2. Fill in the blanks, using the options below, and give examples to show that you picked the right answers.

   (a) The SD of a list is 0. This means ______ .

   (b) The r.m.s. size of a list is 0. This means ______ .

   Options:

   (i) there are no numbers on the list

   (ii) all the numbers on the list are the same

   (iii) all the numbers on the list are 0

   (iv) the average of the list is 0

(a) (ii) $x_i - \bar{x} = 0$ for any i in $1, 2, \ldots, n$

$$SD_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{0}{n}} = 0$$

$\therefore$ (iii) is also right

(iv) $SD_x = \sqrt{\frac{\Sigma x_i^2}{n}} \neq 0$ if at least one of the number in the list is not zero

(b) Recall r.m.s. size : $\sqrt{\text{avg of } (x_i)^2}$

$$= \sqrt{\frac{\Sigma x_i^2}{n}} = 0 \qquad x_i^2 \geq 0$$

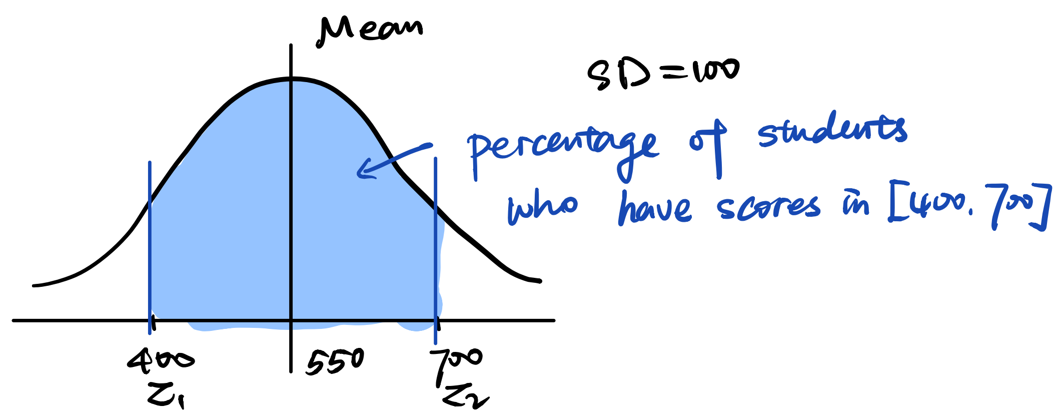
iff all the numbers are 0.

$\therefore$ (iii) is right

4. Among first-year students at a certain university, scores on the Verbal SAT follow the normal curve; the average is around 550 and the SD is about 100.
   (a) What percentage of these students have scores in the range 400 to 700?

   (b) There were about 1,000 students with scores in the range 450–650 on the Verbal SAT. About ______ of them had scores in the range 500 to 600. Fill in the blank; explain briefly.

(a)



$$Z_1 = \frac{400 - \text{mean}}{SD} = \frac{400 - 550}{100} = -1.5$$

$$Z_2 = \frac{700 - \text{mean}}{SD} = \frac{700 - 550}{100} = 1.5$$

. Z_1 and Z_2 are symmetric around average since $|Z_1| = |Z_2|$

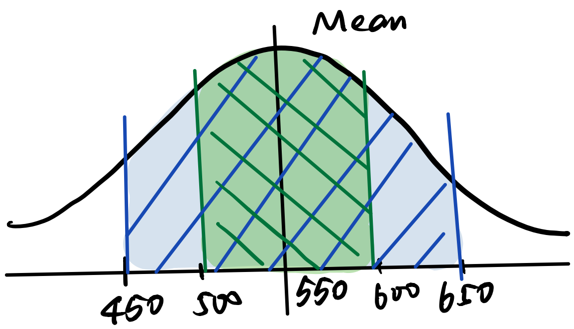
Check the normal table,

Area = 86.64% ⇒ answer

(b) 450 and 650 are also symmetric around avg.

check the normal table:

$$Z = \frac{650 - 550}{100} = 1 \Rightarrow \text{Area} = \underline{68.27\%}$$



Blue area : 68.27%

Green area : 38.29%

$1000 = \text{Area} \times \text{Total \# of students } (N)$

$$\Rightarrow N = \frac{1000}{68.27\%}$$

500 and 600 are also symmetric around avg.

Check the normal table :

$$Z = \frac{600 - 550}{100} = 0.5 \Rightarrow \text{Area} = \underline{38.29\%}$$

$\therefore$ number of students within [500, 600]

$$= \text{Area} \times N$$

$$= 38.29\% \times \frac{1000}{68.27\%}$$

$\therefore$ the percentage is =

$$\frac{\text{number of students within } [500, 600]}{\text{-- -- -- -- -- -- -- -- -- -- -- -- } [450, 650]}$$

$$= \frac{38.29\% \times \frac{1000}{68.27\%}}{1000}$$

$$= \frac{38.29\%}{68.27\%}$$

$$= 56.08\%$$

6. Among entering students at a certain college, the men averaged 650 on the Math SAT, and their SD was 125. The women averaged 600, but had the same SD of 125. There were 500 men in the class, and 500 women.

