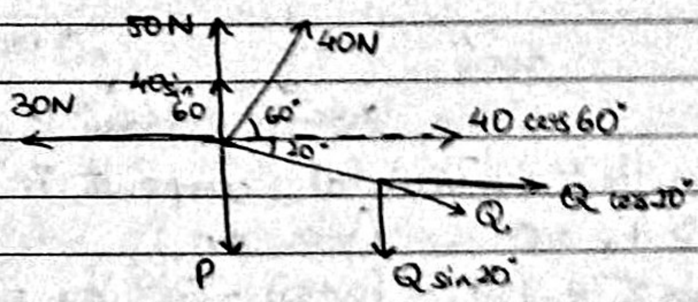
47 _ Yash Sarang

LTCE

Assignment 1

1) Five concurrent coplanar forces act on a body as shown in figure. Find the force P and Q such that the resultant of five forces is zero.

FBD:



Since resultant is zero, forces along X axis and Y axis will be zero.

$\therefore \sum F_x = 0 \quad (\rightarrow +ve)$

$\therefore 40\cos 60^\circ + Q\cos 20^\circ - 30 = 0.$

$\boxed{\therefore Q = 21.2836\ N}$

$\therefore \sum F_y = 0 \quad (\uparrow +ve)$

$50 + 40\sin 60^\circ - Q\sin 20^\circ - P = 0.$

$\boxed{\therefore P = 77.3616 N}$

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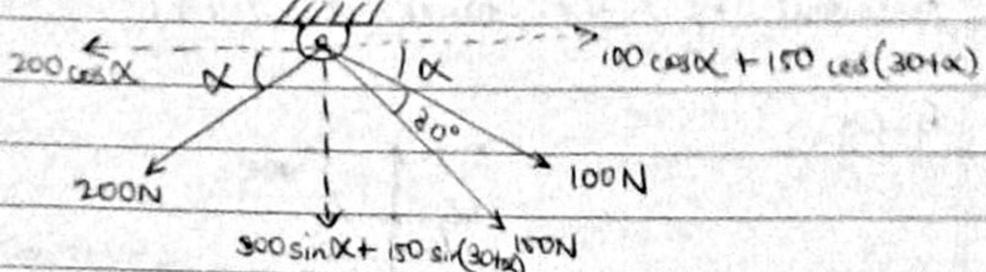
2) For the system shown, determine

i) The required value of α if resultant of three forces is to be vertical

ii) The corresponding magnitude of resultant.



(i) For resultant, horizontal component is zero.

$\therefore \Sigma F_x = 0 \qquad (\rightarrow +ve)$

$\cancel{100}\cos\alpha + 150\cos(\alpha+30) - \cancel{200}\cos\alpha = 0.$

$3\cos(\alpha+30) = 2\cos\alpha.$

$3\cos\alpha \times \frac{\sqrt{3}}{2} - \frac{3\sin\alpha}{2} = 2\cos\alpha$

$\cos\alpha\left(\frac{3\sqrt{3}}{2} - 2\right) = \frac{3}{2}\sin\alpha \qquad \therefore \tan\alpha = \sqrt{3} - \frac{4}{3}.$

$\therefore \alpha = \tan^{-1}\left(\sqrt{3} - \frac{4}{3}\right) = (21.74)^\circ.$

(ii) $\therefore \Sigma F_y = -\left[300\sin(21.74)^\circ + 150\sin(51.74)^\circ\right]$

$= -228.8999\,N$

$\therefore R = 228.8999\,N\ (\downarrow)$

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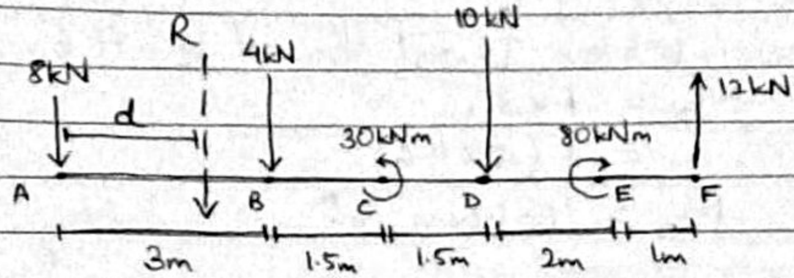
3) Figure shows a parallel system of four forces and two couples.

FBD:



i) Replace it by a single force and obtain its location from point A.

This is a system of 4 parallel forces and two couples.

$\therefore$ Resultant force $R = \Sigma F_y$ ($\uparrow$ +ve)

$= (-8-4-10+12)$

$= -10\,kN \quad = 10\,kN \downarrow$

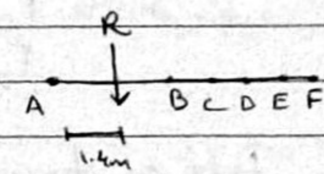
Let us assume that the resultant force is located at a distance d from A.

Using Varignon's theorem,

$\Sigma M_A^F = M_A^R$ ($\circlearrowleft$ +ve)

$\therefore \; -10 \times d = -4\times3 - 10\times6 + 12\times9.$

$\boxed{\therefore d = 1.4m}$ (from right of A)



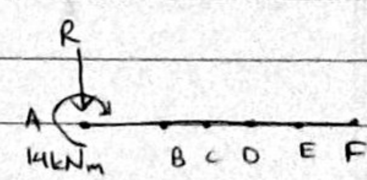
ii) To replace it by a force couple system at point A, we need to shift resultant force R = 10kN to point A by introducing a couple M. The ⊥ distance between point A and force R is 1.4m.

