Assignment 1

Name: Yash Sarang

Roll no.: 47

Class: D1AD

Topic: Self-Study

PART A

1] $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^{-1}$

$\longrightarrow$

let, $z = \log_e x$

$\therefore x = e^z$

$z = \log_e x$

differentiating wrt x

$\frac{dz}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \frac{1}{x}\cdot\frac{dy}{dz}$ — ①

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dx^2}\frac{dz}{dx}$

$= \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$ — ②

Substitute ① & ② in above eqn

$x^2\left[\frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right] - 2x\left(\frac{1}{x}\frac{dy}{dz}\right) + 2y = \frac{1}{e^z}$

$\frac{d^2y}{dz^2} - \frac{dy}{dz} - 2\frac{dy}{dz} + 2y = \frac{1}{e^z}$

$\frac{d^2y}{dz^2} - 3\frac{dy}{dz} + 2y = \frac{1}{e^z}$

But $D = \frac{d}{dz}$

$\therefore (D^2 - 3D + 2)y = e^{-z}$

AE is $D^2 - 3D + 2$

$\therefore \quad D^2 - 3D + 2 = 0$

$(D-2)(D-1) = 0$

$D = 1, 2$

$y_c = c_1 e^z + c_2 e^{2z}$

$$y_p = \frac{1}{f(D)} \cancel{x} e^{-z}$$

$$= \frac{1}{(D^2 - 3D + 2)} \cancel{x} e^{-z}$$

$$\therefore y_p = \frac{1}{1+3+2} e^{-z} = \frac{1}{6} e^{-z} = \frac{1}{6x}$$

$y = y_c + y_p$

$$\therefore y = c_1 e^z + c_2 e^{2z} + \frac{1}{6x}$$

2] $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10(x + x^{-1})$

$\rightarrow$

Let, $z = \log x \qquad \therefore x = e^z$

diff w.r.t x

$$\frac{dz}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{dy}{dz} \cdot \frac{1}{x} \quad -(1)$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right) \quad -(2)$$

Similarly, $$\frac{d^3y}{dx^3} = \frac{1}{x^3}\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right) \quad -(3)$$

Substituting eqn ①, ② & ③ in given eqn

$$x^3\left[\frac{1}{x^3}\left(\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+2\frac{dy}{dx}\right)\right]+2x^2\left[\frac{1}{x^2}\left(\frac{d^2y}{dz^2}-\frac{dy}{dz}\right)\right]$$
$$+2y=10\left(e^z+e^{-z}\right)$$

$$\therefore \frac{d^3y}{dz^3}-3\frac{d^2y}{dz^2}+2\frac{d^2y}{dz^2}-2\frac{dy}{dz}+2y=10\left(e^z+e^{-z}\right)$$

$$\frac{d^3y}{dz^3}-\frac{d^2y}{dz^2}+2y=10\left(e^z+e^{-z}\right)$$

Put $D=\frac{d}{dz}$

$$\therefore (D^3-D^2+2)\,y=10\left(e^z+e^{-z}\right)$$

$$\therefore AE: D^3-D^2+2=0$$
$$(D+1)(D^2-2D+2)=0$$
$$\therefore D=-1,\ 1\pm i$$

$$\therefore y_c=c_1e^{-z}+e^z\left(c_2\cos z+c_3\sin z\right)$$

$$\therefore y_c=\frac{c_1}{x}+x\left[c_2\cos(\log x)+c_3\sin(\log x)\right]$$

$$y_p=\frac{1}{f(D)}\left[10\left(e^z+e^{-z}\right)\right]$$
$$=\frac{1}{(D^3-D^2+2)}\left[10\left(e^z+e^{-z}\right)\right]$$
$$=10\left(\frac{1}{2}e^z+\frac{ze^{-z}}{5}\right)$$
$$=10\left(\frac{1}{2}x+\frac{\log x}{5x}\right)$$

$y = y_c + y_p$

$\therefore y = \frac{c_1}{x} + x[c_2\cos(\log x) + c_3\sin(\log x)] + 10\left[\frac{1}{2}x + \frac{\log x}{5x}\right]$

3] $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 3x = \log x\ (x + x^{-1})^2$

$\rightarrow$

let $z = \log x$ $\quad \therefore x = e^z$

$\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dz}$ — ①

$\frac{d^2y}{dx^2} = \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$ — ②

Substitute ~~in~~ ① & ② in given eqn,

$x^2\left[\frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right] + 5x\left(\frac{1}{x}\frac{dy}{dz}\right) + 3y = z(e^z + e^{-z})^2$

