4. Interlude: Square Roots of Matrices

Exercises due Jul 27, 2021 19:59 EDT

Interlude: Square root of a positive semi-definite matrix

Recall that a matrix $\mathbf{A}$ of size $d \times d$ is **positive semi-definite** if $\mathbf{x}^T\mathbf{A}\mathbf{x} \geq 0$ for all $\mathbf{x} \in \mathbb{R}^d$. Two example classes of positive semi-definite matrices are:

- Diagonal matrices with non-negative entries: $\mathbf{D} = \begin{pmatrix} c_1 & 0 & \dots & 0 \\ 0 & c_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \dots & c_d \end{pmatrix}$ where $c_i \geq 0$ for all i. (You have shown in exercise in a previous lecture that indeed $\mathbf{x}^T\mathbf{D}\mathbf{x} \geq 0$ for all $\mathbf{x}$.
- Matrix products $\mathbf{P}^T\mathbf{D}\mathbf{P}$ where $\mathbf{P}$ is an invertible (square) matrix and $\mathbf{D}$ is a diagonal matrix with non-negative entries (as above). **Proof:** $\mathbf{x}^T\left(\mathbf{P}^T\mathbf{D}\mathbf{P}\right)\mathbf{x} = (\mathbf{P}\mathbf{x})^T\mathbf{D}\left(\mathbf{P}\mathbf{x}\right) = \mathbf{y}^T\mathbf{D}\mathbf{y} \geq 0$ for all vectors $\mathbf{x}$.

The **positive semi-definite square root** (or simply the square root) of a positive semi-definite matrix $\mathbf{A}$ is another positive semi-definite matrix, denoted by $\mathbf{A}^{1/2}$, satisfying $\mathbf{A}^{1/2}\mathbf{A}^{1/2} = \mathbf{A}.$ It is the case that for any positive semi-definite matrix (positive definite matrix, respectively), the positive semi-definite square root (positive definite square root, respectively) is unique.

Square Root of a Matrix

1 point possible (graded)

Using the definition above of the square root of a matrix, find the square root $\mathbf{D}^{1/2}$ of $\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$.

(Enter your answer as a matrix, e.g. by typing **[[1,2],[5,1]]** for the matrix $\begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}$. Note the square brackets, and the commas as separators.)

$\mathbf{D}^{1/2} =$

STANDARD NOTATION

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(Optional): Square Root of a Matrix

0 points possible (ungraded)

Let

$$\mathbf{A} = \mathbf{P}^T\mathbf{D}\mathbf{P} \qquad \text{where} \quad \mathbf{D} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{P} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Note that $\mathbf{P}^T = \mathbf{P}^{-1}$.

Find the square root $\mathbf{A}^{1/2}$ of the matrix $\mathbf{A}$.

Hint: $\mathbf{P}^T\mathbf{B}^2\mathbf{P} = \mathbf{P}^T\mathbf{B}\left(\mathbf{P}\mathbf{P}^T\right)\mathbf{B}\mathbf{P}$.

(Enter your answer as a matrix, e.g. by typing **[[1,2],[5,-1]]** for the matrix $\begin{pmatrix} 1 & 2 \\ 5 & -1 \end{pmatrix}$. Note the square brackets, and commas as separators.)

$\mathbf{A}^{1/2} =$

STANDARD NOTATION

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