Solved problems on de-broglie wavelength

1] Calculate the λ associated with a proton moving with $v = \frac{1}{20} c$.

A $\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^{8}} = 2.645 \times 10^{-14} m$

2] λ associated with an e^- is $0.1 Å$. Find P.D. by which the e^- is associated.

A $\lambda = \sqrt{\frac{150}{V}} Å$ $\left(\lambda = \frac{h}{\sqrt{2mqV}} \right)$

$\therefore V = \frac{150 \times 10^{-20}}{0.1 \times 0.1 \times 10^{-10} \times 10^{-10}} = 15 kV$

3] Calculate λ associated with an alpha particle accelerated through a P.D. of 0.2kV.

A alpha particle : He_2^4

$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$

$\lambda = 7.16 \times 10^{-13} m$

4] Calculate λ associated with an avg. Helium atom in furnance of 400K. Given $K_B = 1.38 \times 10^{-23} J/K$

A $\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 4 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 400}} = 6.299 \times 10^{-11} m$

5] An e^- & a photon have λ of 2Å. What are their momentum and Energy

A $p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{2 \times 10^{-10}}$

$p_{e^-} = p_{photon} = 3.313 \times 10^{-24}$ kg m/s

For e^-,

$\lambda = \frac{h}{\sqrt{2mE}}$, $E = \frac{h^2}{\lambda^2 2m} = \frac{(6.626 \times 10^{-34})^2}{(2 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}}$

$E_e = 6.03 \times 10^{-18}$ J

For photon,

$E = h\nu = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-10}}$

$E = 9.939 \times 10^{-16}$ J

6] Find λ of an e^- in 1^{st} Bohr orbit of H-atom

A $E_n = \frac{-13.6}{n^2}$ eV

1^{st} orbit,

$E = -13.6$ eV $\qquad |E| = 13.6 \times 1.6 \times 10^{-19}$ J

$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 13.6 \times 1.6 \times 10^{-19}}}$

$\lambda = 3.329 \times 10^{-10}$ m

7] Find out the ratio of λ of α-particles & deutron moving in same potential. Find out the ratio of their KE

A $$\lambda = \frac{h \rightarrow \text{const.}}{\sqrt{2mqV}}$$

$V \rightarrow$ same

$2 \rightarrow$ const.

$$\therefore \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\therefore \frac{\lambda_\alpha}{\lambda_d} = \sqrt{\frac{m_d q_d}{m_\alpha q_\alpha}} = \sqrt{\frac{2m_p \times 1e}{4m_p \times 2e}} = \frac{1}{2}$$

$$\therefore \lambda_\alpha : \lambda_d = 1:2$$

$$E = qV \rightarrow \text{same}$$

$$\therefore E \propto q$$

$$\therefore \frac{E_\alpha}{E_d} = \frac{q_\alpha}{q_d} = \frac{2e}{1e} = \frac{2}{1}$$

$$\therefore E_\alpha : E_d = 2:1$$

8] Find out the ratio of λ of H-atom & He-atom at room temperature when they move with terminal velocity. ($T_R = 27^\circ C = 300 K$)

A $$\lambda = \frac{h \rightarrow \text{const.}}{\sqrt{3mkT}}$$

$3 \rightarrow$ const.

$kT \rightarrow$ const.

$$\therefore \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}}{m_H}} = \sqrt{\frac{4m_p}{1m_p}} = \frac{2}{1}$$

$$\therefore \lambda_H : \lambda_{He} = 2:1$$

9] Find out the ratio of λ of neutrons with KE of 1eV & 510eV.

A $\lambda = \frac{h \rightarrow \text{const.}}{\sqrt{2mE}}$

2 → const. m → same

$\therefore \lambda \propto \frac{1}{\sqrt{E}}$

$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{510}{1}} = \frac{22.58}{1}$

$\therefore \lambda_1 : \lambda_2 = 22.58 : 1$

10] A proton & a deutron have same KE. Which has longer λ?

A $\lambda = \frac{h \rightarrow \text{const.}}{\sqrt{2mE}}$

2 → const. E → same

$\therefore \lambda \propto \frac{1}{\sqrt{m}}$

$m_p < m_d$

$\therefore \lambda_p > \lambda_d$

Solved problems on Heisenberg Uncertainty principle

1] An $\bar{e}$ is confined to a potential of width 10nm. Calculate min uncertainity in velocity.

