Ultrasound Image Reconstruction with Denoising Diffusion Restoration Models

DGM4MICCAI - 2023

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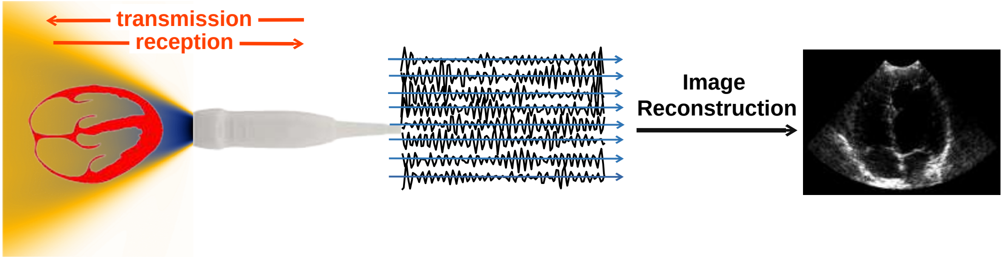
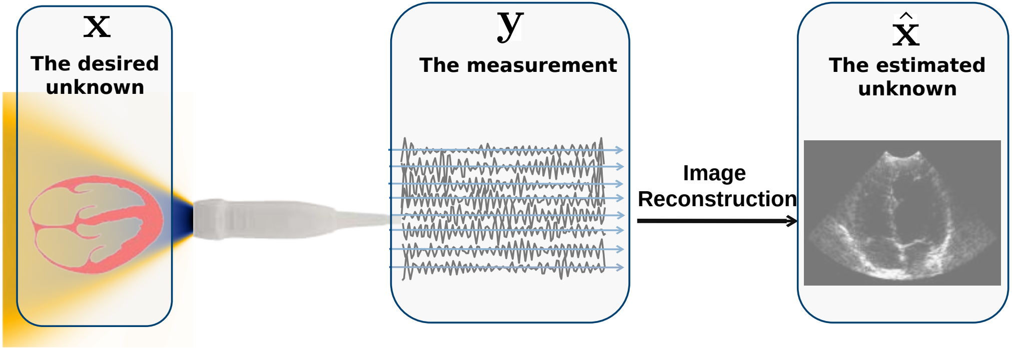
8 - October - 2023

Ultrasound Imaging



Source : https://www.biomecardio.com/files/Tracking_motions_in_the_body.pdf

Ultrasound Imaging

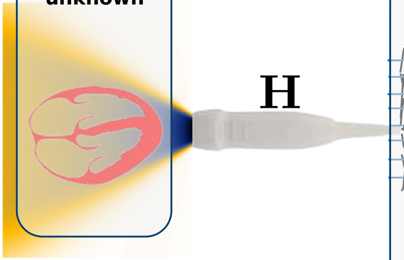


Source : https://www.biomecardio.com/files/Tracking_motions_in_the_body.pdf

Image Reconstruction → an Inverse Problem

$\mathbf{x}$

The desired unknown



$\mathbf{H}$

$\mathbf{y}$

The measurement

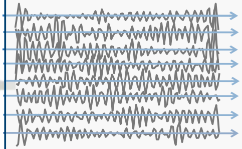


Image Reconstruction →

$\hat{\mathbf{x}}$

The estimated unknown

By solving an Inverse Problem:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Source : https://www.biomecardio.com/files/Tracking_motions_in_the_body.pdf

Inverse Problem Solving

Model-based

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \phi_{\text{reg}}$$

Ozkan et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2018

Goudarzi et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2022

Inverse Problem Solving

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Learning-based

Perdios et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. (accepted)

Model-based

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \phi_{\text{reg}}$$

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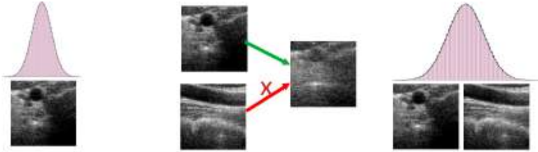
Hybrid



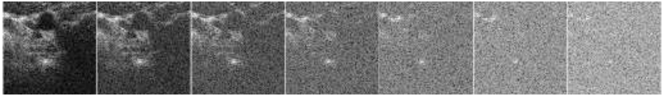
Posterior $(\hat{\mathbf{x}})$ ⟵ Likelihood (Inverse Problem) × Prior (Diffusion Model)

