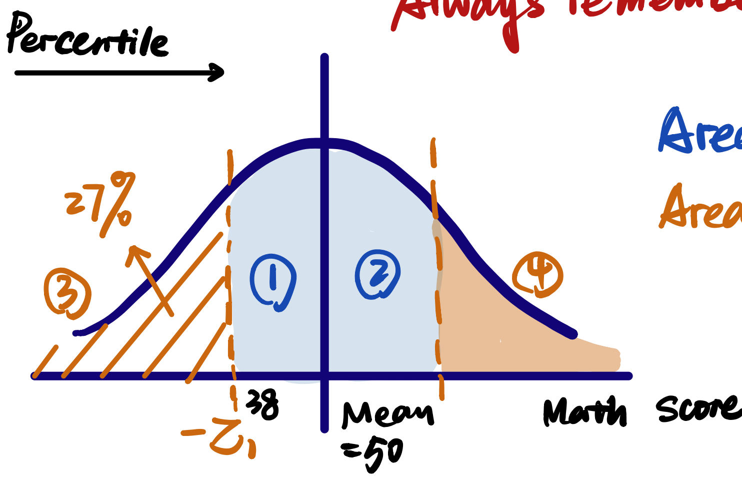
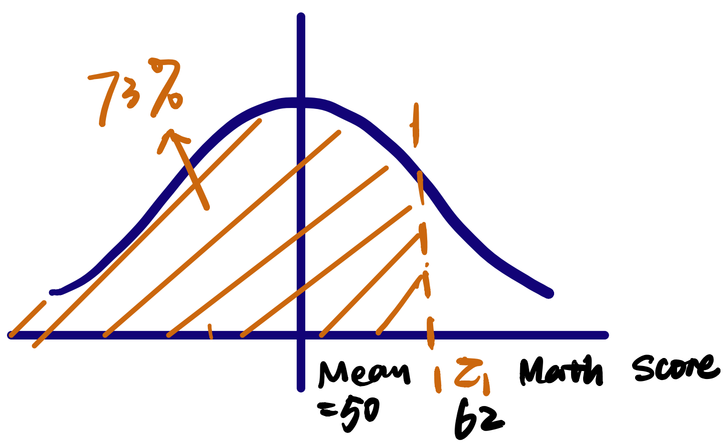
Always remember to use symmetry.



Area ① = Area ②

Area ③ = Area ④

- Math score 38 is 27th percentile means that the score less than 38 accounts for 27%
- - - - - - - - - - - larger than 38 - - - - - - (1−27%) = 73%



- Math score 62 is 73th percentile means that the score less than 62 accounts for 73%
- - - - - - - - - larger than 62 - - - - - (1−73%) = 27%

CHAPTER 4

1. (a) Find the average and SD of the list 41, 48, 50, 50, 54, 57.
   (b) Which numbers on the list are within 0.5 SDs of average? within 1.5 SDs of average?

(a) $\bar{X} = \dfrac{41+48+50+50+54+57}{6}$

$= 50$

SD: step 1: $41-50=-9$ → $X_i - \bar{X}$

$48-50=-2$

$50-50=0$

$50-50=0$

$54-50=4$

$57-50=7$

step 2. $(-9)^2+(-2)^2+0^2+0^2+4^2+7^2=150$ → $\sum (X_i-\bar{X})^2$

step 3. $SD = \sqrt{\dfrac{150}{6}} = \sqrt{25} = 5$ → $\sqrt{\dfrac{\sum (X_i-\bar{X})^2}{n}}$

(b) $0.5\ SD_s = 5 \times 0.5 = 2.5$

$[\text{Average} - 0.5SD_s, \text{Average} + 0.5SD_s]$

$\Rightarrow [50-2.5, 50+2.5] \Rightarrow [47.5, 52.5]$

3. Here is a list of numbers:

0.7	1.6	9.8	3.2	5.4	0.8	7.7	6.3	2.2	4.1
8.1	6.5	3.7	0.6	6.9	9.9	8.8	3.1	5.7	9.1

(a) Without doing any arithmetic, guess whether the average is around 1, 5, or 10.
(b) Without doing any arithmetic, guess whether the SD is around 1, 3, or 6.

