SETSQUARE ACADEMY

Degree Engineering (Mumbai University)

F.E. Semester - I

Previous Year Paper Solutions

(December 2007 - May 2016)

Basic Electrical Engineering

Common for all Branches

Chapter 2: A.C. CIRCUITS

Theory Questions

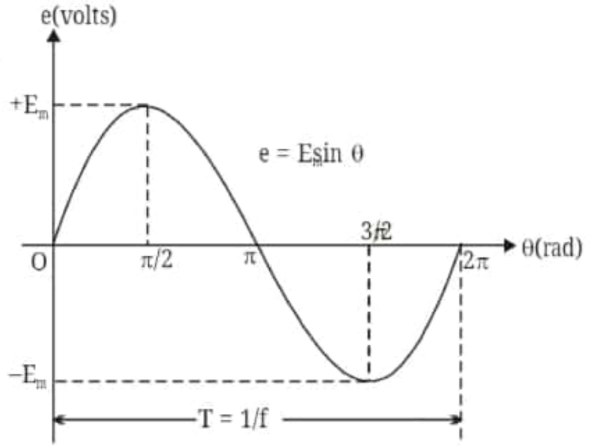
(1) Draw an a.c, waveform, indicate there on and explain (i) instantaneous value, (ii) peak value and , (iii) time period for one cycle of the alternating quantity **[M-15][3]**

Solution:

An alternating quantity e.g. an emf, current or voltage does not have fixed value, its values go on changin instantly. An AC quantity keeps on alternating positively and negatively during one cycle.

A sinusoidally varying and alternating emf can be expressed as:

$$e = E_m \sin(\theta) = E_m \sin(\omega t) = E_m \sin(2\pi f t) = E_m \sin(2\pi \frac{1}{T} t)$$



where,

e = Instantaneous value, in volts
The value at a particular instant is called Instantaneous value

E_m = Peak value in volts,
Maximum value attained by the quantity is called as the peak value.

T = time period in sec.
The time taken by an alternating quantity to complete one cycle is called as the time period(T).

(2) Derive an expression for the average value of a sinusoidally varying current in terms of Peak Value. **[D-13][3]**

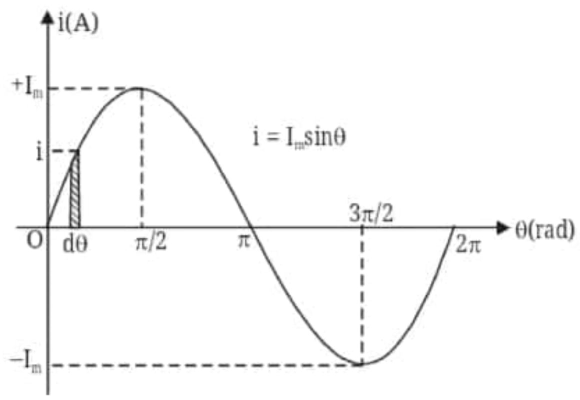
Solution:

The average value (I_{av}) of an AC is represented by that direct current (DC) which will transfer the same amount of charge across any circuit while flowing through the same time as is transferred by the given ac.

Let us now derive the expression for I_{av} in terms of its max value I_m.

Let the sinusoidally varying and alternating current be expressed as $i = I_m \sin\theta$.

Fig. below represents one cycle of its waveform.



Consider an elemental strip with base = $d\theta$.

Let i be its height $\therefore$ area of the strip = $i.d\theta$

$\because$ the waveform is symmetircal

i.e. + 1/2 cycle = –1/2 cycle,

$\therefore$ consider average value over 1/2 cycle only.

Thus, average value = $\dfrac{\text{area under } 1/2 \text{ cycle}}{\theta \text{ for } 1/2 \text{ cycle}}$

$$\therefore I_{av} = \frac{\int_0^{\pi} i\,d\theta}{\pi} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}[-\cos\pi - (\cos 0)] = \frac{I_m}{\pi}[-(-1) - (-1)] = \frac{2 I_m}{\pi}$$

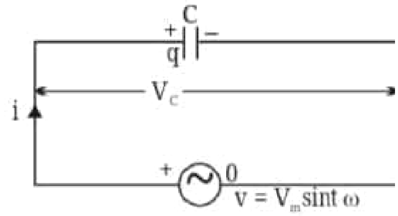
$$\therefore\ I_{av} = \frac{2I_m}{\pi} = 0.637\ I_m$$

Thus, in the case of a sinusoidally varying and alternating quantity (SVAQ) only,

Average value = 0.637 × Peak value.

(3) With proper phase diagrams, explain behaviour of a pure capacitor in a AC circuit. **[D-13][4]**

Solution:-



(a)

(i) **Circuit Diagram:** It is a shown in fig.:

The applied voltage can be expressed as

$v = V_m \sin\omega t$ ....(i)

Let v_c = p.d. across the capacitor plates.

(ii) **To establish the equation of current:**

At any instant, charge on capacitor $q = Cv_c = CV_m \sin\omega t$

$$\therefore\quad i = \frac{dq}{dt} = C\frac{d}{dt}(V_m \sin\omega t) = \omega C V_m \cos\omega t = \frac{V_m}{1/\omega C}\sin(\omega t + 90^\circ)$$

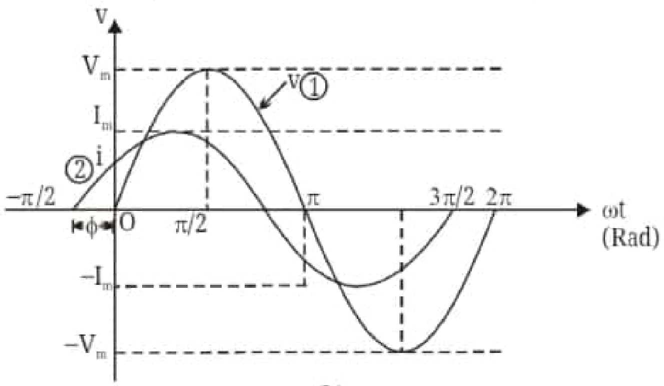
Clearly, i will be max i.e. $I_m = \dfrac{V_m}{1/\omega C}$ when $\sin(\omega t + 90^\circ) = 1$

thus, we get $i = I_m \sin(\omega t + 90^\circ)$ .....(ii)

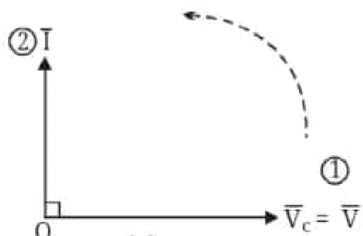
From equations (i) and (ii), it is seen that i leads v by 90°.

(iii) **Waveforms** of v and i are as shown in Figure (b).

(iv) **Phasor Diagrams:** are as shown in Figure (c).



(b)



(c)

(v) **Opposition to current flow:**

Clearly, it is $\dfrac{1}{\omega C} = \dfrac{1}{2\pi f\, C}$ and is represented as X_c and called as capacitive reactance.

It is measured in Ω.

For DC supply as f = 0. $\therefore X_c = \infty$ and hence pure C acts as open circuit.

(vi) **Power factor angle(ϕ)** = +90° as $\bar{I}$ leads $\bar{V}$.

(vii) **Power factor(p.f.)** = cos(90°) = 0 leading $\because \bar{I}$ leads $\bar{V}$

Sometimes it is also denoted as ZPF leading.

(viii) **Power:**

Instantaneous power, $P = v.i = V_m \sin\omega t . I_m \sin(\omega t + 90°) = V_m I_m \sin\omega t . \cos\omega t = \dfrac{V_m I_m}{2} \sin 2\omega t$

$$\therefore \quad \text{Average power } P_{av} = \frac{\int_0^{2\pi} p\,d(\omega t)}{2\pi} = \frac{1}{2\pi}\frac{V_m I_m}{2}\int_0^{2\pi} \sin 2\omega t\,d(\omega t) = 0.$$

$\therefore$ Pure capacitance does not consume any power.

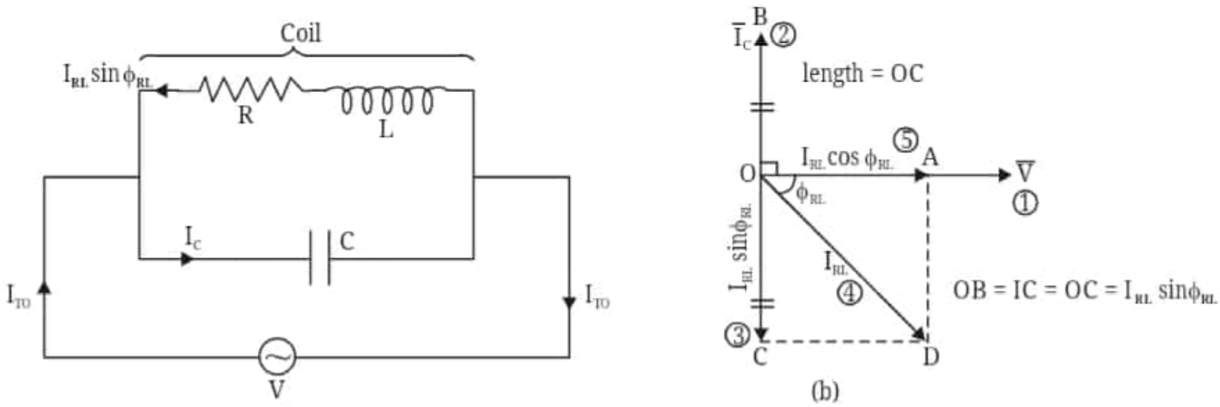
(4) Derive the equation for resonance frequency (f_r) in parallel resonance circuit.

[D-12][2], [M-14][3], [D-10][8]

Solution:

AC Parallel Resonance:

Consider a practical case of a coil in parallel with a capacitor as shown in the following Fig. Carefull note that the direction of I_C and ($I_{RL} \sin\phi_{RL}$) are opposite.



(a) (b)

We note the following points:

To derive the expression for the resonance frequency (f_0).

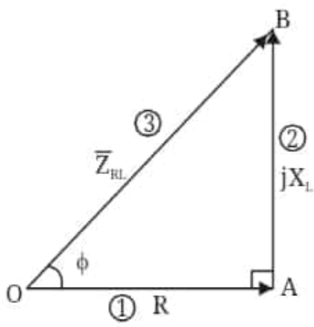
$$I_{RL_O} \sin\phi_{RL_O} = I_{C_O} \quad \text{...(i)}$$

From the circuit diagram during resonance,

$$I_{RL_O} = \frac{V}{Z_{RLO}} \text{ and } I_{C_O} = \frac{V}{Z_{C_O}}$$

From the Z-triangle, $\sin\phi_{RL_O} = \dfrac{X_{L_O}}{Z_{RL_O}}$

Substituting these values of I_{RL_O}, I_{C_O} and $\sin\phi_{RL_O}$ in the equation (i),



$$\frac{V}{Z_{RL_O}} \times \frac{X_{L_O}}{Z_{RL_O}} = \frac{V}{X_{C_O}} \quad \Rightarrow \quad \frac{1}{Z_{RL_O}} \times \frac{X_{L_O}}{Z_{RL_O}} = \frac{1}{X_{C_O}}$$

$$\therefore \quad X_{L_O} \times X_{C_O} = Z^2_{RL_O} \quad \Rightarrow \quad \omega_0 L \times \frac{1}{\omega_0 C} = Z^2_{RL_O}$$

$$\therefore \quad \frac{L}{C} = Z^2_{RL_0} = R^2 + X^2_{L_0} = R^2 + (2\pi f_0 L)^2$$

$$\therefore \quad (2\pi f_0 L)^2 = \frac{L}{C} - R^2 \Rightarrow \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ c/s}$$

(5) Give the comparison between series and parallel resonance circuits. **[M-13][3], [M-15][3], [D-08][5]**

Solution:

Comparison between series and Parallel Resonance :

No.	Parameter	Sereis Circuit (R + L + C)	Parallel Circuit (R + L) ∥ C
1.	Impedance at resonance	Z_0 is min. and = R	Z_d is max. and = L/CR
2.	Current at resonance	I_0 = max. and = V/R	I_{T_0} = min.and= $\frac{V}{L/CR}$
3.	P.f. at resonance (pf_0)	1	1
4.	It magnifies	Voltage	Current
5.	Q-factor	$\frac{\omega_0 L}{R}$	$\frac{\omega_0 L}{R}$
6.	Resonance frequent (f_0)	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

(6) Derive the expressions for resonant frequency, band width, condition for resonance in a series R-L-C circui Show variation of R,L,C,Z and current with respect to frequency. Mark band width also.

