Master in Computer Vision Barcelona

M1 – Pixel-based processing

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Outline

- **Introduction:**
  - **Image model definition**
- **Generic operators:**
  - **Arithmetic/Logic operations**
  - **Range transform operations**
- **Histogram based operators:**
  - **Histogram definition**
  - **Histogram equalization**
- **Summary and Conclusions**

Introduction (I)

Image Model Definition

- In the **pixel-based image model**, the image is understood as a collection of independent picture elements (pixels).
- Operations only take into account the values of the pixels (**point-wise operators**), but neither their position nor the values of their neighbor pixels.
  - Pixel-based image operators will process in the same manner all pixels with the same value.
- Pixel-based image operators can be defined:
  - In a generic way, **without taking into account the specificity of the images** they will be applied to.
  - In a specific way, **adapting the operator to the image pixel statistics**:
    - Pixels in an image are assumed to be realizations of a given random variable.

Introduction (II)

Pixel-based Image Processing Tools

- They are **very fast operators** since they only require accessing at the pixel value of the pixel being processed
  - They are **memory-less operations** since they do not require storing any neighbor pixel values.
  - Other image models require analyzing a neighborhood of the pixel being processed:
    - Space/Frequency image model: Impulse response (convolution mask)
    - Geometrical model: Structuring element
    - Region-based model: Neighborhood connectivity
- There are **three main types** of pixel-based image operators:
  - Arithmetic/Logic operators:
    - May combine various images
  - Range transform operators.
  - Histogram-based operators.

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Generic Operators (I)

Arithmetic operators

- Extension of the basic operators in a **pixel by pixel basis**
  - Addition, Subtraction, ….
  - Implementation issue: Possible representation problems due to the **change of range** of the output image

$$I_{AV}(i,j) = \frac{1}{N} \sum_{k=1,\dots,N} I_k(i,j)$$

$$N_k(i,j) = I_{AV}(i,j) - I_k(i,j)$$

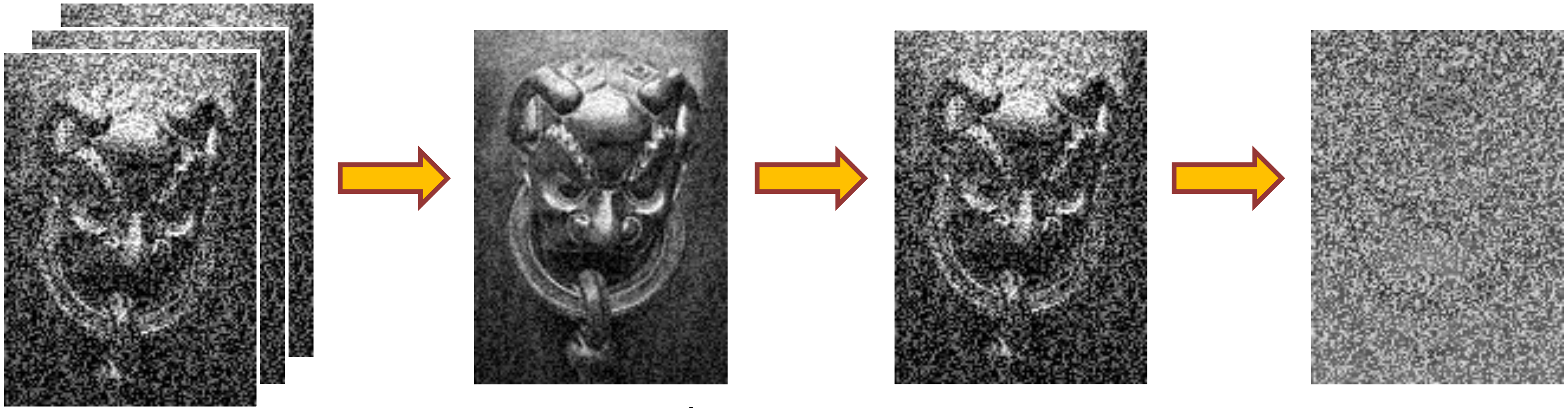


Image Averaging
Noise reduction

Change Detection
Noise estimation

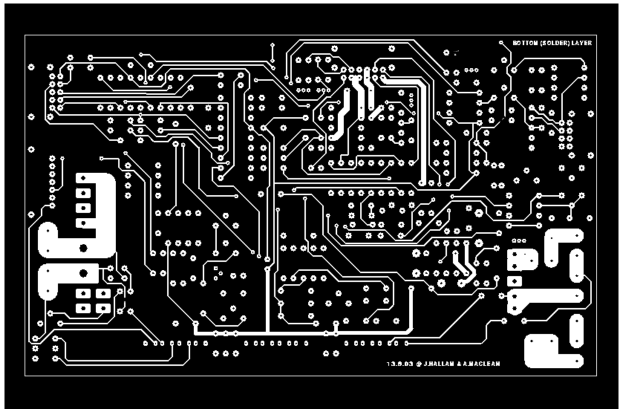
Generic Operators (II)

Logic operators

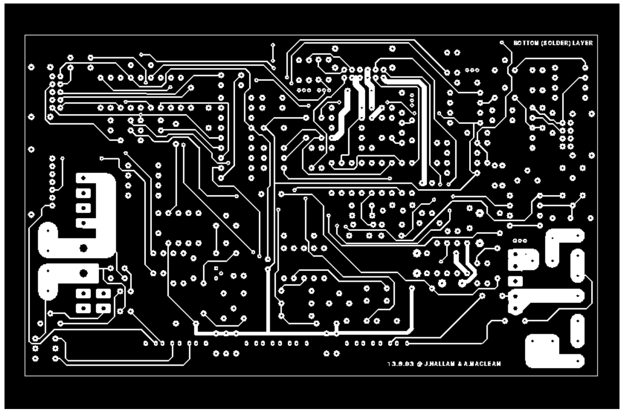
- Extension of the basic operators in a **pixel by pixel basis**
  - ADD, OR, MAX, MIN, ….
  - They are applied over binary images and require a **binarization step**.

