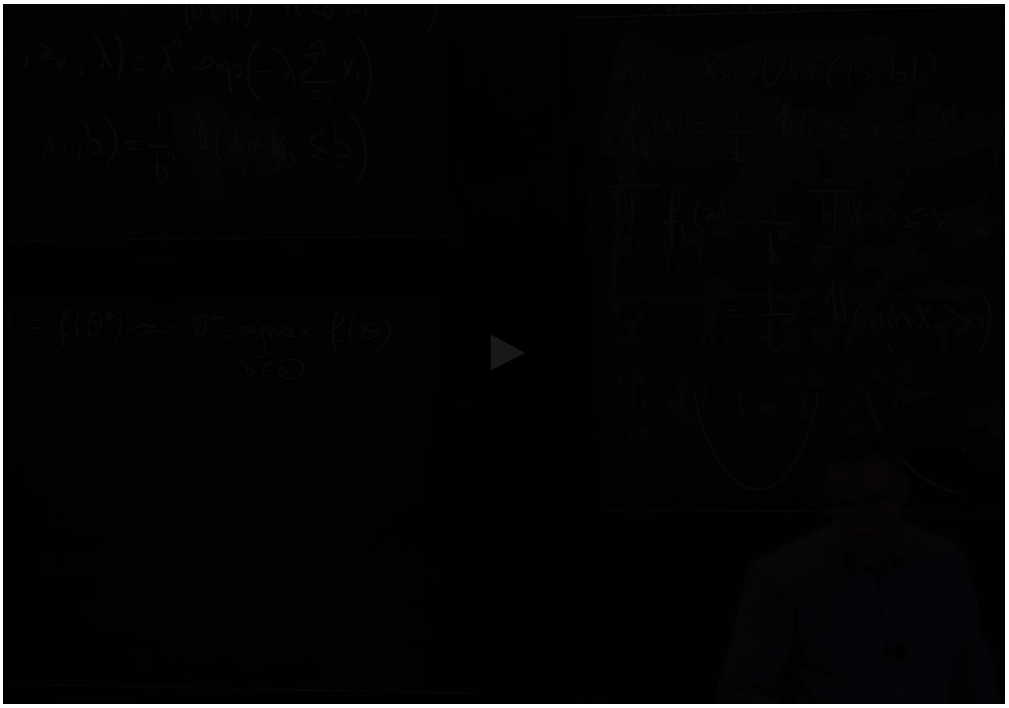
Concavity in 1 dimension



7:09 / 7:09 1.50x

Which is just minus

it's the same as concavity from minus the function.

OK, so a convex function-- so if this is boom, boom, and boom,

just think of what would happen if you put a negative sign.

So this is convex, this is convex, and this is convex.

OK.

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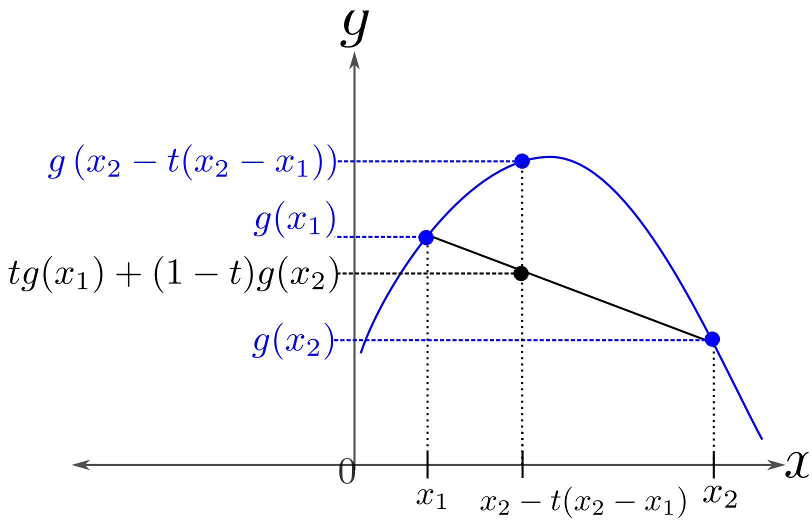
A function $g : I \to \mathbb{R}$ is **concave** (or concave down) on an interval I, if for all pairs of real numbers $x_1 < x_2 \in I$

$$g\left(tx_1 + (1-t)\,x_2\right) \geq tg\left(x_1\right) + (1-t)\,g\left(x_2\right) \qquad \text{for all } 0 < t < 1.$$

Geometrically, this means that for $x_1 < x < x_2$, the graph of g is **above** the secant line connecting the two points $\left(x_1, g\left(x_1\right)\right)$ and $\left(x_2, g\left(x_2\right)\right)$.



