4.1 Introduction

A **transformer** is a static device by means of which electric power from one circuit can be transferred to another circuit without change in frequency. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. Transformer works on the principle of mutual induction.

In electric systems, transformers are used near generating stations for stepping up the voltage. In a typical power system, voltage is generated at the level of 11 kV or 22 kV. It is stepped up to 220 kV or 400 kV by using step-up transformers. High voltage during transmission helps to increase efficiency of the system by reducing line losses[1]. At the end of transmission line, again a step-down transformer is used to reduce the voltage to 11 kV. During distribution of power, again transformers are used to change the voltage level to 400 V (line voltage) or 230 V (phase voltage). Thus, transformers are required at all the stages (see Fig. 4.1).

4.2 Working Principle

A transformer consists of two inductive coils or windings placed on a common core. Windings are electrically insulated from each other. The winding or coil in which electrical energy is fed is called **primary winding** and the other from which electrical energy is drawn out is called **secondary winding** [refer to Fig. 4.2(a)].

When an alternating voltage V_1 is applied to primary coil of a transformer, an alternating current I_1 flows through it producing alternating flux in the core. This flux links with the primary winding and according to Faraday's law of electromagnetic induction, an emf e_1 is induced in the primary coil, which is given by

$$e_1 = -N_1 \frac{d\Phi}{dt} \text{ V} \tag{4.1}$$

where N_1 is the number of turns in the primary coil. The induced emf in the primary coil is nearly equal and opposite to the applied voltage V_1.

[1] For transmission of same amount of power with an increase in voltage level, the current level decreases. Hence, power losses in transmission line (i.e., I^2R losses) decrease.

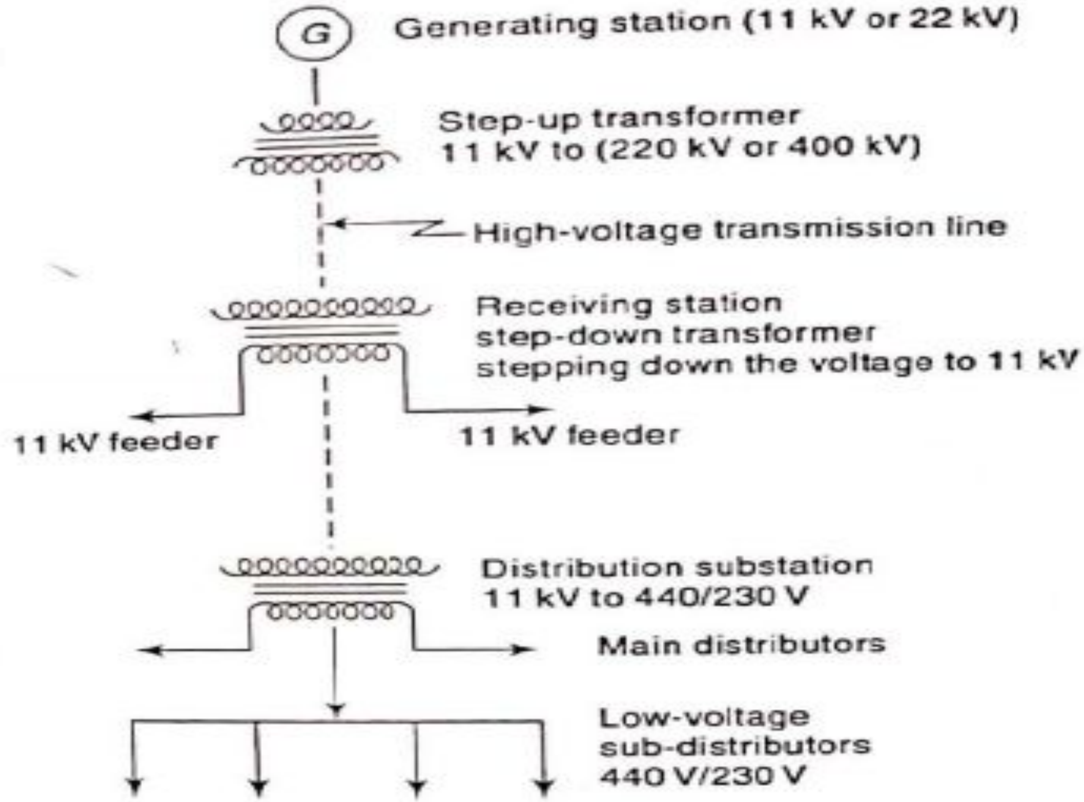


Fig. 4.1 Power system

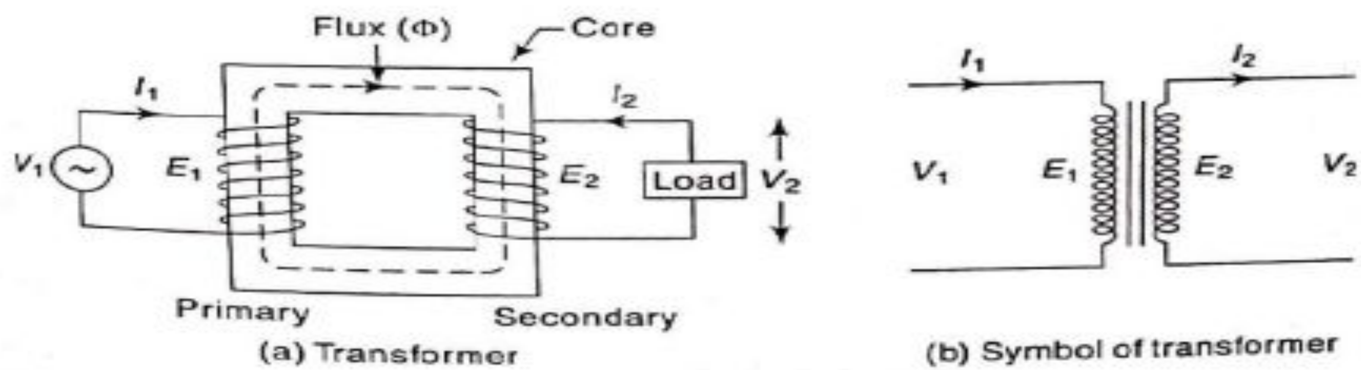


Fig. 4.2 Transformer and symbol of transformer

Assuming leakage flux to be negligible, almost whole flux produced by the primary winding (coil) links with the secondary winding (coil). Hence, an emf e_2 is produced in the secondary coil, which is given by

$$e_2 = -N_2 \frac{d\Phi}{dt} \text{ V} \tag{4.2}$$

where N_2 is the number of turns in the secondary coil. The emf e_1 is called self-induced emf, while the emf e_2 is called mutually induced emf. If the secondary coil is closed through the load, a current I_2 flows through the secondary (coil). Thus, energy is transferred from the primary coil to the secondary coil.

The symbol of a transformer is shown in Fig. 4.2(b). The lines between the two windings represent iron core. If lines are not drawn, then it is air core.

From Eqs (4.1) and (4.2), if N_2 is greater than N_1, then the secondary emf e_2 is higher than the primary emf e_1. Such a transformer is called a step-up transformer. If N_2 is less than N_1, then the secondary emf e_2 is lower than the primary emf e_1. Such a transformer is called a step-down transformer. Thus, the step-up transformer is one that receives electrical energy at one voltage and delivers it at a higher voltage. The step-down transformer is one that receives electrical energy at one voltage and delivers it at a lower voltage.

4.3 Construction Details

A transformer mainly consists of an iron core, primary and secondary windings wound over the core. These are discussed below (see Fig. 4.3).

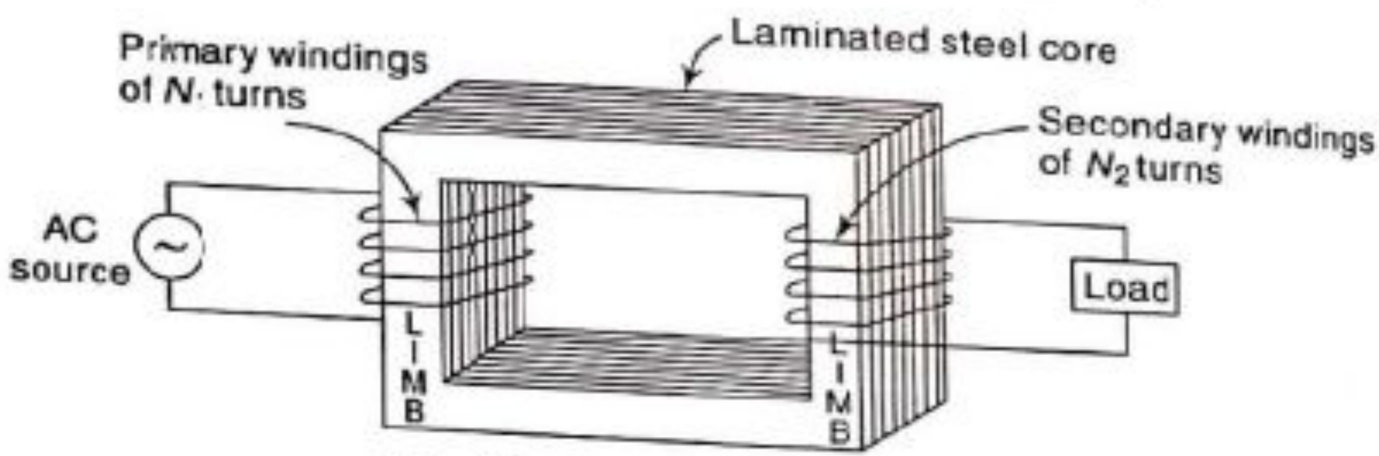


Fig. 4.3 Transformer construction

Fig. 4.3 Transformer construction

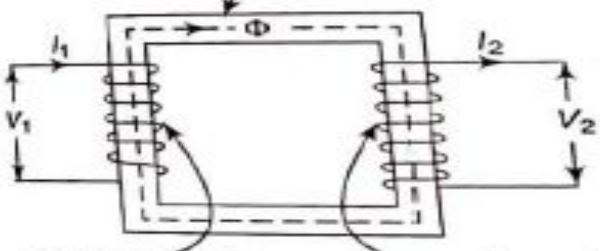
4.3.1 Core

A core is rectangular in shape and is laminated. It is noted that when a core carries alternating flux, loss of energy takes place in the core due to the effect of hysteresis and eddy current. Therefore, the steel used for manufacturing the core is high-grade silicon steel where hysteresis loss is very low. Such steel is called **soft steel**. Due to the alternating flux, certain currents are induced in the core. They are called **eddy currents**. These currents cause considerable loss of power in the core itself, called **eddy current loss**. To minimize this loss, the core is not manufactured as solid core but as stack of laminations, where successive laminations are insulated by a thin layer of varnish. The varnish provides a very high resistance to the flow of eddy currents and thus, the eddy currents get obviously reduced and the eddy current loss gets minimized. The above two losses (i.e., hysteresis and eddy current losses) are together called **core** or **iron losses**.

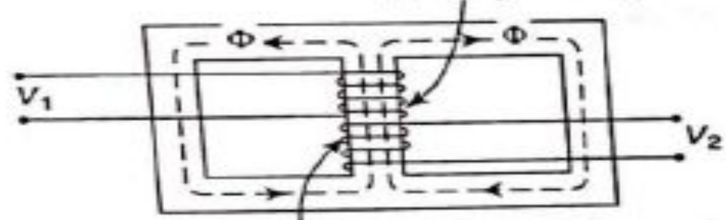
4.3.2 Two Windings

One winding is connected to the source of the electrical energy (called primary winding), while the other is connected to the load (called secondary winding). These windings are coils of different number of turns, wound around the two limbs of the core. Each winding is properly insulated from each other as well as the core.

Depending upon the core and winding arrangement, the transformer construction is classified as (i) core-type construction and (ii) shell-type construction. A core-type transformer is shown in Fig. 4.4(a), where on two limbs of the core, the two windings are placed. Many a time, each winding is divided into two equal parts and each limb carries one part of primary and one part of secondary. The two parts of each winding are electrically connected in series. This arrangement gives good magnetic coupling between the primary and secondary windings. In a shell-type transformer, the arrangement is as shown in Fig. 4.4(b). Here the windings are surrounded by the core. The arrangement has three limbs and both the windings are placed over the central limb.



(a) Core-type transformer



(b) Shell-type transformer

Fig. 4.4 Types of transformer

Table 4.1 compares core-type and shell-type transformers.

Table 4.1 Comparison of core-type and shell-type transformers

Core-type transformer	Shell-type transformer
1. The windings enclose the whole core.	1. The core encloses the windings.
2. Average length of the core is more.	2. Average length of the core is less.
3. Magnetic flux has only one continuous path.	3. Magnetic flux is distributed into two parts.
4. It is more suitable for high-voltage transformers.	4. It is more economical for low voltage transformers.
5. It is easy to repair.	5. It is difficult to repair.

4.4 EMF Equation of a Transformer

Normally, sinusoidal alternating voltage is applied to the primary winding. This produces the flux in the core, which also varies sinusoidally. Let the equation of the sinusoidal alternating flux in the core be

$$\Phi = \Phi_m \sin \omega t \tag{4.3}$$

According to Faraday's law of electromagnetic induction, the self-induced emf in primary winding is given by

$$e_1 = -N_1 \frac{d\Phi}{dt}$$

or $$e_1 = -N_1 \frac{d}{dt}(\Phi_m \sin \omega t)$$

So, $$e_1 = -N_1 \Phi_m \omega \cos \omega t$$

or $$e_1 = N_1 \Phi_m \omega \sin(\omega t - 90°) \tag{4.4}$$

Note: From Eqs (4.3) and (4.4), it may be noted that the self-induced emf (e_1) lags behind the flux (Φ) by 90°.

