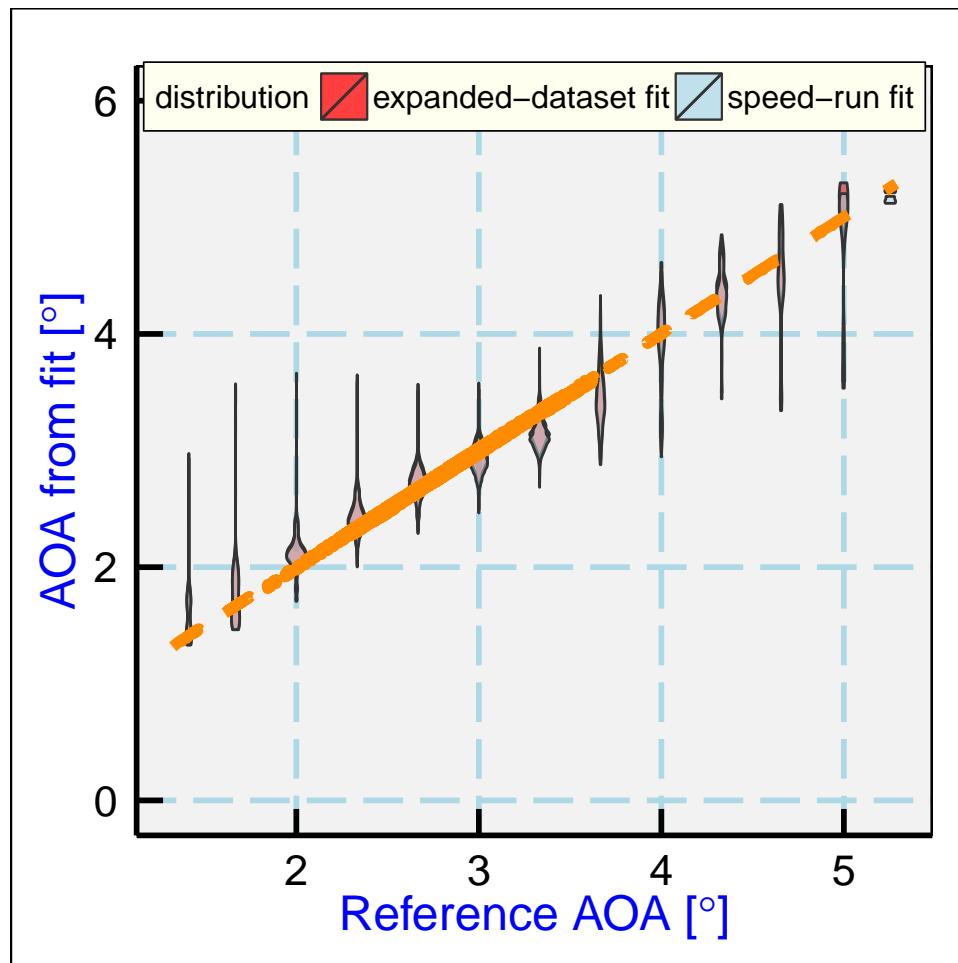


Figure 19: Distributions in the angle of attack determined from the two-coefficient fit to the expanded dataset, plotted with the distributions that would result from use of the equation based on the speed-runs only. The pink color denotes regions covered by both distributions.

ranges, so the fit results are consistent when the data are partitioned into various subsets. The fit results thus appear reasonably characterized by the estimated standard errors from the fit.

Repeating this for different subset sizes N indicated that the standard deviations in the coefficients scaled in a manner consistent with \sqrt{N} behavior, suggesting that the results are not biased by isolated outlier measurements such as would occur from regions of extended non-zero vertical wind. The subsetting also produced resulting values for the individual coefficients that were highly correlated, with correlation coefficient around 0.99. This also supports the partial cancellation of uncertainty as evaluated for the fits.

These estimates of uncertainty characterize how well the sensitivity coefficients characterize the data, but the validity of the result is dependent on the mean vertical wind being zero, as discussed in connection with Eq. (29). A non-zero mean vertical wind of 0.1 m/s will introduce bias into the first sensitivity coefficient of magnitude 0.02° , a value much larger than the estimated uncertainty in angle of attack resulting from the fit. This then is the major uncertainty affecting angle of attack and hence vertical wind, and it is difficult to quantify except by estimates of how large the vertical wind might be. For all 1-s measurements of vertical wind in the DEEPWAVE project, the mean vertical wind was 0.03 m/s without correction for pitch errors and <0.01 m/s with such correction, in both cases with a standard deviation of about 0.5 m/s. When measurements are partitioned into groups within $1\text{--}5^\circ$ longitude upwind and downwind of the island crest, using a slanted dividing line approximately matching the orientation of the island, the two groups had respective mean vertical wind measurements of -0.026 and +0.032 m/s, so even over the South Island of New Zealand there was no significant mean vertical wind at the altitude of research flights. It therefore seems reasonable to use a limit of about 0.05 m/s as applicable to the calibration process, leading to an estimated bias limit on the first sensitivity coefficient of magnitude 0.01° . This is then the dominant uncertainty component affecting the measurement of angle of attack.

One additional test was used to check the consistency of the measurements and to ensure that the project-mean vertical wind would be near zero for research-flight conditions. All DEEPWAVE flights that appeared to provide good data for the calibration procedure were combined into one dataset. The excluded flights were 6 and 7 (where there were problems with ice-blocked lines in the radome), and 15 (the calibration flight, mostly at lower than research flight altitude). Also, the period from 9:50 to 10:30 UTC on flight 23 was excluded because the radome measurements looked suspicious and might have been affected by blockage. Pitch corrections as discussed in the preceding section were applied to these flights, and then the following tests were used to exclude periods not significant for the calibration: a) measurements where the true airspeed was less than 130 m/s, to exclude periods of anomalously high angle of attack, esp. during takeoff and landing when flaps and/or landing gear might be deployed; (b) periods when the roll angle was less than -5 or more than $+5^\circ$, to exclude turns; and (c) periods of flight below 35,000 ft pressure altitude, to emphasize the altitudes most used during research flights. More than 400,000 measurements were available for the fit after these exclusions. The resulting fit is summarized in the table below:

```
## lm(formula = AOAREF ~ AR, data = DataC)
## [1] "Coefficients:"
##                Estimate Std. Error t value Pr(>|t|)
```

```

## (Intercept) 4.435 0.001014 4373      0
## AR          21.166 0.013268 1595      0
## [1] "Residual standard error: 0.115, dof=439552"
## [1] "R-squared 0.853"

```

The resulting coefficients were 4.435° and 21.166° . These coefficients are similar to those determined for the subset data used previously, but they could be used instead if the goal is to minimize the offsets in mean vertical wind for the project because, for the constraints used, they will force a project-mean vertical wind of zero. A test of these coefficients for the segment of flight 12 away from the influence of South Island indeed led to a mean wind very close to zero, and flight-by-flight averages also are reasonably small with these coefficients. This set of coefficients then is a possible alternative to those recommended above (cf. p. 69).

5.1.3 Application to the gust pod

A similar approach was taken for the gust pod, but with variables translated to apply to the gust-pod instead of the radome: $\Delta p_\alpha \rightarrow \Delta p_{\alpha,g} = \text{ADIF_GP}$, $q \rightarrow q_g = \text{QC_GP}$, $p \rightarrow p_g = \text{PS_GP}$, and the Mach number $M \rightarrow M_g$ was that determined from the uncorrected measurements p_g and q_g from the gust pod. Apparently because of the under-wing location in disturbed airflow, a fit involving more terms was needed to represent the reference measurements from Eq. 31. After exploration of various possible terms, the fit selected to represent the gust-pod was the following:

$$\alpha_g = b_0 + \frac{\Delta p_{\alpha,g}}{q_g} (b_1 + b_2 M_g) + b_3 \frac{q_g}{p_g} \quad (35)$$

```

## lm(formula = AOAREF_GP ~ AQR_GP + AQRM_GP + RR2_GP, data = Data2)
## [1] "Coefficients:"
##                   Estimate Std. Error t value
## (Intercept) -0.9033   0.007859 -114.94
## AQR_GP       3.6025   0.068942   52.25
## AQRM_GP      4.2860   0.137571   31.16
## RR2_GP       1.3299   0.033658   39.51
##                   Pr(>|t|)
## (Intercept) 0.000e+00
## AQR_GP      0.000e+00
## AQRM_GP     1.128e-173
## RR2_GP      4.153e-252
## [1] "Residual standard error: 0.099, dof=1975"
## [1] "R-squared 0.989"

```

The results of this formula are compared to the reference data in Fig. 20. The best-fit coefficients were $\{b_i\} = \{-0.903^\circ, 3.602^\circ, 4.286^\circ, 1.330^\circ\}$ and the square of the correlation was 0.989 with residual scatter (residual standard error) of 0.099° . The fit was thus even better than that obtained for the radome for these same speed runs.

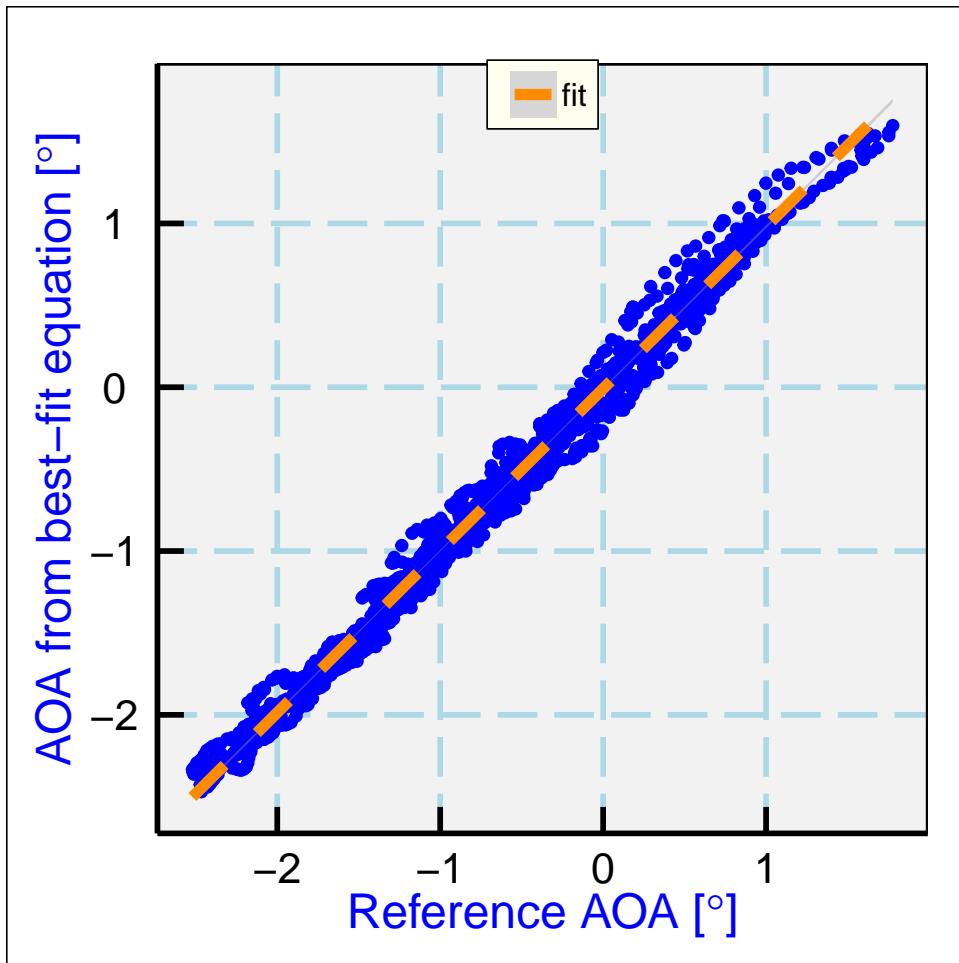


Figure 20: For the gust pod, the angle of attack determined from the fit as a function of the reference angle provided by (31), for the combined four speed runs. Compare to Fig. 17, the corresponding plot for the radome.

As for the radome, an all-project fit was obtained for the gust pod. In this case, different flights were excluded: 2, 3, 4, 11, 15, 17, 18, 19, 23, 24, 25 and the flight period from 8:05–8:40UTC on flight 1. In most cases, this was because the CMIGITS IRU providing gust-pod measurements of pitch seemed to have larger than normal deviations that appeared suspicious, so it was thought preferable to fit without those periods of suspicious measurements. The fit summary is as follows:

```
## lm(formula = AOAREF_GP ~ AQR_GP + AQRM_GP + RR_GP, data = DataC)
## [1] "Coefficients:"
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.8083   0.002366 -341.7    0
## AQR_GP       3.2478   0.030399  106.8    0
## AQRM_GP      5.3507   0.049025  109.1    0
## RR_GP        1.2820   0.007171  178.8    0
## [1] "Residual standard error: 0.163, dof=315554"
## [1] "R-squared 0.861"
```

These coefficients $\{-0.808^\circ, 3.248^\circ, 5.351^\circ, 1.282^\circ\}$ provide an alternative to the coefficients obtained above that might be preferable if it is desirable to minimize the all-project mean vertical wind for the gust-pod measurements.

5.2 Sideslip Angle

5.2.1 Equations underlying the calibration

Calibration of the sideslip angle is more difficult, both because the equations are more complicated and because the maneuver is very hard to fly. Ideally, the yaw maneuver should only change sideslip angle and heading without change in roll, altitude, or angle of attack, but that is impossible to fly. It is practical, however, to minimize roll and change in altitude, and that was how these maneuvers were flown. The three sets of yaw maneuvers on DEEPWAVE flight 15 were at these times: 3:32:00–3:35:30, 4:31:00–4:33:30, and 5:26:40–5:29:30 UTC.¹³ In the case of yaw maneuvers, the calibration is based on the expectation that the horizontal wind remains constant. The first-order equations for the east and north components of the wind, u and v , are:

$$\begin{aligned} u &= -U_a \sin(\Psi + \beta) + u_p \\ v &= -U_a \cos(\Psi + \beta) + v_p \end{aligned} \quad (36)$$

where U_a is the true airspeed, Ψ the heading, β the sideslip angle, and u_p and v_p are the eastward and northward ground-speed components of the aircraft. These two equations lead to the following reference formula for β :

¹³Because the yaw maneuvers on DEEPWAVE flight 11 (10:25:00 to 10:30:00) were flown less well, they were not combined with these new measurements, but it does not appear that the sideslip calibration has any significant dependence on altitude.

$$\beta^* = -\Psi + \arctan\left(\frac{u_p - u}{v_p - v}\right) \quad (37)$$

where the second term represents a correction for the change in direction of motion of the aircraft, which is difficult to avoid in the yaw maneuver. The measurements thus provide β^* , an estimate of the sideslip angle during the yaw maneuvers.

There is, however, a circular component in (37) because it involves the wind components and those require β for their measurement when sideslip angle changes. To reduce the feedback from this term, the horizontal wind components u and v were low-pass-filtered with periods ranging from 5–60 s and the filtered values were used in (37). Filtering made small differences in the fit coefficients while increasing the residual error significantly, but 60-s filtering was still selected because that is a period long in comparison to the yaw maneuvers so it should reduce possible bias in the fit coefficients from use of the older sensitivity coefficients and any associated fluctuations in the wind measurements during maneuvers.

5.2.2 Application to the radome-based and gust-pod systems

For both systems, a relatively simple fit was sufficient, in the following form:

$$\beta = e_0 + e_1 \frac{\Delta p_\beta}{q} \quad (38)$$

where Δp_β is the pressure difference between horizontally separated pressure ports and q the dynamic pressure. For the radome, $q = \text{QCF}$ and $\Delta p_\beta = \text{BDIFR}$; for the gust-pod, $q = \text{QC_GP}$ and $\Delta p_\beta = \text{BDIF_GP}$. The resulting fit for the radome is listed below:

```
## lm(formula = SSREF ~ BQR, data = DataV)
## [1] "Coefficients:"
##              Estimate Std. Error t value
## (Intercept) 0.09187   0.005428 16.92
## BQR        22.30224   0.170344 130.92
##             Pr(>|t|)
## (Intercept) 7.164e-50
## BQR          0.000e+00
## [1] "Residual standard error: 0.114, dof=441"
## [1] "R-squared 0.975"
```

The best-fit coefficients were $\{e\} = \{0.092^\circ, 22.302^\circ\}$, the squared correlation was 0.97 and the residual standard error was 0.11° , as listed above. The plotted measurements and this fit are shown in Fig. 21. The sideslip-angle offset that is represented by the first term will be determined later by other means; cf. Section 7.1.5, where the suggested value is $e_0=0.008^\circ$.

The standard calibration in use for the GV has coefficient $e_1=21.155^\circ$, so this fit has slope about 5% greater than the standard values. These maneuvers were flown with special care to minimize altitude and roll changes, so this value may be preferable to the older values. The standard error of the fit, about 0.1° , is likely much larger than the error in representing the values

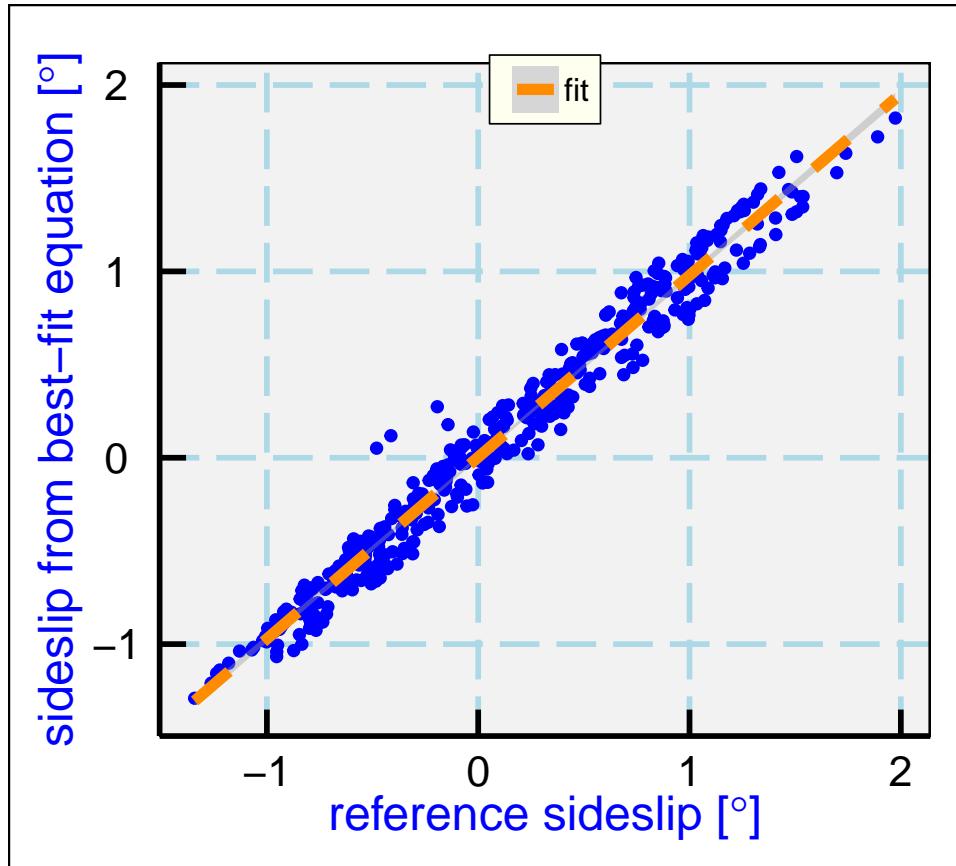


Figure 21: The sideslip attack determined from the fit, as a function of the reference angle provided by Equation (37), for the combination of data from all three yaw maneuvers listed in the text. The gray area underlying the dashed orange line denotes the standard-uncertainty range for the fit.

of sideslip angle via (38) because much of the variability likely arises from real fluctuations in the horizontal wind, which would contribute this magnitude error for wind fluctuations of only about 0.4 m/s. The respective uncertainties in the fit coefficients were about 0.005° and 0.17° , the former leading to a possible bias error and the latter to an error of $0.17/22.3$ or less than 1% in the value of the sideslip angle. Measurements of sideslip angle are seldom larger than 0.2° , so a fractional error of 1% is not significant and will be neglected here. The bias in sideslip angle is intertwined with possible bias in heading, so further study of this offset will be deferred to Section 7.1.5 where it will be determined by a better method that separates it from the offset in heading.

For the gust-pod, the same approach was followed, giving a fit with characteristics as listed below:

```
## lm(formula = SSREF_GP ~ BQR_GP, data = DataV)
## [1] "Coefficients:"
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.621     0.02624 -138.0  0.0e+00
## BQR_GP       12.184    0.13287   91.7 2.5e-289
## [1] "Residual standard error: 0.164, dof=441"
## [1] "R-squared 0.950"
```

The squared correlation of this fit was 0.95 and the residual standard error was 0.164° . The best-fit coefficients were $\{-3.621^\circ, 12.184^\circ\}$. The difference in first coefficient vs the radome is a result of the offset in heading between the aircraft longitudinal axis and the gust pod. The detailed report for the fit is listed above, and the result of applying these sensitivity coefficients to the measurements from the gust pod is shown in Fig. 22.

An additional study is needed to separate a possible offset in sideslip angle from a similar error in heading, because the approach used here assumes that the heading is accurate. In the data files used for this study, an offset in heading of -0.08° was imposed; without that offset, the offset in sideslip angle (coefficient e_0) would be reduced by 0.08° . A method of determining these separate offsets is developed later in this report (Section 7.1), where a set of circles flown with constant roll angle are used to obtain a better estimate of the sideslip-angle offset. That study results in a corrected value for e_0 .

cite

5.3 True Airspeed from the Gust Pod

The measurements ADIF_GP and BDIF_GP, normalized by QC_GP, are used with the calibrations in the preceding section to find the angle of attack and sideslip angle. In addition, to find the relative wind, a measurement of true airspeed is needed. In an effort to allow the gust-pod system to operate independently of the standard radome-based system, it is desirable to determine the true airspeed directly from the gust-pod measurements. However, the standard system has been calibrated to low uncertainty by reference to the laser air-motion sensor, so for the purpose of calibration that true airspeed (TASF) will be used as a reference. Once the gust-pod measurement of true airspeed is calibrated, the gust-pod system still measures wind without further reference to the standard system, but the calibration process links the two systems to have a common reference.

