# Appendix E

## DC Machines

### E.1 Introduction

Electrical energy is one form of energy that is most flexible and can be easily controlled. It can be converted into other forms of energy and converted back from other energy forms. Thus, energy conversion devices are required both at the generating end and the receiving end of the electrical power systems. At the generating end, energy obtained from a natural source (e.g., heat, water, or nuclear energy) is first converted into electrical energy, transmitted to the load centre, and then converted into required form of energy (e.g., heat, sound, light, mechanical, or chemical energy).

In generating stations (such as hydroelectrical and diesel-electric power stations), mechanical or thermal energy is converted into electrical energy with the help of generators. When electrical energy is available and mechanical work is to be done by it, a device called electrical motor is needed, which converts electrical energy into mechanical energy.

Thus, the **electrical machines**—generators and motors—are devices that transform mechanical power into electrical power and vice versa. This chapter incorporates a discussion of electrical machines. A special emphasis is given on their basic principles, working, types, and applications.

### E.2 Principle and Working of DC Machines

Electrical machines related to electrical energy of direct type are called dc machines. These machines are classified as dc generator and dc motor.

A **dc generator** is a machine that converts mechanical energy (or power) into electrical energy (or power), whereas a **dc motor** is a machine that converts electrical energy (or power) into mechanical energy (or power). From construction point of view, there is no basic difference between a dc generator and a dc motor. Any dc machine can act as a dc generator or a dc motor.

## E.2.1 DC generator

### *Working principle*

A dc generator works on the principle of Faraday's law of electromagnetic induction that states that when a conductor cuts the magnetic flux lines, an emf is induced in it, called dynamically induced emf. The direction of the induced emf can be determined by the Fleming's right hand rule and the magnitude of the induced emf is given by

$$e = B\,l\,v\,\sin\theta \text{ volt}$$

where $B$ = flux density in $Wb/m^2$

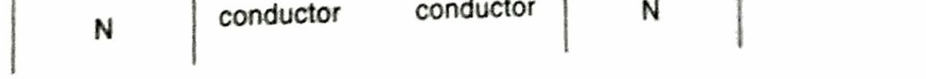
$l$ = active length of the conductor in m

$v$ = relative velocity of the conductor in m/s

$\theta$ = angle between the direction of motion of the conductor and the magnetic field

According to Fleming's right hand rule, if three fingers of right hand, namely thumb, index finger and middle finger, are outstretched so that they are mutually perpendicular to each other, and if the index finger is made to point in the direction of magnetic field, the thumb in the direction of motion of the conductor, then the outstretched middle finger gives the direction of the emf induced in the conductor.

Consider the arrangement as shown in Fig. E.1(a). If we move the conductor in a magnetic field in a direction at right angle to the field as shown in Fig. E.1(a), it cuts the flux lines, and emf is induced in it, called dynamically induced emf (as conductor is in motion). By applying the Fleming's right hand rule, it is found that the direction of the induced current (or induced emf) is out of the plane. This direction is shown by putting the dot inside the cross section of the conductor.




(a) Current coming out

(b) Current going in

**Fig. E.1** Working principle of a dc generator

It is seen that reversal of direction of motion of the conductor reverses the direction of the induced current [see Fig. E.1(b)].

### *Working*

Figure E.2 shows the schematic diagram of a simple dc generator consisting of a rectangular copper coil *ABCD* mounted on a shaft and rotating about its own axis (shaft) in a magnetic field produced by permanent magnets. The coil is rotated

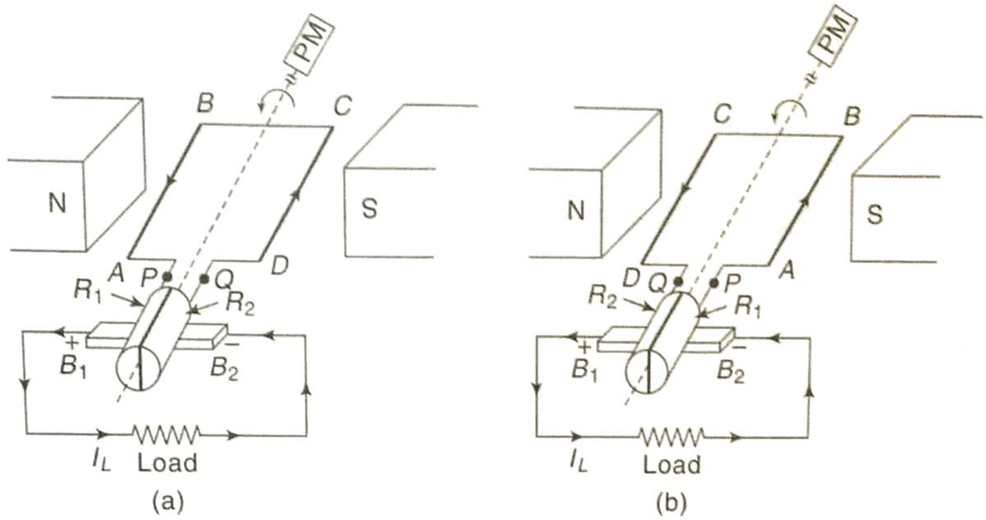



**Fig. E.2** Schematic diagram of a dc generator

with constant angular velocity by means of a prime mover[1] (PM). The coil has two identical conductors $AB$ and $CD$. The two ends of the coil $P$ and $Q$ are connected permanently to two commutator[2] segments $R_1$ and $R_2$ respectively. The commutator segments are well insulated from each other. The commutator is placed on the same shaft and rotates along with the coil. Two stationary carbon brushes $B_1$ and $B_2$ are pressed against the commutator. These brushes are further connected to the external load circuit. Their function is to collect the current induced in the coil and deliver it to the external circuit.

Let the coil be rotated in an anticlockwise direction with constant angular velocity. So, its conductors $AB$ and $CD$ cut the lines of flux and according to Faraday's law of electromagnetic induction, an emf get induced in them.

Now, emf induced in one conductor = $B\,l\,v\,\sin\theta$ volt

By Fleming's right hand rule, it is seen that at any instant, the emf's induced in the two conductors are additive in nature. As the coil has two identical conductors, emf induced in the coil, $v_{PQ} = 2B\,l\,v\,\sin\theta$ volt (E.1)

When $\theta = 90°$, emf induced is maximum. The maximum value of induced emf is expressed by $V_m$. Thus, when $\theta = 90°$, $v = V_m$.

So, $V_m = 2\,B\,l\,v\,\sin 90°$

or $V_m = 2\,B\,l\,v$

Now, Eq. (E.1) becomes

$$v_{PQ} = V_m \sin\theta \text{ volt} \tag{E.2}$$

Equation (E.2) gives the instantaneous value of the voltage induced in the coil, which appears across the coil terminals $P$ and $Q$. This voltage is a sinusoidal alternating voltage as shown in Fig. E.3.

[1]The prime mover, which drives the generator, may be a turbine, a diesel engine, or some type of motor.

[2]A commutator is a cylindrical drum mounted on a shaft. The surface of the drum is made of a large number of segments of harddrawn copper.