Comparing Eq. (4.4) with the standard sinusoidal form [i.e., $e = E_m \sin(\omega t \pm \phi)$], maximum value of the induced emf is given by

$$E_M = N_1 \Phi_m \omega$$

Hence, rms value of the induced emf in primary winding is given by

$$E_1 = \frac{E_M}{\sqrt{2}} = \frac{N_1 \Phi_m \omega}{\sqrt{2}} = \frac{N_1 \Phi_m 2\pi f}{\sqrt{2}}$$

So, $$E_1 = 4.44 f N_1 \Phi_m \text{ V} \tag{4.5}$$

where Φ_m is the maximum flux in Wb and f is the supply frequency.

Similarly, rms value of the induced emf in secondary winding is given by

$$E_2 = 4.44 f N_2 \Phi_m \text{ V} \tag{4.6}$$

Equations (4.5) and (4.6) are called emf equations of the transformer.

If B_m = maximum flux density in the magnetic circuit (core) in tesla (T)

and A = area of cross section of the core in sq m (m^2), then

$$B_m = \frac{\Phi_m}{A}$$

The winding with higher number of turns will have a high voltage and is called the high-voltage winding, and the winding with lower number of turns is called the low-voltage winding.

4.5 Transformation Ratio (K)

Considering the emf equations of the transformer,

$$E_1 = 4.44\, f N_1 \Phi_m \text{ V}$$
$$E_2 = 4.44\, f N_2 \Phi_m \text{ V}$$

Dividing, we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

The ratio N_2/N_1 is called transformation ratio and is denoted by K.

Thus, $$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

Neglecting small primary and secondary voltage drops,

$$V_1 \cong E_1 \quad \text{and} \quad V_2 \cong E_2$$

So, $$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

In transformer, losses are negligible. Hence, input and output can be approximately equated.

$$V_1 I_1 = V_2 I_2$$

or $$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K$$

Thus, the different forms of the transformation ratio are:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

For step-up transformers, $N_2 > N_1$ and so, $K > 1$

For step-down transformers, $N_2 < N_1$ and so, $K < 1$

4.6 Actual (Practical) and Ideal Transformers

In a practical transformer, windings are not ideal, and every winding has some resistance (very small value). Hence, there is power loss (copper loss) in the windings. Due to alternating flux, hysteresis loss and eddy current loss take place in the magnetic core. These two losses are together called **core loss**. Also, there is some leakage in flux present near the windings.

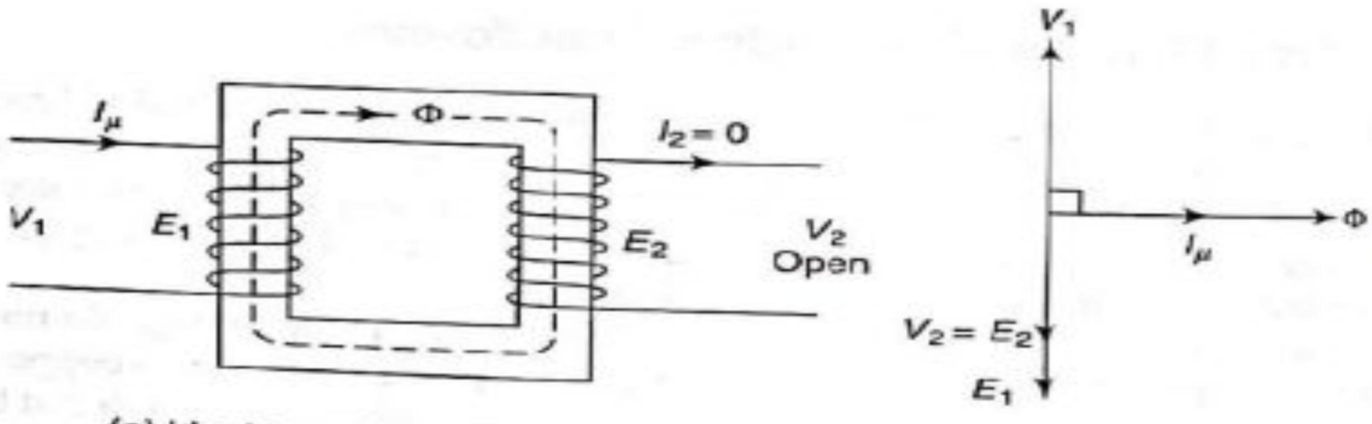
An ideal transformer is one that has no losses, i.e., the windings do not possess any resistance. Hence, there is no voltage drop and power loss (copper loss) in the windings. Also there is no leakage of flux, i.e., all the flux produced by the primary winding links with the secondary winding and all the flux produced by the secondary winding links with the primary winding. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core with no magnetic leakage. It may, however, be noted that it is impossible to realize such a transformer in practice.

Table 4.2 gives a comparison of practical and ideal transformers.

Table 4.2 Comparison of practical and ideal transformers

Practical transformer	Ideal transformer
1. There are copper and eddy current losses.	1. There is no loss.
2. There is leakage of flux.	2. There is no leakage of flux.
3. Its windings contain ohmic resistance.	3. Its windings consist of purely inductive coils, wound on lossless core.
4. Voltage regulation is never 0%.	4. Voltage regulation is 0% (see Section 4.14).
5. Efficiency is 93–97%.	5. Efficiency is 100%.
6. All constructed transformers are practical transformers.	6. It is impossible to construct an ideal transformer.

Consider an ideal transformer as shown in Fig. 4.5(a), whose secondary winding is open (no-load) and primary winding is connected to source of alternating voltage V_1. This potential difference causes an alternating current to flow in the primary winding, called magnetizing current I_μ. Since the primary coil is purely inductive and there is no output (secondary being open), the magnetizing current I_μ lags V_1 by 90° and it is very small in magnitude. The function of this current is to magnetize the core. The alternating current I_μ produces the alternating flux ϕ, which is, at all times, proportional to the current and hence, is in phase with it. This changing flux is linked with both the primary and secondary windings. Therefore, it produces a self-induced emf in the primary winding and a mutually induced emf in the secondary winding. The self-induced emf E_1 is, at every instant, equal to and in opposition to V_1. It is also known as counter emf or back emf of the primary winding. Similarly, in the secondary winding, mutually induced emf E_2 is produced. This emf is antiphase with V_1 and its magnitude is proportional to the secondary number of turns. Figure 4.5(b) shows the phasor diagram of an ideal transformer with no-load.



(a) Ideal transformer (on no-load)

(b) Phasor diagram

Fig. 4.5 Iedal transformer and phasor diagram (on no-load)

The

... and phasor diagram (on no-load)

The power input to the transformer is $V_1 I_\mu \cos \phi$, i.e., $V_1 I_\mu \cos 90°$ or zero. This is because on no-load, output power is zero and for an ideal transformer, there are no losses and thus, input power is zero.

By using the following steps, phasor diagram can be drawn:

Step I: Take magnetizing current I_μ as reference phasor.

Step II: In same phase, draw alternating flux phasor (Φ). The alternating current I_μ produces the alternating flux Φ, and hence, both quantities are in phase.

Step III: Draw phasor E_1, lagging behind flux phasor (Φ) by 90° [see Section 4.4].

Step IV: Draw phasor V_1, equal and antiphase to E_1. [For ideal winding (coil), by Lenz's law, applied voltage (V_1) is equal and opposite to self-induced emf (E_1)].

Step V: Draw phasor E_2, in phase with E_1. Depending upon the type of transformer, i.e., step up or step down, E_2 may be greater than or less than E_1.

Step VI: Draw phasor V_2 in phase with E_2. V_2 is equal in magnitude to E_2.

4.7 Transformer Losses

Since a transformer is a static device, it has no rotational losses. The main losses that occur in a transformer are copper loss and iron loss.

Copper loss (W_{cu})

In a transformer, windings are not ideal, and every winding has some resistance. The loss that takes place due to winding resistance is called **copper loss** (as windings are made of copper).

If R_1 is resistance of the primary winding and R_2 is that of the secondary winding, then

$$\text{primary copper loss} = I_1^2 R_1 \text{ W} \tag{i}$$

$$\text{secondary copper loss} = I_2^2 R_2 \text{ W} \tag{ii}$$

So, $$\text{total copper loss}, W_{cu} = (I_1^2 R_1 + I_2^2 R_2) \text{ W} \tag{iii}$$

From the above equations, it is clear that copper loss is a variable loss and varies with square of the current or output. Thus, copper loss at half load is one-fourth of that at full load. At no load, copper loss is negligible. Copper loss can be calculated by conducting short-circuit test on transformer.

Iron loss or core loss (W_i)

Due to applied alternating voltage, an alternating flux is produced in the core. When any magnetic material is placed in alternating magnetic field, the hysteresis loss and eddy current loss take place in the magnetic material. Thus, the hysteresis loss and eddy current loss take place inside the magnetic core of a transformer. These two losses are together called core loss. Since the core is made of iron, it is also termed as iron loss. The core loss is proportional to the applied voltage[2]. Normally, the applied voltage is constant, so the core loss remains constant for all loads. Core loss (or iron loss) can be calculated by conducting open-circuit test (oc test) on transformer.

Hysteresis loss Since the supply to the primary is alternating, the core gets alternately magnetized and demagnetized causing loss of energy (see Section C.4.2, Appendix C). This loss is given by

$$W_h = \eta B_{\max}^{1.6} fV \text{ W}$$

where W_h = hysteresis loss in W

B_{max} = maximum value of flux density in tesla (T)

f = frequency of magnetic reversal in hertz (Hz)

η = Steinmetz constant

V = volume of the material in m^3

[2] The core loss is due to the alternating flux in the core. The magnitude of alternating flux set in the core is directly proportional to the applied voltage. So, core loss is proportional to the applied voltage.

This loss can be minimized by selecting a material for the core that has low hysteresis coefficient, e.g., silicon steel.

Eddy current loss This loss is due to eddy current induced in the transformer core and is seen in the form of heat in the core (see Section C.5.9, Appendix C). This loss is given by

$$W_e = K B_{max}^2 f^2 t^2 \text{ W}$$

where t is thickness of the lamination. This loss can be minimized by using laminated core.

4.8 Transformer Parameters

In this section, we will discuss parameters of a transformer, namely winding resistance, leakage reactance, and impedance.

4.8.1 Winding Resistance

An ideal transformer is supposed to possess no resistance, but in actual transformer, there is always some resistance of primary and secondary windings. In Fig. 4.6, a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 ohm respectively. The resistances have been shown externall to the windings.

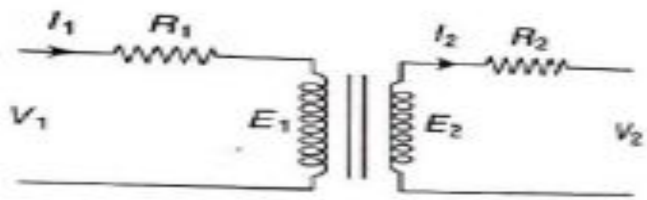


Fig. 4.6 Winding resistance

The resistances of the two windings can be transferred to either of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

It can be proved that a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary[3]. The value R_2/K^2 will be denoted by R_2'—*the equivalent secondary resistance as referred to primary*. In Fig. 4.7(a), secondary resistance has been transferred to primary side leaving secondary circuit without resistance. The resistance $R_1 + R_2' = R_1 + (R_2/K^2)$ is known as the *equivalent resistance of the transformer as referred to primary* and may be denoted by R_{01}.

Thus, $\quad R_{01} = R_1 + R_2' = R_1 + (R_2/K^2)$

Similarly, it can be proved that a resistance of R_1 in primary is equivalent to K^2R_1 in secondary. The value K^2R_1 will be denoted by R_1'—*the equivalent primary resistance as referred to secondary*. In Fig. 4.7(b), primary resistance has been transferred to secondary side leaving primary circuit without resistance. The resistance $R_2 + R_1' = R_2 + K^2R_1$ is known as the *equivalent resistance of the transformer as referred to secondary* and may be denoted by R_{02}.

Thus, $\quad R_{02} = R_2 + R_1' = R_1 + K^2 R_1$

---

[3]The copper loss due to resistance R_2 is equal to $I_2^2 R_2$. When resistance R_2 is transferred to primary side, its value changes. Let it be R_2'. Now, copper loss on primary side will be $I_1^2 R_2'$. Theoretically the above two copper losses must be equal. Thus, $I_2^2 R_2 = I_1^2 R_2'$ So, $R_2' = R_2/K^2$.

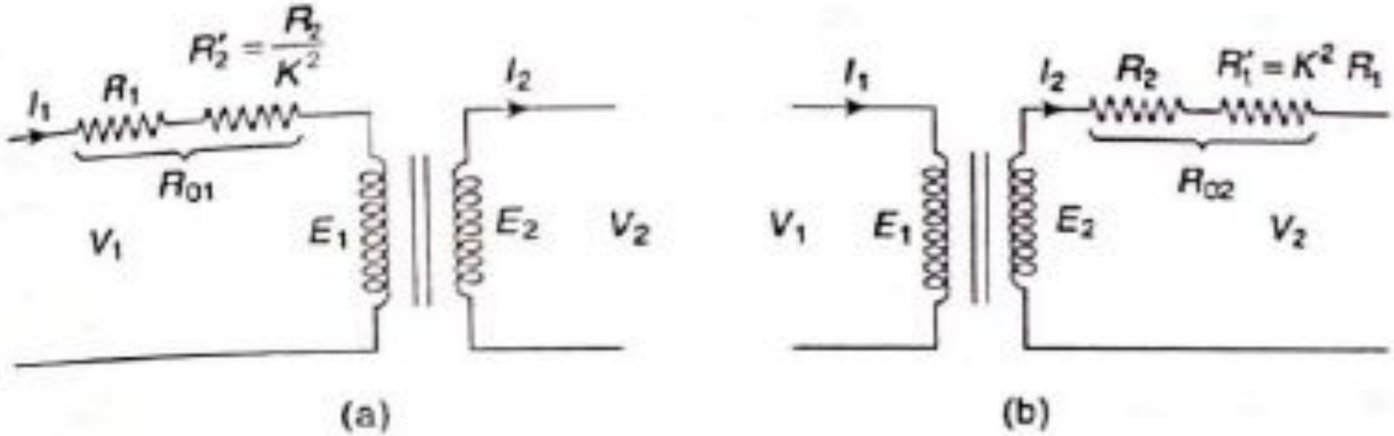


Fig. 4.7 Shifting of winding resistance

The following points may be noted while concentrating the resistances in one winding:

1. When shifting any primary resistance to the secondary winding, multiply it by K^2.
2. When shifting any secondary resistance to the primary winding, divide it by K^2.
3. Concentrating both the resistances in one winding makes calculations very simple and easy. The copper loss from individual resistances (see Fig. 4.6) can be calculated as $W_{cu} = I_1^2 R_1 + I_2^2 R_2$ W.
4. If resistances are concentrated on primary side [see. Fig. 4.7(a)], then the copper loss can be calculated as $W_{cu} = I_1^2 R_{01}$ W.
5. If resistances are concentrated on secondary side [see.Fig. 4.7(b)], then the copper loss can be calculated as $W_{cu} = I_2^2 R_{02}$ W.

copper loss can be calculated as $W_{cu} = I_2^2 R_{02}$ W.