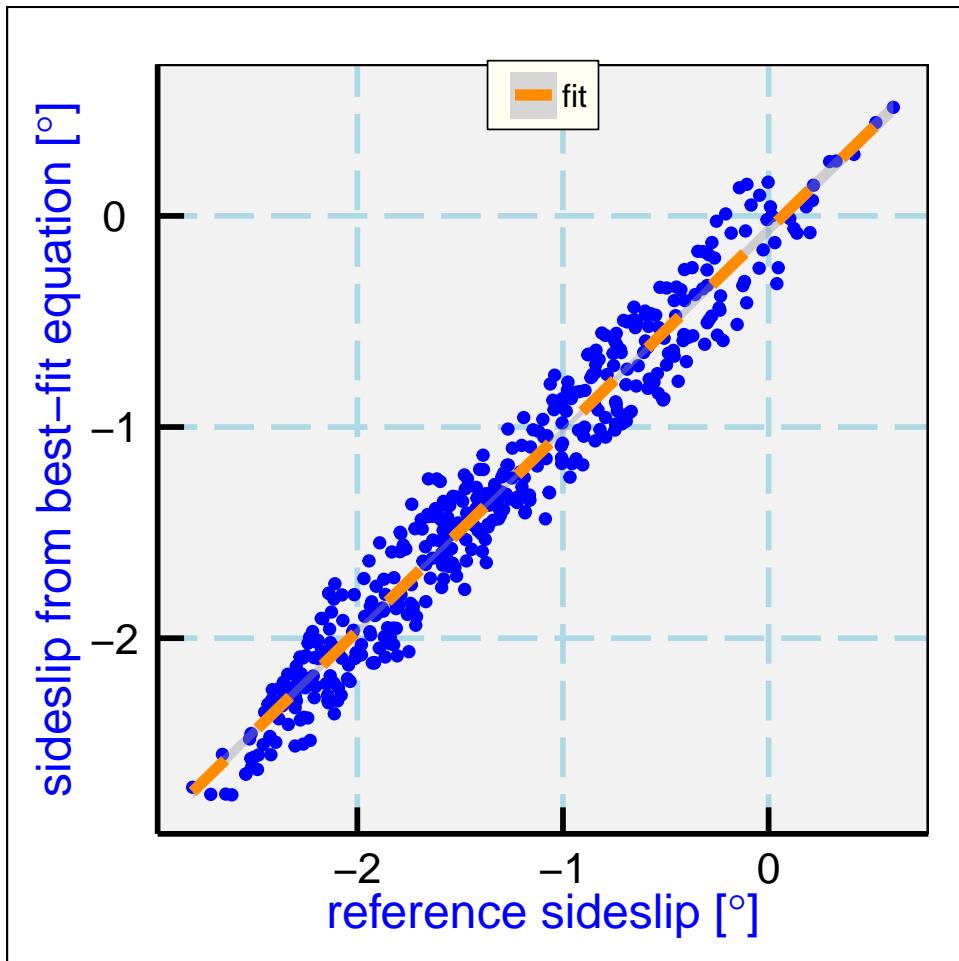


Figure 22: As in the preceding figure but for the gust-pod system.

Therefore, the next step needed to make the gust pod wind-sensing system independent of the standard wind-sensing system is to determine a functional relationship between gust-pod measurements and the true airspeed. The approach taken is to fit the basic pressure ratio q/p as measured by the 858 probe to match the corresponding pressure ratio provided by the conventional measurements of dynamic and static pressure, which incorporate corrections for the static defect. That ratio then can be used in the normal equations for true airspeed (cf. [this document](#)) to determine the airspeed measured by the gust pod.

For this purpose, all the measurements from the DEEPWAVE calibration flight (flight 15) were used qualified only by requiring TASF>130 m/s, absolute value of roll smaller than 5°, and altitude above 5000 m. These restrictions were selected because the DEEPWAVE project mostly needed only such measurements and low uncertainty could only be obtained with some restrictions on the measurements. This is the first use of the gust-pod measurements, so for future projects and different research objectives this approach should be repeated with different restrictions.¹⁴

The measurements entering the resulting fit are as follows: q =QCFC is the reference dynamic pressure, p =PSFC is the reference ambient pressure, q_g =QC_GP is the gust-pod dynamic pressure, p_g =PS_GP is the gust-pod static pressure, M is the Mach number determined from the reference measurements, M_g the Mach number determined from the gust-pod static and dynamic pressure, and $\Delta p_{\alpha,g}$ =ADIF_GP is the pressure difference between top and bottom pressure ports on the gust-pod sensor. The following equation was found to provide a useful representation of the reference ratio q/p in terms of quantities measured only by the gust-probe system:

$$\frac{q}{p} = d_0 + d_1 \frac{q_g}{p_g} + d_2 M_g \frac{q_g}{p_g} + d_3 \left(\frac{q_g}{p_g} \right)^2 + d_4 \frac{\Delta p_{\alpha,g}}{q_g} + d_5 M_g + d_6 \left(\frac{\Delta p_{\alpha,g}}{q_g} \right)^2 + d_7 \frac{\Delta p_{\alpha,g}}{q_g} M_g \quad (39)$$

This complexity in the equation was needed to obtain a good fit (with residual error translating to a typical error in true airspeed of about 0.5 m/s for typical DEEPWAVE research legs), and all terms were indicated to be significant in the fit. The fit details are listed below:

```
## lm(formula = B1 ~ B2 + BxM + I(B2^2) + AQR_GP + MachG + I(AQR_GP^2) +
##      I(MachG * AQR_GP), data = DataV)
## [1] "Coefficients:"
##                               Estimate Std. Error t value
## (Intercept)           1.1940    0.031653 37.72
## B2                  27.2893    0.844564 32.31
## BxM                 -29.0483   0.965719 -30.08
## I(B2^2)              13.6231   0.411740 33.09
## AQR_GP              -0.5936   0.004224 -140.54
## MachG                -7.5812   0.226389 -33.49
## I(AQR_GP^2)          0.1554    0.002811 55.27
## I(MachG * AQR_GP)  1.2607    0.007205 174.97
## Pr(>|t|)
```

¹⁴An example applicable to low-level flight is presented in Appendix B, where an application to characterization of turbulence is discussed.

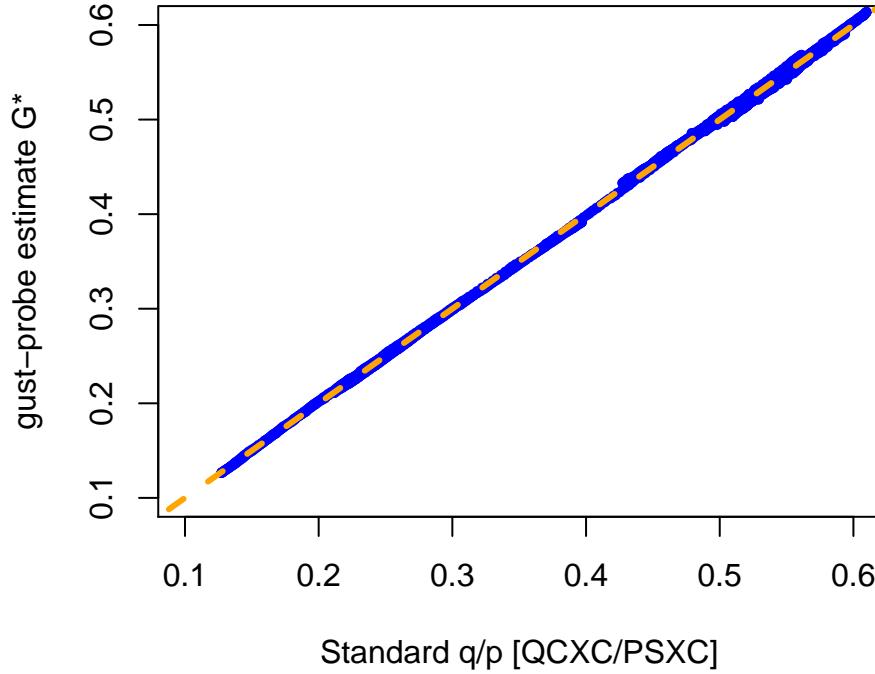


Figure 23: The estimate of q/p obtained from the right side of (39), determined entirely from measurements provided by the gust pod, plotted against the standard measurement obtained from QCXC/PSXC. The coefficients are listed in the text.

```

## (Intercept)      3.365e-305
## B2              1.114e-225
## BxM             3.094e-196
## I(B2^2)         2.311e-236
## AQR_GP          0.000e+00
## MachG           5.543e-242
## I(AQR_GP^2)     0.000e+00
## I(MachG * AQR_GP) 0.000e+00
## [1] "Residual standard error: 0.002, dof=34524"
## [1] "R-squared 0.998"

```

The resulting fit, with coefficients $\{d_0, \dots, d_7\}$ as listed above, is shown in Fig. 23. With this result for G^* from (39) used for q/p , the true airspeed can then be obtained using the usual formula:

$$V = \sqrt{2 \left(\frac{(c_p - R_a)}{R_a} \right) (1 + G^*)^{\frac{R_a}{c_p} - 1} \left(\frac{c_p}{c_v} R_a T_a \right)} \quad (40)$$

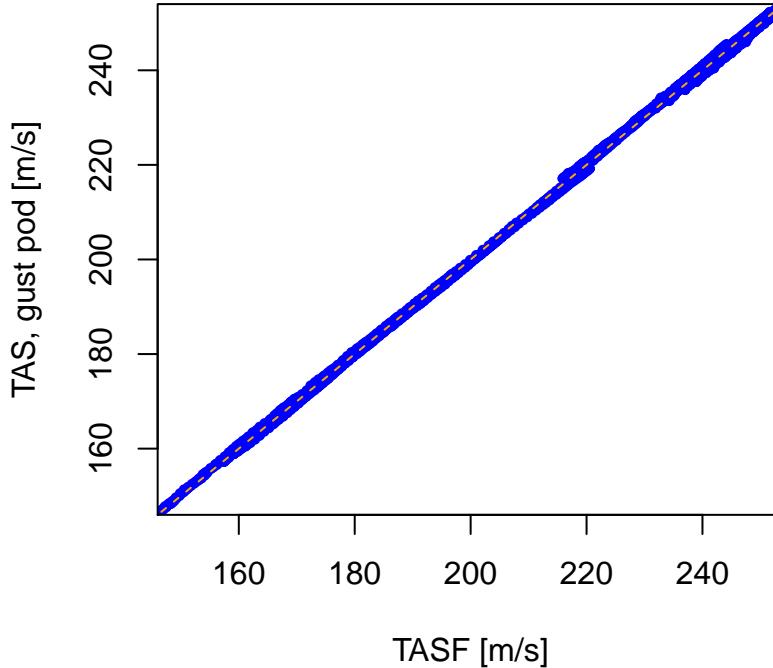


Figure 24: True airspeed obtained from the gust pod, plotted against corresponding measurements from the conventional measurement system (TASX)

where c_p , c_v , and R_a are respectively the specific heat of air at constant pressure, the specific heat of air at constant volume, and the gas constant for air. In this equation, T_a is the absolute temperature.

The resulting true airspeed is compared to the conventional measurement in Fig. 24. The fit provides a reasonable representation of airspeed, with a standard error of about 0.4 m/s. This is not much larger than the estimated uncertainty in the standard measurement of airspeed (about 0.3 m/s), so wind calculations might be based on this value from the gust pod if necessary. However, there is serious danger of over-fitting with this many coefficients and variables, even though the fit was improved significantly with each addition; indeed, the standard error in the q/p fit was reduced by 50% with the addition of the last two variables in the fit. Because of the danger that this variable might not perform as well when used outside the range of this fit (which was $\text{TASF} > 130$, $|\text{ROLL}| < 5$, and $\text{GGALT} > 5000$), it still should be better to base wind measurements from the gust pod on the conventional measurement of airspeed (TASF) combined with the relative-wind angles from the gust pod. However, if it is desirable to have a wind measurement that is completely independent of the standard radome-based system, using true airspeed measured with this fit might provide a useful alternative.

In particular, for two DEEPWAVE flights (flights 6 and 7) where the standard vertical wind

measurement was compromised by an obstruction in the pressure-sensor lines of the radome, the best vertical wind will be that calculated using (35) for the angle of attack but conventional TASF for the true airspeed, because the pressure ports used to determine TASF were not obstructed.

5.4 Summary of sensitivity coefficients

5.4.1 Angle of Attack

Radome: Eq. (34), coefficients $c_{0,1} = \{4.468^\circ, 21.481^\circ\}$.¹⁵

Gust pod: Eq. (35), coefficients $\{b_{i=0,3}\} = \{-0.903^\circ, 3.602^\circ, 4.286^\circ, 1.330^\circ\}$ ¹⁶

5.4.2 Sideslip Angle

Radome and gust pod: Eq. (38), coefficients $\{e_0, e_1\} = \{0.008^\circ, 22.302^\circ\}$ for the radome,¹⁷
 $\{-3.621^\circ, 12.184^\circ\}$ for the gust pod.

5.4.3 True Airspeed, Gust Pod

Equation (40), with (39) and coefficients $\{d_{i=0,7}\} = \{1.194, 27.2893, -29.0483, 13.6231, -0.5936, -7.5812, 0.1554, 1.2607\}$.

¹⁵See also the alternative coefficients $\{4.435^\circ, 21.166^\circ\}$ discussed on page 72.

¹⁶See also the alternative coefficients $\{-0.808^\circ, 3.248^\circ, 5.351^\circ, 1.282^\circ\}$ discussed on page 74.

¹⁷This incorporates the revision of the first coefficient that follows in Section 7.1.6.

6 Studies of the Vertical Wind

6.1 Overview

The preceding sections provided extensive information on how vertical wind is measured and how the radome-based system is calibrated. Here, three additional topics not covered well there are discussed to complement those earlier discussions. The sections here deal with the choice and quality of the variable characterizing the vertical motion of the aircraft, some issues related to the relative timing of the measurements entering the calculation of vertical wind, and a proposed method for using detection of the Schuler oscillation of the IRU to correct the measurement of pitch.

6.2 The vertical velocity of the aircraft

6.2.1 Available measurements

There are several independent measurements of the vertical motion of the aircraft:

Variable	Source
VSPD	Honeywell IRU
VSPD_A	Avionics system (Honeywell IRU)
GGVSPD	GPS receiver, possibly with OmniSTAR corrections
CVSPD_GP	CMIGITS IRU, gust pod
CVSPD_LAMS	CMIGITS IRU, LAMS pod

In standard processing, the nimbus subroutine “gust” calculates wind via the algorithm described in Sect. 1.2. The standard vertical wind calculation depends on a measurement of vertical motion of the aircraft. Past projects have used one of VSPD (from the IRU), VSPD_A, VSPD_G or GGVSPD (from GPS, the first two alternate names for the value provided by the avionics package) or, much earlier, WP3 from a baro-inertial update loop using VSPD. Baro-inertial updating is no longer used because the Honeywell IRU that now provides VSPD already incorporates such updating. For the systems based on the gust pod or LAMS, additional vertical-velocity measurements are provided by their pod-mounted IRUs, respectively CVSPD_GP and CVSPD_LAMS. Changing roll of the aircraft can cause these measurements to differ from the fuselage-based measurements.

Because the measurement VSPD provided by the Honeywell IRU is controlled against the known instability of IRU measurements in the vertical by updating to a reference value provided by pressure altitude, it has some drawbacks. Pressure altitude (cf. the RAF Technical Note on [Processing Algorithms](#), Sect. 3.3) is not a measure of altitude but rather of pressure, so the reference altitude can be biased for flight lets where altitude changes on a constant-pressure surface. Also, the characteristics of the IRU-imposed baro-inertial loop are proprietary to the manufacturer and so have unknown response characteristics and uncertainty, and the value provided by the IRU has inherent filtering. The advantage of an IRU variable over a GPS measurement

has been, until recently, better response at high frequency at the expense of absolute accuracy. GPS measurements have now improved, esp. with OmniSTAR, so it is worth considering what variable or combination of variables should be used in the calculation of vertical wind.

For horizontal wind, the standard solution in use since the early 1990s has been a complementary-filter solution (discussed in Sect. 7.2), where the difference between IRU and GPS measurements is low-pass filtered and the result is added to the IRU measurement. This preserves the high-frequency response of the IRU measurement but causes the low-frequency components to match the GPS measurements, thus providing both absolute accuracy and valid high-frequency measurements. It seems worthwhile to explore a similar approach for the vertical wind, to see if there is an advantage to a variable that is provided by an analogous complementary filter.

In the course of the study discussed here, an additional variable GGVSPDB was sometimes used. That variable is a higher-resolution version of GGVSPD, but there appeared to be no advantage to using that variable, so GGVSPD is used in the analysis presented here. It has sufficient resolution, so there appears to be no advantage to using the higher-resolution variable.

In standard processing, two vertical-wind variables are produced from the radome-based system, WI and WIC. In archived data files, these are called, respectively, “Wind Vector, Vertical Gust Component” and “GPS-Corrected Wind Vector, Vertical Gust Component”. They are calculated by adding the relative wind measured by the radome system to the vertical motion of the aircraft, the latter from either VSPD (WI) or GGVSPD (WIC). The former is directly from the Honeywell IRU; the latter is produced by the GPS receiver and may benefit from OmniSTAR accuracy (flagged by GGQUAL equal to 5).¹⁸¹⁹

6.2.2 Variance spectra for components affecting the vertical wind

For evaluation of these measurements of aircraft velocity, it is useful to compare the variance spectra to evaluate the contribution each makes to the vertical wind. For this purpose, one flight from DEEPWAVE, RF16 (4 July 2014), was used because it provided a good example of relatively intense and prolonged vertical motion, with the core 6 h period of the flight from 6:30:00 – 12:30:00 UTC having a standard deviation in vertical wind of 1.0 m/s. Much of the variation in vertical wind was from waves, so the field was not fully developed turbulence, but the signals to be resolved had enough intensity that noise floors on the measurements were not a problem.

Figure 25 shows variance spectra calculated for the entire 6-h period, for a number of measurements entering the vertical-wind calculations.²⁰ The thick blue trace shows the spectrum

¹⁸In addition, the Honeywell IRU provides a measurement of vertical acceleration. In a special calculation, this was integrated to get velocity and the result compared to VSPD. The results of the integration and the variance spectrum of that result were quite similar to VSPD, provided that a feedback loop was used to avoid exponentially growing errors from positive feedback, so there is no advantage to using that integrated acceleration in place of VSPD. (It was hoped that some of the filtering imposed on VSPD could be avoided.)

¹⁹The names are not really appropriate and may therefore be changed in the future, but they appear in many past datasets. Neither is the gust component; both are the full vertical wind including relative wind and aircraft motion. Also, WIC is not GPS-corrected, it is completely based on the GPS and does not use the IRU at all except for the attitude angles needed to determine the relative wind (the same for both).

²⁰This plot is an exception to the goals of "reproducible analysis" explained in the introduction and in Appendix

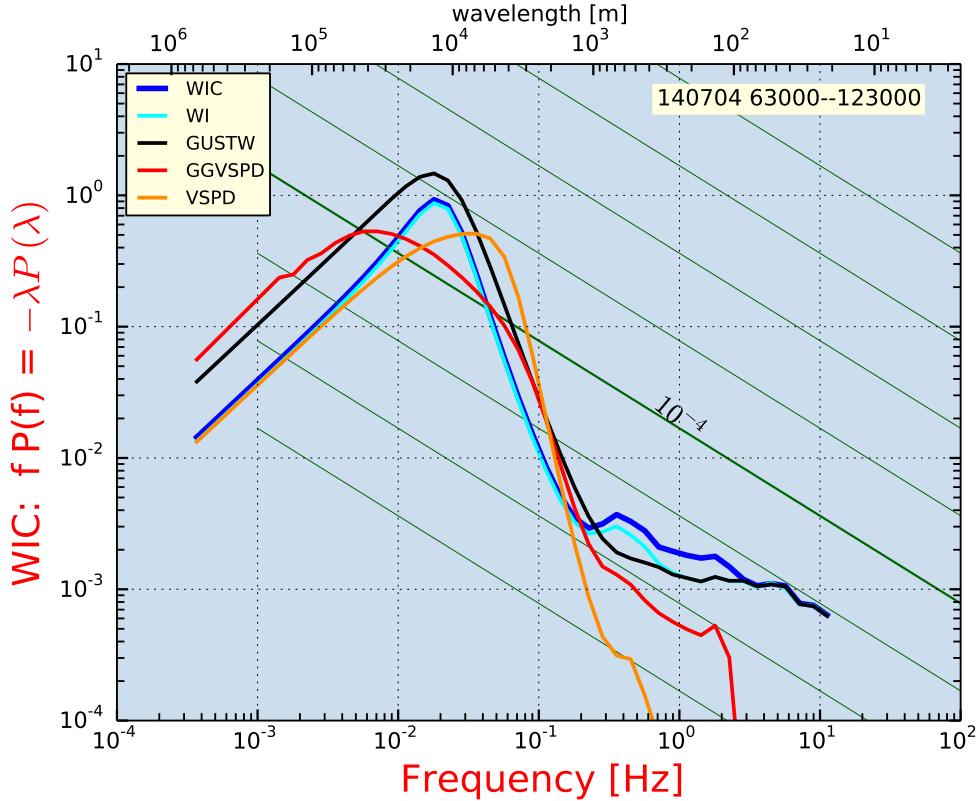


Figure 25: Variance spectra [units: m^2s^{-2} , density functions per logarithmic interval in frequency expressed in Hz] for various components entering vertical-wind calculations. Data are from DEEPWAVE flight 16, 6:30:00–12:30:00 UTC. The thick blue trace is that for the vertical wind variable WIC; others show either measurements of the aircraft vertical motion (VSPD, GGVSPD) or, as GUSTW, the relative-wind contribution to WIC. The green diagonal lines show the slope expected for an inertial subrange; the thicker green line labeled “ 10^{-4} ” corresponds to the spectrum expected for an eddy dissipation rate of $1 \times 10^{-4} \text{ m}^2\text{s}^{-3}$. Other green reference lines are displaced by an order of magnitude in eddy dissipation rate. The calculations are based on the all-poles or maximum-entropy method, with 100 poles; cf. [Press et al. \[1992\]](#). The resolution used was 0.0005, with results smoothed in 50 bins in the logarithm of frequency. Total variance is $1.36 \text{ m}^2\text{s}^{-2}$.

for the standard vertical wind measurement WIC, which has a pronounced broad peak near 10 km wavelength, a relatively small inertial subrange extending only to about 2 km, and (after smoothing in 50 logarithmic intervals across the range of the plot) uncertainty estimates that range from about 5% at the lowest frequencies to <1% at the highest frequencies.

The two measures of vertical aircraft motion, GGVSPD (from the GPS receiver) and VSPD (from the inertial reference unit), have important differences in their properties. At high frequency, VSPD (orange line) decreases rapidly with increasing frequency, so it makes negligible contribution to the vertical wind (WI) at frequencies above 1 Hz. In contrast, GGVSPD (red line) does not show a similar steep drop in variance until after about 2 Hz, and it continues to make an important contribution to WIC over the frequency range from about 0.3 to 3 Hz. The absence of spectral variance above about 0.3 Hz in VSPD appears to be the result of internal filtering of this signal in the inertial unit, and indeed some limited information on the properties of the signals does indicate filtering at about this frequency. Because the aircraft motion clearly has components in this frequency range (based on the feel of the ride in turbulence), it appears that WI should not be used for frequencies above about 0.3 Hz, because part of the high-frequency signal is missing from this variable. It may be removed from future data archives.

As discussed above, the inertial-system variable VSPD is also subject to pressure damping, and this may account for the difference between GGVSPD and VSPD at long wavelength. The aircraft normally flies with reference to pressure altitude, so in a region of varying altitude at a given pressure there are fluctuations in the aircraft geometric altitude (red line) not reflected in the pressure altitude (orange line). Both contribute in ways that result in essentially the same spectral variance for frequencies below 0.3 Hz, as shown by the near coincidence of the blue and cyan lines.

For these reasons, WIC is the preferable variable to use for vertical wind. This also answers the question posed earlier regarding the possibility of using a combination of measurements from inertial and GPS systems to obtain better frequency response. The IRU measurements have poorer frequency response and lead to apparent biases for frequencies around 1 Hz, so this is not a useful combination. Instead, it appears best to use the GPS-provided measurement of vertical aircraft speed directly without further modification.

There is still some reason for concern about the spectral response of GGVSPD at frequencies around 0.5–1 Hz, because there is no evidence that the GPS is providing valid response at this high rate. The GPS unit provided measurements at 5 Hz and these measurements were interpolated and filtered to higher frequency, so the cut-off apparent in Fig. 25 is a result of that sampling and could not provide useful measurements above the Nyquist frequency of 2.5 Hz. This topic will be reconsidered in Appendix B, where variance spectra in more turbulent regions are examined.

