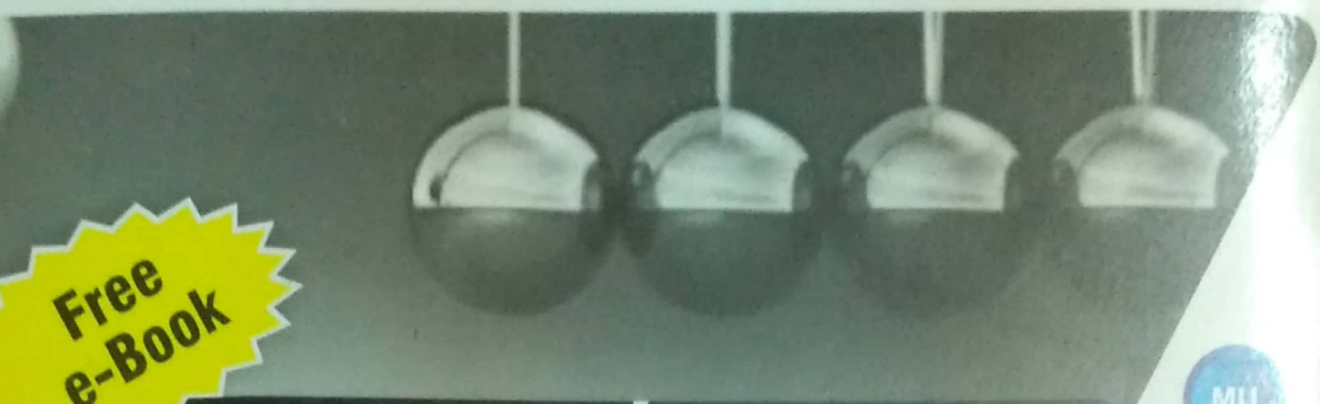

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Syllabus

Module No.	Detailed Contents	Hrs
01 33	**INTERFERENCE AND DIFFRACTION OF LIGHT** Interference by division of amplitude and by division of wave front; Interference in thin film of constant thickness due to reflected and transmitted light; origin of colours in thin film; Wedge shaped film(angle of wedge and thickness measurement); Newton's rings Applications of interference - Determination of thickness of very thin wire or foil; determination of refractive index of liquid; wavelength of incident light; radius of curvature of lens; testing of surface flatness; Anti-reflecting films and Highly reflecting film. Diffraction of Light - Fraunhoffer diffraction at single slit, Fraunhoffer diffraction at double slit, Diffraction Grating, Resolving power of a grating, dispersive power of a grating Application of Diffraction - Determination of wavelength of light with a plane transmission grating. **(Refer chapters 1, 2 and 3)**	14
02 10	**LASERS** Quantum processes as absorption, spontaneous emission and stimulated emission; metastable states, population inversion, pumping, resonance cavity, Einsteins's equations; Helium Neon laser; Nd:YAG laser; Semiconductor laser, Applications of laser- Holography (construction and reconstruction of holograms) and industrial applications (cutting, welding etc), Applications in medical field. **(Refer chapter 4)**	05
03 10	**FIBRE OPTICS** Total internal reflection; Numerical Aperture; critical angle; angle of acceptance; Vnumber; number of modes of propagation; types of optical fiber; Losses in optical fibre(Attenuation and dispersion) Applications of optical fibre - Fibre optic communication system; sensors (Pressure, temperature, smoke, water level), applications in medical field **(Refer chapter 5)**	04
04	**ELECTRODYNAMICS** Cartesian, Cylindrical and Spherical Coordinate system, Scaler and Vector field, Physical significance of gradient, curl and divergence, Determination of Maxwell's four equations. Applications-design of antenna, wave guide, satellite communication etc. **(Refer chapter 6)**	08
05 8	**CHARGE PARTICLE IN ELECTRIC AND MAGNETIC FIELDS** Fundamentals of Electromagnetism, Motion of electron in electric field (parallel, perpendicular, with some angle); Motion of electron in magnetic field (Longitudinal and Transverse); Magnetic deflection; Motion of electron in crossed field; Velocity Selector; Velocity Filter, Electron refraction; Bethe's law; Electrostatic focusing; Magnetostatic focusing; Cathode ray tube (CRT); Cathod ray Oscilloscope (CRO) Application of CRO: Voltage (dc, ac), frequency, phase measurement. **(Refer chapter 7)**	05

Module No.	Detailed Contents	Hrs
06 10	**NANOSCIENCE AND NANOTECHNOLOGY** Introduction to nano-science and nanotechnology, Surface to volume ratio, Two main approaches in nanotechnology -Bottom up technique and top down technique; Important tools in nanotechnology such as Scanning Electron Microscope, Transmission Electron Microscope, Atomic Force Microscope. Nano materials : Methods to synthesize nanomaterials (Ball milling, Sputtering, Vapour deposition, solgel), properties and applications of nanomaterials. **(Refer chapter 8)**	04

Suggested Experiments: (Any five)

1. Determination of radius of curvature of a lens using Newton's ring set up
2. Determination of diameter of wire/hair or thickness of paper using Wedge shape film method.
3. Determination of wavelength using Diffracion grating. (Hg/ Ne source)
4. Determination of number of lines on the grating surface using LASER Sourse.
5. Determination of Numerical Aperture of an optical fibre.
6. Determination of wavelength using Diffracion grating. (Laser source)
7. Use of CRO for measurement of frequency and amplitude.
8. Use of CRO for measurement of phase angle.
9. Study of divergence of laser beam
10. Determination of width of a slit using single slit diffraction experiment (laser source)

The distribution of Term Work marks will be as follows –

4. Attendance (Theory and Practical) : 05 marks
5. Assignments : 10 marks
6. Laboratory work (Experiments and Journal) : 10 marks

Assessment:

Internal Assessment Test:

Assessment consists of two class tests of 15 marks each. The first class test is to be conducted when approx. 40% syllabus is completed and second class test when additional 35% syllabus is completed. Duration of each test shall be one hour.

End Semester Theory Examination:

1. Question paper will comprise of total 06 questions, each carrying 15 marks.
2. Total 04 questions need to be solved.
3. Question No: 01 will be compulsory and based on entire syllabus wherein sub-questions of 2 to 3marks will be asked.
4. Remaining questions will be mixed in nature.(e.g. Suppose Q.2 has part (a) from module 3 then part (b) will be from any module other than module 3)
5. In question paper weightage of each module will be proportional to number of respective lecture hrs as mentioned in the syllabus.

Module 1

Module 2

Chapter 4 : LASER 2·2 **4-1 to 4-39**

Syllabus : Quantum processes as absorption, spontaneous emission and stimulated emission; metastable states, population inversion, pumping, resonance cavity, Einstein's equations; Helium Neon laser ; Nd :YAG laser; Semiconductor laser,

Applications of laser : Holography (construction and reconstruction of holograms) and industrial applications (cutting, welding etc), Applications in medical field.

Module 3

Chapter 5 : Fibre Optics 2·1 **5-1 to 5-32**

Syllabus : Total internal reflection; Numerical Aperture; critical angle; angle of acceptance; V number; number of modes of propagation; types of optical fiber; Losses in optical fibre(Attenuation and dispersion) Applications of optical fibre - Fibre optic communication system; sensors (Pressure, temperature, smoke, water level), applications in medical field.

Module 4

Chapter 6 : Electrodynamics 2·1 **6-1 to 6-46**

Syllabus : Cartesian, Cylindrical and Spherical Coordinate system, Scalar and Vector field, Physical significance of gradient, curl and divergence, Determination of Maxwell's four equations. Applications-design of antenna, wave guide, satellite communication etc.

Module 5

Chapter 7 : Charged Particle in Electric and Magnetic Fields 4-1 **7-1 to 7-53**

Syllabus : Fundamentals of Electromagnetism, Motion of electron in electric field (parallel, perpendicular, with some angle); Motion of electron in magnetic field (Longitudinal and Transverse); Magnetic deflection; Motion of electron in crossed field; Velocity Selector ; Velocity Filter, Electron refraction; Bethe's law; Electrostatic focusing; Magnetostatic focusing; Cathode ray tube (CRT);Cathode ray Oscilloscope (CRO).Application of CRO: Voltage (dc, ac), frequency, phase measurement.

Module 6

Chapter 8 : Nano-Science and Nanotechnology 6.1 **8-1 to 8-16**

Syllabus : Introduction to nano-science and nanotechnology, Surface to volume ratio, Two main approaches in nanotechnology -Bottom up technique and top down technique; Important tools in nanotechnology such as Scanning Electron Microscope, Transmission Electron Microscope, Atomic Force Microscope.

Nano materials : Methods to synthesize nanomaterials (Ball milling, Sputtering, Vapour deposition, Sol Gel), properties and applications of nanomaterials.

□□□

CHAPTER 1

Module 1

Interference in Thin Film

Syllabus

Interference by division of amplitude and by division of wave front; Interference in thin film of constant thickness due to reflected and transmitted light; origin of colours in thin film;

Wedge shaped film (angle of wedge and thickness measurement); Newton's rings

Syllabus Topic

Interference by Division of Amplitude and by Division of Wave Front

➤ **Topics covered :** Introduction

1.1 Introduction

- When two or more waves arrive at a point in a space simultaneously, the disturbance at that point is vector sum of disturbances caused by those waves taken individually.
- This is the general principle of superposition of waves and applies to all kinds of transverse waves. Because of this disturbance, the resultant amplitude and intensity may be very different from the sum of those contributed by the two beams acting simultaneously.
- This modification of intensity obtained by the superposition of two or more waves of light is what we call as **interference**. If the resultant intensity is zero then destructive and if greater then we have constructive interference.
- It is important to know that a very small difference of geometrical spacing of $\lambda/4$ for a to and fro journey of light will create a path difference of $\lambda/2$ and in

turn a bright spot in interference pattern will appear dark and a dark will appear bright.

- This provides us a tool for an accurate measurement of spacing $\lambda/4$. For minimum visible wavelength 4000Å this spacing comes out to be 1000Å i.e. 0.1 Micrometer.
- If a reader compares it with least count of Micrometer screw gauge, will surely agree that optical instrument based upon interference as the principle are excellent as long as their ability to measure small spacing.
- For producing interference, at least two waves are required and the two waves should have identical frequency and infinite extension or they must be coherent. But they do not exist in nature. There are following methods to produce coherent waves.

(a)	Division of wavefront
(b)	Division of amplitude

(a) Division of wavefront

- When light from a source is allowed to pass through different two slits, the original wavefront gets partitioned into two wavefronts.
- The wavefronts travel through different paths and when they are united, they interfere.
- Example Fresnel's Bi-prism, Loyd's mirror.

(b) Division of amplitude

- The incident beam is divided into two or several beams by means of partial reflection at the surface of thin film.
- The amplitude and therefore the intensity of the original wavefront gets divided.
- This beams subsequently follow different optical paths and produce interference when brought together.
- This method is known as **division of amplitude**.

In this chapter we will discuss division of amplitude technique observed in thin films.

Syllabus Topic
Interference in Thin Film of Constant Thickness due to Reflected and Transmitted Light

➤ **Topics covered :** Change of Phase on Reflection or Stoke's Relation, Interference in a Thin Parallel Sided Film, Thin and Thick Films

1.2 Change of Phase on Reflection or Stoke's Relation

- We are all acquainted with the fact that when soap bubbles are kept under sunlight different colours are visible in different directions. The colours appear due to the interference of light, which originates by the multiple reflections in thin transparent soap film.
- This phenomenon was first explained by Newton. According to him interference from thin film takes place because of

(i)	Reflected light
(ii)	Transmitted light.

- When light ray is reflected from a denser medium, a phase change of π and a path difference of $\lambda/2$ is introduced. Consider, a ray of light is incident on a transparent medium.
- It is partially reflected and partially transmitted. Assume 'a' be the incident amplitude and μ_1 and μ_2 are the refractive index of two medium respectively. Say, r is reflection coefficient and t is transmission coefficients from rarer to denser medium and r′ is the reflection coefficient and t′ is transmission coefficients from denser to rarer medium.

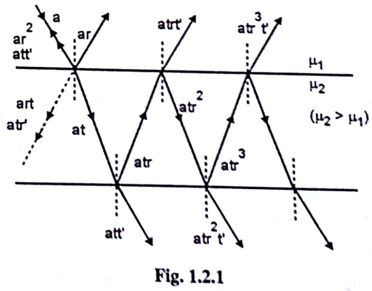


Fig. 1.2.1

- Hence, at the first point of interaction at the interface of rarer and denser medium,

  amplitude of reflected ray is "ar"

  amplitude of transmitted ray is "at"

  Keeping principle of reversibility in mind the, the reflected light ray "ar" has its transmission component "art"

  And the transmitted component "at" has its reflected component atr'

- Since, there is no light along dotted line, so that, we can write

$$art + at\,r' = 0 \Rightarrow r + r' = 0$$

$$\Rightarrow \quad r' = -r \qquad ...(1.2.1)$$

  These equation is known as **Stoke's relation.**

- This relation $r' = -r$ shows that a phase change of π or path difference of $\lambda/2$ occurs because of the reflection of light from the denser medium.
- Therefore, we can conclude that a **phase change of π or path difference $\lambda/2$ occurs when light waves are reflected at the surface of denser medium and no change of phase occurs when light waves are reflected at the surface of rarer medium.**

1.3 Interference in a Thin Parallel Sided Film

MU - Dec. 2013, Dec. 2014, May 2015, Dec. 2016

- Consider a ray AB of monochromatic light of wavelength λ from an extended source incident at B, on the upper surface of a parallel sided thin film of thickness t and refractive index μ as shown in Fig. 1.3.1.
- Let the angle of incidence be i.
- At B, the beam is partly reflected along BR_1 and partly refracted at an angle r along BC.
- At C, it is again partly reflected along CD and partly refracted along CT_1. Similar partial reflections and refractions occur at points D, E etc.
- Thus we get a set of parallel reflected rays BR_1, DR_2 etc. and a set of parallels transmitted rays CT_1, ET_2, etc.
- For a thin film, the waves travelling along BR_1 and DR_2 in the reflected system will overlap.
- These waves originate from the same incident wave AB and hence are coherent.

- Hence they will interfere constructively or destructively according as the path difference between them is integral multiple of λ or odd multiple of $\frac{\lambda}{2}$.

(Reflected system)

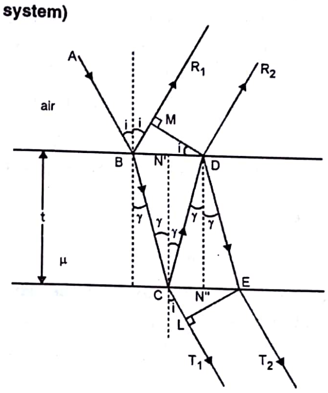


Fig. 1.3.1 : Interference in thin films

- To find the path difference between BR_1, and DR_2, draw DM perpendicular to BR_1. The paths travelled by the beams beyond DM are equal. Hence the optical path difference (optical path difference is obtained by multiplying geometrical path difference by its refractive index) between them is

$$\Delta = \text{Path BCD in film} - \text{Path BM in air}$$
$$= \mu(BC + CD) - BM$$

From Fig. 1.3.1 we have $BC = CD = \frac{t}{\cos r}$

$$\therefore \quad \mu(BC + CD) = \frac{2\mu t}{\cos r}$$

$$\text{and} \quad BM = BD \cdot \sin i = 2\,BN' \sin i \qquad (\because BD = 2\,BN')$$

$$= 2t \cdot \tan r \cdot \sin i \quad (\because BN' = CN' \tan r = t \cdot \tan r)$$

$$\therefore \quad BM = 2t\frac{\sin r}{\cos r} \cdot \sin i$$

$$= \frac{2\mu t}{\cos r} \cdot \sin^2 r \qquad \left(\because \frac{\sin i}{\sin r} = \mu\right)$$

$\therefore$ The optical path difference between the rays is

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r}\sin^2 r = \frac{2\mu t}{\cos r}(1-\sin^2 r) \quad ...(1.3.1)$$

or $$\Delta = 2\mu t \cos r$$

- The film is optically denser than the surrounding medium air. Hence the ray BR_1 originating by reflection at the denser medium suffers a phase change of π or a path change of $\frac{\lambda}{2}$ due to reflection at B. (No such change of phase occurs for ray DR_2 as it is a result of reflection at C)
- Hence the effective path difference between BR_1 and DR_2 is

$$2\mu t \cos r + \frac{\lambda}{2}$$

Condition for maxima and minima in reflected light

(i) The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

or $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$ **where n = 1, 2, 3, 4,...(For maxima)** ...(1.3.2)

or $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ **when n = 0, 1, 2, 3,....**

When this condition is satisfied the film will appear bright in the reflected system.

(ii) The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ ie.

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $$2\mu t \cos r = n\lambda \text{ (For minima)} \quad ...(1.3.3)$$

where n = 0, 1, 2, 3, ...

Transmitted system

- The transmitted rays CT_1 and ET_2 are also derived from the same incident ray AB and hence are coherent. (Fig. 1.3.1)

- When they interfere, they can give the interference pattern in transmitted system. To find the path difference between CT_1 and ET_2 we drop EL perpendicular to CT_1. See Fig. 1.3.1.

$\therefore$ Path difference $\Delta = \mu(CD + DE) - CL$

- It can be calculated the same way as in reflected system and it is found that path difference.

$$\Delta = 2\mu t \cos r$$

- But in this case no phase change occurs due to reflection at C and D. Hence the effective path difference between CT_1 and ET_2 is $2\mu t \cos r$.
- Hence the condition for constructive interference to take place in the transmitted system is that

$$2\mu t \cos r = n\lambda \text{ (Maxima)} \quad ...(1.3.4)$$

and the film appears bright in transmitted system.

- The condition for destructive interference is

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \text{ (Minima)} \quad ...(1.3.5)$$

and the film appears dark in transmitted system.

- Comparison of Equations (1.3.2), (1.3.3), (1.3.4) and (1.3.5) shows that the conditions of maxima and minima in reflected light are just opposite to those in transmitted light.
- Hence **the film which appears bright in reflected light appears dark in transmitted light and vice versa.**

Ex. 1.3.1 : A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 Å.

Soln. :

Given : $i = 30°$, $\mu = 1.46$, $n = 8$, $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : For dark band,

$$2\mu t \cos r = n\lambda \qquad n = 0, 1, 2, 3, ...$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 30°}{1.46} = 0.3424$$

$$\therefore \quad \cos r = \sqrt{1-\sin^2 r} = 0.9395 \quad = \sqrt{1-0.3424^2}$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cdot \cos r} = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times 0.9395}$$

$$\therefore \quad t = 1.7176 \times 10^{-4} \text{ cm.}$$ ....Ans.

1.4 Thin and Thick Films

MU - May 2016

The effective path difference between the interfering rays in reflected light is

$$2\,\mu\, t \cos r + \frac{\lambda}{2}$$

(i) If the film is excessively thin, then its thickness being very small as compared to the wavelength of light, the term $2\,\mu\, t \cos r$ can be neglected as compared to $\frac{\lambda}{2}$. Hence the effective path difference becomes $\frac{\lambda}{2}$ which is the condition for minima. Hence every wavelength in the incident light will be absent in reflected system and the film will appear black in reflected light.

(ii) If the thickness of the film is large enough as compared to the wavelength of light, the path difference at any point of the film will be large. Under these conditions the same point will be a maximum for a large number of wavelengths and the same point will be a minimum for another set of large number of wavelengths. The number of wavelengths sending maximum intensity at a point are almost equal to the number of wavelengths sending minimum intensity. Also these wavelengths sending maximum and minimum intensity will be distributed equally over all colours in white light. Hence the **resultant effect at any point will be the sum of all colours i.e. white in thick film.**

(iii) Hence we define 'thin film' as : The film whose thickness is of the order of wavelength of the light which is used to expose it.

Syllabus Topic : Origin of Colours in Thin Films

➤ **Topics covered :** Production of Colours in Thin Films, Necessity of the Extended Source

1.5 Production of Colours in Thin Films

MU - Dec. 2012, May 2015, Dec. 2016

When a thin film is exposed to white light from an extended source, it shows beautiful colours in the reflected system.

- Light is reflected from the top and bottom surfaces of a thin film and the reflected rays interfere.
- The path difference between the interfering rays depends on the thickness of the film and the angle of refraction r and hence on the inclination of the incident ray.
- White light consists of a continuous range of wavelengths. At a particular point of the film and for a particular position of the eye (i.e. t and r constant) those wavelengths of incident light that satisfy the condition for the constructive interference in the reflected system will be seen in reflected light.
- The colouration will vary with the thickness of the film and inclination of the rays (i.e. with the position of the eye with respect to the film). Hence if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed each time.

1.6 Necessity of the Extended Source

- In case of interference in thin films, a narrow source limits the visibility of the film.
- Consider a thin film and a narrow source of light S as shown in Fig. 1.6.1. The ray 1 produces interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the film at some different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye.
- Similarly rays incident at different angles on the film do not reach the eye. Hence only the portion A of the film is visible and not the rest.

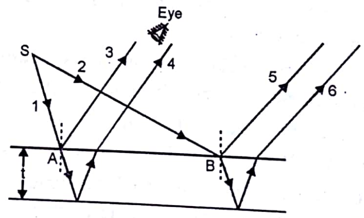


Fig. 1.6.1

- If an extended source of light is used as shown in Fig. 1.6.2, the ray 1, after reflection from the upper and lower surfaces of the film emerges as 3 and 4 which reach the eye.
- Also the other rays incident at different angles on the film enter the eye and the field of view is large.

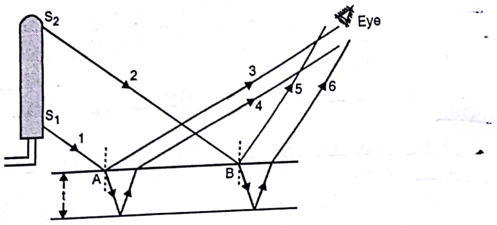


Fig. 1.6.2

- **Hence to observe interference phenomenon in thin films, a broad source of light is required.**

Syllabus Topic

Wedge-Shaped Film (Angle of Wedge and Thickness Measurement)

> **Topics covered :** Film of Non-uniform Thickness (Wedge-Shaped Film), Spacing between Two Consecutive Bright Bands

1.7 Film of Non-uniform Thickness (Wedge-Shaped Film)

MU - Dec. 2013, May 2014, Dec. 2015

- Consider a film of non-uniform thickness as shown in Fig. 1.7.1. It is bound by two surfaces OX and OX′ inclined at an angle θ. The thickness of the film gradually increases from O to X.
- Such a film of non-uniform thickness is known as **wedge shaped film**. The point O at which the thickness is zero is known as the edge of the wedge.
- The angle θ between the surfaces OX and OX′ is known as the angle of wedge. Let be the refractive index of the material of the film.
- Let a beam AB of monochromatic light of wavelength λ be incident at an angle i on the upper surface of the film. It is reflected along BR_1 and is

transmitted along BC. At C also the beam suffers partial reflection and refraction and finally we have the ray DR_2 in the reflected system.

- Thus as a result of partial reflection and refraction at the upper and lower surfaces of the film, we have two coherent rays BR_1 and DR_2 in the reflected system.

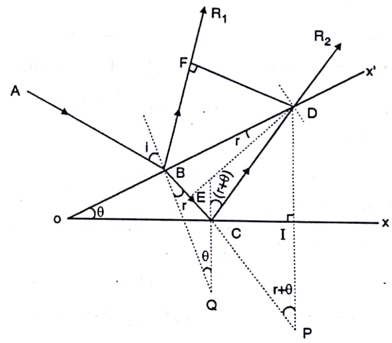


Fig. 1.7.1 : Interference in wedge shaped film

- These rays are not parallel but diverge from each other. These rays interfere constructively or destructively according to whether the path difference between them satisfies the condition for constructive or destructive interference in the reflected system.
- To find the path difference between these two rays, draw DF perpendicular to BR_1.
- The optical path difference between the rays BR_1 and DR_2 is

$$\Delta = \mu (BC + CD) - BF \qquad ...(1.7.1)$$

- Draw a perpendicular to surface OX at point C.
- We have the perpendicular to OX′ at point B. Both these perpendiculars will meet at point Q as shown in diagram.

$$\therefore \angle BQC = \angle XOX' = \theta$$

- Draw a perpendicular DE on BC from D.
- As θ is small enough, BE = EC

- Also from diagram $\angle$ QBE = r = $\angle$ BDE
- Draw a perpendicular from D on OX such that intersects OX at I and B produced at P. Also we get CP = CD.
- Equation (1.7.1) can be written as

$$\Delta = \mu (BC + CD) - BF = \mu (BE + EC + CP) - BF$$

From diagram

$$\mu = \frac{\sin i}{\sin r} = \frac{BF}{BE} \quad \text{or} \quad BF = \mu BE$$

$$\therefore \Delta = \mu (BE + EC + CP) - \mu BE = \mu EP \quad (\text{as } E - C - P)$$

- Now consider ΔDPC

$$\text{as } CP = CD, \angle CPD = r + \theta$$

and Δ DPE is a right angle triangle

$$\therefore \cos (r + \theta) = \frac{EP}{DP}$$

$$\therefore EP = DP \cos (r + \theta) = 2 t \cos (r + \theta)$$

where DP = 2 DI = 2t , t = thickness of film at point D

$$\therefore \Delta = \mu EP = 2 \mu t \cos (r + \theta) \quad ...(1.7.$$

- Due to reflection at B, an additional path change of $\frac{\lambda}{2}$ occurs for the ray BR

Hence the total path difference between the interfering rays i $2 \mu t \cos (r + \theta) + \frac{\lambda}{2}$

- Hence for **maxima**, we have the condition for constructive interference

$$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = n\lambda$$

$$\text{or} \quad 2 \mu t \cos (r + \theta) = (2n - 1) \frac{\lambda}{2} \quad ...(1.7.$$

$$n = 1, 2, 3, 4 .....$$

- For **minima**, we have the condition for destructive interference

$$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = (2n - 1) \frac{\lambda}{2}$$

$$2 \mu t \cos (r + \theta) = n\lambda \quad ...(1.7.$$

$$n = 0, 1, 2, 3,.....$$

- It is clear that for a maximum or a minimum of a particular order, t must remain constant. In case of the wedge shaped film, t remains constant along lines parallel to the thin edge of the wedge.
- Hence the maxima and minima are straight lines parallel to the thin edge of the wedge.
- At the thin edge, t = 0 hence path difference between the rays is $\frac{\lambda}{2}$, a condition for darkness. Hence the edge of the film appears dark. It is called as **zero order band**. That is the reason extensively thin film appears dark (black) in reflected light.
- Beyond the edge for a thickness t for which path difference is λ, we obtain the first bright band. As t increases to a value for which path difference is $\frac{3\lambda}{2}$ we obtain the first dark band.
- Thus as the thickness increases we obtain alternate bright and dark bands; which are equally spaced and equal in width.

(i) For normal incidence and air film, r = 0 and μ = 1

$$\text{Total path difference} = 2t \cdot \cos \theta + \frac{\lambda}{2}$$

$$2t \cos \theta = (2n - 1) \frac{\lambda}{2}, \quad \text{For maxima}$$

$$\text{and} \quad 2t \cos \theta = n\lambda \quad \text{For minima}$$

(ii) For very small angle of the wedge,

As $\theta \to 0$, $\cos \theta \to 1$

$\therefore$ For constructive interference,

$$2t = (2n - 1) \frac{\lambda}{2}$$

and for destructive interference

$$2t = n\lambda$$

$\therefore$ For constructive interference

$$t = (2n - 1) \frac{\lambda}{4} \quad n = 1, 2, 3, ...$$

$$t = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$$

i.e. **every next bright fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.**

Similarly for destructive interference

$$t = \frac{n\lambda}{2} \qquad n = 0, 1, 2, 3, ...$$

$$t = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, ...$$

i.e. **every next dark fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.**

1.8 Spacing between Two Consecutive Bright Bands

- For the wedge shaped film, we have for the n^{th} maximum.

$$2\mu t \cos(r + \theta) = (2n - 1)\frac{\lambda}{2}$$

- For normal incidence and air film,

$$r = 0 \quad \text{and} \quad \mu = 1$$

$$2t\cos\theta = (2n-1)\frac{\lambda}{2} \qquad ...(1.8.1)$$

Where, t is the thickness corresponding to n^{th} bright band.

- Consider Fig. 1.8.1. The n^{th} bright band is produced at a distance x_n from the edge of the wedge.

$$t = x_n \cdot \tan\theta \qquad ...(1.8.2)$$

∴ Substituting for t in Equation (1.8.1) we have,

$$2 \cdot x_n \tan\theta \cos\theta = (2n-1)\frac{\lambda}{2}$$

$$\text{or} \quad 2x_n \cdot \sin\theta = (2n-1)\frac{\lambda}{2} \qquad ...(1.8.3)$$

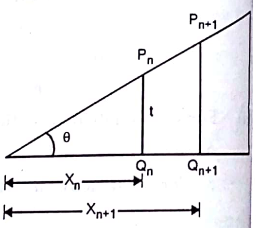


Fig. 1.8.1

- Let $(n + 1)^{th}$ maximum be obtained at a distance x_{n+1} from the thin edge. Then we have

$$\left.\begin{aligned} 2x_{n+1}\sin\theta &= [2(n+1) - 1]\cdot\frac{\lambda}{2} \\ \text{or } 2x_{n+1}\cdot\sin\theta &= (2n+1)\frac{\lambda}{2} \end{aligned}\right\} \qquad ...(1.8.4)$$

- Therefore from Equations (1.8.3) and (1.8.4) we have

$$2(x_{n+1} - x_n)\cdot\sin\theta = \lambda$$

- Therefore the spacing between two consecutive bright bands is

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\sin\theta}$$

$\sin\theta \to \theta$ if θ is small and measured in radians. β is called fringe width.

$$\beta = \frac{\lambda}{2\theta}$$

- For a medium of refractive index μ, we have $\beta = \frac{\lambda}{2\mu\theta}$.

as μ, λ and θ are constant, one can say that fringe width in wedge shaped film is constant. Or **wedge shaped fringes are of constant thick ness**

Ex. 1.8.1 : Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 Å. Calculate the angle of wedge in seconds of an arc. **MU - Dec. 2015, 3 Marks**

Soln. :

Given : $\mu = 1.52$, $\lambda = 5893 \times 10^{-8}$ cm, $\beta = 1$ mm $= 0.1$ cm.

Formula : The fringe spacing is

$$\beta = \frac{\lambda}{2\mu\theta}$$

The angle of wedge in radians is

$$\theta = \frac{\lambda}{2\mu\cdot\beta} = \frac{5893\times10^{-8}}{2\times1.52\times0.1} \text{ radians}$$

$$\therefore \quad \theta = \frac{5893\times10^{-8}}{2\times1.52\times0.1}\times\frac{180}{\pi}\times3600 \text{ seconds}$$

$$\therefore \quad \theta = \textbf{40 seconds of an arc.}$$ **....Ans.**

Syllabus Topic : Newton's Rings

> **Topics covered :** Newton's Rings, Newton's Rings by Transmitted Light, Characteristics of Newton's Rings, Newton's Ring with White Light, Newton's Rings with Bright Centre in Reflected System, Similarities and Dissimilarities between Newton's Rings and Wedge Shaped Films

1.9 Newton's Rings

MU - May 2012, May 2015, Dec. 2016

- When a plano-convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.

- The thickness of the film gradually increases from the point of contact outwards.
- If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film.
- These rings were **first studied by Newton and hence known as Newton's rings.** They can be seen through a low power microscope focussed on the film.

Formation of Newton's Rings

- Newton's rings are formed as a result of interference between the waves reflected from the top and bottom surfaces of the air film formed between the lens and the plate.

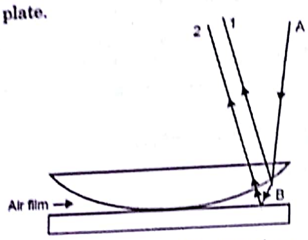


Fig. 1.9.1 : Formation of Newton's rings

- As shown in Fig. 1.9.1, let AB be a beam of monochromatic light of wavelength λ incident normally on the film. As a result of reflection at the top and bottom faces of the film, rays 1 and 2 are the coherent rays which interfere in the reflected system. For constructive interference, the path difference between them should be

$$2\mu t \cdot \cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

where μ = R.I. of the film

t = Thickness at a point under consideration

r = Angle of refraction

θ = Angle of wedge

- The factor $\frac{\lambda}{2}$ accounts for a phase change of π on reflection at the lower surface of the film.

Now for the air film $\mu = 1$

For normal incidence $r = 0$

- For a lens of large radius of curvature, $\theta = 0$ practically. This is the reason why we prefer lens with large radius.

$\therefore$ Path difference between rays 1 and 2 is $2t + \frac{\lambda}{2}$

- At the point of contact of the lens and the plate, $t = 0$.

$$\therefore \quad \text{Path difference} = \frac{\lambda}{2}$$

- This is the condition for minimum intensity. Hence the central spot is dark.

For the n^{th} maximum, we have

$$2t + \frac{\lambda}{2} = n\lambda$$

- Thus a maximum of particular order n will occur for a constant value of t. In the air film t remains constant along a circle and hence the maximum is in the form of a circle.
- Different maxima will occur for different values of 't'. Similarly, it can be shown that the minima are also circular in form.
- The minima occur for path difference $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$ and maxima occur for path difference $\lambda, 2\lambda, 3\lambda, \ldots$, the maxima and minima occur alternately.
- Each fringe is a locus of constant film thickness and hence these are fringes of constant thickness.

Diameter of dark and bright rings

- Let POQ be a plano-convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens surface in contact with the plate. Fig. 1.9.2.

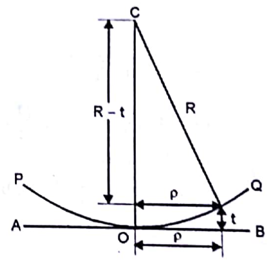


Fig. 1.9.2

- Let ρ be the radius of a Newton's ring corresponding to the constant film thickness 't'. The path difference between the two interfering rays in the reflected system is $2\mu t \cos(r + \theta) + \frac{\lambda}{2}$.

λ = wavelength of incident light.

$\mu = 1$ for air film.

$r = 0$ for normal incidence

$\theta = 0$ for large R.

$$\therefore \quad \text{Path difference} = 2t + \frac{\lambda}{2} \qquad \text{...(1.9}$$

- From Fig. 1.9.2 we see that

$$R^2 = \rho^2 + (R - t)^2$$

$$\text{or} \quad \rho^2 = R^2 - (R - t)^2$$

$$\text{or} \quad \rho^2 = 2Rt - t$$

$t << R$ and hence we have

$$\rho^2 = 2Rt$$

$$\therefore \quad 2t = \frac{\rho^2}{R} \qquad \text{...(1.9}$$

$\therefore$ Path difference between the interfering rays is $\frac{\rho^2}{R} + \frac{\lambda}{2}$.

For dark rings

- The condition to get dark rings is that

$$\text{Path difference} = \frac{\rho^2}{R} + \frac{\lambda}{2}$$

$$= (2n + 1)\frac{\lambda}{2} \qquad (n = 1,2,3,\ldots$$

- *If D is the diameter of Newton's ring, then* $\rho = \frac{D}{2}$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda \qquad \text{where } D_n = \text{Diameter of } n^{th} \text{dark ring}$$

$$\therefore \quad D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \qquad \text{...(1.9}$$

$$D_n \propto \sqrt{n}$$

- **Hence the diameter (and hence radius) of the dark ring is proportional to the square roots of natural numbers.** One can say that rings are unequally spaced, because difference between two consequtive rings represents thickness of ring.

For bright rings

The condition to get bright rings is that the path difference

$$\frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

$$\text{or} \quad \frac{\rho^2}{R} = (2n - 1)\frac{\lambda}{2} \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\therefore \quad \rho^2 = (2n - 1)\frac{\lambda R}{2}$$

Putting $\rho = \frac{D}{2}$, we have,

$$\frac{D_n^2}{4} = (2n - 1)\frac{\lambda R}{2}$$

where D_n = Diameter of n^{th} bright ring.

$$D_n^2 = 2\lambda R \cdot (2n - 1)$$

$$D_n = \sqrt{(2n - 1)}\sqrt{2\lambda R} \qquad \text{...(1.9.4)}$$

$$D_n \propto \sqrt{2n - 1} \qquad \text{where } n = 1, 2, 3, \ldots$$

- **Hence the diameter (and hence radius) of the bright ring is also proportional to the square root of odd natural numbers.**

Ex. 1.9.1 : In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20^{th} ring.

Soln. :

Given : $n = 4$, $n + p = 12$, $D_4 = 0.400$ cm, $D_{12} = 0.700$ cm

$$D_n^2 = 4nR\lambda, \; D_{n+p}^2 = 4(n + p)R\lambda$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \qquad \text{...(1)}$$

$$\text{Similarly, } D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \qquad \text{...(2)}$$

Divide Equation (1) by Equation (2), we get,

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4 \times 8}{4 \times 16} = \frac{1}{2}$$

$$D_{20}^2 = 2\,D_{12}^2 - D_4^2$$

$$= 2\,(0.700)^2 - (0.400)^2$$

$$= 0.98 - 0.16 = 0.82$$

Diameter of 20th ring $= \sqrt{0.82}$ = **0.906 cm** ...**Ans.**

1.10 Newton's Rings by Transmitted Light

- Newton's rings can be seen in reflected light as well as in transmitted light also. (As shown in Fig. 1.10.1). The ray 1′ is transmitted directly through the air film while the ray 2′ suffers two internal reflections (or a phase change of 2π) before emerging out.
- So, two interfering transmitted rays have a phase change of 2π or no phase difference. So, effective path difference

$$\Delta = 2\mu t, \qquad (\mu = 1 \text{ for air film})$$

- For bright fringe, we can write,

$$2\mu t = n\lambda, \qquad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow 2t = n\lambda \qquad (\mu = 1 \text{ for air film}) \qquad \ldots(1.10.1)$$

And for dark fringe

$$2\mu t = (2n-1)\,\lambda/2, \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\Rightarrow 2t = (2n-1)\,\lambda/2 \qquad \ldots(1.10.2)$$

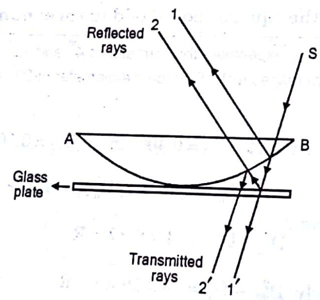


Fig. 1.10.1

- By using the property of circle, we have,

$$2t = \frac{\rho^2}{R} = \frac{D^2}{4R}, \qquad \text{where D is the diameter of the ring.}$$

- For bright rings, we can write,

$$2t = \frac{D^2}{4R} = n\lambda \qquad \Rightarrow D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \qquad \Rightarrow D_n \propto \sqrt{n} \qquad \ldots(1.10.3)$$

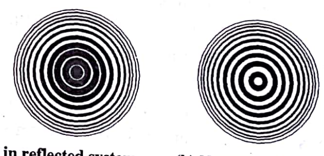
- For dark rings, we can write,

$$2t = \frac{D^2}{4R} = (2n-1)\,\lambda/2$$

$$\Rightarrow D_n^2 = 2R\lambda\,(2n-1)$$

$$D_n = \sqrt{2R\lambda\,(2n-1)} \qquad \Rightarrow D_n \propto \sqrt{2n-1} \qquad \ldots(1.10.4)$$

- In transmitted system of light, the diameters of bright rings are proportional to the square roots of natural numbers; while the diameters of dark rings are proportional to the odd natural numbers and the central ring is bright. The nature of Newton's ring in reflected and transmitted system is shown in Fig. 1.10.2.



(a) Newton's ring in reflected system (b) Newton's ring in transmitted system

Fig. 1.10.2

- **We can conclude that the system of rings in transmitted light is complementary to those seen in reflected light.**

1.11 Characteristics of Newton's Rings

(1) Why centre of Newton's rings is always dark ? **MU - May 2014**

At the point of contact the thickness of air film is zero. Consider the case of thin film at this point i.e. condition for bright spot is given by,

$$2\,\mu t \cos(r + \theta) = (2n + 1)\,\lambda/2 \qquad n = 0, 1, 2, \ldots\ldots$$

and condition for dark spot is given by,

$$2\ \mu t \cos(r + \theta) = n\lambda$$

For Newton's ring set up r = 0 (large radius of lens θ = 0), (for norma incidence r = 0)

$$\therefore \cos(r + \theta) = \cos 0 = 1$$

∴ Condition for bright

$$2\ \mu t = (2n + 1)\ \lambda/2 \qquad n = 0, 1, 2, 3, \ldots$$

∴ Condition for dark

$$2\ \mu t = n\lambda$$

At point of contact, t = 0 and one can see that condition for bright does n get satisfied. But condition for dark gets satisfied for t = 0 and n = 0.

(2) Why Newton's rings are always seen on reflected side ?

- Newton's rings can also be formed in transmitted system due interference between the transmitted rays. The conditions for t bright and dark rings in the transmitted system are opposite to tho in reflected system and hence the rings have a bright centre.
- In the reflected system the intensity of the interference maxima about 15% of the incident intensity and the intensity of minima is ze Hence the contrast between bright and dark rings is good.
- In the transmitted system the intensity of maxima is about 100% a the minima is about 85%. The contrast between bright and dark rin is not good. The visibility of fringes is much higher in reflected syste than in transmitted system. **Hence Newton's rings are seen i reflected system only and not in transmitted system.**

(3) How insertion of liquid affects ring structure ?

- In case of reflected system, the central spot can be made bright if t space between lens surface and glass plate is filled with an oil havi refractive index greater than that of lens and smaller than that plate. Fig. 1.11.1.

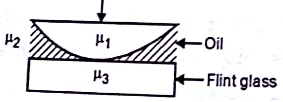


Fig. 1.11.1

- Newton's rings are formed with the lens of crown glass and glass plate of flint glass. The space with air film is then filled with an oil (like oil of sassafaras) having intermediate refractive index.
- The reflections at the upper and lower surfaces of the film take place under similar conditions i.e. at the denser medium.
- Hence there is a phase change of π at both reflections. Hence the phase difference between the interfering rays at the point of contact is zero.
- This is the condition for constructive interference and hence a bright spot is produced at the centre.

4. Why Newton's rings are circular and wedge shaped films are straight ?

MU - May 2014, Dec. 2016

- In both air-wedge film and Newton's ring experiments, each fringe is the locus of points of equal thickness of the film. In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of plano-convex lens and the glass plate as centre. So, the fringes are circular in nature and concentric.
- For wedge shape air film, the locus of points of equal thickness are straight lines parallel to the edge of the wedge. So, fringes appear straight and parallel.

5. What happens if Plano convex lens is lifted up slowly?

- As the lens is lifted up slowly from the flat surface, the order of the ring at a given point decreases. The rings, therefore, come closer and closer until they can no longer be separately observed.
- Also it is important to know that initially as the lens is lifted up by a spacing of $\frac{\lambda}{4}$, total path difference for the ray will be $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$ and center will become dark to bright or bright to dark.
- For a lift of every λ/4, the center will change from dark to bright and bright to dark. Also the order of the ring will reduce by one.

1.12 Newton's Ring with White Light

MU - May 2013

- If a monochromatic light source is used in Newton's ring experiment alternate dark and bright rings are obtained. **If we use white light instead of monochromatic light, a few mixed coloured rings around**

a black centre are observed and beyond it, a uniform illumination is obtained.

- White light consists of seven colours of different wavelength. Diameters of the Newton's ring are proportional to the wavelength of the different colours. As we know that $\lambda_r > \lambda_v$, therefore the diameter of violet ring of the same order will be smallest and those for red ring will be the largest, and the diameters of other coloured rings shall occupy the intermediate positions.
- Due to overlapping of the rings of different colours over each other, only first few coloured rings will be clearly seen while other rings cannot be observed.

1.13 Newton's Rings with Bright Centre in Reflected System

- If Newton's rings are observed in reflected system, the central spot is dark. A liquid of refractive index μ_2 is poured between lens and glass plate.
- The refractive index of the lens, liquid and glass plate μ_1, μ_2, μ_3 are such that $\mu_1 < \mu_2 < \mu_3$, then the central spot is bright. This is possible if oil of sassafras is introduced between lens of crown glass and plate of flint glass.
- Then the reflection of two interfering rays from denser to rarer medium takes place under same condition. Hence the effective path difference between both the interfering rays at the point of contact becomes zero which is the condition of maximum intensity.
- So, the centre of Newton's ring appears bright. [Sassafras oil (μ = 1.57), crown glass lens (μ = 1.50), flint glass plate (μ = 1.65)].

1.14 Similarities and Dissimilarities between Newton's Rings and Wedge Shaped Films

Similarities

(1) Fringes are formed due to thin film enclosed.

(2) Both can be explained only with division of amplitude concept.

(3) Both can be used for determination of optical flatness.

Dissimilarities

Sr. No.	Newton's Rings	Wedge shaped film
1.	We get alternate dark and bright rings.	We get straight alternate dark and bright fringes
2.	Airgap is having its thickness linearly increased	Airgap is non linearly increased.
3.	Popularly used for determination of unknown wavelength.	Popularly used for determination of very small thickness
4.	As we go for higher orders thickness of rings reduces.	Fringe width remains constant.

1.15 Solved Problems

Problems on Thin Film

Ex. 1.15.1 : White light falls at an angle of 45° on a parallel soap film of refractive index 1.33. At what minimum thickness of the film will it appear bright yellow of wavelength 5896 Å in the reflected light ?

Soln. :

Given : $i = 45°, \mu = 1.33, \lambda = 5896\ \text{Å} = 5896 \times 10^{-10}\ \text{m}$ Å

Formula : $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$, $n = 1, 2, 3, \ldots$ For bright fringe

For minimum thickness, n is minimum i.e. n = 1

$$\therefore\ t = \frac{\lambda}{2 \times 2\mu \cos r}$$

$$= \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times \cos r}$$

Now $$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{1.33} = 0.5316$$

$$\therefore\ \cos r = \sqrt{1 - \sin^2 r} = 0.8469$$

$$\therefore\ t = \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times 0.8469}$$

$$t = 1304.5\ \text{Å}$$ ....Ans..

Ex. 1.15.2 : A light of wavelength 5500 Å incident on thin transparent denser medium having refractive index 1.45. Determine the thickness of thin medium if the angle of refraction is 45° (Consider n = 1).

Soln. ;

Here it is not specified whether reflected side or transmitted side is to be considered.

Let us assume reflected side.

Also it is not specified whether dark fringe or bright fringe condition is satisfied.

Let us assume dark fringe

∴ Condition for dark fringe on reflected side

$$2\mu\, t \cos r = n\lambda$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5500 \times 10^{-8}}{2 \times 1.45 \times \cos 45}$$

$$= \mathbf{2.68 \times 10^{-5}\ cm}$$ ...Ans.

Ex. 1.15.3 : Light of wavelength 5880 Å is incident on a thin film of glass of $\mu = 1.5$ such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection. **MU - Dec. 2014, 3 Marks**

Soln. :

$\lambda = 5880 \times 10^{-8}$ cm, $\mu = 1.5$, $r = 60°$, $t = ?$

Condition for film to appear dark is,

$$2\,\mu t \cos r = n\lambda\ .$$

The smallest thickness will be for n = 1.

$$2 \times 1.5 \times t \times \cos 60 = 1 \times 5880 \times 10^{-8}$$

$$t = \frac{5880 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$= \mathbf{3920 \times 10^{-8}\ cm}$$ ...Ans.

Ex. 1.15.4 : A soap film of refractive index 1.43 is illuminated by white light incident at an angle 30°. The refracted light is examined by a spectroscope in which dark band corresponding to wavelength 6×10^{-7} m is observed. Calculate the thickness of the film.

Soln. :

For the thin film, refracted light forms transmitted system and the condition of minima in transmitted system is given by,

$$2\mu t \cos r = (2n - 1)\,\lambda/2$$

$$t = \frac{(2n-1)\,\lambda}{4\mu \cos r}$$

Given : $\mu = 1.43$, $\lambda = 6 \times 10^{-7}$ m, $i = 30°$

Using Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}} = \sqrt{1 - \left(\frac{\sin 30°}{1.43}\right)^2} = 0.9369$$

$$\text{So,}\quad t = \frac{(2n-1) \times 6 \times 10^{-7}}{4 \times 1.43 \times 0.9369} = (2n - 1) \times 1.12 \times 10^{-7}\ \text{m} \qquad n = 1, 2, 3, \ldots$$

For minimum thickness of film, n = 1

Hence, $$t = \mathbf{1.12 \times 10^{-7}\ m}$$ ...Ans.

Ex. 1.15.5 : An oil drop of volume 0.2 c.c. is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the surface and white light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band coincides with wavelength 5.5×10^{-5} cm in air. Find the refractive index of oil.

Soln. :

The oil drop of volume 0.2 c.c. spreads uniformly over 1 m²; hence the thickness of the film so formed is given by,

$$t = \frac{0.2}{(100)^2} = 2 \times 10^{-5}\ \text{cm.}$$

The film appears dark by reflected light.

Hence, $2\mu t \cos r = n\lambda$

For normal incidence $r = 0$ $\qquad \therefore \cos r = 1$

$n = 1$ and $\lambda = 5.5 \times 10^{-5}$ cm.

Refractive index of oil is

$$\mu = \frac{n\lambda}{2t \cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1} = \frac{5.5}{4} = \mathbf{1.375}$$ ....Ans.

Ex. 1.15.6 : A drop of oil of $\mu = 1.20$ floats on water with $\mu = 1.33$ surface and is observed from above by reflected light. The thickness of the oil drop at the edge is very small-almost zero and gradually increases towards the middle of the drop. Answer the following :

(i) Will the thinnest outer region of the drop correspond to a bright or a dark region ? Give reason.

(ii) What will be the thickness of oil drop where wavelength of 4800 Å is intensified in reflected light for the third order ?

Soln. :

(i) The thinnest region of the drop corresponds to a bright region because both the reflected rays, one from the boundary between air and oil and another from the boundary between oil and water are in phase. Hence the condition for brightness is $2\mu t \cos r = n\lambda$

At the edge, $t \approx 0$ and satisfies the condition for maximum intensity.

(ii) The condition for maximum intensity in reflected light is $2\mu t \cos r = n\lambda$

For normal incidence,

$$2\mu t = n\lambda$$

$$\therefore \quad t = \frac{n\lambda}{2\mu}$$

The thickness when $n = 3$,

$\lambda = 4800$ Å and $\mu = 1.2$ is

$$t = \frac{3 \times 4800 \text{ Å}}{2 \times 1.2} = \mathbf{6000 \text{ Å}}$$ ...Ans.

Ex. 1.15.7 : Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear (i) Black (ii) Bright ?

Soln. :

i) In reflected system, the condition of dark is

$$2\mu t \cos r = n\lambda$$

For normal incidence, $r = 0$, $\cos r = 1$

So, $2\mu t = n\lambda$, $t = \dfrac{n\lambda}{2\mu}$

For minimum thickness of the film, $n = 1$

Hence, $$t = \frac{\lambda}{2\mu} = \frac{5893 \times 10^{-8}}{2 \times 1.42} \text{ cm} = \mathbf{2075 \text{ Å}}$$ ...Ans.

ii) In reflected system, the condition of bright is

$$2\mu t \cos r = (2n - 1)\lambda/2$$

For normal incidence, $r = 0$, $\cos r = 1$

For minimum thickness of the film, $n = 1$

So, $$t = \frac{\lambda}{4\mu} = \frac{5893 \times 10^{-8}}{4 \times 1.42} \text{ cm} = \mathbf{1037.5 \text{ Å}}$$ ...Ans.

Ex. 1.15.8 : Light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33. Which wavelength in the visible region will be reflected most strongly ?

MU - Dec. 2012, 5 Marks

Soln. :

The condition of maxima is given by,

$$2\mu t \cos r = (2n - 1)\lambda/2 \quad \text{where } n = 1, 2, 3, ...$$

Given : $t = 5 \times 10^{-5}$ cm, $\mu = 1.33$

$r = 0°$, i.e. $\cos r = 1$

Now, $$\lambda = \frac{4\mu t \cos r}{(2n-1)} = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n-1)}$$

By substituting the values of $n = 1, 2, ...$ we get a series of wavelengths which shall be predominantly reflected by the film.

For $n = 1$, $$\lambda_1 = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1} = 26.66 \times 10^{-5} \text{ cm}$$

Similarly,

For $n = 2$, $\lambda_2 = 8.866 \times 10^{-5}$ cm

For $n = 3$, $\lambda_3 = 5.32 \times 10^{-5}$ cm

For $n = 4$, $\lambda_4 = 3.8 \times 10^{-5}$ cm

Out of these wavelengths 5.320×10^{-5} cm lies in the visible region (4000 Å to 7500 Å).

Hence 5320 Å is the most strongly reflected wavelength. ...Ans.

Ex. 1.15.9 : White light is incident on a soap film at an angle $\sin^{-1}(4/5)$ and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands correspond to wavelengths 6.1×10^{-5} and 6.0×10^{-5} cm. If the refractive index of the film be 4/3, calculate the thickness.

Soln. : We have the condition for dark band in reflected system is,

$$2\mu t \cos r = n\lambda$$

If n and $(n + 1)$ are the orders of consecutive dark bands for wavelengths λ_1 and λ_2 respectively, then,

$$2\mu t \cos r = n\lambda_1, \qquad 2\mu t \cos r = (n+1)\lambda_2$$

$$\Rightarrow \quad 2\mu t \cos r = n\lambda_1 = (n+1)\lambda_2 \quad \text{...(1)}$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Put the value of n in Equation (1), we have,

$$2\mu t \cos r = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}$$

$$t = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)\, 2\mu \cos r}$$

$$= \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)\, 2\mu \cdot \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2}} \quad \text{...(2)}$$

Given : $\mu = 4/3, \ \sin i = 4/5$

as $\mu = \frac{\sin i}{\sin r}$ and $\cos r = \sqrt{1 - \sin^2 r}$

$$\therefore \cos r = \sqrt{1 - \left(\frac{4/5}{4/3}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Given : $\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = 4/3$

Put all these values in Equation (2),

$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 - 6) \times 10^{-5} \times 2 \times 4/3 \times 4/5}$$

$$= \mathbf{0.0017\ cm} \quad \text{...Ans.}$$

Ex. 1.15.10 : A soap film of refractive index $\frac{4}{3}$ and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference band.

Soln. :

Given : $\mu = \frac{4}{3}$, $t = 1.5 \times 10^{-4}$ cm.

$i = 45°$, $\lambda = 5 \times 10^{-5}$ cm

Formula : For dark band,

$$2\mu t \cos r = n\lambda; \quad 2\mu t \cos r = n\lambda$$

$$\text{Now } \mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3}$$

$$\therefore r = \sin^{-1}\left(\frac{\sin 45°}{4/3}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}} \times \frac{3}{4}\right) = 32.02°$$

$$\therefore \cos r = 0.8478$$

$\therefore$ Order of dark band is

$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times 0.8478}{5 \times 10^{-5}}$$

or $n = 6.7$

$\therefore$ Order of dark band is 6. **...Ans.**

Ex. 1.15.11 : A film of refractive index μ is illuminated by white light at an angle of incidence i. In reflected light two consecutive bright fringes of wavelength λ_1 and λ_2 are found overlapping. Obtain expression for thickness of film.

Soln. :

Say, n^{th} bright fringe of λ_1 overlaps with $(n+1)^{th}$ fringe of λ_2

For maxima of λ_1

$$2\mu t \cos r = (2n-1)\frac{\lambda_1}{2} \quad \text{...(1)}$$

And for λ_2

$$2\mu t \cos r = \{2(n+1) - 1\}\frac{\lambda_2}{2} \quad \text{...(2)}$$

So, $(2n-1)\frac{\lambda_1}{2} = \{2(n+1)-1\}\frac{\lambda_2}{2}$

$$\Rightarrow \quad 2n(\lambda_1 - \lambda_2) = \lambda_1 - \lambda_2$$

$$\Rightarrow \quad 2n = \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2}$$

Thickness of the film t, we get from Equation (1)

$$2\mu t \cos r = \left\{\frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2} - 1\right\}\frac{\lambda_1}{2} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$t = \frac{\lambda_1 \lambda_2}{2\mu \cos r\, (\lambda_1 - \lambda_2)} \quad \text{...Ans.}$$

Ex. 1.15.12 : White light is incident at an angle of 45° on a soap film 4×10^{-5} cm thick. Find the wavelength of light in the visible spectrum which will be absent in the reflected light ($\mu = 1.2$).

Soln. :

Here white light is made incident and on reflected side it is expected to find the absent wavelength i.e. the one which will satisfy the condition for dark.

$$2\,\mu t \cos r = n\lambda$$

Here, $t = 4 \times 10^{-5}$ cm

$i = 45°$; $\mu = 1.2$

$$\therefore \quad 1.2 = \frac{\sin 45}{\sin r}$$

$$\therefore \quad r = \sin^{-1}\left(\frac{\sin 45}{1.2}\right) = 36.104°$$

$$\therefore \quad \cos r = 0.8079$$

$\therefore$ Now find λ for various order n such that it remains between 4000 Å to 8000 Å i.e. visible spectrum

$\therefore$ For n = 1, condition for dark fringe

$$1 \times \lambda = 2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079$$

$$\lambda = 7755 \times 10^{-8} \text{ cm}$$

This is in visible range and it will remain absent

Similarly for n = 2

$$\lambda = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{2} = 3877 \times 10^{-8} \text{ cm}$$

This is not in visible range.

$\therefore$ 7755 Å will remain absent. **...Ans.**

Ex. 1.15.13 : A plane wave of monochromatic light falls normally on a uniform thin film of oil which covers a glass plate. The wavelength of the source can be varied continuously. Complete destructive interference is obtained only for wavelengths 5000 Å and 7000 Å. Find the thickness of the oil layer. Given R.I of oil = 1.3 and R.I of glass = 1.5. **MU - May 2013, 7 Marks**

Soln. :

Here the path is from air to oil and oil to glass.

(1) For air to oil :

For destructive interference the condition is, (for normal incidence)

$$2\mu_{oil}\, t_{oil} = (2n+1)\frac{\lambda}{2} \quad ...(1)$$

As the arrangement remains same for both the wavelengths μ_{oil} and t_{oil} will be the same or constant.

$$\therefore \quad \text{const.} = (2n+1)\frac{\lambda}{2}$$

$\therefore$ Order of destructive interference is inversely proportional to wave length.

$\therefore$ For λ_1 = 7000 Å take order n and for next wavelength λ_2 = 5000 Å take next order i.e. n + 1

$\therefore$ For λ_1 = 7000 Å

$$2\,\mu_{oil}\, t_{oil} = (2n+1) \times \frac{7000}{2} \text{ Å} \quad ...(2)$$

For λ_2 = 5000 Å

$$2\,\mu_{oil}\, t_{oil} = (2(n+1)+1) \times \frac{5000}{2} \text{ Å} \quad ...(3)$$

$$\therefore (2n+1) \times \frac{7000}{2} = (2(n+1)+1) \times \frac{5000}{2}$$

$$\therefore \quad \frac{(2n+1)}{(2n+3)} = \frac{5000}{7000}$$

On solving, n = 2

$\therefore$ Substitute in Equation (2)

$$t_{oil} = \frac{(2(2)+1) \times 7000}{2 \times 1.3 \times 2} = 6730.769 \text{ Å}$$ **...Ans.**

Problems on Wedge Shaped Film

Ex. 1.15.14 : A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used.

Soln. :

Given : $\theta = 40 \text{ seconds} = \frac{40}{3600} \text{ degrees} = \frac{40}{3600} \times \frac{\pi}{180}$ radians, $\beta = 0.12$ cm.

Formula : Spacing between the consecutive bright fringes is,

$$\beta = \frac{\lambda}{2\theta}$$

The wavelength is,

$$\lambda = 2\beta \cdot \theta = 2 \times 0.12 \times \frac{40 \times \pi}{3600 \times 180}$$

$$= 4654 \times 10^{-8} \text{ cm}$$

$$\lambda = 4654 \text{ Å}$$ ...Ans.

Ex. 1.15.15 : Light of wavelength 5500 A° falls normally on a thin wedge shaped film of refractive index 1.4 forming fringes that are 2.5 mm apart. Find the angle of wedge in seconds.

Soln. : We have fringe width,

$$\beta = \frac{\lambda}{2\mu\theta}; \qquad \theta = \frac{\lambda}{2\mu\beta}$$

Given : $\lambda = 5500 \times 10^{-8}$ cm, $\mu = 1.4$, $\beta = 0.25$ cm

$$\theta = \frac{5500 \times 10^{-8}}{2 \times 1.4 \times 0.25} = 7.86 \times 10^{-5} \text{ radian}$$

$$\theta = 7.86 \times 10^{-5} \times \frac{180°}{\pi} = 0.0045° = 0.0045° \times 3600$$

$$= \mathbf{16.2 \text{ sec.}}$$ ...Ans.

Problems on Newton's Rings

Ex. 1.15.16 : Newton's rings are obtained with reflected light of wavelength 5500 Å. The diameter of 10th dark ring is 5 mm. Now the space between the lens and the plate is filled with a liquid of refractive index 1.25. What is the diameter of the 10th ring now ?

Soln. :

Given : $\lambda = 5500$ Å $= 5500 \times 10^{-8}$ cm, $D_{10} = 5$ mm $= 0.5$ cm, $\mu = 1.25$

Formula : Diameter of nth dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For the air film, $\mu = 1$

Hence diameter of 10th dark ring is,

$$(0.5)^2 = D_{10}^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1}$$

For the liquid film, the diameter of the 10th dark ring is,

$$D_{10}'^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1.25}$$

$$\frac{D_{10}'^2}{D_{10}^2} = \frac{1}{1.25} \qquad \therefore D_{10}'^2 = \frac{D_{10}^2}{1.25}$$

$$\therefore D_{10}' = \frac{D_{10}}{\sqrt{1.25}} = \frac{D_{10}}{1.1180} = \frac{0.5}{1.118}$$

$$D_{10}' = 0.4472 \text{ cm.}$$

Diameter of 10th dark ring for the liquid film = **4.472 mm** ...Ans.

Ex. 1.15.17 : Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring ?

Soln. :

Let the diameter of the nth dark ring be double the diameter of 40th dark ring.

$$\therefore D_n = 2 D_{40}$$

Now the diameter of nth dark ring is given by the expression

$$D_n^2 = 4 n R \lambda \qquad ...(1)$$

where R = Radius of curvature of lens.

λ = Wavelength of light.

Hence for the 40th dark ring.

$$D_{40}^2 = 4 \times 40 \times R\lambda \qquad ...(2)$$

From Equations (1) and (2) we have

$$D_n^2 = 4 \times D_{40}^2$$

$$\therefore 4 n R \lambda = 4 \times 4 \times 40 \times R\lambda$$

$$n = \mathbf{160}$$ ...Ans.

Ex. 1.15.18 : The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring. **MU - May 2016. 4 Marks**

Soln. :

$$D_n^2 = 4 nR\lambda$$

As Diameter of 5th dark ring = 0.42 cm

Now diameter of 10th dark ring = ?

$$\therefore \frac{D_5^2}{D_{10}^2} = \frac{4(5) R\lambda}{4(10) R\lambda}$$

$$\therefore 2(D_5^2) = D_{10}^2$$

$$D_{10} = \sqrt{2}\,(D_5) = \sqrt{2}\,(0.42)$$
$$= 0.594 \text{ cm}$$

$\therefore$ Diameter of 10^{th} dark ring = 0.594 cm

Ex. 1.15.19 : A Newton's ring arrangement is used with a source emitting two wavelength $\lambda_1 = 6 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm. It is found that the n^{th} dark ring due t λ_1 coincides with $(n + 1)^{th}$ dark ring for λ_2. If the radius of the curved surface i 90 cm, find the diameter of 3^{rd} dark ring for λ_1.

Soln. :

Given : $\lambda_1 = 6 \times 10^{-5}$ cm, $\lambda_2 = 4.5 \times 10^{-5}$ cm, R = 90 cm

$[D_n]_{\lambda_2} = [D_{n+1}]_{\lambda_1}$

Formula : Diameter of n^{th} dark ring is,

$$D_n^2 = 4\,n\,R\,\lambda \qquad (\mu = 1)$$

$\therefore$ For the n^{th} dark ring λ_1

$$\left[D_n^2\right]_{\lambda_1} = 4\,n\,R\,\lambda_1 \qquad \text{...(1}$$

and for the $(n + 1)^{th}$ dark ring λ_2

$$\left[D_{n+1}^2\right]_{\lambda_2} = 4\,(n+1) \cdot R \cdot \lambda_2 \qquad \text{...(2}$$

$$\therefore \quad 4\,n\,R\,\lambda_1 = 4\,(n+1)\,R \cdot \lambda_2 \quad \therefore n\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{0.45 \times 10^{-5}}{(6 - 0.45) \times 10^{-5}}$$

$$\therefore \quad n = 3$$

$\therefore$ Using Equation (1), the diameter of 3^{rd} dark ring for λ_1 is

$$\left[D_3^2\right] = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$\therefore \quad [D_3]_{\lambda_1} = \sqrt{4 \times 3 \times 90 \times 6 \times 10^{-5}} = 0.2545 \text{ cm.}$$

$\therefore$ Diameter of third dark ring for λ_1 is **0.2545 cm.** **...Ans**

Ex. 1.15.20 : Light containing two wavelengths λ_1 and λ_2 falls normally on a convex lens radius of curvature R, resting on a glass plate. Now if the n^{th} dark ring due t λ_1 coincides with $(n + 1)^{th}$ dark ring due to λ_2, then prove that the radius of th n^{th} dark ring due to λ_1 is $\sqrt{\dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \cdot R}$.

Soln. : The diameter of n^{th} dark ring is given by a relation

$$D_n^2 = 4\,n\,R\,\lambda$$

$\therefore$ For n^{th} dark ring of λ_1,

$$D_n^2 = 4\,n\,R\,\lambda_1 \qquad \text{...(1)}$$

and $(n + 1)^{th}$ dark ring of λ_2,

$$D_{n+1}^2 = 4\,(n+1)\,R\,\lambda_2 \qquad \text{...(2)}$$

Now $D_n^2 = D_{n+1}^2$ as per data

$$\therefore \quad 4\,n\,R\,\lambda_1 = 4\,(n+1)\,R\,\lambda_2$$

$$\therefore \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

As per Equation (1), the diameter of n^{th} dark ring of λ_1 is,

$$D_n = 2\sqrt{n\,R\,\lambda_1}$$

$\therefore$ Radius of n^{th} dark ring of λ_1 is,

$$r_n = \frac{D_n}{2} = \sqrt{nR\,\lambda_1}$$

or

$$r_n = \sqrt{\frac{\lambda_2}{\lambda_1 - \lambda_2} \cdot R\lambda_1}$$

$$= \sqrt{\frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \cdot R}$$

Hence proved.

