Thevenin's theorem.

Steps to apply:

① Remove the branch resistance.

② Calulate the voltage across these open circuited terminals.
∴ Find V_{TH}.

③ Replace all sources by their internal resistances to find R_{TH}.

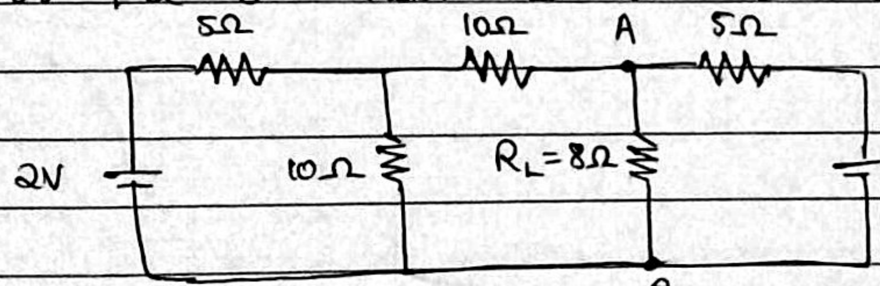
④ Draw Thevenin's equivalent circuit showing source V_{TH} with resistance R_{TH} in series.

⑤ Reconnect the branch resistance.
∴ The required current through the branch is given by

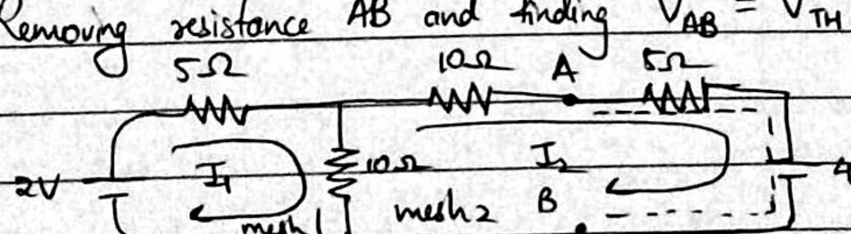
$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

① 1.63 Determine the current through 8Ω resistor in the network.

→ ∴ The 8Ω resistor is the load resistor.



Step 1: Removing resistance AB and finding $V_{AB} = V_{TH}$.



∴ Applying KCL in mesh 1, $-2 + 5I_1 + 10(I_1 - I_2) = 0$.

$\therefore 15I_1 - 10I_2 = 2$ ——— ①

Now, Applying KCL to mesh 2, we got

$$10 I_2 + 5I_2 + 4 + 10(I_2 - I_1) = 0.$$

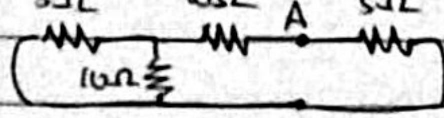
$$10I_1 - 25I_2 = 4 \quad \text{——} ②$$

solving ① and ②, $I_2 = -0.145A.$

Hence $V_{AB} = 4 + 5(I_2) = 4 + 5(-0.145)$

$V_{TH} = 3.275V.$

Now, for R_{TH} (Step 2)



5Ω, 10Ω are in parallel.

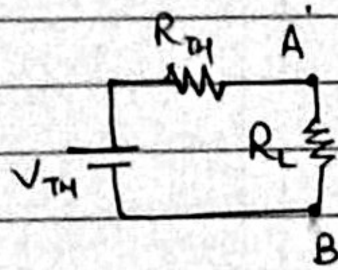
their eq will be 10/3 and it will be in series with 10Ω

and theire eq will be 13.33 Ω which will be then in parallel with 5Ω

$$\therefore R_{eq} = \frac{1}{1/13.33 + 1/5} \qquad \therefore R_{TH} = R_{eq} = 3.636\,\Omega.$$

