SETSQUARE ACADEMY

Degree Engineering (Mumbai University)

F.E. Semester - I

Previous Year Paper Solutions

(December 2007 - May 2016)

Basic Electrical Engineering

Common for all Branches

Theory Questions

(1) Prove that for 3 ϕ, balanced, delta connected load line current is $\sqrt{3}$ times phase current. Also define power triangle in 3 ϕ circuits. **[M-15][2],[D-13][3],[D-12][3],[M-08][10]**

Solution:

Delta Connection:

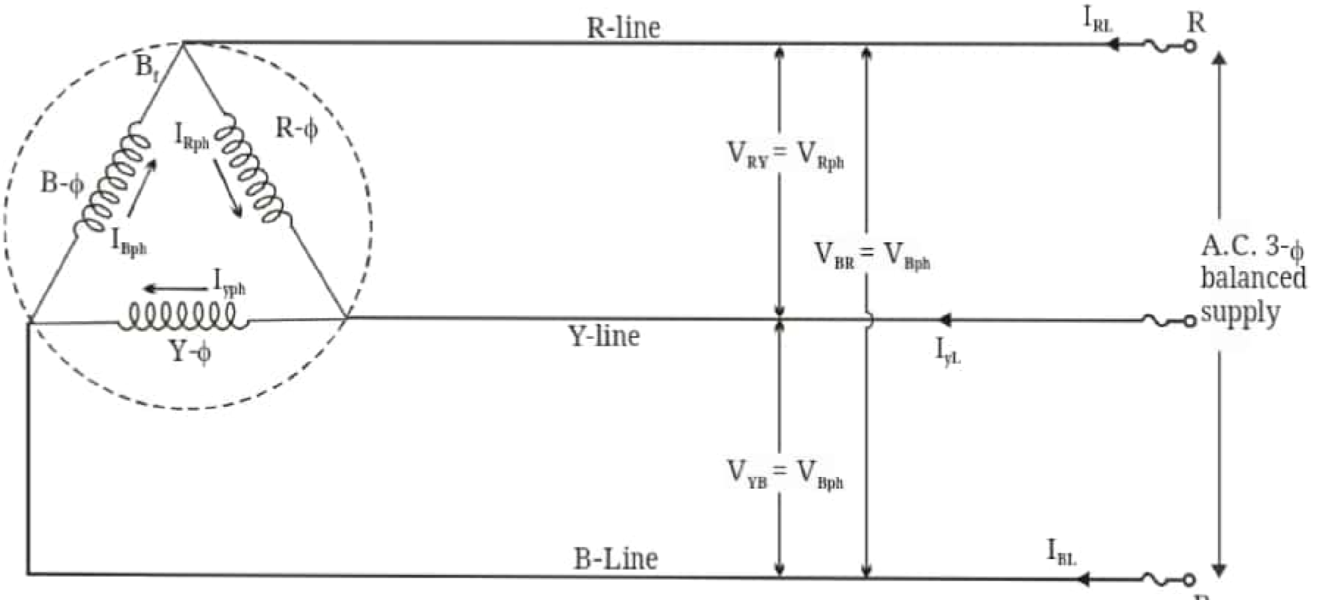


Fig. (a)

Relationship between line and phase voltages:

From the connection diagram, fig.(a) it is seen that phase of the load is connected in parallel with respective two lines.

Thus, $V_L = V_{ph}$

Relationship between line and phase currents:

The phasor diagram is as shown in figure (b).

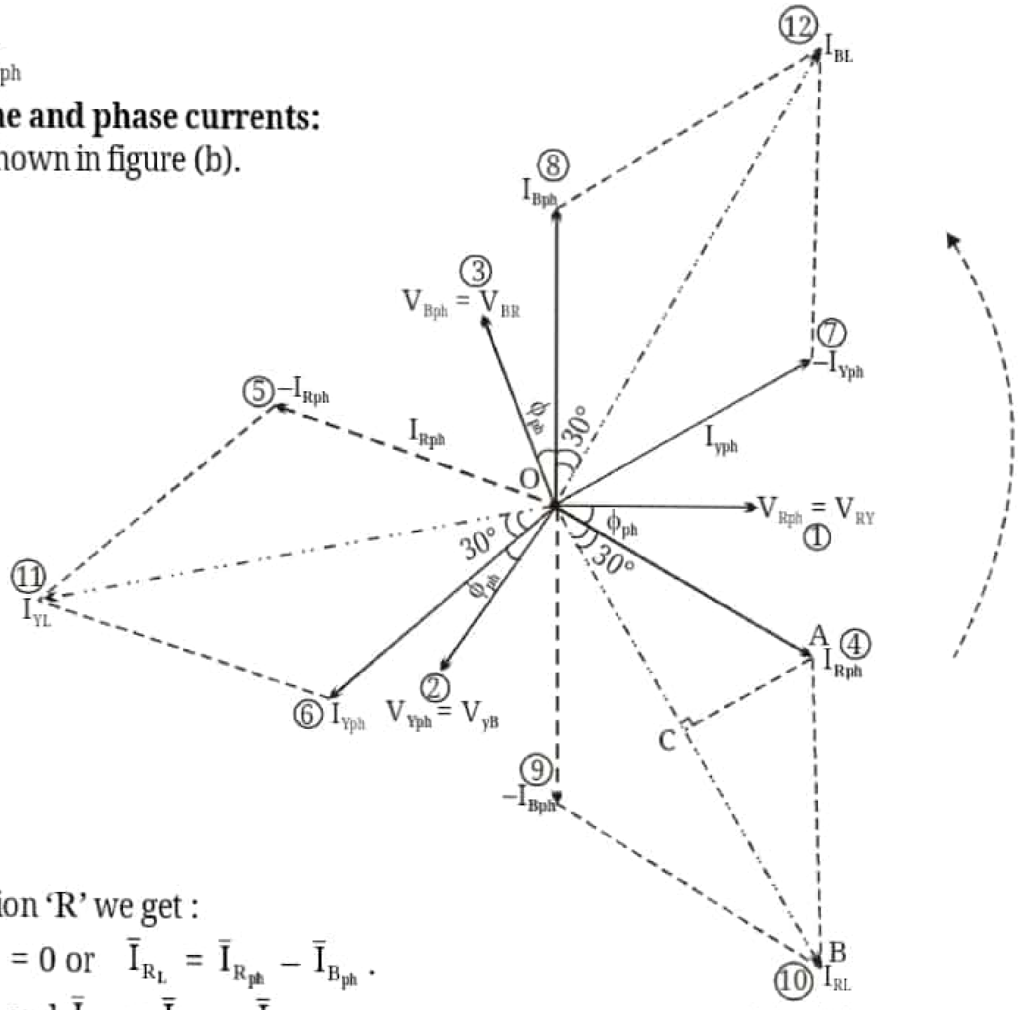


Fig. (b)

Applying KCL at the junction 'R' we get :

$$+\bar{I}_{R_L} - \bar{I}_{R_{ph}} + \bar{I}_{B_{ph}} = 0 \text{ or } \bar{I}_{R_L} = \bar{I}_{R_{ph}} - \bar{I}_{B_{ph}}.$$

Similarly, $\bar{I}_{Y_L} = \bar{I}_{Y_{ph}} - \bar{I}_{R_{ph}}$ and $\bar{I}_{B_L} = \bar{I}_{B_{ph}} - \bar{I}_{y_{ph}}$.

(a) In the phasor diagram OA and OB represent IRph and IRL respectively.

$\therefore$ $OB = I_{RL} = 2(OC) = 2(OA.\cos 30°) = 2(I_{Rph}.\frac{\sqrt{3}}{2}) = \sqrt{3}\, I_{Rph}.$

Thus $I_L = \sqrt{3}\, I_{Ph}.$

(b) From the phasor diagram, it can be seen that I_L lags respective I_{ph} by 30°.

