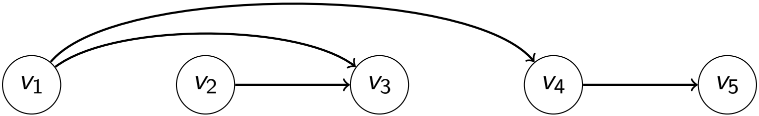
Consecutive Constructions

The problem

Problem

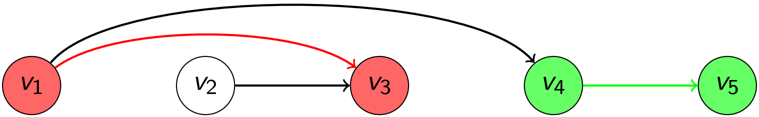
Given a DAG G. Find the maximum number of edges that can be packed in vertex-disjoint paths in G.

Example



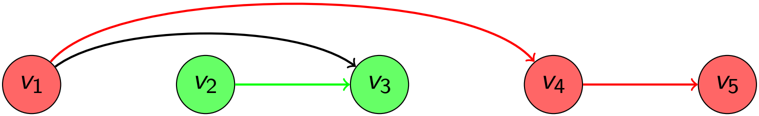
Example – greedy solution

Greedy solution with two edges.



Example – optimal solution

Optimal solution with three edges.



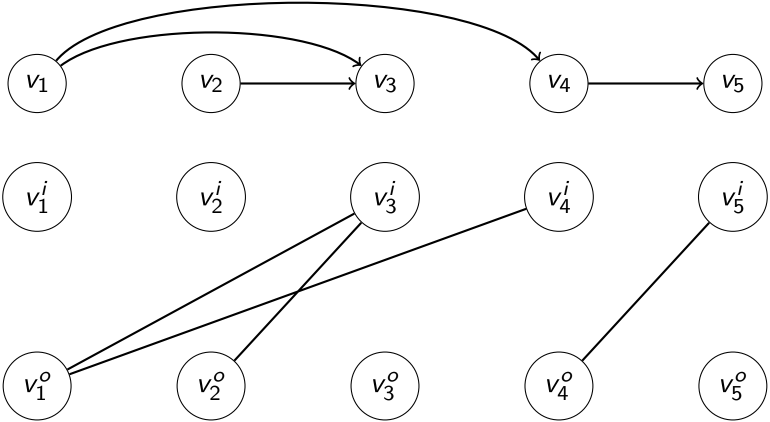
Possible approaches

- Greedy – we've just seen it doesn't work.
- DP – the graph is a DAG, so there is some hope. However, we hit the wall pretty soon (assuming you take the edge v_1 to v_k, you still need a solution on $v_2, \ldots, v_{k-1}, v_{k+1}, \ldots, v_n$ where parts "before" and "after" v_k are not independent).

Matching solution

Let $V_{out} := \{u_{out} : u \in V(G)\}$. Similarly, $V_{in} := \{u_{in} : u \in V(G)\}$.
Finally, let $E' := \{(u_{out}, v_{in}) : (u, v) \in E(G)\}$.
Consider bipartite $G' := (V_{out} \cup V_{in}, E')$.

Matching solution – example



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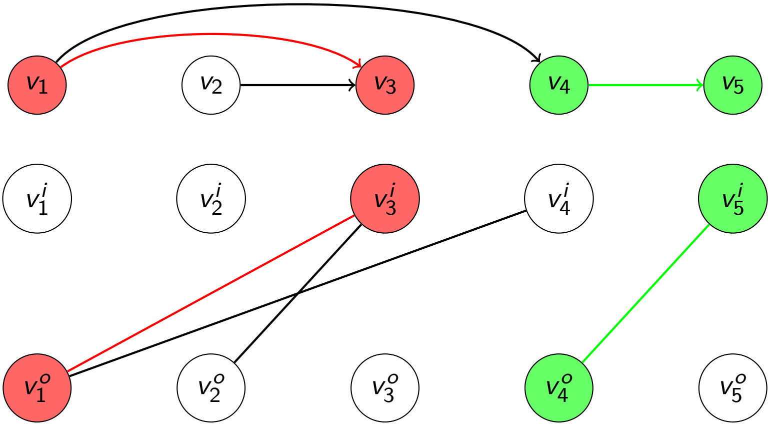
Lemma

There is a 1-1 to correspondence between solutions for G with k edges and matchings in G' of size k.

Therefore, the problem reduces to finding a maximum matching.

Matching solution – example



Matching solution – example