   (a) For the men and the women together, the average Math SAT score was ______.

   (b) For the men and the women together, was the SD of Math SAT scores less than 125, just about 125, or more than 125?

(a) Mean of Men $= \bar{x} = 650 = \dfrac{\Sigma x_i}{n_1}$

Mean of Women $= \bar{y} = 600 = \dfrac{\Sigma y_i}{n_2}$

$n_1 = 500, \quad n_2 = 500$

For men and women ~~together~~,

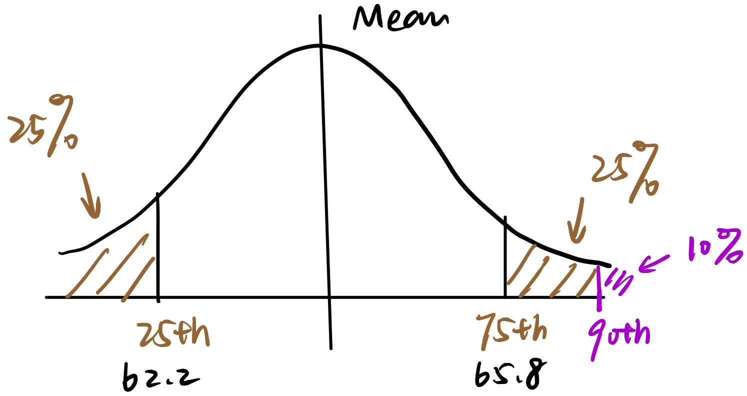
$$\text{Mean} = \frac{\sum_{i=1}^{n_1} x_i + \sum_{j=1}^{n_2} y_j}{n_1 + n_2} = \frac{n_1 \bar{x} + n_2 \bar{y}}{n_1 + n_2}$$

$$= \frac{500 \times 650 + 500 \times 600}{500 + 500} = 625$$

(b) More than 125.

Because the group is more spread out than before.

11. For a certain group of women, the 25th percentile of height is 62.2 inches and the 75th percentile is 65.8 inches. The histogram follows the normal curve. Find the 90th percentile of the height distribution.



① Mean $= \frac{1}{2}(62.2 + 65.8) = 64$

② Z score for SD:

the middle area $= 1 - 2 \times 25\% = 50\%$

check the normal table: Z score ≈ 0.68

$$Z = \frac{65.8 - \text{mean}}{SD} = \frac{65.8 - 64}{SD} = 0.68$$

$$\therefore SD = \frac{65.8 - 64}{0.68} \approx 2.65$$

③ Find Z-score of 90th percentile

the middle area $= 1 - 2 \times 10\% = 80\%$

check the normal table: Z score ≈ 1.3

④ Value: $\frac{\text{Value} - \text{mean}}{SD} = \frac{\text{Value} - 64}{2.65} = Z = 1.3$

$$\therefore \text{Value} = 1.3 \times 2.65 + 64 = 67.445$$

8. True or false, and explain briefly—
   (a) If you add 7 to each entry on a list, that adds 7 to the average.
   (b) If you add 7 to each entry on a list, that adds 7 to the SD.
   (c) If you double each entry on a list, that doubles the average.
   (d) If you double each entry on a list, that doubles the SD.
   (e) If you change the sign of each entry on a list, that changes the sign of the average.
   (f) If you change the sign of each entry on a list, that changes the sign of the SD.

(a) True. $\frac{\sum(x_i+7)}{n} = \frac{7n+\sum x_i}{n} = 7+\bar{x}$

(b) False. $\sqrt{\frac{\sum(7+x_i-(7+\bar{x}))^2}{n}} = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = SD$

So SD keeps the same.

(c) True. $\frac{\sum 2\cdot x_i}{n} = \frac{2\sum x_i}{n} = 2\bar{x}$

(d) True. $\sqrt{\frac{\sum(2x_i-2\bar{x})^2}{n}} = \sqrt{\frac{4\sum(x_i-\bar{x})^2}{n}} = 2\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = 2SD$

(e) False.
1. SD is never negative
2. change the sign of each entry mean times (-1) for each entry, so for (b) we know that SD keeps the same.

Line Calculation Ex.

Two points ((10, 20), (30, 80)) are in a line.

① Find ~~out~~ the equation of this line.

$y = a + bx$

$$\begin{cases} 20 = a + 10x & ① \\ 80 = a + 30x & ② \end{cases}$$

② − ① $\Rightarrow$ $60 = 20x$, $x = 3$

$x = 2$ to ① $\Rightarrow$ $20 = a + 10 \times 3$

$a = -10$

$\Rightarrow$ $y = -10 + 3x$

② Find out if point (22, 46) is above | below the line.

$y_{x=22} = -10 + 3 \times 22 = 56 > 46$

So is above the line.