Ex. 1.15.21 : Show, with clear examples, that separation between two consecutive similar rings, In Newton's rings experiment, goes on reducing as the serial number of ring increases.

Soln. :

According to theory of Newton's ring the diameter of the n^{th} dark ring is given by,

$$D_n = \sqrt{4n\,R\,\lambda}$$

$\therefore D_n \propto \sqrt{n}$, where n is the serial number of the ring.

For example, first calculate the separation of 5^{th} and 4^{th} dark ring,

$$\therefore D_5 \propto \sqrt{5} = 2.236 \text{ and}$$

$$D_4 \propto \sqrt{4} = 2$$

Hence the separation between 5^{th} and 4^{th} ring is

$$D_5 - D_4 \propto 0.236 \text{ in SI unit.}$$

Similarly, the separation between 79^{th} and 80^{th} ring is

$$D_{80} \propto \sqrt{80} = 8.9442 \text{ and } D_{79} \propto \sqrt{79} = 8.8881$$

$$D_{80} - D_{79} \propto 0.0560 \text{ in SI unit.}$$

Hence, we can conclude that,

$$D_{80} - D_{79} < D_5 - D_4 \text{ hence proved.}$$

Important Formulae

1. Intensity in interference pattern where phase difference between two waves is δ

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

2. For maxima, $\delta = 2n\pi$ or path difference

$$\Delta = n\lambda \qquad I_{max} = (a_1 + a_2)^2$$

3. For minima, $\delta = (2n - 1)\pi$, or path difference

$$\Delta = (2n - 1)\lambda/2 \qquad I_{min} = (a_1 - a_2)^2$$

4. Interference in thin parallel films

Reflected light

(1) $2\mu t \cos r = (2n - 1)\dfrac{\lambda}{2}$ (For maxima or bright fringe)

where n = 1, 2, 3, 4, ...

(2) $2\mu t \cos r = n\lambda$ (For minima or dark fringe)

Transmitted light

(1) $2\mu t \cos r = n\lambda$ (For maxima or bright fringe)

(2) $2\mu t \cos r = (2n - 1)\dfrac{\lambda}{2}$ (For minima or dark fringe)

$$\mu = \frac{\sin i}{\sin r}$$

The fringe spacing $\beta = \dfrac{\lambda}{2\mu\theta}$ (For wedge shaped film)

5. Wedge shaped film

Reflected system

Condition of maxima, $2\mu t \cos(r + \theta) = (2n - 1)\lambda/2$

Condition of minima, $2\mu t \cos(r + \theta) = n\lambda$.

6. Newton's ring in reflected light

For n^{th} bright ring $D_n^2 = 2\lambda R \cdot (2n - 1)$

For n^{th} dark ring $D_n^2 = \dfrac{4 n R \lambda}{\mu}$

Diameter of $(n + p)^{th}$ dark ring is $D_{n+p}^2 = \dfrac{4(n + p)R\lambda}{\mu}$

$$\lambda = \frac{\mu\left(D_{n+p}^2 - D_n^2\right)}{4pR}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4(n + p)\lambda}$$

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)}$$

A Quick Revision

- The sources having constant initial phase are called coherent sources.
- Interference of light is the redistribution of intensity in the region of superposition.
- Interference is constructive when the actual path difference between the rays is integral multiple of wavelength λ.
- Interference is destructive when the actual path difference between the rays is odd integral multiple of half the wavelength.
- Interference in thin films is due to division of amplitude of incident beam.
- A path change of λ corresponds to phase change of 2π.
- When reflection occurs at the boundary of denser medium a path change of $\dfrac{\lambda}{2}$ occurs and there is no path change for reflection at a rarer medium.
- When a beam of light travels a thickness t of a medium of R.I. μ, the equivalent path is $\mu \cdot t$.
- The conditions for constructive and destructive interference in reflected and transmitted systems of a thin film are complementary.
- Production of colours in thin films is a result of interference of light.

- To observe the interference in thin films an extended source of light i required.
- Newton's rings are produced as a result of interferences at the wedge shape film.
- Diameters of dark rings are proportional to the square roots of natur numbers and diameters of bright rings are proportional to square roots odd natural numbers.
- The centre of the ring system is dark in reflected system. (Usually)
- The separation between diameters of Newton's rings decreases with increa of order.

Review Questions

Short Answer Questions

Q. 1 What do you mean by coherent sources ?

Q. 2 What do you mean by interference of light ?

Q. 3 Explain why two independent sources can never be coherent.

Q. 4 Can two halves of a 60 W bulb be coherent ?

Q. 5 Write the conditions of maxima and minima in an interference pattern.

Q. 6 Explain redistribution energy in interference pattern.

Q. 7 Name the methods of production of coherent sources.

Q. 8 Write the formula for fringe width.

Q. 9 Explain the conditions of sustained interference of light.

Q. 10 What happens when a very thin film of mica sheet is placed in the path of one the interfering waves ?

Q. 11 What is the effect on fringe width when a thin mica sheet is placed in the path one of the interfering beams ?

Q. 12 What is the effect on interference pattern when monochromatic light is replaced white light in Fresnel biprism experiment ?

Q. 13 Explain the formation of colours in thin films.

Q. 14 A thick film shows no colours in reflected white light, explain.

Q. 15 An excessively thin film appears black in reflected system, explain.

Q. 16 Explain the need of extended source in interference with division of amplitude.

Q. 17 Why is the centre of Newton's ring appears dark in reflected light ?

Q. 18 Why are Newton's rings circular ?

Q. 19 A soap film on a wire loop held in air appears black at its thinnest position when viewed by reflected light. On the other hand a thin oil film floating on water appears bright at its thinnest position when similarly viewed from the air above. Explain.

Ans. :

i) In the first case it appears black because at the contact (t = 0), it satisfy the condition of dark i.e.

Path difference $= 2\mu t \cos r + \lambda/2$

$= 0 + \lambda/2 = \lambda/2$

ii) In the second case, both the reflected rays, one from the boundary between air and oil and another from the boundary between oil and water are in phase. Hence the condition of brightness i.e. $2\mu t \cos r = n\lambda$ is satisfied (at $t \approx 0$) and so it appears bright.

Long Answer Questions

Q. 1 What do you understand by coherent sources ? How are these obtained in practice ?

Q. 2 Describe and explain the phenomenon of interference in thin films. Give the necessary theory.

Q. 3 Why do colours appear in thin films with white light ?

Q. 4 Explain clearly the formation of colours in thin films. Show that the film which appears bright in reflected light appears dark by transmitted light.

Q. 5 With suitable diagram explain why a broad source of light is necessary to observe interference in thin films.

Q. 6 Why a thick film seen by reflected light shows no colours but appears white ?

Q. 7 Describe the fringes observed when a wedge shaped air film is examined by normally reflected sodium light.

Q. 8 Calculate the spacing between two consecutive bright bands in case of interference due to a wedge shaped film.

Q. 9 Describe and explain the formation of Newton's rings in reflected monochromatic light.

Q. 10 Prove that for Newton's rings in reflected light the diameters of dark rings are proportional to the square root of natural numbers and the diameters of bright rings are proportional to the square roots of odd integers.

Q. 11 Account for the perfect blackness of central spot in Newton's rings system. How can you make the central spot bright ?

Problems for Practice

1. A parallel beam of light of wavelength 6000 Å is incident on a parallel glass film of refractive index 1.5 at 45°. Find the minimum thickness of the film for the above light which gives a maximum on reflection.

(Ans. : 1133.89 Å)

2. White light falls normally on a soap film ($\mu = 1.33$) of thickness 5000 Å. Wha wavelength within the visible spectrum (4000 A° to 70000 Å) will be strongl reflected ? **(Ans. : 5320 Å)**

3. Monochromatic light emitted by a broad source of light of wavelength 6×10^{-10} c falls normally on two glass plates which enclose a thin wedge shaped film of ai The plates touch at one end and are separated at a point 15 cm. From the end by wire 0.05 mm in diameter. Find the width between any two consecutive brig fringes. **(Ans. : 0.09 cm**

4. White light is incident on a soap film at an angle $\sin^{-1}\frac{4}{5}$ and the reflected light examination by a spectroscope shows dark bands. Two consecutive dark ban correspond to wavelengths 6.1×10^{-5} cm and 6.0×10^{-5} cm. If the refractive ind of the film be $\frac{4}{3}$, calculate its thickness. **(Ans. : 0.0017 cm**

5. A Newton's rings set up is used with a source emitting two wavelengths $\lambda_1 = 60$ Å and $\lambda_2 = 4500$ Å. It is found that n^{th} dark ring due to 6000 Å coincides wi $(n + 2)^{th}$ dark ring due to 4500 Å. If the radius of curvature of the lens is 90 c find the diameter of the n^{th} dark ring for 6000 Å. **(Ans. : 0.36 cm**

1.16 University Questions

May 2012

Q. 1 Explain why Newton's rings are unequally spaced ?
(Ans. : Refer section 1.9) (3 Marks)

Dec. 2012

Q. 1 Explain why we see beautiful colors in thin film when is exposed to sun light.
(Ans. : Refer section 1.5) (3 Marks)

Q. 2 *Refer Ex. 1.15.8* (5 Marks)

May 2013

Q. 1 Suppose that in experiment on Newton's Rings, first light of red colour is used and then blue light, which set of rings would have larger diameter ? Justify your answer w proper expression.
(Ans. : Refer section 1.12) (3 Mar

Q. 2 *Refer Ex. 1.15.13* (7 Mar

Dec. 2013

Q. 1 Explain why an extensively thin fi appears black in reflected light ?
(Ans. : Refer section 1.7) (3 Mar

Q. 2 Derive the conditions for maxima a minima due to interference of li reflected from thin film of unifo thickness.
(Ans. : Refer section 1.3) (7 Mar

May 2014

Q. 1 Why the Newton's rings are circu and centre of interference patt (reflected) is dark ?
(Ans. : Refer section 1.11(1) and ((3 Mar

Q. 2 Obtain the conditions for maxima and minima due to interference in a wedge shaped film observed in reflected light.
(Ans. : Refer section 1.7) (4 Marks)

Dec. 2014

Q. 1 *Refer Ex. 1.15.3* (3 Marks)

Q. 2 Explain the interference in thin parallel film and derive the expression for path difference between reflected rays, hence obtain the conditions of maxima and minima for interference with monochromatic light.
(Ans. : Refer section 1.3) (7 Marks)

May 2015

Q. 1 Comment on colors in a soap film in sunlight. *(Ans. : Refer section 1.5)* (3 Marks)

Q. 2 Show that the diameter of Newton's n^{th} dark ring is proportional to square root of ring number.
(Ans. : Refer section 1.9) (5 Marks)

Q. 3 Derive the condition for a thin transparent film of constant thickness to appear bright and dark when viewed in reflected light.
(Ans. : Refer section 1.3) (7 Marks)

Dec. 2015

Q. 1 *Refer Ex. 1.8.1* (3 Marks)

Q. 2 Why are the fringes in wedge shaped film straight ? Derive the conditions of maxima and minima for interference in wedge shaped films ?
(Ans. : Refer section 1.7) (7 Marks)

May 2016

Q. 1 Why does an excessively thin film appear to be perfectly dark when illuminated by white light ?
(Ans. : Refer section 1.4) (3 Marks)

Q. 2 *Refer Ex. 1.15.18* (4 Marks)

Dec. 2016

Q. 1 Why the Newton's rings are circular and fringes in wedge shaped film are straight ?
(Ans. : Refer section 1.11(4)) (3 Marks)

Q. 2 For Newton's ring, prove that diameter of n^{th} dark ring is directly proportional to the square root of natural number.
(Ans. : Refer section 1.9) (4 Marks)

Q. 3 Why do we see beautiful colours in thin film when it is exposed to sunlight ?
Obtain expression for path difference between two reflected rays in thin transparent film of uniform thickness and write the conditions of maxima and minima.
(Ans. : Refer sections 1.3 and 1.5) (4 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Interference by Division of Amplitude and by Division of Wave Front

Q. 1 What are the methods of coherent waves ? *(Ans. : Refer section 1.1)*

Syllabus Topic : Interference in Thin Film of Constant Thickness due to Reflected and Transmitted Light

Q. 1 Derive the conditions for maxima and minima due to interference of light reflec from thin film of uniform thickness.
(Ans. : Refer section 1.3) **(Dec. 2013, Dec. 2014, May 2015, Dec. 201**

Q. 2 Why does an excessively thin film appear to be perfectly dark when illuminated white light ? (Ans. : Refer section 1.4) **(May 20**

Syllabus Topic : Origin of Colours in Thin Films

Q. 1 Why do we see beautiful colours in thin film when it is exposed to sunlight ?
(Ans. : Refer section 1.5) **(Dec. 2012, May 2015, Dec. 20**

Syllabus Topic : Wedge-Shaped Film (angle of wedge and thickness measurement)

Q. 1 Why are the fringes in wedge shaped film straight ? Derive the condition maxima and minima for interference in wedge shaped films ?
(Ans. : Refer section 1.7) **(Dec. 2013, May 2014, Dec. 20**

Syllabus Topic : Newton's rings

Q. 1 Explain why Newton's rings are unequally spaced ? **(May 20**
(Ans. : Refer section 1.9)

OR

Show that the diameter of Newton's n^{th} dark ring is proportional to square roo ring number. (Ans. : Refer section 1.9) **(May 2015, Dec. 20**

Q. 2 Why centre of Newton's rings is always dark ? **(May 20**
(Ans. : Refer section 1.11(1))

Q. 3 Why Newton's rings are circular and wedge shaped films are straight ?
(Ans. : Refer section 1.11(4)) **(May 2014, Dec. 20**

Q. 4 Suppose that in experiment on Newton's Rings, first light of red colour is used then blue light, which set of rings would have larger diameter ? Justify your ans with proper expression. (Ans. : Refer section 1.12) **(May 20**

Solved Problems

Q. 1	Refer Ex. 1.8.1	**(Dec. 2015)**	**Q. 4**	Refer Ex. 1.15.13	**(May 20**
Q. 2	Refer Ex. 1.15.3	**(Dec. 2014)**	**Q. 5**	Refer Ex. 1.15.18	**(May 20**
Q. 3	Refer Ex. 1.15.8	**(Dec. 2012)**			

CHAPTER 2

Module 1

Applications of Interference in Thin Film

Syllabus

Applications of interference - Determination of thickness of very thin wire or foil, determination of refractive index of liquid, wavelength of incident light, radius of curvature of lens, testing of surface flatness, Anti-reflecting films, Highly reflecting film.

2.1 Introduction

It is important to know that a very small difference of geometrical spacing of $\lambda/4$ creates for a to and fro journey of light a path difference of $\lambda/2$ and in turn a bright spot in interference pattern will appear dark and a dark will appear bright. This provides us a tool for an accurate measurement of spacing $\lambda/4$. For minimum visible wavelength 4000Å this spacing comes out to be 1000Å i.e. 0.1 Micrometer. If a reader compares it with least count of Micrometer screw gauge, will surely agree that optical instrument designed on the basis of interference in thin film provides excellent ability to measure very small spacing. In this unit we will study some of the applications of interference with the aim of higher degree of ability and unmatched accuracy for small scale measurements.

Syllabus Topic : Determination of Thickness of Very Thin Wire or Foil

> **Topics covered :** Determination of Thickness of Very Thin Wire or Foil.

2.2 Determination of Thickness of Very Thin Wire or Foil

There are many applications where in we need to know the thickness of a very thin wire or an equivalent for example "contact lens". As mentioned in introduction, concept of inference can lead us to an experimental arrangement which gives us accurate measure up to one tenth of micrometer.

Measurement of thickness of very thin wire or foil :

- As seen in the case of "Spacing between two consecutive bright bonds", we have derived fringe width $\beta = \frac{\lambda}{2\theta}$
- The same concept is used for determination of thickness of thin wire. We take two glass slides (optically flat) and put them in touch at one end and at the other we put the wire or foil whose thickness is to be determined. Hence we prepare a wedge with very small angle 'θ'.
- Here we make the use of same set up which is used for Newton's rings. In place of plano convex lens we take the above mentioned wedge.
- On viewing through microscope when illuminated by a monochromatic light of wavelength λ at normal incidence, we get alternate dark and bright lines.
- We take readings of dark lines at the spacing of some interval say 'P' number of lines.
- We draw a graph for Reading → order of dark line and find the fringe width 'β'.
- From experimental set up measure 'l'

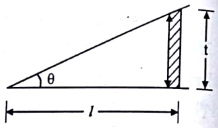


Fig. 2.2.1

$$\therefore \tan\theta = \frac{t}{l} \simeq \theta \quad \text{(as } \theta \text{ is very small)}$$

$$\text{as } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{t}{l} = \frac{\lambda}{2\beta} \qquad \therefore t = \frac{\lambda l}{2\beta} \qquad \ldots(2.2.1)$$

Since λ, l and β are known we can calculate thickness of a foil or thin wire.

Ex. 2.2.1 : Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light at normal incidence, what is the thickness of the wire? (Given λ = 5893 Å)

Soln. :

$$\text{Fringe width } \beta = \frac{\lambda}{2\mu\theta} \ldots\text{(For normal incidence)}$$

$$\text{For air film } \mu = 1, \ \beta = \frac{\lambda}{2\theta}$$

$$\text{Thickness of wire } t = x_n \tan\theta$$

$$= x_n \cdot \theta \text{ (for small } \theta, \tan\theta \simeq \theta)$$

$$= 20 \cdot \beta \cdot \theta$$

As there are 20 fringes between two edge points

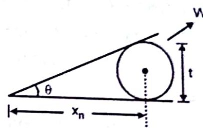


Fig. P. 2.2.1

Putting the value of β, we get,

$$t = 20\left(\frac{\lambda}{2\theta}\right)\cdot\theta = 10\lambda$$

$$= 10 \times 5893 \times 10^{-10} = 5.893 \times 10^{-6} \text{ m}$$

$$= \mathbf{5.893\ \mu m}$$

...Ans.

Syllabus Topic : Wavelength of Incident Light

> **Topics covered :** Determination of Wavelength of Monochromatic Light or Radius of Curvature of Lens by Newton's Rings Method

2.3 Determination of Wavelength of Monochromatic Light or Radius of Curvature of Lens by Newton's Rings Method

Experimental arrangement **MU - May 2013, Dec. 2014**

- A carefully cleaned convex surface of a plano-convex lens L of large radius of curvature is placed on a plane glass plate P. (Fig. 2.3.1).
- Another glass plate G is held at a suitable distance above at an angle 45° with the vertical (to make the normal incidence on the film).
- With a condensing lens, maximum light from source (Sodium lamp) is allowed to fall on G. The inclined plate G reflects light onto the air film between the lens L and plate P.

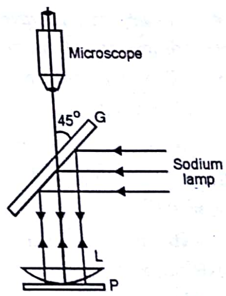


Fig. 2.3.1

- Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.
- They are seen through a low power microscope focussed on the air film where the rings are formed.

Theory

- The effective path difference between the interfering rays is

$$2\mu t \cos(r+\theta)+\frac{\lambda}{2}$$

where μ = R.I. of film.
θ = Angle of film at any point
λ = Wavelength of light

- If D_n is the diameter of n^{th} dark ring then as per the theory of Newton's rings described in Equation (1.9.3),

$$D_n^2 = 4nR\lambda$$

where R = Radius of curvature of lower surface of lens.

Let $D_{n+p}^2 = 4(n+p)R\lambda$ (for $(n+p)^{th}$ ring) ...(2.3.1)

$$\therefore \quad D_{n+p}^2 - D_n^2 = 4pR\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \qquad ...(2.3.2)$$

- Alternately if wavelength of incident monochromatic ray is known, using Equation (2.3.2), we can find R i.e. the radius of curvature.
- For more accurate approach we plot a graph of D_n^2 vs n as shown in Fig. 2.3.2.

Thus by measuring the diameter of n^{th} and $(n+p)^{th}$ dark rings and the radius of curvature R, the wavelength λ can be calculated.

The diameter of the rings is measured with the travelling microscope and the radius of curvature can be determined by using lens equation or by a spherometer.

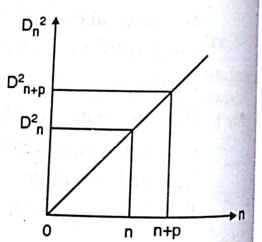


Fig. 2.3.2

Syllabus Topic : Radius of Curvature of Lens

> **Topics covered :** Determination of radius of curvature of lens

2.3.1 Determination of Radius of Curvature of Lens

This can be done easily with the help of spherometer and the formula:

$$R = \frac{l^2}{6h}+\frac{h}{2} \qquad ...(2.3.3)$$

Where l = distance between two legs of spherometer.
h = Difference in the reading when placed on lens as well as when placed on surface.

Ex. 2.3.1 : In Newton's rings experiment the diameter of 5^{th} ring was 0.336 cm and the diameter of 15^{th} ring was 0.590 cm. Find the radius of curvature of plane-convex lens if the wavelength of light used is 5890 Å. **MU - May 2015, 5 Marks**

Soln. :

Given : $D_5 = 0.336$ cm, $D_{15} = 0.590$ cm, $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : $D_{n+p}^2 - D_n^2 = 4pR\lambda$

$$R = \frac{D_{n+p}^2 - D_n^2}{4\cdot p\cdot\lambda} = \frac{(0.590)^2-(0.336)^2}{4\times 10\times 5890\times 10^{-8}} = 99.91 \text{ cm}$$

$$\therefore \quad R = \mathbf{99.91\ cm.}$$

Syllabus Topic : Determination of Refractive Index of a Liquid

> **Topics covered :** Determination of Refractive Index of a Liquid by Newton's Rings

2.4 Determination of Refractive Index of a Liquid by Newton's Rings

MU - May 2014, May 2016

- There is a popular branch of engineering known as " Hydraulics" in which liquid is in motion and transmits energy, Viscosity and refractive index are some of the parameters which can be used to decide the suitability of the oil (Called Grade of the oil). In such applications, the correct measurement of the R.I. is thus an important issue.
- Consider that a transparent liquid whose refractive index is to be determined is placed between the lens L and plate P of the Newton's rings arrangement.

- If the liquid is rarer than glass, a phase change of π will occur at the reflection from the lower surface of liquid film.
- If the liquid is denser than glass, then a phase change of π will occur due to reflection at the upper surface of the film.
- Hence in either case, a path difference of $\lambda/2$ will be introduced between the interfering rays in the reflected system; and hence the effective path difference between them will be $2\mu t \cos(r+\theta) + \frac{\lambda}{2}$

Now $r = 0$ for normal incidence

$\theta = 0$ for large R

$$\therefore \quad \text{Path difference} = 2\mu t + \frac{\lambda}{2}$$

$$2t = \frac{\rho^2}{2}$$

∴ For n^{th} dark ring, we have,

$$\frac{\mu \rho^2}{R} + \frac{\lambda}{2} = (2n+1)\lambda/2 \qquad \therefore \quad \frac{\mu \cdot D_n^2}{4R} = n\lambda$$

$$\therefore \quad D_n^2 = \frac{4nR\lambda}{\mu}$$

Similarly for the $(n+p)^{th}$ dark ring we have,

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu} \qquad \text{...(2.4.1)}$$

$$D_{n+p}^2 - D_n^2 = \frac{4pR\cdot\lambda}{\mu}$$

D_{n+p} = Diameter of $(n+p)^{th}$ dark ring, then we have

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)_{liquid}} \qquad \text{...(2.4.2)}$$

$$\therefore \quad \mu = \frac{4pR\lambda}{\left[D_{n+p}^2 - D_n^2\right]_{air}} = 1 \quad (\because \mu = 1 \text{ for air}) \qquad \text{...(2.4.3)}$$

For more accurate approach we plot a graph of D_n^2 vs n as shown below.

It is similar to Fig. 2.3.2.

∴ From Equations (2.4.2) and (2.4.3) we have,

$$\mu = \frac{\left[D_{n+p}^2 - D_n^2\right]_{air}}{\left[D_{n+p}^2 - D_n^2\right]_{liquid}}$$

- If we obtain ring structure using air as the medium then diameter is given by $D_n^2\,(\text{air}) = 4nR\lambda$. Now slowly if we insert a liquid with R.I. μ then $D_n^2\,(\text{liquid}) = \frac{4nR\lambda}{\mu}$, Take ratio of these two.

$$\mu = \frac{D_n^2\,(\text{air})}{D_n^2\,(\text{liq})} \qquad \text{...(2.4.4)}$$

- Thus to find μ, Newton's rings are formed with air film. The diameters of n^{th} and $(n+p)^{th}$ dark rings are measured for air film.
- Then the transparent liquid is introduced between lens L and plate P.
- The diameters of n^{th} and $(n+p)^{th}$ dark rings are measured with liquid film. Hence using Equation (2.4.4), μ can be calculated.

Ex. 2.4.1 : Newton's rings are formed in reflected light of wavelength 6000 A° with a liquid between the plane and curved surfaces. If the diameter of the 6^{th} bright ring is 3.1 mm and the radius of curvature of the curved surface is 100 cm, calculate the refractive index of the liquid.

Soln. :

The diameter of n^{th} bright ring is,

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2}$$

Given : $n = 6$, $\lambda = 6000 \times 10^{-8}$ cm, $R = 100$ cm, $D_6 = 0.31$ cm

$$\mu = \frac{2(2\times 6-1)\,6000\times 10^{-8}\times 100}{(0.31)^2}$$

$$= \frac{2\times 11\times 6\times 10^{-3}}{(0.31)^2}$$

$$= \mathbf{1.373} \qquad \textbf{...Ans.}$$

Syllabus Topic : Testing of Surface Flatness

➤ **Topics covered :** Determination of Optical Flatness

2.5 Determination of Optical Flatness

Testing the optical flatness of surfaces

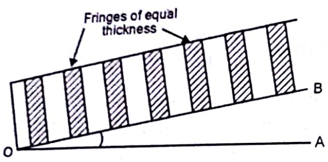


Fig. 2.5.1

- The phenomenon of interference is also used in testing the plainness of the surfaces.
- If two surfaces OA and OB (Fig. 2.5.1) are perfectly plane, the air film between them gradually varies in thickness from O to A. The fringes are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a constant value.
- If the fringes are not of equal thickness, it means that the surfaces are not plane.
- To test the optical flatness of a surface, the specimen surface to be tested (OB) is placed over an optically plane surface (OA).
- The fringes are observed in the field of view. If they are of equal thickness the surface OB is plane. If not, then surface OB is not plane.
- The surface OB is polished and the process is repeated. When fringes observed are of equal width, it means the surface OB is plane.
- The accuracy in this method is far more superior compare to any other technique adopted. The accuracy level is of the order of fraction of micrometer.

Syllabus Topic : Anti Reflecting Films

➤ **Topics covered :** Concept of Anti Reflecting Coating

2.6 Concept of Anti Reflecting Coating

Non reflecting films : MU - May 2012. Dec. 2013

- We are aware that compound microscope, telescope, camera lenses etc. uses a combination of lenses.
- When the light enters the optical instrument at the glass air interface, around 4% of light (for air with $n_1 = 1$ and glass with $n_2 = 1.5$) that too at single reflection is lost by reflection which is highly undesirable. For advance telescopes the total loss comes out to be nearly 30% and can not be tolerated if working under low intensity applications.
- In order to reduce the reflection loss, a transparent film of proper thickness is deposited on the surface. This film is known as "non Reflecting film".
- Popular material used is MgF_2 because its refractive index is 1.38 (i.e. between air and glass). Cryolite ($n_1 = 1.36$) is also used.
- Thickness of the film may be obtained for given purpose as shown below :

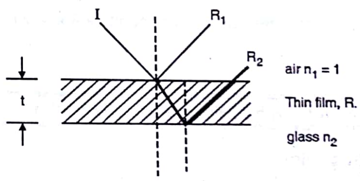


Fig. 2.6.1 : Thin film coating

- Let a ray I incident up on thin film of MgF_2 coated on glass. This ray is reflected from upper surface as R_1 and from lower surface as R_2. The optical path difference between these two rays is n_1 (2t). As the incident ray enters from rarer to denser twice i.e. at air to film and film to glass.

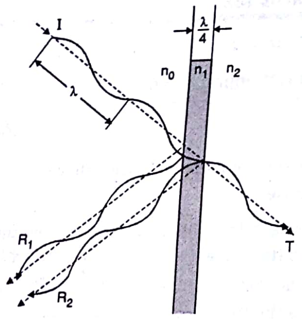


Fig. 2.6.2

- If both the rays R_1 and R_2 interfere with each other and path difference is $(2n + 1)\,\lambda/2$ (for n= 0, 1, 2, ...) then destructive interference will take place.

$$\therefore \quad 2 n_1 t = \frac{\lambda}{2} \qquad \text{(for n = 0)}$$

$$\therefore \quad n_1 t = \frac{\lambda}{4\mu}$$

- It means, in order to have destructive interference a layer of $n_1 t = \frac{\lambda}{4}$ is coated on glass plate.

Syllabus Topic : Highly Reflecting Film

2.7 Highly Reflecting Film

- As shown in the case of non Reflecting film, we have seen that a thin film of thickness $\lambda/4$ will create additional path difference of $\lambda/4 + \lambda/4 = \lambda/2$ or additional phase difference $\frac{\pi}{2} + \frac{\pi}{2} = \pi$.

  This creates destructive interference.

- The same logic is extended by considering a film of thickness $\lambda/2$.

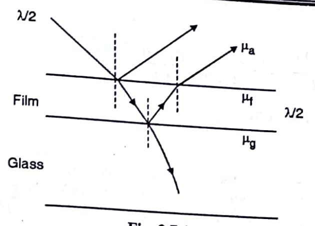


Fig. 2.7.1

In this case the total path difference is $\frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$ or phase difference of $\pi + \pi = 2\pi$

- Thus by the condition of complete constructive interference is satisfied. With this we can make majority of light reflected back from the surface of the glass.

  This principle is used in all kind of sun control films used on car, sun glasses etc.

2.8 Solved Problems

Problems on Application of Newton's Rings

Ex. 2.8.1 : In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and that of 15th ring was 0.59 cm. If the radius of curvature of the plano - convex lens 100 cm. Calculate the wavelength of light.

MU - May 2013, May 2014, 3 Marks

Soln. :

Given : R = 100 cm, D_{15} = 0.59 cm, D_5 = 0.336 cm

Formula :
$$\lambda = \frac{D_{15}^2 - D_5^2}{4 \times n \times R} = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= \frac{0.2352}{4000} = \mathbf{5.88 \times 10^{-5}\ cm} \qquad \text{...Ans.}$$

Ex. 2.8.2 : Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th bright ring is 0.336 cm, what is the wavelength of light used ?

Soln. :

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= 5.880 \times 10^{-5} \text{ cm}$$

$$= \mathbf{5880\ Å}$$

...Ans.

Ex. 2.8.3 : In a Newton's ring experiment the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid. **MU - Dec. 2012, 5 Marks**

Soln. :

$$D_n(\text{air}) = 1.4 \text{ cm}$$

$$D_n(\text{liquid}) = 1.27 \text{ cm}$$

$$\therefore \quad \mu = \frac{D_n^2(\text{air})}{D_n^2(\text{liquid})} = \frac{1.4^2}{1.27^2} = 1.215$$

Ex. 2.8.4 : Newton's rings are formed using light of wavelength 5896 Å in reflected light with a liquid placed between plane and curved surface. The diameter of 7th bright fringe is 0.4 cm and radius of curvature is 1 m. Find the refractive index of liquid. **MU - Dec. 2012, 5 Marks**

Soln. :

For Newton's rings diameter of nth dark ring with air as medium

$$D_n^2 = 4nR\lambda$$

And with liquid having R.I. μ is given by

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$\therefore \quad \mu = \frac{4nR\lambda}{D_n^2} = \frac{4 \times 7 \times 1 \times 5896 \times 10^{-10}}{(0.4 \times 10^{-2})^2}$$

$$= \mathbf{1.038}$$

...Ans.

Ex. 2.8.5 : Newton's rings are formed by light reflected normally from a convex lens of radius of curvature 90 cm and a glass plate with a liquid in between them. The diameter of nth dark ring is 2.25 mm and that of (n + 9)th dark ring is 4.5 mm. Calculate the refractive index of the liquid. Given : λ = 6000 Å.

Soln. :

Given : $R = 90$ cm, $D_n = 2.25$ mm, $D_{n+p} = 4.5$ mm

$\lambda = 6000\ Å = 6000 \times 10^{-8}$ cm

Formula :
$$\lambda = \frac{\mu\,(D_{n+p}^2 - D_n^2)}{4pR}$$

$$\mu = \frac{4\,pR\,\lambda}{(D_{n+p}^2 - D_n^2)} = \frac{4 \times 9 \times 90 \times 6000 \times 10^{-8}}{(0.45)^2 - (0.225)^2}$$

$$= \mathbf{1.28}$$

...Ans.

Ex. 2.8.6 : In a Newton's rings arrangement if a drop of water (μ = 4/3) be placed in between the lens and the plate, the diameter of 10th ring is found to be 0.6 cm. Obtain the radius of curvature of the face of the contact with the plate. The wavelength of light used is 6000 Å.

Soln. :

Given : $\mu = 4/3$, $D_{10}^2 = 0.6$ cm., $\lambda = 6000\ Å = 6000 \times 10^{-8}$ cm

Formula : Diameter of nth dark ring is

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$R = \frac{\mu \cdot D_n^2}{4n\lambda}$$

$$\frac{4 \times (0.6)^2}{3 \times 4 \times 10 \times 6000 \times 10^{-8}} = 200 \text{ cm.}$$

$$\therefore \quad R = \mathbf{200\ cm.}$$

...Ans.

Ex. 2.8.7 : Newton's rings are observed in reflected light of wavelength 6000 A°. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the corresponding air film.

Soln. :

We have, the diameter of dark ring,

$$D_n^2 = 4nR\lambda, \qquad R = \frac{D_n^2}{4n\lambda}$$

Given : $n = 10$, $D_n = 0.5 \times 10^{-2}$ m, $\lambda = 6 \times 10^{-7}$ m

R = Radius of curvature

$$\therefore \quad R = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 6 \times 10^{-7}} = \frac{25}{24} = 1.04 \text{ m} = 104 \text{ cm}$$

Thickness of air film,

$$t = \frac{D^2}{8R}$$

$$D = 0.5 \times 10^{-2}\ \text{m}, \quad R = 1.04\ \text{m}$$

$$t = \frac{(0.5 \times 10^{-2})^2}{8 \times 1.04} = 3.0 \times 10^{-6}\ \text{m} = \mathbf{3.0\ \mu m}$$ ...Ans.

Ex. 2.8.8 : In a Newton's ring arrangement with a film observed with light of wavelength 6×10^{-5} cm, the difference of square of diameters of successive rings are $0.125\ \text{cm}^2$. What will happen to this quantity if :

i) Wavelength of light changed to 4.5×10^{-5} cm.
ii) A liquid of refractive index is 1.33 introduced between the lens and the plate.
iii) The radius of curvature of convex surface of plano-convex lens is doubled.

Soln. :

Using Equation (2.3.2) with $\mu \neq 1$.

i) We have, $$D^2_{n+p} - D^2_n = \frac{4pR\lambda}{\mu}$$

For successive rings p = 1, so,

$$D^2_{n+1} - D^2_n = \frac{4\lambda R}{\mu} \quad ...(1)$$

When wavelength changes to λ', we have,

$$D'^2_{n+1} - D'^2_n = \frac{4\lambda' R}{\mu} \quad ...(2)$$

Divide Equation (2) by Equation (1), we get,

$$\frac{D'^2_{n+1} - D'^2_n}{D^2_{n+1} - D^2_n} = \frac{\lambda'}{\lambda}$$

Given : $\lambda' = 4.5 \times 10^{-5}$ cm, $\lambda = 6 \times 10^{-5}$ cm, $D^2_{n+1} - D^2_n = 0.125$ cm

$$\text{So, } D'^2_{n+1} - D'^2_n = \frac{4.5 \times 10^{-5}}{6.0 \times 10^{-5}} \times 0.125$$

$$= \mathbf{0.0937\ cm^2}$$ ...Ans.

ii) When liquid introduced between the lens and plate, we have,

$$D'^2_{n+1} - D'^2_n = \frac{4\lambda R}{\mu'} \quad ...(3)$$

Divide Equation (3) by Equation (1), we get,

$$\frac{D'^2_{n+1} - D'^2_n}{D^2_{n+1} - D^2_n} = \frac{\mu}{\mu'}$$

Given : $\mu = 1, \mu' = 1.33$,

$$D'^2_{n+1} - D'^2_n = \frac{1}{1.33} \times 0.125 = 0.094\ \text{cm}^2$$

iii) When radius curvature (R) is made doubled, we can write,

$$D'^2_{n+1} - D'^2_n = \frac{4\lambda R'}{\mu} \quad ...(4)$$

where $R' = 2R$

Divide Equation (4) by Equation (1),

$$\frac{D'^2_{n+1} - D'^2_n}{D^2_{n+1} - D^2_n} = \frac{R'}{R} = \frac{2R}{R} = 2$$

$$D'^2_{n+1} - D'^2_n = 2 \times 0.125 = \mathbf{0.250\ cm^2}$$ ...Ans.

Ex. 2.8.9 : In Newton's ring experiment the diameter of n^{th} and $(n + 8)^{th}$ bright rings are and 7 mm respectively. Radius of curvature of lower surface of lens is 2 m. determine the wavelength of light use. **MU - Dec. 2016, 5 Marks**

Soln. :

Given :

(1) Diameter of n^{th} bright ring = 4.2 mm
(2) Diameter of $(n + 8)^{th}$ bright ring = 7 mm
(3) Radius of curvature of plane convex lens = 2 m

To find : Wavelength of monochromatic light.

Formula :

$$D^2_n = 2\lambda R(2n - 1)$$

For $(n + 8)^{th}$ ring

$$D^2_{n+8} = 2\lambda R(2(n + 8) - 1) = (7 \times 10^{-3})^2 \quad ...(1)$$

For n^{th} ring

$$D^2_n = 2\lambda R(2n - 1) = (4.2 \times 10^{-3})^2 \quad ...(2)$$

$\therefore$ Divide Equations (1) by (2)

$$\frac{2n + 15}{2n - 1} = \left(\frac{7}{4.2}\right)^2 = 2.7778$$

$$\therefore \quad n = 5 \quad ...(3)$$

Using this in Equation (2)

$$D_n^2 = 2\lambda R(2n-1) = (4.2\times10^{-3})^2$$

$$\therefore \quad \lambda = 4.9\times10^{-7}\text{ m}$$ ...Ans.

Problems on Concept of Wedge Shaped Film

Ex. 2.8.10 : Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophane whose refractive index is 1.4. The angle of wedge is 20 sec. of arc and the distance between successive fringes is 0.25 cm. Calculate the wavelength of light.

Soln. :

Given : $\beta = 0.25$ cm, $\mu = 1.4$, $\theta = 20\text{ sec} = \dfrac{20}{60\times60}\times\dfrac{\pi}{180}$ radians

Formula : $\beta = \dfrac{\lambda}{2\mu\theta}$

$$\lambda = 2\mu\theta\beta = 2\times1.4\times\frac{\pi}{180\times180}\times0.25$$

$$\lambda = \mathbf{6.7873\times10^{-5}\,cm}$$ ...Ans.

Ex. 2.8.11 : Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.01 mm is introduced between them at one end to form an air film. If the light used has wavelength 5900Å, find the separation between consecutive bright fringes. **MU - May 2014, 4 Marks**

Soln. :

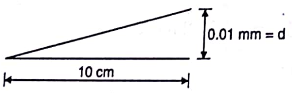


Fig. P. 2.8.11

Let $l = 10$ cm, $d = 0.01$ mm

$\lambda = 5900$ Å

here, $\tan\theta = \dfrac{0.01\times10^{-3}}{10\times10^{-2}} = 1\times10^{-4}$

For very small angles, $\tan\theta \simeq \theta$

Now fringe width $\beta = \dfrac{\lambda}{2\mu\theta}$

For air film, $\mu = 1$

$$\therefore \beta = \frac{5900\times10^{-8}}{2\times1\times10^{-4}} = \mathbf{0.295\ cm}$$

$\therefore$ Separation between two consecutive bright fringes is **0.295 cm** ...Ans.

Ex. 2.8.12 : Two plane rectangular pieces of glass are in contact at one edge and are separated at other end 10 cm away by a wire to form a wedge shaped film. When the film was illuminated by light of wavelength 6000 Å, 10 fringes were observed per cm. Determine the diameter of the wire.

Soln. :

Given : $l = 10$ cm

$\lambda = 6000$ Å

No. of fringes per cm = 10

$\therefore \quad \beta = \dfrac{1}{10} = 0.1$ cm

To find : Diameter of wire

Assuming that (i) wedge angle is very small, (ii) the medium as air (iii) Incidence of light is normal.

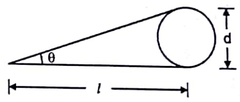


Fig. P. 2.8.12

$$\tan\theta = \frac{d}{l} = \theta \quad \text{(for very small } \theta)$$

$$\text{and} \quad \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{d}{l} = \frac{\lambda}{2\beta}$$

$$\therefore \quad d = \frac{\lambda l}{2\beta} = \frac{6000\times10^{-8}\times10}{2\times0.1}$$

Diameter $= 3\times10^{-3}$ cm ...Ans.

Important Formulae

Interference in thin parallel films :

Wedge shaped films

1. The fringe spacing $\beta = \frac{\lambda}{2\mu\theta}$

2.. Wedge shaped film :

Reflected system :

Condition of maxima, $2\mu t \cos(r+\theta) = (2n-1)\lambda/2$

Condition of minima, $2\mu t \cos(r+\theta) = n\lambda$

Newton's Rings

$$\lambda = \frac{\mu\left(D_{n+p}^2 - D_n^2\right)}{4pR}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4(n+p)\lambda}$$

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)}$$

Anti reflecting film

Condition for Anti reflecting film : Thick ness of the film: $\lambda/4$.

A Quick Revision

- Newton's rings are produced as a result of interferences at the wedge shaped film.
- Diameters of dark rings are proportional to the square roots of natural numbers and diameters of bright rings are proportional to square roots of odd natural numbers.
- The centre of the ring system is dark in reflected system. (Usually)
- Newton's rings method can be used to find the wavelength of monochromatic light, R.I. of liquids.

$$\lambda = \frac{\mu\left(D_{n+p}^2 - D_n^2\right)}{4pR} \quad \text{for air R.I. is} = 1$$

If Wave length λ is known one can find radius of curvature R

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)}$$

- When a drop of water is introduced between plano-convex lens and glass plate, the rings contract.
- The centre of Newton's ring may be made bright by introducing a drop of sassafras oil between crown glass lens and flint glass plate.
- Condition for Anti reflecting film : Thick ness of the film: $\lambda/4$

Review Questions

Long Answer Questions

Q. 1 Describe Newton's rings method for measuring the wavelength of monochromatic light and give the necessary theory. What will happen if a little water is introduced between the lens and the plate ?

Q. 2 Explain how you can determine the refractive index of a liquid by means of Newton's rings.

Q. 3 Write a note on `fringes of equal inclination' and `fringes of equal thickness.'

Q. 4 Explain how a wedge shaped air film formed by glass plates can be used for testing the optical flatness of glass plate.

Q. 5 Explain the use of thin film as anti-reflection coating.

Problems for Practice

1. A convex lens is placed on a slab of plane glass and is illuminated by monochromatic light. The diameter of the 10^{th} dark ring is found to be 0.433 cm. Find the wavelength of light if the radius of curvature of the lower face of the lens is 70 cm. **(Ans. : 6695 Å)**

2. In Newton's rings experiment the diameters of the n^{th} and $(n + 8)^{th}$ bright rings are 4.2 mm and 7.00 mm respectively. Radius of curvature of the lower surface of the lens is 2.0 metres. Determine the wavelength of light. **(Ans. : 4900 Å)**

3. The diameter of the sixth dark Newton's ring formed in reflected light is found to be 1.5 cm. When a liquid is introduced in the space between the lens and the plate, the diameter of the same ring becomes 1.0 cm. Find the refractive index of the liquid used. **(Ans. : 2.25)**

2.9 University Questions

May 2012

Q. 1 Describe in detail the concept of anti reflecting film with a proper ray diagram of thin film interference. Which condition the material should satisfy to act as anti reflecting film? *(Ans. : Refer section 2.6)* (8 Marks)

Dec. 2012

Q. 1 *Refer Ex. 2.8.3* (5 Marks)

Q. 2 *Refer Ex. 2.8.4* (5 Marks)

May 2013

Q. 1 With the help of proper diagram and necessary expressions, explain how Newton's ring experiment is useful to determine the radius of curvature of a plane convex lens. *(Ans. : Refer section 2.3)* (5 Marks)

Q. 2 *Refer Ex. 2.8.1* (3 Marks)

Dec. 2013

Q. 1 What do you understand by anti-reflecting coating? Derive the conditions with proper diagram. *(Ans. : Refer section 2.6)* (8 Marks)

May 2014

Q. 1 With Newton's ring experiment explain how to determine the refractive index of Liquid. *(Ans. : Refer section 2.4)*

Q. 2 *Refer Ex. 2.8.1* (4 Marks)

Q. 3 *Refer Ex. 2.8.11* (4 Marks)

Dec. 2014

Q. 1 With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of planoconvex lens. *(Ans. : Refer section 2.3)* (8 Marks)

May 2015

Q. 1 *Refer Ex. 2.3.1* (3 Marks)

May 2016

Q. 1 How is Newton's ring experiment used to determine refractive index of liquid medium ? *(Ans. : Refer Section 2.4)* (4 Marks)

Dec. 2016

Q. 1 *Refer Ex. 2.8.9* (4 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Determination of Thickness of Very Thin Wire or Foil

Q. 1 Refer Ex. 2.2.1

Syllabus Topic : Wavelength of Incident Light

Q. 1 With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of plane convex lens. *(Ans.: Refer section 2.3)*

(May 2013, Dec. 2014)

Syllabus Topic : Radius of Curvature of Lens

Q. 1 Refer Ex. 2.3.1 **(May 2015)**

Syllabus Topic : Determination of Refractive Index of a Liquid

Q. 1 How is Newton's ring experiment used to determine refractive index of liquid medium ? *(Ans. : Refer section 2.4)* **(May 2014, May 2016)**

Syllabus Topic : Testing of Surface Flatness

Q. 1 Write a short note on : Determination of Optical Flatness. *(Ans. : Refer Section 2.5)*

Syllabus Topic : Anti Reflecting Films

Q. 1 Describe in detail the concept of anti reflecting film with a proper ray diagram of thin film interference. Which condition the material should satisfy to act as anti reflecting film? *(Ans.: Refer Section 2.6)* **(May 2012)**

OR

What do you understand by anti-reflecting coating ? Derive the conditions with proper diagram. *(Ans.: Refer Section 2.6)* **(Dec. 2013)**

Syllabus Topic : Highly Reflecting Film

Q. 1 Write short note on : Highly Reflecting Film. *(Ans.: Refer Section 2.7)*

Solved Problems

Q. 1 Refer Ex. 2.8.1 **(May 2013, May 2014)**

Q. 2 Refer Ex. 2.8.3 **(Dec. 2012)**

Q. 3 Refer Ex. 2.8.4 **(Dec. 2012)**

Q. 4 Refer Ex. 2.8.9 **(Dec. 2016)**

Q. 5 Refer Ex. 2.8.11 **(May 2014)**

❑❑❑

CHAPTER 3

Module 1

Diffraction of Light

Syllabus

Diffraction of Light - Fraunhoffer diffraction at single slit, Fraunhoffer diffraction at double slit, Diffraction Grating, resolving power of a grating, dispersive power of a grating, Application of Diffraction : Determination of wavelength of light with a transmission grating.

3.1 Introduction

MU - Dec. 2015

- To all outward appearance light seems to travel in straight line path. It is a matter of common experience that the path of light entering a dark room through a hole in a window illuminated by sunlight is straight.
- Similarly if an opaque obstacle is placed in the path of light, a sharp shadow is cast on the screen. This shows that light travels in straight lines. This is called rectilinear motion of light.
- But it is also observed that when a beam of light passes through a small narrow opening or close to the edges of an obstacle, it spreads to some extent into the region of geometrical shadow also. This happens due to the bending of light waves round the corners of the obstacle or opening.
- The bending of light is very small when the dimensions of the slit or the obstacle are large as compared to the wavelength of light and it becomes much pronounced when the dimensions of obstacle or slit are comparable with the wavelength of light.

much pronounced when the dimensions of obstacle or slit are comparable with the wavelength of light.

- For example, when light waves diverging from a narrow slit S (Fig. 3.1.1) which is illuminated by a monochromatic source O, pass an obstacle AB with a straight edge A parallel to the slit, the geometrical shadow on the screen is never sharp.
- A small portion of light bends around the edge into geometrical shadow. Outside the shadow, parallel to its edge, several bright and comparatively dark bands are observed.

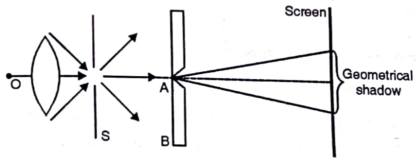


Fig. 3.1.1

- If an opaque object is placed between a point source of light and screen then, as per the rules of geometrical optics, a well defined and distinct shadow of the object should be obtained on the screen. No light should be observed in the region of shadow.
- But the pattern obtained on the screen is in the form of alternate dark and bright rings with a bright spot at the centre of the shadow.
- This spot cannot be explained on the basis of rectilinear propagation of light but can be explained due to the bending of light into the regions of shadow.
- The phenomenon of bending of light round the corners of an obstacle and spreading of light waves into the region of geometrical shadow of the obstacle is called **diffraction. Since very small obstacles are needed to create it, diffraction is not evident in daily life easily.**
- Fresnel explained the phenomenon of diffraction on the basis of Huygen's wave theory of light.

- The luminous border that surrounds the profile of a mountain just before the sun rises behind it; the light streaks that one sees while looking at a strong source of light with half shut eyes; the coloured spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are some examples of diffraction effects.

3.2 Types of Diffraction

MU - Dec. 2015

The phenomenon of diffraction is divided into two types :

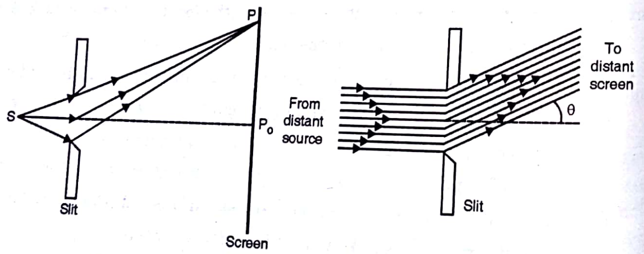
1. Fresnel diffraction
2. Fraunhoffer diffraction

1. **Fresnel diffraction**

Diffraction phenomenon in which the light source and the screen are finite distance from the diffracting aperture is termed as Fresnel diffraction. This is shown in Fig. 3.2.1(a).

2. **Fraunhoffer diffraction**

Diffraction phenomenon in which the light source and the screen are at infinite distance from the diffracting aperture is termed as Fraunhoffer diffraction. This is shown in Fig. 3.2.1(b).



(a) Fresnel Diffraction (b) Fraunhoffer Diffraction

Fig. 3.2.1

The points of difference between these two types of diffraction are as given below :

Sr. No.	Fresnel diffraction	Fraunhoffer diffraction
1.	Distance of source and screen from obstacle is finite.	Source and screen are at infinite distance from obstacle.
2.	No lenses are required to study diffraction in the laboratory.	Two biconvex lenses are required to study diffraction in laboratory.
3.	The wavefront incident on the aperture is either spherical or cylindrical.	The wavefront incident on aperture is plane.
4.	The diffracted wavefront is either spherical or cylindrical.	The diffracted wavefront is plane.
5.	The initial phase of secondary wavelets is different at different points in the plane of aperture.	The initial phase of secondary wavelets is same at all points in the plane of aperture.
6.	It has no importance in optical instruments.	It is very important in optical instruments.

3.3 Difference between Interference and Diffraction

Interference

1. Interference is the result of interaction of light coming from two different wavefronts originating from the same source.
2. Interference fringes in a particular pattern may or may not be of same width.
3. In interference all the bright fringes are of the same or uniform intensity.
4. The points of minimum intensity are perfectly dark. Hence the contrast between the fringes is good.

Diffraction

1. Diffraction is the result of interaction of light coming from different parts of the same source.
2. Diffraction fringes in a particular pattern are not of same width.
3. In diffraction the bright fringes are not of the same intensity.

4. The points of minimum intensity are not perfectly dark. Hence the contrast between fringes is not good.

Syllabus Topic : Fraunhoffer Diffraction at a Single Slit

> **Topics covered :** Fraunhoffer Diffraction at a Single Slit

3.4 Fraunhoffer Diffraction at a Single Slit

Step I

- Consider a single slit of width 'a' illuminated by monochromatic light of wavelength λ as shown in Fig. 3.4.1.
- The incident plane wavefront is diffracted by the slit and is then focussed on the screen by the lens L.
- Every point of the incident wavefront in the plane of the slit acts as a secondary source and sends out secondary waves in all directions.
- The secondary wavelets travelling normally to the slit are brought to focus at point P_o by the lens.
- All these secondary waves travel the same distance through the lens along the direction $\theta = 0$ and hence produce maximum intensity of light.

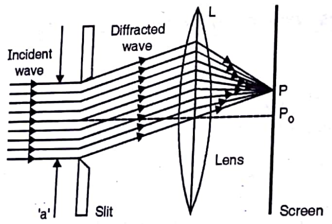


Fig. 3.4.1 : Fraunhoffer Diffraction at Single Slit

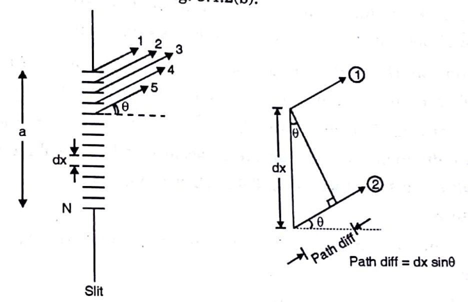
- Consider a point P on the screen at which the secondary wavelets travelling at an angle θ with the normal are focussed.
- The intensity of light at point P will depend upon the path difference between the secondary wavelets originating from the corresponding points of the wavefront.

Step II

- Consider the given slit to be divided into N parallel slits, each of width dx. See Fig. 3.4.2(a).

$$\therefore\ a = dx_1 + dx_2 + dx_3 + \ldots + dx_N \quad \ldots(3.4.1)$$

- One of such slits is shown in Fig. 3.4.2(b).



(a) **Fig. 3.4.2** (b)

- The path difference between the rays diffracted from its upper and lower edges is $dx \cdot \sin\theta$ and hence the corresponding phase difference between them is, (refer Fig. 3.4.2(b)).

$$\Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta \quad \ldots(3.4.2)$$

because

Path difference	Phase difference
λ	2π
$dx\sin\theta$	$\Delta\phi$

$$\therefore\ \Delta\phi = \frac{2\pi}{\lambda} dx\sin\theta$$

- The total path difference between the rays diffracted from the top and bottom edges of the slit of width 'a' is $a\sin\theta$ and the corresponding phase difference between them is,

$$\phi = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta \qquad ...(3.4.3)$$

- Each infinitesimal slit acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at point P on the screen.
- This disturbance we represent by a quantity called as phasor. (Concept of phasor is useful when we are dealing in that area where not only the magnitude but also the phase difference is significant.)
- To find out the total amplitude of disturbance at point P, we have to consider N phasors corresponding to N parallel infinitesimal slits.
- Thus at point P, N phasors with same amplitude, same frequency and same phase difference $\Delta\phi$ between the adjacent members combine to produce the resultant disturbance. Fig. 3.4.2 shows the vector addition of such N phasors.
- The resultant disturbance at P is represented by the arc AB of a circle with radius R.
- Let C be the centre of arc AB.

$$\therefore \quad AC = BC = R$$

Step III

- Join the chord AB and draw CD perpendicular to AB.
- ϕ is the phase difference between the infinitesimal vectors at the ends of the arc AB i.e. it is the phase difference between the rays coming from the top and bottom edges of the slit of width 'a'.

$$\therefore \quad \angle ACB = \phi$$

- Let us represent arc AB = E_m and chord AB = E_θ where $E_m \rightarrow$ Amplitude at the centre and $E_\theta \rightarrow$ resultant amplitude at P.
- From the geometry of Fig. 3.4.3, we have,

$$AD = DB$$

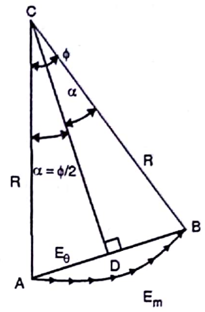


Fig. 3.4.3 : Phasor diagram

$$\angle ACD = \angle BCD$$

$$= \frac{\phi}{2} = \frac{\pi}{\lambda} \cdot a \sin\theta = \alpha \qquad ...(3.4.4)$$

$$\therefore \quad AD = \frac{E_\theta}{2} = R \cdot \sin\frac{\phi}{2}$$

$$\therefore \quad E_\theta = 2R \cdot \sin\frac{\phi}{2} \qquad ...(3.4.5)$$

- Using the relation, angle = $\frac{\text{arc}}{\text{radius}}$, we have,

$$\phi = \frac{E_m}{R} \qquad \therefore R = \frac{E_m}{\phi}$$

- Therefore Equation (3.4.5) gives,

$$E_\theta = 2 \cdot \frac{E_m}{\phi} \cdot \sin\frac{\phi}{2} = E_m \frac{\sin\phi/2}{\phi/2}$$

$$\therefore \quad \boxed{E_\theta = E_m \cdot \frac{\sin\alpha}{\alpha}} \qquad ...(3.4.6)$$

- This gives the amplitude of wave disturbance at point P where the rays diffracted at angle θ meet.
- If $I_\theta \rightarrow$ Resultant intensity of light at P and $I_m \rightarrow$ Maximum intensity at P_o then we have $I_\theta \propto E_\theta^2$ and $I_m \propto E_m^2$

- Hence squaring Equation (3.4.6) gives,

$$I_\theta = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad ...(3.4.7)$$

Step IV : Principal Maximum

- For E_θ to be maximum; all the phasors must be in phase i.e. α i.e. $\alpha = 0$

$$\therefore \quad \alpha = \frac{\pi}{\lambda} \cdot a \sin\theta = 0$$

$$\Rightarrow \quad \sin\theta = 0 \text{ or } \theta = 0$$

- Thus the maximum value of E_θ is E_m and this principal maximum is formed at $\theta = 0$. i.e. this maximum is formed by the parts of the secondary wavelets which travel normally to the slit. Hence it is produced at P_o.

Step V : Minimum Intensity

- The intensity at point P will be zero when $\sin\alpha = 0$.
- The values of α which satisfy this condition are

$$\alpha = \pm m\pi \qquad \text{where} \quad m = 1, 2, 3, 4, ...$$

- Thus the condition for minimum intensity at point P is

$$\alpha = \frac{\pi}{\lambda} \cdot a \sin\theta = \pm m\pi$$

$$\text{or} \qquad a \sin\theta = \pm m\lambda \qquad ...(3.4.8)$$

$$\text{where} \quad m = 1, 2, 3, ...$$

(m = 0 is not possible because then θ becomes zero which corresponds to principal maximum).

- Thus Equation (3.4.8) shows that we have points of minimum intensity on either side of the principal maximum (as there is $\pm$ sign on RHS).

Step VI : Secondary Maxima

- In addition to the principal maximum at $\theta = 0$, there are weak secondary maxima on either side of it. They lie approximately halfway between the two minima.
- Hence the secondary maxima can be obtained for

$$\alpha = \pm\left(m + \frac{1}{2}\right)\pi \qquad \text{where} \quad m = 1, 2, 3, ...$$

- Putting this condition in Equation (3.4.7) we have the relative intensity of secondary maxima as

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(m + \frac{1}{2}\right)\pi}{\left(m + \frac{1}{2}\right)\pi}\right]^2$$

- Putting m = 1, 2, 3, we have

$$\frac{I_\theta}{I_m} = 0.045; 0.016; 0.008; .....$$

- Thus the successive maxima decrease in intensity rapidly.
- The relative intensity distribution in single slit diffraction pattern is shown in Fig. 3.4.4.

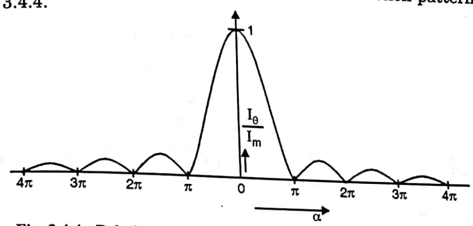


Fig. 3.4.4 : Relative intensity distribution in single slit diffraction

Ex. 3.4.1 : Calculate the angular position in the first minimum in Fraunhofer pattern of a slit 10^{-6} m wide if it is illuminated by light of wavelength 4000 Å.

Soln. :

Given : $m = 1$; $a = 10^{-6}$ m ; $\lambda = 4000 \times 10^{-10}$ m.

Formula : condition of minima $a \sin\theta = m\lambda$

$$\sin\theta = \frac{m\lambda}{a}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{1 \times 4000 \times 10^{-10}}{10^{-6}}\right)$$

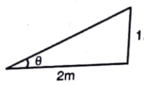
$$= \sin^{-1}(0.4) = \theta = \mathbf{23° \, 35'} \qquad \text{...Ans.}$$

Ex. 3.4.2 : A single slit of width 0.14 mm is illuminated normally with monochromatic light and diffraction bands are observed on a screen 2 m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light.

Soln. : The direction of minima is

$$a \sin\theta = n\lambda \quad ...(1)$$

From diagram as θ is very small, $\sin\theta \approx \theta \approx \tan\theta = \frac{y}{D}$



So, from Equation (1), we get,

$$a\left(\frac{y}{D}\right) = n\lambda \Rightarrow \lambda = \frac{ya}{nD}$$

Given, $n = 2$ (second order), $D = 2m$,

$y = 1.6 \times 10^{-2}$ m, $a = 0.14 \times 10^{-3}$ m

$$\lambda = \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2 \times 2}$$

$$= 5.6 \times 10^{-7} \text{ m} = \mathbf{5600 \text{ Å}}$$ ...Ans.

Syllabus Topic : Fraunhoffer Diffraction at a Double Slit

➤ ***Topics covered :*** *Fraunhoffer Diffraction at a Double Slit, Missing Orders in a Double Slit Diffraction Pattern*

3.5 Fraunhoffer Diffraction at a Double Slit

- Consider a beam of monochromatic light of wavelength λ incident normally on two narrow slits AB and CD as shown in Fig. 3.5.1. Let 'a' be the width of each slit and 'b' be the width of opaque space BC separating the two slits such that a and b are comparable.
- The diffracted light is focussed on the screen by a convex lens L.
- The diffraction pattern is found to consist of equally spaced interference maxima and minima in the region originally occupied by the central maximum in the single slit Fraunhoffer diffraction pattern.
- The central interference maximum possesses maximum intensity while the maxima on either side of it are of gradually decreasing intensity.
- In the region originally occupied by the secondary maxima of single slit diffraction pattern, faint interference maxima and minima are observed

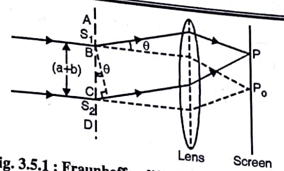


Fig. 3.5.1 : Fraunhoffer diffraction at double slit

Explanation

- When the plane wavefront reaches the plane of the slits, each point in the slit (AB and CD) sends secondary wavelets in all directions.
- From the theory of diffraction at a single slit, we see that the resultant amplitude due to all wavelets diffracted from each slit at an angle θ with the normal is given by,

$$E_\theta = E_m \frac{\sin\alpha}{\alpha} \quad ...(3.5.1)$$

where $E_m \rightarrow$ Maximum amplitude at P_o and $\alpha = \frac{\pi}{\lambda} a \sin\theta$ ...(3.5.2)

- This disturbance due to all waves from a single slit AB can be replaced by a single wave starting from midpoint S_1 of AB and producing a disturbance of amplitude E_θ at P (i.e. in a direction θ to the normal).
- Thus the resultant amplitude at point P will be due to the interference of two waves of same amplitude E_θ and having a phase difference that depends on the path difference between the two.
- From Fig. 3.5.1, the path difference between the waves from S_1 and S_2 at P is,

$$S_2K_1 = S_1S_2 \sin\theta = (a + b)\sin\theta$$

∴ The corresponding phase difference is $\frac{2\pi}{\lambda}(a + b)\sin\theta = 2\beta$ (say) ...(3.5.3)

- The resultant amplitude at P can be calculated by the method of vector addition and intensity at P is found to be proportional to square of amplitude.

Vector addition method

The resultant amplitude at P can be obtained from Fig. 3.5.2.

- QR and RS represent amplitude of two waves originating from S_1 and S_2 and angle ϕ as phase difference between them.
- From Fig. 3.5.2, we can write,

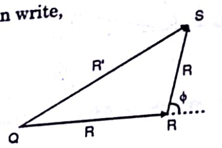


Fig. 3.5.2

(Using formula $|a+b| = \sqrt{a^2 + b^2 + 2ab\cos\phi}$)

$$QS^2 = QR^2 + RS^2 + 2\,(QR)\,(RS)\cos\phi$$

$$R'^2 = R^2 + R^2 + 2R\cdot R\cdot\cos\phi = 2R^2\,(1+\cos\phi)$$

$$R'^2 = 4R^2\cos^2\phi/2$$

- Substitute the following values from theory of single slit

$$R = A\,\frac{\sin\alpha}{\alpha} \text{ and}$$

$$\alpha = \frac{\phi}{2}$$

$$\frac{\pi\,(a+b)\sin\theta}{\lambda} = \beta$$

Hence, $$R'^2 = 4A^2\,\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

- Hence, the resultant intensity at the point P is given by,

$$I_\theta = 4E_m^2\,\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \quad \ldots(3.5.4)$$

$$= I_m\,\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Where, $A^2 = 4\,E_m^2 = \text{constant} = I_m$

- The term $\frac{I_m\cdot\sin^2\alpha}{\alpha^2}$ gives diffraction pattern like that of a single slit. **Hence it is called diffraction componment.** The factor $\cos^2\beta$ gives the interference pattern due to light waves of same amplitude from the two slits. **Hence it is called interference component.**

- It is observed that in case of **single slit diffraction,** the factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ gives a principal maximum at centre i.e. $\theta = 0$.
- On either side of this principal maximum alternate minima and secondary maxima of diminishing intensity are observed as shown in Fig. 3.5.3(a).

