Module 4

JESSY

INTERFERENCE IN THIN FILM

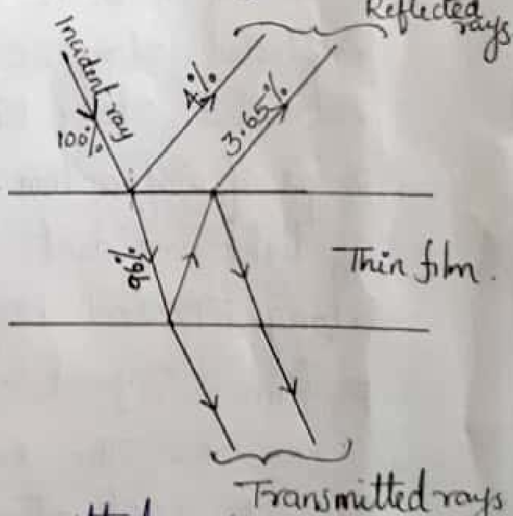
The redestribution of light energy due to the superposition of two or more light waves is called interference

(a) Division of wavefront	(b) Division of amplitude
In this method narrow slit is using as the source and the wavefront is divided. eg. Young's double slit Experiment	In this method amplitude of light beam is divided by Partial reflection or refraction in to two or more beams.

- **Interference by Division of amplitude**

E.g. Interference in Thin films.

A thin film is an optical medium of thickness of the order of 1 wavelength of incident light. A film of thickness in the range of 0.5 μm to 10 μm may be considered as a thin film.



When light is incident on a thinfilm, a small part of it gets reflected from top surface and major part (96%) is transmitted into film. out of the light reaching the bottom surface, again a small part is reflected and the rest is transmitted out of film. A small portion of the light thus gets reflected partially several times with in the film. At each reflection, the intensity and hence the amplitude of light wave is divided into a reflected component and a refracted component. The reflected components (or refracted) travel along different paths and can overlap to produce interference. Hence interference in thin film is called by Division of amplitude

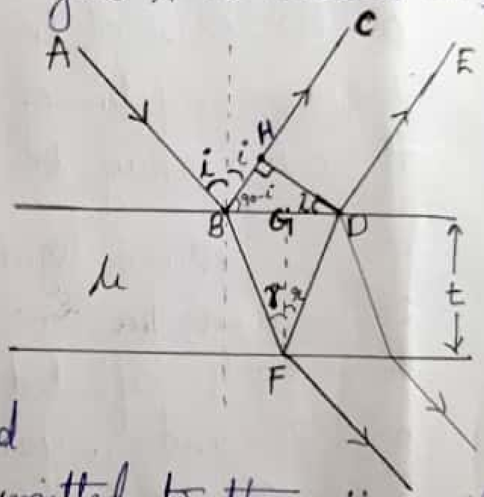
(2) In thin films only the first reflection from the top surface and bottom surface will be of appreciable strength. After two reflections, the intensity will become Insignificantly small.

- Interference in thin Film of constant thickness

A transparent thin film of uniform thickness bounded by two parallel surface is called a Plane parallel thin film.

- (A) Interference due to Reflected Light

Consider a thin transparent film of uniform thickness 't' and refractive index 'μ' bounded by two parallel surfaces surrounded by air. Let monochromatic light of wavelength λ be incident on film. A ray of light AB, is incident on the upper surface at an angle of incidence 'i' is partly reflected along BC and partly refracted along BF. 'r' is the angle of refraction at B. A part of light incident at F is reflected along FD and the other is transmitted to the other side of film. A part of light incident at D emerges out along DE. The rays BC and DE are parallel and close to each other. These rays are from same source (coherent) and produce interference.



Optical path difference between the two reflected rays (along BC and BFDE) is

$$\Delta = \text{path } (BF+FD) \text{ in film} - \text{Path } BH \text{ in air}$$

$$= \mu(BF+FD) - BH$$

In Δ^{le} BFD, $\angle BFG = \angle GFD = r$

$$BF = FD$$
$$BG = GD$$

$\therefore$ Path difference $\Delta = 2\mu(BF) - BH$ —— ①

To find BF ; $\Delta BFG \Rightarrow \cos r = \frac{FG}{BF} = \frac{t}{BF}$

$$\therefore BF = \frac{t}{\cos r} \quad \text{——} ②$$

To find BH ; $\Delta BHD \Rightarrow \sin i = \frac{BH}{BD} = \frac{BH}{2BG}$

$$\sin i = \frac{BH}{2t\tan r}$$
$$BH = 2t \tan r \sin i \quad ③$$

ΔBFG; $\tan r = \frac{BG}{t}$

$BG = t \tan r$

substitute ② & ③ in ①

$$\Delta = 2\mu \frac{t}{\cos r} - 2t \tan r \sin i$$
$$= 2\mu \frac{t}{\cos r} - 2t\left(\frac{\sin r}{\cos r}\right)(\mu \sin r)$$
$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$
$$= \frac{2\mu t}{\cos r}\cos^2 r$$
$$\Delta = 2\mu t \cos r$$

