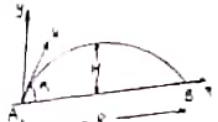
Projectile Motion



u → Initial Velocity

α → Inclination of Trajectory

Equation of Path

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

Height (H)

$$H = \frac{u^2\sin^2\alpha}{2g}$$

Imp Note:

1) Acceleration
$H.M = 0$
$V.M = -9.81\ m/s^2$

2) Time
$H.M = V.M = t$

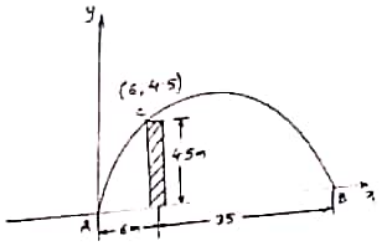
Time (T)

$$T = \frac{2u\sin\alpha}{g}$$

Range (R)

$$R = \frac{u^2\sin 2\alpha}{g}$$

$s = ut + \frac{1}{2}at^2$
$v = u + at$
$v^2 = u^2 + 2as$



$R = 41$

$$R = \frac{u^2\sin 2\alpha}{g}$$

$$41 \times 9.81 = u^2\sin 2\alpha$$

$$\frac{41 \times 9.81}{\sin 2\alpha} = u^2 \quad \text{—— (1)}$$

pt C (6, 4.5)

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$4.5 = 6\tan\alpha - \frac{9.81 \times 6^2}{2u^2\cos^2\alpha}$$

$$4.5 = 6\tan\alpha - \frac{9.81 \times 6^2}{2 \times \frac{41 \times 9.81}{\sin 2\alpha}\cos^2\alpha}$$

$$4.5 = 6\tan\alpha - \frac{9.81 \times 6^2 \times \sin 2\alpha}{2 \times 41 \times 9.81 \times \cos^2\alpha}$$

$$\boxed{\alpha = 41.5^\circ}$$

From eqn (1)

$$\boxed{u = 20.13\ m/s}$$

a-t graph

$x = t^3$ (Cubic)

$v = \frac{dx}{dt} = 3t^2$ (Parabola)

$a = \frac{dv}{dt} = 6t$ (Linear)

ear (In) Parabola (Anh)

(De) Parabola (Clock)

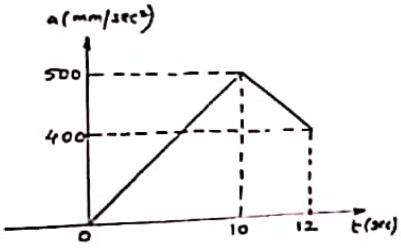
Anh-Clock

under graph

v-t → x-t

v-t → a-t

a-t → v-t → x-t



	0-10	10-12
a-t	Linear (In)	Linear (De)
v-t	Parabola (Anh)	Parabola (Clock)
x-t	Cubic (Anh)	Cubic (Anh)

Velocity

$V_{10} - V_0 = \frac{1}{2} \times 10 \times 500$

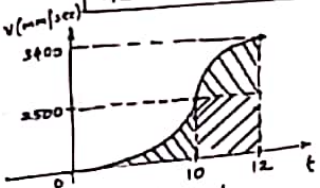
$V_{10} - V_0 = 2500 \quad (V_0 = 0)$

$V_{10} = 2500$ mm/sec

$V_{12} - V_{10} = (2 \times 400) + \left(\frac{1}{2} \times 2 \times 100\right)$

$V_{12} - V_{10} = 900$

$V_{12} = 3400$ mm/sec



v-t graph

Displacement

$x_{10} - x_0 = \frac{1}{3} \times 10 \times 2500$

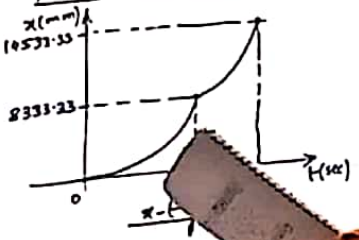
$x_{10} - x_0 = 8333.33 \quad (x_0 = 0)$

$x_{10} = 8333.33$ mm

$x_{12} - x_{10} = (2 \times 2500) + \left(\frac{2}{3} \times 2 \times 900\right)$

