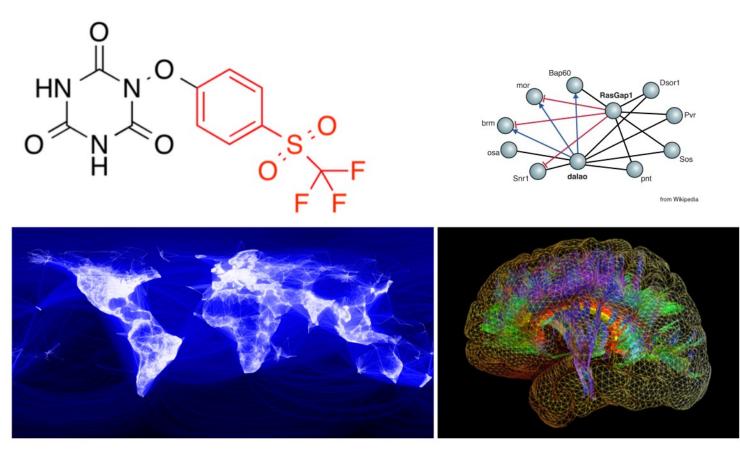
# Neural networks for graphstructured data

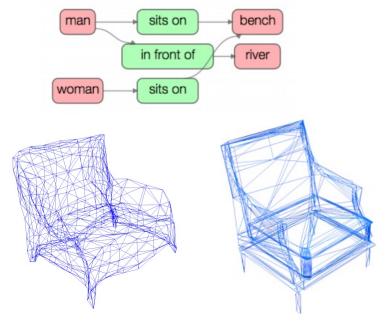
Master in Computer Vision Barcelona Adriana Romero adrianars@fb.com

#### Motivation



Graphs are everywhere!





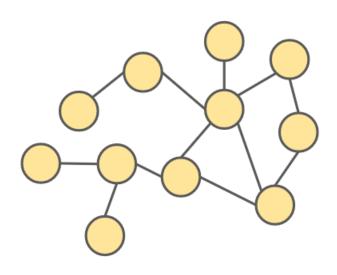
#### Outline

- Notation & problem formulation
- Node Embeddings:
  - Per-node classifier
  - Indirectly injecting structural information
- Graph Neural Networks
  - Neural Message Passing
  - Basic GNN
  - GCN
  - GAT
  - Updates and over-smoothing
- Computer vision applications
- Wrap Up

# Notation & problem formulation

#### Notation (1)

A graph  $\mathcal G$  is defined as a set of nodes (vertices)  $\mathcal V$  and a set of edges  $\mathcal E$  between nodes:  $\mathcal G=(\mathcal V,\mathcal E)$ 

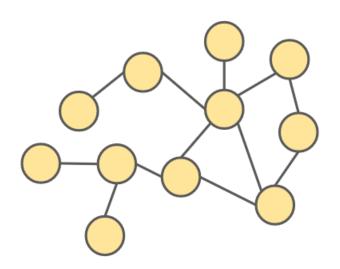


Graphs are often represented by means of an adjacency matrix A:

- A can represent directed and undirected graphs (symmetric).
- $A \in \{0, 1\}^{|\mathcal{V}| \times |\mathcal{V}|}$  to represent presence/absence of edges.
- $A \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}|}$  to represent graphs with weighted edges.
- $A \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}| \times |\mathcal{R}|}$  to represent multirelation graphs (with edges representing types of interactions)

#### Notation (2)

A graph  $\mathcal G$  is defined as a set of nodes (vertices)  $\mathcal V$  and a set of edges  $\mathcal E$  between nodes:  $\mathcal G=(\mathcal V,\mathcal E)$ 

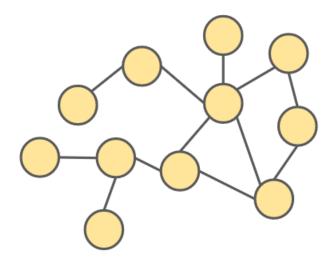


Graphs are often represented by means of an adjacency matrix A.

Nodes may be characterized by a matrix of node features  $\mathbf{F} \in \mathbb{R}^{|\mathcal{V}| \times m}$ , representing e.g. attribute information.

#### Problem formulation

Machine learning with graphs can also follow supervised, unsupervised or reinforcement learning paradigms.

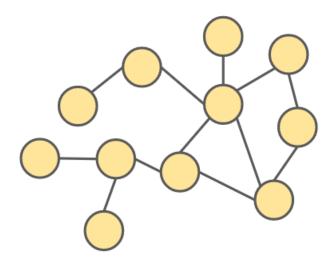


Among the most common tasks, we can find:

- Node classification
- Relation prediction
- Community detection
- Graph classification

#### Problem formulation

Machine learning with graphs can also follow supervised, unsupervised or reinforcement learning paradigms.



