Missing Roads

Problem

- Given: undirected graph G and an integer k
- Find: set of k cities $\{v_1, v_2, \dots, v_k\}$ and edges $\{e_1, e_2, \dots, e_k\}$ such that
  a) each city $\boldsymbol{v_i}$ is **adjacent** to the edge $\boldsymbol{e_i}$
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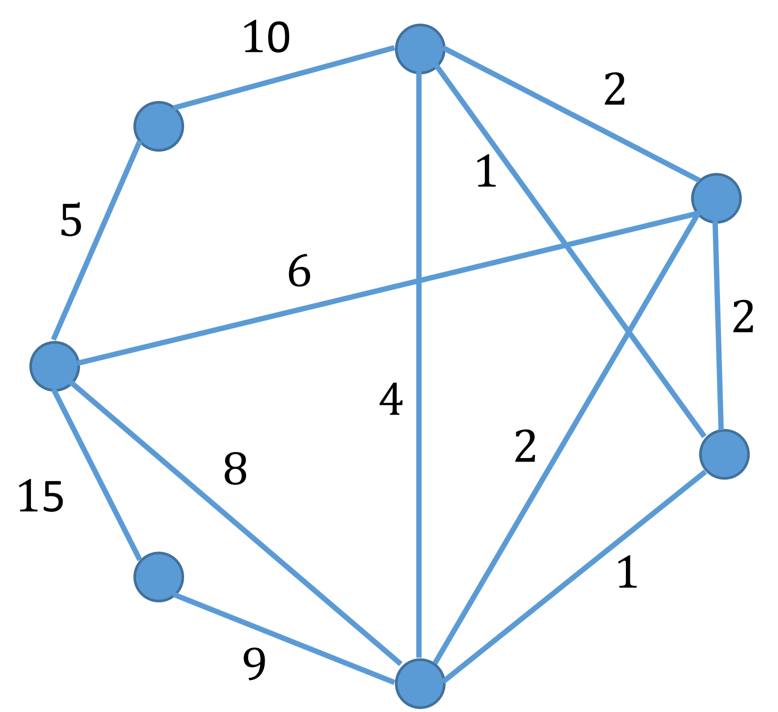

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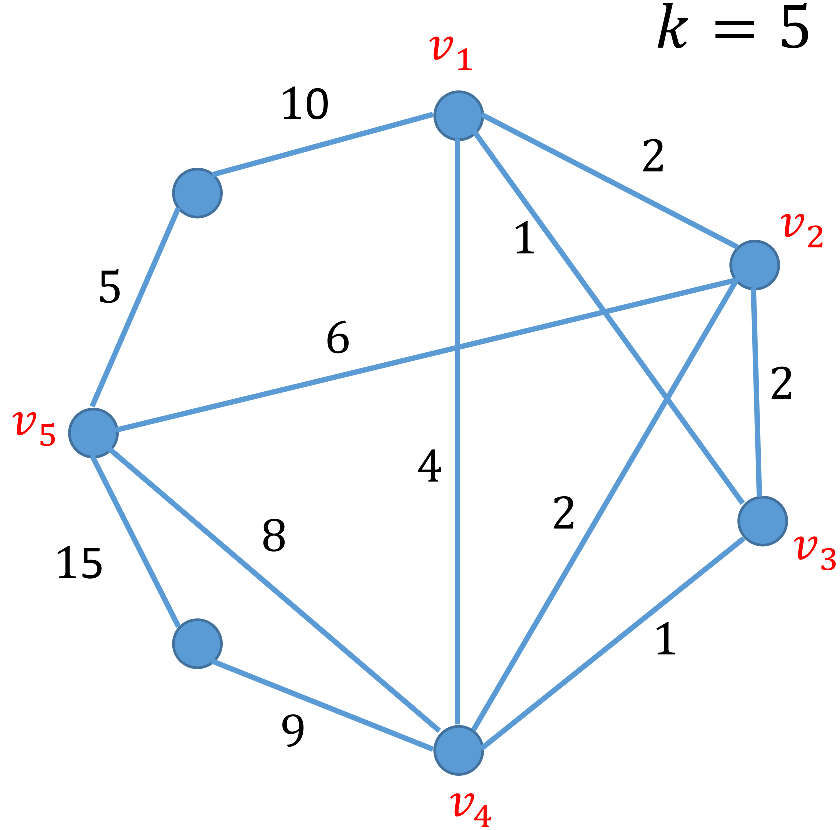

