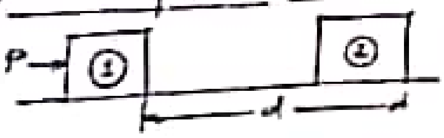
# Kinetics of Particle

## Work & Energy

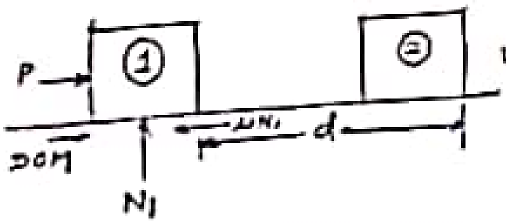
$\boxed{W D = F \times d}$ (Nm or J)

1) W D by External force



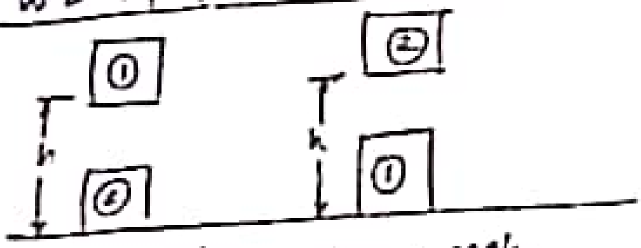

$\boxed{W D = P \times d}$

2) W D by Frictional Force




$W D = -(\mu N_1 \times d)$

3) W.D by Gravitational force




$WD = mgh$ $\quad$ $WD = -mgh$

4) W D by Spring force

$= \frac{1}{2} K (x_1^2 - x_2^2)$

$K_1$ = spring Const

$x_1$ = Deformation of spring at position 1

$x_2$ = Deformation of spring at position 2

Deformation = Change in length

$= \left| \text{Available length} - \text{Free length} \right|$

W-E Principle

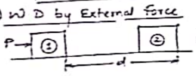
$\Sigma W D = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

# Kinetics of Particle

## Work & Energy

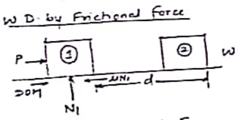
$\boxed{W.D. = F \times d}$ (Nm or J)

**1) W.D by External force**



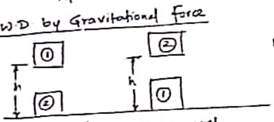

$\boxed{W.D. = P \times d}$

**2) W.D by Frictional force**




$W.D = -(\mu N_1 \times d)$

**3) W.D by Gravitational force**




W.D. = mgh  W.D = −mgh

**4) W.D by Spring force**

$= \frac{1}{2}K(x_1^2 - x_2^2)$

$K$ = spring Const

$x_1$ = Deformation of spring at position 1

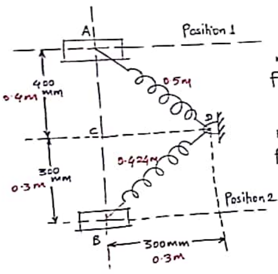
$x_2$ = Deformation of spring at position 2

Deformation = Change in length

$= \left|\text{Available length} - \text{Free length}\right|$

**W-E Principle**

$\Sigma W.D = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$




m = 10 kg

Friction = 0

$v_1 = 0$

$v_2 = ?$

K = 200 N/m

Free length = 200mm = 0.2m

1) W.D by E.F = 0

2) W.D by F.F = 0

3) W.D by G.F = mgh

$= 10 \times 9.81 \times 0.7$

$= 68.67$ J

4) W.D by S.F $= \frac{1}{2}K(x_1^2 - x_2^2)$

$x_1 = 0.5 - 0.2 = 0.3$ m

$x_2 = 0.424 - 0.2 = 0.224$ m

$= \frac{1}{2} \times 200(0.3^2 - 0.224^2)$

$= 3.982$ J

**W-E Principle**

$68.67 + 3.982 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$68.67 + 3.982 = \frac{1}{2} \times 10 \times v_2^2$