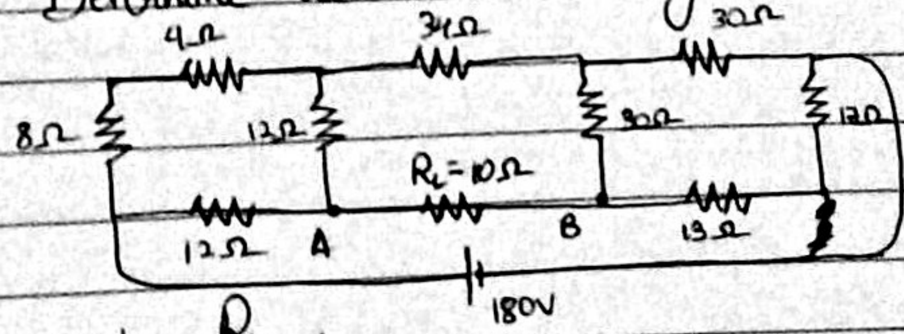
Step 3: Load current.

we know, $$I_L = \frac{V_{TH}}{R_L + R_{TH}} = \frac{3.275}{8 + 3.636}$$



$\therefore I_L = 0.281\ A\ (\downarrow).$

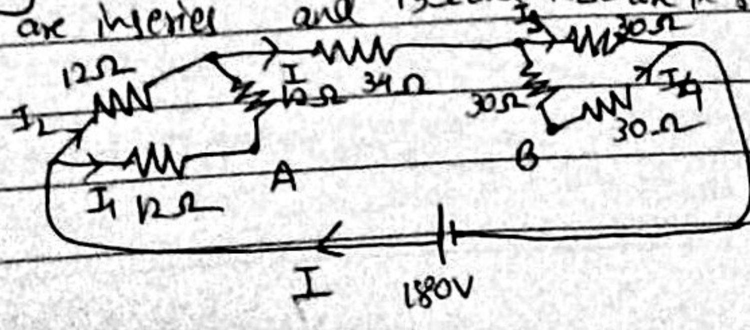
② 1.64 Determine the current through 10Ω.



After removing R_L,

8Ω, 4Ω are in series and 13Ω and 17Ω are in series.

∴



$$R_{eq} = 34 + \frac{1}{\frac{1}{12}+\frac{1}{24}} + \frac{1}{\frac{1}{30}+\frac{1}{60}} = 34 + \frac{\cancel{24}^{8}}{\cancel{3}} + \frac{\cancel{60}^{20}}{\cancel{3}} = 62\,\Omega$$

$$\therefore I_{eq} = \frac{V}{R_{eq}} = \frac{180}{62} \qquad I = I_1 + I_2 \text{ (from diagram)}$$

$$\therefore I_1 = \frac{24}{36} \times I, \quad I_2 = \frac{12\,I}{36}$$

Similarly, $I = I_3 + I_4$

$$I_3 = \frac{60}{90} \times I, \quad I_4 = \frac{30}{90} \times I$$

$$\therefore V_{TH} = V_{AB} = 12 \times I_2 + 34 \times I + 30 \times I_4$$

$$= \cancel{12}^{4} \times \frac{1}{\cancel{3}} I + 34 I + \cancel{30}^{10} \times \frac{1}{\cancel{3}} I$$

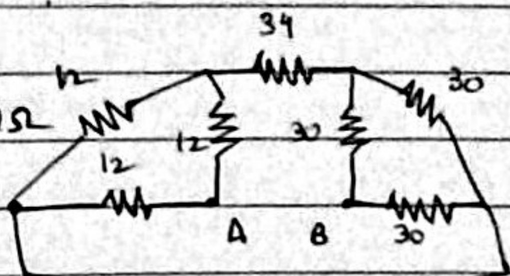
$$= 48 I$$

$$= \frac{180 \times \cancel{48}^{24}}{\cancel{62}\,31}$$

$$\therefore V_{TH} = 139.35\,V$$

Now for R_{TH}.

Converting the 12Ω, 12Ω and 12Ω delta to star.

