Lecture 6:
Language Models and Recurrent Neural Networks

Language Modeling

- The task of predicting what word comes next

ex) 자동완성기능, 검색어 미리보기

- t번째 까지의 단어들을 가지고 t+1번째에 해당 단어가 올 확률을 구하는 방식으로 예측!

$$P(\boldsymbol{x}^{(t+1)} | \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

where $x^{(t+1)}$ can be any word in the vocabulary $V = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_{|V|}\}$

n-gram Language Models

<u>Definition:</u> A *n-gram* is a chunk of *n* consecutive words.

- unigrams: “the”, “students”, “opened”, ”their”
- bigrams: “the students”, “students opened”, “opened their”
- trigrams: “the students opened”, “students opened their”
- 4-grams: “the students opened their”

n의 개수만큼 끊어서 보기!

n-gram Language Models

- (n-1)개의 단어로 마지막 n번째 단어 예측
- **Frequency !**

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\overbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}^{n\text{-1 words}})$$

(assumption)

prob of a n-gram

prob of a (n-1)-gram

$$= \frac{\boxed{P(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}}{\boxed{P(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}}$$

(definition of conditional prob)

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}$$

(statistical approximation)

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their* $\boldsymbol{w}$*"* never occurred in data? Then $\boldsymbol{w}$ has probability 0!

(Partial) Solution: Add small δ to the count for every $\boldsymbol{w} \in V$. This is called *smoothing*.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count(students opened their } \boldsymbol{w})}}{\boxed{\text{count(students opened their)}}}$$

Sparsity Problem 2

Problem: What if *"students opened their"* never occurred in data? Then we can't calculate probability for *any* $\boldsymbol{w}$!

(Partial) Solution: Just condition on *"opened their"* instead. This is called *backoff*.

Note: Increasing *n* makes sparsity problems *worse.* Typically we can't have *n* bigger than 5.

n-gram Language Models

- Sparsity problem

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)}, \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(t-n+2)})}$$

(statistical approximation)

- 분자가 0이 될 경우 -> **smoothing**
- 분모가 0이 될 경우 -> **backoff** (n의 개수를 하나 줄이기)
- N이 커질수록 sparsity problem도 커지기 때문에 n은 5 이하로 하는 것이 좋음

Storage Problems with n-gram Language Models

Storage: Need to store count for all *n*-grams you saw in the corpus.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count}(\text{students opened their } \boldsymbol{w})}}{\text{count}(\text{students opened their})}$$

Increasing *n* or increasing corpus increases model size!

Generating text with a n-gram Language Models

today the price of ________

condition on this

get probability distribution

the	0.072
18	0.043
oil	0.043
its	0.036
gold	0.018
…	

sample

- Conditioning -> sampling -> conditioning -> sampling 반복
- 문맥에 잘 맞게 예측하려면 n 개수를 늘려야 하는데 n 개수를 늘리면 sparsity proble과 storage problem도 함께 커짐

A fixed-window neural Language Model

~~*as the proctor started the clock*~~ *the students opened their* ______

discard

fixed window

A fixed-window neural Language Model

output distribution

$$\hat{\boldsymbol{y}} = \mathrm{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