The black trace (labeled GUSTW) is the vertical component of the relative wind, calculated as in Sect. 1.2.1. The sum of GUSTW and GGVSPD determines WIC, and the alternate measure of vertical wind WI is the sum of GUSTW and VSPD from the inertial reference unit. GUSTW has higher spectral variance than WIC for frequencies below 0.3 Hz, so at these frequencies

C. The calculations used an external spectral-analysis program, not R code, and the specific data are not part of the data archive for the document. See Appendix B for more information on the method used to calculate these variance spectra.

the aircraft motion tends to counter the relative wind and reduce the measured variance. This results in close correspondence of the gust-component GUSTW and the vertical aircraft motion GGVSPD from about 0.07–0.2 Hz, resulting in a vertical wind WIC that is much smaller than either of these contributing components. On the other hand, for frequencies around about 0.5 Hz the spectral variance in the vertical wind exceeds that in the relative wind, showing that the aircraft responds to the vertical gusts only partially and about in phase at this frequency.

The high-frequency variance spectrum has a slope differing a small amount from the expected -5/3 spectrum, and that is cause for some concern, but the turbulence in this region was not very well developed or consistent. Very good agreement with expectations has been seen in cases of boundary-layer measurements where well-developed turbulence at small scales is expected, but in those cases the turbulence at small scales was also more intense. This and the absence of a valid measurement of aircraft response above 2.5 Hz are reasons to continue to be suspicious of the response around 0.5–1 Hz, but additional variance spectra presented in Appendix B appear to conform to the expected -5/3 slope better than this example does.

The key conclusions reached from this study of the vertical aircraft velocity are that WI, based on VSPD, is an inferior measurement that should not be used and that GGVSPD is the best available measure of aircraft vertical motion and should be the basis for wind calculations.

An example where there was very low turbulence is shown in Fig. 26. The intensity of vertical-wind fluctuations was quite small for this flight segment, with a standard deviation in vertical wind of only about 0.2 m/s. Some features of this plot are explained below:

1. The measured vertical wind at high frequency (>1 Hz) is a white-noise spectrum with intensity that can be matched by generating a random-noise signal with peak amplitude of 0.15–0.2 m/s, which would lead to a random error of about $(0.15\text{--}0.2)/\sqrt{12} \simeq 0.05$ m/s. The noise is dominated by the relative-wind contribution; there is essentially no contribution at these frequencies from the motion of the aircraft. The noise arises almost entirely from the angle-of-attack contribution to the relative wind, and specifically from the measurements “ADIFR” and “QCF”, which exhibit noise spectra for frequencies above 1 Hz. In ADIFR, the noise is at a level that would arise from a white-noise signal with peak amplitude of 0.4 hPa or a random error of about 0.1 hPa.²¹ Similar noise is present in both QCR and QCF, but QCF has additional problems at high frequency as discussed in Appendix B; this may arise from the long pressure lines used with the transducer for QCF. The specifications for the pressure transducer that measures ADIFR assert a standard uncertainty of about 0.07 hPa. As sampled and digitized by the data system the resolution is about 0.002 hPa (± 70 hPa for 2^{16} range digital encoding), which is comparable to the sensor resolution, so noise arising from digitation would be much less intense than the measured noise distribution. Thus the white-noise spectrum is consistent with a random error of about the specified uncertainty for the sensor, but this is not the result of digital truncation.
2. The relative-wind contribution (GUSTW) and the aircraft-motion contribution (GGVSPD) both have peaks at about 0.05 Hz, with canceling contributions so that no peak occurs at

²¹This was verified by simulation.

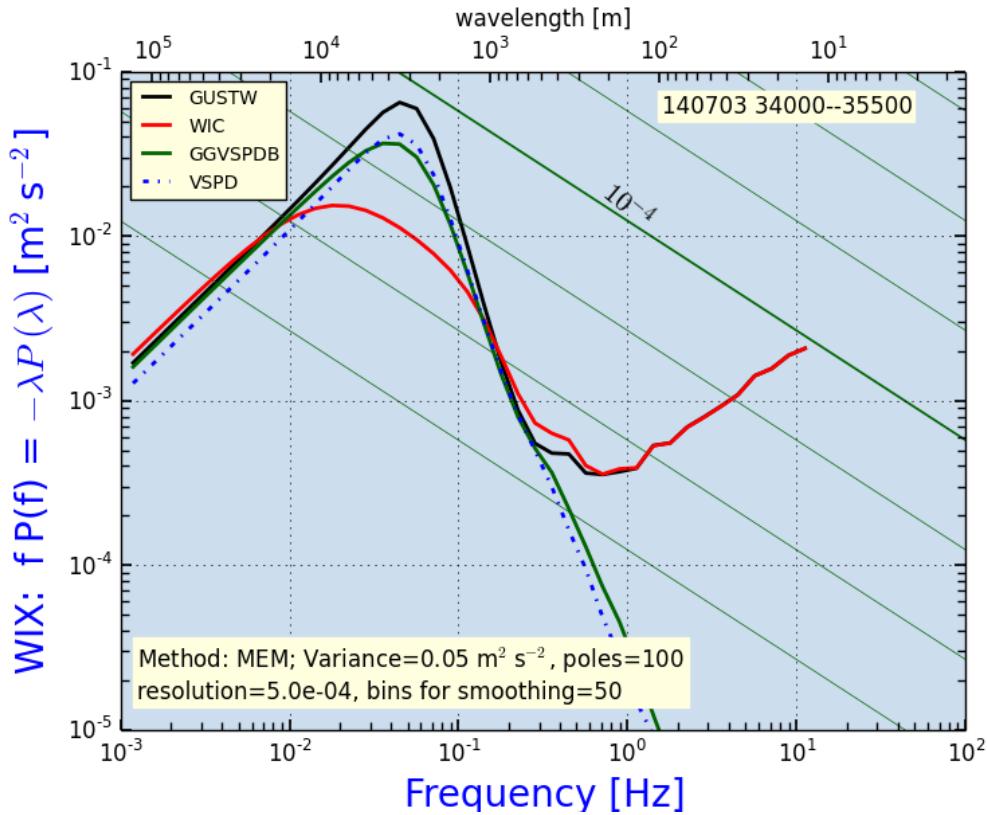


Figure 26: Variance spectra for the vertical wind (here, WIX) and for the contributions to it made by the relative wind (GUSTW) and the aircraft motion (GGVSPDB, essentially the same as GGVSPD). Also shown for comparison is the spectrum for the IRU-provided aircraft motion (VSPD, dashed line). Data from DEEPWAVE flight 15, 3:40:00–3:55:00.

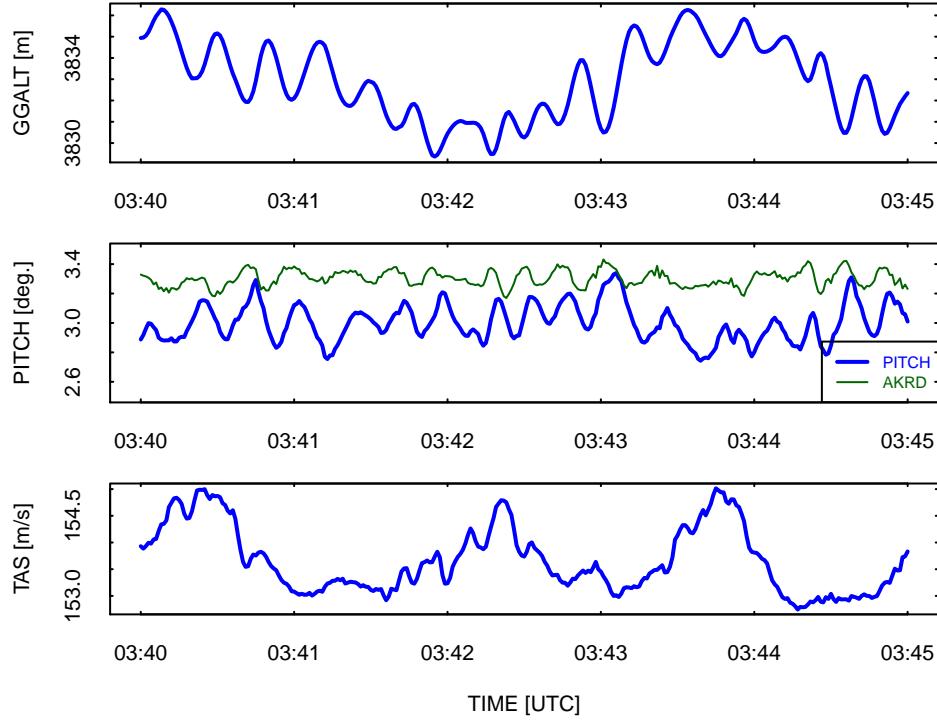


Figure 27: Coupled variations in pitch, altitude, and airspeed for a segment of flight in smooth air from DEEPWAVE flight 15, 3:40:00–3:45:00 UTC, an illustration of normal variations in flight conditions. GGALT = aircraft altitude; PITCH = pitch angle; AKRD = angle of attack; TAS = true airspeed.

that frequency in the resulting vertical wind. These peaks arise from a basic dynamical flight mode of an aircraft called a phugoid, in which altitude, airspeed, and pitch all undergo oscillations that are tracked by the flight management system (FMS) of the aircraft but are not completely removed by it. The angle of attack also oscillates but with smaller amplitude. A regular oscillation with this period is quite evident in Fig. 27 and is especially visible in calm flight conditions where it also manifests itself in small oscillating adjustments by the FMS to the throttle positions. A properly operating wind sensing system should remove this motion from the measured vertical wind, and indeed that is the case for the example shown.

```
## [1] "output rate for this data.frame is 1"
```

3. The disagreement between VSPD and GGVSPD even at low frequency is another indication that the IRU-provided value (VSPD) should not be used, because the GPS-measured value at low frequency is surely a better measurement than is possible from the IRU. (OmniSTAR corrections were present throughout this flight.)
4. The difference between WIC and GUSTW near 0.3 Hz, seen also as a possible effect in Fig. 25, indicates that both the relative wind and the aircraft motion are making contri-

butions that add at these frequencies. The spectrum of vertical wind here may be suspect because it is not clear if the amplitude of this contribution from the GPS measurements should be trusted at these frequencies.

6.3 Timing of measurements

The different measurements entering the calculation of vertical wind should be sampled at the same time. That is particularly difficult in the case of samples from the inertial reference unit and GPS because they produce sample streams according to their own timing and not in response to requests from the aircraft data system. The variables involved in calculating the vertical wind are:

- Angle of attack, from transducers attached to radome ports via lines that can introduce small lags. No lag is currently used in processing. The specifications for the transducer used indicate that it will introduce a lag of about 21 ms and has a response time of 50 ms.
- Pitch measured by the IRU is transferred to the aircraft data system after some delay that must be removed in processing. Standard processing has used a delay of -60 ms; i.e., before use the measurement is moved in time so as to apply to a time *earlier* than when it was received.
- Other attitude angles (heading and roll), which enter in minor ways if the aircraft is not flying a straight-and-level course. The timing of these can probably be neglected for calculations of vertical wind, but the standard variables are also given a time lag of -60 ms.
- True airspeed, measured using the pitot-tube measurement of dynamic pressure, used also with a measurement of temperature. While conventionally no lag is assumed for dynamic pressure, the sensor specifications indicate that a 21 ms lag should be present and the pressure lines probably introduce an additional lag. The reference total temperature for DEEPWAVE is RTRL, for which no time lag is assumed. Although this sensor has slow (ca. 1 s) response, this does not have an important effect on vertical wind because the airspeed enters as a multiplicative factor so effects of even a few percent have no significant effect on the vertical wind.
- The vertical speed of the aircraft, taken for the preferred vertical wind variable from GGVSPD, the variable produced by the Novatel GPS receiver employing OmniSTAR corrections when possible. This variable is only sampled at a rate of 5 Hz, and no time lag is used in processing. The aircraft shows little motion at high frequency, as discussed in Sect. 6.2.2, so small adjustments in GGVSPD are not needed.

It is useful to try to determine appropriate lags from the data and from appropriate maneuvers. For example, in pitch maneuvers (in which the pitch is alternately increased and decreased with typically a 10-s period) if the timing of measurements of pitch and angle of attack are not matched or if the measurement of vertical speed of the aircraft is not timed correctly there will

be a residual measured vertical wind, so these maneuvers are particularly stringent tests of the relative timing of the signals.

An approximate formula for the vertical wind w (cf. (28)) is

$$w = V \sin(\alpha - \theta) + w_p \quad (41)$$

where V is true airspeed, α is angle of attack, θ is pitch and w_p is the vertical velocity of the aircraft. This equation can be used to adjust relative timing among the signals to minimize the variance in vertical wind during the pitch maneuvers. A good example is that from DEEPWAVE flight 15, 3:15–3:18 UTC. As initially processed using standard processing at the time of DEEPWAVE, the standard deviation in vertical wind through the pitch maneuvers was 0.4 m/s while the variance in vertical motion of the aircraft was 5.6 m/s. A long-standing criterion used by NCAR/RAF to check for acceptable pitch maneuvers is for less than 10% of the imposed velocity to enter the vertical wind, so by this criterion the test was successful. However, a plot of the vertical wind shows a clear match to the imposed velocity, so it is worthwhile to see if better results are possible.²²

The relative timing of the measurements entering Eq. 41 can be adjusted to determine if the residual vertical wind can be reduced. For the purpose of this study, to avoid the phase lag produced by filters at the high-frequency limit, Savitzky-Golay filtering was used for GGVSPD (measuring w_p) with 4th-order polynomials applied over 25-measurement intervals to smooth the original measurements, which are only available at a frequency of 5 Hz. The sensors producing V and α are located close together and are processed in the standard manner by the data acquisition system, so these can be assumed to determine the reference time except for possible delays introduced by pressure lines. However, the other variables θ and w_p are determined by independent systems and may have timing offsets from the standard measurements, so these are the variables whose lags are explored here. For the purpose of this study, to avoid the phase lag produced by filters at the high-frequency limit, Savitzky-Golay filtering was used for GGVSPD with 4th-order polynomials applied over 25-measurement intervals to smooth the original measurements.

The timing of the measurement of pitch (θ) relative to that of angle of attack (α) is the most important. The lowest standard deviation in vertical wind resulted from shifting the measured pitch (θ) forward in time by 60 ms; i.e., removing the -60 ms delay previously imposed. With this shift in pitch, the best shift in GGVSPD (w_p) was -40 ms, but the standard deviation was almost as small for no shift (0.23 m/s with no shift vs 0.22 for the shifted values). Therefore, because negative shifts are not physical and there is little high-frequency variation in w_p , it appears best not to impose any no time shifts on these measurements. The IRU specifications state that a delay of "up to" 60 ms should be expected, but this result suggests that either this is an upper limit not characteristic of measurements at the time being analyzed here or else there is a compensating delay in the measurements of angle-of-attack. The latter seems plausible because the known transducer characteristics lead to a delay of 21 ms, a response time of 50 ms, and some likely delay arising from the length of the pressure lines.

²²Speed runs have been used to determine the sensitivity coefficients for determining α from the measured pressure differences on the radome, so the sensitivity coefficients should not be adjusted on the basis of the pitch maneuvers.

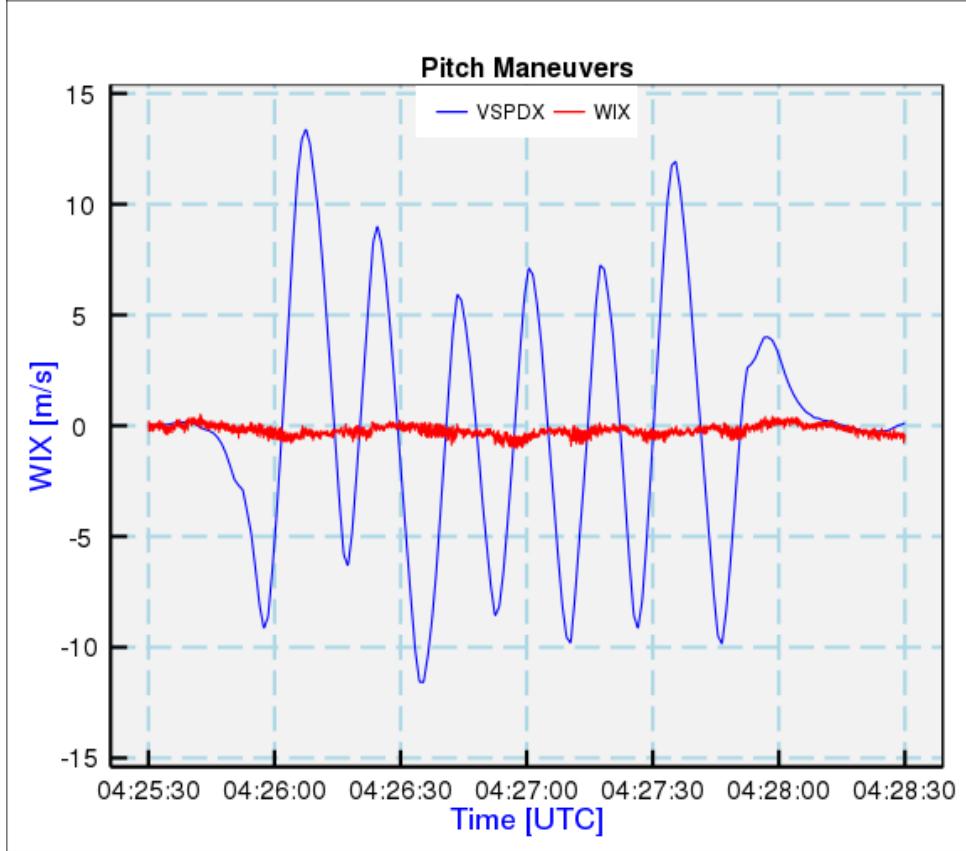


Figure 28: Vertical wind measured during pitch maneuvers. The variable VSPDX (which is GGVSPD interpolated, filtered, and shifted) shows the vertical motion imposed on the aircraft, and WIX shows the resulting measurement of vertical wind.

With the assumed time lag in PITCH removed, the resulting measurement of vertical wind during the pitch maneuver is shown in Fig. 28. The standard deviation in measured wind through the pitch maneuvers is only 4% of the imposed vertical motion of the aircraft, vs. about 7% for the unshifted measurements. Furthermore, there is little signal in the resulting measurements of vertical wind that corresponds to the imposed vertical motion or variations in pitch, and the measured standard deviation in vertical wind is about the same as that for measurements just before and just after the pitch maneuvers. It thus appears that the measuring system is able to remove the effects of the pitch maneuvers with essentially undetectable residual. The suggested conclusion is that the assumed delay in pitch should be reset to zero for processing and no delay should be used with GGVSPD.

6.4 Correcting pitch for the Schuler oscillation

6.4.1 Basis for the correction

The primary weakness in the measurement of vertical wind normally is the error in the measurement of pitch, which is provided by inertial reference units and therefore has the uncertainty

and fluctuations associated with those systems. An inertial system aligns during initialization to detect the local vertical direction and then calculates the new vertical direction as the aircraft moves and accelerates. Any misalignment present at initialization persists but also can oscillate and cause the roll and pitch errors to mix as the aircraft changes flight direction. The standard uncertainty associated with this measurement is 0.05° for flight duration of a few hours (Table 1).

The work of Schuler ([Schuler \[1923\]](#)) demonstrated that coupling among some of these error sources leads to limits on the growth of errors and to simultaneous oscillations in some of the measurement errors. In particular, an error in pitch leads to an error in horizontal acceleration because a component of gravity is resolved as a horizontal acceleration. Integration of that error in horizontal acceleration leads to a position error that grows to compensate for the original error in pitch. However, when the error in pitch is reduced to zero, errors in position and velocity have been accumulated and those lead to growth of the error in pitch in the direction opposite to the original error. The result is a Schuler oscillation having a period of $T_{Sch} = (R_e/g)^{0.5}/(2\pi) \approx 5064\text{ s}$ or 84.4 min, where R_e is the radius of the Earth and g is the acceleration of gravity.

The existence of this coupling allows estimation of the pitch error if the error in horizontal acceleration can be measured. Because high-quality measurements of velocity are available from GPS, those measurements can be compared to the uncorrected measurements from the inertial systems to detect the error in velocity and, from its derivative, the error in acceleration. The specified uncertainty for the IRU, 0.05° , would contribute an uncertainty in measured vertical wind of about 0.2 m/s and so would be the largest contributor to uncertainty in vertical wind. Therefore, improving the measurement of pitch can lead to important reduction in the uncertainty associated with measurement of vertical wind. Furthermore, because the sensitivity coefficients for angle of attack are determined using fits to the measurements of pitch, improved measurements of pitch can reduce the uncertainty in the measurement of angle of attack as well.

For these reasons, this section documents a procedure that can be used to correct for a major part of the error in the primary measurement of pitch by correcting for the Schuler oscillation as determined from errors in the ground-speed components of the aircraft that are measured by the IRU.²³

6.4.2 Simple illustration of Schuler oscillation

The following is standard material, presented here to make the discussion of the Schuler oscillation self-contained and possibly for tutorial purposes.

Consider first a case of straight flight to the north. If, at some starting point at rest, there is a pitch error $\delta\theta$, that will cause gravity to be resolved into a north-south component, as shown in this diagram where a pitch error produces an erroneous northward acceleration of $g \sin(\delta\theta) \simeq g\delta\theta$, as shown in Fig. 29. The result of this erroneous acceleration is growth of a false northward velocity which, integrated, gives a northward error in position. However, the error then causes the integrated position to become too far north, where the calculated direction toward the center

²³This correction procedure applies only to the Honeywell IRU used with the radome-based wind system because the IRUs used with the gust pod or the LAMS already incorporate Kalman-filter corrections that apply similar adjustments to the measurements, and the uncorrected measurements are not available.

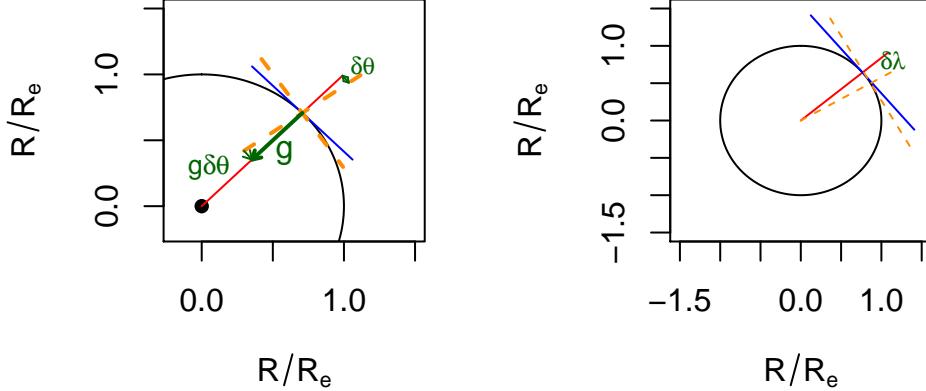


Figure 29: Illustration of how a pitch error leads to a varying false horizontal acceleration: (left) A pitch error of magnitude $\delta\theta$ leads to a false measurement of horizontal acceleration of magnitude $g\delta\theta$. The blue line represents the true level plane tangent to the surface of the Earth and g the magnitude and direction of the gravitational acceleration. (right) An error in the measurement of position, denoted here as an error in latitude $\delta\lambda$, leads to an error in the direction of the vertical axis and so to a changed error in horizontal acceleration.

of the earth becomes biased in the opposite direction, and at some point the calculated offset from being too far north equals the pitch bias, as shown on the right side of Fig. 29.

At the point where the error in latitude cancels the error in pitch, there is an accumulated error in velocity, so the integrated solution for position overshoots the equilibrium position by an amount such that the error in pitch becomes the negative of the original error, at which point the integrated velocity is zero but the new error in pitch now causes a reversal of the position error. The result is that the errors in pitch, velocity and position all oscillate with a period equal to the Schuler period T_{Sch} .