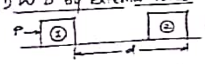
$\boxed{v_2 = 3.81 \text{ m/s}}$

# Kinetics of Particle

## Work & Energy

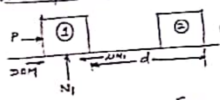
$\boxed{W \cdot D \cdot = F \times d}$ (Nm or J)

1) W.D by External force



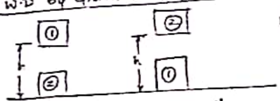

$\boxed{W \cdot D = P \times d}$

2) W.D by Frictional force




$W \cdot D = -(\mu N_1 \times d)$

3) W.D by Gravitational force




$W \cdot D = mgh$ $\quad$ $W \cdot D = -mgh$

4) W.D by Spring force

$= \frac{1}{2}K(x_1^2 - x_2^2)$

$K$ = spring Const.

$x_1$ = Deformation of spring at position 1

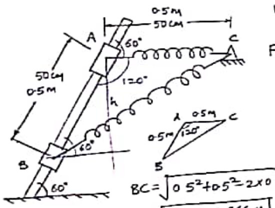
$x_2$ = Deformation of spring at position 2

Deformation = Change in length

$= \left| \text{Available length} - \text{Free length} \right|$

### W-E Principle

$\Sigma W \cdot D = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$




$BC = \sqrt{0.5^2 + 0.5^2 - 2 \times 0.5 \times 0.5 \cos 120^\circ}$

$\boxed{BC = 0.866\ m}$

1 m = 100 cm

0.5 m = 50 cm

W = 50 N

Friction = 0

Free length = 50 cm = 0.5 m

K = ?

$V_B = 2.5$ m/s

$V_A = 0$

1) W.D. by E.F. = 0

2) W.D by F.F = 0

3) W.D. by G.F. = mgh

$\sin 60^\circ = \frac{h}{0.5}$

$h = 0.5 \sin 60^\circ$

$h = 0.433$ m

$= 50 \times 0.433$

$= 21.65$ J

4) W.D by S.F. $= \frac{1}{2}K(x_1^2 - x_2^2)$

$x_1 = 0$

$x_2 = 0.866 - 0.5 = 0.366$ m

$= \frac{1}{2} \times K(0 - 0.366^2)$

$= -0.5 \times K \times 0.366^2$

### W-E Principle

$21.65 + (-0.5 \times K \times 0.366^2) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$

$21.65 - 0.5 \times K \times 0.366^2 = \frac{1}{2} \times \frac{50}{9.81} \times 2.5^2$

$\boxed{K = 85.4\ N/m}$

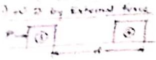
Q.37) Pg-65)

# Kinetics of Particle

## Work & Energy

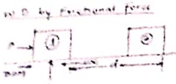
$\boxed{W.D = F \times d}$ (Nm or J)

1) W.D by External force



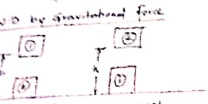
$\boxed{W.D = F \times d}$

2) W.D by Frictional force



$W.D = -(\mu N \times d)$

3) W.D by Gravitational force



$W.D = mgh$ $\quad$ $W.D = -mgh$

4) W.D by Spring force

$= \frac{1}{2}K(x_1^2 - x_2^2)$

$K$ = spring const

$x_1$ = Deformation of spring at position 1

$x_2$ = Deformation of spring at position 2

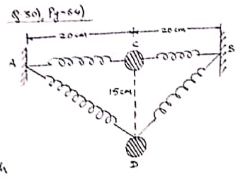
Deformation = Change in length

$= \left| \text{Available length} - \text{Free length} \right|$

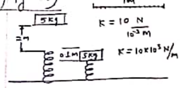
W-E Principle

$\Sigma W.D = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Q.50) Pg-64)




Q.37) Pg-65)

