

```
In [1]: #load "Angara.Charting.Paket.fsx"
#load "Angara.Charting.fsx"

In [2]: Paket.Package ["FSharp.Data"]

Out[2]: <null>

In [3]: Paket.Package ["Angara.Table"]
Paket.Package ["Angara.Statistics"]
#r "packages/Angara.Table/lib/net452/Angara.Table.dll"
#r "packages/Angara.Statistics/lib/net452/Angara.Statistics.dll"
#r "packages/System.Collections.Immutable/lib/portable-net45+win8+wp8+wpa81/System.Collections.Immutable.dll"

open Angara.Data
open Angara.Charting
open Angara.Statistics
open System.Linq

In [4]: type Stat = {
    min: float
    lb95: float
    lb68: float
    median: float
    ub68: float
    ub95: float
    max: float
    sd: float
}
let stat series =
    let summ = series |> summary
    let qsumm = series |> qsummary
    { min = summ.min; lb95 = qsumm.lb95; lb68 = qsumm.lb68; median = qsumm.median;
      ub68 = qsumm.ub68; ub95 = qsumm.ub95; max = summ.max; sd = sqrt(summ.variance) }

let colors = ["blue";"red";"green";"lightblue";"orange";"yellow";"grey";"magenta"]
```

Single VM vs Multiple VMs

Multiple VMs:

```
In [5]: let wctime = [446.3462;438.8182;442.2185;417.7349;444.6729;436.9418;432.9028;402.9171;413.887;440.8813]
stat wctime

Out[5]: {min = 402.9171;
lb95 = 402.9171;
lb68 = 413.7407347;
median = 437.88;
ub68 = 413.7407347;
ub95 = 446.3462;
max = 446.3462;
sd = 14.90006123;}
```

```
In [6]: let x,y = kde2 512 300.0 600.0 wctime
let plot_pdf =
    Plot.line(LineX.Values x, LineY.Values y, displayName = "Multiple VMs",
              titles = Titles.line("Duration (sec)", "Probability"))
```

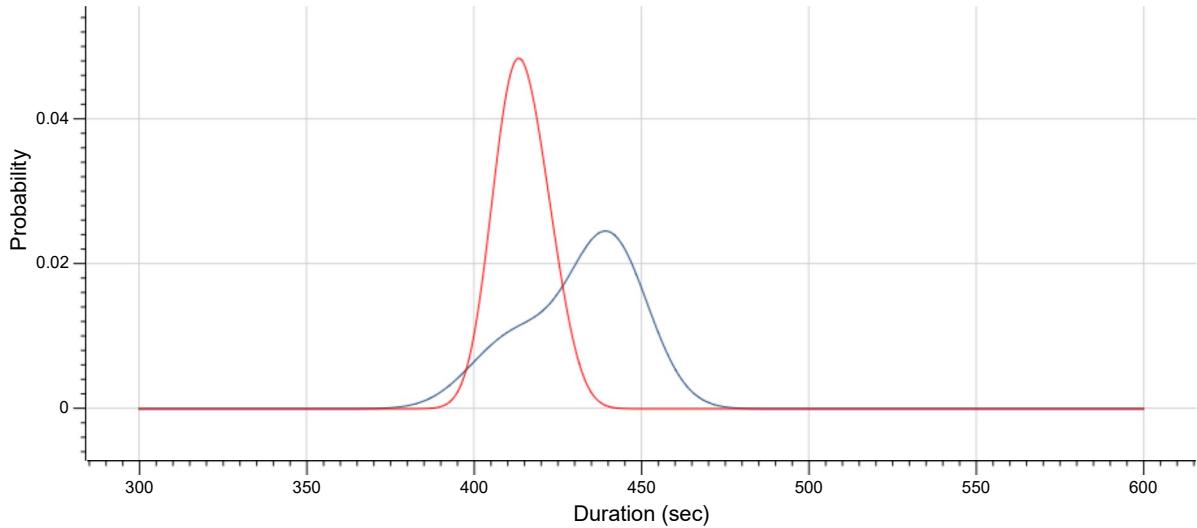
Single VM:

```
In [7]: let wctime_same = [421.4155;408.283;409.7987;422.9174;415.5584;411.1633;413.4903;409.9151;418.3712;410.9465]
stat wctime_same

Out[7]: {min = 408.283;
lb95 = 408.283;
lb68 = 409.7784907;
median = 412.3268;
ub68 = 409.7784907;
ub95 = 422.9174;
max = 422.9174;
sd = 5.167761907;}
```

```
In [8]: let x,y = kde2 512 300.0 600.0 wctime_same
let plot_pdf_same =
    Plot.line(LineX.Values x, LineY.Values y, stroke = "red", displayName = "Single VM",
              titles = Titles.line("Duration (sec)", "Probability"))
[ plot_pdf; plot_pdf_same ] |> Chart.ofList
```

Out[8]:



```
In [9]: // This function returns a float array of the given table column
let col (colName:string) (t:Table) : float[] = t.[colName].Rows.AsReal.ToArray()

let rename (name:string) (c:Column) = Column.Create(name, c.Rows, c.Height)

let loadTable strContent =
    let reader = new System.IO.StringReader(strContent)
    Table.Load(reader,
               { Angara.Data.DelimitedFile.ReadSettings.Default
                 with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let loadTableC strContent =
    let reader = new System.IO.StringReader(strContent)
    Table.Load(reader,
               { Angara.Data.DelimitedFile.ReadSettings.Default
                 with Delimiter = Angara.Data.DelimitedFile.Delimiter.Comma })

let schema (table:Table) = table
    |> Seq.iteri (fun colIdx col ->
                    printfn "%d: %s of type %s" colIdx col.Name
                    (match col.Rows with
                     | RealColumn _ -> "float"
                     | IntColumn _ -> "int"
                     | StringColumn _ -> "string"
                     | DateColumn _ -> "DateTime"
                     | BooleanColumn _ -> "bool"))
```

```
In [10]: let transpose (table:Table) =
    table
    |> Seq.map(fun col -> col.Rows.AsReal.ToArray())
    |> Seq.toArray
    |> MatrixTable.OfMatrix

let takeRows n (table:Table) =
    table
    |> Seq.map(fun col -> Column.Create(col.Name, col.Rows.AsReal.ToArray() |> Array.take n))
    |> Table.OfColumns

let drawPdfColumnsRangeColored table vmin vmax (color: int -> string) =
    table
    |> Seq.map(fun (col:Column) ->
        let name = col.Name
        let vals = col.Rows.AsReal.ToArray()
        let x,y = kde2 512 vmin vmax vals
        name, x, y)
    |> Seq.mapi (fun i (name, x,y) ->
        Plot.line(LineX.Values x, LineY.Values y, stroke = color i,
            displayName = name,
            titles = Titles.line("Duration (sec)", "Probability")))
    |> Seq.toList
    |> Chart.ofList

let defaultColoring i = colors.[i%colors.Length]

let drawPdfColumnsRange table vmin vmax = drawPdfColumnsRangeColored table vmin vmax defaultColoring

let drawPdfColumns table = drawPdfColumnsRange table 300.0 600.0

let toQuantiles (qsums: qsummaryType[]) =
    { median = qsums |> Array.map(fun q -> q.median)
      lower68 = qsums |> Array.map(fun q -> q.lb68)
      upper68 = qsums |> Array.map(fun q -> q.ub68)
      lower95 = qsums |> Array.map(fun q -> q.lb95)
      upper95 = qsums |> Array.map(fun q -> q.ub95) }

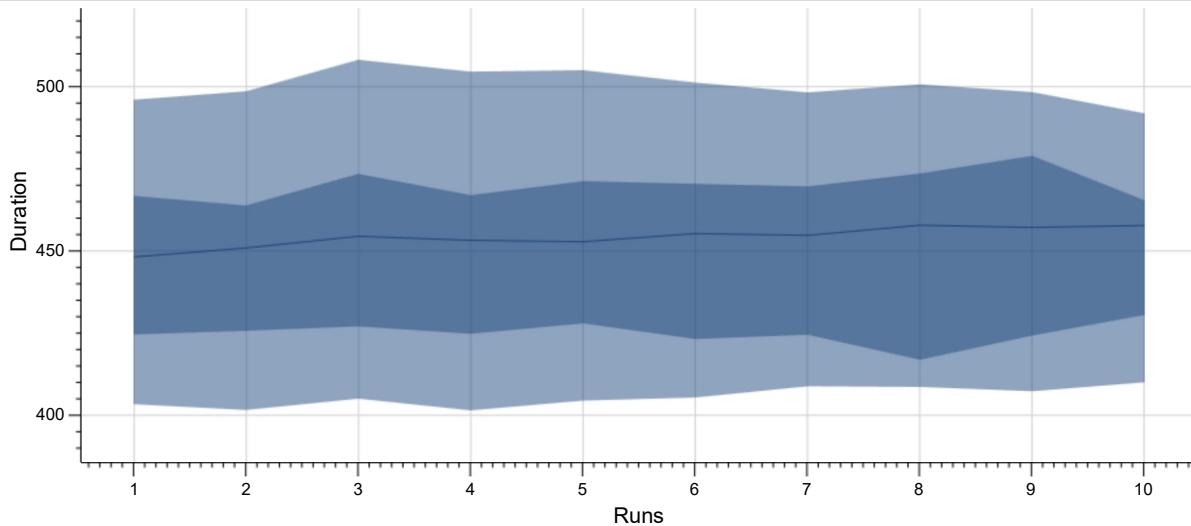
let drawRowsUncertainties table =
    let m = table |> transpose
    let unc = m.Columns |> Seq.map(fun runRes -> qsummary (runRes.ToArray())) |> Seq.toArray
    let x = Array.init m.Count (fun i -> float(i+1))
    let duration = toQuantiles unc
    [ Plot.line(LineX.Values x, LineY.UncertainValues duration,
        titles = Titles.line("Runs", "Duration"),
        displayName = "Duration Distribution") ]
    |> Chart.ofList
```

Multiple runs on multiple machines

```
In [11]: let s = "Run #  VM0      VM1      VM2      VM3      VM4      VM5      VM6      VM7      VM8      VM9      AVERAGE  STD
          0      435.4531  403.5313  495.9219  462.125   466.375  448.2031  453.3906  424.9844
                  448.1875  443.7344  448.1906  23.54834
          1      455.1094  401.7813  498.5     457.1719  463.3594  441.5938  450.9531  4
          26.1406 450.9063  431.7031  447.7219  24.203
          2      457.6094  405.2031  508.0469  461.5     472.9688  443.1094  453.1719  4
          27.4375 455.7656  442.2344  452.7047  25.84747
          3      456.1406  401.6406  504.4531  466.4531  463.4844  441.5469  455.5938
                  425.2031  450.9063  450.9063  451.6328  25.46885
          4      456.6719  404.6094  504.8906  464.5938  470.7656  446.9688  454.6563
                  428.375 451     450.8906  453.3422  24.80039
          5      456.8594  405.5469  501.1406  463.0625  469.9844  442.6406  453.7188
                  423.5469  452.7656  457.9375  452.7203  24.43953
          6      456.7188  408.9688  498.1406  460.9531  469.2188  447.4375  456.1563
                  424.8281  453.3594  451.4375  452.7219  22.70002
          7      458.75   408.7656  500.6094  461.3438  473.1875  444.1875  456.9219  4
          17.125 452.9375  459.4688  453.3297  24.808
          8      456.5781  407.4531  498.2344  462.3438  478.6563  446.5781  457.7188
                  424.6094  455.8125  461.0625  454.9047  24.05242
          9      462.6406  410.1875  491.7813  460.5469  465.0313  450.1094  456.2969
                  430.8906  459.2188  456.0781  454.2781  20.42402
          "
let tableF = s |> loadTable |> Seq.filter(fun c -> c.Name.StartsWith "VM") |> Table.OfColumns
```

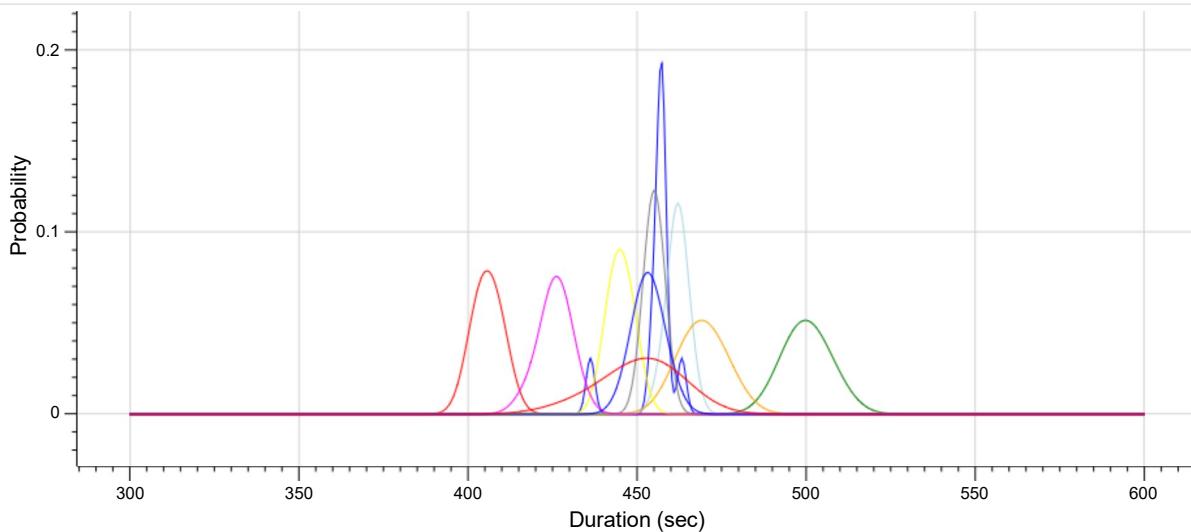
```
In [12]: drawRowsUncertainties tableF
```

```
Out[12]:
```



```
In [13]: drawPdfColumns tableF
```

```
Out[13]:
```



1. Each of the VMs shows quite stable result; standard deviation is about 5 sec.
2. Different VMs have different stable results (location of peaks); it varies between 400 and 500 sec.