$\therefore \frac{d^2y}{dz^2} - \frac{dy}{dz} + 5\frac{dy}{dz} + 3y = z(e^z + e^{-z})^2$

$\frac{d^2y}{dz^2} + 4\frac{dy}{dz} + 3y = z(e^z + e^{-z})^2$

Put $D = \frac{d}{dz}$

$\therefore (D^2 + 4D + 3)y = z(e^z + e^{-z})^2$

AE: $D^2 + 4D + 3 = 0$

$(D+3)(D+1) = 0$

$D = -3, -1$

$$\therefore y_c = c_1 e^{-z} + c_2 e^{-3z}$$

$$\therefore y_c = \frac{c_1}{x} + \frac{c_2}{x^3}$$

$$y_p = \frac{1}{f(D)} \left[z(e^z + e^{-z})^2 \right]$$

$$= \frac{1}{(D+3)(D+1)} \left[z(e^{2z} + e^{-2z} + 2) \right]$$

$$y_{p_1} = \frac{1}{(D+1)(D+3)} ze^{2z} = \left[z - \frac{f'(D)}{f(D)} \right] \left[\frac{1}{f(D)} e^{2z} \right]$$

$$= \left[z - \frac{2D+4}{(D+1)(D+3)} \right] \left[\frac{e^{2z}}{(D+1)(D+3)} \right]$$

$$= \left[z - \frac{2D+4}{(D+1)(D+3)} \right] \left[\frac{e^{2z}}{15} \right]$$

$$= \frac{ze^{2z}}{15} - \frac{2}{15} \left[\frac{D+2}{(D+1)(D+3)} e^{2z} \right]$$

$$= \frac{ze^{2z}}{15} - \frac{2}{15} \frac{(2e^{2z} + 2e^{2z})}{15}$$

$$= \frac{ze^{2z}}{15} - \frac{8e^{2z}}{225}$$

$$\therefore y_{p_1} = \frac{x^2 \log x}{15} - \frac{8x^2}{225}$$

$$y_{p_2} = \frac{1}{(D+1)(D+3)} ze^{-2z}$$

Similar to y_{p_1},

$$y_{p_2} = \left[z - \frac{2D+4}{(D+1)(D+3)} \right] \left[\frac{e^{-2z}}{(D+1)(D+3)} \right] = \left[z - \frac{2D+4}{(D+1)(D+3)} \right] \left[\frac{e^{-2z}}{-1} \right]$$

$$= -ze^{-2z} + \frac{2}{-1}\left(-2e^{-2z} + 2e^{-2z}\right)$$

$$= -te^{-2z}$$

$$\therefore y_{P_2} = -\frac{\log x}{x^2}$$

$$y_{P_3} = \frac{1}{(D+1)(D+3)} 2z = \frac{2}{D+3}\left[\frac{1}{(1+D)} t\right]$$

$$= \frac{2}{D+3}\left[1-2D\right]z$$

$$= \frac{2}{3}\left[\frac{1}{1+D/3}(z-2)\right] = \frac{2}{3}\left[1-\frac{2D}{3}\right](t-2)$$

$$= \frac{2}{3}(t-2) - \frac{4}{9} = \frac{2}{3}t - \frac{4}{3} - \frac{4}{9} = \frac{2}{3}t - \frac{8}{9}$$

$$\therefore y_{P_3} = \frac{2}{3}\left[\log x - \frac{4}{3}\right]$$

$$y_P = y_{P_1} + y_{P_2} + y_{P_3}$$

$$\therefore y_P = \frac{1}{15}\left[x^2\log x - \frac{8}{15}x^2\right] + \frac{2}{3}\left[\log x - \frac{4}{3}\right] - \frac{\log x}{x^2}$$

$$\therefore y = y_c + y_p$$

$$\therefore y = \frac{c_1}{x} + \frac{c_2}{x^3} + \frac{1}{15}\left[x^2\log x - \frac{8}{15}x^2\right] + \frac{2}{3}\left[\log x - \frac{4}{3}\right] - \frac{\log x}{x^2}$$

