

Moment Generating Function of Gaussian Distribution

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Theorem

Let $X \sim N(\mu, \sigma^2)$ for some $\mu \in \mathbb{R}, \sigma \in \mathbb{R}_{>0}$, where N is the Gaussian distribution.

Then the moment generating function M_X of X is given by:

$$M_X(t) = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$$

Proof

From the definition of the Gaussian distribution, X has probability density function:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

From the definition of a moment generating function:

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(x) dx$$

So:

$$\begin{aligned} M_X(t) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(tx - \frac{(x-\mu)^2}{2\sigma^2}\right) dx \\ &= \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp((\sqrt{2}\sigma u + \mu)t - u^2) du && \text{substituting } u = \frac{x-\mu}{\sqrt{2}\sigma} \\ &= \frac{\exp \mu t}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-(u^2 - \sqrt{2}\sigma u t)) du \\ &= \frac{\exp \mu t}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-\left(u - \frac{\sqrt{2}}{2}\sigma t\right)^2 + \frac{1}{2}\sigma^2 t^2\right) du \\ &= \frac{\exp(\mu t + \frac{1}{2}\sigma^2 t^2)}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-v^2) dv && \text{substituting } v = u - \frac{\sqrt{2}}{2}\sigma t \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt{\pi} \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)}{\sqrt{\pi}} \\
 &= \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)
 \end{aligned}$$

Gaussian Integral



Examples

First Moment

The first moment generating function of X is given by:

$$M_X'(t) = (\mu + \sigma^2 t) \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)$$

Second Moment

The second moment generating function of X is given by:

$$M_X''(t) = \left(\sigma^2 + (\mu + \sigma^2 t)^2\right) \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)$$

Third Moment

The third moment generating function of X is given by:

$$M_X'''(t) = \left(3\sigma^2(\mu + \sigma^2 t) + (\mu + \sigma^2 t)^3\right) \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)$$

Fourth Moment

The fourth moment generating function of X is given by:

$$M_X^{(4)}(t) = \left(3\sigma^4 + 6\sigma^2(\mu + \sigma^2 t)^2 + (\mu + \sigma^2 t)^4\right) \exp\left(\mu t + \frac{1}{2} \sigma^2 t^2\right)$$



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