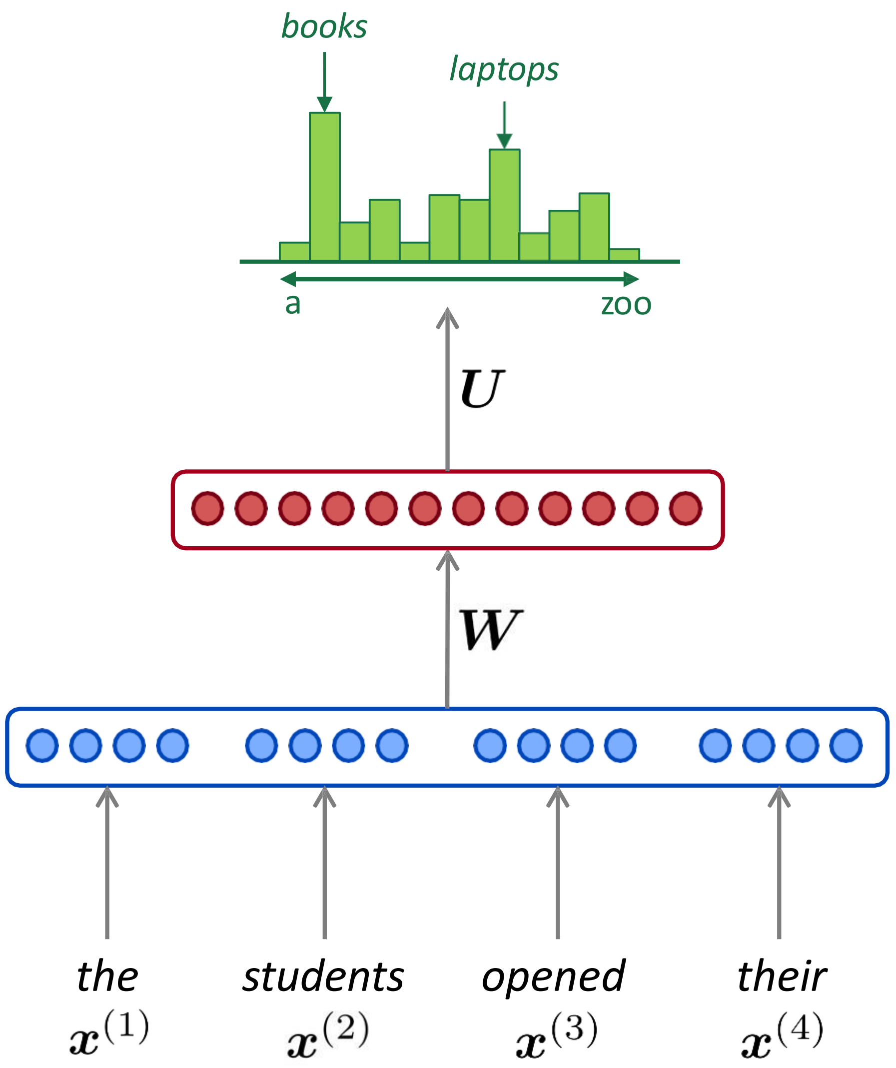
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$

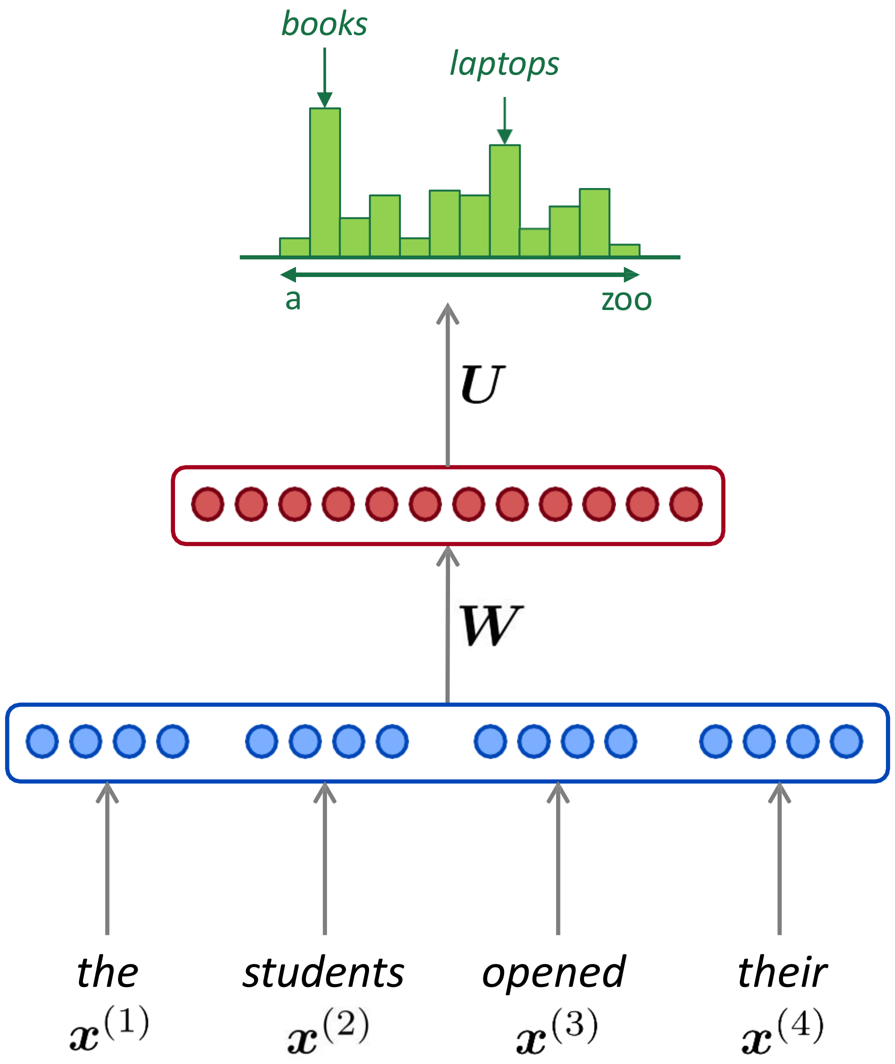


A fixed-window neural Language Model

Improvements over *n*-gram LM:
- No sparsity problem
- Don't need to store all observed *n*-grams

Remaining **problems**:
- Fixed window is too small
- Enlarging window enlarges $\boldsymbol{W}$
- Window can never be large enough!
- $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$ are multiplied by completely different weights in $\boldsymbol{W}$. No symmetry in how the inputs are processed.

