YXHXianYu-XCPC-Template

0. 目录

    - 9. 其他
        - 9.1 树哈希
- 缺：EXSAM、回文自动机、序列自动机

1. 常用算法

1.1 对拍

```
// 随机数生成（不保证环境均可用）
mt19937 rand(chrono::system_clock::now().time_since_epoch().count());
cout << rand() << endl;
// 对拍
while(true) {
    system("Edm.exe");
    system("Eac.exe");
    system("E.exe");
    if(system("fc E.out E.ans")) break;
}
```

1.2 哈希表

```
struct HASH { // 多测记得清空
    #define LLI long long // (maybe) int => signed (take care of SPACE Complexiy)
    const static int maxn = 2e5 + 10;
    const static int hashMaxn = 1 << 23;
    struct Edge {
        LLI a, b, v;
        int nxt;
    }e[maxn];
    int lst[hashMaxn], tot;
    int stk[maxn], top;

    void makeEdge(int h, LLI a, LLI b, LLI v) {
        e[++tot].a = a;
        e[tot].b = b;
        e[tot].v = v;
        e[tot].nxt = lst[h];
        lst[h] = tot;

        stk[++top] = h;
    }
    int getH(LLI a, LLI b) {
        return ((int)(133331ll * a + b) & (hashMaxn - 1));
    }
    void clear() {
        while(top) lst[stk[top]] = 0, top--;
        tot = 0;
    }
    void add(LLI a, LLI b, LLI v, bool ad) {
        int h = getH(a, b);
        if(ad) {
            for(int i = lst[h]; i; i = e[i].nxt) {
                if(e[i].a == a && e[i].b == b) {
                    e[i].v += v;
                    return;
                }
            }
        }
        makeEdge(h, a, b, v);
    }
    LLI query(LLI a, LLI b) {
        int h = getH(a, b);
        for(int i = lst[h]; i; i = e[i].nxt) {
            if(e[i].a == a && e[i].b == b) {
                return e[i].v;
            }
        }
        return 0;
    }
    #undef LLI
} mpn; // 多测记得清空
```

1.3 fread快读

```
static char buf[1000000],*p1=buf,*p2=buf,obuf[1000000],*p3=obuf;
#define getchar() p1==p2&&(p2=(p1=buf)+fread(buf,1,1000000,stdin),p1==p2)?EOF:*p1++
#define putchar(x) (p3-obuf<1000000)?(*p3++=x):(fwrite(obuf,p3-obuf,1,stdout),p3=obuf,*p3++=x)
template<typename item>
inline void read(item &x){
    x=0;int f=1;char c=getchar();
    while(c<'0'||c>'9'){if(c=='-')f=-1;c=getchar();}
    while(c>='0'&&c<='9')x=(x<<3)+(x<<1)+(c^48),c=getchar();
    x*=f;
}
template<typename item>
inline void uread(item &x){
    x=0;char c=getchar();
    while(c<'0'||c>'9')c=getchar();
    while(c>='0'&&c<='9')x=(x<<3)+(x<<1)+(c^48),c=getchar();
}
static char cc[20];
template<typename item>
inline void print(item x){
    int len=0;
    if(x<0)x=-x,putchar('-');
    while(x)cc[len++]=x%10+'0',x/=10;
    while(len--)putchar(cc[len]);
}
template<typename item>
```

```
inline void uprint(item x){
    int len=0;
    while(x)cc[len++]=x%10+'0',x/=10;
    while(len--)putchar(cc[len]);
}
```

2. 计算几何

> 本部分主要学习自杜老师的代码源计算几何课程～(∠・ω<)⌒★!

2.1 基础

```
typedef double db; // 换longdouble时，需要同时替换alpha函数和getTheta函数
const db EPS = 1e-9;

inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }

inline int cmp(db a, db b) { return sign(a - b); }

// 点P
struct P {
    db x, y;
    P(): x(0), y(0) {}
    P(db _x, db _y): x(_x), y(_y) {}
    P operator+(P p) { return {x + p.x, y + p.y}; }
    P operator-(P p) { return {x - p.x, y - p.y}; }
    P operator*(db d) { return {x * d, y * d}; }
    P operator/(db d) { return {x / d, y / d}; }

    bool operator<(P p) const {
        int c = cmp(x, p.x);
        if(c) return c == -1;
        return cmp(y, p.y) == -1;
    }

    // 这里的比较可能不满足传递性，因为这是基于EPS的比较
    bool operator==(P p) const {
        return cmp(x, p.x) == 0 && cmp(y, p.y) == 0;
    }

    db dot(P p) { return x * p.x + y * p.y; }
    db det(P p) { return x * p.y - y * p.x; }
    // P cross(P p) { return {y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x}; }

    db distTo(P p) { return (*this-p).abs(); }
    // return value in [-pi, pi]; longdouble ver is atan2l(y, x)
    db alpha() { return atan2(y, x); }
    void read() { cin >> x >> y; }
    db abs() { return sqrt(abs2()); }
    db abs2() { return x * x + y * y; }
    P unit() { return *this / abs(); }
    P rot90() { return P(-y, x); }
    P rot(db an) { return {x*cos(an) - y*sin(an), x*sin(an) + y*cos(an)}; }
    int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0); }
};

#define cross(p1, p2, p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
#define crossOp(p1, p2, p3) (sign(cross(p1, p2, p3)))
```

2.2 极角排序

```
// 待排序元素必须去除(0, 0)，否则结果会错误!
int n = 50;
vector<P> p(n + 1);
sort(p.begin() + 1, p.end() + n + 1, [&](P& a, P& b) {
    int qa = a.quad(), qb = b.quad();
    if(qa != qb) return qa < qb;
    else return sign(a.det(b)) > 0;
});
```

2.3 点与线段

```
// 判断 直线p1p2 和 直线q1q2 是否严格相交
// = crossOp(p1, p2, q1) * crossOp(p1, p2, q2) < 0

// 判断 直线p1p2 和 直线q1q2 是否相交
bool isLL(P p1, P p2, P q1, P q2) {
    db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
    return sign(a1 + a2) != 0;
}

// 求交点
P getLL(P p1, P p2, P q1, P q2) {
    db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
    return (p1 * a2 + p2 * a1) / (a1 + a2);
}

// 判断区间是否相交
bool intersect(db l1, db r1, db l2, db r2) {
    if(l1 > r1) swap(l1, r1); if(l2 > r2) swap(l2, r2);
    return !( cmp(r1, l2) == -1 || cmp(r2, l1) == -1 );
}

// 线段相交
bool isSS(P p1, P p2, P q1, P q2) {
    return intersect(p1.x, p2.x, q1.x, q2.x)
        && intersect(p1.y, p2.y, q1.y, q2.y)
        && crossOp(p1, p2, q1) * crossOp(p1, p2, q2) <= 0
        && crossOp(q1, q2, p1) * crossOp(q1, q2, p2) <= 0;
}
```

```cpp
// 线段严格相交(不能交于端点)
bool isSS_strict(P p1, P p2, P q1, P q2) {
    return crossOp(p1, p2, q1) * crossOp(p1, p2, q2) < 0
        && crossOp(q1, q2, p1) * crossOp(q1, q2, p2) < 0;
}

// 点m 是否在 a 和 b 之间
bool isMiddle(db m, db a, db b) {
    if(a > b) swap(a, b);
    return cmp(a, m) <= 0 && cmp(m, b) <= 0;
}

// 点m 是否在 点a 和 点b 之间
bool isMiddle(P m, P a, P b) {
    return isMiddle(m.x, a.x, b.x) && isMiddle(m.y, a.y, b.y);
}

// 点q 是否在 直线p1p2 上
// => crossOp(p1, p2, q) == 0

// 点q 是否在 线段p1p2 上
bool onSeg(P p1, P p2, P q) {
    return crossOp(p1, p2, q) == 0 && isMiddle(q, p1, p2);
}

// 点q 是否在 线段p1p2 上(严格)
bool onSeg_strict(P p1, P p2, P q) {
    return crossOp(p1, p2, q) == 0 && sign((q-p1).dot(p1-p2)) * sign((q-p2).dot(p1-p2));
}

// 求 q 到 直线p1p2 的投影(垂足)
P proj(P p1, P p2, P q) {
    P dir = p2 - p1;
    return p1 + dir * (dir.dot(q - p1) / dir.abs2());
}

// 求 q 以 直线p1p2 为轴的反射
P reflect(P p1, P p2, P q) {
    return proj(p1, p2, q) * 2 - q;
}

// 求 q 到 线段p1p2 的最小距离
db nearest(P p1, P p2, P q) {
    if(p1 == p2) return p1.distTo(q);
    P v1 = p2 - p1, v2 = q - p1, v3 = q - p2;
    if(sign(v1.dot(v2)) == -1) return v2.abs();
    if(sign(v1.dot(v3)) == 1) return v3.abs();
    return fabs(v1.det(v2) / v1.abs());
    // if(p1 == p2) return p1.distTo(q); // TLE Version
    // P h = proj(p1, p2, q);
    // if(isMiddle(h, p1, p2)) return h.distTo(q);
    // return min(p1.distTo(q), p2.distTo(q));
}

// 求 线段p1p2 与 线段q1q2 的距离
db disSS(P p1, P p2, P q1, P q2) {
    if(isSS(p1, p2, q1, q2)) return 0;
    return min(
        min(nearest(p1, p2, q1), nearest(p1, p2, q2)),
        min(nearest(q1, q2, p1), nearest(q1, q2, p2))
    );
}
```

2.4 多边形

```
/* 重要约定，每个vector大小必须等于size+1，且index从1开始 */
/* 重要约定，逆时针为正方向 */
```

2.4.1 简单多边形面积

```
// 求简单多边形面积（测过）
db area(vector<P>& ps) {
    int n = ps.size() - 1;
    db ret = 0;
    for(int i = 1; i <= n; i++) {
        ret += ps[i].det(ps[i % n + 1]);
    }
    return ret / 2;
}
```

2.4.2 点包含

```
// 求点包含（测过）
// 2:inside, 1:on_seg, 0:outside
int contain(vector<P>& ps, P p) {
    int n = ps.size() - 1;
    int ret = 0;
    for(int i = 1; i <= n; i++) {
        P u = ps[i], v = ps[i % n + 1];
        if(onSeg(u, v, p)) return 1;
        if(cmp(u.y, v.y) <= 0) swap(u, v);
        if(cmp(p.y, u.y) > 0 || cmp(p.y, v.y) <= 0) continue;
        ret ^= crossOp(p, u, v) > 0;
    }
    return ret * 2;
}
```

2.4.3 凸包

```
// 凸包（测过）
vector<P> convexHull(vector<P>& ps) {
    int n = ps.size() - 1;
    if(n <= 1) return ps;
    sort(ps.begin() + 1, ps.begin() + n + 1); // normal sort
    vector<P> qs(2 * n + 1);
    int cnt = 0;
    for(int i = 1; i <= n; qs[++cnt] = ps[i++]) {
        while(cnt > 1 && crossOp(qs[cnt - 1], qs[cnt], ps[i]) <= 0) cnt--;
    }
    for(int i = n - 1, t = cnt; i >= 1; qs[++cnt] = ps[i--]) {
        while(cnt > t && crossOp(qs[cnt - 1], qs[cnt], ps[i]) <= 0) cnt--;
    }
    qs.resize(cnt); // 去除了最后一个重复的点
    return qs;
}
```

2.4.4 不严格凸包

```
// 不严格凸包(考虑共线)
// 使用前必须对点进行去重
vector<P> convexHullNonStrict(vector<P>& ps) {
    int n = ps.size() - 1;
    if(n <= 1) return ps;
    sort(ps.begin() + 1, ps.begin() + n + 1); // normal sort
    vector<P> qs(2 * n + 1);
    int cnt = 0;
    for(int i = 1; i <= n; qs[++cnt] = ps[i++]) {
        while(cnt > 1 && crossOp(qs[cnt - 1], qs[cnt], ps[i]) < 0) cnt--;
    }
    for(int i = n - 1, t = cnt; i >= 1; qs[++cnt] = ps[i--]) {
        while(cnt > t && crossOp(qs[cnt - 1], qs[cnt], ps[i]) < 0) cnt--;
    }
    qs.resize(cnt); // 去除了最后一个重复的点
    return qs;
}
```

2.4.5 直线切凸多边形

```
// 直线切凸多边形（测过）
// convexCut是用 直线cut出直线的左半边（即叉积为正的点集）
// 所以 convexCut(r, p[i], p[i] o (p[1 - i] - p[i]).rot90());
// 若 o 为 +，则表示 p[1 - i] 到 p[i] 的外侧（不包含 p[1 - i] 的区域）；
// 若 o 为 -，则表示 p[1 - i] 到 p[i] 的内测（包含 p[1 - i] 的区域）；
vector<P> convexCut(const vector<P>& ps, P q1, P q2) {
    vector<P> qs(1);
    int n = ps.size() - 1;
    for(int i = 1; i <= n; i++) {
        P p1 = ps[i], p2 = ps[i % n + 1];
        int d1 = crossOp(q1, q2, p1), d2 = crossOp(q1, q2, p2);
        if(d1 >= 0) qs.push_back(p1);
        if(d1 * d2 < 0) qs.push_back(getLL(p1, p2, q1, q2));
    }
    return qs;
}
```

