2.3 AC Parallel Circuits

An *ac circuit that* has number of branches connected in parallel such that the voltage across them is same, is called an **ac parallel circuit.** The current in any one branch depends upon the impedance of that branch. The total line current supplied to the circuit is the phasor sum of branch currents. Parallel circuits are used more often in practice as all lighting and power circuits are constant voltage circuits and the loads/equipments are connected in parallel.

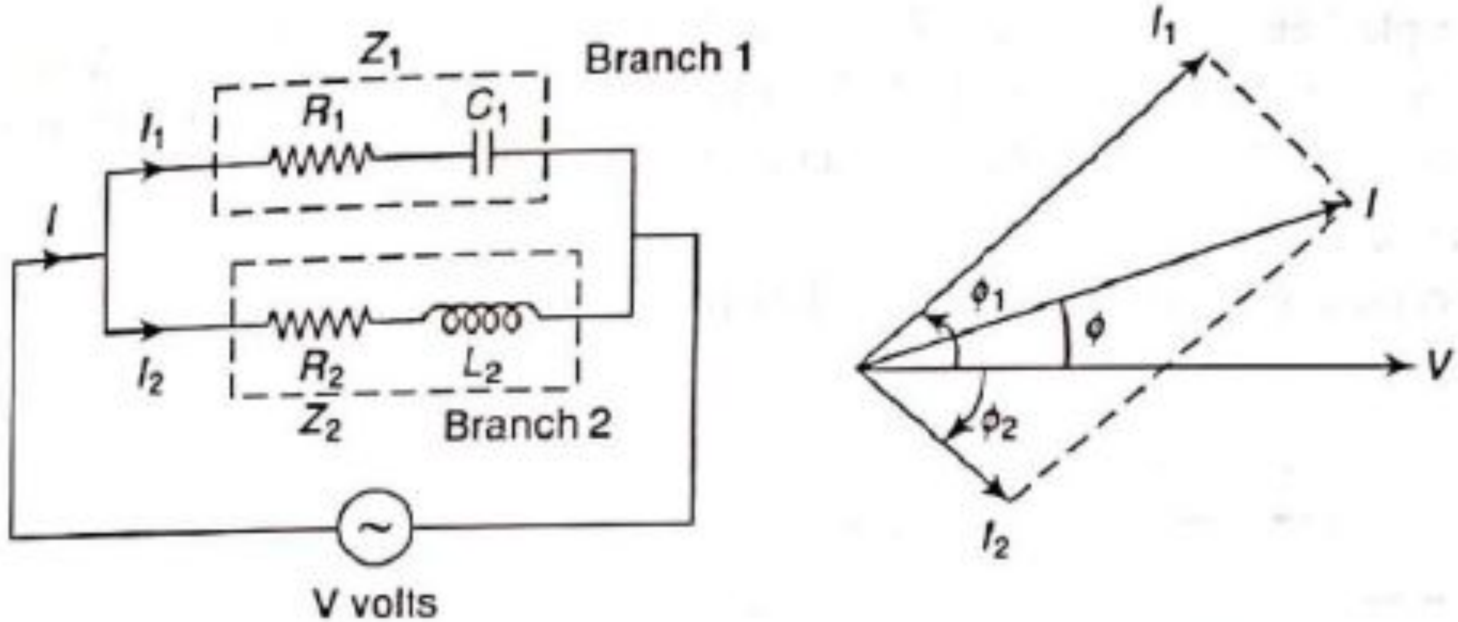
While analysing an ac parallel circuit, two important points must be kept in mind. First, a parallel circuit, in fact, consists of two or more series circuits connected in parallel. Therefore, each branch of the circuit can be analysed separately as a series circuit and then the effect of the separate branches can be combined. Secondly, alternating voltages and currents are phasor quantities. This implies that both magnitudes and phase angles must be taken account while carrying out circuit calculations. There are three methods of solving ac parallel circuits, namely;

1. By phasor diagram
2. By phasor algebra
3. Admittance method

The use of a particular method will depend upon the conditions of the problem. However, in general, that method should be used which yields quick results.

2.3.1 Phasor diagram

In this method, we find the magnitude and phase angle of each branch current. We then draw the phasor diagram taking voltage as reference phasor. The circuit or line current is the phasor sum of the branch currents and can be determined by parallelogram law. Consider a parallel circuit as shown in Fig. 2.122(a), consisting of two branches and connected to an alternating voltage of V volts.



(a) Circuit diagram

(b) Phasor diagram

Fig. 2.122 Analysis of parallel circuit by phasor diagram

Branch 1 $Z_1 = \sqrt{R_1^2 + X_{C1}^2}$; $I_1 = \frac{V}{Z_1}$; $\phi_1 = \tan^{-1}\frac{X_{C1}}{R_1}$

The current I_1 in branch 1 leads the applied voltage V by ϕ_1°.

Branch 2 $Z_2 = \sqrt{R_2^2 + X_{L2}^2}$; $I_2 = \frac{V}{Z_2}$; $\phi_2 = \tan^{-1}\frac{X_{L2}}{R_2}$

The current I_2 in branch 2 lags behind the applied voltage V by ϕ_2°.

The phasor diagram is shown in Fig. 2.122(b). The line current I is the phasor sum of I_1 and I_2.

The phasor diagram method is suitable only when the parallel circuit is simple and contains two branches. However, if the parallel circuit is complex having more than two branches, this method becomes very inconvenient. In such cases, use of phasor algebra is recommended to solve parallel-circuit problems.

2.3.2 Phasor Algebra

In this method, voltages, currents and impedances are expressed in the complex form, i.e., either in the rectangular or polar form. Since complex form includes both magnitude and phase angle, the solution of parallel circuit problems can be obtained mathematically by using the rules of phasor algebra. This eliminates the need of phasor diagram. Referring back to the parallel circuit shown in Fig. 2.122(a), we have

$$\bar{V} = (V - j\,0) = (V \angle 0)$$

$$\bar{Z}_1 = (R_1 - j\,X_{C1}) = (Z_1 \angle -\phi_1)$$

$$\bar{Z}_2 = (R_2 + jX_{L2}) = (Z_2 \angle \phi_2)$$

Branch current, $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1}$

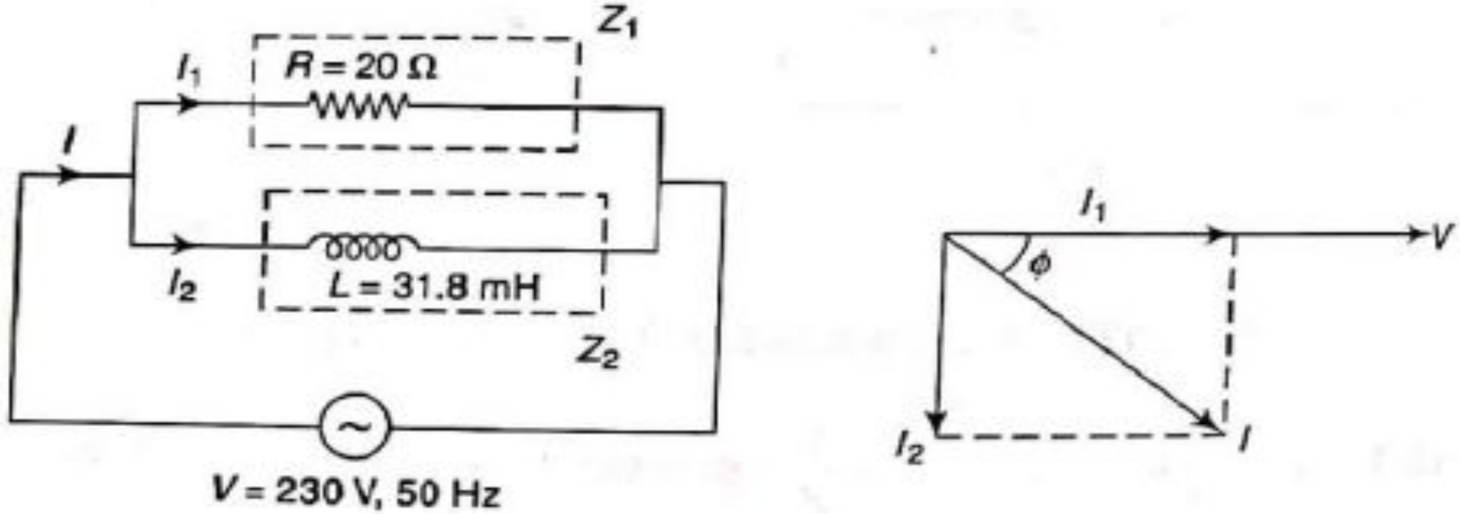
Branch current, $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2}$

Total circuit current, $\bar{I} = \bar{I}_1 + \bar{I}_2$

Example 2.66 A resistance of 20 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 230 V, 50 Hz supply. Find (i) the line current, (ii) power factor, and (iii) power consumed by the circuit.