[M-16][3],[D-13][3],[M-11][10

Solution:

Resonant Frequency

An AC R-L-C series circuit is said to be under resonance when $X_{L_O} = X_{C_O}$

i.e net inductive reactance = net capacitive reactance.

When the given series circuit is in resoance, we observe following:

Let us now calculate resoance frequency f_0.

$$X_{L_O} = X_{C_O}$$

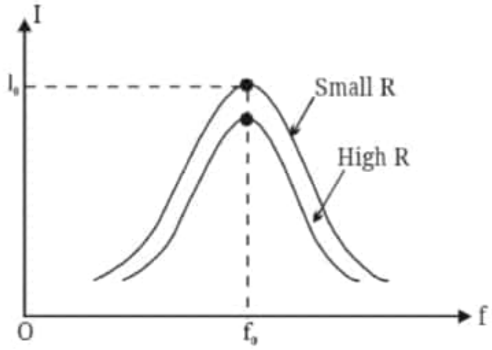
$$\therefore \quad 2\pi f_0 = \frac{1}{2\pi f_0 C} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ c/s} \quad \text{or } \omega_0 = \frac{1}{\sqrt{LC}} \text{ r/s}$$

Band width Δω.

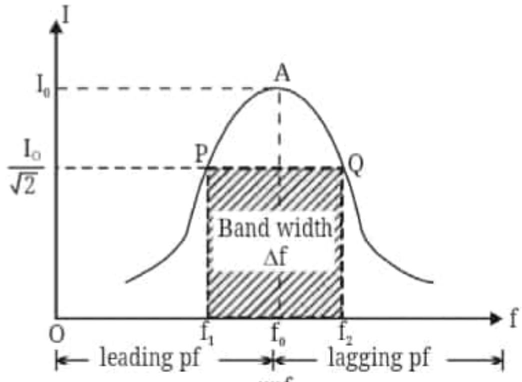
The band of frequency between f_2 and f_1 is called band width of circuit. It is measured in c/s or r/s.

Thus, $BW = \Delta f = (f_2 - f_1)$ c/s or $BW = \Delta\omega = (\omega_2 - \omega_1)$ r/s.

The following figure represents variation of circuit current with variation of supply frequency.



(a)



(b)

To find the values of ω_1, ω_2 and hence band width $\Delta\omega$.

Generally at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{....(i)}$$

At points P and Q,

$$I = \frac{I_0}{\sqrt{2}} \text{ but } I_0 = \frac{V}{R} \qquad \therefore \quad I = \frac{V}{\sqrt{2}R} \quad \text{.....(ii)}$$

From equations (i) and (ii), $\dfrac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \dfrac{V}{\sqrt{2}R}$

Squaring and inverting both the sides we get,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2 \quad \Rightarrow \quad \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\therefore \quad \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2 = R^2 \quad \text{or} \quad \left(\frac{\omega^2 LC - 1}{\omega C}\right) = \pm R$$

$\therefore \quad \omega^2 LC - 1 = \pm R\omega C$

$\therefore \quad LC\omega^2 \mp RC\omega - 1 = 0$

$\therefore \quad$ This is the equation governing AC series resonance.

(7) Define RMS value in alternating waveforms. **[M-12][2], [D-10][2], [M-08][2]**

Solution:

The RMS value (I_{RMS}) of an AC is represented by that direct current (DC) which will produce the same amount of heat while flowing through the same circuit during the same time as is produced by the given AC

$$\therefore \; I_{RMS} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

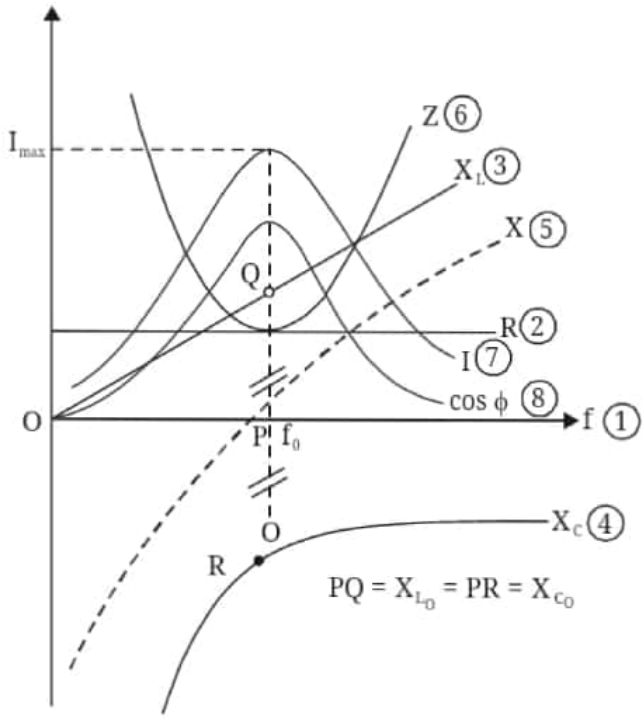
RMS value = 0.707 × max. value

All measuring instruments indicate RMS value, also all electrical apparatus are designed to work on RM values.

(8) Draw the resonance graph for the following:

(i) XL (ii) R (iii) Z (iv) cos ϕ (v) I **[D-11][10]**

Solution:



(9) Explain the quality factor in case of series resonance. **[M-10][4], [M-08][3]**

Solution:

The ratio of p.d. across inductance or capacitance V_{L_0} = (or V_{C_0}) to total voltage applied i.e. V durir AC series resonance is called as Quality factor. Thus,

$$\text{Q-factor} = \frac{V_{L_0}(\text{or } V_{C_0})}{V}$$

$$= \frac{I_0 X_{L_0}}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi L}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right) = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{\text{Resonance frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

(10) Explain the behaviour of ac through pure capacitor. Show that the average power consumed here is zero. **[M-09][5], [D-07][3]**

Solution:

Instantaneous power, $P = v.i = V_m \sin\omega t . I_m \sin(\omega t + 90^\circ)$

$$= V_m I_m \sin\omega t . \cos\omega t$$

$$= \frac{V_m I_m}{2}\sin 2\omega t$$

$\therefore$ Average power $P_{avg} = \dfrac{\int_0^{2\pi} p\,d(\omega t)}{2\pi} = \dfrac{1}{2\pi}\dfrac{V_m I_m}{2}\int_0^{2\pi}\sin 2\omega t\,d(\omega t) = 0.$

$\therefore$ Pure capacitance does not consume any power.

(11) What is the concept of phasor in AC circuits? **[M-08][3]**

Solution:

We know that a vector has only two specifications viz. (a) Magnitude and (b) Polarity or direction. However, in the case of alternating electrical quantities, we need a third specificaton i.e. phase, as they are time-dependant and that is why alternating electrical quantities are known as phasors.

Thus, to summerize, phasor has three specifications:

(a) magnitude, (b) direction or polarity and (c) phase.

Where phase is defined as that fractional part of time period or cycle through which the quantity has advanced from the selected zero position of the reference.

In electrical engineering, we are more interested in relative phases or phase difference beteen different alternating quantities rather than their absolute phases.

Numerical Problems

Type I : Time Expression

(1) An alternating current takes 3.375 ms to reach 15A for the first time after becoming instantaneously zero. The Frequency of the current is 40 Hz. Find the maximum value of the alternating current. **[M-14][3]**

Solution:-

$$i = I_m \sin 2\pi ft$$

$$\text{Here, } f = 40\text{Hz} \quad \Rightarrow \quad 2\pi f = 251.2$$

$$\therefore \quad i = I_m \sin (251.2)t$$

$$t = 3.375\,\text{ms} = 3.375 \times 10^{-3}\,\text{s} \quad \text{and } i = 15\,\text{A}$$

$$\therefore\ 15 = I_m \sin\left(251.2 \times 3.375 \times 10^{-3} \times \frac{180}{\pi}\right)$$

$$\therefore\ I_m = \frac{15}{0.7498} = 20\,\text{Amp}$$

(2) An alternating voltage is given by V = 141.4 sin 314t find: **[D-12][3]**

(i) Frequency (ii) R.M.S. value

(iii) Average value (iv) Instantaneous value of voltage when t is 3 msec.

Solution:-

$$V = 141.4 \sin 314\ \pi ft$$

Comparing with the standard equation ie. $V = V_m \sin 2\pi ft$

Here $V_m = 141.4$ and $2\pi f = 314$

(i) Frequency: $2\pi f = 314 = 50\text{Hz}$

(ii) R.M.S. Value: $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100\text{V}$

(iii) Average value: $V_{avg} = \frac{2V_m}{\pi} = \frac{2x141.4}{\pi} = 90V$

(iv) $V_{t=3msec}$: $V = 141.4\sin(314x3x10^{-3}) = 114.35\ V$

(3) An a.c. current is given by i = 14.14 sin (ωt + π/6). Find the rms value and phase angle of current. **[Dec 09][4]**

Solution:-

$i = 14.14 \sin(\omega t + \pi/6)$

rms value $(I_{rms}) = \frac{Im}{\sqrt{2}} = \frac{14.14}{\sqrt{2}}$

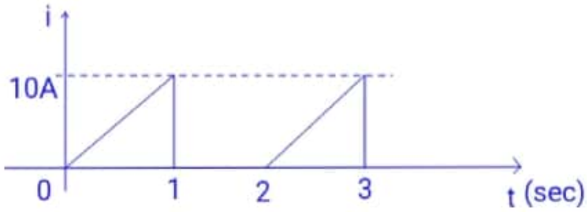
$I_{rms} = 9.998\ A \simeq 10A$

Phase angle $= \frac{\pi}{6} = 30^0$

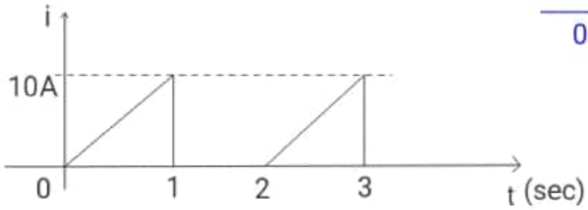
$\therefore$ The $I_{rms} = 9.998 \simeq 10A$ and Phase angle is 30^0

Type II : Wave Forms

(1) Find the rms value for the given waveform **[M-15][5]**



Solution:-



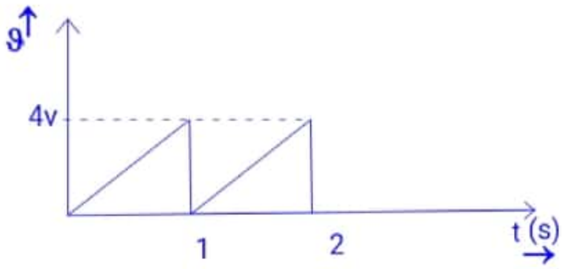
$T = 2,\ I(t) = 10\,t; \quad 0 < t \leq 1$

$= 0; \quad 1 \leq t \leq 2$

$$I_{rms} = \sqrt{\frac{1}{2}\int_0^1 (10t)^2 .dt} = \sqrt{\frac{1}{2}\int_0^1 100t^2 .dt} = \sqrt{50\left(\frac{t^3}{3}\right)_0^1}$$

$$\therefore\ I_{rms} = \sqrt{\frac{50}{3}} = \sqrt{16.6667} = 4.0825\,A$$

(2) Find the average value of the following waveform **[D-14][3]**



Solution:-

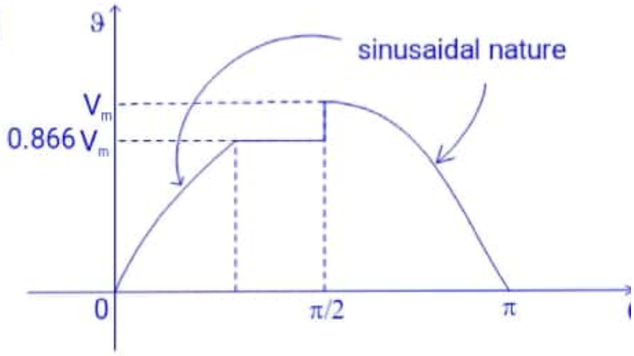
$T = 1$ sec, $V(t) = mt + C$

$m = \frac{4-0}{1-0} = 4,\ C = 0$

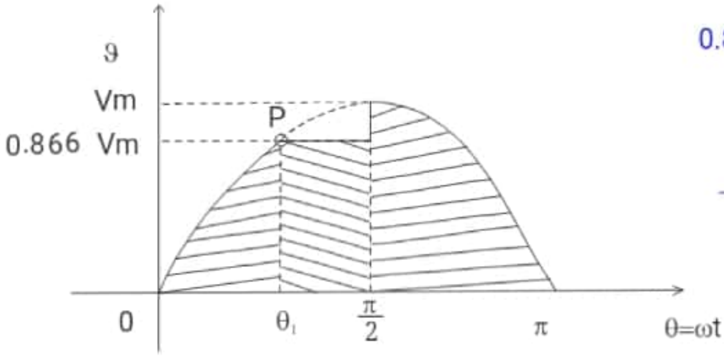
$\therefore\ V = 4t$

$$V_{Avg} = \frac{1}{T}\int_0^T V(t)\,dt = \frac{1}{1}\int_0^1 v.dt = \int_0^1 4t.dt = 4\left(\frac{t^2}{2}\right)_0^1 = 2V$$

