SYLLABUS
ENGINEERING MATHEMATICS - I
F. E. Semester - I
(Mumbai University - Effective From August 2019)

easy – solutions

Engineering Mechanics

Strictly as per the New Choice Based Credit and Grading System syllabus of Mumbai University w.e.f academic year 2016-2017

Semester I
Common to All Branches

1EM164A

Engineering Mechanics
Semester I – Common to All Branches (MU)

Edition 2017

This edition is for sale in India, Bangladesh, Bhutan, Maldives, Nepal, Pakistan, Sri Lanka and designated countries in South-East Asia. Sale and purchase of this book outside of these countries is unauthorized by the publisher.

Printed at : Image Offset, Dugane Ind. Area Survey No. 28/25, Dhayari Near Pari Company, Pune - 41, Maharashtra State, India. E-mail : rahulshahimage@gmail.com

Published by
easy-solutions

Head Office : B/5, First floor, Maniratna Complex, Taware Colony, Aranyeshwar Corner, Pune - 411 009. Maharashtra State, India. Ph : 91-20-24225065, 91-20-24217965.
Fax 020-24228978. Email : info@techmaxbooks.com,
Website : www.techmaxbooks.com

(Book Code : 1EM164A)

INDEX

Table of Contents

□□□

Syllabus

Module	Detailed Contents
01	**1.1 System of Coplanar Forces :** Resultant of concurrent forces, parallel forces, non-concurrent Non-parallel system of forces, Moment of force about a point, Couples, Varignon's Theorem. Force couple system. Distributed Forces in plane. **1.2 Centroid** for plane Laminas.
02	**2.1 Equilibrium of System of Coplanar Forces :** Condition of equilibrium for concurrent forces, parallel forces and non-concurrent non-parallel general forces and Couples. **2.2 Types of support :** Loads, Beams, Determination of reactions at supports for various types of loads on beams.(Excluding problems on internal hinges) **2.3 Analysis of plane trusses :** By using Method of joints and Method of sections. (Excluding pin jointed frames).
03	**3.1 Forces in space :** Resultant of Non-coplanar Force Systems: Resultant of concurrent force system, parallel force system and non-concurrent non-parallel force system. Equilibrium of Non-coplanar Force Systems: Equilibrium of Concurrent force system, parallel force system and non-concurrent non-parallel force system. **3.2 Friction :** Introduction to Laws of friction, Cone of friction, Equilibrium of bodies on inclined plane, Application to problems involving wedges, ladders. **1.3 Principle of virtual work :** Applications on equilibrium mechanisms, pin jointed frames.
04	**4.1 Kinematics of a Particle :** Rectilinear motion, Velocity & acceleration in terms of rectangular co-ordinate system, Motion along plane curved path, Tangential& Normal component of acceleration, Motion curves (a-t, v-t, s-t curves), Projectile motion.

Module	Detailed Contents
05	**5.1 Kinematics of a Rigid Body :** Introduction to general plane motion, Instantaneous center of rotation for the velocity, velocity diagrams for bodies in plane motion.
06	**6.1 Kinetics of a Particle :** Force and Acceleration: -Introduction to basic concepts, D'Alemberts Principle, Equations of dynamic equilibrium, Newton's second law of motion. **6.2 Kinetics of a Particle :** Work and Energy: Principle of work and energy, Law of conservation of energy. **6.3 Kinetics of a Particle :** Impulse and Momentum: Principle of linear impulse and momentum. Law of conservation of momentum. Impact and collision.

□□□

Engineering Mechanics

Chapter 1 : Co-planar Forces

Q. 1 Five concurrent coplanar forces act on a body as shown in Fig. 1. Find forces P and Q such that resultant of the five forces is zero. **Dec. 2009**

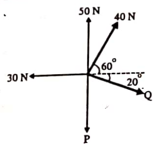


Fig. 1

Ans. :

Simplify the Fig. 1 as shown in Fig. 2.

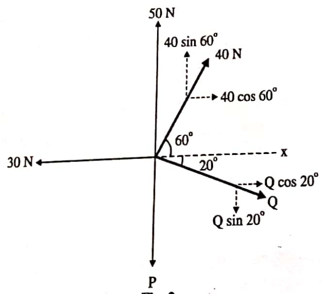


Fig. 2

Given : $R = 0 \quad \therefore \Sigma F_x = 0$ and $\Sigma F_y = 0$

(i) $\Sigma F_x = 0 \Rightarrow 40 \cos 60° - 30 + Q \cos 20° = 0$

$\therefore \quad Q = 10.64\ N$ ...Ans.

(ii) $\Sigma F_y = 0 \Rightarrow 40 \sin 60° + 50 - P - Q \sin 20° = 0$

$\therefore \quad P = 81\ N$ ...Ans.

Q. 2 Four forces and a couple are acting on a plate as shown in Fig. 3. Determine the resultant force and locate it with respect to point A. **Dec. 2015**

Ans. :

$\Sigma F_x = 100 + 200 \cos (36.87°) = 260\ N\ (\rightarrow)$

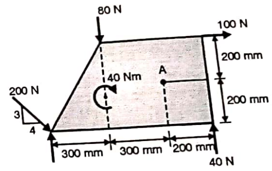


Fig. 3

$\Sigma F_y = -80 + 40 - 200 \sin (36.87°) = -160\ N = 160\ N\ (\downarrow)$

$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 305.29\ N \qquad \tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

$\therefore \quad \theta = $ **31.61° in 4th quadrant.**

By Varignon's theorem,

$R \cdot x = \Sigma M_A = (-100 \times 0.2) + (40 \times 0.2) + (80 \times 0.3) + (200 \sin 36.87° \times 0.6) + (200 \cos (36.87°) \times 0.2) - 40$

$\therefore R.x = 76\ N.m\ (A.C.W) \qquad \therefore x = \dfrac{76}{305.29} \qquad \therefore x = 0.25\ m$ from A

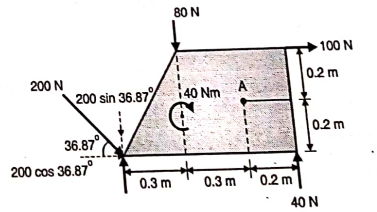


Fig. 4

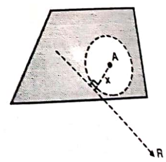


Fig. 5

Q. 3 Determine magnitude and direction of forces F_1 and F_2 when the resultant of given force system is found to be 800 N along x-axis. **May 2006**

Ans. :

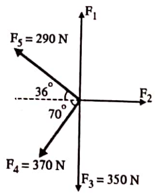


Fig. 6

Simplify the Fig. 6 as shown in Fig. 7 for further resolution.

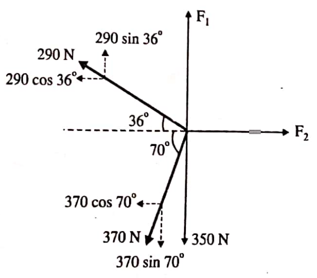


Fig. 7

(i) $\Sigma F_x = 800$ N ; $\quad \therefore \; F_2 - 290 \cos 36^\circ - 370 \cos 70^\circ = 800$

$\therefore \quad F_2 = 1161.16 \text{ N} (\rightarrow)$ **...Ans.**

(ii) $\Sigma F_y = 0$; $\quad \therefore F_1 + 290 \sin 36^\circ - 370 \sin 70^\circ - 350 = 0$

$\therefore \quad F_1 = 527.23 \text{ N} (\uparrow)$ **...Ans.**

Q. 4 Three concurrent forces P = 150N, Q = 250N and S = 300N are acting at 120° with each other. Determine their resultant force magnitude and direction with respect to P. What is their equilibrant ? **Dec. 2015**

Ans. :

$\Sigma F_x = 150 \cos 30^\circ - 250 \cos 30^\circ = -86.6 \text{ N} = 86.6 \text{ N} (\leftarrow)$

$\Sigma F_y = 150 \sin 30^\circ + 250 \sin 30^\circ - 300$

$= -100 \text{ N} = 100 \text{ N} (\downarrow)$

$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 132.28$ N

$\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

$\therefore \theta = 49.1^\circ$ with X-axis in 3rd quadrant

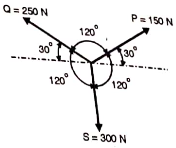


Fig. 8

Simplify the Fig. 8 as shown in Fig. 9 for further resolution of forces.

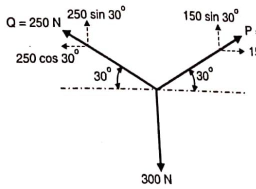


Fig. 9 : Equilibrium

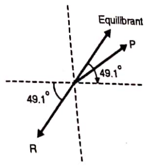


Fig. 10

Q. 5 For the system shown, determine :

(i) The required value of α if resultant of three forces is to be vertical.

(ii) The corresponding magnitude of resultant.

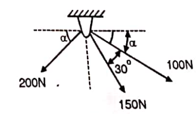


Fig. 11

Ans. : (i) As the resultant is along y axis

$\Sigma F_x = 0$; $\therefore$ Resolving along x axis :

$\Sigma F_x = 0$; $\quad 100 \cos\alpha + 150 \cos (30 + \alpha) - 200 \cos\alpha = 0$

Dividing by 50.

$\therefore 2 \cos\alpha + 3 \cos (30 + \alpha) = 4 \cos\alpha$

$\therefore \; 3 (\cos 30 \cos\alpha - \sin 30 \sin\alpha) = 2 \cos\alpha$

$\therefore \; 3 \times 0.866 \cos\alpha - 3 \times 0.5 \sin\alpha = 2 \cos\alpha$

$\therefore \; (2.598) \cos\alpha - 1.5 \sin\alpha = 2 \cos\alpha$

$\therefore \; 1.5 \sin\alpha = 0.598 \cos\alpha.$

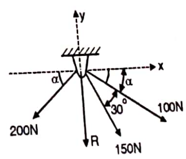


Fig. 12

$\therefore \quad \tan\alpha = \frac{0.598}{1.5} \qquad \therefore \quad \alpha = 21.74°$ ... Ans.

Simplify the given Fig. 12 as shown in Fig. 13 for further resolution.

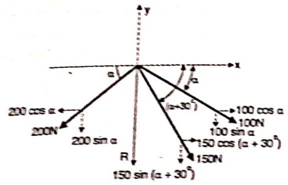


Fig. 13

(ii) Now,

$$R = \Sigma F_y = -200\sin\alpha - 150\sin(30+\alpha) - 100\sin\alpha$$

$$= -200(\sin 21.74°) - 150\sin(51.24°) - 100\sin(21.74°)$$

$\therefore \quad R = -228.9$ N.

$\therefore$ **Resultant R = 228.9 N acting vertically downwards.** ...Ans.

Q. 6 Find the resultant of the force system shown in Fig. 14. **Dec. 2012**

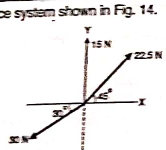


Fig. 14

Ans. : *Simplify the Fig. 14 as shown in Fig. 15*

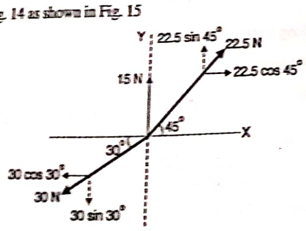


Fig. 15

$\Sigma F_x = 22.5\cos 45° - 30\cos 30° = -10.07\text{ N} = 10.07\text{ N} (\leftarrow)$

$R_y = \Sigma F_y = 22.5\sin 45° + 15 - 30\sin 30°$

$\Sigma F_y = 15.91\text{ N} (\uparrow)$

Magnitude of resultant, $\quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 18.83\text{ N}$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\left|\frac{15.91}{10.07}\right| = 57.67°$$

Point of application of the resultant is point O.

Q. 7 A ring is pulled by three forces as shown in Fig. 16. Find the force F and the angle θ if resultant of these three forces is 100 N acting in vertical direction. **Dec. 2013**

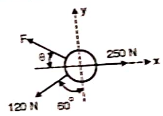


Fig. 16

Ans. : Simplify the Fig. 16 as shown in Fig. 17.

The resultant force is, $R = 100\text{N} (\uparrow)$

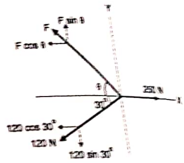


Fig. 17

$\therefore \Sigma F_x = 0, \quad$ and $\quad \Sigma F_y = R$

We have, $\Sigma F_x = 0 \qquad \therefore -F\cos\theta - 120 + 250 = 0$

$\therefore F\cos\theta = 146.08$ ...(1)

$\Sigma F_y = R \qquad \therefore 100 = F\sin\theta - 120\cos 30°$

$\therefore F\sin\theta = 160$ ...(2)

From Equation (1) and (2), $\quad \frac{F\sin\theta}{F\cos\theta} = \frac{160}{146.08}$

$\tan\theta = 1.095 \qquad \therefore \theta = 47.6° \qquad \therefore F = 216.65\text{ N}$

Q. 8 Two concurrent forces P and Q acts at O such that their resultant acts along x-axis. Determine the magnitude of Q and hence the resultant. **May 2014**

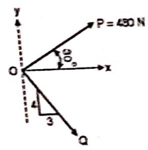


Fig. 18

Ans. :

Simplify the Fig. 18 as sho...

Fig. 19 for further resoluti...

Resultant acts along the ...

$\Sigma F_y = 0$ and ΣF_x ...

Using $\Sigma F_Y = 0$

$\therefore \quad 0 = 480\sin 30° - $ ...

$\therefore \quad Q = 300$ N

$\therefore \quad \Sigma F_x = 480\cos 30°$ ...

But $R_Y = 0 \quad \therefore$ R ...

Q. 9 Find forces P and ...

Ans. : Simplify the ...

Resultant of ...

$\Sigma F_x = 0;$ 40 co...

$\Sigma F_y = 0;$ 40 sin...

Q. 10 Four co...

Ans. :

Simplify the Fig. 18 as shown in Fig. 19 for further resolution.

Resultant acts along the x-axis. Hence

$\Sigma F_y = 0$ and $\Sigma F_X = R$ (Assuming toward right)

Using $\Sigma F_Y = 0$

$\therefore \quad 0 = 480 \sin 30° - Q \sin 53.13° \; ; \; \sin^{-1}\left(\frac{4}{3}\right) = 53.13°$

$\therefore \quad Q = 300$ N

$\therefore \quad \Sigma F_X = 480 \cos 30° + 300 \cos 53.13° = 595.69$ N (→)

But $R_Y = 0 \quad \therefore \quad R = R_X = 595.69$ N (→)

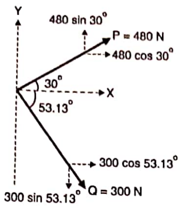


Fig. 19

Q. 9 Find forces P and Q such that resultant of given system is zero. **May 2013**

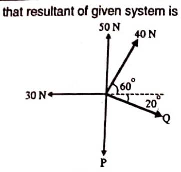


Fig. 20

Ans. :

Simplify the Fig. 20 as shown in Fig. 21 for further resolution.

Resultant of the given concurrent force system is zero.

$\Sigma F_X = 0; \quad 40 \cos 60° - 30 + Q \cos 20° = 0$

$\therefore Q = 10.642$ N

$\Sigma F_Y = 0; \quad 40 \sin 60° + 50 - Q \sin 20° - P = 0$

$P = 81$ N

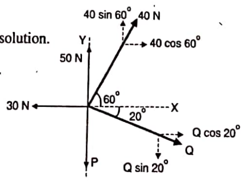


Fig. 21

Q. 10 Four concurrent forces act at a point as shown in Fig. 22, find their resultant. **Dec. 2014**

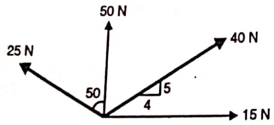


Fig. 22

Ans. :

$\tan^{-1}\left(\frac{3}{4}\right) = 36.87°$

$\Sigma F_X = 15 + 40 \cos 36.87° - 25 \cos 40°$

$= 27.85$ N (→)

$\Sigma F_y = 40 \sin 36.87° + 50 + 25 \sin 40° = 90.07$ N (↑)

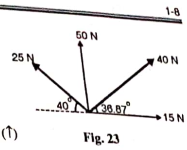


Fig. 23

Simplify the Fig. 23 as shown in Fig. 24 for further resolution.

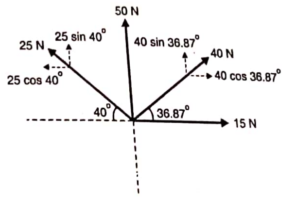


Fig. 24

Magnitude of Resultant $= R = \sqrt{\Sigma F_X + \Sigma F_y} = 94.28$ N

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_X}\right) = 72.82°$ (

Q. 11 The guy cables AB and AC are attached to the top of the transmission tower as shown in Fig. 25. The tension in cable AC is 8 kN. Determine the required tension T in cable AB such that the net effect of the two cable tensions is a downward force at point A. Determine the magnitude R of this downward force. **May 2015**

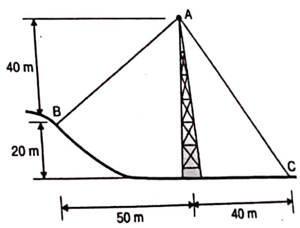


Fig. 25

Ans. :

$$\theta = \tan^{-1}\left(\frac{40}{50}\right) = 38.66°$$

$$\alpha = \tan^{-1}\left(\frac{60}{40}\right) = 56.31°$$

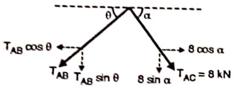


Fig. 26

The resultant is vertical

$\therefore\ \Sigma F_x = 0$ and $\Sigma F_y = -R$ (Assuming down ward)

$-T_{AB}\cos 38.66° + 8\cos 56.31° = 0$

$$\therefore\ T_{AB} = 5.683 \text{ kN}$$

$$R_y = \Sigma F_y = -T_{AB}\sin 38.66° - T_{AC}\sin 56.31°$$

$$-(5.683)\sin 38.66° - (8)\sin 56.31° = -10.207 \text{ kN}$$

$$\therefore\ R = 10.207 \text{ kN } (\downarrow)$$

Q. 12 Determine the resultant of the forces acting as given in Fig. 27. Find the angle which the resultant makes with the positive x-axis. **May 2016**

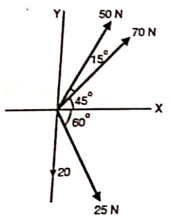


Fig. 27

Ans. :

Simplify the Fig. 27 as shown in Fig. 28 for further resolution.

$$\Sigma F_x = 70\cos 45° + 50\cos 60° + 25\cos 60° = 87 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 70\sin 45° + 50\sin 60° - 20 - 25\sin 60° = 51.15 \text{ N } (\uparrow)$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 100.92 \text{ N}$$

$$\tan\theta = \left|\frac{\Sigma F_y}{\Sigma F_x}\right| \quad \therefore\ \theta = 30.45° \text{ in first quadrant}$$

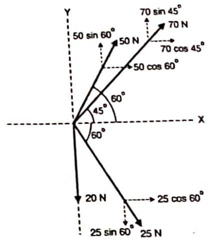


Fig. 28

Q. 13 Determine the resultant of the system of forces shown in Fig. 29. Locate the point where the resultant cuts the base AB. **Dec. 2009**

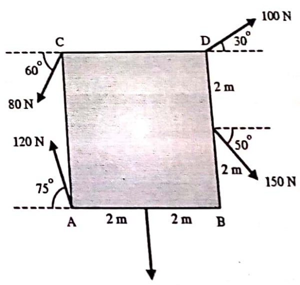


Fig. 29

Ans. :

Simplify the Fig. 29 as shown in Fig. 30 for further resolution of forces.

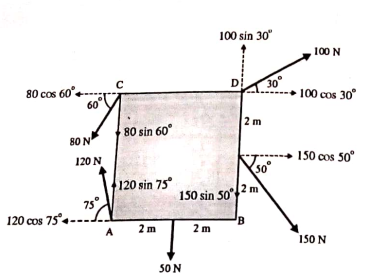


Fig. 30

$$\Sigma F_x = 100 \cos 30° - 80 \cos 60° - 120 \cos 75° + 150 \cos 50° = 111.96 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 100 \sin 30° - 80 \sin 60° + 120 \sin 75° - 50 - 150 \sin 50° = -68.28 \text{ N} = 68.28 \text{ N} \downarrow$$

$$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \qquad \therefore \quad \mathbf{R = 131.14\ N}$$

$$\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right] = \mathbf{31.38° \text{ in } 4^{th} \text{ quadrant}}$$

By Varignon's theorem,

$$R \cdot x = \Sigma M_A$$

$$= (-50 \times 2) - (150 \cos 50° \times 2) - (150 \sin 50° \times 4) - (100 \cos 30° \times 4) + (100 \sin 30° \times 4) + (80 \cos 60° \times 4)$$

$$= -738.87 \text{ N·m}$$

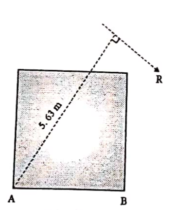


Fig. 31

$$R \cdot x = 738.87 \text{ N·m} \circlearrowleft$$

$$\therefore \quad x = \frac{738.87}{131.14} = \mathbf{5.63\ m\ from\ A}$$

$$\text{or} \quad \frac{\Sigma M_A}{\Sigma F_y} = \frac{738.87}{68.28} = 10.82 \text{ m right of A along line AB}$$

Q. 14 For given system find resultant and its point of application with respect to point O on the x-axis {x intercept}. Force, along CA = 100 N, along OD = 250N, along ED = 150N, along OE = 100N. An clockwise moment of 5000 N-cm is also acting at the point O. **Dec. 2014**

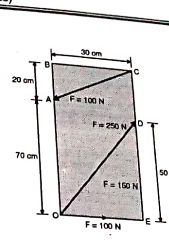


Fig. 32

Ans. :

Simplify the Fig. 32 as shown in Fig. 33 for further resolution.

$$\tan^{-1}\left(\frac{20}{30}\right) = 33.69°, \qquad \tan^{-1}\left(\frac{50}{30}\right) = 59.04°$$

$$\Sigma F_X = 250 \cos 59.04° + 100 - 100 \cos 33.69°$$

$$= 145.4 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 250 \sin 59.04° + 150 - 100 \sin 33.69°$$

$$= 308.9 \text{ N } (\uparrow)$$

$$R = \sqrt{\Sigma F_X + \Sigma F_y} = 341.42 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_X}\right) = 64.79° \; (\measuredangle)$$

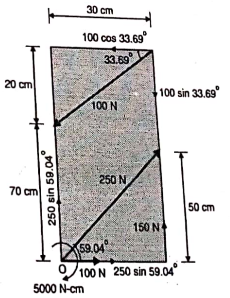


Fig. 33

Using Varignon's theorem,

$$\therefore R\,x = \Sigma M_o \qquad ...(1)$$

$$\Sigma M_o = 150(30) + (100 \cos 33.69°)(70) - 5000$$

$$= 5324.36 \text{ N cm } (\circlearrowleft) \qquad x = 15.59 \text{ cm}$$

x-intercept of resultant force (Horizontal distance)

$$a = \frac{\Sigma M_o}{R_y} = \frac{5324.36}{308.9} = 17.24 \text{ cm (to the right of point O)}$$

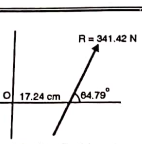


Fig. 34 : (Position of R)

Q. 15 Replace the force system (Fig. 35) by a single force w.r.t. point C. **May 2015**

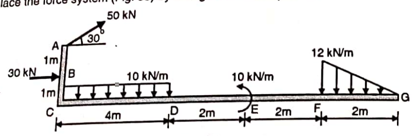


Fig. 35

Ans. : Simplify the Fig. 35 as shown in Fig. 36 for further resolution.

$$\Sigma F_X = 50 \cos 30° + 30$$

$$= 73.3 \text{ kN } (\rightarrow)$$

$$\Sigma F_y = 50 \sin 30° - 40 - 12$$

$$= -27 \text{ kN} = 27 \text{ kN } (\downarrow)$$

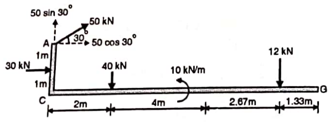


Fig. 36

$$R = \sqrt{\Sigma F_X + \Sigma F_y} = 78.11 \text{ kN}$$

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_X}\right| = 20.22°$$

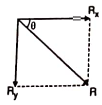


Fig. 37

Using Varignon's theorem,

$$R \cdot x = \Sigma M_c \qquad ...(i)$$

$$\Sigma M_c = -(50 \cos 30°)(2) - 30(1) - 40(2) + 10 - 12(8.69) = -290.64 \text{ kNm}$$

$$= 290.64 \text{ kNm (clockwise)}$$

$$\therefore x\text{-int} = \frac{\Sigma M_c}{R_y} = \frac{290.64}{27} = 10.76 \text{ m}$$

$$y\text{-int} = \frac{\Sigma M_c}{R_x} = \frac{290.64}{73.3} = 3.97 \text{ m}$$

$$x = \frac{\Sigma M_c}{R} = \frac{290.64}{78.11} = 3.72 \text{ m}$$

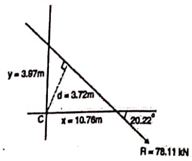


Fig. 38

Q. 16 A system of forces acting on a bell crank is as shown in Fig. 39. Determine the magnitude, direction and the point of application of the resultant w.r.t.'O'. **May 2014**

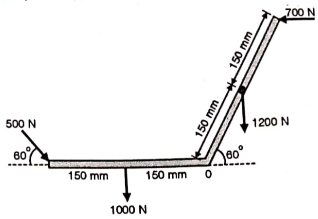


Fig. 39

Ans. : Simplify the given Fig. 39 as shown in Fig. 40 for further resolution.

$$\Sigma F_X = \Sigma F_X = 500 \cos 60° - 700 = -450 \text{ N} = 450 \text{ N } (\leftarrow)$$

$$\Sigma F_y = \Sigma F_Y = -500 \sin 60° - 1000 - 1200 = -2633 \text{ N} = 2633 \text{ N } (\downarrow)$$

$$R = \sqrt{\Sigma F_X + \Sigma F_y} = 2671.18 \text{ N}$$

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_X}\right| = 80.3°$$

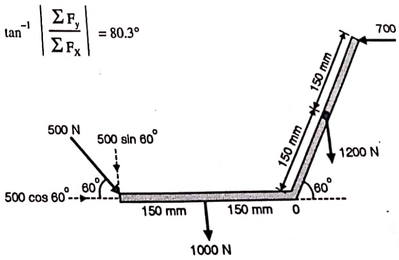


Fig. 40

To locate the resultant,

$$\Sigma M_0 = 500 \sin 60° (300) + 1000 (150)$$

$$- 1200 (150 \cos 60°) + 700 (300 \sin 60°)$$

$$= 371769.1 \text{ Nmm } (\circlearrowleft)$$

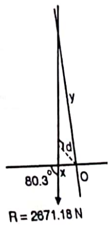


Fig. 41

X-intercept $x = \frac{\Sigma M_0}{R_Y} = \frac{371769.1}{2633} = 141.2$ mm

Y-intercept $y = \frac{\Sigma M_0}{R_X} = \frac{371769.1}{450} = 826.2$ mm

Perpendicular distance, $d = \frac{\Sigma M_0}{R} = \frac{371769.1}{2671.18} = 139.18$ mm

Q. 17 Find out resultant of given (lever) force system w.r.t. "B". **May 2013**

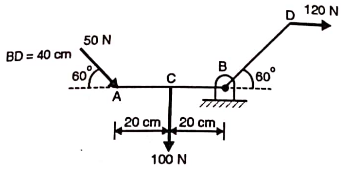


Fig. 42

Ans. :

Simplify the Fig. 42 as shown in Fig. 43 for further resolution.

$\Sigma F_X = 120 + 50 \cos 60° = 145$ N (→)

$\Sigma F_Y = -100 - 50 \sin 60° = -143.3$ N $= 143.3$ N (↓)

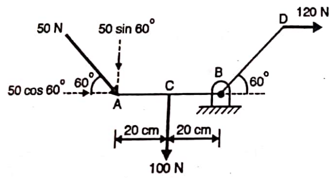


Fig. 43

$\therefore \quad R = \sqrt{\Sigma F_X + \Sigma F_y} = 203.86$ N

$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_X}\right| = \tan^{-1}\left|\frac{143.3}{145}\right| = 44.7°$

$\Sigma M_B = (-120)(40 \sin 60°) + (100)(20) + (5 \sin 60°)(40) = -424.87 = 424.87$ Nm (↷)

Using Varignon's Principle :

X-intercept, $X = \frac{\Sigma M_B}{R_Y} = \frac{424.87}{143.3} = 2.965$ m

Y-intercept, $Y = \frac{\Sigma M_B}{R_X} = \frac{424.87}{145} = 2.93$ m

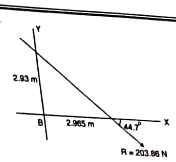


Fig. 44 : Position of R

❑❑❑

Chapter 2 : Coplanar Forces : Equilibrium

Q. 1 A heavy rod AB of length 3 m lies on horizontal ground. To lift the end B off the ground needs a vertical force of 200 N. To lift A end off the ground needs a force of 160 N. Find the weight of the rod and the position of center of mass. **May 2008**

Ans. :

(i) **End 'B' lifted**

Let 'x' be distance of c. g. from A (Fig. 1)

taking moments about 'A',

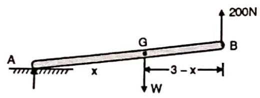


Fig. 1

$\Sigma M_A = 0 ;$ $200 \times 3 - W \times x = 0$ $\therefore$ Wx = 600 ...(i)

(ii) **End 'A' lifted**

Taking moments about B, (Fig. 2)

$\Sigma M_B = 0 ;$ $W(3-x) - 160 \times 3 = 0$

$\therefore$ $3W - Wx = 480$ ...(ii)

$\therefore$ $3W - 600 = 480$ ...from (i)

$\therefore$ $3W = 1080$

$\therefore$ **W = 360 N and x = 1.667 m. ...Ans.**

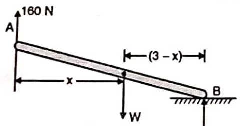


Fig. 2

Q. 2 A man raises a 12 kg joist of length 4m by pulling the rope. Find the tension in the rope and the two reaction at A. **Dec. 2010**

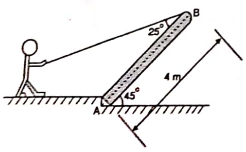


Fig. 3

Ans. : Here joist is in equilibrium under 3 forces.

(i) Tension in rope T (ii) Self weight mg (iii) Reaction from A

For equilibrium, above forces must be concurrent at D.

Using Lami's Theorem at D.

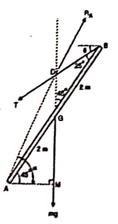


Fig. 4

Geometry

Here, $\theta = 45° - 25° = 20°$

Now, in Δ DGB by sine rule, (Refer Fig. 5)

$$\frac{2}{\sin(110°)} = \frac{DG}{\sin(25°)}$$

$\therefore$ DG = 0.9 m

$\therefore$ In Δ AMD, $\tan\alpha = \frac{DM}{AM} = \frac{DG + GM}{AM} = \frac{0.9 + 2\sin 45°}{2\cos 45°}$

$\alpha = 58.57°$

$\therefore$ by Lami's theorem, (Refer Fig. 6)

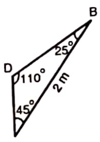


Fig. 5

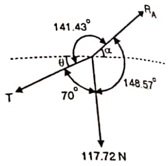


Fig. 6 : FBD at A

$$\frac{R_A}{\sin(70°)} = \frac{117.72}{\sin(141.43°)} = \frac{T}{\sin(148.57°)}$$

$\therefore$ $R_A = 177.43$ N $\therefore$ T = 98.46 N ...Ans.

Q. 3 A bar AB 12 m long rests in horizontal position as shown in Fig. 7 on two smooth planes. Find distance x at which a load P = 100 N is to be placed to keep the bar in equilibrium. Neglect weight of bar. **May 2013**

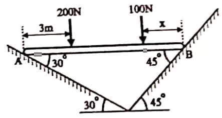


Fig. 7

Ans. :

Step 1 : F.B.D. of bar :

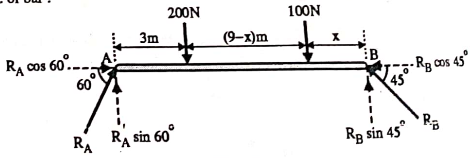


Fig. 8

Step 2 : Conditions of equilibrium :

$\Sigma F_x = 0 ;$ $R_A \cos 60° - R_B \cos 45° = 0$ $R_A = 1.414 R_B$ ...(1)

$\Sigma F_y = 0 ;$ $R_A \sin 60° + R_B \sin 45° - 200 - 100 = 0$

Substituting $R_A = 1.414 R_B$ from

$\therefore$ $1.414 R_B \sin 60° + R_B$

$\therefore$ R_B

$\therefore$ By Equation (1) R_A

$\Sigma M_B = 0 ;$ $(-R_A \cdot \sin 60°$

$\therefore$ (219.6 sin 60° ×

Q. 4 Determine the react

Ans. : Consider F.B.D. of

$\Sigma F_x = 0 ;$ $R_1 \cos 65° -$

$\Sigma F_y = 0 ;$ $R_1 \sin 65° +$

Solving Equations (i) an

we get

Now for cylind

$\Sigma F_y = 0 ;$

$R_1 \sin 65°$

$\Sigma F_x = 0 ;$

$R_1 \cos 6$

$\therefore$ $\alpha = 44.07°$

Q. 5 Two s

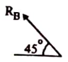
11. D

of B =

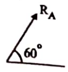
Substituting $R_A = 1.414\, R_B$ from Equation (1)

$\therefore 1.414\, R_B \sin 60° + R_B \sin 45° = 300$

$\therefore \quad R_B = 155.3\ N$



$\therefore$ *By Equation (1)* $\quad R_A = 1.414\,(155.3) = 219.6\ N$



$\Sigma M_B = 0;\quad (-R_A \cdot \sin 60° \times 12) + (200 \times 9) + 100\,x = 0$

$\therefore (219.6 \sin 60° \times 12) - (200 \times 9) = 100\,x$

$\therefore \quad x = 4.82\ m$ ...Ans.