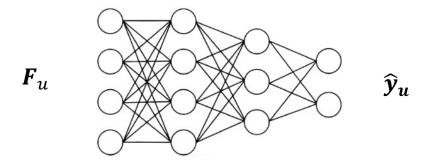
Among the most common tasks, we can find:

- Node classification (transductive vs inductive)
- Relation prediction
- Community detection
- Graph classification

# Node embeddings

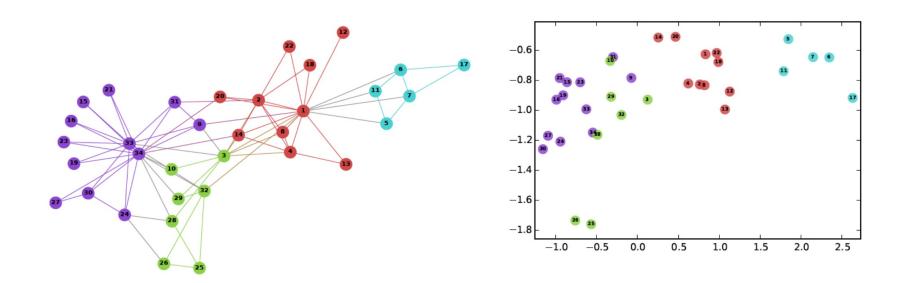
#### Per-node classifier

Classify single node features, ignoring graph structure.



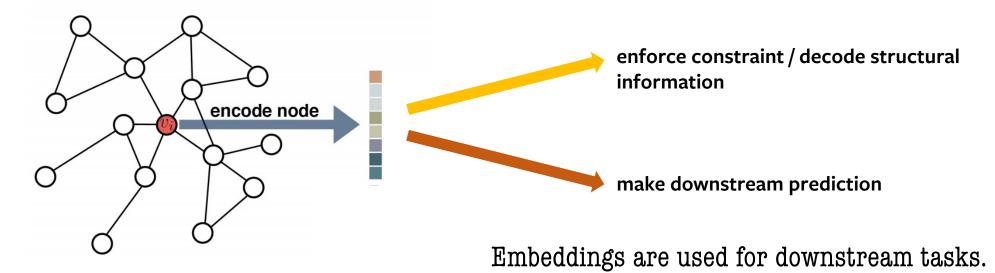
How can we leverage structural information?

Represent each node with a feature vector that preserves the structure of the graph.



#### Node embedding architectures:

- encode nodes into an embedding vector (feature vector).
- enforce some structural constraint in the embedding space or decode node embeddings into information about node and/or its neighborhood



Constraining the learnt node embeddings to be close/distant depending on the presence of edges (Weston et al., 2008, extending Belkin and Niyogi, 2006 to neural networks)

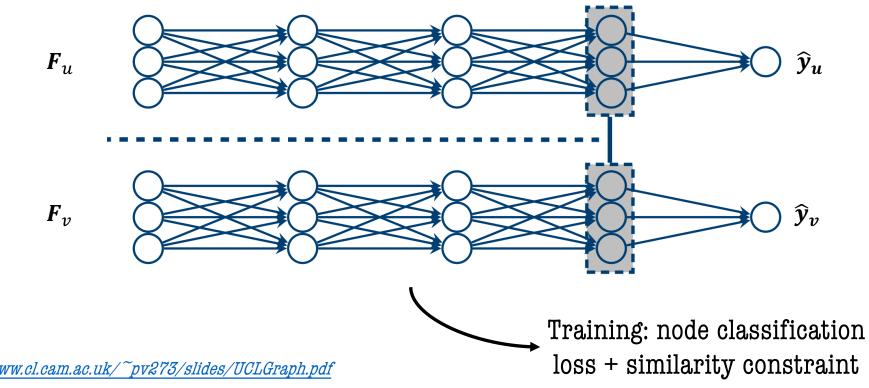
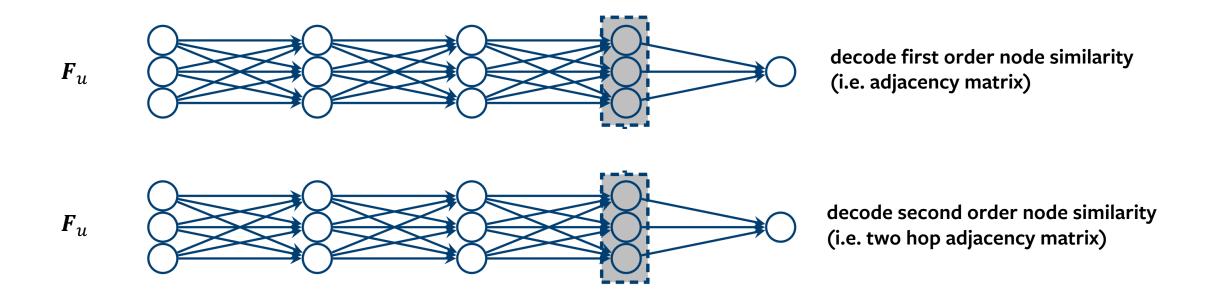
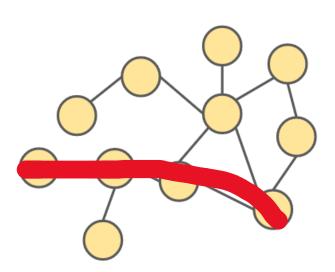