A
$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$\therefore \Delta x \, m \Delta v \geq \frac{\hbar}{2}$$

$$\therefore \Delta v \geq \frac{\hbar}{2m\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times 10 \times 10^{-9} \times 9.1 \times 10^{-31}}$$

$$\therefore \Delta v \geq 5791 \, m/s$$

2] An $\bar{e}$ and a 150g baseball are travelling at a velocity of 220 m/s measured to an accuracy of 0.065%. Calculate uncertainty in position of each.

A
$$v = 220 \, m/s$$

$$\frac{\Delta v}{v} = 0.065\%$$

$$\Delta v = \frac{0.065}{100} \times 220 = 0.143 \, m/s$$

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2m\Delta v}$$

For $\bar{e}$,
$$\Delta x \geq \frac{1.054 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 0.143} = 4.05 \times 10^{-4} \, m$$

For baseball,
$$\Delta x = \frac{1.054 \times 10^{-34}}{2 \times 0.15 \times 0.143} = 2.457 \times 10^{-33} \, m$$

3] The speedometer of a 180 kg vehicle reads 72 km/h at a particular instant. The readings are known to have an accuracy of ± 2 km/hr. What would be the uncertainty in position of vehicle at that instant according to Heisenberg uncertainty principle.

A
$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2m\Delta V}$$

$$\Delta x \geq \frac{1.054 \times 10^{-34}}{2 \times 180 \times 2 \times \frac{5}{18}}$$

$$\Delta x \geq 0.527 \times 10^{-36} \, m$$

4] An e^- has a speed of 300 m/s with uncertainty 0.01%. Find accuracy in its position.

A
$$V = 300 \, m/s$$

$$\frac{\Delta V}{V} = 0.01\%$$

$$\therefore \Delta V = \frac{0.01}{100} \times 300 = 0.03 \, m/s$$

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2m\Delta V}$$

$$\Delta x \geq \frac{1.054 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 0.03}$$

$$\Delta x \geq 1.93 \times 10^{-3} \, m$$

5] The position and momentum of a 1 KeV e are simultaneously measured. If the position is located within 1Å. What is the percentage uncertainity in momentum

A $E = 1 KeV = 10^3 \times 1.6 \times 10^{-19} J = 1.6 \times 10^{-16} J$

$\Delta x = 1Å = 1 \times 10^{-10} m$

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2 \Delta x}$$

$$\Delta p \geq \frac{1.054 \times 10^{-34}}{2 \times 10^{-10}}$$

$$\Delta p \geq 0.527 \times 10^{-24} \ kg\,m/s$$

$$p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}} = 1.706 \times 10^{-23} \ kg\,m/s$$

% uncertainity in momentum $= \frac{\Delta p}{p} \times 100$

$$= \frac{0.527 \times 10^{-24}}{1.706 \times 10^{-23}} \times 100$$

$$= 3.09 \%$$

$$= 3.1 \%$$

6] The uncertainity in the location of a particle is equal to de-Broglie wavelength. Calculate the uncertainity in its velocity.

A $\Delta x = \lambda$

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta v \geq \frac{\hbar}{2m\Delta x}$$

$$\Delta v \geq \frac{h}{4\pi m \lambda}$$

$$\Delta v \geq \frac{h}{4\pi m \frac{h}{p}}$$

$$\Delta v \geq \frac{mv}{4\pi m}$$

$$\therefore \Delta v \geq \frac{v}{4\pi}$$

7] λ can be determined with accuracies of one part in 10^6. What is the uncertainty in the position of 1Å X-ray photon when its λ is simultaneously measured.

A

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6}$$

$$p = \frac{h}{\lambda}$$

$$\therefore \Delta p = \frac{-h}{\lambda^2}\Delta\lambda$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \left(\frac{h}{\lambda^2}\Delta\lambda\right) \geq \frac{h}{4\pi}$$

$$\Delta x \frac{\Delta\lambda}{\lambda} \geq \frac{\lambda}{4\pi}$$

$$\Delta x \cdot \frac{1}{10^6} \geq \frac{10^{-10}}{4\pi}$$

$$\Delta x \geq 7.96 \times 10^{-6} m$$

8] If an excited state of H-atom has a lifetime of 2.5×10^{-14} s. What is the min. error with which the energy of the state can be measured.