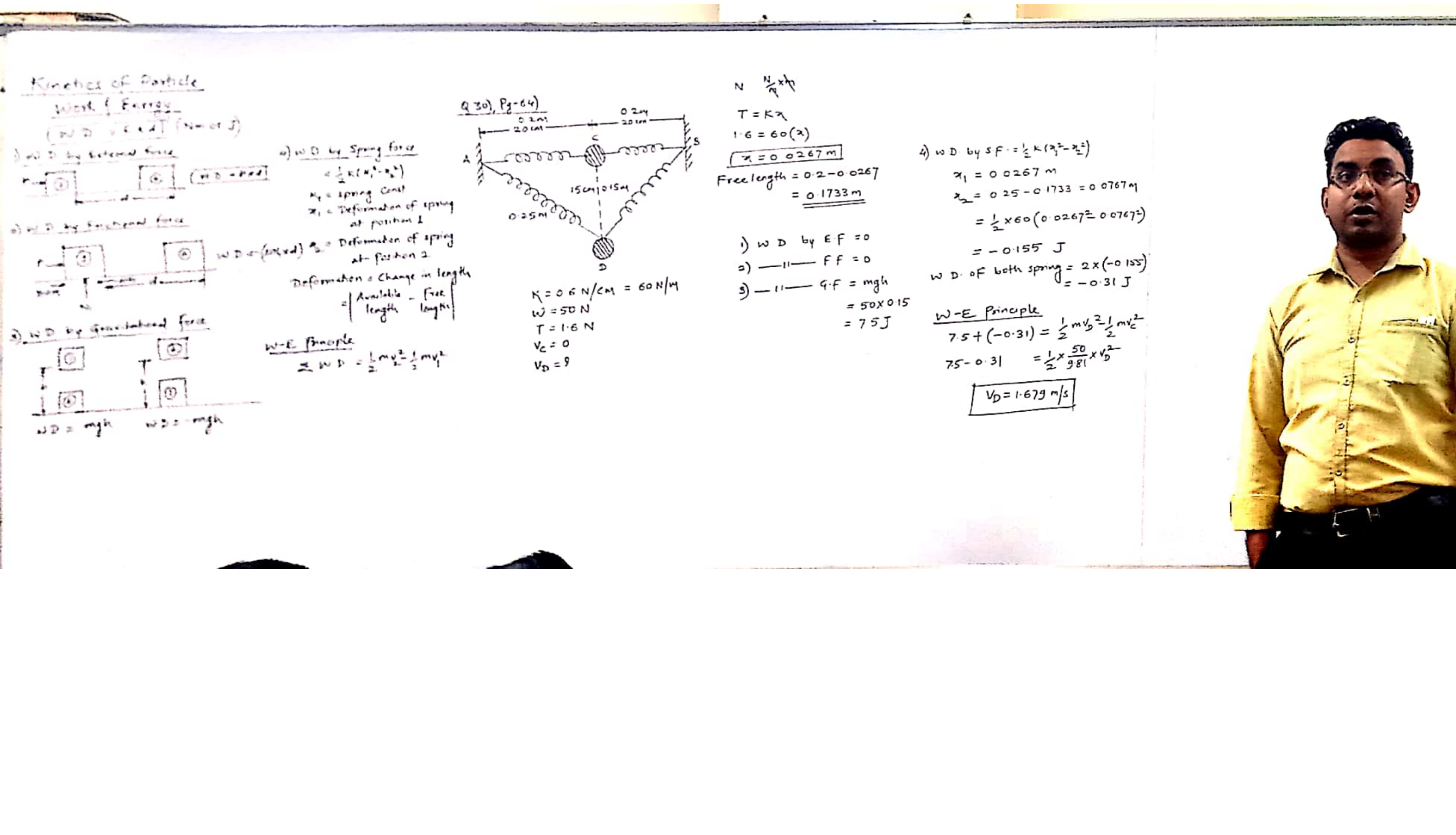



$K = 10$ N/mm $= 10 \times 1000$ N/m

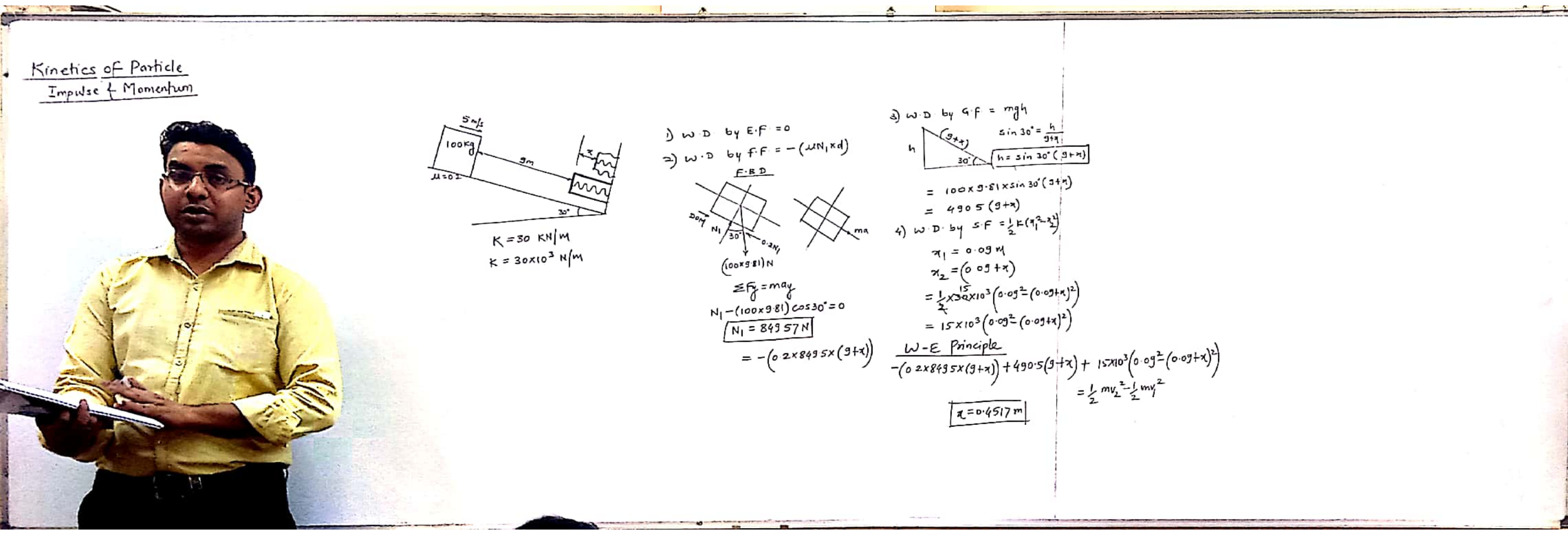
1 m

$K = 10 \frac{N}{10^{-3} m}$

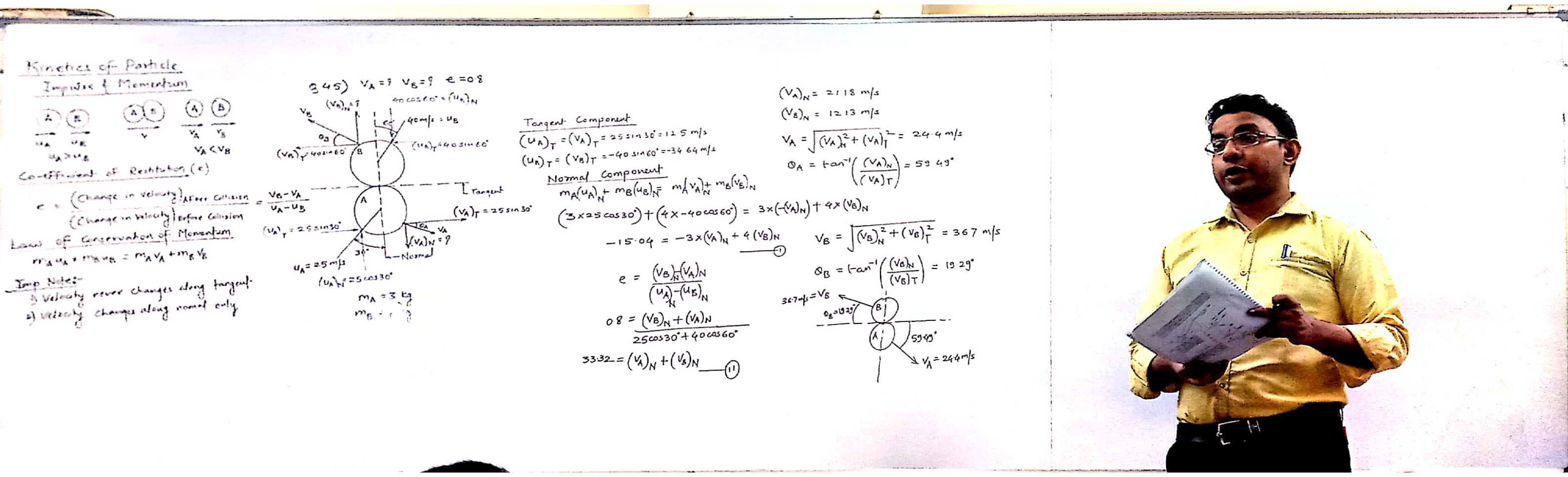
$K = 10 \times 10^3$ N/m



Kinetics of Particle
Work & Energy
W.D = F × d (N-m or J)
1) W.D by External force
(W.D = F×d)
2) W.D by Frictional force
3) W.D by Gravitational force
W.D = mgh