Image based on https://www.cl.cam.ac.uk/~pv273/slides/UCLGraph.pdf

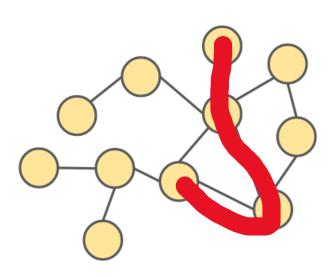
Decoding structural information (LINE – Tang et al. 2015)



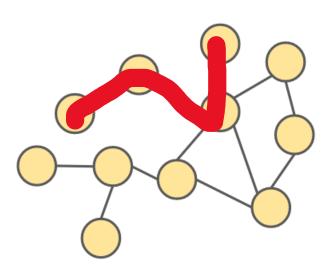
Alternatively to augmenting the loss with structural constraints, learn node structural features from random walks (DeepWalk – Perozzi et al. 2014, Node2Vec – Grover et al. 2016)



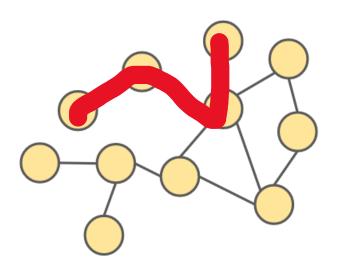
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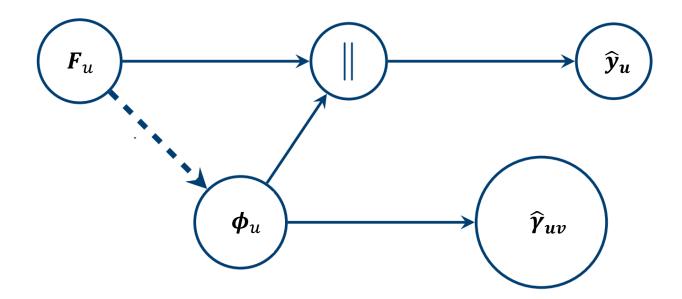
Alternatively to augmenting the loss with structural constraints, learn node structural features from random walks (DeepWalk – Perozzi et al. 2014, Node2Vec – Grover et al. 2016)



- 1. Start with random node features  $\phi_u$
- 2. Sample a random walk  $\mathcal{W}_u$  from u
- 3. Update  $\boldsymbol{\phi}_u$  to maximize  $P(v|\boldsymbol{\phi}_u)$ , where  $v \in \mathcal{W}_u$ .

A good representation of a node should allow us to easily predict the nodes that *surround* it.

What if we had access to node features and labels? (Planetoid – Yang et al. 2016)



 $\hat{\gamma}_{uv}$  predicts whether node u and v have the same label or different label.

All methods covered so far used a classifier that classifies each node independently, with graph structure injected only indirectly, through the learnt embeddings.

How can we directly leverage the graph structure when computing node features?

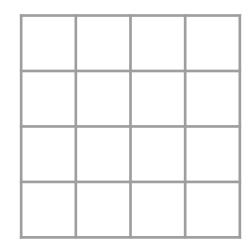
# Deep Learning toolbox (1)

**CNNs?** They work well on data defined in n-D grids



2D grid





1D grid



"I like cats and dogs."

## Deep Learning toolbox (2)

RNNs? They are well-defined over sequences.

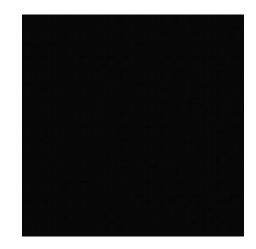
image captioning

A(0.96)

from <u>http://</u>
<u>kelvinxu.github.io/projects/</u>
capgen.html

video frame classification

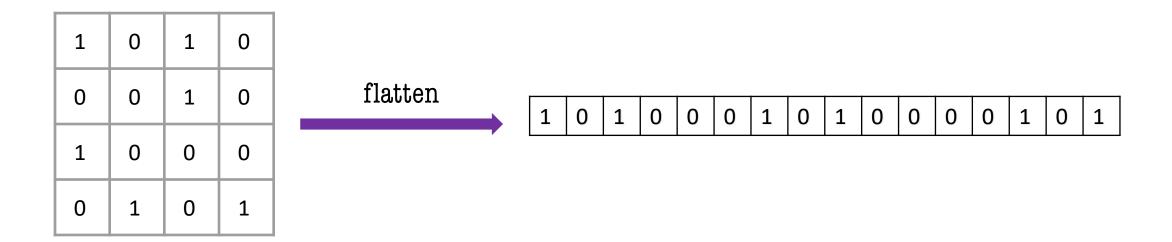
Sports Video Classification image generation (sequential processing of output)



from http://cs.stanford.edu/people/karpathy/deepvideo/ from https://github.com/jbornschein/draw