2.4.6 旋转卡壳

```
// 旋转卡壳（输入必须是一个凸包）
db convexDiameter(vector<P>& ps) {
    int n = ps.size() - 1; if(n <= 1) return 0;
    int is = 1, js = 1;
    for(int k = 2; k <= n; k++) {
        is = ps[k] < ps[is] ? k : is;
        js = ps[js] < ps[k] ? k : js;
    }
    int i = is, j = js;
    db ret = ps[i].distTo(ps[j]);
    do {
        if((ps[i % n + 1] - ps[i]).det(ps[j % n + 1] - ps[j]) >= 0)
            j = j % n + 1;
        else
            i = i % n + 1;
        ret = max(ret, ps[i].distTo(ps[j]));
    } while(i != is || j != js);
    return ret;
}
```

2.4.7 半平面交

```
// 半平面交
db halfPlane(vector<pair<P, P>>& a) {
    int n = a.size() - 1;
    auto compare = [](pair<P, P>& a, pair<P, P>& b) {
        P va = a.second - a.first;
        P vb = b.second - b.first;
        int qa = va.quad();
        int qb = vb.quad();
        if(qa != qb) return qa < qb;
        int v = sign(va.det(vb));
        return v == -1;
    };
    sort(a.begin() + 1, a.begin() + n + 1, [](pair<P, P>& a, pair<P, P>& b) {
        P va = a.second - a.first;
        P vb = b.second - b.first;
        int qa = va.quad();
        int qb = vb.quad();
        if(qa != qb) return qa < qb;
        int v = sign(va.det(vb));
        if(v != 0) return v == -1;
        return crossOp(a.first, a.second, b.second) < 0;
    });
    auto right = [](pair<P,P>& a, pair<P,P>& b, pair<P,P>& c) {
        P p = getLL(b.first, b.second, c.first, c.second);
        return crossOp(a.first, a.second, p) <= 0;
    };
    vector<pair<P, P>> q(n + 1);
    int h = 1, t = 1;
    q[1] = a[1];
    for(int i = 2; i <= n; i++) {
        if(compare(a[i], a[i-1])) continue;
        while(h < t && right(a[i], q[t], q[t-1])) t--;
        while(h < t && right(a[i], q[h], q[h+1])) h++;
        q[++t] = a[i];
    }
    while(h < t && right(q[h], q[t], q[t-1])) t--;

    if(h == t) return 0.0;

    vector<P> ans(1);
    for(int i = h, j = 1; i < t; i++, j++) {
        ans.push_back(getLL(q[i].first, q[i].second, q[i+1].first, q[i+1].second));
        P cp = getLL(q[i].first, q[i].second, q[i+1].first, q[i+1].second);
    }
    ans.push_back(getLL(q[t].first, q[t].second, q[h].first, q[h].second));

    return fabs(area(ans));
    // return ans;
}
```

2.5 圆

2.5.1 圆和圆的关系

```
// 判断两个圆之间的关系
int circleIntersect(P o1, db r1, P o2, db r2) {
    db d = o1.distTo(o2);
    if(cmp(d, r1 + r2) == 1) return 4;      // 相离
    if(cmp(d, r1 + r2) == 0) return 3;      // 外切
    if(cmp(d, abs(r1 - r2)) == 1) return 2; // 相交
    if(cmp(d, abs(r1 - r2)) == 0) return 1; // 内切
    return 0;                               // 内含
}
```

2.5.2 圆和直线的交点

```
// 获取圆和直线的交点
vector<P> getCL(P o, db r, P p1, P p2) {
    if (cmp(abs((o-p1).det(p2-p1)/p1.distTo(p2)),r)>0) return {};
    db x = (p1-o).dot(p2-p1), y = (p2-p1).abs2(), d = x * x - y * ((p1-o).abs2() - r*r);
    d = max(d,(db)0.0); P m = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
    return {m-dr,m+dr}; //along dir: p1->p2
}
```

2.5.3 两个圆的交点

```
// 获取两个圆的交点
// need to check whether two circles are the same
vector<P> getCC(P o1, db r1, P o2, db r2) {
    db d = o1.distTo(o2);
    if (cmp(d, r1 + r2) == 1) return {};
    if (cmp(d,abs(r1-r2))==-1) return {};
    d = min(d, r1 + r2);
    db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
    P dr = (o2 - o1).unit();
    P q1 = o1 + dr * y, q2 = dr.rot90() * x;
    return {q1-q2,q1+q2}; // along circle 1
}
```

2.5.4 两个圆的切线

```
// 求两个圆的切线
// extanCC(r2>0); intanCC(r2<0); tanCP(r2=0)
vector<pair<P, P>> tanCC(P o1, db r1, P o2, db r2) {
    P d = o2 - o1;
    db dr = r1 - r2, d2 = d.abs2(), h2 = d2 - dr * dr;
    if (sign(d2) == 0|| sign(h2) < 0) return {};
    h2 = max((db)0.0, h2);
    vector<pair<P, P>> ret;
    for (db sign : {-1, 1}) {
        P v = (d * dr + d.rot90() * sqrt(h2) * sign) / d2;
        ret.push_back({o1 + v * r1, o2 + v * r2});
    }
    if (sign(h2) == 0) ret.pop_back();
    return ret;
}
```

2.5.5 圆的内心、外心、垂心

```
// 求三角形内心
P inCenter(P A, P B, P C) {
    double a = (B - C).abs(), b = (C - A).abs(), c = (A - B).abs();
    return (A * a + B * b + C * c) / (a + b + c);
}

// 求三角形外心
P circumCenter(P a, P b, P c) {
    P bb = b - a, cc = c - a;
    double db = bb.abs2(), dc = cc.abs2(), d = 2 * bb.det(cc);
    return a - P(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
}

// 求三角形垂心
P othroCenter(P a, P b, P c) {
    P ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.y * ca.y * bc.y,
    A = ca.x * ba.y - ba.x * ca.y,
    x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
    y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
    return {x0, y0};
}
```

2.5.6 最小圆覆盖

```
// 最小圆覆盖
pair<P, db> min_circle(vector<P>& ps) {
    random_shuffle(ps.begin(), ps.end());
    int n = ps.size() - 1;
    P o = ps[0]; db r = 0;
    for(int i = 2; i <= n; i++) if(o.distTo(ps[i]) > r + EPS) {
        o = ps[i]; r = 0;
        for(int j = 1; j < i; j++) if(o.distTo(ps[j]) > r + EPS) {
            o = (ps[i] + ps[j]) / 2; r = o.distTo(ps[i]);
            for(int k = 1; k < j; k++) if(o.distTo(ps[k]) > r + EPS) {
                o = circumCenter(ps[i], ps[j], ps[k]);
                r = o.distTo(ps[i]);
            }
        }
    }
    return {o, r};
}
```

2.5.7 两个向量的夹角

```
// 求两个向量的夹角
db getTheta(P p1, P p2) {
    return atan2(p1.det(p2), p1.dot(p2));
}
```

2.5.8 求圆和三角形的交的有向面积

```
// 求圆和三角形的交的有向面积
// 如果要求圆和简单多边形的面积，需要将圆心置于原点，然后再划分简单多边形求和
db areaCT(db r, P p1, P p2){
    vector<P> is = getCL(P(0,0),r,p1,p2);
    if(is.empty()) return r*r*getTheta(p1,p2)/2;
    bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(), r*r) == 1;
    if(b1 && b2){
        P md=(is[0]+is[1])/2;
        if(sign((p1-md).dot(p2-md)) <= 0)
            return r*r*(getTheta(p1,is[0]) + getTheta(is[1],p2))/2 +
is[0].det(is[1])/2;
        else return r*r*getTheta(p1,p2)/2;
    }
    if(b1) return (r*r*getTheta(p1,is[0]) + is[0].det(p2))/2;
    if(b2) return (p1.det(is[1]) + r*r*getTheta(is[1],p2))/2;
    return p1.det(p2)/2;
}
```

2.5.9 圆的面积并、欧拉公式

```
// 圆的面积并
//    利用格林公式，将面积信息转换为边界信息
// 欧拉公式
//    V - E + F = 1 + C
//    V顶点 E边 F面 C连通块
//    求平面个数都非常困难，通常会将问题转化为求点和边的数量
//    点数：将所有圆的交点求出来，再离散化去重
//    边数：圆上有k个点，那么就有k条边，暴力算出每个圆上有几个交点即可
```

2.6 杂题

2.6.1 平面最近点对

```
void work() {
    int n; n = read();

    vector<P> a(n + 1), b(n + 1);
    for(int i = 1; i <= n; i++) a[i].x=read(), a[i].y=read();

    sort(a.begin() + 1, a.end());
    db ans = a[1].distTo(a[2]), lim = sqrt(ans);

    function<void(int, int)> solve = [&](int L, int R) {
        if(R - L <= 1) return;
        if(R - L <= 4) {
            for(int i = L; i <= R; i++) for(int j = i + 1; j <= R; j++) {
                db v = a[i].distTo(a[j]); if(v < ans) { ans = v; lim = sqrt(ans); }
            }
            sort(a.begin() + L, a.begin() + R + 1, [](const auto& a, const auto& b) { return a.y < b.y; });
            return;
        }

        int mid = L + R >> 1;
        db midx = a[mid].x;
        db midx2 = a[mid+1].x;
        solve(L, mid); solve(mid+1, R);

        int i = L, j = mid+1, k = L;
        while(i <= mid && j <= R) { if(a[i].y < a[j].y) b[k++] = a[i++]; else b[k++] = a[j++]; }
        while(i <= mid) b[k++] = a[i++];
        while(j <= R) b[k++] = a[j++];
        for(int i = L; i <= R; i++) a[i] = b[i];

        vector<P> t;
        for(int i = L; i <= R; i++) {
            if(cmp(abs(a[i].x - midx), lim) <= 0) {
                for(int j = t.size()-1; j >= 0 && cmp(a[i].y-t[j].y,lim) <= 0; j--) {
                    db v = t[j].distTo(a[i]); if(v < ans) { ans = v; lim = sqrt(ans); }
                }
                t.push_back(a[i]);
            }
        }
    };

    solve(1, n);

    cout << ans << endl;
}
```

2.6.2 最小矩形覆盖

```
void work() {
    int n;
    cin >> n;

    vector<P> p(n + 1);
    for(int i = 1; i <= n; i++) {
        cin >> p[i].x >> p[i].y;
    }
    p = convexHull(p);
    n = p.size() - 1;

    p.resize(2 * n + 1);
    for(int i = 1; i <= n; i++) p[i + n] = p[i];

    P d0 = p[2] - p[1];
    int R = 1, L = 1, U = 1;
    for(int i = 2; i <= n; i++) {
        P di = p[i] - p[1];
        while(d0.dot(di) > d0.dot(p[R] - p[1])) R = i;
        while(d0.dot(di) < d0.dot(p[L] - p[1])) L = i;
        while(d0.det(di) > d0.det(p[U] - p[1])) U = i;
    }

    int n2 = 2 * n;
    db ans = 1e18;
    vector<P> result(4);
    for(int i = 1; i <= n; i++) {
        P d = p[i + 1] - p[i];
        while(R + 1 <= n2 && d.dot(p[R + 1] - p[i]) > d.dot(p[R] - p[i])) R++;
        while(L + 1 <= n2 && d.dot(p[L + 1] - p[i]) < d.dot(p[L] - p[i])) L++;
        while(U + 1 <= n2 && d.det(p[U + 1] - p[i]) > d.det(p[U] - p[i])) U++;

        db s1 = d.dot(p[R] - p[L]) / d.abs();
        db s2 = d.det(p[U] - p[i]) / d.abs();
        db S = s1 * s2;
        if(S < ans) {
            ans = S;
            // down: p[i], p[i]+d
            // up  : p[u], p[u]+d
            // righ: p[r], p[r]+d.rot90()
            // left: p[l], p[l]+d.rot90()
            result[0] = getLL(p[i], p[i] + d, p[L], p[L] + d.rot90());
            result[1] = getLL(p[i], p[i] + d, p[R], p[R] + d.rot90());
            result[2] = getLL(p[U], p[U] + d, p[R], p[R] + d.rot90());
            result[3] = getLL(p[U], p[U] + d, p[L], p[L] + d.rot90());
        }
    }

    int mnp = 0;
    for(int i = 1; i <= 3; i++) {
        if(cmp(result[i].y, result[mnp].y) < 0) {
            mnp = i;
        } else if(cmp(result[i].y, result[mnp].y) == 0 && cmp(result[i].x,
result[mnp].x) < 0) {
            mnp = i;
        }
    }
    cout << fixed << setprecision(7) << ans << endl;
    for(int i = 0; i <= 3; i++) {
        cout << result[(i+mnp)%4].x << " " << result[(i+mnp)%4].y << endl;
    }
}
```

2.7 三维计算几何

```
// 点积
db dot(P a, P b) { return a.x*b.x + a.y*b.y + a.z*b.z; }
// 叉积
P det(P a, P b) { return P{a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x};
}
```

---

3. 字符串

```
/* 约定，所有字符串均为1-index */
```

3.1 KMP

```
vector<int> nxt(m + 1);
for(int i = 2, j = 0; i <= m; i++) {
    while(j && b[i] != b[j + 1]) j = nxt[j];
    if(b[i] == b[j + 1]) j++;
    nxt[i] = j;
}

for(int i = 1, j = 0; i <= n; i++) {
    while(j && a[i] != b[j + 1]) j = nxt[j];
    if(a[i] == b[j + 1]) j++;
    if(j == m) cout << i - m + 1 << endl;
}
```

