M4 Project
Week 2: Image Transformations

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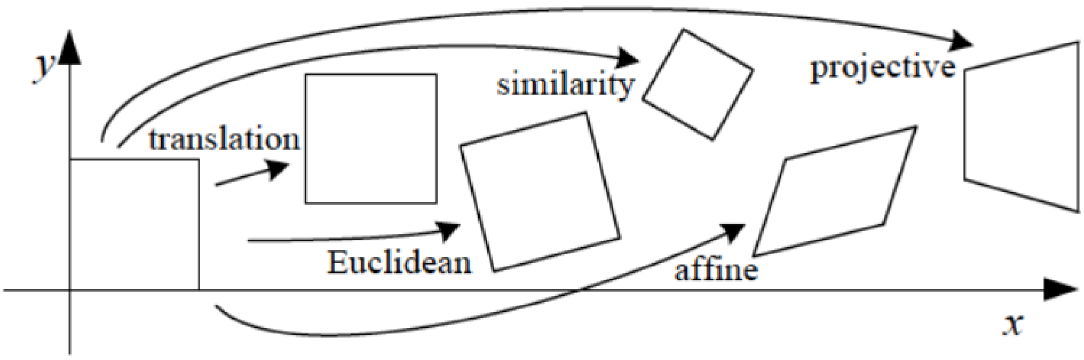
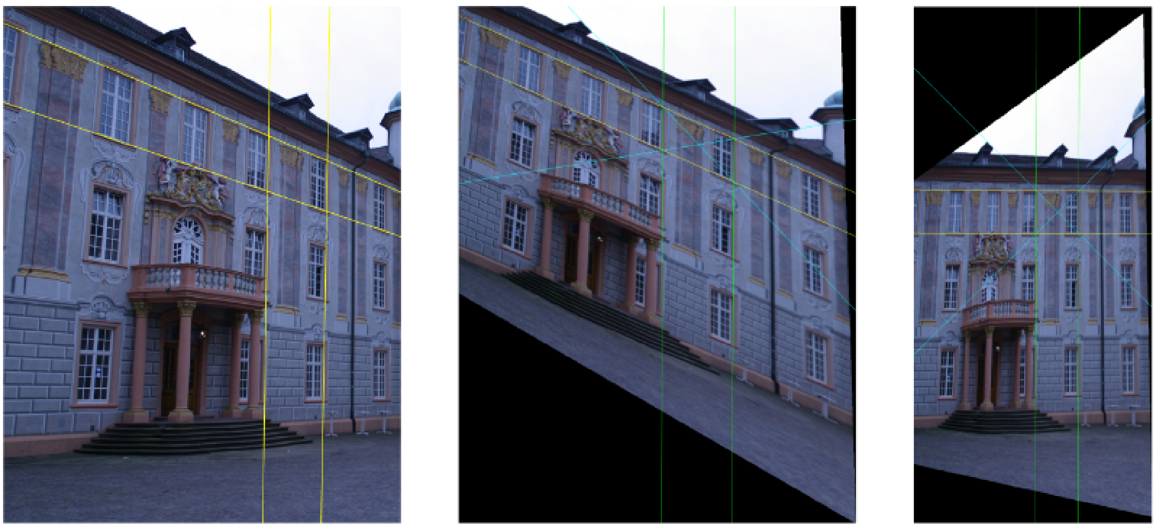
Group ID: 4

Introduction

The goal of the first session of the project is to understand different fundamental image transformations (i.e. planar transformations), and to learn affine and metric rectification as a means to correct perspective distortion and recover parallel lines and straight angles.
We use samples from the EPFL Stretcha Dataset (Facades). We can summarize the tasks:

- Function that applies a given homography to an image.
- Play with the hierarchy of planar transformations.
- Compute a line that passes through two points.
- Compute vanishing points.
- Compute a transformed line.
- Affine rectification of an image.
- Metric rectification of an image.
- (optional) Metric rectification of an image with a single step.





