DIVISION / ROLLNO. - DIAD - 47

Vivekanand Education Society's Institute of Technology
(Academic year 2020-2021)

Subject - Engineering Mathematics - 2.

Semester - II.

TUTORIAL COVER PAGE

TUTORIAL NO - 1.

TUTORIAL TOPIC - MODULE 1

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SIGNATURE OF THE TEACHER -

1) $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}\left(x + xy^2\right) dy = 0$

$\rightarrow$

$$\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \Big/ N = \frac{12 + 12y^2 - 3 - 3y^2}{3x(1+y^2)} = \frac{9(1+y^2)}{3x(1+y^2)} = \frac{3}{x}.$$

$\therefore$ IF $= e^{\int 3/x \cdot dx} = e^{3\log x} = x^3.$

$$\boxed{\begin{aligned} &\therefore \int M.dx = \int (12x^3y + 4x^3y^3 + 6x^5).dx = 3x^4y + x^4y^3 + x^6 \\ &\therefore \int N.dy = 0. \end{aligned}}$$

2) $x\frac{dy}{dx} + y = x^3y^6.$

$\rightarrow$ Dividing by y^6, we get $\frac{x}{y^6}\frac{dy}{dx} + \frac{1}{y^5} = x^3$

Again dividing by x, we get $\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{xy^5} = x^2$

Put $\frac{1}{y^5} = v$, $\therefore -\frac{5}{y^6} = \frac{dv}{dx}$ $\therefore -\frac{1}{5}\frac{dv}{dx} + \frac{1}{x}.v = x^2$

$\therefore \frac{dv}{dx} - \frac{5}{x}.v = -5x^2.$ which is a linear equation.

$$\int P.dx = \int -\frac{5}{x}\, dx = -5\log x = \log x^{-5}$$

$\therefore$ IF $= e^{\int P.dx} = e^{\log x^{-5}} = x^{-5}.$

$\therefore$ The solution is $v.e^{\int P.dx} = \int Q.e^{\int P.dx} + c.$

$$\therefore vx^{-5} = \int -5x^{-5} \cdot x^{2} \cdot dx + c = -5\int x^{-3} \cdot dx + c = \frac{5}{2} x^{-2} + c.$$

Put $v = 1/y^5$, the solution will be.

$$\frac{1}{y^5} \times x^{-5} = \frac{5}{2} \times \frac{1}{x^2} + c. \qquad \boxed{\therefore \frac{1}{y^5} = \frac{5}{2} x^3 + c \cdot x^5.}$$

3) $\cos(x+y) \cdot dx + [3y^2 + 2y + \cos(x+y)]\, dy = 0.$

$$M = \cos(x+y) \qquad \frac{\partial M}{\partial y} = -\sin(x+y)$$

$$N = [3y^2 + 2y + \cos(x+y)] \qquad \frac{\partial N}{\partial x} = -\sin(x+y)$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$ $\therefore$ The equation is exact.

$$\int M \cdot dx + \int N \cdot dy = c$$

$$\int \cos(x+y) \cdot dx + \int (3y^2 + 2y) \cdot dy = c$$

$$\int (\cos x \cos y - \sin x \sin y) dx + y^3 + y^2 = c.$$

$$\therefore \cos y \sin x + \sin y \cos x + y^3 + y^2 = c$$

$$\boxed{\therefore \sin(x+y) + y^3 + y^2 = c.}$$

4) $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y.$

→ Dividing by $\cos^2 y$.

$\sec^2 y \frac{dy}{dx} + x.\sin 2y.\sec^2 y = x^3.$ $\therefore \sec^2 y \frac{dy}{dx} + 2\tan y.x = x^3$ — ①

Put $\tan y = v$ and differentiate w.r.t x,

$\therefore \sec^2 y \frac{dy}{dx} = \frac{dv}{dx}.$

Hence, from ① we get $\frac{dv}{dx} + 2v.x = x^3.$

$\therefore \int P.dx = \int 2x.dx = x^2.$

$\therefore IF = e^{\int P.dx} = e^{\int 2x.dx} = e^{x^2}.$

$\therefore$ The solution is $ve^{x^2} = \int e^{x^2}.x^3\,dx + c.$

To find the internal put $x^2 = t$, $x\,dx = \frac{dt}{2}$

$I = \int e^t.t.\frac{dt}{2} = \frac{1}{2}\left[t.e^t - \int e^t.dt\right]$

$= \frac{1}{2}\left[te^t - e^t\right] = \frac{1}{2}e^{x^2}(x^2-1)$

$\therefore$ The solution is $ve^{x^2} = \frac{1}{2}e^{x^2}(x^2-1) + c.$

$\therefore \tan y.e^{x^2} = \frac{1}{2}e^{x^2}(x^2-1) + c.$

$\boxed{\therefore \tan y = \frac{1}{2}(x^2-1) + ce^{-x^2}.}$

5) $xy(1+xy^2)\frac{dy}{dx} = 1$

$\rightarrow$ We have $\frac{dx}{dy} = xy(1+xy^2)$

$\therefore \frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$

Putting $-\frac{1}{x} = v$ and $\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$, we get $\frac{dv}{dy} + vy = y^3$.

Which is a linear differential equation.

$\therefore e^{\int p.dy} = e^{\int y.dy} = e^{y^2/2}$

$\therefore$ The solution of $ve^{y^2/2} = \int e^{y^2/2} . y^3 dy + c.$

$= e^{y^2/2}(y^2-2)$ $\left[\text{Put } \frac{y^2}{2} = t\right]$

$\therefore$ The solution is $ve^{y^2/2} = e^{y^2/2}(y^2-2) + c.$

$\therefore -\frac{1}{x} e^{y^2/2} = e^{y^2/2}(y^2-2) + c.$

$\therefore -\frac{1}{x} = y^2 - 2 + ce^{-y^2/2}$

$$\boxed{\therefore \frac{1}{x} = 2 - y^2 + c'e^{-y^2/2}.}$$