(V) Power:

The total power in the $3-\phi$ system is the sum of powers in each of the 3 phases. fig. (c) represents the tota power triangle. Clearly,

(a) Total active power

$$P_T = 3V_{ph}I_{ph}\cos\phi_{ph} = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi_p$$

$\therefore$ $P_T = \sqrt{3}$ VLIL $\cos\phi_{ph}$. similarly,

(b) Total reactive power

$Q_{TL} = \sqrt{3}\, V_L I_L \sin f_{ph}$

(c) Total apparent power

$S_T = \sqrt{3}\, V_L I_L$

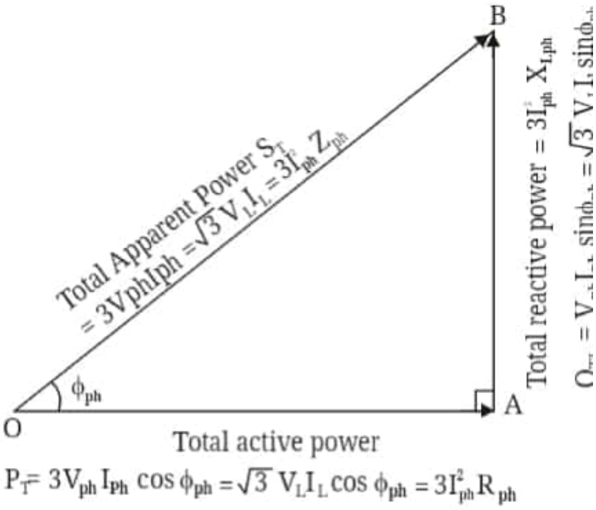
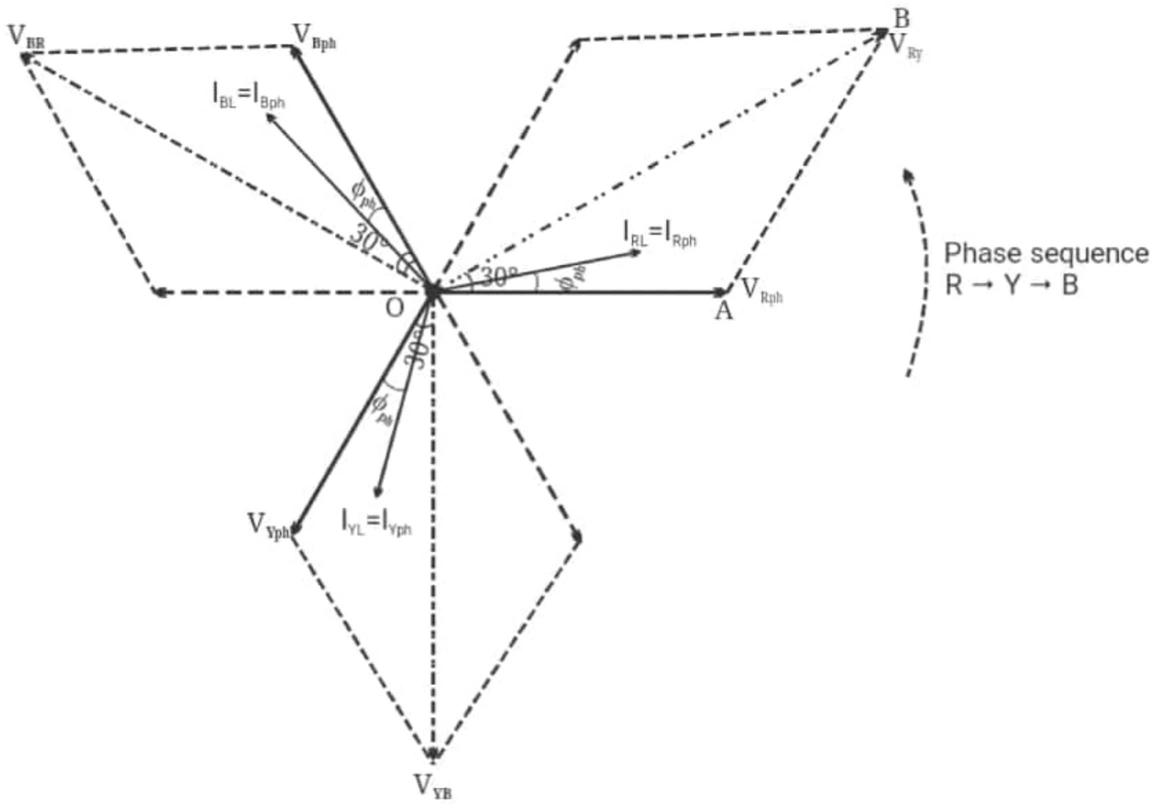


Fig. (c)

(2) Draw the phasor diagram for 3-phase star connected load with a leading power factor. Indicate line and phase voltages and currents. **[D-14][2]**



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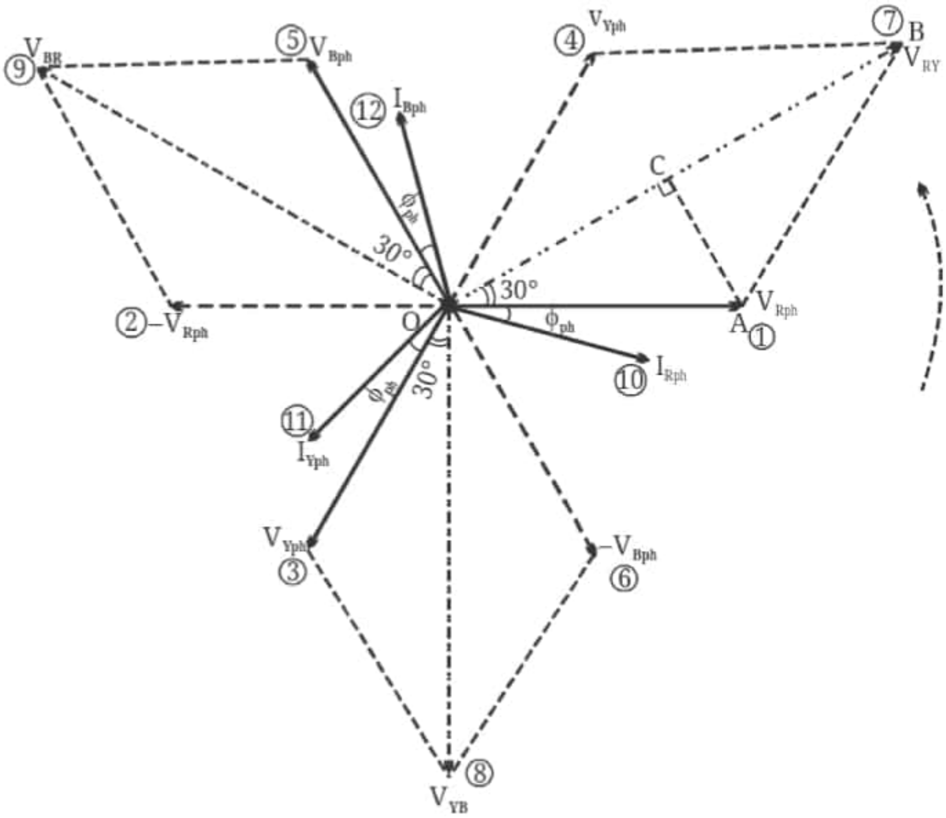
(3) Draw a phasor diagram with 3-phase star connected load with lagging power factor. **[D-15][2],[M-13][**



(4) Give relation between line current and phase current, line voltage and phase voltage in balanced star connected load. **[M-16][2],[D-13][2]**

Solution

Star (⅄) Connection

Relationship between line and phase currents: $I_L = I_{ph}$.

Relationship between line and phase voltages: $V_L = \sqrt{3}\, V_{ph}$.

(5) Derive the relation between power in Delta and Star system. **[D-11][5] [M-10][4]**

Solution:

Case I: ⅄ connection:

Consider that 3 similar impedances Z_{Ph} are connected in ⅄ across a $3-\phi$ balanced AC supply of line voltag V_L.

∴ Total active power consumed $P_{T⅄}$ will be given as:

$$P_{T⅄} = 3I^2_{ph}R_{ph} = 3\left[\frac{V_{Ph}}{Z_{Ph}}\right]^2 R_{ph} = 3V^2_{ph}\frac{R_{Ph}}{Z^2_{ph}} = 3X\left[\frac{V_L}{\sqrt{3}}\right]^2 \cdot \frac{R_{Ph}}{Z^2_{ph}} = V^2_L\frac{R_{ph}}{Z^2_{ph}} \quad .....(i)$$

Case II: Δ connection:

Let us connect the above 3 similar impedances in Δ across the same supply.