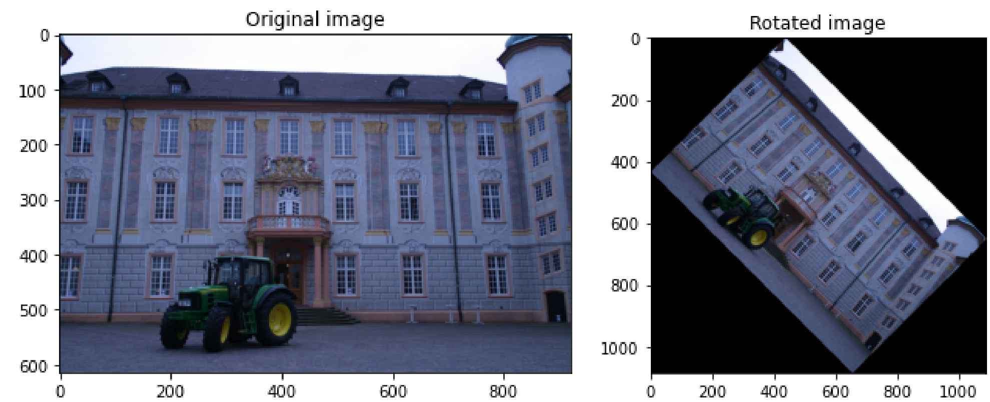
1. Image Transformations

Image transformations are the process where given an input image an output image is produced. In the context of image processing, the term “transformation” refers to a mathematical function that maps from one set X to another set Y after performing some operations (i.e. linear operations via matrix multiplications).
We proceed to define some terms used during this project.

1.1 Similarity

Similarity in the context of image processing refers to the Euclidean geometry definition where two objects are similar if they share the same shape regardless of size difference, or orientation. This means an object can be obtained from the other by scaling, rotating, mirroring, reflecting, or different combinations of multiple of the aforementioned changes.

We show the result of rotating the original image 45 degrees by applying the proper homography matrix H.

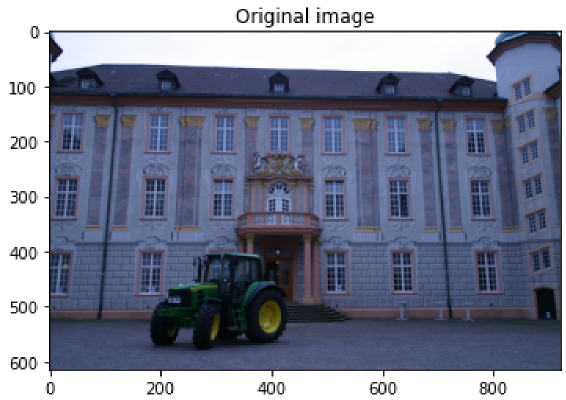


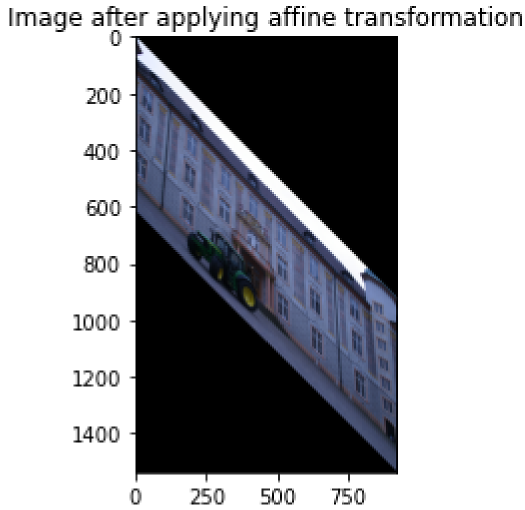
1.2 Affinity

An affine transformation is any transformation that preserves a linear relationship between two variables (collinearity) and distance ratio. Affine transformations are indicative of a special class of projective transformations that do not actually affect the position of the objects relative to the space they are in.

In the context of image processing, affine transformations are changes applied to an image that don’t affect the collinearity nor the distance ratio of an object inside the image in relation to the other objects on the image.

We show the results of an affine transformation that we decompose in four transformations: two rotations, a scale, and a translation.

