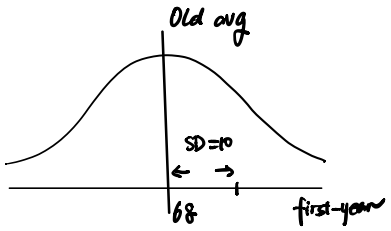
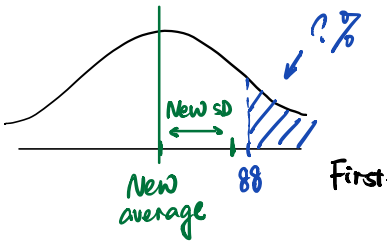
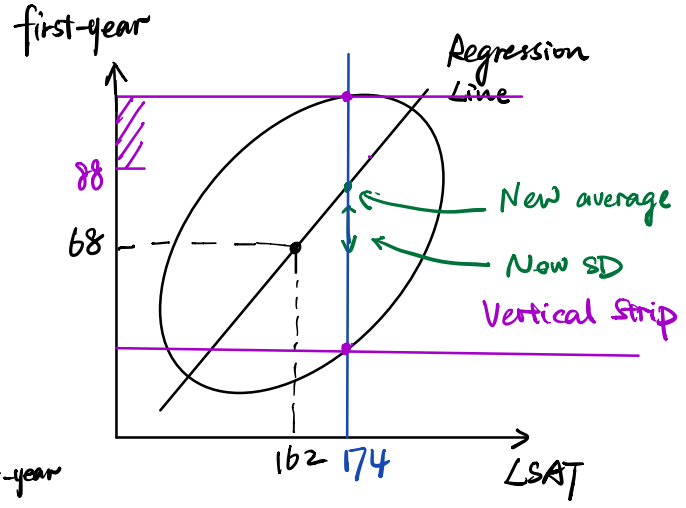
Exercise 1:







① Get the regression line

$\bar{X}_{LSAT} = 162$ $SD_x = 6$ $r = 0.6$

$\bar{y}_{first\text{-}year} = 68$ $SD_y = 10$

$\hat{y} = -94 + x$

Slope: $\frac{r \times SD_y}{SD_x} = \frac{0.6 \times 10}{6} = 1$

intercept: $\bar{y} = \text{slope} \cdot \bar{x} + \text{intercept} \Rightarrow \text{intercept} = 68 - 1 \times 162 = -94$

② Get the new avg & SD for the new normal dist.

r.m.s. error: $\sqrt{1-r^2} \times SD_y = \sqrt{1-0.6^2} \times 10 = 8$

∴ new SD = 8

$\hat{y} = -94 + x_{new} = -94 + 174 = 80$

∴ new average = 80

→ we can get the new normal dist with avg of 80 and SD of 8

③ Find the new Z score of value and get the percentage.

$Z = \frac{88 - 80}{8} = \frac{8}{8} = 1$ ← new avg; 8 ↑ new SD

Percentage = shaded area

$= \frac{1}{2}(1 - 68.7\%)$

Exercise Ⅱ:

① What's the SAT score of the 90th?

$\bar{X} = 550, \quad SD_X = 80.$

Area $= 1 - 2 \times (1 - 90\%) = 80\%$

Check the normal table to get Z score.

$Z \approx 1.3$

$\therefore Z = \frac{SAT - \bar{X}}{SD_X} = 1.3 \qquad SAT = 550 + 1.3 \times 80$

$= 654$

② What is the GPA when SAT score is 654?

→ fit the regression line.

Slope: $\frac{r \times SD_Y}{SD_X} = \frac{0.4 \times 0.6}{80} = 0.003$

intercept: $\bar{y} = 0.003 \times \bar{x} + \text{intercept}$

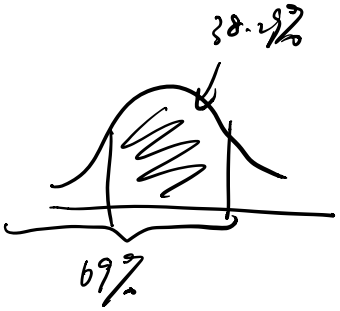
$\text{intercept} = 2.6 - 0.003 \times 550 = 0.95$

$\therefore \hat{y} = 0.003x + 0.95 \Rightarrow$ when $X = 654$, $\hat{y} = 0.003 \times 654 + 0.95$

$= 2.912$

③ What is the percentile of GPA is 2.912?

Z score $= \frac{2.912 - 2.6}{0.6} = 0.52$



check the normal table:

area $= 1 - \frac{1}{2} \times (1 - 38.29\%) = 69\%$.