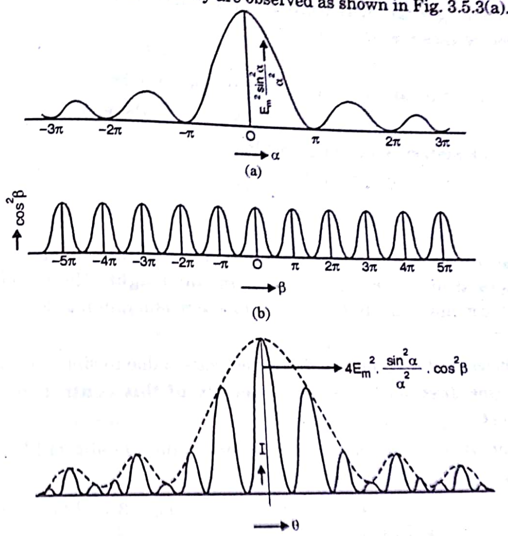


Fig. 3.5.3

- The angular positions of minima (for single slit) are given by,

$$\sin\alpha = 0, \quad \text{but } \alpha \neq 0 \text{ i.e.}$$

- We can write,

$$\alpha = \pm m\pi, \quad \text{where } m = 1, 2, 3, \ldots$$

$$\Rightarrow \quad \frac{\pi}{\lambda}\,a\sin\theta = \pm m\pi$$

$$\Rightarrow \quad a\sin\theta = m\lambda \quad \ldots(3.5.5)$$

- The positions of secondary maxima for single slit due to this term (α) may be written as

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \pm .... \text{ etc.}$$

- According to the interference term, $\cos^2 \beta$, the intensity will be maximum for double slit when,

$$\cos^2 \beta = 1 \quad \text{i.e.}$$

bright fringes are obtained in the directions given by,

$$\beta = \pm n\pi \quad \text{where } n = 0, 1, 2, 3, ...$$

- From Equation (3.5.3), we can write,

$$\frac{\pi (a + b) \sin \theta}{\lambda} = \pm n\pi$$

$$\Rightarrow (a + b) \sin \theta = \pm n\lambda$$

when $n = 0$, $\theta = 0$, i.e. **central maximum of interference pattern is observed along the direction of incident light.** The various maxima corresponding to n = 0, 1, 2, 3, ... are the zero-order, first order, second order maxima ...

- As the central maximum of diffraction pattern due to single slit also lies in the same direction, hence, **the intensity of this central maximum is highest.**
- The intensity of interference pattern **due to double slit** will be minimum, when,

$$\cos^2 \beta = 0 \quad \text{i.e.} \quad \beta = \pm (2n + 1) \pi/2$$

$$\Rightarrow \frac{\pi (a + b) \sin \theta}{\lambda} = (2n + 1) \pi/2$$

$$(a + b) \sin \theta = \pm (2n + 1) \lambda/2$$

where $n = 0, 1, 2, 3, ...$

- The above equation represents that for interference minima the path difference between the parallel diffracted rays originating from any pair of corresponding points within the two slits be odd multiple of $\lambda/2$.
- It can be proved that for small values of θ, the maxima and minima are equally spaced. So, $\cos^2 \beta$ term in Equation (3.5.4) is responsible for interference fringes.

- If the width of slit 'a' kept constant and 'b' is varied, the position of maxima and minima due to diffraction remain unaffected while those due to interference undergo a change.
- The variation of the diffraction term $E_m^2 \frac{\sin^2 \alpha}{\alpha^2}$ with α is shown in Fig. 3.5.3(a). The variation of interference term $\cos^2 \beta$ with β shown in Fig. 3.5.3(b). The resultant intensity distribution due to double slit is shown in Fig. 3.5.3(c).

Conclusions

The entire pattern due to double slit may be considered as consisting of interference fringes due to light from both slits. The intensities of these fringes being governed by diffraction occurring at the individual slit.

Effect of increasing the number of slits

- Instead of two slits, if we increase the number of slits to 3, 5, 6, ... to a large value, each slit being of same width 'a' and for equal separation 'b' between them, the diffraction patterns in each case can be obtained.
- It is found that with the increase in the number of slits, the narrowing of interference maxima takes place.
- With more number of slits, the sharpness of the principal maxima increases.
- **Effect of increasing the slit separation 'b' :** Keeping the slit width a constant, if the slit separation b is increased, then the fringe spacing decreases and the fringes become closer together. Hence, more interference maxima lies within the central maximum.
- **Effect of increasing the wavelength 'λ' :** If the wavelength of the monochromatic light incident on the slit increases, the field of view becomes broader and the fringes move further apart.
- **Effect of increasing the slit width 'a' :** When the width of the slit 'a' is increased, the envelope of the fringe-pattern changes and central peak becomes sharper. The fringe spacing remains unaffected because it depends on slit separation 'b', which is constant in this case. So, the number of interference maxima lies within the central diffraction maximum decreases.

3.6 Missing Orders in a Double Slit Diffraction Pattern

- On keeping the slit width 'a' constant, if the slit spacing 'b' is changed, the fringe width of interference maxima changes. On varying b, some orders of interference maxima are missing. The missing orders in double slit diffraction pattern depend upon the relative values of a and b.

- The direction of interference maxima is given by,

$$(a + b) \sin \theta = n\lambda \quad ...(3.6.1)$$

And the direction of diffraction minima is given by,

$$a \sin \theta = m\lambda \quad ...(3.6.2)$$

- For the same angle of θ, the value of a and b satisfies the Equations (3.6.2) and (3.6.1) simultaneously. Then the position of interference maxima and diffraction minima are same.
- Dividing Equation (3.6.1) by Equation (3.6.2), we get,

$$\frac{(a + b) \sin \theta}{a \sin \theta} = \frac{n\lambda}{m\lambda} \quad ...(3.6.3)$$

$$\frac{a+b}{a} = \frac{n}{m} \quad ...(3.6.4)$$

- **Condition 1 :** If a = b, then,

$$\frac{n}{m} = 2$$

$$n = 2m$$

If m = 1, 2, 3, ..., then n = 2, 4, 6, ...

So, second, fourth, sixth orders of interference maxima are missing in diffraction pattern, because these maxima will coincide with 1^{st}, 2^{nd}, 3^{rd} ... order diffraction minima due to single slit.

- **Condition 2 :** If 2a = b, then,

$$\frac{n}{m} = 3$$

$$n = 3m$$

If m = 1, 2, 3, ... then, n = 3, 6, 9, ...

So, 3^{rd}, 6^{th}, 9^{th} orders of interference maxima are missing in the diffraction pattern, because these maxima will coincide with 1^{st}, 2^{nd}, 3^{rd} ... order of diffraction minima.

- **Condition 3 :** If a + b = a, i.e. b = 0

The two slits are joined and all orders of interference maxima are missing and the diffraction pattern obtained is similar to that of single slit of width 2a.

Ex. 3.6.1 : In a double slit Fraunhoffer diffraction pattern the screen is 170 cm away the slits. The slit widths are 0.08 mm and they are 0.4 mm apart. Calculate the wavelength of light if the fringe spacing is 0.25 cm. Also deduce the missing order.

Soln. : The fringe spacing in the case of interference pattern written as,

$$\beta = \frac{D\lambda}{d} \qquad \Rightarrow \lambda = \frac{\beta d}{D}$$

Where d is the separation between the slits.

Given : d = 0.04 cm, β = 0.25 cm, d = 160 cm

$$\lambda = \frac{0.04 \times 0.25}{170}$$

$$= 5.88 \times 10^{-5} \text{cm} = 5880 \text{ A}^\circ$$

The condition of missing order, is

$$\frac{a+b}{a} = \frac{n}{m}$$

Given : b = 0.40 mm, a = 0.08 mm

So, $\frac{0.08 + 0.40}{0.08} = \frac{n}{m}$

n = 6 m, m = 1, 2, 3, ...

or, n = 6, 12, 18, ...

Hence, 6^{th}, 12^{th} and 18^{th} orders are missing. **...Ans.**

Syllabus Topic : Diffraction Grating

➤ **Topics covered :** Fraunhoffer Diffraction due to N Parallel Equidistant Slits Theory of Diffraction Grating, Condition for Absent Spectra, Highest Possible Orders ,

3.7 Fraunhoffer Diffraction due to N Parallel Equidistant Slits Theory of Diffraction Grating

MU - May 2014, Dec. 2014, Dec. 2016

- It is an arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces, is called **grating.**
- It is obtained by ruling equidistant parallel lines on a glass plate with the help of a fine diamond point.

- The lines act as opaque spaces and the incident light cannot pass through them. The space between the two lines is transparent to light and acts as a slit. The number of lines in a plane transmission grating is of the order of 15,000 to 20,000 per inch.
- The grating was first devised by Fraunhoffer.
- The spacing between the lines is of the order of wavelength of visible light. Hence an appreciable deviation of light is produced. For practical purposes, replicas of original grating are prepared.

Theory of plane transmission grating

Step I

- Consider a plane transmission grating placed perpendicular to the plane of the paper as shown in Fig. 3.7.1.
- Let N be the number of parallel slits each of width 'a' and separated by opaque space 'b'.
- Then the distance between the centres of the adjacent slits is $d = (a + b)$ and is known as the **grating element**.
- Let a plane wavefront of monochromatic light of wavelength λ be incident normally on the grating.
- When the incident wavefront is at the plane of the slits, every point in each slit acts as a source of secondary wavelets which spread out in all directions.
- The secondary wavelets travelling in the same direction as that of the incident light are brought to focus at point P_o on the screen by the lens L.

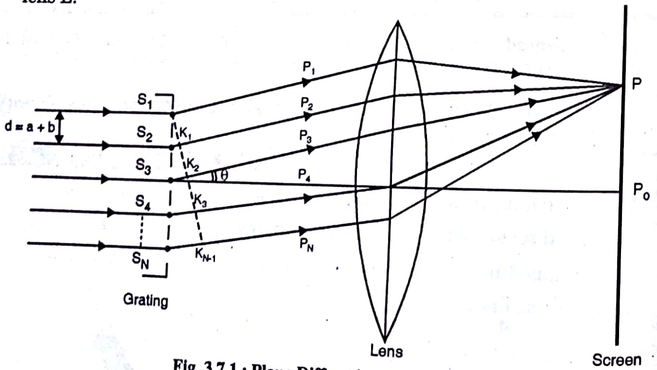


Fig. 3.7.1 : Plane Diffraction Grating

- The screen is placed at the focal plane of the lens.
- The secondary waves travelling in the direction θ with the incident light are focussed at point P on the screen.
- We have to find the resultant intensity of light at P.

Step II

- We have seen that the resultant amplitude of light from a single slit of width 'a' in a direction making an angle θ with the normal is given by,

$$E_\theta = E_m \cdot \frac{\sin \alpha}{\alpha} \qquad \text{...(3.7.1)}$$

Where $\alpha = \frac{\pi}{\lambda} \cdot a \sin \theta$ ...(3.7.2)

$E_m \rightarrow$ Maximum amplitude

- All the secondary wavelets in each slit can be replaced by a single wave of amplitude $E_m \cdot \frac{\sin \alpha}{\alpha}$ starting from the mid - point of the slit and travelling at an angle θ with the normal.
- Let $S_1, S_2, S_3, \ldots S_N$ be the midpoints of the N number of slits in the grating.
- We want to find the resultant effect of these N vibrations at P.

Step III

- From S_1, draw S_1K_{N-1}, perpendicular on the parallel paths of the diffracted rays.
- Then the path difference between S_1P_1 and S_2P_2 is given by,

$$S_2K_1 = (a + b) \sin \theta$$

and the corresponding phase difference is,

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot (a + b) \sin \theta$$

- Similarly, the path difference between S_3P_3 and S_1P_1 is given by,

$$S_3K_2 = 2 (a + b) \sin \theta$$

and the corresponding phase difference is,

$$\frac{2\pi}{\lambda} \cdot 2 (a + b) \sin \theta = 2\Delta\phi$$

- Thus the phase difference between successive vibrations is

$$\Delta\phi = \frac{2\pi}{\lambda} (a + b) \sin \theta$$

- Hence to find the resultant amplitude at P, we have to find the resultant of N vibrations of equal amplitude $E_m \cdot \frac{\sin\alpha}{\alpha}$, equal periods; but the phase difference between the adjacent vibrations being constant and equal to

$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \text{ say} \qquad ...(3.7.3)$$

Step IV

- The resultant amplitude of these N waves can be found out by vector addition method and is given by

$$E_\theta = E_m \frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta} \qquad ...(3.7.4)$$

- The corresponding resultant intensity of light at P is given by,

$$\mathbf{I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}} \qquad ...(3.7.5)$$

- The first factor $I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$ in Equation (3.7.5) gives the intensity distribution in the diffraction pattern due to single slit.
- The second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives intensity pattern due to N slits.
- Reader can try this take N = 1

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$$

- This is the formula for single slit Equation (3.4.7)
- For N = 2

$$I_\theta = 4E_m^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

(As $\sin 2\beta = 2\cos\beta.\sin\beta$)

This the formula for double slit (Refer Equation (3.4.7))

- Thus every slit gives rise to a diffracted beam whose intensity depends upon the slit width and these diffracted beams then interfere to produce the final diffraction pattern. This is shown in Fig. 3.7.2.

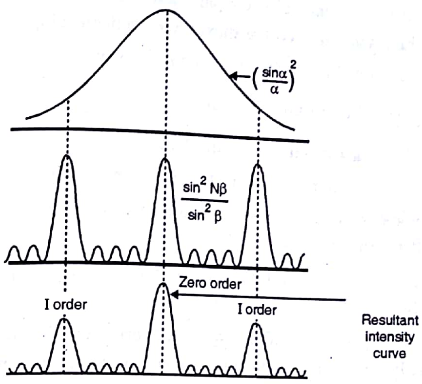


Fig. 3.7.2

Step V : Principal maxima

- The resultant intensity of light at P is given by Equation (3.7.5).
- The intensity is maximum when

$$\sin\beta = 0$$

or $\beta = \pm m\pi$ where m = 0, 1, 2, 3, ...

- But for these values of β, the term $\frac{\sin N\beta}{\sin\beta}$ in Equation (3.7.5) becomes indeterminate. We find its value as,

$$\lim_{\beta\to\pm m\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm m\pi}\frac{d(\sin N\beta)}{d(\sin\beta)} = \lim_{\beta\to\pm m\pi}\frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N\cdot m\pi}{\cos m\pi} = N$$

$$\therefore \lim_{\beta\to\pm m\pi}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

- Thus for $\beta = \pm m\pi$ the intensity of the principal maxima is,

$$I_\theta = N^2\cdot I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{which increases with increasing N.}$$

- The intensity of the central principal maximum is greatest while on either side of it, the intensities of other maxima go on decreasing.
- Thus the condition for principal maxima is $\beta = \pm m\pi$

$$\text{or} \quad \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\text{or} \quad \mathbf{(a+b)\sin\theta = \pm m\lambda} \quad \ldots \qquad \ldots(3.7.6)$$

where $m = 0, 1, 2, 3,$ .....m is called order number.

$m = 0$ corresponds to zero order maximum i.e. the central maximum which occurs at P_o i.e. in a direction $\theta = 0$.

- For $m = 1, 2, 3, \ldots$ we get the first order. Second order, ... principal maxima respectively.
- The $\pm$ sign in Equation (3.7.6) shows that there are two principal maxima of same order lying on either side of the central principal maximum.
- As number of lines on grating increases, (a + b) decreases. Because $a + b = \frac{1}{\text{Number of lines per inch}}$. Also $(a + b) \sin\theta = m\lambda$.
- $\therefore$ With increase in number of lines on grating order decreases(For given θ and λ).

Step VI

Minima

- From Equation (3.7.5) it is seen that the intensity of light at P is minimum when

$$\sin N\beta = 0 \qquad \text{but } \sin\beta \neq 0.$$

$$N\beta = \pm m\pi$$

where m can have all integral values except 0, N, 2N, ...nN; because for these values of m, $\sin\beta$ becomes zero and we get principal maxima.

- Hence putting for β from Equation (3.7.3) we have the condition for minima as

$$N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\text{or} \quad N(a+b)\sin\theta = \pm m\lambda \qquad .....(3.7.7)$$

where m can have all integral values except 0, N, 2N, ...nN.

Important Observation :

- **Effect of width of the ruled surface :** When the width of the ruled surface, i.e., N (a + b) increases, the width of principal maxima decreases i.e. it becomes sharper.
- **Effect of closeness of ruling :** When (a + b) is small, i.e., the lines on the grating are close together, the dispersive power will be large i.e., the angular spacing between maxima increases.
- **Effect of increasing the number of rulings on a grating :** When the number of rulings in a grating increases, the principal maxima become intense and sharp i.e., secondary maxima becomes weaker.

Ex. 3.7.1 : Monochromatic light of wavelength 6560 Å falls normally on a grating 2 cm. wide. The first order spectrum is produced at an angle of 18° 14′ from the normal. Calculate the total number of lines on the grating.

Soln. :

Given : $\lambda = 6560$ Å $= 6560 \times 10^{-8}$ cm, $m = 1$, $\theta = 18° 14'$, Width W = 2 cm

Formula : $(a + b) \sin\theta = m\lambda$

$$\therefore (a+b) = \frac{m\lambda}{\sin\theta} = \frac{1 \times 6560 \times 10^{-8}}{\sin(18° 14')}$$

$$= 2.097 \times 10^{-4} \text{ cm}$$

$$\therefore \text{Number of lines per cm} = \frac{1}{a+b} = \frac{1}{2.097 \times 10^{-4}}$$

$$= 4770$$

$\therefore$ Total number of lines on the grating.

= Number of lines/cm × width of grating

= 4770 × 2 = **9540** **...Ans.**

3.8 Condition for Absent Spectra

MU - May 2013, Dec. 2016

- The intensity of light due to a diffraction grating in a direction making an angle θ with the normal to the grating is given by,

$$I_\theta = I_m\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$$

$$\text{where} \quad \alpha = \frac{\pi}{\lambda} a \sin\theta$$

and $\beta = \frac{\pi}{\lambda}(a + b) \sin \theta$

- The factor $I_m \frac{\sin^2 \alpha}{\alpha^2}$ gives the intensity distribution due to a single slit while the factor $\frac{\sin^2 N\beta}{\sin^2 \beta}$ gives that due to the combined effect or N slits of the grating.
- Now the principal maxima in case of a grating are obtained in the directions given by,

$$(a + b) \sin \theta = m\lambda \quad ...(3.8.1)$$

m being the order of the spectrum.

- Also the minima in case of a single slit are obtained in the directions given by,

$$a \sin \theta = n\lambda \quad ...(3.8.2)$$

$$n = 1, 2, 3, ...$$

- If both the conditions (3.8.1) and (3.8.2) are satisfied simultaneously, a particular maximum of order m will be missing in the grating spectrum. Hence dividing (3.8.1) by (3.8.2) we have

$$\frac{a + b}{a} = \frac{m}{n} \quad ...(3.8.3)$$

which is the condition for absent spectra.

If a = b, then we have from Equation (3.8.3).

$$\frac{m}{n} = 2$$

$$\text{or} \quad m = 2n \qquad \text{where} \quad n = 1, 2, 3, ...$$

i.e. 2^{nd}, 4^{th}, 6^{th}, ... etc. order spectra will be absent.

i.e. the even order spectra will be absent if the width of ruling is equal to width of the slit.

3.9 Highest Possible Orders

Condition for maxima in diffraction grating is given by

$$(a + b) \sin \theta = \pm m\lambda$$

$$\text{or} \quad m = [(a + b) \sin \theta]/\lambda$$

if highest order m_{max} is needed then highest value of sin θ =1 can be used to find maximum value of m.

Alternately make sin θ as the subject of the formula and by substituting various values starting from the minimum, find the highest order till sin θ remains below one. The value of m which provides sin θ > 1 becomes not acceptable.

The following example will help us understand it.

Ex. 3.9.1 : How many orders will be visible if the wavelength of incident radiation is 5000 Å and the number of lines on the grating is 2620 to an inch ?

Soln. : We have, condition of maxima,

$$(a + b) \sin \theta = n\lambda$$

For maximum orders, θ = 90°

$$\text{So,} \quad (a + b) = n\lambda, \qquad n = \frac{(a + b)}{\lambda}$$

Given : $(a + b) = \frac{2.54}{2620}$ cm, $\lambda = 5000 \times 10^{-8}$ cm

$$n = \frac{2.54}{2620 \times 5000 \times 10^{-8}} = 19.4$$

So, maximum number of orders visible are 19. **...Ans.**

Syllabus Topic : Determination of Wavelength of Light with Plane Transmission Grating

> **Topics covered :** Determination of Wavelength of Light using Grating

3.10 Determination of Wavelength of Light using Grating

MU - Dec. 2012, May 2016

- The diffraction grating is often used in the laboratories for the determination of wavelength of light.
- The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in Fig. 3.10.1.

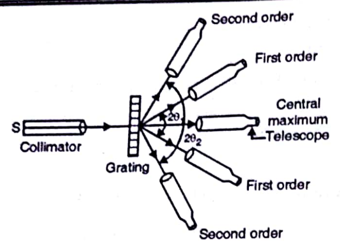


Fig 3.10.1

- The spectrometer is first adjusted for parallel rays. The grating is then placed on the prism table and adjusted for normal incidence.
- In the same direction as that of the incident light, the direct image of the slit or the zero order spectrum can be seen in the telescope.
- On either side of this direct image a symmetrical diffraction pattern consisting of different orders can be seen.
- The angle of diffraction θ for a particular order m of the spectrum is measured.
- The number of lines per inch of grating are written over it by the manufacturers.

Hence the grating element is,

$$(a+b) = \frac{1}{\text{Number of lines/cm}} = \frac{2.54}{\text{Number of lines/inch}}$$

- Thus using the equation

$$(a+b)\sin\theta = m\lambda$$

- The unknown wavelength λ can be calculated by putting the values of the grating element (a + b), the order m and the angle of diffraction θ .

Syllabus Topic : Dispersive Power of a Grating

➤ **Topics covered :** Dispersive Power of a Grating, Resolving Power of an Optical Instrument, Rayleigh's Criterion of Resolution, Rayleigh's Criterion of Resolution

3.11 Dispersive Power of a Grating

- The dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in the wavelength between the two spectral lines.

- It can also be defined as the rate of variation of angle of diffraction with wavelength.
- It is expressed as $\frac{d\theta}{d\lambda}$.
- The m^{th} order principal maximum for wavelength λ is given by,

$$(a+b)\sin\theta = m\lambda \quad ...(3.11.1)$$

where (a + b) = grating element

θ = angle of diffraction for order m.

- Differentiating Equation (3.11.1) w.r.t. λ we have,

$$(a+b)\cos\theta\frac{d\theta}{d\lambda} = m$$

$$\therefore \quad \frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} \quad ...(3.11.2)$$

which is the expression for the dispersive power of grating.

It is seen from Equation (3.11.2) that :

(i) The dispersive power is directly proportional to order m i.e. higher the order greater is the dispersive power.

(ii) The dispersive power is inversely proportional to the grating element. Hence smaller the grating element more widely spread is the spectrum.

(iii) The dispersive power is inversely proportional to cos θ. Larger the value of θ, smaller is the value of cos θ and higher is the dispersive power.

(iv) As the angle of diffraction for red light is greater than that for the violet in a given order spectrum, the dispersion in the red region is greater than that is violet region.

- If the linear spacing of two spectral lines of wavelength λ and λ + dλ is dx in the focal plane of telescope objective of the photographic plate then,

$$dx = f.d\theta$$

where f = focal length of the objective

- Therefore the linear dispersion,

$$\frac{dx}{d\lambda} = f\cdot\frac{d\theta}{d\lambda}$$

$$= f \cdot \frac{m}{(a+b)\cos\theta}$$

$$\text{or } dx = \frac{f \cdot m}{(a+b)\cos\theta} \cdot d\lambda$$

- The linear dispersion is useful in studying the photographs of a spectrum.

Ex. 3.11.1 : For a grating with grating element (a + b) = 18000 Å, calculate the dispersive powers in the first and third order spectra around λ = 5000 Å, assuming normal incidence.

Soln. :

We have,

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} = \frac{1}{\sqrt{\left(\frac{a+b}{m}\right)^2 - \lambda^2}}$$

Given : (a + b) = 18000 Å, λ = 5000 Å, m = 1

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{(18000)^2-(5000)^2}} = \frac{1}{10^3\sqrt{299}}$$

$$= 5.78 \times 10^{-5} \text{ rad/ Å}$$

Similarly for third order, n = 3

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{18000}{3}\right)^2 - (5000)^2}}$$

$$= \frac{1}{\sqrt{(6000)^2-(5000)^2}} = \frac{1}{10^3\sqrt{11}}$$

$$= 3.0 \times 10^{-4} \text{ rad/Å}$$ ...Ans.

3.12 Resolving Power of an Optical Instrument

- When the two objects are very near to each other and they are at very large distance from our eye, the eye may not be able to see them as separate.
- If we want to see them as separate and optical instruments such as telescope, microscope (for close objects) and instruments like prism, grating etc. for spectral lines are employed.
- When light from an object passes through an optical instrument we obtain a diffraction pattern with a bright central maximum and the other secondary maxima having minima in between.

- An optical instrument is said to be able to resolve two point object corresponding diffraction patterns are distinguishable from each other.
- The ability of the instrument to produce just separate diffraction patterns of two close objects is known as its **resolving power.**

3.13 Rayleigh's Criterion of Resolution

MU - May 2014, May 2015, Dec. 2016

- According to Rayleigh's criterion, two closely spaced point sources of light are said to be just resolved by an optical instrument only if the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa.
- In order to illustrate the criterion consider the resolution of two wavelengths λ_1 and λ_2 by an optical instrument. Fig. 3.13.1 shows the intensity curves of the diffraction patterns of the two wavelengths.
- As shown in Fig. 3.13.1(a) the principal maxima of the two wavelengths are widely separated. Hence the two wavelengths are well resolved.

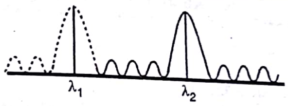
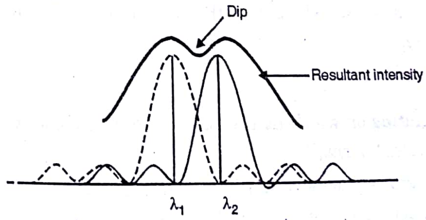


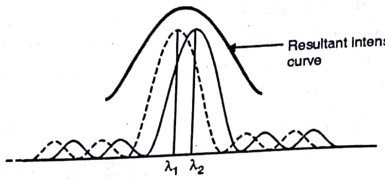
Fig. 3.13.1(a) : Objects Well Resolved

- Fig. 3.13.1(b) shows the intensity curves of two wavelengths whose difference is such that the principal maximum of one coincides with the first minimum of the other and vice versa.
- The resultant intensity curve shows a distinct dip in the middle indicating the presence of two wavelengths. This is the limiting case when the two wavelengths are just resolved.



(b) Objects Just Resolved

Fig. 3.13.1 (contd...)



(c) Objects not Resolved

Fig. 3.13.1

- It however; the two wavelengths are still closer, their principal maxima will be still nearer as shown in Fig. 3.13.1(c).
- The resultant intensity curve shows only one maximum at the centre. The two wavelengths cannot be seen as separate and hence are not resolved.
- Thus the spectral lines can be resolved only upto the limit expressed Rayleigh's Criterion.

Syllabus Topic : Resolving Power of a Grating

> **Topics covered :** Resolving Power of a Grating.

3.14 Resolving Power of a Grating

MU - Dec. 2013, May 2014, May 2015

- The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in the wavelength between this line and a neighbouring line such that the two lines appear to be just resolved.

$$\therefore \quad R.P = \frac{\lambda}{d\lambda}$$

where $\lambda \rightarrow$ wavelength of a line

and $\lambda + d\lambda \rightarrow$ wavelength of the next line that can just be seen as separate.

Step I

- Consider a grating on which light consisting of two close wavelengths λ and $(\lambda + d\lambda)$ is incident normally.
- Let $(a + b) \rightarrow$ grating element and $N \rightarrow$ Total number of lines on the grating.
- These two wavelengths will be just resolved if the principal maximum of $(\lambda + d\lambda)$ falls over the first minimum of λ.

- As per the Rayleigh's criterion, the first minimum of λ adjacent of m^{th} principal maximum should be along $(\theta_m + d\,\theta_m)$ for just resolution.

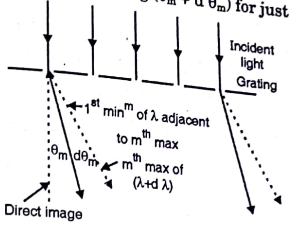


Fig. 3.14.1

Step II

- The m^{th} principal maximum of $(\lambda + d\lambda)$ along $(\theta_m + d\,\theta_m)$ is given by the equation.

$$(a + b) \sin(\theta_m + d\,\theta_m) = m(\lambda + d\lambda) \quad ...(3.14.1)$$

- The equation for minima for λ is

$$N(a + b) \sin\theta_m = m\lambda \quad .....(3.14.2)$$

where m has all integral values except 0, N, 2N, 3N, ...nN.

- Thus the first minimum of λ adjacent to m^{th} principal maximum can occur along $(\theta_m + d\,\theta_m)$ by putting m as $(m\,N + 1)$ in Equation (3.14.2).
- Hence the equation for first minimum of λ in the direction $(\theta_m + d\,\theta_m)$ will be given by,

$$N(a + b) \sin(\theta_m + d\,\theta_m) = (m.N + 1).\lambda \quad ...(3.14.3)$$

Step III

- Multiply Equation (3.14.1) by N

$$\therefore \; N(a + b) \sin(\theta_m + d\,\theta_m) = mN(\lambda + d\lambda) \quad ...(3.14.4)$$

- Therefore from Equations (3.14.3) and (3.14.4) we have,

$$(m \cdot N + 1) \cdot \lambda = mN(\lambda + d\lambda)$$

$$\therefore \; \lambda = m\,N \cdot d\lambda$$

$$\therefore \quad \frac{\lambda}{d\lambda} = m \cdot N$$

$\therefore$ R.P. of grating $= \frac{\lambda}{d\lambda} = m \cdot N$ .....(3.14.5)

Thus from Equation (3.14.5) we see that :

(i) The R.P. of grating increases with the order of spectrum

(ii) The R.P. increases with the increase in total number of lines on the grating.

(iii) The R.P. is independent of the grating element.

Ex. 3.14.1 : Calculate the minimum number of lines in a grating which will just resolve in the first order the lines whose wavelengths are 5890 Å and 5896 Å .

MU - Dec. 2014, 5 Marks

Soln. :

Given : $m = 1$, $\lambda_1 = 5890$ Å, $\lambda_2 = 5896$ Å

$\therefore$ $d\lambda = \lambda_2 - \lambda_1 = 6$ Å

$\therefore$ Mean wavelength $\lambda = 5893$ Å

Formula : R.P. $= mN = \frac{\lambda}{d\lambda}$

$$N = \frac{\lambda}{d\lambda} \cdot \frac{1}{n} = \frac{5893 \times 10^{-10}}{6 \times 10^{-10}} \times \frac{1}{1}$$

$$= \mathbf{983}$$ ...Ans.

3.15 Solved Problems

Problems on Single Slit

Ex. 3.15.1 : A slit of width 'a' is illuminated by white light. For what value of 'a' will the first minimum for red light fall at an angle 30° ? Wavelength of red light is 6500 Å.

Soln. :

Given : $\theta = 30°$, $m = 1$, $\lambda = 6500 \times 10^{-8}$ cm

To find : $a = ?$

Formula : For minima in single slit diffraction pattern.

$$a \sin \theta = m\lambda$$

For first minimum, $m = 1$

$$\therefore \quad a = \frac{\lambda}{\sin \theta} = \frac{6500 \times 10^{-8}}{\sin 30°} = \frac{6500 \times 10^{-8}}{0.5}$$

$$\therefore \quad a = 1.3 \times 10^{4} \times 10^{-8} \text{ cm}$$

$$\therefore \quad a = \mathbf{1.3 \times 10^{-4} \text{ cm.}}$$ ...Ans.

Ex. 3.15.2 : Find the half angular width of the central maximum in the Fraunhoffer diffraction pattern of a slit of width 12×10^{-5} cm, when illuminated by light of wavelength 6000 Å.

Soln. :

Given : $a = 12 \times 10^{-5}$ cm, $\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm

Formula : Half angular width of the first maxima is the angle made by the first minima with the normal to the slit.

$$\therefore \quad a \sin \theta = m\lambda$$

$m = 1$

$$\therefore \quad \sin \theta = \frac{m\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{12 \times 10^{-5}} = \frac{1}{2}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = \mathbf{30°}$$ ...Ans.

Ex. 3.15.3 : Plane waves of wavelength 6000 Å fall normally on a single slit of width 0.2 mm. Calculate the total angular width of the central maximum and also the linear width as observed on a screen placed 2 m. away.

Soln. :

Given : $a = 0.2$ mm $= 2 \times 10^{-2}$ cm, $A = 6000$ Å $= 6000 \times 10^{-8}$ cm

Distance of screen from the slit. (When lens is close to slit) is $f = 2$m $= 200$ cm.

Formula : $a \sin \theta = m\lambda$

Total angular width of the central maximum is twice the angle made by the first minimum with the normal to the slit.

$$\therefore a \sin \theta = m\lambda$$

$$\therefore \quad \sin \theta = \frac{m\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{2 \times 10^{-2}} = 3 \times 10^{-3} = 0.003$$

$$\therefore \quad \theta = \sin^{-1}(0.003) \quad \text{or } \theta = 10.3'$$

$\therefore$ Total angular width $= 2\theta = 20.6'$

The linear width of the central maximum on the screen is 2x where,

$$x = \frac{f \cdot \lambda}{d}$$

$$\therefore \text{ Linear width } = 2x = \frac{2f\lambda}{a} = \frac{2 \times 200 \times 6000 \times 10^{-8}}{2 \times 10^{-2}} = \mathbf{1.2 \text{ cm}}$$ ...Ans.

Ex. 3.15.4 : Calculate the wavelength of light whose first diffraction maximum in the diffraction pattern due to a single slit falls at $\theta = 30°$ and coincides with the first minimum for red light of wavelength 6500 Å.

Soln. :

According to diffraction theory of single slit, the first diffraction maximum is about half way between the first and second minima.

If λ_1 is the wavelength of light, so we can write the position of first maxima is,

$$a \sin\theta = \frac{3}{2}\lambda_1 = 1.5\lambda_1 \quad ...(1)$$

The position of first order (n = 1) minima for red light is,

$$a \sin\theta = n\lambda \Rightarrow a \sin\theta = 1.\lambda \quad ...(2)$$

Divide Equation (1) by (2), we get,

$$1.5\lambda_1 = \lambda \Rightarrow \lambda_1 = \frac{\lambda}{1.5}$$

$$\Rightarrow \lambda_1 = \frac{6500}{1.5} = 4333.3 \text{ Å} \quad \text{...Ans.}$$

Ex. 3.15.5 : A slit of width 0.3 mm is illuminated by a light of wavelength 5890 Å. A lens whose focal length is 40 cm forms a Fraunhofer diffraction pattern. Calculate the distance, between first dark and the next bright fringe form the axis.

MU - May 2015, 5 Marks

Soln. :

Given : $a = 0.3$ mm

$\lambda = 5890$ A°

$f = 40$ cm

To find : $x_2 - x_1$

where x_1 = Distance between centre of fringe pattern and the first order minima

x_2 = Distance between centre of fringe pattern and next bright maxima.

Use the following diagram.

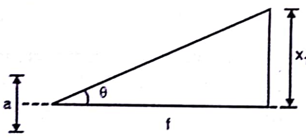


Fig. P. 3.15.5

As θ is very small, $\sin\theta \simeq \theta = \frac{x_1}{f}$

Condition for minima in single slit.

$$a \sin\theta = m\lambda$$

$$\therefore a\left(\frac{x_1}{f}\right) = m\lambda$$

$$\frac{x_1}{f} = \frac{\lambda}{a} \quad (\text{for } m = 1)$$

$$\therefore x_1 = \frac{f \times \lambda}{a} = \frac{5890 \times 10^{-8} \times 40}{0.3 \times 10^{-1}} = 0.0785 \text{ cm} \quad ...(1)$$

Similarly for maxima in single slit

$$a \sin\theta = \left(m + \frac{1}{2}\right)\lambda = \frac{3}{2}\lambda \quad (\text{for } m = 1)$$

Consider above mentioned diagram for maxima

we get $\sin\theta \simeq \theta = \frac{x_2}{f}$

$$\therefore a \times \frac{x_2}{f} = \frac{3}{2}\lambda$$

$$\therefore \frac{x_2}{f} = \frac{3\lambda}{2a}$$

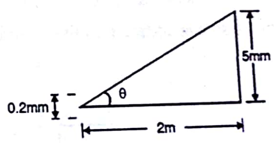
$$x_2 = \frac{3}{2} \times \frac{5890 \times 10^{-8} \times 40}{0.03} = 0.1178 \text{ cm} \quad ...(2)$$

$\therefore$ Dist between the first dark and next bright fringes

$$x_2 - x_1 = 0.0393 \text{ cm} \quad \text{...Ans.}$$

Ex. 3.15.6 : In Fraunhoffer diffration due to a single slit of width 0.2 mm, a screen is placed 2m away from the lens, to obtain the pattern. The first minima lie 5mm on either side of central maximum. Compute the wavelength of light.

Soln. :



here $\tan\theta = \frac{5\text{mm}}{2\text{m}}$

$\therefore$ θ is very small

$$\therefore \tan\theta = \theta \simeq \sin\theta$$

$$\sin\theta = \frac{5\text{mm}}{2\text{m}} = 2.5 \times 10^{-3}$$

$\therefore$ For single slit

$$a \sin\theta = n\lambda$$

here a = slit width = 0.2 mm = 0.2×10^{-3} m

n = order = 1

λ = wavelength

$$\therefore \quad \lambda = \frac{a \sin\theta}{n} = \frac{a}{n}(2.5 \times 10^{-3})$$

$$= 0.2 \times 10^{-3} \times 2.5 \times 10^{-3}$$

$$= 5 \times 10^{-7} \text{ m} = 5000 \text{ Å}$$

...Ans.

Ex. 3.15.7 : Calculate the angle at which the first dark band and next bright band are observed in Fraunhoffer's diffraction pattern due to a slit 0.30 mm wide. The wavelength of the light used in 5890 Å.

Soln. : For minima, $a \sin \theta = n\lambda$

$$\sin \theta = \frac{\lambda}{a}, \qquad (n = 1, \text{ for first order})$$

$$= \frac{5890 \times 10^{-8}}{0.03} = 0.00196$$

$$\theta = \sin^{-1}(0.00196) = 0.112°$$

First bright band on either side of the central maximum is given by

$$a \sin \theta' = \frac{3}{2} \lambda$$

$$\sin \theta' = 1.5 \frac{\lambda}{a} = 1.5 \times 0.00196$$

$$\sin \theta' = 0.00294$$

$$\theta' = \sin^{-1}(0.00294) = \mathbf{0.168°}$$

...Ans.

Problems on Double Slit

Ex. 3.15.8 : Reduce the missing order for a double slit fraunhoffer diffraction pattern if the slit widths are 0.16 mm and are 0.8 mm apart.

Soln. : Using condition for missing order in double slit case

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\therefore \quad \frac{0.16 + 0.8}{0.16} = \frac{n}{m}$$

$$n = 6 \text{ m}$$

$$\therefore \quad \text{for } m = 1, 2, 3....$$

n = 6, 12, 18.... etc will be missing in the diffraction pattern.

Ex. 3.15.9 : Monochromatic light from a helium neon laser (λ = 6328 Å) is incident normally on a diffraction grating 6000 lines/cm. Find the angles at which one would observe the first order maximum, second order maximum etc.

Soln. : We have, the angular position of maxima is,

$$(a + b) \sin \theta = n\lambda$$

Given :

$$(a + b) = \frac{1 \text{ cm}}{6000} = \frac{1}{6} \times 10^{-5} \text{ m}$$

$$\lambda = 6328 \times 10^{-10} \text{ m}$$

So,

$$\sin \theta_1 = \frac{n\lambda}{(a+b)}$$

$$= \frac{1 \times 6328 \times 10^{-10}}{\frac{1}{6} \times 10^{-5}} \qquad (\because n = 1 \text{ for first order})$$

$$= 0.37968$$

$$\theta_1 = \sin^{-1}(0.37968) = 22.3°$$

For second order, $n = 2$, we have,

$$\sin \theta_2 = 2 \times 0.37968 = 0.75936$$

$$\theta_2 = \sin^{-1}(0.75936) = \mathbf{49.408°}$$

...Ans.

Problems on Basics of Diffraction grating

Ex. 3.15.10 : In an experiment with grating, third order spectral line of wavelength λ coincides with the fourth order spectral line of wavelength 4992 Å. Calculate the value of λ.

Soln. :

Using the grating Equation

$$n\lambda = (a + b) \sin \theta$$

for order $n = 3$ and for $\lambda_1 = \lambda$

$$3\lambda_1 = (a + b) \sin \theta$$

For order $n = 4$ and $\lambda_2 = 4992$ Å

$$4\lambda_2 = (a + b) \sin \theta$$

as they coincides

$$3\lambda_1 = 4\lambda_2$$

$$\therefore \quad \lambda_1 = \frac{4\lambda_2}{3} = \frac{4 \times 4992}{3} = 6656 \text{ Å}$$

...Ans.

Ex. 3.15.11 : What is the longest wavelength that can be observed in the fourth order for a transmission grating having 5000 lines per cm ?

Soln. :

Given : $m = 4, (a + b) = \frac{1}{5000}$ cm

To find : $(\lambda)_{max} = ?$

Formula : $(a + b) \sin \theta = m\lambda$

For the longest wavelength, $\sin \theta = 1$

$$\therefore \lambda = \frac{a+b}{m} = \frac{1}{5000 \times 4} = 5 \times 10^{-5} \text{ cm} = 5000 \times 10^{-8} \text{ cm.}$$

$\therefore$ The longest wavelength = **5000 Å** ...**Ans.**

Ex. 3.15.12 : In a plane transmission grating the angle of diffraction for second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines/cm of the grating surface. **MU - May 2013, 3 Marks, May 2014, 5 Marks**

Soln. :

Given : $\lambda = 5 \times 10^{-5}$ cm, $\theta = 30°$, $n = 2$

$$\therefore n\lambda = (a+b) \sin \theta$$

$$\therefore a + b = \frac{n\lambda}{\sin \theta} = \frac{2 \times 5 \times 10^{-5}}{\sin (30°)} = 2 \times 10^{-4}$$

$\therefore$ No. of lines per cm $= \frac{1}{a+b} = 5000$ ...**Ans.**

Problems on Order of Spectrum

Ex. 3.15.13 : A plane transmission grating has 5000 lines/cm.

(a) Find out the highest order of spectrum observed if incident light has $\lambda = 6000$ Å

(b) If the opaque spaces between the slits are exactly 2 times the transparent space and the maximum order observed is three, find which order of spectra will be absent.

Soln. :

Given : $(a + b) = \frac{1}{5000}$ cm.

(a) **Formula :** $(a + b) \sin \theta = m\lambda$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ m}$$

Maximum value of $\sin \theta = 1$

$$\therefore (m)_{max} = \frac{a+b}{\lambda} = \frac{1}{5000 \times 6000 \times 10^{-8}} = 3.33$$

$\therefore$ Third order is the highest order visible.

(b) The condition for the spectrum of order n to be absent is,

$$\frac{a+b}{a} = \frac{m}{n}$$

Given : $b = 2a$

$$\frac{a+2a}{a} = \frac{m}{n}$$

$$\therefore 3 = \frac{m}{n} \qquad \therefore m = 3n$$

$\therefore$ **Third, sixth and ninth order spectrum will be absent.** ...**Ans.**

Ex. 3.15.14 : How many orders will be observed by a grating having 4000 lines per cm, if it is illuminated by light of wavelength in the range 5000 Å to 7500 Å.

Soln. : We have, condition of maxima,

$$(a + b) \sin \theta = n\lambda$$

For maximum order visible in grating,

$$\theta = 90°$$

So, $n_{max} = \frac{(a+b)}{\lambda}$

Given, $(a + b) = \frac{1}{4000}$

$$\therefore n_{max} = \frac{1}{4000\,\lambda}$$

For $\lambda = 5000 \times 10^{-8}$ cm, $n_{max} = \frac{1}{4000 \times 5000 \times 10^{-8}} = \frac{10}{2} = 5$

For $\lambda = 7500 \times 10^{-8}$ cm, $n_{max} = \frac{1}{4000 \times 7500 \times 10^{-8}} = \frac{10}{3} = 3.3$

So, in the wavelength range 5000 Å to 7500 Å, the observed number of orders range between 3 to 5.

Ex. 3.15.15 : What other spectral lines in the range 4000 Å to 7000 Å will coincide with the fifth order line of 6000 Å in a grating spectrum ?

Soln. : If the line of unknown wavelength λ_1 in the order n_1 coincides with the line of known wavelength (λ) in the n^{th} order provided,

$$(a + b) \sin \theta = n\lambda = n_1 \lambda_1$$

$$\Rightarrow \lambda_1 = \frac{n\lambda}{n_1}$$

$\Rightarrow$ Given, $n = 5$, $\lambda = 6000$ Å and λ_1 lies between 4000 Å and 7000 Å

So, $$\lambda_1 = \frac{5 \times 6000}{n_1} = \frac{30000 \text{ Å}}{n_1}$$

If $n_1 = 4$, $\lambda_1 = \frac{30000}{4} = 7500$ Å

If $n_1 = 5$, $\lambda_1 = \frac{30000}{5} = 6000$ Å

If $n_1 = 6$, $\lambda_1 = \frac{30000}{6} = 5000$ Å

If $n_1 = 7$, $\lambda_1 = \frac{30000}{7} = 4285.7$ Å

If $n_1 = 8$, $\lambda_1 = \frac{30000}{8} = 3750$ Å

So, in the grating spectrum the spectral lines with wavelengths 5000 Å and 4285.7 Å in the range 4000 Å to 7000 Å, will coincide with fifth order line of 6000 Å.

Ex. 3.15.16 : What is the highest order spectrum that is visible with light of wavelength 6000 Å by means of a diffraction grating having 5000 lines per cm ?

MU - Dec. 2013, 5 Marks

Soln. :

Given : $\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m $= 6000 \times 10^{-8}$ cm

Number of lines per cm = 5000

$\therefore$ Grating element $(a + b) = \frac{1}{5000}$ cm

The grating equation is $(a + b) \sin \theta = m\lambda$

m is maximum when $\sin \theta = 1$

$$\therefore \quad m_{max} = \frac{a+b}{\lambda} = \frac{1}{5000} \times \frac{1}{6000 \times 10^{-8}}$$

$$= 3.33$$

$\therefore$ Maximum order of spectrum = 3

...Ans.

Ex. 3.15.17 : Show that only first order of spectra is possible if the width of the grating element is less than twice the wavelength of light.

Soln. : For grating, the condition of maximum $(a + b) \sin \theta = n \lambda$,

Given $(a + b) < 2\lambda$

For maximum order $\sin \theta = \sin 90° = 1$

So, n must be less than 2 i.e. only first order is possible.

Ex. 3.15.18 : Calculate the maximum order of diffraction maxima seen from a plane diffraction grating having 5500 lines per cm if light of wavelength 5896 Å falls normally on it.

MU - May 2015. 5 Marks

Soln. :

Given :

Number of lines per cm = 5500

$\lambda = 5896$ A°

$$\therefore \quad a + b = \frac{1}{5500}$$

For maximum order take $\sin \theta = 1$ in

$$n\lambda = (a + b) \sin \theta$$

$$\therefore \quad n = \frac{a+b}{\lambda} \times \frac{1}{\sin \theta}$$

$$= \frac{1}{5500} \times \frac{1}{5896 \times 10^{-8}} \times \frac{1}{1} = 3$$

Problems on Angular Separation in the Spectrum

Ex. 3.15.19 : The wavelengths of visible spectrum are approximately 4000 Å to 7000 Å. Find the angular breadth of the first order visible spectrum produced by a plane grating having 6000 lines per cm, when light is incident normally on the grating.

Soln. :

Given : $\lambda_1 = 4000$ Å $= 4000 \times 10^{-10}$ m,

$\lambda_2 = 7000$ Å $= 7000 \times 10^{-10}$ m,

$a + b = \frac{1}{6000}$ cm ; m = 1

Formula : $(a + b) \sin \theta = m\lambda$

Hence, for λ, we have

$$\sin \theta_1 = \frac{1 \times 4000 \times 10^{-10} \times 6000 \times 10^{+2}}{1} = 0.24$$

$$\therefore \quad \theta_1 = \sin^{-1}(0.24) = 13° \, 53'$$

For λ_2 we have
$$\sin \theta_2 = \frac{1 \times 7000 \times 10^{-10} \times 6000 \times 10^{2}}{1} = 0.42$$

$$\therefore \quad \theta_2 = \sin^{-1}(0.42) = 24° \, 50'$$

Angular breadth of the first order visible spectrum

$$= \theta_2 - \theta_1 = 24° \, 50' - 13° \, 53' = \mathbf{10° \, 57'}$$ ...Ans.

Ex. 3.15.20 : A grating with 15000 rulings per inch is illuminated normally with white light extending from 4000 Å – 7000 Å Show that only the first order spectrum is isolated; but the second and third order overlap.

Soln. :

Given : $(a + b) = \frac{2.54}{15000}$ cm, $\lambda_1 = 4000$ Å $= 4000 \times 10^{-8}$ cm,

$\lambda_2 = 7000$ Å $= 7000 \times 10^{-8}$ cm

Formula : $(a + b) \sin \theta = m\lambda$

$$\sin \theta = \frac{m\lambda}{(a + b)} \qquad \text{For } \lambda = 4000 \times 10^{-8} \text{ cm,}$$

We have
$$\sin \theta = \frac{m \times 4000 \times 10^{-8}}{2.54/15000} = 0.236 \text{ m}$$

$$\therefore \quad \theta = \sin^{-1}(0.236 \text{ m})$$

m = 1, m = 2, m = 3 correspond to the first, second and third order spectra respectively.

$\therefore \theta \approx 13°, 28°, 45°$ for these orders.

For $\lambda = 7000 \times 10^{-8}$ cm,

we have
$$\sin \theta = \frac{m \times 7000 \times 10^{-8}}{2.54/15000} = 0.413 \text{ m}$$

$$\therefore \quad \theta' = \sin^{-1}(0.413 \text{ m})$$

Hence for m = 1, 2, 3, we have $\theta' \approx 24°, 56°, ...$

(ϕ for m = 3, $\theta = \infty$)

Thus the angular positions for 4000 Å and 7000 Å lines in the various order are obtained as follows :

I order	13°	24°	Isolated
II order	28°	56°	
III order	45°	....	Overlap

Ex. 3.15.21 : The visible spectrum ranges from 4000A° to 7000A°. Find the angular breadth of the first order visible spectrum produced by a plane waling having 6000 lines/cm when light is incident normally on the grating. **MU - Dec. 2015, 5 Marks**

Soln. :

Given : Range of wavelength 4000A° to 7000A°

Number of lines / cm = 6000

$$\therefore \quad (a + b) = \frac{1}{6000} \qquad \text{...(1)}$$

Formula $n\lambda = (a + b) \sin \theta$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{n\lambda}{a + b}\right)$$

$\therefore$ For $\lambda_1 = 4000$ A°

$$\theta_1 = \sin^{-1}(4000 \times 10^{-8} \times 6000)$$

$$= 13.89° \qquad \text{...(2)}$$

For $\lambda_2 = 7000$ A°

$$\theta_2 = \sin^{-1}(7000 \times 10^{-8} \times 6000)$$

$$= 24.84° \qquad \text{...(3)}$$

$\therefore$ Angular breadth, 15 = 24.84 – 13.89

= 10.95° ...Ans.

Ex. 3.15.22 : A grating has 6000 lines/cm. Find the angular separation of two yellow lines of mercury of wavelengths 5770 A.U. and 5791 A.U. in the second order.

Soln. :

Given : $\lambda_1 = 5770$ Å $= 5770 \times 10^{-10}$ m, $\lambda_2 = 5791$ Å $= 5791 \times 10^{-10}$ m

$$a + b = \frac{1}{6000} \text{ cm} = \frac{1}{6000} \times 10^{-2} \text{ m}, \quad m = 2$$

Formula : $(a + b) \sin \theta = m\lambda$

Hence for λ_1 we have,

$$\sin\theta_1 = \frac{2 \times 5770 \times 10^{-10} \times 6000 \times 10^2}{1} = 0.6924$$

$$\therefore \quad \theta_1 = \sin^{-1}(0.69)$$

or $\theta_1 = 43°49'$

For λ_2, we have, $$\sin\theta_2 = \frac{2 \times 5791 \times 10^{-10} \times 6000 \times 10^2}{1}$$

$$\sin\theta_2 = 0.6949 \qquad \theta_2 = \sin^{-1}(0.6949)$$

$$\theta_2 = 44°$$

$\therefore$ Angular separation $= \theta_2 - \theta_1 = \mathbf{11'}$ **...Ans.**

Problems on Grating Element and Number of Lines on Grating

Ex. 3.15.23 : *A diffraction grating used at normal incidence gives a yellow line (λ = 6000 Å) in a certain spectral order superimposed on a blue line (λ = 4800 Å) of next higher order. If the angle of diffraction is $\sin^{-1}\left(\frac{3}{4}\right)$, calculate the grating element.*

MU - Dec. 2015, May 2016, 5 Marks

Soln. : We have the direction of principal maxima is

$$(a + b)\sin\theta = n\lambda$$

For yellow line (λ_1 = Å), we can write,

$$(a + b)\sin\theta = n\lambda_1 \qquad ...(1)$$

For blue line (λ_2 = 4800 Å), we can write,

$$(a + b)\sin\theta = (n + 1)\lambda_2 \qquad ...(2)$$

Comparing Equations (1) and (2), we get,

$$n\lambda_1 = (n + 1)\lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4800 \text{ Å}}{(6000 - 4800) \text{ Å}} = 4$$

Given : $$\theta = \sin^{-1}\left(\frac{3}{4}\right) \Rightarrow \sin\theta = \left(\frac{3}{4}\right)$$

Put these values in Equation (1), we get,

$$(a + b) = \frac{n\lambda_1}{\sin\theta} = \frac{4 \times 6 \times 10^{-7}}{3/4} = \mathbf{3.2 \times 10^{-6}\ m}$$ **...Ans.**

Ex. 3.15.24 : A diffraction grating used at normal incidence gives a line 5400 Å in certain order superimposed on another line 4050 Å of the next higher order. If the angle of diffraction is 30°, how many lines/cm are there on the grating ?

Soln. :

Formula $(a + b)\sin\theta = n\lambda$

As n and λ are inv.

$$\therefore \quad (a + b)\sin\theta = n\lambda_1 \quad \text{for } \lambda_1 \qquad ...(1)$$

$$\therefore \quad (a + b)\sin\theta = (n + 1)\lambda_2 \quad \text{for } \lambda_2 \qquad ...(2)$$

$$\therefore \quad n\lambda_1 = (n + 1)\lambda_2$$

$$n(5400) = (n + 1)(4050)$$

$$\therefore \quad \frac{5400}{4050} = \frac{n+1}{n}$$

$$\therefore \quad n = 3 \qquad ...(3)$$

Using Equations (1) and (3)

$$(a + b)\sin 30° = 3 \times (5400 \times 10^{-8})$$

$$\therefore \quad a + b = \frac{3 \times 5400 \times 10^{-8}}{\sin 30} = 3.24 \times 10^{-4}$$

$\therefore$ No. of lines per cm = 3086.41 = 3086 **...Ans.**

Problems on Missing order on Spectrum of Grating

Ex. 3.15.25 : What particular spectra would be absent when the width of the opacity is double than that of the transparency in grating ?

Soln. :

As discussed in section 3.7, using interference component and principal maxima

$$(a + b)\sin\theta = m\lambda \qquad ...(1)$$

And minima in case of a diffraction component

$$a\sin\theta = n\lambda \qquad ...(2)$$

If both the conditions are satisfied simultaneously, a particular maximum order m will be missing in the grating spectrum.

$$\therefore \quad \frac{a+b}{a} = \frac{m}{n} \qquad ...(3)$$

This is the condition for absent spectra.

Now width of opacity is double that of transparency

$$\therefore \quad b = 2a \qquad ...(4)$$

$$\therefore \quad \frac{a+b}{a} = \frac{2a+a}{a} = 3 = \frac{m}{n}$$

$$\therefore \quad m = 3n$$

As n represent minima $\therefore$ for $n = 1, 2, 3, \ldots$

$m = 3(1), 3(2), 3(3), \ldots$

i.e. $= 3, 6, 9, \ldots$ Will be found absent.

Ex. 3.15.26 : In a plane transmission grating, the angle of diffraction for the second order principal maxima for wavelength 5×10^{-5} cm is 35°. Calculate the number of lines/cm on diffraction grating. **MU - Dec. 2016, 5 Marks**

Soln. :

Given : $\theta = 35°$ $\quad \lambda = 5 \times 10^{-5}$ cm $\quad n = 2$

To find : No. of lines / cm

Formula

$$n\lambda = (a+b)\sin\theta$$

$$2 \times 5 \times 10^{-5} = (a+b)\sin 35°$$

$$\therefore \quad (a+b) = \frac{2 \times 5 \times 10^{-5}}{\sin 35}$$

$$\therefore \text{ number of lines per cm} = \frac{1}{a+b} = \frac{\sin 35}{2 \times 5 \times 10^{-5}}$$

$$= \mathbf{5735}$$

...Ans.

Problems on Dispersive Power

Ex. 3.15.27 : For a grating with grating element (a + b) = 18000 Å, calculate the dispersive powers in the first and third order spectra around λ = 5000 Å, assuming normal incidence.

Soln. :

We have, $$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} = \frac{1}{\sqrt{\left(\frac{a+b}{m}\right)^2 - \lambda^2}}$$

Given : $(a+b) = 18000$ Å, $\quad \lambda = 5000$ Å, $\quad m = 1$

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{(18000)^2 - (5000)^2}} = \frac{1}{10^3\sqrt{299}}$$

$$= 5.78 \times 10^{-5} \text{ rad/ Å}$$

Similarly for third order, n = 3

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{18000}{3}\right)^2 - (5000)^2}}$$

$$= \frac{1}{\sqrt{(6000)^2 - (5000)^2}} = \frac{1}{10^3\sqrt{11}}$$

$$= \mathbf{3.0 \times 10^{-4} \text{ rad/ Å}}$$

...Ans.

Ex. 3.15.28 : A diffraction grating having 15000 lines to an inch is used to photograph a spectrum. Calculate the angular dispersion in the second order spectrum of wavelength region 5.9×10^{-5} cm. If the focal length of convex lens is 25 cm, calculate the linear dispersion in the spectrograph and also the separation between the spectral lines 5890 Å and 5896 Å in second order.

Soln. : We have, directions of maxima,

$$(a+b)\sin\theta = m\lambda \qquad \Rightarrow \sin\theta = \frac{m\lambda}{(a+b)}$$

Given : $(a+b) = \frac{2.54}{15000} = 17 \times 10^{-5}$ cm, $m = 2$ and $\lambda = 5.9 \times 10^{-5}$ cm

$$\therefore \quad \sin\theta = \frac{m\lambda}{(a+b)} = \frac{2 \times 5.9 \times 10^{-5}}{17 \times 10^{-5}} = 0.69$$

Hence, $$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(0.69)^2} = 0.72$$

So, the angular dispersion,

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} = \frac{2}{17 \times 10^{-5} \times 0.72}$$

$$= 1.6 \times 10^{4} \text{ rad/cm} = 1.6 \times 10^{4} \text{ rad/ Å}$$

The linear dispersion is given by,

$$\frac{dx}{d\lambda} = f \cdot \frac{d\theta}{d\lambda}$$

Given, f = 25 cm

$$\frac{dx}{d\lambda} = 25 \times 1.6 \times 10^{-4} \text{ rad/Å} = 4 \times 10^{-3} \text{ cm/Å}$$

The separation between the spectral lines 5896 Å and 5890 Å is,

$$dx = 4 \times 10^{-3} \times d\lambda = 4 \times 10^{-3} \times (5896 - 5890) \text{ Å}$$

$$\therefore \quad dx = \mathbf{2.4 \times 10^{-2} \text{ cm}}$$

...Ans.

Ex. 3.15.29 : A diffraction grating which has 4000 lines per cm is used at normal incidence. Calculate the dispersive power of the grating in the third order spectrum of wavelength region 5000 Å.

Soln. :

Given : Number of lines per cm = 4000

$$\therefore \quad (a+b) = \frac{1}{4000}\text{ cm} \qquad m = 3$$

$$\lambda = 5000\text{ Å} = 5000 \times 10^{-8}\text{ cm}$$

Formula : Dispersive power = $\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta}$

The grating formula for order m is $(a+b)\sin\theta = m\lambda$

$$\therefore \quad \sin\theta = \frac{m\lambda}{(a+b)} = \frac{3 \times 5000 \times 10^{-8} \times 4000}{1}$$

$$= 0.6$$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = = \sqrt{1-(0.6)^2}$$

$$= 0.8$$

$$\therefore \quad \frac{d\theta}{d\lambda} = \text{D.P.} = \frac{m}{(a+b)\cos\theta}$$

$$\therefore \quad \text{D.P.} = \frac{d\theta}{d\lambda}$$

$$= \frac{3 \times 4000}{0.8} = 15{,}000$$ ...Ans.

Problems on Resolving Power

Ex. 3.15.30 : A diffraction grating has 5000 lines per cm and the total ruled width is 5 cm. Calculate for λ = 6000 A.U. in second order (i) The resolving power , (ii) The smallest value of λ which can be resolved.

Soln. :

Given : Lines per cm = 5000

$$\therefore \quad (a+b) = \frac{1}{5000}\text{ cm.} \qquad W = 5\text{cm.}$$

Total number of lines on grating are

$$N = 5000 \times 5 = 25000$$

$$\lambda = 6000\text{ A.U.} = 6000 \times 10^{-8}\text{ cm} \qquad m = 2$$

Formula :(i) R.P. = m N, (ii) R.P. = $\frac{\lambda}{d\lambda}$

(i) R.P. = m.N. = $2 \times 25000 = 50{,}000$

(ii) R.P. = $\frac{\lambda}{d\lambda}$

$$\therefore \quad d\lambda = \frac{\lambda}{\text{R.P.}} = \frac{6000 \times 10^{-8}}{50000} = 0.12 \times 10^{-8}\text{ cm} = 0.12\text{ Å}$$ ...Ans.

Ex. 3.15.31 : A plane grating just resolves two lines in the second order. Calculate the grating element if $d\lambda$ = 6 Å, $\lambda = 6 \times 10^{-5}$ cm and the width of the ruled surface is 2 cm.

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Soln. :

Given : $m = 2$, $d\lambda = 6\text{ Å} = 6 \times 10^{-8}$ cm, $\lambda = 6 \times 10^{-5}$ cm, W = 2 cm.

Formula :(i) R.P. = $\frac{\lambda}{d\lambda}$, (ii) R.P. = mN

$$\text{R.P.} = \frac{\lambda}{d\lambda} = mN$$

$$\therefore \quad N = \frac{\lambda}{d\lambda} \cdot \frac{1}{m} = \frac{6 \times 10^{-5}}{6 \times 10^{-8}} \times \frac{1}{2} = 500$$

$\therefore$ Number of lines in a width of 2 cm of grating = 500

$$\therefore \quad \text{Number of lines per cm} = \frac{500}{2} = 250$$

$$\therefore \quad \text{Grating element } (a+b) = \frac{1}{\text{Number of lines/cm}}$$

$$\therefore (a+b) = \frac{1}{250} = 4 \times 10^{-3}\text{ cm}$$ ...Ans.

Ex. 3.15.32 : Find the maximum value of resolving power of a diffraction grating 3 cm wide having 5000 lines per cm. if the wavelength of light used is 5890 Å.

Soln. :

Given : $\lambda = 5890\text{ Å} = 5890 \times 10^{-8}$ cm,

W = 3 cm, Number of lines per cm = 5000

$$\therefore \text{ grating element } (a+b) = \frac{1}{5000}\text{ cm}$$

$$\text{R. P. of grating} = \text{m.N}$$

R.P. is maximum when m is maximum.

The grating equation is $(a+b) \sin\theta = m\lambda$

$$\therefore \quad m_{max} = \frac{(a+b)}{\lambda} \text{(i.e. } \sin\theta = 1)$$

$$= \frac{1}{5000} \times \frac{1}{5890 \times 10^{-8}}$$

$$= \frac{1}{0.2945} = 3.395$$

N = Total number of lines on grating

$$N = 3 \times 5000 = 15000$$

$$\therefore \quad (R.P.)_{max} = m_{max} \times N = 3 \times 15000 = 45,000$$

∴ Maximum resolving power is 45,000. ...Ans.

Ex. 3.15.33 : A diffraction grating has a resolving power $R = \frac{\lambda}{\Delta\lambda} = Nm$. Show that the corresponding frequency range Δv that can be first resolved is given by, $\Delta v = \frac{C}{Nm\,\lambda}$

Soln. : We have, resolving power

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

$$\Delta\lambda = \frac{\lambda}{Nm} \quad ...(1)$$

We have the relation,

$$c = v\lambda$$

Or,

$$v = \frac{c}{\lambda}$$

$$\Delta v = -c \cdot \frac{1}{\lambda^2} \Delta\lambda \quad ...(2)$$

Put Equation (1) in Equation (2),

$$\Delta v = -c \frac{1}{\lambda^2} \cdot \frac{\lambda}{Nm} = -\frac{c}{Nm\,\lambda}$$

Negative sign shows that if wavelength range is λ and $\lambda + \Delta\lambda$, then frequency range will be $v - \Delta v$. Hence, frequency range that can be first resolved by grating is given by,

$$\Delta v = \frac{c}{Nm\,\lambda}$$ ...Proved

Ex. 3.15.34 : Find the minimum number of lines in a plane diffraction grating required to just resolve the sodium doublet (5890 Å and 5896 Å) in the (i) first order (ii) second order.

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Soln. : We have R.P. of grating,

$$\frac{\lambda}{d\lambda} = m \cdot N$$

$$\therefore \quad N = \frac{1}{m}\left(\frac{\lambda}{d\lambda}\right)$$

(i) For the first order, m = 1

$$\text{Average wavelength } \lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5890 + 5896}{2} = 5893 \text{ Å}$$

$$d\lambda = 5896 - 5890 = 6 \text{ Å}$$

$$\therefore \quad N = \frac{5893}{1 \times 6} = 982$$

(ii) For the second order m = 2, the minimum number of lines,

$$N = \frac{5893}{2 \times 6} = 491$$

...Ans.