(4) Find the r.m.s. value for the given waveform **[D-14][5]**



Solution:-



$$V = V_m \sin\theta \qquad 0 < \theta \le \pi/3$$

$$V = 0.866\,V_m \qquad \pi/3 \le \theta \le \pi/2$$

$$V = V_m \sin\theta \qquad \pi/2 \le \theta < \pi$$

$$V_{RMS} = \left[\frac{1}{\pi}\int_0^{\pi} V^2.d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^{\pi/3} V_m^2.\sin^2\theta.d\theta + \frac{1}{\pi}\int_{\pi/3}^{\pi/2}(0.866V_m)^2\,d\theta + \frac{1}{\pi}\int_{\pi/2}^{\pi} V_m^2\sin^2\theta.d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\pi}\int_0^{\pi/3}\frac{1-\cos 2\theta}{2}.d\theta + \frac{0.75V_m^2}{\pi}\int_{\pi/3}^{\pi/2} d\theta + \frac{V_m^2}{\pi}\int_{\pi/2}^{\pi}\frac{1-\cos 2\theta}{2}.d\theta\right]^{\frac{1}{2}}$$

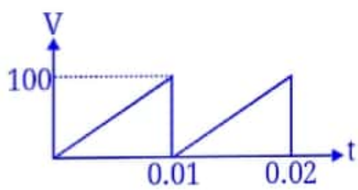
$$= \left[\frac{V_m^2}{2\pi}\left\{\theta - \frac{\sin 2\theta}{2}\right\}_0^{\pi/3} + \frac{0.75V_m^2}{\pi}\left\{\frac{\pi}{2} - \frac{\pi}{3}\right\} + \frac{V_m^2}{2\pi}\left\{\theta - \frac{\sin 2\theta}{2}\right\}_{\pi/2}^{\pi}\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi}\left\{\frac{\pi}{3} - \frac{\sin 2\pi/3}{2}\right\} + \frac{0.75V_m^2}{\pi} x \frac{\pi}{6} + \frac{V_m^2}{2\pi}\left\{\pi - 0 - \frac{\pi}{2} - 0\right\}\right]^{\frac{1}{2}}$$

$$= \left\{\frac{V_m^2}{2\pi}\left\{\frac{\pi}{3} - 0.433\right\} + \frac{0.75V_m^2}{\pi} + \frac{V_m^2}{2\pi} x \frac{\pi}{2}\right\}^{1/2}$$

$$= V_m\{0.09775 + 0.125 + 0.25\}^{1/2} = 0.6876\,V_m$$

(5) Determine the rms value of voltage waveform shown below: **[M-13][3]**



Soluion:-

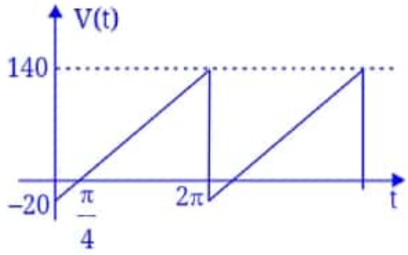
$T = 0.01$ sec, $V(t) = mt + C$ where $m = \frac{100}{0.01} = 10000$, $C = 0$

$$\therefore V = 1000t \qquad 0 < t < 0.01$$

$$\therefore V_{rms} = \sqrt{\frac{1}{0.01}\int_0^{0.01}(10000t)^2 dt} = \sqrt{\frac{1}{0.01} x (10000)^2 \left[\frac{t^3}{3}\right]_0^{0.01}}$$

$$= 57.735\,V$$

(6) Find the average and rms value for the wave form given below. **[May 11][5]**



Solution:

Step 1: Find expression for the waveform:

$$V(t) = mt + C \text{ where } m = \frac{160}{2\pi} \text{ and } C = -20$$

$$\therefore V(t) = \frac{160}{2\pi} \times t - 20$$

Step 2: Find average value:

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{160t}{2\pi} - 20\right\}dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{160t}{2\pi}dt - \frac{20}{2\pi}\int_0^{2\pi}dt = \frac{160}{(2\pi)^2}\left[\frac{t^2}{2}\right]_0^{2\pi} - \frac{20}{2\pi}[t]_0^{2\pi} = \frac{160}{2} - 20 = 60V$$

Step 3: Find RMS value:

$$V_{rms} = \left\{\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{160}{2\pi} - 20\right)^2 dt\right\}^{1/2} = \left\{\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{160}{2\pi}\times t\right)^2 - \frac{2\times 20\times 160}{2\pi}t + (20)^2\right\}^{1/2}$$

$$= \left\{\left[\left(\frac{160}{2\pi}\right)^2 \times \frac{t^3}{3}\times\frac{1}{2\pi} - \frac{1}{2\pi}\times\frac{2\times 20\times 160}{2\pi}\times\frac{t^2}{2} + \frac{(20)^2}{2\pi}\times t\right]_0^{2\pi}\right\}^{1/2}$$

$$= \left\{\frac{(160)^2\times(2\pi)^3}{(2\pi)^2\times 3\times 2\pi} - \frac{1}{4\pi^2}\times\frac{160\times 40}{2}\times 4\pi^2 + \frac{400}{2\pi}\times 2\pi\right\}^{1/2}$$

$$= \left\{\frac{(160)^2}{3} - (160\times 20) + 400\right\}^{1/2} = 75.72 \text{ V}$$

Type III : Phasor Algebra

(1) A circuit consists of three parallel branches. The branch currents are given as $i_1 = 10\sin\omega t$, $i_2 = 20\sin(\omega t + 60^o)$, and $i_3 = 75\sin(\omega t - 30)$. Find the resultant current and express it in the form $i = I_m \sin(\omega t + \phi)$. If the supply frequency is 50Hz, calculate the resultant current when (i) t = 0, (ii) t = 0.001 sec. **[M-14][5]**

Solution:-

$$i_1 = 10\sin\omega t,\ i_2 = 20\sin\left(\omega t + 60^o\right),\ i_3 = 75\sin\left(\omega t - 30\right)$$

Currents in parallel added

$$\therefore I_1 = \frac{10}{\sqrt{2}}\angle 0^o = 7.07\angle 0^o, \quad I_2 = \frac{20}{\sqrt{2}}\angle 60^o = 14.14\angle 60^o \quad I_3 = \frac{75}{\sqrt{2}}\angle -30^o = 53.03\angle -30^o$$

Now $i = i_1 + i_2 + i_3 = \left(7.07\angle 0^o\right) + \left(14.14\angle 60^o\right) + \left(53.03\angle -30^o\right) = 60.065 - j14.27$

$$i = 61.74\angle -13.36^o$$

$$\therefore i = 87.3135 \sin(\omega t - 13.36^\circ) \qquad ...(I_m = 61.74 \times \sqrt{2})$$

Now $f = 50$ Hz

(i) at $t = 0$ $\quad \therefore i = 87.3135 \sin\left(2\pi \times 50 \times 0 - 13.36 \times \frac{\pi}{180}\right) = -0.3552$ Amp

(ii) at $t = 0.001$ sec. $\quad \therefore i = 87.3135 \sin\left(2\pi \times 50 \times 0.001 - 13.36 \times \frac{\pi}{180}\right) = 0.1232$ Amp.

(2) Two current are represented by $I_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right)$ and $I_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$. These currents are fed into common conductor. Find the total current. If the conductor has resistance 50Ω what will be energy loss in 10 hours. **[D-13][5]**

Solution:-

$$I_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right), \qquad I_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\bar{I}_1 = \frac{15}{\sqrt{2}} \angle 60^\circ = 10.606 \angle 60^\circ \qquad \bar{I}_2 = \frac{25}{\sqrt{2}} \angle 45^\circ = 17.793 \angle 45^\circ$$

$$\overline{I_1} = 5.303 + j9.185 \qquad \overline{I_2} = 12.49 + j12.49$$

$$\bar{I} = \overline{I_1} + \overline{I_2} = 17.793 + j21.675 = 28.04 \angle 50.61^\circ$$

$$\therefore I = 28.04 \times \sqrt{2} \sin(\omega t + 50.61^\circ)$$

$$I = 39.65 \sin(\omega t + 50.61^\circ)$$

(3) The voltage drops across four series connected impedances are given **[M-13][5]**
$V_1 = 60\sin(\omega t + \pi/6)$; $V_2 = 75\sin(\omega t - 5\pi/6)$; $V_3 = 100\cos(\omega t + \pi/4)$; $V_4 = V_{4m}\sin(\omega t + \phi_4)$; Calculate the values of V_{4m} and ϕ_4 if the voltage applied across series circuit is $140\sin(\omega t + 3\pi/5)$.

Solution:-

$$v_1 = 60 \sin\left(wt + \frac{\pi}{6}\right) \qquad v_2 = 75 \sin\left(wt - \frac{5\pi}{6}\right)$$

$$v_3 = 100 \cos\left(wt + \frac{\pi}{4}\right) = 100 \sin\left(wt + \frac{3\pi}{4}\right) \qquad v_4 = V_{4m} \sin(wt + \phi_m)$$

$$v = 140 \sin\left(wt + \frac{3\pi}{5}\right)$$

$$\bar{V}_1 = \frac{60}{\sqrt{2}} \angle 30^\circ = 42.43 \angle 30^\circ \text{ V} \qquad \bar{V}_2 = \frac{70}{\sqrt{2}} \angle -150^\circ = 53.03 \angle -150^\circ \text{ V}$$

$$\bar{V}_3 = \frac{100}{\sqrt{2}} \angle 45^\circ = 70.71 \angle 45^\circ \text{ V} \qquad \bar{V} = \frac{140}{\sqrt{2}} \angle 180^\circ = 99 \angle 180^\circ \text{ V}$$

$$\therefore \overline{V} = \overline{V}_1 + \overline{V}_2 + \overline{V}_3 + \overline{V}_4$$

$$\therefore \overline{V}_4 = \overline{V} - \left(\overline{V}_1 + \overline{V}_2 + \overline{V}_3\right) = 99\angle 108^0 - 60.53\angle 47.6^0 = 86.86\angle 145.3V$$

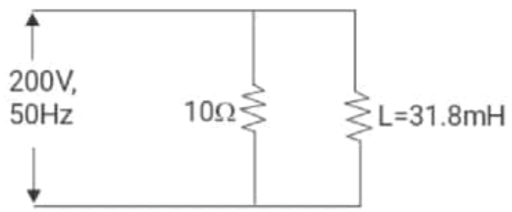
$$\therefore V_4 = 86.86\sqrt{2}\sin(wt + 145.3)^0$$

$$\therefore V_{4m} = 122.84\,V \qquad \phi_m = 145.3^0$$

Type IV : RL

(1) A resistance of 10 Ω and a pure coil of inductance 31.8 mH arc connected in parallel across 200V. 50 Hz supply. Find the total current and power factor. **[M-15][4]**

Solution:-



$$X_1 = 2\pi fL = 9.99\,\Omega$$

$$Z_1 = 10\,\Omega,\ Z_2 = j9.99\,\Omega$$

$$\therefore \overline{Z}_T = \overline{Z}_1 \| \overline{Z}_2 = \frac{\overline{Z}_1 \times \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} = 7.0675\angle 45.03^0\,\Omega$$

$$\bar{I}_T = \frac{\overline{V}}{\overline{Z}_T} = \frac{200\angle 0}{7.0675\angle 45.03^0} = 28.2984\angle -45.03^0\,A$$

$$\text{Power facter} = \cos(45.03^0) = 0.7067\,(\text{Lag})$$

(2) A coil having a resistance of 10 Ω and an inductance of 40 mH is connected to a 200V 50 Hz supply. Calculate the impedance of the coil, current, power factor and power consumed. **[M-15][8]**

