Legged Robots That Balance

The MIT Press Series in Artificial Intelligence
Edited by Patrick Henry Winston and Michael Brady

Artificial Intelligence: An MIT Perspective, Volume I: Expert Problem Solving, Natural Language Understanding, Intelligent Computer Coaches, Representation and Learning edited by Patrick Henry Winston and Richard Henry Brown, 1979

Aritificial Intelligence: An MIT Perspective, Volume II: Understanding Vision, Manipulation, Computer Design, Symbol Manipulation edited by Patrick Henry Winston and Richard Henry Brown, 1979

NETL: A System for Representing and Using Real-World Knowledge by Scott Fahlman, 1979

The Interpretation of Visual Motion by Shimon Ullman, 1979

A Theory of Syntactic Recognition for Natural Language by Mitchell P. Marcus, 1980

Turtle Geometry: The Computer as a Medium for Exploring Mathematics by Harold Abelson and Andrea diSessa, 1981

From Images to Surfaces: A Computational Study of the Human Early Visual System by William Eric Leifur Grimson, 1981

Robot Manipulators: Mathematics, Programming and Control by Richard P. Paul, 1981

Computational Models of Discourse edited by Michael Brady and Robert C. Berwick, 1982

Robot Motion: Planning and Control by Michael Brady, John M. Hollerbach, Timothy Johnson, Tomas Lozano-Pèrez, and Matthew T. Mason, 1982

In-Depth Understanding: A Computer Model of Integrated Processing for Narrative Comprehension by Michael G. Dyer, 1983

Robotics Research: The First International Symposium edited by Michael Brady and Richard Paul, 1984

Robotics Research: The Second International Symposium edited by Hideo Hanafusa and Hirochika Inoue, 1985

Robot Hands and the Mechanics of Manipulation by Matthew T. Mason and J. Kenneth Salisbury, Jr., 1985

The Acquisition of Syntactic Knowledge by Robert C. Berwick, 1985

The Connection Machine by W. Daniel Hillis, 1985

Legged Robots that Balance by Marc H. Raibert, 1986

Legged Robots That Balance

Marc H. Raibert

The MIT Press

Cambridge, Massachusetts
London, England

Library of Congress Cataloging-in-Publication Data

Raibert, Marc H.
Legged robots that balance.

(The MIT Press series in artificial intelligence)
Bibliography: p.
Includes index.
1. Robotics. 2. Artificial intelligence. I. Title.
II. Series.
TJ211.R35 1986 629.8′92 85-23888
ISBN 0-262-18117-7

The MIT Press, Cambridge, Massachusetts

Printed in the United States of America

To Ivan Sutherland

Contents

SERIES FOREWORD

Artificial intelligence is the study of intelligence using the ideas and methods of computation. Unfortunately, a definition of intelligence seems impossible at the moment because intelligence appears to be an amalgam of so many information-processing and information-representation abilities.

Of course psychology, philosophy, linguistics, and related disciplines offer various perspectives and methodologies for studying intelligence. For the most part, however, the theories proposed in these fields are too incomplete and too vaguely stated to be realized in computational terms. Something more is needed, even though valuable ideas, relationships, and constraints can be gleaned from traditional studies of what are, after all, impressive existence proofs that intelligence is in fact possible.

Artificial intelligence offers a new perspective and a new methodology. Its central goal is to make computers intelligent, both to make them more useful and to understand the principles that make intelligence possible. That intelligent computers will be extremely useful is obvious. The more profound point is that artificial intelligence aims to understand intelligence using the ideas and methods of computation, thus offering a radically new and different basis for theory formation. Most of the people doing artificial intelligence believe that these theories will apply to any intelligent information processor, whether biological or solid state.

There are side effects that deserve attention, too. Any program that will successfully model even a small part of intelligence will be inherently massive and complex. Consequently, artificial intelligence continually confronts the limits of computer science technology. The problems encountered have been hard enough and interesting enough to seduce artificial intelligence people into working on them with enthusiasm. It is natural, then, that there has been a steady flow of ideas from artificial intelligence to computer science, and the flow shows no sign of abating.

The purpose of this MIT Press Series in Artificial Intelligence is to provide people in many areas, both professionals and students, with timely, detailed information about what is happening on the frontiers in research centers all over the world.

Patrick Henry Winston
Michael Brady

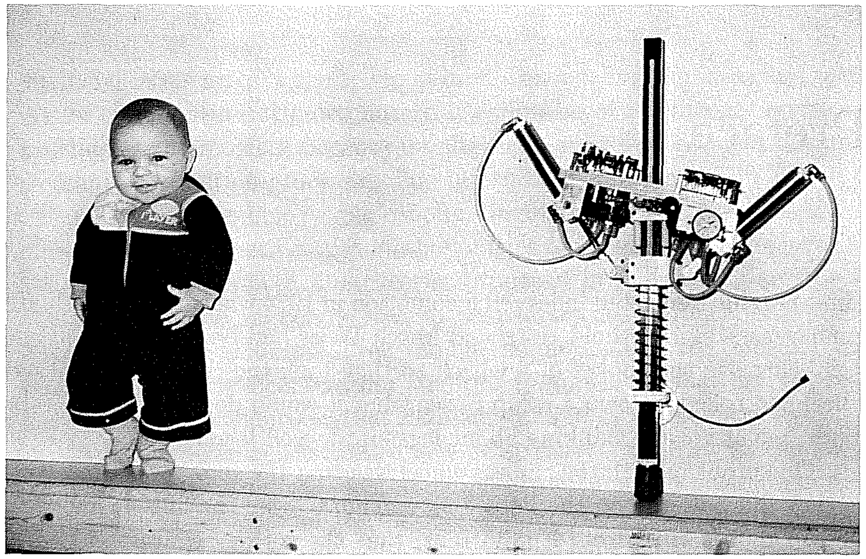
Preface

I first became interested in legged locomotion in 1974 when I was a graduate student at MIT. Berthold Horn and Mitch Weiss had been thinking about legs being like the spokes of a wheel, so they took the rim off of a spoked wheel and rolled it down a ramp to see what it would do. It didn't do very much, but it got me thinking about legs and legged locomotion.

One problem with the rimless wheel was that the spokes were too stiff, so they did not stay on the floor long enough to give the hub a chance to turn. I had learned from Emilio Bizzi that the springy characteristics of muscle and tendon played an important role in controlling animal limb movement, and it seemed to me that the spokes would work better if they were somewhat springy too. Another problem with the rimless wheel was the lack of a means for keeping it upright, so it fell over after one or two steps—it needed a mechanism for balance.

It was not until 1979 that I had an opportunity to pursue the subject further. I was teaching a robotics course as a visitor at Caltech when Ivan Sutherland encouraged me to start a robotics project in his department. I mentioned to him that a computer controlled pogostick would be a good model for learning about control and balance, and that it might lead to a fundamental understanding of legged locomotion. To my astonishment he took the idea seriously. With money from one of Sutherland's slush funds and substantial help from Elmer Szombathy in the machine shop, I built a preliminary machine that was to hop and balance like a pogostick on a single springy leg (see photograph).

From the moment Sutherland and I showed the machine to Craig Fields in 1980—we had carried it to Washington in an old suitcase—it took less than a month for the Defense Advanced Research Projects Agency to support the project and to initiate a national research program on legged vehicles. In 1981 I relocated to Pittsburgh and established the Leg Laboratory at Carnegie-Mellon University where my colleagues and I finished the preliminary machine and went on to build several others. This book



An early one-legged hopping machine (right) as shown to Craig Fields, together with a competitor. This picture was taken in the Spring of 1981 when neither legged system was yet operational.

is about those machines and the progress we have made in using them to understand the fascinating problem of legged locomotion.

Unfortunately, the behavior of a dynamic legged system is difficult to convey through the printed word and still photograph. To compensate for this limitation, I have compiled a videotape that forms an appendix to this book. It includes material from the running machines described in the text and from several of the computer simulations. The tape runs about fifteen minutes and copies are available from the MIT Press.

It is not possible to thank individually all those who have contributed to this book and to the work it reports. I am particularly grateful to the members of the Leg Laboratory who have made contributions of all sorts, especially Ben Brown and Michael Chepponis. Ivan Sutherland helped get this research started and continues to be a rich source of inspiration and good ideas. Stimulating and provocative discussions with Matt Mason cleared up several technical issues.

I am indebted to Craig Fields, Clint Kelly, and Charlie Smith who let the idea of a legged technology capture their imaginations and vision.

They have made this work possible through their support, and the support of their institutions, the Defense Advanced Research Projects Agency and the System Development Foundation.

Many colleagues and students have helped by reading all or parts of the manuscript. They include: Ben Brown, Nancy Cornelius, Matt Mason, Ken Goldberg, Ivan Sutherland, and especially Michael Chepponis and Jessica Hodgins. Ivor Durham's PLOT program generated the graphs used in the text. Ivor made several additions and changes to PLOT specifically for this book, for which I am particularly grateful. The technical illustrations were drawn by Steve Talkington. Michael Ullner wrote the typesetting macros and Roberto Minio helped with the formatting. Sylvia Brahm contributed in many ways.

Finally, I must thank my family, Nancy, Matthew, and Linda, for their loving support throughout this project.

Pittsburgh, Pennsylvania
October 1985

M.H.R.

Legged Robots That Balance

Chapter 1

Introduction

This book is about machines that use legs to run. They are dynamic machines that balance themselves actively as they travel about the laboratory. The purpose of these machines is to learn about the principles of legged locomotion, particularly those underlying control and balance. Such principles can help us to understand animal locomotion and to build useful legged vehicles.

This first chapter explains why legged locomotion is an important problem, it provides the reader with some background on the general topic of legged machines, and it highlights the results reported in the chapters that follow.

Why Study Legged Machines?

Aside from the sheer thrill of creating machines that actually run, there are two serious reasons for exploring the use of legs for locomotion. One reason is mobility. There is a need for vehicles that can travel in difficult terrain, where existing vehicles cannot go. Wheels excel on prepared surfaces such as rails and roads, but most places have not yet been paved. Only about half the earth's landmass is accessible to existing wheeled and tracked vehicles, whereas a much larger fraction can be reached by animals on foot. It should be possible to build legged vehicles that can go to the places that animals are already able to reach.

One reason legs provide better mobility in rough terrain than do wheels or tracks is that they can use isolated footholds that optimize support and traction, whereas a wheel requires a continuous path of support. As a

Figure 1.1. Legged systems do not require a continuous path of support. They can use isolated footholds that are separated by unusable terrain.

consequence, the mobility of a legged system is generally limited by the best footholds in the reachable terrain and a wheel is limited by the worst terrain. A ladder provides a good example—its steepest parts prohibit ascent on wheels, while the flattest parts, the rungs, enable ascent using legs (figure 1.1).

Another advantage of legs is that they provide an active suspension that decouples the path of the body from the paths of the feet. The payload is free to travel smoothly despite pronounced variations in the terrain. A legged system can also step over obstacles. In principle, the performance of legged vehicles can, to a great extent, be independent of the detailed

roughness of the ground. A legged system uses this decoupling to increase its speed and efficiency on rough terrain.

The construction of useful legged vehicles depends on progress in several areas of engineering and science. Legged vehicles will need systems that control joint motions, cycle the use of legs, monitor and manipulate balance, generate motions to use known footholds, sense the terrain to find good footholds, and calculate negotiable foothold sequences. Most of these tasks are not well understood yet, but research is under way. If this research is successful, it will lead to the development of legged vehicles that travel efficiently and quickly in terrain where softness, grade, or obstacles make existing vehicles ineffective. Such vehicles will be useful in industrial, agricultural, and military applications.

A second reason for exploring machines that use legs for locomotion is to understand human and animal locomotion. One need watch only a few instant replays on television to be awed by the variety and complexity of ways athletes can carry, swing, toss, glide, and otherwise propel their bodies through space, maintaining orientation, balance, and speed as they go. Such performance is not limited to professional athletes; behavior at the local playground is equally impressive from a mechanical engineering, sensory-motor integration, or computational point of view. Perhaps most exciting is the sight of one's own child advancing rapidly from creeping and crawling to walking, running, hopping, jumping, and climbing.

Animals also demonstrate great mobility and agility. They move quickly and reliably through forest, swamp, marsh, and jungle, and from tree to tree. Sometimes they move with great speed, often with great efficiency.

Despite excellence in using our own legs for locomotion, we are still at a primitive stage in understanding the control principles that underlie walking and running. What control mechanisms do animals use? One way to learn more about plausible mechanisms for animal locomotion is to build machines that locomote using legs. To the extent that an animal and a machine perform similar locomotion tasks, their control systems and mechanical structures must solve similar problems. By building machines, we can get new insights into these problems, and we can learn about possible solutions. Of particular value is the rigor required to build physical machines that actually work. Concrete theories and algorithms can guide biological research by suggesting specific models for experimental testing and verification. This sort of interdisciplinary approach is already becoming popular in other areas where biology and robotics have a common ground, such as vision, speech, and manipulation.

Dynamics and Balance Improve Mobility

The work reported in this book focuses on a dynamic treatment of legged locomotion, with particular attention to balance. This means that the legged systems studied operate in a regime where the velocities and kinetic energies of the masses are important determinants of behavior. In order to predict and influence the behavior of a dynamic system, one must consider the energy stored in each mass and spring as well as the geometric structure and configuration of the mechanism. Geometry and configuration taken alone do not provide an adequate model when a system moves with substantial speed or has large mass. Consider, for example, a fast-moving vehicle that would tip over if it stopped suddenly with its center of mass too close to the front feet.

The exchange of energy among its various forms is also important in understanding the dynamics of legged locomotion. For example, there is a cycle of activity in running that changes the form of the stored energy several times: the body's potential energy of elevation changes to kinetic energy during falling, then to strain energy when parts of the leg deform elastically during rebound with the ground, then into kinetic energy again as the body accelerates upward, and finally back into potential energy of elevation. This sort of dynamic exchange is central to an understanding of legged locomotion.

A dynamic treatment, however, does not imply an intractable treatment. Although the detailed dynamics of a legged system may indeed be complicated, control techniques that use dynamics may be simple. For example, if hopping is primarily a resonant bouncing motion, then a control system with the task of regulating hopping need not actively servo the body along a specified trajectory. It can stimulate and modulate the bouncing motion by delivering a thrust of the right magnitude just once during each cycle. Control systems can generally be made simpler if they are attuned to the dynamics of the mechanism they control and to the task the mechanism performs. A specific goal of the work reported in this book is to identify and explore control techniques that use dynamics in simple ways.

Dynamics also plays a role in giving legged systems the ability to balance actively.[1] A statically balanced system avoids tipping and the ensuing horizontal accelerations by keeping the center of mass of the body over the polygon of support formed by the feet. The feet and body move according to gait patterns that maintain this support relationship, as shown in figure 1.2. Animals sometimes use this sort of balance when they move slowly, but they usually balance actively.

[1] The terms "active balance," "dynamic balance," and "dynamic stability" are used interchangeably in this book. "Passive balance," "static balance," and "static stability" are also used interchangeably.

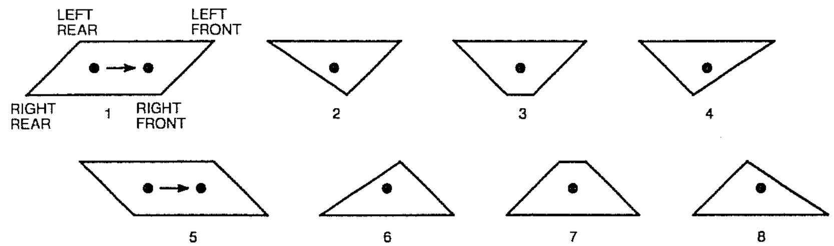
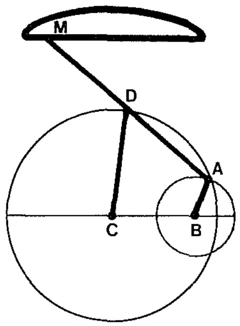


Figure 1.2. Statically stable gait. The diagram shows the sequence of support patterns provided by the feet of a crawling quadruped. The body and legs move to keep the projection of the center of mass within the polygon defined by the feet. A supporting foot is located at each vertex. The dot indicates the projection of the center of mass. Adapted from McGhee and Frank (1968).

A legged system that balances actively can tolerate departures from static equilibrium. Unlike a statically balanced system, which must always operate in or near equilibrium, an actively balanced system is permitted to tip and accelerate for short periods of time. The control system manipulates body and leg motions to ensure that each tipping interval is brief and that each tipping motion in one direction is compensated by a tipping motion in the opposite direction. An effective base of support is maintained over time. A system that balances actively may also permit vertical acceleration, such as the bouncing that occurs when the legs deform elastically and the ballistic travel that occurs between bounces.

The ability of an actively balanced system to depart from static equilibrium relaxes the rules on how legs can be used for support. This leads to improved mobility. For example, if a legged system can tolerate tipping, then it can position the feet far from the center of mass in order to use footholds that are widely separated or erratically placed. If it can remain upright with a small base of support, then it can travel where obstructions are closely spaced or where the path of firm support is narrow. The ability to tolerate intermittent support also contributes to mobility. It allows a system to move all its legs to new footholds at one time, to jump onto or over obstacles, and to use ballistic motions for increased speed. These abilities to use narrow base and intermittent support generally increase the types of terrain a legged system can negotiate. Animals routinely exploit active balance to travel quickly on difficult terrain. Legged vehicles will have to balance actively, too, if they are to move with animal-like mobility and speed.



$$CD = AD = DM = \frac{3+\sqrt{7}}{2}$$

$$BD = \frac{4+\sqrt{7}}{3}$$

Figure 1.3. Linkage used in an early walking machine. When the input crank AB rotates, the output point M moves along a straight path during part of the cycle and an arched path during the other part of the cycle. Two identical linkages are arranged to operate out of phase so at least one provides a straight motion at all times. The body is always supported by the feet connected to the straight-moving linkage. After Lucas (1894).

Research on Legged Machines

Before introducing the main topic of this book, the study of machines that run using active balance, we turn briefly to an account of previous work on legged machines.

The scientific study of legged locomotion began just over a century ago when Leland Stanford, then Governor of California, commissioned Eadweard Muybridge to find out whether or not a trotting horse left the ground with all four feet at the same time. Stanford had wagered that it never did. After Muybridge proved him wrong with a set of stop-motion photographs that would appear in *Scientific American* in 1878, Muybridge went on to document the walking and running behavior of over forty mammals, including humans (Muybridge 1955, 1957). His photographic data are still of considerable value and survive as a landmark in locomotion research.

The study of walking machines also had its origin in Muybridge's time. An early walking model appeared in about 1870. It used a kinematic linkage (figure 1.3) to move the body along a straight horizontal path while the

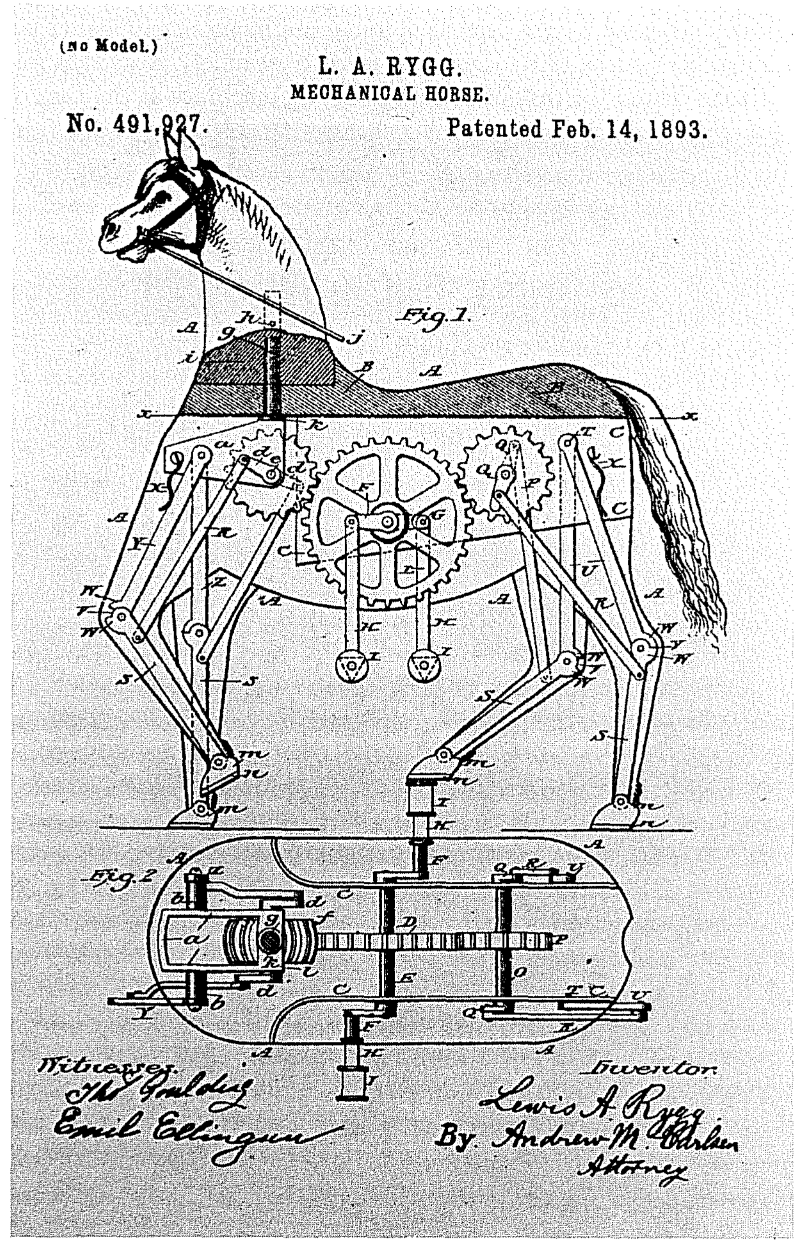


Figure 1.4. Mechanical horse patented by Lewis A. Rygg in 1893. The stirrups double as pedals so the rider can power the stepping motions. The reins move the head and forelegs from side to side for steering. Apparently the machine was never built.

feet moved up and down to exchange support during stepping. The linkage was originally designed by the famous Russian mathematician Chebyshev some years earlier (Lucas 1894). During the eighty or ninety years that followed, workers viewed the task of building walking machines as the task of designing kinematic linkages that would generate suitable stepping motions when driven by a source of power. Many designs were proposed (Rygg 1893, Nilson 1926, Ehrlich 1928, Kinch 1928, Snell 1947, Urschel 1949, Shigley 1957, Corson 1958, Bair 1959, Morrison 1968), but the performance of such machines was limited by their fixed patterns of motion which could not adjust to variations in the terrain. By the late 1950s it had become clear that a linkage providing fixed motion would not do the trick and that useful walking machines would need *control* (Liston 1970).

One approach to control was to harness a human. Ralph Mosher used this approach in building a four-legged walking truck at General Electric in the mid-1960s (Liston and Mosher 1968). The project was part of a decade-long campaign to build better teleoperators, capable of providing better dexterity through high-fidelity force feedback. The machine Mosher built stood 11 ft tall, weighed 3000 lbs, and was powered hydraulically. It is shown in figure 1.5. Each of the driver's limbs was connected to a handle or pedal that controlled one of the truck's four legs. Whenever the driver caused a truck leg to push on an obstacle, force feedback let the driver feel the obstacle as though it were his or her own arm or leg doing the pushing.

After about twenty hours of training Mosher was able to handle the machine with surprising agility. Films of the machine operating under his control show it ambling along at about 5 mph, climbing a stack of railroad ties, pushing a foundered jeep out of the mud, and maneuvering a large drum onto some hooks. Despite its dependence on a well-trained human for control, this walking machine was a landmark in legged technology, and it continues to be a significant advance over many of its successors.

An alternative to human control of legged vehicles became feasible in the 1970s: the use of a digital computer. Robert McGhee's group at Ohio State University was the first to use this approach successfully in 1977 (McGhee 1983). They built an insectlike hexapod that could walk with a number of standard gaits, turn, crab, and negotiate simple obstacles. The computer's primary task was to solve kinematic equations in order to coordinate the eighteen electric motors driving the legs. This coordination ensured that the machine's center of mass stayed over the polygon of support provided by the feet while allowing the legs to cycle through a gait. The machine traveled quite slowly, covering several meters per minute.

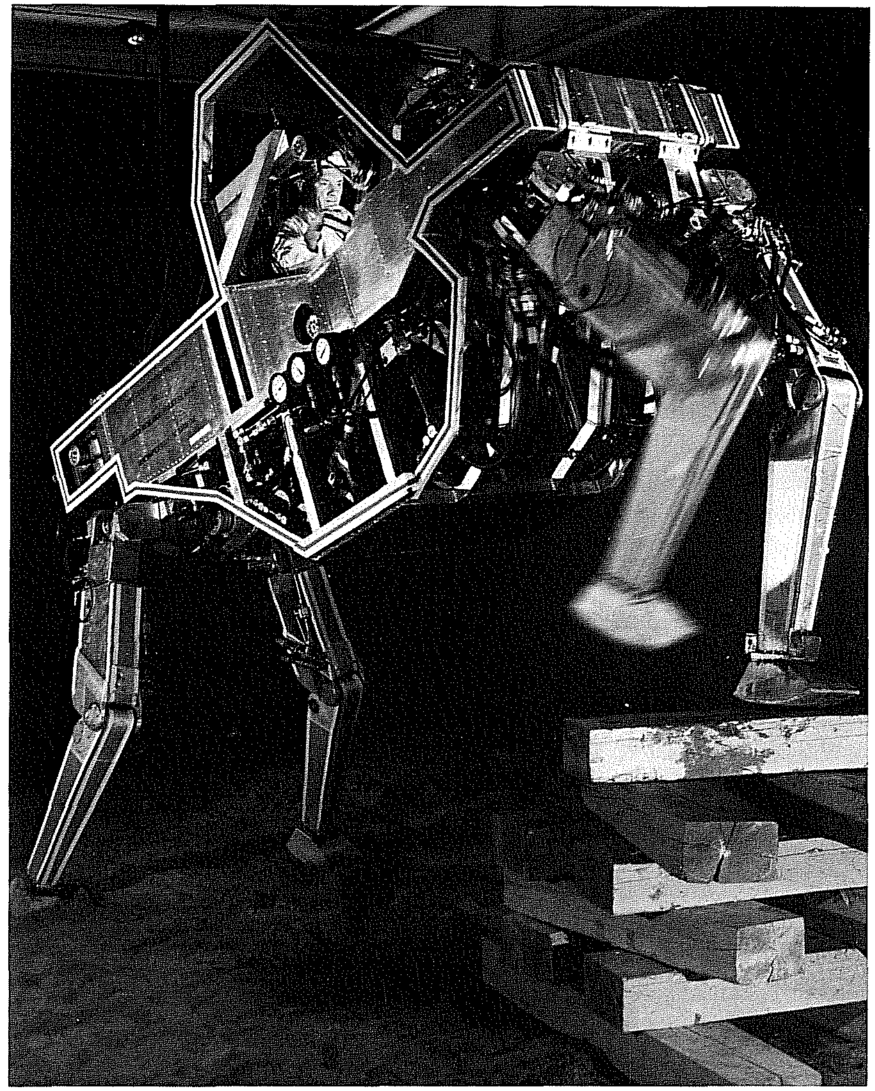


Figure 1.5. Walking truck developed by Ralph Mosher at General Electric in about 1968. The human driver controlled the machine with four handles and pedals that were connected to the four legs hydraulically. Photograph courtesy of General Electric Research and Development Center.

Force and visual sensing provided a measure of terrain accommodation in later developments (McGhee 1980, Klein and Briggs 1980, Ozguner et al. 1984). The hexapod provided McGhee with an excellent opportunity to pursue his earlier theoretical findings on the combinatorics and selection of gait (McGhee 1968, McGhee and Jain 1972, Koozekanani and McGhee 1973). The group at Ohio State is currently building a much larger hexapod, about 3 tons, that is intended to operate on rough terrain with a high degree of autonomy (Waldron et al. 1984).

Gurfinkel and his co-workers in the USSR built a machine with characteristics and performance quite similar to McGhee's at about the same time (Gurfinkel et al. 1981). It used a hybrid computer for control, with heavy use of analog computation for low-level functions.

Hirose realized that linkage design and computer control were not mutually exclusive. His experience designing clever and unusual mechanisms—he had built seven kinds of mechanical snake—led to a special leg that simplified the control of locomotion and could improve efficiency (Hirose and Umetani 1980, Hirose et al. 1984). The leg was a three-dimensional pantograph that translated the motion of each actuator into a pure Cartesian translation of the foot. With the ability to generate x, y, and z translations of each foot by merely choosing an actuator, the control computer was freed from the arduous task of performing kinematic solutions. Actually, the mechanical linkage was helping to perform the calculations needed for locomotion. The linkage was efficient because the actuators performed only positive work in moving the body forward.

Hirose used this leg design to build a small quadruped, about 1 m long. It was equipped with touch sensors on each foot and an oil-damped pendulum attached to the body. Simple algorithms used the sensors to control actions of the feet. For instance, if a touch sensor indicated contact while the foot was moving forward, the leg would move backward a little bit, move upward a little bit, then resume its forward motion. If the foot had not cleared the obstacle, the cycle would repeat. The use of several simple algorithms like this one permitted Hirose's machine to climb up and down stairs and to negotiate other obstacles without human intervention (Hirose 1984).

These three walking machines, McGhee's, Gurfinkel's, and Hirose's, represent a class called *static crawlers*. Each differs in the details of construction and in the computing technology used for control, but they share a common approach to balance and stability. Enough feet are kept on the ground to guarantee a broad base of support at all times, and the body and legs move to keep the center of mass over this broad support base.

The forward velocity is low enough to predict stability based on the spatial configuration of the body and feet, without worrying about stored energy. Each of these machines has been used to study rough terrain locomotion in the laboratory, including experiments on terrain sensing, gait selection, and selection of foothold sequences. Several other machines that fall into this class have been studied in the intervening years (e.g., Russel 1983, Sutherland and Ullner 1984, Ooka et al. 1985).

Research on Active Balance

The last section focused on legged machines that use static techniques for balance. We now turn to the study of dynamic machines that balance actively. The first machines that balanced actively were automatically controlled inverted pendulums. Everyone knows that a human can balance a broom on his finger with relative ease. Why not use automatic control to build a broom that can balance itself?

Claude Shannon was probably the first to do so. In 1951 he used the parts from an erector set to build a machine that balanced an inverted pendulum atop a small powered truck (Shannon 1985). The truck drove back and forth in response to the tipping movements of the pendulum, as sensed by a pair of switches at its base. In order to move from one place to another, the truck first had to drive away from the goal to unbalance the pendulum toward the goal. In order to balance again at the destination, the truck moved past the destination until the pendulum was again upright with no forward velocity. It then moved back to the goal.

It was at Shannon's urging that Cannon and two of his students at Stanford University set about demonstrating controllers that balanced two pendulums at once. In one case the pendulums were mounted side by side on the cart, and in the other case they were mounted one on top of the other (figure 1.6). Cannon's group was interested in the single-input multiple-output problem and in the limitations of achievable balance: how could they use the single force that drove the cart's motion to control the angle of two pendulums as well as the position of the cart? How far from balance could the system deviate before it was impossible to return to equilibrium, given the parameters of the mechanical system, e.g., cart motor strength or pendulum lengths.

Using analysis based on normal coordinates and bang-bang switching curves, they expressed regions of controllability as explicit functions of the physical parameters of the system. Once these regions were found, their

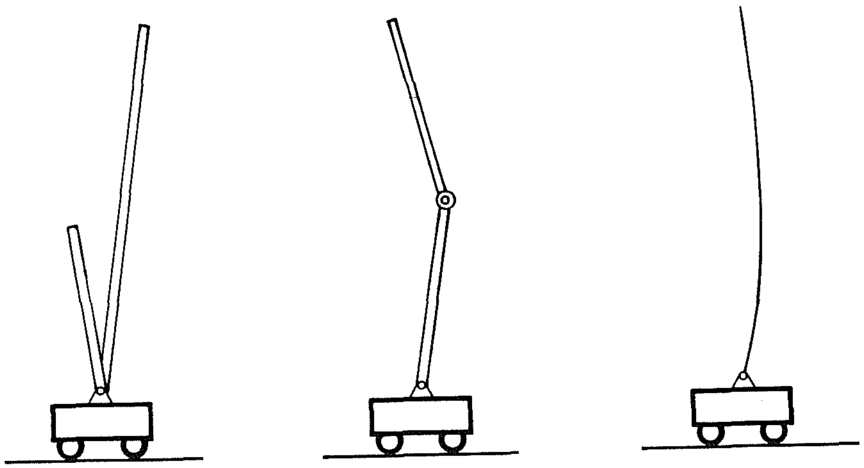


Figure 1.6. Cannon and his students built machines that balanced inverted pendulums on a moving cart. They balanced two pendulums side by side, one pendulum on top of another, and a long limber inverted pendulum. Only one input, the force driving the cart horizontally, was available for control. Adapted from Schaefer and Cannon (1966).

boundaries were used to find switching functions that provided control (Higdon and Cannon 1963, Higdon 1963). Later, they extended these techniques to provide balance for a flexible inverted pendulum (Schaefer 1965, Schaefer and Cannon 1966). These studies of balance for inverted pendulums were important precursors to later work on locomotion. The inverted pendulum model for walking would become the primary tool for studying balance in legged systems (e.g., Hemami and Weimer 1974, Vukobratovic and Stepaneko 1972, Vukobratovic 1973, Hemami and Golliday 1977, Kato et al. 1983, Miura and Shimoyama 1984). It is unfortunate that no one has yet extended Cannon's elegant analytical results to the more complicated legged case.

Mosher's group at General Electric was also interested in balance. Their original intention, before deciding to build the quadruped described earlier, was to build a walking biped that would be controlled by a human who would "walk" in an instrumented harness inside the cockpit. They started with a human factors experiment because they were unsure of the human's ability to adjust to the exaggerated vestibular input that would be experienced when one drives a machine several times taller than one's self. In the experiments the subjects stood on an inverted pendulum about 20 ft tall. The pendulum had pivots like an ankle and hip, one at the floor and one just below the platform that supported the subject. These pivots were servoed to follow the corresponding ankle and hip motions of the subject. All eighty-six people tested learned to balance the machine in less than fifteen minutes, and most learned in just two or three (Liston and Mosher 1968). Although the GE walking truck mentioned earlier could, in principle, operate using purely static techniques, the driver's ability to balance it actively probably contributed to its smooth operation. A GE walking biped was never built.

The importance of active balance in legged locomotion had been widely recognized for some years (e.g., Manter 1938, McGhee and Kuhner 1969, Frank 1970, Vukobratovic 1973, Gubina, Hemami, and McGhee 1974, Beletskii 1975a), but progress in building physical legged systems that employ such principles was retarded by the perceived difficulty of the task. It was not until the late 1970s that experimental work on balance in legged systems got underway.

Kato and his co-workers built a biped that walked with a *quasi-dynamic* gait (Ogo et al. 1980, Kato et al. 1983). The machine had ten hydraulically powered degrees of freedom and two large feet. Generally, this machine was a static crawler, moving along a preplanned trajectory to keep the center of mass over the base of support provided by the supporting foot. Once during each step, however, the machine temporarily destabilized itself to tip forward so that support would be transferred quickly from one foot to the other. Before the transfer took place on each step, the *catching* foot was positioned so that it would return the machine to equilibrium passively, without requiring an active response. A modified inverted pendulum model was used to plan the tipping motion.

In 1984 the machine walked with a quasi-dynamic gait, taking about a dozen 0.5 m long steps per minute (Takanishi et al. 1985). The use of a dynamic transfer phase makes an important point: A legged system can employ complicated dynamic behavior without requiring a very complicated control system.

Miura and Shimoyama (1980, 1984) built the first walking machine that really balanced actively. Their *stilt biped* was patterned after a human walking on stilts. Each foot provided only a point of support, and the machine had three actuators: one for each leg that moved the leg sideways and a third that separated the legs fore and aft. Because the legs did not change length, the hips were used to pick up the feet. This gave the machine a pronounced shuffling gait like Charlie Chaplin's stiff-kneed walk.

Once again, control for the stilt biped relied on an inverted pendulum model of its behavior. Each time a foot was placed on the floor, its position was chosen according to the tipping behavior that was expected from an inverted pendulum. Actually, the problem was broken down as though there were two pendulums, one in the pitching plane and one in the rolling plane. The choice of foot position along each axis took the current and desired state of the system into account. In order to perform the necessary calculations, the control system used tabulated descriptions of planned leg motions together with linear feedback. Unlike Kato's machine, which came to static equilibrium before and after each dynamic transfer, the stilt biped tipped all the time.

Matsuoka was the first to build a machine that ran, where running is defined by periods of ballistic flight with all feet leaving the ground. His goal was to model repetitive hopping in the human. He formulated a model consisting of a body and one massless leg and he simplified the problem by assuming that the duration of the support phase was short compared with the ballistic flight phase. This extreme form of running, for which nearly the entire cycle was spent in flight, minimized the influence of tipping during support. This model permitted Matsuoka to derive a time-optimal state feedback controller that provided stability for hopping in place and for low speed translations (Matsuoka 1979).

To test his method for control, Matsuoka built a planar one-legged hopping machine. The machine operated at low gravity by lying on a table inclined 10° from the horizontal, rolling on ball bearings. An electric solenoid provided a rapid thrust at the foot, so the support period was short. The machine hopped in place at about 1 hop/s and traveled back and forth on the table.

Introduction to Running Machines

Running is a special form of legged locomotion that uses ballistic flight phases to obtain high speed. To study running, my co-workers and I have explored a variety of legged systems and implemented some of them in the form of physical machines. In the course of this work we have identified a number of simple ideas about the control of legged locomotion, and have applied them to demonstrate machines that run and balance. The purpose of this work is to provide a foundation of knowledge that can lead to both the construction of useful legged vehicles and to a better understanding of legged locomotion as it occurs in nature. This section is an overview of this work on running and a summary of the main findings; details are found in the chapters that follow.

Table 1.1. Milestones in legged technology.

When	Who	What
1850	Chebyshev	Designs linkage used in early walking mechanism (Lucas 1894).
1872	Muybridge	Uses stop-motion photography to document running animals.
1893	Rygg	Patents human-powered mechanical horse.
1945	Wallace	Patents hopping tank. Reaction wheels provide stability.
1961	Space General	Eight-legged kinematic machine walks in outdoor terrain (Morrison 1968).
1963	Cannon, Higdon & Schaefer	Control system balances single, double, and limber inverted pendulums.
1968	Frank & McGhee	Simple digital logic controls walking of Phony Pony.
1968	Mosher	GE quadruped truck climbs railroad ties under control of human driver.
1969	Bucyrus-Erie Co.	Big Muskie, a 15,000 ton walking dragline is used for strip mining. It moves in soft terrain at 900 ft/hr (Sitek 1976).
1977	McGhee	Digital computer coordinates leg motions of hexapod walking machine.
1977	Gurfinkel	Hybrid computer controls hexapod walker in USSR.
1977	McMahon & Greene	Human runners set new speed records on *tuned track* at Harvard. Its compliance was adjusted to mechanics of human leg.
1980	Hirose & Umetani	Quadruped machine climbs stairs and over obstacles using simple sensors. The leg mechanism simplifies control.
1980	Kato	Hydraulic biped walks with quasi-dynamic gait.
1980	Matsuoka	Mechanism balances in the plane while hopping on one leg.
1981	Miura & Shimoyama	Walking biped balances actively in three dimensional space.
1983	Sutherland	Hexapod carries human rider. Computer, hydraulics, and human share computing task.
1983	Odetics	Self-contained hexapod lifts and moves back end of pickup truck (Russell 1983).

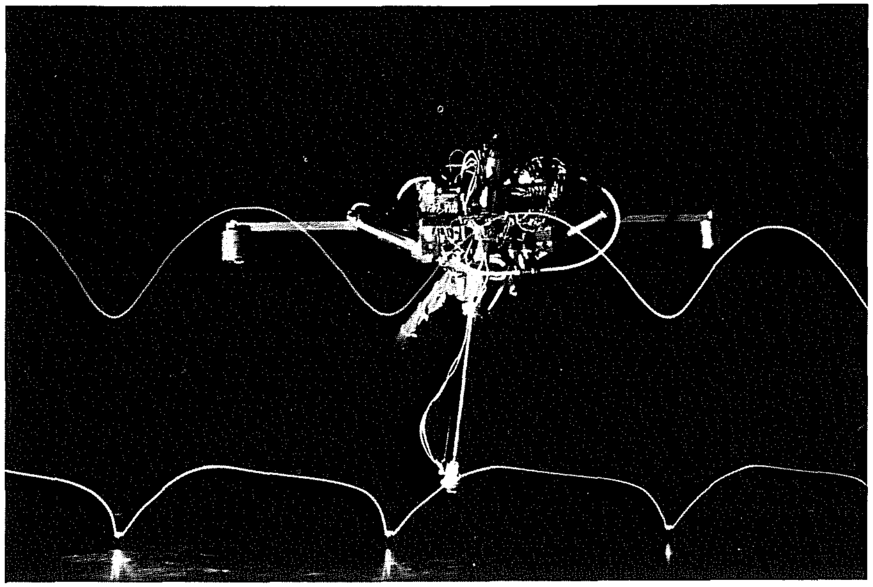


Figure 1.7. Planar hopping machine traveling at about 0.8 m/s (1.75 mph) from right to left. Lines made by light sources attached to the machine indicate paths of the foot and the hip.

It was to study running in its simplest form that we built a running machine that had just one leg. It ran by hopping like a kangaroo, using a series of leaps. A machine with only one leg draws attention to active balance and dynamics while postponing the difficult problems of coordinating the behavior of many legs. Active balance and dynamics are central issues for a one-legged machine, while gait and interleg coordination are of little concern. Gait has dominated thinking about legged locomotion for some years, and one wonders how central it really is. Are there algorithms for walking and running that are independent of gait or that work correctly for any number of legs? Perhaps a machine with just one gait could suggest answers to these questions.

The machine we built to study these problems had two main parts: a body and a leg. The body provided the main structure that carried the actuators and instrumentation needed for the machine's operation. The leg could telescope to change length and was springy along the telescoping axis. Sensors measured the pitch angle of the body, the angle of the hip,

the length of the leg, the tension in the leg spring, and contact with the ground. This first machine was constrained to operate in a plane, so it could move only up and down and fore and aft and rotate in the plane. An umbilical cable connected the machine to power and a control computer.

For this machine running and hopping are the same. The running cycle has two phases. During one phase, called *stance* or *support*, the leg supports the weight of the body and the foot stays in a fixed location on the ground. During stance, the system tips like an inverted pendulum. During the other phase, called *flight*, the center of mass moves ballistically, with the leg unloaded and free to move.

Control of Running Was Decomposed into Three Parts

We were surprised to find that a simple set of algorithms could provide control for this planar one-legged hopping machine. The approach was to consider separately the hopping motion, forward travel, and posture of the body. This decomposition lead to a control system with three parts:

- *Hopping.* One part of the control system excited the cyclic hopping motion that underlies running while regulating how high the machine hopped. The hopping motion is an oscillation governed by the mass of the body, the springiness of the leg, and gravity. During support, the body bounced on the springy leg, and during flight, the system traveled a ballistic trajectory. The control system delivered a vertical thrust with the leg during each support period to sustain the oscillation and to regulate its amplitude. Some of the energy needed for each hop was recovered by the leg spring from the previous hop.
- *Forward Speed.* A second part of the control system for the one-legged hopping machine regulated the forward running speed and acceleration. This was done by moving the leg to an appropriate forward position with respect to the body during the flight portion of each cycle. The position of the foot with respect to the body when landing has a strong influence on the behavior during the support period that follows. Depending on where the control system places the foot, the body will continue to travel with the same forward speed, it will accelerate to go faster, or it will slow down. To calculate a suitable forward position for the foot, the control system takes account of the actual forward speed, the desired speed, and a simple model of the legged system's dynamics. A single algorithm works correctly when the machine is hopping in place, accelerating to a run, running at a constant speed, and slowing to a stationary hop.

- *Posture.* The third part of the control system stabilizes the pitch angle of the body to keep the body upright. Torques exerted between the body and leg about the hip accelerate the body about its pitch axis, provided that there is good traction between the foot and the ground. During the support period there is traction because the leg supports the load of the body. A linear servo operates on the hip actuator during each support period to restore the body to an upright posture.

An important simplification came from breaking running down into the control of the up and down bouncing motion, the forward speed, and body posture. Partitioning the control into these three parts made running much easier to understand and led to a fairly simple control system. The algorithms implemented to perform each part of the control task were themselves simple, although none was optimized for performance. The details of the individual control algorithms are not so important as the framework provided by the decomposition.

Using the three-part control system, the planar one-legged machine hopped in place, traveled at a specified rate, maintained balance when disturbed, and jumped over small obstacles. Top running speed was about 1.2 m/s (2.6 mph). The utility of the decomposition and framework was not limited to planar hopping on one leg—the approach was generalized for controlling a three-dimensional one-legged machine, a planar two-legged machine, and a quadruped.

Locomotion in Three Dimensions

The machine just described was constrained mechanically to operate in the plane, but useful legged systems must balance themselves in three-dimensional space. Can the control algorithms used for hopping in the plane be generalized somehow for hopping in three dimensions? A key to answering this question was the recognition that animal locomotion is primarily a planar activity, even though animals are three-dimensional systems. Films of a kangaroo hopping on a treadmill first suggested this point. One observes the legs sweeping fore and aft through large angles, the tail sweeping in counteroscillation with the legs, and the body bouncing up and down. These motions all occur in the sagittal plane, with little or no motion normal to the plane.

Sesh Murthy realized that the plane in which all this activity occurs can generally be defined by the forward velocity vector and the gravity vector. He called this the *plane of motion* (Murthy 1983). For a legged system without a preferred direction of travel, the plane of motion might

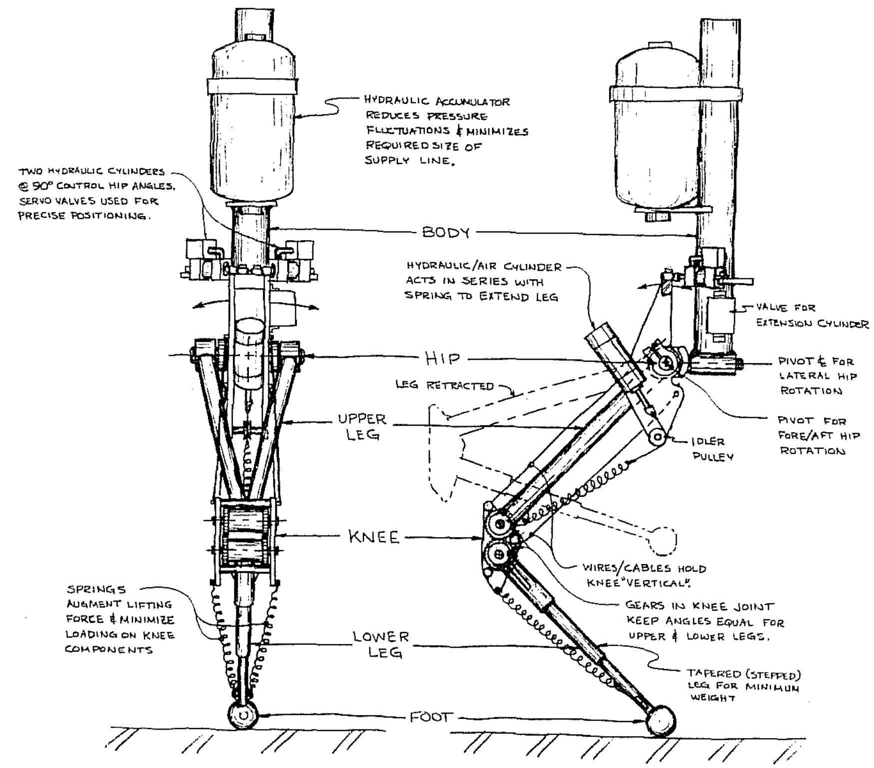


Figure 1.8. Ben Brown and I had this early concept for a one-legged hopping machine that was to operate in three dimensions. This version never left the drawing board.

vary from stride to stride, but it would be defined in the same way. We found that the three-part control system retained its effectiveness when used to control activity within the plane of motion.

We also found, however, that the mechanisms needed to control the remaining *extraplanar* degrees of freedom could be cast in a form that fit into the original three-part framework. For instance, the algorithm for placing the foot to control forward speed became a vector calculation. One component of foot placement determined forward speed in the plane of motion, whereas the other component caused the plane to rotate about a vertical axis, permitting the control system to steer. A similar extension applied to body posture. The result was a three-dimensional three-part control system that was derived from the one used for the planar case, with very little conceptual complication.

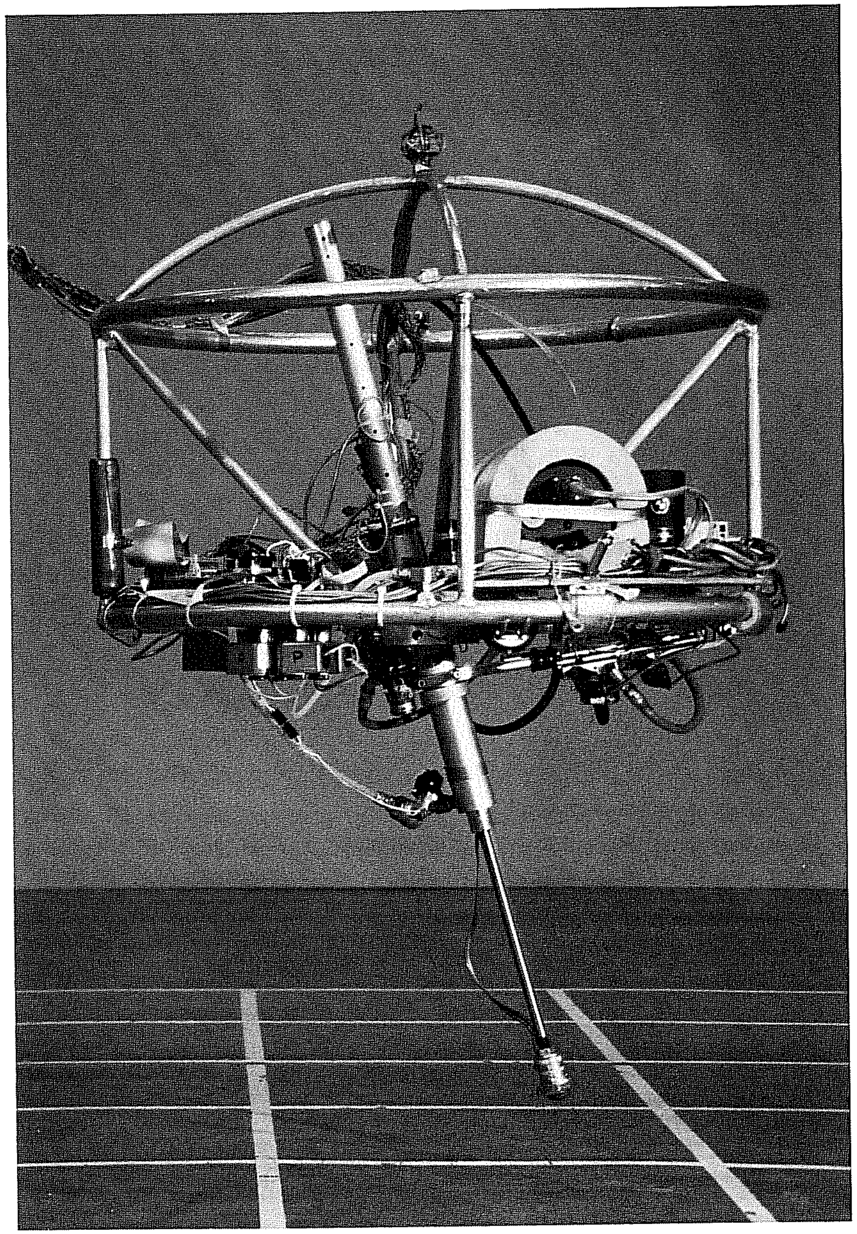


Figure 1.9. Three-dimensional hopping machine used for experiments. The control system operates to regulate hopping height, forward velocity, and body posture. Top recorded running speed was about 2.2 m/s (4.8 mph).

To explore these ideas, we built a second hopping machine, shown in figure 1.9. It had an additional joint at the hip to permit the leg to move sideways as well as fore and aft, and the machine had no external mechanical support. Otherwise, it was similar to the planar hopping machine described earlier. In operation this machine balanced itself as it hopped along simple paths in the laboratory, traveling with a top speed of 2.2 m/s (4.8 mph).

Running on Several Legs

Experiments on machines with one leg were not motivated by an interest in one-legged vehicles. Although such vehicles might very well turn out to have merit,[2] our interest was in getting at the basics of active balance and dynamics in the context of a simplified locomotion problem. In principle, results from machines with one leg could have value for understanding all sorts of legged systems, perhaps with any number of legs.

Given the successful control of machines that run and balance on one leg, can we use what we learned to understand and control machines with several legs? Our study of this problem has progressed in two steps. For a biped that runs like a human, with alternating periods of support and flight, the one-leg control algorithms apply directly. Because the legs are used in alternation, only one leg is active at a time: only one leg is placed on the ground at a time, only one leg thrusts on the ground at a time, and only one leg can exert a torque on the body at a time. We call this sort of running a *one-foot gait*. Assuming that the behavior of the other leg does not interfere, the one-leg algorithms for hopping, forward travel, and posture can each be used to control the active leg. Of course, to make this workable, some bookkeeping is required to keep track of which leg is active and to keep the extra leg out of the way.

Jessica Hodgins and Jeff Koechling demonstrated the effectiveness of this approach by using the one-leg algorithms to control each leg of a planar biped. The machine, shown in figure 1.10, has run at 4.3 m/s (9.5 mph). As one might guess, the biped can also travel by hopping on one leg, and it can switch back and forth between gaits. We found that it was very simple to extend the one-leg algorithms for two-legged running.

In principle, this approach could be used to control any number of legs, so long as just one is made active at a time. Unfortunately, when

[2] Wallace and Seifert saw merit in vehicles with one leg. Wallace (1942) patented a one-legged hopping tank that was supposed to be hard to hit because of its erratic movements. Seifert (1967) proposed the *Lunar Pogo* as a means of efficient travel on the moon.

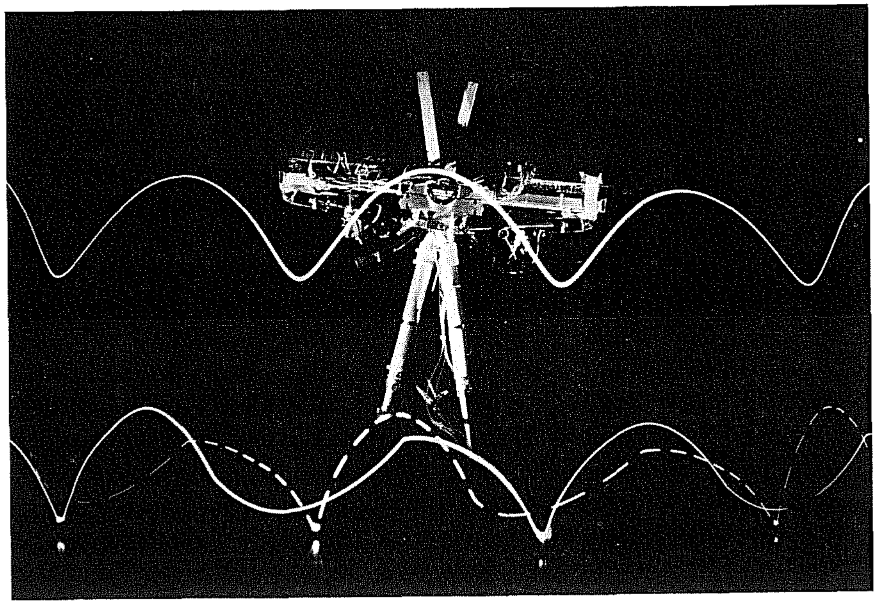


Figure 1.10. The planar biped can run with a gait that uses the legs in alternation like a human or with a hopping gait, and it can switch back and forth between gaits. The two legs counteroscillate during normal running. Top running speed was 4.3 m/s (9.5 mph). The control is based on the three-part decomposition originally used for the one-legged hopping machines. During one-legged hopping, the extra leg acts like a tail, swinging out of phase with the active leg. From Hodgins, Koechling and Raibert (1985).

there are several legs this is usually not feasible. Suppose, however, that a control mechanism coordinates legs that share support simultaneously, making them behave like a single equivalent leg—what Sutherland (1983) has called a *virtual leg*. Suppose further that more than one leg provides support at a time but that all support legs are coordinated to act like a virtual leg. One can then map several multi-legged gaits into *virtual biped one-foot gaits*. For example, the trotting quadruped maps into a virtual biped running with a one-foot gait.

We argue that the trotting quadruped is like a biped, that a biped is like a one-legged machine, and that control of one-legged machines is a solved problem. A control system for quadruped trotting could consist of a servo that coordinates each pair of legs to act like one virtual leg, a three-part control system that acts on the virtual legs, and a bookkeeping mechanism that keeps track. Figure 1.11 is a photograph of a four-legged machine that runs with precisely this sort of control system.

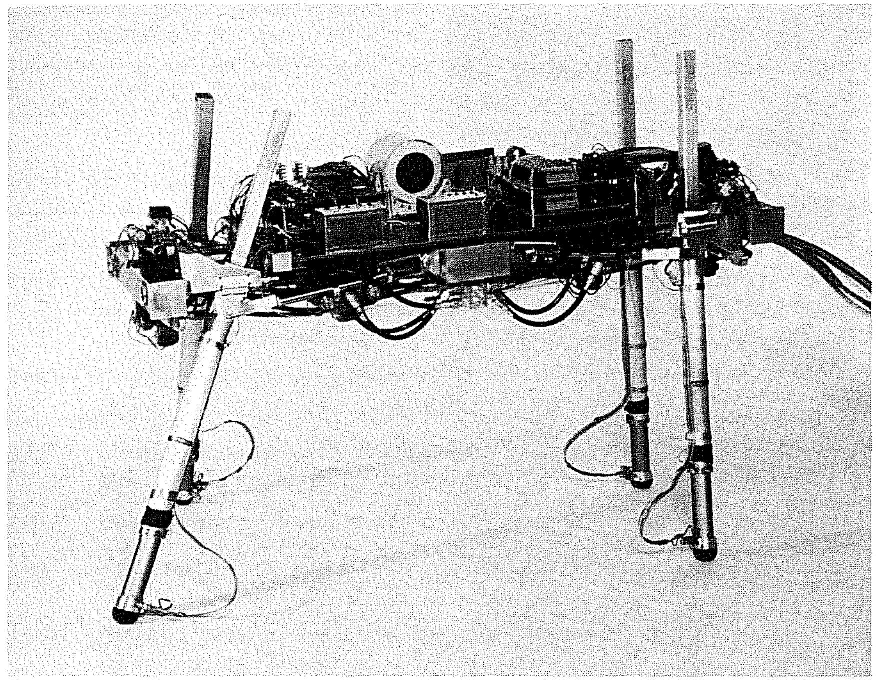


Figure 1.11. Quadruped machine that runs by trotting. *Virtual legs* are used to map trotting into biped running, which in turn is controlled with one-leg algorithms.

Symmetry in Robots and Animals

In order to run at constant forward speed, the instantaneous forward accelerations that occur during a stride must integrate to zero. One way to satisfy this requirement is to organize running behavior so that forward acceleration has an odd symmetry throughout each stride—functions with odd symmetry integrate to zero over symmetric limits.[3] This sort of symmetry was used to control forward speed in all four machines just described. It was accomplished by choosing an appropriate forward position for the foot on each step. In principle, symmetry of this sort can be used to simplify locomotion in systems with any number of legs and for a wide range of gaits.

Can the symmetries developed for legged machines help us to understand the behavior of legged animals? To find out, we have examined film

[3] If $x(t)$ is an odd function of time, then $x(t) = -x(-t)$. If $x(t)$ is even, then $x(t) = x(-t)$.

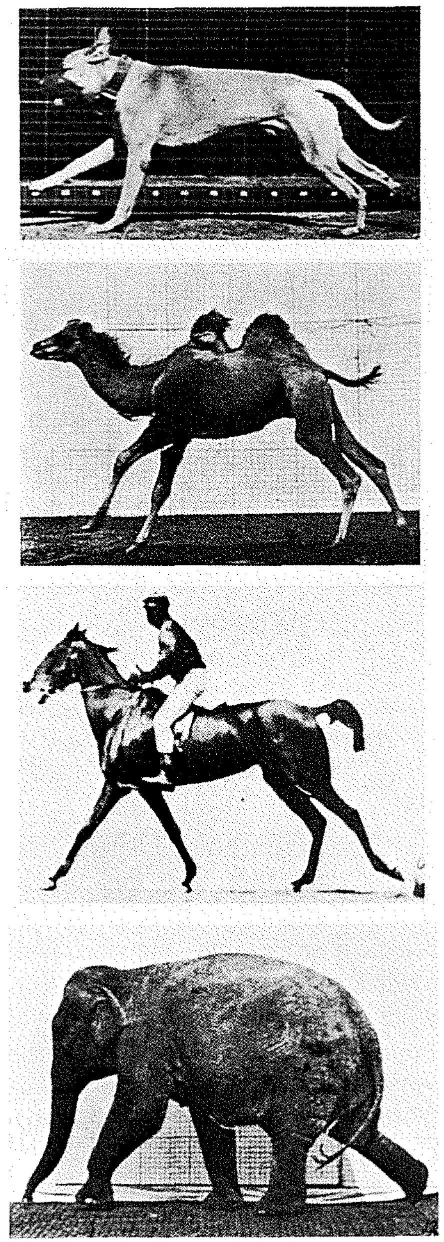


Figure 1.12. (Facing page.) Symmetry in animal locomotion. Animals shown in symmetric configuration halfway through the stance phase for several gaits: rotary gallop (top), transverse gallop (second), canter (third), and amble (bottom). In each case the body is at minimum altitude, the center of support is located below the center of mass, the rearmost leg was recently lifted, and the frontmost leg is about to be placed. Photographs from Muybridge (1957); reprinted with permission from Dover Press.

