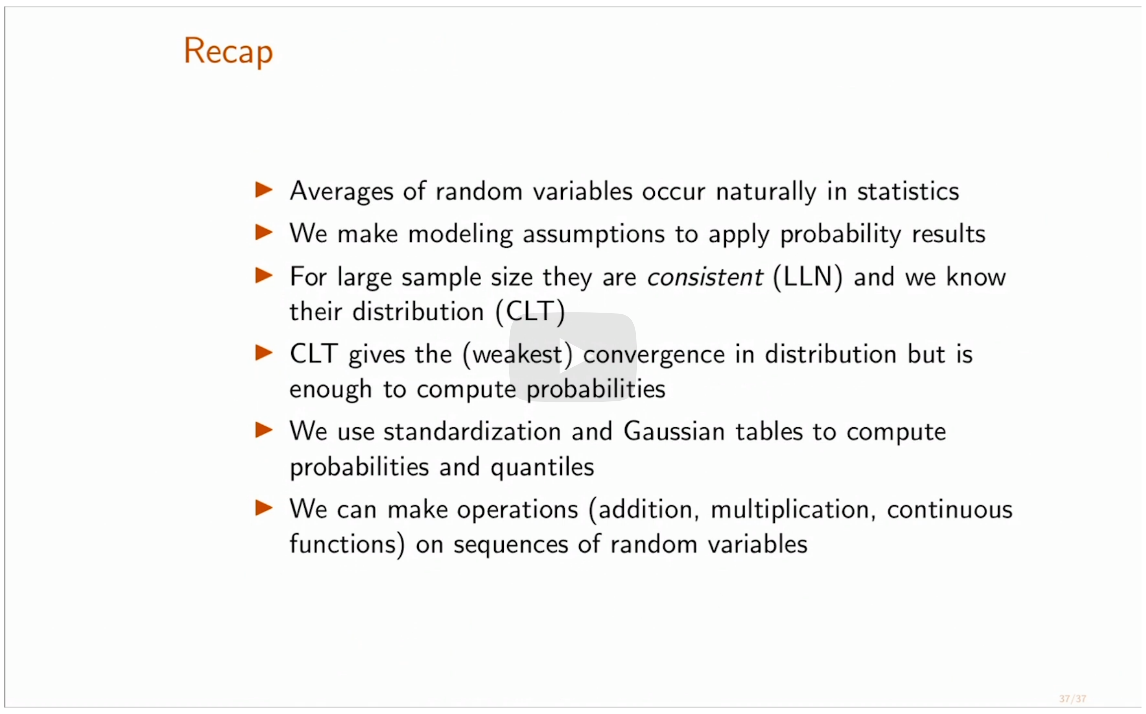
8. Operations on Sequences and Convergence

Bookmark this page

Exercises due May 25, 2021 19:59 EDT

Addition, multiplicaton, division; Slutsky's Theorem; Continuous Mapping Thoerem



7:10 / 7:17 1.50x

OK?

So I'm running out of time.

There's a bit of a recap on what we've been seeing.

I encourage you to take a look.

We saw that averages show up, and therefore central

limit theorem and law of large numbers.

Then we can use standardization and we can make operations.

But for limits and distributions,

we need to use Slutsky and be careful.

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

We restate the theorems discussed in lecture below.

Addition, Multiplication, Division for Convergence almost surely and in probability :

Addition, Multiplication, and Division preserves convergence almost surely $(\mathrm{a.s.})$ and convergence in probability ($\mathbf{P}$).

More precisely, assume

$$T_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} T \qquad \text{and} \qquad U_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} U$$

Then,

- $T_n + U_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} T + U,$

- $T_n U_n \xrightarrow[n\to\infty]{\text{a.s./}\mathbf{P}} TU$,
- If in addition, $U \neq 0$ a.s., then $\frac{T_n}{U_n} \xrightarrow[n\to\infty]{\text{a.s./}\mathbf{P}} \frac{T}{U}$.

