... absorbed in a capacitor is zero.

Example 2.26 A 318 μF capacitor is connected across a 230 V, 50 Hz system. Determine (i) capacitive reactance, (ii) rms value of current, and (iii) equations for voltage and current.

Solution

(i) Capacitive reactance, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 318 \times 10^{-6}} = 10\ \Omega$

(ii) The rms value of current, $I = \dfrac{V}{Z} = \dfrac{V}{X_C} = \dfrac{230}{10} = 23$ A

(iii) $V_m = \sqrt{2} \times V$

$= \sqrt{2} \times 230$

$= 325.27$ V

and $I_m = \sqrt{2} \times I$

$= \sqrt{2} \times 23$

$= 32.53$ A

Hence, equations for voltage and current are:

$$v = 325.27 \sin (2\pi \times 50)t$$

$$v = 325.27 \sin 314t$$

$$i = 32.53 \sin \left(314t + \frac{\pi}{2}\right)$$

Example 2.27 A 10 mH inductor has a current of $i = 5 \cos(2000t)$ A. Obtain the voltage V_L across it.

Solution

Given $L = 10 \times 10^{-3}$ H

and $i = 5 \cos(2000t)$

Converting the current equation into standard sinusoidal form,

$$i = 5 \sin\left(2000t + \frac{\pi}{2}\right) \tag{i}$$

From Eq. (i), $\omega = 2000$ rad/sec and $\phi = \frac{\pi}{2}$ rad $= 90°$

The rms value of current, $I = \frac{5}{\sqrt{2}} = 3.54$ A

Now, $X_L = \omega L = 2000 \times 10 \times 10^{-3} = 20\ \Omega$

So, $V_L = IX_L = 3.54 \times 20 = 70.8$ V

Alternative method:

The equation of the voltage across the inductor (v_L) can be calculated as

$$v_L = L\frac{di}{dt}$$

$$= 10 \times 10^{-3} \frac{d}{dt} [5 \cos(2000t)]$$

$$= -10 \times 10^{-3} \times 5 \times 2000 \sin 2000t$$

or $\quad v_L = 100 \sin(2000t + 180°)$

Comparing with standard sinusoidal form,

$$V_m = 100 \text{ V}$$

So, $\quad V_L = \dfrac{100}{\sqrt{2}} = 70.71 \text{ V}$

Example 2.28 A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current.

Solution

Given: $V = 150$ V

So, $V_{max} = \sqrt{2}\ V = \sqrt{2} \times 150 = 212.13$ V

Supply frequency, $f = 50$ Hz

Inductance, $L = 0.2$ H

Inductive reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

In a pure inductive coil,

if applied voltage, $v = V_m \sin \omega t$,

then $$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

The rms value of current, $I = \dfrac{V}{X_L} = \dfrac{150}{62.83} = 2.39$ A

So, $I_m = \sqrt{2}\, I = \sqrt{2} \times 2.39 = 3.38$ A

Now, $v = 212.13 \sin \omega t$

or $v = 212.13 \sin 2\pi ft$

Hence, $v = 212.13 \sin 314t$

and $$i = 3.38 \sin\left(314t - \frac{\pi}{2}\right)$$

2.2 AC Series Circuits

An ac circuit differs from a dc circuit in many respects. Firstly, in a dc circuit, we consider resistance only, whereas in an ac circuit, in addition to resistance, inductance (L) and capacitance (C) are also considered. The elements L and C offer opposition (i.e., X_L and X_C) to the current flow in an ac circuit. Secondly, the magnitude of the current in an ac circuit is affected by the supply frequency because X_L ($=2\pi fL$) and X_C [$= 1/(2\pi fC)$] are frequency dependent. Thirdly, in a dc circuit, voltages or currents can be added or subtracted arithmatically. But in an ac circuit, there is a phase difference of 90° between the voltage across and the current through L or C. This implies that for the addition or subtraction of alternating voltages or currents, phase difference has to be taken into account. All these features make the analysis of an ac circuit quite different from that of dc circuit.

A circuit in which the same alternating current flows through all the elements (i.e., R, L, C) is called series ac circuit. Here, we will discuss the following series circuits:

1. R-L series circuit
2. R-C series circuit
3. R-L-C series circuit

In the study of the above circuits, our points of interest will be (i) phase angle ϕ between the applied voltage and the circuit current, (ii) nature of the circuit (i.e., whether resistive, inductive, or capacitive), (iii) circuit impedance, and (iv) power consumed. In drawing the phasor diagram, either voltage or current can be taken as the reference phasor to show the phase difference. However, it is always convenient to take that quantity as the reference phasor, which is common in the circuit. Since current is common in a series circuit, it shall be taken as the reference phasor in drawing the phasor diagrams.

2.2.1 *R-L* Series Circuit

Consider a resistor of $R\ \Omega$, and an inductance of L H connected in series across an ac supply (see Fig. 2.53).

Let V = rms value of the applied voltage

I = rms value of the circuit current

Z = impedance of the circuit

By Ohm's law, $V_R = IR$ where V_R is in phase with I

$V_L = IX_L$ where V_L leads I by 90°

$V = IZ$

Circuit diagram

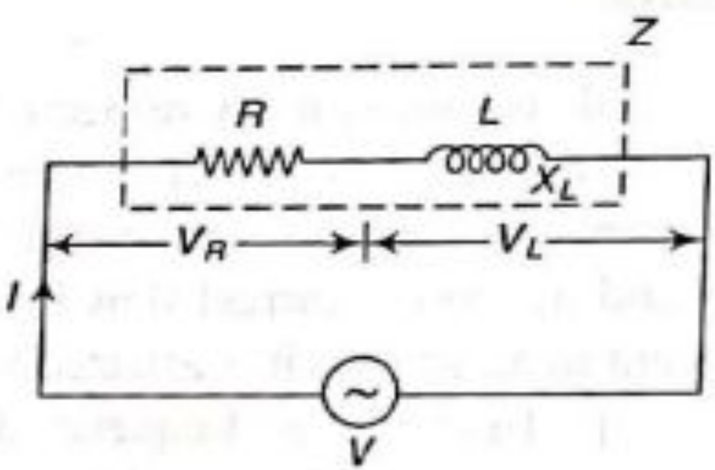


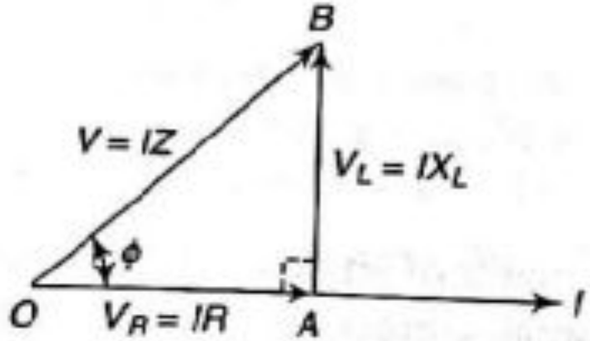
Fig. 2.53 *R-L* series circuit

Phasor diagram

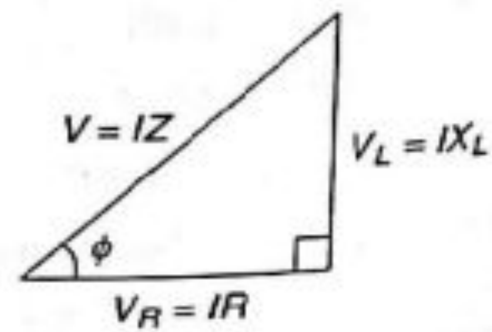
Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.54(a). The voltage V_R is in phase with current I, whereas the voltage V_L leads current I by 90°.

From the circuit diagram, we get the voltage equation as

$$\bar{V} = \bar{V}_R + \bar{V}_L$$



(a) Phasor diagram



(b) Voltage triangle

Fig. 2.54

Note: Phasor diagram can be drawn by using the following steps:

Step I: Write the voltage equation, i.e., $\bar{V} = \bar{V}_R + \bar{V}_L$ from the circuit diagram. According to the voltage equation, to get the supply voltage phasor 'V', we have to take the phasor sum of V_R and V_L.

Step II: Draw the reference phasor, i.e., 'I' from the starting point 'O'.
Step III: Draw 'V_R' from the starting point 'O', which is in phase with 'I'.
Step IV: After phasor 'V_R' place the phasor V_L. Phasor 'V_L' leads 90° to I. Let the finishing terminal be 'B'.
Step V: To get the resultant phasor 'V', join the starting terminal, i.e., 'O' and finishing terminal, i.e., 'B', and put the arrow at finishing terminal.
Step VI: Mark the angle between circuit current 'I' and supply voltage 'V' as $\phi°$ (ϕ is called phase angle of the circuit).

It is clear from the phasor diagram that circuit current I lags behind the applied voltage V by $\phi°$ ($\phi < 90°$). Therefore, nature of the circuit is inductive. The value of phase angle ϕ can be determined from the phasor diagram.

$$\text{Phase angle of the circuit, } \phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\frac{X_L}{R}$$

$$\text{Hence, power factor of the circuit, pf} = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ lagging}$$

(As the circuit current I lags behind the applied voltage V, the nature of the power factor is lagging.)

Impedance (Z)

The total opposition offered to the flow of alternating current is called impedance Z. The voltage triangle of the circuit is shown in Fig. 2.54(b). The impedance has two components, resistance (R) and reactance (X_L). The impedance along with its components can be represented by a right-angled triangle known as impedance triangle. Dividing each of the voltage phasor by I in a voltage triangle, we get the impedance triangle as shown in Fig. 2.55.

