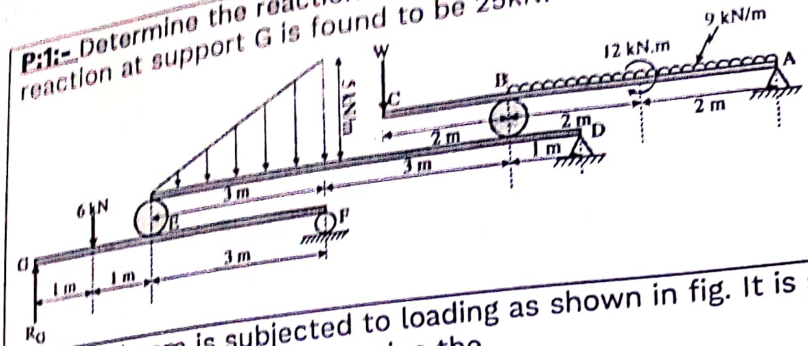
Prof. Manjrekar's
Engineering Group Tuition
Engineering Mechanics

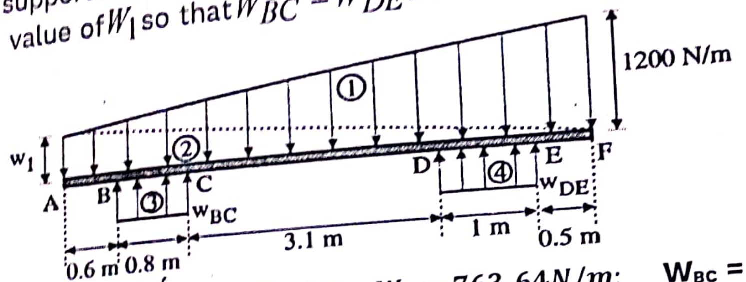
Mechanics Practice Problems

CH 2. SUPPORT REACTION

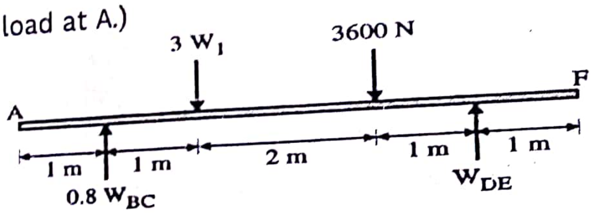
P:1:- Determine the reaction components at A,B,D,E and F . Also find value of load W if reaction at support G is found to be 25kN.



P:2:- A beam is subjected to loading as shown in fig. It is supported by two wide supports BC and DE, Determine the corresponding value of W_{BC} and W_{DE}. Also find value of W_1 so that $W_{BC} = W_{DE}$.

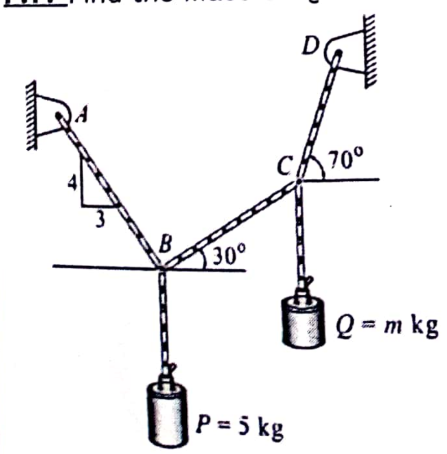


Ans: ∴ $W_{BC} = 3272.72\ N/m;\ W_1 = 763.64 N/m;$ **$W_{BC} = W_{DE}$ =3272.72 N/m** are the intensities of distributed reactions and $W_1 = 763.64 N/m$ is the intensity of distributed load at A.)

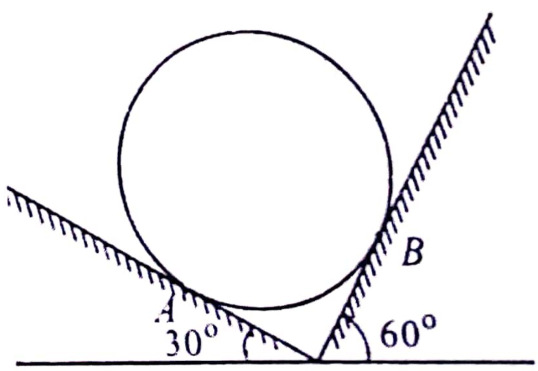


CH 3.EQUILIBRIUM OF COPLANAR FORCES

P:1 :- Find the Mass of Q.



P: 2:- Mass 50kg. Find Reactions at A & B.



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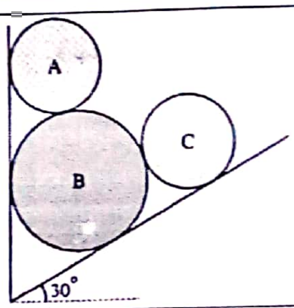
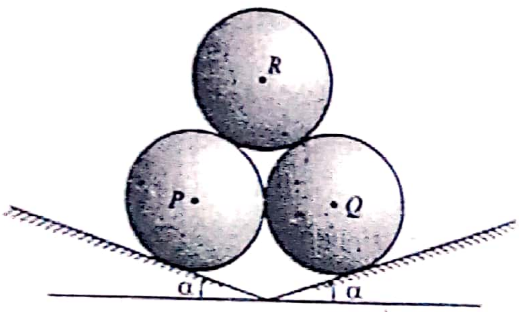
Mechanics Practice Problems

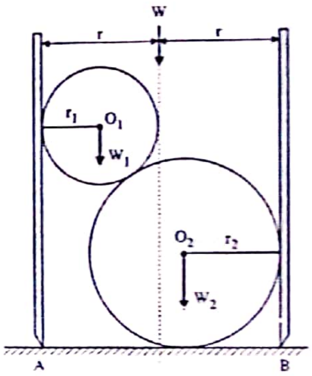
P: 3 :- Three homogenous smooth spheres A,B and C of weight 300 N, 600 N and 300 N and having diameters 800mm, 1200mm and 800mm respectively are placed in a trench as shown in Fig. Determine the reaction at all points of contact.



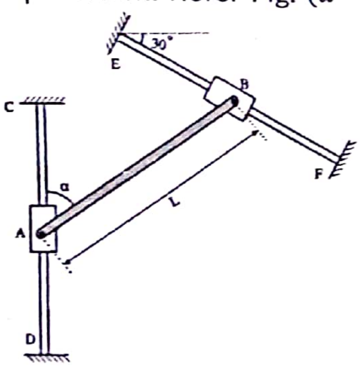
P:-4– Three identical spheres *P, Q, R* of weight *W* are arranged on smooth inclined surface. Determine the angle which will prevent the arrangement from the collapsing. ($\alpha = 10.89°$)



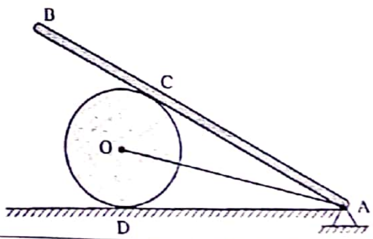
P: 5:-A hollow cylindrical of radius r is open at both ends and rests on a smooth horizontal plane. Two spheres having weight W_1 and W_2 and radii r_1 and r_2 respectively are placed inside the cylinder as shown in Fig. Find minimum weight W of the cylinder in order that it will not tip over.



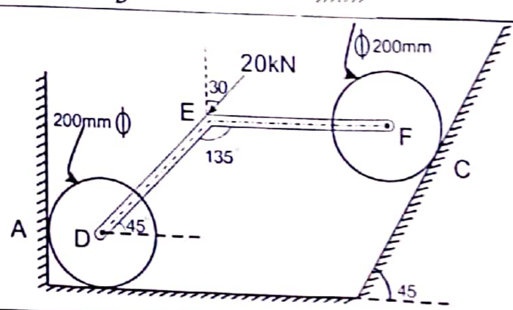
P: 6:-A slender rod of mass 8 kg and length L is attached to collars which may slide without friction along the guides. If rod is in equilibrium find reaction at A and B. Also find angle α.α for equilibrium. Refer Fig. ($\alpha = 49.11°$)



P: 7:- A smooth cylinder of radius r = 500 mm. resting on a horizontal surface supports a bar AB of length 1500 mm. which is hinged at A. The weight of bar is 1000 N. The cylinder is kept from rolling away by a string AO of length 1000 mm. Assuming all surface to be frictionless, find the tension in the string.
(T = 433.01 N)

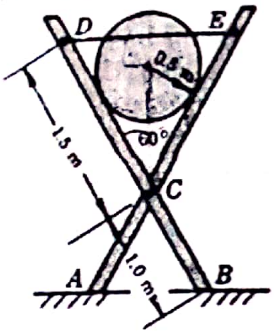


P: 8:- Find the support reaction at A, B, C for the rigid link DEF supported by the cylinder at D and F. The link is loaded by a single force of 20 kN as shown in the fig. neglect friction and self weight of link and cylinders. Take diameter of cylinders as 200mm. and DE=EF=300mm.

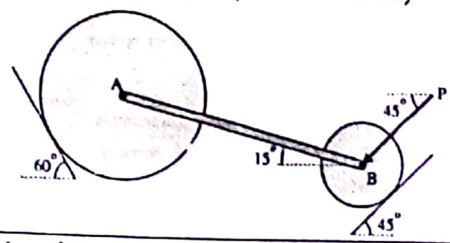


P: 9:- A cylinder 1 m in diameter and of 10-kg mass is lodged between the cross pieces which makes an angle of 60° with each other as shown in Fig. Determine the tension in the

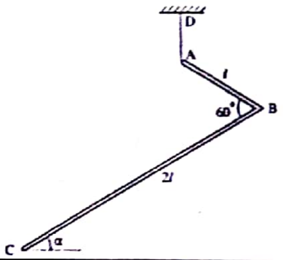
P: 10:- Two cylinders of masses $m_1 = 100kg$ and $m_2 = 50kg$ resting an smooth inclined planes having inclinations $60°$ and $45°$ with horizontal rope DE assuming a smooth floor. (T = 84.2N)



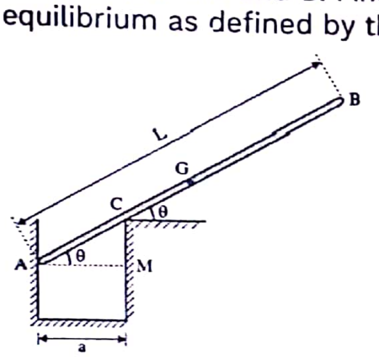
the horizontal respectively as shown in fig. They are connected by a weightless bar AB with hinge connections. The bar AB makes 15° with the horizontal. Find the magnitude of the force 'P' required to hold the system in equilibrium. (P = 253.78N)



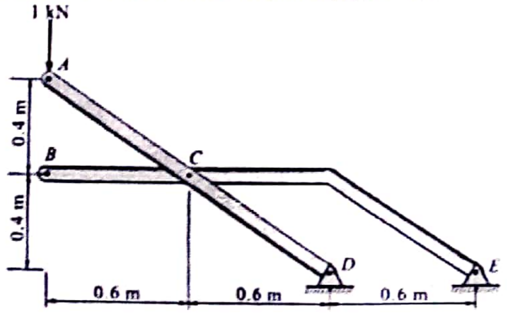
P:11 :- Two bars AB and BC of length 1m and 2m and weights 100N and 200N respectively are rigidly joined at B and suspended by a string AO as shown in fig. Find the inclination θ of bar BC to the horizontal when the system is in equilibrium. ($\alpha = 19.11°$)



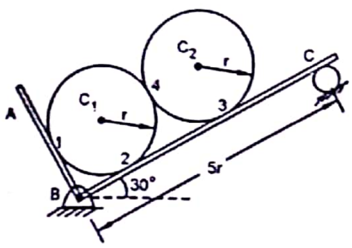
P:12:- A prismatic bar AB of weight W is resting against a smooth vertical wall at A and is supported on a small roller at the point D. if a vertical force F is applied at the end B. Find the position of equilibrium as defined by the angle θ.



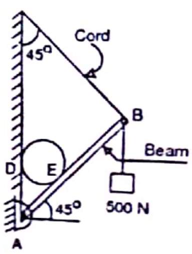
P:-13– Find the support reactions at *D* and *E*, if the frame is loaded at *A* by a load of 1 kN. find reaction at C?



P:14:- Two identical rollers each of weight 500N and radius r are kept on a right-angle frame ABC having negligible weight. Assuming smooth surface, find the reactions induced at all contact surface.

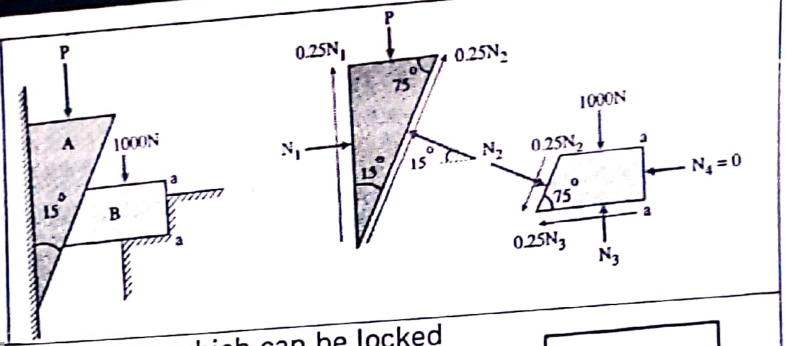


P:15:- A cylinder of diameter 1m and weighing 1000N and another block weighing 500N are supported by a beam of length 7m and weighing 250N with the help of a cord as shown. If the surfaces of contact are frictionless, determine tension in cord and reaction at point of contacts.



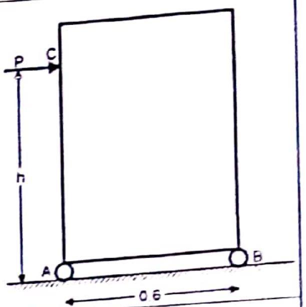
CH 4. FRICTION

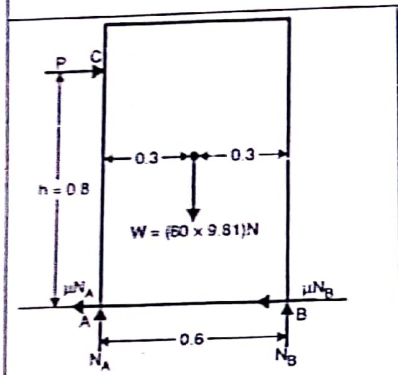
P:-1:- A 15° wedge of negligible weight is to be driven to tighten a body B which is supporting a vertical load of 1000N. If the co-efficient of friction for all surfaces of contact is to be 0·25. Find minimum force P required to drive the wedge. (P = 233.89N)

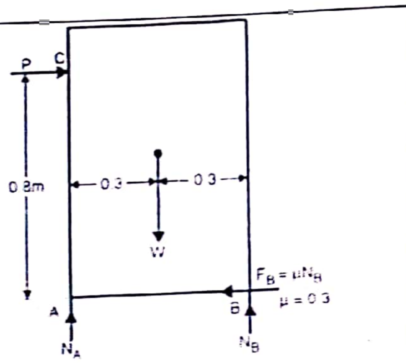


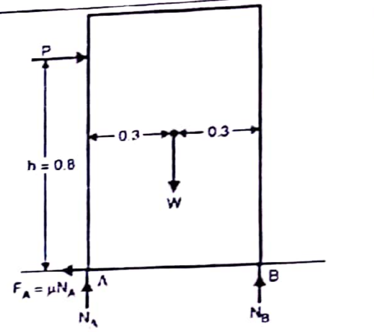
P: 2:- A 60 kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of friction is 0.30. If h= 0.8 m, determine the magnitude of the force P required to move the cabinet to the right: (a) if all casters are locked, (b) if the casters at B are locked and the casters at A are free to rotate, (c) if the casters at A are locked and casters at B are free to rotate.

a) 176.6N b) 147.3N C) 63.1N

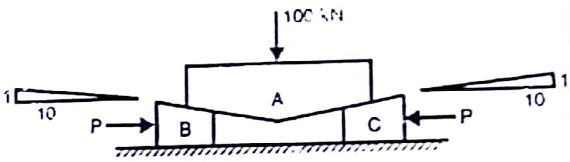




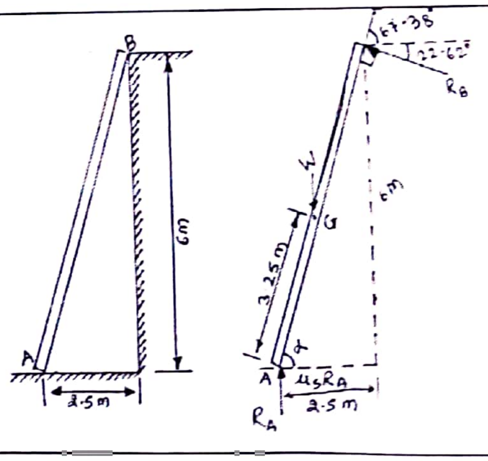




P:-3:- Calculate the magnitude of the horizontal Force P acting on the wedges B & C to raise a load of 100 kN resting on A. Assume μ between the wedges and the ground as 0.25 and between the wedges and A as 0.20 .Also assume symmetry of loading and neglect the weight of A, B and C. Wedges are resting on horizontal surface and their slope is 1:10. (P = **27.88N)**



P:4:-A ladder of length 6.5m leans against a wall. If coefficient of static friction μs is the same at A and B. Determine the value of μ_s for which equilibrium is maintained. (μ_s = 0.183)



CH. 5 CENTRE OF GRAVITY

P:1 Determine centroid of the area ABDEFA.	P:-2– $\bar{X} = 535.946$; $\bar{Y} = 535.946$

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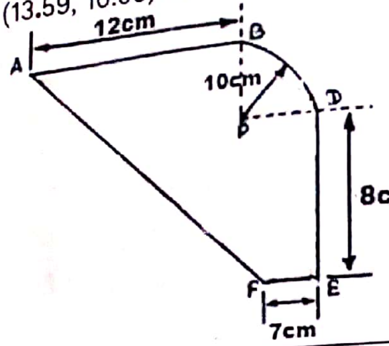
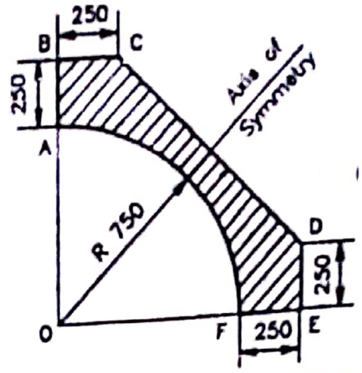
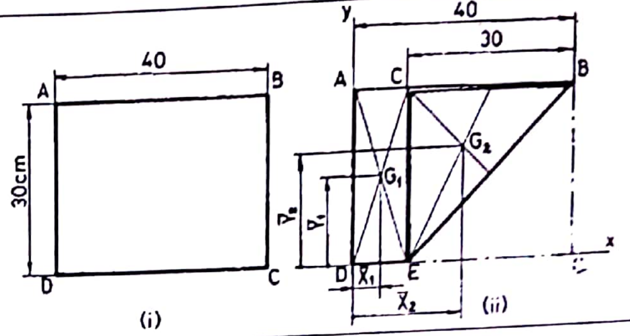
(13.59, 10.08) cm



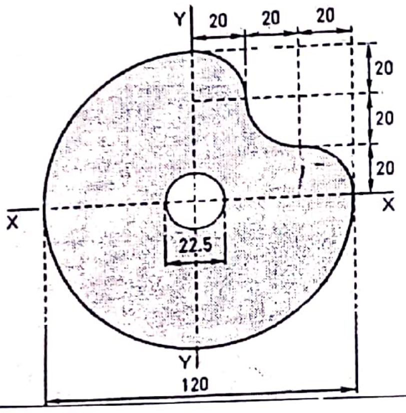


P: 3:- A piece of paper is 40cm long 30cm wide. The shorter side of this paper is folded down such that it fully lies on the longer side. Find the centroid of this folder paper.

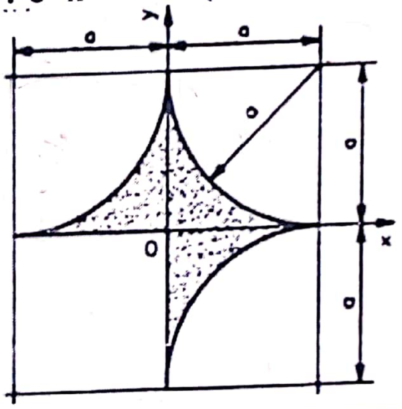
$\bar{x} = 16.25cm$; $\bar{y} = 18.75cm$



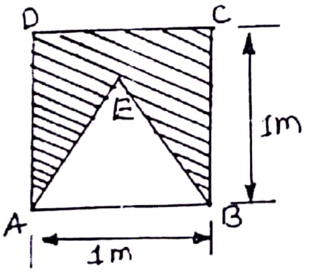
P:4: C(-3.23, -3.23)



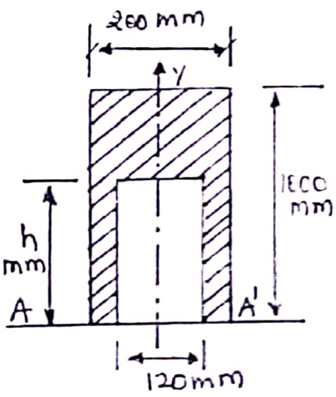
P:-5 $\bar{X} = \bar{Y} = +(0.07446)a$



P: 6:- An isosceles triangle is to be cut from one edge of a square plate of side 1 m such that the remaining part of the plate remains in equilibrium in any position when suspended from the apex of the triangle. Find the area of triangle to be removed. **Ans:** Height of Triangle= 0.639 m, Area of Triangle= 0.319 m^2.



P:7:- Determine the distance 'h' for which the centroid of the shaded area is as high above line AA' as possible. **(Ans:** h=612.57mm)



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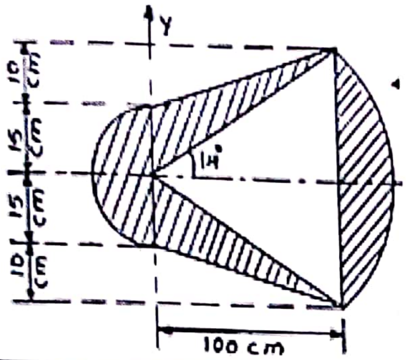
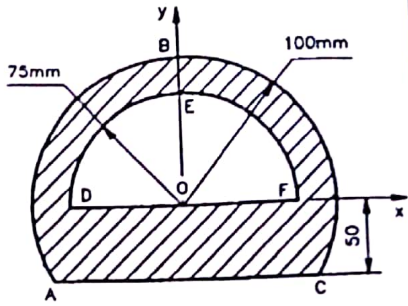
P:8

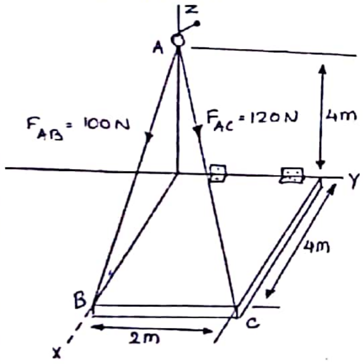


P:9

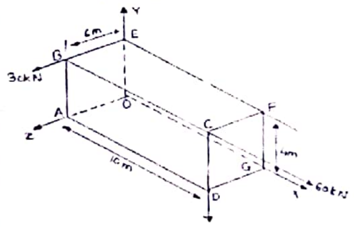


6. SPACE FORCES

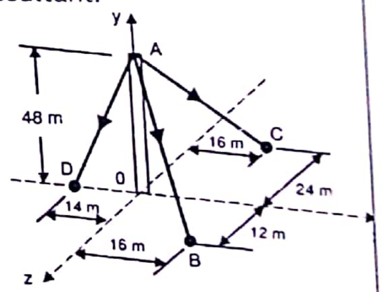
P: 1:- The cable exerts forces FAB =100 N and FAC= 120 N on the ring at A, as shown in the fig. Determine the magnitude of the resultant force acting at A.



P:2:- Three forces act on a rectangular box as shown in fig. Determine the moment of each of the co-ordinate axis point of application of 30kN force is B, point of application of 70 kN force is D. Point of application of 60 kN force is C.



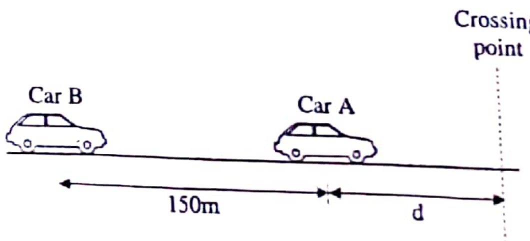
P:3 Knowing that the tension in AC= 20kN, determine tension in wire AB and AD so that resultant of 3 forces applied at A is vertical and find the resultant.