At $x = x_2 - t(x_2 - x_1) = tx_1 + (1-t)x_2$, the y-value of the graph of g is $g(x) = g(tx_1 + (1-t)x_2)$, while the y-value of the secant line is $tg(x_1) + (1-t)g(x_2)$.

If the inequality is strict, i.e. if

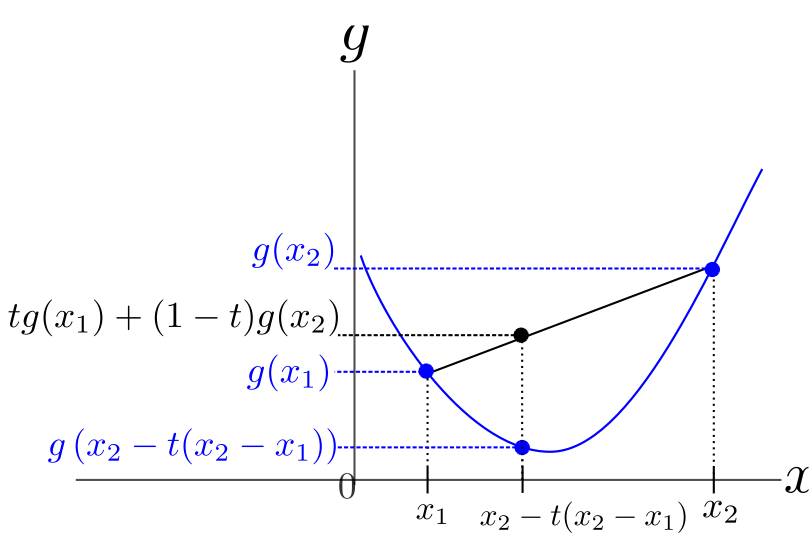
$$g(tx_1 + (1-t)x_2) > tg(x_1) + (1-t)g(x_2) \qquad \text{for all } 0 < t < 1.$$

then g is **strictly concave** .

The definition for **(strictly) convex** is analogous. A function $g : I \to \mathbb{R}$ is **convex** (or concave up), where I is an interval, if for all pairs of real numbers $x_1 < x_2 \in I$

$$g(tx_1 + (1-t)x_2) \leq tg(x_1) + (1-t)g(x_2) \qquad \text{for all } 0 < t < 1.$$

Geometrically, this means that for $x_1 < x < x_2$, the graph of g is **below** the secant line connecting the two points $(x_1, g(x_1))$ and $(x_2, g(x_2))$.



At $x = x_2 - t(x_2 - x_1) = tx_1 + (1-t)x_2$, the y-value of the graph of g is $g(x) = g(tx_1 + (1-t)x_2)$, while the y-value of the secant line is $tg(x_1) + (1-t)g(x_2)$.

If the inequality is strict, i.e. if

$$g\left(tx_1+(1-t)\,x_2\right)<tg\left(x_1\right)+(1-t)\,g\left(x_2\right)\qquad \text{for all } 0<t<1.$$

then g is **strictly convex** .

If in addition g is twice differentiable in the interval I, i.e. $g''(x)$ exists for all $x \in I$, then g is

- **concave** if and only if $g''(x) \leq 0$ for all $x \in I$;
- **strictly concave** if $g''(x) < 0$ for all $x \in I$;
- **convex** if and only if $g''(x) \geq 0$ for all $x \in I$;
- **strictly convex** if $g''(x) > 0$ for all $x \in I$;

Note: In the lecture video and slides, we used these inequality conditions on the second derivative to defined concave functions and strictly concave functions *analytically*. The *synthetic* definition above is slightly more general because it does not require differentiability at every point. For example, the function $x \mapsto x^4$ is strictly convex according to the definition above, but has three vanishing derivatives at the origin $x = 0$.

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