Snell's Law

$$\mu = \frac{\sin i}{\sin r}$$

Correction due to phase change:

When a ray is reflected at the boundary of a rarer to denser medium, the ray suffers a phase change of π which corresponds to a path difference of $\frac{\lambda}{2}$. Therefore BC undergoes a phase change of π, where as no phase change occurs from the bottom surface of film. Hence total path difference is

$$\boxed{\Delta = 2\mu t \cos r - \frac{\lambda}{2}}$$

④ Condition for Maxima – Brightness – Constructive Interference

For brightness path difference must be $m\lambda$

$$2\mu t\cos r - \frac{\lambda}{2} = m\lambda$$

$$2\mu t\cos r = m\lambda + \frac{\lambda}{2}$$

$$\boxed{2\mu t\cos r = (2m+1)\frac{\lambda}{2}} \qquad m = 0,1,2\ldots$$

Condition for Minima — Darkness – Destructive Interference

For darkness path difference $= (2m-1)\frac{\lambda}{2}$

$$2\mu t\cos r - \frac{\lambda}{2} = (2m-1)\frac{\lambda}{2}$$

$$2\mu t\cos r = \frac{\lambda}{2}(1+2m-1)$$

$$\boxed{2\mu t\cos r = m\lambda} \qquad m = 1,2\ldots$$

Film viewed by white light

If white light falls on a film, it consist of all colours. At any point of the film μ, t and r may be such that $2\mu t\cos r = m\lambda$, for a particular colour. Hence in reflected light, due to destructive interference, that colour will be suppressed and all the other colours will be reflected. Hence that point of the film will appear brilliantly coloured with one colour absent. Similar effects may be observed at other points of the film and hence different points will appear differently coloured.

Effect of thickness of film

An excessively thin film appears black in reflected light (thickness $\simeq 0$ $\therefore \Delta = 2\mu t\cos r - \frac{\lambda}{2} = -\frac{\lambda}{2}$ gives darkness condition). However

If the thickness of the film is of the order of many wavelength, it appears uniformly illuminated in reflected light.

Scanned with CamScanner

(B) Interference due to Transmitted Light

When the transmitted rays overlap each other, they can give interference pattern in transmitted system. The path difference can be calculated in the same way as reflected system and it is found that

Path difference $\Longrightarrow$ $\Delta = 2\mu t \cos r$

But in the case of transmitted rays, reflection occurs at the rarer medium (film – air interface) and hence no phase change occurs. Therefore effective path difference between the transmitted rays is

$$\Delta = 2\mu t \cos r$$

Condition for brightness

$$\boxed{2\mu t \cos r = m\lambda}$$

Condition for darkness

$$\boxed{2\mu t \cos r = (2m-1)\frac{\lambda}{2}} \quad m = 1, 2 \ldots$$

Interference pattern produced by reflected and transmitted light are complementary to each other. That is the point or region which appears bright in reflected system appears dark in the transmitted system and vice versa.

Reflected and Transmitted systems in white Light

The conditions for brightness and darkness (maxima and minima) in the reflected and transmitted cases are opposite to each other. Thus with white light, the colours visible in reflected light will be complementary to colours visible in transmitted light. The thickness of film at any point is such that the condition of maxima holds for certain colours in reflected system, the condition for minima will hold at the same point for same colours in transmitted system. That is, the colours which are absent in one system will be present in the other.

The visibility of interference fringes is higher in the reflected system than in transmitted system. When the film is viewed in transmitted light, the interference fringes are seen to be with less intensity. This is because there is a large difference between the amplitudes of transmitted light rays. The intensity gradually decreases in the transmitted system due to successive reflections by division of amplitude

one of the rays is transmitted through film without any reflection, while the other is transmitted after two internal reflections

Fringes of equal inclination & Fringes of equal optical Thickness

In thin film, fringes produced due to change in path difference ($\Delta = 2\mu t \cos r$). The path difference changes with (i) optical thickness μt of film, (ii) angle of refraction 'r' and (iii) wavelength of light λ

Case (I): In films of uniform thickness, optical path μt remains constant. Therefore corresponding to one value of 'r' there will be either a bright fringe or dark fringe. Such fringes are known as fringes of equal inclination

case (II): When thickness of film is rapidly varying, the inclination r and wavelength remains constant. Then path difference depends on optical thickness μt. i.e, the fringes are mainly due to variation in optical thickness of film. Each fringe will be the locus of all the points of same thickness. Such fringes are called fringes of equal optical thickness.
e.g: Newton's ring, straight parallel band in wedge film.