4.8.2 Leakage Reactance

In an ideal transformer, it is assumed that all the flux produced by the primary winding links both the primary and secondary windings. But in practice, it is impossible to realize this condition. However, it is found that all the flux linked with primary does not link with secondary. As shown in Fig. 4.8(a), some part of primary flux, Φ_{L1}, completes its path by passing through air rather than around the core. The flux Φ_{L1} is called primary leakage flux, which links to primary winding and does not link to secondary winding. Similarly, Φ_{L2} is called secondary leakage flux, which link to secondary winding and does not link to primary winding. The flux Φ_{L1} is in time phase with I_1 and induces an emf E_{L1} in primary

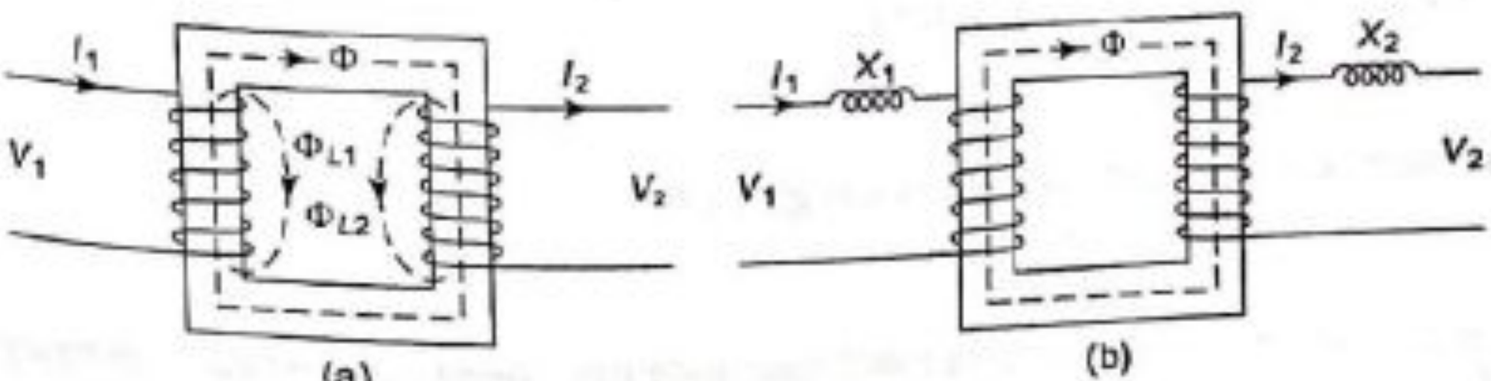


Fig. 4.8 Magnetic leakage and leakage reactance

but not in secondary. Similarly, the flux Φ_{L2} is in time phase with I_2 and induces an emf E_{L2} in secondary but not in primary.

Thus, each leakage flux links one winding only and it is caused by the current in that winding alone. The flux which passes completely through the core and links both windings is called main or mutual flux and is shown by Φ.

It should be noted that the induced voltages E_{L1} and E_{L2} due to leakage fluxes Φ_{L1} and Φ_{L2} are different from induced voltages E_1 and E_2 caused by the main flux Φ. The leakage flux linking with each winding produces a self-induced emf in that winding. Hence, in effect, it is equivalent to a small inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux [refer to Fig. 4.8(b)]. Thus,

$$E_{L1} = I_1 X_1 \quad \text{and} \quad E_{L2} = I_2 X_2$$

The terms X_1 and X_2 are known as primary and secondary leakage reactances respectively. It should be noted that X_1 and X_2 are fictitious quantities introduced as a convenience in representing the effects of primary and secondary leakage fluxes.

4.8.3 Impedance

As discussed in Section 4.6, in practical transformer, windings are not ideal, and every winding has some resistance (very small value). In Fig. 4.9, a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 ohm respectively. The resistances have been shown external to the windings. Also in practical transformer, there is some leakage flux present near the primary and secondary windings. In Fig. 4.9, the effects of primary and secondary leakage fluxes are represented by the reactances X_1 and X_2 respectively.

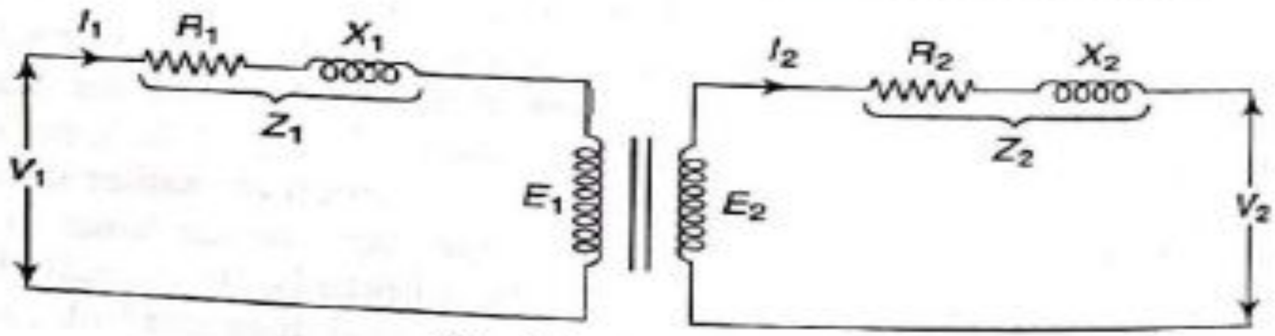


Fig. 4.9 Impedance

The primary impedance is given by

$$Z_1 = \sqrt{R_1^2 + X_1^2} \ \Omega$$

Similarly, the secondary impedance is given by

$$Z_2 = \sqrt{R_2^2 + X_2^2} \ \Omega$$

In Fig. 4.10, secondary resistance and reactance have been transferred to primary side.

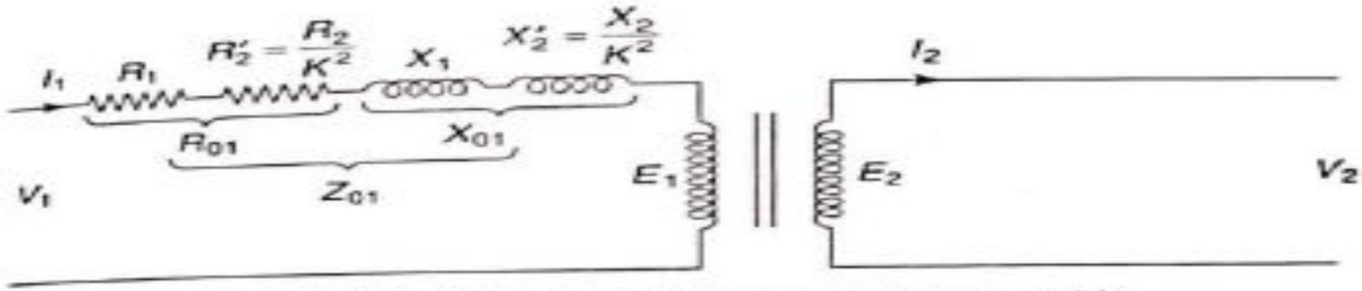


Fig. 4.10 Shifting of parameters to primary side

The equivalent resistance of the transformer as referred to primary is given by

$$R_{01} = R_1 + R_2' = R_1 + (R_2/K^2)$$

Similarly, the equivalent reactance of the transformer as referred to primary is given by

$$X_{01} = X_1 + X_2' = X_1 + (X_2/K^2)$$

The equivalent impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \;\; \Omega$$

In Fig. 4.11, primary resistance and reactance have been transferred to secondary side.

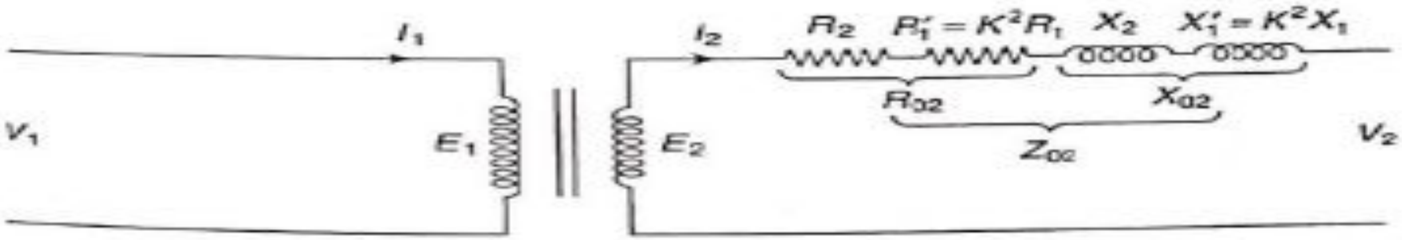


Fig. 4.11 Shifting of parameters to secondary side

The equivalent resistance of the transformer as referred to secondary is given by

$$R_{02} = R_2 + R_1' = R_2 + K^2R_1$$

Similarly the equivalent reactance of the transformer as referred to secondary is given by

$$X_{02} = X_2 + X_1' = X_2 + K^2X_1$$

The equivalent impedance of the transformer as referred to secondary is given by

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} \;\; \Omega$$

4.9 Transformer on No-Load

A transformer is on no-load when its secondary winding is open-circuited, i.e., secondary current I_2 is zero [refer to Fig. 4.12(a)]. Under such a condition, the primary input current has to supply: (i) iron loss in the core (i.e., hysteresis and

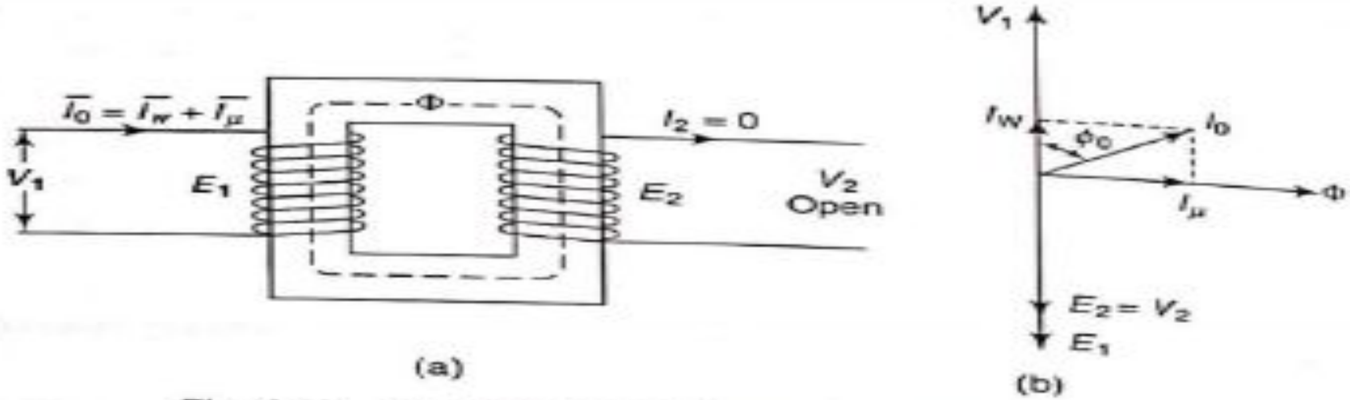


Fig. 4.12 Transformer and phasor diagram (on no-load)

eddy current loss) and (ii) a very small amount of copper loss in the primary winding only (the secondary winding being open involves no copper loss). The primary winding is not a pure winding and has some winding resistance. Hence, the no-load primary input current (I_0) is not at 90° behind V_1 but lags it by an angle $\phi_0 < 90°$.

Now, the primary current (I_0) under no-load condition can be split up into two components [see Fig. 4.12(b)].

(i) Iron loss (or active or working) component (I_W) is in phase with V_1, which supplies the iron losses.
Thus, $I_W = I_0 \cos \phi_0$

(ii) Magnetizing component (I_μ) in quadrature with V_1.
Thus, $I_\mu = I_0 \sin \phi_0$

This component is wattless, and its function is to sustain (or maintain) the alternating flux in the core. The current I_0 is vector sum of I_W and I_μ, i.e.,

Thus, from the phasor diagram,

$$I_0 = \sqrt{I_W^2 + I_\mu^2}$$
$$I_W = I_0 \cos \phi_0$$
$$I_\mu = I_0 \sin \phi_0$$

No-load power input is given by

$$W_0 = V_1 I_0 \cos \phi_0 \text{ W}$$

where $\cos \phi_0$ is power factor at no load. As losses are very small, input power is also very small. Thus, input current I_0 is very small. As I_0 is very small, the no-load primary copper loss is negligibly small, which means that no-load primary input is practically equal to the iron loss in the transformer.

