EM Notebook / 47_ YASH SARANG

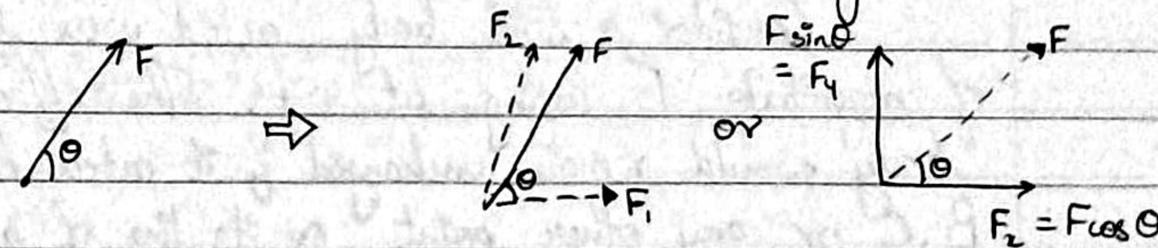
Modules
1. Coplanar forces + Centre of gravity (33%) (27m) 9
2. Equilibrium. (26%) (20/21m) 7
3. Friction. (15%) (12m) 4
4. Kinematics. of Particle. (15%) (12m) 4
5. Kinematics of Rigid body (11%) (8/9m) 3

* Coplanar forces (Module 1)

1.1 System of Coplanar forces.

- **Resolution of a force.**

Resolution or resolving a force implies breaking the force into components, such that the components combined together would have the same (resultant force) effect as the original force.



resolved into force F_1, F_2 and F_x, F_y.

2. @ · **Classification of force systems.**

* System of forces

- Coplanar
  - General
  - Parallel
  - Concurrent
- Non-coplanar (Space forces)
  - When the forces acting in a system do not lie in a single plane

In coplanar,

- Concurrent – All the forces meet at a point.
- Parallel – lines of action of forces are parallel.
- General – Non concurrent, non parallel system of forces which lie in one plane.

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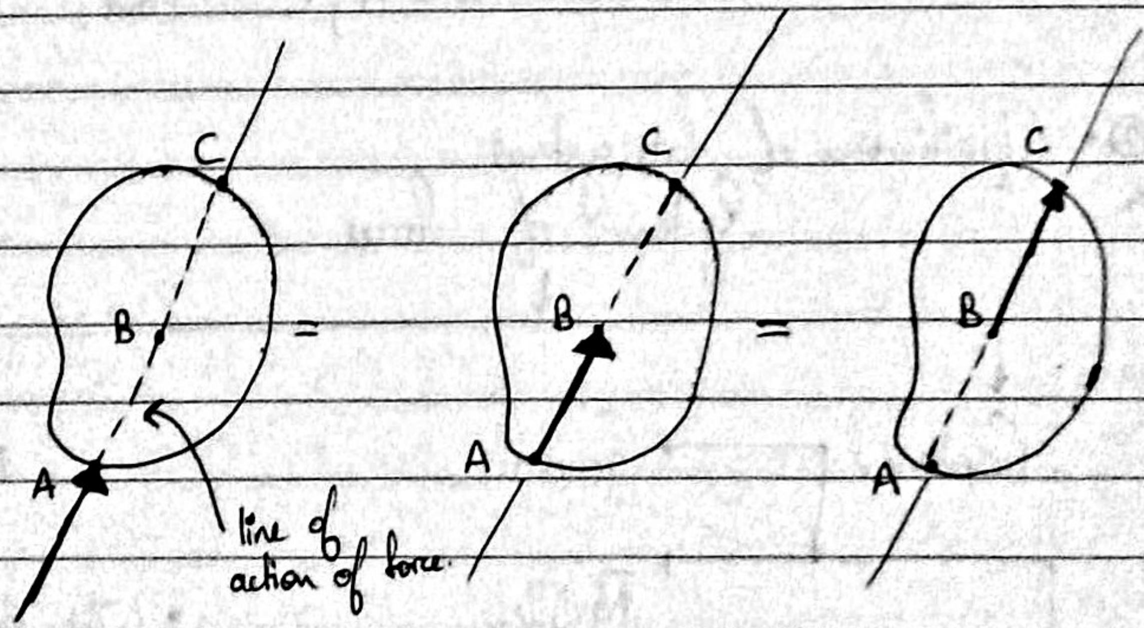
(b) Principle of transmissibility.
V. Imp
Q.
A force being a sliding vector continues to act along its line of action and therefore makes no change if it acts from a different point on its line of action on a rigid body.

Consider a rigid body, acted upon by a force of magnitude F acting at A. The effect on the body would remain unchanged if it acted from point B, C or any other point on its line of action.



1.2 Resultant :

- Resultant of Concurrent Systems using Parallelogram law of forces.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}.$$

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}.$$

- Resultant of Concurrent Systems using method of Resolution.

When more than 2 forces act at a point, the use of method of resolution is made to avoid tedious repetition of parallelogram law of forces to successive forces. The following steps are adopted in the solution-

Step 1. Resolve the inclined forces if any along the horizontal x and vertical y direc

Step 2. a) Add up the horizontal forces to get ΣF_x.

b) Add up the vertical forces to get ΣF_y.

c) Resultant force $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$.

Step 3. The direction of the resultant force is the angle θ made by it with x axis.

$$\tan\theta = \left|\frac{\Sigma F_y}{\Sigma F_x}\right|.$$

Step 4. Decide the quadrant of the resultant force.

Step 5. Draw a diagram showing the resultant

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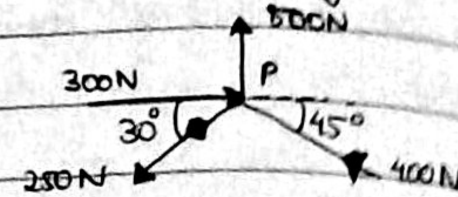
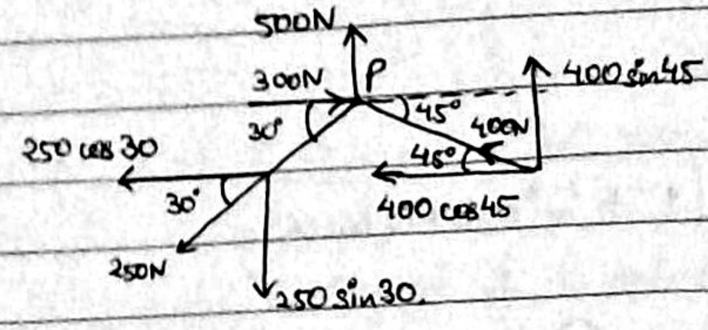
Q. Find the resultant of the four concurrent forces acting on a particle P.



Solution -



$$\therefore \Sigma F_x = 300 + (-400\cos 45) + (-250\cos 30)$$
$$= -199.3\,N$$
$$= 199.3\,N \leftarrow$$

$$\Sigma F_y = 500 + (400\sin 45) + (-250\sin 30)$$
$$= 657.8\,N \uparrow$$

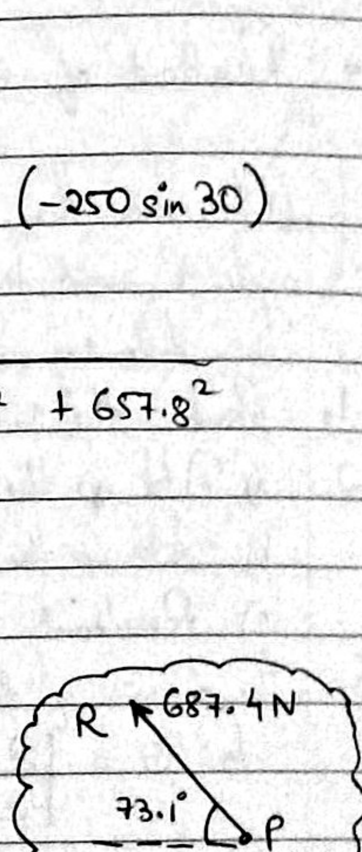
$$\therefore R = \sqrt{F_x^2 + F_y^2} = \sqrt{199.3^2 + 657.8^2}$$

$$R = 687.4\,N$$

also, $\tan\theta = \left|\dfrac{\Sigma F_y}{\Sigma F_x}\right| = \dfrac{657.8}{199.3}$

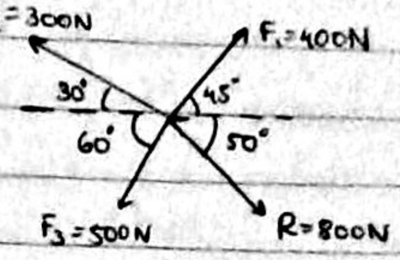
$\therefore \theta = 73.1^\circ$

The arrows of ΣF_x and ΣF_y indicate that the resultant R lies in 2nd quadrant.

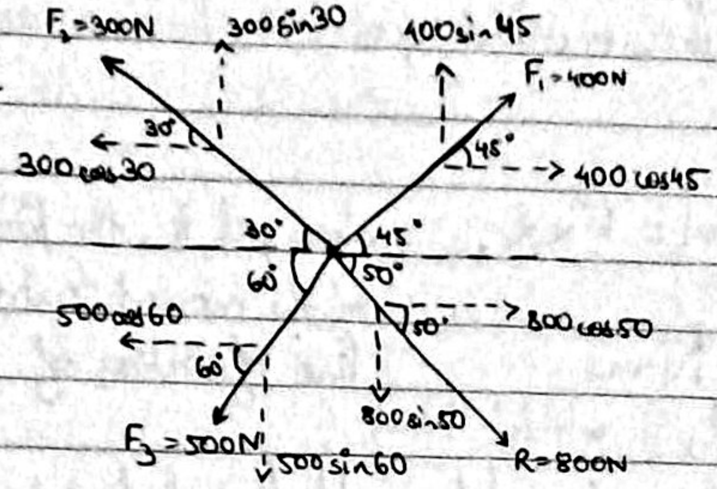


$\therefore$ Resultant force $R = 687.4\,N$ at $\theta = 73.1^\circ$ acts at particle P

Q R = 800N is the resultant of 4 concurrent forces. Find the fourth force F_4.



Solution -



Let F_{4x} and F_{4y} be the perpendicular components of the fourth force.

$\Sigma F_x = 800\cos 50$

$\therefore 800\cos 50 = F_{4x} + (-500\cos 60) + 400\cos 45 + (-300\cos 30).$

$\therefore F_{4x} = 741.2\text{N} \rightarrow$

$\Sigma F_y = (-800\sin 50)$

$\therefore -800\sin 50 = F_{4y} + (-500\sin 60) + 400\sin 45 + 300\sin 30.$

$\therefore F_{4y} = -612.6\text{N}$

$= 612.6\text{N} \downarrow$

Now, $F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = \sqrt{741.2^2 + 612.6^2}$

$F_4 = 961.6\text{N}$

also, $\tan\theta = \left|\frac{(F_4)_y}{(F_4)_x}\right| = \frac{612.6}{741.2}$ $\therefore \theta = 39.6°.$

The ~~direction~~ arrows of F_{4x} and F_{4y} indicates that the force F_4 lies in IV quad.

$\therefore$ The fourth force $F_4 = 961.6\text{N}$ at $\theta = 39.6°$ ↘.

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ⓑ Moment of a force

The rotational effect of a force is known as the moment of the force.

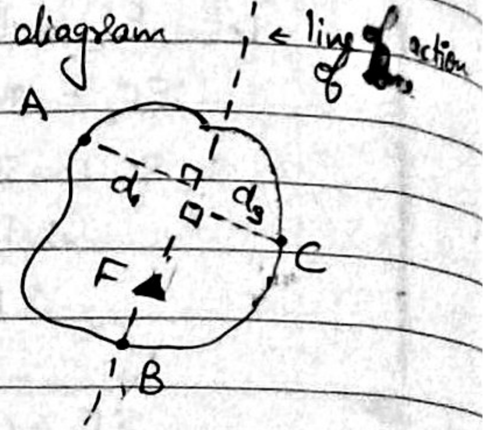
The concerned point is known as the moment centre.

$M = F \times d$. where, d is the perpendicular dist. of the moment centre from the line of action of the force.

Unit - N-m.
(Not J which is Nm)

The tendency to rotate could be clockwise or anticlockwise.

The moment of F in the following diagram



about A $= F \times d_1$ (anti-clockwise)

$\therefore M_A = -(Fd_1)$.

about B $= F \times 0$ (on the line of action of force)

$\therefore M_B = 0$.

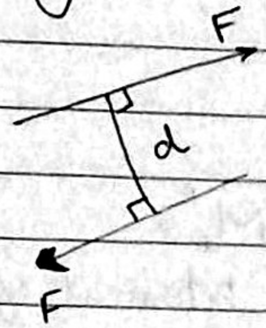
about C $= F \times d_3$ (clockwise)

$M_c = Fd_3$.

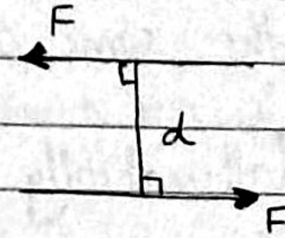
Sign convention: We shall take anti-clockwise moments as positive moments and vice versa.
This shall be indicated as ↺ +ve.

Q. (c) Couple and its properties.

* Couple is a special case of parallel forces.
Two parallel forces of equal magnitudes and opposite ~~speak~~ sense form a couple. The effect of a couple is to rotate the body on which it acts.



$M = F \times d$ ↻
Clockwise couple



$M = F \times d$ ↺ =
Anticlockwise couple

Couples are represented by curved arrows. Units are N-m.

Properties-

1. Couple tends to cause rotation of the body about an axis $\perp$ to the plane containing the two parallel forces.
2. The magnitude of rotation of or moment of a couple is equal to the product of one of the forces and the arm of the couple.
3. Couple is a free vector because of which it can be moved anywhere on the body on which it acts without causing any change.
4. The resultant force of a couple system is zero.
5. To balance a system whose resultant is a couple, another couple of the same magnitude and opposite sense is to be added.
6. To shift a force to a new parallel position, a new couple couple is required to be added to the system.