Then:

1. How to make the results across different machines consistent?
2. What is the reason of different performance of machines?

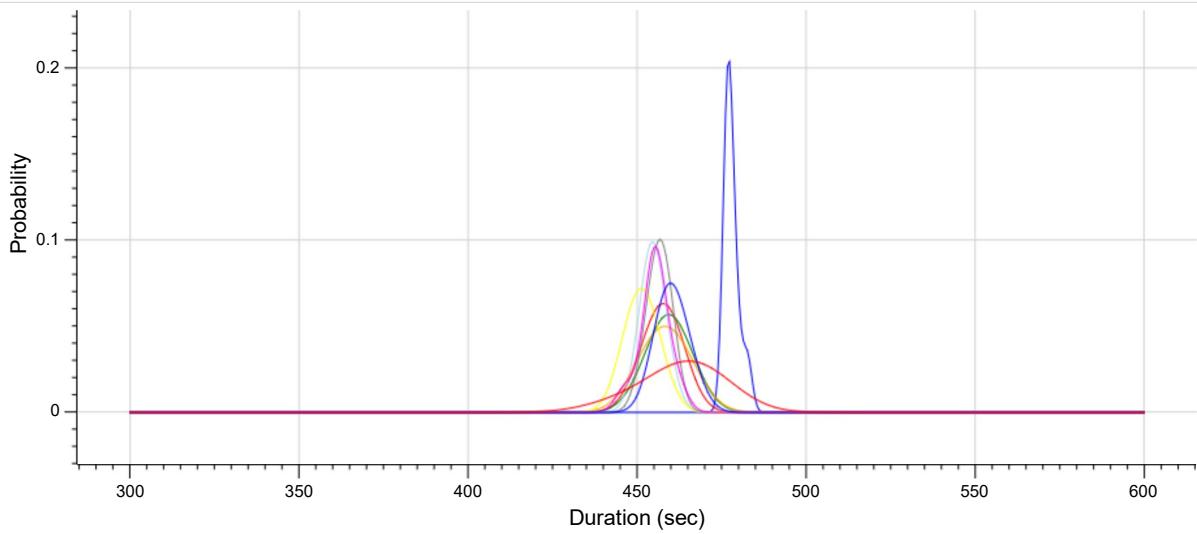
Normalization

Now, if we consider 454.8578 to be 'standard' time, use the first measurement on each VM to compute 'performance coefficient' (standard time / measurement) and multiply other measurement by that coefficient, we'll get

```
In [14]: let s = "VM0    VM1    VM2    VM3    VM4    VM5    VM6    VM7    VM8    VM9
475.39  452.8852  457.2224  449.9826  451.9166  448.1503  452.4124  456.0953
        457.617 442.525
478.0014 456.7423  465.9788  454.2426  461.2887  449.6884  454.6383  4
57.4834 462.5487  453.3202
476.4672 452.7267  462.6826  459.1179  452.0386  448.1027  457.0681  4
55.0919 457.617 462.2095
477.0221 456.0731  463.0839  457.2877  459.14 453.6051  456.1275  458.4868
        457.7122 462.1934
477.218 457.1298  459.6444  455.7806  458.378 449.2127  455.187 453.3193  459.5041
        469.417
477.0711 460.9869  456.8928  453.7044  457.6313  454.0808  457.6324  4
54.6906 460.1066  462.754
479.1928 460.758 459.1572  454.0888  461.5021  450.7826  458.4005  446.446 4
59.6785 470.9866
476.9242 459.2785  456.9788  455.0731  466.8358  453.2087  459.1999  4
54.4564 462.5963  472.6203
483.2568 462.3607  451.06 453.3045  453.5472  456.7924  457.7735  461.1792
        466.0532 467.511
"
let tableFCoeff = s |> loadTable
```

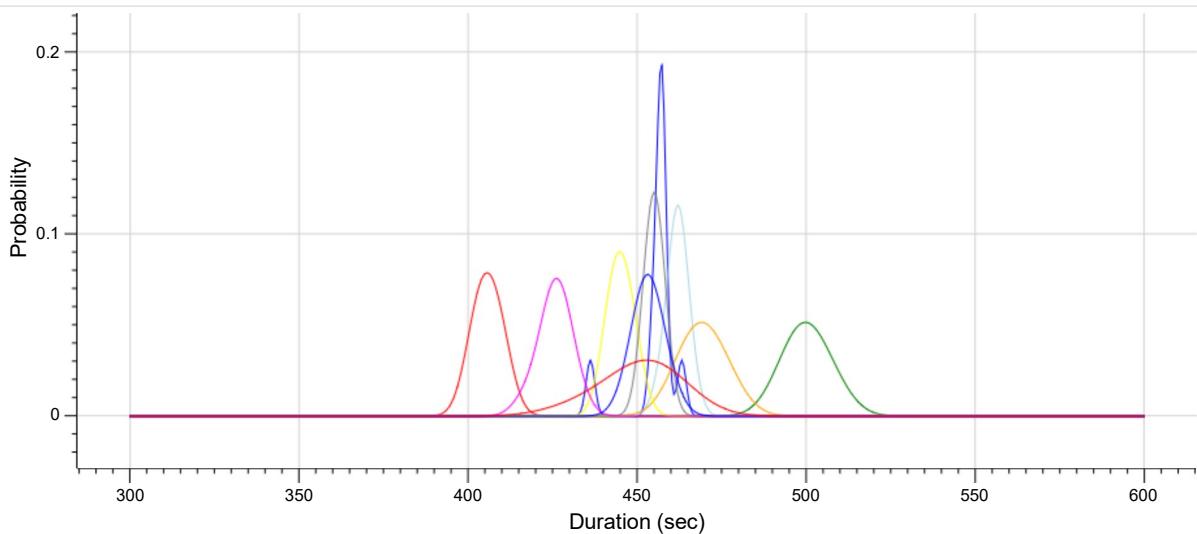
```
In [15]: drawPdfColumns tableFCoeff
```

```
Out[15]:
```



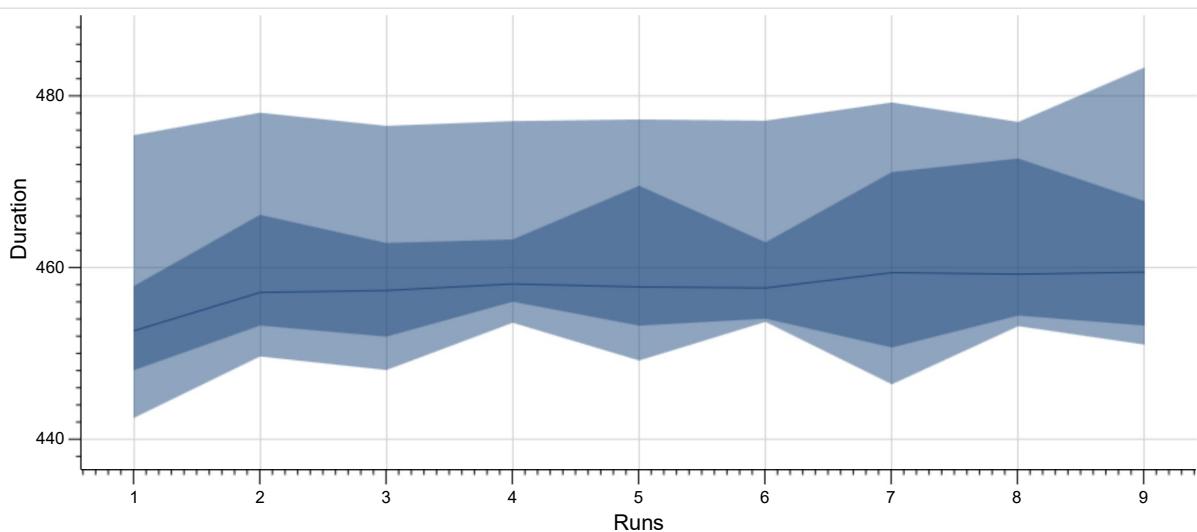
```
In [16]: drawPdfColumns tableF
```

```
Out[16]:
```



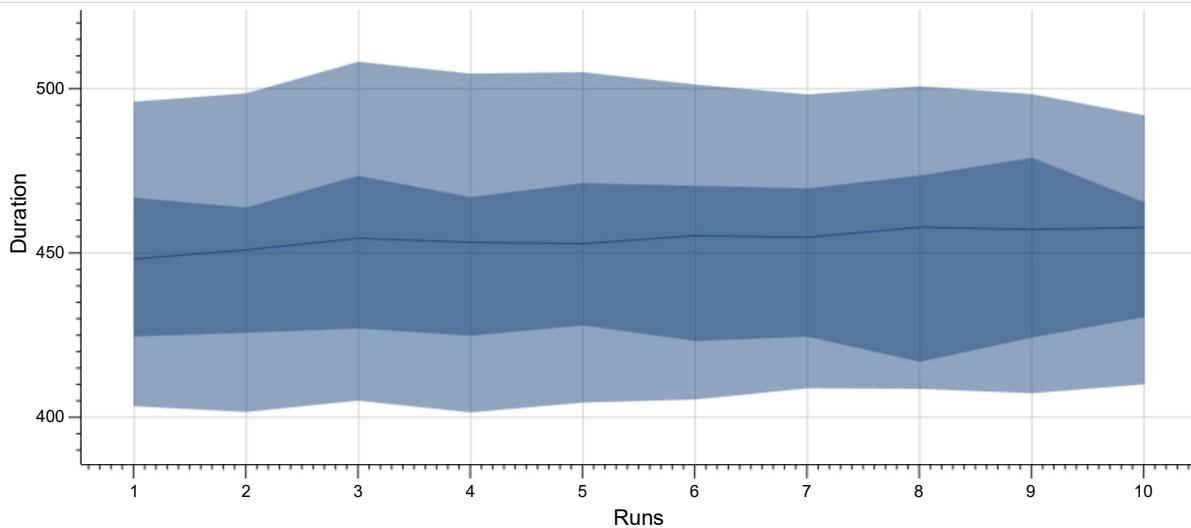
```
In [17]: drawRowsUncertainties tableFCoeff
```

```
Out[17]:
```



```
In [18]: drawRowsUncertainties tableF
```

```
Out[18]:
```



