DIVISION / ROLL NO.: D1AD / 47

Vivekanand Education Society's Institute of Technology

(Academic Year 2020-2021)

Subject: Engineering Mathematics- I

Semester: I

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Assignment NO :- 1

TOPIC:- *Matrices and System of Linear Equations*

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Assignment 1

Topic: Matrices, System of Linear Equations

1] Show that every skew Hermitian matrix can be expressed in the form of $P+iQ$ where P is real skew-symmetric and Q is real symmetric matrix.

→ Let A be a Skew-Hermitian matrix, Then $A^{\theta}=-A$.

Let $P=\frac{1}{2}(A+\bar{A})$ & $Q=\frac{1}{2i}(A-\bar{A})$

We know that if $z=x+iy$ is a complex number then $\bar{z}=x-iy$ and hence $\frac{1}{2}(z+\bar{z})=x$ is real and also $\frac{1}{2i}(z-\bar{z})=\frac{1}{2i}\cdot 2iy=y$ is real. Hence, P & Q are real matrices.

Now we can write,

$$A=\frac{1}{2}(A+\bar{A})+i\left(\frac{1}{2i}(A-\bar{A})\right)=P+iQ$$

But,
$$P'=\frac{1}{2}(A+\bar{A})'=\frac{1}{2}[A'+(\bar{A})']=\frac{1}{2}[A'+A^{\theta}]$$

$$P'=\frac{1}{2}[(-A^{\theta})'+(-A^{\theta})^{\theta}] \quad \because \{A^{\theta}=-A\}$$

$$\therefore P'=\frac{1}{2}[((-\bar{A})')'-A]=\frac{1}{2}(-\bar{A}-A)=-\frac{1}{2}(A+\bar{A})$$

$\therefore P'=-P$

$\therefore$ P is real skew symmetric matrix

And
$$Q'=\left[\frac{1}{2i}(A-\bar{A})\right]'=\frac{1}{2i}(A-\bar{A})'=\frac{1}{2i}(A'-\bar{A}')$$

$$Q'=\frac{1}{2i}(A'-A^{\theta})=\frac{1}{2i}[(-A^{\theta})'-(-A^{\theta})^{\theta}] \quad \because \{A^{\theta}=-A\}$$

$\therefore C' = \frac{1}{2i}[((-\bar{A}))' + A] = \frac{1}{2i}[-\bar{A} + A] = \frac{1}{2i}(A - \bar{A})$

$\therefore C' = C$

$\therefore$ C is real symmetric matrix.

Thus, its proved that Skew-Hermitian matrix P+iQ ~~can~~ where P is real skew-Hermitian Matrix and Q is symmetric matrix.

2] Find rank of matrix by reducing in row echelon form

$$\begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$$

$\rightarrow$

Let,

$$A = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$$

$R_3 \Rightarrow R_3 - 6R_1$

$R_4 \Rightarrow R_4 - 6R_1$

$$\therefore A = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 0 & -18 & -11 & -21 \\ 0 & -24 & -12 & -26 \end{bmatrix}$$

$R_3 \Rightarrow R_3 + 9/2 R_2$

$R_4 \Rightarrow R_4 + 6R_2$

$$\therefore\ A = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 0 & 0 & 5/2 & -15/2 \\ 0 & 0 & 6 & -8 \end{bmatrix}$$

$R_4 \Rightarrow R_4 - \frac{12}{5} R_3$

$$\therefore\ A = \begin{bmatrix} 1 & 6 & 8 & 6 \\ 0 & 4 & 3 & 3 \\ 0 & 0 & 5/2 & -15/2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$r(A)$ = no. of non-zero rows.

$\therefore\ r(A) = 4$

3] Find the rank of the matrix by reducing it to normal form.

$$\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

$\longrightarrow$

Let,

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

$R_2 \Rightarrow R_2 - R_1$

$R_3 \Rightarrow R_3 - 2R_1$

$R_4 \Rightarrow R_4 - R_1$

$$\therefore\ A = \begin{bmatrix} 1 & 2 & -3 & 3 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & -1 & 1 & 1 & 5 \end{bmatrix}$$

$C_2 \Rightarrow C_2 - 2C_1$
$C_3 \Rightarrow C_3 + 3C_1$
$C_4 \Rightarrow C_4 - 3C_1$
$C_5 \Rightarrow C_5 - C_1$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & -1 & 1 & 1 & 5 \end{bmatrix}$$

$R_4 \Rightarrow R_4 + R_2$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

$C_5 \Rightarrow C_5 + C_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 3 & 0 & 2 \end{bmatrix}$$

$R_4 \Rightarrow R_4 - 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -3 & -10 \end{bmatrix}$$

$C_4 \Rightarrow C_4 - C_3$

$C_5 \Rightarrow C_5 - 4C_3$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -10 \end{bmatrix}$$

$R_4 \Rightarrow (-1/3)R_4$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 10/3 \end{bmatrix}$$

$C_5 \Rightarrow C_5 - (10/3)C_4$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$\therefore \quad A = [I_4 \quad O]$

