CM20316 Coursework Report Group 16

ABM: Introduction and Motivation

Auctions have existed for millennia, dating back to the Ancient Greeks. They facilitate a vast swathe of transactions each year, with over 17.6 billion dollars of art and antiques sold via auction in 2020, not to mention the mammoth volume of auction-based transactions in the general financial system (Statista, 2022). Total assets under management for the hedge-fund industry eclipses $4.5 trillion (with a 't') dollars and many of these institutions use auctions for the sale of their assets (BarclayHedge, 2022). The sheer volume of transactions by auctions and the power they wield warrants an investigation into optimising such systems.

We aim to investigate various bidding strategies in a second-price auction system. The goal of such a system is for both buyers and sellers to pay/receive a 'fair' price for the goods on sale - mitigating against the infamous "winner's curse". We introduce two types of bidders: early and so-called 'sniper' bidders, which we model using a Python Agent-Based Modelling framework "Mesa". The characteristic distinction between the two is that whilst early bidders are permitted to bid throughout the auction, with acute scale factors used to update their bids, sniper bidders wait until the end to swoop in and place a large bid in the hope they 'steal' the item at the last moment.

Hypothesis

Our overarching goal is to investigate bidding strategies within second-price auctions. We aim to determine the optimal agent and further explore whether it was rational for the agent to bid the winning price. The agents' performances will be assessed using two main metrics: **win percentage** and **premium paid**. The latter calculates the percentage difference between the average price an agent pays and the overall mean sale price.

Our primary hypothesis is sniper bidders will win a greater proportion of the auctions (i.e, they are the dominant agent of the two). We believe the last minute bidding of snipers will take the early bidders by surprise, who will not have enough time to react and update their bids, culminating in consistent wins for the snipers. In fact, Ely and Hossain (2009) stated that sniping leads to a roughly 5% increase in the probability of winning an auction.

As a secondary hypothesis, we expect sniper bidders to pay a larger premium to win the auction as they are more inclined to exaggerate a last second bid in order to secure an item. Despite Ely and Hossain disagreeing with our claim, we stand by our belief with Ariely and Simonson (2003) also claiming that snipers end up being more attached to the prospect of winning rather than the price. As this is so, we introduce a more intelligent sniper agent which we hypothesise to improve upon its naive equivalent (see Systems section).

A practical example of this is eBay, one of the most famous e-commerce websites. On this website, users can set an automatic bid: they increase the current bid on the item by the set minimum amount, then they set a maximum bid which is the maximum amount they're willing to pay. Groenwegen (2017) claims that this makes eBay a second-price auction system where live results of sniper vs early bidders can be seen.

Systems

Our auction system is built using a model outlined by Mizuta, H. and Steiglitz, K. (2000) as a reference. In our system, an auction consists of the sale of one item, with variables *snipers* and *earlyBidders* dictating the number of bidders of each agent type partaking. It runs for a period of *auctionLength* timesteps, with all agents being notified of the **second highest** bid at each step. Once an auction has ended, the highest bidder wins. This is the general architecture of an **online second-price auction**. Note all variables written in *italics* are user-settable parameters (although we fixed these for our analysis - see Appendix 1.)

Our model incorporates three agents: Auctioneer, EarlyBidder, SniperBidder, which interact as follows. The Auctioneer agent contains a sorted bid history and is accessed by the model to collect metrics about the auction, which are used to perform data analysis of the hypothesis.

The EarlyBidder agent emulates the role of a conventional bidder in an auction - one who observes it intermittently, posting bids occasionally and updating their private valuation by small scalars. These bidders are initialised with a private valuation and bid limit, each drawn from normal distributions (see Appendix 1 for parameters). At each timestep, an EarlyBidder will check if it is outbid (using the second highest bid for reference) with a probability *watchProba*. If so, they will update their valuation according to

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min(valuation * scalar, bid_limit)
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Where the scalar value is drawn from a uniform distribution in the range [1.0, 1.2]. They will then post the bid to the Auctioneer with probability *bidProba.*

On the contrary, our SniperBidder agent emulates the actions of individuals who swoop in at the last moment, hoping to steal the item with a late and high bid - often observed in online auction platforms (eBay). These too are initialised with private valuations and bid limits drawn from normal distributions. Their primary logical difference is that they are not 'activated' (i.e., cannot bid) until the final *bidTimeframe* steps of the auction at which point they update their internal valuation with the same update formula as EarlyBidders, where scalar is drawn from a uniform distribution, except this time the range is [1.2, 2.0]. This valuation is posted as a bid with probability *bidProba*. Notice, however, that the SniperBidder agent does not include a watch probability as they are always observing the auction.

