Exercises 1

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8.1

When $n = 1$, $1^2 + 1 = 2 = 2^1$.
When $n = 2$, $2^2 + 1 = 5 > 2^2 = 4$.
When $n = 3$, $3^2 + 1 = 10 > 2^3 = 8$.
When $n = 4$, $4^2 + 1 = 17 > 2^4 = 16$.

8.4

When $x > 0, y > 0$, $|x| + |y| = x + y = |x + y|$.
When $x > 0, y < 0 and x + y > 0$, $|x| + |y| = x - y > |x + y| = x + y$.
When $x > 0, y < 0 and x + y < 0$, $|x| + |y| = x - y > |x + y| = -x - y$.
When $x < 0, y > 0 and x + y > 0$, $|x| + |y| = -x + y > |x + y| = x + y$.
When $x < 0, y > 0 and x + y < 0$, $|x| + |y| = -x + y > |x + y| = -x - y$.
When $x < 0, y < 0$, $|x| + |y| = x + y = |x + y|$.

8.8

a)

Let $x = y$, $P(x)$ is true. Otherwise, $P(x)$ is false.

b)

$\exists x P(X)$ proves the existence, and $\forall x \forall y (P(x) \land P(y) \to x = y)$ proves the uniqueness.

c)

There is a x makes $P(x)$ is true. At the same time, for all elements y, if $P(y)$ is true only if $x = y$, that proves the uniqueness.