Single Sideband Modulation using Hilbert Transform

Introduction

Single Sideband Modulation (SSB) or Single Sideband Suppressed Carrier Modulation(SSB-SC) is a type of modulation technique mainly used in the transmission of audio signals/voice signals. Single Sideband Modulation is an efficient form of Amplitude Modulation(AM) that uses half the bandwidth used by Amplitude Modulation reducing the power wastage on the carrier but leads to increase in cost of increased device complexity and difficult tuning at receiver end.

Hilbert Transform

The Hilbert Transform of a signal g(t) is given as the convolution of g(t) with signal $(1/\Pi t)$.
Hilbert Transform is represented as H[g(t)] or g'(t) and is given as

$$H[g(t)] = g(t) * (1/\Pi t) = (1/\Pi) \int_{-\infty}^{\infty} (g(\tau)/(t-\tau))d\tau = (1/\Pi) \int_{-\infty}^{\infty} (g(t-\tau)/\tau)d\tau$$

Single Sideband Modulation using Hilbert Transform

Considering a signal g(t)

Now $g_+(t) = (g(t) + jg'(t))/2$ ----- (1)

$g_-(t) = (g(t) - jg'(t))/2$ ----- (2)

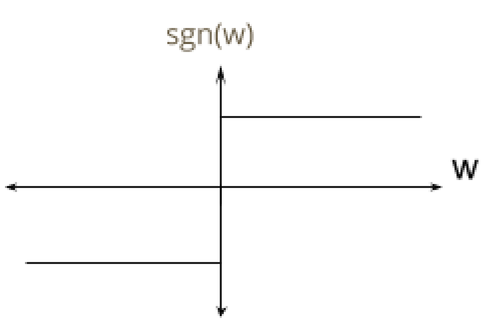
This can be verified by Fourier Transform

$$G_+(\omega) = F(g_+(t)) = (G(\omega) + jG'(\omega))/2$$

$$G_-(\omega) = F(g_-(t)) = (G(\omega) - jG'(\omega))/2$$

Where $G'(\omega)$ is the Fourier Transform of g'(t) and is given by

$$G'(\omega) = -j * sgn(\omega) * G(\omega)$$



$G_+(\omega) = G(\omega)(1 + sgn(\omega))/2$

$G_+(\omega) = G(\omega) * u(\omega)$

Where $u(\omega)$ is unit step function

Similarly,

$G_-(\omega) = F(g_-(t)) = G(\omega) * u(-\omega)$

Spectrum of $g_+(t)$ contains only positive frequencies and $g_-(t)$ contains negative frequencies

Let g(t) be the modulating signal and fc Hz is the frequency of the carrier signal. After modulation, the signal is $\phi(\omega)$

$\phi_{USB}(\omega) = G_+(\omega - \omega c) + G_-(\omega + \omega c)$

$\phi_{USB}(t) = F^{-1}(\phi_{USB}(\omega)) = g_+(t) * e^{j*\omega c*t} + g_-(t) * e^{-j*\omega c*t}$

Substituting values from (1) and (2)

$\phi_{USB}(t) = g(t) * cos(\omega c * t) - g'(t) * sin(\omega c * t)$

Similarly,

$\phi_{LSB}(t) = g(t) * cos(\omega c * t) + g'(t) * sin(\omega c * t)$

Therefore, Single Sideband Modulation can be regarded as Quadrature Amplitude Modulation with modulating signal g(t) placed in in-phase channel and Hilbert Transform of g(t) placed in quadrature channel.