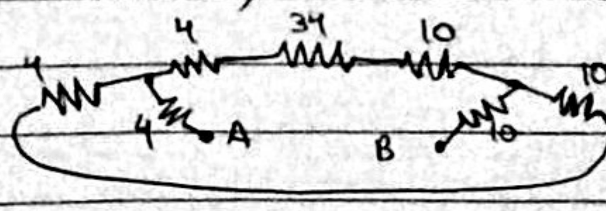


we get

$$R_1 = R_2 = R_3 = \frac{\cancel{12}^{4} \times 12}{\cancel{3 \times 12}} = 4\,\Omega$$

Similarly $R_4 = R_5 = R_6 = \frac{\cancel{30 \times 30}^{10}}{\cancel{3 \times 30}} = 10\,\Omega$ for 30Ω, 30Ω and 30Ω ~~star~~ delta to star.

$\therefore$ New diagram will be



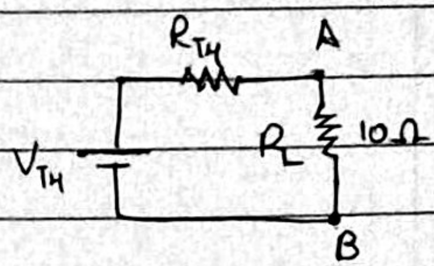
$$\therefore R_{eq} = 4 + 10 + \frac{1}{\frac{1}{14}+\frac{1}{48}}$$

$$R_{eq} = 24.84\,\Omega = R_{TH}$$

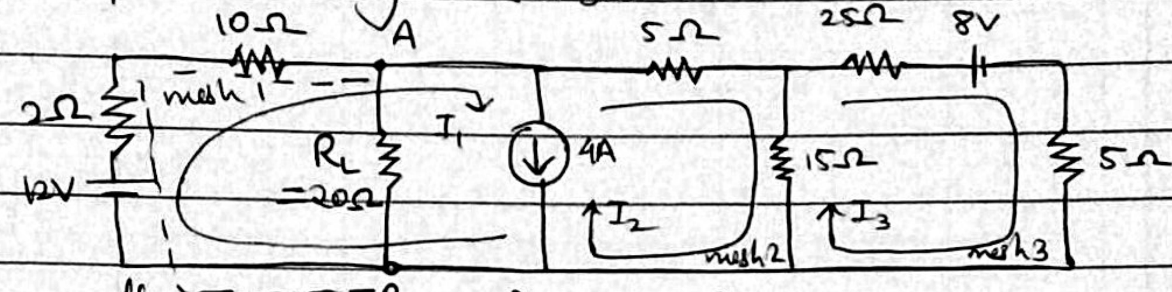
$\therefore$ Now we know,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}} = \frac{139.34}{10 + 24.84}$$

$$I_L = 4A\,(\rightarrow) = I_{10\Omega}.$$



(3) 1.65 Obtain power drawn by 20Ω resistor.



for power through 20Ω, $P_{20\Omega} = I_{20\Omega}^2 R_{20\Omega}$

$$= 20 I_{20\Omega}^2.$$

Step 1: Calculation of V_{TH}.

$$\therefore \quad I_1 - I_2 = 4 \quad \text{—— (1)}$$

Mesh 1 and mesh 2 form a supermesh, Applying KVL

$$12 - 2(I_1) - 10(I_1) - 5(I_2) - 15(I_2 - I_3) = 0.$$

$$\therefore \quad 12I_1 + 20I_2 - 15I_3 = 12 \quad \text{—— (2)}$$

Now, applying KVL to mesh 3,

$$-25(I_3) - 8 - 5(I_3) - 15(I_3 - I_2) = 0.$$

$$\therefore \quad 15I_2 - 45I_3 = 8 \quad \text{—— (3)}$$

$$\therefore \begin{bmatrix} 1 & -1 & 0 \\ 12 & 20 & -15 \\ 0 & 15 & -45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} \quad \text{from (1), (2) and (3).}$$

$$\Delta = 1\,[20 \times -45 - 15 \times (-15)] + 1\,[12 \times -45]$$

$$= -15 \times 45 - 12 \times 45$$

$$\Delta = -27 \times 45 = -1215$$

$$\Delta_1 = \begin{vmatrix} 4 & -1 & 0 \\ 12 & 20 & -15 \\ 8 & 15 & -45 \end{vmatrix} = -3120.$$

$$\therefore \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{-3120}{-1215} = 2.57A.$$

