



Bonus Level



Problem

Score = 0

0 	5	0	10
5	5	0	0
1	0	0	0
0	0	0	1 



Problem

Score = 0

0 	5	0	10
5	5	0	0
1	0	0	0
0	0	0	1 



Problem

Score = 0

0	5	0	10
5 	5	0	0
1	0	0	0
0	0	0	1 

Problem



Score = 0 + 5

0	5	0	10
0 	5	0	0
1	0	0	0
0	0	0	1 

Problem

Score = 0 + 5

0	5	0	10
0	5	0	0
1	0	0	0
0	0	0	1



A 4x4 grid with numerical values and character images. The grid is as follows:

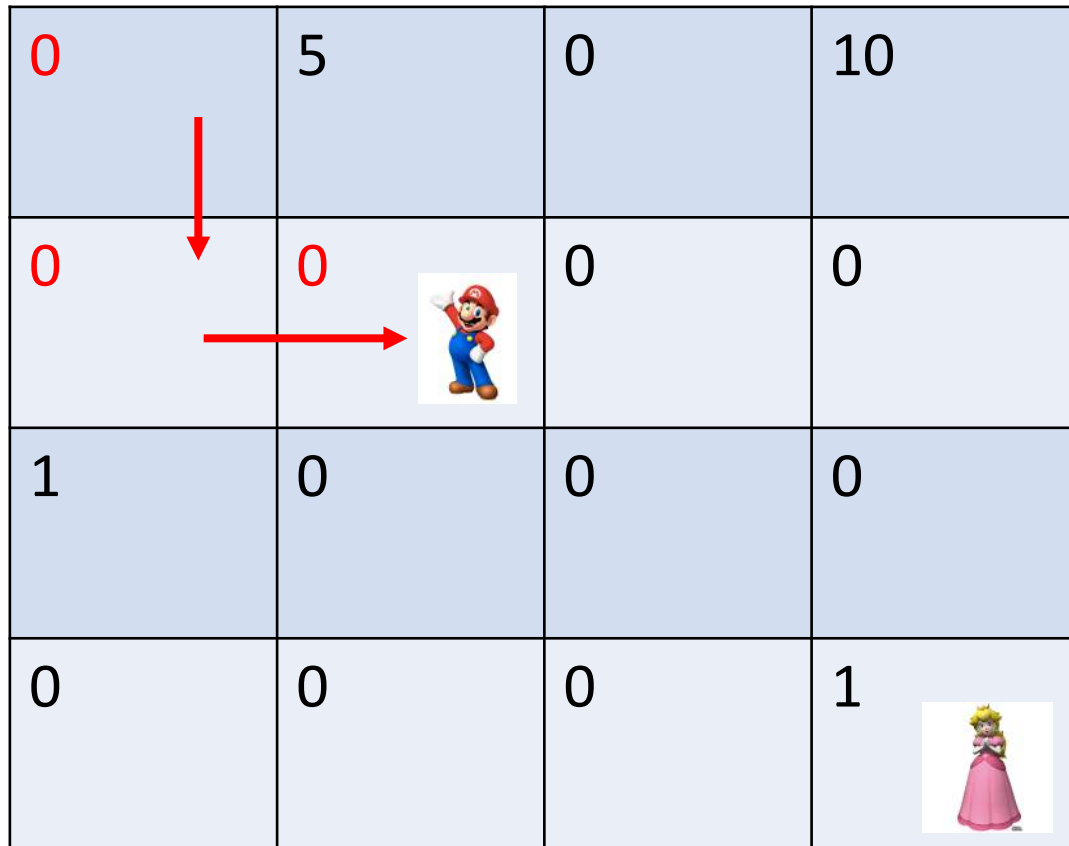
Row \ Column	1	2	3	4
1	0	5	0	10
2	0	5	0	0
3	1	0	0	0
4	0	0	0	1

Red arrows indicate a path from the cell (1,1) to (2,2). The cell (2,2) contains a Mario character. The cell (4,4) contains a Princess Peach character.

Problem

Score = 0 + 5 + 5

0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	1



The grid contains the following values:

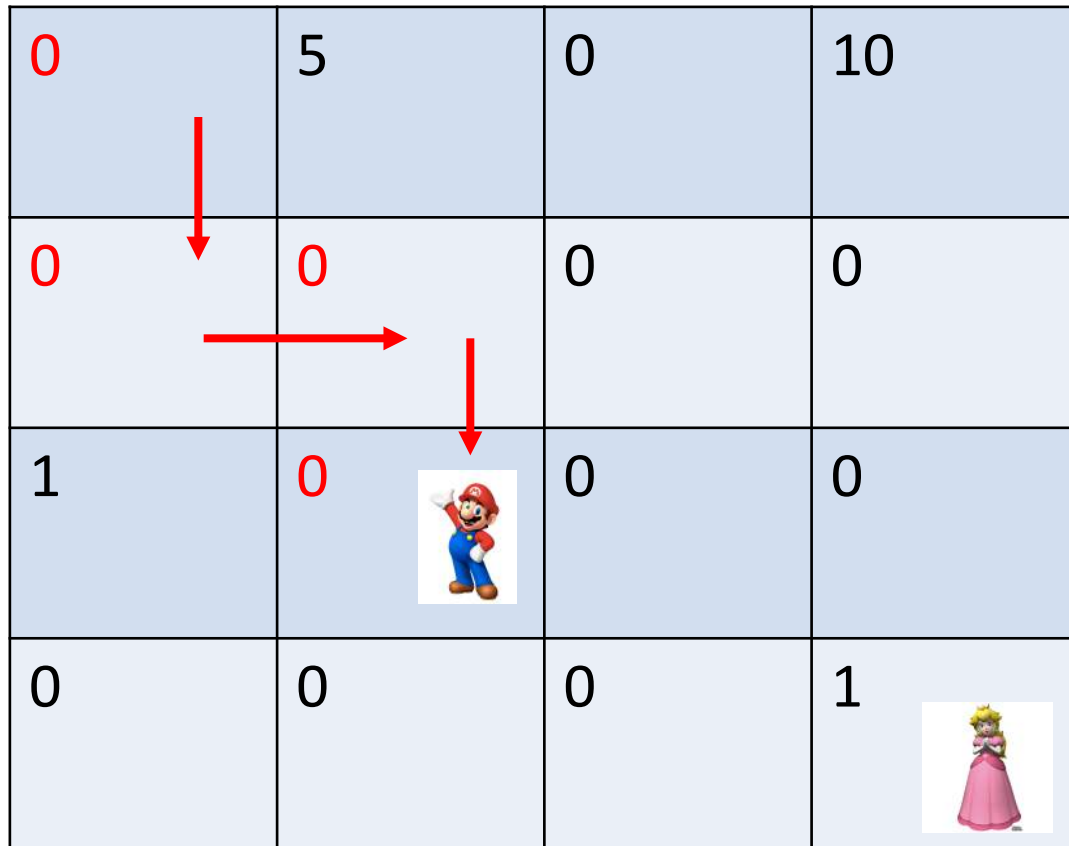
- Row 1: 0, 5, 0, 10
- Row 2: 0, 0, 0, 0
- Row 3: 1, 0, 0, 0
- Row 4: 0, 0, 0, 1

Red arrows indicate a path from the top-left '0' to the '0' in the second row, first column, and then to the '0' in the second row, second column. The Mario character is located in the second row, second column. The Princess Peach character is located in the fourth row, fourth column.

Problem

Score = 0 + 5 + 5

0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	1



The grid contains the following values and images:

- Row 0: [0, 5, 0, 10]
- Row 1: [0, 0, 0, 0]
- Row 2: [1, 0, 0, 0]
- Row 3: [0, 0, 0, 1]

Red arrows indicate a path from the top-left cell (0,0) to the cell below it (1,0), then right to (1,1), and finally down to (2,1). Mario is located in the cell at (2,1). Princess Peach is located in the cell at (3,3).

