

## The Ant and the Grasshopper: A Fable of Algorithms

An algorithm is a set of specific steps or instructions for solving a problem.

One summer day a grasshopper came upon an ant who was collecting grain. The grasshopper watched as the ant struggled to remove a kernel from a fallen stalk. After a few minutes, the grasshopper spoke.

"Little ant, what are you doing?" the grasshopper asked.

"Collecting food for the winter," responded the ant in a weary voice. He was exhausted from a day of hard labor.

"But it is the middle of the summer," said the grasshopper. "Winter is not for months, and the food is plentiful. Why do you spend your day like this?"

The ant paused for a moment while he thought. "It is the algorithm that we use," he finally replied.

"Algorithm?" asked the grasshopper.

"A set of steps or instructions for accomplishing a task," explained the ant. "Like when a carpenter builds a chair, he uses an algorithm that includes measuring, cutting, smoothing, and hammering."

"What task does your algorithm solve?" asked the grasshopper. "Does it solve the problem of having too much time during the summer?" He chuckled out loud at his own joke.

"It accomplishes the task of keeping the colony healthy all year round. Everyday we have a set of tasks that we perform. During the summer we spend the mornings collecting food, the afternoons digging tunnels, and the nights sleeping. It might not sound like much, but it ensures that we have food during the cold of the winter."

"That sounds like a simple algorithm," remarked the grasshopper.

"Algorithms can be simple or complex," explained the ant. "They can even include steps that require other algorithms to solve. For example, when I collect food, I use a special food collection algorithm. It has five steps: 1) walk to the field, 2) search for a wheat stalk with grain on it, 3) remove a kernel of grain from the stalk, 4) carry the grain back to the ant hill, and 5) place the grain in the storage tunnel. I follow those exact steps to collect a giant pile of grain."

"That sounds boring," said grasshopper. "I do not use algorithms. I just do whatever I want, whenever I want. Complete freedom. In fact, I think I am going to climb to the top of the wheat stalk and sing for a while. I bet your algorithm does not let you do that."

The ant shrugged in response. He had his algorithm, and thus his next steps. It had worked for his colony for hundreds of years. While the grasshopper jumped away singing, the ant returned to his task.

### Epilogue:

Six months later, a harsh winter engulfed the land. The grasshopper scavenged the, now bare, wheat field for food. There was not a single kernel to be found.

At the same time, the ant was safe and warm in his colony's tunnels. He was hard at work following his winter day algorithm, which consisted of: digging tunnels, eating, and relaxing. He greatly preferred the winter algorithm to the summer one. As he worked on extended the eastern food tunnel, he paused and thought back to the grasshopper. He wondered if the grasshopper was still spending his days singing in the wheat fields or whether he had learned the value of a good algorithm.

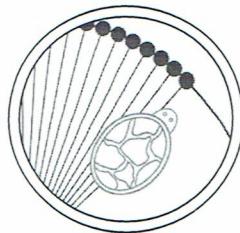
---- The Ant and the Grasshopper

- Modificar el algoritmo mencionado para consumir la hoja de una planta.
- Proponer un algoritmo para la recolección de néctar realizada por abejas.

----- Chapter 6: A Tinker's Trade

- Crear un algoritmo para dibujar un triángulo equilátero.
- Proponer una aplicación para cada uno de los algoritmos de dibujo de un círculo.
- Modificar el segundo algoritmo para obtener un círculo de manera que la tortuga inicie en el centro del círculo.

## CHAPTER 6



# A Tinker's Trade

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When Laurie and Xor were safely inside the town walls, the little lizard popped his head out of Laurie's pocket.

"See what I mean? Let's hope they don't figure out what you did to get in here," Xor said. "So, why are we here?"

"We're looking for information that could help me get home. Maybe we can find a map or something."

"Oh," said Xor. "I was hoping you were going to say food. Why don't we try this place?"

