DIVISION / ROLL NO.: D1AD - 47

Vivekanand Education Society's Institute of Technology

(Academic Year 2020-2021)

Subject: Engineering Mathematics- I

Semester: I

TUTORIAL/SCILAB COVER PAGE

TUTORIAL /SCILAB NO :- 2

TUTORIAL TOPIC:- Hyperbolic functions

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SIGNATURE OF TEACHER :- ____________________

LTCE

Tutorial 11 : Hyperbolic functions and logarithm of Complex Number.

1) Find the positive value of e^x if $5\sinh x - \cosh x = 5$.

2) Separate into real and imaginary parts $\log(3+4i)$.

3) By considering only principal value, express $(1+i)^i$ in the form of $a+ib$.

4) If $\tan\left(\frac{\pi}{8} + i\alpha\right) = x + iy$, prove that $x^2+y^2+2x=1$.

5) Prove that $\sinh^{-1}(\tan\theta) = \log(\sec\theta + \tan\theta)$.

LTCE

1)

$5 \sinh x - \cosh x = 5$

$\therefore 5\left(\frac{e^{x}-e^{-x}}{2}\right) - \left(\frac{e^{x}+e^{-x}}{2}\right) = 5.$

$5e^{x} - 5e^{-x} - e^{x} - e^{-x} = 10.$

$2e^{x} - 3e^{-x} = 5.$

$2e^{2x} - 3 = 5e^{x}.$

$\therefore 2e^{2x} - 5e^{x} - 3 = 0.$

$\therefore$

$e^{x} = \frac{5 \pm \sqrt{25 - 4(2)(-3)}}{2 \times 2}$

$e^{x} = \frac{5 \pm 7}{4}$ $\quad \therefore e^{x} = \frac{5+7}{4}$ or $e^{x} = \frac{5-7}{4}.$

$e^{x} = 3$ or $e^{x} = -1/2$

but $e^{x} > 0.$

$\boxed{\therefore e^{x} = 3.}$ is the positive value of e^{x}.

2) →

$$\log(3+4i) = \frac{1}{2}\log(3^2+4^2) + i\tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore \log(3+4i) = \log 5 + i\tan^{-1}\left(\left(\frac{4}{3}\right) + 2n\pi\right) = x+iy$$

$\therefore$ Real part $x = \log 5.$

$\therefore$ Imaginary part $y = \tan^{-1}\left(\left(\frac{4}{3}\right) + 2n\pi\right)$

3) →

$$1+i = \sqrt{2} \times \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= \sqrt{2} \times e^{i\pi/4}$$

$$\therefore (1+i)^i = \left(\sqrt{2} \times e^{i\pi/4}\right)^i = (\sqrt{2})^i \times e^{-\pi/4} = y = a+ib.$$

$$\log y = \frac{-\pi}{4} + \frac{i}{2}\log 2.$$

$$\therefore y = e^{-\pi/4 + i/2 \log 2} = e^{-\pi/4} \times e^{\frac{\log 2}{2} i}$$

$$= e^{-\pi/4} \times \left[\cos\left(\frac{\log 2}{2}\right) + i\sin\left(\frac{\log 2}{2}\right)\right]$$

$$y = e^{-\pi/4}\cos\left(\frac{\log 2}{2}\right) + i\left[\sin\left(\frac{\log 2}{2}\right) \times e^{-\pi/4}\right]$$

$$\therefore (1+i)^i = \left[e^{-\pi/4}\cos\left(\frac{\log 2}{2}\right)\right] + i\left[e^{-\pi/4}\sin\left(\frac{\log 2}{2}\right)\right]$$

4)

→ $\tan\left(\frac{\pi}{8}+i\alpha\right) = x+iy$. $\therefore \tan\left(\frac{\pi}{8}-iy\right) = x-iy$.

$\therefore \tan\left[\left(\frac{\pi}{8}+i\alpha\right)+\left(\frac{\pi}{8}-iy\right)\right] = \frac{\tan(\pi/8+i\alpha)+\tan(\pi/8-i\alpha)}{1-\tan(\pi/8+i\alpha)\tan(\pi/8-i\alpha)}$

$\therefore \tan\left(\frac{\pi}{4}\right) = \frac{(x+iy)+(x-iy)}{1-(x^2+y^2)} = \frac{2x}{1-(x^2+y^2)}$

$\therefore 1\left(1-x^2-y^2\right) = 2x$.

$\boxed{\therefore\ x^2+y^2+2x = 1.}$

5)

→ $\sinh^{-1}(\cancel{\tan}\,x) = \log\left(x+\sqrt{x^2+1}\right)$

$\therefore \sinh^{-1}(\tan\theta) = \log\left(\tan\theta+\sqrt{1+\tan^2\theta}\right)$

$\boxed{\therefore \sinh^{-1}(\tan\theta) = \log\left(\sec\theta+\tan\theta\right)}$