Table 1.2. Summary of progress in the CMU Leg Laboratory.

When	What
1982	Planar one-legged machine hops in place, travels at specified rate of up to 1.2 m/s (2.6 mph), tolerates mechanical disturbances, and jumps over small obstacles.
1983	One-legged hopping machine runs on open floor, balancing in three dimensions. Top speed about 2.2 m/s (4.5 mph).
1983	Murphy finds passively stabilized bounding gait for simulated quadruped-like model (Murphy 1984).
1984	Cat and human found to run with symmetry like running machines.
1984	Quadruped runs with trotting gait. *Virtual legs* permit use of one-leg control algorithms.
1985	Planar biped runs with one- and two-legged gaits and can change gait while running. Top speed is 4.3 m/s (9.5 mph). (Hodgins, Koechling and Raibert, 1985).

data for running animals and humans. In particular we have looked at a cat trotting and galloping on a treadmill and a human running on a track. The data conform reasonably well to the predicted even and odd symmetries. In some cases the data are remarkably symmetric.

Summary

Here is a brief summary of this introduction to running machines and of the following chapters (table 1.2):

- The goal of this work is to learn more about dynamics and active balance in legged locomotion, both for the purpose of building legged vehicles and to understand animal locomotion.
- A planar one-legged hopping machine can balance actively using separate control algorithms for hopping, forward speed, and posture.

- It is not much harder to provide control and balance for hopping in three dimensions than it is in two—the three-part control decomposition still applies.
- The one-leg control algorithms remain effective for biped running, requiring only some additional bookkeeping.
- Quadruped trotting can be implemented like biped running if the control system has a mechanism to coordinate pairs of legs.
- Symmetry is important for simplifying the control of legged robots and may be important in animal locomotion, too.

One caveat before ending this introductory discussion. Despite the goal of improved vehicular mobility in difficult terrain, legged vehicles have not yet proved themselves by moving out of the laboratory and into the bush. Several researchers are actively working toward this goal, but the research reported in this book avoids the issue of difficult terrain entirely. Although we take motivation from the need to travel on rough terrain, the running experiments reported here have not yet ventured beyond our very flat laboratory floor.

Additional Readings

To gain a broader background and learn more about problems, issues, and progress in legged locomotion, the following additional readings are recommended: Gabrielli and Von Karmen (1950) is the classic paper on the fundamental energetics of vehicular travel. Hirose (1984) follows up on the same theme. Bekker (1969) discusses the general problem of vehicular mobility in rough terrain in great detail. He includes a treatment of soil mechanics, an often neglected but important part of the story.

To learn more about current research on legged robots, see the special issue edited by Raibert (1984b). A companion video tape to the special issue available from the MIT Press shows several walking machines from Japan and the US in action. For a fascinating historical perspective on walking machines, see Liston (1970). To access the large body of theoretical work on legged machines, start with McGhee (1968), McGhee and Frank (1968), and Vukobratovic and Stepaneko (1972). Hemami and Golliday (1977) consider problems in control theory related to legged locomotion.

Margaria (1976) and McMahon (1984) introduce the biomechanics of animal locomotion, and Alexander and Goldspink (1977) and Hoyt and Taylor (1981) provide many interesting details. The story of how animals

use elastic storage in running is particularly relevant: Dawson and Taylor (1973), Cavagna, Heglund and Taylor (1977), McMahon and Greene (1978).

Hildebrand (1960) and Pearson (1976) provide general introductions to animal locomotion. For a collection that covers a wide range of locomotion topics in invertebrates and vertebrates, see Herman et al. (1976). Excellent reviews of research in the relevant neurophysiology are Grillner (1975) and Wetzel and Stuart (1976). Both reviews are brought up to date in the collection by Grillner et al. (1985). To learn more about how robotics and biology can interact productively, see the excellent discussion by Hildreth and Hollerbach (1985) and Marr (1976).

Chapter 2

Hopping on One Leg in the Plane

Running is like the bouncing of a ball (Margaria 1976). A ball falls under the acceleration of gravity until an elastic collision with the ground reverses its direction, sending it upward to be accelerated by gravity once again. During the collision, the ball first deforms to absorb its kinetic energy and then returns the kinetic energy as it recovers its original shape. The exchange dissipates a fraction of the energy. The overall bouncing pattern oscillates between ballistic flight phases and elastic collisions until the ball's energy is dissipated entirely.

In running the body falls ballistically until the feet land on the ground. Then the legs deform elastically to absorb the body's kinetic energy, and they return the energy a short time later to help power the next step. Although a passively bouncing ball must eventually come to rest, a legged system can sustain its oscillation indefinitely by using leg actuators to replace lost energy. Margaria used this bouncing-ball model of running to distinguish running from walking, which, he points out, is better modeled by the rolling of an egg (Margaria 1976).

Elastic storage and recovery of energy is particularly important to the efficiency of the hopping kangaroo (Dawson and Taylor 1973), but it also helps other animals, including the human, to run efficiently (Cavagna et al. 1977, Alexander and Jayes 1978). Perhaps more important than the efficiency of bouncing is the simplicity bouncing gives to the control of the locomotion cycle. Hopping can rely on the passive bouncing oscillation to generate the detailed pattern of the motion, while the control system excites the oscillation and regulates its amplitude.

In addition to bouncing like a ball, running systems tip like an inverted pendulum. An inverted pendulum has an elevated mass that pivots above a support point. When the mass is located directly over the support point, there are no tipping moments, so the system is in equilibrium. Any small displacement of the mass from directly over the support point, however, causes tipping moments that drive the pendulum further from equilibrium. The equilibrium point is unstable. A control system can provide balance for an inverted pendulum by moving the support point back and forth in response to tipping motions. Actually, the balanced inverted pendulum tips all the time, but the control system keeps it from tipping over entirely by ensuring that each tipping motion in one direction is balanced by an equal and opposite tipping motion in the other direction.

Legged systems behave like this, too; they tip about unstable equilibria when their feet are not directly under the body, and they balance by moving their feet in response to tipping motions. Unfortunately, several factors complicate the story for legged systems. For instance, legs usually change length when they are loaded by the body, so the distance between the mass and the support point varies. Also, legs often have large feet that cause the instantaneous support point to move during tipping. Perhaps most important, a system that runs can move the pivot points only when the legs are unloaded during flight. Despite these complications, the concept of an inverted pendulum and our knowledge of its behavior greatly simplifies thinking about balance in running.

In this chapter I describe a machine that runs by hopping on a single springy leg. It can bounce like a ball and tip like an inverted pendulum, making it an ideal vehicle for the study of mechanisms underlying running. With only one leg there is no need to coordinate several legs, so this difficult problem is avoided, whereas the need for active balance is central.

The control system for running that my research group has explored decomposes the control task into three separate parts that regulate hopping, forward travel, and posture. This three-part control system permits the one-legged machine to hop in place, run at a desired rate, travel from place to place, maintain its balance when disturbed mechanically, and leap over small obstacles. Top running speed is 1.2 m/s (2.6 mph). This first hopping machine is restricted to move in the plane, but a three-dimensional version is considered in the next chapter.

A Planar Machine That Hops on One Leg

Figure 2.1 shows the machine we used to study running. Its main parts are a body and a leg, connected by a hinge-type hip. It has only one leg,

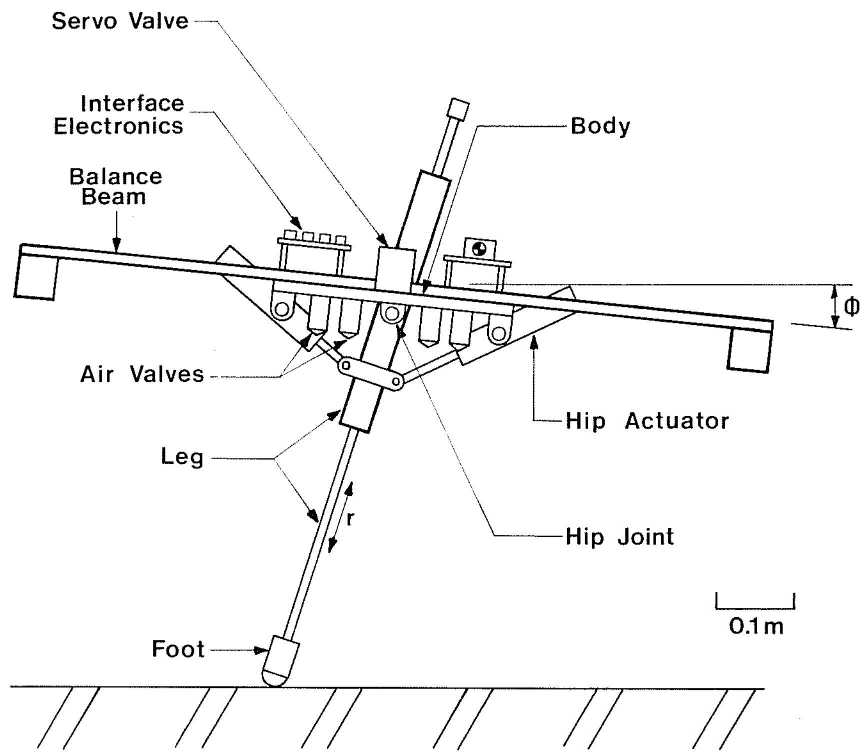


Figure 2.1. Planar one-legged hopping machine. It has two main parts: a body and a leg. The body provides mounting for valves, electronics, and sensors, and it has a weighted beam for increased moment of inertia. The leg is an air cylinder that pivots with respect to the body, with a padded foot at one end. The machine is powered by compressed air. Four on-off solenoid valves control the flow of air to and from the leg cylinder. They can trap air in the leg cylinder to make it act like a spring. A pair of pneumatic actuators exerts a torque between the leg and the body about the hip. These actuators are powered by a proportional pressure-control servo valve. Sensors mounted on the machine measure the length of the leg, the angle of the hip, contact between the foot and the ground, and the pressures in the leg air cylinder.

so hopping is the only gait it can use. The leg is springy, enabling it to use a resonant oscillation for hopping. The body consists of a platform that carries sensors, valves, actuators, and computer interface electronics.

The behavior of the hopping machine was simplified by restricting its motion to the plane. The tether mechanism shown in figure 2.2 constrains the machine to move with just three degrees of freedom; it can move fore and aft and up and down and can rotate about the pitch axis. The tether

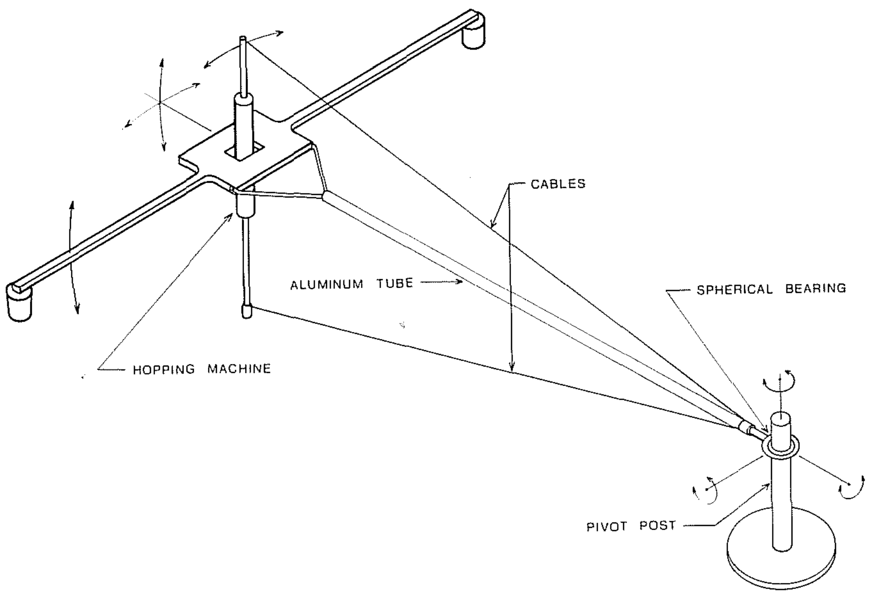


Figure 2.2. The tether mechanism constrains motion of the hopping machine to three degrees of freedom, permitting it to travel on a large circle in the laboratory. The mechanism consists of an aluminum tube, a spherical pivot fixed to the floor, a fork pivot fixed to the hopping machine, and tension cables. This arrangement keeps the machine 2.5 m from the fixed spherical pivot, giving it radial and yaw stability. A pair of nylon cables prevents motion about the roll axis. The cables also keep the foot a nearly constant distance from the spherical pivot as the leg changes length, minimizing radial scrubbing. The tether is instrumented to provide measurements of the machine's three motions: vertical translation, forward translation, and rotation about the axis of the boom.

prevents lateral translation, roll rotation, and yaw rotation. Actually the machine moves on the surface of a large sphere centered at the tether pivot. In earlier experiments the machine was constrained by air bearings that let it float on an inclined table, making its motion truly planar. But when the machine began to run at speed, it quickly traveled the full length of the table, and fell off the end. The tether permits the machine to travel on a continuous circular path with a radius of 2.5 m. No changes in the control were needed to convert from planar to spherical operation.

Sensors mounted on the tether's pivoting base measure the machine's forward position on the circle and the pitch angle of the body. The tether supports an umbilical cable that connects the hopping machine to a source

of compressed air, electrical power supplies, and the control computer.

The body and leg are connected by a hinge joint that forms a hip. A proportional pneumatic pressure-control valve drives a pair of air cylinders that exerts torques about the hip. A potentiometer measures the angle between the body and the leg, the hip angle γ. The control computer servos the hip angle with a simple linear servo:

$$\tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}), \tag{2.1}$$

where

τ	is the actuator torque generated at the hip,
γ	is the hip angle,
γ_d	is the desired hip angle,[1] and
k_p, k_v	are position and velocity feedback gains. Typical values are $k_p = 47\text{N} \cdot \text{m/rad}$, and $k_v = 1.26\,\text{N} \cdot \text{m/(rad/s)}$.

A full 40° sweep of the leg takes approximately 120 ms with a servo rate of 500 hz. The ratio of the moment of inertia of the body to that of the leg is 14:1. This relatively high ratio ensures that the orientation of the leg can change during flight without severely disturbing the attitude of the body. The center of mass of the body is located at the hip, so the only moments acting on the body are those generated by the hip actuator. Table 2.1 gives dimensions and parameters for the machine.

The Leg

The leg consists of a double-acting air cylinder with a padded foot attached to the lower end of the cylinder rod. The foot is narrow, about 20 mm when fully loaded, providing a good approximation to a point of support. The coefficient of friction between the foot and the floor in our laboratory is about 0.6, so the foot does not slip much. The foot has a switch in it that closes whenever it touches the floor. A Rube Goldberg arrangement of aircraft wire, a pulley, and a potentiometer provides a measurement of leg length r, the distance from the hip to the foot.

Four electric solenoid valves control the flow of compressed air to the leg's air cylinder (figure 2.3). The valves connect each chamber of the leg cylinder either to atmospheric pressure through a flow-restricting orifice or to a regulated supply at 90 psi. Sensors monitor the air pressures in both chambers. Delivery of pressurized air to the top chamber of the cylinder

[1] Throughout this book a variable with subscript d specifies the variable's desired value.

drives the piston and rod assembly downward, providing a vertical thrust for hopping.

The leg is made springy by trapping air in the upper chamber of the leg cylinder, with both solenoid valves closed. It is possible to regulate the air pressure in a chamber by charging it to a higher pressure than desired, then exhausting air through the flow-restricting orifice until the desired pressure is reached, and then closing the exhaust valve. The solenoid valves operate in about 10 ms, resulting in pressure regulation to about 1 psi. When the leg shortens under load, the trapped air compresses, acting like a $1/r$ spring. The effective stiffness of the spring is determined by the resting pressure in the chamber.

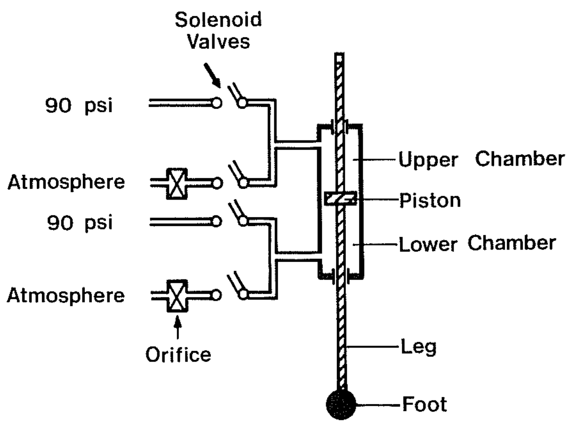


Figure 2.3. The leg actuator is a pneumatic cylinder. Electric solenoid valves control the flow of air to both chambers of the cylinder. When both valves to a chamber are off, trapped air makes the leg springy. Pressure sensors (not shown) measure the air pressure in both chambers.

Operation of the Machine

To initiate hopping, someone has to drop the machine from a shallow height onto the extended leg. Then the control system operates the solenoid valves to excite and sustain the hopping motion. The control system applies thrust by opening the supply solenoid valve to the upper chamber of the leg cylinder during each stance. The duration of this valve's operation is defined as the magnitude of thrust. Once the foot leaves the ground, the control system exhausts the upper chamber of the leg cylinder until it reaches a designated pressure, typically 15 psi. Increasing the pressure during stance

Table 2.1. Physical parameters of planar one-legged hopping machine.

Parameter	Metric Units	English Units
Overall height	0.69 m	27.3 in
Overall width	0.97 m	38.0 in
Hip height	0.5 m	19.5 in
Total mass	8.6 kg	19 lbm
Unsprung leg mass	0.45 kg	1.0 lbm
$\frac{\text{Body mass}}{\text{Unsprung leg mass}}$	18:1	18:1
Body moment of inertia	$0.52\,\text{kg}\cdot\text{m}^2$	$1770\,\text{lbm}\cdot\text{in}^2$
Leg moment of inertia	$0.037\,\text{kg}\cdot\text{m}^2$	$125\,\text{lbm}\cdot\text{in}^2$
$\frac{\text{Body moment of inertia}}{\text{Leg moment of inertia}}$	14:1	14:1
Leg Axial Motion		
Stroke	0.25 m	10.0 in
Static force	360 N @ 620 kPa	80 lb @ 90 psi
Leg Sweep Motion		
Sweep angle	± 0.33 rad	$\pm 19°$
Static torque	$27\,\text{N}\cdot\text{m}$ @ 620 kPa	$240\,\text{lb}\cdot\text{in}$ @ 90 psi

and decreasing it during flight excites the spring-mass/gravity-mass oscillator formed by the leg and the body. Peak to peak amplitude of body oscillation can be varied between 0.04 and 0.3 m, with corresponding hopping frequencies of 3 and 1.5 hops per second. Over this range of frequencies the duration of stance is nearly constant at about 175 ms, with just a few percent variation.

During the hopping cycle, accelerations of the unsprung part of the leg dissipate a fraction of the hopping energy. The mass of the unsprung part of the leg is m_ℓ, and the remaining mass of the system is m. From conservation of linear momentum we find that $m_\ell/(m_\ell+m)$ of the hopping energy is lost each time the foot strikes the ground and each time the foot leaves the ground. This assumes that collisions between the foot and the ground and collisions of the piston with the leg cylinder are plastic, with a coefficient of restitution of zero. The ratio of body mass to unsprung mass in the planar one-legged hopping machine is 18:1, resulting in an 11% energy loss for each hopping cycle. Other losses are due to friction in the leg cylinder. Under ideal testing conditions, frictional losses dissipate about 25% of the hopping energy on each bounce.

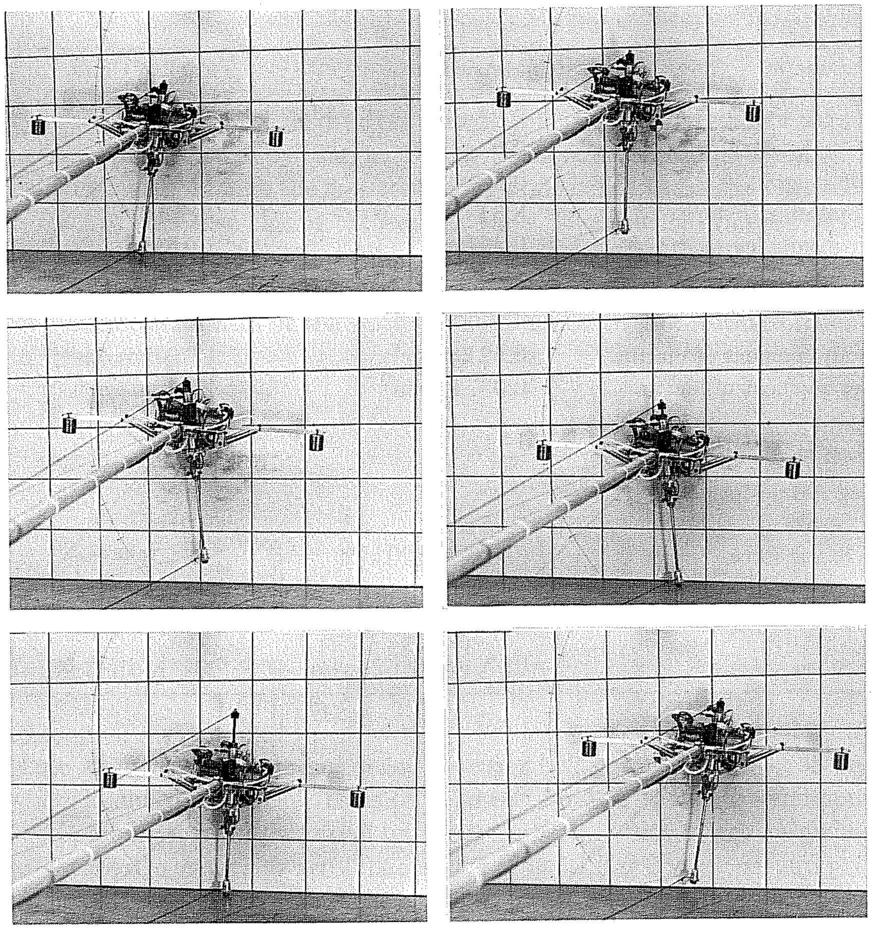


Figure 2.4. One complete stride of hopping at a speed of 0.75 m/s. Stride length was 0.45 m, stride period 0.68 s. Background grid spacing is 0.2 m. Adjacent frames separated by 100 ms. From Raibert and Brown (1984).

Figure 2.4 illustrates the general operation of the machine as it hops. The leg actuator drives the vertical bouncing motion. The control system extends the leg forward during flight according to the rate of forward travel—the faster it is going, the further forward it extends the leg. The control system also servos the hip during stance to keep the body upright. There are four events in this hopping cycle that are useful to name:

1. *Lift-off.* The moment the foot loses contact with the ground.
2. *Top.* The moment in flight when the body has maximum altitude and vertical motion changes from upward to downward.
3. *Touchdown.* The moment the foot makes contact with the ground.
4. *Bottom.* The moment during stance when the body has minimum altitude and its vertical velocity changes from downward to upward.

Control of Running Decomposed into Three Parts

The control system we explored for the planar one-legged machine treats hopping, forward speed, and body attitude as three separate control problems. One part of the control excites the hopping motion and regulates its amplitude by specifying the thrust to be delivered by the leg on each hop. The second part of the control stabilizes the machine's forward speed by extending the foot forward to a position that will provide the needed acceleration during stance. The third part of the control maintains the body in an upright attitude by servoing the hip during stance. These three parts of the control system are synchronized by a finite state machine that tracks the machine's hopping activity. By decomposing the problem in this way we rely on a weak coupling between these motions.

Control Hopping Height

In order for a legged system to operate and to make forward progress, each leg must spend some of its time supporting the weight of the body and some of its time unloaded, with the foot free to move. An alternation between a loaded phase and an unloaded phase is observed in the legs of all legged systems. For the one-legged machine the loaded phase is an elastic collision and the unloaded phase is ballistic flight, just like the bouncing ball mentioned earlier. The overall hopping behavior is an oscillation that is largely passive, with the details of the motion determined by the springiness of the leg, the mass of the body, and gravity.

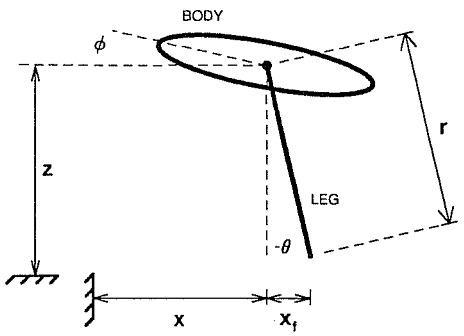


Figure 2.5. Diagram of planar one-leg machine showing variables used for control.

The control system relies on this passive mechanical oscillation to determine the form of the basic hopping motion, whereas leg thrust delivered to the body during each hop determines the amplitude. In principle, the control system could figure out how hard to thrust by comparing the energy needed to reach a desired hopping height with the actual energy, making up the difference with leg thrust. Such a calculation could take into account the kinetic energy of the body, the elastic energy of the leg spring, and the expected energy loss. This approach was quite effective in controlling a hopping model studied by computer simulation (see chapter 6), but a simpler method is used here.

The control system for the hopping machine delivers a fixed thrust during each stance phase. This causes the bouncing motion to come to equilibrium at a hopping height for which the energy injected by thrust just equals the energy lost to friction and accelerating unsprung leg mass. Because these mechanical losses are monotonic with hopping height, a unique equilibrium hopping height exists for each fixed value of thrust, and greater thrust results in greater height. The relationship between thrust and hopping height is not simple. The operator is left with the task of choosing a fixed value for thrust that results in an acceptable hopping height during a set of experiments.

Hopping data recorded from the physical one-legged machine are plotted in figure 2.6. A new thrust was specified every 5 seconds while the machine hopped in place. Each time the setpoint changed, it took the machine four or five hops for the hopping amplitude to stabilize. Data from four cycles of level hopping are replotted in the form of a phase diagram in figure 2.7. The four trajectories overlap precisely in this figure, indicating that the hopping motion was stable. The slight indentation just after

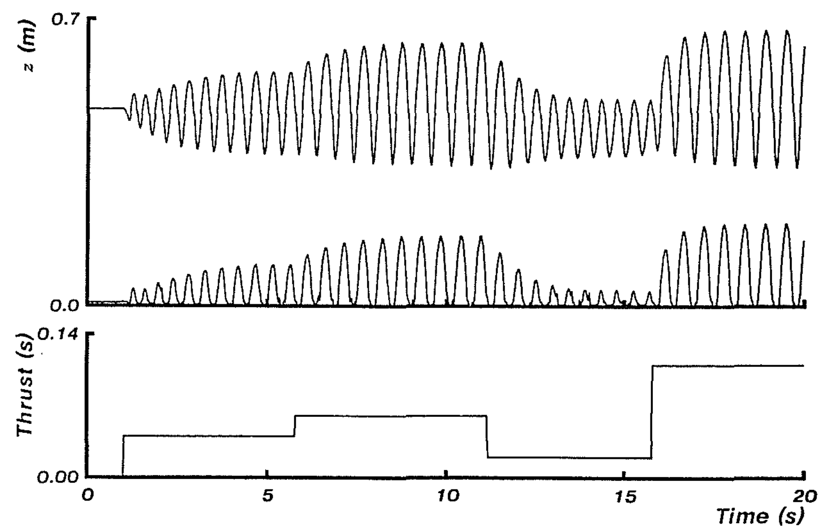


Figure 2.6. Data recorded while the machine hopped in place. Every 5 seconds the duration of vertical thrust was adjusted to change hopping height. In each case it took about 2 seconds and four cycles to arrive at equilibrium. (Top curve) elevation of the hip, z; (middle curve) elevation of the foot, $z - z_f$; and (bottom curve) duration of thrust. From Raibert and Brown (1984)

lift-off is due to the sudden deceleration of the body that occurs when the leg is accelerated to body speed. The trajectory is parabolic in the upper part of this diagram because of gravity's constant acceleration and nearly harmonic in the lower part because of the leg spring. Because the leg spring is not linear—it has a $1/r$ characteristic that makes it a *hard* spring—there is a slight deviation from harmonic behavior.

A State Machine Tracks the Hopping Cycle

An important function of the hopping motion is to provide a regular cycle of activity that synchronizes the control. A state machine keeps track of the hopping motion by switching state when sensory data indicate the occurrence of key events. A new set of control actions takes effect during each state. For example, the state machine switches from COMPRESSION to THRUST when the derivative of leg length changes from negative to positive ($\dot{r} > 0$). The action taken is to begin leg thrust and to servo the body attitude. Figure 2.8 shows the cycle of activity used for one-legged hopping, and table 2.2 provides some additional details.

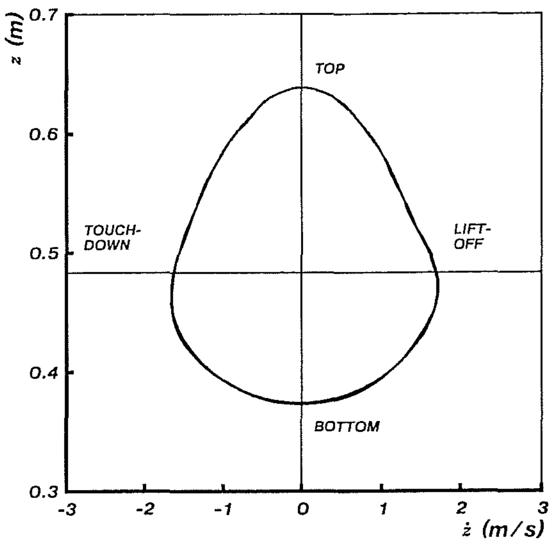


Figure 2.7. Phase plot of vertical hopping. Four cycles of level hopping at a fixed height are replotted in the phase plane. The curves cross the axes at lift-off, top, touchdown, and bottom. Note. Unlike a normal phase plot, position is plotted on the ordinate and velocity on the abscissa, so time progresses in the counterclockwise direction. From Raibert and Brown (1984).

Control Forward Speed

The position of the foot when it first touches the ground at the end of flight has a powerful influence on the accelerations that occur during the ensuing stance phase. The accelerations are like those of an inverted pendulum in that the foot's position with respect to the center of mass determines the tipping moments. In addition, the forward speed of the body influences the accelerations, as do the vertical speed and the axial leg force.

The control system manipulates these accelerations to control forward running speed by choosing a forward position for the foot before each landing. Because the leg is connected to the body, the control system can position the foot with respect to the body any time during flight in order to determine the relative position at touchdown. Once the foot is in position and the stance phase begins, the control system takes no further action for the remainder of the step—the dynamics of the mechanical system consisting of the body, the leg, and the ground govern what happens. For a wide range of conditions, the *net forward acceleration*, the difference between forward speed at touchdown and at lift-off, $\Delta\dot{x} = \dot{x}_{lo} - \dot{x}_{td}$, is a

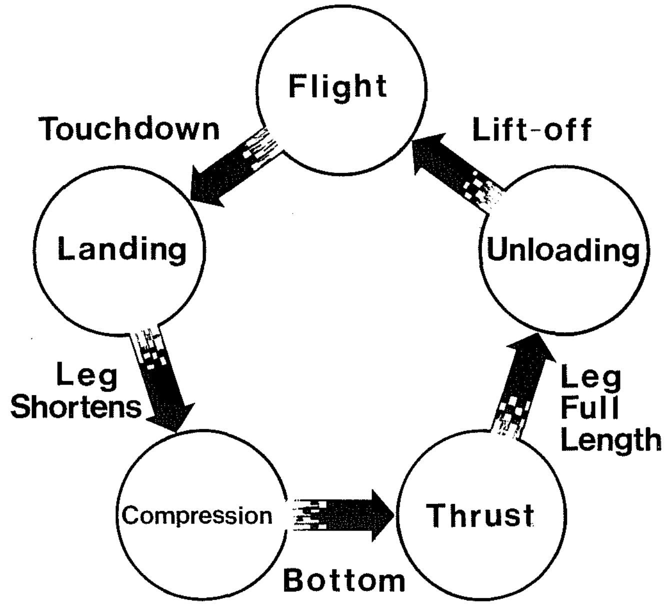


Figure 2.8. A state machine tracks the hopping behavior to synchronize the three parts of the control system. Sensory information triggers transitions between the states, and each state specifies what the control system should do.

Table 2.2. Details of state machine sequence for the hopping cycle. The state shown in the left-hand column is entered when the event in the center column occurs. The control action to be taken is shown in the column on the right. States advance sequentially during normal hopping. States LOADING and UNLOADING help to isolate the stance and flight phases from each other, as described in the text.

State	Trigger Event	Action
1 LOADING	Foot touches ground	Stop exhausting leg Zero hip torque
2 COMPRESSION	Leg shortens	Upper leg chamber sealed Servo body attitude with hip
3 THRUST	Leg lengthens	Pressurize leg Servo body attitude with hip
4 UNLOADING	Leg near full length	Stop thrust Zero hip torque
5 FLIGHT	Foot not touching	Exhaust leg to low pressure Position leg for landing

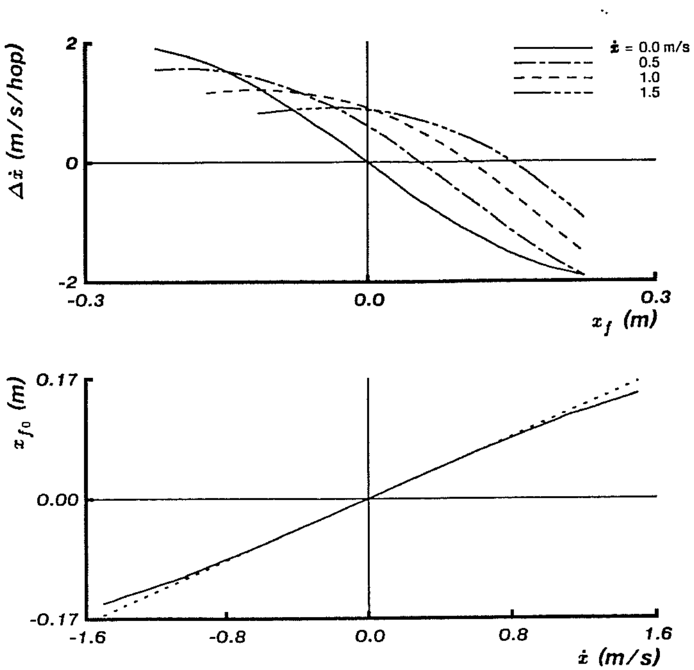


Figure 2.9. (Top) The net forward acceleration varies with forward foot position. The parameter is forward speed. Data are from simulations of a one-legged hopping machine with a linear leg spring. (Bottom) The location of the neutral point varies with forward running speed. The function is nearly linear up to about 1 m/s.

monotonic function of the forward position of the foot at touchdown. The net forward acceleration is a single number that summarizes accelerations that occur throughout the stance phase. It has units of m/s/hop. The forward acceleration is assumed to be zero throughout flight,[2] so accelerations during the stance phase control speed.

For each forward speed there is a unique foot position that results in zero net forward acceleration. We call this the *neutral point*, designated x_{f0}. For hopping in place with no forward travel, the neutral point is located directly under the body, but for nonzero forward speed it is located in front of the body in the direction of travel. The faster the running the further forward the neutral point, as shown in figure 2.9.

[2] Throughout this book I ignore the effects of air resistance during both stance and flight. See Pugh (1971) for measurements of wind drag on humans in walking and running.

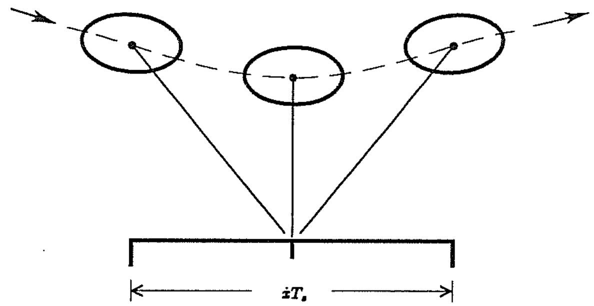


Figure 2.10. Symmetric trajectory. When the foot is placed in the neutral position, there is a symmetric motion of the body with respect to the foot. The figure shows the configuration just before the foot touches the ground (left), the configuration halfway through stance when the leg is vertical and maximally compressed (center), and the configuration just after the foot loses contact with the ground (right). The forward position of the body, the angle of the body, and the angle of the leg have odd symmetry, $x(t) = -x(-t)$, $\phi(t) = -\phi(-t)$, $\theta(t) = -\theta(-t)$, whereas the vertical position of the body and leg length have even symmetry, $z(t) = z(-t)$, $r(t) = r(-t)$. Time and position are defined so that $t = 0$ halfway through the stance phase, and $x(0) = 0$. The locus of points over which the center of gravity travels during stance is called the *CG-print*. It is shown by the horizontal bar at the bottom of the diagram.

Symmetry and Asymmetry

When the foot is placed on the neutral point, the body's center of mass travels over the foot during stance with a symmetric motion described by even and odd functions of time. The schematic in figure 2.10 shows this kind of symmetric behavior. When the system moves with symmetry, the center of mass spends the same amount of time in front of the foot as it spends behind the foot, so forward tipping that occurs during the second half of stance precisely compensates for the backward tipping that occurs during the first half of stance. The horizontal components of the axial leg force also balance because the leg is maximally compressed at the same time the foot is located under the center of mass. This assumes that the axial leg force $f(t)$ is an even function of time during stance, which would be the case if the vertical bouncing motion of the body on the leg were passive with neither losses nor thrust. The forward speed does not change because the horizontal forces acting on the body throughout the stance phase average to zero. Another way to say this is that for a symmetric body motion the tipping moments and horizontal ground forces are odd functions of time during stance. Odd functions integrate to zero over symmetric limits, providing zero net acceleration.

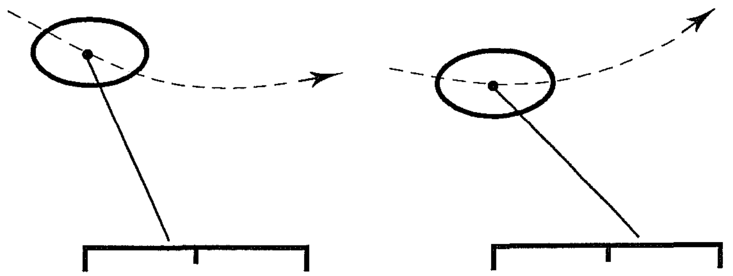


Figure 2.11. Asymmetric trajectories. Displacement of the foot from the neutral position accelerates the body by skewing its trajectory. When the foot is placed behind the neutral point, the body accelerates forward during stance (left). When the foot is place forward of the neutral point, the body accelerates backward during stance (right). Dashed lines indicate the path of the body, and solid horizontal lines under each figure indicate the CG-print.

Displacement of the foot from the neutral point results in body trajectories that are no longer symmetric, as illustrated in figure 2.11. They are skewed according to the sign and magnitude of the foot displacement. The skewed trajectories have nonzero net forward acceleration of the body, and the forward speed changes as a result. By placing the foot forward of the neutral point, the control system creates a net rearward acceleration that slows the machine down. By placing the foot behind the neutral point it creates a net forward acceleration that speeds the machine up. Figure 2.9 shows the functional relationship between the net forward acceleration and displacement from the neutral point. The relationship is nearly linear for small displacements at a single forward speed. Figure 2.12 shows the path of the body during stance for the different foot positions described.

An Algorithm for Foot Placement

To regulate forward speed, the control system must calculate a forward position for the foot based on the state of the machine and the desired behavior. There are several ways to solve this problem. One is to solve the equations of motion for the system to find expressions for the state variables as functions of time. These solutions could be inverted to express foot position as a function of state and desired behavior. A control system could plug the present and desired states of the system into such closed-form solutions to calculate the required forward foot placement. Unfortunately, analytic solutions to the differential equations that describe mechanical systems are known only rarely and in most cases analytic solutions do not even exist. Closed form expressions relating forward foot placement to net

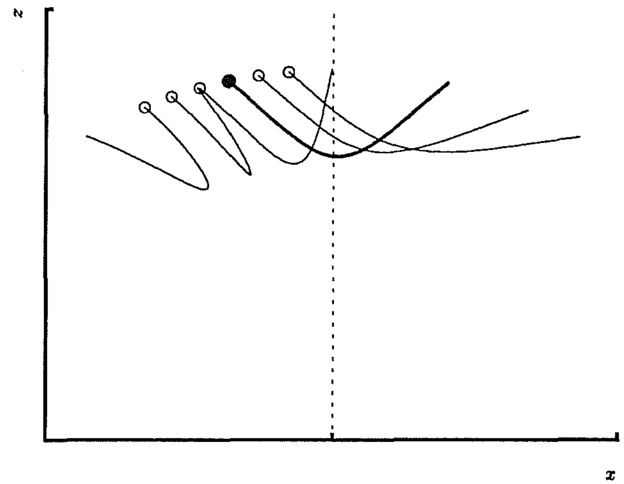


Figure 2.12. Path of the body during stance for several forward foot positions. Only the neutral foot position results in a symmetric body trajectory (bold), whereas those to either side are skewed, either forward or backward. The initial forward speed is the same for each trajectory. The circles indicate the location of the body at touchdown, and the origin is the foot position. These data are from simulations of a model with a linear leg spring. Adapted from Stentz (1983).

forward acceleration for the one-legged machine are not known.

A second approach would be to simulate numerically a large enough set of situations so that the results could be tabulated to provide approximate solutions. We have explored this technique for a simple legged model with encouraging results that are described in chapter 7.

A third approach, the one taken here, is to use closed-form approximations to the solutions. The control system we implemented uses rather crude but simple approximations to estimate the location of the neutral point and to choose a forward position for the foot. Despite several shortcomings, these approximations have proven to be quite effective.

Two factors enter into the calculation of forward foot position as implemented. The measured forward speed is used to approximate the location of the neutral point. The error in forward speed is used to calculate a displacement from the neutral point to accelerate the system. The neutral point and the displacement combine to specify how the control system places the foot.

To calculate the neutral point, the control system estimates the locus of points over which the center of gravity will travel during the next stance phase. We call this locus the *CG-print*; it is analogous to a footprint. From figure 2.10 we see that the center of the CG-print is the neutral point. With the foot located in the center of the CG-print, the values of leg angle and forward body position at touchdown are equal but opposite in sign to their values at lift-off. This satisfies the symmetry described earlier. The length of the CG-print is approximately the product of the forward speed and the duration of stance, $\dot{x}T_s$. To place the foot in the center of the CG-print, the control system extends the leg forward during flight so that the foot is a distance in front of the hip:

$$x_{f0} = \frac{\dot{x}T_s}{2}, \tag{2.2}$$

where

x_{f0}	is the forward displacement of the foot with respect to the the center of mass,
$\dot{x}$	is the forward speed, and
T_s	is the duration of the stance phase.

Because a spring mass system oscillates with a period that is independent of amplitude, the duration of the stance phase is nearly constant for a given leg stiffness. The control system uses the duration of the previous stance phase as the expected duration for the next stance phase. To the extent that the body continues to travel forward during stance with speed $\dot{x}$ and to the extent that the compression of the leg has even symmetry, (2.2) places the foot at the neutral point to provide unaccelerated travel.

To accelerate the machine the control system introduces asymmetry. Acceleration is needed to stabilize the forward speed against errors and external disturbances and to change from one forward speed to another. To accelerate the machine on purpose, the control system displaces the foot from the neutral point (see figure 2.11). The control system uses a linear function of the error in forward speed to find a displacement for the foot:

$$x_{f\Delta} = k_{\dot{x}}(\dot{x} - \dot{x}_d), \tag{2.3}$$

where

$x_{f\Delta}$	is the displacement of the foot from the neutral point,
$\dot{x}_d$	is the desired forward speed, and
$k_{\dot{x}}$	is a feedback gain.

Combining (2.2) and (2.3) yields the algorithm for placing the foot:

$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d). \tag{2.4}$$

Once the control system calculates x_f, kinematics are used to find the required hip angle (see figure 2.5):

$$\gamma_d = \phi - \arcsin\left(\frac{\dot{x}T_s}{2r} + \frac{k_{\dot{x}}(\dot{x} - \dot{x}_d)}{r}\right). \tag{2.5}$$

where γ is the angle between the leg and the body. The servo given by (2.1) drives the hip joint. This algorithm for placing the foot controls forward speed and accelerations when hopping in place, accelerating to a run, running with constant speed, and slowing to a stop. The process of choosing a foot position to control acceleration is the primary mechanism used for balance.

Control Body Attitude

The control system maintains an upright body attitude by exerting torques about the hip during stance. Because angular momentum is conserved during flight, the stance phase provides the only opportunity to change the angular momentum of the whole system. Friction between the foot and the ground during stance permits torques to be applied to the body without causing large accelerations of the leg. These torques are used to servo the body to the desired attitude. The control system does this with a linear servo:

$$\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}), \tag{2.6}$$

where

τ	is the hip torque,
ϕ	is the pitch angle of the body, and
k_p, k_v	are position and velocity feedback gains. Typical values are $k_p = 153\,\mathrm{N \cdot m/rad}$, and $k_v = 14\,\mathrm{N \cdot m/(rad/s)}$.

Friction keeps the foot from slipping on the ground. Its magnitude is proportional to the normal force. Precautions were taken to ensure that there is adequate normal force to hold the foot in place when hip torques are used to correct the body attitude during stance. Two states, LOADING and UNLOADING, were added to the state machine that synchronizes the control system to the hopping behavior. These states prevent the body attitude servo from operating when the leg just begins to accept load after touchdown and when it is nearly unloaded just before lift-off. I think of these as *twilight states* because they suggest that the machine is neither fully in stance nor fully in flight. Another twilight state, ESCAPE (not shown in the diagram), keeps the control system from moving the leg forward just after lift-off until the foot has attained sufficient altitude to clear the ground. Premature movement of the leg would stub the toe.

To summarize this section, the control system operates as three separate parts. One part regulates the hopping motion by delivering a thrust to the body during each support phase. The second part of the control manipulates forward speed by choosing a position for the foot that will provide the required net forward acceleration during the next stance phase. The third part servos the body to an upright posture when friction holds the foot in place during stance. We now turn to experiments to evaluate the approach.

Hopping Experiments

The one-legged hopping machine was used to explore the workability of the three-part control decomposition and to demonstrate balance in a dynamic legged machine. The algorithms for hopping, forward speed, and body attitude and the finite state machine were implemented in a set of computer programs that ran on a minicomputer. These programs controlled the hopping machine and recorded its behavior.

Rate Control

To test regulation of forward speed, the control computer specified a staircase of desired values over a 10-second interval. Before the interval began, the machine hopped in place, with desired forward speed specified from a joystick. The results from the test are plotted in figure 2.13. The machine started by hopping in place, then increased speed up to about 0.9 m/s, then held speed, and finally came to a stop. Throughout the test, error in forward speed was controlled to about ± 0.25 m/s. This accuracy is typical. It was possible to improve the regulation of forward speed at any given speed by adjusting the velocity error gain, $k_{\dot{x}}$ in (2.4). The need for this adjustment has already been suggested by figure 2.9, which shows that the relationship between foot displacement and net forward acceleration depends on forward speed.

During running the leg and body counteroscillate, as shown in the plots of θ and ϕ in figure 2.13. Oscillations of the body are expected because angular momentum must be conserved during flight and because

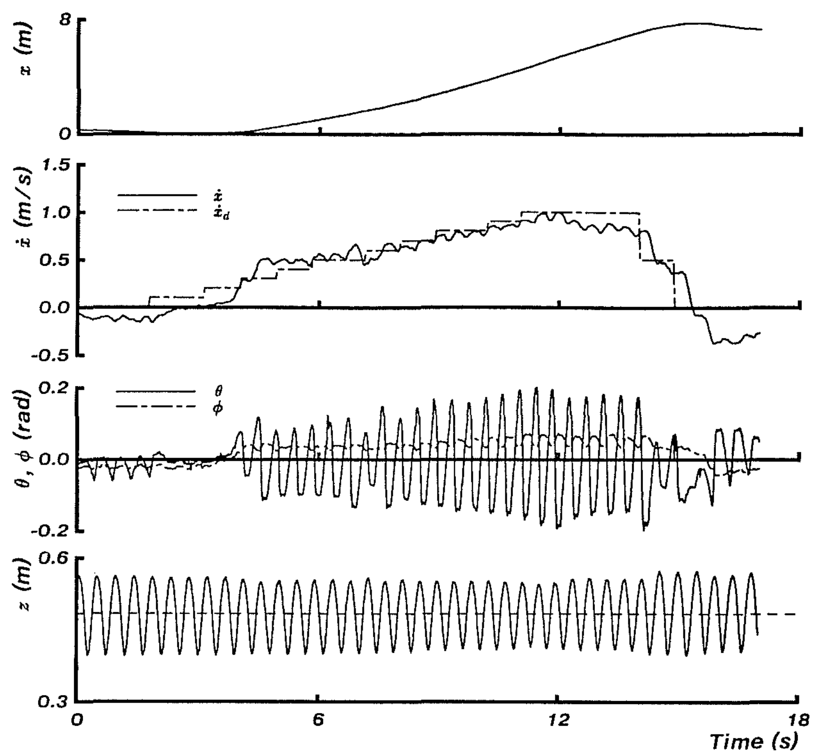


Figure 2.13. Control of forward running speed was tested by varying $\dot{x}_d$, the rate setpoint (dashed-dotted line), from 0 to 1.0 m/s over a 10 s period. Also shown are the forward position of the machine x, the body pitch angle ϕ, the vertical position of the body z, and the leg angle θ. The dashed line on the plot of z (bottom curve) separates stance (below line) from flight (above line). From Raibert and Brown (1984).

the attitude of the body is corrected only during stance. The average body pitch angle deviates from zero in rough proportion to running speed, as indicated by the plot of ϕ in figure 2.13. Hopping height and stride frequency are also affected by running speed. Actually, the relevant factor is not running speed directly but the angle of the leg at touchdown. Faster running results in large deviations of the leg from vertical and therefore shallower hops. These shallower hops have shorter flight time and result in more rapid stepping. When the machine runs at 0.9 m/s, the peak foot clearance is reduced by 20% and stride period is reduced by 8.6%.

During running the leg sweeps back and forth like the legs of running animals. In the case of the hopping machine, these motions were not explicitly programmed. They emerged as a by-product of the interplay between the control of forward speed, which moves the foot to a forward position during flight, and the control of body attitude, which permits the body to coast past the foot during stance.

Position Control

Position control was used to make the machine stay in one place or to translate from place to place. A position controller was built on top of the three-part control system by transforming position errors into desired forward speeds:

$$\dot{x}_d = \min\left[k(x - x_d), \dot{x}_{max}\right], \tag{2.7}$$

where x_d is the target location and $\dot{x}_{max}$ limits the maximum rate of travel when the machine is far from the target. Target positions were sometimes specified by hand with a joystick and sometimes by the control computer according to a programmed sequence. Data obtained from the machine under position control are plotted in figure 2.14. The machine stayed on the specified location with less than ± 0.1 m error.

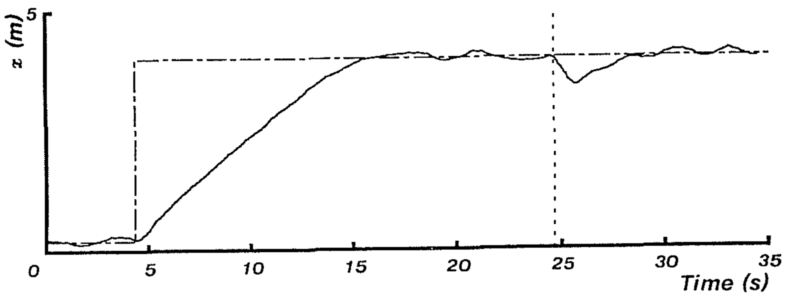


Figure 2.14. Position control. Position errors were transformed into rate setpoints to control the machine's forward position. After 4.3 seconds of stationary hopping, the computer specified a 4-m change in desired position (dot-dashed line). A limit cycle of about ± 0.1 m is present whenever the control system keeps the machine in one place. The experimenter disturbed the machine by delivering a sharp horizontal jab by hand (vertical dotted line). It returned to the setpoint within a few seconds. From Raibert and Brown (1984).

Also shown in figure 2.14 is the response to an external mechanical disturbance. After about 25 seconds the experimenter delivered a sharp horizontal jab to the body as the machine hopped in place. The machine recovered its balance and returned to the commanded position after a few seconds. The control system tolerated fairly strong disturbances, provided

that the forces were primarily horizontal. Disturbances that introduced large rotations of the body usually caused the machine to tip over.

Leaping

A specialized version of the hopping part of the control system makes the machine leap. To demonstrate leaping, the machine approached a small obstacle with a moderate running rate. One step before the obstacle the operator pressed a *leap* button, initiating a preplanned sequence that was synchronized to vertical hopping by the state machine. The sequence began at the start of the next stance phase:

1. Thrust is delayed so that the leg shortens more than normal under load of the body. This is done to prepare for a hop of maximum height. Once thrust begins, it continues until the leg extends fully.
2. Once airborne, the leg shortens, and the sweeping motion is delayed; both provide extra clearance for the foot.
3. At the peak of the hop the leg swings to the correct landing angle. There is less time to swing the leg than normal, but the shorter leg moves more quickly because of the reduced moment of inertia.
4. The leg lengthens in preparation for landing.
5. Upon landing, the standard hopping sequence is re-established.

The control system uses the standard forward speed and body attitude algorithms during leaping.

This procedure was used to leap over stacks of styrofoam blocks, as shown in figure 2.15. Although many leaps were successful, at least as many were failures. The general task of jumping over an obstacle requires that the foot be placed in a suitable location on the ground relative to the obstacle on the step before the leap, that the leap have sufficient altitude, and that the leap have sufficient span. The existing control system does a good job with height and span, but it cannot manipulate the takeoff point.

The task of placing the foot on a particular location is more demanding than merely controlling the forward speed as the existing control system does. In order to position the foot for takeoff, it is necessary to control the stride during a number of steps before the leap. This can be accomplished by adjusting the forward speed while approaching the obstacle, by adjusting leg stiffness, or by adjusting the height of each hop. The need to position feet on specific footholds is an important part of the larger problem of traveling on rough terrain.

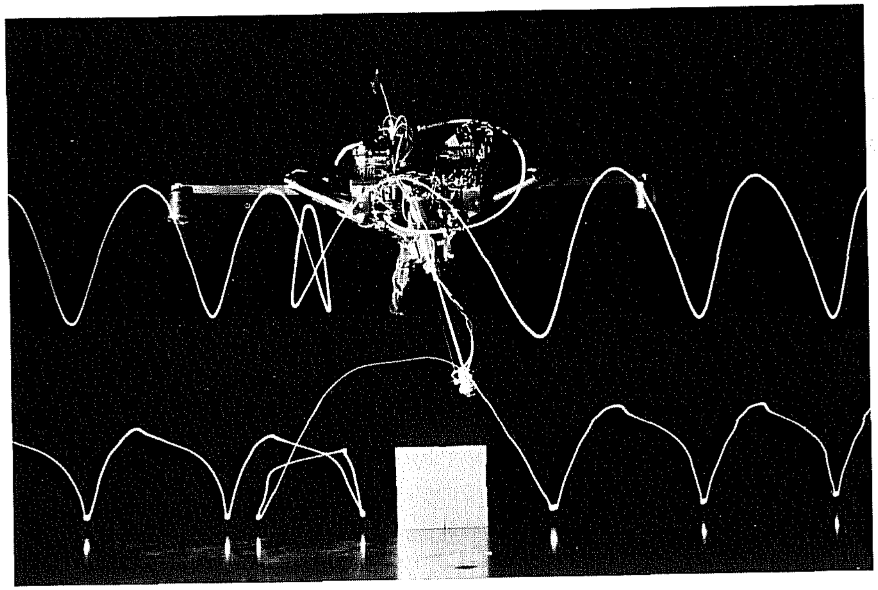


Figure 2.15. Hopping machine leaping over an obstacle. The machine approaches from right and continues to the left after the leap. Lights indicate the paths of the foot and hip. The sequence of operations used to make the leap is described in text. The obstacle is a stack of styrofoam blocks, 0.19 m tall and 0.15 m wide.

Improvements and Limitations

Each part of the control system described in this chapter applies a simple algorithm to a part of the overall locomotion task. None of the particular algorithms is sophisticated, and none were tuned for high performance. In fact, the purpose has been to focus on the shape of the problem and to identify an overall framework within which algorithms with well-defined goals can operate. With a framework and a working control system in place, the task of refining and optimizing the details of the individual components should be straightforward. None of our work has yet focused on this sort of tuning, but the following suggests a few things that are at the top of the list.

The algorithm that controls body attitude produces an asymmetrical oscillation in body attitude, as shown in figure 2.13. The average body angle deviates from zero in rough proportion to running speed, with a sudden uprighting of the body during the first part of stance. Ben Brown has suggested that it should be possible to reduce the asymmetry in these

body oscillations, eliminate the large error at touchdown, and in general, reduce the amount of work required of the servo that controls body attitude. This could be accomplished by designing the control system to permit the body to pitch back and forth in counterrotation with the leg. The idea is to control the average orientation of the body and leg, rather than the body attitude itself. The behavior would be described by

$$J\phi + J_\ell\theta = 0, \tag{2.8}$$

$$J\dot{\phi} + J_\ell\dot{\theta} = 0, \tag{2.9}$$

where J and J_ℓ are the moments of inertia of the body and leg about the hip. For steady-state undisturbed behavior, the control system would not have to exert any torque on the body. The passive pitching behavior would be the same as the nominal pitching behavior.

The approximations used by the forward speed control, both to estimate the length of the CG-print and to generate accelerations, are somewhat crude. The CG-print estimate works fine for a stiff leg and for low forward running speeds because both keep the leg nearly vertical all the time, but the estimate deteriorates when there are large excursions in leg angle. In this latter case, the axial leg force first slows down the body during the first part of stance and then accelerates it, resulting in an average forward speed that is less than the speed during flight and in a CG-print that is shorter than expected. The effect is a steady-state error in running speed that increases with both increasing forward speed and decreasing leg stiffness. Stentz (1983) proposed a more accurate prediction that should reduce the error in predicting the length of the CG-print. It takes into account the vertical speed of the body at touchdown and a model of the leg. The functional relationship between the desired net acceleration and the displacement of the foot, (2.3), is another prime candidate for improvement.

Another example of an optimization that could improve performance is what I call *ground speed matching*. When running at high speed, the foot should not merely be left motionless during touchdown but should accelerate backward with respect to the hip before contact until it is stationary in space. This matches the foot's backward speed to the ground's backward speed before touchdown. At lift-off the foot should continue moving backward until it is fully unloaded. Running animals match their feet to ground speed in this way, but the hopping machine does not. Matching the foot and ground speed at touchdown requires accurate synchronization of the foot's backward acceleration with the precise moment of touchdown.

A difficult problem in locomotion is to measure the external state of the system, such as the position, speed, and orientation of the body. The problem is that there is no permanent place to attach the sensors. For the implementation reported in this chapter, the tether mechanism was instrumented to provide this information. The pitch angle is measured by an electro-optical sensor mounted on the pivoting base of the tether, and the forward position of the body is measured by a potentiometer mounted on the pivot. In a sense, we cheated, because a real vehicle must perform these sensing functions with onboard instrumentation and the tether is not onboard. On the other hand, the planar hopping machine is not a prototype vehicle, but an apparatus for experiments. In any case, in the next chapter I described techniques that solve some of these external sensing problems in a more satisfying way.

Unlike natural legs that fold, the one-legged machine described in this section and the machines described later on in this book all use legs that telescope to change length. Is this important? In terms of the geometry needed to place the foot on a foothold, both sorts of legs have similar capabilities. Both telescoping and folding legs can also be designed to deliver equivalent forces to the body and the ground. The difference comes when considering the dynamics of the leg motion itself. For the intermediate-level view of locomotion we are considering here, these details are not too important. But when one begins to optimize the leg motion, these details will be important. An example is Mochon and McMahon's (1980) study of human leg motion during walking. They found that the leg acts like a compound pendulum that swings freely to move forward. When modeling this level of detail for a particular legged system and when concerned with performance and efficiency, a telescoping leg will not do. However, we find that telescoping legs capture a large part of what is important in legged locomotion while avoiding some of the complication—they are easier to model and to build.

The three-part control system emphasizes the separate actions of controlling the vertical bouncing motion, forward travel, and the attitude of the body. Although there are interactions between these activities, we have found that the dynamics are sufficiently separate to permit the control system to treat them separately—each part of the control system behaves as though it affects only the one variable it is supposed to control, and interactions show up as disturbances. This independence results in a particularly simple control design that is effective when the machine hops in place, translates from one point to another, accelerates to change running speed, and leaps.

An important characteristic of the control system for locomotion that we implemented is its once-per-hop method of operation. The controls for hopping and forward speed take action just once during each hopping cycle, ignoring the servos that work at the joint level. For instance, the control system positions the foot with respect to the center of mass at touchdown. If the forward speed is in error during a step, no action can be taken to correct it until the next step, when the foot lands on the ground again. The step becomes the basic unit of control. A similar description applies to the delivery of leg thrust that drives the hopping motion. This approach to control requires that the control system incorporate knowledge about the intrinsic mechanical behavior of the machine, so that acceptable behavior occurs within each cycle, between control actions. The tipping of an inverted pendulum and the bouncing of a ball represent this sort of knowledge.