In front of them was a storefront with a very fancy sign painted on the window:

N. Veterate Tinker  
Algorithms & Abstractions  
BUY \* SELL \* TRADE

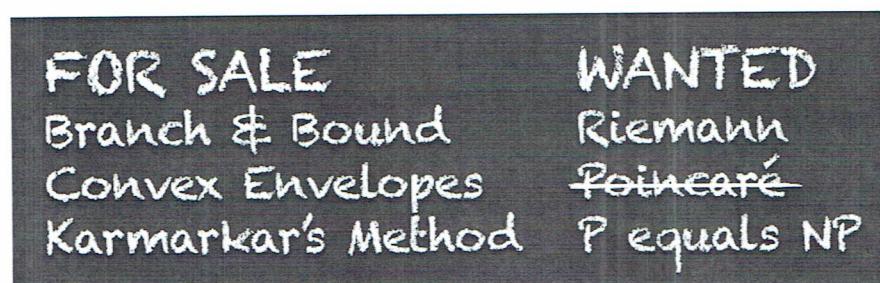
“Al-go-rith-ms. That sounds like a kind of fruit.”

“Are you *always* hungry, Xor?”

“Time flies like an arrow, and fruit flies like a banana. Let’s see if there’s a fruit fly problem I can help them solve.”

A bell jingled as Laurie opened the door. “Hello, hello!” the shopkeeper said. “And welcome to my shop. I’m Tinker, and you are looking for a finely crafted algorithm, am I right?”

Laurie looked at the items listed on the chalkboard, but they didn’t make any sense.



“I’m not sure. What *is* an algorithm? Can you eat it?” asked Laurie.

“What? No, it’s just a fancy way of saying ‘how to do something.’ But *Algorithm* looks more impressive on the sign,” said Tinker.

Xor turned orange with disappointment.

“How to do something,” repeated Laurie. “In that case, I want to find a sensible way to visit every town.”

“That sounds like an interesting problem. What have you been doing so far?”

Laurie told Tinker about her adventure in the Red-Black Forest and her visit with Eponymous Bach.

“A Hamiltonian path, eh?” said Tinker. “That’s a tough one. I hate to say it, because he sounds like a nice person, but the

Wandering Salesman might take a long, long time to finish his tour of all the towns.”

“Oh, no! But why?”

“If you always go to the nearest town you haven’t visited yet, you might miss a town that’s just a little farther away. Then you go to another town that’s closer to you but still farther from the one you missed, and so on. You can end up crisscrossing the whole country to get to the last few towns.”

“That sounds exhausting,” said Laurie. The Wandering Salesman wasn’t so sensible after all! “So how do I find the shortest path?”

“I’ll see what I have in stock. But it might be expensive.”

“I don’t have much money with me,” Laurie said. She took a few quarters from her pocket and showed them to Tinker.

He looked at them with surprise. “Quarter Dollar? I don’t know what a Dollar is, never mind a quarter of one. Is this money where you come from?”

“Of course it’s money! That’s seventy-five cents,” she said.

“Cents? We use Fair Coins here.”

“What’s a Fair Coin?”

“Well, they are a bit bigger than these Quarter Dollars of yours, but not nearly as pretty! You can tell genuine Fair Coins because they always flip heads or tails, fifty-fifty.”

“But you can flip quarters fifty-fifty, too!”

“That may be true, but I can’t just take *your* word for it, can I? Here, all Fair Coins must be certified Fair.”

Laurie was crestfallen.

“Don’t look so sad! I do want to help you,” said Tinker. “Maybe we can do a trade. It so happens I’m in the market for a particular algorithm.”

“But I don’t have any algorithms, either,” said Laurie.

“That’s not a problem,” said Tinker. “You can compose new ones any time you want, with a little bit of thinking.”

“I can? How?”

“Well, everyone develops their own style. You can put little ideas together to make big ideas. Or you put two ideas side by side and compare them. Or you start with big ideas and take them apart.”

“You mean like Eponymous does?”

“Yes, just like her. She’s a great Composer.”