4] $(1+2x)^2 \frac{d^2y}{dx^2} - 2(1+2x)\frac{dy}{dx} - 12y = x^2$

$\rightarrow$

Let $z = \log(1+2x)$ $\qquad \therefore e^z = 1+2x$

diff wrt x $\qquad \therefore x = \frac{e^z - 1}{2}$

$$\frac{dz}{dx} = \frac{2}{1+2x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{dy}{dz}\left(\frac{2}{1+2x}\right) \quad \text{— ①}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{4}{(1+2x)^2}\frac{dy}{dx} + \frac{2}{1+2x}\left(\frac{d^2y}{dz^2}\frac{dz}{dx}\right)$$

$$= -\frac{4}{(1+2x)^2}\frac{dy}{dz} + \left(\frac{2}{1+2x}\right)^2\left(\frac{d^2y}{dz^2}\right)$$

$$= -\left(\frac{-2}{1+2x}\right)^2\frac{dy}{dz}\left(\frac{2}{1+2x}\right)^2\frac{d^2y}{dz^2}$$

$$\therefore \frac{d^2y}{dx^2} = \left(\frac{2}{1+2x}\right)^2\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right] \quad \text{— ②}$$

Substitute ① & ② in given eqn

$$(1+2x)^2\left[\left(\frac{2}{1+2x}\right)^2\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right] - 2(1+2x)\left[\frac{2}{(1+2x)}\frac{dy}{dz}\right] - 12y = x^2$$

$$\therefore 4\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right) - 4\frac{dy}{dz} - 12y = x^2$$

$$4\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right) - 4\frac{dy}{dz} - 12y = \left(\frac{e^z - 1}{2}\right)^2$$

$$4\frac{d^2y}{dz^2} - 8\frac{dy}{dz} - 12y = \left(\frac{e^z-1}{2}\right)^2$$

Put, $D = \frac{d}{dz}$

$$\therefore (4D^2 - 8D - 12)y = \left(\frac{e^z-1}{2}\right)^2$$

AE : $4D^2 - 8D - 12 = 0$

$\cancel{4}D^2 - \cancel{4}2D - 3 = 0$

$(D-3)(D+1) = 0$

$\therefore D = +3, -1$

$$\therefore y_c = C_1 e^{3z} + C_2 e^{-z}$$

$$y_P = \frac{1}{f(D)}\left[\frac{e^z-1}{2}\right]^2 = \frac{1}{4(D+1)(D-3)}\left(\frac{1}{4}e^{2z} - 2e^z + 1\right)$$

$$= \frac{1}{16}\left[\frac{e^{2z}}{(D-3)(D+1)} - \frac{2e^z}{(D-3)(D+1)} + \frac{1}{(D-3)(D+1)}\right]$$

Putting $D = a$, where e^{at} is the func^n

$$= \frac{1}{16}\left[\frac{e^{2z}}{-3} - \frac{2e^z}{-4}\right] + \frac{1}{16}\left[\frac{e^{0z}}{(D-3)(D+1)}\right]$$

$$= \frac{1}{16}\left[\frac{-e^{2z}}{3} + \frac{e^z}{2} + \frac{1}{-3}\right]$$

$$\therefore y_p = \frac{1}{16}\left[\frac{e^t}{2} - \frac{e^{2t}}{3} - \frac{1}{3}\right]$$

$$y = y_c + y_p$$

$$\therefore y = C_1 e^{3z} + C_2 e^{z} + \frac{1}{16}\left[\frac{e^{z}}{2} - \frac{e^{2z}}{3} - \frac{1}{3}\right]$$

$$\therefore y = C_1(1+2x)^3 + \frac{C_2}{1+2x} + \frac{1}{16}\left[\frac{1+2x}{2} - \frac{(1+2x)^2}{3} - \frac{1}{3}\right]$$

5] $(2+3x)^2 \frac{d^2y}{dx^2} + 5(2+3x)\frac{dy}{dx} - 3y = x^2+x+1$

$\rightarrow$

Let $z = \log(2+3x)$ $\quad\therefore e^z = 2+3x$

diff wrt x $\quad\therefore x = \frac{e^z - 2}{3}$

$$\therefore \frac{dz}{dx} = \frac{1}{2+3x}(3)$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \left(\frac{3}{2+3x}\right)\frac{dy}{dz} \quad — ①$$

$$\frac{d^2y}{dx^2} = \frac{-9}{(2+3x)^2}\frac{dy}{dz} + \frac{3}{2+3x}\frac{d^2y}{dz^2}\frac{dz}{dx}$$

$$= -\left(\frac{3}{2+3x}\right)^2\frac{dy}{dz} + \left(\frac{3}{2+3x}\right)^2\frac{d^2y}{dz^2}$$

$$\frac{d^2y}{dx^2} = \left(\frac{3}{2+3x}\right)^2\left(\frac{d^2y}{dz^2} + \frac{dy}{dz}\right) \quad — ②$$