So, Iron loss, $W_i = V_1 I_0 \cos \phi_0$ W

$I_0 \cos \varphi_0$ W

Phasor diagram

Figure 4.12(b) shows the phasor diagram. Since the flux Φ is common to both the windings, Φ is chosen as reference. The self-induced emf E_1 lags behind the flux Φ by 90° (see Section 4.4). Hence, the phasor E_1 lags behind the flux by

90°. The phasor E_2 is in phase with E_1. The magnetizing current I_μ is drawn in phase with the flux Φ. The applied voltage V_1 is drawn equal and opposite to E_1. The active component I_W is drawn in phase with V_1. The phasor sum of I_W and I_μ gives the no-load current I_0. The angle between V_1 and I_0 is marked as ϕ_0 (no-load power factor angle).

4.10 Transformer on Load

When transformer is on no-load (i.e., when its secondary winding is open circuited and secondary current I_2 is zero), primary winding draws a very small current I_0 and Φ is the main primary flux in the core [see Fig. 4.13(a)].

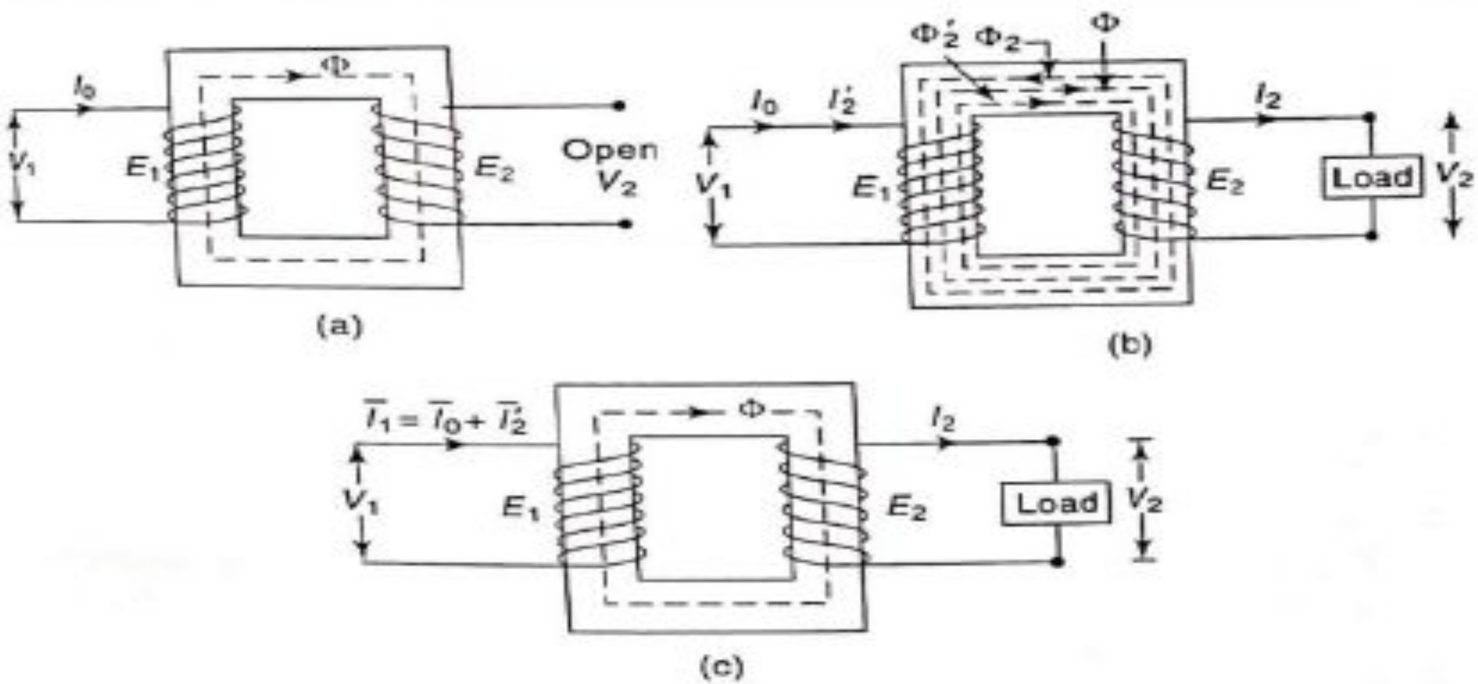


Fig. 4.13 Transformer on load

Fig. 4.13 Transformer on load

When secondary winding is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. The secondary current set up its own flux Φ_2, which is in opposition to main primary flux Φ. (Basically flux Φ_2 is due to mmf N_2I_2.) The opposing secondary flux Φ_2 weakens the primary flux Φ momentarily. Hence, primary back emf E_1 tends to be reduced and for a moment, V_1 becomes greater than E_1. Therefore, additional current flows through primary winding [see. Fig. 4.13(b)].

Let the additional primary current be I_2' [see Fig. 4.13(b)]. It is known as load component of primary current. This current is antiphase with I_2. This current set up its own flux Φ_2', which is in opposition to Φ_2 and is equal to it in magnitude. (Basically flux Φ_2' is due to mmf $N_1 I_2'$.) Hence, the two cancel each other out, and we find that the magnetic effect of secondary current I_2 is immediately neutralized by the additional current I_2'. Hence, whatever the load condition, the net flux passing through the core is approximately the same as at no-load. Therefore, the core loss is also practically the same under all load conditions.

As $\Phi_2 = \Phi_2'$, $\quad N_2 I_2 = N_1 I_2'$,

So, $$I_2' = \frac{N_2}{N_1} \times I_2 = KI_2$$

Hence, when transformer is on load, the primary winding has two currents in it– one is I_0 and the other is I_2', which is antiphase with I_2 and K times in magnitude. The total primary current is the phasor sum of I_0 and I_2'.

4.10.1 Phasor Diagram: Without Considering Winding R...

phasor sum of I_0 and I_2.

4.10.1 Phasor Diagram: Without Considering Winding Resistance and Magnetic Leakage

Neglecting small primary and secondary voltage drops due to winding resistance and leakage reactance, we get

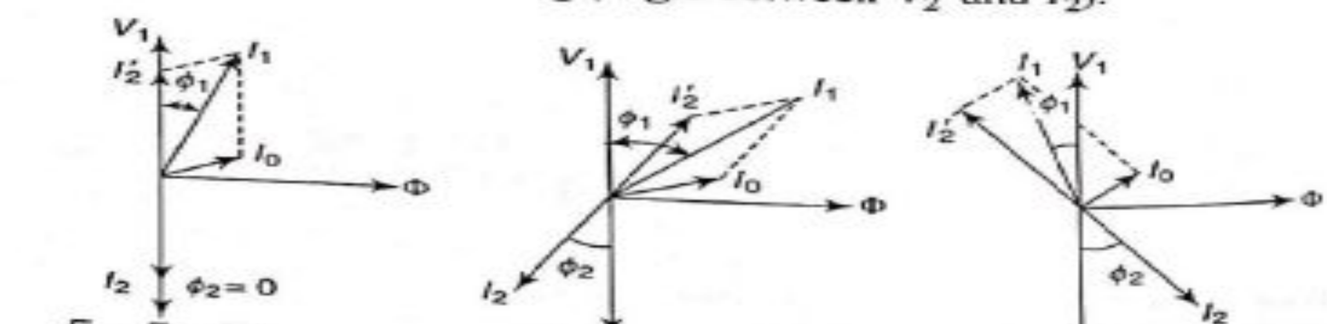
On primary side, $E_1 = V_1$, antiphase with each other

On secondary side, $V_2 = E_2$

Case (i) When load is resistive (unity power factor)

Figure 4.14(a) shows the phasor diagram for resistive load (assuming $K = 1$). Steps for drawing the phasor diagram are as follows:

1. Take flux Φ as reference phasor.
2. Draw $\overline{E}_1$, which lags behind the flux Φ by 90°.
3. Draw $\overline{E}_2$, which is in phase with $\overline{E}_1$. As $K = 1$, E_2 is equal to E_1. $\overline{V}_2$ is equal to $\overline{E}_2$ both in magnitude and phase.
4. Draw $\overline{V}_1$, equal and opposite to $\overline{E}_1$.
5. Draw phasor $\overline{I}_0$, which lags $\overline{V}_1$ by an angle $\phi_0 < 90°$.
6. Draw phasor $\overline{I}_2$ in phase with $\overline{V}_2$ (as load is resistive).
7. The secondary current I_2 causes primary current I_2', which is antiphase with it and equal to it in magnitude (since $K = 1$). Therefore, draw the phasor $\overline{I}_2'$ in antiphase with $\overline{I}_2$ and equal in magnitude.
8. By law of parallelogram, find the resultant of $\overline{I}_0$ and $\overline{I}_2'$. This resultant is the total primary current $\overline{I}_1$.
9. Mark the primary phase angle ϕ_1 (angle between $\overline{V}_1$ and $\overline{I}_1$). Similarly, mark the secondary phase angle ϕ_2 (angle between $\overline{V}_2$ and $\overline{I}_2$).



$E_1 = E_2 = V_2$
(a) Load: resistive (unity pf)

$E_1 = E_2 = V_2$
(b) Load: inductive (lagging pf)

$E_1 = E_2 = V_2$
(c) Load: capacitive (leading pf)

Fig. 4.14 Phasor diagram (on load), without considering winding resistance and magnetic leakage

Case (ii) When load is inductive (lagging power factor)

By using the same steps as in case (i), the phasor diagram can be drawn. Only modification required is that current I_2 lags V_2 by an angle $\phi_2 < 90°$ (as load is inductive). Figure 4.14(b) shows the phasor diagram for inductive load (assuming $K = 1$).

Case (iii) When load is capacitive (leading power factor)

By using the same steps as in case (i), the phasor diagram can be drawn. Only modification required is that current I_2 leads V_2 by an angle $\phi_2 < 90°$ (as load is capacitive). Figure 4.14(c) shows the phasor diagram for capacitive load (assuming $K = 1$).

4.10.2 Phasor Diagram: Considering Winding Resistance and Magnetic Leakage

Consider a transformer with winding resistance and leakage reactance as shown in Fig. 4.15.

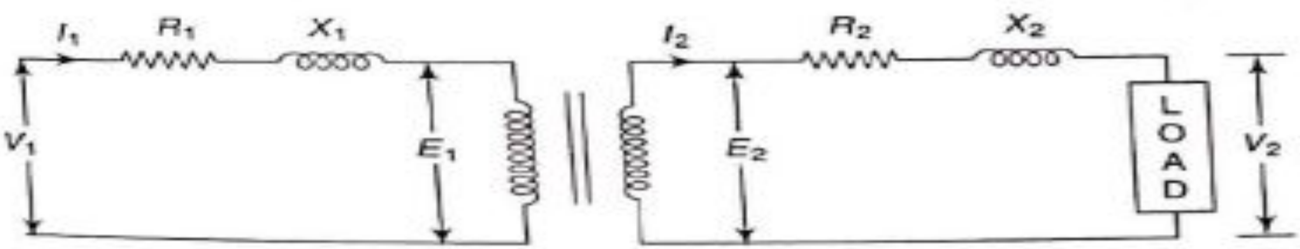


Fig. 4.15 Transformer with parameters

Writing vector equations for primary and secondary sides,

$$\bar{V}_1 = \bar{I}_1\bar{R}_1 + \bar{I}_1\bar{X}_1 + (-\bar{E}_1)$$

and $$\bar{E}_2 = \bar{I}_2\bar{R}_2 + \bar{I}_2\bar{X}_2 + \bar{V}_2$$

where $$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

The phasor diagram of a transformer on load condition is drawn with the help of the above expressions.

Steps for drawing the phasor diagram are as follows:

1. First draw $\bar{V}_2$ and then $\bar{I}_2$. Phase angle between $\bar{I}_2$ and $\bar{V}_2$ will depend on the type of load.
2. To $\bar{V}_2$, add resistive drop $\bar{I}_2\bar{R}_2$ parallel to $\bar{I}_2$ and inductive drop $\bar{I}_2\bar{X}_2$ leading $\bar{I}_2$ ($\bar{I}_2\bar{R}_2$) by 90° such that
$$\bar{E}_2 = \bar{I}_2\bar{R}_2 + \bar{I}_2\bar{X}_2 + \bar{V}_2$$
3. Draw $\bar{E}_1$ in the same phase of $\bar{E}_2$.
4. Draw $-\bar{E}_1$ equal and opposite to $\bar{E}_1$.
5. For drawing $\bar{I}_1$, first draw $\bar{I}_0$ and $\bar{I}_2'$ such that
$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

6. Now add $\overline{I}_0$ and $\overline{I}_2'$ using parallelogram law of vector addition such that

$$\overline{I}_1 = \overline{I}_0 + \overline{I}_2'$$

7. To $-\overline{E}_1$, add resistive drop $\overline{I}_1 \overline{R}_1$, parallel to $\overline{I}_1$ and inductive drop $\overline{I}_1 \overline{X}_1$ leading $\overline{I}_1$ ($\overline{I}_1 \overline{R}_1$) by 90° such that

$$\overline{V}_1 = \overline{I}_1 \overline{R}_1 + \overline{I}_1 \overline{X}_1 + (-\overline{E}_1)$$

8. Draw flux Φ such that Φ leads $\overline{E}_1$ and $\overline{E}_2$ by 90°.

Case (i) When load is resistive (unity power factor)