W.D = -mgh
4) W.D by Spring force
= 1/2 K(x1² - x2²)
K = Spring Const
x1 = Deformation of spring at position 1
x2 = Deformation of spring at position 2
Deformation = Change in length
= | Available length - Free length |
W-E Principle
Σ W.D = 1/2 mv2² - 1/2 mv1²
Q 30), Pg-64)
0.2m 20cm
0.2m 20cm
A
C
S
D
15cm 0.15m
0.25m
K = 0.6 N/cm = 60 N/m
W = 50 N
T = 1.6 N
Vc = 0
VD = ?
T = Kx
1.6 = 60(x)
x = 0.0267 m
Free length = 0.2 - 0.0267
= 0.1733 m
1) W.D by E.F = 0
2) —"— F.F = 0
3) —"— G.F = mgh
= 50×0.15
= 7.5 J
4) W.D by S.F = 1/2 K(x1² - x2²)
x1 = 0.0267 m
x2 = 0.25 - 0.1733 = 0.0767 m
= 1/2 × 60 (0.0267² - 0.0767²)
= -0.155 J
W.D of both spring = 2×(-0.155)
= -0.31 J
W-E Principle
7.5 + (-0.31) = 1/2 mvD² - 1/2 mvc²
7.5 - 0.31 = 1/2 × 50/9.81 × vD²
VD = 1.679 m/s