These equations predicting how this oscillation will occur:

$$v_n = \frac{dx_n}{dt} = R_e \frac{d\lambda}{dt} \quad (42)$$

$$\frac{d^2x_n}{dt^2} = a_n \quad (43)$$

where x_n and v_n are the measured north coordinate of position and northward component of aircraft motion, R_e the radius of the Earth, and a_n is the measured northward acceleration. However, if $a_n = a_n^* + \delta a_n$ where a_n^* is the true northward acceleration of the aircraft and δa_n is the erroneous acceleration that results from pitch and displacement errors, then

$$\delta a_n = g(\delta\lambda - \delta\theta) \quad (44)$$

with λ the latitude, $\delta\lambda = \delta x_n/R_e$ the error in latitude, and $\delta\theta$ the error in pitch. Then

$$\frac{d(\delta v_n)}{dt} = -g\delta\theta \quad (45)$$

$$\frac{d(\delta\theta)}{dt} = \frac{\delta v_n}{R_e} = -\frac{1}{g} \frac{d^2(\delta v_n)}{dt^2} \quad (46)$$

which has the solution:

$$\delta v_n = A_n \cos(\Omega_{Sch}t + \zeta_n) \quad (47)$$

where A_n is the amplitude of the oscillation and $\Omega_{Sch} = \sqrt{g/R_e} \simeq 0.00124$ is the Schuler angular velocity. The error in the north component of the velocity therefore oscillates with the Schuler period and a phase ζ_n . Integrating in time gives

$$\delta x_n = \int \delta v_n dt = \frac{V_n}{\Omega_{Sch}} \sin(\Omega_{Sch}t + \zeta_n) \quad (48)$$

The key to developing a correction to the pitch angle is that both δx_n and δv_n are observable because reference measurements are available from GPS. For example, if LAT and GGLAT are measured variables representing latitude respectively from the IRU and GPS,

$$\begin{aligned} \delta x_n &= R_e(\text{LAT} - \text{GGLAT}) \\ \delta v_n &= \text{VNS} - \text{GGVNS} \end{aligned} \quad (49)$$

These observations can determine A_n and ζ_n in (47). From these, the error in pitch can be found from (45):

$$\delta\theta = -\frac{1}{g} \frac{d(A_n \cos(\Omega_{Sch}t + \zeta_n))}{dt} = \frac{A_n \Omega_{Sch}}{g} \sin(\Omega_{Sch}t + \zeta_n) \quad (50)$$

Analogous equations lead to a similar coupling between the roll angle and the east component of the ground velocity:

$$\delta\phi = \frac{A_e \Omega_{Sch}}{g} \sin(\Omega_{Sch}t + \zeta_e) \quad (51)$$

where ϕ is the roll angle and A_e and ζ_e are determined from fits to the observed error $\delta v_e = \text{VEW} - \text{GGVEW}$.

Thus the observed errors in the components of the aircraft ground speed can be used to find corrections to be applied to the measurements of pitch and roll. Note, however, that "pitch" and "roll" as used here apply to aircraft attitude angles only for northward level flight. Otherwise, transformations are needed to obtain the attitude angles in the reference frame of the aircraft. The appropriate transformation is developed in Sect. 6.4.4.

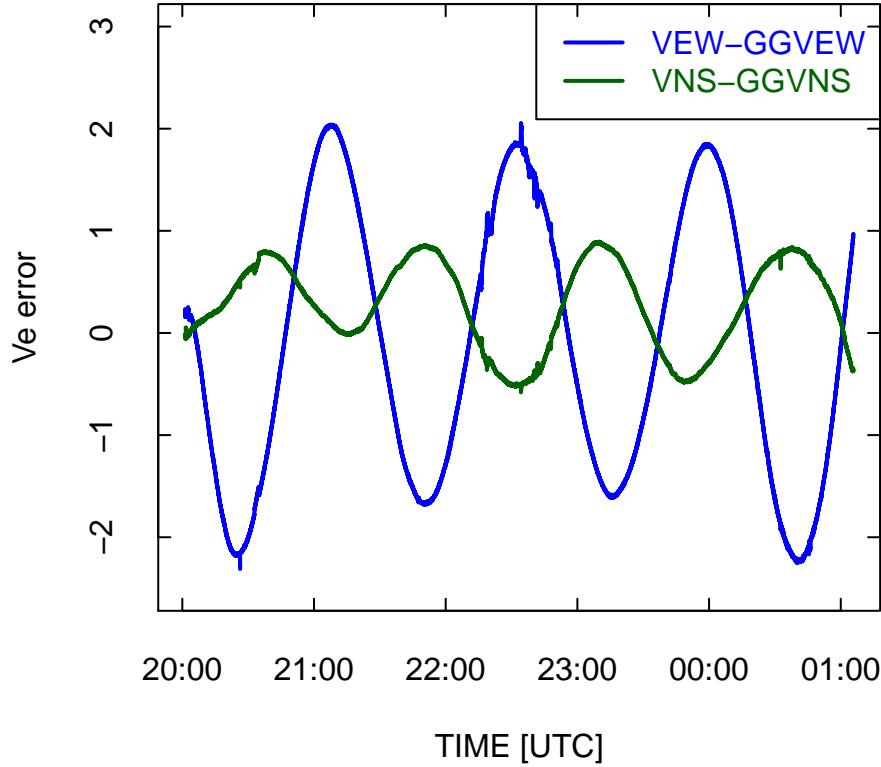


Figure 30: Errors in the NS and EW components of ground speed as determined by comparison to GPS, for DEEPWAVE flight ff02, a ferry flight starting on 1 June 2014 and traveling from Hawaii to Pago-Pago.

6.4.3 Illustrative example

An example from the second ferry flight of DEEPWAVE is shown in Fig. 30. The heading for most of this flight is close to southbound and steady, so to a reasonable approximation the errors in pitch and heading will be given by the respective derivatives in the error terms δv_n and δv_e . The errors can be determined directly from the time-derivatives of the error terms in (45) and the analogous equation for roll, restated as:

$$\delta\theta = -\frac{1}{g} \frac{d(\delta v_n)}{dt} \quad (52)$$

$$\delta\phi = -\frac{1}{g} \frac{d(\delta v_e)}{dt} \quad (53)$$

Therefore, rather than fitting variations like that shown in Fig. 30 to sine and cosine functions,

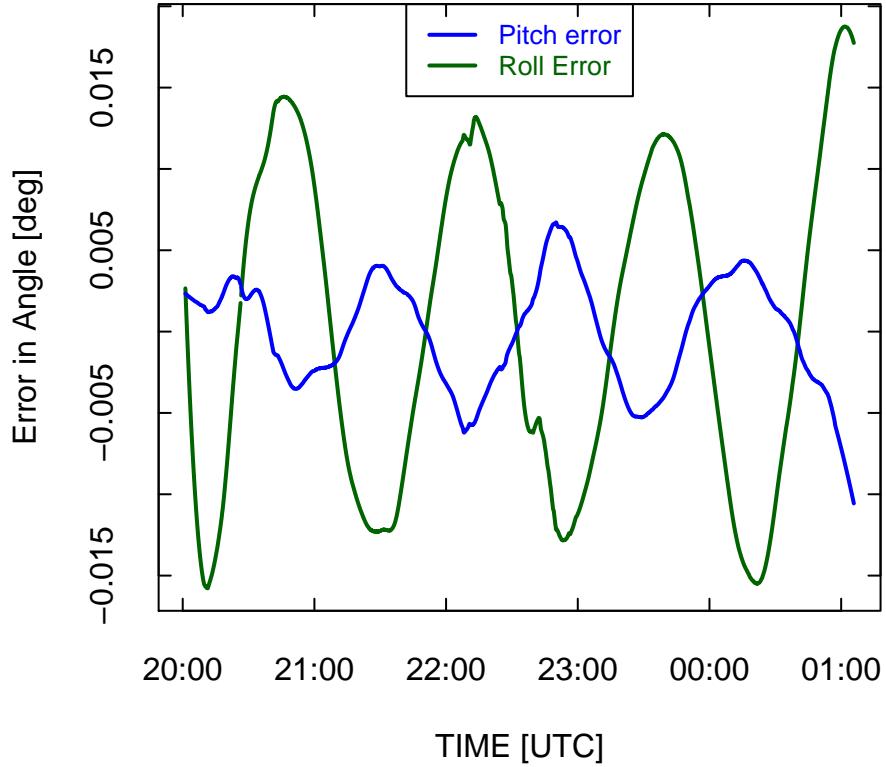


Figure 31: Deduced error in pitch and roll angles for DEEPWAVE ferry flight ff02.

it is possible to obtain an estimate of the time derivatives of the velocity-error terms over some shorter interval and then use that derivative in (52) or (53) to find the errors in pitch and roll. This gives errors in the local reference frame with axes eastward, northward, and upward, here called the l -frame, and these errors will then must be transformed to the aircraft reference frame to get errors in the measured quantities. The choice made here is to estimate the derivatives using fitted Savitzky-Golay polynomials, as shown in Fig. 31. Savitzky-Golay polynomials were chosen because they are computationally efficient, don't introduce a time shift, and can provide derivatives directly. A rather long averaging period of 1009 s, or about 1/5 of a Schuler oscillation, was used to reduce noise in the result, and interpolation filled some gaps in the measurements.

The result is that the estimated pitch error is less than about 0.01° in magnitude for most of this flight, except for the final descent, and the roll error is limited to less than about 0.015° for the same period. This is evidence for low uncertainty in the pitch measurement for this flight, well below the specification of 0.05° . Section 6.4.5 contains further discussion of the errors from the research flights, where the estimated errors sometimes can be larger.

6.4.4 Transformation of attitude angles

In a reference frame called the *l*-frame or ENU frame, where the coordinate axes are local-east, local-north, and upward, the preceding subsection showed that the pitch and roll errors are related, via (52) and (53), to the time-derivatives of the errors in horizontal velocity. Pitch and roll errors as used in these equations will be the respective errors in platform alignment²⁴ in the north-south and east-west directions, so these angles must be transformed to account for the orientation of the aircraft when it is not flying straight-and-level to the north. Coordinates in the body or *b*-frame of the aircraft are obtained from those in the ENU or *l*-frame by applying three rotations to account for the heading, pitch, and roll of the *b*-frame. This transformation leads to pitch errors in the body frame of the aircraft (where pitch and roll are measured and where the pitch measurement affects the calculated vertical wind) that are mixtures of pitch and roll errors in the *l*-frame, with the mixture dependent primarily on the heading. A positive pitch error for northbound level flight will be a negative pitch error for southbound level flight, and for eastbound flight an *l*-frame roll error becomes a *b*-frame pitch error while an *l*-frame pitch error become a negative *b*-frame roll error.

Consider a unit vector $\mathbf{b}^{(l)}$ representing the orientation errors in pitch and roll in the *l*-frame, with components $\{\sin \delta\phi, \sin \delta\theta, \sqrt{1 - \sin^2 \delta\phi - \sin^2 \delta\theta}\}$ or, because the errors are always small, approximately $\{\delta\phi, \delta\theta, 1\}$. The three-angle transformation of this vector from the *l*-frame to the *b*-frame is then represented by the following matrix, with $\{\phi, \theta, \psi\}$ denoting {roll, pitch, heading}:

$$R_l^b = \begin{bmatrix} \cos \psi \cos \phi + \sin \psi \sin \phi \sin \theta & -\sin \psi \cos \phi + \cos \psi \sin \phi \sin \theta & -\cos \theta \sin \phi \\ \sin \psi \cos \theta & \cos \psi \cos \theta & \sin \theta \\ \cos \psi \sin \phi - \sin \psi \sin \theta \cos \phi & -\sin \psi \sin \phi - \cos \psi \sin \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (54)$$

If the roll and pitch angles are small,

$$\mathbf{b}^{(b)} = R_l^b \mathbf{b}^{(l)} \approx \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta\phi \\ \delta\theta \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \psi \delta\phi - \sin \psi \delta\theta \\ \sin \psi \delta\phi + \cos \psi \delta\theta \\ 1 \end{bmatrix} \quad (55)$$

which leads to $\delta\theta^{(b)}$ and $\delta\phi^{(b)}$, the pitch and roll errors in the *b*-frame:

$$\begin{aligned} \delta\theta^{(b)} &\simeq \sin \psi \delta\phi + \cos \psi \delta\theta \\ \delta\phi^{(b)} &\approx \cos \psi \delta\phi - \sin \psi \delta\theta \end{aligned} \quad (56)$$

In turns, the roll angle is no longer negligible, so in general the full transformation matrix (54) should be used. Because this matrix imposes rotations by the measured pitch and roll angles,

²⁴The inertial system used is a strap-down system, so there is no actual motion of the “platform”. Instead, from measured rotations and accelerations, the system calculates the expected orientation if there were a true stabilized platform. The errors referenced here are those relative to that calculated platform orientation.

the unit vector representing the errors in pitch and roll after transformation will have components from which the true pitch and roll can be determined, so subtracting the corrected pitch from the measured pitch gives the error in measured pitch:

$$\delta\theta^{(b)} = \theta - \arctan \frac{b_2^{(b)}}{b_3^{(b)}} \quad (57)$$

This pitch error should then be subtracted from the measured pitch to obtain a corrected value of the pitch for use in the calculation of vertical wind.

6.4.5 Application to research flights

The research flights have frequent changes in heading, with mixing of the roll and pitch errors but also accelerations that affect those errors and introduction of new errors from heading errors. The corrections to pitch therefore appear much less systematic than was the case for the ferry flight. An example with larger than those in most DEEPWAVE flights, DEEPWAVE flight 1, is presented here. Figure 32 shows the measured errors in ground-speed components, and Fig. 33 shows the deduced pitch and roll errors. There are instances where the pitch error abruptly reverses sign; those are cases where the flight direction changes by about 180 deg. During turns, the full transformation leads to a result significantly different from the small-angle-approximation result, as shown by the orange line in Fig. 33, but when not turning the full-transformation results replicate the small-angle-approximation results (blue line), as indicated by the orange dashed line overlapping the blue line. The proposed solution is to use the full transformation to find a corrected pitch variable to be used for calculation of the vertical wind. In straight-and-level flight, the needed corrections for this flight are about $\pm 0.03^\circ$ at some times, and this error can lead (for true airspeed of 220 m/s) to an error in vertical wind of more than ± 0.1 m/s. Correction for this error thus can lead to a useful reduction in the uncertainty associated with the measured vertical wind.

6.4.6 Tests of the correction

Two tests were used to test if these pitch corrections made any significant difference in the measurements of vertical wind. First, wind measurements made before and after level course reversal were compared to see if correcting the pitch reduces the difference in measurements on the two legs before and after turns. Reduction would be expected because if there is a pitch error it would reverse sign between the two legs, increasing their absolute difference. Second, flight-average and project-average vertical wind measurements were compiled without and with the pitch correction.

The following is a tabulation of five instances where the flight track reversed course and remained at the same altitude. A number of other candidates were excluded because conditions were too variable along the legs to produce a small-uncertainty estimate of the vertical wind. In each case, flight periods of about 5 min (sometimes adjusted in times of strong wind to give similar-length segments flown upwind and downwind) are listed before and after the turn, but

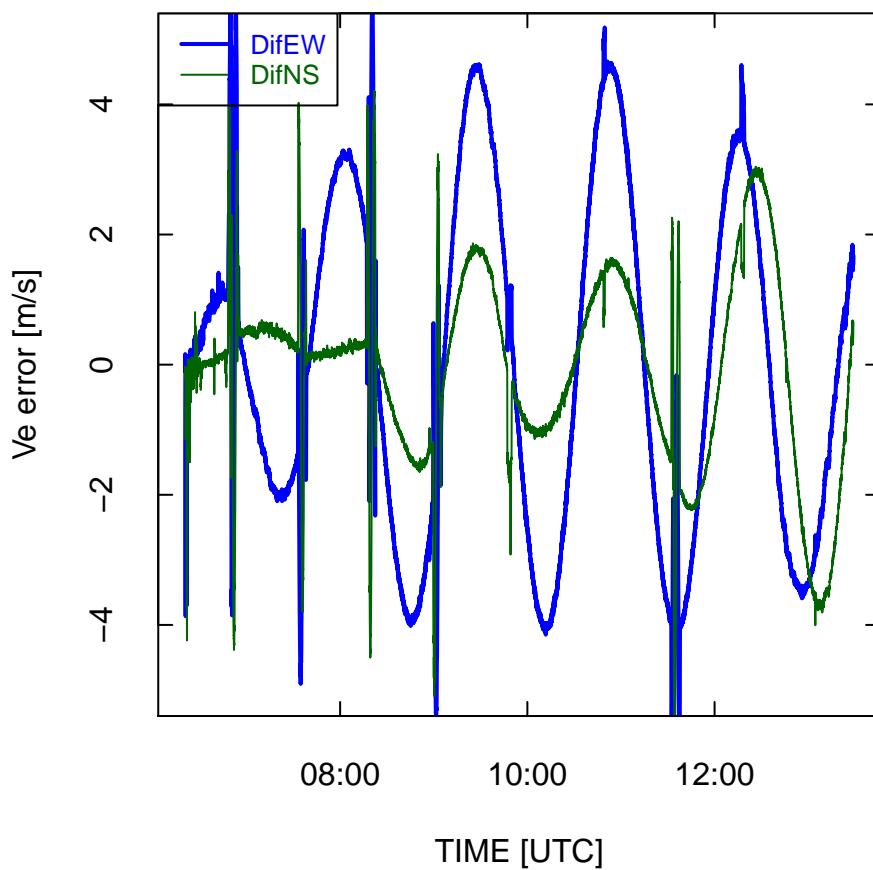


Figure 32: Measured errors in ground-speed components for DEEPWAVE flight 1. VEW and VNS are the east and north components of the ground speed measured by the inertial system, and GGVEW and GGVNS are the corresponding components measured independently by the GPS system.

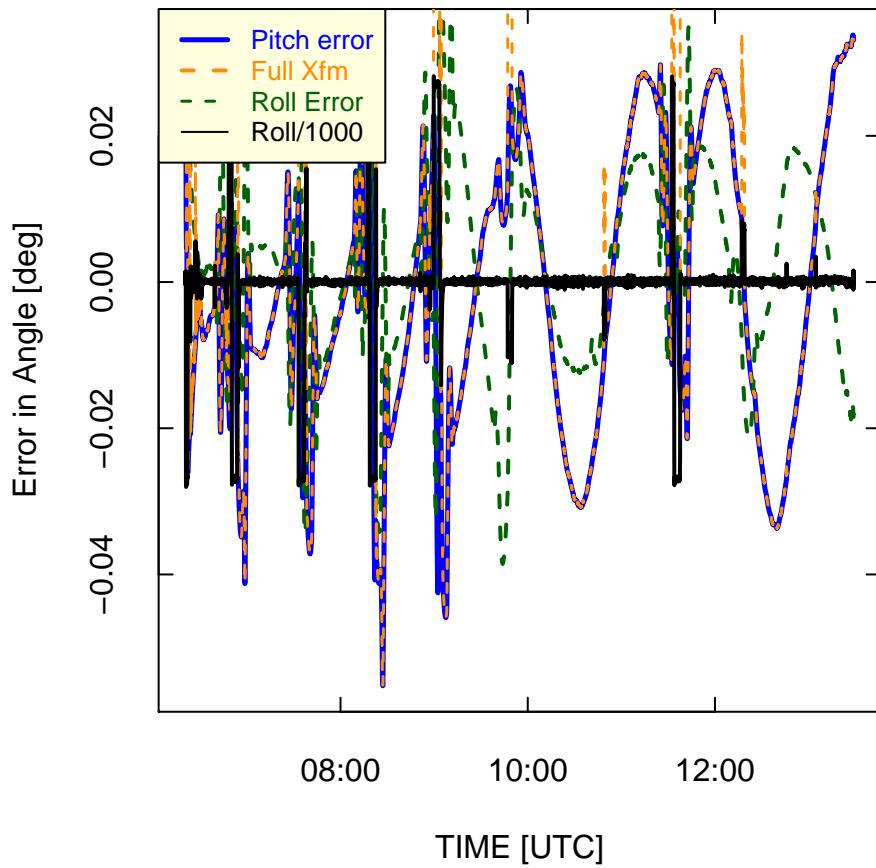


Figure 33: Errors in pitch and roll determined from the measured errors in ground-speed components, after transformation to the reference frame that is the body frame of the aircraft. The orange line labeled 'Full Xfm' uses (55) and (57), while the blue line uses the approximate result (56). The limits ± 0.03 correspond to roll angle of $\pm 30^\circ$ after division by 1000, so the regions with vertical black lines are ones with significant roll.

excluding the turn, to represent approximately overlapping flight segments where it would be expected that the vertical wind would be the same.

Flight	Times before turn	Times after turn
1	81000–81600	82400–83000
2	122500–123000	123700–124300
19	83930–84430	85130–85630
21	85100–85500	90300–90730
21	95630–100130	100800–101300

The difference between average vertical wind measurements for each pair of legs was calculated before and after applying the pitch-correction algorithm developed in this subsection. The results are shown in Fig. 34. The measurements were in good agreement without any pitch correction, with an average absolute value of the difference between opposing legs of 0.09 m s^{-1} . The pitch correction kept the averages quite small and improved the agreement, reducing the mean value of the difference to 0.03 m s^{-1} . More significant than this reduction is that the values were this small even before correction. This is a good indicator of the low uncertainty of the pitch measurement even without correction. A course-reversal difference of 0.1 m s^{-1} would result from a pitch error of less than 0.015° , so this suggests that the inertial system is performing significantly better than its specified uncertainty (0.05°). After correction, the mean difference suggests an error of less than 0.005° for the corrected values.

Table 9 lists the mean vertical wind with and without pitch correction for each of the DEEP-WAVE flights. For each flight, all measurements above 25,000 ft with true airspeed above 130 m/s and roll between -5 and 5° were included to emphasize normal research conditions. Any missing measurements were also excluded from the averages, and measurements from flights 6, 7, and 15 are not included because the first two were cases where the conventional vertical-wind measuring system malfunctioned and flight 15 was a flight devoted to calibration with little upper-level flight and frequent turns including circles.

