Three Phase Circuits

3.1 Introduction

We have seen that a single-phase voltage can be generated by rotating a coil (winding) in a magnetic field. Such an ac machine is called a **single-phase generator**. It has only one armature winding. But if the generator is arranged to have two or more separate windings displaced from each other by equal electrical angles, it is called polyphase generator. 'Poly' means many and 'phase' means windings, branches, or circuits.

Figure 3.1(a) shows an elementary single-phase generator. It has one winding or coil R rotating in anticlockwise direction with a constant angular velocity ω rad/sec in a uniform magnetic field. Across the winding, an alternating voltage is generated, which can be expressed by the equation

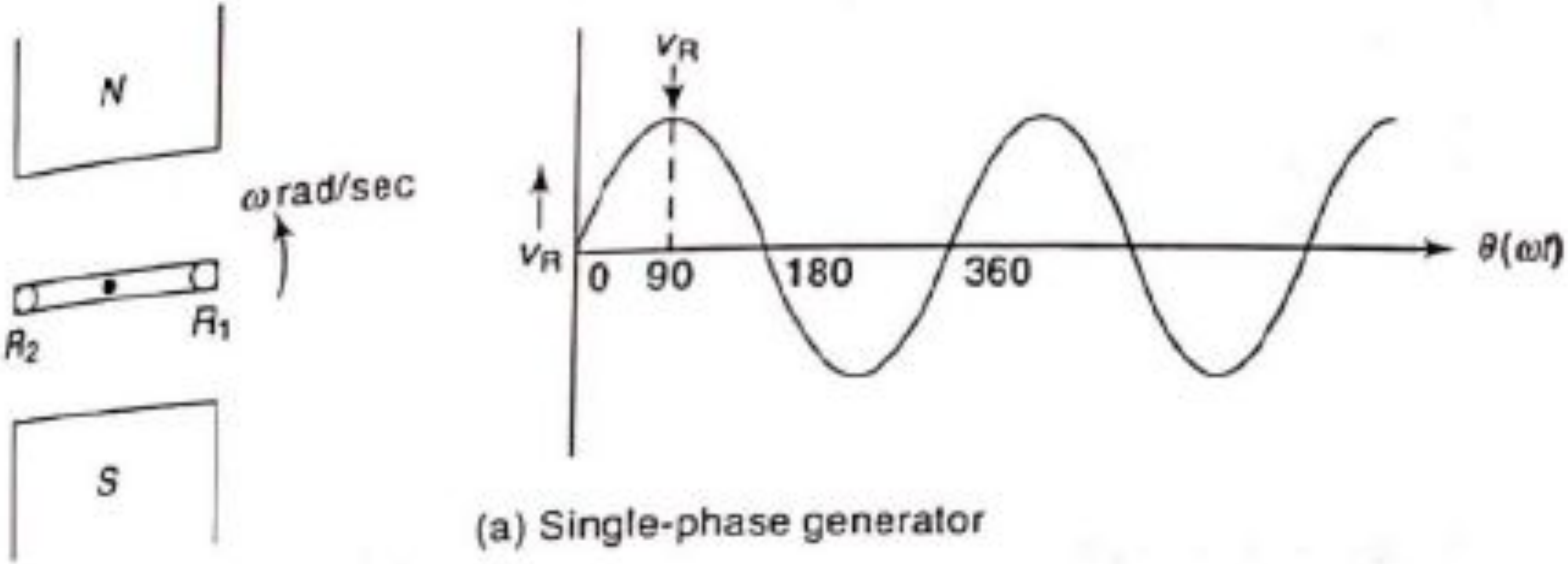
$$v_R = V_m \sin\omega t$$

Figure 3.1(b) shows an elementary two-phase generator. It has two identical windings or coils R and Y displaced by 90° from each other and rotating in anticlockwise direction with a constant angular velocity ω rad/sec in a uniform magnetic field. Here R_1, Y_1 are the start terminals and R_2, Y_2 are the finish terminals of the two coils. Since the two coils are identical and have the same angular velocity, voltages induced in them will be of same magnitude and frequency. However, these voltages will have a phase difference of 90°. The equations of the two voltages are as follows:

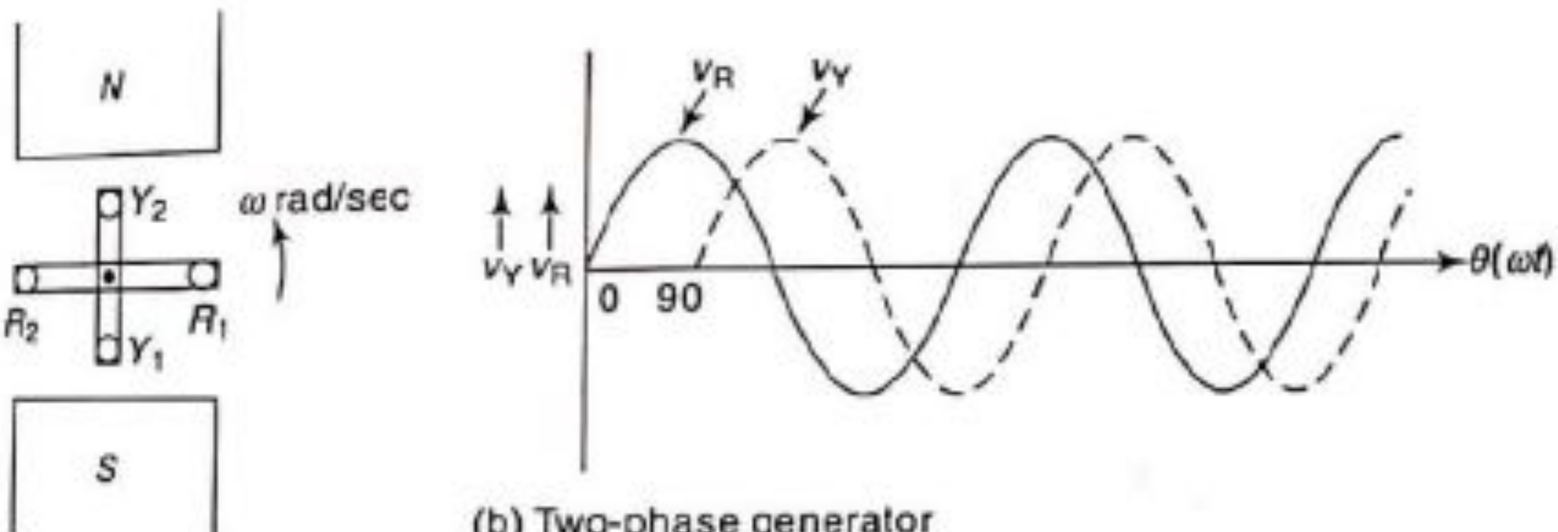
$$v_R = V_m \sin\omega t$$

$$v_Y = V_m \sin(\omega t - 90°)$$

Thus, a two-phase generator has two separate but identical windings that are 90° electrical apart and rotating in a uniform magnetic field. Obviously, such a generator will produce two alternating voltages of same magnitude and frequency having a phase difference of 90°. Similarly, a three-phase generator has three separate but identical windings that are 120° electrical apart and rotating in a uniform magnetic field. A three-phase generator will, therefore, produce three alternating voltages of same magnitude and frequency but displaced by 120° electrical from each other.



(a) Single-phase generator



(b) Two-phase generator

Fig. 3.1

3.2 Generation of Three-Phase Voltages

Figure 3.2(a) shows an elementary three-phase generator. It has three identical windings or coils R, Y and B displaced by 120° from each other and rotating in anticlockwise direction with a constant angular velocity ω rad/sec in a uniform magnetic field. Note that the corresponding start terminals R_1, Y_1, and B_1 are 120° apart. Likewise the finish terminals R_2, Y_2, and B_2 are 120° apart. Since the three coils are identical and have the same angular velocity, voltages induced in them will be of same magnitude and frequency. However, the three voltages will be displaced from each other by 120°. Note that the emf in coil Y lags behind that in coil R by 120° and the emf in coil B lags behind that in coil R by 240°. This is shown by the wave diagram in Fig. 3.2(b). Figure 3.2(c) shows the phasor diagram. The equations of the three voltages are as follows:

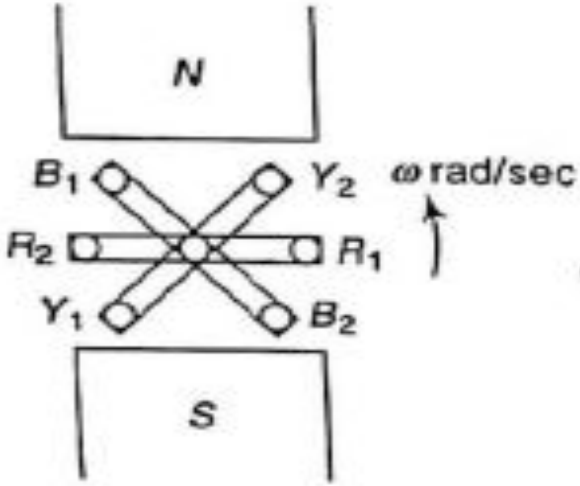
$$v_R = V_m \sin\omega t$$
$$v_Y = V_m \sin(\omega t - 120°)$$
$$v_B = V_m \sin(\omega t - 240°)$$

Resultant $= v_R + v_Y + v_B$

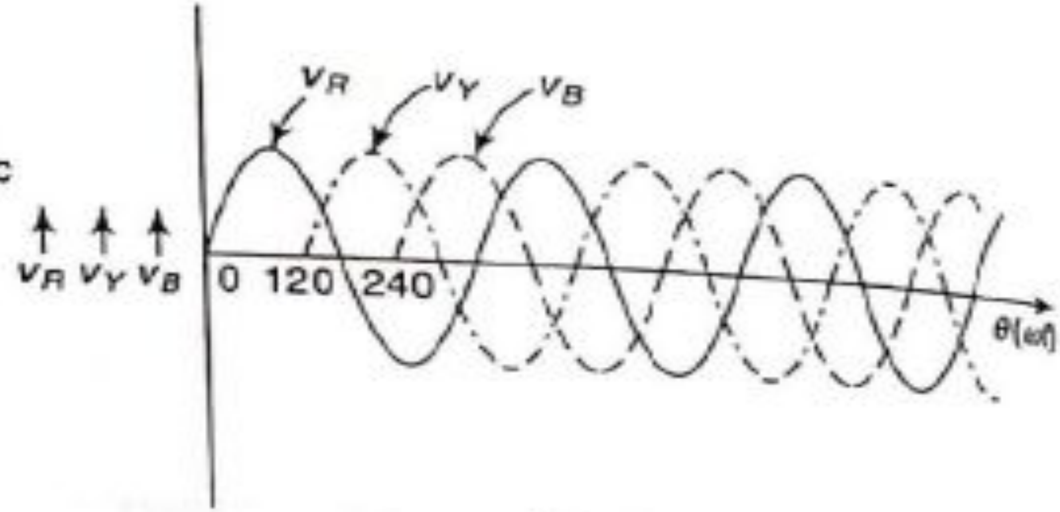
Putting the values, we get

$$= V_m \sin\omega t + V_m \sin(\omega t - 120°) + V_m \sin(\omega t - 240°)$$
$$= V_m [\sin\omega t + 2 \sin(\omega t - 180°) \cos 60°]$$
$$= V_m [\sin\omega t - 2 \sin\omega t \cos 60°]$$
$$= 0$$

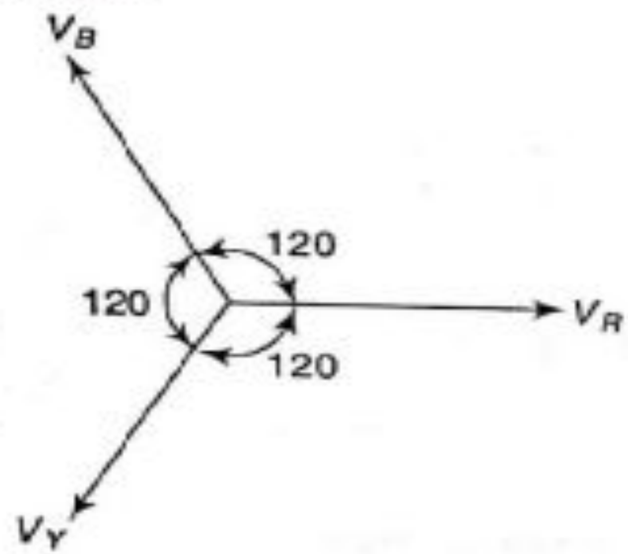
Hence, the sum of the three voltages at every instant is zero.