Solution:-

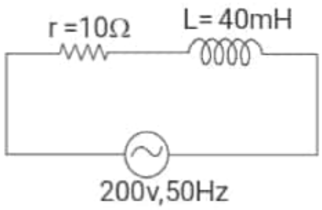
$$X_L = 2\pi fL = 12.5664\,\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = 16.0597\,\Omega$$

$$I = V/Z = 12.4535\,A$$

$$\cos\phi = r/z = 0.6227\,(\text{lag})$$

$$P = V.I.\cos\phi = 1.5509\,kW$$



(3) When a resistor and an inductor in series are connected to a 240V supply, a current of 3A flows lagging 37° behind the supply voltage, while voltage across inductor is 171 volt. Find theresistance of resistor, resistance & reactance of the inductor. **[D-15][8],[M-14][8]**

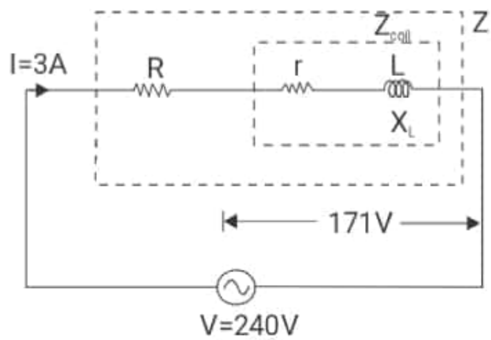
Solution:-

Current of 3A flows lagging 37° behind the supply voltage,

Phase angle, $\phi = 37^0$

$$\therefore V_{coil} = 171V$$

$$\therefore Z_{coil} = \frac{V_{coil}}{I} = \frac{171}{3} = 57\,\Omega$$



$$\text{Total impedance of the circuit, } Z = \frac{V}{I} = \frac{240}{3} = 80\Omega$$

$$(pf)_{circuit} = \frac{R+r}{Z}$$

$$\therefore\ R+r = \cos\phi \times Z = \cos 37^0 \times 80 = 63.89\,\Omega$$

$$\therefore\ Z = \sqrt{(R+r)^2 + X_L^2} \Rightarrow Z^2 = (R+r)^2 + (X_L)^2 \Rightarrow (80)^2 = (63.89)^2 + (X_L)^2$$

$$\therefore\ X_L = 48.15\,\Omega$$

$$\therefore\ Z_{coil} = \sqrt{r^2 + X_L^2} \Rightarrow Z_{coil}^2 = r^2 + X_L^2 \Rightarrow (57)^2 = r^2 + (48.15)^2$$

$$\therefore\ r = 30.51\Omega$$

$$\therefore\ R+r = 63.89 \Rightarrow R = 63.89 - 30.51$$

$$\therefore\ R = 33.38\Omega$$

(4) The voltage and current in a circuit are given by e = 100 sin(ωt + 30°) and i = 50 sin(ωt + 60°). Determine the impedance of the circuit. Assuming the circuit to contain 2 elements in series find resistance, reactance and power factor of the circuit. **[D-14][4]**

Solution:-

$$e = 100\sin\left(\omega t + 30^0\right), \qquad \bar{E} = 70.7107\angle 30^0\ V$$

$$i = 50\sin\left(\omega t + 60^0\right), \qquad \bar{I} = 35.3553\angle 60^0\ A$$

$$\therefore\ \bar{z} = E/I = 2\angle -30^0\,\Omega, = (1.732 - j1.1)\Omega$$

$$\therefore\ R = 1.732\,\Omega$$

$$\therefore\ X = 1^0 1\Omega$$

$$p.f. = \cos\phi = R/z = 0.866(\text{Leading})$$

(5) Two coils A and B are connected in series across 240 V, 50 Hz supply. The resistance of A is 5Ω and the inductance of B is 0.015 H. If the input from supply is 3kW and 2 kVAR. Find the inductance of A and resistance of B. Calculate the voltage across each coil. **[M-13][8]**

Solution:-

Given: $V = 240V$, $f = 50Hz$, $R_A = 5\Omega$, $L_B = 0.015H$, $P = 3KW = 3000watt$
$Q = 2\ K\ VAR = 2000\ VAR$

To find: $L_A = ?$ $R_B = ?$ $V_A = ?$ $V_B = ?$

$$\therefore\ P = VI\cos\phi \quad \text{and} \quad Q = VI\sin\phi$$

$$\therefore\ \frac{Q}{P} = \frac{\sin\phi}{\cos\phi} = \tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$$

$\therefore \cos\phi = 0.8321$ and $\sin\phi = 0.5547$

$\therefore L_B = 0.015H \Rightarrow X_{L_B} = 2\pi f L_B = 4.71\Omega$

$\therefore P = VI\cos\phi \Rightarrow I = \dfrac{P}{V\cos\phi} = \dfrac{3000}{240 \times 0.8321} = 15.02$ Amp.

$Z_T = \dfrac{V}{I} = \dfrac{240}{15.02} = 15.97\Omega$

$\therefore pf = \cos\phi = \dfrac{\text{Total Resistane}}{\text{Total Impedance}} \Rightarrow 0.8321 = \dfrac{R_A + R_B}{15.97}$

$\therefore R_A + R_B = 13.28$

$\therefore R_B = 13.28 - 5 = 8.28\Omega$

$\therefore Z_T = \sqrt{(R_A + R_B)^2 + (X_A + X_B)^2} \Rightarrow (15.97)^2 = (13.28)^2 + (X_A + 4.71)^2$

$\therefore X_A = 4.16$

$\therefore Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.16^2} = 6.5\Omega$

$\therefore Z_B = \sqrt{R_B^2 + X_B^2} = \sqrt{8.28^2 + 4.71^2} = 9.52\Omega$

$\therefore V_A = I.Z_A = 97.63$ volt

$\therefore V_B = I.Z_B = 143.07$ volt

(6) A voltage of 150V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current. **[D-12][5]**

Solution:-

Given: V = 150V, F = 50Hz, L = 0.2H

$\therefore V_{rms} = \dfrac{V_m}{\sqrt{2}}$

$\therefore V_m = \sqrt{2} \times 150 = 150\sqrt{2}$ V

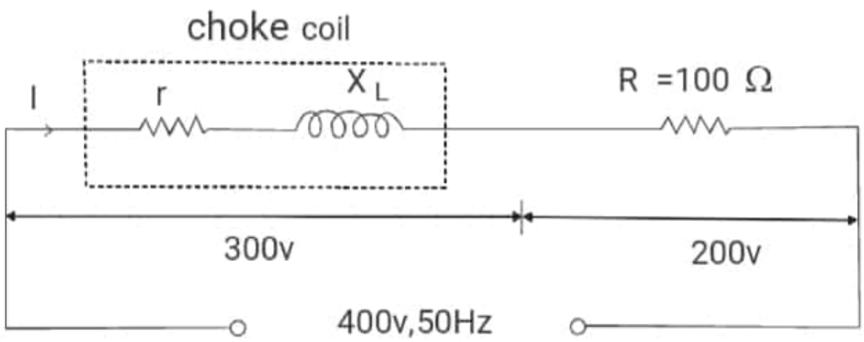
$\therefore V = V_m \sin\omega t = 150\sqrt{2}\sin 100\pi t$

$\therefore I = I_m \sin(\omega t - 90) = \dfrac{V_m}{\omega L}\sin(\omega t - 90) = \dfrac{150\sqrt{2}}{100\pi \times 0.2}\sin(\omega t - 90)$

$\therefore I = 3.77\sin(100\pi t - 90)$

(7) A 100Ω resistor is connected in series with a choke coil. When a 400V, 50Hz supply is applied to this combination, the voltage across the resistance and the choke coil are 200V and 300V respectively. Find the power consumed by the choke coil. Also calculate the power factor of choke coil and power factor of the circuit. **[D-12][8]**

Solution:-



$$I = \frac{200}{100} = 2\,\text{Amp}.$$

$$Z_{coil} = r + jX_L = \frac{300}{2} = 150\,\Omega$$

$$\therefore Z_{coil} = \sqrt{r^2 + X_L^2} = 150\,\Omega$$

$$\therefore r^2 + X_L^2 = 22500 \qquad ...(1)$$

$$\therefore Z_{Total} = \frac{400}{2} = 200\Omega$$

$$\therefore Z = \sqrt{(R+r)^2 + X_L^2} = 200 \Rightarrow (100+r)^2 + X_L^2 = 40000 \qquad ...(2)$$

Substracting equation (1) from equation (2)

$$\therefore (100+r)^2 - r^2 = 17500 \quad \Rightarrow r = 37.5\,\Omega$$

Substituting $r = 37.5\,\Omega$ in equation (1)

$$\therefore 37.5^2 + X_L^2 = 22500 \quad \Rightarrow X_L = 145.24\Omega$$

$$\therefore P_{coil} = I^2 r = 2^2 x 37.5 = 150\text{watt}$$

$$(P.F.)_{coil} = \frac{r}{Z_{coil}} = \frac{37.5}{150} = 0.25\,(\text{lagging})$$

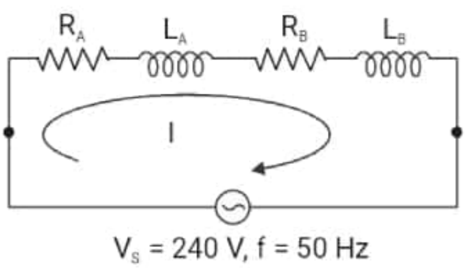
$$(P.L.)_{Total} = \frac{R+r}{Z_{Total}} = \frac{100+37.5}{200} = 0.6875\ (\text{lagging})$$

(8) Two practical coils A and B are connected in series and excited by single phase ac supply of 240V, 50 Hz. Input from the supply to the circuit is 3KW and 2KVAR. If resistance of coil A is 5 ohms and inductance of coil B is 15 mH then calculate.
(i) Inductance of coil A (ii) Resistance of coil B (iii) Voltages across both the coils **[M-12][10]**

Solution:

Given: Two coils A and B in series, $V_S = 240V$, $f = 50$ Hz,
$P = 3kW$, $Q = 2$ kVAR, $R_A = 5\Omega$, $L_B = 15$ mH.

Step 1: Find I and ϕ:



$$P = VI\cos\phi$$

$$\therefore 3000 = 240 \times I \times \cos\phi$$

$$\therefore I\cos\phi = 3000/240 = 12.5 \qquad ...(1)$$

$$Q = VI\sin\phi$$

$\therefore\ 2000 = 240\ I \sin\phi \quad \Rightarrow \quad I \sin\phi = 8.33 \qquad ...(2)$

Divide Equation (2) by Equation (1),

$$\frac{I\sin\phi}{I\cos\phi} = \frac{8.33}{12.5} \Rightarrow \tan\phi = 0.666 \Rightarrow \phi = \tan^{-1} 0.666 = 33.69^\circ \qquad ...(3)$$

Now $I \cos 33.69 = 12.5$

$$\therefore\ I = \frac{12.5}{\cos 33.69} = \frac{12.5}{0.832} = 15 \text{ Amp.} \qquad ...(4)$$

Step 2: Find the total impedance R_A and L_n:

$$|Z| = \frac{V_S}{I} = \frac{240}{15} = 16\,\Omega$$

$\therefore\ Z = 16\angle + \phi = 16 \angle + 33.69^\circ\ \Omega$

$\therefore\ Z = (13.3 + j\ 8.88)\Omega$

Now $Z = R + j\,X_L = (R_A + R_B) + j(X_A + X_B)$

$\therefore\ R_A + R_B = 13.3 \quad \Rightarrow \quad 5 + R_B = 13.3$

$\therefore\ R_B = 8.3\ \Omega$

But $X_B = 2\pi f\, L_B = 2\pi \times 50 \times 15 \times 10^{-3} = 4.71\Omega$

$\therefore\ X_A = 8.88 - 4.71 = 4.17\ \Omega$

$$\therefore\ L_A = \frac{4.17}{2\,\pi f} = \frac{4.17}{2\pi \times 50} = 0.01327 \text{ H or } 13.27 \text{ mH}$$

Step 3: Voltage across both the coils:

$Z_A = R_A + j\,X_{LA} = (5 + j\,4.17)\Omega = 6.51 \angle 39.83^\circ\ \Omega$

$Z_B = R_B + j\,X_{LB} = (8.3 + j\,4.71)\Omega = 9.54 \angle 29.57^\circ\ \Omega$

$I = 15 \angle -33.69^\circ$ Amp.