$\therefore V_{TH} = V_{AB}$

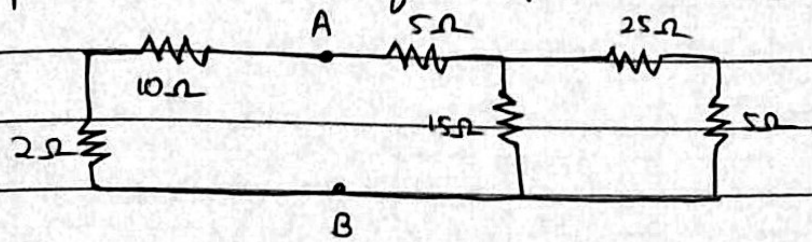
$= -10(I_1) - 2(I_1) + 12$

$= -12 \times 2.57 + 12.$

$V_{TH} = -18.84V.$

Now,

Step II : Calculation of R_{TH}.



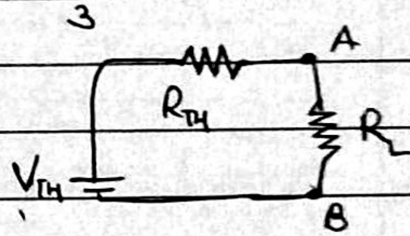
25Ω and 5Ω become 30Ω since they are in series. and then 30Ω and 15Ω act as a parallel connection whose equivalent resistance would be 10Ω.

10Ω and 5Ω will be in series and also 10Ω and 2Ω.

∴ 15Ω and 12Ω will be in parallel as a result.

$\therefore R_{eq} = R_{TH} = \frac{1}{1/15 + 1/12} = \frac{1}{1/3\left(\frac{4+5}{20}\right)} = \frac{20}{3} = 6.67\Omega.$

$\therefore R_{TH} = 6.67\Omega.$



∴ Step 3 : Calculation of load current.

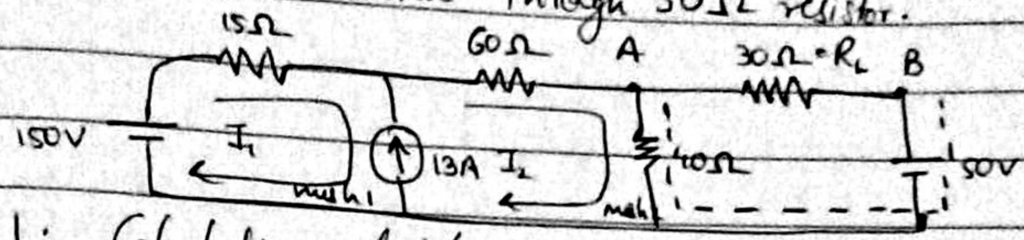
$I_L = \frac{V_{TH}}{R_L + R_{TH}} = \frac{-18.84}{6.67 + 20} = 0.706A(\uparrow).$

$\therefore I_{20\Omega} = 0.706A.$

$\therefore P_{20\Omega} = 20 \times (0.706)^2$

$= 9.97 W.$

4 1.66 Determine current through 30Ω resistor.



Step 1: Calculation of V_{TH}

mesh 1 and 2 form a super mesh.

also $I_1 - I_2 = -13$ —— ①.