The primary reason for using a one-legged apparatus was not to lay the groundwork for a one-legged vehicle. It was to focus on the general problem of active balance in dynamic legged locomotion in a way that could later be generalized for multilegged systems. If we ignore the third dimension, generalizing from the one-legged machine to the kangaroo hopping on two legs is straightforward. A direct comparison can be made between the motions of the hopping machine's one leg and kangaroo's pair of legs. The primary difference is that the kangaroo uses its tail to help compensate for the large sweeping motions of the legs so that the body need not react by pitching so much on each hop. A control system for a kangaroo might still regulate hopping height, body attitude, and velocity as before.

The generalization to multilegged systems that do not hop is also not too hard to imagine. Many characteristics of the running biped are similar to the running of a one-legged machine, including the alternation between stance and flight, the regular vertical oscillations, and the support provided by one leg at a time. In the case of the biped, the two legs always sweep in opposite directions, making rotations of the body unnecessary even without a tail. Think of a biped as a hopping machine that substitutes a different leg on each stride. The three-part decomposition can be employed as before. For a limited set of gaits, this approach can also be used to control running in the quadruped. The details of adapting these one-leg techniques and algorithms to multilegged systems are the topic of chapter 4.

Summary

In this chapter I described a machine that uses a particularly simple form of running: hopping on one leg. Study of this machine was motivated by three points: the importance of balance, the requirement that the legs be springy, and the difficulty of leg coordination. We found that control of the one-legged hopping machine can be decomposed into three separate parts. One part controls hopping height by delivering a fixed leg thrust during each hopping cycle. A second part of the control system regulates the forward rate of travel by placing the foot a specified distance in front of the hip as the machine approaches the ground on each step. The third part of the control system corrects the attitude of the body by servoing the hip during stance. A state machine provides the glue that synchronizes the control actions to the ongoing hopping behavior. The control system that results from the decomposition is simple:

Hopping:

Thrust for specified duration during stance.

Exhaust to specified pressure during flight.

Forward Speed:

Choose foot position $x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d).$

Convert to hip angle $\gamma_d = \phi - \arcsin\left(\frac{x_f}{r}\right).$

Servo hip angle $\tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}).$

Body Attitude:

Servo body angle $\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}).$

Experiments show that these algorithms provide good control of the machine. They maintain consistent hopping height, reaching equilibrium after a change within a few hopping cycles. The machine can run at speeds of up to 1.2 m/s, with speed regulation to about ±0.25 m/s, and can travel from place to place. A modification to the hopping control enabled the machine to leap over small obstacles.

Chapter 3

Hopping in Three Dimensions

At first glance, the running of an animal, say a horse, a human, or a kangaroo, appears to be a planar activity. The legs swing fore and aft while the body bobs up and down. Depending on the gait and the animal, the body may also pitch back and forth. Motions of the legs propel the animal forward and upward so the feet can recover to new footholds further along the path of travel, and they allow the animal to balance itself so that it does not tip over. Despite the appearance of planarity, however, animal locomotion takes place in three dimensions, where motions occur with six degrees of freedom.

The appearance of planarity in animal running led us to wonder if the techniques used for locomotion in the plane could be extended for locomotion in three dimensions. The dynamics for straight line running might be largely determined by motion occurring in the sagittal plane, with negligible influence from motion normal to the plane. If so then control systems for locomotion in three dimensions could avoid the complexity of three-dimensional dynamics.

To consider this question, we built a second one-legged hopping machine that operates without external support. It balances itself as it travels freely about the laboratory. It can hop in place, travel from point to point under velocity or position control, and maintain its balance when pushed (figure 3.1). The techniques used to control this machine are direct extensions of those described in the previous chapter for planar hopping, with surprisingly little extra complication. In particular, they preserve the decomposition of running into vertical hopping, forward travel, and posture. Control of this three-dimensional machine is the topic of this chapter.

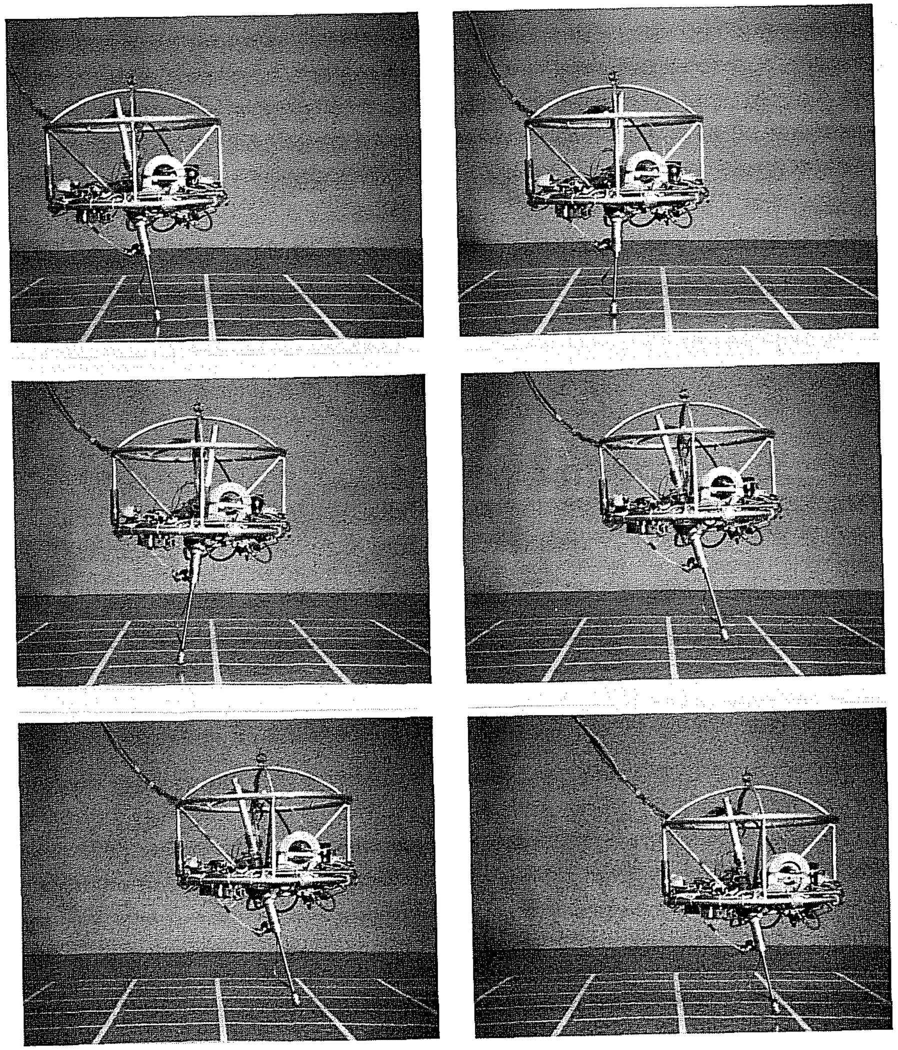


Figure 3.1. Hopping in three dimensions. Sequence of photographs showing the 3D one-legged hopping machine during a complete stride. Travel is from left to right. Running speed is about 1.75 m/s, with stride length 0.63 m and stride period 380 ms. The grid indicates 0.5-m intervals. Adjacent frames are separated by 76 ms. From Raibert, Brown, and Chepponis (1984).

Balance in Three Dimensions

During each cycle of locomotion the feet collide with the ground providing an opportunity to maintain and modify the forward velocity. In the last chapter I described how the forward position of the foot at the beginning of this collision, the stance phase, is used to control the planar hopping machine's forward speed. The approach is to boil down the details of the time-varying behavior during stance into a single outcome, the net forward acceleration. An entire cycle of stepping is treated as the basic unit of speed adjustment. The control system finds a suitable location to place the foot on each step by combining a neutral component with an acceleration component.

In three dimensions the task of controlling forward velocity and of providing balance for a one-legged system is essentially the same as it was in two. Broadly speaking, zero net acceleration results when the foot is positioned so that it is *under* the body during support. The difficulty lies in predicting where the body will be and how it will move during the stance phase. This is the same prediction problem faced in planar locomotion. Ideally, these predictions would be made by solving the equations of motion for a model of the system, and then expressing foot position as a function of the present state and the desired acceleration. Even for very simple models, however, analytic solutions to the equations of motion are not known and may not exist. The control system for the 3D one-legged hopping machine uses algorithms based on simple approximations to the solutions.

The neutral point has the same general meaning as before, but its location is specified by a vector with components that span the horizontal plane. When the foot lands on the neutral point, the body traverses a symmetric trajectory during support, with zero net acceleration. The speed and heading remain unchanged from one step to the next. When the foot lands on a point some distance from the neutral point, the body accelerates according to the magnitude and direction of the foot's displacement. The foot's displacement and the resulting acceleration are each given by vectors that span the horizontal plane.

For the special case of hopping in place with zero forward velocity, the neutral point is located directly under the body. Placing the foot out from under the body accelerates the body away from the foot. The pattern of net accelerations is shown in figure 2.9. It has a circular symmetry that is predicted by the inverted pendulum model. For each foot placement the acceleration vectors point toward the neutral point, which is also the origin. When the forward velocity is nonzero, the pattern of net accelerations is similar to the zero velocity case, in that the vectors all point in the general direction of the neutral point. To first order, fore-aft displacement of the foot causes fore-aft acceleration, and lateral displacement causes lateral acceleration (figure 3.2). There is, however, a distortion in the magnitude of net acceleration associated with forward speed.

When the body has a forward velocity, the problem is simplified by recognizing that the center of mass of a three-dimensional system traverses a planar trajectory during flight. The plane in which the motion occurs during flight is defined by the forward velocity vector, the gravity vector, and the location of the center of mass (figure 3.3). Sesh Murthy calls this the *plane of motion* (Murthy 1983). The behavior of a three-dimensional one-legged machine within the plane of motion is identical to the behavior of a comparable planar machine. Figure 2.9 was used in the previous chapter to describes behavior of the planar machine, but it also describes the behavior of a three-dimensional machine in the plane of motion. The figure plots the relationships between foot placement and net acceleration and between the location of the neutral point and forward speed machine. So long as the foot lands in the plane of motion, the direction of the forward velocity and the orientation of the plane of motion remain fixed from one step to the next.[1]

When the foot is displaced laterally from the plane of motion, the net acceleration includes a lateral component, giving the machine a lateral velocity with respect to the previous heading. The plane of motion for the next flight phase will have a new orientation determined by the sum of the velocity vector at previous touchdown and the net acceleration developed during the stance phase:

$$\dot{\mathbf{x}}_{i+1} = \dot{\mathbf{x}}_i + \Delta\dot{\mathbf{x}}, \tag{3.1}$$

where

$\dot{\mathbf{x}}$ is the forward velocity and
$\Delta\dot{\mathbf{x}}$ is the net forward acceleration vector.

[1] The discussion assumes that gyroscopic forces caused by rotation of the body about the yaw axis are small enough to be treated as disturbances. The hip torques used to control body attitude may influence the heading, even when the foot lands in the plane of motion. However, the control system also treats this sort of interaction between attitude control and forward velocity control as a disturbance.

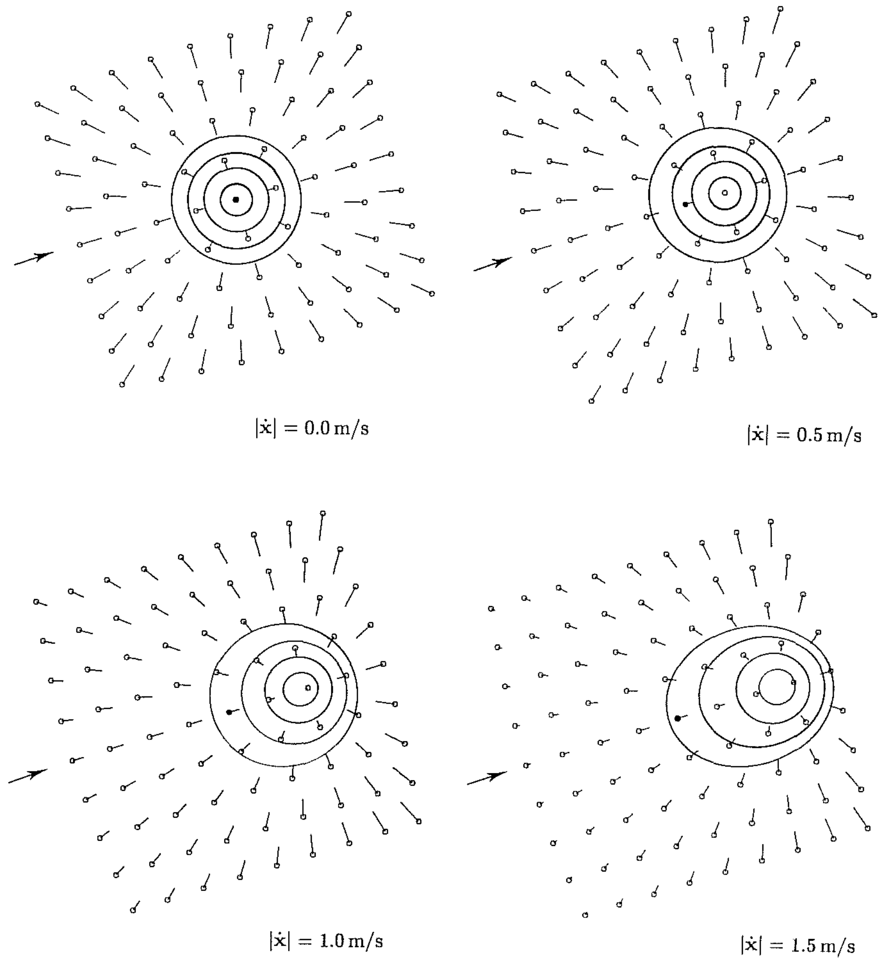


Figure 3.2. Placement of the foot at touchdown influences the net acceleration during stance. Each line segment indicates the direction and relative magnitude of the net acceleration that results when the foot is placed on the corresponding circle before landing. The contours are lines of constant net acceleration for $|\Delta\dot{\mathbf{x}}| = 0.25$, 0.5, 0.75, and 1.0 m/s/hop. Each plot is for a different initial forward speed. The arrow indicates the direction of travel and the projection of the plane of motion, and • indicates the location of the center of mass at touchdown. The patterns are symmetric about the plane of motion. Data are from computer simulations of the 3D one-legged machine, modeled with a linear leg spring. At touchdown $\dot{z} = -1\,\mathrm{m/s}$. Adapted from Murthy and Raibert (1983).

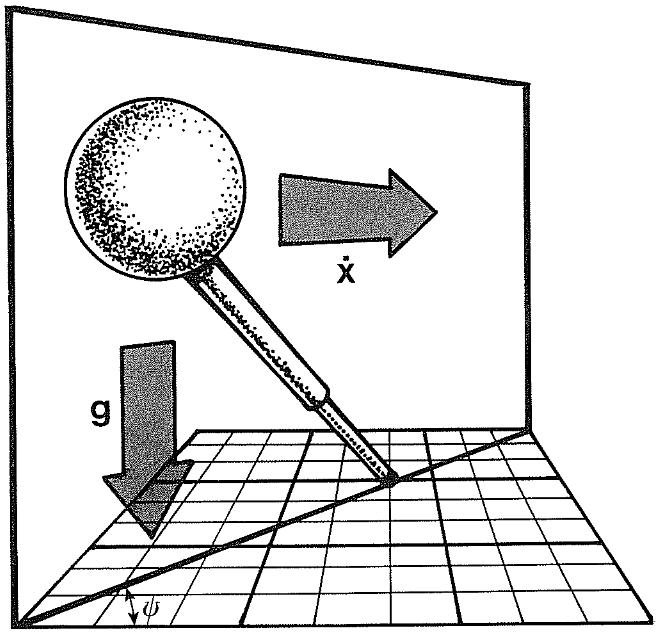


Figure 3.3. The plane of motion is defined by the forward velocity vector, the gravity vector, and the location of the machine's center of mass.

It is possible to identify the foot positions that accelerate either the speed or the direction, but not both. The *neutral speed locus* is the collection of foot positions that preserve the running speed while providing for changes in heading. Such loci for several forward speeds are plotted in figure 3.4. The *neutral heading locus,* on the other hand, preserves heading while speed is adjusted. These two loci intersect at the neutral point, as one might expect.

For straight-line travel, the 3D one-legged machine behaves like a planar machine. When the foot is placed in the plane of motion, there is no lateral acceleration, so the machine travels straight ahead. The ballistic motion of the body during flight, the bouncing motion of the body during stance, and the sweeping motion of the leg that moves the foot from one foothold to the next all lie in the plane of motion. The behavior is reminiscent of the apparently planar behavior observed in animal running.

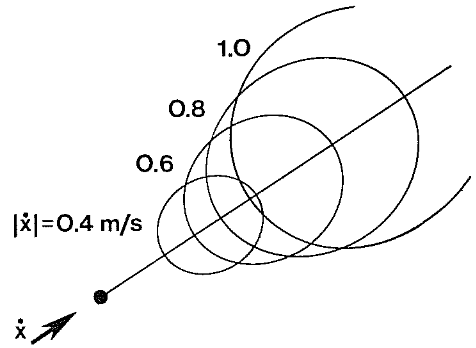


Figure 3.4. Neutral speed loci. For each forward speed there is a locus of foot positions that alters the running direction while leaving the running speed unchanged. Such loci are shown for $\dot{x}_{td} = 0.4$, 0.6, 0.8, and 1.0 m/s, with larger contours corresponding to higher speeds. The vertical velocity at touchdown for all loci is $\dot{z} = -1$ m/s.

3D One-Legged Hopping Machine

The hopping machine we built to study locomotion in three dimensions (figure 3.5) is just like the planar hopping machine, with two basic differences. One difference is the lack of a tether mechanism to constrain the motion, so the body can move with six degrees of freedom. The other difference is in the hip. It is a gimbal joint with an extra degree of freedom that permits the leg to swing sideways with respect to the body, as well as fore and aft. Aside from these basic differences and several implementation details, the 3D and planar one-legged hopping machine are similar.

The body consists of a lightweight platform and roll cage, on which are mounted sensors, valves, actuators, and interface electronics. A pair of free gyroscopes measures the roll, pitch, and yaw orientation of the body in space. The roll cage protects the hopping machine when it falls over and provides convenient handles during experiments.

The leg is a pneumatic cylinder just like the one used for the planar machine. The efficiency of the leg was improved by using the lower chamber of the cylinder to deliver power while the upper chamber acts as an air spring. The pneumatic circuit is illustrated in figure 3.6. The control system can operate this circuit to vary the peak to peak amplitude of body oscillations between 0.02 and 0.5 m.

A pair of hydraulic actuators operate at the hip to exert torques between the body and the leg. The arrangement is shown in figure 3.7. The

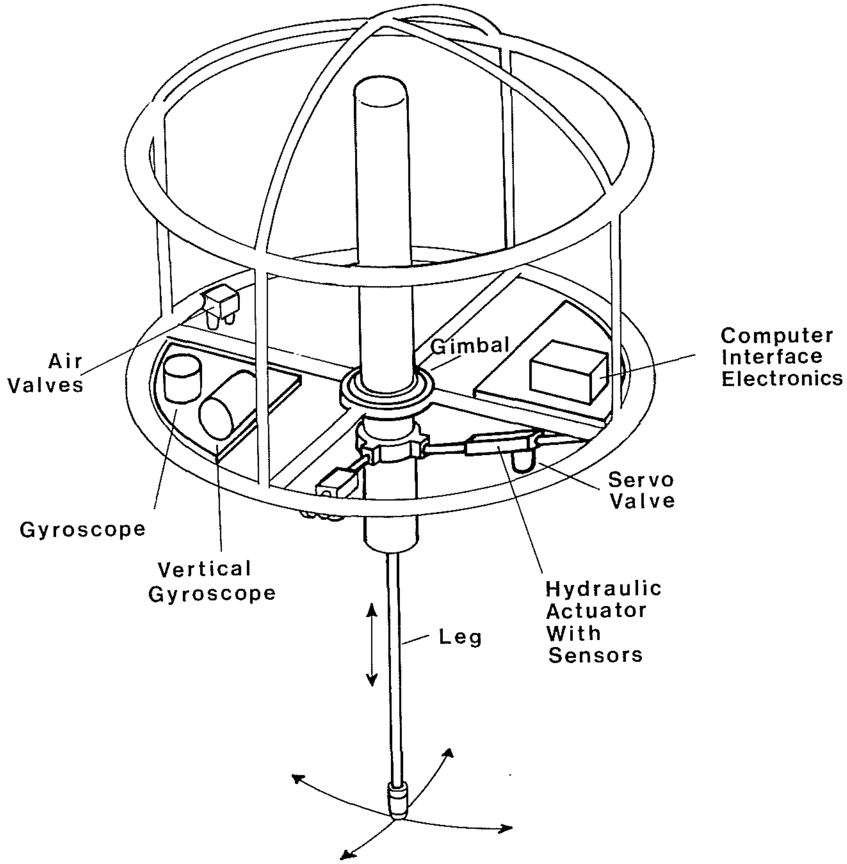


Figure 3.5. Diagram of 3D one-legged hopping machine. It has two primary parts: a body and a leg. The body is made of an aluminum frame, on which are mounted hip actuators, valves, gyroscopes, and computer interface electronics. The leg is a pneumatic cylinder with a padded foot at one end and a linear potentiometer at the other end. Two on-off pneumatic valves control the flow of compressed air to and from the lower end of the leg actuator. A pressure regulator and check valve control the pressure in the upper end of the leg actuator. The leg is springy because air trapped in the leg actuator compresses when the leg shortens. The leg is connected to the body by a gimbal-type hip that allows ±30° swing in one direction and ±20° swing in the perpendicular direction. A pair of low friction hydraulic actuators controlled by pressure servo valves acts between the leg and body to determine the hip angles. Sensors measure the length of the leg, the length and velocity of each hydraulic actuator, contact between the foot and the floor, pressures in the leg air cylinder, and the pitch, roll, and yaw orientation of the body. Analog measurements are digitized on the machine and transmitted to the control computer over a digital bus. An umbilical cable connects the machine to the hydraulic, pneumatic, and electrical power supplies and to the control computer, all of which are located nearby in the laboratory.

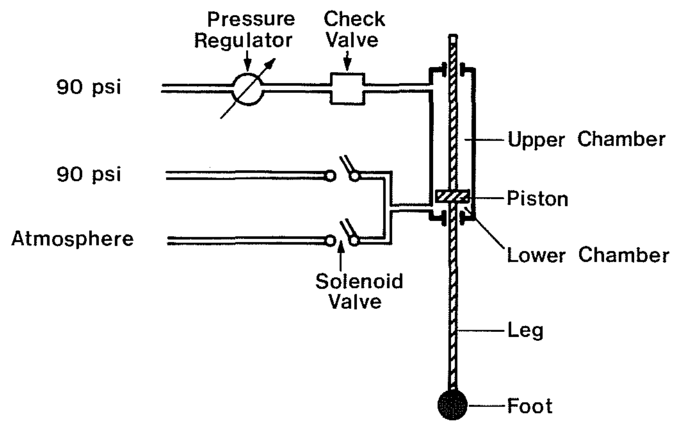


Figure 3.6. Pneumatic circuit for leg. The upper chamber is used as a spring, and the lower chamber is used as an actuator. A pressure regulator maintains pressure in the upper chamber of the leg cylinder, keeping it above a preset limit, typically 45 psi. A check valve permits the pressure to increase when the leg shortens, without forcing air back through the regulator. On-off solenoid valves connect the lower chamber of the leg cylinder to an 80 psi supply or to atmosphere. High pressure in the lower chamber moves the piston upward, shortening the leg. Low pressure moves the piston downward, lengthening the leg. During hopping, the control system pressurizes the lower chamber during flight and exhausts it during stance. The thrust magnitude is defined as the length of time the intake valve is open during flight. This method for leg actuation uses less air to drive hopping than the method described for the planar machine, resulting in better efficiency. If the piston and rod seals did not leak, then the upper chamber could be charged permanently, eliminating the need for the pressure regulator and the check valve.

control computer servos these actuators with a linear servo:

$$\tau_i = -k_p(w_i - w_{i,d}) - k_v(\dot{w}_i), \tag{3.2}$$

where

τ_i	is the valve signal for the ith actuator,
w_i, $\dot{w}_i$	are the length and velocity of the ith actuator, and
k_p, k_v	are position and velocity feedback gains.

A full 60° sweep of the leg takes approximately 70 ms. The servo operates at 250 hz.

The foot is small, so it exerts negligible torques on the ground about all three axes. Friction prevents the foot from slipping horizontally on the ground under normal conditions, but the torsional traction is quite small.

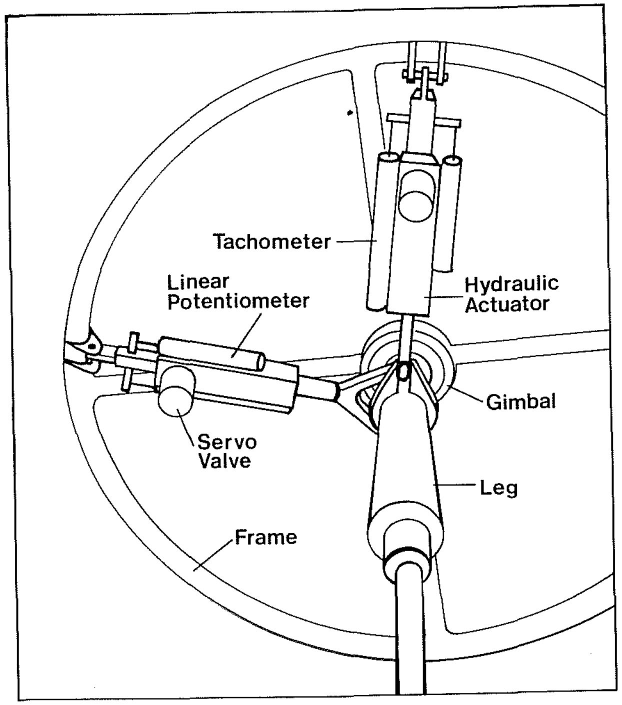


Figure 3.7. Bottom view of the hip. Two hydraulic actuators position the leg with respect to the body, providing ±30° leg motion on one axis and ±20° on the other. Each hydraulic actuator has a position sensor, a velocity sensor, and a pressure control servo valve. The servo valve makes the differential pressure across the actuator proportional to the control signal. Friction in the actuators was kept low by using clearance seals on the piston and rod, with low-pressure O-ring seals and leakage drains connected to the return line for backup.

An early design decision for the machine was that it should not have a preferred direction of travel. The body has no front or back, and the leg can move about equally well in all directions. A consequence of this decision is that control of heading does not have to take the facing direction of the machine into account—steering does not require turning. I use "heading" to mean the direction of travel, and "facing" to mean the orientation of the body about the vertical axis. The difference between heading and facing directions is the crab angle. Heading angle is $\psi = \arctan(\dot{y}/\dot{x})$, where the velocity is $\dot{\mathbf{x}} = [\dot{x}\ \dot{y}]^T$. The facing angle is ϕ_Y. For automobiles, airplanes, and boats, the heading and facing directions are roughly the same, but the two are unrelated for the 3D one-legged machine.

Table 3.1. Physical parameters for 3D one-legged hopping machine.

Parameter	Metric Units	English Units
Overall height	1.10 m	43.5 in
Overall width	0.76 m	30.0 in
Hip height	0.58 m	23.0 in
Total mass	17 kg	38 lbm
Unsprung leg mass	0.91 kg	2.0 lbm
$\frac{\text{Body mass}}{\text{Unsprung leg mass}}$	18:1	18:1
Body moment of inertia	$0.709\,\mathrm{kg \cdot m^2}$	$2420\,\mathrm{lbm \cdot in^2}$
Leg moment of inertia	$0.111\,\mathrm{kg \cdot m^2}$	$380\,\mathrm{lbm \cdot in^2}$
$\frac{\text{Body moment of inertia}}{\text{Leg moment of inertia}}$	6.4:1	6.4:1
Leg Axial Motion		
Stroke	0.25 m	10.0 in
Static force	630 N @ 620 kPa	140 lb @ 90 psi
Leg Sweep Motion		
Sweep angle	±0.5 rad/ ± 0.35 rad	±28°/ ± 21°
Static torque in x	$90\,\mathrm{N \cdot m}$ @ 14 MPa	$800\,\mathrm{lb \cdot in}$ @ 2000 psi
Static torque in y	$136\,\mathrm{N \cdot m}$ @ 14 MPa	$1200\,\mathrm{lb \cdot in}$ @ 2000 psi

Control System for 3D One-Legged Machine

The control system we tested for the 3D one-legged machine decomposes the task into three independent parts, one each for forward velocity, body attitude, and hopping height. The hopping height control needs no discussion because the problem and the solution we implemented are just the same as for the planar case. The control of forward velocity and of body attitude are based on the solutions used for the planar case.

Control Forward Velocity

The control system for forward velocity chooses a forward position for the foot during each flight phase that will provide the desired net acceleration during the impending stance phase. The algorithm implemented for 3D one-legged hopping uses the same principles used by the planar machine. The forward velocity determines a position for the foot designed to place it on the neutral point, and the error in forward velocity determines a displacement of the foot that provides the proper acceleration. The neutral foot position and the displacement combine to specify where the control system actually places the foot. The forward velocity controller servos the hip angles during flight to position the foot. Once the foot touches the ground, the attitude control takes over operation of the hip, and the forward motion develops passively.

The forward velocity is given by the vector $\dot{\mathbf{x}}$, and the forward position of the foot with respect to the body is another vector, $\mathbf{x}_f$. These vectors are expressed in a coordinate system whose origin moves with the center of mass of the body but whose axes have a fixed orientation in space. The location of the neutral point is found from the CG-print:

$$\mathbf{x}_{f0} = \frac{\dot{\mathbf{x}} T_s}{2},$$

where

$\mathbf{x}_{f0}$ is the neutral foot position and
T_s is the duration of stance.

This approximation to the CG-print has the same difficulties as for the planar case, namely that the approximation overestimates the distance traveled during support at higher running speeds because it ignores the deceleration and reacceleration that occurs during support.

The control system approximates the relationship between the foot's displacement from the neutral point $\mathbf{x}_{f\Delta}$, and net acceleration $\Delta\dot{\mathbf{x}}$, with a linear function $\Delta\dot{\mathbf{x}} = -\mathbf{x}_{f\Delta}/k_{\dot{\mathbf{x}}}$, where $k_{\dot{\mathbf{x}}}$ is a constant. This approximation ignores the variations with speed and orientation that are apparent in figure 3.2. The control system uses this approximation to convert forward velocity errors into foot displacements:

$$\mathbf{x}_{f\Delta} = k_{\dot{\mathbf{x}}}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d). \tag{3.3}$$

By combining the neutral foot position with the displacement, we arrive at the equation for calculating forward foot position during flight:

$$\mathbf{x}_f = \frac{\dot{\mathbf{x}} T_s}{2} + k_{\dot{\mathbf{x}}}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d). \tag{3.4}$$

The duration of stance T_s is governed by the springiness of the leg, so it is largely independent of hopping height and nearly constant for a given leg stiffness. The control system uses the measured duration of the previous stance phase as the expected duration of the next stance phase. The desired forward velocity $\dot{\mathbf{x}}_d$ is usually provided by a two-axis joystick that is manipulated by a human operator. Sometimes a position control algorithm generates velocity setpoints, as will be described later.

Keep in mind that (3.4) specifies only the horizontal position for the foot. The leg length, and therefore the foot's relative altitude, is determined by the hopping part of the control system.

Once the control system calculates a desired foot position with (3.4), it uses a kinematic transformation to determine the actuator lengths that will correctly position the foot. $\mathbf{F}$ is a function that transforms the hip actuator lengths and the leg length into a foot position. It is an implicit function of the body orientation in space, $\Phi = [\phi_P \ \phi_R \ \phi_Y]^T$, where the subscripts indicate rotations about the pitch, roll, and yaw axes. $\mathbf{F}$ and its inverse $\mathbf{F}^{-1}$ are given in appendix 3A. Once the actuator lengths are known, the hip servos given by (3.2) position the foot.

For $\mathbf{x} = [x \ y]^T$, (3.4) can be expressed as two scalar equations:

$$\begin{aligned} x_f &= \frac{\dot{x} T_s}{2} + k_{\dot{\mathbf{x}}}(\dot{x} - \dot{x}_d), \\ y_f &= \frac{\dot{y} T_s}{2} + k_{\dot{\mathbf{x}}}(\dot{y} - \dot{y}_d). \end{aligned} \tag{3.5}$$

This is equivalent to having two planar algorithms that operate at right angles to one another in a fixed coordinate system.

Estimate Forward Velocity from Leg Motion

The control system can control forward velocity only if it can measure it. The behavior of the body during stance provides the information needed to estimate $\dot{\mathbf{x}}$. Because the foot does not move with respect to the ground during stance, we can infer the hip's motion with respect to the ground from the measured motion of the foot with respect to the hip:

$$\dot{\mathbf{x}} = -\dot{\mathbf{x}}_f. \tag{3.6}$$

The position of the foot with respect to the hip can be found from measurements of the hip actuator lengths, the leg length, and the gyroscope angles using a kinematic transformation:

$$\mathbf{x}_f = \mathbf{F}^{-1}(\mathbf{w}). \tag{3.7}$$

The control system estimates the body's velocity during stance by numerically differentiating the foot position as determined from (3.7). Any change in forward velocity that occurs during flight is neglected.

Control Body Attitude

The control system maintains the body in an upright posture by exerting torques between the leg and the body about the hip during stance. Both the pitch and roll must be controlled.

The gyroscope that measures the pitch and roll orientation of the body for the 3D hopping machine is aligned so that the pitch axis is parallel to one axis of the hip and the roll axis is parallel to the other axis of the hip. This allows the attitude control servo to use the untransformed gyroscope measurements of pitch and roll to servo the hip actuators:

$$\begin{aligned} \tau_1 &= -k_p(\phi_P - \phi_{P,d}) - k_v(\dot{\phi}_P), \\ \tau_2 &= -k_p(\phi_R - \phi_{R,d}) - k_v(\dot{\phi}_R), \end{aligned} \tag{3.8}$$

where

τ_1, τ_2	are control signals for the two hip actuators,
ϕ_P, ϕ_R	are the pitch and roll angles of the body, and
k_p, k_v	are gains.

Orientation about the yaw axis is the third degree of freedom in the body's attitude, the facing direction. Although pitch and roll are relatively easy to control, yaw is more difficult. The difficulty is an artifact of our decision not to provide the 3D machine with an actuator that exerts torque directly about the axis of the leg and a foot capable of developing torsional traction. An actuator operating about the leg axis and an extended foot would provide torque that could correct yaw motion, but they would have also substantially complicated the machine. Because the 3D one-legged machine has no preferred direction of travel anyway, we decided to ignore this problem and to permit the machine to rotate freely about its yaw axis. The control system keeps track of the machine's facing direction as measured by one of the gyroscopes and uses the measurement in performing the coordinate transformations $\mathbf{F}$ and $\mathbf{F}^{-1}$.

Although the 3D hopping machine was not designed to exert torque on the foot about the leg axis, the required torque can, in principle, be generated using another technique. If the control system exerts a pitch torque about the hip with the foot displaced laterally from the plane of motion, then a moment is developed about the yaw axis of the system. The torque is proportional to the pitch torque and the magnitude of foot displacement. A control system could manipulate this moment to stabilize the yaw behavior of the system. To compensate for the induced pitching motion and lateral displacement, the control system could use compensating pairs of displacements on alternate steps. For instance it could pitch forward with the leg displaced to the left during one step and pitch backward with the leg to the right during the next step. This would produce compensating pairs of disturbances on alternate steps, but a counterclockwise yaw torque on every step.

Unfortunately, the maximum yaw torque that can be generated in this manner is substantially smaller than the disturbances generated by the hopping machine's umbilical cable. Therefore the control system was not able to generate adequate yaw torque to control the facing direction.

Hopping Experiments in Three Dimensions

Rate Control

We examined the control of forward velocity by having the control computer specify a ramp in $\dot{x}_d$ with $\dot{y}_d = 0$. The data are plotted in figure 3.8. They show the machine hopping in place, then running at increasing rates up to about 1.6 m/s. The velocity was controlled to an error of ± 0.2 m/s. This accuracy is typical. When the desired velocity was set to zero at $t = 5.3$ s, it took about 0.5 s for the velocity to adjust. This delay occurred because the change in $\dot{x}_d$ was not felt until the next time the foot touched the ground.

During running, the leg and body counteroscillate, as shown in the plots of x_f, ϕ_P, and ϕ_R in figure 3.8. The relative magnitudes of the pitch and roll oscillations varied as the machine rotated about its yaw axis. Once again, the back and forth sweeping motion of the leg was not explicitly programmed but resulted from interactions between the forward velocity control that positions the foot during flight and the attitude control that operates to erect the body during stance.

In another experiment, the desired speed was held constant while the desired direction was abruptly changed by 90°. The results are shown in figure 3.9. It took about 1 second, two steps, for the hopping machine to change direction, after which it continued running. The machine lost speed during the turn, but regained it again afterwards.

Position Control

Position control is used to make the hopping machine hop in one place and to translate from place to place. We implemented a position control

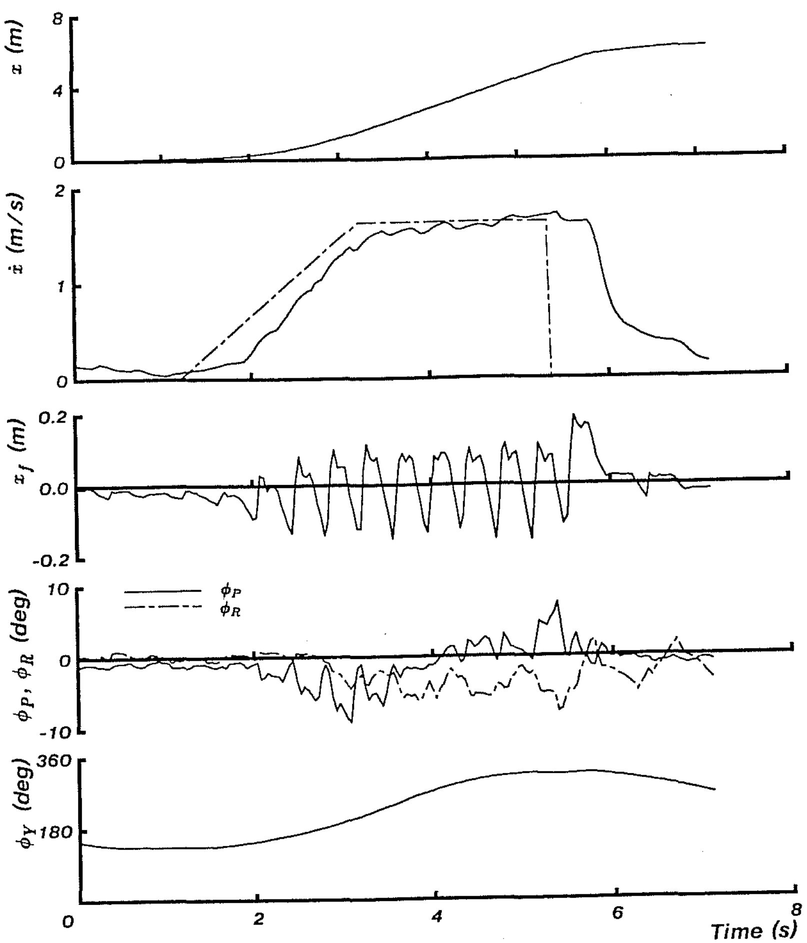


Figure 3.8. Velocity control was examined by varying the desired velocity in the x direction from 0.0 to 1.6 m/s with an acceleration of 1.0 m/s^2, then holding the setpoint constant for about 2 s, and then setting the rate setpoint to zero (dash-dot line in second plot). The facing direction of the body ϕ_Y was measured but not controlled. Also shown are (top) the position of the machine in the room, (middle) the position of the foot with respect to the hip, and (bottom) the yaw orientation of the body. From Raibert, Brown, and Chepponis (1984).

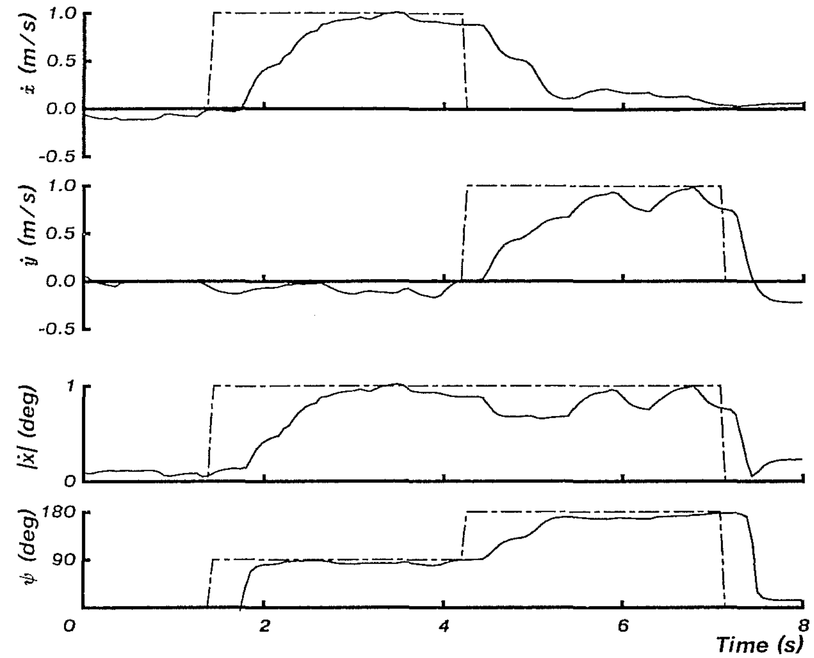


Figure 3.9. A step change in desired direction generates a right-angle turn while holding desired speed constant. The top two curves plot the desired and measured velocities. The bottom two curves show the speed and heading. The turn was completed in two steps. From Raibert, Brown, and Chepponis (1984).

algorithm that transforms position errors into desired velocities:

$$\dot{\mathbf{x}}_d = \min\{-k_p(\mathbf{x} - \mathbf{x}_d) - k_v\dot{\mathbf{x}}\,, \quad \dot{\mathbf{x}}_{max}\}, \tag{3.9}$$

where

k_p, k_v	are position and velocity feedback gains and
$\dot{\mathbf{x}}_{max}$	is a limit on allowable velocity.

In order to control the machine's position the control system must have a sense of where the machine is. The control system estimates the machine's position in the room in two ways. One way is to integrate the forward velocity estimate, $\dot{\mathbf{x}}$. Another way to estimate the machine's position is to use an electro-optical sensor mounted on the ceiling of the laboratory to

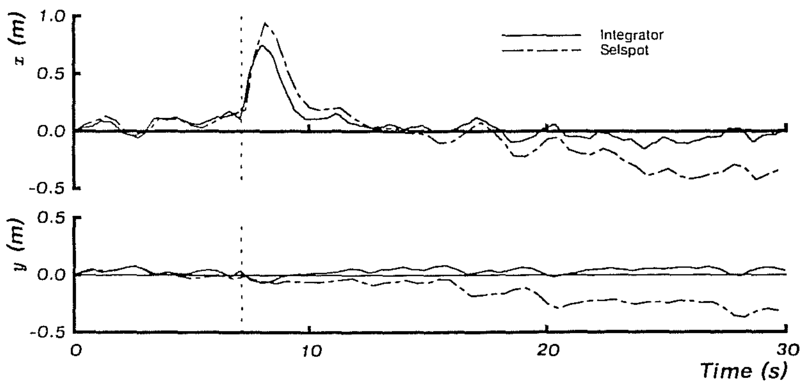


Figure 3.10. The 3D one-legged machine hopping in place under position control. The control system integrates forward velocity to determine and control the machine's position in the room. The accuracy is ± 0.2 m. An electro-optical system mounted on the ceiling provides an independent measurement of the machine's position. Divergence between electro-optical and integrator data indicates drift in the integrator. Vertical dotted line indicates instant at which the experimenter disturbed the machine by delivering a sharp horizontal jab to the frame with his hand. The machine returned to the position setpoint within a few seconds. From Raibert, Brown, and Chepponis (1984).

track a light source on the machine's body. This sensor serves the same role as the geosynchronous satellites used for global navigation. Our satellite has a very low orbit.

Figure 3.10 is a plot of the machine's position as it hopped in place, using the integrator position estimates for control. The machine stayed within ± 0.2 m of the setpoint. Deviations of this magnitude were typical for stationary hopping. The position as measured by the electro-optical system is also plotted. The discrepancy between the data from the integrator and the electro-optical measurement shows that the integrator drifts by roughly 5 mm/hop. This means that if the machine were instructed to hop on one spot and were using the integrator for control, it might drift a meter in one minute. Informal experiments with blindfolded humans hopping on one leg indicate that they often drift by similar amounts. In the case of the hopping machine, the primary sources of drift are gyroscope calibration errors and unmodeled forces exerted on the machine by the umbilical cable. The umbilical accelerates the machine during flight when the forward velocity is expected to be constant.

Figure 3.10 also shows the response to an external disturbance. After about 7 seconds, the experimenter delivered a sharp horizontal jab to the machine's body as it hopped in place (dotted vertical line in figure 3.10).

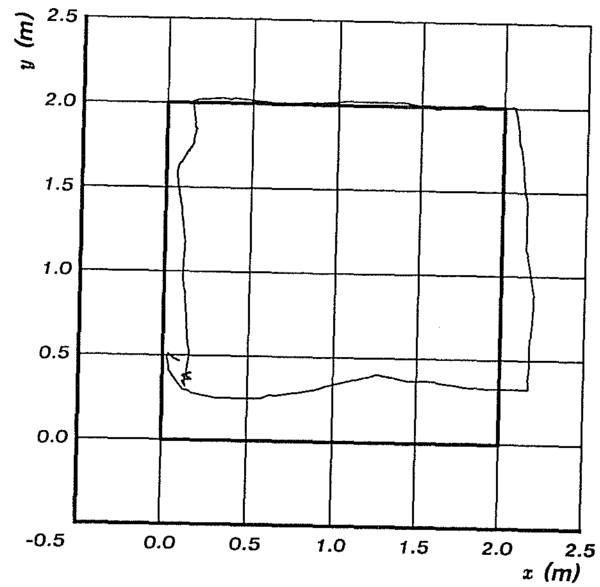


Figure 3.11. Data recorded while 3D hopping machine traversed a square path. The control system integrated forward velocity estimates to determine and control the machine's position in the room. Desired and measured path of machine plotted in x-y plane. The desired path is indicated by the bold line. It starts at (0,0) and moves through (0,2), (2,2), (2,0), and (0,0). From Raibert, Brown, and Chepponis (1984).

The machine maintained its balance and returned to the position setpoint after a few seconds. The system also kept its balance when given substantial torsional disturbances about the roll and pitch axes.

For another measurement of performance under position control, the control computer specified a preplanned sequence of positions setpoints. Each time an operator pressed a button, the control system advanced to the next position setpoint in the sequence. The sequence specified a square path, 2 m on a side. The data are shown in figure 3.11. When $y_d = 0$ there was a fixed position error of about 0.3 m caused by the umbilical cable, which was just long enough to permit the machine to reach $y = 0$ but short enough to exert some tension. The system came to equilibrium where the force exerted by the umbilical cable equaled the accelerations produced by the control system. Figure 3.12 is a photograph made during a traverse of the same square path, but the electro-optical system provided the position information used for control.

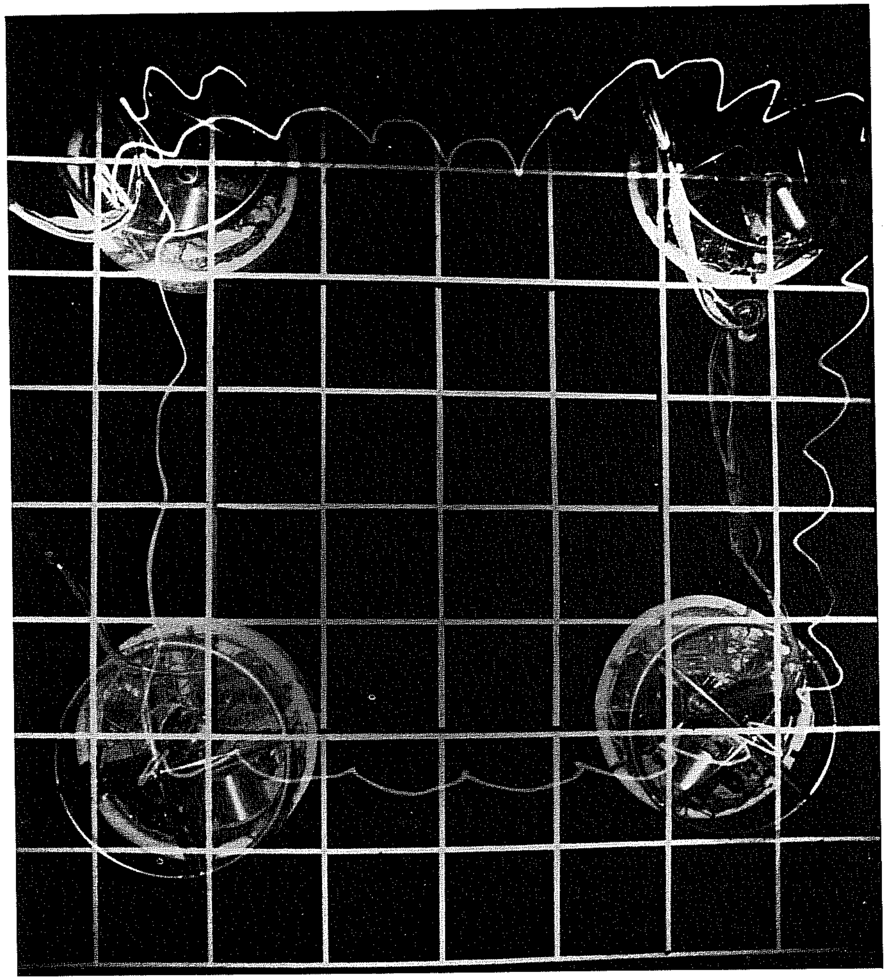


Figure 3.12. A bird's-eye view of the 3D one-legged machine as it traversed a square path. Each time the operator pressed a sequencing button, the machine advanced from one predefined position setpoint to the next. An electro-optical sensor mounted on the ceiling provided position measurements that were used by the position-control servo. It took about 14 s to traverse the path. The camera was mounted on the ceiling, looking down. The scalloped white line in the photograph was made by a small light source attached to the top of the body. From Raibert, Brown, and Chepponis (1984).

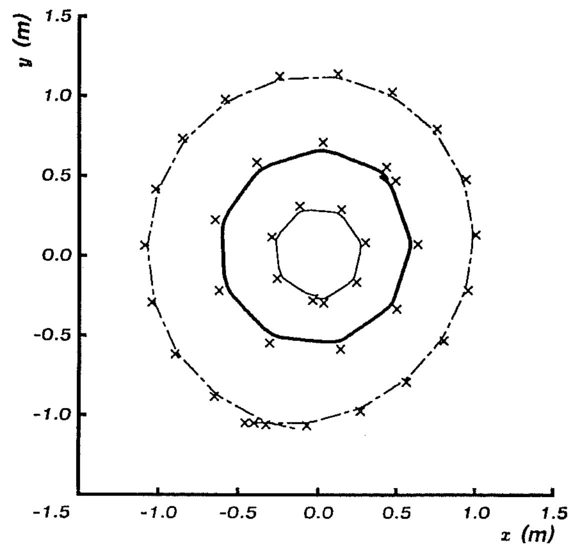


Figure 3.13. Circular path. A simulated one-legged hopping machine travels along a circular path when the forward velocity is kept constant with a fixed lateral displacement of the foot on each step. The curves show the path of the center of gravity, and the 'x's mark the footholds. The radius of curvature varies with both the forward speed and the lateral acceleration. (Bold: $|\dot{\mathbf{x}}| = .6\,\text{m/s}$, $x_{f\Delta} = .04\,\text{m}$; Solid: $|\dot{\mathbf{x}}| = .4\,\text{m/s}$, $x_{f\Delta} = .04\,\text{m}$; Stippled: $|\dot{\mathbf{x}}| = .6\,\text{m/s}$, $x_{f\Delta} = .02\,\text{m}$.) From Murthy (1983).

Running in Circles

Displacement of the foot from the plane of motion introduces a lateral acceleration and a change of heading. The control system can manipulate the lateral displacement of the foot to run along a contour with a particular curvature. The change in heading on a single hop is a function of the forward speed and the net acceleration vector:

$$\Delta\psi = \arccos\left(\frac{\dot{\mathbf{x}} \cdot \Delta\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|\,|\Delta\dot{\mathbf{x}}|}\right). \qquad (3.10)$$

The control system can adjust the lateral foot displacement and the running speed to control the amount of turning.

Figure 3.13 plots data from a computer simulation of a 3D one-legged machine running in circles. The desired forward velocity and the lateral foot displacement were held constant for each circle. The radius of curvature was changed by varying the lateral foot displacement in one case, and the speed in the other. The ability to change the radius of curvature of the path of travel, as demonstrated in these simulations, suggests the

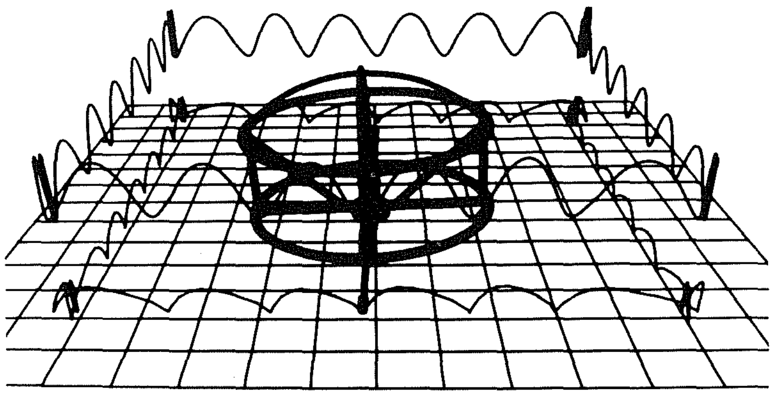


Figure 3.14. Computer simulation of 3D machine traversing a square path. The upper trace marks the path of the hip and the lower trace marks the path of the foot. From Murthy and Raibert (1983).

feasibility of travel on paths of arbitrary planar contour. Figure 3.14 shows a simulated hopping machine traversing a square path.

Summary

The techniques that provided control for the planar one-legged machine generalized to three dimensions with surprising ease. The three-part decomposition used for the planar machine survived the generalization and led to a simple implementation. The control of forward velocity requires a vector calculation to determine a suitable position for the foot that will provide balance and acceleration. The effects of foot placement are kept simple by looking at velocities and accelerations in the context of a plane of motion. To control body attitude the control system servos the pitch and roll axes of the body while permitting the facing direction of the machine to rotate freely.

Experiments showed that the 3D hopping machine balanced without external support while hopping in place and while traveling about the laboratory. It tracked a ramp in desired velocity and sudden changes in desired direction with $\pm 0.25\,\mathrm{m/s}$ accuracy. At higher speeds the system consistently ran slower than specified because of inaccuracies in estimating the length of the CG-print, and therefore the location of the neutral point. Top recorded running speed was $2.2\,\mathrm{m/s}$ ($4.8\,\mathrm{mph}$).

The control system can determine the position of the machine in the laboratory by integrating the estimated forward velocity. With a stationary position setpoint, the machine can hop in place with about $\pm 0.2\,\mathrm{m}$ accuracy. The machine also traversed a square path, but the accuracy was not very good because the umbilical cable exerted a large disturbance force on one side of the square. The machine kept its balance when disturbed manually.

Appendix 3A. Kinematics of 3D One-Legged Machine

We define three coordinate frames $\{\mathbf{W}\}$, $\{\mathbf{H}\}$, and $\{\mathbf{B}\}$. Frame $\{\mathbf{W}\}$ is the world coordinate frame, which is fixed in the laboratory. The origin of frame $\{\mathbf{H}\}$ moves with the hip but its orientation remains parallel to those of $\{\mathbf{W}\}$. Think of frame $\{\mathbf{H}\}$ as attached to the innermost gimbal of the gyroscope. For $\{\mathbf{W}\}$ and $\{\mathbf{H}\}$, $\mathbf{z}$ is aligned with the gravity vector and positive upward. Frame $\{\mathbf{B}\}$ is fixed to the body. Its origin also moves with the hip, but $\{\mathbf{B}\}$ changes orientation with respect to $\{\mathbf{W}\}$ and $\{\mathbf{H}\}$. The Euler angles that specify the orientation of $\{\mathbf{B}\}$ are (ϕ_Y, ϕ_R, ϕ_P). The hip and leg actuators determine the position of the foot in frame $\{\mathbf{B}\}$.

Let the vector ${}^P\mathbf{x}$ be expressed in coordinate frame $\{\mathbf{P}\}$. The transformation from coordinate frame $\{\mathbf{B}\}$ to coordinate frame $\{\mathbf{H}\}$ is

$$ {}^H\mathbf{x} = {}^H_B\mathbf{T}\,{}^B\mathbf{x}, \tag{3.11}$$

$${}^H_B\mathbf{T} =$$

$$\begin{bmatrix} \cos\phi_P\cos\phi_Y - \sin\phi_P\sin\phi_R\sin\phi_Y & -\cos\phi_R\sin\phi_Y & -\cos\phi_P\sin\phi_R\sin\phi_Y - \cos\phi_Y\sin\phi_P & 0 \\ \cos\phi_P\sin\phi_Y + \cos\phi_Y\sin\phi_P\sin\phi_R & \cos\phi_R\cos\phi_Y & \cos\phi_P\cos\phi_Y\sin\phi_R - \sin\phi_P\sin\phi_Y & 0 \\ \cos\phi_R\sin\phi_P & -\sin\phi_R & \cos\phi_P\cos\phi_R & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The transformation from coordinate frame $\{\mathbf{H}\}$ to $\{\mathbf{B}\}$ is

$$ {}^B\mathbf{x} = {}^B_H\mathbf{T}\,{}^H\mathbf{x}, \tag{3.12}$$

$${}^B_H\mathbf{T} =$$

$$\begin{bmatrix} \cos\phi_P\cos\phi_Y - \sin\phi_P\sin\phi_R\sin\phi_Y & \cos\phi_P\sin\phi_Y + \cos\phi_Y\sin\phi_P\sin\phi_R & \cos\phi_R\sin\phi_P & 0 \\ -\cos\phi_R\sin\phi_Y & \cos\phi_R\cos\phi_Y & -\sin\phi_R & 0 \\ -\cos\phi_P\sin\phi_R\sin\phi_Y - \cos\phi_Y\sin\phi_P & \cos\phi_P\cos\phi_Y\sin\phi_R - \sin\phi_P\sin\phi_Y & \cos\phi_P\cos\phi_R & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

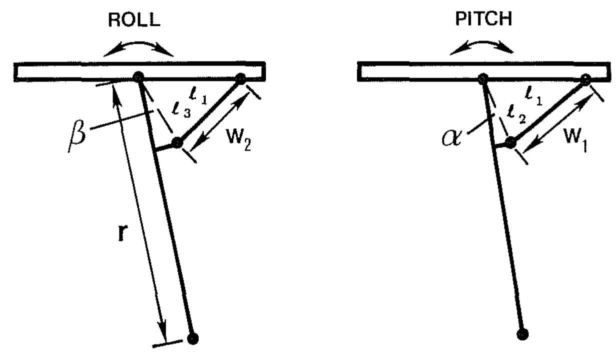


Figure 3.15. Diagram that shows kinematics of actuators, hip, and leg. Hip actuator lengths are represented by w_1 and w_2 and leg length by r. $l_1 = 0.345\,\mathrm{m}$, $l_2 = 0.0508\,\mathrm{m}$, $l_3 = 0.0762\,\mathrm{m}$, $\alpha = 8.46°$, $\beta = 27.28°$.

Figure 3.15 shows the linkage that connects the hip actuators to the leg. The forward transformation from actuator lengths to the position of the foot with respect to the hip expressed in the body frame $\{\mathbf{B}\}$ is

$$ {}^B\mathbf{x}_f = {}^B_A\mathbf{T}(\mathbf{w}) \quad = \quad \begin{cases} x_f = r\cos\left\{\arccos\left(\dfrac{w_1^2 - l_1^2 - l_2^2}{-2l_1l_2}\right) + \alpha\right\} \\ y_f = r\cos\left\{\arccos\left(\dfrac{w_2^2 - l_1^2 - l_3^2}{-2l_1l_3}\right) + \beta\right\} \\ z_f = \sqrt{r^2 - x_f^2 - y_f^2} \end{cases} \tag{3.13}$$

The inverse transformation from foot position to actuator lengths is

$$ \mathbf{w} = {}^A_B\mathbf{T}({}^B\mathbf{x}_f) \quad = \quad \begin{cases} w_1 = \sqrt{l_1^2 + l_2^2 - 2l_1l_2\cos\left\{\arccos\left(\dfrac{x_f}{-r}\right) - \alpha\right\}} \\ w_2 = \sqrt{l_1^2 + l_3^2 - 2l_1l_3\cos\left\{\arccos\left(\dfrac{y_f}{-r}\right) - \beta\right\}} \\ r = -\sqrt{x_f^2 + y_f^2 + z_f^2} \end{cases} \tag{3.14}$$

The overall forward and reverse transformations between actuator variables and foot position are

$$ {}^H\mathbf{x}_f = {}^H_B\mathbf{T}\,{}^B_A\mathbf{T}(\mathbf{w}) = \mathbf{F}^{-1}(\mathbf{w}), \tag{3.15}$$

$$ \mathbf{w} = {}^A_B\mathbf{T}({}^B_H\mathbf{T}\,{}^H\mathbf{x}_f) = \mathbf{F}({}^H\mathbf{x}_f). \tag{3.16}$$

Chapter 4

Biped and Quadruped Running

The machines described in the last two chapters were used to study control of running for the simple case of one leg. Machines with one leg permitted us to focus on the issues of active balance and resonant bouncing in running while avoiding the problem of coordinating the interactions of many legs. Gait was not an issue because a one-legged machine can only hop. Can we now use the results from the study of running on one leg to understand running on several legs?