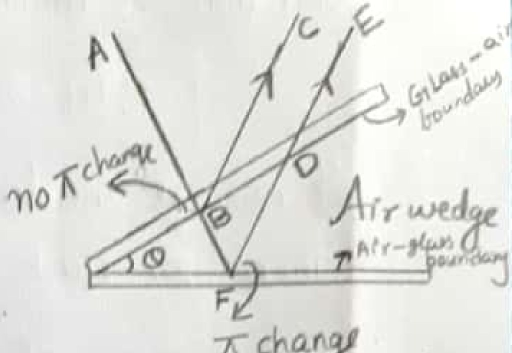
Origin of colours in Thin film

The colours exhibited in reflection by thin films of oil, mica, soap bubbles etc are due to interference from an extended source such as sky. The eye looking at the thin film receives light waves reflected from the top and bottom surface of the film. The reflected rays are close to each other and interfere. The optical path difference between the rays is $\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

The path difference depends upon the thickness t of the film, wavelength λ and angle r, which is related to angle of incidence of light on film. White light consists of a range of wavelengths and for specific values of t and r, waves of only certain colours constructively interfere. Therefore only those colours are present in the reflected light. The other colours interfere destructively and are absent from reflected light. Hence the film at a particular point appears coloured.

In the case of a film of oil or soap film, the thickness of film continuously changes. Therefore different colours are intensified at different places.

WEDGE SHAPED FILM



A wedge shaped film is a thin film of varying thickness, which has surfaces inclined at a small angle. It has zero thickness at one end and progressively increasing its thickness to the other end.

A thin wedge of air film can be formed by two glass slides resting on each other at one edge and separated by a thin spacer at other end. The film is illuminated by a parallel beam of monochromatic light. If the film is viewed from above the wedge in reflected light, alternate bright and dark fringes are observed. This is due to the interference between the rays reflected at the upper and lower surfaces of the film. The darkness or brightness of fringes observed on top surface of film depends on the path difference between the rays reflected from upper and lower surface of film.

Optical path difference between rays BC and FE is

$$\boxed{\Delta = 2\mu t\cos r - \frac{\lambda}{2}}$$

where $\frac{\lambda}{2}$ is due to the phase change of π radian for the reflected ray from bottom surface of film. (Air to glass)

Condition of Maxima – Brightness – Constructive Interference

$$2\mu t\cos r - \frac{\lambda}{2} = m\lambda$$

$$\boxed{2\mu t\cos r = (2m+1)\frac{\lambda}{2}}$$

9 Condition of minima — Destructive Interference — Darkness

$$\boxed{2\mu t \cos r = m\lambda}$$

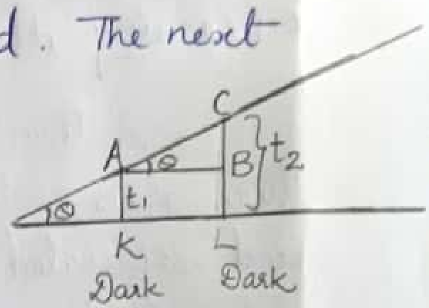
Similar to plane film

If normal incidence is assumed, then $r = 0$ & $\cos r = 1$

Then for dark fringe $2\mu t = m\lambda$

Expression of fringe width β

Let at A dark fringe occured. The next dark fringe occur at C. The thickness of air film at A & C be 't_1' and 't_2'. Apply the condition for darkness at A & C (for normal incidence)



At 'A' :⇒ $2\mu t_1 = m\lambda$ —— (1)

At 'C' :⇒ $2\mu t_2 = (m+1)\lambda$ —— (2)

(2) − (1) ⇒ $2\mu(t_2 - t_1) = \lambda$

$2\mu(BC) = \lambda$

$2\mu(AB \tan\theta) = \lambda$

$t_2 - t_1 = BC$

ΔABC

$\tan\theta = \frac{BC}{AB}$

AB is the distance between successive dark fringes and is called fringe width β

$AB = \beta$

∴ $2\mu\beta\tan\theta = \lambda$

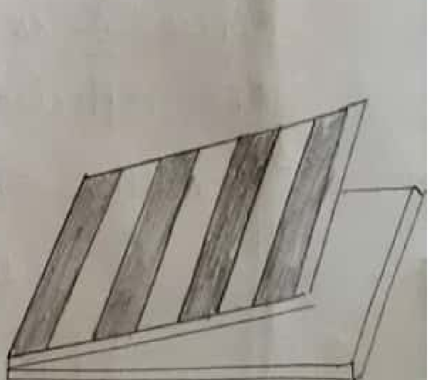
θ is very small

∴ $\tan\theta = \theta$

$2\mu\beta\theta = \lambda$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Features of wedge shape — Interference