Ex. 3.15.35 : Find the minimum number of lines that a diffraction grating would need to have in order to resolve in first order the red doublet given by a mixture of hydrogen and deuterium. The wavelength difference is 1.8 Å at λ = 6553 Å.

Soln. : We have resolving power to grating,

$$\frac{\lambda}{d\lambda} = mN$$

$$\Rightarrow \quad N = \frac{1}{m}\left(\frac{\lambda}{d\lambda}\right)$$

Given : λ = 6553 Å, $d\lambda$ = 1.8 Å and m = 1

$$N = \frac{6553}{1 \times 1.8} = 3641$$ ...Ans.

Ex. 3.15.36 : A diffraction-grating is just able to resolve two lines of wavelengths 5140.34 Å and 5140.85 Å in the first order. Will it resolve the lines 8037.2 Å and 8037.5 A in second order ?

Soln. :

$$R.P. = \frac{\lambda}{d\lambda} = nN$$

$$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$$

In the given problem, $\lambda = \frac{5140.34 + 5140.85}{2} = 5140.595$ Å

$$d\lambda = 5140.85 - 5140.34$$

$$= 0.51 \text{ Å}, n = 1 \text{ (first order)}$$

$$N = \frac{1}{1}\left(\frac{5140.595}{0.51}\right) = 10080$$

Hence, the resolving power ($\lambda/d\lambda$) of a grating in second order should be

$$nN = 2 \times 10080 = 20160.$$

In this case, $\lambda = \frac{8037.20 + 8037.50}{2} = 8037.35$ Å

$$d\lambda = 8037.50 - 8037.20 = 0.30$$

$$\therefore \quad R.P. = \frac{8037.35}{0.30} = 26791.17$$

So, the grating will not be able to resolve the lines 8037.20 Å and 8037.50 Å in the second order because the required resolving power (26791.17) is greater than the actual resolving power (20160).

Ex. 3.15.37 : *Light incident on a grating of 0.5 cm wide with 3000 lines. Find the angular separation in 2nd order of two sodium lines 5893 Å and 5896 Å. Check whether those two lines are resolved in 2nd order or not?*

Soln. :

For 0.5 cm $\rightarrow$ 3000 lines

$\therefore$ for 1 cm $\rightarrow$ 6000 lines

$$\therefore (a + b) = \frac{1}{\text{Number of lines per cm}} = \frac{1}{6000} = 1.667 \times 10^{-4} \quad ...(1)$$

$\therefore$ Angular separation for 2^n order and $\lambda_1 = 5893$ Å

$$\sin \theta_1 = \frac{2 \times 5893 \times 10^{-8}}{1.667 \times 10^{-4}} = 0.70701$$

$$\therefore \quad \theta_1 = 44.9928 \quad ...(2)$$

And for λ_2, $\therefore \sin \theta_2 = \frac{2 \times 5896 \times 10^{-8}}{1.667 \times 10^{-4}}$

$$\therefore \sin \theta_2 = 0.707378$$

$$\theta_2 = 45.022$$

$\therefore$ Angular separation = 0.0292 ...(3)

$\therefore$ As discussed in section 3.14 condition for resolving power ...(4)

$$R.P. = \frac{\lambda}{d\lambda} = m.N$$

$$\therefore \text{ Now } d\lambda = 5896 - 5893 = 3 \text{ Å} \quad ...(5)$$

$$\text{And } \lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5896 + 5893}{2} = 5894.5 \text{ Å}$$

$$\therefore R.P. = \frac{\lambda}{d\lambda} = \frac{5894.5}{3} = 1964.833$$

$\therefore$ Number of lines per cm on grating which will just resolve in the second order is given by

$$\therefore N = \frac{\lambda}{d\lambda} \cdot \frac{1}{m} = \frac{1964.833}{2} = 982.41 \quad ...(5)$$

But number of lines per cm on grating is 6000.

$\therefore$ Number of lines on grating per cm is much higher than the number of lines per cm needed for resolution.

$\therefore$ Both the lines i.e. 5893 Å and 5896 Å will be well resolved in 2nd order.

Ex. 3.15.38 : A grating has 620 rulings/mm and is 5.05 mm wide. What is the smallest wavelength interval that can be resolved in the third order at λ = 481 nm ?

MU - May 2016, 3 Marks

Soln. :

$$\text{No. of lines / mm} = 620$$

$$\therefore \text{No. of lines / cm} = N = 6200$$

$$\therefore \text{Order } m = 3$$

$$\lambda = 481 \times 10^{-9} \text{ m} = 418 \times 10^{-7} \text{ cm}$$

$$\text{as } \frac{\lambda}{d\lambda} = mN$$

$$\therefore d\lambda = \frac{\lambda}{mN} = \frac{481 \times 10^{-9}}{3 \times 6200}$$

$$= 2.58 \times 10^{-9} \text{ cm}$$

Smallest wavelength interval = 2.58×10^{-9} cm ...Ans.

Important Formulae

1. **Single slit diffraction**

Direction of principal maxima $\theta = 0$

Direction of minima $a \sin \theta = \pm n\lambda$, $n = 1, 2, 3, \ldots$

Direction of secondary maxima $a \sin \theta = \pm (2n + 1) \lambda/2$, $n = 1, 2, 3, \ldots$

Total angular width of central maximum $2\theta = 2 \sin^{-1} \frac{\lambda}{a}$

Linear half width on screen at a distance D from slit $y = \frac{\lambda D}{a}$

2. **Diffraction grating**

Directions of principal maxima are $(a + b) \sin \theta = \pm m\lambda$, $m = 0, 1, 2, 3, \ldots$

The directions of minima are $N (a + b) \sin \theta = \pm m\lambda$

(m may have all values except 0, 1N, 2N, 3N, ...)

Angular half width

$$d\theta_n = \frac{\lambda}{N (a + b) \cos \theta_n}$$

Condition of absent spectra $\frac{n}{m} = \frac{a + b}{a}$

Dispersive power of diffraction grating $\frac{d\theta}{d\lambda} = \frac{m}{(a + b) \cos \theta}$

3. **Resolving power of grating :** $\frac{\lambda}{d\lambda} = mN$

A Quick Revision

- Diffraction is the phenomenon of bending of light round the corners of an obstacle and spreading of light in the region of geometrical shadow.
- Diffraction is observable only when the width of the slit or the dimensions of obstacle are comparable with the wavelength of wave.
- The diffraction pattern of a rectangular slit consists of central maximum as well as minima and secondary maxima on either side.

- The diffraction pattern due to a circular aperture consists of central bright disc surrounded by alternate dark and bright concentric rings.
- A diffraction grating gives the effect of a large number of slits together.
- Grating forms multiple spectra.
- For resolution of spectral lines Rayleigh's criterion is to be applied.
- The R. P. of telescope is $\frac{1}{\theta}$ where θ is in radians.
- The resolving power of a telescope defined as its ability to form separate images of two distant point objects situated close to each other.
- The limit of resolution of telescope is defined by the angle subtended at its objective by two nearly distant objects whose images are just resolved.
- Resolving power of microscope increases with decrease of wavelength of light used.
- Resolving power of grating increases with increase of total number of runling and order of spectrum.

Review Questions

Short Answer Questions

Q. 1 What do you mean by diffraction ?

Q. 2 What are the types of diffraction and differentiate between them ?

Q. 3 What are the difference between interference and diffraction ?

Q. 4 Write the conditions of maxima and minima in diffraction pattern due to single slit ?

Q. 5 What is the ratio of relative intensities of successive principal maxima in diffraction due to a single slit ?

Q. 6 What is the width of central maxima in the diffraction due to a single slit ?

Q. 7 Explain the effects to the diffraction pattern of a single slit, if the slit width (a) decreases.

Q. 8 Explain the difference between single slit and double slit diffraction pattern.

Q. 9 What is grating and grating element ?

Q. 10 Explain the process to design a grating.

Q. 11 Write the condition of principal maxima and secondary maxima in the case of diffraction pattern of a grating.

Q. 12 Define dispersive power of a grating.

Q. 13 Write the expression of dispersive power of a grating.

Q. 14 Define resolving power of an optical instrument.

Q. 15 Write Rayleigh's criterion of resolution of two point objects.

Q. 16 Write factors on which resolving power of grating depends.

Long Answer Questions

Q. 1 What do you understand by diffraction of light ?

Q. 2 Explain the difference between interference and diffraction.

Q. 3 What are the types of diffraction ? Distinguish between them.

Q. 4 Describe Fraunhoffer diffraction obtained by a narrow slit illuminated by a parallel beam of monochromatic light.

Q. 5 Explain the theory of plane diffraction grating. Obtain the condition for n^{th} order maxima.

Q. 6 Explain the use of grating in determining the wavelength of light.

Q. 7 Explain the formation of multiple spectra with a grating.

Q. 8 Derive the condition for absent spectra in grating.

Q. 9 Define the term dispersive power of a grating. Obtain an expression for it. On what factors does it depend ?

Q. 10 Define resolving power of an optical instrument.

Q. 11 State and explain Rayleigh's criterion of resolution.

Q. 12 Obtain an expression for the resolving power of a grating. On what factors it depends ?

Q. 13 Discuss and explain the phenomenon observed when monochromatic light falls on (i) single slit (ii) double slit. Compare the patterns obtained in the two cases and establish a distinction between interference and diffraction of light.

Q. 14 Give the construction and theory of plane transmission grating and explain the formation of spectra by it. Explain what are absent spectra in the grating.

Q. 15 What is Rayleigh's criterion for the limit of resolutions ? Derive an expression for the limit of resolution of a telescope. How is it related to its resolving power.

Q. 16 Explain the Rayleigh's criterion for resolution and discuss it in relation to the resolving power of a microscope.

Q. 17 Two gratings A and B have the same width of the ruled surface but A has greater number of lines. Giving reasons compare in the two cases the resolving power of gratings.

Problems for Practice

1. In a single slit diffraction pattern the distance between the first minimum on the right and first minimum on the left is 5.2 mm. The screen on which the pattern is displayed is 80 cm from the slit and the wavelength of light used is 5460 Å. Calculate the width of the slit.

(Ans. : 2.1×10^{-3} cm)

2. A grating has 6000 lines per cm. How many orders of light of wavelength 4500 Å can be seen ?

(Ans. : 3)

3. In a Fraunhoffer diffraction pattern due to a single slit, the screen is at a distance of 100 cm. from the slit and the slit is illuminated by monochromatic light of wavelength λ = 5893 Å. The width of the slit is 0.1 mm. Calculate the separation between the central maximum and the first minimum.

4. In a grating spectrum, which spectral line in the fourth order will overlap with the third order line of λ = 5416 Å ?

(Ans. : λ = 4095.75 Å)

5. A single slit Fraunhoffer diffraction pattern is formed using white light. For what wavelength of light does the second minimum coincide with the third minimum for the wavelength 4000 Å ?

(Ans. : 6000 Å)

6. When the plane waves from a monochromatic source fall normally on a plane grating having 5000 lines per mm, it is observed that the second order spectrum is deviated through 30°. Find the wavelength of the source of light.

(Ans. : 5×10^{-5} cm)

7. What is the minimum number of lines in a plane transmission grating of width 2.5 cm so that yellow lines of sodium of wavelength 5890 Å and 5896 Å are just resolved in the first order spectra ?

(Ans. : 982)

8. Angular separation between two stars is 10^{-6} radians. Calculate the minimum aperture of the objective of a telescope, which can resolve the images of the stars for the light of wavelength 5000 Å.

(Ans. : 61 cm)

9. A parallel beam of light is incident on a plane transmission grating having 10600 lines per inch and a second order spectral line is observed at an angle of 30°. Calculate the wavelength of the line.

(Ans. : 5975×10^{-8} cm)

10. Examine if two spectral lines of wavelengths 5890 Å and 5896 Å can be clearly resolved in the (i) first order and (ii) second order by a diffraction grating 2 cm. wide and having 425 lines per cm.

11. Find the half angular width of the central bright maximum in the Fraunhoffer diffraction pattern of a slit of width 12×10^{-5} cm, when the slit is illuminated by monochromatic light of wavelength 6000 Å.

(Ans. : 0.5 rad)

12. Calculate the wavelength of light whose diffraction maximum in the diffraction pattern due to a single slit falls at θ = 30° and coincides with the first minimum for red light of wavelength 6500 Å.

(Ans. : 4333 Å)

13. A grating with 15000 rulings per inch is illuminated normally with white light extending from 4000 Å – 7000 Å. Show that only the first order spectrum is isolated but the second and third orders overlap.

(Ans. : I order 13°, 24° isolated, II order 28°, 56° overlap III order 45°)

14. A plane transmission grating having 6000 lines per cm is used to obtain a spectrum of light from a neon lamp in the first order. Calculate the angular separation between the neon lines whose wavelength are 5882 Å and 585 Å.

(Ans. : 1.8×10^{-3} radian)

15. A diffraction grating having 4000 lines per cm, is illuminated normally by light of wavelength 5000 Å. Calculate the angular dispersion in the third order spectrum.

(**Ans. :** $\frac{d\theta}{d\lambda}$ **= 15000 rad/cm**)

16. Light is incident normally on a grating 0.5 cm wide with 5000 lines, find the angular deviations of the two sodium lines in the first order spectrum. The wavelengths are 5890.2 Å and 5896.4 Å. Would you expect these two lines to be distinguished ?

(**Ans. :** 7896×10^{-3} **rad, yes**)

17. A plane grating just resolves two lines in the second order. Calculate the grating element if $\Delta\lambda = 6$ Å, $\lambda = 6000$ Å and width of the ruled surface is 2 cm.

(**Ans. :** $\frac{1}{250}$ **cm**)

18. Determine, if 1 inch grating having 3000 lines per cm can resolve the sodium D-lines (5890 and 5896 Å) in the (i) first order (ii) second order spectrum.

(**Ans. : (a) No, (b) yes**)

19. In a plane transmission grating with 5000 lines/cm and for wavelength 6000 Å, if the opaque spaces are exactly 2.0 times the transparent spaces, find out which order of spectra will be absent ?

(**Ans. : 3rd, 6th, 9th, etc. orders**)

3.16 University Questions

May 2012

Q. 1 Explain how the number of lines on grating decides the maximum number of orders of diffraction ? *(Ans. : Refer section 3.7)* (5 Marks)

Dec. 2012

Q. 1 Explain the experimental method to determine the wavelength of spectral line using diffraction grating. *(Ans. : Refer section 3.10)* (5 Marks)

May 2013

Q. 1 Refer Ex. 3.15.12 (3 Marks)

Q. 2 Refer Similar to Ex. 3.7.1 (5 Marks)

Q. 3 Derive condition for absent spectra in grating. *(Ans. : Refer section 3.8)* (5 Marks)

Q. 4 Refer Ex. 3.15.31 (5 Marks)

Dec. 2013

Q. 1 How do you increase the resolving power of a diffraction grating. *(Ans. : Refer section 3.14)* (3 Marks)

Q. 2 Refer Ex. 3.15.16 (5 Marks)

May 2014

Q. 1 What is Rayleigh's criteria of resolution? What is resolving power of diffraction grating ? *(Ans. : Refer sections 3.13 and 3.14)* (3 Marks)

Q. 2 What is grating element? Derive condition for maximum diffraction at diffraction grating. *(Ans. : Refer section 3.7)* (5 Marks)

Q. 3 Refer Ex. 3.15.12 (5 Marks)

Dec. 2014

Q. 1 What is grating and grating element ? *(Ans. : Refer section 3.7)* (3 Marks)

Q. 2 For plane transmission grating prove that d sin θ = nλ, n = 1, 2, 3... *(Ans. : Refer section 3.7)* (5 Marks)

Q. 3 Refer Ex. 3.14.1 (5 Marks)

Q. 4 Refer Ex. 3.15.34 (5 Marks)

May 2015

Q. 1 What is Rayleigh's criterion of resolution ? Define resolving power of a grating. *(Ans. : Refer sections 3.13 and 3.14)* (3 Marks)

Q. 2 Refer Ex. 3.15.18 (5 Marks)

Q. 3 Refer Ex. 3.15.5 (5 Marks)

Dec. 2015

Q. 1 What is meant by diffraction? State its types and differentiate them. *(Ans. : Refer sections 3.1 and 3.2)* (3 Marks)

Q. 2 Refer Ex. 3.15.23 (5 Marks)

Q. 3 Refer Ex. 3.15.21 (5 Marks)

May 2016

Q. 1 Refer Ex. 3.15.38 (3 Marks)

Q. 2 Explain the experimental method to determine the wavelength of spectral line using diffraction grating ? *(Ans. : Refer section 3.10)* (5 Marks)

Q. 3 Refer Ex 3.15.23 (5 Marks)

Dec. 2016

Q. 1 What is Rayleigh's criteria of resolution ? How to increase resolving power of diffraction grating ? *(Ans. : Refer section 3.13)* (3 Marks)

Q. 2 What is grating element ? Derive condition for absent spectra in plane transmission grating and explain with example. *(Ans. : Refer sections 3.7 and 3.8)* (5 Marks)

Q. 3 Refer Ex. 3.15.26 (5 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Fraunhoffer Diffraction at a Single Slit

Q. 1 Explain Fraunhoffer Diffraction at a Single Slit. *(Ans.: Refer section 3.4)*

Syllabus Topic : Fraunhoffer Diffraction at a Double Slit

Q. 1 Explain vector addition method. *(Ans.: Refer section 3.5)*

Q. 2 Explain the effect of increasing the number of slits. *(Ans.: Refer section 3.5)*

Q. 3 Explain the missing order in double slit diffracton. *(Ans.: Refer section 3.6)*

Syllabus Topic : Diffraction Grating

Q. 1 Explain how the number of lines on grating decides the maximum number of orders of diffraction ? *(Ans.: Refer section 3.7)* **(May 2012)**

OR

What is grating element ? Derive condition for maximum diffraction at diffraction grating. *(Ans.: Refer section 3.7)* **(Dec. 2014)**

OR

What is grating and grating element ? *(Ans.: Refer section 3.7)* **(Dec.2014)**

OR

For plane transmission grating prove that $d \sin \theta = n\lambda$, n = 1, 2, 3... *(Ans.: Refer section 3.7)* **(Dec.2014)**

Q. 2 Derive condition for absent spectra in grating. *(Ans.: Refer section 3.8)* **(May 2013)**

Syllabus Topic : Determination of Wavelength of Light with plane transmission Grating

Q. 1 Explain the experimental method to determine the wavelength of spectral line using diffraction grating. *(Ans. : Refer section 3.10)* **(Dec. 2012, May 2016)**

Syllabus Topic : Dispersive Power of a Grating

Q. 1 What is Rayleigh's criteria of resolution ? *(Ans. : Refer section 3.13)* **(May 2014, May 2015, Dec. 2016)**

Syllabus Topic : Resolving Power of a Grating

Q. 1 How to increase resolving power of diffraction grating ? *(Ans. : Refer section 3.14)* **(Dec. 2013, Dec. 2016)**

Q. 2 What is resolving power of diffraction grating ? *(Ans. : Refer section 3.14)* **(May 2014, May 2015, Dec. 2016)**

Solved Problems

Q. 1	Refer Ex. 3.14.1	**(Dec. 2014)**
Q. 2	Refer Ex. 3.15.5	**(May 2015)**
Q. 3	Refer Ex. 3.15.12	**(May 2013, May 2014)**
Q. 4	Refer Ex. 3.15.16	**(Dec. 2013)**
Q. 5	Refer Ex. 3.15.18	**(May 2015)**
Q. 6	Refer Ex. 3.15.21	**(Dec. 2015)**
Q. 7	Refer Ex. 3.15.23	**(Dec. 2015, May 2016)**
Q. 8	Refer Ex. 3.15.26	**(Dec. 2016)**
Q. 9	Refer Ex. 3.15.31	**(May 2013)**
Q. 10	Refer Ex. 3.15.34	**(Dec. 2014)**
Q. 11	Refer Ex. 3.15.38	**(May 2016)**

□□□

CHAPTER 4

Module 2

LASER

Syllabus

Quantum processes as absorption, spontaneous emission and stimulated emission; metastable states, population inversion, pumping, resonance cavity, Einstein's equations; Helium Neon laser ; Nd :YAG laser; Semiconductor laser, Applications of laser : Holography (construction and reconstruction of holograms) and industrial applications (cutting, welding etc), Applications in medical field.

4.1 Introduction

MU - May 2014

- LASER is acronym for Light Amplification by Stimulated Emission of Radiation. A laser beam is highly parallel coherent beam of light of very high intensity. Production of laser light is a particular consequence of interaction of radiation as a rule with matter. The interpretation of the interaction is done on the basis of ideas related to energy levels of the concerned system from which light is derived. In case of gases, it may be noted that, though different gases have different energy level patterns, any gas as a rule will have discrete energy levels, the energy quantization rules always hold good. Hence matter, irrespective of its state of existence, is referred to as a quantized system.
- Since LASER is high energy beam, at times it is compared with x-rays. But both of them differ completely. Some points at which LASER differs from x-rays are :

(1) LASER is highly coherent where an x-ray is not.

(2) LASER has it wavelength of the order of visible spectrum, where as x-rays has very small wavelength.

(3) Stimulated emission is essential for LASER where as x-ray needs high energy electron and their retardation.

4.2 Interaction of Radiation with Matter

- The understanding of the working principle of laser requires an appreciation of quantum process that takes place in a material when it is exposed to radiation. A material medium is composed of identical atoms or molecules each of which is characterized by a set of discrete allowed energy states. An atom when it receives or releases an amount of energy equal to the energy difference between those two states, is termed as a transition.
- For sake of simplicity, we will restrict our attention to two energy levels E_2, an excited state and E_1, a lower energy state. Let a monochromatic radiation of frequency ν be incident on the medium. The radiation may be viewed as a stream of photons, each photon carrying an energy hν. If $h\nu = E_2 - E_1$, the interaction of radiation with atoms leads to following distinct processes.

Syllabus Topic : Quantum Processed as Absorption

➤ **Topics covered :** Absorption

4.2.1 Absorption

- An atom in lower energy state E_1 may absorb the incident photon and may be excited to E_2 as shown in Fig. 4.2.1. This transition is known as stimulated absorption corresponding to each transition made by an atom one photon disappears from the incident beam.

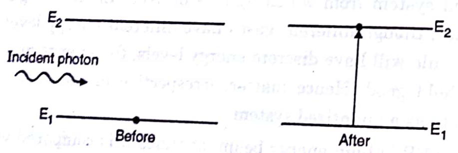


Fig. 4.2.1 : Induced absorption

The transition may be written as

$$A + h\nu = A^* \quad \text{...(4.2.1)}$$

Where A = Atom in lower energy state

A^* = Atom in excited state

- The number of atoms N_{ab} excited during the time Δt is given by

$$N_{ab} = B_{12} N_1 Q \Delta t \quad \text{...(4.2.2)}$$

Where N_1 = Number of atoms in state E_1

Q = Energy density of the incident beam

B_{12} = Probability of an absorption transition.

Syllabus Topic : Spontaneous Emission

➤ **Topics covered :** Spontaneous Emission

4.2.2 Spontaneous Emission

- Excited state with higher energy is inherently unstable because of a natural tendency of atoms to seek out lowest energy configuration. Therefore excited atoms do not stay in the excited state for a relatively longer time but tend to return to the lower state by giving up the excess energy $h\nu = E_2 - E_1$ in the form of spontaneous emission or stimulated emission.
- The excited atom in the state E_2 may return to the lower state E_1 on its own out of natural tendency to attain the minimum potential energy condition. During the transition the excess energy is released as a photon of energy $h\nu = E_2 - E_1$. This type of process in which photon emission occurs without any external agency is called **spontaneous** or **natural emission**. Fig. 4.2.2 represents natural emission and shows the transition.

$$A^* \rightarrow A + h\nu \quad \text{...(4.2.3)}$$

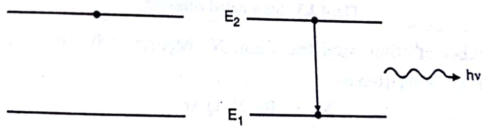


Fig. 4.2.2 : Natural or Spontaneous emission

- The number of spontaneous transitions N_{sp} taking place in the medium during time Δt depends only on the number of atoms N_2 lying in the excited state E_2. It is given by,

$$N_{sp} = A_{21} N_2 \Delta t \quad \text{...(4.2.4)}$$

where, N_2 = Number of atoms in the state E_2

A_{21} = Probability of a spontaneous emission.

Syllabus Topic : Stimulated Emission

➤ **Topics covered :** Stimulated Emission

4.2.3 Stimulated Emission

MU - Dec. 2013, Dec. 2014, Dec. 2015, May 2016

- An atom in excited state need not wait for spontaneous emission to occur. There exists an additional possibility according to which an excited atom can make a downward transition and emit a radiation. A photon of energy $h\nu = E_2 - E_1$ can induce the excited atom to make a downward transition releasing the energy in the form of a photon. Thus the interaction of a photon with an excited atom triggers the excited atom to drop to the lower energy state giving up a photon.
- This phenomenon is called **forced emission** or **stimulated emission** as shown in Fig. 4.2.3. The process may be represented as

$$A^* + h\nu = A + 2h\nu \quad \text{...(4.2.5)}$$

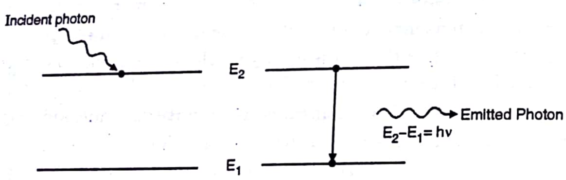


Fig. 4.2.3 : Stimulated emission

- The number of stimulated transition N_{st} occurring in the material during time Δt may be written as

$$N_{st} = B_{21} N_2 Q \Delta t \quad \text{...(4.2.6)}$$

Where B_{21} = Probability of a stimulated emission.

- Einstein had predicted this probability and it is considered as one of the essential requirements of laser. The main features are :
- The emitted photon is identical to the incident photon in all respects. It has the same frequency ν as that of incident photon. Both the photons travel in the same direction.

- The process is controllable from outside.
- Multiplication of photons takes place in the process. One photon induces an atom to emit a second photon, these two travelling along the same direction de excite two atoms in their path producing a total four atoms which in turn stimulates four atoms generating eight photons and so on. It suggests that electromagnetic waves of the number as atoms in light source are very large, coherent emission leads to enormously high intense light than incoherent emission.

Difference between Spontaneous emission and Stimulated emission :

Table 4.2.1

Spontaneous emission	Stimulated emission
1) It is a natural process.	1) Artificial, induced process.
2) Can not be controlled.	2) Can be controlled effectively.
3) No multiplication of photons takes place.	3) Multiplication of photons takes place.
4) Not useful for LASER.	4) Essential for LASER.

4.3 Active Medium

A medium in which light gets amplified is called an active medium. The medium may be a solid, liquid or gas. Therefore the medium where we get population inversion and laser as output is active medium.

Solid : Ruby laser, Nd-YAG laser

Gas : He-Ne laser, CO_2 laser

Liquid : Dye laser

Syllabus Topic : Population Inversion

➤ **Topics covered :** Population Inversion

4.4 Population Inversion

MU - May 2013, May 2016

In order to understand population inversion, we must know what do we mean by,

(a) Population of energy level

(b) Boltzmann factor

(a) Population of energy level

- For the sake of simplicity, let us consider the atomic state of matter, where we have set of allowed energy levels for an atom. The set of allowed energy levels is the energy level scheme is same for all the atoms of same type.
- If we consider an assembly of identical atoms, then we can compare the energies of all those atoms in a single energy level scheme. Normally, the numbers of atoms are required to possess one or the other energy values which are permitted only in this energy scheme, each of energy states will be having many atoms as its members. The number of atoms in a particular state is referred to as its **population**.

(b) Boltzmann factor

- The population of different energy states of any physical system are related to each other, provided, the system is in thermal equilibrium. The relation is given by Boltzmann factor. Among the various energy states, if we consider any two energy states E_1 and E_2 with population N_1 and N_2 respectively, and if $E_2 > E_1$, then Boltzmann factor is the ratio (N_2/N_1) given by,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/KT} \quad \text{...(4.4.1)}$$

Where K is Boltzmann constant.

Since $E_2 > E_1$, $e^{-(E_2-E_1)/KT} < 1$

$\therefore N_2 < N_1$

Hence for a system in thermodynamic equilibrium, it is mandatory that the population of any higher state is always lesser than that in any of the lower states.

- **Population inversion is the state of a system at which the population of a particular energy state is more than that of a specified lower energy state** i.e. $N_2 > N_1$. Under normal condition population inversion condition does not exist. However it is possible to achieve the population inversion condition in certain system, which has metastable states.
- It is considered as a precondition of LASER. It makes LASER possible with the help of metastable state.

Syllabus Topic : Metastable States

> **Topics covered :** Metastable States

4.5 Metastable States

- By providing energy, if an atom is made to go to one of its excited states, it stays there over a brief interval of time not exceeding 10^{-8} sec., and then returns to one of the lower energy states. In case the state to which the atom is excited is a metastable state, then the atom stays there for unusually long time, which is of the order of 10^{-3} to 10^{-2} seconds. **This property is essential for achieving population inversion.**
- Metastable states may be considered as a special privilege enjoyed by atoms of some specific elements. Such states are not created. Infact, existence of metastable states helps us decide which element is useful for population inversition and which one is not. Like He is useful in He-Ne, Nd is useful in Nd – YAG etc.

Syllabus Topic : Pumping

> **Topics covered :** Pumping, Pumping Schemes, Three Level Pumping Scheme, Four Level scheme, Two Level scheme

4.6 Pumping and Pumping Types

MU - Dec. 2016

The process of raising large number of atoms from lower energy level to a higher energy level is called pumping.

Types of pumping

- **Optical pumping :** Which uses strong light source for excitation.
- **Electrical pumping :** Which uses electron impact for excitation.
- **Chemical pumping :** Which uses chemical reactions for excitation.
- **Direct pumping :** Which uses direct conversion of electric energy into light.

4.7 Pumping Schemes

Any atom has large number of energy levels but for pumping process only few are of some use. Important pumping schemes which are famous.

(a) Three level	(b) Four level

4.7.1 Three Level Pumping Scheme

Consider the case of three energy levels taking part E_1, E_2 (Metastable state) and E_3 as shown in Fig. 4.7.1.

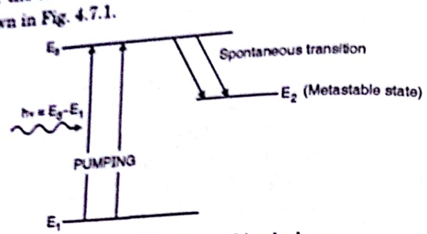


Fig. 4.7.1 : Pumping in 3 level scheme

- For pumping we select a radiation with frequency satisfying $h\nu = E_3 - E_1$. As E_3 is not a metastable state, spontaneous emission will take place between $E_3 \rightarrow E_2$.
- Laser materials are selected such that energy levels will have very small probability for transition $E_3 \rightarrow E_1$. (These probabilities are calculated through selection rules which are described in terms of quantum numbers.) As E_2 is a metastable state, probability of transition $E_2 \rightarrow E_1$ is very small. (Pumping is never done between $E_1 \rightarrow E_2$ as it describes a two level pumping scheme which is not used due to reasons given at later stage). As pumping continues, E_2 gets filled up and population inversion takes place between E_2 and E_1.
- As E_1 is ground state, a large number of atoms must be pumped to E_2 to have population inversion, hence a very high pumping power is needed for this scheme.
- A photon with energy $h\nu = E_2 - E_1$ may trigger the stimulated emission process as shown in Fig. 4.7.2.

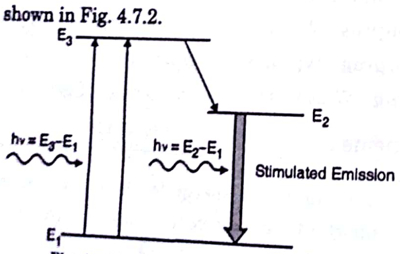


Fig. 4.7.2 : Stimulated emission in 3 levels

4.7.2 Four Level Scheme

- Consider the following case, where four energy levels are taking part into laser emission process.

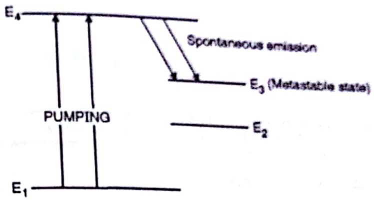


Fig. 4.7.3

- Pumping is created between E_1 and E_4 and as E_4 is not a metastable state, spontaneous emission will transfer atoms to level E_3 which is a metastable state. As the pumping continues, E_3 also gets atoms from E_4. Population inversion between E_3 and E_2 is achieved. As there is no pumping from E_1 to E_2, we have E_2 virtually empty and hence population inversion between E_3 and E_2 is achieved. As there is no pumping from E_1 to E_2, we have E_2 virtually empty and hence population inversion between E_3 and E_2 is some what easier than that of 3 level scheme.
- A photon with energy $h\nu = E_3 - E_2$ triggers stimulated emission as shown in Fig. 4.7.4.
- After reaching to E_2, through stimulated emission, atoms will generate spontaneous emission to go to E_1 i.e. the ground state.

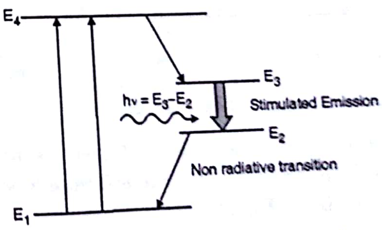


Fig. 4.7.4 : Stimulated emission in 4 level scheme

4.7.3 Two Level Scheme

Selection rules which are described in terms of quantum numbers, and the relation discussed in chapter linked to quantum mechanics.

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$$

One can find that two level pumping scheme is not suitable for population inversion.

Syllabus Topic : Resonance Cavity

> **Topics covered :** Resonance Cavity

4.8 Resonant Cavity

- A laser device consists of an active medium bound between two mirrors or highly reflecting surfaces as shown in Fig. 4.7.1. These surfaces reflect the photons to and fro through the active medium. A photon moving in a particular direction represents a light wave moving in the same direction. Thus the two mirrors along with the active medium form a cavity inside which two types of waves exists, one type comprises of waves moving to the right, and the other one to the left Figs. 4.8.2 and 4.8.3.

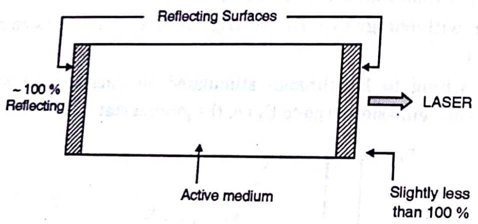


Fig. 4.8.1 : Resonant cavity

- The two waves interfere constructively if there is no phase difference between the two (Fig. 4.8.2) But, their interference becomes destructive if the phase difference is $\pi/2$ (Fig. 4.8.3).

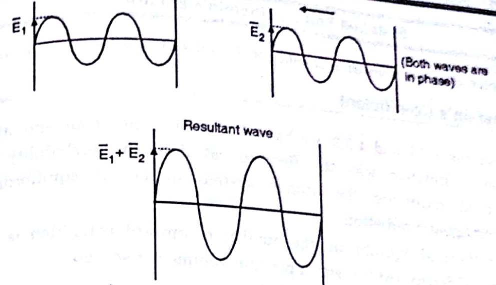


Fig. 4.8.2 : Constructive interference

- For destructive interference.

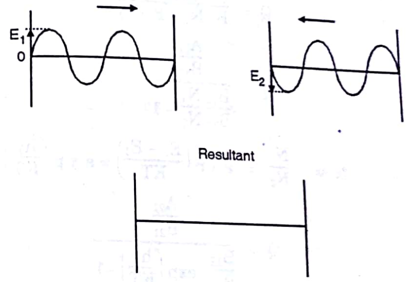


Fig. 4.8.3 : Destructive interference

- In order to arrange for constructive interference, the distance 'L' between the two reflecting surfaces should be such that the cavity should support an integral number of half wavelengths.

$$\therefore \; L = m\frac{\lambda}{2} \qquad \text{...(4.8.1)}$$

- Where, m is an integer and $m > 0$, and λ is the wavelength of the laser light. In such a case, a standing wave pattern is established within the cavity, and the cavity is said to be resonant at wavelengths.

$$\therefore \; \lambda = \frac{2L}{m} \qquad \text{...(4.8.2)}$$

Syllabus Topic : Einstein's Equation

➤ **Topics covered :** Einstein's Co-efficient, Important Characteristics of Laser Beam

4.9 Einstein's Co-efficient

- In sections 4.2.2 and 4.2.3, we have discussed spontaneous and stimulated emission. Einstein was the first to calculate the probability of such transition assuming the atomic system to be in equilibrium with electromagnetic radiation.
- Under thermal equilibrium the number of upward transition is equal to number of downward transition per unit volume per second.

$$\therefore\ A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q \quad \dots(4.9.1)$$

From Equation (4.9.1) we get,

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$Q = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}}\frac{N_1}{N_2} - 1}$$

$$\text{Now}\quad \frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{KT}\right) = \exp\left(\frac{h\nu}{KT}\right)$$

$$\therefore\quad Q = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}} \cdot \exp\left(\frac{h\nu}{KT}\right) - 1} \quad \dots(4.9.2)$$

- Equation (4.9.2) must agree with Planck's energy distribution formula which is given by,

$$Q = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp\left(\frac{h\nu}{KT}\right) - 1} \quad \dots(4.9.3)$$

- Now by comparing Equations (4.9.2) and (4.9.3), we get,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \dots(4.9.4)$$

and

$$\frac{B_{12}}{B_{21}} = 1$$

or

$$B_{12} = B_{21} \quad \dots(4.9.5)$$

4.10 Important Characteristics of Laser Beam

Highly coherent

MU - May 2015

- The light is coherent with waves all exactly in phase with another. It means an interference pattern can be obtained by using two laser sources.
- **There are two types of coherences, (i) Temporal (ii) Spatial coherences :**
  (i) **Temporal coherence :** In the wave propagation system correlation between the waves at one place at different times, is called **"temporal coherence"**.
  (ii) **Spatial coherences :** In the propagation wave correlation between different places but not along the path is called **"spatial coherence"**.
- From the above description it is clear that these two types of coherence are independent of each other. That is an electromagnetic wave with partial temporal coherence can have perfect spatial coherence.

Highly monochromatic

- A laser produces light in more or less single wavelength i.e. the line width associated with laser beams are extremely narrow.
- In general traditional (conventional) sources of light are not strictly monochromatic. But it is observed that the laser light is almost perfectly monochromatic (one only). However in all practical practices the laser light is not perfectively monochromatic, but its degree of monochromaticity is very high. Its divergence from monochromaticity is due to ***Doppler effect*** of the fast moving atoms or molecules from the laser source.
- Let us consider that such a monochromatic light of wave length λ and its spreading is denoted by $\Delta\lambda$ which is called **line width,** e.g. sodium vapor monochromatic source of light gives two bands (spectrum) of wave length $\Delta\lambda_1$ and $\Delta\lambda_2$ which *are 5890 A° and 5893 A° respectively.*
- Mathematically **degree of non-monochromaticity 'τ'** is defined as the ratio of line width $\Delta\lambda$ to the original wavelength λ that is $\tau = \frac{\Delta\lambda}{\lambda}$.
- The reciprocal of τ is known as degree of monochromaticity and hence it is clear that $1/\tau$ is high when beam spreading is low.

- In general lasers beam generate light in a very narrow band. The degree of monochromoticity is described in terms of *wavelength bandwidth* or *frequency bandwidth*. It is clear that the narrow *line width* of laser, gives higher degree of the monochromocity laser light.
- However, narrowness of *line width* is depends on the type of laser, and special techniques used to improve monochromaticity.
- There are certain factors responsible for increasing its monochromaticity :
  (i) Laser light originally emitted by stimulated emission from single set of *atomic energy levels*. This is its basic principle; hence from origin its monochromaticity is very high.
  (ii) Here electromagnetic waves of frequency $\nu = (E_2 - E_1)$ only can be amplified, and it has a particular range which is called *line width*. The line width is decided by factors like Doppler effect of moving atoms and molecules.
  (iii) In the system of laser generation the *laser cavity* forms a *resonant system* in which *laser oscillation* is sustained only at the resonant frequencies of the cavity. This phenomenon leads to the further descries in *laser line width*. Therefore laser light is usually very pure in wavelength.

Highly directional

- A laser beam diverges hardly at all. Such a beam sent from the earth to a mirror left by the Apollo-II expedition, remained narrow enough to be detected on its return to the earth. (Distance between moon and earth is around 3,84,000 km).
- This is one of the important properties of the laser due to its high directionality. Directionality or Collimation means it does not spread out much. A conventional light source emits light in all directions. On the other hand, laser emits light only in one direction without spreading or very little divergence of it.
- The width of laser beam is extremely narrow and hence a laser beam can travel to long distances without spreading. The directionality of laser beam is due to its laser cavity system in which very nearly parallel front back mirrors arrangement is made. The perfectively collimated beam is never expanding at all. Its divergence angle is zero.

- It can be brought to focus extremely sharp. Further Laser beam is well defined wave-front therefore it is highly directional. Its high directionality allows us to focus it into a point by passing the beam through a suitable convex lens.
- In the laser system diffraction plays an important role in fixing the size of laser spot which can be focused at a given distance.
- A narrow laser beam produced in the resonator cavity that diverges at some angle depending on the design of resonator, output aperture's size, and resulting diffraction effects on it. These diffraction effects generally called as beam-spreading effect.
- Finally we get the high directionality laser beam due to the diffraction effects which can make minimum divergence and spot size of the beam. High directionality laser beam is the prime demand in the laser based devices and instruments.

Brightness

The laser beam is highly intense as compared to ordinary source of light.

Highly energetic

The laser beam is highly intense. To understand it clearly here is an example : To achieve an equal energy density to that in laser beam, a hot object would have to be at temperature of 10^{30} K. This makes laser suitable for applications like cutting, drilling and welding.

Difference between LASER and ordinary light :

LASER	Ordinary light
It is highly monochromatic.	It is poly chromatic.
It is highly coherent.	It is not coherent.
Stimulated emission is responsible for it.	Spontaneous emission is responsible for it.
Highly direction.	Not directional.
Highly energetic.	Poor energy is associated.
Example : He-Ne, Nd : YAG etc.	Example : Sunlight, LED

Syllabus Topic : Helium Neon Laser

> **Topics covered :** He- Ne Laser

4.11 He-Ne Laser

MU - May 2013, Dec. 2014, May 2015

Principle

- Gas lasers are usually employ a mixture of two gases say A and B where atoms of type A are initially excited by electron impact and they in turn transfer their energy to atoms of type B which are actual active centers. Here the energy transfer is done by atomic collisions between A and B where two of their energy levels are equal. It is also known as resonance transfer energy. It can be expressed as

$$e_1 + A = e_2 + A^*$$

$$A^* + B = A + B^*$$

- Here A^* and A represented the energy values of the atom of type A in metastable state and ground state respectively. B and B^* represent the energy values of the atom of type B in ground state and excited state respectively.

Construction

- The He-Ne laser consists of a long and narrow discharge tube which is filled with Helium and Neon in the ratio 10 : 1 with pressure of 1 mm of mercury. Flat glass quartz plates which function as Brewster windows are sealed to the tube at both ends. Two optically plane mirrors are fixed on either side of the tube normal to its axis as shown in Fig. 4.11.1.

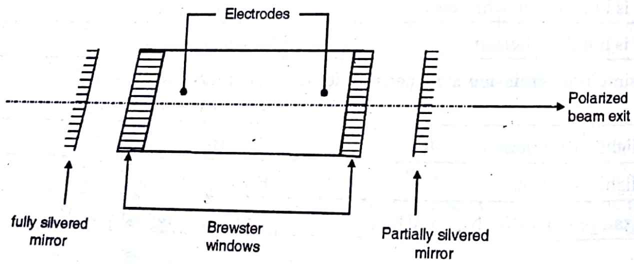


Fig. 4.11.1 : He-Ne laser tube (schematic presentation)

- One of the mirror is fully silvered with 100% reflectivity, where as the silvering of the other is slightly less so that 1% of the incident laser beam could be trapped by transmission.

Working

Energy level diagram :

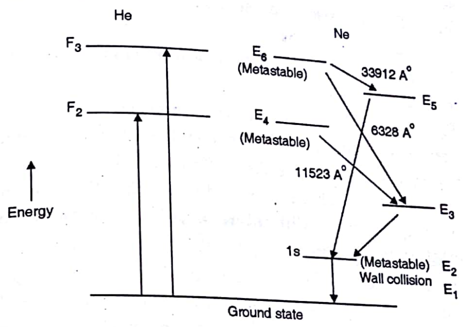


Fig. 4.11.2 : Energy level diagram of He-Ne

- When a voltage of about 1 kV is applied across the tube, a slow discharge of the gases is initialized in the tube. During the discharge, many electrons are rendered free from the gas atoms and are accelerated towards positive electrode and collide with Helium. Since He atoms are large in number, excitation of He takes place and as discussed in the principle above, we get

$$e_1 + He = e_2 + He^*$$

He atoms are excited to two energy levels F_2 and F_3 which are non-metastable states which leads to an increase of population in each of them.

- For Ne energy states metastable states E_6 and E_4 are very close to F_3 and F_2 of He atom*. Therefore when He atoms collides with a Neon atoms, because of the matching of energy levels, resonant transfer of energy takes place from He to Ne atoms. As a result, the Ne atoms get elevated to the E_6 and E_4 levels, whereas the He-atoms returns to ground state. This is represented as

$$He^* + Ne = He + Ne^*$$

The population increases rapidly and population inversion takes place between E_6 and E_4 with respect to E_5 and E_3.

Three main transitions become available.

(a) E_6 to E_5 gives rise to a radiation of wavelength 33912A° which is in infra-red region and hence not visible.

(b) E_6 to E_3 gives rise to a radiation of wavelength 6328 A°, which is visible and of red colour.

(c) E_4 to E_3 gives rise to a radiation of wavelength 11523 A° which is also in infra-red region.

- From E_5 and E_3 levels, atoms undergo spontaneous transitions to E_2 level at much faster rate. But E_2 level is metastable for Ne. The atoms will come down to ground state by wall collision. This effect is inversely proportional to the diameter of the discharge tube and hence **the diameter of the He-Ne laser is only of few millimeters in diameter to enable efficient depopulation of E_2 level.**
- Since the discharge in the tube is maintained continuously, the cycle of events also takes place continuously and the emission of laser is also continuous, because of which He-Ne laser is referred as a **continuous wave laser.**

Some important points regarding He-Ne laser

(a) In He-Ne laser, E_6 level is common source for both 6328 A° and 33912 A° radiations. Since the longer wavelength have higher stimulated emission probability (B_{21} is high), the 33912A° captures major share from 3s population. Also it tends to prevent 6328A°, 33912 A° must be suppressed. One of the way to do so to fill the space between Brewster window and mirror by methane which is opaque for 33912A° but transparent to 6328A°.

(b) Why Helium is needed? When lasing action is taking place in Ne, only Ne is subjected to discharge, then it is E_5 state most probable to experience upward transitions, but it is not a metastable. As far as E_6 state is concerned, it appears that, it can not be populated to any great degrees by collision between the Ne atoms themselves. The only way to populate E_6 is use of He.

- **Merits**

(a) Continuous output.

(b) Highly monochromatic.

(c) Highly stable.

(d) No separate cooling is needed.

(e) As gases are found in pure form their optical properties are well defined.

- **Demerits**

Very low output power.

- **Applications**

(a) In holography.

(b) Research activities.

(c) Communication.

Syllabus Topic : Nd :YAG Laser

> **Topics covered :** Nd:YAG Laser

4.12 Nd : YAG Laser

MU - May 2016

- It is a solid state laser.
- Nd represents Neodymium (Nd^{+3} ions are used).
- YAG represents Yittrium Aluminium Garnet ($Y_3 Al_5 O_{12}$).
- Some of the Y^{+3} ions are replaced by Nd^{+3}. The crystal atoms of YAG do not take part into lasing action, but serve as a host lattice in which Nd^{+3} resides.

Construction

- As shown in Fig. 4.12.1, an elliptically cylinder reflector with both of its axis occupied by a flash lamp and Nd : YAG rod respectively. The light leaving one focus of the ellipse will certainly pass through the other focus after reflection from reflecting surface. Hence entire light generated by flash tube is focussed on the Nd : YAG rod.
- Optical resonator is formed by highly silvered reflecting surfaces as shown in Fig. 4.12.1.

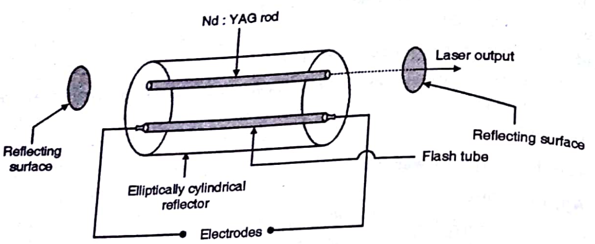


Fig. 4.12.1 : Nd : YAG laser

Function

- In Fig. 4.12.2 we have energy levels E_1, E_2 and E_3 of Nd along with many other levels of YAG. E_1 is ground state and E_3 offers metastable state.
- Pumping takes place with light of wavelength 5000 A° to 8000 A° which excites Nd^{+3} ions to higher states. The metastable state E_3 rapidly gets populated due to downward transitions from higher energy levels as none of them is metastable. Population inversion takes place between E_3 and E_2. A continuous laser of 10600 A° in infrared region is given out due to stimulate emission taking place between E_3 and E_2.

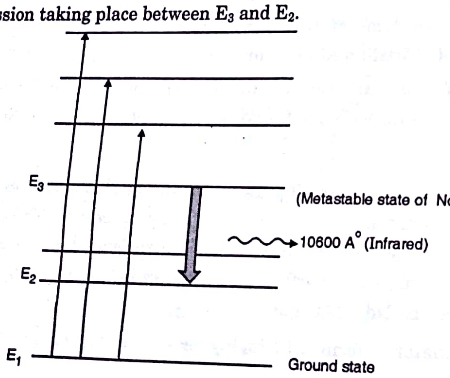


Fig. 4.12.2 : Energy levels of Nd along with YAG

Applications

Due to steady and reliable source of laser energy the Nd-YAG laser has number of applications. Few major applications are :

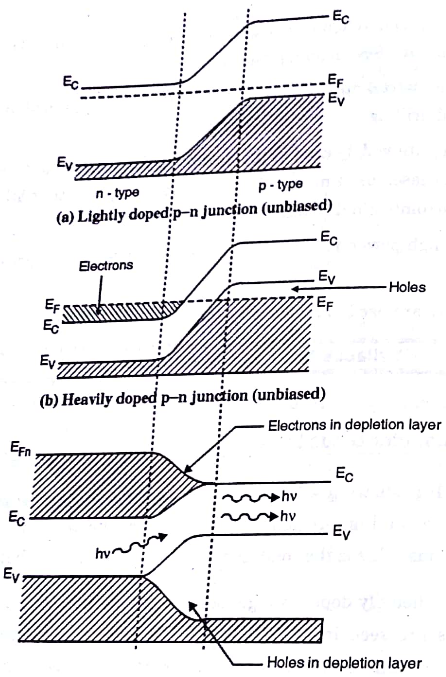
1. The main industrial applications are in the mechanical process like cutting welding and drilling.
2. These lasers are widely used in the medical field. The special wavelength of 1.064 μm of laser used in the fiber optics endoscopes with YAG lasers are used to gastrointestinal treatment.
3. Due to its high power YAG lasers are used as range finders operation in the military.
4. These lasers are used in the laser fusion process due to its extra high power.

Syllabus Topic : Semiconductor Laser

> **Topics covered :** Semiconductor Diode Laser

4.13 Semiconductor Diode Laser

- Fig. 4.13.1(a) shows a scheme of an ordinary p-n junction semiconductor. The valence band in p-region has holes and conduction band in n region has free electrons. This is the condition when semiconductor is lightly doped.
- When it is heavily doped, we get some electrons shifted to conduction band and holes are seen in valence bands. But this does not create population inversion at all (Fig. 4.13.1(b)). The Fermi level on n-side is found on conduction band and on p-side it is found on valence band but it is in equilibrium at both sides.
- When a forward biased is applied, energy level diagram gets altered as shown in Fig. 4.13.1(c). Electrons from conduction band of n-type and holes from valence band of p-type are injected into depletion layer.



(a) Lightly doped p–n junction (unbiased)

(b) Heavily doped p–n junction (unbiased)

(c) A forward biased and heavily doped p-n junction above threshold value.

Fig. 4.13.1 : Energy bands in semiconductor laser

- A threshold current (A minimum forward current) is defined below which electron-hole recombination will have spontaneous emission and p-n juction works as LED. A forward current above the threshold value, carrier concentration in depletion region will reach very high values which describes population inversion state. The emission of light due to recombination of electrons and holes will be stimulated emission and produces laser.

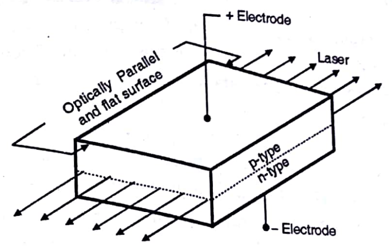


Fig. 4.13.2 : Schematic construction of a semiconductor diode

- Some of the semiconductor lasers and their wavelength.
  - (a) GaAs : 8400 A° (at room temperature)
  - (b) GaAsP : 6500 A° (at room temperature)
- **Merits**
  - (a) Simple and compact.
  - (b) Requires very little power and more efficient.
  - (c) Output can be controlled by controlling the junction current.
  - (d) Metastable state is not required.
- **Demerits**
  - (a) Highly temperature sensitive.
  - (b) Less monochromatic.
- **Applications**
  - (a) Laser printers and copiers.
  - (b) CD players.
  - (c) Optical communication (as light source).

Syllabus Topic : Application of Lasers – Holography (Construction and Reconstruction of Holograms)

> **Topics covered :** Application of Lasers, Holography, Memory Reading and Writing

4.14 Application of Lasers

4.14.1 Holography

MU - May 2014, Dec. 2015, Dec. 2016

- The advent of lasers has made the art of holography possible. Photography can be thought of as a new approach to the problem of generating images. An ordinary photograph represents a two dimensional recording of a three dimensional scene. The emulsion on the photographic plate is sensitive only to intensity variations. In this process the phase information carried by the electromagnetic wave scattered from the object is lost. Since only the intensity pattern is recorded, the 3D character of the object is lost.
- The principle behind the holograph is "During the recording process one superimposes on the scattered wave (emanating from the object) another coherent wave (called reference beam) of the same wavelength". These two waves interfere in the plane of the recording medium and produce interference fringes. This is known as recording process. The interference fringes-characteristic of the object is formed. The recording medium records the intensity distribution in the interference pattern. This interference pattern has recorded in it not only the amplitude distribution but also the phase of the electromagnetic wave scattered from the object. Since the recorded intensity pattern has both the amplitude and phase recorded in it has been called "hologram" (Holos in Greek means whole).
- The holograms have little resemblance to the object. It has in it a coded form of a wavefront. The reproduction of the image is known as reconstruction in which a wave identical to the one used as reference wave is used. When hologram is illuminated by the reconstruction wave, two waves are produced. One wave appears to diverge from the object and provides the virtual image of the object. The second wave converges to form a second image which is real.

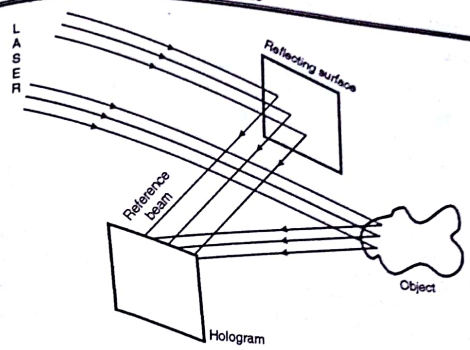


Fig. 4.14.1(a) : Construction of hologram

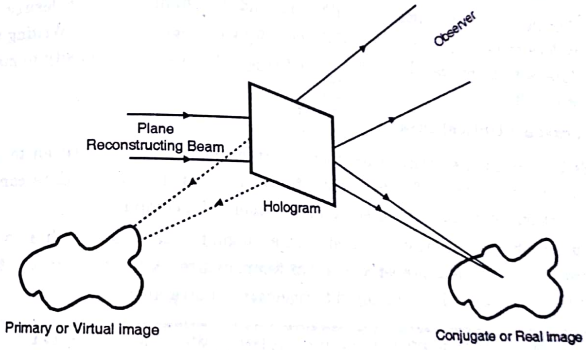


Fig. 4.14.1(b) : Reconstruction of image

4.14.2 Memory Reading and Writing

Information is written or read from the optical disk using laser beam.

Types of optical disks :

(a) CD-ROM : Compact Disk - Read Only Memory
(b) WORM : Write Once Read Many
(c) Erasable Optical Disk

(a) CD ROM

It is an optical ROM. Pre-recorded data can be read out. The manufacturer writes data on CD-ROMs. The disk is made up of a resin such as polycarbonate. It is coated with a material which will change when a high density laser beam is focused on it. The coating material is highly reflective usually Aluminium. The high intensity laser beam forms a tiny pit along a trace to represent logic 1. For reading the data, laser beam of less intensity is employed. The reflected laser is sensed by photodiode to read data. The intensity of the reflected light of laser changes as it encounters the pit. A pit spreads light so that photodiode receives little reflect light. But the surface without pit reflects sufficient light to photodiode. Thus the change in reflected light is sensed and converted into electrical signals for data reading purpose.

(b) WORM

The user can write data on WORM and read it as many times as desired. To reduce the access time the disk is rotated at a constant speed. Writing the data is as mentioned before i.e. by using a laser of modest density to make pits on it.

(c) Erasable Optical Disk

It is a read/write optical disk memory. Information can be written to and read from the disk. The disk contents can be erased and new data can be rewritten. So it can serve as secondary memory of a computer.

An erasable optical disk is coated with a magnetic material which does not change its magnetic property at room temperature. A laser beam together with a magnetic field is employed to read/write information.

Syllabus Topic : Industrial Application (Cutting, Welding), Application in Medical Field

> **Topics covered :** Industrial and Medical Applications of Laser

4.14.3 Industrial and Medical Applications of Laser

Lasers widely used in the various industrial production purposes and material production. It is also used in the medical instruments. Few explained are as follows.

In industries, lasers are applied to a larger extent for the following processes.

(1) **For welding and melting :** In very short time metal can be melt and then it will be evaporating. Thus accurate welding and melting of the hard material can be done very easily. Using laser perfect and non porous joints of metals are possible. The typical laser welding machine is as shown in the Fig. 4.14.2.

- Due to increased power output it is possible to use this as a welding tool. Generally CO_2 gas laser is used as cutting tool. Thus advantages of laser welding are. Very high welding rate is possible.
- It is possible to weld dissimilar metals. Complex counters can be welded using easy turning of laser beam.
- Working material is not stressed because of non-contact method.

The typical laser welding machine is as shown in Fig. 4.14.2.

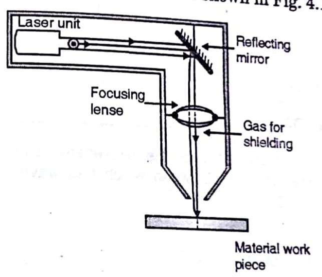


Fig. 4.14.2 : Laser welding machine

(2) **For cutting and drilling holes :**

- For some engineering applications accurate cutting and holes are required it is difficult by normal cutting and drilling machines. Generally high power pulse Nd: YAG laser is used for such operations. Pulsed laser is allowed to fall on the metal. The laser pulse increases the temperature of surface of metal. During the sort period its temperature exceeds above the boiling point of the metal, which results in accurate whole formation.

- The laser cutting of the materials is the common and useful application of the laser systems. During the cutting process the energy, absorbed by the material in the area in which the laser beam has immersed. The power densities about 10^5-10^8 W/cm² is transformed in to heat. This heat locally useful to a quick increase of the temperature of the metal piece; the fusion and/or the vaporization of the interaction zone determine the formation of accurate hole. If the passing hole has moved along the piece in working, it gives the separation of the two cutting pieces.
- **Also LASER finds its applications in :**
  1) Cutting, drilling of hard metals and diamonds .
  2) In surgery, these are used because due to the production of localized heat, they cut and seal all blood vessels simultaneously. This is known as bloodless surgery. To repair detached retina, lasers are very useful.
  3) Automatic control of rockets and satellites.
  4) Used in warfare of detecting and destroying missiles, tanks etc.
  5) High speed photography.

4.15 Solved Problems

Ex. 4.15.1 : Find the ratio of population of the two energy states of the active medium producing laser transition between which has wavelength 694.3 nm. Assume temperature 27°C.

Soln. :

Data : $\lambda = 694.3 \times 10^{-9}$ m, T = 300 °K

To find : $\frac{N}{N_0}$

Let ΔE be the energy difference between the two energy states

$$\therefore \quad N = N_0\, e^{-\Delta E/KT}$$

$$\therefore \quad \frac{N}{N_0} = e^{-\Delta E/KT}$$

$$\text{Now} \quad \Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{694.3 \times 10^{-9}}$$

$$= 2.865 \times 10^{-19}$$

$$\therefore \quad \frac{N}{N_0} = e^{-\frac{2.865 \times 10^{-19}}{138 \times 10^{-23} \times 300}}$$

$$= e^{-69.2}$$

$$\frac{N}{N_0} = \mathbf{8.82 \times 10^{-31}}$$

...Ans.

Ex. 4.15.2 : A He-Ne laser is emitting a laser beam with an average power of 4.5 mW. Find the number of photons emitted per second by the laser. The wavelength emitted is 6328 A°.

Soln. :

Data : Wavelength $\lambda = 6328 \times 10^{-10}$ m, Power = 4.5 mW = 4.5×10^{-3} W

To find : Number of photons emitted/second

We know that energy difference

$$\Delta E = h\nu = \frac{hc}{\lambda} = \frac{6.634 \times 10^{-34} \times 3 \times 10^{8}}{6328 \times 10^{-10}}$$

$$= 3.143 \times 10^{-19} \text{ J}$$

This energy difference becomes the energy of each of the emitted photon. If N is the number of photons emitted per second to give a power output of 4.5 mW then,

$$N \times \Delta E = 4.5 \times 10^{-3} \text{ W} = 4.5 \times 10^{-3} \frac{\text{J}}{\text{sec}}$$

$$= \frac{4.5 \times 10^{-3}}{3.143 \times 10^{-19}} = 1.43 \times 10^{16}$$

∴ Number of photons emitted per second is 1.43×10^{16}

...Ans.

Ex. 4.15.3 : A pulsed laser emits photons of wavelength 780 nm with 20 mW average power/pulse. Calculate the number of photons contained in each pulse if pulse duration is 10 ns.

Soln. :

Data : Wavelength of the photon $\lambda = 780 \times 10^{-9}$ m

Power of each pulse P = 20×10^{-3} J/sec.

Duration of each pulse = t = 10 ns = 10×10^{-9} s

To find : Number of photon in each pulse N = ?

$$\text{Energy of each photon} = \Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{780 \times 10^{-9}}$$

$$= 2.55 \times 10^{-19} \text{ J}$$

Energy of each pulse = power × time = $20 \times 10^{-3} \times 10 \times 10^{-9}$

$= 2 \times 10^{-10}$ J

Now $N \times \Delta E = E$

$$\therefore N = \frac{E}{\Delta E} = \frac{2 \times 10^{-10}}{2.546 \times 10^{-19}} = 7.86 \times 10^{8}$$

∴ Number of photons in each pulse is 7.86×10^8 ...Ans.

Ex. 4.15.4 : A laser beam of intensity 7.68 × 106 mW/m². It has an aperture of 5×10^{-3} m and it emits light of wavelength 7000 A°. The beam is focused with a lens of focal length 0.1 m, calculate the area and power of the image.

Soln. :

Given : $\lambda = 7200 \times 10^{-8}$ m, f = 0.1 m, $a = 5 \times 10^{-3}$ m, $I = 7.68 \times 10^{6}$ mW/m².

Formula : *Hint : for a wavelength λ of laser light, let a be the radius of beam and f is the focal length of the lens, then the incoming beam get focused into a region of area of radius r is given by,*

$$r = \frac{\lambda f}{a}, \quad P = I \pi r^2$$

where, 'λ' is wavelength of laser beam light, 'a' is the radius of beam.

f is the focal length of the lens and r is the region of radius in which the incoming beam focused.

Substituting the values we get,

$$r = \frac{7200 \times 10^{-8} \times 0.1}{5 \times 10^{-3}}$$

$$r = 1.44 \times 10^{-3} \text{ m}$$

Intensity I of beam is = 7.68×10^{6} mW/m².

$$P = 7.68 \times 10^{6}\ 3.14\ (1.44 \times 10^{-3})^2$$

$$I = 50.0 \text{ mW}$$

$$P = 50 \text{ mW}$$

Power of intense laser beam is beam is P = 50 mW. ...Ans.

Ex. 4.15.5 : Energy transitions in the a laser have two states at different temperature one is at 300 °K and another at 500 °K. If it emits light of wavelength 7000 A°, then determine the relative population.

Soln. : Given : $T_1 = 350$ K, $T_2 = 550$ K, $\lambda = 7000$

Formula : $E_2 - E_1 = \Delta E = h\nu = hc/\lambda$, Population ratio is $\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT}$

Substituting the given values with standard physical data we get,

$$\Delta E = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{7000 \times 10^{-10}}$$

$$\Delta E = 2.839 \times 10^{-19} \text{ J}$$

$$\Delta E = 1.772 \text{ eV}$$

Population at 350 °K,

$$\frac{N_2}{N_1} = \exp\left[\frac{-1.772}{8.6 \times 10^{-5} \times 350° K}\right]$$

$$= e^{-58}$$

$$\frac{N_2}{N_1} = 6.4 \times 10^{-26}$$

Population at, 550 °K

$$\frac{N_2}{N_1} = \exp\left[\frac{-1.77}{8.6 \times 10^{-5} \times 550° K}\right] = e^{-37.52}$$

$$= 5.07 \times 10^{-7}$$

Ans. : Population at 350 °K, is 6.4×10^{-26} and at 550 °K is 5.07×10^{-7}

Ex. 4.15.6 : A laser emits radiation of wavelength 8000 A°, in this phenomenon 2.8×10^{19} total number of ions (photons) are contributed. Calculate total energy per laser pulse, if energy of emitted photon is 1.55 eV.

Soln. :

Given : $\lambda = 8000 \times 10^{-10}$ m

Total photons $= 2.8 \times 10^{19}$

$E = 1.55 \text{ eV} = 1.55 \times 1.6 \times 10^{-19}$ J.

Formula :

Energy per pulse = Energy of single photon / ion × Total number of ions

Substituting the values we get,

Energy / pulse = $1.55 \times 1.6 \times 10^{-19} \times 2.8 \times 10^{19}$

$= 1.55 \times 1.61 \times 2.8 = 6.98$ J

Ans. : Energy per pulse = 6.98 Joule.