CH 7 A.KINEMATICS OF PARTICLES

P:1 A motorist is traveling at 80 kmph, when he observes a traffic light 200m ahead of him turns red. The traffic light is timed to stay red for 10 sec. If the motorist wishes to pass the light without stopping, just as it turns green. Determine (i) The required uniform deceleration of the motor. (ii) The speed of the motor as it passes the light. (V = 64kmph)

P:2 Two cars moving in the same directions are 150 m apart, car A being ahead of car B. At this instant, velocity of A is 3m/s and constant acceleration is $1.2 m/s^2$. While the velocity of B is 30 m/s and its uniform retardation is 0.6 m/s^2. How many times do the cars cross each other? Find when and where they cross w.r.t. given position of A. (d= 375.6m)



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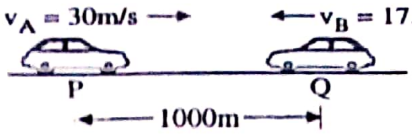
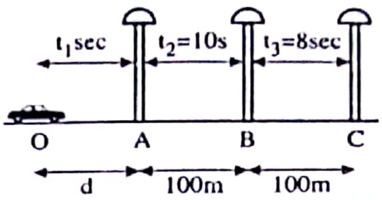
P:3 Two cars A and B are moving towards each other. At t =0, A and B are 1 km. apart with speeds 30m/s and 17.5m/s respectively and they are at points P and Q. Knowing that A passes point Q 40 sec after B was there and that B passes point P 42 sec after A was there, determine (a) uniform accelerations of A and B. (b) when two cars pass each other (c) the speed of B at that time. **$a_A = -0.25\ m/s^2$ $a_B = -0.3\ m/s^2$**

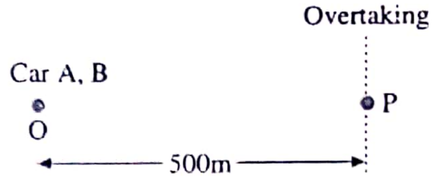


P:4 :- Three vertical poles A, B and C distance of 100m along a straight road. A car starting from rest and accelerating uniformly passes pole A and take 10 seconds to reach pole B and further 8 sec to reach pole C. Calculate (1) acceleration of car (2) Velocity of A and B (3) Starting position of car (Va = 8.68 m/s Vb = 11.48 m/s)

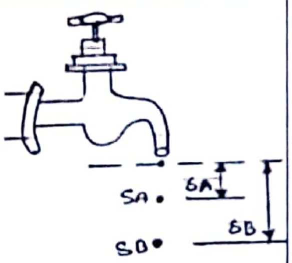


P:5 :- Two cars A and B traveling in the same direction on same adjacent lanes are stopped at a traffic signal. As a signal turns green car A accelerates at a constant rate of $2m/s^2$. 3 seconds later car B starts and accelerates at $3.6m/s^2$. Find(1) When and where B will overtake A (2) The speed of each car at that time. (Va = 23.56 m/s Vb = 31.61 m/s)

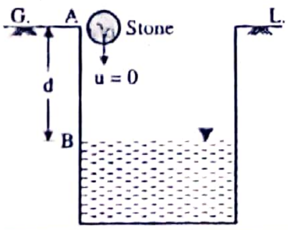
P:6 :- A car 'A' starts from rest and accelerates uniformly on a straight highway. A car 'B' starts from the same point 6 sec later with initial velocity zero and accelerates uniformly at $6m/s^2$. If both cars overtake at 500m from starting point, determine the acceleration of car A. Also find velocity of each car at the time of overtaking. (a_a = 2.796 m/s2 , V_a = 52.94 m/s and V_b = 77.46 m/s)



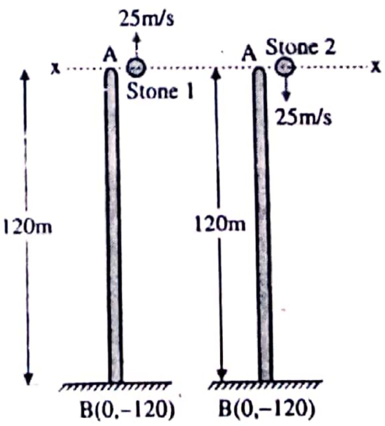
P:7:- Water drips from a faucet at the rate of 5 drops per second as shown in fig. Determine the vertical separation between two consecutive drops after the lower drop has attained a velocity of 3m/sec. (t =0.2 sec)



P:8;-A stone is dropped in to a well and sound of splashed is heard 3.3 sec. after the stone is dropped. If velocity of sound is 350 m/s.Find depth of well upto water level. (d =48.93 m)



P:9;-A stone is projected from top of a building 120m high with initial velocity of 25 m/s. A second stone is projected vertically downwards with the same velocity. Find time taken by each object to reach the ground. At what height the first stone must be just released from rest in order the two stones may hit the ground simultaneously? (h = 44.57m)



P:10:- A body A is projected vertically upwards from the top of a tower with a velocity of 40 m/sec, the tower being 180m high. After 't' sec, another body B is allowed to fall from the same point. Both the bodies reach the ground simultaneously. Calculate
(1) 't' of A and B; and (2) the velocities of A and B on reaching the ground.

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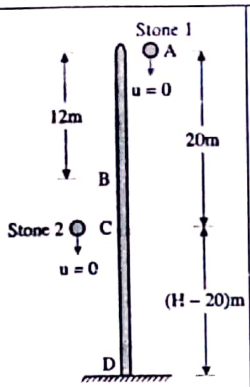
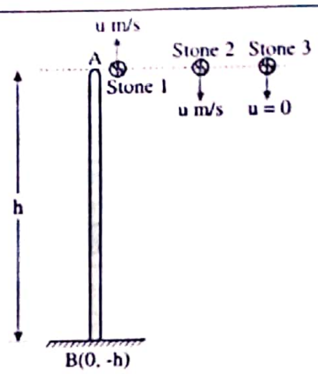
(Va = -71.64 m/s and Vb = -59.45 m/s)

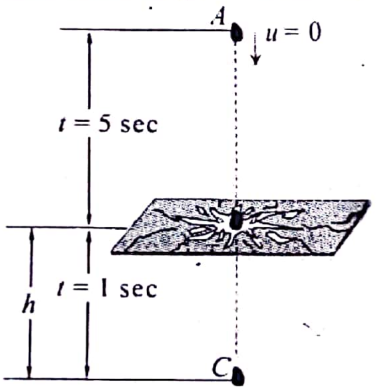
P:11:- A stone has fallen a distance of 12 m after being dropped from the top of a tower, another stone is dropped from a point 20 m below the top of tower. If both stones reach the ground together. Find height of tower.
(H = 21.33 m)



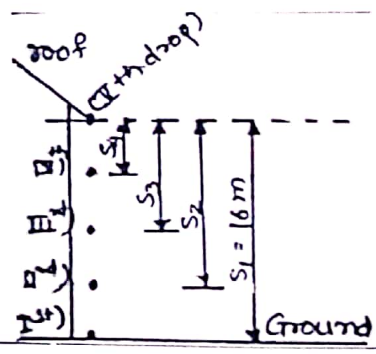
P:12:- From the top of building, two stones are projected, one vertically upwards and the other vertically downwards with same velocity. If t_1 and t_2 are the times taken by two stones to reach the ground, then proved that the time taken by any one of the two stones if simply dropped is given by $t = \sqrt{t_1 \cdot t_2}$



P:13– A stone after falling 5 sec from rest breaks a glass pan and in breaking it losses 30% of its velocity. How far will it fall in the next second? (H = -39.24 m)



P:14 :- Drops of water fall from the roof of a building 16 m high at regular intervals of time, the first drop reaching the earth at the instant the fifth drop stats its fall. Find the distance between the individual the first drop in the air at the instant the first drop reaches the earth

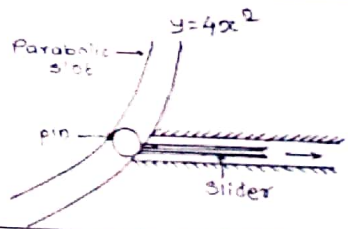


CH 7 B.KINEMATICS OF PARTICLES

P:1:- $x = A\sin(pt + \phi)$ when $t = 0; v = v_0; x = x_0$

Prove :- $(I)\tan\phi = \dfrac{x_0 p}{v_0}$ $\quad (II) A = \sqrt{x_0^2 + \left(\dfrac{v_0}{p}\right)^2}$

P:2:- A pin is moving in parabolic slot whose equation is given by $y = 4x^2$. The slider moves to right with constant speed of 20 m/s. Find velocity and acceleration of pin C (in any position x,y) if p is at origin when t = 0. Also express position vector as a function of time.



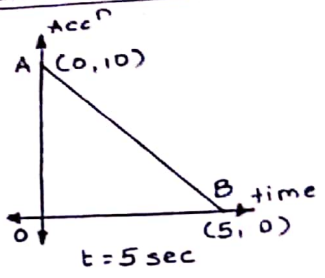
P:3:- A particle moves along a straight line. Its displacement x at time t is $x = x_0\left(2e^{-kt} - e^{-2kt}\right)$ where x_0 is initial displacement and k is constant. Find maximum velocity reached by the particle and corresponding time't'.

Mechanics Practice Problems

P:4:- A particle moving in positive x-direction has an acceleration $a = (100 - 4v^2)\,m/\sec^2$ where v is m/sec. Determine (I) the time interval and displacement of particle when speed changes from 1 m/sec to 3 m/sec. (II) The speed of the particle at t = 0·05 sec.

P:5:- A point moves in the plane xy according to the law x = kt and y = kt(1-αt), where k and t are positive constants and t is time. Find (I) The equation of the points trajectory y(x). (II) The velocity v and acceleration of the point as function of time.

P:6:- A particle is moving in a direction of a given line AB starting from rest at point A with an initial acceleration of $10m/\sec^2$. The acceleration is uniformly reduced continuously with the time t elapsed and is zero at t = 5 seconds. (I) determine the distance traveled and the velocity after 5 seconds from the start.(II) Compute also maximum distance traveled in its initial direction of motion.



P:7:- Position of particle is given as $x = t^3 - 6t^2 - 15t + 40$ find (a) t when v = 0, (b) position and distance traveled at that time, (c) Distance traveled from t = 4 to t = 6 seconds.

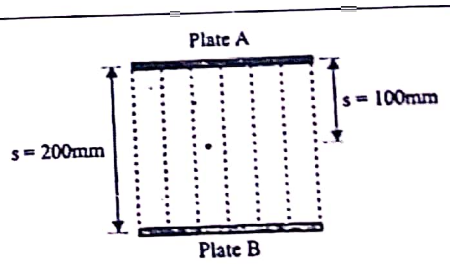
P:8:- Motion of a particles along a straight line is defined by $V^3 = 64S^2$ where V is m/sec and S is in m. Determine (a)Velocity when distance covered is 8 m. (b) acceleration when the velocity is 9m/sec.

P:9:- The displacement (x) of a particle moving in one direction, under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$ where x is in meters and t is in seconds. Find the displacement of a particle when its velocity is zero.

P:10:- The velocity of a particle moving along a straight line is given by $V = 2t^3 + 5t^2$ where V is in m/sec and t is in seconds. What distance does it travels while its velocity increases from 7m/s to 99m/s?

P:11:- Initial velocity of particle is 12m/s on a straight line deceleration is given as a = (A−Bt). Find expression for velocity and position also find A & B if particle covers 60m in 5 sec, then stops.

P:12:- A metallic particle is subjected to the influence of magnetic field such that it travels vertically downwards through the fluid that extends from plate A to plate B. If the particle is released from rest at s =100mm and acceleration is measured as a =(4s) m/s^2 (where s in m) determine the velocity of particle when it reaches plate B, that is s = 200mm and time needed from plate A to plate B.

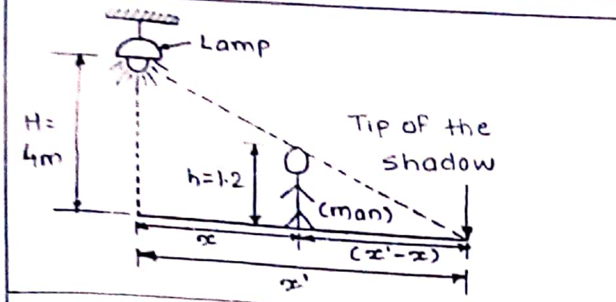


P:13:- The velocity of a particle is given by the equation $v = (8 - 0{\cdot}02s)\,m/\sec$. where v is velocity in m/sec. and (s) is the displacement in meters. Knowing data at t=0, s=0, determine (I) Distance traveled before the particle comes to rest. (II) Acceleration at start. (III) Time required for 100m displacement.

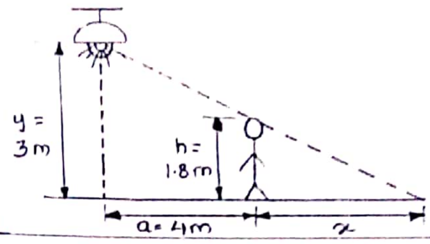
P:14:- A man of height 1·2 meters walks away from a lamp hanging at a height of 4 meters above ground level. If the man walks with a speed of 2·8 m/sec. Determine the speed of the tip of man's shadow.

P:15:- A man of height h = 1·8m stands at a distance of x = 4m from a lamp, which is suspended from the top. The lamp is lowered down at a uniform velocity of 1·0m/sec. Find the velocity and acceleration of tip of the

Mechanics Practice Problems



man's shadow when the lamp is at a height of 3·0m above the ground.

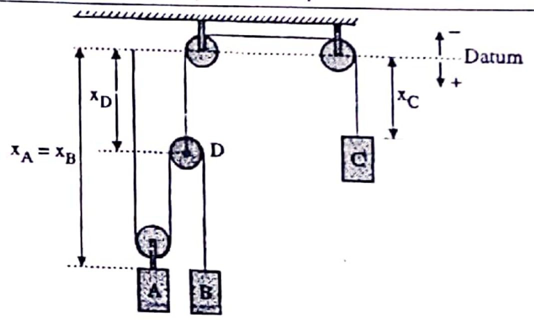


P:16:-The motion of the particle is defined by the equation $x = 4t^4 - 6t$ and $y = 6t^3 - 2t^2$ where x and y are in mm and't' in seconds. Determine velocity and acceleration when t=1sec.

P:17:-A bus starts from rest on a curve of radius 250m & acceleration at constant rate of $0.6 m/s^2$ Determine distance and time that bus will ravel before magnitude of total acceleration becomes $0.75 m/s^2$

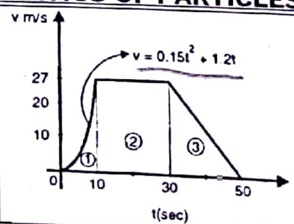
P:18:-A particle moves along the path $\bar{r} = \left(8t^2\right)\hat{i} + \left(t^3 + 5\right)\hat{j}$ m. where t is in sec. Determine magnitudes of particle's velocity and acceleration when t=3sec. Also determine the equation $y = f(x)$ of the path.

P:19:-For given arrangement block C is moving up at a constant speed of 8 m/s. If elevations of blocks A and B are always equal. Determine velocity of block B.

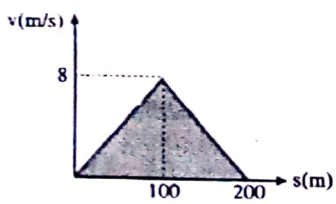


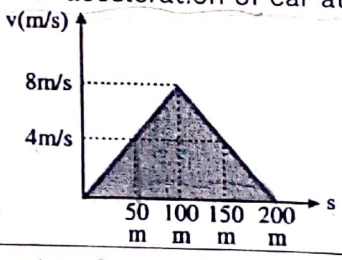
CH 7.C.KINEMATICS OF PARTICLES

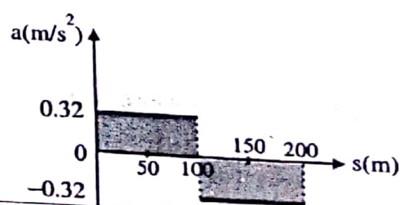
P:1:-From v –t diagram, find : (i) distance travelled in 10 seconds, (ii) total distance travelled in 50sec, (iii) retardation.



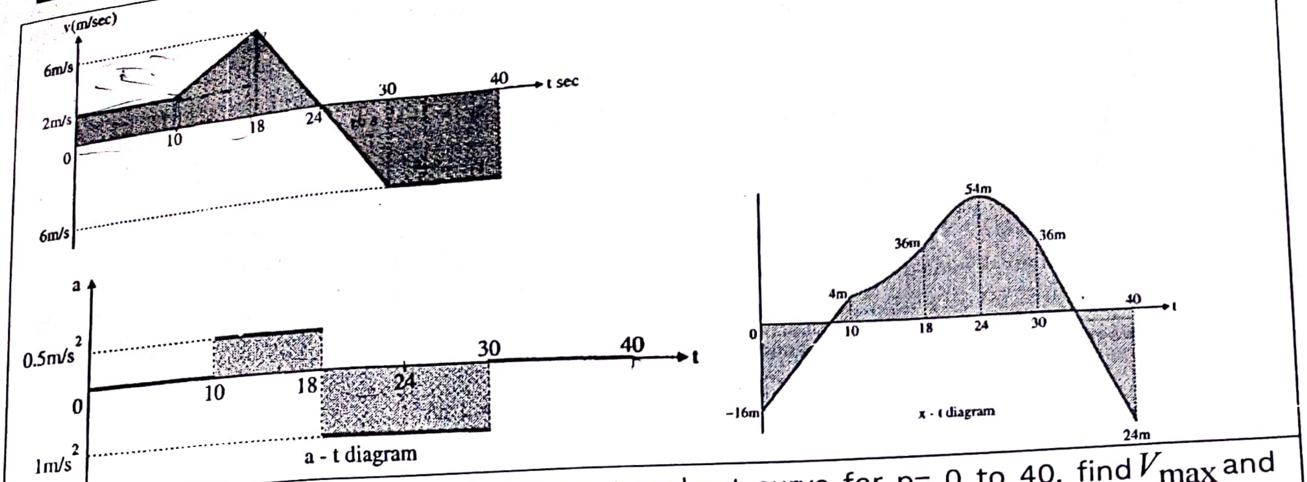
P:2:- v-s curve is given. find acceleration of car at s = 50 , and s = 150 m. draw a-s graph



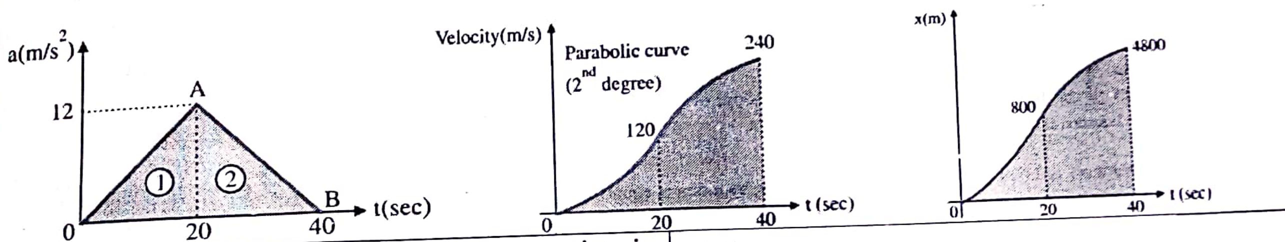




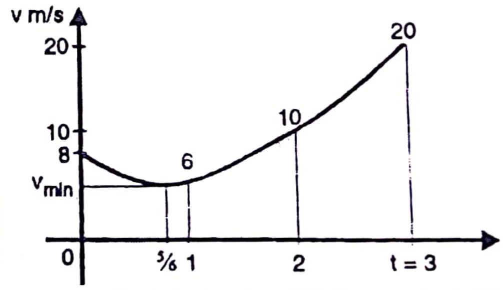
P:3:- v-t diagram is given at t= 0 , x= -16, plot a-t and x-t curve for 0 to 40 sec. find X_{max} and time t, when particle is at 36m from origin.



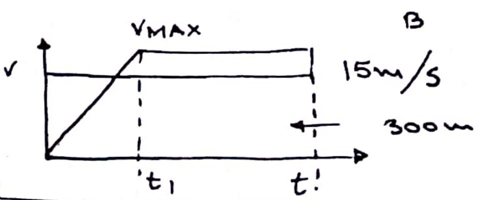
P:4:- a-t graph is given, at t=0, v=0, draw v-t and x-t curve for p= 0 to 40. find V_{max} and X_{max} during that interval .



P:5:- Velocity of a particle in rectilinear motion is given by $v=\left(3t^2 = 5t+8\right)m/s.$ Knowing that $x=15m.$ at $t=0$ draw v-t diagram for the motion up to $t=3\sec$ Hence or otherwise, find the values of acceleration velocity and position co-ordinates x when the velocity is minimum.



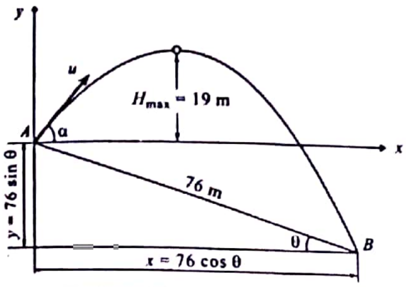
P:6:- A series of city traffic signals is timed so that an automobile traveling at a constant speed of 54 km/h will reach each signal just as it turns green. A motorist misses a signal and is stopped at signal A. Knowing that the next signal B is 300m ahead and that the maximum acceleration of the automobile is 1.8m/s², determine what the motorist should do to keep the maximum speed as small as possible, yet reach signal B just as it turns green. What is the maximum speed reached? Ans: Motorist should accelerate for 11.84sec. and then continue at speed of 76.7 km/hr.



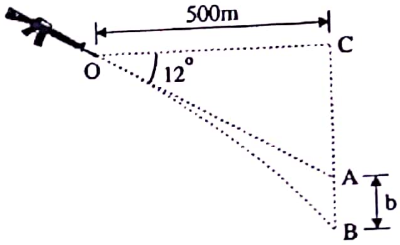
CH 7.D. KINEMATICS OF PARTICLES

Mechanics Practice Problems

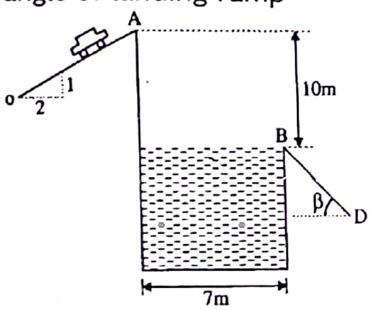
P:1:- A ball rebounds at A and strikes the incline plane at point B at a distance 76 m. If the ball rises to a maximum height h = 19m above the point of projection. Compute the initial velocity and the angle of projection α. Given (θ = 18.44)



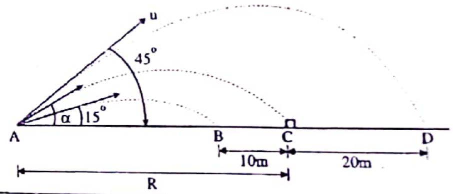
P:2:- The rifle is aimed at point A and fired, Calculate the distance b below A to the point 'B' where the bullet strikes, The muzzle velocity of bullet is 800m/s.



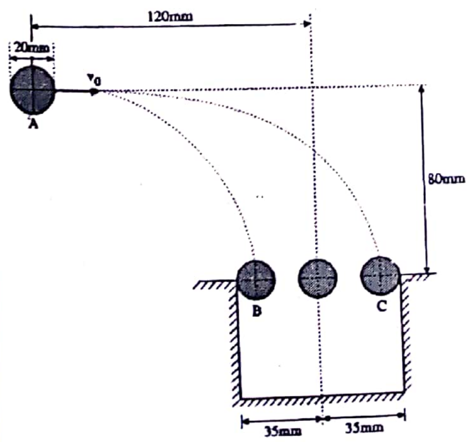
P:3:- A stuntman is to drive an auto across the water filled gap as shown in fig. Determine auto's minimum take off velocity and the angle of landing ramp



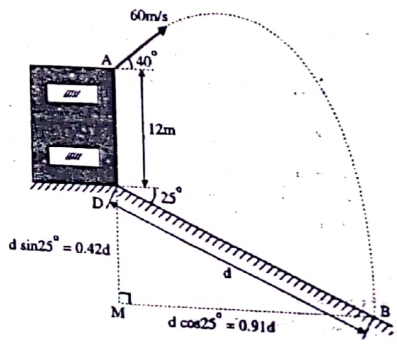
P:4:- A Projectile is the aimed at an object on the horizontal plane through the point of projection and falls 10m short when angle of projection is 15° , while it overshoots the object by 20m when angle of projection is 45°. Determine the angle of projection to hit the object correctly.



P:5:- A steel ball of diameter 20mm is thrown horizontally and it falls through 70mm diameter hole as shown in fig. Calculate range of velocity which enables the ball to enter the hole.

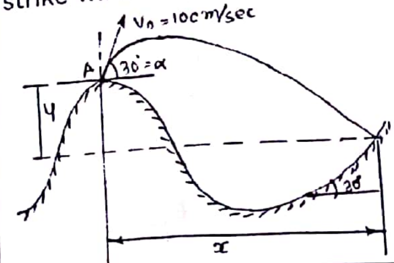


P:6:- A building 12m high is situated at top of a sloping ground which makes an angle of 25° and slopes downward from foot of the building. A projectile is fired from top of the building with a velocity of 60 m/s at an angle of projection of 40°. Find when and where it strikes the ground.