Problem

Score = 0 + 5 + 5

0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	1

The grid contains the following values:

- Row 0: 0, 5, 0, 10
- Row 1: 0, 0, 0, 0
- Row 2: 1, 0, 0, 0
- Row 3: 0, 0, 0, 1

Red arrows indicate a path from the top-left cell (0,0) to the cell containing Mario (1,3). The path consists of the following moves:

- Down from (0,0) to (1,0)
- Right from (1,0) to (1,1)
- Down from (1,1) to (1,2)
- Right from (1,2) to (1,3)

Mario is located in the cell at row 2, column 3. Peach is located in the cell at row 3, column 4.

Problem

Score = 0 + 5 + 5

0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	1

The grid shows a path of red arrows starting from the top-left cell (0,0) and ending at the bottom-right cell (3,3). The path consists of the following cells: (0,0), (1,0), (2,0), (3,0), (3,1), and (3,2). The grid contains the following values:

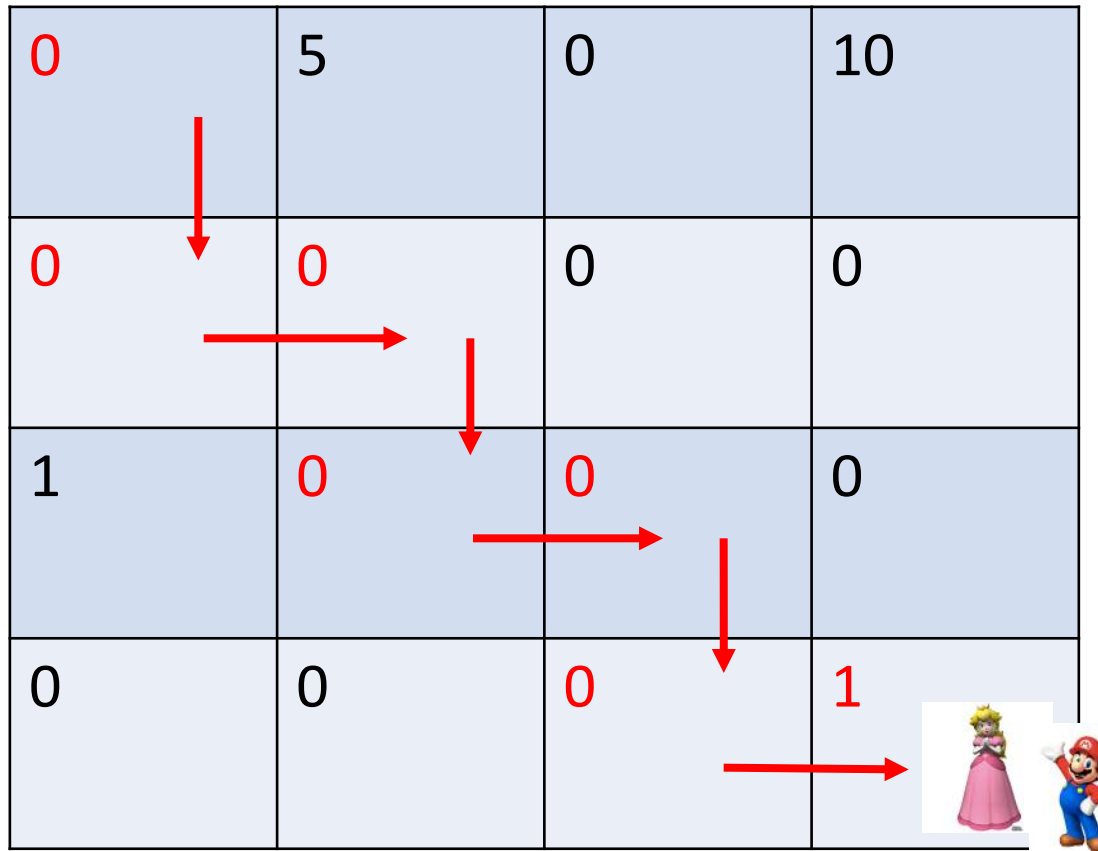
- Row 0: 0, 5, 0, 10
- Row 1: 0, 0, 0, 0
- Row 2: 1, 0, 0, 0
- Row 3: 0, 0, 0, 1

The path ends at the cell containing the Mario character, which has a value of 0. The cell containing the Princess Peach character has a value of 1.

Problem

Score = 0 + 5 + 5

0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	1

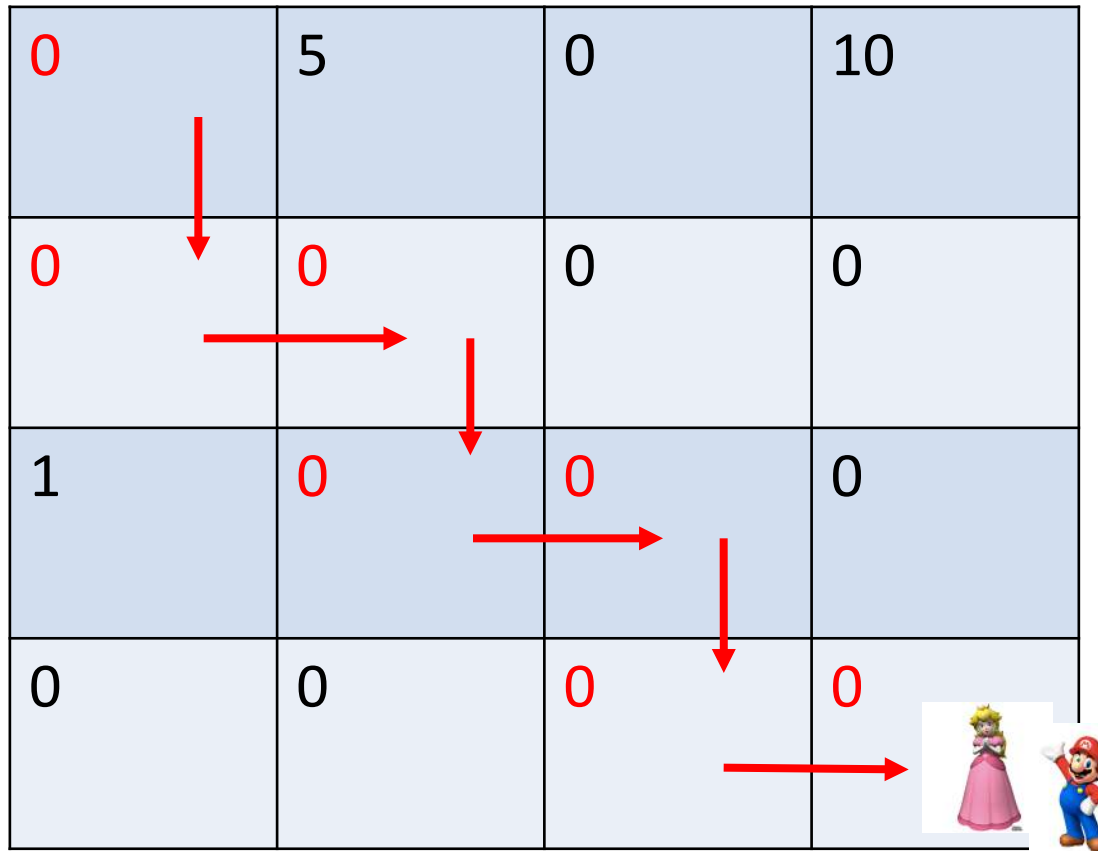


The grid shows a path of red arrows starting from the top-left cell (0,0) and ending at the bottom-right cell (3,3). The path consists of the following cells: (0,0), (1,0), (1,1), (2,1), (2,2), (3,2), and (3,3). The values in the cells are: (0,0)=0, (0,1)=0, (0,2)=1, (0,3)=0, (1,0)=5, (1,1)=0, (1,2)=0, (1,3)=0, (2,0)=0, (2,1)=0, (2,2)=0, (2,3)=0, (3,0)=0, (3,1)=0, (3,2)=0, (3,3)=1. The final cell (3,3) contains a small image of Princess Peach and Mario.