Substituting ① & ② in given eqn

$$(2+3x)^2\left[\left(\frac{3}{2+3x}\right)^2\left(\frac{d^2y}{dz^2}-\frac{dy}{dz}\right)\right]+5(2+3x)\left[\frac{3}{2+3x}\left(\frac{dy}{dz}\right)\right]$$

$$-3y = x^2+x+1$$

$$\therefore 9\frac{d^2y}{dz^2}-9\frac{dy}{dz}+15\frac{dy}{dz}-3y=\left(\frac{e^z-2}{3}\right)^2+\left(\frac{e^z-2}{3}\right)+1$$

$$\therefore 9\frac{d^2y}{dz^2}+6\frac{dy}{dz}-3y=\frac{e^{2z}-4e^z+4}{9}+\frac{e^z-2}{3}+1$$

$$\therefore 9\frac{d^2y}{dz^2}+6\frac{dy}{dz}-3y=\frac{e^{2z}-e^z+7}{9}$$

Put $D=\frac{d}{dz}$

$$\therefore (9D^2+6D-3)y=\frac{1}{9}(e^{2z}-e^z+7)$$

AE: $9D^2+6D-3=0$

$3D(D+1)-1(D+1)=0$

$D=-1, 1/3$

$$\therefore y_c = c_1e^{-z}+c_2e^{z/3} = c_1\left(\frac{1}{2+3x}\right)+c_2(2+3x)^{1/3}$$

$$y_p=\frac{1}{9(9D^2+6D-3)}(e^{2z}-e^z+7)$$

$$=\frac{1}{9}\left[\frac{1}{(3D-1)(D+1)}(e^{2z}-e^z+7e^{0z})\right]$$

$$=\frac{1}{9}\left[\frac{e^{2z}}{15}-\frac{e^z}{4}-7\right]$$

$$\therefore y_p = \frac{1}{9}\left[\frac{(2+3x)^2}{15} - \frac{(2+3x)}{4} - 7\right]$$

$$\therefore y = y_c + y_p$$

$$\therefore y = \frac{c_1}{2+3x} + c_2(2+3x)^{1/3} + \frac{1}{9}\left[\frac{(2+3x)^2}{15} - \frac{(2+3x)}{4} - 7\right]$$

6] $(-1+2x)^3 \frac{d^3y}{dx^3} + 2(-1+2x)\frac{dy}{dx} - 2y = 0$

$\rightarrow$ Let $z = \log(-1+2x)$ $\quad \therefore e^z = -1+2x$

diff w.r.t x $\quad \therefore x = \frac{e^z+1}{2}$

$$\frac{dz}{dx} = \frac{1}{-1+2x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \left(\frac{1}{2x-1}\right)\frac{dy}{dz} \quad — ①$$

$$\therefore \frac{d^2y}{dx^2} = \left(\frac{2}{-1+2x}\right)^2\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right] \quad — ②$$

$$\therefore \frac{d^3y}{dx^3} = \left(\frac{2}{-1+2x}\right)^3\left[\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right] \quad — ③$$

Substituting ①, ② & ③ in given eqn

$$2^3\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right) + 2\frac{dy}{dz} - 2y = 0$$

$$8\frac{d^3y}{dz^3} - 24\frac{d^2y}{dz^2} + 16\frac{dy}{dz} + 2\frac{dy}{dz} - 2 = 0$$

$$\therefore \quad 8\frac{d^3y}{dz^3} - 24\frac{d^2y}{dz^2} + 18\frac{dy}{dz} - 2y = 0$$

Put, $D = \frac{d}{dz}$

$$\therefore \quad (8D^3 - 24D^2 + 18D - 2)\,y = 0$$

AE : $8D^3 - 24D^2 + 18D - 2 = 0$

$$(D-1)(4D^2 - 8D + 1) = 0$$

$\therefore D = 1$ $\qquad$ $4D^2 - 8D + D = 0$

$$\therefore D = \frac{2+\sqrt{3}}{2}, \frac{2-\sqrt{3}}{2}$$

$$\therefore \quad y_c = C_1 e^{z} + C_2 e^{\left(\frac{2+\sqrt{3}}{2}\right)z} + C_3 e^{\left(\frac{2-\sqrt{3}}{2}\right)z}$$

