Name : Yash Sarang

Seat no : 7128542

Roll no : 47 Class : DIAD

Subject : Engineering Mathematics 1

Signature : Sarangyash

Page no : 1/5

Q2.

A. If $\cos 6\theta = a\cos^6\theta + b\cos^4\theta\sin^2\theta + c\cos^2\theta\sin^4\theta + d\sin^6\theta$. find a,b,c,d.

→

$$(\cos\theta + i\sin\theta)^6 = {}^6C_0\cos^6\theta + {}^6C_1\cos^5\theta\sin\theta\, i$$
$$- {}^6C_2\cos^4\theta\sin^2\theta - {}^6C_3\cos^3\theta\sin^3\theta\, i$$
$$+ {}^6C_4\cos^2\theta\sin^4\theta + {}^6C_5\cos\theta\sin^5\theta\, i$$
$$- {}^6C_6\cos^0\theta\sin^6\theta.$$

$$= (\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta)$$
$$+ i(6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta)$$

$$\therefore \cos 6\theta + i\sin 6\theta = (\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\sin^4\theta - \sin^6\theta) + i(6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta)$$

$\therefore$ from real parts,

$$\cos 6\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta.$$

comparing with $\cos 6\theta = a\cos^6\theta + b\cos^4\theta\sin^2\theta + c\cos^2\theta\sin^4\theta + d\sin^6\theta$, we get

$$\boxed{a=1;\ b=-15;\ c=15;\ d=-1.}$$

Q2.
D Find a, b, c and A^{-1} if $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal. (Correction)

→ Given that A is orthogonal. $\therefore AA^T = I$.

$$\frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{9}\begin{bmatrix} 5+a^2 & 4+ab & ac-2 \\ 4+ab & 5+b^2 & 2+bc \\ ac-2 & 2+bc & 8+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating diagonal elements of both the matrices,

$\frac{5+a^2}{9} = 1$. $\therefore a = \pm 2$; $\frac{5+b^2}{9} = 1$ $\therefore b = \pm 2$;

$\frac{8+c^2}{9} = 1$ $\therefore c = \pm 1$.

Equating other elements, we get $4+ab=0$; $-2+ac=0$; $2+bc=0$.

$\therefore ab=-4$; $ac=2$; $bc=-2$.

$\therefore$ If $a=2$, then $b=2$ and $c=1$.

If $a=-2$, then $b=2$ and $c=-1$.

Since A is orthogonal, $A^{-1} = A$.

$$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \text{ or } \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

Q2.
F. If $u = f(x^2-y^2, y^2-z^2, z^2-x^2)$, then prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$.

→ let, $x^2-y^2 = p$; $y^2-z^2 = q$; $z^2-x^2 = r$.

$\therefore u = f(p,q,r)$ and p,q,r are functions of x,y,z.

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\times\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\times\frac{\partial q}{\partial x} + \frac{\partial u}{\partial r}\times\frac{\partial r}{\partial x}$$

$$= 2x\cdot\frac{\partial u}{\partial p} + -2x\frac{\partial u}{\partial r}$$

$$\therefore \frac{1}{x}\frac{\partial u}{\partial x} = 2\left(\frac{\partial u}{\partial p} - \frac{\partial u}{\partial r}\right) \quad \text{— ①}$$

Similarly, we can get

$$\frac{1}{y}\frac{\partial u}{\partial y} = 2\left(\frac{\partial u}{\partial q} - \frac{\partial u}{\partial p}\right) \quad \text{— ②}$$

$$\frac{1}{z}\frac{\partial u}{\partial z} = 2\left(\frac{\partial u}{\partial r} - \frac{\partial u}{\partial q}\right) \quad \text{— ③}$$

Now, adding ①, ② and ③,

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 2\left(\frac{\partial u}{\partial p} - \frac{\partial u}{\partial r} + \frac{\partial u}{\partial q} - \frac{\partial u}{\partial p} + \frac{\partial u}{\partial r} - \frac{\partial u}{\partial q}\right)$$

$$\boxed{\therefore \frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0}$$

Q2. E. Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum, using Lagrange's method.

→

Let, the three parts be x, y, z, respectively.

Let, $u = f(x,y,z) = xy^2z^3$ ——— ①

and

$\phi = x+y+z-24 = 0$ ——— ②

for Lagrange's function,

$$F = u + \lambda\phi$$

$$F = xy^2z^3 + \lambda(x+y+z-24)$$

$\therefore \frac{\partial F}{\partial x} = 0, \quad y^2z^3 + \lambda = 0$ ——— ③

$\frac{\partial F}{\partial y} = 0, \quad 2xyz^3 + \lambda = 0$ ——— ④

$\frac{\partial F}{\partial z} = 0, \quad 3xy^2z^2 + \lambda = 0$ ——— ⑤

$\therefore y^2z^3 = 2xyz^3 = 3xy^2z^2$

$\therefore \frac{1}{x} = \frac{2}{y} = \frac{3}{z} = k. \qquad \therefore x = 1/k, \; y = 2/k, \; z = 3/k$ ——— ⑥

$\therefore \frac{1}{k} + \frac{2}{k} + \frac{3}{k} = 24$ ——— (from ⑥ and ②) $\therefore k = \frac{1}{4}$

$\therefore x = 4, \; y = 8, \; z = 12.$

$\therefore$ The three required parts are 4, 8 and 12.