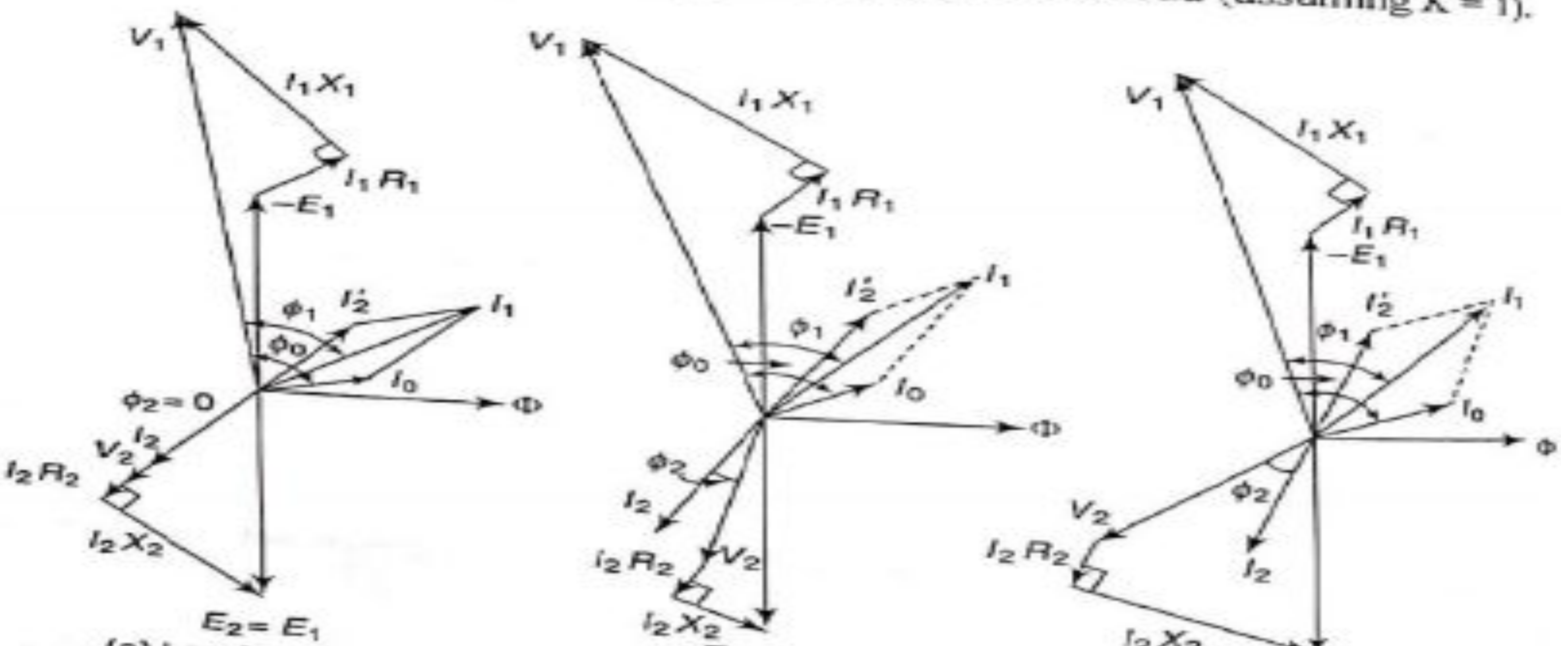
Figure 4.16(a) shows the phasor diagram for resistive load (assuming $K = 1$).

Case (ii) When load is inductive (lagging power factor)

Figure 4.16(b) shows the phasor diagram for inductive load (assuming $K = 1$).

Case (iii) When load is capacitive (leading power factor)

Figure 4.16(c) shows the phasor diagram for capacitive load (assuming $K = 1$).



(a) Load: resistive (unity pf)

(b) Load: inductive (lagging pf)

(c) load: capacitive (leading pf)

Fig. 4.16 Phasor diagram (on load), considering winding resistance and magnetic leakage

4.11 Transformer Ratings

...diagram (on load), considering winding resistance and magnetic leakage (leading pf)

4.11 Transformer Ratings

Two types of ratings are used for transformer—voltage rating and kVA rating.

Voltage rating

It indicates the normal or rated primary and secondary voltages. For example, for the voltage rating given as 200/400 V,

primary rated voltage, $E_1 \cong V_1 = 200$ V

secondary rated voltage, $E_2 \cong V_2 = 400$ V

kVA rating

During operation of a transformer, power losses take place in the windings and core of the transformer. These power losses appear in the form of heat, which increases the temperature of the device. This temperature must be maintained below a certain limiting value as it is always harmful to the transformer.

The output of a transformer is expressed in kVA (i.e., kilo-volt ampere). The rated transformer output is limited by heating and hence losses in the transformer, i.e., copper loss and core loss. These losses depend upon the voltage and current, and are almost unaffected by the power factor of the load. Therefore, the transformer rated output is expressed in kVA and not in kW.

At a zero power factor also (i.e., delivering zero power), a transformer can be made to operate at rated kVA.

The kVA rating is given by

$$\text{kVA rating} = \frac{E_1(I_1)_{FL}}{1000} = \frac{E_2(I_2)_{FL}}{1000}$$

where $(I_1)_{FL}$ = full-load primary current

$(I_2)_{FL}$ = full-load secondary current

From kVA rating, we can calculate full-load currents on primary and secondary windings. This is the safe maximum current limit, which may keep temperature rise below its limiting value. So,

$$(I_1)_{FL} = \frac{\text{kVA rating} \times 1000}{E_1}$$

$$(I_2)_{FL} = \frac{\text{kVA rating} \times 1000}{E_2}$$

The full-load primary and secondary currents indicate the safe maximum values of currents that the transformer windings can carry. These values indicate how much maximum load can be connected to a given transformer of a specified kVA rating.

Example 4.1 A 50 kVA, single-phase transformer has 600 turns on the primary winding and 40 turns on the secondary winding. The primary winding is connected to a 2.2 kV, 50 Hz supply. Determine (i) secondary voltage at no load and (ii) primary and secondary currents at full load.

Solution

(i) $$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

So, $$E_2 = \frac{N_2}{N_1} \times E_1 = \frac{40}{600} \times 2200 = 146.67 \text{ V}$$

(ii) $$(I_1)_{FL} = \frac{\text{kVA rating} \times 1000}{E_1}$$

$$= \frac{50 \times 1000}{2200}$$

$$= 22.73 \text{ A}$$

$$(I_2)_{FL} = \frac{\text{kVA rating} \times 1000}{E_2}$$

$$= \frac{50 \times 1000}{146.67}$$

$$= 340.9 \text{ A}$$

$= 340.9$ A

Example 4.2 A 80 kVA, 3200/400 V, single-phase, 50 Hz, transformer has 111 turns on the secondary winding. Calculate (i) number of turns on primary winding, (ii) secondary full-load current, and (iii) cross-sectional area of the core, if the maximum flux density is 1.2 tesla.

Solution

(i) We know that

$$\frac{N_2}{N_1} = \frac{E_2}{E_1}$$

So, $N_1 = \frac{E_1}{E_2} \times N_2 = \frac{3200}{400} \times 111 = 888$ turns

(ii) $(I_2)_{FL} = \frac{\text{kVA rating} \times 1000}{E_2}$

$$= \frac{80 \times 1000}{400}$$

$= 200$ A

(iii) The emf equation of a transformer is:

$$E_2 = 4.44\, f N_2\, \Phi_m \text{ V}$$

or $E_2 = 4.44\, f N_2\, B_m A$ V $\qquad (\because \Phi_m = B_m \times A)$

where B_m = maximum flux density in Wb/m^2 or tesla

A = cross-sectional area of the core in m^2

So, $400 = 4.44 \times 50 \times 111 \times 1.2 \times A$

or cross-sectional area, $A = 0.0135$ m^2

Example 4.3 The required no-load ratio in a single-phase 50 Hz core-type transformer is

Example 4.5 A single-phase transformer has primary voltage of 230 V. No-load primary current is 5 A. No-load pf is 0.25. Number of primary turns is 200 and frequency is 50 Hz. Calculate (i) maximum value of flux in the core, (ii) core loss, and (iii) magnetizing current.

Solution

Given:

$$E_1 \approx V_1 = 230 \text{ V}$$
$$I_0 = 5 \text{ A}$$
$$\cos \phi_0 = 0.25$$
$$N_1 = 200 \text{ turns}$$
$$f = 50 \text{ Hz}$$

(i) The emf equation of a transformer is given by

$$E_1 = 4.44 f N_1 \Phi_m \text{ V}$$

So, $230 = 4.44 \times 50 \times 200 \times \Phi_m$

which gives $\Phi_m = 5.18$ mWb

(ii) At no load, copper loss is negligible.

So, input power = iron loss (W_i)

Thus, core loss, $W_i = V_1 I_0 \cos \phi_0$

$$= 230 \times 5 \times 0.25 = 287.5 \text{ W}$$

(iii) Magnetizing current,

$$I_\mu = I_0 \sin \phi_0$$
$$= 5 \times 0.97 \qquad (\because \cos \phi_0 = 0.25 \text{ and } \sin \phi_0 = 0.97)$$
$$= 4.85 \text{ A}$$

Example 4.9 A 40 kVA, single-phase transformer has full-load copper loss equal to 2500 W. Determine (i) copper loss at 60% full-load condition and (ii) copper loss at half-load condition.

Solution

(i) Copper loss at 60% full-load condition,

$$[W_{cu}]_{60\%FL} = (0.60)^2 [W_{cu}]_{FL}$$
$$= (0.60)^2 \times 2500$$
$$= 900 \text{ W}$$

(ii) Copper loss at half-load condition,

$$[W_{cu}]_{HL} = (0.5)^2 [W_{cu}]_{FL}$$
$$= (0.5)^2 \times 2500$$
$$= 625 \text{ W}$$

Example 4.10 For a transformer, copper loss at 75% full-load condition is equal to 1200 W. What is the copper loss at half load?

Solution

Given: $[W_{cu}]_{75\% FL} = 1200$ W

Required: $[W_{cu}]_{HL}$

First we will calculate the copper loss at full load.

We have, $[W_{cu}]_{75\% FL} = (0.75)^2 [W_{cu}]_{FL}$

or $1200 = (0.75)^2 [W_{cu}]_{FL}$

Hence, $[W_{cu}]_{FL} = 2133.33$ W

Now, $[W_{cu}]_{HL} = (0.5)^2 [W_{cu}]_{FL}$

$$= (0.5)^2 \times 2133.33$$
$$= 533.33 \text{ W}$$

4.12 Equivalent Circuit

The complete details of the currents and their components, voltages, winding parameters of primary as well as secondary windings of a transformer are represented in one common electric circuit called **equivalent circuit** of the transformer. The equivalent circuit of a transformer is quite helpful in predetermining the behaviour of the transformer under various conditions of operation. In equivalent circuit, the actual transformer is assumed to be equivalent to an ideal transformer along with the additional parameters inserted between supply and primary winding and between secondary winding and load (see Fig. 4.18).

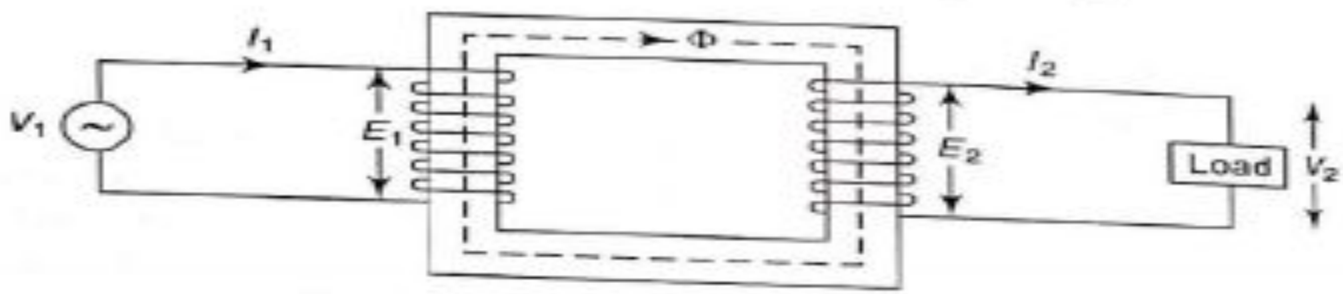


Fig. 4.17 Practical or actual transformer

The practical or actual transformer shown in Fig. 4.17 can be represented as an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding (refer to Fig. 4.18). We know that the no-load current I_0 has two components I_W and I_μ. The current I_W is in phase with voltage V_1, whereas I_μ lags behind the voltage V_1 by 90°. Therefore, in equivalent circuit, I_0 is simulated by pure inductance X_0 taking the magnetizing component I_μ and a resistance R_0 taking the working component I_W connected in parallel across the primary circuit. The resistance R_0 represents the core loss and so, the current I_W which supplies core loss, is shown passing through it. Thus, $I_W^2 R_0$ = core loss of the actual transformer.

core loss of the actual transformer.