(i) Fringe at apex is dark
(ii) Fringes are equidistant
(iii) Fringes are straight and parallel
(iv) Fringes of equal thickness
(v) Fringes are localised.

(i) Fringe at the apex is dark

The thickness of air film at the contact edge (apex) of glass sides is negligible. That is $t \simeq 0$

$\therefore$ optical path difference $\Delta = 2\mu t - \frac{\lambda}{2} = -\frac{\lambda}{2}$

A path difference of $\frac{\lambda}{2}$ occurs for destructive interference and hence fringe at apex is always dark.

(ii) Equidistant fringes

Fringe width $\beta = \frac{\lambda}{2\theta}$

As λ and θ are constants, β is also constant.

$\therefore$ Fringes are equidistant.

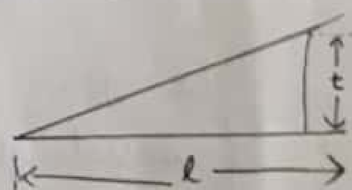
(iii) Straight and parallel fringes

The rays reflected from the region of wedge having same thickness interfere and produce fringe pattern. The locus of points having same thickness lies along line parallel to contact edge. Since the fringes are equidistant, they will be parallel.

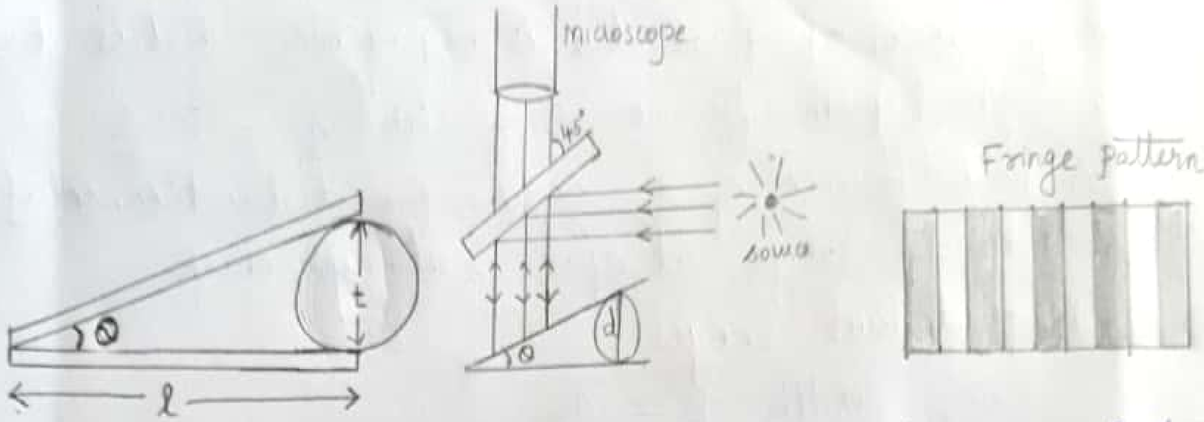
Number of dark fringes in air wedge.

If there are N dark fringes observed in airwedge of length 'l'

Then $l = N\beta$



(11) Determination of thickness of very thin wire or foil



The given wire whose thickness is to be measured is placed between two optically flat glass slides such that the glass slides are in contact with each other at one end. Then wedge shaped air film is formed between the glass slides. 'θ' be the angle of wedge. If 't' is the thickness (diameter) of wire, then

$$\tan\theta = \frac{t}{l}$$

As θ is very small $\tan\theta \simeq \theta$

$$\theta = \frac{t}{l}$$

The film is illuminated by a monochromatic light of wavelength λ. The interference pattern produced is viewing through microscope and fringe width β is measured.