∴ Total active power consumed $P_{TΔ}$ will be given as:

$$P_{TΔ} = 3I^2_{ph}R_{ph} = 3\left[\frac{V_{ph}}{Z_{ph}}\right]^2 R_{ph} = 3V^2_{ph}\frac{R_{ph}}{Z^2_{ph}} = 3V^2_L\frac{R_{ph}}{Z^2_{ph}} \quad .....(ii)$$

∴ From (i) and (ii) $P_{T⅄} = \frac{1}{3}P_{TΔ}$. Similarly, $Q_{T⅄} = \frac{1}{3}Q_{TΔ}$ & $S_{T⅄} = \frac{1}{3}S_{TΔ}$.

(6) With the help of a neat circuit diagram and phasor diagram explain the 2 wattmeter method to measure power in a 3ϕ balanced star connected load. **[M-16][6], [D-15][4],[M-15][6],[M-14][6],[M-13][4], [D-12][6],[M-12][5],[M-11][8],[M-09][6],[M-08][6]**

Solution: **Assumptions:**

(i) The $3-\phi$ system considered here is a load, which is ⅄ connected.

(ii) The phase sequence is $R \rightarrow Y \rightarrow B$

(iii) It is a balanced load.

(iv) The p.f. of each phase is lagging.

Connection: It is as shown in figure (a) below:

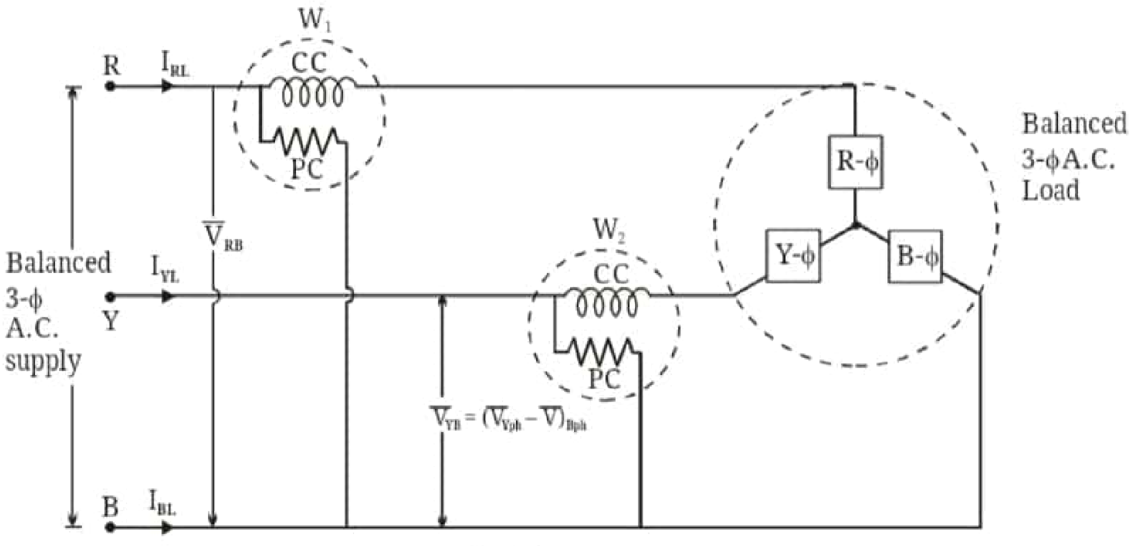


Fig. (a)

Proof:

The phasor diagram is as shown in figure (b) below:

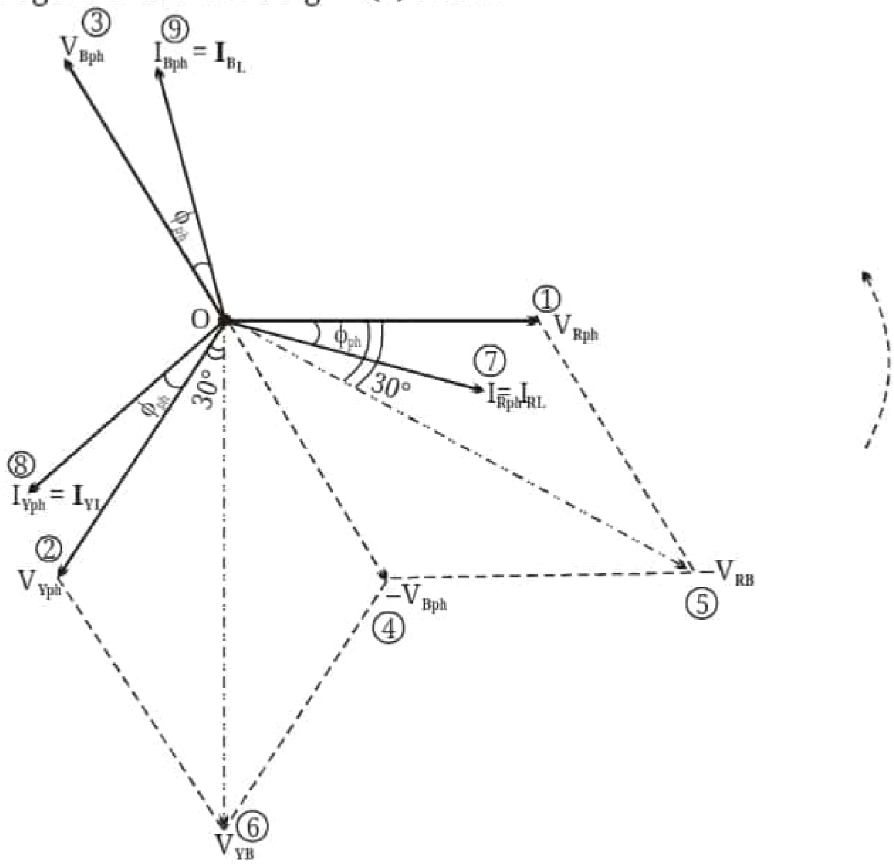


Fig. (b)

(i) Reading of wattmeter W_1

$W_1 = V_{RB}.I_{RL} \cos(V_{RB}, I_{RL}) = V_L I_L \cos(30° - \phi_{ph})$

(ii) Reading of wattmeter W_2

$W_2 = V_{YB}.I_{YL} \cos(V_{YB}, I_{YL}) = V_L I_L \cos(30° + \phi_{ph})$

(iii) $\therefore \quad (W_1 + W_2) = V_L I_L \{\cos(30° - \phi_{ph}) + \cos(30° + \phi_{ph})\}$

$$= \left\{2\cos\left(\frac{60°}{3}\right)\cos\left(-\frac{2\phi_{ph}}{2}\right)\right\} = V_L I_L \{2\cos 30°.\cos\phi_{ph}\} = V_L I_L \left\{2 \times \frac{\sqrt{3}}{2}.\cos\phi_{ph}\right\}$$

$$= \sqrt{3}\, V_L I_L \cos\phi_{ph}$$

= Total power consumed by the $3-\phi$ load and hence the proof.

Note: In the above proof lagging p.f. is considered. However for leading p.f., the readings of wattmeters are interchanged. i.e. $W_1 = V_L I_L \cos(30° + \phi_{ph})$ and $W_2 = V_L I_L \cos(30° - \phi_{ph})$.

(7) With the help of a neat circuit diagram and phasor diagram explain the 2-wattmeter method to measure power in a 3-phase balanced delta connected load. **[D-14][6], [D-09][10]**

Solution:

Assumptions:

(i) The $3-\phi$ system considered here is a load, which is delta connected.

(ii) The phase sequence is $R \rightarrow Y \rightarrow B$

(iii) It is a balanced load.

(iv) The p.f. of each phase is lagging.