Solution

The conditions in the example are shown in Fig. 2.123(a).



(a) Circuit diagram

(b) Phasor diagram

Fig. 2.123

Given: $V = 230$ V

$f = 50$ Hz

$L = 31.8 \times 10^{-3}$ H $\Rightarrow$ $X_L = 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Branch current, $I_1 = \frac{V}{R} = \frac{230}{20} = 11.5 \text{ A}$

The current I_1 is in phase with the applied voltage.

Branch current, $I_2 = \frac{V}{X_L} = \frac{230}{10} = 23 \text{ A}$

The current I_2 lags behind the applied voltage by 90°.

So, line current, $I = \sqrt{I_1^2 + I_2^2}$

$$= \sqrt{(11.5)^2 + (23)^2}$$

$$= 25.71 \text{ A}$$

From phasor diagram,

$$\text{pf} = \cos \phi = \frac{I_1}{I} = \frac{11.5}{25.71} = 0.447 \text{ lag}$$

Power consumed, $P = VI\cos\phi$

$$= 230 \times 25.71 \times 0.447$$

$$= 2643 \text{ W}$$

Alternative method:

Taking V as reference phasor,

$$\bar{V} = (230\angle 0) \text{ V}$$

$$\bar{Z}_1 = (20 + j0)\ \Omega = (20\angle 0)\ \Omega$$

$$\bar{Z}_2 = (0 + j10)\ \Omega = (10\angle 90)\ \Omega$$

By Ohm's law, $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1} = \dfrac{(230\angle 0)}{(20\angle 0)} = (11.5\angle 0) \text{ A}$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{(230\angle 0)}{(10\angle 90)} = (23\angle -90) \text{ A}$$

Total current, $\bar{I} = \bar{I}_1 + \bar{I}_2$

$$= (11.5\angle 0) + (23\angle -90)$$

$$= (11.5 + j0) + (0 - j23)$$

$$= (11.5 - j23) \text{ A}$$

$$= (25.71\angle -63.43) \text{ A}$$

So, $\text{pf} = \cos\phi$

$$= \cos 63.43$$

$$= 0.447 \text{ lagging}$$

Power, $P = VI\cos\phi$

$$= 230 \times 25.71 \times 0.447$$

$$= 2643 \text{ W}$$

$= 2643$ W

Example 2.67 For a circuit shown in Fig. 2.124, determine:

(i) Total impedance of the circuit and total current

(ii) Branch current I_1 and I_2

(iii) Power factor of each branch and total power factor

(iv) Power consumed by each branch

Solution

Assuming the different impedances, we get Fig. 2.125.

Taking applied voltage as reference phasor,

$$\bar{V} = (230 \angle 0)$$

$$L = 10 \times 10^{-3} \text{ H}; X_L = 2\pi f L = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\ \Omega$$

$$C = 50 \times 10^{-6} \text{ F}; X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 500 \times 10^{-6}} = 6.37\ \Omega$$

$$\bar{Z}_1 = (10 + j3.14)\ \Omega = (10.48 \angle 17.43)\ \Omega$$

$$\bar{Z}_2 = (10 - j6.37)\ \Omega = (11.86 \angle -32.5)\ \Omega$$

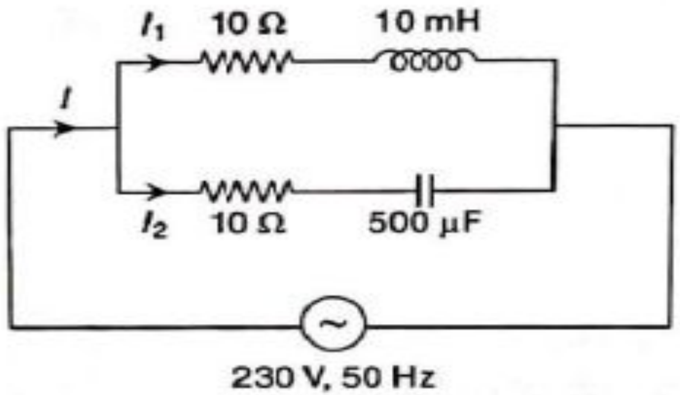


Fig. 2.124

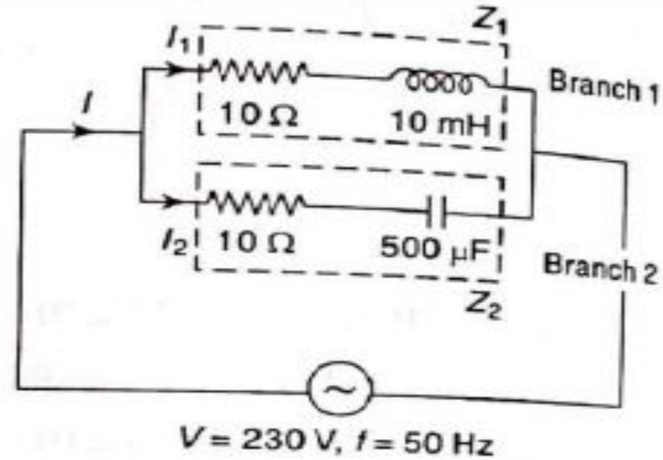


Fig. 2.125

(i) Total impedance of the circuit,

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(10.48 \angle 17.43)\,(11.86 \angle -32.5)}{(10 + j3.14) + (10 - j6.37)}$$

$$= \frac{(124.29 \angle -15.07)}{(20 - j3.23)}$$

$$= \frac{(124.29 \angle -15.07)}{(20.26 \angle -9.17)}$$

$$= (6.13 \angle -5.9)\ \Omega$$

$$\text{Total current, } I = \frac{\bar{V}}{\bar{\bar{Z}}} = \frac{(230 \angle 0)}{(6.13 \angle -5.9)} = (37.52 \angle 5.9) \text{ A}$$

(ii) Branch current, $$\bar{I}_1 = \frac{\bar{V}}{\bar{\bar{Z}}_1} = \frac{(230 \angle 0)}{(10.48 \angle 17.43)} = (21.95 \angle -17.43) \text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{\bar{Z}}_2} = \frac{(230 \angle 0)}{(11.86 \angle -32.5)} = (19.39 \angle 32.5) \text{ A}$$

(iii) Power factor of branch 1,

$$(\text{pf})_1 = \cos \phi_1$$

where ϕ_1 is the phase angle of branch 1. From polar form of impedance Z_1, $\phi_1 = 17.43$. Branch 1 is inductive.

So, $$(\text{pf})_1 = \cos 17.43$$
$$= 0.954 \text{ lagging}$$

Power factor of branch 2,

$$(pf)_2 = \cos \phi_2$$

where ϕ_2 is the phase angle of branch 2. From polar form of impedance Z_2,
$\phi_2 = -32.5$. Branch 2 is capacitive.

So, $(pf)_2 = \cos(-32.5)$

$= 0.843$ leading

Total power factor.

$$pf = \cos \phi$$

where ϕ is the phase angle total circuit. From polar form of impedance Z,
$\phi = -5.9$. As phase angle is negative total circuit is capacitive.

So, $pf = \cos(-5.9)$

$= 0.995$ leading

(iv) Power consumed by branch 1,

$$P_1 = I_1^2 R = (21.95)^2 \times 10 = 4.818 \text{ kW}$$

Power consumed by branch 2,

$$P_2 = I_2^2 R = (19.39)^2 \times 10 = 3.76 \text{ kW}$$

Example 2.69 Two impedances, $\overline{Z}_1 = (10 + j15)\ \Omega$ and $\overline{Z}_2 = (6 - j8)\ \Omega$ are connected in parallel. The total current supplied is 15 A. What is the power taken by each branch?

