DIVISION / ROLL NO.: 47 -D1AD

Vivekanand Education Society's Institute of Technology

(Academic Year 2020-2021)

Subject: Engineering Mathematics- I

Semester: I

TUTORIAL/~~SCILAB~~ COVER PAGE

TUTORIAL /~~SCILAB~~ NO :- 1

TUTORIAL TOPIC:- Complex Numbers

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SIGNATURE OF TEACHER :- ____________________

Tutorial 1 (DIAD) : EM-1 : Module 1: Complex Numbers.

1) Prove that $\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)^{10}+\left[\frac{1}{\sqrt{2}}-i\left(\frac{1}{\sqrt{2}}\right)\right]^{10}=0.$

2) Find all the values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ and show that the continued product of all the values of 1.

3) Solve the following equations with the help of D.M.T
$x^7+x^4+i(x^3+1)=0.$

4) If $\sin^5\theta = a\sin 5\theta + b\sin 3\theta + c\sin\theta$, then find the values of a, b, c.

5) Expand $\cos 6\theta$ and $\sin 6\theta$ in terms of $\cos\theta$ and $\sin\theta$.

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1)

→ $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\therefore \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{10}$$

$$= \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{10} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{10}$$

$$= \cos\frac{10\times\pi}{4} + \cancel{i\sin\frac{10\pi}{4}} + \cos\frac{10\pi}{4} - \cancel{i\sin\frac{10\times\pi}{4}}$$

$$= 2\cos\frac{5\pi}{2}$$

$$= 2\cos\left(2\pi + \frac{\pi}{2}\right)$$

$$= 0.$$

$$\boxed{\therefore \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{10} = 0.}$$

2)

→ $$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4} = \left[\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3}\right]^{1/4}$$

$$= \left(\cos\frac{\cancel{3}\times\pi}{\cancel{3}} + i\sin\frac{\cancel{3}\times\pi}{\cancel{3}}\right)^{1/4}$$

$$= \left(\cos\pi + i\sin\pi\right)^{1/4}$$

$$\therefore \left(\cos\pi + i\sin\pi\right)^{1/4} = \left[\cos(2k\pi + \pi) + i\sin(2k\pi + \pi)\right]^{1/4}$$

$$= \cos\left(\frac{2k\pi + \pi}{4}\right) + i\sin\left(\frac{2k\pi + \pi}{4}\right) \quad \text{— By DMT}$$

now, $k = 0, 1, 2, 3.$

for $k=0$,

$$x_0 = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = e^{i\pi/4}$$

$$\boxed{x_0 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}}$$

for $k=1$,

$$x_1 = \cos\left(\frac{2\pi+\pi}{4}\right) + i\sin\left(\frac{2\pi+\pi}{4}\right) = e^{i3\pi/4}$$

$$\boxed{x_1 = \frac{-1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}}$$

for $k=2$,

$$x_2 = \cos\left(\frac{4\pi+\pi}{4}\right) + i\sin\left(\frac{4\pi+\pi}{4}\right) = e^{i5\pi/4}$$

$$\boxed{x_2 = \frac{-1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}}$$

for $k=3$,

$$x_3 = \cos\left(\frac{6\pi+\pi}{4}\right) + i\sin\left(\frac{6\pi+\pi}{4}\right) = e^{i7\pi/4}$$

$$\boxed{x_3 = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}}$$

$\therefore$ Product of x_1, x_2, x_3 and x_0 $= e^{i\pi/4} \times e^{i3\pi/4} \times e^{i5\pi/4} \times e^{i7\pi/4}$.

$$= e^{i(16\pi/4)}$$

$$= e^{i4\pi}$$

$$= \cos 4\pi + i\sin 4\pi.$$

$$\boxed{\therefore x_0 \times x_1 \times x_2 \times x_3 = 1}$$

$\boxed{\therefore \text{The continuos product of all values is 1.}}$

3)

$$x^7 + x^4 + i(x^3+1) = 0.$$
$$x^4(x^3+1) + i(x^3+1) = 0.$$
$$\therefore (x^4+i)(x^3+1) = 0.$$

$\therefore \quad x^4 = -i \quad$ and $\quad x^3 = -1.$

$x = (-i)^{1/4} \quad$ and $\quad x = (-1)^{1/3}$

$x = \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)^{1/4}$ ① and $x = (\cos\pi + i\sin\pi)^{1/3}$ ——— ②

$x = \cos\left(\frac{2k\pi + 3\pi/2}{4}\right) + i\sin\left(\frac{2k\pi + 3\pi/2}{4}\right)$ ——— from ① and D.M.T.