Quality control on printed circuit board

Circuit board

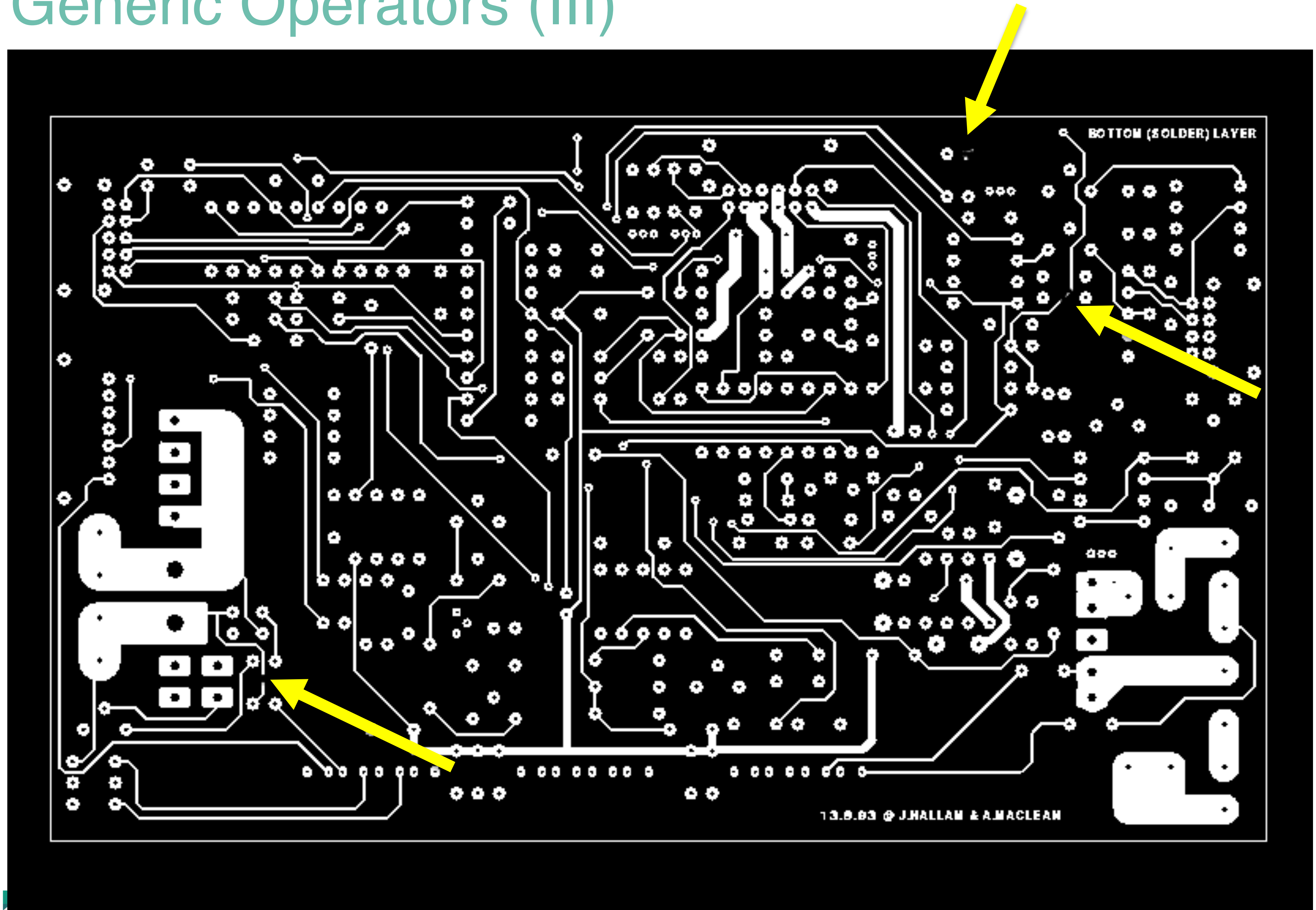


Ideal Circuit board



Logical difference

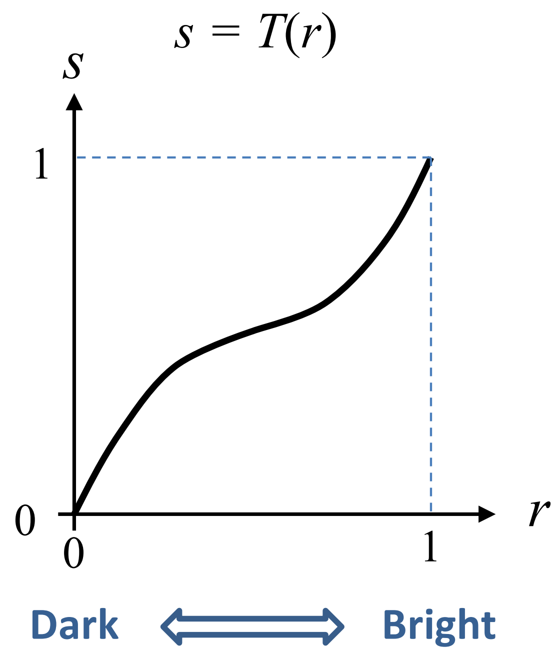
Generic Operators (III)



Generic Operators (III)

Range transform operators

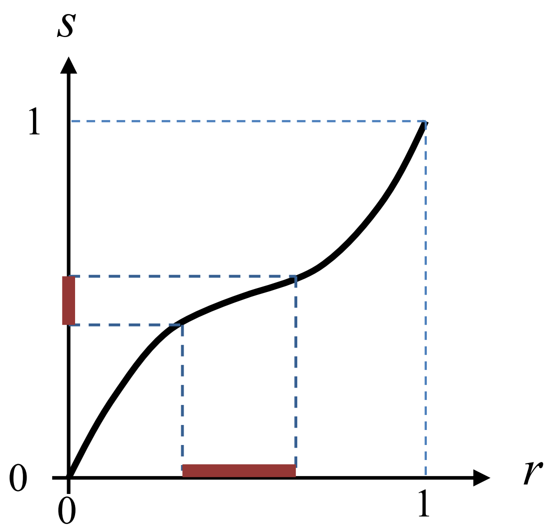
- Define a transformation or **mapping** ($T(.)$) on the range of values of the input image (r) onto the range of values of the output image (s)
  - In the examples, **ranges are normalized [0, 1]** but they may represent different real ranges.