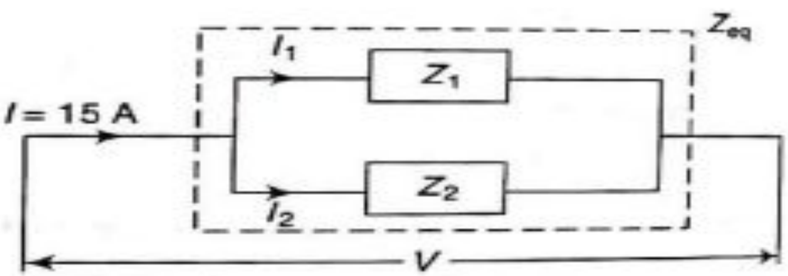


Fig. 2.127

Solution

The conditions in the example are shown in Fig. 2.127.

$$\overline{Z}_1 = (10 + j15)\ \Omega = (18.03\ \angle 56.31)\ \Omega$$

$$\overline{Z}_2 = (6 - j8)\ \Omega = (10\ \angle -53.13)\ \Omega$$

Total or equivalent impedance of the circuit,

$$\overline{Z}_{eq} = \frac{\overline{Z}_1 \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2}$$

$$= \frac{(18.03\ \angle 56.31)\ (10\ \angle -53.13)}{(10 + j15) + (6 - j8)}$$

$$= \frac{(180.3\ \angle 3.18)}{(16 + j7)}$$

$$= \frac{(180.3\ \angle 3.18)}{(17.46\ \angle 23.63)}$$

$$= (10.33\ \angle -20.45)\ \Omega$$

By Ohm's law,

Voltage across the circuit, $V = IZ_{eq}$

$$= 15 \times 10.33$$

$$= 154.95\ \text{V}$$

Branch current, $I_1 = \frac{V}{Z_1} = \frac{154.95}{18.03} = 8.59\text{ A}$

Branch current, $I_2 = \frac{V}{Z_2} = \frac{154.95}{10} = 15.495\text{ A}$

Hence,

Power taken by the first branch, $P_1 = I_1^2 R_1 = (8.59)^2 \times 10 = 737.88\text{ W}$

Power taken by the second branch, $P_2 = I_2^2 R_2 = (15.495)^2 \times 6 = 1440.57\text{ W}$

Example 2.70 Two circuits A and B are connected in parallel across a 200 V, 50 Hz mains. Circuit A consists of a resistance of 10 Ω and an inductance of 0.12 H connected in series. Circuit B consists of a resistance of 20 Ω in series with a capacitor of 40 μF. Calculate (i) current in each branch and (ii) power factor. Draw phasor diagram.

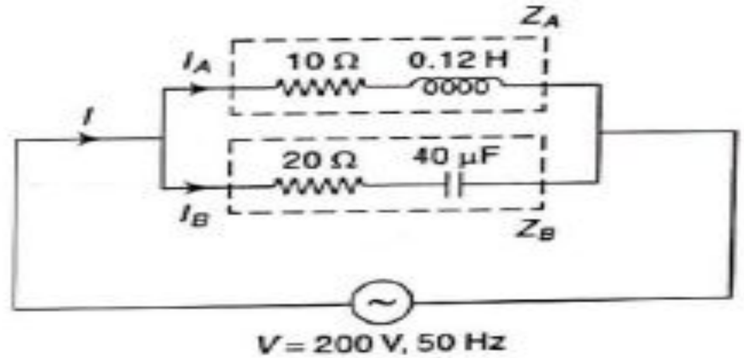


Fig. 2.128

Solution

The conditions in the example are shown in Fig. 2.128. Assuming the various impedances, currents and voltage, we have,

$$L = 0.12 \text{ H}; X_L = 2\pi fL = 2\pi \times 50 \times 0.12 = 37.7 \ \Omega$$

$$C = 40 \times 10^{-6} \text{ F}; X_C = \frac{1}{2\pi FC} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}} = 79.58 \ \Omega$$

$$\overline{Z}_A = (10 + j37.7) \ \Omega = (39 \angle 75.14) \ \Omega$$

$$\overline{Z}_B = (20 - j79.58) \ \Omega = (82.05 \angle -75.89) \ \Omega$$

Total or equivalent impedance of the circuit,

$$Z_{eq} = \frac{\overline{Z}_A \overline{Z}_B}{\overline{Z}_A + \overline{Z}_B} = \frac{(39 \angle 75.14)(82.05 \angle -75.89)}{(10 + j37.7)(20 - j79.58)}$$

$$= \frac{(3200 \angle -0.75)}{(30 - j41.88)} = \frac{(3200 \angle -0.75)}{(51.52 \angle -54.38)}$$

$$= (62.11 \ \angle 53.63) \ \Omega$$

Taking applied voltage as reference, we have

$$\overline{V} = (200 \angle 0) \text{ V}$$

(i) Branch current, $\bar{I}_A = \dfrac{\bar{V}}{\bar{Z}_A} = \dfrac{(200\angle 0)}{(39\angle 75.14)} = (5.13\angle -75.14)\text{ A}$

Branch current, $\bar{I}_B = \dfrac{\bar{V}}{\bar{Z}_B} = \dfrac{(200\angle 0)}{(82.05\angle -75.89)} = (2.44\angle 75.89)\text{ A}$

(ii) pf = cos 53.63
= 0.59 lagging

Phasor diagram (Fig. 2.129):

Taking applied voltage as reference phasor.

From circuit diagram, $\bar{I} = \bar{I}_A + \bar{I}_B$

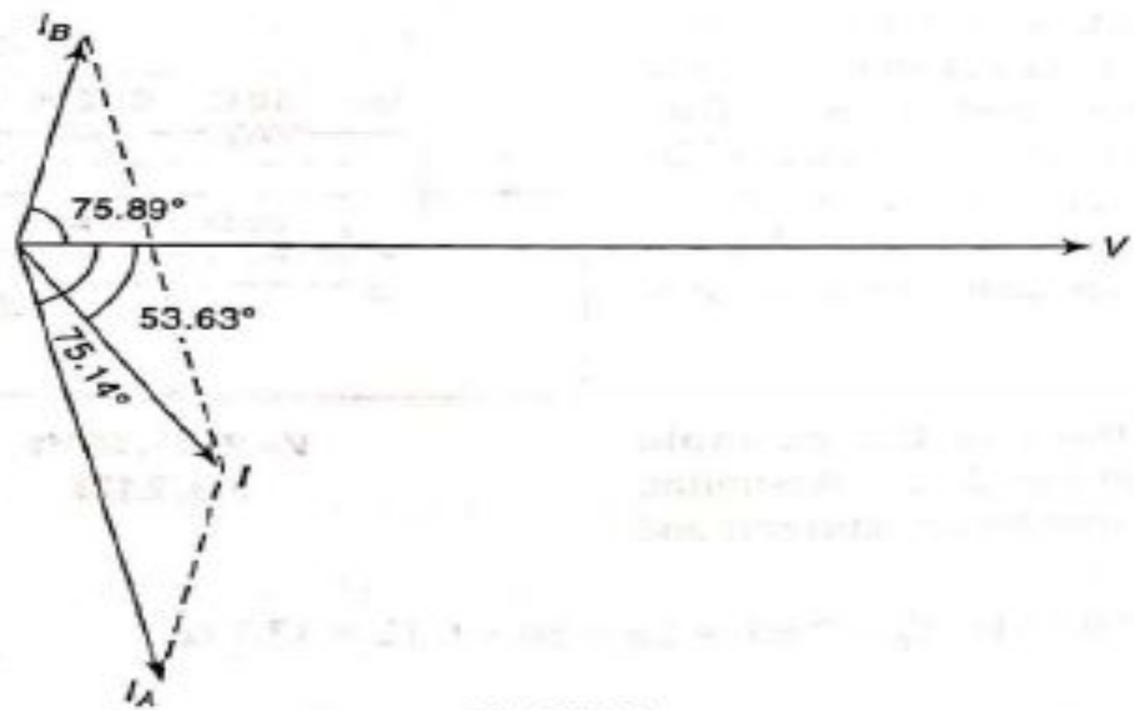


Fig. 2.129

Fig. 2.129

Example 2.71 Two impedances $(20 \angle -45)\ \Omega$ and $(30 \angle 30)\ \Omega$ are connected in series across a certain ac supply and the resulting current is drawn to be 10 A. If the supply voltage remains unchanged, calculate the supply current, when the two impedances are connected in parallel.