3.2 EXKMP

```
vector<int> z(m + 1);
int l = 0, r = 0;
for(int i = 2; i <= m; i++) {
    z[i] = i <= r ? min(z[i - l + 1], r - i + 1) : 0;
    while(i + z[i] <= m && b[z[i] + 1] == b[i + z[i]]) z[i]++;
    if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
}

vector<int> p(n + 1);
l = 0, r = 0;
for(int i = 1; i <= n; i++) {
    p[i] = i <= r ? min(z[i - l + 1], r - i + 1) : 0;
    while(i + p[i] <= n && b[p[i] + 1] == a[i + p[i]]) p[i]++;
    if(i + p[i] - 1 > r) l = i, r = i + p[i] - 1;
}
```

3.3 Manacher

```
int m = (n << 1) + 1;
string t(m + 1, ' ');
for(int i = 1; i <= n; i++) {
    t[(i << 1) - 1] = '#';
    t[(i << 1)] = s[i];
}
t[m] = '#';

vector<int> f(m + 1);
int md = 0, r = 0, j, k;
for(int i = 1; i <= m; i++) {
    if(i <= r) f[i] = min(r - i, f[(md << 1) - i]);
    while(i-f[i]-1 >= 1 && i+f[i]+1 <= m && t[i-f[i]-1] == t[i+f[i]+1]) f[i]++;
    if(i + f[i] - 1 > r) md = i, r = i + f[i] - 1;
}

int ans = 1;
for(int i = 1; i <= m; i++) ans = max(ans, f[i]);
cout << ans << endl;
```

3.4 AC自动机

```
/**
 * n个模式串si 和 一个文本串t，求每个模式串的在文本串里的出现次数
 * ① 建Trie
 * ② 从根开始Bfs
 * long long 对效率存在影响，建议不开启 #define int long long
*/

struct ACAutomaton { // 默认无多测
    const static int maxn = 1e6 + 10; // 字符串总长
    const static int M = 26;          // 字符集
    struct Node {
        int son[M], go[M], fail;
        int cnt;
    } t[maxn];
    int tot, rt, cur;

    int newNode() {
        tot++;
        memset(t[tot].son, 0, sizeof(t[tot].son));
        memset(t[tot].go, 0, sizeof(t[tot].go));
        t[tot].fail = t[tot].cnt = 0;
        return tot;
    }
    void init() { tot = 0; rt = newNode(); }

    int insert(const string& s) { // 1-index
        int n = s.size() - 1;
        int p = rt;
        for(int j = 1; j <= n; j++) {
            int e = s[j] - 'a';
            if(!t[p].son[e]) t[p].son[e] = newNode();
            p = t[p].son[e];
        }
        return p;
    }

    int q[maxn + 1];
    int tl;
    void build() {
        tl = 0;
        q[++tl] = rt;
        for(int i = 1; i <= tl; i++) {
            int p = q[i];
            for(int j = 0; j < M; j++) {
                if(t[p].son[j]) {
                    t[p].go[j] = t[p].son[j];
                    t[t[p].son[j]].fail = (p == rt ? rt : t[t[p].fail].go[j]);
                    q[++tl] = t[p].son[j];
                } else {
                    t[p].go[j] = (p == rt ? rt : t[t[p].fail].go[j]);
                }
            }
        }
    }

    void query(const string& s) { // 1-index
        int n = s.size() - 1;
        int p = rt;
        for(int i = 1; i <= n; i++) {
            int e = s[i] - 'a';
            p = t[p].go[e];
            t[p].cnt += 1;
        }
        for(int i = tl; i >= 1; i--) {
            int p = q[i];
            t[t[p].fail].cnt += t[p].cnt;
        }
    }
};

ACAutomaton ac; // 多测记得清空
```

3.5 后缀数组 - 倍增 O(nlogn)

```
// long long 对效率存在影响，建议不开启 #define int long long
// all vec is 1-index; ht[2..n]
void buildSA(string& s, vector<int>& sa, vector<int>& rk, vector<int>& ht, int M = 128) {
    int n = s.size() - 1;
    sa = rk = ht = vector<int>(n + 1);
    vector<int> x(n + 1), y(n + 1), c(max(n, M) + 1);
    for(int i = 1; i <= n; i++) c[x[i] = s[i]]++;
    for(int i = 2; i <= M; i++) c[i] += c[i - 1];
    for(int i = n; i >= 1; i--) sa[c[x[i]]--] = i;
    for(int k = 1; k <= n; k <<= 1) {
        int p = 0;
        for(int i = n - k + 1; i <= n; i++) y[++p] = i;
        for(int i = 1; i <= n; i++) if(sa[i] > k) y[++p] = sa[i] - k;
        for(int i = 1; i <= M; i++) c[i] = 0;
        for(int i = 1; i <= n; i++) c[x[i]]++;
        for(int i = 2; i <= M; i++) c[i] += c[i - 1];
        for(int i = n; i >= 1; i--) sa[c[x[y[i]]]--] = y[i];
        swap(x, y);
        p = 1; x[sa[1]] = 1;
        for(int i = 2; i <= n; i++)
            x[sa[i]] = y[sa[i-1]] == y[sa[i]] && y[sa[i-1]+k] == y[sa[i]+k] ? p : ++p;
        if(p == n) break;
        M = p;
    }
    for(int i = 1; i <= n; i++) rk[sa[i]] = i;
    s.push_back(0);
    for(int i = 1, k = 0; i <= n; i++) {
        k = max(k - 1, 0ll);
        if(rk[i] == 1) continue;
        int j = sa[rk[i] - 1];
        while(s[i + k] == s[j + k]) k++;
        ht[rk[i]] = k;
    }
}

// n=1e5时，倍增241ms，SAIS 178ms (dls-oj)
// n=1e6时，SAIS 0.3s (reference)
// n=3e5，define int long long 影响 < 100ms
void work() {
    string s;
    cin >> s;
    int n = s.size();
    s = " " + s;

    vector<int> sa, rk, ht;
    buildSA(s, sa, rk, ht); // 1-index
    // ST-LCP 按需
    int lim = __lg(n);
    vector<vector<int>> st(lim + 1, vector<int>(n + 1));
    for(int i = 1; i <= n; i++) st[0][i] = ht[i];
    for(int k = 1; k <= lim; k++) {
        int lim2 = n - (1 << k) + 1;
        for(int i = 1; i <= lim2; i++) {
            st[k][i] = min(st[k-1][i], st[k-1][i+(1<<k-1)]);
        }
    }
    auto query = [&](int L, int R) {
        int k = __lg(R - L + 1);
        return min(st[k][L], st[k][R-(1<<k)+1]);
    };
    auto LCP = [&](int u, int v) {
        if(u == v) return n - u + 1;
        if(rk[u] > rk[v]) swap(u, v);
        // RMQ(ht, rk[u] + 1, rk[v])
        return query(rk[u] + 1, rk[v]);
    };

    // input: abaabaaabaaaab
    // output: 10 11 6 12 7 3 13 8 4 1 14 9 5 2
    //         3 4 2 3 6 1 2 5 4 0 1 4 3
    for(int i = 1; i <= n; i++) cout << sa[i] << " "; cout << endl;
    // for(int i = 2; i <= n; i++) cout << ht[i] << " "; cout << endl;
    // for(int i = 2; i <= n; i++) cout << LCP(sa[i-1], sa[i]) << " "; cout << endl;
}
```

3.6 后缀数组 - SAIS O(n)

```
// long long 对效率存在影响，建议不开启 #define int long long
struct SA{
    static const int MAX_LEN=1e6+5;
    static const int SIGMA=128;
    int str[MAX_LEN<<1];
    int type[MAX_LEN<<1];
    int LMS[MAX_LEN<<1];
    int sa[MAX_LEN];
    int rank[MAX_LEN];
    int cnt[MAX_LEN];
    int cur[MAX_LEN];
    void inducedsort(int *str,int len,int sigma,int *LMS,int LMSC,int *type){
        #define PUSH_S(x) (sa[cur[str[x]]--] = x)
        #define PUSH_L(x) (sa[cur[str[x]]++] = x)
        memset(sa,-1,sizeof(int)*(len+2));memset(cnt,0,sizeof(int)*(sigma+2));
        for(int i=0;i<len;i++)++cnt[str[i]];
        for(int i=1;i<sigma;i++)cnt[i]+=cnt[i-1];
        for(int i=0;i<sigma;i++)cur[i]=cnt[i]-1;
        for(int i=LMSC-1;i>=0;--i)PUSH_S(LMS[i]);
        for(int i=1;i<sigma;i++)cur[i]=cnt[i-1];
        for(int i=0;i<len;i++)sa[i]>0&&type[sa[i]-1]==0&&PUSH_L(sa[i]-1);
        for(int i=0;i<sigma;i++)cur[i]=cnt[i]-1;
        for(int i=len-1;i>=0;--i)sa[i]>0&&type[sa[i]-1]&&PUSH_S(sa[i]-1);
        #undef PUSH_S
        #undef PUSH_L
    }
    void sais(int *str,int len,int sigma,int *LMS,int *type){
        type[len-1]=1;
        for(int i=len-2;i>=0;--i)type[i]=(str[i]==str[i+1]?type[i+1]:str[i]<str[i+1]);
        int LMSC=0;
        rank[0]=-1;
        for(int i=1;i<len;i++)rank[i]=(type[i]&&!type[i-1])?(LMS[LMSC]=i,LMSC++):-1;
        inducedsort(str,len,sigma,LMS,LMSC,type);
        int tot=-1;
        int *s1=str+len;
        for(int i=0,now,last;i<len;i++){
            if(-1==(now=rank[sa[i]]))continue;
            if(tot<1||LMS[now+1]-LMS[now]!=LMS[last+1]-LMS[last])++tot;
            else for(int j=LMS[now],k=LMS[last];j<LMS[now+1];++j,++k)if((str[j]<<1|type[j])!=(str[k]<<1|type[k])){
                ++tot;
                break;
            }
            s1[last=now]=tot;
        }
        if(tot+1<LMSC)sais(s1,LMSC,tot+1,LMS+LMSC,type+len);
        else for(int i=0;i<LMSC;i++)sa[s1[i]]=i;
        for(int i=0;i<LMSC;i++)s1[i]=LMS[sa[i]];
        inducedsort(str,len,sigma,s1,LMSC,type);
    }
    // 0-index
    void buildSA(string& s, vector<int>& sa, vector<int>& rk, vector<int>& ht){
        int n = s.size();
        sa = rk = ht = vector<int>(n + 1);
        for(int i = 0; i < n; i++) str[i] = s[i];
        sais(str, n + 1, SIGMA, LMS, type);
        for(int i = 1; i <= n; i++) sa[i] = this->sa[i] + 1;
        for(int i = 1; i <= n; i++) rk[sa[i]] = i;
        s.push_back(0);
        for(int i = 1, k = 0; i <= n; i++) {
            k = max(k - 1, 0);
            if(rk[i] == 1) continue;
            int j = sa[rk[i] - 1];
            while(s[i + k] == s[j + k]) k++;
            ht[rk[i]] = k;
        }
    }
}S;

void work() {
    string s;
    cin >> s;
    int n = s.size();

    vector<int> sa, rk, ht;
    S.buildSA(s, sa, rk, ht); // 0-index

    for(int i = 1; i <= n; i++) cout << sa[i] << " "; cout << endl;
}
```

3.7 后缀自动机

```
struct SAM {
    static const int M = 26;
    struct Node {
        int len, link;
        int nxt[M];
    };
    vector<Node> t;
    int N, lst;

    Node& operator[] (int i) { return t[i]; }

    SAM(int n = 0) { init(n); }
    void init(int n) {
        t.resize(2 * n + 1); // 2n-1 Points, 3n-4 Edges
        N = 0;
        t[++N].len = 0;
        t[N].link = 0;
        memset(t[N].nxt, 0, sizeof(t[N].nxt));
        lst = N;
    }

    void add(char c) {
        c -= 'a';
        if(t[lst].nxt[c] && t[lst].len + 1 == t[t[lst].nxt[c]].len) { // for exSAM
            lst = t[lst].nxt[c];
            return;
        }

        int x = ++N;
        t[N].len = t[lst].len + 1;
        t[N].link = 0;
        memset(t[N].nxt, 0, sizeof(t[N].nxt));

        int p = lst;
        lst = x;
        for(; p && !t[p].nxt[c]; p = t[p].link) t[p].nxt[c] = x;
        if(!p) {
            t[x].link = 1;
        } else {
            int q = t[p].nxt[c];
            if(t[p].len + 1 == t[q].len) {
                t[x].link = q;
            } else {
                int cl = ++N;
                t[cl] = t[q];
                t[cl].len = t[p].len + 1;
                if(t[cl].len == t[x].len) lst = cl;
                for(; p && t[p].nxt[c] == q; p = t[p].link) t[p].nxt[c] = cl;
                t[q].link = t[x].link = cl;
            }
        }
    }
};
```