Connection: It is as shown in figure (a):

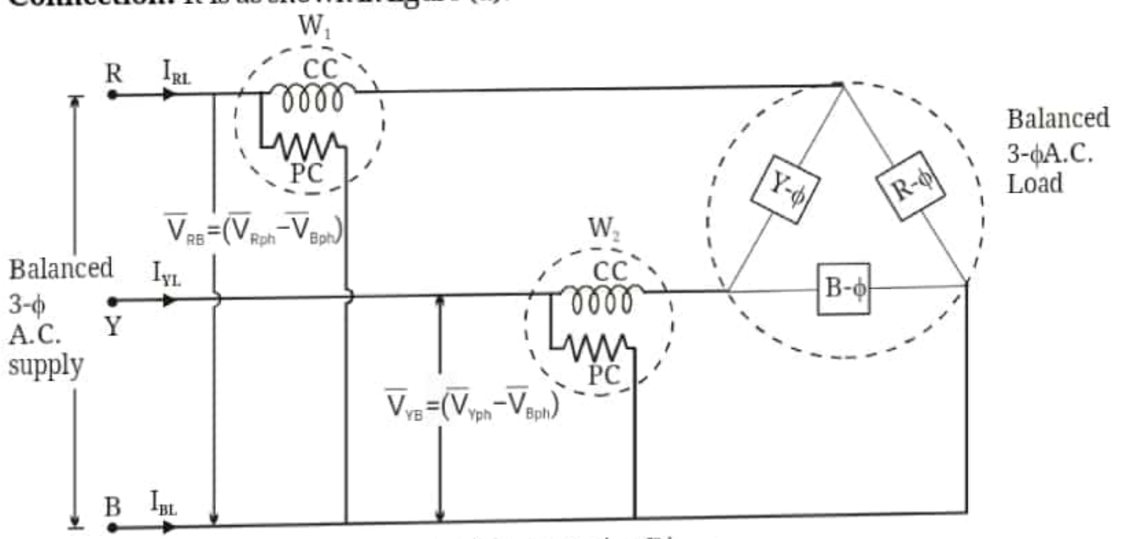
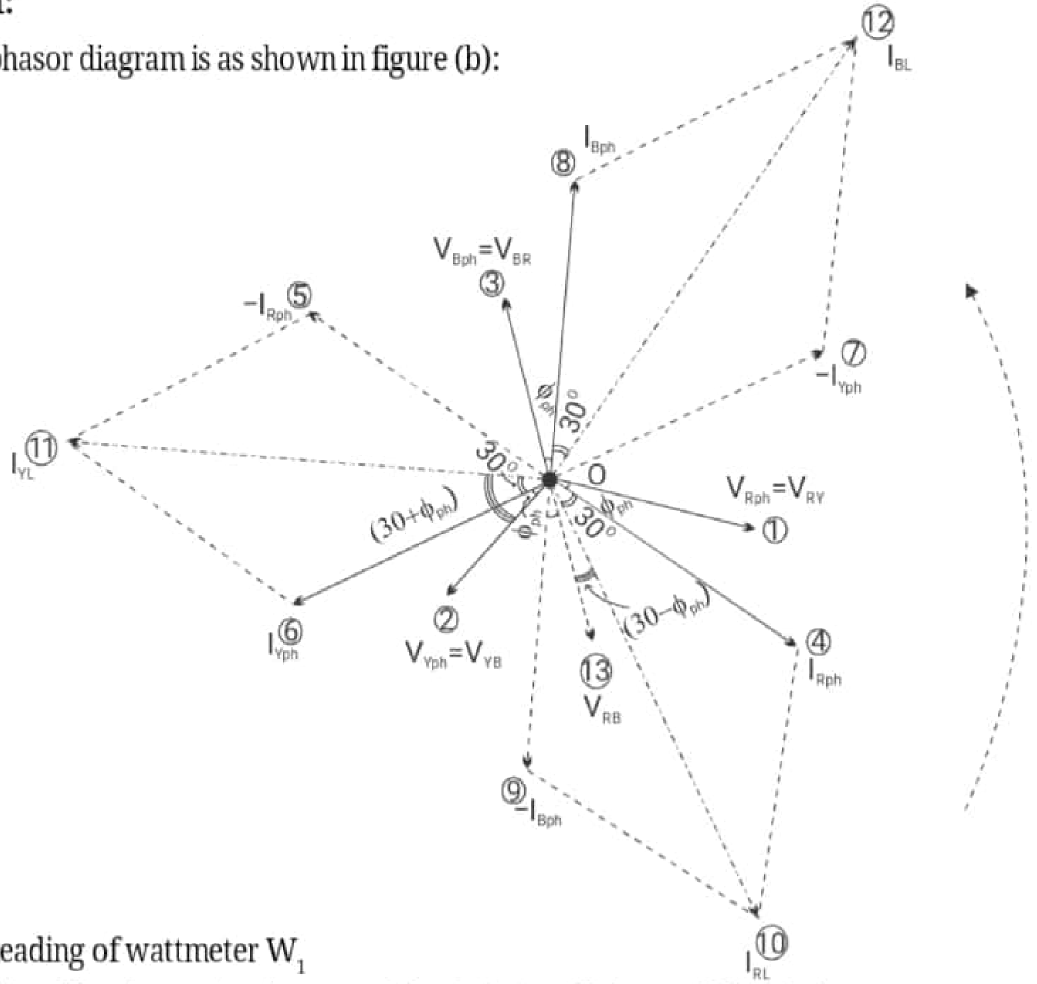


Fig. (a) Connection Diagram

Proof:

The phasor diagram is as shown in figure (b):



(i) Reading of wattmeter W_1

$W_1 = V_{RB}.I_{RL}\cos(\angle \text{between } V_{RB} \,\&\, I_{RL}) = V_L I_L \cos(30° - \phi_{ph})$

(ii) Reading of wattmeter W_2

$W_2 = V_{YB} I_{YL} \cos(\angle \text{between } V_{YB} \,\&\, I_{YL}) = V_L I_L \cos(30° + \phi_{ph})$

(iii) $\therefore (W_1 + W_2) = V_L I_L \{\cos(30° - \phi_{ph}) + \cos(30° + \phi_{ph})\}$

$$= \left\{2\cos\left(\frac{60°}{3}\right)\cos\left(-\frac{2\phi_{ph}}{2}\right)\right\} = V_L I_L \{2\cos 30°.\cos\phi_{ph}\}$$

$$= V_L I_L \left\{2 \times \frac{\sqrt{3}}{2}.\cos\phi_{ph}\right\} = \sqrt{3}\, V_L I_L \cos\phi_{ph}$$

= Total power consumed by the $3-\phi$ load .

Note: In the above proof lagging p.f. is considered. However for leading p.f., the readings of wattmeter are interchanged. i.e. $W_1 = V_L I_L \cos(30° + \phi_{ph})$ and $W_2 = V_L I_L \cos(30° - \phi_{ph})$.

(8) In a three phase power measurement by two walttmeter method, both the wattmeters read the same value. What is the power factor of the load? Justify your answer. **[D-13][4]**

Solution:

For lagging p.f.,

$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi_{ph}$ ......(i)

Similarly, $W_1 - W_2 = V_L I_L \sin\phi_{ph}$ ......(ii)

Dividing (ii) by (i), $\tan\phi_{ph} = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

When both wattmeters show same value, i.e. $W_1 = W_2$

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\left[\frac{0}{W_1 + W_2}\right] = 0$$

Power factor = $\cos\phi$ = $\cos 0 = 1$

(9) In a balanced three phase circuit, power is measured by two wattmeters, the ratio of two wattmeter readings is 2:1. Determine the power factor of the system. **[D-12][4]**

Solution:

Given: $\frac{W_1}{W_2} = \frac{2}{1}$ i.e. $W_1 = 2W_2$

To find: Power factor

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\frac{\sqrt{3}W_2}{3W_2}$$

$\phi = \tan^{-1}(0.557) = 30^\circ$

Power factor = $\cos\phi = \cos 30^\circ = 0.8661$

(10) State the advantages over other methods of 3-phase power measurement. **[M-13][4],[May 08][4]**

Solution:

Given below are some important advantages of the two wattmeter method:

(i) It can be used for balanced as well as unbalanced loads.

(ii) The two wattmeter can be simply connected between the two lines externally without disturbing the 3 – system.

(iii) It does not require the neutral wire $\therefore$ can be used for $\curlywedge$ or Δ connected system.

(iv) Apart from providing the total true power consumed $= (W_1 + W_2)$, it also furnishes following additiona information in the case of a balanced system

(a) the p.f. can be indirectly found as $\tan\phi_{ph} = \pm\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$ + for lagging p.f.

(b) the total reactive power $= \pm\sqrt{3}\,(W_1 - W_2)$ – for leading p.f.