# Deep Learning toolbox (3)

#### MLPs?



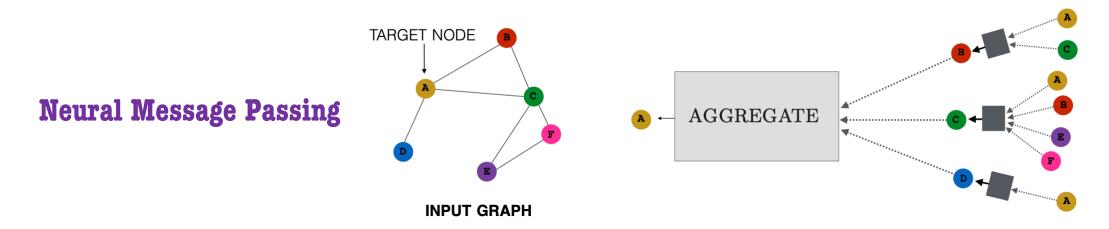
Depends on ordering of nodes in the adjacency matrix, the model would not be permutation equivariant.

# Graph Neural Networks (GNN)

## GNN (1)

Key idea: compute node representations that depend on the structure of the graph as well as node features. (Gori et al. 2005, Scarselli et al. 2009)

<u>Note:</u> The same model has been derived as a generalization of convolutions to non-Euclidean data (Bruna et al., 2014).



## GNN (2)

Formally, message passing updates are expressed as

Note that we can reformulate with self loops by considering  $\boldsymbol{H}_{u}^{(k-1)}$  within  $AGGREGATE^{(k)}$ .

$$\mathbf{H}_{u}^{(k)} = \mathit{UPDATE}^{(k)}(\mathbf{H}_{u}^{(k-1)}, \mathit{AGGREGATE}^{(k)}(\left\{\mathbf{H}_{v}^{(k-1)}, \forall v \in \mathcal{N}(u)\right\})$$

$$\mathsf{message}$$

where *UPDATE* and *AGGREGATE* are arbitrary differentiable functions (e.g. neural nets).

As iterations progress, each node contain more information from further reaches of the graph.

#### GNN (3)

Formally, message passing updates are expressed as

$$\mathbf{H}_{u}^{(k)} = \mathit{UPDATE}^{(k)}(\mathbf{H}_{u}^{(k-1)}, \mathit{AGGREGATE}^{(k)}(\left\{\mathbf{H}_{v}^{(k-1)}, \forall v \in \mathcal{N}(u)\right\})$$

$$\mathsf{message}$$

where *UPDATE* and *AGGREGATE* are arbitrary differentiable functions (e.g. neural nets).

For basic GNNs: 
$$H_u^{(k)} = \sigma(W_{self}^{(k)}H_u^{(k-1)} + W_{neigh}^{(k)}\sum_{v\in\mathcal{N}(u)}H_v^{(k-1)} + \boldsymbol{b}^{(k)})$$
 Sum of neighbor features

#### GNN (3)

Formally, message passing updates are expressed as

$$\mathbf{H}_{u}^{(k)} = \mathit{UPDATE}^{(k)}(\mathbf{H}_{u}^{(k-1)}, \mathit{AGGREGATE}^{(k)}(\left\{\mathbf{H}_{v}^{(k-1)}, \forall v \in \mathcal{N}(u)\right\})$$

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For basic GNNs: 
$$H^{(k)} = \sigma((A+I)H^{(k-1)}W^{(k)})$$

#### GNN (3)

Formally, message passing updates are expressed as

$$\mathbf{H}_{u}^{(k)} = \mathit{UPDATE}^{(k)}(\mathbf{H}_{u}^{(k-1)}, \mathit{AGGREGATE}^{(k)}(\left\{\mathbf{H}_{v}^{(k-1)}, \forall v \in \mathcal{N}(u)\right\})$$

$$\mathsf{message}$$

where *UPDATE* and *AGGREGATE* are arbitrary differentiable functions (e.g. neural nets).

features

For basic GNNs: 
$$H_u^{(k)} = \sigma(W_{self}^{(k)}H_u^{(k-1)} + W_{neigh}^{(k)}\sum_{v\in\mathcal{N}(u)}H_v^{(k-1)} + b^{(k)})$$
 Sum of neighbor

highly sensitive to node degree

#### Graph Convolutional Networks (GCN)

Key idea: neighborhood normalization motivated by spectral graph theory, resulting in first order approximation of a spectral graph convolution. (Kipf and Welling, 2017)

Message defined as: 
$$\sum_{v \in \mathcal{N}(u)} \frac{H_v^{(k-1)}}{\sqrt{|\mathcal{N}(u)||\mathcal{N}(v)|}}$$

and so, 
$$H_u^{(k)} = \sigma \left( W^{(k)} \left( \sum_{v \in \mathcal{N}(u) \cup \{u\}} \frac{H_v^{(k-1)}}{\sqrt{|\mathcal{N}(u)||\mathcal{N}(v)|}} \right) \right)$$

This formulation gives less weight to messages from neighbors with higher degrees.

#### Other aggregate functions

Key idea: use any permutation invariant function as aggregator (e.g. set aggregators).