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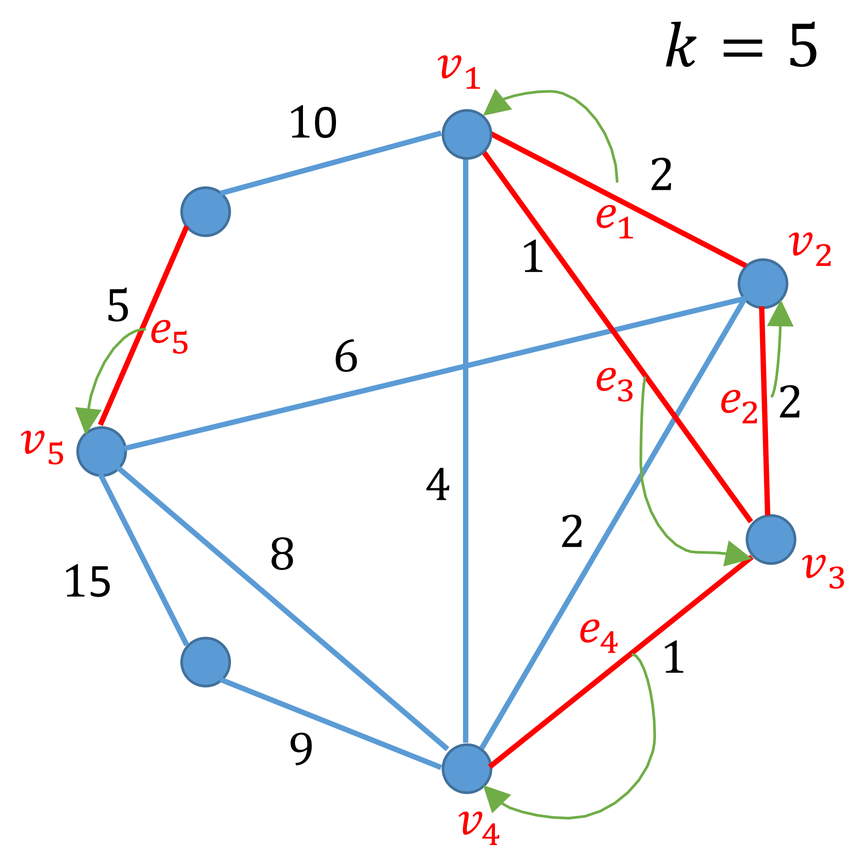

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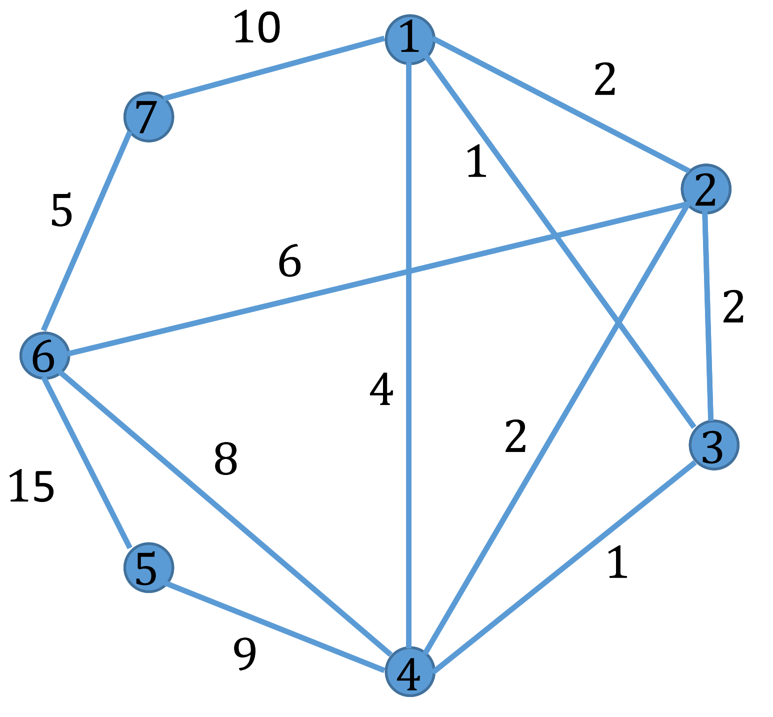