Q ① Varignon's theorem
V. Imp

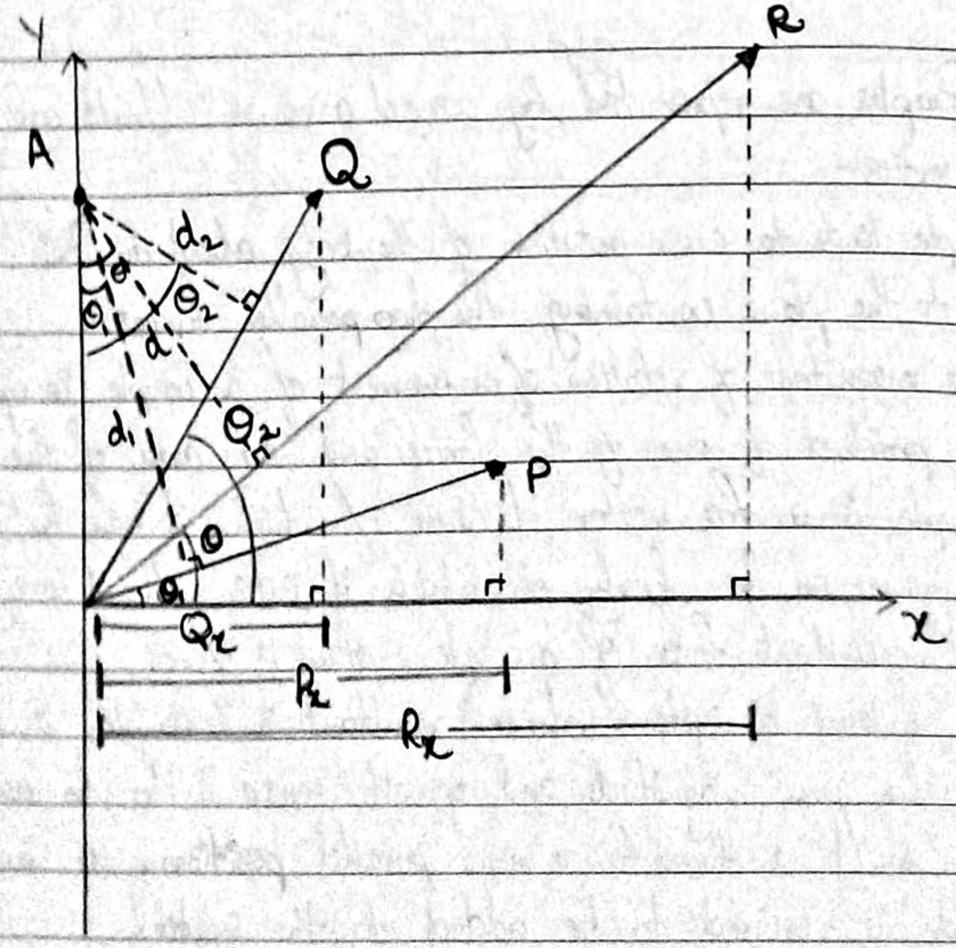
The algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point.

Mathematically it is written as,

$$\sum M_A^F = M_A^R.$$

Sum of moments of all forces about any point, Eg. point A = Moment of resultant about the same point A.

Diagram



Cont.

Proof. Let P and Q be two concurrent forces at O, making angle θ_1 and θ_2 with the x-axis, respectively.
Let R be their resultant making an angle θ with x-axis.

Let A be a point on y-axis about which we shall find the moments of P, Q and R.
Let d_1, d_2 and d be the moment arm of P, Q and R from moment centre A.

Let the x components of forces P, Q and R be P_x, Q_x and R_x, respectively.

Now, Moment of P about A $= M_A^P = P \times d_1$ ——— ①
Moment of Q about A $= M_A^Q = Q \times d_2$ ——— ②
Moment of R about A $= M_A^R = R \times d$
$= R \times (OA \cos\theta)$
$= OA \cdot R_x$ ——— ③

Adding equations ① and ②, we have

$$M_A^P + M_A^Q = Pd_1 + Qd_2$$

or sum of moments $\sum M_A^F = P \times (OA \cos\theta_1) + Q \times (OA \cos\theta_2)$
$= OA\ (P.\cos\theta_1 + Q.\cos\theta_2)$
$= OA\ (P_x + Q_x)$

$\therefore \sum M_A^F = OA \cdot (R_x)$ ——— ④

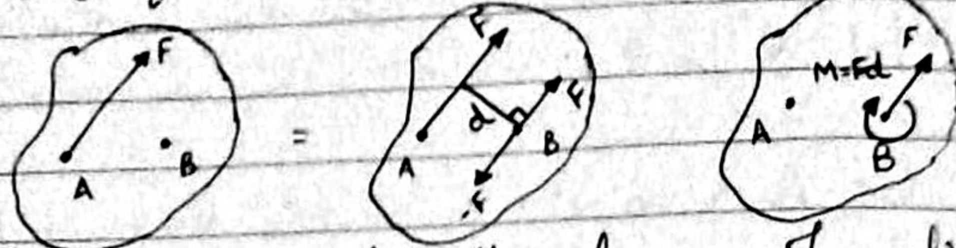
$P_x + Q_x = R_x$ since the resultant of forces in the 'x' direction is the sum of components of forces.

Comparing eqn ③ and ④, we get

$$\sum M_A^F = M_A^R$$

The above equation can be similarly extended for more than two forces in the system.

- Shifting of a force at a new parallel position.



force originally at A — two collinear forces added at B. — Force shifted to B with a couple added.

- Resultant of a parallel system.

To find the Result of Parallel Force System follow

Step 1: The forces are directed in one direction, they can be simply added up using a sign convention for the ~~sense~~ sense of the force. i.e $R = \Sigma F$.

Step 2: Location of the resultant force forms an important step. The point of application of the resultant force is found out using Varignon's theorem.

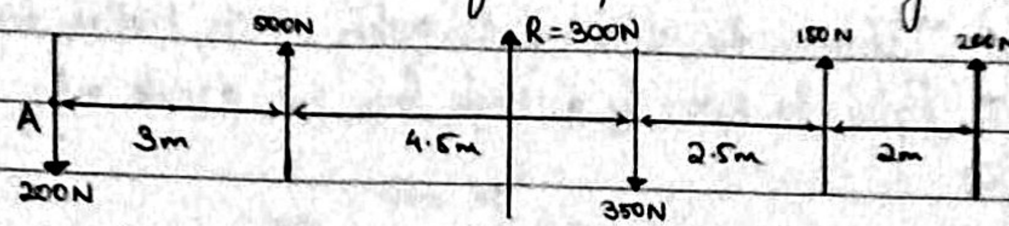
The result is initially assumed to act either to the right or left of the referance point at a $\perp$ distance d.

$\Sigma M_A^F = M_A^R$ is used. If a positive value of d is obtained then the assumption made was right.

- Resultant of General force system.

Combination of steps for concurrent and parallel force systems.

Numericals ④ Determine the resultant of the parallel force system.



Solution- This is a parallel system of five forces

Resultant force $R = \sum F$ $\uparrow$ +ve.

$\therefore R = (-200) + 500 + (-350) + (150) + 200$

$\therefore R = 300N \uparrow$

Let the resultant force be at a ⊥ distance 'd' to the right of 200N force

Let, A be a point on the line of force of 200N.

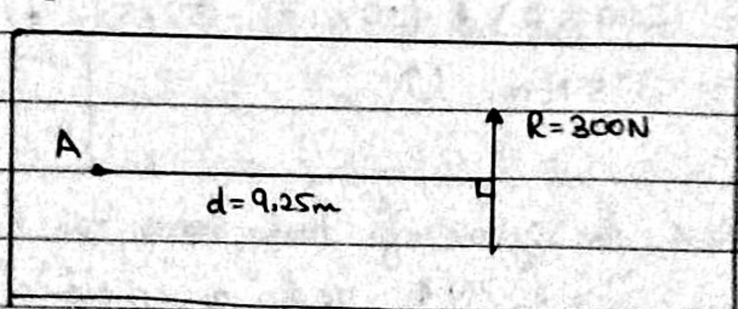
Using Varignon's force theorem,

$$\sum M_A^F = M_A^R \quad \circlearrowleft +ve.$$

$$0 + (500 \times 3) - [350 \times (4.5+3)]$$
$$+ 150 \times (4.5+2.5+3) + 200(3+2+4.5+2.5) = 300 \times d$$

$$\therefore d = 9.25m.$$

Hence, the resultant force $R = 300N \uparrow$ lies at a ⊥ distance $d = 9.25m$ to the right of point A.



Resultant - force.

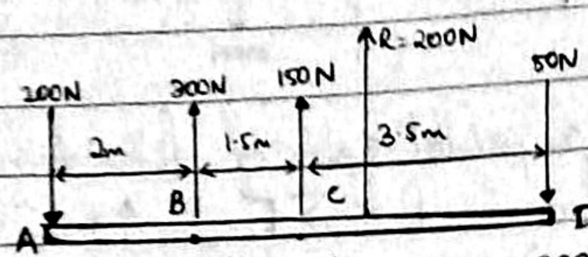
⑤ Figure shows two parallel forces acting on a beam ABCD.

i) Determine the resultant of the system and its location from A.

ii) Replace the system by a single force and a couple acting at point B.

iii) point D.

Solution →



Resultant $R = \sum F$ ↑+ve. $= -200 + 300 + 150 - 50$

$\therefore R = 200N \uparrow$

Location of resultant force from A.

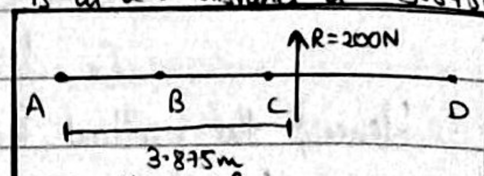
Let, the resultant force be located at a ⊥ distance 'd' from point A.

Using Varignon's theorem, $\sum M_A^F = M_A^R$ ↺ +ve.

$\therefore 0 + (300 \times 2) + [150 \times (1.5+2)] - 50 \times (2+1.5+3.5) = 200 \times d$

$\therefore d = 3.875m.$

$\therefore$ The resultant force $R = 200N \uparrow$ is at a ⊥ distance $d = 3.875m$ ~~from~~ the right of point A.

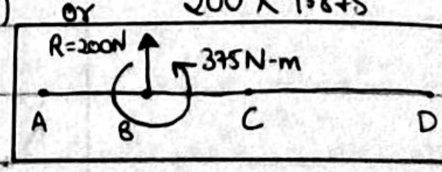


Resultant - force.

(ii) $\sum M_B$ ↺ +ve

$= (200 \times 2) + (150 \times 1.5) - (50 \times 5)$ or 200×1.875

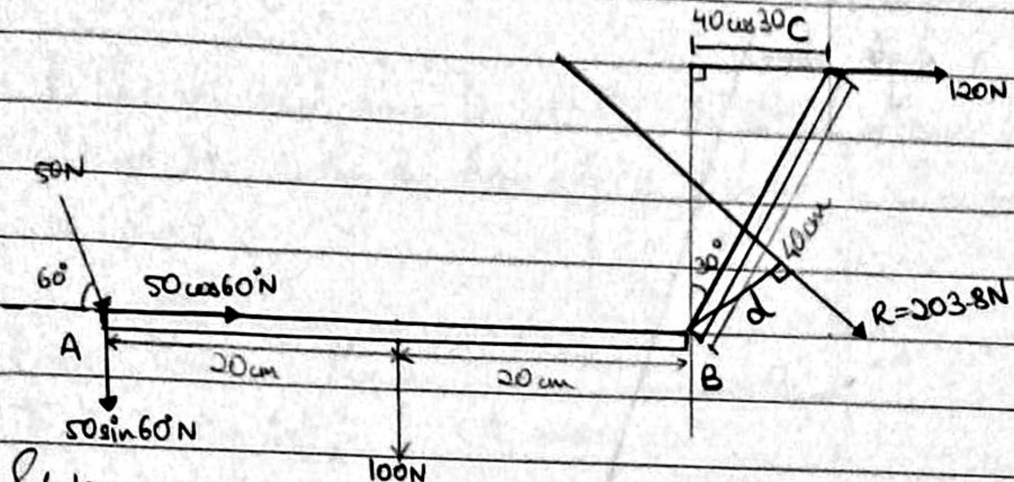
$= 375$ N-m. ↺



Hence, the system of four forces can be replaced by a single force $R = 200N \uparrow$ and a couple of 375 N-m ↺ at B.

This is ~~reffer~~ ~~ref~~ referred to as a force couple system at B.

⑥ Find the resultant of the forces acting on the bell crank lever shown. Also locate its position w.r.t hinge B.



Solution-

This is a general system of 3 forces acting on the bell crank.

$\sum F_x \rightarrow +ve$

$= 120 + 50\cos 60$

$\therefore \sum F_x = 145\,N \rightarrow$

$\sum F_y \uparrow +ve$

$= (-100) + (-50\sin 60) = -143.3\,N$

$\sum F_y = 143.3\,N \downarrow$

Using $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{145^2 + 143.3^2}$

$R = 203.8\,N$ also $\tan\theta = \left|\dfrac{\Sigma F_y}{\Sigma F_x}\right| = \dfrac{143.3}{145}$ $\therefore \theta = 44.66°$

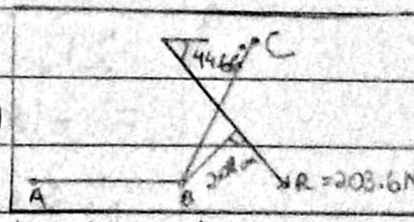
The arrows of ΣF_x and ΣF_y, indicates that the resultant force lies in the IVth quadrant.

Let us assume, the resultant force R is located at a perpendicular distance 'd' to the right of point B. Using Varignon's theorem.

$\sum M_B^F = M_B^R$ ↺ +ve

$(50\sin 60 \times 40) + (100\times 20) - (120 \times 40\cos 30) = -(203.8 \times d)$

$\therefore d = 2.08$ cm. to the right of B



Hence, the resultant force $R = 203.8N$ at $\theta = 44.66°$ is located at $\perp$ distance $d = 2.08$ cm to the right of B.

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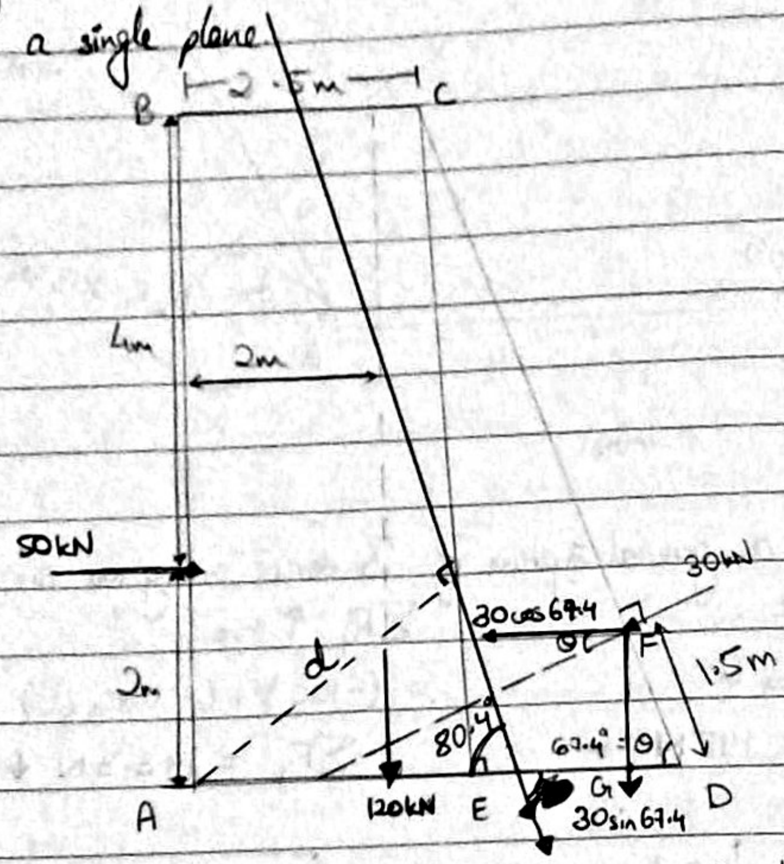
⊕ A dam is subjected to three forces, 50 KN on the upstream face AB, 30 KN force on the down stream inclined face and its own weight of 120 kN as shown. Determine the single force and locate its point of intersection with the base AD assuming all the forces to lie in a single plane.