$x_{12} - x_{10} = 6200$

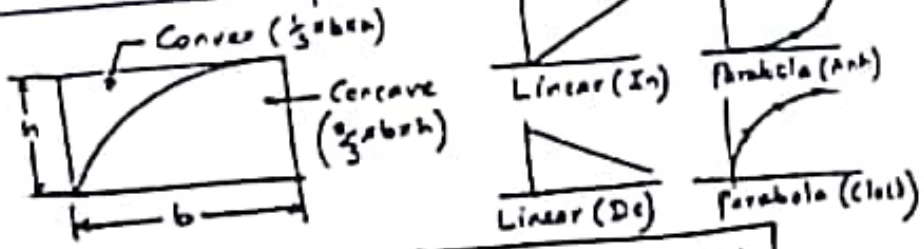
$x_{12} = 14533.33$ mm



x-t

Relation between x-t, v-t & a-t graph

$x = t$ (Linear)	$x = t^2$ (Parabola)	$x = t^3$ (Cubic)
$v = \frac{dx}{dt} = 1$ (Const)	$v = \frac{dx}{dt} = 2t$ (Linear)	$v = \frac{dx}{dt} = 3t^2$ (Parabola)
$a = \frac{dv}{dt} = 0$ (zero)	$a = \frac{dv}{dt} = 2$ (const)	$a = \frac{dv}{dt} = 6t$ (Linear)



* Cubic → Anti-Clock

Displacement

Change in displacement within time interval = Area under v-t graph

v-t → x-t, a-t

Velocity

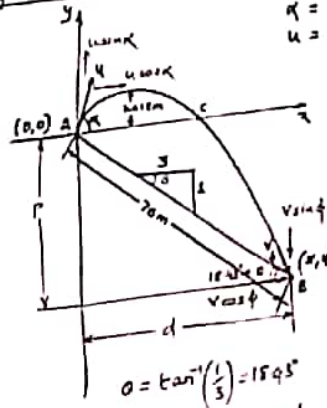
Change in Velocity within time interval = Area under a-t graph

a-t → v-t → x-t

Acceleration

Acceleration within time interval = Slope of v-t graph.

Pg-47, 9-84)



$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$

$\sin\theta = \frac{P}{76}$ | $\cos\theta = \frac{d}{76}$

$P = 24.02\,m$ | $d = 72.1\,m$

$x = 72.1$

$y = -24.02$

$\kappa = ?$

$u = ?$

	HM	VM
u	$u\cos\kappa$	$u\sin\kappa$
v	$v\cos\phi$	$-v\sin\phi$
s	72.1	-24.02
a	0	-9.81
t	t	t

HM

$s = ut + \frac{1}{2}at^2$

$72.1 = u\cos\kappa \times t$ —(i)

$v = u + at$

$v\cos\phi = u\cos\kappa$ —(ii)

VM

$s = ut + \frac{1}{2}at^2$

$-24.02 = u\sin\kappa \times t - \frac{1}{2} \times 9.81 \times t^2$ —(iii)

$v = u + at$

$-v\sin\phi = u\sin\kappa - 9.81 \times t$ —(iv)

$v^2 = u^2 + 2as$

$(-v\sin\phi)^2 = (u\sin\kappa)^2 + 2(-9.81)(-24.02)$ —(v)

Motion from A to C

$H = 19\,m$

$H = \frac{u^2\sin^2\kappa}{2g}$

$19 \times 2 \times 9.81 = u^2\sin^2\kappa$

$19.3 = u\sin\kappa$ —(vi)

put eqn (vi) into (iii)

$-24.02 = 19.3 \times t - \frac{1}{2} \times 9.81 \times t^2$

$t = 4.92\,sec$

From eqn (i)

$u\cos\kappa = 14.65$ —(vii)

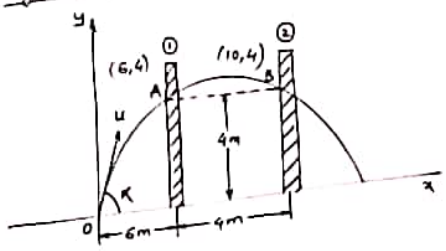
eqn (vi) ÷ (vii)

$\kappa = 52.79°$

From eqn (vii)

$u = 24.23\,m/s$

Pg - 44, Q-71)



eqn of path

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

pt A (6,4)

$$4 = 6\tan\alpha - \frac{g\,81\times 6^2}{2u^2\cos^2\alpha} \quad \text{—①}$$

pt B (10,4)

$$4 = 10\tan\alpha - \frac{g\,81\times 10^2}{2u^2\cos^2\alpha} \quad \text{—②}$$

$$\frac{g\,81\times 10^2}{2u^2\cos^2\alpha} = 10\tan\alpha - 4$$

$$\frac{g\,81}{2u^2\cos^2\alpha} = \frac{10\tan\alpha - 4}{10^2}$$

from eqn ①

$$4 = 6\tan\alpha - \left(\frac{10\tan\alpha - 4}{10^2}\right)6^2$$

$$\boxed{\alpha = 46.84^\circ}$$

from eqn ①

$$4 = 6\tan(46.84^\circ) - \frac{g\,81\times 6^2}{2u^2\cos^2(46.84^\circ)}$$

$$\boxed{u = 12.54\ m/s}$$

v-t & a-t graph

$x = t^3$

$v = \frac{dx}{dt} =$

$a = \frac{dv}{dt} =$

Linear (In) Parabola (Anti)

