SECTION 7

Chapter 26 and 27

CHAPTER 26

Tests of Significance

- The Null and the Alternative
- Test Statistics
- Significance Level

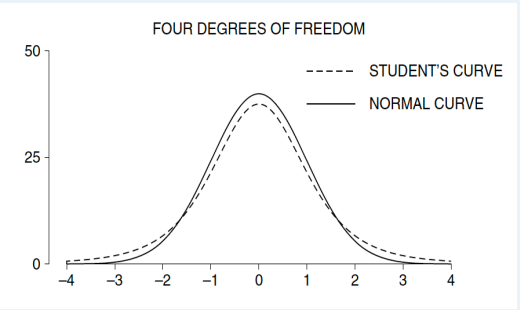
The Null and the Alternative

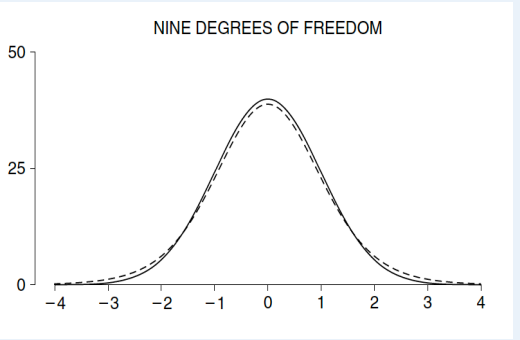
Test of significance: another type of inference (review of confidence interval)
A procedure to provide evidence for comparing observed data with a claim.

	Null Hypothesis	Alternative Hypothesis
Notation	H_0	H_1 or H_a
Definition	what someone believed to be true / a basis for argument	what someone sets out to prove
Idea	The observed difference is due to chance.	The observed difference is real.
One-sided	A parameter is equal to the null hypothesis value. (no difference)	A parameter is larger or smaller than the null value (one direction)
Two-sided	A parameter is equal to the null hypothesis value. (no difference)	A parameter is different from the null value (smaller or larger than the null value)
How to prove	Prove the null hypothesis is true.	Prove the null hypothesis is false.

Test Statistics

	z-statistics	t-statistics
SD	Known Standard Deviation	Unknown Standard Deviation
Applie when	The number of observations is large;	The number of observations is small; Current data is approximately normal
Definition	$\frac{Observed\ -Expected}{\boldsymbol{SE}}$	$\frac{Observed\ -Expected}{\boldsymbol{SE}}$
Test mean	$\frac{\overline{\boldsymbol{x}}-\mu_0}{\boldsymbol{\sigma}/\sqrt{\boldsymbol{n}}}$	$\frac{\overline{\boldsymbol{x}}-\mu_0}{\boldsymbol{S}/\sqrt{\boldsymbol{n}}}$ $S=\boldsymbol{\sigma}\sqrt{\boldsymbol{n}/(\boldsymbol{n}-\mathbf{1})}$ With number of measurement – 1 (n-1) degree of freedom





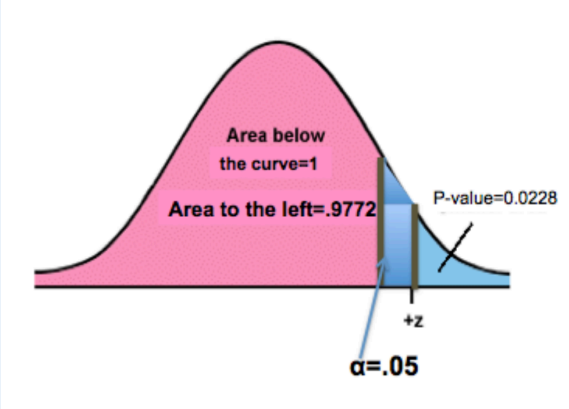
For t-distribution:
Less piled up in the middle and more spread out.

P-value (The 'Observed' Significance Level)

P-value: Given the null hypothesis is true, the probability of getting a test statistic as extreme or more than the oberved one.

α : The significance level. The probability of rejecting H_0 when H_0 is true. (Or the probability of Type I error)

- When P-value is **very small** (or if smaller than α)
  reject the null hypothesis and conclude that the alternative hypothesis is true
  (under the significance level of α)
- The smaller P-value, the stronger evidence of reject H_0
- When P-value is **NOT very small** (or if larger than α)
  CANNOT reject the null hypothesis and conclude that the null hypothesis is true
- Typical values for α are 0.1, 0.05, 0.01



(Assuming H_0 is true)

Procedures of Tests of Significance

1. State the null and alternative hypothsis (One-sided or Two-sided)
2. Calculate the test statistic (z-statistic or t-statistic)
3. Get the P-value
4. Set up the significant level and compare the P-value with the significant level α. Determine if we can accept or reject the null hypothesis.

CHAPTER 27

More Tests for Averages

- Standard Error for a Difference
- Two-sample Z Tests

Standard Error for a Difference

If the SE for the sample A is a, the SE for the sample B is b:

If the sample A and sample B are independent:

Apply the square root law: SE_diff = $\sqrt{SE_a^2 + SEb^2}$

Two-sample Z Tests

$$Z = \frac{Observed\ difference - Expected\ difference}{SE\ for\ difference}$$

$$= \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{10} - \mu_{20})}{\sqrt{SE_a{}^2 + SEb^2}}$$

Apply the same procedures a the one-sample z tests.

Note:
If the data come from a randomized controlled experiment, we can apply the procedure even the samples are dependent.

The SE for the difference between the two sample averages can be conservatively estimated as follows:

(i) compute the SEs for the averages as if the draws were made with replacement;
(ii) combine the SEs as if the samples were independent.

Exercises

5. A newspaper article says that on the average, college freshmen spend 7.5 hours a week going to parties.[14] One administrator does not believe that these figures apply at her college, which has nearly 3,000 freshmen. She takes a simple random sample of 100 freshmen, and interviews them. On average, they report 6.6 hours a week going to parties, and the SD is 9 hours. Is the difference between 6.6 and 7.5 real?
   (a) Formulate the null and alternative hypotheses in terms of a box model.
   (b) Fill in the blanks. The null says that the average of the box is _____. The alternative says that average of the box is _____.
   (c) Now answer the question: is the difference real?

3. The Gallup poll asks respondents how they would rate the honesty and ethical standards of people in different fields—very high, high, average, low, or very low.[22] The percentage who rated clergy "very high or high" dropped from 60% in 2000 to 54% in 2005. This may have been due to scandals involving sex abuse; or it may have been a chance variation. (You may assume that in each year, the results are based on independent simple random samples of 1,000 persons in each year.)

   (a) Should you make a one-sample z-test or a two-sample z-test? Why?
   (b) Formulate the null and alternative hypotheses in terms of a box model. Do you need one box or two? Why? How many tickets go into each box? How many draws? What do the tickets show? What do the null and alternative hypotheses say about the box(es)?
   (c) Can the difference between 60% and 54% be explained as a chance variation? Or was it the scandals? Or something else?