DIVISION / ROLL NO: D1AD/47

Vivekanand's Education Society's Institute of Technology
(Academic year 2020-21)

Subject - Engineering Mathematics 2

Semester II

Tutorial Cover Page.

TUTORIAL NO: 3.

TUTORIAL TOPIC: Beta and Gamma Functions.

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Signature of the Teacher:

1) $\int_0^\infty y^4 e^{-y^6}\, dy.$

Let $y^6 = t$, $y = t^{1/6}$, $\therefore dy = \frac{1}{6} t^{-5/6}\, dt.$

when $y=0$, $t=0$ and $y=\infty$, $t=\infty$.

$\therefore I = \int_0^\infty (t^{1/6})^4 e^{-t} \times \frac{1}{6} t^{-5/6}\, dt.$

$= \frac{1}{6} \int_0^\infty t^{-1/6} . e^{-t}\, dt$

$= \frac{1}{6} \Gamma\left(\frac{-1}{6}+1\right) = \frac{1}{6} \Gamma\frac{5}{6}$

$\therefore \int_0^\infty y^4 e^{-y^6} = \frac{1}{6} \Gamma\frac{5}{6}.$

2) $\int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy = \Gamma n$ (To prove) $(n>0)$

→ Let $\log \frac{1}{y} = x$, $\frac{1}{y} = e^x$ $\therefore y = e^{-x}$ $dy = e^{-x} dx$

$$\int_0^1 (\log 1/y)^{n-1} \cdot dy = \int_\infty^0 (x)^{n-1} (-e^{-x})\, dx$$

$$= + \int_0^\infty x^{n-1} e^{-x} \cdot dx$$

$$= \Gamma n$$

3) Show that $\int_0^{\pi/2} \sqrt{\cot\theta} \cdot d\theta = \frac{\pi}{\sqrt{2}}$

→ $L.H.S = \int_0^{\pi/2} \cos^{1/2}\theta \sin^{-1/2}\theta \cdot d\theta = \frac{1}{2} \beta\left(\frac{1/2+1}{2}, \frac{-1/2+1}{2}\right)$

$$= \frac{1}{2} \beta\left(\frac{3}{4}, \frac{1}{4}\right) = \frac{1}{2} \frac{\Gamma\frac{3}{4} \cdot \Gamma\frac{1}{4}}{\Gamma 1}$$

$$= \frac{1}{2} \Gamma\frac{1}{4} \times \Gamma 1 - \frac{1}{4} = \frac{1}{2} \frac{\pi}{\sin \pi/4}$$

$$= \frac{1}{2} \frac{\pi}{1/\sqrt{2}} = \frac{\pi}{\sqrt{2}} = R.H.S$$

$$\therefore \int_0^{\pi/2} \sqrt{\cot\theta} \cdot d\theta = \frac{\pi}{\sqrt{2}}$$

4) $\int_0^1 x^5 \sin^{-1}x . dx = I.$

→ Integrating by parts, we have

$$I = \left[\sin^{-1}x . \frac{x^6}{6}\right]_0^1 - \int_0^1 \frac{x^6}{6} . \frac{1}{\sqrt{1-x^2}} . dx$$

$$= \frac{\pi}{2} \times \frac{1}{6} - \frac{1}{6} \times \int_0^1 \frac{x^6}{\sqrt{1-x^2}} . dx$$

put $x = \sin\theta \quad \therefore dx = \cos\theta \, d\theta.$

$$= \frac{\pi}{12} - \frac{1}{6} \int_0^{\pi/2} \frac{\sin^6\theta}{\cancel{\cos\theta}} \times \cancel{\cos\theta} . d\theta = \frac{\pi}{12} - \frac{1}{6} \int_0^{\pi/2} \sin^6\theta . d\theta.$$

$$= \frac{\pi}{12} - \frac{1}{6} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} - \frac{5\pi}{192}$$

$$= \frac{11}{192}\pi.$$

$$\therefore \int_0^1 x^5 \sin^{-1}x . dx = \frac{11}{192}\pi.$$

5) $I = \int_0^{\pi/6} \cos^6 3\theta . \sin^2 6\theta . d\theta$

Put $3\theta = t$, $\quad I = \int_0^{\pi/2} \cos^6 t . 4\sin^2 t \cos^2 t \frac{dt}{3}$

$$I = \frac{4}{3} \int_0^{\pi/2} \cos^8 t \sin^2 t \, dt = \frac{7}{384}\pi.$$

$$\therefore \int_0^{\pi/6} \cos^6 3\theta . \sin^2 6\theta . d\theta = \frac{7}{384}\pi.$$