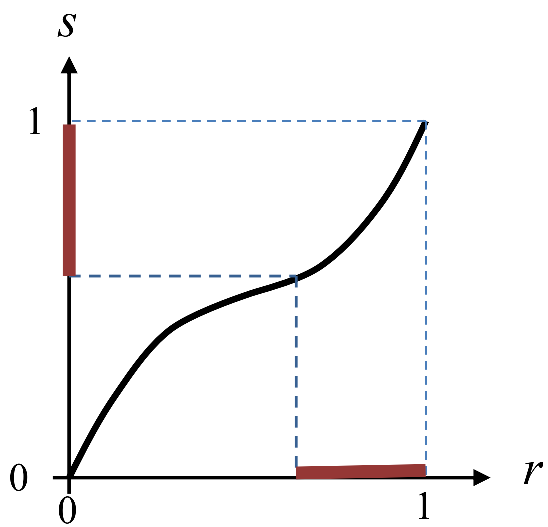
Generic Operators (IV)

Range transform operators: Grey-level mapping

- Different segments of the input range are **expanded or compressed** depending on the transform characteristics



Compression



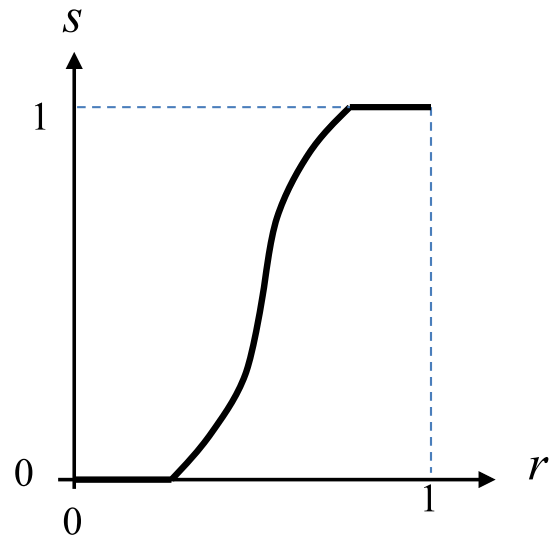
Expansion

- The segments of r where the (magnitude of the) **derivative** of $T(r)$ is greater than 1 are expanded and vice versa.

Generic Operators (V)

Range transform operators: Contrast mapping

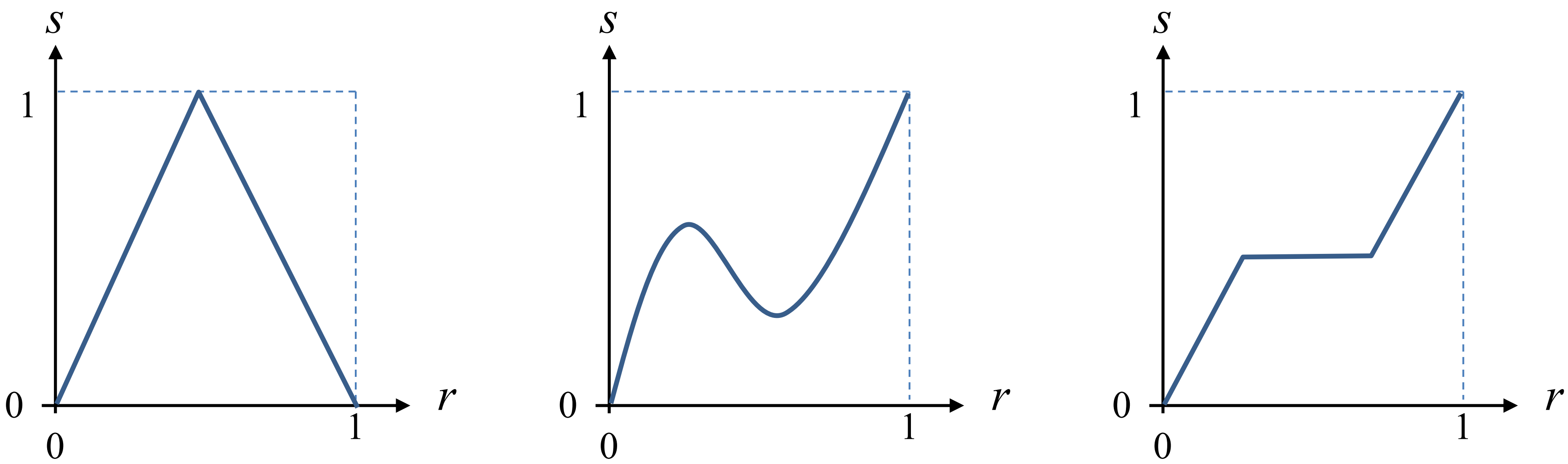
- **Expands** (stretches) a range of the input image, mapping it into the whole output image range.



- **Clipping**: A set of values of r are mapped into a single value of s.
  - Typically, lowest and highest values of r are mapped to the minimum and maximum values of s, respectively.
- This is a **non-reversible transform**: It is not bijective.

Generic Operators (VI)

Range transform operators: Invertibility

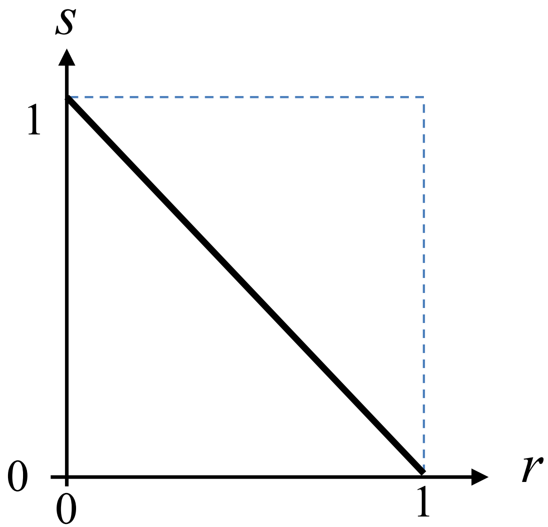


Examples of non-bijective range transform

Generic Operators (VI)

Range transform operators: Negative mapping

- **Inverts** the range of values of the input image creating a negative version of it.