Ex. 4.15.7 : If wave length of the laser beam is 6550 A°. Find out energy and momentum of the photon.

Soln. :

Given : $\lambda = 6550 \times 10^{-10}$ m., E = ... eV or J.

(Momentum) p = ... kg-m/s.

Formula : $E = h\nu = \frac{hc}{\lambda}$, $p = \frac{E}{c}$

Substituting the values we get,

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{6550 \times 10^{-10}}$$

$$= \frac{6.63 \times 3 \times 10^{-34} \times 10^{8} \times 10^{10}}{6550}$$

$$= 2.986 \times 10^{-3} \times 10^{-16}$$

$$E = 2.986 \times 10^{-19}\ J$$

Momentum $p = \frac{2.986 \times 10^{-19}}{3 \times 10^{8}} = \frac{2.986}{3} \times 10^{-27}$

$$= 0.995 \times 10^{-27}\ \text{kg-m/s}$$

$$p = 9.95 \times 10^{-26}\ \text{kg-m/s}$$

Ans. : Energy of photon $E = 2.986 \times 10^{-19}$ J and

Momentum of the photon $p = 9.95 \times 10^{-26}$ kg-m/s.

Ex. 4.15.8 : Find the ratio of population states i.e. upper level and lower level energy states. When optical pumping mechanism is carried out at 27°C. Emission of photons takes place along with its wave length 6982×10^{-8} m.

Soln. :

Given : $T = 27°C = 300\ K$, $\lambda = 6982 \times 10^{-8}\ m$, $K = 1.38 \times 10^{-23}\ J/K$

Formula : $E_2 - E_1 = \Delta E = \frac{hc}{\lambda}$

$$\frac{N_2}{N_1} = e^{\frac{-\Delta E}{KT}}$$

Substituting the values we get,

$$\Delta E = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{6982 \times 10^{-8}}$$

$$\Delta E = \frac{6.625 \times 3 \times 10^{-34} \times 10^{16}}{6982}$$

$$\Delta E = 2.846 \times 10^{-3} \times 10^{-34} \times 10^{16}$$

$$\Delta E = 2.846 \times 10^{-21}\ J$$

Also we have $\frac{N_2}{N_1} = e^{-\left(\frac{2.846 \times 10^{-21}}{1.38 \times 10^{-23} \times 300}\right)}$

$$= e^{-\left(\frac{2.846}{1.38 \times 300} \times 10^{2}\right)} = e^{-\left(\frac{2.846}{414} \times 10^{2}\right)}$$

$$= e^{-6.874 \times 10^{-1}} = e^{-68.743}$$

$$\frac{N_2}{N_1} = 1.39773 \times 10^{-30}$$

Ans. : Ratio of population of two energy states is $\frac{N_2}{N_1} = 1.39773 \times 10^{-30}$.

Ex. 4.15.9 : A certain laser beam is focused on target, a small spot of radius 1.44×10^{-5} m is observed. It emits 4.5×10^{-3} Joule of energy in the time 10^{-9} sec. Calculate the power per unit area in this laser system.

Soln. :

Given : $r = 1.44 \times 10^{-5}\ m$, $E = 4.5 \times 10^{-3}\ J$ (Energy), $t = 10^{-9}$ sec.

Formula : $P = \frac{E}{t}$, $A = \pi r^2$, $I = \frac{P}{A}$

[**Hint :** Power per unit area is intensity I]

Substituting the values we get,

$$P = \frac{4.5 \times 10^{-3}}{10^{-9}}$$

$$= 4.5 \times 10^{6}$$

(Power in Watt) $P = 4.5 \times 10^{6}\ W.$

$$\therefore A = 3.14 \times (1.44)^2 \times 10^{-10}\ m^2$$

$$= 6.511 \times 10^{-10}\ m^2$$

$\therefore$ Power per unit area is I

$$I = \frac{4.5 \times 10^{6}}{6.511 \times 10^{-10}} = 0.6911 \times 10^{16}$$

$$I = 6.911 \times 10^{16}\ W/m^2$$

Ans. : Power per unit area is $I = 6.911 \times 10^{16}\ W/m^2$.

Ex. 4.15.10 : For a source, the coherence time is 10^{-9} sec. Find out the degree of non-monochromaticity for its original wave length 5891×10^{-10} m.

Soln. :

Given : $T_c = 10^{-9}$ sec., $\lambda_0 = 5891 \times 10^{-10}\ m$, $\Delta\nu = ?$ Hz

Degree of non-monochromaticity

Formula : $\Delta\nu = \frac{1}{T_c}$ ...($\because$ ν = frequency)

$\frac{\Delta\nu}{\nu_0}$ = Degree of non-monochromaticity

$$\nu_0 = \frac{C}{\lambda_0}$$

Substituting the values we get,

$$\Delta\nu = \frac{1}{10^{-9}}$$

$$\Delta\nu = 10^9 \text{ Hz}$$

Now for $\lambda_0 = 5891 \times 10^{-10}$

$$\nu_0 = \frac{3 \times 10^8}{5891 \times 10^{-10}}$$

$$\nu_0 = \frac{3}{5891} \times 10^{18}$$

$$= 5.0925 \times 10^{-4} \times 10^{18}$$

$$\nu_0 = 5.0925 \times 10^{14}$$

$$\therefore \frac{\Delta\nu}{\nu_0} = \frac{10^9}{5.0925 \times 10^{14}}$$

$$\therefore \frac{\Delta\nu}{\nu_0} = \frac{\cdot 10^{-5}}{5.0925} = 0.19636 \times 10^{-5}$$

$$= 19.636 \times 10^{-3}$$

Non-monochromaticity $= 19.636 \times 10^{-3}$

Ans. : Non-monochromaticity of given source is 19.636 $\times 10^{-3}$

A Quick Revision

- The transition of electrons from lower energy E_1 to a higher excited state after absorption of an incident photon is known as stimulated absorption.
- If an excited electron returns to the lower state on its our accord by releasing an energy equivalent to $h_2 = E_2 - E_1$ then the process is known as spontaneous emission.
- If an electron in excited state returns of the lower energy level due to the aid of an external agency then it is know as stimulated emission. The electron jumps to the lower energy level by giving out an energy equivalent to $E_2 - E_1$.
- Characteristics of stimulated emission :

(i) The emitted photon is identical to the incident photon in all respects.
(ii) Both the photon has same frequency.
(iii) Both the photon travel in the same direction.
(iv) Multiplication of photons takes place in process.

- A medium in which light gets amplified and where we get the condition of population inversion and hence laser action is called an active medium.
- Normally the number of atoms in the ground state will be more than that of the higher energy state. But by due to some method if the number of atoms in the higher energy state is made higher than the ground state then that condition is known as population inversion.
- The process of raising large number of atoms from lower energy level to a higher energy level is called pumping.
- Some types of pumping are :
  (i) Optical pumping
  (ii) Electrical pumping
  (iii) Chemical pumping
  (iv) In elastic-atom-atom collisions
  (v) Direct pumping
- Laser light is highly directional, monochromatic coherent, intense and bright.
- Life time is the maximum amount of time for which an electron can stay in an excited state.
- Metastable state is the state where an electron stays for maximum amount of time.
- The conditions of laser action are :
  (i) Population inversion
  (ii) Rate of stimulated emission should be more than the rate of absorption.
  (iii) There should be a upper excited level having long life time and lower level that decays fastly to the ground level.
  (iv) Active medium, pump and optical resonance are essential.
- The principle behind a gas laser :

Two gases A and B whose metastable state are same are mixed. One is pumped electrically which in turn collides with the other and raises it also to the metastable state. Now this state is stimulated externally resulting in laser action. This is known as atomic collision and resonance transfer of energy.

- When forward bias is applied to the pn diode like GaAs the recombination of electrons and holes within the junction region results in a radiation. This becomes a laser when the forward bias is increased.
- Electron energy level, each electron energy level is associated with nearly equally spaced vibrational levels and each vibrational level in turn has a number of rotational levels.
- The three modes of vibrations are symmetric stretching mode, bending mode and asymmetric stretching mode.
- Helium improves the conductivity of heat to the walls of the tube and also depopulates the lower levels.
- Holography is a type of photography which consists of a recorded intensity pattern which has both the amplitude and phase recorded in it.
- Optical resonator consists of two mirrors facing each other. The active medium is placed in the cavity. One of the mirrors is 100% reflecting the other mirror is partially reflecting.
- It is used for amplification of laser.

Review Questions

Q. 1 Derive Einstein's relation for stimulated emission and hence explain for the existence of stimulated emission?

Q. 2 What is a gas laser? Explain the working of He-Ne laser with relevant diagram?

Q. 3 What is a molecular gas laser? Explain with a neat sketch the construction and working of a CO_2 laser?

Q. 4 What is a Semiconductor laser ? Explain the construction and working of a Ga-As laser ?

Q. 5 Explain the terms :

(a) Spontaneous emission (b) Stimulated emission

(c) Metastable state (d) Population inversion.

Q. 6 Write a note on Nd : YAG laser.

Q. 7 Discuss holography as an application of laser.

4.16 University Questions

May 2013

Q. 1 What is a population inversion state? Explain its significance in the operation of LASER.
(*Ans. : Refer section 4.4*) (3 Marks)

Q. 2 With a neat energy level diagram describe the construction and working of He-Ne laser. What are its merits and demerits.
(*Ans. : Refer section 4.11*) (8 Marks)

Dec. 2013

Q. 1 What is the difference between spontaneous and stimulated emission?
(*Ans. : Refer section 4.2.3*) (3 Marks)

Q. 2 What is the difference between holography and photograph? Discuss the construction and reconstruction of image in holography with neat diagram.
(*Ans. : Refer section 4.14*) (8 Marks)

May 2014

Q. 1 What is acronym of 'LASER' ? How are they different than X-rays ?
(*Ans. : Refer section 4.1*) (3 Marks)

Q. 2 What is holography? Explain its construction and reconstruction with neat diagram.
(*Ans. : Refer section 4.14.1*) (8 Marks)

Dec. 2014

Q. 1 Differentiate spontaneous and stimulated emission of radiation.
(*Ans. : Refer section 4.2.3*) (3 Marks)

Q. 2 With neat energy level diagram describe the construction and working of He-Ne Laser.
(*Ans. : Refer section 4.11*) (8 Marks)

May 2015

Q. 1 Compare light from ordinary source with laser light.
(*Ans. : Refer section 4.10*) (3 Marks)

Q. 2 Explain construction and working of He-Ne laser. What are its merits ?
(*Ans. : Refer section 4.11*) (8 Marks)

Dec. 2015

Q. 1 Differentiate spontaneous and stimulated emission process related to laser operation.
(*Ans. : Refer section 4.2.3*) (3 Marks)

Q. 2 What is Holography ? Explain the construction and reconstruction of Hologram with neat diagrams.
(*Ans. : Refer section 4.14.1*) (8 Marks)

May 2016

Q. 1 Explain the term stimulated emission and population inversion. *(Ans. : Refer sections 4.2.3 and 4.4)* (3 Marks)

Q. 2 With neat energy level diagram, explain principle, construction and working of Nd :YAG laser. *(Ans. : Refer section 4.12)* (8 Marks)

Dec. 2016

Q. 1 What is pumping in LASER ? Give the types of pumping. *(Ans. : Refer section 4.6)* (5 Marks)

Q. 2 What is the fundamental principle of a Hologram ? How is it produced and how the real image is constructed by it. *(Ans. : Refer section 4.14.1)* (8 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Quantum Processed as Absorption

Q. 1 What is acronym of 'LASER' ? How are they different than X-rays ? *(Ans.: Refer section 4.1)* **(May 2014)**

Q. 2 Write short note on : Absorption. *(Ans.: Refer section 4.2.1)*

Syllabus Topic : Spontaneous Absorption

Q.1 Explain the terms Spontaneous emission. *(Ans.: Refer section 4.2.2)*

Syllabus Topic : Stimulated Emission

Q. 1 Explain the term stimulated emission and population inversion. *(Ans. : Refer section 4.2.3)* **(May 2016)**

Q. 2 What is the difference between spontaneous and stimulated emission ? *(Ans. : Refer section 4.2.3)* **(Dec. 2013)**

OR

Differentiate spontaneous and stimulated emission of radiation. *(Ans. : Refer section 4.2.3)* **(Dec. 2014, Dec. 2015)**

Syllabus Topic : Population Inversion

Q. 1 What is a population inversion state ? Explain its significance in the operation of LASER. *(Ans. : Refer section 4.4)* **(May 2013)**

OR

Explain the term population inversion. *(Ans. : Refer section 4.4)* **(May 2016)**

Syllabus Topic : Metastable States

Q. 1 Write short note on : Metastable states. *(Ans. : Refer section 4.5)*

Syllabus Topic : Pumping

Q. 1 What is pumping in LASER ? Give the types of pumping. *(Ans. : Refer section 4.6)* **(Dec. 2016)**

Syllabus Topic : Resonance Cavity

Q. 1 Write short note on : Resonance Cavity. *(Ans. : Refer section 4.8)*

Syllabus Topic : Einstein's Equation

Q. 1 Derive Einstein's relation for stimulated emission. *(Ans. : Refer section 4.9)*

Q. 2 Compare light from ordinary source with laser light. *(Ans. : Refer section 4.10)* **(May 2015)**

Syllabus Topic : Helium Neon Laser

Q. 1 With a neat energy level diagram describe the construction and working of He-Ne laser. What are its merits and demerits. *(Ans. : Refer section 4.11)* **(May 2013, Dec. 2014, May 2015)**

Syllabus Topic : Nd : YAG Laser

Q. 1 With neat energy level diagram, explain principle, construction and working of Nd : YAG laser. *(Ans. : Refer section 4.12)* **(May 2016)**

Syllabus Topic : Semiconductor Laser

Q. 1 What are the merits and demerits of Semiconductor Diode Laser ?

Syllabus Topic : Application of Lasers – Holography (Construction and Reconstruction of holograms)

Q. 1 What is holography ? Explain its construction and reconstruction with neat diagram. *(Ans. : Refer section 4.14.1)* **(May 2014, Dec. 2015)**

OR

Q. 2 What is the fundamental principle of a Hologram ? How is it produced and how the real image is constructed by it. *(Ans. : Refer section 4.14.1)* **(Dec. 2016)**

Syllabus Topic : Industrial Application (Cutting, Welding)

Q.1 Explain Industrial and Medical Applications of Laser

Solved Problems

Q. 1	Refer Ex. 4.15.1	Q. 4	Refer Ex. 4.15.4
Q. 2	Refer Ex. 4.15.2	Q. 5	Refer Ex. 4.15.5
Q. 3	Refer Ex. 4.15.3	Q. 6	Refer Ex. 4.15.6

CHAPTER 5

Module 3

Fibre Optics

Syllabus

Total internal reflection; Numerical Aperture; critical angle; angle of acceptance; V number; number of modes of propagation; types of optical fiber; Losses in optical fibre (Attenuation and dispersion) Applications of optical fibre - Fibre optic communication system; sensors (Pressure, temperature, smoke, water level), applications in medical field.

5.1 Introduction

With the advent of fibre optics, data can be very easily and rapidly transmitted over long distances through glass fibres of the size of human hair. Keeping latest technology and development in mind, it is expected that very soon it will become as common as electrical wiring in the house or in automobile. Fibre optics will take its own time to developfully, however, it is increasingly replacing wire transmission lines in communication systems.

5.2 What are Optical Fibres ?

Physically, an optical fibre is a very thin and flexible medium, having a cylindrical shape consisting of three sections :

(a) Core

(b) Cladding

(c) Buffer coating or Jacket or Sheath

Optical fibre is light equivalent of microwave waveguides with additional advantage of a very thin bandwidth.

5.3 Propagation of Light in Different Media

Propagation of light is based on an important parameter which determines the optical characteristics of the material. This parameter is the refractive index of the material in which it passes.

The refractive index is expressed as

$$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in material}} \quad ...(5.3.1)$$

As we know that $\mu > 1$ for all known materials, light travels more slowly in a material than in vacuum.

Velocity of light in a substance is expressed as

$$v_{sub} = \frac{c}{\mu} \quad ...(5.3.2)$$

Where, c = velocity of light in vacuum

Syllabus Topic : Total Internal Reflection

> **Topics Covered :** Total Internal Reflection, Basic Construction of Optical Fibre

5.3.1 Total Internal Reflection

MU - Dec. 2014, Dec. 2015

- If light wave enters at one end of a fibre, most of it propagates down the length of the fibre and comes out from the other end of the fibre.
- There may be some loss due to leakages through side walls of the fibre. One should know how light gets guided through the fibre. The reason for confining the light beam inside the fibre is **"Total Internal Reflection"** (TIR). Fibre obeys laws of *"reflection and refraction of light waves"*.
- The light ray which enters at one end of a fibre at a small angle to the axis of the fibre, follows a zig-zag path due to series of reflections down the line of the fibre.
- The total internal reflection at the wall of fibre can occur only if :
  - (a) The glass around the axis of the fibre should have higher refractive index (μ_1) than that of the material (cladding), surrounding the fibre (μ_2).
  - (b) The light should be incident at an angle of incidence θ which is greater than the critical angle θ_c.

$$\therefore \sin \theta_c = \frac{\mu_2}{\mu_1} \quad ...(5.3.3)$$

- Refraction and total internal reflection are shown in Fig. 5.3.1.

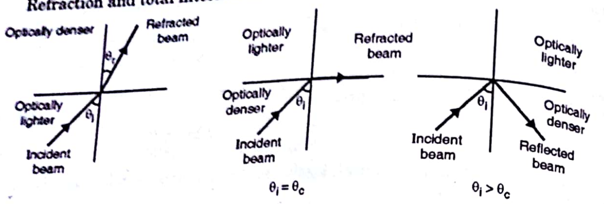


Fig. 5.3.1 : Total internal reflection

5.4 Basic Construction of Optical Fibre

- As seen in Section 5.3, light entered in optical fibre undergoes successive total internal reflections and follows zig-zag path. Hence it is essential to construct optical fibre so that all the conditions required for TIR are satisfied.
- In Fig. 5.4.1 we have cross-sectional view of optical fibre. In fact it is represented by three co-axial regions.

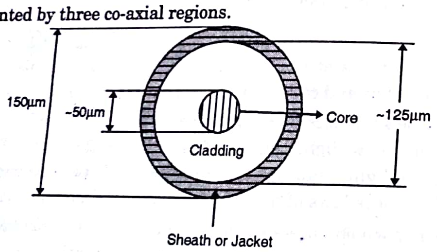


Fig. 5.4.1 : Cross sectional view of optical fibre

- The innermost region is known as core of R.I. μ_1, it is surrounded by a solid dielectric material having R.I μ_2 such that $\mu_1 > \mu_2$ so that TIR takes place.
- Cladding is added upto :
  1. Reduce scattering loss at core surface.
  2. Enhance mechanical strength of fibre.
  3. Protect the core from absorbing surface contaminants with which it could come in contact.

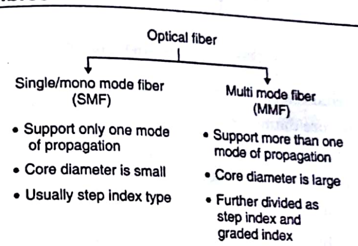
- Most of the fibres are encapsulated in an abrasion resistant plastic material called **buffer coating, sheath** or **jacket.** This is necessary to add up further strength to the fibre and to isolate it from small geometrical irregularities, distortions or roughness of adjacent surface.

Syllabus Topic : Types of Optical Fibres

> **Topics Covered :** Types of Fibres, Step Index Fibre, Graded Index Fibre, Comparison between SI and GRIN Fibre, Difference between SI and GRIN Fibre

5.5 Types of Fibres

MU - Dec. 2016



5.5.1 Step Index Fibre

- As shown in Fig. 5.5.1, the simplest type of an optical fibre consists of a thin cylindrical structure of transparent glossy material of uniform refractive index μ_1, surrounded by a cladding of another material of uniform but slightly lower refractive index μ_2.
- These fibres are referred to as step index fibres due to the discontinuity of the index profile at the core cladding interface.

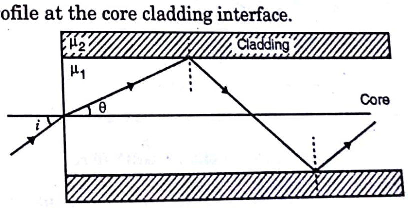


Fig. 5.5.1

- If the core radius is 'a' and the cladding thickness is 'b' then the corresponding refractive index distribution is given by

$$\mu(r) = \mu_1 \quad (0 < r < a) \quad \text{core}$$
$$= \mu_2 \quad r > a \quad \text{cladding}$$

- Refractive index profile is given as shown in Fig. 5.5.1(a).

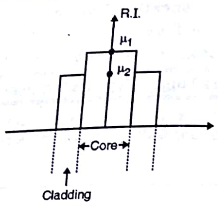


Fig. 5.5.1(a) : Refractive index profile

- The important characteristics of step index fibre are :
  1. Very small core diameter.
  2. Low numerical aperture.
  3. High attenuation with respect to GRIN
  4. Very high bandwidth.

5.5.2 Graded Index Fibre

- In contrast to step index fibre, the graded index fibre has its core refractive index gradually decreasing in a nearly parabolic manner from a maximum value at the center of the core to a content value at the core-cladding interface, as shown in Fig. 5.5.2.

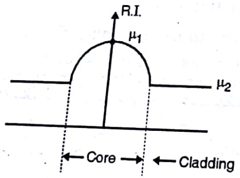


Fig. 5.5.2 : R.I. profile for GRIN fibre

- The variation in refractive index is achieved by using concentric layers of different refractive indices. Such a profile causes a periodic focusing of the light propagating through the fibre.
- It can be proved that in a parabolic index medium, the ray paths are sinusoidal. The rays making larger angles with the axis traverses a larger path length in a region of lower refractive index (and hence larger speed).

- The longer path length is compensated by a greater average speed such that all rays take the same time in traversing the fibre. This is shown in Fig. 5.5.3.

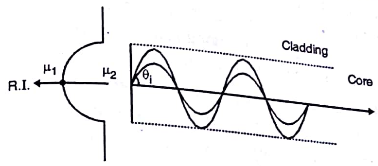


Fig. 5.5.3 : Propagation in GRIN fibre

- This is considered to be the main difference with compare to SI fibre. In SI fibre, the various light waves, travelling along the core, will have propagation paths of different lengths.
- Hence they will take different times to reach a given destination. Thus a distortion is produced and is called a transit-time dispersion.

5.5.3 Comparison between SI and GRIN Fibre

1. Attenuation
2. Mode dispersion
3. Numerical Aperture (NA)

1. Attenuation

- GRIN fibre tends to have generally lower attenuation than SI fibre. There is no fundamental reason why a SI fibre should have higher attenuation than a GRIN fibre.
- Atleast theoretically they are made of the exact same kind of glass with same amount of energy absorption. But as a practical matter, in SI fibre, there may be some irregularities at the interface between the core and the cladding which in turn will contribute to mode conversion and some reflection loss. But in GRIN fibre there is no such interface.
- Hence, the GRIN fibre has no real counterpart for the loss mechanism.

2. Mode dispersion

- There is an inherent advantage in mode dispersion for the graded index fibre as compared to the SI fibre.

- If a wavefront had originally crossed the fibre axis with smaller inclinations, it would have followed a path with a smaller sinusoidal amplitude. As a result it would have spent more time wading through glass with higher index and would have traveled more slowly.
- The rays having wide swinging actually travel further, but they spend most of their time traveling at a faster rate.

3. **Numerical Aperture (NA) : (Discussed further in Section 5.6)**
   - It has been observed that for a given fibre diameter, the N.A. of GRIN fibre is generally smaller than the one on a SI fibre.
   - For a GRIN fibre, with attenuation between 5 and 10 dB/km, the NA will tend to run between 0.16 and 0.2. Whereas for SI fibre of the same physical size, with a loss of the order of 12 dB/km will tend to have NA of the order of 0.2 to 0.35.

5.5.4 Difference between SI and GRIN Fibre

MU- May 2014

Sr. No.	SI	GRIN
1.	Discontinuity of index profile at core-cladding junction.	R.I of core decreases gradually to attain R.I of cladding at core-cladding junction.
2.	R.I. of core is constant.	R.I. of core decreases nearly in parabolic manner.
3.	High attenuation	Low attenuation
4.	For a given diameter the N.A. is greater	For a given diameter the N.A. is lesser compare to SI.

Syllabus Topic : Numerical Aperture, Critical Angle, Angle of Acceptance

> **Topics Covered :** Acceptance Angle and Numerical Aperture of a Fibre, Numerical Aperture for a Graded Index Fibre

5.6 Acceptance Angle and Numerical Aperture of a Fibre

MU - Dec. 2013, May 2014, Dec. 2014, May 2015, Dec. 2015

- We know that in SI fibre any light wave which travels along the core and meets the cladding, with $\theta > \theta_c$, will be totally reflected. This reflected ray will then meet the opposite surface of the cladding again at the $\theta > \theta_c$ and so it will be again totally reflected. Any other light wave, which is meeting the core cladding interface at or above θ_c, will also be totally reflected and hence will propagate along the core.

- However, any light wave meeting the core cladding interface at any angle below θ_c will pass into and be absorbed by cladding. For a fibre for which $\mu_1 > \mu_2$, light impinging on the core within a critical angle θ_c, is coupled into the fibre and will propagate.
- If the external incident angle is θ_0 corresponding to the critical angle of incidence θ_c at the core-cladding interface of the fibre as shown in Fig. 5.6.1, the light will stay in the fibre.
- So any light wave impinging on the core within this maximum external incident angle θ_0 is coupled into the fibre and will propagate. This angle is called as **angle of acceptance.**

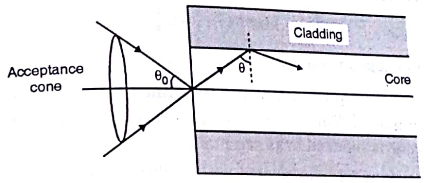
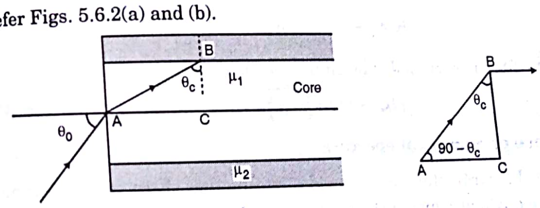


Fig. 5.6.1

- Let us now derive a mathematical relation for the acceptance angle θ_0. Refer Figs. 5.6.2(a) and (b).



(a) (b)

Fig. 5.6.2

- Now from Figs. 5.6.2(a) and (b) and applying law of refraction at A and B we get,

$$\mu_0 \sin\theta_0 = \mu_1 \sin(90-\theta_c) \quad ...(5.6.1)$$

$$\mu_1 \sin\theta_c = \mu_2 \sin 90 \quad ...(5.6.2)$$

$$\sin\theta_c = \frac{\mu_2}{\mu_1} \quad \text{and} \quad \mu_0 \sin\theta_0 = \mu_1 \cos\theta_c$$

$$\mu_0 \sin\theta_0 = \mu_1\sqrt{1-\sin^2\theta_c}$$

$$\mu_0 \sin\theta_0 = \mu_1\sqrt{1-\frac{\mu_2^2}{\mu_1^2}} = \sqrt{\mu_1^2-\mu_2^2}$$

$$\therefore \sin\theta_0 = \frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0} \qquad ...(5.6.3)$$

$$\text{Acceptance angle } \theta_0 = \sin^{-1}\frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0} \qquad ...(5.6.4)$$

If fibre has surrounding medium as air, $\mu_0 = 1$

$$\therefore \text{Acceptance angle} \quad \theta_0 = \sin^{-1}\sqrt{\mu_1^2-\mu_2^2} \qquad ...(5.6.5)$$

Significance of acceptance angle

- The light which travels within a cone defined by the acceptance angle is trapped and guided. This is fundamental property of light propagation in a fibre. This cone is referred to as acceptance cone.
- Another important term associated with a fibre is the numerical aperture – sometimes called as the **figure of merit for optical fibres.** Numerical aperture of an optical fibre is defined as (using Equation 5.6.3).

$$\text{N.A.} = \sin\theta_0\,(\text{max}) = \frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0} \qquad ...(5.6.6)$$

If the fibre is surrounded by air ($\mu_0 = 1$) then

$$\text{NA} = \sqrt{\mu_1^2-\mu_2^2} \qquad ...(5.6.7)$$

Significance of numerical aperture

- *NA is the parameter which provides information about the acceptance angle i.e. the angle at which if the light ray enters the fibre it is sure to have Total Internal Reflection experienced, that too in terms of parameters associated with fibre i.e. refractive indices of core and cladding.*
- *Generally μ_1 is only a few percentage greater than μ_2.*

$$\therefore \text{NA} = \sqrt{(\mu_1+\mu_2)(\mu_1-\mu_2)}$$

$$\simeq \sqrt{2\mu_1(\mu_1-\mu_2)} \qquad (\mu_1+\mu_2 \simeq \mu_1)$$

- On defining $\Delta = \frac{\mu_1-\mu_2}{\mu_1}$ = Fractional refractive index change

$$\therefore \text{NA} = \sqrt{2\mu_1^2 \cdot \frac{\mu_1-\mu_2}{\mu_1}} = \mu_1\sqrt{2\Delta} \qquad ...(5.6.8)$$

Fig. 5.6.3 shows the variation of NA with acceptance angle.

- It has been observed that NA for the fibres used in short distance communication are in range of 0.4 to 0.5, where as for long distance communications NA are in the range of 0.1 to 0.3. Now one question may come to readers mind – **which NA is expected for better performance. Smaller NA is not an advantage because it makes it harder to launch power in to the fibre.**

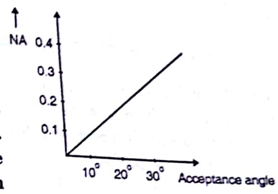


Fig. 5.6.3 : NA → Acceptance angle

Ex. 5.6.1 : The N.A. of an optical fibre is 0.5 and core R.I. is 1.54. Find R.I. of cladding.

MU - Dec. 2014, 3 Marks

Soln. :

N. A. = 0.5, n_1 = 1.54

To find n_2

$$\text{Formula N.A} = \left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore n_2 = \left(\sqrt{n_1^2 - (\text{N.A})^2}\right) = \sqrt{1.54^2 - 0.5^2} = 1.4565 \qquad \textbf{... Ans.}$$

5.7 Numerical Aperture for a Graded Index Fibre

- So far we have discussed and calculated the NA of a step index fibre which is very simple in calculation.
- In GRIN fibre NA is a function of position across the core, whereas the same is constant across the core for step index fibre. Now to get an idea about NA for GRIN fibre, we are to introduce the term local NA which is a function of radius. From the consideration of the geometrical optics it is evident that light incident on the fibre core at position r will propagate as a guided mode only if it is within the local numerical aperture NA (r) at that point.

NA (r) = local numerical aperture at r is expressed as

$$\text{NA}(r) = [\mu_1^2(r) - \mu_2^2(r)]^{1/2}$$

$$\simeq NA(0)\sqrt{1-\left(\frac{r}{a}\right)^p} \quad \text{for } r \le a$$

$$= 0 \text{ for } r > 0 \qquad ...(5.7.1)$$

Where, NA (0) = axial numerical aperture i.e. NA at center of fibre core.

$$NA(0) = [\mu_1^2 - \mu_2^2]^{1/2}$$

$$\simeq \mu_1\sqrt{2\Delta} \qquad ...(5.7.2)$$

where $\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$ (fractional refractive index change)

- From Equation (5.7.1) it is evident that for a graded index fibre NA decreases from axial numerical aperture NA (0) to zero as r increases from zero to core radius a. Fig. 5.7.1 represents NA for graded index fibre for various (r/a) profile.

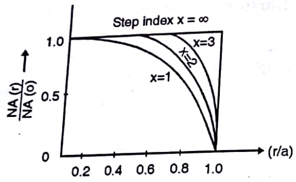


Fig. 5.7.1 : Comparison of the NA for fibres having various (r/a) profile

Syllabus Topic : V-number

> *Topics Covered :* V-Number

5.8 V-Number

MU - Dec. 2016

- In optical fibre the light propagates in the same way as electromagnetic wave propagates. When confined to a duct or guide, it propagates like electromagnetic wave, but at a much higher frequency. In optical fibre communication, two bands are presently famous for their low losses.
  (1) 800 nm to 900 nm
  (2) 1200 nm to 1400 nm
- The propagation is possible through number of modes like TE, TM, hybrid HE and hybrid EH (Discussion beyond the scope of this syllabus). It is also important to know that all modes will not be there in a single fibre. The number of modes supported by a fibre is determined by an important parameter called "cut off parameter". It is also termed as "normalized frequency" or "cut off" or "V-number".

- The mathematical expression for the V-number is,

$$V = \pi\frac{d}{\lambda_0}\sqrt{\mu_1^2-\mu_2^2} \qquad ...(5.8.1)$$

Where λ_0 = free space wavelength

d = 2a = diameter of the core (a = radius of the core)

$$\therefore V = \frac{\pi d}{\lambda}(NA)$$

$$= \frac{\pi d}{\lambda}\mu_1\sqrt{2\Delta} \qquad ...(5.8.2)$$

- The maximum number of modes N_m supported by SI fibre is given by

$$N_m \simeq \frac{1}{2}V^2 \quad \text{(provided V-number is considerably larger than unity)}$$

Significance

- Each of the mode mentioned above has a particular value of V-parameter, below which the mode will be cut off.
- For V < 2.405 the fibre can support only one mode and hence it is Single Mode Fibre (SMF), for V > 2.405 it is MMF (Multi Mode Fibre) as it can support many modes.

Ex. 5.8.1 : Relative R.I. of fibre is 0.055, when core R.I. is 1.48. Find N.A., cladding R.I., acceptance angle, normalized frequency (V) and the number of guided modes, when wavelength of light propagated is 1μm and radius of the core is 50 μm.

Soln. :

$$\text{Relative R.I.} = \Delta = \frac{n_1 - n_2}{n_1} = 0.055$$

as $n_1 = 1.48$, $\qquad 0.055 = \frac{1.48 - n_2}{n_1}$

$\therefore n_2 = 1.398$

now N.A. $= n_1\sqrt{2\Delta} = 0.49$

acceptance angle $= \sin^{-1}(NA) = 29.34$

$$V \text{ number} = \frac{2\pi a\,(NA)}{\lambda}$$

$$= \frac{2\pi\,(50 \times 10^{-6})\,(0.49)}{1 \times 10^{-6}} = 153.93$$

$$\text{Number of guided mode} = \frac{V^2}{2} = 11848$$

Syllabus Topic : Losses in Optical Fibre(Attenuation and Dispersion)

> **Topics Covered :** Losses in Fibres, Attenuation, Dispersion

5.9 Losses in Fibres

5.9.1 Attenuation

- Attenuation means "loss of optical power" in fibre itself. As the ray propagates ahead in fibre, it gets attenuated. It is defined as ratio of optical output power from a fibre of the optical fibre of length L to the input optical power.

  ∴ Fibre attenuation is given as

$$\alpha = \frac{10}{L} \log \frac{P_{in}}{P_{out}} \frac{dB}{km} \qquad \ldots(5.9.1)$$

- Attenuation is wavelength dependent, and therefore it is mentioned along with wavelength. It is observed that attenuation → wavelength curve shows minimum attenuation at around a particular band of optical wavelengths. The band of the wavelengths at which the attenuation remains minimum is called an "optical window". Such windows are selected for data transmission through fibre.

5.9.2 Dispersion

MU - May 2013

- The light pulses that propagate through a fibre suffer several dispersion effects. The dispersion effects spread the output pulse and changes its shape, so that it may merge into the succeeding or previous pulses as shown in Fig. 5.9.1. Hence pulses are separated out, affecting the maximum bit rate.
- There are three distinct types of dispersions, using separate mechanism are observed :

(a) Intermodal dispersion
(b) Waveguide dispersion
(c) Material dispersion

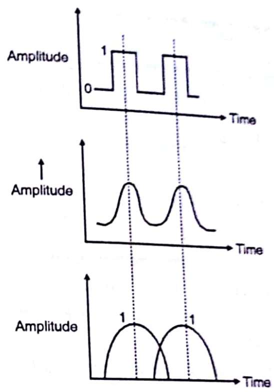


Fig. 5.9.1 : Variation in waveform

(a) Intermodal dispersion

The rays following zig-zag paths while traveling in an optical fibre, and hence the rays, under different modes will travel different total lengths of path and emerge at the far end at slightly different times. This is known as **intermodal dispersion.**

(b) Waveguide dispersion

The effective refractive index for any mode varies with wavelength, which causes pulse spreading just like the variation in refractive index. This is known as **waveguide dispersion.**

(c) Material dispersion (or material chromatic dispersion)

- As the refractive index of a material is a function of wavelength (i.e. colour) of light.

- The refractive index of silica is different for different frequency of wave and hence different speeds of light. Waves with shorter wavelengths travel slower than long wavelengths.
- This makes narrow pulses of light broaded as they travel down the optical fibre. This is known as **material dispersion.**

Syllabus Topic : Applications of fibre : Fibre Optic Communication System, Sensors(Pressure, Temperature, Water Level), Applications in Medical Field

> **Topics Covered :** Applications of Fibre, Solved problems

5.10 Applications of Fibre

MU - May 2015, May 2016

Advantages of optical fibre

By now reader must have concluded that :

(i) Through optical fibre light rays passes using concept of Total Internal Reflection (TIR) and hence there is no loss up to few kilometers.

(ii) Diameter of the fibre is very small and hence network made of optical fibre requires very less space.

(iii) The power required to operate it is very small compare to conventional copper wire.

(iv) Since optical fibres are made of either silica or plastic and hence it is very cheap compare to its metallic equivalent.

(v) Optical fibres are not much affected by parameters like pressure, temperature, twist, salinity etc. (Except for specially designed fibres)

And hence optical fibres have found many applications by now and we are getting more and more added every day. Few of them are listed below :

(1) Communication system
(2) Medicine
(3) In aircrafts cabling
(4) Data security
(5) Communication network in high voltage region
(6) Entertainment applications
(7) Optical fibre sensor

(1) **Communication system**

- The fibre optical commutations system is very much similar to that of a traditional communication system. Fig. 5.10.1 shows optical fibre communication system Light waves which are carrier waves, are generated by a LED or a laser diode.
- The intensity of light is proportional to the current passing through it. When the message in electrical form is fed to the optic source, the light output from it follows the variations of message. In order to have efficient transmission through the fibre the frequency of operation is chosen to lie in the optical window region of the fibre.
- The optical signal traveling through the fibre will become progressively attenuated and distorted because of various losses and dispersion in the fibre. Hence repeaters are used in the transmission line to amplify and reshape the signal.

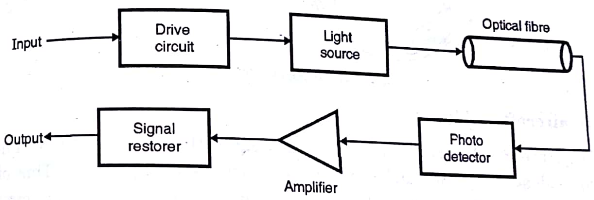


Fig. 5.10.1

- At the received end, an output coupler directs the light emerging from the fibre channel to a photo detector. The photo detector converts the light wave into an electric current. The message is contained in the detector output current.
- The detector output current is filtered to extract the message and it is amplified if needed.
- It is then fed to a suitable transducer to convert into an audio or video form.

(2) **Medicine**

- As shown in Fig. 5.10.2 bundle of fibres, which forms a flexible cable of few millimeter thickness is inserted into human body through certain openings or by drilling a small hole.

- Usually the outer cables carry light from external source to the required place and inner cables carry the image of that illuminated part to the observer.
- The image carrier has great many applications in medicine to investigate the interior of body. One can have a direct magnified view of internal parts of anatomy. The instruments used in medical endoscopy are Bronchioscopes, Oesophagoscopes, Vaginascopes etc.

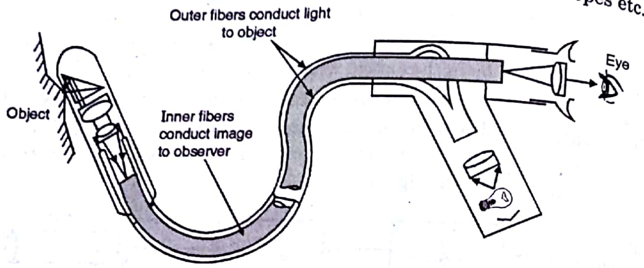


Fig. 5.10.2 : Image Carrier

(3) In aircrafts cabling

They are much smaller in size and weight than an electrical line of equivalent bandwidth and thus occupy much less duct space. This offers distinct advantages over the heavy, bulky wire cables in crowded underground city ducts. This is also of importance in aircraft where small light weight cables are advantageous.

(4) Data security

Fibre to fibre cross talk is very low since optical signals are well confined within the waveguide. This is of great importance in defense communication network.

(5) Communication network in high voltage region

Fibres are composed of dielectric materials therefore they are totally immune to extraneous interfering electromagnetic signals. Therefore they can be safely used in high voltage environment. For example, power station and can be laid alongside metallic cables.

***(6)* Entertainment applications**

A coherent optical fibre bundle is used to enlarge the image displayed on a TV screen. Conventional optical projection system is bulky and expensive.

(7) Optical fibre sensor

In these particular applications the inherent physical property of the fibre material is utilized. The variation in refractive index of the fibre under the influence of external forces leads to the possibility of an optical fibre being used as an transducer.

Types of Optic Sensors

Extrinsic or Hybrid or Passive Sensors	Intrinsic or Active Sensor
• Here the interaction between the light and quantity under measurement takes place outside the fibre.	• Here the measuring quantity directly acts on fibre itself and produces a change in the transmission characteristics.
• Fibre acts merely on waveguide. The sensed optical signal will be transferred to the measurement point with low attenuation.	• Liquid level sensors, Faraday sensors are of this type.
• These sensors can be used for the measurement of voltages, current, temperature, pressure, displacement etc.	—

(i) Pressure sensors :

- Microbending sensors are based on measurement of the light losses caused by controlled microbending. A fibre is mounted in between a pair of plates containing parallel groves.
- When pressure is applied over the plates, it causes microbends in the fibre and hence results in leakage of light thereby reduction in the power of transmitted light.
- The measurement and calibration of this output power provides the amount of pressure acted over the fibre. The process is explained in the Fig. 5.10.3.

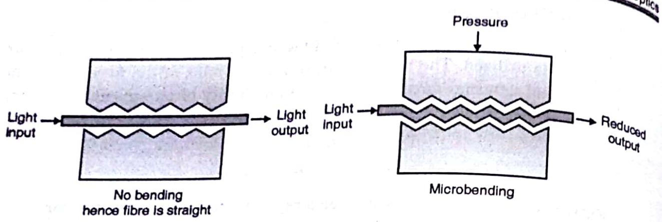


Fig. 5.10.3

(ii) Temperature sensor :

- If the fibre is subjected to heating, the temperature causes a change in the refractive index of the fibre. As temperature increases, the difference between the refractive indices of core and cladding reduces, leading to the leakage of light into the cladding.
- A simple thermometer can be built by using a LED as a light source, a coil of fibre as heat sensing element and a photo detector to measure the intensity of light. Temperature in the range from 80° to 700° C are measured using such a thermometer.

(iii) Smoke detector :

- A smoke detector and pollution detector can be built using fibre. A beam of light radiating from one end of a fibre can be collected by another fibre.
- If foreign particles are present they scatter light and the variation in intensity of the collected light reveals their presence.

(iv) Level detector :

- A loop of fibre can be used to determine the level of liquid in a container. A part of the cladding is scraped and the loop is suspended above the liquid level.
- Light is directed to pass through the fibre and its intensity is measured at the output. A bare core losses more light when it is immersed in liquid than when it is exposed to air. A sudden change of out coming light intensity indicates the liquid level.
- These sensors are used to monitor the filling of petroleum tanks.

5.11 Solved Problems

Problems on Numerical Aperture and Angle of Acceptance

Ex. 5.11.1 : Calculate the numerical aperture of a fibre with core index $n_1 = 1.55$ and cladding index, $n_2 = 1.51$.

MU - May 2013. 3 Marks

Soln. :

Given : $n_1 = 1.55, \quad n_2 = 1.51$

To find : NA = ?

Formula :

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.51^2} = \mathbf{0.35}$$ ...Ans.

Ex. 5.11.2 : An optical glass fibre of R.I. 1.50 is to be clad with another glass to ensure internal reflection that will contain light travelling within 5° of the fibre axis. What minimum index of refraction is allowed for the cladding?

MU - May 2014. 3 Marks

Soln. : $\theta_{max} = 5°, \quad n_1 = 1.5$

To find : n_2

Formula : $\theta_m = \sin^{-1}(N.A) = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

$$\therefore \sin\theta_m = \left(\sqrt{n_1^2 - n_2^2}\right)$$

$$n_2 = \left(\sqrt{n_1^2 - \sin^2\theta_m}\right) = 1.4974$$ ... Ans.

Ex. 5.11.3 : Calculate the numerical aperture and hence the acceptance angle for an optical fibre given that refractive indices of the core and the cladding are 1.45 and 1.40 respectively.

Soln. : $n_1 = 1.45, \quad n_2 = 1.40$

$$NA = \left(n_1^2 - n_2^2\right)^{1/2} = (0.1425)^{1/2} = \mathbf{0.3775}$$ ...Ans.

Acceptance angle

$$= \sin^{-1}\sqrt{n_1^2 - n_2^2} = \sin^{-1} 0.3775 = \mathbf{22.18}$$ ...Ans.

Ex. 5.11.4 : A fibre cable has an acceptance angle of 30° and core index of R.I. 1.4. Calculate R.I. of cladding.

MU - Dec. 2016. 3 Marks

Soln. :

$$\theta = 30°, \quad n_1 = 1.4$$

To find : $n_2 = ?$

$$\text{Acceptance angle } \theta = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore \quad \sin 30° = \sqrt{n_1^2 - n_2^2}$$

$$0.5^2 = 1.4^2 - n_2^2$$

$$\therefore \quad n_2 = \mathbf{1.308}$$

...Ans.

Ex. 5.11.5 : Compute the NA, acceptance angle, and the critical angle of the fibre having μ_1 (core refractive index) = 1.50 and the refractive index of the cladding = 1.45.

Soln. :

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = \frac{1.5 - 1.45}{1.5} = 0.033$$

$$NA = \mu_1 \sqrt{2\Delta} = 1.5\sqrt{2 \times .033} = \mathbf{0.387}$$

$$\theta_0 = \text{Acceptance angle} = \sin^{-1} NA = \mathbf{22°}$$

...Ans.

$$\theta_c = \sin^{-1}\frac{\mu_2}{\mu_1} = \sin^{-1}\frac{1.45}{1.5} = \mathbf{75.2°}$$

...Ans.

Ex. 5.11.6 : Calculate the refractive indices of the core and cladding material of a fibre from the following data :

NA = 0.22 $\quad \Delta = 0.012$

Soln. :

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = 0.012$$

$$NA = \mu_1 \sqrt{2\Delta} \quad \text{or} \quad \mu_1 = \frac{NA}{\sqrt{2\Delta}} = \mathbf{1.42}$$

...Ans.

$$0.012 = \frac{1.42 - \mu_2}{1.42} \quad \text{or} \quad \mu_2 = \mathbf{1.40}$$

...Ans.

Ex. 5.11.7 : A glass clad fibre is made with core glass of refractive index 1.5 and the cladding is doped to give a fractional index difference of 0.0005.

Find : (a) The cladding index

(b) The critical internal reflection angle

(c) The external critical acceptance angle

(d) The numerical aperture

Soln. : $n_1 = 1.5, \; \Delta = 0.0005$

(a) Let the refractive index cladding be n_2

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$0.0005 = \frac{1.5 - n_2}{1.5}$$

$$n_2 = 1.5 - 1.5 \times 0.0005 = 1.499925$$

...Ans.

(b) Let the critical internal reflection angle be ϕ_c

$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\phi_c = \left[\frac{n_2}{n_1}\right] = \sin^{-1}\left[\frac{1.4925}{1.5}\right]$$

$$= \sin^{-1}(0.9995) = \mathbf{88.2}$$

...Ans.

(c) Let the external critical acceptance angle be

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2} = \sin^{-1}[(1.5)^2 - (1.49925)^2]^{1/2} = \mathbf{2.720}$$

...Ans.

(d) Numerical Aperture $NA = n_1\sqrt{2\Delta}$

$$NA = 1.5\sqrt{2 \times 0.0005} = 1.5\,(0.03162) = \mathbf{0.0474}$$

...Ans.

Ex. 5.11.8 : An optical fibre has a NA of 0.20 and a refractive index of cladding is 1.59. Determine the acceptance angle for the fibre in water which has a refractive index of 1.33.

Soln. :

$$NA = \sqrt{n_1^2 - n_2^2}$$

When the fibre is in air $n_0 = 1$

$$NA = \sqrt{n_1^2 - n_2^2} = 0.20$$

$$n_1 = \sqrt{(NA)^2 + n_2^2} = \sqrt{(0.20)^2 + (1.59)^2} = 1.6025$$

When the fibre is in water $n_0 = 1.33$

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{\sqrt{(1.6025)^2 - (1.59)^2}}{1.33} = 0.15$$

...Ans.

$$\theta_0\,(\max) = \sin^{-1}(NA) = \sin^{-1}(0.15) = \mathbf{8.6°}$$

...Ans.

Ex. 5.11.9 : A step index fibre has a numerical aperture of 0.16, a core, a core refractive index of 1.45 and core diameter of 90 mm calculate.

a) The acceptance angle θ_c $\quad$ b) The refractive index of the cladding.

Soln. : $NA = 0.16, \quad n_1 = 1.45$

(a) $$NA = \sqrt{n_1^2 - n_2^2}$$

$$0.16 = \sqrt{1.45^2 - n_2^2}$$

$$0.0256 = 1.45^2 - n_2^2$$

$$n_2^2 = 2.0769$$

$$n_2 = 1.441$$

...Ans.

(b) $$\sin \theta_c = \frac{NA}{n_0} = \frac{0.16}{1}$$

$$\theta_c = 9° 12$$

...Ans.

Problems on V Number and Number Modes

Ex. 5.11.10 : The core diameter of multimode step Index fibre is 50 μm. The numerical aperture is 0.25. Calculate the no. of guided modes at an operating wavelength of 0.75 μm.

MU - Dec. 2015, 3 Marks

Soln. :

Given : $d = 50\ \mu m$

$NA = 0.25$

$\lambda_0 = 0.75\ \mu m$

To find : N = Number of modes

$$N = \frac{V^2}{2}$$

$$\text{Here } V = \frac{\pi d}{\lambda_0}(NA) = 52.36$$

$$\therefore \text{Number of modes} = \frac{52.36^2}{2} = 1371$$

...Ans.

Ex. 5.11.11 : Compute the maximum radius allowed for a fibre having core refractive index 1.47 and cladding refractive index 1.46. The fibre is to support only one mode at a wavelength of 1300 nm.

MU - May 2013, 3 Marks

Soln. :

The condition for single mode is $V < 2.405$.

$$\text{or} \quad \frac{2\pi a}{\lambda}\sqrt{\mu_1^2 - \mu_2^2} < 2.405$$

$$\text{or} \quad a < \frac{2.405}{2\pi\sqrt{\mu_1^2 - \mu_2^2}}\lambda$$

$$\text{or} \quad a < \frac{5.82}{2}\ \mu m$$

The maximum radius = 2.91 μm

Ex. 5.11.12 : Calculate V number for an optical fiber having numerical aperture 0.25 and core diameter 20 μm if it is operated at 1.55 μm.

MU - May 2015, 3 Marks

Soln. :

$NA = 0.25$

Core diameter $= 20\ \mu m$

$\lambda_0 = 1.55\ \mu m$

Formula

$$V = \frac{\pi d}{\lambda_0} \times (NA) = \frac{\pi \times 20 \times 10^{-6}}{1.55 \times 10^{-6}} \times 0.25$$

$$V = 10.13$$

...Ans.

Ex. 5.11.13 : Compute the cut-off parameter and the number of modes supported by a fibre μ_1 (core) = 1.53 and μ_2 (cladding) = 1.5; core radius 50 μm and operating wavelength is 1 μm.

MU - Dec. 2013, 3 Marks

Soln. :

$$V = \frac{2\pi a}{\lambda} \cdot NA \text{ (cut off parameter)}$$

$$NA = \sqrt{\mu_1^2 - \mu_2^2} = 0.3015$$

$$V = 94.72$$

$$\text{The number of modes} = \frac{V^2}{2} = 4485.85$$

...Ans.

Ex. 5.11.14 : An optical fibre has core diameter of 6 μm and its core R.I. is 1.45. The critical angle is 87°. Calculate (i) R.I. of cladding (ii) Acceptance angle (iii) The number of modes propagating through fibre when wavelength of light is 1 μm.

MU - May 2016, 7 Marks

Soln. :

Given : $n_1 = 45$, $d = 6\ \mu m$

Critical acceptance angle $\theta_c = 87°$

$\therefore$ R.I. of cladding,

$$\text{as} \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\therefore \sin(87) = \frac{n_2}{n_1}$$

$\therefore\ n_2 = n_1 \sin(87°) = 1.45 \times \sin 87°$

$= 1.448$

$\therefore$ R.I. of cladding = **1.448** ...Ans.

Now acceptance angle θ_0

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

$$= \sin^{-1}\left(\sqrt{1.45^2 - 1.448^2}\right)$$

$$= \mathbf{4.366°}$$ ...Ans.

As number of modes propagating through fibre is $N = \frac{V^2}{2}$

$$\text{Here } V = \frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore\ N = \frac{1}{2}\left[\frac{\pi^2 d^2}{\lambda_0^2} \times \left(n_1^2 - n_2^2\right)\right] = \frac{1}{2}\left[\pi^2\left[\frac{6}{1}\right]^2 \times (1.45^2 - 1.448^2)\right]$$

$$= \mathbf{1.029}$$ ...Ans.

Ex. 5.11.15 : Calculate the number of modes for an optical fibre of core diameter 40 μm will transmit as its core and cladding R.I. are 1.5 and 1.46 respectively. Wavelength of light used is 1.5 μm. **MU - Dec. 2016, 7 Marks**

Soln. :

Given : $n_1 = 1.5$, $n_2 = 1.46$, $\lambda_0 = 1.5\ \mu m$, $d = 40\ \mu m$

as $$N = \frac{V^2}{2}$$

and $$V = \frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore\ N = \frac{1}{2}\left[\frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)^2\right]$$

$$= \frac{1}{2}\left[\pi \times \left(\frac{40}{1.5}\right) \times \sqrt{(1.5)^2 - (1.46)^2}\right]^2$$

$$= \mathbf{415.48}$$ ...Ans.

Ex. 5.11.16 : A step index fibre has a core diameter of 29×10^{-6} m. The refractive indices of core and cladding are 1.52 and 1.5189 respectively. If the light of wavelength 1.3 μm is transmitted through the fibre, determine.

(1) Normalised frequency of the fibre.

(2) The number of modes the fibre will support.

Soln. :

Data :

Core diameter, $d = 29 \times 10^{-6}$ m

Refractive index of core = 1.52

Refractive index of cladding = 1.5189

Free space wavelength $\lambda_o = 1.3 \times 10^{-6}$ m

To find :

(i) Normalised frequency of the fibre or V-number.

(ii) Number of modes

$$V = \frac{\pi d}{\lambda_o}\sqrt{\mu_1^2 - \mu_2^2}$$

$$= \pi \times \frac{29 \times 10^{-6}}{1.3 \times 10^{-6}}\sqrt{1.52^2 - 1.5189^2}$$

$$= 4.052$$

The maximum number of modes N_m supported by SI fibre is

$$N_m \simeq \frac{1}{2} V^2$$

$$= \frac{1}{2}(4.052)^2$$

$$= 8 \text{ (take only integer)}$$ ...Ans.

Ex. 5.11.17 : SI fibre has core refractive index 1.466, cladding refractive index 1.46. The cut off parameter is 2.4. Calculate the core radius, NA and spot size at an operating wavelength of 800 nm. Also find the divergence angle, spot size at 10 m.

Soln. :

$$\text{Numerical aperture} = \sqrt{\mu_1^2 - \mu_2^2} = 1.13$$

$$\text{Cut off parameter} = 2.4 = \frac{2\pi a\sqrt{\mu_1^2 - \mu_2^2}}{\lambda}$$

or $$a = \frac{2\pi \times \lambda}{2\lambda\sqrt{\mu_1^2 - \mu_1^2}} = \frac{2.4 \times 0.8}{6.28 \times 0.13} = 2.35\ \mu m.$$

$$V = 2.4 \text{ so } \frac{\omega}{a} = 1.1$$

$$\therefore\ \text{Spot size} = 1.1 \times 2.35 = 2.6\ \mu m.$$

$$\text{Divergence angle } \theta = \frac{2\lambda}{\pi w} = \frac{2 \times 0.8}{3.14 \times 2.6} = 0.2 \text{ radian} = \mathbf{11.5^\circ}$$ ...Ans.

$$\text{The spot size at 10 m} = \frac{\lambda \times \text{distance}}{\pi \times \text{beam size}} = \frac{0.8 \times 10^{-6} \times 10}{3.14 \times 2.6 \times 10^{-6}} \text{ metres}$$

$$= 0.98 \text{ m} = \mathbf{98 \text{ cm}}$$ ...Ans.

Ex. 5.11.18 : Compute the maximum value of $\Delta = \left(\frac{\mu_1 - \mu_2}{\mu_1}\right)$ and μ_2 (cladding) of a single mode fibre of core diameter 10 μm and core refractive index 1.5. The fibre is coupled to a light source with a wavelength of 1.3 μm. V cut-off for single mode propagation is 2.405. Also calculate the acceptance angle.

Soln. :

We know that $V \text{ cut-off} = \frac{2\pi a}{\lambda} \text{NA (cut off)}$

$$\text{NA (cut off)} = \frac{V \times \lambda}{2\pi a} = \frac{2.405 \times 1.3}{3.14 \times 10} = 0.09957$$

So acceptance angle $= \sin^{-1}(0.09957) = 5.71^\circ$

We know also that $NA = \mu_1 \sqrt{2\Delta}$

$$\therefore \quad \Delta = \left(\frac{NA^2}{\mu_1}\right)/2 = 2.2 \times 10^{-3}$$

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} \qquad \therefore \mu_2 = \mu_1 - \Delta\,\mu_1 = \mu_1 (1 - \Delta)$$

$$\therefore \quad \mu_2 = 1.5 (1 - 2.2 \times 10^{-3}) = \mathbf{1.496.}$$ ...Ans.

A Quick Revision

- *Optical fibres are glass or plastic as thin as human hair designed to guide light waves along their length.*
- *An optical fibre works on the principle of total internal reflection. When light enters through the end of the fibre it under goes successive total internal reflections from the side walls and travels down the length of the fibre along a Zig Zag path.*
- *The refractive index of cladding is always lower than that of the core.*
- *Light launched into the core and striking the core to cladding interface at an angle greater than critical angle will be reflected back into the core.*
- *The numerical aperture is defined as the sine of the acceptance angle.*

$$NA = \sqrt{n_1^2 - n_2^2}$$

- Numerical aperture determines the light gathering ability of the fibre. It is a measure of the amount of light that can be accepted by the fibre.
- Optical fibres are classified as
  i) Single mode fibre (SMF) has a smaller core diameter and can support only one mode of propagation. Multi mode fibre (MMF) has larger core diameter and supports a number of modes.
  ii) According to refractive index profile it is divided as step index fibre and graded index fibre.
- The refractive index changes abruptly at the core cladding boundary and hence it is known as step index fibre.
- The refractive index of the core varies with distance from the fibre axis. It has a high value at the centre and falls off with increasing radial distance from the axis.
- Absorption, Rayleigh scattering and geometric effects attribute to fibre losses.
- The band of wavelengths at which the attenuation is minimum are called as optical windows or transmission windows.
  - **First window** has wavelength between 0.8 μm to 0.9 μm.
  - **Second window** has wavelength between 1.3 μm to 1.6 μm.
- A light pulse launched into a fibre either decreases in the fibre or spreads during its travel or the output pulse becomes wider than the input pulse. These are due to dispersion of light in the fibre.
- Laser light is much preferred than LED because it is more directional, highly coherent, faster rise time and narrow spectral width than LED.
- Couplers are needed for signal distribution (i.e.) either to combine or separate from multiple fibres.
- Connectors are used for connecting optical fibres. Connectors allow direct transition of optical power from one core to another so they should have insertion loss of 1 db.
- Splices are permanent joints in fibres just like soldered joints in an electrical system.
- Photo detectors convert optical signal into an electrical one. They are required to have,
  i) High quantum efficiency
  ii) Adequate frequency response
  iii) Low signal dependence

- WDM stand for "Wavelength division multiplexing" several light sources of different wavelength is transmitted down a single fibre through a WDM device.

Review Questions

Q. 1 What is dispersion in optical fibres ? Explain the different mechanisms that contribute to dispersion ?

Q. 2 What are the advantages of an optical fibre communication system over the conventional ones ?

Q. 3 Draw the block diagram of an optical fibre communication system and explain the function of each block.

Q. 4 Derive expression of Numerical aperture for fibre optics cable and explain how it controls the optical communication.

Q. 5 Explain the significance of (a) Acceptance angle (b) cone of acceptance.

Q. 6 What is monomode and multimode fibre ? How N.A. is involved with them?

Q. 7 Explain the term V – number.

Q. 8 Write a note on the advantage of using optical fibre.

Q. 9 Explain the difference between step index and graded index fibre.

Q. 10 Explain the principle behind optical fibre.

Problems for Practice

1. Calculate Δ for a step index fibre having core and cladding refractive indices 1.48 and 1.46 respectively. **[Ans. : 1.351 %]**
2. Calculate the number of modes in the graded index fibre at an operating wavelength of 850 nm. **[Ans. : 504 modes]**
3. If $\mu = 1.5$, calculate the velocity of light in glass.
4. Find μ_2 if $\Delta = 1\%$ and $\mu_1 = 1.48$. What is the critical angle for this material if the light travels from glass into air ?
5. 1 meter below a water-air interface point source of light is situated. Find the radius of the light circle sun by an observer positioned over the source outside the water. The refractive index of water is 1.333.
6. A single mode fibre with V = 2.3 and mode of fused silica (core) of $\mu_1 = 1.458$. The numerical aperture of the fibre is 0.10.
   Now compute the following :
   i) μ_2 and radius of the fibre core.
   ii) Number of modes in the fibre for operation at 820 nm.
7. Compute the number of modes in a 50/125 graded index fibre having a parabolic index of 2.0, $\mu_1 = 1.485$, and $\mu_2 = 1.46$ at an operating wavelength of 820 nm and at 1300 nm. Also calculate the number of modes in an equivalent step index fibre at both wavelengths.

8. Compute the core diameter required to ensure single mode operation of a step index fibre with $\mu_1 = 1.485$, and $\mu_2 = 1.480$ at a wavelength 820 and 1300 nm. What is the NA and θ_{max} for this fibre ?
9. Calculate the numerical aperture of a step index fibre having $\mu_1 = 1.48$ and $\mu_2 = 1.46$. What is the maximum entrance angle for this fibre if the outer layer is air with $\mu = 1$?
10. Determine the normalized frequency at 0.82 μm for a step-index fibre having a 25 μm core radius, $\mu_1 = 1.48$ and $\mu_2 = 1.46$. How many modes propagate in this fibre at 0.82 μm ? How many modes propagate at a wavelength of 1.3 μm ? What percentage of the optical power flows in the cladding in each case ?
11. Determine the core radius necessary for single-mode operation at 0.82μm of a step-index fibre with $\mu_1 = 1.480$ and $\mu_2 = 1.478$. What is the numerical aperture and maximum acceptance angle of this fibre ?

5.12 University Questions

May 2013

Q. 1 Refer *Ex. 5.11.1* (3 Marks)

Q. 2 What is dispersion in optical fibres? Mention any three dispersion you have studied and explain one in detail.
(Ans. : Refer section 5.9.2) (4 Marks)

Q. 3 *Refer Ex. 5.11.11* (3 Marks)

Dec. 2013

Q. 1 Refer *Similar to Ex. 5.11.1* (3 Marks)

Q. 2 What is NA ?
(Ans. : Refer section 5.6) (3 Marks)

Q. 3 *Refer Ex. 5.11.13* (5 Marks)

May 2014

Q. 1 *Refer Ex. 5.11.2* (3 Marks)

Q. 2 Differentiate between S.I. fibre and GRIN fibre. Derive the expression for N.A. for step Index fibre.
Ans. : Refer sections 5.5.4 and 5.6) (7 Marks)

Dec. 2014

Q. 1 Define the terms :
(i) Total internal reflection
(ii) Numerical Aperature
(iii) Acceptance angle.
(Ans. : Refer sections 5.3.1 and 5.6) (3 Marks)

Q. 2 Derive the expression for numerical Aperature for a step index fibre.
(Ans. : Refer sections 5.6)

Q. 3 *Refer Ex. 5.6.1* (7 Marks)

May 2015

Q. 1 *Refer Ex. 5.11.12* (3 Marks)

Q. 2 Derive an expressions for numerical aperture of step index optical fiber. What are the advantages of using an optical fiber ?
(Ans. : Refer sections 5.6 and 5.10) (7 Marks)

Dec. 2015

Q. 1 Refer Ex. 5.11.10 (3 Marks)

Q. 2 Define :

(i) Numerical aperture

(ii) Total internal reflection

(iii) Acceptance angle

Derive the expression for numerical aperture of step index fibre.
(Ans. : Refer sections 5.3.1 and 5.6) (6 Marks)

May 2016

Q. 1 Why would you recommend use of optical fibre in communication system ? *(Ans. : Refer Section 5.10)* (3 Marks)

Q. 2 Refer Ex. 5.11.14 (7 Marks)

Dec. 2016

Q. 1 Refer Ex. 5.11.4 (7 Marks)

Q. 2 What is monomode and multimode fibre? Explain the term V-number. *(Ans. : Refer Sections 5.5 and5.8)*

Q. 3 Refer Ex. 5.11.15 (7 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Total Internal Reflection

Q. 1 Define : Total internal reflection *(Ans. : Refer sections 5.3.1)* **(Dec. 2014, Dec. 2015)**

Syllabus Topic : Types of Optical Fibre

Q. 1 What is monomode and multimode fibre ? *(Ans. : Refer sections 5.5)* **(Dec. 2016)**

Q. 2 Differentiate between S.I. fibre and GRIN fibre. *(Ans. : Refer sections 5.5.4)* **(May 2014)**

Syllabus Topic : Numerical Aperture, Angle of Acceptance

Q. 1 What is NA ? *(Ans. : Refer sections 5.6)* **(Dec. 2013, Dec. 2014, Dec. 2015)**

Q. 2 Derive the expression for N.A. for step Index fibre. *(Ans. : Refer sections 5.6)* **(May 2014, May 2015)**

Q. 3 Define the terms : Acceptance angle. *(Ans. : Refer sections 5.6)* **(Dec. 2014, Dec. 2015)**

Q. 4 Refer Ex. 5.6.1 **(Dec. 2014)**

Syllabus Topic : V Number

Q. 1 Explain the term V-number. *(Ans. : Refer sections 5.8)* **(Dec. 2016)**

Syllabus Topic : Losses in Optical Fibre (Attenuation and Dispersion)

Q. 1 What is dispersion in optical fibers' ? Mention any three dispersion you have studied and explain one in detail. *(Ans. : Refer sections 5.9.2)* **(May 2013)**

Syllabus Topic : Applications of fibre : Fibre Optic Communication System, Sensors(Pressure, Temperature, Water Level), Applications in Medical Field

Q. 1 What are the advantages of using an optical fiber ? *(Ans. : Refer sections 5.10)* **(May 2015)**

OR

Why would you recommend use of optical fibre in communication system ? *(Ans. : Refer Section 5.10)* **(May 2016)**

Solved Problems

Q. 1	Refer Ex. 5.11.1	**(May 2013)**	Q. 6	Refer Ex. 5.11.12	**(May 2015)**
Q. 2	Refer Ex. 5.11.2	**(May 2014)**	Q. 7	Refer Ex. 5.11.13	**(Dec. 2013)**
Q. 3	Refer Ex. 5.11.4	**(Dec. 2016)**	Q. 8	Refer Ex. 5.11.14	**(May 2016)**
Q. 4	Refer Ex. 5.11.10	**(Dec. 2015)**	Q. 9	Refer Ex. 5.11.15	**(Dec. 2016)**
Q. 5	Refer Ex. 5.11.11	**(May 2013)**			

❑❑❑

CHAPTER 6

Module 4

Electrodynamics

Syllabus

Cartesian, Cylindrical and Spherical Coordinate system, Scalar and Vector field, Physical significance of gradient, curl and divergence, Determination of Maxwell's four equations. Applications-design of antenna, wave guide, satellite communication etc.