The approach used in this chapter is to decompose the behavior of biped and quadruped running systems into components that we understand from the one-legged systems. We argue that biped running has the same essential features as one-legged running and that certain gaits of the quadruped are, in turn, like those of the biped. The first step is to show that a system with two legs can run as though it had just one. This is possible when the running gait uses just one leg for support at a time, with the other leg elevated and out of the way. The legs are like the barrels of a Gatling gun, with just one *firing* at a time. Humans run this way. The control algorithms for this mode of running can be like those used to control systems with one leg.

The second step in the approach considers the quadruped gaits that use the legs in pairs: the trot, the pace, and the bound. If the control system can make the two legs that form a pair work together as though they were one leg, then the quadruped can be controlled like an equivalent biped. Once this is done, the Gatling approach and the one-legged control algorithms can be used again. The result is a control system for running on four legs built upon the algorithms originally used to control hopping on one leg. We have examined the feasibility of this approach with experiments on a biped that operates in the plane (figures 4.1 and 4.2) and a four-legged machine that runs in three dimensions by trotting (figure 4.10).

One-Foot Gaits

For the purpose of this discussion, let us assume that the three-part control system described in previous chapters is effective for controlling machines that have a springy leg, a body, and a suitable collection of actuators and sensors. It is not difficult to imagine a control system that uses these same three control algorithms for human-type biped running.

Despite a number of differences in detail, running on two legs and running on one leg are remarkably similar.[1] In both cases only one leg provides support at a time, only one foot is held in place by friction during support at a time, and only one leg recovers to a forward position during flight. We define a class of running gaits called *one-foot gaits* for which: 1) one leg provides support at a time, and 2) support phases and flight phases proceed in strict alternation. This is the Gatling gait I mentioned earlier. Adult humans normally run with a one-foot gait, as do the one-legged machines we have built.

[1] There are several important differences in the running behavior of systems with one and two legs:

1. Two legs permit running with substantially reduced pitching of the body. The recovery motion for a one-legged system can occur only during flight, when the angular momentum of the system is fixed. So every recovery motion of the leg must be compensated by a pitching motion of the body. A biped can overlap the backward motion of the supporting leg with the forward motion of the recovery leg, resulting in zero net angular momentum of the legs. The stance leg, whose foot is held in place by friction, can absorb any residual angular momentum by exerting a pitching torque on the body.
2. When the stance and recovery motions overlap in time, the duration of flight does not uniquely determine the time available for the recovery motion. Therefore, if the time it takes to recover a leg limits the running speed, then a biped can run faster than a comparable one-legged system.
3. If a biped is to recover a leg during stance, then it must be able to shorten that leg during recovery. The recovery leg must be substantially shorter than the support leg if it is to clear the ground without stubbing. Therefore a biped must have a mechanism that will permit the leg to shorten substantially during recovery, and to lengthen again in time for landing. The one-legged systems we explored did not have this requirement, because the recovery motion occurred when the body achieved peak height during flight.
4. A biped can generate yaw moments on the body when the legs counteroscillate, swinging fore and aft. This was not a problem with the one-legged systems because the leg swung in the plane containing the center of mass.

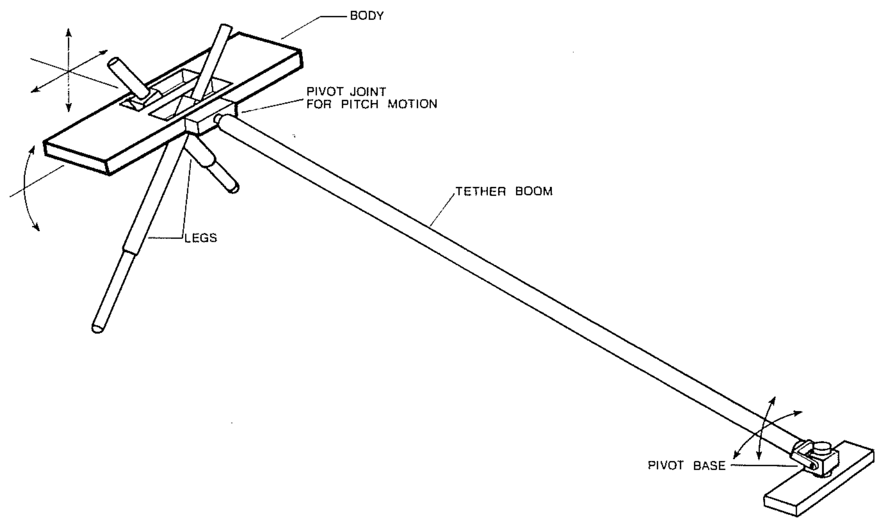


Figure 4.1. A tether mechanism constrains motion of the biped to three degrees of freedom. It can travel on a 2.5-m radius circle in the laboratory. The tether mechanism is instrumented to provide measurements of the machine's three motions: vertical translation, forward translation, and rotation about the axis of the boom. The tether also supports an umbilical cable that carries hydraulic connections, DC power, and a connection to the control computer.

The one-foot gaits provide an opportunity to control running on two legs with the same algorithms used to control hopping on one leg. Because there is just one active leg at a time the three control algorithms work correctly when applied to each leg of a biped running with a one-foot gait. The thrust the leg delivers during support, the torque generated at the hip during support, and the forward placement of the foot during flight all influence the system as they do in the one-legged case. The one-leg algorithms, therefore, can continue to operate effectively to regulate hopping height, body posture, and forward speed.

To make this approach workable the control system would need a sequencing mechanism that kept track of the legs and assigned each of the three control functions to the right leg at the right time. The sequencing mechanism would select the next leg to provide support so that the leg

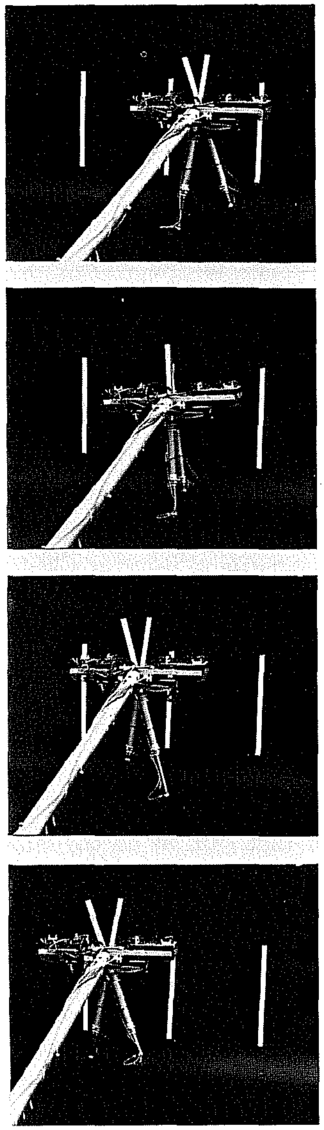


Figure 4.2. Planar biped during one half stride. The control system uses the same three-part decomposition that was described for the one-legged machines. In addition, the extra leg shortens during recovery to avoid touching the ground when it swings forward. The machine can also hop on one leg and it can switch between one- and two-legged gaits while running. A tether boom constrains the machine so that it moves with three degrees of freedom. The vertical lines in the background indicate 0.5 m intervals. Frames are separated by 80 msec. From Hodgins, Koechling, and Raibert (1985).

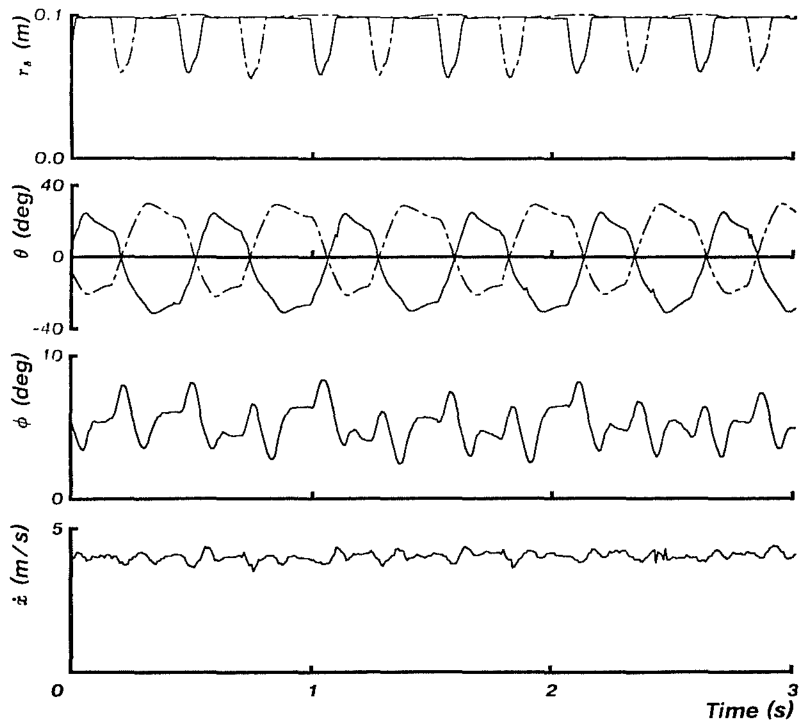


Figure 4.3. Data recorded from the biped as it runs. The top curve shows the alternating compression of the air springs for the two legs. The second set of curves are the angles of the legs with respect to the vertical. The legs sweep back and forth out of phase. The third curve shows the body's pitch angle and the bottom curve shows the forward running speed, which averages about 4 m/s (8.8 mph). Top recorded running speed is 4.3, m/s (9.5 mph).

could move to a forward position for landing, and it would assign the hopping height and attitude control functions to the leg currently providing support. The control system would also shorten the recovery leg to keep it from touching the ground while the other leg provides support.

Planar Biped Running Experiments

Jessica Hodgins and Jeff Koechling have used this approach to control the planar biped shown in figures 4.1and 4.2 (Hodgins, Koechling, and Raibert 1985). The machine's sensors, hydraulic actuators, and electronics are like those used for the 3D one-legged machine, and it is constrained to the plane by a tether mechanism like the planar one-legged machine. The legs, shown in figure 4.4, have long-stroke hydraulic actuators that provide rapid retraction and substantial clearance during recovery, as well as thrust during stance. The legs also have air springs for vertical bouncing.

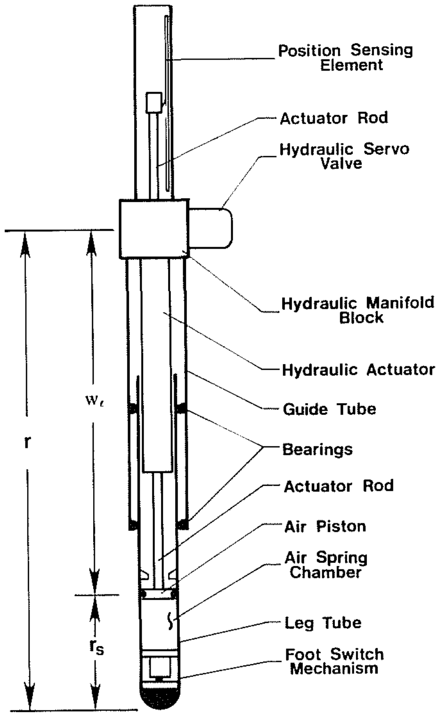


Figure 4.4. Diagram of leg used in biped and quadruped running machines. A hydraulic actuator acts in series with an air spring. The hydraulic actuator is used to drive resonant bouncing motion of the machine and to retract the leg during flight. It also acts in conjunction with the air spring to determine the axial force the leg exerts on the ground. Sensors measure the hydraulic length, the overall leg length, the air pressure in the spring, and loading on the foot. Table 4.2 gives the physical parameters of the machine.

The sequencing mechanism that assigns control functions to legs is a state machine that traverses ten states during each stride (figure 4.5 and table 4.1). Each state prescribes a set of sensor conditions that triggers transition into the state and a set of control actions to be taken during the state. The state transitions track the behavior of the machine and synchronize the control functions—vertical thrust, attitude control, and foot placement—to the ongoing behavior.

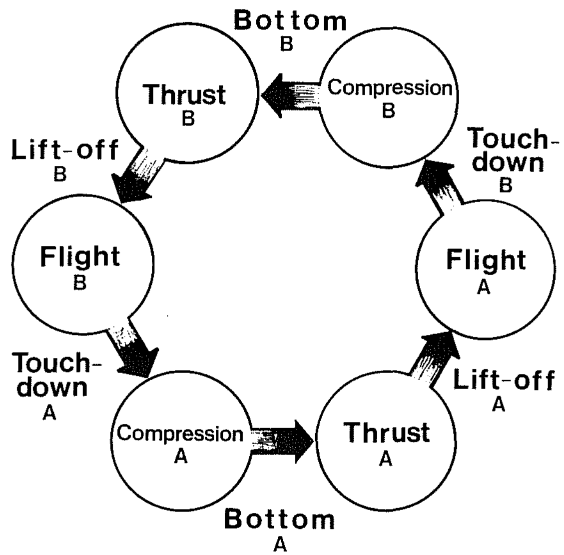


Figure 4.5. Simplified state diagram used to sequence biped gait. Leg $\mathbf{B}_\ell$ recovers while leg $\mathbf{A}_\ell$ provides support, and vice versa. State transitions are determined by events related to the support leg. See table 4.1 for the details.

Data recorded during running are shown in figure 4.3. They show that the machine can run at a good clip, and does so without much cyclic pitching of the body. Top recorded running speed is 4.3 m/s (9.5 mph). In addition to running with a gait that uses the legs in alternation, the machine can hop on either leg and it can change back and forth between running and hopping (Hodgins, Koechling, and Raibert 1985).

Quadruped One-Foot Gaits?

The definition of a one-foot gait given earlier avoids limiting it to systems with one or two legs. One-foot gaits can, in principle, exist for systems with any number of legs. A quadruped performing a one-foot gait would cycle through use of the legs, perhaps in a regular order. One might propose {LF;RR;RF;LR;} and {LF;LR;RF;RR;} as plausible quadruped one-foot gaits; one rotary and one transverse. The sequencing mechanism could be like the one used for a biped, but there would be more choice in deciding which leg would become active next. The sequencing mechanism would also keep the inactive legs out of the way until they were needed.

A practical problem with this sort of running for quadrupeds is the difficulty of attaching the legs close enough to the center of the body to

Table 4.1. Finite state sequence used to synchronize legs for one-foot gait. The state shown in the left column is entered when the event listed in the center column occurs. During normal running states advance sequentially. For biped running $\mathbf{A}_\ell$ refers to leg 1 and $\mathbf{B}_\ell$ refers to leg 2. For quadruped trotting $\mathbf{A}_\ell$ refers to the virtual leg that uses the left front and right rear physical legs, and $\mathbf{B}_\ell$ refers to the virtual leg that uses the left rear and right front physical legs. During states 1–5, $\mathbf{A}_\ell$ is in support and $\mathbf{B}_\ell$ is in recovery. During states 6–10, these roles are reversed.

State	Trigger Event	Action
1 LOADING $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ touches ground	Zero hip torque $\mathbf{A}_\ell$ Shorten $\mathbf{B}_\ell$ Don't move hip $\mathbf{B}_\ell$
2 COMPRESSION $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air spring shortened	Erect body with hip $\mathbf{A}_\ell$ Shorten $\mathbf{B}_\ell$ Position $\mathbf{B}_\ell$ for landing
3 THRUST $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air springs lengthening	Extend $\mathbf{A}_\ell$ Erect body with hip $\mathbf{A}_\ell$ Keep $\mathbf{B}_\ell$ short Position $\mathbf{B}_\ell$ for landing
4 UNLOADING $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air spring near full length	Shorten $\mathbf{A}_\ell$ Zero hip torque $\mathbf{A}_\ell$ Keep $\mathbf{B}_\ell$ short Position $\mathbf{B}_\ell$ for landing
5 FLIGHT $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ not touching ground	Shorten $\mathbf{A}_\ell$ Don't move hip $\mathbf{A}_\ell$ Lengthen $\mathbf{B}_\ell$ for landing Position $\mathbf{B}_\ell$ for landing

States 6–10 repeat states 1–5, with $\mathbf{A}_\ell$ and $\mathbf{B}_\ell$ interchanged.

permit the feet to reach footholds that would provide balance. Each foot must be placed near the neutral point, so that the average point of support during the support interval is under the center of mass. It is not hard to attach one or two legs near the center of mass, but the design problem becomes severe with more legs. It is also difficult to keep motions of many legs from interfering with one another when the legs are mounted close together. Nature has eased this problem somewhat by providing some animals with bodies that are flexible enough to permit the feet to reach under the center of mass, even though the hips are located far from the center of mass. For example, see Muybridge's photograph of a running dog,

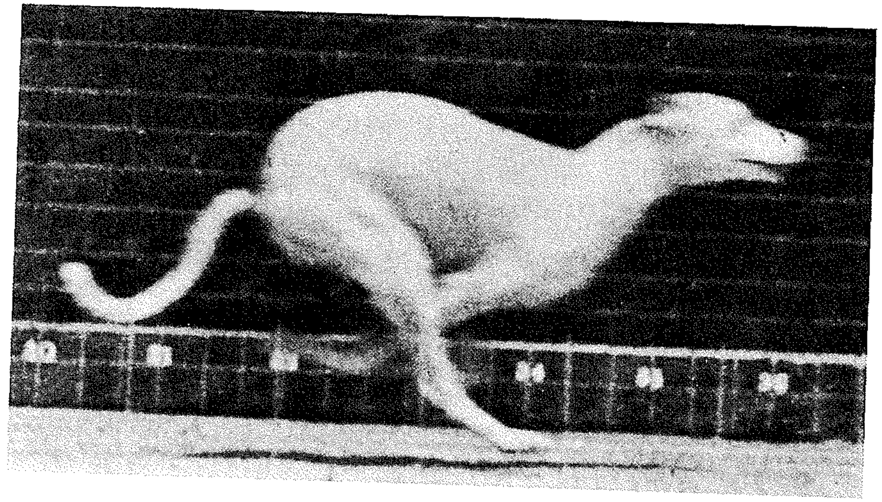


Figure 4.6. Photograph of dog in crossed phase of bound. Flexibility in the back enables the feet to sweep under the center of mass, even with hips located far from the center of mass. Photograph from Muybridge (1957), Plate 121. Reprinted with permission from Dover Press.

reproduced in figure 4.6. Despite this adaptation, it is not clear that any natural quadruped employs a one-foot gait. Duikers and muntjaks would be the best candidates (Hildebrand 1985).

Even without the hips near the center of mass and without a flexible spine, a quadruped could use a one-foot gait if it were not required that each step by itself provide perfect balance. The feet could be positioned on either side of the center of mass on alternate steps. The system would tip in one direction during one stance phase and in the opposite direction during the next stance phase. For such a system to balance, the step rate would have to be high compared to the tipping rate so that one step did not cause the system to tip over entirely before the next step. In chapter 5 I discuss this sort of balance, which relies on symmetric sequences of steps to generate complementary pitching motions of the body.

To summarize this section so far, the one-foot is a class of running gaits for which only one leg provides support at a time and in which a flight phase occurs between each stance phase. In principle, one-foot gaits can be defined for systems with any number of legs, but leg interference becomes a practical problem when there are more than about two legs. The control algorithms for running with a multilegged one-foot gait can be like those used for hopping, provided that the control system also has a sequencing mechanism that assigns the control functions to the legs and retracts the legs during recovery.

The next step in generalizing from one leg to several legs is to consider legs that work together in pairs. The practical limitations of the quadruped one-foot gaits, namely the difficulty of placing the feet on the neutral point when the hips are separated by feasible distances, can be reduced or eliminated when legs are used in pairs.

Virtual Legs

Suppose a pair of legs were coordinated to work together, like one equivalent leg. Then a quadruped gait that used the four legs in two pairs can be viewed as an equivalent biped. Quadruped gaits that can be understood in this way are the trot, the pace, and the bound. In each of these gaits the legs operate in pairs. The members of a pair strike the ground in unison and they leave the ground in unison. While one pair of legs provides support, the other pair of legs swings forward in preparation for the next step. Diagonal legs form pairs in the trot, lateral legs form pairs in the pace, and the front legs and rear legs each form a pair in the bound.

An important piece of background comes from Sutherland's work on the construction of a human-carrying walking machine (Sutherland 1983, Sutherland and Ullner 1984). In order to design the hydraulic circuits that would coordinate the load-bearing operation of the machine, Sutherland introduced the idea of a *virtual leg*. He made two or more physical legs act like a single equivalent virtual leg. For instance, when two legs move downward to lift the machine, each leg supports the same load because a parallel hydraulic circuit equalizes the oil pressures. The two physical legs act like one virtual leg located between them, as shown in figure 4.7. The forces and torques exerted on the body by the set of physical legs and by the virtual leg are equal, so the behavior of the body is the same in both cases. The virtual leg concept permitted Sutherland to express and analyze the complicated behavior of a machine with six legs in simpler terms.

The idea of the virtual leg separates the task of generating quadruped running gaits that use legs in pairs into two simpler tasks. One task is to control the physical legs of a pair, so that the virtual leg they form behaves as desired. A set of operations that coordinate the physical legs to produce desired virtual behavior are

- *Positioning.* Choose positions for the physical feet that place the virtual foot in the desired location.

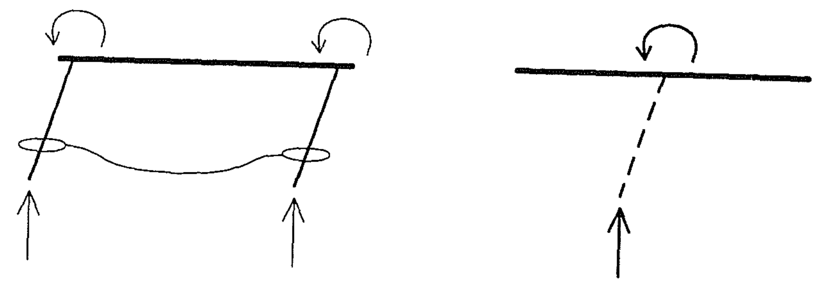


Figure 4.7. Virtual legs. When two legs act in unison, they can be thought of as a functionally equivalent *virtual leg*. The original pair of legs and the virtual leg exert the same forces and moments on the body, so they both produce the same behavior of the body. A *force-equalizing* virtual leg is shown here. It locates the virtual leg halfway between the two physical legs it represents. Sutherland (1983) introduced the concept of the virtual leg to simplify the design of a six-legged walking machine.

- *Synchronization.* Ensure that both legs of a pair strike the ground in unison and leave the ground in unison.
- *Force Equalization.* Ensure that both legs of a pair develop equal axial force to the ground.

In order to implement each of these operations, the control system must coordinate the low-level behavior of the physical legs.

The second task in generating quadruped running gaits that use legs in pairs is to provide locomotion algorithms that specify the desired behavior for the virtual system. Because the trot, the pace, and the bound reduce to *virtual biped one-foot gaits,* as shown in figure 4.8, the results of the previous section apply. The three-part control algorithms can be used to specify the behavior of each virtual leg. The same sequencing mechanism that was used for the biped (figure 4.5 and table 4.1) can be used for the quadruped, with the understanding that it now specifies virtual legs rather than physical legs.

The *force-equalizing* virtual leg is a special case of Sutherland's more general concept. When the control system equalizes the axial forces that the legs deliver to the ground, it moves the effective support point halfway between the physical feet. This choice makes the resulting behavior simple to analyze and similar to that of the one-legged systems we have already studied. It is possible to implement virtual legs that do not equalize axial forces, with correspondingly more complicated behavior. Equations that define the general virtual leg are given in appendix 4A. We consider only the force-equalizing type of virtual leg here.

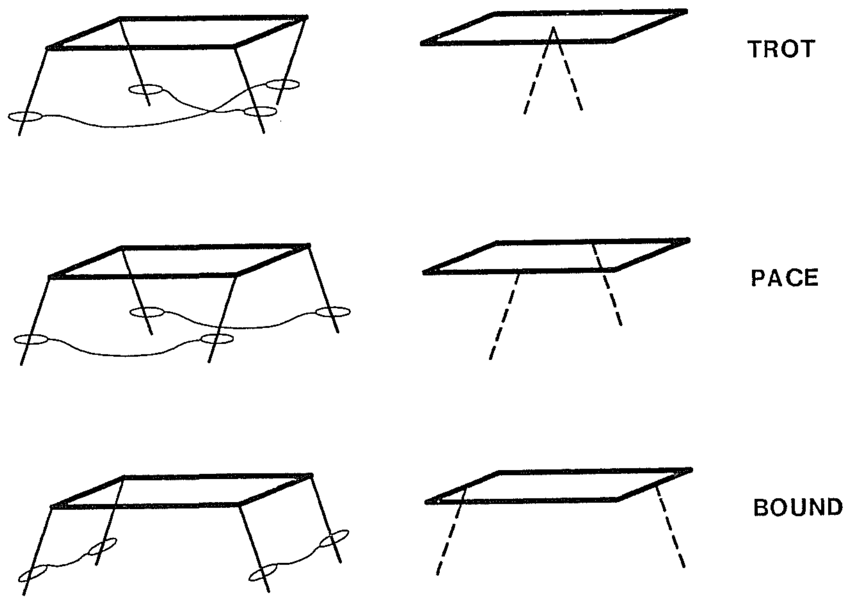


Figure 4.8. Three quadruped gaits move the legs in pairs. In the trot diagonal pairs of legs act in unison, as shown by the shackles. They strike the ground at the same time, they leave the ground at the same time, and they swing forward at the same time. In the pace lateral pairs of legs act in unison. In the bound the front legs act in unison, as do the rear legs. When the virtual leg idea is used, each of the gaits shown on the left reduces to the virtual biped one-foot gait shown on the right.

Relationships between the physical and virtual legs permit the control system to convert the desired behavior of the virtual leg into control signals for each physical leg. For instance, if the physical hips were separated a distance $2d$ in the fore-aft direction, then the control system could add d to the desired virtual foot position to find the desired position for the forward physical foot of the pair and could subtract d to find the desired position for the rear physical foot. A similar procedure could position the feet sideways.

The one-foot running gaits described in the last section would be difficult for a quadruped to use because it is difficult to locate four hips close enough to the center of mass to position the feet properly while avoiding leg collisions. When pairs of legs act together, however, the effective point of support is located close to the center of mass even though the physical legs are located a substantial distance from the center of the body. The effective point of support provided by a virtual leg lies halfway along the line connecting the two physical feet. In the trot, the gait involving diagonal

support pairs, the midpoints of the lines connecting the feet of each pair are near the center of the body, which we assume is near the center of mass. In the pace, the gait involving lateral support pairs, the lines connecting the feet may pass under the center of mass if the legs are angled inward during stance or they may pass quite close to the center of mass if the body is narrow.

In the bound the virtual legs do not provide support under the center of mass. If the body is elongated in the fore-aft direction, the virtual leg formed by the front legs may not be able to reach under the center of the body. This is the same problem described earlier for the one-foot gaits. For this reason it may be incorrect to include the bound in the same class with the trot and pace because part of the control task will be to stabilize the pronounced pitching motion of the body. Once again, a flexible spine would help to provide a solution, or the control system might be extended to tolerate the pitching motions that occur when the virtual feet are not placed far enough under the body.

To summarize, the concept of a virtual leg separates the task of making a quadruped run into two simpler tasks. One task is to couple the behavior of two physical legs so their combined behavior conforms to the desired behavior of an equivalent virtual leg. The second task is to provide control algorithms for virtual biped one-foot gaits. This approach applies to three quadruped gaits, the trot, the pace, and perhaps the bound.

Quadruped Trotting Experiments Using Virtual Legs

In order to explore the idea that algorithms for systems with one leg can be extended to systems with four legs, we built a four-legged running machine (Raibert, Chepponis, and Brown 1985). Figures 4.9 describes the machine, and figure 4.10 shows a photograph of it running with a trotting gait.

We designed the control system for these experiments along the lines discussed in the previous two sections. The control system uses the one-leg algorithms to specify desired behavior for each virtual leg, it sequences the virtual legs with a state machine, and it coordinates the behavior of pairs of physical legs to act like virtual legs.

One-Leg Algorithms

The algorithms used previously to control the one-legged hopping machines were implemented to control the virtual legs of the four-legged running ma-

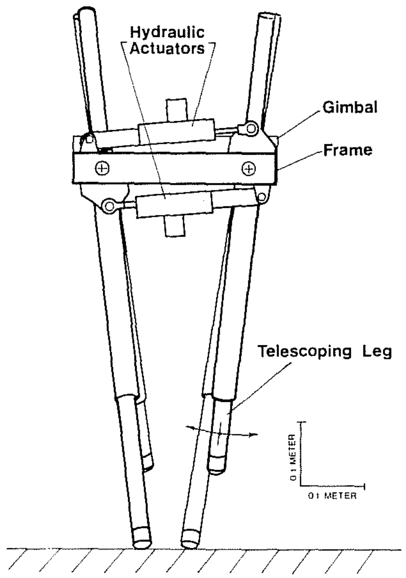


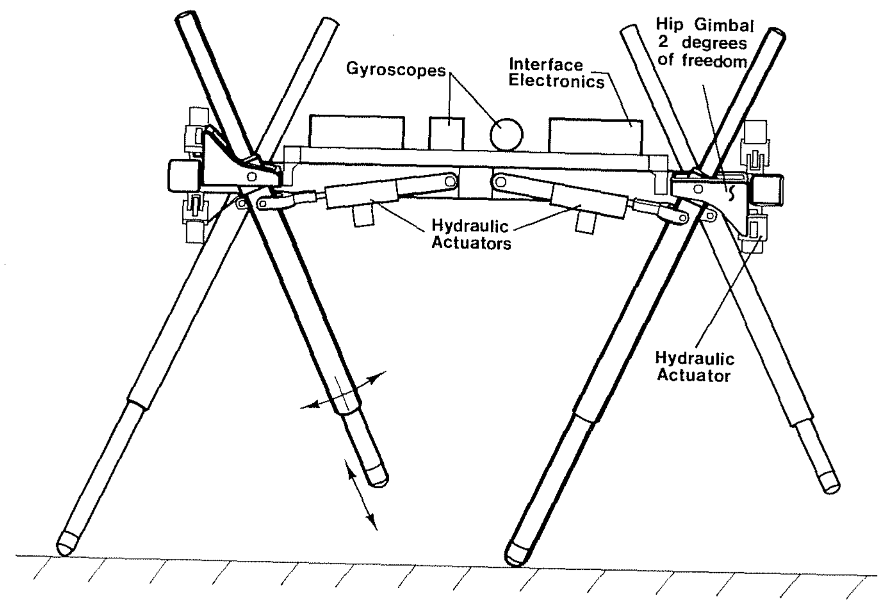
Figure 4.9. Diagram of running machine used for experiments. The body is an aluminum frame on which are mounted hip actuators, gyroscopes, and computer interface electronics. Each hip has two low friction hydraulic actuators that position a leg fore and aft and sideways. An actuator within each leg changes the leg's length and an air spring makes the leg springy in the axial direction (see figure 4.4). Sensors measure the lengths of the legs, the positions, velocities, and pressures of the hydraulic hip actuators, pressures in the leg air springs, contact between the feet and the floor, and the pitch and roll angles of the body. Fore-aft hip spacing is 0.78 m, and lateral hip spacing is 0.24 m.

chine. To control the forward running velocity, the control system positions the virtual legs with respect to the center of mass of the body during each flight phase:

$$\mathbf{x}_{f,d} = \frac{\dot{\mathbf{x}}T_s}{2} + \mathbf{k}_{\dot{\mathbf{x}}}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d), \tag{4.1}$$

where

$\mathbf{x}_{f,d}$	is the desired displacement of the virtual foot with respect to the projection of the center of mass,
$\dot{\mathbf{x}}$	is the forward velocity in machine coordinates, and
$\mathbf{k}_{\dot{\mathbf{x}}}$	is a diagonal gain matrix.



To control the pitch and roll attitude of the body during stance, the control system applies torques about the virtual hips with linear servos:

$$\tau_x = -k_{p,x}(\phi_P - \phi_{P,d}) - k_{v,x}(\dot{\phi}_P), \tag{4.2}$$

$$\tau_y = -k_{p,y}(\phi_R - \phi_{R,d}) - k_{v,y}(\dot{\phi}_R), \tag{4.3}$$

where

τ_x, τ_y	are virtual hip torques,
ϕ_P, ϕ_R	are the pitch and roll angles of the body, and
k_p, k_v	are gains.

In the experiments reported here, an operator used a two-axis joystick to specify the desired forward running velocity, $(\dot{x}_d, \dot{y}_d)$. The control system estimates the forward velocity of the body using the assumption that the feet do not move with respect to the ground during stance. Under this assumption, the backward motion of a foot with respect to the body is equal to the forward motion of the body with respect to the ground. We

Table 4.2. Physical parameters of four-legged running machine used for experiments.

Parameter	Metric Units	English Units
Overall length	1.05 m	41.2 in
Overall height	0.95 m	37.5 in
Overall width	0.35 m	13.8 in
Hip height (max)	0.668 m	26.31 in
Hip spacing (x)†	0.776 m	30.56 in
Hip spacing (y)	0.239 m	9.40 in
Leg sweep angle (x)	$\pm$0.565 rad	$\pm 32.4°$
Leg sweep angle (y)	$\pm$0.384 rad	$\pm 22.0°$
Leg stroke (hydraulic)	0.229 m	9.0 in
Leg stroke (spring)	0.102 m	4.0 in
Body mass	25.2 kg	55.4 lb
Body moment of inertia (x)	$0.257\,\mathrm{kg \cdot m^2}$	$880\,\mathrm{lb \cdot in^2}$
Body moment of inertia (y)	$1.60\,\mathrm{kg \cdot m^2}$	$5470\,\mathrm{lb \cdot in^2}$
Body moment of inertia (z)	$0.86\,\mathrm{kg \cdot m^2}$	$6340\,\mathrm{lb \cdot in^2}$
Leg mass, total	1.40 kg	3.08 lb
Leg mass, unsprung	0.286 kg	0.63 lbm
Leg moment of inertia (about hip)	$0.14\,\mathrm{kg \cdot m^2}$	$480\,\mathrm{lb \cdot in^2}$
Leg spring stiffness @20 psi	2100 N/m	12 lbf/in
Hip torque, @2000 psi (x)	111 N · m	983 in · lbf
Hip torque, @2000 psi (y)	77.6 N · m	687 in · lbf
Leg thrust, @2000 psi	765 N	172 lbf

† x—fore and aft, y—sideways, z—up and down.

also assume that the forward running velocity does not change during flight. The control system measures the duration of stance T_s during each stance phase, and it uses the most recent value for control.

The control system adjusts the hydraulic length of the virtual leg several times throughout the running cycle to control the hopping motion. The length of a physical leg is the sum of the hydraulic length w_ℓ plus the spring length r_s, as shown in figure 4.4. The hydraulic length of a virtual leg is the average of the hydraulic lengths for the two physical legs. When a virtual leg is in recovery, the desired hydraulic length is shortened ($w_{\ell,d} = L_1$) to keep the feet from touching the ground. This keeps the virtual leg out of the way. When a virtual leg is preparing for landing or compressing under load of the body, the desired hydraulic length is set to an intermediate value ($w_{\ell,d} = L_2$). During the second part of stance, when the legs delivers a thrust to the body, the desired hydraulic length is increased, ($w_{\ell,d} = L_3$). The operator specifies L_1, L_2, and L_3 from a control box, with $L_1 < L_2 < L_3$.

Implementing Virtual Legs

In order to make the legs work together in pairs, the control system coordinates positioning of the physical legs, synchronizes ground contact, and equalizes the axial leg force. The geometry of the body simplifies the task of positioning the physical legs, especially for trotting. Because the hips are located in symmetric positions about the center of mass, an x-y displacement of both physical feet from the projection of their hips results in a comparable x-y displacement of the virtual foot from the projection of the center of mass. Therefore the desired position of the virtual foot with respect to the center of mass is used to specify the desired position of the physical legs with respect to their hips:

$$x_{h,i,d} = x_{h,j,d} = x_{f,d}\ , \tag{4.4}$$

$$y_{h,i,d} = y_{h,j,d} = y_{f,d}\ , \tag{4.5}$$

where

$x_{h,i,d}$, $y_{h,i,d}$	is the desired displacement of the ith foot with respect to the projection of the ith hip,
$x_{f,d}$, $y_{f,d}$	is the desired displacement of the virtual foot with respect to the projection of the center of mass, and
i, j	designate the physical legs that form a virtual leg.

Once these desired foot displacements are known, transformations based on the kinematics of the legs, hips, and actuators are used to find actuator lengths that will position the foot as desired. These transformations, which are given in appendix 4B, take into account the pitch and roll orientations of the body and the lengths of the legs.

To synchronize the instant of ground contact for the two legs forming a virtual leg, the control system servos the leg lengths during flight, so that both feet have the same altitude. This adjustment affects only the difference in the lengths of the legs, whereas L_2 determines the average length. Pitch and roll measurements and a kinematic calculation are required to perform these adjustments.

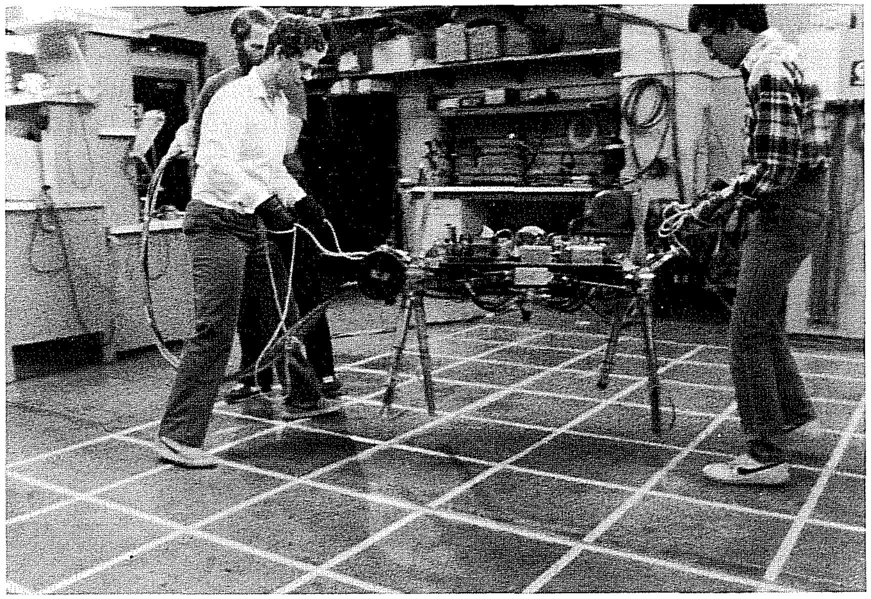


Figure 4.10. Quadruped machine trotting. The trot is a gait that uses diagonal pairs of legs in synchrony. This photograph was taken during the flight phase as a pair of legs prepared for landing. Running speed was about 0.75 m/s. During these experiments *wranglers* held safety ropes attached to the machine.

This approach to synchronizing ground contact works correctly on a flat floor, but it would fail if used with a support surface that was not flat. An alternative approach that might better tolerate variations in the altitude of the terrain would be to servo the axial leg forces in anticipation of ground contact, perhaps using (4.6). When one foot struck the ground, its leg would retract and the other extend. When both feet touch the ground, the stance phase would proceed. This method would rely on fast response from the actuators that retract the legs.

The control system servos the differential length of the leg actuators to equalize the axial forces delivered by the legs during stance:

$$w_{\ell,i,d} = w_{\ell,i} + \frac{r_{s,i} - r_{s,j}}{2}, \tag{4.6}$$

where

$w_{\ell,i}$	is the hydraulic length of the ith leg,
$r_{s,i}$	is the air spring length of the ith leg, and
r_i	is the length of the ith leg, $r_i = w_{\ell,i} + r_{s,i}$.

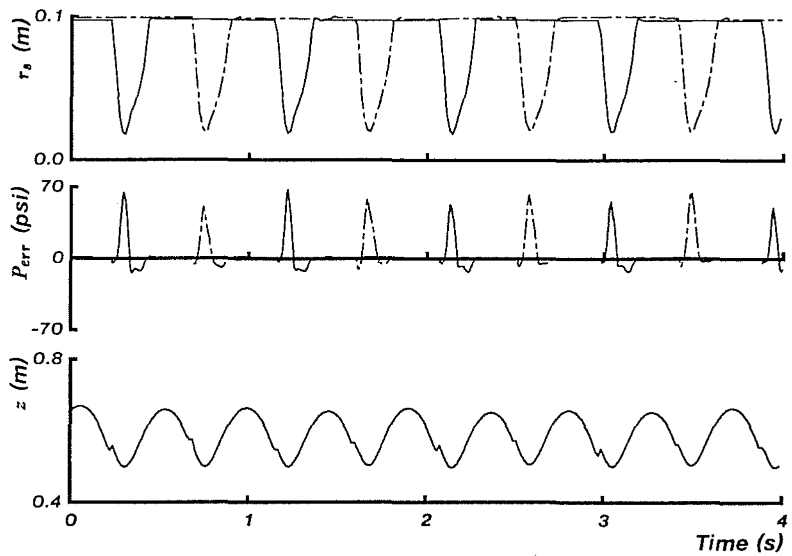


Figure 4.11. Quadruped trotting data. These graphs show the bouncing motion that underlies the machine's running. The top curve shows the alternating compression of the air springs for legs 1 and 4. The second curve shows the error in force equalization for the legs that form a virtual leg. One curve (solid) shows the difference in air spring pressures for legs 1 and 3, and the other curve (dash-dots) shows the pressure difference for legs 2 and 4. The bottom curve shows the altitude of the body above the floor, as estimated by the control system. The discontinuities in z are due to errors in estimating the vertical velocity of the body when the feet leave the ground.

This differential adjustment forces the air springs to assume equal lengths and therefore to generate equal axial force. Once again, values for L_2 and L_3 determine the average length of the hydraulic actuators contributing to the virtual leg.

Twelve linear servos act on the hydraulic actuators to position the hips and leg lengths once the desired actuator lengths are known:

$$f_i = -k_p(w_i - w_{i,d}) - k_v(\dot{w}_i), \tag{4.7}$$

where

f_i	is the servovalve output signal for the ith actuator,
w_i, $w_{i,d}$, $\dot{w}_i$	are the position, desired position, and velocity of the ith actuator, and
k_p, k_v	are position and velocity gains.

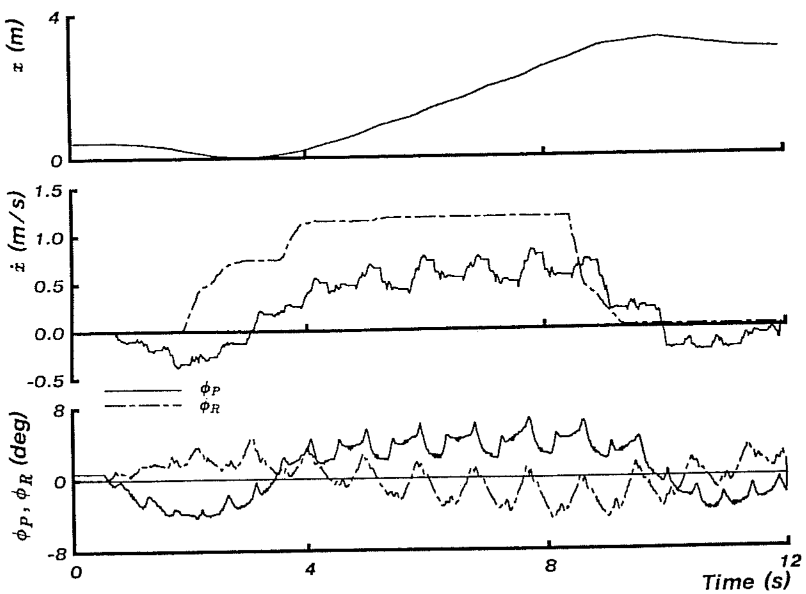


Figure 4.12. Forward running. When the joystick specifies a desired forward velocity, the machine accelerates forward. The forward running velocity is not controlled with high precision, as shown by the plots of desired and actual running velocity, $\dot{x}$ and $\dot{x}_d$. The body tips in the direction of running, as shown by the pitch and roll angles. Positive pitch indicates nose down.

Data recorded during trotting experiments are shown in figures 4.11 and 4.12. They show that diagonal pairs of legs are used for support in alternation, as required for trotting. The synchronization of foot impacts and equalization of axial leg forces are controlled with good precision as shown by the small differences in axial forces between the legs of a pair. The vertical bouncing motion of the body is regular and quite smooth.

The control system's ability to regulate forward running velocity is rather poor, as shown in figure 4.12. We observe only a rough proportionality between the desired and actual running velocities. These errors may be due to the limitations of the velocity control algorithm described in the previous two chapters. During forward running the inclination of the body about the pitch axis ϕ_P deviates from the desired value by as much as 8°. The magnitude and sign of this error are generally related to the forward running velocity. The control system keeps error about the roll axis to about 5° in these experiments, although control of this degree of freedom proves to be the most difficult task for the control system to perform.

Discussion of Quadruped Experiments

We consider only force-equalizing virtual legs. The force-equalizing virtual leg has the virtue of keeping the effective point of support, the virtual foot, located halfway between the physical points of support. This makes the task of positioning the point of support comparable to the one-legged system's task of positioning the foot. This similarity permits us to bring our experience with one-legged systems to bear on the quadruped control problem.

A consequence of using the force-equalizing virtual leg, however, may be to limit the control system's ultimate performance. We have given up a degree of freedom in the control by restricting the control system to this special case. Without this restriction the control system could adjust the differential axial leg force and the hip torque to manipulate the location of the virtual foot *during* the support interval. Such adjustments could correct the forward running velocity with finer temporal resolution than the once-per-step method described. However, this would be done at the expense of simplicity in the control. Such additional control might be particularly useful at low stepping rates, or for systems that tip over quickly because of short legs.

Another consequence of using force-equalizing virtual legs is the loss of passive stability that a set of legs might otherwise provide. A table resists tipping when unevenly loaded, because the legs near the load generate more force than the legs that are remote from the load. If a table had force-equalizing legs, then an uneven load would cause the legs near the load to shorten, the legs remote from the load to lengthen, and the surface to tip. This force-equalized behavior should be expected, since it is precisely the behavior of a table with just one leg located in the middle. This one-legged behavior is what we set out to accomplish in the first place.

Our experiments have shown that this approach which discards passive stability of the legs, is workable, but it leaves us in a philosophical quandary. On the one hand, the force-equalizing virtual leg permits relatively sophisticated behavior with an extremely simple implementation, largely because it permits us to build on previous results. On the other hand, we believe that a well-engineered control system should take advantage of the intrinsic mechanical properties of the mechanism. If the machine is cleverly designed, its intrinsic mechanical behavior will be the desired behavior. The control system need only fine tune this correct system behavior, not having to fight the mechanism to make it obey. Such an approach that splits responsibility for good behavior between the mechanical design and the control design leads to simpler, hardier, and more efficient machines. Because the control system that uses the force-equalizing virtual leg discards the passive stability available from the legs rather than somehow harnessing it, we expect that it will eventually be replaced by a better method.

Despite these limitations, it is entirely possible that four-legged animals use force equalization when they trot, pace, or bound. One might find out by measuring the axial forces that develop in the legs of running quadrupeds, perhaps using sets of force platforms. The experiment would disturb one of the feet during stance by shifting the support surface upward or downward. If force equalization were in effect, the difference in axial leg force would not be affected by the manipulation. Exact force equalization is unlikely to be found, because the distribution of mass in animals' bodies is skewed by the asymmetric placement of their heads and often by unequal lengths of the fore and hind legs. One might expect, therefore, to find that the forces delivered by the legs vary in proportion to the loading caused by distribution of mass in the body.

There are two quadruped running gaits in addition to the trot that should yield to the virtual leg approach. They are the pace and the bound. These gaits use pairs of legs in unison, as does the trot, but they also involve pitching and rolling motions of the body. We have not yet done experiments with these gaits, but we anticipate that they will require somewhat more sophisticated methods for controlling the attitude of the body and the forward running velocity.

For instance, if the running motion has a rhythmic rolling or pitching motion of the body, it does not seem wise to servo the body to a fixed upright posture during stance. Likewise, running velocities in both the pace and the bound would normally vary on alternate steps, so the control system should systematically accommodate those variations. While the additional elements needed for control of pacing and bounding are not provided by the existing control system, they represent minor additions that can fit into the existing locomotion framework without major renovation.

So far, we have considered virtual legs that represent the behavior of physical legs acting in unison. A plausible extension of the concept might represent the behavior of physical legs that act in sequence, possibly overlapped in time. The simplest approach would separate each support interval into subintervals, during which a fixed set of legs would provide support. Then the entire support interval might be represented by a sequence of virtual support phases. For a rotary gallop the sequence of phases would be (1) right rear, (2) right rear and left rear, (3) left rear, (4) left rear and left front, (5) left front, (6) left front and right front, and (7) right front. A key challenge in this problem is to find a mechanism that can mediate the smooth exchange of support from one leg to another without disrupting the bouncing motion of the body. With such an approach one might understand and produce galloping by using control techniques no more complicated than those already described.

I have discussed methods for generating several gaits but have been silent on the issue of choosing which gait to use. In animals, the energetic cost of a gait seems to be an important factor in its selection. Animals change gait as they change speed in order to minimize the cost of transportation (Hoyt and Taylor 1981). The geometry of the animal may also enter into gait selection. At low running speeds, for instance, long-legged animals use a pace rather than a trot, presumably to avoid interference between the front and rear legs on each side (Hildebrand 1968). Other factors, such as the range of leg motion and stiffness, may also be important. Despite these potential factors, we do not yet have any clear criteria for selecting one gait over another.

Summary

- Locomotion control algorithms originally developed for use with one leg are extended for use with several legs.
- There is a class of gaits called the one-foot, for which only one foot touches the ground at a time and each stance phase is separated by a flight phase. In principle, the three-part control algorithms can be used to control a variety of systems executing one-foot gaits, independent of the number of legs.
- Control systems for N-legged one-foot gaits need mechanisms to sequence the use of the legs, and they must be able to shorten the idle legs to keep them out of the way.
- Experiments with a planar biped demonstrated the feasibility of using the three-part control decomposition to produced a multilegged one-foot gait.
- The behavior of a pair of legs acting in unison can be represented by an equivalent virtual leg (Sutherland 1983). Virtual legs are used to map several quadruped running gaits—the trot, the pace, and the bound—into virtual biped one-foot gaits.

- The control system for a four-legged running machine uses the one-leg algorithms, a finite state machine, and virtual legs to make the machine run with a trotting gait. Pacing and bouncing should be achievable with similar methods.
- The virtual leg approach to quadruped control discards the passive stability that four legs can provide, in favor of active stability.

Appendix 4A. Equations for Virtual Leg

This appendix describes the relationships between a general virtual leg and the two physical legs it represents. The equations given below were developed by Ben Brown to express the angular position θ, axial force f, and hip torque τ for the virtual leg in terms of variables that describe the behavior of the two physical legs. The analysis is for the static case. The configuration and variables are defined in figure 4.13.

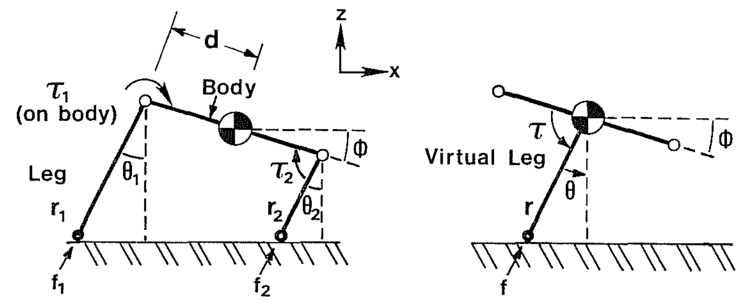


Figure 4.13. Model used for analysis of virtual leg.

We assume that the virtual foot is located on a line connecting the physical feet. This yields the geometric constraint

$$r_1 \cos\theta_1 + r_2 \cos\theta_2 = 2r\cos\theta = 2A. \tag{4.8}$$

Body angle ϕ is not independent of θ_1, θ_2, r_1, r_2, and d, so

$$r_1 \cos\theta_1 - r_2 \cos\theta_2 = 2d\sin\phi. \tag{4.9}$$

By summing forces and moments we obtain

$$\begin{aligned} \sum F_x &= f_1 \sin\theta_1 + f_2 \sin\theta_2 - \frac{\tau_1}{r_1}\cos\theta_1 - \frac{\tau_2}{r_2}\cos\theta_2 \\ &= f\sin\theta - \frac{\tau}{r}\cos\theta \\ &= B, \end{aligned} \tag{4.10}$$

$$\begin{aligned} \sum F_z &= f_1 \cos\theta_1 + f_2 \cos\theta_2 + \frac{\tau_1}{r_1}\sin\theta_1 + \frac{\tau_2}{r_2}\sin\theta_2 - mg \\ &= f\cos\theta - \frac{\tau}{r}\sin\theta - mg \\ &= C, \end{aligned} \tag{4.11}$$

$$\sum M_{cg} = -f_1 d\cos(\theta_1 - \phi) + f_2 d\cos(\theta_2 - \phi) - \frac{\tau_1}{r_1}(r_1 + d\sin(\theta_1 - \phi)) - \frac{\tau_2}{r_2}(r_2 - d\sin(\theta_2 - \phi)) = -\tau = -D. \tag{4.12}$$

Using the lumped variables A, B, C, and D, we can solve (4.8)–(4.12) for θ, r, τ, and f:

$$r = \frac{A}{\cos\theta}, \tag{4.13}$$

$$\theta = \arctan\left(\frac{B + D/A}{C}\right), \tag{4.14}$$

$$\tau = D, \tag{4.15}$$

$$f = \frac{B + (\tau/r)\cos\theta}{\sin\theta}. \tag{4.16}$$

How do these results square with the force-equalizing virtual leg described in the text? For $f_1 = f_2$, $r_1 \sin\theta_1 = r_2 \sin\theta_2$, and small ϕ,

$$r \approx \frac{r_1 + r_2}{2}, \tag{4.17}$$

$$\theta \approx \frac{\theta_1 + \theta_2}{2}, \tag{4.18}$$

$$f \approx 2f_1, \tag{4.19}$$

$$\tau \approx \tau_1 + \tau_2. \tag{4.20}$$

This is the desired result.

Appendix 4B. Kinematics for Four-Legged Machine

Machine coordinates $\{\mathbf{M}\}$. A Cartesian coordinate system that moves with the machine as follows:

1. The origin is fixed to the machine's center of mass.
2. The x-axis points forward toward the front of the machine but always lies in the horizontal plane. Positive x is forward.
3. The y-axis points toward the left-hand side of the machine but always lies in the horizontal plane. Positive y is to the machine's left.
4. The z-axis points up along the gravity vector. Positive z is up.

Think of this coordinate system as fixed to the inner gimbal of the vertical gyroscope. It moves about the room as the machine moves, it turns about the yaw axis as the machine turns, but the x- and y-axes always remains in the horizontal plane. This is roughly how your eyes behave when you walk and run.

Hip coordinates $\{\mathbf{H}_i\}$. A Cartesian coordinate system affixed to the ith hip:

1. The origin is located at the hip.
2. The x-, y-, and z-axes are parallel to the corresponding axes of $\mathbf{M}$. Positive x is forward, positive y is to the machine's left, and positive z is up. The x-y plane is always horizontal.

Body coordinates $\{\mathbf{B}_i\}$. A Cartesian coordinate system fixed to the body of the machine at the ith hip:

1. The origin is located at the ith hip.
2. The x-axis lies in the plane of the body, pointing forward. Positive x is forward.
3. The y-axis lies in the plane of the body, pointing to the left side. Positive y is to the machine's left.
4. The z-axis completes the right-hand set, pointing normal to the top surface of the body. Positive z is up.

The angles between $\{\mathbf{B}_i\}$ and $\{\mathbf{H}_i\}$ are the pitch and roll angles indicated by the vertical gyroscope.

Actuator coordinates $\{\mathbf{A}_i\}$. One can think of the foot's position expressed in terms of the actuator lengths. It is a vector of actuator lengths for the ith leg ($w_{x,i}$, $w_{y,i}$, $r_i = w_{\ell,i} + r_{s,i}$) that correspond to a given foot position with respect to the body.

Compass coordinates $\{\mathbf{C}\}$. A Cartesian coordinate system whose origin moves with the machine's center of mass but with a fixed orientation in space. The axes are parallel to those of $\{\mathbf{W}\}$. It would ride on the inner gimbal of a vertically stabilized compass. (Not used.)

World coordinates $\{\mathbf{W}\}$. A Cartesian coordinate system with a fixed position and orientation in the laboratory:

1. The x- and y-axes lie in the horizontal plane with z vertical.
2. The x- and y-axes are aligned with the walls in the laboratory.

(Not used.)

Coordinate Transformations

Kinematically, each hip of the quadruped machine is nearly identical to the hip of the 3D one-legged hopping machine (see figure 3.15). The kinematic parameters are $l_1 = 0.3194\,\text{m}$, $l_2 = 0.0444\,\text{m}$, $l_3 = 0.2648\,\text{m}$,$l_4 = 0.0635\,\text{m}$, $\alpha = 8.0°$, $\beta = 13.872°$, fore-aft hip separation $L = 0.775\,\text{m}$, lateral hip separation $W = 0.23876\,\text{m}$.

$\{\mathbf{A}_i\} \Rightarrow \{\mathbf{B}_i\}$. Function AtoB$_i$ determines where the ith foot is with respect to the ith hip:

$$(x_f, y_f, z_f)_{\{\mathbf{B}\},i} = AtoB_i[(w_x, w_y, r)_i], \tag{4.21}$$

$$x_{f,\{\mathbf{B}\},i} = r_i \sin\left\{\arcsin\left(\frac{A\,w_{x,i}^2 - l_1^2 - l_2^2}{-2l_1 l_2}\right) - \alpha\right\}, \tag{4.22}$$

$$y_{f,\{\mathbf{B}\},i} = \sqrt{r_i^2 - x_{f,\{\mathbf{B}\},i}^2}\quad \sin\left\{\arcsin\left(\frac{-A\,w_{y,i}^2 - l_3^2 - l_4^2}{2l_3 l_4}\right) + \beta\right\}, \tag{4.23}$$

$$z_{f,\{\mathbf{B}\},i} = -\sqrt{r_i^2 - x_{f,\{\mathbf{B}\},i}^2 - y_{f,\{\mathbf{B}\},i}^2}\,, \tag{4.24}$$

$$\text{where}\quad A = [-1, 1, 1, -1] \quad \text{for } i = 1, 2, 3, 4.$$

$\{\mathbf{B}_i\} \Rightarrow \{\mathbf{A}_i\}$. Function BtoA$_i$ determines the actuator positions that will place the foot at the specified point:

$$w_{x,i} = \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \sin\left(\arcsin \frac{A\,x_{f,\{\mathbf{B}\},i}}{r_i} + \alpha\right)}, \tag{4.25}$$

$$w_{y,i} = \sqrt{l_3^2 + l_4^2 + 2l_3 l_4 \sin\left(\arcsin \frac{-Ay_{f,\{\mathbf{B}\},i}}{\sqrt{w_{l,i}^2 - x_{f,\{\mathbf{B}\},i}^2}} - \beta\right)}, \tag{4.26}$$

$$r_i = \sqrt{x_{f,\{\mathbf{B}\},i}^2 + y_{f,\{\mathbf{B}\},i}^2 + z_{f,\{\mathbf{B}\},i}^2}. \tag{4.27}$$

$\{\mathbf{B}_i\} \Rightarrow \{\mathbf{H}_i\}$. Function BtoH$_i$ transforms from body coordinates at the ith hip to hip coordinates:

$$\begin{aligned} x_{f,\{\mathbf{H}\}} &= x_{f,\{\mathbf{B}\}} \cos\theta_P + y_{f,\{\mathbf{B}\}} \sin\theta_P \sin\theta_R \\ &\quad + z_{f,\{\mathbf{B}\}} \sin\theta_P \cos\theta_R, \end{aligned} \tag{4.28}$$

$$y_{f,\{\mathbf{H}\}} = y_{f,\{\mathbf{B}\}} \cos\theta_R - z_{f,\{\mathbf{B}\}} \sin\theta_R, \tag{4.29}$$

$$\begin{aligned} z_{f,\{\mathbf{H}\}} &= -x_{f,\{\mathbf{B}\}} \sin\theta_P + y_{f,\{\mathbf{B}\}} \cos\theta_P \sin\theta_R \\ &\quad + z_{f,\{\mathbf{B}\}} \cos\theta_P \cos\theta_R. \end{aligned} \tag{4.30}$$

$\{\mathbf{H}_i\} \Rightarrow \{\mathbf{B}_i\}$. Function HtoB$_i$ is the inverse of BtoH$_i$:

$$x_{f,\{\mathbf{B}\}} = x_{f,\{\mathbf{H}\}} \cos\theta_P - z_{f,\{\mathbf{H}\}} \sin\theta_P, \tag{4.31}$$

$$\begin{aligned} y_{f,\{\mathbf{B}\}} &= x_{f,\{\mathbf{H}\}} \sin\theta_P \sin\theta_R + y_{f,\{\mathbf{H}\}} \cos\theta_R \\ &\quad + z_{f,\{\mathbf{H}\}} \cos\theta_P \sin\theta_R, \end{aligned} \tag{4.32}$$

$$\begin{aligned} z_{f,\{\mathbf{B}\}} &= x_{f,\{\mathbf{H}\}} \sin\theta_P \cos\theta_R - y_{f,\{\mathbf{H}\}} \sin\theta_R \\ &\quad + z_{f,\{\mathbf{H}\}} \cos\theta_p \cos\theta_R. \end{aligned} \tag{4.33}$$

$\{\mathbf{B}_i\} \Rightarrow \{\mathbf{M}\}$. Function BtoM_i transforms from body coordinates at the ith hip to machine coordinates:

$$
\begin{aligned}
x_{f,\{\mathbf{M}\}} = &\left(x_{f,\{\mathbf{B}\}} + \frac{BL}{2}\right)\cos\theta_P \\
&+ \left(y_{f,\{\mathbf{B}\}} + \frac{CW}{2}\right)\sin\theta_P \sin\theta_R \\
&+ z_{f,\{\mathbf{B}\}} \sin\theta_P \cos\theta_R,
\end{aligned}
\tag{4.34}
$$

$$
y_{f,\{\mathbf{M}\}} = \left(y_{f,\{\mathbf{B}\}} + \frac{CW}{2}\right)\cos\theta_R - z_{f,\{\mathbf{B}\}} \sin\theta_R,
\tag{4.35}
$$

$$
\begin{aligned}
z_{f,\{\mathbf{M}\}} = &-\left(x_{f,\{\mathbf{B}\}} + \frac{BL}{2}\right)\sin\theta_P \\
&+ \left(y_{f,\{\mathbf{B}\}} + \frac{CW}{2}\right)\cos\theta_P \sin\theta_R \\
&+ z_{f,\{\mathbf{B}\}} \cos\theta_P \cos\theta_R,
\end{aligned}
\tag{4.36}
$$

$$
\begin{aligned}
B &= [1, -1, -1, 1] \quad \text{for } i = 1, 2, 3, 4, \\
C &= [1, 1, -1, -1] \quad \text{for } i = 1, 2, 3, 4.
\end{aligned}
$$

This is equivalent to $\mathrm{BtoH}[(x_f + \frac{BL}{2}, y_f + \frac{CW}{2}, z_f)]$.