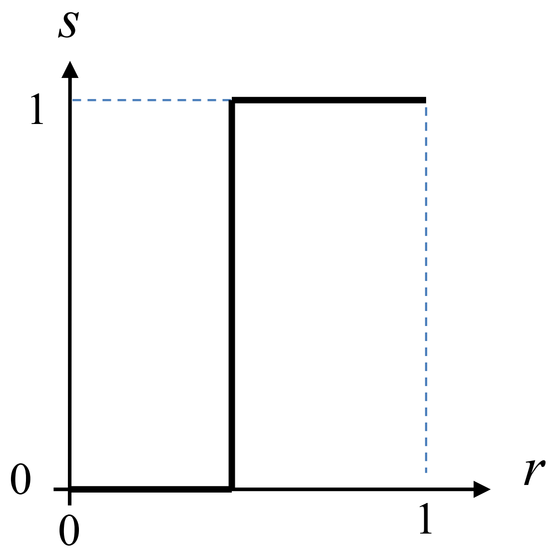


- Negative mappings **do not change the contrast** of the image:
  - The difference between two neighbor pixels remain the same.
- **The magnitude of the derivative of $T(r)$ is equal 1** in the whole range of the input image.

Generic Operators (VII)

Range transform operators: Binarization mapping

- **Binarizes** the image by clipping all values below a given threshold to 0 and all values above this threshold to 1.



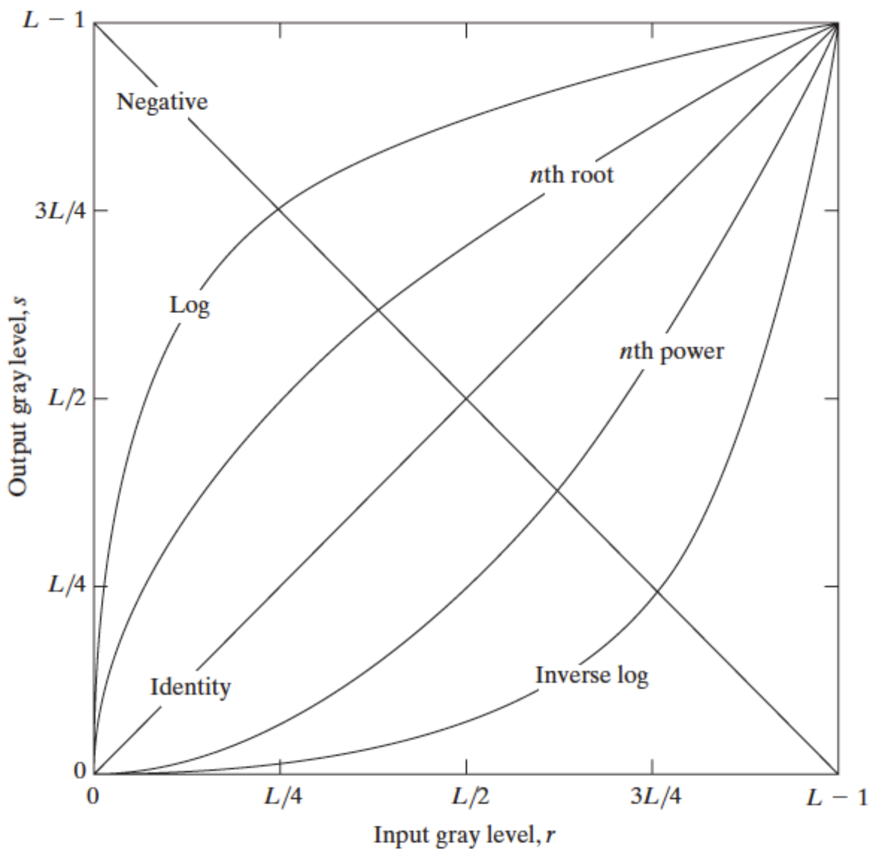
- It is commonly known as **thresholding**.
- It is a non-reversible operation, since it is based on two clippings.

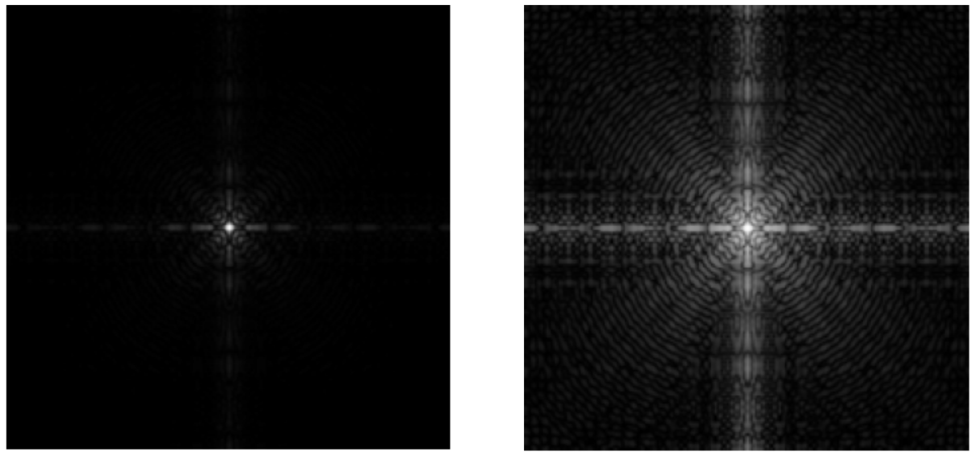
Generic Operators (VIII)