Warning: In general, these rules **do not** apply to convergence in distribution (d).

Slutsky's Theorem

For convergence in distribution, the Slutsky's Theorem will be our main tool.

Let $(T_n), (U_n)$ be two sequences of r.v., such that:

- $T_n \xrightarrow[n\to\infty]{(d)} T$
- $U_n \xrightarrow[n\to\infty]{\mathbf{P}} u$

where T is a r.v. and u is a given real number (deterministic limit: $\mathbf{P}(U = u) = 1$). Then,

- $T_n + U_n \xrightarrow[n\to\infty]{(d)} T + u$,
- $T_n U_n \xrightarrow[n\to\infty]{(d)} Tu$,
- If in addition, $u \neq 0$, then $\frac{T_n}{U_n} \xrightarrow[n\to\infty]{(d)} \frac{T}{u}$.

Continuous Mapping Theorem

If f is a continuous function:

$$T_n \xrightarrow[n\to\infty]{\text{a.s./}\mathbf{P}/(d)} T \quad \Rightarrow \quad f(T_n) \xrightarrow[n\to\infty]{} f(T).$$

Convergence in distribution

4 points possible (graded)

Let X_n be a sequence of random variables that are converging **in probability** to another random variable X. Let Y_n be a sequence of random variables that are converging **in probability** to another random variable Y.

For each of the statements below, choose true (“This statement is always true") or false (“This statement is sometimes false"). Keep in mind that “convergence in probability" is stronger than “convergence in distribution".

- $X_n + Y_n \longrightarrow X + Y$ in distribution.

True

False

- $X_n Y_n \longrightarrow XY$ in distribution.

True

False

- $X_n/Y_n \longrightarrow X/Y$ in distribution, provided Y is constant.

True

False

- $X_n^2 - 2X_n + 5 \longrightarrow X^2 - 2X + 5$ in distribution.

True

False

Submit You have used 0 of 2 attempts

Applying Slutsky's and the Continuous Mapping theorems

1 point possible (graded)

Given the following:

- $Z_1, Z_2, \ldots, Z_n, \ldots$ is a sequence of random variables that converge in distribution to another random variable Z;
- $Y_1, Y_2, \ldots, Y_n, \ldots$ is a sequence of random variables each of which takes value in the interval $(0, 1)$, and which converges in probability to a constant c in $(0, 1)$;
- $f(x) = \sqrt{x(1-x)}$.

Does $Z_n \dfrac{f(Y_n)}{f(c)}$ converge in distribution? If yes, enter the limit in terms of Z, Y and c; if no, enter **DNE**.

$$Z_n \frac{f(Y_n)}{f(c)} \xrightarrow{\text{d}}$$

Submit You have used 0 of 3 attempts

Discussion

Hide Discussion

Topic: Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 8. Operations on Sequences and Convergence

Add a Post

Show all posts by recent activity

? Having trouble understading this topic
So convergence in probability implies convergence in distribution apparently. Then why do the notes to the video state that "Addition... 4

(staff) missing mode of convergence in text for this page
The text in this page in the "Continuous Mapping Theorem" section is missing the mode of convergence over the limit arrow. Please f... 2

Applying Slutsky's and the Continuous Mapping theorems
Hi Does The Answer Should Be In Terms Of Y? There's No Y Variable To Converge. 9

? [Staff] Possible Early Release of New Content? 16

If possible, could the staff consider releasing new content a bit earlier than scheduled (while keeping the due dates the same)? It's ni...

How to pronounciate Slutsky
You should read it as "*S-loot-ski*", Евге́ний Евге́ньевич Слу́цкий (Yevgenyi Yevgenyevich Slutskyi). [Source][1] for Russian speakin... 2

Generating Speech Output

Convergence in distribution 2

< Previous | Next >

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Connect

Blog
Contact Us
Help Center
Media Kit
Donate

GET IT ON Google play
Download on the App Store