$\{\mathbf{M}\} \Rightarrow \{\mathbf{B}_i\}$. Function MtoB_i is the inverse of BtoM_i:

$$
x_{f,\{\mathbf{B}\}} = x_{f,\{\mathbf{M}\}} \cos\theta_P - z_{f,\{\mathbf{M}\}} \sin\theta_P - \frac{BL}{2},
\tag{4.37}
$$

$$
\begin{aligned}
y_{f,\{\mathbf{B}\}} = &\; x_{f,\{\mathbf{M}\}} \sin\theta_P \sin\theta_R + y_{f,\{\mathbf{M}\}} \cos\theta_R \\
&+ z_{f,\{\mathbf{M}\}} \cos\theta_P \sin\theta_R - \frac{CW}{2},
\end{aligned}
\tag{4.38}
$$

$$
\begin{aligned}
z_{f,\{\mathbf{B}\}} = &\; x_{f,\{\mathbf{M}\}} \sin\theta_P \cos\theta_R - y_{f,\{\mathbf{M}\}} \sin\theta_R \\
&+ z_{f,\{\mathbf{M}\}} \cos\theta_P \cos\theta_R.
\end{aligned}
\tag{4.39}
$$

$\{\mathbf{H}_i\} \Rightarrow \{\mathbf{A}_i\}$. Function HtoA_i is used to find desired actuator lengths when desired foot placement is known:

$$
(w_x, w_y)_i = \mathrm{BtoA}_i[\mathrm{HtoB}_i[(x_f, y_f)_i]].
\tag{4.40}
$$

$\{\mathbf{A}_i\} \Rightarrow \{\mathbf{H}_i\}$. Function AtoH_i is used to find the foot position that corresponds to a given set of actuator lengths. This can be used during stance to measure the velocity of the hip:

$$
(x_f, y_f, z_f)_i = \mathrm{BtoH}_i[\mathrm{AtoB}_i[(w_x, w_y, r)_i]].
\tag{4.41}
$$

$\{\mathbf{H}_i\} \Rightarrow \{\mathbf{M}\}$. Function HtoM_i transforms from hip to machine coordinates:

$$
(x, y, z)_{\{\mathbf{M}\}} = \mathrm{HtoM}_i[(x, y, z)_{\{\mathbf{H}\}}],
\tag{4.42}
$$

$$
x_{\{\mathbf{M}\}} = x_{\{\mathbf{H}\}} + \frac{BL}{2}\cos\theta_P + \frac{CW}{2}\sin\theta_P \sin\theta_R,
\tag{4.43}
$$

$$
y_{\{\mathbf{M}\}} = y_{\{\mathbf{H}\}} + \frac{CW}{2}\cos\theta_R,
\tag{4.44}
$$

$$
z_{\{\mathbf{M}\}} = z_{\{\mathbf{H}\}} - \frac{BL}{2}\sin\theta_P + \frac{CW}{2}\cos\theta_P \sin\theta_R,
\tag{4.45}
$$

$\{\mathbf{M}\} \Rightarrow \{\mathbf{H}_i\}$ Function MtoH_i transforms from machine coordinates to hip coordinates:

$$
(x, y, z)_{\{\mathbf{H}\}} = \mathrm{MtoH}_i[(x, y, z)_{\{\mathbf{M}\}}],
\tag{4.46}
$$

$$
x_{\{\mathbf{H}\}} = x_{\{\mathbf{M}\}} - \frac{BL}{2}\cos\theta_P - \frac{CW}{2}\sin\theta_P \sin\theta_R,
\tag{4.47}
$$

$$
y_{\{\mathbf{H}\}} = y_{\{\mathbf{M}\}} - \frac{CW}{2}\cos\theta_R,
\tag{4.48}
$$

$$
z_{\{\mathbf{H}\}} = z_{\{\mathbf{M}\}} + \frac{BL}{2}\sin\theta_P - \frac{CW}{2}\cos\theta_P \sin\theta_R,
\tag{4.49}
$$

$\{\mathbf{A}_i\} \Rightarrow \{\mathbf{M}\}$. Function AtoM_i is obtained by concatenating AtoB_i and BtoM_i. It determines where the feet are with respect to the hip and the velocity of the machine's center of mass.

$$
(x, y, z)_{\{\mathbf{M}\}} = \mathrm{BtoM}[\mathrm{AtoB}[(w_x, w_y, r)]].
\tag{4.50}
$$

$\{\mathbf{M}\} \Rightarrow \{\mathbf{A}_i\}$. Function MtoA_i is obtained by concatenating MtoB_i and BtoA_i. It is used to put a foot in a particular place with respect to the machine's center of mass.

$$
(w_x, w_y, r)_i = \mathrm{BtoA}[\mathrm{MtoB}[(x, y, z)]].
\tag{4.51}
$$

Chapter 5

Symmetry in Running

Running is a series of bouncing and ballistic motions that exert forces on the body during every stride. The bouncing motions are caused by the vertical rebound of the body when the legs push on the ground, and the ballistic motions occur between bounces when the body is airborne. If a legged system is to keep its forward running speed fixed and its body in a stable upright posture despite these motions, then the net acceleration of the body must be zero over each entire stride. This requires that the torques and horizontal forces exerted on the body by the legs must integrate to zero over each stride and that the vertical forces must integrate to the body's weight times the duration of the stride. This is equally true for running machines and for running animals.

Although there are many patterns of body and leg motion that can satisfy these requirements, a particularly simple solution arises when each variable changes with an even or odd symmetry during stance:

$$\begin{array}{c}\text{Body}\\ \text{Symmetry}\end{array} \quad \left\{ \begin{array}{l} x(t) = -x(-t) \\ z(t) = z(-t) \\ \phi(t) = -\phi(-t) \end{array} \right. \tag{5.1}$$

$$\begin{array}{c}\text{Leg}\\ \text{Symmetry}\end{array} \quad \left\{ \begin{array}{l} \theta(t) = -\theta(-t) \\ r(t) = r(-t) \end{array} \right. \tag{5.2}$$

where x, z, and ϕ are the forward position, vertical position, and pitch angle of the body and θ and r are the angle and length of the leg, all measured in the sagittal plane (see figure 5.1). For simplicity, t and x are defined so that $t=0$ halfway through the stance phase, and $x(0) = 0$.

These symmetry equations specify that forward body position, body pitch angle, and leg angle are each odd functions of time throughout the stance phase, and that body elevation and axial leg length are even functions of time.

The symmetry also requires that the actuators operate with even and odd symmetry:

$$\begin{array}{l} \text{Actuator} \\ \text{Symmetry} \end{array} \left\{ \begin{array}{l} f(t) = f(-t) \\ \tau(t) = -\tau(-t) \end{array} \right. \qquad (5.3)$$

where τ is the torque exerted about the hip and f is the force exerted along the leg axis.

These symmetries are significant because they result in accelerations of the body that are odd functions of time throughout a stride—odd functions integrate to zero over symmetric limits, leaving the forward running speed, body elevation, and body pitch angle unchanged from one stride to the next.

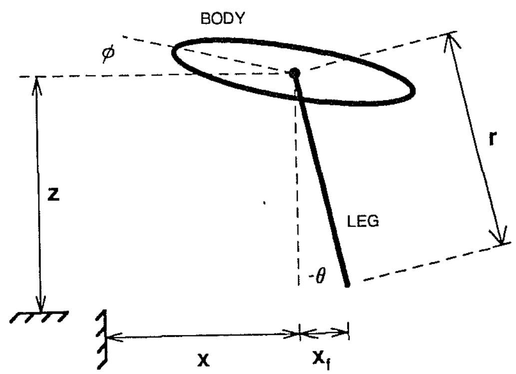


Figure 5.1. Definition of variables used in symmetry equations. Positive τ acts about the hip to accelerate the body in the positive ϕ direction. Positive f acts along axis of leg and pushes body away from the ground.

Control algorithms based on symmetry control the machines described in the previous three chapters. Symmetry is particularly simple for one-legged machines because only one leg provides support at a time, each support interval is isolated in time by periods of ballistic flight, and the hip is located at the center of mass. In this chapter we explore symmetry in more complicated circumstances. After reviewing the one-leg case, we consider motions that span several support intervals and that use several legs for support during a single support interval. The same symmetry equations apply for one, two, and four legs, and for gaits that use legs singly and in combination.

In addition to suggesting simple control for legged robots, the symmetries developed in this chapter may help us to understand the control mechanisms at work in running animals. To explore this possibility, we have examined data for the trotting and galloping cat and for the running human. Results described later in this chapter show that these biological legged systems sometimes move as the symmetries predict.

Mechanics of Symmetry

A number of simplifications ease the analysis of symmetry in running. We assume that motion is restricted to the plane, that the legs are massless, and that there are no losses anywhere in the system. The body is a rigid object that moves fore and aft and up and down and that pitches in the plane, with position and orientation given by $[x\ z\ \phi]$. Each leg is a single massless member that pivots about its hip at a hinge-type joint and that lengthens and shortens by telescoping. The length of the leg and its angle with respect to the vertical are given by $[r\ \theta]$. At the end of each leg is a foot that provides a single point of support. Friction between a foot and the ground prevents the foot from sliding when there is contact. A foot in contact with the ground acts mechanically like a hinge joint.

Each leg actuator exerts a force f along the leg's axis between the body and the ground. Positive f accelerates the body away from the ground, and, because the feet are not sticky, $f \geq 0$. When there is no contact between the foot and the ground, this force is zero. Normally, we think of the leg as being springy in the axial direction, in which case f is a function of leg length. A second actuator acts at the hip, generating a torque τ between the leg and the body. Positive τ accelerates the body in the positive ϕ direction. Equations of motion for this sort of model are given in appendix 5A.

Symmetric Motion with One Leg

Imagine that at time $t = 0$ the foot of a one-legged system is located directly below the center of mass, the body is upright, and the velocity of the body is purely horizontal: $\theta = 0$, $\phi = 0$, and $\dot{z} = 0$. Figure 5.2 shows this configuration. Because it has left-right symmetry and there are no losses, the system's expected behavior proceeding forward in time is precisely the same as its past behavior receding backward in time, but with a reflection

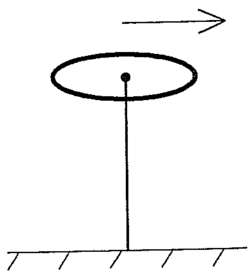


Figure 5.2. Symmetric configuration of one-legged system halfway through stance, when it has fore-aft symmetry (left-right as shown in diagram), as well as symmetry moving forward and backward in time. The vertical velocity is zero, the support point is located directly under the center of mass, and the body is upright: $\dot{\theta}(0) = x_f(0) = \phi(0) = 0$.

about the line $x = 0$. This behavior is described by the body symmetry equations (5.1): $x(t)$ and $\phi(t)$ are odd functions of time and $z(t)$ is an even function. Because the body moves along a symmetric trajectory with respect to the origin and because the foot is located at the origin during support, (5.1) implies that the foot's motion is symmetric with respect to the body, which gives the leg symmetry equations (5.2).

Symmetric motion of the body and legs requires symmetric actuation, as given in (5.3). From the equations of motion (appendix 5A) we see that hip torque is the only influence on body pitch angle, so odd ϕ implies odd τ. With the evenness and oddness of the other variables specified, f must be even to satisfy the equations of motion.

A locomotion system operates in steady state when the state variables, measured at the same time during each stride cycle, do not vary from stride to stride. The state variables of interest are the body's forward velocity, vertical position, vertical velocity, pitch angle, and pitch rate. With the state vector $\mathbf{S}$ representing these variables, $\mathbf{S} = [\dot{x}\ z\ \dot{z}\ \phi\ \dot{\phi}]$, steady state is defined by

$$\mathbf{S}(t) = \mathbf{S}(t+T), \tag{5.4}$$

where T is the duration of one stride.

Symmetric body and leg motion results in steady-state locomotion.[1]

[1] When I say that a trajectory provides steady-state motion, I mean that the trajectory provides a nominal motion that would repeat from cycle to cycle if there were no disturbances. I do not mean that there are restoring forces that will return the system to the trajectory if it deviates as the result of a disturbance. A mechanism that generates such restoring forces is also required for stability once the nominal trajectory has been determined. Asymmetry in the motion is a source of such restoring forces.

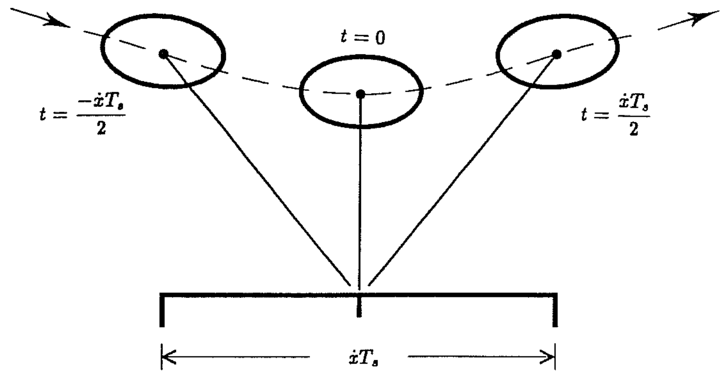


Figure 5.3. When the foot is placed on the neutral point, there is a symmetric motion of the body. The figure depicts running from left to right. The left-most drawing shows the configuration just before the foot touches the ground the center drawing shows the configuration halfway through stance when the leg is maximally compressed and vertical, and the right-most drawing shows the configuration just after the foot loses contact with the ground.

For the forward speed to remain unchanged from stride to stride the horizontal force f_x acting on the body must integrate to zero over a stride:

$$\int_{stride} f_x \, dt = 0. \tag{5.5}$$

Assume that $f_x = 0$ during flight and that the forward speed does not change. From the equations of motion we have that $f_x = f \sin\theta - (\tau/r)\cos\theta$ during stance, which is an odd function because f and r are even and τ and θ are odd. Therefore

$$\dot{x}(t_{lo}) - \dot{x}(t_{td}) = \int_{t_{td}}^{t_{lo}} f_x \, dt = 0. \tag{5.6}$$

This confirms that symmetric motion provides no net horizontal force on the body, and running speed proceeds in steady state from stride to stride.

The vertical position and velocity also proceed in steady state for a symmetric motion. The elevation of the body is an even function of time during stance, so $z(t_{lo,i}) = z(t_{td,i})$ and $\dot{z}(t_{lo,i}) = -\dot{z}(t_{td,i})$. During flight the body travels a parabolic trajectory which is also even, if we specify $t = 0$ halfway through flight: $z(t_{td,i+1}) = z(t_{lo,i})$ and $\dot{z}(t_{td,i+1}) = -\dot{z}(t_{lo,i})$. Consequently, $z(t_{td,i}) = z(t_{td,i+1})$ or $z(t_{td}) = z(t_{td+T})$, which is the steady-state condition on z, and $\dot{z}(t_{td}) = \dot{z}(t_{td+T})$, which is the steady-state condition on $\dot{z}$.

The torque acting on the body is zero during flight and an odd function during stance, so the body pitch rate undergoes zero net acceleration during stance, $\dot{\phi}(t_{lo}) = \dot{\phi}(t_{td})$. This satisfies the steady-state condition on $\dot{\phi}$. For the pitch angle of the body to proceed in steady-state, it's value at the end of flight must be equal and opposite to its value at the beginning of the flight phase. Assuming that symmetry holds during stance so that $\phi_{lo} = -\phi_{td}$ and that no torques act on the body during flight, a repeating pattern requires that

$$\frac{\dot{z}(t)}{-g} = \frac{\phi(t)}{\dot{\phi}(t)}, \tag{5.7}$$

where g is the acceleration of gravity. This constraint prescribes the relationship among pitch angle, pitch rate, and vertical velocity needed for steady-state running. It is trivially satisfied when there is no pitching motion, $\phi(t) = 0$ and $\dot{\phi}(t) = 0$, as found in human running and quadruped trotting. Equation (5.7) results in a second symmetric configuration that occurs during flight. This configuration, given by $f = 0$, $\dot{z} = 0$, and $\phi = 0$, ensures that the body's behavior is symmetric during flight.

A further conclusion is that for a one-legged system, symmetric body motion can be obtained *only* from symmetric leg motion—if x, z, and ϕ obey the symmetries of (5.1), then r must be even and θ must be odd. The proof is given in appendix 5B.

Pairs of Antisymmetric Steps

Motion symmetry need not be confined to just one step. Although we have concentrated on symmetry that applies on a step by step basis, the symmetries apply equally well when pairs of steps produce complementary accelerations, with the symmetry distributed over more than one support interval. This case is discussed next.

Suppose that single support periods deviate from symmetry but that two sequential support periods each deviate from symmetry in a complementary fashion. Figure 5.4 shows a sequence of such antisymmetric steps. The trajectory of the body during each step is asymmetric and the system accelerates because the foot is displaced from the neutral point. If the foot position on the next step compensates, however, then the body motions on successive steps balance with equal and opposite accelerations. Equations (5.1)–(5.3) still describe the behavior of the body and leg, provided that we define $t = 0$ at the point halfway between the two steps.

So far we have assumed that the forward running speed is nonzero, but it need not be. Antisymmetric pairs of steps can apply to running in place, with no forward speed. For instance, if the foot were placed so that the

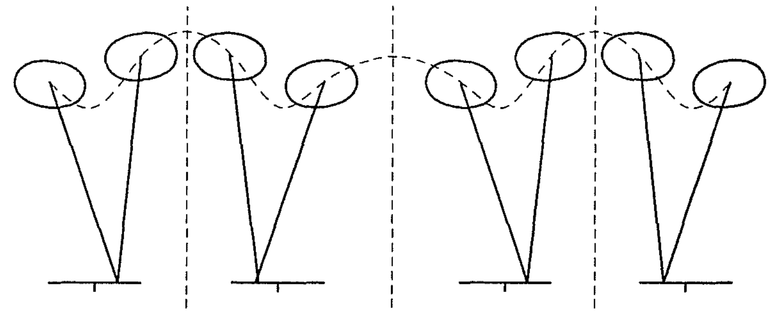


Figure 5.4. Pairs of antisymmetric steps. If the foot is positioned behind the neutral point on one step, and in front of it on the next step, then the pair of steps may have symmetry that stabilizes the forward running speed, even though motion during each step is no longer symmetric. We redefine the stride to include the symmetric pair of steps. The body and leg are drawn once for each touchdown and lift-off. The vertical dashed lines indicate the planes of symmetry, which occurs halfway through the strides and between the strides.

horizontal component of the body velocity is just reversed during support and this were done on each step, then the average forward running speed would be zero and the system would bounce back and forth on each step. This is just the sort of behavior observed in the frontal plane of the human and the pacing quadruped.

Figure 5.5 presents data from a physical demonstration of symmetry distributed over a pair of steps for which the forward running speed is zero. To generate these data, we modified the control algorithm for a physical one-legged hopping machine to add an offset Δx to the desired foot placement on every even-numbered hop and to subtract Δx on every odd numbered hop. For small values of Δx, the system hopped from side to side with no net forward acceleration. The system maintained its balance, provided that the offset of the foot was small enough so that the system did not tip over entirely before the next step.

Symmetry with Several Legs

A system with two legs can run with a variety of gaits. The two legs can operate precisely in phase, precisely out of phase, or with intermediate phase. Figure 5.6 shows several examples that differ with regard to the amount of body pitching, the variation in forward running speed within a stride, and the degree of temporal overlap in the support provided by the two legs. In each case, however, symmetric body and leg motion results in steady-state locomotion.

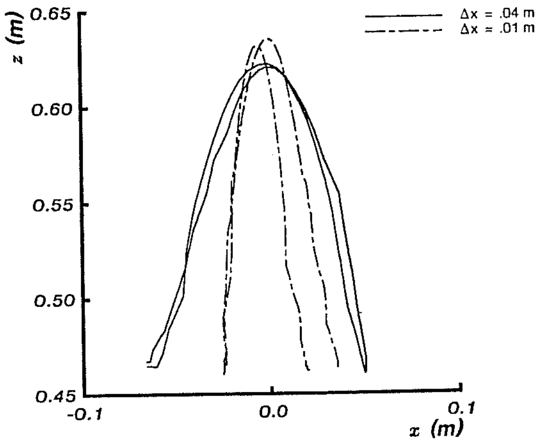


Figure 5.5. Symmetric pairs of steps. The curves show the recorded path of the body for a physical one-leg hopping machine hopping in place. The control algorithms were those described by Raibert, Brown, and Chepponis (1984), but an offset, Δx, was added to the foot position on even-numbered hops and subtracted on odd-numbered hops. The magnitude of Δx was set to two different values, shown separately in the two curves. The plot shows the motion of the body in a vertical plane that contains the foot offset.

The body symmetries for a system with several legs are the same as for one leg, but the leg and actuator symmetries are modified slightly. Each leg and actuator variable, θ, r, τ, and f, have the same meanings as before but with subscripts to distinguish among the individual legs:

$$\theta_j(t) = -\theta_k(-t), \tag{5.8}$$
$$r_j(t) = r_k(-t), \tag{5.9}$$
$$\tau_j(t) = -\tau_k(-t), \tag{5.10}$$
$$f_j(t) = f_k(-t). \tag{5.11}$$

For a system with two legs, $j = 1$ and $k = 2$. For four legs two pairings are possible: $j =[1, 4]$ and $k =[2, 3]$ or $j =[1, 4]$ and $k =[3, 2]$, depending on the gait, where 1 is left front, 2 is left rear, 3 is right rear, and 4 is right front.

Symmetric body motion no longer requires an individual leg to move with a symmetry of its own. Instead, the behavior of one leg is linked to the behavior of another leg, so that they operate with reciprocating symmetry. This frees the variables describing any one leg to take on arbitrary functions of time while preserving the symmetric forces and moments impinging on the body during support. These motion symmetries apply when legs operate in unison, when legs have different but overlapping support periods,

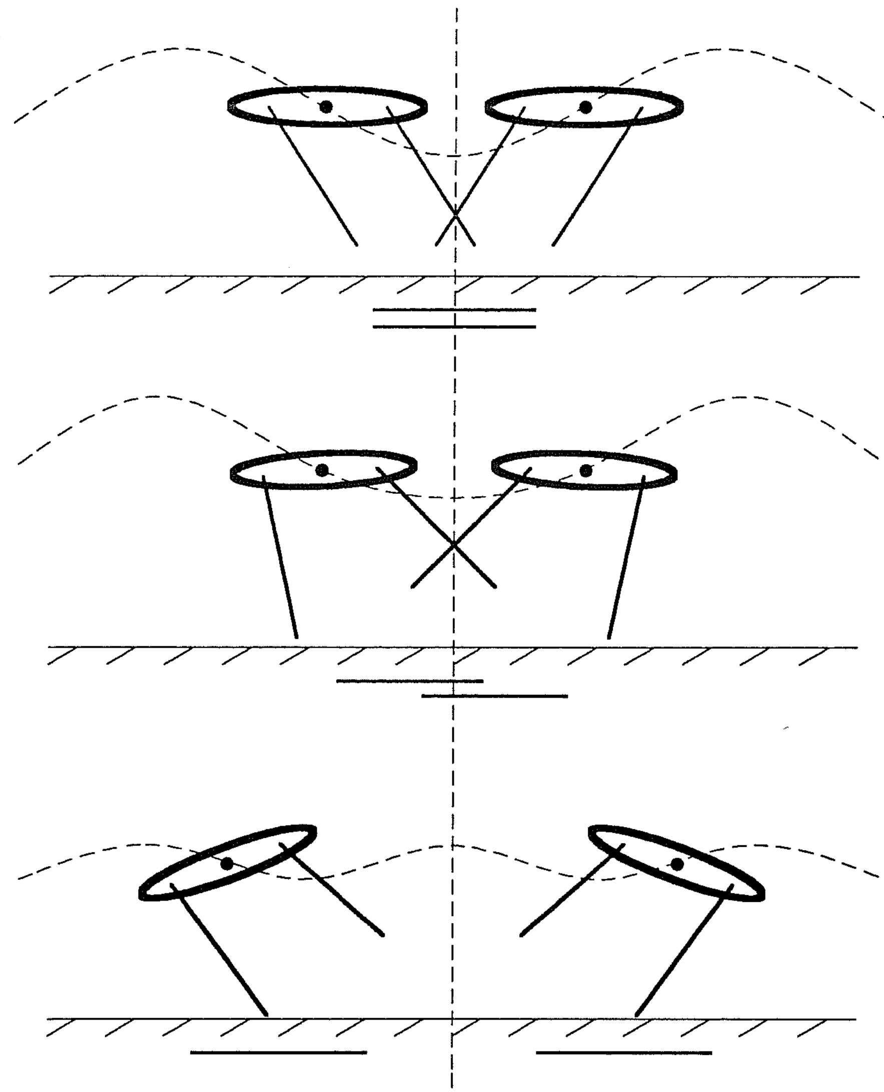


Figure 5.6. Running with two legs separated by a long body. Symmetry can be achieved when both feet provide support simultaneously, when there is partial overlap in the support periods, and when the legs provide support in sequence. These three cases are distinguished by the phasing of the legs. It may be difficult to place the feet on the neutral points when the hips are widely separated. Displacements of the feet from the neutral points influence pitching of the body and the duration of each flight phase.

and when the legs provide support separately. As before, the equations that describe leg motion apply only when $f > 0$, so it does not matter how the legs move when they are not touching the ground. Equations (5.8)–(5.11) reduce to the one-legged case when $j = k = 1$.

For a system with two legs the conditions for symmetric behavior are nearly the same as for one leg, but it is no longer required that an individual foot be located under the center of mass. For instance, figure 5.7 shows two symmetric configurations that place no feet directly under the center of mass. In both cases the *center of support* is located under the center of mass. It is also possible to have no support in the symmetric configuration, as the bottom gait in figure 5.6 suggests. The antisymmetric activity of the two legs operating as a pair produces symmetric motion of the body when measured over the stride. This is very much like the behavior of the one-legged system when it uses pairs of steps to achieve symmetry.

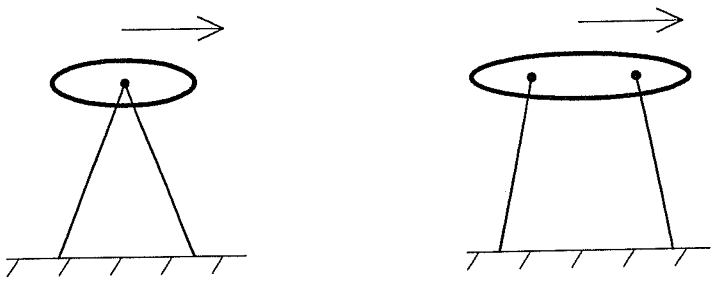


Figure 5.7. Symmetric configuration during support for two legs. Configuration of two-legged systems halfway through the support interval. The *center of support* is located under the center of mass, vertical velocity is zero, and the body is upright: $\theta_i + \theta_j = \dot{z} = \phi = 0$.

A characteristic of locomotion when pairs of legs work in reciprocation is that the individual feet need not be placed on the neutral point to achieve steady-state behavior. This is important because it may be difficult for a legged system to reach far enough under the center of mass when the hips and shoulders are located at the extremes of a long body. This situation arises in the sagittal plane for the quadruped bound and gallop and to a lesser extent in the frontal plane for the quadruped pace.

Symmetry in Animal Running

The importance of symmetry in the control of legged robots raises the question of what role it might play in the behavior of running animals.

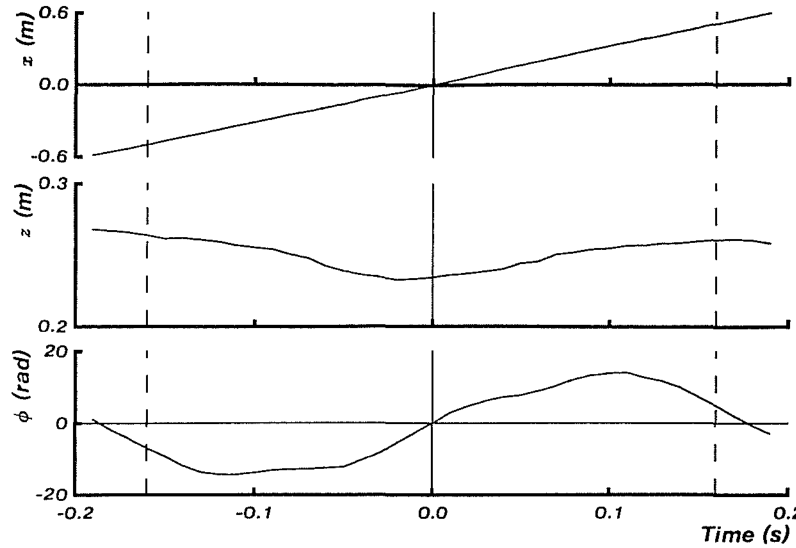


Figure 5.8. Body motion of the galloping cat. Data are shown for one stride of a cat running on a treadmill with a rotary gallop. According to symmetry theory, forward body position x and body pitch angle ϕ should each have odd symmetry and body height z should have even symmetry. The symmetry displayed in these plots is good. Dashed vertical lines indicate the beginning and end of the stance phase. Solid vertical line indicates the symmetry point, when $t = 0$.

Can the symmetries developed for legged robots help us to describe and understand the running behavior of legged animals?

Hildebrand first recognized the importance of symmetry in animal locomotion about two decades ago when he observed that the left half of a horse often uses the same pattern of footfalls as the right half, but 180° out of phase (Hildebrand 1965, 1966, 1968, 1976, 1977). He devised a simple and elegant characterization of the symmetric walking and running gaits for a variety of quadrupeds, using just two parameters: the phase angle between the front and rear legs and the duty cycle of the legs. By mapping each observation of symmetric behavior into a point in phase/duty-cycle space, Hildebrand was able to classify systematically gaits for over 150 quadruped genera.

Rather than look at relationships between the footfalls of the left and right legs, as Hildebrand did, I considered the trajectories of the feet with respect to the body, and the trajectory of the body through space, all measured in the sagittal plane. I analyzed data for a cat trotting and galloping

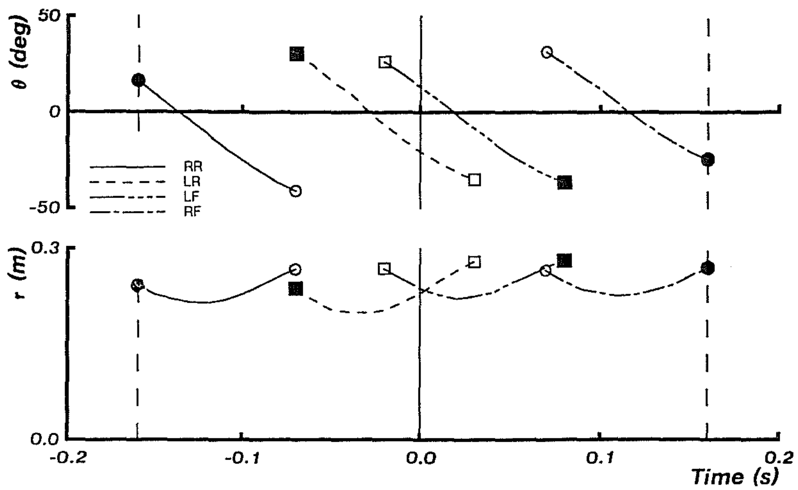


Figure 5.9. Leg motion for galloping cat. Leg angle θ should have odd symmetry and leg length r should have even symmetry. Symmetry in behavior of the legs is found when they are considered in reciprocating pairs, e.g., $\theta_{RR}(t) = -\theta_{RF}(-t)$ and $r_{RR}(t) = r_{RF}(-t)$. Symbols indicate pairs of points that should have symmetric positions with respect to the origin (for odd symmetry), or the z-axis (for even symmetry). Both leg angle and leg length show very good symmetry. Data for each leg are shown only when its foot touches the support surface. Dashed vertical lines indicate the beginning and end of the stance phase. Solid vertical line indicates the symmetry point, when $t = 0$. The data are from the same stride as in figure 5.8.

on a treadmill, and for a human running on an outdoor cinder track. The cat data were obtained by digitizing 16 mm, 100 fps film provided by Wetzel, Atwater, and Stuart (1976). Each frame showed a side view of the cat on the treadmill and a 1-ms counter used to calibrate the film speed. Treadmill markers spaced at 0.25-m intervals provided a scale of reference and permitted registration of each frame. Small circular markers attached to the cat's skin made the digitizing easier. Running speeds with respect to the treadmill surface were about 2.2 m/s for trotting and 3.1 m/s for galloping.

The human measurements were made by digitizing 16 mm film of a runner on the semicircular section of an outdoor cinder track. The camera was mounted on a tripod located at the center of the semicircle and panned to track the runner. Ground markers spaced at 1.0 m intervals provided scale and registration as before. Running speed was about 3.8 m/s.

In digitizing both the cat and the human data, the point of support provided by each foot was estimated visually. A straight line from this

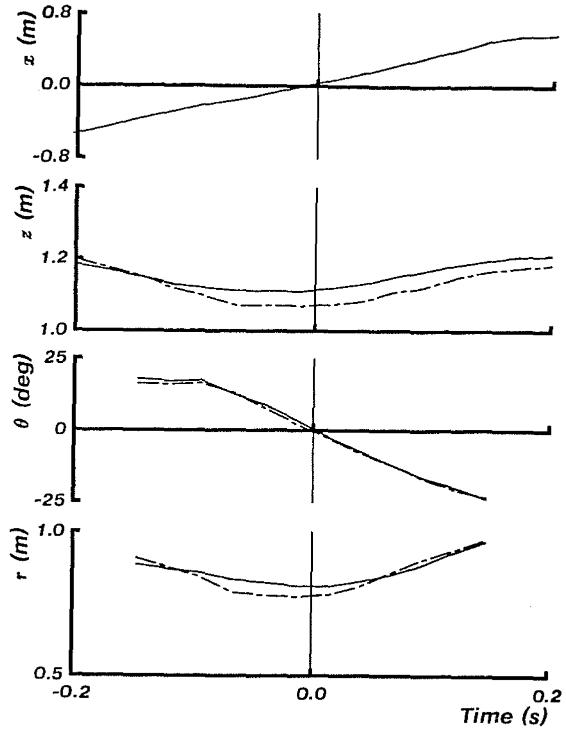


Figure 5.10. Data for one stride of human running on outdoor cinder track. Data for the right (stippled) and left legs are superimposed. Running speed was 3.8 m/s. Subject MHR.

point to the hip, or shoulder for the cat's front legs, was used to find the leg length r and the leg angle θ. The center of mass of the cat was taken as the midpoint between the shoulder and the hip. The pitch angle of the body was the angle between the line connecting shoulder to hip and the horizontal, offset so that $\phi(0) = 0$. These measurements provided three parameters of the body's motion, its forward position, vertical position, and pitch angle $[x\ z\ \phi]$, and two parameters of each leg's motion, its length and angle with respect to the vertical $[r\ \theta]$. In addition to information about the timing of footfalls, these measurements provided information about where on the ground the feet were placed with respect to the body and how the body itself moved.

Data for one stride of the gallop and human run are plotted in figures 5.8, 5.9, and 5.10. In each case the data are in good agreement with the even and odd symmetries predicted by the symmetry equations. The cat data for galloping show a remarkable degree of symmetry.

Some of the data examined reveal a bias in foot position toward the rear of the animal. The timing diagram of figure 5.11 illustrates such a bias. For each leg, $|\theta(t_{lo})| > |\theta(t_{td})|$, and the last leg providing support to the body stayed in contact with the ground longer than the first leg providing support. According to the principles outlined in this chapter, such bias or skew might mean that a net forward force was generated on the body. Such a force could accelerate the system forward, compensate for an external disturbance, or compensate for losses occurring elsewhere in the system.

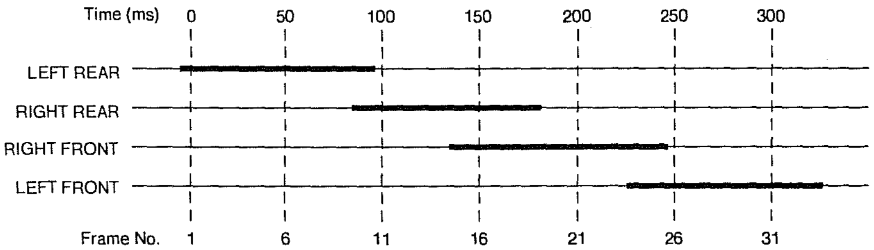


Figure 5.11. Pattern of foot contacts in the rotary gallop of a cat. Horizontal bars indicate that foot is in contact with the support surface. The duration of an entire stride is 350 ms. Vertical dotted lines indicate the seven frames used in figure 5.12.

Another explanation, however, might be that the axial leg force does not obey (5.11). For instance, because the legs are not massless, their collision with the ground on each step results in an asymmetric force. One might also expect the legs to deliver thrust actively during support, in order to make up for losses and to maintain the vertical bouncing motion. This active thrusting would result in violation of (5.11). Without knowing the actual force that each leg exerts on the ground, it is difficult to draw definite conclusions regarding the implication of the observed asymmetry in foot position.

These running symmetries can be visualized graphically. The symmetry equations imply that if we reverse both the direction of forward travel and the direction of time, $x = -x$, $t = -t$, then the pattern of forward body movement and of footfalls should not be affected: $x(t) = -x(-t)$. This invariance is illustrated in figure 5.12. Of particular interest is the precise overlap of the footfalls for the forward and reverse running sequences. This overlap was predicted by the symmetry equations.

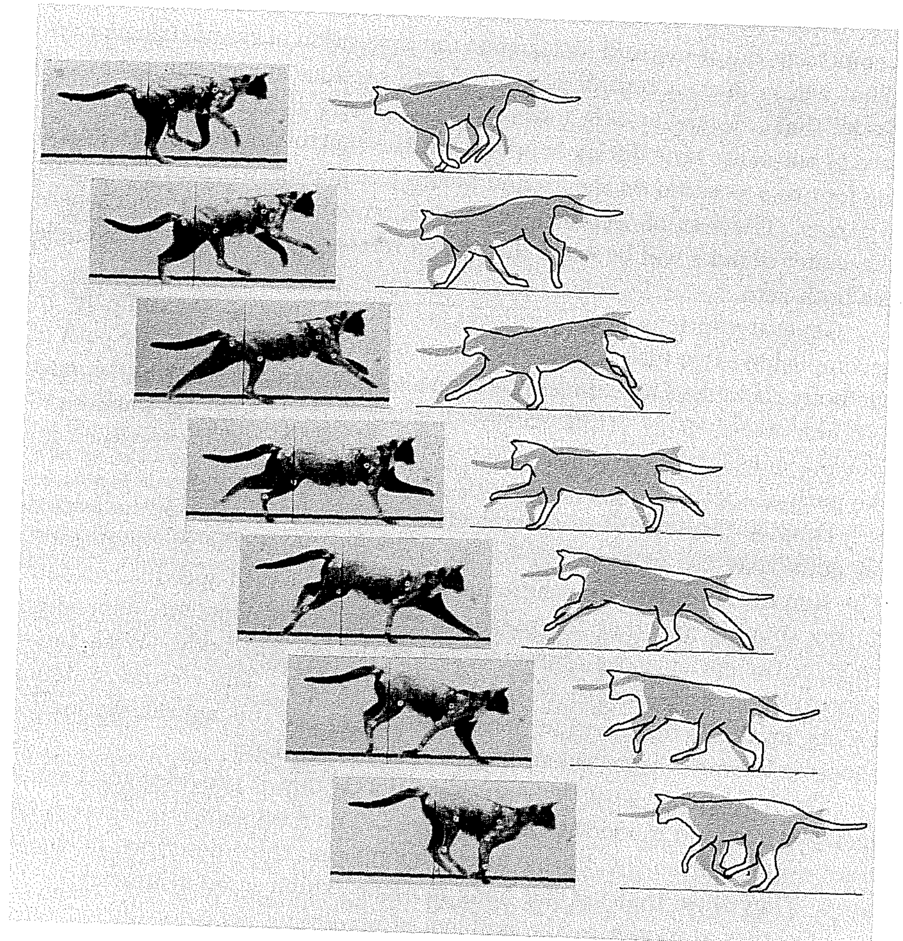


Figure 5.12. Graphical interpretation of symmetry in the galloping cat. (Left) Photographs of galloping cat taken at 50-ms intervals. (Right) The shaded figures show the forward translation and the configuration of the cat during normal running, and the outlines show reverse running. The outlines were made from the same photographs as the shaded figures, but were reflected about the vertical axis and are presented in reverse sequential order, $x(t) = -x(-t)$ Therefore the outline at the top was made from the photograph at the bottom after reversing its orientation. The positions of supporting feet and the rightward motion of the body correspond quite well in the two sequences, as predicted by symmetry. (Diagram Construction) The relative placement of the figures for each sequence—the photographs, the shaded silhouettes, and the outlines—accurately reflects the forward progress of the cat with respect to the surface of the treadmill. After each set of figures was assembled according to the forward travel, the three sets were positioned relative to one another. Photographs are from film provided by Wetzel, Atwater, and Stuart (1976).

Generating Symmetric Motions

The discussion has focused on the nature and value of symmetric motion in locomotion, but it has said little about the generation of symmetric motion. What action must a control system take to produce symmetric behavior? Recall that a legged system moves with symmetry if θ, $\dot{z}$, and ϕ all equal zero at the same time during support, but the control system must commit the foot to a position on the ground before touchdown, when neither $\dot{z}$ nor ϕ is zero. The task of orchestrating such a rendezvous is to predict where the center of mass will be when the body's vertical velocity and pitch angle are both zero.

General solutions to this problem are not yet known. The difficulty in accomplishing this task is that placement of the foot influences the path of the body. If we had an expression for the body's trajectory during support as a function of $\dot{x}(t_{td})$, $\dot{z}(t_{td})$, and $\theta(t_{td})$, then we could solve for the desired foot placement. We have not, however, found a closed form expression for the path of the body during support, even for simple models.

Despite the lack of a general solution, approximate solutions exist for gaits that use just one leg for support at a time, the one-foot gaits. The simplest approximate solution assumes that forward speed is constant during support and that the period of support T_s is constant, depending only on the spring mass characteristics of the leg and body. With these approximations the length of the CG-print is $\dot{x}T_s$. The control systems for the machines described in previous chapters use this approximation to choose a forward displacement for the foot during flight.

Our experience with this approximation is that it provides good symmetry at low and moderate running speed. The data shown in figure 5.13 were recorded from the 3D one-legged hopping machine described in chapter 3. They show that leg and body motions exhibit good symmetry.

Another approach to the problem of predicting behavior during support might be to use a tabulated solution to the equations of motion. A table could be indexed by forward, vertical, and desired velocities, and it would provide the necessary leg touchdown angle. Depending on the size of the table, this approach could provide any desired degree of accuracy. Tabular solutions are discussed more fully in chapter 7.

Scissor Symmetry

When a human runs, the two legs form roughly symmetric angles with respect to a vertical axis passing through the hip. The angle formed between

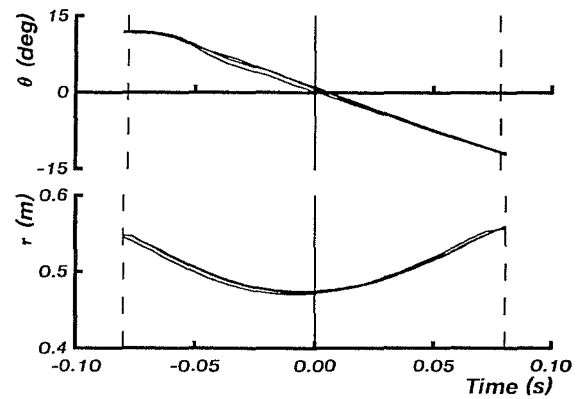


Figure 5.13. Symmetry data recorded from physical 3D one-legged hopping machine. The behavior of the machine obeys the symmetry equations when the foot is placed on the neutral point. Data for three consecutive support intervals are superimposed. The leg is longer at lift-off than at touchdown because it lengthens during support to provide thrust that compensates for various mechanical losses in the system. The time axes were adjusted so that $t = 0$ halfway through the support interval, and the x-origin was adjust so that $x(t{=}0) = 0$. Running speed is about 1.6 m/s. Dashed vertical lines indicate touchdown and lift-off.

the hip and foot of the forward leg and the vertical axis, is about equal and opposite to the corresponding angle for the rearward leg. This symmetry is largely independent of the speed, bounce, stride, and other parameters of the gait. This behavior reminds one of the way one orients the blades of a scissors to the paper they cut.

A consequence of scissor symmetry is that the angle of the leg to be placed is about equal to the angle of the leg that was just lifted. Can this symmetry be used to formulate an algorithm that correctly places the foot on each step, eliminating the need for a calculation that depends on forward running speed and the duration of support? A scissor algorithm would specify that

$$\theta(t_{td,i+1}) = -\theta(t_{lo,i}), \tag{5.12}$$

where

$\theta(t_{lo,i})$	is the angle of the lift-off leg on step i, and
$\theta(t_{td,i+1})$	is the angle of the landing leg on step $(i+1)$.

The scissor algorithm of (5.12) could be used to specify foot placement for systems with any number of legs, provided that the gait used only one leg for support at a time. What sort of behavior would result?

When running with constant forward speed $\dot{x}$ and uniform stance duration T_s, the foot moves a distance $\dot{x}T_s$ backward with respect to the hip during the support interval. For a given landing angle of the leg $\theta(t_{td,i})$, the lift-off angle is

$$\theta(t_{lo,i}) = \arcsin\left(\frac{\dot{x}T_s + r\sin\theta_{td,i}}{r}\right). \tag{5.13}$$

Combining (5.12) and (5.13), we obtain

$$\begin{aligned}\theta(t_{td,i+2}) &= \arcsin\left(\frac{\dot{x}T_s + r\sin\left(-\arcsin\left(\frac{\dot{x}T_s + r\sin\theta(t_{td,i})}{r}\right)\right)}{r}\right)\\ &= \theta(t_{td,i}).\end{aligned} \tag{5.14}$$

During running at constant speed, the algorithm generates pairs of steps that have symmetry, like those discussed earlier. A pattern of paired antisymmetric steps provides balance, provided that the degree of asymmetry is relatively small and the step rate is large. When $\theta(t_{lo,i}) = \arcsin(\dot{x}T_s/2)$, the scissor algorithm generates the same foot placement on every step, and the placements are the same as those produced by using the CG-print calculation for the neutral point.

The scissor algorithm can also work properly during forward accelerations. Suppose that during the support interval an external disturbance accelerates the system forward. The result is that the stance leg sweeps farther back and the lift-off angle of the stance leg is larger than it would have been without the disturbance. The other leg is placed correspondingly further forward, compensating for the increased velocity. A decelerating disturbance works in a corresponding manner. The acceleration need not be due to an external disturbance but could be caused by actions of the hip actuator that are intended to stabilize the body attitude. They might be caused by the driving or swinging actions of other legs in a more complicated system.

One way to look at the scissor algorithm is that it provides an alternative method for estimating the length of the CG-print. The lift-off angle of the leg serves to indicate both the forward velocity and ground time. The faster the body moves forward relative to the ground, the further backward the foot moves during stance. The foot also moves backward further when the system spends more time on the ground. Therefore the angle of the leg at lift-off is determined by the product of the average forward velocity and the duration of stance. The scissor algorithm is attractive because it is difficult to estimate the length of the CG-print accurately. It avoids the need to measure explicitly the forward velocity of the body and the duration of stance.

There are several difficulties with the scissor algorithm. First, the leg angle at touchdown is also influenced by the leg angle at lift-off. The product of average velocity and ground time relates only the change in leg angle, so the starting angle of the leg at touchdown determines where it is at lift-off. In principle, the algorithm can generate a sequence of uniform symmetric steps; in practice, there is no mechanism to keep from drifting to antisymmetric pairs of skewed steps with diverging skew.

This problem might be overcome by somehow damping the foot placement excursions or by using information from previous steps to filter the two-step oscillations. Another alternative might be to take both the touchdown and lift-off angles into account when calculating the next foot placement:

$$\theta(t_{td,i+1}) = \frac{\theta(t_{td,i}) - \theta(t_{lo,i})}{2}. \tag{5.15}$$

Another problem with the scissor algorithm is that it may not be responsive to sudden changes in the forward speed of the body. The forward speed that determines the next foot placement is the average from the entire previous support interval. The latency inherent in this indirect measurement could result in sluggish response to disturbances.

Asymmetry in Running

Despite the value of symmetry, there are several reasons why one should not expect to see perfect symmetry in the behavior of legged machines and animals. One reason for asymmetry is that legs are not lossless. The arguments used to motivate the relationship between symmetric motion and steady-state behavior do not apply in the presence of friction. In particular, the behavior of the system moving forward in time is no longer symmetric to its behavior moving backward in time. The details of the discrepancy depend on the details of the losses and on the geometry of the system. Another energy loss contributing to asymmetric motion is due to unsprung mass in the legs. Each time a foot strikes or leaves the ground, the system loses a fraction of its kinetic energy. In order to maintain stable locomotion, the control system must resupply energy on each cycle to compensate for these losses. For instance, the leg lengthens during the support interval and shortens during flight to maintain a stable hopping height. This can be done only by delivering asymmetric forces and torques through the actuators.

Another reason for asymmetric behavior is asymmetry in the mechanical system. Most animals have large heavy heads at one end of their bodies that are not counterbalanced by large heavy tails at the other end. Front and rear legs often vary in size, and the hips and shoulders may not be equally spaced about the center of mass. Each of these factors may induce asymmetry in the motions that can provide balanced steady-state behavior. This is less of a problem for laboratory machines because we can design them to conform to whatever mechanical symmetry we require.

Naturally, we shouldn't expect to see symmetric motion when the control system purposely skews the motion to change running speed. In this case asymmetry in the motion provides the forces that accelerate the body. An external load, such as that produced by wind resistance, or a draw-bar load would also require a component of asymmetry in motion of the body and legs. A runner at the start of a footrace and the driver of a jinrikisha demonstrate these sorts of asymmetric behavior.

Perhaps a better view is to think of locomotion in terms of the sum of a symmetric part and an asymmetric part. The symmetric part of the motion during each stride maintains steady-state behavior. Deviations from symmetry compensate for losses and provide acceleration.

The symmetry discussed in this chapter postulates that each body variable, each leg variable, and each actuator variable has an even or odd symmetry. The net result of their interaction is to constrain the forces acting on the body throughout a stride so that they preserve the body's forward speed, elevation, and pitch angle. One might imagine a less complete symmetry that does not require symmetry of the basic variables individually but requires symmetry only in the net forces and torques acting on the body:

$$\begin{aligned} f_x(t) &= -f_x(-t), \\ f_y(t) &= f_y(-t), \\ \tau_\phi(t) &= -\tau_\phi(-t). \end{aligned} \tag{5.16}$$

Another way to say this is that the body moves with symmetry while the legs do not. We have proved that this cannot be the case when only one leg is used for support at a time. The proof is given in appendix 5B. However such solutions may be workable with additional legs.

What Does Symmetry Mean?

We can interpret symmetry in several ways. First, it helps us to control legged machines. The strategy used to control running machines was built

around symmetry, and symmetry may play a role in achieving more complicated running behavior in the future. For instance, reciprocating leg symmetry is important in making a quadruped gallop.

Symmetry also helps us to characterize and understand the behavior we observe in animals. The analysis of symmetry in the cat and human shows that it describes how animals move when they trot, gallop, and run, and we expect to find that the same symmetries describe the motions of other animals running with other gaits. Perhaps most important is the idea that symmetry and balance give us tools for dealing with a dynamic system without requiring detailed solutions to complex formulations. Symmetry implies that each motion has two parts with opposing effects, just as balance requires equal and compensating forces and torques.

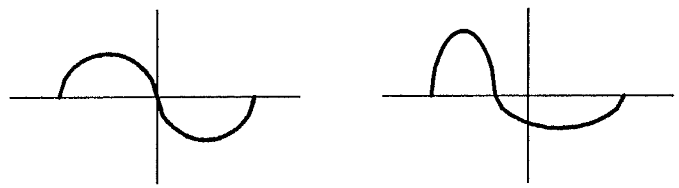


Figure 5.14. Two functions that integrate to zero. One is symmetric and one is not. Symmetry provides a sufficient condition for zero net forward acceleration, not a necessary condition.

In certain respects, these symmetries are limited. They do not specify the details of a particular body motion that provides locomotion but merely give a broad classification that embodies several interesting features of the motion. The symmetries provide only sufficient conditions for successful locomotion, not necessary conditions (figure 5.14). So far as we have been able to determine, the behavior of a legged system may violate the motion symmetries we have described with impunity, without limiting its ability to run and balance. Finally, these symmetries do not yield a specific prescription for control. They suggest only how the system should ultimately move and hint at possible avenues of attack.

In other respects, the symmetries described here are quite powerful. Three simple equations outline plausible body motions for systems with any number of legs engaged in a wide variety of gaits. Another small set of equations describes how the legs move. Although the symmetries do not specify individual motions or how to produce them, they provide rules that govern a large class of successful motions and suggest a wide variety of experiments.

This work on symmetry falls into a broader context that splits responsibility for control between the control system and the mechanical system being controlled. In this context, the control of locomotion is a low bandwidth activity that takes advantage of the intrinsic properties of the mechanical system. Rather than use a high bandwidth servo to move each joint of the legged system along a prescribed trajectory at high rate, the control system makes adjustments just once per stride. Once the foot has been positioned on each step, the mechanical system passively determines the details of the motion for the remainder of the stride. This approach depends on having a passive nominal motion that is close to the desired behavior. In the present context symmetry is the means of achieving the nominal motion. This sort of approach may have value only for systems that perform repetitive behaviors. For instance, aside from juggling and handwriting (Hollerbach 1980), robot manipulation may be unsuited to this approach.

Summary

Symmetric motions of the body in space and of the feet with respect to the body provide nominal motions for steady-state locomotion. A control system for running can produce steady-state behavior by choosing motions of the legs that give $x(t)$ and $\phi(t)$ odd symmetry and $z(t)$ even symmetry. The leg motions chosen are themselves described by odd and even symmetries. This method applies to a number of legged configurations and helps to describe the behavior of running animals.

The significance of these symmetric motions is that they permit a control system to manipulate the symmetry and skewness of the motion, rather than the detailed shape of the motion. When the system's behavior conforms to (5.1)–(5.3), all forces acting on the body integrate to zero throughout one stride, so the body experiences no net acceleration. When behavior deviates from symmetry, the net acceleration of the system deviates from zero in a manageable way. The control task becomes one of manipulating these deviations.

The conditions for symmetric body motion can be stated simply: at a single point in time during the support period, the center of support must be located under the center of mass, the pitch angle of the body must be zero, and the vertical velocity of the body must be zero, i.e., $\theta_j(0) + \theta_k(0) = 0$, $\phi(0) = 0$, and $\dot{z}(0) = 0$. The body follows a symmetric trajectory during stance when these conditions are satisfied.

Symmetric running motions may have great generality. In principle, a wide variety of natural running gaits can be achieved using body and leg motions that exhibit the symmetries described. These include the trot, the pace, the canter, the gallop, the bound, and the pronk, as well as the intermediate forms of these gaits. Although we have plotted symmetry data only for the cat and human, we expect to find a wide variety of natural legged systems using nearly symmetric motions when they run.

Appendix 5A. Equations of Motion for Planar Systems

Equations of Motion for Planar One-Legged System

The equations of motion for a planar one-legged model, with massless leg and the hip located at the center of mass, as shown in figure 5.15, are:

$$m\ddot{x} = f\sin\theta - \frac{\tau}{r}\cos\theta, \tag{5.17}$$

$$m\ddot{z} = f\cos\theta + \frac{\tau}{r}\sin\theta - mg, \tag{5.18}$$

$$J\ddot{\phi} = \tau, \tag{5.19}$$

where

x, z, ϕ	are the horizontal, vertical, and angular positions of the body,
r, θ	are the length and orientation of the leg,
τ	is the hip torque (positive τ accelerates body in the positive ϕ direction),
f	is the axial leg force (positive f accelerates the body away from and ground),
m	is the body mass,
J	is the body moment of inertia, and
g	is the acceleration of gravity.

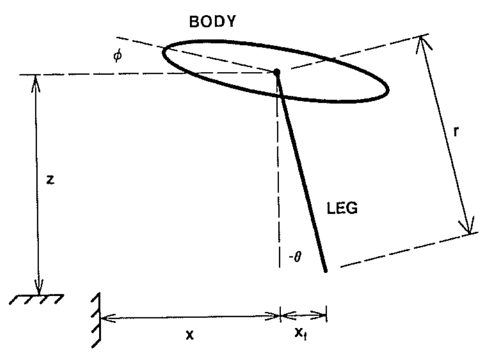


Figure 5.15. Model of planar one-legged system with massless leg.

Equations of Motion for Planar Two-Legged System

The equations of motion for planar model with two massless legs and hips located a distance d from the body's center of mass, as shown in figure 5.16, are:

$$m\ddot{x} = f_1\sin\theta_1 + f_2\sin\theta_2 - \frac{\tau_1}{r_1}\cos\theta_1 - \frac{\tau_2}{r_2}\cos\theta_2, \tag{5.20}$$

$$\begin{aligned} m\ddot{z} =& f_1\cos\theta_1 + f_2\cos\theta_2 \\ &+ \frac{\tau_1}{r_1}\sin\theta_1 + \frac{\tau_2}{r_2}\sin\theta_2 - mg, \end{aligned} \tag{5.21}$$

$$\begin{aligned} J\ddot{\phi} =& f_1 d\cos(\theta_1 - \phi) - \frac{\tau_1 d}{r_1}\sin(\phi - \theta_1) + \tau_1 \\ &- f_2 d\cos(\theta_2 - \phi) + \frac{\tau_2 d}{r_2}\sin(\phi - \theta_2) + \tau_2. \end{aligned} \tag{5.22}$$

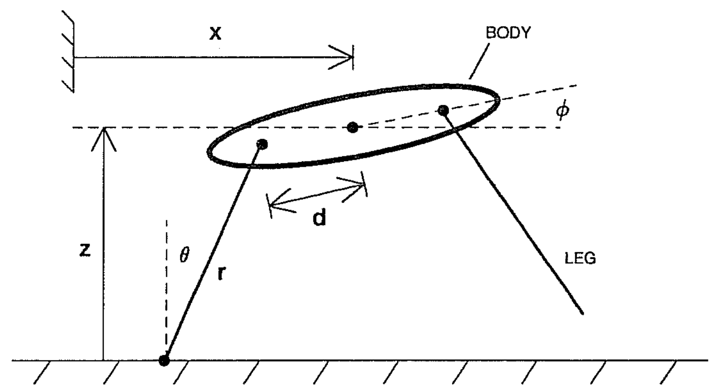


Figure 5.16. Model of two-legged planar system with separated hips. It can represent the lateral half of a quadruped projected onto the sagittal plane or a biped projected onto the frontal plane.

Appendix 5B. Proof of Symmetric Leg Motion

In this appendix we prove that symmetric body motion requires symmetric leg motion for the one-legged case. Body and leg motions are symmetric when $x(t)$, $\phi(t)$, and $\theta(t)$ are odd and $z(t)$ and $f(t)$ are even.