A

$$\Delta E . \Delta t \geq \frac{\hbar}{2}$$

$$\Delta E \geq \frac{\hbar}{2\Delta t}$$

$$\Delta E \geq \frac{1.054 \times 10^{-34}}{2 \times 2.5 \times 10^{-14}}$$

$$\Delta E \geq 2.11 \times 10^{-21}\ J$$

9] An excited atom has an avg. life time of 10^{-8}s. During this time period it emits a photon & returns to ground state. What is the min uncertainty in ν of this photon?

A

$$\Delta E\ \Delta t \geq \frac{\hbar}{2}$$

$$h \Delta \nu\ \Delta t \geq \frac{h}{4\pi}$$

$$\Delta \nu \geq \frac{1}{4\pi \Delta t} = \frac{1}{4\pi \times 10^{-8}}$$

$$\Delta \nu \geq 7.96 \times 10^{6}\ s^{-1}$$

10] Compare the uncertainties in velocity of a proton & an e^- contained in a 20Å box.

A

$$\Delta x \cdot \Delta p \geq \hbar/2$$

$$\Delta V \geq \frac{\hbar \rightarrow \text{const.}}{\text{const} \leftarrow 2 m \Delta x \rightarrow \text{same}} \quad \therefore \Delta V \propto \frac{1}{m}$$

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_e}{m_p} = \frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} = 5.449 \times 10^{-4}$$

11] The speed of an e^- is measured to be 5×10^3 m/s to an accuracy of 0.003%. Find the uncertainty in position of e^-

A

$$\frac{\Delta V}{V} = 0.003\%$$

$$\Delta V = \frac{0.003}{100} \times 5 \times 10^3 = 0.15 \text{ m/s}$$

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar}{2m\Delta V} = \frac{1.054 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 0.15}$$

$$\Delta x \geq 3.861 \times 10^{-4} \text{ m}$$

12] An e^- has a momentum 5.4×10^{-26} kgm/s with accuracy of 0.05%. Find ~~max~~ min uncertainty in location of e^-.

A

$$\frac{\Delta p}{p} = 0.05\%$$

$$\Delta p = \frac{0.05}{100} \times 5.4 \times 10^{-26} = 0.27 \times 10^{-28} \text{ kg m/s}$$

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar}{2\Delta p} = \frac{1.054 \times 10^{-34}}{2 \times 0.27 \times 10^{-28}}$$

$$\Delta x \geq 1.952 \times 10^{-6} \text{ m}$$

13] A H-atom has a diameter of 0.53 Å. Estimate the min energy an e^- can have in this atom.

A

$$(\Delta x)_{max} = 0.53 \text{ Å}$$

$$p_{min} = \Delta p_{min} \geq \frac{\hbar}{2\Delta x_{max}} = \frac{1.054 \times 10^{-34}}{2 \times 0.53 \times 10^{-10}}$$

$$p_{min} = 9.94 \times 10^{-25} \text{ kgm/s}$$

$$E_{min} = \frac{p_{min}^2}{2m}$$

$$E_{min} = \frac{(9.94 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}$$

$$E_{min} = 5.43 \times 10^{-19} \text{ J}$$

Solved problems on energy of a particle trapped in 1D potential well.

1. Compute the lowest energy of a neutron confined to a the nucleus which is considered as a box with size of 10^{-14} m.

A
$$E_1 = \frac{n^2h^2}{8mL^2} = \frac{1 \times (6.626 \times 10^{-34})^2}{8 \times 1.67 \times 10^{-27} \times (10^{-14})^2}$$
$$E_1 = 3.286 \times 10^{-13}\ J$$

2. The min energy possible for a particle trapped in 1D box is 3.2×10^{-18} J. What are the next 3 energies in eV the particle can have?

A
$$E_1 = \frac{3.2 \times 10^{-18}}{1.6 \times 10^{-19}}\ eV = 20\ eV$$
$$E_n = n^2 E_1$$
$$\therefore E_2 = 2^2 E_1 = 80\ eV\ ;\ E_3 = 3^2 E_1 = 180\ eV\ ;\ E_4 = 4^2 E_1 = 320\ eV$$

3. Calculate the energy required for an e^- to jump from ground state to 2nd excited state in a potential well of width L.