In Δ CED, CE = 6m, ED = 2.5m. $\therefore$ $\theta = \tan^{-1}(6/2.5)$ $\therefore \theta = 67.4°$

In Δ DFG, $\theta = 67.4°$, FD = 1.5m. $\therefore$ $\sin\theta = \frac{FG}{FD}$. $\cos\theta = \frac{GD}{FD}$

$\therefore$ FG = 1.38m. $\therefore$ GD = 0.5767m.

$\therefore$ AG = AD - GD = 4.423m.

This is a general system of three coplanar forces acting on the dam.

$\Sigma F_x \rightarrow +ve$

$= 50 + (-30\cos 67.4°) = 22.3$ kN $\rightarrow$

$\Sigma F_y \uparrow +ve$

$= -120 - 30\sin 67.4° = -131.5$ kN $= 131.5$ kN $\downarrow$.

$\therefore \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{131.5}{22.3}\right)$ $\therefore \theta = 80.4°$

The arrows of ΣF_x and ΣF_y indicates that the resultant force lies in the 4^{th} quadrant.

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{22.3^2 + 131.5^2} = 133.4\,kN.$$

Point of intersection with the base AD.

Let the resultant force lie at a perpendicular distance 'd' to the right of A, cutting the base at a distance x from end A, as shown.

Using Varignon's theorem,

$$\Sigma M_A^F = M_A^R \qquad \circlearrowleft +ve$$

$$-(50 \times 2) + (-120 \times 2) + (30\cos 67.4 \times 1.38) + (-30\sin 67.4 \times 4.423)$$
$$= -(133.4 \times d)$$
$$\therefore d = 2.64\,m.$$

from geometry; $\sin 80.4^\circ = \dfrac{d}{x} = \dfrac{2.64}{x}$

$$x = \frac{2.64}{\sin 80.4} = \frac{2.64}{0.986} = 2.68\,m.$$

Hence, Resultant force $R = 133.4\,kN$ at $\theta = 80.4^\circ$

lies at a $\perp$ distance $d = 2.64\,m$ right of A and cuts the base AD at $x = 2.68\,m$.

* * * * *

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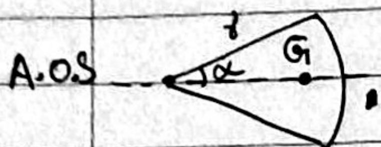
1.3 Centroid.

ⓐ First moment of Area.

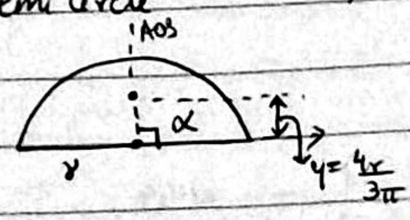
Rectangle Area - $b \times d$. $\bar{x} = b/2$ $\bar{y} = d/2$.

Right angled triangle Area - $\frac{1}{2} b \times h$ $\bar{x} = b/3$ $\bar{y} = h/3$.

Sector Area - $r^2\alpha$ $\bar{x} = \frac{2r}{3}\frac{\sin\alpha}{\alpha}$. $\bar{y} = 0$

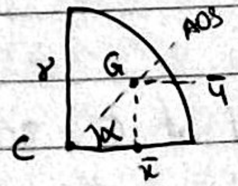


Semi circle Area - $\frac{\pi r^2}{2}$ $\bar{x} = 0$ $\bar{y} = \frac{4r}{3\pi}$.



Quarter circle

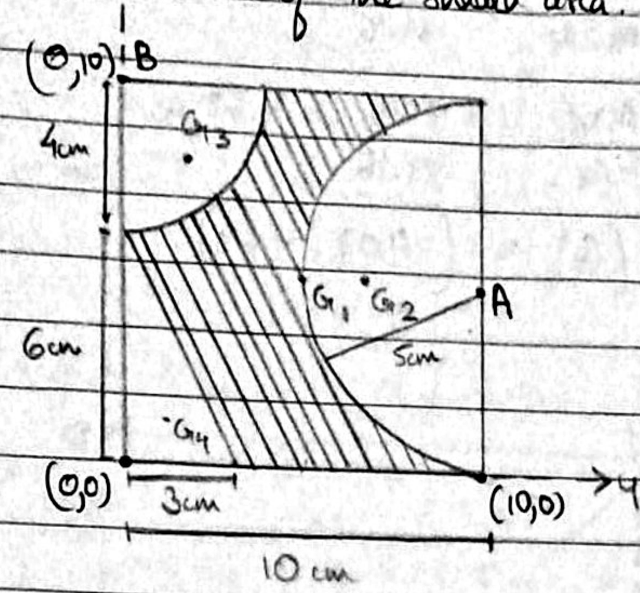
Area = $\frac{\pi r^2}{4}$ $\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$.



Centroid - $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i}, \quad \bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

Numerical Q2. Find centroid of the shaded area.



for triangle,

$G_4 = \left(\frac{l}{3}, \frac{h}{3}\right)$

$G_4 = (1, 2)$

Part	Area (A_i) cm^2	x_i (cm)	y_i (cm)	$A_i x_i$	$A_i y_i$
Square	100	5	5	500	500
Semi-circle	-39.27	7.878	5	-309.37	-196.35.
Quarter circle	-12.57	1.6[illegible]97	8.302	-21.32	-104.33
Triangle.	-9	1	2	-9	-18
	$\boxed{\Sigma A_i = 39.16}$			$\boxed{\Sigma A_i x_i = 160.31}$	$\boxed{181.32}$

for semicircle, area $= \pi r^2/2 = 25\pi/2$

Centroid $G_2 = (\bar{x}_2, \bar{y}_2)$ (from A) $\quad \bar{x}_2 = \frac{2r \sin 90}{3 \quad \pi/2} = \left(\frac{4r}{3\pi}\right) \times -1.$

from A. $G_2 = (-20/3\pi, 0) \quad \bar{x}_2 = (-20/3\pi)$

$\therefore$ from origin, $G_2 = (10 + (-20/3\pi), 5+0)$

$= (30\pi - 20/3\pi, 5) = (7.878, 5)$

Similarly for quarter circle,

$G_2 = \left(0 + \frac{4r}{3\pi}, 10 - \frac{4\pi}{3\pi}\right)$

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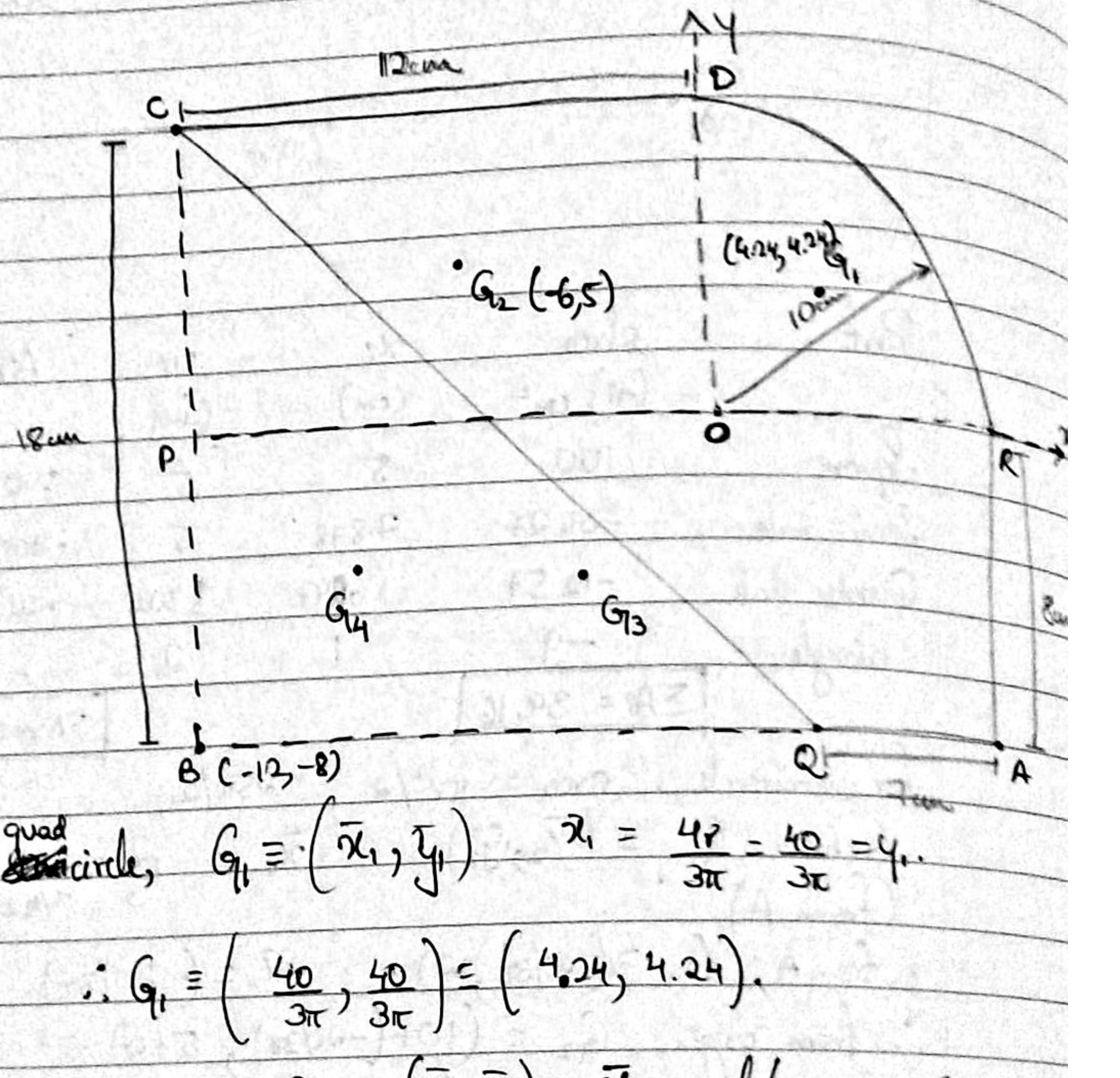
$\therefore \quad \bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{160.31}{39.16} = 4.09 \text{ cm.}$

$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{181.32}{39.16} = 4.63 \text{cm.}$

$\therefore$ Centroid $(G) \equiv (4.09, 4.63)$.

Q3.



for quad circle, $G_1 \equiv (\bar{x}_1, \bar{y}_1)$ $\quad x_1 = \frac{4r}{3\pi} = \frac{40}{3\pi} = y_1$.

$\therefore G_1 \equiv \left(\frac{40}{3\pi}, \frac{40}{3\pi}\right) \equiv (4.24, 4.24)$.

for rectangle CPOD, $G_2 \equiv (\bar{x}_2, \bar{y}_2)$ $\quad \bar{x}_2 = -l/2 = -6 \text{ cm}$

$= (-6, 5)$ $\quad \bar{y}_2 = b/2 = 5 \text{cm}$

for rectangle PRAB, $G_3 \equiv (\bar{x}_3, \bar{y}_3)$ w.r.t B. $\bar{x}_3 = l/2 = \cancel{22/2}\, 11cm$

$\bar{y}_3 = b/2 = 4cm.$

(w.r.t B) $G_3 \equiv (11, 4).$

$\therefore$ (w.r.t O) $G_3 \equiv (11-12, 4-8)$

$\equiv (-1, -4)$

for triangle CBQ, $G_4 \equiv (\bar{x}_4, y_4)$ w.r.t B. $\bar{x}_4 = b/3 = (22-7)/3 = 5cm$

$\bar{y}_4 = h/3 = 6cm.$

(w.r.t B) $G_4 \equiv (5, 6)$

$\therefore$ (w.r.t O) $G_4 \equiv (5-12, 6-8)$

$G_4 \equiv (-7, -2).$

Shape	Area A_i	Coordinate x_i	y_i	$A_i x_i$	$A_i y_i$
Quad circle	78.54	4.24	4.24	333	333
Rectangle 1	120	−6	5	−720	600
Rectangle 2	176	−1	−4	−176	−704
Triangle	−135	−7	−2	945	270

$\boxed{\Sigma A_i = 239.54}$ $\boxed{\Sigma A_i x_i = 382}$ $\boxed{\Sigma A_i y_i = 499}$

$\therefore$ Centroid $(G) \equiv (\bar{x}, \bar{y})$ $\bar{x} = \dfrac{\Sigma A_i x_i}{\Sigma A_i}$ $\bar{y} = \dfrac{\Sigma A_i y_i}{\Sigma A_i}$

$= 382/239.54$ $= 499/239.54$

$\bar{x} = 1.59\,cm$, $\bar{y} = 2.08\,cm.$

$\therefore$ Centroid $(G) \equiv (1.59, 2.08)\,cm.$
of the given figure

47_YASH SARANG

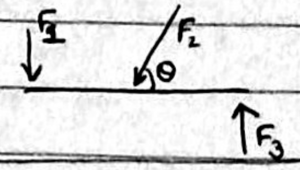
Equilibrium. (Module 2)

A body is said to be in equilibrium if it is in state of rest or uniform motion.

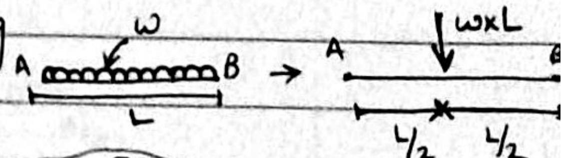
- Conditions of Eq. (COE) ① $\sum F_x = 0$,
  ② $\sum F_y = 0$, ③ $\sum M = 0$.

- Types of Load. - On Beams.

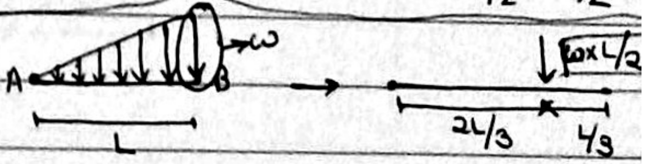
① Point load. (concentrated at one point) Eg.



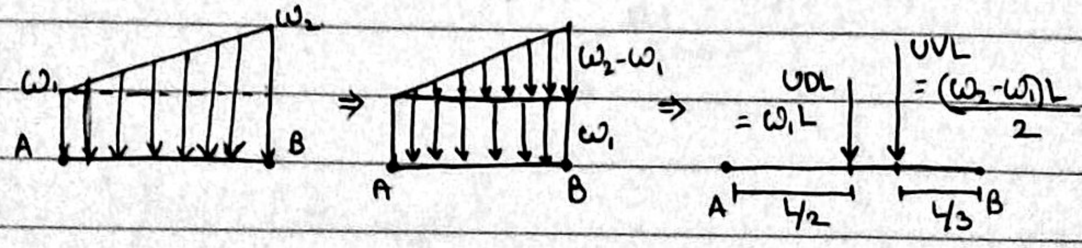
② Uniformly distributed load. UDL. Eg



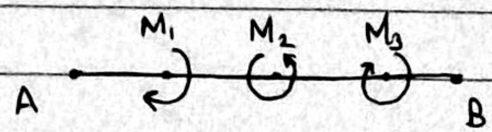
③ Uniformly varying load UVL Eg.