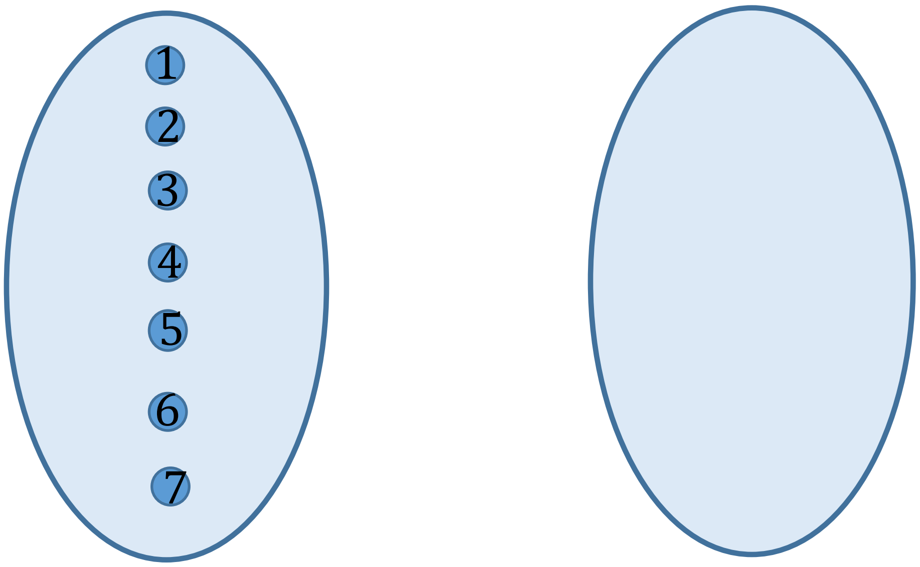

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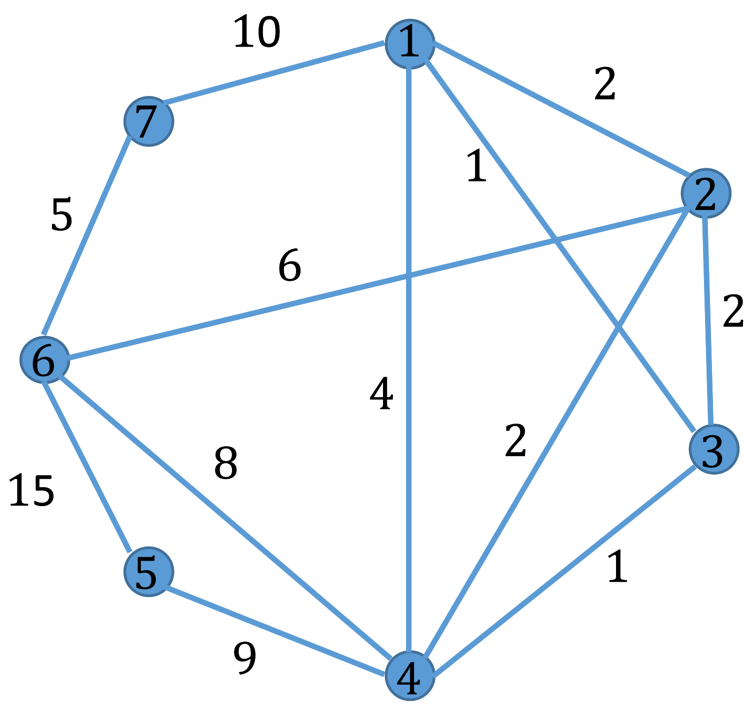