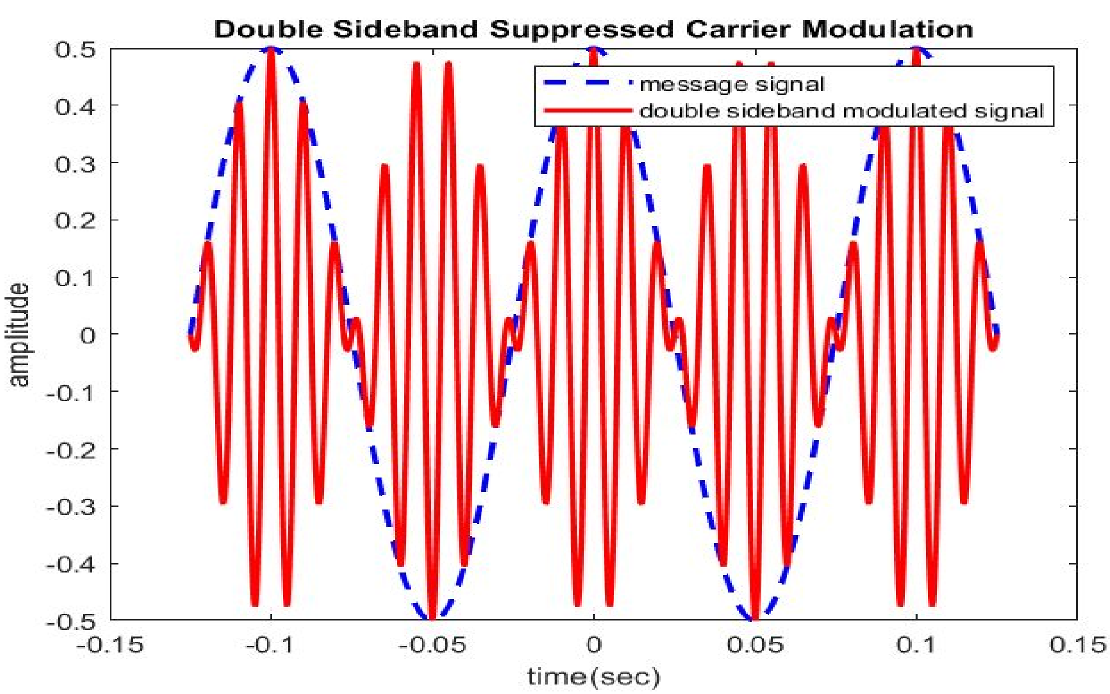
Simulation Using MATLAB

In this project, we attempt to plot the modulated signals after both Double Sideband Suppressed Carrier Modulation and Single Sideband Suppressed Carrier Modulation using MATLAB 2020a.

Below is the code and results of simulation.