④ Trapezoidal load (Combination of UDL and UVL)

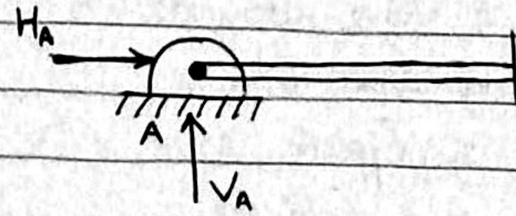


⑤ Couple loads Eg.

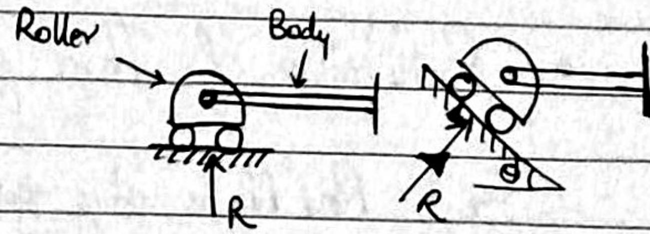


Types of Supports

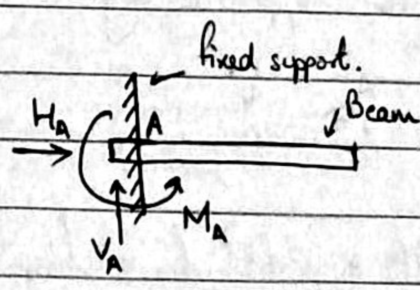
1) Hinge Support.



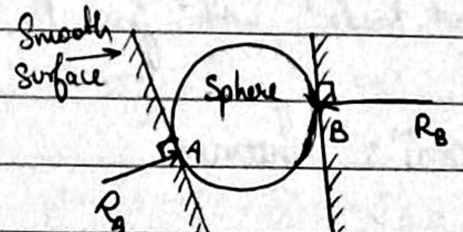
2) Roller Support.



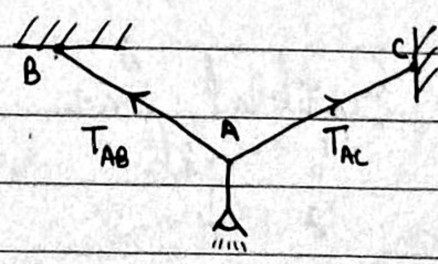
3) Fixed support



4) Smooth Surface Support.



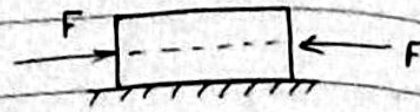
5) Rope / String / Cable support.



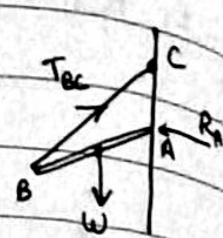
47_YASH SARANG

* Equilibrium of Two force body.

Characteristics
1) Same magnitude.
2) Same direction.
3) Opposite sense.
4) Collinear.

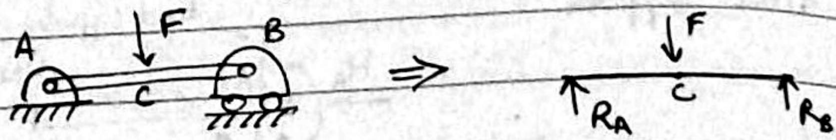


Equilibrium of Three force body.



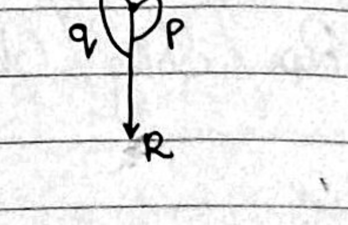
Ex.1. Rod AB is resting against a smooth wall and supported by a string.

Ex 2.



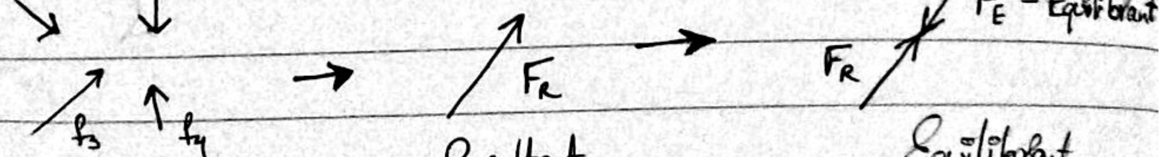
Beam AB supported by a hinge & roller ($\because H_A = 0,\ V_A = R_A$)
and loaded with force F.

Lami's theorem.



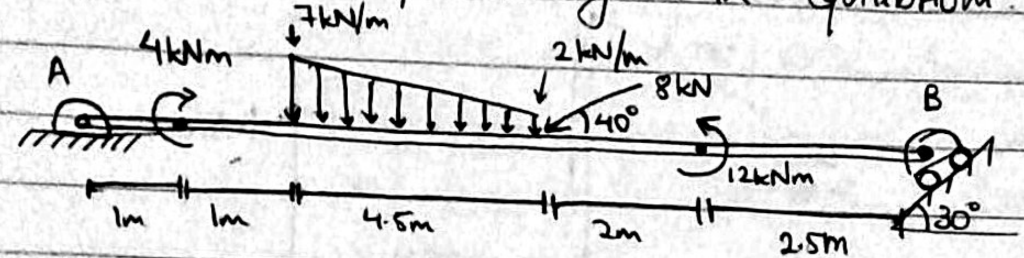
$$\frac{P}{\sin p} = \frac{Q}{\sin q} = \frac{R}{\sin r}$$

Equilibrant force.



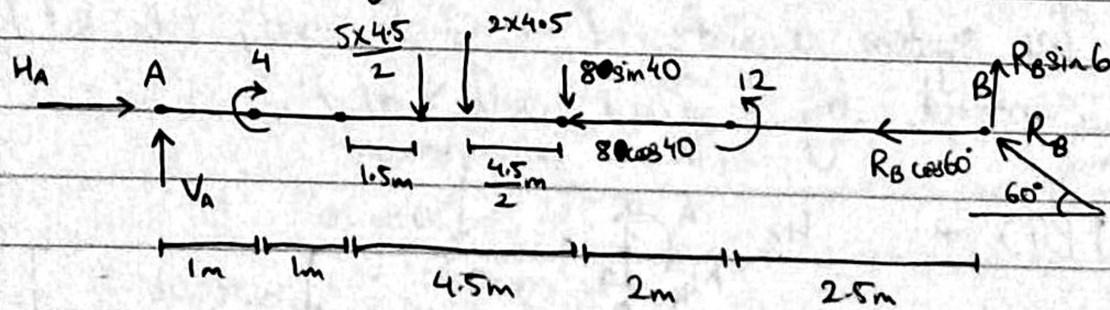
System of forces — Resultant (not equilibrium) — Equilibrant (System in equilibrium)

Q1. The beam AB is loaded by forces and couples as shown. Find the reaction force offered by the supports to keep the system in equilibrium.



→

Consider FBD of Beam AB,



Since Beam AB is in equilibrium

$\Sigma M_A = 0$

So applying COE, (Taking ↑, → and ↺ as +ve)

$$-4 - \left[\left(\frac{5\times4.5}{2}\right)\times 3.5\right] - (9\times4.25) - (8\sin40°\times6.5) + 12 + R_B\sin60°\times11 = 0.$$

$\therefore R_B = 10.82\,kN$ (60°)

Now,

$\Sigma F_x = 0.$ and $\Sigma F_y = 0.$

$H_A - 8\cos40° - 10.82\cos60° = 0$

$\therefore H_A = 11.54\,kN\,(\rightarrow)$

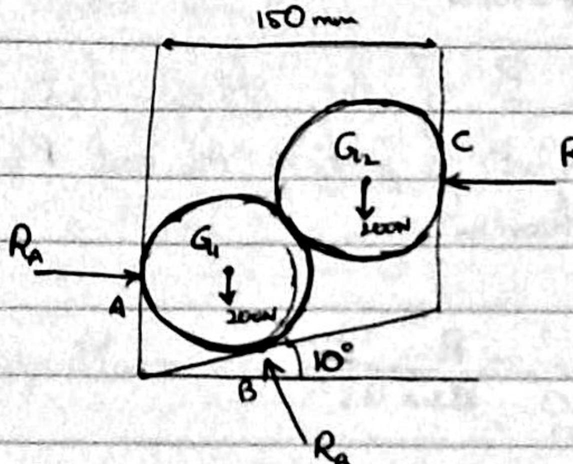
$V_A - \frac{5\times4.5}{2} - 9 - 8\sin40° + 10.82\sin60° = 0$

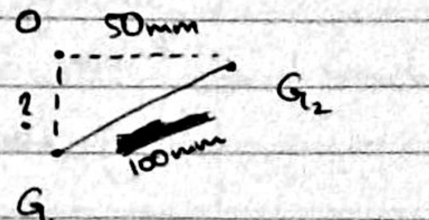
$\therefore V_A = 16.02\,kN\,(\uparrow)$

Q 3. Two cylinders each of diameter 100 mm and each weighing 200 N are placed as shown in figure. Assuming that all the contact surfaces are smooth. Find the reactions at A, B and C.

→

Let R_A, R_B and R_C be the reactions at three supports. The FBD of the system.





$\therefore G_1O = 86.6$mm.

Applying COE to the system

$\therefore \Sigma M_{G_1} = 0$ ($\circlearrowleft$ +ve) $\qquad \therefore -(200 \times 50) + R_C \times 86.6 = 0.$

$\therefore R_C = 115.47$ [redacted] N ($\leftarrow$)

$\therefore \Sigma F_x = 0$ ($\rightarrow$ +ve) and $\qquad \Sigma F_y = 0$. ($\uparrow$ +ve)

$R_A - R_B \cos 80^\circ - R_C = 0$, $\qquad R_B \sin 10^\circ - 400 = 0$

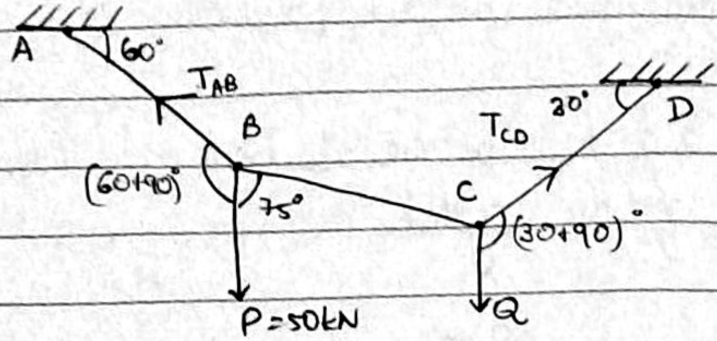
$\therefore R_B = 406.17$N (80°)

$R_A - 406.17 \times \cos 80 - 115.47 = 0$

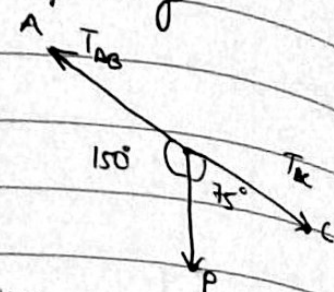
$\therefore R_A = 186$N ($\rightarrow$)

47_Yash SARANG

Q 5) A string ABCD carries two loads P and Q. If P = 50kN, find force Q and tensions in different positions of string.



→ Isolating joint B of the string. Let T_{AB} and T_{BC} be the tension in the string portions AB and BC respectively. Using Lami's theorem.



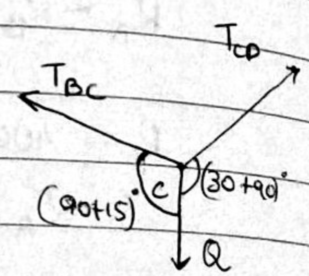
$$\frac{T_{AB}}{\sin 75} = \frac{T_{BC}}{\sin 150} = \frac{P}{\sin 135^\circ}$$

$$\therefore\ T_{AB} = 50\,\frac{\sin 75}{\sin 135},\quad T_{BC} = 50 \times \frac{\sin 150}{\sin 135}$$

$$\therefore\ T_{AB} = 68.3\,kN,\quad \therefore\ T_{BC} = 35.35\,kN.$$

Now isolating joint C of the string. Let T_{CD} be the tension in the string position CD.

Using Lami's theorem,



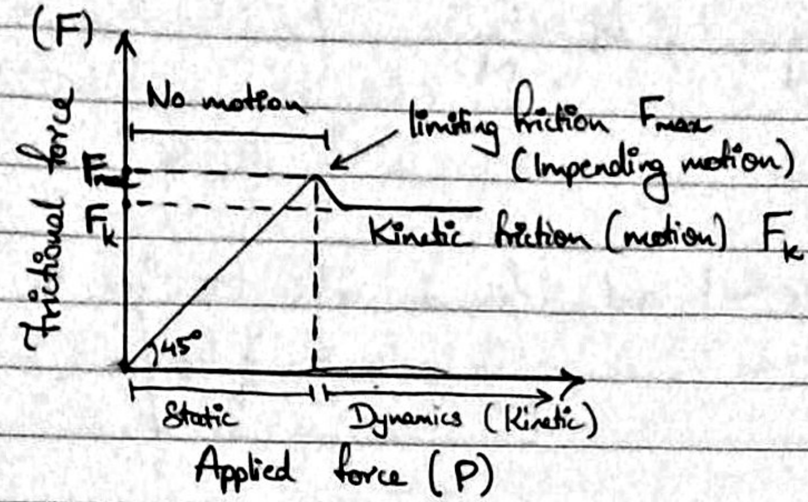
$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 105^\circ} = \frac{Q}{\sin 135^\circ},$$

$$\therefore\ T_{CD} = T_{BC}\,\frac{\sin 105}{\sin 120},\quad Q = T_{BC}\,\frac{\sin 135^\circ}{\sin 120^\circ}$$

$$\therefore\ T_{CD} = 39.43\,kN,\quad Q = 28.86\,kN.$$

* * * * * *

Module 3. FRICTION



We have 3 possibilities,

① Static → $\boxed{F_{max} > P \text{ ie } F}$ Frictional force (F) = Applied force (P)
Body is purely in rest.