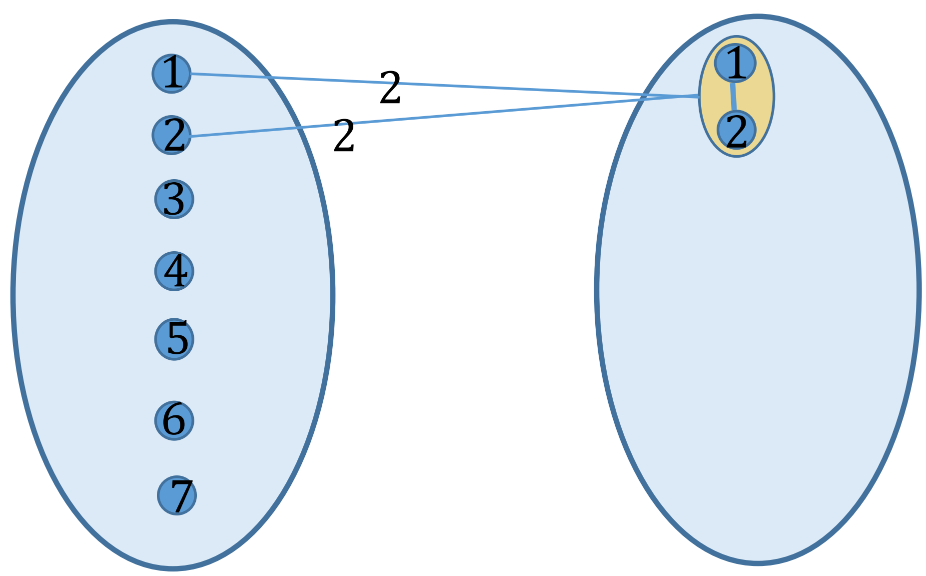

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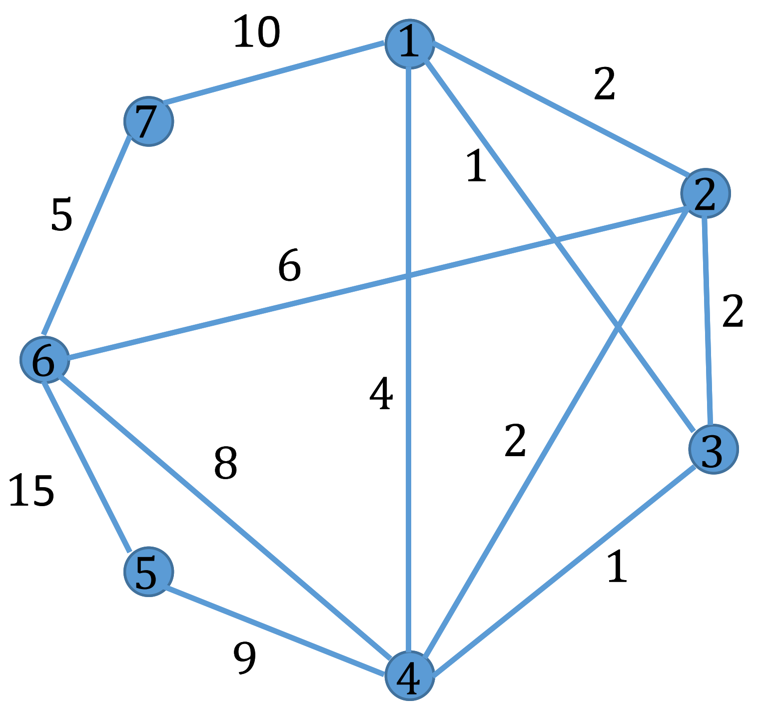

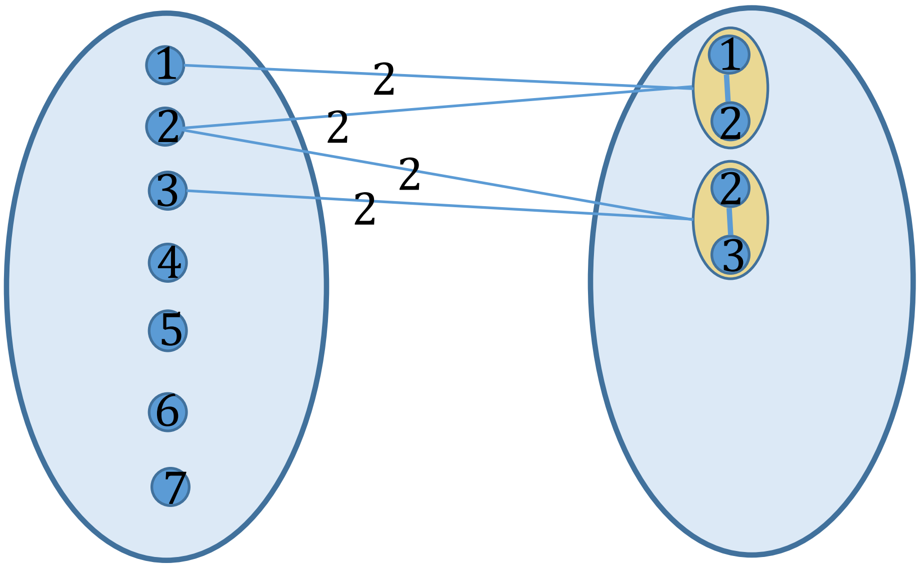

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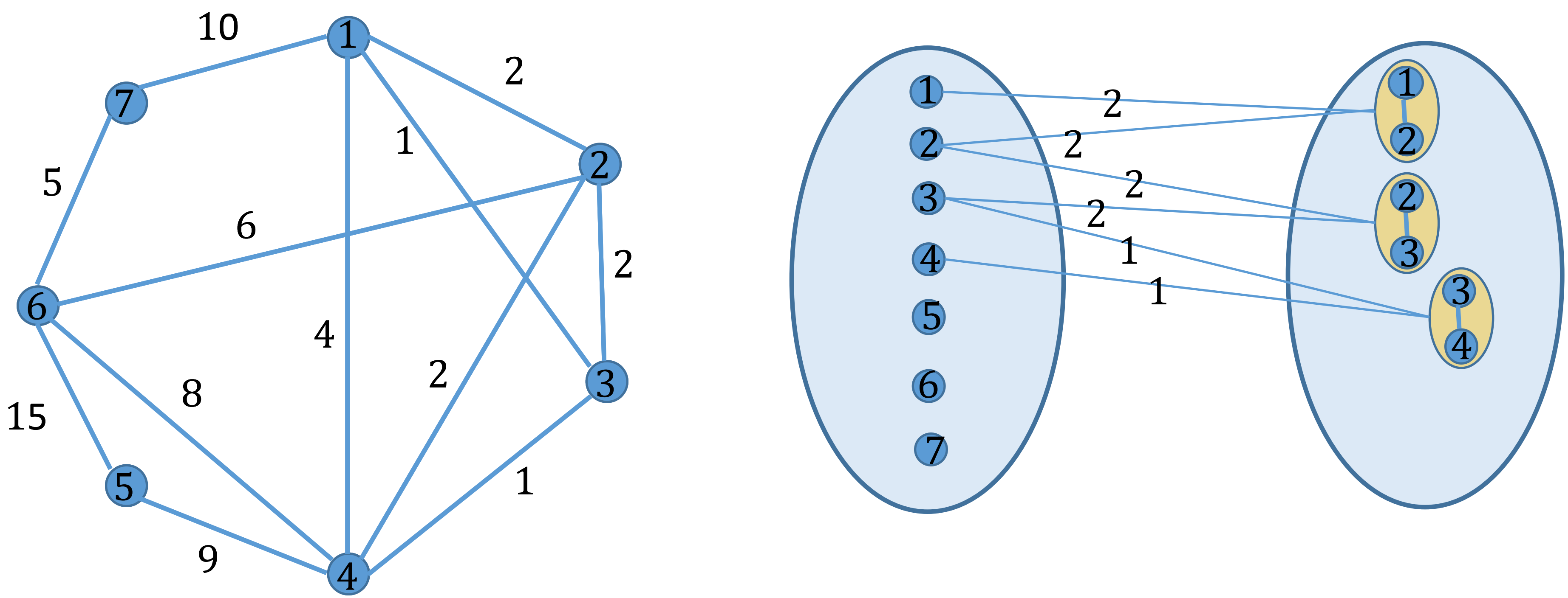