Introduction

It is expected that the reader is aware about the ordinary matter which is made up of atoms having positively charged nuclei and negatively charged electrons surrounding them. Charge is quantized in terms of the electronics charge – e. One electron is suppose to carry as charge $e = 1.6 \times 10^{-19}$ C. Separated charges produce electric field, where as the motion of charges generates current and hence the magnetic field. When these fields are time varying, they are coupled with each other through Maxwell's equations. When Maxwell's equations are properly studied one can derive wave equation. Based upon it propagation of electromagnetic waves through different media can be investigated. This can be applied to transmission line, waveguides, antenna, satellite communication etc.

Syllabus Topic : Cartesian

> **Topics covered :** Co-ordinate Systems, Cartesian Coordinate System

6.1 Co-ordinate Systems

To understand the vectors it is important to know various coordinate systems. It is also important to know that laws of electromagnetic are invariant with co-ordinate system. Even a simplest step where we require to define a point or position of charge we need coordinates.

There are number of co-ordinate systems are present like rectangular (or Cartesian), cylindrical, Spherical, Elliptical, Spherical, Paraboidal etc., but most commonly used are

1. Cartesian or rectangular co-ordinate system
2. Cylindrical coordinate system
3. Spherical coordinate system

6.1.1 Cartesian Coordinate System

(I) Point in Cartesian system :

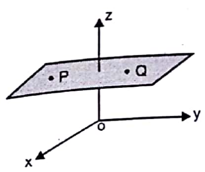


Fig. 6.1.1 : Z = Constant plane

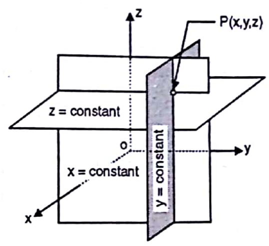


Fig. 6.1.2 : Point is defined by three planes

- Consider a plane perpendicular to the z-axis as show in Fig. 6.1.1. As, z-axis is vertical, a plane normal to z is horizontal, also xy plane is horizontal. Therefore xy plane and plane under consideration are parallel.
- The perpendicular distance of any point on this plane from xy plane will be the same. The distance from xy plane is nothing but z coordinate. Thus any point on the plane perpendicular to z-axis is having the same z coordinate. In other words we can say that for a plane perpendicular to z-axis, z coordinate of any point on the plane is constant or simply the plane is defined as **z = constant** plane.
- Similarly, a plane perpendicular to x-axis can be defined as **x = constant** plane and a plane perpendicular to y-axis is **y = constant** plane. Remember, x = constant and y = constant planes are vertical planes, while z = constant plane is horizontal plane.

- Consider, x = constant and y = constant planes. These two vertical planes intersect and the intersection is a vertical line. To obtain a point take the third plane, i.e. z = constant plane i.e. horizontal plane. The vertical line and horizontal plane intersects and the intersection is a point [point P in Fig. 6.1.2].
- Thus, point P is a intersection of x = constant, y = constant and z = constant planes and coordinates of it can be written as (x, y, z).

Any point in Cartesian system is the intersection of x = constant, y = constant and z = constant planes.
Point in Cartesian system : (x, y, z)

- These planes are mutually perpendicular to each other. This is a very important concept. So, if the coordinates of a point are (2, –1, 3) it means, it is the intersection of x = 2, y = –1 and z = 3 planes.

(ii) Unit Vectors in Cartesian System :

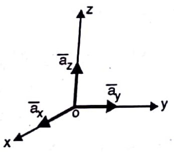


Fig. 6.1.3 : Unit vectors

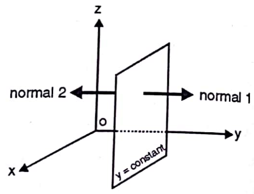


Fig. 6.1.4 : Unit vector $\bar{a}_y$

- Unit vectors in Cartesian system are $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$. These vectors are in the positive x, y and z directions as shown in Fig. 6.1.3. Instead of $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$ many times different notations are used like or $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.
- We are using first notations for unit vectors in Cartesian system throughout this book. There is one more way to describe directions of unit vectors.

Unit vectors in Cartesian system : $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$

Syllabus Topic : Cylindrical Coordinate System

Topics covered : Cylindrical Coordinate System, Conversion between Cartesian and Cylindrical Co-ordinates

6.1.2 Cylindrical Coordinate System

(I) Point in cylindrical system :

- Just like in Cartesian system, as we obtain a point by intersection of three mutually perpendicular plane surfaces, in this system also a point is obtained by intersection of three surfaces mutually perpendicular to each other.
- But the difference is that all three surfaces are not plane surfaces, one is cylindrical and other two are plane surfaces.
- In the cylindrical system again we are taking x, y, z-axis for reference. Now imagine a hollow vertical cylinder of radius r placed such that axis of the cylinder coincides with z-axis. If you take any point on the cylindrical surface it is at a same distance r from the axis therefore we define the cylindrical surface as **r = constant surface.**
- Consider now a plane vertical surface of which one edge coincides with z-axis. We can rotate this plane about z-axis. The angle of rotation ϕ is measured from xz plane as shown in Fig. 6.1.5. When we take any point on this plane, every time the angle of the point with xz plane is ϕ. Therefore we define this plane as **ϕ = constant plane.**
- Thus we have two new surfaces, one is r = constant and the other is ϕ = constant surface. The intersection of these two surfaces is a vertical line.
- Remember we want a point. The situation is similar to in Cartesian system. Hence we need one more surface, take **z = constant plane.**
- The intersection of vertical line with z = constant plane is a point. This point is on r = constant, ϕ = constant and z = constant surfaces and this point can be written as (r, ϕ, z). The three surfaces in cylindrical system are again mutually perpendicular to each other.
- One important thing about angle ϕ is, ϕ = constant plane can make full rotation about z-axis i.e. ϕ can vary from 0 to 360°.

Point in cylindrical system : (r, ϕ, z)

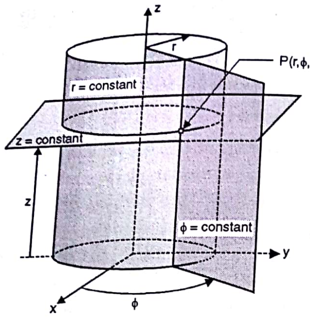


Fig. 6.1.5 : Cylindrical system

(II) Unit Vectors in Cylindrical System :

- The unit vectors in this system are, $\bar{a}_r$, $\bar{a}_\phi$, and $\bar{a}_z$. These unit vectors are not along any axis because there is no r or ϕ axis in cylindrical system. **The unit vectors are normal to the respective surfaces and in the increasing directions of the coordinates.**
- Thus the unit vector $\bar{a}_r$ is normal to r = constant surface i.e. normal to the cylinder at given point and in the increasing direction of r. As r increases away from axis of the cylinder (i.e. z-axis), the unit vector is away from z-axis.
- The unit vector $\bar{a}_\phi$ is normal to ϕ = constant surface and in the increasing direction of ϕ. Angle ϕ is increasing away from x-axis in anticlockwise direction.
- Separate explanation for $\bar{a}_z$ is not required as it is similar to $\bar{a}_z$ in Cartesian.
- Notice that the vectors $\bar{a}_r$ and $\bar{a}_\phi$ both are horizontal as the r = constant and ϕ = constant surfaces are vertical. You can get a perfect idea of $\bar{a}_r$ and $\bar{a}_\phi$ if we obtain top view of the coordinate

system. Refer Fig. 6.1.6. The vector $\bar{a}_z$ is vertical at point P. Thus, $\bar{a}_r$, $\bar{a}_\phi$ and $\bar{a}_z$ are perpendicular to each other.

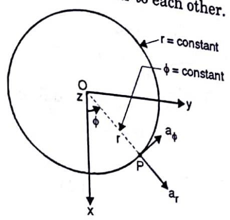


Fig. 6.1.6 : Showing $\bar{a}_r$, $\bar{a}_\phi$ vectors

Unit vectors in cylindrical system : $\bar{a}_r$, $\bar{a}_\phi$, $\bar{a}_z$

6.1.3 Conversion between Cartesian and Cylindrical Co-ordinates

(I) Cartesian from cylindrical coordinates :

- In the beginning of this article, we have seen that cylindrical system is built by taking x, y, z axes for reference. When we write a point as P (r, ϕ, z) it must have some Cartesian coordinates associated with it. Here we are going to find relation between r, ϕ, z in cylindrical and x, y, z in Cartesian.
- To find x and y corresponding to P (r, ϕ, z), project pt. P on xy plane i.e. point Q. Project OQ on x and y axes. The length of projection on x-axis gives x coordinate (length OR) and length of projection on y axis (OS) gives y coordinate. In Fig. 6.1.7.

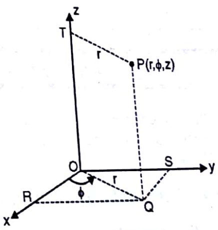


Fig. 6.1.7 : To show co-ordinate transformation

$$OR = OQ \cos\phi$$

i.e. $$x = r\cos\phi \quad ...(6.1.1)$$

and $$OS = r\cos(90-\phi)$$

i.e. $y = r \sin \phi$ ...(6.1.2)

And z in Cartesian is same as z in cylindrical

$\therefore \quad z = z$ ...(6.1.3)

(II) Cylindrical from Cartesian coordinates :

You can also make a reverse transformation i.e. point in Cartesian can be transformed in cylindrical as follows :

Squaring Equations (6.1.1) and (6.1.2) and adding

$$x^2 + y^2 = r^2 (\cos^2\phi + \sin^2\phi) = r^2$$

or $$r = \sqrt{x^2 + y^2} \quad ...(6.1.4)$$

Dividing Equation (6.1.2) by Equation (6.1.1)

$$\frac{y}{x} = \frac{r \sin \phi}{r \cos \phi} = \tan \phi$$

$$\therefore \quad \phi = \tan^{-1}\frac{y}{x} \quad ...(6.1.5)$$

and z in Cartesian = z in cylindrical

$$\therefore \quad z = z \quad ...(6.1.6)$$

Table 6.1.1 : Conversion between Cartesian and Cylindrical

(i) Cartesian from cylindrical	(ii) Cylindrical from Cartesian coordinates.
$x = r \cos \phi$	$r = \sqrt{x^2 + y^2}$
$y = r \sin \phi$	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$
$z = z$	$z = z$

Syllabus Topic : Spherical Coordinate System

> **Topics covered :** Spherical Coordinate System, Conversion between Cartesian and Spherical Coordinates, Transformation of Vector from Cartesian to Spherical or Vice Versa

6.1.4 Spherical Coordinate System

(I) Point in spherical system :

- For spherical coordinate system also x, y, z axes are used for reference. Imagine a sphere of radius r with center at origin. Any point on the sphere is at the same distance r from origin, therefore the spherical surface is defined as **r = constant** surface.

- Now consider a line from origin making angle θ with z-axis. Rotate this line about z-axis fixing the end at the origin. This forms a cone with angle θ, this conical surface is defined as a **θ = constant** surface.
- When a sphere with center at origin intersects with a vertical cone with vertex at origin, the intersection is a horizontal circle with radius equal to r sin θ [see Fig. 6.1.8].
- We want to locate a point in spherical coordinate system. Imagine a ϕ = constant plane similar to in cylindrical system. A horizontal circle with center on z-axis, intersects ϕ = constant plane. The intersection is a point.

Point in spherical system : (r, θ, ϕ)

- Because r = constant, θ = constant and ϕ = constant surface intersects at a point, the point is defined as (r, θ, ϕ). In spherical system variation of angle θ is from 0 to 180° and variation of ϕ is from 0 to 360°.

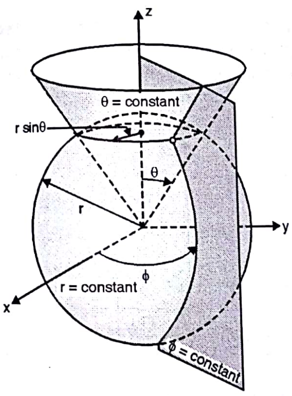


Fig. 6.1.8 : Spherical Coordinates

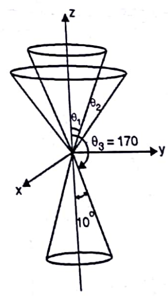


Fig. 6.1.9 : Variation of θ from 0 to 180° only

About the variation of angle θ :

- Above we mentioned that the variation of angle θ is from 0 to 180° only, why ? The range of limits is from 0 to 180°, this fact can be best understood from the Fig. 6.1.9.
- By increasing angle θ slowly from zero, separate cones are formed for different angles. This is shown in Fig. 6.1.9. Let $\theta = \theta_3 = 170°$, the cone formed is shown in the figure. Now when θ is made 190°, the cone formed is similar to cone formed with θ = 170°. Thus, the cone repeats, for any angle greater than 180°. Hence there is no point in increasing θ above 180°.

Unit vectors in spherical system :

Unit vectors are $\bar{a}_r$, $\bar{a}_\theta$ and $\bar{a}_\phi$ perpendicular to r = constant, θ = constant and φ = constant surfaces respectively.

> **Unit vectors in spherical system : $\bar{a}_r$, $\bar{a}_\theta$, $\bar{a}_\phi$**

6.1.5 Conversion between Cartesian and Spherical Coordinates

(1) Cartesian from Spherical Coordinates :

The unit vectors in spherical system are $\bar{a}_r$, $\bar{a}_\theta$ and $\bar{a}_\phi$. These unit vectors are perpendicular to r = constant, θ = constant, and φ = constant surface respectively and in the increasing directions of the r, θ and φ respectively. In Fig. 6.1.10 only the first octant is shown for simplicity, and the unit vectors are also shown in Fig. 6.1.10.

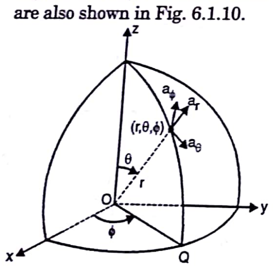


Fig. 6.1.10 : Unit vectors in spherical System

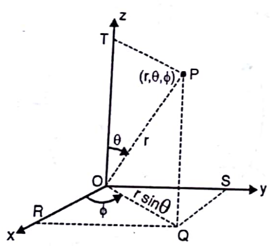


Fig. 6.1.11 : Relation between Spherical and Cartesian coordinates

When point P (r, θ, φ) is present in Cartesian system, it must have corresponding Cartesian coordinates. Here we are going to find relation between spherical coordinates and Cartesian coordinates.

To find x and y corresponding to point P, project point P in xy plane (Fig. 6.1.11). The projection is OQ = r sin θ. Now its projection on x and y axis gives x and y coordinates corresponding to point P.

Projection of OQ on x-axis is

$$OR = OQ \cos\phi = r \sin\theta \cos\phi$$

$$\therefore \quad x = r \sin\theta \cos\phi \qquad ...(6.1.7)$$

Projection of OQ on y-axis is

$$OS = OQ \sin\phi = r \sin\theta \sin\phi$$

$$\therefore \quad y = r \sin\theta \sin\phi \qquad ...(6.1.8)$$

Projection of OP on z-axis gives z coordinates of P.

Projection of OP on z-axis = OT = OP cos θ

$$\therefore \quad z = r \cos\theta \qquad ...(6.1.9)$$

(2) Spherical from Cartesian Coordinates :

The reverse transformation is obtained from Equations (6.1.7), (6.1.8) and (6.1.9), it is,

$$r = \sqrt{x^2+y^2+z^2}\ ; \phi = \tan^{-1}\left(\frac{y}{x}\right)\ ; \theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$$

Table 6.1.2 : Conversion between Cartesian and Spherical

i) Cartesian from Spherical Coordinates. $x = r \sin\theta \cos\phi$ $y = r \sin\theta \sin\phi$ $z = r \cos\theta$	ii) Spherical from Cartesian coordinates. $r = \sqrt{x^2+y^2+z^2}$ $\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$ $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

6.1.6 Transformation of Vector from Cartesian to Spherical or Vice Versa

- To transform a vector in Cartesian into spherical or for reverse transformation, use the procedure explained in Section 6.1.5 and a Table 6.1.3.

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Table 6.1.3 : Table of dots products

	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$
$\bar{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\bar{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\bar{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Syllabus Topic : Scalar and Vector Field

> **Topics covered :** Scalar and Vector Field, Scalar Field, Vector Field, Difference between a Scalar and Vector Field

6.2 Scalar and Vector Field

6.2.1 Scalar Field

- A scalar field associates a scalar value to every point in a space. It may be representing either a mathematical number or a physical quantity.
- Scalar fields are required to be coordinate independent means if any two observers using the same units will essentially have same value of the scalar field at the same absolute point in space regardless of their respective points of origin.
- For example, temperature, or pressure distribution throughout the space.

6.2.2 Vector Field

- A vector field can be imagined by assigning a vector to individual points.
- A vector field in the plane can be regarded as collection of arrows with a given magnitude and direction attached to a point in the plane.
- Vector fields are often used to (i) to model the speed and direction of moving fluid throughout the space (ii) to represent the strength and direction of some force such as magnetic or gravitational force as it changes from point to point.

6.2.3 Difference between a Scalar and Vector Field

Difference between a scalar and vector field is not that "a scalar is just one number and vector is several numbers". The difference is in : how their co-ordinates respond to co-ordinate transformations discussed in Section 6.1.

Syllabus Topic : Physical Significance of Gradient, Curl and Divergence

> **Topics covered :** Del Operator, Significance of Gradient, Significance of Divergence, Curl of a Vector

6.3 Del Operator

- Now we will introduce some elementary mathematical operations in vector calculus to understand same concepts of electromagnetism.
- As per conversions, any position vector is defined as :

$$r = \hat{i}x + \hat{j}y + \hat{k}z \text{ or } r = a_x \cdot x + a_y \cdot y + a_z \cdot z$$

Where, $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors in the specific directions, x, y, z are rectangular coordinates and r is a position vector.

- The vector differential operator ∇ (del) is defined as,

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ or } \nabla = a_x\frac{\partial}{\partial x} + a_y\frac{\partial}{\partial y} + a_z\frac{\partial}{\partial z}$$

- It is not a vector in itself, but when operates on a scalar function it provides the resultant as vector. The del operator can operate in different ways.
- For example, when it acts on a scalar function F, the resultant ∇F is called the gradient of a scalar function. When it acts on a vector function $\bar{A}$ via the dot product resultant is $\nabla . \bar{A}$ which is called divergence of a vector $\vec{A}$. Whereas when it acts via cross product $\nabla \times \bar{A}$, the resultant is a vector and called curl of a vector $\bar{A}$.
- For cylindrical co ordinate system it is given by

$$\nabla = \hat{r}\frac{\partial}{\partial s} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$$

- For spherical co-ordinate system it is given by

$$\nabla = \hat{r}\frac{\partial}{\partial x} + \frac{\hat{\phi}}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

6.3.1 Significance of Gradient

- A gradient is a directional derivative. In a simple terms, it is the rate of change of a function in a specified direction. The gradient of a scalar function is a vector quantity. The magnitude of the vector quantity is the maximum directional derivative at the point being considered and its direction is given by the directional derivative at that point.

- Mathematically, if ϕ (x, y, z) is differentiable at each point (x, y, z) in a certain region of space then :

$$\nabla\phi\,(x, y, z) = \frac{\hat{i}\,\partial\phi}{\partial x} + \frac{\hat{j}\,\partial\phi}{\partial y} + \frac{\hat{k}\,\partial\phi}{\partial z} \text{ or } \nabla\phi\,(x, y, z) = \frac{a_x\,\partial\phi}{\partial x} + \frac{a_y\,\partial\phi}{\partial y} + \frac{a_z\,\partial\phi}{\partial z} \quad ...(6.3.1)$$

- If we think of derivative of a function of one variable we notice that it simply tells us how fast the function varies if we move a small distance. It means the gradient is the rate of change of a quantity with distance.
- For example : Temperature gradient in a metal bar is the rate of change of temperature along bar. But for a function of three variables the situation is more complicated, as it will depend on what direction we choose to move.

Gradient in Cartesian Coordinate System :

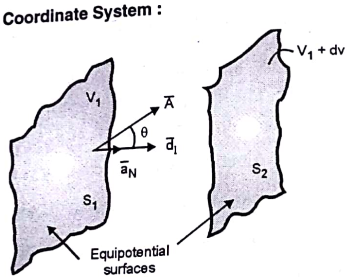


Fig. 6.3.1 : gradient in Cartesian

- Consider S_1 and S_2 are two surfaces on which the value of the V are V_1 and V_1 + dV respectively. At any point on S_1 the value of V is V_1 which is a constant. Then S_1 is called as equipotential surface, similarly S_2. The change in potential between these surfaces

$$dV = \frac{\partial V}{\partial x}\,dx + \frac{\partial V}{\partial y}\,dy + \frac{\partial V}{\partial z}\,dz$$

- The right side of it can written as do product of two vectors,

$$dV = \left(\frac{\partial V}{\partial x}\,\bar{a}_x + \frac{\partial V}{\partial y}\,\bar{a}_y + \frac{\partial V}{\partial z}\,\bar{a}_z\right)\cdot\left(dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z\right)$$

- Let $\bar{A}$ is Cartesian is the first bracket

i.e. $$\bar{A} = \frac{\partial V}{\partial x}\,\bar{a}_x + \frac{\partial V}{\partial y}\,\bar{a}_y + \frac{\partial V}{\partial z}\,\bar{a}_z$$

and we know the second bracket is $d\bar{l}$ in Cartesian,

$$d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$\therefore dV = \bar{A}\cdot d\bar{l} = A\,dl\cos\theta$$

$$\text{or } \frac{dV}{dl} = A\cos\theta$$

- The maximum value of dV/dl is obtained when $\theta = 0$ i.e. $\bar{A}$ is in the direction of $\bar{a}_N$ (unit normal vector to the surfaces)

$$\therefore \left.\frac{dV}{dl}\right|_{max} = A = \frac{dV}{dN}$$

- Thus the magnitude and direction of A is those of the maximum rate of change of V. then by definition,

$$\boxed{\text{grad } V = \nabla V = \frac{\partial V}{\partial x}\,\bar{a}_x + \frac{\partial V}{\partial y}\,\bar{a}_y + \frac{\partial V}{\partial z}\,\bar{a}_z}$$

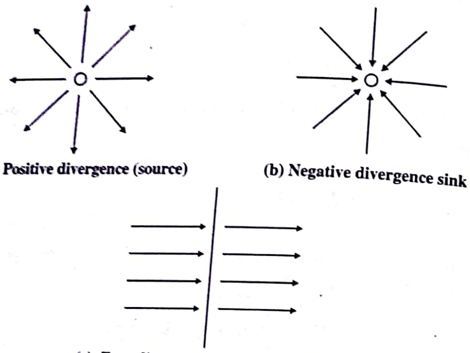
6.3.2 Significance of Divergence

- It is known that divergence of vector field $\bar{A}$ is expressed as $\nabla\cdot\bar{A}$. It is given by,

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- Clearly the divergence of a vector is a scalar. Also the divergence of a scalar cannot be obtained.
- The simple meaning can be expressed by considering net flux $\oint \bar{A}\cdot d\bar{s}$ of a vector field $\bar{A}$ from a closed surface S. The divergence of $\bar{A}$ is defined as the net outward flux per unit volume over a closed surface.
- In Fig. 6.3.2(a), at point O the divergence of vector field is positive as the vector spreads out which also represents source.

- In Fig. 6.3.2(b), the field is converging and hence divergence at point O is negative which represents sink. In Fig. 6.3.2(c), one can notice that divergence of vector field is zero.
- A water fountain, or a tyre which has just been punctured by a nail where air is expanding and forming a net outflow are some more examples which may be used to represent divergence.



(a) Positive divergence (source)

(b) Negative divergence sink

(c) Zero divergence (Incompressible)

Fig. 6.3.2

- For completeness, expressions for divergence in cylindrical and spherical co-ordinates are given as follows :

$$\operatorname{div}\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{A}_r\right) + \frac{1}{r}\frac{\partial \bar{A}_\phi}{\partial \phi} + \frac{\partial \bar{A}_z}{\partial z}$$

(For cylindrical co-ordinates)

$$\operatorname{div}\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2\bar{A}_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\bar{A}_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\cdot\frac{\partial \bar{A}_\phi}{\partial\phi}$$

(For spherical co-ordinates)

If $\nabla \times \bar{A} = 0$, the vector filed is said to be solenoidal.

- A detailed explanation of divergence of a vector field $\bar{A}$ is as given below.

(A) Divergence of a vector in Cartesian :

- Let us consider a rectangular box of edges with differential lengths Δx, Δy and Δz and defined by six surfaces $x = x$, $x = x + \Delta x$, $y = y$, $y = y + \Delta y$, $z = z$ and $z = z + \Delta z$ as shown in the Fig. 6.3.3.

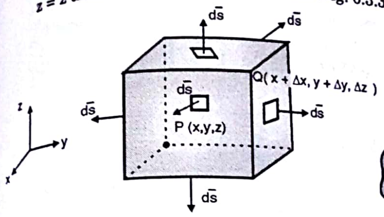


Fig. 6.3.3 : Divergence in Cartesian

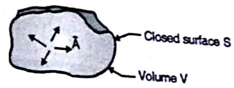


Fig. 6.3.4 : divergence of a vector

- We wish to find the total flux through a closed surface. This can be obtained by adding flux through each surface forming a closed surface. The closed surface in the Fig. 6.3.3 consists of six surfaces namely front, back, left, right, top and bottom. Thus,

$$\oint_S \bar{A}\cdot d\bar{s} = \int_{\text{front}} \bar{A}\cdot d\bar{s} + \int_{\text{Back}} + \int_{\text{Left}} + \int_{\text{Right}} + \int_{\text{Top}} + \int_{\text{Bottom}}$$

- Let us solve each integral separately.

$$\int_{\text{front}} \bar{A}\cdot d\bar{s} = \int_z^{z+\Delta z}\int_y^{y+\Delta y} [A_x]_{x+\Delta x}\bar{a}_x\cdot dy\,dz\,\bar{a}_x$$

$$= \int_z^{z+\Delta z}\int_y^{y+\Delta y} [A_x]_{x+\Delta x}\,dy\,dz$$

$$= [A_x]_{x+\Delta x}\,\Delta y\,\Delta z$$

- The term $[A_x]_{x+\Delta x}$ we read as component of $\bar{A}$ in x direction (i.e. A_x) at the surface $x + \Delta x$ which is a front surface.

$$\int_{\text{Back}} \bar{A}\cdot d\bar{s} = \int_z^{z+\Delta z}\int_y^{y+\Delta y} [A_x]_x\bar{a}_x\cdot dy\,dz\,(-\bar{a}_x) = -[\bar{A}_x]_x\,\Delta y\,\Delta z$$

- The negative sign in $d\bar{s}$ expression is due to normal to back surface is in negative x-direction (out of the box). Similarly we find other integrals as,

$$\int_{\text{Left}} \bar{A}\cdot d\bar{s} = -[A_y]_y \Delta x\, \Delta z$$

$$\int_{\text{Right}} \bar{A}\cdot d\bar{s} = [-A_y]_{y+\Delta y} \Delta x\, \Delta z$$

$$\int_{\text{Top}} \bar{A}\cdot d\bar{s} = [A_z]_{z+\Delta z} \Delta x\, \Delta y$$

$$\int_{\text{Bottom}} \bar{A}\cdot d\bar{s} = [A_z]_z \Delta x\, \Delta y$$

- Adding all these integral values we get total flux through closed surface.

$$\oint \bar{A}\cdot d\bar{s} = \{[A_x]_{x+\Delta x} - [A_x]_x\}\,\Delta y\, \Delta x + \{[A_y]_{y+\Delta y} - [A_y]_y\}\,\Delta x\,\Delta z - \{[A_z]_{z+\Delta z} - [A_z]_z\}\,\Delta x\,\Delta y$$

$$\text{Now,}\quad \nabla\cdot\bar{A} = \lim_{\Delta v\to 0}\frac{\oint \bar{A}\cdot d\bar{s}}{\Delta v} = \lim_{\Delta v\to 0}\frac{\oint \bar{A}\cdot d\bar{s}}{\Delta x\ \Delta y\ \Delta z}$$

- Substituting the value of $\oint \bar{A}\cdot d\bar{s}$ and cancelling common terms,

$$\nabla\cdot\bar{A} = \lim_{\Delta x\to 0}\frac{[A_x]_{x+\Delta x} - [A_x]_x}{\Delta x} + \lim_{\Delta y\to 0}\frac{[A_y]_{y+\Delta y} - [A_y]_y}{\Delta y}$$

$$= + \lim_{\Delta z\to 0}\frac{[A_z]_{z+\Delta z} - [A_z]_z}{\Delta z}$$

- The term $[A_x]_{x+\Delta x} - [A_x]_x$ indicates change in A_x between front and back. Similarly other terms, so we write

$$\boxed{\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial_x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}}$$

- Remember the definition of divergence

$$\text{div}\bar{A} = \nabla\cdot\bar{A} = \lim_{\Delta v\to 0}\frac{\oint_s \bar{A}\cdot d\bar{s}}{\Delta v}$$

- From which for some finite volume enclosed by surface S we write

$$\boxed{\oint_s \bar{A}\cdot d\bar{s} = \int_v (\nabla\cdot\bar{A})\,dv}$$

- This is called as **Divergence Theorem**. It is true for any vector field. Divergence theorem relates the closed surface integral of a vector field to the volume integral of the divergence of that vector field.

(B) Word statement of Divergence Theorem :

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.

6.3.3 Curl of a Vector

- Curl is simply defined as circulation per unit area where a closed path is vanishingly small. It is represented as $\nabla\times\bar{A}$ for a vector field $\bar{A}$. Mathematically,

$$\nabla\times\bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

- Curl is a vector quantity as it has magnitude and direction both. In order to make its meaning clear, we consider the circulation of a vector field $\bar{A}$ around a closed path i.e. $\oint \bar{A}\cdot d\bar{l}$.

- It is evident that the curl of $\bar{A}$ is a rotational vector. Its magnitude would be the maximum circulation of $\bar{A}$ per unit area. Its direction is the normal direction of the area when the area is oriented so as to make the circulation maximum. For completeness, expressions for curl in cylindrical and spherical co-ordinates respectively area as follows :

$$\nabla \times \bar{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$$

(For cylindrical)

$$\nabla \times \bar{A} = \frac{1}{r \sin \theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin \theta) - \frac{\partial A_\phi}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(r A_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$$

If $\nabla \times \bar{A} = 0$, the vector filed is called irrotational.

- Let us consider a vector in Cartesian system having all three components (x, y and z), each of them depending on all three coordinates (x, y, and z).

$$\bar{A}(x, y, z) = A_x(x, y, z)\,\bar{a}_x + A_y(x, y, z)\,\bar{a}_y + A_z(x, y, z)\,\bar{a}_z$$

- Let us have three infinitesimal rectangular paths in planes parallel to the three mutually orthogonal planes of the Cartesian coordinate system, as shown in Fig. 6.3.5.

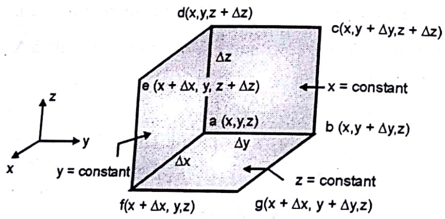


Fig. 6.3.5 : Infinitesimal rectangular paths

Now, $\nabla \times \bar{A}$ has three components, so

$$\nabla \times \bar{A} = (\nabla \times \bar{A})_x \bar{a}_x + (\nabla \times \bar{A})_y \bar{a}_y + (\nabla \times \bar{A})_z \bar{a}_z$$

- Where $(\nabla \times \bar{A})_x$ indicates component of $\nabla\times\bar{A}$ is x-direction and so on. To find x-component, consider a plane normal to x-direction, which is x = constant plane. Taking integral of $\bar{A} \cdot d\bar{l}$ for infinitesimal rectangular path (abcda) we

get $(\nabla \times \bar{A})_x$. Similarly for $(\nabla \times \bar{A})_y$ and $(\nabla \times \bar{A})_z$ we will consider infinitesimal paths in y = constant and z = constant planes (adefa and afgba) respectively.

$$\oint_{abcda} \bar{A} \cdot d\bar{l} = \int_{ab} \bar{A} \cdot d\bar{l} + \int_{bc} \bar{A} \cdot d\bar{l} + \int_{cd} \bar{A} \cdot d\bar{l} + \int_{da} \bar{A} \cdot d\bar{l}$$

Each of these integral we will solve separately

- $$\int_{ab} \bar{A}\cdot d\bar{l} = \int_{y}^{y+\Delta y} [A_y]_{ab}\,\bar{a}_y \cdot dy\,\bar{a}_y = [A_y]_{ab}\,\Delta y = [A_y]_{(x,z)}\,\Delta y$$

- Where $[A_y]_{ab} \Rightarrow [A_y]_{(x,z)}$ we read as value of A_y along ab where x and z are constants. Similarly other notations.

$$\int_{bc} \bar{A} \cdot d\bar{l} = \int_{z}^{z+\Delta z} [A_z]_{bc}\,\bar{a}_z \cdot d_z\,\bar{a}_z = [A_z]_{bc}\,\Delta z = [A_z]_{(x,y+\Delta y)}\Delta z$$

$$\int_{cd} \bar{A} \cdot d\bar{l} = \int_{y+\Delta y}^{y} [A_z]_{cd}\,\bar{a}_y \cdot d_y\,\bar{a}_y = -[A_y]_{cd}\,\Delta y = -[A_y]_{(x,z+\Delta z)}\Delta y$$

$$\int_{da} \bar{A} \cdot d\bar{l} = \int_{z+\Delta z}^{z} [A_z]_{da}\,\bar{a}_z \cdot d_z\,\bar{a}_z = -[A_z]_{da}\,\Delta z = -[A_z]_{(x,y)}\Delta z$$

- Adding all we get,

$$\oint \bar{A}\cdot d\bar{l} = \left\{[A_z]_{(x,y+\Delta y)} - [A_z]_{(x,y)}\right\}\Delta z + \left\{[A_y]_{(x,z)} - [A_y]_{(x,z+\Delta z)}\right\}\Delta y$$

- Using the definition of curl $\bar{A}$,

$$(\text{curl } A)_x = (\nabla\times\bar{A})_x = \lim_{\Delta y\,\Delta z \to 0}\left(\frac{\oint_{abcda} \bar{A}\cdot d\bar{l}}{\text{area } \Delta y\,\Delta z}\right)$$

$$= \lim_{\Delta y \to 0}\frac{[A_z]_{(x,y+\Delta y)} - [A_z]_{(x,y)}}{\Delta y} - \lim_{\Delta z \to 0}\frac{[A_y]_{(x,z+\Delta z)} - [A_y]_{(x,z)}}{\Delta z}$$

$$= \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \qquad \text{...(6.3.2)}$$

- Similarly by taking area adef i.e. taking a path adefa and area afgb (path afgba) gives y and z components of $\nabla \times \bar{A}$.

$$(\nabla \times \bar{A})_y = \lim_{\Delta x\, \Delta z \to 0} \left(\frac{\oint_{adefa} \bar{A} \cdot d\bar{l}}{\text{area } \Delta x\, \Delta z} \right) = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \qquad ...(6.3.3)$$

$$(\nabla \times \bar{A})_z = \lim_{\Delta x\, \Delta y \to 0} \left(\frac{\oint_{afgba} \bar{A} \cdot d\bar{l}}{\text{area } \Delta x\, \Delta y} \right) = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \qquad ...(6.3.4)$$

- Combining Equations (6.3.2), (6.3.3) and (6.3.4),

$$\nabla \times \bar{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \bar{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \bar{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \bar{a}_z$$

$$\text{or,} \quad \boxed{\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}} \qquad ...(6.3.5)$$

- Note that curl of $\bar{A}$ is defined at a point. But when the area S is some finite area using definition of curl we can prove that,

$$\oint_C \bar{A} \cdot d\bar{l} = \int_S (\nabla \times \bar{A}) \cdot d\bar{s}$$

- This is called as Stoke's theorem. It relates closed line integral of the field with surface integral.

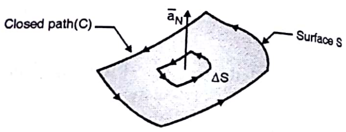


Fig. 6.3.6 : Curl of a vector

Word statement of Stoke's theorem :

It states that the circulation of a vector field $\bar{A}$ around a closed path C is equal to the surface integral of the curl of $\bar{A}$ over the open surface S bounded by C provided that $\bar{A}$ and $\nabla \times \bar{A}$ are continuous on S.

Syllabus Topic : Derivation of Maxwell's Four Equation

> **Topics covered :** Maxwell's Equations, Derivation of Maxwell's First Equation, Derivation of Maxwell's Second Equation, Derivation of Maxwell's Third Equation, Derivation of Maxwell's Fourth Equation

6.4 Maxwell's Equations in Differential or Point Form

- The fundamental law of electricity and their connection with magnetic field was studied by many Physicists and the outcome as laws of electromagnetism is given as under.

(a) Gauss law (for static electric field)

$$\nabla . \bar{E} = \frac{\rho}{\epsilon_0}$$

Where ρ = Charge density and

ϵ_0 = Permittivity of free space

(b) Gauss law (for static magnetic field)

$$\nabla . \bar{B} = 0$$

(c) Faraday's law (for electromagnetic field)

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

(d) Ampere's law (For magneto electric field)

$$\nabla . \bar{B} = \mu_0 \bar{J}$$

Where $\bar{J}$ = current density

- After the discovery of electromagnetic laws, James Clark Maxwell set out to formulate mathematical equations based on the laws of electromagnetism.
- There are two forms of each Maxwell equation namely integral form and differential or point form.
- Integral form govern the interdependence of certain field (E, D, B, H) and source quantities (charge and current) associated with regions in space differential form of Maxwell's equations relate characteristics of the field vector a given point to one another and to the source densities at that point.
- These equations provide us the mathematical background for the study of electromagnetic waves, transmission lines and antenna.

6.4.1 Derivation of Maxwell's First Equation

- Let us assume an arbitrary surface's' bounding an arbitrary volume V in a dielectric medium. For any dielectric medium, the total charge density present is the sum of free charge density (ρ) and polarized charged density (ρ_p).
- The total electric flux ψ crossing the closed surface is equal to the total charge enclosed by that surface (Gauss law of electrostatics)

$$\iint \bar{E}\cdot ds = \oint_s \bar{E}\cdot ds = \frac{1}{\epsilon_0}\oint_v (\rho+\rho_p)\, dV$$

Where $q = \rho V$ and $\rho_p = -\nabla\cdot\bar{P}$

$$\oint_s \epsilon_0 \bar{E}\, ds = \oint_v (\rho-\nabla\cdot\bar{P})\, dV$$

- Using divergence theorem convert surface integral to volume integral

$$\oint_s \epsilon_0 E\, ds = \oint_v \nabla\cdot(\epsilon_0\bar{E})\, dV = \oint_v (\rho-\nabla\cdot\bar{P})\, dV$$

$$\therefore \oint_v \nabla\cdot(\epsilon_0\bar{E}+P)\, dV = \oint_v \rho\, dV$$

- As electric displacement vector $\bar{D} = \epsilon_0\bar{E}+P$

$$\therefore \oint_v \nabla\cdot\bar{D}\, dV = \oint_v \rho\, dV$$

$$\text{Or} \oint_v (\nabla\cdot\bar{D}-\rho)\, dV = 0 \qquad \therefore \nabla\cdot\bar{D}-\rho = 0$$

$$\nabla\cdot\bar{D} = \rho$$

- This is Point from of Maxwell's first equation.

6.4.2 Derivation of Maxwell's Second Equation

- As the magnetic lines of force are closed (or go off to infinity), the number of magnetic lines of flux entering any surface is exactly the same as leaving. (The occurrence of an isolated magnetic pole has no physical significance).

$$\therefore \oint_v \bar{B}\cdot ds = 0$$

- Using Gauss divergence theorem, convert surface integral to volume integral.

$$\therefore \oint_v \bar{B}\cdot ds = \oint_v \nabla\cdot\bar{B}\, dv = 0$$

$$\therefore \nabla\cdot\bar{B} = 0$$

- This is point form of Maxwell's second equation.

6.4.3 Derivation of Maxwell's Third Equation

- According to Faraday's law, electromagnetic force induced in a closed loop is negative rate of charge of the magnetic flux.

$$e = \frac{-d\phi}{dt} \qquad \text{...(6.4.1)}$$

- Total magnetic flux on any arbitrary surface S

$$\phi = \oint_s \bar{B}\, ds \qquad \text{...(6.4.2)}$$

$$\therefore e = \frac{-d}{dt}\left[\oint_s \bar{B}\cdot ds\right] = -\oint_s \left[\frac{d\bar{B}}{dt}\right] ds$$

- The electromotive force is the work done in carrying a unit charge around the closed loop.

$$\therefore e = \oint_l \bar{E}\cdot dl$$

$$\therefore \oint_l \bar{E}\cdot dl = -\oint_s \left(\frac{d\bar{B}}{dt}\right) ds$$

- By using Stoke's theorem contour integration can be converted to surface integration as

$$\oint_l \bar{E}\cdot dl = \oint_s (\bar{\nabla}\times\bar{E})\, ds$$

$$\therefore \oint_s (\bar{\nabla}\times\bar{E})\, ds = -\oint_s \frac{d\bar{B}}{dt}\, ds$$

$$\therefore \oint_s \left(\nabla\times\bar{E}+\frac{d\bar{B}}{dt}\right) ds = 0$$

$$\therefore \nabla\times\bar{E} = \frac{-d\bar{B}}{dt}$$

This is Maxwell's third equation.

6.4.4 Derivation of Maxwell's Fourth Equation

- Consider the definition of work done by the magnetic field around the closed closed loop and
- Using Ampere's circuital law

$$\oint \bar{H} \cdot dl = I = \oint \bar{J} \cdot ds$$

Where $\bar{J}$ = current density

- Use Stoke's theorem to convert the contour integration to surface integrative.

$$\therefore \oint \bar{J} \cdot ds = \oint (\nabla \times \bar{H})\, ds$$

$$\therefore \oint (\bar{J} - \nabla \times \bar{H})\, ds$$

$$\therefore \bar{J} = \nabla \times \bar{H} \quad \text{...(6.4.3)}$$

- This equation needs to be verified for time varying field as its validity holds for steady state only. Because if it is correct then conservation of charge will be violated. The reason is very simple : div of curl is zero.

$$\text{div } J = \text{div}(\nabla \times \bar{H}) = 0$$

- This is in contrast with the continuity equation.

$$\text{div } \bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{or div } \bar{J} = -\frac{\partial \rho}{\partial t} \quad \text{...(6.4.4)}$$

- To correct this, Maxwell suggested the total current density needs an additional component i.e. $\bar{J}$.

$$\nabla \times \bar{H} = \bar{J} + \bar{J}'$$

- Now, if divergence is taken then using Equation (6.4.3).

$$\text{div}(\nabla \times \bar{H}) = \text{div}(\bar{J} + \bar{J}')$$

$$0 = \text{div}(\bar{J} + \bar{J}')$$

$$\therefore \text{div } \bar{J} = \text{div } \bar{J}$$

- From Equation (6.4.4).

$$\text{div } \bar{J} = -\frac{\partial \rho}{\partial t}$$

- But from Maxwell's first Equation

$$\nabla \cdot \bar{D} = \rho$$

$$\therefore \text{div } \bar{J} = \frac{\partial}{\partial t}(\nabla \cdot \bar{D}) = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

$$\text{Hence, } \quad J' = \frac{\partial \bar{D}}{\partial t}$$

- ∴ Maxwell's fourth Equation is

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

- The additional term $\frac{\partial \bar{D}}{\partial t}$ is called Maxwell's correction and it is known as displacement current.

6.5 Maxwell's Equations in Integral Form

As in science and technology many times it happen that compare to differential form, Integral form of Maxwell's equation is found easy for applications. They can be derived and they are represented as

6.5.1 Maxwell's First Equation

$$\text{As } \nabla \cdot \bar{D} = \rho$$

Take integration over complete volume V.

$$\therefore \int_v \nabla \cdot \bar{D}\, dv = \int_v \rho\, dv$$

Using Gauss's divergence theorem

$$\int_v \nabla \cdot \bar{D}\, ds = \oint_s \bar{D} \cdot ds$$

$$\therefore \oint_s \bar{D} \cdot ds = \int_V \rho\, dv$$

As $\rho\, dv = q$

$$\oint \bar{D} \cdot ds = q$$

6.5.2 Maxwell's Second Equation

As $\nabla \cdot \bar{B} = 0$

Again using Gauss's divergence theorem and procedure adopted above one can arrive at

$$\oint_s \bar{B} \cdot ds = 0$$

6.5.3 Maxwell's Third Equation

Using differential form of Maxwell's Third Equation

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Integrating over a surface

$$\int_s (\nabla \times \bar{E}) \cdot ds = -\int_s \frac{\partial \bar{B}}{\partial t} ds$$

Using Stoke's theorem, convert surface integral to line integral.

$$\oint_c \bar{E} \cdot dl = -\frac{\partial}{\partial t} \int \bar{B} \cdot ds$$

6.5.4 Maxwell's Fourth Equation

Using differential form of Maxwell's Equation

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

Using Stoke's theorem and procedure adopted in Section 6.5.3 one can arrive at

$$\oint_c \bar{H} \cdot dl = \int_c \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) ds$$

Syllabus Topic : Applications - Design of Antenna, Wave Guide, Satellite Communication

6.6 Applications - Design of Antenna, Waveguide, Satellite Communication

- In the present era of communication we are heavily dependent on various communication devices, like cell phone, radio, TV, satellite etc. Looking at the scope of communication field one can divide this into at least two part (i) Wired communication (ii) Wireless communication.
- For wired communications we need something like transmission line or waveguide. For wireless communication we require antenna to transmit or receive signals.
- In both the modes we badly require a complete knowledge of electromagnetism and Maxwell's equation. To carry electromagnetic wave when we make use of wired mode we make use of transmission lines or cables. For point to point transmission, the source energy must be directed for this purpose transmission line is use.
- Other way to make electromagnetic wave propagate is waveguide. A structure that can "guide" waves is called waveguide. The original and the most common meaning of waveguide is a hollow metal pipe used for guiding the waves.
- The electromagnetic waves in such waveguides may be imagined as waves travelling down the guide in a zig-zag path as these waves are repeatedly reflected between opposite walls of guide. In order to analyze the mode propagation in the waveguide, we solve the Maxwell's equation.
- For wireless or satellite communications we essentially require antenna. While designing antenna, all the inputs like distance, medium, energy density, wavelength etc. are considered.

6.7 Solved Problems

Problems on Basic Vector Algebra

Ex. 6.7.1 : Two vectors are represented by

$\bar{A} = 2\bar{a}_x + 2\bar{a}_y + 0\bar{a}_z$; $\bar{B} = 3\bar{a}_x + 4\bar{a}_y - 2\bar{a}_z$

Find $\bar{A} \times \bar{B}$ and show that $\bar{A} \times \bar{B}$ is at right angle to $\bar{A}$.

Soln. :

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & 2 & 0 \\ 3 & 4 & -2 \end{vmatrix} = \bar{a}_x\,(-4-0) - \bar{a}_y\,(-4-0) + \bar{a}_z\,(8-6)$$

$$= -4\bar{a}_x + 4\bar{a}_y + 2\bar{a}_z$$

The vector $\bar{A} \times \bar{B}$ is at right angle to $\bar{A}$ then

$$(\bar{A} \times \bar{B}) \cdot \bar{A} = 0$$

$$\text{L.H.S} = (-4\,\bar{a}_x + 4\,\bar{a}_y + 2\bar{a}_z) \cdot (2\bar{a}_x + 2\,\bar{a}_y) = -8 + 8 = 0 = \text{R.H.S.}$$

$\therefore$ $\bar{A} \times \bar{B}$ is at right angle to $\bar{A}$.

Ex. 6.7.2 : Given vectors

$\bar{A} = 3\,\bar{a}_x + 4\,\bar{a}_y + \bar{a}_z$, $\bar{B} = 2\,\bar{a}_y - 5\,\bar{a}_z$

Find the angle between $\bar{A}$ and $\bar{B}$ using (i) dot product (ii) cross product.

Soln. :

The magnitudes of vectors are,

$$A = |\bar{A}| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26}$$

$$B = |\bar{B}| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$$

i) Using dot product :

$$\bar{A} \cdot \bar{B} = AB \cos\theta$$

Here the dot product is obtained using

$$\bar{A} \cdot \bar{B} = (3 \times 0) + (4 \times 2) + (1 \times -5) = 3$$

$$\therefore 3 = \sqrt{26}\sqrt{29}\cos\theta \rightarrow \cos\theta = \frac{3}{\sqrt{26}\sqrt{29}} = 0.1093$$

$$\theta = \cos^{-1}(0.1093) = 83.73°$$

ii) Using cross product :

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix} = \bar{a}_x(-20-2) - \bar{a}_y(-15-0) + \bar{a}_z(6-0)$$

$$= -22\,\bar{a}_x + 15\,\bar{a}_y + 6\,\bar{a}_z$$

We have, $\bar{A} \times \bar{B} = AB \sin\theta\, \bar{a}_z$

Taking mod of both sides,

$$|\bar{A} \times \bar{B}| = AB \sin\theta$$

$$\text{or} \quad \sin\theta = \frac{|\bar{A} \times \bar{B}|}{AB} = \frac{\sqrt{(-22)^2 + (15)^2 + 6^2}}{\sqrt{26}\sqrt{29}} = \frac{\sqrt{745}}{\sqrt{26}\sqrt{29}} = 0.994$$

$$\text{or} \quad \theta = \sin^{-1}(0.994) = 83.73°$$

Ex. 6.7.3 : Find area of rectangle in z = 5 plane with $-1 \le x \le 3$ and $0 \le y \le 5$.

Soln. :

As $z = 5$, $dz = 0$, $\therefore$ $ds = dx\,dy$

The total area is,

$$S = \int ds = \int_0^5 \int_{-1}^3 dx\,dy$$

$$= [x]_{-1}^{3}\,[y]_0^5 = 4 \times 5 = 20\ \text{m}^2$$

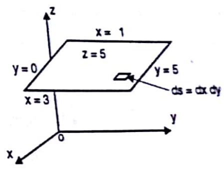


Fig. P. 6.7.3

Ex. 6.7.4 : Find volume of a closed surface bounded by $0 \le x \le 2$, $0 \le y \le 1$ and $0 \le z \le 5$.

Soln. :

Differential volume is $dv = dx\,dydz$

$\therefore$ The total volume is,

$$V = \int dv = \int_0^5\int_0^1\int_0^2 dx\,dydz = [x]_0^2\,[y]_0^1\,[z]_0^5 = 2 \times 1 \times 5 = 10\ \text{m}^3.$$

Ex. 6.7.5 : Express the unit vector which is directed towards the origin from an arbitrary point on the plane $z = -3$.

Soln. :

The point P on the $z = -3$ plane can have any x and y coordinate but z coordinate is -3. The general coordinates of P can be written as $(x, y, -3)$.

The vector from P to origin is,

$$\bar{R} = (0-x)\ \bar{a}_x + (0-y)\ \bar{a}_y + (0-(-3))\ \bar{a}_z$$

$$= -x\,\bar{a}_x - y\,\bar{a}_y + 3\ \bar{a}_z$$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-x\,\bar{a}_x - y\,\bar{a}_y + 3\,\bar{a}_z}{\sqrt{x^2+y^2+9}}$$

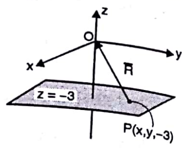


Fig. P. 6.7.5

Ex. 6.7.6 : Three points P_1, P_2, and P_3 are given by (2, 3, – 2), (5, 8, 3) and (7, 6, 2) respectively. Obtain

(a) the vector drawn from P_1 to P_2;

(b) the straight line distance from P_2 to P_3; and

(c) the unit vector along the line from P_1 to P_3.

Soln. : From the given points

(a) $\overline{P_1 P_2} = (5-2)\ \bar{a}_x + (8-3)\ \bar{a}_y + [3-(-2)]\ \bar{a}_z = 3\ \bar{a}_x + 5\ \bar{a}_y + 5\ \bar{a}_z$

(b) To find the straight line distance, either distance formula can be used or find length of vector $\overline{P_2 P_3}$.

$$\overline{P_2 P_3} = (7-5)\ \bar{a}_x + (6-8)\ \bar{a}_y + (2-3)\ \bar{a}_z = 2\ \bar{a}_x - 2\ \bar{a}_y - \bar{a}_z$$

Now, $P_2 P_3 = \left|\overline{P_2 P_3}\right| = \sqrt{2^2+(-2)^2+(-1)^2} = 3$

(c) The vector from P_1 to P_3 is,

$$\overline{P_1 P_3} = (7-2)\ \bar{a}_x + (6-3)\ \bar{a}_y + [2-(-2)]\ \bar{a}_z$$

$$= 5\ \bar{a}_x + 3\ \bar{a}_y + 4\ \bar{a}_z$$

$$\bar{a}_{P_1P_3} = \frac{\overline{P_1 P_3}}{|P_1P_3|} = \frac{5\ \bar{a}_x + 3\ \bar{a}_y + 4\ \bar{a}_z}{\sqrt{25+9+16}}$$

$$= 0.71\ \bar{a}_x + 0.42\ \bar{a}_y + 0.57\ \bar{a}_z$$

Problems on Conversion of Coordinates System

Ex. 6.7.7 : Find the area of circular disk with radius of 2 m.

Soln. :

For simplicity place the circular disk with centre at origin in xy place. This circular disk is a part of cylinder in cylindrical system. It is horizontal, therefore

z = constant = 0 or $dz = 0$

$\therefore$ differential area is $ds = r\,dr\,d\phi$

And total area is obtained by integrating this,

$$\therefore S = \int ds = \int_0^{2\pi}\int_0^{2} r\,dr\,d\phi = \left[\frac{r^2}{2}\right]_0^2 [\phi]_0^{2\pi}$$

$$= 2 \times 2\pi = 4\pi\ (m^2)$$

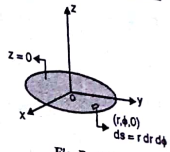


Fig. P. 6.7.7

Even if you consider this disk as a top plate of the cylinder, there is no problem, the answer will be same.

Ex. 6.7.8 : Find the circumference of the circle with radius of 3 m.

Soln. :

Imagine the circle as a part of cylinder in cylindrical system either top or bottom of the cylinder. This is horizontal. For any point on the circle

r = constant = 3m, and z = constant = 0

$\therefore$ $dr = 0$, $dz = 0$

Then, $dl = r\,d\phi$

The total length i.e. circumference is,

$$= \int dl = \int_0^{2\pi} r\,d\phi = 3 \times [\phi]_0^{2\pi}$$

$$= 3 \times 2\pi = 6\pi\ (m) \quad \text{...since } r = 3$$

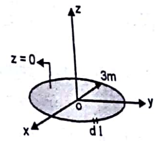


Fig. P. 6.7.8

Ex. 6.7.9 : Find area of the cylindrical surface with height = 2 m and radius = 1 m.

Soln. :

For the cylindrical surface,

r = constant = 1 m.

$\therefore dr = 0$ then $ds = r\,d\phi dz$

And $$S = \int ds = \int_0^{2}\int_0^{2\pi} r\,d\phi dz$$

$$= 1 \times [\phi]_0^{2\pi} \times [z]_0^2$$

$$= 1 \times 2\pi \times 2 = 4\pi\ (m^2)$$

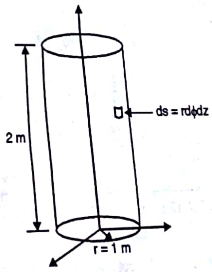


Fig. P. 6.7.9

Ex. 6.7.10 : *Use of the cylindrical coordinate system to find the area of a curved surface on the right circular cylinder of radius 2 m, height 8 m and $45° \le \phi \le 90°$.*

Soln. :

The given surface is a part of cylinder with r = 2m. then

$$dS = r\, d\phi dz = 2\, d\phi dz$$

The total area is obtained by integrating it.

$$S = \int_S dS = 2 \int_0^8 \int_{\pi/4}^{\pi/2} d\phi\, dz$$

$$= 2 \times (\phi)_{\pi/4}^{\pi/2} (z)_0^8$$

$$= 2 \times \frac{\pi}{4} \times 8 = 4\pi\ (m^2)$$

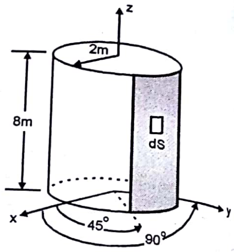


Fig. P. 6.7.10

Ex. 6.7.11 : Find the volume of the cylinder with height = 3 m and radius = 2 m.

Soln. :

The differential volume in cylindrical system is,

$$dv = r\, dr\, d\phi dz$$

Then, the total volume is obtained by integrating,

$$V = \int dv = \int_0^3 \int_0^{2\pi} \int_0^2 r\, dr\, d\phi dz$$

$$= \left[\frac{r^2}{2}\right]_0^2 [\phi]_0^{2\pi} [z]_0^3$$

$$= \frac{1}{2}(4-0) \times (2\pi - 0) \times (3-0)$$

$$= 12\pi\ (m^3)$$

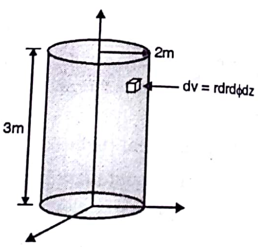


Fig. P. 6.7.11

Ex. 6.7.12 : Convert the following points specified in Cartesian into cylindrical coordinates (a) $(0, -2, 2)$, (b) $(\sqrt{3}, 1, -1)$ and (c) $(-\sqrt{2}, \sqrt{2}, 3)$.

Soln. : To convert the points from cartesian to cylindrical, use relations

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

(a) The given point is $(0, -2, 2)$

then, $r = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2$; $\phi = \tan^{-1}\left(\frac{-2}{0}\right) = -\frac{\pi}{2}$

but as $y = -2$, the angle is $3\pi/2$ and $z = 2$.

Therefore, the point in cylindrical is $(2, 3\pi/2, 2)$

(b) The given point is $(\sqrt{3}, 1, -1)$

then, $r = \sqrt{(\sqrt{3})^2 + (\sqrt{1})^2} = 2$, $\phi = \tan^{-1}\frac{1}{3} = \frac{\pi}{6}$ and $z = -1$

therefore the point in cylindrical is $(2, \pi/6, -1)$

(c) the given point is $(-\sqrt{2}, \sqrt{2}, 3)$

then, $r = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = 2$, $\phi = \tan^{-1}\frac{\sqrt{2}}{-\sqrt{2}} = -\frac{\pi}{4}$ and $z = 3$

but as x coordinate is – ve, the angle is $3\pi/4$.

Therefore, the point in cylindrical is $(2, 3\pi/4, 3)$.

Ex. 6.7.13 : Convert the following points specified in cylindrical into Cartesian coordinates (a) $(2, 5\pi/3, -2)$; (b) $(4, \pi/6, 1)$

Soln. :

The required relations to convert points from cylindrical into Cartesian are

$$x = r\cos\phi\,; \quad y = r\sin\phi\,; \quad z = z$$

(a) The given point is $(2, 5\pi/3, -2)$

then, $x = 2\cos\left(\frac{5\pi}{3}\right) = 1$, $y = 2\sin\left(\frac{5\pi}{3}\right) = -1.73$ and $z = -2$

Therefore, the point in Cartesian is $(1, -1.73, -2)$

(b) The given point is $(4, \pi/6, 1)$

then, $x = 4 \times \cos\left(\frac{\pi}{6}\right) = 3.46$, $y = 4 \times \sin\left(\frac{\pi}{6}\right) = 2$ and $z = -2$

Therefore, the point in cartesian is $(3.46, 2, 1)$.

Ex. 6.7.14 : Given points A (x = 2, y = 3, z = – 1) and B (ρ = 4, φ = – 50°, z = 2) find the distance A to B.

Soln. : Here note that ρ stands for r in cylindrical, converting B into Cartesian

$$x = \rho \cos\phi = 4\cos(-50°) = 0.643$$

$$y = r\sin\phi = 4\sin(-50°) = -0.766$$

$$z = z = r$$

The distance between A, B is,

$$AB = \sqrt{(2-0.643)^2 + (3-(-0.766))^2 + (-1-2)^2} = \sqrt{25.02} = 5.$$

Ex. 6.7.15 : Transform $\bar{B} = y\,\bar{a}_x - x\,\bar{a}_y + z\,\bar{a}_z$ into cylindrical system.

Soln. :

Let $\bar{B} = B_r\,\bar{a}_r + B_\phi\,\bar{a}_\phi + B_z\,\bar{a}_z$ be a vector in cylindrical system where B_r, B_ϕ and B_z are length of projections of given vector in the directions $\bar{a}_r$, $\bar{a}_\phi$ and $\bar{a}_z$.

The length of projection of given $\bar{B}$ in the direction $\bar{a}_r$ is

$$B_r = \bar{B}\cdot\bar{a}_r = y\,\bar{a}_x\cdot\bar{a}_r - x\,\bar{a}_y\cdot\bar{a}_r + z\,\bar{a}_z\cdot\bar{a}_r$$

Using the table of dot product and

$$x = r\cos\phi; \quad y = r\sin\phi; \quad z = z$$

We get, $\quad B_r = r\sin\phi\,(\cos\phi) - r\cos\phi\,(\sin\phi) + z\,(0) = 0$

Similarly, $\quad B_\phi = \bar{B}\cdot\bar{a}_\phi = y\,\bar{a}_x\cdot\bar{a}_\phi - x\,\bar{a}_y\cdot\bar{a}_\phi + z\,\bar{a}_z\cdot\bar{a}_\phi$

$$= r\sin\phi\,(-\sin\phi) - r\cos\phi\,(\cos\phi) + 0$$

$$= -r\sin^2\phi - r\cos^2\phi = -r$$

and $\quad B_z = \bar{B}\cdot\bar{a}_z = y\,\bar{a}_x\cdot\bar{a}_z - x\,\bar{a}_y\cdot\bar{a}_z + z\,\bar{a}_z\cdot\bar{a}_z = 0 - 0 + z = z$

Then the vector in cylindrical system is obtained by using the values of B_r, B_ϕ and B_z as,

$$\bar{B} = -r\,\bar{a}_\phi + z\,\bar{a}_z$$

Ex. 6.7.16 : Transform given vector $\bar{A}$ into cylindrical system.

$$\bar{A} = y\,\bar{a}_x + x\,\bar{a}_y + \frac{x^2}{\sqrt{x^2+y^2}}\,\bar{a}_z$$

Soln. :

Let vector in cylindrical system be $\bar{A} = A_r\,\bar{a}_r + A_\phi\,\bar{a}_\phi + A_z\,\bar{a}_z$

$$A_r = \bar{A}\cdot\bar{a}_r = \left(y\,\bar{a}_x + x\,\bar{a}_y + \frac{x^2}{\sqrt{x^2+y^2}}\,\bar{a}_z\right)\cdot\bar{a}_r$$

$$= y\,\bar{a}_x\cdot\bar{a}_r + x\,\bar{a}_y\cdot\bar{a}_r + \frac{x^2}{\sqrt{x^2+y^2}}\,\bar{a}_z\cdot\bar{a}_r$$

$$= y\cos\phi + x\sin\phi + 0$$

To express in terms of cylindrical use the relations $x = r\cos\phi$, $y = r\sin\phi$ .....(Using Table 6.1.1)

$$\therefore \quad A_r = r\sin\phi\cos\phi + r\cos\phi\cdot\sin\phi = 2r\sin\phi\cos\phi = r\sin 2\phi$$

$$A_\phi = \bar{A}\cdot\bar{a}_\phi = y\,\bar{a}_x\cdot\bar{a}_\phi + x\,\bar{a}_y\cdot\bar{a}_\phi + \frac{x^2}{\sqrt{x^2+y^2}}\,\bar{a}_z\cdot\bar{a}_\phi$$

$$= y(-\sin\phi) + x(\cos\phi) + 0 = r(\cos^2\phi - \sin^2\phi) = r\cos 2\phi$$

$$A_z = \bar{A}\cdot\bar{a}_z = y\,\bar{a}_x\cdot\bar{a}_z + x\,\bar{a}_y\cdot\bar{a}_z + \frac{x^2}{\sqrt{x^2+y^2}}\,\bar{a}_z\cdot\bar{a}_z$$

$$= 0 + 0 + \frac{x^2}{\sqrt{x^2+y^2}} = \frac{(r\cos\phi)^2}{\sqrt{r^2}} = r\cos^2\phi$$

Hence, $\bar{A} = r\sin 2\phi\,\bar{a}_r + r\cos 2\phi\,\bar{a}_\phi + r\cos^2\phi\,\bar{a}_z$ is a vector in cylindrical coordinate system.

