Task Discussion
Tetris

Andreas Bärtschi
based on slides by Jan Hązła

ETH Zürich

December 16, 2015

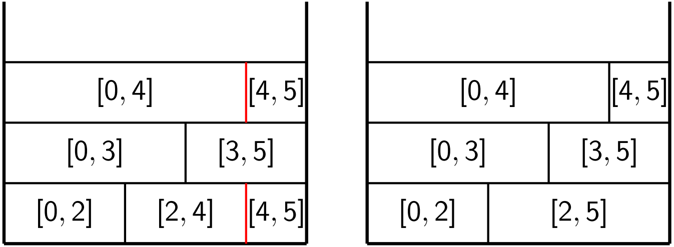
The problem

Given:

- A playing field of width w.
- n (height 1) rectangles with coordinates $[a_i, b_i]$.

Use the rectangles to create as many full lines $[0, w]$ as possible.

$w = 5$



Each rectangle can be used at most once. Moreover, each coordinate can be used as a **splitting point** between two rectangles at most once.

Problem size $\rightarrow$ Running time

$w < 500, \ n < 200'000$.
w small, n large.

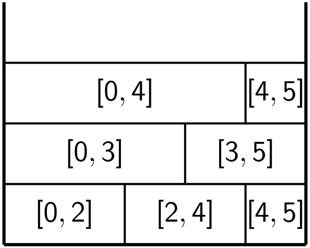
Acceptable running time for the algorithm:

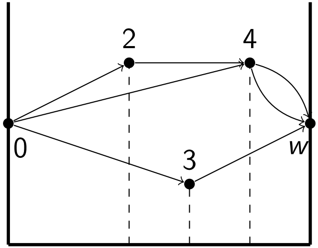
- $n \cdot w$,
- $n \cdot \log n$,
- w^3.

Solution – simplified case

What if there is no "splitting point" restriction?

Make a graph G with vertices $\{0, \ldots, w\}$.
For each rectangle $[a, b]$ create a directed edge (a, b) **with capacity 1.**





Each possible full line corresponds to 0-w path in G.
Therefore, we want to maximise the number of edge-disjoint 0-w paths.
But this is the **maximum 0-w flow** in G!

Solution & Implementation

What about the **splitting points restriction?**

It means that for each coordinate in $\{1, \ldots, w-1\}$
the flow through its vertex should be at most 1.

But this can be implemented with a standard trick:
replace each w with two vertices and an "internal" edge.

Implementation:

A single flow call to a graph with $2w$ vertices and $2w - 2 + n$ edges.

- Edmonds-Karp: $O(|f| \cdot |E|) = O(w \cdot n)$.
- Push-Relabel: $O(|V|^3) = O(w^3)$.
- Watch out for bordercases: There can be multiple $[0, w]$ edges.

Food for thought: Can you solve it when $w = 10^9$, and $n = 200$?