3.8 广义后缀自动机

```
/* ----- ex suffix automaton ----- */
struct state {
    int len, link;
    int nxt[26];
} t[maxn]; // need double space
int N, lst;

int newNode() {
    ++N;
    for(int j = 0; j <= 25; j++) t[N].nxt[j] = 0;
    t[N].link = 0;
    t[N].len = 0;
    return N;
}

void samInit() {
    N = 0;
    newNode();
}

void trieBuild(const char *s, int n) {
    int p = 1;
    for(int i = 1; i <= n; i++) {
        int e = s[i] - 'a';
        if(t[p].nxt[e] == 0)
            t[p].nxt[e] = newNode();
        p = t[p].nxt[e];
    }
}

int samAdd(int lst, int c) { // lst is father; c is edge
    int x = t[lst].nxt[c];
    if(t[x].len) return x;

    t[x].len = t[lst].len + 1;
    int p = t[lst].link;
    while(p && !t[p].nxt[c]) {
        t[p].nxt[c] = x;
        p = t[p].link;
    }
    if(!p) {
        t[x].link = 1;
        return x;
    }
    int q = t[p].nxt[c];
    if(t[p].len + 1 == t[q].len) {
        t[x].link = q;
        return x;
    }
    int cl = newNode();
    for(int i = 0; i <= 25; i++)
        t[cl].nxt[i] = t[t[q].nxt[i]].len != 0 ? t[q].nxt[i] : 0;
    t[cl].len = t[p].len + 1;
    while(p && t[p].nxt[c] == q) {
        t[p].nxt[c] = cl;
        p = t[p].link;
    }
    t[cl].link = t[q].link;
    t[x].link = cl;
    t[q].link = cl;
    return x;
}

void samBuild() {
    queue<pair<int, int> > q;
    for(int i = 0; i <= 25; i++)
        if(t[1].nxt[i])
            q.push(make_pair(1, i));
    while(!q.empty()) {
        auto pr = q.front(); q.pop();
        int lst = samAdd(pr.first, pr.second);
        for(int i = 0; i <= 25; i++)
            if(t[lst].nxt[i])
                q.push(make_pair(lst, i));
    }
}

/* ----- ex suffix automaton ----- */

int n;
char s[maxn];

void work() {

    cin >> n;
    samInit();
    for(int i = 1; i <= n; i++) {
        cin >> (s + 1);
        int m = strlen(s + 1);
        trieBuild(s, m);
    }
    samBuild();

    int ans = 0;
    for(int i = 2; i <= N; i++)
        ans += t[i].len - t[t[i].link].len;
    cout << ans << endl;
}
```

3.9 例题：SAM+线段树合并+树上倍增

```
/**
 * 2022广东省赛I题：https://codeforces.com/gym/103650/problem/I
 * 原题：给一个字符串S和Q次询问，每次询问查询 一个最短的串使得S[l..r]是该串的Border，输出串长
 *
 * 线段树合并，维护后缀自动机上每个结点的endpos集合
 * 线段树维护了endpos集合中相邻两个非0元素的最近距离
 * 树上倍增可以logn找到一个子串对应的节点
*/

// 单点修改，区间最短相邻1，合并
const int INF = 1 << 30;
struct SGT {
    struct Node {
        int L, R, ans;
        Node(): L(0), R(0), ans(INF) {}
        Node(int _L, int _R, int _ans): L(_L), R(_R), ans(_ans) {}
        Node operator+(const Node& p) const {
            if(L == 0 && p.L == 0) return Node{0, 0, INF};
            if(L == 0) return p;
            if(p.L == 0) return *this;
            Node s;
            s.ans = min(p.L - R, min(ans, p.ans));
            s.L = L;
            s.R = p.R;
            return s;
        }
    };
    vector<Node> t;
    vector<int> lc, rc;
    int N;

    #define LC (lc[x])
    #define RC (rc[x])

    SGT(int n = 0) { init(n); }
    void init(int n) {
        t.resize(n);
        lc.resize(n);
        rc.resize(n);
        N = 0;
    }

    void pushup(int x) {
        t[x] = t[LC] + t[RC];
    }
    void modify(int &x, int l, int r, int pos) {
        if(!x) x = ++N;

        if(pos <= l && r <= pos) { t[x] = {l, l, INF}; return; }
        if(pos < l || r < pos) return;

        int mid = l + r >> 1;
        if(pos <= mid) modify(LC, l, mid, pos);
        else modify(RC, mid+1, r, pos);
        pushup(x);
    }
    // 线段树合并，每次不新建节点
    int merge(int x, int y) {
        if(!x || !y) return x + y;
        lc[x] = merge(lc[x], lc[y]);
        rc[x] = merge(rc[x], rc[y]);
        pushup(x);
        return x;
    }
    // 线段树合并，每次新建节点（空间复杂度需要开2倍）
    // int merge(int y, int z) {
    //  if(!y || !z) return y + z;
    //  int x = ++N;
    //  lc[x] = merge(lc[y], lc[z]);
    //  rc[x] = merge(rc[y], rc[z]);
    //  pushup(x);
    //  return x;
    // }
};

const int M = 26;
struct SAM {
    struct Node {
        int len, link;
        int nxt[M];
    };
    vector<Node> t;
    int N, lst;

    vector<int> endpos;
    vector<int> pos;

    SAM(int n = 0) { init(n); }
    void init(int n) {
        t.resize(2 * n + 1);
        N = 1;
        t[N] = t[0];
        lst = N;

        endpos.resize(2 * n + 1);
        pos.resize(2 * n + 1);
    }

    void add(int c, int id) {
        c -= 'a';
        int x = ++N;
        t[x] = t[0];
        t[x].len = t[lst].len + 1;

        endpos[N] = id;
        pos[id] = N;

        int p = lst;
        lst = x;
        for(; p && t[p].nxt[c] == 0; p = t[p].link) t[p].nxt[c] = x;

        if(!p) {
            t[x].link = 1;
        } else {
            int q = t[p].nxt[c];
            if(t[p].len + 1 == t[q].len) {
                t[x].link = q;
            } else {
                int cl = ++N;
                t[cl] = t[q];
                t[cl].len = t[p].len + 1;
                if(t[cl].len == t[x].len) lst = cl;
                for(; p && t[p].nxt[c] == q; p = t[p].link) t[p].nxt[c] = cl;
                t[q].link = t[x].link = cl;
            }
        }
    }

    void solve() {
        /* 1. build edges */
        vector<vector<int>> e(N + 1, vector<int>());
        for(int i = 2; i <= N; i++) e[t[i].link].push_back(i);

        /* 2. ST */
        vector<vector<int>> st(log2(N) + 1, vector<int>(N + 1));
        int lim = log2(N);
        for(int i = 1; i <= N; i++) st[0][i] = t[i].link;
        for(int i = 1; i <= lim; i++) {
            for(int j = 1; j <= N; j++) {
                st[i][j] = st[i - 1][st[i - 1][j]];
            }
        }

        auto getPos = [&](int L, int R) {
            int len = R - L + 1;
            int p = pos[R];
            for(int i = lim; i >= 0; i--) {
                if(t[st[i][p]].len >= len) {
                    p = st[i][p];
                }
            }
            return p;
        };

        /* 3. endpos & solve */
        int Q;
        cin >> Q;

        vector<int> ans(Q + 1);
        vector<vector<int>> q(N + 1, vector<int>());
        for(int i = 1; i <= Q; i++) {
            int L, R;
            cin >> L >> R;
            int p = getPos(L, R);
            ans[i] = R - L + 1;
            q[p].push_back(i);
        }

        vector<int> rt(N + 1);
        SGT sgt(N * (log2(N) + 2) + 10);

        function<void(int)> dfs2 = [&](int x) {
            if(endpos[x]) sgt.modify(rt[x], 1, N, endpos[x]);
            for(auto y: e[x]) {
                dfs2(y);
                rt[x] = sgt.merge(rt[x], rt[y]);
            }
            int v = sgt.t[rt[x]].ans;
            for(auto i: q[x]) {
                ans[i] = (v == INF ? -1 : ans[i] + v);
            }
        };
        dfs2(1);

        /* 5. output */
        for(int i = 1; i <= Q; i++) cout << ans[i] << endl;
    }
};

void work() {
    string s;
    cin >> s;
    int n = s.size();
    s = " " + s;

    SAM sam(n);
    for(int i = 1; i <= n; i++) sam.add(s[i], i);
    sam.solve();
}
```

3.10 最小表示法

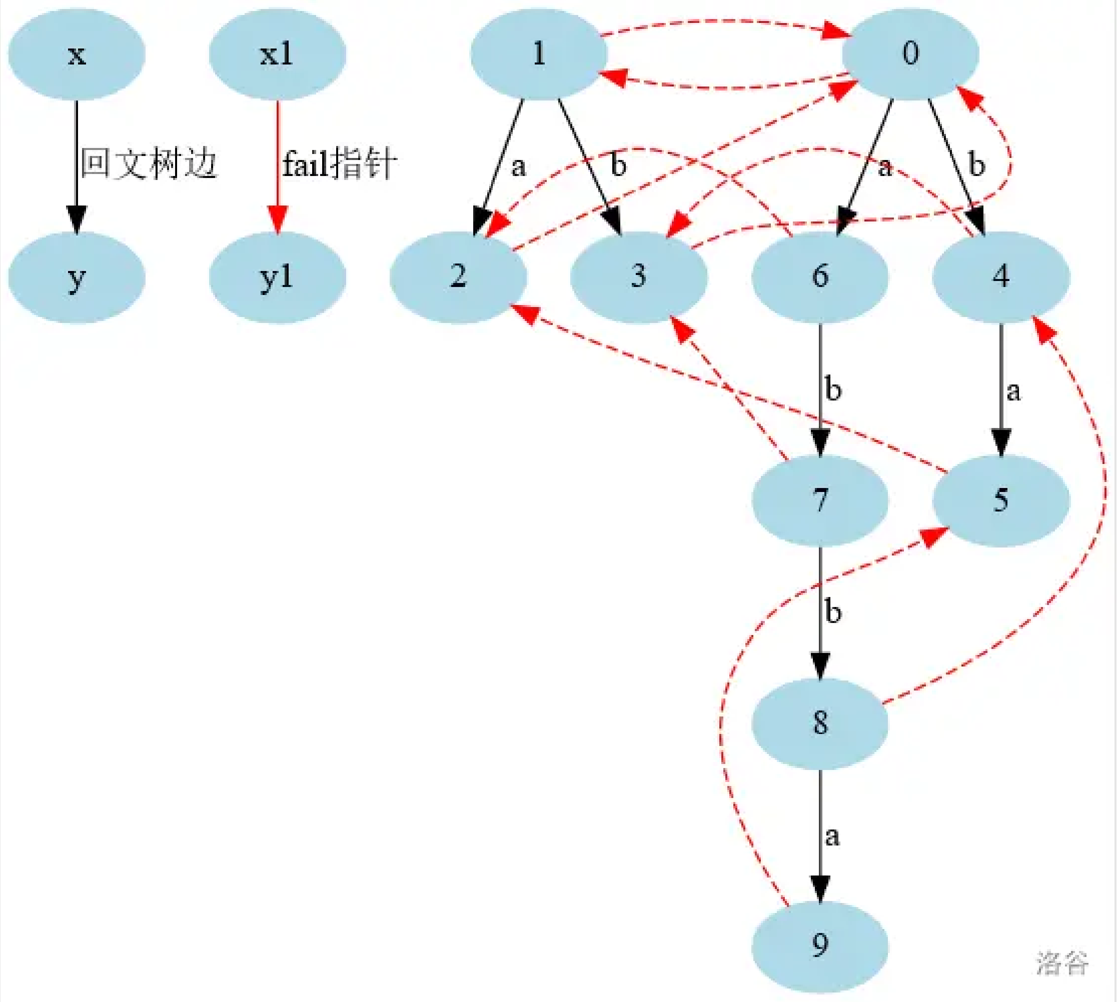
```
// C++ Version
int k = 0, i = 0, j = 1;
while (k < n && i < n && j < n) {
  if (sec[(i + k) % n] == sec[(j + k) % n]) {
    k++;
  } else {
    sec[(i + k) % n] > sec[(j + k) % n] ? i = i + k + 1 : j = j + k + 1;
    if (i == j) i++;
    k = 0;
  }
}
i = min(i, j);
```

3.11 杂题

- JSOI 2007 文本生成器（AC自动机+DP）
  - 题意：给定 m 个字符串 s_i，求长度为 n 且包含至少一个 s_i 的字符串的方案数。
  - 数据范围：$1 \le n \le 60,\ 1 \le m \le 100,\ 1 \le |s_i| \le 100$
  - 题解：AC自动机上DP，$dp[i][j][0/1]$: 填了 i 个字符，在AC自动机上转移到第 j 个点，是否经过任意一个 s_i 的方案数
- POI 2000 病毒（AC自动机）
  - 题意：给定 n 个 01 串 s_i，求是否存在一个无限长的 01 串，使得该串不包含任意一个 s_i
  - 数据范围：$1 \le n \le 2000,\ \sum |s_i| \le 5e4$
  - 题解：若AC自动机上存在一个不经过 **标记点** 的环，则存在一个合法无限长 01 串。
    - 标记点：表示一个完整 s_i 的节点。
    - 找环：直接dfs，若下一个点，已经访问过且在栈中，则找到一个环。
- 阿狸的打字机（Fail树）
  - 题意：给定一个Trie树，有 m 次询问，每次求一个节点表示的字符串，在另一个节点表示的字符串中出现了几次。
  - 题解：（应该是）等价于，在Fail树上，这两个节点必须满足一个是另一个的祖先，而次数即为两个点的距离。
- AHOI 2013 差异（后缀数组）
  - 题意：求 $\sum_{1\le i<j\le n} len(T_i) + len(T_j) - 2 * LCP(T_i,\ T_j)$
  - 题解：后缀数组 + 笛卡尔树上计数（或并查集等经典模型）