A
$$E_n = \frac{n^2 h^2}{8mL^2}$$
$$E_3 - E_1 = \frac{h^2(3^2 - 1^2)}{8mL^2} = \frac{8h^2}{8mL^2} = \frac{h^2}{mL^2}$$

4. The lowest energy of an e^- trapped in a potential well is 38 eV. Calculate the width of the well.

A
$$E_n = \frac{n^2h^2}{8mL^2}$$
$$L = \sqrt{\frac{n^2 h^2}{8mE}} = \frac{h}{\sqrt{8mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}} = 9.959 \times 10^{-11}\ m$$
$$L = 0.996\ Å$$

5] An e^- is constrained to move in a 1D box of length 0.1nm. Find the first 3 energy eigen values of the corresponding de-Broglie wavelength Calculate the energy difference b/w the ground state & 1st excited state.

A $$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2 \times (6.626\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-10})^2} = n^2\times6.03\times10^{-18}\,J$$

$$E_1 = 6.063\times10^{-18}J \quad ; \quad E_2 = 2^2E_1 = 24.12\times10^{-18}J$$

$$E_3 = 3^2E_1 = 54.27\times10^{-18}J$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\frac{n^2h^2}{8mL^2}}} = \frac{2L}{n}$$

$$\lambda_1 = 2L = 2\text{Å}$$

$$\lambda_2 = L = 1\text{Å} \quad ; \quad \lambda_3 = \frac{2L}{3} = 0.667\text{Å}$$

$$E_2 - E_1 = (24.12 - 6.03)\times10^{-18} = 18.09\times10^{-18}J$$

6] State the value of momentum & energy of particle in 1D box which has impenetrable walls. Find their values for an e^- in a box of length 1Å for n=1 & n=2 energy states.

A $$p = \sqrt{2mE} = \sqrt{2m\frac{n^2h^2}{8mL^2}} = \frac{nh}{2L}$$

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(6.626\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-10})^2} = n^2(6.03\times10^{-18})J$$

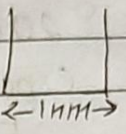
$$E_1 = 6.03\times10^{-18}J \quad ; \quad E_2 = 24.12\times10^{-18}J$$

$$p = \frac{n\times6.626\times10^{-34}}{2\times10^{-10}} = n\times3.313\times10^{-24}\,kgm/s$$

$$p_1 = 3.313\times10^{-24}\,kgm/s \quad ; \quad p_2 = 6.626\times10^{-24}\,kgm/s$$

7] An e^- is confined to move b/w 2 rigid walls seprated by 1nm. Find the de-Broglie wavelength representing the ~~1st~~ ~~two~~ first 2 allowed energy states.

A $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\frac{n^2h^2}{8mL^2}}} = \frac{2L}{n}$



$\lambda = \frac{2L}{n}$

$\lambda_1 = 2nm \quad ; \quad \lambda_2 = \cancel{2}\,1nm$

8] The energy of an e^- constrained to move in a 1D box of length 4A° is 9.664×10^{-17} J. Find out the order of excited state and momentum of e^- in that state.

A $P_n = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 9.664 \times 10^{-17}}$

$P_n = 1.326 \times 10^{-23}$ kgm/s

$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (4 \times 10^{-10})^2} = n^2 \times 3.77 \times 10^{-19}$

$n^2 = \frac{E_n}{3.77 \times 10^{-19}} = \frac{9.664 \times 10^{-17}}{3.77 \times 10^{-19}} = 256.34$

$\therefore n = 16$

9] An e^- is confined to an infinite potential well of width 5A°. Calculate the energy & λ of an emitted photon if the e^- makes a transition from 1st excited state to ground state.

A Energy emitted by photon $= E_2 - E_1$

$n=2$

$n=1$ $\rightsquigarrow E = ?, \lambda = ?$

$E_n = \frac{n^2h^2}{8mL^2}$

$E_2 - E_1 = \frac{(2^2-1^2)(6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (5 \times 10^{-10})^2} = 7.237 \times 10^{-19}$ J

$E = h\nu = \frac{hc}{\lambda}$

$\therefore \lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{7.237 \times 10^{-19}} = 2.747 \times 10^{-17}$ m

10] Evaluate the ~~1st~~ first 3 energy levels of an e^- enclosed in a box of width 10Å. Compare it with those of glass marble of mass 1g contained in a box of width 20 cm. Can we measure these levels of marble experimentally?