(c) Merely from the two wattmeter reading W_1 and W_2, we can know the nature of the load e.g. if W_1 = W then $\tan\phi_{ph} = 0$

$\therefore \phi_{ph} = 0$ $\therefore \cos\phi_{ph} = 1$ $\therefore$ resistive load and so on.

Numerical Problems

Type I : Three Phase Connection

(1) Three similar coils each having a resistance of 10 Ω and inductance of 0.04 H are connected in star across a 3 phase, 50 Hz, 200V supply. Calculate the line current, total power absorbed, reactive volt amperes and total volt amperes. **[M-15][8]**

Solution:-

$$R = 10\Omega,\ L = 0.04\,H;\ X_L = 2\pi fL = 12.5664\Omega$$

Star connection $f = 50\,Hz,\ V_L = 200\,V$

$$V_{ph} = V_L/\sqrt{3} = 115.47\,V$$

$$Z_{ph} = \sqrt{R^2 + X_L^2} = 16.0597\Omega$$

$$\phi_{ph} = \tan^{-1}\frac{X_L}{R} = 51.488 \Rightarrow pf = \cos\phi_{ph} = 0.62$$

$$I_{ph} = = V_{ph}/z_{ph} = 7.19\,A$$

$$I_L = I_{ph} = 7.19\,A$$

$$P = 3|V_{ph}||I_{ph}|pf = 1.551\,kw$$

$$Q = 3|V_{ph}||I_{ph}|\sin\phi_{ph} = 1.9489\,kVAR$$

$$S = 3|V_{ph}||I_{ph}| = 2.4907\,kVA$$

(2) A balanced 3-phase load consists of 3 coils, each of resistance 4 Ω and inductance 0.02 H. It is connected to a 440 V, 50 Hz, 3ϕ supply. Find the total power consumed when the load is connected in star and the total reactive power when the load is connected in delta. **[D-14][8]**

Solution:-

$$R_{ph} = 4\Omega,\ L_{ph} = 0.02\,H,\ X_{Lph} = 2\pi fL_{ph} = 6.2832\Omega,\ V_L = 440\,V$$

Star Connection	Delta Connection
$V_{ph} = V_L/\sqrt{3} = 254.034\,V$	$V_{ph} = V_L = 440\,V$
$z_{ph} = \sqrt{R_{ph}^2 + X_{Lph}^2} = 7.4484\angle 57.52^\circ\,\Omega$	$z_{ph} = \sqrt{R_{ph}^2 + X_{Lph}^2} = 7.4484\angle 57.52^\circ\,\Omega$
$\bar{I}_{ph} = \bar{V}_{ph}/\bar{z}_{ph} = 34.1058\angle -57.52^\circ\,A$	$\bar{I}_{ph} = \bar{V}_{ph}/\bar{z}_{ph} = 59.0731\angle -57.52^\circ\,A$
$P = 3V_{ph}I_{ph}.\cos\phi_{ph} = 13.9574\,kw$	$Q = 3V_{ph}I_{ph}\sin\phi_{ph} = 65.7793\,KVAR$

(3) Three identical coils each [4.2 + j 5.6] ohm are connected in star across 415 V, 3 phase 50Hz supply. Determine (i) V_{ph} (ii) I_{ph} (iii) Power factor **[M-14][2]**

Solution:-

Given: $\bar{Z}_{ph} = 4.2 + j5.6\Omega,\ V_L = 415V,\ f = 50Hz$

For a star connection,

$$\therefore V_L = \sqrt{3}V_{ph} \Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}}$$

$\therefore\ V_{ph} = 239.6\,V$

$\therefore\ \overline{Z}_{ph} = 4.2 + j5.6 = 7\angle 53.13$

$\therefore\ Z_{ph} = 7\,\Omega$ and $\phi = 53.13^{o}$

$$\therefore\ I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{7} = 34.23\,\text{Amp.}$$

Power Factor, $\cos\phi = \cos(53.13^{o})$

$\therefore$ p.f. = 0.6

(4) Three similar coils connected in star, take a total power of 18KW at a power factor of 0.866 lagging from a three phase 400 volts. 50Hz system. Calculate the resistance and inductance of each coil. Also draw the phasor diagram showing the current and voltages. **[M-14][8]**

Solution:-

Given: $V_L = 400\,V,\ P = 18\,KW,\ \cos\phi = 0.866$

For star connect

$$\text{Phase Voltage, } V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\,V$$

$\therefore\ P = \sqrt{3}\,V_L I_L \cos\phi \ \Rightarrow\ 1.8 x 10^{3} = \sqrt{3}\,x\,400\,x\,I_L\,0.866$

$\therefore\ I_L = 30\,\text{Amp}$

$I_{ph} = I_L = 30$ Amp.

$$\text{By Ohm's law, } Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254.03}{30} = 8.467\Omega$$

$\therefore\ \phi = \cos^{-1}(0.866) = 30^{o}$

$\therefore\ \overline{Z}_{ph} = 8.467\ \angle 30^{o}\ \Omega$

$\therefore\ \overline{Z}_{ph} = 7.33 + j4.23$

$\therefore\ R = 7.33\Omega,\ X_L = 4.23,\ f = 50\,Hz$

$$\therefore\ X_L = 2\pi f_L \ \Rightarrow\ L = \frac{X_L}{2\pi f}$$

$\therefore\ L = 0.01347\,H$

Phasor Diagram:

$V_{ph} = 254.03\,V,$

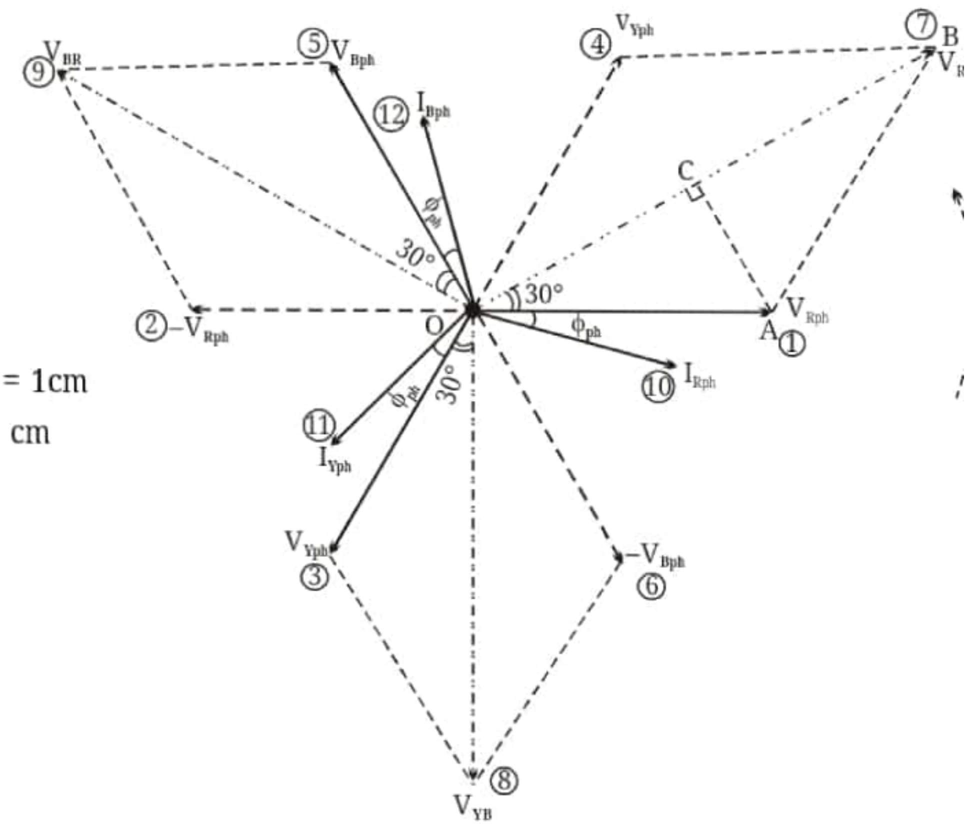
$I_{ph} = 30\,Amp,$

$V_L = 440\,V,$

$\phi = 30^O$

Scale : For voltsge, 100 V = 1cm

and For current, 10 A = 1 cm



(5) Find the value of circuit elements and reactive voltampere drawn for a balanced 3 phase load connected in delta and draws a power 12kW at 440V. The power factor is 0.7 leading..**[D-13][8]**