Problem

$$\text{Score} = 0 + 5 + 5 + 1$$

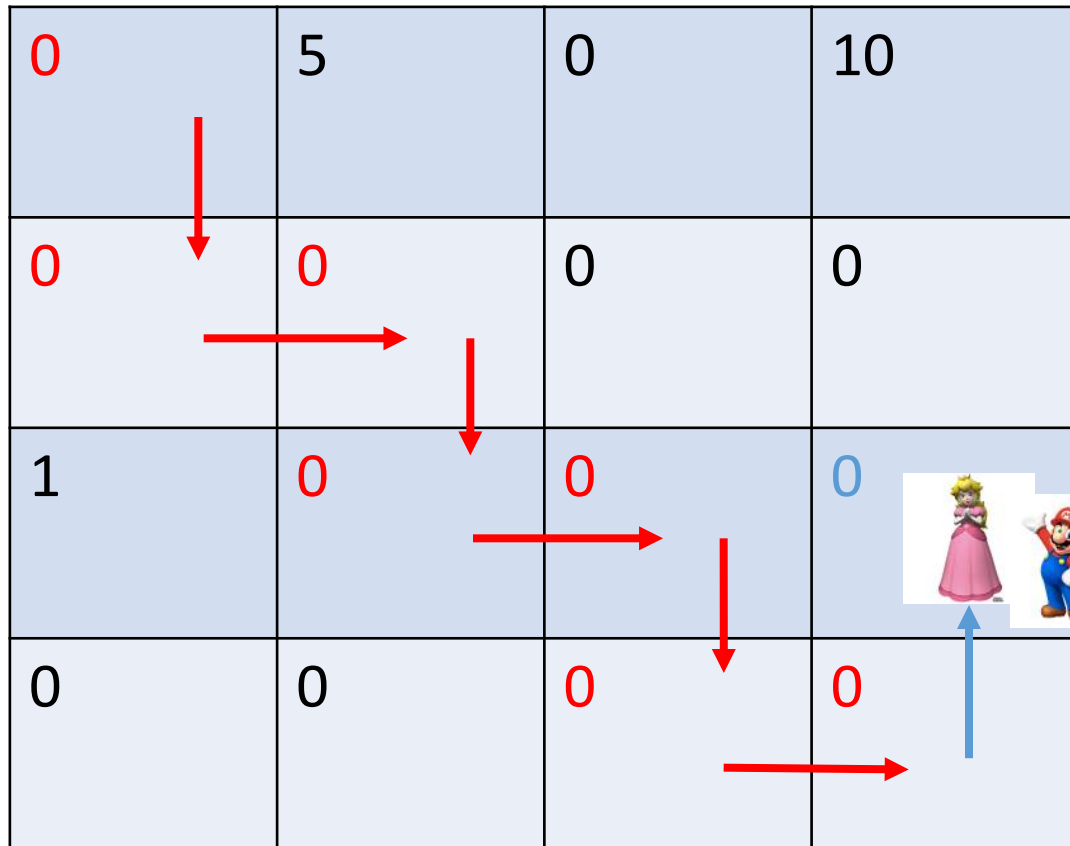
0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	0



Problem

$$\text{Score} = 0 + 5 + 5 + 1$$

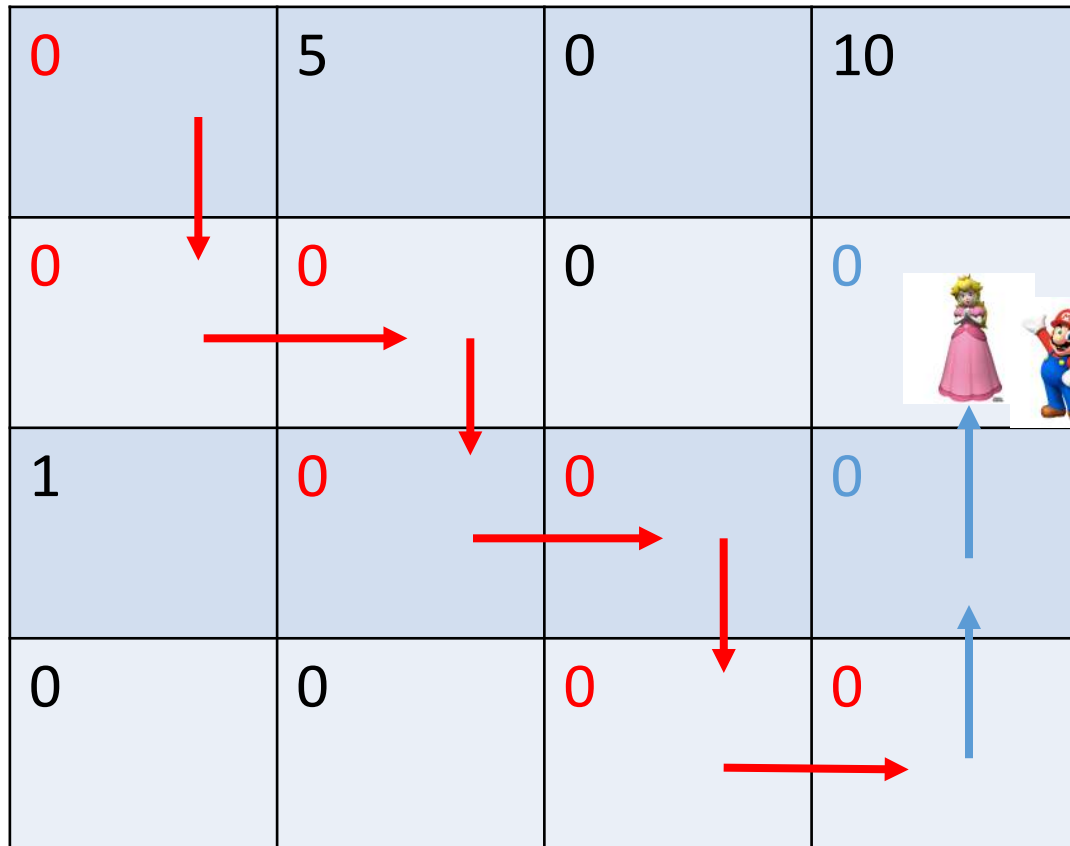
0	5	0	10
0	0	0	0
1	0	0	0
0	0	0	0



The grid shows a path of red arrows starting from the top-left cell (0,0) and ending at the bottom-right cell (3,3). The path consists of the following sequence of cells: (0,0) → (1,0) → (1,1) → (2,1) → (2,2) → (3,2) → (3,3). The values in these cells are 0, 0, 0, 0, 0, 0, and 0 respectively. The value in the top-right cell (0,3) is 10. The value in the cell (2,3) is 0. The value in the cell (3,0) is 1. The value in the cell (1,2) is 0. The value in the cell (2,0) is 5. The value in the cell (3,1) is 0. The value in the cell (0,1) is 0. The value in the cell (1,3) is 0. The value in the cell (2,3) is 0. The value in the cell (3,0) is 1. The value in the cell (3,1) is 0. The value in the cell (3,2) is 0. The value in the cell (3,3) is 0. A blue arrow points up to the cell (3,3). In the cell (3,3), there is an image of Princess Peach and Mario.