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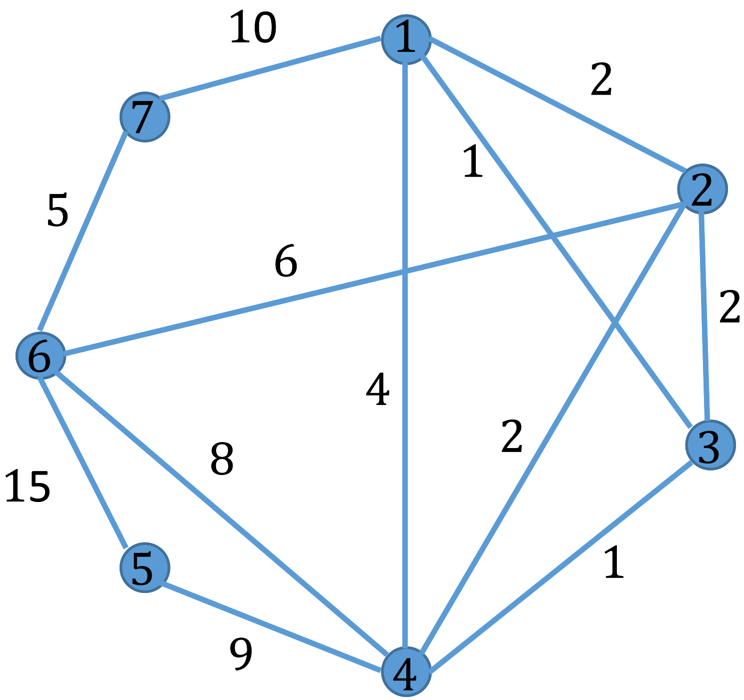