In particular, the message could be defined by means of a set pooling operation (Zaheer et al., 2017)

$$MLP_{\theta}\left(\sum_{v \in \mathcal{N}(u)} MLP_{\phi} \left(\boldsymbol{H}_{v}^{(k-1)}\right)\right)$$

by combining a set of node features into a single embedding.

Note that the sum could be replaced by max or min operation. (Qi et al., 2017)

#### Graph Attention Networks - GAT (1)

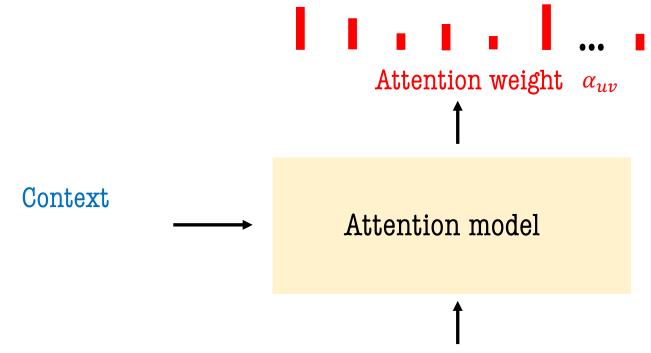
Key idea: weigh the neighbors influence at aggregation time, by learning attention weights. (Velickovic et al., 2018)

Message defined as:  $\sum_{v \in \mathcal{N}(u) \cup \{u\}} \alpha_{uv} \mathbf{H}_{v}^{(k-1)}$ 

How do we compute  $\alpha_{uv}$  ?

#### GAT (2)

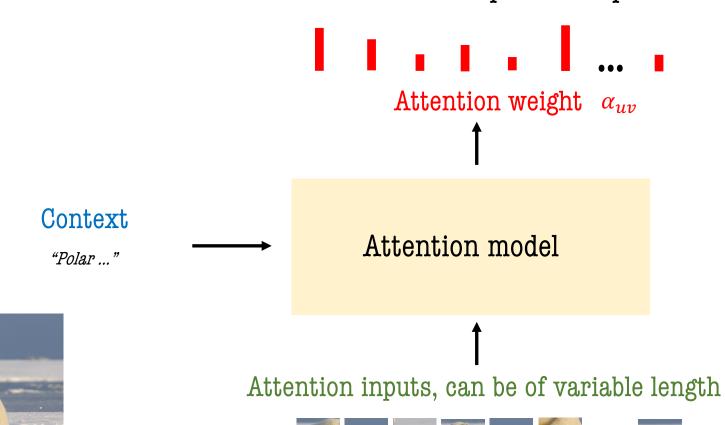
Attention allows us to focus and select the most pertinent pieces of information.



Attention inputs, can be of variable length

#### **GAT (2)**

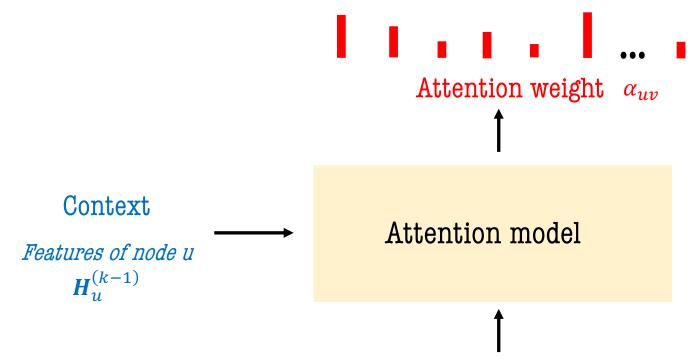
Attention allows us to focus and select the most pertinent pieces of information.



(Bahdanau et al., 2015; Vaswani et al., 2017)

#### GAT (2)

Attention allows us to focus and select the most pertinent pieces of information.

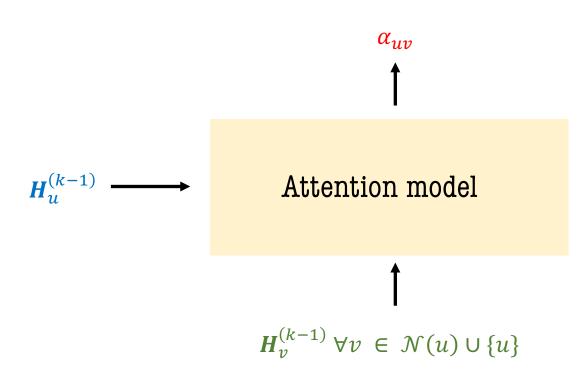


Attention inputs, can be of variable length

Features of neighboring nodes v $\mathbf{H}_{v}^{(k-1)} \, \forall v \in \mathcal{N}(u) \cup \{u\}$ 

(Velickovic et al., 2018)

#### GAT (3)



1. Combine attention input and context information

$$\boldsymbol{s}_{uv} = f_{\theta} \left( \boldsymbol{H}_{u}^{(k-1)}, \boldsymbol{H}_{v}^{(k-1)} \right)$$

2. Apply softmax to obtain attention weights  $\alpha_{uv}$ 

$$\alpha_{uv} = \frac{\exp(s_{uv})}{\sum_{v' \in \mathcal{N}(u) \cup \{u\}} \exp(s_{uv'})}$$