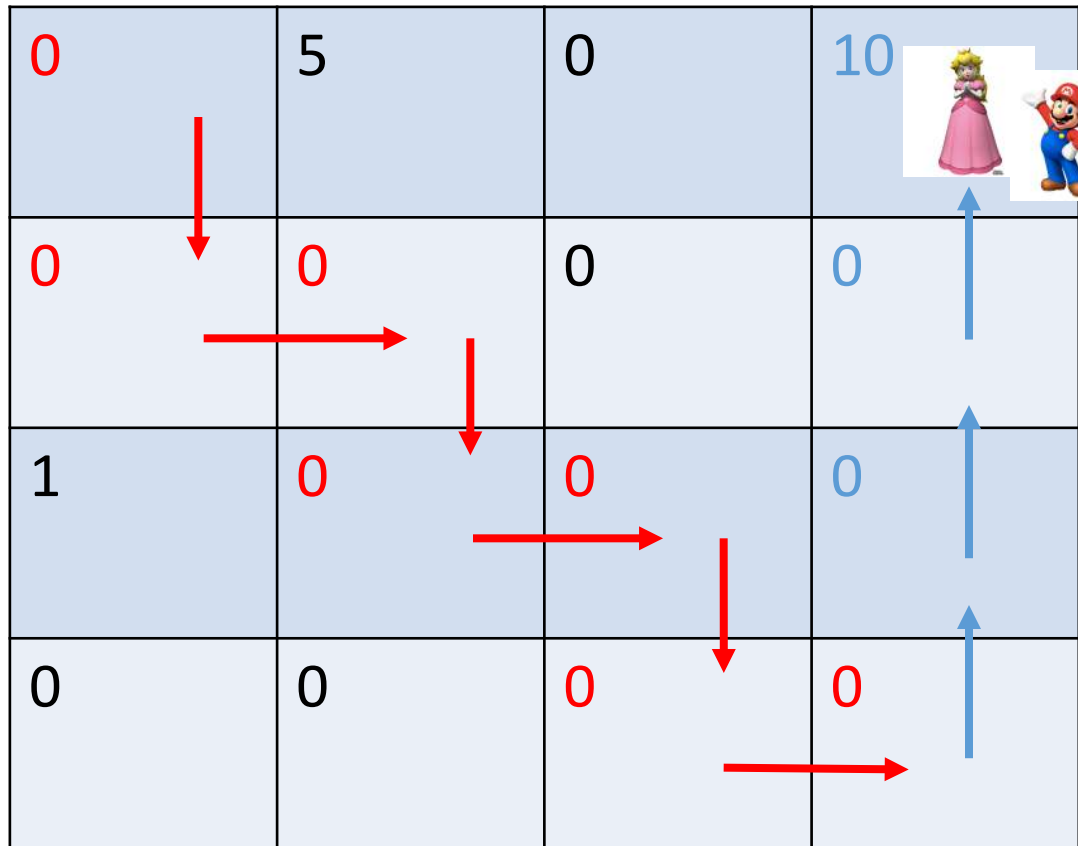
Problem

$$\text{Score} = 0 + 5 + 5 + 1$$



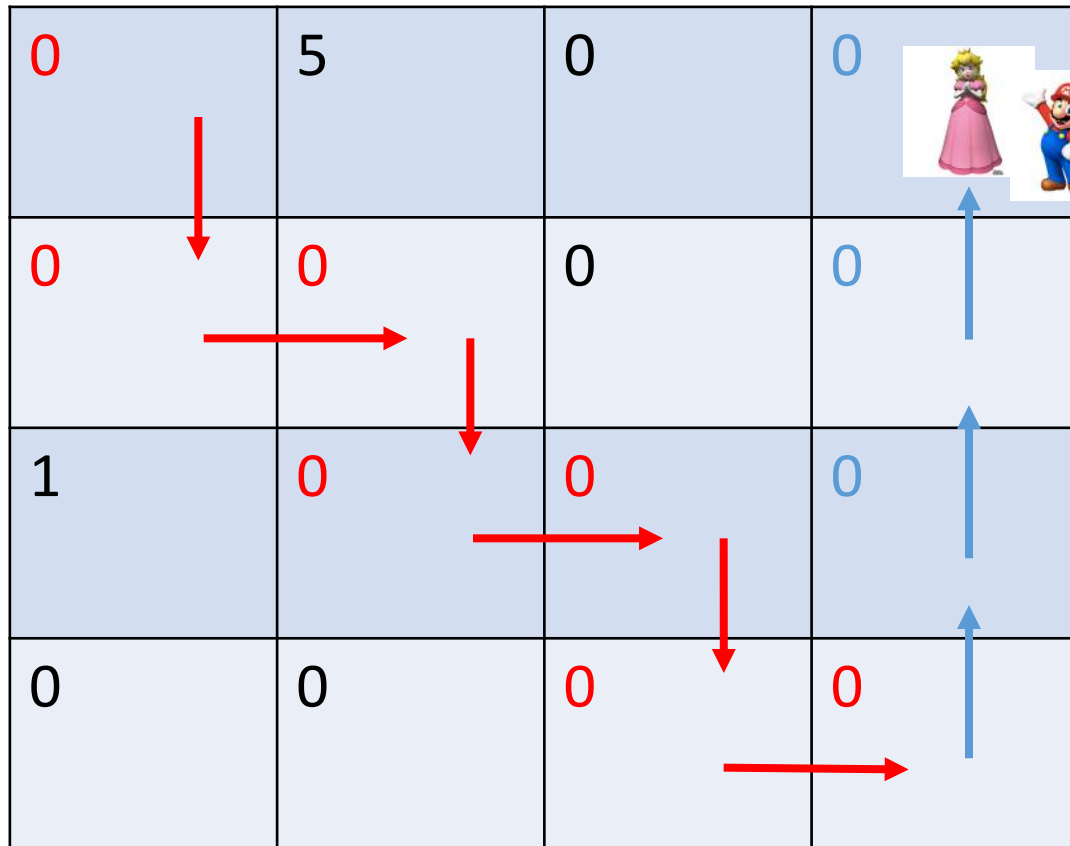
Problem

Score = 0 + 5 + 5 + 1



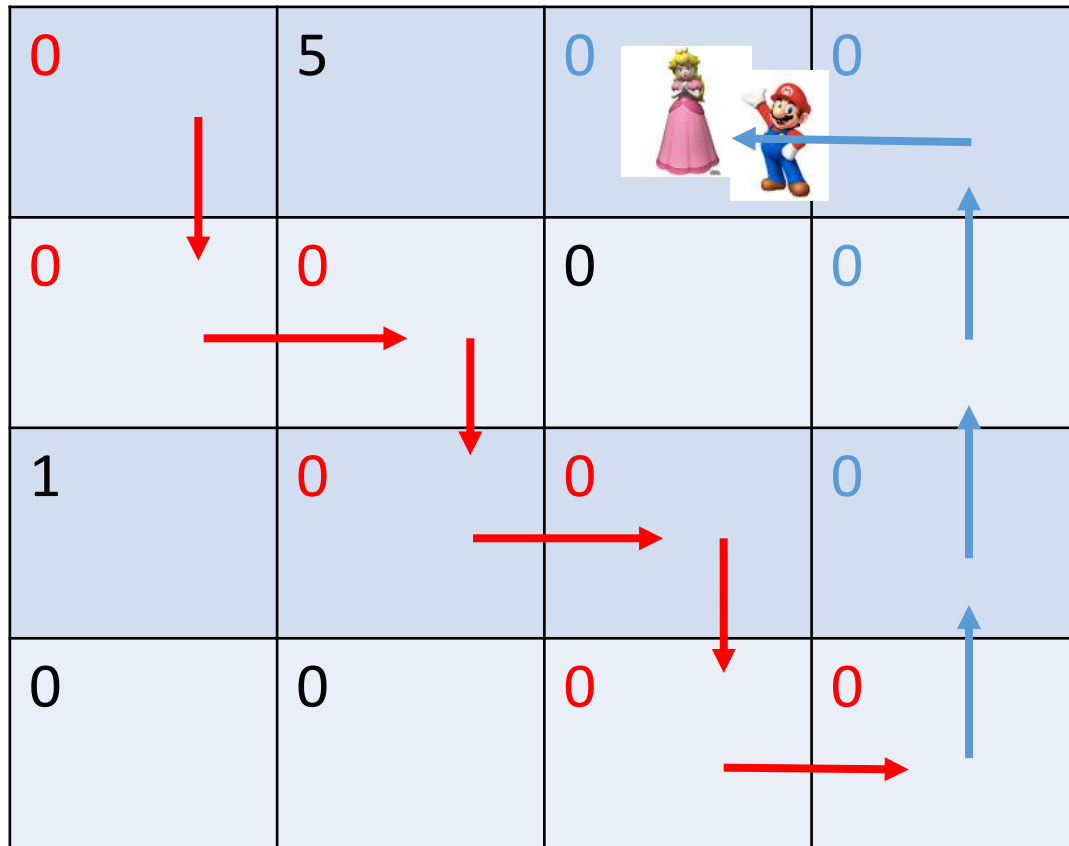
Problem

Score = 0 + 5 + 5 + 1 + 10



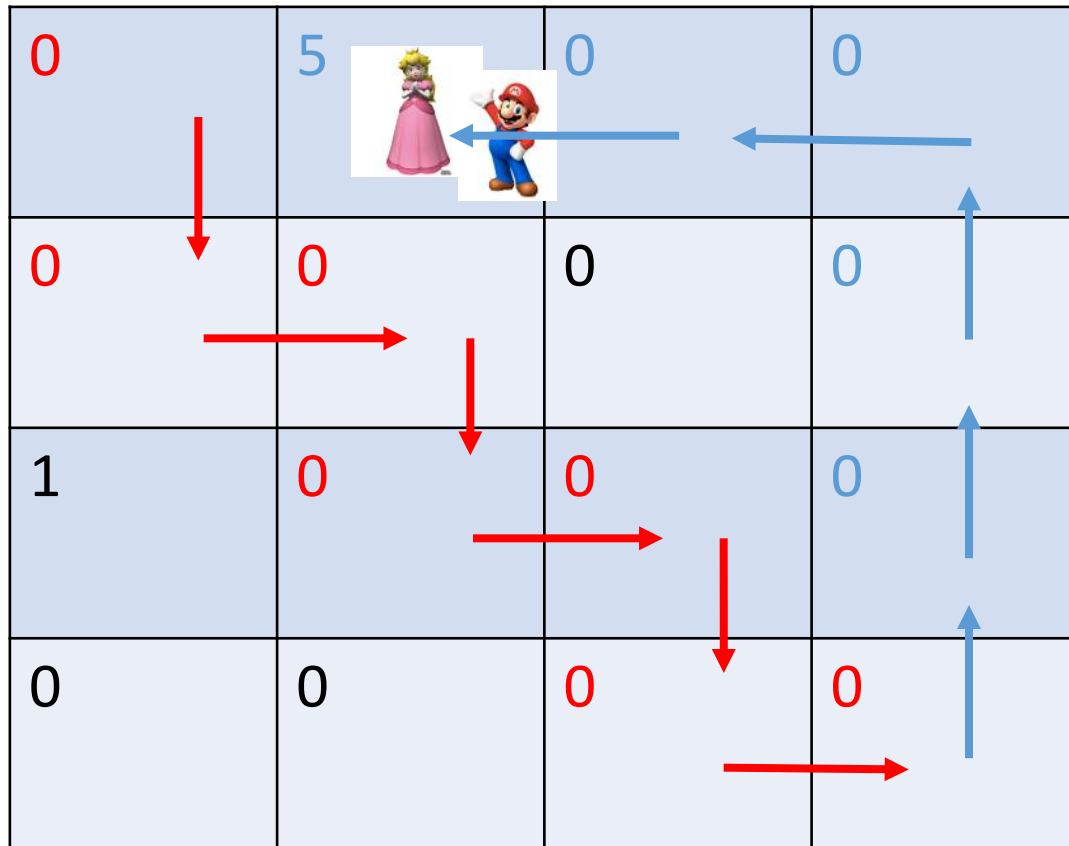
Problem

$$\text{Score} = 0 + 5 + 5 + 1 + 10$$



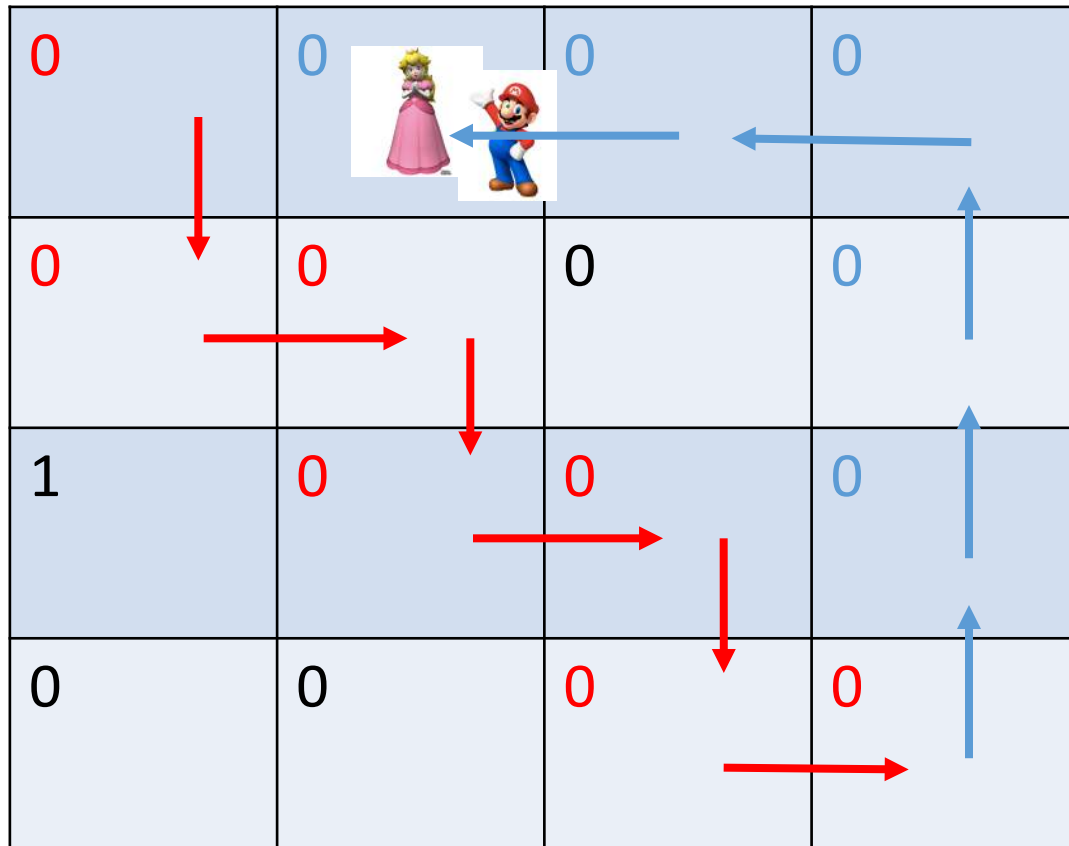
Problem

Score = 0 + 5 + 5 + 1 + 10