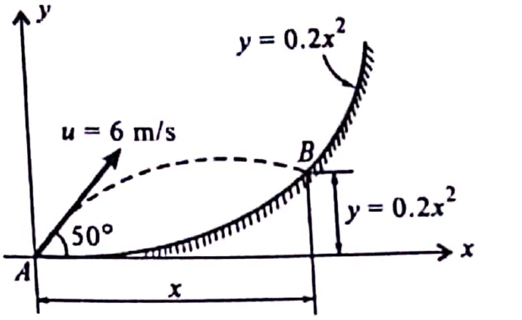


Mechanics Practice Problems

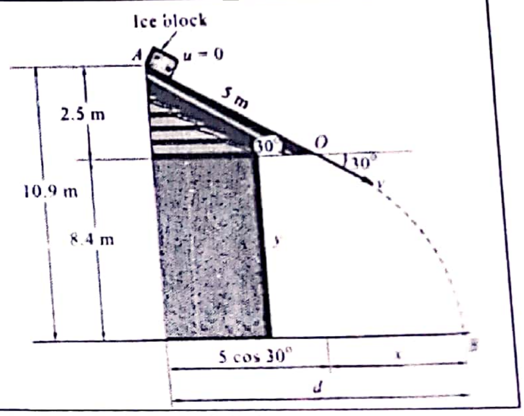
P:7:- A ball is thrown upward from a high cliff with a velocity of 100m/sec at an angle of elevation of 30° with the horizontal. The ball strikes the inclined ground at right angle. If inclination of ground is 30° as shown, determine: (1) Time after which the ball strikes the ground (2) velocity with it strikes the ground. (3) Co-ordinates (x,y) of a point of strike w.r.t. to point of projection..



P:8:- The water sprinkler positioned at the base of hill releases a stream of water with a velocity of 6 m/s. Determine the point *B(x, y)* where the water particles strike the ground on the hill.

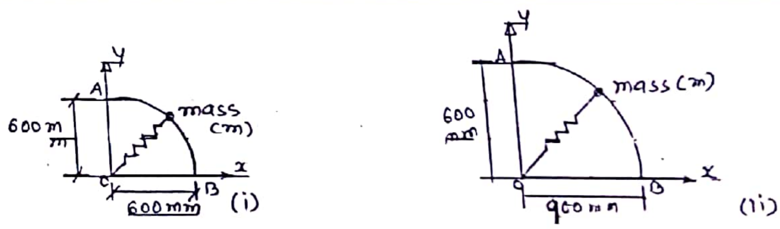


P:9:- A block of ice starts sliding down from the top of the inclined roof of a house along a line of maximum slope. At what horizontal distance from the starting point will the block hit the ground? (Neglect friction)



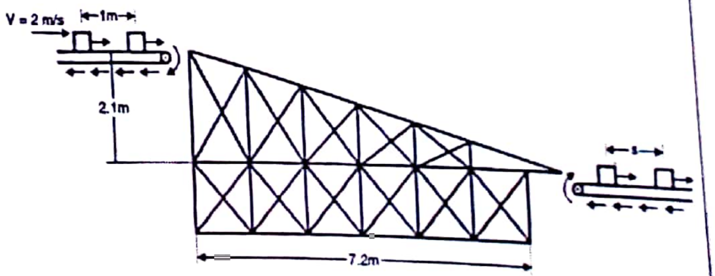
CH 8. A.KINETICS OF PARTICLES

P: 1:- The mass m = 1·8 kg slides from rest at A along the frictionless rod bent in to a quarter circle. The spring with modulus K = 16 N/M has an unscratched length of 400 mm. 1. Determine the speed of m at B 2. If the path is elliptical, what is the speed at B? (Vb = 3.15 m/s)



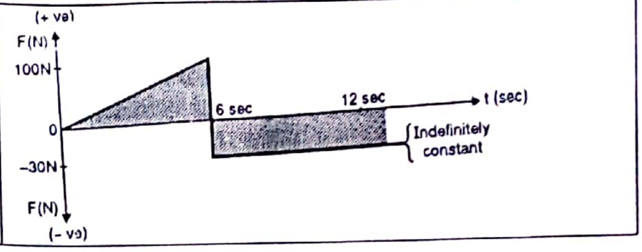
P:-2:- A gun of a battery is nested by three springs each of stiffness = 250 kN/cm as shown in fig. It fires a 500 kg shell with a muzzle velocity of 1000m/sec. Calculate the total recoil and the maximum force developed in each spring if the gun has a mass of 80,000 kg. (F = 5100 N)

P:-3:- Packages having a mass of 5 kg. are transferred horizontally from one conveyor to the next using a ramp for which the coefficient of kinetic friction is $\mu k = 0{\cdot}15$, The top conveyer is moving at 2 m/sec, and packages are speed 1 m. determine the required speed of bottom conveyer so that no slipping occurs when packages come horizontally in contact



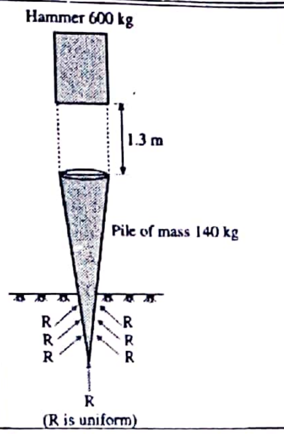
with it. What is the spacing S between the packages on the bottom conveyer? Use work Energy principle. (S =2.45 m)

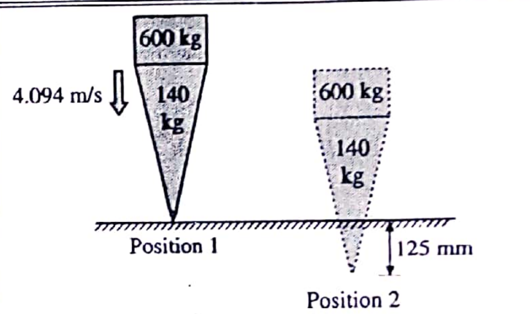
P:-4:- A body which is initially at rest, at the origin is subject to a force varying with time as shown in fig:-find the time :- (I)When the body again comes to rest. (II) When it comes again to its original position.
(T = 27.83 sec)



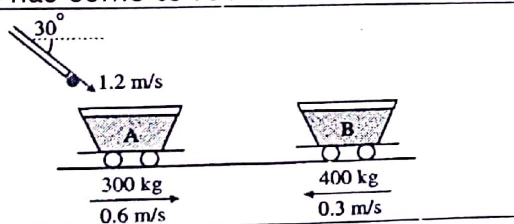
CH 8. B.KINETICS OF PARTICLES
IMPACT OF SOLID BODIES AND D'ALEMBERT PRINCIPLE

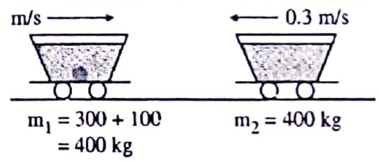
P-1 :- A 600 kg hammer is dropped from a height of 1·3 m on to the top of a pile of mass 140 kg. The pile is driven 125mm. in to the ground because of the impact. What is the average resistance to penetration offered by the ground after the impact? Assume perfectly plastic impact.



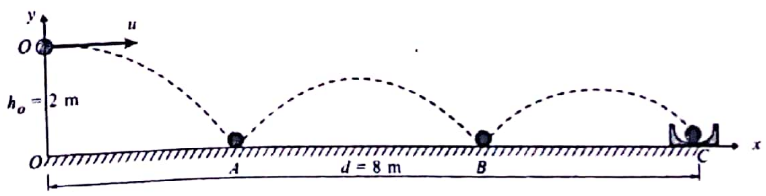


P-2:- The 300 kg and 400 kg mine cars are rolling in opposite directions along track with respective speeds of 0.6 m/s and 0.3 m/s. Upon impact the cars become coupled together. Just before the impact, a 100 kg stone leaves the delivery chute with velocity as shown in Fig. and lends in 300 kg car. Calculate the velocity v of the system after the stone has come to rest relative to the car. (V = 0.2049 m/s)

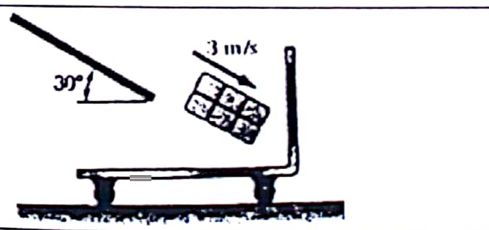




P:3– A small steel ball is to be projected horizontally such that it bounces twice on the surface and lands into a cup placed at a distance of 8 m. If e= 0.8, determine the velocity of projection '*u*' of the ball. (u = 3.232 m/s)



P-4:- A 10 kg package drops from a chute into a 25 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely. Determine (a) the final velocity of the cart, (b) the impulse exerted by the cart on the package, (c) the fraction of the initial energy lost in the impact.

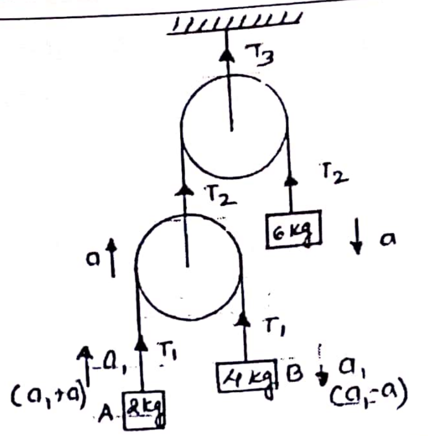


Mechanics Practice Problems

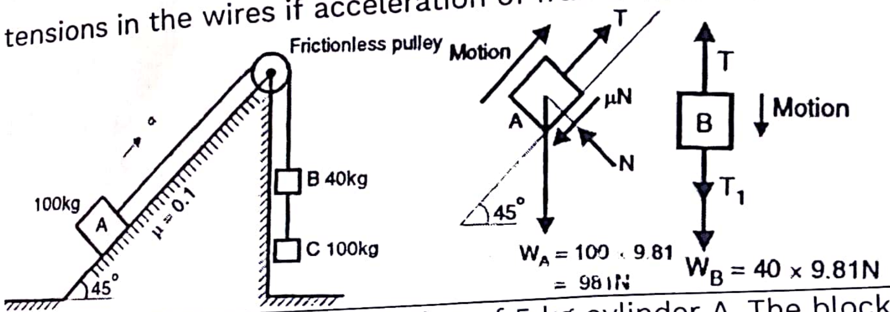
P:5:- In the system of pulleys, masses and connecting inextensible cables. Shown in fig, the pulleys and cables are considered mass less and frictionless. Mass of A=2 kg. , mass of B=4 kg, and mass of C=6 kg. If the system is released from rest, find : (I) Tension in each of the three cables

(II) Acceleration of each of the three masses.

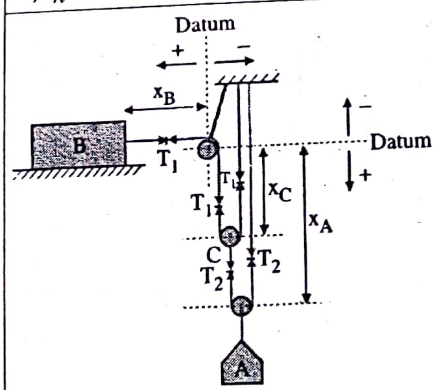
Present your answer in tabular form.

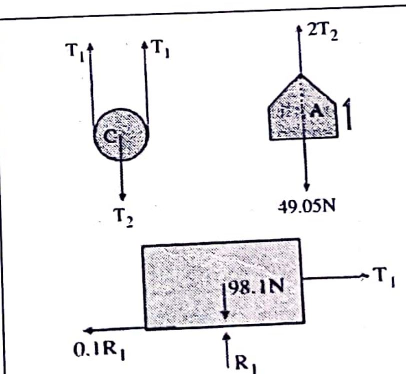


P:6:- A ball of mass m is suspended from accelerating frame by two wires A and B. Determine tensions in the wires if acceleration of frames is 2.5m/s^2

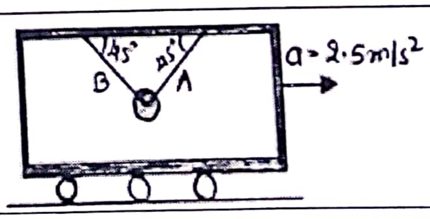


P:7:- Determine the acceleration of 5 kg cylinder A. The block B has a mass of 10 kg. $\mu_k = 0.5$ (a_A = 0.0596 m/s2)

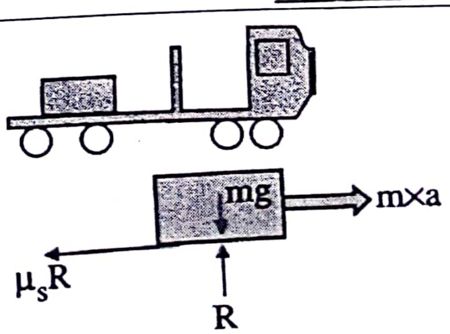




P:8:- A ball of mass m is suspending from accelerating frame by two wires A and B. Find tensions in the wires if acceleration of frame is 2.5m/s^2. (Ta = 8.7 N, Tb = 5.165 N)

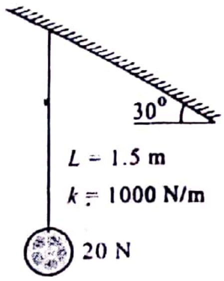


P:9:- The coefficient of frictions are μ_s =0.3 and μ_k =0.25 between the flat bed of the truck and the crate. Determine the minimum stopping distance and corresponding time which the truck can have from a speed of 70kmph with constant deceleration if the crate is not to slip forward.

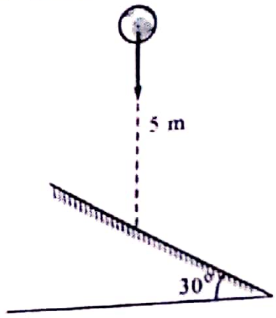


Mechanics Practice Problems

P:10– A ball of weight 20 N is suspended from an elastic cord tied to an inclined ceiling. It is stretched vertically down by 0.3 m and then released. The ball travels vertically up and strikes the ceiling. Find the speed of the ball (i) just before impact and (ii) just after impact. Take e = 0.7

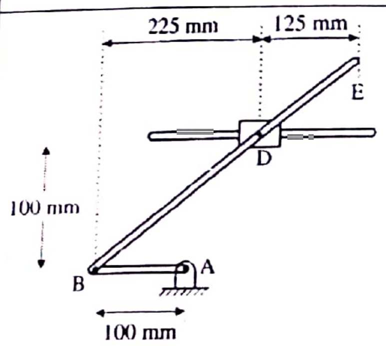


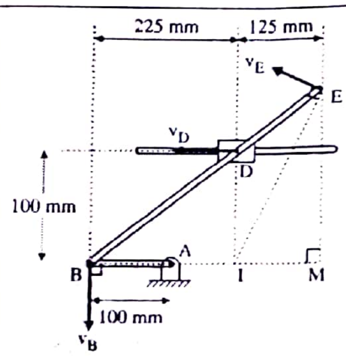
P:11– A ball is dropped from a height of 5 m on an inclined surface of 30°. Find the velocity of ball after impact, take e = 0.8.



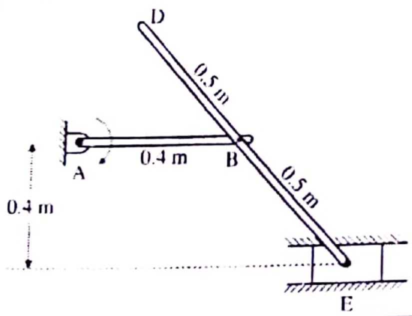
CH 9 .KINEMATICS OF RIGID BODIES

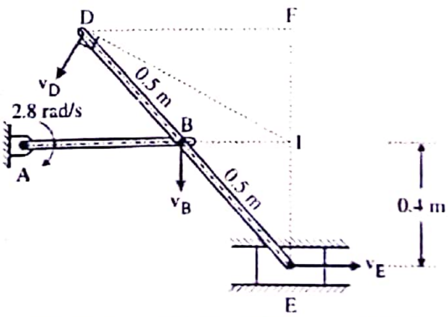
P: 1:- Knowing that at the instance shown in fig. the velocity of collar D is 120mm/s to the left. Determine a) Angular velocities of crank AB & rod BE. b)The velocity of point E



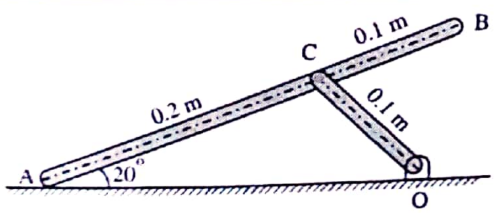


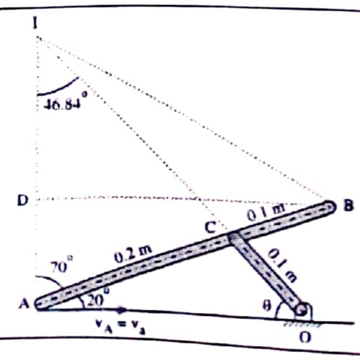
P: 2:-At the position shown, crank AB 0.4m long has angular velocity of 2.8rad/sec. If DB=BE=0.5m and if AB is horizontal, determine velocity of slider E and velocity of point VE= 1.492m/s (perpendicular to EI)





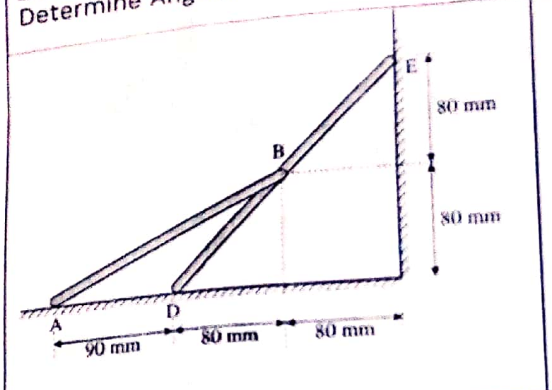
P: 3:-The end A of a slender bar AB follows the horizontal straight line OA while point C is attached to crack OC. Find velocity of B in terms of VA which is known velocity of point A.

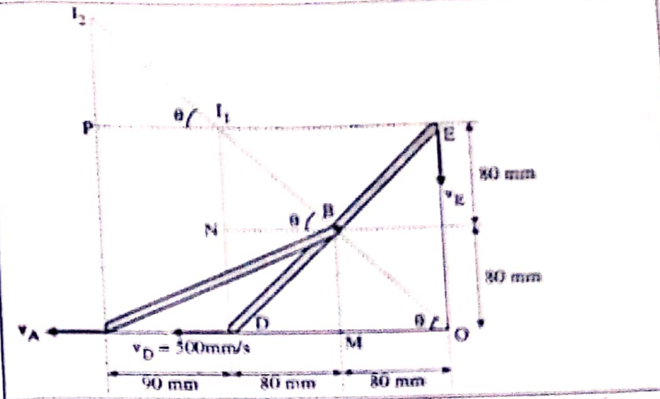




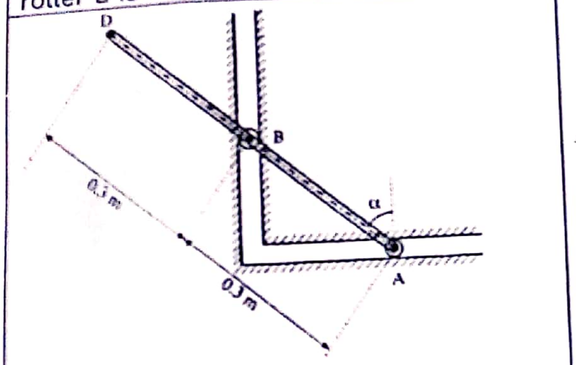
Mechanics Practice Problems

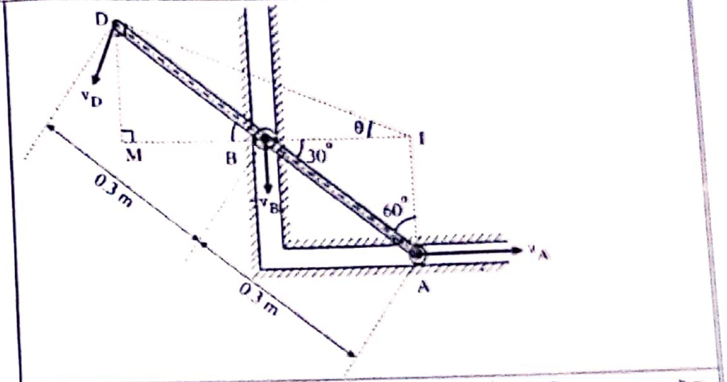
P: 4:- Two rods AB and DE are shown in fig. The point D moves to the left with 500 mm/sec. Determine Angular velocity of each rod. Velocity of point A.



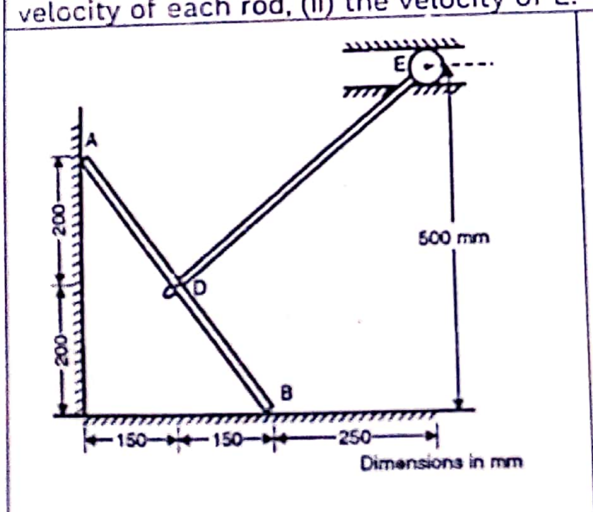


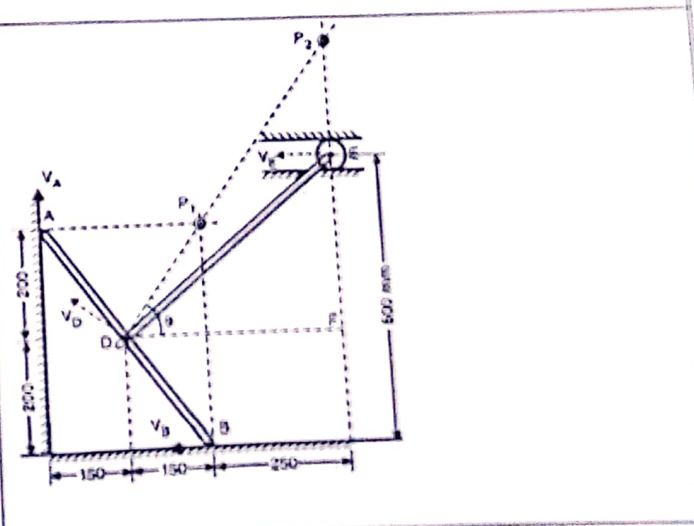
P:5:- A rod ABD is guided by rollers at A and B. At the instant when α = 60° & velocity of roller B is 0.8 m/s . Determine: - 1. Angular velocity of the rod. 2. The velocity of point D.





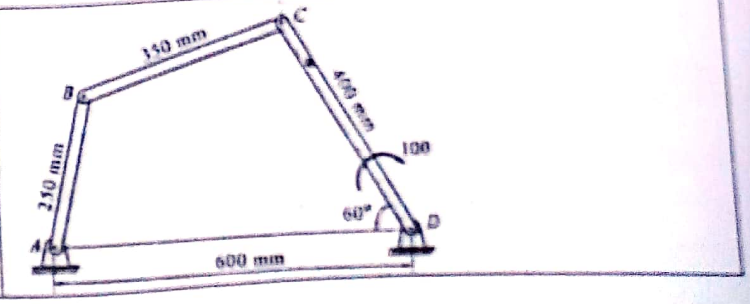
P: 6:- Two 500 mm rods are pin-connected at D as shown in fig. Knowing that B moves to the left with constant velocity of 360 mm/s. determine at the instant shown (i) the angular velocity of each rod, (ii) the velocity of E.



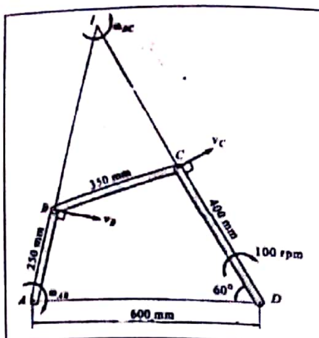


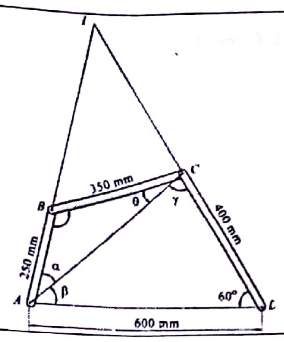
Note:- 3 link mechanism excluded from syllabus.

P:-7- Three bars AB, BC and CD hinged together to form a mechanism. If bar *CD* has angular velocity of 100rpm in clockwise sense. Determine velocities of points *B* and *C*. What is the angular velocity of bar *AB*?



