Algolab 2015
Radiation Therapy Solution

Antonis Thomas

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  $(1 \leq h + t \leq 50)$
- **Polynomial** p**:** of minimal degree d, s.t.
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Example: Polynomial of degree 2

$$1c_0 + x_1c_1 + x_2c_2 + x_3c_3 + x_1^2c_4 + x_1x_2c_5 + x_1x_3c_6 + x_2^2c_7 + x_2x_3c_8 + x_3^2c_9$$

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- Generating the polynomials — nested *for* loops
- Find the best degree — binary search on d

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- **Cycling**
  Use `CGAL::QP_BLAND`

Generating the polynomials

- ```
  for (int i = 0; i <= degree; ++i)
    for (int j = 0; j <= degree-i; ++j)
      for (int k = 0; k <= degree-i-j; ++k) {
        /* term x_1^i * x_2^j * x_3^k */
        pw(p.x1,i)*pw(p.x2,j)*pw(p.x3,k);
      }
  ```

For efficiency, you can also precompute the values of `pw(p.x[1-3],i)` for $i = 0, \ldots, 30$
  ```

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