3.12 回文自动机

3.12.1 总览

- 每个本质不同的回文串个数
    - 节点数-2（除去0节点与1节点）
- 以某个节点为结尾的回文串个数
    - 见 `3.12.2`
- 统计某个回文串的出现次数
    - 见 `3.12.3`
    - 需要在fail树上从底向上累计
- 公共回文子串数量
    - 对两个串建立PAM，如果有相同的状态，就是找到了相同的回文串
- 最小回文划分
    - 题意：将字符串S拆成 k 个回文串，使得 k 数值最小
        - 显然可以划分成 n 个回文串
    - 解法：考虑dp[i]: 前缀i的最小回文划分
- 字符串 `abbaabba` 的PAM
    - 

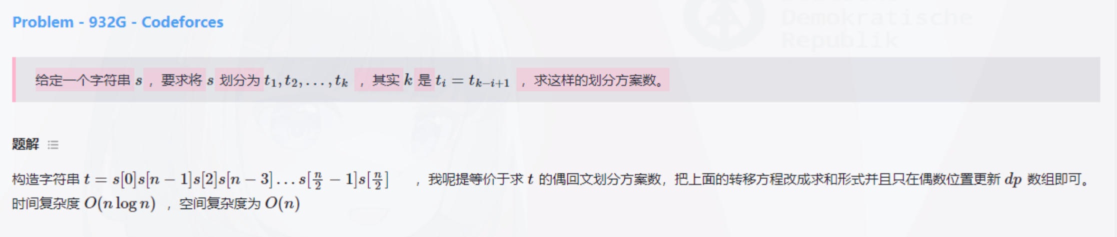
3.12.2 每个前缀的回文串个数

```cpp
// 求每个前缀的回文串个数
struct PAM {
    struct Node {
        int fail, len, num, fa; // num: 该回文串的后缀的回文串个数
        int son[26];
    };
    vector<Node> t;
    vector<int> f; // f[i]: 以第i个字符结尾的回文串个数
    int N;

    PAM(int n = 0) {
        t.resize(n + 2);
        f.resize(n + 2);
        t[0].fail = 1;
        t[1].len = -1;
        N = 1;
    }

    void build(string& s) {
        int n = s.size() - 1;
        auto getfail = [&](int x, int i) {
            while(s[i - t[x].len - 1] != s[i]) x = t[x].fail;
            return x;
        };
        int lst = 0;
        for(int i = 1; i <= n; i++) {
            /* For LuoguP5496 */ if(i >= 2) s[i] = (s[i] + f[i - 1] - 'a') % 26 + 'a';
            int e = s[i] - 'a';
            int x = getfail(lst, i);
            if(!t[x].son[e]) {
                t[++N].fail = t[getfail(t[x].fail, i)].son[e];
                t[x].son[e] = N;
                t[N].fa = x;
                t[N].len = t[x].len + 2;
                t[N].num = t[t[N].fail].num + 1;
            }
            lst = t[x].son[e];
            f[i] = t[lst].num;
        }
    }
};

void work() {
    string s;
    cin >> s;
    int n = s.size();
    s = " " + s;
    PAM pam(n);
    pam.build(s);

    for(int i = 1; i <= n; i++) cout << pam.f[i] << " ";
    cout << endl;
}
```

3.12.3 每个本质不同的回文串个数

```
struct PAM {
    struct Node {
        int fail, len, num, fa;
        int son[26];
    };
    vector<Node> t;
    int N = 0;

    PAM(int n) {
        t.resize(n + 2);
        t[0].fail = 1;
        t[1].len = -1;
        N = 1;
    }

    int build(string& s) {
        int n = s.size() - 1;

        auto getfail = [&](int x, int i) {
            while(s[i - t[x].len - 1] != s[i]) x = t[x].fail;
            return x;
        };

        int lst = 0;
        vector<int> f(n + 2); // f[i]: 第i个点的出现次数
        for(int i = 1; i <= n; i++) {
            int e = s[i] - 'a';
            int x = getfail(lst, i);
            if(!t[x].son[e]) {
                t[++N].fail = t[getfail(t[x].fail, i)].son[e];
                t[x].son[e] = N;
                t[N].fa = x;
                t[N].len = t[x].len + 2;
                t[N].num = t[t[N].fail].num + 1;
            }
            lst = t[x].son[e];
            f[lst] += 1;
        }

        vector<int> id(N);
        for(int i = 2; i <= N; i++) id[i - 1] = i;
        sort(id.begin() + 1, id.end(), [&](int i, int j) {
            return t[i].len > t[j].len;
        });
        int ans = 0;
        for(int i = 1; i < N; i++) {
            int x = id[i];
            ans = max(ans, f[x] * t[x].len);
            f[t[x].fail] += f[x]; // 类似后缀自动机，往fail树父亲累计贡献
        }
        return ans;
    }
};
```

3.12.4 最小回文划分相关(无整理纯复制)

- Codeforces 932G
  - 

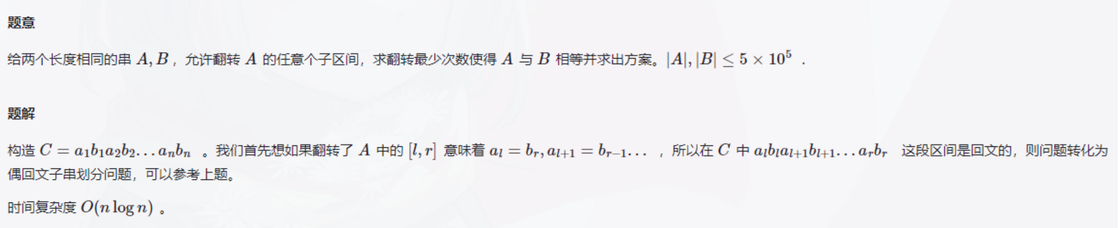
    Problem - 932G - Codeforces

    给定一个字符串 s ，要求将 s 划分为 $t_1, t_2, \dots, t_k$ ，其实 k 是 $t_i = t_{k-i+1}$ ，求这样的划分方案数。

    题解

    构造字符串 $t = s[0]s[n-1]s[2]s[n-3]\dots s[\frac{n}{2}-1]s[\frac{n}{2}]$ ，我呢提等价于求 t 的偶回文划分方案数，把上面的转移方程改成求和形式并且只在偶数位置更新 dp 数组即可。时间复杂度 $O(n\log n)$ ，空间复杂度为 $O(n)$

```
#include <iostream>
#include <cstdio>
#include <cstring>
using namespace std;
typedef long long ll;
const int MOD=1000000007,N=1000006;
struct PAM{
    int s[N];
    int tot,siz,las;
    int ch[N][26],fail[N],len[N],dif[N],slink[N];
    inline nwnode(int l){
        len[++tot]=l;
        memset(ch[tot],0,sizeof(ch[tot]));
        fail[tot]=0;
        return tot;
    }
    PAM(){
        tot=-1,las=0;
        s[siz=0]='#';
        nwnode(0);
        nwnode(-1);
        fail[0]=1;
    }
    inline int getfail(int x){
        while(s[siz-len[x]-1]!=s[siz])x=fail[x];
        return x;
    }
    inline void inser(char c){
        s[++siz]=c;
        int u=getfail(las);
        if(!ch[u][c-'a']){
            int v=nwnode(len[u]+2);
            fail[v]=ch[getfail(fail[u])][c-'a'];
            ch[u][c-'a']=v;
            dif[v]=len[v]-len[fail[v]];
            if(dif[v]==dif[fail[v]])
                slink[v]=slink[fail[v]];
            else slink[v]=fail[v];//不满足等差数列，直接赋成现在的fail
        }
        las=ch[u][c-'a'];
    }
}am;
int n;ll dp[N],g[N];
char s[N],t[N];
int main(){
    scanf("%s",s+1);
    n=strlen(s+1);
    for(int i=1,p=0;i<=n;++i)
        t[++p]=s[i],t[++p]=s[n-i+1];
    dp[0]=1;
    for(int i=1;i<=n;++i){
        am.inser(t[i]);
        for(int x=am.las;x>1;x=am.slink[x]){
            g[x]=dp[i-am.len[am.slink[x]]-am.dif[x]];
            if(am.dif[x]==am.dif[am.fail[x]])g[x]=(g[x]+g[am.fail[x]])%MOD;
            if(i%2==0)dp[i]=(dp[i]+g[x])%MOD;
        }
    }printf("%lld\n",dp[n]);
    return 0;
}
```

- Codeforces 906E
    - 
      题意

      给两个长度相同的串 A,B，允许翻转 A 的任意个子区间，求翻转最少次数使得 A 与 B 相等并求出方案。$|A|,|B|\le 5\times 10^5$.

      题解

      构造 $C=a_1b_1a_2b_2\dots a_nb_n$ 。我们首先想如果翻转了 A 中的 $[l,r]$ 意味着 $a_l=b_r,a_{l+1}=b_{r-1}\dots$ ，所以在 C 中 $a_lb_la_{l+1}b_{l+1}\dots a_rb_r$ 这段区间是回文的，则问题转化为偶回文子串划分问题，可以参考上题。

      时间复杂度 $O(n\log n)$ 。

```
#include <iostream>
#include <cstdio>
#include <cstring>
typedef long long ll;
using namespace std;
const int N=1000006;
int n;
int dp[N],g[N],pre[N];
struct PAM{
    int s[N];
    int siz,tot,las;
    int fail[N],ch[N][26],len[N],dif[N],slk[N];
    inline int nwnode(int l){
        len[++tot]=l;
        memset(ch[tot],0,sizeof(ch[tot]));
        fail[tot]=0;
        return tot;
    }
    PAM(){
        tot=-1,las=0;
        s[siz=0]=-1;
        nwnode(0);
        nwnode(-1);
        fail[0]=1;
    }
    inline int getfail(int x){
        while(s[siz-len[x]-1]!=s[siz])x=fail[x];
        return x;
    }
    inline void inser(int c){
        s[++siz]=c;
        int u=getfail(las);
        if(!ch[u][c]){
            int v=nwnode(len[u]+2);
            fail[v]=ch[getfail(fail[u])][c];
            ch[u][c]=v;
            dif[v]=len[v]-len[fail[v]];
            if(dif[v]==dif[fail[v]])
                slk[v]=slk[fail[v]];
            else slk[v]=fail[v];
        }las=ch[u][c];
    }
    inline void solve(){
        if(siz%2==0&&s[siz]==s[siz-1]&&dp[siz]>dp[siz-2])
            dp[siz]=dp[siz-2],pre[siz]=siz-2;
        for(int x=las;x;x=slk[x]){
            g[x]=siz-len[slk[x]]-dif[x];
            if(dif[fail[x]]==dif[x]&&dp[g[x]]>dp[g[fail[x]]])
                g[x]=g[fail[x]];
            if(siz%2==0&&dp[siz]>dp[g[x]]+1)
                dp[siz]=dp[g[x]]+1,pre[siz]=g[x];
        }
    }
}am;
char s1[N],s2[N],s[N*2];
int main(){
    scanf("%s%s",s1+1,s2+1);
    n=strlen(s1+1);
    for(int i=1;i<=n;++i){
        //printf("%c%c",s1[i],s2[i]);
        s[i*2-1]=s1[i],s[i*2]=s2[i];
    }
    n*=2;
    memset(dp,0x3f,sizeof(dp));
    dp[0]=0;
    for(int i=1;i<=n;++i){
        am.inser(s[i]-'a');
        am.solve();
    }
    if(dp[n]>=0x3f3f3f3f)return puts("-1"),0;
    printf("%d\n",dp[n]);
    for(int i=n;i>=1;i=pre[i]){
        if(i-pre[i]>2)
            printf("%d %d\n",pre[i]/2+1,i/2);
    }
    return 0;
}//  1.33s /  133.78MB /  1.62KB C++14 O2
```

3.12.5 一个串的所有前缀的双回文子串个数

```
/**
 * 设A是任意一个回文串，则形如AA的串，就叫做双回文串
 * 统计一个串的所有前缀的双回文子串个数
*/
struct PAM {
    struct Node {
        int fail, len, fa, sum;
        int son[26];
        int cnt = 0;
    };
    vector<Node> t;
    vector<vector<int>> fa;
    int lim;
    int N = 0;

    PAM(int n) {
        lim = log2(n + 1);
        t.resize(n + 2);
        fa.resize(lim + 1, vector<int>(n + 2));
        t[0].fail = 1;
        t[1].len = -1;
        N = 1;
    }

    void solve(string& s) {
        int n = s.size() - 1;

        auto getfail = [&](int x, int i) {
            while(s[i - t[x].len - 1] != s[i]) x = t[x].fail;
            return x;
        };

        int lst = 0;
        int ans = 0;
        for(int i = 1; i <= n; i++) {
            int e = s[i] - 'a';
            int x = getfail(lst, i);
            if(!t[x].son[e]) {
                t[++N].fail = t[getfail(t[x].fail, i)].son[e];
                t[x].son[e] = N;
                t[N].fa = x;

                // 树上倍增
                fa[0][N] = t[N].fail;
                for(int k = 1; k <= lim; k++) fa[k][N] = fa[k - 1][fa[k - 1][N]];

                t[N].len = t[x].len + 2;
            }
            lst = t[x].son[e];
            t[lst].cnt = t[t[lst].fail].cnt;

            // 找fail树祖先中是否存在长度为当前回文串一半的节点
            if(~t[lst].len & 1) {
                int y = lst;
                int tar = t[lst].len / 2;
                for(int k = lim; k >= 0; k--) {
                    if(t[fa[k][y]].len == tar) {
                        y = fa[k][y];
                        break;
                    } else if(t[fa[k][y]].len > tar) {
                        y = fa[k][y];
                    }
                }
                if(tar > 0 && t[y].len == tar) {
                    t[lst].cnt += 1;
                }
            }

            ans += t[lst].cnt;
            cout << ans << " ";
        }
        cout << endl;
    }
};
```