A
$$E_n = \frac{n^2h^2}{8mL^2}$$

For e^-,
$$E_n = \frac{n^2 (6.626\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-9})^2} = n^2\times6.03\times10^{-20}\,J$$

$E_1 = 6.03\times10^{-20}J$; $E_2 = 24.12\times10^{-20}J$; $E_3 = 54.27\times10^{-20}J$

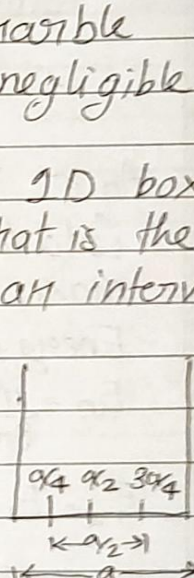
For glass marble
$$E_n = \frac{n^2(6.626\times10^{-34})^2}{8\times10^{-3}\times(0.2)^2} = n^2\times1.372\times10^{-63}\,J$$

$E_1 = 1.372\times10^{-63}J$; $E_2 = 5.488\times10^{-63}J$; $E_3 = 12.348\times10^{-63}J$

We cannot measure energy levels of marble experimentally since their magnitude is negligible (10^{-63})

11] A quantum ~~physics~~ particle is confined to an 1D box of width 'a' in its first excited state what is the probability of finding the particle over an interval of a/2 at the centre of box.

A
$\cancel{P = \int |\psi|^2 dx}$ $\qquad \psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$



$$P = \int_{a/4}^{3a/4} |\psi|^2 dx = \int_{a/4}^{3a/4}\left(\frac{2}{a}\sin^2\frac{2\pi x}{a}\right)dx$$

$$= \frac{2}{a}\int_{a/4}^{3a/4}\left(1-\cos\frac{4\pi x}{a}\right)dx = \frac{1}{a}\left[x - 8\,\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\pi/a}\right]_{a/4}^{3a/4}$$

$$= \frac{1}{a}\left[\left(\frac{3a}{4}-\frac{a}{4}\right) - (0-0)\right] = \frac{1}{2} = 0.5$$

$P = 0.5 = 50\%$.

Problems on de-Broglie Wavelength

1] Compute the de-Broglie wavelength of 10 KeV neutron

A $\lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{10 \times 10^{3} \times 1.6 \times 10^{-19}} = 1.2424 \times 10^{-10} m = 1.2424 A^{\circ}$

2] Find the energy of neutron in units of e volt whose de-Broglie wavelength is 1 A°

A $E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1 \times 10^{-10} \times 1.6 \times 10^{-19}} = 12423.7793 = 12.423 kJ$

3] Calculate de-Broglie wavelength of an e^- which has been accelerated from rest through a P.D. of 100V.

A $\lambda = \sqrt{\frac{150}{100}}\ A^{\circ} = 1.2247 A^{\circ}$

4] Calculate the ratio of de-Broglie waves associated with a proton & e^- each having KE as 20 MeV.

A $\lambda = \frac{\overset{\rightarrow \text{const.}}{h}}{\sqrt{\underset{\text{const}}{2m}\,\underset{\rightarrow \text{same}}{E}}}$

$\therefore \quad \lambda \propto \frac{1}{\sqrt{m}}$

$\therefore \quad \frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}}} = 0.0233$

$\therefore \quad \lambda_p : \lambda_e = 0.0233 : 1$

5] Calculate the KE of a proton & an e^- so that the de-Broglie wavelengths associated with ~~to~~ them is the same and equal to 5000 A°.

A

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{2m\lambda^2}{h^2} = \frac{2m(5000\times10^{-10})^2}{(6.626\times10^{-34})^2} = m\times 1.1388\times10^{54}$$

$$E_p = m_p \times 1.1388\times10^{54} = 1.9048\times10^{27}J$$

$$E_e = m_e \times 1.1388\times10^{54} = \cancel{1.1388\times10^{3}}\ 1.0374\times10^{24}J$$

6] Each of a photon & an e^- has an energy of 1keV. Calculate their corresponding wavelength.

A

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For photon,

$$\lambda = \frac{h}{\sqrt{2m_p\times10^3\times\frac{1}{1.6\times10^{-19}}}} = 1.4491\times10^{-31}m$$

For e^-,

$$\lambda = \frac{h}{\sqrt{2m_e\times10^3\times\frac{1}{1.6\times10^{-19}}}} = 6.2095\times10^{-30}m$$

7] Find the energy of neutron having de-Broglie wavelength 10^{-14}m. Give answer in eV.