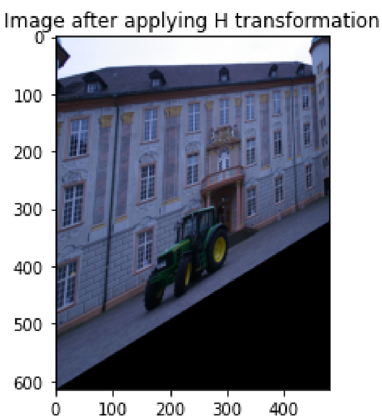


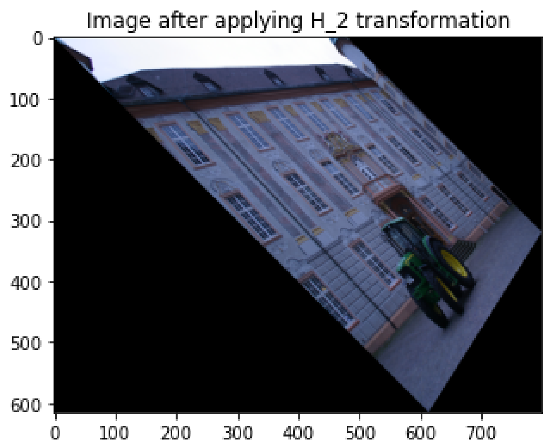


1.3 Homography (Projective Transformation)

Homography (also known as projective transformation) is the relation of any two images in the same planar space. In image processing, from an input image, an output image is generated with a line-to-line mapping, meaning that the lines on the input image correspond to the lines in the output image.

We apply different projective transformations to the original image below:

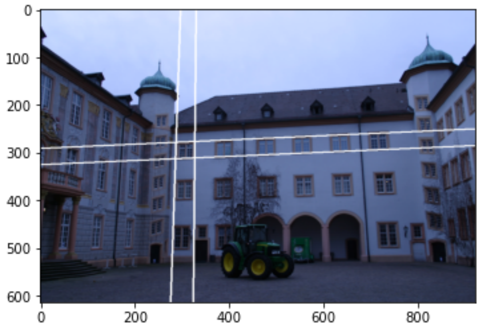


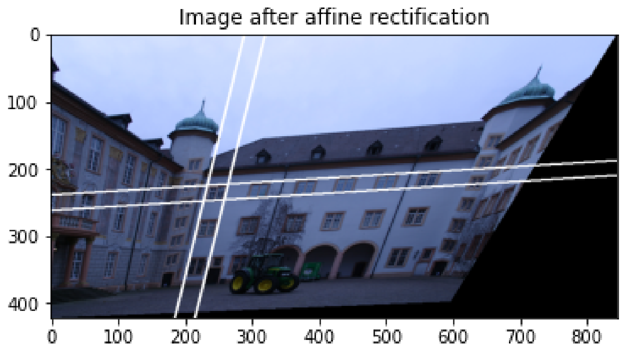


2. Affine Rectification

Affine rectification is the process where, using affine transformations, a target image's projection is changed to match a desired projection.

This step is needed in order to rectify an image in a stratified way, where we first perform affine rectification and then metric rectification (i.e. 2-step rectification).
We follow the algorithm and compute the lines l1, l2, l3, l4, that are parallel in the real world, then we find the vanishing line and build the homography that affinely rectifies the image, we compute the new transformed lines lr1, lr2, lr3, lr4 and check that the lines are now parallel.



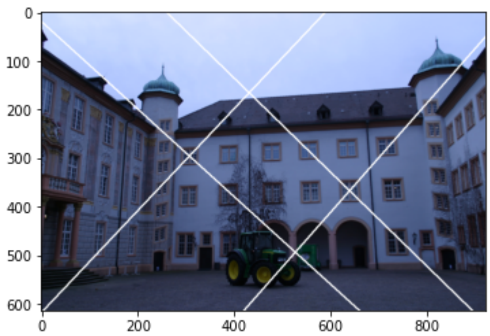


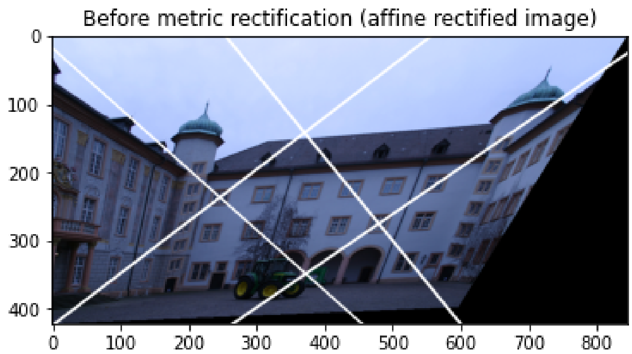
3. Metric rectification

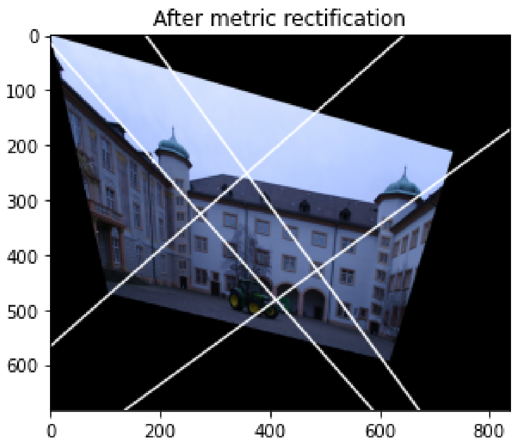
Projective and affine distortions are removed from camera images.
In the Hartley-Zisserman book (pages 55-57) we find 2 methods of metric rectification (examples 2.26 and 2.27). We refer to the fist one as the stratified method or 2-step, and the second one as the 1-step method.