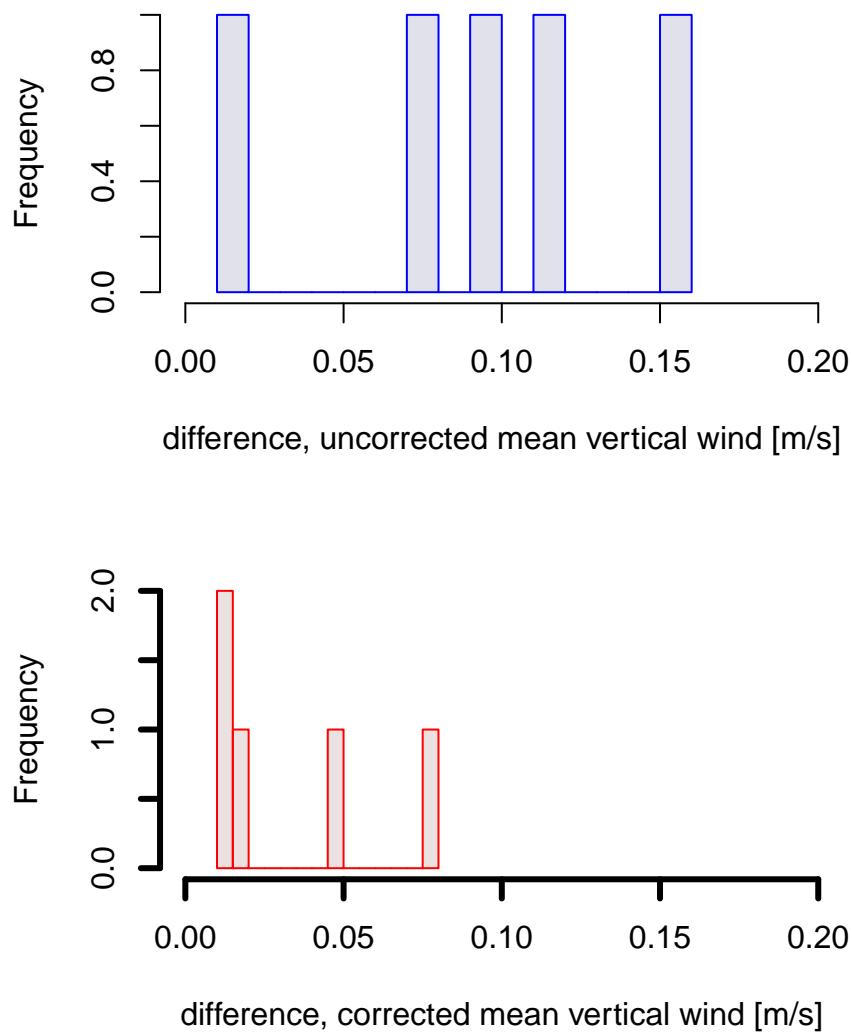


Figure 34: Absolute difference in vertical wind for flight segments before and after level course-reversal maneuvers. The top panel shows the uncorrected measurements and the bottom panel shows the result of applying the pitch correction developed in this subsection.

flight	mean WIC	mean corrected WIC
1	-0.01	-0.02
2	0.03	0.02
3	-0.07	-0.07
4	-0.09	-0.14
5	0.03	-0.02
8	0.01	-0.03
9	0.02	0.01
10	0.15	0.13
11	0.08	0.08
12	0.07	0.04
13	0.02	-0.01
14	0.08	0.08
16	0.07	0.05
17	0.02	0.01
18	-0.07	-0.07
19	0.08	0.08
20	0.07	0.06
21	0.11	0.08
22	0.02	0
23	0.07	0.07
24	0.03	0.03
25	-0.02	-0.02
26	0.03	0.03

Table 9: The average vertical wind for each flight, before and after application of the pitch-correction algorithm developed in this subsection. The data restriction applied was that the true airspeed be above 130 m/s, absolute value of the roll less than 5° , and the flight level above 25,000 ft (to emphasize normal research measurement conditions in the DEEPWAVE project). Flights 6, 7, and 15 are also missing, as explained in the text.

The mean value of the vertical wind, for all flights combined, was 0.03 m/s for the uncorrected measurements and 0.02 m/s for the corrected measurements, with standard deviations of

0.06 m/s, so both are well within expected tolerances. The pitch correction has little effect on these mean measurements or the overall standard deviation. However, the small offset obtained with the pitch corrections applied does not indicate that the measurements are only uncertain within these limits, because most flights are long compared to the Schuler-oscillation period of about 84 min. Flights will average over an oscillating correction and that average may be small compared to the correction applied. The standard deviation of the applied correction is 0.02° when calculated for the entire project. That indicates that the correction to vertical wind arising from application of the pitch-correction algorithm introduces changes with standard deviation of about 0.09 m s⁻¹ project-wide. Studies of individual flights show that this varies significantly from flight to flight. This uncertainty, however, is a significant contributor to the uncertainty in vertical wind. Without pitch correction, measurements of vertical wind will have an error with typical period of the Schuler oscillation that, for measurements spanning much shorter periods, will appear as a slowly varying bias.

The correction procedure developed here is not applied in normal processing of data files because it fits to the entire sequence of ground-speed measurements to find the corrections while the normal processor is sequential and has no access to future measurements while processing. To apply these corrections, an additional processing step is required. Code for this purpose has been developed to add values of pitch and vertical wind after correction, and this code was used for all the analyses reported here including the determinations of sensitivity coefficients in Sect. 5. The code can be accessed in the reference file for this document, WindUncertainty.Rnw; for details, see Appendix C.

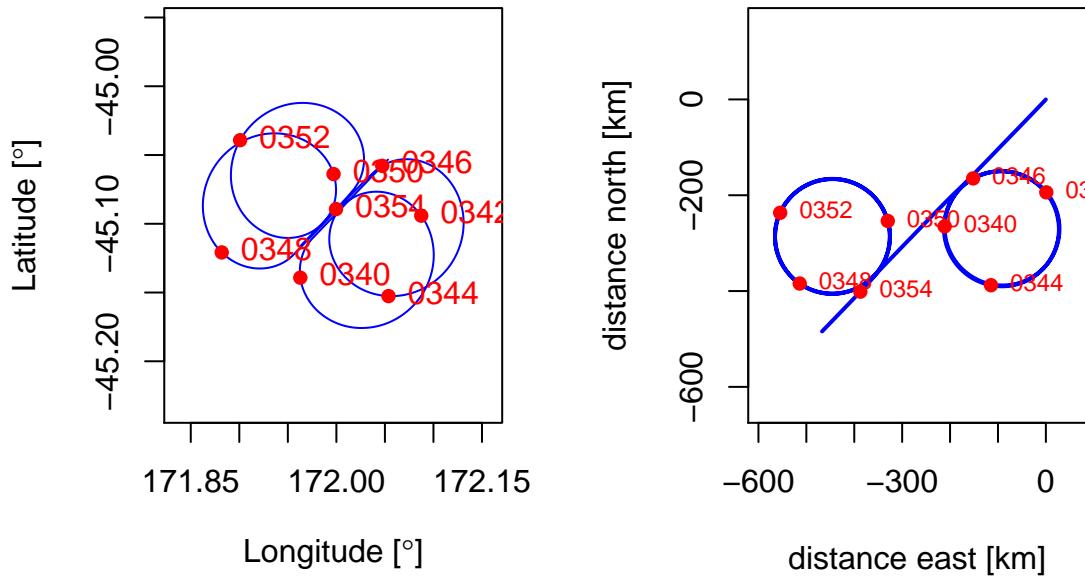


Figure 35: An example of circle flight pattern, from DEEPWAVE flight 15, 3:38:30–3:54:30 UTC. Left side: normal flight track referenced to ground coordinates; right side, flight track plotted in a Lagrangian reference frame drifting with the horizontal wind.

7 Studies of the Horizontal Wind

7.1 Analysis of circle maneuvers

7.1.1 Data Used

During the DEEPWAVE project, several circle patterns were flown to check the wind measurements. In these maneuvers, a constant roll angle was maintained so that the flight track drifted with the wind. That drift alone provides a measurement of mean horizontal wind that is dependent only on the measurement of position from the GPS. The first example used here is from DEEPWAVE flight 15, 3:38:30–3:54:30 UTC, during which two circles were flown with left roll and then two with right roll, as shown in Fig. 35.

7.1.2 Constraints arising from the assumption that the wind is steady

The circle maneuver is a stringent test of the wind measurements because, in steady conditions, the measured wind should remain constant around the circles. From the patterns of deviations with orientation in the turn, it is possible to detect an error in true airspeed or an offset in heading or sideslip angle. A time offset in measured ground speed from the GPS relative to the IRU can

also be detected because it produces a deduced heading offset that changes sign when left-turn and right-turn circles are compared.

For these patterns, the wind should be relatively steady and non-turbulent and the roll angle should be constant. For this flight segment, the left-turn circles had a roll angle of $-26.92 \pm 0.17^\circ$ and the right-turn circles had roll angle $27.08 \pm 0.12^\circ$, while the mean true airspeed for these circles was 153.5 ± 0.5 m/s. Most of the standard deviation in true airspeed arose from the normal fluctuations created by the flight management system, as discussed in Sect. 6.2.2 (cf. Fig. 27). The steadiness of these measurements indicates that this maneuver was flown with good precision and symmetrically, so the following is a good illustration of what can be learned from this maneuver.

7.1.3 Basic equations

These tests use the measured relative wind in Earth-based coordinates, with east and north components given approximately by $\{V \sin \psi', V \cos \psi'\}$ where $\psi' = \psi + \beta \cos \phi - \alpha \sin \phi$ with ψ the heading, β the sideslip angle, ϕ the roll angle, and α the angle of attack. In the circle maneuver, $\phi \approx 27^\circ$ so the last two terms in the expression for ψ' do not simplify with the small-angle approximation. If the east and north components of the horizontal wind are v_x and v_y , the corresponding components of the ground speed of the aircraft ($v_{p,x}$ and $v_{p,y}$) are

$$\begin{aligned} v_{p,x} &= V \sin \psi' - v_x \\ v_{p,y} &= V \cos \psi' - v_y \end{aligned} \quad (58)$$

and the difference between the motion of the aircraft expressed as (58) and the ground-speed components measured by GPS ($v_{g,x}$, $v_{g,y}$) is

$$\begin{aligned} \delta v_x &= V \sin \psi' - v_x - v_{g,x} \\ \delta v_y &= V \cos \psi' - v_y - v_{g,y} \end{aligned} \quad . \quad (59)$$

If error terms for true airspeed (δV) and for the adjusted heading angle ($\delta \psi'$) are introduced so that the true values are $V = V_m + \delta V$ and $\psi' = \psi'_m + \delta \psi'$ where subscript m refers to the measured quantity, and if it is assumed that the wind components $\{v_x$ and $v_y\}$ are steady around the circles, then estimates for the four fit parameters $\{\delta V, \delta \psi', v_x, v_y\}$ can be found by minimizing the errors given by (59).

Once the average wind direction (λ) and wind speed (v) have been determined either by the above fit or from the mean of measurements around the circles,²⁵ the error δv_m in the measurement of wind speed (v_m) can be expressed as

$$\delta v_m = v_m - v = -\delta V \cos \xi - V \delta \psi' \sin \xi \quad (60)$$

²⁵With equal weighting for measurements with all orientations relative to the wind, averaging measurements should give correct values even if there are errors in the individual terms

where $\xi = \psi' - \lambda$ is the angle between the relative wind (in the direction ψ' which is the heading adjusted for the roll angle by components from the sideslip angle and angle of attack) and the direction of the wind relative to the Earth (λ).

This equation is justified as follows. Because the relative wind transformed to an Earth reference frame is added to the ground-speed vector to find the wind, an error δV in true airspeed leads to an error of the same magnitude in the component of measured wind along the longitudinal axis of the aircraft. The measured wind therefore changes by $-2\delta V$ when the aircraft changes from an upwind to a downwind flight direction, with the negative sign arising from the convention that wind direction is specified as the direction from which the wind blows. The error in true airspeed thus can be determined from the difference between wind measured while flying upwind and that measured while flying downwind. For other flight directions, this error projects to the wind direction as $-\delta V \cos \xi$. Similarly, when flying crosswind the wind measurement is determined by the sum of the ground-speed component along the wind direction and the component of the relative wind lateral to the aircraft. If the wind is from the port side of the aircraft (e.g., for an east flight direction with wind from the north), a positive heading error leads to a relative wind component opposing the wind and so to a measurement error of $-V \delta \psi'$. For wind from the starboard side of the aircraft, the sign reverses, and for other angles the error contribution is $-V \delta \psi' \sin \xi$.

Fitting to minimize the deviations expressed by (60) can then give estimates of the three fit parameters δV , $\delta \psi'$, and v , or a fixed value can be used for v as given by the mean of measurements or by a previous fit to (59). These fits to (59) or (60) should give consistent results, but a fit to (60) is particularly illustrative because plots of the error as a function of flight direction relative to the wind clearly reveal the magnitude and source of the deviations. The expected pattern is shown in Fig. 36.

7.1.4 Finding the wind from the GPS ground track

From the definitions (59), the mean wind can be found by adjusting the fit parameters v_x , v_y , δV and $\delta \psi$ to minimize the error measure $\chi^2 = \sum(\delta v_x^2 + \delta v_y^2)$. The results, with wind converted to wind direction λ and wind speed v , are shown in Table 10 for the full circle maneuver and also separately for the right-turn circles and left-turn circles.

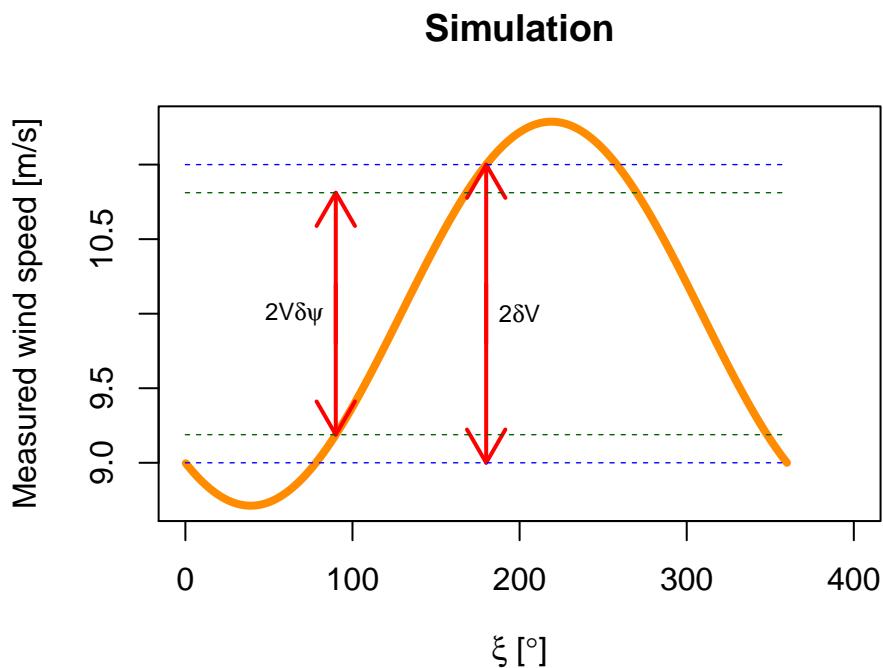


Figure 36: Illustration of the expected variation in measured wind speed with flight angle relative to the wind direction, for assumed errors of $\delta V = 1$ m/s in true airspeed and $\delta\psi' = 0.3^\circ$ in heading and for an assumed true wind speed of 10 m/s. The assumed true airspeed is 155 m/s. The red arrows show the differences in measured wind speed between 90° and 270° directions and between 0° and 180° directions.

	λ [°]	v [m/s]	δV [m/s]	$\delta \psi'$ [°]	residual error [m/s]
all turns	222.8	17.7	0.4	-0.05	1.0
mean of measurements	222.8	18.0			
left turns	222.9	18.3	0.5	-0.05	1.0
left-turn measurements	222.8	18.5			
right turns	222.6	17.2	0.3	-0.06	0.4
right-turn measurements	222.8	17.5			

Table 10: The best-fit parameters that minimize the errors given by (59) for the first circle maneuver from DEEPWAVE flight 15, 3:38:30–3:56:30 UTC, at an altitude of about 4.1 km (13,500 ft). The lines "mean of measurements", "left-turn measurements" and "right-turn measurements" are based on the wind measurements from the aircraft data system.

There is an apparently significant difference of about 1 m/s in mean wind speed, both measured and resulting from the fits, between the right-turn circles and the left-turn circles. This seems to be a real difference between wind conditions in the regions where the circles were flown and is supported by the difference between maximum and minimum ground speed for the two sets of circles. Conditions are more uniform for the right-turn circles, as reflected in the lower residual error for those circles, so more weight will be given here to the results from the right-turn circles.

The fits point to a very small heading error that is consistent for the two turn directions. That result is very sensitive to the timing of signal acquisition: Any difference in timing between the ground-speed components from the GPS and the heading measurement from the IRU will produce an indicated error in heading that changes sign with turn direction. Shifting either pitch or ground-speed components by 40 ms changes the indicated errors in heading by 0.07°, so these lags are very sensitive to the assumed delay. The results here were obtained for a heading shift of –50 ms, slightly different from the conventional shift of –80 ms, and with no shift imposed on the GPS measurements of ground-speed components. Also, no offset was applied to heading. For documentation, the attributes for the heading variable as processed for this study are listed here:

```
## [1] "attributes for variable"
## [1] "_FillValue: -32767"
## [1] "units: degree_T"
## [1] "long_name: IRS Aircraft True Heading Angle"
## [1] "standard_name: platform_orientation"
## [1] "valid_range: c(0, 360)"
## [1] "actual_range: c(0.00287811458110809, 359.997497558594)"
## [1] "Category: Analog"
## [1] "SampledRate: 25"
## [1] "TimeLag: -50"
```

```
## [1] "TimeLagUnits: milliseconds"
## [1] "DataQuality: Preliminary"
## [1] "CalibrationCoefficients: c(0, 1)"
## [1] "modulus_range: c(0, 360)"
```

Two other circle patterns were flown on this same flight, at 4:35:00–4:53:00 and 5:30:00–5:53:00 UTC. Results from fits to those circle maneuvers are shown in Tables 11 and 12. In these two cases and in Table 10, indicated airspeed corrections from individual circles varied from -0.6 to 0.6 m/s and angular corrections (combining heading and sideslip-angle errors) varied from 0.01 to 0.23° , with mean values of 0.2 ± 0.2 m/s and $0.13 \pm 0.03^\circ$. The fits thus support the low uncertainty limit claimed for airspeed but suggest that there is an error in the combination of heading and sideslip that should be corrected.

	λ [$^\circ$]	v [m/s]	δV [m/s]	$\delta\psi'$ [$^\circ$]	residual error [m/s]
all turns	230.7	22.8	0.5	-0.01	0.6
mean of measurements	230.4	22.2			
left turns	231.3	22.6	0.5	-0.02	0.6
left-turn measurements	231.2	22			
right turns	230.1	23	0.6	-0.01	0.4
right-turn measurements	229.6	22.5			

Table 11: The best-fit parameters that minimize the errors given by (59) for the second circle maneuver from DEEPWAVE flight 15, 4:35:00–4:53:00 UTC, at an altitude of about 6.1 km (20,000 ft). The lines "mean of measurements", "left-turn measurements" and "right-turn measurements" are based on the wind measurements from the aircraft data system.

	λ [°]	v [m/s]	δV [m/s]	$\delta \psi'$ [°]	residual error [m/s]
all turns	253.7	7	-0.2	-0.04	1.3
mean of measurements	253.7	7.2			
left turns	256.6	7.1	-0.6	-0.14	1.4
left-turn measurements	256.4	7.6			
right turns	250.9	6.9	0.2	0.05	0.8
right-turn measurements	251.1	6.8			

Table 12: The best-fit parameters that minimize the errors given by (59) for the third circle maneuver from DEEPWAVE flight 15, 5:30:30–5:53:00 UTC, at an altitude of about 9.1 km (30,000 ft). The lines "mean of measurements", "left-turn measurements" and "right-turn measurements" are based on the wind measurements from the aircraft data system.

7.1.5 Minimizing the variation in measured wind speed in circle maneuvers

The fit results in Tables 10–12 and also the mean measurements of the wind in those tables provide reference angles for the wind direction, so those angles can be used in fits that minimize (60). Because there is significant variation among the circles, the tabulated values for fitted values for the appropriate set (all turns, left turns, or right turns) will be used as the value of λ when fitting. An appropriate error function to minimize to find values for the parameters δV , $\delta \psi'$, and v in (60) is $\chi^2_2 = \sum \delta v_m^2$.

Tables 13–15 show the fit results for the three circles:

	v [m/s]	δV [m/s]	$\delta \psi'$ [°]	residual error [m/s]
all turns	18	0.5	-0.15	0.6
left turns	18.5	0.7	-0.20	0.2
right turns	17.5	0.2	-0.10	0.2

Table 13: The best-fit parameters that minimize the errors given by (60) for the first circle maneuver from DEEPWAVE flight 15, 3:38:30–3:56:30 UTC.

	v [m/s]	δV [m/s]	$\delta \psi'$ [$^\circ$]	residual error [m/s]
all turns	22.3	0.7	-0.08	0.3
left turns	22	0.8	-0.07	0.2
right turns	22.5	0.7	-0.08	0.2

Table 14: The best-fit parameters that minimize the errors given by (60) for the second circle maneuver from DEEPWAVE flight 15, 4:35:00–4:53:00 UTC.

	v [m/s]	δV [m/s]	$\delta \psi'$ [$^\circ$]	residual error [m/s]
all turns	7.1	-0.5	0.17	1.0
left turns	7.5	-1.1	0.37	0.7
right turns	6.8	0.2	0.00	0.5

Table 15: The best-fit parameters that minimize the errors given by (60) for the third circle maneuver from DEEPWAVE flight 15, 5:30:00–5:53:00 UTC.

Figures 37–39 show the mean measurements of wind speed as a function of the angle between the mean wind direction and the adjusted heading representing the motion of the aircraft relative to the air. The two turn directions are shown separately in each figure because they often require a different mean wind speed, as also shown in Tables 13–15 where the fit results are tabulated. The fits to (60), found by minimizing the defined χ^2 , are also shown in this figures (orange lines).

Some of the fits, notably the left-turn circles from the third maneuver, don't conform very well to the expected sinusoidal error pattern. However, if all six are averaged, the mean value for the error in airspeed is 0.25 ± 0.32 m/s and the indicated error in adjusted heading ψ' is $-0.01 \pm 0.09^\circ$, where the indicated ranges are the standard deviations estimated for the mean values. These are useful constraints on the uncertainty associated with these key contributors to uncertainty in measured wind.

Offset in sideslip angle In the preceding, the error $\delta \psi'$ was discussed as an error in adjusted heading, but that adjustment includes the sideslip angle so the error could also be one in sideslip angle. These errors are difficult to separate, and normal sideslip calibration (Sect. 5.2) even with reverse-heading maneuvers does not provide a separation.

The error term determined as in the above tables should be represented by $\delta \psi'$ given by

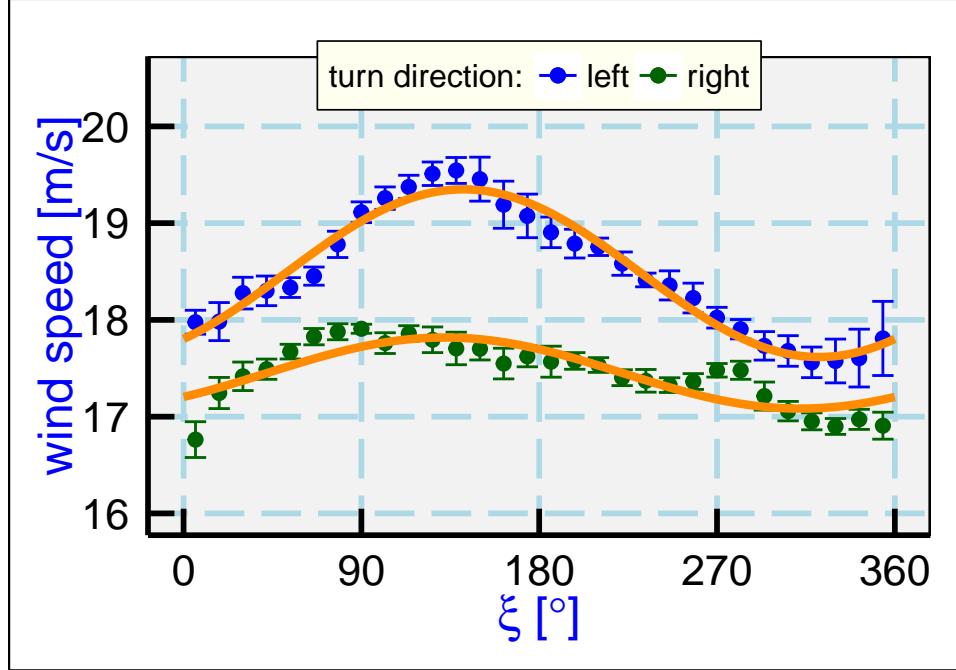


Figure 37: Measured wind speed from the left-turn and right-turn circles in the circle pattern shown in Fig. 35, as a function of ξ , the difference between the adjusted heading and the mean wind direction. Orange lines are the results of fitting (60) to the measurements, with results as listed in Table 13.

$$\delta\psi' = \delta\psi + \cos\phi\delta\beta \quad (61)$$

where ϕ is the roll angle and $\delta\psi$ and $\delta\beta$ are the offsets in heading and sideslip angle.²⁶ Because the dependence in (61) is on the cosine of the roll which is an even function, left and right turns are affected in the same way and also cannot distinguish the two terms in the equation. Even though three different altitudes were used for these circle maneuvers, the GV flight management system maintains nearly the same roll angle for all three maneuvers so changes in roll also cannot be used to distinguish the contributions from heading offset and sideslip offset.