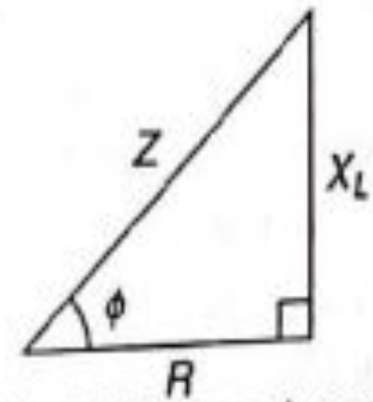


Fig. 2.55 Impedance triangle

From impedance triangle, impedance $Z = \sqrt{R^2 + X_L^2}\ \Omega$

The impedance Z can be expressed in rectangular and polar forms as

$$\bar{Z} = (R + jX_L)\ \Omega$$

and $$\bar{Z} = (Z \angle\phi)\ \Omega$$

Power (P)

The purpose of passing the current through any circuit is to transfer the power from one place to another. The power that is actually consumed in the circuit is called active or true power (in case of resistance R). The circulating power is called reactive power (in case of inductance L or capacitance C). In case of resistance, current is in phase with voltage, while in case of inductance and capacitance, current is 90° out of phase with voltage. Thus, the current in phase with voltage produces the active power, while current 90° out of phase with voltage produces the reactive power, i.e.,

Active power, P = Voltage × Current in phase with voltage
Reactive power, Q = Voltage × Current 90° out of phase with voltage

For R-L series circuit,
Taking V as reference phasor, the phasor diagram can be drawn as shown in Fig. 2.56 (only voltage phasor and current phasor are drawn).

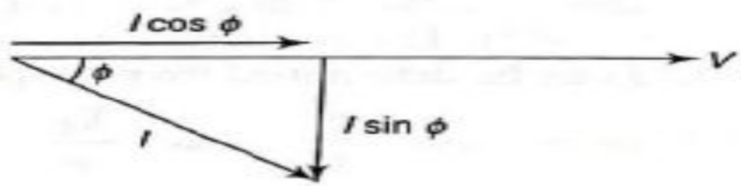


Fig. 2.56 Phasor diagram (taking V-reference)

The circuit current I can be resolved into two components:
(i) $I\cos\phi$, in phase with voltage
(ii) $I\sin\phi$, 90° out of phase with voltage

Thus, Active power, $P = VI\cos\phi$ W or kW
Reactive power, $Q = VI\sin\phi$ VAR or kVAR
and Apparent power, $S = VI$ VA or kVA

In Fig. 2.56, if we multiply each of current phasor by V, we get the power triangle as shown in Fig. 2.57.

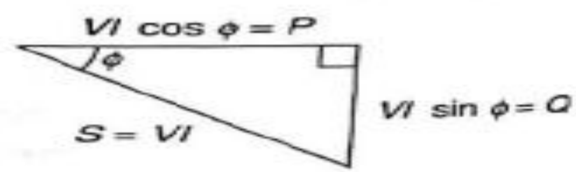


Fig. 2.57 Power triangle

From power triangle,

(Active power)^2 + (Reactive power)^2 = (Apparent power)^2

i.e., $P^2 + Q^2 = S^2$

2.2.2 *R-C* Series Circuit

Consider a resistor of R Ω and a capacitor of capacitance C F are connected in series across an ac supply (see Fig. 2.58).

Let V = rms value of the applied voltage

I = rms value of the circuit current

Z = impedance of the circuit

By Ohm's law,

$V_R = IR$ where V_R is in phase with I

$V_C = IX_C$ where V_C lags behind I by 90°

$V = IZ$

Circuit diagram

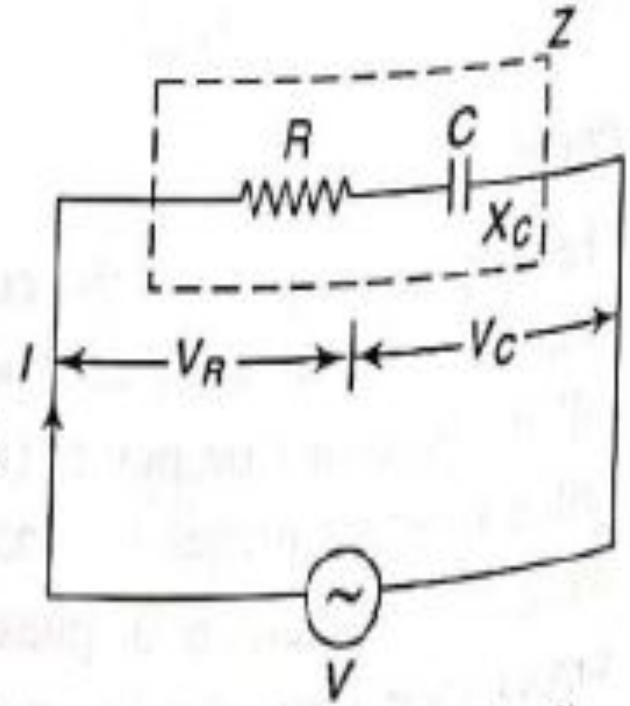
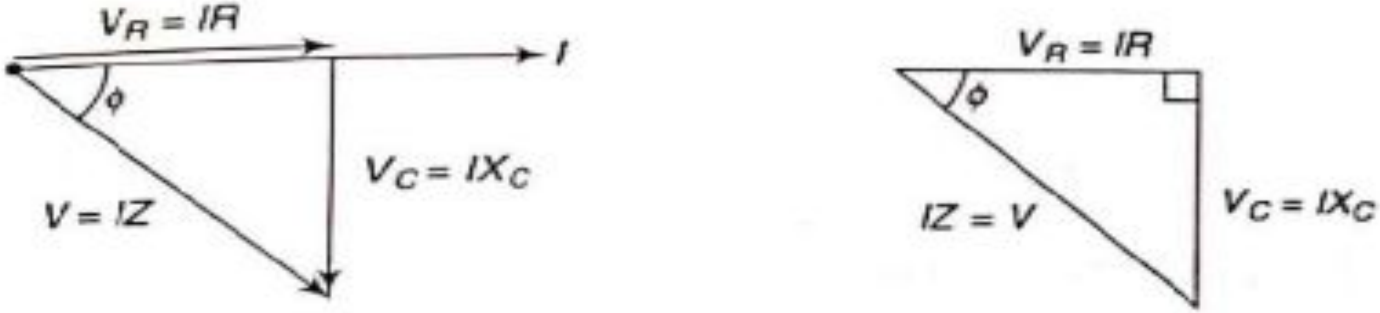


Fig. 2.58 *R-C* series circuit

Phasor diagram

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.59(a). Voltage V_R is in phase with current I, whereas voltage V_C lags behind current I by 90°.



(a) Phasor diagram

(b) Voltage triangle

Fig. 2.59

From the circuit diagram, we get the voltage equation as $\bar{V} = \bar{V}_R + \bar{V}_C$

It is clear from the phasor diagram that current I leads the applied voltage V by $\phi°$ ($\phi < 90$). Therefore, nature of the circuit is capacitive. The value of the phase angle ϕ can be determined from the phasor diagram.

Phase angle of the circuit, $\phi = \tan^{-1}\dfrac{V_C}{V_R} = \tan^{-1}\dfrac{X_C}{R}$

Hence, power factor of the circuit, $\text{pf} = \cos\phi = \dfrac{V_R}{V} = \dfrac{IR}{IZ} = \dfrac{R}{Z}$ leading

(As the circuit current 'I' leads the applied voltage 'V', the nature of the power factor is leading.)

Impedance (Z)

The voltage triangle of the circuit is shown in Fig. 2.59(b). Dividing each of voltage phasor by 'I' in a voltage triangle, we get the impedance triangle as shown in Fig. 2.60.

From impedance triangle,

Circuit impedance, $Z = \sqrt{R^2 + X_C^2}\ \Omega$

The impedance can be expressed in rectangular and polar forms as

$$\bar{Z} = (R - jX_C)\ \Omega$$

and

$$\bar{Z} = (Z \angle -\phi)\ \Omega$$

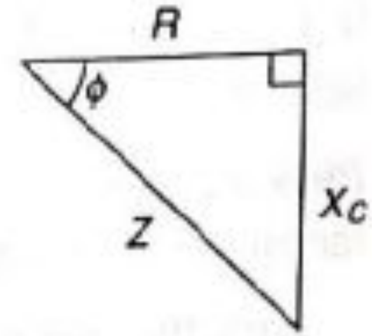


Fig. 2.60 Impedance triangle

Power (P)

For R-C series circuit, taking V as reference, the phasor diagram can be drawn as shown in Fig. 2.61 (only voltage phasor and current phasor are drawn).

The circuit current can be resolved into two components:

(i) $I\cos\phi$, is in phase with voltage

(ii) $I\sin\phi$, 90° out of phase with voltage

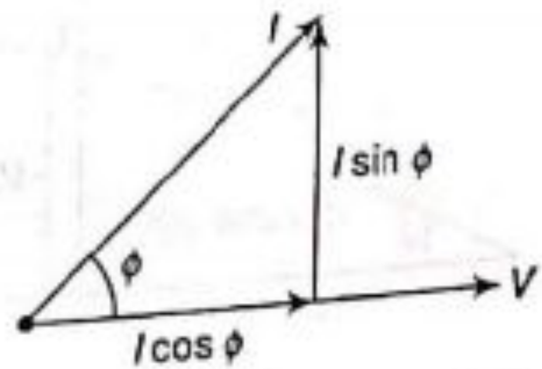


Fig. 2.61 Phasor diagram (taking V-reference)

In Fig. 2.61, if we multiply each of current phasor by V, we get the power triangle as shown in Fig. 2.62.