Q. 4 Determine the reaction at point of contact. Assume smooth surfaces. **Dec. 2011**

Ans. : Consider F.B.D. of both cylinder together,

$\Sigma F_x = 0;\quad R_1 \cos 65° - R_3 \cos 75° = 0$

$\therefore \quad R_1 = 0.612\, R_3$ ...(i)

$\Sigma F_y = 0;\quad R_1 \sin 65° + R_3 \sin 75° - 39.24 - 9.81 = 0$ ...(ii)

Solving Equations (i) and (ii)

we get

$R_1 = 19.73\ N$

$R_3 = 32.22\ N$

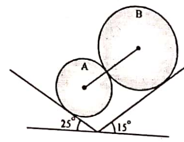


Fig. 9

Now for cylinder A

$\Sigma F_y = 0;$

$R_1 \sin 65° - R_2 \sin \alpha - 9.81 = 0$

$\therefore \quad R_2 \sin \alpha = 8.071$ ...(iii)

$\Sigma F_x = 0;$

$R_1 \cos 65° - R_2 \cos \alpha = 0$

$\therefore \quad R_2 \cos \alpha = 8.34$ ...(iv)

$\therefore \quad \dfrac{R_2 \sin \alpha}{R_2 \cos \alpha} = \dfrac{8.071}{8.34}$

$\therefore \alpha = 44.07°$ $\qquad \therefore R_2 = 11.604\ N$

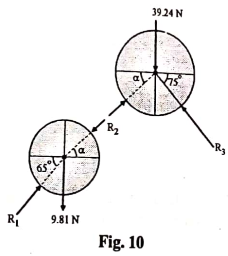


Fig. 10

Q. 5 Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in the Fig. 11. Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A = 400 mm and radius of B = 300 mm. **May 2011**

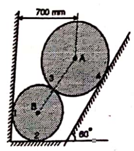


Fig. 11

Ans. :

From geometry $\quad \cos \alpha = \dfrac{400\ (PQ)}{700\ (PR)}$

$\therefore \alpha = 55.15°$

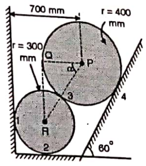


Fig. 12

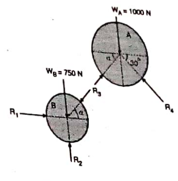


Fig. 13 : F.B.D. of spheres A and B

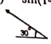
For A :

Use Lami's theorem

$$\frac{R_3}{\sin(120°)} = \frac{1000}{\sin(94.85°)} = \frac{R_4}{\sin(145.15°)}$$

$\therefore \quad R_3 = 869.14\ N$

$R_4 = 573.48\ N$



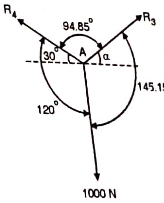


Fig. 14 : FBD at A

For B : (applying conditions of equilibrium)

$\Sigma F_y = 0;\quad R_2 - 750 - R_3 \sin \alpha = 0$

$\therefore \quad R_2 = 1463.26\ N \uparrow$ ...Ans.

$\Sigma F_x = 0;\quad R_1 - R_3 \cos \alpha = 0$

$\therefore \quad R_1 = 496.65\ N \rightarrow$ ...Ans.

Q. 6 Three cylinders are piled up in a rectangular channel as shown in Fig. 15. Determine the reactions at point 6 between the cylinder A and the vertical wall of the channel. (Cylinder A : radius = 4 cm, m = 15 kg, Cylinder B : radius = 6 cm, m = 40 kg, Cylinder C : radius = 5 cm, m = 20 kg). **May 2010**

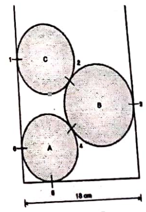


Fig. 15

Ans. : Given :

Mass of A $= m_1 = 15$ kg

Mass of B $= m_2 = 40$ kg

Mass of C $= m_3 = 20$ kg

In Fig. 16.

$$AB = (4+6) = 10 \text{ cm}$$

$$BD = \text{Radius} = 6 \text{ cm} \qquad AD = 18 - 4 - 6 = 8 \text{ cm}$$

$\therefore$ ADB is right angled triangle.

$$\therefore \sin\theta = \frac{6}{10} = 0.6 \qquad \cos\theta = \frac{8}{10} = 0.8$$

All the forces acting are shown.

If the system of three spheres and a channel is considered, external forces acting on the system are :

Reactions : R_1, R_3, R_5, R_6 and weights m_1g, m_2g and m_3g

Reactions R_2 and R_9 are between the spheres. They are internal to the system.

Considering equilibrium of the system of spheres and channels.

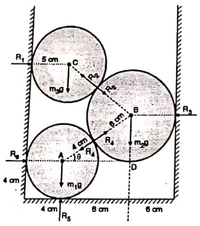


Fig. 16

$$\Sigma F_y = 0 ; \qquad R_5 - m_1g - m_2g - m_3g = 0$$

$$\therefore R_5 = g(m_1 + m_2 + m_3) = 9.81 \times (15 + 40 + 20)$$

$$\therefore R_5 = 735.75 \text{ N} \uparrow$$

Now consider equilibrium of sphere A. Forces acting on it are :

Reaction : R_4, R_5, R_6 and weight m_1g

$$\Sigma F_y = 0 ; \; R_5 - m_1g - R_4 \sin\theta = 0 \qquad \therefore 735.75 - 15 \times 9.81 - R_4 \times 0.6 = 0$$

$$\therefore R_4 = 981 \text{ N}$$

$$\Sigma F_x = 0 ; \qquad R_6 - R_4 \cos\theta = 0 \qquad R_6 = 981 \times 0.8 \qquad R_6 = 784.8 \text{ N} \longrightarrow$$

$\therefore$ Reaction between vertical wall and sphere A at point 6 is.

R_6 = 784.8 N horizontal. ...Ans.

Q. 7 A prismatic bar AB of length 6m and weight 3 kN is hinged to a wall and supported by a cable BC. Find hinge reaction and tension in cable BC. Refer Fig. 17. **Dec. 2015**

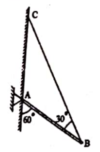


Fig. 17

Ans. :

Concept : Here bar is **in equilibrium under three forces (non-parallel)** RA, T and W. Hence they must be concurrent. For concurrency produce two forces which are known in directions to meet a point and for equilibrium third force must pass through that point.

Step 1 : F.B.D. of bar AB in equilibrium position :

It is recognized that weight and tension intersect at O so the reaction R_A which is unknown in direction must pass through O as shown in Fig. 18.

Geometry : Here AG $=$ GB $= l/2$ and Δ OBG is isosceles Δ

$\therefore \angle GOB = 30°$

Also $\angle AGO = 60°$ and $AG = OG = l/2$

$\therefore \Delta$ AOG is an equilateral triangle.

$\therefore \angle AOG = 60°$

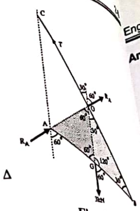


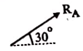
Fig. 18

Step 2 : To find out forces use Lami's theorem at O.

$$\therefore \frac{R_A}{\sin 150°} = \frac{3}{\sin 90°} = \frac{T}{\sin 120°}$$

$$\therefore R_A = \frac{3 \sin 150°}{\sin 90°} = 1.5 \text{ kN}$$

$$\therefore R_A = 1.5 \text{ kN}$$



...Ans.

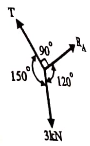
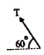


Fig. 19

$$T = \frac{3 \sin 120°}{\sin 90°} = 2.6 \text{ kN}$$

$$T = 2.6 \text{ kN}$$



Q. 8 A cylinder of weight 500 N is kept on two inclined planes as shown in the Fig. 20. Determine the reactions at the contact points A and B. **Dec. 2014**

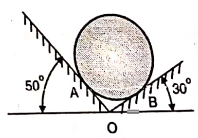


Fig. 20

Ans. :

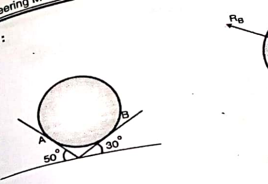


Fig. 21

F.B.D. of roller and use Lami's theorem

$$\frac{R_A}{\sin(150)} = \frac{500}{\sin(80)} =$$

$\therefore R_A = 253.85$ N

Q. 9 Two spheres A and B are kept in ... the contact surfaces. Consider ... $W_A = 500$ N and $W_B = 200$N.

Ans. : For entire system, apply th...

$\Sigma F_x = 0$; $R_1 - R_3$ =

$\therefore R_1 = R_3$

$\Sigma F_y = 0$; $R_2 - 500 - 200$

$\therefore$ R

$\Sigma M_A = 0$; $R_3(BM) - 200$ (

where A

$\therefore R_3(49) - 200($

$\therefore$

$\therefore$ From Equation (i)

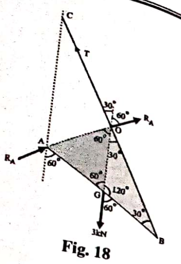


Fig. 18

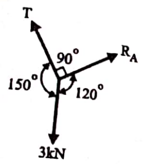


Fig. 19

in the Fig. 20. Determine the

Dec. 2014

Ans. :

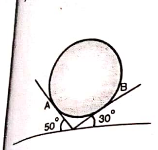


Fig. 21

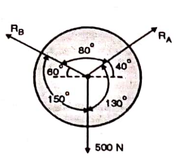


Fig. 22

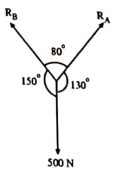


Fig. 23

F.B.D. of roller and use Lami's theorem at centre

$$\frac{R_A}{\sin(150)} = \frac{500}{\sin(80)} = \frac{R_B}{\sin(130)}$$

$\therefore$ $R_A = 253.85$ N $40°$ $\quad R_B = 388.93$ N $60°$

Q. 9 Two spheres A and B are kept in a horizontal channel. Determine the reactions coming from all the contact surfaces. Consider the radius of A and B are 40mm and 30mm respectively. Take $W_A = 500$ N and $W_B = 200$N. **May 2016**

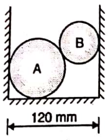


Fig. 24

Ans. : For entire system, apply three condition of equilibrium

$\Sigma F_x = 0$; $R_1 - R_3 = 0$

$\therefore R_1 = R_3$ ...(i)

$\Sigma F_y = 0$; $R_2 - 500 - 200 = 0$

$\therefore$ $R_2 = 700$ N ↑

$\Sigma M_A = 0$; $R_3(BM) - 200(AM) = 0$

where $AM = 80 - 30 = 50$ mm

$\therefore R_3(49) - 200(50) = 0$

$BM = \sqrt{(AB)^2 - (AM)^2}$

$\therefore$ $R_3 = 204.08$ N ←

$= \sqrt{(70)^2 - (50)^2}$

$= 48.98$ mm $\cong 49$ mm

$\therefore$ From Equation (i) $R_1 = 204.08$ →

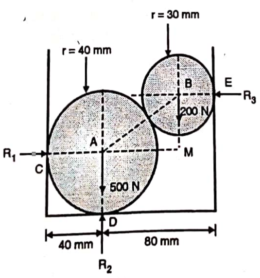


Fig. 25

Q. 10 Two spheres rest in a smooth trough as shown in Fig. 26. Find forces at points of contacts. **May 2012**

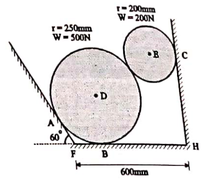


Fig. 26

Ans. :

F.B.D. of both spheres.

Geometry to find angle α.

Join centers D and E.

$\therefore DE = r_1 + r_2 = 250 + 200 = 450$ mm

Let $DM = x = BN$, $\therefore FB = 400 - x$

Now join DB and DA $\angle B = \angle A = 90°$

If we join DF it will be an angle bisector which is bisecting $\angle AFB = 120°$

$\therefore$ In Δ DBF, $\tan 60° = \frac{BD}{BF} = \frac{250}{400 - x}$

$\therefore x = 255.66$ mm

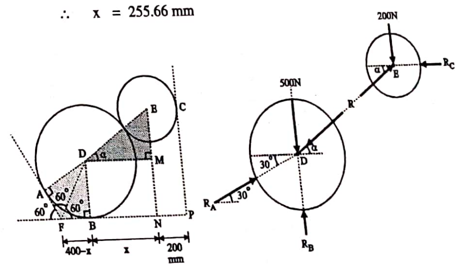


Fig. 27

$\therefore$ In Δ DME, $\cos\alpha = \frac{DM}{DE} = \frac{255.66}{450}$

$\therefore \alpha = 55.38°$

Apply Lami's theorem at E

$$\therefore \frac{R_C}{\sin(90+\alpha)} = \frac{200}{\sin(180-\alpha)} = \frac{R}{\sin 90}$$

$$\therefore R_C = 138.07\ \text{N} \leftarrow \quad \text{...Ans.}$$

$$R = 243.03\ \text{N}$$

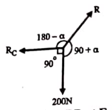
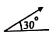


Fig. 28 : FBD at E

At D solving the forces and apply conditions of equilibrium.

$$\Sigma F_x = 0; \quad R_A \cos 30° - R\cos(55.38°) = 0$$

$$\therefore R_A = 159.43\ \text{N} \quad \text{...Ans.}$$



$$\Sigma F_y = 0; \quad R_A \sin 30° + R_B - 500 - R\cdot\sin\alpha = 0$$

$$\therefore R_B = 500 + 243.03 \sin 55.38° - 159.43 \sin 30°$$

$$\therefore R_B = 620.28\ \text{N} \uparrow \quad \text{...Ans.}$$

Q. 11 A 30 kg pipe is supported at 'A' by a system of five cords. Determine the force in each cord for equilibrium. **May 2009**

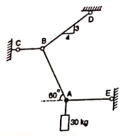


Fig. 29

Ans. : Lami's theorem at A and B

where $\tan\theta = \frac{3}{4}$ $\quad\therefore\ \theta = 6.87°$

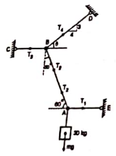


Fig. 30

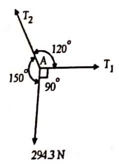


Fig. 31: FBD at A

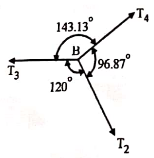


Fig. 32 : FBD at B

At A

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 90°} = \frac{294.3}{\sin 120°}$$

$$\therefore T_1 = 169.91\ \text{N}$$

$$T_2 = 339.83\ \text{N} \quad \text{...Ans.}$$

At B

$$\frac{T_2}{\sin 143.13°} = \frac{T_4}{\sin 120°} = \frac{T_3}{\sin 96.87°}$$

$$\therefore T_3 = 490.5\ \text{N}$$

$$\therefore T_4 = 562.30\ \text{N} \quad \text{...Ans.}$$

Q. 12 Determine the tensions in cords AB and BC for equilibrium of 30 kg block (Fig. 33). **May 2013**

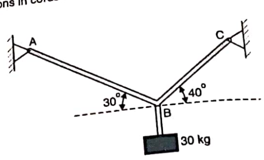


Fig. 33

Ans. :

Apply Lami's theorem at B.

$$\frac{T_{AB}}{\sin(130)} = \frac{294.3}{\sin(110)} = \frac{T_{BC}}{\sin(120)}$$

$$\therefore T_{AB} = 239.91\ \text{N}$$

$$T_{BC} = 271.23\ \text{N}$$

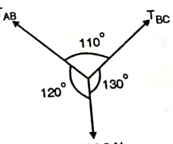


Fig. 34 : FBD

Q. 13 If the cords suspend the two buckets in the equilibrium position as shown in Fig. 35. Determine the weight of bucket B. Bucket A has a weight of 60N. **Dec. 2005**

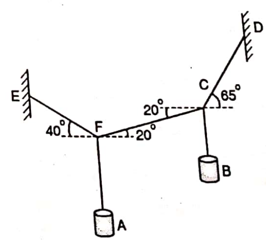


Fig. 35

Ans. : Simplify the given Fig. 35 a
Applying Lami's theorem a

(I) FBD, Fo

At F :

At C :

Q. 14 A uniform be
using a strin
in Fig. 37. F

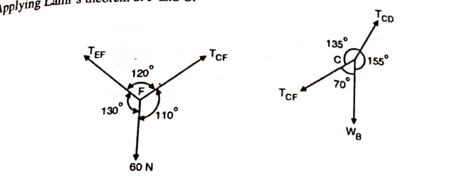
Ans. :

F.B.I

For beam

$\Sigma M_A = 0$

Ans. : Simplify the given Fig. 35 as shown in Fig. 36.
Applying Lami's theorem at F and C.



(i) FBD, Force acting at F (ii) FBD, Force acting at C

Fig. 36

At F : $\dfrac{T_{EF}}{\sin(110)} = \dfrac{60}{\sin(120)} = \dfrac{T_{CF}}{\sin(130)}$

$\therefore\ T_{EF} = 65.1\text{ N}$ $\qquad T_{CF} = 53.07\text{ N}$

At C : $\dfrac{T_{CD}}{\sin(70)} = \dfrac{W_B}{\sin(135)} = \dfrac{T_{CF}}{\sin(155)}$

$\therefore\ T_{CD} = 118\text{ N}$ $\qquad W_B = 88.79\text{ N}$

Q. 14 A uniform beam AB hinged at A is kept horizontal by supporting and setting a 100 N weight by using a string tied at B and passing over a smooth pulley at C The beam also loaded as shown in Fig. 37. Find the reactions at A and C. **May 2016**

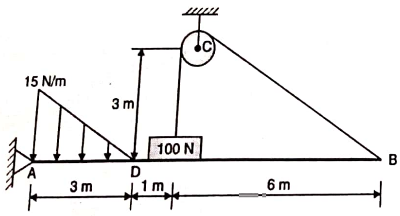


Fig. 37

Ans. :

F.B.D. of beam AB and block

For beam AB

$\Sigma M_A = 0;$ $\qquad (-22.5\times 1) - (R\times 4) + (T\sin 26.56^\circ \times 10) = 0$

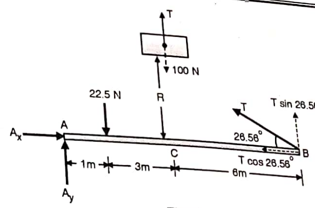


Fig. 38

$4.47\,T - 4R = 22.5$

$\Sigma F_x = 0;\quad A_x - T\cos(26.56^\circ) = 0$ ...(1)

$\therefore\ A_x = 0.89\,T$

$\Sigma F_y = 0;\quad A_y - 22.5 - R + T\sin(26.56^\circ) = 0$ ...(2)

$\therefore\ A_y = 22.5 + R - 0.45\,T$ ...(3)

For block : $\Sigma F_y = 0$ $\quad T + R - 100 = 0$

$\therefore\ T + R = 100$ ...(4)

Solving equations (1) and (4) we get

$T = 49.88\text{ N}$

$R = 50.12\text{ N}$

Substituting in equations (2) and (3)

$A_x = 44.39\text{ N}\rightarrow$

$A_y = 50.17\text{ N}\uparrow$

Q. 15 Find the reactions at supports A and E for the beam loaded as shown in the Fig. 39. **May 2016**

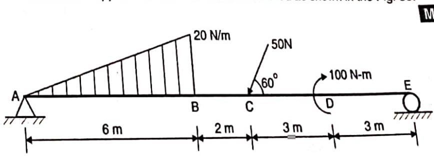


Fig. 39

Ans. :

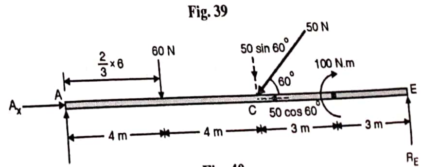


Fig. 40

F.B.D. of beam AE

Applying conditions of equilibrium

$\Sigma M_A = 0;$ $(-60 \times 4) - (50 \sin 60° \times 8) - 100 + (R_E \times 14) = 0$ $\quad \therefore R_E = 49\text{ N} \uparrow$

$\Sigma F_y = 0;$ $A_y - 60 - 50 \sin 60° + R_E = 0$ $\quad \therefore A_y = 54.3\text{ N} \uparrow$

$\Sigma F_x = 0;$ $A_x - 50 \cos 60° = 0$ $\quad \therefore A_x = 25\text{ N} \rightarrow$

Q. 16 Find the reactions at supports B and F for the beam loaded as shown in the Fig. 41 below **Dec. 2014**

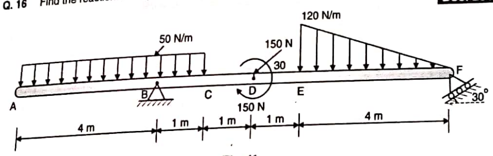


Fig. 41

Ans. :

F.B.D. of beam :

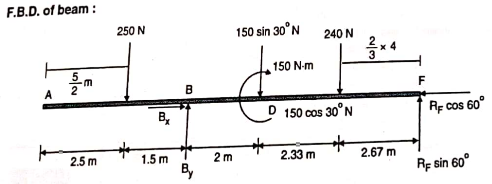


Fig. 42

Applying conditions of equilibrium

$\Sigma M_B = 0;$ $(250 \times 1.5) - (150 \sin 30° \times 2) - 150 - (240 \times 4.33) + (R_F \sin 60° \times 7) = 0$

$\therefore R_F = 159.05\text{ N}$ $60°$

$\Sigma F_x = 0;$ $B_x - 150 \cos 30° - R_F \cos 60° = 0 \quad \therefore B_x = 209.43\text{ N} \rightarrow$

$\Sigma F_y = 0;$ $-250 + B_y - 150 \sin 30° - 240 + R_F \sin 60° = 0 \quad \therefore B_y = 427.26\text{ N} \uparrow$

$\therefore R_B = \sqrt{B_x^2 + B_y^2} = 475.83\text{ N} \qquad \theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = 63.89°$ in 1st quadrant.

Q. 17 Find the support reactions for the beam (Fig. 43). **May 2015**

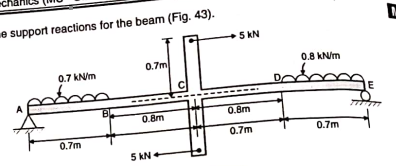


Fig. 43

Ans. :

F.B.D. of beam

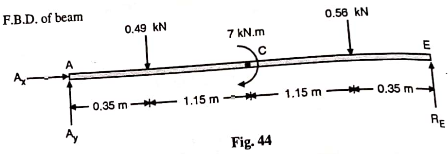


Fig. 44

Applying conditions of equilibrium.

$\Sigma M_A = 0;$ $(-0.49 \times 0.35) - 7 - (0.56 \times 2.65) + (R_E \times 3) = 0$ $\quad R_E = 2.885\text{ kN} \uparrow$

$\Sigma F_x = 0;$ $A_x = 0$

$\Sigma F_y = 0;$ $A_y + R_E = -0.49 - 0.56 = 0$

$\therefore A_y = -1.835\text{ kN}$ $\quad \therefore A_y = 1.835\text{ kN} \downarrow$

Q. 18 Find analytically the support reaction at B and load P for the beam shown in Fig. 45 it reaction at support A is zero. **Dec. 2008, May 2014**

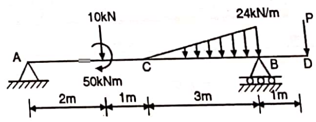


Fig. 45

Ans. :

Reaction at A : $R_A = 0$ ...(given)

Total load on CB = W = Area of Δ

$\therefore W = \frac{1}{2} \times 3 \times 24 = 36\text{ kN } (\downarrow)$ $\qquad$ acting at $\frac{1}{3} \times 3 = 1\text{m}$ from B

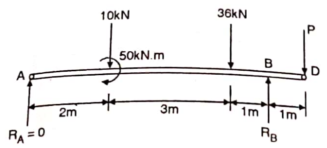


Fig. 46

$\Sigma M_B = 0$; $-(P \times 1) + (36 \times 1) + (10 \times 4) - 50 = 0$

$\therefore$ P = 26 kN $\qquad \therefore$ **Load P = 26 kN** ...Ans.

Now, for the entire beam.

$\Sigma F_y = 0$; $R_A + R_B - 10 - 36 - P = 0$

$\therefore$ $0 + R_B - 10 - 36 - 26 = 0$ $\qquad \therefore R_B = 72$ kN

$$\begin{bmatrix} P & = & 26\text{ kN} & \downarrow \\ R_B & = & 72\text{ kN} & \uparrow \end{bmatrix}$$ ...Ans.

Q. 19 Find the reactions at the supports of the beam loaded as shown in Fig. 47. **Dec. 2009**

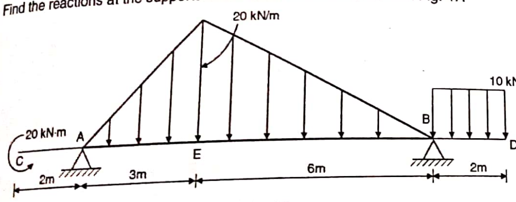


Fig. 47

Ans. :

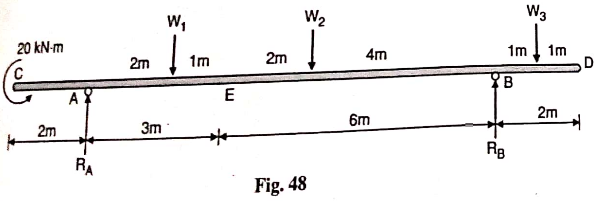


Fig. 48

In Fig. 48 :

Total load on portion AE is area of the load triangle

$\therefore$ $W_1 = \frac{1}{2} \times 3 \times 20 = 30$ kN acting at 2m from A $(\because \frac{2}{3} \times 3 = 2\text{m})$

Total load on portion EB is area of the load triangle

$\therefore$ $W_2 = \frac{1}{2} \times 6 \times 20 = 60$ kN acting at $\frac{2}{3} \times 6 = 4$m from B

Total u. d. l. on overhang BD is

$W_3 = 2 \times 10 = 20$ kN acting at mid point of BD

Taking moments about A,

$\Sigma M_A = 0$; $20 + R_B \times 9 - 30 \times 2 - 60 \times 5 - 20 \times 10 = 0$

$\therefore$ $9 R_B - 540 = 0$ $\qquad \therefore$ **R_B = 60 kN ↑**

Now $\Sigma F_y = 0$; $R_A + R_B - W_1 - W_2 - W = 0$

$\therefore$ $R_A + 60 - 30 - 60 - 20 = 0$

R_A = 50 kN ↑

Reaction at support A = R_A = 50 kN

Reaction at support B = R_B = 60 kN ...Ans.

Q. 20 Find the reactions at the supports of the beam applying conditions of equilibrium. **May 2010**

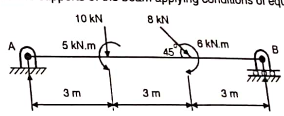


Fig. 49

Ans. :

Load of 8 kN is inclined. (see Fig. 50)

$\therefore$ Resolving along x axis :

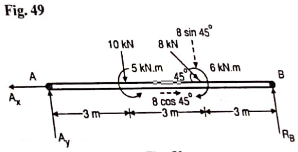


Fig. 50

$\Sigma F_x = 0$;

$8 \cos 45° - A_x = 0$

$\therefore$ **A_x = 5.657 kN ⟵**

Taking moments about A,

$\Sigma M_A = 0$;

$9 \times R_B - 10 \times 3 - (8 \sin 45° \times 6) + 5 - 6 = 0$

[Note that 5 kNm moment is c.c.w so +ve 6 kNm moment is c.w. so $-^{ve}$ moment is not to be multiplied by distance.]

$\therefore$ **R_B = 7.216 kN ↑**

Now, $\Sigma F_y = 0$;

$A_y + R_B - 10 - 8 \sin 45° = 0$

$\therefore$ $A_y + 7.216 - 15.657 = 0$

$\therefore$ **A_y = 8.441 kN ↑**

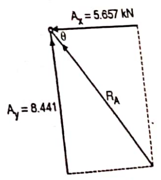


Fig. 51

$\therefore$ Support Reactions

At support A $\quad A_x$ = 5.657 kN

$\qquad A_y$ = 8.441 kN

$\therefore \; R_A = \sqrt{A_x^2 + A_y^2} = 10.161 \text{ kN}$

$\tan\theta = \dfrac{8.441}{5.657} \quad \therefore \; \theta = 56.17°$

Q. 21 Find the reactions at the supports of the beam AB loaded as shown in the Fig. 52. **May 2011**

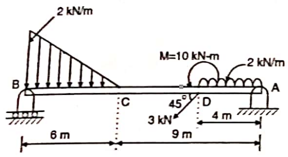


Fig. 52

Ans. :

F. B. D. of beam

$\Sigma M_A = 0; \quad (8 \times 2) + 10 + (3 \sin 45° \times 4) + (6 \times 13) - R_B \times 15 = 0 \qquad \therefore R_B = 7.5 \text{ kN} \uparrow$

$\Sigma F_y = 0; \quad R_B + A_y - 6 - 8 - 3 \sin 45° = 0 \qquad \therefore A_y = 8.62 \text{ kN} \uparrow$

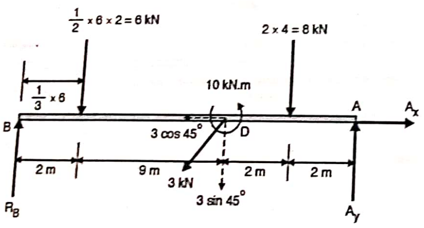
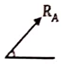


Fig. 53

$\Sigma F_x = 0; \quad A_x - 3 \cos 45° = 0 \qquad \therefore A_x = 2.12 \text{ kN} \rightarrow$

$R_A = \sqrt{A_x^2 + A_y^2} = 8.88 \text{ kN}$

$\theta = \tan^{-1}\left[\dfrac{A_y}{A_x}\right] = 76.18°$



...Ans.

Q. 22 Find the reactions at 'A'. **May 2012**

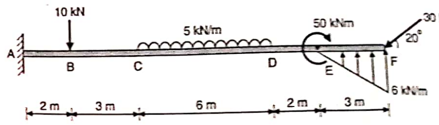


Fig. 54

Ans. : Draw FBD of beam as shown in Fig. 55.

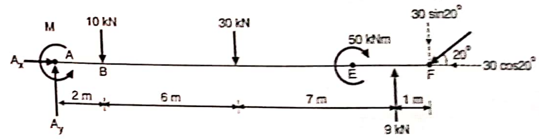


Fig. 55

$\Sigma M_A = 0;$

$M - (10 \times 2) - (30 \times 8) - 50 + (9 \times 15) - (30 \sin 20° \times 16) = 0$

$M = 339.17 \text{ kN.m} \circlearrowleft$

$\Sigma F_y = 0;$

$A_y - 10 - 30 - 30 \sin 20° + 9 = 0 \qquad A_y = 41.26 \text{ kN} \uparrow$

$\Sigma F_x = 0;$

$A_x - 30 \cos 20° = 0 \qquad \therefore A_x = 28.19 \text{ kN} \rightarrow$

$\therefore R_A = \sqrt{A_x^2 + A_y^2} \qquad \therefore R_A = 49.97 \text{ kN}$

$\tan\theta = \dfrac{A_y}{A_x} \qquad \therefore \theta = 55.67°$

Q. 23 Determine the intensity of distributed load W at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B. **May 2008**

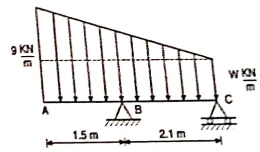


Fig. 56

I have classified solved examples in three classes; a, b, c. Class 'a' contains short examples for 3 or 4 marks, class 'b' contains medium examples for 6 marks and class 'c' contains lengthy and intricate examples for 8 marks. In the same manner, examples given in the ... carefully classified in three classes a, b, c as above.
... Prof. A. N. Nakra, M. Sc; P. G. (BARC); M. S. ... Prof. Nelson Pereira, M. Sc.

SYLLABUS
ENGINEERING MATHEMATICS - I
F. E. Semester - I
(Mumbai University - Effective From August 2019)

Module 1 : Complex Numbers
... : Review of Complex Numbers, Algebra of Complex ... Cartesian, ... number.

Ans. : Dividing given loading to rectangle and triangle

$A_1 = 3.6\,W$ kN (↓) acting at 1.8 m from A

$A_2 = \frac{1}{2} \times 3.6\,(9 - W) = 1.8\,(9 - W)$ kN (↓) acting at

$\frac{1}{3} \times 3.6 = 1.2$ m from A

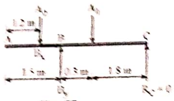


Fig. 57

$\Sigma F_y = 0 : B_y + R_c = A_1 + A_2$

$B_y = 3.6\,W + 1.8\,(9 - W)$ ...(i)

$\Sigma M_B = 0 : A_2\,(0.3) - A_1\,(0.3) = 0$

$1.8\,(9 - W)\,(0.3) - 3.6\,W\,(0.3) = 0$

$W = 3$ kN/m

∴ by Equation (i) $B_y = 21.6$ kN ↑ ...Ans.

Q. 24 Find reactions at A and B for a bent beam ABC loaded is as shown in Fig. 58. **May 2006**

Fig. 58

Ans. :

F.B.D. of beam

Dividing load acting on BC into two parts

∴ W_1 = Area of part 1 = 4×3

= 12 kN ← acting at 1.5 m from B.

W_2 = Area of part 2

$= \frac{1}{2} \times 4 \times 3$

= 6 kN ← acting at $\frac{1}{3} \times 3 = 1$ m from B.

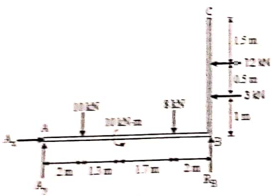


Fig. 59

Applying conditions of equilibrium :

$\Sigma M_A = 0 : \quad -(10 \times 2) - 10 - (8 \times 6) + R_B\,(7) + 12 \times 1.5 + 6 \times 1 = 0$

∴ $R_B = 4.86$ kN

$R_B = 4.86$ kN ↓ ...Ans.

$\Sigma F_x = 0 : \quad A_x - 12 - 6 = 0$

∴ $A_x = 18$ kN →

$\Sigma F_y = 0 : \quad A_y - 10 - 8 + R_B = 0$

$A_y = 18 + 4.86 \quad A_y = 22.86$ kN ↑

Q. 25 Find analytically the support reaction at B and load P for the beam as shown in Fig. 60 if reaction at support 'A' is zero. **Dec. 2011**

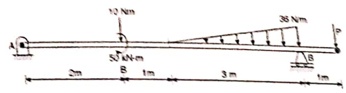


Fig. 60

Ans. :

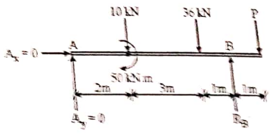


Fig. 61

$\Sigma M_A = 0 : \; R_B\,(6) - (10 \times 2) - 50 - (36 \times 5) - (P \times 7) = 0$

∴ $6\,R_B - 7P = 250$ ...(i) $\quad \Sigma F_y = 0 : \; R_B - P = 46$ ...(ii)

Solving we get

$R_B = 72$ kN ↑ $\qquad P = 26$ kN ↓ ...Ans.

Q. 26 Find the support reactions at hinge A and roller B. **May 2013**

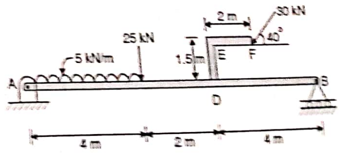


Fig. 62

Ans. :

F.B.D of beam

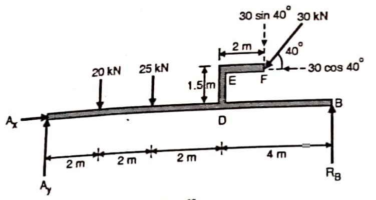


Fig. 63

Apply conditions of equilibrium.

