47_ Yash Sarlung

Assignment 6 : Kinematics of Particles

1) The motion of a particle moving in straight line is given by a relation $s = t^3 - 3t^2 + 2t + 5$ where 's' is displacement in metres and 't' is time in seconds. Determine
i) velocity and acceleration at $t = 4sec$
ii) maximum or minimum velocity and corresponding displacement
iii) time at which velocity is zero.

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$s = t^3 - 3t^2 + 2t + 5$

$v = \frac{ds}{dt} = 3t^2 - 6t + 2$, $a = \frac{dv}{dt} = 6t - 6$

i) $\therefore$ at $t = 4s$, $v = 3(4^2) - 6(4) + 2$, $a = 6(4) - 6$

$\boxed{v_4 = 26\ m/s}$ $\boxed{a_4 = 18\ m/s^2}$

ii) for maximum or minimum velocity, acceleration should be zero.

$a = 6t - 6 = 0$ $\boxed{\therefore t = 1s}$

$\boxed{\therefore v_1 = -1\ m/s}$ is the minimum velocity

also displacement, $s = 1^3 - 3(1)^2 + 2(1) + 5$

$\boxed{s = 5m}$ at $t = 1s$.

iii) when $v = 0$, $3t^2 - 6t + 2 = 0$

$\therefore t = 1.577s$ or $t = 0.422s$.

$\therefore$ velocity is zero at $0.422s$ and $1.577s$.

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Q2) A particle is moving in x-y plane and its position is defined by $\bar{r} = \left(\frac{3}{2}t^2\right)\hat{i} + \left(\frac{2}{3}t^3\right)\hat{j}$. Find radius of curvature when $t = 2s$.

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$\therefore \bar{v} = \frac{d\bar{r}}{dt} = 3t\hat{i} + 2t^2\hat{j}$

$\bar{a} = \frac{d\bar{v}}{dt} = 3\hat{i} + 4t\hat{j}$.

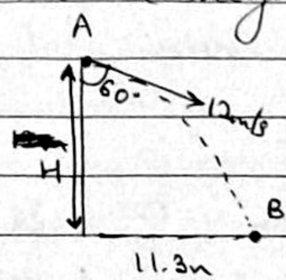
at $t = 2s$, $\bar{v} = 6\hat{i} + 8\hat{j}$, $\bar{a} = 3\hat{i} + 8\hat{j}$.

$v = |\bar{v}| = 10 m/s$

$\therefore$ Radius of curvature $\rho = \left|\frac{v^3}{a_x v_y - a_y v_x}\right| = \left|\frac{1000}{3\times8 - 8\times6}\right|$

$\rho = \left|\frac{\cancel{1000}\ 125}{-\cancel{8}\times3}\right| = 41.667 m$

Q3) A ball thrown with a speed of 12m/s at an angle of 60° with building strikes the ground 11.3m horizontally from the foot of the building as shown. Determine the height of building.



Projectile motion (A-B)

Horizontal motion

$u_x = 12 \sin 60° = 10.39 m/s$

$s_x = 11.3 m$ $\therefore s_x = u_x t$ $\therefore t = \frac{11.3}{10.39} s = 1.087 s$.

Vertical motion. $u_y = -12\cos 60° = -6 m/s$, $v_y = 0$, $a = -g$, $s = H$.

$s = u_y t + \frac{1}{2} a_y t^2$, $\therefore H = -6\times1.087 - \frac{1}{2} g \times (1.087)^2$

$\boxed{\therefore H = 12.32m}$

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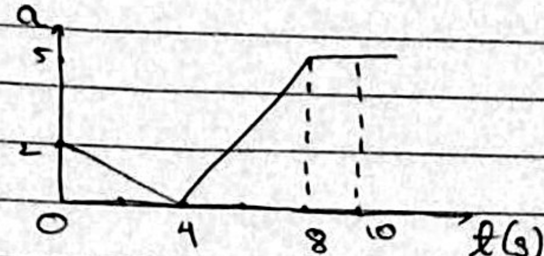
Q4) A particle is projected with an initial velocity of 2m/s along a straight line. The relation between acceleration and time is given.

→ A-T graph :



Area under a-t curve = change in velocity

for $0 \le t \le 4$, $\frac{1}{\cancel{2}} \times 4 \times \cancel{2} = V_4 - V_0$ $\quad V_0 = 2m/s$

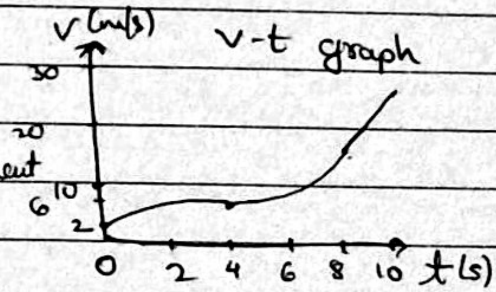
$\therefore V_4 = 6m/s.$

for $4 \le t \le 8$, $\frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 5 = V_8 - V_4$

$V_8 = 16m/s$

for $8 \le t \le 10$, $2 \times 5 = V_{10} - V_8$

$\therefore V_{10} = 26m/s.$



v-t graph :

Area under v-t graph = change in displacement

for $0 \le t \le 4$,

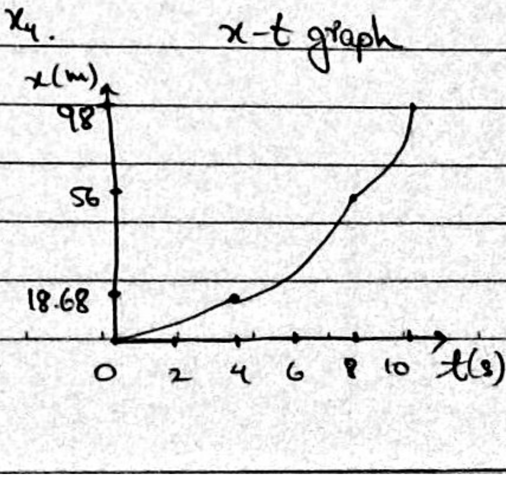
$(2 \times 4) + \frac{2 \times 4 \times 4}{3} = x_4 - x_0.$

$x_0 = 0.$

$\therefore x_4 = 18.68m.$

for $4 \le t \le 8$, $(6 \times 4) + \frac{2}{3} \times 4 \times 10 = x_8 - x_4.$

x-t graph



$\therefore x_8 = 56m$

for $8 \le t \le 10$, $\frac{1}{2} \times 10 \times 2 + 16 \times 2 = x_{10} - x_8$

$\boxed{\therefore x_{10} = 98m}$