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Projection Matrix

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A projection matrix \mathbf{P} is an $n \times n$ square matrix that gives a vector space projection from \mathbb{R}^n to a subspace \mathcal{W} . The columns of \mathbf{P} are the projections of the standard basis vectors, and \mathcal{W} is the image of \mathbf{P} . A square matrix \mathbf{P} is a projection matrix iff $\mathbf{P}^2 = \mathbf{P}$.

A projection matrix \mathbf{P} is orthogonal iff

$$\mathbf{P} = \mathbf{P}^*, \quad (1)$$

where \mathbf{P}^* denotes the adjoint matrix of \mathbf{P} . A projection matrix is a symmetric matrix iff the vector space projection is orthogonal. In an orthogonal projection, any vector \mathbf{v} can be written $\mathbf{v} = \mathbf{v}_{\mathcal{W}} + \mathbf{v}_{\mathcal{W}^\perp}$, so

$$\langle \mathbf{v}, \mathbf{P} \mathbf{w} \rangle = \langle \mathbf{v}_{\mathcal{W}}, \mathbf{P} \mathbf{w} \rangle = \langle \mathbf{P} \mathbf{v}, \mathbf{w} \rangle. \quad (2)$$

An example of a nonsymmetric projection matrix is

$$\mathbf{P} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad (3)$$

which projects onto the line $y = x$.

The case of a complex vector space is analogous. A projection matrix is a Hermitian matrix iff the vector space projection satisfies

$$\langle \mathbf{v}, \mathbf{P} \mathbf{w} \rangle = \langle \mathbf{v}_{\mathcal{W}}, \mathbf{P} \mathbf{w} \rangle = \langle \mathbf{P} \mathbf{v}, \mathbf{w} \rangle, \quad (4)$$

where the inner product is the Hermitian inner product. Projection operators play a role in quantum mechanics and quantum computing.

Any vector in \mathcal{W} is fixed by the projection matrix $\mathbf{P} \mathbf{w} = \mathbf{w}$ for any \mathbf{w} in \mathcal{W} . Consequently, a projection matrix \mathbf{P} has norm equal to one, unless $\mathbf{P} = \mathbf{0}$,

$$\|\mathbf{P}\| = \sup_{\|\mathbf{x}\|=1} \|\mathbf{P} \mathbf{x}\| \geq 1. \quad (5)$$

Let \mathcal{A} be a C^* -algebra. An element $p \in \mathcal{A}$ is called projection if $p^* = p$ and $p^2 = p$. For example, the real function f defined by $f(x) = 0$ on G_1 and $f(x) = 1$ on G_2 is a projection in the C^* -algebra $C(X)$, where X is assumed to be disconnected with two components G_1 and G_2 .

SEE ALSO:

Idempotent, Inner Product, Map Projection, Orthogonal Set, Projection, Projection Operator, Pseudoinverse, Symmetric Matrix, Vector Space Projection, Vertical Perspective Projection

.999 with 123 repeating



THINGS TO TRY:

= .999 with 123 repeating

= GF(8)

= inverse of quaternion $1+0i+0j+2k$
** $(-1i+3+4j+3k)$

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