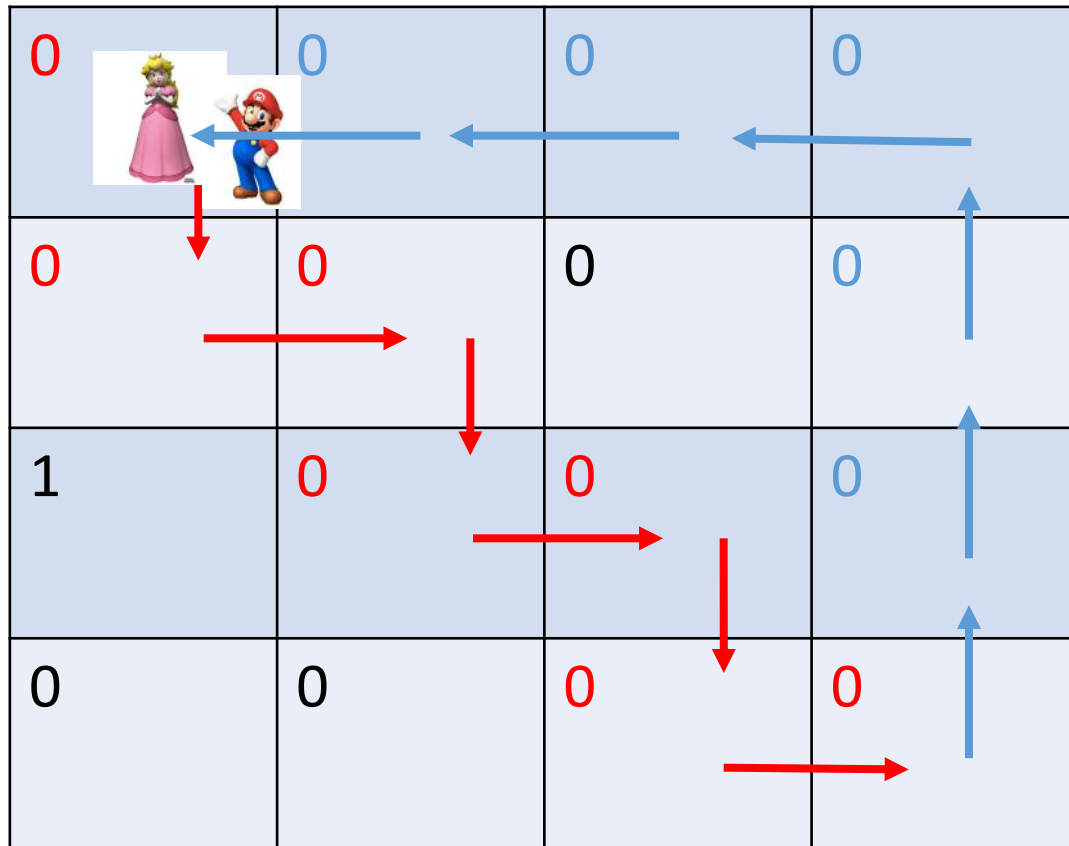
Problem

Score = 0 + 5 + 5 + 1 + 10 + 5



Problem

$$\text{Score} = 0 + 5 + 5 + 1 + 10 + 5 = 26$$



Problem

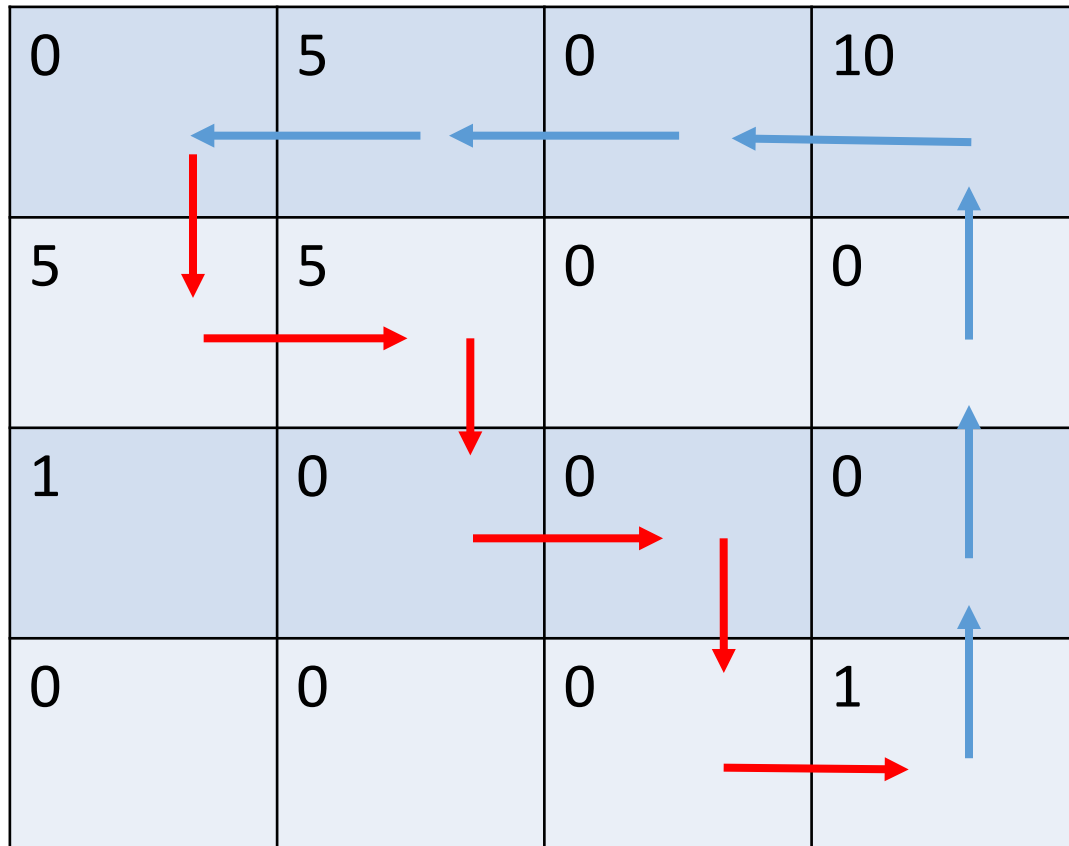
$$\text{Score} = 0 + 5 + 5 + 1 + 10 + 5 = 26$$

0	5	0	10
5	5	0	0
1	0	0	0
0	0	0	1

The diagram shows a 4x4 grid with numerical values in each cell. Red arrows indicate a path starting from the top-left cell (0,0) and ending