$\Sigma M_A = 0;\ (-20 \times 2) - (25 \times 4) - (30 \sin 40° \times 8) + (30 \cos 40° \times 1.5) + (R_B \times 10) = 0$

$\therefore\ R_B = 25.98\ kN \uparrow$

$\Sigma F_x = 0;\quad A_x - 30 \cos 40 = 0 \qquad \therefore\ A_x = 22.98\ kN \rightarrow$

$\Sigma F_y = 0;\quad A_y - 20 - 25 - 30 \sin 40 + R_B = 0\ \therefore\ A_y = 38.3\ kN \uparrow$

$\therefore\ R_A = \sqrt{A_x^2 + A_y^2} = 44.67\ N$

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = 59.04°$ in 1[st] quadrant.

Q. 27 Find the support reactions at A and B for the beam shown in Fig. 64. **Dec. 2013**

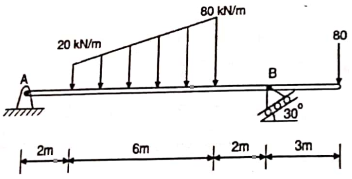


Fig. 64

Ans. :

F.B.D. of beam

Dividing given load into rectangle (1) and Triangle (2)

$A_1 = 20 \times 6 = 120\ KN \downarrow$ acting at 5m from A

$A_2 = \frac{1}{2} \times 6 \times 60 = 180\ kN \downarrow$ acting at $2 + \left(\frac{2}{3} \times 6\right) = 6$ m from A.

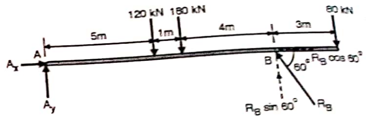


Fig. 65

Applying conditions of equilibrium.

$\Sigma M_A = 0;\quad (-120 \times 5) - (180 \times 6) + (R_B \sin 60° \times 10) - (80 \times 13) = 0$

$\therefore\ R_B = 314.07\ kN$ 60°

$\Sigma F_y = 0;\ A_y - 120 - 180 + R_B \sin 60 - 80 = 0\ \therefore\ A_y = 108\ kN \uparrow$

$\Sigma F_x = 0;\ A_x - R_B \cos 60 = 0\ \therefore\ A_x = 157.035\ N \rightarrow$

Chapter 3 : Friction

Q. 1 Determine minimum value of co-efficient of friction so as to maintain the position shown in Fig. 1. Length of Rod AB is 3.5 m and it weighs 250 N. **Dec. 2007**

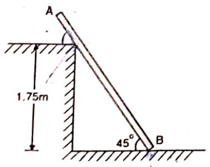


Fig. 1

Ans. :

Conditions of equilibrium :

$\Sigma F_x = 0$; $-\mu R_2 + R_1 \cos 45° - \mu R_1 \cos 45° = 0$

$\therefore \mu R_2 = R_1 (0.707 - 0.707\mu)$ ...(1)

n/BC.

$\Sigma F_y = 0$; $R_2 + R_1 \sin 45° + \mu R_1 \sin 45° - 250 = 0$

$\therefore R_2 = 250 - R_1 (0.707 + 0.707\mu)$ ...(2)

$\Sigma M_B = 0$; $(250 \times 1.75 \cos 45°) - (R_1 \times BC) = 0$

Here $BC = \dfrac{1.75}{\sin 45°} = 2.47$ m

$$\therefore R_1 = \frac{250 \times 1.75 \cos 45°}{2.47}$$

$\therefore R_1 = 125.25$ N.

Substituting in Equations (1) and (2) and by solving we get

$\mu = 0.414$

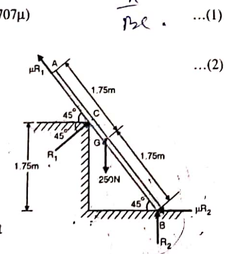


Fig. 2

Q. 2 A uniform ladder 3m long weighs 200 N, it is placed against a wall 60º with floor as shown in Fig. 3. Coefficient of friction between the wall and the ladder is 0.3 and that between the floor and the ladder is 0.4. The ladder, in addition to its own weight, has to support a man weighing 800 N at its top at A. **May 2007**

(i) Calculate the horizontal force F to be applied to the floor level to prevent slipping.

(ii) If the force F is not applied, what would be the minimum inclination of the ladder with the horizontal so that there is no slipping of it with the man at the top ?

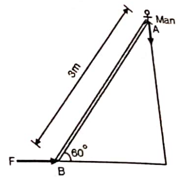


Fig. 3

Ans. :

Case (i) :

$\Sigma F_x = 0$

$F + 0.4 R_3 - R_A = 0$ ...(1)

$\Sigma F_y = 0$

$R_B - 200 - 800 + 0.3 R_A = 0$

$\therefore R_B + 0.3 R_A = 1000$ ...(2)

$\Sigma M_B = 0 \quad = -200 \times 1.5 \cos 60° - 800 \times 3 \cos 60° + R_A \times 3 \sin 60° + 0.3 R_A \times 3 \cos 60° = 0$

$\therefore R_A = 442.9 \text{ N} \leftarrow$

$\therefore$ by Equation (2) $R_B = 867.13 \text{ N} \uparrow$

$\therefore$ by Equation (1) $F = 96.048\text{N} \rightarrow$ ...Ans.

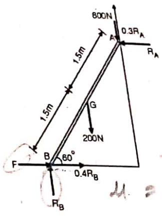


Fig. 4

μ = 0.3.

μ = 0.4

Case (ii) : When force F is not applied

$\Sigma F_x = 0 \quad 0.4R_B - R_A = 0$

$\therefore 0.4 R_B = R_A$ ...(1)

$\Sigma F_y = R_B + 0.3R_A = 1000$ ...(2)

Solving Equation (1) and Equation (2) we get

$R_A = 357.14 \text{ N} R_B = 892.86 \text{ N}$

$\Sigma M_B = 0$

$-200 \times 1.5 \cos\theta \times 1071.42 \sin\theta + 321.43 \cos\theta - 800 \times 3 \cos\theta + 1071.42 \sin\theta + 321.43 \cos\theta = 0$

$\therefore 2700 \cos\theta = 1071.42 \sin\theta + 321.43 \cos\theta$

$\therefore 2378.57 \cos\theta = 1071.42 \sin\theta$

$\therefore \theta = 65.75°$

Q. 3 A ladder AB of length 3m and weight 25 kg is resting against a vertical wall and a horizontal floor. The ladder makes an angle 50 degrees with the floor. A man of weight 60 kg tries to climb the ladder. How much distance along the ladder he will be able to climb if the coefficient of friction between ladder and floor as 0.2 and that between ladder and wall as 0.3. Also find the angle the ladder should make with the horizontal such that the man can climb till the top of the ladder. **Dec. 2014**

Ans. : **F.B.D. of Ladder**

Apply conditions of equilibrium

$\Sigma F_x = 0;$ $R_A - 0.2\,R_B = 0$ ...(i)

$\therefore R_A = 0.2\,R_B$

$\Sigma F_y = 0;$ $R_B + 0.3\,R_A - 588.6 - 245.25 = 0$

$\therefore 0.3\,R_A + R_B = 833.85$ ...(ii)

Solving equations (i) and (ii) we get

$R_A = 157.33\text{ N}$ $R_B = 786.65\text{ N}$

$\Sigma M_B = 0$ $(-R_A \times 2.3) - (0.3\,R_A \times 1.93)$

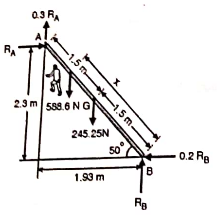


Fig. 5

$+ (245.25 \times \frac{1.93}{2}) + (588.6 \times \cos 50 = 0)$

$\therefore x = 0.571\text{ m}$

Case II : When man can reach the top

$\Sigma F_x = 0$ $R_A - 0.2\,R_B = 0$ ...(i)

$\Sigma F_y = 0$ $0.3\,R_A + R_B = 833.85$ ...(ii)

$\therefore R_A = 157.33\text{ N}$

$R_B = 786.65\text{ N}$

$\Sigma M_A = 0$ $(R_B \times 3\cos\theta) - (0.2\,R_B \times 3\sin\theta)$

$- (245.25 \times 1.5\cos\theta) = 0$; On solving we get $\theta = 76.67°$

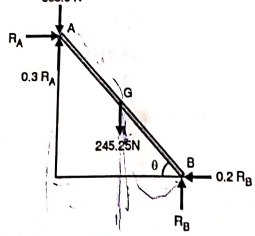


Fig. 6

Q. 4 A ladder of 4 m length weighing 200 N is placed as shown in Fig. 7. μ_B = 0.25 and μ_A = 0.35. Calculate the minimum horizontal force to be applied at A to prevent slipping. **May 20..**

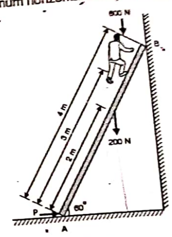


Fig. 7

Ans. :

$\Sigma F_x = 0;$ $P - R_B + 0.35\,R_A = 0$ ...(i)

$\Sigma F_y = 0;$ $R_A + 0.25\,R_B - 200 - 600 = 0$

$\therefore R_A + 0.25\,R_B = 800$ ...(ii)

$\Sigma M_B = 0;$ $R_B(3.46) + 0.25\,R_B(2) - (200 \times 1)$

$- (600 \times 3\cos 60°) = 0$

$\therefore R_B = 277.78\text{ N}$

$\therefore R_A = 730.55\text{ N}$

(From equation ii)

$\therefore P = 22.08\text{ N} \rightarrow$

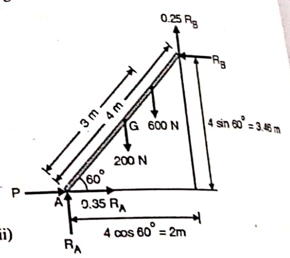


Fig. 8

Q. 5 A heavy metal bar AB rests with its lower end A on a rough horizontal floor having coefficient of friction μ_F and the other end B on a rough vertical wall having coefficient of friction μ_w. If the centre of gravity of the bar is at distances a and b from the ends A and B respectively, show that at impending motion, the inclination of the bar with the horizontal will be :

$$\theta = \tan^{-1}\left(\frac{1}{\mu_F}\,\frac{a - b\mu_F\,\mu_W}{a + b}\right)$$

May 2016

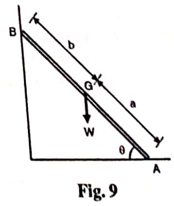


Fig. 9

Engineering Mechanics (MU ...

Ans. : $R_B - \mu_F$ ...

$\Sigma F_x = 0$ ...

$\Sigma F_y = 0$ $\mu_w R_B + R_A$ ...

$\therefore \mu_w(\mu_F R_A)$ ...

$R_A(1 + \mu$ ...

$\Sigma M_B = 0$ $R_A(a+b)$ c...

$\therefore R_A(a+b)$ c...

$R_A(1 + \mu_F \cdot \mu$ ...

$(a+b)\cos\theta$ ...

$\cos\theta\,[a+b$ ...

Q. 6 There... coeffi... maxi...

Ans. : ...

slide ... three ... (i) ... (ii)

Ans. :

$\Sigma F_x = 0 \qquad R_B - \mu_F R_A = 0$

$\therefore R_B = \mu_F R_A \quad ...(i)$

$\Sigma F_y = 0 \qquad \mu_w R_B + R_A - W = 0$

$\therefore \mu_w (\mu_F R_A) + R_A = W$

$R_A (1 + \mu_F \cdot \mu_w) = W \quad ...(ii)$

$\Sigma M_B = 0 \quad R_A (a + b) \cos\theta - \mu_F R_A (a + b) \sin\theta - W (b \cos\theta) = 0$

$\therefore R_A (a + b) \cos\theta - \mu_F R_A (a + b) \sin\theta$

$R_A (1 + \mu_F \cdot \mu_W) b \cos\theta$

$(a + b) \cos\theta - (1 + \mu_F \mu_w) b \cos\theta = \mu_F (a + b) \sin\theta$

$\cos\theta [a + b - b - b \mu_F \mu_W] = \mu_F (a + b) \sin\theta$

$\therefore \frac{\sin\theta}{\cos\theta} = \frac{a - b\mu_F \cdot \mu_w}{(a+b)\mu_F} \qquad \tan\theta = \frac{1}{\mu_F} \times \frac{a - b\mu_w \cdot \mu_F}{(a+b)}$

$\therefore \theta = \tan^{-1}\left[\frac{1}{\mu_F} \frac{a - b\mu_F \mu_W}{(a+b)}\right]$ proved.

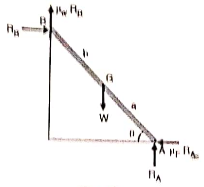


Fig. 10

Q. 6 There blocks are placed on the surface one above the other as shown in Fig. 11. The static coefficient of friction between the blocks and block C and surface is also shown. Determine the maximum value of P that can be applied before any slipping takes place.

May 2008

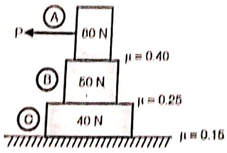


Fig. 11

Ans. :

Here there are three possibilities :

Block 'A' may slide over B. 'A' and 'B' together may slide over 'C'. 'A, B and C' together may slide over the floor. For each of these three cases value of 'P' is to be calculated. Minimum of these three is the maximum value of 'P' that can be applied before any slipping takes place.

(i) 'A' slipping over 'B'.

Here $N_1 = W_1 = 80$ N

$\therefore P_1 = \mu_1 N_1 = 0.40 \times 80$

$\therefore P_1 = 32$ N ...(i)

(ii) 'A' and 'B' together slipping over 'C'.

Here $N_2 = W_1 + W_2 = (80 + 50) = 130$ N

$\therefore P_2 = \mu_2 N_2 = 0.25 \times 130$ N

$\therefore P_2 = 32.5$ N ...(ii)

(iii) A, B and C together slipping over the floor.

Here $N_3 = W_1 + W_2 + W_3$

$\therefore N_3 = (80 + 50 + 40) = 170$ N

$\therefore P_3 = \mu_3 N_3 = 0.15 \times 170$

$\therefore P_3 = 25.5$ N

$\therefore$ The maximum value of 'P' that can be applied without causing any slipping is **25.5 N** ...(iii)

Q. 7 A block of weight 200 N rests on a horizontal surface. The co-efficient of friction between the block and the horizontal surface is 0.4. Find the frictional force acting on the block if a horizontal force of 40 N is applied to the block.

Dec. 2009

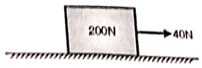


Fig. 12

Ans. : See Fig. 13

Normal reaction = 200 N

$\mu = 0.4$

$\therefore$ Limiting value of frictional force is $\mu \times$ Normal reaction

$= 0.4 \times 200 = 80$ N

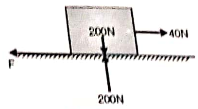


Fig. 13

However the applied force is only 40 N.

$\therefore$ The block is not in limiting equilibrium state.

$\therefore$ Frictional force developed.

= Applied force = 40 N

Frictional force developed at the common surfaces of contact is 40 N to the left. **...Ans.**

Q. 8 The mass of A is 23kg and mass of B is 36 kg. The coefficient of friction are 0.4 between A and B, and 0.2 between ground and block B. Assume smooth drum. Determine the maximum mass of M at impending motion.

May 2014

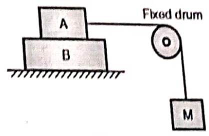


Fig. 14

Ans. :

Case I : When only black A moves

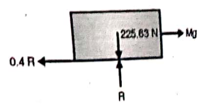


Fig. 15

$\Sigma F_y = 0 \qquad R = 225.63$ N

$\Sigma F_x = 0 \qquad Mg - 0.4 R = 0$

$$\therefore \ M = \frac{0.4 \times 225.63}{9.81}$$

$$\therefore \ M = 9.2 \text{ kg}$$

Case II : When both blocks move together

$\Sigma F_y = 0$ $\qquad R_1 = 578.79 \text{ N}$

$\Sigma F_x = 0$ $\qquad Mg - 0.2 R_1 = 0$

$$\therefore \ M = \frac{0.2 \times 578.79}{9.81} = 11.8 \text{ kg}$$

So, Maximum Mass when motion impends is 9.2 kg

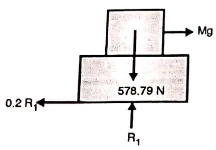


Fig. 16

Q. 9 Three blocks A, B and C are placed as shown . Determine the maximum value of P that can be applied before any slipping takes place. **May 2008**

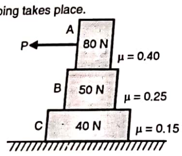


Fig. 17

Ans. :

Case I : When only block A slips.

$\Sigma F_Y = 0$ $\qquad R_1 = 80 \text{ N}$

$\Sigma F_X = 0$ $\qquad P = 0.4 R_1$

$\therefore \ P = 32 \text{ N}$

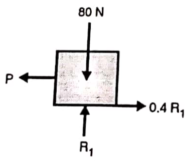


Fig. 18

Case II : When blocks A and B move together on block C

$\Sigma F_x = 0$ $\qquad P = 0.25 R_2$

$\Sigma F_y = 0$ $\qquad R_2 = 130 \text{ N}$

$\therefore \ P = 0.25 (130)$

$P = 32.5 \text{ N}$

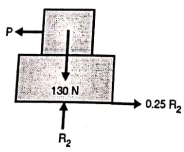


Fig. 19

Case III : When blocks A, B and C move together on surface

$\Sigma F_y = 0;$ $\qquad R_3 = 170 \text{ N}$

$\Sigma F_x = 0;$ $\qquad P = 0.15 R_3 \qquad \therefore P = 0.15 (170)$

$P = 25.5 \text{ N}$

$\therefore$ Max force 'P' that can be applied is 25.5 N when all 3 blocks move together.

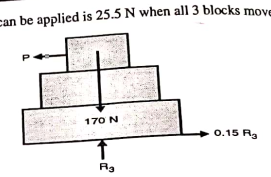


Fig. 20

Q. 10 A block of weight 1000 N is kept on a rough inclined surface. Find out range of P for which the block will be in equilibrium. **May 2013**

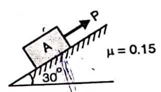


Fig. 21

Ans. :

Case (i) : When block tends to move up the plane.

$\Sigma F_y = 0 \quad R_1 - 1000 \cos 30° = 0$

$\therefore \ R_1 = 866.025 \text{ N}$

$\Sigma F_x = 0 \quad P - 1000 \sin 30° - 0.15 R_1 = 0$

$\therefore \ P = 629.9 \text{ N}$

Case (ii) : When block tends to move down (only frictional force 0.15 R_1 acts up the plane)

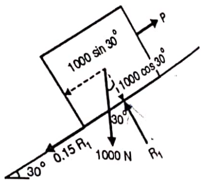


Fig. 22

$$\Sigma F_x = 0 \quad P - 1000 \sin 30° + 0.15 R_1 = 0$$

$$\therefore \ P = 370 \text{ N}$$

For equilibrium, $\qquad 370\text{N} \le P \le 629.9 \text{ N}$ ...Ans.

Q. 11 For the block shown in the Fig. 23 find the minimum value of P, which will just disturb the equilibrium of the system. **Dec. 2012**

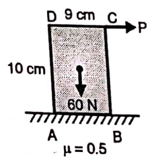


Fig. 23

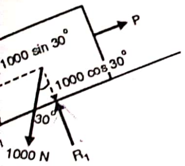
I have classified solved examples in three classes; a, b, c. Class 'a' contains short examples for 3 or 4 marks, class 'b' contains medium examples for 6 marks and class 'c' contains lengthy and intricate examples for 8 marks. In the same manner, examples given in the ... classified in three classes a, b, c as above.

SYLLABUS
ENGINEERING MATHEMATICS
F. E. Semester - I

all 3 blocks move together.

$0.15\ R_B$

...ce. Find out range of P for which the
May 2013



...ich will just disturb the
Dec. 2012

Ans. :

Case I : When block tends to slip

$\Sigma F_y = 0$ $\quad R_1 = 60\ N$

$\Sigma F_x = 0$ $\quad P = 0.5\ R_1$

$\therefore P = 0.5 \times 60$

$P_{(Slip)} = 30\ N$

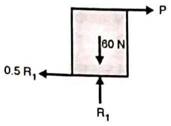


Fig. 24

Case II : When block tends to overturn

$\Sigma M_B = 0$ $\quad (-P \times 10) + (60 \times 4.5) = 0$

$\therefore P_{(coverturn)} = 27\ N$

Here $P_{(overturn)} > P_{(slip)}$

So, block will over turn first at a force of 27 N

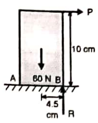


Fig. 25

Q. 12 Two 6° wedge are used to push a block horizontally as shown. Calculate the minimum force required to push the block of weight 10 kN. Take μ = 0.25 for all contact surfaces **Dec. 2008**

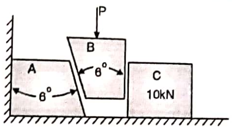
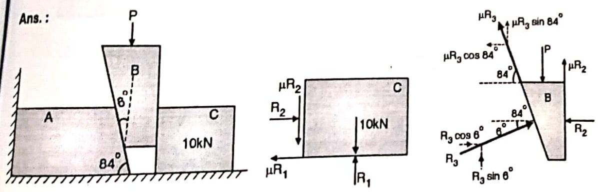


Fig. 26

Ans. :



(a) (b) Forces on block C (c) forces on wedge B

Fig. 27

Consider equalities of block C :

For the block (Fig. 27): R_2 is reaction between B and C.

$$\Sigma F_x = R_2 - \mu R_1 = 0$$

$\therefore\ R_2 = 0.25\ R_1$ ... (i)

$\Sigma F_y = R_1 - \mu R_2 - 10 = 0$

$\therefore\ R_1 = 0.25\ R_2 + 10$

$= 0.25 \times 0.25\ R_1 + 10$ ... (ii)

$\therefore\ R_1 = 10.667\ kN \uparrow$

$\therefore\ R_2 = 0.25\ R_1 = 0.25 \times 10.667 = 2.6668\ kN$

Now consider equilibrium of the wedge. 'R_3' is reaction between A and B. Forces acting on wedge are shown in Fig. 27(c).

$\Sigma F_x = R_3 \cos 6^\circ - \mu R_3 \cos 84^\circ - R_2 = 0.$

$\therefore\ R_3 \cos 6^\circ - 0.25\ R_3 \cos 84^\circ = 2.6668$

$\therefore\ R_3 (\cos 6^\circ - 0.25 \cos 84^\circ) = 2.6668$

$\therefore\ R_3 = 2.7538\ kN$

$\Sigma F_y = R_3 \sin 6^\circ + \mu R_3 \sin 84^\circ + \mu R_2 - P = 0.$

$\therefore\ P = (2.7538 \times \sin 6^\circ) + (0.25 \times 2.7538 \sin 84^\circ) + (0.25 \times 2.6669) = 1.6392\ kN.$

$\therefore$ Minimum value of P to push the block is.

$P = \mathbf{1.6392\ kN}$

Q. 13 Determine the force P required to move the block A of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees. **May 2011**

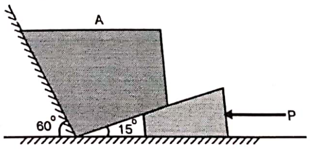


Fig. 28

Ans. : Draw F. B. D. of wedge and block.

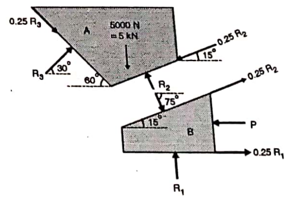
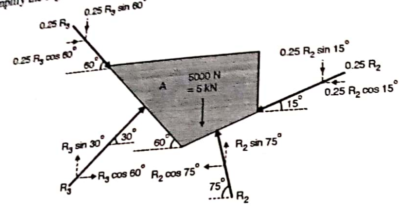


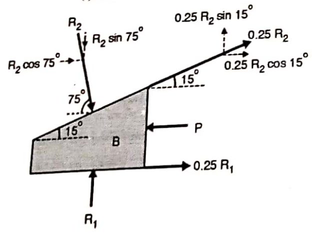
Fig. 29

Scanned by CamScanner

Simplify the Fig. 29(a) as shown in Fig. 30(b).



(i) Force Acting on block A



(ii) Force Acting on wedge B
Fig. 30

Solving forces acting on block 'A'

$$\Sigma F_x = 0 R_3 \cos 30° + 0.25\, R_3 \cos 60° - 0.25\, R_2 \cos 15° - R_2 \cos 75° = 0$$

$$R_3 - 0.5\, R_2 = 0 \qquad ...(1)$$

$$\Sigma F_y = 0 R_3 \sin 30° - 0.25\, R_3 \sin 60° - 0.25\, R_2 \sin 15° + R_2 \sin 75° - 5 = 0$$

$$\therefore \quad 0.28\, R_3 + 0.9\, R_2 = 5 \qquad ...(2)$$

Solving Equation (1) and (2) we get,

$$R_3 = 2.404 \text{ kN}, \quad \text{and} \quad R_2 = 4.807 \text{ kN}$$

Now, solving forces acting on 'B'

$$\Sigma F_y = 0 \quad R_1 + 0.25\, R_2 \sin 15° - R_2 \sin 75° = 0$$

$$\therefore \quad R_1 = 4.332 \text{ kN}$$

$$\Sigma F_x = 0 \quad -P + 0.25\, R_1 + 0.25\, R_2 \cos 15° + R_2 \cos 75° = 0$$

$$\therefore \quad P = 3.488 \text{ kN} \leftarrow \qquad ...\text{Ans.}$$

Q. 14 A block of weight 800 N is acted upon by a horizontal force P as shown in Fig. 31. If the coefficient of friction between the block and incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the value of P for impending motion up the plane. **Dec. 2015**

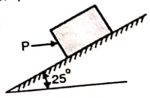


Fig. 31

Ans. :

Simplify the Fig. 32 as shown in Fig. 33.

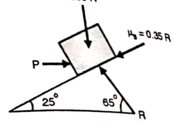


Fig. 32

$$\Sigma F_y = 0 \quad R \sin 65° - 0.35\, R \sin 25° - 800 = 0$$

$$\therefore R = 1054.86 \text{ N}$$

$$\Sigma F_x = 0 \quad P - R \cos 65° - 0.35\, R \cos 25° = 0$$

$$\therefore P = 780.41 \text{ N} \rightarrow$$

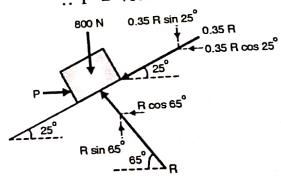


Fig. 33

Q. 15 Find force requires to pull block B as shown. Coefficient of fricition between A and B is 0.3 and between B and floor is 0.25. Mass of A = 40kg and B = 60kg. **Dec. 2015**

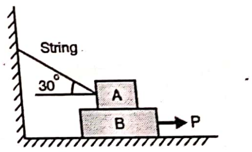


Fig. 34

Ans. :

For block A

$$\Sigma F_x = 0 \qquad 0.3\, R_1 - T \cos 30° = 0$$

$$\Sigma F_y = 0 \qquad R_1 + T \sin 30° = 392.4$$

Solving, $\quad R_1 = 334.47$ N $\qquad T = 115.86$ N

I have classified solved examples in three classes; a, b, c. Class 'a' contains short examples for 3 or 4 marks, class 'b' contains medium examples for 6 marks and class 'c' contains lengthy and intricate examples for 8 marks. In the same manner, examples given in the ... also carefully classified in three classes a, b, c as above.

... Prof. A. N. Nakra, M. Sc; P. G. (BARC); M. S.

SYLLABUS

ENGINEERING MATHEMAT

F. E. Semester - I

(Mumbai University - Effective From

...tal force P as shown in Fig. 31. If the ... $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the ...

Dec. 2015

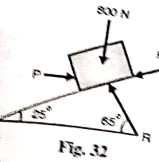


Fig. 32

...between ...

...(i)

...(ii)

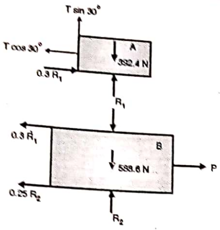


Fig. 35

For block B :

$\Sigma F_y = 0$ $\qquad R_2 - R_1 - 588.6 = 0$ ...(i)

$\therefore R_2 = 923.07$ N

$\Sigma F_x = 0$ $\qquad P - 0.3\,R_1 - 0.25\,R_2 = 0$ $\qquad \therefore$ **P = 331 N →**

□□□

Chapter 4 : Truss

Q. 1 Determine the force in each member of given truss.

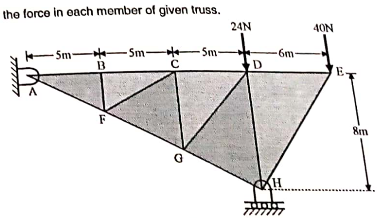


Fig. 1

Ans. :

Step 1 : Stability check

Number of joints J = 8 $\qquad$ Number of members n = 13

$2j - 3 = 2(8) - 3 = 13$ $\qquad \therefore n = 2j - 3$ so given truss is perfect truss.

Step 2 : F.B.D. of truss and conditions of equilibrium :

$\Sigma M_A = 0$; $\qquad (-24 \times 15) - (40 \times 21) + (R_H \times 15) = 0$ $\qquad \therefore R_H = 80$ N ↑

$\Sigma F_y = 0$; $\qquad A_y - 24 - 40 + R_H = 0$

$\therefore A_y = 24 + 40 - 80 = -16$ N $\qquad A_y = 16$ N ↓

$\Sigma F_x = 0$; $\qquad \therefore A_x = 0$

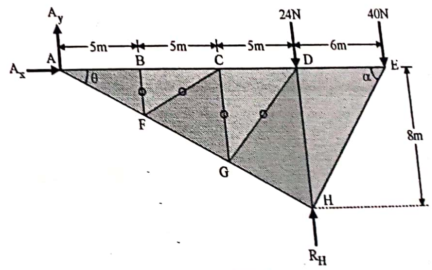


Fig. 2

Step 3 : Forces by observation

1) By observing joint B,
$F_{BF} = 0$ and $F_{AB} = F_{BC}$ ---result (1)

2) By observing joint F,
$F_{CF} = 0$ and $F_{AF} = F_{GF}$ ---result (2)

3) By observing joint C,
$F_{CG} = 0$ and $F_{BC} = F_{CD}$ ---result (3)

4) By observing joint G,
$F_{GD} = 0$ and $F_{FG} = F_{GH}$ ---result (4)

5) By observing joint D [four forces with two pairs of collinear forces]
$F_{DH} = -24$ kN and $F_{CD} = F_{DE}$ ---result (5)

Step 4 : Equilibrium of joints

a) Consider joint A :

Apply, $\Sigma F_y = 0;$ $-16 - F_{AF} \cdot \sin\theta = 0$

$\therefore F_{AF} = \dfrac{-16}{\sin(28.072°)}$ $F_{AF} = -34$ N

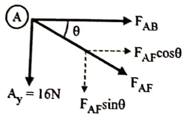


Fig. 3

$\Sigma F_x = 0;$ $F_{AB} + F_{AF}\cos\theta = 0$

$\therefore F_{AB} - 34\cos(28.072°) = 0$ $\therefore F_{AB} = 30$ N

$\therefore$ By result (2) and (4) $F_{AF} = F_{FG} = F_{GH} = -34$ N

and by result (1) and (3) $F_{AB} = F_{BC} = F_{CD} = F_{DE} = -30$ N

$\tan\theta = 8/15$ $\therefore \theta = 28.072°$

b) Consider joint E :

Apply, $\Sigma F_y = 0;$ $-40 - F_{EH} \cdot \sin\alpha = 0$

$\therefore F_{EH} = \dfrac{-40}{\sin(53.13°)}$

$F_{EH} = -50$ N

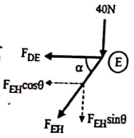


Fig. 4

$\tan\alpha = 8/6$

$\alpha = 53.13°$

$\Sigma F_x = 0;$ $-F_{DE} - F_{EH} \cdot \cos\alpha = 0$

$\therefore -F_{DE} - (-50) \cdot \cos 53.13° = 0$

$\therefore F_{DE} = 30$ N

Step 5 : Force Table :

Sr. No.	Member	Magnitude (N)	Nature
1.	AB, BC, CD, DE	30	T
2.	AF, FG, GH	34	C
3.	BF, CF, CG, GD	0	–
4.	DH	24	C
5.	EH	50	C

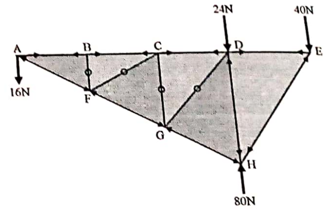


Fig. 5

Q. 2 Determine the forces in members GH, CG and CD.

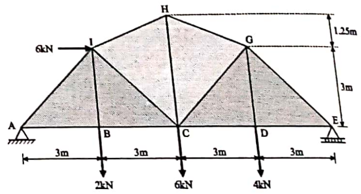


Fig. 6

Ans. :

Step 1 : Stability check :

Number of joints J = 8 Number of members n = 13 and R = 4

$\therefore$ 2j – R = 2(8) – 3 = 13 $\therefore$ n = 2j – R so given truss is perfect.

Step 2 : F.B.D. and conditions of equilibrium :

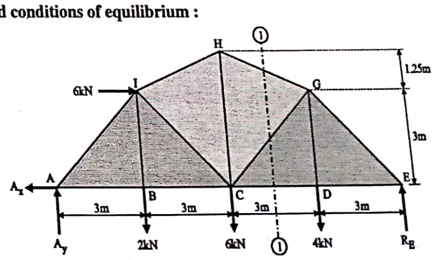


Fig. 7

$\Sigma M_A = 0;\quad (R_E \times 12) - (2\times3) - (6\times6) - (4\times9) - 6\times3 = 0 \qquad \therefore R_E = 8\ kN \uparrow$

$\Sigma F_y = 0;\quad A_y + R_E - 2 - 6 - 4 = 0 \qquad \therefore A_y = 4\ kN \uparrow$

$\Sigma F_x = 0;\quad -A_x + 6 = 0 \qquad \therefore A_x = 6\ kN \leftarrow$

Step 3 : Select a section

Select section (1) - (1) cutting three members GH, CG and CD and draw F.B.D. of right or left part of truss w.r.t. section. We shall consider right part here as shown in Fig. 8.

