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## 7. Concluding Remarks on MLE for GLMs

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## Optimization for General Link Functions and Prediction using Estimated Beta

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Just to conclude, as part of those concluding remarks, if I

talk about--

so this is actually--

let's actually switch the second point.

I'll come back to the first.

In general, there's closed form for the maximum likelihood estimator.



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## Gradient Descent

Let  $\ell_n(\beta)$  be the likelihood function as a function of  $\beta$  for a given  $\mathbb{X}, \mathbf{Y}$ . Recall from [Lecture 9](#) the gradient of a real-valued function  $f(\mathbf{x}), \mathbf{x} \in \mathbb{R}^d$ .

We can use **gradient descent** to find a local minimum of the negative of the log-likelihood function. The gradient descent optimization algorithm, in general, is used to find the local minimum of a given function  $f(\mathbf{x})$  around a starting initial point  $\mathbf{x}_0$ .

Let  $\ell_{n,1}(\beta) = -\ell_n(\beta)$ .

Given a starting point  $\beta$ , **repeat**

1.  $\Delta\beta = -\nabla\ell_{n,1}(\beta)$ .
2. Choose step size  $t$ .
3. Update  $\beta := \beta + t\Delta\beta$ .

**until** a stopping criterion is satisfied.

The **stopping criterion** for gradient descent is usually of the form  $\|\nabla\ell_n(\beta)\| \leq \epsilon$  for some very small  $\epsilon$ .

The analysis of gradient descent and the choice of step size  $t$  in every iteration is beyond the scope of this class, but the implementation of this algorithm requires one to compute gradients of the function  $\ell_n(\beta)$  at various points as given in Step 1 of the algorithm. Hence, the computational complexity of gradient descent boils down to the complexity of evaluating the gradient of the function  $\ell_n(\beta)$ .

**Note:** The above algorithm is a **descent** algorithm to **minimize** and find a local minimum of a given function. This is the reason why we used the conversion  $\ell_{n,1}(\beta) = -\ell_n(\beta)$ . If one were to re-write the algorithm without this conversion, we would have maximized  $\ell_n(\beta)$  and Step 1 of the algorithm would be  $\Delta\beta = \nabla\ell_n(\beta)$ . Such an algorithm is called a **gradient ascent** algorithm. It is more common in literature in optimization to use the descent version rather than the ascent version.

# One Step of Gradient Ascent for the Poisson GLM

2/2 points (graded)

Let  $\boldsymbol{\beta} \in \mathbb{R}^1$  and let  $\ell_2(\boldsymbol{\beta}) = \sum_{i=1}^2 \frac{Y_i X_i^T \boldsymbol{\beta} - e^{X_i^T \boldsymbol{\beta}}}{\phi} + c$ , for some constant  $c$ . For the Poisson GLM, recall that  $\phi = 1$  and we have  $b(\theta) = e^\theta$ .

What is  $\nabla \ell_2(\boldsymbol{\beta})$  for any  $\boldsymbol{\beta}$ ?

Use **X\_i** for  $X_i$  and **Y\_i** for  $Y_i$ .

$\sum_{i=1}^2$

STANDARD NOTATION

Let

1.  $X_1^T = X_1 = 0.1, Y_1 = 1,$
2.  $X_2^T = X_2 = 0.2, Y_2 = 2,$
3.  $\boldsymbol{\beta} = 0,$
4. Step size  $t = 0.01.$

What is the new  $\boldsymbol{\beta} = \boldsymbol{\beta} + t \cdot \nabla \ell_2(\boldsymbol{\beta})$ ?

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You have used 1 of 3 attempts