$\therefore$ for $k = 0, 1, 2, 3.$

$\underset{(k=0)}{x_0} = \cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}, \quad \underset{(k=1)}{x_1} = \cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}.$

$\underset{(k=2)}{x_2} = \cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}, \quad \underset{(k=3)}{x_3} = \cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}$

Now,

$x = \cos\left(\frac{2k\pi + \pi}{3}\right) + i\sin\left(\frac{2k\pi + \pi}{3}\right)$ ——— from ② and D.M.T.

for $k = 0, 1, 2.$

$\underset{(k=0)}{x_0} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}, \quad \underset{(k=1)}{x_1} = \cos\pi + i\sin\pi, \quad \underset{(k=2)}{x_2} = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}.$

$\therefore$ The solutions of the given equation are

$\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right), \left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right), \left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right), \left(\cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}\right),$

$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right), (\cos\pi + i\sin\pi)$ and $\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right).$

4)

→ $z = (\cos\theta + i\sin\theta)$, $1/z = \cos\theta - i\sin\theta$.

$\therefore z + \frac{1}{z} = 2\cos\theta$, $z - \frac{1}{z} = 2i\sin\theta$

$\therefore (z^n + 1/z^n) = 2\cos n\theta$, $(z^n - 1/z^n) = 2i\sin n\theta$.

$\therefore (2i\sin\theta)^5 = \left(z - \frac{1}{z}\right)^5$

$$2^5 i \sin^5\theta = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$$

$$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$$

$$2^4 \times 2i\,(\sin^5\theta) = 2i\sin 5\theta - 5\,2i\sin 3\theta + 10\,2i\sin\theta$$

$$\sin^5\theta = \frac{1}{2^4}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$$

$\therefore \sin^5\theta = \frac{1}{2^4}\sin 5\theta - \frac{5}{2^4}\sin 3\theta + \frac{10}{2^4}\sin\theta$.

$\therefore$ Comparing with $a\sin 5\theta + b\sin 3\theta + c\sin\theta$.

we get,

$\therefore a = \frac{1}{2^4} = \frac{1}{16}$, $\therefore b = \frac{-5}{2^4} = \frac{-5}{16}$, $c = \frac{10}{2^4} = \frac{5}{8}$.

$\boxed{\therefore \text{The values of } a, b \text{ and } c \text{ are } \frac{1}{16}, \frac{-5}{16} \text{ and } \frac{5}{8}, \text{ respectively.}}$

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5)

$\rightarrow$ $z = (\cos\theta + i\sin\theta)$

$\therefore z^6 = (\cos\theta + i\sin\theta)^6.$

$\therefore (\cos\theta + i\sin\theta)^6 = \cos^6\theta + i6\cos^5\theta\sin\theta$
$- 15\cos^4\theta\sin^2\theta - i\,20\cos^3\theta\sin^3\theta$
$+ 15\cos^2\theta\sin^4\theta + i6\cos\theta\sin^5\theta$
$- \sin^6\theta.$

$\therefore \cos 6\theta + i\sin 6\theta = (\cos^6\theta + 15\cos^2\theta\sin^4\theta - 15\cos^4\theta\sin^2\theta - \sin^6\theta)$
$+ i(6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta$
$+ 6\cos\theta\sin^5\theta)$

$\boxed{\therefore \cos 6\theta = \cos^6\theta + 15\cos^2\theta\sin^4\theta - 15\cos^4\theta\sin^2\theta - \sin^6\theta}$

$\boxed{\therefore \sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta.}$

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