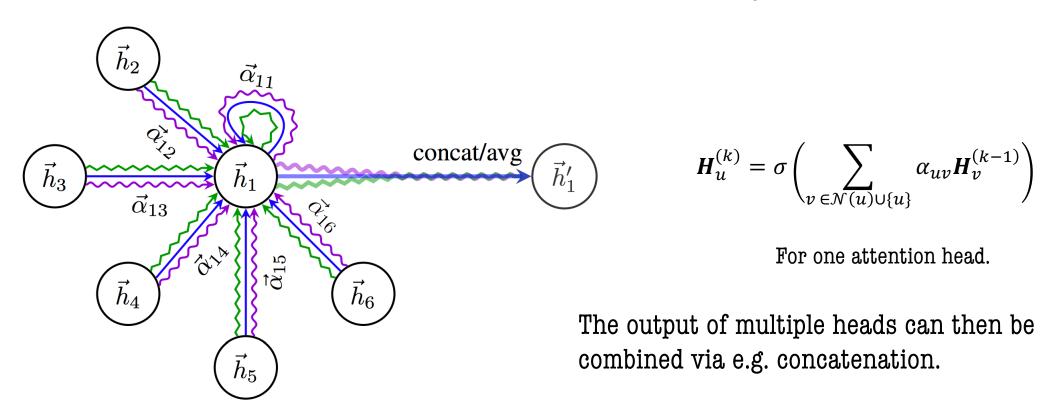
3. Apply attention coefficients to input and aggregate

$$\sum_{v \in \mathcal{N}(u) \cup \{u\}} \alpha_{uv} H_v^{(k-1)}$$

(Velickovic et al., 2018)

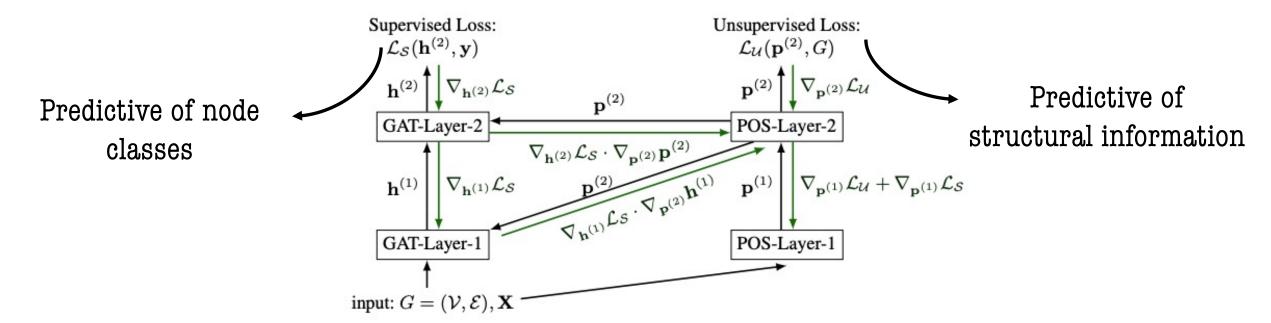
### **GAT (4)**

Following the Transformer (Vaswani et al., 2017), GAT employs multiple attention heads:



#### **GAT-POS**

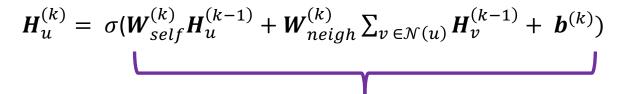
Note that GAT formulation treats nodes in a neighborhood as a set, and so structural information of the nodes is lost. GAT-POS augments GAT with positional information.



(Ma et al., 2021)

#### UPDATE & over-smoothing

Recall the basic GNN update:



Linear combination of the aggregated neighboring information and the previous representation of the node

May result in over-smoothing.

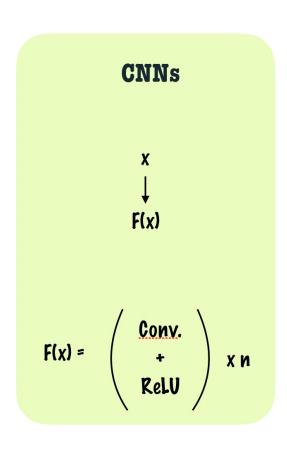
How can we address over-smoothing?

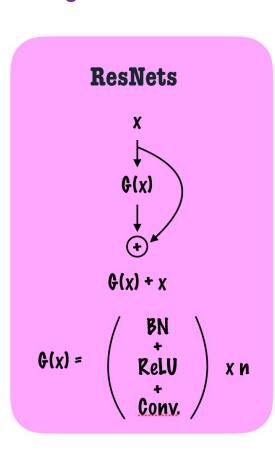
### Skip connections (1)

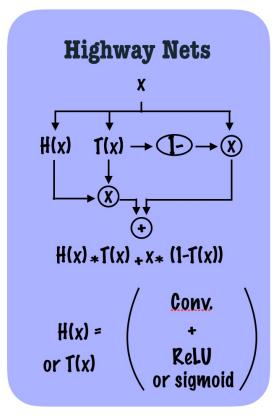
To preserve information from previous message passing iterations in the update function.

