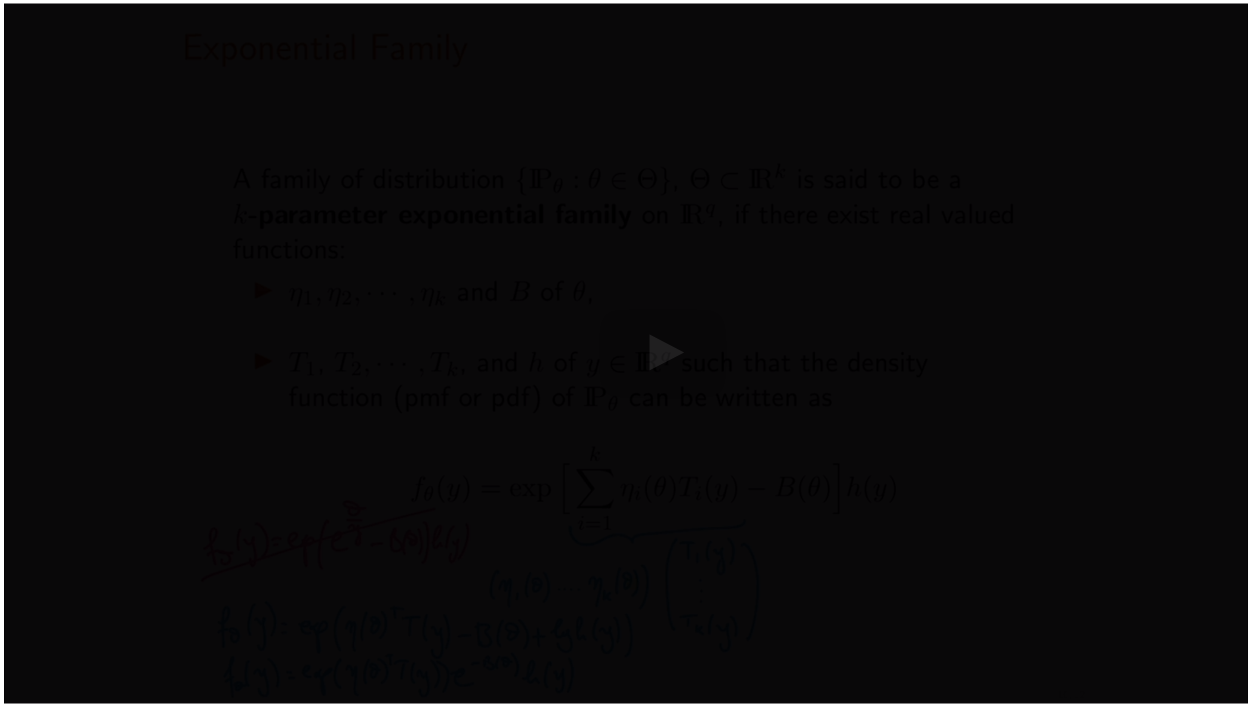
Exercises due Aug 31, 2021 19:59 EDT

Exponential Families: Definition



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happen.

So we'll say there's a bunch of limitations

to this very general family.

It's actually quite general, but it does not allow everything.

But the fact that I can take any function eta here

and any function t here just gives me a lot of flexibility.

It's not uniquely defined, right?

I could multiply my etas by 2 and divide my t by 2,

and that would give me the same guy.

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Recall from lecture that a family of distribution $\{\mathbf{P}_{\boldsymbol{\theta}} : \boldsymbol{\theta} \in \Theta\}$, where the parameter space $\Theta \subset \mathbb{R}^k$ is k-dimensional, is called a k**-parameter exponential family** on $\mathbb{R}^q$ if the pmf or pdf $f_{\boldsymbol{\theta}} : \mathbb{R}^q \to \mathbb{R}$ of $\mathbf{P}_{\boldsymbol{\theta}}$ can be written in the form

$$f_{\boldsymbol{\theta}}(\mathbf{y}) = h(\mathbf{y}) \exp\left(\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y}) - B(\boldsymbol{\theta})\right) \qquad \text{where} \begin{cases} \boldsymbol{\eta}(\boldsymbol{\theta}) = \begin{pmatrix} \eta_1(\boldsymbol{\theta}) \\ \vdots \\ \eta_k(\boldsymbol{\theta}) \end{pmatrix} & : \mathbb{R}^k \to \mathbb{R}^k \\ \mathbf{T}(\mathbf{y}) = \begin{pmatrix} T_1(\mathbf{y}) \\ \vdots \\ T_k(\mathbf{y}) \end{pmatrix} & : \mathbb{R}^q \to \mathbb{R}^k \\ B(\boldsymbol{\theta}) & : \mathbb{R}^k \to \mathbb{R} \\ h(\mathbf{y}) & : \mathbb{R}^q \to \mathbb{R}. \end{cases}$$

When $k = q = 1$, this reduces to

$$f_\theta(y) = h(y) \exp\left(\eta(\theta) T(y) - B(\theta)\right).$$

Note: The following exercises are similar to what will be presented in lecture, but we encourage you to first attempt these yourselves.

Practice: Decomposing the exponent

4 points possible (graded)

For the two following pmfs with one parameter θ that are written in the form

$$f_\theta(y) = h(y)\, e^{w(\theta, y)},$$

first decompose $w(\theta, y)$ as

$$w(\theta, y) = \eta(\theta) T(y) - B(\theta),$$

then enter the product $\eta(\theta) T(y)$ below. Select the distribution that f_θ defines.

1. For $f_\theta(y) = e^{w(\theta, y)}$ where

$$w(\theta, y) = y \ln(\theta) + (1-y)\ln(1-\theta)$$

and $y = 0, 1,\ \theta \in (0,1)$:

$\eta(\theta) T(y) =$

What distribution does the pmf $f_\theta(y)$ define?

- $\mathcal{N}(\theta, 1)$
- $\mathcal{N}(1, \theta)$
- $\text{Ber}(\theta)$
- $\text{Poiss}(\theta)$
- none of the above

2. For $f_\theta(y) = \frac{1}{y!} e^{w(\theta, y)}$ where $w(\theta, y) = -\theta + y \ln(\theta)$, and $y = 0, 1, 2, \ldots,\ \theta \in (0,1)$:

$\eta(\theta)\, T(y) =$

What distribution does the pmf $f_\theta(y)$ define?

- $\mathcal{N}(\theta, 1)$
- $\mathcal{N}(1, \theta)$
- $\mathsf{Ber}(\theta)$
- $\mathsf{Poiss}(\theta)$
- none of the above

STANDARD NOTATION

Submit You have used 0 of 3 attempts

Practice: Normal distribution with known variance

1 point possible (graded)

The normal distribution $\mathcal{N}(\theta, 1)$ with with mean θ and known variance $\sigma^2 = 1$ has pdf

$$f_\theta(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\theta)^2}{2}}.$$

Rewrite f_θ in the form

$$f_\theta(y) = h(y)\, e^{\eta(\theta)T(y)-B(\theta)} \qquad \text{where } \eta(\theta),\, T(y) : \mathbb{R} \to \mathbb{R},$$

and enter the product $\eta(\theta)\,T(y)$ below.

$\eta(\theta)\,T(y) =$

STANDARD NOTATION

Submit You have used 0 of 3 attempts

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Topic: Unit 7 Generalized Linear Models:Lecture 21: Introduction to Generalized Linear Models; Exponential Families / 6. The Exponential Family

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Hints for the exercise please

I took my best shot and got them wrong. I have no idea how to approach this.

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