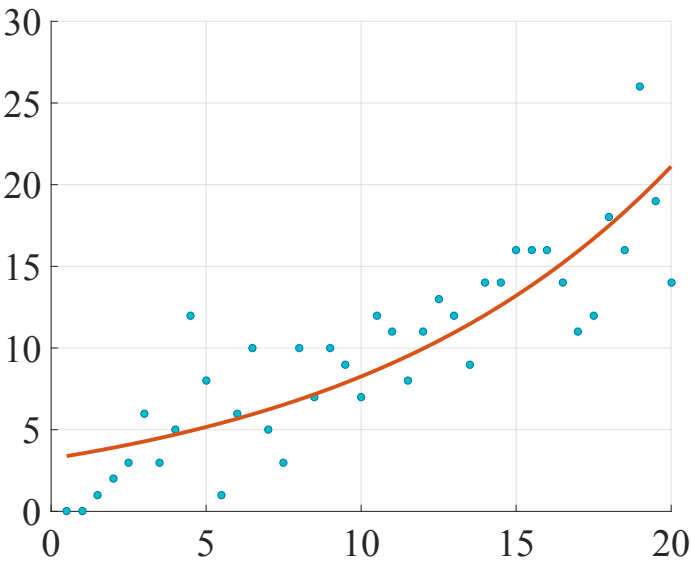
✓ Correct (2/2 points)

## Prediction

1/1 point (graded)

The following figure shows

- scatter plot of  $(x_i, y_i),$
- a generalized linear model assuming  $Y$  is Poisson and using the canonical link function for the Poisson exponential family.



The estimated vector  $\hat{\boldsymbol{\beta}} = [1.1723 \ 0.0939]$ , where we assume  $g(\mu(x)) = \beta_1 + x\beta_2$ .

What is the predicted value  $\hat{\mu}(x)$  for a new  $x = 22$ ? Provide an answer with at least 3 decimals.

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You have used 1 of 2 attempts

✓ Correct (1/1 point)

## Why Choose the Canonical Link?

0/1 point (graded)

Each choice is an optimistic statement about the canonical link function in the context of GLMs and parameter estimation. Which of the following statements are **correct** ? Choose all that apply.

- ☐ Given an exponential family, there is always a parametrization that gives a canonical link that is increasing and invertible.
- ☐ The log-likelihood function is concave if  $\phi > 0$ .
- ☐ If the family is Gaussian, then the MLE for  $\beta$  is the LSE for linear regression,  $(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$ .
- ☐ There is always a nice formula for the maximum likelihood estimator of  $\beta$ .

### Solution:

The only false statement here is the last one: **'There is always a nice formula for the maximum likelihood estimator of  $\beta$ .** Of the examples we have seen so far, the Gaussian is the only nice one (as demonstrated by the third option). In general, one needs to use optimization algorithms on the computer to numerically compute the MLE.

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You have used 2 of 2 attempts

## Asymptotic Normality

Let  $(\mathbf{X}_i, Y_i), i = 1, \dots, n$  be iid with a distribution from an exponential family such that

$$g(\mu_i) = \mathbf{X}_i^T \beta,$$

where  $\mu_i = \mathbb{E}[Y_i | \mathbf{X}_i], \mu_i = b'(\theta_i)$ , and  $g(\cdot)$  is a link function.  $\theta_i$  is the canonical parameter of the exponential family for each  $i$ . With the transformation that  $\theta_i = h(\mathbf{X}_i^T \beta^*)$ , let  $\beta^*$  denote the true underlying parameter of the observed iid samples  $(\mathbf{X}_i, Y_i), i = 1, \dots, n$ .

Let  $\hat{\beta}_{n,\text{MLE}}$  denote the maximum likelihood estimator of  $\beta^*$ .




Then,  $\hat{\beta}_{n,\text{MLE}}$  is asymptotically normal if the statistical model and the parameter space of  $\beta$  satisfy conditions required for asymptotic normality of the ML estimator as stated previously in the [lecture on maximum likelihood estimation](#).

## Discussion

**Topic:** Unit 7 Generalized Linear Models:Lecture 22: GLM: Link Functions and the Canonical Link Function / 7. Concluding Remarks on MLE for GLMs

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	<a href="#">BIG UPS to JefferyGLi and staff, Community TAs and fellow learners!</a> <a href="#">Massive respect to Jeffery, who seemed to be on every question in the forum (man, do you sleep?). I'm sure there are other staff on here and I...</a>	7
	<a href="#">Clarification on canonical link</a> <a href="#">Can a function from the exponential family have a canonical link even if it isn't a canonical function itself? I presumed the answer is No. If the ans...</a>	2
	<a href="#">Transpose operator?</a> <a href="#">Seems like the answer to the first question should require a transpose operator. But the grader accepts without it.</a>	3

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