$\therefore$ rank of A = order of I

$\therefore \quad r(A) = 4$

4] Test the consistency and solve those which are solvable

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1$$
$$2x_1 - x_2 + 3x_3 + 4x_5 = 2$$
$$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$$
$$x_1 + x_3 + 2x_4 + x_5 = 0$$

→ Let,

$$AX = B$$

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & -1 & 3 & 0 & 4 \\ 3 & -2 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 3 & 0 & 4 & 2 \\ 3 & -2 & 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 0 \end{array}\right]$$

$\therefore \cancel{[A|B]}$

$$R_2 \Rightarrow R_2 - 2R_1$$
$$R_3 \Rightarrow R_3 - 3R_1$$
$$R_4 \Rightarrow R_4 - R_1$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 1 & 0 & 3 & 0 & -1 \end{array}\right]$$

$$R_3 \leftrightarrow R_4$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 & -1 \\ 0 & 1 & -1 & 4 & -2 & -2 \end{array}\right]$$

$$R_3 \Rightarrow R_3 - R_2$$
$$R_4 \Rightarrow R_4 - R_3$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & -1 & 1 & -2 & -1 \\ 0 & 0 & -1 & 1 & -2 & -1 \end{array}\right]$$

$$R_4 \Rightarrow R_4 - R_3$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & -1 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ The rank of the coefficient matrix is equal to the rank of the augmented matrix $= 3$.
Hence, the equations are consistent. But the rank of $(A) (=3)$ is less than the number of unknowns $(=5)$

$\therefore$ The number of parameters $= 5-3 = 2$

$\therefore$ Hence, the equations have doubly infinite solutions

Now, $x_1 - x_2 + x_3 - x_4 + x_5 = 1$

$$x_2 + x_3 + 2x_4 + 2x_5 = 0$$
$$-x_3 + x_4 - 2x_5 = -1$$

If $x_4 = t_1$, $x_5 = t_2$

$-x_3 + t_1 - 2t_2 = -1$ $\quad \therefore x_3 = 1 + t_1 - 2t_2$

Now, $x_2 + x_3 + 2x_4 + 2x_5 = 0$ gives ~~$x_2 + 1 + t_1 - 2t_2 + 2t_1 + 2t_2 = 0$~~

$$x_2 + 1 + t_1 - 2t_2 + 2t_1 + 2t_2 = 0$$
$$\therefore x_2 = -1 - 3t_1$$

And $x_1 + 1 + 3t_1 + t_1 - 2t_2 - t_1 + t_2 = 1$

$$\therefore x_1 = -1 - 3t_1 + t_2$$

$\therefore (x_1, x_2, x_3, x_4, x_5) = (-1-3t_1+t_2, -1-3t_1, 1+t_1-2t_2, t_1, t_2)$

5] For what value of λ and μ, the system has no solution, a unique solution, infinite ~~solution~~ numbers of solution.

$$3x - 2y + z - \mu = 0$$
$$5x - 8y + 9z - 3 = 0$$
$$2x + y + \lambda z + 1 = 0$$

$\rightarrow$

$3x - 2y + z = \mu$; $5x - 8y + 9z = 3$; $2x + y + \lambda z = -1$

$$AX = B$$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & \lambda \end{bmatrix} \quad B = \begin{bmatrix} \mu \\ 3 \\ -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & -2 & 1 & \mu \\ 5 & -8 & 9 & 3 \\ 2 & 1 & \lambda & -1 \end{array}\right]$$

$$R_2 \Rightarrow R_2 - 5/3 R_1$$
$$R_3 \Rightarrow R_3 - 2/3 R_1$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & -2 & 1 & \mu \\ 0 & -14/3 & 22/3 & \frac{9-5\mu}{3} \\ 0 & 7/3 & \frac{3\lambda-2}{3} & \frac{-3-2\mu}{3} \end{array}\right]$$

$$R_3 \Rightarrow R_3 + 1/2 R_2$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & -2 & 1 & \mu \\ 0 & -14/3 & 22/3 & \frac{9-5\mu}{3} \\ 0 & 0 & \frac{3\lambda+9}{3} & \frac{3-9\mu}{3} \end{array}\right]$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & -2 & 1 & \mu \\ 0 & -14/3 & 22/3 & \frac{9-5\mu}{3} \\ 0 & 0 & \lambda+3 & \frac{1-3\mu}{2} \end{array}\right]$$

(i) No solution:

if $r(A) \neq r(A|B)$

which is only possible when

$$\lambda+3=0 \quad \& \quad \frac{1-3\mu}{2} \neq 0$$

i.e. $\lambda=-3$ & $\mu \neq \frac{1}{3}$

(ii) Unique solution:

if $r(A) = r(A|B) =$ no. of unknows $=2$

only possible when $\lambda+3 \neq 0$ & $\frac{1-3\mu}{2} \in R$

i.e. $\lambda \neq -3$ & $\mu \in R$

(iii) Infinite number of solution:

if $r(A) = r(A|B) <$ no. of unknowns i.e. 2

which is only possible when

$\lambda+3=0$ & $\frac{1-3\mu}{2}=0$

~~$\lambda = 3$~~

i.e. $\lambda=-3$ & $\mu=\frac{1}{3}$

6] In the following system has trivial solution? Obtain the non-trivial solution if exist

$$3x_1+4x_2-x_3-9x_4=0$$
$$2x_1+3x_2+2x_3-3x_4=0$$
$$2x_1+x_2-14x_3-12x_4=0$$
$$x_1+3x_2+13x_3+3x_4=0$$