From power triangle,

(Apparent power)^2 = (Active power)^2
+ (Reactive power)^2

i.e., $$P^2 + Q^2 = S^2$$

So, $$\text{pf} = \cos\phi = \frac{P}{S}$$

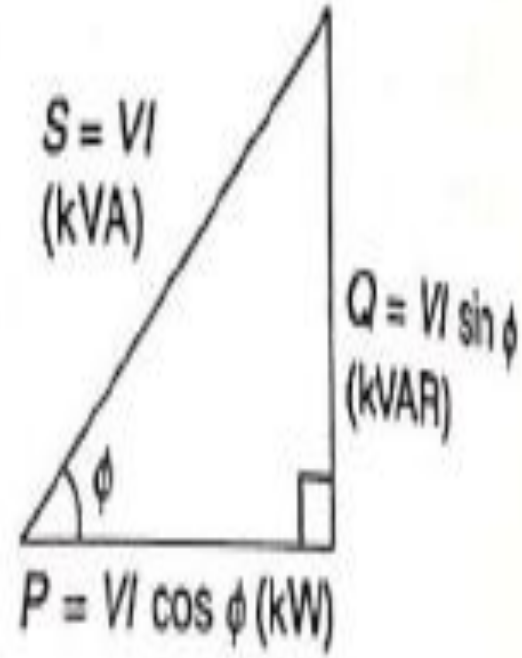


Fig. 2.62 Power triangle

2.2.3 *R-L-C* Series Circuit

Consider an ac circuit containing a resistor of resistance R ohm, inductor of inductance L henry, and a capacitor of capacitance C farad, all connected in series across an ac supply (see Fig. 2.63).

Let V = rms value of the applied voltage

I = rms value of the circuit current

Z = impedance of the circuit

By Ohm's law,

$V_R = IR$, where V_R is in phase with I

$V_L = IX_L$, where V_L leads I by 90°

$V_C = IX_C$, where V_C lags behind I by 90°

$V = IZ$

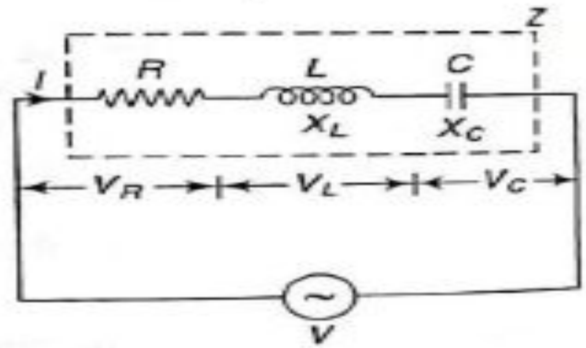


Fig. 2.63 *R-L-C* series circuit

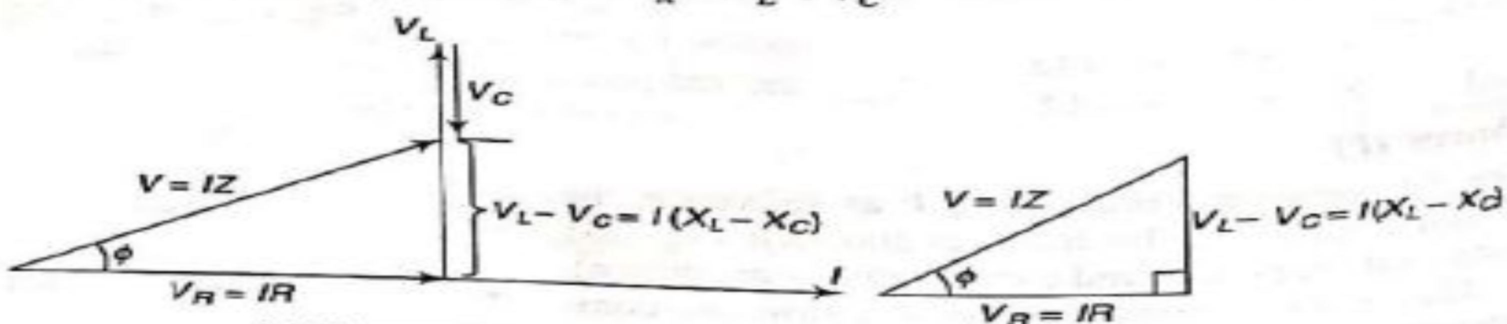
Figure 2.63 shows the *R-L-C* series circuit. The *R-L-C* series circuit can be effectively inductive, capacitive, or resistive depending upon the values of X_L and X_C.

Case (i) $X_L > X_C$

If $X_L > X_C$, then $V_L > V_C$. The capacitive effect gets neutralized and the circuit behaves like a *R-L* circuit.

Phasor diagram Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.64(a).

From the circuit diagram, $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$



(a) Phasor diagram

(b) Voltage triangle

Fig. 2.64

It is clear from the phasor diagram that the current I lags behind the applied voltage V by $\phi°$ ($\phi° < 90$). Therefore, nature of the circuit is inductive.

Phase angle of the circuit, $\phi = \tan^{-1} \dfrac{V_L - V_C}{V_R} = \tan^{-1} \dfrac{I(X_L - X_C)}{IR}$

$$= \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

So, power factor of the circuit, $\text{pf} = \cos\phi = \dfrac{V_R}{V} = \dfrac{IR}{IZ} = \dfrac{R}{Z}$ lagging

Impedance (Z) The voltage triangle of the circuit is shown in Fig. 2.64(b). Dividing each of voltage phasor by 'I' in a voltage triangle, we get the impedance triangle as shown in Fig. 2.65.

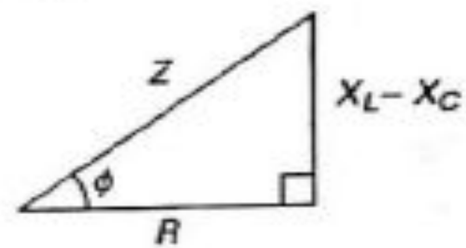


Fig. 2.65 Impedance triangle (when $X_L > X_C$)

From the impedance triangle,

Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$

The impedance can be expressed in rectangular and polar forms as

$$\bar{Z} = [R + j(X_L - X_C)]\ \Omega$$

and $$\bar{Z} = (Z \angle \phi)\ \Omega$$

Case (*ii*) $X_C > X_L$

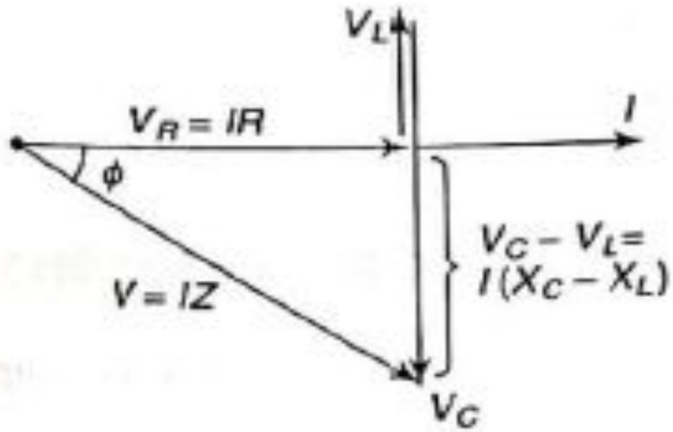
If $X_C > X_L$, then $V_C > V_L$. The inductive effect gets neutralized and the circuit behaves like a R-C circuit.

Case (ii) $X_c > X_L$

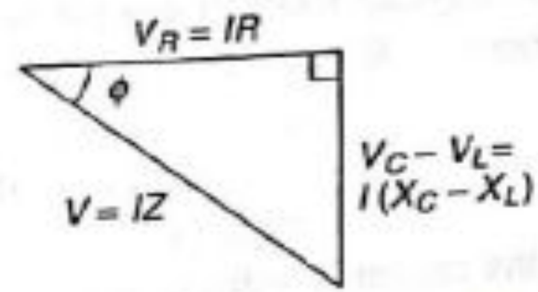
If $X_C > X_L$, then $V_C > V_L$. The inductive effect gets neutralized and the circuit behaves like a *R-C* circuit.

Phasor diagram Taking current as reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.66(a).

From circuit diagram, $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$



(a) Phasor diagram



(b) Voltage triangle

Fig. 2.66

It is clear from the phasor diagram that the current I leads the applied voltage V by $\phi°$ ($\phi° < 90°$). Therefore, nature of the circuit is capacitive.

Phase angle of the circuit, $\phi = \tan^{-1} \dfrac{V_C - V_L}{V_R} = \tan^{-1} \dfrac{I(X_C - X_L)}{IR}$

$$= \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

So, power factor of the circuit, $\text{pf} = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$ leading

Impedance (Z)

The voltage triangle of the circuit is shown in Fig. 2.66(b). Dividing each of voltage phasor by I, we get the impedance triangle as shown in Fig. 2.67.

From the impedance triangle, circuit impedance,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}\ \Omega$$

The impedance can be expressed in rectangular and polar forms as

$$\bar{Z} = [R - j(X_C - X_L)]\ \Omega$$

and $$\bar{Z} = (Z \angle -\phi)\ \Omega$$

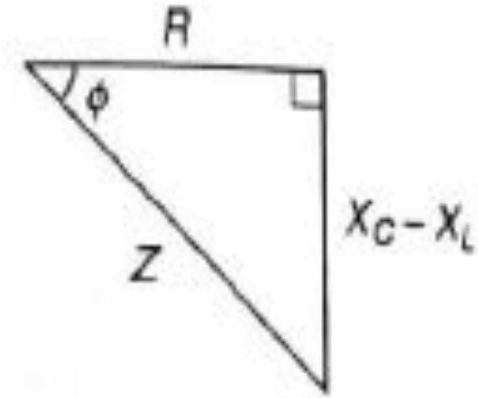


Fig. 2.67 Impendance triangle (when $X_C > X_L$)

2.2.4 Analysis of AC Series Circuit

The following concepts are useful for analysis of an ac series circuit:

1. While analysing the ac series circuits, we need to express the circuit impedance into its rectangular and polar forms.