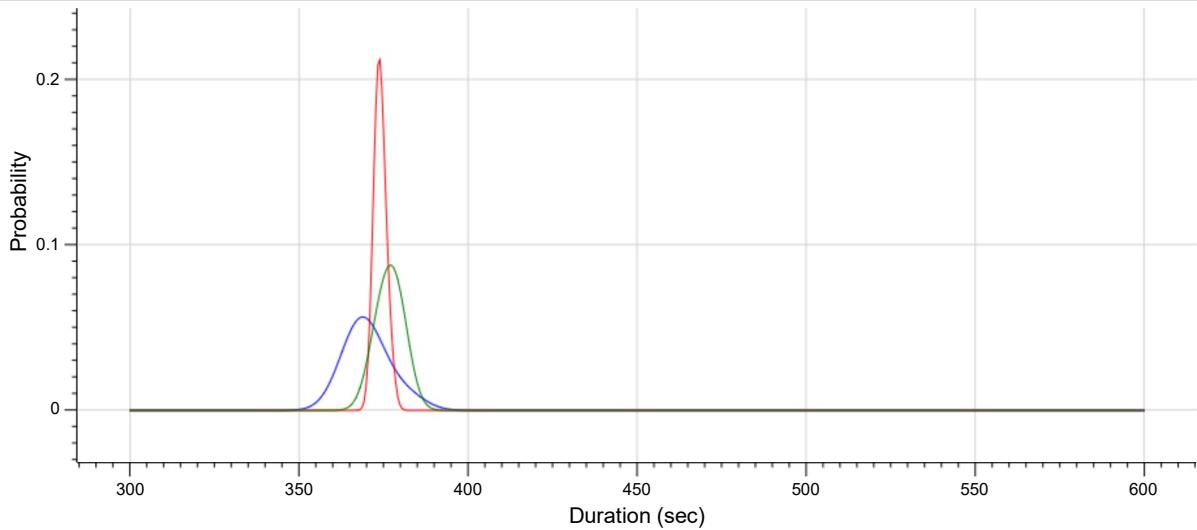
Effect of sharing same physical machine

```
In [19]: let share_1 = "Run #    Result code    Return value    CPUTime WCTime    Memory
0      OK      0      374.46875     374.4983537    448053248
1      OK      0      373.328125    373.3428245    462352384
2      OK      0      372.171875    372.1924532    456261632
3      OK      0      372.453125    372.4779358    456318976
4      OK      0      373.375 373.3988532  454131712
5      OK      0      374.25   374.2604102  462417920
6      OK      0      374.796875    374.8168565    456237056
7      OK      0      377.890625    377.9116777    454197248
8      OK      0      373.453125    373.4841731    462446592
9      OK      0      372.265625    372.2689823    454057984
"
let share_2 = "Run #    Result code    Return value    CPUTime WCTime    Memory
0      OK      0      379.171875    383.9745942    448053248
1      OK      0      376.59375     382.5917482    462352384
2      OK      0      374.75   379.3451419    456261632
3      OK      0      372.609375    378.4968077    456318976
4      OK      0      376.25   381.1666909    454131712
5      OK      0      378.109375    382.640683     462417920
6      OK      0      378.0625     382.2941934    454144000
7      OK      0      381.109375    384.8785986    462446592
8      OK      0      378.03125     381.9148036    456298496
9      OK      0      372.3125     376.4337885    454025216
"
let excl = "Run #    Result code    Return value    CPUTime WCTime    Memory
0      OK      0      368.421875    374.9458616    232374272
1      OK      0      369.203125    375.735785     232259584
2      OK      0      370.140625    377.652724     232243200
3      OK      0      374.34375     378.1208825    230219776
4      OK      0      368.15625     373.7346988    230203392
5      OK      0      367.046875    371.729211     232296448
6      OK      0      362.859375    365.9769345    230060032
7      OK      0      376.5   380.5797761    232366080
8      OK      0      369.125 372.6886823    230232064
9      OK      0      383.25 389.6150868    230236160
"
let takeAndRename oldName newName table =
  table
  |> Seq.choose(fun (c:Column) -> if c.Name = oldName then Some(c |> rename newName) else None)
  |> Seq.exactlyOne

let cpu_share1 = loadTable share_1 |> takeAndRename "CPUTime" "Shared 1"
let cpu_share2 = loadTable share_2 |> takeAndRename "CPUTime" "Shared 2"
let cpu_Excl = loadTable excl |> takeAndRename "CPUTime" "Exclusive"
let table_F = [ cpu_Excl; cpu_share1; cpu_share2 ] |> Table.OfColumns
```

```
In [20]: drawPdfColumns table_F
```

```
Out[20]:
```



Running experiment one per physical machine and then two simultaneously on different cores, shows that stuff going on physical cores other than those we run experiment on **does not affect the result** any more than random factors within the machine.

```
In [21]: let task1 = @"run # \ VM #,0,1,2,3,4,5,6,7,8,9
0,0.953125,0.921875,0.921875,1.03125,1.171875,0.828125,0.9375,0.9375,1.078125,0.9375
1,0.90625,0.921875,0.890625,0.9375,0.984375,0.828125,0.921875,0.921875,0.96875,0.9375
2,0.90625,0.90625,0.90625,0.9375,1.046875,0.890625,0.921875,0.90625,1,0.90625
3,0.890625,0.9375,0.875,0.9375,1.03125,0.890625,0.921875,0.96875,0.984375,0.90625
4,0.890625,0.921875,0.84375,0.96875,1.03125,0.84375,0.9375,0.984375,1,0.9375
5,0.859375,0.9375,0.84375,0.953125,1.015625,0.890625,0.9375,0.9375,1,0.9375
6,0.828125,0.9375,0.859375,0.96875,1.109375,0.875,0.921875,1,0.984375,0.921875
7,0.875,0.9375,0.859375,0.96875,1.09375,0.8125,0.9375,0.90625,1,0.90625
8,0.96875,0.90625,0.828125,0.9375,1.03125,0.828125,0.921875,0.90625,1.015625,0.9375
9,0.9375,0.9375,0.875,0.9375,1.03125,0.890625,0.9375,0.953125,1.03125,0.9375"
let task2 = @"run # \ VM #,0,1,2,3,4,5,6,7,8,9
0,1.140625,1.1875,1.125,1.203125,1.296875,1.140625,1.203125,1.1875,1.25,1.171875
1,1.109375,1.171875,1.09375,1.28125,1.28125,1.046875,1.203125,1.203125,1.171875
2,1.15625,1.203125,1.078125,1.203125,1.296875,1.125,1.203125,1.28125,1.296875,1.15625
3,1.140625,1.15625,1.125,1.1875,1.265625,1.15625,1.1875,1.21875,1.234375,1.203125
4,1.1875,1.15625,1.09375,1.1875,1.390625,1.171875,1.203125,1.265625,1.234375,1.1875
5,1.171875,1.15625,1.109375,1.1875,1.296875,1.234375,1.1875,1.171875,1.21875,1.171875
6,1.1875,1.1875,1.125,1.296875,1.296875,1.140625,1.171875,1.171875,1.234375,1.125
7,1.15625,1.171875,1.109375,1.125,1.28125,1.1875,1.21875,1.1875,1.234375,1.140625
8,1.21875,1.15625,1.125,1.140625,1.4375,1.09375,1.1875,1.28125,1.265625,1.171875
9,1.109375,1.15625,1.125,1.15625,1.25,1.171875,1.1875,1.234375,1.21875,1.140625"
let task3 = @"run # \ VM #,0,1,2,3,4,5,6,7,8,9
0,533.375,558.8125,541.640625,588.9375,624.796875,545.234375,566.734375,580.078125,603.640625,559.015625
1,536.359375,505.3125,565.53125,585.140625,628.78125,546.609375,576.328125,575.625,592.15625,564.09375
2,567.21875,502.84375,553.9375,575.484375,632.53125,541.140625,575.25,576.03125,605.640625,565.03125
3,577.578125,501.46875,530.265625,585.71875,633.140625,541.03125,572.34375,580.34375,606.515625,552.3125
4,577.671875,497.703125,536.75,581.953125,627.671875,546.796875,569.1875,572.015625,599.296875,557.96875
5,576.890625,499.046875,562.15625,587.375,616.109375,550.90625,570.984375,577.375,593.890625,558.5
6,567.625,503.1875,577.28125,587.828125,620.46875,536.453125,569.5,571.4375,592.46875,555.21875
7,585.0625,496.28125,576.859375,590.546875,628.203125,540.09375,567.484375,572.453125,588.828125,553.9375
8,588.640625,558.125,548.203125,589.15625,628.484375,535.15625,569.265625,576.140625,600.1875,562.296875
9,586.875,560.171875,557.015625,595.96875,621.3125,540.234375,567.6875,570.578125,591.90625,560.109375"
let task4 = @"run # \ VM #,0,1,2,3,4,5,6,7,8,9
0,95.359375,92.421875,93.140625,94.5,103.078125,88.28125,91.703125,93.75,95.75,91.484375
1,91.484375,91.15625,92.296875,95.203125,103.953125,85.453125,91.734375,92.265625,95.875,90.65625
2,88.96875,92.203125,91.96.359375,102.421875,86.09375,91.8125,94.140625,96.171875,91.09375
3,95.515625,93.89.296875,97.859375,101.890625,86.59375,92.984375,93.46875,96.4375,89.234375
4,95.40625,91.5,89.97.578125,101.234375,86.265625,92.609375,93.625,97.8125,90.921875
5,95.3125,92.0625,90.875,95.921875,102.890625,87.9375,92.65625,93.546875,96.65625,91.859375
6,95.34375,92.421875,91.328125,96.75,101.6875,89.359375,92.84375,93.765625,97.1875,90.515625
7,95.734375,92.03125,92.890625,94.9375,102.21875,87.734375,92.796875,93.984375,97.609375,92.328125
8,94.046875,92.21875,94.5,96.8125,102.4375,87.9375,92.625,92.125,96.734375,91.96875
9,95.25,92.453125,94,95.671875,102.015625,86.5625,93.4375,92.1875,96.53125,92.890625"
let task5 = @"run # \ VM #,0,1,2,3,4,5,6,7,8,9
0,446.390625,411.859375,436.640625,450.1875,466.421875,405.84375,430.984375,430.375,449.0625,427.65625
1,452.15625,379.15625,430.9375,449.546875,467.84375,418.71875,429.453125,433.671875,448.921875,426.046875
2,448.21875,371.671875,409.421875,448.421875,471,413.71875,425.171875,428.71875,442.125,422.1875
3,428.671875,375.234375,405.96875,448.671875,490.03125,417.71875,424.8125,432.90625,443.953125,425.5625
4,434.078125,376.421875,429.765625,446.140625,481.6875,418.234375,431.578125,427.78125,457.625,421.96875
5,429.90625,374.75,409.578125,435.96875,479,415.734375,428.09375,428.65625,449.5625,427.75
6,431.9375,372.234375,401.0625,437.796875,483.734375,409.828125,429.75,432.28125,454.84375,425.625
7,435.203125,375.96875,416.140625,434.875,482.46875,410.03125,427.75,435.015625,455.15625,424.984375
8,430.328125,371,434.609375,436.796875,481.421875,409.375,425.40625,430.46875,449.71875,421.59375
9,432.265625,377.484375,430.5625,437.296875,475.78125,407.859375,428.9375,430.25,448.453125,422.0625"

let tableT1 = task1 |> loadTableC |> Seq.filter(fun c -> not (c.Name.StartsWith "run")) |> Table.OfColumns
let tableT2 = task2 |> loadTableC |> Seq.filter(fun c -> not (c.Name.StartsWith "run")) |> Table.OfColumns
let tableT3 = task3 |> loadTableC |> Seq.filter(fun c -> not (c.Name.StartsWith "run")) |> Table.OfColumns
let tableT4 = task4 |> loadTableC |> Seq.filter(fun c -> not (c.Name.StartsWith "run")) |> Table.OfColumns
let tableT5 = task5 |> loadTableC |> Seq.filter(fun c -> not (c.Name.StartsWith "run")) |> Table.OfColumns
```

```
In [22]: let allTables = [| tableT1; tableT2; tableT3; tableT4; tableT5 |]
let uncertaintyR data = let s = stat data in s.ssd / s.median
let uncertaintyP data = (uncertaintyR data) * 100.0
allTables |> Array.map (fun t -> t |> Array.ofSeq |> Array.map (fun c -> uncertaintyP c.Rows.AsReal) |> Array.reduce max)
```

```
Out[22]: [|5.309039404; 4.649950274; 5.614944324; 2.35291249; 3.189734557|]
```

```
In [23]: //let averages3T1 = tableT1 |> Seq.map (fun c -> c.Rows.AsReal |> Seq.mapi (fun i v -> (v + c.Rows.AsReal.[(i + 1) % c.Height] + c.Rows.AsReal.[(i + 2) % c.Height]) / 3.0) |> Array.ofSeq) |> Array.ofSeq
//let coefs1 = averages3T1 |> Array.map (fun a -> a.[0])
let coefs1 = tableT1 |> Seq.map (fun c -> 0.9375 / (qsummary c.Rows.AsReal).median) |> Array.ofSeq
coefs1
```