(a) 5 can't be 10: no values > 10 & can't be 1: lots of numbers around 9

(b) 3 can't be 6 12 6+6 68% (but not value > 12) & can't be 1: got extreme values: 0.6, 9.9

7. A study on college students found that the men had an average weight of about 66 kg and an SD of about 9 kg. The women had an average weight of about 55 kg and an SD of 9 kg.

   (a) Find the averages and SDs, in pounds (1 kg = 2.2 lb). change the scale
   (b) Just roughly, what percentage of the men weighed between 57 kg and 75 kg?
   (c) If you took the men and women together, would the SD of their weights be smaller than 9 kg, just about 9 kg, or bigger than 9 kg? Why?

(a) For men:

① Average: $\bar{X}_{old} = \frac{\sum X_{i\,old}}{n} = 66$, $\Rightarrow \bar{X}_{new} = \frac{\sum 2.2 X_{i\,old}}{n} = \frac{2.2 \sum X_{i\,old}}{n} = 2.2 \times 66 \approx 145\text{ lb}$

② SDs: $SD_{new} = \sqrt{\frac{\sum (X_{i\,new} - \bar{X}_{new})^2}{n}}$

$$= \sqrt{\frac{\sum (2.2 X_{i\,old} - 2.2 \bar{X}_{old})^2}{n}}$$

$$\Rightarrow \sqrt{\frac{2.2^2 \sum (X_{i\,old} - \bar{X}_{old})^2}{n}} \Rightarrow 2.2 SD_{old} = 2.2 \times 9 \approx 20$$

(b) $57 = 66 - 9 = \text{Average} - 1SD$

$75 = 66 + 9 = \text{Average} + 1SD$

$\therefore$ Average $\pm 1SD$ $\sim 68\%$

(c) Bigger. Spread is larger

CHAPTER 5

1. The following list of test scores has an average of 50 and an SD of 10:

39	41	47	58	65	37	37	49	56	59	62	36	48
52	64	29	44	47	49	52	53	54	72	50	50	

(a) Use the normal approximation to estimate the number of scores within 1.25 SDs of the average.

(b) How many scores really were within 1.25 SDs of the average?

(a) ① total # scores : 25 = n

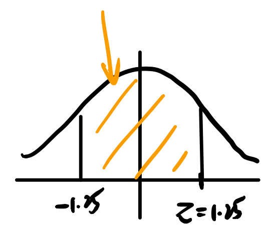
② percentage of the scores population within the 1.25 SDs.

1.25 SDs : z score

Area : 78.87%

③ the number of scores within :

$25 \times 78.87\% \approx 20$



(b) 1 SD = 10 , Average = 50

$\Rightarrow$ 1.25 SDs = $1.25 \times 10 = 12.5$

Range of score : [Average − 1.25 SDs, Average + 1.25 SDs]

$\Rightarrow [50 - 12.5, \quad 50 + 12.5]$

$\Rightarrow [37.5, \ 62.5]$

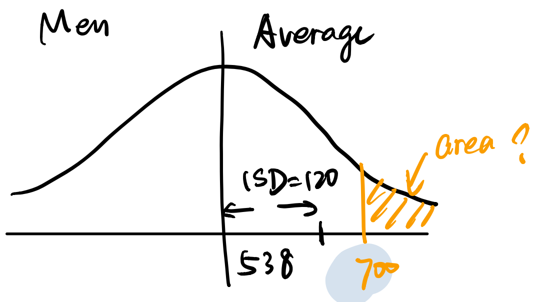
4. On the Math SAT, men have a distinct edge. In 2005, for instance, the men averaged about 538, and the women averaged about 504.

   (a) Estimate the percentage of men getting over 700 on this test in 2005.

   (b) Estimate the percentage of women getting over 700 on this test in 2005.