A

$$\lambda = \frac{h}{\sqrt{2m_nE}}$$

$$E = \frac{2m_n\lambda^2}{h^2} = \frac{2\times m_n\times(10^{-14})^2}{1.6\times10^{-19}(6.626\times10^{-34})^2} = 4.7686\times10^{30}eV$$

Unsolved Numericals

1] An e^- of mass 9.1×10^{-31} kg has a speed of 1 m/s with an accuracy of 0.05%. Calculate the uncertainty with which the position of the e^- can be located.

A
$$\frac{\Delta v}{v} = 0.05\%$$

$$\Delta v = \frac{0.05}{100}\times 1 = 5\times10^{-4}\ m/s$$

$$\Delta x\,\Delta p \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar}{2m\Delta v} = \frac{1.054\times10^{-34}}{2\times9.1\times10^{-31}\times5\times10^{-4}}$$

$$\Delta x \geq 1.15\times10^{-4}\ m$$

2] The e^- in H-atom may be confined to a nucleus of radius 5×10^{-11} m. Find out the min uncertainty in the momentum of the e^- & also find out the min KE of the e^-.

A
$$\Delta x_{max}\,\Delta p_{min} \geq \hbar/2$$

$$\Delta p_{min} \geq \frac{\hbar}{2\Delta x} = \frac{1.054\times10^{-34}}{2\times2\times5\times10^{-11}}$$

$$\Delta p_{min} \geq 5.2728\times10^{-25}\ kgm/s$$

$$E_{min} = \frac{p^2_{min}}{2m} = \frac{(5.2728\times10^{-25})^2}{2\times9.1\times10^{-31}} = \cancel{\frac{28.9418.8963}{1.6\times10^{-19}}}\ \frac{1.526\times10^{-19}}{1.6\times10^{-19}}$$

$$E_{min} = \cancel{1.5088\times10^{-24}\ eV}\ 0.95375$$

3] The speed of a bullet of mass 50g is measured to be 300 m/s with an uncertainty of 0.01%. With what accuracy can we locate the position of the bullet if it is measured simultaneously with its speed.

A
$$\frac{\Delta v}{v} = 0.01\%$$

$$\Delta v = \frac{0.01}{100}\times300 = 0.03\ m/s$$

$$\Delta x \cdot \Delta p \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar}{2m\Delta V} = \frac{1.054\times10^{-34}}{2\times50\times10^{-3}\times0.03}$$

$$\Delta x \geq 3.5152\times10^{-32}\ m$$

4] Life time of a nucleus in the excited state is 10^{-12} s. Calculate the probable uncertainty in energy & frequency of a γ-ray photon emitted by it.

A
$$\Delta E\,\Delta t \geq \hbar/2$$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054\times10^{-34}}{2\times10^{-12}} = 5.2728\times10^{-23}\ J$$

$$h\Delta\nu\,\Delta t \geq \frac{h}{4\pi}$$

$$\Delta\nu \geq \frac{1}{4\pi\Delta t} = \frac{1}{4\pi\times10^{-12}}$$

$$\Delta\nu \geq 7.9577\times10^{10}\ Hz$$

5] Compute the energy difference b/w the ground state & 1st excited state for an e^- in 1D rigid box of length 10^{-8} cm.

A
$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_2 - E_1 = \frac{(4-1)(6.626\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-10})^2} = 1.80739\times10^{-17}\ J$$

$$E_2 - E_1 = \frac{1.80739\times10^{-17}}{1.6\times10^{-19}} = 112.9619\ eV$$

6] Calculate the value of lowest energy of an e^- in 1D force free region of length 4Å.

A.
$$E_1 = \frac{n^2h^2}{8mL^2} = \frac{(1)^2(6.626\times10^{-34})^2}{8\times9.1\times10^{-34}\times(4\times10^{-10})^2} = \frac{3.7654\times10^{-19}}{1.6\times10^{-19}} = 2.35\ eV$$

7] The lowest energy possible for a certain particle entrapped in a box is 40eV. What are next 3 higher ~~energy~~ energies the particle can have?

A $E_n = n^2 E_1$

$E_2 = 2^2(40) = 160\,eV$; $E_3 = 3^2(40) = 360\,eV$

$E_4 = 4^2(40) = 640\,eV$

8] Find the energy levels of an e^- in a box 1mm wide. Mass of e^- is 9.1×10^{-31} kg. Also find the energy levels of 10g marble in a box 10 cm wide.