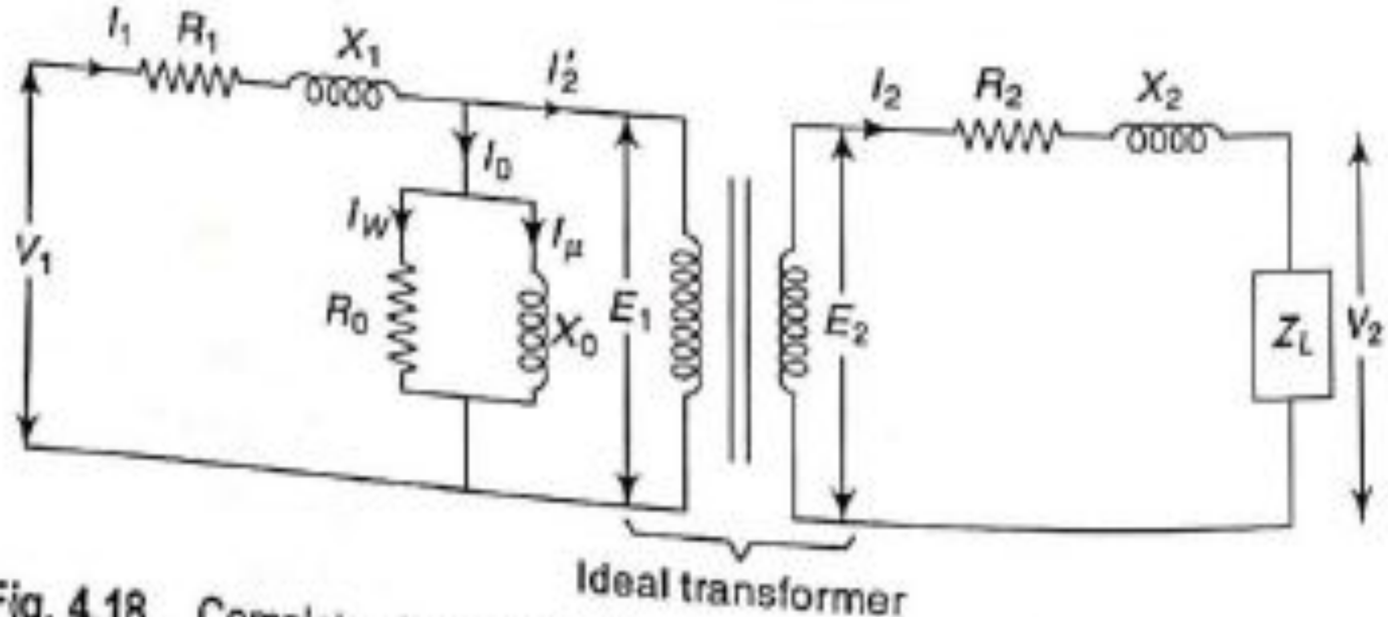


Fig. 4.18 Complete circuit model (equivalent circuit) of a practical transformer

Figure 4.18 shows the complete equivalent circuit of a transformer. An exact equivalent circuit of a transformer referred to the primary side can be obtained by transferring (reflecting) the entire secondary circuit to primary side as follows:

(i) All secondary resistances and reactances are transferred (reflected) to the primary by dividing them by the square of the transformation ratio. That is, the quantities R_2, X_2 and Z_L in the secondary become R_2/K^2, X_2/K^2 and Z_L/K^2 respectively, when referred to the primary.

(ii) All voltages are transferred (reflected) to the primary by dividing them by the transformation ratio. That is, the quantities V_2 and E_2 in the secondary become V_2/K and E_2/K respectively, when referred to the primary.

(iii) The secondary current is transferred (reflected) to the primary by multiplying it by the transformation ratio. That is, the current I_2 in the secondary becomes KI_2, when referred to the primary.

Figure 4.19 shows the equivalent circuit with all secondary values referred (reflected) to the primary. This circuit is called the **exact equivalent circuit of the transformer referred to the primary.**

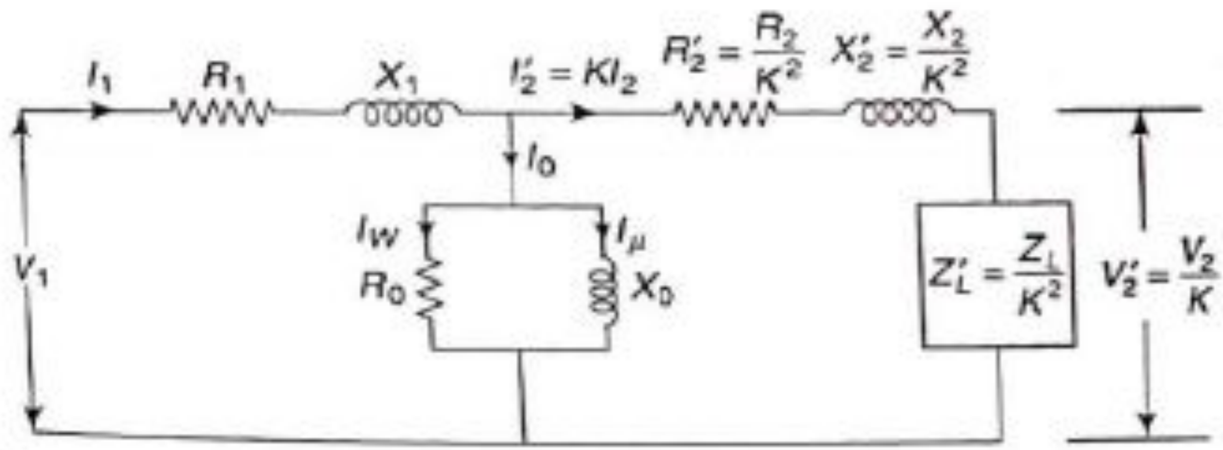


Fig. 4.19 Exact equivalent circuit of a transformer referred to the primary

With the equivalent circuit shown in Fig. 4.19, it is not possible to add directly the primary impedance $(R_1 + jX_1)$ to the secondary impedance transferred (reflected) to the primary side.

Figure 4.20 shows the approximate equivalent circuit referred to the primary. The justification for approximation is as follows:

The no-load current I_0 is usually less than 5% of the full-load primary current. The voltage drop produced by I_0 in $(R_1 + jX_1)$ is negligible for practical purposes. Therefore, it is immaterial that the shunt branch $R_0 \parallel X_0$ is connected before the primary series impedance $(R_1 + jX_1)$ or after it. The currents I_W and I_μ are not much affected. Therefore, the parallel branch $R_0 \parallel X_0$ is connected to the input terminals as shown in Fig. 4.20.

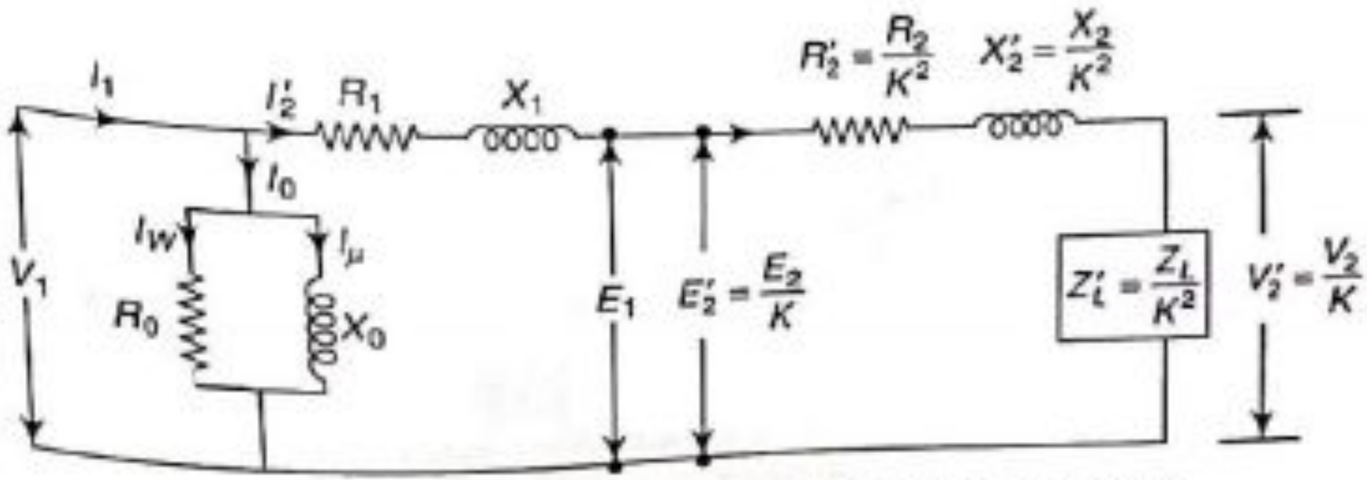


Fig. 4.20 Shifting of parallel branch $R_0 \parallel X_0$ to input side

Due to this approximation, the primary and secondary impedances referred (reflected) to the primary can be added conveniently.

That is, $R_1 + R_2' = R_{01}$

and $\quad X_1 + X_2' = X_{01}$

Thus, we get the approximate equivalent circuit of the transformer as referred to the primary side as shown in Fig. 4.21.

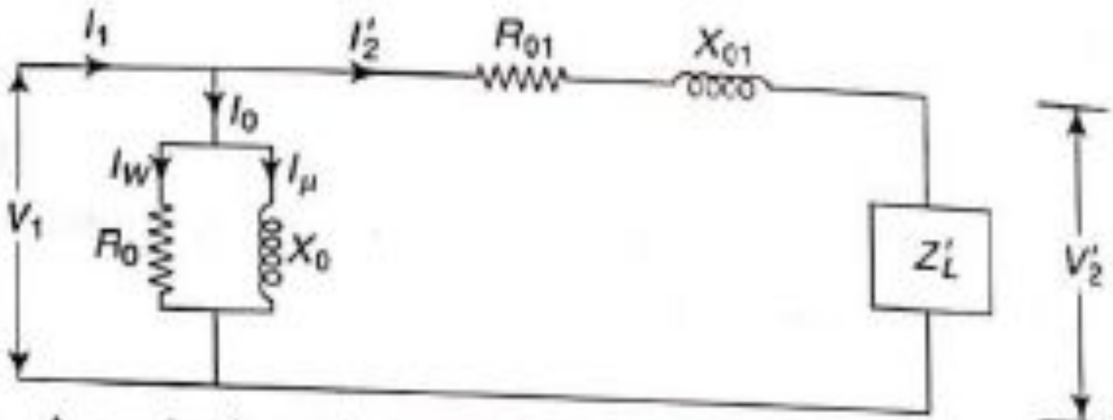


Fig. 4.21 Approximate equivalent circuit of a transformer referred to the primary

The main parameters of the equivalent circuit are:

R_{01} : equivalent resistance of transformer as referred to primary side
X_{01} : equivalent reactance of transformer as referred to primary side
R_0 : equivalent core loss resistance
X_0 : magnetizing reactance

4.13 Transformer Tests

The performance of a transformer can be calculated on the basis of its equivalent circuit, which contains four main parameters—R_{01} (equivalent resistance as referred to primary), X_{01} (equivalent leakage reactance as referred to primary), R_0 (equivalent core-loss resistance), and X_0 (magnetizing reactance). These constants or parameters can be easily determined by two tests: (i) open-circuit test and (ii) short-circuit test. These tests are very economical and convenient because they furnish the required information without actually loading the transformer.

4.13.1 Open-Circuit Test (No-Load Test)

The purpose of this test is to determine core loss (W_i), equivalent core loss resistance (R_0), magnetizing reactance (X_0), and no-load current (I_0). The circuit diagram of open-circuit test is shown in Fig. 4.22.

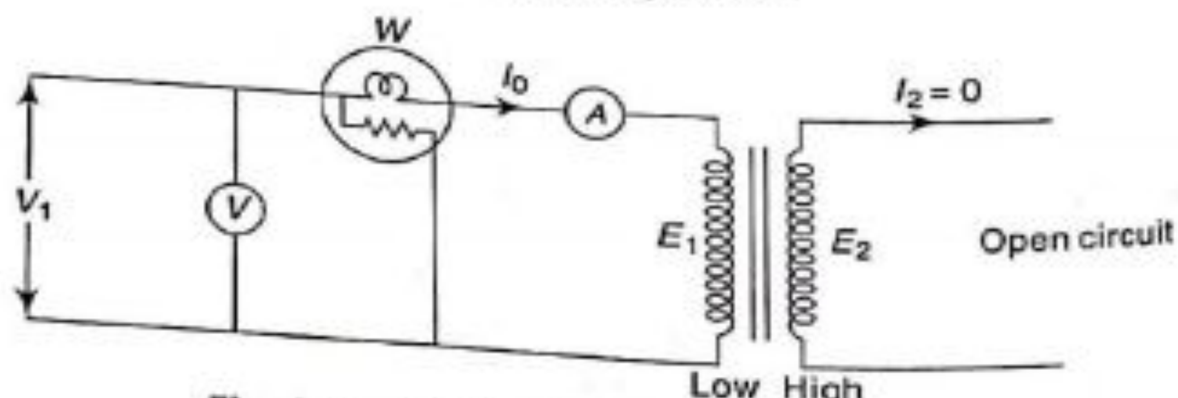


Fig. 4.22 Circuit diagram of open-circuit test

One winding of the transformer, whichever is convenient but usually the high-voltage winding[4], is left open and other is connected to the supply of normal voltage and frequency. A wattmeter, a voltmeter and an ammeter are connected in the low-voltage winding, i.e., the primary winding in the present case. With rated/normal voltage applied to the primary, normal flux will be set up in the core, and hence, normal iron losses will occur, which are recorded by the wattmeter. As the primary no-load current I_0 is small, copper loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition. Ammeter indicates the no-load primary current I_0.

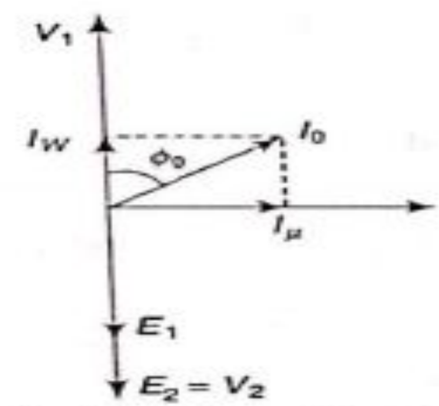


Fig. 4.23 Phasor diagram under no-load condition (open circuit)

Let wattmeter reading = W watt

ammeter reading = I_0 ampere

voltmeter reading = V_1 volt

Figure 4.23 shows the phasor diagram under no-load condition (open circuit).

The equivalent circuit of a transformer under no-load condition is shown in Fig. 4.24. This equivalent circuit is derived from the approximate equivalent circuit of Fig. 4.21. The approximate equivalent circuit is modified according to the open-circuit test condition, i.e., as secondary is open, the load is removed and the load component of the primary current I_2' is made zero.