Conditional Diffusion Sampling

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- Song Y et al. Solving inverse problems in medical imaging with score-based generative models. ICLR, 2022
- Song J et al. Pseudoinverse-guided diffusion models for inverse problems. ICLR, 2023
- Chung H et al. Score-based diffusion models for accelerated MRI. Med Image Anal. 2022
- Chung H et al. Diffusion posterior sampling for general noisy inverse problems. ICLR, 2023
- Kawar B et al. Denoising diffusion restoration models. NeurIPS. 2022 **(DDRM)**

$\hat{\mathbf{x}}$ ← **Diffusion sampling process** — **noise**

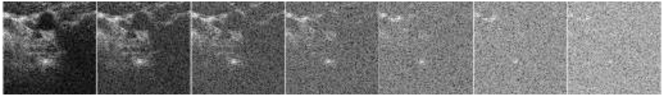


$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\boldsymbol{\Sigma}^{\dagger}\mathbf{U}^{T}\mathbf{y} = \mathbf{V}^{T}\mathbf{x} + \boldsymbol{\Sigma}^{\dagger}\mathbf{U}^{T}\mathbf{n}$$

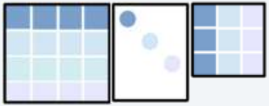
$$\overline{\mathbf{y}} = \overline{\mathbf{x}} + \overline{\mathbf{n}}$$

$\hat{\mathbf{x}}$ ← **Diffusion sampling process** **noise**



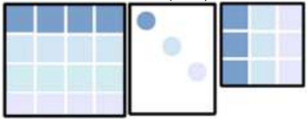
DDRM runs "denoising and/or inpainting" in the space transformed by **Singular Value Decomposition**

$$\mathbf{H} = \mathbf{U} \quad \boldsymbol{\Sigma} \quad \mathbf{V}^{T}$$



DDRM (Kawar et al. NeurIPS 2022) initially for **natual images**

$\mathrm{SVD}(\mathbf{H})$

$\mathbf{y} =$  $\mathbf{x} +$ white $\mathbf{n}$

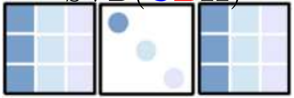
data compressing

$\mathrm{SVD}(\mathbf{BH})$

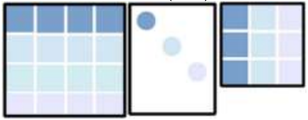
$\mathbf{By} =$  $\mathbf{x} +$ colored $\mathbf{Bn}$

noise whitening

$\mathrm{SVD}(\mathbf{CBH})$

$\mathbf{CBy} =$  $\mathbf{x} +$ white $\mathbf{CBn}$

$\mathrm{SVD}(\mathbf{H})$

$\mathbf{y} =$  $\mathbf{x} +$ white $\mathbf{n}$

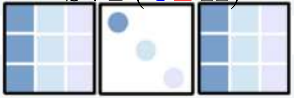
data compressing

$\mathrm{SVD}(\color{red}{\mathbf{B}}\color{black}{\mathbf{H}})$

$\color{red}{\mathbf{B}}\color{black}{\mathbf{y}} =$  $\mathbf{x} +$ colored $\color{red}{\mathbf{B}}\color{black}{\mathbf{n}}$

noise whitening

$\mathrm{SVD}(\color{blue}{\mathbf{C}}\color{red}{\mathbf{B}}\color{black}{\mathbf{H}})$

$\color{blue}{\mathbf{C}}\color{red}{\mathbf{B}}\color{black}{\mathbf{y}} =$  $\mathbf{x} +$ white $\color{blue}{\mathbf{C}}\color{red}{\mathbf{B}}\color{black}{\mathbf{n}}$

Natural Images **VS** Ultrasound Images (SIGNED)

Pre-trained on :

Figure – the ImageNet dataset (1,281,167 images) (?)

Fine-tuned on :

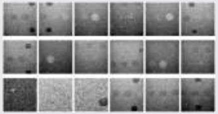


Figure – Examples of the self-acquired dataset (800 images)

Test set : PICMUS dataset (?) gives the observation $\mathbf{y}$.