Mechanics Practice Problems





(i) $\cos 60° = \frac{0.6^2+0.4^2-AC^2}{2 \times 0.6 \times 0.4}$ $AC = 0.53$m;

$$\cos\alpha = \frac{0.25^2 + 0.53^2 - 0.35^2}{2(0.25)(0.53)} \quad \alpha = 33.53°$$

$$\cos\theta = \frac{0.35^2 + 0.53^2 - 0.25^2}{2(0.25)(0.53)}; \qquad \theta = 23.24°$$

Using sine rule, we get

$$\frac{IB}{\sin 77.64°} = \frac{IC}{\sin 56.77°} = \frac{0.350}{\sin 45.59°};$$

$$IB = 0.48\text{m}; \quad IC = 0.41\text{m};$$

$$\omega_{CD} = 100\, x \left(\frac{2\pi}{60}\right) \text{rad sec}; \ \omega_{CD} = 100 \text{ x } \left(\frac{2\pi}{60}\right) \text{ rad/sec}$$

$$\cos\beta = \frac{0.6^2 + 0.53^2 - 0.4^2}{2(0.6)(0.53)}; \ \beta = 40.88°$$

$$\gamma = 180° - (60° + \beta) = 180° - 60° - 40.88°$$

$$\gamma = 79.12°$$

ii) Bar CD (Performs rotational motionabout point D)

$$\therefore v_c = (CD)(\omega_{CD}) = 0.400 \text{ x } \left[100\, x \frac{2\pi}{60}\right]$$

$$v_C = 4.189 \text{ m/sec}(\nearrow)(30°)$$

(iii) **Rod BC**(Performs general plane motion)At the given instant point I is the ICR

$$v_C = (IC)(\omega_{BC}); \ \omega_{BC} = \frac{4.189}{0.41}; \omega_{BC} = 10.22\text{rad/sec}(\circlearrowleft)$$

$$v_B = (IB)(\omega_{BC}) = 0.48\, x\, 10.22; \ v_B = 4.906 \text{ m/s}$$

(iv) **Rod AB**(Performs rotational motion about point A)

$$v_B = (AB)(\omega_{AB}); \ \omega_{AB} = \frac{4.906}{0.25}$$

$$\omega_{AB} = 19.62 \text{ rad/sec}(\curvearrowright)$$

2. Support Reaction

P:1:-

Since there is no horizontal force,

$\therefore A_X = D_X = 0$ $\therefore R_E = 33.67KN$

$\sum M_F = -(R_E \times 3) - (6\times 4) + (25\times 5) = 0$ $\therefore R_F = 14.67KN(\uparrow)$

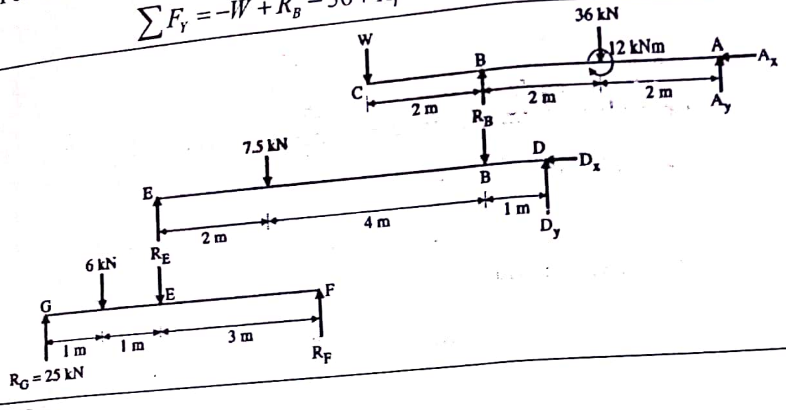
For beam GF, $\sum F_Y = -R_E - 6 + 25 + R_F = 0$

For beam ED, $\sum M_D = -(R_B \times 1) - (7.5\times 5) + (R_E \times 7) = 0$ $\therefore R_B = 198.19KN$

$\sum F_Y = -R_B + R_E - 7.5 + D_Y = 0$ $\therefore D_Y = 172.02KN(\uparrow)$

For beam CA, $\sum M_A = -(W\times 6) + (R_B \times 4) - (36\times 2) + 12 = 0$ $\therefore W = 122.13KN(\downarrow)$

$\sum F_Y = -W + R_B - 36 + A_Y = 0$ $\therefore A_Y = 40.06(\downarrow)$



P:2:-

$\sum F_Y = 0.8W_{BC} - 3W_1 - 3600 + W_{DE} = 0$

But $W_{DE} = W_{BC}$ (Given)

$\therefore 1.8W_{BC} - 3W_1 = 3600....(i)$

$\sum M_A = (3W_1 \times 2) + (3600\times 4) - (0.8W_{BC}\times 1) - (W_{DE}\times 5) = 0$

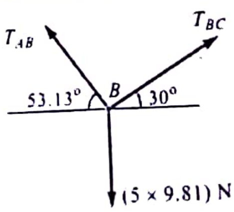
$\therefore 6W_1 - 5.8W_{BC} = -14400...(ii)$

$\therefore W_{DE} = W_{BC} = 3272.73N/m$

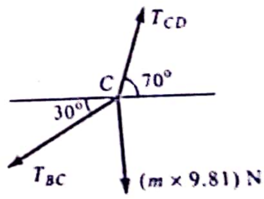
$W_1 = 763.64N/m$

3. Equilibrium of Coplaner Forces

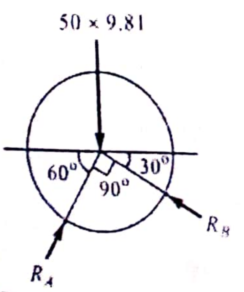
P: 1:-
ApplyLami's theorem & find T_{AB}&T_{BC}



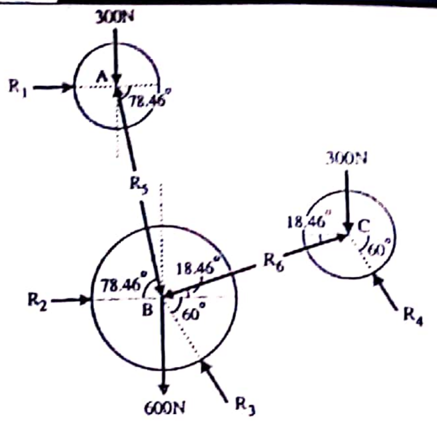
ApplyLami's theorem & find m.

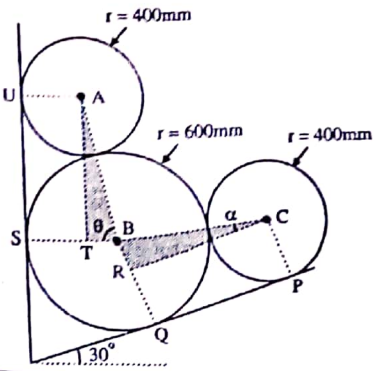


P:2 Apply Lami's theorem & find R_A&R_B



P: 3





For A: By Lami's theorem:

$$\frac{R_1}{\sin 168.46} = \frac{300}{\sin 101.54} = \frac{R_5}{\sin 90}$$

$\therefore R_1 = 61.25N$ $\qquad$ $\therefore R_5 = 306.19N$

For C: By Lami's theorem:

$$\frac{R_4}{\sin 108.46} = \frac{300}{\sin 101.54} = \frac{R_6}{\sin 150}$$

$\therefore R_4 = 290.43N$ $\qquad$ $\therefore R_6 = 153.095N$

For B:

$\sum F_x = 0$

$\therefore R_2 - R_3 \cos 60 - R_6 \cos 18.43 + R_5 \cos 78.46 = 0$

$\therefore R_2 - 0.5R_3 = 83.98$

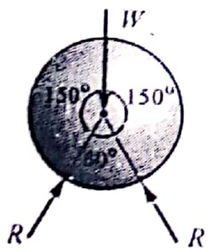
$\sum F_y = 0$

$-R_5 \sin 78.46 - R_6 \sin 18.43 + R_3 \sin 60 - 600 = 0$

$\therefore R_3 = 1095.12N$

$\therefore R_2 = 631.54N$

P: 4



Consider the F.B.D. of upper sphere *R*.

$$\frac{W}{\sin 60°} = \frac{R}{\sin 150°} \therefore R = 0.577\,W$$

Consider the F.B.D. of any one lower sphere (say *P*)

The reaction at contact between two spheres will be zero because at the required angle α the arrangement is about to collapse.

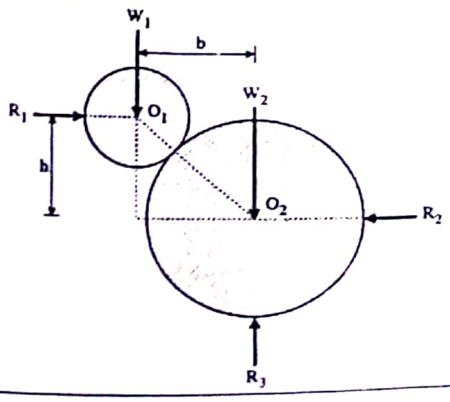
$\sum F_x = 0;\ R_1 \sin\alpha = 0.577\ W\cos 60° \ldots. (\text{I})$

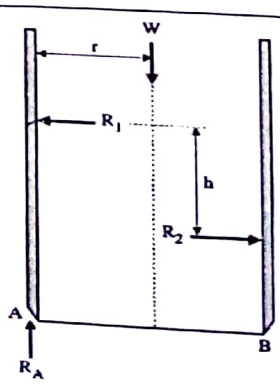
$\sum F_y = 0;\ R_1 \cos\alpha = W + 0.577\ W\sin 60°$

Dividing Eq. (I) by Eq. (II)

$$\tan\alpha = \frac{0.577\,W\cos 60°}{W + 0.577\,W\sin 60°} \therefore \alpha = 10.89°$$

P:5:-





$\sum F_x = 0 \rightarrow R_1 - R_2 = 0$ $\qquad \therefore R_1 = R_2$

$\sum F_y = 0 \rightarrow R_3 = W_1 + W_2$

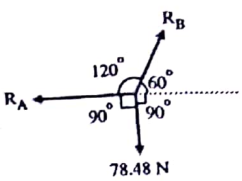
$\sum M_m = 0 \rightarrow -(W_1.b) + (R_1.h) = 0$ $\qquad \therefore R_1 = \dfrac{W_1.b}{h}$ $\qquad \therefore R_2 = \dfrac{W_1.b}{h}$

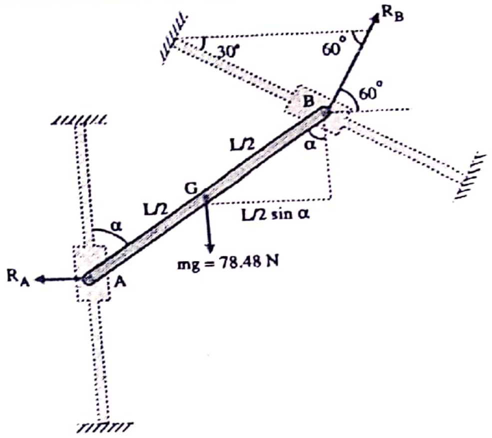
From F.B.D of cylinder, we see that R_1 and R_2 form a couple which tends to overturn the cylinder about point A.

$\sum M_A = 0 \rightarrow W.r - R_1 h = 0$ $\qquad \therefore W.r = R_1 h$ $\qquad \therefore W = \dfrac{W_1 b}{h}.\dfrac{h}{r} = \dfrac{W_1 b}{r}$

But $b = 2r - r_1 - r_2$ $\qquad \therefore W = \dfrac{W_1}{r}(2r - r_1 - r_2)$

P:6:-



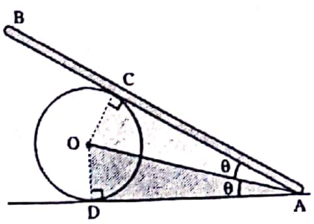


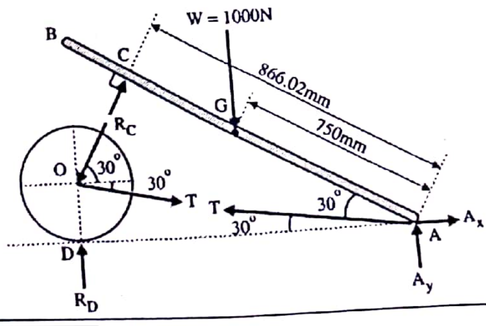
When a sliding collar is given, reaction must be perpendicular to the guide along which the collar slides.

$\dfrac{R_B}{\sin 90} = \dfrac{78.48}{\sin 120} = \dfrac{R_A}{\sin 150}$ $\qquad \therefore R_A = 45.31N(\leftarrow)$ $\qquad \therefore R_B = 90.62N\angle 60$

$\sum M_B = 0 \rightarrow -mg \times \dfrac{L}{2}\sin\alpha + R_A.L\cos\alpha = 0$ $\qquad \therefore \tan\alpha = 45.31/39.24$ $\qquad \therefore \alpha = 49.11^\circ$

P:7:-





In ΔACO, $AC = \sqrt{(AO)^2 - (OC)^2}$

$= \sqrt{(1000)^2 - (500)^2}$

$= 866.02mm$

Also $\sin\theta = \dfrac{OC}{OA} = \dfrac{500}{1000} = \dfrac{1}{2}$

$\therefore \theta = 30^\circ$

Now for cylinder use Lami's theorem,

$\therefore \frac{T}{\sin 120^\circ} = \frac{R_D}{\sin 120^\circ} = \frac{R_C}{\sin 120^\circ}$

$\therefore T = R_D = R_C$ ......(1)

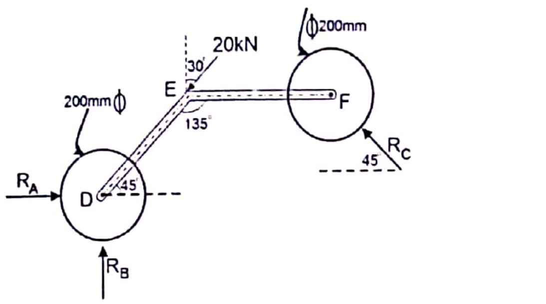
For bar use,

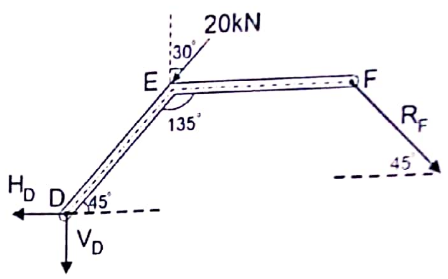
$\sum M_A = 0 \qquad \therefore 1000 \times (750 \cos 60^\circ) - R_C \times 866.02 = 0$

$\therefore R_C = 433.01 N$

$\therefore$ By Equation (1), T = 433.01 N

P:8:-





In equilibrium using two forces principle system, $R_C = R_F$ Step I :- Applying the cond of equilibrium,

$\Sigma M_D = 0$

$-(20 \times \sin 30)(300 \sin 45) + (20 \cos 30)(300 \cos 45)$

$-(R_F \sin 45)(300 \cos 45 + 300) - (R_F \cos 45)(300 \sin 45) = 0$

$\therefore R_F = 3 \cdot 03 kN$

$\Sigma H = 0 \therefore H_D - 20 \sin 30 - R_F \cdot \cos 45 = 0$

$\therefore H_D = 12 \cdot 143 kN$

$\Sigma V = 0 \therefore V_D - 20 \cos 30 + R_F \cdot \sin 45 = 0$

$V_D = 20 \cos 30 - 3 \cdot 03 \sin 45 \qquad \therefore V_D = 15 \cdot 18 kN$

Step II :- Applying the condition of equilibrium, for the cylinders, we get,

$\Sigma H = 0 \quad \therefore R_A - H_D = 0 \qquad \Sigma V = 0 \quad \therefore R_B - V_D = 0$

$\therefore R_A - 12 \cdot 143 = 0 \qquad \therefore R_B - 15 \cdot 18 = 0$

$\therefore R_A = 12 \cdot 143 kN \qquad \therefore R_B = 15 \cdot 18 kN$

ANS:- $R_C = R_F = 3 \cdot 03 kN$, $R_A = 12 \cdot 143 kN$, $R_B = 15 \cdot 18 kN$

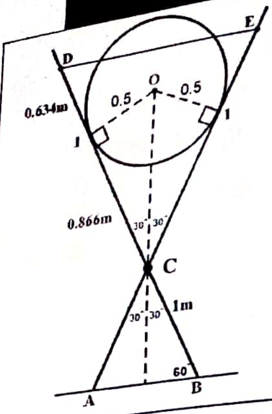
P:9:-

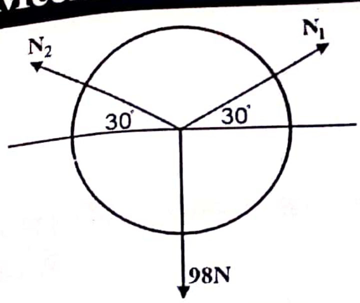
Because of symmetry it is apparent that $R_A = R_B = 49N$ vertically upwards (gravitational force is 10*9.8=98N)

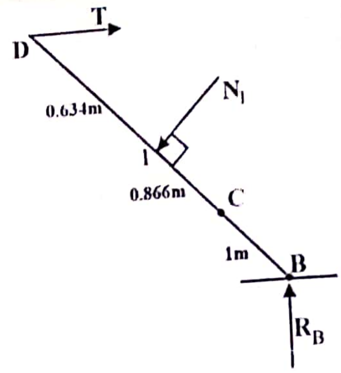
$\sum F_y = 0 = 2N_1 \sin 30 - 98 \qquad \therefore N_1 = 98N$

Perpendicular distance between 1 to C is 0.5/(tan30) = 0.866m

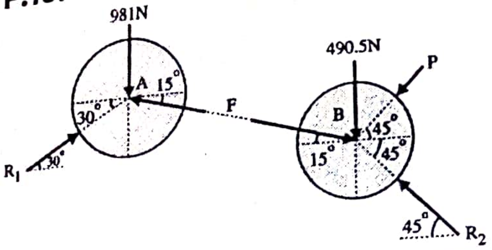
$\sum M_c = 0 = -T \times 1.5 \cos 30 + 98 \times 0.866 + 49 \times 1 \sin 30 \qquad \therefore T = 84.2N$







P:10:-



Consider the Cylinder A
Applying conditions of equilibrium, we get,

$\sum H = 0;$

$\therefore R_1 \cos 30 - F_{AB} \cos 15 = 0$

$\therefore R_1 = \dfrac{\cos 15}{\cos 30} F_{AB}$

$\sum V = 0;$

$\therefore R_1 \sin 30 + F_{AB} \sin 15 = 981$

$\therefore \dfrac{\cos 15}{\cos 30} F_{AB} \sin 30 + F_{AB} \sin 15 = 981$

$\therefore F_{AB} = 1196.34 \quad N$

Consider Cylinder B,
Applying conditions Of equilibrium,

$\sum H = 0;$

$\therefore F_{AB} \cos 15 - R_2 \cos 45 - P \cos 45 = 0$

$\therefore R_2 = \dfrac{F_{AB} \cos 15 - P \cos 45}{\cos 45}$

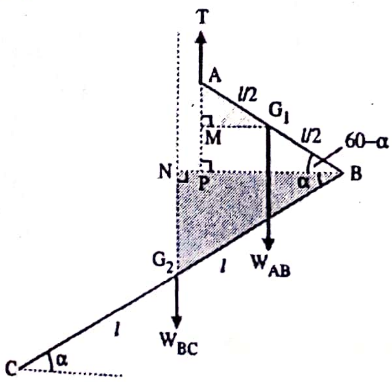
$\sum V = 0;$

$\therefore -F_{AB} \sin 15 - P \sin 45 + R_2 \sin 45 = 490.5$

$\therefore P = \dfrac{F_{AB}\left[\cos 15 . \tan 45 - \sin 15\right] - 490.5}{2 \sin 45}$

$\therefore P = 253.78 N$

P:11:-



$BN = l \times \cos \alpha$

$BP = l \times \cos(60 - \alpha)$

$NP = BN - BP$

$= l \times \cos \alpha - l \times \cos(60 - \alpha)$

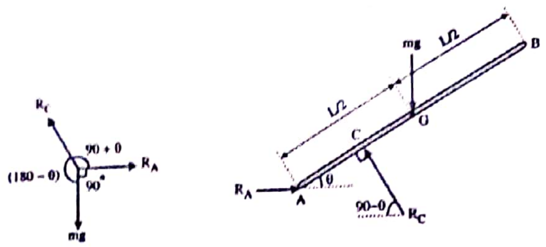
$= l\left[\cos \alpha - \cos(60 - \alpha)\right]$

Using the condition of equilibrium

$\Sigma M_A = 0$

$\tan \alpha = \dfrac{75}{216 \cdot 51}$ $\qquad \alpha = 19 \cdot 11°$

P:12:-



$\cos\theta = \frac{a}{AC}; \therefore AC = a\sec\theta$

Applying Conditions of Equilibrium

$\sum V = 0;$

$\therefore R_C \cdot \cos\theta - W - F = 0$

$\therefore R_C = \frac{W+F}{\cos\theta}$

$\sum M_A = 0;$

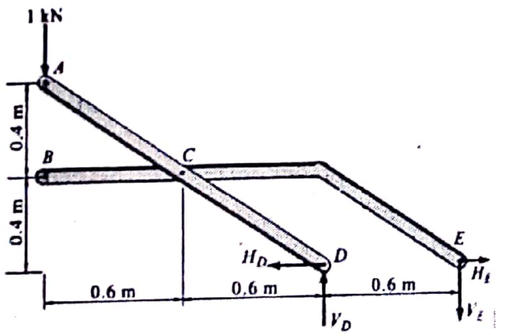
$-R_C \times a\sec\theta + W\left(\frac{l}{2}\cos\theta\right) + F\left(l\cos\theta\right) = 0$

$\therefore \left(\frac{W+F}{\cos\theta}\right) \times a\sec\theta = l\cos\theta\left[\frac{W+2F}{2}\right]$

$\therefore a(W+F) = l\cos^3\theta\left[\frac{W+2F}{2}\right]$

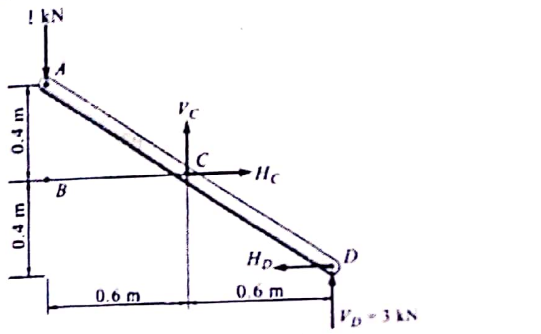
$\therefore \theta = \cos^{-1}\left[\frac{2a(W+F)}{l(W+2F)}\right]^{\frac{1}{3}}$

P:13:-



Consider the F.B.D. of entire frame

$V_E = 2$ kN (↓); $V_D = 3$ kN (↑); $H_E - H_D = 0$ ... (I)



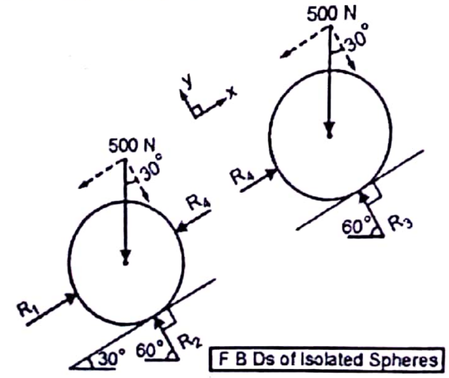
Consider the F.B.D. of *ACD*

$H_D = 6$kN (←); $V_C = 2$kN(↓); $H_C = 6$kN (→)

From (I)

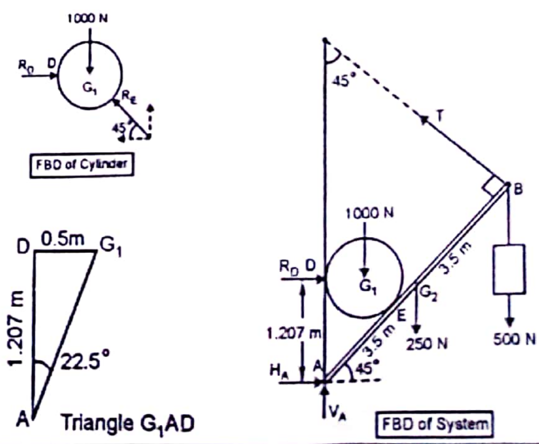
$H_E - H_D = 0;\ H_E = 6$kN (→)

P:14:- R_3= 433, R_4= 250; R_2=433;R_1=500



F B Ds of Isolated Spheres

P:15:- R_E=1414.2N; R_D=1000N; T=685.8N



FBD of Cylinder

Triangle G_1AD

FBD of System

4. Friction

P:1:-

For block B:-

$\sum F_x = 0 \rightarrow -0.25N_3 - 0.25N_2 \cos 75 + N_2 \cos 15 = 0$ $\therefore 0.25N_3 = 0.9N_2$ $\therefore N_3 = 3.6N_2$

$\sum F_y = 0 \rightarrow -1000 - 0.25N_2 \sin 75 + N_3 - N_2 \sin 15 = 0$

Substituting the value of the N3 in the above equation, we get $N_2 = 322.61N$

For wedge A:

$\sum F_x = 0 \rightarrow N_1 + 0.25N_2 \cos 75 - N_2 \cos 15 = 0$ $\therefore N_1 = 0.9N_2 = (322.61)N_2 = 290.35N$

$\sum F_y = 0 \rightarrow 0.25N_2 \sin 75 + 0.25N_1 + N_2 \sin 15 - P = 0$ $\therefore P = 233.89N$

P:2:-

Step I : (a) All casters are locked. motion is impending consider the F.B.D of as shown in Fig.

Applying the equation of equilibrium i.e. $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Note : Here F_{max} acts at both the casters.