One approximate test is to compare the sideslip measurement in left vs right turns. The measurements of sideslip angle²⁷ are shown in Fig. 40. In the turns, some sideslip is introduced as the aircraft configuration remains slightly nose-up during the turn, and by symmetry that sideslip angle should reverse sign when the flight pattern changes from left-turn to right-turn circles. That figure shows that this is approximately characteristic of the measurements, but examination of the pitch and roll angles show small variations so it is useful to obtain a better estimate.

A better approach is to take into account the influence of all angles on vertical wind. The transformation equations from [Lenschow and Spyres-Duran \[1989\]](#) (see also [Lenschow \[1972\]](#)),

²⁶The angle of attack was determined by separate calibration in Sect. 5.1.2 so that contribution to ψ' is not included in the error term.

²⁷The sensitivity coefficients as determined in Sect. 5.2 have been used to construct this figure.

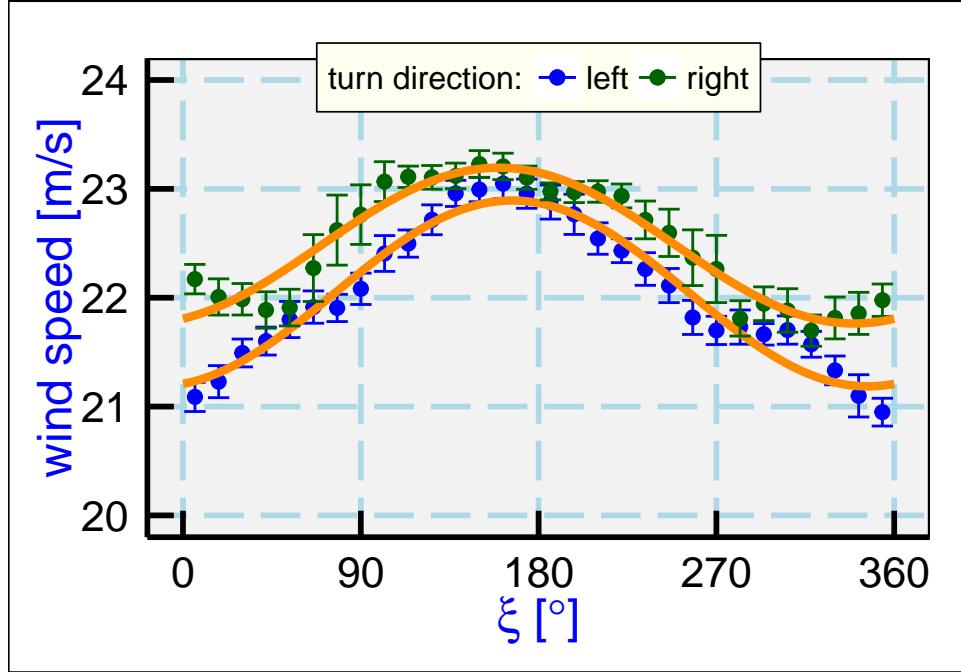


Figure 38: Measured wind speed from the left-turn and right-turn circles in the second circle pattern as a function of ξ , the difference between the adjusted heading and the mean wind direction. Orange lines are the results of fitting (60) to the measurements, with results as listed in Table 14.

with some small-angle simplifications, lead to this equation for the vertical wind:

$$u_z = w_p + V(\sin \phi \tan \beta + \cos \phi \tan \alpha - \sin \theta) \quad (62)$$

where w_p is the vertical motion of the aircraft, V its true airspeed, ϕ the roll angle, β the sideslip angle, α the angle of attack, and θ the pitch angle. If it is assumed that the vertical wind is zero, in the circle maneuver this equation predicts that the sideslip angle will be β^* given by

$$\beta^* \approx \frac{\theta - \alpha \cos \phi - (w_p/V)}{\sin \phi}. \quad (63)$$

This is not useful in straight flight where the roll is near zero, but it provides a valid equation for sideslip angle in the case of steady turns. The key assumption is that the vertical wind is zero; a vertical wind of 0.1 m/s will typically increase the deduced sideslip angle from this equation by about 0.06° , so it is important that the circle maneuver be flown where there is no mean updraft. It is then possible to determine the offset in sideslip angle by comparing this prediction to the measured sideslip angle β_m :

$$\delta\beta = \beta_m - \beta^*. \quad (64)$$

```
## [1] "mean error in sideslip: 0.084 +/-0.001"
```

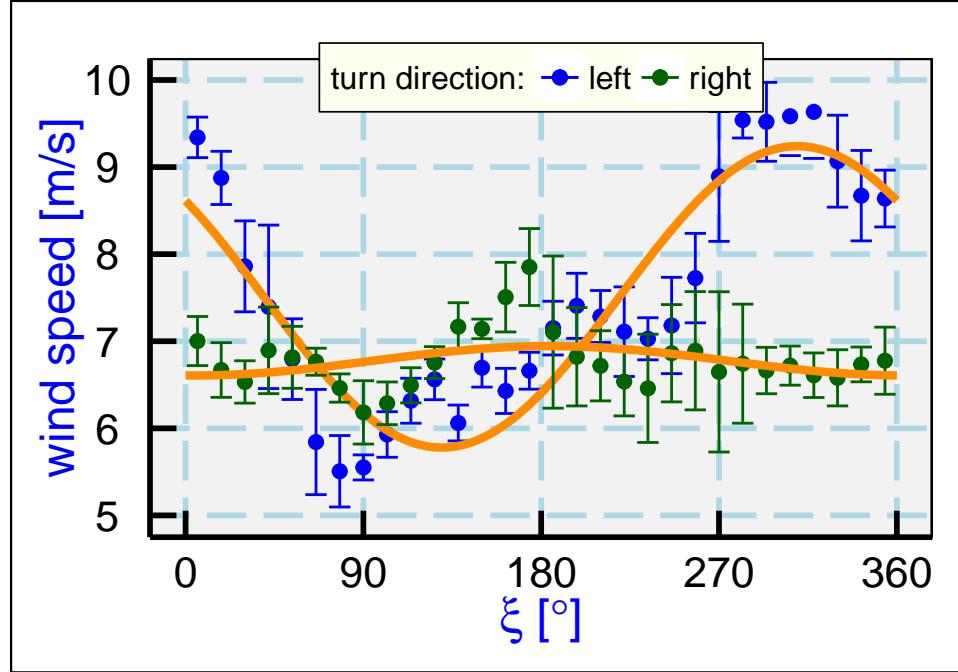


Figure 39: Measured wind speed from the left-turn and right-turn circles in the third circle pattern as a function of ξ , the difference between the adjusted heading and the mean wind direction. Orange lines are the results of fitting (60) to the measurements, with results as listed in Table 15.

Measurements of this estimate of the sideslip error are shown in Fig. 41. The mean error is 0.084° , which should be subtracted from the sideslip measurements.²⁸ This indicates that the sideslip sensitivity coefficients should be modified from those initially found in Sect. 5.2.2 to these revised values: $\{e_0, e_1\} = \{0.008, 22.302\}$. Because this result is dependent on the vertical wind being zero, an uncertainty of at least 0.03° should be assigned to the first coefficient to recognize that the vertical wind might typically be ± 0.05 m/s in a region such as this.

The results of the circle analyses thus point to a combined error from heading and sideslip angle of $-0.01 \pm 0.09^\circ$, so the mean heading error calculated from (61) is $-0.09 \pm 0.09^\circ$. The required heading correction is then the negative of this value, and the result of this calibration is to make compensating adjustments in the offsets for heading and sideslip.

7.1.6 Summary

The results obtained from analysis of the circle maneuvers are these:

1. The circle maneuvers indicate that the measured true airspeed (TASF) with current LAMS-based pressure corrections is accurate to within expected uncertainty. The fits indicate an error in true airspeed of 0.25 ± 0.32 m/s, which is within the expected (± 0.3 m/s) uncertainty limits for TAS deduced using the LAMS calibration (Cooper et al. [2014]). The

²⁸The value is the measurement error so the required correction is the negative of that value.

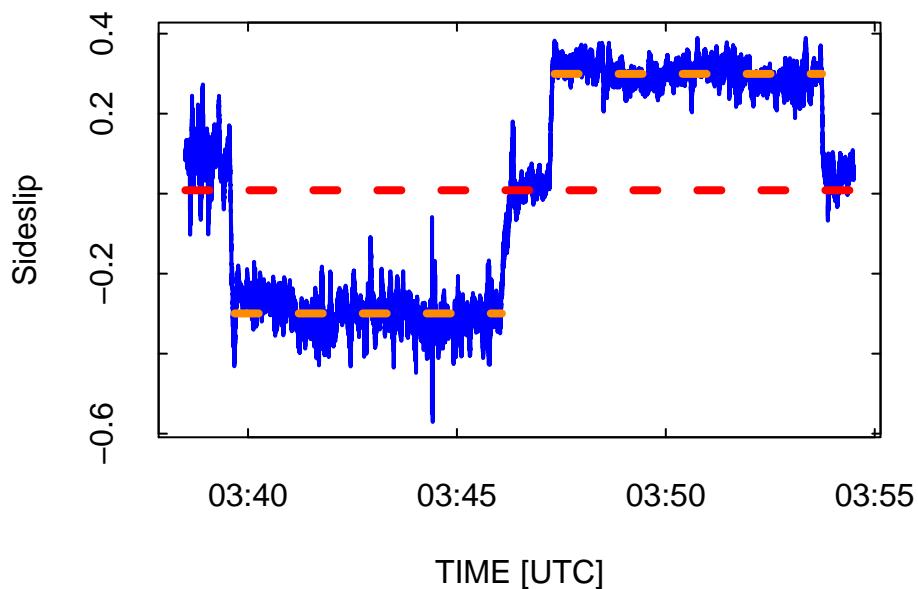


Figure 40: Measurement of sideslip during the first circle maneuver from DEEPWAVE flight 15, with left-turn circles from 3:40:00 – 3:46:00 UTC, followed by a straight segment and then right-turn circles 3:47:30 – 3:53:40 UTC.

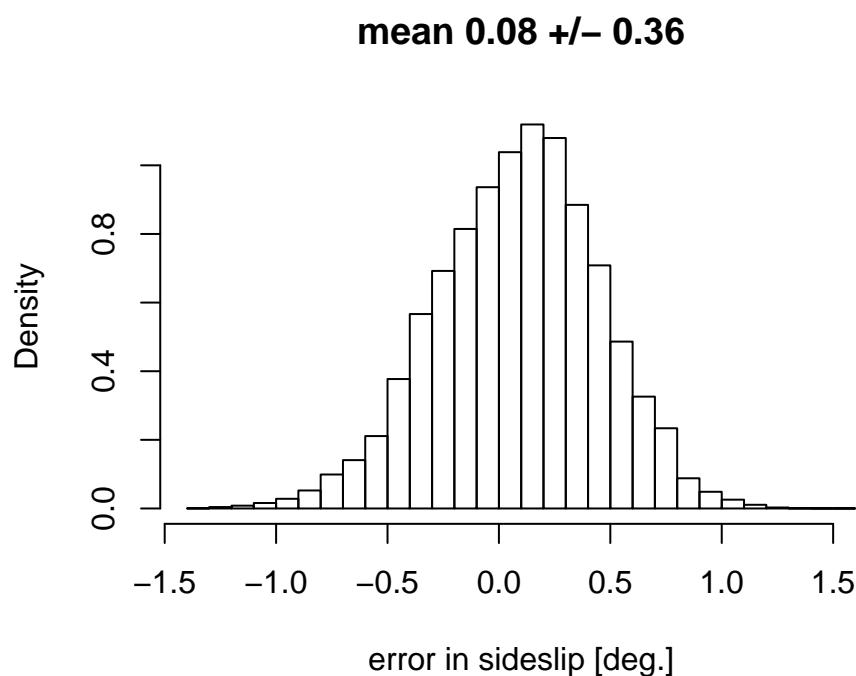


Figure 41: Distribution of measurements of sideslip error determined from (64) for all measurements in turns from the three circle maneuvers.

uncertainty range covers the calibrated value, so it appears better to make no adjustment to true airspeed.

2. The indicated error in sideslip is 0.084° , so the first sensitivity coefficient should be reduced by this amount. That leads to sensitivity coefficients for the radome of $\{0.008, 22.302\}$. Cf. the discussion in Section 5 and the summary section 5.4.2 at the end of that section. It appears reasonable to consider that the bias in this measurement may be uncertain by about 0.03° because the mean offset is determined with this standard deviation from the combination of the available circle maneuvers and this is also the sensitivity to 0.05 m/s vertical wind during the maneuvers.
3. With the preceding sensitivity coefficients, an offset should be introduced to heading of magnitude $0.09 \pm 0.09^\circ$. The evidence from the circle fits is that the combined bias associated with the heading and sideslip offsets is $-0.01 \pm 0.09^\circ$. The heading and sideslip offsets are coupled, so the two offsets indicated by the circle fits should be used together.
4. The results are consistent with no time offset in the GPS measurements of ground speed (GGVEW and GGVNS).

7.2 The complementary filter

Wind measurements combine a measurement of relative wind with a measurement of aircraft motion to determine the air motion relative to the ground. The aircraft motion is measured independently by an IRU and also by a GPS receiver. These have complementary strengths: The IRU provides very good information on short-term motion but drifts with a characteristic period of more than an hour, while the GPS provides good absolute accuracy but sometimes is unable to receive the GPS signals and (except in differential-GPS mode) can have short-term errors that make short segments of the track look jagged. To take advantage of the strengths of each, a complementary-filter calculation was developed and implemented in the 1980s. This section provides documentation of a procedure that has been in use for many years.

To combine these measurements, a low-pass filter, $F_L(\{GVNS, GVEW\})$, is applied to the GPS measurements of ground speed, $\{GVNS, GVEW\}$, which are assumed to be valid for frequencies at or lower than the cutoff frequency f_c of the filter. Then the complementary high-pass filter, denoted $(1 - F_L)(\{VNS, VEW\})$, is applied to the IRU measurements of ground speed, $\{VNS, VEW\}$, which are assumed valid for frequencies at or higher than f_c . Ideally, the transition frequency would be selected where the GPS errors (increasing with frequency) are equal to the IRU errors (decreasing with frequency). The filter used historically was a three-pole Butterworth low-pass filter, coded following the algorithm described in Bozic [1980], p. 49. This has now been changed to use the Butterworth filter coefficients given by the R routine 'butter' for the corresponding filter function. The digital filter used is recursive, not centered, to permit calculation during a single pass through the data. If the cutoff frequency lies where both the GPS and INS measurements are valid and are almost the same, then the detailed characteristics of the filter in the transition region (e.g., phase shift) do not matter because the complementary filters have cancelling effects when applied to the same signal. The transition frequency f_c was chosen to be $(1/600)$ Hz (but this value can be overridden via the "defaults" file). The

Butterworth filter was chosen because it provides flat response away from the transition. The resulting variables for aircraft motion, {VNSC,VEWC}, are then each the sum of two filtered signals, calculated as described in the following box:

VEW = IRU-measured east component of the aircraft ground speed
 VNS = IRU-measured north component of the aircraft ground speed
 GVEW = GPS-measured east component of the aircraft ground speed
 GVNS = GPS-measured north component of the aircraft ground speed
 $F_L()$ = three-pole Butterworth low-pass recursive digital filter

$$\{VNSC\} = \{VNS\} + F_L(\{GVNS\} - \{VNS\})$$

$$\{VEWC\} = \{VEW\} + F_L(\{GVEW\} - \{VEW\})$$

This is straightforward and effective when both sets of measurements (IRU and GPS) are available. The approach in use becomes more complicated when the GPS signals are lost, as sometimes happens in sharp turns. Then some means is needed to avoid sudden discontinuities in velocity (and hence wind speed), which would introduce spurious effects into variance spectra and other properties dependent on a continuously valid measurement of wind. To extrapolate measurements through periods when the GPS measurements are not available, a fit is determined to the difference between the best-estimate variables {VNSC, VEW} and the IRU variables {VNS, VEW} for the period before GPS reception was lost, and that fit is used to extrapolate through periods when GPS reception is not available. The procedure is described in section 3.4 of [this document on processing algorithms](#).

A Uncertainty-analysis conventions

So that this document might serve as a template for future analyses of uncertainty in NCAR/RAF measurements, this appendix documents some of the conventions followed here and suggested for standardized use.

A.1 Why perform analyses of uncertainty?

When measurements are made to test scientific theories, provide input to models, or characterize nature, they are only useful if accompanied by some sense of their reliability. A key use of uncertainty analysis is to provide this sense, in as quantitative terms as can be justified. A quoted value should be considered incomplete unless accompanied by some sense of the associated uncertainty, preferably in the form of estimated confidence limits to be associated with measurements or to be propagated to final scientific results. Although it is usually impossible in a strict statistical sense to provide formal estimates of confidence limits, this target still underlies approaches to uncertainty analysis. If those who make measurements don't characterize their reliability, others must make their own (probably less informed) evaluations.

There are additional benefits of analyzing measurement uncertainty. If an uncertainty analysis is done before an experiment, it may suggest ways to refine the experiment to minimize critical uncertainty contributions, and it should be possible to judge if the desired uncertainty is attainable. An uncertainty analysis also highlights the dominant sources of error and so can guide efforts to improve instruments.

A.2 Error, accuracy, and uncertainty

The *error* in a measurement is the difference between the measurement and the correct value of the measurand. A measurement is of little use unless there is some way of estimating how large this error may be. This estimate is called the *uncertainty*.²⁹ The uncertainty usually can be estimated in some way from knowledge of the performance of an instrument or from calibrations, intercomparisons, or statistical analysis of repeated measurements of the same quantity.

The term *accuracy* is often used erroneously where *uncertainty* would be appropriate. *Accuracy* is determined by the presence or absence of error, not uncertainty; a measurement may by chance be accurate and still have a large uncertainty. *Measurement uncertainty* is the correct term for an estimate of the limits to the experimental error; it is incorrect to refer to this as the measurement *accuracy*, although that is unfortunately common usage. *Accuracy* is sometimes

²⁹Results are sometimes classified according to their use: *indication* based only on primary measures such as sample means or correlation coefficients; *determination* based on primary and secondary statistics, so that some estimate of uncertainty is obtained; and *inference*, in which a specific mathematical model is used to assess uncertainty quantitatively. Often, a considerable amount of information about the underlying distribution must be known (or assumed) before statistical inference is possible. Experimental results are usually appropriately quoted as determinations.

used to refer to error, not uncertainty, but because accuracy is an absolute term even this usage is best avoided. A measurement will either be accurate or not.

A.3 Standards for evaluating uncertainty

Many different measures are used to characterize measurement error, often making it difficult to determine which interpretation should be associated with a quoted uncertainty. However, there is now an established international consensus, defined by the International Organization for Standardization (ISO) and by the US National Institute of Standards and Technology (NIST), and this or modified forms have also been adopted by many engineering societies. After decades of debate within engineering societies and among international groups, this methodology finally reached standardization through the recommendations of the International Committee on Weights and Measures. The two key publications now defining these standards are the [Guide to the Expression of Uncertainty in Measurement](#)(often referred to as the “GUM”) and NIST Technical Note 1297 [1994 revision], [“Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results.”](#)The latter is available in full from the web link and will be the primary reference followed in this document.³⁰

A.4 Classification of sources of error and of uncertainty

A.4.1 Errors

Errors are often classified as “systematic” or “random,” the former arising from consistent and repeatable sources (like an offset in calibration) and the latter from fluctuations about the measurand that are expected to average to zero in a repeated series of measurements. The former are also called “biases” when they arise from characteristics of an instrument. It is straightforward to differentiate these error classes by this test: Random errors are reduced when an experiment is repeated many times and the results averaged together, while systematic errors remain the same. Systematic errors can be reduced by better equipment or better calibration or better experimental procedures. Figure 42 illustrates these terms.

³⁰While the methodology described here is consistent with recommendations from those publications, it seems appropriate in addition to advocate separate estimation of the uncertainty associated with systematic errors because the validity of such estimates often depends on judgment and so is much harder to defend than in the case of random error.

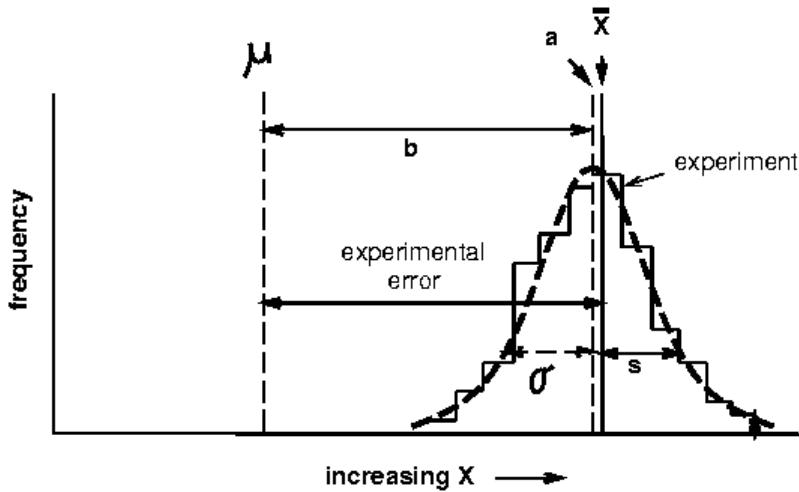


Figure 42: Illustration of the separate effects of bias errors and random errors. The true population mean is μ , but an instrument is used that has a bias b and measures with random error (in each observation) σ . The resulting estimate of the mean, obtained from \bar{x} , is in error because of the separate contributions of the bias error b and the random error in the measurement of the mean, in this case $(\bar{x} - a)$. The precision of the instrument is σ , so the estimated random error in the mean is $\sigma/N^{1/2}$. The actual error in an experiment is the difference between the true value μ and the measured value \bar{x} . The histogram represents a frequency distribution measured in a particular experiment, with mean shown as the solid line labeled \bar{x} . In a large number of observations, it would be expected that the results would tend toward the smooth dashed curve with mean $a = \mu + b$. The measured standard deviation is s , but the limiting value for a large number of measurements is expected to be σ .

A.4.2 Uncertainties

Like errors, estimates of *uncertainty* are also classified into two categories, “Type-A” (evaluated by statistical methods) and “Type-B” (evaluated by other means).

A Type-A evaluation of uncertainty is based on statistical analysis of repeated measurements or knowledge of the statistical character of the observations. Standard statistical measures and approaches including the standard deviation, analysis of variance, propagation of error, etc., can provide the needed estimate. For example, the standard deviation s_i in repeated measurements of the same quantity leads to an estimate of standard uncertainty $u_i = s_i$. Statistical means can also provide the required estimate of the degrees of freedom associated with the standard uncertainty.