Ex. 6.7.17 : Convert $\bar{B} = -r\,\bar{a}_\phi + z\,\bar{a}_z$ in rectangular system.

Soln. :

Let vector in cartesian system be $\bar{B} = B_x\,\bar{a}_x + B_y\,\bar{a}_y + B_z\,\bar{a}_z$ ...(i)

Where

$$B_x = \bar{B}\cdot\bar{a}_x = (-r\,\bar{a}_\phi + z\,\bar{a}_z)\cdot\bar{a}_x = -r\,\bar{a}_\phi\cdot\bar{a}_x + z\,\bar{a}_z\cdot\bar{a}_x = r\sin\phi \quad ...(ii)$$

$$B_y = \bar{B}\cdot\bar{a}_y = -r\,\bar{a}_\phi\cdot\bar{a}_y + z\,\bar{a}_z\cdot\bar{a}_y = -r\cos\phi \quad ...(iii)$$

$$B_z = \bar{B}\cdot\bar{a}_z = z \quad ...(iv)$$

Using the relation $\phi = \tan^{-1}\frac{y}{x}$, draw a triangle as shown in figure and from the triangle

$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}}, \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$

Putting these values in Equations (ii), (iii) and (iv) and using $r = \sqrt{x^2+y^2}$

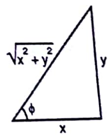


Fig. P. 6.7.17

we get,

$$B_x = \sqrt{x^2+y^2} \cdot \frac{y}{\sqrt{x^2+y^2}} = y\ ; \quad B_y = -x\ ; \quad B_z = z$$

Then Equation (i) becomes

$$B = y\,\bar{a}_x - x\,\bar{a}_y + z\,\bar{a}_z$$

In this problem, instead of using the triangle. We can use directly the result

$$x = r\cos\phi, \qquad y = r\sin\phi$$

Ex. 6.7.18 : Find area of spherical surface of radius 1 m using spherical coordinates.

Soln. :

Though we can find area of sphere as $4\pi r^2$ the following procedure illustrate the use of spherical coordinate system.

The given surface can be placed in coordinate system as shown in figure.

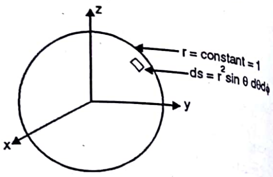


Fig. P. 6.7.18

For the spherical surface,

$r = \text{constant} = 1$ m, $\therefore\ dr = 0$

Hence, $\quad dS = r^2 \sin\theta\, d\theta d\phi$

$$S = \int dS$$

$$= \int_0^{2\pi}\int_0^{\pi} 1^2 \sin\theta\, d\theta d\phi \quad \ldots \because r = 1$$

$$= [-\cos\theta]_0^{\pi}\,[\phi]_0^{2\pi} = 4\pi\ (m)^2$$

Ex. 6.7.19 : Find the volume of above sphere.

Soln. :

The differential volume is spherical coordinate system is

$$dv = r^2 \sin\theta dr\, d\theta d\phi$$

$$\therefore \quad v = \int dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} r^2 \sin\theta dr\, d\theta d\phi$$

$$= \left[\frac{r^3}{3}\right]_0^1 [-\cos\theta]_0^{\pi}\,[\phi]_0^{2\pi} = \frac{1}{3}\times 2\times 2\pi = \frac{4}{3}\pi\ (m^3)$$

Ex. 6.7.20 : Use spherical coordinates to find area of strip $\alpha \le \theta \le \beta$ on a sphere of radius a. What is the result when $\alpha = 0°$ and $\beta = \pi$.

Soln.: Figure gives the clear idea of the strip in the problem.

For the given surface

$r = \text{constant} = a \qquad \therefore dr = 0$

$$dS = r^2 \sin\theta\, d\theta d\phi$$

$$S = \int_0^{2\pi}\int_{\alpha}^{\beta} r^2 \sin\theta\, d\theta d\phi = a^2[-\cos\theta]_{\alpha}^{\beta}\,[\phi]_0^{2\pi}$$

$$= a^2\,[\cos\alpha - \cos\beta]\times 2\pi = 2\pi a^2\,(\cos\alpha - \cos\beta)\ m^2$$

When $\alpha = 0$ and $\beta = \pi$

$$S = 2\pi a^2\,(\cos 0 - \cos\pi) = 4\pi a^2\ (m^2)$$

The result is quite obvious because the given limits convert the strip into sphere and the area of sphere is $4\pi a^2$.

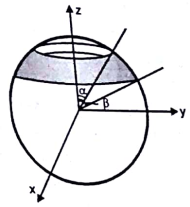


Fig. P. 6.7.20

Ex. 6.7.21 : Find areas (1) and (2) for the surfaces shown in Fig. P. 6.7.21.

Soln. : For surface (1) : $\quad \phi = \text{constant} = 60°, \quad \therefore\ d\phi = 0$

$$dS = r\,dr\,d\theta$$

$$S = \int dS = \int_0^{\pi/2}\int_0^{2} r\,dr\,d\theta = \left[\frac{r^2}{2}\right]_0^2 [\theta]_0^{\pi/2}$$

$$= 2\times\frac{\pi}{2} = \pi\ (m^2)$$

For surface (2) : $\quad r = \text{constant} = 2, \quad \therefore\ dr = 0$

Hence, $\quad dS = r^2 \sin\theta\, d\theta d\phi$

$$S = \int dS = \int_{\pi/3}^{\pi/2}\int_0^{\pi/2} 2^2 \sin\theta\, d\theta d\phi$$

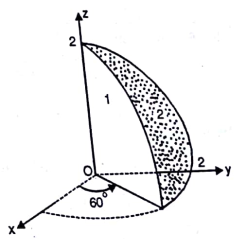


Fig. P. 6.7.21

... since variation of ϕ is from 60° to 90° i.e. $\frac{\pi}{3}$ to $\frac{\pi}{2}$ and θ varies from 0° to 90° i.e. 0 to $\frac{\pi}{2}$

$$\therefore \quad S = 4\,[-\cos\theta]_{\pi/3}^{\pi/2}\,[\phi]_{\pi/3}^{\pi/2} = \frac{2}{3}\pi\ (m^2)$$

Ex. 6.7.22 : Convert P (8, π / 3, π / 6) in cartesian coordinates.

Soln. :

The given point is in spherical system then using the relations,

$$x = r \sin\theta \cos\phi = 8 \sin(\pi/3) \cos(\pi/6) = 6,$$

$$y = r \sin\theta \sin\phi = 8 \sin(\pi/3) \sin(\pi/6) = 3.46,$$

$$z = r \cos\theta = 8 \cos(\pi/3) = 4$$

Thus the point in cartesian is (6, 3.64, 4).

Ex. 6.7.23 : Convert P (10, π / 6, π / 3) in cylindrical.

Soln. :

Since direct conversion from spherical to cylindrical is not derived, we go step by step by first converting point P into Cartesian using relations.

$$x = r \sin\theta \cos\phi = 10 \sin(\pi/6) \cos(\pi/3) = 2.5 \text{ m},$$

$$y = r \sin\theta \sin\phi = 10 \sin(\pi/6) \sin(\pi/3) = 4.33 \text{ m},$$

$$z = r \cos\theta = 10 \cos(\pi/6) = 8.66 \text{ m}.$$

Now convert into cylindrical coordinates using

$$r = \sqrt{x^2 + y^2} = \sqrt{(2.5)^2 + (4.33)^2} = 5$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(4.33/2.5) = 59.99°,$$

$$z = z = 8.66 \text{ m}$$

Ex. 6.7.24 : Show that the vector fields

$$\vec{A} = \hat{a}_r \frac{\sin 2\theta}{r^2} + 2\,\hat{a}_0 \frac{(\sin 2\theta)}{r^2} \text{ and } \vec{B} = r\cos\hat{a}_r + r\,\hat{a}_0$$

Are everywhere parallel to each other.

Soln. :

For the parallel vectors their cross product must be zero.

i.e. $\bar{A} \times \bar{B} = 0$

Taking cross product of the given vectors,

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_r & \bar{a}_\theta & \bar{a}_\phi \\ \dfrac{\sin 2\theta}{r^2} & \dfrac{2\sin\theta}{r^2} & 0 \\ r\cos\theta & r & 0 \end{vmatrix}$$

$$= \bar{a}_r (0) - \bar{a}_\theta (0) + \bar{a}_\phi \left[\frac{\sin 2\theta}{r^2} \cdot r - \frac{2\sin\theta}{r^2} \cdot r\cos\theta\right]$$

$$= \bar{a}_\phi \left[\frac{\sin 2\theta}{r} - \frac{2\sin\theta\cos\theta}{r}\right] = \bar{a}_\phi \left[\frac{\sin 2\theta}{r} - \frac{\sin 2\theta}{r}\right] = 0$$

Thus $\bar{A} \times \bar{B}$ are parallel.

Ex. 6.7.25 : Convert $\overline{G} = (xz/y)\,\bar{a}_x$ into spherical.

Soln. :

Let $G = G_r\,\bar{a}_r + G_\theta\,\bar{a}_\theta + G_\phi\,\bar{a}_\phi$ be a vector in spherical system. Where,

$$G_r = \overline{G} \cdot \bar{a}_r = \left(\frac{xz}{y}\right) \bar{a}_x \cdot \bar{a}_r = \frac{r\sin\theta\cos\phi \times r\cos\theta}{r\sin\theta\sin\phi} \times \sin\theta\cos\phi$$

$$= r\sin\theta\cos\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \overline{G} \cdot \bar{a}_\theta = \left(\frac{xz}{y}\right) \bar{a}_x \cdot \bar{a}_\theta = r\cos^2\theta \frac{\cos^2\phi}{\sin\phi}$$

and $$G_\phi = \overline{G} \cdot \bar{a}_\phi = \left(\frac{xz}{y}\right) \bar{a}_x \cdot \bar{a}_\phi = -r\cos\theta\cos\phi$$

Putting these values in $\overline{G}$ in spherical system

$$\overline{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\bar{a}_r + \cos\theta\cot\phi\,\bar{a}_\theta - \bar{a}_\phi)$$

Ex. 6.7.26 : Convert $\overline{A} = (3\,\bar{a}_x + 4\,\bar{a}_y + 5\,\bar{a}_z)$ at the point (3, 4, 5) in spherical coordinates.

Soln. :

Let $$\overline{A} = A_r\,\bar{a}_r + A_\theta\,\bar{a}_\theta + A_\phi\,\bar{a}_\phi \quad \text{...(i)}$$

be a vector in spherical coordinate system, where,

$$A_r = \bar{A} \cdot \bar{a}_r = (3\,\bar{a}_x + 4\,\bar{a}_y + 5\,\bar{a}_z) \cdot \bar{a}_r = 3\,\bar{a}_x \cdot \bar{a}_r + 4\,\bar{a}_y \cdot \bar{a}_r + 5\,\bar{a}_z \cdot \bar{a}_r$$

Using Table 6.1.3 of dot products

$$A_r = 3\sin\theta\cos\phi + 4\sin\theta\sin\phi + 5\cos\theta \quad \text{...(ii)}$$

Similarly, $$A_\theta = \bar{A} \cdot \bar{a}_\theta = (3\,\bar{a}_x + 4\,\bar{a}_y + 5\,\bar{a}_z) \cdot \bar{a}_\theta = 3\,\bar{a}_x \cdot \bar{a}_\theta + 4\,\bar{a}_y \cdot \bar{a}_\theta + 5\,\bar{a}_z \cdot \bar{a}_\theta$$

$$= 3\cos\theta\cos\phi + 4\cos\theta\sin\phi - 5\sin\theta \quad \text{...(iii)}$$

And $$A_\phi = \bar{A} \cdot \bar{a}_\phi = (3\,\bar{a}_x + 4\,\bar{a}_y + 5\,\bar{a}_z) \cdot \bar{a}_\phi = 3\,\bar{a}_x \cdot \bar{a}_\phi + 4\,\bar{a}_y \cdot \bar{a}_\phi + 5\,\bar{a}_z \cdot \bar{a}_\phi$$

$$= -3\sin\phi + 4\cos\phi + 0 \quad \text{...(iv)}$$

The given point (3, 4, 5) can be converted into spherical and then putting these spherical coordinates into (ii), (iii) and (iv) we get A_r, A_θ and A_ϕ.

We know $r = \sqrt{x^2+y^2+z^2} = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$ m

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos^{-1}\frac{5}{5\sqrt{2}} = 45^\circ$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{4}{3} = 53.13^\circ$$

$\therefore \cos\theta = 0.707;$ $\sin\theta = 0.707$; $\cos\phi = 0.6$; $\sin\phi = 0.78$

Putting these values in Equations (ii), (iii) and (iv)

$A_r = 7.07$; $A_\theta = 0$; $A_\phi = 0$

$\therefore$ The vector in spherical coordinate system using (A) is $\bar{A} = 7.07\ \bar{a}_r$

Ex. 6.7.27 : Express the field $\vec{E} = \frac{A\hat{a}_r}{r^2}$ in (i) rectangular components, (ii) cylindrical components.

Soln. :

Table P. 6.7.27(a) : Table of dots products

	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$
$\bar{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\bar{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\bar{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

i) To convert in rectangular components :

Let the vector in rectangular be,

$$\bar{E} = E_x\bar{a}_x + Ey\bar{a}_y + E_2\bar{a}_2$$

where, $E_x = \bar{E}\cdot\bar{a}_x = \frac{A}{r^2}\bar{a}_r\cdot\bar{a}_x = \frac{A}{r^2}\sin\theta\cos\phi$ (Using Table P. 6.7.27(a))

We have $x = r\sin\theta\cos\phi \rightarrow \sin\theta\ \cos\phi = x/r$

$$\therefore\ E_x = \frac{A}{r^2}\left(\frac{x}{r}\right) = \frac{A\,x}{r^3}$$

In spherical $r = \sqrt{x^2+y^2+z^2}$

$$\therefore\ E_x = \frac{Ax}{(x^2+y^2+z^2)^{3/2}}$$

Similarly, $E_y = \bar{E}\cdot\bar{a}_y = \frac{A}{r^2}\bar{a}_r\cdot\bar{a}_y = \frac{A}{r^2}\sin\theta\sin\phi$

but $y = r\sin\theta\sin\phi \rightarrow \sin\theta\sin\phi = y/r$

$$\therefore\ E_y = \frac{A\,y}{r^3} = \frac{A\,y}{(x^2+y^2+z^2)^{3/2}}$$

Now $E_z = \bar{E}\cdot\bar{a}_z = \frac{A}{r^2}\bar{a}_r\cdot\bar{a}_z = \frac{A}{r^2}\cos\theta$

but $z = r\cos\theta \rightarrow \cos\theta = z/r$

$$\therefore\ E_z = \frac{A\,z}{r^3} = \frac{A\,z}{(x^2+y^2+z^2)^{3/2}}$$

Thus the vector in Cartesian is,

$$\bar{E} = \frac{A}{(x^2+y^2+z^2)^{3/2}}(x\,\bar{a}_x + y\,\bar{a}_y + z\,\bar{a}_z)$$

ii) To convert in cylindrical :

Let the vector in cylindrical is,

$$\bar{E} = E_\rho\bar{a}_\rho + E\phi + E_z\bar{a}_z$$

Note that, since r appears in both cylindrical and spherical, to avoid confusion, r in cylindrical is replaced by ρ. Also direct conversion from spherical to cylindrical is difficult, so Cartesian vector is converted to cylindrical.

Table P. 6.7.27(b) : Conversion between Cartesian and Cylindrical

(i) Cartesian from cylindrical	(ii) Cylindrical from Cartesian coordinates.
$x = r\cos\phi$	$r = \sqrt{x^2+y^2}$
$y = r\sin\phi$	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$
$z = z$	$z = z$

$$E_\rho = \bar{E}\cdot\bar{a}_\rho = \frac{A}{(x^2+y^2+z^2)^{3/2}}(x\,\bar{a}_x + y\,\bar{a}_y + z\,\bar{a}_z)\cdot\bar{a}_\rho$$

$$= \frac{A}{(x^2+y^2+z^2)^{3/2}}(x\,\bar{a}_x\cdot\bar{a}_\rho + y\,\bar{a}_y\cdot\bar{a}_\rho + 2\,\bar{a}_z\cdot\bar{a}_\rho)$$

$$= \frac{A}{(x^2+y^2+z^2)^{3/2}}(x\cos\phi + y\sin\phi) \qquad (\because \bar{a}_z\cdot\bar{a}_\rho = 0)$$

We have (Refer Table P. 6.7.27(b)),

$$x = \rho\cos\phi,\ y = \rho\sin\phi \text{ and } x^2+y^2 = \rho^2$$

$$\therefore \quad E_\rho = \frac{A}{(\rho^2+z^2)^{3/2}} (\rho \cos^2\phi + \rho \sin^2\phi) = \frac{A\rho}{(\rho^2+z^2)^{3/2}}$$

$$E_\phi = \bar{E} \cdot \bar{a}_\phi = \frac{A}{(x^2+y^2+z^2)^{3/2}} (x\, \bar{a}_x \cdot \bar{a}_\phi + y\, \bar{a}_y \cdot \bar{a}_\phi + z\, \bar{a}_z \cdot \bar{a}_\phi)$$

$$= \frac{A}{(\rho^2+z^2)^{3/2}} [x(-\sin\phi) + y(\cos\phi) + 0]$$

$$= \frac{A}{(\rho^2+z^2)^{3/2}} [-r\cos\phi \sin\phi + r \sin\phi \cos\phi] = 0$$

Now, $\quad E_2 = \bar{E} \cdot \bar{a}_z$

$$= \frac{A}{(x^2+y^2+z^2)^{3/2}} [x\, \bar{a}_x \cdot \bar{a}_z + y\, \bar{a}_y \cdot \bar{a}_z + z\, \bar{a}_z \cdot \bar{a}_z]$$

$$\therefore \quad E_z = \frac{A \cdot Z}{(\rho^2+z^2)^{3/2}} \quad (\bar{a}_x \cdot \bar{a}_z = \bar{a}_y \cdot \bar{a}_z = 0)$$

Thus the vector in cylindrical is,

$$\bar{E} = \frac{A}{(\rho^2+z^2)^{3/2}} (\rho \bar{a}_\rho + z\, \bar{a}_z)$$

Ex. 6.7.28 : Find the gradient of function ϕ : (i) $\phi = \cosh xyz$; (ii) $\phi = x^2 + y^2 + z^2$

Soln. :

(i) $\phi = \cosh xyz$

$$\text{Now } \nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$$

$$\text{Here, } \frac{\partial\phi}{\partial x} = yz \sinh xyz \,; \frac{\partial\phi}{\partial y} = xz \sinh xyz \,; \frac{\partial\phi}{\partial z} = xy \sinh xyz.$$

$$\nabla\phi = \sinh xyz \, (yz\bar{a}_x + xz\bar{a}_y + xy\bar{a}_z)$$

(ii) $\phi = x^2 + y^2 + z^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$$

$$\text{Here, } \frac{\partial\phi}{\partial x} = 2x \,; \frac{\partial\phi}{\partial y} = 2y \,; \frac{\partial\phi}{\partial z} = 2z$$

$$\therefore \nabla\phi = 2x\, \bar{a}_x + 2^y \bar{a}_y + 2^z \bar{a}_z = 2(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z)$$

Problems of Gradient, Diversion, Curl

Ex. 6.7.29 : If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $(1, -2, -1)$.

Soln. : The gradient of ϕ is given by,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$$

$$\frac{\partial\phi}{\partial x} = 6xy; \quad \frac{\partial\phi}{\partial y} = 3x^2 - 3y^2z^2 \,; \frac{\partial\phi}{\partial z} = -2y^3 z.$$

Thus, $\quad \nabla\phi = 6xy\, \bar{a}_x + (3x^2 - 3y^2z^2)\, \bar{a}_y - (2y^3 z)\, \bar{a}_z$

at $(1, -2, 1)$,

$$\nabla\phi = 6 \times 1 \times (-2)\, \bar{a}_x + [3 \times (1)^2 - 3(-2)^2(-1)^2]\, \bar{a}_y + [2(-2)^3(-1)]\, \bar{a}_z$$

$$= -12\, \bar{a}_x - 9\, \bar{a}_y - 16\, \bar{a}_z$$

Ex. 6.7.30 : If $\bar{A} = x^2 z\, \bar{a}_x - 2y^2 z^2\, \bar{a}_y + xy^2 z\, \bar{a}_z$, find $\nabla \cdot \bar{A}$ at the point $(1, -1, 1)$.

Soln. : The divergence of $\bar{A}$ is given by,

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{Here, } \frac{\partial A_x}{\partial x} = 2xz \,; \frac{\partial A_y}{\partial y} = -4yz^2 \,; \frac{\partial A_z}{\partial z} = xy^2$$

Then $\quad \nabla \cdot \bar{A} = 2xz - 4yz^2 + xy^2$

At $(1, -1, 1)$, $\quad \nabla \cdot \bar{A} = 2(1)(1) - 4(-1)(1)^2 + (1)(-1)^2 = 7$

Ex. 6.7.31 : Find the divergence of the vector function $\bar{A} = x^2\, \bar{a}_x + x^2y^2\, \bar{a}_y + 24\, x^2 y^2 z^3\, \bar{a}_z$

Evaluate the integral of $\nabla \cdot \bar{A}$ throughout the volume of a unit cube centered at the origin.

Soln. : For $\quad \bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

$$\text{Div. } \bar{A} = \nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{Here, } \frac{\partial A_x}{\partial x} = 2x; \quad \frac{\partial A_y}{\partial y} = 2x^2y; \quad \frac{\partial A_z}{\partial z} = 72x^2y^2z^2$$

$$\therefore \nabla \cdot \bar{A} = 2x + 2x^2y + 72x^2y^2z^2$$

Since the unit cube is centered at the origin, the x, y, z coordinates varies each from $-(1/2)$ to $(1/2)$. Now the volume integral of $\nabla \cdot \bar{A}$ is

$$\int_V (\nabla \cdot \bar{A})\, dv = \int_{-1/2}^{1/2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} (\nabla \cdot \bar{A})\, dx\, dydz$$

$$= \int_{-1/2}^{1/2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} (2x + 2x^2y + 72x^2y^2z^2)\, dx\, dydz$$

$$= 2\left[\frac{x^2}{2}\right]_{-1/2}^{1/2} [y]_{-1/2}^{1/2} [z]_{-1/2}^{1/2} + 2\left[\frac{x^3}{3}\right]_{-1/2}^{1/2}\left[\frac{y^2}{2}\right]_{-1/2}^{1/2} [z]_{-1/2}^{1/2}$$

$$+ 72\left[\frac{x^3}{3}\right]_{-1/2}^{1/2}\left[\frac{y^3}{3}\right]_{-1/2}^{1/2}\left[\frac{z^3}{3}\right]_{-1/2}^{1/2}$$

$$= 41.67 \times 10^{-3}$$

Ex. 6.7.32 : Determine the 'curl' of these vector field.

(i) $\bar{A} = \bar{a}_x\, (2x^2 + y^2) + \bar{a}_y\, (xy - y^2)$ (ii) $\bar{A} = yz\, \bar{a}_x + 4xy\, \bar{a}_y + y\, \bar{a}_z$

Soln. :

(i) $$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 + y^2 & xy - y^2 & 0 \end{vmatrix}$$

$$= \bar{a}_x\, (0 - 0) - \bar{a}_y\, (0 - 0) + \bar{a}_z\, (y - 2y) = -y\, \bar{a}_z$$

(ii) $$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 4xy & y \end{vmatrix} = \bar{a}_x\, (1 - 0) - \bar{a}_y\, (0 - y) + \bar{a}_z\, (4y - z)$$

$$= \bar{a}_x + y\, \bar{a}_y + (4y - z)\, \bar{a}_z$$

Review Questions

Q. 1 Derive necessary expressions to convert cylindrical co-ordinates to Cartesian co-ordinates.

Q. 2 Explain spherical co-ordinate system and hence explain its conversion to Cartesian system.

Q. 3 Explain physical significance of divergence.

Q. 4 What is curl of a vector ? Explain its significance.

Q. 5 Derive point form of all Maxwell's equations.

Q. 6 Write integral form of all Maxwell's equations.

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Cartesian

Q. 1 Explain Cartesian Coordinate System. *(Ans. : Refer Section 6.1.1)*

Syllabus Topic : Cylindrical Coordinate System

Q. 1 Derive necessary expressions to convert cylindrical co-ordinates to Cartesian co-ordinates. *(Ans. : Refer Section 6.1.3)*

Syllabus Topic : Spherical Coordinate System

Q. 1 Explain spherical co-ordinate system and hence explain its conversion to cartesian system. *(Ans. : Refer Sections 6.1.4 and 6.1.5)*

Syllabus Topic : Scalar and Vector Field

Q. 1 Explain Scalar and Vector Field. *(Ans. : Refer Sections 6.2.1 and 6.2.2)*

Syllabus Topic : Physical significance of gradient, curl and divergence

Q. 1 Explain physical significance of divergence. *(Ans. : Refer Section 6.3.2)*

Q. 2 What is curl of a vector ? Explain its significance. *(Ans. : Refer Section 6.3.3)*

Syllabus Topic : Derivation of Maxwell's Four Equation

Q. 1 Derive point form of all Maxwell's equations. *(Ans. : Refer Section 6.4)*

Q. 2 Write integral form of all Maxwell's equations. *(Ans. : Refer Section 6.5)*

Syllabus Topic : Applications - Design of Antenna, Wave Guide, Satellite Communication

Q. 1 Write short note on : Design of Antenna, Wave Guide, Satellite Communication.

Solved Problems

Q. 1 Refer Ex. 6.7.1

Q. 2 Refer Ex. 6.7.2

Q. 3 Refer Ex. 6.7.3

Q. 4 Refer Ex. 6.7.4

Q. 5 Refer Ex. 6.7.7

Q. 6 Refer Ex. 6.7.8

Q. 7 Refer Ex. 6.7.9

Q. 8 Refer Ex. 6.7.10

Q. 9 Refer Ex. 6.7.29

Q. 10 Refer Ex. 6.7.30

Q. 11 Refer Ex. 6.7.31

Q. 12 Refer Ex. 6.7.32

CHAPTER 7

Charged Particle in Electric and Magnetic Fields

Module 5

Syllabus

Fundamentals of Electromagnetism, Motion of electron in electric field (parallel, perpendicular, with some angle); Motion of electron in magnetic field (Longitudinal and Transverse); Magnetic deflection; Motion of electron in crossed field; Velocity Selector ; Velocity Filter, Electron refraction; Bethe's law; Electrostatic focusing; Magnetostatic focusing; Cathode ray tube (CRT);Cathode ray Oscilloscope (CRO).Application of CRO: Voltage (dc, ac), frequency, phase measurement.

Syllabus Topic : Fundamentals of Electromagnetism, Motion of Electron in Electric Field (Parallel)

➤ **Topic covered :** Motion of electron in electric field (parallel)

Fundamentals of Electromagnetism

Electricity and magnetism through existing independently but at the same time if seen together can make us understand many revolutionary concepts of present industries. The entire credit goes to electromagnetism.

To further understand it let us understand the most fundamental points.

7.1 Motion of Electron in Electric Field (Parallel)

- Motion of electron in electric field such that electron is moving parallel to external electric field.

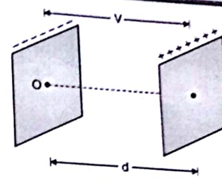


Fig. 7.1.1

- As electron is a negatively charged particle with $e = 1.6 \times 10^{-19}$ C and mass $m_e = 9.1 \times 10^{-31}$ kg ; for the arrangement shown in Fig. 7.1.1 we can derive the following.
- In the Fig. 7.1.1 two plates are separated by spacing 'd' and potential difference V. The plate on RHS is positively charged.
- The electric field intensity is

$$E = \frac{V}{d} \qquad ...(7.1.1)$$

and the acceleration experienced by electron entering at point '0' is

$$F = ma = eE$$

$$a = \left(\frac{e}{m}\right)E \qquad ...(7.1.2)$$

- The terminal velocity of an electron that has travelled through a potential difference V is given by

$$\frac{1}{2} m\upsilon^2 = eV$$

$$\therefore \upsilon = \sqrt{\frac{2 eV}{m}} \qquad ...(7.1.3)$$

Syllabus Topic : Motion of Electron in Electric Field (Perpendicular)

➤ **Topic covered :** Motion of electron is perpendicular to the applied electric field.

7.2 Motion of Electron is Perpendicular to the Applied Electric Field

In Fig. 7.2.1 we have two plates with length '*l*' and separated by a distance 'd'. The upper plate is having positive charge and the lower plate is having negative charge.

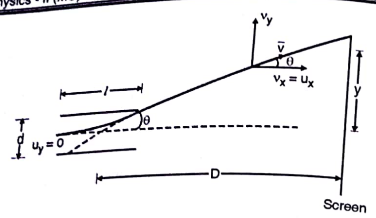


Fig. 7.2.1

- The potential difference between these two plates is V. An electron is made to enter this perpendicular field with initial velocity u_x ($u_y = 0$) due to accelerating potential V_A. Electron will experience perpendicular electric field by a force F_y.
- The path of electron will become parabolic till it takes an exist and later move on straight line path with velocity v, (with $u_x = v_x$) and hit the screen at deflection 'y' as shown in Fig. 7.2.1.
- The force acting on Y direction is

$$F_y = ma_y = \in \left(\frac{E_d}{d}\right)$$

as $F_y = eV_d = (V_d = \text{deflecting voltage})$

$$\therefore a_y = \frac{eE_y}{m}$$

Now $u_y = 0$, displacement at any instant t in the y-direction is given by

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{e\,E_y}{m}\right) t^2$$

- To find t consider the following :

As velocity in x-direction is $u_x = v_x$

$$\therefore x = u_x t$$

$$\therefore t = \frac{x}{u_x}$$

$$\therefore y = \frac{1}{2}\left(\frac{e\,E_y}{m}\right) \cdot \left(\frac{x}{u_x}\right)^2$$

$$= \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{E_y}{u_x^2}\right) x^2$$

$$y = k\,x^2$$

∴ An electron travels a parabolic path in transverse electric field. ...(7.2.1)

- After exit from perpendicular electric field, electron moves an straight line path at angle 'θ'.

∴ From Fig. 7.2.1

$$\therefore \text{Slope} = \tan\theta = \frac{y}{D} = \frac{v_y}{u_x}$$

$$\therefore y = D\tan\theta = D\frac{v_y}{u_x}$$

$$\text{as } v_y = u_y + a_y t = a_y t \qquad (\text{as } u_y = 0)$$

$$\therefore y = D \cdot \frac{a_y t}{u_x}$$

$$\text{Now, } t = \frac{l}{u_x} \quad \text{and} \quad a_y = \frac{e}{m}(E_y)$$

$$y = D \cdot \left(\frac{e}{m}\right) E_y \cdot \frac{l}{(u_x^2)}$$

$$\text{By taking KE } = \frac{1}{2} m u_x^2 = e V_A$$

$$u_x^2 = \frac{2e\,V_A}{m}$$

$$\therefore y = D \cdot \left(\frac{e}{m}\right) \cdot \frac{l}{E_y \cdot \left(\frac{2e}{m}\right)} V_A$$

$$= \frac{D \cdot E_y \cdot l}{2 \cdot V_A}$$

$$\text{Now } E_y = \frac{V_d}{d}$$

$$\therefore y = \frac{D}{2} \cdot \left(\frac{V_d}{V_A}\right) \cdot \frac{l}{d}$$

Syllabus Topic : Motion of Electron in Magnetic Field (Longitudinal and Transverse)

➤ **Topics covered :** Motion of Electron in Magnetic Field(Longitudinal and Transverse), Motion of Electron in Magnetic Field (Longitudinal), Motion of Electron in Magnetic Field (Transverse)

7.3 Motion of Electron in Magnetic Field (Longitudinal and Transverse)

7.3.1 Motion of Electron in Magnetic Field (Longitudinal)

- Consider the fact that magnetic force experienced by charged particle when it enters a magnetic field B with velocity v is given by

$$\vec{F} = q\left(\vec{B} \times \vec{v}\right)$$

$$= q\,B\,v\,\sin\theta$$

- For longitudinal magnetic field, $\theta = 0$ or $180°$ hence $\sin\theta = 0$ and $F = 0$. As magnetic force is zero, motion of charged particle remains unaffected.

7.3.2 Motion of Electron in Magnetic Field (Transverse)

- When an electron or any charged particle travels through magnetic field of strength B with velocity v then the force is given by

$$\vec{F}_m = q\left(\vec{B} \times \vec{v}\right)$$

$$= q\,B\,v\,\sin\theta$$

- If B and v are in same direction then $\theta = 0$ $\quad \therefore F_m = 0$
- If B and v are perpendicular to each other (transverse field) then $\theta = 90$

$$\therefore \sin\theta = 1$$

$$\therefore F_m = q\,B\,v$$

- Since B, v, q = e all are constants in magnitude, $\overline{F}_m$ is constant in magnitude and perpendicular to the direction of motion.
- Due to this type of force, electron will be continuously deflected by Fleming's left hand rule and finally results in motion in a circular path with constant speed within the transverse magnetic field. The force is similar to centripetal force. If this circular path has radius R, then

$$F_m = \frac{mv^2}{R}$$

$$\text{as } F_m = Bev = \frac{mv^2}{R}$$

$$\therefore R = \left(\frac{m}{e}\right)\cdot\frac{v}{B} \qquad ...(7.3.1)$$

Syllabus Topic : Magnetic Deflection

➤ **Topic covered :** Magnetic Deflection

7.4 Magnetic Deflection

- If an electron enters a magnetic field at an angle θ to its initial velocity, the particle describes a helical path in the field. The axis of helix will be the direction of magnetic field as shown in Fig. 7.4.1.

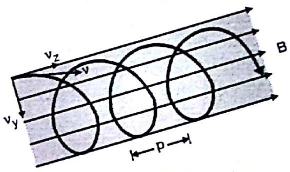


Fig. 7.4.1 : Electrostatic focussing

- The pitch p of the helix is given by $P = \frac{2\pi}{B}\left(\frac{m}{e}\right) v \cos\theta$ ...(7.4.1)
- If T is the time period to complete one revolution then

$$T = \frac{\text{Circumference}}{v} = \frac{2\pi R}{v}$$

Using Equation (7.4.1)

$$T = \frac{2\pi}{v}\left(\frac{m}{e}\right)\cdot\frac{v}{B}$$

$$T = \frac{2\pi}{B}\left(\frac{m}{e}\right)\cdot v \qquad ...(7.4.2)$$

- When field is acting normally but over a limited extension then the path will define an arc for a limited distance l or till the electron is in the field.
- Once out of magnetic field, it will follow a tangential path and reach the screen which is at distance of D from centre of magnetic field.

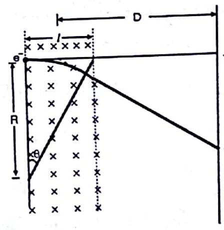


Fig. 7.4.2

the deflection y on screen is given by

$$y = D l B \sqrt{\frac{e}{2\,m \cdot V_A}} \quad \ldots(7.4.3)$$

Syllabus Topic : Motion of Electron in Crossed Field

> **Topic covered :** Motion of Electron in Crossed Field

7.5 Motion of Electron in Crossed Field

- Consider an electron moving with an initial velocity v entre in a crossed electric and magnetic field. The main feature of cross field is to cancel the deflection caused by electric and magnetic fields.
- Within crossed electric and magnetic fields, if the electron is made to travel along x-direction then due to electric field alone it will defect along positive y direction.
- Whereas due to magnetic field alone it will deflect along negative y direction.
- Now if force acting on electron by electric and magnetic fields are simultaneously same, then there will not be any deflection.

as force due to electric field

$$F_E = eE$$

and due to magnetic field

$$F_E = Bev$$

$$\therefore \cancel{e}E = B\cancel{e}v$$

$$v = \frac{E}{B} \quad \ldots(7.5.1)$$

Syllabus Topic : Velocity Selector and Velocity Filter

> **Topics covered :** Velocity Selector and Velocity Filter

7.6 Velocity Selector and Velocity Filter

As seen in case of crossed field,

$$v = \frac{E}{B}$$

$\therefore$ Using this formula one can design an arrangement which makes selection of monoveloicty charged particles.

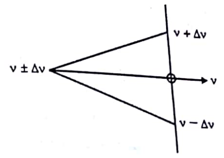


Fig. 7.6.1

As shown in Fig. 7.6.1 if there is a beam of charged particles with velocity v $\pm \Delta v$ (i.e. mixed velocities to same extent) then particles will be filtered for $v + \Delta v$ and $v - \Delta v$ as the given values of E and B are such that only those charges with velocity 'v' will be able to go on straight path (selection) where as $v + \Delta v$ will experience higher F_E hence it will go above straight path and $v - \Delta v$ will experience higher F_m hence it will go below straight path (filtration).

Syllabus Topic : Electrostatic Focussing, Electron Refraction, Bethe's Law

> **Topics covered :** Electrostatic Focusing, Calculation of Refractive Index, Case of More than One Equipotential Surface, Electrostatic Lens

7.7 Electrostatic Focussing

MU - May 2013, Dec. 2013, May 2016

- As we know, in geometrical optics, focussing of light ray by a lens is due to phenomenon of refraction more popularly Snell's law.
- Let us find out how electron optics provided focussing of electron beam. Before we proceed, let us take idea of **"equipotential surface".**

Equipotential surface :

- It is defined as a surface in an electric field where the electric potential remains the same.
- Consider Fig. 7.7.1 where an electron is at rest and placed in an electric field produced by two plates A and B.

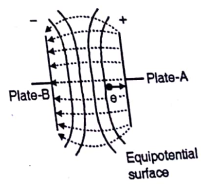


Fig. 7.7.1 : Electric field between two plates

- If an electron at rest placed in an electric field produced by two plates A and B, the force experienced by electron is

$$F = -eE$$

$$= -\frac{eV}{d} \quad \left(\because E = \frac{v}{d}\right) \qquad ...(7.7.1)$$

- The negative sign indicates that force acts in the opposite direction to that of field. This discussion is common but valid only if field is uniform (i.e. within the area of plates A and B). When areas near boundaries of plates are concerned, the field is no more uniform.
- The field intensity is low at both the ends as shown with spreading of field lines. In the diagram we have equipotential surfaces shown. **The force on electron is normal to the equipotential surface.**

Equipotential surface and refraction and Bethe's law :

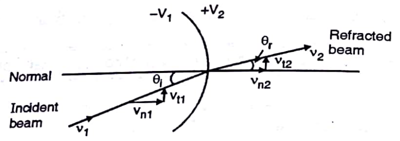


Fig. 7.7.2 : Refraction of a beam

- In Fig. 7.7.2, we have an equipotential surface. It is considered as a boundary across which the potential V_1 abruptly changes to potential V_2.
- Let an electron through a region on LHS of equipotential surface with a uniform velocity v_1 and enter the region of RHS.
- Because of this, it velocity alters. Since the force acts in a direction normal to the surface, its normal component of velocity that is altered. If $V_2 > V_1$, $v_{n2} > v_{n1}$ and if $V_1 > V_2$, $v_{n2} < v_{n1}$. We select $V_2 > V_1$. The electron path is therefore bent nearer to the normal.
- Since there is no force acting tangentially, tangential component will remain the same.

$$\therefore \; v_{t1} = v_{t2} \qquad ...(7.7.2)$$

From Fig. 7.7.2

$$v_{t1} = v_1 \sin \theta_i, \qquad v_{t2} = v_2 \sin \theta_r$$

$$\therefore \; v_1 \sin \theta_i = v_2 \sin \theta_r$$

$$\therefore \; \frac{\sin \theta_i}{\sin \theta_r} = \frac{v_2}{v_1} \qquad ...(7.7.3)$$

Where v_2 = final velocity after leaving surface

v_1 = initial velocity with which electron approches

- Equation (7.7.3) is identical of the Snell's law in optics which describes refraction. This is also referred as Bethe's law.

7.8 Calculation of Refractive Index

- On LHS of equipotential surface, velocity is v_1 and potential is V_1.

$$\therefore \; eV_1 = \frac{1}{2} mv_1^2$$

$$\therefore \; \sqrt{V_1} \propto v_1 \qquad ...(7.8.1)$$

- Similarly on RHS, velocity is v_2 and potential V_2.

$$\therefore \; eV_2 = \frac{1}{2} mv_2^2$$

$$\therefore \; \sqrt{V_2} \propto v_2 \qquad ...(7.8.2)$$

$$\text{R.I.} = \frac{v_2}{v_1} = \frac{\sqrt{V_2}}{\sqrt{V_1}}$$

$$\therefore \; \frac{\sin \theta_i}{\sin \theta_r} = \sqrt{\frac{V_2}{V_1}}$$

$$\therefore \; \textbf{Refractive index} = \frac{\sin \theta_i}{\sin \theta_r} = \left[\frac{V_2}{V_1}\right]^{1/2} \qquad ...(7.8.3)$$

> **Note : Difference between optical refraction (Snell's law) and electron refraction (Bethe's law) :** When light ray enters denser medium, it slows down and bends towards the normal, where as electrons get faster and bends toward the normal.

7.9 Case of More than One Equipotential Surface

- If we take a non uniform electric field represented by more than one equipotential surfaces separating equipotential regions of potentials V_1, V_2, V_3, V_4 ... etc. as shown in Fig. 7.9.1, a curved path of electron motion can be observed.

- This is analogous to bending of optical ray.

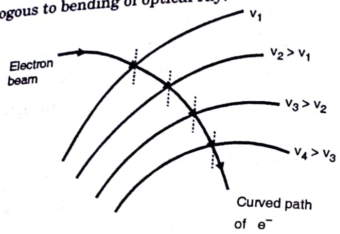


Fig. 7.9.1 : Curved path of electron beam

7.10 Electrostatic Lens

- *Consider two co-axial cylinders A and B at connected to supply potentials V_1 and V_2 respectively such that $V_2 > V_1$. A high equipotential ring R is placed in the gap between cylinders A and B.*
- *A non uniform electric field is produced in the gap as a result of different potentials applied to both the cylinders. In Fig. 7.10.1, we have equipotential surfaces and field lines depicted.*
- *It should be noted that equipotential surfaces are perpendicular to field lines. The electric field within hollow space of cylinder is weak enough and can be neglected.*

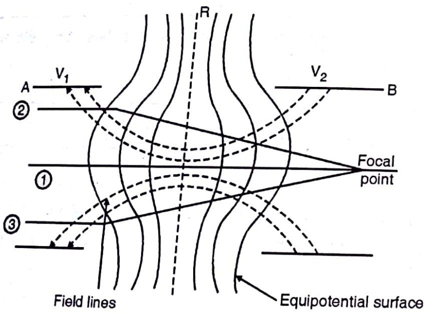


Fig. 7.10.1 : Electrostatic focussing

Let us understand the operation :

- Consider electron beam (1) :

  From Fig. 7.10.1 it is clear that beam (1) remain normal to all equipotential surfaces and hence it is simply accelerated and no deviation takes place on its path.
- Consider electron beam (2) :

  Electron beam (2) will have two different type of effects.
- Consider the equipotential surface on LHS i.e. convex type surfaces. In this case electron beam will have two components of the force experienced A parallel component F_P will drag the electron further where as normal component (F_N) will be in such a direction that electrons are deflected down towards the axis as shown in Fig. 7.10.2 (a).
- When it crosses the mid plane R, the equipotential surface will appear concave (Fig. 7.10.2 (b)). Here, again the force at any concave equipotential surface will be resolved in to two components. A parallel component F_P will drag it further where as now normal component F_N will be in such a direction that the beam will experience divergence.
- It is important to note that $V_2 > V_1$ Hence under the strong influence of V_2, parallel component will become much stronger and this will make it focus at focal point.

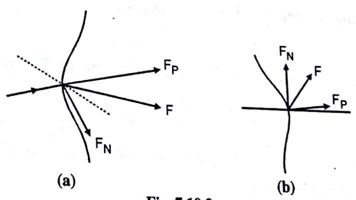


Fig. 7.10.2

- Consider electron beam (3) :

  Electron beam (3) will have its path as shown in Fig. 7.10.1, with the explanation as shown for beam (2).
- So, the focussing of electron beam under electrostatic field is very much similar to focussing of optical ray through a lens. Yet there are some differences as listed below :

Table 7.10.1

Sr. No.	Optical lens	Electron lens
1.	Bending takes place only at boundary of lens.	Bending takes place through its path in field.
2.	Focal length can not be altered.	Focal length can be altered by altering V_1 and V_2.

This type of focussing is mainly used in CRO.

Syllabus Topic : Magnetostatic Focussing

> **Topics covered :** Magnetic Focusing, Longitudinal Magnetic Field Focussing, Transverse Magnetic Field Focussing

7.11 Magnetic Focussing

- Focussing can also be achieved using magnetic field produced by solenoid etc. called magnetostatic focussing.
- Magnetostatic focussing is divided in to two categories viz (a) longitudinal magnetic field focussing (b) transverse magnetic field focussing.

7.11.1 Longitudinal Magnetic Field Focussing

Principle :

Path of electron in a uniform field would be a helix. The pitch P is given by

$$P = \frac{2\pi m}{eB} v \cos\theta \quad ...(7.11.1)$$

If the divergence angle is small, pitch P remains constant.

Let us understand the principle.

In the given Fig. 7.11.1 we have an electron with velocity v entering a magnetic field of strength B. Such that angle between $\bar{v}$ and $\bar{B}$ is θ.

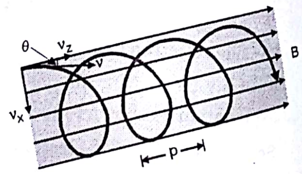


Fig. 7.11.1 : An electron in Magnetic field

$$\therefore v_z = v\cos\theta, \quad v_x = v\sin\theta \quad ...(7.11.2)$$

The pitch P is defined as the distance travelled by the particle along its axis in one revolution.

$$\therefore P = v_z \cdot T \quad ...(7.11.3)$$

Where T = time taken to complete one revolution

If there is only one component $v_x = v \sin\theta$, then electron will move in a circular orbit of radius r such that

$$\text{Centripetal force} = \text{Magnetic force}$$

$$\frac{m (v\sin\theta)^2}{r} = Bev\sin\theta$$

$$\therefore v\sin\theta = \frac{Ber}{m} \quad ...(7.11.4)$$

Time take for one revolution T is given by

$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi rm}{Ber} = \frac{2\pi m}{Be} \quad ...(7.11.5)$$

$$\therefore P = v_z T$$

$$P = v\cos\theta \cdot \frac{2\pi m}{Be}$$

If the angle of divergence θ is small (θ < 10°), cos θ is unity

$$\therefore P = \frac{2\pi m}{Be} \quad ...(7.11.6)$$

$$\therefore P = \text{constant} \quad ...(7.11.7)$$

This Equation (7.11.7) helps us understand the magnetostatic focussing.

Let us consider a beam of electron with many rays with velocity v, originating at point O. All the rays have assumed to be different angle of divergence but not more than 10°. Such that cos θ → 1. Any ray leaving at point O again intersects the field lines after the time interval T. Therefore, all the electron rays leaving at point O at small angles will again cross the same field line at a common point P after the time period T.

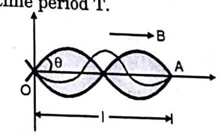


Fig. 7.11.2 : Focusing of charged particle in magnetic field

The distance OA = l can be calculated as follows :

$$l = v\cos\theta \times t$$

$$= v\cos\theta \times \frac{2\pi m}{Be}$$

as θ is small, $\cos\theta \rightarrow$ unity

$$\therefore l = \frac{2\pi mv}{Be} \quad \ldots(7.11.8)$$

As T is independent of θ, v and r, so the electrons starting from O can diverges in any direction, but will be forced to focus at point A by the uniform magnetic field. Thus magnetic field just acts as a lens to bring them on focus.

This type of focussing is used in electron microscope.

7.11.2 Transverse Magnetic Field Focussing

- Consider the motion of charged particles in a transverse magnetic field. Suppose a beam of charged particles with the velocity perpendicular to a magnetic field. Originates at point O (Fig. 7.11.3).
- The angular divergence is sufficiently small. The beam is focussed after a rotation of 180° or a semicircle.

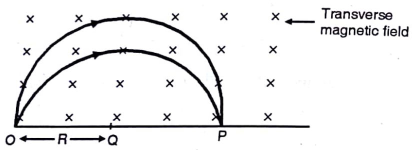


Fig. 7.11.3 : Focussing in transverse magnetic field

- This type of focussing is used in mass spectroscopy where isotopes are separated. Isotopes are having same number of protons and electrons but they differ by number of neutrons. Since Z is same, chemical reactions can not separate them. By maintaining v same for all the particles, entering the transverse magnetic field, centripetal force is given by

$$\frac{mv^2}{R} = Bqv$$

$$\therefore R = \frac{mv}{Bq}$$

$$\therefore R \propto m \quad \ldots(7.11.9)$$

(as v, B, q are same)

Hence as number of neutron differs, mass differs, hence the radius.

Ex. 7.11.1 : Electrons accelerated by a potential of 250 V enter the electric field at an angle of incidence 50° and get refracted at an angle 30°. Find the potential difference between the two regions.

Soln. :

Data : $V_1 = 250$ V, $\theta_i = 50°$, $\theta_r = 30°$

Formula : Using Equation (7.7.3)

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_2}{v_1} = \sqrt{\frac{V_2}{V_1}}$$

$$\therefore \sqrt{V_2} = \sqrt{250} \cdot \frac{\sin 50°}{\sin 30°} = 24.22 \text{ V}$$

$$\therefore V_2 = 586.6 \text{ volt}$$

∴ Potential difference

$$V_2 - V_1 = 586.6 - 250$$

$$= \mathbf{336.8 \text{ volt}}$$

...Ans.

Ex. 7.11.2 : An electron travels with a velocity of 2.5×10^6 m/s in vacuum in a uniform magnetic field strength of 0.94×10^{-4} wb/m², such that velocity vector makes an angle of 30° with the field direction. Determine the radius of revolution of electron path and the distance covered along the magnetic induction lines in five such revolutions.

Soln. : Radius of helical path

$$r = \frac{mv\sin\theta}{Be} = \frac{9.11\times10^{-31}\times2.5\times10^6\times\sin 30°}{0.94\times10^{-4}\times1.6\times10^{-19}}$$

$$= \mathbf{75.71 \text{ mm}}$$

...Ans.

The distance along induction lines in one revolution represents pitch P of the helix formed.

$$P = \frac{2\pi m}{Be}\cdot v\cos\theta$$

$$= \frac{2\pi(9.11\times10^{-31})\times2.5\times10^6\times\cos 30°}{0.94\times10^{-4}\times1.6\times10^{-19}}$$

$$= 0.824 \text{ m}$$

∴ Distance covered in 5 revolutions.

$$5P = 5(0.824) = \mathbf{412 \text{ m}}$$

...Ans.

Ex 7.11.3 : An electron enters a uniform magnetic field B = 0.23 wb/m² at 45 angle to B. Determine radium and pitch of helical path. Assume electron speed to be 3×10^7 m/sec.

Soln. :

$$r = \frac{wv\sin\theta}{eB} = \frac{9.1\times10^{-31}\times3\times10^7\sin 45}{1.6\times10^{-19}\times0.23}$$

= 0.52 mm ...Ans.

$$P = \frac{2\pi\, mv \cos\theta}{eB} = \frac{2 \times 3.14 \times 9.1 \times 10^{-31} \times 3 \times 10^{7} \times \cos 45}{1.6 \times 10^{-19} \times 0.23}$$

$$= 3.29 \times 10^{-3}\ \text{m}$$

$$P = 3.29\ \text{mm}$$ ...Ans.

Syllabus Topic : Cathode Ray Tube (CRT)

> **Topic covered :** Cathode Ray Tube (CRT)

7.12 Cathode Ray Tube (CRT)

MU - May 2014, May 2015, Dec. 2015

A cathode ray tube is a specially designed vacuum tube in which an electron beam controlled by electric or magnetic fields are used to visual display of input electrical signal on the screen which is coated with fluorescent materials.

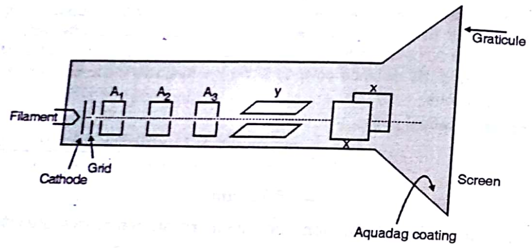


Fig. 7.12.1 : Schematic diagram of CRT

A CRT is a complex arrangement it is described by various parts like,

(a) Electron gun (b) Deflection system
(c) Fluorescent screen (d) Glass envelope

(a) Electron gun :

- The focussed and accelerated electron beam is given out by the electron gun.
- It consists of a heater, a cathode, a grid, a pre accelerating anode, a focussing anode and an accelerating anode.

Cathode :

Indirectly heated cathode which is usually coated with barium and strontium oxide which generates thermionic electrons.

Control gird :

- Usually a nickel cylinder with a centrally located hole, co axial with CRT axis. We know that intensity of electron beam depends upon the number of electrons emitted from the cathode.
- The control grid with its negative bias, controls the number of electrons emitted from the cathode and **hence the intensity is controlled by the grid.**

Pre accelerating anode and accelerating anode :

The electrons emitted from the cathode and passing through the grid are accelerated by the high positive potential which is applied to pre accelerating and accelerating anodes. Electrons coming out from focussing anode are further accelerated through accelerating anode.

Focussing anode :

Focussing of electron beam is achieved through focussing anode. Focussing anode has its potential around 500 V, which is adjustable. The potential of focussing anode is smaller than accelerating anode. Electrostatic focussing is used for focussing.*

Electrostatic focussing is discussed in section 7.7.

(b) Deflection system :

On refering article **"Pre requisite"** and points 6, 7 and 8 over there we get a general picture of electrostatic deflection. There are two pairs of strictly parallel metal plates, two of them are parallel to X-axis called y plates as the causes deflection along Y-axis where as other two which are parallel to Y-axis and called x-plates as they causes deflection along X-axis, as shown in Fig. 7.12.2(a) and (b). The deflected beam generates as bright spot on the screen.

Table 7.12.1 : Location of spot on screen

1.	Upper y-plate positive	On positive y-axis
2.	Lower y-plate positive	On negative y-axis
3.	LHS x-plate positive	On negative x-axis
4.	RHS x-plate positive	On positive x-axis
5.	Upper y-plate and RHS x-plate positive with equal potential	In the first quarter at 45°

* For large screens like TV or RADAR, electromagnetic focussing is used.

As shown in 'pre requisite' point-8 the deflection on Y-axis is given by

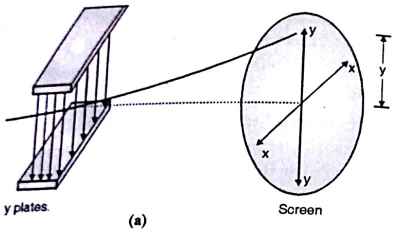
$$y = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{V}{V_A} \quad ...(7.12.1)$$

Where y = Deflection on y-screen

l = Length of electric field applied

d = Spacing between two y-plates

D = Distance of point of screen apparent origin to the screen (apparent origin is at the center of deflection plates)



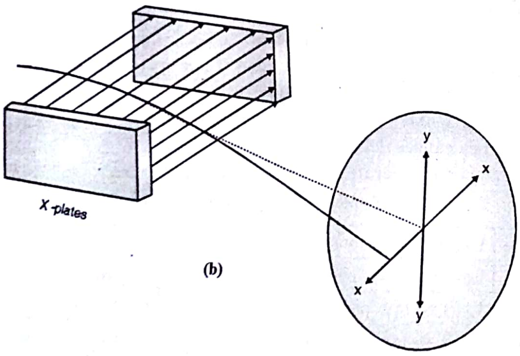


Fig. 7.12.2 : Deflection in CRT

From Equation (7.12.1) it is clear that,

(1) For a given acceleration potential and for particular dimensions of CRT, the deflection of the electron beam is directly proportional to the deflecting voltage.

$$y \propto V$$

∴ CRT may be used as indicating device.

(2) Since deflecting voltage is usually a time varying, image on the screen will follow a variation of the deflecting voltage in a linear manner.

Deflection sensitivity of CRT :

It is defined as the deflection of the screen per unit deflection voltage.

$$\therefore \text{Deflection sensitivity} = S = \frac{y}{V}$$

$$\therefore S = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{1}{V_A} \quad ...(7.12.2)$$

Unit of S is $\frac{m}{V}$

Deflection factor :

It is defined as reciprocal of S

$$\text{i.e. } G = \frac{1}{S} = \frac{2}{D} \cdot \frac{d}{l} \cdot V_A \quad ...(7.12.3)$$

(c) Fluorescent screen :

- The interior surface of circular front face of the CRT is coated with a thin translucent layer of phosphors.
- This consists of very pure inorganic crystalline phosphor crystal and with very small quantity of other elements like silver, manganese, chromium etc, called activators are found affecting characteristics of phosphors such as luminous efficiency, spectral emission etc.
- When the high energy electron beam strikes the phosphor coating, if glows at that point and continues to glow for a short period of time even after the electron beam is discontinued. This is known as *persistence or phosphorescence.*
- Aluminium in form of a thin foil is usually deposited on non-viewing side of the phosphor. It has following effects :

(i) Fast moving electron when strike the screen they generate heat. It works as heat sink.

(ii) The scattering of light is reduced and hence brightness is increased.

(iii) The electron striking the screen release secondary electrons. Though acquadag is used, under certain conditions they remain on the screen and a negative voltage is build up. Aluminium layer prevents such charge build up.

Acquadag :

When fast moving electrons strikes the screen, secondary electrons are released. These secondary electrons are collected by an aqueous solution of graphite called acquadag which is connected to the second anode. Secondary emitted electrons must be collected otherwise CRT screen will loose the state of electrical equilibrium.

7.13 Applications of CRT

7.13.1 Lissajous Pattern

When sinusoidal voltages are applied simultaneously on X and Y plates the resultant pattern called **"Lissajous pattern"** on the screen is very interesting as it is used to find the phase and frequency.

Theory of Lissajous pattern :

Two SHM (sine wave) of equal time period and acting at right angles :

Let $$x = a \sin(\omega t + \alpha) \quad \text{...(7.13.1)}$$

$$y = b \sin \omega t \quad \text{...(7.13.2)}$$

Represent the displacement of a particle along x and y axes due to the influence of two SHM from Equation (7.13.2)

$$\therefore y/b = \sin \omega t$$

$$\cos \omega t = \sqrt{1 - y^2/b^2}$$

$\therefore$ Equation (7.13.1), when expanded

$$\frac{x}{a} = \sin(\omega t + \alpha)$$

$$= \sin \omega t \cdot \cos \alpha + \cos \omega t \sin \alpha$$

$$= \frac{y}{b} \cos \alpha + \sqrt{1 - \frac{y^2}{b^2}} \cdot \sin \alpha$$

$$\therefore \frac{x}{a} - \frac{y}{b} \cos \alpha = \sqrt{1 - \frac{y^2}{b^2}} \cdot \sin \alpha$$

on squaring both sides

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} \cos^2 \alpha - \frac{2xy}{ab} \cos \alpha = \left(1 - \frac{y^2}{b^2}\right) \sin^2 \alpha$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} (\cos^2 \alpha + \sin^2 \alpha) - \frac{2xy}{ab} \cos \alpha = \sin^2 \alpha$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \alpha = \sin^2 \alpha \quad \text{...(7.13.3)}$$

This represents a general equation of an ellipse.

Analytical Approach to Lissajous Pattern :

The resultant pattern will depend upon the value of α.

Take following cases

(1) If $\alpha = 0 \text{ or } 2\pi$

$$\therefore \cos \alpha = 1, \sin \alpha = 0$$

$\therefore$ Equation (7.13.3) becomes

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\therefore \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \qquad \therefore \frac{x}{a} = \frac{y}{b}$$

or $$y = \frac{b}{a} x \quad \text{...(7.13.4)}$$

This represents, an equation of straight line, (even if both the potentials have the same value, i.e. a = b) as shown in Fig. 7.13.1(a).

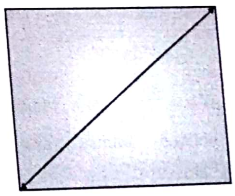


Fig. 7.13.1(a)

(2) if $\alpha = \pi \ \therefore \ \sin \alpha = 0$

and $\cos \alpha = -1$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\therefore \left(\frac{x}{y} + \frac{y}{b}\right)^2 = 0$$

or $$y = \frac{-b}{a} x \quad \text{...(7.13.5)}$$

Scanned by CamScanner

This represents equation of straight line with negative slope as shown in Fig. 7.13.1(b).

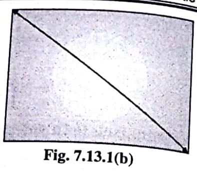


Fig. 7.13.1(b)

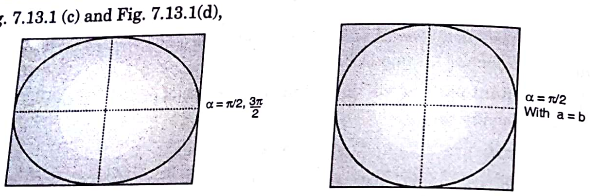
(3) If $\alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\therefore \sin\alpha = 1, \quad \cos\alpha = 0$

$\therefore$ Equation (7.13.3) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(7.13.6)$$

This represents the equation of the ellipse with 'a' and 'b' as semi major and semi-minor axes. If a = b it represents a circle with radius r = a = b Refer Fig. 7.13.1 (c) and Fig. 7.13.1(d),



(c) (d)

Fig. 7.13.1

(4) If $\alpha = \pi/4$ or $\frac{7\pi}{4}$, the resultant is an oblique ellipse as shown in Fig. 7.13.1(e)

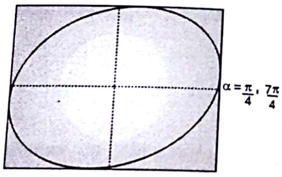


Fig. 7.13.1(e)

(5) if $\alpha = \frac{3\pi}{4}, \frac{5\pi}{4}$, ... the resultant is an oblique ellipse as shown in Fig. 7.13.1(f).

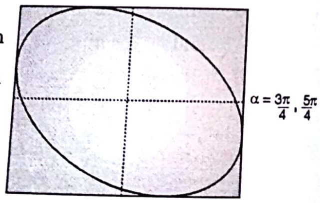


Fig. 7.13.1(f)

7.13.2 Application of CRT for Measurements

(A) Phase measurements :

MU - Dec. 2016

(i) With equal voltage and zero phase difference (Fig. 7.13.2.) In this particular case the resultant on the screen is a straight line.

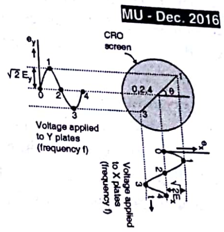


Fig. 7.13.2

(ii) With equal voltage and 90° phase difference In this case, on CRO screen we get a circle as shown in Fig. 7.13.3.

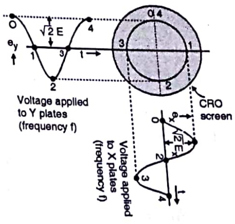


Fig. 7.13.3

(iii) When two equal voltages of equal frequencies but with a phase difference of ϕ ($\phi \neq 0°$ or 90°) are applied to X and Y plates.

In this case, we get an ellipse as shown in Fig. 7.13.4.

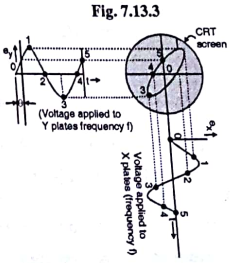


Fig. 7.13.4

Fig. 7.13.5 shows the observed pattern on CRO screen when two voltage of equal magnitude and frequency are applied to x and y plates and they differ only in terms of phase.

Regardless of the two amplitudes of the applied voltage the ellipse provides a simple means of finding phase difference between two voltages. Refer Fig. 7.13.6, the sine of the phase angle between the voltages in given by,

$$\theta = \sin\left(\frac{y}{x}\right) \quad ...(7.13.7)$$

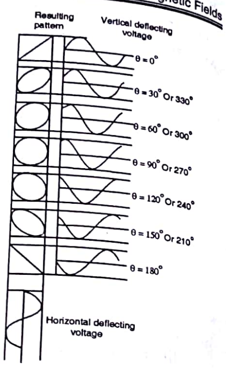


Fig. 7.13.5

If the ellipse is the result of two sine waves with phase difference θ, they can be represented as,

$$v_x = V \sin \omega t$$

$$v_y = V \sin (\omega t + \theta)$$

At $t = 0$

$$v_x = 0$$

$$v_y = V \sin \theta$$

The distance $X = 2 V \sin \theta$ and

$Y =$ peak to peak voltage $V_{p-p} = 2 V$

$$\therefore \sin \theta = y / x \text{ or } \theta = \sin^{-1}\left(\frac{y}{x}\right)$$

It is also clear that if the slope of major axis of ellipse is positive refer Fig. 7.13.6(a) the phase difference is between 0° to 90° or between 270° to 360° and if the slope of minor axis is negative Fig. 7.13.6(b) the phase difference is to between 90° to 180° or 180° to 270°.

(B) Frequency measurement : MU - Dec. 2016

- Frequency measurement Lissajous patterns may be used for accurate measurement of frequency.

The method is as follows :

- The known and unknown signal sources are connected to X and Y plates respectively.
- By varying the frequency of the unknown source a stable loop pattern is displayed.
- Horizontal and vertical tangents to the loops are drawn.
- The lines on the graticul may be used instead the number of points at which the loops are touching the horizontal tangent and vertical tangents are noted.
- The unknown signal frequency f_y is given by,

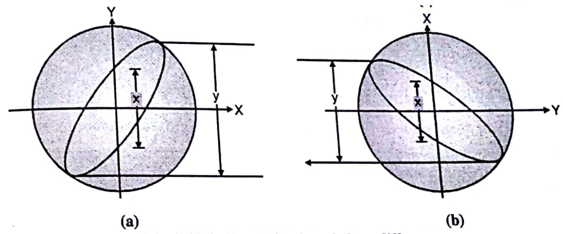


Fig. 7.13.6 : Determination of phase difference

$$f_y = f_x\left[\frac{L_H}{L_V}\right] \quad ...(7.13.8)$$

where f_x = Frequency applied to X plate (known)

L_H = Number of contact points on horizontal tangent

L_V = Number of contact points on vertical tangent

Here $L_H = 2, L_V = 1$

$$\therefore \frac{f_y}{f_x} = 2/1$$

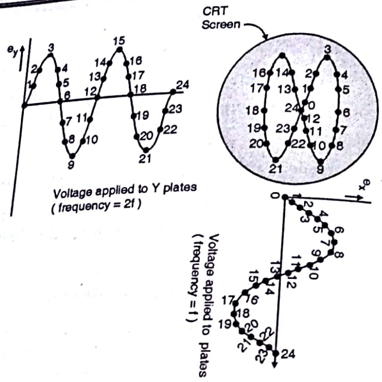


Fig. 7.13.7 : Determination of frequency

Few other possible Lissajous figures for frequency measurements are,

Case-I Here $L_H = 3, L_V = 1$

$$\therefore f_y = f_x\left(\frac{3}{1}\right)$$

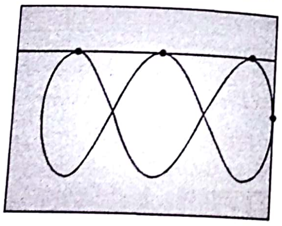


Fig. 7.13.8(a)

Therefore frequency of the Y plate is thrice that of horizontal voltage,

Case-II Here $L_H = 3, \ L_V = 2$

$$\therefore f_y = \frac{3}{2} f_x$$

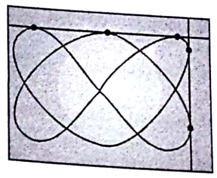


Fig. 7.13.8(b)

i.e. frequency of the Y plate voltage is 1.5 times to that of horizontal voltage.