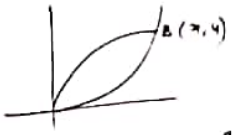
Linear (De) Parabola (Clock)

Cubic → Anti-Clock

= Area under v-t graph

rea under -t graph

ope of -t graph



$u = 6$ m/s

$\alpha = 50°$

$y = 0.2x^2$

$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$

$0.2x^2 = x\tan(50°) - \frac{9.81 \times x^2}{2 \times 6^2 \times \cos^2 50°}$

$x = 2.25$ m

$y = 1.003$ m

t graph

$x = t^3$ (Cubic)

$v = \frac{dx}{dt} = 3t^2$ (Parabola)

$a = \frac{dv}{dt} = 6t$ (Linear)

(In) Parabola (Anti)

Parabola (Clock)

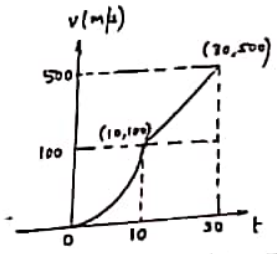
Anti-Clock

der

graph

v-t → x-t, a-t

a-t → v-t → x-t



	0-10	10-30
v-t	Parabola (Anti)	Linear (In)
x-t	Cubic (Anti)	Parabola (Anti)
a-t	Linear (In)	Const.

Displacement

$x_{10} - x_0 = \frac{1}{3} \times 10 \times 100$

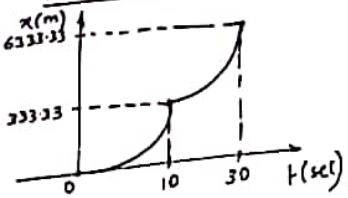
$x_{10} - x_0 = 333.33\ m \quad (x_0 = 0)$

$\boxed{x_{10} = 333.33\ m}$

$x_{30} - x_{10} = (20 \times 100) + \left(\frac{1}{2} \times 20 \times 400\right)$

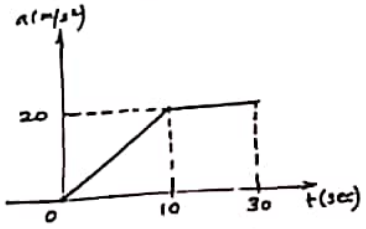
$x_{30} - x_{10} = 6000$

$\boxed{x_{30} = 6333.33\ m}$



Acceleration

$a_{10-30} = \frac{500 - 100}{30 - 10} = 20\ m/s^2$

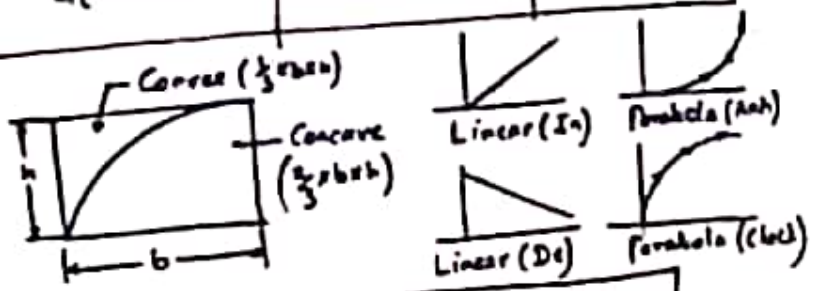


(a-t graph)

Motion Curves

Relation between x-t, v-t & a-t graph

$x = t$ (Linear)	$x = t^2$ (Parabola)	$x = t^3$ (Cubic)
$v = \frac{dx}{dt} = 1$ (Const)	$v = \frac{dx}{dt} = 2t$ (Linear)	$v = \frac{dx}{dt} = 3t^2$ (Parabola)
$a = \frac{dv}{dt} = 0$ (zero)	$a = \frac{dv}{dt} = 2$ (const)	$a = \frac{dv}{dt} = 6t$ (Linear)