We believed this agent was too naive and decided to further implement an alternate strategy for the SniperBidder. This variation is more intelligent, observing the whole auction and calculating the average bid increase at each time step. Once the agent is activated (i.e., the auction is in the final *bidTimeframe* steps), this agent will now update its internal valuation by summing the posted second highest bid, its calculated average bid increase (over the whole auction) and an arbitrary scalar. This bid is posted with probability *bidProba* like before.

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min(second_highest_price + average_bid_increase + x, bid_limit)
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Analysis of hypothesis and discussion of results

To evaluate our hypothesis we first produced a dataset of auctions by batch running **10,000 epochs** of our model (see Figure 1.) using the original EarlyBidder and SniperBidder agents, producing a dataset of roughly 1 million entries which we could analyse. To our surprise, our overarching hypothesis was instantly dismissed as the win percentage for Early and Sniper bidders was **60.70%** and **39.30%** respectively, showing the EarlyBidder was the dominant agent. Whilst this was unanticipated, we theorise this outcome was due to bids eclipsing the average agents' internal bid limit early on in the auction (as all agents' internal bid limits are drawn from identical normal distributions), leaving no bidding power for the snipers when they were activated.

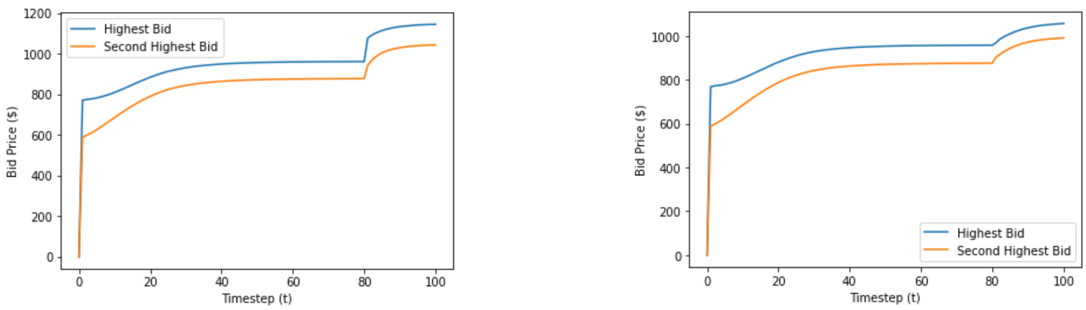

Figure 1. a) Mean price curve over 10,000 epochs using original Agents b) Mean price curve over 10,000 epochs using 'optimal' average bid increase SniperAgent

Furthermore, when we calculated the mean winning bid for each agent, we found that SniperBidders had to pay **$1247.65** versus only **$1077.60** for EarlyBidders, which was a significant **9.02%** premium in comparison to the mean sale price. This confirmed our secondary hypothesis that the original SniperBidder agent could be improved (i.e., it overpays) and led us to perform a similar analysis for a SniperBidder agent which utilised the average bid increase of the auction (described in the Systems section above).

For this more intelligent sniper agent, we generated a new dataset consisting of **10,000 epochs** of our new model to analyse its effects. Our analysis showed the intelligent sniper agent paid on average **$1045.16** which was a **1.00%** discount in comparison to the mean sales price of **$1055.73**. Whilst this was an astounding result, garnering a **10.02%** improvement over the naive sniper agent, the win rate of the SniperBidder dropped from **39.30%** to **13.18%** showing that although the agents' sales were better, they won auctions much less frequently. Thus, if this strategy were to be employed one would have to take into account its low success rate.

We observed the average sale price is significantly lower when sniper bidders utilise bid history information (shown in Figure 1.b). This is especially noticeable around the 80th timestep onwards, as this was when the snipers were activated. In Figure 1.a), there is a big spike pushing the price towards **$1200**, whereas in Figure 1.b) the spike is smoother and the price only reaches **$1000**. These price differences are further exemplified with histogram plots of the distribution of sale prices as shown in Figure 3, showing that the intelligent (average bid increase) sniper acts in a more rational manner (for itself) compared to the naive agent.

Using both datasets we calculated what we will call the "winner's curse" - the percentage difference between the 1st and 2nd highest bids - for each epoch and agent. We use the term "winner's curse" due to the phenomenon in Auction theory whereby the winning bidder is suddenly overwhelmed by a feeling of worry that they have overpaid for their item.