Rewrite the equations of motion expressing each element of the leg motion as the sum of an even and odd part. For instance, the angle of the leg with respect to the vertical is $\theta = {}^e\theta + {}^o\theta$, where ${}^e\theta$ represents the even part and ${}^o\theta$ represents the odd part. Also, replace r with $1/({}^ez + {}^oz)$:

$$m\ddot{x} = ({}^ef + {}^of)\sin({}^e\theta + {}^o\theta) - \tau({}^ez + {}^oz)\cos({}^e\theta + {}^o\theta), \tag{5.23}$$

$$m\ddot{z} = ({}^ef + {}^of)\cos({}^e\theta + {}^o\theta) + \tau({}^ez + {}^oz)\sin({}^e\theta + {}^o\theta) - mg, \tag{5.24}$$

$$J\ddot{\phi} = {}^o\tau + {}^e\tau. \tag{5.25}$$

τ must be odd from (5.25) and because ϕ is odd by assumption making $\ddot{\phi}$ odd. To specify that the body moves with the desired symmetry, set the even part of the right-hand side of (5.23) to zero and the odd part of the right-hand side of (5.24) to zero:

$$\begin{aligned} 0 =& {}^ef \sin {}^e\theta \cos {}^o\theta + {}^of \cos {}^e\theta \sin {}^o\theta \\ &+ \tau\, {}^ez \sin {}^e\theta \sin {}^o\theta - \tau\, {}^oz \cos {}^e\theta \cos {}^o\theta. \end{aligned} \tag{5.26}$$

$$\begin{aligned} 0 =& -\, {}^ef \sin {}^e\theta \sin {}^o\theta + {}^of \cos {}^e\theta \cos {}^o\theta, \\ &+ \tau\, {}^ez \sin {}^e\theta \cos {}^o\theta + \tau\, {}^oz \cos {}^e\theta \sin {}^o\theta. \end{aligned} \tag{5.27}$$

Solutions to (5.26) and (5.27) require that

$$\tan {}^e\theta = \frac{\tau\, {}^oz}{{}^ef} \quad \text{and} \quad \tan {}^e\theta = -\frac{{}^of}{\tau\, {}^ez}. \tag{5.28}$$

During the support interval, the foot remains stationary with respect to the ground, so motion of the body with respect to the ground determines motion of the foot with respect to the body. Therefore the symmetries of (5.1) and the solutions to (5.28) also govern the trajectory of the foot with respect to the body. They require that $x_f(t) - x_f(0) = -x_f(-t) + x_f(0)$ and $z_f(t) = z_f(-t)$. The leg motion is symmetric if $x_f(0) = 0$.

Because odd functions equal zero when $t = 0$, (5.28) requires that ${}^e\theta(t{=}0) = 0$, implying that $x_f(0) = 0$. Hence ${}^e\theta = {}^or = {}^of = 0$, leaving θ odd and r and f even. They obey the leg symmetries given by (5.2).

Ken Goldberg simplified the proof as follows. The foot remains stationary with respect to the ground during the support interval, so motion of the body with respect to the ground determines the motion of the foot with respect to the body. Therefore the symmetries of (5.2) govern the trajectory of the foot with respect to the body:

$$x_f(t) - x_f(0) = -x_f(-t) + x_f(0), \tag{5.29}$$

$$z_f(t) = z_f(-t). \tag{5.30}$$

The leg motion is symmetric if $x_f(0) = 0$.

Let f_x and f_z be the horizontal and vertical forces between the foot and the ground. The torque at the hip can be written as

$$\tau = -f_x z_f + f_z x_f. \tag{5.31}$$

From the equations of motion we know that τ and f_x are odd and that f_z is even, so (5.31) requires that x_f be odd. Therefore leg angle $\theta = \arctan(x_f/z_f)$ is odd and leg length $r = \sqrt{x_f^2 + z_f^2}$ is even. Axial leg force $f = (f_x x_f + f_z z_f)/r$, which is even.

Chapter 6

Alternatives for Locomotion Control

In synthesizing control systems for legged machines, one faces the infinite space of design alternatives. Several functions must be performed, and each function can be accomplished in several different ways. There may be hundreds of combinations of plausible functions and methods in all. The experimental challenge is to eliminate most of the possibilities, to make specific implementation decisions, and to home in on the few alternatives that seem most promising. Only then is experimentation feasible.

The preceding chapters focused on the function and performance of one particular set of control algorithms that were the result of many design and implementation decisions. A succession of machines and experiments were based on these decisions, including the three-part decomposition, the use of foot placement and symmetry to control forward running speed, and the use of virtual legs. This chapter puts these design decisions into context by identifying and examining a few of the many alternative control algorithms that are possible.

More on the Control of Bouncing

The running machines described in this book use a fixed thrust during each step to drive the vertical bouncing motion. The mass of the body and the springiness of the leg form a passive oscillator that is excited by operation of the leg actuator during stance. When the amplitude of the oscillation exceeds a certain magnitude, the machines leave the ground and the entire system becomes a spring-mass/gravity-mass oscillator. For a given fixed energy injected on each cycle the system comes to equilibrium at an equilibrium hopping height because the energy lost during each hopping cycle is a monotonic function of hopping height.

In this section we explore a method for controlling hopping height that adjusts the energy injected on each hop. The idea is to measure the energy in the vertical motion of the system during the stance phase and to operate the leg actuator to inject or extract energy, depending on the amount needed for the desired hopping height. For a given hopping height there is a specific energy needed, so the task is to leave the ground with the correct energy.

This approach should have several advantages over the fixed thrust method. For one thing, it should allow the control system to specify a specific hopping height rather than just a relative height such as "higher" or "lower." It should also be less susceptible to frictional variations in the leg mechanism because the actuator can continue to servo the energy right up until lift-off. Finally, the fixed thrust method does not take account of the sweeping motion of the legs, so the thrust is not purely vertical. The energy method can correct for sweeping of the leg by servoing the energy resulting from the vertical component of the motion. These advantages come at the cost of a somewhat more complicated control.

The Model

Figure 6.1 illustrates the model used for analysis and simulations. The essential features of the model are a body of mass m, a compliant leg of mass m_ℓ, and a compliant support surface. The leg has a sliding joint which lets it change length, a spring, and an actuator. The actuator is an ideal position source that operates in series with the spring. The actuator and spring act together to lengthen and shorten the leg, and to exert forces between the foot and the body.

The leg spring absorbs energy by shortening under load of the body and returns energy by lengthening, accelerating the body upward. A mechanical stop prevents the leg spring from extending beyond a maximum length. The stop is modeled as a stiff, damped spring in tension. The spring and mechanical stop are arranged so that the spring tends to lengthen the leg when it is in compression, $r_s < r_{s0}$, and the mechanical stop tends to shorten the leg when it is in tension, $r_s > r_{s0}$. The stiffness and damping of the mechanical stop, k_{stop} and b_{stop}, are chosen so that vibrations between the body and leg at lift-off decay within a few cycles. The leg mass m_ℓ represents that portion of the leg that is functionally *below* the spring, the rest being included in m.

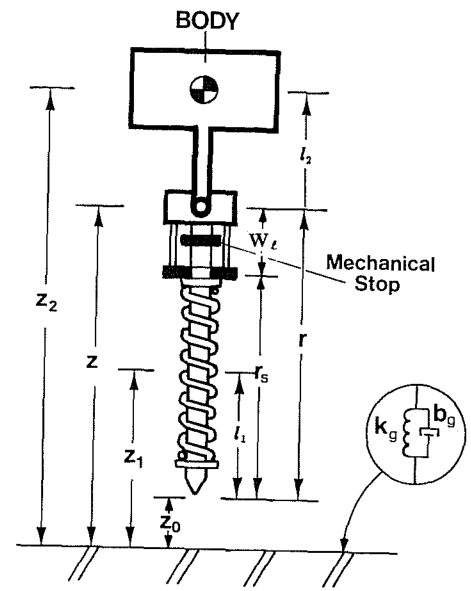


Figure 6.1. Model used to study control of vertical hopping. The body has mass m and the leg has unsprung mass m_ℓ. A sliding joint permits the leg to change length under the influence of a spring and position source. The spring of stiffness k_ℓ acts in series with a position source between the mass of the body and the lower part of the leg. A mechanical stop keeps the leg from extending longer than the rest length of the leg spring, r_{s0}. The support surface is modeled as a spring of stiffness k_g with damping b_g. The center of mass of the leg is located a distance l_1 from the foot, and the center of mass of the body is located a distance l_2 above the hip. Values for simulation parameters are given in table 6.1.

When the actuator changes length, it does work on the leg spring to increase or decrease its stored energy. The arrangement of actuator, leg spring, and mechanical stop permits the model to hop. During support, the actuator excites the spring-mass system formed by the body and the leg spring. As the leg reaches maximum length, the mechanical stop permits a fraction of the kinetic energy to transfer from the body to the leg, enabling the foot to leave the ground.

The support surface has stiffness k_g and damping b_g. The damping coefficient is chosen to keep the foot from bouncing on the ground during touchdown and lift-off. This compliance in the support surface represents the compliances of both the support surface and of the foot. We assume that the stiffness of the ground is much greater than the stiffness of the leg, $k_g \gg k_\ell$.

Table 6.1. Parameters of planar one-legged model used to study bouncing and sweep control.

Parameter	Symbol	Value
Body mass	m	10 kg
Unsprung leg mass	m_ℓ	1 kg
Body moment of inertia	J	$10\,\mathrm{kg}\cdot\mathrm{m}^2$
Leg moment of inertia	J_ℓ	$1\,\mathrm{kg}\cdot\mathrm{m}^2$
Leg center of mass	l_1	0.5 m
Body center of mass	l_2	0.4 m
Leg spring rest length	r_{s0}	1 m
Leg stiffness	k_ℓ	$10^3\,\mathrm{N/m}$
Mechanical stop stiffness	k_{stop}	$10^5\,\mathrm{N/m}$
Mechanical stop damping	b_{stop}	$125\,\mathrm{N}\cdot\mathrm{s/m}$
Ground stiffness	k_g	$10^4\,\mathrm{N/m}$
Ground damping	b_g	$75\,\mathrm{N}\cdot\mathrm{s/m}$

Hopping

Lengthening the actuator when the leg provides support does positive work on the system by compressing the leg spring and accelerating the body mass upward. Shortening the actuator during support does negative work on the system. Energy is injected into the system over a number of hopping cycles by lengthening the position actuator during support and shortening it during flight. Energy is removed by shortening the leg during support, doing negative work on the leg spring, and lengthening the leg during flight, when it is unloaded.

The following analysis applies to repetitive hopping in which periods of support alternate with periods of flight. During the part of the hopping cycle when the leg provides support, the model is a spring-mass oscillator with natural frequency

$$\omega_n = \sqrt{\frac{k_\ell}{m}}. \tag{6.1}$$

If we assume that one half cycle of oscillation occurs during stance,[1] then

[1] McMahon points out that this assumption holds when the *Marx number*, given by $\dot{z}\omega_n/g$, is large compared to 1 (McMahon 1985). For humans the Marx number is normally close to 1, but typical values for the model described here are 1.5 to 5.

during repetitive hopping each support interval has duration

$$T_s = \frac{\pi}{\omega_n} = \pi\sqrt{\frac{m}{k_\ell}}. \tag{6.2}$$

During flight the system moves along a parabolic trajectory determined by the acceleration of gravity. The period of flight is

$$T_f = \frac{2\dot{z}}{g} = \sqrt{\frac{8H}{g}}, \tag{6.3}$$

where

g is the acceleration of gravity and
H is the hopping height measured at the foot.

The period of a full hopping cycle is just the sum of T_s and T_f:

$$T = \pi\sqrt{\frac{m}{k_\ell}} + \sqrt{\frac{8H}{g}}. \tag{6.4}$$

Hopping Energy

When a system hops vertically, its energy takes several forms. Both the body and the leg have potential and kinetic energy as a result of the vertical position and velocity of their masses, and the leg stores energy in elastic deformation of the spring. In principle, energy can also be stored in elastic deformation of the ground, but we assume that the running surface is sufficiently damped to make that energy impossible to recover—the coefficient of restitution is zero. Running on a tuned track (McMahon and Greene 1978) or hopping on a trampoline would violate this assumption. The total vertical energy at any time in the hopping cycle is given by

$$E = m_\ell g z_1 + m g z_2 + \frac{1}{2}m_\ell \dot{z}_1^2 + \frac{1}{2}m\dot{z}_2^2 + \frac{1}{2}k_\ell(r_{s0} - r + w_\ell)^2. \tag{6.5}$$

The expressions for potential energy were chosen to be zero when the model stands vertically with the leg spring extended to its rest length and with the foot just touching the ground. From (6.5) we see that energy can be stored in the leg spring, in the motion of the body and leg masses, and in the position of the body and leg masses.

Energy is lost to air resistance throughout the hopping cycle, but such losses are generally small (Pugh 1971). Significant energy losses occur at two events in the hopping cycle, *touchdown* and *lift-off*. At touchdown the leg dissipates its kinetic energy to ground damping when it is suddenly brought to rest. The energy lost is the kinetic energy of the unsprung mass of the leg

$$\Delta E_{td} = \frac{1}{2} m_\ell \dot{z}^2_{1,td-}\,, \tag{6.6}$$

where $\dot{z}^2_{1,td-}$ is the vertical velocity of the leg just before touchdown. The energy lost at touchdown is a fixed fraction of the total kinetic energy during flight, $m_\ell/(m + m_\ell)$.

The leg's mechanical stop dissipates this same fraction of kinetic energy at lift-off. The leg's vertical velocity during stance is zero, and the body's is $\dot{z}_{2,lo-}$. After lift-off $\dot{z}_{1,lo+} = \dot{z}_{2,lo+}$. By equating linear momentum before and after lift-off, we obtain

$$m\dot{z}_{2,lo-} = (m + m_\ell)\dot{z}_{2,lo+}\,, \tag{6.7}$$

$$\dot{z}_{2,lo+} = \frac{m}{m_\ell + m}\dot{z}_{2,lo-}\,. \tag{6.8}$$

The kinetic energies before and after lift-off are found by substituting (6.8) into (6.5). The energy loss when accelerating the leg upward at lift-off is

$$\Delta E_{lo} = -\frac{m_\ell\, m}{2(m_\ell + m)}\dot{z}^2_{2,lo-}\,. \tag{6.9}$$

The fraction $m/(m_\ell + m)$ represents a fundamental efficiency of the leg. The efficiency is highest when the unsprung mass of the leg is small compared with the leg's total mass.

The control system operates the leg actuator to increase or decrease the total hopping energy. When the leg actuator changes length from w_ℓ to $w_\ell + \Delta w_\ell$, the change in hopping energy is given by

$$\Delta E_{w_\ell} = k_\ell\left(\frac{1}{2}\Delta w_\ell^2 + \Delta w_\ell r_{s\Delta}\right), \tag{6.10}$$

where $r_{s\Delta}$ is the displacement of the leg spring from its rest length, $r_{s\Delta} = r_{s0} - r + w_\ell$. The control system removes energy by making Δw_ℓ negative. For a given Δw_ℓ the magnitude of ΔE depends on the length of the leg and the position actuator. This applies during stance. When the leg actuator changes length during flight, it changes the separation between the body and leg masses. Although there is a temporary increase in stored energy, it is quickly dissipated in the mechanical stop.

Lengthening the actuator at bottom and shortening it during flight causes the total hopping energy to increase. Shortening the actuator at bottom and lengthening during flight causes the total hopping energy to decrease, eventually to zero.

During stance, the control system can calculate the total hopping energy for the next flight phase by combining (6.9) and (6.5):

$$E_f = \frac{m}{m_\ell + m}\Bigg(m_\ell g z_1 + m g z_2 + \frac{1}{2}m_\ell \dot{z}_1^2 + \frac{1}{2}m\dot{z}_2^2 + \frac{1}{2}(k_\ell r_{s\Delta}^2) + \frac{1}{2}k_g \dot{z}_0^2\Bigg)\,. \tag{6.11}$$

For the system to hop to a height H measured at the foot, the total vertical energy must be

$$E_H = m_\ell g(H + l_1) + mg(H + r_{s0} + l_2)\,. \tag{6.12}$$

This assumes that at peak elevation the leg and the body have zero vertical velocity and that no energy is stored in the leg. Therefore at peak elevation all energy is expressed as potential energy of elevation. The energy change needed to produce a hop of height H can be found from (6.12) and (6.5). To supply or remove a specific energy ΔE, the leg actuator must extend by

$$\Delta w_\ell = -r_{s\Delta} + \sqrt{r_{s\Delta}^2 + \frac{2\,\Delta E}{k_\ell}}\,. \tag{6.13}$$

An important parameter in the mechanical design of a legged system is the distance the leg spring must compress and the leg shorten during stance. The maximum compression of the leg spring during stance is a function of body and leg mass, leg stiffness, and hopping height. It can be found from (6.12) and (6.13):

$$\Delta r = \frac{mg}{k_\ell} + \sqrt{\frac{m^2 g^2}{k_\ell^2} + \frac{2(m_\ell + m)^2 gH}{m k_\ell}}\,. \tag{6.14}$$

Simulation Results for Hopping

A computer simulation was used to test the regulation of hopping height through manipulation of vertical energy. The equations of motion, given in appendix 6A, were integrated numerically to see behavior as a function of time. Figure 6.2 shows results from a simple test. The model started at

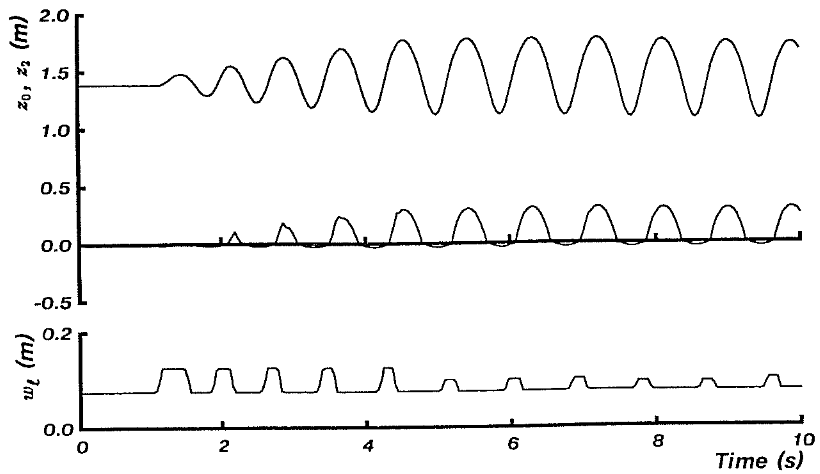


Figure 6.2. Vertical hopping. Starting from rest, vertical energy was increased until desired hopping height was attained. (Top) Elevation of hip. (Middle) Elevation of foot. (Bottom) Length of leg actuator. Note difference in vertical scales. From Raibert (1984a).

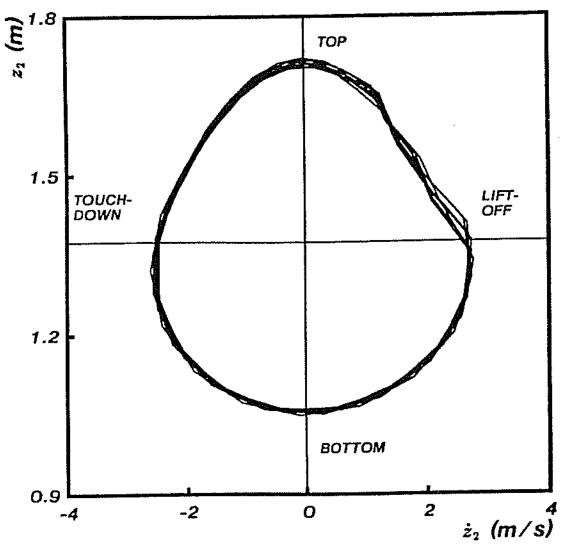


Figure 6.3. Phase plot for vertical hopping. Four synchronization events are indicated where curve crosses axes. Data are from stable part of figure 6.2. The rough part of the curves between lift-off and top indicate the damped vibration that occurred when the mechanical stop was hit. Note that position is plotted on the ordinate, velocity is on the abscissa, and the action advances in a counterclockwise direction. From Raibert (1984a).

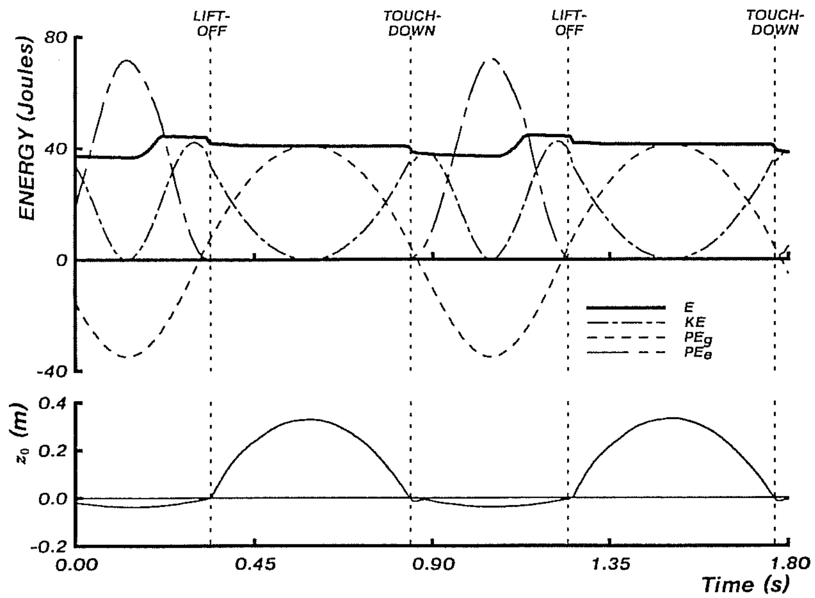


Figure 6.4. Hopping energy for two cycles at constant hopping height. Total energy, kinetic, gravitational potential, and elastic potential energies are shown. Sudden energy loss occurs at touchdown and lift-off, indicated by the vertical dotted lines. Total energy increases during stance as leg actuator extends. From Raibert (1984a).

rest, then the leg actuator injected energy on each cycle until the desired value was reached. To represent the finite response time of a physical actuator, w_ℓ increases and decreases as a quadratic function of time:

$$w_\ell(t) = w_{\ell,0} + kt^2, \tag{6.15}$$

where $w_{\ell,0}$ is the initial length of the position actuator and k is a timing constant. The stroke of the actuator is limited to $w_{\ell,min} < w_\ell < w_{\ell,max}$, so a maximum energy can be injected on a single cycle. Several cycles were required to achieve the desired hopping height.

A portion of the data in figure 6.2 is replotted in the phase plane in figure 6.3. The velocity of the body is plotted on the abscissa, and the elevation is plotted on the ordinate. The parabolic trajectory during flight was caused by constant gravitational acceleration, and the harmonic trajectory during stance was due to the spring.

The exchange of energy among its various forms, as indicated by (6.11), is shown in figure 6.4. It shows the kinetic, gravitational, strain, and total energies during two cycles of hopping to a fixed height. There are three

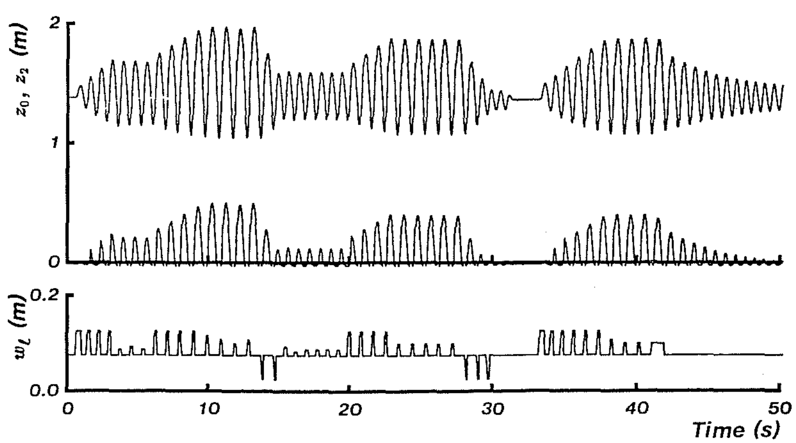


Figure 6.5. Vertical hopping sequence. At times t = 1, 6, 13, 20, 28, 33, 40 desired hopping height H = 1.7, 2.0, 1.6, 1.9, 1.4, 1.9, 1.4. The descent beginning at $t = 28$ was actively damped, while the descent at $t = 40$ was passive. (Top) Elevation of hip. (Middle) Elevation of foot. (Bottom) Length of leg actuator.

events of interest in the total energy curve: the loss at touchdown when the unsprung mass of the leg is suddenly accelerated to a stop, the loss at lift-off when the unsprung mass of the leg is suddenly accelerated up to flight speed, and the increase in energy during stance when the leg actuator does work to inject energy.

Figure 6.5 shows control of vertical hopping. The control system adjusted hopping height several times throughout the sequence. The ability to increase hopping height by increasing energy (t = 1, 6, 20, 33) and to decrease height by removing energy (t = 13, 28) are both shown. The descent employing active damping ($t = 28$) was much more rapid than the one that relied on passive dissipation of the hopping energy ($t = 40$).

Hopping Strategies

The time at which the leg shortens during the hopping cycle can be manipulated to optimize a variety of criteria:

1. If the leg shortens at lift-off, then ground clearance of the foot during flight is maximized. This is important when terrain is uneven or when high-speed travel requires large horizontal leg travel during flight. If the leg is not short during swing, it may *stub its toe*. Shortening at lift-off also reduces the leg's moment of inertia during flight, so the leg can swing forward faster and with less angular effect on the body.

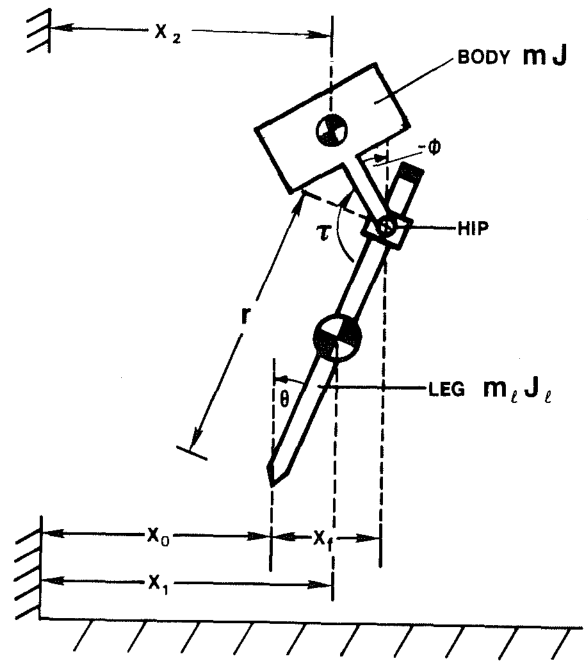


Figure 6.6. Planar one-legged model. A body and leg are connected by a hinge joint at the hip. The leg consists of a spring in series with a position actuator, as described in the previous section (see figure 6.1). In addition to their masses, the leg and body have moments of inertia J_ℓ and J. A hip actuator can exert a torque τ at the hip between the body and the leg. The model is restricted to motion in the plane. See appendix 6A for equations of motion and table 6.1 for model parameters used in simulations. Leg angle θ, leg actuator length w_ℓ, and hip actuator torque τ are shown positive in the figure. Body angle ϕ, is shown negative.

2. If the leg shortens at top, then the time between vertical actuations is maximized. This strategy could permit use of a slower actuator.
3. If the leg shortens during the initial part of touchdown, then the ground impact forces on the foot are minimized. The period over which the foot is accelerated to ground speed is increased.

It is also possible for the leg to shorten at lift-off, lengthen again just before the next touchdown, and shorten during the landing. This strategy, apparently used by humans when running, maximizes ground clearance and simultaneously minimizes impact forces on the foot. However, it is accomplished at the expense of an extra lengthening and shortening of each leg during each step.

An Alternative Three-Part Control

In order to formulate the three-part decomposition that was used to control running for the machines described in this book, it was necessary to assign a control action to each variable in need of control. Forward foot placement was assigned to the control of forward velocity, hip torque during stance was assigned to the control of body attitude, and leg thrust was assigned to hopping height. A control action might, in fact, have several effects on the behavior of the system, but all effects are ignored except for those on the variable assigned. For instance, hip torque during stance generates a horizontal ground force that accelerates the body forward, but this is ignored and the control system focuses on the task of regulating the attitude of the body.

Because control actions have multiple effects, no given set of assignments is unique. One could design a control system for legged locomotion with other assignments than those described in the preceding chapters. This section demonstrates this point. It reports a control system that uses foot placement to regulate body attitude and hip torque during stance to control forward running speed. The assignments are tested with computer simulations of a planar one-legged model that is similar to the machine described in chapter 2.

Leg Sweeping Algorithm

There must be no horizontal force acting on the body if a legged system is to run at constant speed. I have already described how symmetry can ensure that the horizontal force on the body is an odd function that averages to zero over each stride. The leg's thrust and angle combine to generate a backward force on the body during the first half of stance and a forward force during the second half of stance. The average force is zero over a stride, so the running speed does not change from stride to stride.

Suppose we want the forward running speed to be constant throughout the stride, not just across strides. In that case, no horizontal force should act on the body. The supporting feet should move backward with respect to the body at the same constant rate that the body moves forward, $\dot{x}_f = -\dot{x}_d$. The horizontal motion of the supporting feet should be independent of the vertical motion of the body, which, during support, is dominated by compression and extension of the leg.

The *sweep control algorithm* we implemented avoids generating horizontal forces on the body by sweeping the leg and foot backward during stance at the desired rate of travel. The control system adjusts the hip angle to sweep the leg while taking the instantaneous length of the leg into account. Leg length varies as the bouncing motion proceeds throughout stance. The effect is that under nominal conditions the resultant force acting between the foot and the ground is vertical. When the body's forward speed lags behind the desired value, the hip servo sees an error in the foot's backward progress. It counteracts the error by exerting hip torques that encourage the foot and the body to catch up.

The attitude of the body varies as the system tips during support. The control system can regulate the attitude of the body by choosing a forward position for the foot before landing that will provide net tipping during stance. Once again, symmetry determines the neutral point, where the foot should be placed to provide zero net tipping. If the foot is placed on the neutral point, then backward tipping during the first half of stance is balanced by forward tipping during the second half of stance. Unlike the three-part control algorithms described in previous chapters, $\dot{x}T_s$ now accurately predicts the length of the CG-print and the location of the neutral point because the forward speed is constant.

Displacement of the foot from the neutral point results in a net angular acceleration of the body. The control system can use a linear function of errors in body attitude and body attitude rate to determine a displacement of the foot from the neutral point.

Implementation of the Algorithm

The model used to test the leg sweeping algorithm is shown in figure 6.6. It incorporates the leg model shown in figure 6.1, which was described in the previous section. The leg pivots with respect to the body about a hinge-type hip that is driven by a linear servo:

$$\tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}), \tag{6.16}$$

where

τ	is the torque exerted about the hip,
γ	is the hip angle, and
k_p, k_v	are feedback gains. Typical values are $k_p = 1800$ N·m/rad, $k_v = 200$ N·m·s/rad during stance and $k_p = 1200$ N·m/rad, $k_v = 60$ N·m·s/rad during flight.

The model assumes that friction keeps the foot from slipping on the ground during support. In order to calculate the horizontal forces acting between the foot and the ground, the ground is modeled as a stiff, damped spring in the horizontal direction (k_g, b_g). Each time the foot touches the ground, the rest position of this spring is reset to the point at which the foot first touches. This springiness represents the combined compliances of the support surface and the foot.

Viewing the problem in a coordinate system that moves forward with the hip, we see that the task is to make the foot sweep backward at the same rate as the ground. The leg sweeping algorithm accomplishes this task by calculating a target angle for the hip at each moment during stance. The target angle is based on the rate of forward travel, the instantaneous length of the leg, the time since touchdown, and the placement of the foot relative to the hip at touchdown.

If the duration of stance, the forward speed of the body, and the geometry of the system are known, then the control system can calculate an appropriate leg angle and sweeping function. For the model used here, the body mass is separated from the hip, so the location of the center of gravity is a function of the body and leg angle. The forward position of center of mass with respect to the hip is

$$x_{cg} = \frac{(l_1 - r)m_\ell \sin(\theta) + l_2 m \sin(\phi)}{m_\ell + m}. \tag{6.17}$$

Placement of the foot on the neutral point provides no net angular acceleration of the body attitude. The location of the neutral point with respect to the center of mass is

$$x_{f0} = \frac{\dot{x}T_s}{2}. \tag{6.18}$$

To accelerate the attitude of the body the foot is displaced from the neutral point by a linear function of errors in body attitude and body attitude rate:

$$x_{f\Delta} = k_\phi(\phi - \phi_d) + k_{\dot{\phi}}\dot{\phi}, \tag{6.19}$$

where k_ϕ, $k_{\dot{\phi}}$ are gains. At touchdown the forward position of the foot is

$$x_f = x_{cg} + x_{f0} + x_{f\Delta}. \tag{6.20}$$

Equation (6.20) provides the forward position of the foot with respect to the center of mass at touchdown. During stance, the foot should move backward with respect to the center of mass at the fixed speed $\dot{x}_d$. The horizontal trajectory for the foot with respect to the center of mass during support is then

$$x_f(t) = x_{cg} + x_{f0} + x_{f\Delta} - \dot{x}_d(t - t_{td}), \tag{6.21}$$

where t_{td} is the time at touchdown. To limit the maximum acceleration when changing speed, the desired rate of travel is restricted on each step so that $|(\dot{x} - \dot{x}_d)| < \Delta\dot{x}_{max}$. This expression for positioning the foot with respect to the hip, (6.21), incorporates terms for the forward location of the center of mass, the neutral point, the desired displacement, and the leg sweeping function. The kinematics of the leg, $x_f = -r\sin\theta$, allow us to eliminate the dependence on leg angle from the right-hand side of (6.21):

$$-r\sin\theta = \frac{(l_1 - r)m_\ell \sin(\theta) + l_2 m \sin(\phi)}{m_\ell + m} + x_{f0} + x_{f\Delta} - \dot{x}_d(t - t_{td}). \tag{6.22}$$

Leg, body, and hip angles are related by $\gamma = \phi - \theta$. Solving for the desired hip angle to place the foot, we obtain

$$\gamma_d = \phi + \arcsin\left(\frac{l_2 m \sin(\phi) + (m_\ell + m)\left(x_{f0} + x_{f\Delta} - \dot{x}_d(t - t_{td})\right)}{l_1 m_\ell + rm}\right). \tag{6.23}$$

The hip servo given in (6.16) is used to move the hip to this angle.

The vertical hopping motion is controlled with the energy regulation technique described in the previous section.

Simulation Results from Leg Sweep Algorithm

Figure 6.7 shows running at constant speed using the leg sweep algorithm. Forward speed was well controlled, with only small errors within each cycle. The attitude of the body was not controlled with high precision, but it was kept generally upright, with a maximum error of about 0.15 rad. The body attitude had a distinct oscillation at the stepping rate, about 0.3 hz. This oscillation was caused by the need to swing the leg forward during flight.

Figure 6.8 shows the response to a change in desired running speed. Although the control system successfully maintains the model's balance, the body attitude is poorly regulated during the starting and stopping accelerations (t = 2.5, 8). The accelerations required to change running speed disturb the body attitude. These disturbances can be predicted from (6.21), which sweeps the leg backward according to the desired velocity. Only after the body is disturbed by the sweeping motion, an entire hop later, does the control system adjust the foot placement to compensate. In this example the interactions between forward speed and body attitude are large, so the three-part decomposition nearly breaks down.

One remedy to this problem is to provide a term in the foot placement calculation that adjusts for acceleration. Equation (6.19) could be replaced with

$$x_{f\Delta} = k_\phi(\phi - \phi_d) + k_{\dot{\phi}}\dot{\phi} + k_{\dot{x}}(\dot{x} - \dot{x}_d), \tag{6.24}$$

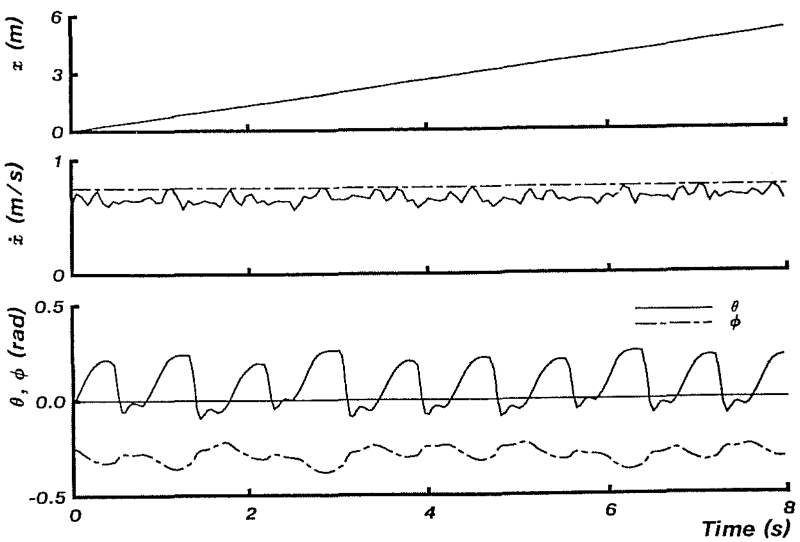


Figure 6.7. Running at constant forward speed is generated by the leg sweep algorithm. Rate of travel is 0.75 m/s. $k_\phi = 0.1\,\mathrm{m/rad}$, $k_{\dot\phi} = 0.15\,\mathrm{m/rad/s}$.

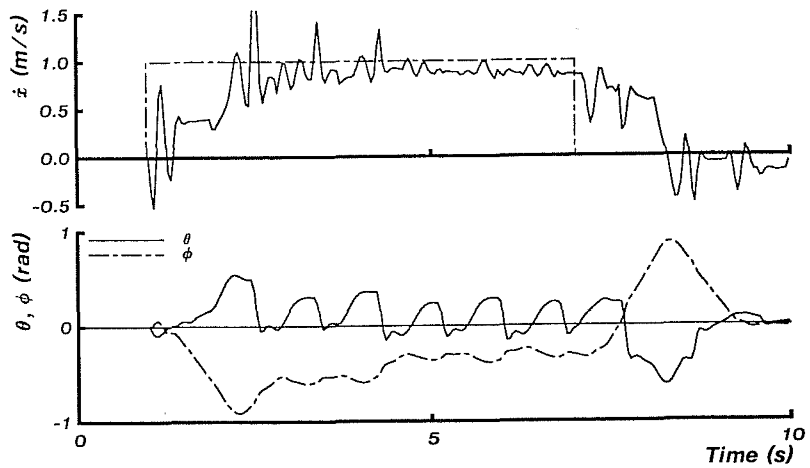


Figure 6.8. Sweep control algorithm attempting to accommodate a step in desired running speed. The acceleration needed to change speed disturbs control of body attitude. $k_\phi = 0.1\,\mathrm{m/rad}$, $k_{\dot\phi} = 0.15\,\mathrm{m/rad/s}$, $\Delta\dot{x}_{max} = 0.45\,\mathrm{m/s}$.

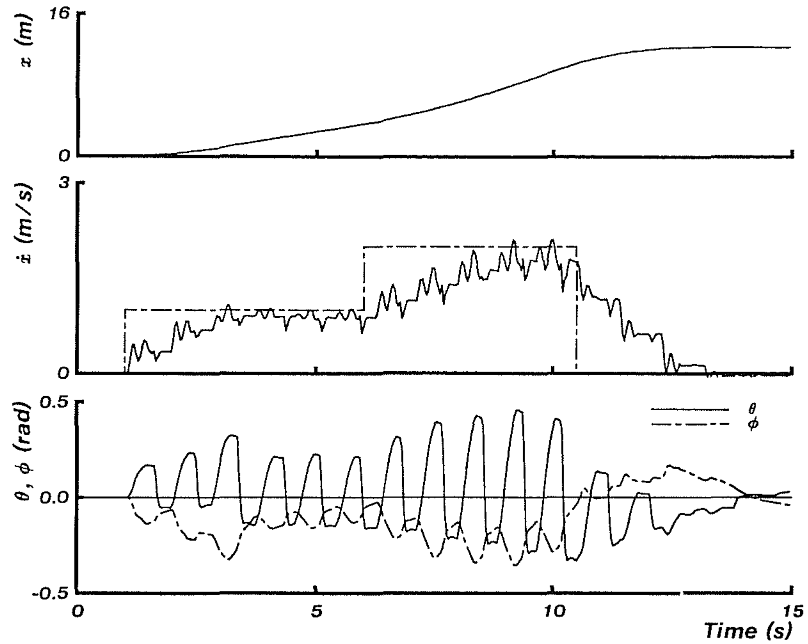


Figure 6.9. Leg sweeping algorithm controls running as desired velocity, $\dot{x}_{2,d}$ (shown stippled in second plot), changed in steps. $k_{\dot x} = 0.2$, $k_\phi = 0.1\,\mathrm{m/rad}$, $k_{\dot\phi} = 0.1\,\mathrm{m/rad/s}$, $\Delta\dot{x}_{max} = 0.45\,\mathrm{m/s}$. From Raibert (1984a).

Figure 6.9 shows the the behavior of the modified sweep control algorithm. The model starts hopping in place, then accelerates to 1 m/s, then to 2 m/s, and then slows to a stop. The running speed is controlled with good precision and body attitude is controlled with substantially better precision than before. Acceleration data for another run are shown as a cartoon in figure 6.10, where the pattern of motion can be visualized more easily.

This modified sweep control algorithm improves performance at the cost of compromising the three-part decomposition. The algorithm is a compromise in that foot placement is no longer chosen solely on the basis of controlling one variable, in this case body attitude. In general, I expect more advanced control systems to require compromises of this sort, trading off simplicity for improvements in performance. The ultimate compromise would be to make foot placement, hip torque, and leg thrust nonlinear functions of all the relevant state variables. The next chapter describes such a control system.

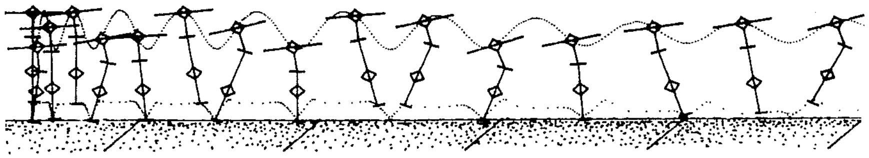


Figure 6.10. Cartoon of running controlled by leg sweeping algorithm. Model accelerates from standing start to about 2.2 m/s in 10 s. The dotted lines represent the paths of hip and foot (20 ms/dot, 600 ms per stick figure). Maximum speed is limited by clearance between foot and ground. From Raibert (1984a).

Taxonomy of Control for Quadruped Running

Of the running machines described in the preceding chapters, the quadruped provides the richest opportunity for exploring variations in the control of locomotion. This section outlines some of the alternatives for quadruped control and lists the specific decisions that led to the control. The others have not yet been explored.

Machine Coordinates versus World Coordinates

Should the control system for a quadruped machine operate like that of a car or an airplane, where desired motions are expressed relative to the position and orientation of the machine, or like that of the 3D one-legged hopping machine, where desired velocities are expressed relative to a coordinate system fixed to the room? Room coordinates are natural for the hopping machine because it has no preferred orientation and because we did not control its facing direction. Machine coordinates seem natural for a quadruped because it has a preferred direction of travel, there is asymmetry in the forward and lateral behavior, and undesirable yaw motions are less of a problem.

◇ *For the first round of experiments we used machine coordinates. The central coordinate system used for the quadruped described in chapter 4 translates with the body and changes its facing direction with the body, but is unaffected by the pitch and roll of the body.*

The Hopping Cycle

How should events in the locomotion cycle govern switching between control actions? One approach would be to couple the interactions among legs loosely. It might be possible to treat each leg as though it were the only leg on the machine. Each leg could thrust according to the progress of its own hopping cycle, each leg could be positioned during flight according to its own assessment of when flight started and its own estimate of the forward velocity, and each leg could exert torques to right the body during stance when the leg provides support. For this case the control would be fully independent. Donner (1984) experimented with independent leg control for a six-legged walking machine.

Another approach would be to couple the activities of the legs tightly. Assume that each gait specifies sets of legs that act in unison. The joint state of all legs acting together in a set would determine events in the locomotion cycle. For example, in a trot touchdown would occur when both legs of the diagonal pair are in contact with the ground. Leg thrust used to control hopping height and hip torque used to control body attitude would be coordinated to the joint hopping cycle. Velocity control would depend on the estimated velocity of the machine's center of gravity.

Between these two extremes there are many intermediate possibilities. For instance, legs in a set could work together to provide vertical thrust, but separately to control body attitude. Movement of the legs in a set for velocity control could be timed according to the separate activities of the legs, whereas the position to which they move might be determined by the overall velocity of the body. Presumably, experiments will give us clues for finding a rational basis for choosing among the many possibilities.

◇ *For the first round of experiments we tightly coupled the sequence of activities for all legs.*

Mechanisms for Coordinating Leg Thrusts

The key to getting the four-legged machine to run is to coordinate the thrust delivered by legs that act on the ground simultaneously so that variations do not tip the machine over. Attitude variations of the body during flight cause the feet to touchdown at different times, complicating the task of coordinating the thrust their legs deliver. Another problem is that differential thrust of the legs accelerates the attitude of the body. Because feet can only push on the ground, there may also be restrictions on how differential thrust can be controlled. There are several plausible approaches to controlling leg thrust:

1. Servo leg lengths during flight so the clearance of each foot above the ground is the same for all legs acting together in a set. The feet lie in a horizontal plane and all touch down simultaneously.

2. Servo the length of all legs providing support so that they generate the same axial force. This is complicated for gaits that have the feet touch down one at a time. As soon as a foot touches the ground, the leg suddenly extends to assume its share of the load. In a system with a central notion of gait, the control system could delay the thrust servo until all the legs in a set are touching the ground or until they are bearing a substantial load.
3. Servo the lengths of all legs providing support to keep the body level. The servo must be careful not to pick up the feet, because glue does not hold the feet to the ground during stance. A linear combination of length error and air spring pressure might work correctly:

$$r_{d,i} = L - k(p_i - p_{ave}), \tag{6.25}$$

where

$r_{d,i}$	is the desired length for the ith leg,
L	is the leg length setpoint determined by the hopping algorithm (it may change during stance),
p_i	is the air spring pressure in the ith leg,
p_{ave}	is the average air spring pressure, and
k	is a gain.

If leg length is used to control body attitude, then the control system must take both hopping control and attitude control into account when calculating setpoints for the leg lengths.
4. Thrust independently according to the separate state of each leg without doing anything explicit to coordinate. There is something quite satisfying about the way the body bounces passively on the leg in the one-legged hopping machines. It would be interesting to do this for a machine with several multiple legs.

The problem of coordinating leg thrust so that it provides attitude stability for the body or so that it does not interfere with body attitude stability is the central problem in controlling the locomotion of a system with four legs.

◇ *For the first round of experiments the control system servoed the length of all legs providing support to generate equal axial forces.*

Velocity Control

Should the legs operate together or independently in controlling the running velocity of the machine? Assuming that the control system can determine the machine's forward velocity by combining information from all legs in support and from the gyroscopes, there are two major alternatives: The velocity of the machine's center of mass can determine where the feet are placed, or the velocity of each hip can determine where each foot is placed. The second approach might be good if we wanted to do something fancy with yaw rotations. For instance, we might like the front end of the machine to remain stationary while the rear end moves sideways. For instance, the driver might use a fancy joystick to specify a center of turning.

An entirely different mechanism for controlling forward running velocity is to make hip torque during stance a function of errors in forward speed. If this approach is uses, then body attitude must rely on some mechanism other than hip torque during stance. The *sweep control* method described earlier in this chapter falls into this class of mechanism. Murphy's simulations of a planar model with two legs also uses a mechanism of this sort (Murphy and Raibert 1985). During flight, the legs are positioned according to the actual forward velocity, independent of the desired forward velocity. During stance, hip torque is used to correct velocity errors. Body attitude is either controlled passively or through differential thrust of the legs.

◇ *For the first round of experiments we tightly coupled the positioning of legs during flight according to the velocity of the center of mass.*

◇ *For the first round of experiments we used a velocity control algorithm that positions the foot during flight, incorporating an error term. We did not use sweep control.*

Attitude Control

There are three ways to control body attitude:

1. *Hip torque.* Hip torques during stance are a function of errors in the pitch and roll attitude of the body. This method depends on a favorable interaction between the hip torques and the springiness of the legs. For instance, if the attitude correction is to pitch the body forward, i.e. nose down, then the front legs touching the ground must shorten and the rear legs must lengthen. This method is like that used in the planar and 3D one-legged machines, but it is more complicated when there are several legs.

2. *Leg thrust.* Modulate the differential thrust of legs providing support to reorient the body. Murphy (1984) used this technique to control trotting in a planar two-legged simulation. The technique is limited by the legs providing support for a given gait. For instance, when two legs on the same side of a quadruped provide support, their differential thrust has no affect on the roll angle of the body.
3. *Limit-cycle.* Let the machine seesaw in a limit cycle. This might be considered a form of using leg thrust to control body attitude, where the thrusting legs act at different times in the hopping cycle, rather than at the same time. The limit cycle might be accomplished by manipulating the thrust delivered by the legs in response to motion of the body or it might be accomplished passively. Murphy's model uses such a passive method to stabilize the pitch angle of the body when the model bounds (Murphy 1984).

The attitude of the body must be controlled about two axes, pitch and roll. The methods of attitude control that use hip torque and leg thrust are called *leveling controls*, whereas limit-cycle attitude control is called *rocking control.* If attitude control is implemented separately about the roll and pitch axes, then it is not necessary to use the same method for both axes. This may be a factor underlying the gaits normally observed in animals (table 6.2). For example:

- The pronk, a quadruped gait that uses all four legs in synchrony, uses leveling control on both the pitch and roll axes.
- The trot may use leveling control on both pitch and roll axes or it may use rocking attitude control on both axes. A third alternative is that it does attitude control about the diagonal axes; about one diagonal axis on one step and the other diagonal axis on the next step.
- The pace uses rocking control about the roll axis and leveling control about the pitch axis.
- The bound uses leveling control about the roll axis and rocking control about the pitch axis.
- The gallop clearly uses rocking control about pitch, but it is not clear what stabilizes roll. In a rotary gallop does an animal rock from side to side in addition to pitching? Does it wobble about the vertical axis?

◇ *For the first round of experiments we used hip torque for leveling attitude control about both the pitch and roll axes.*

Table 6.2. Conjecture as to mechanism providing attitude control for each axis for several gaits.

Gait	Pitch attitude	Roll attitude
Pronk	leveling	leveling
Trot	leveling	leveling
Pace	leveling	rocking
Bound	rocking	leveling
Gallop	rocking	rocking?

Given that hip torque is used to control attitude of the body, there remains the question of how to time the hip torquing actions of the legs providing support. There are two alternatives:

1. All legs in a set begin to correct body attitude errors when all legs in the set bear adequate load. Adequate load is required so that the legs do not slip when the attitude control torques generate tangential forces at the feet. All legs in the set terminate attitude control when any one of the legs in the set no longer bears adequate load (tightly coupled).
2. The ith leg begins to correct body attitude errors when it first bears adequate load. The ith leg stops correcting body attitude errors when it no longer bears adequate load (loosely coupled).

The other way attitude control could be tightly or loosely coupled has to do with the calculation of specific values for hip torque at each hip. This is orthogonal to the timing question just discussed. For instance, even if the load on each leg determines when it should generate hip torque, the value of hip torque could be a function of how many legs are contributing to attitude control at the moment.

On the other hand, the friction between a foot and the ground is probably the limitation on how much attitude control torque can be used to right the body. An algorithm that lets each leg generate torque when it bears a load results in an attitude control torque that is roughly proportional to the available friction.

◇ *For the first round of experiments we permitted a leg to generate hip torque for leveling attitude control when all the legs working together is a set bear adequate load.*

◇ *For the first round of experiments, we let the value of hip torque that each leg provides for leveling attitude control be independent of which other legs are providing support.*

Recovery

Recovery is the process that lifts a leg and moves it forward when it is not providing support. For a one-legged system recovery is simple: when the foot is clear of the ground, the leg moves to the angle required for landing. There is a strict alternation between stance and recovery.

In a system with more than one leg there is an opportunity to recover some legs while others are providing support. This has two advantages:

1. It is not necessary for the flight interval to be long enough to recover a leg. The other way to say this is that the leg actuators need not be fast enough to recover the leg during flight. This is a bandwidth and speed issue.
2. Because recovery legs can move forward while stance legs move backward, the system need not pitch so much during running as the one-legged systems do.

On the other hand, recovery is more complicated than before. The most important problem is to ensure that recovery legs are short enough to clear the ground when other legs are providing support. Also, if a leg needs to shorten to recover without stubbing, then there must be some coordination among the various phases of the recovery motion: leg shortening, leg sweeping, leg lengthening, ballistic flight of the body, and ground rebound of the body. The need for coordination of this sort is one of the primary reasons for providing hydraulic servo actuation of leg length in the four-legged running machine described in chapter 4.

One approach to providing coordination would be to servo the lengths of all legs in recovery so that their feet are a distance H_{min} above the ground. The recovery motion would have three phases:

1. Once lift-off occurs, shorten the leg until the foot clears the ground by at least H_{min}.
2. Once a ground clearance of H_{min} is obtained, move the leg to a forward position according to the velocity control algorithm. Continue to servo leg length to maintain ground clearance.
3. Once forward position is obtained, lengthen leg to landing length.

A limitation of this method is its reliance on estimates of z, which depend on accurate estimates of $\dot{z}(t_{lo})$.

Another approach would be to shorten the leg to a fixed recovery length, to move the leg forward a fixed time interval after lift-off, and to lengthen the leg a fixed time later. This is *type I recovery.* This approach can be modified slightly by letting the lift-off event for the stance legs trigger lengthening of the recovery legs. This variation, called *type II recovery,* has the advantage that it provides a means for synchronizing the two sets of legs in the system—the recovery legs are synchronized to the active legs.

◇ *For the first round of experiments we used type II recovery.*

There are really two parts to a recovery algorithm. The first part specifies when to shorten the leg, when to swing the leg, and when to lengthen the leg. The other part specifies the magnitude and trajectory for shortening, swinging, and lengthening the leg. One approach to timing the recovery motions is to synchronize them to the ongoing motions of the stance legs. Suppose that legs in a set act in perfect unison. The recovery set could begin to swing when the stance legs touch down. The recovery legs could lengthen when the stance legs lift off.

An approach to the magnitude and trajectory part might also coordinate the recovery legs with the stance legs. The recovery algorithm could servo the hip angles of the recovering legs to the negative of the hip angles of the stance legs. Of course, some time before touchdown the recovery feet would have to move to the position determined by the velocity control algorithm, even if it were different from the negative of the stance leg angle.[2] Both these recovery methods would work correctly only for the gaits that use the legs in pairs: the trot, the pace, and the bound.

In this discussion recovery has been described as a three part motion that shortens the leg, swings it forward, and then lengthens it. There can be another action related to recovery that accelerates the foot backward relative to the body just before it touches the ground. When done correctly, this causes the foot speed to match the ground speed just before impact, so the foot is not suddenly accelerated backward by impact with the ground. This means less wear on the foot and perhaps a smoother forward running speed. This recovery action, called *prestance ground speed matching,* or just *ground matching,* is a refinement that is important for high-speed and high-efficiency locomotion.

I believe that accurate ground matching will be difficult to achieve.

[2] Unless the scissor algorithm of chapter 5 were in use.

The problem is that the backward motion of the foot just before contact must satisfy two conditions: that the foot be in the right place when contact occurs and that the backward speed of the foot match the forward speed of the body. This requires a fine coordination among the vertical motion of the body, the forward motion of the body, and the backward motion of the leg.

Design of the recovery algorithm involves two sorts of specification: the timing of the leg actions and the method of the actions themselves. The algorithm must specify when to stop shortening the leg in order to begin sweeping it and when to stop sweeping the leg in order to start lengthening it. The recovery algorithm must also specify a method for shortening, sweeping, and lengthening. It would make sense to decompose the particular recovery algorithms suggested into timing specifications and method specifications.

Leg Coordination

We must choose methods for coordinating actions of the various legs. Although we have made decisions that suggest the actions of legs be loosely or tightly coupled under various circumstances, we have not provided mechanisms for phasing the actions of the legs, whether or not their actions are coordinated.

One approach is to do nothing; left to itself the system may find some sort of stable leg phasing on its own. For instance, if the control system stabilizes the attitude of the body successfully and servos the legs to the same length, then as the level body falls downward the feet all strike the ground at the same time (assuming that the support surface is flat and level).

There are three classes of gait, each requiring a different kind of leg synchronization:

- *Pronk.* All legs act together.
- *Pair gaits.* The legs form two pairs. Within each pair the legs move in phase, and between the pairs the legs move in antiphase. The trot, pace, and bound employ such pairwise operation. Two synchronizations are necessary:
    - Each leg should work in near synchrony with another leg. The four legs form two pairs.
    - The two pairs of legs operate out of phase.

- *Gallops.* One leg strikes the ground at a time, one leg leaves the ground at a time, and one, two, or three feet provide support during stance.

The following is an approach to providing synchronization for the pair gaits. The two legs in a pair that operate in phase are called *mates*. The two legs that move out of phase are called *opposites*:

- For each leg assign a mate for each gait, as listed in table 6.3.
- For each leg assign an opposite for each gait, as shown in table 6.3. In order to force the pronk into this framework, we define a phantom fifth leg, indicated by *. For the fifth leg, touchdown, bottom, and lift-off are always true.
- During recovery, a leg lengthens after its opposite has lifted off. The definition of *after* is tricky. What is important is that the lengthening leg not strike the ground while the machine is still moving upward and that it be fully lengthened by the time the machine is ready to land.
- During stance, a leg thrusts in synchrony with its mate. They both thrust after both legs reach bottom.

Table 6.3. A leg and its mate work together in phase as a pair. For each gait there is a different pairing of mates. A leg and its opposite work out of phase. LF, left front; LR, left rear; RF, right front; RR, right rear.

	Mate			
Leg	Trot	Pace	Bound	Pronk
LF	RR	LR	RF	LR
LR	RF	LF	RR	RR
RH	LF	RF	LR	RF
RF	LR	RR	LF	LF
	Opposite			
Leg	Trot	Pace	Bound	Pronk
LF	RF	RF	LR	*
LR	RR	RR	LF	*
RR	LR	LR	RF	*
RF	LF	LF	RR	*

Summary

Chapters 2 through 5 painted a narrow picture of how one might control running. They described an approach, a set of control algorithms, and some implementions. The purpose of this chapter was to broaden the picture somewhat by looking at the factors used to arrive at an implementation and at alternative control algorithms.

The first and second sections described control algorithms for running that differ from those described earlier. They make the point that the design space for locomotion control systems is large and that there is nothing sacred about the algorithms we have chosen to study.

The third section of this chapter considers some of the implementation decisions that were made for the quadruped once the general form of the control algorithm had been specified. Many of these decisions were made on intuitive grounds or were purely arbitrary. The need for such decisions is typical of the experimental approach, and good experiments help to make better design decisions.

Summary of Quadruped Control Decisions

Here is a brief summary of the decisions made for the quadruped experiments reported in chapter 4. The quadruped control system:

- ◇ *Used machine coordinates.*
- ◇ *Tightly coupled the sequence of activities for all legs.*
- ◇ *Tightly coupled each leg's thrust with a force-equalizing servo.*
- ◇ *Positioned legs for velocity control dependently, based on the velocity of the machine's center of mass.*
- ◇ *Used a velocity control algorithm that positioned the foot only during flight. It did not use sweep control.*
- ◇ *Used hip torque for leveling attitude control:*
  - ◇ *Permitted a leg to generate hip torque for leveling attitude control when all the legs working together is a set bear adequate load.*
  - ◇ *Let the value of hip torque that each leg provides for leveling attitude control be independent of which other legs are providing support.*
- ◇ *Used type II recovery.*

Appendix 6A. Equations of Motion for Planar One-Legged Model

The following equations apply to the model shown in figure 6.1 and 6.6; they were derived from free body diagrams of the leg and body using d'Alembert's principle:

$$\ddot{z}_1 = \ddot{z}_0 - l_1(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta), \tag{6.26}$$

$$\ddot{x}_1 = \ddot{x}_0 + l_1(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta), \tag{6.27}$$

$$\ddot{z}_2 = \ddot{z}_0 + \ddot{r}\cos\theta - r(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) - l_2(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi) - 2\dot{r}\dot{\theta}\sin\theta, \tag{6.28}$$

$$\ddot{x}_2 = \ddot{x}_0 + \ddot{r}\cos\theta + r(\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta) + l_2(\ddot{\phi}\sin\phi - \dot{\phi}^2\cos\phi) + 2\dot{r}\dot{\theta}\sin\theta, \tag{6.29}$$

$$m_\ell\ddot{z}_1 = F_z - F_t\cos\theta + F_n\sin\theta - m_\ell g, \tag{6.30}$$

$$m_\ell\ddot{x}_1 = F_x - F_t\sin\theta - F_n\cos\theta, \tag{6.31}$$

$$J_\ell\ddot{\theta} = -F_x l_1\cos\theta + F_z l_1\sin\theta - F_n(r - l_1) - \tau, \tag{6.32}$$

$$m\ddot{z}_2 = F_t\cos\theta - F_n\sin\theta - mg, \tag{6.33}$$

$$m\ddot{x}_2 = F_t\sin\theta + F_n\cos\theta, \tag{6.34}$$

$$J\ddot{\phi} = F_t l_2\sin(\phi - \theta) - F_n l_2\cos(\phi - \theta) + \tau, \tag{6.35}$$

where

(x_0, z_0)	are coordinates of the foot,
(x_1, z_1)	are coordinates of the leg's center of mass,
(x_2, z_2)	are coordinates of the body's center of mass,
(F_x, F_z)	are the horizontal and vertical forces on the foot, and
(F_t, F_n)	are forces acting at the hip between the leg and the body. F_t acts tangent to the leg and F_n acts perpendicular to the leg.