Laurie had never thought that *she* could do things like that herself. But Tinker seemed to think it was normal.

“So what do I do?”

“The algorithm I’m looking for is how to draw a circle,” Tinker said. “It’s a tough one, so you’ll have to use your imagination. I’ve asked all the adults and even Ponens and Tollens already, but all they do is mutter about  $x$  squared plus  $y$  squared and never get anywhere.”

“Take a look at this.” He handed Laurie a wind-up toy animal. It had a Shell, and was Round and Green. “This turtle can do three things: it can move forward or backward, it can turn, and it can draw a little dot on the paper.”

“Hey, that’s pretty neat!”

“Yes, but the thing is, it doesn’t know how to do anything else. That’s where the algorithm comes in.” Tinker took out a piece of paper and wrote what looked like a little poem:

Go forward one inch,  
make a mark,  
repeat five times.

Then he wound up the turtle and placed it on the poem. It went *zzzrbt bzzaap whuzzzsh*, and so on. Then it drew a line of dots, just like the poem said:



"You see? If you put little ideas together, you can make bigger ones," Tinker said. "And you can compose *those* ideas into even bigger and bigger ones."

"How do you do that?" asked Laurie.

"By giving them a name. You can use the name like a handle: you'd carry a pot of soup by the handle, and you can move around an entire idea just by writing its name. Here, let's call the first idea *LINE*. Then you can put four lines together to make a square."

**LINE:**

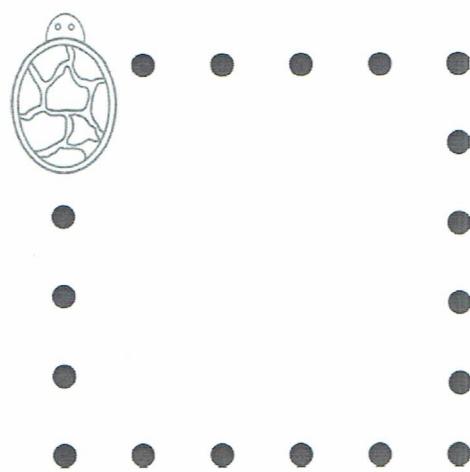
Go forward one inch,  
make a mark,  
repeat five times.

**SQUARE:**

make a LINE,  
make a right turn,  
repeat four times.

make a SQUARE.

The little turtle *zzzrbted* and *whuzzshed* and *bzzaaped*, then drew this:



Laurie was amazed. It was like magic, but every step made sense.

"So, knowing what the turtle can do, can you teach it how to draw a circle?" Tinker asked.

"I don't know," Laurie said, "but I want to try!"

"That's good enough for me. Here, you can work at my desk. There's plenty of paper and compasses and things like that."

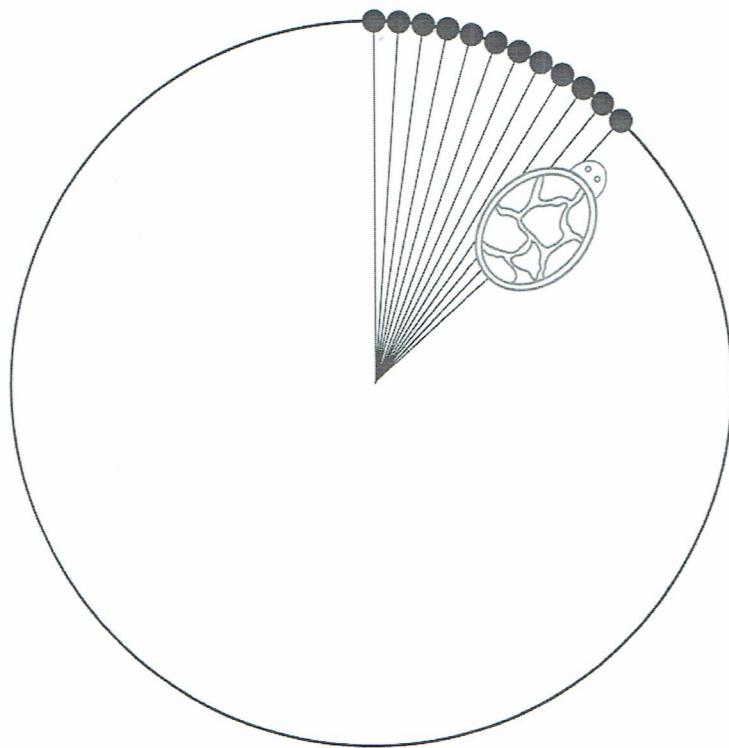
Laurie sat down at Tinker's desk. She doodled with the compass and played with the turtle for a while, trying to remember what she knew about circles.