Solution:-

$P = 12\,k\omega,\ V_L = 440\,V,$ pf = 0.7 (leading) i.e. load is capacitive

For delta connection,

$V_L = V_{ph} \quad \Rightarrow \quad V_{ph} = 440\,V$

Now, $P = \sqrt{3}\,V_L\,I_L\cos\phi \quad \Rightarrow \quad 12 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.7$

$\therefore I_L = 22.49\,Amp$ But $I_L = \sqrt{3}\,I_{ph}$

$$\therefore I_{ph} = \frac{22.49}{\sqrt{3}} = 12.98\,Amp$$

By Ohm's slow, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{440}{12.98} = 33.88\,\Omega$

$\phi = \cos^{-1}(0.7) = -45.57°$ ....($\because$ leading pf)

$\therefore \bar{Z}_{ph} = 33.88\angle -45.57^O$

$\bar{Z}_{ph} = 23.72 - j24.19$

$\therefore R = 23.72\,\Omega$ and $X_C = 24.19\,\Omega$

Reactive volt - ampere drawn

$$Q = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3} \times 440 \times 22.49 \times \sin(-45.57^{o})$$

$$Q = -12.24 \text{ KVAR}$$

(6) Each phase of a delta connected load consist of a 50 mH inductor in series with a parallel combination of 5Ω resistor and 5μF capacitor. The load is connected to a three phase, 550V, 50Hz ac supply. Find (i) Phase current (ii) Line current (iii) Power drawn (iv) Power factor, (v) Reactive power and kVA rating of the load. **[M-13][8]**

Solution:-

Given : $L = 50\,mH$, $R = 5\Omega$, $C = 5\mu f$, $V_L = 550V$, $f = 50Hz$

For a delta connected load,

$$\therefore V_L = V_{ph} = 550V$$

$$\therefore X_L = 2\pi f L = 15.71\Omega$$

$$\therefore X_C = \frac{1}{2\pi f L} = 636.62\,\Omega$$

$$\therefore \overline{Z}_{ph} = jX_L + \frac{R(-jX_C)}{R - jX_C} = j15.71 + \frac{5(-j636.62)}{5 - j636.62} = 5 + j15.67$$

$$\therefore \overline{Z}_{ph} = 16.45\angle 72.30^{o} = 16.45\,\Omega \quad \text{and} \quad \phi = 72.3^{o}$$

$$\therefore I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{550}{16.45} = 33.43 \text{ Amp}$$

$$\therefore I_L = \sqrt{3}\, I_{ph} = 57.91 \text{Amp}$$

$$\therefore P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 550 \times 57.91 \times \cos(72.3) = 16.78 \text{ KW}$$

$$\therefore pf = \cos\phi = \cos(72.3) = 0.304 \text{ lagging}$$

$$\therefore Q = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3} \times 550 \times 57.91 \times \sin(72.3) = 52.55 \text{ KVAR}$$

$$\therefore S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 550 \times 57.91 = 55.17 \text{ KVA}$$

(7) Three similar coils, connected in star, take a total power of 1.5 kW at a p.f. of 0.2 lagging from a three phase, 440 V, 50 Hz supply. Calculate the resistance and inductance of each coil. **[D-12][8]**

Solution:-

Given : $V_L = 440V$, $P = 1.5KW$, $\cos\phi = p.f. = 0.2$

For a star connected load

$$V_L = \sqrt{3}\, V_{ph}$$

$$\therefore V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03V$$

$$\therefore P = \sqrt{3} V_L I_L \cos\phi$$

$$\therefore I_L = \frac{P}{\sqrt{3} V_L \cos\phi} = \frac{1.5\text{x}10^{3}}{\sqrt{3}\text{x}440\text{x}0.2} = 9.841 \text{ Amp}$$

$\therefore\ I_L = I_{ph} = 9.841$ Amp.

$$\therefore Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254.03}{9.841} = 25.814\Omega$$

$$\therefore \cos\phi = 0.2 \Rightarrow \phi = 78.46^0$$

$$\overline{Z_{ph}} = 25.814\angle 78.46 = 5.164 + j25.29$$
$$R = 5.164\Omega$$
$$X_L = 25.29 \Rightarrow L = 0.0805H$$

(8) Each of the star connected load consists of a non-reactive resistance of 100Ω in parallel with a capacitance of 31.8μf. Calculate the line current, power absorbed, the total KVA and power factor when connected to a 416V, 3 phase, 50 Hz supply. **[M-11][6]**

Solution:

Given: 3 phase star connected load with each branch having R = 100Ω parallel with C = 31.8 μF, Power supply = 416 Volts 3 phase 50 Hz

To find: Line current, powder absorbed, total kVA and p.f.

Step 1: Find the voltage V_{ph}

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{416}{\sqrt{3}} = 240.18$$

Step 2: Find capacitive reactance X_C

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 31.8} = 100\,\Omega$$

Step 3: Find current through resistor I_R, power absorbed by resistor and total power absorbed

$$I_R = \frac{V_{ph}}{R} = \frac{240.18}{100} = 2.401$$

Power absorbed by resistor = $P_1 = I_R^2 R = (2.401)^2 \times 100 = 576.48$ Watts

$\therefore$ Total power absorbed = $3 \times P_1 = 3 \times 576.48 = 1729.4$ Watts

Step 4: Find current through the capacitor I_C:

$$I_C = \frac{V_{ph}}{X_C} = \frac{240.18}{100} = 2.4018 \text{ Amp}$$

Step 5: Find total current. Since the two currents are at right angles to each other:

$$\text{Total current } I_T = \sqrt{I_R^2 + I_C^2} = \sqrt{(2.401)^2 + (2.4018)^2}$$

$\therefore$ Line current = $I_L = 3.396$ Amp

Step 6: Find kVA per phase and toal kVA:

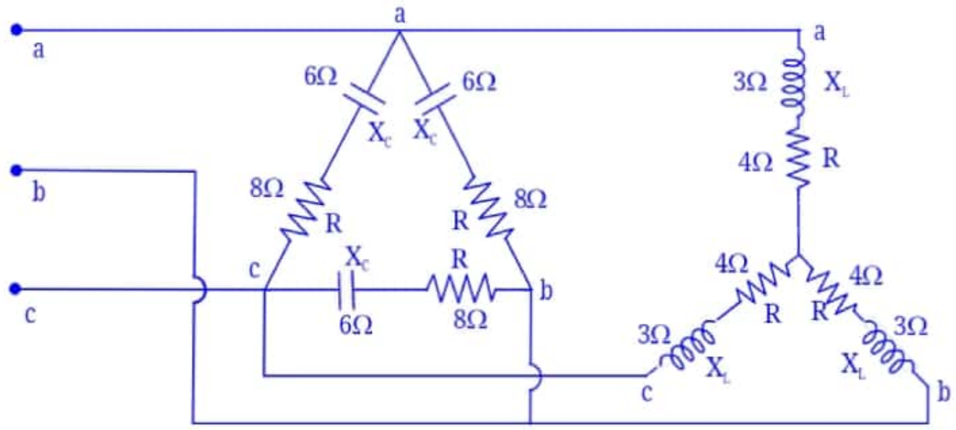
kVA per phase = $V_{ph} \times I_L = 240.18 \times 3.396 = 815.65$

$\therefore$ Total kVA = 3 × kVA per phase = 3 × 815.65 = 244.95

Step 7: Find power factor

$$\text{P.f.} = \frac{kW}{kVA} = \frac{1729.4}{2446.95} = 0.7067$$

(9) If 3-ϕ 400 V, 50 Hz is supplied to the circuit. Calculate line current, phase current, power factor, active power and reactive power. **[D-10][12]**



Solution:

Given: Supply 3 phase 400 V, 50 Hz, connected to star load R = 4Ω, X_L = 3Ω and delta load R = 8Ω ar
$X_L = 6\Omega$

To find: Line currents, phase currents, power factor, active powder and reactive power