(a) Three-phase generator



(b) Waveforms



(c) Phasor diagram

Fig. 3.2

3.2.1 Advantages of Three-Phase System

In a three-phase system, the generator has three windings and it produces three independent alternating voltages. These voltages have same magnitude and frequency but have a phase difference of 120° between them. Such a three-phase system has the following advantages over a single-phase system:

1. In a single-phase system, the instantaneous power is a function of time, i.e., fluctuating and, hence, causes considerable vibrations in the single-phase motor. Hence, performance of single-phase motors is poor, while instantaneous power in symmetrical three-phase systems is constant. Also three-phase systems give steady output.
2. Power factor of single-phase motors is poor than three-phase motors of same rating.
3. The output of a three-phase machine is always greater than that of a single-phase machine of same size. So, for a given size and voltage, a three-phase generator occupies less space and has less cost than a single-phase having same rating.
4. For transmission and distribution, a three-phase system needs less copper or conducting material than a single-phase system for given volt amperes and voltage rating. So, transmission becomes very much economical in case of three-phase systems.
5. It is possible to produce rotating magnetic field with stationary coils by using a three-phase system. Hence, three-phase motors are self-starting.

6. Single-phase supply can be obtained from a three-phase system but not vice versa.
7. For converting machines such as rectifiers, the dc output voltage becomes smoother if the number of phases is increased.

It is found that the optimum number of phases required to get all the above said advantages is three. Any further increase in number of phases causes a lot of complications. Hence, three-phase system is universally accepted as standard system.

3.2.2 Some Concepts

The terms that we come across while analysing a three-phase system are briefly discussed below.

Phase sequence The order in which the voltages in the three phases reach their maximum positive value is called the phase sequence or phase order. From the waveforms of three-phase system [Fig. 3.2(b)], it is clear that the voltage in coil R attains maximum positive value first, next coil Y and then coil B. Hence, the phase sequence is R-Y-B.

Phase voltage The voltage induced in each winding is called phase voltage.

Phase current The current flowing through each winding is called phase current.

Line voltage The voltage available between any pair of terminals or lines is called line voltage.

Line current The current flowing through each line is called line current.

Balanced system A three-phase system is said to be balanced if

(a) The voltages in the three phases are equal in magnitude and differ in phase from each other by 120°.

(b) The currents in the three phases are equal in magnitude and differ in phase from each other by 120°.

(c) The loads connected across the three phases are identical i.e. all the loads have same magnitude and power factor.

Symmetrical system It is possible in a polyphase system that magnitudes of different alternating voltages are different. But a three-phase system in which the three voltages are of same magnitude and frequency and differ from each other by 120° phase angle, is defined as a symmetrical system.

is defined as a symmetrical system.

3.3 Interconnection of Three Phases

A three-phase generator has three identical windings. Each winding has two terminals, viz. 'start' and 'finish'. If a separate load is connected across each winding as shown in Fig. 3.3, six conductors are required to transmit and distribute power. This will make the system complicated and expensive. In order to reduce the number of conductors, the three windings are connected in the following two ways:

1. Star or wye connection
2. Delta or mesh connection

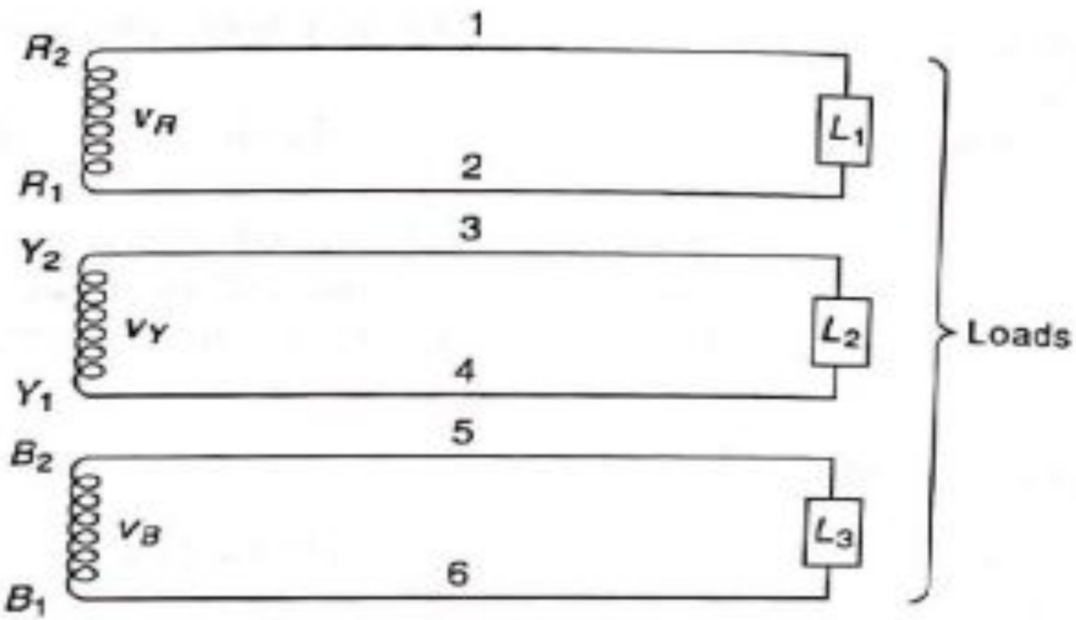


Fig. 3.3 Three-phase system with six wires used for connection

3.3.1 Star or Wye Connection

In this type of connection, similar terminals (start or finish) of the three windings are joined together as shown in Fig. 3.4. The common point is called star or neutral point. The three conductors (lines) are taken out from the three phases.

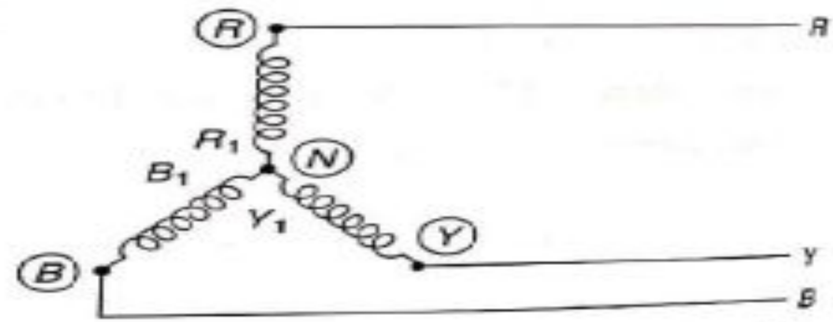


Fig. 3.4 Star-connected generator (supply)

Figure 3.5 shows a three-phase, four-wire, star-connected system. When three identical loads are connected in star, we get a three-phase, Y-connected balanced load[1]. As shown in the figure, the balanced load is connected across a supply using four wires and the current flowing through the neutral wire is the sum of the three currents I_R, I_Y and I_B. Since the load is balanced, these currents are equal in magnitude but

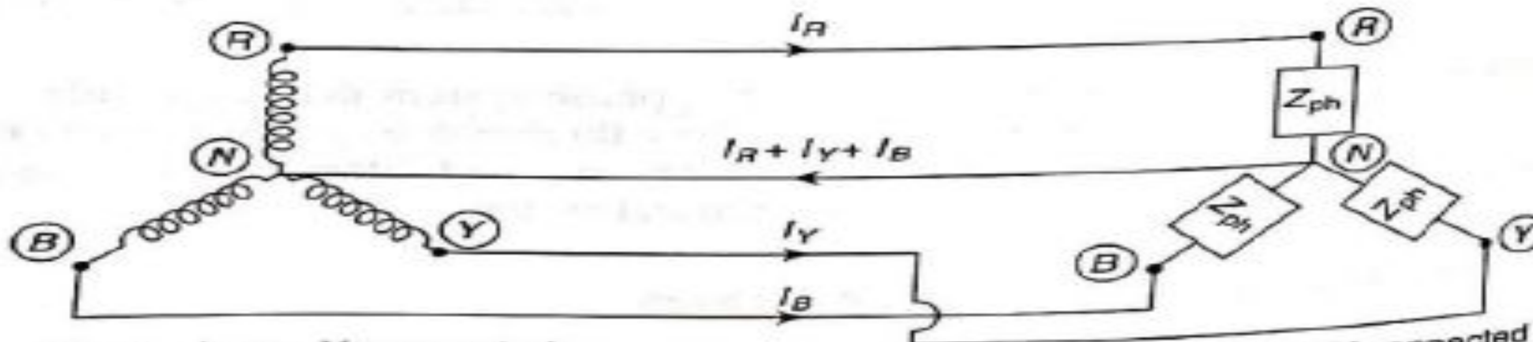


Fig. 3.5 Three-phase, four-wire, star-connected system

[1]A balanced load is a load in which magnitudes of all impedances connected to the load are equal (Z_{ph} each) and the phase angles of them are also equal and of same type (i.e., inductive, resistive, or capacitive).

differ in phase from one another by 120°. Thus, the three line currents are given as follows:

$$i_R = I_m \sin \omega t$$
$$i_Y = I_m \sin (\omega t - 120°)$$
$$i_B = I_m \sin (\omega t - 240°)$$

The current flowing through the neutral wire can be calculated as follows:

$$i_R + i_Y + i_B = I_m \sin \omega t + I_m \sin (\omega t - 120°) + I_m \sin (\omega t - 240°) = 0$$

Thus, if the load is balanced, the neutral wire carries zero current.

We can, therefore, remove the neutral wire without affecting the voltages or the currents in the circuit as shown in Fig. 3.6. This constitutes three-phase, three-wire system. If the load is not balanced, the neutral wire carries some current.

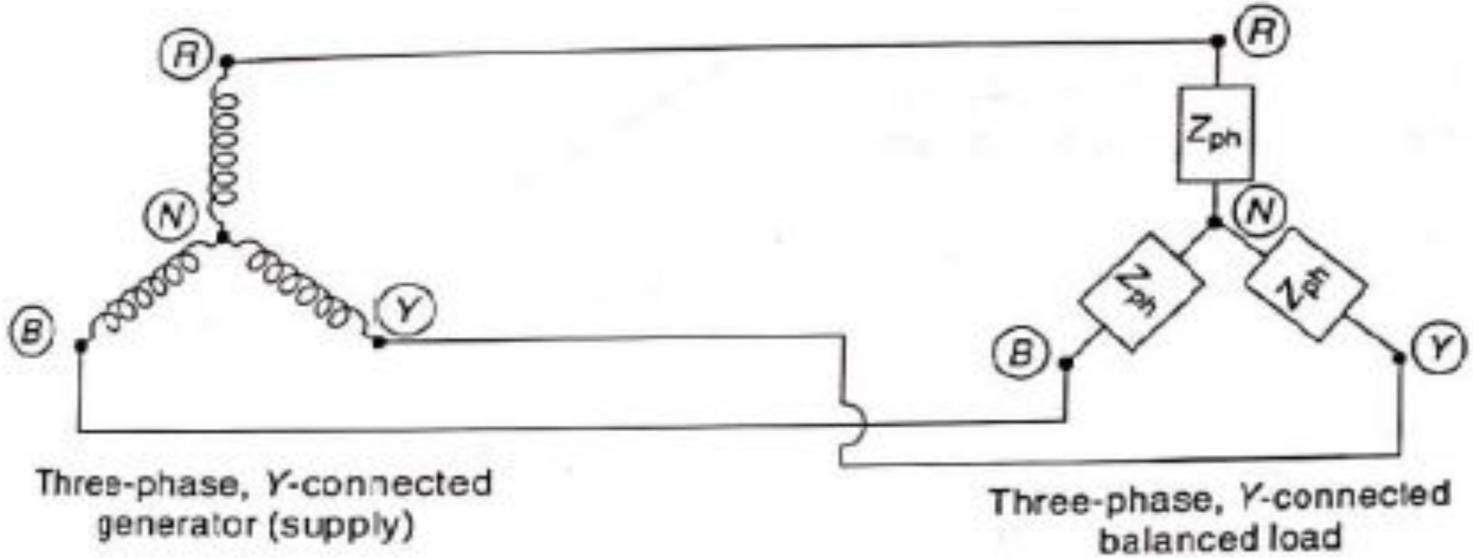


Fig. 3.6 Three-phase, three-wire, star-connected system

Relationship between line voltage and phase voltage

Figure 3.7 shows a balanced three-phase Y-connected system.