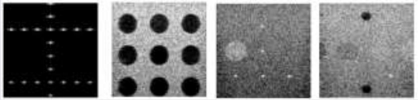
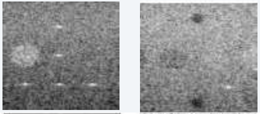
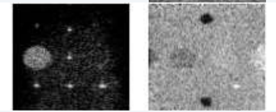


Figure – Examples of PICMUS reconstructed ultrasound images

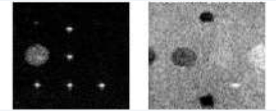
1 transmission (Fast acquisition)



Baseline (DAS1)

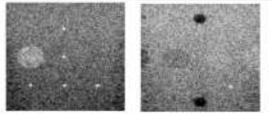


DRUS **(ours)**
$(\mathbf{\color{red}{B}y} = \mathbf{\color{red}{B}Hx} + \mathbf{\color{red}{B}n})$



WDRUS **(ours)**
$(\mathbf{\color{blue}{C}\color{red}{B}y} = \mathbf{\color{blue}{C}\color{red}{B}Hx} + \mathbf{\color{blue}{C}\color{red}{B}n})$

75 transmissions (Slow acquisition)



Golden standard (DAS75)

	Resolution (FWHM [mm]↓)		Contrast (CNR[dB] ↑)
	Axial	Lateral	
Baseline	0.51	1.21	8.15
DRUS	0.26	0.69	**12.9**
WDRUS	**0.25**	0.62	11.95
Golden standard	0.49	0.59	12.05

Take-home message

Diffusion Inverse Problem Solver

Model-based ✓

Learning-based ✓

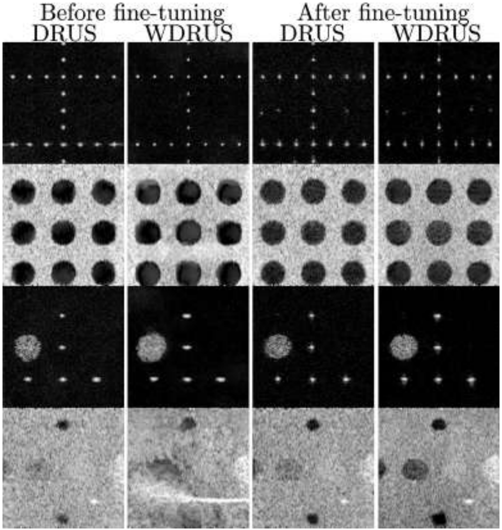
Ultrasound Inverse Problem Model

noise whitened ← **data compressed** ← **original**

Fine-Tuning from a Natural-Image Diffusion Model

Thank you !

Fine-tuning

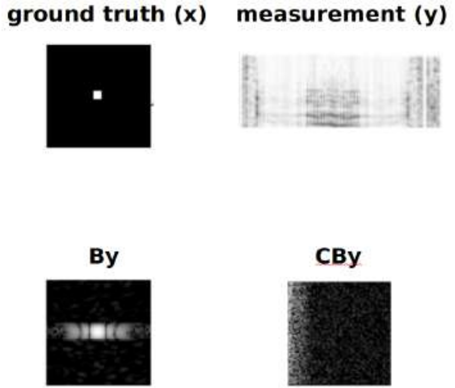


B and C in a simple case

*$\color{red}{\mathbf{B}} = \mathbf{H}^{\mathrm{t}}$

*$\color{blue}{\mathbf{C}} = \mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{V}^{\mathrm{t}}$, where $\mathrm{eig}(\mathbf{BB}^{\mathrm{t}}) = \mathbf{V\Lambda V}^{\mathrm{t}}$

$\mathrm{Cov}(\mathbf{CBn}) = \mathrm{E}\left[\mathbf{CBnn}^{\mathrm{t}}\mathbf{B}^{\mathrm{t}}\mathbf{C}^{\mathrm{t}}\right] = \gamma^2\mathbf{CBB}^{\mathrm{t}}\mathbf{C}^{\mathrm{t}} = \gamma^2\mathbf{CV\Lambda V}^{\mathrm{t}}\mathbf{C}^{\mathrm{t}} = \gamma^2\mathbf{I}_M$



In summary

$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$

|___ $\mathbf{By} = \mathbf{BHx} + \mathbf{Bn}$ (DRUS)

|___ $\mathbf{CBy} = \mathbf{CBHx} + \mathbf{CBn}$ (WDRUS)