$$\beta = \frac{\lambda}{2\mu\theta}$$

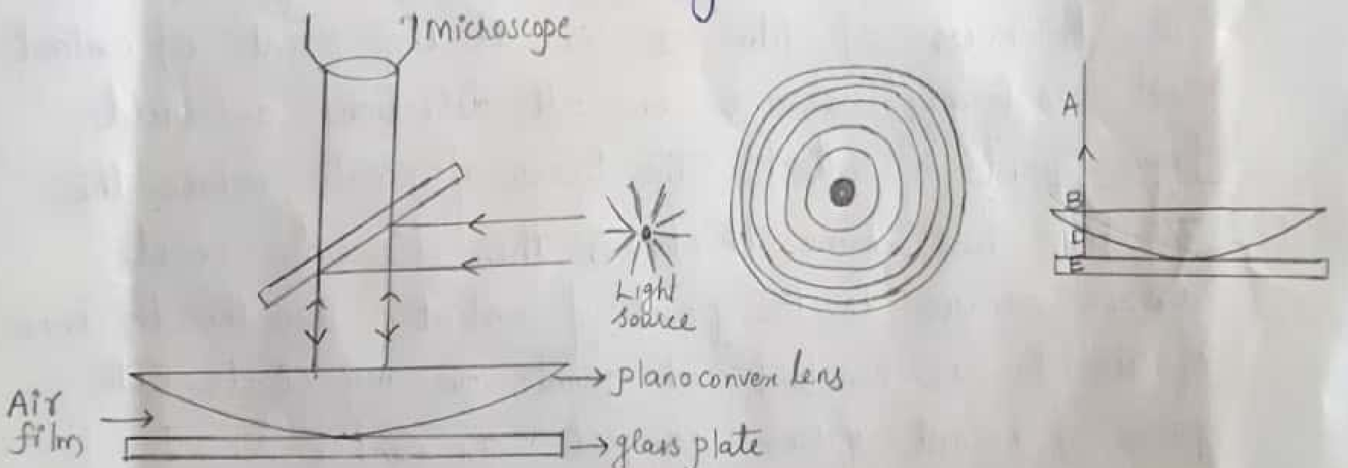
$$\beta = \frac{\lambda l}{2\mu t}$$

Thickness $t = \dfrac{\lambda l}{2\mu\beta}$

for air wedge $\mu = 1$ $\boxed{t = \dfrac{\lambda l}{2\beta}}$

Newton's Rings.

When a plano convex lens of large radius of curvature is placed ~~below~~ on a plane glass plate, a thin air-film of variable thickness is formed between lower surface of lens and upper surface of glass plate. The thickness of air is almost zero at the point of contact and gradually increases from the point of contact outwards. When monochromatic light is allowed to fall normally on the film, a system of bright and dark rings with dark spot at centre of ring system is seen in reflected light. These interference fringes of equal thickness are called Newton's rings.



Newton's rings are produced due to interference between the light waves reflected from upper and lower surface of air film between lens and glass plate (DBA and EDBA rays).

optical path difference between the rays is

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

where t is the thickness of film at that point (E).

The phase change of π occurs for the ray reflected from ~~botto~~ lower surface of film.

13 **Conditions for bright and dark ring.**

For bright fringe — Constructive interference

$$2\mu t \cos r = (2m+1)\frac{\lambda}{2}$$

For dark fringe — Destructive Interference

$$2\mu t \cos r = m\lambda$$

For normal incidence of light $\cos r = 1$ & for air $\mu = 1$

$\therefore$ for bright ring $\Rightarrow$ $2t = (2m+1)\frac{\lambda}{2}$

for dark ring $\Rightarrow$ $2t = m\lambda$

Newton's ring fringes are Circular.

In Newton's ring arrangement, an air film is formed between plano convex lens and glass plate. The thickness of film is zero at the point of contact and uniformly increases in all directions outward from point of contact. The locus of points where the air film has same thickness then lie on a circle whose centre is the point of contact. Thus the thickness of air film is constant at points on any circle with plane of contact of lens–glass plate as centre.

$\therefore$ Newton's fringes are circular.

Centre of ring system is dark

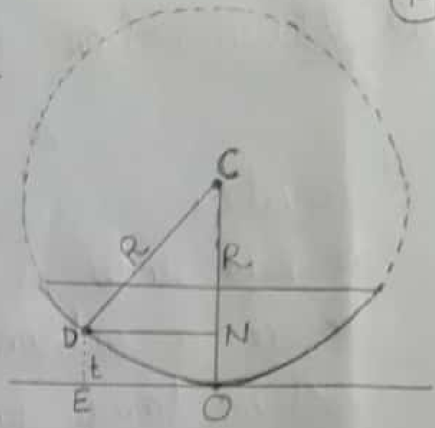
At the point of contact of lens and glass plate, thickness of air film is zero.

$\therefore$ path difference $= 2\mu t \cos r - \frac{\lambda}{2} = -\frac{\lambda}{2}$

This is the condition for darkness.

$\therefore$ centre of ring system appears dark.

