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Created, developed, and nurtured by Eric Weisstein at Wolfram Research

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Projection Matrix



A projection matrix P is an $n \times n$ square matrix that gives a vector space projection from \mathbb{R}^n to a subspace W. The columns of P are the projections of the standard basis vectors, and W is the image of P. A square matrix P is a projection matrix iff $P^2 = P$.

A projection matrix P is orthogonal iff

$$P = P^*, (1)$$

where P* denotes the adjoint matrix of P. A projection matrix is a symmetric matrix iff the vector space projection is orthogonal. In an orthogonal projection, any vector v can be written $v = v_W + v_{W^+}$, so

$$\langle v, P w \rangle = \langle v_W, P w \rangle = \langle P v, w \rangle.$$
 (2)

An example of a nonsymmetric projection matrix is

$$\mathsf{P} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},\tag{3}$$

which projects onto the line v = x.

The case of a complex vector space is analogous. A projection matrix is a Hermitian matrix iff the vector space projection satisfies

$$\langle v, P w \rangle = \langle v_W, P w \rangle = \langle P v, w \rangle,$$
 (4)

where the inner product is the Hermitian inner product. Projection operators play a role in quantum mechanics and quantum computing.

Any vector in W is fixed by the projection matrix $P_{w} = w$ for any w in W. Consequently, a projection matrix P has norm equal to one, unless P = 0,

$$\|P\| = \sup_{|x|=1} |Px| \ge 1. \tag{5}$$

Let A be a C^* -algebra. An element $p \in A$ is called projection if $p^* = p$ and $p^2 = p$. For example, the real function fdefined by f(x) = 0 on G_1 and f(x) = 1 on G_2 is a projection in the C^* -algebra C(X), where X is assumed to be disconnected with two components G_1 and G_2 .

SEE ALSO:

Idempotent, Inner Product, Map Projection, Orthogonal Set, Projection, Projection Operator, Pseudoinverse, Symmetric Matrix, Vector Space Projection, Vertical Perspective Projection

.999 with 123 repeating



THINGS TO TRY:

- = .999 with 123 repeating
- = GF(8)
- = inverse of quaternion 1+0i+0j+2k ** (-1i+3+4j+3k)







Linear Transformations and **Basic Computer Graphics**



Projecting Images from a Point Cloud Portions of this entry contributed by Mohammad Sal Moslehian

Portions of this entry contributed by Todd Rowland

REFERENCES:

Kadison, R. V. and Ringrose, J. R. Fundamentals of the Theory of Operator Algebras, Vol. 1: Elementary Theory. Providence, RI: Amer. Math. Soc., 1997.

Murphy, G. J. C-*-Algebras and Operator Theory. New York: Academic Press, 1990.

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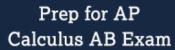
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