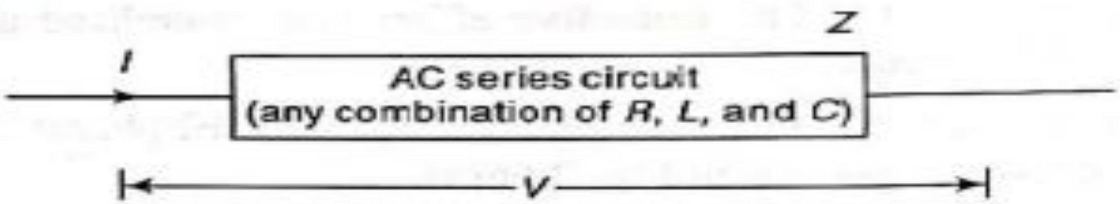


Fig. 2.68(a)

Consider the ac series circuit shown in Fig. 2.68(a).

Rectangular form of circuit impedance, $\overline{Z} = (R \pm jX)\ \Omega$

where R = Real part of impedance, which is the total or equivalent resistance of the circuit

X = Net reactance of the circuit, i.e., $|X_L - X_C|$, imaginary part of impedance

If the circuit is inductive, i.e., $X_L > X_C$, the imaginary part of the impedance will be +ve.

If the circuit is capacitive, i.e., $X_C > X_L$, the imaginary part of impedance will be −ve.

For example, consider an ac series circuit as shown in Fig. 2.68(b).

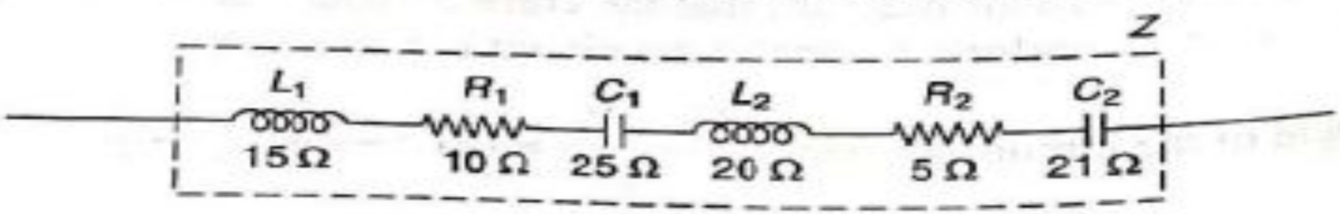


Fig. 2.68(b)

Total resistance of the circuit, $R = R_1 + R_2 = 10 + 5 = 15\ \Omega$
Total inductive reactance, $X_L = X_{L1} + X_{L2} = 15 + 20 = 35\ \Omega$
Total capacitive reactance, $X_C = X_{C1} + X_{C2} = 25 + 21 = 46\ \Omega$
Hence, net reactance, $X = X_C - X_L = 46 - 35 = 11\ \Omega$
Now, rectangular form of circuit impedance can be written as

$$\overline{Z} = (15 - j11)\ \Omega$$

(As $X_C > X_L$, imaginary part of $\overline{Z}$ is negative.)

2. For any unknown circuit, if rectangular form of impedance is given, then we get the following information:
   1. The effective components of the circuit, i.e., real part of $\overline{Z}$ is the total or equivalent resistance of the circuit and imaginary part of $\overline{Z}$ is the net reactance of the circuit.
   2. Nature of the circuit, i.e., if imaginary part is +ve, then circuit is inductive and if imaginary part is –ve, then circuit is capacitive.
   3. Phase angle of the circuit can be calculated as

$$\phi = \tan^{-1} \frac{\text{imaginary part of } \overline{Z}}{\text{real part of } \overline{Z}}$$

For example, consider the ac series circuit shown in Fig. 2.68(c). From rectangular form of impedance, we get

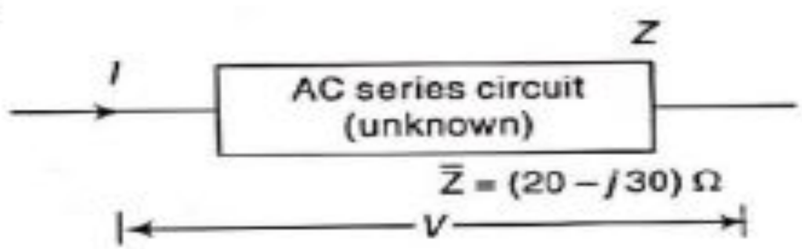


Fig. 2.68(c)

1. $R_{\text{effective}} = 20\ \Omega$, $X_{\text{effective}} = 30\ \Omega$
2. Nature of the circuit is capacitive.
3. Phase angle, i.e., $\phi = \tan^{-1} \dfrac{30}{20} = 56.31°$

3. For any unknown circuit, if polar form of circuit impedance is given, then we get the following information:
   1. The magnitude of impedance (Z).
   2. Phase angle of the circuit, i.e., angle between voltage across the circuit (V) and current through the circuit (I).
   3. Nature of the circuit, i.e., if angle is +ve, then circuit is inductive and if angle is –ve, then it is capacitive.

For example, consider the ac series circuit shown in Fig. 2.68(d).

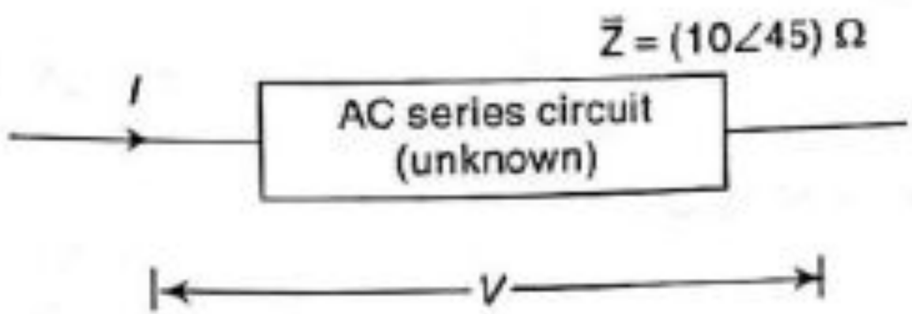


Fig. 2.68(d)

From polar form of impedance, we get the following information:
1. Magnitude of impedance, $Z = 10\ \Omega$.
2. Phase angle, $\phi = 45°$.
3. Nature of the circuit is inductive (as angle is +ve).

4. By using Ohm's law, we can calculate the unknown quantities, i.e.,

Circuit impedance, $Z = \frac{V}{I}$ ohm or $\bar{Z} = \frac{\bar{V}}{\bar{I}}$ ohm

Circuit current, $I = \frac{V}{Z}$ ampere or $\bar{I} = \frac{\bar{V}}{\bar{Z}}$ ampere

Voltage across the circuit, $V = I\,Z$ volt or $\bar{V} = \bar{I}\,\bar{Z}$ volt

Example 2.29 A coil having a resistance of 7 Ω and an inductance of 31.8 mH is connected to a 230 V, 50 Hz supply. Calculate (i) circuit current, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution

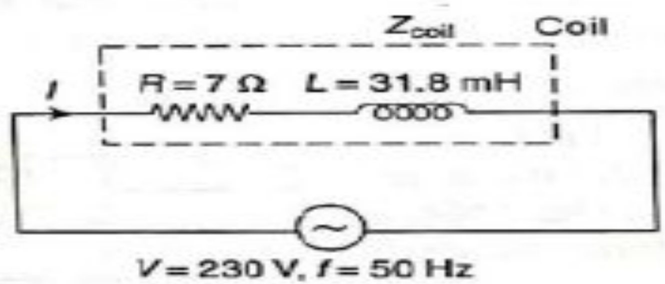


Fig. 2.69

We have Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Impedance of coil, $Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{(7)^2 + (10)^2} = 12.21\ \Omega$

(i) By Ohm's law,

Circuit current, $I = \frac{V}{Z_{coil}} = \frac{230}{12.21} = 18.84$ A

(ii) Phase angle, $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{10}{7} = 55°$

(iii) Power factor, PF = $\cos\phi = \cos 55° = 0.574$ lagging
(As circuit is inductive, pf is lagging.)

(iv) Power consumed, $P = VI\cos\phi$

$= 230 \times 18.84 \times 0.574 = 2487.26$ W

Example 2.30 For a circuit in Fig. 2.70, determine the (i) circuit impedance (ii) circuit current (iii) power factor (iv) active power (v) reactive power (vi) apparent power.