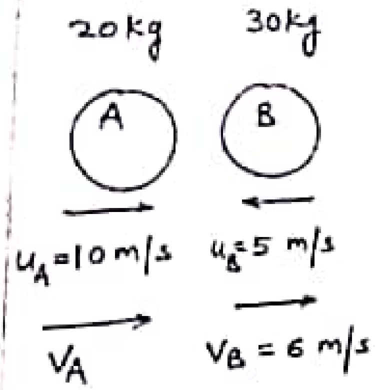


Kinetics of Particle
Impulse & Momentum
5 m/s
100kg
9m
μ=0.2
x
30°
K = 30 KN/M
K = 30×10³ N/M
1) W.D by E.F = 0
2) W.D by F.F = −(μN₁×d)
F.B.D
DOM
N₁
30°
0.2N₁
(100×9.81)N
ma
ΣFy = may
N₁ − (100×9.81) cos30° = 0
N₁ = 849.57 N
= −(0.2×849.5×(9+x))
3) W.D by G.F = mgh
h
(9+x)
30°
sin 30° = h/(9+x)
h = sin 30° (9+x)
= 100×9.81×sin 30° (9+x)
= 490.5 (9+x)
4) W.D by S.F = ½ K(x₁² − x₂²)
x₁ = 0.09 m
x₂ = (0.09 + x)
= ½ × 30×10³ (0.09² − (0.09+x)²)
= 15×10³ (0.09² − (0.09+x)²)
W-E Principle
−(0.2×849.5×(9+x)) + 490.5(9+x) + 15×10³(0.09² − (0.09+x)²)
= ½ mv₂² − ½ mv₁²
x = 0.4517 m



Kinetics of Particle
Impulse & Momentum
$u_A > u_B$
$v_A < v_B$
Co-efficient of Restitution (e)
$e = \frac{(\text{Change in velocity})_{\text{After Collision}}}{(\text{Change in velocity})_{\text{Before Collision}}} = \frac{v_B - v_A}{u_A - u_B}$
Law of Conservation of Momentum
$m_A u_A + m_B u_B = m_A v_A + m_B v_B$
Imp Note:-
1) Velocity never changes along tangent.
2) Velocity changes along normal only.
3.45) $v_A = ?$ $v_B = ?$ $e = 0.8$
$40\cos 60° = (u_B)_N$
$40 m/s = u_B$
$(u_B)_T = 40\sin 60°$
Tangent
Normal
$(v_A)_T = 25\sin 30°$
$(v_A)_N = ?$
$(u_A)_T = 25\sin 30°$
$u_A = 25 m/s$
$(u_A)_N = 25\cos 30°$
$m_A = 3$ kg
Tangent Component
$(u_A)_T = (v_A)_T = 25\sin 30° = 12.5$ m/s
$(u_B)_T = (v_B)_T = -40\sin 60° = -34.64$ m/s
Normal Component
$m_A(u_A)_N + m_B(u_B)_N = m_A(v_A)_N + m_B(v_B)_N$
$(3\times 25\cos 30°) + (4\times -40\cos 60°) = 3\times(-(v_A)_N) + 4\times(v_B)_N$
$-15.04 = -3\times(v_A)_N + 4(v_B)_N$ — (I)
$e = \frac{(v_B)_N - (v_A)_N}{(u_A)_N - (u_B)_N}$
$0.8 = \frac{(v_B)_N + (v_A)_N}{25\cos 30° + 40\cos 60°}$
$33.32 = (v_A)_N + (v_B)_N$ — (II)
$(v_A)_N = 21.18$ m/s
$(v_B)_N = 12.13$ m/s
$v_A = \sqrt{(v_A)_N^2 + (v_A)_T^2} = 24.4$ m/s
$\theta_A = \tan^{-1}\left(\frac{(v_A)_N}{(v_A)_T}\right) = 59.49°$
$v_B = \sqrt{(v_B)_N^2 + (v_B)_T^2} = 36.7$ m/s
$\theta_B = \tan^{-1}\left(\frac{(v_B)_N}{(v_B)_T}\right) = 19.29°$
36.7 m/s $= v_B$
$\theta_B = 19.29°$
59.49°
$v_A = 24.4$ m/s

**Q.44)**




L.C M

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$(20 \times 10) + (30 \times -5) = 20 V_A + 30 \times 6$$