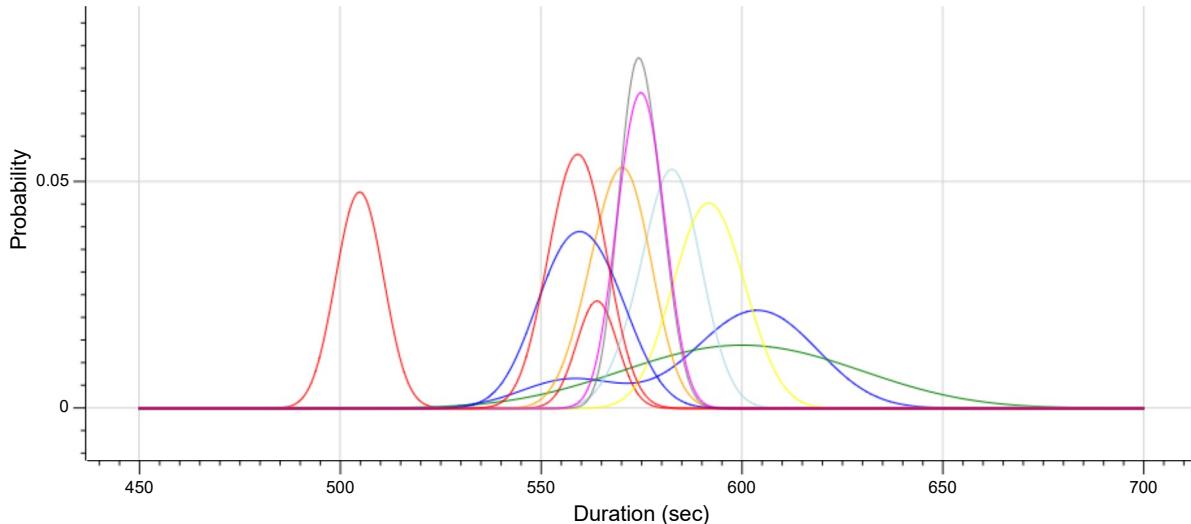
```
Out[23]: [|1.043478261; 1.008403361; 1.081081081; 0.9917355372; 0.9090909091; 1.090909091;
1.008403361; 1.0; 0.9375; 1.0|]
```

```
In [24]: let tableT3a = tableT3 |> Seq.mapi (fun i c -> c.Rows.AsReal |> Seq.map (fun v -> v * coefs1.[i]) |> Array.ofSeq)
    |> Array.ofSeq
let tT3a = tableT3 |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs1.[i])) |> List.ofSeq |> Table.OfColumns
tT3a
```

```
Out[24]: seq
[0|10]: Array is not evaluated yet; 1|10]: Array is not evaluated yet;
2|10]: Array is not evaluated yet; 3|10]: Array is not evaluated yet; ...]
```

```
In [25]: drawPdfColumnsRange tt3a 450.0 700.0
```

```
Out[25]:
```



Analysis of relative standard deviation of medians among multiple runs of a test

Here we run a small problem for 2000 times on each of 10 VMs of our cluster. Then, we combine runs within each machine into equal sized groups (each run belongs to only one group), compute median value for each group and then compute relative standard deviation of those median value. This is done separately for each VM. So, for every group size (ranging from 1 to 200) we obtain 10 relative standard deviations (one per VM). We visualize them as a band plot.

```
In [26]: let tableSmalls = Table.Load ("smalls2k.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = A ngara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls
let smalls = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableSmalls |> Table.Filter ["Task #"] (fun (x: float) -> x = float i)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns
smalls |> Array.ofSeq |> Array.map (fun c -> uncertaintyP c.Rows.AsReal)
```

```
Out[26]: [|4.401407015; 2.677162547; 5.327866042; 4.377607359; 4.769894385; 2.934385546;
3.529012758; 4.642639781; 3.785091269; 6.074913539|]
```

```
In [27]: let blur k (data: System.Collections.Generic.IList<float>) =
    let c = data.Count
    [| 0 .. c - 1 |] |> Array.map (fun i -> (|[| i .. i + k - 1 |] |> Array.map (fun j -> data.[j % c])) |> qsummary y).median)

let blurT k (table: Table) = table |> Seq.mapi (fun i c -> Column.Create (c.Name, blur k c.Rows.AsReal)) |> List. ofSeq |> Table.OfColumns
```

```
In [28]: let smallsB = blurT 10 smalls
smallsB |> Array.ofSeq |> Array.map (fun c -> uncertaintyP c.Rows.AsReal)
```

```
Out[28]: [|1.558699626; 1.432006232; 2.249972518; 1.214148978; 2.357909631; 1.304095235;
1.318277488; 1.639993701; 1.372957626; 2.332889617|]
```

```
In [29]: let blurGroups k (data: System.Collections.Generic.IList<float>) =
    let c = data.Count
    [| 0 .. c / k - 1 |] |> Array.map (fun i -> (|[| i * k .. i * k + k - 1 |] |> Array.map (fun j -> data.[j % c])) |> qsummary y).median)

let blurGroupsT k (table: Table) = table |> Seq.mapi (fun i c -> Column.Create (c.Name, blurGroups k c.Rows.AsReal)) |> List. ofSeq |> Table.OfColumns
```

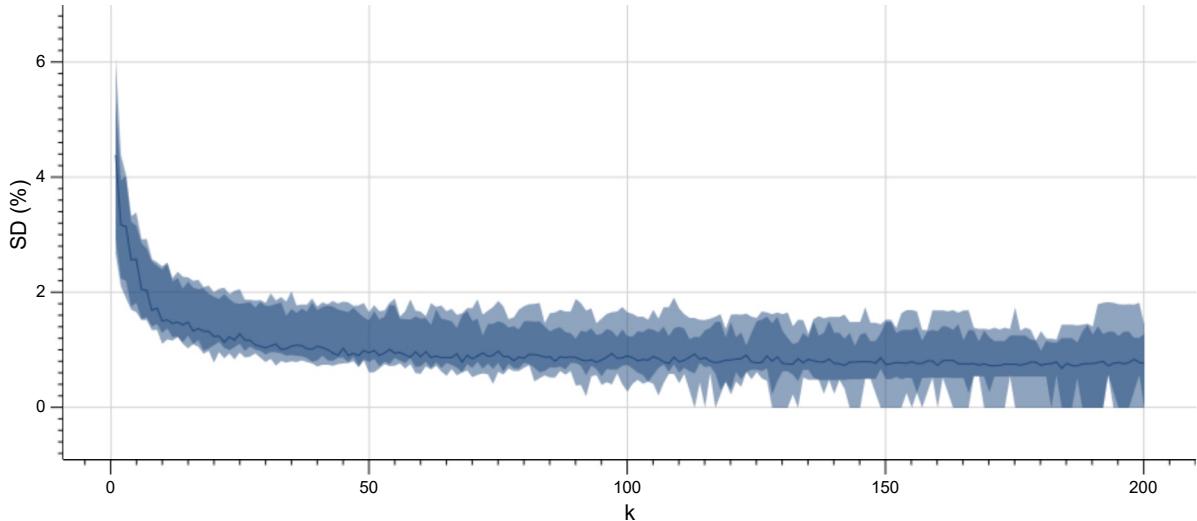
```
In [30]: let smallsBG = blurGroupsT 50 smalls
smallsBG |> Array.ofSeq |> Array.map (fun c -> uncertaintyP c.Rows.AsReal)
```

```
Out[30]: [|0.9653347394; 1.065755067; 1.426853362; 0.7969931839; 1.658535029;
0.6171766131; 0.8996990601; 0.9435641951; 0.9162086768; 1.515441649|]
```

```
In [31]: let smallsBGsd k = blurGroupsT k smalls |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary
//let smallsBGarr = [|1..200|] |> Array.map (fun i -> smallsBGsd i)

//let smallsBGquants = toQuantiles smallsBGarr
[Plot.line(LineX.Values ([|1..200|] |> Array.map float),
    LineY.UncertainValues ([|1..200|] |> Array.map (fun i -> smallsBGsd i) |> toQuantiles),
    titles = Titles.line("k", "SD (%))") ] |> Chart.ofList
```

Out[31]:

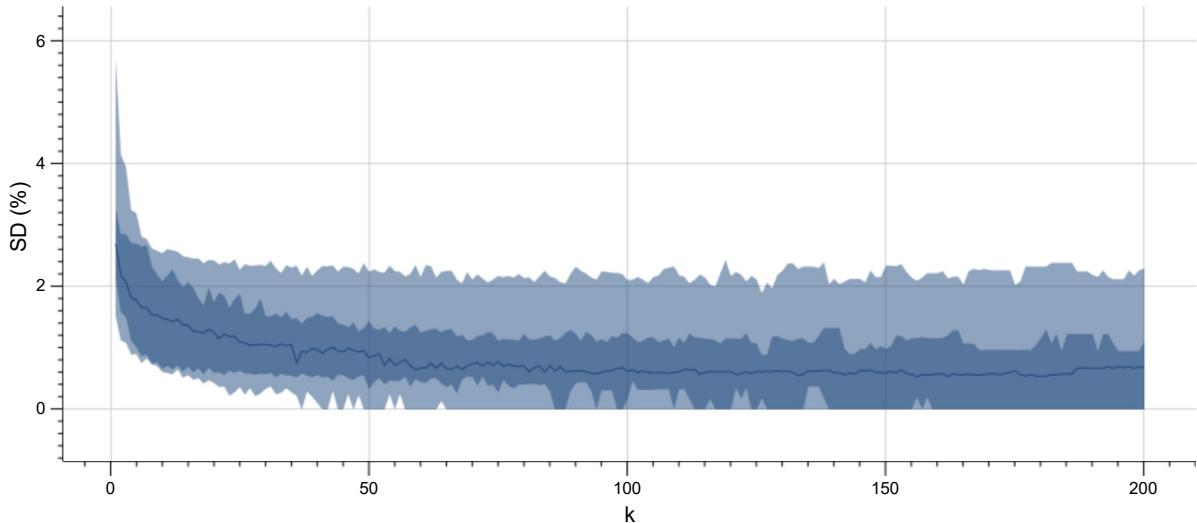


Dependence between relative standard deviation of median values of the groups of test runs and the size of the group. Plot's uncertainty (band) shows range of relative SD obtained on different VMs.

```
In [32]: let tableSmalls2 = Table.Load ("smalls2k2.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter =
    Angara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls2
let smalls2 = [|0..9|] |> List.map (fun i -> Column.Create (i.ToString(), (tableSmalls2 |> Table.Filter ["Task #"])
    (fun (x: float) -> x = float i)).["CPUTime"].Rows.AsReal) |> Table.OfColumns
let smalls2BGsd k = blurGroupST k smalls2 |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary
//let smallsBGarr = [|1..200|] |> Array.map (fun i -> smallsBGsd i)

//let smallsBGquants = toQuantiles smallsBGarr
[Plot.line(LineX.Values ([|1..200|] |> Array.map float),
    LineY.UncertainValues ([|1..200|] |> Array.map (fun i -> smalls2BGsd i) |> toQuantiles),
    titles = Titles.line("k", "SD (%))") ] |> Chart.ofList
```

Out[32]:



Same but for a different test task.

```
In [33]: let drawRowsUncertaintiesP table =
    let m = table |> transpose
    let unc = m |> Seq.map (fun runRes -> uncertaintyP (runRes.Rows.AsReal)) |> Seq.toArray
    let x = Array.init m.Count (fun i -> float(i+1))
    //let duration = toQuantiles unc
    [ Plot.line(LineX.Values x, LineY.Values unc,
        titles = Titles.line("Runs", "SD (%))",
        displayName = "SD / Median (%)") ]
    |> Chart.ofList
```

```
In [34]: let tableExpl = Table.Load ("expl.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls
let smallTask = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExpl |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float) -> x = float i && y = 0.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns
let largeTask = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExpl |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float) -> x = float i && y = 1.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns

let etalonicTime = 0.9375

let coefByN n = blurGroupsT n smallTask |> Seq.map (fun c -> etalonicTime / c.Rows.AsReal.[0]) |> Array.ofSeq
let coefs = coefByN 30

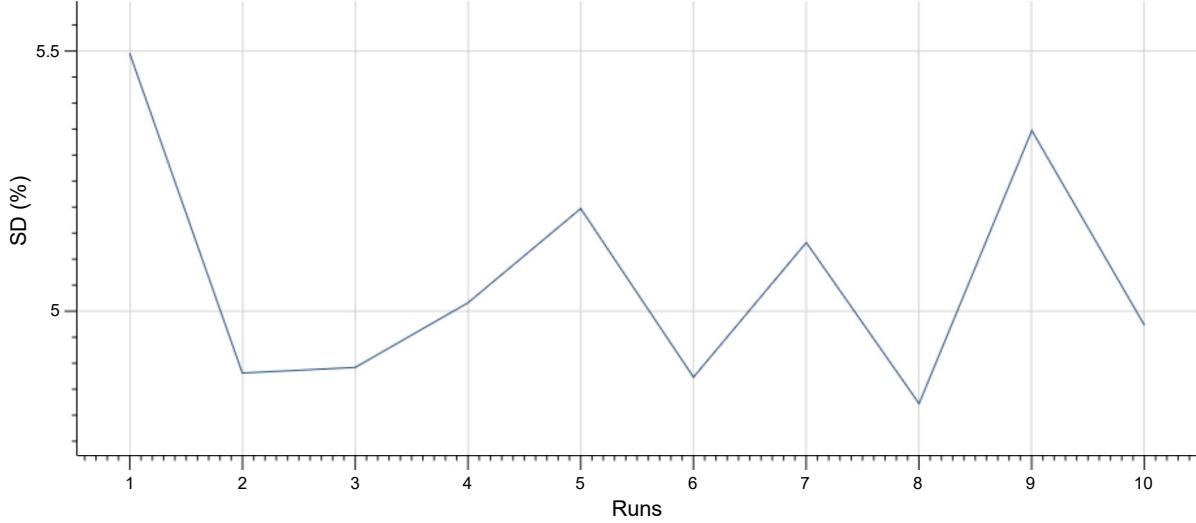
let adjustedLargeTask = largeTask |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs.[i]))) |> List.ofSeq |> Table.OfColumns

adjustedLargeTask.[9].Rows.AsReal |> Array.ofSeq
```

```
Out[34]: [|95.19886364; 96.35795455; 96.06818182; 95.30113636; 94.53409091; 95.25;
95.76136364; 96.13636364; 95.53977273; 96.46022727|]
```