As load is balanced, all the three phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and 120° apart. By phase sequence, V_{YN} lags behind that of V_{RN} by 120° and V_{BN} lags behind that of V_{RN} by 240°. The magnitude of each phase voltage is denoted by V_{ph}. Thus,

$$V_{ph} = V_{RN} = V_{YN} = V_{BN}$$

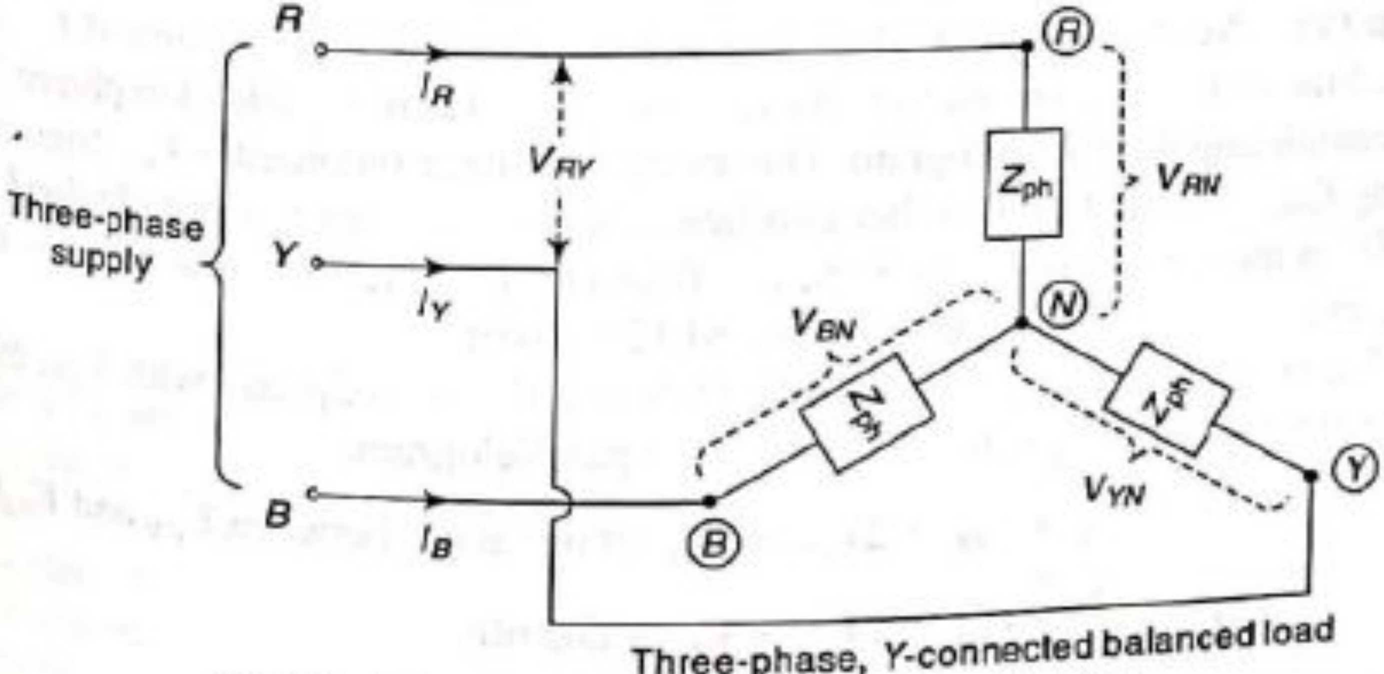


Fig. 3.7 Three-phase, Y-connected balanced system

The three line voltages are V_{RY}, V_{YB} and V_{BR}. From the circuit diagram, it is clear that line voltage is not same as phase voltage. However, for a balanced system, all three line voltages must also be equal and the magnitude of each line voltage is denoted by V_L. Thus,

$$V_L = V_{RY} = V_{YB} = V_{BR}$$

From the circuit diagram, the line voltage V_{RY} can be written in terms of the phase voltages as

$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY} \quad (3.1)$$

Similarly, $$\bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} \quad (3.2)$$

$$\bar{V}_{BR} = \bar{V}_{BN} + \bar{V}_{NR} \quad (3.3)$$

Now, magnitude of the line voltage V_{RY} in terms of the phase voltages can be calculated by using phasor diagram as shown in Fig. 3.8.

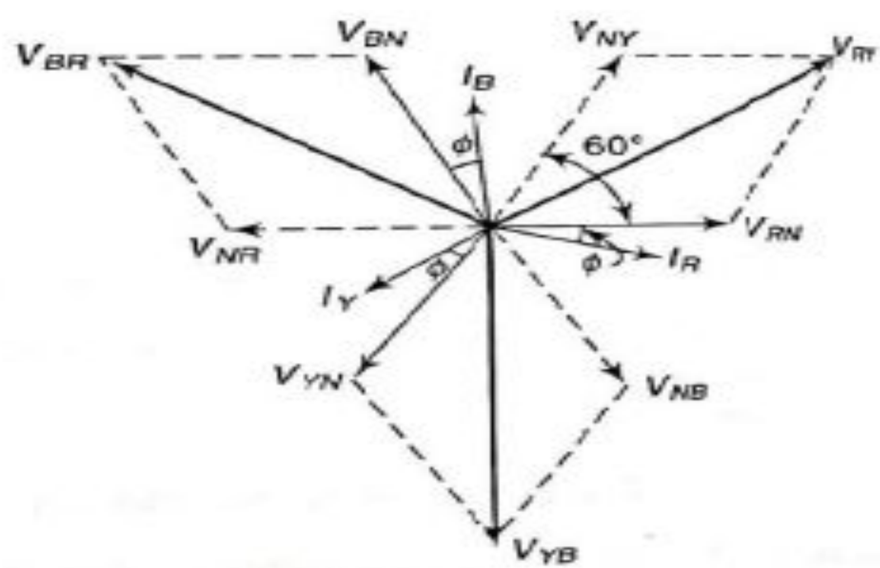


Fig. 3.8 Phasor diagram

Phasor diagram can be drawn by using the following steps:

Step I: Take the first phase voltage V_{RN} as reference and draw the three phase voltages V_{RN}, V_{YN} and V_{BN} equal in magnitude and 120° apart.

Step II: As load is balanced, all the three phase currents I_R, I_Y and I_B are equal in magnitude and 120° apart. Assuming inductive load, draw the three phase currents. All the three phase currents are equal in magnitude in current scale, each phase current lagging behind its respective phase voltage by an angle ϕ.

... for star connection. Hence,

voltage by an angle ϕ.

Step III: Line currents are the same as phase currents for star connection. Hence, separate line currents need not be shown.

Step IV: According to Eq. (3.1), line voltage V_{RY} is the phasor sum of V_{RN} and V_{NY}. Phasor V_{YN} is reversed to make it phasor V_{NY}. Then it is added in phasor V_{RN} by completing the parallelogram. The resultant voltage obtained is V_{RY}. Similarly, using Eqs (3.2) and (3.3), other two line voltages V_{YB} and V_{BR} are obtained.

From the symmetry of diagram, it is found that all the three line voltages V_{RY}, V_{YB} and V_{BR} are equal in magnitude and 120° apart.

Angle between V_{RN} and V_{YN} is 120°. Since V_{NY} is antiphase with V_{YN}, angle between V_{RN} and V_{NY} is 60°. Using law of parallelogram,

$$V_{RY}^2 = V_{RN}^2 + V_{NY}^2 + 2V_{RN} \times V_{NY} \times \cos \text{ (angle between } V_{RN} \text{ and } V_{NY})$$

or
$$V_{RY}^2 = V_{RN}^2 + V_{NY}^2 + 2V_{RN} \times V_{NY} \times \cos 60^\circ$$

But V_{RY} is the line voltage and V_{RN} and V_{NY} are the phase voltages. So, we have

$$V_L^2 = V_{ph}^2 + V_{ph}^2 + 2V_{ph} \times V_{ph} \times \cos 60° = 3V_{ph}^2$$

$$V_L = \sqrt{3}\, V_{ph}$$

Hence, in a balanced three-phase star connection,

1. Line voltage, $V_L = \sqrt{3}\, V_{ph}$
2. All line voltages are equal in magnitude but 120° apart from each other
3. Line voltage leads by 30° to their respective phase voltage

Relationship between line current and phase current

In a star connection, each line conductor is connected in series to a separate phase as shown in Fig. 3.7. Therefore, current in the line conductor is same as that in the phase to which the line conductor is connected, i.e.,

Line current, $I_L = I_{ph}$

Power

In a three-phase star-connected balanced load, power consumed in each load phase is the same, as all three phases are identical. Therefore, total power in the circuit is the sum of powers in the three-phases, i.e.,

Total power, $P = 3 \times$ power in each phase

or $\quad P = 3 \times V_{ph} I_{ph} \cos \phi$

or $\quad = 3 V_{ph} I_{ph} \cos \phi \qquad (3.4)$

For a star connection, $V_L = \sqrt{3}\, V_{ph}, \quad I_L = I_{ph}$

So, $\quad P = 3 \times \dfrac{V_L}{\sqrt{3}} \times I_L \cos \phi$

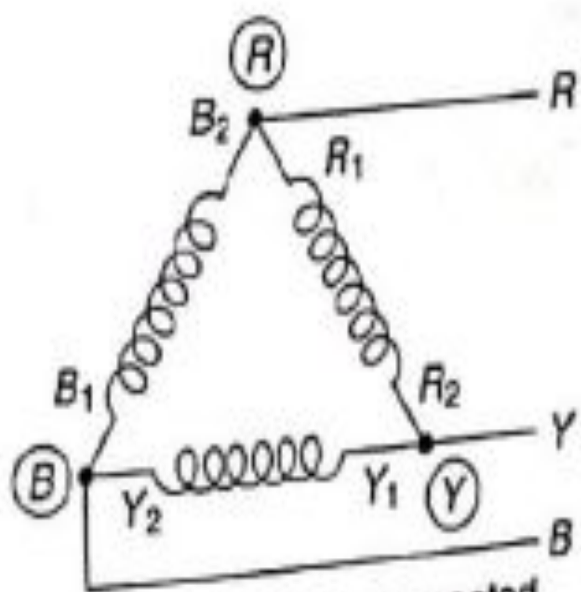
$$= \sqrt{3}\, V_L I_L \cos \phi \qquad (3.5)$$

Either of relations (3.4) and (3.5) can be used to determine the power. It may be noted that ϕ is the phase difference between the phase voltage and the corresponding phase current but not between the line current and the corresponding line voltage.

3.3.2 Delta Connection

In this type of connection, dissimilar terminals of the three windings are joined together, i.e., 'finish' terminal of one winding is connected to 'start' terminal of other and so on as shown in Fig. 3.9. The three line conductors are taken from the three junctions of the delta connection.