   You may assume (i) the histograms followed the normal curve, and (ii) both SDs were about 120.[4]

(a)



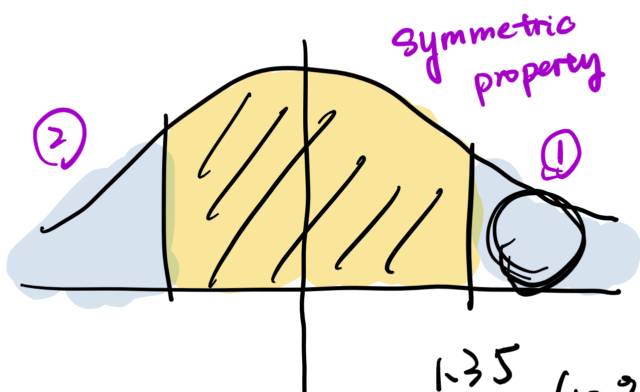
① Find the Z score of 700: $\frac{\text{Number} - \text{Mean}}{\text{SD}}$

$$Z = \frac{700 - 538}{120} = 1.35$$

② Check the Normal table for the Area

Area = 82.3%

③ $\frac{1 - \text{Area}(82.3\%)}{2} = 8.85\%$



Area = 82.3%

$\therefore$ ① = $\frac{1}{2}$(   )

(100% − 82.3%) = ① + ②

6. Among applicants to one law school, the average LSAT score was about 169, the SD was about 9, and the highest score was 178. Did the LSAT scores follow the normal curve?

Assume it followed the normal curve:

the highest score is around 3 SDs above the average (99.7%)

$169 + 3\times 9 = 196$, much larger than 178

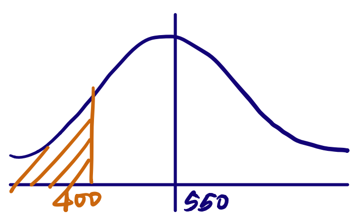
So it did not follow the normal curve.

7. Among freshmen at a certain university, scores on the Math SAT followed the normal curve, with an average of 550 and an SD of 100. Fill in the blanks; explain briefly.

   (a) A student who scored 400 on the Math SAT was at the ______ th percentile of the score distribution.

   (b) To be at the 75th percentile of the distribution, a student needed a score of about ______ points on the Math SAT.

(a)



① Find the z score:

$$\frac{400-550}{100} = -1.5$$

With symmetric property, we can check the Normal table with positive value: 1.5

② Check the Normal table

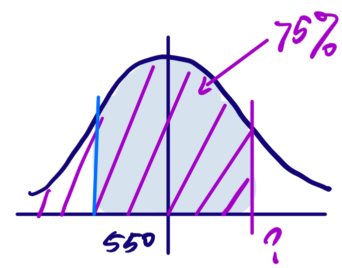
Area = 86.64%

③ Get the percentile:

$$\frac{1-86.64\%}{2} = 6.68\%$$

∴ 6.68th percentile

(b)



① Get the middle area by percentile:

$100\% - (100\% - 75\%)\times 2$

$= 50\%$

② Check the Normal table for z score

$z = 0.68$

③ Get the value of SAT score

$$0.68 = \frac{\text{Score value} - 550}{100}$$

so the score value $= 550 + 68$

$= 618$

8. True or false, and explain briefly—
   (a) If you add 7 to each entry on a list, that adds 7 to the average.
   (b) If you add 7 to each entry on a list, that adds 7 to the SD.
   (c) If you double each entry on a list, that doubles the average.
   (d) If you double each entry on a list, that doubles the SD.
   (e) If you change the sign of each entry on a list, that changes the sign of the average.
   (f) If you change the sign of each entry on a list, that changes the sign of the SD.

(a) True. $\frac{\sum(x_i+7)}{n} = \frac{7n+\sum x_i}{n} = 7+\bar{x}$

(b) False. $\sqrt{\frac{\sum(7+x_i-(7+\bar{x}))^2}{n}} = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = SD$

So SD keeps the same.

(c) True. $\frac{\sum 2\cdot x_i}{n} = \frac{2\sum x_i}{n} = 2\bar{x}$

(d) True. $\sqrt{\frac{\sum(2x_i-2\bar{x})^2}{n}} = \sqrt{\frac{4\sum(x_i-\bar{x})^2}{n}} = 2\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = 2SD$

(e) False. 1. SD is never negative

2. change the sign of each entry mean times (-1) for each entry, so for (b) we know that SD keeps the same.