Solution

Let $\overline{Z}_1 = (20 \angle -45)\ \Omega = (14.14 - j14.14)\ \Omega$

$\overline{Z}_2 = (30 \angle 30)\ \Omega = (26 + j15)\ \Omega$

When impedances are in series, current is 10 A (Fig. 2.130):

Total impedances,

$$\overline{Z}_{eq} = \overline{Z}_1 + \overline{Z}_2$$

$$= (14.14 - j14.14) + (26 + j15)$$

$$= (40.14 + j0.86)\ \Omega$$

$$= (40.15 \angle 1.23)\ \Omega$$

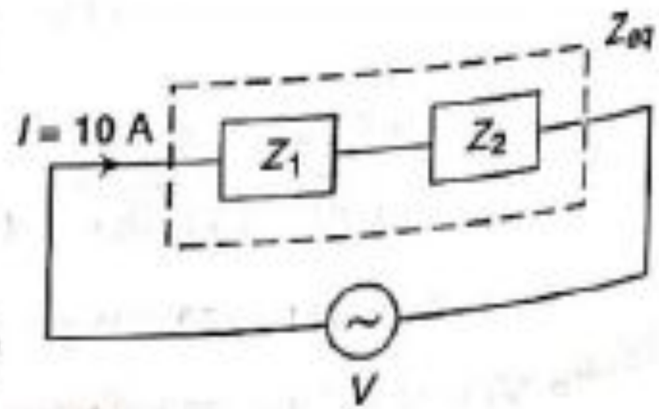


Fig. 2.130

Now, taking 'I' as reference, $I = (10 \angle 0)$

$\therefore$ Supplied voltage $= \bar{I}\bar{Z}_{eq}$

$$= (10 \angle 0)\,(40.15 \angle 1.23)$$
$$= (401.5 \angle 1.23)\ \text{V}$$

When impedances are in parallel across the same supply (Fig. 2.131):

Total impedances,

$$\bar{Z}_{eq} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(20\angle -45)\,(30\angle 30)}{(14.14 - j14.14) + (26 + j15)}$$

$$= \frac{(600\angle -15)}{(40.14 + j0.86)}$$

$$= \frac{(600\angle -15)}{(40.15\angle 1.23)}$$

$$= (14.94\angle -16.23)\,\Omega$$

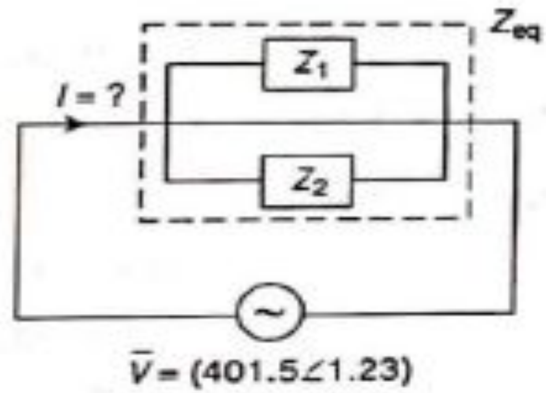


Fig. 2.131

Hence, Supply current, $I = \dfrac{\bar{V}}{\bar{Z}_{eq}}$

$$= \frac{(401.5\angle 1.23)}{(14.94\angle -16.23)}$$

$$= (26.87 \angle 17.46)\ \text{A}$$

Fig. 2.147

2.3.3 Admittance (Y)

The admittance is defined as the reciprocal of impedance, i.e.,

$$\text{Admittance, } Y = \frac{1}{Z} = \frac{I}{V} \quad \text{or} \quad \overline{Y} = \frac{1}{\overline{Z}} = \frac{\overline{I}}{\overline{V}}$$

The unit of admittance is mho (ohm spelt backward) and its symbol is ℧. The admittance of the circuit may be considered as a measure of the ease with which a circuit can conduct alternating current.

Thus, a circuit with higher value of admittance will have a higher value of current.

Consider a parallel circuit as shown in Fig. 2.148.

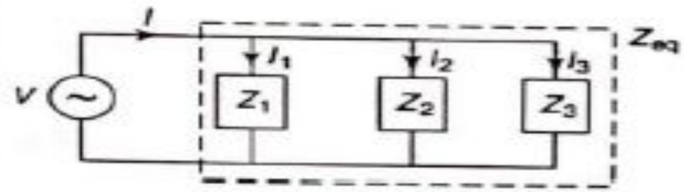


Fig. 2.148 Parallel circuit

By law of parallel circuit,

$$\frac{1}{\overline{Z}_{eq}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3}$$

Since admittance is reciprocal of impedance, we have

$$\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3$$

where Y_1, Y_2, and Y_3 are the individual admittances of the parallel branches and Y_{eq} is the total or equivalent admittance of the circuit.

So, Line current, $\overline{I} = \dfrac{\overline{V}}{\overline{Z}_{eq}} = \overline{V}\,\overline{Y}_{eq}$

Components of admittance

The impedance of the circuit can be expressed in the complex form as $\overline{Z} = (R + jX_L)\,\Omega$ or $\overline{Z} = (R - jX_C)\,\Omega$ depending upon the nature of reactance, where R is the resistance or in phase component of Z while X_L or X_C is the reactive or quadrature component of Z. The reciprocal of impedance (i.e., admittance) will also have a complex form.

Therefore, admittance $\overline{Y}$ can be expressed as

$$\overline{Y} = (G - jB_L)\,℧ \quad \text{or} \quad \overline{Y} = (G + jB_C)\,℧$$

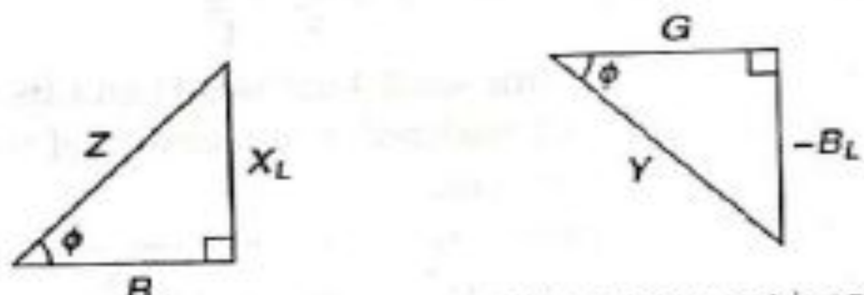
where G is called conductance and represents the in-phase component of $\overline{Y}$, while B is called the susceptance and represents the quadrature component of $\overline{Y}$. The susceptance of an inductance is specially called inductive susceptance B_L whereas that of capacitance is called capacitive susceptance B_C. Note that B_L is always negative while B_C is always positive.

The units of G and B will also be mho (℧).

Admittance triangle

Since admittance has in-phase component (i.e., G) as well as quadrature component (i.e., B_L or B_C), it can be represented by a right-angled triangle, called admittance triangle.

(i) For an inductive circuit, the impedance and admittance triangle are shown in Fig. 2.149.



(a) Impedance triangle (b) Admittance triangle

Fig. 2.149

Note that admittance angle is equal to the impedance angle but is negative.

Conductance, $G = Y\cos\phi = \frac{1}{Z} \times \frac{R}{Z}$

So, $$G = \frac{R}{Z^2} = \frac{R}{R^2 + X_L^2}\ \mho$$

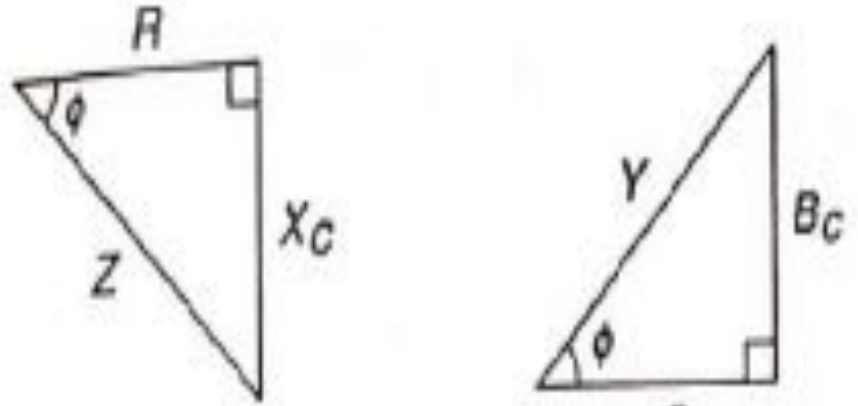
Susceptance, $B_L = Y\sin\phi = \frac{1}{Z} \times \frac{X_L}{Z}$

So, $$B_L = \frac{X_L}{Z^2} = \frac{X_L}{R^2 + X_L^2}\ \mho$$

(ii) For a capacitive circuit, the impedance and admittance triangle are shown in Fig. 2.150.

Conductance, $G = Y\cos\phi = \frac{1}{Z} \times \frac{R}{Z}$

So, $$G = \frac{R}{Z^2} = \frac{R}{R^2 + X_C^2}\ \mho$$



(a) Impedance triangle (b) Admittance triangle

Fig. 2.150

Susceptance, $B_C = Y \sin\phi = \dfrac{1}{Z} \cdot \dfrac{X_C}{Z}$

So, $$B_C = \frac{X_C}{Z^2} = \frac{X_C}{R^2 + X_C^2}\ \mho$$

Example 2.82 Calculate the admittance ($\bar{Y}$) and draw the admittance triangle of the circuit shown in Fig. 2.151.

