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Q3.

A Find the continued product of the roots of $x^4 = 1+i$.

→ given that,

$$x^4 = 1+i. \qquad x^4 = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$\therefore \quad x = \sqrt{2}^{1/4}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{1/4}$$

$$= \sqrt{2}^{1/4}\left(\cos\left(2k\pi + \frac{\pi}{4}\right) + i\sin\left(2k\pi + \frac{\pi}{4}\right)\right)^{1/4}$$

$$= 2^{1/8}\left[\cos\frac{8k\pi+\pi}{16} + i\sin\frac{8k\pi+\pi}{16}\right]$$

where $k = 0, 1, 2, 3.$

$$\therefore \quad x_0 = 2^{1/8}\left[\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right] = 2^{1/8}e^{i\pi/16}$$

$$x_1 = 2^{1/8}\left[\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right] = 2^{1/8}e^{i9\pi/16}$$

$$x_2 = 2^{1/8}\left[\cos\frac{17\pi}{16} + i\sin\frac{17\pi}{16}\right] = 2^{1/8}e^{i17\pi/16}$$

$$x_3 = 2^{1/8}\left[\cos\frac{25\pi}{16} + i\sin\frac{25\pi}{16}\right] = 2^{1/8}e^{i25\pi/16}$$

$\therefore$ Continued product of the roots $= x_0 . x_1 . x_2 . x_3$

$$= \left(2^{1/8}\right)^4 e^{i\pi/16 + i9\pi/16 + i17\pi/16 + i25\pi/16} = 2^{1/2} e^{i52\pi/16}$$

$$= \sqrt{2}\, e^{i13\pi/4} = \sqrt{2}\, e^{i(2\pi + 5\pi/4)}$$

$$\boxed{x_0 . x_1 . x_2 . x_3 = \sqrt{2}\, e^{i5\pi/4}.}$$

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Q3. D. If $x = \cosh\left(\frac{1}{m}\log y\right)$, then prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$

→

$x = \cosh\left(\frac{1}{m}\log y\right)$ $\therefore \cosh^{-1}x = \frac{1}{m}\log y$

$\log y = m\cosh^{-1}x$. $\therefore y = e^{m\cosh^{-1}x}$

$y_1 = e^{m\cosh^{-1}x} \times \frac{1}{\sqrt{x^2-1}}$ $\therefore \sqrt{x^2-1}\, y_1 = \cancel{e^{m\cosh^{-1}}}\ me^{m\cosh^{-1}x}$

$= my$.

$\therefore \sqrt{x^2-1}\, y_1 = my$

differenciating w.r.t x,

$$\sqrt{x^2-1}\, y_2 + \frac{xy_1}{\sqrt{x^2-1}} = \frac{m^2 y}{\sqrt{x^2-1}}.$$

$$(x^2-1)y_2 + xy_1 = m^2 y.$$

Applying Leibnitz's theorem,

$$(x^2-1)y_{n+2} + n(2x)y_{n+1} + n(n-1)y_n + xy_{n+1} + ny_n = m^2 y_n.$$

$$\boxed{\therefore (x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0.}$$

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Q3

B. Prove that $2e^{2x} = \cosh 2v - \cos 2u$, where $e^{z} = \sin(u+iv)$ and $z = x+iy$.

→

we have, $e^{z} = \sin(u+iv)$

$$e^{x+iy} = \sin(u+iv) \qquad e^{x} \times e^{iy} = \sin(u+iv)$$

$$e^{x}(\cos y + i\sin y) = \sin u \cos iv + \cos u \sin iv.$$

$$= \sin u \cosh v + i\cos u \sinh v$$

⇒ Equating real and imaginary parts,

$$e^{x}\cos y = \sin u . \cosh v \text{ and } e^{x}\sin y = \cos u . \sinh v,$$

Squaring and adding,

$$e^{2x}(\cos^2 y + \sin^2 y) = \sin^2 u . \cosh^2 v + \cos^2 u \sinh^2 v.$$

$$= (1-\cos^2 u)\cosh^2 v + \cos^2 u(\cosh^2 v - 1)$$

$$= \cosh^2 v - \cancel{\cos^2 u . \cosh^2 v} + \cancel{\cos^2 u . \cosh^2 v} - \cos^2 u$$

$$= \cosh^2 v - \cos^2 u . = \frac{1}{2}(1+\cosh 2v) - \frac{1}{2}(1+\cos 2u)$$

⇒

$$= \frac{1}{2}(1+\cosh 2v - 1 - \cos 2u)$$

$$\boxed{\therefore\ 2e^{2x} = \cosh 2v - \cos 2u}$$

Q3
E. If $u = \log r$, and $r^2 = x^2 + y^2$, then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial y^2} + = 0$.

$\rightarrow$ $u = \log r$ $\qquad$ $r^2 = x^2 + y^2$.

$\therefore e^u = r$. $\qquad$ $\therefore \frac{\partial r}{\partial x} \times 2r = 2x$ $\qquad$ $\therefore \frac{\partial r}{\partial y} \times 2r = 2y$

$\boxed{n = 1}$

for homogenous eqn.

$\therefore \frac{\partial r}{\partial x} = \frac{x}{r}$; $\frac{\partial r}{\partial y} = \frac{y}{r}$.

but for $u = \log r$

$\boxed{n = -1}$

$x\left(\frac{\partial y}{\partial x}\right) + y\left(\frac{\partial y}{\partial y}\right) = 1 \cdot \frac{e^0}{e^u} = 1$. $\qquad$ $\therefore g(u) = 1$.

$\therefore$ We know,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial y^2} = g(u) \times (g'(u) - 1)$$

$$= 1 \times [0 - 1]$$

$$= -1$$

$$\boxed{\therefore x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 y}{\partial y \partial x} + y^2 \frac{\partial^2 y}{\partial y^2} + 1 = 0}$$