DIVISION / ROLL NO - D1AD / 47

Vivekanand Education Society's Institute of Technology
(Academic year 2020-2021)

Subject - Engineering Mathematics 2

Semester II

TUTORIAL COVER PAGE

TUTORIAL NO - 2

TUTORIAL TOPIC - DIFFERETIAL EQUATIONS 2 - MODULE 2.

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NAME OF THE STUDENT - YASH SARANG

SIGNATURE OF THE TEACHER -

Linear Differential Equations of Higher Order

1) $(D^3+3D^2+D-5)y=0$ where $D=\frac{d}{dx}$

→ A.E is $D^3+3D^2+D-5=0$, $D=1$ is one of the solutions.

$$\begin{array}{c|cccc} 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

$\therefore D^3+3D^2+D-5=(D-1)(D^2+4D+5)$

$$D=\frac{-4\pm\sqrt{16-4(1)(5)}}{2} = -2\pm i$$

$D=1, -2+i, -2-i.$

$$\boxed{\therefore y = c_1e^{x}+e^{-2x}\left[c_2\cos x+c_3\sin x\right]}$$

2)

→ we have, $(D^2+4)y=\cos 2x+e^{3x}$

A.E is $D^2+4=0$ $\therefore D=\pm 2i.$

$$y_c=e^{0x}\left[A_1\cos 2x+B_1\sin 2x\right]$$

$$y_p=\frac{1}{D^2+4}\cos 2x+\frac{1}{D^2+4}e^{3x}$$

$$y_p=x\frac{1}{2D}\cos 2x+\frac{1}{(3^2+4)}e^{3x}$$

$$y_p=\frac{x}{2}\times\frac{\sin 2x}{2}+\frac{e^{3x}}{13} = x\sin 2x/4+e^{3x}/13$$

$$\boxed{\therefore y_p = \frac{x \sin 2x}{4} + \frac{e^{3x}}{13}}$$

3)

$\rightarrow$ We have $(D^2 - 2D + 1)y = x^2 e^{3x}$

A.E is $D^2 - 2D + 1 = 0$. $\therefore D = 1, 1$.

$\therefore CF = [C_1 + C_2 x] e^x$

$\therefore PI = \frac{1}{f(D)} \times X = \frac{1}{D^2 - 2D + 1} \times x^2 e^{3x} (a = 3)$

Substituting $f(D) = f(D+3)$.

$$PI = \frac{1}{(D-1)^2} x^2 e^{3x} = e^{3x} \frac{1}{[(D+3)-1]^2} x^2$$

$$= \frac{e^{3x}}{(D+2)^2} \times x^2 = \frac{e^{3x} \times x^2}{D^2 + 4D + 4} = \frac{e^{3x}}{4} \frac{\times x^2}{(D^2 + 4D + 4)}$$

$$= \frac{e^{3x}}{4} \times x^2 \times (1 + D + D^2/4)^{-1}$$

$$= \frac{e^{3x}}{4} \left(1 - \left(D + \frac{D^2}{4} \right) + \left(D + \frac{D^2}{4} \right)^2 + \dots \right) x^2$$

$$= \frac{e^{3x}}{4 \times 4} (4x^2 + 6 - 2x)$$

$$y_p = \frac{e^{3x}}{16} (4x^2 + 6 - 2x)$$

$$\boxed{\text{Complete solution } y = (C_1 + C_2 x) e^x + \frac{e^{3x}}{16} (4x^2 + 6 - 2x)}$$

4)

→ We have $(D^2+1)y = \frac{1}{1+\sin x}$

A.E is $m^2+1=0$ and hence $\boxed{m = \pm i}$

$\therefore$ CF $= C_1 \cos x + C_2 \sin x$

$y = A(x)\cos x + B(x)\sin x$ —— ①

be the complete solution of the given differential equation.

We have $y_1 = \cos x\, y_2 = \sin x$,

$y_1' = -\sin x\, y_2' = \cos x$.

$W = y_1 y_2' - y_2 y_1' = 1$, Also $\phi(x) = 1/(1+\sin x)$.

Now, $A' = \frac{-y_2\, \phi(x)}{W}$ and $B' = \frac{y_1\, \phi(x)}{W}$.

i.e $A' = -\sin x/(1+\sin x)$ and $B' = \cos x/(1+\sin x)$

consider

$$A' = \frac{-(1+\sin x - 1)}{1+\sin x} = -1 + \frac{1}{\sin x}$$

$$A = \int\left(-1 + \frac{1}{1+\sin x}\right).dx + k_1 = -x + \int \frac{1-\sin x}{\cos^2 x}.dx + k_1$$

$$= -x + \int(\sec^2 x - \sec x \tan x)\,dx + k_1.$$

$\therefore A = -x + \tan x - \sec x + k_1$ —— ②

Also, $B' = \frac{\cos x}{1+\sin x} = \frac{\cos x(1-\sin x)}{\cos^2 x} = \frac{1-\sin x}{\cos x}$

$$B = \int \frac{1-\sin x}{\cos x}.dx + k_2 = \int(\sec x - \tan x).dx + k_2$$

$= \log(\sec x + \tan x) + \log(\cos x) + k_2$

$= \log\left(\frac{1+\sin x}{\cos x}\right) + \log(\cos x) + k_2.$

$B = \log(1+\sin x) + k_2$ ——— (3)

Using equations (1), (2) and (3),

$y = [-x + \tan x - \sec x + k_1]\cos x + [\log(1+\sin x) + k_2]\sin x$

i.e

$$\boxed{y = k_1\cos x + k_2\sin x - x\cos x + \sin x - 1 + \sin x[\log(1+\sin x)]}$$

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