In a balanced delta-connected system, the three phase voltages are equal in magnitude and differ in phase from each other by 120°. Therefore, sum of these voltages around the closed mesh is zero.



Three-phase, Δ-connected generator (supply)

Fig. 3.9

Relationship between line voltage and phase voltage

As load is balanced, all the three-phase voltages V_{RY}, V_{YB} and V_{BR} are equal in magnitude and 120° apart.

So, $\quad V_{ph} = V_{RY} = V_{YB} = V_{BR}$

In the delta connection, each phase is connected across a pair of line as shown in Fig. 3.10. Therefore, phase voltage is same as line voltage across which the phase is connected

or $\quad$ Phase voltage, $V_{ph} = V_L$

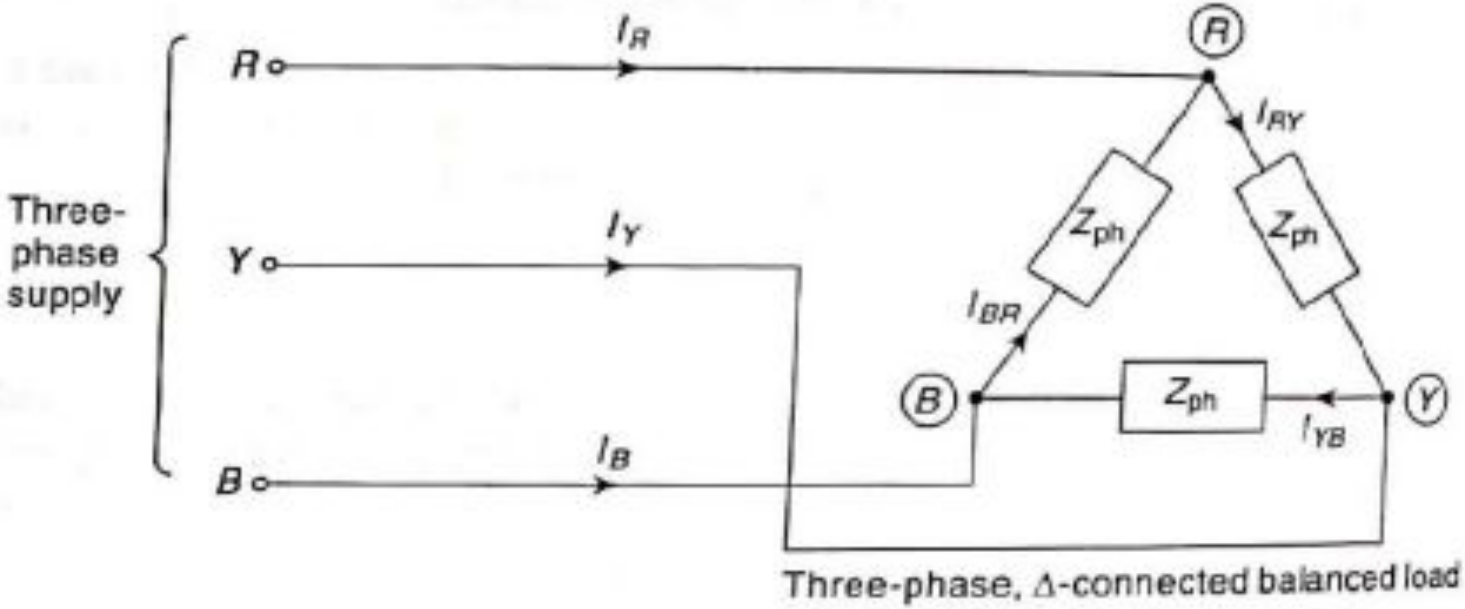


Fig. 3.10 Three-phase, Δ-connected balanced system

Relationship between line current and phase current

Relationship between line current and phase current

Figure 3.10 shows a balanced three-phase, Δ-connected system.

As load is balanced, all the three phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude and 120° apart from each other. By phase sequence, I_{YB} lags behind I_{RY} by 120° and I_{BR} lags behind I_{RY} by 240°. The magnitude of each phase current is denoted by I_{ph}.

So, $\quad I_{ph} = I_{RY} = I_{YB} = I_{BR}$

The three line currents are I_R, I_Y and I_B. From the circuit diagram, it is clear that line current is not same as phase current. However, for a balanced system, all the three line currents must also be equal and the magnitude of each line current is denoted by I_L.

So, $\quad I_L = I_R = I_Y = I_B$

Consider the first line current I_R. From the circuit diagram, by applying KCL at node R, the line current I_R can be written in terms of phase currents as

$$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY} \tag{3.6}$$

$$\bar{I}_R = \bar{I}_{RY} + (-\bar{I}_{BR})$$

Similarly, by applying KCL at node Y,

$$\bar{I}_Y + \bar{I}_{RY} = \bar{I}_{YB} \tag{3.7}$$

$$\bar{I}_Y = \bar{I}_{YB} + (-\bar{I}_{RY})$$

By applying KCL at node B,

$$\bar{I}_B + \bar{I}_{YB} = \bar{I}_{BR} \tag{3.8}$$

$$\bar{I}_B = \bar{I}_{BR} + (-\bar{I}_{YB})$$

Now, magnitude of the line current I_R in terms of phase currents can be calculated by using phasor diagram as shown in Fig. 3.11.

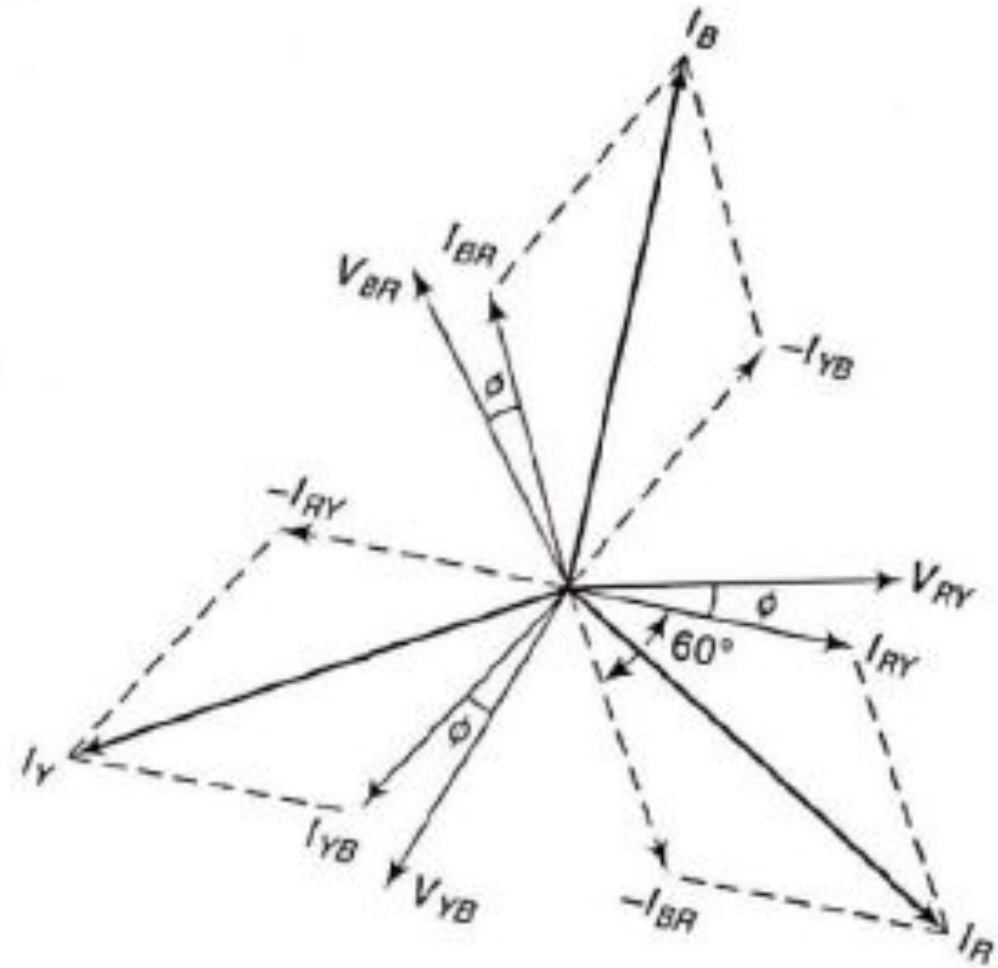


Fig. 3.11 Phasor diagram

Phasor diagram can be drawn by using the following steps:

Step I: Take the first phase voltage V_{RN} as reference and draw the three phase voltages V_{RN}, V_{YN} and V_{BN} equal in magnitude and 120° apart.

Step II: As load is balanced, all the three phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude and 120° apart. Assuming inductive load, draw three phase currents. All the three phase currents should be equal in magnitude in current scale, each current lagging behind its respective phase voltage by an angle ϕ.

Step III: Line voltages are the same as phase voltages for delta connection. Hence, separate line voltages need not be shown.

Step IV: According to Eq. (3.6), line current I_R is the phasor sum of I_{RY} and $(-I_{BR})$. Phasor I_{BR} is reversed to make it phasor $(-I_{BR})$. Then it is added in phasor I_{RY} by completing the parallelogram. The resultant current obtained is I_R. Similarly, using Eqs (3.7) and (3.8), other two line currents I_Y and I_B are obtained.

From the symmetry of diagram, it is found that all the three line currents I_R, I_Y and I_B are equal in magnitude and 120° apart.

Angle between I_{RY} and I_{BR} is 120°. Since $(-I_{BR})$ is antiphase with I_{BR}, angle between I_{RY} and $(-I_{BR})$ is 60°. Using law of parallelogram,

$$I_R^2 = I_{RY}^2 + I_{BR}^2 + 2I_{RY} \times I_{BR} \cos [\text{angle between } I_{RY} \text{ and } (-I_{BR})]$$

where I_R is the line current and I_{RY} and I_{BR} are the phase currents.

So, $$I_L^2 = I_{ph}^2 + I_{ph}^2 + 2I_{ph} \times I_{ph} \times \cos 60° = 3I_{ph}^2$$

or $$I_L = \sqrt{3} I_{ph}$$

Hence, in a balanced three-phase delta-connection,

1. Line current, $I_L = \sqrt{3} I_{ph}$.
2. All the line currents are equal in magnitude but 120° apart from each other.
3. Line current lags behind its respective phase current by 30°.

Power

In a three-phase delta connected balanced load, as all the three phases are identical, power consumed in each load phase is the same. Therefore, total power in the circuit is the sum of powers in the three phases.

Total power, $\quad P = 3 \times \text{power in each phase}$

$$= 3 \times V_{ph} I_{ph} \cos \phi$$

$$= 3 V_{ph} I_{ph} \cos \phi \tag{3.9}$$

For a delta connection, $V_L = V_{ph}, \quad I_L = \sqrt{3} I_{ph}$

So, $$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos \phi$$

$$= \sqrt{3} V_L I_L \cos \phi \tag{3.10}$$

Either of relations (3.9) and (3.10) can be used to determine the power. It may be noted that ϕ is the phase difference between the phase voltage and the corresponding phase current and not between the line current and the corresponding line voltage.