Radii of Newton's Rings



Let $R \rightarrow$ Radius of curvature of lens
$C \rightarrow$ Centre of curvature
$O \rightarrow$ point of contact
Let at $E \rightarrow m^{th}$ dark fringe formed
Then $OE \rightarrow r_m$
At E, thickness of air film is $DE = t$

ΔDCN;
$$DC^2 = DN^2 + CN^2$$
$$R^2 = r_m^2 + (R-t)^2$$
$$R^2 = r_m^2 + (R^2 - 2Rt + t^2)$$
$$r_m^2 = 2Rt - t^2$$
But $R >> t$, $\therefore 2Rt >> t^2$
t^2 can be neglected
$$r_m^2 = 2Rt$$

Cond^n for dark ring at E is : $2t = m\lambda$
$$\therefore r_m^2 = mR\lambda$$
$$\boxed{r_m = \sqrt{mR\lambda}}$$
$m = 1, 2, 3 \cdots$

$r_m \propto \sqrt{m}$ $\therefore$ Radius of Newton's rings is proportional to square root of natural numbers.

Diameter of ring $D_m = 2\sqrt{mR\lambda}$
$$\boxed{D_m^2 = 4mR\lambda}$$

Newton's rings are unequally spaced.

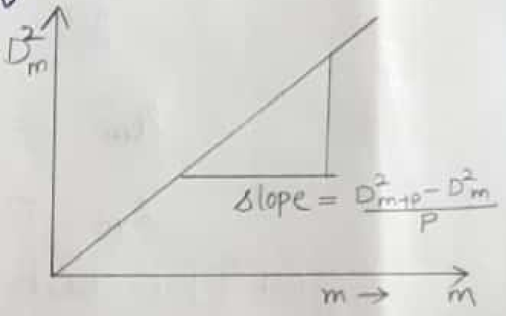
The diameter of rings $D_m = 2\sqrt{mR\lambda}$
The diameters of rings are proportional to square root of natural numbers. Therefore diameter of the ring does not increase in the same proportion as order of ring. $\therefore$ Spacing between fringes is not even.

$$D_1 : D_2 : D_3 : \ldots\ldots = 1 : \sqrt{2} : \sqrt{3} : \ldots\ldots$$

(15) Determination of wavelength of light

Draw diagram

A plano-convex lens of large radius of curvature is placed on a plane glass plate. The system is held under a travelling microscope arranged before a sodium vapour lamp. The light from sodium lamp falls on a glass plate held at 45° above this ~~as~~ lens arrangement. Then Newton's rings are produced due to the interference of light reflected from top and bottom surface of air film between lens and glass plate



The position of various dark rings is ~~mea~~ noted on LHS & RHS ~~raby~~ making the vertical cross wire is tangential to the rings. Thereby diameter of each ring can be calculated as the difference of LHS & RHS reading

Let diameter of m^{th} ring: $D_m^2 = 4mR\lambda$ ——— (1)

diameter of $(m+P)^{th}$ ring: $D_{m+P}^2 = 4(m+P)R\lambda$ ——— (2)

(2) — (1) $\quad D_{m+P}^2 - D_m^2 = 4PR\lambda$

$$\therefore \boxed{\lambda = \frac{D_{m+P}^2 - D_m^2}{4PR}}$$

A graph of D_m^2 versus ring number m gives a straight line. The slope of graph is $\frac{(D_{m+P}^2 - D_m^2)}{P}$.

$$\therefore \lambda = \frac{slope}{4PR}.$$

- Radius of Curvature of Lens

$$R = \frac{D_m^2}{4m\lambda}$$

$$R = \frac{D_{m+P}^2 - D_m^2}{4P\lambda}$$

- Determination of Refractive Index of Liquid (μ)

The liquid whose refractive index is to be determined is filled between lens and glass plate. The ring system is formed for the liquid film.

Let Diameter of m^{th} dark ring : $[D_m^2]_L = \frac{4mR\lambda}{\mu}$ —— ①

Diameter of $(m+P)^{th}$ dark ring : $[D_{m+P}^2]_L = \frac{4(m+P)R\lambda}{\mu}$ —— ②

② − ① $\quad [D_{m+P}^2]_L - [D_m^2]_L = \frac{4PR\lambda}{\mu}$

But $[D_{m+P}^2]_{air} - [D_m^2]_{air} = 4PR\lambda$

$$\therefore \boxed{\mu = \frac{(D_{m+P}^2)_{air} - (D_m^2)_{air}}{(D_{m+P}^2)_{Liq} - (D_m^2)_{Liq}}}$$

note: From eqn ① it is clear that diameter of ring decreases for liquid film compared to air film. Therefore the ring system shrinks for liquid film.