$\therefore\ V_A = Z_A \times I = (6.51 \angle 39.83^\circ) \times (15 \angle -33.69^\circ)$

$\therefore\ V_A = 97.65 \angle 6.14^\circ$ Volts

And $\quad V_B = Z_B \times I = (9.54 \angle 29.57^\circ) \times (15 \angle -33.69^\circ)$

$\therefore\ V_B = 143.1 \angle -4.12^\circ$ Volts

(9) Coil A takes 2 Amps at a power factor of 0.8 lagging with an applied voltage of 10 Volts. A second coil B takes 2 Amps with a power factor of 0.7 lagging with an applied voltage of 5 Volts. What voltag will be required to produce a total current of 2 Amps — **[D-08][12]**

(i) With A and B in series (ii) With A and B in parallel.

Solution:-

Given: Coil A: $I_A = 2A\ \cos\phi_A = 0.8(\text{lag})$ for $V_s = 10V$

Coil B: $I_B = 2A\ \cos\phi_B = 0.7(\text{lag})$ for $V_s = 5V$

Part I : A and B in series:

Step 1 : Calcuating of Z_A, Z_B and Z:

$$Z_A = \frac{10V}{2A} = 5\,\Omega, \qquad \phi_A = \cos^{-1} 0.8 = 36.87^{\circ}$$

$$\therefore Z_A = 5\angle 36.87^{\circ}\,\Omega = (4 + j3)$$

$$Z_B = \frac{5V}{2A} = 2.5\,\Omega, \qquad \phi_B = \cos^{-1} 0.7 = 45.57^{\circ}$$

$$\therefore Z_B = 2.5\angle 45.57^{\circ}\,\Omega = (1.75 + j1.79)$$

$\therefore$ Total impedance $Z = Z_A + Z_B = (4 + j3) + (1.75 + j1.79) = 5.75 + j4.79 = 7.48\angle 39.8^{\circ}\,\Omega$

Step 2 : Calculation of V_s:

Given that the current through series cobination is 2A.

$$\therefore V_S = 2 \text{ x } Z = 2 \text{ x } 7.48 \approx 15V$$

Part II : A and B in parallel:

Step 1 : Calculation of total impedance Z :

$$Z = \frac{Z_A Z_B}{Z_A Z_B} = \frac{(5\angle 36.87) \text{ x } (2.5\angle 45.57)}{7.48\angle 39.8} = \frac{12.5\angle 82.44}{7.48\angle 39.8}$$

$$\therefore Z = 1.67\angle 42.64^{\circ}\,\Omega$$

Step 2 : Calculation of V_s :

$$V_s = 2 \text{ x } Z = 2 \text{ x} 1.67 = 3.34V$$

(10) A voltage of 125V at 50Hz is applied across a non-inductive resistance connected in series with a capacitance. The current is 2.2A. The power loss in resistance is 96.8 watts. Find R and C. **[M-11][5]**

Solution:

V = 125 V, P = 96.8 W, I = 2.2 A, F = 50 Hz

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.818\,A$$

$$P = I^2R \Rightarrow 96.8 = (2.2)^2 \times R$$

$$\therefore R = 20\Omega$$

$$X_c = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ W$$

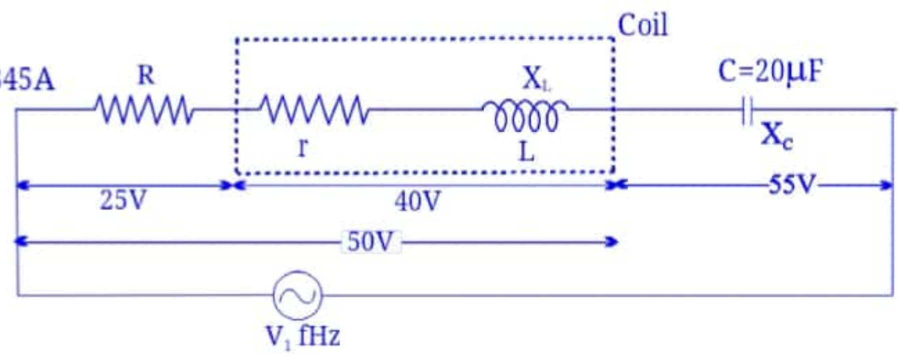
$$X_c = \frac{1}{2\pi FC} \Rightarrow 53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$\therefore C = \frac{1}{2\pi \times 50 \times 53.18} = 59.85\ \mu F$$

Type V : RLC

(1) For the circuit shown determine the **[D-13][8]**

(i) Supply frequency (f)
(ii) Coil resistance (r)
(iii) Supply Voltage (v)



Solution:-

$$I = 0.345\,A,\ C = 20\,\mu F = 20 \times 10^{-6}\,F,\ V_R = 25\,V,\ V_{coil} = 40\,V,\ V_C = 55\,V,\quad V_{R+coil} = 50\,V$$

Resistance of resistor, $R = \dfrac{V_R}{I} = \dfrac{25}{0.345} = 72.46\,\Omega$

Impedance of coil, $Z_{coil} = \dfrac{V_{coil}}{I} = \dfrac{40}{0.345} = 115.94\,\Omega$

Rectance of capacitor, $X_C = \dfrac{V_C}{I} = \dfrac{55}{0.345} = 159.42\,\Omega$

$$Z_{R+coil} = \frac{V_{R+coil}}{I} = \frac{50}{0.345} = 144.927\,\Omega$$

$$\therefore Z_{coil} = \sqrt{r^2 + X_L^2} \quad \Rightarrow \quad \therefore 115.94 = \sqrt{r^2 + X_L^2}$$

$$\therefore r^2 + X_L^2 = 13442.08 \qquad \text{...(i)}$$

$$\therefore Z_{R+coil} = \sqrt{(R+r)^2 + X_L^2} \quad \Rightarrow \quad 144.927 = \sqrt{R^2 + r^2 + 2Rr + X_L^2}$$

$$\therefore 21003.84 = R^2 + 2Rr + (r^2 + X_L^2) = 5250.45 + 13442.08 + 2Rr \qquad \text{..........from (1)}$$

$$2311.31 = 2 \times 72.46 \times r$$

$$\therefore r = 15.94\,\Omega \leftarrow \text{ Coil resistance}$$

From (1) $\quad X_L^2 = 13187.71 \Rightarrow X_L = 114.82\,\Omega$

$$Z_{Total} = \sqrt{(R+r)^2 + (X_L - X_C)^2} = \sqrt{(72.46 + 15.94)^2 + (114.82 - 159.92)^2} = 99\,\Omega$$

$$V = I \times Z = 0.345 \times 99 = 34.16\,V$$

$$\therefore \cos\phi = \frac{\text{Total Resistance}}{\text{Total Impedance}} = \frac{R+r}{Z_{Total}} = \frac{72.36 + 16.07}{99.04} = 0.8928$$

$$\therefore \phi = \cos^{-1}(0.8928) = 26.76^{\circ}$$

$$V = 34.16 \angle 26.76^{\circ}$$

$$\therefore X_C = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi C X_C} = \frac{1}{2 \times \pi \times 20 \times 10^{-6} \times 159.42} = 49.94 \approx 50$$

$$\therefore f = 50\,Hz$$

(2) A 46 mH inductive coil has a resistance of 10Ω. (i) How much current will it draw if connected acros a 100V, 60 Hz supply? (ii) What is the power factor of the coil? (iii) Determine the value of capacitanc that must be connected across the coil to make the power factor of overalll circuit units **[M-13][4**

Solution:-

Given: L = 46 mH, f = 60 Hz, R = 10 Ω, V = 100V

$$X_L = 2\pi fL = 2\pi \times 60 \times 46 \times 10^{-3} = 17.34\Omega$$

$$\therefore Z_{coil} = \sqrt{R^2 + X_L^2} = 20\Omega$$

$$\therefore I = \frac{V}{Z_{coil}} = \frac{100}{20} = 5A$$

$$\therefore pf = \cos\phi = \frac{R}{Z_{coil}} = \frac{10}{20} = 0.5$$

$$\therefore \text{To make, pf} = 1 \Rightarrow X_L = X_C$$

$$\therefore X_C = \frac{1}{2\pi fC} = 17.34$$

$$\therefore C = 152.97\,\mu F$$

(3) In RLC series circuit the voltage across the resistor, inductor and capacitor are 10V, 15V and 10V respectively. What is the power factor of the circuit? **[D-10][4]**

Solution:

$V_R = 10$ V, $V_L = 15$ V, $V_C = 10$ V

$V = V_R + V_L + V_C = 10 + 15 + 10 = 35V$

PF = $\cos\phi$

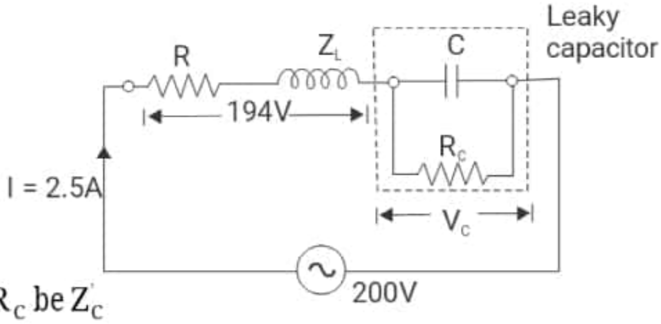
$$PF = \frac{V_R}{V} = \frac{10}{35} = 0.285771$$

(4) A leaky capacitor Z_c = 74.5 ohm is in series with a coil Z_L = 40 ohm and a resistor r = 56 ohms. When a voltage V = 200 volts is applied, I = 2.5A and the p.d. across R and Z_L combined is 194 V. Find the loss in the capacitor. **[D-07][4]**

Solution:-

Given : Z_c = 74.5 ohm, Z_L = 40 ohm, R = 56 ohm, V = 200 volts, I = 2.5A

Step 1 : To find V_C



$$V_L = 194V$$

$$V^2 = V_L^2 + V_C^2 \Rightarrow (200)^2 = (194)^2 + V_C^2$$

$$\therefore V_C = 48.62 \text{ Volt}$$

Step 2 : To find R_C :

Let the parallel combination of Z_C and R_C be Z_C'

$$\therefore Z_C' = \frac{V_C}{I} = \frac{48.62}{2.5} = 19.448\Omega.$$

$$\text{But } Z_C' = \frac{R_C Z_C}{\left(R_C^2 + Z_C^2\right)^{1/2}} \Rightarrow (19.448) = \frac{R_C Z_C}{\left(R_C^2 + Z_C^2\right)^{1/2}}$$

$$378.22\left(R_C^2 + 74.5^2\right) = 74.5^2 R_C^2$$

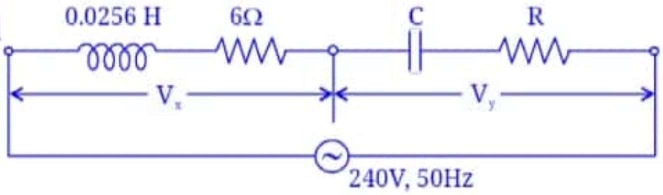
$$\therefore 378.22 R_C^2 + 5550.25 = 5550.25 R_C^2$$

$\therefore R_C = 1.073\Omega$ This is the value of leakage resistance of capaciter.

Step 3 : Loss in the capacitor :

$$\text{Loss} = \frac{V_C^2}{R_C} = \frac{(48.62)^2}{1.073} = 2202.8\,W$$

(5) Find the value of R and C so that $V_b = 3V_a$ and V_b and V_a are in quadrature. Find also the phase relation between V and V_b; V_a and I. Draw phasor diagram. **[D-07][8]**



Solution:-

Given : $V_b = 3\,V_a$, V_b leads V_a by 90

Find V_a and V_b :

$$V_a^2 + V_b^2 = V^2$$

substitute $V_b = 3V_a$ to get

$$V_a^2 + 9V_b^2 = 240^2$$

$$\therefore V_a = 75.89 \text{ Volts and } V_b = 3V_a = 227.68 \text{ Volts}$$

Find current I :

Let R_1 and L together form the impedance Z_b

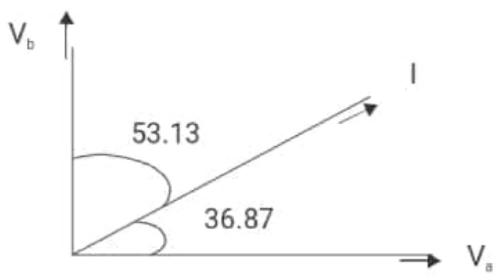
$\therefore Z_b = R_1 + j\,X_{L1} = 6 + j\,(2\pi \times 50 \times 0.0255) = 6 + j8$ ohms

$$Z_b = \sqrt{6^2 + 8^2}\,\angle \tan^{-1}(8/6) = 10\angle 53.13\Omega$$

$$I = \frac{V_b}{Z_b} = \frac{227.68\angle 0}{10\angle 53.13} = 2.2768\angle -53.13\,\text{Amp}$$

Thus current I lags V_b by 53.13 as shown in fig.