3.4 Power Triangle for Three-Phase Load

In ac circuits, we have already defined the different types of power as

1. Apparent power S (volt ampere VA)
2. Active power P (Watts)
3. Reactive power Q (volt ampere reactive VAR)

All these are applicable to the three-phase circuits as follows:

1. In a three-phase balanced system (Y-connected or Δ-connected), apparent power can be calculated as

   Total apparent power, $S = 3 \times$ apparent power per phase

   $$S = 3\, V_{ph} I_{ph} \qquad \text{VA or kVA}$$

   $$S = \sqrt{3} V_L I_L \qquad \text{VA or kVA}$$

2. In a three-phase balanced system (Y-connected or Δ-connected), active power is given as

   Total active power, $P = 3\, V_{ph} I_{ph} \cos\phi \quad$ W or kW

   $$P = \sqrt{3} V_L I_L \cos\phi \quad \text{W or kW}$$

3. In a three-phase balanced system (Y-connected or Δ-connected), reactive power is given as

Total reactive power,

$$Q = 3\,V_{ph} I_{ph} \sin\phi \quad \text{VAR or kVAR}$$

$$Q = \sqrt{3}\,V_L\,I_L\,\sin\phi \quad \text{VAR or kVAR}$$

The power triangle for a three-phase system is as shown in Fig. 3.12, which shows that

$$S = \sqrt{P^2 + Q^2}$$

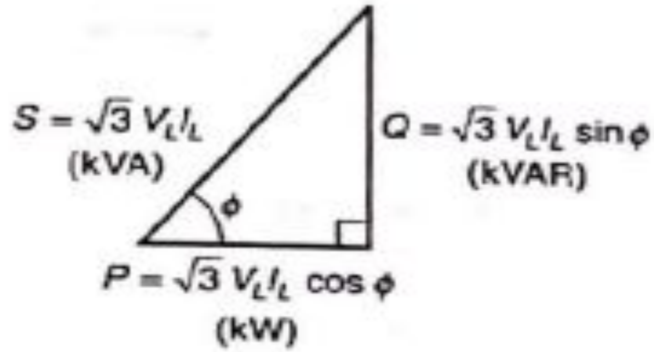


Fig. 3.12 Power triangle

3.5 Comparison between Star Connection and Delta Connection

Star connection	Delta connection
$V_L = \sqrt{3}\,V_{ph}$	$V_L = V_{ph}$
$I_L = I_{ph}$	$I_L = \sqrt{3}\,I_{ph}$
Line voltage leads the respective phase voltage by 30°.	Line current lags behind the respective phase current by 30°
Power in star connection is one-third of power in delta connection.	Power in delta connection is three times of power in star connection
Three-phase, three-wire and three-phase, four-wire systems are possible.	Only three phase-three wire system is possible
Phasor sum of all the phase currents is zero	Phasor sum of all the phase voltages is zero

Steps to be followed while solving the problem on three-phase circuits:

(i) Assume supply voltage as line voltage.

(ii) Indentify the type of load, i.e., whether star or delta connected and determine the phase voltage.

(iii) Determine the phase current as, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}}$.

(iv) Determine the line current depending on whether the load is star or delta connected.

(v) The phase angle, ϕ is the angle between V_{ph} and I_{ph}. Calculate its value from Z_{ph}.

Example 3.1 Three similar coils, each of resistance 20 Ω and inductance 0.5 H, are connected in (a) the star, (b) the delta to a three-phase 50 Hz, 400 V supply. Calculate the line current and the total power absorbed.

Solution

(a) *Star connection:*

When three similar coils are connected in star, we get a three-phase, *Y*-connected balanced load as shown in Fig. 3.13.

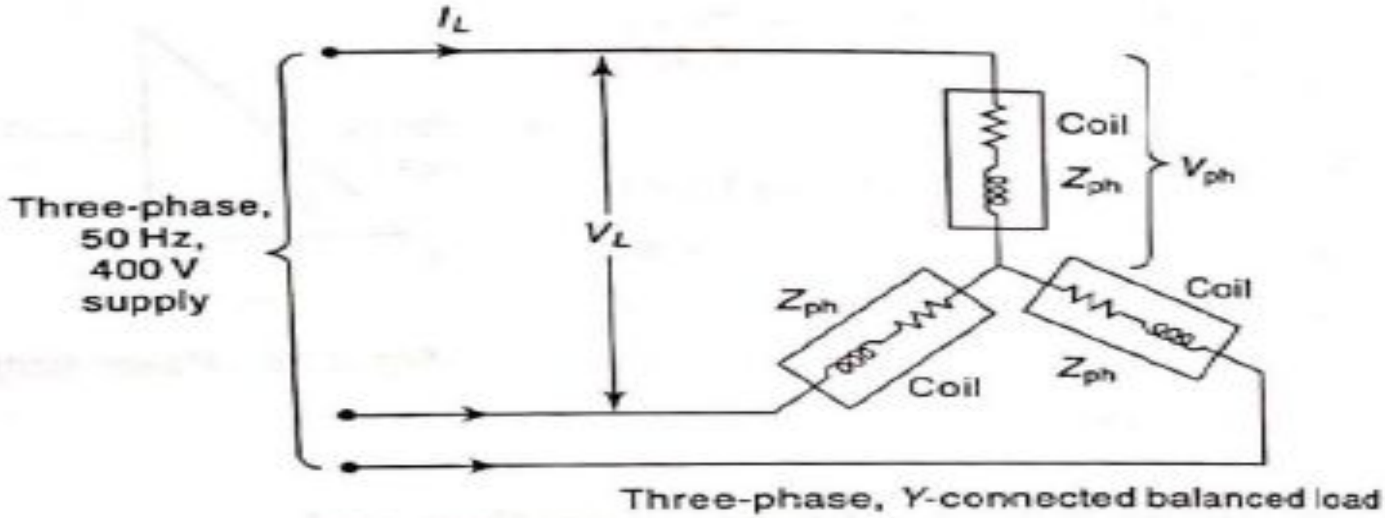


Fig. 3.13

Let the impedance of each coil is Z_{ph}.

Z_{ph} can be calculated as follows:

Resistance of each coil, $R_{ph} = 20\ \Omega$

Inductance of each coil, $L = 0.5$ H

Reactance of each coil, $X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157\ \Omega$

So, Rectangular form of Z_{ph}, $\overline{Z}_{ph} = (20 + j157)\ \Omega$

and Polar form of Z_{ph}, $\overline{Z}_{ph} = (158.27\angle 82.74)\ \Omega$

From polar form, $Z_{ph} = 158.27\ \Omega$ and $\phi = 82.74°$

$$\text{Phase voltage,} \quad V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$\text{Phase current,} \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{158.27} = 1.46 \text{ A}$$

In Y-connection, $I_L = I_{ph}$
So, $I_L = 1.46$ A

Total power absorbed, $P = \sqrt{3} V_L I_L \cos \phi$

$$= \sqrt{3} \times 400 \times 1.46 \times \cos 82.74°$$
$$= 127.83 \text{ W}$$

(b) *Delta connection:*
When three similar coils are connected in delta, we get three phase, delta-connected balanced load as shown in Fig. 3.14.
As the same three coils are connected in delta, Z_{ph} is same.

So, $\bar{Z}_{ph} = (158.27\angle 82.74)\ \Omega$

In delta-connection, $V_L = V_{ph} = 400$ V

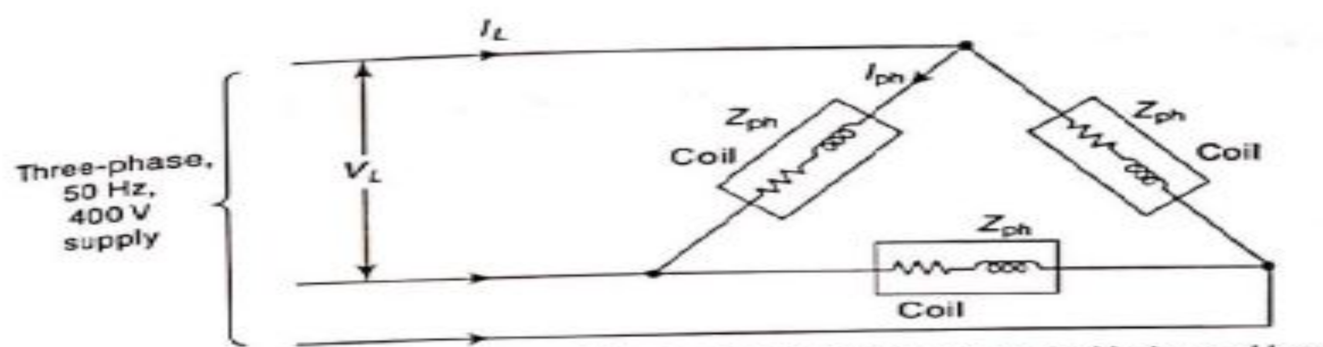


Fig. 3.14

Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{158.27} = 2.53$ A

Line current, $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 2.53 = 4.38$ A

Total power absorbed, $P = \sqrt{3} V_L I_L \cos \phi$

So, $P = \sqrt{3} \times 400 \times 4.38 \times \cos 82.74°$

or $P = 383.48$ W

Example 3.2 Three similar coils, each of resistance 8 Ω, and inductance 0.02 H, are connected in star across a three-phase 50 Hz, 230 V supply. Calculate the line current, total power absorbed, reactive volt amperes, and total volt amperes.

Solution

We have $R_{ph} = 8\ \Omega$

$L = 0.02$ H

So, $X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$

$V_L = 230$ V

$f = 50$ Hz

For star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79\ \text{V}$$

$$\bar{Z}_{ph} = (8 + j6.28)\ \Omega = (10.17\angle 38.13)\ \Omega$$

So, $Z_{ph} = 10.17\ \Omega$ and $\phi = 38.13$

Phase current, $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{10.17} = 13.06\ \text{A}$$

Line current, $I_L = I_{ph} = 13.06$ A

Total power absorbed, $P = \sqrt{3} V_L I_L \cos \phi$

$$= \sqrt{3} \times 230 \times 13.06 \times \cos 38.13°$$
$$= 4092 \text{ W}$$
$$= 4.092 \text{ kW}$$

Reactive volt ampere, $Q = \sqrt{3} V_L I_L \sin \phi$

$$= \sqrt{3} \times 230 \times 13.06 \times \sin 38.13°$$
$$= 3.21 \text{ kVAR}$$

Total volt ampere, $S = \sqrt{3} V_L I_L$

$$= \sqrt{3} \times 230 \times 13.06$$
$$= 5.2 \text{ kVA}$$

$= 5.2 \text{ kVA}$

Example 3.3 Three identical coils, each having resistance of 15 Ω and inductance of 0.03 H, are connected in delta across a three-phase 50 Hz, 230 V supply. Calculate the phase current, line current, and total power absorbed. Draw phasor diagram.

Solution

We have $R_{ph} = 15\ \Omega$

$L = 0.03$ H, $X_L = 2\pi fL = 2\pi \times 50 \times 0.03 = 9.42\ \Omega$

So, impedance of each coil, $\bar{Z}_{ph} = (15 + j\,9.42)\ \Omega = (17.71\angle 32.13)\ \Omega$

From polar form, $Z_{ph} = 17.71\ \Omega$ and $\phi = 32.13$

For delta-connected load, $V_L = V_{ph} = 230$ V

Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230}{17.71} = 12.99$ A

Line current, $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 12.99 = 22.5$ A

Total power absorbed,

$$P = \sqrt{3}\, V_L I_L \cos\phi$$
$$= \sqrt{3} \times 230 \times 22.5 \times \cos 32.13^\circ$$
$$= 7.59 \text{ kW}$$

Phasor diagram

$V_{ph} = 230$ V

$I_{ph} = 12.99$ A

$\phi = 32.13^\circ$

$I_L = 22.5$ A

Scale:

For voltage, 100 V = 1 cm

For current, 5 A = 1 cm

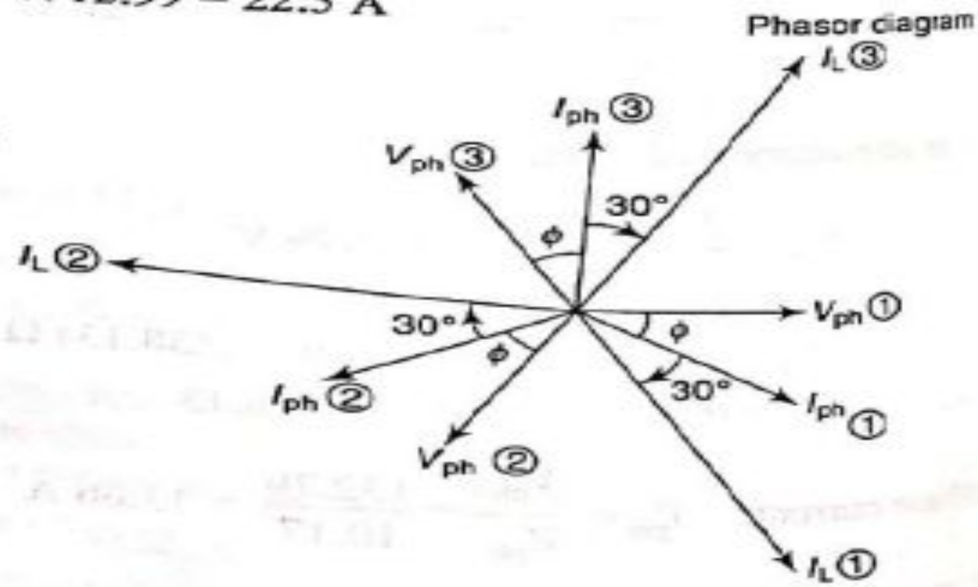


Fig. 3.15

Steps for drawing phasor diagram:

1. First draw the three phase voltages. Take the first phase voltage, i.e., $V_{ph}(1)$ as reference. All the three phase voltages are equal in magnitude and 120° apart.
2. As the load is inductive, the phase current lags behind the respective phase voltage by $\phi°$. All the phase currents are equal in magnitude. Thus, draw the three phase currents.
3. Line voltages are same as phase voltages for delta connection. Hence, separate line voltages need not be shown.
4. In delta connections, the line current lags behind the respective phase current by 30° [for example, $I_L(1)$ lags behind $I_{ph}(1)$ by 30° and so on). Thus, draw all the three line currents. All the line currents are equal in magnitude.

draw all the three line currents. All the line currents are equal in magnitude.

Example 3.4 A balanced delta-connected load has impedance of $(14.151 - j200)\ \Omega$, in each branch. Determine branch current, line current, total power taken if balanced 3-ϕ, 400 V, 50 Hz supply is used. How much power is consumed in each branch of delta? Find also reactive element in suitable unit.

Solution

For delta connection, $V_{ph} = V_L = 400$ V

The impedance in each branch (phase), $\overline{Z}_{ph} = (14.151 - j200)\ \Omega$

So, $\overline{Z}_{ph} = (200.5\angle{-85.95})\ \Omega$.

From polar form, $Z_{ph} = 200.5\ \Omega$ and $\phi = -85.95$

Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{200.5} = 1.995$ A

Line current, $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 1.995 = 3.455$ A

Total power, $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 3.455 \times \cos(-85.95) = 169$ W

Power in each branch (phase) of delta connection $= V_{ph} I_{ph} \cos\phi$

$$= 400 \times 1.995 \times \cos(-85.95)$$
$$= 56.36 \text{ W}$$

Rectangular form of Z_{ph} indicates that it is capacitive impedance. (imaginary part of $\overline{Z}_{ph}$ is negative).

Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

or

$$200 = \frac{1}{2\pi \times 50 \times C}$$

So,

$$C = 15.91 \times 10^{-6} \text{ F}$$
$$= 15.91\ \mu\text{F}$$

3.6 Three-Phase Power Measurement

A wattmeter is an instrument used for the measurement of power. The power consumed in a dc circuit is the product of the voltage and the current, and for an ac circuit, it is the product of the rms voltage, the rms current, and the power

factor of the circuit. Therefore, an ac circuit requires the use of wattmeter to measure the power.

The wattmeter has two coils, the current coil and the potential coil or the pressure coil. The current coil has two terminals marked as M and L. M stands for mains and L is for load. The potential or pressure coil has two terminals marked as C and V. C is the common terminal and V indicates the voltage rating.

Figure 3.24 shows the common method of connecting the wattmeter to measure a power in a single-phase circuit. Current coil senses the current and always connected like an ammeter in series with the load to carry the load current. Similar to ammeter, the resistance of the current coil is as small as possible and hence its cross-sectional area is large and it has less number of turns. Care is required to be taken while connecting the current coil, i.e., terminal M should be connected to mains (i.e., towards source) and terminal L to load. Pressure coil senses the voltage and always to be connected like a voltmeter so that potential across it is equal to the voltage across the load. Similar to voltmeter, the resistance of the pressure coil is very large.

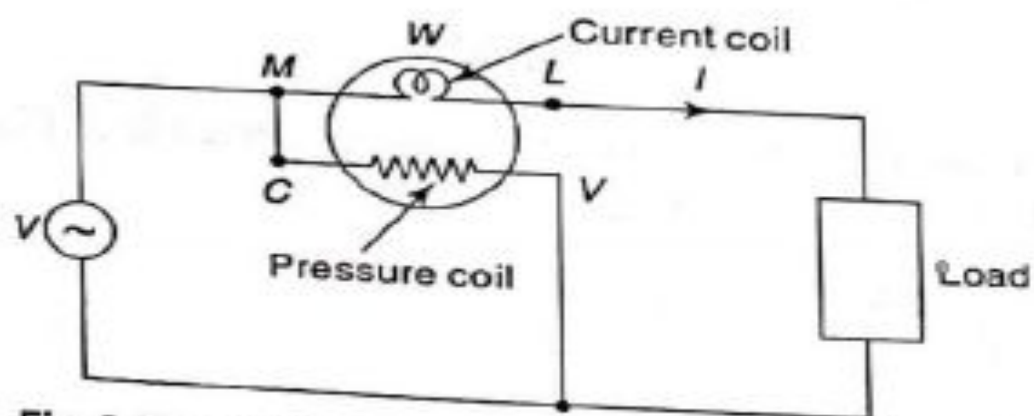


Fig. 3.24 Power measurment in a single-phase circuit

The wattmeter

Fig. 3.24 Power measurment in a single-phase circuit

The wattmeter reading depends on its connection. It is important to note that wattmeter senses the angle between the current phasor which is sensed by its current coil and the voltage phasor which is sensed by its pressure coil. In general, if I_C is the current through the current coil and V_p is the voltage across the pressure coil then wattmeter reading is given by

$$W = V_P \times I_C \times \cos (V_P \wedge I_C) \text{ W} \tag{3.11}$$

There are three methods to measure the power in a three-phase circuit:

1. One-wattmeter method
2. Two-wattmeter method
3. Three-wattmeter method

...attmeter method

3.6.1 One-Wattmeter Method

One-wattmeter method is used for balanced load (Y-connected or Δ-connected) only. When load is balanced, the total power can be calculated as

$$P = 3V_{ph}I_{ph} \cos \phi \tag{3.12}$$

where ϕ is the angle between the phase voltage and the corresponding phase current.

In a three-phase star-connected balanced load, power can be measured by connecting the current coil of the wattmeter in any one phase and its pressure coil between the same phase and the neutral as shown in Fig. 3.25.

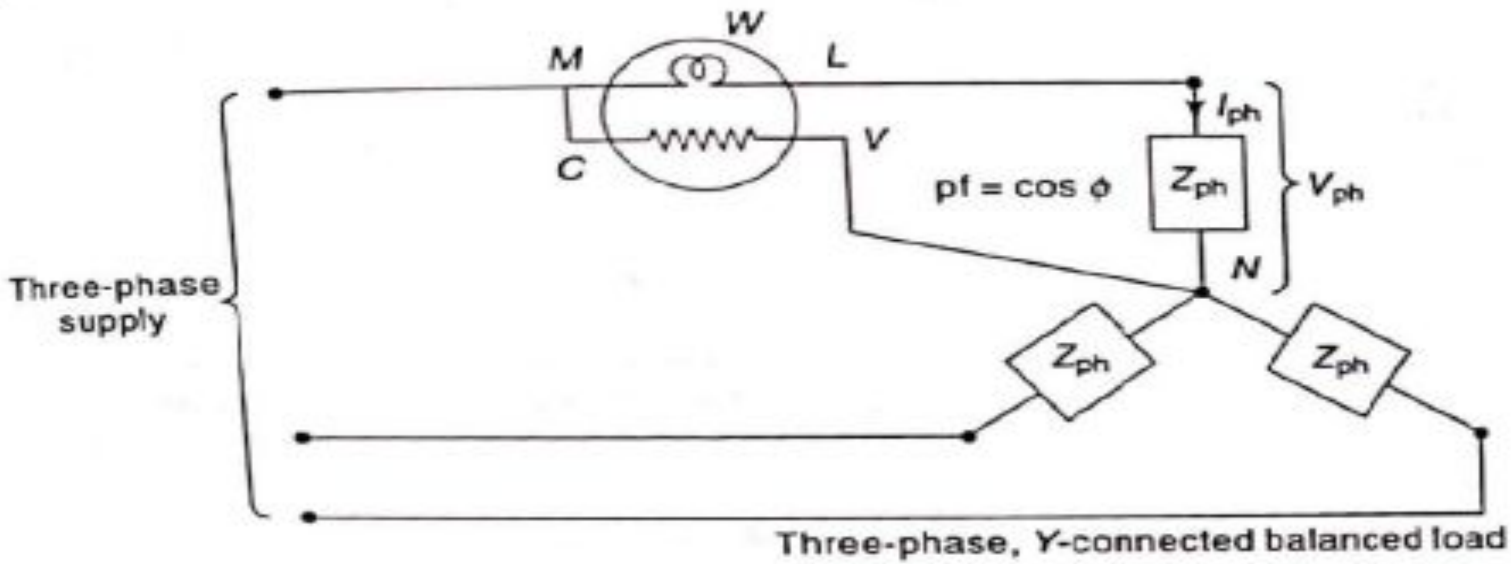


Fig. 3.25 One-wattmeter method

From the circuit diagram, the wattmeter reading is given as

$$W = V_{ph} I_{ph} \cos \phi$$

From Eq. (3.12), the total power is given as

$$P = 3 \times V_{ph} I_{ph} \cos \phi$$

or $$P = 3 \times W$$

Thus, the reading shown by the wattmeter is the active power per phase. The total active power of the star-connected balanced load is equal to three times the wattmeter reading.

...meter reading.

3.6.2 Two-Wattmeter Method

Two-wattmeter method is quite convenient for measuring power in the star or delta-connected balanced or unbalanced load of a three-phase system. In this method, the current coils of the two wattmeters are connected in any two lines while voltage coil of each wattmeter is connected to the third line. For example, if the current coils are inserted in the lines R and B then the pressure coils are connected between R-Y for one wattmeter and B-Y for other wattmeter, as shown in Fig. 3.26.

The connections are same for the star- or delta-connected load. It can be prove that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power consumed by the three-phase load.

Proof

Consider a three-phase star-connected balanced load as shown in Fig. 3.26. Load may be assumed to be inductive, i.e., power factor of the load is $\cos \phi$ lagging.