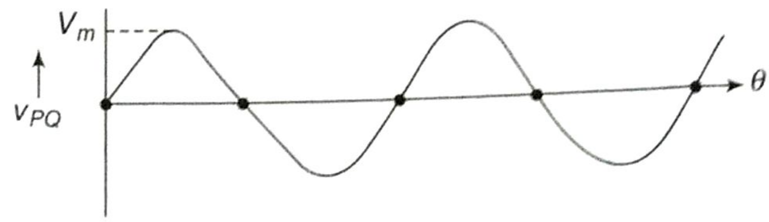



**Fig. E.3** Voltage induced in the coil of a dc generator

The direction of the induced emf can be determined by Fleming's right hand rule. In Fig. E.2(a), as the conductor $AB$ of the coil $ABCD$ moves downward and $CD$ moves upward, the direction of the induced emf in the coil is along $DCBA$. The coil terminal $P$ (i.e., commutator segment $R_1$) is positive and the coil terminal $Q$ (i.e., commutator segment $R_2$) is negative. The current in the external load circuit flows from brush $B_1$ to brush $B_2$. The direction of the current remains the same for half revolution of the coil starting from its vertical position. In the next half revolution [see Fig. E.2(b)], the direction of the induced current is reversed and the coil terminal $Q$ (i.e., commutator segment $R_2$) is positive and the coil terminal $P$ (i.e., commutator segment $R_1$) is negative. As the commutator segments interchange their positions, the current direction in the external load circuit remains same, i.e., from brush $B_1$ to brush $B_2$.

The commutator segments are so arranged that during half revolution of the coil, each segment remains in contact with a particular brush, whereas during the next half cycle, when the current is reversed, the same segment is in contact with the other brush. It is seen that in the first half revolution [Fig. E.2(a)], brush $B_1$ in contact with segment $R_1$ acts as the positive end of the supply and brush $B_2$ acts as the negative end. In the next half revolution [Fig. E.2 (b)], the direction of the induced current in the coil has reversed. But at the same time, the positions of segments $R_1$ and $R_2$ are also reversed with the result that brush $B_1$ comes in contact with the segment that is positive, i.e., segment $R_2$ in this case, and brush $B_2$ comes in contact with the segment that is negative, i.e., segment $R_1$ in this case. Hence, the current in the external load circuit remains unchanged.

The nature of current in the external load circuit with the rotation of the coil, i.e., with time, is shown in Fig. E.4. This current is unidirectional but not constant like a pure direct current.

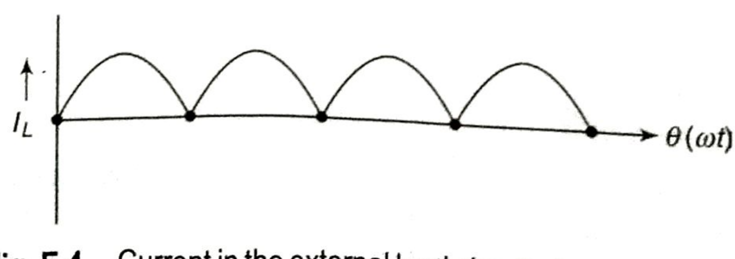



**Fig. E.4** Current in the external load circuit of a dc generator

Thus, the current induced in the coil is alternating but due to the commutator and the brushes, the current flowing through the external circuit is unidirectional. In other words, the commutator acts as a rectifier.

## E.2.2 DC Motor

### *Working principle*

The working of a dc motor is based on the principle that when a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force, whose direction is given by Fleming's left hand rule and magnitude is given by

$$F = B\,I\,l\,\sin\theta \text{ newton}$$

where $F$ = mechanical force experience by the conductor in N
$B$ = flux density in Wb/m$^2$
$l$ = active length of the conductor in m
$I$ = current through the conductor in A
$\theta$ = angle between the direction of the current and the magnetic field

Consider a single conductor placed in a magnetic field produced by permanent magnets as shown in Fig. E.5(a). If current is passed through the conductor in the direction shown in Fig. E.5(b), i.e., into the plane, then according to the basic principle, the conductor will experience a mechanical force. If the force is sufficient, then the conductor will move in the direction of the force. By Fleming's left hand rule, it is observed that the direction of motion of the conductor or the force is towards the right direction (from left to right).

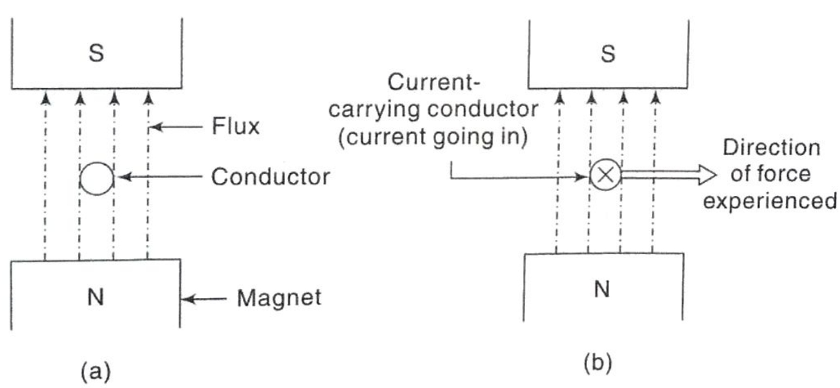



**Fig. E.5** Working principle of a dc motor

Fleming's left hand rule is as follows:

Outstretch the three fingers of left hand, namely thumb, index finger and middle finger, such that they are mutually perpendicular to each other. If the index finger is made to point in the direction of magnetic field, middle finger in the direction of the current, then the thumb gives the direction of the force experienced by the conductor.

Apply the above rule to verify the direction of force experienced by a single conductor placed in a magnetic field as shown in Figs E.6 (a), (b), (c), and (d).

It can be seen that if the direction of magnetic field is reversed without changing the direction of current through the conductor, then the direction of force experienced also gets reversed [see Figs E.6(a) and (c)]. Similarly, keeping the direction of magnetic field same, if the direction of current through the conductor is reversed, then also the direction of force experienced by the conductor gets

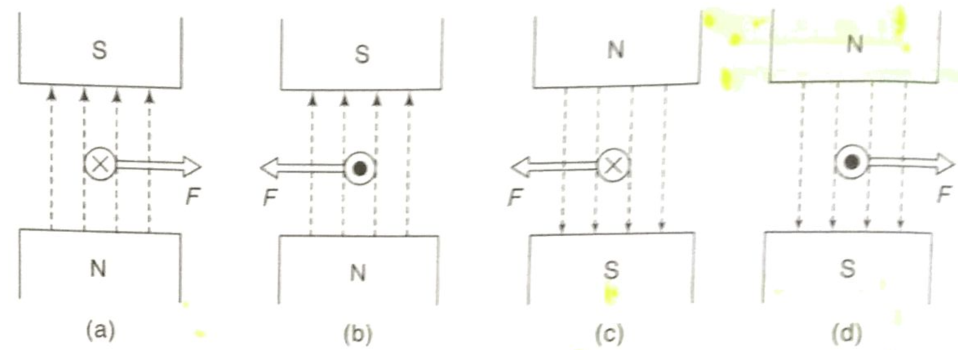



**Fig. E.6** A conductor placed in a magnetic field under different situations

reversed [see Figs E.6(a) and (b)]. But if directions of both magnetic field and current through the conductor are reversed, then the direction of force experienced by the conductor remains unchanged [see Figs E.6(a) and (d)].