We need a neural architecture that can process *any length input*



A RNN Language Model

$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$

output distribution

$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$

hidden states

$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h\boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e\boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$

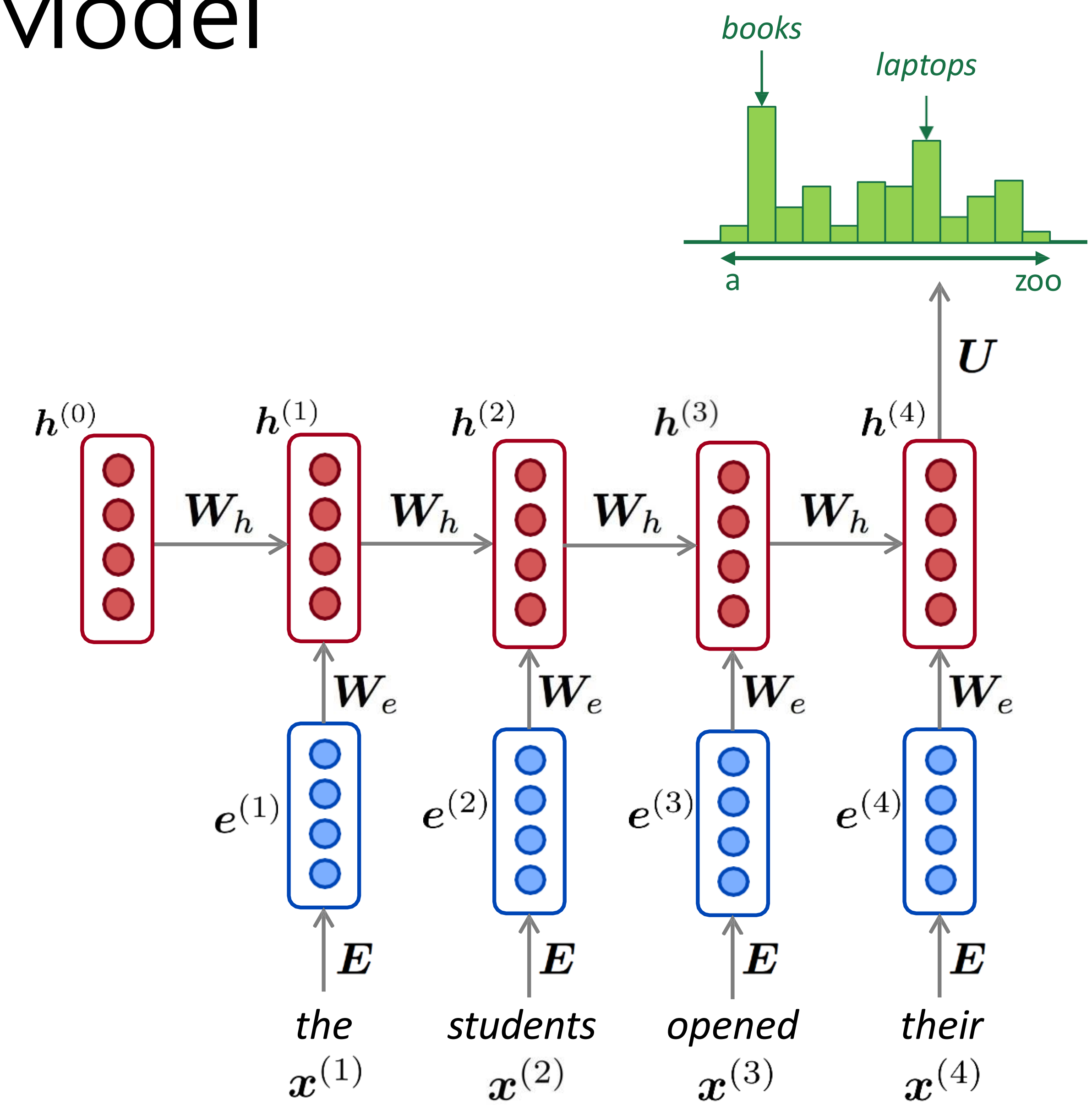
$\boldsymbol{h}^{(0)}$ is the initial hidden state