at the bottom-right cell (3,3). The path follows the sequence of cells: (0,0) → (1,0) → (1,1) → (2,1) → (2,2) → (3,2) → (3,3). Blue arrows indicate another path starting from the top-right cell (3,0) and ending at the bottom-right cell (3,3). The path follows the sequence of cells: (3,0) → (3,1) → (3,2) → (3,3).

Problem

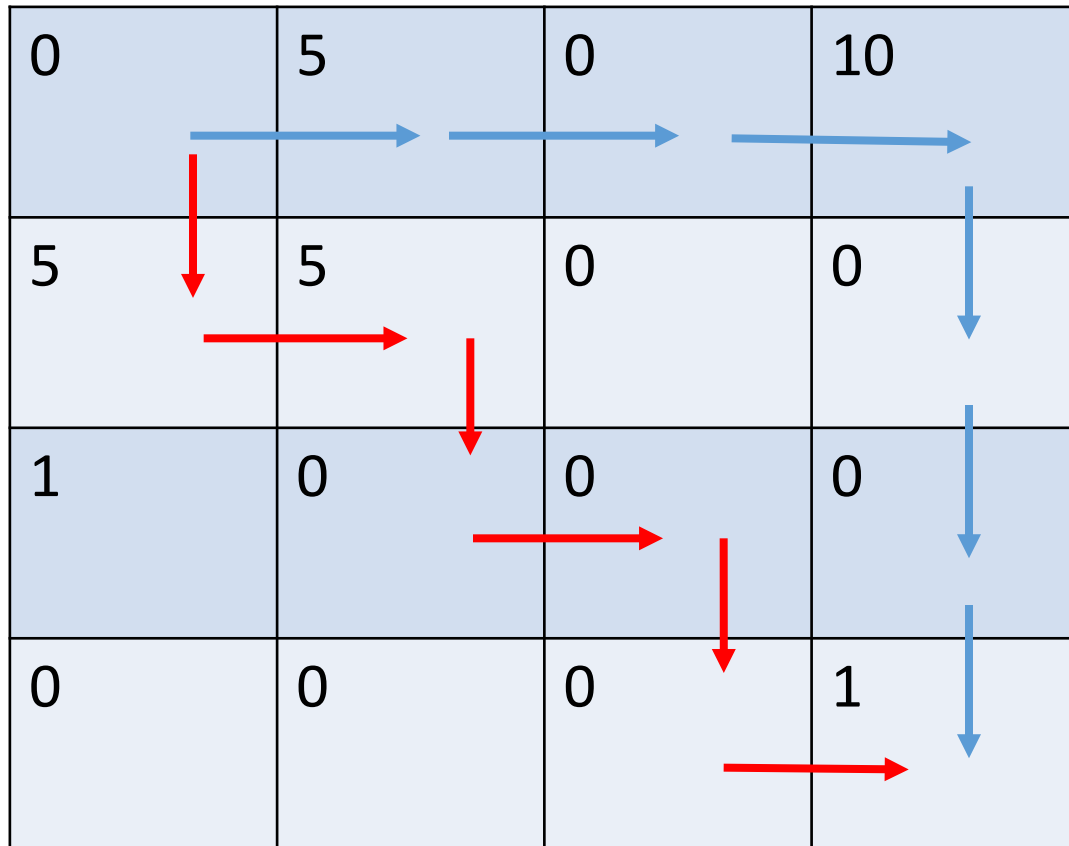
$$\text{Score} = 0 + 5 + 5 + 1 + 10 + 5 = 26$$