Step 1 : Find star impedance Z_1 and delta impedance Z_2:

$$Z_1 = 4 + j3 = Z = \sqrt{4^2 + 3^2} = 5$$

$$\cos\phi = \frac{4}{5} = 0.8, \quad \therefore \quad \phi = 36.87$$

$$\therefore \; Z_1 = 5 \angle 36.87^\circ$$

$$Z_2 = 8 - j6, \qquad \therefore \quad Z = \sqrt{8^2 + 6^2} = 10$$

$$\cos\phi = \frac{8}{10} = 0.8 = \phi = 36.87,$$

$$\therefore \; Z_2 = 10 \angle -36.87^\circ$$

Step 2 : Find I_{L1} = Line current for star load:

$$I_{L1} = I_{ph} = \frac{V_L}{\sqrt{3}\,Z_1} = \frac{400\angle 0^\circ}{\sqrt{3}\times 5 \angle 36.87^\circ} = 46.19\angle -36.87^\circ$$

$$\therefore \; I_{L1} = 46.19 \angle -36.87 = 36.95 - j27.71$$

Step 3 : Find I_{L2} = Line current for delta load:

$$I_{L2} = \sqrt{3}\times I_{ph} = \frac{\sqrt{3}\,V_L}{Z_2} = \frac{\sqrt{3}\times 400\angle 0}{10 \angle -36.87} = 69.28\angle -36.87^\circ$$

$$\therefore \; I_{L2} = 69.28 \angle 36.87 = 55.42 + j41.57$$

Step 4 : Find total current $I_L = I_{L1} + I_{L2}$:

$$I_L = 36.95 - j\,27.71 + 55.42 + j\,41.57$$
$$= 92.37 + j13.86 = 93.40 \angle 8.53^\circ$$

$$\therefore \; \cos\phi = 0.9889 \text{ and } \sin\phi = 0.1483$$

$$\cos\phi = 0.9889 \text{ (lead)}$$

Active power = $\sqrt{3}\, V_L\, I_L \cos\phi$

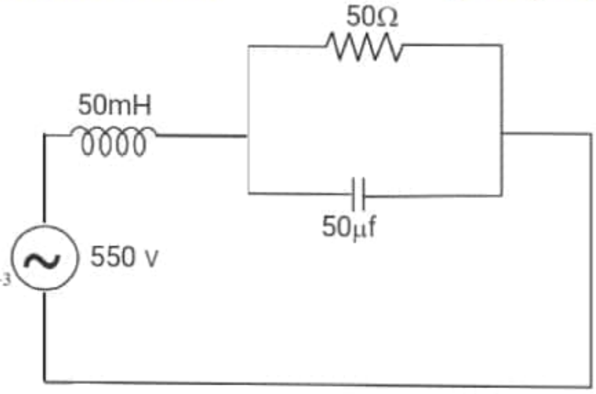
$$= \sqrt{3} \times 400 \times 93.4 \times 0.9889 = 63.991 \text{ kW}$$

Reactive power $= \sqrt{3}\, V_L I_L \times \sin\phi$

$$= \sqrt{3} \times 400 \times 93.4 \times 0.1483 = 9.596 \text{ KVA}$$

(10) Each phase of a delta connected load consists of a 50 mH inductor in series with a parallel combination of 50Ω resistor and 50μF capacitor. The load is connected to 3 phase 550 V, 800 rad/sec. a.c. supply. Find (a) Phase current (b) Line current (c) Power drawn (d) Power factor (e) Reactive power (f) kVA rating of the load. **[D-09][10]**

Solution:



Given : $L = 50 \times 10^{-3}$ H

$R = 50\ \Omega$

$C = 50 \times 10^{-6}$ F

$V_L = 550$ V

$\omega = 800$ rad/sec

Inductive reactance $X_L = \omega_L = 800 \times 50 \times 10^{-3}$

$\therefore X_L = 40\ \Omega$

Capacitive reactance $X_C = \dfrac{1}{\omega_C} = \dfrac{1}{800 \times 50 \times 10^{-6}}$

$\therefore X_C = 25\ \Omega$

Admittance of parallel circuit, $y_p = \dfrac{1}{R} + j\dfrac{1}{X_C} = \dfrac{1}{50} + j\dfrac{1}{25}$

$$y_p = 0.02 + j\,0.04 = (0.045 \angle 63.43^0)\ S$$

$$Z_p = \frac{1}{y_p} = \frac{1}{0.045\angle 63.43^0}$$

$$Z_p = 22.22 \angle -63.43^0$$

$$Z_p = (9.938 - j\,19.87)\ \Omega$$

Total impedance of the circuit

$$Z_{ph} = (0 + jX_L) + Z_p = (0 + j40) + 9.938 - j19.87$$

$$Z_{ph} = (9.938 + j20.13)\ \Omega$$

$$Z_{ph} = 22.44 \angle 63.72^0$$

(i) For phase current : $I_{ph} = \dfrac{V_L}{Z_{ph}} = \dfrac{550}{22.44\angle 63.72}$

$\therefore I_{ph} = 24.50 \angle -63.72$ A

(ii) Line current : $I_L = \sqrt{3} \times I_{ph} = \sqrt{3} \times (24.50)$

$\therefore I_L = 42.43$ A

(iii) Power drawn :$P = 3 \times V_{Ph} \times I_{ph} \times \cos\phi = 3 \times 550 \times 24.50 \times \cos(-63.72)$

$\therefore P = 17898.50$ W$= 17.898$ kW

(iv) Power factor : $P.f. = \cos(\phi) = \cos(-63.72)$

$\therefore$ $P.f. = 0.4427$

(v) Reactive power $Q = \sqrt{3} \times V_L \times I_L \times \sin\phi$ $Q = \sqrt{3} \times 550 \times 42.43 \times \sin(63.72)^0$

$\therefore$ $Q = 36.\ 24$ kVAR

(11) A balanced 3 phase delta connected load draws 10 A of line current and 3 kw at 220 V. Determine the value of resistance and reactance of each phase of load. **[M-09][8]**

Solution:

Given: Delta load, $I_L = 10$ A, $V_L = V_{Ph} = 220$ V, $P = 3$kW

Step 1: Calculation of $\cos\phi$ (Power factor) :

$$P = \sqrt{3}\, V_L I_L \cos\phi \Rightarrow 3kW = \sqrt{3} \times V_L I_L \cos\phi = \sqrt{3} \times 220 \times 10 \times \cos\phi$$

$\therefore \cos\phi = 0.787$

Step 2 : Calculation of I_{ph} :

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.77 \text{ Amp}$$

Step 3: Calculation of Z_{ph}, R_{ph} and X_{ph}:

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{220}{5.77} = 38.10\,\Omega$$

$$R_{ph} = Z_{ph}\cos\phi = 38.10 \times 0.787 = 29.98\,\Omega$$

$$\cos\phi = 0.707 \Rightarrow \phi = 45^0 \Rightarrow \sin\phi = 0.707$$

$$\therefore X_{ph} = Z_{ph}\sin\phi = 38.10 \times 0.787 = 29.98\,\Omega$$

Assuming that power factor is a lagging

$\therefore$ The inductance per phase is given by,

$$X_{ph} = 2\pi f L_{ph} \Rightarrow 29.98 = 2\pi \times 50 L_{ph}$$

$\therefore L_{ph} = 95.42$ mH

Type II : Three Phase Power Measurement by Two Wattmeter Method

(1) Two wattmeters are used to measure power in a 3ϕ balanced delta connected load using two wattmeter method. The readings of the 2 wattmeters are 500 W and 2500W respectively. Calculate the total power consumed by the 3ϕ load and the power factor **[M-15][4]**

Solution:-

$W_1 = 500$ W, $W_2 = 2500$ W

$\therefore$ Total Power absorbed, $P = W_1 + W_2 = 3000$ W

Assume lagging p.f.

$$\therefore \text{Power factor angle } \phi = \tan^{-1}\left\{\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right\}$$

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}(2000)}{3000}\right\}$$

$$\therefore \phi = 49.1066^o$$

$\therefore$ Power facter, $\cos\phi = 0.6546$ (lagging)

(2) Two wattmeters are used to measure power in a 3ϕ balanced star connected load using the two wattmeter method. The readings of the 2 wattmeters are 8 kW and 4 kW respectively. Calculate the total power consumed by the 3ϕ load and the power factor. **[D-14][4]**

Solution:-

$W_1 = 8kW = 8000W, \quad W_2 = 4kW = 4000\,W$

$\therefore$ Total power consumed, $P = W_1 + W_2 = 12kW$

Assume lagging p.f.

$$\therefore \phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right] = \tan^{-1}\left[\sqrt{3}\left(\frac{4000}{12,000}\right)\right]$$

$$\therefore \phi = 30^0$$

Power factor,

$$\therefore \cos\phi = 0.866 \text{ lagging}$$

(3) A 3 phase, 10KVA lead has power factor of 0.342. The power is measured by two wattmeter method. Find the reading of each wattmeter when.