Case-III Here $L_H = 1, \quad L_V = 2$

$$\therefore f_y = \frac{1}{2} f_x$$

i.e. frequency of the Y plate voltage is 0.5 time that of X plate voltage.

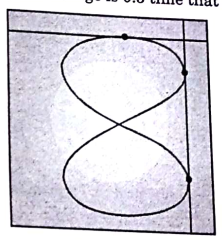


Fig. 7.13.8(c)

7.13.3 Determination of e/m of an Electron by Thomson's Method

In 1887, J.J. Thomson determined the specific charge of electron.

Principle :

Its working is based on the fact that if a beam of electrons is subjected to the transverse electric and magnetic fields, it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these two fields are equal and opposite, the beam remains un deflected.

Construction :

- The modernized version of Thomson's apparatus is shown in Fig. 7.13.9. It consists of a highly evacuated discharge tube G in which F is a filament of tungsten coated with a layer of BaO or SrO.

- It emits large number of electrons when heated with a current from low tension (L.T.) battery.
- The emitted electrons are accelerated by the high voltage V applied between anode A the cathode C.
- The accelerated electrons emerge through a hole in the anode in the form of narrow beam.
- P and Q are the two metal plates through which the electric field can be applied perpendicular to the path of electron beam, in the plane of paper.
- A magnetic field (shown by dotted circle) can also be applied on the electron beam at the same place where electric field is acting. Its direction is perpendicular to the plane of paper, directed outwards.
- S is a fluorescent screen coated with a florescent material like zinc sulphide.
- When the electron beam meets the screen, luminous spot is seen at that point.

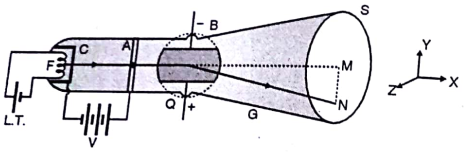


Fig. 7.13.9 : Arrangement for Thomson's experiment

Working and Theory :

- If no electric or magnetic field is applied, the electron beam meets the screen at M.
- Apply the electric field, and note that the spot on screen is deflected downwards.
- Then adjust the strength of the magnetic field B such that the electron beam meets the screen S at its undeflected position M.
- In this situation the forces on the electron due to electric field and magnetic field balance each other.
- Let m and e be mass and charge of electron and v be the velocity of the electron when it comes out of the hole of the anode.
- Force on the electron due to electric field = E e

- Force on the electron due to magnetic field = B e v
- For the undeflected position of the spot on the screen S, the two forces must be equal and opposite, therefore,

$$E e = B e v \quad \text{or} \quad v = E/B$$

- As the electron beam is accelerated from cathode to anode, its potential energy at the cathode appears as gain in its kinetic energy at the anode.
- If V is the potential difference between cathode and anode, then potential energy of electron at

Cathode = Charge × Potential difference = e V

Gain in kinetic energy of electron at anode $= \frac{1}{2} m v^2$

So, we have $e V = \frac{1}{2} m v^2$

Or

$$\frac{e}{m} = \frac{1}{2}\frac{v^2}{V} = \frac{1}{2V} = \left(\frac{E}{B}\right)^2 = \frac{E^2}{2VB^2}$$

Knowing the value of E, B, and V, the value of e/m of electron can be determined.

Thomson found that the value of e/m of electron comes out to be 1.77×10^{11} C kg^{-1}. The accurate value of e/m for electron is found to be 1.7589×10^{11} C kg^{-1}.

Syllabus Topic : Cathode Ray Oscilloscope (CRO)

> **Topics covered :** Cathode Ray Oscilloscope (CRO), CRT, Saw tooth Sweep Generator, Display of Signal on Screen, Synchronization, Power Supply

7.14 Cathode Ray Oscilloscope (CRO)

MU - Dec. 2014

CRO is an important instrument in any laboratory where different current and voltage signals are studied. Even non electrical quantities once converted to electrical equivalent using transducers can be visualized using CRO.

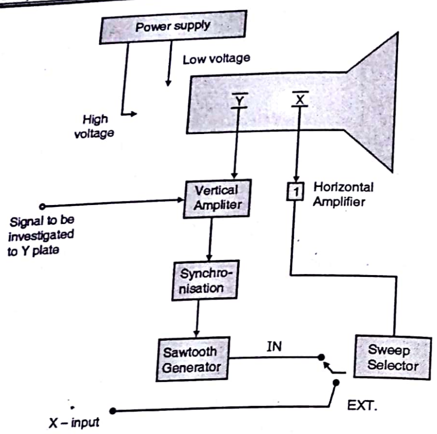


Fig. 7.14.1 : A diagram of general purpose CRO

Let us see a block diagram of a general purpose CRO, with important sections on it.

The Fig. 7.14.1 has details of important section of a general purpose CRO. Let us discuss their role step by step.

7.14.1 CRT

- CRT as mentioned in Section 7.13 is heart of any CRO. It generates electrons, make them accelerated, focuss them and cause deflection as per requirement with the help of other electronic circuits.
- The signal under investigation needs to reach on screen and it should develop visual image of such signal.
- This is possible if electron beam is moving on X-Y plane on screen that is it should move along X and Y axes simultaneously. This is done with following circuit sections

Vertical deflection system :

- The signals to be examined is usually applied to Y deflection plates through an amplifier or attenuator. The signals if not strong enough to produce measurable deflection on the CRT screen, an amplifier is essential.

- At the same time when high voltage signals are to be examined, they must be alternated to bring them within the range of vertical amplifier.
- The vertical amplifier output is also applied to the synchronizing amplifier stage through synchronizer selector switch when internal Horizontal sweep circuit is to be triggered.

Horizontal deflection system :

- The function of this stage is to spread the signal along x-axis. Let us see it through following diagram shown in Fig. 7.14.2.
- If a sine wave is applied to Y plates it is expected that on screen a sine wave is observed. But if there is no signal on X-deflection plate, we will get the image on screen as shown in Fig. 7.14.2 that is a vertical line.

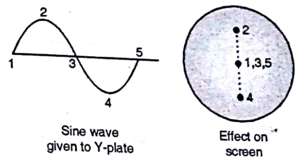


Fig. 7.14.2 : No x-plate signal applied

- This problem is sorted out by using horizontal deflection system which is responsible to provide a sweep signal which provides a time base. The horizontal plates are supplied through an amplifier but they can be fed directly when voltages are of sufficient magnitudes.

  Signal to the horizontal (X) plates can be supplied by two different ways.

  (i) Using external signal

  (ii) Using internal signal generator
- This choice is available on CRO panel as sweep selector switch when the sweep selector switch is in the 'internal' position, the horizontal amplifier receives an input form the saw tooth sweep generator which is triggered by the synchronizing amplifier.

7.14.2 Sawtooth Sweep Generator

The voltage signal given to horizontal deflection plates should have following characteristics :

- The electron beam should have linear displacement with voltage applied to x-plates.
- At the end of the horizontal motion, the beam should return back starting point to repeat the motion.

All the qualities are possessed by a saw tooth wave or ramp voltage as shown in Fig. 7.14.3.

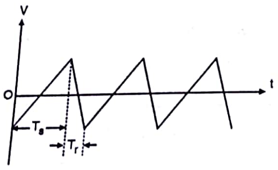


Fig. 7.14.3 : Typical sawtooth waveform

- Ramp voltage increases linearly with time and decrease rapidly. It can be divided into two parts that is T_S = Sweep time or **trace time** and T_R = Retrace time, ($T_S >> T_R \simeq T_S$). Retrace time is also called as **flyback time.**

Blanking pulse :

- The retrace should be eliminated or blanked out that is the beam when goes back to its start point form right to left it should remain invisible.
- If there is no voltage applied across Y deflection plates, the image created on screen is shown in Fig. 7.14.4.

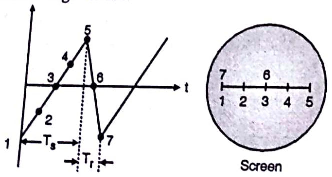


Fig. 7.14.4 : Ramp voltage and its effect on CRO screen

It is easy to see that the length of the trace on the screen is measure of the period of the oscillator frequency in seconds. The x-axis of the screen not only denotes the amount of horizontal deflection but also the time elapsed. Due to this reason, the ramp voltage is also known as time base. Therefore the x-axis on the screen is calibrated in milliseconds and microseconds.

Since signals of different frequencies are to be observed with the CRO, the sweep rate must be adjustable. We can change the sweep rate in steps. **The front panel control for this adjustment is marked time/div or sec/div.**

7.14.3 Display of Signal on Screen

How a signal gets displayed on CRO screen on Fig. 7.14.5 with the help of two cases,

(i) where $T_{signal} = T_{sweep}$.

(ii) $T_{signal} = 2\, T_{sweep}$

Case - I : $T_{signal} = T_{sweep}$

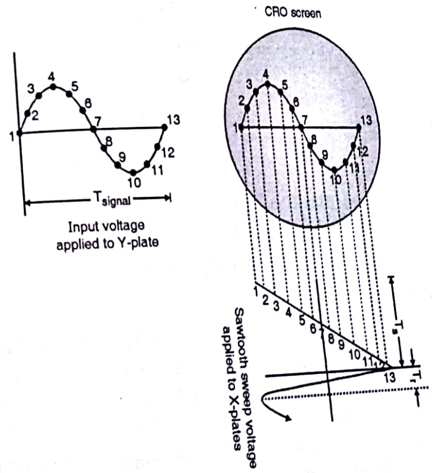


Fig. 7.14.5(a) : When $T_{signal} = T_{sweep}$

Case – II : $T_{signal} = T_{sweep}$

As $T_{signal} = 2\,T_{sweep}$, only half the signal is visible on CRO screen.

Above mentioned cases explain that when the signal applied to Y – plate and time base signal on x-plate are considered simultaneously. The beam will experience two force acting in perpendicular direction. The deflection of the beam at any instant occurs along the direction of the resultant of the two forces. Both the cases give a complete brief explanation as how a signal gets it's image constructed on CRO screen.

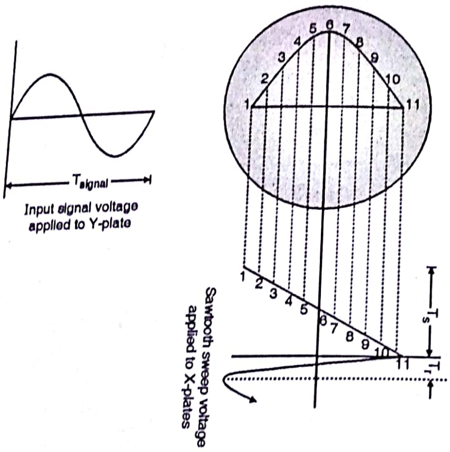


Fig. 7.14.5(b) : When $T_{signal} = 2\,T_{sweep}$

7.14.4 Synchronization

- In Fig. 7.14.5(a) and Fig. 7.14.5(b) it must have been observed by the reader that horizontal sweep signal to x-plate deflection must be applied at the same time when input signal on vertical deflection (Y plate) in order to have stationary wave pattern.
- In this case sweep signal is said to be **synchronized** with input signal. If this type of synchronization is not achieved, wave pattern on CRO screen continuously drifts either on right or left of the screen.

Trigger circuit :

- An electronic circuit is required to lock the frequency of time based circuit with the frequency of signal under investigation which is applied to Y – plates is called triggered circuit.
- In trigger mechanism, a part of the output obtained from vertical amplifier is fed to trigger generator. When a predetermined level is reached, trigger generator will produce a pulse and this pulse which acts as a command to generate one sweep cycle of the time base.

7.14.5 Power Supply

Various sections of CRO require different magnitudes of electric potentials. Hence various power supply circuits are designed. Basically these circuits are divided into two main categories,

(a) Low voltage supply :

- It provides electric supply to electronic circuits such as amplifiers trigger generator, time base generator etc.
- All these section require few tens of volts.

(b) High voltage power supply :

- It provides voltages to electrodes in the CRT i.e. heater, accelerating and focussing mechanism.
- Magnitude of voltage supplied are around few kilo volts.

Syllabus Topic : Applications of CRO : Voltage (dc, ac), Frequency, Phase Measurement

➤ **Topics covered :** Applications of CRO, Measurement of D.C. Voltage, Measurement of A.C. Voltage and Frequency, Measurement of Phase Difference

7.15 Applications of CRO

MU - Dec. 2013

CRO is basically used for measuring certain parameters of an electrical signal such as amplitude, phase and frequency if AC signal is used. it is also used to measure phase difference between two signals.

Various applications are :

(a) Measurement of D.C. Voltage
(b) Measurement of A.C. Voltage and Frequency
(c) Measurement of Phase Difference

7.15.1 Measurement of D.C. Voltage

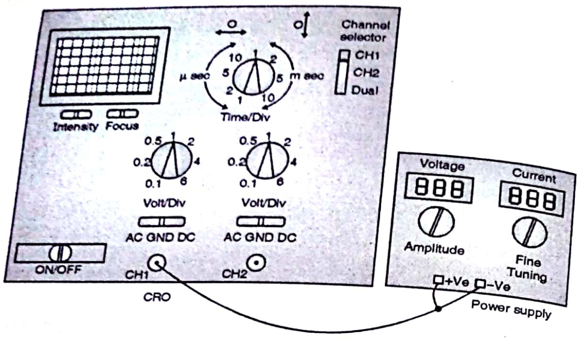


Fig. 7.15.1 : DC volts measurement set up

(1) Initially the input coupling switch is kept in D.C. position. The trace on the screen is made to coincide with the central horizontal line of the graticule.

(2) CRO probe connected to CH 1 on one end is made to connect to the power supply at the other end. Make sure that the red banana pin is connected to the positive terminal and black banana pin is connected to the negative terminal of the power supply.

(3) The volt/div knob is kept at 1 volt/div. and the D.C. power supply is switched ON. The output voltage of power supply is varied and its effect on the trace is noted. The trace moves upward with increase in voltage and moves down with decrease in voltage.

(4) The amount of deflection in terms of number of divisions on the graticule are noted. By multiplying it with the value at volt/div knob that is currently 1 volt/div. we get the value of the D.C. voltage. The measurement is illustrated in Fig. 7.15.2.

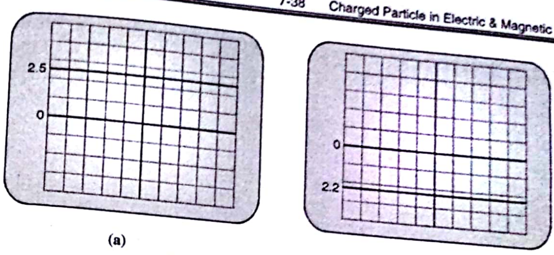


Fig. 7.15.2 : DC volts measurement

7.15.2 Measurement of A.C. Voltage and Frequency

MU - May 2013, May 2014, May 2016

(1) The input coupling switch is kept in A.C. position. The function generator's mode is kept as sinusoidal. The output of the function generator is connected to CH 1 of CRO using BNC connectors.

(2) The trace on the screen is made to coincide with the central horizontal line of the graticule. The function generator is then turned ON. A signal of a particular frequency is generated using the frequency selector knobs of the generator. The amplitude is set using the amplitude knob.

(3) By adjusting the vertical amplifier sensitivity and time/div knob on the CRO the wave size on the graticule is adjusted to get a sufficiently large display without going beyond the limits of the screen.

(4) The vertical length of the wave from the negative maximum to the positive maximum is read on the graticule. It is multiplied by the value set on the volt/div knob. This will give us the peak to peak value (amplitude) of the A.C. signal. Then observe the length of one full cycle of the wave on the CRO in the horizontal direction. Multiplying this value with the value set on the time/div knob we get the time period of the A.C. signal. Reciprocal of this value will give us the frequency of the A.C. signal (Refer Fig. 7.15.3).

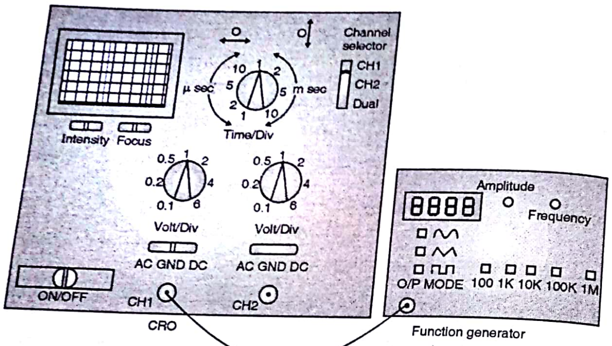


Fig. 7.15.3 : AC volts measurement

Vertical displacement = 5.6 div

Volt/div knob is at 1 volt/div

$\therefore$ *Amplitude of the applied signal* = 5.6 div × 1 volt / div

= 5.6 Vp.p

horizontal displacement of one complete cycle = 3.2 div.

$\therefore$ *time period of the signal* = 3.2 div × 2 m sec/div

= 6.4×10^{-3} sec

$\therefore$ *Frequency of the signal* = $1/6.4 \times 10^{-3} = 156.25$ Hz

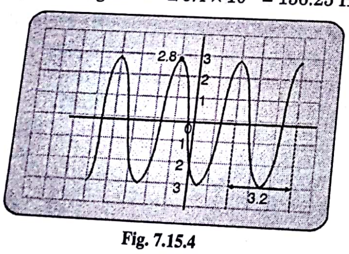
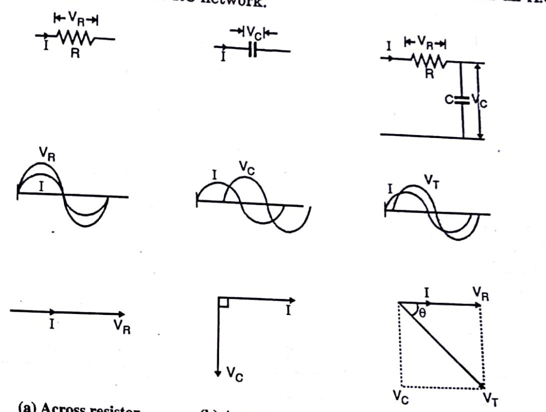


Fig. 7.15.4

7.15.3 Measurement of Phase Difference

MU - May 2015, Dec. 2015

- The circuit connection for performing the phase shift experiment shown in Fig. 7.15.4. A resistor is connected in series with a capacitor and an A.C. voltage is applied to the RC network.



(a) Across resistor (b) Across capacitor (c) Across RC network

Fig. 7.15.5 : Phase relationships between voltage and current

- The voltage form the A.C. source is directly applied at CH1 and voltage across the capacitor is applied at CH2. The A.C. voltages have the same frequency but differ in their phases as the RC network introduces a finite phase difference between them.
- The resistance R may be made variable so that the phase difference between the voltages can be varied. Usually a decade resistor box is employed for the purpose.

(I) Direct method :

(1) The phase shift circuit is connected into the circuit along with the CRO and signal generator as shown in the Fig. 7.15.6. The RC network introduces a phase difference between the signals fed to CH1 and CH2.

(2) The CRO is kept is duel mode and the function generator which is kept in sinusoidal mode is switched ON. The signal from the function

generator is kept at a particular amplitude and frequency using the frequency selector knob.

(3) Two signals of the same frequency are displayed on the screen. It is seen that the phases of the signals are different.

(4) The vertical position settings of CH1 and CH2 are adjusted such that the two waves overlap on each other.

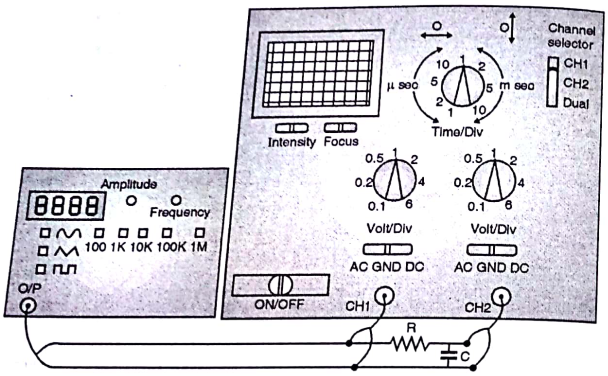


Fig. 7.15.6

(5) The sweep speed in initially adjusted such that the period T of the sine wave is measured (same as the procedure discussed earlier). Fig. 7.15.7(a).

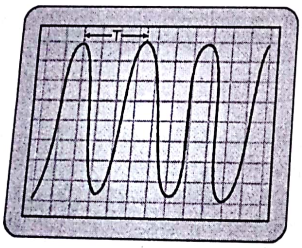


Fig. 7.15.7(a)

(6) The sweep speed is increased and the delay time T_d between the two sine waves in accurately measured. Fig. 7.15.7(b).

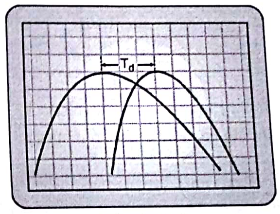


Fig. 7.15.7(b)

(i) $\theta_{exp} = \frac{T_d}{T} \times 360°$

(ii) $\theta_{the} = \tan^{-1}(2\pi fCR)$

(Where θ_{exp} is the measured phase difference and θ_{the} is the calculated phase difference)

(7) The resistance value is changed by altering the setting of the dial of the decade box. The phase difference is again determined and measured.

(II) XY method :

(1) This method makes use of lissajous pattern.

(2) The time/div knob is set to X-Y EXT state. In this state, the signal given to CH1 acts as the sweep signal.

(3) The vertical mode switch is kept in X-Y position. An ellipse such as the one illustrated in Fig. 7.15.8 is obtained on the CRO screen.

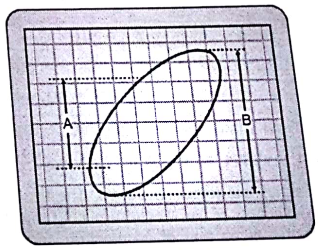


Fig. 7.15.8

(4) The AC – GND – D.C. switch is set to GND position.

(5) The vertical lengths. A and B of the ellipse are measured.

$$\theta = \sin^{-1}(A/B)$$
$$= \sin^{-1}(2.4/4)$$
$$= 36.86°.$$

(6) The value of the resistor R is varied by changing the dial setting on the decade box and measurements are taken one again.

> **Note :** If phase difference between two different signals is to be measured, then connect one signal to CH1 and the other to CH2. there is then no need for any RC circuit.

7.16 Solved Problems

Ex. 7.16.1 : In a CRT the distance from screen to the center of the coils is 0.2 m. The length of the magnetic field along the axis is 5 cm. Calculate the flux density B required to produce a deflection of 1 cm on the screen. If the anode voltage is 1000 volt.

Soln. :

Data :
$$D = 0.2 \text{ cm}$$
$$l = 5 \times 10^{-2} \text{ cm}$$
$$Y = 1 \times 10^{-2} \text{ cm}$$
$$V_A = 1000 \text{ V}$$

Formula : $$Y = DlB\sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}$$

$$\therefore \; B = \frac{Y}{Dl} \cdot \frac{1}{\sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}}$$

$$= \frac{1 \times 10^{-2}}{0.2 \times 5 \times 10^{-2}} \; \frac{1}{\sqrt{\frac{1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31} \times 1000}}}$$

$$\therefore \; B = \mathbf{1.066 \times 10^{-4} \; Wb/m^2}$$ ...Ans.

Ex. 7.16.2 : A CRT, employing magnetic deflection has a length of field equal to 3 cm along tube axis and flux density equal to $1 \times 10^{-4} \frac{Wb}{m^2}$. The distance form the centre of the field to the screen is 20 cm. The final anode voltage is 800 volts. Calculate the deflection of an electron in cm. Also find the displacement if a particle with charge twice that of an electron and mass 7344 times as large. Given $e/m = 1.76 \times 10^{11}$ C/kg.

Soln. :

Data :
$$l = 3 \text{ cm}$$
$$B = 1 \times 10^{-4} \text{ Wb/m}^2$$
$$D = 20 \text{ cm}$$
$$V_A = 800 \text{ Volts}$$
$$e/m = 1.76 \times 10^{11} \text{ C/kg}$$

Formula :

(i) $$Y = DlB\sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}$$

$$= 20 \times 10^{-2} \times 3 \times 10^{-2} \times 1 \times 10^{-4} \times \sqrt{\frac{1.76 \times 10^{11} \times 1}{2 \times 800}}$$

$\therefore$ Deflection $= \mathbf{6.3 \times 10^{-3} \; m}$ ...Ans.

(ii) For the charge particle with charge twice that of electron and mass 7344 times.

$\therefore$ Using the same formula

$$Y = DlB\sqrt{\frac{2e}{2(7344)m} \cdot \frac{1}{V_A}}$$

$$= 20 \times 10^{-2} \times 3 \times 10^{-2} \times 1 \times 10^{-4} \times \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{2 \times 7344 \times 9.1 \times 10^{-31} \times 800}}$$

$$= 1.038 \times 10^{-4} \text{ m}$$

$\therefore$ Displacement = **0.01 cm** ...Ans.

Ex. 7.16.3 : An electrically deflected CRT has a anode voltage 2000 V and parallel deflecting plates 1.5 cm long and 5 mm apart. If the screen is 50 cm from the center of deflecting plates find (a) Beam speed (b) The deflection sensitivity of the tube and (c) Deflection factor of tube.

Soln. :

Data :
$$V_A = 2000 \text{ V}$$
$$l = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$$
$$d = 5\text{mm} = 5 \times 10^{-3} \text{ m}$$
$$D = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$$

(I) Velocity of beam :

Using concept $KE = \frac{1}{2} mv^2 = eV_A$

$$\therefore \quad v = \sqrt{\frac{2eV_A}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2000}{9.1 \times 10^{-31}}}$$

$$\text{Velocity} = 26.5 \times 10^6 \text{ m/sec} \qquad \text{...Ans.}$$

(ii) Deflection sensitivity :

$$S = \frac{y}{D} = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{1}{V_A}$$

$$= \frac{0.5 \times 1.5 \times 10^{-2}}{2 \times 5 \times 10^{-3} \times 2000}$$

$$= 3.75 \times 10^{-4} \text{ m/volt}$$

$$\therefore \text{Deflection sensitivity} = 0.375 \text{ mm/volt} \qquad \text{...Ans.}$$

(iii) Deflection factor :

$$D = \frac{1}{S} = \frac{1}{0.375} = \mathbf{2.67\ V/mm} \qquad \text{...Ans.}$$

Ex. 7.16.4 : A CRT has anode voltage 1000V and parallel deflecting plates 2cm long and 5 mm apart. the screen is 30cm from the center of the plates. Find the input voltage required to deflect the beam 1.5 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.

Soln. :

Data : $V_A = 1000$ V

$l = 2 \text{ cm} = 2 \times 10^{-2}$ m

$d = 5 \text{ mm} = 5 \times 10^{-3}$ m

$D = 30 \text{ cm} = 0.3$ m

$Y = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m

Formula : $$Y = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{V}{V_A}$$

$$\therefore \quad V = \frac{2dV_A}{Dl} \cdot Y$$

$$= \frac{2 \times 5 \times 10^{-3} \times 1000 \times 1.5 \times 10^{-2}}{0.3 \times 2 \times 10^{-2}} = 25 \text{ Volt}$$

$$\text{Input voltage required for a deflection of 1.5 cm} = \frac{V}{\text{gain}} = \frac{25}{100}$$

$$= \mathbf{0.25\ Volt} \qquad \text{...Ans.}$$

Ex. 7.16.5 : A CRT has anode voltage of 1.6 kV and parallel deflection plates are 1.8 cm long and 4.2 mm apart. The screen is at 2.8 cm from the centre of deflecting plates. Find the input voltage required to deflect the beam by 1.5 cm. The input voltage is applied to the deflection plates through the amplifiers having overall gain 60.

Soln. :

$$y = \frac{D}{2} \cdot \frac{l}{d} \frac{V}{VA}$$

$$\therefore \quad V = \frac{2yd\,V_A}{D \cdot L} = \frac{1.5 \times 2 \times 4.2 \times 1.6 \times 10^3 \times 10^{-3}}{2.8 \times 1.8}$$

$$= 4V$$

$$\text{As} \quad \frac{V}{V_{app}} = \text{gain}$$

$$\therefore \quad V_{app} = \frac{V}{\text{gain}} = \frac{4}{60} = 0.0667 \text{ V} \qquad \text{...Ans.}$$

Ex. 7.16.6 : The sketches shown below display Lissajous patterns for cases where voltages of same frequency and of different phases are connected to Y and X plates of the CRO. Find the phase difference in each case. The spot generating the pattern moves in clockwise direction. Calculate the angles the spot generating the patterns moves in the anticlock wise direction.

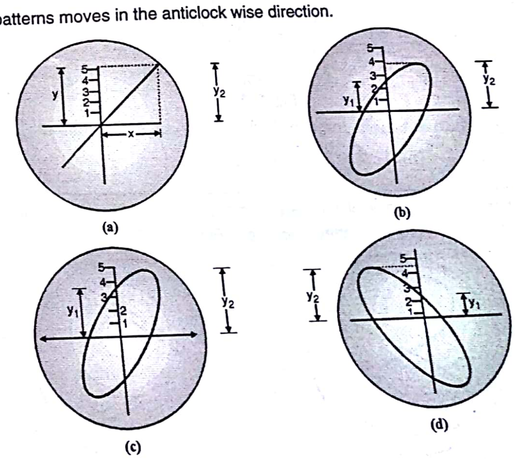


Fig. P. 7.16.6

Soln. : The spot generating the patterns moves in clockwise direction

(a) $\sin\phi = \frac{y_1}{y_2} = \frac{0}{5} = 0$

$\therefore \phi = 0$

(b) $\sin\phi = \frac{2.5}{5} = 0.5$

$\therefore \phi = 30°$

(major axis is in 1st and 3rd quadrants)

(c) $\sin\phi = \frac{3.5}{5} = 0.7$

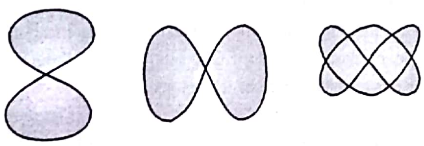
(d) $\sin\phi = \frac{2.5}{5} = 0.5$

$\phi = 180° - 30° = 150°$

(major axis is in 2nd and 4th quadrant)

If the spot generating the pattern moved in the anticlockwise direction the angles would be (a) 180° (b) – 30° (c) – 45° (d) 180 + 30 = 210°

Ex. 7.16.7 : Calculate the ratio of vertical to horizontal frequencies for an oscilloscope which displays the following Lissajous figure



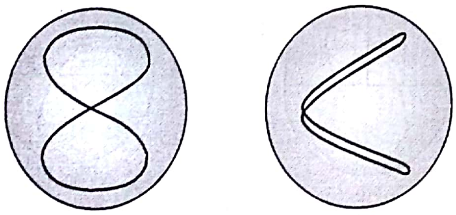
(a) (b) (c)

Fig. P. 7.16.7

Soln. : Using Equation (7.7.7) one can find answer as shown below :

(a) 2 : 1 (b) 1 : 2 (c) 2 : 3

Ex. 7.16.8 : Find the frequency of the vertical plates if the frequency applied to horizontal plate is 50Hz for the patterns shown in Fig. P. 7.16.8.



(a) (b)

Fig. P. 7.16.8

Soln. : As $\frac{f_y}{f_x} = \frac{\text{Number of contact points on horizontal}}{\text{Number of contact points on vertical}}$

For Fig a and b it is same i.e. $= \frac{1}{2}$

But $f_X = 50$ Hz given $\therefore$ $f_y = \frac{1}{2} \times f_x \times = \frac{50}{2} = 25$ **Hz** **...Ans.**

Ex. 7.16.9 : In phase measurement by Lissajous pattern ellipse is obtained with major axis of 2cm and minor axis of 0.8cm. Calculate phase change.

MU - Dec. 2014, 3 Marks

Soln. :

$$\text{Phase angle} = \sin^{-1}\left(\frac{\text{Minor axis}}{\text{Major axis}}\right) = \sin^{-1}\left(\frac{y_1}{y_2}\right)$$

$$= \sin^{-1}\left(\frac{0.8}{2}\right)$$

$$= 23.57°$$

A Quick Revision

- $\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_2}{v_1}$ is identical of the Snell's law in optics which describes refraction.
- Refractive Index of electrostatic lens is given by,

  $\text{Refractive index} = \frac{\sin\theta_i}{\sin\theta_r} = \left[\frac{V_2}{V_1}\right]^{1/2}$
- Path of electron in a uniform field would be a helix. The pitch P is given by,

  $P = \frac{2\pi m}{eB} v\cos\theta$
- A cathode ray tube is a specially designed vacuum tube in which an electron beam controlled by electric or magnetic fields are used to visual display of input electrical signal on the screen which is coated with fluorescent materials.
- Deflection sensitivity $S = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{1}{V_A}$
- When sinusoidal voltages are applied simultaneously on X and Y plates the resultant pattern called **"Lissajous pattern"**.
- The unknown signal frequency f_y is given by,

  $$f_y = f_x\left[\frac{L_H}{L_V}\right]$$

  where f_x = Frequency applied to X plate (known)

Exercise

Q. 1 The deflection sensitivity of an oscilloscope is 35V/cm. If the distance from the deflection plates of the CRT screen is 16 cm, the length of the deflection plates is 2.5 cm, and the distance between the deflection plates is 1.2cm. What is the acceleration anode voltage ?

Ans. : [583 V]

Q. 2 In a cathode ray tube the distance between the deflecting plates is 1.0 cm, the length of the deflecting plates is 4.5 cm and the distance of the screen from the centre of the deflecting plates is 33 cm. If the accelerating voltage supply is 300 volt, calculate deflecting sensitivity of the tube

Ans. : [2.48 mm/v]

Q. 3 An electrostatic ally deflected cathode ray tube has plane parallel deflecting plates which are 2.5 cm long and 0.5 cm apart, and the distance from their centre to the screen is 20cm. The electron beam is accelerated by a potential difference of 2500 V and is projected centrally between the plates. Calculate the deflecting voltage required to cause the beam to strike at 4 cm on screen.

Ans. : [200 V]

Q. 4 Voltage E_1 is applied to the horizontal input and E_2 to the vertical input of a CRO. E_1 and E_2 have same frequency. The trace on the screen is an ellipse. The slope of major axis is negative. The maximum vertical value is 3 divisions and the point where the ellipse crosses the vertical axis is 2.6 divisions. The ellipse is symmetrical about horizontal and vertical axis. Determine the possible phase angle of E_2 with respect to E_1.

Ans. : [120° or 210°]

Q. 5 The wave form shown in Fig. 7.36 is observed on a CRT screen. If the time/div switch is set to 10μsec and the volt/div. switch is set 20 to 200 mV. Determine the frequency and peak to peak amplitude of the signal.

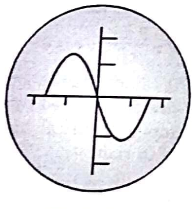


Fig. Q. 5

Ans. : [33.33 kHz; 600 mV]

Review Questions

Q. 1 What is meant by 'electron optics' ? Explain its significance.

Q. 2 What is 'equipotential surface' ? Why this concept is considered important in electron optics ?

Q. 3 Using concept of equipotential surface, explain the concept of refraction in electron optics.

Q. 4 Write a short note on electrostatic focussing. Also derive the formula for refractive index.

Q. 5 What are the differences between electron optics and geometrical optics.

Q. 6 Draw a path of electron beam when it passes through non uniform electric field. Take help of equipotential surfrace.

Q. 7 Explain construction and function of electrostatic lens. Write few points regarding the difference between electron lens and optical lens. Where it is used ?

Q. 8 What is magnetostatic focusing ? What are the types ?

Q. 9 Explain in detail the principle behind longitudinal magnetostatic focusing what is pitch ?

Q. 10 Show that in uniform magnetic field pitch remains constant.

Q. 11 Write a note on longitudinal magnetostatic focusing.

Q. 12 Write a note on transverse magnetic field focusing.

Q. 13 List various important parts of a CRT and explain their role in short.

Q. 14 Write the formula for vertical displacement on the screen giving meaning of each term used in it.

Q. 15 Define sensitivity of CRT. What is its unit ?

Q. 16 What is the significance of sensitivity of CRT ?

Q. 17 What is aquadag ? Why it is used ?

Q. 18 What is Lissajous pattern ? Explain its theory.

Q. 19 Explain Lissajous pattern forming a straight line, an ellipse and a circle.

Q. 20 What is CRO ? What are various sections ? Draw its block diagram.

Q. 21 What is time base circuit ? What type of output it generates ?

Q. 22 Why sawtooth or ramp voltage is used as time base ?

Q. 23 What is a blanking pulse ?

Q. 24 What is the need of synchronisation on CRO ?

Q. 25 Explain formation of sine wave pattern on CRO screen. Use different sweep frequencies.

Q. 26 How a CRO can be used to measure frequency and phase angle between tow AC signals ?

Q. 27 How a CRO can be used to generate Lissajous pattern ?

7.17 University Questions

May 2013

Q. 1 Explain the measurement of frequency of AC signal using CRO.
(Ans. : Refer section 7.15.2) (3 Marks)

Q. 2 Derive Bethe's law for electron refraction.
(Ans. : Refer section 7.7) (5 Marks)

Dec. 2013

Q. 1 Write any two applications of CRO.
(Ans. : Refer section 7.15)
(3 Marks)

Q. 2 Write a short note on electrostatic focussing. *(Ans. : Refer section 7.7)*
(5 Marks)

May 2014

Q. 1 Explain measurement of frequency of AC signal using CRO.
(Ans. : Refer section 7.15.2)
(3 Marks)

Q. 2 With neat diagram explain construction and working of CRT.
(Ans. : Refer section 7.12)
(5 Marks)

Dec. 2014

Q. 1 *Refer Ex. 7.16.9* (3 Marks)

Q. 2 Explain construction and working of Cathode Ray Oscilloscope.
(Ans. : Refer section 7.14)
(5 Marks)

May 2015

Q. 1 How phase difference between two signals is measured using CRO ?
(Ans. : Refer section 7.15.3)
(3 Marks)

Q. 2 Draw a labelled diagram and explain construction and working of CRT. *(Ans. : Refer section 7.12)*
(5 Marks)

Dec. 2015

Q. 1 How is phase difference between two A.C. signals measured by CRO ? *(Ans. : Refer section 7.15.3)*
(3 Marks)

Q. 2 Explain the principle, construction and working of CRT with neat diagram.
(Ans. : Refer section 7.12)
(5 Marks)

May 2016

Q. 1 Explain measurement of frequency of ac signal using CRO.
(Ans. : Refer section 7.15.2)
(3 Marks)

Q. 2 Derive Bethe's law for electron refraction ?
(Ans. : Refer section 7.7) (5 Marks)

Dec. 2016

Q. 1 How Lissajous figures are used to measure unknown frequency ?
(Ans. : Refer section 7.13.2(B))
(3 Marks)

Q. 2 Explain how Lissajous figures are used to determine the phase difference between two ac signals. ?
(Ans. : Refer section 7.13.2(A))
(08 Marks)

e-book

Note :

- *Exact syllabus topic wise answers to the following questions are available in e book.* ***Please download as per instructions given.***

Syllabus Topic : Motion of Electron in Electric Field (Parallel)

Q. 1 Describe Motion of Electron in Electric Field (Parallel). *(Ans.: Refer section 7.1)*

Syllabus Topic : Motion of Electron in Electric Field (Perpendicular)

Q. 1 Describe motion of electron is perpendicular to the applied electric field.
(Ans.: Refer section 7.2)

Syllabus Topic : Motion of Electron in Magnetic Field(Longitudinal and Transverse)

Q. 1 Write a note on longitudinal and transverse magnetic field.
(Ans.: Refer sections 7.3.1 and 7.3.2)

Syllabus Topic : Magnetic Deflection

Q. 1 Explain the term magnetic deflection. *(Ans.: Refer section 7.4)*

Syllabus Topic : Motion of Electron in Crossed Field

Q. 1 Write short note on : Motion of electron in crossed field. *(Ans.: Refer section 7.5)*

Syllabus Topic : Velocity Selector and Velocity Filter

Q. 1 Write short note on : Velocity selector and velocity filter.
(Ans.: Refer section 7.6)

Syllabus Topic : Electrostatic Focusing, Electron Refraction, Bethe's Law

Q. 1 Derive Bethe's law for electron refraction.
(Ans.: Refer section 7.7) **(May 2013, May 2016)**

Q. 2 Write a short note on electrostatic focussing.
(Ans.: Refer section 7.7) **(Dec. 2013)**

Syllabus Topic : Magnetic Focussing

Q. 1 Define magnetic focussing. *(Ans.: Refer section 7.11)*

Syllabus Topic : Cathode Ray Tube (CRT)

Q. 1 With neat diagram explain construction and working of CRT.
(Ans. : Refer section 7.12) **(May 2014, May 2015)**

Q. 2 Explain the principle, construction and working of CRT with neat diagram.
(Ans. : Refer section 7.12) **(Dec. 2015)**

Syllabus Topic : Cathode Ray Oscilloscope (CRO)

Q. 1 Explain construction and working of cathode ray oscilloscope.
(Ans. : Refer section 7.14) **(Dec. 2014)**

Syllabus Topic : Applications of CRO : Voltage (dc, ac), Frequency, Phase Measurement

Q. 1 Write any two applications of CRO. *(Ans. : Refer section 7.15)* **(Dec. 2013)**

Q. 2 Explain the measurement of frequency of AC signal using CRO.
(Ans. : Refer section 7.15.2) **(May 2013, May 2014, May 2016)**

Q. 3 How phase difference between two signals is measured using CRO ?
(Ans. : Refer section 7.15.3) **(May 2015, Dec. 2015)**

Solved Problems

Q. 1	Refer Ex. 7.16.1	**Q. 4**	Refer Ex. 7.16.4
Q. 2	Refer Ex. 7.16.2	**Q. 5**	Refer Ex. 7.16.5
Q. 3	Refer Ex. 7.16.3	**Q. 6**	Refer Ex. 7.16.9 **(Dec. 2014)**

□□□

CHAPTER 8

Module 6

Nano-Science and Nanotechnology

Syllabus

Introduction to nano-science and nanotechnology, Surface to volume ratio, Two main approaches in nanotechnology -Bottom up technique and top down technique; Important tools in nanotechnology such as Scanning Electron Microscope, Transmission Electron Microscope, Atomic Force Microscope.

Nano materials : Methods to synthesize nanomaterials (Ball milling, Sputtering, Vapour deposition, Sol Gel), properties and applications of nanomaterials.

Syllabus Topic : Introduction to Nano-science and Nanotechnology

➤ **Topics covered :** Introduction

8.1 Introduction

MU - Dec 2016

- We know all materials are composed of atoms with different sizes. If we take a material in which the atoms do not move away from each other and with size in the range of 1 to 100 nano meters, these materials are called nano materials.
- The technology emerged out of this called nanotechnology. Using these highly sophisticated latest technology nano materials can be formed from metals, ceramics polymers and even from liquids.
- Hence we can describe nano science as the study and nano technology as the exploitation of the unique properties exhibited by particles of the size few nano meters.
- Nanotechnology which is multidisciplinary emerging area of fusion between Applied science and Engineering. Nanotechnology is very diverse, ranging from extensions of conventional device physics to completely new approaches based upon molecular with self assembly, from developing new materials

with dimensions on the nano scale to investigating whether we can directly control matter on the atomic scale.

- Even through discussions related to nano technology were in air from 1959 (Richard Feynman) to 1986 (Discovery of buckminster fullerence which was later referred as "bucky balls") but it didnot become an experimental science as there were no experimental tools available to the scientists to handle materials at atomic level.
- But the scenario changed with the development of quantum mechanics and invention of STM and AFM (Atomic Force Microscope).
- In the hot pursuit of fabricating devices that serve the human needs and were hitherto eluded the attempts to secure them some expect that nanotechnology could be one of the largest manufacturing sector in the world.

Syllabus Topic

Surface to Volume Ratio, Two Main Approaches in Nanotechnology - Bottom Up Technique and Top Down Technique

> **Topics covered :** Important Features of Nanotechnology, Surface to Volume Ratio, Bottom Up and Top Down Approach

8.2 Important Features of Nanotechnology

8.2.1 Surface to Volume Ratio

- The properties of nano-materials are very much different from those at larger scale. One of the factor which causes this change is "Increased surface area" or "Increased surface to volume ratio".
- This can change or enhance properties such as reactivity, severity and electrical properties.
- Nano material have a relatively large surface to volume ratio compared to same volume of the material produced in larger form.
- Let us consider a sphere of radius 'r'.

$\therefore$ Its surface area $= 4\pi r^2$

and its volume $= \frac{4}{3}\pi r^3$

$\therefore$ Surface area to volume ratio

$$= \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$$

- When the radius of the sphere decreases, it's surface to volume ratio increases.
- This makes nano materials more chemically reactive.

8.2.2 Bottom Up and Top Down Approach

MU - May 2015, Dec. 2015

In nano science, we are suppose to arrive at nano scale assembly. This can be obtained by two different approaches, namely

(i) Bottom up approach :

In this, nano materials are made by building atom by atom or molecule by molecule.

(ii) Top down approach :

In this a bulk material is broken or reduce in size or pattern. The techniques developed under this tile are modified or improved one what we have in use to fabricate micro-processors, Micro-Electro-Mechanical Systems (MEMS) etc.

Syllabus Topic

Important Tools in Nanotechnology such as Scanning Electron Microscope, Transmission Electron Microscope, Atomic Force Microscope.

> **Topics covered :** Tools used in Nanotechnology, Electron Microscope, SEM (Scanning Electron Microscope), Scanning Tunneling Microscope (STM), Atomic Force Microscope (AFM), Differences between SEM and AFM

8.3 Tools used in Nanotechnology

As described in introduction, it is highlighted that until the discovery of certain microscopy tools, development in this field were at theoretical level and not at experimental level. The reasons are very clear that we were having limitations to reach up to atomic level.

8.3.1 Electron Microscope

- Electron microscope makes use of electron – a negatively charged particle, in terms of its wave like properties (quantum mechanical aspects). Electrons possess two main advantages for use in microscopy. Electron Microscope
- According to quantum physics, the wavelength associated with electron varies inversely as its energy.

$$\lambda = \frac{h}{\sqrt{2 mE}}$$

i.e. with increase in the energy, wavelength can be reduced. This brings in higher resolving power and hence better magnification, (nearly 1000 times that of optical microscope).

- Electron can be focussed by means of magnetic lenses unlike electromagnetic radiations.
- The first electron microscope which was a TEM (Transmission Electron Microscope) was schematically as shown below. For a student, comparison between electron microscope and optical microscope is advisable.
- Fig. 8.3.1 shows schematic diagram of TEM.

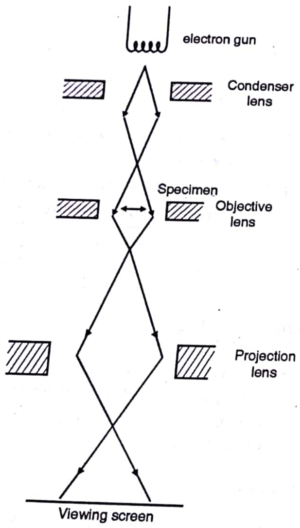


Fig. 8.3.1

- The source of electrons will provide a flow of electrons whose energy and hence the wavelength can be varied.
- The beam path, the placement of the lenses, the specimen, the aperture etc. follow the plan of the light microscope. The entire arrangement is placed in high vacuum to avoid extraneous scattering and absorption of the electrons by air.

- The lenses are magnetic lenses and the magnified image is projected on a fluorescent screen. The extra aperture called the objective aperture is placed in electron microscope which not there in optical microscope.
- In an optical microscope light is transmitted or absorbed by the specimen. This creates a contrast between different parts of the image. Here incident beam is passed through the specimen with very little absorption.

8.3.2 SEM (Scanning Electron Microscope)

MU - May 2013, Dec. 2013, May 2014, Dec. 2014

- Unlike TEM mentioned above, SEM (Scanning Electron Microscope) is used to obtain images of surface of thickness. Also a thin specimen can be studied.

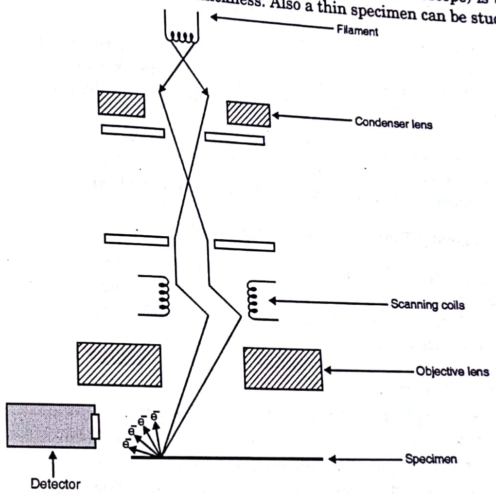


Fig. 8.3.2 : Schematic diagram of SEM

- Construction of TEM includes an arrangement that makes it possible for an electron beam to scan the specimen similar to that we have in TV picture tube.

- Construction of SEM is shown in Fig. 8.3.2. Here electron beam is obtained from electron gun and it is made to pass through condenser lens. Next stage is of scanning coil which is used to focus the electron beam on a small spot on specimen surface and also to scan the surface like electron beam scans in TV picture tube.
- Electrons, on reaching the surface of specimen, will be scattered. The scattered beam will form the image signals. Detector is kept on the same side where other components of SEM are kept.
- Image formation in SEM is due to two main combining aspects.
  (a) Scattering of electron beam is because of atoms on the surface of the specimen and these atoms have different scattering power.
  (b) Topographical variations of the surface.

  Actually, the aspects mentioned above are also responsible for the contrast which is essential for image formation.
- During the scanning of atoms by electron beam, the scattered electrons intensities are measured by detector and then displayed on the screen. If the scattering is high at particular point during the scanning, the corresponding point on the viewing screen will be bright and for low scattering, the corresponding point on the screen will be dark. This develops required contrast for a clear image of the specimen.
- Specimen as small as 50 A° size may be resolved by SEM.

8.3.3 Scanning Tunnelling Microscope (STM)

- This is one of the most advanced microscopes working on the concept of quantum mechanics, discussed in chapter 6. i.e. **Tunnelling of electron** which describes a non zero probability to find an electron on the other side of potential barrier even if electron does not possess the required energy to overcome the potential barrier. This is possible only if a tunnel is created through the barrier.
- STM is non optical microscope that scans the surface through an electrical probe to detect a weak current flowing between tip and the surface.
- Construction of tunnelling electron microscope has a very sharp needle (size of the tip equivalent to an atom) which is brought at few Angstroms distance to the surface to be imaged.
- At this very small distance one can think of a potential barrier between surface and the needle and the surface electrons forming a tunnel across the barrier. The tunnelling of electrons form a tunnelling current in the needle.

- The magnitude of current though it is very small of the order of few nano-amperes, is used to determine the distance between tip of the needle and the surface (or precisely the atom on the surface).
- Since the electron beam can be focussed at a very small area i.e. at an atom, the scanning across the surface by adjusting the spacing between tip of the needle and the surface in order to maintain the tunnelling current at constant value provides complete topographical information. This can be converted into a visual image.

Limitations :

Use of STM is restricted as the required conductivity for the specimen is a big hurdle. Hence insulators can not be scanned through it.

8.3.4 Atomic Force Microscope (AFM)

MU - May 2015. May 2016. Dec. 2016

- Atomic Force Microscope is high resolution type of Scanning probe microscope with resolution of 1 A.U. Because of this it is one of the foremost tool in the field of nano science.
- Atomic Force Microscope (AFM) is a modified TEM to overcome limitations of TEM. The needle mentioned in TEM which works as the probe is kept in touch with sample using a micro scale cantilever.
- When the tip is brought in touch with the sample surface, forces between the tip and the sample lead to the deflection of the cantilever.
- The force present in the tip is kept constant and the scanning is done. As the scanning continues, the tip will have vertical movements depending upon topography of the sample.
- The tip has a mirror on top of it, a laser beam is used to have the record of vertical movement of the needle. Interferometer is also used for best of accuracy.
- The information is later converted to a visible one.
- It overcomes the difficulty of TEM i.e. the problem associated with non conducting material as AFM does not generate any current.
- Depending on the situation forces that are measured in AFM include mechanical contact force, Vander Walls force, capillary forces, electrostatic and magnetic forces.

Limitations :

Since the tip of the needle is in direct touch with the sample surface, it is likely to distort the biological samples.

8.3.5 Differences between SEM and AFM

Sr. No.	SEM	AFM
1.	Provides a two dimensional projection.	Provides 3D profile of the specimen.
2.	Requires a special treatment to the sample which has the potential to either damage the sample or change some of the property.	Samples viewed by AFM does not require special treatments like coating etc.
3.	Requires a vacuum environment for its operation.	Can work very well in air.
4.	Living cells can not be studied due to requirement of vacuum.	Living cells can be studied.
5.	The quality of the resolution is better.	The quality of the resolution is restricted by radius of curvature of the tip of the needle.

Syllabus Topic : Method to Synthesize Nanomaterials

➤ **Topics covered :** Synthesize of Nano Materials

8.4 Synthesize of Nano Materials

MU - May 2013

Various techniques are adopted for the synthesis of nano materials that too in various forms like nano particles, nano powder, nano crystals, nano films, nano wires, nano tubes, nano dots. These methods includes:

1. Ball Milling
2. Sputtering
3. Vapor deposition
4. Sol gel technique.
5. Electro deposition.
6. Mechanical crushing or Ball milling
7. LASER synthesis
8. Inert gas condensation

We will take few of them for study.

Syllabus Topic : Ball Milling

➤ **Topics covered :** Ball Milling

8.4.1 Ball Milling

MU - May 2013, May 2014, Dec. 2016

- It is a process where a powder mixture placed in the ball mill is subjected to high energy collision from the balls.

- Planetary ball mill is most frequently used system for mechanical alloying since only a very small amount of powder is required. In simple language, a ball mill consists of a hollow cylindrical shell rotating about its axis.
- The axis of the shell may be either horizontally or at a small angle to the horizontal. It is partially filled with balls which makes grinding media and made up of steel, stainless steel or ceramic.
- The inner surface of the shell is made up of an abrasion resistant material. When continuously operated, the shell rotates and lifts the balls up and drops them from near the top of the shell which causes the grinding of the particles inside.

Syllabus Topic : Sputtering

➤ **Topics covered :** Sputtering

8.4.2 Sputtering

MU - May 2014, Dec. 2016

- Sputtering is a process whereby particles are ejected from a solid target material due to bombarding of target by energetic particles. It is necessary to have kinetic energy of incoming particles much greater than conventional thermal energies.
- In this technology, the substrate is placed in a vacuum chamber with source material, named target, and an inert gas(such as Argon) is introduced at low pressure. A gas plasma is struck using an RF power source, causing the gas to become ionized.
- The ions are accelerated towards the surface of the target, causing atoms of the source material to break from the target in vapour form and condense on all surfaces.

Syllabus Topic : Vapour Deposition

➤ **Topics covered :** Vapour Deposition

8.4.3 Vapour Deposition

- This method is used to prepare nano powder.
- In this technique initially the material is heated to form a gas and is allowed to deposit on a solid surface under vacuum condition which forms nano powders on the surface of the solid.

Syllabus Topic : Sol Gel

> **Topics covered :** SOL - Gel Technique, Electro – Deposition

8.4.4 SOL - Gel Technique

- Nano particles and nano powder is obtained using this technique.
- In general, sol - gel technique is based on the hydrolysis of liquid precursors and formation of colloidal solutions.
- Out of few more processes, hydro - dynamic cavitation is often used, in which nanoparticles can be generated through creation and release of gas bubbles inside the sol - get solution.
- Here, the sol - get solution is taken in a drying chamber and thoroughly mixed by applying enormous pressure, high temperature and further exposing it to cavitational disturbances.
- This process creates hydrodynamic bubbles in the sol - gel. These bubbles will undergo nucleation, growth and then it quenches to form nano particles.

8.4.5 Electro – Deposition

- This technique is developed generally in electroplating and in the nano films.
- By using conventional way of immersing two electrodes in electrolyte i.e. an aqueous solution of salts. When current is passed through the electrodes, a certain mass of substance is liberated from one electrode and is deposited on the other in form of a thin film. The thickness of the film is controlled by current passing through and the time for which it is allowed to pass.

Syllabus Topic : Properties and Applications of Nanomaterials

> **Topics covered :** Properties and Applications of Nano Particles, Applications in Mechanical Engineering, Applications in Electrical, Electronic and Communication Engineering, Applications in Computer Science Engineering and IT, Applications in Bio - Medical and Chemical Energy, Risks of Nanotechnology.

8.5 Properties and Applications of Nano Particles

8.5.1 Properties of Nano Particles

Nano materials are materials possessing grain size of the order of a billionth of the meter. Few of the properties are

1. Because of very small size, nano particles cannot be further divided into smaller ones and it cannot have any dislocations. Therefore we can describe them with (a) hard and (b) wear resistant
2. They are ductile at high temperature.

3. Exhibit very low wear and tare
4. Active for chemical reactions.
5. Less corrosion.

Though nano particles are very small, they are the important materials to built the future world. They have applications almost in all engineering field.

8.5.2 Applications in Mechanical Engineering

(1) Since they are stronger, lighter etc., they are used to make hard metals.
(2) Smart magnetic fluids are used in vacuum seals, magnetic separators etc.
(3) They are also used in Giant Magneto Resistance (GMR) spin valves.
(4) Nano MEMS (Micro Electro Mechanical Systems) are used in optimal switches, pressure sensors, mass sensors.

8.5.3 Applications in Electrical, Electronic and Communication Engineering

(1) Orderly assembled nano materials are used as quantum electronic devices and used in photonic crystals.
(2) Some of the nano materials are used as sensing elements. Especially the molecular nano materials are used to design robots, assembler etc.
(3) They are used in energy storage device such as hydrogen storage device in ionic batteries.
(4) In magnetic recording devices.

8.5.4 Applications in Computer Science Engineering and IT

(1) To make CDs, and semiconductor LASER
(2) To make smaller chips few information storage.
(3) In mobile phones, Lap - tops etc.
(4) Nano dimensional photonic crystals and quantum electronic crystals and quantum electronic devices play a vital role in recently developed computers.

8.5.5 Applications in Bio - Medical and Chemical Energy

(1) Consolidated state nano particles are used as catalysts, electrodes in solar and fuel cells.
(2) Bio- sensitive nano particles are used in the production of DNA chips, bio-sensors etc.
(3) Nano - structured ceramic materials are used in synthetic bones.
(4) Few nano materials are used in absorbents, self cleaning glass fuel additives, drugs, ferro fluids etc.
(5) Nano metallic colloids are used as film precursors.

8.6 Risks of Nanotechnology

- Because of the far ranging claims that have been made about potential applications of nanotechnology, a number of serious concerns have been raised about what effects these will have on our society of realized, and what action if any is appropriate to mitigate these risks.
- There are possible dangers that arise with the development of nanotechnology. The Center for Responsible Nanotechnology suggests that new development could result, among other things, in untraceable weapons of mass destruction, networked cameras for use by the government, and weapons developments fast enough to destabilize arms races.

Health and environmental concerns

- Some of the recently developed nanoparticle products may have unintended consequences. Researchers have discovered that silver nanoparticles used in socks only to reduce foot odor are being released in the wash possible negative consequences.
- Silver nanoparticles, which are bacteriostic, may then destroy beneficial bacteria which are important for breaking down organic matter in waste treatment plants or farms.
- A study at the University of Rochester found that when rats breathed in nanoparticles, the particles settled in the brain and lungs, which led to significant increases in biomarkers for inflammation and stress response. A study in china indicated that nanoparticles induce skin aging through oxidative stress in hairless mice.
- A two - year study at UCLA's school of Public Health found lab mice consuming nano - titanium dioxide showed DNA and chromosome damage to a degree" linked to all the big killers of man, namely cancer, heart disease, neurological disease and aging."
- A major study published more recently in Nature Nanotechnology suggests some forms of carbon nanotubes a poster child for the "nanotechnology revolution"- could be as harmful as asbestos if inhaled in sufficient quantities.
- Anthony Seaton of the Institute of Occupational Medicine in Edinburgh, Scotland, who contributed to the article on carbon nanotubes said " We know that some of them probably have the potential to cause mesothelioma. So those sorts of materials need to be handled very carefully."
- In the absence of specific nano - regulation forthcoming from governments, Paull and Lyons (2008) have called for an exclusion of engineered nanoparticles from organic food. A newspaper article reposts that workers in a paint factory developed serious lung disease and nanoparticles were found in their lungs.

A Quick Revision

- There are two different approaches to obtain nano particlesnamely Bottom up approach, Top down approach
- Electron microscope makes use of electron – a negatively charged particle, in terms of its wave like properties
- According to quantum physics, the wavelength associated with electron varies inversely as its energy.

$$\lambda = \frac{h}{\sqrt{2\,mE}}$$

- SEM (Scanning Electron Microscope) is used to obtain images of surface of thickness. Also a thin specimen can be studied.
- Image formation in SEM is due to two main combining aspects.

  Scattering of electron beam is because of atoms on the surface of the specimen and these atoms have different scattering power and topographical variations of the surface.
- STM is non optical microscope that scans the surface through an electrical probe to detect a weak current flowing between tip and the surface
- Atomic Force Microscope is high resolution type of Scanning probe microscope with resolution of 1 A.U. Because of this it is one of the foremost tools in the field of nano science.
- Nano particles can also be produced by generating plasma using radio frequency heating coils."
- Sol - gel technique is based on the hydrolysis of liquid precursors and formation of colloidal solutions.

Review Questions

Q. 1 What is the need of Nano technology ?

Q. 2 What are various approaches adopted to synthesis nano particles ?

Q. 3 Draw the schematic diagram to explain construction and function of SEM.

Q. 4 Explain the principle and function of STM. What is the significance of word tunneling over here?

Q. 5 Explain the principle and function of AFM.

Q. 6 Explain the difference between SEM and AFM.

Q. 7 What are the various methods available to manufacture nano materials ? Explain one of them in detail.

8.7 University Questions

May 2013

Q. 1 What are different techniques to synthesis nanomaterial? Explain one of them in detail.
(Ans. : Refer sections 8.4 and 8.4.1) (5 Marks)

Q. 2 Draw the schematic diagram of SEM and explain its construction and working.
(Ans. : Refer section 8.3.2) (5 Marks)

Dec. 2013

Q. 1 Explain the working of SEM with a neat diagram.
(Ans. : Refer section 8.3.2) (5 Marks)

May 2014

Q. 1 With neat diagram explain construction and working of Scanning Electron Microscope.
(Ans. : Refer section 8.3.2) (5 Marks)

Q. 2 Explain the physical methods for synthesis of Nanoparticles.
(Ans. : Refer sections 8.4.1 and 8.4.2) (5 Marks)

Dec. 2014

Q. 1 Explain with neat diagram principle and working of SEM. (5 Marks)
(Ans. : Refer section 8.3.2)

May 2015

Q. 1 Explain construction and working of atomic force microscope.
(Ans. : Refer section 8.3.4) (5 Marks)

Q. 2 Explain top down and bottom up approaches to prepare nanomaterials.
(Ans. : Refer section 8.2) (5 Marks)

Dec. 2015

Q. 1 Explain the top down approach and. bottom up approach to prepare nanomaterials.
(Ans. : Refer section 8.2) (5 Marks)

May 2016

Q. 1 With neat diagram, explain construction and working of atomic force microscope.
(Ans. : Refer section 8.3.4) (5 Marks)

Dec. 2016

Q. 1 With neat diagram the explain construction and working of Atomic Force Microscope.
(Ans. : Refer section 8.3.4) (5 Marks)

Q. 2 What are nano materials ? Explain any two methods for synthesis of nano particles.
(Ans. : Refer sections 8.1, 8.4.1 and 8.4.2) (5 Marks)

Note : **e-book**

- *Exact syllabus topic wise answers to the following questions are available in e book. **Please download as per instructions given.***

Syllabus Topic : Introduction to Nano-science and Nanotechnology

Q. 1 What are nano materials ? *(Ans. : Refer section 8.1)* **(Dec. 2016)**

Syllabus Topic : Surface to Volume Ratio, Two Main Approaches in Nanotechnology -Bottom Up Technique and Top Down Technique

Q. 1 Explain the top down approach and bottom up approach to prepare nanomaterials.
(Ans. : Refer section 8.2.2) **(May 2015, Dec. 2015)**

Syllabus Topic : Important Tools in Nanotechnology such as Scanning Electron Microscope, Transmission Electron Microscope, Atomic Force Microscope.

Q. 1 With neat diagram, explain construction and working of transmission electron microscope.*(Ans. : Refer section 8.3.1)*

Q. 2 Draw the schematic diagram of SEM and explain its construction and working.
(Ans. : Refer section 8.3.2) **(May 2013, Dec. 2013, May 2014, Dec. 2014)**

Q. 3 With neat diagram, explain construction and working of atomic force microscope.*(Ans. : Refer section 8.3.4)* **(May 2015, May 2016, Dec. 2016)**

Q. 4 Explain the difference between SEM and AFM. *(Ans. : Refer section 8.3.5)*

Q. 5 What are different techniques to synthesis nanomaterial?
(Ans. : Refer section 8.4) **(May 2013)**

Syllabus Topic : Ball Milling

Q. 1 Explain Ball Milling technique in detail for synthesis of nano particles.
(Ans. : Refer section 8.4.1) **(May 2013, Dec. 2016)**

OR

Explain the Physical Methods for synthesis of Nanoparticles.
(Ans. : Refer section 8.4.1) **(May 2014)**

Syllabus Topic : Sputtering

Q. 1 Explain Sputtering technique in detail for synthesis of nano particles.
(Ans. : Refer section 8.4.2) **(Dec. 2016)**

OR

Explain the Physical Methods for synthesis of Nanoparticles.
(Ans. : Refer section 8.4.2) **(May 2014)**