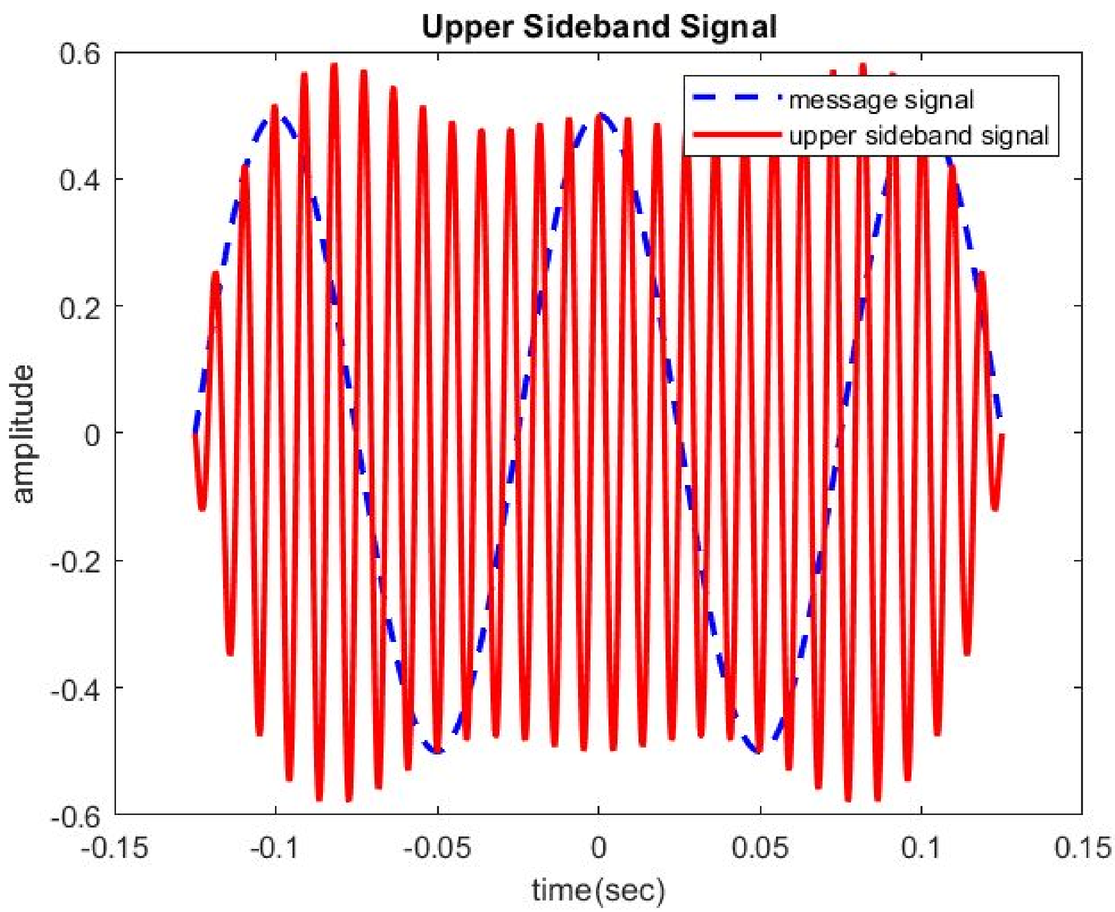
Firstly defining frequencies of message and carrier signals and defining the message signal and modulated signal

```
% frequency of message signal in Hz
fm = 10;

%frequency of carrier signal in Hz
fc = 100;

t = linspace(-0.125, 0.125, 1000);

%message signal
m = 0.5*cos(2*pi*fm*t);

%modulated signal/signal after Modulation
f = m.*cos(2*pi*fc*t);

%plotting message signal and modulated signal
plot(t, m, "b--", "LineWidth", 2)
hold on
plot(t, f, "r-", "LineWidth", 2)
xlabel("time(sec)")
ylabel("amplitude")
title("Double Sideband Suppressed Carrier Modulation")
legend("message signal", "double sideband modulated signal")
hold off
```