Range transform operators: Log & Power-law

- **Log transformation**: mainly used to **compress** the dynamic range

$$s = c \log(r+1)$$





Log transform of
2D Fourier transform

R. Gonzalez & R. Woods, Digital Image Processing, Prentice Hall.

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Histogram-based Operators (I)

Histogram definition

- The histogram $h(r_k)$ of a grey-level image with range [0 … L-1] is a discrete function that stores for each possible image value (r_k) **the number of occurrences** of that value in the image; that is, the number of pixels in the image with a given grey-level value.

$$h(r_k) = \#\, pixels\ with\ value\ \ r_k = n_k \quad \forall r_k \in [0 \ldots L-1]$$

- The histogram information is related to the probability of occurrence of a given value in the image. The **normalized histogram** $p(r_k)$ is an estimation of the probability density function (pdf) of a random variable associated to the grey-level values of the image pixels.

$$p(r_k) = \frac{h(r_k)}{\sum_{k=0}^{L-1} h(r_k)} = \frac{n_k}{\sum_{k=0}^{L-1} n_k} = \frac{n_k}{n} \quad \forall r_k \in [0 \ldots L-1]$$

Histogram-based Operators

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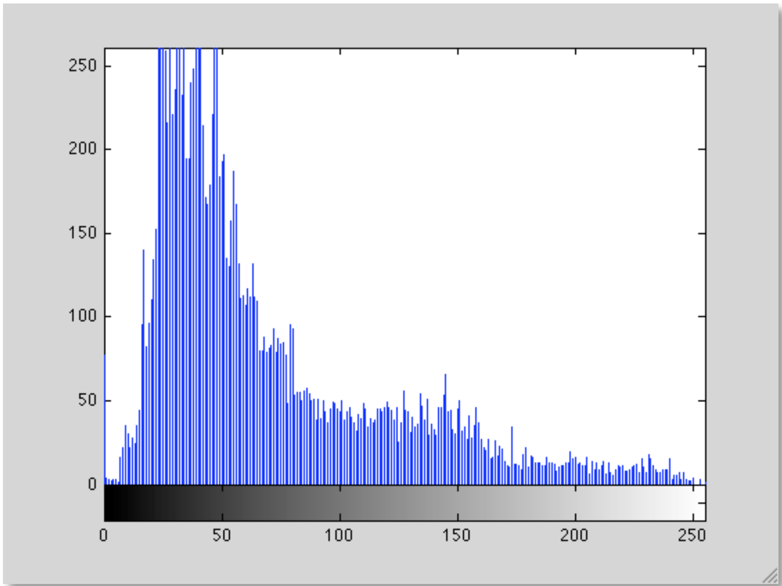

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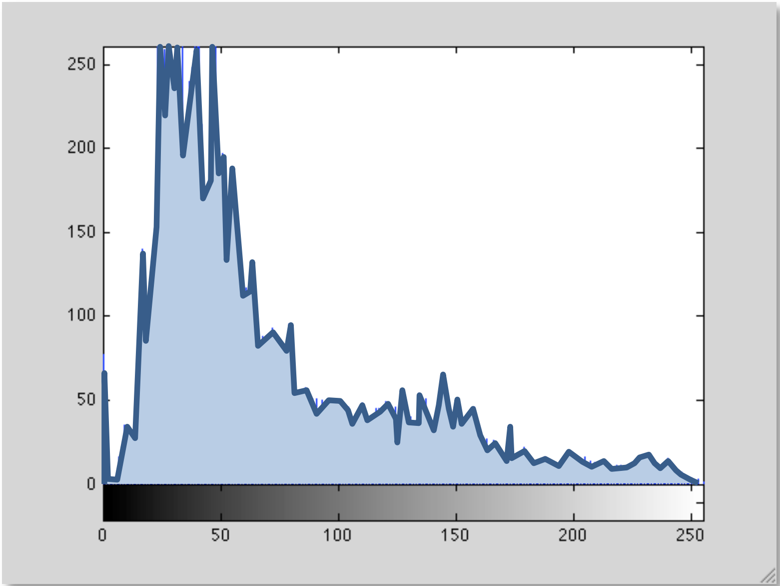
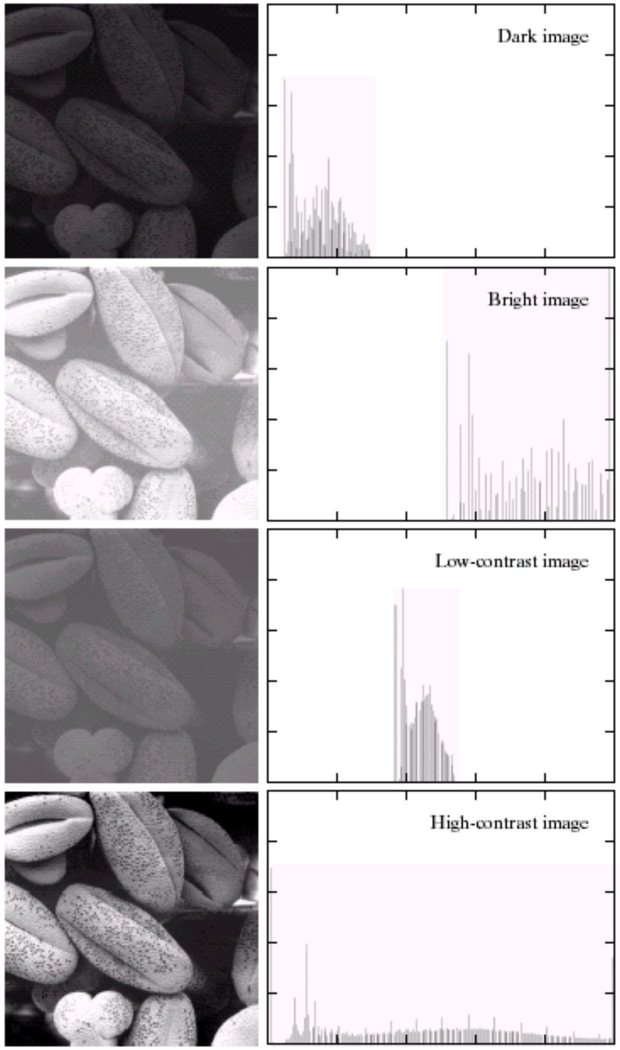