Assume tensile forces in each member.

by geometry $\tan\theta = \frac{1.25}{3} \qquad \therefore \theta = 22.62°$

$\tan\alpha = 3/3 \qquad \therefore \alpha = 45°$

Step 4 : Conditions of equilibrium $\Sigma M = 0$

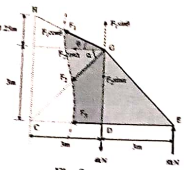


Fig. 8

1) $\Sigma M_G = 0;$ [As two forces F_1 and F_2 are passing through point G, we can find F_3 easily]

$(-F_3 \times 3) + (8 \times 3) = 0$

$\therefore F_3 = 8\ kN$

2) $\Sigma M_C = 0;\quad (-4\times3) + (8\times6) + (F_1 \cos\theta \times 4.25) = 0$ [Resolving F_1 at point H]

$\therefore F_1 = -9.176\ kN$

3) $\Sigma M_D = 0;\quad (8\times3) + (F_2 \cos\alpha \times 3) + (F_1\cos\theta \times 3) = 0$ [Resolving F_1 at G and F_2 at G]

$\therefore 24 + F_2 \cos 45° \times 3 - 9.176 \cos 22.62° \times 3 = 0$

$\therefore F_2 = 0.664\ kN$

Step 5 : Force Table :

Sr. No.	Member	Magnitude (kN)	Nature
1	GH	9.176	C
2	CG	0.664	T
3	CD	8	T

□□□

Chapter 5 : Centroid and Centre of Gravity

Q. 1 Determine centroid of the shaded area shown in Fig. 1. **May 2014**

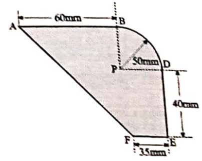


Fig. 1

Ans. :

Step 1 : Selecting co-ordinate axis and divide the area into parts :

Here select x-axis and y-axis as shown in Fig. 2 and divide the given Fig.2 (area) into five parts.

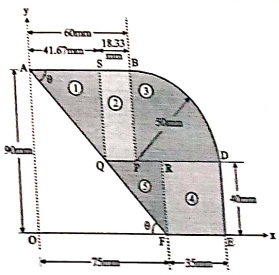


Fig. 2

Step 2 : Area calculations :

$$A_1 = \frac{1}{2} \times AS \times SQ = \frac{1}{2} \times 41.67 \times 50 = 1041.75\ mm^2$$

$$A_2 = 18.33 \times 50 = 916.5\ mm^2$$

$$A_3 = \frac{\pi r^2}{4} = \frac{\pi (50)^2}{4} = 1963.5 \text{ mm}^2$$

$$A_4 = 40 \times 35 = 1400 \text{ mm}^2$$

$$A_5 = \frac{1}{2} \times QR \times RF = \frac{1}{2} \times 33.33 \times 40 = 666.66 \text{ mm}^2$$

Also, $PR = 50 - 35 = 15$ mm

And $QP = SB = 18.33$ mm

Geometry :

From Δ AOF $\equiv$ Δ ASQ

$$\tan\theta = \frac{90}{75} = \frac{50}{AS}$$

$\therefore$ AS = 41.67 mm

$\therefore$ SB = 60 − 41.67 = 18.33 mm

$\therefore$ QR = 18.33 + 15 = 33.33 mm

Step 3 : Calculation of centroidal distances :

$$x_1 = \frac{2}{3} \times AS = \frac{2}{3} \times 41.67 = 27.78 \text{ mm}$$

$$x_2 = 41.67 + \frac{18.33}{2} = 50.835 \text{ mm}$$

$$x_3 = 60 + \frac{4r}{3\pi} = 60 + \frac{4(50)}{3\pi} = 81.2 \text{ mm}$$

$$x_4 = 75 + 35/2 = 92.5 \text{ mm}$$

$$x_5 = 41.67 + \left(\frac{2}{3} \times QR\right) = 41.67 + \left(\frac{2}{3} \times 33.33\right) = 63.89 \text{ mm}$$

OR $x_5 = 75 - \frac{1}{3} \times QR = 63.89$ mm

$$y_1 = 40 + \frac{2}{3} \times QS = 40 + \left(\frac{2}{3} \times 50\right) = 73.33 \text{ mm}$$

$$y_2 = 40 + 50/2 = 65 \text{ mm}$$

$$y_3 = 40 + \frac{4r}{3\pi} = 40 + \frac{4(50)}{3\pi} = 61.2 \text{ mm}$$

$$y_4 = \frac{40}{2} = 20 \text{ mm}$$

$$y_5 = \frac{2}{3} \times 40 = 26.67 \text{ mm}$$

Step 4 : Calculation of $\bar{x}$ and $\bar{y}$:

$$\bar{x} = \frac{(A_1x_1) + (A_2x_2) + (A_3x_3) + (A_4x_4) + (A_5x_5)}{A_1 + A_2 + A_3 + A_4 + A_5} = 67.97 \text{ mm}$$

$$\bar{y} = \frac{(A_1y_1) + (A_2y_2) + (A_3y_3) + (A_4y_4) + (A_5y_5)}{A_1 + A_2 + A_3 + A_4 + A_5} = 50.415 \text{ mm.}$$

$\therefore$ **Co-ordinates of centroid C (67.97, 50.415)** ...Ans.

Q. 2 Find centroid of the shaded area. **May 2013**

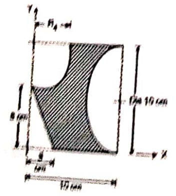


Fig. 3

Ans. :

Step 1 : Divide the area into four parts :

Step 2 : Area calculations :

A_1 (square) $= 10 \times 10 = 100 \text{ cm}^2$

A_2 (Qt. circle) $= \frac{\pi r^2}{4} = \frac{\pi (4)^2}{4} = 12.57 \text{ cm}^2$

A_3 (Triangle) $= \frac{1}{2} \times 3 \times 6 = 9 \text{cm}^2$

A_4 (semi. circle) $= \frac{\pi r^2}{2} = \frac{\pi (5)^2}{2} = 39.27 \text{ cm}^2$

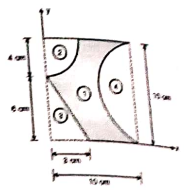


Fig. 4

Step 3 : Calculation of censorial distances :

$x_1 = 5$cm

$x_2 = \frac{4r}{3\pi} = \frac{4(4)}{3\pi} = 1.696$ cm

$x_3 = \frac{1}{3} \times 3 = 1$ cm

$x_4 = 10 - \frac{4r}{3\pi} = 10 \frac{4(5)}{3\pi} = 7.88$ cm

$y_1 = 5$cm

$y_2 = 10 - \frac{4r}{3\pi} = 10 - \frac{4(4)}{3\pi} = 8.3$ cm

$y_3 = \frac{1}{3} \times 6 = 2$ cm

$y_4 = 5$ cm

Step 4 : Calculation of $\bar{x}$ and $\bar{y}$ [co-ordinates of centroid] :

$$\bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4}{A_1 - A_2 - A_3 - A_4} = 4.09 \text{ cm}$$

$$\bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4}{A_1 - A_2 - A_3 - A_4} = 4.63 \text{ cm}$$

$\therefore$ **co-ordinates of centroid C(4.09, 4.63)**

Q. 3 Locate the centroid of the shaded area. **Dec. 2013**

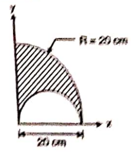


Fig. 5

Ans. : Step 1 : Divide the area into two parts as shown in Fig. 6 :

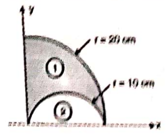


Fig. 6

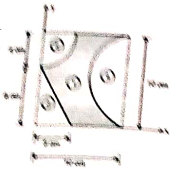


Fig. 4

Step 2 : Area Calculation :

$$\underset{\text{(Q. circle)}}{A_1} = \frac{\pi r^2}{4} = \frac{\pi(20)^2}{4} = 314.15\ cm^2$$

$$\underset{\text{(Semi circle)}}{A_2} = \frac{\pi r^2}{2} = \frac{\pi(10)^2}{2} = 157.08\ cm^2$$

Step 3 : Calculation of centroidal distance :

$$x_1 = \frac{4r}{3\pi} = \frac{4(20)}{3\pi} = 8.48\ cm \qquad y_1 = \frac{4r}{3\pi} = 8.48\ cm$$

$$x_2 = 10\ cm \qquad y_2 = \frac{4r}{3\pi} = \frac{4(10)}{3\pi} = 4.24\ cm$$

Step 4 : Calculation of $\bar{x}$ and $\bar{y}$ [co-ordinates of centroid] :

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} \qquad \bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$\bar{x} = 6.96\ cm \qquad \bar{y} = 12.72\ cm$$

$\therefore$ **Co-ordinates of centroid c (6.96, 12.72)**

Q. 4 Find centroid of shaded area. **Dec. 2014**

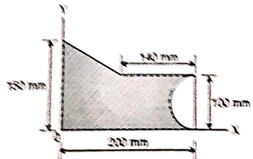


Fig. 7

Ans. :

Step 1 : Divide the area into three parts as shown :

Step 2 : Area Calculations :

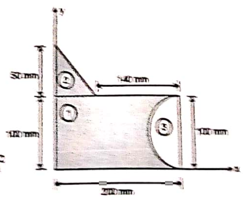


Fig. 8

$$\underset{\text{(Rectangle)}}{A_1} = 200 \times 100 = 20{,}000\ mm^2$$

$$\underset{\text{(Triangle)}}{A_2} = \frac{1}{2} \times 60 \times 50 = 1500\ mm^2$$

$$\underset{\text{(Semi Circle)}}{A_3} = \frac{\pi r^2}{2} = \frac{\pi 50^2}{2} = 3927\ mm^2$$

Step 3 : Calculation of centroidal distances :

$x_1 = 100\ mm$	$y_1 = 50\ mm$
$x_2 = \frac{1}{3} \times 60 = 20\ mm$	$y_2 = 100 + \left(\frac{1}{3} \times 50\right) = 116.67\ mm$
$x_3 = 200 - \frac{4r}{3\pi} = 178.8\ mm$	$y_3 = 50\ mm$

Step 4 : Calculation of $\bar{x}$ and $\bar{y}$ (co-ordinate of centroid) :

$\bar{x} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$	$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$
$\bar{x} = 75.56\ mm$	$\bar{y} = 55.69\ mm$

$\therefore$ **Co-ordinates of centroid C (75.56, 55.69)**

Q. 5 Locate the centroid of the shaded portion w.r.to ox and oy axes (Fig. 9). **May 2015**

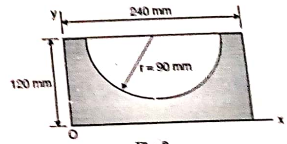


Fig. 9

Ans. :

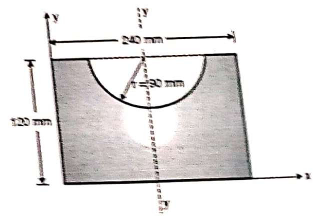


Fig. 10

Step 1 : Divide area into two parts as shown :

Also Area is symmetrical about y – y axis $\quad \therefore \bar{x} = 120\ mm$

Step 2 : Area calculations :

$$\underset{\text{(Rectangle)}}{A_1} = 240 \times 120 = 28800\ mm^2$$

$$\underset{\text{(Semi Circle)}}{A_2} = \frac{\pi r^2}{2} = \frac{\pi 90^2}{2} = 12723.45\ mm^2$$

Step 3 : Calculation of centroidal distances :

$y_1 = 60\text{ mm}$; $y_2 = 120 - \left(\frac{4r}{3\pi}\right) = 120 - \frac{4(90)}{3\pi} = 81.84\text{ mm}$

Step 4 : Calculation of $\bar{y}$

$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$ $\therefore$ $\bar{y} = 42.715\text{ mm}$

$\therefore$ **Centroid C (120, 42.715) mm**

Q. 6 Find centroid of shaded area with reference to X and Y axes. **May 2016**

Ans. :

Step 1 : Divide the area into three parts as shown

Step 2 : Area Calculations :

A_1 (square) $= (20 \times 20)$

$= 400\text{ cm}^2$

A_2 (semi-circle) $= \frac{\pi r^2}{2} = \frac{\pi(10)^2}{2}$

$= 157.08\text{ cm}^2$

A_3 (Quarter-circle) $= \frac{\pi r^2}{4} = \frac{\pi(20)^2}{4} = 314.15\text{ cm}^2$

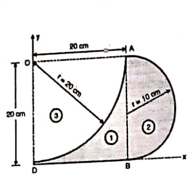


Fig. 11

Step 3 : Calculation of centroidal distance

$x_1 = 10\text{ cm}$	$y_1 = 10\text{ cm}$
$x_2 = 20 + \frac{4r}{3\pi} = 24.24\text{ cm}$	$y_2 = 10\text{ cm}$
$x_3 = \frac{4(20)}{3\pi}$	$y_3 = r - \frac{4r}{3\pi}$
$= 8.48\text{ cm}$	$= 20 - \frac{4(20)}{3\pi} = 11.52\text{ cm}$

Step 4 : Calculation of $\bar{x}$ and $\bar{y}$

$\bar{x} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$	$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$
$\bar{x} = 21.17\text{ cm}$	$\bar{y} = 8.034\text{cm}$

$\therefore$ **Co-ordinates of centroid C(21.77,8.034)**

Q. 7 Find the centroid of shaded area of the semicircle of diametesr 100 cm **May 2011**

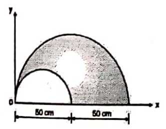


Fig. 12

Ans. . :

Step 1 : Area calculations :

$A_1 = \frac{\pi r^2}{2} = \frac{\pi(50)^2}{2} = 3927\text{ cm}^2$

$A_2 = \frac{\pi r^2}{2} = \frac{\pi(25)^2}{2} = 981.75\text{ cm}^2$

Step 2 : Centroidal distance :

$x_1 = 50\text{ cm}$	$y_1 = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2\text{ cm}$
$x_2 = 25\text{ cm}$	$y_2 = \frac{4r}{3\pi} = \frac{4(25)}{3\pi} = 10.6\text{ cm}$

Step 3 : Co-ordinates of centroid 'C' :

$\therefore \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$	$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$
$\bar{x} = 58.33\text{ cm}$	$\bar{y} = 24.73\text{ cm.}$

$\therefore$ **Centroid C (58.33, 24.73)** ...Ans.

Q. 8 Determine the Centroid of the shaded area. All dimensions are in mm. **Dec. 2015**

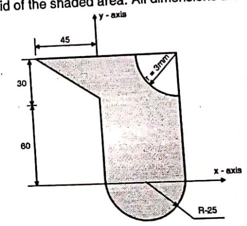


Fig. 13

Ans. :

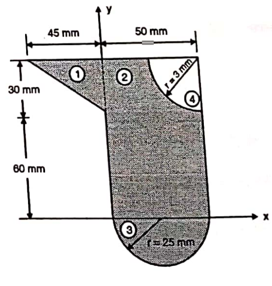


Fig. 14

Area calculation :

A_1 (Triangle) $= \frac{1}{2} \times 45 \times$

A_3 (semi circle) $= \frac{\pi r^2}{2} = \frac{\pi(}{}$

Calculation of centroidal

$x_1 =$

$x_2 =$

$x_3 =$

$x_4 =$

co-ordinates of ce

$\bar{x} = \frac{A_1 x}{}$

$\bar{x} = 20.$

$\therefore$ **Centroid C**

Area calculation :

$$\underset{\text{(Triangle)}}{A_1} = \frac{1}{2} \times 45 \times 30 = 675 \text{ mm}^2 \qquad \underset{\text{(Rectangle)}}{A_2} = 50 \times 90 = 4500 \text{ mm}^2$$

$$\underset{\text{(semi circle)}}{A_3} = \frac{\pi r^2}{2} = \frac{\pi (25)^2}{2} = 981.75 \text{ mm}^2 \qquad \underset{\text{(Qt circle)}}{A_4} = \frac{\pi r^2}{4} = \frac{\pi (3)^2}{4} = 7.068 \text{ mm}^2$$

Calculation of centroidal distances

$$x_1 = -\frac{1}{3} \times 45 = -15 \text{mm} \qquad y_1 = 90 - \left(\frac{1}{3} \times 30\right) = 80 \text{ mm}$$

$$x_2 = 25 \text{ mm} \qquad y_2 = 45 \text{ mm}$$

$$x_3 = 25 \text{ mm} \qquad y_3 = -\frac{4(25)}{3\pi} = 10.6 \text{ mm}$$

$$x_4 = 50 - \frac{4r}{3\pi} = 48.73 \text{ mm} \qquad y_4 = 90 - \frac{4r}{3\pi} = 88.73 \text{ mm}$$

co-ordinates of centroid $\left(\bar{x} \text{ and } \bar{y}\right)$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 - A_4} \qquad \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - A_4 y_4}{A_1 + A_2 + A_3 - A_4}$$

$$\bar{x} = 20.58 \text{ mm} \qquad \bar{y} = 43.3 \text{ mm}$$

$\therefore$ **Centroid C (20.58, 43.3) mm**

□□□

Chapter 6 : Space Forces

Q. 1 A rectangular parallelepiped carries three forces shown in Fig. 1. Reduce the force system to a resultant force applied at the origin and a moment around origin. **Dec. 2014**

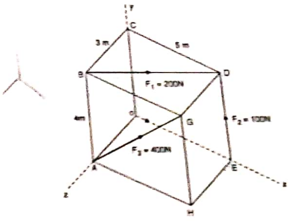


Fig. 1

Ans. :

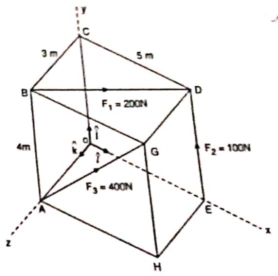


Fig. 2

Force vectors

(1) $$\overline{F_1} = F_1 \cdot \overline{e}_{B \to D} = 200 \left[\frac{5\hat{i} + (0)\hat{j} - 3\hat{k}}{\sqrt{(5)^2 + (3)^2}} \right]_{B \to G \to D}$$

$$\overline{F_1} = 171.5\,\hat{i} - 102.9\,\hat{k}$$

(2) $$\overline{F_2} = F_2 \cdot \overline{e}_{E \to D} = 100\,\hat{j}$$

(3) $\bar{F}_3 = F_3 \cdot \bar{e}_{A\to G} = 400\left[\frac{5\hat{i}+4\hat{j}+0\hat{k}}{\sqrt{(5)^2+(4)^2}}\right]$

$\therefore \bar{F}_3 = 312.35\hat{i} + 249.87\hat{j} + 0\hat{k}$

Resultant $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

$\bar{R} = 483.85\hat{i} + 349.87\hat{j} - 102.9\hat{k}$ ...Ans.

Moment about origin

$$M_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 171.5 & 0 & -102.9 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ 0 & 100 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 312.35 & 249.87 & 0 \end{vmatrix}$$

$\therefore \hat{i}[-338.01] - \hat{j}[-1451.55] + \hat{k}[-186]$

$\bar{M}_0 = -338.01\hat{i} + 1451.55\hat{j} - 186\hat{k}$ ...Ans.

Q. 2 A force of 100 N acts at a point P(− 2, 3, 5) m has its line of action passing through Q(10, 3, 4) m. calculate moment of this force about origin (0,0,0). **Dec. 2014**

Ans. :

Force vector $\bar{F}_{PQ} = 100\left[\frac{10-(-2)\hat{i}+(3-3)\hat{j}+(4-5)\hat{k}}{\sqrt{(12)^2+(0)^2+(-1)^2}}\right] = 99.65\hat{i} + 0\hat{j} - 8.304\hat{k}$

$\therefore$ Moment about origin

$$\bar{M}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 3 & 4 \\ 99.65 & 0 & -8.304 \end{vmatrix} = -24.91\hat{i} + 481.7\hat{j} - 298.98\hat{k} \text{ N.m}$$ ...Ans.

Q. 3 A force $\bar{F} = 80i + 50j - 60k$ passes through a point A (6, 2, 6). Compute its moment about point B (8, 1, 4). **May 2015**

Ans. : Moment of force about point B.

$$\bar{M}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (6-8) & (2-1) & (6-4) \\ 80 & 50 & -40 \end{vmatrix} = -150\hat{i} + 60\hat{j} - 180\hat{k}$$ ...Ans.

Q. 4 *A force of 10 kN acts at a point P (2, 3, 5) m and gas its line of action passing through Q (10, − 3, 4) m. Calculate moment of this force about a point S (1, − 10, 3) m.* **Dec. 2013**

Ans. :

Force vector $\bar{F}_{PQ} = F_{PQ} \cdot \bar{e}_{P\to Q} = 10\left[\frac{(10-2)\hat{i}+(-3-3)\hat{j}+(4-5)\hat{k}}{\sqrt{(10-2)^2+(-3-3)^2+(4-5)^2}}\right]$

$= 10\left[\frac{8\hat{i}-6\hat{j}-\hat{k}}{\sqrt{101}}\right]$

Moment of force about point s(1, − 10, 3)

$$\bar{M}_S = \frac{10}{\sqrt{101}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (2-1) & (3+10) & (5-3) \\ 8 & -6 & -1 \end{vmatrix} = \frac{10}{\sqrt{101}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 13 & 2 \\ 8 & -6 & -1 \end{vmatrix}$$

$\bar{M}_S = -0.995\hat{i} + 16.91\hat{j} - 109.45\hat{k}$ (kN.m) ...Ans.

Q. 5 The lines of action of three forces concurrent at origin 'O' pass respectively through points A(−1, 2, 4), B(3, 0, −3) and C(2, 2, 4) m. The magnitudes of forces are 40N, 10N and 30N respectively. Determine the magnitude and direction of their resultant. **May 2014**

Ans. :

Force vectors

$\bar{F}_A = F_A \cdot \bar{e}_{0\to A} = 40\left[\frac{-1\hat{i}+2\hat{j}+4\hat{k}}{\sqrt{(1)^2+(2)^2+(4)^2}}\right] = -8.728\hat{i} + 17.45\hat{j} + 34.91\hat{k}$

$\bar{F}_B = F_B \cdot \bar{e}_{0\to B} = 10\left[\frac{3\hat{i}+0\hat{j}-3\hat{k}}{\sqrt{(3)^2+0+(3)^2}}\right] = 7.07\hat{i} + 0\hat{j} - 7.07\hat{k}$

$\bar{F}_C = F_C \cdot \bar{e}_{0\to C} = 30\left[\frac{2\hat{i}+2\hat{j}+4\hat{k}}{\sqrt{(2)^2+(2)^2+(4)^2}}\right] = 12.247\hat{i} - 12.247\hat{j} + 24.49\hat{k}$

$\therefore \bar{R} = \bar{F}_A + \bar{F}_B + \bar{F}_C$ $\quad\therefore \bar{R} = 10.589\hat{i} + 5.203\hat{j} + 52.33\hat{k}$

$\therefore$ Magnitude $|\bar{R}| = \sqrt{(10.589)^2+(5.203)^2+(52.33)^2}$

$\therefore |\bar{R}| = 53.64$ N

Direction cosines

$\cos\theta_x = \pm\frac{R_x}{R} = \frac{10.589}{53.64} \Rightarrow \theta_x = 78.61°$; $\quad \cos\theta_y = \pm\frac{R_y}{R} = \frac{5.203}{53.64} \Rightarrow \theta_y = 84.43°$

$\cos\theta_z = \pm\frac{R_z}{R} = \frac{52.33}{53.64} \Rightarrow \theta_z = 12.69°$

Q. 6 A pole is held in place b in Fig. 3. Determine the

Ans. :

Force vectors :

Q. 7 Force 5 along B about

Ans. :

Force vecto

Q. 6 A pole is held in place by three cables. If the force of each cable acting on the pole is as shown in Fig. 3. Determine the resultant. **Dec. 2013**

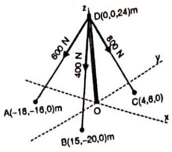


Fig. 3

Ans. :

Force vectors :

$$\overline{F}_{DA} = F_{DA} \cdot \overline{e}_{D\to A} = 600\left[\frac{(-18-0)\hat{i}+(-16-0)\hat{j}+(0-24)\hat{k}}{\sqrt{(18)^2+(16)^2+(24)^2}}\right]$$

$$\therefore \overline{F}_{DA} = -317.65\,\hat{i} - 282.35\,\hat{j} - 423.53\,\hat{k}$$

$$\overline{F}_{DB} = F_{DB} \cdot \overline{e}_{D\to B} = 400\left[\frac{(15-0)\hat{i}+(-20-0)\hat{j}+(0-24)\hat{k}}{\sqrt{(15)^2+(20)^2+(24)^2}}\right]$$

$$\therefore \overline{F}_{DB} = 173.13\,\hat{i} - 230.84\,\hat{j} - 277.01\,\hat{k}$$

$$\overline{F}_{DC} = F_{DC} \cdot \overline{e}_{D\to C} = 800\left[\frac{(4-0)\hat{i}+(6-0)\hat{j}+(0-24)\hat{k}}{\sqrt{(4)^2+(6)^2+(24)^2}}\right]$$

$$\overline{F}_{DC} = 127.69\,\hat{i} - 191.54\,\hat{j} - 766.16\,\hat{k}$$

$$\therefore \overline{R} = \overline{F}_{DA} + \overline{F}_{DB} + \overline{F}_{DC}$$

$$\therefore \overline{R} = -16.83\,\hat{i} - 321.65\,\hat{j} - 1466.7\,\hat{k}$$

$$\therefore \text{Magnitude } |\overline{R}| = \sqrt{(16.83)^2+(321.65)^2+(1466.7)^2}$$

$$\therefore |\overline{R}| = 1501.65\text{ N}$$ ...Ans.

Q. 7 Force 5kN is acting along AB where A(0,0, – 1)m and B(5, -2, -4)m. Another force 8kN is acting along BC where C(3, 3, 4)m. Find resultant of two forces and find moment of resultant force about a point D(0, 3, – 2) m. **Dec. 2015**

Ans. :

Force vectors

$$\overline{F}_1 = F_1\,\overline{e}_{A\to B} = 5\left[\frac{(5-0)\hat{i}+(-2-0)\hat{j}+(-4+1)\hat{k}}{\sqrt{(5)^2+(-2)^2+(-3)^2}}\right]$$

$$\overline{F}_1 = 4.056\,\hat{i} - 1.62\,\hat{j} - 2.43\,\hat{k} \qquad \text{...(i)}$$

$$\overline{F}_2 = F_2\,\overline{e}_{B\to C} = 8\left[\frac{(3-5)\hat{i}+(3+2)\hat{j}+(4+4)\hat{k}}{\sqrt{(2)^2+(5)^2+(8)^2}}\right]$$

$$\overline{F}_2 = -1.66\,\hat{i} + 4.15\,\hat{j} + 6.64\,\hat{k}$$

$$\therefore \overline{R} = \overline{F}_1 + \overline{F}_2$$

$$\overline{R} = 2.396\,\hat{i} + 2.53\,\hat{j} + 4.21\,\hat{k}$$

Moment of $\overline{R}$ about point D(0, 3, – 2)

$$\overline{M}_D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -5 & -2 \\ 2.396 & 2.53 & 4.21 \end{vmatrix}$$

$$\overline{M}_D = -16\,\hat{i} - 25.84\,\hat{j} + 24.63\,\hat{k}$$

Q. 8 A force acts at the origin in a direction defined by the angles $\theta_y = 65°$ and $\theta_z = 40°$ Knowing that the x-component of the force is – 750N, determine (i) the other components (ii) magnitude of the force (ii) the value of θ_x. **Dec. 2015**

Ans. :

$\theta_y = 65°$ $\qquad \theta_z = 40°$ $\qquad F_x = -750$ N

using property of DCS

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\therefore \cos^2\theta_x = 1 - (\cos 65°)^2 + (\cos 40°)^2$$

$$\cos^2\theta_x = 0.234 \qquad \therefore \cos\theta_x = \pm 0.484$$

As $F_x < 0$ $\qquad \cos\theta_x = -0.484 \qquad \therefore \theta_x = 118.95°$

Now, $\qquad F_x = F\cos\theta_x \qquad -750 = F(-0.484)$

$$\therefore F = 1549.59\text{ N}$$

Now other components

$$F_y = F\cos\theta_y \quad \text{and} \quad F_Z = F\cos\theta_z$$

$$= 1549.59\cos(65°) \qquad = 1549.59\cos(40°)$$

$$F_y = 654.88\text{ N} \qquad F_Z = 1187.05\text{ N}$$

□□□

Chapter 7 : Virtual Work

Q. 1 The parallelogram linkage shown in Fig. 1 carries forces P, F and Q applied at the mid points of the links. For a given value of θ, determine force Q in terms of P and F to maintain equilibrium of the linkage. (Refer Fig. 1)

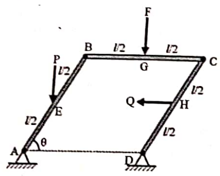


Fig. 1

Ans. : Consider fixed point 0 as origin.

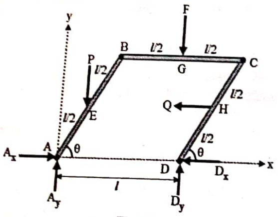


Fig. 2

Using

Active Force	Co-ordinate of point of application	Virtual displacement
$-P$	$y_E = \frac{l}{2}\sin\theta$	$\frac{l}{2}\cos\theta \cdot d\theta$
$-F$	$y_G = l\sin\theta$	$l\cos\theta \cdot d\theta$
$-Q$	$x_H = l + \frac{l}{2}\cos\theta$	$-\frac{l}{2}\sin\theta \cdot d\theta$

Principle of virtual work

$$\Sigma \text{ virtual work } = 0$$

$$-P\left(\frac{l}{2}\cos\theta \cdot d\theta\right) - F\,(l\cos\theta \cdot d\theta) - Q\left(-\frac{l}{2}\sin\theta\, d\theta\right) = 0$$

$$\frac{-P}{2}\cos\theta - F\cos\theta + \frac{Q}{2}\cdot\sin\theta = 0$$

$$\therefore \quad \frac{Q}{2}\sin\theta = (F + P/2)\cos\theta$$

$$\therefore \quad Q = (2F + P)\cot\theta$$

...Ans.

Q. 2 Find Force P that will support the cylinder of weight W. Neglect friction. Refer Fig. 3.

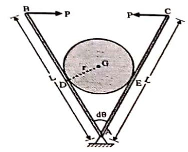


Fig. 3

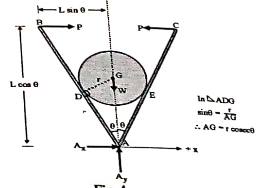


Fig. 4

Ans. :

Step 1 : F.B.D. of system shown in Fig. 4 showing forces.

Step 2 : Select co-ordinate axis at A

Step 3 :

The active forces are P acting at B, P acting at C, weight of cylinder (W) acting from G. No work is done by A_x, A_y and reactions at D and E.

Active Force	Co-ordinate of point of application	Virtual displacement
P	$x_B = -L\sin\theta$	$-L\cos\theta \cdot d\theta$
$-P$	$x_C = L\sin\theta$	$L\cos\theta \cdot d\theta$
$-W$	$y_G = r\,\text{cosec}\,\theta$	$-r\,\text{cosec}\,\theta \cdot \cot\theta \cdot d\theta$

$\therefore$ By principle of virtual work $\quad du = 0$

$$\therefore P(-L\cos\theta \cdot d\theta) - P(L\cos\theta \cdot d\theta) - W(-r\,\text{cosec}\,\theta \cdot \cot\theta \cdot d\theta) = 0$$

$$\therefore \quad 2PL\cos\theta = Wr \cdot \text{cosec}\,\theta \cdot \cot\theta\,; \qquad 2PL\cos\theta = Wr\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}$$

$$\therefore \quad 2PL = \frac{Wr}{\sin^2\theta} \qquad P = \frac{Wr}{2L\sin^2\theta}$$

...Ans.

Q. 3 For given frame find force P to maintain equilibrium. Take L = 0.6 m and θ = 60°. Refer Fig. 5.

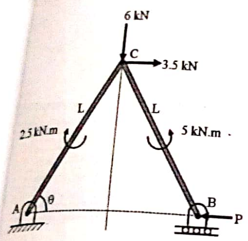


Fig. 5

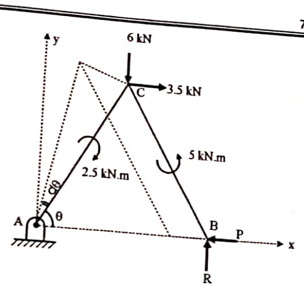


Fig. 6

Ans. :

Step 1 : Draw F.B.D. of frame as shown in Fig. 6 and select co-ordinate axis at A.

Step 2 : Now give virtual displacement to given frame (dθ).

The new position of frame is shown by dotted line.

Active force/couple	Co-ordinate of point of application	Virtual displacement
$-P$	$x_B = 2L\cos\theta$	$-2L\sin\theta\, d\theta$
$+3.5$	$x_C = L\cos\theta$	$-L\sin\theta\, d\theta$
-6	$y_C = L\sin\theta$	$L\cos\theta\, d\theta$
-2.5	–	$d\theta$
-5	–	$d\theta$

by principle of virtual work, $du = 0$

$$-P(-2L\sin\theta\, d\theta) + 3.5(-L\sin\theta\, d\theta) - 6(L\cos\theta\, d\theta) - 2.5\, d\theta - 5\, d\theta = 0$$

$$\therefore \quad 2PL\sin\theta - 3.5L\sin\theta - 6L\cos\theta = 7.5$$

substitute $L = 0.6\text{ m}, \theta = 60°$

we have, $2P(0.6)\sin 60° - 3.5(0.6)\sin 60° - 6(0.6)\cos 60° = 7.5$

$$\therefore \quad P = 10.7\text{ kN}$$

...Ans.

