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Assignment 07

- Information Theory -

1) $P(y|x)$

$P(X,Y) =$

X-age \ y-symptoms	Alkinesia y_1	Tremor y_2	Total
X_1 below 50	$0.1 \times 0.9 =$ 0.09	$0.1 \times 0.1 =$ 0.01	0.1
X_2 above 50	$0.9 \times 0.25 =$ 0.225	$0.9 \times 0.75 =$ 0.675	0.9
Total	0.315	0.685	1

$P = 0.5 \to E = 0.5$

$P(y) = \sum P(x) \cdot P(y|x)$

$P(y|x) = \frac{P(y \cap x)}{P(x)}$

$H(x)$ –

$H_{max} = \log_2 M$

$H_{max} = \log_2 2 = 1$ bit

$H(X,Y) = -(0.09 \log 0.09 + 0.01 \log 0.01 + 0.225 \log 0.225 + \log 0.675) = 1.246$

a) $H(Y) = -\sum P(y_i) \log P(y_i) = -(0.315 \log 0.315 + 0.685 \log 0.685) = 0.899$ bit/message

$H(X) = 0.469 = -(0.1 \log 0.1 + 0.9 \log 0.9)$

b) $H(Y|X) = -\sum_x \sum_y P(y,x) \log P(y|x) = H(X,Y) - H(X) = 1.246 - 0.469 = 0.777$

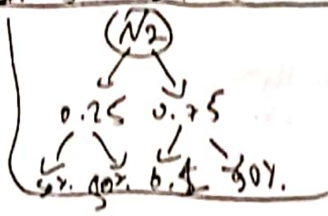
$= -(0.09 \log 0.9 + 0.01 \log 0.1 + 0.225 \log 0.25 + 0.675 \log 0.75) = 0.777$

c) $I(X,Y) = \sum_{x,y} P(x,y) \log \frac{P(x,y)}{P(x) \cdot P(y)} = H(Y) - H(Y|X) = 0.899 - 0.777 = 0.122$

2) upright — Neurons

$P_0(S=U) = 0.75 \to N_1 =$

$P_0(S=D) = 0.25 \to N_2 =$



a) $H(P_0) = 0.811$

$-(0.75 \log 0.75 + 0.25 \log 0.25)$

$H_{max} \log 2 = 1$

	X: R_{N_1} (100)	X: R_{N_1} (0)	Y: R_{N_2} (100)	Y: R_{N_2} (0)	
X_1-U	80% 0.6	20% 0.15	90% 0.68	10% 0.08	0.75
X_2-D	30% 0.08	70% 0.18	50% 0.13	50% 0.13	0.25
	x_1	x_2	y_1	y_2	1

$H(X) = -\sum P(x_i) \log P(x_i) = -[(0.8 \cdot 0.75 + 0.3 \cdot 0.25) \log (\;) + (0.2 \cdot 0.75 + 0.7 \cdot 0.25) \log (\;)]$

$H(N_1) = 1.573$

$H(Y) = -\sum P(y_i) \log P(y_i) = -[(0.9 \cdot 0.75 + 0.5 \cdot 0.25) \log (\;) + (0.1 \times 0.75 + 0.5 \times 0.75) \log (\;)]$

$H(N_2) = 1.413$

$H(X,Y) = 0$

$H(N_1) = 1.573$

$H(N_2) = 1.413$

b) $I(X,Y) = H(X) + H(Y) - H(X,Y) = 2.986$

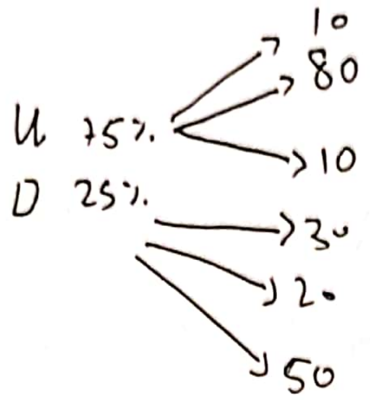
$1.573 + 1.413$

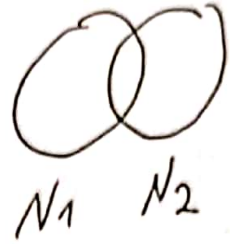
Since they fire differently!