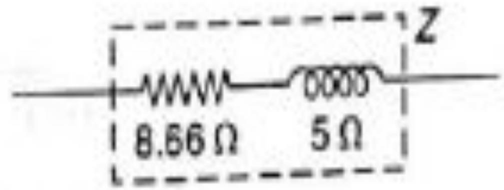


Fig. 2.151

Solution

Circuit impedance, $\bar{Z} = (8.66 + j5)\ \Omega$

$$= (10 \angle 30)\ \Omega$$

So, Circuit admittance, $\bar{Y} = \dfrac{1}{\bar{Z}}$

$$= \frac{1}{(10 \angle 30)}$$

$$= (0.1 \angle -30)\ \mho$$

$$= (0.0866 - j0.05)\ \mho$$

From polar form of admittance, $Y = 0.1\ \mho$ and $\phi = -30^\circ$.

From rectangular form of admittance,

$$G = 0.0866\ \mho \text{ and } B_L = 0.05\ \mho$$

The admittance triangle is shown in Fig. 2.152.

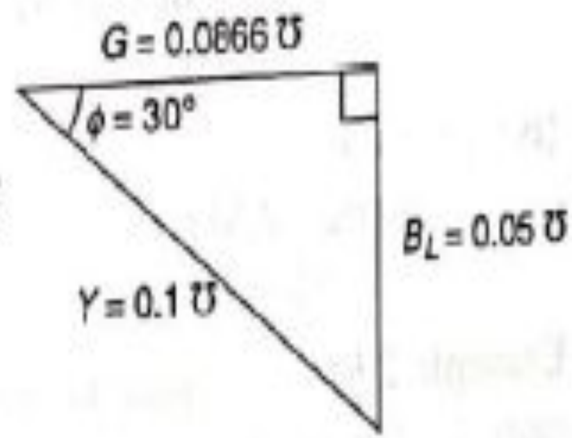


Fig. 2.152

...ance triangle is shown in Fig. 2.152.

Example 2.83 Three loads are placed across 230 V, 50 Hz supply. The loads are $(10\angle{-30})\,\Omega$; $(20\angle 60)\,\Omega$ and $(40\angle 0)\,\Omega$. Determine (i) admittance, (ii) equivalent impedance, (iii) power consumed, and (iv) power factor.

Solution

The conditions in the example are shown in Fig. 2.153.

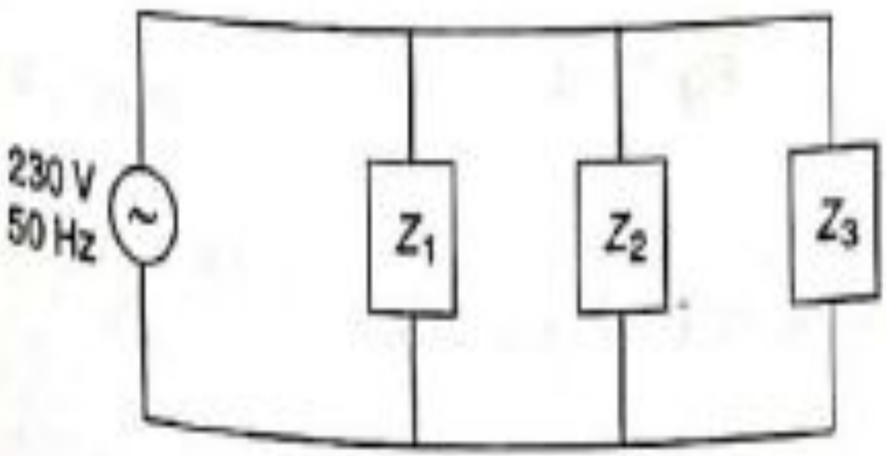


Fig. 2.153

Let

$\bar{Z}_1 = (10\angle{-30})\Omega$

$\bar{Z}_2 = (20\angle 60)\Omega$

$\bar{Z}_3 = (40\angle 0)\Omega$

(i) $\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{(10 \angle -30)} = (0.1 \angle 30)\ ℧$

$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(20 \angle 60)} = (0.05 \angle -60)\ ℧$

$\bar{Y}_3 = \frac{1}{\bar{Z}_3} = \frac{1}{(40 \angle 0)} = (0.025 \angle 0)\ ℧$

The total admittance of the circuit,

$$\begin{aligned}\bar{Y} &= \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 \\ &= (0.1 \angle 30) + (0.05 \angle -60) + (0.025 \angle 0) \\ &= (0.0866 + j0.05) + (0.025 - j0.0433) + (0.025 + j0) \\ &= (0.1366 + j0.0067)\ ℧ \\ &= (0.137 \angle 2.81)\ ℧\end{aligned}$$

(ii) Equivalent impedance of the circuit,

$$\bar{Z} = \frac{1}{\bar{Y}} = \frac{1}{(0.137 \angle 2.81)} = (7.3 \angle -2.81)\ \Omega$$

(iii) Total current, $I = \frac{V}{Z} = \frac{230}{7.3} = 31.51$ A

Power consumed by the circuit,

$$\begin{aligned}P &= VI \cos \phi \\ &= 230 \times 31.51 \times \cos(-2.81) \\ &= 7238 \text{ W}\end{aligned}$$

(iv) $$\begin{aligned}\text{pf} &= \cos \phi \\ &= \cos(-2.81) \\ &= 0.998 \text{ leading}\end{aligned}$$

Example 2.84

= 0.998 leading

Example 2.84 Compute the equivalent impedance Z_{eq} and equivalent admittance Y_{eq} for a circuit shown in Fig. 2.154. Also calculate the total current.

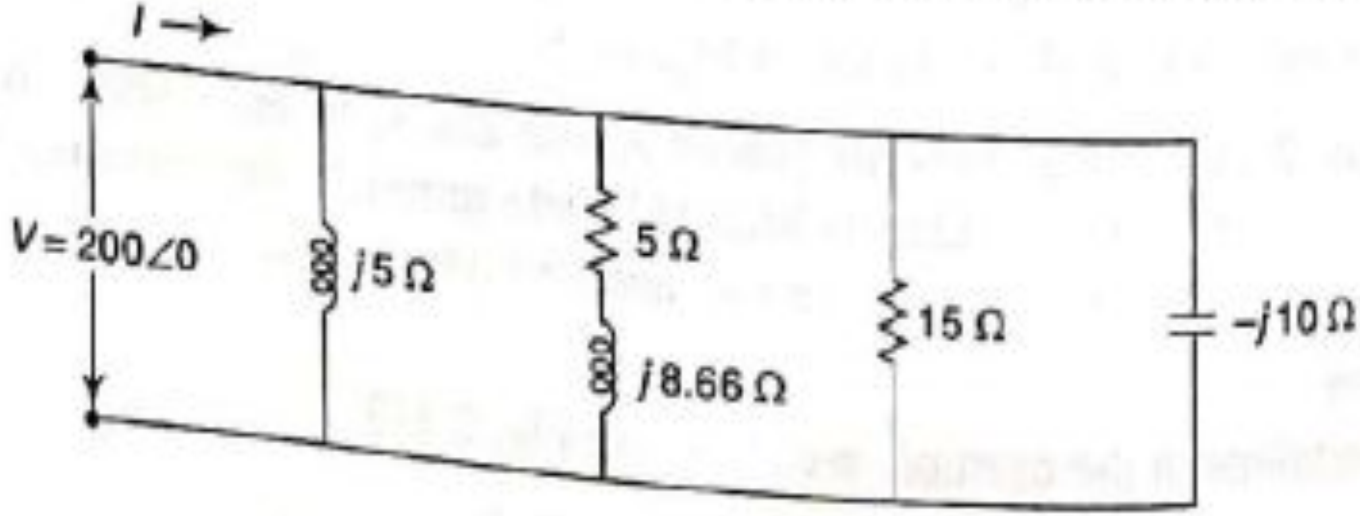


Fig. 2.154

Solution

Marking the various impedances, we get Fig. 2.155.