Chapter 8 : Kinematics of Particles

Q. 1 Acceleration of a particle moving along a straight line is represented by the relation $\alpha = 30 - 4.5x^2$ m/s². The starts with zero initial velocity at x = 0. Determine (a) the velocity when x = 3 m(b) the position when the velocity is again zero (c) the position when the velocity is maximum. **Dec. 2014**

Ans. :

$$a = 30 - 4.5x^2$$

$$\therefore \quad v\frac{dv}{dx} = 30 - 4.5x^2 \quad ...(1)$$

$$\therefore \quad \int v\,dv = \int (30 - 4.5x^2)\,dx$$

$$\therefore \quad \frac{v^2}{2} = 30x - \frac{4.5x^3}{3} + C$$

At $x = 0, v = 0$

$$\therefore \quad C = 0$$

$$\therefore \quad \frac{v^2}{2} = 30x - 1.5x^3 \quad ...(2)$$

a) When $x = 3$m,

$$\frac{v^2}{2} = 30(3) - 1.5(3)^3$$

$$\therefore \quad v = \pm 9.95\text{ m/s}$$

b) When $v = 0$,

$$0 = 30x - 1.5x^3 = x(30 - 1.5x^2)$$

$$\therefore \quad x = 0 \quad \text{or} \quad 30 - 1.5x^2 = 0$$

$$\therefore \quad x = \pm 4.47\text{ m}$$

c) For v_{max}, $a = 0$

$$\therefore \quad 30 - 4.5x^2 = 0$$

$$\therefore \quad x = \pm 2.582\text{ m}$$

Q. 2 Two cars start towards each other from stop X and stop Y at 1:36 PM, the first car reaches stop Y, travelling 8 km path at 1:44 PM. Second car reaches stop X at 1:46 PM. If they move at uniform velocity, determine their time of meeting and their distance from stop X. **May 2015**

Ans. :

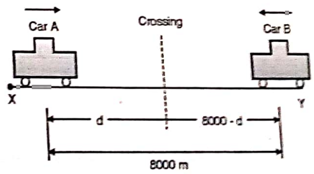


Fig. 1

For car A : (U. M) Distance = Velocity × time

$$8000 = V_A \times 8 \times 60$$

$$\therefore \quad V_A = 16.67\ \frac{m}{s} \rightarrow$$

For car B (U. M) Distance = Velocity × time

$$8000 = V_B \times 10 \times 60$$

$$\therefore \quad V_B = 13.33\ \frac{m}{s} \rightarrow$$

Let 'd' be the distance covered by car A to reach crossing point in time 't'

$\therefore$ **For A : (U. M)** $\quad d = V_A \times t \qquad \therefore \quad d = 16.67\,t$ ...(i)

For B (U. M) $\quad 8000 - d = 13.33\,t$ ...(ii)

From (i) and (ii) $\quad$ **$d = 4444.4$ m** $\qquad$ **$t = 266.67$ sec** **...Ans.**

Q. 3 Car A starts from rest and accelerates uniformly on a straight road. Another car B starts from the same place 5 seconds later with initial velocity zero and it accelerates uniformly at 5 m/sec^2. If both the cars overtake at 500 m from the starting place, find the acceleration of car A. **May 2016**

Ans. :

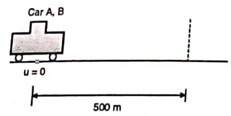


Fig. 2

For car A : $\quad u = 0$

(U.A.M) $\quad s = 500$ m

time $= t$ sec

For car A (UAM) $\quad \therefore s = ut + \frac{1}{2} a_A t^2$

$$500 = 0 + \frac{1}{2} a_A t^2 \quad ...(i)$$

For car B (U.A.M)

$u = 0 ; s = 500$ m

$a_B = 5$ m/s^2

time $= (t - 5)$ sec

$$\therefore s = ut + \frac{1}{2} a_B t^2$$

$$500 = 0 + \frac{1}{2}(5)(t-5)^2$$

$$200 = (t-5)^2$$

$$\therefore 14.14 = t - 5$$

$$\therefore \mathbf{t = 19.14\ sec.}$$

Substitute in (1)

$$500 = \frac{1}{2} a_A (19.14)^2$$

$$\therefore \mathbf{a_A = 2.73\ m/s^2} \quad \textbf{...Ans.}$$

Q. 4 During a test the car moves in a straight line such that its velocity is defined by v = 0.3 (9t^2 + 2t) m/s where 't' is in seconds. Determine the position and acceleration when t = 3 sec. Take at t = 0, x = 0 **May 2012**

Ans. :

Given :

$$v = 0.3(9t^2 + 2t) \qquad \therefore \quad \mathbf{v = 2.7\,t^2 + 0.6\,t} \quad ...(i)$$

$$\text{Use } v = \frac{dx}{dt} \qquad \therefore \quad x = \int v\,dt = \int (2.7\,t^2 + 0.6\,t)\,dt$$

$$x = 0.9\,t^3 + 0.3\,t^2 + C_1$$

Now, at $t = 0$, $\quad x = 0 \qquad \therefore C_1 = 0$

$$\therefore \quad \mathbf{x = 0.9\,t^3 + 0.3\,t^2} \quad ...(ii)$$

From Equation (i) $\quad a = \frac{dv}{dt} \quad \Rightarrow \quad \mathbf{a = 5.4\,t + 0.6}$ ...(iii)

At $t = 3$ sec.

Position $x = 0.9(3)^3 + 0.3(3)^2 \qquad \therefore$ **$x = 27$ m** **...Ans.**

acceleration $a = 5.4(3) + 0.6 =$ **16.8 m/s^2** **...Ans.**

Q. 5 The acceleration of the particle is defined by the relation a = 25 – 3x^2 mm/s^2. The particle starts with no initial velocity at the position x = 0. (a) Determine the velocity when x = 2 mm. (b) The position when velocity is again zero. (c) Position where the velocity is maximum and the corresponding maximum velocity. **Dec. 2010**

Ans. :

Given equation : $\quad \mathbf{a = 25 - 3x^2}$ ...(i)

Conditions : $t = 0, v = 0, x = 0$

$$\text{use} \quad a = v \cdot \frac{dv}{dx} \qquad \therefore v\,dv = (25 - 3x^2)\,dx$$

Integrating $\quad \int v\,dv = \int (25 - 3x^2)\,dx$

$$\frac{v^2}{2} = 25x - x^3 + C_1 \quad \text{but at } v = 0,\ x = 0,\ \text{so } C_1 = 0$$

$$\therefore \mathbf{v^2 = 50x - 2x^3} \quad ...(ii)$$

(a) When x = 2 mm

$$v^2 = 50(2) - (2)^3 \qquad \therefore \quad \mathbf{v = \pm 9.165\ mm/sec.} \quad \textbf{...Ans.}$$

(b) When v = 0

$$0 = 50x - 2x^3 \qquad \therefore 50 = 2x^2$$

$$\mathbf{x = \pm 5\ mm} \quad \textbf{...Ans.}$$

(c) When velocity is maximum $\quad a = 0$

$$\therefore \quad 25 - 3x^2 = 0 \qquad \therefore \quad \mathbf{x = 2.886\ mm} \quad \textbf{...Ans.}$$

And corresponding maximum velocity by Equation (ii)

$$v^2 = 50(2.886) - 2(2.886)^3 \qquad \therefore \quad \mathbf{v = 9.81\ mm/sec} \quad \textbf{...Ans.}$$

Q. 6 A small block rests on a turn table, 0.5m away from its centre. The turn table, starting from rest, is rotated in such a way that the block undergoes a constant tangential acceleration. Determine the angular velocity of the turn table at the instant when the block starts slipping. μ = 0.4. **Dec. 2015**

Ans. : When block starts slipping velocity is given by,

$$V = \sqrt{\mu_s\,g \cdot r} = \sqrt{0.4 \times 9.81 \times 0.5} = 1.4\ m/s$$

$$\therefore \text{Angular velocity of turn table } \omega = \frac{V}{r} = \frac{1.4}{0.5}$$

$$\therefore \mathbf{\omega = 2.8\ rad/sec}$$

Q. 7 The velocity of a particle travelling in a straight line is given by $v = 6t - 3t^2$ m/s. Where t is in seconds ? If s = 0 when t = 0, determine the particle's deceleration and position when t = 3s. How far has the particle travelled during the 3 second time interval and what is its average speed ? **Dec. 2010**

Ans. :

Velocity $v = 6t - 3t^2$ ...(i)

Given : Conditions : when $t = 0$, $s = 0$

Use, $v = \frac{ds}{dt} = 6t - 3t^2$

$\therefore \; s = \int (6t - 3t^2)\,dt = 3t^2 - t^3 + C_1$

When $t = 0$, $s = 0 \;\therefore C_1 = 0$

So, expression $s = 3t^2 - t^3$ ...(ii)

again $v = 6t - 3t^2$

$\therefore \; \frac{dv}{dt} = 6 - 6t$

$\therefore \; a = 6 - 6t$ ...(iii)

Now, at t = 3 sec,

$a = 6 - 6(3) = -12 \text{ m/s}^2$ ...Ans.(i)

And $s = 3(3)^2 - (3)^3$

$s = 0$ ...Ans.(ii)

Now, distance travelled in 3 sec,

Checking first $v = 0$ $\quad 6t - 3t^2 = 0 \quad \therefore t = 2$ sec.

$\therefore$ distance $d = |s_3 - s_2| + |s_2 - s_0|$ $\quad$ where, $s_2 = 3(2)^2 - (2)^3 = 12 - 8 = 4$ m

$= |0 - 4| + |4 - 0|$

$d = 8$ m ...Ans.(iii)

Average speed $= \frac{\text{distance travelled}}{\text{time period}} = \frac{8}{3} = 2.67$ m/sec. ...Ans.(iv)

Q. 8 For a particle in rectilinear motion $a = -0.05V^2 \text{ m/s}^2$ at v = 20 m/s, x = 0. Find x at v = 15 m/s and acceleration at 50 m. **May 2013**

Ans. :

$a = -0.05\,v^2$ ...(1)

$\therefore \; v\frac{dv}{dx} = -0.05\,v^2$ $\qquad \therefore \int \frac{dv}{v} = -0.05 \int dx$

$\therefore \; \log_e^{v} = -0.05\,x + C$

At $x = 0$, $v = 20$ $\quad \log_e^{20} = 0 + C$ $\qquad \therefore \log_e^{v} = -0.05\,x + \log_e^{20}$ ...(2)

At $v = 15$ m/s $\quad \log_e^{15} = -0.05\,x + \log_e^{20}$

on solving, $x = 5.754$ m

To find acceleration when x = 50 m

Using Equation (2), $\log_e^{v} = -0.05(50) + \log_e^{20} = 0.4957$

$v = e^{0.4957} = 1.642$ m/s

Using Equation (1) $\quad a = -0.05(1.642)^2 = -0.1348 \text{ m/s}^2$

Q. 9 A hot air balloon starts rising vertically up from the ground with an acceleration of 0.2m/s^2, 12 seconds later the man sitting inside the balloon releases a stone. Find the time taken by the stone to hit the ground. **Dec. 2015**

Ans. :

Height attained by the ballon in 12 sec.

$s = ut + \frac{1}{2}at^2 \qquad s = 0 + \frac{1}{2}(0.2)(12)^2 \qquad s = 14.4$ m

velocity of balloon after 12 sec, $V = u + at$

$V = 0 + 0.2(12) = 2.4$ m/s ↑

When stone is released from balloon its initial velocity is also 2.4 m/s ↑

For stone (M.U.G) $\quad s_y = u_y t - \frac{1}{2}gt^2 \qquad -14.4 = 2.4t - 4.905\,t^2$

$\therefore 4.905\,t^2 - 2.4t - 14.4 = 0 \qquad t = 1.975$ sec

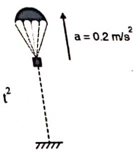


Fig. 3

Q. 10 In an Asian games event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete complete the race in 10.4 seconds, determine his initial acceleration and his maximum velocity. **May 2011**

Ans. :

Plotting v-t diagram for given motion.

Here, $s_1 = \frac{1}{2} \times t_1 \times v_{max}$

$4 = \frac{1}{2} \times t_1 \times v_{max}$

$\therefore \; v_{max} \cdot t_1 = 8$ ...(1)

Now, $s_2 = v_{max}(10.4 - t_1)$ ...(2)

and slope $= a = \frac{v_{max}}{t_1}$

$\therefore \; v_{max} = a\,t_1$ ...(3)

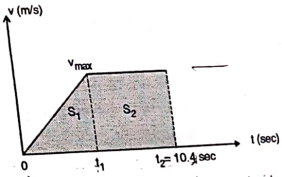


Fig. 4

Now, assuming 100 m race [Note : In question it is not specified]

$\therefore \; s_1 + s_2 = 100$ m $\therefore \; s_2 = 96$ m substitute in Equation (2)

$96 = v_{max}(10.4 - t_1)$

but $v_{max} = \frac{8}{t_1}$ ...from (1)

$\therefore \; 96 = \frac{8}{t_1}(10.4 - t_1) \qquad \therefore t_1 = 0.8$ sec. $\qquad \therefore v_{max} = \frac{8}{t_1} = 10$ m/s

$v_{max} = 10$ m/s ...Ans.

$\therefore$ by Equation (3) $a = \frac{v_{max}}{t_1} = \frac{10}{0.8} = 12.5\ m/s^2$

$$a = 12.5\ m/s^2 \qquad ...Ans.$$

Q. 11 The race car starts from rest and travels along a straight road until it reaches a speed of 42 m/s in 50 seconds as shown by v – t graph. Determine the distance travelled by race car in 50 seconds. Draw x – t and a – t graph. **May 2008**

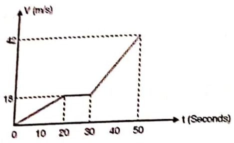


Fig. 5

Ans. : Form the (v-t) diagram.

Area $A_1 = \frac{1}{2} \times 10 \times 9 = 45$

$A_2 = \frac{1}{2}(9 + 18) \times 10 = 135$

$A_3 = 18 \times 10 = 180$

$A_4 = \frac{1}{2}(18 + 30) \times 10 = 240$

$A_5 = \frac{1}{2}(30 + 42) \times 10 = 360$

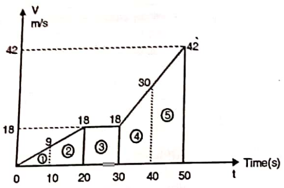


Fig. 6

$\therefore$ Distance in 50 (s) $= \Sigma A = (45 + 135 + 180 + 240 + 360) = 960$ m.

To draw (x-t) diagram.

(Fig. 7)

Distance in 10 (s) $= A_1 = 45$ m.

Distance in 20 (s) $= A_1 + A_2 = 180$ m.

Distance in 30 (s) $= 180 + 180 = 360$ m.

Distance in 40 (s) $= 360 + 240 = 600$ m.

Distance in 50 (s) $= 600 + 360 = 960$ m.

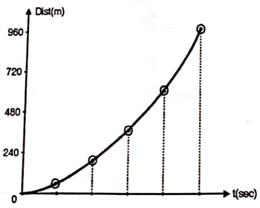


Fig. 7

To draw (a-t) diagram. (Fig. 8)

Acceleration from 0 to 20 (sec) $= \frac{18}{20} = 0.9\ m/s^2$ (slop of v-t diagram $(0 \rightarrow 20$ sec))

Acceleration from 20 to 30 (sec) = 0 as velocity is constant

Acceleration from 30 to 50 (sec) $= \frac{(42 - 18)}{20} = 1.2\ m/s^2$.

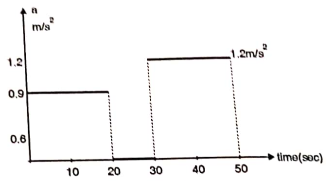


Fig. 8

Q. 12 The car starts form rest and travels along a straight track such that it accelerates at a constants rate for 10 seconds and then decelerates at a constant rate. Draw the v-t and s-t graphs and determine the time t' needed to stop the car. How far has the car travelled ? **May 2009**

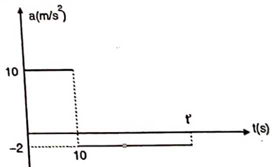


Fig. 9

Ans. : The car starts from rest and comes to rest

$\therefore$ **change in velocity = area of (a – t) diagram**

$\therefore v_t' - v_0 = $ Area

$\therefore 10 \times 10 - (t' - 10) \times 2 = 0 \qquad \therefore t' = 60$ (s)

$\therefore$ time required to stop the car is 60 (s).

(ii) Initial velocity = 0

In 10(s) velocity increases to (10 × 10) = 100 m/s from (a-t) diagram. Then in 50(s) it uniformity decreases to zero. v-t diagram is drawn as in Fig. 11.

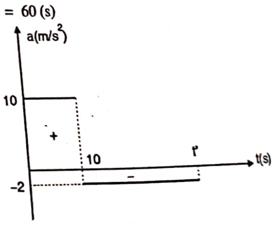


Fig. 10

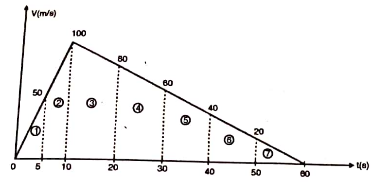


Fig. 11

Sr. No.	x Time (s)	Calculations	Area	y Cumulative distance
0	0	–	0	0
1	5	$A_1 = \frac{1}{2} \times 5 \times 50 =$	125	125 m
2	10	$A_2 = \frac{1}{2}(50 + 100) \times 5 =$	375	500 m
3	20	$A_3 = \frac{1}{2}(100 + 80) \times 10 =$	900	1400 m
4	30	$A_4 = \frac{1}{2}(80 + 60) \times 10 =$	700	2100 m
5	40	$A_5 = \frac{1}{2}(60 + 40) \times 10 =$	500	2600 m
6	50	$A_6 = \frac{1}{2}(40 + 20) \times 10 =$	300	2900 m
7	60	$A_7 = \frac{1}{2} \times 20 \times 10 =$	100	3000 m

From this table s-t or x-t diagram is constructed as in Fig. 12.

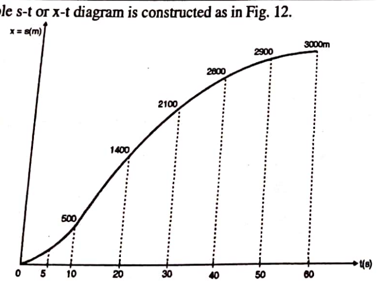


Fig. 12

Q. 13 Figure shows an plot of a_x versus time for a particle moving along x-axis. What is the speed and distance covered by the particle after 50 seconds ? **May 2010**

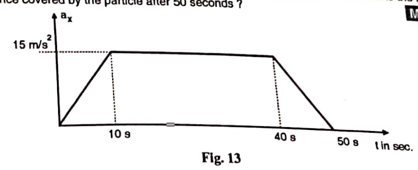


Fig. 13

Ans. :

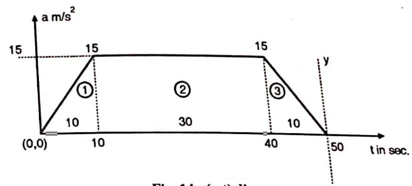


Fig. 14 : (a-t) diagram

velocity (speed) at t = 50 (s)

it is given by the area of (a – t) diagram from 0 → 50 sec

$$\therefore \; v = A_1 + A_2 + A_3$$

$$= \left(\frac{1}{2} \times 10 \times 15\right) + (30 \times 15) + \left(\frac{1}{2} \times 10 \times 15\right) = 75 + 450 + 75$$

$$= 600 \text{ m/s}$$

$\therefore$ velocity at t = 50 (s) **is 600 m/s.**

Distance moved in 50 (s) is given by the moment of area of (a – t) diagram about a vertical line at t = 50 (s) (see Fig. 14)

$$\therefore \text{ Distance } x = A_1x_1 + A_2x_2 + A_3x_3$$

$$\therefore \; x = \left[\frac{1}{2} \times 10 \times 15 \times \left(50 - \frac{2}{3} \times 10\right)\right]$$

$$+ \left[30 \times 15 \times \left(10 + \frac{30}{2}\right)\right] + \left[\frac{1}{2} \times 10 \times 15 \times \frac{2}{3} \times 10\right]$$

$$= \mathbf{15000 \text{ m}}$$

Q. 14 A car moves along a straight road such that its velocity is described by the graph shown in Fig. 15. For the first 10 seconds the velocity variation is parabolic and between 10 seconds to 30 seconds the variation is linear. Construct s-t and a-t graphs for the time period $0 \le t \le 30$s. **Dec. 2010**

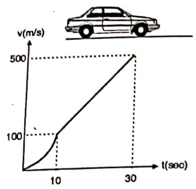


Fig. 15

Ans. :

From given v-t diagram

Area between two instants = change in position

From $0 \rightarrow 10$ sec,

$$x_{10} - x_0 = \frac{1}{3} \times \text{base} \times \text{height}$$

$$x_{10} - 0 = \frac{1}{3} \times 10 \times 100$$

$$\therefore \; x_{10} = \mathbf{333.33\ m}$$

From 10 to 30 sec,

$$x_{30} - x_{10} = \left[\frac{100 + 500}{2}\right] \times 20$$

$$x_{30} - 333.33 = 6000$$

$$\therefore \; x_{30} = \mathbf{6333.33\ m}$$

From given v-t diagram

Slope between two points = Acceleration

From $0 \rightarrow 10$ sec parabolic curve is given

If Equation parabola is $y = K x^2$ then at t = 10 sec.

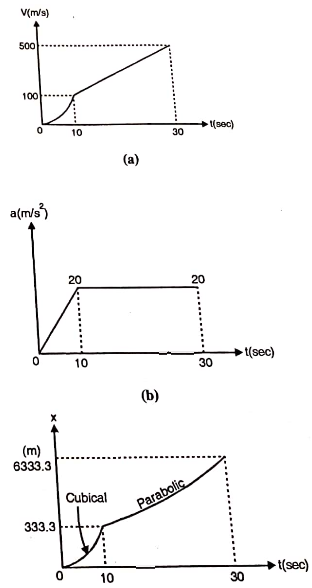
$$100 = K(10)^2 \qquad \therefore \; k = 1$$

$$\therefore \text{ Equation is } \quad y = x^2 \qquad \therefore \; \frac{dy}{dx} = 2x$$

$$\therefore \; \text{Slope} = 2x = 2(10) = 20$$

Acceleration at t = 10 sec is 20 m/s^2

$$\text{From 10 to 30 sec,} \quad \text{Slope} = \frac{400}{20} = 20\ m/s^2$$



(a)

(b)

(c)

Fig. 16

Q. 15 Velocity-time graph for a particle moving along a straight line is given below. Draw displacement-time and acceleration-time graphs. Also find the maximum displacement of the particle. **May 2012**

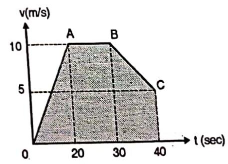


Fig. 17

Ans. :

From v-t diagram,

Slope of v-t diagram =

$\therefore$ From $0 \rightarrow 20$ sec, a =

From $20 \rightarrow 30$ sec, a =

From $30 \rightarrow 40$ sec, a

From a-t diagram

By moment equation,

$x_t = x_0 + v_0 t + M_t$

$x_{20} = 0 + 0 + (0.5 \times$

$x_{20} = \mathbf{100\ m}$

$x_{30} = 0 + 0 + (0.5$

$x_{30} = \mathbf{200\ m}$

$x_{40} = 0 + 0 + [(0$

$x_{40} = \mathbf{275\ m}$

$\therefore$ **Maximum displ**

Q. 16 Fig. 20 shows ac displacement – t of 5 m/sec. from

Ans. : Area under

Now,

From momen

Ans. :

From v-t diagram,

Slope of v-t diagram = acceleration

$\therefore$ From 0 → 20 sec, a = $\frac{10}{20} = 0.5\ m/s^2$

From 20 → 30 sec, a = 0 ($\because$ v constant)

From 30 → 40 sec, a = $\frac{-5}{10} = -0.5\ m/s^2$

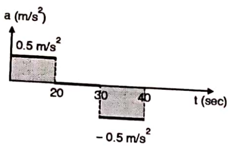


Fig. 18

From a-t diagram

By moment equation,

$$x_t = x_0 + v_0 t + M_t$$

$$x_{20} = 0 + 0 + (0.5 \times 20 \times 10)$$

$$x_{20} = 100\ m$$

$$x_{30} = 0 + 0 + (0.5 \times 20 \times 20)$$

$$x_{30} = 200\ m$$

$$x_{40} = 0 + 0 + [(0.5 \times 20 \times 30) - (0.5 \times 10 \times 5)]$$

$$x_{40} = 275\ m$$

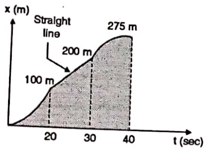


Fig. 19

$\therefore$ **Maximum displacement =** $x_{max} = 275\ m$ **...Ans.**

Q. 16 Fig. 20 shows acceleration – time diagram of rectilinear motion. Construct velocity – time and displacement – time diagrams for the motion assuming that the motion starts with initial velocity of 5 m/sec. from starting point. **Dec. 2011**

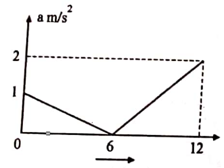


Fig. 20

Ans. : Area under a – t diagram = change in velocity

$$\therefore\ v_6 - v_0 = \frac{1}{2} \times 6 \times 1$$

$$\therefore\ v_6 - v_5 = 3$$

$$\therefore\ v_6 = 8\ m/s$$

Now, $v_{12} - v_6 = \frac{1}{2} \times 6 \times 2$ $\quad\therefore\ v_{12} - 8 = 6$ $\quad\therefore\ v_{12} = 14\ m/s$

From moment equation, $x_6 = x_0 + v_0 t + M_t$

$$= 0 + 5\,(6) + \left[\frac{1}{2} \times 6 \times 1 \times \frac{2}{3} \times 6\right] = 42\ m \qquad \therefore\ x_6 = 42\ m$$

$$x_{12} = 0 + 5\,(12) + \left[\frac{1}{2} \times 6 \times 1 \times \left(6 + \frac{2}{3} \times 6\right) + \left(\frac{1}{2} \times 6 \times 2 \times \frac{1}{3} \times 6\right)\right]$$

$$\therefore\ x_{12} = 102\ m$$

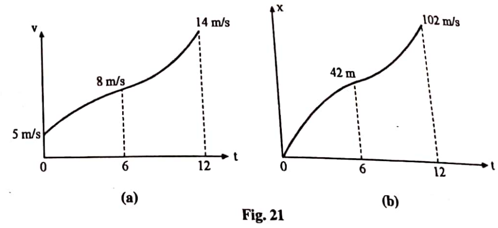


Fig. 21

Q. 17 Fig. 22 shows the v-t diagram for the motion of a train as it moves from station A to station B. Draw a-t graph and find the average speed of the train and the distance between the stations. **May 2015**

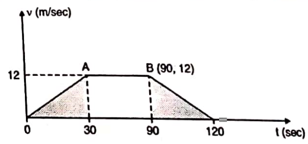


Fig. 22

Ans. :

From v – t diagram :

From 0 → 30 sec, slope = $a = \frac{12}{30} = 0.4\ m/s^2$

From 30 → 90 sec, slope = a = 0

From 90 → 120 sec, slope = $a = \frac{-12}{30} = -0.4\ m/s^2$

The distance covered = Area under v – t diagram

$$= \left(\frac{1}{2} \times 30 \times 12\right) + (60 \times 12) + \frac{(30 \times 12)}{2} = 1080\ m \quad \text{...Ans.}$$

$\therefore$ Average speed = $\frac{\text{Distance}}{\text{time}} = \frac{1080}{120} = 9\ m/s$ **...Ans.**

a – t diagram

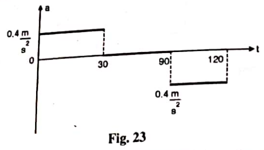


Fig. 23

Q. 18 For a vehicle moving along a straight line, v-t diagram is as shown in Fig. 24. Plot a-t and s-t diagrams for the given time period. **May 2016**

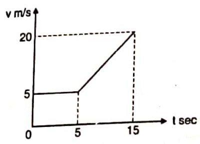


Fig. 24

Ans. :

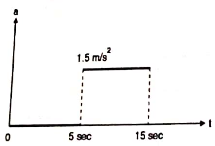


Fig. 25

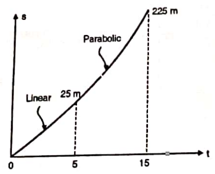


Fig. 26

a – t diagram

From 0 → 5 sec, v constant ∴ a = 0

From 5 → 15 sec,

$$\text{Slope} = a = \frac{20-5}{15-5} = 1.5 \text{ m/s}^2$$

x – t OR s – t diagram

From a – t diagram, by moment equation

$$x_5 = x_0 + v_0 t + M_t = 0 + 5(5) + 0 = 25 \text{ m}$$

$$x_{15} = 0 + 5(15) + (1.5 \times 10 \times 10) = 225 \text{ m}$$

Q. 19 A train leaves station A and attains speed at the rate of 4 m/s² for 6 seconds and then 6 m/s² till it reaches a velocity of 48 m/s. Further the velocity remains constant, then brakes are applied giving the train a constant deceleration stopping it in 6 seconds. If the total running time between the 2 station is 40 sec. Plot a – t, v – t and x – t curve. Determine the distance between two station. **May 2012**

Ans. :

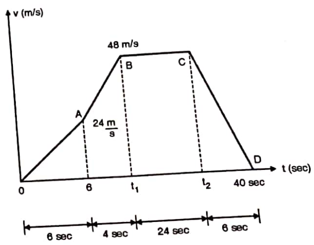


Fig. 27

From 0 → 6 sec, a = 4 m/s²

$$\therefore \text{Slope} = \frac{V_6}{6} = 4 \qquad \therefore V_6 = 24 \text{ m/s}$$

From A to B, slope = a = 6 m/s² given

$$\therefore 6 = \frac{48-24}{t_1 - 6} \qquad \therefore t_1 = 10 \text{ sec}$$

The deceleration time from C to D is given 6 sec

$$\therefore t_2 = 40 - 6 = 34 \text{ sec}$$

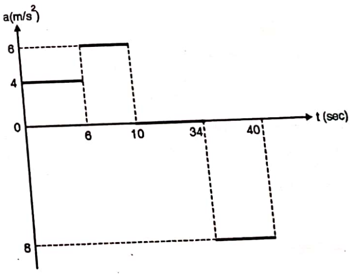


Fig. 28

Engineering Mechanics (MU - Dyn

Total distance between tw

= Area under v – t diagra

S = 1512 m

To plot x – t di

Q. 20 A fighter p from an a ground. ground.

Ans. :

Conside

Cor

...nds and then 6 m/s² till
...hen brakes are applied
...l running time between
... distance between two

May 2012

Total distance between two stations

$$= \text{Area under } v - t \text{ diagram} = \left(\frac{1}{2}\times 6\times 24\right) + \frac{1}{2}(24+48)\,4 + (48\times 24) + \left(\frac{1}{2}\times 6\times 48\right)$$

$$S = 1512 \text{ m}$$

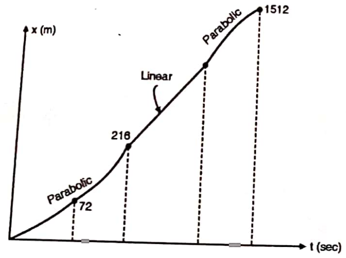


Fig. 29

To plot x – t diagram

$$x_6 = 0+0+(6\times4\times3) = 72 \text{ m}$$

$$x_{10} = 0+0+(6\times4\times7)+(6\times4\times2) = 216 \text{ m}$$

$$x_{34} = 0+0+(6\times4\times31)+(6\times4\times26) = 1368 \text{ m}$$

$$x_{40} = 0+0+(6\times4\times37)+(6\times4\times32)-(6\times8\times3) = 1512 \text{ m}$$

Q. 20 A fighter plane moving horizontally with a constant velocity of 200 m/seconds releases a bomb from an altitude of 400 m. Find the velocity and direction of the bomb just before it strikes the ground. Also determine the distance travelled by the plane before the bomb just strikes the ground. **May 2016**

Ans. :

Consider y-motion from A to B (MUG)

$$s_y = u_y t - \frac{1}{2}gt^2$$

$$-400 = 0 - \frac{1}{2}\times 9.81\, t^2$$

$$\therefore t = 9.03 \text{ sec}$$

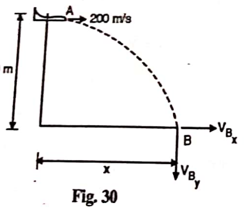


Fig. 30

Consider x – motion (A → B) (U.M)

$$\therefore S_x = V_x \times t$$

$$x = 200\,(9.03)$$

$$x = 1806 \text{ m}$$

At B, velocity when bomb strikes the ground is in 4th quadrant but x – motion is uniform so, V_{B_x} = 200 m/s →

To find V_{By} consider y – motion from A → B

$$\therefore V = u_y - gt \qquad V_{By} = 0 - 9.81\,(9.03)$$

$$\therefore V_{By} = 88.58\,\frac{m}{s} \downarrow$$

$\therefore$ Striking velocity at B, $V_B = \sqrt{(V_{Bx})^2 + (V_{By})^2} = 218.74$ m/s

$$\theta = \tan^{-1}\left(\frac{V_{By}}{V_{Bx}}\right) = 23.89° \text{ in 4th quadrant.}$$

Distance covered by plane before bomb strikes the ground is 1806 m

Q. 21 A stone is thrown with a velocity (u) m/sec at an angle of 20° with horizontal from a point 2 m above the ground. The stone strikes the ground 5 m away from the original position. The motion of stone is subjected to gravitational acceleration and wind resistance of 0.82 m/sec², opposing the horizontal motion. Determine the time of flight of the stone. **May 2016**

Ans. : Consider x - motion (A → B) (U.A.M)

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$5 = (u\cos 20)\,t + \frac{1}{2}(-0.82)\,t^2$$

$$5 = 0.94\,ut - 0.41\,t^2 \qquad \text{...(i)}$$

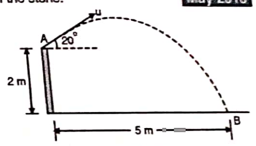


Fig. 31

Consider y - motion (A → B) (MUG)

$$s_y = u_y t - \frac{1}{2}gt^2$$

$$-2 = u\sin 20°\,t - 4.905\,t^2 \qquad -2 = 0.34\,ut - 4.905\,t^2 \qquad \text{...(ii)}$$

$$4.905\,t^2 = 0.34\,ut + 2 \qquad \therefore t^2 = \frac{1}{4.905}(0.34\,ut + 2) \qquad \text{...(iii)}$$

Substitute in equation (i)

$$5 = 0.94\,ut - 0.41\times\frac{1}{4.905}(0.34\,ut+2)$$

$$5.167 = 0.91\,ut \qquad \therefore ut = 5.68$$

Substitute in equation (iii)

$$t = 0.895 \text{ sec}$$

Q. 22 A particle is projected from the top of a tower of height 50 m with a velocity of 20 m/sec at an angle 30 degrees to the horizontal. Determine :

(1) Horizontal distance AB it travel from the foot of the tower.
(2) The velocity with which it strikes the ground at B
(3) Total time take to reach point B. **Dec. 2014**

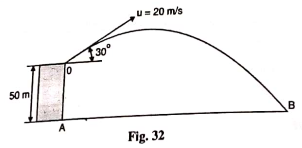


Fig. 32

Ans. :

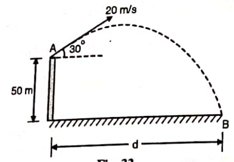


Fig. 33

y - motion (A → B) (MUG)

$$S_y = u_y t - \frac{1}{2} g t^2 \quad -50 = (20 \sin 30)\, t - 4.905\, t^2$$

$$\therefore 4.905\, t^2 - 10\, t - 50 = 0$$

$$t = 4.37 \text{ sec}$$

x - motion (A → B) (U. M.)

$$S_x = V_x \times t$$

$$d = 20 \cos 30° \times 4.37$$

$$d = 75.69 \text{ m}$$

At B : $V_{B_x} = 20 \cos 30° = 17.32 \frac{m}{s} \rightarrow$ $V_{B_y} = (20 \sin 30°) - 9.81\,(4.37) = 32.87 \frac{m}{s} \downarrow$

$$\therefore V_B = 37.15 \text{ m/s}, \qquad \theta = \tan^{-1}\left(\frac{32.87}{17.32}\right) = 62.21°$$

Q. 23 With what minimum horizontal velocity (u) can a bcy throw a rock at A and have it just clear the obstruction at B? Refer Fig. 34. **May 2015**

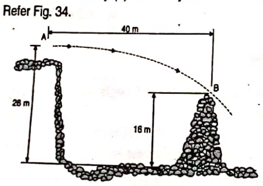


Fig. 34

Ans. :

Using equation of trajectory

$$y = \frac{-gx^2}{2u^2} \text{ (as velocity is horizontal)}$$

$$-10 = \frac{-9.81\,(40)^2}{2u^2}$$

$$\therefore u = 28.01 \text{ m/s} \rightarrow$$

...Ans.