Eliminating x_1, z_1, x_2, z_2, F_n, and F_t, we can express these equations in terms of the state variables θ, ϕ, x_0, z_0, r, and by substituting $R = r - l_1$, we obtain

$$
\begin{aligned}
(mRr + J_\ell)\ddot{\theta}\cos\theta m + ml_2R\ddot{\phi}\cos\phi + mR\ddot{x}_0 + mRr\sin\theta = \\
Rm(\dot{\theta}^2 R\sin\theta - 2\dot{\theta}\dot{r}\cos\theta + l_2\dot{\phi}^2\sin\phi + l_1\dot{\theta}^2\sin\theta) \\
- l_1F_x\cos^2\theta + (l_1F_z\sin\theta - \tau)\cos\theta + F_kR\sin\theta,
\end{aligned}
\tag{6.36}
$$

$$
\begin{aligned}
(mRr + J_\ell)\ddot{\theta}\sin\theta + ml_2R\ddot{\phi}\sin\phi - mR\ddot{z}_0 + mR\ddot{r}\cos\theta = \\
- Rm(\dot{\theta}^2 R\cos\theta + 2\dot{\theta}\dot{r}\sin\theta + l_2\dot{\phi}^2\cos\phi l_1\dot{\theta}^2\cos\theta - g) \\
- l_1F_x\cos\theta\sin\theta - \sin\theta(l_1F_z\sin\theta - \tau) + F_kR\cos\theta,
\end{aligned}
\tag{6.37}
$$

$$
\begin{aligned}
(m_\ell l_1 R - J_\ell)\ddot{\theta}\cos\theta + m_\ell R\ddot{x}_0 = \\
R(m_\ell L_1\dot{\theta}^2\sin\theta - F_k\sin\theta + F_x) - (F_zl_1\sin\theta - F_xl_1\cos\theta - \tau)\cos\theta,
\end{aligned}
\tag{6.38}
$$

$$
\begin{aligned}
(J_\ell - m_\ell l_1 R)\ddot{\theta}\sin\theta + m_\ell R\ddot{z}_0 = \\
R(m_\ell l_1\dot{\theta}^2\cos\theta - F_k\cos\theta + F_z - m_\ell g) - (F_zl_1\sin\theta - F_xl_1\cos\theta - \tau)\sin\theta,
\end{aligned}
\tag{6.39}
$$

$$
\begin{aligned}
J_\ell l_2\ddot{\theta}\cos(\phi - \theta) - Jr\ddot{\phi} = \\
R(F_kl_2\sin(\theta - \phi) - \tau) + l_2\cos(\phi - \theta)(l_1F_z\sin\theta - l_1F_x\cos\theta - \tau),
\end{aligned}
\tag{6.40}
$$

where

$$
F_k = \begin{cases} k_\ell r_{s\Delta} & \text{for } r_{s\Delta} > 0 \\ k_{stop}r_{s\Delta} - b_{stop}\dot{r} & \text{otherwise} \end{cases}, \tag{6.41}
$$

$$
F_x = \begin{cases} k_g(x_0 - x_{td}) - b_g\dot{x}_0 & \text{for } z_0 < 0 \\ 0 & \text{otherwise} \end{cases}, \tag{6.42}
$$

$$
F_z = \begin{cases} k_g z_0 - b_g\dot{z}_0 & \text{for } z_0 < 0 \\ 0 & \text{otherwise} \end{cases}. \tag{6.43}
$$

Chapter 7

Tabular Control of Running[1]

This chapter discusses the use of large tables of precomputed data to control forward running speed. The tabular approach takes advantage of the regular cyclic character of legged behavior by partitioning the system state variables into two sets: a set that varies in a fixed pattern on each cycle and a set that varies freely from cycle to cycle. Only this second set of variables determines the size of the table. Stored data were computed by numerically simulating a legged model as it progressed through the stance portion of the running cycle. Repeated simulations were used to characterize the model for different landing conditions.

Because the size of tables used for control can be prohibitively large, polynomial surfaces were used to approximate the tabular data. Good approximations to the tabular data were obtained with just a few dozen terms. Simulations showed the feasibility of using the tabular and polynomial methods to control balance and forward running speed in a planar model with one leg.

Background on the Use of Tables for Control

Control algorithms that use carefully organized tabular data offer the promise of good control for complicated dynamic systems. Tabular tech-

[1] This chapter is based on a paper entitled "Tabular control of balance in a dynamic legged system," by M. H. Raibert and F. C. Wimberly, which originally appeared in *IEEE Transactions on Systems, Man, and Cybernetics* SMC-14:2, 1984.

niques are powerful because they use the results of arbitrarily complicated calculations for control, but the time penalty of actually doing the calculation is incurred off-line. The run-time calculation is typically quite simple, permitting execution at high speed. In a comparison of techniques for computing robot manipulator dynamics, Hollerbach showed that a tabular method required the fewest run-time operations when applied to a manipulator with fewer than nine joints (Hollerbach 1980).

Another advantage of tabular control methods is that tables make it easy to implement simple forms of learning and adaptation. A tabular controller typically performs a simple computation on the state variables to determine appropriate control values. The computation is based on a representation of the dynamics of the system to be controlled. Because the computations are simple, it is usually easy to determine values for the coefficients of the computation, provided that the form of the control computation is already known. Learning occurs when values for the coefficients of the computation are determined from data obtained by observing the behavior of the system to be controlled (Albus 1975a, 1975b, Raibert 1978, Miura and Shimoyama 1980).

The main problem with tabular control methods is the size of the tables they require—table size grows exponentially with the number of state variables and control inputs needed to characterize the dynamic system (Raibert 1977). Researchers have attacked this problem in a number of ways. Albus used a hashing function that mapped tables of astronomical size into the available memory of his computer in order to control a robot manipulator (Albus 1975a, 1975b). His hashing functions were designed to use knowledge of the manipulator's dynamics in order to minimize hashing collisions. Hashing worked in that case because the controller, rather than having the potential of producing all possible motions, dealt only with the subset of manipulator motions that had been learned. Horn and Raibert reduced the size of the tables needed to control a manipulator by striking a balance between computation and tabularization (Raibert and Horn 1978). They found that for most manipulators with n joints, an $(n-1)$-dimensional *configuration space* table would be sufficient. Simons et al. (1982) reduced the size of the tables they use for control of a manipulator by finding an optimum quantization of the state inputs.

In this chapter we describe a tabular controller that maintains balance and regulates forward running speed in a locomotion system that hops on one leg. The task of finding a useful table of moderate size is accomplished, not by manipulating the form of the table, but by partitioning the problem into parts that can be solved separately. Once the problem is partitioned,

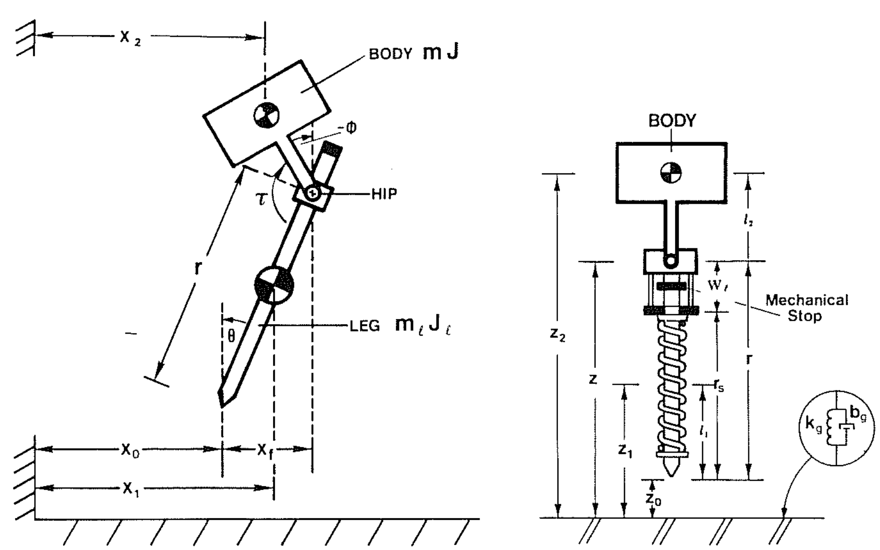


Figure 7.1. Planar one-legged model used to study tabular control. The body has mass m and moment of inertia J, and and the leg has mass m_ℓ and moment of inertia J_ℓ. They are connected by a hinge-type hip, about which an actuator generates a torque τ. The leg consists of a spring of stiffness k_ℓ and rest length r_{s0}, in series with a position actuator of length w_ℓ. The center of mass of the leg is located a distance l_1 from the foot. The body is rigid with the center of mass located l_2 above the hip. The ground is modeled as a damped spring, with parameters k_g, b_g. Rhythmic activation of the leg actuator causes the system to leave the ground periodically in a hopping motion. Motion of the entire system is restricted to the plane. The equations of motion are given in appendix 6A, and the values for simulation parameters are in table 6.1.

the table deals with only a subset of the state variables. The method uses the repetitive cyclic nature of locomotion to find a simple partitioning. We also show that multivariate polynomials of low degree can effectively approximate the tabular data. Such polynomials require substantially less data than a table, but somewhat more run-time computation. Data are presented that show the results of using both methods to control the simulated one-legged hopping machine.

The Problem

During hopping in place, placement of the foot on each step determines how a legged system will balance and it influences the system's forward velocity. Consider the planar one-legged model described in chapter 6 and illustrated in figure 7.1. It has a body, a springy leg, a hip driven by an actuator that provides torque τ, and a small foot. More details of the model are given in chapter 6, along with the equations of motion. The model tips and balances like an inverted pendulum. If the foot is placed to the left, the system tips and accelerates to the right. If the foot is placed to the right, the system tips and accelerates to the left. If the foot is placed directly under the body, the system neither tips nor accelerates. A corresponding set of rules applies when the system is traveling with a forward velocity. For each forward velocity there is a forward position for the foot that neither tips the system nor changes the rate of forward travel.

The effects of foot placement are important because the position of the foot with respect to the body can be directly controlled by torquing the hip during flight, because the foot cannot be moved once placed, and because the foot's position strongly affects balance. For the present problem we think of the foot's position when the system first touches the ground not as a state variable but as a control input.

Once a cycle of stepping activity has been established, the problem of controlling balance and forward velocity is one of choosing a place to put the foot on each cycle that will take the system to the desired state. We are primarily concerned with the forward velocity and the attitude of the body. More specifically, the control task is to find a position for the foot before *touchdown*, the moment there is contact between the foot and the ground, so as to minimize state errors at *lift-off*, the moment the foot next leaves the ground. The state errors of interest are those in forward velocity $\dot{x}$, body angle ϕ, and body angular rate $\dot{\phi}$. Assume that during flight the leg angle is adjusted by a linear servo of the form

$$\tau = -k_p(\theta - \theta_d) - k_v(\dot{\theta}), \tag{7.1}$$

where k_p, $k_v$are feedback gains.

Further assume that during stance the angle between the leg and the body, $(\theta - \phi)$, is held constant by a linear servo similar to (7.1). The problem is to find θ_{td} that minimizes a performance index PI, given the state at touchdown, $\mathbf{x}_{td}$:

$$PI = Q_1(\dot{x}_{lo} - \dot{x}_d)^2 + Q_2(\phi_{lo} - \phi_d)^2 + Q_3(\dot{\phi}_{lo} - \dot{\phi}_d)^2, \tag{7.2}$$

where Q_1, Q_2, Q_3 are weights.

A Tabular Method for Control

In order to minimize (7.2), a relationship Γ is needed that relates the state of the system at lift-off to the state at touchdown:

$$\mathbf{x}_{lo} = \Gamma(\mathbf{x}_{td}), \tag{7.3}$$

where

$$\mathbf{x}_{lo} = [\dot{x}_{lo}\ \phi_{lo}\ \dot{\phi}_{lo}]$$
$$\mathbf{x}_{td} = [x\ \dot{x}\ z\ \dot{z}\ \phi\ \dot{\phi}\ w\ \dot{w}\ \theta\ \dot{\theta}]$$

In general, behavior of the system during stance, the period between touchdown and lift-off, is influenced by the entire state vector at touchdown. The regular nature of the stepping cycle permits us to partition the state variables into two groups, those that vary from one stepping cycle to the next and those that do not. We assume that the values of z, $\dot{z}$, w, and $\dot{w}$ vary along the same trajectory from one cycle to the next. The values of these variables are important to the relationship expressed in (7.3), but their effect is nearly constant from hop to hop. Therefore these variables need not appear as independent variables in (7.3), which can be expressed as a function of a subset of the state variables

$$\mathbf{x}_{lo} = \Gamma(\mathbf{x}'_{td}), \tag{7.4}$$

where $\mathbf{x}'_{td} = [\dot{x}_{td}\ \phi_{td}\ \dot{\phi}_{td}|\theta_{td}]$.

We call the vector $\mathbf{x}_{td} = [\dot{x}_{td}\ \phi_{td}\ \dot{\phi}_{td}]$ the touchdown state vector, $\mathbf{x}'_{td} = [\dot{x}_{td}\ \phi_{td}\ \dot{\phi}_{td}|\theta_{td}]$ the augmented state vector, and $\mathbf{x}_{lo} = [\dot{x}_{lo}\ \phi_{lo}\ \dot{\phi}_{lo}]$ the lift-off state vector. We define a vector field Λ such that there is a dimension of Λ that corresponds to each component of $\mathbf{x}'_{td}$, and for each point in Λ there is a unique value of $\mathbf{x}_{lo}$.

For this problem we think of θ as a control input because it can be changed during flight at will and the remaining components of $\mathbf{x}_{td}$ as state. In general there are i control inputs and n state variables.

The vector field Λ is approximated by a multidimensional table. One dimension of the table corresponds to each dimension of Λ, and all dimensions are quantized to M levels. For n variables and i control inputs, each quantized to M values, there are M^{n+i} hyperregions in the table, each storing an n-vector. M must be chosen to quantize the table finely enough to capture the variations in $\mathbf{x}_{lo}$.

We have used such a table to control the model in simulation. Forward velocity $\dot{x}$, body angle ϕ, and body angular rate $\dot{\phi}$ are the state variables used to address the table, so $n = 3$. Leg angle θ is a control input, $i = 1$. These variables index a four-dimensional space. Each dimension of the

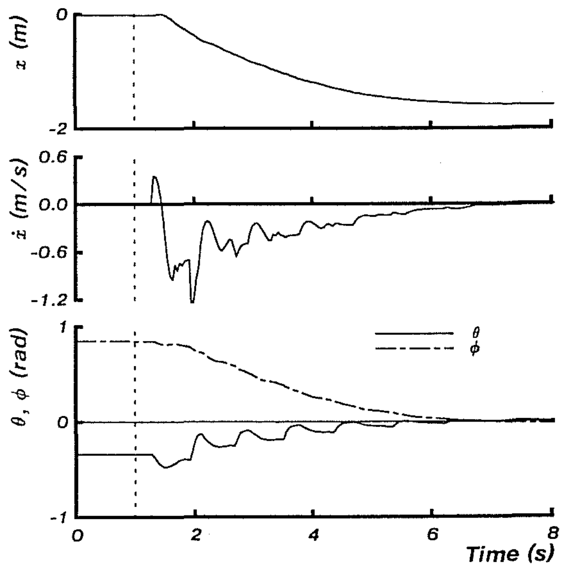


Figure 7.2. Orientation of the body was corrected using tabular method. At $t = 0$ initial error in body attitude was 0.8 rad. At $t = 1$ the system was dropped from 0.3 m and hopping began (dotted line). State errors approached zero about 6 s later. Horizontal position was not controlled, so position changed without correction. $Q_1 = 1.5$, $Q_2 = 5.0$, $Q_3 = 1.0$.

table is quantized to nine levels, $M = 9$, requiring that $nM^{i+n} = 19,683$ values be stored. To compensate for the coarseness of the quantization, the function that accesses the table, $\mathbf{T}(\mathbf{x}'_{td})$, performs a linear interpolation among the 2^{n+1} stored values that bound the desired value.

Tabular data were obtained by simulating a large set of locomotion cycles with systematically varied initial conditions. For these simulations, the angle between leg and body was held constant during stance by the servo in (7.1), just as it would be when controlled. In order to minimize (7.2), the table was searched along a path determined by varying θ through its entire range, with $\mathbf{x} = \mathbf{x}_{td}$. The details of the search are given in appendix 7A. The leg was moved to the minimizing value of θ before touchdown.

This tabular controller was tested in simulation for a simple balance problem. The task was to return the one-legged system to a balanced posture after starting with the body in an inclined position. Figure 7.2 plots the body angle and horizontal position of the system for the test in which the system was dropped from a height of 0.3 m with an initial body angle error of 0.8 rad. Setpoints were $\dot{x}_d = 0$, $\phi_d = 0$, $\dot{\phi}_d = 0$. A vertical

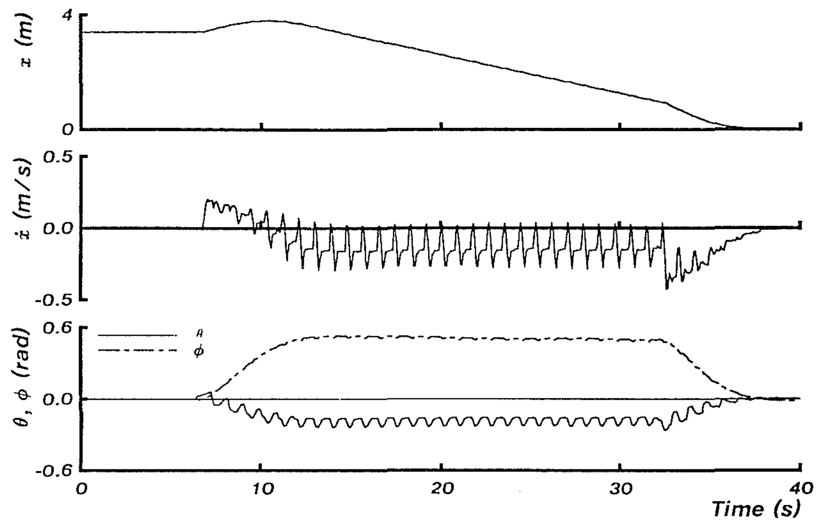


Figure 7.3. Lateral step controlled by tabular method. $Q_1 = 1.0$, $Q_2 = 5.0$, $Q_3 = 1.0$.

posture with no horizontal motion was attained in about 6 seconds. In this test no attempt was made to control horizontal position x.

The same algorithm was used to control forward velocity while the system traveled from one point to another. Figure 7.3 shows data from the resulting translation in which position was controlled indirectly through rate control. Forward velocity was rather low, but precisely controlled with no limit cycles like those caused by a linear controller used in previous experiments (Raibert 1981). The low rate of forward travel was an artifact of the restricted motion of the hip during stance, as required by the simple foot placement algorithm. It is not an inherent attribute of the tabular control method.

In the example given here, the control input θ was not explicitly varied during the interval between touchdown and lift-off. Hip angle was fixed during stance. In general, it is not necessary that the control inputs be constant, only that they do not vary with more degrees of freedom than are represented in the table. This means that rich variations in the control signals are acceptable, provided that all variations are completely determined by the augmented state vector that is available when (7.3) is used.

The table just described is used to evaluate (7.3). Control is actually accomplished by finding the value of θ_{td} that minimizes (7.2) for given values of $\mathbf{x}_{td}$ and $\mathbf{x}_d$. Because only θ varies during this minimization, with

$\mathbf{x}_{td}$ and $\mathbf{x}_d$ fixed, only a one-dimensional search through the tabulated data is required to find the minimizing value. A closed form minimization procedure is used within each quantized region of the tabular data, as described in appendix 7A. This procedure requires about $(n(M+1)-1)2^n + (M-1)(7n+1)$ multiplies for one control input, or about 408 multiplies per hop for $n=3$, $M=9$.

In certain circumstances it is possible to create a table that need not be searched at run time to satisfy (7.2). This can be done when specific values for $\dot{x}_d$, ϕ_d, $\dot{\phi}_d$, Q_1, Q_2, and Q_3 are known at the time the table is created. For this special case control can proceed without a run-time minimization. We did not experiment with this idea.

Polynomial Approximations to Tabular Data

The data of the last section show that the tabular method can effectively control a nonlinear dynamic system with few state variables and control inputs. Even when the problem is partitioned, however, the memory requirements for this approach become severe in larger applications. In this section we show that the tabular data can be approximated by polynomials in the state variables. The polynomials can have many fewer coefficients than entries in the original table, at the expense of additional run-time computation.

For a system with n state variables to be controlled and i control inputs, n polynomials are constructed, each a function of $N = n + i$ state variables and control inputs. Each of the n polynomials minimizes the total square error for the variable it approximates across all data points in the table.

Let A be a matrix in which each row contains values of the N state variables and control inputs; let B be the matrix that contains future values (that is, at lift-off) of the state variables in corresponding rows. The matrices A and B then form a data structure for the table. Given a sequence of distinct terms of the form

$$[x_1^{\alpha_{11}} x_2^{\alpha_{12}} x_3^{\alpha_{13}} x_4^{\alpha_{14}},\ x_1^{\alpha_{21}} x_2^{\alpha_{22}} x_3^{\alpha_{23}} x_4^{\alpha_{24}},\ \ldots,\ x_1^{\alpha_{M1}} x_2^{\alpha_{M2}} x_3^{\alpha_{M3}} x_4^{\alpha_{M4}}] \qquad (7.5)$$

determine a row of the M-column matrix C by evaluating these terms at the values defined by the same row of A. Then

$$C^T C X = C^T B \qquad (7.6)$$

is a linear system whose solution X contains, in each column, the coefficients of a least squares polynomial that estimates the values in the corresponding column of B (Stewart 1973). The polynomial is determined by the choice of the exponents in the above sequence, many of which are set to zero.

Using such polynomials, (7.2) can be minimized in closed form. The details of the procedure are given in appendix 7B. For the case of the one-legged machine, the components of $\mathbf{x}'_{td}$, namely $\dot{x}_{td}$, ϕ_{td}, $\dot{\phi}_{td}$, and θ_{td}, are the independent variables of the polynomials, and the components of $\mathbf{x}_{lo}$, that is $\dot{x}_{lo}$, ϕ_{lo}, and $\dot{\phi}_{lo}$, are the variables to be approximated.

Several polynomials have been tested using the same simulation procedure described earlier in figure 7.2, as well as other similar tests. In each case, the resulting behavior was compared to that obtained with the tabular data. One measure of comparison is the area between the ϕ trajectory produced by the table and the trajectories produced by each of the polynomials. The trajectories used are shown in figure 7.4, and the areas are given in the following list:

- The *24-term polynomial* consists of all odd terms of degree 3 or less. The body angle did not reach the setpoint after 20 seconds. The area between ϕ trajectories for the table and for the 24-term polynomial is 3.24 rad · s.
- The *40-term polynomial* consists of all terms of degree 1 and 3 with 16 terms of degree 5. Behavior was similar to the table but with slightly less rapid convergence. The area between ϕ trajectories for the table and for the 40-term polynomial is 1.12 rad · s.
- The *68-term polynomial* consists of all terms of degree 1, 3, and 5. Convergence is slightly faster than for the 40-term polynomial. The area between ϕ trajectories for the table and for the 68-term polynomial is 1.05 rad · s.
- The *625-term polynomial* consists of all terms such that the exponents of each of the independent variables is less than or equal to 4. Behavior is similar to the original tabulated data. The area between ϕ trajectories for the table and for the 625-term polynomial is 0.23 rad · s.

The approximation error for each polynomial is listed in table 7.1. The area measure given earlier allows a more realistic assessment of the polynomial than the global fit measure, because it takes into account which parts of the table are used for control and which are not.

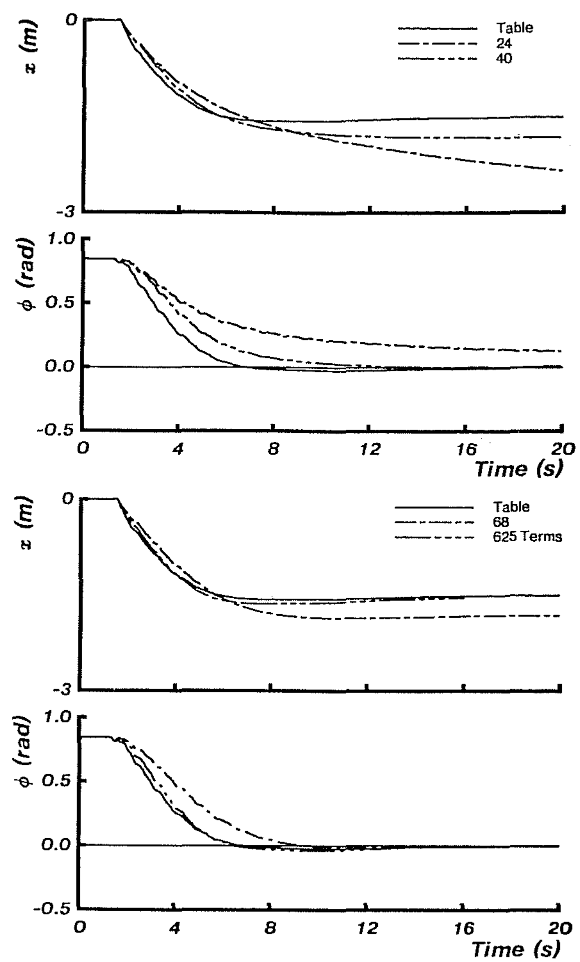


Figure 7.4. Polynomial approximations are compared to tabular data in test that corrects orientation of body, using same procedure as in figure 7.2. The responses for the tabular data and for four different polynomial approximations are plotted. See text for description of how terms of polynomial were chosen.

To use the table we must search it at run-time to find the entry that minimizes (7.2). Specifically this is a search in which $\dot{x}$, ϕ and $\dot{\phi}$ are fixed and θ can vary. The table is quantized with respect to θ at nine equally spaced points in the interval $-1 < \theta < 1$. In each subinterval we derive a linear interpolation formula for each element of $\mathbf{x}_{lo}$. Because θ is the

Table 7.1. Error for each polynomial for each of three state variables.

	Mean Square Error		
No. of Terms	$\dot{x}$	ϕ	$\dot{\phi}$
24	16.6	0.602	4.53
40	13.6	0.527	4.39
68	10.5	0.359	4.25
625	9.55	0.308	3.94

only free variable, we substitute for $\dot{x}$, ϕ, and $\dot{\phi}$ in (7.2), differentiate with respect to θ, set the result equal to zero, and solve for θ. Eight candidates are obtained. We select the one that minimizes globally. The details of this procedure are given in appendix 7A.

A similar approach is used in applying the polynomial. Because $\dot{x}$, ϕ, and $\dot{\phi}$ are fixed during the search for θ, multivariate polynomials in four variables become simpler polynomials in one variable. For instance, consider the 68-term polynomials discussed above. The highest power of θ occurring in each of the three polynomials is θ^5. When computing a control signal, we evaluate six coefficients for each of the three polynomials; these are determined by the given values of $\dot{x}$, ϕ, $\dot{\phi}$ as well as by the 68 original coefficients. The six-term polynomials are algebraically substituted into (7.2), and the resulting polynomial of degree 10 is differentiated with respect to θ. The result is a single polynomial of degree 9 in θ. Its zeros are found by using Laguerre's method (Dahlquist et al. 1974). The performance index PI is explicitly evaluated for each real zero in the interval $-1 < \theta < 1$, and the smallest of these values determines the globally optimum θ. The details of this procedure are given in appendix 7B.

This procedure for minimizing (7.2) does not require finding the zeros of a large polynomial in four variables. The 24 term polynomials require finding zeros of a single 5th degree polynomial in θ, while the 40-, 68-, and 625-term polynomials require finding roots of 9th, 9th, and 7th degree polynomials in θ, respectively.

The computational cost of using the approximating polynomials cannot be determined precisely for the general case, because the cost depends on which terms are included in the polynomials and because an iterative method is used to find roots. In order to get a rough idea of the computational cost of approximating tabular data with polynomials, we measured the number of multiplies required to use each of the four polynomials men-

Table 7.2. Comparison of cost for each polynomial and for table.

Number of terms	Multiplies to find polynomial in ϕ	Multiplies to find roots	Total multiplies
24	196	360	556
40	343	1035	1378
68	460	1035	1495
625	2822	875	3697
Table			408

tioned earlier. The cost is broken into two parts: the number of multiplies needed to apply the methods of appendix 7B in order to convert the original multivariate polynomials into polynomials in one variable, θ, and the number of multiplies needed to find the roots of this polynomial using Laguerre's method. These rough operation counts, shown in table 7.2, suggest that polynomials with a few dozen terms can be used with only three or four times the run-time computing cost of the table. The 625-term polynomial requires almost 10 times the run-time computing that the table requires.

Conclusions

Tables can be useful for control of nonlinear dynamic systems when simple inverse descriptions are not available, or when evaluation of such descriptions requires unacceptably large amounts of run-time computation. The limitation of using tabulated data for control is in the size of the tables needed. In this chapter we approached this problem in two ways.

- First we took advantage of the particular characteristics of legged locomotion in order to partition the state variables of the system into two sets: those that varied in a predictable, stereotyped manner and those that varied freely. Only the freely varying subset contributed to the size of the table.
- Second, multivariate polynomials were used to approximate the tabular data. This resulted in a substantial reduction of memory requirements at the expense of additional run-time computing requirements. Two of the polynomials we examined, those with 40 and 68 terms, gave good performance with low storage and run-time computing cost.

One way to interpret the use of tabular data in the present method is that they are used to make predictions about the future state of the system based on the present state. Specifically for the locomotion problem, the tabular data provide a prediction of state at lift-off based on an augmented state vector at touchdown. Such a prediction provides the same sort of information as would a forward integration of the system's equations of motion, but much faster and with less computation.

Appendix 7A. Performance Index Minimization for Table

Given a state vector at touchdown $[\dot{x}, \theta, \dot{\theta}]$, the value of ϕ that minimizes the performance index $PI([\dot{x}_{lo}\ \theta_{lo}\ \dot{\theta}_{lo}])$ is required, where $[\dot{x}_{lo}\ \theta_{lo}\ \dot{\theta}_{lo}] = \mathbf{T}([\phi\ \dot{\mathbf{x}}\ \theta\ \dot{\theta}])$ is the state vector at next lift-off; $\mathbf{T}$ denotes the vector function that implements the table lookup with linear interpolation, and T_i is the ith component of $\mathbf{T}$. Recall that

$$PI([\dot{x}_{lo}\ \theta_{lo}\ \dot{\theta}_{lo}]) = Q_1(\dot{x}_{lo} - \dot{x}_d)^2 + Q_2(\theta_{lo} - \theta_d)^2 + Q_3(\dot{\theta}_{lo} - \dot{\theta}_d)^2, \quad (7.7)$$

where Q_1, Q_2, Q_3 are weights and $\dot{x}_{lo}$, θ_{lo}, $\dot{\theta}_{lo}$ are desired values.

Assume that $\dot{x}_A \leq \dot{x} \leq \dot{x}_B$, $\theta_A \leq \theta \leq \theta_B$, and $\dot{\theta}_A \leq \dot{\theta} \leq \dot{\theta}_B$, where the A and B subscripts indicate adjacent values stored at quantized locations in the table. Also, assume that $\phi_A \leq \phi \leq \phi_B$. Consider as an example the first term on the right-hand side of (7.7), which can be written in terms of the tabulated data:

$$Q_1(\dot{x}_{lo} - \dot{x}_d)^2 = Q_1\left(\frac{(\phi - \phi_A)\dot{x}_B - (\phi_B - \phi)\dot{x}_A}{(\phi_B - \phi_A)} - \dot{x}_d\right)^2, \quad (7.8)$$

where $\dot{x}_A = T_1[(\phi_A\ \dot{x}\ \theta\ \dot{\theta})]$ and $\dot{x}_B = T_1[(\phi_B\ \dot{x}\ \theta\ \dot{\theta})]$. In other words, once interpolation has been completed for the three state variables, two adjacent values in the table that bracket ϕ, ϕ_A and ϕ_B, are substituted into the linear interpolation formula. Equation (7.8) represents the linear interpolation in the table for the ϕ dimension.

Now the right-hand side of (7.8) can be rewritten as

$$Q_1(\dot{x}_{lo} - \dot{x}_d)^2 = Q_1(\phi Q_A + Q_B)^2, \quad (7.9)$$

where

$$Q_A = \frac{\dot{x}_B - \dot{x}_A}{\phi_B - \phi_A} - \dot{x}_d \quad \text{and} \quad Q_B = \frac{\dot{x}_A\phi_B - \dot{x}_B\phi_A}{\phi_B - \phi_A} - \dot{x}_d.$$

An identical treatment of the other two terms of (7.7) yields

$$PI = Q_1[Q_A\phi + Q_B]^2 + Q_2[Q_C\phi + Q_D]^2 + Q_3[Q_E\phi + Q_F]^2. \quad (7.10)$$

By differentiating with respect to ϕ, setting the result equal to zero, and solving for ϕ, we have a closed expression for that value of ϕ that minimizes PI in the interval $\phi_A \leq \phi \leq \phi_B$:

$$\phi = -\frac{Q_1Q_AQ_B + Q_2Q_CQ_D + Q_3Q_EQ_F}{Q_1Q_A^2 + Q_2Q_C^2 + Q_3Q_E^2}. \quad (7.11)$$

To obtain the global minimum this computation is performed for each of the $M - 1$ subintervals determined by the quantization of ϕ.

Appendix 7B. Performance Index Minimization for Polynomial

The general form of the K-term polynomials that approximate the tabulated data is

$$\theta_{lo} = f_{1,1}\phi^{\alpha_{11}}\theta^{\alpha_{12}}\dot{\theta}^{\alpha_{13}}\dot{x}^{\alpha_{14}} + \ldots + f_{1,K}\phi^{\alpha_{K1}}\theta^{\alpha_{K2}}\dot{\theta}^{\alpha_{K3}}\dot{x}^{\alpha_{K4}},$$

$$\dot{\theta}_{lo} = f_{2,1}\phi^{\alpha_{11}}\theta^{\alpha_{12}}\dot{\theta}^{\alpha_{13}}\dot{x}^{\alpha_{14}} + \ldots + f_{2,K}\phi^{\alpha_{K1}}\theta^{\alpha_{K2}}\dot{\theta}^{\alpha_{K3}}\dot{x}^{\alpha_{K4}},$$

$$\dot{x}_{lo} = f_{3,1}\phi^{\alpha_{11}}\theta^{\alpha_{12}}\dot{\theta}^{\alpha_{13}}\dot{x}^{\alpha_{14}} + \ldots + f_{3,K}\phi^{\alpha_{K1}}\theta^{\alpha_{K2}}\dot{\theta}^{\alpha_{K3}}\dot{x}^{\alpha_{K4}}, \quad (7.12)$$

where, again, lo denotes values at next lift-off and $[\theta\ \dot{\theta}\ \dot{x}]$ is a state vector at touchdown. Because ϕ is the only free variable, (7.12) can be recast as

$$\theta_{lo} = F_{1,1}\phi^{\beta_0} + \ldots + F_{1,N}\phi^{\beta_N},$$

$$\dot{\theta}_{lo} = F_{2,1}\phi^{\beta_0} + \ldots + F_{2,N}\phi^{\beta_N},$$

$$\dot{x}_{lo} = F_{3,1}\phi^{\beta_0} + \ldots + F_{3,N}\phi^{\beta_N}, \quad (7.13)$$

where $\beta_0 \geq 0$ and β_N is the highest power of ϕ in (7.12). Substituting these equations into (7.13) yields a polynomial of degree $2\beta_N$ that expresses PI as a function of ϕ:

$$PI = G_0 + \theta_d^2 + \dot{\theta}_d^2 + \dot{x}_d^2 + G_1\phi^{2\beta_0} + \ldots + G_N\phi^{2\beta_N}. \quad (7.14)$$

The real zeros of the derivative of this polynomial are found using Laguerre's method. The global minimum is found by evaluating (7.13) for each zero and substituting the results into (7.7).

Chapter 8

Research on Animals and Vehicles

The goal of the research reported in this book is to develop a body of theoretical and experimental results that will help us to discover principles underlying legged locomotion. Such principles can give an improved understanding of legged behavior as it occurs in nature and enable the construction of useful legged vehicles.

In this chapter we shift attention from legged machines to animals and vehicles. The following section ties results from the study of legged machines to the study of animal locomotion. It suggests animal experiments that are motivated by the ideas and concepts already described. The subsequent section considers the progress required to make legged locomotion a useful technology.

Experiments in Animal Locomotion

The task of understanding natural legged locomotion requires experiments. This section lists experiments designed to probe the control mechanisms and algorithms that living systems use to walk and run. Each is based on observations or ideas that arose in the course of studying legged machines. Detailed knowledge of working locomotion algorithms, like those embodied in running machines, should help to formulate good experimental questions to ask of biological legged systems.

The study of natural phenomena should also stimulate and enhance the development of sensorimotor control principles for robotics. Biology provides us with clear evidence of what is possible and with data obtained from working systems of superior performance. Research addressing these two problems, that of building legged vehicles with better mobility and that of understanding the mobility of legged animals, can interact productively.

Animal Experiments Motivated by Robot Experiments

Algorithms for Balance. The legged machines and computer simulations we have studied all use a specific algorithm for determining the landing position of the foot with respect to the body's center of mass. The foot is advanced a distance $x_f = T_s\dot{x}/2 + k_{\dot{x}}(\dot{x} - \dot{x}_d)$. This calculation is based on the expected symmetric tipping behavior of an inverted pendulum.

Do animals use this algorithm to position their feet? To find out, one must measure small changes in the forward running speed, in the angular momentum of the body during the flight phase, and in the placement of the feet. Rather than look at average behavior across many steps, as is done in studies of energetics and neural control, we must look at error terms within each step, using simultaneous kinematic and force plate data.

Symmetry in Balance. The control systems for the running machines we built use symmetric motions to provide balance. The symmetry specifies that motion of the body in space and of the feet with respect to the body are even and odd functions of time during each support period. The same sort of symmetry is suggested by data for cat and human running. In principle, symmetry can be independent of the number of legs and of the gait. Despite this apparent generality, symmetry provides only sufficient conditions for balance, not necessary conditions. Many questions arise:

- How universal is the use of symmetric trajectories for motion of the support points in humans and animals?
- What is the precision of the symmetry?
- Does the symmetry extend to angular motion of the body as the theory predicts?
- How do asymmetries in the mechanical structure of the body and legs influence the symmetry in observed motion?
- Does springiness in the legs encourage symmetry by limiting the available degrees of freedom in axial leg thrust?

Algorithms for Foot Placement. In order to traverse irregular terrain, a legged system must choose footholds with respect to variations and obstacles, and it must position its feet on the chosen footholds. Figure 8.1

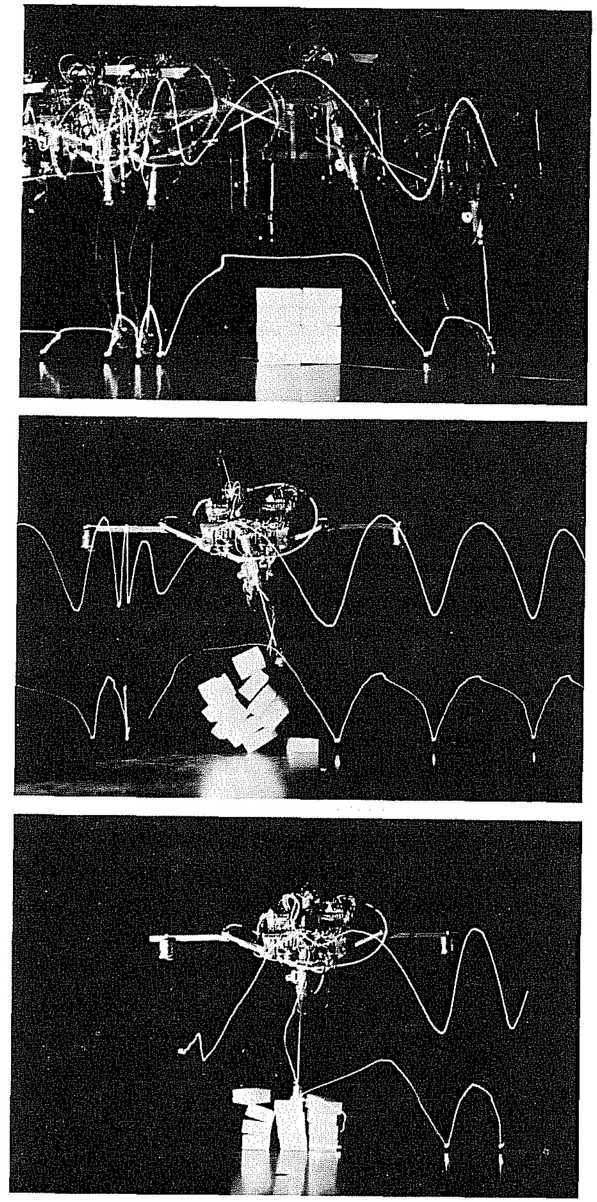


Figure 8.1. Foot placement is important for locomotion on rough terrain. To leap over an obstacle successfully, a legged system must use footholds that are properly positioned. The foot location used for the leap shown in the top photograph was good, but the foot was too close to the blocks in the middle photograph, and too far away in the bottom photograph.

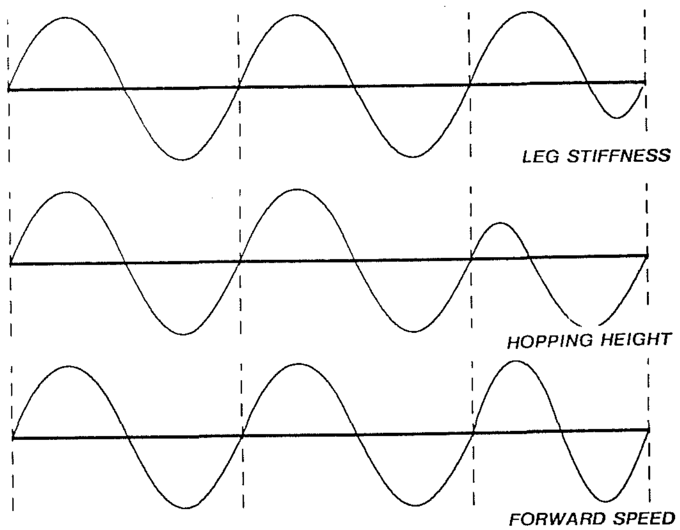


Figure 8.2. Three competing strategies for adjusting stride. The solid curves represent the path of the body during running. When it is below the horizontal line, the system is in the stance phase, and when above it is in flight. The vertical dotted lines separate strides. (Top) The control system manipulates leg stiffness to vary the duration of stance and, therefore, the distance traveled during stance. (Middle) The control system adjusts hopping height to vary the duration of flight. (Bottom) The control system adjusts forward velocity. In all three cases the two strides on the left are of equal length and the stride on the right is shorter. Adapted from Hodgins (1985).

demonstrates the failure to do so. In an effort to give laboratory machines the ability to negotiate simple irregular terrain, we have identified three competing strategies for adjusting stride. Each adjusts a different parameter of the gait:

- forward speed,
- duration of stance, and
- duration of flight.

Figure 8.2 characterizes these mechanisms for controlling stride length and, indirectly, where the feet are placed. Which of these mechanisms, if any, is at work in natural systems? To find out, one could look at animals jumping over obstacles or at humans running in a task where the foot must be placed on a particular spot. Data from Lee, Lishman, and Thomson (1982) and from Warren, Lee, and Young (1985) suggest that humans alter flight time to make these adjustments.

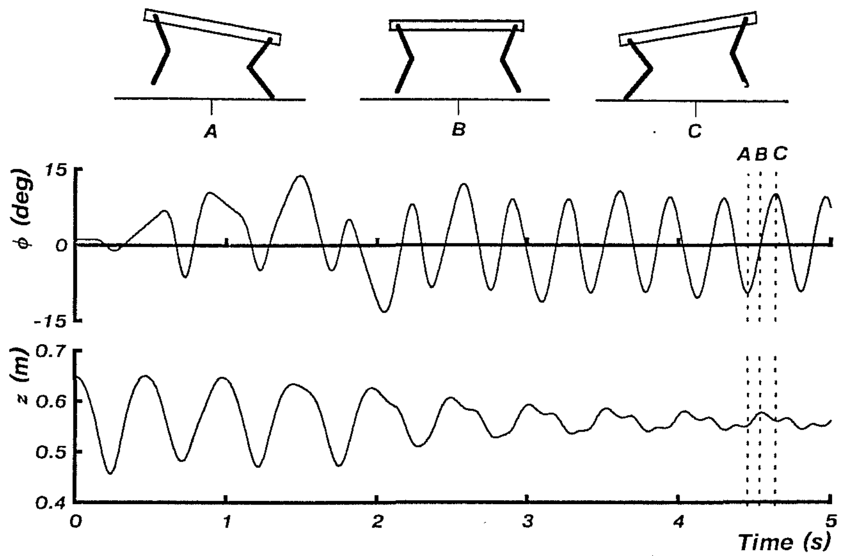


Figure 8.3. Planar two-legged model bounding with passively stabilized body attitude. Initially the rocking pattern is random, but it soon stabilizes. The three configurations shown in the cartoon correspond to the data indicated by the dotted vertical lines. From Murphy and Raibert (1984).

Distribution of Body Mass. Karl Murphy discovered that the distribution of mass in the body can have a profound influence on the behavior of a running system (Murphy and Raibert 1984). He defined the dimensionless group that represents the normalized moment of inertia of the body, $j = J/(md^2)$, where J is the moment of inertia of the body, m is the mass of the body, and d is half the hip spacing. When $j = 1$, the hips are located at the centers of percussion of the body. Murphy found that when $j < 1$ the attitude of the body can be passively stabilized in a bounding gait. When $j > 1$, stabilization is not so easily obtained. (See figures 8.3 and 8.4.)

This finding has straightforward implications for the mechanical structure and behavior of animals. Could it be that bounding animals do not have to worry about actively controlling pitching motion of their bodies but only about controlling forward running speed and direction? A first step toward answering this question is to measure values of j for a variety of quadrupedal animals and to relate the measurements to the animals' preferred modes of running. For instance, does the value of j for a quadruped vary with its trot to gallop transition speed? Anatomical measurements

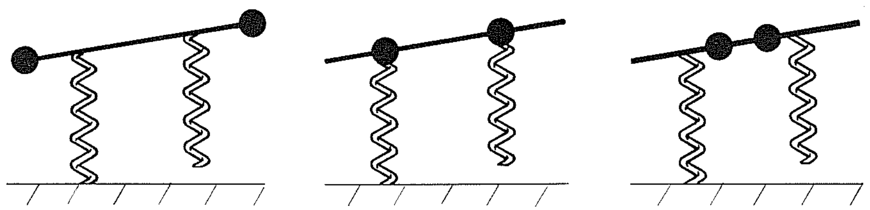


Figure 8.4. The dimensionless moment of inertia, $j = J/(md^2)$, predicts passive stability for the body's pitching motion. (Left) When $j < 1$, an upward force on the left leg causes the right hip to accelerate downward. The model has passive pitch stability. (Center) When $j = 1$, the system acts as two separate oscillators, with neutral stability. (Right) When $j > 1$, a vertical force on the left foot causes the right hip to accelerate upward. The model does not have passive pitch stability.

like those of Fedak, Heglund, Taylor (1982) will provide much of the data needed to answer this question.

Yaw Control. How do human runners keep themselves from rotating about the yaw axis? Control of this degree of freedom in the one-legged hopping machine is difficult because it does not have a foot that can exert a torsional torque on the ground. But humans have long feet that can, in principle, develop substantial torsional traction on the ground about the yaw axis. A first step in exploring this question would be to measure the torsional torque humans exert on the ground during running and to relate the measurements to yaw motions and disturbances of the body.

Conserving Angular Momentum. Newtonian mechanics dictates that a legged system must conserve its angular momentum during the flight phase. How nearly do legged systems conserve angular momentum during the stance phase? It is not required for momentum to be conserved during stance because the legs can exert forces on the ground to accelerate the system. However, a control system that keeps the angular momentum constant during stance could achieve higher efficiency and better performance.

Qualitatively, the legs move as they should to maintain constant angular momentum during support. Angular momentum for one leg is the product of its angular velocity times its moment of inertia. We observe that the swing leg rotates faster than the support leg, but the support leg has a larger moment of inertia than the swing leg. Quantitative measurements made from kinematic data would reveal how precisely angular momentum is preserved during stance.

Virtual Legs. We have built a four-legged machine that runs with a trotting gait. It uses the virtual leg idea to simplify control, as described in chapter 4. By using virtual legs we divide the problem of generating quadruped running gaits that use legs in unison into two simpler problems. One problem is to provide locomotion algorithms that operate correctly to control the virtual legs. The other problem is to control each physical leg so that the virtual legs behave as desired.

Do animals couple the behavior of their legs as the virtual leg model would suggest? The experiment needed to answer this question would make measurements of ground force during trotting and pacing, simultaneously in both supporting legs of a quadruped. An important test would be to disturb one or both of the feet during support and to measure the force response of the legs.

Philosophy

An attractive approach to research in locomotion and manipulation is to combine work on the study of animals and the construction of machines. Biological systems provide both great motivation by virtue of their striking performance, and guidance with the details of their actions. They are existence proofs that give us an indication of what is possible. Unfortunately, biological systems are more complicated to study than we would like—there are many variables, precise measurement is difficult, there are limitations on the experimenter's ability to manipulate the preparation, and perhaps, an inherent difficulty in focusing on the information-level of a problem.

On the other hand, simple laboratory robots are easy to build. Precisely controlled experiments are possible, as are careful measurements and manipulations, and the "subject" can be redesigned when necessary. However, the behavior of these experimental robot systems is impoverished when compared with the biological counterpart. They are easy to study, but they do not perform as well as biological systems.

Analysis of living systems and synthesis of laboratory systems are, therefore, complementary activities, each with its own strengths and weaknesses. Together, these activities can strengthen one another, leading to fundamental principles that elucidate the domain of both problems, independent of the particular implementation details. Because machines face the same physical laws and environmental constraints as those faced by biological systems performing similar tasks, the solutions they use may embrace similar principles. In solving the problem for the machine, we generate a set of plausible algorithms for the biological system. In observing the biological behavior, we explore plausible behaviors for the machine.

In its grandest form, this approach lets the study of robotics contribute to both robotics and biology and lets the study of biology contribute to both biology and robotics.

The Development of Useful Legged Robots

The running machines described in this book should be thought of as experimental laboratory apparatuses that are used to explore ideas about legged locomotion. They are not useful vehicles or even prototypes for such vehicles. Each was designed to look at a particular aspect of legged locomotion, often avoiding important practical considerations that must be solved before useful working vehicles are realized.

There are a variety of difficult problems that must be solved before useful legged vehicles can be built. Some of these problems require scientific progress in understanding fundamental principles in legged locomotion; others require substantial engineering effort. This section lists the key technical developments needed for legged machines to become useful.

The numbers in brackets give a coarse assessment of the difficulty of making substantial progress in each area (1 being relatively easy and 5 being relatively hard), taking into account the current state of advancement. These ratings assume that the goal is an autonomous legged system in the sense that it can receive information from humans only intermittently with a time resolution of several minutes.

Walk and Run on Flat Floor [1]

Solutions to this problem are essentially in hand, provided that the system uses externally supplied specifications of desired direction and rate of travel.

Travel on Rough Terrain [3]

Even with complete knowledge of the terrain, there are substantial control and planning problems that must be solved before legged systems can negotiate difficult terrain. There are several ways that terrain becomes rough and therefore difficult to negotiate:

- not level
- limited traction (slippery)
- areas of poor or nonexistent support (holes)
- vertical variations:
    - minor vertical variations in available footholds (less than about one-half the of leg length)
    - major vertical variations in available footholds (footholds separated vertically by distances comparable to the dimensions of the legged system)
- large obstacles between footholds (poles)
- intricate footholds (e.g., rungs of a ladder).

Solutions to this sort of terrain problem involve the mechanics of locomotion, control, planning, and heavy doses of geometric representation and reasoning. Although medium-grain knowledge of the terrain is important, so are techniques that make a legged system inherently insensitive to minor variations. Substantial mechanical design of new mechanisms will be involved in building legged systems that can negotiate rough terrain.

Speed and Position Measurement [2]

The problem of providing a mobile system with information about where it is, the direction of its motion, and the speed of its motion are important. Although this problem is common to a variety of mobile technologies, the bandwidth needed for legged systems, as we currently envision them, are particularly severe. The satellite-based systems I know about do not provide the kind of fine-grain information needed. Specialized techniques may prove acceptable in specialized environments.

Terrain Sensing and Perception [4]

The single most important barrier to achieving useful legged systems that do not have direct human control is the task of determining the detailed shape of the terrain along the paths of interest. This problem involves sensing, perception, and spatial representation. The difficulty of this problem is related to the generality of the situations in which the system will be expected to work. The speed of locomotion may also be a factor.

Self-Contained Power [3]

Real legged systems will need to provide their own power. Although it does not appear to be too hard to carry a power supply at large scale, say gross vehicle weights in excess of a ton, it is a difficult engineering problem at smaller scale, say human size.

Optimize Payload, Range, and Speed [1]

If a locomotion system is to be a transportation system, it must carry a payload, and the distance that it can carry the payload without intervention is of considerable importance. The speed of transport is also important. Part of the payload may be needed for locomotion, such as computing equipment, communication equipment, and fuel. It usually will be desirable to transport something that is not a part of the locomotion system itself, such as a pair of manipulators or a sensory system.

The trade-offs among payload capacity, range, and speed are not new to transportation, but an evaluation of the implications for legged vehicles will require additional study. Consideration of these factors is an optimization problem for the system architect and the mechanical designer. Special control strategies may also be important.

Running Is Like Juggling

In an informal lecture given at Carnegie-Mellon University in July 1983, Claude Shannon presented some thoughts on juggling:

Definition: The Common Juggle. The time an object spends in contact with a hand is the same for all objects and all hands. The flight times are equal for all objects thrown from all hands.

For a Common Juggle, the following equation is satisfied:

$$\frac{N}{H} = \frac{D+F}{D+V},$$

where

N is the number of objects,
H is the number of hands,
D is the dwell time, the time each object spends in contact with each hand,
F is the flight time, the time each object spends in the air, and
V is the vacant time the time a hand contains no object.

In aid of building machines that juggle, Shannon has formulated a theory of juggling that relies on transformations of planar ellipses to represent the motions of each hand. The phase, location, plane, and aspect ratio of the ellipses determine the type and shape of juggle. These ideas about

juggling make me think that juggling and dynamic legged locomotion have much in common:

1. Both are cyclic, repetitive activities, for which the dynamics of the systems determine the rhythms.
2. The terms *dwell time, vacant time,* and *flight time* for juggling could correspond to *stance time*, *swing time*, and *flight time* for legged locomotion. Note that for a one-legged machine swing time equals flight time and for a one-ball/one-hand juggle, vacant time equals flight time.
3. In both juggling and running there are intermittent periods of support. In locomotion each foot touches the ground for a fraction of the leg's step cycle. In juggling, each juggled object touches a hand for only a fraction of the object's trip cycle.
4. In both there are bodies that move in a ballistic motion part of the time. Because it is not possible to change the trajectory of a body's motion during the ballistic phase, precise control just before launch is important.
5. Shannon has a description of juggling that uses transformations of ellipses that are connected by parabolic arcs. We have a theory of balance in locomotion that decomposes the problem into a planar part and an extraplanar part. Both deal with three-dimensional phenomena in terms of planar ideas.

Consider a game of volleyball and its relationship to locomotion. Imagine two people warming up for a game. The two players stand across the net from each other and hit the ball back and forth. Because they are just warming up, they agree to hit the ball to the each other, rather than to a location where it is hard to get. Each time the ball is in flight, it travels a planar, parabolic path. Each time the ball is handled, its direction is reversed so that it will land in the hands of the other player.

This arrangement of players and ball is similar to the arrangement of legs and body found in a locomotion system with two legs. The rocking motion of the body that characterizes biped walking and running are modeled here by the back and forth motion of the ball. Player contact with the ball is like leg contact with the ground. The primary difference between the two cases is that in volleyball the legs do not travel with the ball but remain fixed to the ground.

This model also works for a quadruped running with a one-foot gait. Suppose four players stand at the corners of a rectangle and hit the ball to each other. Each player hits the ball in a fixed sequence. Each time the ball is in flight, it travels a planar, parabolic path. Each time the ball is handled, the ball is redirected so that it will land in the hands of the next player. The point is that in both the volleyball warmup and quadruped locomotion, the mass of the system must travel back and forth over the support points in order to sustain the activity.

What I have described so far corresponds to running in place, but it is not hard to image the volleyball players all progressing in the same direction at the same rate while they hit the ball back and forth. In this case the forward motion is superimposed on the up-and-down bouncing motion and on the rocking motion that permits spacing of the players.

Do Locomotion and Manipulation Have a Common Ground?

What does the study of legged locomotion have to offer those who study robot manipulation? What can the study of locomotion gain from progress in robot manipulation?

On the one hand, locomotion is a much harder problem than manipulator control. In particular, the most important state variables in a locomotion system cannot be measured directly using simple means. Of course, the internal variables, such as the length or orientation of a leg with respect to the body, can be measured easily, but the orientation of a locomotion system in space and its position in the room can be determined only by using indirect methods. In contrast, there is usually not much trouble determining where the hand of a manipulator is in space or what its orientation is. One merely starts at the fixed base and proceeds from link to link, using joint angle sensors and kinematic transformations. Furthermore, although the manipulator control system typically uses a separate motor to govern directly the action of each manipulator joint, the translation and orientation of a locomotion system, at least a dynamic one, can be controlled only indirectly by making the system bounce, tip, and fall in the desired direction.

On the other hand, manipulator control is a much harder problem than locomotion. The work on locomotion reported in this book relies on a simple, restricted set of leg and body motions. The hopping motion is just the oscillation that occurs when springy legs and a body with mass are excited. During flight the leg is positioned with regard to its final end-point, without concern for the trajectory along which the leg moves. Despite these limitations, the behavior captures important features of legged locomotion.

In contrast, a manipulator control technique that applies to only a restricted subclass of possible motions or one that works correctly in just a portion of a manipulator's working volume would be received with nothing short of hostility. Techniques for kinematics, dynamics, trajectory control, and the use of sensors are expected to be general solutions that work everywhere in the work space for the full range of possible joint motions.

One can reconcile these differences between locomotion and manipulation if one recognizes that locomotion is a task and that manipulator control is a tool. Locomotion is the task of transporting a system and its contents from one point to another. Typically, the detailed leg motions required to accomplish the locomotion task are only of indirect interest, perhaps as they affect the time or efficiency of transport. The motions of the legs are used to accomplish the locomotion goal. Likewise, the task of manipulation is to cause parts to be stacked, placed, inserted, assembled, painted, etc. We should not care what motions the manipulator makes, provided that they accomplish the task.

When one thinks of manipulation in this way, in terms of the task the manipulator and its motions are to accomplish, then the problems of indirect sensing and control are precisely the same for manipulation as they are for locomotion. The manipulation system has the same problem determining the relative positioning between hand and workpiece as the locomotion system has in figuring out where the payload is with respect to the destination. The manipulation system has the same problem controlling the motions of the parts as the locomotion system has in controlling motions of the body.

In the same vein, once one thinks of manipulation in terms of the task to be accomplished, one is free to use specialized control methods. Such methods produce stereotyped motions that are only subsets of all possible motions the manipulator can make. This sort of specialization and stereotyped motion has helped to simplify the study of legged locomotion. Perhaps a comparable approach can be used successfully in manipulation too.

Bibliography

Agarwal, G. C., Berman, B. M., Stark, L., Lohnberg, P., Gottleib, G. L. 1970. Studies in postural control systems: Parts I, II, and III. *IEEE Trans. Systems Science and Cybernetics* 6:116–132.

Albus, J. S. 1975a. A new approach to manipulator control: the Cerebullar Model Articulation Controller (CMAC). *J. Dynamic Systems, Measurement, and Control* 97:220–227.

Albus, J. S. 1975b. Data storage in the Cerebellar Model Articulation Controller (CMAC). *J. Dynamic Systems, Measurement, and Control* 97:228–233.

Aleshinsky, Zatsiorsky 1978. Human locomotion in space analyzed biomechanically through a multi-link chain model. *J. Biomechanics* 11:101–108.

Alexander, R. McN. 1974. The mechanics of jumping by a dog. *J. Zoology (London)* 173:549–573.

Alexander, R. McN. 1976. Mechanics of bipedal locomotion. In *Perspectives in Experimental Biology*, P. Spencer-Davies (ed.). Oxford: Pergamon Press, 493–504.

Alexander, R. McN. 1984. The gaits of bipedal and quadrupedal animals. *International J. Robotics Research* 3:49–59.

Alexander, R. McN., Goldspink, G. 1977. *Mechanics and Energetics of Animal Locomotion*, London: Chapman and Hall.

Alexander, R. McN., Jayes, A. S. 1978. Vertical movements in walking and running. *J. Zoology (London)* 185:27–40.

Alexander, R. McN., Jayes, A. S. 1980. Fourier analysis of forces exerted in walking and running. *J. Biomechanics* 13:383–390.

Alexander, R. McN., Vernon, A. 1975. The mechanics of hopping by kangaroos (Macropodidas). *J. Zoology (London)* 177:265–303.