Newton's Rings by Transmitted Light

Reflected and transmitted ring patterns are complimentary. That is the ring system due to transmitted light is just opposite to that we observe in reflected light. The parts of ring system which appear bright in reflected pattern are dark in transmitted pattern and viceversa. At center, bright spot is appeared in transmitted light. However the rings in transmitted light are much poor in contrast.

$2t = m\lambda \Rightarrow$ bright

$2t = (2m+1)\frac{\lambda}{2} \Rightarrow$ dark.

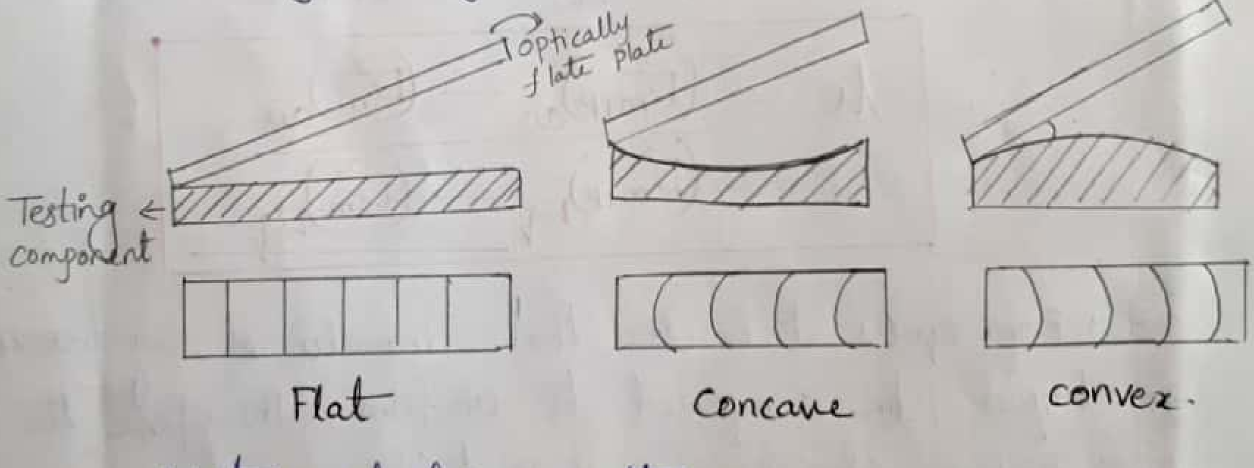
17. Applications of Interference

- (i) Determination of thickness of very thin wire ⇒ Wedge shape film
- (ii) Determination of refractive index of liquid
- (iii) Wavelength of incident light
- (iv) Radius of curvature of Lens

(ii)–(iv) ⇒ Newton's Rings

- (v) Testing of Surface Flatness

The surface irregularities of machine components are detected using thin film interference. The smoothness of a surface can be quickly inspected visually by keeping an optically flat plate on the component to be tested at an angle and illuminating it with a monochromatic light. The air wedge formed between component to be tested and flat plate produces an interference pattern.



Nature of fringe pattern:

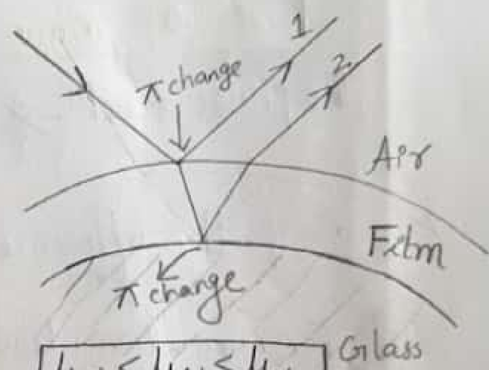
(i) Straight and equidistant fringes if component surface is smooth

(ii) Curved fringes towards contact edge, if surface is concave

(iii) Fringes curved away from contact edge, if surface is convex.

(vi) Antireflecting films

When light is incident on the lens of optical instruments, some part of light is lost due to reflection. In solar cell, the loss of light energy due to reflection at the cell surface leads to less production of electrical energy. It is found that coating the surface with a thin transparent film of suitable refractive index and specific thickness can reduce such reflective loss of light energy at surface. Such coatings are called antireflection (AR) coatings.



$\boxed{\mu_{air} < \mu_{film} < \mu_{glass}}$

The conditions for AR film

(i) Phase condition: The waves reflected from top and bottom surface of film are in opposite phase such that their overlapping leads to destructive interference.

(ii) Amplitude condition:

The waves have equal amplitude.