### *Working*

Figure E.7 shows the schematic diagram of a simple dc motor consisting of a rectangular copper coil *ABCD* mounted on a shaft and placed in a magnetic field produced by permanent magnets. The coil has two identical conductors *AB* and *CD*. Two ends of the coil are connected permanently to two commutator segments $R_1$ and $R_2$ respectively. The commutator segments are well insulated from each other. The commutator is placed on the same shaft along with the coil. Two stationary carbon brushes $B_1$ and $B_2$ are pressed against the commutator.


**Fig. E.7** Schematic diagram of a dc motor

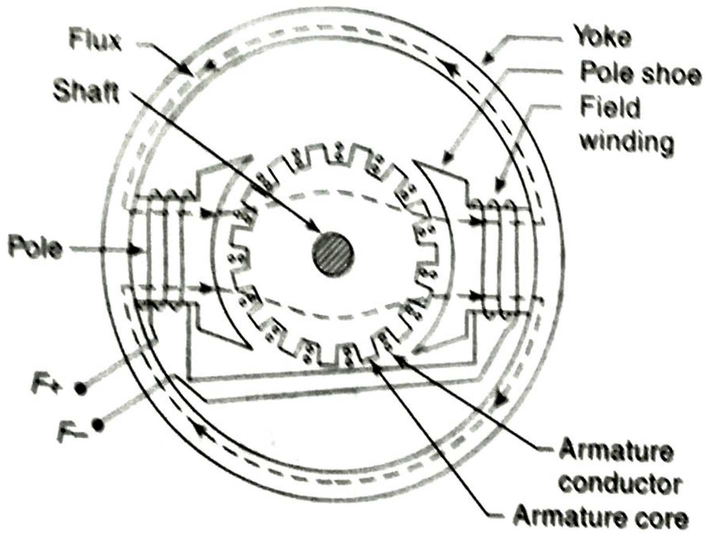
When the dc supply connected across the brushes $B_1$ and $B_2$ is switched on, current flows through the coil *ABCD* as shown in Fig. E.7(a). As a result, each conductor of the coil experiences a mechanical force. By Fleming's left hand rule, the conductor *AB* experiences a force in downward direction, while the conductor *CD* experiences a force in upward direction. These forces collectively produce a torque, and the coil rotates about its own axis (shaft) in anticlockwise direction.

As soon as half a rotation of the coil is completed, segment $R_1$ of the commutator comes in contact with brush $B_2$ and segment $R_2$ with brush $B_1$, thereby reversing the direction of current in the coil as shown in Fig. E.7(b). Since the position of the conductors *AB* and *CD* of the coil are also interchanged, the direction of rotation of the coil remains unchanged and the coil keeps on rotating in the same direction.

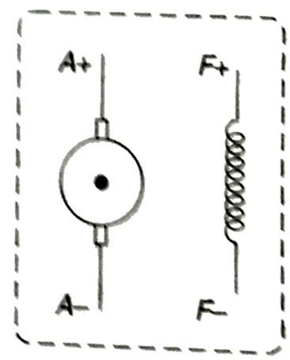
Thus, function of the commutator is to reverse the direction of the current in each conductor as it passes from one pole to another. It helps to develop a continuous and unidirectional torque.

## E.3 Construction of DC Machine

Whether the dc machine is a generator or a motor, the basic construction remains the same. Figure E.8(a) shows the cross-sectional view showing various parts of a two-pole dc machine. Figure E.8(b) shows the equivalent circuit of the dc machine.




(a) Cross-sectional view




(b) Equivalent circuit

**Fig. E.8** DC machine

A dc machine consists of the following parts:

Stationary parts:

(i) Yoke

(ii) Pole

- (a) Pole shoes
- (b) Pole core
- (c) Field winding

(iii) Brushes

Rotating parts:

(i) Armature

- (a) Armature core
- (b) Armature winding

(ii) Commutator

(iii) Bearings

Let $P$ = Number of poles of generator
$\Phi$ = Flux produced by each pole in weber (Wb)
$N$ = Speed of armature in rpm
$Z$ = Total number of armature conductors
$A$ = Number of parallel paths in which the total number of conductors are divided.

For lap type of winding, $A = P$
For wave type of winding, $A = 2$
According to Faraday's law of electromagnetic induction,

$$\text{Average value of emf induced in single conductor} = \frac{d\Phi}{dt} \qquad (\because N = 1)$$

Now, consider one revolution of a conductor. In one revolution, the conductor will cut the total flux produced by all the poles ($= P\Phi$).
Flux cut by the conductor in one revolution, $d\Phi = P\Phi$ weber

$$\text{Time required to complete one revolution, } dt = \frac{60}{N} \text{ sec}$$

$$\text{Hence, average value of emf induced in single conductor} = \frac{d\Phi}{dt} = \frac{P\Phi}{\left(\frac{60}{N}\right)} = \frac{P\Phi N}{60} \text{ volt}$$

This is the emf induced in one conductor. Now, the conductors in one parallel path are always in series. There are $Z$ conductors with $A$ parallel paths. Hence, $Z/A$ number of conductors are always in series and emf remains same across all the parallel paths.
So, total emf can be expressed as

$$E_g = \frac{P\Phi N}{60} \times \frac{Z}{A} \text{ volt}$$

This equation is called emf equation of the dc generator.

We can also write $E_g = \dfrac{\Phi Z N}{60} \times \dfrac{P}{A}$ volt

where $A = P$ for lap winding
$A = 2$ for wave winding

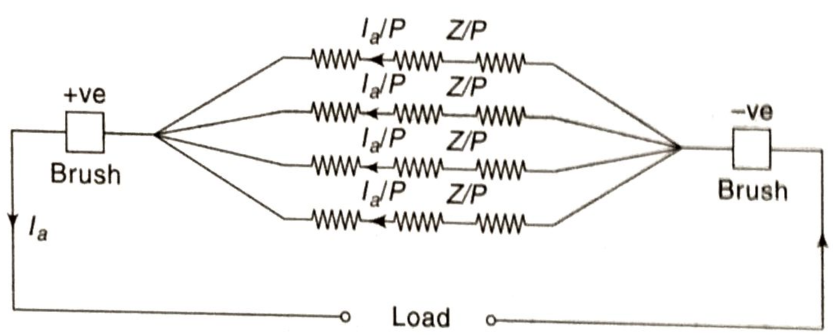
## E.5 Types of DC Generators

The symbolic representation of a dc generator is shown in Fig. E.14. The armature is denoted by a circle with two brushes. The armature is driven by prime mover with speed $N$ rpm. The two ends of the armature are denoted as $A+$ and $A-$. The field winding is shown near armature and the two ends are denoted by $F+$ and $F-$.

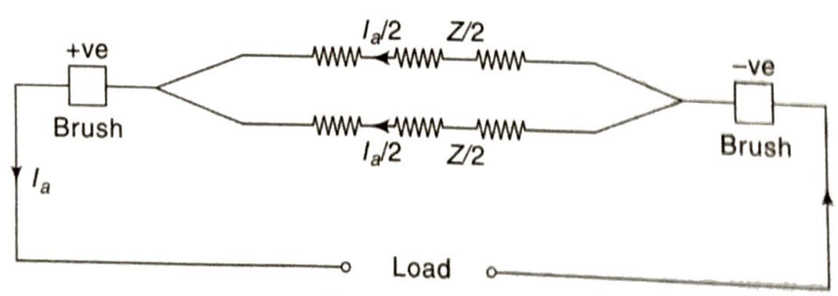
current. Due to the presence of a large number of parallel paths, lap winding is more suitable for generating large currents. Figure E.13(a) shows the internal connections in armature of lap winding.

