2. Two important probability tools

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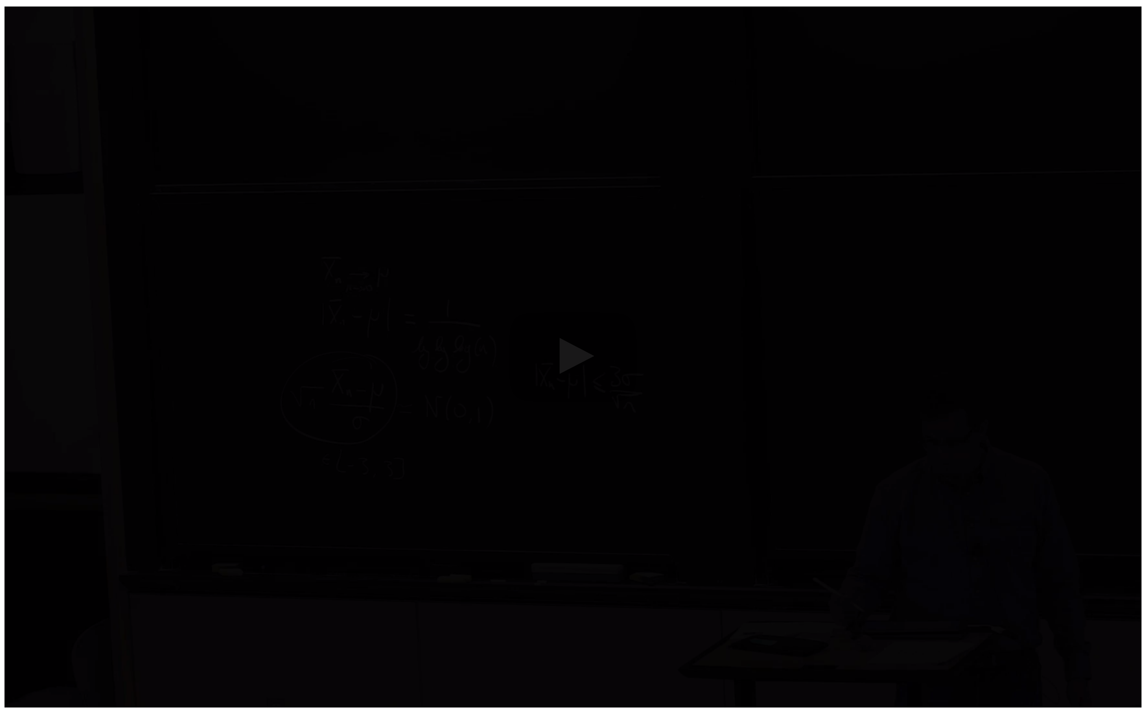
Exercises due May 25, 2021 19:59 EDT

Two important probability tools



8:53 / 8:53 1.50x

And so, what is n close to infinity?

Well, for this class, the rule of thumb

will be n larger than or equal to 30.

OK?

So of course, those numbers depend

on how sure you want to be about your results.

If this was a life and death situation,

you might want to go to n larger than 50, but n larger

than 30 will be fine enough for the purpose of this class.

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Averages of random variables: Laws of Large Numbers and Central Limit Theorem

Let $X, X_1, X_2, \ldots, X_n$ be i.i.d. random variables, with $\mu = \mathbb{E}[X]$ and $\sigma^2 = \text{Var}[X]$.

- Laws (weak and strong) of large numbers (LLN):

$$\bar{X}_n := \frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow[n\to\infty]{\mathbf{P},\text{ a.s.}} \mu$$

where the convergence is in probability (as denoted by $\mathbf{P}$ on the convergence arrow) and almost surely (as denoted by a.s. on the arrow) for the weak and strong laws respectively.

- Central limit theorem (CLT):

$$\sqrt{n}\,\frac{\bar{X}_n - \mu}{\sigma} \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$$

$$\text{or equivalently,} \quad \sqrt{n}\left(\bar{X}_n - \mu\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0, \sigma^2)$$

where the convergence is in distribution, as denoted by (d) on top of the convergence arrow.

We will revisit the different modes of convergence near the end of this lecture.

Note : In *6.431x: Probability–the Science of Uncertainty and Data*, we used yet another equivalent formulation of the CLT:

$$\frac{S_n - n\mu}{\sqrt{n}\sigma} \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0, 1)$$

where $S_n = \sum_{i=1}^{n} X_i$ is the sum (not the average) of X_i.

Average of Gaussians

3 points possible (graded)

Let $X_1, X_2, \ldots, X_n$ be i.i.d. **standard normal random variables**. For a finite n, what is the distribution of

$$\overline{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n}?$$

- A Gaussian.
- A χ^2-distribution.
- Cannot be determined for finite n, but asymptotically Gaussian.

In terms of n, what are the variance and mean of $\overline{X}_n$?

Var $(\overline{X_n}) =$

$\mathbb{E}[\overline{X_n}] =$

Submit You have used 0 of 3 attempts

CLT Concept Check

1 point possible (graded)

Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. random variables with $\mathbb{E}[X] = \mu$, and Var $(X) = \sigma^2$. Assuming that n is very large, according to the Central Limit Theorem, what is the best approximate characterization of the distribution of $\overline{X}_n$?

- $N(0, 1)$.
- $N(\mu, \sigma^2/n)$.
- $N(0, \sigma^2/n)$

$\mathcal{N}(0, \sigma^2/n)$.

Depends on the distribution of X.

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Not allowed to use sigma or X
In the section Gaussian Average, I can not use sigma and X in the answer. Can anyone give me some hints on how to tackle the ques... 5

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