Exercises 4

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1.18

1) Suppose there are $2n$ people at first, and after first stage, there will still n people alive. obviously, The *i-th* person in new circle is the $(2i-1)$-*th* person in last circle. Let $J(n) = k$, so that $J(2n) = 2k-1 = 2J(n)-1$.

2) Suppose there are $2n+1$ people at first, there will $n+1$ people alive after first stage. Further more, the first person will be killed. At this time, the *i-th* person in new circle (with n people) is the $(2i+1)$-*th* person in last circle. Let $J(n) = k$, so that $J(2n) = 2k+1 = 2J(n)+1$.

2.1

a) $k = 3$. **b)** It is not a linear homogeneous recurrence relations with constant coefficient. **c)** $k = 4$. **d)** It is not a linear homogeneous recurrence relations with constant coefficient. **e)** It is not a linear homogeneous recurrence relations with constant coefficient. **f)** $k = 2$ **g)** It is not a linear homogeneous recurrence relations with constant coefficient.

2.10

$r^3 - 3r^2 - 3r - 1 = (r-1)(r+1)^2 = 0 \Rightarrow r = -1, 1, 1$

$$\begin{aligned} a_n &= \alpha_1 \cdot (-1)^n + (\alpha_{2,0} + \alpha_{2,1} n) \cdot 1^n \\ 5 &= \alpha_1 + \alpha_{2,0} \\ -9 &= -\alpha_1 + \alpha_{2,0} + \alpha_{2,1} \\ 15 &= \alpha_1 + \alpha_{2,0} + 2\alpha_{2,1} \end{aligned} \tag{1}$$

$\alpha_1 = \frac{19}{2} \quad \alpha_{2,0} = -\frac{9}{2} \quad \alpha_{2,1} = 5$
$a_n = \frac{19}{2} \cdot (-1)^n + (-\frac{9}{2} + 5n) \cdot 1^n$

2.15

a) $a_n = \alpha \cdot 2^n + 3^{n+1}$

b) $a_1 = 2\alpha + 9 = 5 \Rightarrow \alpha = -2 \Rightarrow a_n = -2 \cdot 2^n + 3^{n+1}$

4.4

a) $a_0 = -64, a_1 = 144, a2 = -108, a_3 + 27$ and $a_n = 0$ for all $n \geq 4$.

b) $a_0 = 1, a_3 = 3, a_6 = 3, a_9 = 1$ and $a_n = 0$ for others.

c) $a_n = 5^n$

d) $a_n = (-3)^{n-3}$ for $n \geq 3$, and $a_{0,1,2} = 0$.

e) $a_0 = 8, a_1 = 3, a_2 = 2$, and $a_n = 0$ for other odd n while $a_n = 1$ for other even n.

f) $a_4 n = 1$ for $n \geq 1$, $a_{1,2,3} = -1$ and $a_0 = 0$.

g) $a_n = n - 1$ for $n \geq 2$ and $a_{0,1} = 0$

h) $a_n = \frac{2^{n+1}}{n!}$

6.2

6 solutions.