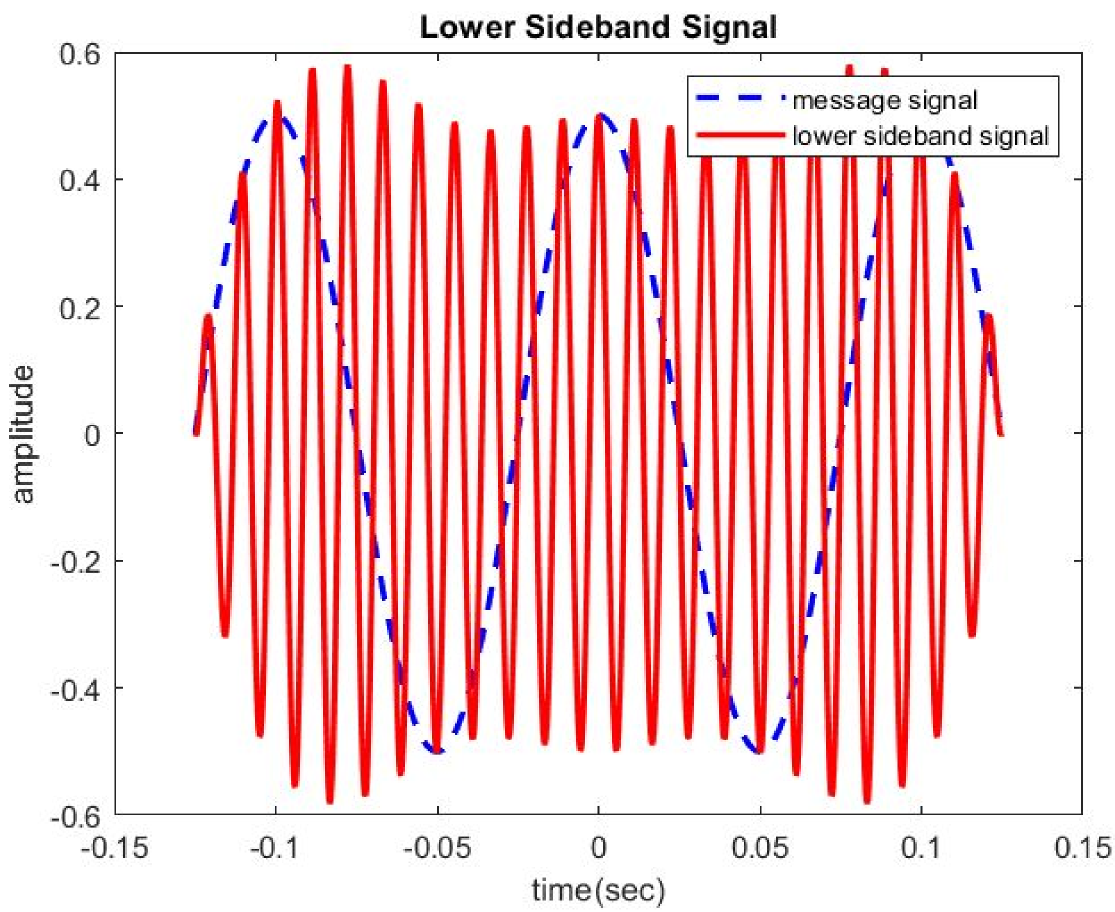
Calculating the Hilbert Transform of the message signal. As the hilbert(m) function of MATLAB produces a complex signal not only the imaginary part. Therefore extracting the imaginary part of the output in the first step. And then plotting the message signal and modulated signal.

```
%calculating Hilbert Transform of message signal
mh = imag(hilbert(m));

%Upper Sideband Signal
ssb = m.*cos(2*pi*fc*t) - mh.*sin(2*pi*fc*t);

%plotting message signal and upper sideband signal
plot(t, m, "b--", "LineWidth", 2)
hold on
plot(t, ssb, "r-", "LineWidth", 2)
xlabel("time(sec)")
ylabel("amplitude")
title("Upper Sideband Signal")
legend("message signal", "upper sideband signal")
hold off
```



