Module: **M6. 3D Vision** **Final exam**
Date: April 28, 2016
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- Books, lecture notes, calculators, phones, etc. are not allowed.
- All sheets of paper should have your name.
- Answer each problem in a separate sheet of paper.
- All results should be demonstrated or justified.

Problem 1 *1 Point*

(a) *(0.25 points)* How do we represent a planar projective transformation in the projective space? How many degrees of freedom does it have?

(b) *(0.5 points)* How does a planar projective transformation act on points and lines?

(c) *(0.25 points)* Which are the geometric invariants for a planar projective transformation?

Problem 2 *1 Point*

Consider the problem of computing a 2D homography H between two image views of a plane objet. Let $\mathbf{x}_i$ in $\mathbb{P}^2$, $i = 1, \ldots, n$, be a set of points on the first image and let $\mathbf{x}'_i$ in $\mathbb{P}^2$, $i = 1, \ldots, n$, be a set of points on the second image such as, in pairs, they correspond: $\mathbf{x}_i \longleftrightarrow \mathbf{x}'_i$, $\forall i = 1, \ldots, n$.

(a) *(0.25 points)* What is the minimum value of n? More precisely, how many corresponding points in general position do you need to compute H such that $\mathbf{x}'_i = H\mathbf{x}_i$, $\forall i = 1, \ldots, n$?
(Recall that general position means that no three points are collinear).

(b) *(0.75 points)* Describe the Normalized Direct Linear Transformation (Normalized-DLT) algorithm to compute H.

Problem 3 *1 Point*

Consider an image of a 3D scene containing flat objects.

(a) *(0.5 points)* Explain the method of affine rectification via the vanishing line.

(b) *(0.5 points)* Explain the method of metric rectification via orthogonal lines.

Problem 4 *0.5 Points*

What is the general form of a finite projective camera matrix P? Describe its internal and external parameters.

Problem 5 *2 Points*

We are studying a system of two cameras where the intrinsic parameters of both cameras are $K = K' = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the estimated fundamental matrix of the system is $F = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ and two possible pixel correspondences are $p_1 = (5, 0)$, $p_1' = (10, 1)$ and $p_2 = (1, 1)$, $p_2' = (1, 10)$. Answer the following questions:

(a) Explain briefly the main difference between the fundamental matrix F and the essential matrix E.

(b) State very briefly the steps to estimate the fundamental matrix F using the 8-point algorithm.

(c) Justify if $e = (0, -1)$ is an epipole. What does it mean if one of the coordinates of the epipole is negative?

(d) Justify if any of the two correspondences p_1, p_1' or p_2, p_2' is an outlier.

(e) Find the essential matrix E of the system.

Let us suppose now that the matrix E of the system could be decomposed in:

$$E = UDV^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T$$

(f) Obtain the two possible translation vectors of the system.

(g) Obtain the two possible rotation matrices of the system (Note: you might need the matrices $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $Z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$).

(h) Explain briefly how would you decide the correct T and R from the 4 possible solutions obtained above.

Problem 6 *0.75 Points*

Triangulation methods.

(a) *(0.25 points)* Describe the triangulation problem, what are the unknowns and the available data.

(b) *(0.25 points)* Which is the minimization problem we need to solve when we use the homogenoeus linear method and how its solution is computed?

(c) *(0.25 points)* Which is the minimization problem we need to solve when we use the geometric method?

Problem 7 *1.5 Point*

Consider the *structure from motion* problem.

(a) *(0.25 points)* Describe the problem, what are the unknowns and the available data.

(b) *(0.25 points)* Why is there a projective ambiguity in the reconstruction if we don't assume any further information?

(c) *(0.25 points)* What are the main steps of a stratified reconstruction method?

(d) *(0.5 points)* Explain the main idea of the factorization method that allows us to estimate a solution of the structure from motion problem (just explain the essence of the algorithm, we are not asking for a detailed pseudo-code containing all the technical and numerical details).

(e) *(0.25 points)* What is the principal limitation of the data we require in the factorization method?

Problem 8 *0.75 Points*

Describe which is the projective transformation we need so as to update a projective reconstruction to an affine one. Explain how we can estimate the elements of this transformation.

Problem 9 *0.5 Points*

Depth sensors

The first generation of depth sensors (Kinect1, Asus, Structure, Orbbec ...) can be classified as non-contact distance measurement methods. Among the following alternatives, could you choose the correct sub-classification for first generation depth sensors and, for each alternative, state a reason for your choice in a few words?

(a) non-optical/optical

(b) passive/active

(c) stereo/structure from motion/shape from silhouette/light coding/time-of-flight

(d) triangulation/active stereo

Problem 10 *0.5 Points*

Point clouds

A point cloud obtained from a depth sensor can be explained as a non-regular sampling of 3D space. Why? Define what is an organized point cloud and discuss the advantages of organized point clouds over unorganized point clouds in terms of 3D processing and analysis.

Problem 11 *0.5 Points*

Depth scans and meshing

Scanning a whole object with a single sensor requires either rotating it or moving the sensor around it. In either option, describe the additional computer vision techniques you need to apply to the depth images or point clouds resulting from the sequence of scanned frames to generate a full watertight mesh of the scanned object