4. 数据结构

4.1 线性基

```
struct LB {
    int n;
    vector<int> a;
    LB(int n = 0) { init(n); }
    void init(int n) { this->n = n; a.resize(n + 1); }
    bool insert(int x) {
        for(int i = n; i >= 0; i--) if(x >> i & 1) {
            if(a[i] == 0) {
                a[i] = x;
                return true;
            }
            x ^= a[i];
        }
        return false;
    }
    void flush() { // 让线性基形成最简行阶梯矩阵
        for(int i = 0; i <= n; i++) {
            if(a[i] == 0) continue;
            for(int j = i + 1; j <= n; j++) {
                if(a[j] == 0) continue;
                if(a[j] >> i & 1) a[j] ^= a[i];
            }
        }
    }
    int queryMin(int x) {
        for(int i = n; i >= 0; i--) {
            x = min(x, x ^ a[i]);
        }
        return x;
    }
    int queryMax(int x) {
        for(int i = n; i >= 0; i--) {
            x = max(x, x ^ a[i]);
        }
        return x;
    }
};

void work() {
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) cin >> a[i];

    LB b(60);
    int cnt0 = 0;
    for(int i = 1; i <= n; i++) {
        if(!b.insert(a[i])) {
            cnt0++;
        }
    }

    // 为什么是向下取整呢？因为有第0小的数，所以是向下取整
    k = k / (1LL << cnt0);
    // k >>= cnt0;
    int ans = 0;
    int cnt = 0;
    for(int i = 0; i < B; i++) {
        if(b.a[i] == 0) continue;
        cnt++;
    }
    cnt--;
    for(int i = B - 1; i >= 0; i--) {
        if(b.a[i] == 0) continue;
        if(k >= (1LL << cnt)) {
            k -= 1LL << cnt;
            ans = max(ans, ans ^ b.a[i]);
        } else {
            ans = min(ans, ans ^ b.a[i]);
        }
        cnt--;
    }

    cout << ans << endl;
}
```

4.2 雨天的尾巴：树链剖分+线段树合并+树上差分

```
/* 线段树 单点加 区间最大值 线段树合并*/
struct ST {
    struct Node {
        int lc, rc;
        array<int, 2> s;
    };
    vector<Node> t;
    int tot;
    #define LC (t[x].lc)
    #define RC (t[x].rc)

    ST(int n = 0) { init(n); }
    void init(int n) { t.resize(n); tot = 0; }

    int newNode() { return ++tot; }
    void pushup(int x) {
        t[x].s = max(t[LC].s, t[RC].s);
    }

    void modify(int &x, int l, int r, int pos, int v) {
        if(!x) x = newNode();
        if(pos <= l && r <= pos) {
            if(t[x].s[1] == 0) t[x].s = {v, -l};
            else t[x].s[0] += v;
            return;
        }
        if(pos < l || r < pos) return;
        int mid = l + r >> 1;
        if(pos <= mid) modify(LC, l, mid, pos, v);
        else modify(RC, mid+1, r, pos, v);
        pushup(x);
    }

    array<int, 2> query(int x, int l, int r, int L, int R) {
        if(!x) return {0, 0};
        if(L <= l && r <= R) return t[x].s;
        if(R < l || r < L) return {0, 0};
        int mid = l + r >> 1;
        return max(query(LC, l, mid, L, R), query(RC, mid+1, r, L, R));
    }

    int merge(int x, int y, int l, int r) {
        if(!x || !y) return x + y;
        if(l == r) {
            t[x].s = {t[x].s[0] + t[y].s[0], -l};
            return x;
        }
        int mid = l + r >> 1;
        LC = merge(LC, t[y].lc, l, mid);
        RC = merge(RC, t[y].rc, mid+1, r);
        pushup(x);
        return x;
    }
};

void work() {
    int n, m;
    cin >> n >> m;

    vector<vector<int>> e(n + 1, vector<int>());
    for(int i = 1; i < n; i++) {
        int u, v;
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }

    /* 树链剖分 */
    vector<int> fa(n + 1), sz(n + 1), son(n + 1), dep(n + 1),
                dfn(n + 1), dfnR(n + 1), top(n + 1), rk(n + 1);

    function<void(int)> dfs1 = [&](int x) {
        dep[x] = dep[fa[x]] + 1;
        sz[x] = 1;
        for(auto y: e[x]) {
            if(y == fa[x]) continue;
            fa[y] = x;
            dfs1(y);
            sz[x] += sz[y];
            if(sz[y] > sz[son[x]]) son[x] = y;
        }
    };
    function<void(int)> dfs2 = [&](int x) {
        dfn[x] = ++dfn[0];
        rk[dfn[0]] = x;
        if(!top[x]) top[x] = x;
        if(son[x]) top[son[x]] = top[x], dfs2(son[x]);
        for(auto y: e[x]) if(y != fa[x] && y != son[x]) dfs2(y);
        dfnR[x] = dfn[0];
    };
    auto getLca = [&](int x, int y) {
        while(top[x] != top[y]) {
            if(dep[top[x]] > dep[top[y]]) swap(x, y);
            y = fa[top[y]];
        }
        if(dep[x] > dep[y]) swap(x, y);
        return x;
    };

    dfs1(1);
    dfs2(1);

    /* 线段树（调用） */
    int N = 1e5;
    ST st(5 * m * __lg(N));
    vector<int> rt(n + 1);

    for(int i = 1; i <= m; i++) {
        int x, y, z;
        cin >> x >> y >> z;

        int lca = getLca(x, y);

        st.modify(rt[x], 1, N, z, 1);
        st.modify(rt[y], 1, N, z, 1);
        st.modify(rt[lca], 1, N, z, -1);
        st.modify(rt[fa[lca]], 1, N, z, -1);
    }

    /* 树上差分 */

    vector<int> ans(n + 1);
    function<void(int)> dfs3 = [&](int x) {
        for(auto y: e[x]) {
            if(y == fa[x]) continue;
            dfs3(y);
            rt[x] = st.merge(rt[x], rt[y], 1, N);
        }
        ans[x] = -st.query(rt[x], 1, N, 1, N)[1];
    };
    dfs3(1);
    for(int i = 1; i <= n; i++) cout << ans[i] << endl;
}
```

4.3 线段树

```
struct ST {
    struct Node {
        int s, lzt;
    };
    vector<Node> t;
    #define LC (x << 1)
    #define RC (x << 1 | 1)

    ST(int n = 0) { init(n); }
    void init(int n) { t.resize(n << 2); }

    void pushup(int x) {
        t[x].s = t[LC].s + t[RC].s;
    }
    void pushdown(int x, int l, int r) {
        if(t[x].lzt) {
            int mid = l + r >> 1;

            t[LC].s += t[x].lzt * (mid - l + 1);
            t[LC].lzt += t[x].lzt;

            t[RC].s += t[x].lzt * (r - mid);
            t[RC].lzt += t[x].lzt;

            t[x].lzt = 0;
        }
    }
    void build(int x, int l, int r, vector<int>& a) {
        if(l == r) {
            t[x].s = a[l];
            return;
        }

        int mid = l + r >> 1;
        build(LC, l, mid, a);
        build(RC, mid+1, r, a);
        pushup(x);
    }
    void modify(int x, int l, int r, int L, int R, int va) {
        if(L <= l && r <= R) {
            t[x].s += va * (r - l + 1);
            t[x].lzt += va;
            return;
        }
        if(R < l || r < L) return;
        pushdown(x, l, r);
        int mid = l + r >> 1;
        modify(LC, l, mid, L, R, va);
        modify(RC, mid+1, r, L, R, va);
        pushup(x);
    }
    int query(int x, int l, int r, int L, int R) {
        if(L <= l && r <= R) return t[x].s;
        if(R < l || r < L) return 0;

        pushdown(x, l, r);
        int mid = l + r >> 1;
        return query(LC, l, mid, L, R) + query(RC, mid+1, r, L, R);
    }
};


void work() {
    int n, m;
    cin >> n >> m;

    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) cin >> a[i];

    ST st(n);
    st.build(1, 1, n, a);

    for(int i = 1; i <= m; i++) {
        int opt;
        cin >> opt;
        if(opt == 1) {
            int x, y, k;
            cin >> x >> y >> k;
            st.modify(1, 1, n, x, y, k);
        } else if(opt == 2) {
            int x, y;
            cin >> x >> y;
            cout << st.query(1, 1, n, x, y) << endl;
        } else assert(false);
    }
}
```

4.4 可持久化数组

```
#define LLI long long
struct PST {
    struct Node {
        int lc, rc;
        LLI v;
    };
    vector<Node> t;
    int N;

    PST(int n) {
        t.resize(n);
        N = 0;
    }

    #define LC (t[x].lc)
    #define RC (t[x].rc)

    void build(int &x, int l, int r, const vector<LLI>& a) {
        x = ++N;

        if(l == r) {
            t[x].v = a[l];
            return;
        }

        int mid = l + r >> 1;
        build(LC, l, mid, a);
        build(RC, mid+1, r, a);
    }
    void modify(int &x, int y, int l, int r, int pos, LLI va) {
        x = ++N;
        t[x] = t[y];

        if(pos <= l && r <= pos) {
            t[x].v = va;
            return;
        }
        if(pos < l || r < pos) return;

        int mid = l + r >> 1;
        if(pos <= mid) modify(LC, t[y].lc, l, mid, pos, va);
        else modify(RC, t[y].rc, mid+1, r, pos, va);
    }
    LLI query(int x, int l, int r, int pos) {
        if(!x) return 0;
        if(pos <= l && r <= pos) return t[x].v;
        if(pos < l || r < pos) return 0;

        int mid = l + r >> 1;
        if(pos <= mid) return query(LC, l, mid, pos);
        else return query(RC, mid+1, r, pos);
    }
};


void work() {
    int n, m;
    cin >> n >> m;
    vector<LLI> a(n + 1);
    for(int i = 1; i <= n; i++) cin >> a[i];

    vector<int> rt(m + 1);

    PST pst((n + m) * 20);

    pst.build(rt[0], 1, n, a);

    for(int i = 1; i <= m; i++) {
        int v, opt, pos;
        cin >> v >> opt >> pos;

        if(opt == 1) {
            int va;
            cin >> va;

            pst.modify(rt[i], rt[v], 1, n, pos, va);

        } else if(opt == 2) {
            rt[i] = rt[v];

            cout << pst.query(rt[v], 1, n, pos) << endl;

        } else assert(false);
    }
}

signed main() {
    ios::sync_with_stdio(0);
    cin.tie(0);

    int t = 1;
    // cin >> t;
    while(t--) {
        work();
    }

    return 0;
}
```

4.5 静态区间第k小

```
/*
 * 区间第k小
 * 可持久化线段树，值域，持久化维护区间，支持单点修改，区间和
 */
struct PST {
    struct Node {
        int s, lc, rc;
    };
    vector<Node> t;
    int N;

    PST(int n) { t.resize(n); N = 0; }

    #define LC (t[x].lc)
    #define RC (t[x].rc)

    void pushup(int x) {
        t[x].s = t[LC].s + t[RC].s;
    }
    void modify(int &x, int y, int l, int r, int pos, int va) {
        x = ++N;
        t[x] = t[y];

        if(pos <= l && r <= pos) { t[x].s += va; return; }
        if(pos < l || r < pos) return;

        int mid = l + r >> 1;
        if(pos <= mid) modify(LC, t[y].lc, l, mid, pos, va);
        else modify(RC, t[y].rc, mid+1, r, pos, va);
        pushup(x);
    }
    int query(int L, int R, int l, int r, int k) {
        if(l == r) return l;
        int mid = l + r >> 1;
        int tmp = t[t[R].lc].s - t[t[L].lc].s;
        if(tmp >= k)
            return query(t[L].lc, t[R].lc, l, mid, k);
        else
            return query(t[L].rc, t[R].rc, mid+1, r, k - tmp);
    }
};

void work() {
    int n, m;
    cin >> n >> m;

    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) cin >> a[i];

    // 离散化是为了避免被卡空间
    auto b = a;
    sort(b.begin() + 1, b.end());
    b.erase(unique(b.begin() + 1, b.end()), b.end());
    int M = b.size() - 1;
    map<int, int> f;
    for(int i = 1; i <= M; i++) f[b[i]] = i;
    for(int i = 1; i <= n; i++) a[i] = f[a[i]];

    PST pst((n + m) * 20);
    vector<int> rt(n + 1);

    for(int i = 1; i <= n; i++)
        pst.modify(rt[i], rt[i - 1], 1, M, a[i], 1);

    for(int i = 1; i <= m; i++) {
        int L, R, k;
        cin >> L >> R >> k;
        cout << b[pst.query(rt[L - 1], rt[R], 1, M, k)] << endl;
    }
}
```

