47_ YASH SHARANG_ DIAD PHYSICS ASSIGNMENT

Q1. Show that divergence of curl of vector is zero.

→ To show $\bar{\nabla} \cdot (\bar{\nabla} \times V) = 0$.

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ V_y & V_z \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ V_x & V_z \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ V_x & V_y \end{vmatrix}\right)$$

$$= \frac{\partial}{\partial x}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ V_y & V_x \end{vmatrix} - \frac{\partial}{\partial y}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ V_x & V_z \end{vmatrix} + \frac{\partial}{\partial z}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ V_x & V_y \end{vmatrix}$$

$$= \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix} = 0.$$

$\therefore$ The value of the determinant of is zero because two rows are identical.

Q2. Derive all Maxwell's equations in ~~force~~ free space in differential forms.

→ (a) First equation.

$$\iint \bar{E}.ds = \oint_a \bar{E}.ds = \frac{1}{\varepsilon_0} \oint_v (\rho + \rho_p)\, dV$$

where $q = \rho V$ and $\rho_p = \bar{\nabla}.\bar{p}$

$$\oint_a E\, \varepsilon_0\, ds = \oint_V \bar{\nabla}.(\varepsilon_0 \bar{E})\, dV = \oint_V (\rho - \bar{\nabla}.\bar{p}).dV$$

$$\therefore \oint_v \bar{\nabla}.(\varepsilon_0 \bar{E} + \rho)\, dV = \oint_V \rho\, dV$$

As $\bar{D} = \varepsilon_0 \bar{E} + P$ $\quad \therefore \oint_v \bar{\nabla}.\bar{D}\, dV = \oint_v \rho\, dV.$

$$\therefore \bar{\nabla}.\bar{D} - \rho = 0.$$

$$\bar{\nabla}.\bar{D} = \rho.$$

This the Point form of Maxwell's first equation.

(b) Second equation

$$\oint B.ds = 0 \quad , \quad \oint B.ds = \oint_v \bar{\nabla}.\bar{B}.dV = 0.$$

$$\therefore \bar{\nabla}.\bar{B} = 0.$$

This is the point form of Maxwell's second equation

c) Third equation.

$$e = -\frac{d\phi}{dt} \qquad \phi = \oint_s \bar{B}.ds$$

$$e = -\frac{d}{dt}\left(\oint_s \bar{B}.ds\right) = -\oint_s \left(\frac{d\bar{B}}{dt}\right) ds$$

$$e = \oint_l \bar{E}.dt \qquad \oint_l \bar{E}.dl = -\oint_s \left(\frac{d\bar{B}}{dt}\right) ds$$

$$\oint_l \bar{E}.dl = \oint_s (\bar{\nabla} \times \bar{E})\, ds.$$

$$\oint_s (\bar{\nabla} \times \bar{E})\, ds = -\oint_s \left(\frac{d\bar{B}}{dt}\right). ds.$$

$$\therefore \oint_s \left[(\bar{\nabla} \times \bar{E}) + \frac{d\bar{B}}{dt}\right] ds = 0.$$

$$\therefore \bar{\nabla} \times \bar{E} = -\frac{d\bar{B}}{dt}$$

This is Maxwell's third equation.

(d) Fourth equation.

$$\oint \bar{H}.dl = I = \oint \bar{J}.ds.$$

$$\oint \bar{J}.ds = \oint (\bar{\nabla} \times \bar{H})\, ds.$$

$$\oint (\bar{J} - \bar{\nabla} \times \bar{H})\, ds = 0.$$

$$\therefore \bar{J} = \bar{\nabla} \times \bar{H}.$$

$$\text{div}\,\bar{J} = \text{div}\,(\bar{\nabla}\times H) = 0.$$

$$\text{div}\,\bar{J} + \frac{\partial \rho}{\partial t} = 0. \quad \text{or} \quad \text{div}\,\bar{J} = -\frac{\partial \rho}{\partial t} \quad \text{——} \ (2)$$

To correct this, Maxwell suggested that the total current density needs ~~to~~ an additional component i.e $\bar{J}$.

$$\bar{\nabla}\times\bar{H} = \bar{J} + \bar{J}'$$

$$\text{div}\,(\bar{\nabla}\times\bar{H}) = \text{div}\,(\bar{J}+\bar{J}')$$

$$0 = \text{div}\,(\bar{J}+\bar{J}')$$

$$\therefore\ \text{div}\,\bar{J} = \text{div}\,\bar{J}'$$

from eqn (2),

$$\text{div}\,\bar{J} = -\frac{\partial \rho}{\partial t}$$

$$\bar{\nabla}\cdot\bar{D} = \rho \qquad \therefore\ \text{div}\,\bar{J} = \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}) = \bar{\nabla}\cdot\frac{\partial \bar{D}}{\partial t}$$

$$\text{Hence,} \quad \bar{J}' = \frac{\partial \bar{D}}{\partial t}.$$

$\therefore$ Maxwell's fourth equation is

$$\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}.$$

The additional term $\frac{\partial \bar{D}}{\partial t}$ is called Maxwell's correction and is known as displacement current.

Q3. State Stoke's theorem with expression.

→ It states that the circulation of a vector field $\bar{A}$ around a closed path $\bar{C}$ is equal to the surface integral of the curl of $\bar{A}$ over the open surface $\bar{S}$ bounded by $\bar{C}$ provided that $\bar{A}$ and $\bar{\nabla}\times\bar{A}$ are continuos on $\bar{S}$.

$$\oint_C \bar{A}\cdot dl = \int_S (\bar{\nabla}\times\bar{A})\cdot ds.$$

It relates closed line integral of the field with surface integral.

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