M2 – Optimisation in Computer Vision
Part 2 – Poisson Editing

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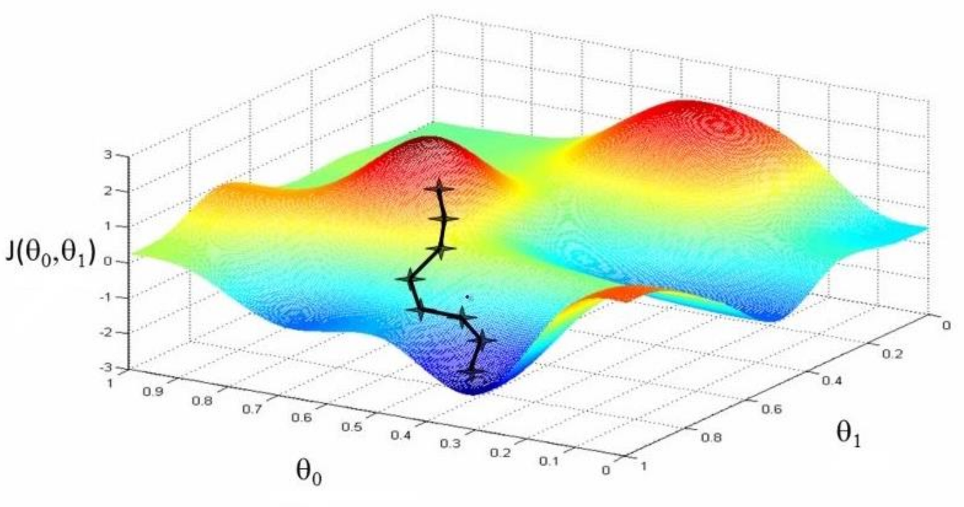
Optimisation



- The act of making something as good as possible (dictionary)
- Making the best out of situation
- Finding maximum or minimum of a function (mathematics)
- Finding the best possible solution, given some criteria

Optimisation

- **Well-posed problem**:
  1. A solution exists
  2. The solution is unique
  3. The solution's behaviour changes continuously with the initial conditions

- In computer vision, we often deal with complex **ill-posed problems**.
- Analytical solutions are often not available.
- Optimisation can help find an acceptable solution to an ill-posed problem

Optimisation in Computer Vision

1. Define a set of **criteria** to solve the computer vision problem
2. Define each criterion **semantically**
3. Then, define each criterion **mathematically**
4. Find a solution that is an “optimal” compromise of the different criteria
5. For example, find a method to minimise or maximise an energy function that is the sum of multiple terms corresponding to the different criteria

Image Inpainting

- Inpainting is the process of producing a complete image from an image with damaged, deteriorating, or missing parts

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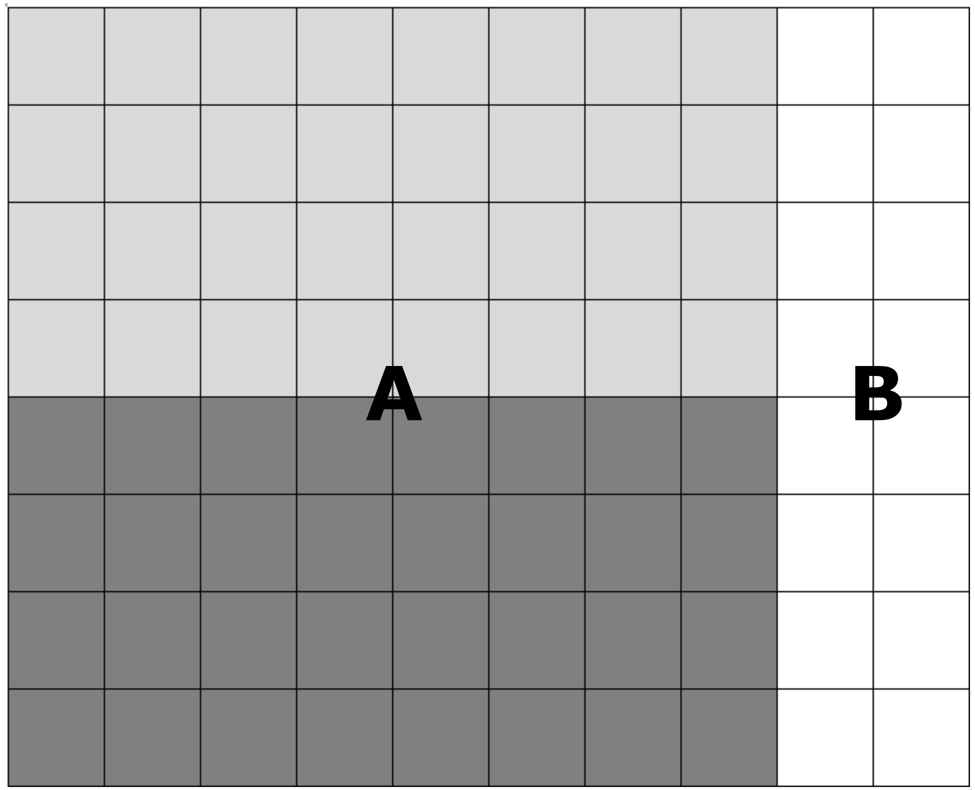
Image Inpainting