The current I leads V_a by (90 – 53.13) = 36.87



Calculate Z_a, R and C :

Let $Z_a = R - jX_L$

$$\text{But } Z_a = \frac{V_a}{I} = \frac{75.89\angle 0^\circ}{2.2768\angle 36.87^\circ} = 33.33\angle -36.87$$

$\therefore Z_a = 33.33\cos(-36.87) + j\,33.33\sin(-36.87) = 26.67 - j20$

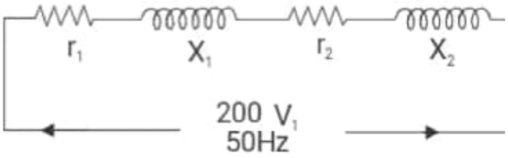
$\therefore R = 26.67,\ X_C = 20\,\Omega$

$$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{11}{2\pi \times 50 \times 20} = 159.15\,\mu f$$

Type VII : Parallel & Series Parallel

(1) Two coils are connected in series across a 200 V, 50 Hz ac supply. The power input to the circuit is 2 kW and 1.15 kVAR. If the resistance and the reactance of the first coil are 5 Ω. and 8 Ω respectively, calculate the resistance and reactance of the second coil. Calculate the active power and reactive power for both the coils individually. **[D-14][8]**

Solution:-



$r_1 = 5\Omega,\ x_1 = 8\Omega,\ \ P = 2000\,\omega,\ Q = 1150\,VAR\ \ S = 2307.05\,VA$

$$\cos\phi = \frac{P}{S} = 0.8669$$

$\therefore\ I = 11.5353\,A$

$Z = V / I = 17.3381\Omega$

$$\frac{r_T}{Z} = 0.8669 \qquad \therefore\ r_T = 15.0305\Omega \qquad \therefore\ r_2 = 10.0305\Omega$$

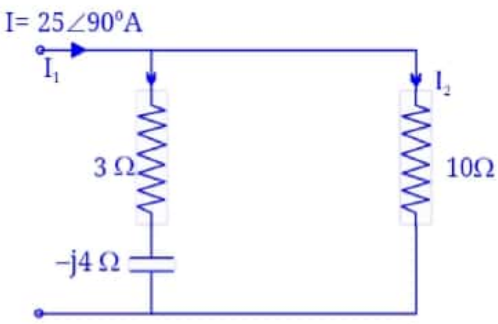
$$\frac{x_T}{z} = \frac{Q}{S} = 0.4985 \qquad \therefore\ x_T = 8.6426\Omega \qquad \therefore\ x_2 = 0.642\Omega$$

$P_1 = I^2 r_1 = 665.3157\ \Omega$	$\phi_1 = \tan^{-1}(X_1/r_1) = 57.99^0$
$P_2 = I^2 r_2 = 1334.6899\ \Omega$	$\phi_2 = \tan^{-1}(X_2/r_2) = 3.66^0$

$V_1 = 108.8238\angle 57.99^0\ V \qquad V_2 = 115.942\angle 3.6656^0\ V$

$Q_1 = V_1\,I.\sin 57.99^0 = 1.064\,KVAR \qquad Q_2 = V_2\,I.\sin 3.6656^0 = 85.5058\,VAR.$

(2) Find current I_1 and I_2 shown in figure. **[M-14][4]**



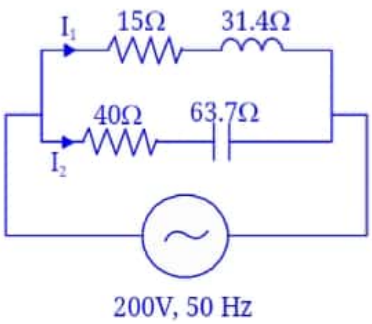
Solution:-

By current division Rule,

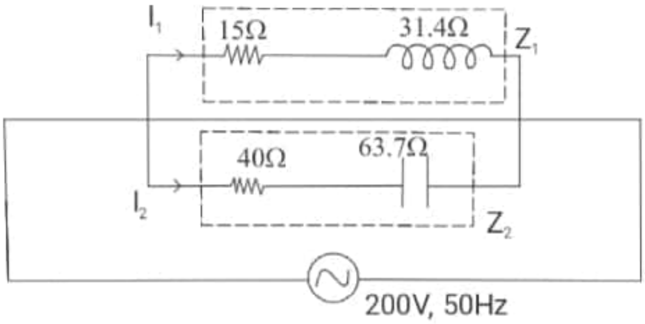
$$\bar{I}_1 = \frac{10}{10+3-j4} \times 25\angle 90^0 = 18.38\angle 107.10^0\ Amp$$

$$\bar{I}_2 = \frac{3-j4}{10+3-j4} \times (25\angle 90^0) = 9.19\angle 53.97^0$$

(3) Calculate the branch current I_1 and I_2 for the circuit shown in figure. **[D-12][4]**



Solution:-



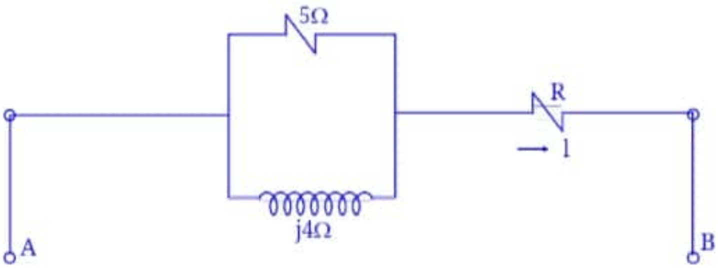
$$\overline{Z_1} = (15 + j31.4)\Omega = 34.8\angle 64.47^\circ \Omega$$

$$\overline{Z_2} = (40 - j63.7) = 75.22\angle -57.87^\circ \Omega$$

$$\therefore \overline{I_1} = \frac{\overline{V}}{\overline{Z_1}} = \frac{200\angle 0^\circ}{34.8\angle 64.47^\circ} = 5.75\angle -64.47^\circ \text{ A}$$

$$\therefore \overline{I_2} = \frac{\overline{V}}{\overline{Z_2}} = \frac{200\angle 0^\circ}{75.22\angle -57.87^\circ} = 2.66\angle 57.87^\circ \text{ A}$$

(4) If a voltage of 150 V applied between terminals A and B produces a current of 32 A, for the circuit shown in figure, find the value of resistance R and the power factor of the circuit. **[D-11][10]**



Solution:

Given: V_{AB} = 150V, I = 32A. To find: R and cos ϕ.

Step 1: Find the value of R:

From the given circuit

$Z = (5\Omega \,||\, j\,4\Omega) + R$

But $5\Omega = 5 + j0 = 5\angle 0^\circ$

$j\,4\Omega = 0 + j4 = 4\angle 90^\circ$

$$\therefore\ (5 + j0)\,||\,(j4) = \frac{5\angle 0^\circ \times 4\angle 90^\circ}{5 + j0 + 0 + j4} = \frac{20\angle 90^\circ}{5 + j4} = \frac{20\angle 90^\circ}{6.4\angle 36.66^\circ}$$

$$= 3.125\angle 53.35^\circ = (1.87 + j2.5)\ \Omega$$

$\therefore\ Z = (1.87 + j2.5) + (R + j0) = (R + 1.87) + j2.5\Omega$

$\therefore\ Z = [(R + 1.87)^2 + (2.5)^2]^{1/2}$ ...(1)

$$\text{Also } Z = \frac{V_{AB}}{I} = \frac{150}{32} = 4.6875\Omega \qquad ...(2)$$

Equating the RHS of Equations (1) and (2)

$[(R + 1.87)^2 + (2.5)^2]^{1/2} = 4.6875$

$\therefore\ R = 2.1\Omega$

Step 2: Find PF:

$Z = R + 1.87 + j\,2.5$

Substituting the value of R

$Z = 2.1 + 1.87 + j2.5 = 3.97 + j2.5$ ...(3)

$$\therefore\ \phi = \tan^{-1}\left[\frac{2.5}{3.97}\right] = 32.2^\circ$$

$\therefore$ PF = cos ϕ = cos (32.2°) = 0.846 (lagging)

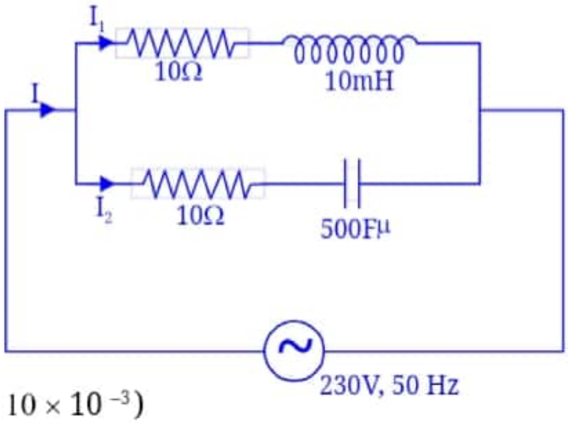
(5) Determine:

(i) Total impedance of the circuit and total current

(ii) Branch current I_1 and I_2.

(iii) Power factor of each branch and total power factor.

(iv) Power consumed by each branch. **[D-09][10]**



Solution:

Step 1: (i) Calculate impedances Z_1 and Z_2:

$$Z_1 = R_1 + jX_1 = 10 + j(2\pi fL) = 10 + j(2 \times 50 \times 10 \times 10^{-3})$$
$$= 10 + j3.141$$

$$|Z_1| = \sqrt{10^2 + 3.141^2} = 10.482\,\Omega$$

$$\theta_1 = \tan^{-1}(3.14\frac{1}{10}) = 17.43^0$$

$$Z_1 = 10.482 \angle 17.43^0\,\Omega$$
$$Z_2 = R_2 + jX_2$$

$$X_2 = \frac{1}{2\pi fC} \quad \Rightarrow \quad X_2 = \frac{1}{2 \times \pi \times 50 \times 500 \times 10^{-6}}$$

$$X_2 = 6.366$$
$$Z_2 = 10 - j6.366$$
$$|Z_2| = 11.85$$

$$\theta_2 = \tan^{-1}\left(\frac{-6.366}{10}\right) = -32.48^0$$

$$Z_2 = 11.85 \angle -32.48^0\,\Omega$$

Total impedances of circuit = $Z_1 || Z_2$

$$Z = \frac{Z_1 . Z_2}{Z_1 + Z_2}$$

$$Z = \frac{|Z_1| \times |Z_2| \angle \theta_1 - \theta_2}{(10 + j3.141) + (10 - j6.366)} = \frac{124.117 \angle -15.05}{20 - j3.225}$$

$$Z = \frac{124.117 \angle -15.05}{20.258 \angle -6.266}$$

$$Z = 6.126 \angle -8.784^0\,\Omega$$

(ii) For total current we habve to find out the separate current

$$\text{Current } I_1 = \frac{V}{Z_1} = \frac{230 \angle 0^0}{10.482 \angle 17.43^0}$$
$$= 21.942 \angle -17.43^0 = 20.934 - j\,6.572$$

$$\text{Current } I_2 = \frac{V}{V_2} \quad \frac{230 \angle 0^0}{11.85 \angle -32.48^0}$$
$$= 19.409 \angle 32.48^0 = 16.373 + j10.4227$$

$\therefore$ Total current (I) = $I_1 + I_2 = 20.934 - j6.572 + 16.373 + j10.4227$

$I = 37.307 + j3.8507^0$ Amp

(iii) Power factor of each branch and total power factor :

Total power factor :

We know that , $I = 37.307 + j3.8507 = 37.50 \angle 5.893^0$ Amp

that means $\phi = 5.893$

$\therefore$ Total power factor = $\cos\phi = \cos(5.893) = 0.9947$

$\therefore$ Total power factor = 0.9947 leading

Power factor of branch having current I_1 :

