Вариант 1

Зад. ①

а)

(1) $\varphi_{e}(x) \leftrightharpoons p(x, x, x)$.

(5) $\varphi_{=}(x, y) \leftrightharpoons \forall z \forall t (p(z, t, x) \Leftrightarrow p(z, t, y))$.

(2) $\varphi_{\%3}(x) \leftrightharpoons \exists y_1 \exists y_2 \exists y_3 (r(y_1, y_2) \& r(y_2, y_3) \& r(y_1, y_3) \& \exists z (p(y_1, y_2, z) \& p(y_3, z, x)))$.

(3) $\varphi_{\Sigma}(x) \leftrightharpoons \forall y \forall z (p(y, z, x) \Rightarrow \varphi_e(y) \vee \varphi_e(z)) \& \& \neg \varphi_e(x)$.

(4) $\varphi_{diff}(x) \leftrightharpoons \exists y \exists y' \exists z \exists z' (p(y, y', x) \& p(z', z, x) \& \varphi_{\Sigma}(y) \& \varphi_{\Sigma}(z) \& \neg \varphi_{=}(y, z))$.

b)

$\varphi \leftrightharpoons \exists x \exists y \exists z (\varphi_{\Sigma}(x) \& \varphi_{\Sigma}(y) \& \varphi_{\Sigma}(z) \& \neg \varphi_{=}(x, y) \& \neg \varphi_{=}(x, z) \& \neg \varphi_{=}(y, z))$.

зад (2)

За Γ_1 модел е следната стр. (а и за $\Gamma \subseteq \Gamma_1$):

$$\mathcal{A} = \langle \Sigma^*, p^{\mathcal{A}}, e^{\mathcal{A}}, c^{\mathcal{A}} \rangle$$

за някоя азбука Σ с поне 2 букви

$$p^{\mathcal{A}}(u,v,w) \leftrightharpoons u \circ v = w$$

$$e^{\mathcal{A}} \leftrightharpoons \varepsilon$$

$e^{\mathcal{A}}(u)$ може за е $\leftrightharpoons u = \varepsilon$ / $\leftrightharpoons u \in \Sigma$

Св-ще за Γ модел е

$$\mathcal{B} = \langle \mathbb{N}, p^{\mathcal{B}}, e^{\mathcal{B}} \rangle$$

$$p^{\mathcal{B}}(n,m,k) \leftrightharpoons n+m=k$$

$$c^{\mathcal{B}} \leftrightharpoons 0$$

Вариант 2

Зад 1 а)

(1) $\varphi_e(x) \leftrightharpoons f(x,x) \doteq x.$

(2) $\varphi_{\%3}(x) \leftrightharpoons \exists y_1 \exists y_2 \exists y_3 (g(y_1) \doteq g(y_2)\ \&$

$g(y_2) \doteq g(y_3)\ \&\ g(y_1) \doteq g(y_3)\ \&\ f(f(y_1, y_2), y_3) \doteq x).$

(3) $\varphi_{\overline{2}}(x) \leftrightharpoons \forall y \forall z\, (f(y,z) \doteq x \Rightarrow \varphi_e(y) \vee \varphi_e(z)) \&$
$\neg \varphi_e(x).$

(4) $\varphi_{diff}(x) \leftrightharpoons \exists y \exists z \exists t\, (f(y, f(z,t)) \doteq x\ \&$

$\varphi_{\overline{2}}(y)\ \&\ \varphi_{\overline{2}}(t)\ \&\ \neg(y \doteq t)).$

(5) $\varphi_{(5)}(x) \leftrightharpoons \forall z \forall t\, (f(z,t) \doteq x\ \&\ \neg\varphi_e(z)\ \&\ \neg(t \doteq z) \Rightarrow$
$\Rightarrow \neg \exists t' (f(t', z) \doteq y))$

Зад 2 Модел за P_1, Q и за $P \subseteq P_1$

$\mathcal{A} = \langle \Sigma^*; f^{\mathcal{A}}, e^{\mathcal{A}}, h^{\mathcal{A}} \rangle$

$f^{\mathcal{A}}(u, v) \leftrightharpoons u \circ v$

$e^{\mathcal{A}} \leftrightharpoons \varepsilon$

$h^{\mathcal{A}}(u) \leftrightharpoons u$