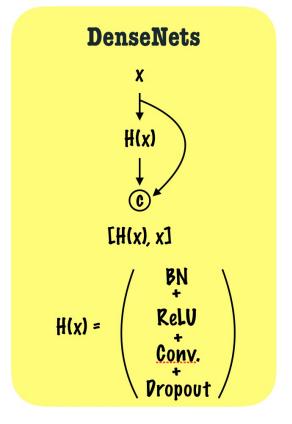
### Skip connections (2)

Review of image classification architectures with skip connections



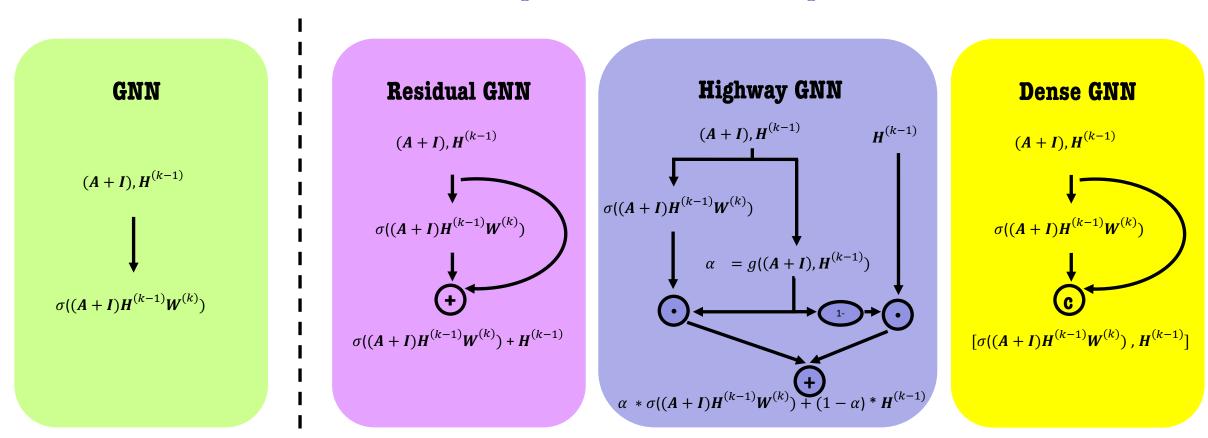






### Skip connections (3)

GNN also have their skip connection-based counterparts.



(Pham et al., 2017) (Hamilton et al., 2017 a& Xu et al., 2018)

### Zero-neighbor update

Key idea: Update only a part of the node's features with the neighbors' information.

Update defined as: 
$$\mathbf{H}^k = \sigma([\mathbf{A}\mathbf{H}'_{0:i}^k || \mathbf{I}\mathbf{H}'_{i:}^k] + \mathbf{b}^k)$$

where

$$\boldsymbol{H}^{\prime k} = \boldsymbol{H}^{(k-1)} \boldsymbol{W}^{(k)}$$



Providing a soft middle ground between full exchange of information and no

communication

#### Graph Isomorphism Networks (GIN)

Key idea: maps different node neighborhoods to different feature vectors to obtain high discriminative/representational power.

Update defined as: 
$$H_u^{(k)} = \text{MLP}^{(k)}((1 + \epsilon^{(k)})H_u^{(k-1)} + \sum_{v \in \mathcal{N}(u)} H_v^{(k-1)})$$
  
Learnable parameter.

#### RNN-based updates

Key idea: GNN updates formulated in terms of RNN-based updates, where

- the RNN hidden state  $h_t$  becomes the node's hidden state
- the RNN observation (input)  $x_t$  becomes the aggregated message

(Li et al., 2016 and Selsam et al., 2018)

$$\boldsymbol{H}_{u}^{(k)} = RNN^{(k)}(\boldsymbol{H}_{u}^{(k-1)}, AGGREGATE^{(k)}(\{\boldsymbol{H}_{v}^{(k-1)}, \forall v \in \mathcal{N}(u)\})$$

- RNN update function eg LSTM/GRU update
- Parameters are shared across nodes (same RNN cell used to update each node)
- Update function shared across (message passing) layers

#### Notes

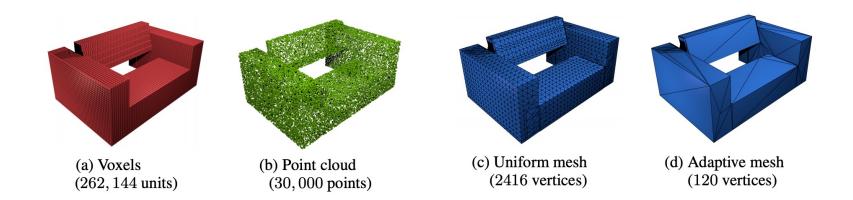
- All GNNs presented thus far iteratively update node features that can be used for any downstream task.
- In the case of node classification:
  - the non-linearity  $\sigma$  used in the last update (layer) is the softmax.
  - the loss function to train the model is analogous to the semantic segmentation one: per node cross-entropy loss.
- In the case of graph classification,
  - a pooling operation is often used to aggregate the information across all nodes.
  - the pooled features are fed to a classifier.
  - the loss function to train the model is analogous to the one of image classification: per graph cross-entropy loss.
- Most of the presented models can be augmented to handle multi-relational graphs, by separately aggregating information across different edge types.