### *Wave winding*

In this type of winding, the armature conductors are divided into two parallel paths. Thus, the armature current entering the negative brush finds two parallel paths while going to the positive brush. Hence, each parallel path carries a current of $I_a/2$, where $I_a$ is the total armature current. In wave winding, the first conductor (say under N-pole) is connected directly to another conductor, which occupies a similar position, but under the opposite polarity pole (i.e., S-pole in the above example). The winding advances forward to the next N-pole and so on [see Fig. E.12(b)]. This type of winding is so named because it travels like a progressive wave. As the number of parallel paths is less, it is preferable for low-current, high-voltage capacity generators. Figure E.13(b) shows the internal connections in armature of wave winding.



(a) Lap winding



(b) Wave winding

**Fig. E.13** Internal connections in armature winding

## E.4 EMF Equation of DC Generator

When field winding is excited, magnetic field is established in the dc machine. To use this machine as a generator, the armature is rotated with constant angular velocity with the help of prime mover. When the armature rotates, its conductors cut the magnetic flux lines and according to Faraday's law of electromagnetic induction, emf induced in the conductors. The equation of total induced emf in a dc generator can be calculated as follows.

Let $P$ = Number of poles of generator
$\Phi$ = Flux produced by each pole in weber (Wb)
$N$ = Speed of armature in rpm
$Z$ = Total number of armature conductors
$A$ = Number of parallel paths in which the total number of conductors are divided.

For lap type of winding, $A = P$
For wave type of winding, $A = 2$
According to Faraday's law of electromagnetic induction,

$$\text{Average value of emf induced in single conductor} = \frac{d\Phi}{dt} \quad (\because N = 1)$$

Now, consider one revolution of a conductor. In one revolution, the conductor will cut the total flux produced by all the poles ($= P\Phi$).
Flux cut by the conductor in one revolution, $d\Phi = P\Phi$ weber

$$\text{Time required to complete one revolution, } dt = \frac{60}{N} \text{ sec}$$

$$\text{Hence, average value of emf induced in single conductor} = \frac{d\Phi}{dt} = \frac{P\Phi}{\left(\frac{60}{N}\right)} = \frac{P\Phi N}{60} \text{ volt}$$

This is the emf induced in one conductor. Now, the conductors in one parallel path are always in series. There are $Z$ conductors with $A$ parallel paths. Hence, $Z/A$ number of conductors are always in series and emf remains same across all the parallel paths.
So, total emf can be expressed as

$$E_g = \frac{P\Phi N}{60} \times \frac{Z}{A} \text{ volt}$$

This equation is called emf equation of the dc generator.

$$\text{We can also write } E_g = \frac{\Phi Z N}{60} \times \frac{P}{A} \text{ volt}$$

where $A = P$ for lap winding
$A = 2$ for wave winding

## E.5 Types of DC Generators

The symbolic representation of a dc generator is shown in Fig. E.14. The armature is denoted by a circle with two brushes. The armature is driven by prime mover with speed $N$ rpm. The two ends of the armature are denoted as $A+$ and $A-$. The field winding is shown near armature and the two ends are denoted by $F+$ and $F-$.

The poles of the machine are electromagnets. By passing the current through the field winding, the magnetic field is produced in the generator. Hence, this current is called **exciting current**. Depending on the way of deriving the field current or exiting current, dc generator is basically divided into two categories: (i) separately excited generator and (ii) self-excited generator. The self-excited generator is further classified depending upon the way of field winding connection with armature as (i) shunt generator, (ii) series generator, and (ii) compound generator.

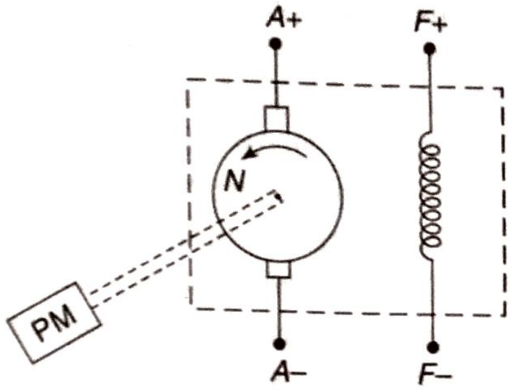



**Fig. E.14** Symbol of a dc generator

## E.5.1 Separately Excited DC Generator

When the field winding is supplied from external, separate dc supply, i.e., excitation of the field winding is separate, the generator is called separately excited dc generator. Schematic representation of separately excited dc generator is shown in Fig. E.15.

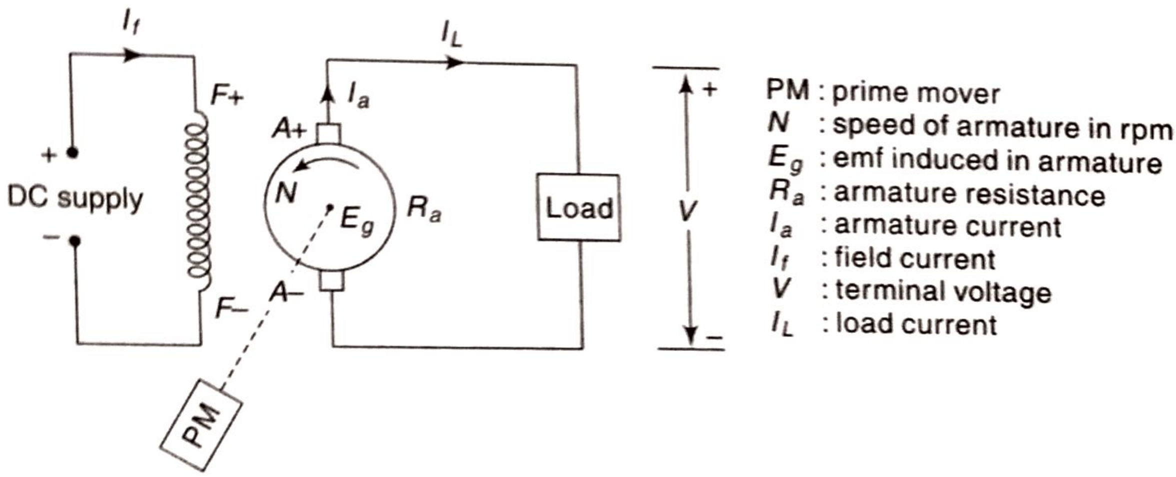



**Fig. E.15** Separately excited dc generator

### *Voltage and current relations*

The prime mover rotates the armature at $N$ rpm. The generator induces the emf $E_g$. Since the field winding is excited separately, the field current depends on supply voltage and resistance of the field winding. For armature side, we can see that it is supplying a load demanding a load current $I_L$ at a voltage of $V$, which is called **terminal voltage**.

Now, $I_a = I_L$ (E.3)

Equation (E.3) is called **current equation**.