Observation: reverse blue edges

Problem

$$\text{Score} = 0 + 5 + 5 + 1 + 10 + 5 = 26$$

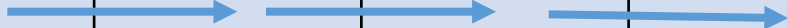


Observation: reverse blue edges

Problem

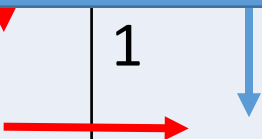
$$\text{Score} = 0 + 5 + 5 + 1 + 10 + 5 = 26$$

0	5	0	10
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Find two (\rightarrow , \downarrow)-paths which maximize the number of points

0	0	0	1
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Observation: reverse blue edges

Solution

- Model the problem using **flows** (min cost max flow)
- Dynamic programming

Solution

- Model the problem using **flows** (min cost max flow)
- **Dynamic programming**

Solution - DP

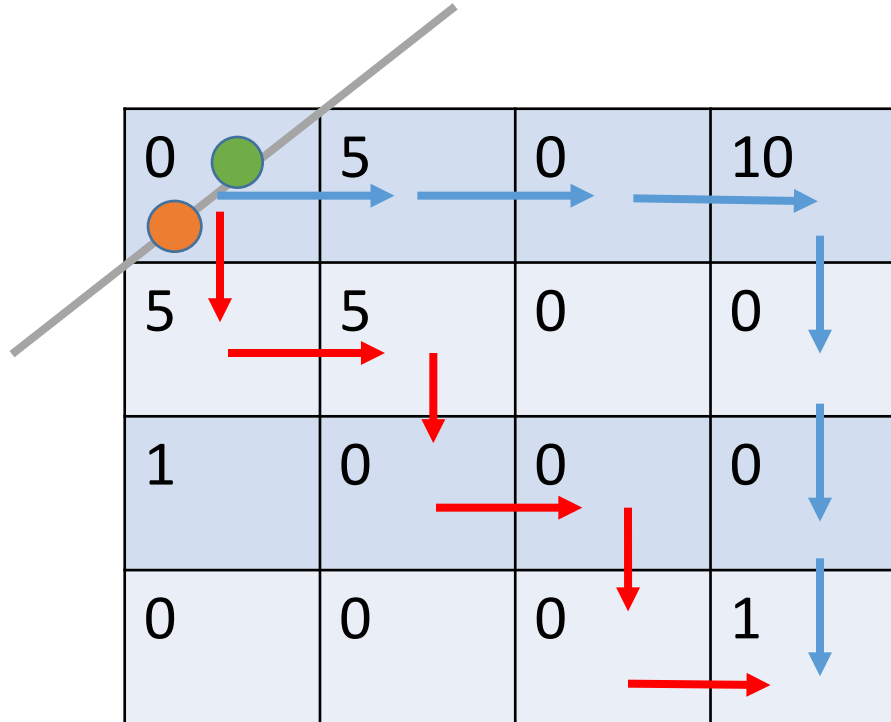
- If we move along paths at the **same time**, we are always on the **same diagonal**

0	5	0	10
5	5	0	0
1	0	0	0
0	0	0	1

The diagram shows a 4x4 grid with values. Blue arrows indicate a path from (0,0) to (0,3) to (1,3) to (2,3) to (3,3). Red arrows indicate a path from (0,0) to (1,0) to (2,0) to (3,0). Both paths are on the same diagonal (i-j = 0).

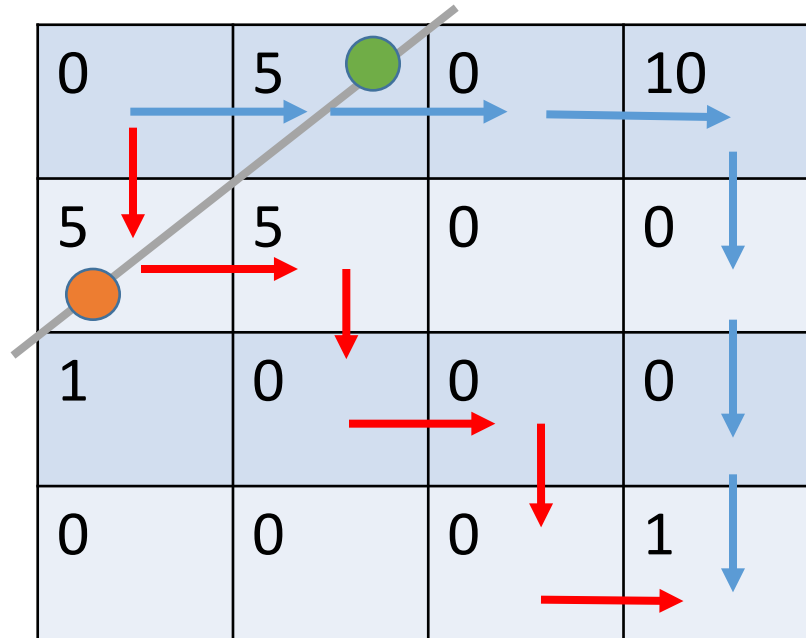
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



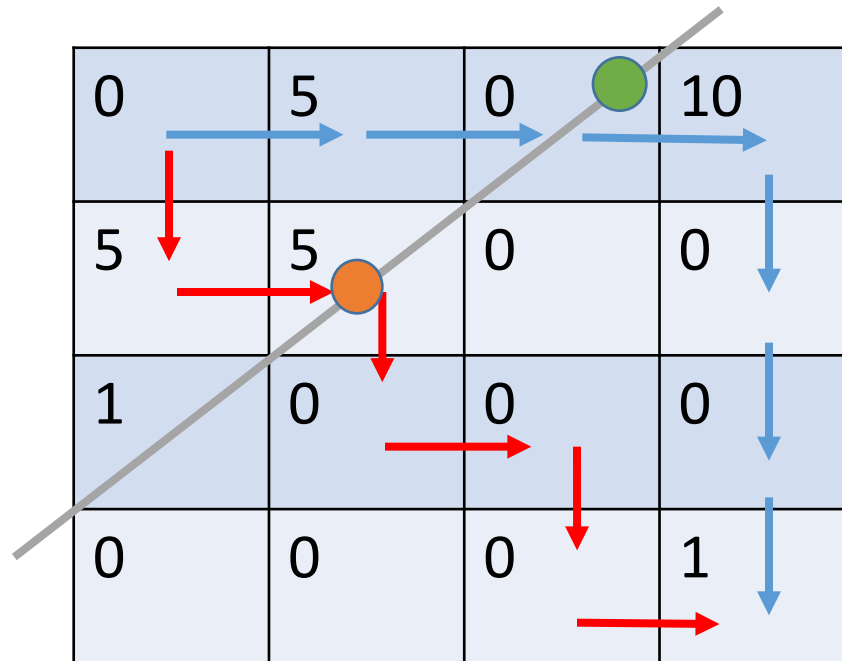
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



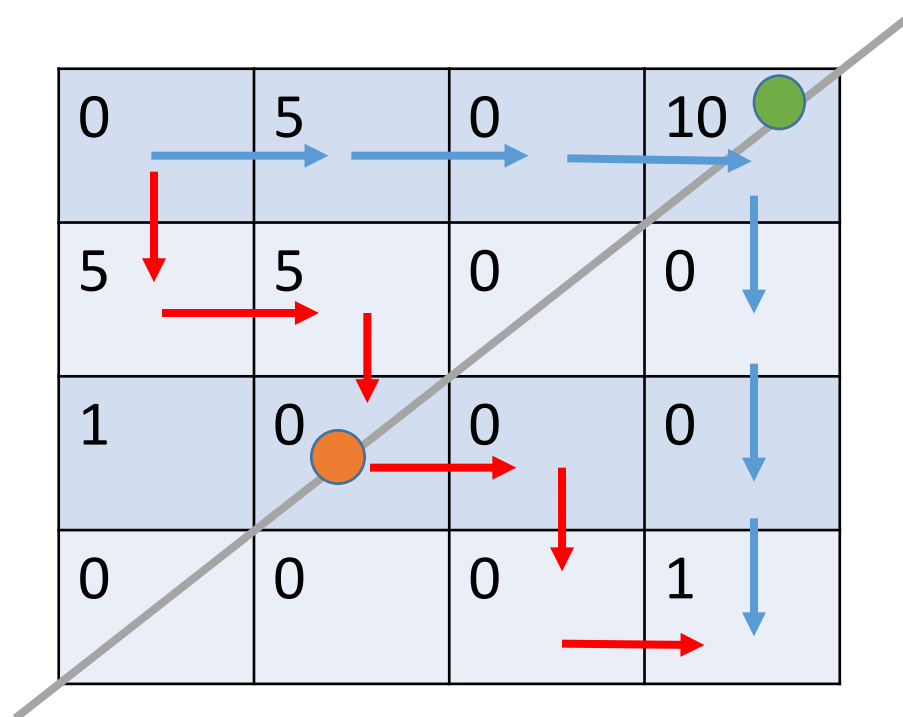
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