$\therefore$ Couple $M = F \times d$

$= 10 \times (-1.4)$

$= -14\,kNm$

$\therefore M = 14\,kNm \circlearrowright$



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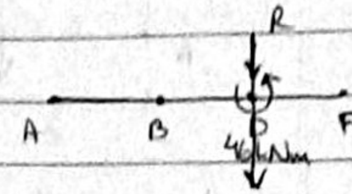
iii) To replace it by a force couple system at point D, we ~~me~~ need to shift force $R = 10kN$ to point D by introducing a couple M_D.
The $\perp$ distance between D and force R is 4.6m.

$\therefore$ Couple $M_D = F \times d$

$= +(10 \times 4.6)$

$M_D = 46\,kNm$ ↺



iv) To replace it by two parallel forces at B and D.
The force couple system at D is shown.
The couple of 46kNm at D can be replaced by two parallel forces at B and D, equal in magnitude and opposite in sense.

Couple $M_D = F \times d$

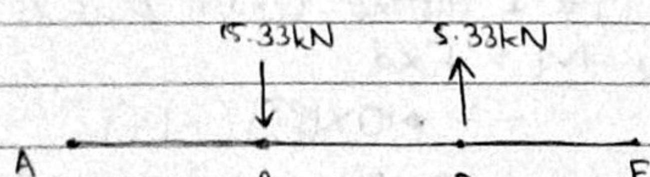
$46 = F \times 3$ (dist btw B and D is 3m)

$\therefore F = 15.33N.$

$\therefore$ Force $F_B = 15.33N \downarrow$ at B and

$F_D = 15.33N \uparrow$ at D can replace the couple of 46kNm.

Adding forces at D i.e $-10 + 15.33 = 5.33\,kN$, we get the two parallel components as $15.33kN (\downarrow)$ at B and $5.33kN (\uparrow)$ at D as shown.



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4) Determine the result of the system of forces shown in figure. Locate the point where the resultant cuts the base AB.

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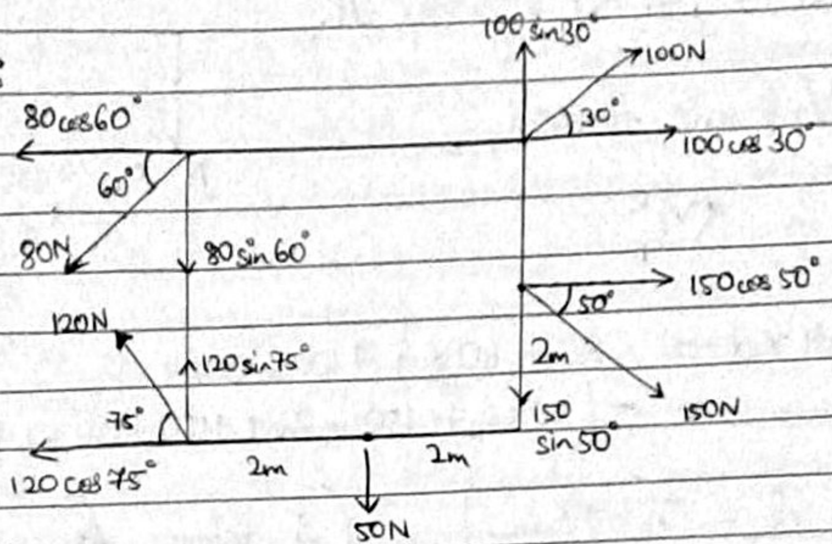

This is a system of five general forces.
Using method of resolution,

$\Sigma F_x = 100 \cos 30° + 150 \cos 50° - 120 \cos 75° - 80 \cos 60°$.
($\rightarrow$ +ve)

$\Sigma F_x = 111.96N \; (\rightarrow)$

$\Sigma F_y = 100 \sin 30° + 120 \sin 75° - 80 \sin 60° - 150 \sin 50° - 50$.
($\uparrow$ +ve)

$\therefore \Sigma F_y = 68.28N \; (\downarrow)$

Now,
$R = \sqrt{F_x^2 + F_y^2} = \sqrt{111.96^2 + 68.28^2}$

$\therefore R = 131.14N.$

$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{68.28}{111.96}$

$\therefore \theta = 31.38°$

$\therefore$ Resultant is located in IV^{th} quadrant.
$R = 131.14N \; (31.38°)$

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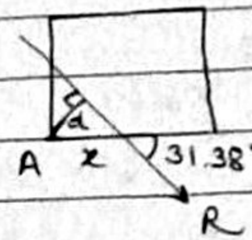
for location of resultant,
Let us assume the resultant is at a perpendicular distance 'd' from point A, to the right of A.



Using Varignon's theorem,

$$\Sigma M_A^F = \Sigma M_A^R$$

$$-131.14 \times d = 80\cos 60 \times 4 + 100\sin 30 \times 4 - \left[50 \times 2 + -150\sin 50 \times 4 + 150\cos 50^\circ \times 2 + 100\cos 30^\circ\right]$$

$\boxed{\therefore d = 5.623\,m.}$ ($\perp$ distance from right of A).

Let the resultant force R cut the base AB at a distance x from A.

$\therefore$ from geometry, $\sin\theta = d/x$.

$$\sin 31.38 = 5.623/x.$$

$\boxed{\therefore x = 10.8\,m}$