A For e^-,

$$E = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} = n^2 6.0246 \times 10^{-20}\,J$$

$E_1 = 6.0246 \times 10^{-20}\,J$; $E_2 = 24.0984 \times 10^{-20}\,J$; $E_3 = 54.2214 \times 10^{-20}\,J$

For glass marble,

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (6.626 \times 10^{-34})^2}{8 \times (10^{-2}) \times (0.1)^2} = 5.488 \times 10^{-64}\,J$$

$E_1 = 5.488 \times 10^{-64}\,J$; $E_2 = 21.952 \times 10^{-64}\,J$; $E_3 = 49.392 \times 10^{-64}\,J$

Numerical Problems

1] Find the energy of a neutron (in eV) whose de-Broglie wavelength is 1Å.

A
$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{2m\lambda^2}{h^2} = \frac{2 \times m_n \times (10^{-10})^2}{(6.626 \times 10^{-34})^2} = 7.6298 \times 10^{19}\,J$$

2] Estimate the amount of accelerating voltage to which e^- are to be subjected in order to associate them with wavelength of 0.50Å

A ~~Both /e~~, ~~$\lambda =$~~

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$V = \frac{2mq\lambda^2}{h^2} = \frac{2 \times 9.1 \times 10^{-31} \times 1.6022 \times 10^{-19} \times (0.5 \times 10^{-10})^2}{(6.626 \times 10^{-34})^2}$$

$$V = 1.6621 \times 10^{-3}\ V$$

3] A nucleon is confined to a nucleus of diameter 5×10^{-14} m. Calculate the min uncertainty in the momentum of the nucleon & its min KE.

A

$$\Delta x_{max} \Delta p_{min} \geq \hbar/2$$

$$\Delta P_{min} \geq \frac{\hbar}{2\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times 5 \times 10^{-14}}$$

$$\Delta P_{min} \geq 1.0546 \times 10^{-21}\ kgm/s$$

$$E_{min} = \frac{P_{min}^2}{2m} = \frac{(1.0546 \times 10^{-21})^2}{2 \times 1.67 \times 10^{-27}} = 3.32 \times 10^{-16} J$$

4] A H-atom is 0.53 Å in radius. Estimate the min energy that an e^- can have in this atom.

A

$$\Delta x_{max} \Delta P_{min} \geq \hbar/2$$

$$\Delta P_{min} \geq \frac{\hbar}{2\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times 2 \times 0.53 \times 10^{-10}}$$

$$\Delta P_{min} \geq 5.2458 \times 10^{-59}\ kg\,m/s$$

$$E_{min} = \frac{p_{min}^2}{2m} = \frac{(5.2458 \times 10^{-59})^2}{1.6 \times 10^{-19} \times 2 \times 9.1 \times 10^{-31}} = 13.6 eV$$

5] Compute the energy of the 3 lowest states of an e^- in a square well of width 6Å.

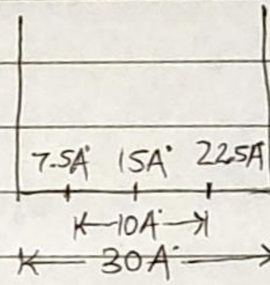
A
$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (6.626\times10^{-34})^2}{8\times(9.1\times10^{-31})(6\times10^{-10})^2} = n^2(1.6735\times10^{-19})\,J$$

$$E_n = n^2(1.0445)\,eV$$

$E_1 = 1.0445\,eV$; $E_2 = 4.178\,eV$; $E_3 = 9.4005\,eV$

6] A particle is moving in a 1D potential box of width 30Å. Calculate the probablity of locating the particle within an interval of 10Å at the centre of the box in its lowest energy state.

A
$$P = \int_{7.5}^{22.5} |\Psi|^2\,dx = \int_{7.5}^{22.5}\left(\frac{2}{30}\sin^2\frac{2\pi x}{30}\right)dx$$



$$= \frac{1}{15\times2}\int_{7.5}^{22.5}\left(1-\cos\frac{4\pi x}{30}\right)dx$$

$$= \frac{1}{30}\left[x - \frac{\sin(4\pi x/30)}{4\pi/30}\right]_{7.5}^{22.5}$$

$$= \frac{1}{30}\left[(22.5-7.5) - \left(\frac{\sin\left(\frac{4\pi\,22.5}{30}\right)}{4\pi/30} - \frac{\sin\left(\frac{4\pi\,7.5}{30}\right)}{\frac{4\pi}{30}}\right)\right]$$

$$= \cancel{\frac{2}{15}\left[15+0\right]}\ \frac{1}{30}\left[15-0\right]$$

$$= 0.5$$