→ Let $AX=B$

where,

$$A = \begin{bmatrix} 3 & 4 & -1 & -9 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -12 \\ 1 & 3 & 3 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{cccc|c} 3 & 4 & -1 & -9 & 0 \\ 2 & 3 & 2 & -3 & 0 \\ 2 & 1 & -14 & -12 & 0 \\ 1 & 3 & 3 & 3 & 0 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 2/3 R_1$

$R_3 \Rightarrow R_3 - 2/3 R_1$

$R_4 \Rightarrow R_4 - 1/3 R_1$

$$[A|B] = \left[\begin{array}{cccc|c} 3 & 4 & -1 & -9 & 0 \\ 0 & 1/3 & 8/3 & 3 & 0 \\ 0 & -5/3 & -40/3 & -6 & 0 \\ 0 & 5/3 & 40/3 & 6 & 0 \end{array}\right]$$

$R_3 \Rightarrow R_3 + 5 R_2$

$R_4 \Rightarrow R_4 - 5 R_2$

$$[A|B] = \left[\begin{array}{cccc|c} 3 & 4 & -1 & -9 & 0 \\ 0 & 1/3 & 8/3 & 3 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & -9 & 0 \end{array}\right]$$

$R_4 \Rightarrow R_4 + R_3$

$$[A|B] = \left[\begin{array}{cccc|c} 3 & 4 & -1 & 9 & 0 \\ 0 & 1/3 & 8/3 & 3 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$r(A|B) = 3$ no. of unknows $= 4$

∴ Non-trivial

∴ no. of parameters $= 4-3=1$

$x_4 = 0 \quad , \quad x_3 = t$

$$3x_1 + 4x_2 - x_3 + 9x_4 = 0 \quad \&$$
$$\tfrac{1}{3}x_2 + \tfrac{8}{3}x_3 + 3x_4 = 0$$

$x_2 + x_3 + 9x_4 = 0$

$x_2 + 8t + 0 = 0$

$x_2 = -8t$

$3x_1 - 32t - t + 0 = 0$

$x_1 = \frac{33t}{3}$

$x_1 = 11t$

$$(x_1, x_2, x_3, x_4) \equiv (11t, -8t, t, 0)$$

7] For what value of λ, the following system of equations poses a non-trivial solution. Obtain the solution for real value of λ.

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z$$

$\rightarrow$

$$(1-\lambda x) + 2y + 3z = 0 \; ; \quad 3x + (1-\lambda)y + 2z = 0$$
$$2x + 3y + (1-\lambda)z = 0$$

Let, $A = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix}$ & $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1+\lambda^2-2\lambda-6)-2(-3-3\lambda-4)+3(9-2+2\lambda)=0$$

$$(1-\lambda)(\lambda^2-2\lambda-5)+2(1+3\lambda)+3(7+2\lambda)=0$$

$$\lambda^2-2\lambda-5-\lambda^3+2\lambda^2+5\lambda+2+6\lambda+21+6\lambda=0$$

$$-\lambda^3+3\lambda^2+15\lambda+18=0$$

$$\lambda^3-3\lambda^2-15\lambda-18=0$$

$$\lambda_1=6 \quad ; \quad \lambda_2=\frac{-3}{2}+\frac{\sqrt{3}}{2}i \quad ; \quad \lambda_3=\frac{-3}{2}-\frac{\sqrt{3}}{2}i$$

i.e. λ_2 & λ_3 are not real they are not taken into consideration.

$\therefore \lambda=6$

$$[A|B]=\left[\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 3 & -5 & 2 & 0 \\ 2 & 3 & -5 & 0 \end{array}\right]$$

$R_2 \Rightarrow R_2+3/5 R_1$

$R_3 \Rightarrow R_3+2/5 R_1$

$$\therefore [A|B]=\left[\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & -19/5 & 19/5 & 0 \\ 0 & 19/5 & -19/5 & 0 \end{array}\right]$$

$R_3 \Rightarrow R_3+R_2$

$$\therefore [A|B]=\left[\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & -19/5 & 19/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$r(A|B)=2$ no. of unknows $=3$

$\therefore$ no. of parameters $=3-2=1$

Let, $z = t$

$$-\frac{19}{5} y + \frac{19}{5} z = 0$$

$$\therefore \quad y = t$$

$$-5x + 2y + 3z = 0$$

$$\therefore \quad -5x + 2t + 3t = 0$$

$$\therefore \quad 5x = 5t$$

$$\therefore \quad x = t$$

$$\therefore \quad \lambda = 6 \quad \& \quad (x, y, z) = (t, t, t)$$