The internally induced emf $E_g$ supplies the voltage to the load. Hence, terminal voltage $V$ is a part of $E_g$. But it is not equal to $V$ while supplying a load. This is because when armature current $I_a$ flows through armature winding, due to armature winding resistance $R_a$ ohm, there is a voltage drop across armature winding equal to $I_aR_a$ volt. The induced emf has to supply this drop, along with the terminal voltage $V$. To keep $I_aR_a$ drop to minimum, the resistance $R_a$ is designed to be very small. In addition to this drop, there is some voltage drop at the contacts of the brush, called **brush contact drop**. But this drop is negligible and hence, generally neglected. When armature carries current $I_a$, it produces its own flux called armature flux. This flux has a tendency to disturb the pattern of main useful flux produced by field winding. This distortion produced by armature flux reacting with field flux is called **armature reaction**. Due to this armature reaction, there is a drop in voltage.

Hence, in all the induced emf, $E_g$ has to overcome $I_aR_a$ drop, brush contact drop and armature reaction drop to produce the terminal voltage $V$ at the load. Thus, the voltage equation for the generator is

$$E_g = V + I_aR_a + V_{\text{brush}} + \text{Armature reaction drop} \tag{E.4}$$

Equation (E.4) is called **voltage equation.**

By using the voltage equation, induced emf $E_g$ or terminal voltage $V$ can be determined if other drops are known. The brush contact drop is generally specified as per brush drop. As there are two brushes, total brush drop is twice the drop per brush.

### E.5.2 Self-excited DC Generator

Self-excited dc generator is one whose field windings are excited by the current produced by the generator itself. Now, without generated emf, field cannot be excited in such a generator and without excitation, there cannot be generated emf. So, one may obviously wonder how this type of generator works. The answer to this is residual magnetism possessed by the field poles under normal condition. This enables armature to develop small emf, which circulates current through field, which further increases the flux produced. Because of this cumulative process, the generator ultimately produces its rated voltage.

There are three types of self-excited generators named according to the manner in which their field windings are connected to the armature as: (i) shunt generator, (ii) series generator, and (iii) compound generator.

***Shunt generator***

When the field winding is connected across or in parallel with the armature, the generator is called shunt generator. Since the field winding has large number of turns of thin wire, it has high resistance compared to the armature winding. Let $R_{sh}$ be the resistance of the field winding and $I_{sh}$ be the current through the field winding. Schematic representation of shunt generator is shown in Fig. E.16(a).

Before loading a shunt generator, it is allowed to build up its voltage. Assume that the generator in Fig. E.16(a) has no load connected to it and the armature is driven at a certain speed by a prime mover. Usually, there is always present some residual magnetism in the poles; hence, a small emf is produced initially. This emf circulates a small current in the field circuit, which increases the pole flux.

When flux is increased, generated emf is increased, which further increases the flux and so on. As shown in Fig. E.16(b), '$E_1$' is the induced emf due to residual magnetism, which appears across the field circuit and causes the field current '$I_{sh1}$' to flow. This current aids residual flux and hence produces a larger induced emf '$E_2$'. In turn, this increased emf '$E_2$' causes an even larger current '$I_{sh2}$', which creates more flux for a still larger emf and so on. This process of voltage build-up continues. The effect of magnetic saturation in the pole faces limits the terminal voltage of the generator to a steady state value ($E_g$).

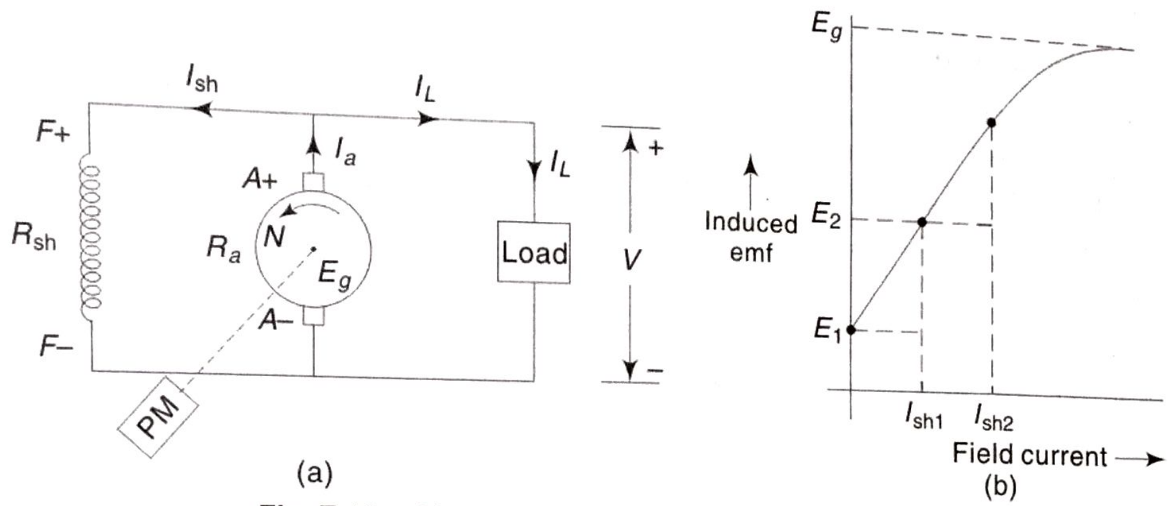



**Fig. E.16** Shunt generator and build-up of a generator

***Voltage and current relations*** From the circuit shown in Fig. E.16(a), we can write the current equation as

$$I_a = I_L + I_{sh} \tag{E.5}$$

Now, voltage across load is $V$, which is same across field winding as both are in parallel with each other.

So, $$I_{sh} = \frac{V}{R_{sh}}$$

While induced emf $E_g$ still requires to supply voltage drop $I_aR_a$, brush contact drop and armature reaction drop. Thus, we get the voltage equation as

$$E_g = V + I_aR_a + V_{\text{brush}} + \text{Armature reaction drop} \tag{E.6}$$

Since the shunt field winding has large number of turns of thin copper, its cross-sectional area is small. Its resistance $R_{sh}$ is high. This is because the load current should not disturb the field current $I_{sh}$ and remains constant for the operation range of generator.

***Load characteristics of shunt generator*** The relation between the terminal voltage $V$ and the load current $I_L$ is called load characteristics or performance characteristics of the generator.

From the voltage equation, we can see that as load current $I_L$ increases, the armature current $I_a$ increases to satisfy the load demand. Thus, the armature voltage drop $I_aR_a$ also increases. Hence, the terminal voltage $V = E_g - I_aR_a$ decreases, neglecting other drops. But as $R_a$ is very small, though $I_L$ changes from no load to full load, the drop in the terminal voltage is very small. This is shown in Fig. E.17. Hence, dc shunt generator is also called constant voltage generator.

***Application of shunt generator*** Due to constant voltage characteristics, shunt generators are commonly used for battery charging, ordinary lighting and power supply purposes.

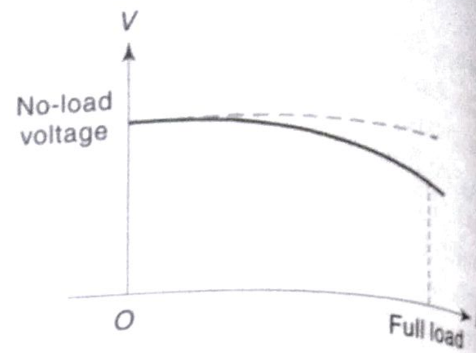



Fig. E.17 Load current vs terminal voltage

### *Series generator*

When the field winding is connected in series with the armature winding while supplying the load, the generator is called series generator. It is shown in Fig. E.18. The field winding resistance is denoted by $R_{se}$. The resistance $R_{se}$ is very small and hence, naturally it has less number of turns of a wire of thick cross section.