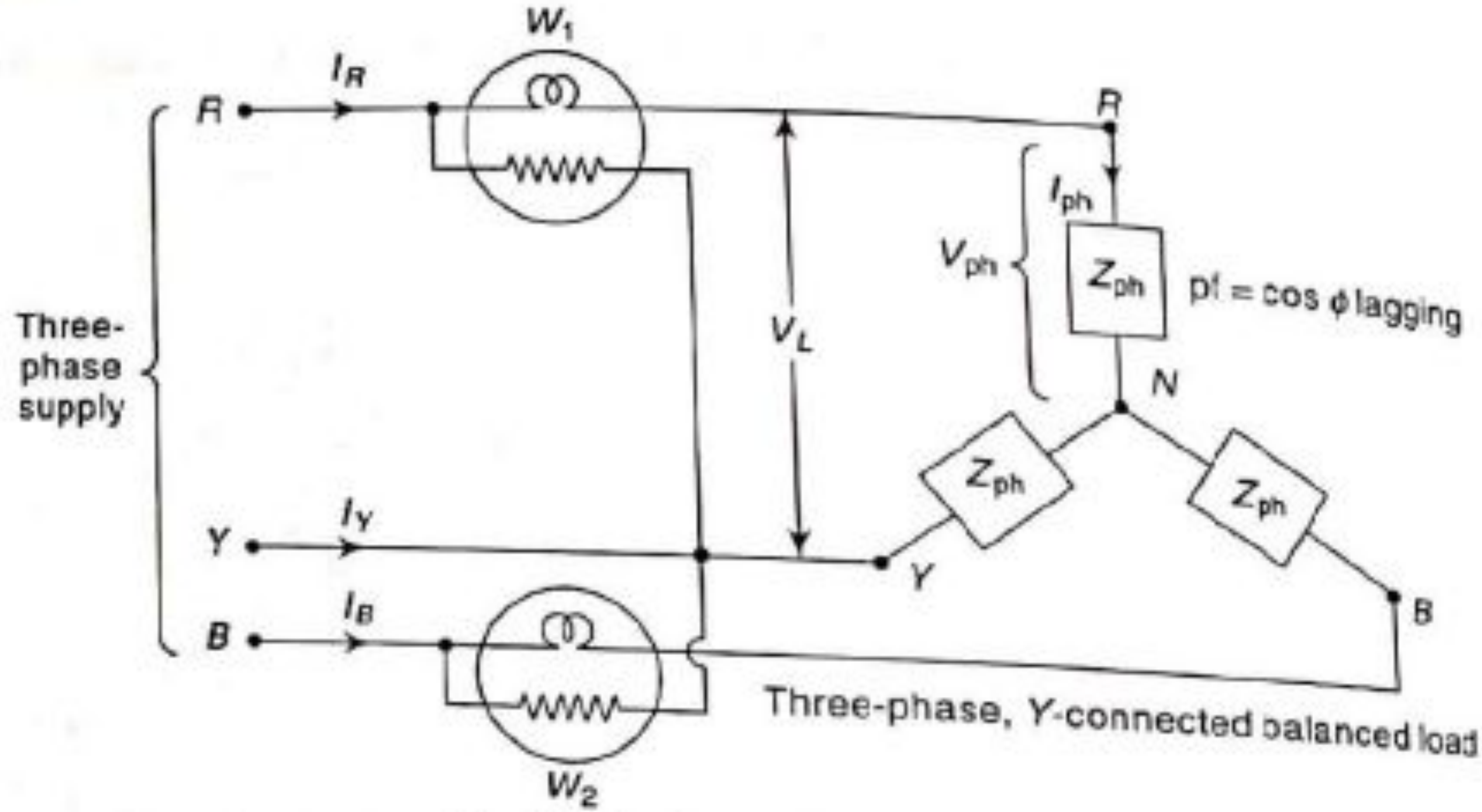


Fig. 3.26 Two-wattmeter method

As load is balanced, all the three phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and 120° apart. By phase sequence, V_{YN} will be 120° behind V_{RN} and V_{BN} will be 240° behind that of V_{RN}. The magnitude of each phase voltage is denoted by V_{ph}.

So, $\quad V_{ph} = V_{RN} = V_{YN} = V_{BN}$

The three-phase currents are I_R, I_Y and I_B,

$\therefore \quad I_{ph} = I_R = I_Y = I_B$

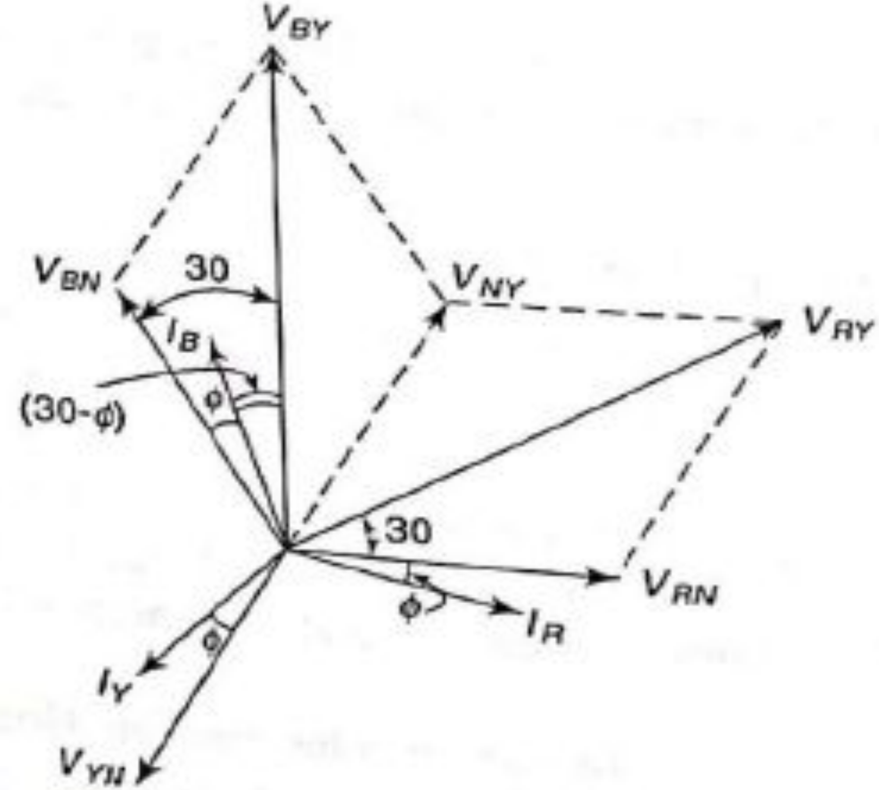


Fig. 3.27 Phasor diagram

According to the connection shown in Fig. 3.27, wattmeter reading W_1 is given by

$$W_1 = V_{RY} I_R \cos \text{(angle between } \bar{V}_{RY} \text{ and } \bar{I}_R)$$

or
$$W_1 = V_{RY} I_R \cos (\bar{V}_{RY} \wedge \bar{I}_R)$$

From the circuit diagram shown in Fig. 3.27,
voltage across the voltage coil of wattmeter W_1, $\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY}$.
From the phasor diagram, it is clear that angle between V_{RY} and I_R is $(30 + \phi)$.
So, wattmeter reading, $W_1 = V_{RY} I_R \cos(30 + \phi)$
As $V_{RY} = V_L$ and $I_R = I_L$,
wattmeter reading,

$$W_1 = V_L I_L \cos(30 + \phi) \tag{3.13}$$

According to the connection shown in Fig. 3.27, wattmeter reading W_2 is given by

$$W_2 = V_{BY} I_B \cos(\bar{V}_{BY} \wedge \bar{I}_B)$$

Again, from the circuit diagram shown in Fig. 3.27,
voltage across the voltage coil of wattmeter W_2, $\bar{V}_{BY} = \bar{V}_{BN} + \bar{V}_{NY}$
From the phasor diagram, it is clear that angle between V_{BY} and I_B is $(30 - \phi)$.
So, wattmeter reading, $W_2 = V_{BY} I_B \cos(30 - \phi)$
As $V_{BY} = V_L$ and $I_B = I_L$,
wattmeter reading,

$$W_2 = V_L I_L \cos(30 - \phi) \tag{3.14}$$

By adding the Eqs. (3.13) and (3.14), we get

$$\begin{aligned} W_1 + W_2 &= V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi) \\ &= V_L I_L [\cos(30 + \phi) + \cos(30 - \phi)] \\ &= V_L I_L [2 \cos 30 \cos \phi] \\ &= \sqrt{3}\, V_L I_L \cos \phi \end{aligned} \tag{3.15}$$

$$= \sqrt{3}\, V_L I_L \cos\phi$$

RHS of Eq. (3.15) indicates the active power of the three-phase circuit. Thus, it is proved that when two wattmeters are connected as shown in Fig. 3.27, the algebraic sum of the two wattmeter readings gives the total power consumed by the three-phase load.

Subtracting Eq. (3.13) from (3.14), we get

$$\begin{aligned} W_2 - W_1 &= V_L I_L \cos(30-\phi) - V_L I_L \cos(30+\phi) \\ &= V_L I_L [\cos(30-\phi) - \cos(30+\phi)] \\ &= V_L I_L [2 \sin 30 \sin\phi] \\ &= V_L I_L \sin\phi \end{aligned}$$

Multiplying both sides by $\sqrt{3}$, we get

$$\sqrt{3}(W_2 - W_1) = \sqrt{3}\, V_L I_L \sin\phi \qquad (3.16)$$

RHS of Eq. (3.16) indicates the total volt-ampere reactive power for a three-phase circuit.

Thus, the reactive power, $Q = \sqrt{3}(W_2 - W_1)$

Now, dividing Eq. (3.16) by Eq. (3.15), we get

$$\frac{\sqrt{3}(W_2 - W_1)}{(W_1 + W_2)} = \frac{\sqrt{3}\, V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi} = \tan\phi$$

or $$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

Now, $$\text{pf} = \cos\phi = \cos\left\{\tan^{-1}\left(\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right)\right\} \qquad (3.17)$$

Thus, various advantages of the two-wattmeter method are as follows:

1. The method is applicable for balanced as well as unbalanced loads.
2. From wattmeter readings, the reactive power can be calculated.
3. If the load is balanced, not only the power but also the power factor can be determined.
4. Neutral point for star connected load is not necessary to connect the wattmeters.

3.6.3 Effect of Load Power Factor on Wattmeter Readings

We have seen that for lagging load (balanced), power factor of $\cos\phi$, the two wattmeter readings are:

$$W_1 = V_L I_L \cos(30 + \phi)$$
$$W_2 = V_L I_L \cos(30 - \phi)$$

It is clear that readings of the two wattmeters depend upon the load power factor angle ϕ.

(i) When power factor is unity (i.e., $\phi = 0$):

$$W_1 = V_L I_L \cos(30 + 0) = V_L I_L \cos 30^\circ$$
$$W_2 = V_L I_L \cos(30 - 0) = V_L I_L \cos 30^\circ$$

Both the wattmeters indicate equal and positive readings.

(ii) When power factor is 0.5 (i.e., $\phi = 60$):

$$W_1 = V_L I_L \cos(30 + 60) = 0$$
$$W_2 = V_L I_L \cos(30 - 60) = V_L I_L \cos 30^\circ$$

Total power is measured by W_2 alone.

(iii) When power factor is less than 0.5 but greater than 0 (i.e., $90 > \phi > 60$):

W_1 = negative reading

W_2 = positive reading

(iv) When power factor is zero (i.e., $\phi = 90$):

Such a case will occur when the load consists of pure inductance or pure capacitance.

$$W_1 = V_L I_L \cos(30 + 90) = -V_L I_L \sin 30^\circ$$
$$W_2 = V_L I_L \cos(30 - 90) = V_L I_L \sin 30^\circ$$

Thus, the two wattmeters show equal and opposite readings.

So, $W_1 + W_2 = 0$

The above facts are summarized in the tabular form below:

ϕ	0°	60°	More than 60°	90°
$\cos\phi$	1	0.5	< 0.5	0
W_2	Positive	Positive	Positive	Positive
W_1	Positive	0	Negative	Negative
Conclusion	$W_1 = W_2$ Total power $= W_1 + W_2$	$W_1 = 0$ Total power $= W_2$	Total power $= W_1 + W_2$	$W_2 = -W_1$ Total power $= 0$

The following points may be noted carefully:

1. The wattmeter whose deflection is proportional to $(30 - \phi)$ (i.e., W_2 in this case) is always positive and is the higher reading wattmeter.
2. The wattmeter whose deflection is proportional to $(30 + \phi)$ (i.e., W_1 in this case) is the lower reading wattmeter.

In discussing two-wattmeter method for measuring power in a three-phase balanced load, we have considered lagging power factor load, i.e., power factor angle ϕ is positive. For leading power factor, angle ϕ becomes negative. Therefore, by putting the value of load pf angle as $-\phi$ in the readings of the two wattmeters, we have

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

Note that effect of leading power factor is that the readings of the two wattmeters are interchanged. Now, wattmeter W_1 has become the higher reading wattmeter.