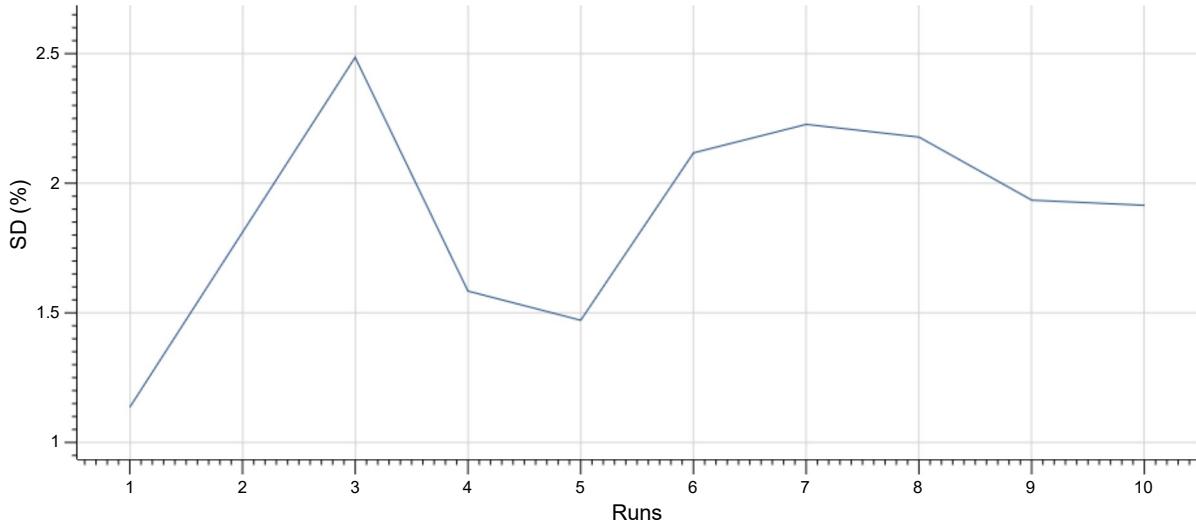
```
In [35]: drawRowsUncertaintiesP largeTask
```

```
Out[35]:
```



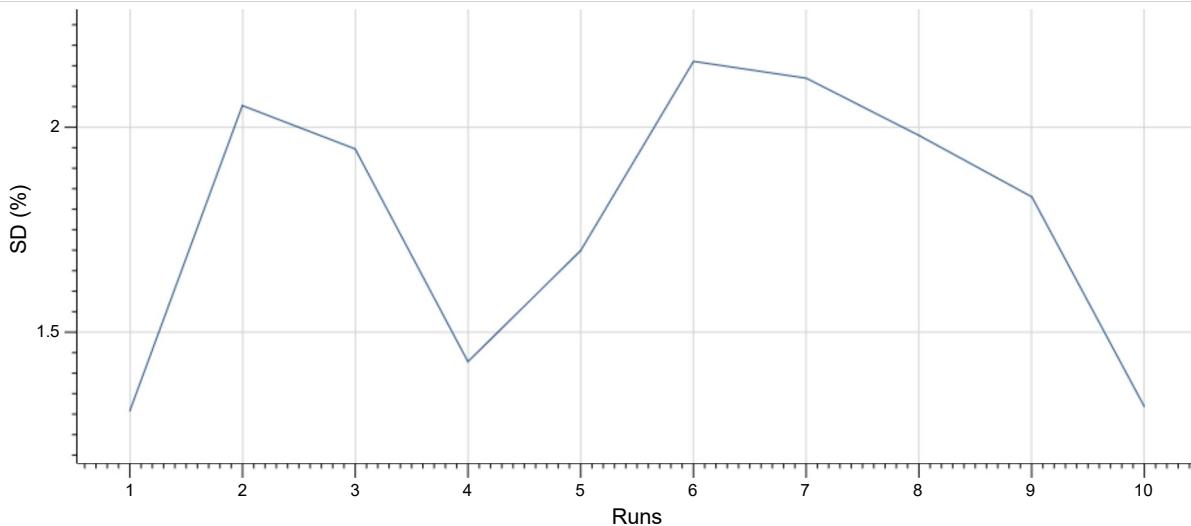
```
In [36]: drawRowsUncertaintiesP adjustedLargeTask
```

```
Out[36]:
```



```
In [37]: let largeTaskB = blurT 2 largeTask
let adjustedLargeTaskB = largeTaskB |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs.[i]) )) |> List.ofSeq |> Table.OfColumns
drawRowsUncertaintiesP adjustedLargeTaskB
```

Out[37]:



Computing Performance Coefficient of a VM using median result of 30 test task runs

```
In [38]: let tableExp2 = Table.Load ("exp2.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls
let smallTask2 = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExp2 |> Table.Filter ["Task #"; "File #"]) (fun (x: float) (y: float) -> x = float i && y = 0.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns
let largeTask2 = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExp2 |> Table.Filter ["Task #"; "File #"]) (fun (x: float) (y: float) -> x = float i && y = 1.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns

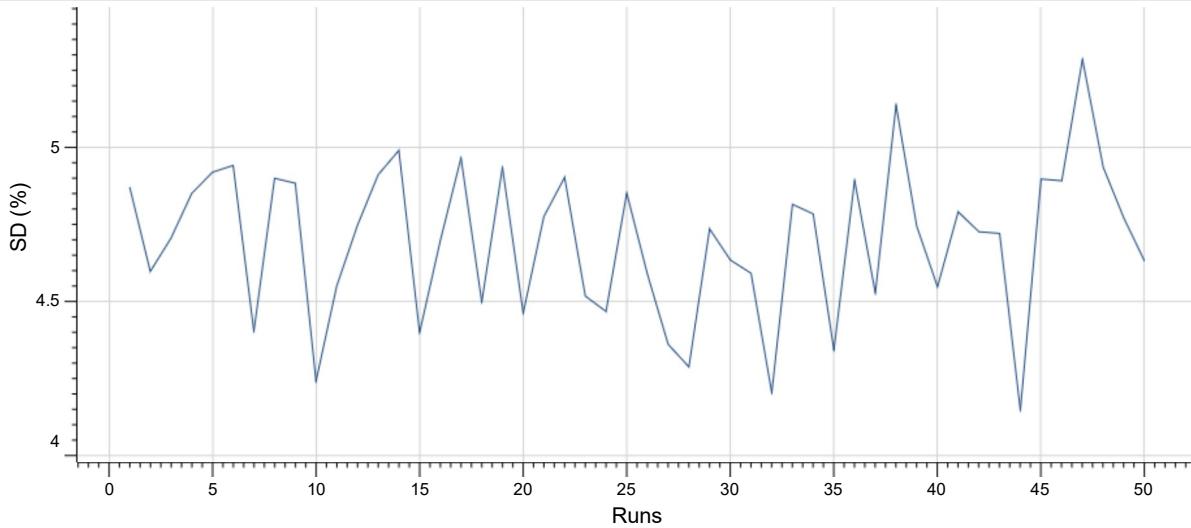
let etalonicTime2 = 0.9375

let coefByN2 n = blurGroupsT n smallTask2 |> Seq.map (fun c -> etalonicTime2 / c.Rows.AsReal.[0]) |> Array.ofSeq
let coefs2 = coefByN2 30

let adjustedLargeTask2 = largeTask2 |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs2.[i]) )) |> List.ofSeq |> Table.OfColumns
```

Out[39]: drawRowsUncertaintiesP largeTask2

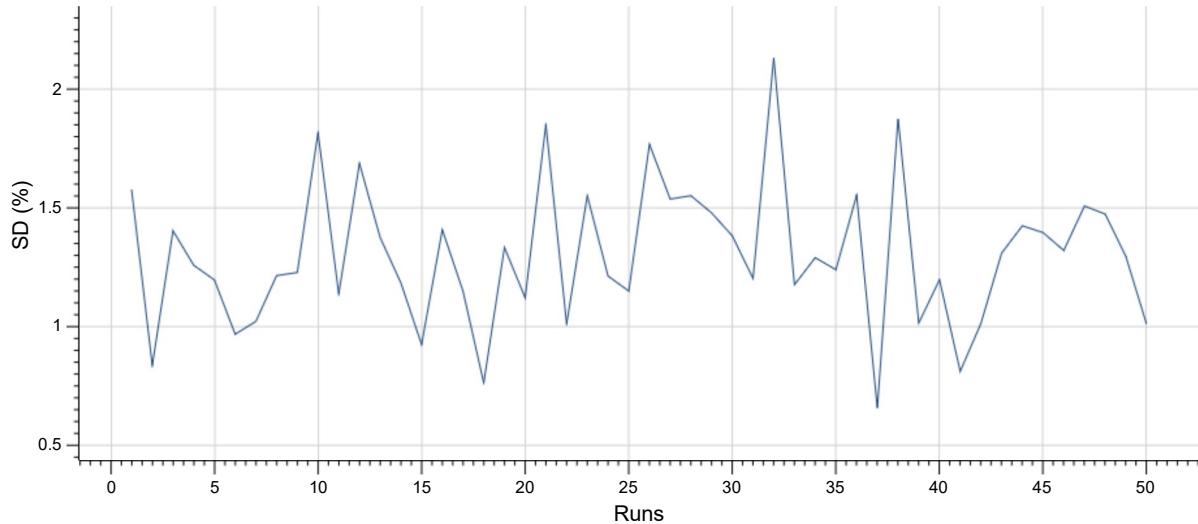
Out[39]:



Relative uncertainty between different VMs.

```
In [40]: drawRowsUncertaintiesP adjustedLargeTask2
```

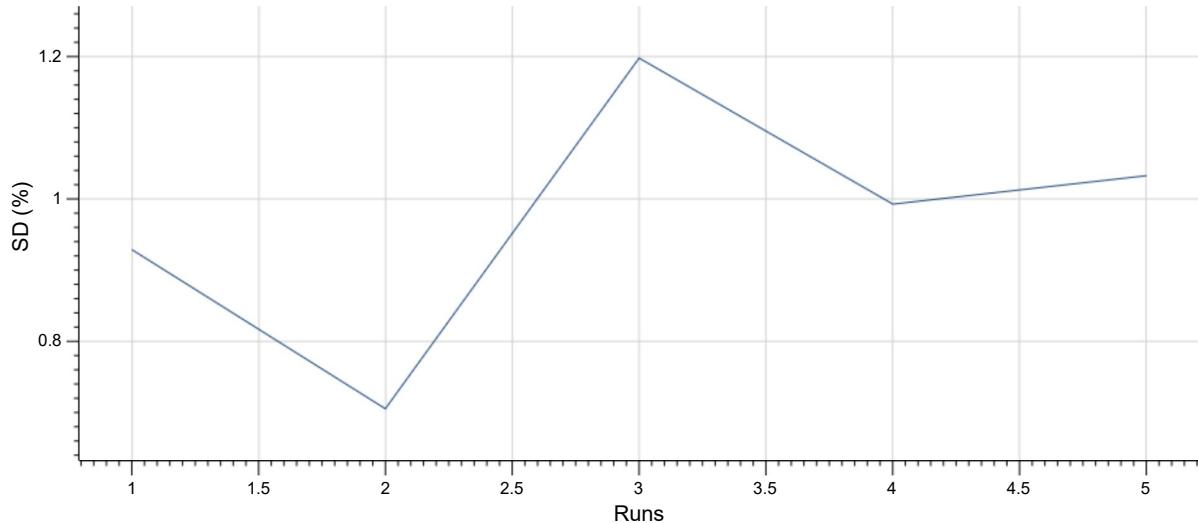
```
Out[40]:
```



Relative uncertainty between different VMs after application of performance coefficient. About 2-3 times lower than it used to be.

```
In [41]: let largeTaskB2 = blurGroupsT 10 largeTask2
let adjustedLargeTaskB2 = largeTaskB2 |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs2.[i])) |> List.ofSeq |> Table.OfColumns
drawRowsUncertaintiesP adjustedLargeTaskB2
```

```
Out[41]:
```