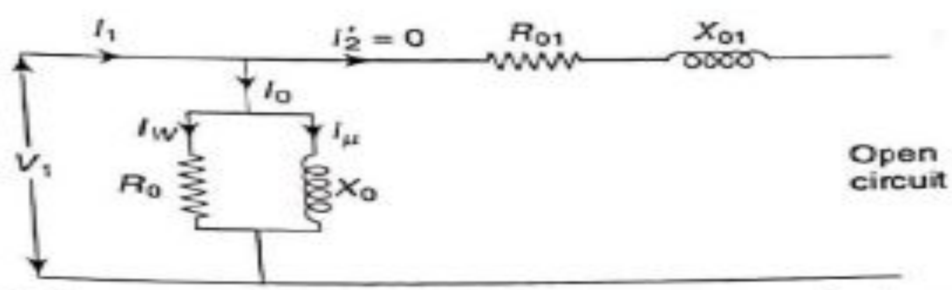


Fig. 4.24 Equivalent circuit of a transformer under no-load condition

Calculation for R_0 and X_0

The wattmeter reading indicates the input power, i.e.,

$$W = V_1 I_0 \cos \phi_0 \text{ W}$$

where W, V_1 and I_0 are the wattmeter, voltmeter and ammeter readings respectively.

So,
$$\cos \phi_0 = \frac{W}{V_1 I_0}$$

[4]The rated voltage on the low-voltage side is lower than that on the high-voltage side. This voltage can be safely applied and measured with the available laboratory voltmeters.

Now, $I_W = I_0 \cos \phi_0$

and $I_\mu = I_0 \sin \phi_0$

From equivalent circuit of Fig. 4.24,

$$R_0 = \frac{V_1}{I_W} \ \Omega$$

and
$$X_0 = \frac{V_1}{I_\mu} \ \Omega$$

4.13.2 Short Circuit (SC) Test

This is an economical method to determine copper loss (W_{cu}) at full load (and at any desired load), equivalent resistance (R_{01} or R_{02}), equivalent reactance (X_{01} or X_{02}), and equivalent impedance (Z_{01} or Z_{02}). The circuit diagram of short-circuit test (conducted on the primary side) is shown in Fig. 4.25.

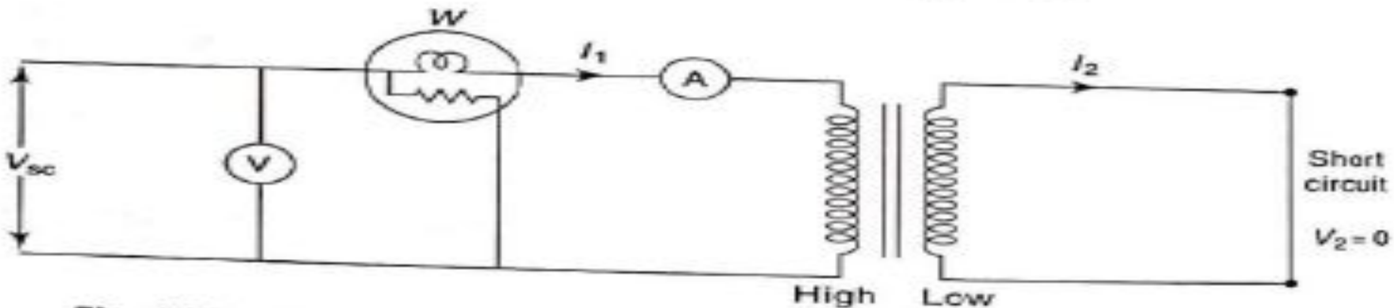


Fig. 4.25 Circuit diagram of short-circuit test (conducted on primary side)

In this test, one winding, usually the low-voltage winding[5], is short circuited by thick conductor as shown in Fig. 4.25. A low voltage is applied to the primary and is cautiously increased till full-load currents are flowing both in the primary and the secondary (as indicated by the ammeter). Since in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced is also a small percentage of its normal value. Hence, core losses are very small with the result that the wattmeter reading represents the full-load copper loss of the transformer.

Let wattmeter reading = W watt
ammeter reading = I_1 ampere
voltmeter reading = V_{sc} volt

The equivalent circuit of a transformer under this condition is shown in Fig. 4.26. This equivalent circuit is derived from the approximate equivalent

[5]The rated current on the high-voltage side is lower than that on the low-voltage side. This current can be safely measured with the available laboratory ammeters. Also since the applied voltage is less than 5% of the rated voltage of the winding, greater accuracy in the reading of the voltmeter is possible when the test is conducted on the high-voltage side.

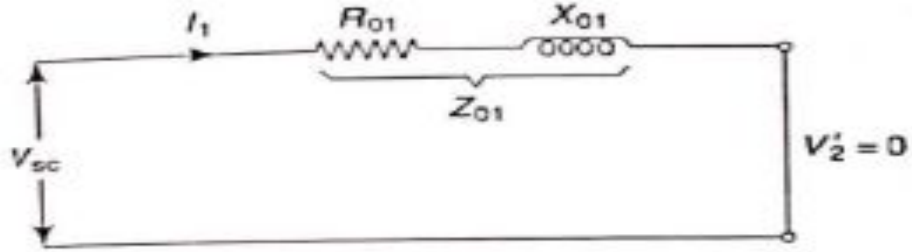


Fig. 4.26 Equivalent circuit of a transformer under short-circuit test condition

circuit of Fig. 4.21. The approximate equivalent circuit is modified according to the short-circuit test condition, i.e., as secondary is shorted, the secondary voltage is made zero. As core loss is negligible, the equivalent core loss resistance R_0 is removed. Therefore, the value of I_0 becomes very small, and so it is neglected in the equivalent circuit.

Calculation for R_{01}, X_{01} and Z_{01}

The wattmeter reading indicates the copper loss, i.e.,

$$W = I_1^2 R_{01}$$

So, $$R_{01} = \frac{W}{I_1^2}\ \Omega$$

From equivalent circuit shown in Fig. 4.26, we get

$$Z_{01} = \frac{V_{sc}}{I_1}\ \Omega$$

where W, V_{sc} and I_1 are the wattmeter, voltmeter and ammeter readings respectively.

Now, $$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}\ \Omega$$

In the above case, sc test is conducted on the primary side. If it is conducted on the secondary side, then the corresponding circuit diagram and calculation of the parameters would be as follows:

parameters would be as follows:

The circuit diagram of short-circuit test (conducted on the secondary side) is shown in Fig. 4.27.

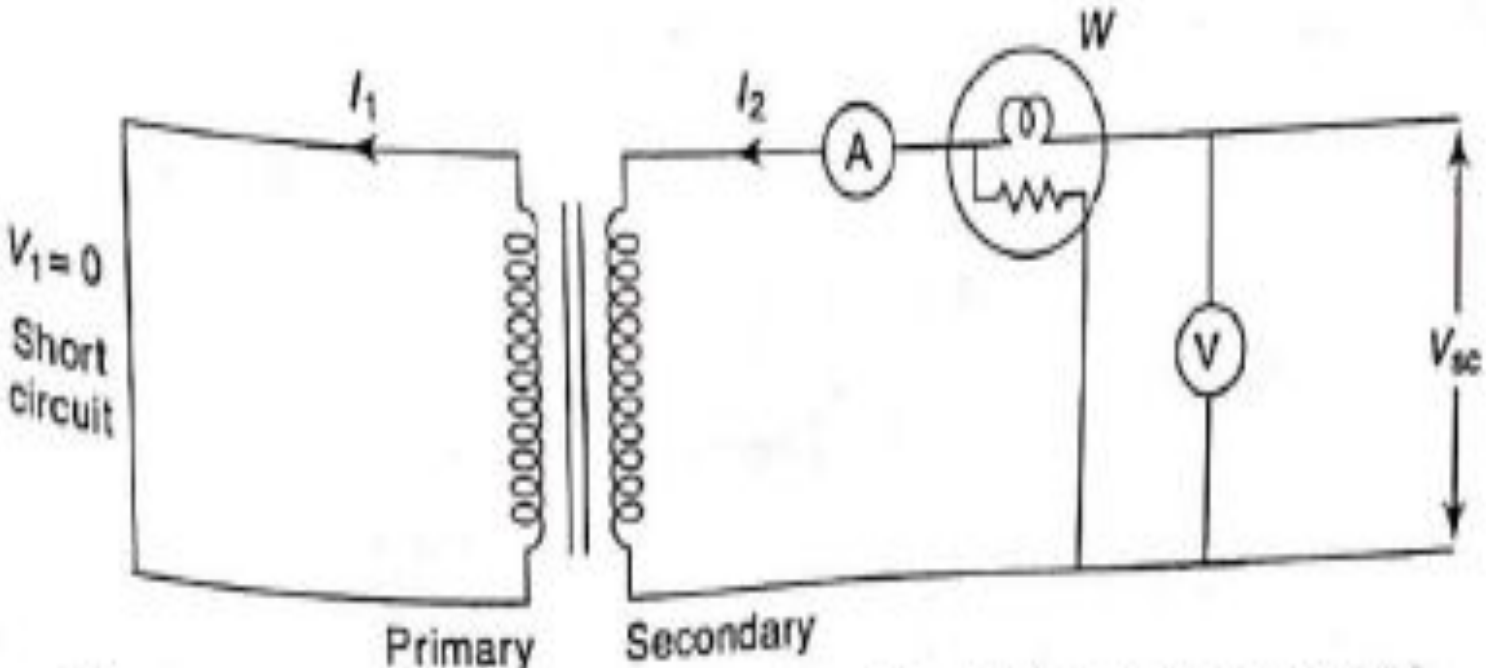


Fig. 4.27 Circuit diagram of short-circuit test (conducted on secondary side)

Let wattmeter reading = W watt
ammeter reading = I_2 ampere
voltmeter reading = V_{sc} volt

The constants or parameters can be calculated as follows:

The wattmeter indicates the Cu loss, i.e.,

$$W = I_2^2 R_{02}$$

or
$$R_{02} = \frac{W}{I_2^2}\ \Omega$$

and
$$Z_{02} = \frac{V_{sc}}{I_2}\ \Omega$$

where W, V_{sc} and I_2 are the wattmeter, voltmeter and ammeter readings respectively.

Now,
$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}\ \Omega$$

Also, R_{01} and X_{01} can be calculated as follows:

$$R_{01} = \frac{R_{02}}{K^2}\ \Omega \quad \text{and} \quad X_{01} = \frac{X_{02}}{K^2}\ \Omega$$

4.14 Regulation of a T

4.14 Regulation of a Transformer

When a transformer is loaded with a constant primary voltage, because of the voltage drop across the primary and secondary impedances, it is observed that secondary terminal voltage drops from its no-load value (E_2) as load current increases.

This change in secondary terminal voltage from no load to the given load conditions, expressed as a fraction of the no-load secondary terminal voltage is called **regulation of the transformer.**

Let E_2 = secondary terminal voltage on no load

V_2 = secondary terminal voltage on the given load

Then mathematically voltage regulation at the given load can be calculated as

$$\%\text{ regulation} = \frac{\begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on no load}\end{pmatrix} - \begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on the}\\ \text{given load}\end{pmatrix}}{\begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on no load}\end{pmatrix}} \times 100$$

or $$\%\text{ regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of power factor of the load.

The voltage regulation is a figure of merit of a transformer. The smaller the voltage regulation, the better is the performance of the transformer. For an ideal transformer, as the primary and secondary impedances supposed to be zero, there are no voltage drops, so regulation remains zero.

Expression for voltage regulation

The simplified equivalent circuit of a transformer with resistance and reactance referred to the secondary winding has been shown in Fig. 4.28. The phasor diagram of the secondary side of the equivalent circuit is drawn in Fig. 4.29(a) for lagging power factor load, which is self-explanatory. In order to find out approximate formula for the voltage regulation of the transformer, the phasor diagram is modified as shown in Fig. 4.29(b), using the following steps:

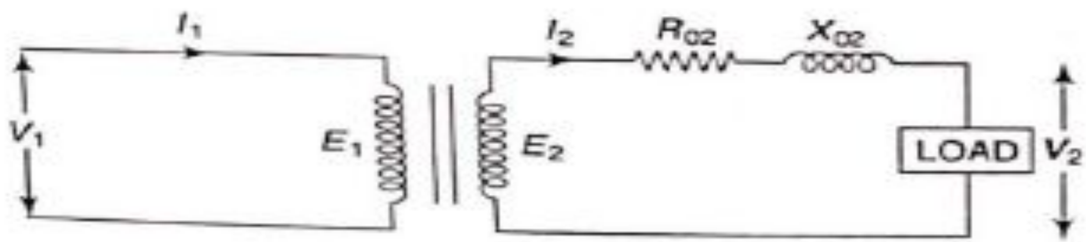


Fig. 4.28 Simplified equivalent circuit

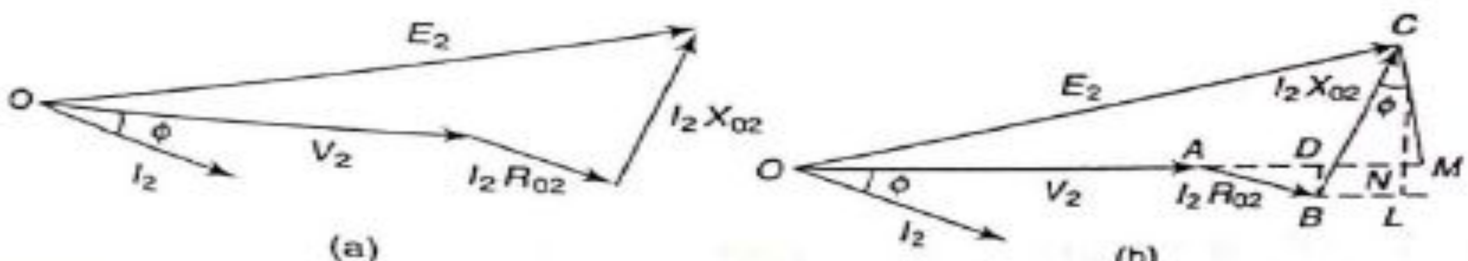


Fig. 4.29 Phasor diagram

1. Draw...

Fig. 4.29 Phasor diagram

1. Draw an arc with centre O and radius OC so as to cut the line OA produced at M.
2. From the point C, drop a perpendicular CN on the line OM.
3. Draw a perpendicular from the point B on the line ON.
4. Draw BL perpendicular to CN produced.