$I_1 = 21.942 \angle -17.43 \Rightarrow \phi = -17.43$

Power factor = $\cos\phi = \cos(-17.43)$

Power factor = 0.9540 lagging

Power factor of branch having current I_2 :

$I_2 = 19.409 \angle 32.48^0 \Rightarrow \phi = 32.48$

Power factor = $\cos\phi = \cos(32.48)$

Power factor = 0.8435 leading

(iv) Power consumed by each branch

Power consumed by the first branch

$P_1 = I_1^2 \times R_1 = (21.942)^2 \times 10$

$P_1 = 4814.5$ watt

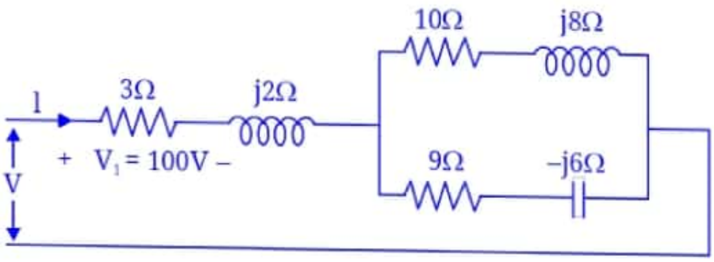
Power consumed by the second branch

$P_2 = I_2^2 \times R_1 = (19.409)^2 \times 10$

$P_2 = 3767.09$ watt

(6) Find I, I_1, I_2 and V in the following figure:

[M-09][8]



Solution:

Step 1: Calculation of all impedances :

$Z_1 = 3 + j2 = [(3)^2 + (2)^2]^{1/2} \angle \tan^{-1}(2/3)$

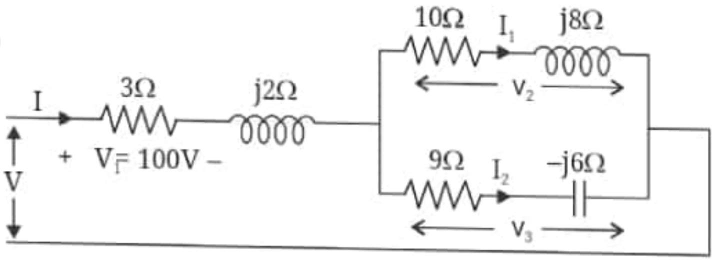
$= 3.6 \angle 33.69^0 \Omega$

$Z_2 = 10 + j8 = [(10)^2 + (8)^2]^{1/2} \angle \tan^{-1}(8/10)$

$= 12.8 \angle 38.65^0 \Omega$

$Z_3 = 9 - j6 = [(9)^2 + (6)^2]^{1/2} \angle \tan^{-1}(6/9)$

$= 10.81 \angle -33.69^0 \Omega$



Step 2 : Calculation of all impedances :

Z_2 and Z_3 are in parallel. Therefore their effective parallel combination is obtained by using the admittance notation. The admittances Y_2 and Y_3 will be in series with each other.

$$Y' = Y_2 + Y_3 = \frac{1}{Z_2} + \frac{1}{Z_3} \Rightarrow Y' = \frac{1}{12.8\angle 38.65^0} + \frac{1}{10.81\angle -33.69^0}$$

$$= 0.078 \angle -38.65^0 + 0.092 \angle 33.69^0$$

Converting into rectangular form,

$$Y' = 0.0609 - j\,0.048 + 0.0765 + j\,0.05103$$

$$\therefore \quad Y' = 0.1374 + j\,0.0285$$

Converting it in to polar form, $Y' = 0.14 \angle 11.71^0$ mho

Hence corresponding impedance, $Y' = Z_2 || Z_3 = 1/Y'$

$$Z' = \frac{1}{0.14\angle 11.71^0} = 7.14\angle -11.71^0\,\Omega$$

$$\therefore Z' = 6.99 - j1.449$$

Total impedance $Z = Z' + Z_1$

$$\therefore Z = 6.99 - j1.449 + 3 + j2 = (9.99 + j\,0.551)\,\Omega$$

Step 3 : Calculation of I :

$$I = \frac{V_1}{Z_1} = \frac{100\angle 0^0}{3.6\angle 33.69} = 27.77\angle -33.69^0 \text{ Amp}$$

Step 4 : Calculation of I_1 and I_2:

The current I gets divided into I_1 and I_2 So using current division rule between two parallel impedances

$$I_1 = \frac{Z_3}{Z_2 + Z_3} \times I \qquad \text{and} \qquad I_2 = \frac{Z_2}{Z_2 + Z_3} \times I$$

$$Z_2 + Z_3 = 10 + j8 + 9 - j6 = 19 + j2$$

$$Z_2 + Z_3 = 19.10 \angle 6^0$$

Substituting to get,

$$I_1 = \frac{10.81\angle -33.69^0}{19.10\angle 6^0} \times 27.77\angle -33.69^0 \text{ Amp}$$

$$= 15.71\angle -73.38 \text{ Amp}$$

$$I_2 = \frac{12.8\angle -38.65^0}{19.10\angle 6^0} \times 27.77\angle -33.69^0$$

$$= 18.61 \angle -1.04^0 \text{ Amp.}$$

Step 5 : Calculation of V_1 and V_3

$V_2 = V_3$ ($\because$ As Z_2 and Z_3 are in parallel)

$V_2 = I_1 \times Z_2 = 15.71\angle -73.38 \times 12.8 \angle 38.65$

$\therefore \quad V_2 = V_3 = 201 \angle -34.73^0\,\Omega$

$V = V_1 + V_2 = 100 \angle 0^0\Omega + 201 \angle -34.73^0\,\Omega$

Converting V_1 and V_2 into rectangular form,

$V = 100 + j0 + 165.19 - j\,114.5 = 265.19 - j\,114.5$

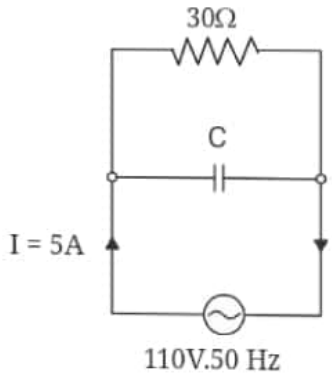
$= 288.85 \angle -23.35$ Volt.

(7) A resistor of 30Ω and a capacitor of unknown value are connected in parallel across a 110V, 50 Hz, supply. The combination draws a current of 5A from the supply. Find the value of the unknown capacitance. This combination is again connected across a 110V supply of unknown frequency. It is observed that the total current drawn from the mains falls to 4 Amps. Determine the frequency of the supply. **[D-08][10]**

Solution:

Part I : Calculation of C

Step 1 : Calculation of |Z| and X_C



$$|Z| = \frac{110V}{5A} = 22\Omega$$

$$R = 30\angle 0 = 30 + j0$$

$$X_C = 0 - jX_C = X_C\angle -90^0$$

$$\therefore |Z| = R \| X_C = \frac{RX_C}{R + X_C} = \frac{(30\angle 0^0)(X_C\angle -90^0)}{(30 + j0)(0 - jX_C)}$$

$$|Z| = \frac{30X_C}{30 - jX_C} = \frac{30X_C}{\sqrt{900 + X_C^2}} \Rightarrow (22)^2 = \frac{900\,X_C^2}{900 + X_C^2}$$

$$\therefore X_C = 32.36\Omega$$

Step 2 : Calculation of C :

$$X_C = \frac{1}{2\pi fC} \Rightarrow 32.36 = \frac{1}{2\pi \times 50C}$$

$$\therefore C = \frac{1}{2\pi \times 50 \times 32.36} = 9.8367 \times 10^{-5}\text{ F or } 98.367\,\mu F$$

Part II : Calculation of f' :

Step 1 : Calculation of new value of impedance |Z'| :

$$|Z'| = \frac{110V}{4A} = 27.5\Omega$$

Step 2 : Calculation of new reactace X_C' :

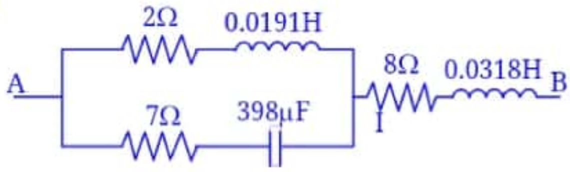
$$|Z'| = \frac{30X_C'}{\sqrt{900 + X_C'^2}} \Rightarrow (27.5)^2 = \frac{900X_C'^2}{900 + X_C'^2}$$

$$\therefore X_C' = 68.8\Omega$$

$$\therefore \text{New frequency } f' : X_C' = \frac{1}{2\pi f'C} \Rightarrow 68.8 = \frac{1}{2\pi \times f' \times 98.367 \times 10^{-6}}$$

$$\therefore f' = 23.51\text{Hz}$$

(8) Find the applied voltage V_{AB} so that 10 A current may flow through the capacitor. Assume frequency of 50Hz. **[M-08][10]**



Solution:

Calculate total impedance :

$Z_1 = R_1 + j\,X_{L1} = 2 + j\,(2\Omega \times 50 \times 0.0191) = 2 + j6 = 6.32\angle 71.57^o$

$$Z_2 = R_2 - jX_{C2} = 7 - \frac{j}{2\pi \times 50 \times 398 \times 10^{-6}} = 7 - j8 = 10.63\angle -11.84^o$$

$Z_3 = R_3 + j\,X_{L3} = 8 + j\,(2\pi \times 50 \times 0.0318) = 8 + j10 = 12.8\angle 51.34$

$$Z_1 \| Z_2 = \frac{Z_1.Z_2}{Z_1 + Z_2} = \frac{6.32 \times 10.63\angle(71.57 - 11.84)}{(2 + j6) + (7 - j8)} = \frac{67.18\angle 59.73}{(9 - j2)}$$

$$Z_1 \| Z_2 = \frac{67.18\angle 59.73}{9.22\angle -12.53} = 7.286\angle 72.26\,\Omega = 2.22 + j6.93\,\Omega$$

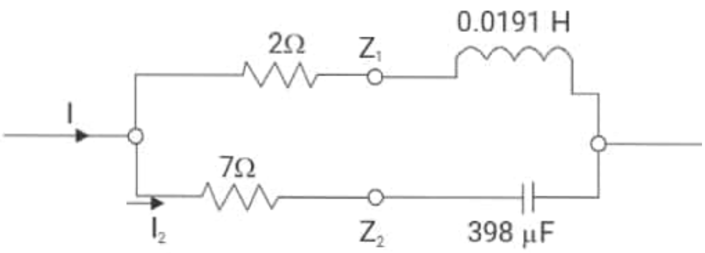
$$Z = (Z_1 \| Z_2) + Z_3 = 2.22 + j6.93 + 8 + j10 = 10.22 + j16.93$$

$$Z = 19.9\angle 58.88^o\ \Omega$$

Calculate the total current I :

I_2 = 10 A (Given)

$$\text{But } I_2 = \frac{Z_1}{Z_1 + Z_2} x \Rightarrow I = \frac{Z_1 + Z_2}{Z_1} x I_2$$



$$I = \frac{(9 - j2) \times 10}{6.32\angle 71.57} = \frac{9.22\angle -12.53}{6.32\angle 71.57} x10$$

$\therefore$ I = 14.59 Amp

Calculate V_{AB} :

$V_{AB} = I \times |z| = 14.59 \times 19.77 = 288.42$ Volts

Type VII : Series Resonannce

(1) A series RLC circuit has the following parameter values: R= 10 Ω, L=0.014H, C= 100 μF. Compute the resonant frequency, quality factor, bandwidth, lower cut off frequency and upper cut-off frequency. **[M-15][7]**

Solution:-

$R = 10\,\Omega,\ L = 0.0144\,H,\ C = 100\,\mu F$

(i) Resonant frequency, $fr = \dfrac{1}{2\pi\sqrt{LC}} = 134.51\,Hz$

(ii) Quality factor, $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = 1.1832$

(iii) Bandwidth $B_W = \dfrac{R}{2\pi L} 113.6821\,Hz \text{ or } 714.28\,r/s$

(iv) Lower cut-off frequency $f_1 = f_r - \frac{BW}{2} = 77.67\,Hz$

Upper cut-off frequancy $f_2 = f_r + \frac{BW}{2} = 191.35\,Hz$

(2) For a series RLC circuit having R = 10 Ω, L = 0.01 H and C = 100 μF, find the resonant frequency, quality factor and bandwidth. **[D-14][3]**