Using Performance Coefficient to Compare Results Obtained on VMs of Different Types (A2 and D2_V2)

```
In [42]: let tableExp3 = Table.Load ("exp3a.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls
let smallTask3 = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExp3 |> Table.Filter ["Task #"; "File #"]) (fun (x: float) (y: float) -> x = float i && y = 0.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns
let largeTask3 = [0..9] |> List.map (fun i -> Column.Create (i.ToString(), (tableExp3 |> Table.Filter ["Task #"; "File #"]) (fun (x: float) (y: float) -> x = float i && y = 1.0)).["CPUTime"].Rows.AsReal)) |> Table.OfColumns

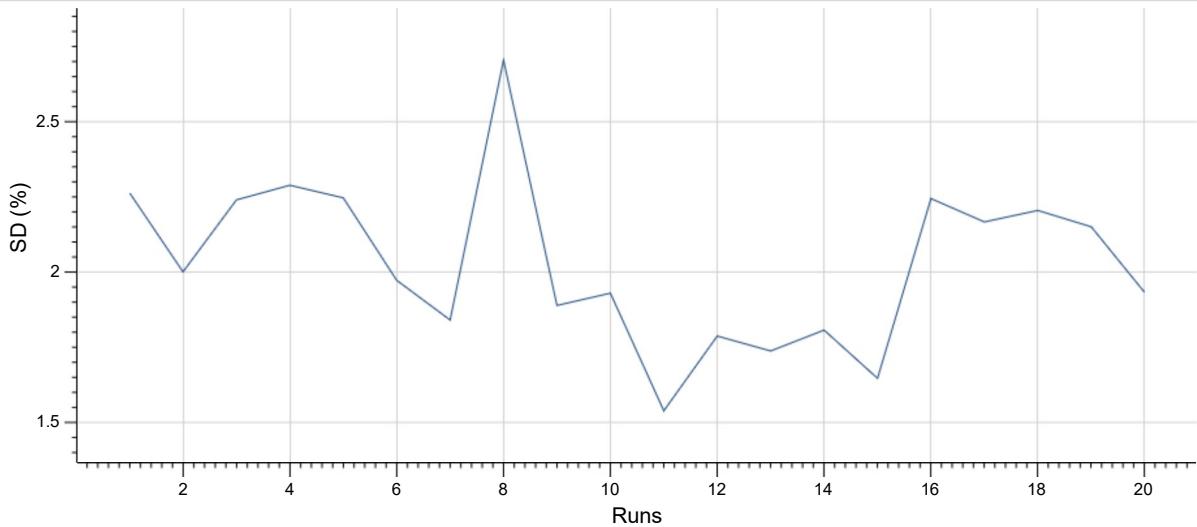
let etalonicTime3 = 0.9375

let coefByN3 n = blurGroupsT n smallTask3 |> Seq.map (fun c -> etalonicTime3 / c.Rows.AsReal.[0]) |> Array.ofSeq
let coefs3 = coefByN3 30

let adjustedLargeTask3 = largeTask3 |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map (fun v -> v * coefs3.[i])) |> List.ofSeq |> Table.OfColumns
```

```
In [43]: let shortenedAdjLarge2 = takeRows 20 adjustedLargeTask2
let diffMachinesLargeTask = shortenedAdjLarge2 |> Seq.append adjustedLargeTask3 |> Table.OfColumns
drawRowsUncertaintiesP diffMachinesLargeTask
```

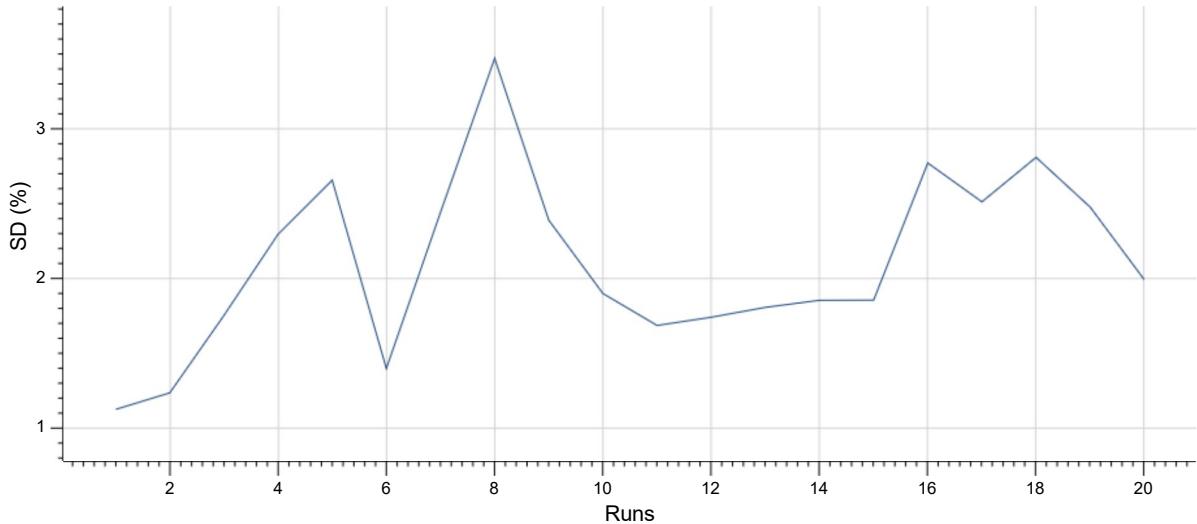
Out[43]:



Relative SD for 10 d2_v2 and 10 a2 machines (performance coefficient applied).

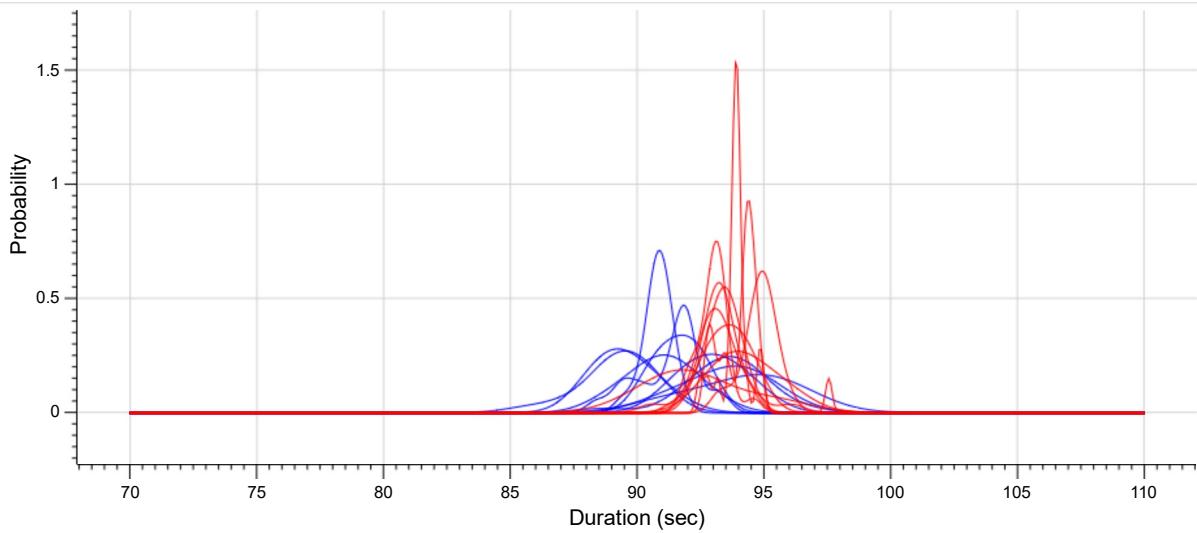
```
In [44]: drawRowsUncertaintiesP adjustedLargeTask3
```

Out[44]:



```
In [45]: drawPdfColumnsRangeColored diffMachinesLargeTask 70.0 110.0 (fun i -> if i < 10 then "blue" else "red")
```

Out[45]:



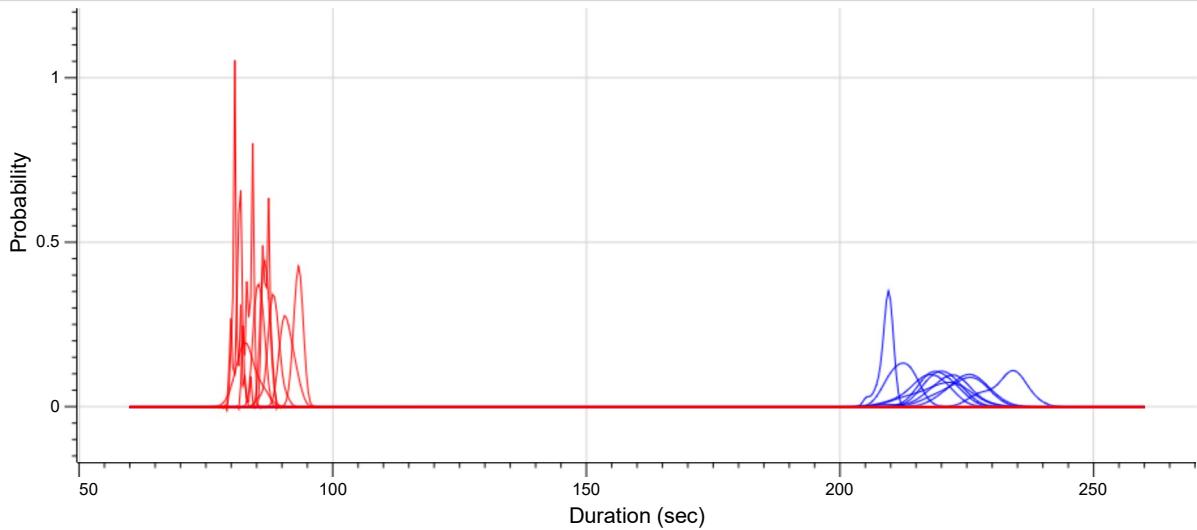
PDFs for results obtained on different VMs adjusted by performance coefficient (red - d2_v2, blue - a2).

```
In [46]: let shortenedLarge2 = takeRows 20 largeTask2
```

```
let diffMachinesLargeTaskNA = shortenedLarge2 |> Seq.append largeTask3 |> Table.OfColumns
```

```
In [47]: drawPdfColumnsRangeColored diffMachinesLargeTaskNA 60.0 260.0 (fun i -> if i < 10 then "blue" else "red")
```

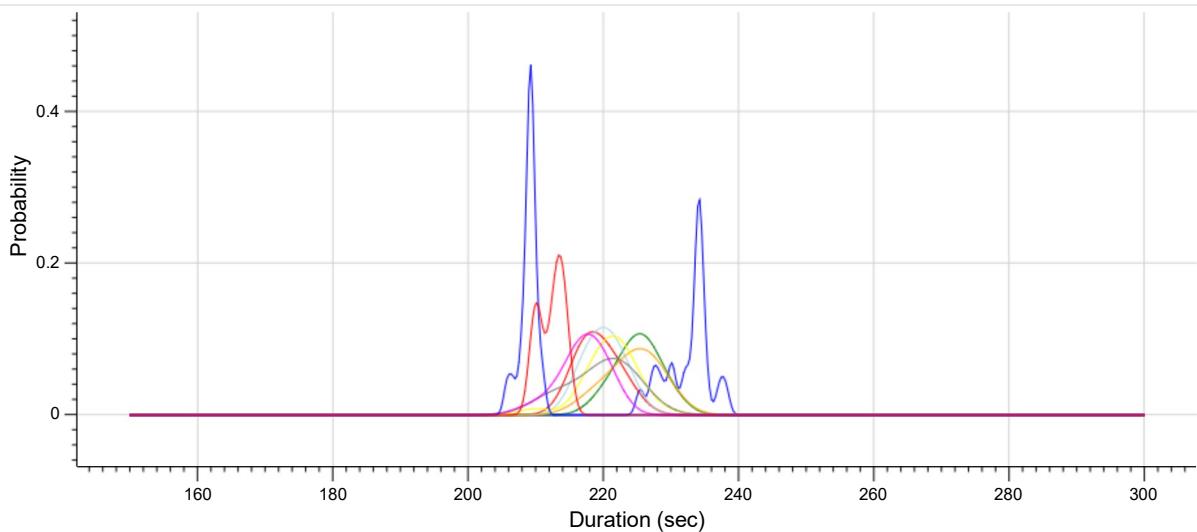
```
Out[47]:
```



PDFs for results on different VMs without adjustment.