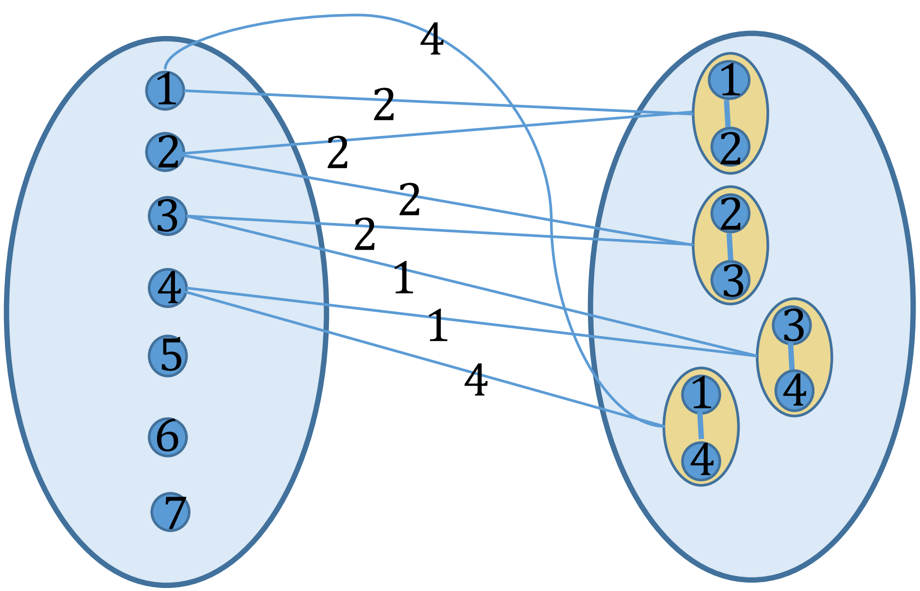

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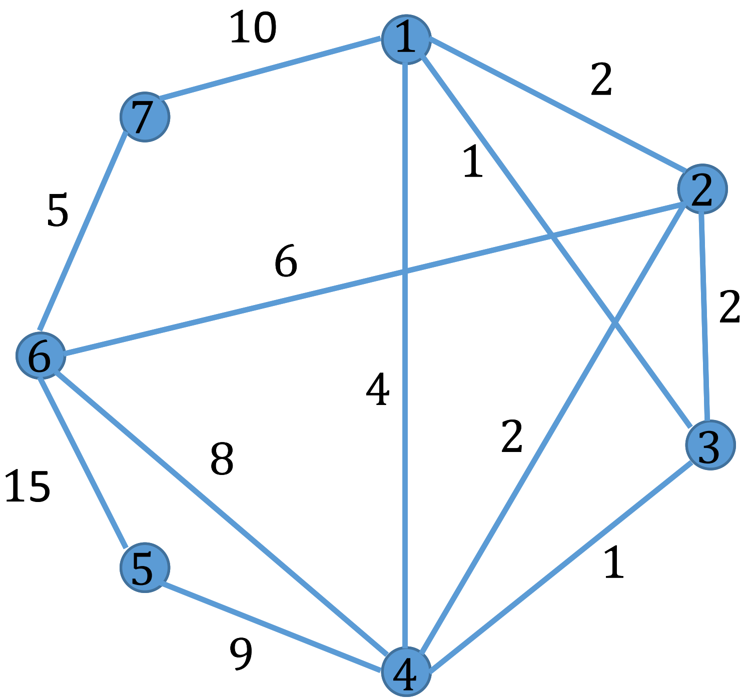

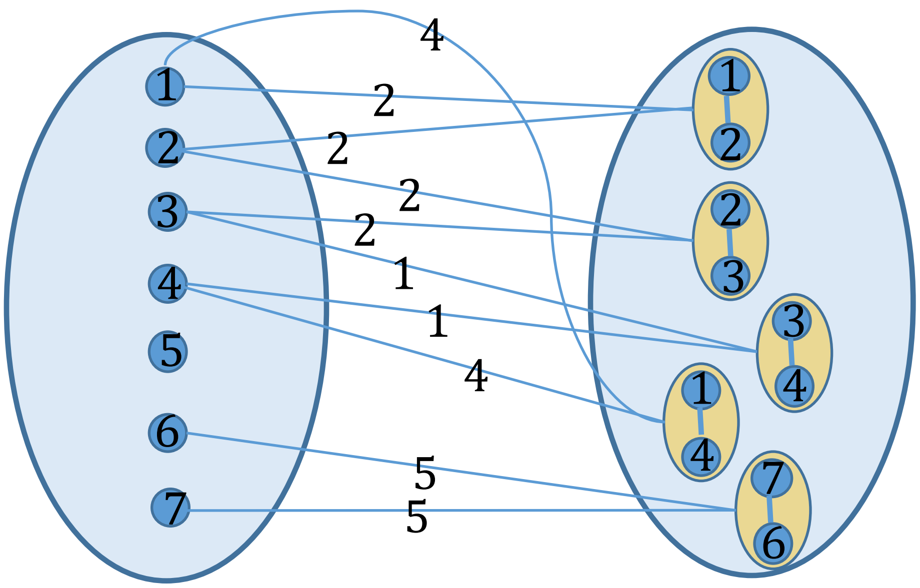

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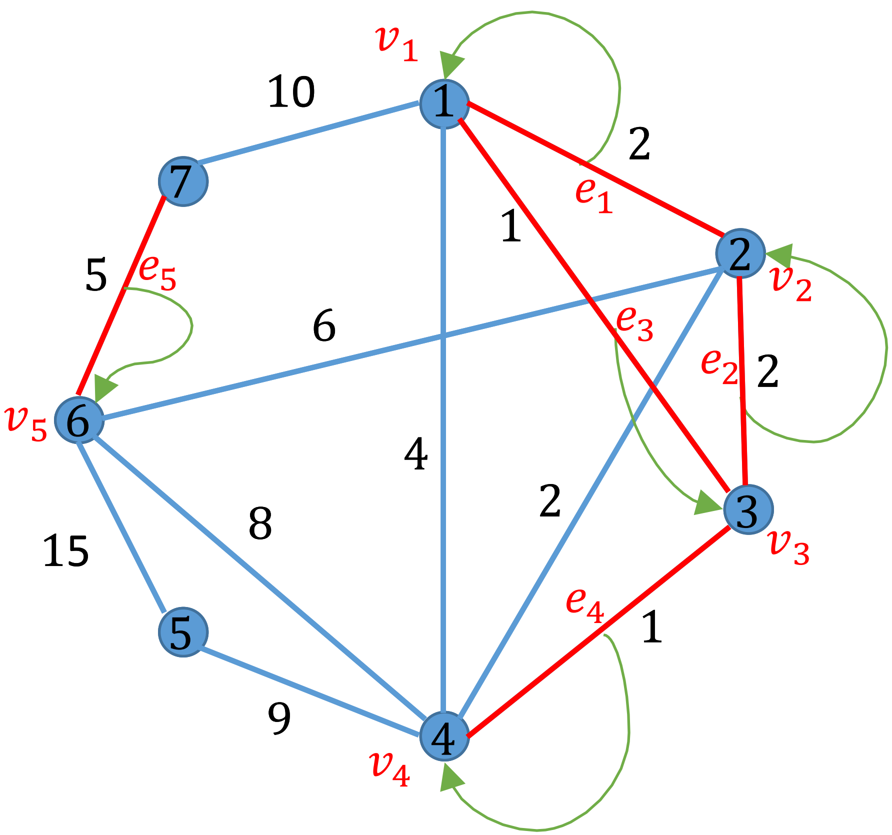