*A circle is round. No, not just round—perfectly round. You put the pin in the center, and the pencil spins around. To make a bigger one, you open the compass; to make a smaller one, you close the compass. If you change the width of the compass when it's spinning, it doesn't make a circle . . .*

Suddenly an idea, or maybe a memory, popped into her head: *a circle is all of the points that are exactly the same distance from the center. Hmm, what if you . . .*

Go forward one inch,  
make a mark,  
go back one inch,  
turn right a tiny bit,  
then repeat!

After Laurie wrote out her poem, she wound up the little turtle again and placed it on the paper. It buzzed and burbled for a moment, then drew this:



"It's working!" she called to Tinker. "Hey, it's not stopping." The turtle was drawing over dots it had already drawn.

"I think it's because you told it to repeat, but not how many times," said Tinker.

"Well, it should stop when the circle is done," Laurie said.

"It doesn't really understand circles," Tinker said. "It's just a toy turtle, remember? You have to teach it."

Laurie thought a little more, then rewrote her poem:

CIRCLE:

Go forward one inch,  
make a mark,  
go back one inch,  
turn right one degree,  
repeat three hundred sixty times.

Then she realized that she could make circles of any size she wanted. It was just like opening the compass wider.

TWO-CIRCLE:

Go forward two inches,  
make a mark,  
go back two inches,  
turn right one degree,  
repeat three hundred sixty times.

"This is interesting. You're working really hard!" Tinker scratched his head. "But as it is, it's no good."

"Why?"

"People want to make lots of different circles," he said. "I'll have to keep a lot of algorithms of different sizes, just in case someone wants three-and-nine-thirteenths inches or four-and-three-quarters inches."

"Well, what if you tell the turtle how big to make the circle?" she said. "Maybe like this."

**ANY-CIRCLE (how-big?):**

Go forward **how-big?** inches,  
make a mark,  
go back **how-big?** inches,  
turn right one degree,  
repeat three hundred sixty times.

"And *then*," she said, "instead of ONE-CIRCLE or TWO-CIRCLE, you can say ANY-CIRCLE(one), or (two), or even (one-and-eleventy-sevenths)!"

"Good idea, Laurie. That's a lot simpler," said Tinker. "I was worried you were going to fill my shop with circles!"

"You know, the turtle is drawing really slowly. Not like when it was drawing the square," she said.

It was true. The turtle would crawl all the way to the edge of the circle, then make a mark, then crawl all the way back to the center, 360 times. With small circles it wasn't too bad, but big circles took a lot longer.

"Hmm," Tinker said. "It spends a *lot* more time running back and forth than it does making marks. Do you think you can reduce the running time?"

*It makes sense, but it isn't sensible.* Laurie thought and doodled, and doodled and thought, but she couldn't figure out how to make it more sensible. The turtle has to go back to the center, right? How else could it know where the edge of the circle was?

Laurie let her eyes wander around the room. Xor was staring at a moth that was flying in lazy loops around a lightbulb. His skin was slowly fading from red to yellow and back to red. The moth went around and around. It was hypnotic. Around and around and around and . . .