word embeddings

$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$

words / one-hot vectors

$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$



__Note__: this input sequence could be much longer, but this slide doesn't have space!

A RNN Language Model

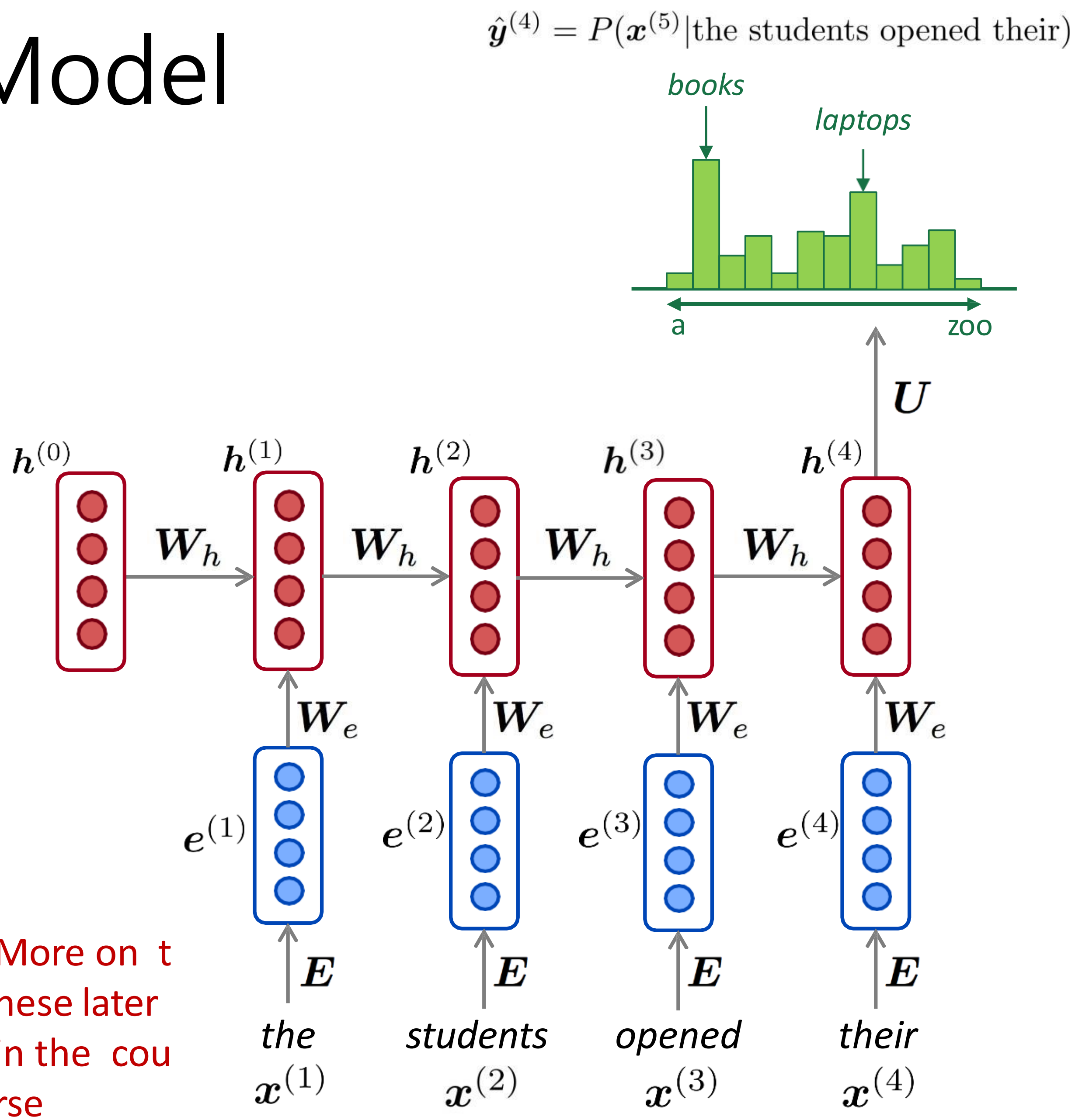
RNN **Advantages**:

- Can process any length input
- Computation for step *t* can (in theory) use inf ormation from many st eps back
- Model size doesn't increase for longer input
- Same weights applied on every timestep, so there is symmetry in how inputs a re processed.