$\Sigma F_x = P - \mu N_A - \mu N_B$ .. [by sign convention $\xrightarrow{+}$ $\xleftarrow{-}$]

$0 = P - \mu N_A - \mu N_B$ $\therefore$ $P = \mu(N_A + N_B)$ ... (i)

$\Sigma F_y = N_A + N_B - W$ .. [by sign convention $\overset{+}{\uparrow}\overset{-}{\downarrow}$]

$0 = N_A + N_B - W$

$0 = N_A + N_B - 60 \times 9.81$ ... [since W = (60 × 9.81) N] ... (ii)

$\therefore$ $N_A + N_B = 589$

From (i). $P = \mu(N_A + N_B)$

$P = 0.30(589)$ ... [$\mu = 0.30$ and $N_A + N_B = 589$ from (ii)] ... **Ans.**

$\therefore$ $P = 176.6\ N\ (\rightarrow)$

Check that cabinet does not tip. We take moment about B.

$\therefore$ $\Sigma M_B = P \times 0.8 - (60 \times 9.81) \times 0.3 + N_A \times 0.6$ .. [by sign convention]

$0 = 176.6 \times 0.8 - (60 \times 9.81) \times 0.3 + 0.6\ N_A$ ... [since $\Sigma M_B = 0$, equation of equilibrium]

$\therefore$ $N_A = 58.9\ N$ Since N_A is positive; and hence contact at A remains hence no tipping.

Step II : (b) Casters locked at B and free at A :

Assume motion impends: Hence F_{max} acts at B and since the point A is smooth no friction at A. Consider F.B.D as shown in Fig.

step-III

Applying the equation of equilibrium i.e. $\Sigma F_x = 0$; $\Sigma F_y = 0$ and $\Sigma M = 0$

$\Sigma F_x = P - \mu N_B$ ... [by sign convention $\xrightarrow{+}$ $\xleftarrow{-}$]

$0 = P - 0.3\ N_B$ $\therefore$ $N_B = 3.33\ P$ ... (iii)

$\Sigma M_A = P \times 0.8 + (60 \times 9.81) \times 0.3 - N_B \times 0.6$ ... [by sign convention]

$0 = 0.8\ P + (60 \times 9.81) \times 0.3 - 3.33\ P \times 0.6$ ... [since $N_B = 0.3\ P$ from (iii)] ... **Ans.**

$\therefore$ $P = 147.3\ N\ (\rightarrow)$

Step III : (c) Casters locked at A and free at B :

Assume that motion impends $F_{max} = \mu N_A$ acts at A and since B is free no frictional force at B. Consider the F.B.D. of as shown in Fig.

Applying the equation of equilibrium i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$:

$$\Sigma F_x = P - \mu N_A \quad \text{... [by sign convention } \underset{\leftarrow -}{\rightarrow +} \text{]}$$

$$0 = P - 0.3\, N_A \quad \text{... [since } \mu = 0.3 \text{ (given)]}$$

$$\therefore \quad N_A = 3.33\ P \qquad \text{... (iv)}$$

$$\Sigma M_A = P \times 0.8 - (60 \times 9.81) \times 0.3 + N_A \times 0.6 \quad \text{... [by sign convention]}$$

$$0 = P \times 0.8 - (60 \times 9.81) \times 0.3 + 3.33\ P \times 0.6 \quad \text{... [since } N_A = 3.33\ P \text{ from (iv)]}$$

$$\therefore \quad P = 63.1\ \text{N} \ (\rightarrow) \qquad \text{... Ans.}$$

P:3:-

Soln. :

Step I : F.B.D. of block A and wedge C are as shown in Fig. Ex. 5.33(a).

For slope 1:10, $\theta = \tan^{-1}\left(\frac{1}{10}\right) = 5.71^\circ$ with horizontal.

Step II : For block A : Refer to Fig. Ex. 5.33(a)

$$\Sigma F_y = 0 = 2\, N_A \cos 5.71^\circ - 100 - 2\,(0.2\, N_A \cos 5.71^\circ) \quad \text{...[by sign convention } \overset{+\ -}{\uparrow \downarrow} \text{]}$$

$$\therefore \quad N_A = 51.282\ \text{kN} \qquad \text{...(i)}$$

Step III : For wedge C : Refer to Fig. Ex. 5.33(a)

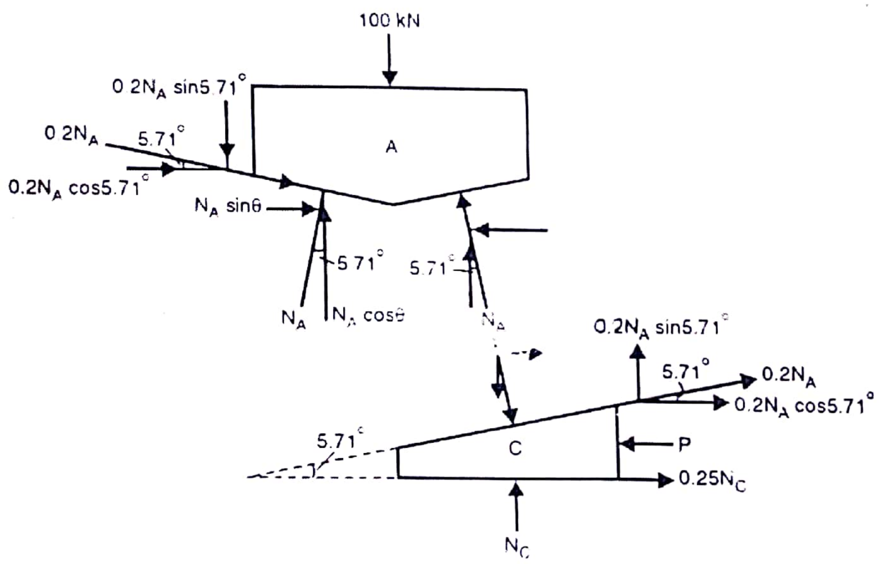


Fig. Ex. 5.33(a) : F.B.D. of block A and wedge C

$$\Sigma F_y = 0 = N_A \cos 5.71^\circ + N_C + 0.2\, N_A \sin 5.71^\circ \quad \text{...[by sign convention } \overset{+\ -}{\uparrow \downarrow} \text{]}$$

$$0 = 51.282 \cos 5.71^\circ + N_C + 0.2 \times 51.282 \sin 5.71^\circ \quad \text{...[} N_A = 51.282 \text{ kN from (i)]}$$

$$\therefore \quad N_C = 50\ \text{kN} \qquad \text{...(ii)}$$

$$\Sigma F_x = 0 = -P + 0.25\, N_C + N_A \sin 5.71^\circ + 0.2\, N_A \cos 5.71^\circ \quad \text{...[by sign convention } \underset{\leftarrow -}{\rightarrow +} \text{]}$$

$$0 = -P + 0.25 \times 50 + 51.282 \sin 5.71^\circ + 0.2 \times 51.282 \cos 5.71^\circ$$

...[since $N_A = 51.282$ kN and $N_C = 50$ kN from (i) and (ii)]

$$\therefore \quad P = 27.885\ \text{kN} \qquad \text{...Ans.}$$

P:4:-

Apply conditions of equilibrium.

$\Sigma F_x = 0 \quad \mu_s \cdot R_A + \mu_s \cdot R_B \cdot \cos 67.38^\circ - R_B \cos 22.62^\circ = 0$

$$\mu_s \cdot R_A = R_B (0.923 - 0.384 \mu_s) \quad \ldots(1)$$

$\Sigma F_y = 0 \quad R_A + \mu_s \cdot R_B \sin 67.38^\circ + R_B \sin 22.62^\circ - W = 0$

$$R_A = W - 0.923 \mu_s \cdot R_B - 0.384 R_B$$

$$R_A = W - R_B (0.923 \mu_s + 0.384) \quad \ldots(2)$$

Substituting this value in Equation (1)

$$\mu_s [W - R_B (0.923 \mu_s + 0.384)] = R_B (0.923 - 0.384 \mu_s)$$

$$\mu_s W = \mu_s R_B (0.923 \mu_s + 0.384) + R_B (0.923 - 0.384 \mu_s)$$

$$= R_B [0.923 \mu_s^2 + 0.384 \mu_s + 0.923 - 0.384 \mu_s]$$

$$= R_B [0.923 \mu_s^2 + 0.923]$$

$$\mu_s \cdot W = 0.923 R_B (1 + \mu_s^2)$$

$$\therefore W = 0.923 R_B \frac{(1 + \mu_s^2)}{\mu_s} \quad \ldots(3)$$

$\Sigma M_A = 0$

$$R_B \times 6.5 - W (1.25) = 0$$

$$6.5 R_B = 1.25 W$$

$$\therefore W = 5.2 R_B \quad \ldots(4)$$

Equating Equation (3) and Equation (4)

$$0.923 R_B \frac{(1 + \mu_s^2)}{\mu_s} = 5.2 R_B$$

$$\therefore \frac{(1 + \mu_s^2)}{\mu_s} = 5.63$$

$$\therefore \mu_s^2 - 5.63 \mu_s + 1 = 0$$

Solving above quadratic equation

We get, $\mu_s = 0.183$ and 5.446

But $\mu_s < 1$ $\quad \therefore \mu_s = $ **0.183**

5. Centre Of Gravity

P:1:

No.	Shape	Area (cm^2)	$\bar{x}$ (cm)	$\bar{y}$ (cm)
1.	Rectangle AGEC	(18)(22)=396	22/2=11	18/2=9
2.	Triangle AGF	$-\left(\frac{1}{2}\right)\times 18\times 15 = -135$	$\frac{1}{3}\times 15 = 5$	$\frac{1}{3}\times 18 = 6$
3.	Rectangle BCDP	-(10)(10)= -100	12+(10/2)=17	8+(10/2)=13
4.	Arc EFG	$\frac{\pi\times 10^2}{4} = 78.5$	$12+\left(\frac{4\times 10}{3\pi}\right)=16.25$	$8+\left(\frac{4\times 10}{3\pi}\right)=12.25$

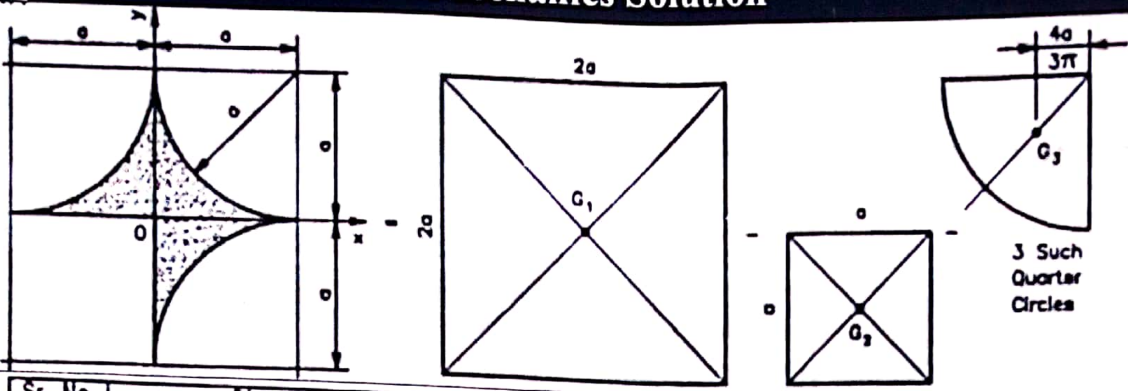
$$\therefore \bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = 13.59cm; \quad \therefore \bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3} = 10.08cm$$

P:3:

No.	Paper	Area (cm^2)	$\bar{x}$ (cm)	$\bar{y}$ (cm)
1.	ACED	(30) (10) = 300	10/2 = 5	30/2 = 15
2.	2 (CBE)	$2\times\left(\frac{1}{2}\right)\times 30\times 30 = 900$	$10+\left(\frac{1}{3}\times 30\right) = 20$	$\frac{2}{3}\times 30 = 20$

Now, $\bar{x} = 16.25cm; \quad \bar{y} = 18.75cm$

P:5– Find the co-ordinates of the centroid of shaded area shown in the Fig.



Sr. No.	Shape	Area	Nature	$\bar{x}$	$\bar{y}$
1.	Large square	$+4a^2$	+ve	+ 0	+ 0
2.	Small square	$-a^2$	−ve	$-\frac{a}{2}$	$-\frac{a}{2}$
3.	1^{st} Quadrant's $\frac{1}{4}$ circle	$-\frac{\pi a^2}{4}$	−ve	$+\left[a-\frac{4a}{3\pi}\right]$	$+\left[a-\frac{4a}{3\pi}\right]$
4.	2^{nd} Quadrant's $\frac{1}{4}$ circle	$-\frac{\pi a^2}{4}$	−ve	$-\left[a-\frac{4a}{3\pi}\right]$	$+\left[a-\frac{4a}{3\pi}\right]$
5.	4^{th} Quadrant's $\frac{1}{4}$ circle	$-\frac{\pi a^2}{4}$	−ve	$+\left[a-\frac{4a}{3\pi}\right]$	$-\left[a-\frac{4a}{3\pi}\right]$

$\bar{x} = \bar{y} = +(0.07446)a$

P:6:-

Step :1:- The remaining part of the plate remains in equilibrium when suspended from apex E of the tringle; ∴ CG of the remaining part of the plate must lie at the apex only. Let h be the height of triangle. ∴ $\bar{Y} = h\ w.r.t.$ base AB.

Step :2:- Split up the given fig. in to square plate say 1. And triangle say 2. As shown in fig.
Let A_1 = Area of square plate = $(1)^2 = 1m^2$
A_2 = Area of triangle = ½ x 1 x h=h/2 m^2
Y_1 = ½ = 0.5m; Y_2 = 1/3 x h.

We know, $\bar{Y} = \frac{A_1Y_1 - A_2Y_2}{A_1 - A_2}$ $\therefore h = \frac{1\times 0.5 - \frac{h}{2}\times\frac{h}{3}}{1-\frac{h}{2}}$ $\therefore h = \frac{0.5-0.1667h^2}{1-0.5h}$ $\therefore h(1-0.5h) = 0.5 - 0.1667h^2$ $\therefore 0.333h^2 - h + 0.5 = 0$ (h=2.39m or h= 0.639m)
Hence h=2.39m is not possible. H=0.639m.
Area of triangle to be removed = ½ (1) (0.639) = 0.319m^2

P:7:-

$$\bar{y} = \frac{A_1Y_1 - A_2Y_2}{A_1 - A_2} = \frac{200000\times 500 - (120h)(0.5h)}{200000 - 120h}$$

$\therefore 60h^2 - 200000h + 10^8 = 0$ $\therefore h = 612.57$ mm.

Now, $\bar{y} = \frac{10^8 - 60h^2}{200000 - 120h}$

Now for $\bar{y}$ to be maximum, $\frac{d\bar{y}}{dh} = 0$

$$\therefore \frac{d\bar{y}}{dh} = \frac{(200000-120h)(-120h) - (10^8 - 60h^2)(-120)}{200000 - 120h}$$

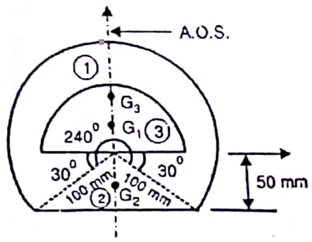
$\therefore \frac{d\bar{y}}{dh} = 0$ $\therefore h = 612.57$ mm.

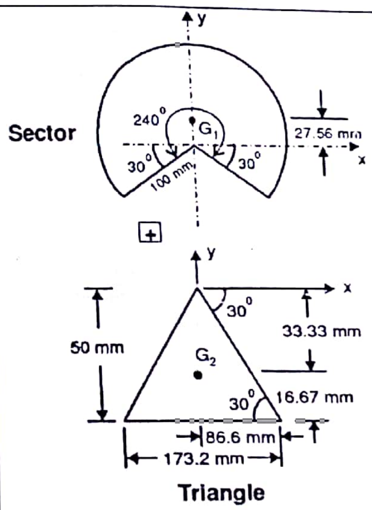
P:8:

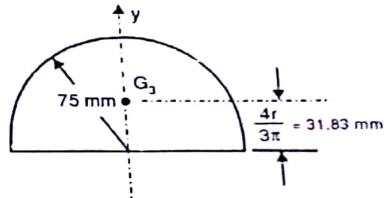
No.	Shape	Area (cm^2)	$\bar{x}$ (x co-ordinate) (cm)
1.	Semicircle A_1	$\frac{\pi \times 15^2}{2} = 353.43$	$-\left(\frac{4 \times 15}{3\pi}\right) = -(6.366)cm$
2.	Rectangle A_2	$100 \times 30 = 3000$	$100/2 = 50$
3.	Triangle A_3	$\frac{1}{2} \times 100 \times 10 = 500$	$(2/3)(100) = 66.67$
4.	Sector A_4	$R^2\alpha = (103.08)^2\left(\frac{14 \times \pi}{180}\right)$	$\frac{2R\sin\alpha}{3\alpha} = 68.04$
5.	Triangle A_5	$\frac{1}{2} \times 100 \times 50 = 2500$	$(2/3)(100) = 66.67$

$$\therefore \bar{x} = \frac{A_1x_1 + A_2x_2 + 2(A_3x_3) + A_4x_4 - 2A_5x_5}{A_1 + A_2 + 2A_3 + A_4 - 2A_5} = 29.61cm; \therefore \bar{y} = 0cm$$

P:9:







PART	AREA A_i, mm²	Y_i mm	A_iY_i mm³
1. SECTOR	20944	27.56	577217
2. TRIANGLE	½ × 173.2 × 50 = 4330	– 33.33	– 144319
3. SEMI-CIRCLE	– ½ π (75)² = – 8835.7	31.83	– 281241
	ΣA_i = 16438.3		$\Sigma A_i.Y_i$ = 151657

Using $\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{151657}{16438.3} = 9.22$ mm $\quad \therefore (\bar{X}, \bar{Y}) = (0, 9.22)$ mm .... **Ans.**

6. Space forces

Mechanics Solution

P:1

$\bar{F}_{AB} = 100\left[\frac{4i - 4k}{5.66}\right] = 70.7i - 70.7kN.$

$\therefore \bar{F}_{AC} = 120\left[\frac{4}{6}i + \frac{2}{6}j - \frac{4}{6}k\right] = (80i + 40j + 80k)N.$

$\bar{R} = \bar{F}_{AB} + \bar{F}_{AC} = (70.7i - 70.7k) + (80i + 40j - 80k)$

$= (150.7i + 40j - 150.7k)N.$

Magnitude R of $\bar{R}$.

$= \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2} = 217N.$

P:-2

$B \equiv (0,4,6); D \equiv (10,0,6); C \equiv (10,4,6).$

Let $\bar{F}_1 = 30k, \bar{F}_2 = -70j$ and $\bar{F}_3 = 60i$

$(\overline{m_0})_1 = 120i - j(0) + k(0)$

$(\overline{m_0})_2 = i(+420) - j(0) + k(-700)$

$(\overline{m_0})_3 = i(0) - j(-360) + k(-240)$

Hence, movement of F_1 *about x, y and z axes are* $(120KN - m; 0$ *and* $0)$.

$\rightarrow$ *movement of* F_2 *about x, y and z axes are* $(420KN - m; 0$ *and* $-700kN - m)$.

$\rightarrow$ *movement of* F_3 *about x, y and z axes are* $(0; 0360KN - m$ *and* $240kN - m)$.

P:3 A(0,48,0) ;B (16,0,12); C (16,0,-24); D(-14,0,0)

$$\bar{T}_{AC} = T_{AC}\cdot\hat{e}_{AC}$$
$$= 20\left[\frac{16\mathbf{i} - 48\mathbf{j} - 24\mathbf{k}}{\sqrt{16^2 + 48^2 + 24^2}}\right]$$
$$\therefore \bar{T}_{AC} = 5.714\,\mathbf{i} - 17.143\,\mathbf{j} - 8.57\,\mathbf{k}\ \text{kN}$$

$$\bar{T}_{AB} = T_{AB}\cdot\hat{e}_{AB}$$
$$= \bar{T}_{AB}\left[\frac{16\mathbf{i} - 48\mathbf{j} + 12\mathbf{k}}{\sqrt{16^2 + 48^2 + 12^2}}\right]$$
$$\therefore \bar{T}_{AB} = \bar{T}_{AB}(0.3077\,\mathbf{i} - 0.923\,\mathbf{j} + 0.2307\,\mathbf{k})\ \text{kN}$$

$$\bar{T}_{AD} = T_{AD}\cdot\hat{e}_{AD}$$
$$= \bar{T}_{AD}\left[\frac{-14\mathbf{i} - 48\mathbf{j}}{\sqrt{14^2 + 48^2}}\right]$$
$$\therefore \bar{T}_{AD} = \bar{T}_{AD}(-0.28\,\mathbf{i} - 0.96\,\mathbf{j})\ \text{kN}$$

The resultant force $\bar{R} = \bar{T}_{AC} + \bar{T}_{AB} + \bar{T}_{AD}$

Also since the resultant force is vertical, i.e. along y axis, implies that $\Sigma F_y = R$, $\Sigma F_x = 0$ and $\Sigma F_z = 0$,

Using $\Sigma F_x = 0$

$$5.714 + 0.3077\, T_{AB} - 0.28\, T_{AD} = 0 \quad \text{......... (1)}$$

Using $\Sigma F_z = 0$

$$-8.57 + 0.2307\, T_{AB} = 0 \quad \text{......... (2)}$$

Solving equations (1) and (2), we get

$T_{AB} = 37.15$ kN and $T_{AD} = 61.23$ kN .... **Ans.**

Using $\Sigma F_y = R$

$$-17.143 - 0.923\, T_{AB} - 0.96\, T_{AD} = R$$

$\therefore$ $R = -17.143 - 0.923(37.15) - 0.96(61.23)$

$\therefore$ $R = -110.2$ kN Or $\bar{R} = -110.2\,\mathbf{j}$ kN .... **Ans.**

7.A. KINEMATICS OF PARTICLES

P:1:-

Initial Velocity = 80 kmph = 22.22 m/s; Time = 10s

s = distance = 200 m

$$s = ut + \frac{1}{2}at^2$$

$$200 = 22.22 \times 10 + \frac{1}{2} \times a \times 10^2; \therefore a = -0.444 m/s^2$$

$$v = u + at$$

$$\therefore v = 64 kmph$$

P:2:-

Car A

$$d = 3t + \frac{1}{2}(1.2)t^2 \text{................}(1)$$

$$d + 150 = 30t + 1/2(-0.6)t^2$$

$$\therefore t = 7.36 \sec, 22.64 \sec$$

Thus, the cars cross each other two times (First car B overtakes A and then A overtakes B)

For t = 7.36 sec, by eqn(1)

d = 54.58 m

For t = 22.64 sec, by eqn(1),

we get

d = 375.46 m

P:3

For car A

$$1000 = 30(40) + \frac{1}{2}a_A(40)^2$$

$$\therefore a_A = -0.25 m/s^2 = 0.25 m/s^2 (\leftarrow)$$

For car B

$$1000 = 17.5(42) + \frac{1}{2}a_B(42)^2$$

$$\therefore a_B = 0.3 m/s^2 = 0.3 m/s^2 (\leftarrow)$$

Now, let 'd' be the distance travelled by car A in time 't' just before crossing.

Therefore, corresponding distance travelled by car B will be (1000-d) in time 't' only

For Car A:-

$$d = 30t + \frac{1}{2}(-0.25)t^2$$

$$\therefore d = 30t - 0.125t^2$$

For Car B:-

$$(1000 - d) = 17.5t + \frac{1}{2}(0.3)t^2$$

$$\therefore d = 1000 - 17.5t - 0.15t^2$$

Solving we get, t = 20.82 sec, d = 570.4 m

$v_B = 17.5 + (0.3)(20.82) = 23.75 m/s = 85.48 kmph$

P:4 Case I:- O to A $d = 0 + \frac{1}{2}a(t_1)^2$ Case II:- O to B $(d+100) = 0 + \frac{1}{2}a(t_1+10)^2$ $\therefore (d+100) = \frac{1}{2}a(t_1^2 + 20t_1 + 100)$ $\therefore 10at_1 + 50a - 100 = 0$ Case III:- O to C	$(d+200) = 0 + \frac{1}{2}a(t_1+18)^2$ $\therefore (d+100) = \frac{1}{2}a(t_1^2 + 36t_1 + 324)$ $\therefore 18at_1 + 162a - 200 = 0$ $a = 0.28 m/s^2$ $t_1 = 31 sec$ $d = 134.54 m$ Consider motion (O to A) $\therefore v_A = u + at = 0 + 0.28(31) = 8.68 m/s$ (O to B) $\therefore v_B = 0 + 0.28(31+10) = 11.48 m/s$
P:5:- Car A $s_A = ut + \frac{1}{2}at^2$ $\therefore s_A = 0 + \frac{1}{2}(2)(t)^2$ $\therefore s_A = t^2$	Car B $\therefore s_B = 0 + \frac{1}{2}(3.6)(t-3)^2$ $\therefore s_B = 1.8(t^2 - 6t + 9)$ $\therefore s_B = 1.8t^2 - 10.8t + 16.2$
$t^2 = 1.8t^2 - 10.8t + 16.2$ $\therefore t = 11.78 sec \quad and \quad 1.72 sec$ But t = 1.72 sec is less than 3 sec so it is not valid.	$\therefore s_A = 138.77 m$ $v_A = u_A + a_A t_A = 0 + 2(11.78) = 23.56 m/s$ $v_B = u_B + a_B t_B = 0 + 3.6(11.78 - 3) = 31.61 m/s$
P:6 :- Car A $s = ut + \frac{1}{2}at^2$ $\therefore 500 = 0 + \frac{1}{2}a_A(t)^2$ $\therefore a_A t^2 = 1000$	Car B $\therefore 500 = 0 + \frac{1}{2}(6)(t-6)^2$ $\therefore (t-6)^2 = 500/3$ $\therefore t = 18.91 sec$

$a_A(18.91)^2 = 1000; \therefore a_A = 2.796 m/s^2$

$v_A = 0 + 2.8(18.91) = 52.94 m/s$

$v_B = 0 + 6(18.91 - 6) = 77.46 m/s$

P:7:- Drops / sec = 5; Time taken for one drop = 1/5 = 0.2 sec ___ Time interval = 0.2 sec

For Drop 'B' Initial velocity = u = 0 $s = -s_B$ $v = -3 m/s$ $v = u + at$ $\therefore -3 = 0 + (-9.81)(t)$ $\therefore t = 0.31 sec$ $s = ut + 1/2 at^2$ $-s_B = (0) + 0.5(-9.81)(0.31)^2$ $\therefore s_B = 0.47 m$	For Drop 'A' Initial velocity = u = 0 $s = -s_A$ $a = -9.81 m/s^2$ $s = ut + 1/2 at^2$ $-s_A = (0) + 0.5(-9.81)(0.31 - time \quad interval)^2$ $\therefore s_A = 0.06 m$ Vertical Separation Between two Drops = s_B - s_A = 0.41 m

P:8:-

Let t_1 = time taken by stone to reach water level t_2 = time taken by sound from water level to G.L.