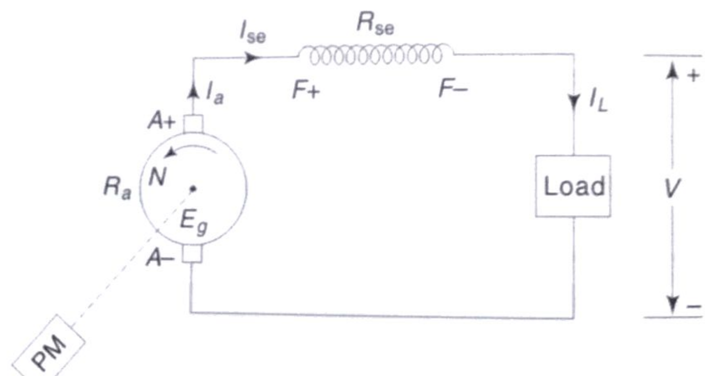



Fig. E.18 Series generator

***Voltage and current relations*** As armature, field winding and load, all are in series, they carry the same current. So, the current equation can be written as

$$I_a = I_{se} = I_L \tag{E.7}$$

where $I_{se}$ = current through series field winding

Now, in addition to drop $I_aR_a$, induced emf has to supply voltage drop across series field winding too.

The voltage drop across series field winding $= I_{se}R_{se} = I_aR_{se}$ $\quad (\because I_a = I_{se})$

Thus, voltage equation can be written as

$$E_g = V + I_aR_a + I_aR_{se} + V_{\text{brush}} + \text{Armature reaction drop}$$

or $$E_g = V + I_a(R_a + R_{se}) + V_{\text{brush}} + \text{Armature reaction drop} \tag{E.8}$$

***Load characteristics of series generator*** In series generator, as $I_a = I_{se} = I_L$, when $I_L$ increases, $I_{se}$ also increases. The flux Φ is directly proportional to $I_{se}$. So, the flux increases. As induced emf $E_g$ is directly proportional to the flux, $E_g$ also increases. For load characteristics, the drop $I_a(R_a + R_{se})$ increases as $I_a$ increases. But this drop is small compared to increase in $V$ due to increase in $E_g$.

and so, graph of $V$ versus $I_L$ is rising in nature as shown in Fig. E.19. On no load, there exists some voltage due to residual flux retained by the field winding, and the characteristics do not pass through origin.

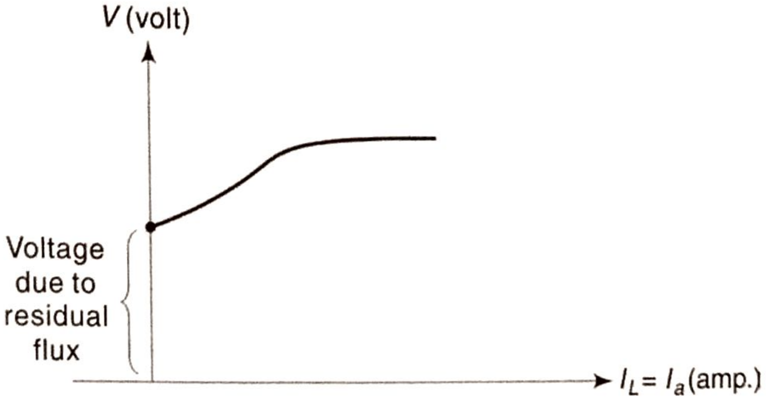



**Fig. E.19** Terminal voltage vs load current

***Application of series generator*** Due to the rising characteristics, series generators are used as boosters on dc feeders and as constant current generators for welding generators and lamps.

### *Compound generator*

In compound generator, the poles of the machine are excited by the two independent field windings, i.e., shunt field winding and series field winding. The shunt field winding is connected in parallel and the series field winding is connected in series, with the armature winding. The shunt field winding is stronger than the series field winding. If series field aids the shunt field, i.e., the magnetizing effect of the two windings is cumulative, the generator is called cumulative compound generator [see Fig. E.20(a)]. If series field opposes the shunt field, the generator is said to be differential compound generator [see Fig. E.20(b)].

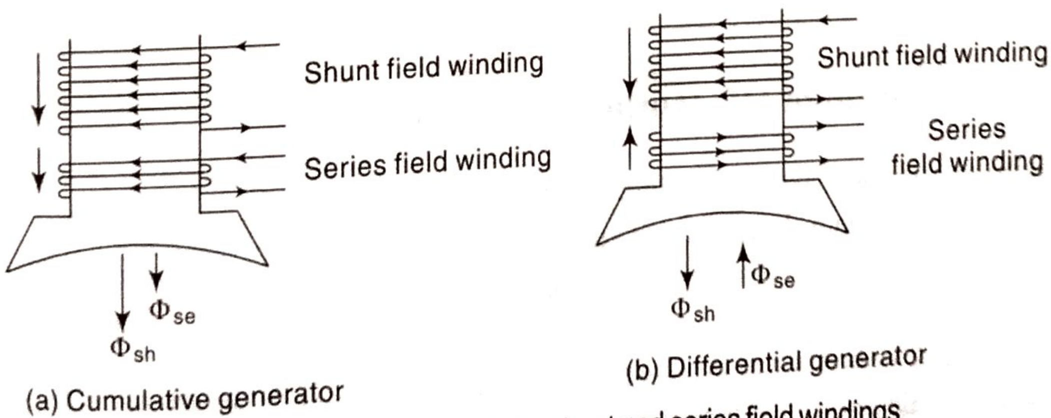



**Fig. E.20** Excitation of pole by shunt and series field windings

The compound generator can be either short shunt or long shunt as shown in Figs E.21(a) and (b) respectively. So, the cumulative or differential compound generator can be either short shunt or long shunt.

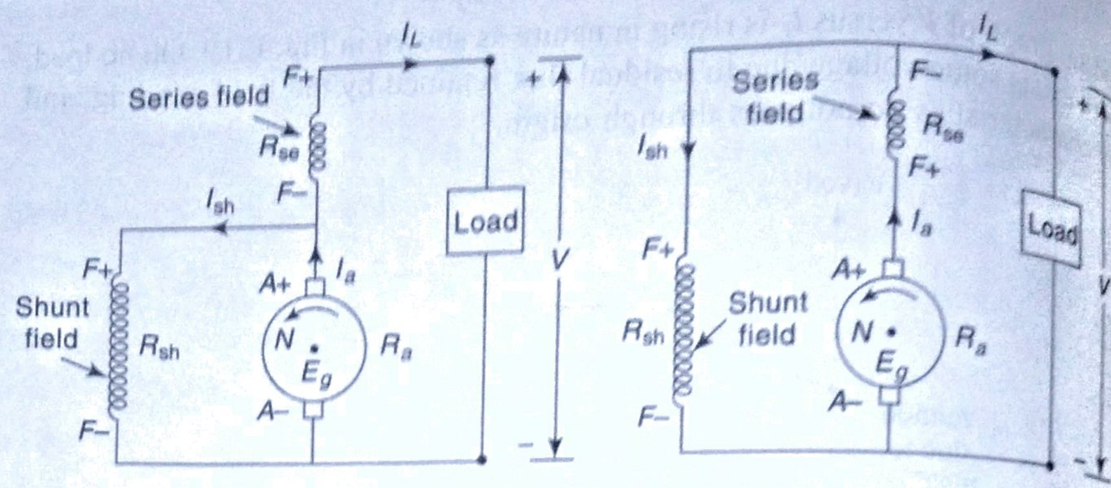



**Fig. E.21** Compound generator

***Load characteristics*** In Fig. E.21, if it is imagined that the series field winding is absent, it is simple-shunt generator and its load characteristics will be same as those shown in Fig. E.17. These characteristics are of drooping nature. For the cumulative compound generator, series field aids the shunt field, and so, it gives characteristics of boosting nature. But for differential compound generator, as series field winding opposes the shunt field, it now gives negative boosting characteristics. The load characteristics of compound generator are shown in Fig. E.22.