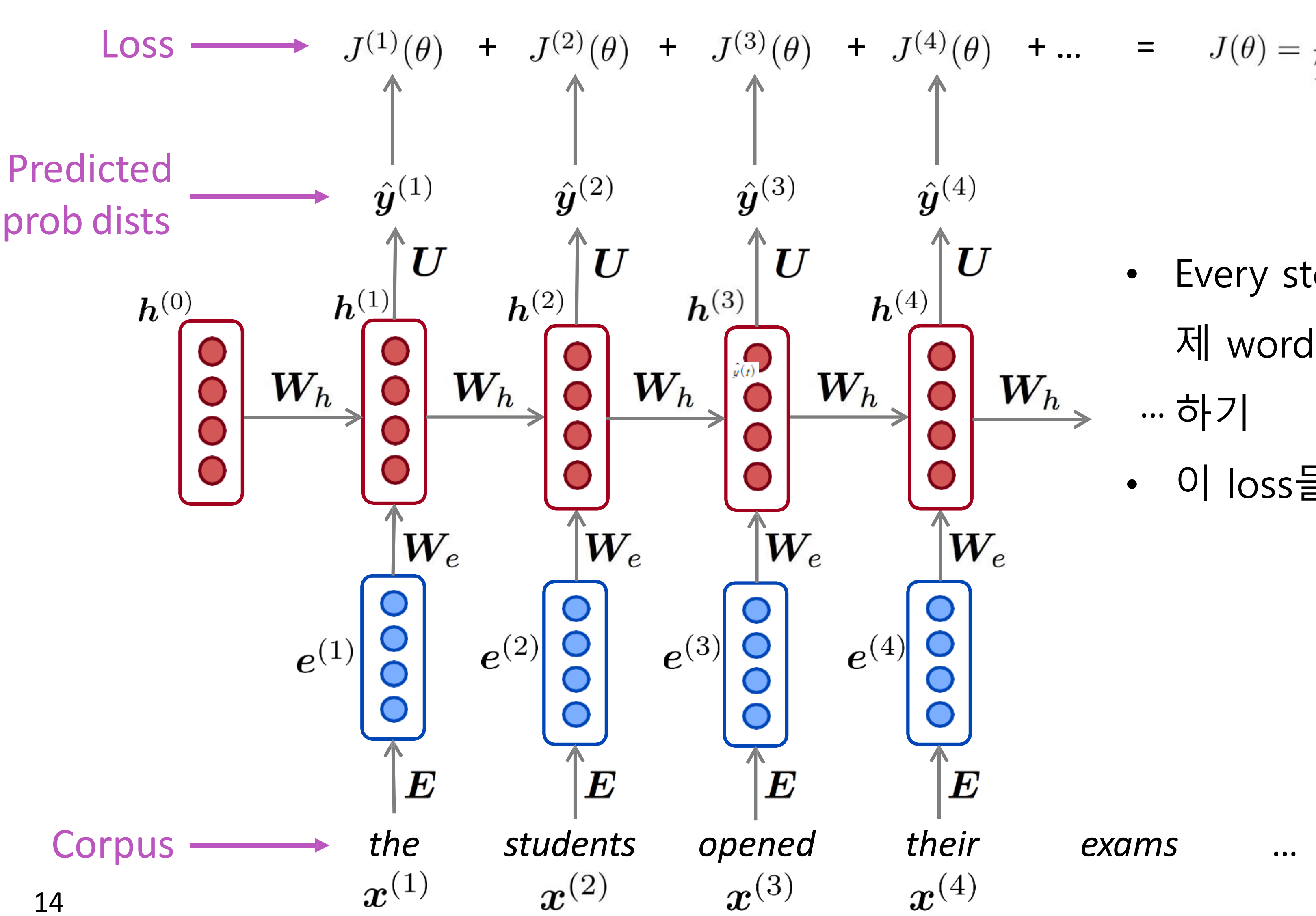
RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to a ccess information from many steps back

More on t hese later in the cou rse



Training a RNN Language Model



$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J^{(t)}(\theta)$$

- Every step에서 나온 결과와 실제 word를 비교해서 loss를 구 … 하기
- 이 loss들을 평균낸게 final loss

Evaluating Language Models

- **Perplexity !**

The standard evaluation metric for Language Models is perplexity.

$$\text{perplexity} = \underbrace{\prod_{t=1}^{T} \left(\frac{1}{P_{\text{LM}}(\boldsymbol{x}^{(t+1)} | \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})} \right)}_{\text{Inverse probability of corpus, according to Language Model}}^{1/T}$$

Normalized by number of words

- **Lower perplexity is better !**
- RNN이 n-gram language model보다 성능 좋음