Total voltage drop $= E_2 - V_2 = OC - OA = OM - OA$
$= AM = AN + NM$

approximate voltage drop $\cong AN$ (since NM is very small)
$= AD + DN = AD + BL$
$= I_2R_{02} \cos \phi + I_2X_{02} \sin \phi$

So,

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2} \times 100$$

For leading pf, it can be proved that

$$\text{approximate voltage drop} = I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{E_2} \times 100$$

Hence, in general,

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

'+' sign is used for lagging pf and '–' sign is used for leading pf.

On the primary side, the regulation can be expressed as

$$\% \text{ regulation} = \frac{I_1 R_{01} \cos\phi \pm I_1 X_{01} \sin\phi}{E_1} \times 100$$

We can also express percentage regulation as

$$\% \text{ regulation} = \frac{100\, I_2 R_{02}}{E_2} \cos\phi \pm \frac{100\, I_2 X_{02}}{E_2} \sin\phi$$

$$= v_r \cos\phi \pm v_x \sin\phi$$

where $v_r = \dfrac{100\, I_2 R_{02}}{E_2}$ = percentage resistive drop

$v_x = \dfrac{100\, I_2 X_{02}}{E_2}$ = percentage reactive drop

4.15 Efficiency of a Transformer

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied. The **efficiency of any device** is defined as the ratio of the output power to the input power. So, for a transformer, the efficiency can be expressed as

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{\text{Output power}}{\text{Output power} + \text{Total losses}}$$

$$= \frac{\text{Output power}}{\text{Output power} + W_i + W_{cu}}$$

where W_i = iron loss and W_{cu} = copper loss

Thus, in general, the efficiency at any load is given by

$$\% \eta = \frac{(x \times \text{full-load kVA} \times \text{pf})}{(x \times \text{full-load kVA} \times \text{pf}) + W_i + x^2 [W_{cu}]_{FL}} \times 100$$

where W_i = iron loss in kW

$[W_{cu}]_{FL}$ = copper loss at full load in kW

x = ratio of given load (actual) to full load

The explanation of the above expression of efficiency is given below:

For transformer, output power $= V_2 I_2 \cos\phi$ W

$$= \frac{V_2 I_2}{1000} \cos\phi \text{ kW}$$

But $\cos\phi = \text{pf}$ and $\dfrac{V_2 I_2}{1000} = \text{kVA}$

So, output power = (kVA × pf) kW

The output power at any given load can be calculated as follows:

Output power at the given load = (kVA at given load × pf) kW

Output power at the given load = (x × full-load kVA × pf) kW

Since kVA at the given load = x × full-load kVA

where x is the ratio of given load (actual) to full load

Similarly, copper loss at any given load is given by

$$[W_{cu}]_{\text{at given load}} = x^2 [W_{cu}]_{FL}$$

So,

$$\% \eta = \frac{[x \times \text{full-load kVA} \times \text{pf}]}{[x \times \text{full-load kVA} \times \text{pf}] + W_i + x^2 [W_{cu}]_{FL}} \times 100$$

4.16 Condition for Maximum Efficiency

When a transformer works on a constant input voltage and frequency, efficiency varies with load. As load increases, the efficiency also increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further, the efficiency starts decreasing. The graph of efficiency versus load current I_2 is shown in Fig. 4.30. The load current at which the efficiency attains the maximum value is denoted by I_{2max} and the maximum efficiency is denoted by η_{max}.

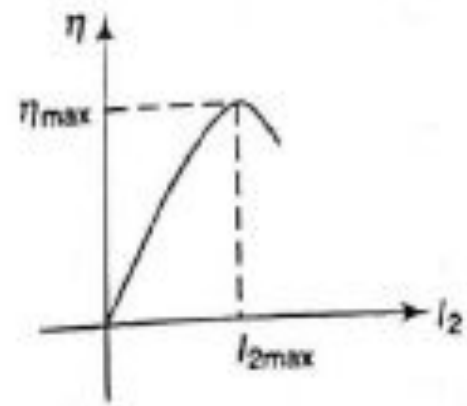


Fig. 4.30 Efficiency vs load current

We know that

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

or

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \quad \because \text{Output power} = \text{Input power} - \text{Losses}$$

Let $\quad W_{cu} = I_1^2 R_{01}$ or $W_{cu} = I_2^2 R_{02}$

Iron (core) loss = W_i

Input power = $V_1 I_1 \cos \phi$

So, $$\eta = \frac{V_1 I_1 \cos \phi - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos \phi} - \frac{W_i}{V_1 I_1 \cos \phi}$$

The current I_1 varies according to the load. Therefore, in the above expression, the efficiency is a function of current I_1 assuming $\cos \phi$ as a constant. The applied voltage V_1 is also assumed as a constant.

For η_{max}, $\dfrac{d\eta}{dI_1} = 0$

So, $$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi} + \frac{W_i}{V_1 I_1^2 \cos \phi} = 0$$

or $$\frac{W_i}{V_1 I_1^2 \cos \phi} = \frac{R_{01}}{V_1 \cos \phi}$$

or $\quad W_i = I_1^2 R_{01}$

or $\quad W_i = I_2^2 R_{02} \quad (\because\ I_1^2 R_{01} = I_2^2 R_{02})$

Thus, the condition to achieve maximum efficiency is[6]

Iron loss = Copper loss

Iron loss = Copper loss

Load current at η_{max}

For η_{max}, $I_2^2 R_{02} = W_i$ but $I_2 = I_{2max}$

So, $$I_{2max}^2 R_{02} = W_i$$

or $$I_{2max} = \sqrt{\frac{W_i}{R_{02}}}$$

This is value of the load current at η_{max}.
Dividing both sides by $(I_2)_{FL}$,

$$\frac{I_{2max}}{(I_2)_{FL}} = \frac{1}{(I_2)_{FL}}\sqrt{\frac{W_i}{R_{02}}}$$

[6]For a transformer, iron loss is constant. So, when the copper loss becomes equal to the iron loss, the maximum efficiency occurs.

or $$\frac{I_{2\max}}{(I_2)_{FL}} = \sqrt{\frac{W_i}{(I_2)_{FL}^2 R_{02}}}$$

or $$I_{2\max} = (I_2)_{FL} \times \sqrt{\frac{W_i}{[W_{cu}]_{FL}}}$$

This is the load current at η_{max} in terms of the full-load current.

kVA at η_{max}

$$\text{kVA at } \eta_{max} = \frac{V_2 I_{2\max}}{1000}$$

Substituting the value of $I_{2\max}$ in the above expression, we get

$$\text{kVA at } \eta_{max} = \frac{V_2 (I_2)_{FL}}{1000} \times \sqrt{\frac{W_i}{[W_{cu}]_{FL}}}$$

$$= (\text{full-load kVA}) \times \sqrt{\frac{W_i}{[W_{cu}]_{FL}}}$$

This is the kVA at η_{max}.

Now, the maximum efficiency η_{max} can be calculated as follows:

$$\% \ \eta_{max} = \frac{\text{Output power at } \eta_{max}}{\text{Output power at } \eta_{max} + \text{Total losses}} \times 100$$

$$= \frac{(\text{kVA at } \eta_{max} \times \text{pf})}{(\text{kVA at } \eta_{max} \times \text{pf}) + 2W_i} \times 100 \qquad [\because W_i = W_{cu}]$$

Load at η

$$(\text{kVA at } \eta_{max} \times \text{pf}) + 2W_i$$

Load at η_{max}

Let at 'x' times full load, the efficiency is maximum.

So, copper loss at $\eta_{max} = x^2 \times [W_{cu}]_{FL}$

But at η_{max}, iron (or core) loss = copper, loss

$$W_i = x^2 \times [W_{cu}]_{FL}$$

or

$$x = \sqrt{\frac{W_i}{[W_{cu}]_{FL}}}$$

We get the expression for η_{max} as

$$\% \, \eta_{max} = \frac{(x \times \text{full-load kVA} \times \text{pf})}{(x \times \text{full-load kVA} \times \text{pf}) + 2W_i} \times 100 \quad [\because \text{at } \eta_{max}, W_i = W_{cu}]$$

where $$x = \sqrt{\frac{W_i}{[W_{cu}]_{FL}}}$$

4.17 All-Day Efficiency

The primary winding of a distribution transformer is connected to the line for all the 24 hours of the day. Thus, the core (iron) losses occur for the whole 24 hours, whereas copper losses occur only when transformer is on load. Distribution transformers operate well below the rated power output for most of the time. These transformers normally operate on a varying load during 24 hours of the day. So, copper losses are different during different periods of the day. The performance of a distribution transformer is more appropriately represented by all-day or energy efficiency. Energy efficiency of a transformer is defined as the ratio of total energy output for a certain period to the total energy input for the same period. The energy efficiency can be calculated for any specific period. When the energy efficiency is calculated for a day of 24 hours, it is called the all-day efficiency. **All-day efficiency** is defined as the ratio of the energy output to the energy input taken over a 24-hour period.

Consequently, all-day efficiency of the transformer is given by

$$\%\ \text{all-day efficiency} = \frac{\text{Output energy in kWh over 24 hours}}{\text{Input energy in kWh over 24 hours}} \times 100$$

or

$$\%\ \eta_{\text{all day}} = \frac{\text{Output energy in kWh over 24 hours}}{\begin{pmatrix}\text{Output energy}\\ \text{in kWh over}\\ \text{24 hours}\end{pmatrix} + \begin{pmatrix}\text{Energy spent}\\ \text{in iron loss}\\ \text{for 24 hours}\end{pmatrix} + \begin{pmatrix}\text{Energy spent}\\ \text{in copper loss}\\ \text{for 24 hours}\end{pmatrix}} \times 100$$

For a given kVA loading, the all-day efficiency of a transformer is less than its commercial efficiency.

Example 4.12 A single-phase, 440/220 V, 10 kVA, 50 Hz transformer has a resistance of 0.2 Ω and a reactance of 0.6 Ω on high-voltage side. The corresponding values of low-voltage side are 0.04 Ω and 0.14 Ω. Calculate the percentage regulation on full load for (i) 0.8 lagging pf, (ii) 0.8 leading pf, and (iii) unity pf.

Solution

The given values are:

$$E_1 = 440\text{ V}, E_2 = 220\text{ V}, R_1 = 0.2\ \Omega, X_1 = 0.6\ \Omega, R_2 = 0.04\ \Omega, X_2 = 0.14\ \Omega$$

Secondary current at full load,

$$(I_2)_{FL} = \frac{10 \times 1000}{220} = 45.45\text{ A}$$

Transformation ratio, $K = \frac{E_2}{E_1} = \frac{220}{440} = 0.5$

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = 0.04 + (0.5)^2 \times 0.2 = 0.09\ \Omega$$

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.14 + (0.5)^2 \times 0.6 = 0.29\ \Omega$$

(i) Percentage regulation at full-load 0.8 pf lagging:

$\cos\phi = 0.8$

So, $\sin\phi = 0.6$

We have

$$\% \text{ regulation} = \frac{I_2 R_{02}\cos\phi + I_2 X_{02}\sin\phi}{E_2}\times 100$$

$$= \frac{45.45\times 0.09\times 0.8 + 45.45\times 0.29\times 0.6}{220}\times 100$$

$$= 5.082$$

(ii) Percentage regulation at full-load 0.8 pf leading:

$\cos\phi = 0.8$

So, $\sin\phi = 0.6$

$$\% \text{ regulation} = \frac{I_2 R_{02}\cos\phi - I_2 X_{02}\sin\phi}{E_2}\times 100$$

$$= \frac{45.45\times 0.09\times 0.8 - 45.45\times 0.29\times 0.6}{220}\times 100$$

$$= -2.107$$

(iii) Percentage regulation at full-load unity pf:

$$\% \text{ regulation} = \frac{I_2 R_{02}\cos\phi \pm I_2 X_{02}\sin\phi}{E_2}\times 100$$

$$= \frac{I_2 R_{02}\cos\phi}{E_2}\times 100 \qquad [\because \sin\phi = 0]$$

$$= \frac{45.45\times 0.09\times 1}{220}\times 100$$

$$= 1.859$$

Example 4.13 A [illegible] transformer has a primary

$$= 1.859$$

Example 4.13 A 250/125 V, 5 kVA single-phase transformer has a primary resistance of 0.2 Ω and a reactance of 0.75 Ω. The secondary resistance is 0.05 Ω and reactance is 0.2 Ω. Determine (i) the regulation while supplying full load on 0.8 leading pf, and (ii) the secondary terminal voltage on full load and 0.8 leading pf.

Solution

The given values are:

$$R_1 = 0.2\ \Omega, X_1 = 0.75\ \Omega, R_2 = 0.05\ \Omega, X_2 = 0.2\ \Omega, \text{pf} = 0.8 \text{ leading}$$

Now, $$K = \frac{E_2}{E_1} = \frac{125}{250} = 0.5$$

$$(I_2)_{FL} = \frac{\text{kVA rating} \times 1000}{E_2} = \frac{5 \times 1000}{125} = 40\ \text{A}$$

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = 0.05 + (0.5)^2 \times 0.2 = 0.1\ \Omega$$

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.2 + (0.5)^2 \times 0.75 = 0.3875\ \Omega$$

(i) Regulation on full-load 0.8 leading pf:

$\cos\phi = 0.8$ leading

So, $\sin\phi = 0.6$

$$\%\ \text{regulation} = \frac{I_2 R_{02}\cos\phi - I_2 X_{02}\sin\phi}{E_2} \times 100$$

$$= \frac{40 \times 0.1 \times 0.8 - 40 \times 0.3875 \times 0.6}{125} \times 100$$

$$= -4.88$$

(ii) The secondary terminal voltage on full-load 0.8 leading pf:

By using basic expression of regulation,

$$\%\ \text{regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

At full-load 0.8 leading pf, regulation is –4.88%.

So, $-4.88 = \dfrac{125 - V_2}{125} \times 100$

or $-6.1 = 125 - V_2$

or $V_2 = 131.3$ V