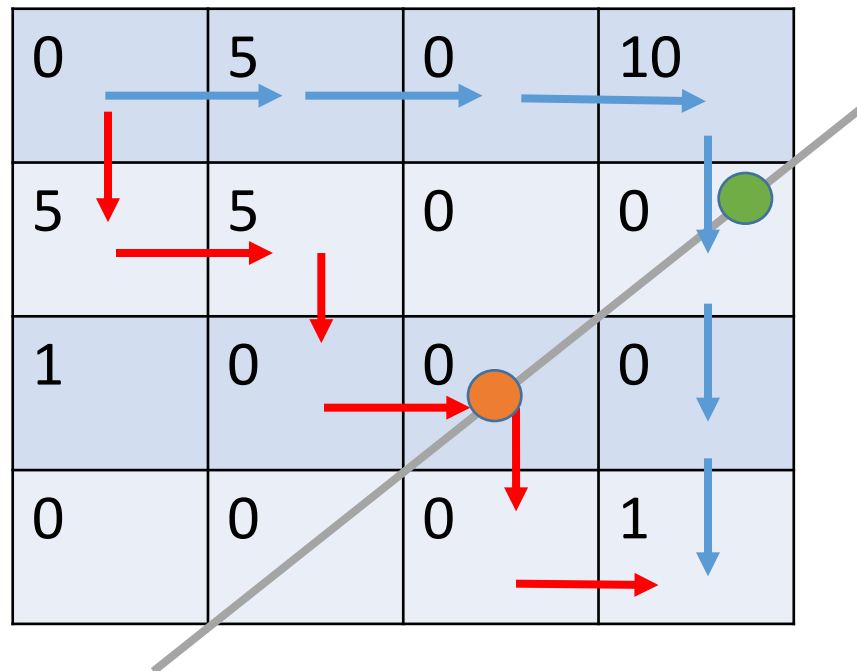
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



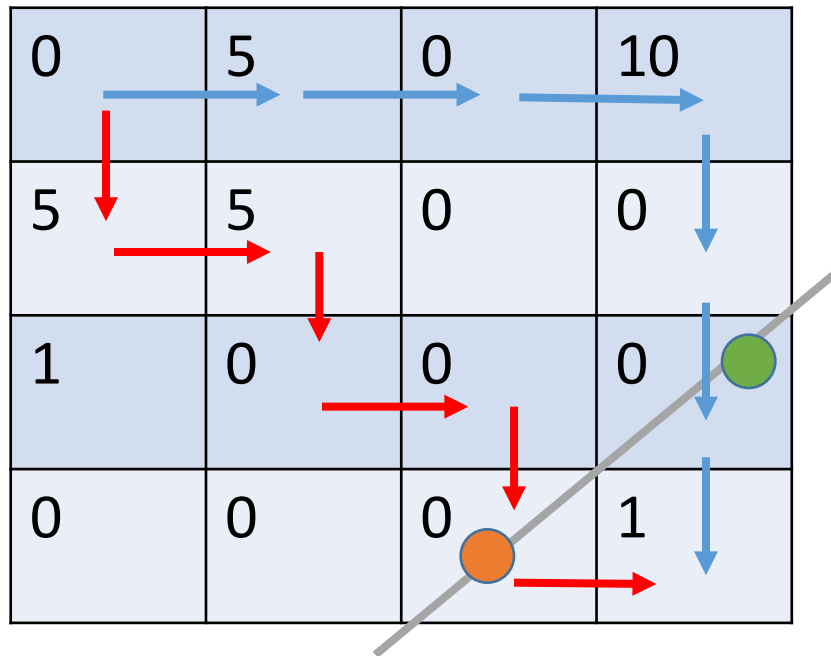
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



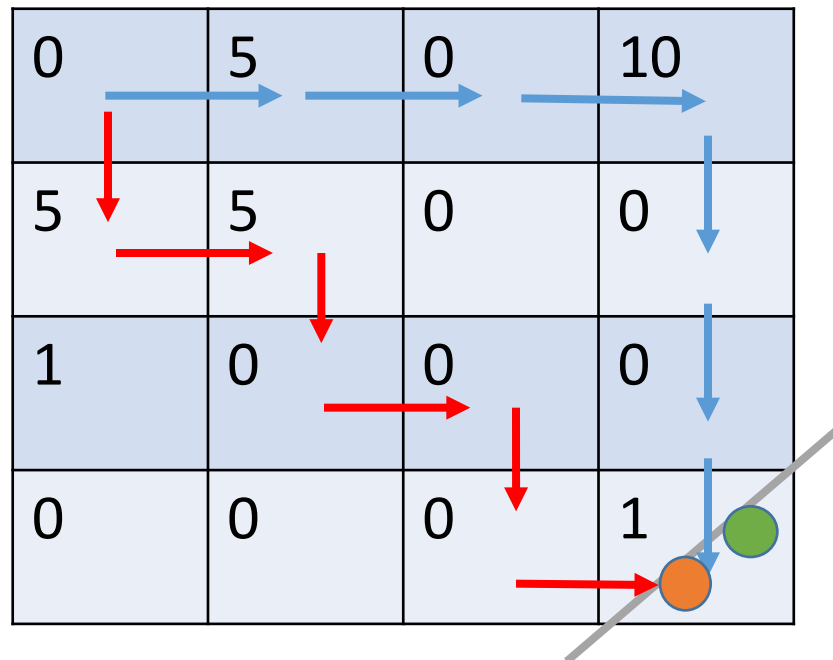
Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**



Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**
- Therefore, the tokens can only intersect at the same time

Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**
- Therefore, the tokens can only intersect at the same time

DP[d, r1, r2] = maximum number of coins we can collect if we start from the position:

- i. the first token is on the cell which intersects diagonal **d** and row **r1**, and
- ii. the second token is on the cell which intersects diagonal **d** and row **r2**

Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**
- Therefore, the tokens can only intersect at the same time

maximum number of coins we can collect if we start from the position:

DP[d, r1, r2] = i. the first token is on the cell which intersects diagonal **d** and row **r1**, and
ii. the second token is on the cell which intersects diagonal **d** and row **r2**

$$\text{DP}[d, r1, r2] = \begin{aligned} &\text{coins}(d, r1) + (\text{coins}(d, r2) \text{ if } r2 \neq r1) + \\ &\max(\quad \text{DP}[d + 1, r1, r2], \\ &\quad \text{DP}[d + 1, r1 + 1, r2], \\ &\quad \text{DP}[d + 1, r1, r2 + 1], \\ &\quad \text{DP}[d + 1, r1 + 1, r2 + 1] \quad) \end{aligned}$$

Solution - DP

- If we move along paths at the **same time**, we are always on the **same diagonal**
- Therefore, the tokens can only intersect at the same time

maximum number of coins we can collect if we start from the position:

DP[d, r1, r2] = i. the first token is on the cell which intersects diagonal **d** and row **r1**, and
ii. the second token is on the cell which intersects diagonal **d** and row **r2**

DP[d, r1, r2] = coins(d, r1) + (coins(d, r2) if r2 ≠ r1) +

max(DP[d + 1, r1, r2],

DP[d + 1, r1 + 1, r2],

DP[d + 1, r1, r2 + 1],

DP[d + 1, r1 + 1, r2 + 1])

Running time: $O(n^3)$