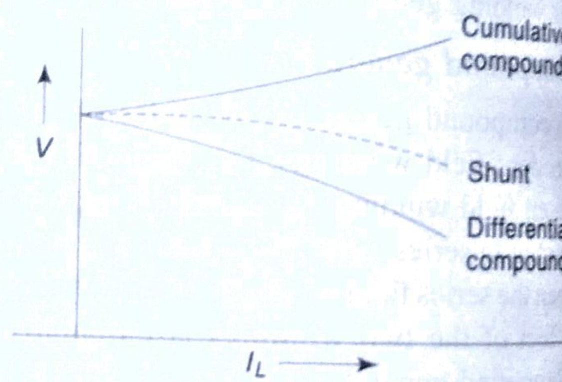



**Fig. E.22** Load characterstics of compound generator

## E.6 Operation of DC Motor and Back EMF

We know that constructionally there is no basic difference between a dc generator and a dc motor. In fact, the same machine can be used interchangeably as a generator or as a motor.

Figure E.23 shows the cross-sectional view of two-pole dc motor. When its field magnets are excited and dc voltage is applied to the motor, current flows through the armature conductors. Armature conductors under the N-pole are assumed to carry the current downwards (shown by crosses) and those under S-pole to carry current upwards (shown by dots).

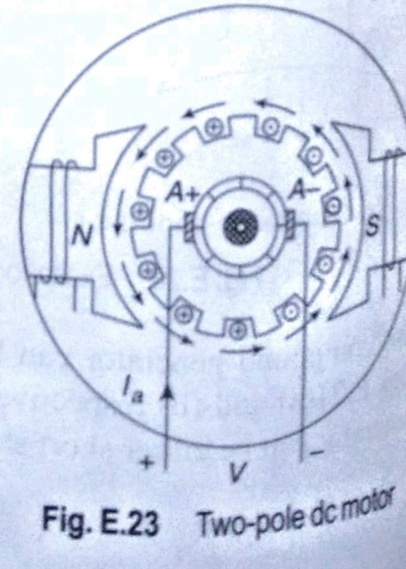



**Fig. E.23** Two-pole dc motor

By basic principle, each conductor will experience a mechanical force. According to Fleming's left hand rule, each conductor will experience a force in anticlockwise direction, which is shown by small arrows placed above each conductor. These forces collectively produce a driving torque, which sets the armature rotating in anticlockwise direction.

It should be noted that the function of commutator is to reverse the direction of current in each conductor as it passes from one pole to another. It helps to develop a continuous and unidirectional torque.

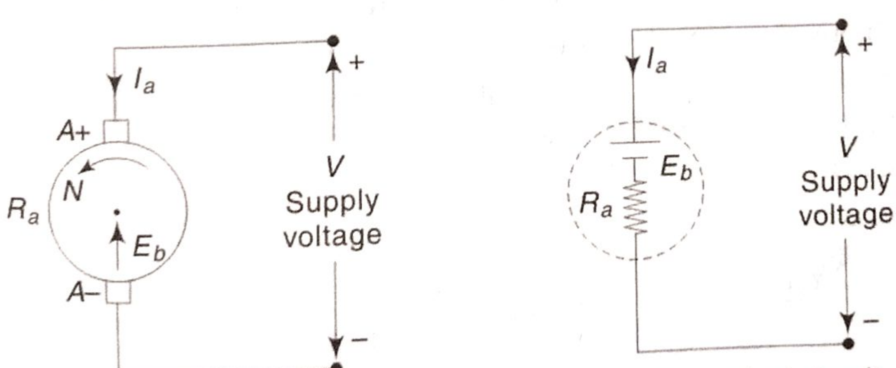
It is seen in the generator action that when an armature conductor cuts the lines of flux, emf gets induced in it. In a dc motor, after a motoring action, there exists a generator action. When the armature rotates, the conductor cuts the magnetic flux lines and according to Faraday's law of electromagnetic induction, emf gets induced in it. This induced emf in the armature always acts in the opposite direction of the supply voltage. This is according to Lenz's law, which states that the direction of induced emf is always so as to oppose the cause producing it. In a dc motor, electrical input, i.e., the supply voltage, is the cause and hence, this induced emf opposes the supply voltage. This emf tries to set up a current through the armature in the opposite direction, which supplies voltage forcing through the conductor.

So, as this emf always opposes the supply voltage, it is called **back emf** and denoted by $E_b$. Though it is denoted as $E_b$, basically it gets generated by the generating action that we have seen earlier. So, its magnitude can be determined by the emf equation derived earlier. Thus,

$$E_b = \frac{\Phi ZN}{60} \times \frac{P}{A} \text{ volt}$$

where all symbols carry the same meaning as in case of generators.

The back emf is shown schematically in Fig. E.24(a). If $V$ is the supply voltage and $R_a$ is the value of the armature resistance, the equivalent circuit will be as shown in Fig. E.24(b). In equivalent circuit, back emf is represented by a battery of emf $E_b$ with polarity such that it opposes the supply voltage.




(a) Back emf (b) Equivalent circuit

**Fig. E.24** Armature circuit

Applying KVL to the equivalent circuit shown in Fig. E.24(b), we get the voltage equation of the dc motor as

$$V = E_b + I_a R_a \tag{E.9}$$

Thus, in case of dc motor, supply voltage $V$ has to overcome back emf $E_b$ (which opposes $V$) and also armature resistance drop $I_aR_a$. In fact, the electrical work done in overcoming the back emf gets converted into the mechanical energy developed in the armature.

The back emf is always less than the supply voltage. The net voltage across the armature is the difference between the supply voltage and the back emf, which decides the armature current. Hence, from the voltage equation, we can write

$$\text{Armature current, } I_a = \frac{V - E_b}{R_a}$$

Multiplying both sides of the voltage equation, Eq. (E.9), by $I_a$, we get

$$VI_a = E_bI_a + I_a^2R_a \tag{E.10}$$

where $VI_a$ = Electrical power input to the armature

$I_a^2R_a$ = Power loss due to the armature resistance called armature copper loss

$E_bI_a$ = Electrical equivalent of gross mechanical power developed in the armature

Equation (E.10) is called **power equation** of the dc motor.

Hence, some of the armature input is wasted in $I^2R$ loss and rest is converted into mechanical power within the armature.