Q. 24 A ball rebounds at A and strikes the incline plane at point B at a distance 76 m as shown in Fig. 35. If the ball rises to a maximum height h = 19 m above the point of projection, compute the initial velocity and the angle of projection α. **May 2009, May 2012**

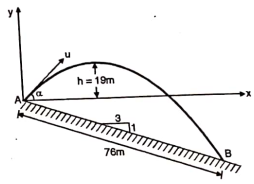


Fig. 35

Ans. :

From Fig. 36 ;

$$\text{Horizontal distance} \quad AD = \frac{76 \times 3}{\sqrt{10}} = 72.10 \text{ m}$$

$$\text{Vertical distance} \quad DB = \frac{76 \times 1}{\sqrt{10}} = 24.03 \text{ m}$$

Initial horizontal velocity $= u \cos \alpha$

Initial vertical velocity $= u \sin \alpha$

For trajectory ACE, maximum height h = 19 m

$$\therefore \frac{u^2 \sin^2 \alpha}{g} = 19$$

$$\therefore u^2 \sin^2 \alpha = 19 \times 9.81$$

$$\therefore u \sin \alpha = 13.652 \ldots \text{(i)}$$

For the projectile ACB,

Horizontal distance x = AD = 72.1 m

Vertical distance y = DB = – 24.03 m

Considering vertical motion from A to B

$$y = ut - \frac{1}{2} g t^2 \quad \therefore \quad -24.03 = (13.652)\, t - \frac{1}{2} \times 9.8 \times t^2$$

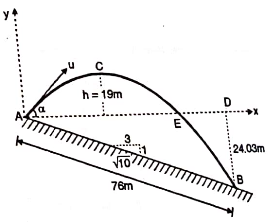


Fig. 36

[Here u = initial vertical velocity = u

$\therefore$ (4.905) t^2 – (13.652) t – 24.

solving,

During this time horizontal distan

velocity

$\therefore$ Horizontal velo

$\therefore$ u c

u

$\therefore$ α = 37.14

[$\therefore$ **Initial velocity of projection = 22**

[Here u = initial vertical velocity = u sin α = 13.652 m/s]

$\therefore$ $(4.905)\,t^2 - (13.652)\,t - 24.03 = 0$

solving, $t = 4.0$ (Seconds)

During this time horizontal distance covered is x = AD = 72.1 m with constant horizontal velocity

$\therefore$ Horizontal velocity $= (u\cos\alpha) = \frac{72.1}{4}$

$\therefore$ $u\cos\alpha = 18.025$

$u\sin\alpha = 13.652$ From…(i)

$\therefore$ $\alpha = 37.14°$ and $u = 22.61$ m/s.

[∴ Initial velocity of projection = 22.61 m/s angle of projection α = 37.14°] **…Ans.**

❑❑❑

Chapter 9 : Kinetics of Particles

Q. 1 Two blocks A and B are held on an inclined plane 5 m apart as shown in Fig. 1. For A, μ = 0.2 and for B, μ = 0.1. If the blocks begin to slide down the plane simultaneously calculate the time and distance travelled by each block before collision.

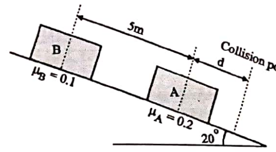


Fig. 1

Ans. : Let a_A and a_B be the acceleration of blocks A and B respectively. Consider dynamic equilibrium of A and B.

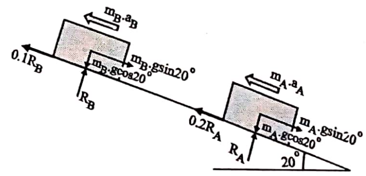


Fig. 2

F.B.D. of A and B

For block A

$\Sigma F_x = 0;\quad m_A \cdot g\sin 20° - 0.2\,R_A - m_A \cdot a_A = 0$ …(1)

$\Sigma F_y = 0;\quad R_A - m_A \cdot g\cos 20° = 0 \qquad \therefore\ R_A = m_A \cdot g \cdot \cos 20°$

Substituting in Equation (1),

$\therefore\ m_A \cdot g\sin 20° - 0.2\,(m_A \cdot g \cdot \cos 20°) - m_A \cdot a_A = 0$

$\therefore\ a_A = g\sin 20° - 0.2\,g\cos 20°$ …(2)

$\therefore\ \mathbf{a_A = 1.51\ m/s^2} \searrow$

Similarly for block B only value of μ is different.

$\therefore$ refer Equation (2), $a_B = g\sin 20° - 0.1\,g\cos 20°$ $\quad \mathbf{a_B = 2.43\ m/s^2} \searrow$

Here $a_B > a_A$ so blocks will collide.

Let d be the distance travelled by block A just before collision in time 't'.

$\therefore$ Distance travelled by block B will be (d + 5) meter in time 't' using kinematic equations,

For A : $S_A = u_A \cdot t + \frac{1}{2} a_A \cdot t^2$

$\therefore \quad d = 0 + \frac{1}{2} \times 1.51 \cdot t^2 \qquad \therefore \quad d = 0.755\, t^2$ ...(3)

For B : $S_B = u_B \cdot t + \frac{1}{2} a_B \cdot t^2$

$(d + 5) = 0 + \frac{1}{2} \times (2.43)\, t^2 \qquad \therefore \quad d + 5 = 1.215\, t^2$

Substituting value of d from Equation (3),

We get, $0.755\, t^2 + 5 = 1.215\, t^2 \qquad \therefore \quad 0.46\, t^2 = 5$

$\therefore \quad t = 3.3$ **sec**

Substituting in Equation (3), $\therefore \quad d = 0.755\,(3.3)^2 \qquad$ **d = 8.22 m**

∴ Distance travelled by A before collision is 8.22 m and distance travelled by B is (8.22 + 5) = 13.22 m. ...Ans.

Q. 2 The system shown is released from rest. Neglecting masses of pulleys and effect of friction determine acceleration of each block.

Ans. : Relation of acceleration : [Refer Fig. 4]

By using concept of dependent motion,

$L_1 = x_A + x_D$

$L_2 = (x_C - x_D) + (x_B - x_D)$

Differentiating twice, Differentiating twice

$\therefore \quad a_A + a_D = 0$ ...(1) $\qquad a_C + a_B - 2\, a_D = 0$

substituting $a_D = -a_A$ from Equation (1)

we get, $\therefore \quad a_C + a_B + 2\, a_A = 0$ ...(2)

Fig. 3

Now assuming accelerations a_A, a_B, a_C and a_D in downward (+ sense) direction.

F.B.D. of blocks and pulley with D'Alembert force as shown in Fig. 5.

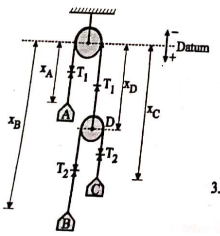


Fig. 4

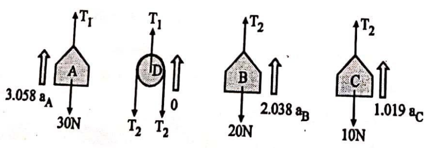


Fig. 5

For A : $\Sigma Fy = 0$; $\quad T_1 + 3.058\, a_A - 30 = 0 \qquad \therefore \quad a_A = \frac{30 - T_1}{3.058}$ ...(3)

For D (pulley which is mass less)

$\Sigma Fy = 0$; $\quad T_1 - 2\, T_2 = 0 \qquad T_1 = 2\, T_2$ ...(4)

For B : $\Sigma Fy = 0$; $\quad T_2 + 2.038\, a_B - 20 = 0 \qquad \therefore \quad a_B = \frac{20 - T_2}{2.038}$ ...(5)

For C : $\Sigma Fy = 0$; $\quad T_2 + 1.019\, a_C - 10 = 0 \qquad \therefore \quad a_C = \frac{10 - T_2}{1.019}$ ...(6)

Substituting all values of a_A, a_B and a_C in Equation (2)

$$a_C + a_B + 2\, a_A = 0 \qquad \frac{10 - T_2}{1.019} + \frac{20 - T_2}{2.038} + 2\left[\frac{30 - T_1}{3.058}\right] = 0$$

substituting $T_1 = 2\, T_2$ and calculating.

We get, $278\, T_2 = 39.25$

$\therefore \quad T_2 = 14.12$ **N** $\qquad \therefore$ by Equation (4) $T_1 = 28.24$ **N**

$a_A = \frac{30 - 28.24}{3.058} = 0.575$ **m/s² ↓** ...Ans.

$\therefore \quad a_B = \frac{20 - 14.12}{2.038} = 2.885$ **m/s² ↓** ...Ans.

$\therefore \quad a_C = \frac{10 - 14.12}{1.019} = -4.043\ \text{m/s}^2 = 4.043$ **m/s² ↑** ...Ans.

Q. 3 A block of mass M_1 = 150 kg resting on inclined plane is connected by a string with another block of mass M_2 = 100 kg as shown in Fig. 6. If μ = 0.2 find tension in the string.

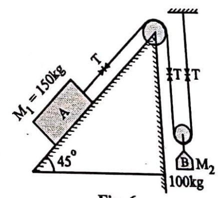


Fig. 6

Ans. :

When frictional force involves it is important to decide direction of motion.

Assume that B is stationary $\therefore T = \frac{100 \times 9.81}{2} = 490.5$ N

For block A, net force acting along the plane is

T-mg sin θ = 490.5 − 150 × 9.81 × sin 45° = − 550 N = 550 N

Now, maximum frictional force developed = $F_{max} = \mu_s \cdot R = 0.2 \cdot mg \cdot \sin\theta = 208.1$ N.

Here net force is greater that max. frictional force, so **block A will move down.**

So acceleration of block A is down the plane.

Now, by direct string law,

$$N_A\,a_A = N_B \cdot a_B$$

$$1 \cdot a_A = 2 \cdot a_B$$

$$\therefore\ a_A = 2\,a_B \qquad \ldots(1)$$

Consider F.B.D. of blocks A and B.

For A : For dynamic equilibrium,

$\Sigma Fx = 0$; (along the plane)

$$T - 150 \times 9.81 \sin 45^\circ + 0.2\,R + 150\,a_A = 0$$

$$\therefore T - 1040.51 + 0.2\,R + 150\,a_A = 0 \qquad \ldots(2)$$

$\Sigma Fy = 0$; $\quad R - 150 \times 9.81 \cos 45^\circ = 0$

$$\therefore R = 1040.51 \text{ N}$$

Substituting in Equation (2),

$$\therefore T - 1040.51 + 0.2\,(1040.51) + 150\,a_A = 0$$

$$\therefore T + 150\,a_A = 832.41 \qquad \ldots(3)$$

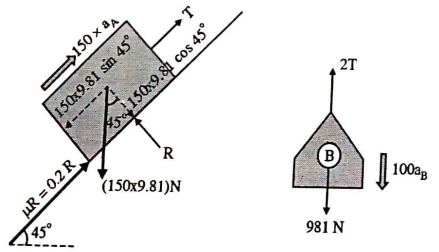


Fig. 7

For block B : For dynamic equilibrium,

$\Sigma Fy = 0$; $\quad 2T - 100\,a_B - 981 = 0 \qquad \therefore\ 2T - 100\,a_B = 981$

but from Equation (1), $\quad a_B = a_A/2 \qquad \therefore\ 2T - 100\,(a_A/2) = 981$

$$\therefore\ 2T - 50\,a_A = 981 \qquad \ldots(4)$$

Solving Equation (3) and (4) we get,

$$a_A = +1.95 \text{ m/s}^2$$

$$\therefore\ \mathbf{a_A = 1.95 \text{ m/s}^2} \swarrow \quad \text{and} \quad \mathbf{T = 539.4 \text{ N}} \qquad \ldots\textbf{Ans.}$$

Chapter 10 : Work-Energy Principle

Q. 1 A 2 kg collar M is attached to a spring and slides without friction in a vertical plane along the curved rod ABC as shown in Fig. 1. The spring has an un-deformed length of 100 mm and it's stiffness k = 800 N/m. If the collar is released from rest at A, determine its velocity (i) as it passes through B (ii) as it reaches C. Refer Fig. 1. **Dec. 99**

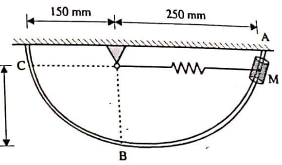


Fig. 1

Ans. :

The spring is undeformed when collar is at C.

$\therefore$ Unstretched length of spring $l_0 = OC = 150$ mm

(i) Now to find velocity of collar at B,

Use work-energy principle from A to B.

Work-done calculations : (A → B)

1) W.D. by gravity $= mgh = 2 \times 9.81 \times 0.2 = 3.924$ J

2) W.D. by spring force $= \frac{1}{2} k\,(x_1^2 - x_2^2)$ here $l_1 = OA = 250$ mm

$$l_2 = OB = 200 \text{ mm (when collar is passing B)}$$

$$\therefore\ x_1 = l_1 - l_0 = 250 - 150 = 100 \text{ mm}$$

$$x_2 = l_2 - l_0 = 200 - 150 = 50 \text{ mm}$$

$$\therefore\ \text{W.D} = \frac{1}{2} \times 600\,[(0.1)^2 - (0.05)^2] = 2.25 \text{ J}$$

Total work-done from A to B.

$$U_{A \to B} = 3.924 + 2.25 = 6.174 \text{ J}$$

Energy calculations : $KE_A = 0 \quad KE_B = \frac{1}{2} m\,v_B^2 = \frac{1}{2} \times 2\,v_B^2 = v_B^2$

Work-energy principle (A → B)

$$U_{A \to B} = KE_B - KE_A \qquad \therefore\ 6.174 = v_B^2 - 0 \quad \therefore\ v_B = 2.48 \text{ m/s}$$

(ii) To find velocity when it reaches C

apply work-energy principle from A to C or B to C.

We shall apply the same from (A $\rightarrow$ C)

Work-done calculations (A $\rightarrow$ C)

1) W.D. by gravity = 0 (A and C are at the same level)

2) W.D. by spring force $= \frac{1}{2} k [x_A^2 - x_C^2]$

$$x_A = l_A - l_0 = 250 - 150 = 100 \text{ mm}$$

$$x_C = l_C - l_0 = 150 - 150 = 0 \text{ (No deformation in spring when collar is at C)}$$

$$\therefore \text{W.D. by spring force} = \frac{1}{2} \times 600 [(0.1)^2 - 0] = 3 \text{ J}$$

Energy calculations : $KE_A = 0$

$$KE_C = \frac{1}{2} m v_C^2 = \frac{1}{2} \times 2 \times v_C^2 = v_C^2$$

Work energy principle from A to C.

$$U_{A \rightarrow C} = KE_C - KE_A$$

$$\therefore \quad 3 = v_C^2 - 0$$

$$v_C = \mathbf{1.732 \text{ m/s}}$$

Q. 2 Block A and B of mass 6kg and 12kg respectively are CONNECTED by a string passing over a smooth pulley. Neglect mass of pull. If coefficient of kinetic friction between the block A and the inclined surface is 0.2, determine the acceleration of block A and block B. **Dec. 2015**

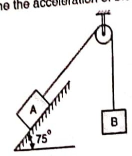


Fig. 2

Ans. : **D'Alembert's principle**

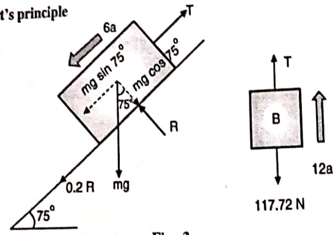


Fig. 3

For block B :

$$T + 12a = 117.72 \quad \text{...(i)}$$

For block A :

$$R = mg \cos 75°$$

$$R = 6 \times 9.81 \cos (75°)$$

$$R = 15.23 \text{ N}$$

$\Sigma F_x = 0$

$$T - mg \sin 75° - 0.2 R - 6a = 0$$

$$\therefore T - 6a = 59.9 \quad \text{...(ii)}$$

From (i) and (ii)

$$\mathbf{T = 79.17 \text{ N}}$$

$$\mathbf{a = 3.21 \text{ m/s}^2}$$

$$\therefore \mathbf{a_A = 3.21 \text{ m/s}^2} \text{ up the plane}$$

$$\mathbf{a_B = 3.21 \text{ m/s}^2 \downarrow}$$

□□□

...(i)

...(ii)

❑❑❑

Chapter 11 : Impulse - Momentum Method

Q. 1 *Two balls with masses 20kg and 30kg are moving towards each other with velocities 10m/s and 5m/s respectively. If after impact the ball having mass 30kg reverses its direction of motion and moves with velocity 6m/s, then determine the coefficient of restitution between the two balls.*

Ans. : **Dec. 2015**

$m_A = 20$ kg $m_B = 30$ kg

$u_A = 10\ \frac{m}{s}\ (\rightarrow)$ $u_B = -5$ m/s

$v_A = ?$ $v_B = 6$ m/s

Using Law of conservation of momentum

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(20 \times 10) + 30(-5) = 20 v_A + 30(6) \quad \therefore v_A = -6.5 \text{ m/s}$$

$$\therefore \text{co-efficient of restitution } e = \frac{v_B - v_A}{u_A - u_B} = \frac{6-(-6.5)}{10-(-5)}$$

$$\therefore \quad \mathbf{e = 0.833}$$

❑❑❑

Chapter 12 : Kinematics of Rigid Bodie

Q. 1 A wheel is rolling along a straight path without slipping. Determine velocity of points A,B and P. OP = 600mm , ω = 4 rad/sec, Vo = 4m/s **May 2016**

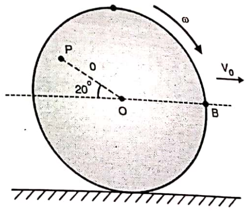


Fig. 1

Ans. :

Here roller rolls without slipping so point C is ICR

$\therefore V_C = 0$ consider points O and C

$$\bar{v}_O = \bar{v}_C + [\bar{\omega} \times \bar{R}_{co}]$$

$$4\hat{i} = o + [-4\hat{k} \times r\hat{j}]$$

$$4\hat{i} = 4r\ \hat{i}$$

$$\therefore r = 1\text{m}$$

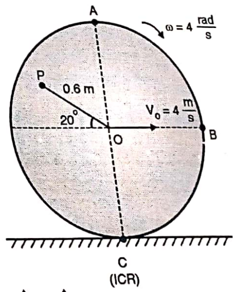


Fig. 2

Consider points A and O

$$\bar{v}_A = \bar{v}_o + [\bar{\omega} \times \bar{R}_{o\to A}]$$

$$= 4\hat{i} + [(-4)\hat{k} \times 1\hat{j}] = 4\hat{i} + 4\hat{i}$$

$$\bar{v}_A = 8\hat{i} \qquad \therefore \bar{v}_A = \frac{8m}{S} \rightarrow$$

Consider points O and B

$$\bar{v}_B = \bar{v}_O + [\bar{\omega} \times \bar{R}_{OB}] = 4\hat{i} + [(-4)\hat{k} \times 1\hat{i}] = 4\hat{i} - 4\hat{j}$$

$$|\bar{v}_B| = \sqrt{32} \text{ m/s}, \theta = 45°$$

Consider points O and P

$$\bar{v}_p = \bar{v}_o [\bar{\omega} \times \bar{R}_{op}] = 4\hat{i} + [(-4)\hat{k} \times (-0.6\cos 20)\hat{i} + 0.6\sin 20\hat{j}]$$

$$= 4\hat{i} + 2.26\hat{j} + 0.82\hat{i}$$

$$= 4.82\hat{i} + 2.26\hat{j} \qquad \therefore \left|\bar{v}_P\right| = 5.32 \text{ m/s}, \theta = 25.12°$$

Q. 2 Find velocity of C and point D at the instant shown ω_{AB} = 3 rad /sec clockwise. AB = 400mm. **May 2016**

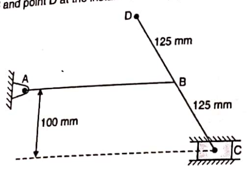


Fig. 3

Ans. :

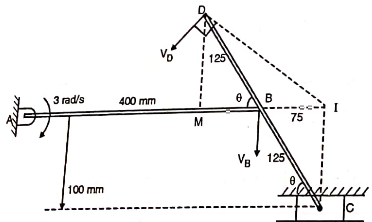


Fig. 4

For Link AB : (ICR is A)

$\therefore \quad V_A = 0$

$\therefore \quad V_B = r_{BA} \cdot \omega_{AB}$

$= 0.4 \times 3$

$= 1.2 \text{ m/s} \downarrow$

Geometry :

$\sin\theta = \dfrac{100}{125} \quad \therefore \theta = 53.13°$

$\therefore$ DM = 100 mm and

BM = 125 cos θ

= 75 mm

$\therefore$ DI $= \sqrt{(100)^2 + (150)^2} = 180.27$ mm

For link CBD, (ICR is I)

$\therefore V_C = rCI\,\omega CD$

$\therefore V_C = 0.1 \times 16$

$\therefore V_C = 1.6 \text{ m/s} \rightarrow$

$V_B = r_{BI}\,\omega_{CD}$

$1.2 = 0.075\,\omega_{CD}$

$\therefore \omega_{CD} = 16 \text{ rad/s} \circlearrowleft$

$V_D = r_{DI}\,\omega_{CD}$

$= 0.18 \times 16$

$\therefore V_D = 2.88$ m/s

Q. 3 The crank BC of a slider crank mechanism is rotating at constant speed of 30 rpm clockwise. Determine the velocity of the piston A at the given instant. **Dec. 2015**

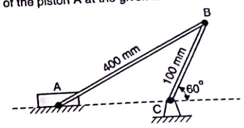


Fig. 5

Ans. : For Link CB (ICR is C)

$\therefore v_c = 0$

$v_B = r_{CB} \cdot \omega_{CB}$

$= 100 \times \pi$

$= 100\,\pi \dfrac{\text{mm}}{\text{s}}$

$\omega_{CB} = 30$ r.p.m

$= 30 \times \dfrac{2\pi}{60}$

$= \pi$ rad/s

Geometry for distance calculation ;

In Δ ABC,

$$\frac{400}{\sin 120°} = \frac{100}{\sin\theta} \qquad \therefore \theta = 12.5°$$

Again $\dfrac{AC}{\sin(47.5)} = \dfrac{400}{\sin(120)}$

$\therefore$ AC = 340.53 mm

In Δ CAI, $\tan 60° = \dfrac{AI}{340.53}$

$\therefore$ AI = 589.8 mm $\qquad \therefore$ CI = 681.06 mm

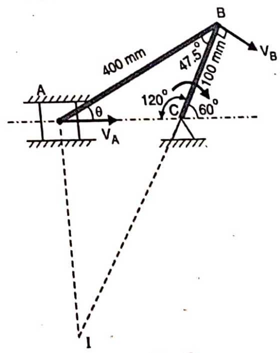


Fig. 6

For AB (ICR is I)

Q. 4 Due to slipping ... and V_b = 3m/s ... point D on this

Ans. : Conside

...nstant speed of 30 rpm clockwise. **Dec. 2015**

For AB (ICR is I)

$\therefore v_B = r_{BI} \cdot \omega$ $\qquad v_A = r_{AI} \cdot \omega$

$100\pi = 781.06\, \omega_{AB}$ $\qquad = 589.8 \times 0.402$

$\therefore \omega_{AB} = 0.402 \frac{\text{rad}}{\text{s}} \curvearrowright$ $\qquad = 237.1 \text{ mm/sec} \rightarrow$

Q. 4 Due to slipping, points A and B on the rim of the disk have the velocities V_a = 1.5m/s to the right and V_b = 3m/s to the left as shown in Fig. 7. Determine the velocities of the centre point C and point D on this rim at the instant. Take radius of disk 0.24m. **Dec. 2015**

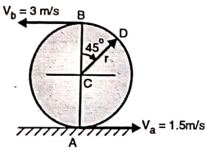


Fig. 7

Ans. : Consider points A and B

$\bar{v}_A = \bar{v}_B + (\bar{\omega} \times \bar{R}_{BA})$ $\qquad 1.5\,\hat{i} = -3\,\hat{i} + [(\omega)\,\hat{k} \times (-0.48)\,\hat{j}]$

$1.5\,\hat{i} = -3\,\hat{i} + 0.48\,\omega\,\hat{i}$ $\qquad 4.5\,\hat{i} = 0.48\,\omega\,\hat{i}$

$\therefore \omega = 9.375$ **rad/s**

Now consider points A and C

$\bar{v}_C = \bar{v}_A + (\bar{\omega} \times \bar{R}_{AC})$

$= 1.5\,\hat{i} + [9.375\,\hat{k} \times (0.24\,\hat{j})]$

$= 1.5\,\hat{i} - 2.25\,\hat{i} = -0.75\,\hat{i}$

$|\bar{v}_C| = 0.75$ **m/s** $\leftarrow$

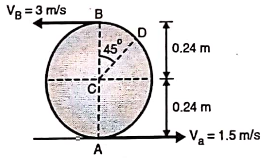


Fig. 8

Now consider points D and C

$\bar{v}_D = \bar{v}_c + (\bar{\omega} \times \bar{R}_{CD}) = -0.75\,\hat{i} + [9.375\,\hat{k}\,(0.24 \cos 45\,\hat{i} + 0.24 \sin 45\,\hat{j})]$

$= -0.75\,\hat{i} + 1.6\,\hat{j} - 1.6\,\hat{i}$ $\qquad \bar{v}_D = -2.35\,\hat{i} + 1.6\,\hat{j}$

$|\bar{v}_D| = 2.84$ **m/s** $\theta = 34.25°$ in 2^{nd} quadrant.

□□□

Engineering Mechanics

Statistical Analysis

Chapter No.	Dec. 2016	May 2017
Chapter 1	10 Marks	06 Marks
Chapter 2	16 Marks	24 Marks
Chapter 3	13 Marks	12 Marks
Chapter 4	12Marks	08 Marks
Chapter 5	06 Marks	08 Marks
Chapter 6	11 Marks	10 Marks
Chapter 7	05 Marks	04 Marks
Chapter 8	21 Marks	16 Marks
Chapter 9	06 Marks	04 Marks
Chapter 10	06 Marks	10 Marks
Chapter 11	04 Marks	06 Marks
Chapter 12	10 Marks	12 Marks
Repeated Questions	–	–

Dec. 2016

Chapter 1 : Co-planar Forces [Total Mark - 10]

Q. 1(a) Find the force F_4, so as to give the resultant of the force system shown below. **(4 Marks)**

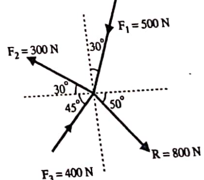


Fig. 1-Q.1(a)

Ans. : Assuming force F_4 to be acting in 4^{th} quadrant (Fig. 2-Q. 1(a))

Here Resultant is inclined

$\therefore$ x-component of R $= \Sigma F_x$

$\therefore 800 \cos 50° = F_4 \cos\theta - 500 \cos 60° - 300 \cos 30° + 400 \cos 45°$ ...(1)

$\therefore F_4 \cos\theta = 741.19$

Scanned by CamScanner

y – component of R $= \Sigma F_y$

$-800 \sin 50° = -F_4 \sin\theta - 500 \sin 60° + 300 \sin 30° + 400 \sin 45°$ ...(2)

$\therefore F_4 \sin\theta = 612.66$

From (1) and (2)

$\frac{F_4 \sin\theta}{F_4 \cos\theta} = \tan\theta = \frac{612.66}{741.19}$

$\therefore \theta = 39.58°$

From (2) $F_4 = \frac{612.66}{\sin(39.58)}$

$\therefore F_4 = 961.55$ N

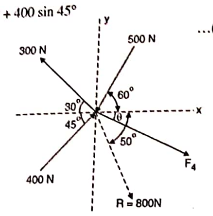


Fig. 2-Q.1(a)

Q. 4(c) A machine part is subjected to forces as shown. Find the resultant of force in magnitude and direction. Also locate the point where resultant cuts the centre line of the bar AB. **(6 Marks)**

Ans. :

$\Sigma F_x = 5\text{kN} \rightarrow$

$\Sigma F_y = -15$ kN

$= 15 \text{ kN} \downarrow$

$\therefore R = \sqrt{(5)^2 + (15)^2}$

$= 15.81$ kN

$\tan\theta = \frac{15}{5}$

$\tan\theta = 71.56°$ in 4^{th} quadrant

now, by varignon's theorem,

$R x = \Sigma M_A$

$= (20 \times 1) - 15 \times (1.5 \cos 60 + 0.5) - 5 \times 3 \sin 60°$

$= -11.74$ kN.m

$R x = 11.74$ kNm (clockwise)

$\therefore x = \frac{11.74}{15.81}$

$\therefore x = 0.74$ m (perpendicular distance from A)

In Δ AMP, $\sin(48.44°) = \frac{AM}{AP} = \frac{0.74}{AP}$

$\therefore AP = 0.988$ m

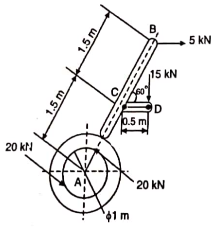


Fig. 1-Q.4(c)

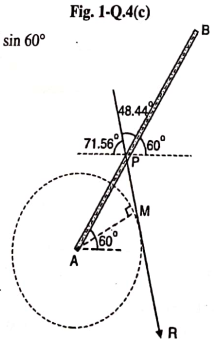


Fig. 2-Q.4(c)

Chapter 2 : Co-Planar forces : Equilibrium [Total Mark - 16]

Q. 2(a) Fig. 1-Q. 2(a) shows a beam AB hinged at A and roller supported at B. The L shaped portion is welded at D to the beam AB. For the loading shown, find the support reactions. **(8 Marks)**

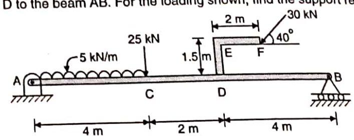


Fig. 1-Q.2(a)

Ans. :

F.B.D of beam AB

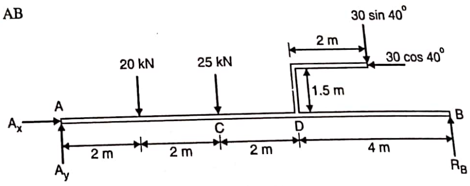


Fig. 2-Q.2(a)

Apply Conditions of equilibrium,

$\Sigma M_A = 0$

$-(20 \times 2) - (25 \times 4) - (30 \sin 40° \times 8) + (30 \cos 40° \times 1.5) + (R_B \times 10) = 0$

$\mathbf{R_B = 25.98 \text{ kN} \uparrow}$

$\Sigma F_y = 0 \quad A_y - 20 - 25 - 30 \sin 40° + R_B = 0$

$\therefore \mathbf{A_y = 38.3 \text{ kN} \uparrow}$

$\Sigma F_x = 0 \quad A_x - 30 \cos 40° = 0$

$\mathbf{A_x = 22.98 \text{ kN} \rightarrow}$

Q. 3 (a) Two spheres A and B of weight 1000N and 750N respectively are kept as shown in Fig. 1-Q.3(a). Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A is 400 mm and Radius of B is 300mm. **(8 Marks)**

Engineering Mecha

Ans. :

Equilibrium

Ans. :

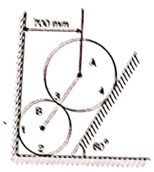


Fig. 1-Q.3(a)

Equilibrium :

$$PQ = 400 + 300 = 700 \text{ mm}$$

$$MQ = 700 - 300 = 400 \text{ mm}$$

$$\therefore \cos \alpha = \frac{MQ}{PQ} = \frac{400}{700}$$

$$\therefore \alpha = 55.15°$$

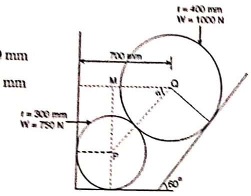


Fig. 2-Q.3(a)

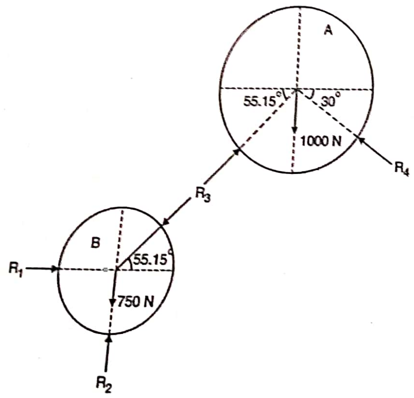


Fig. 3-Q.3(a)

At A :

By Lami's theorem,

$$\frac{R_3}{\sin (120°)} = \frac{1000}{\sin (94.85°)} = \frac{R_4}{\sin (145.15°)}$$

$$\therefore R_3 = 869.14 \text{ N}$$

$$R_4 = 573.48 \text{ N}$$

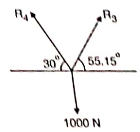


Fig. 4-Q.3(a)

At B :

$$\Sigma F_y = 0$$

$$R_2 - 750 - R_3 \sin (55.15°) = 0$$

$$\therefore R_2 = 1463.26 \text{ N}$$

$$\Sigma F_x = 0$$

$$R_1 - R_3 \cos (55.15°) = 0$$

$$\therefore R_1 = 496.65 \text{ N}$$

Chapter 3 : Friction [Total Marks - 13]

Q. 5 (a) Two blocks A and B are resting against the wall and floor as shown in the Fig.1-Q.5(a). Find minimum value of P that will hold the system in equilibrium. Take μ = 0.25 at the floor, μ = 0.3 at the wall and μ = 0.2 between the blocks. **(8 Marks)**

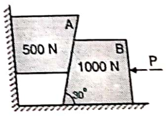


Fig. 1-Q.5(a)

Ans. :

F.B.D. of blocks A and B.

Here, horizontal force P is required to hold or to maintain equilibrium of system.

Hence, impending motion of A must be downward and that of B towards right.

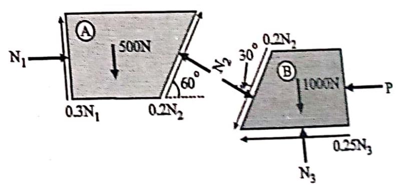


Fig. 2-Q.5(a)

Neglecting size of blocks redraw the forces for convenience.

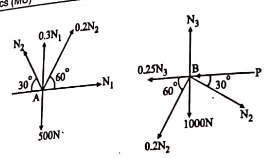


Fig. 3-Q.5(a)

Apply conditions of equilibrium.