(i) Power factor is leading (ii) Power factor is lagging **[M-14][4]**

Solution:-

Given : 10 kVA, pf = 0.342, S = 10 kVA

$S = \sqrt{3}\, V_L I_L = 10 \times 10^3$

$$\therefore V_L I_L = \frac{10 \times 10^3}{\sqrt{3}} = 5773.5 \text{ VA}$$

$$pf = 0.342 \Rightarrow \cos\phi = 0.342 \Rightarrow \phi = \cos^{-1}(0.342)$$

$$\therefore \quad \phi = 70$$

(i) pf is leading, then

$$W_1 = V_L I_L \cos(30 - \phi) = 5773.5 \cos(30 - 70) = 4422.76 \text{ watt}$$

$$\therefore W_1 = 4.42 \text{ kW}$$

$$W_2 = V_L I_L \cos(30 + \phi) = 5773.5 \cos(30 + 70) = -1002.55 \text{ watt}$$

$$W_2 = -1.00255 \text{ kW}$$

(ii) pf is lagging, then

$$W_1 = V_L I_L \cos(30 + \phi) = 5773.5 \times \cos(30 + 70)$$

$$\therefore W_1 = -1.00255 \text{ kW}$$

$$W_2 = V_L I_L \cos(30 - \phi) = 5773.5 \cos(30 - 70)$$

$$\therefore W_2 = 4.42 \text{ kW}$$

(4) Two wattmeters are connected to measure power in a phase circuit. The reading of one of the wattmete is 7 kW when load power factor is unity. If the power factor of the load is changed to 0.707 lagging withou changing the total input power, calculate the readings of the two wattmeters. **[D-13][6]**

Solution:-

Given: $W_1 = 7\,kW,\ pf = 1$

when $pf = 1$ then, $W_1 = W_2$

$\therefore W_2 = 7kW$

i.e. Total Power $\Rightarrow P = W_1 + W_2 = 7 + 7 = 14kW = 14000$ watt

$$\therefore\ P = \sqrt{3}\,V_L I_L \cos\phi$$

$$\therefore\ V_L I_L = \frac{P}{\sqrt{3}\cos\phi} = \frac{14000}{\sqrt{3}\,x1} = 8082.90\ \text{volt} - \text{amp}$$

Now $pf = 0.707$ lagging $\Rightarrow \cos\phi = 0.707$

$$\therefore\quad \phi = \cos^{-1}(0.707) = 45^o$$

$$\therefore\quad W_1 = V_L I_L \cos(30+\phi) = 8082.9\cos(30+45) = 2092\ W$$

$$W_1 = 2.092\ kW$$

$$W_2 = V_L I_L \cos(30-\phi) = 8082.9\cos(30-45) = 7807.48\ \text{watt}$$

$$W_2 = 7.8075\ kW$$

(5) The input power of 3-phase motor was measured by two wattmeter method. The reading of two wattmete are 5.2 kW and –1.7 kW and the line voltage is 415V. Calculate the total Active Power, Power facto and line current. **[M-13][6]**

Solution:-

$W_1 = 5.2\,kW \quad W_2 = -1.7\,kW \quad V_L = 415\,V$

$P = ? \quad pf = ? \quad I_L = ?$

$P = W_1 + W_2 = 5.2 + (-1.7) = 3.5\,kM$

$$pf = \cos\left\{\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\} = \cos\left\{\tan^{-1}\sqrt{3}\left(\frac{5.2-(-1.7)}{3.5}\right)\right\}$$

$$\therefore pf = \cos\left\{\tan^{-1}\sqrt{3}\left(\frac{6.9}{3.5}\right)\right\} = 0.2811\ \text{lagging}$$

$$\therefore P = \sqrt{3}\,V_L I_L \cos\phi$$

$$\therefore I_L = \frac{p}{\sqrt{3}\,V_L\cos\phi} = \frac{3.5\,x\,1000}{\sqrt{3}\,x\,415\,x\,0.2811}$$

$$\therefore\ I_L = 17.32\ \text{Amp.}$$

(6) A 3-phase RYB system had effecitve line voltage 173.2V. Wattmeter's in lines 'R' and 'Y' red 301 W and 1327 W respectively. Find the impedance of the balanced star connected load. **[D-08][6]**

Soluion:-

Given : $V_L = 173.2V,\ W_1 = 301W,\ W_2 = 1327W$, star load

To find : Z_{ph}.

Step 1 : Calculation of $\cos\phi$ and total power

Total power $W = W_1 + W_2 = 301 + 1327 = 1628\,W$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(301-1327)}{1628}\right]\right\} = \cos(-47.5)$$

$\therefore \cos\phi = 0.6755$

Step 2 : Calculation of I_L and I_{ph} :

$$P = \sqrt{3}\,x\,V_L I_L \cos\phi \quad\Rightarrow\quad 1628 = \sqrt{3}\,x\,173.2\,x\,I_L\,x\,0.6755$$

$\therefore I_L = 8.034$ Amp.

$\therefore I_{ph} = I_L = 8.034$ Amp.

Step 3 : Calculation of Z_{ph}

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{V_L/\sqrt{3}}{I_{ph}} = \frac{173.2}{\sqrt{3}\,x\,8.034}$$

$\therefore\quad Zph = 12.45\ \Omega$

(7) Calculate the total power and readings of two Wattmeters connected to measure power in three phase balanced load, if the reactive power is 15 KVAR, and the load p.f. is 0.8 lagging. **[D-08][8]**

Solution:-

Given : $Q = 15\,x\,10^3$ VAR, $\cos\phi = 0.8(\text{lag})$

Step 1: Calculation of W_1 and W_2 :

Reactive power $Q = \sqrt{3}(W_1 - W_2)$

$$\therefore \frac{15\,x\,10^3}{\sqrt{3}} = W_1 - W_2 \quad\Rightarrow\quad W_1 - W_2 = 8660.25 \qquad ...(1)$$

$$\text{Also}\ \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right\} \quad\Rightarrow\quad 0.8 = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right\}$$

$$\therefore 0.75 = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad\Rightarrow\quad 0.75W_1 + 0.75W_2 = \sqrt{3}\,W_1 - \sqrt{3}W_2$$

$$\therefore 0.982\,W_1 = 2.482\,W_2 \qquad ...(2)$$

From Equation (2), $W_1 = \frac{2.482}{0.982} W_2 \Rightarrow W_1 = 2.528 W_2$ ...(3)

Subtituting into Equation (1) to get

$$2.528W_2 - W_2 = 8660.25$$

$$\therefore W_2 = 5669.38 \text{ Watt}$$

$$\text{and } W_1 = 2.528 \times 5669.38 = 14332.2 \text{ Watt}$$

Step 2 : Calculation of total power

$$P = W_1 + W_2 = 14332.2 + 5669.38$$

$$\therefore \; P = 20001.58 \text{ Watt} = 20 \text{ kW}$$

(8) Two wattmeters connected to measure the power input to 3–ϕ circuit, indicate 2500 W and 500 W respectively. Find the power factor of the circuit.

(i) When both readings are positive and

(ii) When later reading is obtained after reversing the connection to the current coil of one Instrument. **[D-09][10], [D-07][8]**

Solution:-

Given : W_1 = 2500 Watts and W_2 = $\pm$500W

To find power factor

Case I: Both reading positive

W_1= 2500 and W_2 = 500

$$\cos\phi = \cos\left(\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right) = \cos\left(\tan^{-1}\left[\frac{\sqrt{3}(2500-500)}{2500+500}\right]\right) = \cos(49.1)$$

$\therefore$ Power factor, $\cos\phi$ = 0.6546 (leading)

Case II: W_1 = 2500 and W_2 = –500

$$\therefore \; \cos\phi = \cos\left(\tan^{-1}\left[\frac{\sqrt{3}(2500+500)}{2500-500}\right]\right) = \cos(68.95)$$

$\therefore$ Power factor, $\cos\phi$ = 0.3592 (leading)