Histogram-based Operators (II)

Histogram Definition: Grey-level examples



Dark Image: Grey level values concentrated in the lowest range

Bright Image: Grey level values concentrated in the highest range

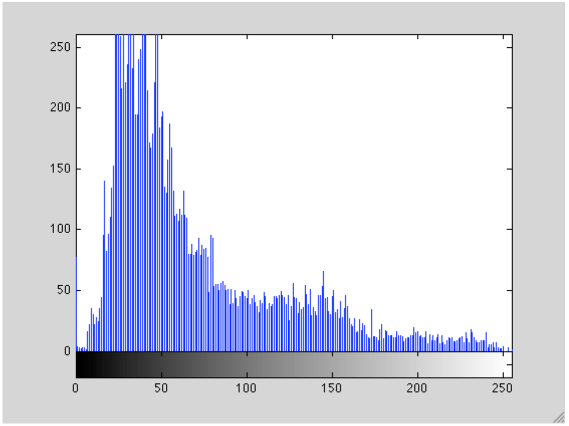
Low contrast Image: Grey level values concentrated in a small range

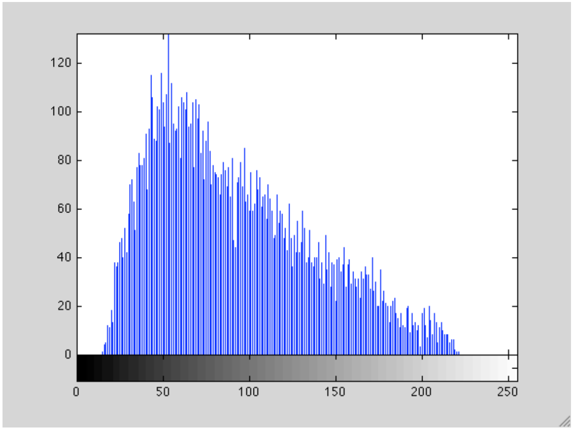
High contrast Image: Grey level values concentrated in a large range

R. Gonzalez & R. Woods, Digital Image Processing, Prentice Hall.

Histogram-based Operators (III)

Histogram Definition: Grey-level examples



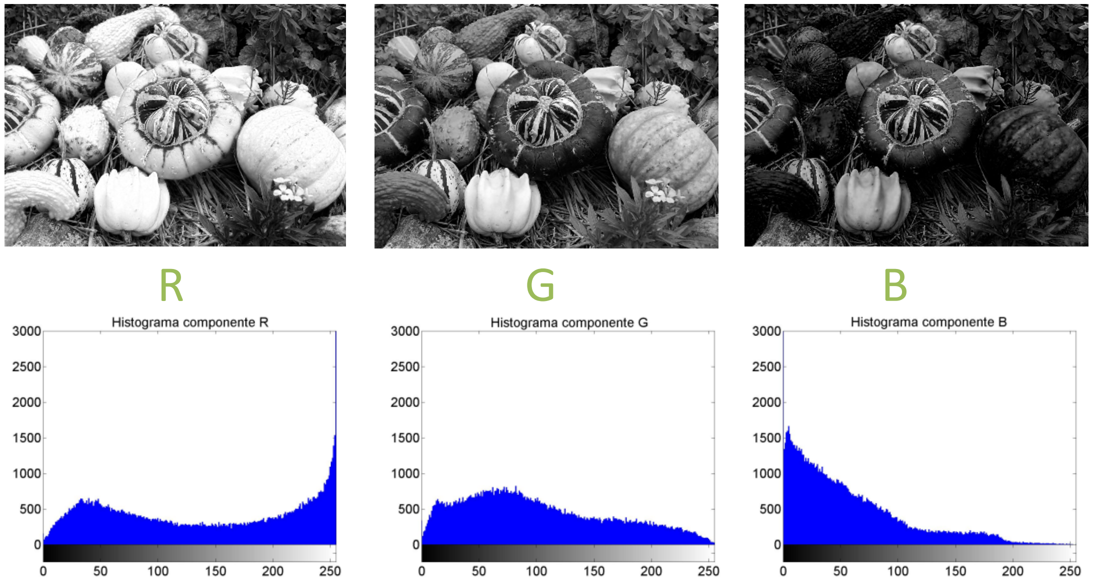


Histogram-based Operators (IV)

Histogram Definition: Color image case

- The histogram of a color image can be defined in several ways:
  1. A separate histogram for each component
  2. A 3D histogram (joint histogram)
  3. A luminance 1D histogram + a joint chrominance 2D histogram

1. A separate **1D histogram for each component**
   - It does not represent the joint probability



Histogram-based Operators

Histogram Definition: Color image case

2. A 3D histogram (**joint histogram**):
   - To count all the occurrences of every possible color (c_1, c_2, c_3)
   - A matrix of L^3 elements is created; typically, 256x256x256 (=16.777.216)

Histogram-based Operators

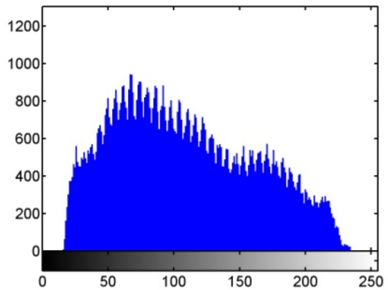
Histogram Definition: Color image case

3. A **luminance 1D histogram** + a joint **chrominance 2D histogram**

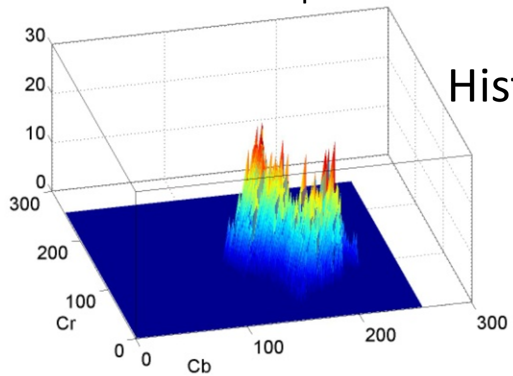
Y

C_b

C_r



Hist. Y



Hist. C_bC_r

Histogram-based Operators (VI)