## Computer Vision Applications

#### 3D vision (1)

Key idea: Exploit mesh representations in the context of 3D visual understanding.

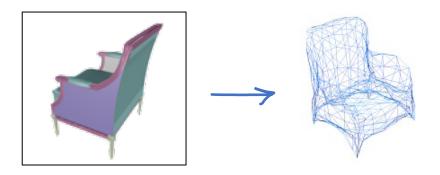
Why meshes?



GNNs for mesh reconstructions: see e.g. Wang et al. 2018, Smith et al. 2019, Gkioxari et al. 2019, Smith et al. 2021.

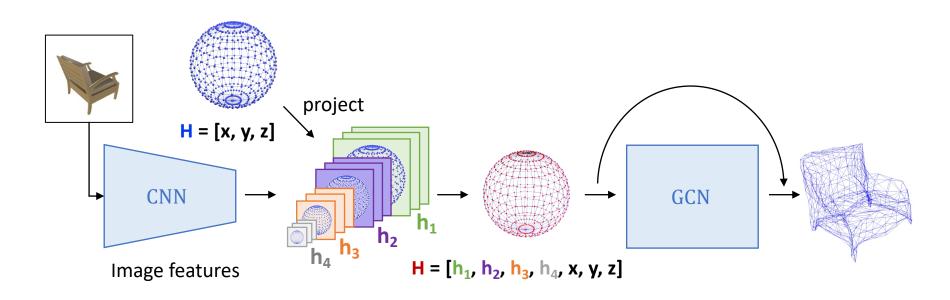
#### 3D vision (2)

Often posed as shape reconstruction from a **single view** image:



Goal: Inferring 3D shape from a single view image of an object.

#### 3D vision (3)



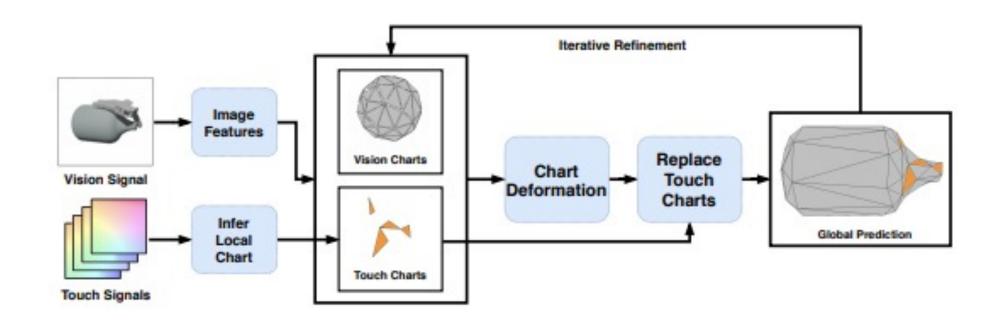
**Feature extraction** 

**Mesh deformation** 

Trained to minimize the Chamfer loss:

$$\sum_{p \in S} \min_{q \in \hat{S}} \|p - q\|_2^2 + \sum_{q \in \hat{S}} \min_{p \in S} \|p - q\|_2^2$$

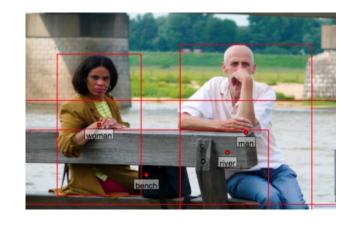
#### 3D vision (4)

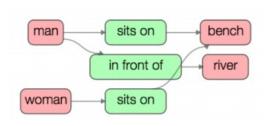


Goal: Inferring 3D shape from a single view image of an object and tactile information.

#### Relational representations

Key idea: Represent the visual world relations (e.g. positional, conceptual or from attributes) by means of graph.





Tasks: Infer relations (graph) from images or videos, exploit relations to improve model generalization, and generate images from graphs, among others.

# Wrap Up

### Wrap Up

#### Problem formulation

- What are graphs?
- How do we represent them?
- What kind of problems can we tackle?

#### Node embeddings

- Per-node classifier (ignoring graph structure)
- Indirect injection of structural information

#### **Graph Neural Networks**

- Neural Message Passing
  - Basic GNN
  - GCN
  - GAT

#### Different aggregate functions.

- Skip connections
- 0-neighbor updates
- GIN
- RNN updates

#### Different update functions.

- How to train those models?
- Multi-relational graphs

#### **Computer Vision Applications**

- 3D Vision
- Relational representations
  - Inferring visual knowledge graph from data
  - Generating visual content from scene graphs

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# Neural networks for graphstructured data

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