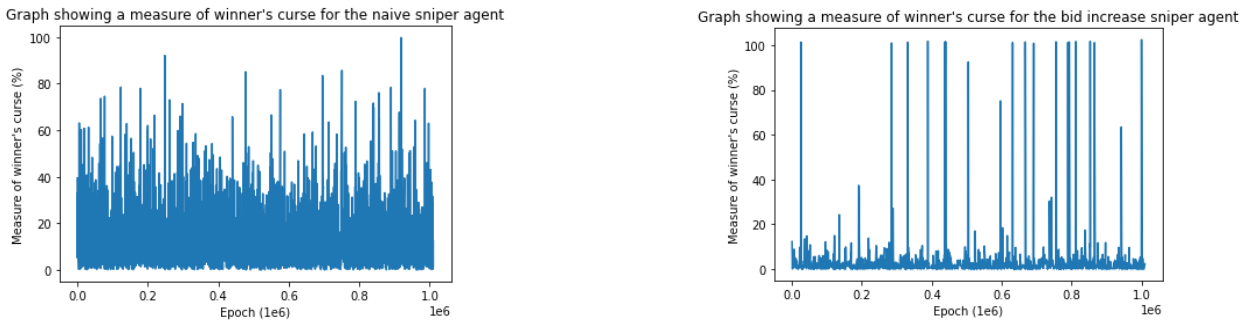

Figure 2. A comparison of the distribution of winner's curse (% difference between 1st and 2nd highest bid at end of the auction) between different sniper strategies

From Figure 2 it is apparent that the naive agent consistently overpays (with an average of **13.01%)**, whilst the intelligent agent rarely spikes above such a value, with only **32** such occurrences and a mean of only **3.49%**. The minimal variance in our metric "winner's curse" for the intelligent agent is reflected in the mean price paid by the agent (**$1045.16**), as noted before, further validating our hypothesis that it is possible to create a more optimal SniperBidder than the naive implementation.

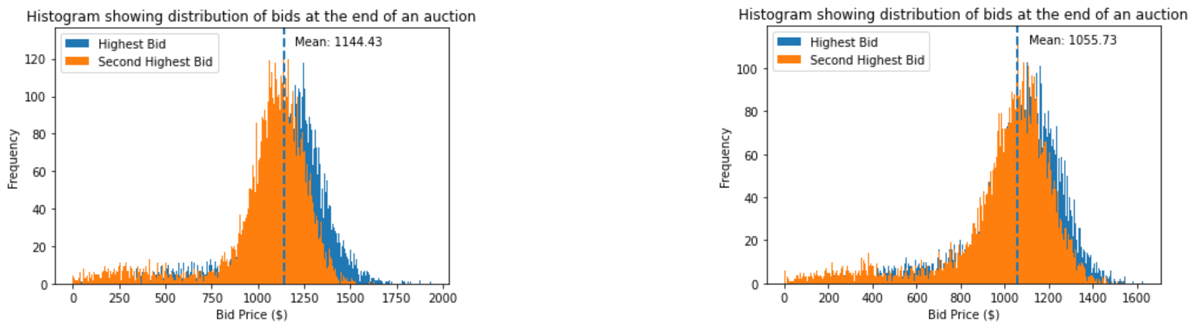

Figure 3. A comparison of histograms showing the distribution of winning bids for each agent a) Using naive agent b) Using intelligent average bid increase agent

In an investigation into reducing the win percentage of early bidders, we increased the number of sniper bidders making them equal to the early bidders (7 vs 7 compared to the previous 3 vs 7). With this variation sniper bidders now won **60.60%** of the time although, even when they took the average bid increase into account, they still paid a premium over the early bidders. We are inclined to say this validates both of our hypotheses, however, are tentative to do so as we feel it is unrealistic to assume there is an equal or greater number of sniper agents to conventional (early) bidders in typical auctions.

Summary and Future work

Our original hypothesis supposed sniper bidders were the optimal agent, which was quickly dismissed by our analysis when using a naive sniper agent. However, we further assumed that whilst SniperBidders would win, they would have to pay a premium for the honour. This assumption was in fact correct, with the data showing sniper agents had to pay a steep premium on average. Our introduction of a more intelligent sniper agent - one which considers the bids of agents in the past and finds the average bid increase - culminated in a sniper agent that paid a discounted price (in relation to the mean sale price), however only won a small minority of the auctions.

In light of our analysis, we had to reject our primary hypothesis, however, our secondary and tertiary hypotheses that our initial implementation of a sniper agent was too naive and not representative of how snipers would act in a real-world auction were correct. In future work, we would like to explore more variations of our model (as for our analysis we used a fixed set of parameters and varied the agents' strategies) as it is very likely that the model can be tuned to be more accurate, as shown by varying the number of each agent type in our analysis. Furthermore, it would be interesting to investigate how different probability distributions for generating initial valuations and bid limits for the agents affect the behaviour of the agents in the model.

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Appendices

User Settable Parameter	Description	Value
snipers	Number of sniper agents	3
earlyBidders	Number of conventional agents	7
maxValueStandardDeviation	Standard deviation of the normal distribution for drawing agents' initial private valuations and bid limits	250
bidProba	Probability an agent places a bid	0.75
watchProba	Probability an agent observes the auction	0.75
bidTimeframe	Window of time (from the end of the auction) where the sniper agents are activated (i.e., can bid)	20
auctionLength	Length of auction	100

Appendix 1. User settable parameters of the model (and the corresponding values used in the data analysis)