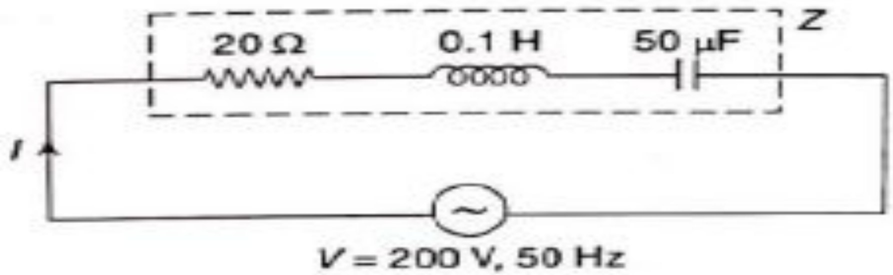


Fig. 2.70

Solution

Given: $V = 200$ V, $f = 50$ Hz, $R = 20\ \Omega$

$L = 0.1\ \text{H} \Rightarrow X_L = 2\pi fL = 31.42\ \Omega$

$$C = 50 \times 10^{-6}\ \text{F} \Rightarrow X_C = \frac{1}{2\pi fC} = 63.66\ \Omega$$

Net reactance, $X = X_C - X_L = 32.24\ \Omega$

As $X_C > X_L$, circuit is capacitive.

Taking applied voltage as reference,

$$\overline{V} = (200 \angle 0)\ \text{V}$$

$$\overline{Z} = (20 - j32.24)\ \Omega$$

or $$\overline{Z} = (37.94 \angle -58.19)\ \Omega$$

(i) From polar form of impedance, $Z = 37.94\ \Omega$

(ii) By Ohm's law,

$$\text{Circuit current, } \overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{(200\angle 0)}{(37.94\angle -58.19)} = (5.27 \angle 58.19)\ \text{A}$$

$\bar{Z} \quad (37.94\angle -58.19)$

(iii) Power factor, pf $= \cos\phi$

$= \cos(-58.19°)$

$= 0.527$ leading

[As circuit is capacitive ($X_C > X_L$), pf is leading]

(iv) Active power, $P = VI\cos\phi$

$= 200 \times 5.27 \times \cos(-58.19°)$

$= 555.46$ W

(v) Reactive power, $Q = VI\sin\phi$

$= 200 \times 5.27 \times \sin(-58.19°)$

$= -895.69$ VAR

(vi) Apparent power, $S = VI$

$= 200 \times 5.27$

$= 1054$ VA

Example 2.31 For a circuit shown in Fig. 2.71, determine (i) circuit current, (ii) voltage drop V_1, and (iii) voltage drop V_2.

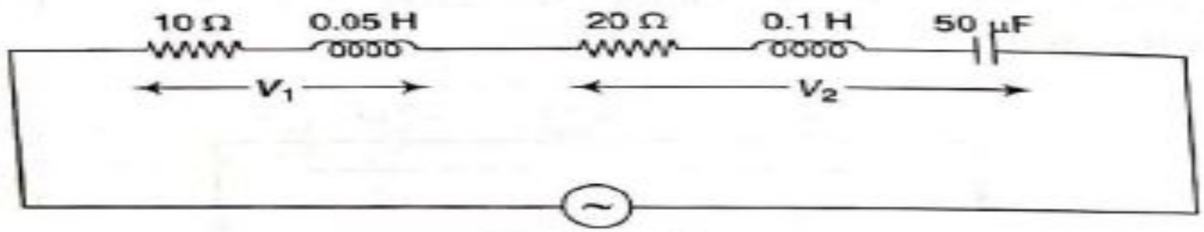


Fig. 2.71

Solution

Let us redraw then the given circuit and assume the different unknown quantities. We then get the circuit as shown in Fig. 2.72.

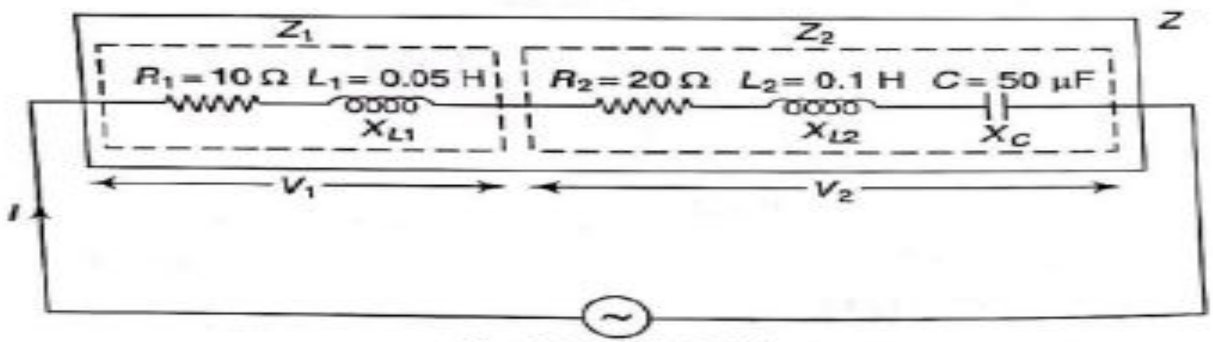


Fig. 2.72

Taking applied voltage as reference,

$$\bar{V} = (200 \angle 0)\ \text{V}, \qquad R = R_1 + R_2 = 10 + 20 = 30\ \Omega$$

$$L_1 = 0.05\ \text{H} \Rightarrow X_{L_1} = 2\pi f L_1 = 15.71\ \Omega$$

$$L_2 = 0.1\ \text{H} \Rightarrow X_{L_2} = 2\pi f L_2 = 31.42\ \Omega$$

So, $\quad X_L = X_{L_1} + X_{L_2} = 47.13\ \Omega$

$$C = 50 \times 10^{-6}\ \text{F} \Rightarrow X_C = \frac{1}{2\pi f C} = 63.66\ \Omega$$

Net reactance, $X = X_C - X_L = 16.53\ \Omega$

As $X_C > X_L$, circuit is capacitive.

Total impedance of the circuit,

$$\bar{Z} = (30 - j16.53)\ \Omega$$

$$\bar{Z} = (34.25 \angle -28.85)\ \Omega$$

By Ohm's law,

Circuit current, $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200 \angle 0}{(34.25 \angle -28.85)} = (5.84 \angle 28.85)$ A

From circuit diagram,

Impedance, $\bar{Z}_1 = (R_1 + j X_{L_1})\ \Omega$

$$= (10 + j15.71)\ \Omega$$

$$= (18.62 \angle 57.52)\ \Omega$$

Impedance, $\bar{Z}_2 = [R_2 - j(X_C - X_{L_2})]\ \Omega$

$$= [20 - j(63.66 - 31.42)]\ \Omega$$

$$= (20 - j32.24)\ \Omega$$

$$= (37.94 \angle -58.19)\ \Omega$$

By Ohm's law,

$$\bar{V}_1 = \bar{I}\bar{Z}_1 = (5.84 \angle 28.85)(18.62 \angle 57.52) = (108.74 \angle 86.37)\ \text{V}$$

and $\bar{V}_2 = \bar{I}\bar{Z}_2 = (5.84 \angle 28.85)(37.94 \angle -58.19) = (221.57 \angle -29.34)$ V

Example 2.33 A voltage $\bar{V} = (150 + j180)$V is applied across an impedance and the current is found to be $\bar{I} = (5 - j4)$A. Determine (i) scalar impedance, (ii) reactance, and (iii) power consumed.

Solution

Given:
$$\bar{V} = (150 + j180) \text{ V}$$
$$= (234.31 \angle 50.19) \text{ V}$$
$$\bar{I} = (5 - j4) \text{ A}$$
$$= (6.4 \angle -38.66) \text{ A}$$

Circuit impedance,

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(234.31 \angle 50.19)}{(6.4 \angle -38.66)}$$
$$= (36.61 \angle 88.85)\ \Omega$$
$$= (0.73 + j36.6)\ \Omega$$

(i) From polar form of impedance, scalar impedance, $Z = 36.61\ \Omega$

(ii) From rectangular form of impedance, reactance = $36.6\ \Omega$

(iii) Power consumed,

$$P = VI \cos \phi$$
$$= 234.31 \times 6.4 \times \cos 88.85°$$
$$= 30.096 \text{ W}$$

$= 30.096\ W$

Example 2.34 The voltage applied to a series circuit consisting of two pure elements is given by $v = 180 \sin \omega t$ and the resulting current is given by $i = 2.5 \sin (\omega t - 45)$. Find the average power taken by the circuit and values of the elements.

Solution

The conditions of the example are shown in Fig. 2.74.

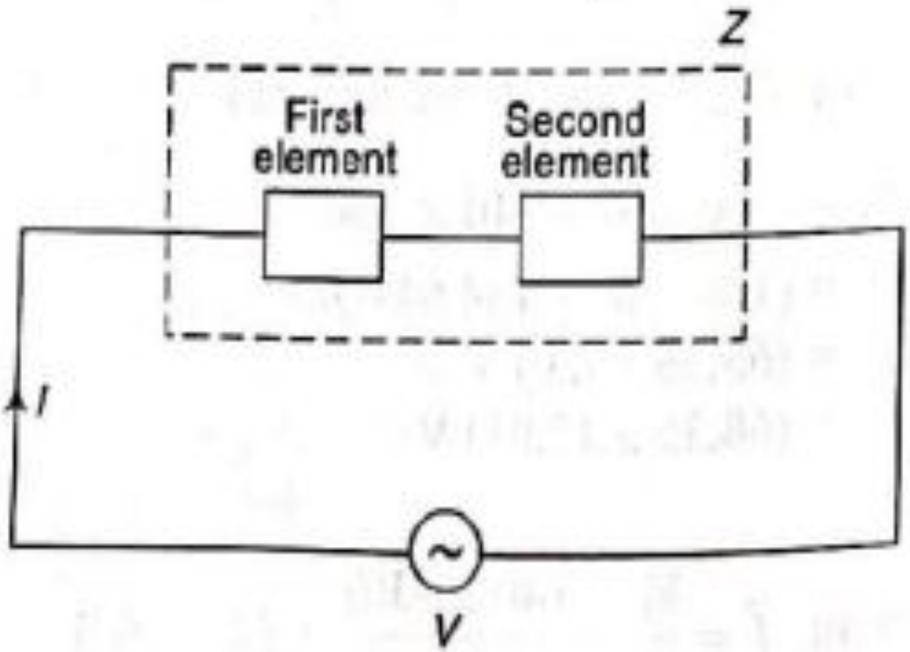


Fig. 2.74

Given:

$$v = 180 \sin \omega t$$

So, $\bar{V} = (127.28 \angle 0)$ V

$$i = 2.5 \sin (\omega t - 45)$$

Also given:

$$\bar{I} = (1.77 \angle -45) \text{ A}$$

So, By Ohm's law, circuit impedance can be calculated as

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(127.28 \angle 0)}{(1.77 \angle -45)}$$

$$= (71.91 \angle 45)\ \Omega$$

$$= (50.85 + j50.85)\ \Omega$$

Power, $P = VI \cos \phi$

$$= 127.28 \times 1.77 \times \cos 45^\circ$$

$$= 159.3 \text{ W}$$

From rectangular form of impedance, values of the circuit elements are:

$$R = 50.85\ \Omega, X_L = 50.85\ \Omega$$

Fig. 2.106

2.2.5 Series Resonance

In many electrical circuits, resonance is very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of a radio receiver to select a certain frequency transmitted by the station and to eliminate frequencies from other stations is based on the principle of resonance. In a series *R-L-C* circuit, the current either lags behind or leads the applied voltage depending upon the values of X_L and X_C. X_L causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive.

Consider a series *R-L-C* circuit as shown in Fig. 2.107(a). The net reactance of the circuit is

$$X = (X_L - X_C)\ \Omega$$

where $X_L = 2\pi f L\ \Omega$ and $X_C = \dfrac{1}{2\pi f C}\ \Omega$

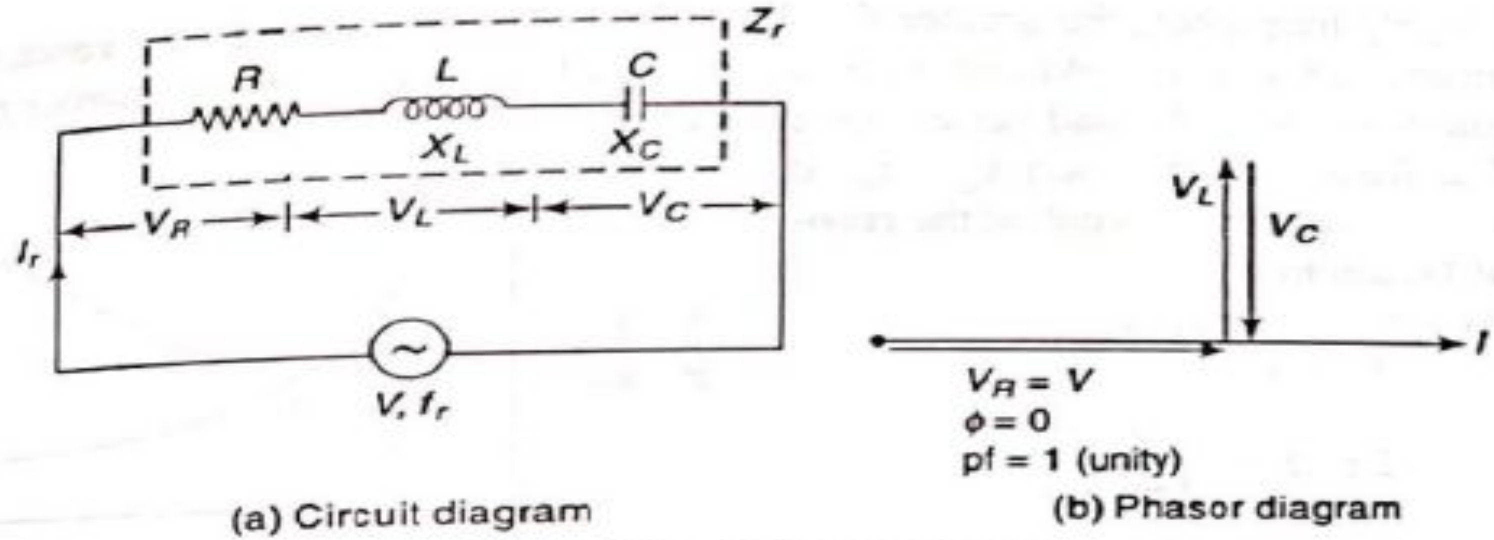


Fig. 2.107 Series resonance

The inductive reactance is directly proportional to the supply frequency, whereas the capacitive reactance is inversely proportional to the supply frequency. At a certain supply frequency, called resonance frequency (f_r), the inductive reactance becomes equal to the capacitive reactance, and the net reactance (X) becomes zero. Therefore, impedance (Z) of the circuit becomes purely resistive (i.e., $Z = R$). In other words, the whole circuit behaves as a purely resistive circuit and the current remains in phase with the applied voltage (pf = 1). This condition is said to be the condition for electrical resonance. Figure 2.107(b) shows the phasor diagram at resonance.

A circuit containing reactive elements (L and C) is resonant when the circuit power factor is unity, i.e., the applied voltage and the circuit current are in phase. If such condition occurs in a series circuit, it is termed as **series resonance.**

For a series resonance, the circuit power factor must be unity, which is possible only if the net reactance of the circuit is zero, i.e., $X_L - X_C = 0$ or $X_L = X_C$.

Effects of series resonance

1. The net reactance of the circuit is zero. Therefore, impedance of the circuit is minimum and is equal to the resistance of the circuit, i.e., $Z_r = R\ \Omega$.
2. The current in the circuit is maximum as it is limited by the resistance of the circuit alone. So, $I_r = \dfrac{V}{Z_r} = \dfrac{V}{R}$
3. As the current is at its maximum value, the power absorbed by the circuit will also be at its maximum value.
4. Since at series resonance, the current flowing in the circuit is very large, the voltage drop across L and C are also very large. In fact, these drops are much greater than the applied voltage. However, voltage drop across L-C combination as a whole will be zero because these drops are equal in magnitude but 180° out of phase with each other.

Resonant frequency (f_r)

The series resonance (i.e., $X_L = X_C$) can be achieved by changing the supply frequency because X_L $(=2\pi fL)$ and X_C $(=1/2\pi fC)$ are frequency dependent. Higher the supply frequency, the greater the X_L and smaller the X_C and vice versa. This indicated in Fig. 2.108. At certain frequency, called the resonant frequency f_r, X_L becomes equal to X_C and series resonance occurs.

The frequency at which $X_L = X_C$ in a *R-L-C* series circuit is called the **resonant frequency** f_r.

At series resonance,

$$X_L = X_C$$

or $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

So, $$f_r^2 = \frac{1}{4\pi^2 LC}$$

or $$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ hertz} \tag{2.12}$$

or $$2\pi f_r = \frac{1}{\sqrt{LC}}$$

Hence, $$\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec} \tag{2.13}$$

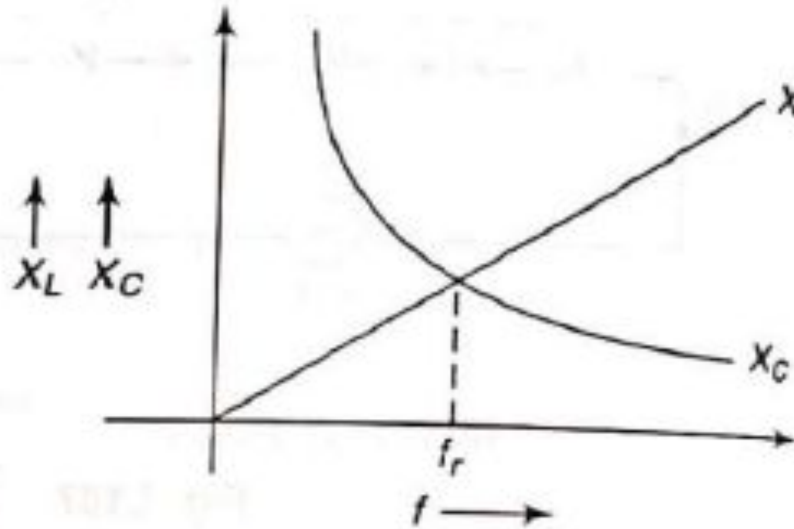


Fig. 2.108 Variation of reactances with frequency

Resonance curve

The curve between current and frequency is known as **resonance curve**. Figure 2.109 shows the resonance curve of the typical R-L-C series circuit. Note that current reaches at its maximum value at the resonant frequency (f_r), falling off rapidly on either side at that point. It is because if the frequency is below f_r, $X_C > X_L$ and the net reactance is no longer zero. If the frequency is above f_r, then $X_L > X_C$ and the net reactance is again not zero. In both the cases, the circuit impedance will be more than the impedance Z_r ($=R$) at resonance.