Histogram Equalization: Continuous case

- Histogram equalization implements a pixel-based transform aiming at producing a **flat histogram output image**.
  - The transform depends on the input image histogram

- Consider the **pdf mapping** defined by: $s = T(r) = \int_0^r p_r(w)\,dw$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{p_r(r)}\right| = 1$$

$$\frac{ds}{dr} = p_r(r)$$

A mapping using the curve of the accumulated probability of r produces an output image **with a uniform pdf** (use equally all gray levels).

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Histogram-based Operators (VII)

Histogram Equalization: Discrete case

- The mapping for the continuous case has to be **adapted to the discrete case**:

$$s = T(r) = \int_0^r p_r(w)\,dw \iff s_k = T(r_k) = \sum_{j=0}^{k} p(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$

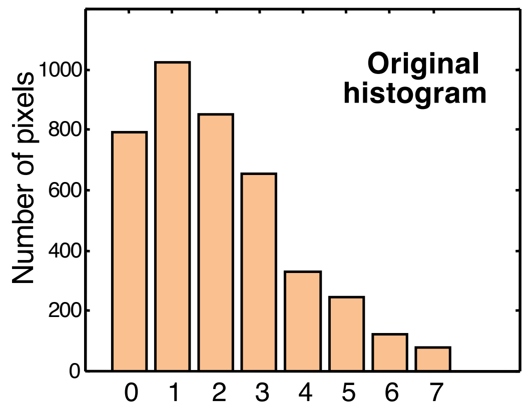
- The resulting values (s_k) are defined on the range $[0 \ldots 1]$. In order to have the values in the range $[0 \ldots L\text{-}1]$, they should be scaled and rounded. One possible approach is

$$t_k = round((L-1)\cdot s_k)$$

- The final equalization maps all pixels with value r_k into the value t_k.