For A : $\Sigma F_x = 0\ N_1 + 0.2\ N_2 \cos 60° - N_2 \cos 30° = 0$ ...(1)

$\therefore\ N_1 = 0.766\ N_2$

$\Sigma F_y = 0\ 0.2\ N_2 \sin 60° + 0.3\ N_1 + N_2 \sin 30° - 500 = 0$

Substituting value of N_1 from Equation (1)

$\therefore\ [0.2 \sin 60° + 0.3\ (0.766) + \sin 30°]\ N_2 = 500$

$\therefore\ N_2 = 553.71\ N$

Substituting in Equation (1) $\therefore\ N_1 = 424.14\ N$

For B : $\Sigma F_y = 0\ N_3 - 0.2\ N_2 \sin 60° - 1000 - N_2 \sin 30° = 0$

$\therefore\ N_3 = 1000 + 0.673\ N_2$

Substituting value of N_2 $\therefore\ N_3 = 1000 + 0.673\ (553.71) = 1372.65\ N$

$\Sigma F_x = 0\ -P - 0.25\ N_3 - 0.2\ N_2 \cos 60° + N_2 \cos 30° = 0$

$\therefore\ P = -0.25\ (1372.65) - 0.2\ (553.71) \cos 60° + 553.71 \cos 30°$

$P = 81\ N$ ...Ans.

$\therefore$ Minimum horizontal force required to hold the system is 81 N.

Q 6(a) Block A of weight 2000N is kept on an inclined plane at 35°. It is connected to weight B by an inextensible string passing over smooth pulley. Determine the weight of pan B so that B just moves down. Assume μ = 0.2. **(5 Marks)**

Ans. :

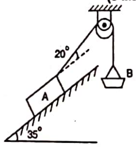


Fig. 1-Q.6(a)

$\Sigma F_y = 0$

$R - mg \cos 35° + T \sin 20° = 0$

$\therefore\ R - T \sin 20° = 1638.3$ ...(i)

$\Sigma F_x = 0$

$T \cos 20° - 0.2R - mg \sin 35° = 0$

$\therefore\ -0.2R + T \cos 20° = 1147.15$ ...(ii)

Solving Equations (i) and (ii) we get,

$R = 2217.23\ N$

$T = 1692.67\ N$

$\therefore$ Weight of pan B is equal to tension T

$\therefore\ W_B = 1692.67\ N$

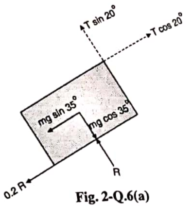


Fig. 2-Q.6(a)

Chapter 4 : Truss [Total Marks - 12]

Q. 1(d) What is Zero force member in a Truss? With examples state the conditions for a zero force member. **(4 Marks)**

Ans. :

If any member of the truss do not carry any force, the member is called zero force member. The zero force member is not useless but it requires for stability and to support weight of the truss.

For example in a truss shown in Fig. 1-Q.1(d).

The joint A is subjected to three forces P, F_{AB} and F_{AC}

To maintain equilibrium force in member AB should be zero and F_{AC} should be equal to P

$\therefore\ F_{AB} = 0$

$F_{AC} = -P$

Here member is AB is known as zero force member.

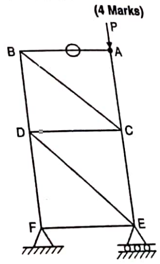


Fig. 1-Q.1(d)

Q. 4 (a) A truss is loaded and supported as shown. Determine the following: **(8 Marks)**

(i) Identify the zero force members if any.

(ii) Find the forces in members EF, ED and FC by method of joints.

(iii) Find the forces in members GF, GC and BC by method of sections.

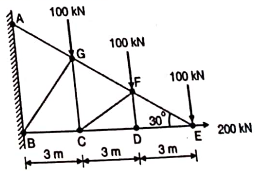


Fig. 1-Q.4(a)

Ans. :

Trusses

(i) By observing joint D

(ii) Consider joint E

$\Sigma F_y = 0$

$-100 + F_{EF} \sin 30° =$

$\therefore\ F_{EF} =$

$\Sigma F_x = 0$

$200 - F_{EF} \cos 30° - F_{DE}$

F_{DE}

$\therefore\ F_{CD}$

Consider joint

$\Sigma F_x = 0$

$-F_{GF} \cos 30° - F_{CF}$

$\Sigma F_y = 0$

$F_{GF} \sin 30° - F_{CF}$ si

Solving Eq

(iii) Select a s and consi

$\Sigma M_E = 0 (100 \times$

$\Sigma F_y = 0 - 100 -$

$\Sigma F_x = 0\ 200 -$

Ans. :

Trusses

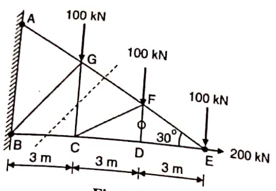


Fig. 2-Q.4(a)

(i) By observing joint D, $F_{DF} = 0$, $F_{CD} = F_{DE}$

(ii) **Consider joint E**

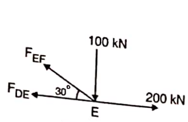


Fig.3-Q.4(a)

$\Sigma F_y = 0$

$-100 + F_{EF} \sin 30° = 0$

$\therefore$ $\mathbf{F_{EF} = 200\ kN}$

$\Sigma F_x = 0$

$200 - F_{EF} \cos 30° - F_{DE} = 0$

$\mathbf{F_{DE} = 26.79\ kN}$

$\therefore$ $\mathbf{F_{CD} = 26.79\ kN}$

Consider joint F

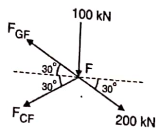


Fig. 4-Q.4(a)

$\Sigma F_x = 0$

$-F_{GF} \cos 30° - F_{CF} \cos 30° + 200 \cos 30° = 0$ ...(i)

$\Sigma F_y = 0$

$F_{GF} \sin 30° - F_{CF} \sin 30° - 200 \sin 30° - 100 = 0$ ...(ii)

Solving Equations (i) and (ii)

$\mathbf{F_{GF} = 300\ kN}$

$\mathbf{F_{CF} = -100\ kN}$

(iii) Select a section which cuts members GF, GC and BC and consider right part w.r.t. section

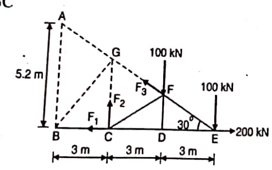


Fig. 5-Q.4(a)

$\Sigma M_E = 0 (100 \times 3) - F_2 \times 6 = 0$

$\therefore$ $\mathbf{F_2 = 50\ kN\ (CG)}$

$\Sigma F_y = 0 - 100 - 100 + F_2 + F_3 \sin 30° = 0$

$\mathbf{F_3 = 300\ kN\ (FG)}$

$\Sigma F_x = 0\ 200 - F_1 - F_3 \cos 30° = 0$

$\therefore$ $F_1 = 200 - 300 \cos 30°$

$\therefore$ $\mathbf{F_1 = -59.8\ kN\ (BC)}$

Chapter 5 : Centroid and Center of Gravity [Total Marks - 06]

Q. 3(b) A circle of diameter 1.5m is cut from a composite plate. Determine the centroid of the remaining area of the plate. **(6 Marks)**

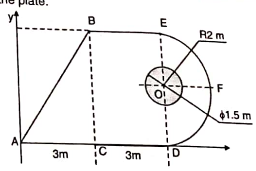


Fig. 1-Q. 3(b)

Ans. :

Centroid :

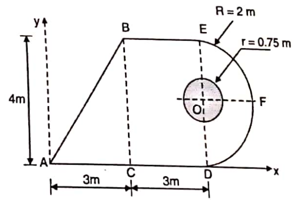


Fig. 2-Q.3(b)

	Part	Area	x	y
1.	Triangle ACB	$\frac{1}{2} \times 3 \times 4 = 6\ m^2$	$\frac{2}{3} \times 3 = 2\ m$	$\frac{1}{3} \times 4 = 1.33\ m$
2.	Rectangle CDEB	$3 \times 4 = 12\ m^2$	$3 + 3/2 = 4.5\ m$	2 m
3.	Semi-circle DFE	$\frac{\pi r^2}{2} = \frac{\pi (2)^2}{2} = 6.28\ m^2$	$6 + \frac{4r}{3\pi} = 6.848\ m$	2 m
4.	Circular hole	$\pi r^2 = \pi (0.75)^2 = 1.767\ m^2$	6 m	2 m

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 - A_4} = 4.37\ m$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - A_4 y_4}{A_1 + A_2 + A_3 - A_4} = 1.82\ m$$

$\therefore$ **Centroid (4.37, 1.82) m**

Chapter 6 : Space Forces [Total Marks - 11]

Q. 2(c) *The resultant of three concurrent space forces at A is $\bar{R}$ = (– 788) N. Find the magnitude of F1, F2 and F3 forces.* **(6 Marks)**

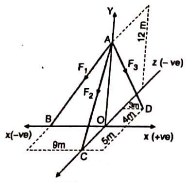


Fig. 1-Q.2(c)

Ans. :

Force vectors :

$$\bar{F_1} = F_1\,\bar{e}_{A\to B} = F_1\left[\frac{-9\hat{i} - 12\hat{j} + 0\hat{k}}{\sqrt{(9)^2 + (12)^2 + 0}}\right] = F_1\left[\frac{-9}{15}\hat{i} - \frac{12}{15}\hat{j} + 0\hat{k}\right]$$

$$\bar{F_2} = F_2\,\bar{e}_{A\to C} = F_2\left[\frac{0\hat{i} - 12\hat{j} + 5\hat{k}}{\sqrt{0 + (12)^2 + (5)^2}}\right] = F_2\left[0\hat{i} - \frac{12}{13}\hat{j} + \frac{5}{13}\hat{k}\right]$$

$$\bar{F_3} = F_3\,\bar{e}_{A\to D} = F_3\left[\frac{3\hat{i} - 12\hat{j} - 4\hat{k}}{\sqrt{(3)^2 + (12)^2 + (4)^2}}\right] = F_3\left[\frac{3}{13}\hat{i} - \frac{12}{13}\hat{j} - \frac{4}{13}\hat{k}\right]$$

Now, Resultant $\bar{R} = -788\,\hat{j}$ i.e. along negative y-axis

$\therefore \Sigma F_x = 0 \qquad -\frac{9}{15}F_1 + (0)\,F_2 + \frac{3}{13}F_3 = 0$ ...(i)

$\Sigma F_y = R \qquad -\frac{12}{15}F_1 - \frac{12}{13}F_2 - \frac{12}{13}F_3 - 788 = 0$ ...(ii)

$\Sigma F_z = 0 \qquad (0)\,F_1 + \frac{5}{13}F_2 - \frac{4}{13}F_3 = 0$ ...(iii)

Solving Equations (i), (ii) and (iii) we get

$F_1 = 153.9$ N **$F_2 = 320.125$ N** **$F_3 = 400.15$ N**

Q. 6(d) *A T-shaped rod is suspended using three cables as shown. Neglecting the weight of the rods, find the tension in each cable.* **(5 Marks)**

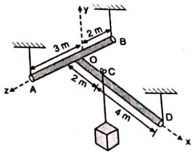


Fig. 1-Q.6(d)

Ans. :

Force Vectors :

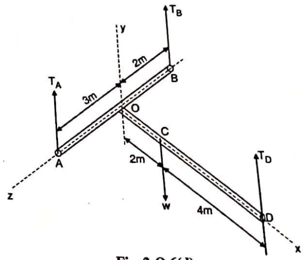


Fig. 2-Q.6(d)

$\bar{T_A} = T_A\hat{j} \qquad \bar{T_B} = T_B\hat{j}$

$\bar{T_D} = T_D\hat{j} \qquad \bar{W} = -W\hat{j}$

Apply conditions of equilibrium

$\Sigma F_y = 0 T_A + T_B + T_C - W = 0$ ...(i)

$\Sigma M_0 = 0$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 0 & T_A & 0 \end{bmatrix} + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -2 \\ 0 & T_B & 0 \end{bmatrix} + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & 0 \\ 0 & T_D & 0 \end{bmatrix} + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -W & 0 \end{bmatrix} = 0$$

Expanding, $\hat{i}$ and $\hat{k}$ co-efficients

$-3T_A + 2T_B + (0)\,T_D + 0 = 0$ ...(ii)

$(0)\,T_A + (0)\,T_B + 6\,T_D - 2W = 0$ ...(iii)

Solving Equations (i), (ii), (iii) we get

$T_A = 0.27$ W **$T_B = 0.4$ W** **$T_C = 0.33$ W**

Note : **Here load W is not given so finding tensions in terms of W**

Scanned by CamScanner

Chapter 7 : Virtual Work [Total Marks - 05]

Q. 6(c) A rod AD of length 40cm is suspended from point D as shown in Fig. 1-Q.6(c). If it has a weight of 25 N and also supports a 40N load, find the tension in the cable using the method of virtual work. Take AC = 30 cm. **(5 Marks)**

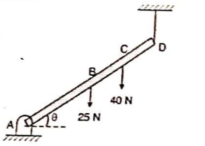


Fig. 1-Q.6(c)

Ans. :

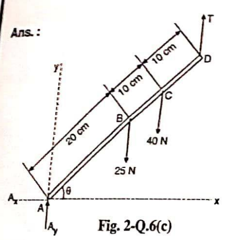


Fig. 2-Q.6(c)

Active force	Co-ordinate	Virtual displacement
− 25 N	$y_B = 20 \sin\theta$	$20 \cos\theta\, d\theta$
− 40 N	$y_C = 30 \sin\theta$	$30 \cos\theta\, d\theta$
+ T	$y_D = 40 \sin\theta$	$40 \cos\theta\, d\theta$

By principle of virtual work

$$dU = 0$$

$$-25\,(20 \cos\theta\, d\theta) - 40\,(30 \cos\theta\, d\theta) + T\,(40 \cos\theta\, d\theta) = 0$$

$$\therefore T = 42.5 \text{ N}$$

Chapter 8 : Kinematics of Particles (Rectilinear Motion) [Total Marks - 21]

Q. 1(b) A particle starts from rest from origin and its acceleration is given by, $a = \frac{k}{(x+4)^2}$ m/s^2. Knowing that V = 4 m/s when x = 8m, find (i) value of k and (ii) Position when V = 4.5 m/s. **(4 Marks)**

Ans. :

Given $\quad a = \frac{k}{(x+4)^2}$

Given condition : (i) $t = 0, v = 0, x = 0$

(ii) $v = \frac{4m}{s}, x = 8$ m

Using $\quad a = v\frac{dv}{dx} = \frac{k}{(x+4)^2} \qquad \therefore \int v\, dv = k\int \frac{1}{(x+4)^2}\, dx$

$$\frac{v^2}{2} = \frac{-k}{(x+4)} + C_1$$

When $v = 0, x = 0$

$$\therefore 0 = \frac{-k}{4} + C_1 \qquad \text{...(1)}$$

When $v = 4$ m/s, $x = 8$m

$$\therefore \frac{(4)^2}{2} = \frac{-k}{12} + C_1 \qquad \therefore 8 = \frac{-k}{12} + C_1 \qquad \text{...(2)}$$

Solving (1) and (2) we get

$$k = 48 \frac{m^3}{sec^2} \qquad C_1 = 12 \qquad \therefore \frac{v^2}{2} = \frac{-48}{(x+4)} + 12$$

$$\therefore v^2 = \frac{-96}{(x+4)} + 24 \qquad \text{...(3)}$$

Now to find position 'x' when v = 4.5 m/s

From equation (3)

$$(4.5)^2 = \frac{-96}{(x+4)} + 24 \qquad \therefore 3.75 = \frac{96}{x+4} \qquad \therefore x = 21.6 \text{ m}$$

Q. 2(b) The acceleration-time diagram for linear motion is shown. Construct velocity-time diagram and displacement-time diagram for the motion assuming that the motion starts with initial velocity of 5m/s from starting point. **(6 Marks)**

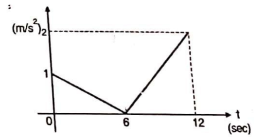


Fig. 1-Q.2(b)

Ans. :

Given that at $t = 0$, $v_0 = 5$m/s, $x_0 = 0$

(i) v-t diagram

Property : Area under a-t diagram = change in velocity

$$v_6 - v_0 = \frac{1}{2} \times 6 \times 1$$

$$\therefore v_6 - 5 = 3$$

$$\therefore v_6 = 8 \text{ m/s}$$

also, $$v_{12} - v_6 = \frac{1}{2} \times 6 \times 2$$

$$\therefore v_{12} - 8 = 6$$

$$\therefore v_{12} = 14 \text{ m/s}$$

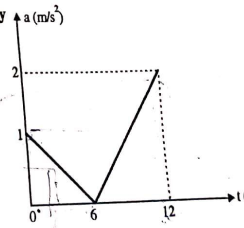


Fig. 2-Q.2(b)(a)

(ii) **x-t diagram**

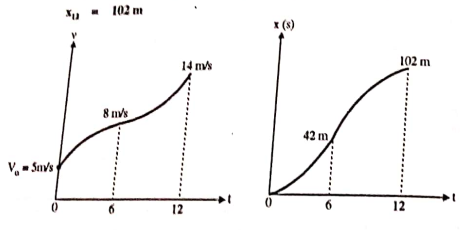
By moment Equation

$$x_6 = 0+5(6)+\frac{1}{2}\times 6\times 1\times \frac{2}{3}\times 6$$

$$x_6 = 42 \text{ m}$$

$$x_{12} = 0+5(12)+\frac{1}{2}\times 6\times 1\times\left(6+\frac{2}{3}\times 6\right)+\frac{1}{2}\times 6\times 2\times\left(\frac{1}{3}\times 6\right)$$

$$x_{12} = 102 \text{ m}$$



(b) (c)

Fig. 2-Q.2(b)

Q. 6(b) A particle falling under gravity travels 25m in a particular second. Find the distance travelled by it in next three seconds. **(5 Marks)**

Ans. :

Rectilinear motion (Motion under gravity)

Let A be the initial position

Let t = time to travel from A → B

And time to travel from B → C is 1 sec

Motion from A → B

$$S = ut-\frac{1}{2}gt^2$$

$$-d = 0-\frac{1}{2}\times 9.81\, t^2$$

$$\therefore d = 4.905\, t^2 \quad \ldots(1)$$

Motion from A → C

$$S = ut - 1/2\, gt^2$$

$$-(d+25) = 0-\frac{1}{2}\times 9.81\,(t+1)^2$$

$$\therefore d+25 = 4.905\,(t+1)^2$$

$$\therefore 4.905\, t^2+25 = 4.905\,(t+1)^2$$

$$t = 2.048 \text{ sec}$$

∴ From A → C

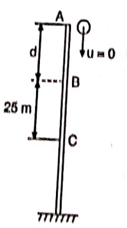


Fig. 1-Q.6(b)

$$V = u-gt$$

$$V_C = 0-9.81\,(3.048) = 29.9 \text{ m/s} \downarrow$$

∴ Distance travelled in next 3 sec,

$$S = ut-\frac{1}{2}gt^2$$

$$-S = -29.9\,(3)-\frac{1}{2}(9.81)\,(3)^2$$

$$\therefore S = 133.845 \text{ m}$$

Q. 5(b) A shot is fired with a bullet with an initial velocity of 20 m/s from a point 10m infront of a vertical wall 5m high. Find the angle of projection with the horizontal to enable to shot to just clear the wall. Also find the range where the bullet falls on the ground. **(6 Marks)**

Ans. :

From A → C, use equation of trajectory

$$y = x\tan\alpha-\frac{gx^2}{2u^2\cos^2\alpha}$$

$$5 = 10\tan\alpha-\frac{9.81\,(10)^2}{2\,(20)^2\cos^2\alpha}$$

$$5 = 10\tan\alpha-1.226\,(1+\tan^2\alpha)$$

$$\therefore 1.226\tan^2\alpha-10\tan\alpha+6.226 = 0$$

$$\therefore \tan\alpha = 7.48 \text{ and } \tan\alpha = 0.679$$

$$\alpha = 82.38°, \quad \alpha = 34.18°$$

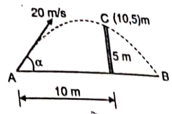


Fig. 1-Q.5(b)

When $\alpha = 82.38°$ Range $R = \frac{u^2\sin 2\alpha}{g} = \frac{(20)^2\sin(2\times 82.38)}{9.81} = 10.718$ m

When $\alpha = 34.18°$ Range $R = \frac{u^2\sin 2\alpha}{g} = \frac{(20)^2\sin(2\times 34.18)}{9.81} = 37.9$ m

∴ Range of shot is 10.718 m ≤ R ≤ 37.9 m

Chapter 9 : Kinetics of Particles (D'Alembert's Principle) [Total Marks - 06]

Q. 5(c) Three blocks A, B and C of masses 3 kg, 2 kg and 7 kg respectively are connected as shown. Determine the acceleration of A, B and C. Also find the tension in the strings. **(6 Marks)**

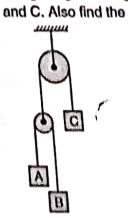


Fig. 1-Q.5(c)

elocity of 20 m/s from a point 10m infront of a
ction with the horizontal to enable to shot to just
ullet falls on the ground. **(6 Marks)**

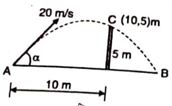


Fig. 1-Q.5(b)

= 10.718 m

37.9 m

lembert's Principle)

g respectively are connected as shown.
tension in the strings. **(6 Marks)**

Ans. :

$L_1 = x_C + x_D$

Differentiate twice

$a_C + a_D = 0$ ...(i)

$L_2 = (x_A - x_D) + (x_B - x_D)$

Differentiate twice

$a_A + a_B - 2a_D = 0$

Substitute $a_D = -a_C$

$\therefore a_A + a_B + 2a_C = 0$ ...(ii)

Assume acceleration of all particles in positive i.e. downward

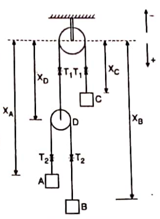


Fig. 2-Q.5(c)

For A $T_2 + 3a_A - 29.43 = 0$

$\therefore a_A = \frac{29.43 - T_2}{3}$

For B $T_2 + 2a_B - 19.62 = 0$

$\therefore a_B = \frac{19.62 - T_2}{2}$

For C $T_1 + 7a_C - 68.67 = 0$

$\therefore a_C = \frac{68.67 - T_1}{7}$

For D $T_1 = 2T_2$

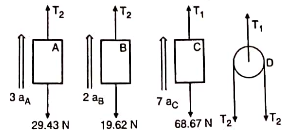


Fig. 3-Q.5(c)

Substitute all values in (ii)

$$\frac{29.43 - T_2}{3} + \frac{19.62 - T_2}{2} + 2\left[\frac{68.67 - 2T_2}{7}\right] = 0$$

$T_2 = 27.93$ N $\quad \therefore T_1 = 55.86$ N $\quad \therefore a_A = 0.5\ m/s^2 \downarrow$

$a_B = -4.155\ m/s^2 \quad \therefore a_B = 4.155\ m/s^2 \uparrow \quad a_C = 1.83\ m/s^2 \downarrow$

Chapter 10 : Work Energy Principle [Total Marks - 06]

Q. 4(b) A cylinder has a mass of 20 kg and is released from rest when h = 0 shown in the Fig. 1-Q.4(b). Determine its speed when h = 3m. The springs each have an unstretched length of 2 m Take k = - 40 N/m. **(6 Marks)**

Ans. :

Work energy principle :

For each spring unstretched length of spring $L_0 = 2$ m

Length of spring at position 1, $L_1 = 2$ m

Length of spring at position 2, $L_2 = \sqrt{(2)^2 + (3)^2} = 3.605$ m

$\therefore x_1 = L_1 - L_0, \quad x_2 = L_2 - L_0$

$\therefore x_1 = 0 \quad x_2 = 3.605 - 2$

$x_2 = 1.605$ m

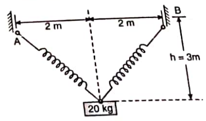


Fig. 1-Q.4(b)

(i) Workd one by spring force, (For two springs)

$= 2\left[\frac{1}{2}K\left(x_1^2 - x_2^2\right)\right] = 40\,[0 - (1.605)^2] = -103.04$ J

(ii) Workdone by gravity $=$ mgh $= 20 \times 9.81 \times 3 = 588.6$ J

$K \cdot E_1 = 0$ (When h = 0)

$K E_2 = \frac{1}{2}MV_2^2 = \frac{1}{2} \times 20\,V_2^2 = 10\,V_2^2$ joules

By work energy principle,

$U_{1 \to 2} = K E_2 - K E_1$

$-103.04 + 588.6 = 10\,V_2^2 - 0 \quad \therefore V_2 = 6.97$ m/s $\downarrow$

Chapter 11 : Impulse - Momentum Method [Total Marks - 04]

Q.1(e) A glass ball is dropped onto a smooth horizontal floor from which it bounces to a height of 9m. On the second bounce it rises to a height of 6m. From what height the ball was dropped and what is the coefficient of restitution between glass and the floor? **(4 Marks)**

Ans. :

At A, $e = \sqrt{\frac{h_2}{h_1}}$

$\therefore e^2 = \frac{9}{h}$ ...(1)

at B, $e = \sqrt{6/9}$

$\therefore e = 0.816$

$\therefore$ by Equation (1)

$h = \frac{9}{(0.81)^2}$

$h = 13.516$ m

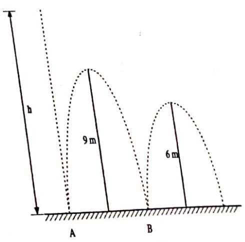


Fig. 1-Q.1(e)

Chapter 12 : Kinematics of Rigid Bodies [Total Marks - 10]

Q. 1(c) Rod AB of length 3m is kept on smooth planes as shown in Fig. 1-Q.1(c). The velocity of end A is 5m/s along the inclined plane. Locate the ICR and find the velocity of end B. **(4 Marks)**

gv_kumbhojkar@yahoo.co.in

...ithout proof) and problems.

Self learning topics : Jacobian's of two and three indep... problems).

Module 5 : Matrices

Pre-requisite : Inverse of a matrix, addition, multipl...

Ans. :

Distance calculations for AI and BI by sine rule in Δ AIB,

$$\frac{AI}{\sin 70^\circ} = \frac{BI}{\sin 80^\circ} = \frac{3}{\sin 30^\circ}$$

$$\therefore \quad AI = 5.64 \text{ m}$$

$$BI = 5.91 \text{ m}$$

Now, **for rod AB (ICR is I)**

$$\therefore \quad v_A = r_{AI} \cdot \omega_{rod}$$

$$5 = 5.64 \cdot \omega_{rod}$$

$$\therefore \quad \omega_{rod} = 0.89 \text{ rad/sec.}$$

$$\text{And} \quad v_B = r_{BI} \cdot \omega_{rod} = 5.91 \times 0.89 = \mathbf{5.26 \text{ m/s}} \rightarrow$$

...Ans.

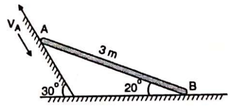


Fig. 1-Q.1(c)

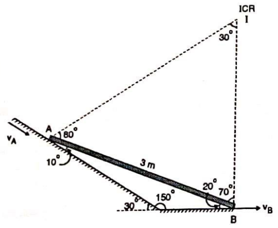


Fig. 2-Q.1(c)

Q. 3(c) A rod AB has an angular velocity of 2 rad/sec, counter clockwise as shown. End C of rod BC free to move on a horizontal surface. Determine (i) angular veloctity of BC and (ii) Velocity of C. **(6 Marks)**

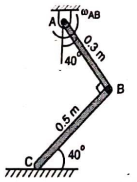


Fig. 1-Q.3(c)

Ans. :

For link AB : (ICR is A)

$$V_A = 0 \quad V_B = r_{BA} \cdot \omega_{AB}$$

$$V_B = 0.3 \times 2 \quad V_B = 0.6 \text{ m/s } (\perp \text{ to AB})$$

For link BC : (ICR is I)

$$V_B = r_{BI} \cdot \omega_{BC}$$

$$0.6 = 0.595\, \omega_{BC}$$

$$\therefore \quad \omega_{BC} = \mathbf{1.008 \text{ rad/s}}$$

$$V_C = r_{CI} \cdot \omega_{BC} = 0.78 \times 1.008$$

$$V_C = \mathbf{0.786 \text{ m/s}} \rightarrow$$

In IBC, sine Rule,

$$\frac{BC}{\sin 40} = \frac{CI}{\sin 90} = \frac{BI}{\sin 50}$$

$$\frac{0.5}{\sin 40} = \frac{CI}{\sin 90} = \frac{BI}{\sin 50}$$

$$\therefore \quad CI = 0.78 \text{ m}$$

$$BI = 0.595 \text{ m}$$

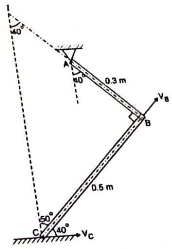


Fig. 2-Q.3(c)

□□□

Q. 2(a) Comput interse

Ans. :

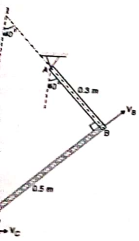


Fig. 2-Q.3(c)

May 2017

Chapter 1 : Co- Planar Forces [Total Marks - 06]

Q. 2(a) Compute the resultant of the three forces acting on the plate shown in Fig. 1-Q.2(a) locate its intersection with AB and BC. **(6 Marks)**

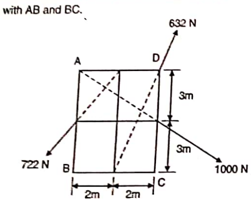


Fig. 1-Q.2(a)

Ans. :

From geometry,

$\tan\theta_1 = \left(\frac{3}{4}\right)$

$\therefore \theta_1 = 36.87°$

$\tan\theta_2 = \left(\frac{3}{2}\right)$

$\therefore \theta_2 = 56.31°$

$\tan\theta_3 = \left(\frac{6}{2}\right)$

$\therefore \theta_3 = 71.56°$

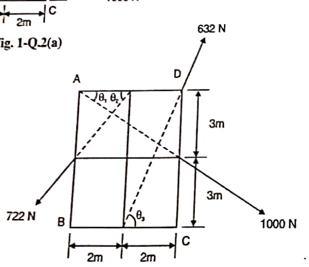


Fig. 2-Q.2(a)

$\Sigma F_x = 1000\cos\theta_1 - 722\cos\theta_2 + 632\cos\theta_3 = 599.41\text{ N} (\rightarrow)$

$\Sigma F_y = -1000\sin\theta_1 - 722\sin\theta_2 + 632\cos\theta_3 = -601.19\text{ N} = 601.19\text{ N} (\downarrow)$

$\therefore$ Magnitude $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\therefore R = 848.96\text{ N}$

$\therefore$ Direction $\tan\alpha = \left|\frac{\Sigma F_y}{\Sigma F_x}\right|$

$\therefore \alpha = 45.08$ **in 4th quadrant.**

Now apply Varignon's theorem at B

$\therefore R x = \Sigma M_B$

$= -(1000\cos\theta_1 \times 6) + (722\cos\theta_2 \times 3) + (632\sin\theta_3 \times 2)$

$= -3198.7$ N.m

$R x = 3198.7$ Nm (clockwise)

$\therefore x = 3.77$ m ($\perp$r distance from B)

So, horizontal distance where 'R' cuts BC is

$$a = \frac{\Sigma M_B}{\Sigma F_y} = \frac{3198.7}{601.19} = 5.32\text{ m}$$

Vertical distance where 'R' cuts BA is

$$b = \frac{\Sigma M_B}{\Sigma F_x} = \frac{3198.7}{599.41} = 5.34\text{ m}$$

Chapter 2 : Coplanar Forces - Equilibrium [Total Marks - 24]

Q. 1(a) In the rocket arm shown in Fig. 1-Q.1(a) the moment of 'F' about 'O' balances that 'P' = 280 N find 'F'. **(4 Marks)**

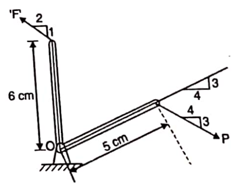


Fig. 1-Q.1(a)

Ans. :

$\Sigma M_D = 0, \quad F\cos(63.43) \times 6 - P\sin(73.74) \times 5 = 0$

Where, $P = 250$ N

$\therefore$ **F = 447.14 N** **...Ans.**

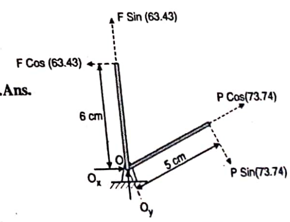


Fig. 2-Q.1(a)

Q. 1(b) State Lami's theorem. State the necessary condition for application of Lami's theorem. **(4 Marks)**

Ans. : **Special Condition of Equilibrium under Three Forces :**

It states 'If a body is in equilibrium under 3 coplanar concurrent forces, each force is proportional to the 'sine' of the angle between other two'.

Proof of Lami's Theorem :

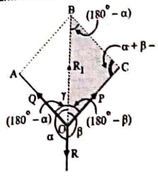


Fig. 1-Q.1(b)

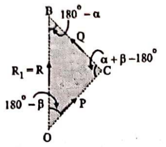


Fig. 2-Q.1(b)

Let P, Q and R be the 3 forces acting at point 'O' in equilibrium. (Refer Fig. 1-Q.1(b)).

Let R_1 be the resultant of two forces P and Q. Now point O is subjected to only two forces R and R_1. As per equilibrium under 2 forces R and R_1 must be equal, opposite and collinear.

Now, by sine rule in Δ OBC,

$$\frac{P}{\sin(180° - \alpha)} = \frac{Q}{\sin(180° - \beta)} = \frac{R_1 = R}{\sin[\alpha + \beta - 180°]}$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin[360° - \gamma - 180°]}$$

$$\ldots[\text{As } \alpha + \beta + \gamma = 360° \therefore \alpha + \beta = 360° - \gamma]$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin(180° - \gamma)}$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

This is mathematical form of Lami's theorem.

Q. 2(b) Two cylinders 1 and 2 are connected by a rigid bar of negligible weight hinged to each cylinder and left so rest in equilibrium in the position shown under the application of force 'P' applied at the center of cylinder 2. Determine the magnitude of force 'P' if the weights of the cylinders 1 and 2 are 100 N and 50N respectively. **(8 Marks)**

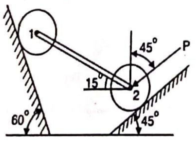


Fig. 1-Q.2(b)

Ans. :

Consider F.B.D. of two cylinders.

Let F be the compressive force in rigid bar AB.