Two important points need to be made in regard to Type-A evaluations:

1. “Type-A” refers to how the estimate is obtained, not to the type of error. Type-A evaluations often result in estimates of systematic error. For example, a random error affecting calibration of an instrument can subsequently produce a systematic error when the instrument is used, but the uncertainty can still be estimated via a Type-A evaluation. This

uncertainty component is therefore Type-A, even though the associated error being characterized is systematic. This has sometimes been called “fossilization” of error: The random error in the calibration procedure becomes a bias when that calibration is used.

2. Variability in a measurement may result from random measurement error, but it also may result from variability in the quantity being measured. Variations in a measurement arising from true variation in the quantity being measured cannot be used to estimate random measurement error, although they may place upper limits on that error. When using a standard deviation in repeated measurements to estimate standard uncertainty, it is necessary to correct for any contribution from natural variability.

A particularly clear example of a Type-A evaluation leading to a proper estimate of precision is that where the measurement consists of counting discrete events, such as cloud droplets or particles. The uncertainty in such measurements is expected to be characterized by Poisson statistics if the events occur at times determined from random distribution of the droplets in space.

A Type-B evaluation of standard uncertainty is more dependent on judgment and experience so it is harder to defend than a Type-A evaluation. Some guidance can be obtained from the following, but it must be acknowledged that these are imperfect and not quantitative, so another analyst with different judgment and experience could well disagree with the estimate. In place of statistical measures, information obtained from intercomparisons with other instruments, performance against standards, repeated calibrations, stability of the measurements, and specifications of components can all contribute to Type-B estimates of uncertainty. Nevertheless, it is useful to attempt to make Type-B evaluations that are as far as possible comparable to Type-A evaluations. For example, it is a goal that the “coverage” of the estimate be comparable to a standard deviation. The following may provide some guidance when developing such estimates:

- If the error source is expected to be within the limits $\pm a$ 50% of the time, then $u_j \approx 1.5a$.
- If it is expected to be within those limits about 2/3 of the time, then $u_j \approx a$.
- If the quantity is expected to be within those limits 100% of the time, but equally probable anywhere in this range, then use $u_j \approx a/\sqrt{3}$.
- If the limits are interpreted as 3-standard-deviation limits, then $u_j \approx a/3$. (The NIST TN and GUM provide other examples also.)

A.5 The composite or net uncertainty

The tabulated standard uncertainties should then be combined to a single standard uncertainty, u_c , which incorporates all sources of uncertainty. Where possible, degrees of freedom should also be provided. The recommended uncertainty to quote with results is the standard uncertainty u_c ; this is a departure from earlier practice, favoring two standard deviations or 95% confidence

limits.³¹ NIST continues to accept such estimates also, and uses the term *expanded uncertainty* (symbol U) such that $U = 2u_c$.

Error contributions thought to be random may really be systematic, and evaluating their associated uncertainty via Type-A methods may not reveal that dependence. An example often cited as a possible source of random error is a dependence of an instrument on line voltage, causing fluctuations in the response function of an instrument during an experiment. However, line voltage fluctuations are seldom random, and are probably biased in a particular direction relative to the conditions at the time of calibration, so it is likely that in a given experiment or series of experiments such fluctuations will introduce a bias. Furthermore, such errors are likely to be correlated in time, so the usual procedure of assuming random error contributions to be independent for different measurements will not be valid. Estimating the associated uncertainty via standard Type-A methods can thus be misleading in such a case. Close inspection of other common sources of error shows that they are often biases, and this increases the importance of estimating the associated uncertainty appropriately. Other examples will be given in later sections.

A.6 Recommended Guidelines

These are central features of the methodology recommended and used here:

1. Components introducing uncertainty are classified into two categories, Type-A and Type-B (as defined in section A.4.1), and *standard uncertainties* are estimated for each component. The estimated coverage associated with these evaluations is, in the case of Type-A components, that corresponding to one standard deviation. This is not quantifiable in a manner that can be defended rigorously in the base of Type-B errors, but estimating a standard uncertainty remains the goal.
2. To obtain the combined evaluation of uncertainty resulting from the net effects of many uncorrelated sources, the standard uncertainties are combined in quadrature,³² and the number of degrees of freedom in the combined uncertainty is estimated from the Welch-Satterthwaite equation (cf. (??)). For cases with correlations among components, methods that treat these correlations must be used, as specified in a subsequent section. A complete uncertainty report should also include an estimate of the number of degrees of freedom associated with the result.
3. If some standard uncertainties are asymmetrical, the positive and negative values should be combined separately to obtain separate upper and lower composite values.
4. The recommended uncertainty to be reported is the combined standard uncertainty, evaluated to represent a single standard deviation. Other estimates (e.g., that covering a specified level of confidence) can be obtained readily from this, provided that the number of degrees-of-freedom in the result is also reported.

³¹“Confidence limits” should only refer to Type-A evaluations; the term “coverage probability” is sometimes used to emphasize the difference between Type-B evaluations and those obtained via statistical analysis.

³²i.e., $s^2 = \sum_i s_i^2$.

5. [Not part of the standard:] The uncertainty report should also include separate estimates of the limits to precision and bias in the result.

An uncertainty report will normally include a tabulated list of sources of uncertainty, which should have separate sections for distinct influences like those arising from calibration, data collection, and data analysis. It is also useful to include associated estimates of precision, degrees of freedom, and bias for each contribution shown in the table. Such tabulations make it possible to isolate major sources of error, to consider the validity of other investigators' estimates of error sources, and to repeat the analyses for a new case when only one of the contributions has changed.

An important aspect of this methodology is that the degrees of freedom associated with cited estimates should be calculated and quoted. This becomes important when the number of degrees of freedom in the result is small, so that error limits and propagated errors have non-Gaussian character. Even if it is assumed that the individual measurements are distributed according to a Gaussian error distribution, the true standard deviation for an average of n samples, σ_n , is not known and must be estimated from the observations. The test statistic $t = (\bar{x} - \xi)/S_n$ (where \bar{x} is the average of n measurements, ξ is the true value of x , and S_n is the estimated standard deviation of the average \bar{x} about ξ , determined from $S_n = [\sum_{i=1}^n (x_i - \bar{x})^2 / (n(n-1))]^{1/2}$) will not be Gaussian distributed. The appropriate distribution for such averages is the Student-t distribution. The difference between the Gaussian and Student-t distributions is generally insignificant when the number of degrees of freedom³³ exceeds about thirty, but for small sample sizes the differences can be quite important. For this reason, when $n < 30$, the confidence limits used should be taken from the Student-t distribution rather than from the normal distribution.

The format advocated and followed here for an analysis of uncertainty includes these components:

³³In the case of an average of n values, the number of degrees of freedom is $n - 1$.

1. *Description of the measurement system with discussion of the limits within which the analysis to be presented is valid.* For example, the uncertainty in measurements of wind for a research aircraft might be specified for straight-and-level flight within three hours of take-off (because of drift of the inertial navigation system), perhaps within some altitude range. This description should discuss the calibration procedures, tests to characterize measurement uncertainty, data processing, and propagation of uncertainty to derived quantities. In this report, the measurement systems are discussed in detail in Sect. 2, calibration procedures in Sect. 3, data processing and associated uncertainties in Sect. ??, and tests of measurement uncertainty in individual components contributing to the measurement of wind are discussed in Sects. 6 and 7.
2. *Tabulation and classification of the elemental sources of uncertainty.* Each elemental source should be listed with its associated standard uncertainty u_i and, for Type-A sources, the number of degrees of freedom (n_i). It is also convenient to tabulate the effect of the error source on the final measurement Y by including entries for $u_i(\partial Y / \partial x_i)$ in the tables, where u_i is the standard uncertainty in the uncertainty-component x_i . This simplifies propagation to the final result, although special treatment is still needed in cases where the contributions are correlated. In this report, this tabulation is contained in Tables 4–7.
3. *Discussion of each elemental source of uncertainty in the table(s) along with a description of the basis for the evaluation.* These discussions should reflect the evidence for the tabulated values. Section 4 contains these discussions.
4. *Summarize the results and the uncertainty limitations of the measurement.* It is helpful here to highlight the main sources of error and possible actions that could improve the measurements. See Table 8 and related discussion in this report.

A.7 Monte Carlo techniques

Sometimes the functional relationships are so complex or non-linear that the preceding analytical formulas are unwieldy. In such cases, an alternative is to employ what is conventionally called a *Monte Carlo* technique. In this approach, the measured quantities are varied randomly in ways that represent the experimental uncertainties, and the calculations leading to the final answer are repeated with these artificial quantities. This is done repeatedly, and the variances and covariances in the resulting final answers are calculated. Random number generators are available on computer systems that generate variables having zero mean, unity variance, and a Gaussian probability distribution. Correlated fluctuations can be represented by defining linear combinations of such independent variables. In cases where the error propagation is especially complex (e.g., where the final answer might depend on non-linear fits to the input data), Monte Carlo techniques may be the only feasible way of determining the uncertainty in the final result.

B Measurements of turbulence

B.1 Scope of this appendix

Distributions showing variance vs. frequency for measured vector components of the wind are needed to characterize turbulence.³⁴ This appendix discusses the high-rate measurements of wind on the NSF/NCAR GV and proposes new variables for use in studies of turbulence.³⁵

B.2 High-rate sampling

The normal high-rate output for wind components is 25 Hz. However, the different measurements entering the calculation of wind are available at various sampling frequencies, so these must be converted to a common rate. The most important contributors to high-frequency variance are the measurements of dynamic pressure (QCF) and the pressure differences (ADIFR and BDIFR) measured at the radome. These are measured at 50 Hz by digital sensors having response times adequate for such sampling.³⁶ Measurements of attitude angles (pitch, roll, and heading) are available from the inertial system at 50 Hz, but it provides ground speeds only at 13 Hz and the GPS system provides ground-speed components at 5 Hz. The aircraft ground speed shows little high-frequency variance, so this does not affect the final measurements, but these samples are all converted to 25 Hz for processing by interpolating all samples with sample rates lower than 25 Hz to 25 Hz, then applying a digital filter with 25 Hz cutoff to reduce aliasing. However, in parts of this appendix 50-Hz samples will be used to illustrate the nature of the unfiltered measurements at the maximum sample rate.

Evidence from Sect. 6.2.2 (cf. Fig. 26) indicates that sensor noise introduces a lower limit to the intensity of turbulence that can be measured that is about $4 \times 10^{-4} \text{ m}^2 \text{s}^{-2} \text{ Hz}^{-1}$ for lateral wind components (arising from the transducers used for measuring ADIFR and BDIFR) and about $1 \times 10^{-3} \text{ m}^2 \text{s}^{-2} \text{ Hz}^{-1}$ for the longitudinal component. These are important limitations to the ability of the wind-sensing system to measure turbulence when the turbulent intensity is low.

B.3 Standard variance spectra

Some examples from the DEEPWAVE project have been shown in Sect. 6.2. That project emphasized studies of waves and often measured in regions where waves rather than turbulent

³⁴For wind, the variance spectrum has units of, e.g., $\text{m}^2 \text{s}^{-2}$ per frequency interval which is energy per mass per frequency interval, so this is best interpreted as an energy spectrum, not a power spectrum (which would have units of (energy/time) per frequency interval or, e.g., Watts/Hz instead of Joules/Hz for unit mass. For this reason, “variance spectrum” or “energy spectrum” or, perhaps best, “spectral density of variance” seem preferable to “power spectrum” for the present application.

³⁵The tools for spectral analysis used here are discussed further in Cooper [1994]. In particular, the “all poles” or “maximum entropy” method of spectral analysis (cf. Press et al. [1992]) will be the main method used for generating plots of the spectral density of variance from the measurements of wind. Because this is commercial software, this code is an exception to the effort to provide reproducible analysis in this report. Construction of the variance spectra shown here is dependent on the routine described on p. 572 of the cited reference.

³⁶However, the sensors introduce a delay of about one sample period or 20 ms in the measurement.

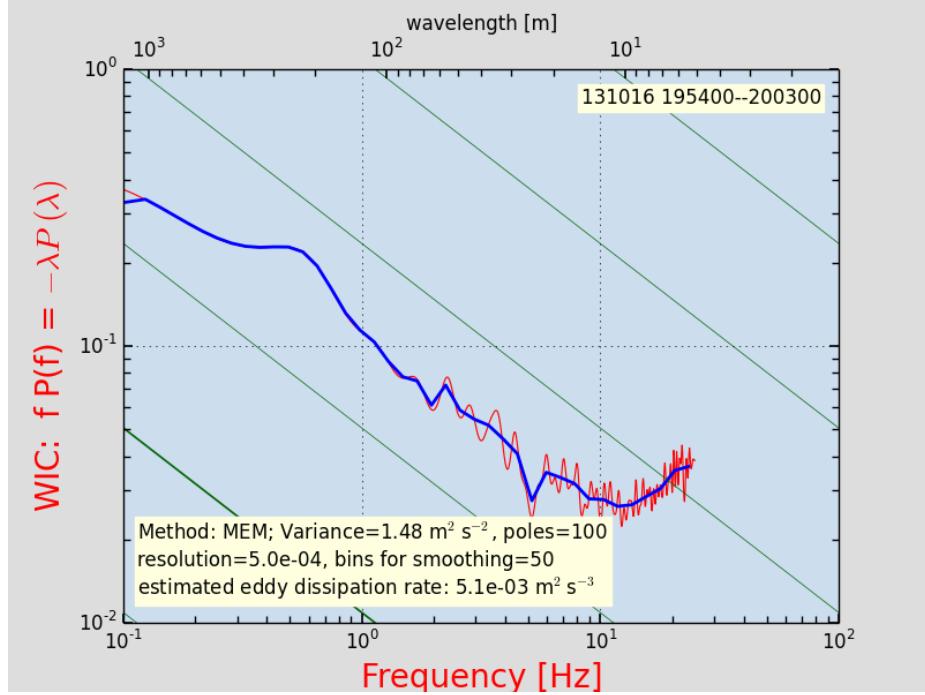


Figure 43: Variance spectra for the vertical wind for the flight segment from 19:51:01–20:12:00 UTC on 16 Oct. 2013.

air motions were predominant, so here different measurements from lower-level flight will be used here. In particular, a flight segment from two test programs flown on Oct 16 2013 and Nov. 20 2014 provide good cases for study of boundary-layer turbulence. In the former, a leg was flown from 19:54:00 to 20:12:00 on Oct. 16, 2013 at an altitude of about 5500 ft (1700 m) over terrain having an approximate elevation of 3700 ft (1130 m). In the latter project the leg to be used was flown from 21:02:00–21:09:99 UTC on Nov./ 24, 2014. It will be used here because both LAMS and the gust pod were also operational.

Because the measurements affecting high-frequency measurements of wind components are all sampled at 50 Hz, variance spectra can in principle be calculated up to a Nyquist frequency of 25 Hz. However, there are some limitations expected, primarily arising from expected effects in the pressure-sampling lines (Iberall [1950]) and the absence of filtering of the signals. Figures 43–45 show examples of the three components of the variance spectra for the first flight leg cited above. The lateral spectra (WIC and VYC) are reasonably similar, have similar variances through the range of frequencies, and show similar indications of noise for frequencies above 10 Hz. However, the longitudinal variance spectrum (variable UXC, Fig. 44) indicates that there is a problem with this measurement at frequencies above about 2 Hz. This problem appears directly in the true airspeed measurement used for determining the longitudinal component of the wind and in the measurement of dynamic pressure (QCF) used for that measurement.

The longer pressure lines used for the measurement of dynamic pressure QCF and the larger random uncertainty in that sensor in comparison to the sensors for angle of attack and sideslip pressure differences may contribute to the apparent problems in the distribution shown in Fig. 44. The lateral components are determined from pressure measurements made using lines about

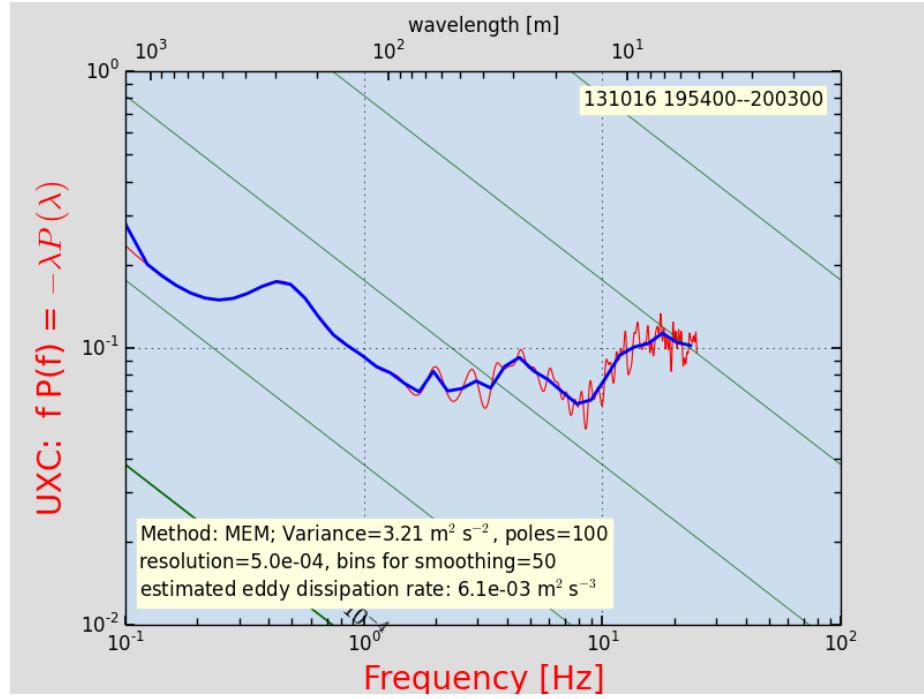


Figure 44: Variance spectra for the longitudinal component of the wind (along the aircraft centerline, and so essentially also the spectrum for the true airspeed) for the flight segment from 19:51:01–20:12:00 UTC on 16 Oct. 2013.

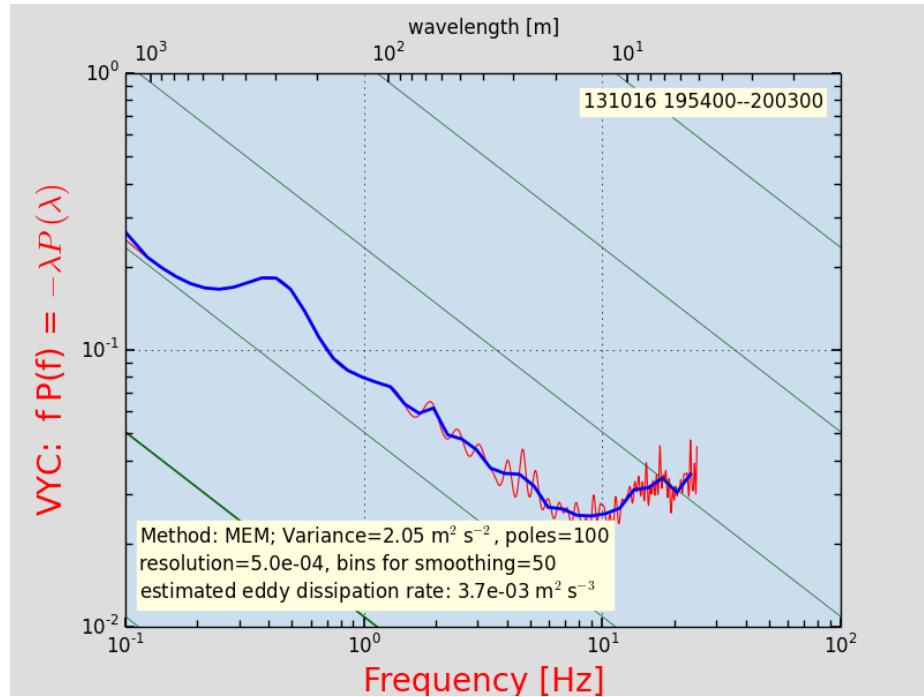


Figure 45: Variance spectra for the lateral component of the horizontal wind for the flight segment from 19:51:01–20:12:00 UTC on 16 Oct. 2013.

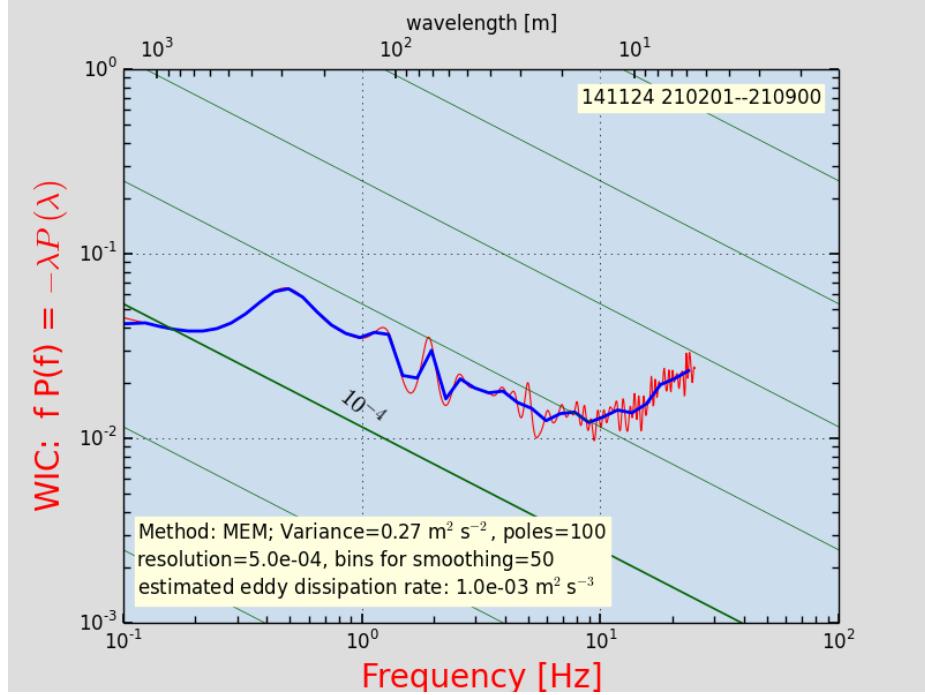


Figure 46: Variance spectra for the vertical component of the wind (WIC) for the flight segment from 21:02:00 to 21:09:00 UTC on 24 Nov. 2014.

2 m long connected to the radome pressure ports, but QCF is the difference between the total pressure at a pitot tube and the static pressure delivered by static buttons on the fuselage, so these necessarily use longer pressure lines which, for the reference static pressure delivered to the lower-pressure port of the QCF sensor, are about 35 ft in length.

If the measurements from UXC at frequencies from about 0.4–2 Hz are representative of the same inertial subrange characterized by the lateral components of the wind at about 0.4–10 Hz, then the intensities of turbulence for all components are reasonably consistent with expectations for an inertial subrange. The reference lines on Figs. 43–45 that indicate expectations for different values of the eddy dissipation rate are adjusted to account for the expected 4:3 ratio between lateral and longitudinal variance spectra. Thus all three components are reasonably consistent with the same eddy dissipation rate of about $5 \times 10^{-3} \text{ m}^2 \text{s}^{-3}$. In this comparison, the horizontal lateral component VYC is lower than the vertical lateral component by about 25%. This would arise if, for example, the second sensitivity coefficient for sideslip angle is too low by about 12%.