② Limiting equilibrium condition. → Impending motion.
ie on the verge of motion.
Frictional force (F) = Limiting frictional force (F_{max})
and $F_{max} = \mu_s N$. $\boxed{P = F_{max}}$

③ Dynamic or Kinetic → Frictional force (F) = Kinetic friction (F_k)
$\boxed{P > F_{max}}$ $F_k = \mu_k N$

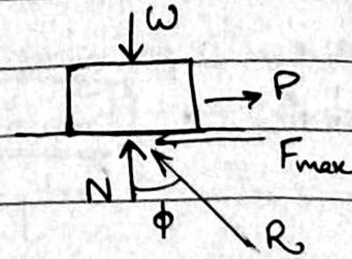
$\mu_s = \frac{F_{max}}{N}$, $\mu_k = \frac{F_k}{N}$ $F_k < F_{max}$,

$\therefore \mu_k < \mu_s$

Note: $\mu < 1$ and μ_k is 25% less than μ_s

$$\boxed{\therefore \mu_k = \frac{3}{4}\mu_s}$$

* Angle of friction (ϕ)



$$R = \sqrt{N^2 + F_{max}^2} = \sqrt{N^2 + N^2\mu_s^2} = N\sqrt{1+\mu_s^2}$$

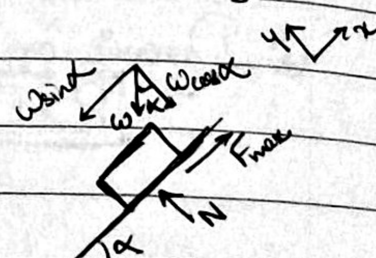
$$\therefore R = N\sqrt{1+\mu_s^2}$$

Now, $\tan\phi = \frac{F_{max}}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$

$$\therefore \phi = \tan^{-1}(\mu_s) \quad , \quad R = N\sqrt{1+\mu_s^2}.$$

* Angle of Repose (α)

minimum angle of inclination of plane with horizontal for which body will just slide down on it without any external force.



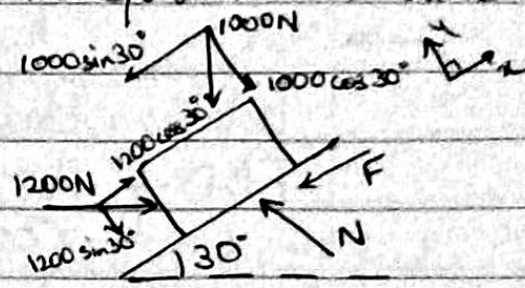
$$\tan\alpha = \mu_s$$

$$\therefore \alpha = \phi$$

Angle of repose is independent of weight of the block.

Numericals ① Problems on Simple Blocks.

① If a horizontal force of 1200N is applied to a block of 1000N, then the block will be held in equilibrium or slide down or move up? $\mu = 0.3$.



→

let F be the frictional force acting down the plane be required to keep the block in equilibrium.

Here we cannot take $F = \mu N$, since the block may not be on the verge of motion.

$\therefore \sum F_y = 0$, $N - 1000\cos 30° - 1200 \sin 30° = 0$
$(\uparrow +ve)$
$\therefore N = 1466N.$

$\therefore \sum F_x = 0$, $-F - 1000\sin 30° + 1200\cos 30° = 0$
$\therefore F = 539.2N.$ i.e $F_{required} = 539.2N.$

F_{req} is the force required to keep to keep the block in equilibrium.
Now, the maximum friction force the contact surface can produce

$F_{max} = \mu N = 0.3 \times 1466.$

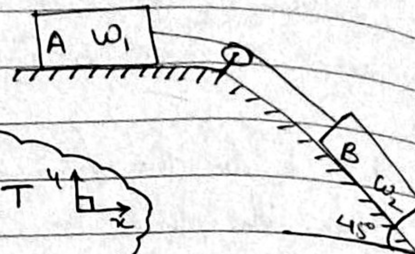
$\therefore F_{max} = 439.8N.$

Since $F_{req} > F_{max}$, the block will not be in equilibrium, but is moving up since F_{max} is directed down.

47_Yash Sarang

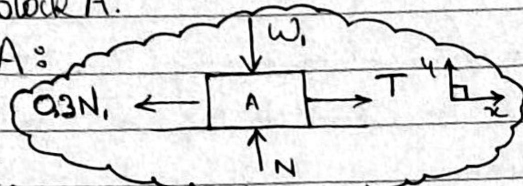
(2) Problems on Connected Bodies.

- $\mu = 0.3$. find the minimum ratio of W_1/W_2 required to maintain equilibrium.



→ Isolating block A.

FBD of A:



Applying COE,

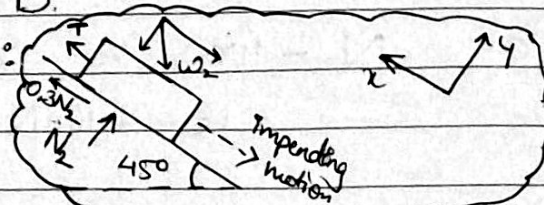
$\therefore \Sigma F_x = 0$ (→ +ve) $T = 0.3N_1$

$\Sigma F_y = 0$ (↑ +ve) $N = W_1$

$\boxed{\therefore T = 0.3W_1}$

Isolating block B.

FBD of B:



Applying COE,

$\therefore \Sigma F_x = 0$, $\therefore T + 0.3N_2 - W_2 \sin 45° = 0$.

$\therefore \Sigma F_y = 0$. $\therefore -W_2 \cos 45° + N_2 = 0$. $\boxed{\therefore N_2 = 0.707W_2}$

$$\therefore T = W_2 \sin 45 - 0.3(0.707W_2)$$

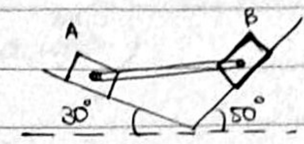
$$\therefore T = 0.4949\,W_2$$

$$\therefore 0.3W_1 = 0.4949W_2$$

$$\therefore \frac{W_1}{W_2} = \frac{0.4949}{0.3}$$

$$\boxed{\therefore \frac{W_1}{W_2} = 1.65}$$

* Block A weighs 1500N, determine maximum weight of Block B for which equilibrium is maintained. Angle of friction for all surfaces of contact is 15°.



→ $15^\circ = \phi = \tan^{-1} \mu_s$.

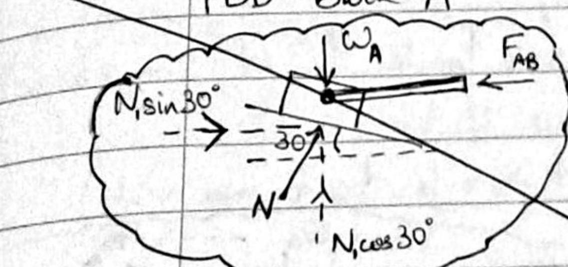
$\therefore \mu_s = \tan\phi = \tan 15^\circ$.

$\therefore \mu_s = 0.268$

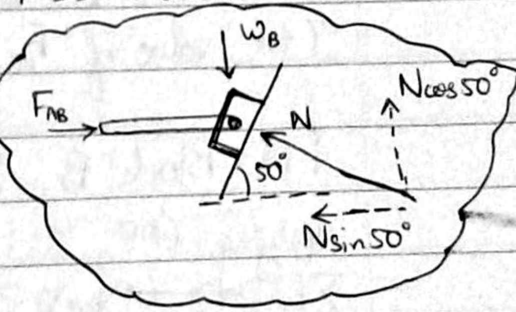
For block B, when its weight W_B is maximum, it will tend to slide down the slope causing block A to be pushed up its plane.

let us isolate the system by ~~cutt~~ cutting the rod. Assume force F_{AB} in the rod is comprehensive in nature.

FBD - Block A



F.BD - Block B.



Applying COE,

$\sum F_y = 0 \; (\uparrow +ve)$

$\therefore N_1 = 2049 N$

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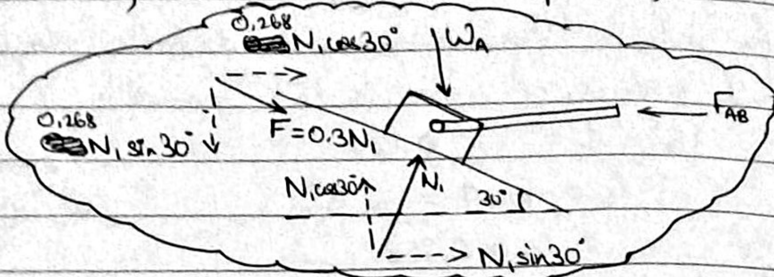
FBD Block A, (tends to move ~~up~~ upwards)



Applying COE, $\Sigma F_y = 0$ (↑ +ve)

$N_1\cos 30 - \cancel{0.3N_1\sin 30} - W_A - 0.268N\sin 30° = 0.$

$\boxed{\therefore N_1 = 2049\text{ N}}$

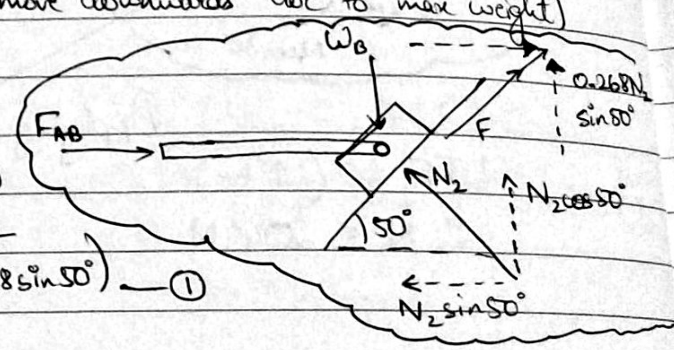
$\Sigma F_x = 0$ (→ +ve)

$0.268 N_1\cos 30° + N_1\sin 30° - F_{AB} = 0.$

$\boxed{\therefore F_{AB} = 1500\text{N}}$

(+ve value of F_{AB} indicates that force F_{AB} in the rod is comprehensive)

FBD Block B, (tends to move downwards due to max weight)



Applying COE,

$\Sigma F_y = 0$ (↑ +ve)

$-W_B + N_2(\cos 50° + 0.268\sin 50°) = 0$

$W_B = N_2(\cos 50° + 0.268\sin 50°)$ — ①

$\therefore \Sigma F_x = 0$ (→ +ve)

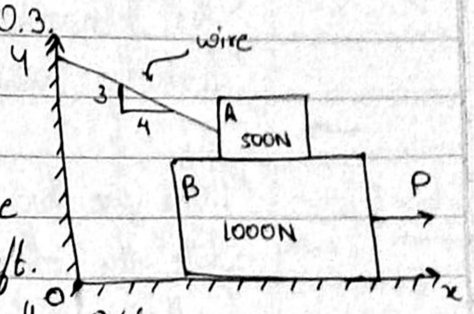
$F_{AB} - N_2\sin 50° + 0.268N_2\cos 50°.$

$\boxed{\therefore N_2 = 2526.2\text{N}}$

$\boxed{\therefore W_{B(max)} = 2142.4\text{ N.}}$

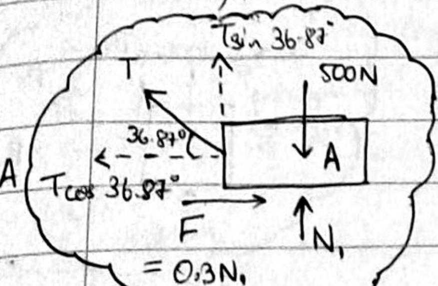
③ Problems on Combined Bodies

* The upper block is tied to a vertical wall by a wire. Determine the horizontal force P required to just pull the lower block. Coefficient of friction for all ~~to~~ surfaces is 0.3.



→ figure shows the blocks isolated.
Since block B tends to move to the right the friction force acts to the left. Hence, for the block A, friction acts to the right.

FBD Block A

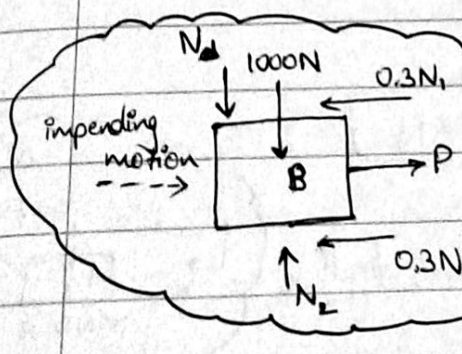


$\Sigma F_y = 0$ (↑ +ve) i) $\boxed{\therefore N_1 + T\sin 36.87^\circ - 500 = 0}$

$\therefore \Sigma F_x = 0$ (→ +ve) $\therefore 0.3N_1 - T\cos 36.87^\circ = 0$

ii) $\boxed{N_1 = \dfrac{T\cos 36.87^\circ}{0.3}}$

from ① and ②, we get $\boxed{N_1 = 408.2N}$



Applying COE,

$\Sigma F_y = 0$ (↑ +ve)

$\therefore N_2 - N_1 - 1000 = 0$

$\therefore \boxed{\therefore N_2 = 1408.2N}$

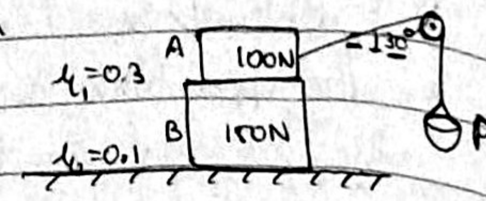
$\therefore \Sigma F_x = 0$ (→ +ve) $P - 0.3N_1 - 0.3N_2 = 0$

$\boxed{\therefore P = 544.9N}$

PTR: When force is applied to upper block for combined blocks check for num of possibilities = no. of ~~blocks~~ blocks.

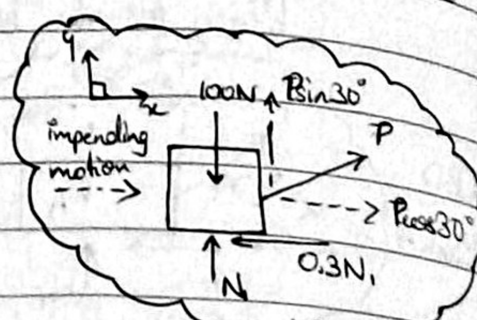
47_YASH SARANG

* Blocks A and B are resting on ground as shown. μ between ground and block B is 0.1 and that between A and B is 0.3.
Find the minimum value of P in the pan so that motion starts.



→

There are two possibilities, One is that block A move over block B, while the other possibility is that ~~b~~ both blocks A and B move together over the ground.



1st possibility : Block A move over block B.

Applying COE,

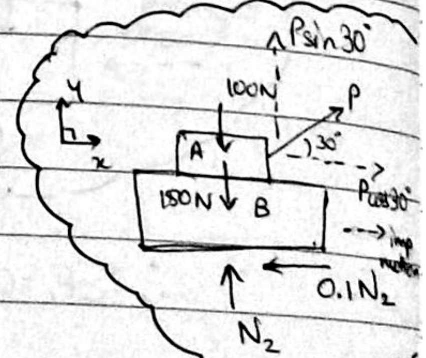
$\Sigma F_x = 0$. ($\rightarrow$ +ve)

$P\cos 30° - 0.3 N_1 = 0$ —— ①

$\Sigma F_y = 0$ ($\uparrow$ +ve)

$P\sin 30° + N_1 - 100 = 0$ —— ②

from ① and ②, $\boxed{P = 29.53 N}$



2nd possibility : Block A and B move together

$\therefore \Sigma F_x = 0$ ($\rightarrow$ +ve)

$P\cos 30° - 0.1 N_2 = 0$ —— ①

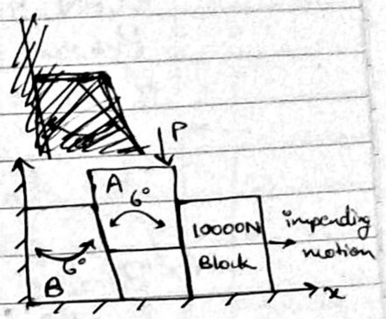
$\Sigma F_y = 0$ ($\uparrow$ +ve)

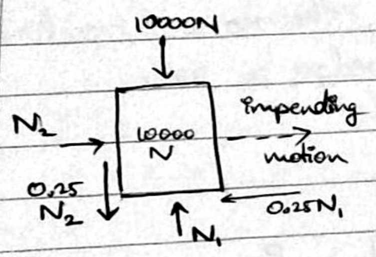
$N_2 + P\sin 30 - 250 = 0$ —— ②.