$\therefore t_1 + t_2 = 3.3\sec$ ....(1)

Motion Of stone :- A to B

$s = ut + \frac{1}{2}at^2$

$\therefore -d = 0 + 0.5(-9.81)(t_1)^2$

$\therefore d = 4.905t_1^2$ ....(2)

Motion Of Sound :- B to A

Distance = (Velocity)(Time)

$\therefore d = 350t_2$

$\therefore d = 350(3.3 - t_1)$..........(3)

Equating(2)*and*(3),

$\therefore t_1 = 3.16\sec$

$\therefore d = 48.93m$

P:9:-

Case I:- For stone 1 (Motion from A to B)

$s = ut + \frac{1}{2}at^2$

$\therefore -120 = 25t - \frac{1}{2}(9.8)(t^2)$

$\therefore 4.9t^2 - 25t - 120 = 0$

$\therefore t = 8.12\sec$

Case II:- For stone 2 (Motion from A to B)

$s = ut + \frac{1}{2}at^2$

$\therefore -120 = -25t - \frac{1}{2}(9.8)(t^2)$

$\therefore 4.9t^2 + 25t - 120 = 0$

$\therefore t = 3.016\sec$

Now, let h be the height at which stone 1 should be released from rest so that both stone may reach the ground simultaneously

$s = ut - \frac{1}{2}gt^2$

$\therefore -h = 0 - \frac{1}{2}(9.8)(3.016)^2$

$\therefore h = 44.57m$

P:10:-Body A

Initial velocity = 40 m/s

$s = -180$

$time = t_A$

$s = ut + \frac{1}{2}at^2$

$\therefore -180 = 40t_A + \frac{1}{2}(-9.81)t_A{}^2$

$\therefore t_A = 11.38\sec$

$v = u + at_A$

$\therefore v = 40 + (-9.81)(11.38)$

$\therefore v = -71.64m/s(\downarrow)$

Body B

Initial velocity = 0 m/s

$s = -180$

$time = t_B \sec$

$s = ut + \frac{1}{2}at^2$

$\therefore -180 = 0 + \frac{1}{2}(-9.81)t_B{}^2$

$\therefore t_B = 6.06\sec$

$v = u + at_B$

$\therefore v = 0 + (-9.81)(6.06)$

$\therefore v = -59.45m/s(\downarrow)$

Total time = 11.38 – 6.06 = 5.32 sec

P:11-

The stone 1 is dropped from top of the tower, when it reaches 12 m below the top(at point B) its velocity is given by,

$v_B^2 = u_1^2 - 2gs = 0 - 2\times 9.8(-12) = 15.34 m/s (\downarrow)$

Let 't' be the time taken stone 2 to reach the ground (C to D)

Therefore time taken by stone 1 from B to D must be 't'sec

For stone 1:- (B toD)

$$s = ut + \frac{1}{2}at^2$$

$$\therefore -(H-12) = -15.34t - \frac{1}{2}(9.8)(t^2)$$

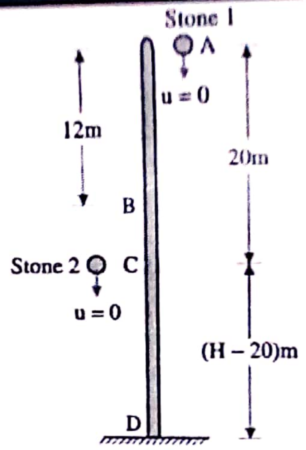
$$\therefore H = 15.34t + 4.9t^2 + 12 \text{.....................(1)}$$

For stone 2:- (C to D)

$$s = ut + \frac{1}{2}at^2$$

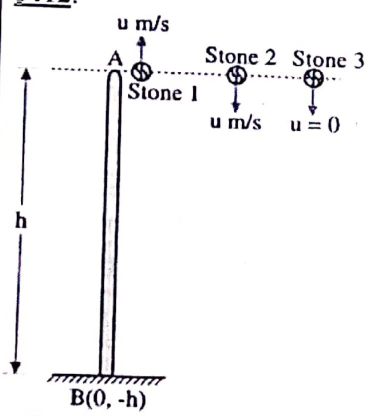
$$\therefore -(H-20) = 0 - \frac{1}{2}(9.8)(t^2)$$

$$\therefore H = 20 + 4.9t^2 \text{.....................(2)}$$



Equating eqn(1) and (2) , we get,
t = 0.52 sec
H = 21.33 m

P:12:-



Stone No.1

Initial velocity = tu

$s = -h$

$time = t_1$

$s = ut + 1/2at^2$

$-h = ut_1 + 0.5(-9.81)(t_1)^2$

$$\therefore -\frac{h}{t_1} = u - \frac{9.81t_1}{2} \text{............(1)}$$

Stone No.2

Initial velocity = - u

$s = -h$

$time = t_2$

$s = ut + 1/2at^2$

$-h = ut_2 + 0.5(-9.81)(t_2)^2$

$$\therefore -\frac{h}{t_2} = -u - \frac{9.81t_2}{2} \text{............(2)}$$

Adding (1) and (2), we get,

$h = 4.905t_1t_2$................(3)

For a simply dropped stone,

$u = 0, s = -h, time = t$

$$s = ut + \frac{1}{2}at^2$$

$$\therefore -h = 0 + \frac{1}{2}(-9.81)t^2$$

$\therefore h = 4.905t^2$..............(4)

Equating (3) and (4), we get,

$\therefore t^2 = t_1t_2$

$\therefore t = \sqrt{t_1t_2}$

Hence proved

P:13

(i) Velocity of stone before glass pan is broken = V_1

Velocity of stone after glass pan is broken = V_2

(ii) Motion of stone from rest to glass pan before braking

$t = 5$ sec; $u = 0$; $v = v_1$; $v = u - gt \rightarrow v_1 = 0 - 9.81 \times 5 \therefore v_1 = -49.05$ m/s

(iii) Loss of 30% of velocity after breaking the glass pan

$v_2 = -34.335 \frac{m}{s} \therefore v_2 = -(49.05) \times (0.7)$

(iv) Motion of stone after breaking of glass pan for next 1 sec.

$u = v_2$; $t = 1$ sec; displacement = h; $h = ut - \frac{1}{2}gt^2 = 34.335 \times 1 - \frac{1}{2} \times 9.81 \times 1^2$; $h = -39.24$m

P:14:-

Let the time interval be t.

First Drop	Second Drop
$s_1 = -16m$ $Time = 4t$ $s = ut + 1/2at^2$ $-16 = (0)(4t) + 0.5(-9.81)(4t)^2$ $\therefore t = 0.45 sec$	$Time = 3t$ $s = ut + 1/2at^2$ $s_2 = (0)(3t) + 0.5(-9.81)(3t)^2$ $\therefore s_2 = -8.94m$
Third drop	**Fourth Drop**
$Time = 2t$ $s = ut + 1/2at^2$ $s_3 = (0) + 0.5(-9.81)(2t)^2$ $\therefore s_3 = -3.97m$	$Time = t$ $s = ut + 1/2at^2$ $s_4 = (0)(t) + 0.5(-9.81)(t)^2$ $\therefore s_4 = -1m$

Distance Between :-

I st and II nd Drop = 16 – 8.94 = 7.06 m

II nd and III rd Drop = 8.94 – 3.97 = 4.97 m

III rd and IV th Drop = 3.97 – 1 = 2.97 m

IV th and V th Drop = 1 m

7.B. KINEMATICS OF PARTICLES

P:1:-

$$\frac{dx}{dt} = v = Ap\cos(pt+\phi)$$

At t = 0,

$x_0 = A\sin\phi;\quad v_0 = Ap\cos\phi$

$\therefore \frac{x_0}{v_0} = \frac{1}{p}\tan\phi; \therefore \tan\phi = \frac{x_0 p}{v_0}$

$\therefore \frac{v_0}{p} = \frac{x_0}{\tan\phi}$

$R.H.S = x_0\sqrt{1+\cot^2\phi} = x_0 \operatorname{cosec}\phi = \frac{x_0}{\sin\phi} = A = L.H.S$

P:2:-

$let \quad \bar{r} = x\hat{i} + y\hat{j} = x\hat{i} + 4x^2\hat{j}$

$\therefore v = \frac{d\bar{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \left(8x\frac{dx}{dt}\right)\hat{j}; \frac{dx}{dt} = 20m/s$

$\therefore \bar{v} = 20\hat{i} + 160x\hat{j}$

$\therefore \bar{a} = 160\frac{dx}{dt}\hat{j} = 3200\hat{j}$

$dx = 20dt$

$\therefore x = 20t + c_1$

$at \quad t = 0, x = 0; \therefore c_1 = 0$

$\therefore x = 20t; y = 4(20t)^2$

$\therefore r = (20t)\hat{i} + (1600t^2)\hat{j}$

P:3:

$x = x_0\left[2e^{-kt} - e^{-2kt}\right]$

$\frac{dx}{dt} = v = x_0\left[-2ke^{-kt} + 2ke^{-2kt}\right]$

$\frac{dv}{dt} = a = x_0\left[2k^2e^{-kt} - 4k^2e^{-2kt}\right]$

For max velocity : a = 0

$\therefore 0 = x_0\left[2k^2e^{-kt} - 4k^2e^{-2kt}\right]$

$\therefore x_0 2k^2e^{-kt}\left[1 - 2e^{-kt}\right] = 0$

$\therefore 1 - 2e^{-kt} = 0; \therefore e^{-kt} = \frac{1}{2}; \therefore \frac{1}{e^{kt}} = \frac{1}{2}; \therefore e^{kt} = 2$

$\therefore kt = \ln(2); \therefore t = \frac{\ln(2)}{k}$

$\therefore v_{max} = x_0\left[-2k\left(\frac{1}{2}\right) + 2k\left(\frac{1}{4}\right)\right] = x_0\left[-k + \frac{k}{2}\right] = \frac{-kx_0}{2}$

P:4:-

(I) $a = 100 - 4v^2$

$$\frac{dv}{dt} = 4\left(25 - v^2\right)$$

$$\therefore \frac{dv}{25 - v^2} \quad 4 \cdot dt$$

Integrating both side, we get,

$$\frac{1}{2\times 5}\left[\log_e\left[\frac{5+v}{5-v}\right]\right]_1^3 = 4\left[t\right]_{t_1}^{t_2}$$

$$\frac{1}{10}\left[\log_e\left[\frac{5+3}{5-3}\right] - \log_e\left[\frac{5+1}{5-1}\right]\right] = 4\left[t_2 - t_1\right]$$

$$\therefore t_2 - t_1 = \frac{1}{40}\log_e\left[\frac{8}{3}\right]$$

time interval $= t_2 - t_1 = 0\cdot0245$ sec.

$$v\cdot\frac{dv}{ds} = 4\left[25 - v^2\right]$$

$$\therefore \frac{v}{5^2 - v^2}dv = 4\,ds.$$

Integrating both sides,

$$\frac{-1}{2}\left[\log\left(25 - v^2\right)\right]_1^3 = 4\left[S\right]_{S_1}^{S_2}$$

$$\therefore S_2 - S_1 = \frac{-1}{8}\left[\log\left(25 - 3^2\right) - \log\left(25 - 1^2\right)\right]$$

$$\therefore S_2 - S_1 = \log_e\left(\frac{16}{24}\right)$$

$$\therefore S_2 - S_1 = 0\cdot0506\,m.$$

(II) $a = 100 - 4v^2$

At $t = 0\cdot05$ sec.

$v = ?$

$$\frac{dv}{dt} = 4\left[25 - v^2\right]$$

$$\therefore \frac{dv}{5^2 - v^2} = 4\,dt$$

Integrating we get,

$$\int\frac{dv}{5^2 - v^2} = 4\int dt$$

$$\frac{1}{2\times 5}\log_e\left(\frac{5+v}{5-v}\right) = 4t + c$$

Assuming $t = 0$, $v = 0$; $\therefore c = 0$

$$\frac{1}{10}\log_e\left(\frac{5+v}{5-v}\right) = 4t$$

$$\therefore \frac{5+v}{5-v} = e^{40t}$$

$$\therefore \frac{5+v}{5-v} = e^{40(0\cdot05)} \qquad \therefore v = 3\cdot81\ m/sec.$$

P:5:-

(I) Given $x = kt$

$$\therefore t = \frac{x}{k}$$

$$y = kt\left(1 - \propto t\right)$$

$$y = k\left(x/k\right)\left[1 - \propto\left(x/k\right)\right]$$

$$y = x\left[1 - \frac{\propto x}{k}\right]$$

$$y(x) = x\left[1 - \frac{\propto x}{k}\right]$$

$$\therefore y(x) = x\frac{-\propto x^2}{k}$$

(II) $x = kt$

$\frac{dx}{dt} = k$

$\therefore v_x = k$

Further, $\frac{d^2x}{dt^2} = 0$

$\therefore a_x = 0$

$y = kt(1-\propto t)$

$\frac{dy}{dt} = k(1-2\propto t)$

$v_y = k(1-2\propto t)$

$\frac{d^2y}{dt^2} = -2k\propto$

$\therefore a_y = -2k\propto$

$\therefore v = \sqrt{(v_x)^2 + (v_y)^2}$

$= \sqrt{(k)^2 + [k(1-2\propto t)]^2}$

$v = k\sqrt{1+(1-2\propto t)^2}$

$\therefore a = \sqrt{(a_x)^2 + (a_y)^2}$

$= \sqrt{(0)^2 + (-2k\propto)^2}$

$a = 2k\propto$

P:6:-

$\therefore \text{slope} = \frac{0-10}{5-0} = -2, a = -2t+10$

(I) $a = -2t+10$

$\frac{dv}{dt} = -2t+10 \qquad \therefore dv = (-2t+10)dt$

$\int dv = \int(-2t+10)dt$

$\therefore v = \frac{-2t^2}{2} + 10t + c_1$

$v = -t^2 + 10t + c_1$

At rest, $t = 0$ & $v = 0$,

$c_1 = 0$

$\therefore v = -t^2 + 10t$

At $= t = 5$ sec

$a = -2(5) + 10$

$a = 0$

At $t = 5$ sec.

$v = -(5)^2 + 10(5)$

$v = 25m/\text{sec}$.

Integrating both sides, we get,

$\int ds = \int(-t^2 + 10t)dt$

$s = \frac{-t^3}{3} + 5t^2 + c_2$

At $t = 0, s = 0, c_2 = 0$

$s = \frac{-t^3}{3} + 5t^2$

At $t = 5$ sec.

$s = \frac{-5^3}{3} + 5(5)^2$

$s = 83\cdot33m.$

(II) For $v = 0$

$0 = -t^2 + 10t \therefore t = 10$ sec.

At $t = 10$ sec.

$s = -(10)^{3}/3 + 5(10)^2 = 166\cdot67m.$

P:7:-

(a) $v = \frac{dx}{dt} = 3t^2 - 12t - 15 = 0$

$\therefore t^2 - 4t - 5 = 0$

$\therefore t = 5, -1$

$\therefore t = 5\sec$

(b) $a = \frac{dv}{dt} = 6t - 12 = 6(t-2)$

$at \quad t = 5\sec,$

$x_5 = -60, Initial \quad position \quad x_0 = 40$

Distance travelled $= x_0 - x = 40 - (-60) = 100$

(c) $x_4 = -52; x_5 = -60; x_6 = -50$

Total Distance travelled $= |x_5 - x_4| + |x_6 - x_5| = 8 + 10 = 18m$

P:8 :-

$V^3 = 64S^2, \quad At \quad S = 8m; \quad V = 16m/s$

(b) $3V^2 \frac{dV}{dt} = 64(2S)\frac{dS}{dt}$

$\therefore a = \frac{128S}{3v}$

$At \quad S = 27m; \quad V = 36m/s$

$\therefore a = 32m/s^2$

(c) $V^3 = 64S^2$

$At \quad V = 9m/s; \quad S = 3.375m$

$\therefore a = 16m/s^2$

P:9:-

$t = \sqrt{x} + 3; \therefore \sqrt{x} = t - 3$

$\therefore x = (t-3)^2 = t^2 - 6t + 9$

Differentiating,

$\frac{dx}{dt} = 2t - 6; \therefore v = 2t - 6$

$when \quad v = 0; t = 3\sec$

$At \quad t = 3\sec; \quad x = 0$

Again diff. w.r.to t, we get,

$\frac{dy}{dt} = \frac{-ah}{x^2} \cdot \frac{dx}{dt}$

$\frac{d^2y}{dt^2} = -ah \cdot \left[\frac{1}{x^2} \cdot \frac{dx}{dt}\right]$

$\frac{d^2y}{dt^2} = -ah\left[\frac{1}{x^2}\frac{d^2x}{dt^2} + \frac{dx}{dt}(-2)^{-3} \times \frac{doc}{dt}\right]$

$\therefore \frac{d^2x}{dt^2} = a = 8 \cdot 33m/\sec^2$

P:10:-

$v = 2t^3 + 5t^2;$

$\therefore \frac{ds}{dt} = 2t^3 + 5t^2; \therefore ds = (2t^3 + 5t^2)dt$

Integrating,

$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} (2t^3 + 5t^2)dt$

$\therefore s_2 - s_1 = \left[\frac{2t^4}{4} + \frac{5t^3}{3}\right]_{t_1}^{t_2}$ ....(1)

$v = 2t^3 + 5t^2;$

$7 = 2t_1^3 + 5t_1^2;$

$\therefore t_1 = 1\sec$

$v = 2t^3 + 5t^2;$

$\therefore 99 = 2t_2^3 + 5t_2^2;$

$\therefore t_2 = 3\sec$

$\therefore s_2 - s_1 = 83.33m$

P:11 :-

$a = A - Bt, t = 0, v_0 = 12, x_5 = 60, v_5 = 0$

$a = \frac{dv}{dt} = A - Bt$

$\therefore \int_{12}^{v} dv = \int_0^t (A - Bt)dt$

$\therefore v = At - \frac{Bt^2}{2} + 12 \ldots\ldots (1)$

$\therefore \int_0^x dx = \int_0^t (At - \frac{Bt^2}{2} + 12)dt$

$\therefore x = \frac{At^2}{2} - \frac{Bt^3}{6} + 12t \ldots\ldots\ldots (2)$

$when \quad x = 60, t = 5$

$\therefore 15A - 25B = 0$

$v = 0, t = 5$

$\therefore 3(5A - 12.5B = -12)$

$\therefore B = 2.88, A = 4.8$

$\therefore v = 4.8t - 1.44t^2 + 12$

$\therefore x = 2.4t^2 - 0.48t^3 + 12t$

P:12:-

Given, $a = 4s$

$\therefore v \cdot \frac{dv}{ds} = 4s \therefore v \cdot dv = 4s \cdot ds$

Integrating both sides, we get, $\therefore \frac{v^2}{2} = 4 \cdot \frac{s^2}{2} + c,$

At $s = 0 \cdot 1m,\quad v = 0 \quad \therefore 0 = \frac{4}{2}(0 \cdot 1)^2 + c_1 \quad \therefore c_1 = -2(0 \cdot 1)^2$

$\therefore \frac{v^2}{2} = 2s^2 - 2(0 \cdot 1)^2 \quad v^2 = 4\left(s^2 - 0 \cdot 1^2\right) \quad v = 2\sqrt{s^2 - 0 \cdot 1^2}$

At $s = 0 \cdot 2m \quad v = 2\sqrt{(0 \cdot 2)^2 - (0 \cdot 1)^2} \quad \therefore v = 0 \cdot 3464 m/\sec$

Also, $v = \frac{ds}{dt} \quad \therefore \frac{ds}{dt} = 2\sqrt{s^2 - 0 \cdot 1^2} \quad \therefore \frac{ds}{\sqrt{s^2 - 0 \cdot 1^2}} = 2 \cdot dt$

Integrating both sides, we get,

$\left[\ln\left(s + \sqrt{s^2 - 0 \cdot 1^2}\right)\right]_{0.1}^{0.2} = 2[t]_0^t$

$\therefore \ln\left[0 \cdot 2 + \sqrt{0 \cdot 2^2 - 0 \cdot 1^2}\right] - \ln\left[0 \cdot 1 + \sqrt{0 \cdot 1^2 - 0 \cdot 1^2}\right] = 2 \cdot t$

$\therefore t = 0 \cdot 6585 \sec.$

P:13:-

(I) Distance traveled before the particle comes to rest :-

Particle at rest means v = 0

$0 = 8 - 0 \cdot 025 \qquad \therefore S = 400m.$

(II) Acceleration :- $a = \frac{dv}{dt} \therefore a = \frac{dv}{ds} \times \frac{ds}{dt} \therefore a = v \cdot \log \frac{dv}{ds}$

$\therefore a = (8 - 0 \cdot 02S) \cdot \frac{d}{ds}(8 - 0 \cdot 02S)$

$= (8 - 0 \cdot 02S) \cdot (-0 \cdot 02)$

$= -0 \cdot 16 + 0 \cdot 0004 \cdot S$

Now at start $S=0$, putting value of $S=0$.