Another example of the high-frequency variance spectrum of the vertical wind is shown in Fig. 46, and the corresponding variance in the longitudinal component of the horizontal wind is shown in Fig. 47. The intensity of turbulence is lower in this case than in the first case presented, and the level of noise appearing for frequencies above 10 Hz in the vertical-wind spectrum is of lower intensity. The spectrum for the lateral component of the horizontal wind, not shown, is quite similar to that for the vertical wind in regard to shape and intensity. Both indicate that, for normal processing at 25 Hz with digital filtering, these variance spectra will appear reasonable. However, the longitudinal component (Fig. 47) again appears to have spurious variance

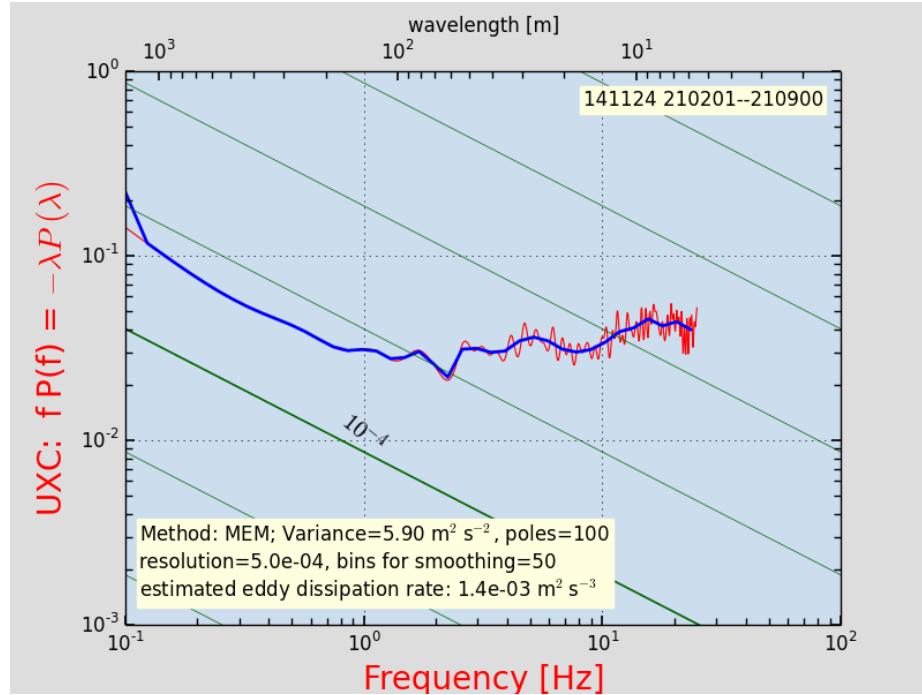


Figure 47: Variance spectra for the longitudinal component of the horizontal wind for the flight segment from 21:02:00 to 21:09:00 UTC on 24 Nov. 2014.

at frequencies above about 1–2 Hz. This will pose problems not only with measurements of the spectrum of turbulence but also with high-frequency contributions to momentum fluxes or Reynolds stresses. Therefore, in the next subsection a new variable constructed from combinations of the measurements that does not show these problems is discussed.

B.4 "Blended" variables

The other wind-sensing systems on the GV can help provide valid measurements of high-frequency components of the variance spectra of winds. The gust pod has very short pressure lines and a sensing head that is much smaller than the radome, and the LAMS also can measure the longitudinal component of the relative wind at high frequency. In the case of the gust pod, this report concludes that the horizontal wind measurements have serious weaknesses arising primarily from the inertial reference unit used for its measurements, but those give slowly varying biases so it might be expected that the high-frequency spectrum for the longitudinal component of the wind would be measured better by the gust pod than by the radome.

Figure 48 shows the variance spectra for the gust-pod dynamic pressure (QC_GP) and for the radome-system dynamic pressure (QCF). This comparison shows that there appears to be noise in the radome-system measurement at high frequency but the gust-pod measurement looks reasonable up to about 10 Hz. (The reduced amplitude for QC_GP at frequencies lower than about 0.1 Hz arises from the reduction in dynamic pressure measured under the wing because of disturbed airflow. This is corrected when the gust-pod true airspeed TAS_GP is calculated.) To

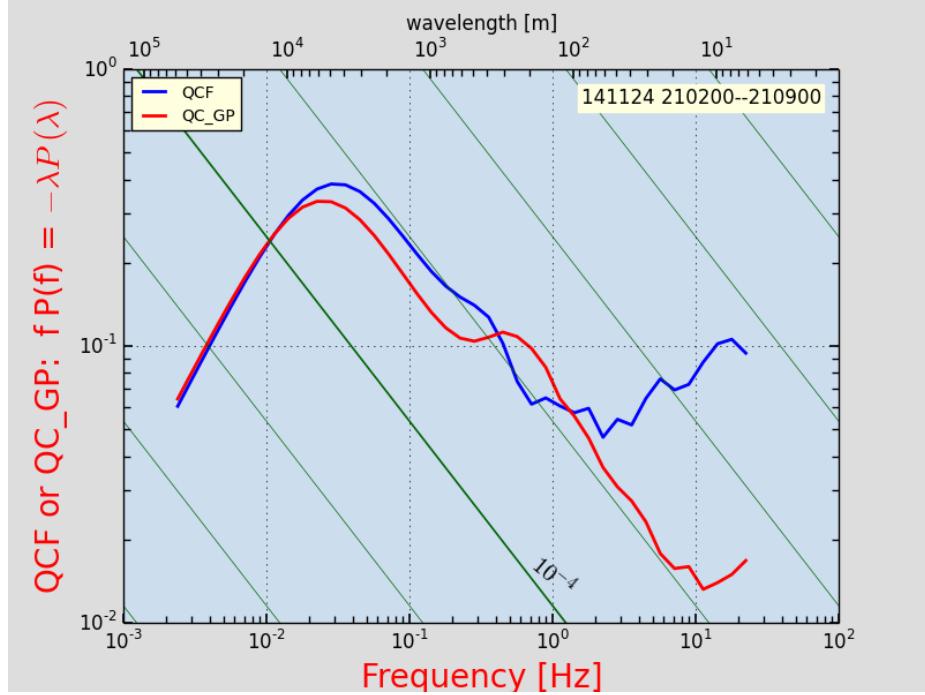


Figure 48: Variance spectra for the dynamic pressure measurements from the radome-based system (QCF) and from the gust-pod system (QC_GP) for the flight segment from 21:02:00 to 21:09:00 UTC on 24 Nov. 2014.

see how this is reflected in the wind measurements and in a proposed new variable that blends measurements from the gust-pod and radome systems, these steps were used:

1. Develop a calibration for airspeed from the gust pod, similar to that developed in Sect. 5.3. There the fit was restricted to measurements at high flight levels because that was of primary interest to DEEPWAVE, but for measurements in the boundary layer this needs to be repeated. The same equation was used to represent the fit, and the fit used the measurements from flight 3 of the HCRTEST program were used, but all measurements with conventionally measured true airspeed (TASF) above 130 m/s and magnitude of the roll angle less than 5° were used. A fit of (39) to these measurements then produced coefficients from which the ratio of dynamic to static pressure could be determined.
2. As in Sect. 5.3, this ratio of pressures then was used to calculate the true airspeed (TAS_GP) in the standard way.
3. This airspeed, with the other measurements from the gust pod, was then used to recalculate the relative wind and then, as explained earlier in this document, the wind direction and magnitude and the various wind components including vertical and horizontal lateral and longitudinal components.
4. Next, new wind variables were calculated, labeled with suffix "M" for "merged", that combined each component of the measurements from the gust-pod and the radome systems, using the technique described in Sect. 7.2. For 50-Hz measurements, the time

constant used was 150 samples or 3 s. This then resulted in measurements that combined the small low-frequency uncertainty of the radome measurements and the good high-frequency response of the gust-pod measurements. Because the complementary-filter technique uses a cross-over between components at a frequency where both signals are thought to be valid and show high coherence and similar amplitudes, no phase shift or other spurious effect on the variance spectra at the cross-over frequency is expected.

The result was that a variable TAS_GP representing the true airspeed from the gust pod was obtained that, for measurements where TASF exceeded 130 m/s and the absolute value of the roll was less than 5°, matched the mean TASF from the radome-based system with a standard error of 0.26 m/s. This also led to new variables representing the wind from the gust pod and also a new longitudinal-wind variable (UXM) obtained by combining the conventional-system variable UXC and the new gust-pod variable UX_GP via complementary filtering.

Because this all involves the same approaches documented in the body of this report, the code will not be documented again, but if it is of interest it can be found in the 'Rnw' file for this report. (See the reproducibility discussion at the end of this report).

The result is shown in Fig. 49, where the longitudinal component of the horizontal wind is calculated as described above by combining measurements from the gust-pod and the radome-based systems. The lateral components have been scaled by a factor of 3/4 because the variance spectra for longitudinal and lateral components are expected to be in the ratio 3:4 in an inertial subrange (e.g., [Batchelor \[1953\]](#)) so this adjustment should make the spectra coincide. From 1–10 Hz all indeed appear consistent with an eddy dissipation rate of about $5 \times 10^{-4} \text{ m}^2\text{s}^{-3}$.

The conclusion from this appendix is that the lateral components of the wind measurements appear reasonable up to a frequency of about 10 Hz, but the longitudinal wind component should not be used at frequencies above about 1 Hz. For higher frequencies, a useful measurement of the longitudinal wind can be obtained by calculating a new true airspeed obtained by combining the high-frequency response from the gust pod with the low-frequency response from the standard radome system and then calculating the wind using this blended variable. There is evidence of noise in all wind measurements for frequencies above about 10 Hz.

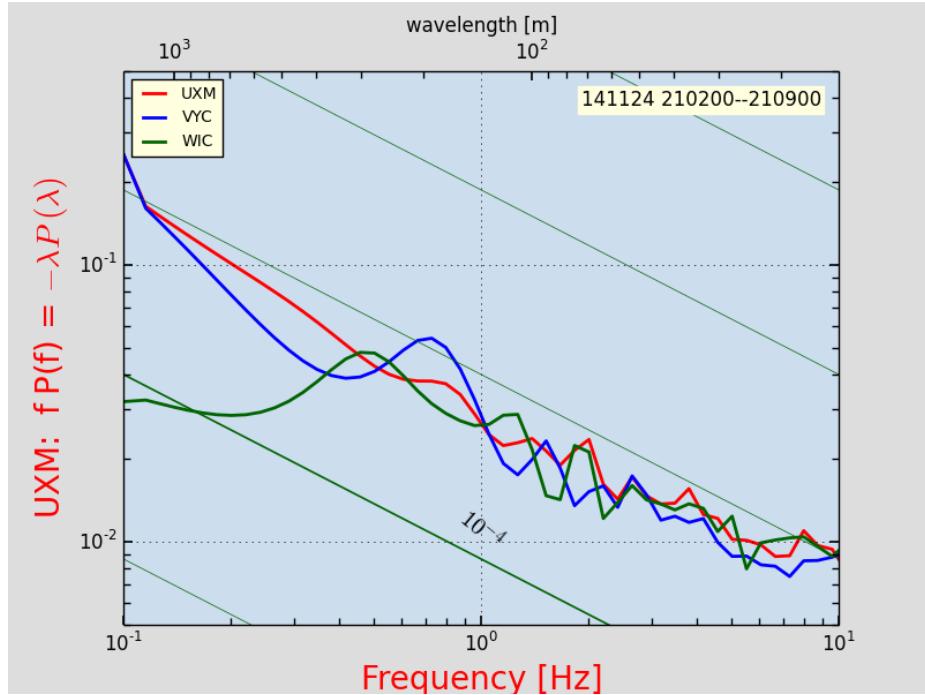


Figure 49: Variance spectra for the three components of the wind represented by UXM (longitudinal component as recalculated by combining the gust-pod true airspeed and the radome-based true airspeed, as discussed in the text), VYC (the conventional lateral component of the horizontal wind), and WIC (the conventional measurement of vertical wind). The two lateral components (VYC and WIC) have been multiplied by 0.75 to make them comparable to the longitudinal component in relation to the indicated reference lines for eddy dissipation rate (green lines), which are shown with magnitudes applicable to the longitudinal component. Data from HCRTEST flight 3, 24 Nov. 2014.

C Reproducibility

This document is constructed in ways that support duplication of the study. The processing programs are incorporated into the same file that generates this document, using principles and techniques described by Xie [2013] as implemented in the R package 'knitr' (Xie [2014]). The core program, 'WindUncertainty.Rnw', is archived on 'GitHub' in the directory at [this URL](#). There is some supplemental material in that directory, like the bibliography and many code segments saved in the 'chunks' subdirectory, so the full directory should be downloaded in order to run the program. The calculations use the programming language R (R Core Team [2013]) and were run within RStudio (RStudio [2009]), so this is the most straightforward way to replicate the calculations and the generation of this document.

A package named Ranadu, containing auxillary functions, is used extensively in the R code. It is available on GitHub as <https://github.com/WilliamCooper/Ranadu.git>. The specific version used for calculations in this report is included in the 'zip' archive listed below.

The data files used are also preserved in the NCAR High Performance Storage System (HPSS) in files that are available, and they can be provided via a request to <mailto:raf-dm@eol.ucar.edu>. The original files representing the data as produced by the NCAR Earth Observing Laboratory, Research Aviation Facility, were in netCDF format (cf. [this URL](#)), but in many cases special reprocessing was used and the files may change after reprocessing so a separate archive is maintained for this document. The data files in this archive represent R data.frames and are preserved as binary-format 'Rdata' files via R 'save' commands. The code in the GitHub archive has appropriate 'load' commands to ingest these data files from a subdirectory named 'Data' but this is not part of the GitHub repository because it is too large to be appropriate there. To reproduce this research, those data files have to be transferred separately from the NCAR HPSS to the 'Data' directory.

Extensive use has been made of attributes assigned to the data.frames and the variables in those data.frames. All the attributes from the original netCDF files have been transferred, so there is a record of how the original data were processed, for example recording calibration coefficients and processing chains for the variables. Once the data.frames are loaded into R, these attributes can be viewed and provide additional documentation of what data were used. Key information like the processing date, the program version, and the selection of primary variables is thus preserved.

PROJECT:	WindUncertainty
ARCHIVE PACKAGE:	WindUncertainty.zip
CONTAINS:	attachment list below
PROGRAM:	WindUncertainty.Rnw
ORIGINAL DATA:	/scr/raf_data/HCRTEST/
GIT:	https://github.com/WilliamCooper/WindUncertainty.git

Attachments: WindUncertainty.Rnw
WindUncertainty.pdf
chunks/*
SessionInfo
Ranadu_2.1-15-3-8.tar.gz

Acknowledgments

The analyses reported here were mostly performed using R³⁷R Core Team [2013], with RStudio³⁸ RStudio [2009] and knitr³⁹ Xie [2013, 2014]. Substantial use also was made of the ggplot2 package⁴⁰ Wickham [2009] for R.

³⁷R Core Team (2014). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL <http://www.R-project.org/>.

³⁸RStudio (2012). RStudio: Integrated development environment for R (Version 0.98.879) [Computer software]. Boston, MA. Available from <http://www.rstudio.org/>

³⁹Xie, Y. (2013), “knitr: A general-purpose package for dynamic report generation in R. R package version 1.3,” Version 1.6 was used for this work. See also Xie, Y (2014), “Dynamic documents with R and knitr,” CRC Press, Chapman and Hall, 190 pp.

⁴⁰H. Wickham. ggplot2: elegant graphics for data analysis. Springer, New York, 2009.

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List of Symbols

A_e =amplitude of east component of the Schuler oscillation in ground speed, 99	δp_β =pressure difference between starboard and port sources on the radome or gust pod [hPa], 68
α =angle of attack, 6	d_i =coefficients for fit representing the gust-pod airspeed, 81
α_{fit} =empirical function representing angle of attack, 65	e_0, e_1 =empirical coefficients used to obtain the sideslip angle, 55
α_g =angle of attack, gust pod, 74	ε =eddy dissipation rate, 41
α^* =angle-of-attack representation for calibration, 65	f_c =cutoff frequency for filters used for complementary filtering, 124
α^* =reference angle of attack for calibration, 65	F_H =high-pass filter applied to IRU ground-speed measurements, 124
A_n =amplitude of north component of the Schuler oscillation in ground speed, 99	F_L =low-pass filter applied to GPS ground-speed measurements, 124
a_n =northward component of acceleration, 98	g =acceleration of gravity, 95
b_0, b_1, b_2, b_3 =empirical coefficients relating angle of attack to pressure measurements from the gust pod [deg], 53	G^* =gust-pod fit for q/p, 81
$\mathbf{b}^{(b)}$ =unit vector representing errors in attitude angles in the b -frame, 102	k =wave number, 41
β =sideslip angle, 6	λ =latitude, 95
$\mathbf{b}^{(l)}$ =unit vector representing errors in attitude angles in the l -frame, 102	\mathbf{v}_E =vector wind relative to the Earth reference frame, 8
c_0, c_1 =coefficients leading to angle of attack [deg.], 50	\mathbf{v}_r =vector relative wind, 8
c_p =specific heat of moist air at constant pressure, 81	M_g =Mach number, 74
c' =alternate-formula coefficients for calculating the angle of attack [deg], 68	M_g =Mach number, from gust-pod pressures, 80
c'_0, c'_1, c'_3 =normal coefficients used for angle of attack, alternates to $\{c_0, c_1\}$, 66	m_i =measured quantities used in empirical functions, 65
c_0^*, c_1^* =empirical coefficients for angle of attack, alternates to $\{c_0, c_1\}$, 66	Ω_{Sch} =angular velocity, 98
c_v =specific heat of moist air at constant volume, 81	p_g =ambient pressure measurement, gust pod, 80
δ =measurement uncertainty, 41	ϕ =roll angle, 6
$\Delta p_{\alpha,g}$ =gust pod pressure difference, bottom vs top, 80	ϕ_L =LAMS azimuthal pointing angle, 32
Δp_α =pressure difference between bottom and top ports on the radome or gust pod [hPa], 68	φ_T =azimuth angle, telescope mount, LAMS calibration, 35
	ψ =heading, 6
	ψ' =heading angle adjusted in turns for sideslip angle and angle of attack, 111
	q =dynamic pressure, 11

q_g =dynamic pressure, gust pod, 80

\hat{r} =reference direction, LAMS receiving telescope, for LAMS angle calibration, 35

$\mathbf{R}(\theta, \hat{v})$ =rotation matrix used for LAMS calibration, 35

R_a =gas constant for moist air, 81

R_e =radius of the Earth, 95

ρ_a =density of air, 11

R_l^b =rotation matrix from l -frame to b -frame, 102

T_a =temperature, absolute units, 83

θ =pitch angle, 6

θ_L =LAMS pointing angle from longitudinal axis, 32

ϑ_T =altitude angle, telescope mount, LAMS calibration, 35

T_{Sch} =period of the Schuler oscillation (84.4 min), 95

u =component of wind along the x-axis, 41

V =airspeed, 6

\mathbf{v} =wind vector, 41

v =component of wind along the y-axis, 41

v =wind speed (relative to the Earth), 112

v_n =northward component of velocity, 98

V^* =longitudinal component of airspeed, 6

v_y =component of the horizontal wind lateral to the aircraft axis, 48

w =component of wind along the z-axis, 41

w_p =upward velocity of the aircraft [m/s], 46

x =northward coordinate in an Earth reference frame, 98

ξ =angle between the relative wind and Earth-referenced wind, 112

ξ =wind direction, 62

Variable Names

ADIF_GP, 59	DPX=dew point [deg C], 9
ADIF_GP=pressure difference between bottom and top sources, 858 probe [hPa], 20	GGALT=geometric altitude above the Earth from GPS [m], 83
ADIFR, 55	GGLAT=latitude measured by the GPS receiver, 98
ADIFR=pressure difference, bottom minus top source on radome [hPa], 12	GGQUAL=variable indicating GPS quality of measurement; 5 if OmniSTAR, 86
AK_GP=angle of attack determined using the gust pod [deg], 52	GGVEW=east component of ground speed, GPS [m/s], 8
AKRD=angle of attack from the standard radome based system [deg], 48	GGVNS=north component of ground speed, GPS [m/s], 8
ATTACK=angle of attack selected for standard wind processing, usually AKRD, 48	GGVSPD, 85, 88, 89
ATX, 56	GGVSPD=vertical component of the aircraft motion, GPS [m/s], 8
ATX=ambient temperature [deg C], 9	GGVSPDB=high-resolution version of GGVSPD, 86
BDIF_GP, 57	GUSTW=special-use variable representing the relative wind [m/s], 88
BDIF_GP=pressure difference between starboard and port sources, 858 probe [hPa], 20	LAT=latitude measured by the Honeywell inertial reference unit, 98
BDIFR, 54	PCOR=correction applied to measured ambient and dynamic pressures, 57
BDIFR=pressure difference, starboard minus port source on radome [hPa], 12	PITCH, 56
CPITCH_GP, 53	CTHDG_GP=heading from the CMIGIT-SIII IRU [deg.], 7
CPITCH_GP=pitch angle from the CMIGIT-SIII IRU [deg.], 7	CVEW_GP=east component of ground speed, gust-pod CMIGITSIII [m/s], 8
CTHDG_GP, 59	CVEW_LAMS=east component of ground speed LAMS SDN500 [m/s], 8
CTHDG_GP=heading from the CMIGITSIII IRUPITCH=pitch angle measured by the Honeywell IRU [deg.], 7	CVNS_GP=north component of ground speed, gust-pod CMIGITSIII [m/s], 8
CVEW_LAMS=east component of ground speed PSF, 56	CVNS_LAMS=north component of ground speed, LAMS SDN500 [m/s], 8
CVSPD_GP, 85	QC_GP=pressure difference between forward and static ports, 858 probe [hPa], 20
CVSPD_GP=vertical component of aircraft motion, gust-pod CMIGITSIII [m/s], 8	QCF, 56, 91
CVSPD_LAMS, 85	QCF=dynamic pressure measured using a fuselage-mounted pitot tube [hPa], 11
CVSPD_LAMS=vertical component of aircraft motion, LAMS SDN500 [m/s], 8	QCFC=corrected dynamic pressure [hPa], 65
	QCR, 91

QCR=dynamic pressure measured at the center port on the radome [hPa], 11	VSPD_A=vertical motion of the aircraft as provided by the avionic system, 85
ROLL=roll angle measured by the Honeywell [deg], 7	VSPD_G=alternate name used for VSPD_A in some projects, 85
ROLL_GP=roll angle from CMIGITSIII IRU [deg.], 7	VSPDx=upward component of the aircraft speed, 'x' system, or {GGVSPD, CVSDP_GP, CVSPD_LAMS}, 8
SS_GP, 58	VYC=horizontal-wind component lateral to the aircraft axis, 143
SSLIP=sideslip variable selected for standard wind processing, usually SSRD, 48	WDx=wind direction from the 'x' measuring system, 'x'={C, _GP, _LAMS}, 8
SSRD, 55	WI, 89
SSRD=sideslip measurement from the standard radome-based system [deg], 48	WI=vertical wind calculated using VSPD [m/s], 86
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THDG=heading measured by the Honeywell IRU [deg.], 7	recommendation, 88
UX_GP, 148	WIC=vertical wind calculated using GGVSPD [m/s], 86
UXC=horizontal-wind component along the aircraft axis, 143	WIX=vertical wind recalculated with pitch correction [m/s], 90
UXM=new variable representing the "blended" longitudinal component of the horizontal wind, 148	WP3=obsolete vertical-motion variable calculated using baro-inertial updating, 85
VEW, 125	
VEW=east component of ground speed, Honeywell IRU [m/s], 7	
VEWC, 124, 125	
VEWC=east component of ground speed, blended VEW and GGVEW [m/s], 8	
VEWx=east component of the ground speed, 'x' system, or {VEWC, CVEW_GP, CVEW_LAMS}, 8	
VNS, 125	
VNS=north component of ground speed, Honeywell IRU [m/s], 7	
VNSC, 125	
VNSC=north component of ground speed, blended VNS and GGVNS [m/s], 8	
VNSx=north component of the ground speed, 'x' system, or {VNSC, CVNS_GP, CVNS_LAMS}, 8	
VSPD, 85, 88	
VSPD=vertical velocity of the aircraft, Honeywell IRU [m/s], 7	

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