$$\therefore \cancel{y_c = c_1(-1+2x) + c_2(-1+2x)(c_2(-1+2x)^{\sqrt{3}/2} + c_3(-1}$$

$$\therefore y_c = C_1(-1+2x) + C_2(-1+2x)^{\frac{2+\sqrt{3}}{2}} + C_3(-1+2x)^{\frac{2-\sqrt{3}}{2}}$$

$$y_p = 0$$

$$\therefore y = y_c + y_p = y_c$$

$$\therefore y = C_1(-1+2x) + C_2(-1+2x)^{\frac{2+\sqrt{3}}{2}} + C_3(-1+2x)^{\frac{2-\sqrt{3}}{2}}$$

$$\therefore y = (-1+2x)\left[C_1 + C_2(-1+2x)^{\sqrt{3}/2} + C_3(-1+2x)^{-\sqrt{3}/2}\right]$$

7] Solve $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ for the case in which the circuit has the initial current i_0 at time $t=0$ and the emf impressed is given by $E = E_0 e^{-kt}$

→ $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ is of the type $\frac{dy}{dx} + Py = Q$

$$\therefore IF = e^{\int P dx} = e^{\int R/L \, dt} = e^{Rt/L}$$

Soln : $ie^{Rt/L} = \frac{E}{L}\int e^{Rt/L}\, dt + c$

$$\therefore ie^{Rt/L} = \frac{E}{L}\frac{L}{R} e^{Rt/L} + c$$

$$\therefore ie^{Rt/L} = \frac{E}{R} e^{Rt/L} + c$$

when $t=0$, $i_0 = \frac{E_0}{R} + c$

$$\therefore c = i_0 - \frac{E_0}{R}$$

$$\therefore ie^{Rt/L} = \frac{E}{R} e^{Rt/L} + i_0 - \frac{E_0}{R}$$

$$\therefore i = \frac{E}{R} e^{-Rt/L}\left[e^{Rt/L} + i_0 R - E_0\right]$$

8] Solve $L\frac{di}{dt} + Ri = E\sin\omega t$ for I, where $i=0$ for $t=0$

→ $L\frac{di}{dt} + Ri = E\sin\omega t$ is of form $\frac{dy}{dx} + Py = Q$

$$IF = e^{\int P dx} = e^{\int R/L\, dt} = e^{Rt/L}$$

Soln: $$i e^{Rt/L} = \frac{E}{L} \int e^{Rt/L} \sin wt \, dt + c$$

(we know $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$)

$$\therefore i e^{Rt/L} = \frac{E}{L} \cdot \frac{e^{Rt/L}}{(R/L)^2 + (\omega^2)} \left[\frac{R}{L} \sin \omega t - \omega \cos \omega t \right] + c$$

$$\therefore i = \frac{E}{R^2+\omega^2 L^2} [R \sin \omega t - \omega L \cos \omega t] + c e^{-Rt/L}$$

at $t=0$, $i=0$

$$c = \frac{E \omega L}{R^2 + \omega^2 L^2}$$

$$\therefore i = \frac{E}{R^2+\omega^2 L^2} [R \sin \omega t - \omega L \cos \omega t] + e^{-Rt/L} \frac{E \omega L}{R^2+\omega^2 L^2}$$

9] An uncharged condenser of capacity C is charged by applying an emf $E \sin nt$ through the leads of an inductance L and negligible resistance. The charge Q on the plate of the condenser satisfies the differential eqn $L\frac{d^2Q}{dt} + \frac{Q}{C} = E \sin nt$. Prove that charge at any time t is given by

$$Q = \frac{EC}{2} (\sin nt - nt \cos nt) \text{ where } n^2 = \frac{1}{LC}$$

$$\rightarrow \frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L}\sin nt$$

putting $n^2 = \frac{1}{LC}$, $\frac{d}{dt} = D$

$$(D^2 + n^2)Q = \frac{E}{L}\sin nt$$

AE : $D^2 + n^2 = 0$

$\therefore D = \pm ni$

$\therefore y_c = c_1 \cos nt + c_2 \sin nt$

$$y_p = \frac{1}{f(D)} X = \frac{E}{L}\frac{1}{(D^2+n^2)}\sin nt$$

putting $D^2 = -n^2$

$$y_p = \frac{E}{L}\frac{\sin t}{0}$$

Differentiating $f(D)$ & multiplying by t

$$y_p = \frac{E}{L} t \frac{1}{2D}\sin nt = \frac{Et}{L}\frac{D}{2D^2}\sin nt$$

again putting $D^2 = -n^2$

$$y_p = \frac{Et}{L}\frac{D(\sin nt)}{-2n^2} = \frac{-Et}{2n^2L}\frac{d}{dt}(\sin nt)$$