$a = -0 \cdot 16 + 0 \cdot 0004(0)$

$\therefore a = -0 \cdot 16 m/s^2$

(III) $v = \frac{ds}{dt}$

$\therefore (8 - 0 \cdot 02S) = \frac{ds}{dt}$

$\frac{\log(8 - 0 \cdot 02S)}{(-0 \cdot 02)} = t + c$

put $= 0$ and $S = 0$

$\frac{\log(8 - 0 \cdot 02(0))}{-0 \cdot 02} = 0 + c$

$\therefore c = -103 \cdot 972$

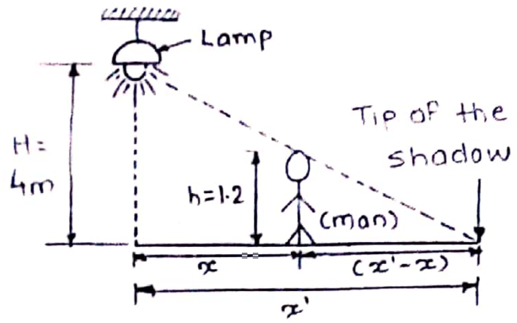
$\therefore \frac{\log(8 - 0 \cdot 02S)}{-0 \cdot 02} = t - 103 \cdot 972$

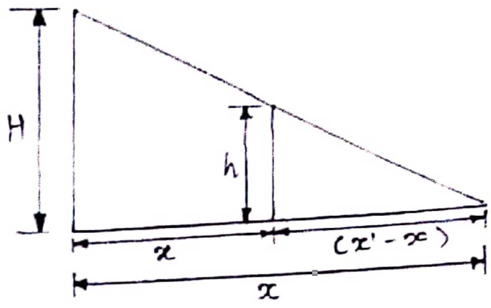
At $t = ?\qquad s = 100m$

$\frac{\log_e(8 - 0 \cdot 02(100))}{-0 \cdot 02} = t - 103 \cdot 972$

$\therefore t = 14 \cdot 384 \sec.$

P:14:-





From similar triangles,

$$\frac{x^1 - x}{h} = \frac{x^1}{H}$$

$$\therefore x^l = \frac{x}{\left[1-\frac{h}{H}\right]}$$

Velocity of man $= \frac{dx}{dt} = v = 2\cdot 8m/\sec$

Velocity of tips shadow $= v^l = \frac{dx^l}{dt}$

$$\therefore \frac{dx^1}{dt} = \frac{1}{\left[1-\frac{h}{H}\right]}\cdot\frac{dx}{dt} \qquad \therefore v^1 = \frac{1}{\left[1-\frac{h}{H}\right]}.v \qquad \therefore v^1 = \frac{1}{\left[1-\frac{1.2}{4}\right]}\times 2.8$$

$\therefore v^1 = 4m/\sec$

P:15:-

$$\frac{y}{a+x} = \frac{h}{x}$$

$$y = h\left(1+\frac{a}{x}\right)$$

$$3 = 1\cdot 8\left(1+\frac{4}{x}\right)$$

$\therefore x = 6m$

At $y = 3m, \frac{dy}{dt} = -1m/\sec, \frac{d^2y}{dt^2} = 0$

$$y = h\left(1+\frac{a}{x}\right)$$

Diff. w.r.to t, we get,

$$\frac{dy}{dt} = h\left[0 + a\left(\frac{-1}{x^2}\right)\cdot\frac{dx}{dt}\right]$$

$$\frac{dx}{dt} = \frac{(6)^2}{4\times 1\cdot 8}$$

$v = \frac{dx}{dt} = 5m/\sec.$

Again diff. w.r.to t, we get,

$$\frac{dy}{dt} = \frac{-ah}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = -ah\cdot\left[\frac{1}{x^2}\cdot\frac{dx}{dt}\right]$$

$$\frac{d^2y}{dt^2} = -ah\left[\frac{1}{x^2}\cdot\frac{d^2x}{dt^2} + \frac{dx}{dt}\cdot(-2)x^{-3}\times\frac{dx}{dt}\right]$$

$$\therefore \frac{d^2x}{dt^2} = a = 8\cdot 33m/\sec^2$$

P:16:-

Acceleration :–

$a_x = \frac{dv_x}{dt} = 48t^2$; $a_y = \frac{dv_y}{dt} = 36t - 4$

$\therefore At \quad t = 1\sec,$

$a_x = 48mm/s^2$; $a_y = 32mm/s^2$

$\therefore a = \sqrt{a_x^2 + a_y^2} = 57.69mm/s^2$;

$\tan\alpha = \frac{a_y}{a_x}; \therefore \alpha = 33.69^\circ$

$x = 4t^4 - 6t; y = 6t^3 - 2t^2$

Velocity:-

$\therefore v_x = \frac{dx}{dt} = 16t^3 - 6$ $\therefore v_y = \frac{dy}{dt} = 18t^2 - 4t$

$\therefore At \quad t = 1\sec$

$\therefore v_x = 10mm/s(\rightarrow); v_y = 14mm/s(\uparrow)$

Magnitude $v = \sqrt{v_x^2 + v_y^2} = 17.2mm/s$

$\tan\theta = \frac{v_y}{v_x}; \therefore \theta = 54.46^\circ$

P:17 :-

$R = 250, a_t = 0.6m/s^2, a_{total} = 0.75$

$a_n = \sqrt{{a_{total}}^2 - {a_t}^2} = 0.45$

$a_n = \frac{v^2}{R}$

$\therefore v = 10.6066m/s$

$u = 0$

$v = u + at$

$\therefore 10.6066 = 0 + 0.6t$

$\therefore t = 17.67\sec$

$v^2 = u^2 + 2as$

$\therefore s = 93.75m$

P:18:-

$x = 8t^2;$

$\therefore v_x = \frac{dx}{dt} = 16t$

$a_x = \frac{dv_x}{dt} = 16$

$\therefore At \quad t = 3\sec$

$\therefore v_x = 48m/s(\rightarrow)$

$a_x = 16m/s^2$

$y = t^3 + 5$

$\therefore v_y = \frac{dy}{dt} = 3t^2$

$\therefore a_y = \frac{dv_y}{dt} = 6t$

$\therefore At \quad t = 3\sec$

$v_y = 27m/s(\uparrow)$

$a_y = 18m/s^2$

Magnitude $v = \sqrt{v_x^2 + v_y^2} = 55.07m/s$

$\therefore a = \sqrt{a_x^2 + a_y^2} = 24.08m/s^2(\downarrow)$

To find path eqn y = f (x) eliminate 't',

$x = 8t^2$

$\therefore t^2 = \frac{x}{8}; \therefore t = \left(\frac{x}{8}\right)^{1/2}$ ....(1)

Substituting in $y = t^3 + 5$

$\therefore y = \left(\frac{x}{8}\right)^{3/2} + 5$

P:19:-

Solution:

Length of string connecting blocks A and B

$L_1 = x_A + (x_A - x_D) + (x_B - x_D)$

$\therefore 2x_A + x_B - 2x_D = constant$

$\Theta\ x_A = x_B; \therefore 3x_B - 2x_D = constant$

differentiating

$3v_B - 2v_D = 0; \therefore v_B = 2v_D/3$

Length of string connecting C and D:

$L_2 = x_C + x_D$

$\therefore v_C + v_D = 0; \therefore v_D = -v_C$

$\therefore v_B = 2/3(-v_C) = 2/3(8) = 16/3m/s \downarrow$

7.C. KINEMATICS OF PARTICLES

P:1:

(i) Note that the distance travelled during t=0 to t=10s, is the area under v-t curve for t = 0, t = 10.

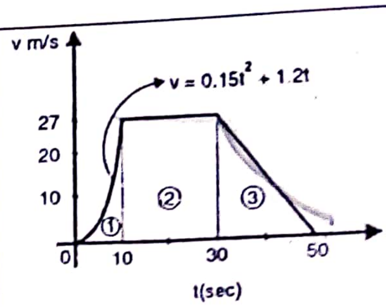
Now,

$$x_{10} - x_0 = \int_0^{10} v\,dt = \int_0^{10} \left(0.15t^2 + 1.2t\right) dt = \left[0.15\frac{t^3}{3} + 1.2\frac{t^2}{2}\right]_0^{10} = 110m$$

(ii) To find the total distance travelled in 50 sec, is the area under the curve t = 0 to t = 50s.

Area = Area(1) + Area (2) +Area (3)

$$= 110 + (27\times 20) + \frac{1}{2}(27\times 20) = 920m$$



(iii) From t = 30s to t = 50s , gradient is decreasing.

And retardation = gradient of v-t curve

$$= \frac{27}{20} = 1.35m/s^2$$

P:2: As $a = v\dfrac{dv}{ds}$

Where v is the velocity at any point $\dfrac{dv}{ds}$ is the slope of tangent at any point on v – s diagram

When s = 50 m, v = 4 m/s and Slope $\dfrac{dv}{ds} = \dfrac{4}{50}$ $\therefore acceleration(a) = v.dv/ds = 0.32m/s^2$

When s = 50 m, v = 4 m/s and Slope $\dfrac{dv}{ds} = -\dfrac{4}{50}$ $\therefore acceleration(a) = v.dv/ds = -0.32m/s^2$

P:3:- For a-t diagram From 0 to 10 sec, a = 0 (v is const.)

t = 10 to t = 18 sec	t = 18 to t = 24 sec	t = 24 to t = 30 sec	t = 30 to t = 40sec
$a = \dfrac{6-2}{18-10}$ $\therefore a = 0.5m/s^2$	$a = \dfrac{0-6}{24-18}$ $\therefore a = 1m/s^2$	$a = \dfrac{-6-0}{30-24}$ $\therefore a = -1m/s^2$	$a = 0$ $\Theta\ v\ \ is\ \ cons\tan t$

For x-t diagram

Change in x = Area under v-t diagram

$\Rightarrow x_{10} - x_0 = 2(10) = 20m;$ $\quad \therefore x_{10} = 20 + (-16) = 4m$

$\Rightarrow x_{18} - x_{10} = \left(\dfrac{2+6}{2}\right)8 = 32m;$ $\quad \therefore x_{18} = 32 + 4 = 36m$

$\Rightarrow x_{24} - x_{18} = \dfrac{1}{2}\times 6\times 6 = 18m;$ $\quad \therefore x_{24} = 36 + 18 = 54m$

$\Rightarrow x_{30} - x_{24} = -\dfrac{1}{2}\times 6\times 6 = -18m;$ $\quad \therefore x_{30} = 54 + (-18) = 36m$

$\Rightarrow x_{40} - x_{30} = -6(10) = -60m;$ $\quad \therefore x_{40} = 36 + (-60) = -24m$

$\therefore x_{max} = 54m$

At t = 18 sec and t = 30 sec particle is at a distance of 36 m from origin.

P:4:- Given $t = 0, v_0 = 0, x_0 = 0$

Area under a – t diagram = change in velocity

$\therefore v_{20} - v_0 = A_1$

$\therefore v_{20} - 0 = \frac{1}{2} \times 20 \times 12$

$\therefore v_{20} = 120 m/s$

$v_{40} - v_{20} = \frac{1}{2} \times 20 \times 12$

$\therefore v_{40} = 240 m/s$

Maximum speed at t = 40 sec, $v_{max} = 240 m/s$

Position at any instant from a – t diagram $x_t = x_0 + v_0 t + M_t$ ....*Moment equation*

$\therefore x_{20} = 0 + 0 + \frac{1}{2} \times 20 \times 12 \times \frac{1}{3} \times 20 = 800m$

$x_{40} = 0 + 0 + \frac{1}{2}(40)(12)(20) = 4800m$

P:5:-

$v = 3t^2 - 5t + 8$

$\therefore a = \frac{dv}{dt} = (6t - 5) m/s^2$

When velocity is minimum acceleration is zero.

$6t - 5 = 0;$ $\therefore t = \frac{5}{6}$ sec...................(1)

$\therefore v_{min} = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 8 = 5.91 m/s$..........(2)

$now, x - x_0 = \int_0^t v dt = \int_0^t (3t^2 - 5t + 8) dt$

$\therefore x = t^3 - \frac{5t^2}{2} + 8t + 15$............(3)

$At \quad t = \frac{5}{6}$ sec, $x = 20.51m$............(4)

We plot v-t curve , for t = 0 to t = 3 sec

$v = 3t^2 - 5t + 8$ m/s

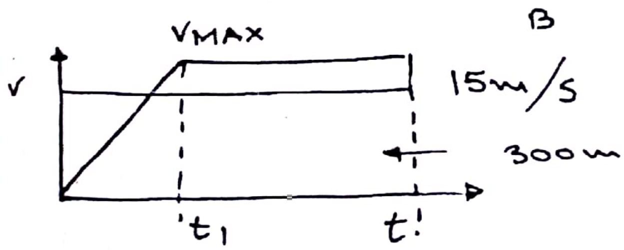
The eqn represents a parabola convex downwards for 0 < t < 3 in v-t curve

at t = 0 , $v_0 = 8m/s$;

at t = 3, $v_3 = 8m/s$

At t = 5/6 velocity is minimum and its value is 5.91 m/s.

P:6:-



$1.8 = \frac{v_{max}}{t_1}$; $\therefore t_1 = \frac{v_{max}}{1.8}$ ....(1)

Now areas are equal,

$\therefore v_{max}(t - t_1) + \frac{1}{2}(t_1)(v_{max}) = 300$ $[t = 300/15 = 20]$

$\therefore v_{max}\left(t - \frac{v_{max}}{1.8}\right) + \frac{1}{2}\left(\frac{v_{max}}{1.8}\right)(v_{max}) = 300$

$\therefore v_{max} = 21.315 = 76.73 kmph$

$\therefore t_1 = \frac{v_{max}}{1.8} = \frac{21.315}{1.8} = 11.84 sec$

7.D. KINEMATICS OF PARTICLES

P:1

$H_{max} = \dfrac{u^2 \sin^2\alpha}{2g}$; $19 = \dfrac{u^2 \sin^2\alpha}{2 \times 9.81}$; $u \sin\alpha = 19.31 \ldots (I)$

Consider vertical motion from A to B (under gravity) $h = ut + \frac{1}{2}gt^2$ and from Eq. (I) we have

$-76 \sin 18.44^c = u \sin\alpha\, t - \frac{1}{2} \times 9.81 \times t^2$; $-24.04 = 19.31 \times t - 4.905t^2$;

$4.905t^2 - 19.31 \times t - 24.04 = 0$;

Solving quadratic equation, we get $t = 4.93$ sec;

Consider horizontal motion with constant velocity,

we have displacement = Velocity x Time

$x = u \cos\alpha \times t$; $76 \cos 18.44^c = u \cos\alpha \times 4.93$; $u \cos\alpha\, t = 14.62 \ldots (II)$

Dividing Eq. (I) by Eq. (II), we get

$\tan\alpha = \dfrac{19.31}{14.62}$ $\therefore$ $\alpha = 52.87^c; u = 24.22(\ )$

P:2

x = 500 m; y = -d;

$\alpha = -12^\circ$ (*angle of depression*)

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\therefore -d = 500\tan(-12) - \frac{9.81 \times (500)^2}{2(800)^2\cos^2(-12)}$$

$$\therefore d = 108.28m$$

$$Now, \tan 12 = \frac{AC}{500}$$

$$\therefore AC = 106.28m$$

$$\therefore b = d - AC = 2m$$

P:3:-

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\therefore -10 = 7\tan(26.56) - \frac{9.81 \times (7)^2}{2(v_0)^2\cos^2(26.56)}$$

$$\therefore v_0 = 4.72m/s$$

$$\therefore (v_B)_x = 4.72\cos 26.56 = 4.22m/s$$

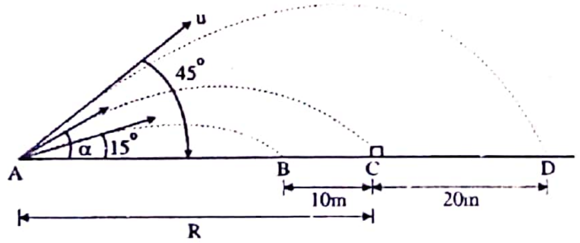
$$\therefore (v_B)^2_y = (4.72\sin 26.56)^2 - 2(9.81)(-10)$$

$$\therefore (v_B)_y = 14.165m/s$$

$$\therefore v_B = 14.78m/s$$

$$\therefore \tan\beta = \frac{(v_B)_y}{(v_B)_x}; \therefore \beta = 73.41^\circ$$

P:4 :-



Case I:-

$\alpha = 15^\circ, Range = R - 10 (R \rightarrow Actual\ \ Range)$

$$\therefore R - 10 = \frac{u^2}{g}\sin(2 \times 15^\circ)$$

$$\therefore 2R - 20 = \frac{u^2}{g}$$

Case II:-

$\alpha = 45^\circ, Range = R + 20$

$$\therefore R + 20 = \frac{u^2}{g}\sin(90^\circ)$$

$$\therefore R + 20 = \frac{u^2}{g}$$

2R - 20 = R + 20

R = 40 m

$$\therefore \frac{u^2}{g} = 60$$

$$\therefore Actual\ \ Range\ \ for\ \ AC = R = \frac{u^2}{g}\sin 2\alpha$$

$$\therefore 40 = 60\sin 2\alpha$$

$$\therefore \alpha = 20.905^\circ$$

P:5 :-

For Projectile A to B

For Projectile A to C

x = 120 + 35 - radius (centre to centre distance to allow ball to enter hole B)

$x = 120 - 35 +$ radius (centre to centre distance to allow ball to enter hole B)

$= 120 - 35 + 10 = 95$ mm $= 0.095$ m

$y = -80$ mm $= -0.08$ m, $\alpha = 0$

By eqn of trajectory,

$$y = \frac{-gx^2}{2u^2}; \qquad (u = v_0)$$

$\therefore v_0 = 0.743 m/s$

$= 120 + 35 - 10 = 145$ mm $= 0.145$ m

$y = -80$ mm $= -0.08$ m, $\alpha = 0$

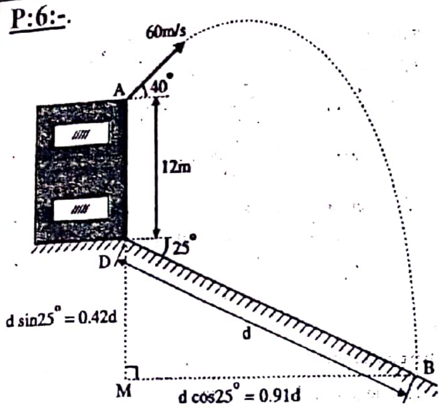
By eqn of trajectory,

$$y = \frac{-gx^2}{2u^2}; \qquad (u = v_0)$$

$\therefore v_0 = 1.135 m/s$

$\therefore$ *Range of velocity* $\Rightarrow 0.743 \le v_0 \le 1.135$

P:6:-



Solution:

$DM = d\sin 25 = 0.42d; BM = d\cos 25 = 0.91d$

$B \equiv [0.91d, -(12 + 0.42d)]$

$$\therefore y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}; \alpha = 40; u = 60$$

$\therefore d = 623.95m$

Therefore the projectile will strike the ground at horizontal distance of ,

0.91d = 0.91(623.95) = 567.79m from A

Horizontal motion from A to B, (U.M)

H.distance = (H.velocity)(time)

$\therefore 567.79 = 60\cos 40 \times t$

$\therefore t = 12.35\sec$

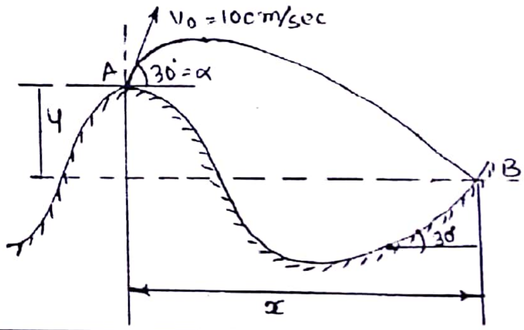
The projectile will strike the ground after 12.35 sec at pt.B = (567.76m , -274.06 m) w.r.t origin A.

P:7:-

(t) *is given by* $\tan\alpha_t = \dfrac{v_0\sin\alpha - gt}{v_0\cos\alpha}$

$$\therefore \tan(-60) = \frac{100\sin 30 - (9.81)(t)}{100\cos 30}$$

$\therefore t = 20.39\sec$



$x = (v_0\cos\alpha)(t)$

$\therefore x = 1765.83m$

$$y = (v_0\sin\alpha)t - \frac{1}{2}gt^2$$

$\therefore y = -1019.76m$

Velocity of the ball after the time t is given by,

$$v = \sqrt{(v_0\cos\alpha)^2 + (v_0\sin\alpha - gt)^2}$$

$\therefore v = 173.23 m/\sec$

P:8:-

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta); y = x\tan 50 - \frac{9.81\ x\, x^2}{2\ x\, 6^2}(1 + \tan^2 50);$$

$y = 1.192\,x - 0.33x^2;\quad 0.2x^2 = 1.192\,x - 0.33x^2;\quad 0.53x^2 = 1.192x;\quad x = 2.25\text{m}$

$\therefore y = 1.192 \times 2.25 - 0.33 \times 2.25^2 = 1.0114\text{m};\quad B(2.25,\ 1.0114)\text{m}$

P:9

Motion of ice block from A to O

$v^2 = u^2 + 2as;\ v^2 = 0 + 2 \times 9.81 \sin 30° \times 5;\ v = 7 m/s$

Motion from O to B is projectile motion. By general equation of projectile motion, we have

$y = x \tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$

$-8.4 = x \tan(-30)° - \frac{9.81 \times (x)^2}{2 \times 7^2}[1 + \tan^2(-30)°];\ x = 6.06m$

Distance from starting point

$d = 5 \cos 30° + x;\quad d = 5 \cos 30° + 6.06;\quad d = 10.39m$

8.A. KINETICS OF PARTICLES

P:1-

$L_0 = 0.4m, L_1 = 0.6m, L_2 = 0.6m$

$x_1 = L_1 - L_0 = 0.2m$

$x_2 = L_2 - L_0 = 0.2m$

$KE_1 = \frac{1}{2} \times 1.8 \times 0^2 = 0$ and $KE_2 = \frac{1}{2} \times 1.8 \times V_B^2 = 0.9V_B^2$

$Change\ \ in\ \ K.E = KE_2 - KE_1 = 0.9V_B^2 J$

Total work done,

$U_{1-2} = (1.8 \times 9.81)(0.6) + \left[\frac{1}{2} \times 16\left(0.2^2 - 0.2^2\right)\right]$

$\therefore U_{1-2} = 10.5948 N.m$

Using work energy principle we get

$0.9V_B^2 = 10.5948 \qquad \therefore V_B = 3.43 m/sec$

$(ii)\ L_0 = 0.4m, L_1 = 0.6m, L_2 = 0.9m$

$x_1 = L_1 - L_0 = 0.2m$

$x_2 = L_2 - L_0 = 0.5m$

$KE_1 = \frac{1}{2} \times 1.8 \times 0^2 = 0$ and $KE_2 = \frac{1}{2} \times 1.8 \times V_B^2 = 0.9V_B^2$

$Change\ \ in\ \ K.E = KE_2 - KE_1 = 0.9V_B^2 J$

Total work done,

$U_{1-2} = (1.8 \times 9.81)(0.6) + \left[\frac{1}{2} \times 16\left(0.2^2 - 0.5^2\right)\right]$

$\therefore U_{1-2} = 8.9148.m$

Using work energy principle we get

$0.9V_B^2 = 8.9148 \qquad \therefore V_B = 3.15 m/sec$

P:2:-

Initial momentum $= 0$

Final Momentum $= m_g v_g + m_s v_s$

$I.M. = F.M.$

$KE_1 = \frac{1}{2}mv^2 = \frac{1}{2}(80000)(6.25)^2$

$KE_2 = 0$

$U_{1-2} = \frac{1}{2}(250 \times 10^5)(0^2 - x^2) \times 3 = -\frac{3}{2} \times (250 \times 10^5)x^2$

$\therefore 0 = m_g v_g + m_s v_s$

$\therefore 0 = (80000)v_g + (500)(1000)$

$\therefore v_g = 6.25 m/s$

This is the velocity with which gun starts moving towards left. This is position 1. At Position 1 all 3 springs are at free length. Position 2 is where gun stops because of 3 springs.

$U_{1-2} = KE_2 - KE_1$

$$\therefore \left[\frac{1}{2} \times 250 \times 10^5 \times (0 - X_2^2)\right] \times 3$$

$$= 0 - \frac{1}{2}(80000)(6.25^2)$$

$\therefore X_2 = 0.2041\text{m}.$

F = kx = (250)(0.2041)100=5100kN

This is the spring force in each spring.

P:3:

Position 1:

$$KE_1 = \frac{1}{2}mv^2 = \frac{1}{2}(5)(2)^2 = 10 \text{ J or Nm}$$

Position 2:

$$KE_2 = \frac{1}{2}mv^2 = \frac{1}{2}(5)V^2 = 2.5V^2\text{J}$$

$$cos\theta = {}^{7.2}/_{7.5} = 0.96$$

$$R = 5 \times 9.812 \times cos\theta$$

$$U_{1-2} = (5 \times 9.812 \times 2.1) - (0.15 \times 5 \times 9.812 \times cos\theta \times 7.5) = 50.013$$

$$U_{1-2} = KE_2 - KE_1 \therefore V = 4.9m/s$$

$$\frac{1m}{2m/s} = \frac{s}{4.9m/s} \qquad \therefore s = 2.45m$$

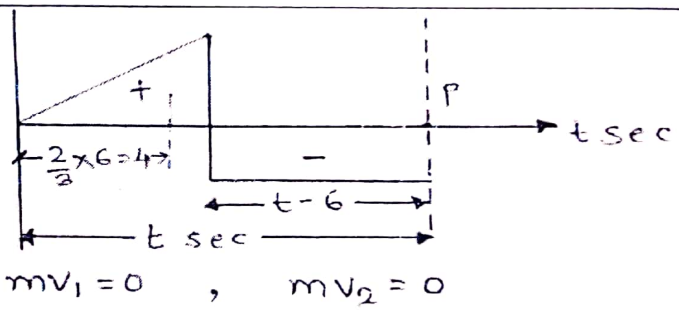
P:4:-

(i) $\int F.dt = 0$

Area under f-t curve:

$$\frac{1}{2} \times 6 \times 100 + (-30)(t-6) = 0$$

$\therefore t = 16\text{sec};$ It will come to rest.



(ii) Let T be time when displacement is 0

$\therefore$ Displacement = Moment of area of a-t curve between O and P about P

$$\therefore 0 = \left(\frac{1}{2} \times 6 \times 100\right)(T-4) - 30(T-6)\left(\frac{T-6}{2}\right) \qquad \therefore T = 27.83\sec$$

8 B. KINETICS OF PARTICLES

P-1:-

Hammer is free falling from 1 to 2,

$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.3} = 5.05$ (downwards)

At position (1)It's a plastic impact

Conservation of momentum $m_h u_h + m_p u_p = (m_h + m_p)(v_h \text{ or } v_p)$

$\therefore (600 \times -5.05) + 0 = (600 + 140) \times V_p \qquad \therefore V_p = 4.094m/s.$

velocity with which pile and hammer starts moving down From 2 to 3, and comes to rest due to frictional resistance offered by the ground.

$$\therefore KE_2 = \frac{1}{2}(600 + 140)(-4.094)^2 = 6201.509 J \quad and \quad KE_3 = 0$$

$U_{2-3} = [(600 + 140) \times 9.812 \times 0.125 - F(0.125)$

By work energy principle, $U_{2-3} = KE_3 - KE_2$

$\therefore 0 - 6201.509 = 907.61 - 0.125 \times F \quad \therefore F = 56872.952N$

This is the resistance offered by ground.