4.6 线段树优化建图

```
#define LLI long long
#define DBL double
const bool DEBUG = false;
const int maxn = 1e5 + 10;

int n;

// --- Edge ---
struct Edge {
    int to, next, va;
}e[maxn * 50];
int tot, last[maxn << 2];
// there are 4*n points including intree and outtree
void addedge(int fm, int to, int va) {
    e[++tot].to = to;
    e[tot].va = va;
    e[tot].next = last[fm];
    last[fm] = tot;
    //printf(" - addedge %d -> %d with %d\n", fm, to, va);
}

// --- Segment Tree ---
// its diffrent from normal segment tree.
// in nature, it just use the shape and though of segment tree.
// it is only a graph (after build the segment tree).
// in other words, it is no need to support query method.
//   ------ but we need the range!
//   my thought:
// in this algorithm, we need to connect each pair leaves.
// why shall we combine each pair leaves to one vertex?
// so, we let vertex 1~n be the true vertex 1~n.

struct Node {
    int l, r,
        lc, rc;
} t[maxn << 2];
int cnt,
    root[2];

#define LC t
#define RC t[x].rc

// return the index of its child
void buildIntree(int &x, int l, int r) {
    // no need to build a new vertex
    if(l == r) {
        x = l;
        t[x].l = t[x].r = l;
        return;
    }
    // you ONLY set t[x].l & t[x].r AFTER set X's value !!!
    x = ++cnt;
    t[x].l = l;
    t[x].r = r;
    int mid = l + r >> 1;
    buildIntree(t[x].lc, l, mid);
    buildIntree(t[x].rc, mid + 1, r);
    addedge(t[x].lc, x, 0);
    addedge(t[x].rc, x, 0);
}
void buildOuttree(int &x, int l, int r) {
    if(l == r) {
        x = l;
        t[x].l = l;
        t[x].r = r;
        return;
    }
    x = ++cnt;
    t[x].l = l;
    t[x].r = r;
    int mid = l + r >> 1;
    buildOuttree(t[x].lc, l, mid);
    buildOuttree(t[x].rc, mid + 1, r);
    addedge(x, t[x].lc, 0);
    addedge(x, t[x].rc, 0);
}

void modify1to2(int x, int l, int r, int u, int v) { // u => [l, r]
    //printf(" - modifying, %d, [%d, %d], %d\n", x, l, r, u);
    if(t[x].r < l || r < t[x].l) return;
    if(l <= t[x].l && t[x].r <= r) {
        addedge(u, x, v);
        return;
    }
    int mid = t[x].l + t[x].r >> 1;
    if(l <= mid) modify1to2(t[x].lc, l, r, u, v);
    if(mid < r) modify1to2(t[x].rc, l, r, u, v);
}
void modify2to1(int x, int l, int r, int u, int v) {
    if(t[x].r < l || r < t[x].l) return;
    if(l <= t[x].l && t[x].r <= r) {
        addedge(x, u, v);
        return;
    }
    int mid = t[x].l + t[x].r >> 1;
    if(l <= mid) modify2to1(t[x].lc, l, r, u, v);
    if(mid < r) modify2to1(t[x].rc, l, r, u, v);
}

#include<queue>
std::priority_queue<std::pair<LLI, int> > q;

const LLI INF = 1000000000000000000ll;
LLI f[maxn << 2];
bool vis[maxn << 2];
void dijkstra(int start) {

    for(int i = 1; i <= cnt; i++) // attention ! n->cnt here !
        f[i] = INF;


    f[start] = 0;
    q.push(std::make_pair(-0, start));

    int fm, to;
    while(!q.empty()) {
        fm = q.top().second;
        q.pop();
        if(vis[fm]) continue;
        vis[fm] = true;
        //printf(" - exploring %d\n", fm);
        for(int i = last[fm]; i; i = e[i].next) {
            to = e[i].to;
            //printf(" - explore %d -> %d\n", fm, to);
            //printf(" - - f[fm]=%lld, f[to]=%lld\n", f[fm], f[to]);
            if(f[to] <= f[fm] + e[i].va) continue;
            f[to] = f[fm] + e[i].va;
            //printf(" - - and add it\n");
            q.push(std::make_pair(-f[to], to));
        }
    }
}


int main() {

    int q, s;

    scanf("%d%d%d", &n, &q, &s);

    cnt = n;
    buildIntree(root[0], 1, n);
    buildOuttree(root[1], 1, n);

    //printf("%d, %d\n", root[0], root[1]);

    int opt, u, v, l, r, w;
    for(int i = 1; i <= q; i++) {
        scanf("%d", &opt);
        if(opt == 1) {
            scanf("%d%d%d", &u, &v, &w);
            addedge(u, v, w);
        } else if(opt == 2) {
            scanf("%d%d%d%d", &u, &l, &r, &w);
            modify1to2(root[1], l, r, u, w);
            // outtree modify - 1 to mul
        } else if(opt == 3) {
            scanf("%d%d%d%d", &u, &l, &r, &w);
            modify2to1(root[0], l, r, u, w);
            // intree modify - mul to 1
        }
    }

    dijkstra(s);

    for(int i = 1; i <= n; i++)
        if(f[i] == INF)
            printf("-1 ");
        else
            printf("%lld ", f[i]);
    printf("\n");

    return 0;
}
```

5. 图论

5.1 2-SAT

- 问题
    - 有 $x_1, x_2, \ldots, x_n$ 这 n 个 $bool$ 变量
    - 有多个形如 $x_1\ or\ (not\ x_2) = 1$ 的限制
        - $x_1 = 0/1,\ x_2 = 0/1$ 至少一个成立
    - 是否有一组解
- 限制的拓展
    - $x_1\ \&\ x_2 = 0$
        - $x_1 = 0,\ x_2 = 0$ 至少一个成立
        - $(not\ x_1)\ or\ (not\ x_2) = 1$

  - $x_1 \ \& \ x_2 = 1$
    - $x_1 = 1, \ x_2 = 1$ 至少一个成立 且 $x_1 = 1, \ x_2 = 0$ 至少一个成立 且 $x_1 = 0, \ x_2 = 1$ 至少一个成立
  - $x_1 = x_2$
    - $x_1 = 1, \ x_2 = 0$ 至少一个成立 且 $x_1 = 0, \ x_2 = 1$ 至少一个成立
  - $x_1 = 1$
    - $x_1 = 1, \ x_1 = 1$ 至少一个成立
  - 二元形式都可以表示成这种形式

```cpp
void work() {
    int n, m;
    cin >> n >> m;

    // 0...2n-1: 2*i, 2*i+1
    // 0 index for x^1
    int N = n << 1;
    vector<vector<int>> e(N + 1, vector<int>());
    for(int i = 1; i <= m; i++) {
        char c1, c2;
        int u, v;
        cin >> c1 >> u >> c2 >> v;

        u--;
        v--;
        u = u * 2 + (c1 == 'h');
        v = v * 2 + (c2 == 'h');
        // u or v = 1
        e[u^1].push_back(v);
        e[v^1].push_back(u);
    }

    // tarjan
    int dfnCnt = 0, sccCnt = 0, top = 0;
    vector<int> dfn(N + 1), low(N + 1), scc(N + 1), stk(N + 2, -1);

    function<void(int)> tarjan = [&](int x) {
        dfn[x] = low[x] = ++dfnCnt;
        stk[++top] = x;
        for(auto y: e[x]) {
            if(!dfn[y]) tarjan(y);
            if(!scc[y]) low[x] = min(low[x], low[y]);
        }
        if(dfn[x] == low[x]) {
            sccCnt++;
            for(; stk[top + 1] != x; top--) {
                scc[stk[top]] = sccCnt;
            }
        }
    };
    for(int i = 0; i < N; i++) if(!dfn[i])
        tarjan(i);

    for(int i = 0; i < n; i++) {
        // scc[i << 1] < scc[i << 1 | 1] => choose i<<1
        if(scc[i << 1] == scc[i << 1 | 1]) {
            cout << "BAD" << endl;
            return;
        }
    }
    cout << "GOOD" << endl;
}
```

5.2 DFS序

```
/**
 * DFS序1：单点修改、查询子树和、查询根到任意点路径权值之和
*/
void work() {
    int n, q;
    cin >> n >> q;

    vector<vector<int>> e(n + 1, vector<int>());
    for(int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        e[u].push_back(v); e[v].push_back(u);
    }

    vector<int> dfn(n + 1), dfnR(n + 1);

    function<void(int, int)> dfs = [&](int x, int fa) {
        dfn[x] = ++dfn[0];
        for(auto y: e[x]) {
            if(y == fa) continue;
            dfs(y, x);
        }
        dfnR[x] = dfn[0];
    };
    dfs(1, 0);

    // bit
    /* Codes about BIT 1 */
    // bit2
    /* Codes about BIT 2 */

    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) {
        cin >> a[i];
        add1(dfn[i], a[i]);
        add2(dfn[i], a[i]);
        add2(dfnR[i] + 1, -a[i]);
    }

    for(int i = 1; i <= q; i++) {
        int opt;
        cin >> opt;
        if(opt == 1) {
            int x, y;
            cin >> x >> y;
            add1(dfn[x], -a[x]);
            add2(dfn[x], -a[x]);
            add2(dfnR[x] + 1, a[x]);
            a[x] = y;
            add1(dfn[x], a[x]);
            add2(dfn[x], a[x]);
            add2(dfnR[x] + 1, -a[x]);
        } else if(opt == 2) {
            int x;
            cin >> x;
            cout << query1(dfnR[x]) - query1(dfn[x] - 1) << " ";
            cout << query2(dfn[x]) << endl;
        } else assert(false);
    }
}

/**
 * DFS序2：单点修改、查询子树和、换根
 * DFS序的换根分三种情况考虑：① root=x，整颗树；
 * ② root在x的子树中，
*/
void work() {
    int n, q;
    cin >> n >> q;

    vector<vector<int>> e(n + 1, vector<int>());
    for(int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        e[u].push_back(v); e[v].push_back(u);
    }

    int lim = __lg(n);
    vector<int> dfn(n + 1), dfnR(n + 1), dep(n + 1);
    vector<vector<int>> fa(lim + 1, vector<int>(n + 1));

    function<void(int)> dfs = [&](int x) {
        dfn[x] = ++dfn[0];
        for(auto y: e[x]) {
            if(y == fa[0][x]) continue;
            dep[y] = dep[x] + 1;
            fa[0][y] = x;
            dfs(y);
        }
        dfnR[x] = dfn[0];
    };

    dep[1] = 1;
    dfs(1);

    for(int k = 1; k <= lim; k++) {
        for(int i = 1; i <= n; i++) {
            fa[k][i] = fa[k-1][fa[k-1][i]];
        }
    }

    // bit
    /* Codes about BIT */


    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) {
        cin >> a[i];
        add(dfn[i], a[i]);
    }

    int root = 1;
    for(int i = 1; i <= q; i++) {
        int opt;
        cin >> opt;
        if(opt == 1) {
            int x, y;
            cin >> x >> y;
            add(dfn[x], -a[x]);
            a[x] = y;
            add(dfn[x], a[x]);
        } else if(opt == 2) {
            int x;
            cin >> x;
            int ans;
            if(x == root) {
                ans = query(n);
            } else if(dfn[x] <= dfn[root] && dfn[root] <= dfnR[x]) {
                // root在x的子树内
                int p = root;
                for(int j = lim; j >= 0; j--) {
                    if(dep[fa[j][p]] > dep[x])
                        p = fa[j][p];
                }
                ans = query(n) - (query(dfnR[p]) - query(dfn[p] - 1));
            } else {
                ans = query(dfnR[x]) - query(dfn[x] - 1);
            }
            cout << ans << endl;
        } else if(opt == 3) {
            cin >> root;
        } else assert(false);
    }
}
```

5.3 欧拉序求LCA O(1)

```
void work() {
    int n; cin >> n;
    vector<vector<int>> e(n + 1, vector<int>());
    for(int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        e[u].push_back(v); e[v].push_back(u);
    }

    int N = (n << 1) - 1;
    vector<int> euler(N + 1), pos(n + 1), dep(n + 1);
    function<void(int, int)> dfs = [&](int x, int fa) {
        euler[++euler[0]] = x;
        pos[x] = euler[0];
        for(auto y: e[x]) {
            if(y == fa) continue;
            dep[y] = dep[x] + 1;
            dfs(y, x);
            euler[++euler[0]] = x;
        }
    };
    dep[1] = 1;
    dfs(1, 0);
    assert(N == euler[0]);

    // st
    int lim = __lg(N);
    vector<vector<array<int, 2>>> st(lim + 1, vector<array<int, 2>>(N + 1));
    for(int i = 1; i <= N; i++) {
        st[0][i] = {dep[euler[i]], euler[i]};
    }
    for(int k = 1; k <= lim; k++) {
        int lim2 = N - (1 << k) + 1;
        for(int i = 1; i <= lim2; i++) {
            st[k][i] = min(st[k-1][i], st[k-1][i+(1<<k-1)]);
        }
    }
    auto query = [&](int L, int R) {
        int k = __lg(R - L + 1);
        return min(st[k][L], st[k][R-(1<<k)+1])[1];
    };
}
```

5.4 Tarjan求LCA

```
void work() {
    int n, m;
    cin >> n >> m;

    vector<vector<array<int, 2>>> e(n + 1, vector<array<int, 2>>());
    for(int i = 1; i < n; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        e[u].push_back({v, w});
        e[v].push_back({u, w});
    }

    vector<int> dis(m + 1);
    vector<int> lca(m + 1);
    const int INF = 1LL << 30; // 注意maxans爆int

    vector<int> f(n + 1);
    for(int i = 1; i <= n; i++) f[i] = i;
    function<int(int)> find = [&](int x) {
        return (f[x] == x ? x : f[x] = find(f[x]));
    };

    vector<vector<array<int, 2>>> q(n + 1, vector<array<int, 2>>());
    for(int i = 1; i <= m; i++) {
        int u, v;
        cin >> u >> v;
        if(u == v) {
            dis[i] = 0;
            lca[i] = u;
        } else {
            q[u].push_back({v, i});
            q[v].push_back({u, i});
            dis[i] = INF;
        }
    }

    vector<int> v(n + 1), d(n + 1);
    function<void(int)> tarjan = [&](int x) {
        v[x] = 1;
        for(auto [y, w]: e[x]) {
            if(v[y]) continue;
            d[y] = d[x] + w;
            tarjan(y);
            f[y] = x;
        }
        for(auto [y, id]: q[x]) {
            if(v[y] == 2) {
                lca[id] = find(y);
                dis[id] = min(dis[id], d[x] + d[y] - 2 * d[lca[id]]);
            }
        }
        v[x] = 2;
    };

    int rt = 1;
    tarjan(rt);

    for(int i = 1; i <= m; i++) cout << lca[i] << endl;
    for(int i = 1; i <= m; i++) cout << dis[i] << endl;
}
```