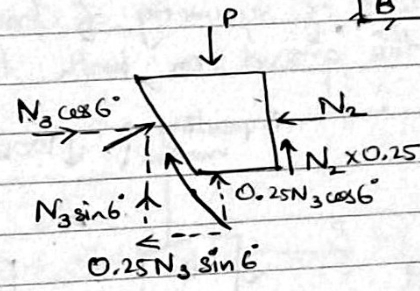
$\boxed{\therefore P = 27.29 N}$

④ Problems on Wedges.

• Two 6° wedges are used to push the block horizontally as shown. Calculate the minimum force P required to push the block of weight 10000N, $\mu = 0.25$ for all surfaces.







Applying COE,

$\sum F_x = 0 \; (\rightarrow +ve)$

$N_2 - 0.25N_1 = 0$ —— ①

$\sum F_y = 0 \; (\uparrow +ve)$

$N_1 - N_2 - 10000 = 0$ —— ②

from ① and ②,

$\boxed{N_2 = 2666.7N}$

$\sum F_x = 0 \; (\rightarrow +ve)$

$N_3 \cos 6^\circ - 0.25 N_3 \sin 6^\circ - N_2 = 0.$

$\boxed{\therefore N_3 = 2753.7N}$

$\sum F_y = 0 \; (\uparrow +ve)$

$0.25 N_2 + 0.25 N_3 \cos 6^\circ + N_3 \sin 6^\circ - P = 0$

$\boxed{\therefore P = 1639.2N}$

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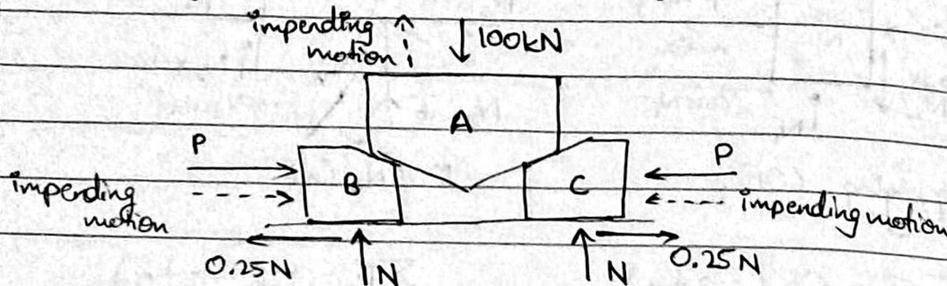
Q8) Calculate P acting on wedges B and C to raise a load of 100kN resting on A. μ between wedge and ground is 0.25. Between wedges and A is 0.2. Also assume symmetry of loading and neglect weights of A, B and C. Slope of wedges 1:10

→ Slope $\tan\theta = \frac{1}{10}$ $\therefore \theta = \tan^{-1}(1/10)$
$= 5.71^\circ$

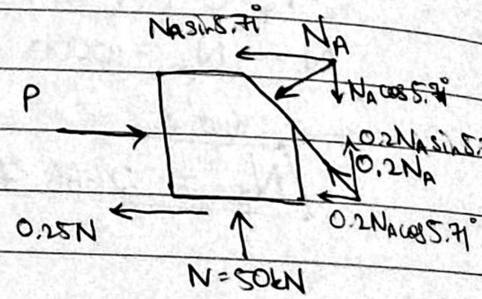
Because of symmetry of loading, the normal reaction offered by the ground on both the wedges is same.



$\therefore \Sigma F_y = 0$ ($\uparrow$ +ve).

$\therefore 2N = 100kN$ $\boxed{\therefore N = 50kN}$

Isolating wedge B,



$\therefore \Sigma F_y = 0$ ($\uparrow$ +ve)

$50 + 0.2N_A \sin 5.71^\circ - N_A \cos 5.71^\circ = 0$

$\boxed{\therefore N_A = 51.27N}$

$\therefore \Sigma F_x = 0$ ($\rightarrow$ +ve)

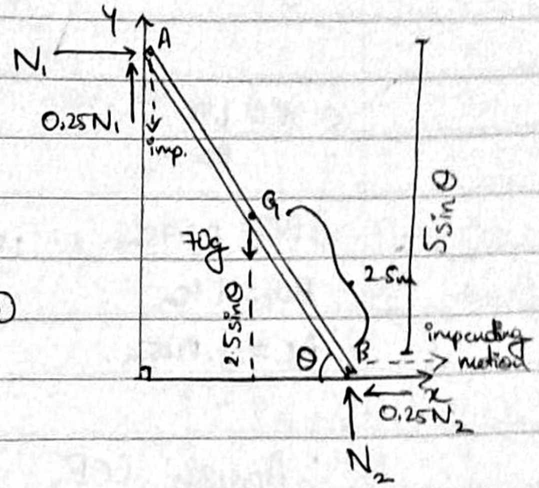
$P - 0.25 \times 50 - 0.2N_A \cos 5.71 - N_A \sin 5.71 = 0$

$\boxed{\therefore P = 27.8 kN}$

⑤ Problems on Ladders.

Q 9) Find minimum θ for equilibrium. $\mu = 0.25$.
$AB = 5m$, mass $= 70kg$.

→



Applying COE,

$\sum F_y = 0$ ($\uparrow$ +ve)

$0.25N_1 - 70 \times 9.81 + N_2 = 0$ — ①

$\sum F_x = 0$ ($\rightarrow$ +ve)

$N_1 - 0.25N_2 = 0$ — ②

from ① and ②, $N_2 = 646.3N$ and $N_1 = 161.6N$.

also,

$\sum M_B^F = 0$ ($\circlearrowleft$ +ve)

$70 \times 9.81 \times 2.5\sin\theta - 0.25N_1 \times \overset{5\cos\theta}{\cancel{5\sin\theta}} - N_1 5\sin\theta = 0$

$\therefore \tan\theta \approx \dfrac{1514.75}{808}$

$\therefore \theta = 61.92°$.

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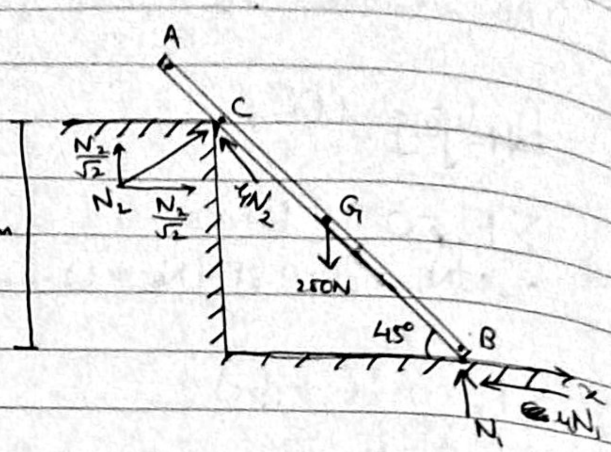
Q 10) Determine minimum μ so as to maintain the position in figure. length of rod AB is 3.5m and it weighs 250N.

$\sin 45^\circ = \frac{1.75}{BC}$

$\therefore BC = 2.475m$

$BG = 1.75m$

$AC = 1.025m.$



$\therefore$ Applying COE,

$$\Sigma F_x = 0 \quad (\rightarrow +ve)$$

$$-\mu N_1 + \frac{N_2}{\sqrt{2}} - \frac{\mu N_2}{\sqrt{2}} = 0.$$

$$N_1 = \frac{N_2}{\sqrt{2}}\left(\frac{1-\mu}{\mu}\right) \quad \text{— ①}$$

$$\Sigma F_y = 0 \quad (\uparrow +ve)$$

$$N_1 - 250N + \frac{N_2}{\sqrt{2}} + \frac{\mu N_2}{\sqrt{2}} \quad \text{— ②}$$

$$\Sigma M_B^F = 0 \quad (\circlearrowleft +ve)$$

$$250 \times \frac{1.75}{\sqrt{2}} - \cancel{\frac{\mu N_2}{\sqrt{2}} \times 1.75} + \cancel{\frac{\mu N_2}{\sqrt{2}} \times 1.75} - N_2 \times 2.475 = 0.$$

$$\therefore N_2 = 125N.$$

$\therefore$ Substituting N_2 and solving eqn ① and ②, we get

$$\mu = 0.414 \text{ or } 2.414.$$

Since $\mu < 1$, $\boxed{\mu = 0.414}$

Kinematics (Module 4)
of Particles

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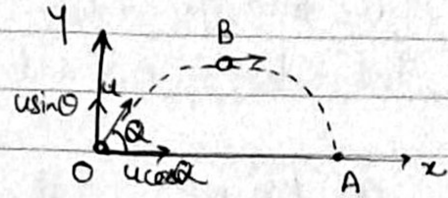
Horizontally,

$s = ut + \frac{1}{2}at^2$



$\boxed{x = u\cos\theta\, t + 0}$

Vertically, $x = u\sin\theta + \frac{1}{2}(-g)t^2$, $0 = u\sin\theta + (-g)t$

$t = \frac{u\sin\theta}{g}$ (O to B)

$\therefore x = \frac{u\cos\theta \times 2u\sin\theta}{g}$

$\boxed{\therefore T = \frac{2u\sin\theta}{g}}$

$\boxed{R = \frac{u^2\sin 2\theta}{g}}$

$0 = (u\sin\theta)^2 + 2(-g)s.$

$\boxed{\therefore H = \frac{u^2\sin^2\theta}{2g}}$

$\boxed{y = x\tan\theta + \frac{1}{2}gx^2 \times \frac{1}{u^2\cos^2\theta}}$

$\boxed{v = \frac{dx}{dt}}$, $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

$\boxed{a = v\frac{dv}{dx}}$

a) Normal Kinematics

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Numericals: 1) $a = kt^2\ m/s^2$. $v = -24\ m/s$ when $t = 0$, $v = 48\ m/s$ when $t = 4$ sec, Also $x = 0$ at $t = 3$ sec find k, x, v and a at $t = 2$ sec.

→

$a = kt^2\ m/s^2$ $\therefore v = \frac{kt^3}{3} + c. = \frac{kt^3}{3} - 24.$

$\therefore$ at $t = 0$, $v = -24 = c.$

at $t = 4$ sec, $v = 48 = \frac{k(4)^3}{3} - 24.$

$\therefore 48 \times 3 = 64k - 72$

$\cancel{144} + \cancel{72} = \cancel{64}k$

$\frac{\cancel{54}\ 27}{\cancel{16}\ 8} = k$

$\boxed{\therefore k = \frac{27}{8}} = 3.375.$

$x = \frac{kt^4}{12} - 24t + c$, at $t = 3$,

$0 = \frac{27}{8} \times \frac{3^4}{12} - 24 \times 3 + c.$

$8 - \frac{81}{32} = c.$ $\therefore x = \frac{kt^4}{12} - 24t + 8 - \frac{81}{32}.$

$\therefore$ at $t = 2$ sec, $x = \frac{27}{8} \times \frac{2 \times 2 \times 2 \times 2}{12} - 24 \times 2 + 8 - \frac{81}{32}$

$x =$

$$\therefore \quad x = \frac{k}{3} \times \frac{t^4}{4} - 24t + c. = 0.2813\,t^4 - 24t + c.$$

at $t = 3$, $x = 0$ $\quad \therefore 0 = 0.2813 \times 3^4 - 24 \times 3 + c.$

$$\therefore c = 49.22.$$

$$\therefore x = 0.2813t^4 - 24t + 49.22$$

$$v = 1.125\,t^3 - 24$$

$$a = \cancel{16}\ 3.375\,t^2.$$

at $t = 2\,sec$,

$$\boxed{\begin{aligned} x &= 5.721\ m \\ v &= -15\ m/s \\ a &= 13.5\ m/s^2 \end{aligned}}$$

Q2) $v = 6t - 3t^2\ m/s$, If $s = 0$ when $t = 0$, find a and x when $t = 3sec$. And also s i.e distance travelled by the particle in the 3 sec time interval and also its average speed.

→ $v = 6t - 3t^2\ m/s$, $a = 6 - 6t\ m/s^2$,

$$x = -\frac{\cancel{3}t^3}{\cancel{3}} + \frac{\overset{3}{\cancel{6}}t^2}{\cancel{2}} + c.$$

when $t = 0$, $x = 0$, $0 = c$. $\quad \therefore x = -t^3 + 3t^2\ m.$

$\therefore$ when $t = 3$, $x = -(3)^3 + 3(3)^2$ and $a = 6 - 6t = 6 - 6(3)$

$\boxed{x = 0m}$ $\qquad \boxed{a = -12\,m/s^2}$

Since distance travelled in 3 sec is also asked, we need to find whether the particle reverses its sense of motion during the 3 sec motion. For a particle to reverse, its velocity should first become zero. Therefore equating $v = 0$,

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$v = -3t^2 + 6t = 0$

$\therefore t = 0$ or $t = 2$.

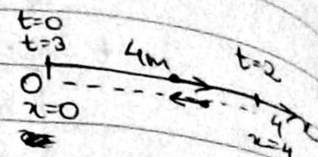
$\therefore$ at $t = 2$sec, the particle reverses its sense of motion

Position calculation,

at $t=0$, $x_0 = 0$.

at $t=2$, $x_2 = (3)(2)^2 - 2^3 = 4\,m$.

at $t=3$, $x_3 = 0$.



$\therefore$ Total distance travelled $= 4+4 = 8m$.

Also, average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{8}{3} = 2.66\,m/s$.

3) $a = -0.05v^2\ m/s^2$, $v = 20\,m/s$ at $x=0$.

find x at $v = 15\,m/s$, ~~and~~ a at $x = 50m$.

$a = v\dfrac{dv}{dx} = -0.05v^2 \quad \therefore \dfrac{dv}{dx} = -0.05v$.

$\therefore \dfrac{1}{v}dv = -0.05\,dx$

Integrating taking lower limits as $v = 20\,m/s$ and $x=0$.

$$\int_{20}^{v} \frac{1}{v}\,dv = \int_{0}^{x} -0.05\,dx.$$

$$\left[\log_e v - \log_e 20\right] = -0.05\,(x).$$

$\therefore x = -20\log_e\left(\dfrac{v}{20}\right)$. Substituting $v = 15\,m/s$, $\boxed{x = 5.745m}$

Substituting $x = 50m$, $\boxed{v = 1.642\,m/s}$ $\boxed{\therefore\ a = -0.1348\,m/s^2}$

B) Projectile motion.