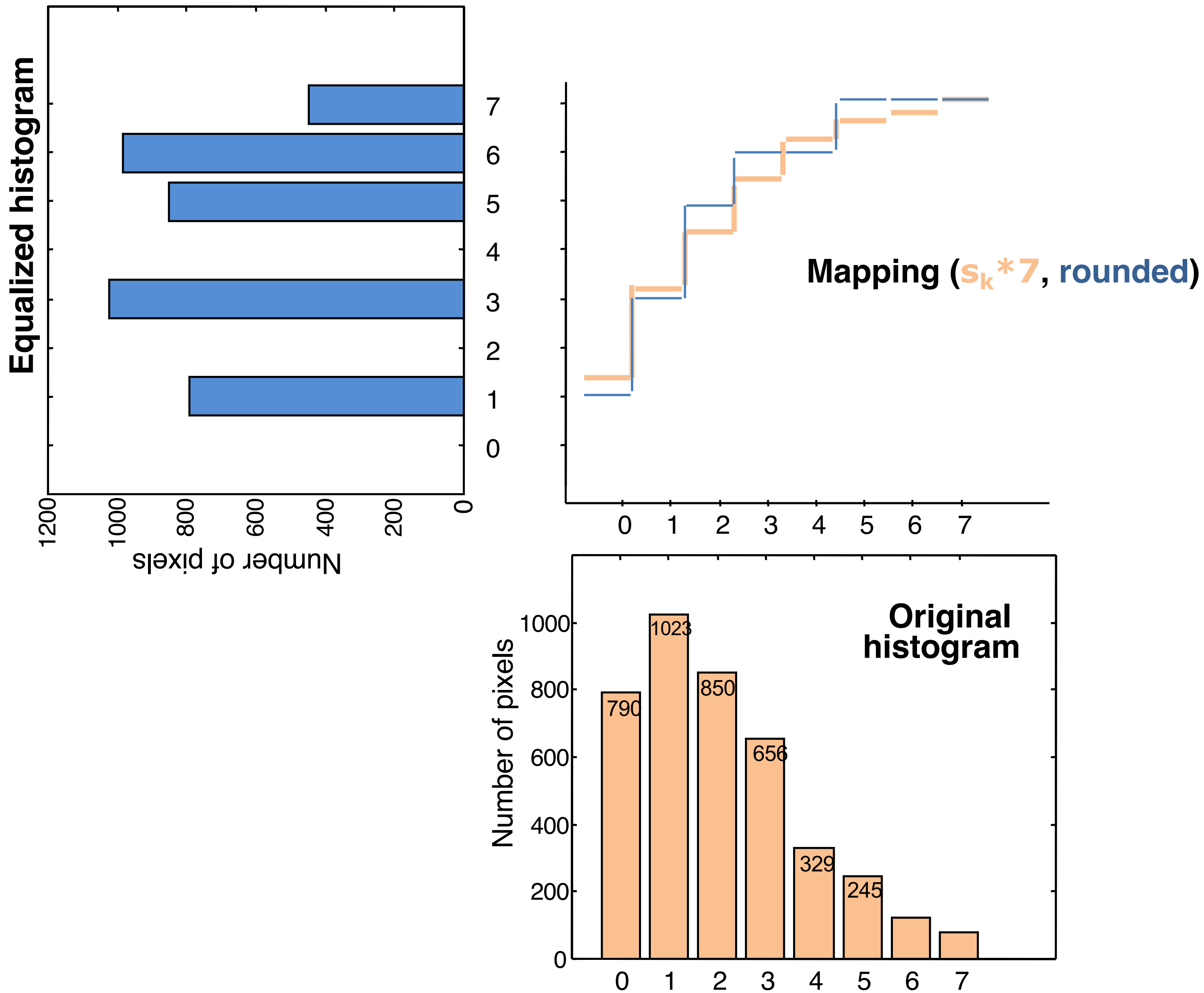
Histogram-based Operators

Discrete Histogram Equalization: Example



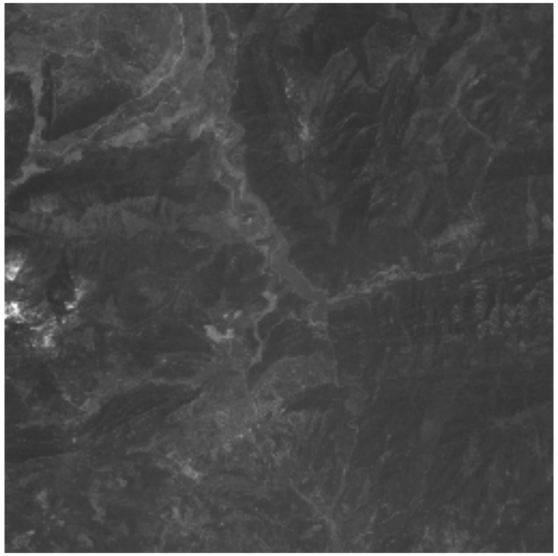
r_k	Initial level	Number of pixels	$P(r_k)$	Accumulated probability	s_k	s_k*7	Final level	t_k
r_o	0	790	0.19	s_0 = 0.19=	0.19	1,33	1	t_0
r_1	1	1023	0.25	s_1 = 0.19+0.25=	0.44	3,08	3	t_1
r_2	2	850	0.21	s_2 =0.44+0.21=	0.65	4,55	5	t_2
r_3	3	656	0.16	s_3 =0.65+0.16=	0.81	5,67	6	t_3
r_4	4	329	0.08	s_4 =0.81+0.08=	0.89	6,23	6	t_4
r_5	5	245	0.06	s_5 =0.89+0.06=	0.95	6,65	7	t_5
r_6	6	122	0.03	s_6 =0.95+0.03=	0.98	6,86	7	t_6
r_7	7	81	0.02	s_7 =0.98+0.02=	1	7	7	t_7

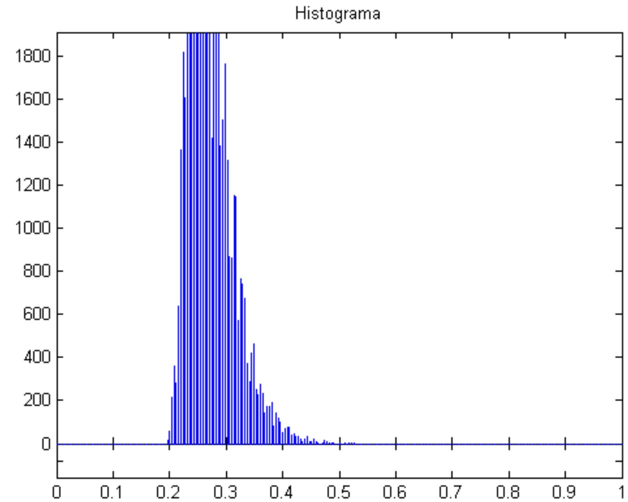
Histogram-based Operators (IX)

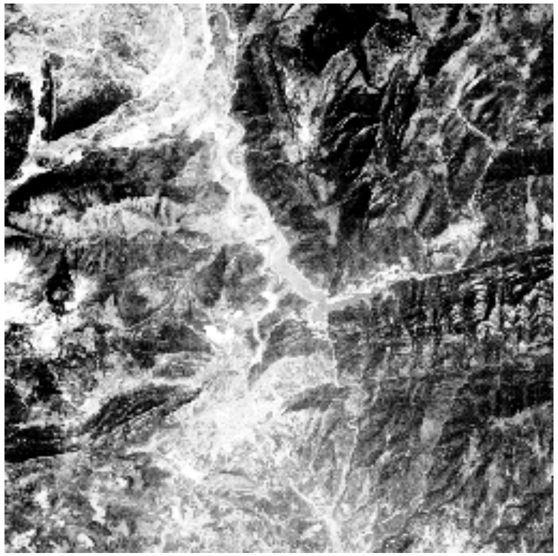


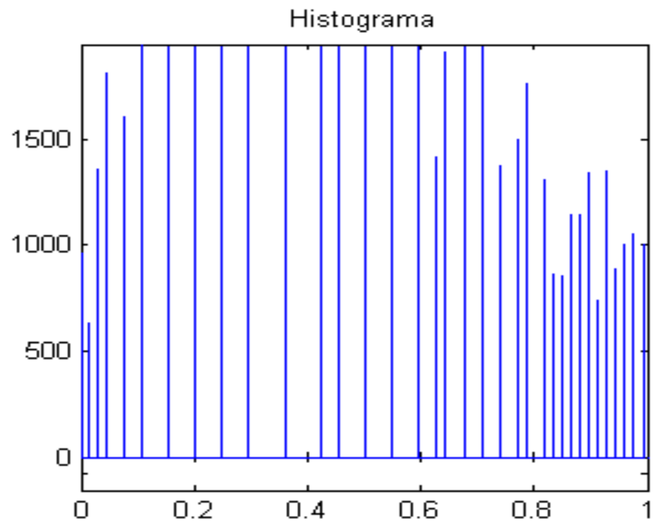
Histogram-based Operators (X)

Discrete Histogram Equalization: Example









Summary and Conclusions

- In the pixel-based image model, operations only take into account the values of the pixels (**point-wise operators**), but neither their position nor the values of their neighbor pixels.
- In range transform operations, a **mapping** ($T(.)$) is defined on the range of values of the input image (r) onto the range of values of the output image (s)
  - The mapping **expands/contracts** segments of the input range depending on the magnitude of the derivative of the transform.
  - If the mapping is not bijective, it **cannot be inverted**.
- The **histogram information** is related to the probability of occurrence of a given value in the image.
  - If the histogram of an image is known, **specific transforms** (such as the equalization transform) **can be defined** for that image.