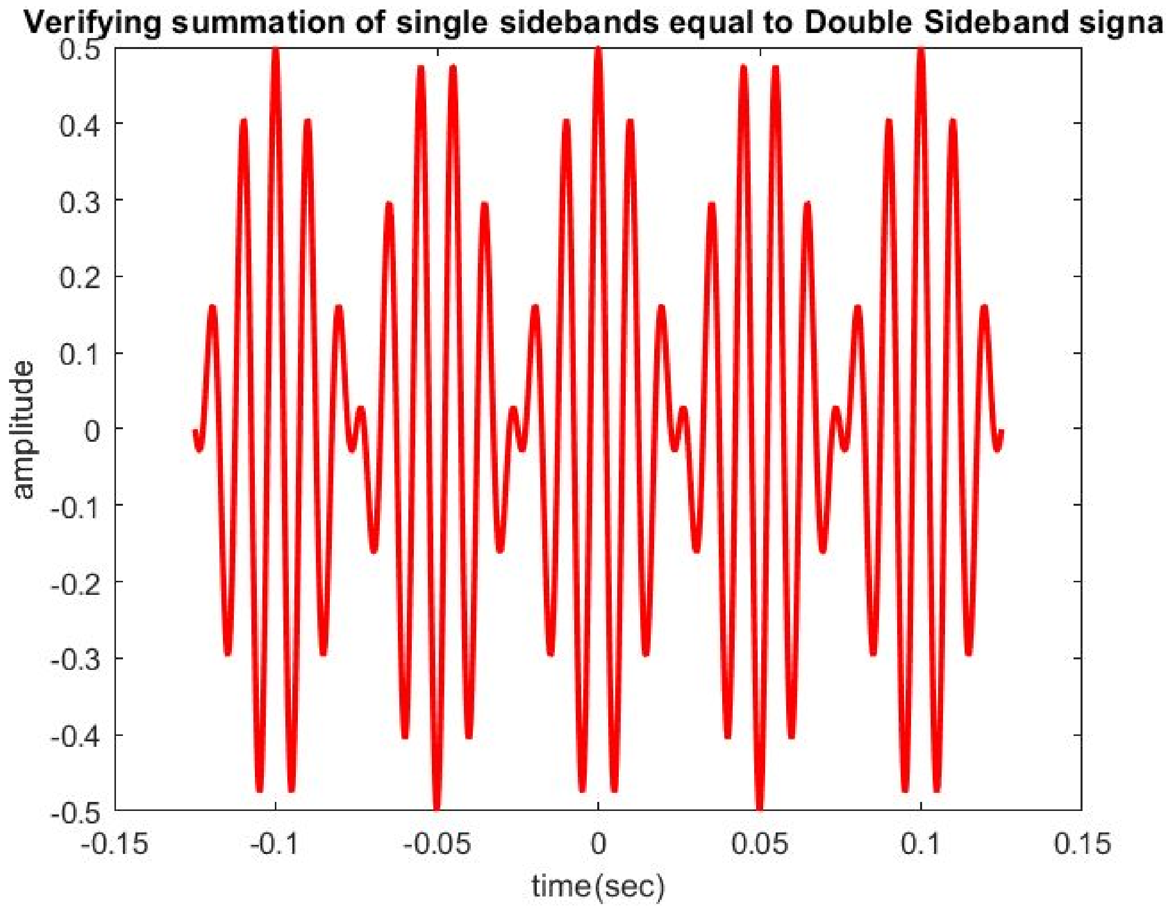
Plotting Lower Sideband Signal and original message signal

```
%plotting message signal and lower sideband signal
slsb = m.*cos(2*pi*fc*t) + mh.*sin(2*pi*fc*t);
plot(t, m, "b--", "LineWidth", 2)
hold on
plot(t, slsb, "r-", "LineWidth", 2)
xlabel("time(sec)")
ylabel("amplitude")
title("Lower Sideband Signal")
legend("message signal", "lower sideband signal")
hold off
```



Now verifying that summation of upper sideband and lower sideband signals is equal to twice of Double sideband modulated signal or summation of lower sideband and upper sideband signals divided by 2 is equal to double sideband modulated signal.

```
%verifying that summation of upper sideband and lower sideband is equal to
%double sideband modulated signal
plot(t, (ssb+slsb)/2, "r-", "LineWidth", 2)
xlabel("time(sec)")
ylabel("amplitude")
title("Verifying summation of single sidebands equal to Double Sideband
signal")
```



Advantages of Single Sideband Modulation

- Lesser band width required as compared to Double Sideband Modulation.
- Reduces wastage of power in carrier signal
- Transmission of more number of signals is possible
- Noise is less

Disadvantages

Generation of Single Sideband signals is complicated and complex.

Conclusion

In this project we have plotted both Double Sideband Modulated signal as well as Single Sideband Modulated signal(Upper Sideband signal as well as Lower Sideband Signal) and also presented the use of Hilbert Transform in producing Single Sideband Modulated signals. In the end some advantages and disadvantages of Single Sideband Modulation are presented. So Single Sideband Suppressed Carrier Modulation provides several advantages over Double Sideband Suppressed Carrier Modulation along with increased complexity in signal generation.