

Module: 3D Vision

Project: 3D recovery of urban scenes

Session 3

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Goal: compute the fundamental matrix that relates two images

#### Algorithms:

- Normalized 8-point algorithm (algebraic method).
- Robust normalized 8-point algorithm.

**Application:** Photo-sequencing.

#### Mandatory tasks:

- Function that estimates the fundamental matrix F with the normalized 8-point algorithm.
- Compute the theoretical fundamental matrix that relates two images with correponding camera matrices P = [I|0], and P' = [R|t].
- Function that robustly estimates F using the previous function and RANSAC (you can use as a basis the provided function in lab 2: 'Ransac\_DLT\_homography').

The inliers are obtained with a threshold on the first order approximation of the geometric error: **Sampson distance**,

$$\sum_{i} \frac{(x_{i}^{\prime T} F x_{i})^{2}}{(F x_{i})_{1}^{2} + (F x_{i})_{2}^{2} + (F^{T} x_{i}^{\prime})_{1}^{2} + (F^{T} x_{i}^{\prime})_{2}^{2}}$$

- Compute the epipolar lines of the matching points in both images.
- Apply the theoretical concepts to do photo-sequencing.









#### Geometric distance

(used for determining the inliers in the RANSAC function)

$$\sum_{i} d([x_{i}], [\hat{x}_{i}])^{2} + d([x'_{i}], [\hat{x}'_{i}])^{2} \text{ s. t. } \hat{x}_{i}^{\prime T} F \hat{x}_{i} = 0 \ \forall i$$

where the different matchings  $x_i \longleftrightarrow x_i'$  are the data, [.] is the projection operator to Euclidean coordinates.

#### Geometric distance

A variant is (we use the distance of a point to a line  $d(x, l) = |x^T l|/||l||$ ):

$$\sum_{i} d(x'_{i}, Fx_{i})^{2} + d(x_{i}, F^{T}x'_{i})^{2}$$

$$= \sum_{i} (x_{i}^{\prime T} F x_{i})^{2} \left( \frac{1}{(F x_{i})_{1}^{2} + (F x_{i})_{2}^{2}} + \frac{1}{(F^{T} x_{i}^{\prime})_{1}^{2} + (F^{T} x_{i}^{\prime})_{2}^{2}} \right)$$

We will use the **Sampson error** (1st order approx. of the geometric distance)

$$\sum_{i} \frac{(x_{i}^{\prime T} F x_{i})^{2}}{(F x_{i})_{1}^{2} + (F x_{i})_{2}^{2} + (F^{T} x_{i}^{\prime})_{1}^{2} + (F^{T} x_{i}^{\prime})_{2}^{2}}$$



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#### **Photo Sequencing**

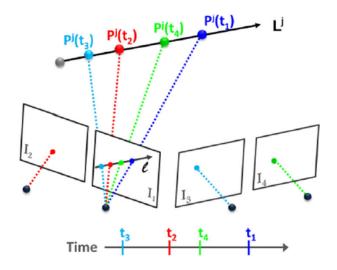
Tali Dekel (Basha) · Yael Moses · Shai Avidan

Given a set of images of a dynamic scene taken at different viewpoints and different time instants, the photo-sequencing algorithm establishes an ordering of the images according to the time they were taken.



There are two underlying hypothesis:

- Object trajectories can be approximated by straight lines.
- Two of the images are taken from approximately the same position.



Computing static and dynamic features (thanks to hypothesis 2)



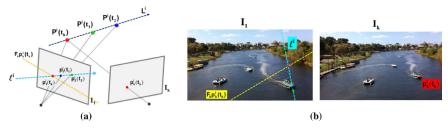
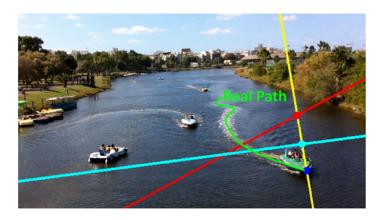
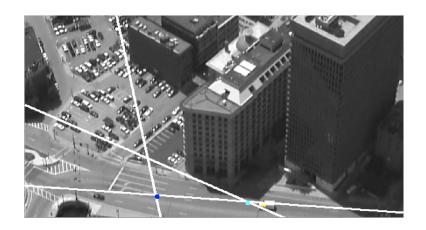


Fig. 5 (a) The projection of the trajectory,  $L^l$ , of the point  $P^l$ , forms the line  $\ell^l$  on image  $I_1$ . The feature points  $p_1^l(t_1)$ ,  $p_2^l(t_2)$ , in image  $I_1$ , and  $p_\ell^l(t_k)$  in image  $I_k$ , are corresponding dynamic features. The line  $\ell^l$  intersects the epipolar line (in yellow), which corresponds to  $p_k^l$ . The intersection point,  $p_l^l(t_k)$ , is the projection of  $P^l$  onto  $I_1$  at time

step  $t_k$ . The spatial order of  $p_1^i(t_1)$ ,  $p_2^i(t_2)$ , and  $p_1^i(t_k)$ , along  $\ell^i$ , defines the temporal order between  $I_1$ ,  $I_2$  and  $I_k$ . (b) The computation on real images: the projected trajectory,  $\ell^i$ , in cyan; the *epipolar line* in *yellow*; the intersection in *red* 



**Fig. 6** Linear Motion Assumptions: In *green*, the real path of the green boat; in yellow, the approximated 2D image line. The epipolar lines intersect both the real path and the 2D image line. The spatial order of both sets of intersections is the same



Language: Python

#### To Do:

- Complete the code in lab3.ipynb as indicated in the same file (e.g. ground-truth F, epipolar lines)
- Write the function fundamental\_matrix
- Write the function Ransac\_fundamental\_matrix
- Complete the code on photo-sequencing, dynamic feature given

### **Evaluation**

To deliver **before 9am of the day before** the next lab session:

- Code deliverable:
  - READY TO BE LAUNCHED on the provided images
- Short document (10 pages):
  - Results
  - Problems and comments, conclusions

### **Evaluation**

### **Grading:**

• Report: 2 points

• Normalized 8-point algorithm: 2 points

• F from P1 and P2: 1 point

• Robust 8-point algorithm (RANSAC): 2 points

• Epipolar lines: 1 point

• Photo-sequencing: **2 points**