3.1 Stratified method

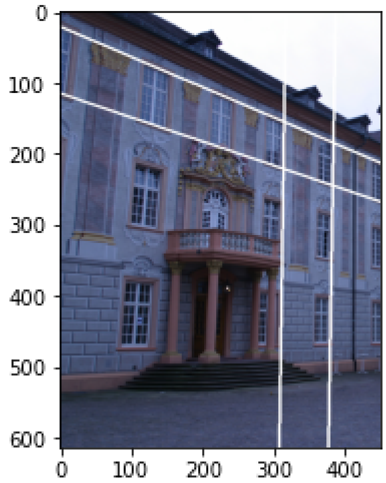
The first method is a two-step approach, in which the projective distortion is removed first and subsequently the affine distortions. Therefore, we first perform affine rectification and then metric rectification. We follow the proposed algorithm as it is straightforward.







We try the same method on the left facade of image 0001.



Original image with the orthogonal lines selected to compute the metric rectificat

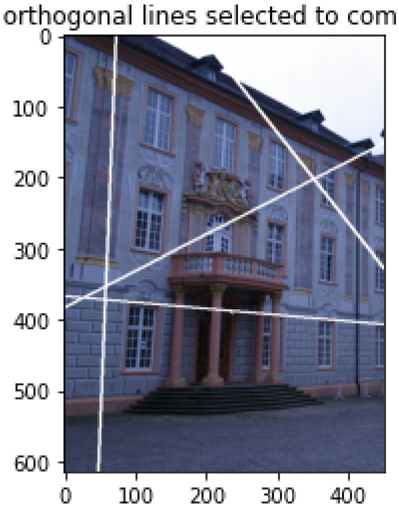
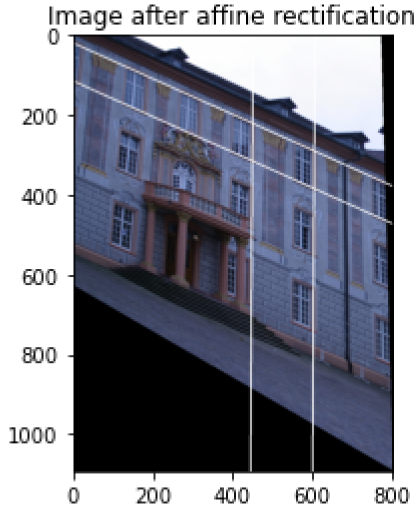
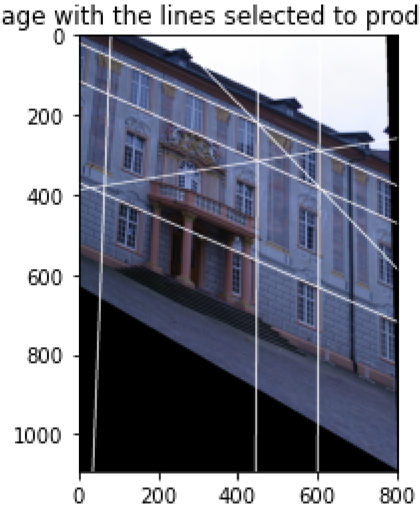


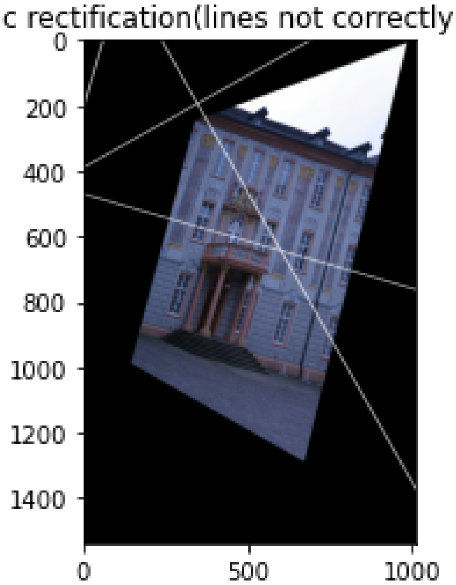
Image after affine rectification



Affine rectified image with the lines selected to produce both rectifications



After metric rectification(lines not correctly transformed)



3.1 Single step method

The single-step metric rectification method explained in the Hartley-Zisserman book (pages 55-57), "*Example 2.27. Metric rectification II*", is the metric rectification using C∗∞. As stated in the textbook, the dual degenerate conic C∗∞ contains all the information for a metric rectification. It determines both the projective and affine components of a projective transformation H and leaves only similarity distortions.