Solution:-

$R = 10\Omega,\ L = 0.01H\ \text{ and }\ C = 100\mu F$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 x 100 x 10^{-6}}} = 159.154\,Hz$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.01}{100 x 10^{-6}}} = 1$$

$$B.W. = \frac{R}{2\pi L} = \frac{10}{2\pi x 0.01} = 159.154\,Hz$$

(3) A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 230V, 50 Hz supply, the maximum current obtained by varying the inductance is 2A. The voltage across the capacitor is 500V. Calculate the resistance, inductor and capacitor of the circuit. **[D-12][7]**

Solution:-

Given : V = 230V, f_o = 50Hz, I_{max} = 2A, V_c = 500V

$$\therefore R = \frac{V}{I_{max}} = \frac{230}{2} = 115\Omega$$

$$\therefore X_c = \frac{V_c}{I_{max}} = \frac{500}{2} = 250\Omega$$

$$\therefore X_c = \frac{1}{2\pi f_o C} \Rightarrow 250 = \frac{1}{2\pi x 50 x C}$$

$\therefore C = 12.73\mu f$

At resonance, $X_C = X_L$

$\therefore X_L = 250 \quad \Rightarrow \quad 2\pi f_o L = 250$

$$\therefore L = \frac{250}{2\pi x 50} = 0.795\,H$$

(4) An inductor having a resistance of 25Ω and Q_0 of 10 at a resonant frequency of 10kHz is fed from $100\underline{|0^o}$ supply. Calculate (i) Value of series capacitance required to produce resonance with the coil. (ii) The inductance of the coil (iii) Q_0 using L/C ratio (iv) Voltage across capacitor (v) Voltage across coil. **[M-13][7]**

Solution:-

Given : $R = 25\,\Omega,\ Q_0 = 10,\ f_0 = 10\,KHz,\ V = 100\angle 0^o$,

To find: $C = ?\quad L = ?\quad Q_0$ by using L / C ratios $\quad V_C = ?\quad V_{coil} = ?$

$$Q_0 = \frac{\omega_0 L}{R} \Rightarrow 10 = \frac{2\pi x 10 x 10^3 x L}{25}$$

$$\therefore L = 3.97 mH$$

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f_0^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore C = \frac{1}{4\pi^2 f_0^2 L} = 63.8\,\mu f$$

$$\therefore I^0 = \frac{V}{R} = \frac{100}{25} = 4\,Amp$$

$$\therefore X_{CO} = \frac{1}{2\pi f_0 C} = 249.46\,\Omega$$

$$\therefore V_{CO} = I_0\,X_{CO} = 4 x 249.46 = 997.83\,volt$$

$$Z_{coil} = \sqrt{R^2 + X_L^2} = 250.69\,\Omega$$

$$\therefore V_{coil} = I_0\,Z_{coil} = 250.69 x 4 = 1002.77\,volt$$

(5) A series circuit with R = 5Ω, C = 20μf and a variable inductor has an applied voltage of 10V with frequency of 1000 rad/sec. The inductor is adjusted until voltage across resistance is maximum. Find voltage across each element. **[M-11][8]**

Solution:

When the frequency reaches its resonant value f_r the impedance is equal to R and hence the current reaches its maximum value and voltage across resistance is maximum.

$$\omega_L = \frac{1}{\omega c} \qquad ...(i)$$

We know $R = 5\Omega,\ C = 20\ \mu F,\ \omega = 1000$ rad/sec

From equation (i) $\omega_L = \frac{1}{\omega c}$

$$L = \frac{1}{\omega^2 c} = \frac{1}{1000^2 \times 20 \times 10^{-6}}$$

L= 50 mH

A resonance condition, $I = \frac{V}{R} = \frac{10}{5} = 2\ \text{Amp}$

Voltage across each element

i. Inductors $V_L = I \cdot \omega L = 2 \times 1000 \times 50 \times 10^{-3} = 100V$

ii. Capacitor $V_c = \frac{I}{\omega_c} = \frac{2}{1000 \times 20 \times 10^{-6}} = 100V$

Under resonance, series circuit acts as voltage amplifier where Q_0 is amplification factor or magnification factor

$V_L = Q_0 \cdot V.$

$V_c = Q_0 \cdot V \Rightarrow 100 = Q_0 \cdot 10 \Rightarrow Q_0 = 10$

Magnification factor $Q_0 = 10$

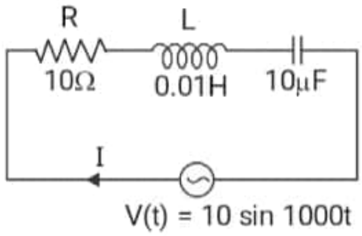
(6) A series RLC circuit has the following parameters; R = 10 ohms, L = 0.01 H, C = 100μf, voltage source v(t) = 10 sin 1000t. Find i) Circuit impedance (ii) Power dissipated in the circuit (iii) Resonant frequency (iv)Band width (v) Quality factor **[M-09][8]**

Solution:

Given : $R = 10\Omega$, $L = 0.01$ H, $C = 100\ \mu F$

$V(t) = 10 \sin 1000\, t, \quad \therefore \quad \omega = 1000$

$\therefore \quad V_m = 10, V_{rms} = \frac{10}{\sqrt{2}} = 7.7071\angle 0^0$



Step 1 : Calculation of circuit impedance

$X_L = 2\pi f L$

[As $\omega = 2\pi f \Rightarrow 1000 = 2\pi f \Rightarrow f = 159$ Hz]

$\therefore \quad X_L = 2\pi \times 159 \times 0.01 = 9.99\ \Omega$

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 159 \times 100 \times 10^{-6}}$$

$\therefore \quad X_C = 10\Omega$

Total impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(10)^2 + (9.99 - 10)^2} = 10$

$\phi = \tan^{-1}(-0.01/10) = -23.35^0$

$Z = 10\ \angle -23.35^0\ \Omega$

Step 2 : Calculation of power dissipated in the circuit:

The current folwing through circuit ,

$$I = \frac{V}{Z} = \frac{7.071\angle 0^0}{10\angle -23.35^0} = 0.707\angle 23.35^0\ \text{Amp}$$

Power consumed , $P = I^2 R = (0.707)^2 \times 10 = 4.99$ Watt

Step 3: Calculation of resonating frequency :

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}}$$

$f_r = 159.15 H_z$

Step 4: Calculation of quality factor :

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.01}{100 \times 10^{-6}}}$$

$Q = 1$

Step 5 : Calculation of Bandwidth :

$$BW = \frac{f_r}{Q} = \frac{159.15}{1} = 159.15\,Hz$$

(7) A 250–V series RLC circuit resonates at 50 Hz. The current is then 1 Amps and the Potential difference across the capacitor is 500–V. Calculate –

(i) the resistance (ii) the inductance (iii) the capacitance (iv) Bandwidth **[D-08][10]**

Solution:-

Given: $V_s = 250V, f_r = 50Hz, I = 1A, V_C = 500V$

To find : R = ?, L = ?, C = ?, BW = ?

Step 1: Calculation of Q :

At resonance the voltage across C is Q times the supply voltage.

$$\therefore V_C = QV_s \Rightarrow 500 = Q \times 250$$

$$\therefore Q = 2$$

Step 2: Calculation of R :

$$\text{At resonance } R = |Z| = \frac{V_s}{I} = \frac{250}{1} = 250\,\Omega$$

Step 3 : Calculation of L and C :

$$Q = \frac{2\pi f_r L}{R} \Rightarrow 2 = \frac{2\pi \times 50 \times L}{250}$$

$$\therefore L = 1.5915\,H$$

$$\text{Also } 2\pi f_r L = \frac{1}{2\pi f_r C} \Rightarrow 2\pi \times 50 \times 1.5915 = \frac{1}{2\pi \times 50 \times C}$$

$$\therefore C = 6.366\,\mu F$$

Step 4 :Calcuation of Bandwidth :

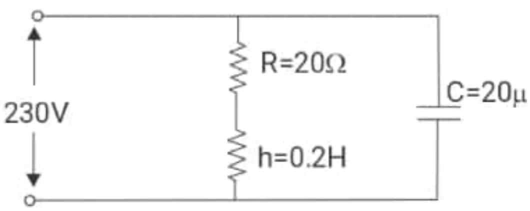
$$B.W. = \frac{f_r}{Q} = \frac{50}{2} = 25Hz$$

Type IX : Parallel Resonannce

(1) An inductive coil having a resistance of 20 Ω. and inductance of 0.2 H is connected in parallel with a 20 μF capacitor with variable frequency and 230 V supply. Find the frequency at which the total current drawn from supply is in phase with the supply voltage. Find the value of the current and the impedance of the circuit at this frequency. **[D-14][7]**

Solution:-

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 20 \times 10^{-6}} - \frac{20^2}{0.2^2}} = 77.9697\,Hz$$



$$y_T = y_1 + y_2 = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$y_T = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C} = \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

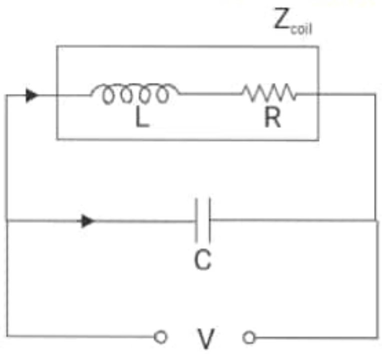
Total admittance at resonance, $y_T = \dfrac{R}{R^2 + X_L^2}$

$$\therefore \quad Z_T = \frac{R^2 + X_L^2}{R} = \frac{20^2 + (2\pi x\, 77.9697\, x\, 0.2)}{20} = 500\,\Omega$$

$$\therefore \quad I_T = V / Z_T = 0.46\, A$$

(2) A coil of inductance 31.8mH with resistance of 12Ω is connected in parallel in with a capacitor across 250 volts, 50 Hz supply. Determine the value of capacitance, if no reactive current is taken from the supply. **[M-14][7]**

Solution:-



Given : Coil $\Rightarrow$ $L = 31.8MH$, $R = 12\Omega$, $V = 220\, V$, $f = 50\, Hz$

$\therefore$ Reactive current is zero. It mens, it is a parallel resonance circuit.

$$\therefore f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \Rightarrow 50 = \frac{1}{2\pi}\sqrt{\frac{1}{31.8x10^{-3}} - \frac{12^2}{(31.8x10^{-3})^2}}$$

$C = 3.22\, x10^{-9}\, F$

(3) An inductive coil of resistance 10Ω and inductance 0.1H is connected in parallel with 150μF capacitortc a variable frequency, 200V supply. Find the resonance frequency at which the total current taken from supply is in Phase with supply voltage. Also find value of this current. Draw the phasor diagram **[D-13][7]**

Solution:-

Given: $R = 10\Omega$ series inductive coil, $L = 0.1H$ series inductive coil

$C = 150\,\mu F = 150\, x10^{-6}\, F$ in parallel with inductive coil, $V = 200\, V$

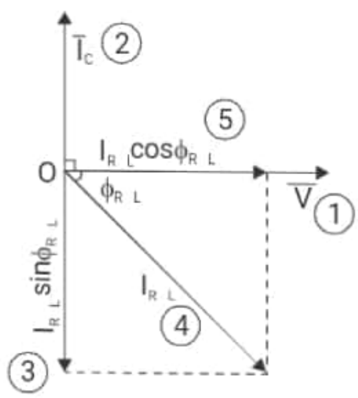
$$\text{Resonant frequency, } f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.1x150\, x10^{-6}} - \frac{10^2}{(0.1)^2}} = 37.9\, Hz$$

$$\text{Dynamic impedance, } Z_r = \frac{L}{CR} = \frac{0.1}{150\, x10^{-6}\, x10} = 66.67\,\Omega$$

$$\text{Circuit current at resonance, } I_r = \frac{V}{Z_r} = \frac{200}{66.67} = 3\, Amp$$

$$\text{For phasor diagram, } I_L = \frac{V}{Z_L} = \frac{200}{\sqrt{10^2 + 2\, x\, \pi\, x\, 37.9\, x\, 0.1}} = 17.97\, Amp$$

$$I_C = \frac{V}{X_C} = \frac{200}{\dfrac{1}{2\pi\, x\, 37.9\, x\, 150\, x10^{-6}}} = 7.14\, Amp$$



$$\text{Phase angle of the coil, } \phi_L = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi f_r L}{R} = \tan^{-1}\left(\frac{2\pi\, x\, 37.9\, x\, 0.1}{10}\right) = 67.21^{o}$$