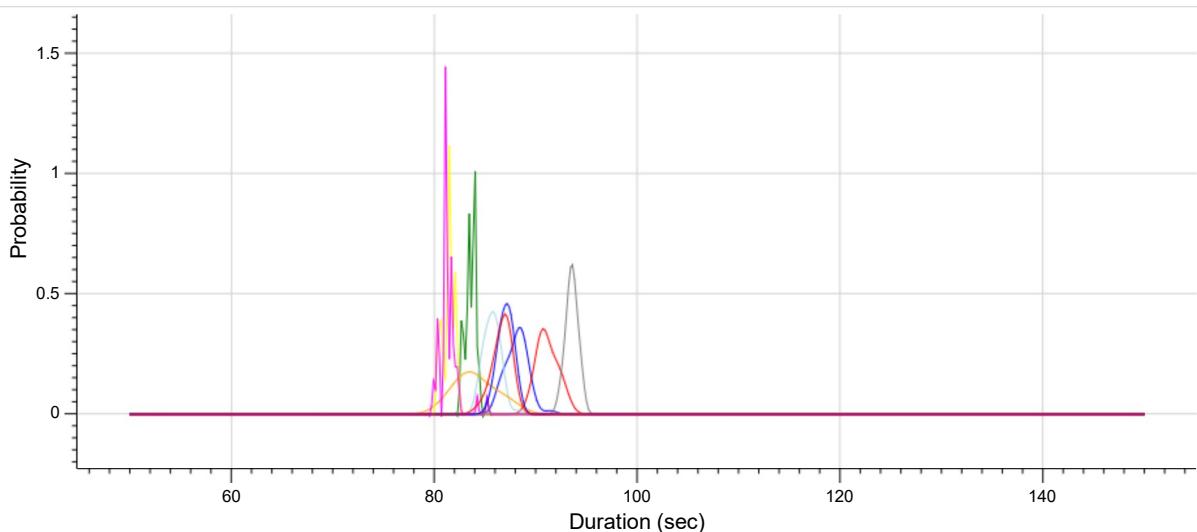
```
In [48]: drawPdfColumnsRange largeTask3 150.0 300.0
```

```
Out[48]:
```



```
In [49]: drawPdfColumnsRange largeTask2 50.0 150.0
```

```
Out[49]:
```



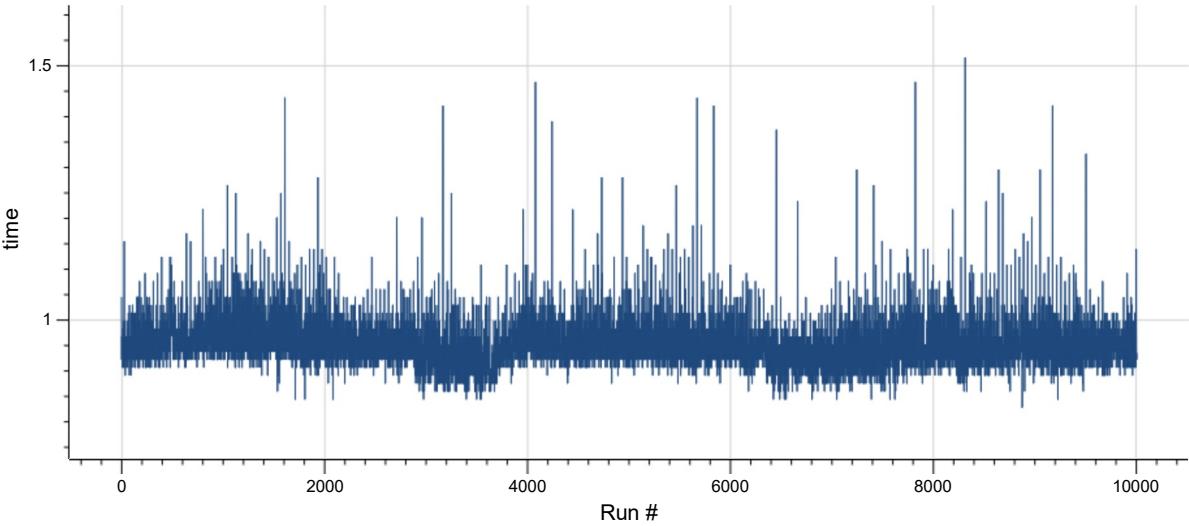
Impact of Long Uptime on Performance of a VM

Here we research, how performance will vary on a VM during a long (~10 hours) period of time. We need this to determine, how often (if at all) should we re-evaluate performance coefficient of a VM.

```
In [50]: let tablelr = Table.Load ("longrun.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
//tableSmalls
let smallTasklr = [ Column.Create ("0", (tablelr |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float)
-> x = 0.0 && y = 0.0)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns
let largeTasklr = [ Column.Create ("0", (tablelr |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float)
-> x = 0.0 && y = 1.0)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns
```

```
In [51]: [Plot.line(LineX.Values ([|1..smallTasklr.[0].Rows.AsReal.Length|] |> Array.map float),
LineY.Values (smallTasklr.[0].Rows.AsReal.ToArray()),
titles = Titles.line("Run #", "time"))] |> Chart.ofList
```

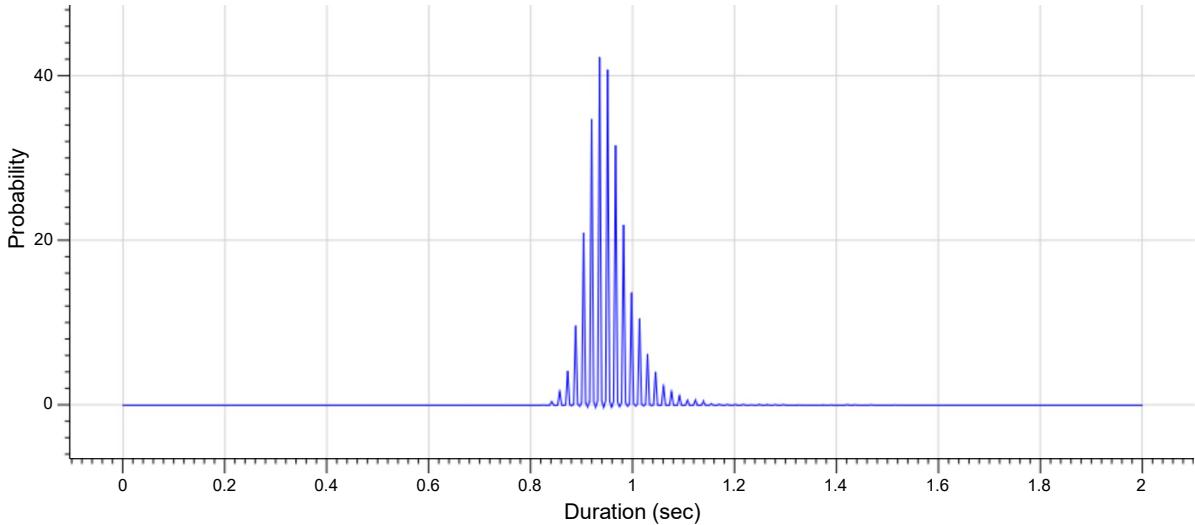
Out[51]:



Running times for a small task on a ~3 hour time interval.

```
In [52]: drawPdfColumnsRange smallTasklr 0.0 2.0
```

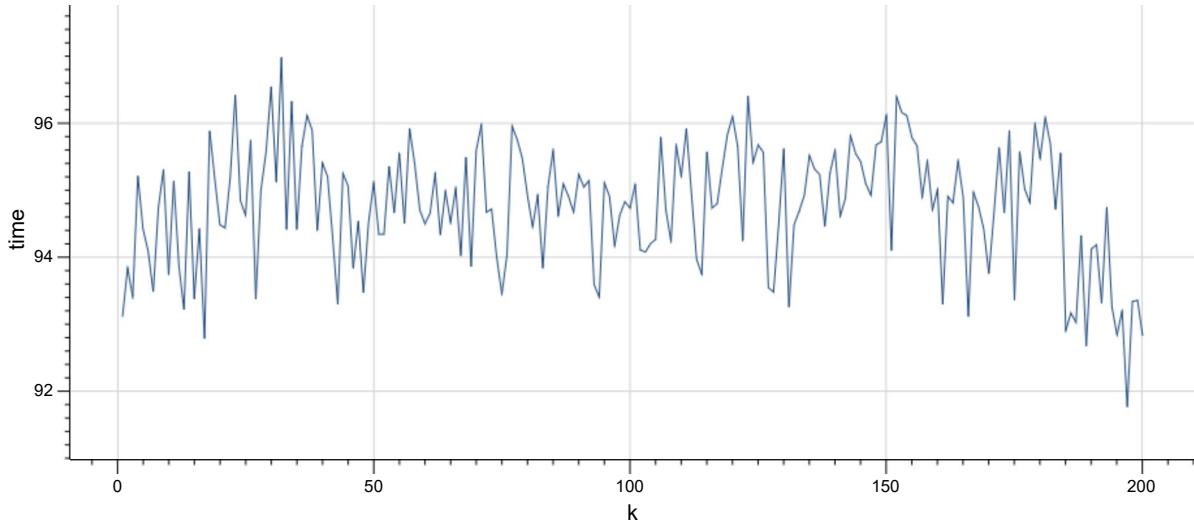
Out[52]:



PDF for 10000 runs of a small task.

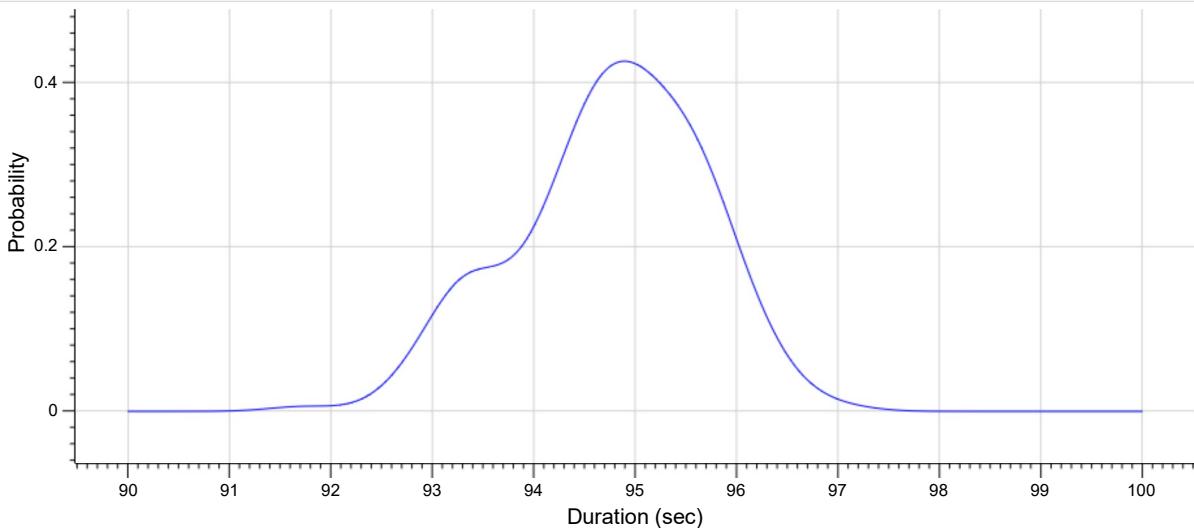
```
In [53]: [Plot.line(LineX.Values [|1..largeTasklr.[0].Rows.AsReal.Length|] |> Array.map float),
          LineY.Values (largeTasklr.[0].Rows.AsReal.ToArray()),
          titles = Titles.line("k", "time")) ] |> Chart.ofList
```

Out[53]:



```
In [54]: drawPdfColumnsRange largeTasklr 90.0 100.0
```

Out[54]:



Same for a large (~90 sec.) task.