We know that for lagging pf load, from wattmeter readings, the phase angle ϕ can be calculated as

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\} \qquad \text{for lagging pf}$$

... wattmeter, the for-

$$(W_1 + W_2)$$

As for leading pf, wattmeter W_1 becomes the higher reading wattmeter, the formula will modify as

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right\} \qquad \text{for leading pf}$$

Thus, for calculation of phase angle from wattmeter readings, the following general formula (for both lagging and leading pf) can be used:

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

We have already discussed that higher-reading wattmeter always read positive. The lower-reading wattmeter may read positive or negative depending upon the load power factor.

For example, in the two-wattmeter method if the two wattmeter readings are 12.5 and –4.8 kW, then

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,[(12.5)-(-4.8)]}{(-4.8)+(12.5)}\right\} = \tan^{-1}3.89 = 75.6$$

So, Power factor, $\cos\phi = \cos 75.6° = 0.2487$

Note that we can find only the magnitude of power factor with the help of the above formula. It does not tell us about the nature of the load, i.e., whether the pf is lagging or leading. In many cases, this can not be judged from the nature of the load. Also note that a wattmeter can not give negative reading. Negative reading is obtained after reversing the connections of either potential or current coil.

Example 3.14 Three identical

potential or current coil.

Example 3.14 Three identical coils, each having a resistance of 10 Ω and an inductive reactance of 10 Ω are connected in (a) star, (b) delta connection, across 400 V, three-phase supply. Find in each case the line current and the readings on each of the two wattmeters connected to measure the power.

Solution

(a) Star connection:

A 400 V, three-phase voltage is applied to three star-connected identical coils, as shown in Fig. 3.28.

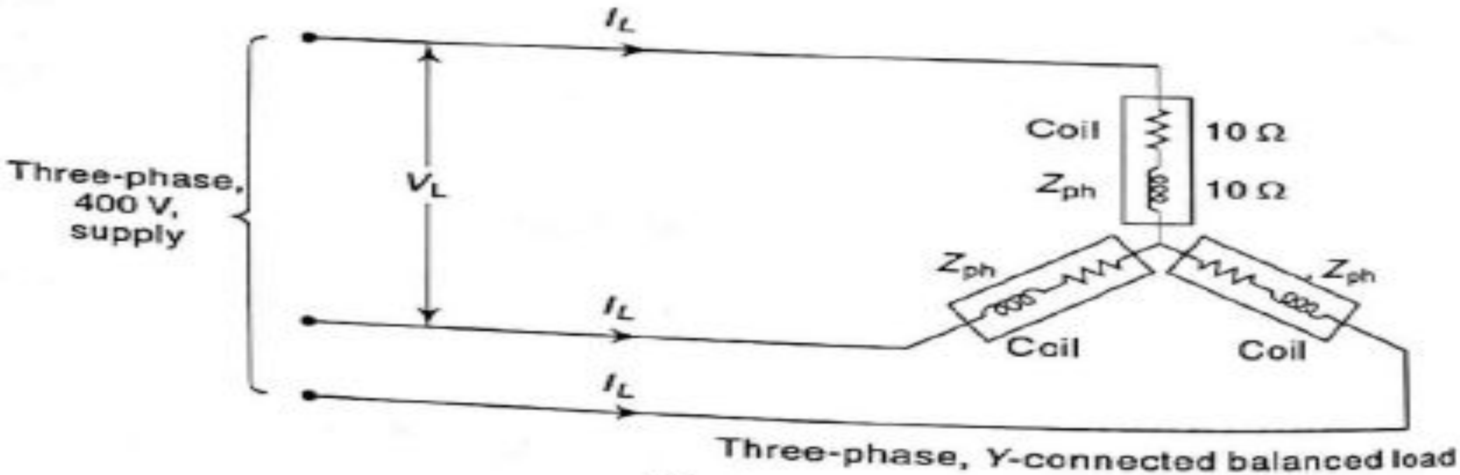


Fig. 3.28

Given:

$$V_L = 400 \text{ V}, R = 10\ \Omega, X_L = 10\ \Omega$$

Impedance of coil, i.e., phase impedance is

$$\bar{Z}_{ph} = (10 + j10)\ \Omega = (14.14\angle 45)\ \Omega$$

So, $\quad Z_{ph} = 14.14\ \Omega \quad$ and $\quad \phi = 45°$

For star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230.94}{14.14} = 16.33$ A

Line current, $I_L = I_{ph} = 16.33$ A

Total power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$$= \sqrt{3} \times 400 \times 16.33 \times \cos 45$$

$$= 8000 \text{ W}$$

Let W_1 and W_2 be the wattmeter readings, W_2 being the higher-reading wattmeter. In the two-wattmeter method, the algebraic sum of the two wattmeter readings gives the total power.

So, $\quad W_1 + W_2 = 8000 \qquad$ (i)

Also

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

So, $$\tan 45° = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

or $$1 = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{8000}\right\}$$

or $\quad W_2 - W_1 = 4619 \qquad$ (ii)

Solving Eqs (i) and (ii), we get

$$W_2 = 6309.5 \text{ W}, \quad W_1 = 1690.5 \text{ W}$$

$$W_2 = 6309.5 \text{ W}, \quad W_1 = 1690.5 \text{ W}$$

(b) Delta connection:

The same three coils are connected in delta across the same supply. Thus,

$$V_L = 400 \text{ V}, \quad Z_{ph} = 14.14\ \Omega, \quad \text{and} \quad \phi = 45$$

For delta-connected load,

$$V_{ph} = V_L = 400 \text{ V}$$

$$\text{Phase current, } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{14.14} = 28.29 \text{ A}$$

$$\text{Line current, } I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 28.28 = 49 \text{ A}$$

$$\text{Total power, } P = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 49 \times \cos 45^\circ$$

$$= 24004 \text{ W}$$

Let W_1 and W_2 be the wattmeter readings, W_2 being the higher-reading wattmeter. In the two-wattmeter method, the algebraic sum of the two wattmeter readings gives the total power.

So, $\quad W_1 + W_2 = 24004$ (iii)

Also

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

or $$\tan 45° = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

or $$1 = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{24004}\right\}$$

or $\quad W_2 - W_1 = 13858.7$ (iv)

Solving Eqs (iii) and (iv), we get

$$W_2 = 18931.35\ \text{W},\ W_1 = 5072.65\ \text{W}$$

Example 3.15 Each phase ...

$W_2 = 18931.35$ W, $W_1 = 5072.65$ W

Example 3.15 Each phase of a three-phase delta-connected load has an impedance of $\bar{Z}_{ph} = (50\angle 60)\ \Omega$. The line voltage is 400 V. Calculate the total power. What will be the readings of the two wattmeters connected to measure the power?

Solution

$$V_L = 400 \text{ V}, \quad Z_{ph} = 50\ \Omega, \quad \text{and} \quad \phi = 60^\circ$$

For delta-connected load,

$$V_{ph} = V_L = 400 \text{ V}$$

Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{50} = 8$ A

Line current, $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 8 = 13.86$ A

Total power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$$= \sqrt{3} \times 400 \times 13.86 \times \cos 60^\circ$$

$$= 4801.2 \text{ W}$$

Let W_1 and W_2 be the wattmeter readings, W_2 being the higher-reading wattmeter. In the two-wattmeter method, the algebraic sum of the two wattmeter readings gives the total power.

So, $\quad W_1 + W_2 = 4801.2$ W (i)

Also

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

or

$$\tan 60^\circ = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}\right\}$$

or

$$\tan 60^\circ = \left\{\frac{\sqrt{3}\,(W_2 - W_1)}{4801.2}\right\}$$

So, $W_2 - W_1 = 4801.2$ (ii)

Solving Eqs (i) and (ii), we get

$$W_2 = 4801.2\ \text{W},\ W_1 = 0\ \text{W}$$

Since the pf is 0.5, the reading of W_1 (i.e., lower-reading wattmeter) must be zero.

Example 3.16 Two wattmeters connected to measure the input to balanced three-phase circuit indicates 2500 and 500 W, respectively. Find the total power supplied, and the power factor of the circuit: (i) when both the readings are positive, and (ii) when the latter reading is obtained after reversing the connections to the current coil.

Solution

(i) When both the readings are positive:

$$W_2 = 2500 \text{ W and } W_1 = 500 \text{ W}$$

$$P = W_1 + W_2 = 500 + 2500 = 3000 \text{ W}$$

$$\text{Phase angle, } \phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{3}\,(2500 - 500)}{500 + 2500}\right\} = \tan^{-1}(1.1547) = 49.11^\circ$$

So, power factor $= \cos\phi = \cos 49.11^\circ = 0.6546$

(ii) When W_1 is negative:

$$W_2 = 2500 \text{ W and } W_1 = -500 \text{ W}$$

$$P = W_1 + W_2 = -500 + 2500 = 2000 \text{ W}$$

$$\text{Phase angle, } \phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{3}\,[2500 - (-500)]}{-500 + 2500}\right\} = \tan^{-1}(2.598) = 68.95^\circ$$

So, power factor $= \cos\phi = \cos 68.95^\circ = 0.3592$

Example 3.17 The readings of two wattmeters connected to measure the total power in a three-phase star-connected 400 V system are 3000 W and 5000 W. Find the power factor, the total power, and the line current.

Solution

We have $V_L = 400$ V, $W_1 = 3000$ W, $W_2 = 5000$ W

$$\text{Phase angle, } \phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{3}\,(5000 - 3000)}{3000 + 5000}\right\} = \tan^{-1}(0.4330) = 23.41°$$

So, power factor $= \cos\phi = \cos 23.41° = 0.9177$

Total power, $P = W_1 + W_2 = 3000 + 5000 = 8000$ W

As total power, $P = \sqrt{3}\, V_L I_L \cos\phi$,

$$8000 = \sqrt{3} \times 400 \times I_L \times 0.9177$$

Hence, $I_L = 12.58$ A

Example 3.18 The power input to a three-phase induction motor running on a 400 V, 50 Hz supply was measured by two-wattmeter method, and the readings were 3000 W and –1000 W. Calculate (i) total input power, (ii) power factor, and (iii) line current.

Solution

A three-phase induction motor can be treated as a three-phase star or delta-connected balanced load with lagging power factor. It receives the electrical power from the three-phase supply and converts it into rotational (mechanical) power.

We have $V_L = 400$ V, $W_1 = -1000$ W, $W_2 = 3000$ W

$$\text{Phase angle, } \phi = \tan^{-1}\left\{\frac{\sqrt{3}\,(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{3}\,[3000-(-1000)]}{-1000+3000}\right\} = \tan^{-1}(3.464) = 73.90^\circ$$

So, power factor = $\cos\phi = \cos 73.90^\circ = 0.2773$ lagging

Total power, $P = W_1 + W_2 = -1000 + 3000 = 2000$ W

As total power, $P = \sqrt{3}\,V_L I_L \cos\phi$,

$$2000 = \sqrt{3} \times 400 \times I_L \times 0.2773$$

Hence, $I_L = 10.41$ A

Example 3.21 In a balanced three-phase circuit, power is measured by two wattmeters, and the ratio of two wattmeter readings is 2:1. Determine the power factor of the system.

Solution

Let $\dfrac{W_2}{W_1} = \dfrac{2}{1}$ or $W_2 = 2W_1$

$$\text{Phase angle, } \phi = \tan^{-1}\left\{\frac{\sqrt{3}(\text{Higher reading} - \text{Lower reading})}{\text{Lower reading} + \text{Higher reading}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2}\right\} = \tan^{-1}\left\{\frac{\sqrt{3}(2W_1 - W_1)}{W_1 + 2W_1}\right\}$$

$$= \tan^{-1}\left\{\frac{1}{\sqrt{3}}\right\} = 30°$$

Hence, power factor $= \cos\phi = \cos 30° = 0.866$