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12. Expectation in terms of the Canonical Parameter

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Expectation in Terms of the Canonical Parameter

Expected value

Note that

$$\ell(\theta) = \frac{Y\theta - b(\theta)}{\phi} + c(Y;\phi).$$

Therefore

$$\frac{\partial \ell}{\partial \theta} = \frac{Y - b'(\theta)}{\phi}$$

It yields

$$0 = \mathbb{E}\left(\frac{\partial \ell}{\partial \theta}\right) = \frac{\mathbb{E}(Y) - b'(\theta)}{\phi}$$

which leads to

$$\mathbb{E}(Y) = b'(\theta)$$

So clearly I can cancel this phi right here.
It goes away.
And so this is just telling me that the expectation of y is just b prime of theta.
So if I give you b and I tell you, oh, here-- well, actually, if I tell you here's a distribution that comes from a canonical exponential family, I don't even need to tell you what the normalizing factor is.
I don't need to tell you what the dispersion parameter is.
If you want the expectation, you just need b.
You take its derivative, and that gives you the expectation.
Just the word dispersion should sort of guide you to the fact that it should not really affect how

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Recall from the lecture that the log-likelihood function satisfies the identities

$$\mathbb{E}\left[\frac{\partial \ell}{\partial \theta}\right] = 0$$
$$\mathbb{E}\left[\frac{\partial^2 \ell}{\partial \theta^2}\right] + \mathbb{E}\left[\left(\frac{\partial \ell}{\partial \theta}\right)^2\right] = 0.$$

The first identity is true since:

$$\begin{aligned} \mathbb{E}\left[\frac{\partial \ell}{\partial \theta}\right] &= \int_Y \frac{\partial \ell}{\partial \theta} f_{\theta}(y) \, dy \\ &= \int_Y \frac{\partial \ln f_{\theta}(y)}{\partial \theta} f_{\theta}(y) \, dy \\ &= \int_Y \frac{1}{f_{\theta}(y)} \frac{\partial f_{\theta}(y)}{\partial \theta} f_{\theta}(y) \, dy \\ &= \frac{\partial}{\partial \theta} \int_Y f_{\theta}(y) \, dy = \frac{\partial}{\partial \theta} 1 = 0. \end{aligned}$$

Try deriving the second identity yourself or recall from [lecture 11](#).

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