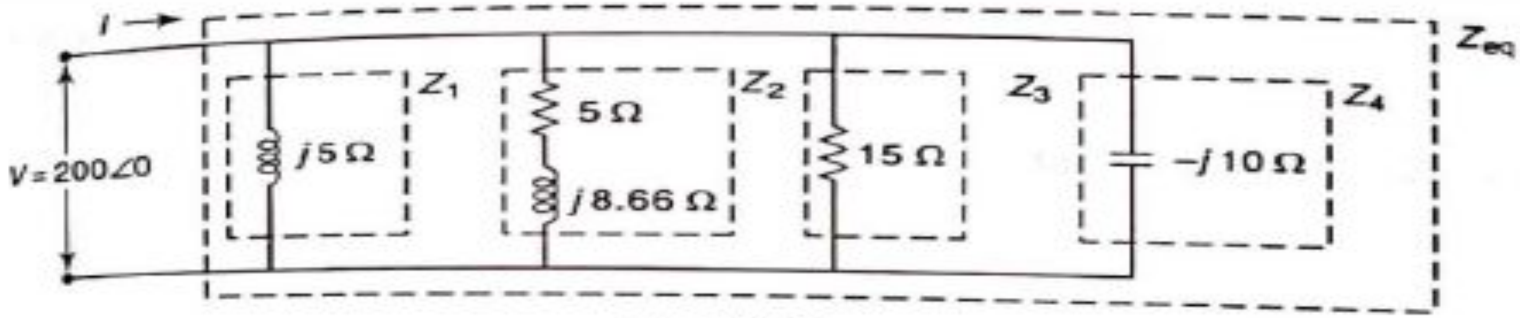


Fig. 2.155

Given: $\bar{Z}_1 = (0 + j5)\ \Omega = (5\ \angle 90)\ \Omega$

$\bar{Z}_2 = (5 + j8.66)\ \Omega = (10\ \angle 60)\ \Omega$

$\bar{Z}_3 = (15 + j0)\ \Omega = (15\ \angle 0)\ \Omega$

$\bar{Z}_4 = (0 - j10)\ \Omega = (10\ \angle -90)\ \Omega$

The individual admittances of the parallel branches are:

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{(5\angle 90)} = (0.2\ \angle -90)\ \mho$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(10\angle 60)} = (0.1\ \angle -60)\ \mho$$

$$\bar{Y}_3 = \frac{1}{\bar{Z}_3} = \frac{1}{(15\angle 0)} = (0.0667\ \angle 0)\ \mho$$

$$\bar{Y}_4 = \frac{1}{\bar{Z}_4} = \frac{1}{(10\angle -90)} = (0.1\ \angle 90)\ \mho$$

The equivalent admit

$$Z_4 \quad (10 \angle -90) = (0.1 \angle 90)\ \mho$$

The equivalent admittance of the circuit,

$$\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \bar{Y}_4$$

$$= (0.2 \angle -90) + (0.1 \angle -60) + (0.0677 \angle 0) + (0.1 \angle 90)$$

$$= (0 - j0.2) + (0.05 - j0.0866) + (0.0677 + j0) + (0 + j0.1)$$

$$= (0.1177 - j0.1866)\ \mho$$

$$= (0.22 \angle -57.76)\ \mho$$

The equivalent impedance of the circuit,

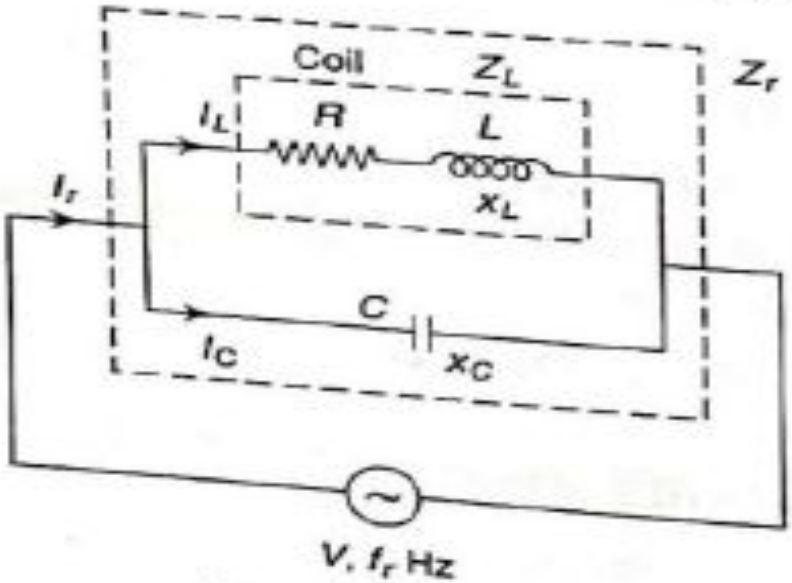
$$\bar{Z}_{eq} = \frac{1}{\bar{Y}_{eq}} = \frac{1}{(0.22 \angle -57.76)} = (4.545 \angle 57.76)\ \Omega$$

The total current,

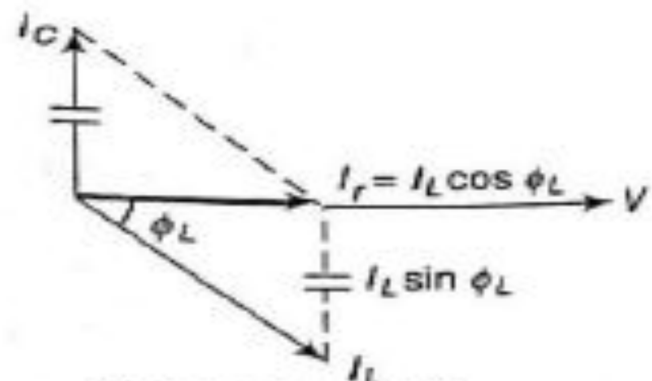
$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \bar{V}\,\bar{Y}_{eq} = (200 \angle 0)\,(0.22 \angle -57.76) = (44 \angle -57.76)\ \text{A}$$

2.3.4 Parallel Resonance

Like series resonance, a parallel circuit containing reactive elements (L and C) is resonant when the circuit power factor is unity, i.e., applied voltage and the supply current are in phase. It is called a parallel resonance because it concerns a parallel circuit. The most common resonant parallel circuit is an inductor (coil) in parallel with a pure capacitor C as shown in Fig. 2.163(a). The phasor diagram of this parallel circuit is shown in Fig. 2.163(b). The coil current I_L has two



(a) Circuit diagram



Phase angle, $\phi = 0$
$pf = 1$ (unity)
$I_C = I_L \sin \phi_L$
and $I = I_L \cos \phi_L$

(b) Phasor diagram (when $I_C = I_L \sin \phi_L$)

Fig. 2.163 Parallel resonance

components, viz active component $I_L \cos \phi_L$ and reactive component $I_L \sin \phi_L$. This parallel circuit will resonant when the circuit power factor is unity. This is possible only when the net reactive component of the circuit is zero,

i.e., $\quad I_C - I_L \sin \phi_L = 0$

or $\quad I_C = I_L \sin \phi_L$

or Reactive component of I_C = Reactive component of I_L

Resonance in a parallel circuit can be achieved by changing the supply frequency. Increase of frequency increase the reactance of the coil and consequently the coil impedance increases. The coil current I_L, therefore, decreases and lags behind the applied voltage V by a progressively greater angle. The capacitive branch current on the other hand, increases although it will always lead V by 90°. At some frequency f_r (called resonant frequency), I_C becomes equal to $I_L \sin \phi_L$ and resonance occurs.

(ii) *Resonance/resonant frequency (f_r)*

The supply frequency at which parallel resonance occurs (i.e., reactive component of circuit current becomes zero) is called the resonant frequency.

At parallel resonance,

$$I_C = I_L \sin\phi_L$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L}$$

$$\frac{1}{X_C} = \frac{X_L}{Z_L^2}$$

$$Z_L^2 = \frac{L}{C}$$

$$R^2 + X_L^2 = \frac{L}{C} \tag{2.25}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{2.26}$$

If R is very very small,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{2.27}$$

(ii) Impedance at resonance (Z_r)

At parallel resonance,

$$I = I_L \cos \phi_L$$

or $$\frac{V}{Z_r} = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

or $$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

or $$Z_r = \frac{Z_L^2}{R}$$

From Eq. (2.25), $Z_L^2 = \dfrac{L}{C}$

So, Circuit impedance, $Z_r = \dfrac{L}{CR}\,\Omega$ (2.28)

Thus, at parallel resonance ...

CR (2.28)

Thus, at parallel resonance, the circuit impedance is equal to L/CR. This is known as equivalent or dynamic impedance of the parallel circuit at resonance.

Z_r is the pure resistance because in the expression of Z_r, frequency term is not present. Also Z_r is very high because the ratio L/C is very large at parallel resonance.

As supply frequency changes, value of circuit impedance also varies. If we plot impedance–frequency graph for a parallel circuit shown in Fig. 2.163(a), the shape of the curve will be as shown in Fig. 2.164. Note that impedance of the circuit is maximum at resonance. As the frequency changes from resonance, the circuit impedance decreases very rapidly.

