Name : Yash Sarang

Roll No: 47 Seat No: AIDSA47

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Signature: Sarangyash

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Q1. B) $(D^4 - 1)y = e^x + \cos x \cos 3x$.

→ The auxiliary eqn is $D^4 - 1 = 0$. $\therefore (D^2-1)(D^2+1) = 0$.

$\therefore D = 1, -1, +i, -i$.

$\therefore$ The C.F is $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$.

$$P.I = \frac{1}{D^4 - 1}\left[e^x + \frac{1}{2}(\cos 4x + \cos 2x)\right]$$

$$= \frac{1}{(D-1)(D+1)(D^2+1)} \cdot e^x + \frac{1}{2} \times \frac{1}{(D^4-1)} \cos 4x + \frac{1}{2(D^4-1)} \cos 2x$$

$$= \frac{xe^x}{4} + \frac{1}{510}\cos 4x + \frac{1}{30}\cos 2x$$

$\therefore$ Complete Solution = C.F + P.I.

Q.1 E) Find area between parabola $y = x^2 - 6x + 3$ and $y = 2x - 9$.

→ To find intersections, $x^2 - 6x + 3 = 2x - 9$.

$x^2 - 8x + 12 = 0$.

$(x-6)(x-2) = 0$. $x = 2, 6$.

when $x = 6$, $y = 3$ and $x = 2$, $y = -5$

$\therefore$ (6,3) and (2,-5) are the points of intersection

The interval is $2 \le x \le 6$.

$\therefore$ The integral of the area is $= \left|\int_2^6 x^2 - 6x + 3 - (2x - 9)\right|$

$$= \left|\int_2^6 x^2 - 8x + 12\right|$$

$$= \left|\left[x^3/3 - 8x^2/2 + 12x\right]_2^6\right|$$

$$= \left| \left(\frac{6\times6\times8^2}{8} - 4\times6\times6 + 12\times6 \right) - \left(\frac{2^3}{3} - 4\times4 + 12\times2 \right) \right|$$

$$= \left| (72 - 144 + 72) - \left(\frac{8}{3} - 16 + 24 \right) \right|$$

$$= \left| - \left(\frac{8}{3} + 8 \right) \right|$$

Area $= |-32/3| = 32/3$ units

$\therefore$ The area between the parabola and the line is $32/3$ units

Q1. A $\quad (3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0.$

Comparing with $M(x,y)\,dx + N(x,y)\,dy = 0.$

$M(x,y) = 3x^2y^4 + 2xy$ and $N(x,y) = 2x^3y^3 - x^2$

$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x \neq \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$

$\therefore$ Given Differential Equation is not exact.

Consider $\frac{\partial N/\partial x - \partial M/\partial y}{M} = \frac{6x^2y^3 - 2x - 12x^2y^3 - 2x}{3x^2y^4 + 2xy} = \frac{-2x}{4x}\left(\frac{3xy^3+2}{3xy^3+2}\right)$

$= -2/y = g(y)$

$\int g(y)\,dy = -2\int \frac{1}{y}\,dy = -2\log y = \log y^{-2}$

$\therefore$ Integrating factor $= e^{\int g(y)\,dy} = e^{\log y^{-2}} = y^{-2} = 1/y^2$

Multiplying given eqn by Integrating factor,

$$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right).dy = 0.$$

This is an exact differential equation.

It's solution is $\int_{y\,\text{constant}} \left(3x^2y^2 + \frac{2x}{y}\right).dx + \int_{\substack{\text{terms not}\\ \text{containing } x}} (0).dy = c.$

$$\cancel{3}y^2\left(\frac{x^3}{\cancel{3}}\right) + \frac{\cancel{2}}{y}\left(\frac{x^2}{\cancel{2}}\right) = c.$$

$\boxed{\therefore\; x^3y^2 + \frac{x^2}{y} = c}$ is the solution of the given differential equation.

Q1. D)
→ We transform the given integral to cylinderical polar coordinates by putting $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ and $dx\,dy\,dz = r\,dr\,d\theta\,dz$.

With the change of coordinate system, the equation the sphere becomes $r^2 + z^2 = a^2$ and of the cylinder becomes $r^2 = ar\cos\theta$ ie $r = a\cos\theta$.

The volume of integration is bounded by the sphere and the cylinder. Thus, z varies from $z = -\sqrt{a^2 - r^2}$ to $z = \sqrt{a^2 - r^2}$, r varies from $r = 0$ to $r = a\cos\theta$ and θ varies from $\theta = -\pi/2$ to $\theta = \pi/2$.

$$\boxed{\therefore I = \int_{-\pi/2}^{\pi/2} \int_0^{a\cos\theta} \int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} z^2 . r .\, dr\, d\theta\, dz}$$