Following the method, 5 pairs of orthogonal lines have to be selected in order to solve a linear system with 5 degrees-of-freedom and 5x6 matrix. Similarly to the stratified method, we will find the conic matrix by solving the following equation:

$$\mathbf{l}^{\mathsf{T}}\mathrm{C}^{*}_{\infty}\mathbf{m} = 0$$

We can define C^{*}_{∞} as a 6 component vector **c** = (a,b,c,d,e,f) to then rewrite the previous equation to:

$$\vec{l}'^{T} C'^{*}_{\infty} \vec{m}' = (l'_1, l'_2, l'_3) \begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix} \begin{pmatrix} m'_1 \\ m'_2 \\ m'_3 \end{pmatrix}$$

$$= \left(\; l'_1 m'_1, \frac{l'_1 m'_2 + l'_2 m'_1}{2}, l'_2 m'_2, \frac{l'_1 m'_3 + l'_3 m'_1}{2}, \frac{l'_2 m'_3 + l'_3 m'_2}{2}, l'_3 m'_3 \; \right) \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = 0.$$

equiv:

$$(l_1 m_1, (l_1 m_2 + l_2 m_1)/2, l_2 m_2, (l_1 m_3 + l_3 m_1)/2, (l_2 m_3 + l_3 m_2)/2, l_3 m_3)\, \mathbf{c} = 0$$

where l_i and m_i are the components of two orthogonal lines in the real world $\vec{l} \perp \vec{m}$.
By imposing **f =1** we can then solve the linear system of equations to determine the rest of the values of the vector **c** and therefore C^{*}_{∞} as it is related to the values in **c** by this expression:

$$C^{*}_{\infty}{}' = \begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix}$$

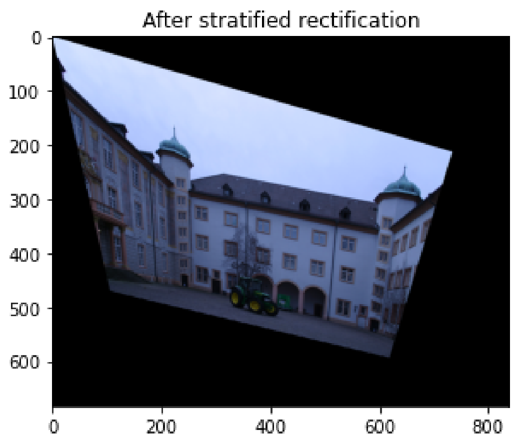
Once we have the values of the conic, we can then calculate the homography to perform the rectification by using matrix decomposition and the fact that,

$$C^{*}_{\infty} = \begin{bmatrix} KK^{T} & KK^{T}\mathbf{v} \\ \mathbf{v}^{T}KK^{T} & \mathbf{v}^{T}KK^{T}\mathbf{v} \end{bmatrix}$$

the matrix K can be calculated using the SVD method on the elements at the 2x2 matrix of the first two rows and columns. The **v** vector can be obtained from multiplying the first and second row of the last column elements that represent $KK^{T}v$, with K^{-1} and $(K^{T})^{-1}$. With this two values, we can then build the homography that rectifies the projective distortion, which will be as it follow:

$$H = \begin{pmatrix} K & 0 \\ v^{t} & 1 \end{pmatrix}$$

Below we show the results after applying each rectification:



5. Conclusions

The proper application of the matrix H is fundamental to perform any transformation.
With respect to metric rectification, the hardest part of this session, as we can see from the results, both methods remove the projective and affine distortions while preserving the similarity. Despite the obtained qualitative results look pleasant, authors declare that the first method (stratified or 2-steps) is more robust by separating it into two steps. The single step method is very sensitive to the **choice** of orthogonal line pairs that a single change of the selected pixel position value may degrade the reconstruction performance.
The main drawback of the applied method is the requirement of having parallel and orthogonal lines, which have to be previously detected, and depending on the selected lines to compute the transformations the final results may vary.

Besides the recommended additional material, we found very useful the material “Image Rectification: Remove Projective and Affine Distortions” by Rong Zhang for the course ECE661 at the University of Purdue.