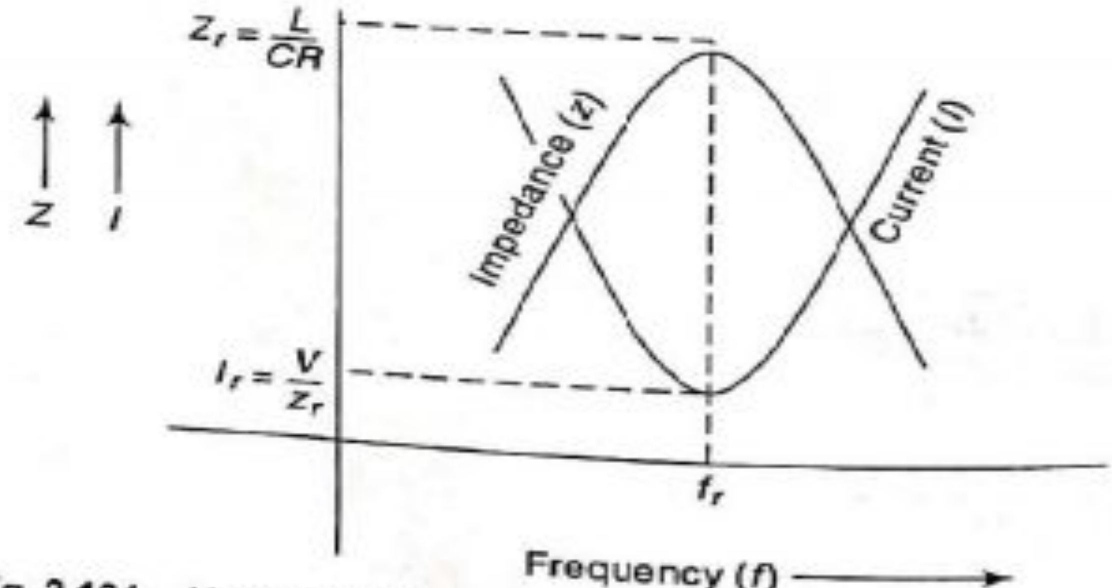


Fig. 2.164 Variation of circuit impedance and current with frequency

(iii) Circuit current at resonance (I_r)

At parallel resonance, the circuit impedance (Z_r) becomes maximum and the circuit current (I_r) is at minimum value. The current–frequency curve of the parallel circuit is shown in Fig. 2.143. Note that the value of line current is minimum at resonance.

At parallel resonance, the impedance of each branch (i.e., X_C and Z_L) is relatively small compared with circuit impedance (Z_r). Therefore, current flowing through the capacitor and coil are much greater than the line current I_r. But these currents are approximately 180° out of phase, therefore resultant current I_r is extremely small.

(iv) Q-factor of parallel resonance circuit

At parallel resonance, the current circulating between the two branches is many times greater than the line current. The current amplification produced by the resonance is termed as Q-factor of the parallel resonant circuit, i.e.,

$$Q\text{-factor} = \frac{I_L \text{ or } I_C}{I_r} \qquad (\because I_L \equiv I_C) \tag{2.29}$$

Let $\quad Q\text{-factor} = \dfrac{I_C}{I_r}$

Now, $\quad I_C = \dfrac{V}{X_C} = 2\pi f_r CV \quad$ and $\quad I_r = \dfrac{V}{(L/CR)}$

So, $\quad Q\text{-factor} = \dfrac{2\pi f_r CV}{\dfrac{V}{\left(\dfrac{L}{CR}\right)}}$

$$= \frac{2\pi f_r L}{R} \tag{2.30}$$

Now,

Now, $f_r = \frac{1}{2\pi\sqrt{LC}}$ (2.30)

By substituting the value of f_r in Eq. (2.30), we get

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad (2.31)$$

2.3.5 Comparison of Series and Parallel Resonant Circuits

Particular	Series circuit	Parallel circuit
1. Impedance at resonance	Minimum ($Z_r = R$)	Maximum ($Z_r = L/CR$)
2. Current at resonance	Maximum $\left(I_r = \frac{V}{R}\right)$	Minimum $\left(I_r = \frac{V}{Z_r}\right)$
3. Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz
4. Q-Factor	$= \frac{V_L \text{ or } V_C}{V}$	$= \frac{I_L \text{ or } I_C}{I}$
5. It magnifies	Voltage	Current
6. When $f < f_r$	Circuit is capacitive.	Circuit is inductive.
7. When $f > f_r$	Circuit is inductive.	Circuit is capacitive.

Example 2.89 A parallel circuit consists of a 2.5 μF capacitor and a coil whose resistance and inductance are 15 Ω and 260 mH, respectively. Determine (i) the resonant frequency, (ii) *Q*-factor of the circuit at resonance, and (iii) dynamic impedance of the circuit.

Solution

(i) Resonant frequency (f_r),

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{260\times10^{-3}\times2.5\times10^{-6}} - \frac{(15)^2}{(260\times10^{-3})^2}}$$

$$= 197.19 \text{ Hz}$$

(ii) $Q\text{-factor} = \frac{2\pi f_r L}{R} = \frac{2\pi\times197.19\times260\times10^{-3}}{15} = 21.48$

(iii) Dynamic impedance (Z_r),

$$Z_r = \frac{L}{CR} = \frac{260\times10^{-3}}{2.5\times10^{-6}\times15} = 6933.33\ \Omega$$

0.7 H is

Example 2.90 An inductive coil of resistance 20 Ω and inductance 0.2 H is connected in parallel with 200 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with supply voltage. Also find the value of this current. Draw the phasor diagram.

Solution

The conditions in the example are shown in Fig. 2.165.

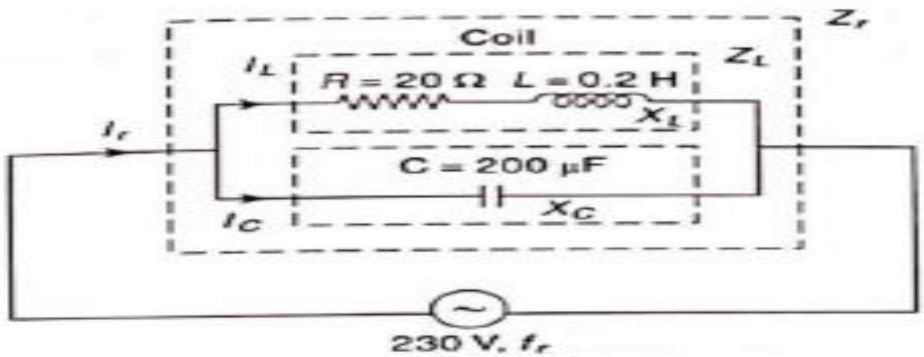


Fig. 2.165

Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{Hz}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 200 \times 10^{-6}} - \frac{20^2}{(0.2)^2}}$$

$$= \frac{1}{2\pi}\sqrt{25000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{15000}$$

$$= 19.49 \text{ Hz}$$

Dynamic impedance of the circuit,

$$Z_r = \frac{L}{CR}$$

$$= \frac{0.2}{200 \times 10^{-6} \times 20}$$

$$= 50\ \Omega$$

Circuit

$$= 50\ \Omega$$

Circuit current at resonance,

$$I_r = \frac{V}{Z_r} = \frac{230}{50} = 4.6\ \text{A}$$

Phasor diagram (see Fig. 2.145):

For phasor diagram, we need to calculate the values of branch currents (I_L and I_C) and phase angle of the coil (ϕ_L).

Now,
$$I_L = \frac{V}{Z_L} = \frac{230}{\sqrt{(20)^2 + (2\pi \times 19.49 \times 0.2)^2}} = \frac{230}{31.62} = 7.274\ \text{A}$$

$$I_C = \frac{V}{X_C} = \frac{230}{\left(\dfrac{1}{2\pi \times 19.49 \times 200 \times 10^{-6}}\right)} = \frac{230}{40.83} = 5.63 \text{ A}$$

Phase angle of the coil,

$$\phi_L = \tan^{-1} \frac{X_L}{R}$$

$$= \tan^{-1} \frac{(2\pi f_r L)}{R}$$

$$= \tan^{-1} \frac{(2\pi \times 19.49 \times 0.2)}{20}$$

$$= \tan^{-1} \frac{24.49}{20}$$

So, $\phi_L = 50.76$

Take V as reference phasor.

We know that $\bar{I}_r = \bar{I}_L + \bar{I}_C$.

