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Q2. A) $\cos x \frac{dy}{dx} + y \sin x = \sqrt{y} \sec x$

$\frac{1}{\sqrt{y}} \cos x \frac{dy}{dx} + \sqrt{y} \sin x = \sqrt{\sec x}$

$\frac{1}{\sqrt{y}} \frac{dy}{dx} + \sqrt{y} \tan x = (\sec x)^{3/2}$

let $\sqrt{y} = u$, $\frac{1}{2\sqrt{y}} \frac{dy}{dx} = \frac{dy}{dx}$

$\therefore 2\frac{du}{dx} + u \tan x = (\sec x)^{3/2}$

This is a linear equation of the form $\frac{dy}{dx} + Pu = Q$

$I.F = e^{\int \frac{\tan x}{2} . dx} = e^{\frac{1}{2}\log \sec x} = (\sec x)^{1/2}$

$\therefore$ Solution of differential equation.

$y\sqrt{\sec x} = \int \frac{1}{2} \sec^2 x . dx.$

$y\sqrt{\sec x} = \frac{1}{2} \tan x + c.$

$\boxed{\therefore y = \frac{1}{2} \tan x \sqrt{\cos x} + c\sqrt{\cos x}}$ is the solution.

Q2. F) $\rightarrow$ $\int_0^{2\pi} \sin^2\theta (1+\cos\theta)^4 . d\theta = 21\pi/8$ (To prove)

Using $\int_0^{2a} F(x).dx = 2\int_0^a F(2a-x).dx$

$\therefore \int_0^{2\pi} \sin^2\theta (1+\cos\theta)^4 . d\theta = 2\int_0^{\pi} \sin^2(2\pi-\theta) [1+\cos(2\pi-\theta)]^4 . d\theta.$

$$= 2\int_0^{\pi} \sin^2\theta\,(1+\cos\theta)^4\,d\theta \qquad \left[\because \begin{array}{l}\cos(2\pi-\theta) = \cos\theta \\ \sin(2\pi-\theta) = -\sin\theta\end{array}\right]$$

$$= 2\int_0^{\pi}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2\left(2\cos^2\frac{\theta}{2}\right)^4 . d\theta$$

$$= 2\int_0^{\pi} 4\sin^2\frac{\theta}{2}\cos^{10}\frac{\theta}{2}\times 2^4 . d\theta$$

$$= 2^7\int_0^{\pi}\sin^2\frac{\theta}{2}\cos^{10}\frac{\theta}{2} . d\theta \qquad \text{let } \frac{\theta}{2} = u \quad d\theta = 2du.$$

as $\theta \to 0$, $u \to 0$ and $\theta \to \pi$, $u \to \pi/2$

$$= 2^7\int_0^{\pi/2}\sin^2 u . \cos^{10} u . 2du.$$

$$= 2^7\left[2\int_0^{\pi/2}\sin^2 u . \cos^{10} u . du\right] = 2^7\,\beta\left(\frac{3}{2},\frac{11}{2}\right)$$

$$= 2^7\,\frac{\Gamma\frac{3}{2}\,\Gamma\frac{11}{2}}{\Gamma\frac{14}{2}} = \cancel{2^7}\,\frac{1/\cancel{2}\,\Gamma\frac{1}{2}\times 9/\cancel{2}\times 7/\cancel{2}\times \cancel{5}/\cancel{2}\times \cancel{3}/\cancel{2}\times 1/\cancel{2}\times\Gamma\frac{1}{2}}{6\times\cancel{5}\times 4\times\cancel{3}\times\cancel{2}}$$

$$= \frac{\sqrt{\pi}\times \overset{3}{\cancel{9}}\times 7\times\sqrt{\pi}}{2\cancel{6}\times 4} = \frac{21\pi}{8}.$$

$$\therefore \int_0^{2\pi}\sin^2\theta\,(1+\cos\theta)^4\,d\theta = \frac{21\pi}{8}.$$

Q2. c) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

A.E is $(D-2)^2 = 0$ where $D = 2, 2$.

$C.F = (C_1 + C_2 x)\, e^{2x}$

$$P.I = \frac{1}{(D-2)^2}\left[8(e^{2x} + \sin 2x + x^2)\right]$$

$$= 8\left[\frac{e^{2x}}{(D-2)^2} + \frac{\sin 2x}{D^2-4D+4} + \frac{x^2}{(D-2)^2}\right]$$

$$= 8\left[\frac{xe^{2x}}{2D-4} + \frac{\sin 2x}{\cancel{-4}-4D\cancel{+4}} + \frac{x^2}{4[1-D/2]^2}\right]$$

$$= 8\left[\frac{x^2e^{2x}}{2} - \frac{1}{4D}\sin 2x + \frac{1}{4}x^2[1-D/2]^{-2}\right]$$

$$= 4\cancel{8}\left[\frac{x^2e^{2x}}{\cancel{2}} - \frac{1}{4}\left(\frac{-\cos 2x}{\cancel{2}}\right) + \frac{x^2}{2\cancel{4}}\left[1 - 2\left(\frac{-D}{2}\right) + \frac{(-\cancel{2})(-3)}{\cancel{2}}\left(\frac{D}{2}\right)^2 \ldots\right]\right]$$

$$= 4\left[x^2e^{2x} + \frac{\cos 2x}{4} + \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)\right]$$

Complete Solution = C.F + P.I

$$\boxed{\therefore \text{Complete Solution} = (C_1 + C_2 x)e^{2x} + 4x^2e^{2x} + \frac{\cos 2x}{4} + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}.}$$

Q2. B)
→ Using method of variation of parameters.

$(D^2+1)y = \sec x \tan x$

$D^2+1=0$ $\qquad$ $\sec x \tan x = 0$.

$R = \sec x \tan x$

$C.F = c_1 \cos x + c_2 \sin x$.

$y_1 = \cos x \qquad y_2 = -\sin x$

$y_1' = -\sin x \qquad y_2' = \cos x$.

$W = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ $\qquad \therefore W = 1$.

$u = -\int \frac{y_2 R}{W} \, dx = -\int \frac{\sin x}{\cos x} \times 1 \cdot \tan x \, dx$

$= -\int \tan^2 x \, dx = -\int (\sec^2 x - 1) \, dx$

$\boxed{u = -\tan x + x}$

$v = \int \frac{y_1 R}{W} \, dx = \int \frac{\cos x \cdot \sec x \tan x}{1} \cdot dx$

$v = -\log |\cos x|$

By variation, $P.I = uy_1 + vy_2 = \cos x(-\tan x + x) + (-\log(\cos x))\sin x$

$\therefore P.I = \cos x\,(x - \tan x) + \sin x\,[-\log(\cos x)]$

$\therefore$ Complete solution $= C.F + P.I$

$= c_1 \cos x + c_2 \sin x + \cos x\,(x - \tan x) + \sin x\,[-\log(\cos x)]$

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