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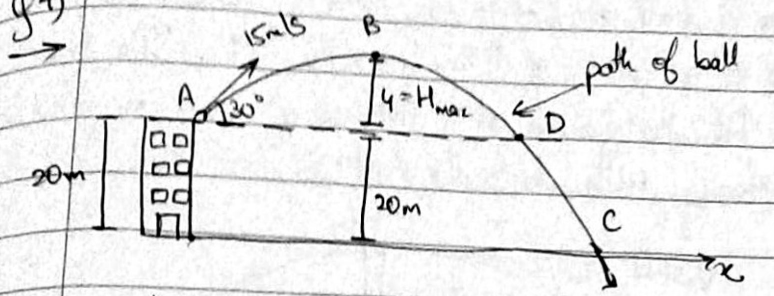
(Q4)



While projectile motion (A-D)

$$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(15\cos 30)^2 \times (\sin 30)^2}{2\times 9.81}$$

$H_{max} = 2.867m.$

$\therefore$ Total maximum height $= H_{max} + 20$

$= 22.867m.$

Projectile motion A-C,

Horizontal motion, $u_x = V_x = 15\cos 30$ — ① $= 13m/s.$

$s = ut$

$\therefore \frac{x}{t} = 13m/s.$ — ①

Vertical motion, $V_y = 15\sin 30 - gt.$

$s = 15\sin 30 t - \frac{g}{2}t^2.$

$-20 = 15\sin 30 t - \frac{9.81}{2}t^2.$ $\therefore t = 2.924$ sec.

$\therefore V_y = 21.18 m/s.$

$\therefore x = 13 \times 2.924 = 38.01 m.$

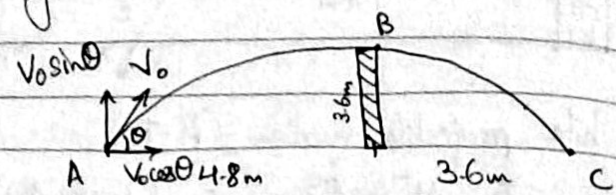
$\therefore$ Velocity of landing at C will be $V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{13^2 + 21.18^2}$

$\therefore \tan\alpha = \frac{V_y}{V_x} = \frac{21.18}{13} = 58.46°$ $\therefore V = 24.85 m/s.$

Q 5) A boy throws a ball so that it may clear a 3.6m high wall. ~~A~~ The boy is at a distance of 4.8m from the wall. The ball was found to hit the ground at a distance of 3.6m on other side. Find least velocity with which the ball can be thrown.

→



at $x = 4.8m$,

$y = 3.6m$. $\quad R = (3.6 + 4.8)m = 8.4m$.

~~$y = xt$~~ $\quad \therefore 8.4 = \dfrac{u^2 \sin 2\theta}{g}$ — ①

$\therefore y = x\tan\theta + \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\theta}$

$3.6 = 4.8\tan\theta + \dfrac{1}{2}\dfrac{g(4.8)^2}{u^2\cos^2\theta}$ — ②

for ①,

$$8.4 = \frac{u^2}{g}\,\frac{2\sin\theta\cos\theta \times \cos\theta}{\cos\theta}$$

$$\frac{8.4 \times g}{2\tan\theta} = u^2\cos^2\theta \text{ — ③}$$

from ② and ③, $\quad 3.6 = 4.8\tan\theta + \dfrac{(4.8)^2 \cancel{g}}{2} \times \dfrac{2\tan\theta}{8.4 \times \cancel{g}}$

$$\therefore \tan\theta\left(4.8 - \frac{4.8 \times 4.8 \times \cancel{2}}{8.4 \times \cancel{2}}\right) = 3.6.$$

$\tan\theta\,(4.8)\left(\dfrac{\cancel{3.6}}{8.4 \times \cancel{3.6}}\right) = 1 \qquad \therefore \theta = \tan^{-1}\left(\dfrac{8.4}{4.8}\right)$

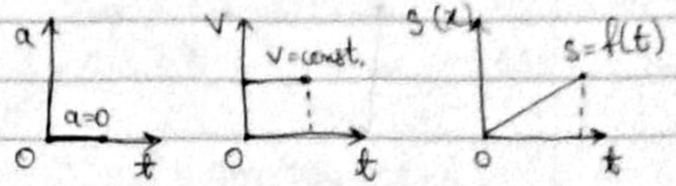
$\therefore \theta = 60.26°$.

$\therefore$ from ③, $\frac{8.4 \times 9.81}{2 \times 8.4} \times 4.8 = u^2 (\cos 60.26)^2$.

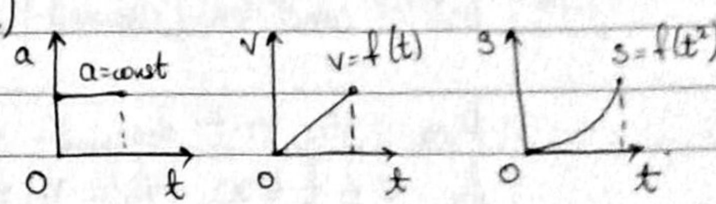
$\therefore u = 9.776 \, m/s$.

c) Motion Curves.

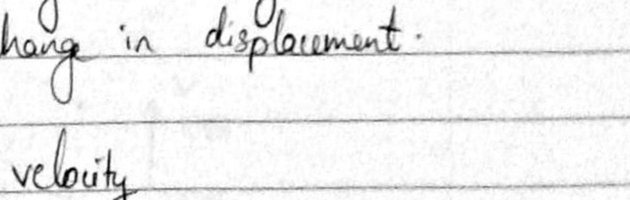
① uniform velocity ($a = 0$)



② uniform acceleration (a = const.)



③ variable acceleration ($a = f(t)$)



area under a-t curve = change in velocity
v-t curve = change in displacement.

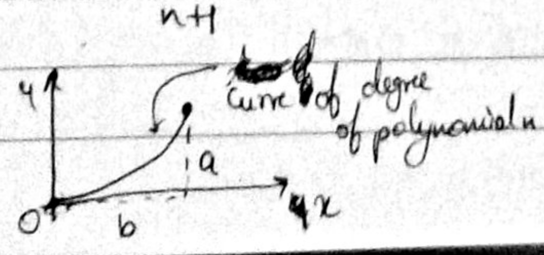
slope of x-t curve = velocity
v-t curve = acceleration.

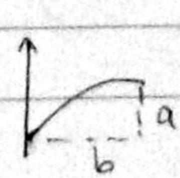
for inward curve, area

$\therefore$ Area $A = \frac{ab}{n+1}$

for outward curve,

$\therefore$ Area $A = \frac{n \, ab}{n+1}$





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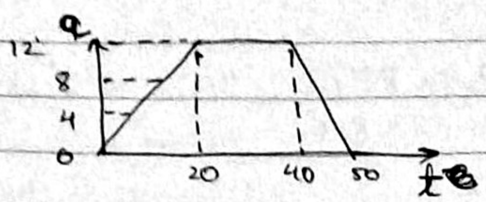
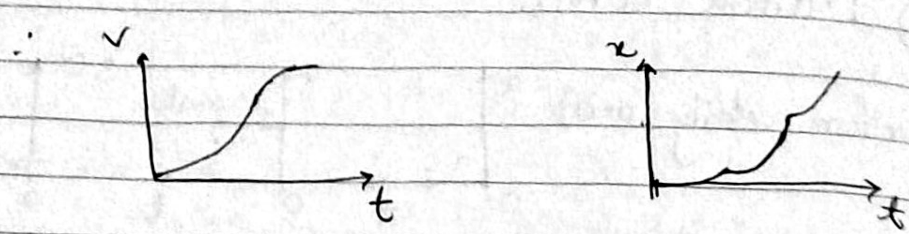
Q6)



find speed and distance covered after 50sec. maximum speed at the time. Draw v-t and x-t diagrams.



→ Initial condition (assumed) $V_0 = 0$, $x_0 = 0$.

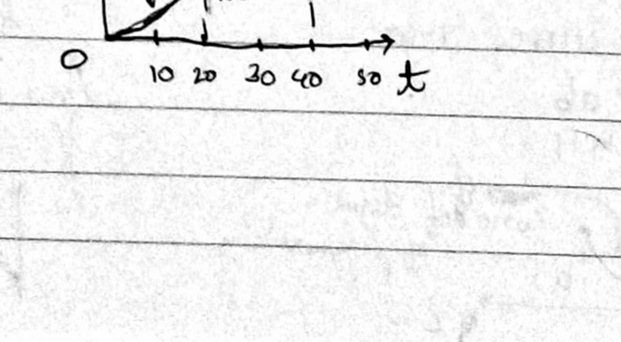
Area under a-t diagram = change in velocity. (Δv).

for $0 \le t \le 20$ $A_1 = 1/2 \times 20 \times 12 = 120 = V_{20} - V_0$

$20 \le t \le 40$ $A_2 = 20 \times 12 = 240 = V_{40} - V_{20}$

$40 \le t \le 50$ $A_3 = 1/2 \times 10 \times 12 = 60 = V_{50} - V_{40}$.

$\therefore V_0 = 0$, $V_{20} = 120\ m/s$, $V_{40} = 360\ m/s$, $V_{50} = 420\ m/s$



using v-t diagram,

Area under v-t diagram = Change in displacement.

for $0 \le t \le 20$, $A_1 = \frac{a \times b}{n+1}$, $a = 20$, $b = 120$ and $n = 2$

$20 \le t \le 40$

$40 \le t \le 50$

$= \frac{20 \times \cancel{120}^{40}}{\cancel{3}} = 800 = x_{20} - x_0$

$\therefore x_{20} = 800m.$ since $x_0 = 0$ (assumed)

$\therefore A_2 = \frac{1}{2} a \times b + c \times d = \frac{1}{\cancel{2}} \times 20 \times \cancel{240}^{120} + 20 \times 120 = 4800 = x_{40} - x_{20}$

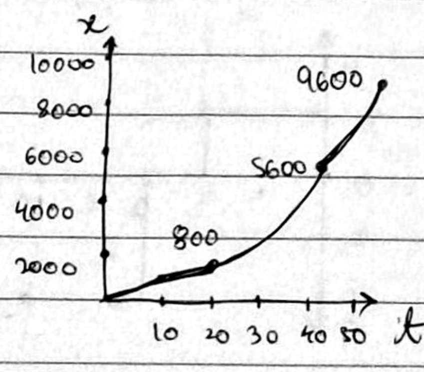
$\therefore x_{40} = \cancel{4000m}.\ 5600m$

$\therefore A_3 = \frac{n}{n+1} a \times b + 10 \times 360$, $a = 10$, $b = 60$, $n = 2$.

$= \frac{2 \times 10 \times \cancel{60}^{20}}{\cancel{3}} + 10 \times 360 = 4000 = x_{50} - x_{40}$

$\therefore x_{50} = 4000 + x_{40}$

$= 9600m.$



$\therefore$ Maximum speed of the particle occurs at $t = 50s$ and $V_{max} = 420 m/s$

Also,

maximum distance covered is 9600 m.

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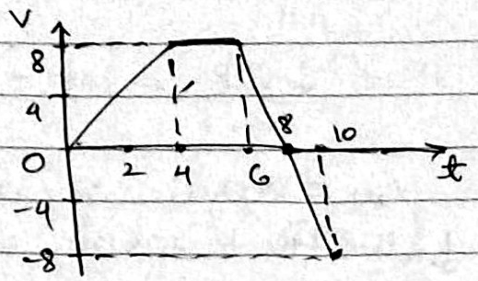
Q 7) Given v-t diagram. Draw a-t and x-t diagrams. At $t = 2s$, $x = 20m$, Also calculate displacement during 6-10s and find total distance covered.



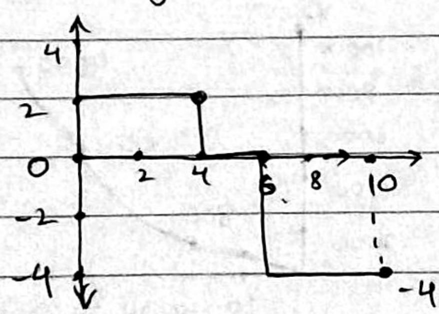
Slope of v-t diagram = acceleration.

for $0 \leq t \leq 4$, slope $= 8/4 = 2 = a$

for $4 \leq t \leq 6$, slope $= a = 0 m/s^2$.

for $6 \leq t \leq 10$, slope $= a = -16/4 = -4 m/s^2$.

~~Area of~~ $\therefore$ a-t diagram would be.



at $t = 2$, $x_2 = 20m$. $\therefore$ Area of v-t for $0 \leq t \leq 2$ = change in dis

$\therefore \frac{1}{\cancel{2}} \times \cancel{2} \times 4 = x_2 - x_0$

$\boxed{\therefore x_0 = 16m}$

$\therefore$ The particle starts at a distance $x = 16m$ from origin.

Now,

for

for

for

fo

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Now, Area of v-t curve = Change is displacement.

for $0 \le t \le 4$, $\frac{1}{\cancel{2}} \times \cancel{4}^{2} \times 8 = x_4 - x_0$

$\therefore x_4 = 32m$

for $4 \le t \le 6$, $2 \times 8 = x_6 - x_4$.

$x_6 = 48m$

for $6 \le t \le 8$, $\frac{1}{\cancel{2}} \times \cancel{2} \times 8 = x_8 - x_6$

$\therefore x_8 = 56m$.

for $8 \le t \le 10$, $-\frac{1}{\cancel{2}} \times \cancel{2} \times 8 = x_{10} - x_8$

$\therefore x_{10} = 48m$

Now, displacement during $t=6$ to $t=10$ $= x_{10} - x_6 = 0$.

$\therefore$ Total distance travelled $= |A_1| + |A_2| + |A_3| + |A_4| = 48m$.

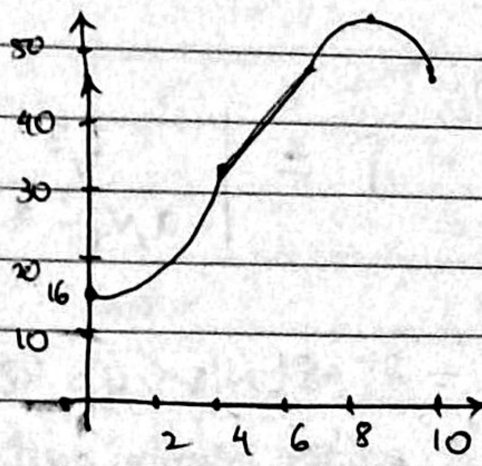
$x_0 = 16m$

$x_4 = 32m$

$x_6 = 48m$.

$x_8 = 56m$

$x_{10} = 48m$.



D Curvillinear motion.