* Cubic → Anti-Clock

Displacement

Change in displacement within time interval = Area under v-t graph

v-t → x-t, a-t

Velocity

Change in Velocity within time interval = Area under a-t graph

a-t → v-t → x-t

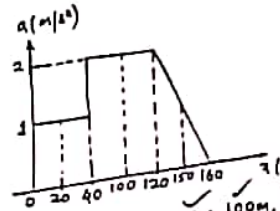
Acceleration

Acceleration within time interval = Slope of v-t graph

$u = 6$ m/s

$\alpha = 50°$

$y = 0.2x^2$



Find v at 20m, 100m, 150m

$v = \frac{dx}{dt}$

$v = \frac{dv}{dt} \cdot \frac{dx}{dv}$

$v = a \frac{dx}{dv}$

Motion From 0 to 20m

$u = 0$
$v = V_{20m}$
$s = 20m$
$a = 1\ m/s^2$
$t = t$

$v^2 = u^2 + 2as$

$V_{20m}^2 = 0 + 2 \times 1 \times 20$

$\boxed{V_{20m} = 6.32\ m/s}$

Motion From 20m to 40m

$u = 6.32\ m/s$
$v = V_{40m}$
$s = 20m$
$a = 1\ m/s^2$
$t = t$

$v^2 = u^2 + 2as$

$V_{40m}^2 = 6.32^2 + 2 \times 1 \times 20$

$\boxed{V_{40m} = 8.94\ m/s}$

Motion From 40 to 100m

$u = 8.94\ m/s$
$v = V_{100m}$
$s = 60m$
$a = 2\ m/s^2$
$t = t$

$v^2 = u^2 + 2as$

$V_{100m}^2 = 8.94^2 + 2 \times 2 \times 60$

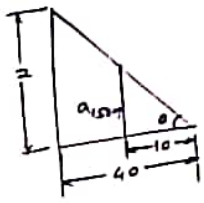
$\boxed{V_{100m} = 17.88\ m/s}$

Motion From 100m to 120m

$u = 17.88\ m/s$
$v = V_{120m}$
$s = 20m$
$a = 2\ m/s^2$
$t = t$

$v^2 = u^2 + 2as$

$\boxed{V_{120m} = 20\ m/s}$



$\tan\theta = \frac{2}{40}$ $\tan\theta = \frac{a_{150m}}{10}$

$\frac{2}{40} = \frac{a_{150m}}{10}$

$\boxed{a_{150m} = 0.5\ m/s^2}$

Motion From 120 to 150m

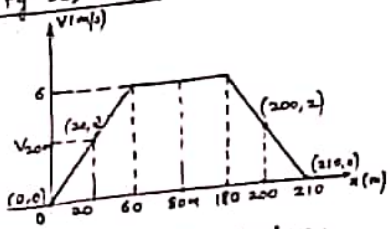
$u = 20\ m/s$
$v = V_{150m}$
$s = 30m$
$a = 0.5\ m/s^2$
$t = t$

$v^2 = u^2 + 2as$

$V_{150m}^2 = 20^2 + 2 \times 0.5 \times 30$

$\boxed{V_{150m} = 20.73\ m/s}$

Pg-35, Q-17)



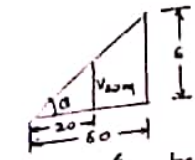
Find a at 20m, 80m & 200m

$$a = \frac{dv}{dt}$$

$$a = \frac{dx}{dt}\frac{dv}{dx}$$

$$a = v\left(\frac{dv}{dx}\right)$$

$\frac{dv}{dx}$ → slope



$\tan\theta = \frac{6}{60}$ $\tan\theta = \frac{V_{20m}}{20}$

$$\frac{6}{60} = \frac{V_{20m}}{20}$$

$$\boxed{V_{20m} = 2\ m/s}$$

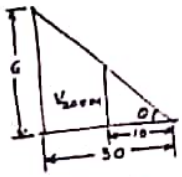
$$a_{20m} = V_{20m}\frac{dv}{dx}$$

$$a_{20m} = 2\left(\frac{2-0}{20-0}\right)$$

$$\boxed{a_{20m} = 0.2\ m/s^2}$$

60 m to 180 m Velocity const

$\therefore \boxed{a_{80m} = 0}$



$\tan\theta = \frac{6}{30}$ $\tan\theta = \frac{V_{200m}}{10}$

$$\frac{6}{30} = \frac{V_{200m}}{10}$$

$$\boxed{V_{200m} = 2\ m/s}$$

$$a_{200m} = V_{200m}\frac{dv}{dx}$$

$$= 2\left(\frac{0-2}{210-200}\right)$$

$$\boxed{a_{200m} = -0.4\ m/s^2}$$