P-2:-

Initial momentum of the car A = mass * velocity =180Nm $(\rightarrow)$

When stone is dropped, momentum of stone = 100*1.2cos30=103.92Nm $(\rightarrow)$

Momentum of car = 180 + 103.92 = 283.92 Nm $(\rightarrow)$

Now, velocity of car A (along with stone) is,

$(M+m)V_1 = 283.92 \qquad \therefore V_1 = 0.7098 m/s$

By law of conservation of momentum,

$(400 \times 0.7098) + (400)(-0.3) = v(400+400)$

$\therefore v = 0.2049 m/s$

P:3 Solution:

(i) Consider motion from *O* to *A*

$u_{1y} = \sqrt{2 \times 9.81 \times 2} \quad \therefore u_{1y} = 6.264 \frac{m}{s} (\downarrow)$

$v_{1yA} = ?(\uparrow); \quad u_{2y} = v_{2y} = 0$ (velocity of flow before and after impact)

(ii) Impact at A

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right] = -\left[\frac{0 - v_{1yA}}{0 - (-6.264)}\right] = 0.8; \qquad v_{1yA} = 5.01 \frac{m}{s} (\uparrow)$$

(iii) Impact at B

$$e = -\left[\frac{v_{2y} - v_{2y}}{u_{2y} - u_{2y}}\right] = -\left[\frac{0 - v_{1yB}}{0-(-5.01)}\right] = 0.8; \qquad v_{1yB} = 4\frac{m}{s}(\uparrow)$$

(iv) Time from *O to A*

$h = ut + \frac{1}{2}gt^2; \quad 2 = 0 + \frac{1}{2} \times 9.81 \times t_1^2; \quad t_1 = 0.6386$ sec

(v) Time from *A to B*

$h = ut + \frac{1}{2}gt^2; 0 = 5.01 \, x \, t_2 - \frac{1}{2} x \, 9.81 \, x \, t_2^2; \, t_2 = 1.021$ sec

(vi) Time from B to C

$h = ut + \frac{1}{2}gt^2; \quad 0 = 4 \times t_3 - \frac{1}{2} \times 9.81 \times t_3^2; \quad t_3 = 0.8155$ sec

(vii) Total Time

$t = t_1 + t_2 + t_3 = 0.6386 + 1.021 + 0.8155; \qquad t = 2.475$ sec

(viii) For projectile motion and oblique impact component of velocity in horizontal direction through the motion remains constant.

$\therefore$ Displacement = Velocity x Time 8 = $u \, x \, 2.475$

$\therefore u = 3.232 \frac{m}{s} (\rightarrow)$

P-4:-

$$m_p v_1 + \sum \text{Im} \, p_{1-2} \rightarrow 2 = (m_p + m_c) v_2$$

$\therefore x$ components: $m_p v_1 \cos 30° + 0 = (m_p + m_c) v_2$

$(10kg)(3m/s)\cos 30^\circ = (10kg + 25kg)v_2 \qquad v_2 = 0.742 m/s(\rightarrow)$

$m_p v_1 + \sum \text{Im}\, p_{1-2} = m_p v_2$

$\therefore xcomponents: (10kg)(3m/s)\cos 30 + F_x \Delta t = (10kg)(0.742 m/s)$

$\therefore F_x \Delta t = -18.56 N.s$

$\therefore ycomponents: -m_p v_1 \sin 30 + F_y \Delta t = 0$

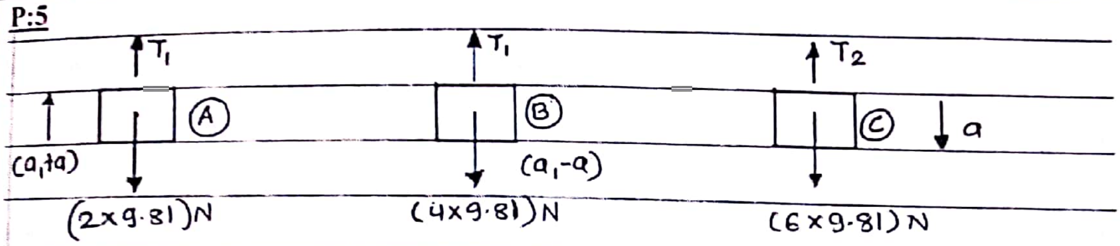
$\therefore -(10kg)(3m/s)\sin 30 + F_y \Delta t = 0 \qquad \therefore F_y \Delta t = +15 N.s \qquad \therefore F\Delta t = 23.9 N \angle 38.9$

$$T_1 = \frac{1}{2} m_p v_1^2 = \frac{1}{2}(10kg)(3m/s)^2 = 45J$$

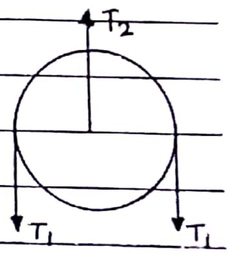
$$T_2 = \frac{1}{2}(m_p + m_c)v_2^2 = \frac{1}{2}(10kg + 25kg)(0.742 m/s)^2 = 9.63J$$

The fraction of energy lost is $\dfrac{T_1 - T_2}{T_1} = \dfrac{45J - 9.63J}{45J} = 0.786$

P:5



Applying Newton second law, we get,



$\Sigma F_y = m.a_y \; ; \; -2\times9.81 + T_1 = 2\times(a_1+a)$.

$\Sigma F_y = m.a_y \; ; \; 4\times9.81 - T_1 = 4\times(a_1-a)$.

$\Sigma F_y = m.a_y \; ; \; 6\times9.81 - T_2 = 6\times a$.

$a = 0.577 m/s^2, \quad T_1 = 27.699 N \qquad T_2 = 2T_1$

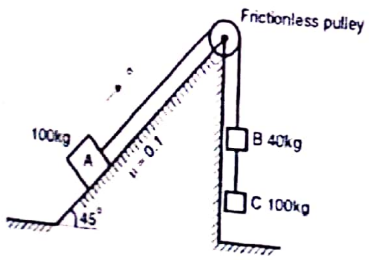
$T_2 = 55.398 N, \; a_1 = 3.463 m/s^2$.

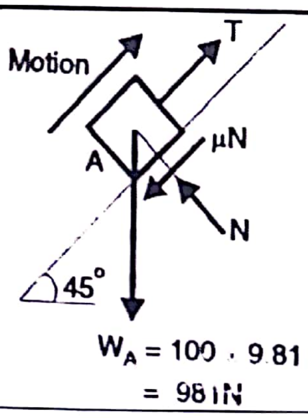
Acc^n of block 'A' = $(a_1+a) = 3.463 + 0.577 = 4.04 m/s^2$.

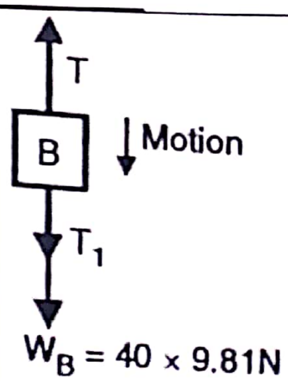
Acc^n of block 'B' = $(a_1-a) = 3.463 - 0.577 = 2.89 m/s^2$.

Acc^n of block 'C' = $(a = 0.577 m/s^2$.

P:6







$F = \mu N \qquad (\mu = 0.1)$

Motion of A is upwards.

Force acting on A upwards is $T - W_A \sin 45 - F$

$\therefore T - W_A \sin 45 - F = m.a \qquad \therefore T = 100a + 763.03.............(i)$

Net force acting on B in downward direction is $W_B + T_1 - T$

$\therefore 392.4 + T_1 - T = 40a$............(ii)

Net force acting on C in downward direction is $\therefore 100 \times 9.81 - T_1 = 100a$..........(iii)

$(ii)+(iii) \rightarrow \quad 1373.4 - T = 140a$........(iv)

From (i) and (iv) $a = 3.12 m/s^2$

Thus before touching the ground $a = 3.12 m/s^2$

Velocity of A after traveling 2m $\rightarrow v^2 = u^2 + 2ax \quad \therefore v = 3.533 m/s$

When C touches the ground, it rests on ground n hence mass will not be effective

If a1 is the new acceleration then $40 \times 9.81 - T_2 = 40 \times a_1 \quad \therefore T_2 = 392 - 40a_1$.........(v)

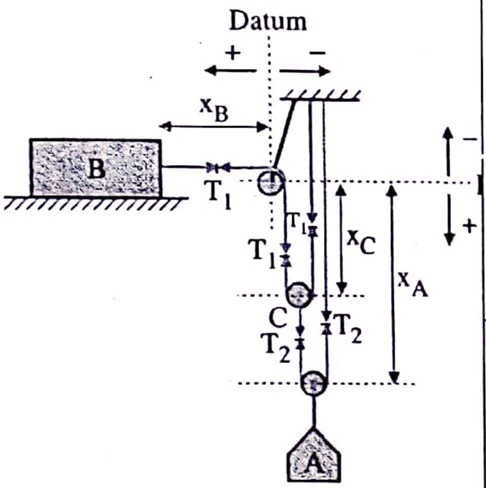
Now for Block A : $T_2 = 100a_1 + 763.03$............(vi)

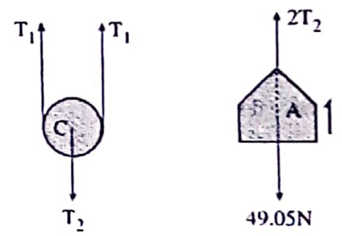
From (v) and (vi) $a_1 = -2.65 m/s^2$

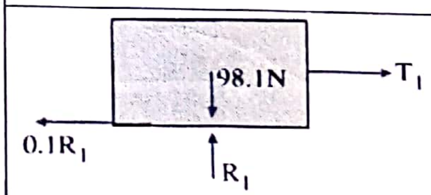
If x is the distance through which A moves up: then $\rightarrow v^2 = u^2 + 2ax \quad \therefore v = 2.355 m/s$

Hence total distance traveled by A is 2+2.355=4.355m

P:7







$L_1 = x_B + 2x_C \quad \therefore a_B + 2a_C = 0$

$L_2 = (x_A - x_C) + x_A = -x_C + 2x_A \quad \therefore -a_C + 2a_A = 0 \quad \therefore a_C = 2a_A$

$\therefore a_B + 4a_A = 0$

For block B:

$\sum F_x = 0 \rightarrow T_1 - 10(-a_B) - 0.1R_1 = 0 \quad \therefore T_1 + 10a_B = 0.1(98.1) \quad \therefore a_B = \dfrac{9.81 - T_1}{10}$

For pulley C: $\sum F_y = 0 \rightarrow 2T_1 = T_2$

For block A: $\sum F_y = 0 \rightarrow 2T_2 + 5a_A - 49.05 = 0 \quad \therefore a_A = \dfrac{49.05 - 4T_1}{5}$

Substituting the values

$\therefore \dfrac{9.81 - T_1}{10} + 4\left[\dfrac{49.05 - 4T_1}{5}\right] = 0 \quad \therefore T_1 = 12.188N \quad \therefore a_A = 0.0596 m/s^2$

P:8

$\sum F_x = 0 \rightarrow T_A \cos 45° - T_B \cos 45° - ma = 0 \qquad \therefore T_A - T_B = 3.535a$

$\sum F_y = 0 \rightarrow T_A \cdot \sin 45° + T_B \sin 45° - mg = 0 \qquad \therefore T_A + T_B = 13.87\,(m)$

$\therefore T_A = 8.7(m)$ and $T_B = 5.165(m)$.

P:9

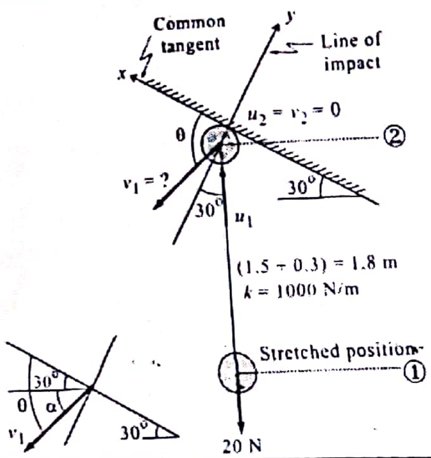
Let a be de-acceleration of truck.

$\sum F_x = 0 \quad \therefore$ ma- μ_s.R = 0 $\quad \therefore$ ma = $\mu_s \times$mg $\quad \therefore a = \mu_s g = 2.943 m/s^2$

(i) $v^2 = u^2 + 2as$ $\quad$ But $u = 70 \times \frac{5}{18} = 19.44 m/s$ $\quad \therefore$ s = 64.205m

(ii) v= u+at $\quad \therefore$ 0=19.44 + (-2.943)t $\quad \therefore$ t = 6.605 sec

P:10



(i) By work energy principal,

$\frac{1}{2} \times 1000 \times (0.3^2 - 0^2) - 20 \times 1.8 = \frac{1}{2} \times \frac{20}{9.81} \times u_1^2 - 0;$

$9 = 1.0194 u_1^2;$

$u_1 = 2.971 \frac{m}{s} (\uparrow)$ (velocity before impact)

(ii) Coefficient of restitution along the line of impact gives

$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right]; \qquad 0.7 = -\left[\frac{0 - (-v_{1y})}{0 - 2.971 \cos 30°}\right]; \qquad v_{1y} = 1.8$

(iii) Component of velocity along common tangent before and after impact is same

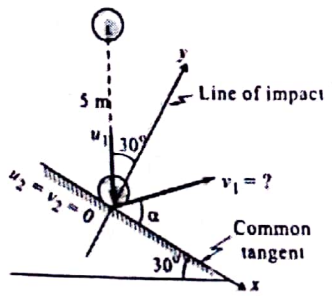
$v_{1x} = 2.971 \sin 30° \therefore v_{1x} = 1.486 \frac{m}{s}; \quad \tan\theta = \frac{v_{1y}}{v_{1x}} = \frac{1.8}{1.486} \quad \therefore \theta = 50.46°$

$\alpha = \theta - 30 = 50.46 - 30 \quad \therefore \alpha = 20.46°$

$v_1 = \sqrt{v_{1x^2} + v_{1y^2}} = \sqrt{1.486^2 + 1.8^2}; \qquad \therefore v_1 = 2.334 \frac{m}{s}$

The ball after impact rebounds with a velocity 2.334 m/s

P:11



Solution:

(i) Velocity before impact

$u_1 = \sqrt{u^2 + 2gh} = \sqrt{0 + 2 \times 9.81 \times 5} = 9.905 \frac{m}{s} (\downarrow)$

$u_{1y} = u_1 \cos 30° = 90.905 \cos 30° = 8.57 \frac{m}{s} (\downarrow)$

(ii) By coefficient of restitution along the line of impact

$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right] \Rightarrow 0.8 = -\left[\frac{0 - v_{1y}}{0 - (-8.578)}\right]; \quad v_{1y} = 6.862\, m/s$

(iii) velocity along common tangent before and after impact is same

$v_{1x} = u_1 \sin 30^\circ = 9.905 \sin 30^\circ = 4.953$ m/s

$\tan\alpha = \frac{v_{1y}}{v_{1x}} = \frac{6.862}{4.953}$ $\therefore \alpha = 54.18^\circ$ but $\alpha = \theta + 30^\circ \therefore \theta = 24.18^\circ$

$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = \sqrt{4.953^2 + 6.862^2}$

$\therefore v_1 = 8.463 \frac{m}{s}$ (velocity of the ball after impact)

9. KINEMATICS OF RIGID BODIES

P:1:-

(1) $V_A = 0$; (2) $V_B = r_{BA} \cdot \omega_{AB}$ i.e. $V_B = 100\,\omega_{AB}$ —— (i).

[Draw perpendicular to V_D & V_B, the intersection point is ICR I]

$\therefore$ (1) $V_B = r_{BI} \cdot \omega_{BE}$ i.e. $\boxed{V_B = 225 \cdot \omega_{BE}}$ $V_B = 225 \times 1.2 = 270$

(2) $\boxed{V_D = r_{DI} \cdot \omega_{BE}}$ $\therefore 120 = 100 \cdot \omega_{BE}$

$\therefore \boxed{\omega_{BE} = 1.2 \text{ rad/sec}}$

Substituting this value in Equation (2) we get

$V_B = 225 \times 1.2$ $\therefore V_B = 270$ mm/s

Substituting value of V_B in Equation (1)

we have $270 = 100\,\omega_{AB}$ i.e. $\boxed{\omega_{AB} = 2.7 \text{ rad/sec}}$

(3) Velocity of E

$V_E = r_{EI} \cdot \omega_{BE}$

$\therefore V_E = 199.55 \times 1.2$

$V_E = 239.46$ mm/s

To Calculate EI

from $\Delta EMB \cong \Delta DIB$,

$\frac{EM}{350} = \frac{100}{225}$ $\therefore EM = 155.55$ mm

Now in ΔEMI, $EI = \sqrt{EM^2 + MI^2} = \sqrt{(155.55)^2 + (125)^2}$

$EI = 199.55$ mm

P:2:-

For AB : (ICR is A) (external hinge) :

(1) $v_A = 0$

(2) $v_B = r_{BA} \cdot \omega_{AB} = 0.4 \times 2.8$ $v_B = 1.12$ m/s (perpendicular to AB)

For link DBE (ICR is I) :

Since slider E always move along horizontal track, the velocity v_E is horizontal. To locate ICR 'I' of link DBE, draw perpendicular to v_B and perpendicular to v_E, the intersection point is ICR(I)

(1) $v_B = r_{BI} \cdot \omega_{DBE}$

$1.12 = 0.3 \cdot \omega_{DBE}$

$\therefore \omega_{DBE} = 3.73 \text{ rad/sec}$ ↶

(2) $v_D = r_{DI} \cdot \omega_{DBE}$

$= 0.72 \times 3.73$

$v_D = 2.685 \text{ m/s}$ (perpendicular to DI)

(3) $v_E = r_{EI} \cdot \omega_{DBE}$

$= 0.4 \times 3.73$

$v_E = 1.492 \text{ m/s}$

(perpendicular to EI) ...Ans.

Calculation of distances DI, BI and EI

In $\triangle$ BIE, $BI = \sqrt{(BE)^2 - (EI)^2} = \sqrt{(0.5)^2 - (0.4)^2}$

$\therefore$ **BI = 0.3 m**

Now, from $\Delta DFE \cong \Delta BIE$,

$$\frac{BE}{EI} = \frac{DE}{EF}$$

$$\frac{0.5}{0.4} = \frac{1}{EF} \quad \therefore EF = 0.8 \text{ m} \quad \therefore FI = 0.4 \text{ m}$$

Now in Δ DFE. $DF = \sqrt{1^2 - 0.8^2} = 0.6 \text{ m}$

$\therefore DI = \sqrt{DF^2 + FI^2} = \sqrt{(0.6)^2 + (0.4)^2}$

DI = 0.72 m

EI = 0.4 m

P:3:-

For link ACB. (ICR is I) :

(To locate I, draw a perpendicular to v_a and produce center line of OC to meet point I)

(1) $v_A = r_{AI} \cdot \omega_{ACB}$

$v_A = 0.245 \cdot \omega_{ACB}$ ...(2)

(2) $v_B = r_{BI} \cdot \omega_{ACB}$

$= 0.313 \times \omega_{ACB}$

$= 0.313 \times \frac{v_a}{0.245}$ (from Equation (2))

$v_B = 1.28\, v_a$ ...Ans.

To calculate distances AI, CI and BI

In Δ ACO, by sine rule,

$$\frac{0.2}{\sin\theta} = \frac{0.1}{\sin 20^\circ}$$

$\therefore \sin\theta = 0.684$

$\theta = 43.16^\circ$

Now in $\triangle$ OAI,

$\angle I = 90^\circ - \theta = 46.84^\circ$

In Δ ACI, $\angle C = 180^\circ - 70^\circ - 46.84^\circ$

$\angle C = 63.16^\circ$

$\therefore$ by sine rule, $\frac{AI}{\sin 63.16^\circ} = \frac{AC}{\sin 46.84^\circ} = \frac{CI}{\sin 70^\circ}$

$\therefore AI = \frac{0.2 \cdot \sin 63.16^\circ}{\sin 46.84^\circ} = 0.245 \text{ m}$

$CI = \frac{0.2 \cdot \sin 70^\circ}{\sin 46.84^\circ} = 0.257 \text{ m}$

To calculate BI :

In $\triangle$ ADB.AD $= 0.3 \cos 70^\circ = 0.103 \text{ m}$

$\therefore DI = AI - AD = 0.245 - 0.103$

$= 0.142 \text{ m}$

and $DB = 0.3 \sin 70^\circ = 0.28 \text{ m}$

$\therefore BI = \sqrt{(DI)^2 + (BD)^2}$

$= \sqrt{(0.142)^2 + (0.28)^2}$

$BI = 0.313 \text{ m}$

P:4:-

For rod DE (ICR is I_1) :

To locate ICR of rod DE, draw a perpendicular to v_D and a perpendicular to v_E, the intersection point is ICR I_1.

1) $v_D = r_{DI_1} \cdot \omega_{DE}$

$500 = 160 \cdot \omega_{DE}$

$\therefore \omega_{DE} = 3.125 \text{ rad/sec.}$ ↷ ...Ans.

2) $v_E = r_{EI_1} \cdot \omega_{DE} = 160 \times 3.125$

$= 500 \text{ mm/s} \downarrow$

3) $v_B = r_{BI_1} \cdot \omega_{DE}$

$= 113.14 \times 3.125$

$= 353.56 \text{ mm/s}$ (perpendicular to BI_1)

Calculation of distances DI_1, EI_1 and BI_1

$DI_1 = 160 \text{ mm}$

$EI_1 = 160 \text{ mm}$

$BI_1 = \sqrt{80^2 + 80^2}$

$= 113.14 \text{ mm}$

For rod AB : (ICR is I_2)

Since, velocity of end A is along the horizontal surface, to locate ICR I_2 draw a perpendicular to v_A and perpendicular to v_B (produce BI_1).

1) $v_A = r_{AI_2} \cdot \omega_{AB}$

$v_A = 250 \cdot \omega_{AB}$ ...(1)

2) $v_B = r_{BI_2} \cdot \omega_{AB}$

$353.56 = 240.42 \cdot \omega_{AB}$

$\therefore \omega_{AB} = 1.47$ **rad/sec.** ↷ ...**Ans.**

Substituting this value in Equation (1)

$v_A = 250 \times 1.47$

$v_A = 367.5$ mm/s ← ...**Ans.**

Calculation of distances AI_2 and BI_2.

In Δ OMB, $\tan\theta = \frac{80}{80}$ $\therefore \theta = 45°$

$\therefore$ In Δ BNI_1, $BI_1 = \sqrt{80^2 + 80^2} = 113.14$ mm.

Also in Δ I_1PI_2, $\cos\theta = \frac{90}{I_1I_2}$ $\therefore I_1I_2 = 127.28$ mm

and $PI_2 = 90$ mm

$\therefore AI_2 = AP + PI_2 = 160 + 90 = 250$ mm

and $BI_2 = BI_1 + I_1I_2 = 113.14 + 127.28$

$BI_2 = 240.42$ mm

P:5:-

Soln. : Here velocity of roller B is 0.8 m/s ↓. So, corresponding velocity of roller A must be towards right along the horizontal guide.

Therefore, to locate ICR of rod ABD, draw perpendicular to v_A and perpendicular to v_B as shown in Fig. Ex. 3.B.23(a).

Calculations for distances. AI, BI and DI.

$AI = 0.3 \cos 60° = 0.15$ m

$BI = 0.3 \sin 60° = 0.26$ m

Now, $BM = 0.3 \cos 30° = 0.26$ m

and $MD = 0.3 \sin 30° = 0.15$ m

$\therefore MI = MB + BI$

$= 0.26 + 0.26 = 0.52$ m

$\therefore$ In Δ DMI, DI

$= \sqrt{(0.15)^2 + (0.52)^2}$

$DI = 0.54$ m

For rod ABD (ICR is I) :

$\therefore$ 1) $v_A = r_{AI} \cdot \omega_{rod}$

$v_A = 0.15\, \omega_{rod}$ ...(1)

2) $v_B = r_{BI} \cdot \omega_{rod}$

$0.8 = 0.26\, \omega_{rod}$

$\therefore \omega_{rod} = 3.076$ rad/sec ↶

Substituting this value in Equation (1)

We have. $v_A = 0.15 \times 3.076 = 0.46$ m/s →

3) $v_D = r_{DI} \cdot \omega_{rod} = 0.54 \times 3.076$

$v_D = 1.66$ **m/s perpendicular to DI**

$\tan\theta = \frac{MD}{MI} = \frac{0.15}{0.52}$

$\therefore \theta = 16.09$

$\therefore$ Angle of v_D with horizontal is $90 - \theta = 73.91°$

P:6:-

$$v_B = r\omega_{AB} \quad where \quad r = I_iB = 400mm$$

$$\therefore 360 = 400\omega_{AB} \qquad \therefore \omega_{AB} = 0.9 rad/\sec(\lrcorner)$$

$$v_D = P_1D.\omega_{AB} = 225mm/\sec$$

$$\tan\theta = \frac{200}{150} = \frac{P_2F}{DF}$$

$$\therefore P_2F = \frac{4}{3}DF = \frac{4}{3}(150+250) = \frac{1600}{3}mm$$

$$DF = 400mm$$

$$P_2D = \sqrt{(P_2F)^2 + (DF)^2} = 2000/3mm$$

$$P_2E = P_2F + 200 - 500 = \frac{700}{3}mm$$

$$v_D = P_2D.\omega_{DE} \qquad \therefore \omega_{DE} = 0.3375 rad/\sec(\lrcorner)$$

$$v_E = P_2E\omega_{DE} \qquad \therefore v_E = 78.75mm/\sec$$