$$= \frac{-Et}{2n^2L} n\cos nt$$

$\therefore y_p = \frac{-Et}{2nL} \cos nt$

$Q = y_c + y_p$

$\therefore Q = c_1 \cos nt + c_2 \sin nt - \frac{Et}{2nL} \cos nt$

since $Q=0$ at $t=0$, we get $c_1 = 0$

Also, $i = \frac{dQ}{dt} = 0$ at $t=0$, we get $c_2 = \frac{EC}{2}$

$\therefore Q = \frac{EC}{2}[\sin nt - nt\cos nt]$

10] An electric circuit consists of condenser of capacity C, an inductance and emf $E = E_0 \cos\omega t$. The charge Q satifies the differential eqn $\frac{d^2Q}{dt} + \frac{Q}{CL} = \frac{E_0 \cos\omega t}{L}$. If $\omega = \frac{1}{\sqrt{CL}}$ and initially $Q = Q_0$ and current $I = I_0$ at $t=0$, show that $Q = Q_0 \cos\omega t + \frac{I_0}{\omega} \sin\omega t + \frac{E_0 t \sin\omega t}{2L\omega}$

→ Putting $\omega^2 = \frac{1}{LC}$, given eqn can be written as

$(D^2+\omega^2)Q = \frac{E_0}{L} \cos\omega t$ where $D = \frac{d}{dt}$

AE: $D^2+\omega^2 = 0$

i.e. $D = \pm \omega i$

$\therefore y_c = c_1 \cos\omega t + c_2 \sin\omega t$

$y_p = \frac{1}{f(D)} X = \frac{E_0}{L} \frac{1}{(D^2+\omega^2)} \cos\omega t$

if we put $D^2 = -\omega^2$

$y_p = \frac{E_0}{L} \frac{1}{0} \cos\omega t$

$\therefore$ Differentiating $f(D)$ & multiplying by t,

$y_p = \frac{E_0 t}{L} \frac{1}{2D} \cos\omega t = \frac{E_0 t}{L} \frac{D(\cos\omega t)}{-2\omega^2}$

(putting $D^2 = -\omega^2$)

$\therefore y_p = \frac{-E_0 t}{2L\omega^2} \frac{d}{dt}(\cos\omega t) = \frac{E_0 t}{2L\omega^2} \omega \sin\omega t$

$\therefore y_p = \frac{E_0 t}{2L\omega} \sin\omega t$

$\therefore Q = y_c + y_p = c_1 \cos\omega t + c_2 \sin\omega t + \frac{E_0 t}{2L\omega} \sin\omega t$

Since, $Q = Q_0$ at $t=0$, we get $c_1 = Q_0$

$I = I_0 = \frac{dQ_0}{dt} = 0$ at $t=0$, we get $c_2 = \frac{I_0}{\omega}$

$\therefore Q = Q_0 \cos\omega t + \frac{I_0}{\omega} \sin\omega t + \frac{E_0 t}{2L\omega} \sin\omega t$

PART - B

e] $\frac{dy}{dx} = (0.1)(x^3+y^2)$; $y(0)=1$

→

The Taylor series is given by,

$$y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \dots$$ where $x_0=0, y_0=1$

Now,

$$y' = (0.1)(x^3+y^2)$$

$$\therefore y_0' = (0.1)(0^3+1^3) = 0.1$$

$$y'' = (0.1)(3x^2+2yy')$$

$$\therefore y_0'' = (0.1)(3(0)^2+2(1)(0.1)) = 0.02$$

$$y''' = (0.1)(6x+2(yy''+y'^2))$$

$$\therefore y_0''' = (0.1)(6(0)+2[(1)(0.02)+(0.1)^2]) = 0.006$$

$$y^{iv} = (0.1)(6+2(y'y''+yy'''+2y'y''))$$

$$\therefore y_0^{iv} = (0.1)(6+2[(0.1)(0.02)+(1)(0.005)+2(0.1)(0.02)])$$

$$= 0.60148$$

$$\therefore y = 1+0.1x+\frac{0.02}{2!}x^2+\frac{0.006}{3!}x^3+\frac{0.60148}{4!}x^4+\dots$$

$$\therefore y = 1+0.1x+0.01x^2+0.001x^3+0.0251x^4+\dots$$