$H_{max}(4) = 2$ bit

$H(N_1) > H(N_2)$ … 1.573 … N_1 … I.D.

c)





$H(N_1,N_2) = H(0.6+0.08+0.075) + H(0.08, 0.05, 0.125) = 1.874$ //

~~H(N1N2)~~

$I(N_1,N_2) = H(N_1) + H(N_2) - H(N_1,N_2) = 1.112$ //

$1.573 + 1.413 - 1.874$

כשהם בת"ל, מתנצאים <u>יותר</u> מידע מאשר כאשר תלויים. וכן נקבל
יותר דויים במודע כפי לקוקע אותם!

3) $\frac{\overset{x}{TS}}{0.5\%}$ | $\frac{\overset{y}{OC}}{1\%}$ $P(\overset{y}{OCD} | \overset{x}{TS}) = 40\% = \frac{P(TS/OCD)}{}$

a) $H(\overset{x}{TS}) = H(0.7, 99.3) = 0.06$ bit // $P(OCD,TS) = 40\% \cdot 0.5\% = 0.2\%$

$H(\overset{y}{OCD}) = H(1.2\%, 98.8\%) = 0.094$ bit //

0.147 bit $= H(TS,OCD) = H(98.3\%, 1\%, 0.5\%, 0.2\%)$

TS	OCD	P
−	−	98.3%
−	+	1%
+	−	0.5%
+	+	0.2%

b) $H(TS|OCD) =$ ~~H(TS) H(TS)~~

$\underbrace{1.2\% H(0.1\%/0.012, 0.2\%/0.012)}_{0.0078} + \underbrace{98.8\% \cdot H(0.983/0.988, 0.5\%/0.988)}_{0.0453} = 0.0531$ bit Avg

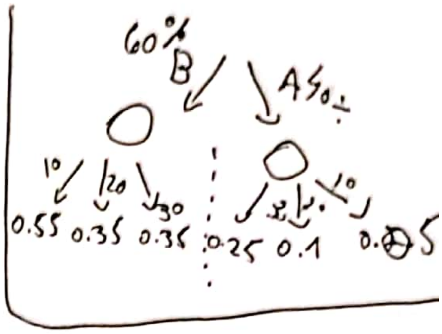
c) $I(TS,OCD) = H(TS) + H(OCD) - H(TS,OCD) =$

$0.06 + 0.094 - 0.147 = 0.007$

כמצופה אין מידע משותף בין המחלות

a) (i) $H(S,y) = 0.4\,H(0.25, 0.1, 0.25) + 0.6\,H(0.55, 0.35, 0.55)$

$0.533 + 0.887 = 1.42$ bit



$H(S) = -0.6\log 0.6 - 0.4\log 0.4 = 0.971$ bit

~~Spike count $= \sum x\,P(x) = 0.2 \times 1 + 0.1$~~

(ii) $H(\text{spike count})$ (10ms, 20ms, 30ms) $= H(0.6\times0.55 + 0.4\times0.05) + H(0.6\times0.35 + 0.4\times0.1) + H(0.6\times0.35 + 0.4\times0.25) = 1.5531$ bit Avg

(0.5301, 0.5, 0.523)

b) $H(\text{count} \mid S\ \text{Pattern}) = H(X,Y) - H(X)$

$H(S,SC) - H(S) = 1.42 - 0.971 = 0.449$ bit Avg

c) $I(10, \text{Stim}) = H(10) + H(S) - H(10,S) = 0.652$ bit

($0.371 + 0.971 - 0.69$)

$H(10,S) = H(0.55) + H(0.05) = 0.69$ bit

$H(10) = 0.6\times H(0.55) + 0.4\times H(0.05) = 0.371$

$I(30, \text{Stim}) = H(30) + H(S) - H(30,S) = 0.459$

($0.518 + 0.971 - 1.03$)

$H(30) = 0.6\times H(0.35) + 0.4\times H(0.25) = 0.518$

$H(30,S) = H(0.35) + H(0.25) = 1.03$

נראה ש 10ms נותן יותר מידע משותף (שונות גבוהה יותר) על הגירוי.

Scanned with CamScanner