Projection Matrix

A projection matrix P is an $n\times n$ square matrix that gives a vector space projection from $\mathbb{R}^n$ to a subspace W. The columns of P are the projections of the standard basis vectors, and W is the image of P. A square matrix P is a projection matrix iff $\mathsf{P}^2=\mathsf{P}$.

A projection matrix P is orthogonal iff

$$\mathsf{P}=\mathsf{P}^*, \quad (1)$$

where P^* denotes the adjoint matrix of P. A projection matrix is a symmetric matrix iff the vector space projection is orthogonal. In an orthogonal projection, any vector v can be written $v=v_W+v_{W^\perp}$, so

$$\langle v, \mathsf{P}\,w\rangle=\langle v_W, \mathsf{P}\,w\rangle=\langle \mathsf{P}\,v, w\rangle. \quad (2)$$

An example of a nonsymmetric projection matrix is

$$\mathsf{P}=\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad (3)$$

which projects onto the line $y=x$.

The case of a complex vector space is analogous. A projection matrix is a Hermitian matrix iff the vector space projection satisfies

$$\langle v, \mathsf{P}\,w\rangle=\langle v_W, \mathsf{P}\,w\rangle=\langle \mathsf{P}\,v, w\rangle, \quad (4)$$

where the inner product is the Hermitian inner product. Projection operators play a role in quantum mechanics and quantum computing.

Any vector in W is fixed by the projection matrix $\mathsf{P}\,w=w$ for any w in W. Consequently, a projection matrix P has norm equal to one, unless $\mathsf{P}=0$,

$$||\mathsf{P}||=\sup_{|x|=1}|\mathsf{P}\,x|\geq 1. \quad (5)$$

Let A be a C^*-algebra. An element $p\in A$ is called projection if $p^*=p$ and $p^2=p$. For example, the real function f defined by $f(x)=0$ on G_1 and $f(x)=1$ on G_2 is a projection in the C^*-algebra $C(X)$, where X is assumed to be disconnected with two components G_1 and G_2.

SEE ALSO:
Idempotent, Inner Product, Map Projection, Orthogonal Set, Projection, Projection Operator, Pseudoinverse, Symmetric Matrix, Vector Space Projection, Vertical Perspective Projection

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