$\cdot\ I_1 = I_2 - 13$

Using KVL on supermesh,

$$\overset{30}{\cancel{150}} - \overset{3}{\cancel{15}}(I_1) - \overset{12}{\cancel{60}}(I_2) - \overset{8}{\cancel{40}}(I_2) = 0.$$

$$30 - 20 I_2 - 3(I_2 - 13) = 0.$$

$$69 - 23 I_2 = 0.$$

$$\therefore I_2 = 3A(\rightarrow) \therefore I_1 = -10A = 10A(\leftarrow)$$

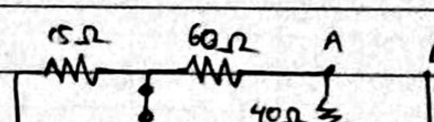
$\therefore V_{TH} = V_{AB}$

$= 50 - 40(I_2)$

$= 50 - 40 \times 3.$

$V_{TH} = -70V.$

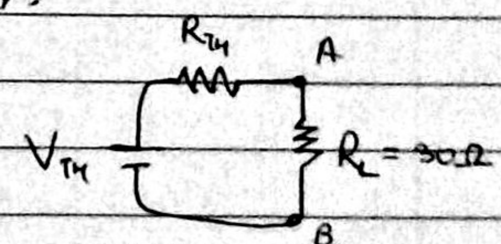
Step 2: Calculation of R_{TH}



15Ω and 60Ω act are in ~~parallel~~ series.

$\therefore$ 75Ω and 40Ω act as in parallel.

$$\therefore R_{eq} = \frac{1}{1/75 + 1/40} = 26.09\,\Omega.$$

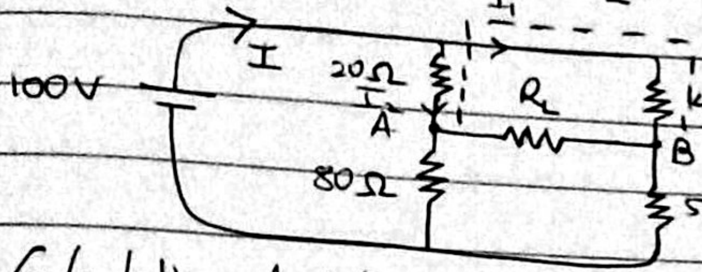


$\therefore R_{TH} = 26.09\,\Omega$

Step 3. $I_L = \dfrac{V_{TH}}{R_L + R_{TH}} = \dfrac{70}{30 + 26.09} = 1.248\,A(\rightarrow)$

$\therefore$ The current through 30Ω resistor is 1.248A ($\rightarrow$)

(5) 1.67 Find the current in R_L when R_L takes values 5Ω, 10Ω and 20Ω.



Calculation of V_{TH}.

After removing R_L.

$$I = I_1 + I_2$$

$$\therefore I_1 = \frac{80+20}{(80+20)+(40+50)} = \frac{100}{190} I.$$

$$\therefore I_2 = \frac{90}{190} \times I.$$

$$R_{eq} = \frac{1}{1/90 + 1/100} = \frac{900}{19} = 47.37\,\Omega.$$

$$\therefore I = V/R = 100/47.37$$

$$I = 2.11\,A$$

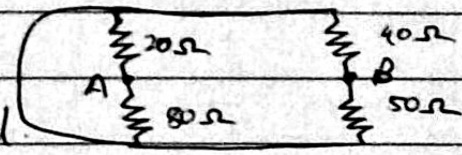
$$\therefore I_2 = 1\,A, \quad I_1 = 1.11\,A.$$

$$\therefore V_{TH} = -20(I_2) + 40(I_1)$$

$$= 20(2I_1 - I_2)$$

$$V_{TH} = 24.44\,V$$

Calculation of R_{TH}.

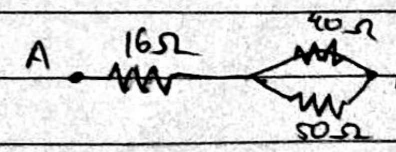


20Ω and 80Ω will be in parallel.

$$\therefore R_1 = \frac{1}{1/20 + 1/80} = 16.$$

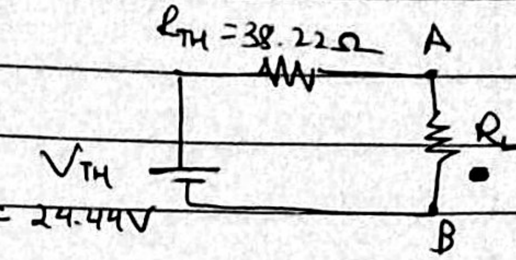
40Ω and 50Ω will also be in parallel.