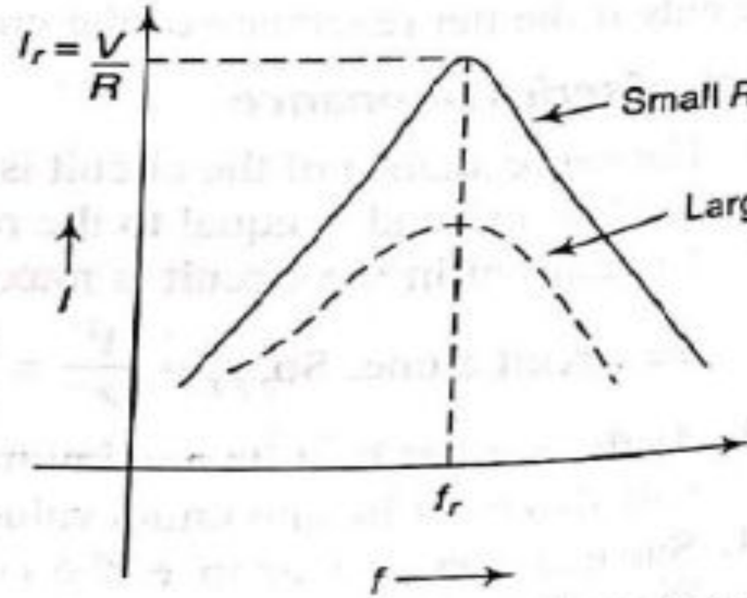


Fig. 2.109 Variation of circuit current with frequency

The result is that the magnitude of the circuit current decreases rapidly as the frequency changes from the resonant frequency.

The shape of the resonance curve depends upon the value of resistance (R). The smaller the resistance, the greater the current at resonance and sharper the curve. On the other hand, the greater the resistance, the lower the resonant peak and flatter the curve.

Q-factor of series resonance circuit

At series resonance, the voltage drops across L or C are very large. In fact, these drops are much greater than the applied voltage. This voltage magnification produced by resonance is termed as Q-factor of the series resonant circuit (Q stands for quality), i.e.,

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}} \tag{2.14}$$

$$= \frac{V_L \text{ or } V_C}{V}$$

$$= \frac{V_L}{V_R} \qquad (\because \text{ At resonance, } V = V_R)$$

$$= \frac{I_r X_L}{I_r R}$$

$$= \frac{X_L}{R}$$

So, $$Q\text{-factor} = \frac{2\pi f_r L}{R} \tag{2.15}$$

We know that $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Substituting the value of f_r in Eq. (2.15), we get

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2.16}$$

Bandwidth of series resonance circuit

Bandwidth (BW) of a series resonance circuit is defined as the range of frequency over which circuit current is equal to or greater than 70.7% of maximum current (i.e., I_r, current at resonance).

The current in series *R-L-C* circuit changes with frequency. Referring to the resonance curve in Fig. 2.110, it is clear that for any frequency lying between f_1 and f_2, the circuit current is equal to or greater than 70.7% of maximum current (i.e., $I_r = V/R$). Therefore, $(f_2 - f_1)$ is the bandwidth of the circuit, i.e.,

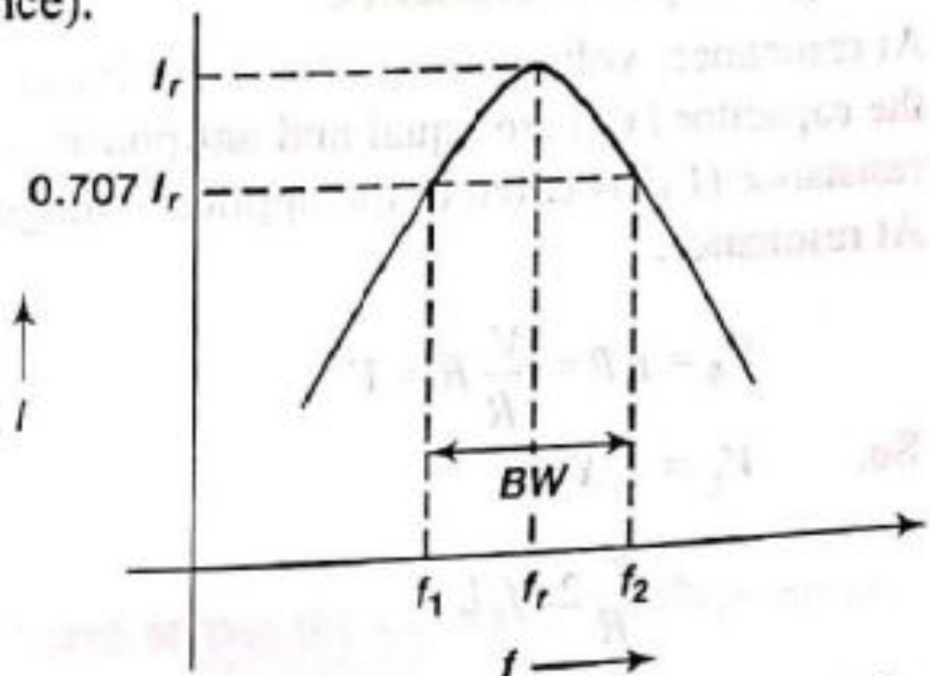


Fig. 2.110 Bandwidth of series resonance circuit

Bandwidth, $\text{BW} = (f_2 - f_1)$ Hz

or

$$\text{BW} = (\omega_2 - \omega_1) \text{ rad/sec} \tag{2.17}$$

It can be proved that, for a series resonance circuit,

$$\text{BW} = \frac{R}{2\pi L}\,\text{Hz} \tag{2.18}$$

or $$\text{BW} = \frac{R}{L}\ \text{rad/sec}$$

Note that f_1 and f_2 are the limiting frequencies at which current is exactly equal to 70.7% of the maximum value. The frequency f_1 is called lower cut-off frequency and the frequency f_2 is called upper cut-off frequency. The resonant frequency is sufficiently centred w.r.t. the two cut-off frequencies (f_1 and f_2). Then

$$f_2 = f_r + \frac{\text{BW}}{2} \tag{2.19}$$

$$f_1 = f_r - \frac{\text{BW}}{2}$$

In case of angular frequency (ω), we have

$$\omega_2 = \omega_r + \frac{\text{BW}}{2}$$

$$\omega_1 = \omega_r - \frac{\text{BW}}{2} \tag{2.20}$$

2

Relation between Q-factor and bandwidth (BW)

We know that

$$Q\text{-factor} = \frac{2\pi f_r L}{R}$$

or $$Q\text{-factor} = \frac{2\pi L}{R} f_r$$

or $$Q\text{-factor} = \frac{f_r}{BW} \qquad \left(\because \frac{2\pi L}{R} = \frac{1}{\text{BW}}\right)$$

or $$f_r = Q\text{-factor} \times \text{BW} \tag{2.21}$$

Voltage drops at resonance

At resonance, voltage drop across the inductance (V_L) and voltage drop across the capacitor (V_C) are equal and antiphase with each other. The drop across the resistance (V_R) is equal to the applied voltage (V).

At resonance,

$$V_R = I_r R = \frac{V}{R} R = V \tag{2.22}$$

So, $$V_L = I_r X_L$$

$$= \frac{V}{R} 2\pi f_r L$$

$$= \frac{V}{R} 2\pi \frac{1}{2\pi\sqrt{LC}} L \qquad \left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{V}{R}\sqrt{\frac{L}{C}} \tag{2.23}$$

Also $V_C = I_r X_C$

$$= \frac{V}{R} \cdot \frac{1}{2\pi f_r C}$$

$$= \frac{V}{R} \cdot \frac{1}{2\pi \frac{1}{2\pi\sqrt{LC}} C} \qquad \left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{V}{R}\sqrt{\frac{L}{C}} \tag{2.24}$$

Example 2.57 A series R-L-C circuit has the following parameter values: $R = 10\ \Omega$, $L = 0.014$ H, $C = 100\ \mu$F.

Compute the following:

(a) Resonance frequency in rad/sec

(b) Quality factor of the circuit

(c) Bandwidth

(d) Lower and upper frequency points of the bandwidth

(e) Maximum value of the voltage appearing across the capacitor if the voltage $v = 1\sin 1000t$ is applied to the R-L-C circuit.

Solution

(a) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.014 \times 100 \times 10^{-6}}} = 845.15$ rad/sec

(b) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{0.014}{100 \times 10^{-6}}} = 1.183$

(c) $\text{BW} = \dfrac{R}{L} = \dfrac{10}{0.014} = 714.29$ rad/sec

(d) Lower and upper frequency points of the bandwidth

$$\omega_1 = \omega_r - \frac{\text{BW}}{2} = 845.15 - \frac{714.29}{2} = 488 \text{ rad/sec}$$

$$\omega_2 = \omega_r + \frac{\text{BW}}{2} = 845.15 + \frac{714.29}{2} = 1202.3 \text{ rad/sec}$$

(e) Applied voltage, $v_1 = 1 \sin 1000t$

So, $\quad V = \dfrac{V_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = 0.707$

At resonance, voltage that appears across the capacitor is maximum and given by

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{0.707}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 0.837 \text{ V}$$

Example 2.58 A voltage of $v = 10 \sin \omega t$ is applied to R-L-C circuit. At the resonance frequency of the circuit, the maximum voltage across the capacitor is found to be 500 V. Moreover, the bandwidth is known to be 400 rad/sec. and impedance at resonance is 100 Ω.

(i) Find the resonant frequency.
(ii) Compute the upper and lower limits of the bandwidth.
(iii) Determine the value of L and C for this circuit.

Solution

A voltage of $v = 10 \sin \omega t$ is applied to R-L-C circuit as shown in Fig. 2.111.