Well-posed problem



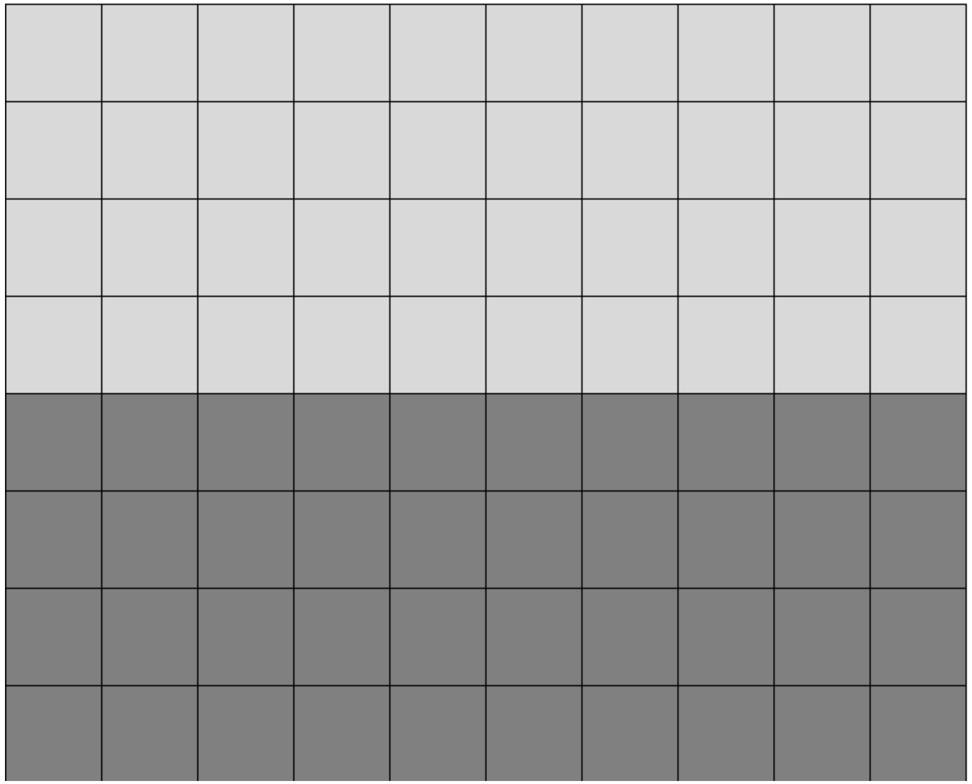
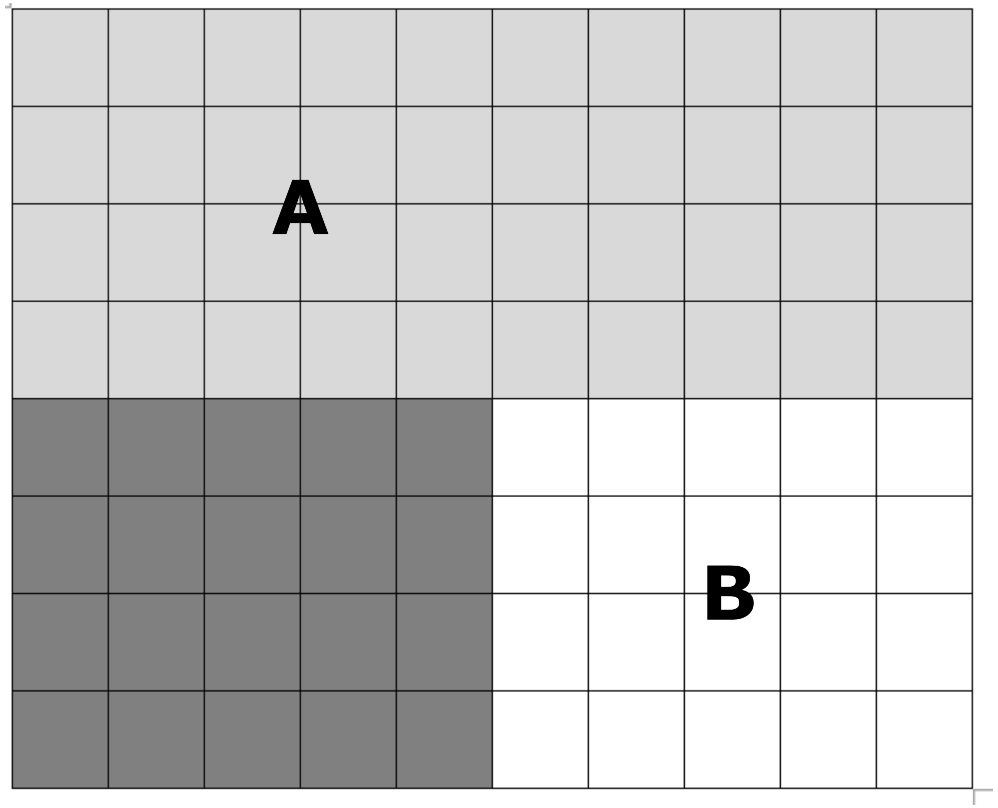