$$R_2 = \frac{1}{1/40 + 1/50} = 22.22\,\Omega.$$



$$\therefore R_{TH} = R_{eq} = 16 + 22.22 = 38.22\,\Omega.$$

Step 3: Calculation of load current.



$$\therefore I_L = \frac{V_{TH}}{R_L + R_{TH}}.$$

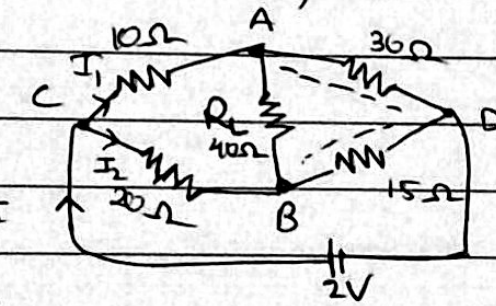
Hence,

when $R_L = 5\Omega$, $I_L = \frac{24.44}{5+38.22} = 0.565A\ (\rightarrow)$

$R_L = 10\Omega$, $I_L = \frac{24.44}{10+38.22} = 0.507A\ (\rightarrow)$

$R_L = 20\Omega$, $I_L = \frac{24.44}{20+38.22} = 0.4198\ (\rightarrow)$

(6) 1.68 By Thevenin's theorem, find the current in 40Ω resistor.



Calculation of V_{TH}.

After removing R_L, $R_{eq} = \frac{1}{\frac{1}{20+15} + \frac{1}{10+30}} = 18.67\Omega$.

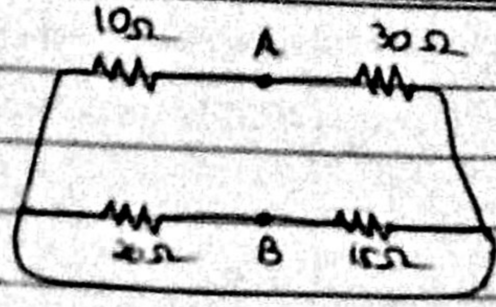
$\therefore I = 0.107A$. $\therefore I_1 = \frac{20+15}{20+15+10+30} \times I = \frac{7}{15} I$

$\therefore I_2 = \frac{10+30}{20+15+10+30} \times I = \frac{8}{15} I$.

$\therefore V_{TH} = V_{AB} = 30(I_1) - 15(I_2) = 15 \times \frac{1}{15} I \left(2\times 7 - 8\right) = 6I.$

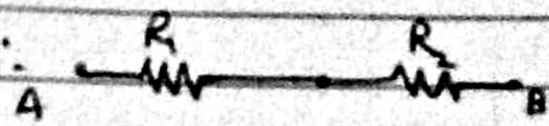
$V_{TH} = 0.642V$

∴ Calculation of R_{TH}.



∴ 10Ω and 30Ω and 20Ω and 15Ω would be in parallel.

∴

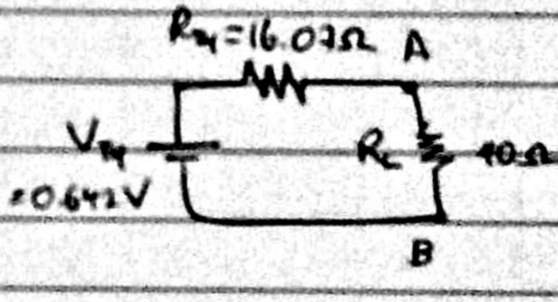


$$\therefore R_1 = \frac{1}{1/10 + 1/30} = 7.5\,\Omega.$$

$$R_2 = \frac{1}{1/20 + 1/15} = 8.57\,\Omega.$$

$$\therefore R_{TH} = R_1 + R_2 = 16.07\,\Omega.$$

∴ Calculation of load current



$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

$$= \frac{0.642}{40 + 16.07}$$

$$I_L = 0.01145\,A$$

$$\therefore I_L = 11.45\,mA\,(\downarrow)$$

* * *