Now, use Lami's theorem for cylinder A.

$$\therefore \quad \frac{R_1}{\sin 105°} = \frac{981}{\sin 135°} = \frac{F}{\sin 120°}$$

$$\therefore R_1 = 1340.07 \text{ N}$$

$$F = 1201.47 \text{ N (compressive)}$$

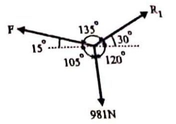


Fig. 2-Q.2(b)

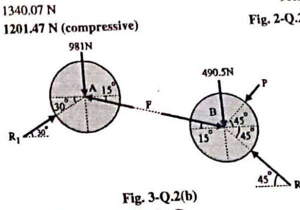


Fig. 3-Q.2(b)

Apply conditions of equilibrium for cylinder B.

$\Sigma F_x = 0 \quad F\cos 15° - R_2\cos 45° - P\cos 45° = 0$

$$\therefore P = 1641.24 - R_2 \quad \ldots(1)$$

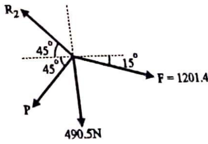
$\Sigma F_y = 0$

$R_2 \sin 45° - P\sin 45° - 490.5 - 1201.47 \sin 15° = 0$

$$\therefore R_2 \sin 45° - P \sin 45° = 801.46$$

Divide by sin 45°

$$\therefore R_2 = P + 1133.435 \quad \ldots(2)$$



(Forces on cylinder B)

Fig. 4-Q.2(b)

Substituting in Equation (1),

$$\therefore P = 1641.24 - (P + 1133.435)$$

$$2P = 507.805$$

P = 253.9 N ...Ans.

Q. 4(a) A boom AB is supported as shown in Fig. 1-Q.4(a) by a cable runs from 'C' over a small smooth pulley at D. Compute the tension T in cable and reaction at A. Neglect the wt of the boom and size of the pulley. **(8 Marks)**

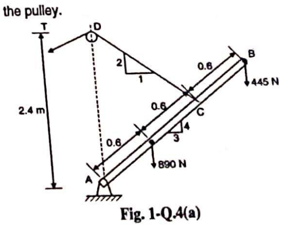


Fig. 1-Q.4(a)

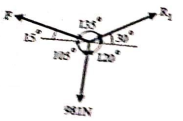
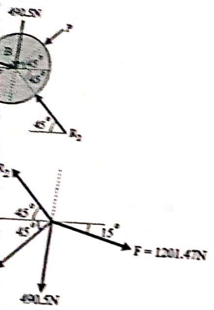


Fig. 2-Q.2(b)



...es on cylinder B)
Fig. 4-Q.2(b)

...Ans.

...ble runs from 'C' over a small ...tion at A. Neglect the wt of the

(8 Marks)

Ans. :

F.B.D. of boom AB :

Apply conditions of equilibrium

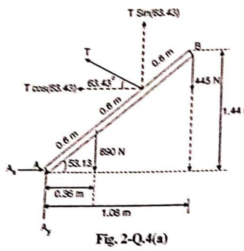


Fig. 2-Q.4(a)

$\Sigma M_A = 0$

$-(890 \times 0.36) - (445 \times 1.08)$
$+ (T \cos 63.43°)(1.2 \sin 53.13°)$
$+ (T \sin 63.43°)(1.2 \cos 53.13°) = 0$

$\therefore 0.43\,T + 0.64\,T = 801$

$\therefore \mathbf{T = 748.6\ N}$

$\Sigma F_x = 0$

$A_x - T\cos(63.43°) = 0$

$\therefore A_x = \mathbf{334.8\ N \rightarrow}$

$\Sigma F_y = 0\ A_y - 890 - 445 + T\sin(63.43°) = 0$

$\therefore A_y = \mathbf{665.46\ N \uparrow}$

$\therefore R_A = \sqrt{A_x^2 + A_y^2} = 744.95\ N$

$\tan\theta = \dfrac{A_y}{A_x}$

$\therefore \theta = 63.29°$ in 1st quadrant

Chapter 3 : Friction [Total Marks - 12]

Q. 1(c) *A homogeneous cylinder 3m diameter and weighting 400 N is resting on two rough inclined surfaces as shown. If the angle of friction is 15° find couple 'C' applied to the cylinder that will start it rotating clockwise.* **(4 Marks)**

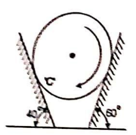


Fig. 1-Q.1(c)

Ans. :

Angle of friction $\phi = 15°$ $\therefore \mu = \tan\phi$ $\therefore \mu = 0.27$

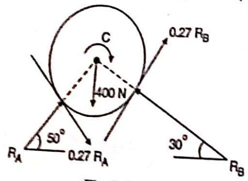


Fig. 2-Q.1(c)

$\Sigma F_x = 0,\quad R_A \cos 50° + 0.27\,R_A \cos 40° - R_B \cos 30° + 0.27\,R_B \cos 60° = 0$

$\therefore 0.85\,R_A - 0.73\,R_B = 0$ ...(1)

$\Sigma F_y = 0,\quad R_A \sin 50° - 0.27\,R_A \sin 40° + R_B \sin 30° + 0.27\,R_B \sin 60° - 400 = 0$

$\therefore 0.59\,R_A + 0.73\,R_B = 400$ ...(2)

Solving Equations (1) and (2) we get,

$\mathbf{R_A = 277.78\ N}$

$\mathbf{R_B = 323.44\ N}$

$\Sigma M_c = 0\quad (0.27\,R_A \times 1.5) + (0.27\,R_B \times 1.5) - C = 0$

$\therefore$ **Couple C = 243.5 N.m**

Q. 6(b) Ref to Fig. 1-Q.6(b) If the coefficient of friction is 0.60 for all contact surfaces and $\theta = 30°$, what force 'P' applied to the block 'B' acting down and parallel to the incline will start motion and what will be the tension in the cord parallel to inclined plane attached to 'A'? Take $W_A = 120N$ and $W_B = 200N$. **(8 Marks)**

Ans. :

For block A :

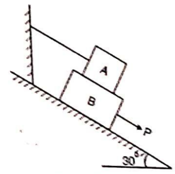


Fig. 1-Q.6(b)

$\Sigma F_x = 0,$ (Parallel to plane) $120 \sin 30° - T + 0.6\,R_2 = 0$ ...(1)

$\Sigma F_y = 0,$ (Perpendicular to plane) $R_2 - 120 \cos 30° = 0$

$\therefore \mathbf{R_2 = 103.92\ N}$

Substitute in Equation (1)

$\mathbf{T = 122.35\ N}$

For block B :

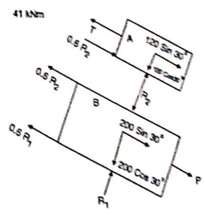


Fig. 2-Q.6(b)

$\Sigma F_y = 0,$ ($\perp$r to plane)

$R_1 - 200 \cos 30° - R_2 = 0$

$\therefore \mathbf{R_1 = 277.125\ N}$

$\Sigma F_x = 0,\quad P + 200 \sin 30° - 0.6\,R_1 - 0.6\,R_2 = 0$

$\therefore P = 0.6\,(277.125) + 0.6\,(103.92) - 200 \sin 30°$

$\mathbf{P = 128.63\ N}$

Chapter 4 : Trusses [Total Marks - 08]

Q. 5(a) In the truss shown in Fig. 1-Q.5(a) compute the forces in each member. **(8 Marks)**

...yahoo.co.in ...and problems. ...of standard functions.
Self learning topics : Jacobian's of two ...
problems).

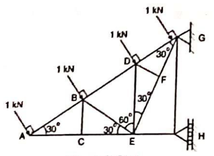


Fig. 1-Q.5(a)

Ans. :

Here dimensions are not given so

Assume AB = BD = DG = a

$\therefore GH = 1.5\,a$

$\Sigma M_G = 0$

$(1 \times a) + (1 \times 2a) + (1 \times 3a) - R_H \times 1.5a = 0$

$\therefore R_H = 4\text{ kN} \leftarrow$

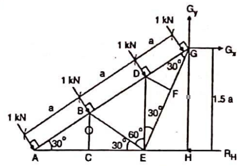


Fig. 2-Q.5(a)

(1) Joint A

$$\Sigma F_y = 0$$

$F_{AB} \sin 30° - 1 \sin 60° = 0$

$\therefore F_{AB} = 1.732\text{ kN}$

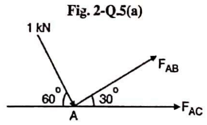


Fig. 3-Q.5(a)

$\Sigma F_x = 0 \qquad F_{AC} + F_{AB} \cos 30° + 1 \cos 60° = 0$

$\therefore F_{AC} = -2\text{ kN} \qquad \therefore F_{CE} = -2\text{ kN}$

(2) Joint B

$$\Sigma F_x = 0$$

$F_{BD} \cos 30° + F_{BE} \cos 30° + 1 \cos 60° - 1.732 \cos 30° = 0$

$\therefore F_{BD} \cos 30° + F_{BE} \cos 30° = 1 \qquad \ldots(1)$

$\Sigma F_y = 0$

$F_{BD} \sin 30° - F_{BE} \sin 30° - 1 \sin 60° - 1.732 \sin 30° = 0$

$\therefore F_{BD} \sin 30° - F_{BE} \sin 30° = 1.732 \qquad \ldots(2)$

Solving Equation (1) and (2)

$F_{BD} = 2.31\text{ kN}$

$F_{BE} = -1.15\text{ kN}$

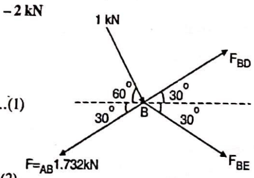


Fig. 4-Q.5(a)

(3) Joint H, by observation

$F_{EH} = -4\text{ kN}$

$F_{GH} = 0$

(4) Joint E

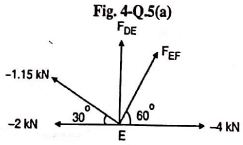


Fig. 5-Q.5(a)

$\Sigma F_x = 0 \quad -4 + F_{EF} \cos 60° + 1.15 \cos 30° + 2 = 0$

$F_{EF} = 2\text{ kN}$

$\Sigma F_y = 0 \quad F_{EF} \sin 60° + F_{DE} - 1.15 \sin 30° = 0$

$\therefore F_{DE} = -1.15\text{ kN}$

(5) **Joint F :** by observation

$F_{DF} = 0, F_{EF} = F_{GF} = 2\text{ kN}$

(6) **Joint D :**

$\Sigma F_x = 0 \quad F_{DG} \cos 30° + 1 \cos 60° - 2.31 \cos 30° = 0$

$F_{DG} = 1.732\text{ kN}$

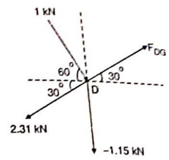


Fig. 6-Q.5(a)

Chapter 5 : Centroid and Center of Gravity [Total Marks - 08]

Q. 3(a) Find the centroid of the shaded portion of the plate shown in the Fig. 1-Q.3(a). **(8 Marks)**

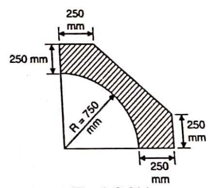


Fig. 1-Q.3(a)

Ans. :

Step 1 : Area calculation :

$A_{1\,(square)} = 1000 \times 1000 = 10^6\text{ mm}^4$

$A_{2\,(triangle)} = \frac{1}{2} \times 750 \times 750 = 281250\text{ mm}^2$

$A_{3\,(quarter\ circle)} = \frac{\pi r^2}{4} = \frac{\pi (750)^2}{4}$

$= 441786.47\text{ mm}^2$

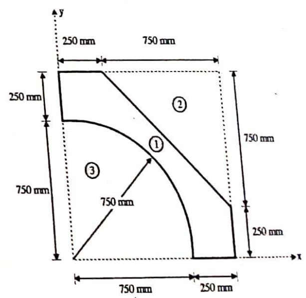


Fig. 2-Q.3(a)

Step 2 : Centroidal distances calculations :

$x_1 = 1000/2 = 500\text{ mm}$

$x_2 = 250 + (2/3 \times 750) = 750\text{ mm}$

$x_3 = \frac{4r}{3\pi} = \frac{4\,(750)}{3\pi} = 318\text{ mm}$

Step 3 : Co-ordinates of centriod 'C' :

$$\therefore \bar{x} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3} = \frac{(10^6 \times 500) - (281250 \times 750) - (441786.47 \times 318)}{10^6 - 281250 - 441786.47}$$

$= 536.44$ mm

Due to symmetry $\bar{y} = 536.44$ mm

$\therefore$ **Centroid C (536.44, 536.44) mm** ...**Ans.**

Chapter 6 : Space Forces[Total Marks - 10]

Q. 3(b) Co-ordinate distance are in 'm' units for the space frame in Fig. 1-Q.3(b) there are 3 members AB AC and AD. There is a force W-10 KN acting at A in a vertically upward direction Determine the tension in AB, AC and AD. **(6 Marks)**

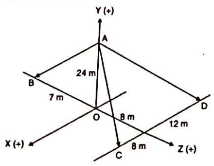


Fig. 1-Q.3(b)

Ans. :

Force Vectors :

$$\bar{F}_{AB} = F_{AB} \cdot \bar{e}_{A\to B} = F_{AB}\left[\frac{0\,\hat{i} - 24\hat{j} - 7\hat{k}}{\sqrt{(0)^2 + (24)^2 + (7)^2}}\right]$$

$$= F_{AB}\left[0\,\hat{i} - \frac{24}{25}\hat{j} - \frac{7}{25}\hat{k}\right]$$

$$\bar{F}_{AC} = F_{AC} \cdot \bar{e}_{A\to C} = F_{AC}\left[\frac{8\,\hat{i} - 24\hat{j} + 8\hat{k}}{\sqrt{(8)^2 + (24)^2 + (8)^2}}\right]$$

$$= F_{AC}\left[\frac{8}{\sqrt{704}}\hat{i} - \frac{24}{\sqrt{704}}\hat{j} + \frac{8}{\sqrt{704}}\hat{k}\right]$$

$$\bar{F}_{AD} = F_{AD} \cdot \bar{e}_{A\to B} = F_{AD}\left[\frac{-12\,\hat{i} - 24\hat{j} + 8\hat{k}}{\sqrt{(12)^2 + (24)^2 + (8)^2}}\right]$$

$$= F_{AD}\left[\frac{-12}{28}\hat{i} - \frac{24}{28}\hat{j} + \frac{8}{28}\hat{k}\right]$$

$$\bar{W} = 10\,\hat{j}\ (kN)$$

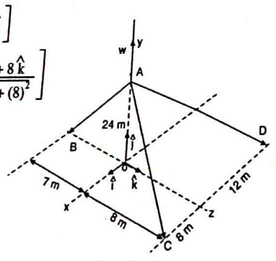


Fig. 2-Q.3(b)

Apply conditions of equilibrium

$$\Sigma F_x = 0, \quad (0)\,F_{AB} + \frac{8}{\sqrt{704}}F_{AC} - \frac{12}{28}F_{AD} + 0 = 0 \quad ...(i)$$

$$\Sigma F_y = 0, \quad \frac{-24}{25}F_{AB} - \frac{24}{\sqrt{704}}F_{AC} - \frac{24}{28}F_{AD} + 10 = 0 \quad ...(ii)$$

$$\Sigma F_z = 0, \quad \frac{-7}{25}F_{AB} + \frac{8}{\sqrt{704}}F_{AC} + \frac{8}{28}F_{AD} + 0 = 0 \quad ...(iii)$$

Solving (i), (ii) and (iii) we get,

$$\mathbf{F_{AB} = 5.55\ kN \quad F_{AC} = 3.095\ kN \quad F_{AD} = 2.178\ kN}$$

Q 6(a) A force of 140 KN passes through point C (– 6, 2, 2) and goes to point B (6, 6, 8) calculate moment of force about origin. **(4 Marks)**

Ans. : F = 140 kN passes through points C(– 6, 2, 2) to B (6, 6, 8).

$\therefore$ Force vector

$$\bar{F} = F \cdot \bar{e}_{C\to B} = 140\left[\frac{(6+6)\,\hat{i} + (6-2)\,\hat{j} + (8-2)\,\hat{k}}{\sqrt{(12)^2 + (4)^2 + (6)^2}}\right]$$

$$\bar{F} = \mathbf{120\,\hat{i} + 40\,\hat{j} + 60\,\hat{k}}$$

Moment of force about origin

$$\bar{M}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 8 \\ 120 & 40 & 60 \end{vmatrix} = \hat{i}\,[40] - \hat{j}\,[-600] + \hat{k}\,[-480]$$

$$\bar{M}_0 = 40\,\hat{i} + 600\,\hat{j} - 480\,\hat{k}$$

$$\therefore |\bar{M}_0| = \mathbf{769.41\ kN.m}$$

Chapter 7 : Virtual Work [Total Marks - 04]

Q. 6(c) Determine the required stiffness 'K' so that the uniform 7Kg bar AC in equilibrium when θ = 30°. Due to the collar guide at B the spring remains vertical and is unscratched when θ = 0 Use Principle of Virtual Work. **(4 Marks)**

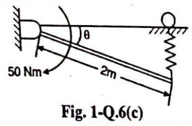


Fig. 1-Q.6(c)

Ans. :

Active force / couple	Co-ordinate	Virtual displacement
– mg	$-1\sin\theta$	$-\cos\theta \cdot d\theta$
+ F	$-2\sin\theta$	$-2\cos\theta \cdot d\theta$
M	θ	$d\theta$

$\therefore$ for equilibrium

$$dU = 0$$

$$\therefore -mg(-\cos\theta\, d\theta) + F(-2\cos\theta\, d\theta) + M\, d\theta = 0$$

$$\therefore Mg\cos\theta + M = 2F\cos\theta$$

$$\therefore F = \frac{mg}{2} + \frac{M}{2\cos\theta} = \frac{7\times 9.81}{2} + \frac{50}{2\cos 30^\circ} = 63.2\ N$$

Now spring force $F = k\delta$

$$63.2 = k\times 2\sin 30^\circ$$

$$k = 63.2\ N/m$$

Chapter 8 : Kinematics of Particles (Rectilinear Motion) [Total Marks - 16]

Q. 1(d) From (v-t) diagram find (i) distance travelled in 10 sec. (ii) total distance travelled in 50 sec. (iii) Retardation. **(4 Marks)**

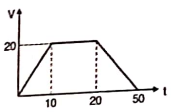


Fig. 1-Q.1(d)

Ans. :

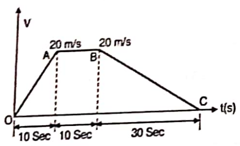


Fig. 2-Q.1(d)

(i) Distance travelled in 10 sec is the area of v – t diagram (from $0 \to 10$ sec)

$$\therefore s = \frac{1}{2}\times 10\times 20 = 100\ m$$

(ii) Total distance travelled in 50 sec $= \left(\frac{1}{2}\times 10\times 20\right) + (20\times 10) + \left(\frac{1}{2}\times 30\times 20\right) = 600\ m$

(iii) Retardation $=$ slope BC $= -\frac{20}{30} = -0.67\ m/s^2$

Q. 4(b) The acceleration of the train starting from rest at any instant is given by the expression $a = \frac{8}{(V^2+1)}$ where V is the velocity of train in m/s. Find the velocity of the train when its displacement is 20 m and its displacement when velocity is 64.8 kmph. **(6 Marks)**

Ans. :

Given relation $\quad a = \frac{8}{v^2+1}$

Using $\quad a = v\frac{dv}{ds} = \frac{8}{(v^2+1)}$

$$\therefore v(v^2+1)\, dv = 8\, ds$$

$$(v^3+v)\, dv = 8\, ds$$

Integrating both sides, $\int (v^3+v)\, dv = 8\int ds$

$$\frac{v^4}{4} + \frac{v^2}{2} = 8s + C_1$$

At $t = 0, v = 0, s = 0 \quad \therefore c_1 = 0$

$$\therefore 8s = \frac{v^4}{4} + \frac{v^2}{2} \quad ...(1)$$

Velocity when displacement is 20 m.

$$8(20) = \frac{v^4}{4} + \frac{v^2}{2}$$

$$160 = \frac{v^4 + 2v^2}{4}$$

$$v^4 + 2v^2 = 640$$

Solving we get $v^2 = 24.32$

$$\therefore v = 4.93\ m/s \quad ...Ans.$$

Displacement when velocity is 64.8 kmph = 18 m/s

$$8s = \frac{(18)^4}{4} + \frac{(18)^2}{2} \qquad \therefore s = 3300.75\ m$$

Q. 5(b) Determine the speed at which the basket ball at 'A' must be thrown at an angle of 30° so that it makes it to the basket at B. Also find at what speed it passes through the hoop. **(6 Marks)**

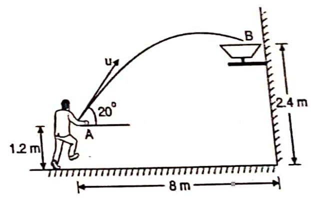


Fig. 1-Q.5(b)

Ans. :

Consider motion from $A \to B$

$$x = 8\ m$$

$$y = -(2.4 - 1.2) = -1.2\ m$$

$$\alpha = 30^\circ$$

Using equation of trajectory

$$y = x\tan\alpha - \frac{g x^2}{2u^2\cos^2\alpha}$$

...(1)

...Ans.

.75 m

n angle of 30° so that if
e hoop. (6 Marks)

$$\therefore -1.2 = 8\tan(30°) - \frac{9.81\,(8)^2}{2u^2(\cos 30)^2}$$

$$\therefore \quad -1.2 = 4.62 - \frac{418.56}{u^2}$$

$$\therefore \quad \frac{418.56}{u^2} = 4.62 + 1.2 \qquad \therefore\ u = 8.48\ \text{m/s}$$

Velocity at B when it passes Hoop at B.

x component of velocity $V_{B_x} = 8.48\cos 30° = 7.34\ \text{m/s} \rightarrow$

To find y component of velocity at B first finding time from $A \rightarrow B$

$$S_x = u_x \times t \qquad 8 = 8.48\cos 30° \times t \qquad \therefore t = 1.089\ \text{sec}$$

Now consider y – motion from $A \rightarrow B$

$$v_y = u_y - gt$$

$$\underset{(B)}{v_y} = 8.48\sin 30° - 9.81\,(1.089)$$

$$\underset{(y)}{v_B} = -6.44\ \text{m/s} = 6.44\ \text{m/s} \downarrow$$

$$\therefore V_B = \sqrt{(V_{Bx})^2 + (V_{By})^2} = \sqrt{(7.34)^2 + (6.44)^2} = 9.76\ \text{m/s}$$

Chapter 9 : Kinetics of Particles (D'Alembert's Principle) [Total Marks - 04]

Q. 6(d) The system shown in Fig. 1-Q. 6(d) is initially at rest. Neglecting friction determine the force 'P' required if the velocity of the collar B is m/s after 2 sec and corresponding tension in the cable. **(4 Marks)**

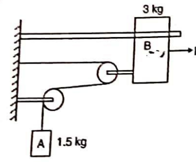


Fig. 1-Q. 6(d)

Ans. :

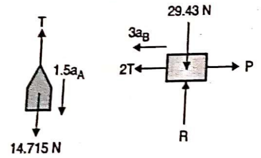


Fig. 2-Q.6(d)

For B :

Initial velocity $u = 0$ Final velocity $v = 5$ m/s

Time $t = 2$ sec $v = u + at$

$5 = 0 + a_B\,(2)$ $\therefore a_B = 2.5\ \text{m/s}^2$

Also $a_A = 2\,a_B$ $\therefore a_A = 5\ \text{m/s}^2$

For A :

$\Sigma F_y = 0 \qquad T - 1.5\,a_A - 14.715 = 0$

$\therefore T = 14.715 + 1.5(5) \qquad T = 22.215\ \text{N}$

For B :

$\Sigma F_x = 0 P - 2T - 3a_B = 0$

$\therefore P = 2(22.215) + 3\,(2.5) \qquad P = 51.93\ \text{N} \rightarrow$

Chapter 10 : Work Energy Principle [Total Marks - 10]

Q. 1(e) Block P_1 and P_2 are connected by inextensible string. Find velocity of block P_1, if it falls by 0.6m starting from rest. The coefficient of ariction is 0.2, pulley is friction less. **(4 Marks)**

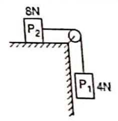


Fig. 1-Q.1(e)

Ans. :

Using work Energy principle for entire system

W.D. calculation :

For block P_1 :

(i) W.D. by gravity $= mgh = 4 \times 0.6 = 2.4$ Joules

For block P_2 :

(ii) W.D. by friction $= -\mu\, mgs = -0.2 \times 8 \times 0.6 = -0.96$ Joules

K.E. Calculation :

Initial K.E. of system $= 0$

$$\text{Final K.E. of system} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}\times\frac{4}{9.81}v_1^2 + \frac{1}{2}\times\frac{8}{9.81}v_1^2\ [\because v_2 = v_1]$$

$$= 0.61 v_1^2$$

Now, by work – energy principle,

Total workdone = Final KE – Initial K.E.

$$\therefore 0.61\,v_1^2 = 2.4 - 0.96$$

$$\therefore v_1 = 1.536\ \text{m/s} \downarrow$$

...Ans.

Q. 3(c) A 50N collar slides without friction along a smooth rod which is kept inclined at 60° to the horizontal. The spring is attached to the collar and the support 'C'. The spring is unstretched when the collar is at 'A' (AC is horizontal) Determine the value of spring constant 'K' given that the collar has a velocity of 2.5 m/s when it has moved 0.5m along the rod as shown in Fig.1-Q.3(c). **(6 Marks)**

gv_kumbhojkar@yahoo.co.in

...fferentiation : n^{th} derivative of standard functions. Le...
(without proof) and problems.
Self learning topics : Jacobian's of two and three independent v...
problems).
Modul...

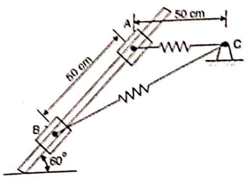


Fig. 1-Q.3(c)

Ans. :

Work-energy principle :

Applying work energy principle between A (position 1) and B (position 2).

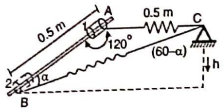


Fig. 2-Q.3(c)

Workdone calculation :

(i) W.D. by gravity $= mgh = 50 \times 0.435 = 21.75\ J$

(ii) W.D. by spring force

$= \frac{1}{2} k \left[x_1^2 - x_2^2 \right]$

Where, $x_1 = L_1 - L_0 = 0.5 - 0.5 = 0$

$x_2 = L_2 - L_0 = 0.87 - 0.5 = 0.37\ m$

$\therefore$ W.D. $= \frac{1}{2} k [(0) - (0.37)^2]$

$= -0.06845\ k$ Joules

$kE_1 = 0$ BC

$kE_2 = \frac{1}{2} mv_2^2 = \frac{1}{2} \times \frac{50}{9.81} (2.5)^2$

$= 15.93\ J$

using $U_{1\to 2} = kE_2 - kE_1$

$21.75 - 0.06845\ k = 15.93 - 0$

$\therefore$ **k = 85.02 N/m**

Geometry :

In Δ CAB, by sine rule,

$$\frac{0.5}{\sin \alpha} = \frac{0.5}{\sin (60 - \alpha)} = \frac{BC}{\sin 120°}$$

$\therefore \sin (60 - \alpha) = \sin \alpha$

$60 - \alpha = \alpha$

$\therefore$ **α = 30**

$= 0.87\ m$

Height $h = BC \sin 30°$

$= 0.87 \sin 30° = 0.435\ m$

Chapter 11 : Impulse and Momentum [Total Marks - 06]

Q. 2(c) Just before they collide, two disks on a horizontal surface have velocities shown in Fig. 1-Q.2(c) Knowing that 90N disk 'A' rebounds to the left with a velocity of 1.8 m/s. Determine the rebound velocity of the 135 N. disk 'B' Assume the impact is perfectly elastic. **(6 Marks)**

Ans. :

Mass of disk A $m_A = \frac{90}{9.81} = 9.174$ kg

Mass of disk B $m_B = \frac{135}{9.81} = 13.76$ kg

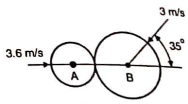


Fig. 1-Q.2(c)

Before impact :

Velocity of A in the direction of impact $u_A = 3.6$ m/s →

Velocity of B in the direction of impact $u_B = -3 \cos 35° = -2.46$ m/s (←)

After impact :

Velocity of A in the direction of impact $v_A = -1.8$ m/s

$\therefore$ By the principle of conservation of momentum

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(9.174 \times 3.6) + 13.76 (-2.46) = 9.174 (-1.8) + 13.76\ v_B$$

$$\therefore v_B = 1.14 \text{ m/s} \rightarrow$$

In tangential direction

For A $v_y = 0$ This remains same after impact

For B $v_x = 1.14$ m/s →

and $v_y = -3 \sin 35° = -1.72$ m/s

$\therefore v_B = \sqrt{v_x^2 + v_y^2}$

v_B **= 2.063 m/s**

Chapter 12 : Kinematics of Rigid Bodies [Total Marks - 12]

Q. 4(c) Angular velocity of connector BC is 4 r/s in clockwise direction. What are the angular velocities of cranks AB and CD. **(6 Marks)**

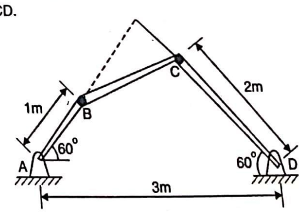


Fig. 1-Q.4(c)

Ans. :

For AB (ICR is A)

$v_A = 0,\ v_B = r_{BA}\ \omega_{AB}$ $\therefore v_B = 1 \times \omega_{AB}$ ...(1)

For BC : (ICR is I)

$$\therefore \quad v_B = r_{BI}\,\omega_{BC}$$
$$\therefore \quad v_B = 2(4)$$
$$v_B = 8 \text{ m/s}$$

$$v_c = r_{CI}\,\omega_{BC}$$
$$\therefore \quad v_c = 1 \times 4 = 4 \text{ m/s}$$

for CD (ICR is D)

$$\therefore \quad v_D = 0,$$

$$v_C = r_{CD}\,\omega_{CD}$$

$$4 = 2 \times \omega_{CD}$$
$$\omega_{CD} = \frac{2 \text{ rad}}{s} \circlearrowleft$$

From Equation (1) $8 = 1 \times \omega_{AB}$

$$\therefore \quad \omega_{AB} = 8 \text{ rad/s} \circlearrowleft$$

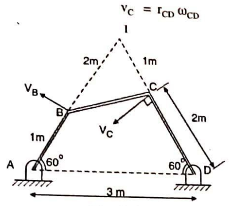


Fig. 2-Q.4(c)

Q. 5(c) Fig. 1-Q. 5(c) shows a collar B which moves upwards with constant velocity of 1.5 m/s. At the instant when θ = 50° determine (i) The Angular velocity of rod pinned at B and freely resting at A against 25 ° sloping ground and (ii) the velocity of end A of the rod. **(6 Marks)**

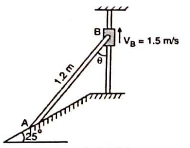


Fig. 1-Q.5(c)

Ans. :

The velocity of collar B is $V_B = 1.5 \frac{m}{s} \uparrow$ so, the corresponding velocity of end A must be along the plane (up the plane). To locate ICR of rod AB draw perpendiculars to v_A and v_B

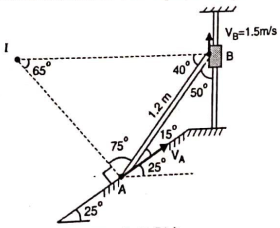
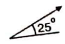


Fig. 2-Q.5(c)



For rod AB (ICR is I)

(i) $v_A = r_{AI}\,\omega_{AB}$ ...(1)

Substitute in Equation (1)

$$\therefore \quad v_A = 0.85\,\omega_{AB}$$
$$v_A = 0.9945 \text{ m/s}$$

(ii) $V_B = r_{BI}\,\omega_{AB}$ ...(2)

Calculation for distance AI, BI

$$1.5 = 1.28\,\omega_{AB}$$

In ΔAIB, by sine rule

$$\omega_{AB} = 1.17 \frac{\text{rad}}{s} \circlearrowleft$$

$$\frac{AI}{\sin 40°} = \frac{1.2}{\sin 65°} = \frac{BI}{\sin 75°}$$

$$\therefore \quad AI = 0.85 \text{ m}$$
$$BI = 1.28 \text{ m}$$

□□□

Dec. 2016

Q. 1(a) Find the force F_4, so as to give the resultant of the force system shown below. **(4 Marks)**

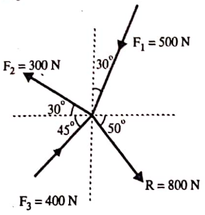


Fig. 1

(b) A particle starts from rest from origin and its acceleration is given by, $a = \frac{k}{(x+4)^2}$ m/s^2. Knowing that V = 4 m/s when x = 8m, find (i) value of k and (ii) Position when V = 4.5 m/s. **(4 Marks)**

(c) Rod AB of length 3m is kept on smooth planes as shown in Fig. The velocity of end A is 5m/s along the inclined plane. Locate the ICR and find the velocity of end B. **(4 Marks)**

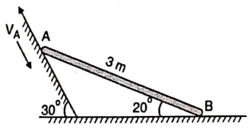


Fig. 2

(d) What is Zero force member in a Truss, With exampoles state the conditions for a zero force member. **(4 Marks)**

(e) A glass ball is dropped onto a smooth horizontal floor from which it bounces to a height of 9m. On the second bounce if rises to a height of 6m. from what height the ball was dropped and wha is the coefficient of restitution between glass and the floor. **(4 Marks)**

Q. 2 (a) Figure shows a beam AB hinged at A and roller supported at B. The L shaped portion is welded at D to the beam AB. For the loading shown, find the support reactions. **(6 Marks)**

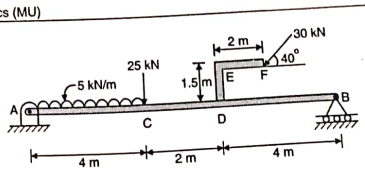


Fig. 3

(b) The acceleration-time diagram for linear motion is shown. Construct velocity-time diagram and displacmenet-time diagram for the motion assuming that the motion starts with initial velocity of 5m/s from starting point.

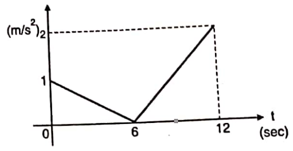


Fig. 4

(c) The resultant of three concurrent space forces at A is $\bar{R} = (-788)$ N. Find the magnitude of F1, F2 and F3 forces. **(6 Marks)**

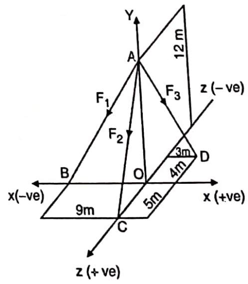


Fig. 5

Q. 3(a) Two spheres A and B of weight 1000N and 750N respectively are kept as shown in Fig. Determine the reactions at all contact points 1,2,3 and 4. Radius of A is 400 mm and Radius of B is 300mm. **(8 Marks)**