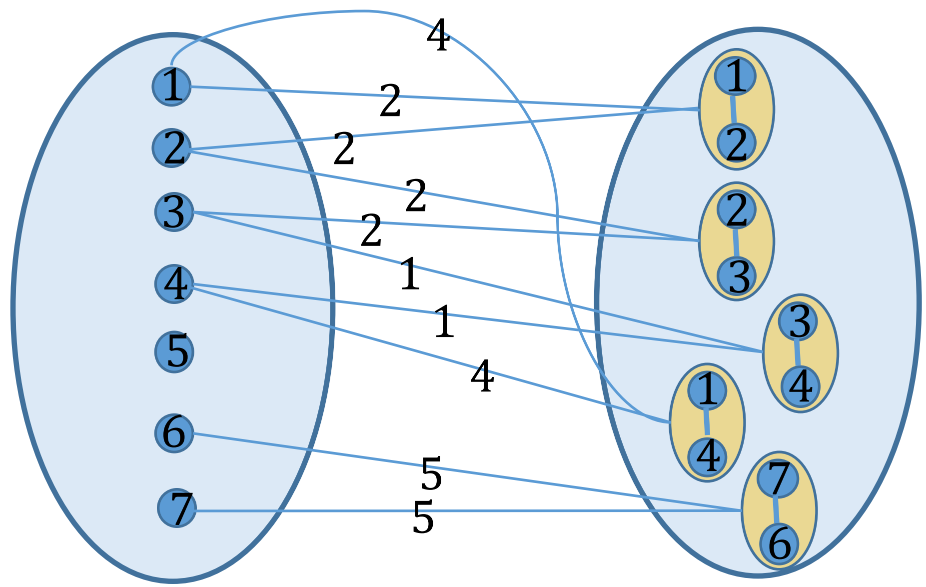

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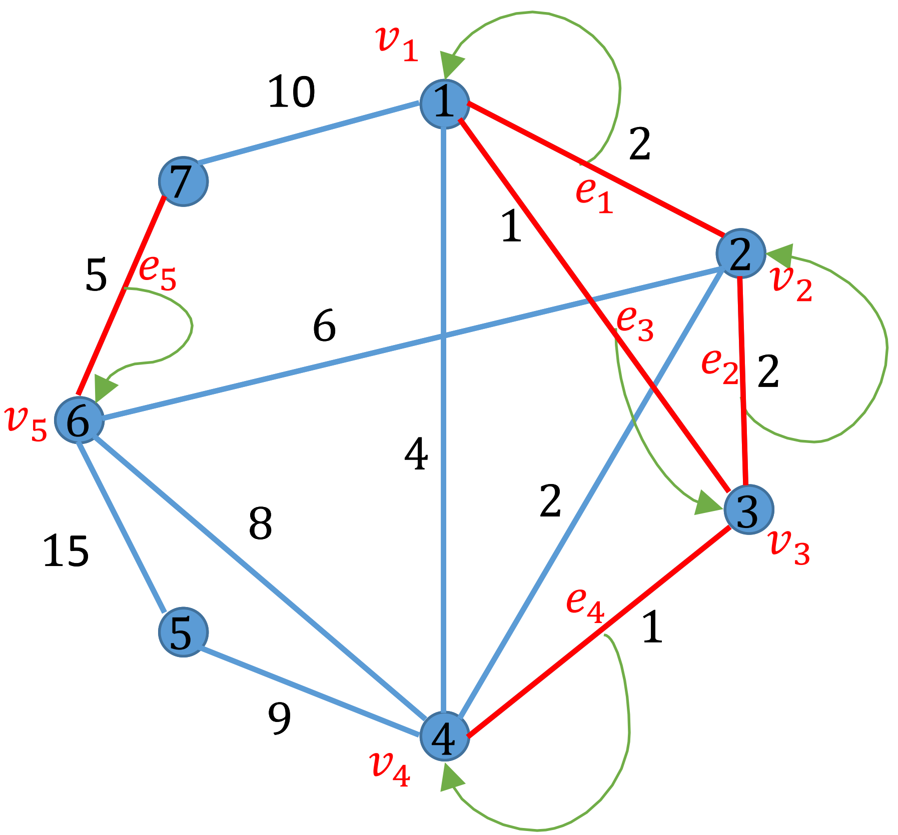