The phase condition gives the required thickness of the film

$$\boxed{t_{min} = \frac{\lambda}{4\mu_f}}$$

The optical thickness of AR coating should be one-quarter of wavelength to suppress the reflection.

The amplitude condition implies that ⇒ the refractive of thin film should be less than that of substrate and possibly nearer to its square root.

$$\boxed{\mu_f = \sqrt{\mu_g}}$$

(19) In case of glass, $\mu_g = 1.5$ $\therefore \mu_f = \sqrt{\mu_g} = 1.22$

The materials which have refractive index nearer to this value are MgF_2 and Cryolite. Apart from refractive index, the film should adhere well, should be durable, scratch proof etc. Magnesium fluoride is cheaper compared cryolite and hence widely used as AR coating. The minimum thickness condition $\left(t_{min} = \frac{\lambda}{4\mu_f}\right)$ is satisfied only at one particular wavelength. Normally the wavelength chosen is 5500 A° (Yellow-green) for which eye is most sensitive.

~~(vii) Highly Reflecting Films~~ Extra

Proof of minimum thickness

The phase condⁿ ⇒ reflected ray from top & bottom surface of thin film is 180° out of phase.

$\therefore$ path difference $\Delta = 2\mu_f t \cos r - \underbrace{\frac{\lambda}{2}}_{\lambda \text{ change at top}} - \underbrace{\frac{\lambda}{2}}_{\lambda \text{ change at bottom surface}}$

For normal incidence ($\cos r = 1$)

$\Delta = 2\mu_f t - \lambda$

$\Delta = 2\mu_f t$

[The addition or subtraction of a full wave (λ) does not affect phase relation ⇒ λ neglected

For destructive interference $\Delta = (2m+1)\frac{\lambda}{2}$

$$2\mu_f t = (2m+1)\frac{\lambda}{2}$$

For film to be transparent thickness should be minimum i.e value of m should be minimum

$$2\mu_f t_{min} = \frac{\lambda}{2}$$

$$\boxed{\therefore t_{min} = \frac{\lambda}{4\mu_f}}$$

Proof of refractive Index.

Amplitude condition requires that amplitude of rays 1 & 2 and equal

$$E_1 = E_2$$

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2 \qquad \mu_a = 1$$

$$\left(\frac{\mu_f - 1}{\mu_f + 1}\right)^2 = \left(\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right)^2$$

Take square root

$$\frac{(\mu_f - 1)}{(\mu_f + 1)} = \frac{(\mu_g - \mu_f)}{(\mu_g + \mu_f)}$$

$$(\mu_f - 1)(\mu_g + \mu_f) = (\mu_g - \mu_f)(\mu_f + 1)$$

$$\cancel{\mu_f \mu_g} + \mu_f^2 - \mu_g - \cancel{\mu_f} = \cancel{\mu_g \mu_f} + \mu_g - \mu_f^2 - \cancel{\mu_f}$$

$$2\mu_f^2 = 2\mu_g$$

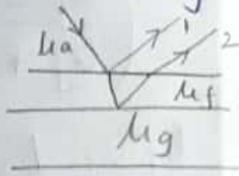
$$\mu_f^2 = \mu_g$$

$$\boxed{\mu_f = \sqrt{\mu_g}}$$

Dr. Jessy P.J

• (vii) Highly Reflecting Film

These films are used on window glasses and sun glasses to reduce the transmission of light by increasing its reflection.



If the refractive index of the film coated on glass is higher than that of glass ($\mu_f > \mu_g$), then the reflectivity of glass surface increases. The Rays 1 & 2 should constructively interfere if the reflection is to be more from surface.

∴ Cond^n ⟹ path difference $\Delta = m\lambda$

$$\Delta = 2\mu_f t - \frac{\lambda}{2}$$

$$2\mu_f t - \frac{\lambda}{2} = m\lambda$$

$$2\mu_f t = (2m+1)\frac{\lambda}{2}$$

For minimum thickness of coating m should minimum

$$2\mu_f t = \frac{\lambda}{2}$$

$$\boxed{t_{min} = \frac{\lambda}{4\mu_f}}$$

Thus the optical thickness of high reflectivity film is again $\frac{\lambda}{4}$, provided $\boxed{\mu_f > \mu_g}$

Thus on a glass plate a $\frac{\lambda}{4}$ thickness film of refractive index more than glass is deposited, the surface reflectivity is enhanced

eg: Titanium oxide ($\mu = 2.8$) & or Zin sulphide ($\mu = 2.3$)

× ———————— ×

youtube: Physics Jesry

Best wishes

Dr. Jesry P. J