Alexander, R. McN., Langman, V. A., and Jayes, A. S. 1977. Fast locomotion of some African ungulates. *J. Zoology, (London)* 183:291–300.

Alexander, R. McN., Maloiy, G. M. O., Ker, R. F., Jayes, A. S., Warui, C. N. 1982. The role of tendon elasticity in the locomotion of the camel (Camelus romedarius). *J. Zoology (London)* 198:293–313.

Anderson, F. 1967. Registration of the pressure power (the force) of the body on the floor during movements, especially vertical jumps. *Biomechanics* 1:87–89.

Arsavskii, Y. I., Kots, Y. M., Orlovskii, G. N., Rodionov, I. M., Shik, M. L. 1965. Biophysics of complex systems and mathematical models—investigation of the biomechanics of running by the dog. *Biofizika* 10:665–672.

Asatryan, D. G., Feldman, A. G. 1965. Functional tuning of nervous system with control of movement or maintenance of a steady posture. *Biofizika* 10:837–846.

Badler, N. I., Smoliar, S. W. 1977. *The Representation of Human Movements Using a Digital Computer*, Moore School of Science, University of Pennsylvania, Philadelphia, MS-CIS-78-4.

Bair, I. R. 1959. Amphibious Walking Vehicle. Patent Number 2,918,738.

Barclay, O. R. 1953. Some aspects of the mechanics of mammalian locomotion. *J. Experimental Biology* 30:116–120.

Bassmajian, J. V., Tuttle, R. 1973. Engineering of locomotion in gorilla and man. In *Control of Posture and Locomotion*, R. B. Stein, K. G. Pearson, R. S. Smith, J. B. Redford (eds.). New York: Plenum Press, 500–609.

Bavarian, B., Wyman, B. F, Hemami, H. 1983. Control of the constrained planar simple inverted pendulum. *Int. J. Control* 37:741–753.

Beckett, R., Chang, K. 1968. An evaluation of the kinematics of gait by minimum energy. *J. Biomechanics* 1:147–159.

Bekker, M. G. 1960. *Off-The-Road Locomotion.* Ann Arbor: University of Michigan Press.

Bekker, M. G. 1961. Is the wheel the last word in land locomotion? *New Scientist* 17:406–410.

Bekker, M. G. 1961. The evolution of locomotion: A conjecture into the future of vehicles. *ASME Winter Annual Meeting* WA-303.

Bekker, M. G. 1962. *Theory of Land Locomotion.* Ann Arbor: University of Michigan Press.

Bekker, M. G. 1969. *Introduction to Terrain-Vehicle Systems.* Ann Arbor: University of Michigan Press.

Beletskii, V. V. 1975a. Biped locomotion dynamics I. *Izv. AN SSSR. Mekhanika Tverdogo Tela* 10(3):3–14.

Beletskii, V. V. 1975b. Dynamics of two-legged walking, II. *Izv. AN SSSR. Mekhanika Tverdogo Tela* 10(4):3–13.

Beletskii, V. V., Chudinov, P. S. 1977. Parametric optimization in the problem of biped locomotion. *Izv. AN SSSR. Mekhanika Tverdogo Tela* 12(1):25–35.

Beletskii, V. V., Chudinov, P. S. 1980. Control of motion of a bipedal walking robot. *Izv. AN SSSR. Mekhanika Tverdoga Tela* 15(3):30–38.

Beletskii, V. V., Kirsanova, T. S. 1976. Plane linear models of biped locomotion. *Izv. AN SSSR. Mekhanika Tverdogo Tela* 11(4):51–62.

Bennet-Clark, H. C. 1975. The energetics of the jump of the locust, *Schistocerca gregaria. J. Experimental Biology* 63:53–83.

Bessonov, A. P., Umnov, N. V. 1973. The analysis of gaits in six-legged vehicles according to their static stability. In *First Symposium on Theory and Practice of Robots and Manipulators.* Amsterdam: Elsevier Scientific Publishing Co.

Beuter, A., Garfinkel, A. 1984. Phase plane analysis of limb trajectories in non-handicapped and cerebral palsied subjects. *J. Motor Behavior.*

Bramble, D. M., Carrier, D. R. 1983. Running and breathing in mammals. *Science* 219:251–256.

Buckett, J. 1977. *Design of an On-board Electronic Joint Control System for a Six-legged Vehicle.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Burns, M. D. 1973. The control of walking in orthoptera, I. Leg movements in normal walking. *J. Experimental Biology* 58:45–58.

Camana, P. C. 1977. *A Study of Physiologically Motivated Mathematical Models for Human Postural Control.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Camana, P. C., Hemami, H., Stockwell, C. W. 1977. Determination of feedback for human posture control without physical intervention. *J. Cybernetics* 7.

Cannon, R. H., Jr. 1962. Some basic response relations for reaction-wheel attitude control. *ARS J.* 2:61–74.

Cappozzo, A., Figura, F., Marchetti, M. 1976. Interplay of muscular and external forces in human ambulation. *J. Biomechanics* 9:35–43.

Cappozzo, A., Leo, T., Pedotti, A. 1975. A general computing method for the analysis of human locomotion. *J. Biomechanics* 8:307–320.

Cavagna, G. A. 1970. Elastic bounce of the body. *J. Applied Physiology* 29:279–282.

Cavagna, G. A. 1975. Force platforms as ergometers. *J. Applied Physiology* 39:174–179.

Cavagna, G. A., Kaneko, M. 1977. Mechanical work and efficiency in level walking and running. *J. Physiology (London)* 268:467–481.

Cavagna, G. A., Margaria, R. 1966. Mechanics of walking. *J. Applied Physiology* 21:271–278.

Cavagna, G. A., Heglund, N. C., Taylor, C. R. 1977. Mechanical work in terrestrial locomotion: Two basic mechanisms for minimizing energy expenditure. *American J. Physiology* 233:R243–R261.

Cavagna, G. A., Komarek, L., Mazzoleni, S. 1971. The mechanics of sprint running. *J. Physiology* 217:709–721.

Cavagna, G. A., Saibene, F. P., Margaria, R. 1964. Mechanical work in running. *J. Applied Physiology* 19:249–256.

Cavagna, G. A., Thys, H., Zamboni, A. 1976. The sources of external work in level walking and running. *J. Physiology* 262:639–657.

Cavagna, G. A., Zamboni, A., Faraggiana, T., Margaria, R. 1972. Jumping on the moon: Power output at different gravity values. *Aerospace Medicine* 43:408–414.

Cavanagh, P. R. 1979. Ground reaction forces in distance running. *J. Biomechanics* 13:397–406.

Ceranowicz, A. Z. 1979. *Planar Biped Dynamics and Control.* Ph.D Thesis, Department of Electrical Engineering, The Ohió State University, Columbus, Ohio.

Ceranowicz, A. Z., Syman, B. F., Hemami, H. 1980. Control of constrained systems of controllability index two. *IEEE Trans. Automatic Control* AC-25.

Chow, C. K., Jacobson, D. H. 1971. Studies of human locomotion via optimal programming. *Mathematical Biosciences* 10:239–306.

Chow, C. K., Jacobson, D. H. 1972. Further studies of human locomotion: Postural stability and control. *Mathematical Biosciences* 15:93–108.

Corliss, W. R., Johnson, E. G. 1968. *Teleoperator Controls*, NASA SP-5070.

Corson, P. E. 1958. Walking Tractor. Patent Number 2,822,878.

Coss, L., Chan, A. K., Goslow, G. E., Jr., Rasmussen, S. 1978. Ipsilateral limb variation in cats during overground locomotion. *Brain Behav. Evol.* 15:85–93.

Cotes, J. E., Meade, F. 1960. The energy expenditure and mechanical energy demand in walking. *Ergonomics.*

Cruse, H. 1979. A new model describing the coordination pattern of the legs of a walking stick insect. *Biological Cybernetics* 32:107–113.

Dahlquist, G., Bjorck, A., Anderson, N. 1974. *Numerical Methods.* Englewood Cliffs, NJ: Prentice-Hall.

Dawson, T. J. 1976. Energetic cost of locomotion in Australian hopping mice. *Nature* 259:305–307.

Dawson, T. J. 1977. Kangaroos. *Scientific American* 237:78–89.

Dawson, T. J., Taylor, C. R. 1973. Energetic cost of locomotion in kangaroos. *Nature* 246:313–314.

Donner, M. 1984. *Control of Walking: Local Control and Real Time Systems.* Ph.D Thesis, Carnegie-Mellon University, Pittsburgh, Pennsylvania.

Dougan, S. 1924. The angle of gait. *American J. Physiological Anthropology* 7:275–279.

Drillis, R., Contini, R. 1966. *Body Segment Parameters.* New York University, New York, Technical Report No. 1166.03.

Duysens, J., Loeb, G. E. 1980. Modulation of ipsi- and contralateral reflex responses in unrestrained walking cats. *J. Neurophysiology* 44:1024–1037.

Ehrlich, A. 1928. Vehicle Propelled by Steppers. Patent Number 1,691,233.

Eisenstein, B. L., Postillion, F. G., Norgren, K. S., Seelhorst, E., Wetzel, M. C., 1977. Kinematics of treadmill galloping by cats II. Steady-state coordination under positive reinforcement control. *Behavioral Biology* 21:89–106.

Elftman, H. 1934. A cinematic study of the distribution of pressure in the human Foot. *Anatomical Record* 59:481–490.

Elftman, H. 1939. Forces and energy changes in the leg during walking. *American J. Physiology* 125:339–356.

Elftman, H. O. 1951. The basic pattern of human locomotion. *Annals NY Acadamy of Science Record* 51:1207–1212.

Elftman, H. O. 1967. Basic function of the lower limbs, *Biomedical Engineering*, 342–345.

English, A. W. 1979. Interlimb coordination during stepping in the cat: An electromyographic analysis. *J. Neurophysiol.* 42:229–243.

Farnsworth, R. 1975. *Gait Stability and Control of a Five Link Model of Biped Locomotion.* MS Thesis, The Ohio State University, Columbus, Ohio.

Fedak, M. A., Heglund, N. C., Taylor, C. R. 1982. Energetics and mechanics of terrestrial locomotion II. Kinetic energy changes of the limbs and body as a function of speed and body size in birds and mammals. *J. Experimental Biology* 97:23–40.

Fedak, M. A., Pinshow, B., Schmidt-Nielsen, K. 1977. Energy cost of bipedal running. *American J. Physiology* 227:1038–1044.

Fenn, W. O. 1930a. Frictional and kinetic factors in the work of sprint running. *American J. Physiology* 92:583–611.

Fenn, W. O. 1930b. Work against gravity and work due to velocity changes in running. *American J. Physiology* 93.

Fitch, J. M., Templer, J., Corcoran, P. 1974. The dimensions of stairs.

Scientific American 231.

Flowers, W. C., Mann, R. W. 1977. An electrodydraulic knee-torque controller for a prosthesis simulator. *J. Bioengineering* 99.

Foley, C. D., Quanbury A., Steinke, T. 1979. Kinematics of normal child locomotion. *J. Biomechanics* 12:1–6.

Fomin, S. V., Gurfinkel, V. S., Feldman, A. G., Shtilkind, T. I. 1976. Movements of the joints of human legs during walking. *Biophysics* 21:572–577.

Fomin, S. V., Shtilkind, T. I. 1972. The concept of equilibrium of systems having legs. *Biofizika* 17:131–134.

Frank, A. A. 1968. *Automatic Control Systems for Legged Locomotion Machines.* Ph.D Thesis, University of Southern California, Los Angeles, California.

Frank, A. A. 1970. An approach to the dynamic analysis and synthesis of biped locomotion machines. *Medical and Biological Engineering* 8:465–476.

Frank, A. A. 1971. On the stability of an algorithmic biped locomotion machine. *J. Terramechanics* 8:41–50.

Frank, A. A., McGhee, R. B. 1969. Some considerations relating to the design of autopilots for legged vehicles. *J. Terramechanics* 6:23–25.

Fu, C. C., Paul, B. 1969. Dynamic stability of a vibrating hammer. *J. Engineering for Industry*, November, 1175–1179.

Fu, C. C., Paul, B. 1972. Dynamic characteristics of a vibrating plate compactor. *J. Engineering for Industry*, May, 629–636.

Gabrielli, G., Von Karmen, T. H. 1950. What price speed? *Mechanical Engineering* 72:775–781.

Galiana, H. L., Milsum, J. H. 1968. A mathematical model for the walking human leg. In *Proceedings of 21st ACEMB*, Houston, Texas.

Gannett, W. R. 1967. *The Peripod.* U.S. Army Weapons Command, Rock Island, Illinois.

Garg, D. P. 1976. Vertical mode human body vibration transmissibility. *IEEE Transaction Systems* SMC-6.

Goddard, R. E., Jr., Hemami, H., Weimer, F. C. 1981. Biped side step in the frontal plane. *ASME Winter Annual Meeting*, 147–155.

Golliday, C. L., Jr., Hemami, H. 1976. Postural stability of the two degree-of-freedom biped by general linear feedback. *IEEE Trans. Automatic Control* AC-21:74–79.

Golliday, C. L., Jr., Hemami, H. 1977. An approach to analyzing biped locomotion dynamics and designing robot locomotion controls. *IEEE Trans. Automatic Control* AC-22:963–972.

Graham, D. 1979a. Effects of circum-oesophageal lesion on the behavior of the stick insect, *Carausius morosus* I. Cyclic behaviour patterns. *Biological Cybernetics* 32:139–145.

Graham, D. 1979. Effects of circum-oesophageal lesion on the behavior of the stick insect *Carausius morosus* II. Changes in walking co-ordination. *Biological Cybernetics* 32:147–152.

Gray, J. 1968. *Animal Locomotion.* New York: Norton.

Greene, P. R., McMahon, T. A. 1979a. Running in circles. *The Physiologist* 22:S-35, S-36.

Greene, P. R., McMahon, T. A. 1979b. Reflex stiffness of man's anti-gravity muscles during kneebends while carrying extra weights. *J. Biomechanics* 12:881–891.

Grillner, S. 1972. The role of muscle stiffness in meeting the changing postural and locomotor requirements of force development by the ankle extensors. *Acta Physiology (Scandanavia)* 86:92–108.

Grillner, S. 1975. Locomotion in vertebrates: Central mechanisms and reflex interaction. *Physiology Review* 55:247–304.

Grillner, S. 1976. Some aspects on the descending control of the spinal circuits generating locomotor movements. In *Neural Control of Locomotion*, R. N. Herman, S. Grillner, P. S. Stein, D. G. Stuart (eds.). New York: Plenum Press.

Grillner, S., Zangger, P. 1979. On the central generation of locomotion in the low spinal cat. *Experimental Brain Research* 34:241–261.

Grillner et al. 1985. *Proceedings of Wenner-Gren Center International Symposium on Neurobiology of Vertebrate Locomotion.* June, Stolckholm.

Grimes, D. L. 1979. *An Active Multi-Mode Above Knee Prosthetic Controller.* Ph.D Thesis, Massachusetts Institute of Technology.

Grimes, D. L., Flowers, W. C., Donath, M. 1977. Feasibility of an active control scheme for above-knee prostheses. *J. Biomechanical Engineering* 99K.

Grundman, J., Seireg, A. 1976. Computer control of a multitask exoskeleton for paraplegics. In *Second Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Amsterdam: Elsevier Scientific Publishing Co., 241–248.

Gubina, F., Hemami, H., McGhee, R. B. 1974. On the dynamic stability of biped locomotion. *IEEE Trans. Biomedical Engineering* BME-21:102–108.

Gurfinkel, E. V. 1973. Physical foundation of the stabilography. In *Symposium International de Posturographic Smolenile.*

Gurfinkel, V. S., Shik, M. L. 1973. The control of posture and locomotion.

In *Motor Control*, A. A. Gydikou, N. T. Tankou, D. S. Kosarov (eds.). New York: Plenum, 217–234.

Gurfinkel, V. S., Gurfinkel, E. V., Shneider, A. Yu., Devjanin, E. A., Lensky, A. V., Shitilman, L. G. 1981. Walking robot with supervisory control. *Mechanism and Machine Theory* 16:31–36.

Halbertsma, J. M., Miller, S., van der Meché, F. G. A. 1976. Basic programs for the phasing of flexion and extension movements of the limbs during locomotion. In *Neural Control of Locomotion*, R. N. Herman, S. Grillner, P. S. Stein, D. G. Stuart (eds.). New York: Plenum Press, 489–518.

Hartrum, T. C. 1971. *Biped Locomotion Models.* The Ohio State University, Columbus, Ohio, Technical Report 272.

Hartrum, T. C. 1973. *Computer Implementation of a Parametric Model for Biped Locomotion Kinematics.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Harvey, K. M. 1968. *Investigation of a Hopping Transporter Concept for Lunar Exploration.* Ph.D Thesis, Stanford University, Stanford, California.

Hatze, H. 1977. A complete set of control equations for the human musculo-skeletal system. *J. Biomechanics* 10:799–805.

Heglund, N. C., Taylor, C. R., McMahon, T. A. 1974. Scaling stride frequency and gait to animal size: Mice to horses. *Science* 186:1112–1113.

Hemami, H. 1976. Reduced order models for biped locomotion. In *Proceedings of 7th Pittsburgh Conference on Modeling and Simulation.* Pittsburgh, Pennsylvania.

Hemami, H. 1980. A feedback on-off model of biped dynamics. *IEEE Trans. Systems, Man, and Cybernetics* SMC-10:376–383.

Hemami, H., Camana, P. C. 1976. Nonlinear feedback in simple locomotion systems. *IEEE Trans. Automatic Control* 21.

Hemami, H., Chen, B. 1984. Stability analysis and input design of a two-link planar biped. *International J. Robotics Research* 3:93–100.

Hemami, H., Cvetkovic, V. S. 1976. Postural stability of two biped models via Lyapunov second method. In *Proceedings of the Joint Automatic Control Conference.* West Lafayette, Indiana.

Hemami, H., Farnsworth, R. L. 1977. Postural and gait stability of a planar five link biped by simulation. *IEEE Trans. Automatic Control* AC-22:452–458.

Hemami, H., Golliday, C. L., Jr. 1977. The inverted pendulum and biped stability. *Mathematical Biosciences* 34:95–110.

Hemami, H., Katbab, A. 1982. Constrained inverted pendulum model for evaluating upright postural stability. *J. Dynamic Systems, Measurement, and Control* 104:343–349.

Hemami, H., Weimer, F. C. 1974. Further considerations of the inverted pendulum. In *Proceedings of Fourth Iranian Conference on Electrical Engineering*, Pahlavi University, Shiraz, Iran, pp. 697–708.

Hemami, H., Weimer, F. C. 1981. Modeling of nonholonomic dynamic systems with applications. *J. Applied Mechanics* APM-12:1–6.

Hemami, H., Wyman, B. F. 1979. Modeling and control of constrained dynamic systems with application to biped locomotion in the frontal plane. *IEEE Trans. Automatic Control* AC-24:526–535.

Hemami, H., Zheng, Y. 1982. Initiation of walk and tiptoe of a planar nine-link biped. *Mathematical Biosciences* 61:163–189.

Hemami, H., Robinson, D. J., Ceranowicz, A. Z. 1980. Stability of planar biped models by simultaneous pole assignment and decoupling. *International J. Systems Science* 11:65–75.

Hemami, H., Wall, C., III, Black, F. O. 1979. Single inverted pendulum biped experiments. *J. Interdisciplinary Modeling and Simulation* 2:211–227.

Hemami, H., Weimer, F. C., Koozekanani, S. H. 1973. Some aspects of the inverted pendulum problem for modeling of locomotion systems. *IEEE Trans. Automatic Control* AC-18:658–661.

Hemami, H., Hines, M. J., Goddard, R. E., Friedman, B. 1982. Biped sway in the frontal plane with locked knees. *IEEE Trans. Systems, Man, and Cybernetics* SMC-12:577–582.

Hemami, H., Weimer, F. C., Robinson, C. S., Stockwell, C. W., Cvetkovic, V. S. 1977. Analysis of some derived models of otoliths and semicircular canals. In *Proceedings of Joint Automatic Control Conference.*

Hemami, H., Weimer, F. C., Robinson, C. S., Stockwell, C. W., Cvetkovic, V. S. 1978. Biped stability considerations with vestibular models. *IEEE Trans. Automatic Control* AC-23:1074–1079.

Herman, B., Cook, T., Cozzens, B., Freeman, W. 1973. Control of postural reaction in man: The initiation of gait. In *Control of Posture and Locomotion*, R. B. Stein, K. G. Pearson, R. S. Smith, J. B. Redford (eds.). New York: Plenum Press, 353–388.

Herman, R. N., Grillner, S., Stein, P. S., Stuart, D. G. (eds.). 1976. *Neural Control of Locomotion.* New York: Plenum Press.

Hicks, J. H. 1961. The three weight bearing mechanisms of the foot. In *Biomechanical Studies of the Musculo-Skeleton System*, F. G. Evans, C. C. Thomas (eds.). Springfield: C. C. Thomas.

Higdon, D. T., 1963. *Automatic control of inherently unstable systems with bounded control inputs.* Ph.D Thesis, Stanford University, Stanford,

California.
Higdon, D. T., Cannon, R. H., Jr. 1963. On the control of unstable multiple-output mechanical systems. In *ASME Winter Annual Meeting.*
Hildebrand, M. 1959. Motion of the running cheetah and horse. *J. Mammalogy* 40:481–495.
Hildebrand, M. 1960. How animals run. *Scientific American* 148–157.
Hildebrand, M. 1961. Further studies on locomotion of the cheetah. *J. Mammalogy* 42:84-91.
Hildebrand, M. 1965. Symmetrical gaits of horses. *Science* 150:701–708.
Hildebrand, M. 1966. Analysis of the symmetrical gaits of tetrapods. *Folia Biotheoretica* 4:9–22.
Hildebrand, M. 1967. Symmetrical gaits of primates. *J. Anthropology* 26:119–130.
Hildebrand, M. 1968. Symmetrical gaits of dogs in relation to body build. *J. Morphology* 124:353–359.
Hildebrand, M. 1976. Analysis of tetrapod gaits: General considerations and symmetrical gaits. In *Neural Control of Locomotion*, R. N. Herman, S. Grillner, P. S. Stein, D. G. Stuart (eds.). New York: Plenum Press, 203–236.
Hildebrand, M. 1977. Analysis of asymmetrical gaits. *J. Mammalogy* 58:131–156.
Hildebrand, M. 1980. The adaptive significance of tetrapod gait selection. *Amer. Zool.* 20:255–267.
Hildebrand, M. 1985. Private communication.
Hildreth, E. C., Hollerbach, J. M. 1985. The computational approach to vision and motor control. In *Handbook of Physiology*, F. Plum (ed.). American Physiological Society.
Hill, J. C. 1969. A model of the human postural control system. In *Proceedings of 8th IEEE Symposium on Adaptive Processes: Decision and Control.*
Hirose, S. 1984. A study of design and control of a quadruped walking vehicle. *International J. Robotics Research* 3:113–133.
Hirose, S., Umetani, Y. 1980. The basic motion regulation system for a quadruped walking vehicle. *ASME Conference on Mechanisms.*
Hirose, S., Umetani, Y. 1981. *A Cartesian Coordinates Manipulator with Articulated Structure.* Tokyo Institute of Technology.
Hirose, S., Nose, M., Kikuchi, H., Umetani, Y. 1984. Adaptive gait control of a quadruped walking vehicle. *International J. Robotics Research* 1:253–277.
Hirose, S., Masui, T., Kikuchi, H., Fukuda, Y., Umetani, Y. 1985. Titan

III: A quadruped walking vehicle. In *Second International Symposium on Robotics Research*, Tokyo Institute of Technology, Japan.
Hodgins, J. 1985. Control of Foot Placement, In *Dynamically Stable Legged Locomotion-Fourth Annual Report, CMU-LL-4-1985*, Raibert et al., Carnegie-Mellon University.
Hodgins, J., Koechling, J., Raibert, M. H. 1985. Running experiments with a planar biped. *Third International Symposium on Robotics Research*, Cambridge: MIT Press.
Hoffer, J. A., O'Donovan, M. J., Pratt, C. A., Loeb, G. E. 1981. Discharge patterns of hindlimb motoneurons during normal cat locomotion. *Science* 213:466–468.
Hollerbach, J. M. 1980. A recursive formulation of lagrangian manipulator dynamics. *IEEE Trans. Systems, Man, and Cybernetics* 10:730–736.
Horn, B. K. P., Raibert, M. H. 1977. Configuration space control. *MIT Artificial Intelligence Laboratory Memo* no. 458.
Howell, R. 1944. *Speed in Animals.* Chicago: University of Chicago Press.
Hoyt, D. F., Taylor, C. R. 1981. Gait and the energetics of locomotion in horses. *Nature* 292:239–240.
Hristic, D., Vukobratovic, M. 1971. A new approach to the rehabilitation of paraplegic persons. *Automatika* 1.
Hughes, G. M. 1952. The coordination of insect movements. *J. Experimental Biology* 29:267–285.
Ikeda, K., et al. 1973. Finite state control of qudruped walking vehicle—Control by hydraulic digital actuator (in Japanese). *Biomechanism* 2:164–172.
Jaswa, V. C. 1975. *Dynamic Stability and Control of a Three Link Model of a Human Being.* MS Thesis, Ohio State University, Columbus, Ohio.
Jaswa, V. C. 1978. *An Experimental Study of Real-time Computer Control of a Hexapod Vehicle.* Ph.D Thesis, Ohio State University, Columbus, Ohio.
Jayes, A. S., Alexander, R. McN. 1980. The gaits of chelonians: Walking techniques for very low speeds. *J. Zoology (London)* 191:353–378.
Jones, F. W. 1949. *Structure and Function as Seen in the Foot.* London: Bailliere, Tindall and Cox, second edition.
Jones, R. L. 1941. The human foot—An experimental study of its mechanics and the role of its muscle and ligaments in the support of the arch. *American J. Anatomy* 68:1–39.
Juricic, D., Vukobratovic, M. 1972. Mathematical modeling of bipedal walking system. *ASME Publication 72-WA* BHF-13.
Kaneko, M., Abe, M., Tachi, S., Nishizawa, S., Tanie, K., Komoriya, K.

1985. Legged locomotion machine based on the consideration of degrees of freedom. In *Fifth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Cambridge: MIT Press, 403–410.

Kamon, E. 1971. Electromyographic kinesiology of jumping. *Archives of Physical Medicine and Rehabilitation* 52:152–157.

Kato, I., Tsuiki, H. 1972. The hydraulically powered biped walking machine with a high carrying capacity. In *IV Symposium on External Control of Human Extremities*, Yugoslav Committee for Electronics and Automation, Dubrovnik, Yugoslavia.

Kato, I., Matsushita, S., Kato, K. 1969. A model of human posture control system. In *Advances in External Control of Human Extremities*, M. M. Gavrilovic, A. B. Wilson Jr. (eds.). Belgrade: Yugoslav Committee for Electronics and Automation, 443–464.

Kato, T., Takanishi, A., Jishikawa, H., Kato, I. 1983. The realization of the quasi-dynamic walking by the biped walking machine. In *Fourth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Warsaw: Polish Scientific Publishers. 341–351.

Katoh, R., Mori, M. 1984. Control method of biped locomotion giving asymptotic stability of trajectory. *Automatica* 20:405–515.

Keith, A. 1929. The history of the human foot and its bearing on orthopedic practice. *J. Bone and Joint Surgery* 11:10–32.

Keller, J. B. 1973. A theory of competative running. *Physics Today* 26:42–47.

Khadelwal, B. M., Frank, A. A. 1974. On the dynamics of an elastically coupled multi-body biped locomotion model. In *Proceedings of Joint Automatic Control Conference*, Austin, Texas.

Khosravi-Sichani, B. 1985. *Control of Multi-Linkage Planar Systems in the Air and on the Ground.* PhD Thesis, The Ohio State University, Columbus, Ohio.

Kinch, E. A. 1928. Vehicle Propelling Device. Patent Number 1,669,906.

Klein, C. A., Briggs, R. L. 1980. Use of active compliance in the control of legged vehicles. *IEEE Trans. Systems, Man, and Cybernetics* SMC-10:393–400.

Kljajic, M., Trnkoczy, A. 1978. A study of adaptive control principle orthoses for lower extremities. *IEEE Trans. Systems, Mans, and Cybernetics* SMC-8:313–321.

Koozekanani, S. H., McGhee, R. B. 1973. Occupancy problems with pairwise exclusion constraints—an aspect of gait enumeration. *J. Cybernetics* 2:14–26.

Kuee, B. 1975. Movement of two legged system of pendulum type. *ISV. AN SSR, Mekhanika Tverdogo Tela* 2:58–61.

Kugushev, E. I., Jaroshevskij, V. S. *Problems of Selecting a Gait for an Integrated Locomotion Robot.* Institute of Applied Mathematics, USSR Academy of Sciences, Moscow.

Larin, V. B. 1976. Stabilization of biped walking apparatus. *Izv. AN SSSR, Mekhanika Tverdogo Tela* 11:4–13.

Larin, V. B. 1979. Control of a jumping robot I. Choice of programmed trajectory. *Izv. AN SSSR. Mekhanika Tverdoga Tela* 14:27–32.

Lee, D. N. 1974. Visual information during locomotion. In *Perception: Essays in Honor of James J. Gibson*, MacLeod and Pick (eds.), Ithaca, NY: Cornell University Press.

Lee, D. N., Lishman, J. R., Thomson, J. A. 1982. Regulation of gait in long jumping. *J. Experimental Psychology* 3:448–459.

Lisin, V. V., Frankstein, S. I., Rechtmann, M. B. 1973. The influence of locomotion on flexor reflex of the hind limb in cat and man. *Experimental Neurology* 38:180–183.

Liston, R. A. 1965. Walking machine. In *Proceedings Annual Meeting of the American Society of Agricultural Engineers.* St. Joseph, MI: American Society of Agricultural Engineers.

Liston, R. A. 1970. Increasing vehicle agility by legs: The quadruped transporter. Presented at *38th National Meeting of the Operations Research Society of America.*

Liston, R. A., Mosher, R. S. 1968. A versatile walking truck. In *Proceedings of the Transportation Engineering Conference.* Institution of Civil Engineers, London.

Lloyd, B. B. 1966. The energetics of running: An analysis of world records. *Advancement of Science* 22:515–530.

Loeb, G. E. 1981. Somatosensory unit input to the spinal cord during normal walking. *Canadian J. Physiology and Pharmacology* 59:627–635.

Lucas, E. 1894. Huitieme recreation—la machine a marcher. *Recreations Mathematiques* 4:198–204.

Mackerle, J. 1968. Walking riders. *T-68* 12:776–777.

Mann, M. 1960. Tanks that walk and jump. *Popular Science* July, 51–54, 216.

Manter, J. 1938. Dynamics of quadrupedal walking. *J. Experimental Biology* 15:522–539.

Marey, E. J. 1874. *Animal Mechanism: A Treatise on Terresrial and Aerial Locomotion.* New York: Appleton.

Margaria, R. 1976. *Biomechanics and Energetics of Muscular Exercise.* Oxford: Oxford University Press.

Marr, D. 1976. Artificial intelligence—A personal view. *MIT Artificial Intelligence Laboratory Memo* 355.

Matsuoka, K. 1976. Study on robot legs. *Bulletin of Mechanical Engineering Department* 11:65–69.

Matsuoka, K. 1979. A model of repetitive hopping movements in man. In *Proceedings of Fifth World Congress on Theory of Machines and Mechanisms.* International Federation for Information Processing.

Matsuoka, K. 1980. A mechanical model of repetitive hopping movements. *Biomechanisms* 5:251–258.

McGhee, R. B. 1967. Finite state control of quadruped locomotion. *Simulation* 5:135–140.

McGhee, R. B. 1968. Some finite state aspects of legged locomotion. *Mathematical Biosciences* 2:67–84.

McGhee, R. B. 1970. A mathematical theory for legged locomotion systems. In *Proceedings of the 1970 Midwest Symposium on Circuit Theory.*

McGhee, R. B. 1977. Control of legged locomotion systems. *Proceedings of Joint Automatic Control Conference*, San Francisco, 205–215.

McGhee, R. B. 1980. Robot locomotion with active terrain accommodation. In *Proceedings of National Science Foundation Robotics Research Workshop*, University of Rhode Island.

McGhee, R. B. 1983. Vehicular legged locomotion. In *Advances in Automation and Robotics*, G. N. Saridis (ed.). JAI Press.

McGhee, R. B., Buckett, J. R. 1977. Hexapod. *Interface Age* 2:29–54.

McGhee, R. B., Frank, A. A. 1968. On the stability properties of quadruped creeping gaits. *Mathematical Biosciences* 3:331–351.

McGhee, R. B., Iswandhi, G. I. 1979. Adaptive locomotion of a multilegged robot over rough terrain. *IEEE Trans. Systems, Man, and Cybernetics* SMC-9:176–182.

McGhee, R. B., Jain, A. K. 1972. Some properties of regularly realizable gait matricies. *Mathematical Biosciences* 13:179–193.

McGhee, R. B., Kuhner, M. B. 1969. On the dynamic stability of legged locomotion systems. In *Advances in External Control of Human Extremities*, M. M. Gavrilovic, A. B. Wilson, Jr. (eds.). Jugoslav Committee for Electronics and Automation, Belgrade, 431–442.

McGhee, R. B., Orin, D. E. 1973. An interactive computer-control system for a quadruped robot. In *Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Amsterdam: Elsevier Scientific Publishing Co.

McGhee, R. B., Pai, A. L. 1972. An approach to computer control for legged vehicles. *J. Terramechanics* 11:9–27.

McGhee, R. B., Sun, S. S. 1974. On the problem of selecting a gait for a legged vehicle. In *Proceedings of the Sixth IFAC Symposium on Automatic Control in Space.* Erevan USSR: International Program Committee, Tsakhkadzor, Armenian SSR, USSR.

McKenney, J. D. 1961. Investigation for a walking device for high efficiency lunar locomotion. In *Proceedings of American Rocket Society Annual Meeting*, Philadelphia.

McMahon, T. A. 1975. Using body size to understand the structural design of animals: Quadrupedal locomotion. *J. Applied Physiology* 39:619–627.

McMahon, T. A. 1976. Allometry. In *Yearbook of Science and Technology.* New York: McGraw-Hill, 49–57.

McMahon, T. A. 1979. Gravitational scale effects. *The Physiologist* 22:S-5–S-6.

McMahon, T. A. 1980. Scaling physiological time. *American Mathematical Society* 13:131–163.

McMahon, T. A. 1984a. *Muscles, Reflexes, and Locomotion.* Princeton: Princeton University Press.

McMahon, T. A. 1984b. Mechanics of locomotion. *International J. Robotics Research* 3:4–28.

McMahon, T. A. 1985. Personal communication.

McMahon, T. A., Greene, P. R. 1978. Fast running tracks. *Scientific American* 239:148–163.

McMahon, T. A., Greene, P. R. 1979. The influence of track compliance on running. *J. Biomechanics* 12:893–904.

Melvill-Jones, G. and Watt, D. O. D. 1971a. Observations on the control of stepping and hopping movements in man. *J. Physiology* 219:709–727.

Melvill-Jones, G. and Watt, D. O. D. 1971b. Muscular control of landing from unexpected falls in man. *J. Physiology* 219:729–737.

Mihajlov, D., Chang, C. W., Bekey, G. A., Perry, J. 1977. Computer graphics in the study of normal and pathological human gait. *Medinfo* 77:561–564.

Miller, D. I., Nelson, R. C. 1973. *Biomechanics of Sport.* Philadelphia: Lea and Febiger.

Miller, S., van der Meché, F. G. A. 1976. Coordinated stepping of all four limbs in the high spinal cat. *Brain Research* 109:395–398.

Miller, S., Ruit, J. B., van der Meché, F. G. A. 1977. Reversal of sign of long spinal reflexes dependent on the phase of the step cycle in the high decerebrate cat. *Brain Research* 128:447–459.

Miller, S., van der Burg, J., van der Meché, F. G. A. 1975a. Coordination of movements of the handlimbs and forelimbs in different forms of locomotion in normal and decerebrate cats. *Brain Research* 91:217–237.

Miller, S., van der Burg, J., van der Meché, F. G. A. 1975b. Coordination of movements of the hindlimbs and forelimbs in different forms of locomotion in normal and decerebrate cats. *Brain Research* 91:217–237.

Miller, S., van der Burg, J., van der Meché, F. G. A. 1975c. Locomotion in the cat: basic programmes of movement. *Brain Research* 91:239–253.

Miura, H., Shimoyama, I. 1980. Computer control of an unstable mechanism (in Japanese). *J. Fac. Eng.* 17:12–13.

Miura, H., Shimoyama, I. 1984. Dynamic walk of a biped. *International J. Robotics Research* 3:60–74.

Miura, H., Shimoyama, I., Mitsuishi, M., Kimura, H. 1985. Dynamical walk of quadruped robot (Collie-1). In *Second International Symposium on Robotics Research*, H. Hanafusa, H. Inoue (eds.). Cambridge: MIT Press, 317–324.

Miyazaki, F., Arimoto, A. 1984 A design method of control for biped walking machine. In *Fourth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Warsaw: Polish Scientific Publishers. 317–327.

Miyazaki, Fumio. 1980. A control theoretic study and dynamic linked locomotion. *J. Dynamic Systems of Measurements and Control* 102:233–239.

Mocci, U., Petternella, M., Salinari, S. 1972. *Experiments with Six-legged Walking Machines with Fixed Gait*, Institute of Automation Report 2-12. University of Rome.

Mochon, S. 1981. A mathematical model of human walking. *Lectures on Mathematics in the Life Sciences* 14:193–213.

Mochon, S., McMahon, T. A. 1980. Ballistic walking. *J. Biomechanics* 13:49–57.

Mochon, S., McMahon, T. A. 1981. Ballistic walking: An improved model. *Mathematical Biosciences* 52:241–260.

Morrison, J. B. 1970. The mechanics of muscle function in locomotion. *J. Biomechanics* 3:431–461.

Morrison, R. A. 1968. Iron mule train. In *Proceedings of Off-Road Mobility Research Symposium.* International Society for Terrain Vehicle Systems, Washington, 381–400.

Morton, D. J. 1935. *The Human Foot.* New York: Columbia University Press.

Morton, D. J. 1952. *Human Locomotion and Body Form.* Baltimore: Williams and Wilkins.

Mosher, R. S. 1965. *Design and Fabrication of a Full-scale, Limited Motion Pedipulator.* General Electric report.

Mosher, R. S. 1968. Test and evaluation of a versatile walking truck. In *Proceedings of Off-Road Mobility Research Symposium*, International Society for Terrain Vehicle Systems, Washington, 359–379.

Murphy, K. N. 1984. *Trotting and Bounding in a Simple Planar Model.* MS Thesis, Carnegie-Mellon University, Pittsburgh, Pennsylvania.

Murphy, K. N., Raibert, M. H. 1985. Trotting and bounding in a planar two-legged model. In *Fifth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Cambridge: MIT Press, 411–420.

Murray, M. P. 1967. Gait as a total pattern of movement. *American J. Physical Medicine* 46:290–333.

Murray, M. P., Drought, A. B., Kovry, R. C. 1964. Walking patterns of normal men. *J. Bone and Joint Surgery* 46-A:335–360.

Murray, M. P., Seirig, A., Scholz, R. C. 1967. Center of gravity, center of pressure and supportive forces during human activities. *J. Applied Physiology* 23:831–838.

Murthy, S. S. 1983. Path control. In *Dynamically Stable Legged Locomotion—Third Annual Report*, Raibert et al. Robotics Institute, Carnegie-Mellon University, CMU-RI-TR-83-20, pp.

Murthy, S. S., Raibert, M. H. 1983. 3D balance in legged locomotion: modeling and simulation for the one-legged case. In *Inter-Disciplinary Workshop on Motion: Representation and Perception*, ACM.

Muybridge, E. 1955. *The Human Figure in Motion.* New York: Dover Publications. First edition, 1901 by Chapman and Hall, Ltd., London.

Muybridge, E. 1957. *Animals in Motion.* New York: Dover Publications, First edition, 1899 by Chapman and Hall, Ltd., London.

Nakano, E., et al. 1982. Methodology of energy saving for autonomous mobile robots (in Japanese). *Biomechanism* 6:223–231.

Napier, J. 1967. The antiquity of human walking. *Scientific American* 216:56–66.

Narinyani, A. S., Patkin, V. P., Kim, P. A. *Walking Robot: A Non-deterministic Model of Control.* Computing Center, USSR Acadamy of Sciences, Siberian Branch, Novosibirsk.

Nashner, L. M. 1971. A model describing vestibular detection of body sway motion. *Acta Otolaryng.* 72:429–436.

Nashner, L. M. 1973. Vestibular and reflex control of normal standing. In *Control of Posture and Locomotion*, R. B. Stein, K. G. Pearson, R. S.

Smith, J. B. Redford (eds.). New York: Plenum Press, 291–308.

Nashner, L. M. 1976. Adapting reflexes controlling the human posture. *Experimental Brain Research* 26:59–72.

Nashner, L. M. 1977. Fixed patterns of rapid postural responses among leg muscles during stance. *Experimental Brain Research* 30:13–24.

Nashner, L. M. 1978. Visual contribution to rapid motor responses during postural control. *Brain Research* 150:403–407.

Nashner, L. M. 1980. Balance adjustments of humans perturbed while walking. *J. Neurophysiology* 44:650–664.

Nashner, L. M., Woollacott, M., Tuma, G. 1979. Organization of rapid responses to postural and locomotor-like perturbations of standing man. *Experimental Brain Research* 36:463–476.

Nilson, F. A. 1926. Supporting and Propelling Mechanism for Motor Vehicles. Patent Number 1,574,679.

Norgren, K. S., Seelhorst, E., Wetzel, M. C., 1977. Kinematics of treadmill galloping by cats I. Steady-state coordination under aversive control. *Behavial Biology* 21:66–88.

Ogo, K., Ganse, A., Kato, I. 1980. Dynamic walking of biped walking machine aiming at completion of steady walking. In *Third Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Amsterdam: Elsevier Scientific Publishing Co.

Okhotsimskii, D. E., Platonov, A. K. 1973. Control algorithm of the walker climbing over obstacles. In *International Joint Conference on Artificial Intelligence*, Stanford, California.

Okhotsimskii, D. E., Gurfinkel, V. S., Devyanin, E. A., Platonov, A. K. 1977. Integrated walking robot development. In *Conference on Cybernetic Models of the Human Neuromuscular System*. Engineering Foundation.

Okhotsimskii, D. E., Platonov, A. K., et al. 1973. Control algorithms of legged vehicle capable of mastering obstacles. In *Proceedings of 5th IFAC Symposium on Automatic Control in Space*, Geneva.

O'Leary, D. P., Ravasio, M. J. 1984. Simulation of vestibular semicircular canal afferent responses during righting movements of a freely falling cat. *Biological Cybernetics* 50:1–7.

Ooka, A., Ogi, K., Wada, Y., Kida, Y., Takemoto, A., Okamoto, K., Yoshida, K. 1985. Intelligent robot system II. In *Second International Symposium on Robotics Research*, H. Hanafusa, H. Inoue (eds.). Cambridge: MIT Press, 341–347.

Orin, D. E. 1976. *Interactive Control of a Six-legged Vehicle with Optimization of Both Stability and Energy.* Ph.D Thesis, The Ohio State

University, Columbus, Ohio.

Orin, D. E. 1981a. *Supervisory Control of a Multilegged Robot.* Electrical Engineering Department Report 23, Ohio State University.

Orin, D. E. 1981. *Three-axis joystick control of a hexapod vehicle over constant-slope terrain*, Digital Systems Lab, Report 23, Ohio State University.

Orin, D. E., McGhee, R. B., Vukobratovic, M. 1979. Kinematic and kinetic analysis of open-chain linkages utilizing Newton-Euler methods. *Mathematical Biosciences* 43:107–130.

Ozguner, F., Tsai, S. J., McGhee, R. B. 1984. An approach to the use of terrain-preview information in rough-terrain locomotion by a hexapod walking machine. *International J. Robotics Research* 3:134–146.

Pai, A. L. 1971. *Stability and Control of Legged Locomotion Systems.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Park, W. T. 1972. *Control of Multilegged Vehicles.* Ph.D Thesis, University of Pennsylvania, Philadelphia, Pennsylvania.

Pasternack, S., Jr. 1971. *Attitude Control of Hopping Vehicles.* Ph.D Thesis, Stanford University, Stanford, California.

Pearson, K. 1976. The control of walking. *Scientific American* 72–86.

Pearson, K. G., Franklin, R. 1984. Characteristics of leg movements and patterns of coordination in locusts walking on rough terrain. *International J. Robotics Research* 3:101–112.

Pedley, T. J. 1977. *Scale Effects in Animal Locomotion.* London: Academic Press.

Pedotti, A. 1977. A study of motor coordination and neuromuscular activities in human locomotion. *Biological Cybernetics* 26:53–62.

Pedotti, A., et al. 1978. Optimization of muscle-force sequencing in human locomotion. *Mathematical Bioscience* 28:57–76.

Pennycuick, C. J. 1975. On the running of the gnu (Connochaetes taurinus) and other animals. *J. Experimental Biology* 63:775–799.

Petternella, M., Salinari, S. 1974. *Six Legged Walking Vehicles.* University of Rome, Report 74-31.

Phillips, W. G., Wait, J. V., Wetzel, M. C. 1977. Motor-driven treadmill for studying locomotion in cats. *American J. Physical Medicine* 56:12–20.

Philippson, M. 1905. L'autonomie et la centralisation dans le system des animaux. *Trav. Lab. Physiol. Inst. Solvay (Bruxelles)* 7:1–208.

Plagenhoef, S. 1971. *Patterns of Human Locomotion: A Cinematographic Analysis.* Englewood Cliffs, NJ:Prentice Hall.

Pugh, L. G. C. E. 1971. The influence of wind resistance in running and walking and the mechanical efficiency of work against horizontal or vertical forces. *J. Physiology* 213:255–276.

Raibert, M. H. 1977. Analytical equations vs. table look-up for manipulation: A unifying concept. In *Proceedings of Conference on Decision and Control*, IEEE, New Orleans, 576–579.

Raibert, M. H. 1978. A model for sensorimotor control and learning. *Biological Cybernetics* 29:29–36.

Raibert, M. H. 1981. Dynamic stability and resonance in a one-legged hopping machine. In *Fourth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Warsaw: Polish Scientific Publishers. 352–367.

Raibert, M. H. 1984a. Hopping in legged systems—Modeling and simulation for the 2D one-legged case. *IEEE Trans. Systems, Man, and Cybernetics* 14:451–463.

Raibert, M. H. (ed.). 1984b. Special Issue on Legged Systems. *International J. Robotics Research* 3:75–92.

Raibert, M. H. 1985. Four-legged running with one-legged algorithms. In *Second International Symposium on Robotics Research*, H. Hanafusa, H. Inoue (eds.). Cambridge: MIT Press, 311–315.

Raibert, M. H., Brown, H. B., Jr. 1984. Experiments in balance with a 2D one-legged hopping machine. *ASME J. Dynamic Systems, Measurement, and Control* 106:75–81.

Raibert, M. H., Horn, B. K. P. 1978. Manipulator control using the configuration space method. *The Industrial Robot* 5:69–73.

Raibert, M. H., Sutherland, I. E. 1983. Machines that walk. *Scientific American* 248:44–53.

Raibert, M. H., Wimberly, F. C. 1984. Tabular control of balance in a dynamic legged system. *IEEE Trans. Systems, Man, and Cybernetics* 14:334–339.

Raibert, M. H., Brown, H. B., Jr., Chepponis, M. 1984. Experiments in balance with a 3D one-legged hopping machine. *International J. Robotics Research* 3:75–92.

Raibert, M. H., Brown, H. B., Jr., Murthy, S. S. 1984. 3D balance using 2D algorithms? In *First International Symposium of Robotics Research*, M. Brady, R. P. Paul (eds.). Cambridge: MIT Press, 279–301.

Raibert, M. H., Brown, H. B., Jr., Chepponis, M., Hastings, E., Murthy, S. S., Wimberly, F. C. 1983. *Dynamically Stable Legged Locomotion.* CMU–RI–TR–83–1, Robotics Institute, Carnegie-Mellon University.

Raibert, M. H., Brown, H. B., Jr., Chepponis, M., Hastings, E., Shreve, S. T., Wimberly, F. C. 1981. *Dynamically Stable Legged Locomotion.* CMU–RI–81–9, Robotics Institute, Carnegie-Mellon University.

Raibert, M. H., Brown, H. B.,Jr., Chepponis, M., Hastings, E., Koechling, J., Murphy, K. N., Murthy, S. S., Stentz, A. 1983. *Dynamically Stable Legged Locomotion—Third Annual Report.* CMU–RI–TR–83–20, Robotics Institute, Carnegie-Mellon University.

Raibert, M. H., Brown, H. B., Jr., Chepponis, M., Hodgins, J., Koechling, J., Miller, J., Murphy, K. N., Murthy, S. S., Stentz, A. J. 1985. *Dynamically Stable Legged Locomotion—Fourth Annual Report.* CMU–LL–4–1985, Carnegie-Mellon University.

Ralston, H. J. 1958. Energy-speed relation and optimal speed during level walking. *Internationale Zeitshrift für Angewante Physiologie Einschliesslich Arbeits Physiologie* 17:277–283.

Ralston, H. J. 1976. Energetics of human walking. In *Neural Control of Locomotion*, R. N. Herman, S. Grillner, P. S. Stein, D. G. Stuart (eds.). New York: Plenum Press, 77–98.

Ramey, M. R. 1970. Force relationship of the running long jump. *Medicine and Science in Sports* 2:146–151.

Roberge, J. K. 1960. *The Mechanical Seal.* S.B. Thesis, Massachusets Institute of Technology, Cambridge, Mass.

Roberts, W. M., Levine, W. S., Zajac, F. E., III. 1979. Propelling a torque controlled baton to a maximum height. *IEEE Trans. Automatic Control* AC-24:779–782.

Russell, M., Jr. 1983. Odex I: The first functionoid. *Robotics Age* 5:12–18.

Rygg, L. A. 1893. Mechanical Horse. Patent Number 491,927.

Saund, E. 1982a. The physics of one-legged mobile robots I. A general discussion. *Robotics Age*, Sept./Oct., 38–45.

Saund, E. 1982b. The physics of one-legged mobile robots II. The mathematical description. *Robotics Age*, Nov./Dec., 35–37, 53.

Saunders, J. B., Imma, V. T., Eberhart, H. D. 1953. The major determinants in normal and pathological gait. *The J. Bone and Joint Surgery* 35-A:543–558.

Schaefer, J. F. 1965. *On the Bounded Control of Some Unstable Mechanical Systems.* Ph.D Thesis, Stanford University, Stanford, California.

Schaefer, J. F., Cannon, R. H., Jr. 1966. On the control of unstable mechanical systems. *International Federation of Automatic Control.* London, 6c.1–6c.13.

Schneider, A. Yu., Gurfinkel, E. V., Kanaev, E. M., Ostapchuk, V. G. 1974. *A System for Controlling the Extremities of an Artificial Walking Apparatus.* Physio-Technical Institute, Report 5.

Seifert, H. S. 1967. The lunar pogo stick. *J. Spacecraft and Rockets* 4:941–943.

Seifert, H. S. 1968. *Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode.* Stanford University Dept. Aeronautics and Astronautics, Report 359.

Seifert, H. S. 1969. *Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode.* Stanford University Dept. Aeronautics and Astronautics, Report 377.

Seifert, H. S. 1970. *Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode.* Stanford University Dept. Aeronautics and Astronautics, Report 397.

Seireg, A., Arvikar, R. J. 1973. A mathematical model for the evaluation of forces in lower extremities of the musculo-skeletal system. *J. Biomechanics* 6:313–326.

Severin, F. V., Orlovskii, G. N., Shik, M. L. 1967. Work of the muscle receptors during controlled locomotion. *Biophysics* 12:575–586.

Shannon, B. 1985. Personal communication.

Shigley, R. 1957. *The Mechanics of Walking Vehicles.* Land Locomotion Laboratory, Report 7, Detroit, Michigan.

Shigley, R. 1960. *The Mechanics of Walking Vehicles.* Land Locomotion Laboratory, Report 71, Detroit, Michigan.

Shik, M. L., Orlovskii, G. N. 1965. *Coordination of the Limbs During Running of the Dog.* Institute of Biological Physics, Academy of Sciences, Moscow, U.S.S.R.

Shik, M. L., Orlovsky, G. N., Severin, F. V. 1968. Locomotion of the mesencephalic cat elicited by stimulation of the pyramids. *Biofizika* 13:127–135.

Silver, W. M. 1981. On the equivalence of Lagrangian and Newton-Euler dynamics for manipulators. In *Proceedings of Joint Automatic Control Conference.*

Simons, J., Van Brussel, H., De Schutter, J., Verhaert, J. 1982. A self-learning automaton with variable resolution for high precision assembly by industrial robots. *IEEE Trans. Automatic Control* AC-27:1109–1112.

Sindall, J. N. 1964. The wave mode of walking locomotion. *J. Terramechanics* 1:54–73.

Sitek, G. 1976. Big Muskie. *Heavy Duty Equipment Maintenance* 4:16–23.

Snell, E. 1947. Reciprocating Load Carrier. Patent Number 2,430,537.

Speckert, G. 1976. A computerized look at cat locomotion or one way to scan a cat. *MIT Artificial Intelligence Laboratory Memo*, 374.

Stein, J. L. 1983. *Design Issues in the Stance Phase Control of Above-Knee Prostheses.* Ph.D Thesis, Massachusetts Institute of Technology, Cambridge, Mass.

Steindler, A. 1935. *Mechanics of Normal and Pathological Locomotion.* Springfield: Charles C. Thomas.

Stentz, A. 1983, Behavior during stance. In *Dynamically Stable Legged Locomotion—Third Annual Report*, M. H. Raibert et al. Robotics Institute, Carnegie-Mellon University, CMU-RI-TR-83-20, pp. 106–110.

Stewart, G. W. 1973. *Introduction to Matrix Computations.* New York: Academic Press.

Stuart, D. G., Withey, T. P., Wetzel, M. C., Goslow, G. E., Jr. 1973. Time constraints for inter-limb coordination in the cat during unrestrained locomotion. In *Control of Posture and Locomotion*, R. B. Stein, K. G. Pearson, R. S. Smith, J. B. Redford (eds.). New York: Plenum Press, 537–560.

Sun, S. S. 1974. *A Theoretical Study of Gaits for Legged Locomotion Systems.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Sutherland, I. E. 1983. *A Walking Robot.* Pittsburgh: The Marcian Chronicles, Inc.

Sutherland, I. E., Ullner, M. K. 1984. Footprints in the asphalt. *International J. Robotics Research* 3:29–36.

Takanishi, A., Naito, G., Ishida, M., Kato, I. 1985, Realization of plane walking by a biped walking robot WL-10R. In *Fifth Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Cambridge: MIT Press, 383–394.

Taylor, C. R., Rowntree, V. J. 1973. Running on two or four legs: Which consumes more energy? *Science* 179:179–187.

Taylor, C. R., Heglund N. C., McMahon, T. A., Looney, T. R. 1980. Energetic cost of generating muscular force during running. *J. Experimental Biology* 86:9–18.

Taylor, C. R., Shkolnik, A., Dmpel, R., Baharav, D., Borut, A. 1974. Running in cheetahs, gazelles, and goats: Energy cost and limb configuration. *American J. Physiology* 227:848–850.

Terhune, C. H., Jr., Fowler, D. R. 1978. *Proposal for Development of Unmanned Remotely Controlled Vehicles to Assist in Mine Rescue and Recovery Operations*, US Bureau of Mines, Department of Interior, Report 5020-84.

Todd, D. J., 1985. *Walking Machines: An Introduction to Legged Robots.* New York: Chapman and Hall.

Tomovic, R. 1965. On the synthesis of self-moving automata. *Automation and Remote Control*, 26.

Tomovic, R., Karplus, W. R. 1961. Land locomotion—simulation and control. In *Third International Analogue Computation Meeting*, Opatija, Yugoslavia, 385–390.

Tomovic, R., McGhee, R. B. 1966. A finite state approach to the synthesis of bioengineering control systems. *IEEE Trans. Human Factors in Electronics* 7:65–69.

Trnkoczy, A., Bajd, T., Malezic, M. 1976. A dynamic model of the ankle joint under functional electrical stimulation in free movement and isometric conditions. *J. Biomechanics* 9:509–519.

Urschel, W. E. 1949. Walking Tractor. Patent Number 2,491,064.

Vukobratovic, M. 1972. Contributions to the study of anthropomorphic systems. *Kibernetika* 2.

Vukobratovic, M. 1973. Dynamics and control of anthropomorphic active mechanisms. In *First Symposium on Theory and Practice of Robots and Manipulator Systems*, A. Morecki, G. Bianchi, K. Kedzior (eds.). Amsterdam: Elsevier Scientific Publishing Co., 313-332.

Vukobratovic, M. 1975a. *Legged Locomotion Robots and Anthropomorphic Systems*. Belgrade: Institute Mihialo Pupin.

Vukobratovic, M. 1975b. *Legged Locomotion Robots: Mathematical Models, Control Algorithms and Realizations*, Beograd: M. Pupin.

Vukobratovic, M., Frank, A. A. 1969. Legged locomotion studies. In *Advances in External Control of Human Extremities*, M. M. Gavrilovic, A. B. Wilson Jr. (eds.). Belgrade: Yugoslav Committee for Electronics and Automation, 407–430.

Vukobratovic, M., Juricic, D. 1969. Contribution to the synthesis of biped gait. *IEEE Trans. Biomedical Engineering* BME-16.

Vukobratovic, M., Okhotsimskii, D. E. 1975. Control of legged locomotion robots. *Proceedings of the International Federation of Automatic Control* Plenary Session.

Vukobratovic, M., Stepaneko, Y. 1972. On the stability of anthropomorphic systems. *Mathematical Biosciences* 14:1–38.

Vukobratovic, M., Frank, A. A., Juricic, D. 1970. On the stability of biped locomotion. *IEEE Trans. Biomedical Engineering* BME-17.

Vukobratovic, M., Hristic, D., Stojoljkovic, Z. 1974. Development of active anthropomorphic exoskeletons. *Medical and Biological Engineering* 12:66–80.

Vukobratovic, M., Juricic, D., Frank, A. A. 1970. On the control and stability of one class of biped locomotion systems. *ASME Trans. Basic Engineering* 328–332.

Waldron, K. J., Vohnout, V. J., Pery, A., McGhee, R. B. 1984. Configuration design of the adaptive suspension vehicle. *International J. Robotics Research* 3:37–48.

Wallace, H. W. 1942. Jumping Tank Vehicle. Patent Number 2,371,368.

Warren, W. H., Jr., Lee, D. N., Young, D. S. 1985. Visual control of running over irregular terrain. Unpublished report, Brown University.

Wetzel, M. C., Stuart, D. G. 1976. Ensemble characteristics of cat locomotion and its neural control. *Progress in Neurobiology* 7:1–98.

Wetzel, M. C., Atwater, A. E., Stuart, D. G. 1976. Movements of the hindlimb during locomotion of the cat. In *Neural Control of Locomotion*, R. N. Herman, S. Grillner, P. S. Stein, D .G. Stuart (eds.). New York: Plenum Press.

Whitney, W. M. 1974. Human vs. autonomous control of planetary roving vehicles. In *IEEE Symposium on Systems, Man and Cybernetics*, Dallas, 1–4.

Williams, M., Lissner, H. R. 1962. *Biomechanics of Human Motion.* Philadelphia: W.B. Saunders Company.

Wilson, D. M. 1966. Insect walking. *Annual Review of Entomology* 11:103–121.

Wilson, D. M. 1967. Stepping patterns in tarantula spiders. *J. Experimental Biology* 47:133–151.

Winter, D. A., Robertson, D. G. E. 1978. Joint torque and energy patterns in normal gait. *Biological Cybernetics* 29.

Witt, D. C. *A Feasibility Study on Automatically Controlled Powered Lower-limb Prosthesis.* University of Oxford, Department of Engineering Science Report.

Wongchaisuwat, C., Hemami, H., Buchner, H. J. 1984. Control of sliding and rolling at natural joints. *J. Biomechanical Engineering* 106:368–375.

Yang, P. 1976. *A Study of Electronically Controlled Orthotic Knee Joint Systems.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Zheng, Y. 1980. *The Study of a Nine Link Biped Model with Two Feet.* Ph.D Thesis, The Ohio State University, Columbus, Ohio.

Zheng, Y. 1984. Impact effects of biped contact with the environment. *IEEE Trans. Systems, Man, and Cybernetics* 14:437–443.

Index

Robots That Run is a fifteen-minute videotape prepared by author Marc Raibert to accompany *Legged Robots That Balance*. The tape opens with a brief historical overview that serves to set the stage for a closer look at the author's current work. It then goes on to chronicle the development and behavior of four machines designed by the author and his co-workers at Carnegie-Mellon University, including a planar one-legged hopping machine, a 3D one-legged hopping machine, a planar biped machine, and a quadruped machine. The concepts of dynamic control and active balance are illustrated by these legged robots performing a variety of behaviors, including hopping in place, running at a desired speed, leaping over small obstacles, and maintaining balance when disturbed.

Order Information

Prices: $50.00 for ¾″ U-matic format; $45.00 for ½″ VHS format. Beta and PAL/SECAM formats available by special order. Be sure to specify format when ordering. Prepayment required. Check, money order, VISA/Mastercard, or purchase order accepted. Please allow three weeks for delivery. Distributed by The MIT Press, exclusive licensee.

Mail to: *Robots That Run*
The MIT Press
28 Carleton Street
Cambridge, MA 02142
or phone: (617) 253–2884