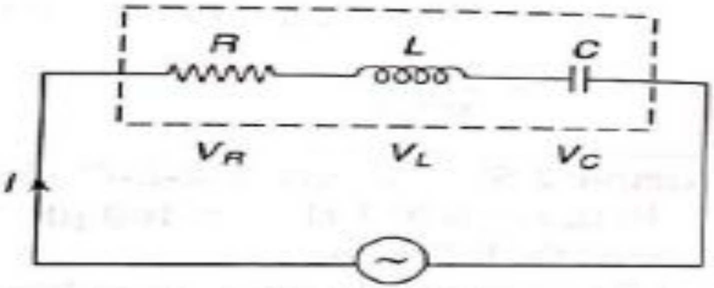


Fig. 2.111

Given: $v = 10 \sin \omega t$

RMS value of the applied voltage,

$$V = \frac{10}{\sqrt{2}} = 7.071 \text{ V}$$

At resonance, $V_C = 500$ V

Bandwidth, BW = 400 rad/sec

or $$\text{BW} = \frac{400}{2\pi} = 63.66 \text{ Hz}$$

Impedance at resonance, $Z_r = R = 100\ \Omega$

(i) $$Q\text{-factor} = \frac{V_C}{V} = \frac{500}{7.071} = 70.71$$

Resonance frequency, $f_r = \text{BW} \times Q\text{-factor}$
$= 63.66 \times 70.71$
$= 4501.4$ Hz

(ii) Lower limit of the bandwidth,

$$f_1 = f_r - \frac{\text{BW}}{2} = 4501.4 - \frac{63.66}{2} = 4469.57 \text{ Hz}$$

Now, upper limit of the bandwidth

$$f_2 = f_r + \frac{\text{BW}}{2} = 4501.4 + \frac{63.66}{2} = 4533.23 \text{ Hz}$$

(iii) $\text{BW} = \dfrac{R}{2\pi L}$ Hz

or $\quad 63.66 = \dfrac{100}{2\pi \times L}$

or $\quad L = 0.25\ \text{H}$

Now, $\quad f_r = \dfrac{1}{2\pi\sqrt{LC}}$

So, $4501.4 = \dfrac{1}{2\pi\sqrt{0.25 \times C}}$

or $\quad C = 5 \times 10^{-9}\ \text{F}$

or $\quad C = 5\ \text{nF}$

Example 2.59 A constant voltage of a frequency of 1 MHz is applied to a coil (inductor) in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value, while it is reduced to one half when the capacitance is 600 pF. Find (i) the resistance of the coil, (ii) the inductance of the coil, and (iii) the Q-factor of the circuit.

Solution

A coil (inductor) is connected in series to a variable capacitor. The resultant circuit is a series R-L-C circuit (see Fig. 2.112).

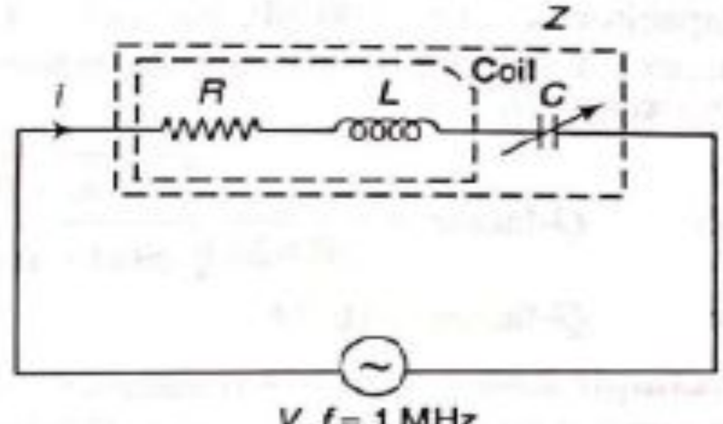


Fig. 2.112

Case (a) When $C = 500$ pF, current has its maximum value, i.e., series resonance occurs.

At series resonance,

Circuit current, $I_r = \dfrac{V}{R}$

At series resonance,

$$X_L = X_C$$

So, $$X_L = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1 \times 10^6 \times 500 \times 10^{-12}}$$

or $X_L = 318.31\ \Omega$

or $2\pi f L = 318.31$

or $L = 50.66 \times 10^{-6}$ H

or $L = 50.66\ \mu$H

$\therefore \quad L = 50.66\ \mu H$

Case (b) When $C = 600$ pF, current reduces to one-half, i.e., circuit current $= I_r/2$.
By Ohm's law, the circuit current can be written as

$$\frac{I_r}{2} = \frac{V}{Z}, \quad \text{where } Z \text{ is impedance of the circuit.}$$

So, $$\frac{I_r}{2} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad \left(\because Z = \sqrt{R^2 + (X_L - X_C)^2}\right)$$

or $$\frac{V}{2 \times R} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad \left(\because I_r = \frac{V}{R}\right)$$

or $$\frac{1}{2 \times R} = \frac{1}{\sqrt{R^2 + \left[(2\pi \times 1 \times 10^6 \times 50.66 \times 10^{-6}) - \dfrac{1}{(2\pi \times 1 \times 10^6 \times 600 \times 10^{-12})}\right]^2}}$$

Hence, $R = 30.623\ \Omega$

Now, Q-factor can be calculated as

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The current magnification produced at resonance is known as **Q-factor**. When capacitor is set to 500 pF, the current has its maximum value, i.e., resonance occurs. Therefore, in the above expression of Q-factor, we have to substitute $C = 500 \times 10^{-12}$ F.

So, $$Q\text{-factor} = \frac{1}{30.623}\sqrt{\frac{50.66 \times 10^{-6}}{500 \times 10^{-12}}}$$

or $$Q\text{-factor} = 10.39$$

Example 2.60 A coil of resistance 2 Ω and inductance of 0.01 H is connected in series with a capacitor across 230 V mains. What must be the capacitance, in order that maximum current occurs at a frequency of 50 Hz? Find also the current and the voltage across the capacitor.

Solution

A coil is connected in series with a capacitance. The resultant circuit is a series *R-L-C* circuit (see Fig. 2.113).

The value of capacitance is required. So, at 50 Hz, maximum current flows in the circuit (i.e., resonance occurs in the circuit). Thus, resonant frequency, $f_r = 50$ Hz.

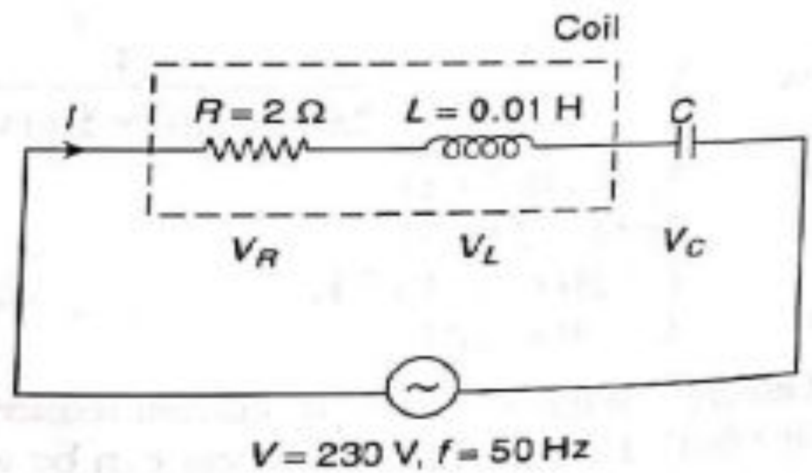


Fig. 2.113

Now, $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or $$50 = \frac{1}{2\pi\sqrt{0.01 \times C}}$$

or $$C = 1.013 \times 10^{-3} \text{ F}$$

At resonance circuit current, $I_r = \dfrac{V}{R} = \dfrac{230}{2} = 115$ A

At resonance, voltage across the capacitor can be calculated as

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{230}{2}\sqrt{\frac{0.01}{1.013 \times 10^{-3}}} = 361.32 \text{ V}$$

Example 2.64 A 20 Ω resistor is connected in series with an inductor and a capacitor, across a variable frequency, 25 V supply. When the frequency is 400 Hz, the current is at its maximum value of 0.5 A and the potential difference across the capacitor is 150 V. Calculate the resistance and inductance of the inductor.

Solution

The conditions of the example are shown in Fig. 2.117.

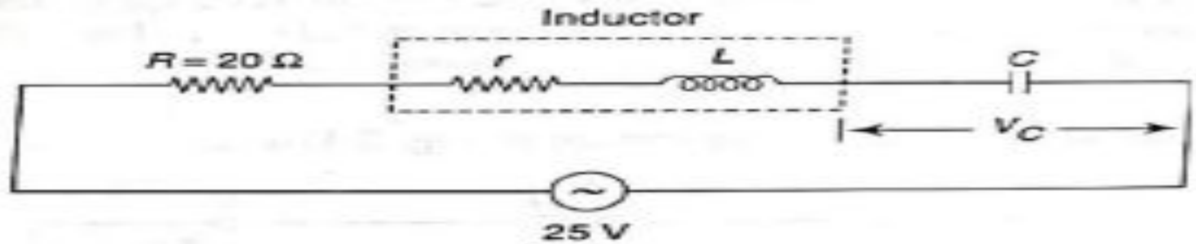


Fig. 2.117

When frequency is 400 Hz, the current is at the maximum value of 0.5 A and potential difference across the capacitor is 150 V. This is resonance condition. So, at resonance,

$$I_r = \frac{V}{(R+r)}$$

$$\therefore \quad 0.5 = \frac{25}{(R+r)}$$

$$\therefore \quad (R+r) = 50\ \Omega$$

$$\therefore \quad (20+r) = 50$$

$$\therefore \quad r = 30\ \Omega$$

Now, $V_C = I_r X_C$

$$150 = 0.5 X_C$$

$$\therefore \quad X_C = 300\ \Omega$$

Now, at resonance,

$$X_L = X_C$$

$$\therefore \quad 2\pi \times 400 \times L = 300$$

$$\therefore \quad L = 119.37 \text{ mH}$$