## E.7 Torque Equation of a DC Motor

In general, torque is the turning or twisting movement of a force about an axis. The torque, angular speed and power are related as

Mechanical power developed, $P = T \times \omega$

where $T$ = Torque in Nm

$\omega$ = Angular speed in rad/sec

In case of motor, each armature conductor experiences a force, and these forces collectively produce a torque ($T_a$).

Let $T_a$ be the torque developed by the armature of the motor running at $N$ rpm.

So, the mechanical power developed, $P = T_a \times \omega$ (E.11)

From Eq. (E.10) above (i.e., power equation of a motor), we know that

Mechanical power developed = $E_bI_a$ (E.12)

Equating Eqs (E.11) and (E.12),

$$T_a \times \omega = E_bI_a$$

$$\text{or} \quad T_a \times \frac{2\pi N}{60} = E_bI_a \qquad \left(\because \omega = \frac{2\pi N}{60}\right)$$

$$\text{or} \quad T_a \times \frac{2\pi N}{60} = \left(\frac{\Phi ZN}{60} \times \frac{P}{A}\right)I_a \qquad \left(\because E_b = \frac{\Phi ZN}{60} \times \frac{P}{A}\right)$$

$$\text{or} \quad T_a = \frac{1}{2\pi}\Phi ZI_a\left(\frac{P}{A}\right) \text{ Nm}$$

This is the **torque equation** of the dc motor. So, for a dc motor,

$$T_a \propto \Phi\, I_a$$

## E.8 Types of DC Motor

Similar to the dc generators, the dc motors are classified depending upon the way of connecting the field winding with the armature winding as shunt motor and series motor.

### E.8.1 Shunt Motor

In this type of dc motor, the field winding is connected across the armature winding and the combination is connected across the supply as shown in Fig. E.25.

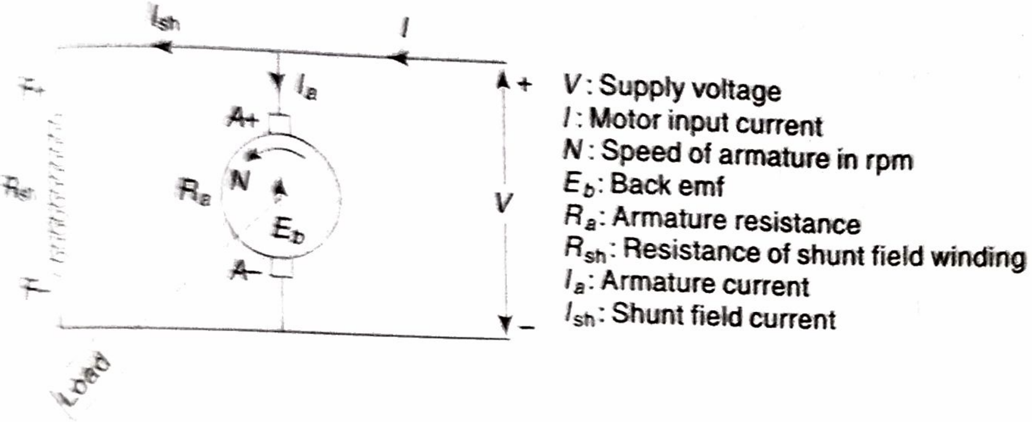



**Fig. E.25** Shunt motor

The value of $R_a$ is very small while $R_{sh}$ is quite large. Hence, shunt field winding has more number of turns with less cross-sectional area.

The voltage across the armature and the field winding is same—equal to the supply voltage $V$.

From circuit diagram, the total current drawn from the supply can be written as

$$I = I_a + I_{sh} \tag{E.13}$$

Equation (E.13) is called **current equation** of the shunt motor.

where shunt current, $I_{sh} = \dfrac{V}{R_{sh}}$

The supply voltage $V$ has to overcome back emf $E_b$ (which is opposing $V$) and also armature resistance drop $I_aR_a$. So, we get the voltage equation of shunt motor as

$$V = E_b + I_aR_a \tag{E.14}$$

Now, the flux produced by the field winding is proportional to the current passing through it, i.e.,

$$\Phi \propto I_{sh}$$

As long as the supply voltage is constant, which is generally so in practice, the flux produced is constant. Hence, dc shunt motor is also called **constant flux motor**.

***Applications of shunt motor***

Shunt motor is a constant speed motor having medium starting torque. The speed of a shunt motor can be adjusted over a wide range. Therefore, shunt motor can be used in

(i) various machine tools such as lathe machines, drilling machines, milling machines, etc.,
(ii) centrifugal and reciprocating pumps,
(iii) blowers and fans, and
(iv) printing machinery and paper machines.

## E.8.2 Series Motor

In this type of dc motor, the series field winding is connected in series with the armature winding and the supply as shown in Fig. E.26.

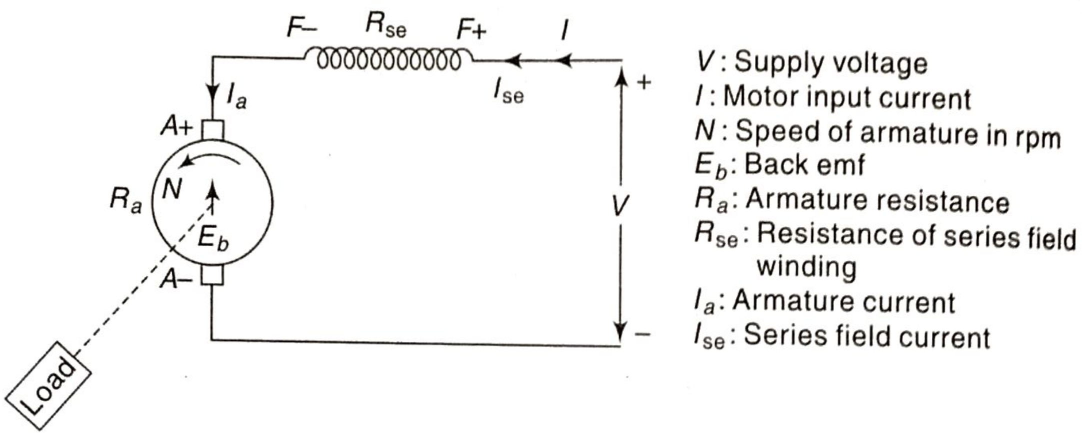



**Fig. E.26** Series motor

The value of $R_{se}$ is very small and it is made of small number of turns having large cross-sectional area.

As all armature, field winding and supply are in series, they carry the same current. So, the current equation can be written as

$$I = I_{se} = I_a \tag{E.15}$$

where $I_{se}$ = current through the series field winding

The supply voltage $V$ has to overcome the drop across series field winding, in addition to the back emf $E_b$ (which is opposing $V$) and the armature resistance drop $I_aR_a$. So, we get the voltage equation of series motor as

$$V = E_b + I_aR_a + I_aR_{se}$$

$$V = E_b + I_a(R_a + R_{se}) \tag{E.16}$$

In this motor, entire armature current is passing through the series field winding. So, flux produced is proportional to the armature current, i.e.,

$$\Phi \propto I_{se} \propto I_a$$

***Applications of series motor***

Series motor has very high starting torque and good accelerating torque. The speed is adjustable and varying. The motor has very low speed at high loads and dangerously high speed at low loads. Therefore, series motor can be used in

(i) electric trains,