5.5 虚树

- 消耗战

```
void work() {
    int n;
    cin >> n;

    vector<vector<array<int, 2>>> e(n + 1, vector<array<int, 2>>());
    for(int i = 1; i < n; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        e[u].push_back({v, w});
        e[v].push_back({u, w});
    }

    /* 树链剖分 LCA */
    const int INF = 1LL << 30;
    vector<int> sz(n + 1), fa(n + 1), son(n + 1), dep(n + 1);
    vector<int> dfn(n + 1), dfnR(n + 1), rk(n + 1), top(n + 1);
    vector<int> dis(n + 1);

    function<void(int)> dfs1 = [&](int x) {
        dep[x] = dep[fa[x]] + 1;
        sz[x] = 1;
        for(auto [y, w]: e[x]) {
            if(y == fa[x]) continue;
            fa[y] = x;
            dis[y] = min(dis[x], w);
            dfs1(y);
            sz[x] += sz[y];
            if(sz[son[x]] < sz[y]) son[x] = y;
        }
    };
    function<void(int)> dfs2 = [&](int x) {
        dfn[x] = ++dfn[0];
        rk[dfn[0]] = x;
        if(!top[x]) top[x] = x;
        if(son[x]) top[son[x]] = top[x], dfs2(son[x]);
        for(auto [y, w]: e[x]) if(y != fa[x] && y != son[x]) dfs2(y);
        dfnR[x] = dfn[0];
    };

    dis[1] = INF;
    dfs1(1);
    dfs2(1);

    auto getLca = [&](int x, int y) {
        while(top[x] != top[y]) {
            if(dep[top[x]] > dep[top[y]]) swap(x, y);
            y = fa[top[y]];
        }
        if(dep[x] > dep[y]) swap(x, y);
        return x;
    };

    // 虚树

    vector<int> mp(n + 1);
    vector<int> tag(n + 1);

    int m;
    cin >> m;

    for(int i = 1; i <= m; i++) {
        int N;
        cin >> N;
        vector<int> b(N);
        for(int j = 0; j < N; j++) cin >> b[j];

        auto a = b;
        for(auto x: a) tag[x] = 1;

        // build 虚树

        sort(b.begin(), b.end(), [&](int u, int v) {
            return dfn[u] < dfn[v];
        });
        b.push_back(1);
        for(int j = 1; j < N; j++) b.push_back(getLca(b[j - 1], b[j]));
        sort(b.begin(), b.end(), [&](int u, int v) {
            return dfn[u] < dfn[v];
        });

        b.erase(unique(b.begin(), b.end()), b.end());
        N = b.size();
        vector<vector<array<int, 2>>> E(N + 1, vector<array<int, 2>>());

        for(int i = 0; i < N; i++) mp[b[i]] = i + 1;

        // for(int i = 0; i < N; i++) cout << b[i] << " "; cout << endl;
        for(int i = 1; i < N; i++) {
            int u = b[i - 1];
            int v = b[i];
            int lca = getLca(u, v);
            if(lca == v) continue;
            E[mp[lca]].push_back({mp[v], dis[v]});
            E[mp[v]].push_back({mp[lca], dis[v]});
        }

        // solve

        vector<int> f(N + 1);
        function<void(int, int)> dfs = [&](int x, int fa) {
            for(auto [y, w]: E[x]) {
                if(y == fa) continue;
                if(tag[b[y - 1]]) {
                    f[x] += w;
                } else {
                    dfs(y, x);
                    f[x] += min(w, f[y]);
                }
            }
        };

        dfs(1, 0);
        cout << f[1] << endl;

        for(auto x: a) tag[x] = 0;
    }
}
```

5.6 支配树

- 对于一张有向图，我们确定一个起点 S
- 对于一个点 k，x 支配 k，当且仅当删去点 x 后，S 无法到达 k。
- 对于一个点 k，x 可能有多个，取离它最近的 x 连边，得到一个树形结构，就是支配树。

```
const int maxn = 2e5 + 10;
const int maxm = 3e5 + 10;

vector<int> e[maxn];
vector<int> ei[maxn];
vector<int> dfsT[maxn];
vector<int> dfsTi[maxn];
vector<int> dom[maxn];

int n, m;
int fa[maxn];
int dfn[maxn];
int id[maxn]; // dfn[id[x]] == x
int tot;

void dfs(int x) {
    dfn[x] = ++tot;
    id[tot] = x;

    for(auto y: e[x]) {
        if(dfn[y]) continue;
        fa[y] = x;
        dfs(y);
        dfsT[x].push_back(y); // add a edge on dfs tree
    }
}

int sdom[maxn];
int mn[maxn];
int gfa[maxn];
int find(int x) {
    if(x != gfa[x]) {
        int t = gfa[x];
        gfa[x] = find(gfa[x]);
        if(dfn[sdom[mn[x]]] > dfn[sdom[mn[t]]])
            mn[x] = mn[t];
    }
    return gfa[x];
}
void tarjan() {
    for(int i = 1; i <= n; i++) {
        gfa[i] = i;
        sdom[i] = i;
        mn[i] = i;
    }
    for(int j = n; j >= 2; j--) {
        int x = id[j];
        if(!x) continue;

        int pos = j;
        for(auto y: ei[x]) {
            if(!dfn[y]) continue;
            if(dfn[y] < dfn[x]) {
                pos = min(pos, dfn[y]);
            } else {
                find(y);
                pos = min(pos, dfn[sdom[mn[y]]]);
            }
        }
        sdom[x] = id[pos];
        gfa[x] = fa[x];
        dfsT[sdom[x]].push_back(x);
    }
}
```

```
int dep[maxn];
int dp[25][maxn];
int getLCA(int x, int y) {
    if(dep[x] < dep[y]) swap(x, y);
    int del = dep[x] - dep[y];
    for(int k = 0; k <= 20; k++)
        if((1 << k) & del)
            x = dp[k][x];
    if(x == y) return x;
    for(int k = 20; k >= 0; k--) {
        if(dp[k][x] != dp[k][y]) {
            x = dp[k][x];
            y = dp[k][y];
        }
    }
    return dp[0][x];
}
void buildDom(int x) {
    int to = dfsTi[x][0];
    for(auto y: dfsTi[x]) {
        to = getLCA(to, y);
    }
    dep[x] = dep[to] + 1;
    dp[0][x] = to;
    dom[to].push_back(x);

    for(int i = 1; i <= 20; i++)
        dp[i][x] = dp[i - 1][dp[i - 1][x]];
}

int indeg[maxn];
void topoSort() {
    for(int x = 1; x <= n; x++) {
        for(auto y: dfsT[x]) {
            indeg[y]++;
            dfsTi[y].push_back(x);
        }
    }
    for(int i = 1; i <= n; i++) {
        if(!indeg[i]) {
            dfsT[0].push_back(i);
            dfsTi[i].push_back(0);
        }
    }

    queue<int> q;
    q.push(0);
    while(!q.empty()) {
        int x = q.front(); q.pop();
        for(auto y: dfsT[x]) {
            if(--indeg[y] <= 0) {
                q.push(y);
                buildDom(y);
            }
        }
    }
}

int idom[maxn];
void dfsDom(int x) {
    idom[x] = 1;
    for(auto y: dom[x]) {
        dfsDom(y);
        idom[x] += idom[y];
    }
}


void work() {

    cin >> n >> m;
    for(int i = 1; i <= m; i++) {
        int x, y;
        cin >> x >> y;
        e[x].push_back(y);
        ei[y].push_back(x);
    }

    dfs(1);
    tarjan();
    topoSort();
    dfsDom(0);

    for(int i = 1; i <= n; i++) cout << idom[i] << " "; cout << endl;
}
```

6. 数论

6.1 逆元

```
//线性求1~n逆元
inv[i] = (MOD - MOD / i) * inv[MOD % i] % MOD;
//线性求n个不同数的逆元
s[1] = a[1];
for(int i = 2; i <= n; i++) s[i] = s[i - 1] * a[i] % MOD;
inv[n] = Pow(s[n], MOD - 2);
for(int i = n - 1; i >= 1; i--) inv[i] = inv[i + 1] * a[i + 1] % MOD;
for(int i = 2; i <= n; i++) inv[i] = inv[i] * s[i - 1] % MOD;
//卢卡斯定理
// - 若p为质数，则
// - C(n, m) % MOD = C(n / MOD, m / MOD) * C(n % MOD, m % MOD) % MOD;
// - 第一部分递归处理，第二部分逆元处理
```

6.2 EXGCD

```
function<int(int, int, int&, int&)> exgcd = [&](int n, int m, int &x, int &y) -> int
{
    if(m == 0) {
        x = 1;
        y = 0;
        return n;
    }
    int g = exgcd(m, n % m, x, y);
    int z = x;
    x = y;
    y = z - n / m * y;
    return g;
};
```

6.3 实数高斯消元

```
typedef double db;
const db EPS = 1e-9;

inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
inline int cmp(db a, db b) { return sign(a - b); }

void work() {
    int n;
    cin >> n;

    int lim = n + 1;
    vector<vector<db>> a(n + 1, vector<db>(lim + 1));
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= lim; j++)
            cin >> a[i][j];
    }

    auto gauss = [](int n, vector<vector<db>>& f) {
        int i, j, k; db t;
        int lim = n + 1;
        for(k = 1; k <= n; k++) {
            for(i = k; i <= n; i++) //row
                if(sign(f[i][k]) != 0)
                    break;
            if(i > n) return false; // 多解
            if(i != k)
                for(j = 1; j <= lim; j++) // col
                    std::swap(f[i][j], f[k][j]);
            t = f[k][k];
            for(i = 1; i <= lim; i++) // col
                f[k][i] /= t;
            for(i = 1; i <= n; i++) { // row
                if(i == k) continue;
                t = f[i][k];
                for(j = 1; j <= lim; j++)
                    f[i][j] -= f[k][j] * t;
            }
        }
        return true;
    };

    if(!gauss(n, a)) {
        cout << "No Solution" << endl;
        return;
    }

    for(int i = 1; i <= n; i++)
        cout << fixed << setprecision(2) << a[i][lim] << endl;
}
```

9. 其他

9.1 树哈希

```
// O(m*logm * n)
// O(n) per query
const int maxn = 100 + 10;

int n;
vector<int> e[maxn];

int f[maxn];
int sz[maxn];
int pri[10010];
bool isNPri[10010];

void getPri() {
    for(int i = 2; i <= 10000; i++) {
        if(!isNPri[i]) pri[++pri[0]] = i;
        for(int j = 1; i * pri[j] <= 10000 && j <= pri[0]; j++) {
            isNPri[i * pri[j]] = 1;
            if(i % pri[j] == 0) break;
        }
    }
}

int rt[3];
int szm[maxn];
int sum;
void get_rt(int x, int fa) {
    sz[x] = 1;
    szm[x] = 0;
    for(auto y: e[x]) {
        if(y == fa) continue;
        get_rt(y, x);
        sz[x] += sz[y];
        szm[x] = max(szm[x], sz[y]);
    }
    szm[x] = max(szm[x], sum - sz[x]);
    if(szm[x] < szm[rt[1]]) rt[1] = x;
    else if(szm[x] == szm[rt[1]]) rt[2] = x;
}

void dfs(int x, int fa) {
    sz[x] = 1;
    f[x] = 1;
    for(auto y: e[x]) {
        if(y == fa) continue;
        dfs(y, x);
        sz[x] += sz[y];
        f[x] += f[y] * pri[sz[y]];
    }
}

map<pair<int, pair<int, int> >, int> hsh;
void work() {
    getPri();

    int m;
    cin >> m;
    for(int T = 1; T <= m; T++) {
        cin >> n;
        for(int i = 1; i <= n; i++) e[i].clear();
        for(int i = 1; i <= n; i++) {
            int x; cin >> x;
            if(x) { e[x].push_back(i); e[i].push_back(x); }
        }

        sum = n;
        rt[1] = 0;
        rt[2] = 0; // 下面要检测所以要赋初值
        szm[0] = n + 1;
        get_rt(1, 0);

        pair<int, int> fvalue;

        if(szm[rt[1]] != szm[rt[2]]) {
            dfs(rt[1], 0);
            fvalue = make_pair(f[rt[1]], 0);
        } else {
            dfs(rt[1], 0);
            fvalue = make_pair(f[rt[1]], 0);
            dfs(rt[2], 0);
            fvalue.second = f[rt[2]];
            if(fvalue.first < fvalue.second)
                swap(fvalue.first, fvalue.second);
        }

        pair<int, pair<int, int> > va = make_pair(n, fvalue);
        if(hsh[va] == 0) {
            hsh[va] = T;
        }
        cout << hsh[va] << endl;
    }
}
```