Image Inpainting

Ill-posed problem



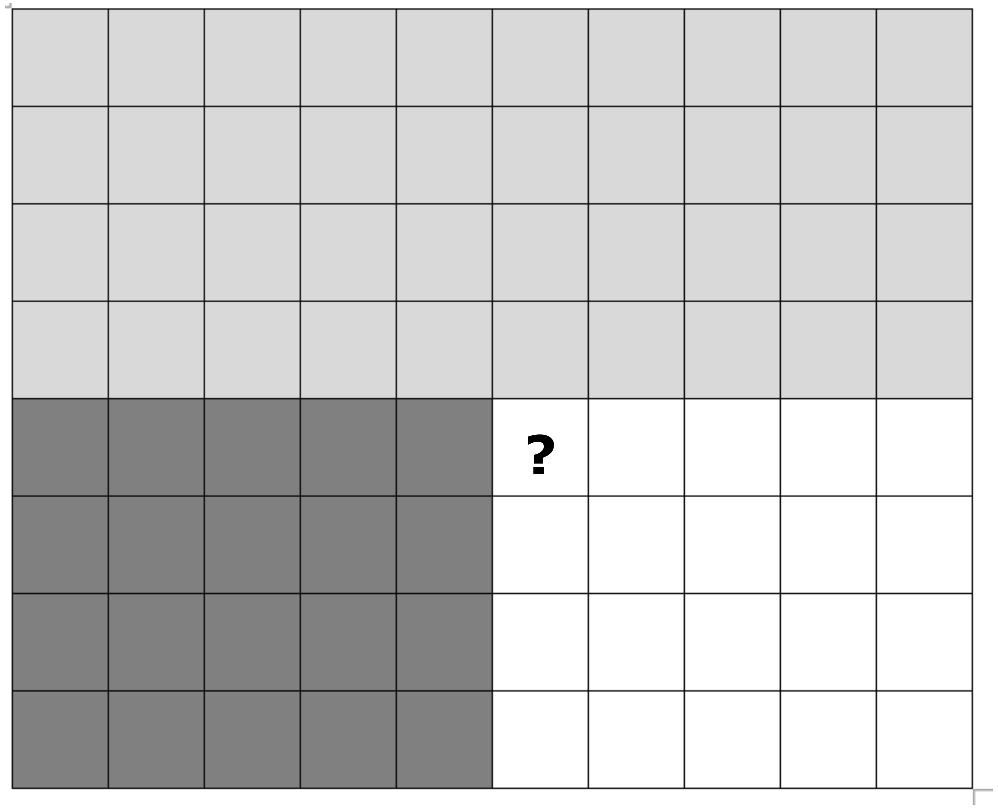


Image Inpainting

- We have an image U = (A , B), we produce a new image V

- Criteria:

  1. In A, the inpainted image should be the same in V as in U
  2. In B, the inpainted image should be smooth

$$V(x, y) = U(x, y) \text{ at each } (x, y) \text{ in A}$$

$$4V(x, y) - (V(x-1, y) + V(x-1, y) + V(x, y-1) + V(x, y+1)) = 0 \text{ at each } (x, y) \text{ in B}$$