Now, $I_L = 7.274$ A

$I_C = 5.63$ A

$I_r = 4.6$ A

$\phi_L = 50.76°$

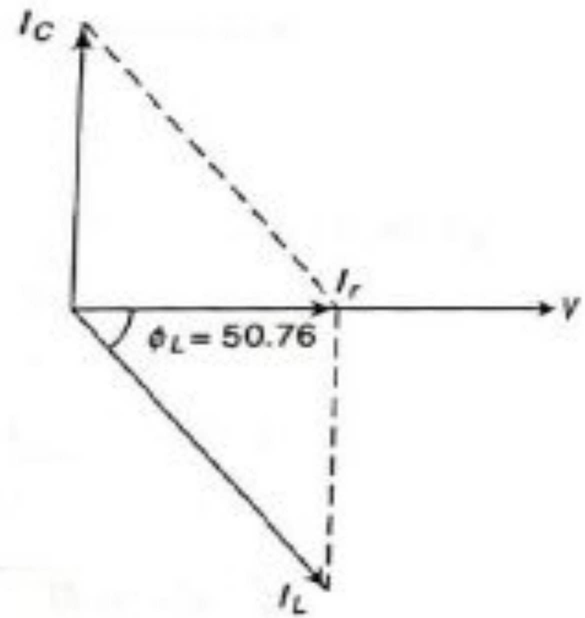


Fig. 2.166

Taking scale 1 cm = 1 A, the phasor diagram can be drawn as shown in Fig. 2.166.

Example 2.91 A coil takes a current of 1 A at 0.3 pf when connected to a 100 V, 50 Hz supply. Determine the value of the capacitance, which when connected in parallel with the coil, will reduce the line current to a minimum. Calculate the impedance of the parallel circuit at 50 Hz.

Solution

When a coil is connected to 100 V, 50 Hz supply, it takes a current of 1 A at 0.3 pf (see Fig. 2.167).

$$(\text{pf})_{\text{coil}} = 0.3$$

or $\cos \phi_L = 0.3$

or $\phi_L = 72.542$ (ϕ_L is the phase angle of the coil)

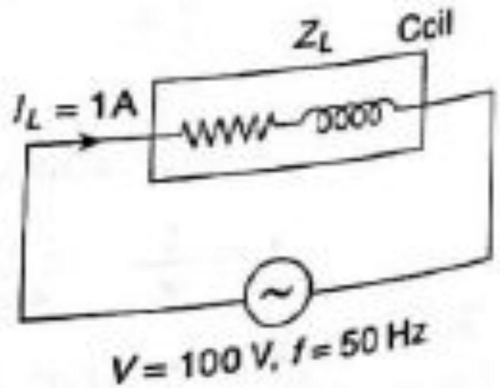


Fig. 2.167

Now, capacitor is connected across the coil (see Fig. 2.168). We need a value of C, so that the line current reduces to a minimum value, i.e., parallel resonance occurs at $f = 50$ Hz.

At resonance, $I_C = I_L \sin \phi_L$

So, $I_C = 1 \times \sin (72.542)$

or $I_C = 0.954$ A

$$\text{Now, } X_C = \frac{V}{I_C} = \frac{100}{0.954} = 104.82\ \Omega$$

So, $$\frac{1}{2\pi f_r C} = 104.82$$

or $$\frac{1}{2\pi \times 50 \times C} = 104.82$$

or $$C = 30.37 \times 10^{-6}\ \text{F}$$
$$= 30.37\ \mu\text{F}$$

At resonance,

$$I_r = I_L \cos\phi_L$$

or $$I_r = 1 \times \cos 72.542$$

or $$I_r = 0.3\ \text{A}$$

Dynamic impedance of the circuit,

$$Z_r = \frac{V}{I_r} = \frac{100}{0.3} = 333.33\ \Omega$$

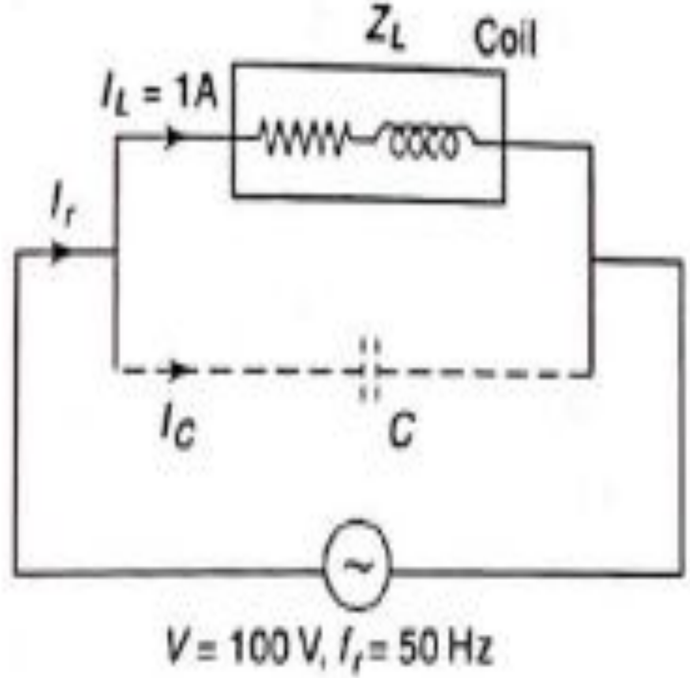


Fig. 2.168

Example 2.92 A circuit has $X_L = 20\ \Omega$ at 50 Hz, its resistance being 15 Ω. For an applied voltage of 200 V at 50 Hz, calculate (i) the pf, (ii) the current, (iii) the value of shunting capacitance to bring the resultant current into phase with the applied voltage, and (iv) the resultant current in case (iii).

Solution

Impedance of the circuit, $Z_L = \sqrt{(15)^2 + (20)^2}$

$= 25\ \Omega$

(i) $(\text{pf})_{\text{coil}} = \cos \phi_L = \dfrac{R}{Z_L} = \dfrac{15}{25}$

$= 0.6$ lagging

So, $\phi_L = \cos^{-1} 0.6 = 53.15$

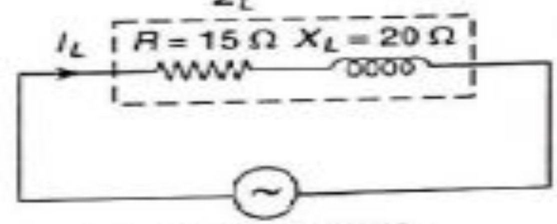


Fig. 2.169

(ii) Current, $I_L = \dfrac{V}{Z_L} = \dfrac{200}{25} = 8$ A

(iii) Now, capacitance is connected in parallel with the above circuit (see Fig. 2.170).

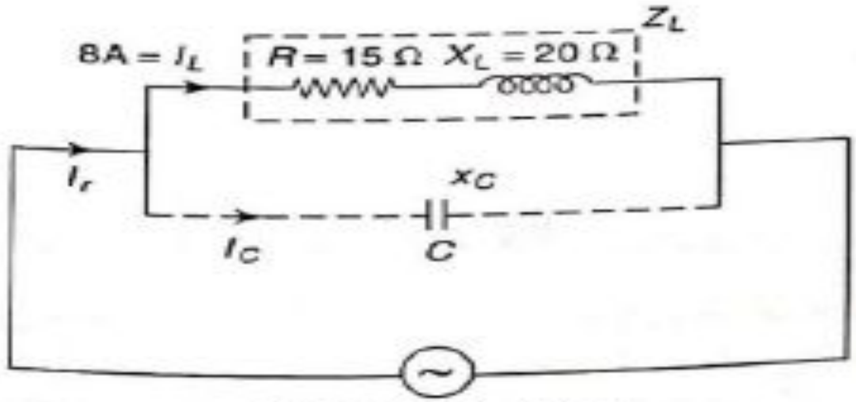


Fig. 2.170

We need a value of C, so that resulting current comes in phase with the applied voltage, i.e., pf = 1 (unity). In other words, parallel resonance occurs at $f = 50$ Hz.

At resonance,

$$I_C = I_L \sin \phi_L$$

So, $I_C = 8 \times \sin(53.13)$

or $I_C = 6.4$ A

Now, $$X_C = \frac{V}{I_C} = \frac{200}{6.4} = 31.25\ \Omega$$

So, $$\frac{1}{2\pi f_r C} = 31.25$$

or $$\frac{1}{2\pi \times 50 \times C} = 31.25$$

So, $$C = \frac{1}{2\pi \times 50 \times 31.25}$$

or $$C = 101.859 \times 10^{-6}\ \text{F}$$
$$= 101.859\ \mu\text{F}$$

(iv) The circuit current at resonance,

$$I_r = I_L \cos \phi_L$$

or $I_r = 8 \times 0.6$

or $I_r = 4.8$ A