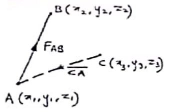
$$\boxed{V_A = -6.5 \text{ m/s}}$$

$$e = \frac{V_B - V_A}{u_A - u_B}$$

$$= \frac{6-(-6.5)}{10-(-5)}$$

$$\boxed{e = 0.833}$$

## Forces in Space




**Force Vector**

$$\overline{F_{AB}} = F_{AB}\left(\frac{(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}\right)$$

$$\overline{F_{AB}} = (F_x i + F_y j + F_z k)$$

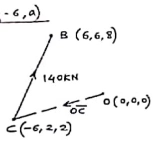
**Position Vector**

$$\overline{CA} = (x_1-x_3)i+(y_1-y_3)j+(z_1-z_3)k$$

**Moment Vector**

$$\overline{M}_C^{AB} = \overline{CA}\times\overline{F_{AB}} = \begin{vmatrix} i & j & k \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \\ F_x & F_y & F_z \end{vmatrix}$$

**Pg-77, Q-6, a)**




$$\overline{F_{CB}} = 140\left(\frac{12i+4j+6k}{\sqrt{12^2+4^2+6^2}}\right)$$

$$= 10(12i+4j+0k)$$

$$\boxed{\overline{F_{CB}} = (120i+40j+60k)\ \text{KN}}$$

**Position Vector**

$$\overline{OC} = (-6i+2j+2k)\ \text{m}$$

**Moment Vector**

$$\overline{M}_O^{CB} = \overline{OC}\times\overline{F_{CB}} = \begin{vmatrix} i & j & k \\ -6 & 2 & 2 \\ 120 & 40 & 60 \end{vmatrix}$$

$$\overline{M}_O^{CB} = (40i+600j-480k)\ \text{KNm}$$

**Pg-72, Q-6 a)**

**Q.44)**

20kg

A

B

$u_A = 10$ m/s

$V_A$

$V_B$

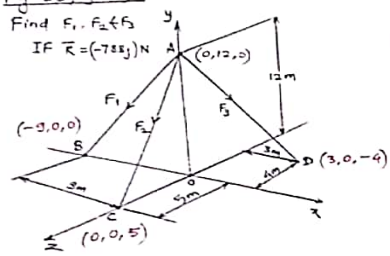
L.C.M

$e =$

Pg-85, Q-2c)

Find $F_1$, $F_2$ & $F_3$

IF $\overline{R} = (-788j)$ N




$$\overline{F_1} = F_1\left(\frac{-9i - 12j + 0k}{\sqrt{(-9)^2 + (-12)^2}}\right)$$

$$= 0.066 F_1 (-9i - 12j + 0k)$$

$$\overline{F_1} = (-0.594 F_1 i - 0.792 F_1 j + 0k)$$

$$\overline{F_2} = F_2\left(\frac{0i - 12j + 5k}{\sqrt{(-12)^2 + 5^2}}\right)$$

$$= 0.076 F_2 (0i - 12j + 5k)$$

$$\overline{F_2} = (0i - 0.912 F_2 j + 0.38 F_2 k)$$

$$\overline{F_3} = F_3\left(\frac{3i - 12j - 4k}{\sqrt{3^2 + (-12)^2 + (-4)^2}}\right)$$

$$= 0.076 F_3 (3i - 12j - 4k)$$

$$\overline{F_3} = (0.228 F_3 i - 0.912 F_3 j - 0.304 F_3 k)$$

$$\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3}$$

$$0i - 788j + 0k = (-0.594 F_1 + 0.228 F_3) i + (-0.792 F_1 - 0.912 F_2 - 0.912 F_3) j + (0.38 F_2 - 0.304 F_3) k$$

$$-0.594 F_1 + 0.228 F_3 = 0$$

$$-0.792 F_1 - 0.912 F_2 - 0.912 F_3 = -788$$

$$0.38 F_2 - 0.304 F_3 = 0$$

$F_1 = 155.46$ N

$F_2 = 324.01$ N

$F_3 = 405.01$ N