Image Inpainting

- We must solve this for m by n pixels (e.g. 128 * 128 = 16384)
- We will have m by n equations and unknowns (system of linear equations)
- But we will work with (m+2) by (n+2) images (ghost boundaries)

$$
\begin{pmatrix}
2 & -1 & 0 & \dots & 0 & -1 & 0 & & & & \dots & & & & & 0 \\
 & & & & & & & & \dots & & & & & & & \\
0 & & & \dots & & & & 0 & 1 & 0 & & & \dots & & & 0 \\
 & & & & & & & & \dots & & & & & & & \\
0 & \dots & 0 & -1 & 0 & \dots & 0 & -1 & 4 & -1 & 0 & \dots & 0 & -1 & 0 \; \dots & 0 \\
 & & & & & & & & \dots & & & & & & & \\
0 & & & & & \dots & & & & & 0 & -1 & 0 & \dots & 0 \;\; -1 & 2
\end{pmatrix}
*
\begin{pmatrix}
v_1 \\ \dots \\ v_{i,j} \\ \dots \\ \dots \\ \dots \\ v_{(m+2)*(n+2),(m+2)*(n+2)}
\end{pmatrix}
=
\begin{array}{l}
\begin{pmatrix}
0 \\ \dots \\ u_{i,j} \\ \dots \\ 0 \\ \dots \\ 0
\end{pmatrix}
\end{array}
\begin{array}{l}
\leftarrow \text{North boundary} \\ \\ \leftarrow \text{Region A} \\ \\ \leftarrow \text{Region B} \\ \\ \leftarrow \text{South boundary}
\end{array}
$$

Size = ((m+2)*(n+2) , (m+2)*(n+2))

Size = ((m+2)*(n+2) , 1)

Size = ((m+2)*(n+2) , 1)

Code

```
A=sparse(idx_Ai, idx_Aj, a_ij, ???, ???); %??? and ???? is the size of matrix A

x=mldivide(A,b);      u_ext=reshape(x, ni+2, nj+2);


%Inner points
for j=2:nj+1
    for i=2:ni+1

        %from image matrix (i,j) coordinates to vectorial (p) coordinate
        p = (j-1)*(ni+2)+i;

        if (dom2Inp_ext(i,j)==1) %If we have to inpaint this pixel

            %Fill Idx_Ai, idx_Aj and a_ij with the corresponding values and
            %vector b
            %TO COMPLETE
```

Goal

Image A

Goal

Image B

Goal

Image H
(Incorrect)

Goal

Image H
(Correct)

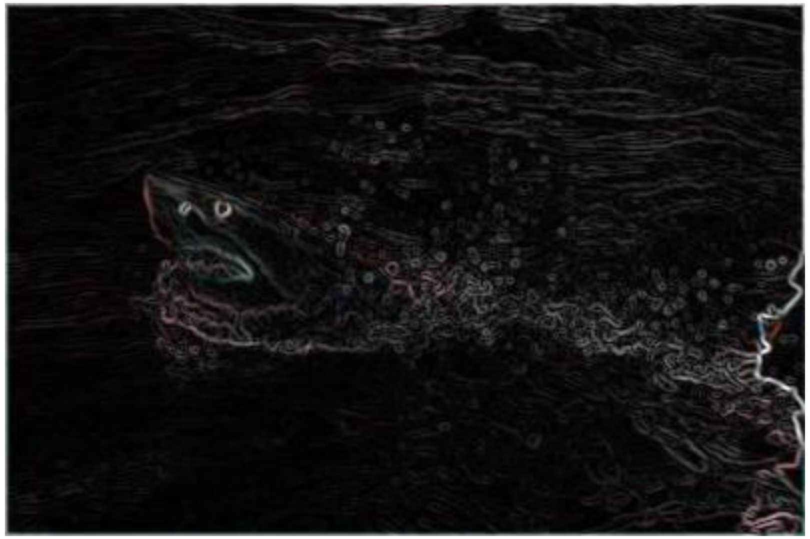
Image H (new)

Method

Image B

Method

∇B (gradient)



Method

- Criterion 1: We want the transition between A and B to be smooth

$$H(x, y) = A(x, y) \text{ at each } (x, y) \text{ of the boundary of B}$$

- Criterion 2: We want to keep the details of image B, i.e. the gradients at each point

$$\nabla H(x, y) = \nabla B(x, y) \text{ at each } (x, y) \text{ inside B}$$

Method

- Criterion 1: We want the transition between A and B to be smooth

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$$4H(x, y) - \sum H(x + dx, y + dy) = 4B(x, y) - \sum B(x + dx, y + dy)$$