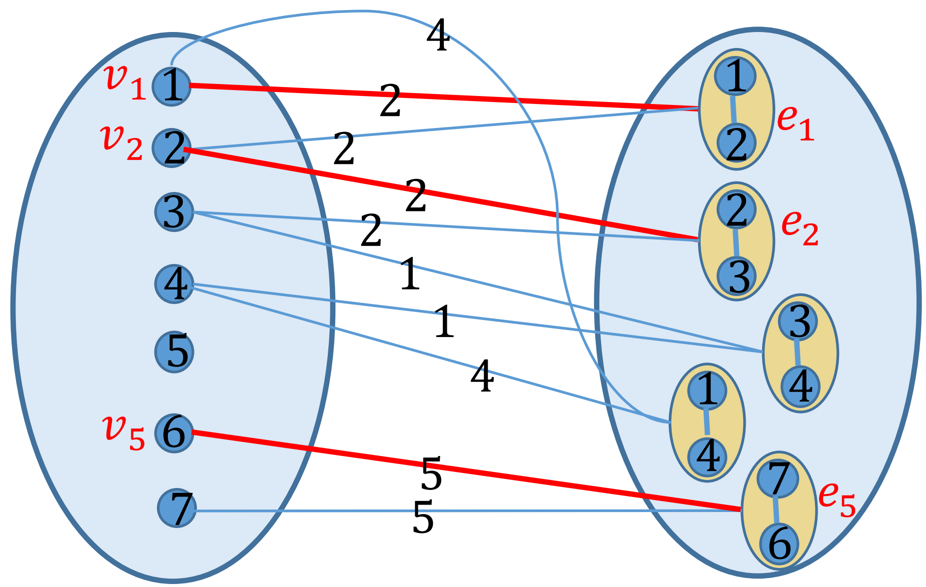

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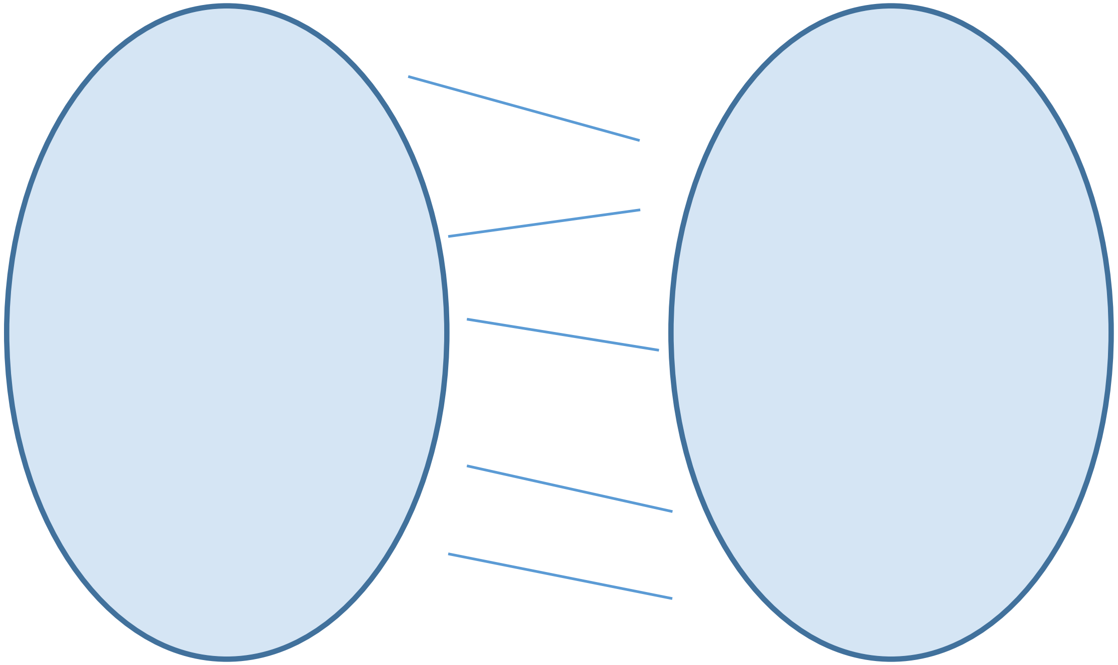

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