- Rectangular system

$$\bar{r} = x\hat{i} + y\hat{j}$$

$$\bar{v} = \frac{d\bar{r}}{dt} = v_x\hat{i} + v_y\hat{j}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = a_x\hat{i} + a_y\hat{j}$$

- Normal tangent (N-T) systems.

$$a_n = \frac{v^2}{\rho}$$

$$\boxed{\rho = \frac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2}}$$

$a_t \rightarrow$ tangential
$a_n \rightarrow$ normal component.

Also

$$\rho = \left| \frac{v^3}{a_x v_y - a_y v_x} \right|$$

Q8. $v_x = 25 - 8t$ m/s, $y = 48 - 3t^2$ m. at $t=0$, $x=0$.
find position, velocity, acceleration vectors at $t = 4$sec.
Also their magnitudes.

$\rightarrow$ $v_x = 25 - 8t$. $\therefore$ $x = 25t - \cancel{8}^{4}t^2/\cancel{2} + c$. at $t=0, x=0$

$\cancel{\frac{dx}{dt}} = \cancel{25}$ $\therefore$ $0 = c$.

$\therefore x = -4t^2 + 25t$. also $v_y = \frac{dy}{dt} = -6t$ m/s.

$$\therefore a_x = \frac{dv_x}{dt} = -8 \text{ m/s}^2, \quad a_y = \frac{dv_y}{dt} = -6 \text{ m/s}^2.$$

$$\therefore a = \sqrt{a_x^2 + a_y^2} = 10 \text{ m/s}^2.$$

$\therefore$ at $t=4$, $V_x = 25-8\times4$, $V_y = -6\times4$.

$V_x = -7m/s$ $\quad V_y = -24m/s$

$\therefore V = \sqrt{V_x^2 + V_y^2} = 25m/s$.

at $t=4$, $x = -4\times4^2 + 25\times4$ and $y = 48 - 3\times4^2$

$x = 36m$ $\quad y = 0m$.

$\therefore \bar{r} = 36\hat{i}$, $r = 36m$.

$\bar{v} = -7\hat{i} - 24\hat{j}$, $v = 25m/s$.

$\bar{a} = -8\hat{i} - 6\hat{j}$, $a = 10m/s^2$.

Q 9) $\bar{r} = \frac{1}{4}t^3\hat{i} + 3t^2\hat{j}$ m. determine ρ and N-T comp at $t=2sec$.

$\rightarrow$ $\bar{v} = \frac{3}{4}t^2\hat{i} + 6t\hat{j}$ m/s. , $\bar{a} = \frac{3}{2}t\hat{i} + 6\hat{j}$ m/s^2.

at $t=2sec$,

$\bar{r} = 2\hat{i} + 12\hat{j}$ m.

$\bar{v} = 3\hat{i} + 12\hat{j}$ m/s. $\quad \therefore v = 12.369$ m/s

$\bar{a} = 3\hat{i} + 6\hat{j}$ m/s^2 $\quad a = 6.708$ m/s.

we know,

$$\rho = \left|\frac{v^3}{a_x v_y - a_y v_x}\right| = \left|\frac{(12.369)^3}{3\times12 - 6\times3}\right|$$

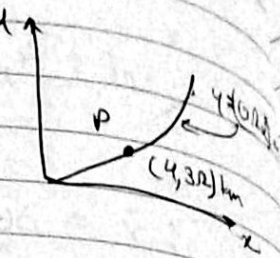
$$\boxed{\rho = 105.1m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.369)^2}{105.1} = 1.456 m/s^2.$$

also, $a_t = \sqrt{a^2 - a_n^2} = 6.548 m/s^2$.

Q 10) At P, speed $= 360\ km/h. = \frac{\cancel{360}^{20} \times 5}{\cancel{18}} = 100\ m/s$.
increasing at a rate of $0.5\ m/s^2$ ~~a = 0.5m/s²~~.
Determine total magnitude of acceleration at P.

→ Given eqn of path as $y = 0.2x^2$.



$$\frac{dy}{dx} = 0.4x \qquad \left(\frac{dy}{dx}\right)_{x=4km} = 1.6.$$

$$\frac{d^2y}{dx^2} = 0.4 \qquad \left(\frac{d^2y}{dx^2}\right)_{x=4km} = 0.4$$

$$\rho = \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{d^2y/dx^2} = \frac{\left(1 + (1.6)^2\right)^{3/2}}{0.4} = 16.792\ km = 16792\ m$$

$$a_n = \frac{v^2}{\rho} = \frac{(100)^2}{16792} = 0.595\ m/s^2$$

$$a_t = 0.5\ m/s^2 \qquad (\text{Given})$$

$$\therefore\ a_{Total} = \sqrt{a_t^2 + a_n^2} = 0.777\ m/s^2.$$

* * * * *

Kinematics

* Instantan

v = rω.

linear vel

linear vel

ICR.
(A & B)

Use

47_YASH SARANG

Kinematics of Rigid bodies (Module 5)

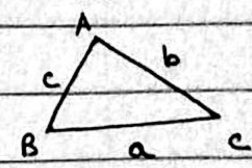
* Instantaneous Centre of Rotation (ICR)

$v = r\omega$. $\omega = 2\pi n$.

linear velocity (v) is diff for different points.
linear velocity and radius of rotation are always $\perp$ to each other.

ICR is obtained by intersection of lines joining starting points of arrows (A & B) and tips of arrows.

formulae - $\omega = \dfrac{V_A}{IA} = \dfrac{V_B}{IB}$



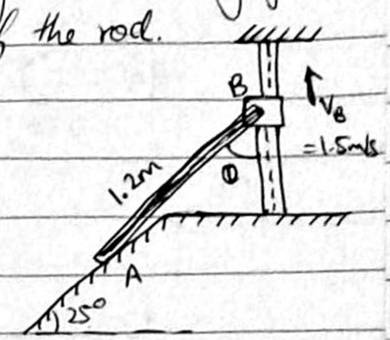
Use sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule ~~cos~~ $a = \sqrt{b^2+c^2-2bc\cos A}$

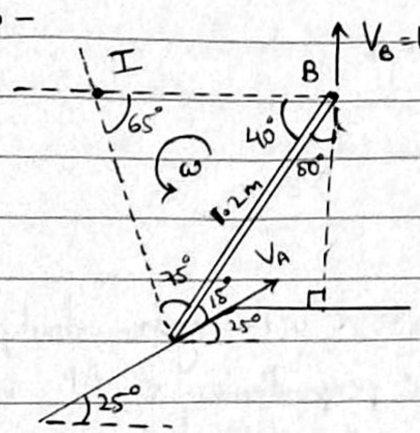
Numericals.

1) Type I : When body slides on two surfaces.

Figure shows a collar B which moves upwards with constant velocity of 1.5 m/s. At the instant when $\theta = 50^\circ$, determine i) the angular velocity of rod pinned at B and freely resting at A against 25° sloping ground. and ii) the velocity of end A ~~against 25° s~~ of the rod.



F.B.D -



When collar moves up with velocity 1.5 m/s, point B also moves with velocity 1.5 m/s upwards then point A moves with velocity V_A along incline. ICR is a point of intersection of V_A and V_B as shown in F.B.D.

$\omega = \dfrac{V_A}{IA} = \dfrac{V_B}{IB}$ $\qquad \therefore \dfrac{V_A}{IA} = \dfrac{1.5}{IB}$

from sine rule, $\dfrac{1.2}{\sin 65^\circ} = \dfrac{IB}{\sin 75^\circ} = \dfrac{IA}{\sin 40^\circ}$ $\qquad \therefore IA = 0.851\,m.$

$IB = 1.279\,m.$

$\therefore \omega = 1.5/1.279 = 1.173\,r/s$ (↻) $\qquad \therefore V_A = 0.998\,m/s$ (V_A ∠25°)

$\therefore$ Angular velocity of rod $(\omega) = 1.173$ rad/s (↻)

$\therefore$ Velocity of end A of rod $(V_A) = 0.998$ m/s (∠25°).

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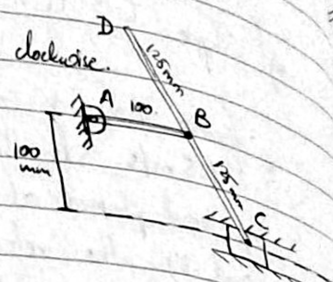
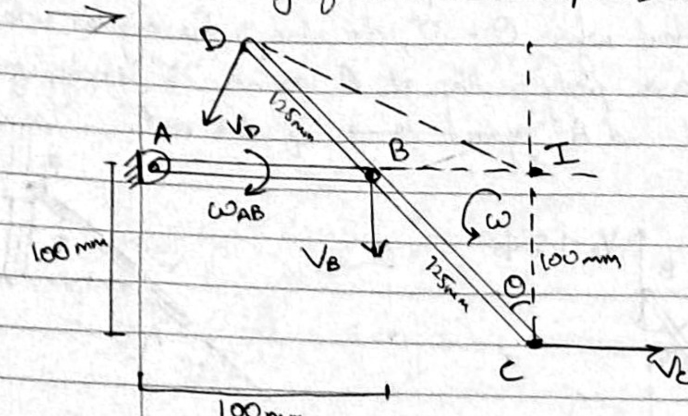
2) Type 2: When one part of the body slides and another part rotates about a hinge point.

The crank AB has an angular velocity of 3 rad/s clockwise. Find velocity of the slider C and point D.





Crank AB rotates about A with angular velocity $\omega_{AB} = 3$ rad/s (c
Velocity of point B (V_B) is perpendicular to AB in downward direction
Slider at C moves horizontally with velocity V_c.

$V_B = l(AB) \times \omega_{AB}$

$= 100 \times 10^{-3} \times 3 = 0.3$ m/s $(\downarrow)$

Also $V_B = IB \times \omega = 0.3$ ___ (1)

$V_c = IC \times \omega = 0.1\omega$ ___ (2)

$V_D = ID \times \omega$ ___ (3)

$\cos\theta = \frac{100}{125}$ $\quad \boxed{\therefore \theta = 36.87^\circ}$ $\quad \sin 36.87 = \frac{IB}{125}$ $\quad \therefore IB = 0.075$ m.

$\therefore \omega = 4$ rad/s. (from (1)) $\quad \therefore V_c = 0.4$ m/s $(\rightarrow)$

$V_D = 0.7212$ m/s ($\perp^r$ to ID)

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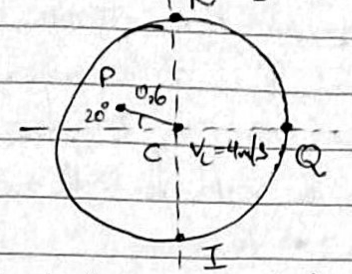
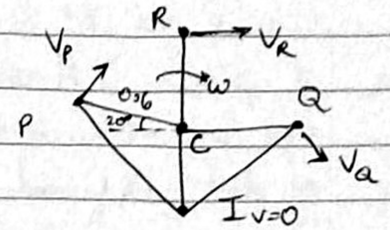
3) Type 3: ~~A~~ wheel of 2m diameter rools without slipping on a flat surface. The centre of the wheel is moving with a velocity of 4 m/s towards right. Determine the angular ~~vers~~ velocity of the wheel and velocity of P, Q and R shown on wheel.





→ Point I which is in contact with the flat surface becomes instantaneous centre of rotation. Hence, velocity of point I i.e $V_I = 0$. Let the wheel rotate with angular velocity ω. To determine velocities of points P, Q and R join these points with I.

$$\therefore \omega = \frac{V_Q}{IQ} = \frac{V_R}{IR} = \frac{V_c}{IC} = \frac{V_P}{IP}.$$

$$\therefore \omega = \frac{V_c}{IC} = 4 \text{ rad/s}. \quad \therefore 4 = \frac{V_Q}{IQ} = \frac{V_R}{IR} = \frac{V_P}{IP}.$$

$\therefore V_Q = 4IQ$, $V_R = 4IR$, $V_P = 4IP$.

$\therefore IQ = \sqrt{1^2+1^2} = 1.4142$m.

In △IPC, using cosine rule $\quad IP^2 = \sqrt{IC^2 + PC^2 - IC \times PC \times \cos 110°}$

$= \sqrt{1^2 + 0.6^2 - 0.6 \cos 110°}$.

$\therefore IP = 1.3305$m.

$IR = 2$m.

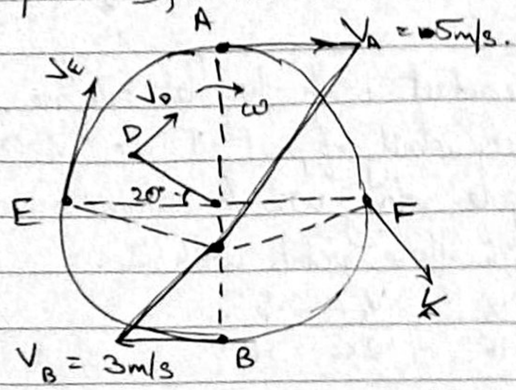
$\therefore V_R = 8$ m/s (→)

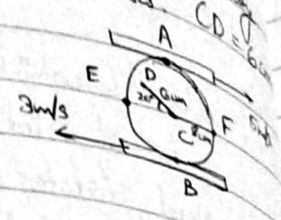
$V_P = 5.322$ m/s ($\perp^r$ to IP)

$V_Q = 5.6568$ m/s ($\perp^r$ to IQ)

4) Type 4: When body lies between two moving surfaces

A roller of radius 8cm rides between two horizontal bars moving in opposite direction as shown. i) locate the instantaneous centre of velocity and give its distance from B. Assume no slip conditions at points A and B. ii) Do the same, when both bars are moving in the same direction. iii) Also find the velocity of point D, E and F on the roller for both the cases. CD = 6cm





→ i) Since disc rolls without slipping, points A and B of disc will have same velocity as that of the horizontal bars. Since both velocities are parallel, instantaneous centre lies on a line drawn perpendicular to these velocities. Intersection point is obtained by joining tips of velocity vectors as shown. After locating I, join points D, E and F with I.

$$\omega = \frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \frac{V_D}{ID} = \frac{V_E}{IE} = \frac{V_F}{IF} \quad ; \quad \frac{V_A}{IA} = \frac{V_B}{IB}, \; \boxed{5IB = 3IA}$$

$IA + IB = 16$.

$\therefore IB = 6cm, IA = 10$

$$\therefore \omega = \frac{V_A}{IA} = \frac{5}{10} = 0.5 \text{ rad/s}.$$

By cosine rule,

$$ID = \sqrt{IC^2 + CD^2 - IC \times CD \cos(90+20)^\circ}$$

$$= \sqrt{2^2 + 6^2 - 2 \times 6 \cos 110^\circ}$$

$$ID = 6.943 \text{ cm}.$$

from ΔCIE, $IE = \sqrt{CE^2 + IC^2} = \sqrt{8^2 + 2^2}$

$\therefore IE = 8.246 \text{ cm} = IF$

$\therefore$ Velocity of point D is $V_D = \omega \times ID = 3.4715 \text{ m/s}$.

point E is $V_E = \omega \times IE = 4.1231 \text{ m/s}$.

point F is $V_F = \omega \times IF = 4.1231 \text{ m/s}$.

. . . . .