*Oh! If the moth doesn't have to go to the center of the lightbulb to fly around it in a circle, then why does the turtle need to go back to the center to draw one?*

Laurie reached for a fresh piece of paper before the idea got away. *Don't let a new thing out of your sight without a name.*

MOTH-CIRCLE (how-big?):

Go forward how-big? inches,  
make a mark,  
turn right one degree,  
repeat three hundred sixty times.

make a MOTH-CIRCLE (one).

The turtle went *bzzaap* and *zzzrbt* and *whuzzzsh* and then it started to draw. It moved one inch, made a dot, then turned a tiny bit, then moved one inch, then made another dot . . .

"Whoops. It's making a *huge* circle! Let me try a small number." Laurie didn't have a small number handy, so she borrowed one she had heard from Tortoise: one thirty-second of an inch.

"That's better," Laurie said.

"Let me see," Tinker said. "Wow, look at the little guy run!"



"That was fun," said Laurie. "I didn't know you could just make up new ways to do things."

"Of course you can. Often you aren't the first to think of something, but if it works, who cares? Now, for my end of the trade."

"Did you find the shortest path?" Laurie asked.

"Not exactly. The bad news is that what you are trying to do is impossible."

"It's impossible?"

"Well, highly improbable. There are many different ways to visit all the towns. It seems like you could write an algorithm for the turtle to try each one and find the shortest, right?"

"Sure, why not?" said Laurie.

"There are twenty-one towns in Userland. How many paths do you think there are?" Tinker said.

“I don’t know,” said Laurie. “A hundred?”

“Way more.”

“Um, a million?” Laurie said.

“More like a million million times that!” said Tinker.

“But how can that be?”

“Let’s say there are only three towns: A, B, and C,” Tinker said. “You are already standing in A, so you have to worry only about B and C. How many ways can you go?”

“Well,” she said, “I could go from B to C, or go to C and then B. That’s two.”

“That’s right! But BC is the same as CB, just backward. Every path has a mirror image, so with three towns there is really only one possible path that visits them all. What if there were four towns, A, B, C, and D?”

Laurie counted on her fingers. “I could go BCD, or BDC, or CBD, or CDB, or DCB, or . . . DBC. Six! No, three.”

“That’s three times as many. Add another town, and you have twelve times as many,” Tinker said. “Add a sixth town and there are *sixty* different paths through all of them. With seven towns there are *three hundred sixty paths*. As you add more towns, the number of paths gets very big!”

$$3 \text{ towns: } 2 \div 2 = 1$$

$$4 \text{ towns: } 2 \times 3 \div 2 = 3$$

$$5 \text{ towns: } 2 \times 3 \times 4 \div 2 = 12$$

$$6 \text{ towns: } 2 \times 3 \times 4 \times 5 \div 2 = 60$$

$$7 \text{ towns: } 2 \times 3 \times 4 \times 5 \times 6 \div 2 = 360$$

$$8 \text{ towns: } 2 \times 3 \times 4 \times 5 \times 6 \times 7 \div 2 = 2,520$$

$$9 \text{ towns: } 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \div 2 = 20,160$$

“For twenty-one towns you have to multiply one times two times three times four, all the way up to twenty. It makes a HUGENORMOUS number!”

$$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times \\ 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \div 2 =$$

1,216,451,004,088,320,000

“!” said Laurie.

“Indeed!” Tinker said. “All of that ‘one times two times three’ stuff takes too long to write. So you can use the exclamation point as a shorthand.”

$20! \div 2 = 1,216,451,004,088,320,000$

“But that’s . . .” Laurie said, counting the commas, “over one million million *million* paths!”

“One of those umpty-million paths is the shortest,” Tinker said. “I don’t know of any way to find it quickly.”

“I’ll be old before we check them all! Isn’t there a better way to do it?”

“Ah, that’s the good news!” Tinker said. “I deal only in Exact answers. But there is a brilliant Composer who lives in Permute, named Hugh Rustic. He deals in Good Enough answers. I send him all of my hardest cases. I’ll write an IOU that you can take to him.”