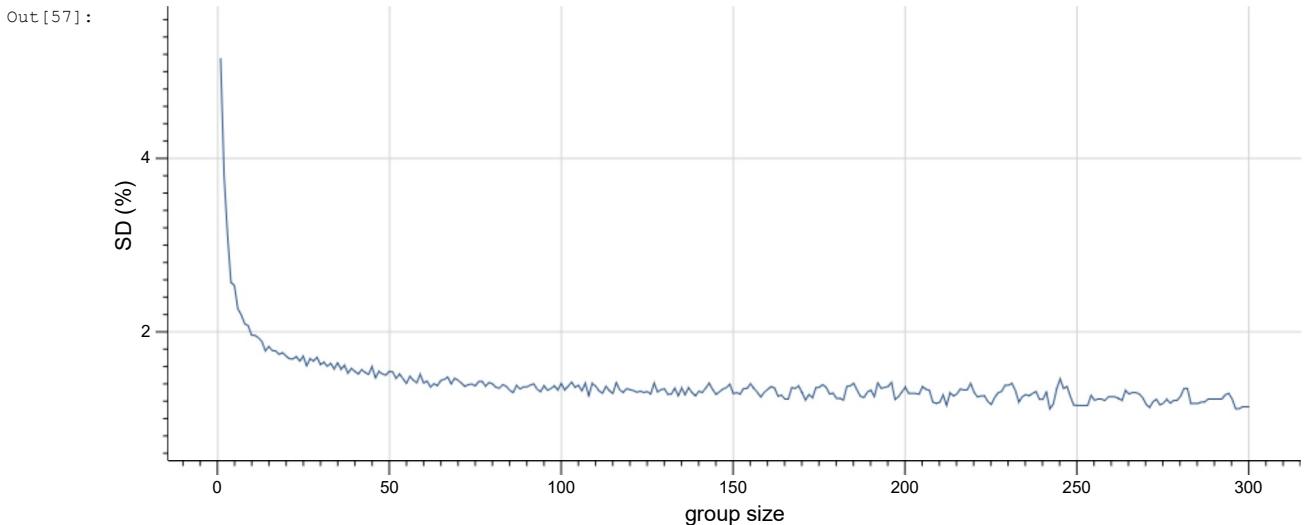
```
In [55]: stat (smallTasklr.[0].Rows.AsReal)
```

```
Out[55]: {min = 0.828125;
          lb95 = 0.875;
          lb68 = 0.921875;
          median = 0.953125;
          ub68 = 0.921875;
          ub95 = 1.0625;
          max = 1.515625;
          sd = 0.04908684442;}
```

```
In [56]: stat (largeTasklr.[0].Rows.AsReal)
```

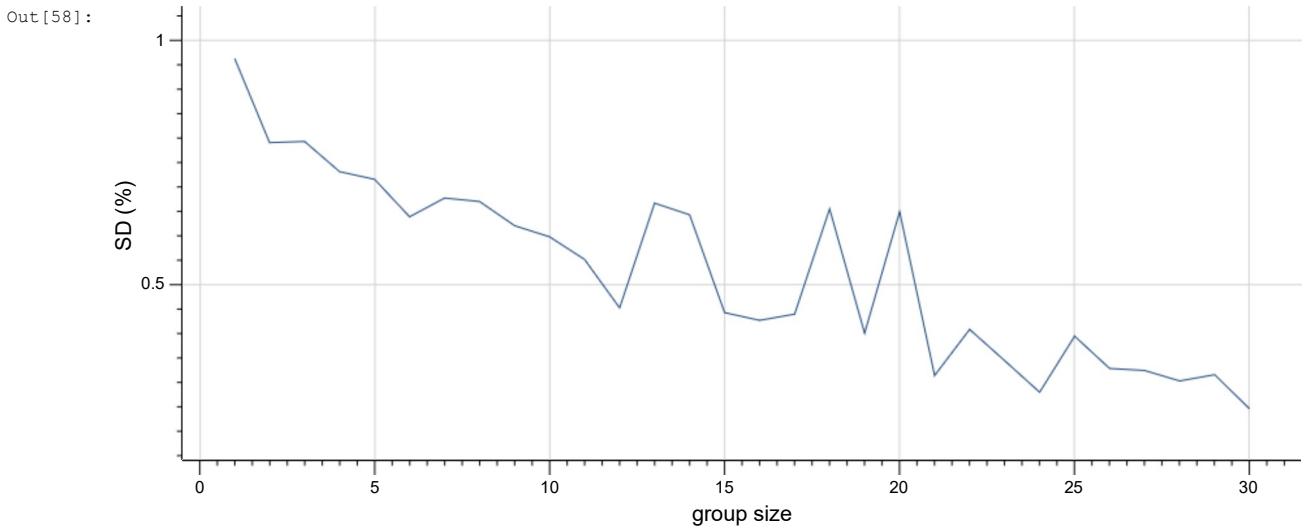
```
Out[56]: {min = 91.765625;
          lb95 = 92.86510417;
          lb68 = 93.75;
          median = 94.8359375;
          ub68 = 93.75;
          ub95 = 96.36927083;
          max = 96.984375;
          sd = 0.9126280627;}
```

```
In [57]: let groupedSmallLr k = blurGroupsT k smallTasklr
[Plot.line(LineX.Values [|1..300|] |> Array.map float),
 LineY.Values [| for i in 1..300 -> uncertaintyP ((groupedSmallLr i).[0].Rows.AsReal) |],
 titles = Titles.line("group size", "SD (%))") ] |> Chart.ofList
```



Relative SD for median value of a group of runs depending on group size (see above for explanations). Small task. It appears, running a small task 20-50 times and taking median value significantly reduces uncertainty.

```
In [58]: let groupedLargeLr k = blurGroupsT k largeTasklr
[Plot.line(LineX.Values [|1..30|] |> Array.map float),
 LineY.Values [| for i in 1..30 -> uncertaintyP ((groupedLargeLr i).[0].Rows.AsReal) |],
 titles = Titles.line("group size", "SD (%))") ] |> Chart.ofList
```

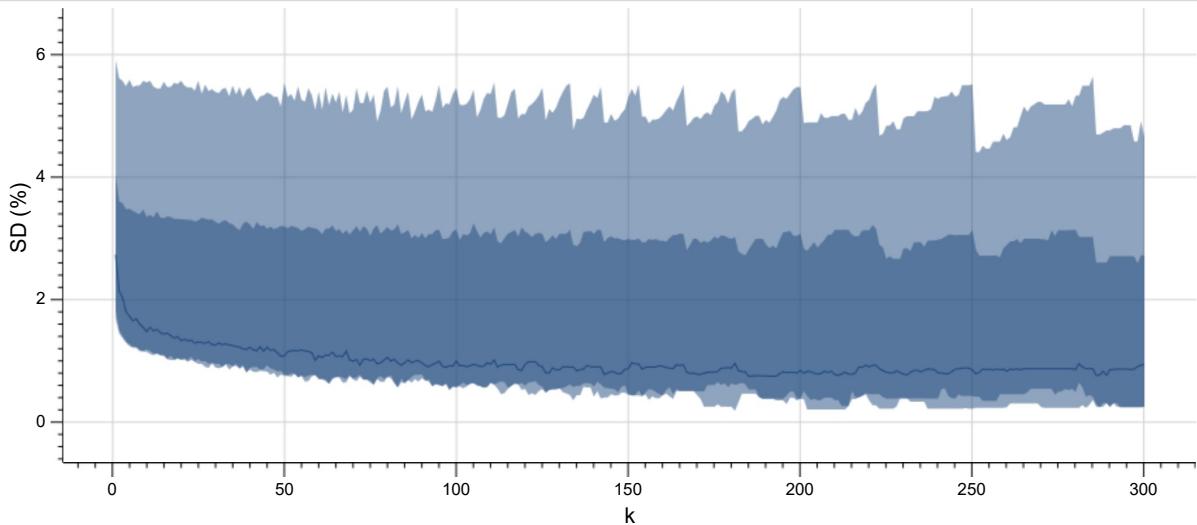


Same for large task. Relative SD is below 1% from the beginning, which is much better, than that for a small task.

```
In [59]: let tablelrA2 = Table.Load ("longrunA2.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
let smallTasklrA2 = [ for i in 0..9 -> Column.Create (i.ToString(), (tablelrA2 |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float) -> x = float i && y = 0.0)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns
let largeTasklrA2 = [ for i in 0..9 -> Column.Create (i.ToString(), (tablelrA2 |> Table.Filter ["Task #"; "File #"] (fun (x: float) (y: float) -> x = float i && y = 1.0)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns
```

```
In [60]: let groupedSmallLrA2 k = blurGroupsT k smallTasklrA2 |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary
[Plot.line(LineX.Values ([|1..300|] |> Array.map float),
LineY.UncertainValues ([|1..300|] |> Array.map (fun i -> groupedSmallLrA2 i) |> toQuantiles),
titles = Titles.line("k", "SD (%))") ] |> Chart.ofList
```

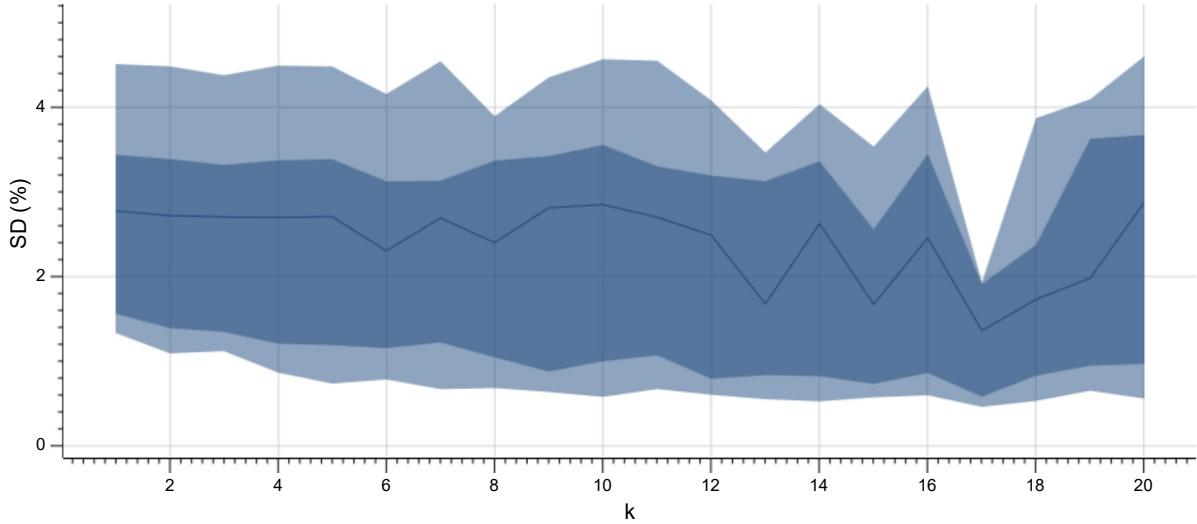
Out[60]:



Dependence between relative standard deviation of median values of the groups of test runs and the size of the group. A2 VMs. Plot's uncertainty (band) shows range of relative SD obtained on different VMs. It appears, A2 machines produce quite unstable results.

```
In [61]: let groupedLargeLrA2 k = blurGroupsT k largeTasklrA2 |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary
[Plot.line(LineX.Values ([|1..20|] |> Array.map float),
LineY.UncertainValues ([|1..20|] |> Array.map (fun i -> groupedLargeLrA2 i) |> toQuantiles),
titles = Titles.line("k", "SD (%))") ] |> Chart.ofList
```

Out[61]:



Same for a large task. (A2)

Comparison of Run Statistics (z3 -st) Obtained by Running Same Tests on Different VMs

```
In [62]: let tableExpSt = Table.Load ("expst.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let smallTaskSt = tableExpSt |> Table.Filter ["File #"] (fun (x: float) -> x = 0.0)
let largeTaskSt = tableExpSt |> Table.Filter ["File #"] (fun (x: float) -> x = 1.0)
```

In [63]: schema smallTaskSt

Out[63]: <null>

```
In [64]: [|| for i in 4..36 -> (smallTaskSt.[i].Name, (smallTaskSt.[i].Rows.AsReal |> Seq.distinct |> Seq.length) = 1) ||]

Out[64]: [{"Return value", true}; {"CPUTime", false}; {"WCTime", false};
("Memory", false); {"binary_propagations", true}; {"conflicts", true};
("decisions", true); {"del_clause", true};
("dyn-subsumption-resolution", true); {"elim-blocked-clauses", true};
("elim-bool-vars", true); {"elim-clauses", true}; {"elim-literals", true};
("eliminated-applications", true); {"eliminated-vars", true};
("gc-clause", true); {"max-memory", true}; {"memory", true};
("minimized-lits", true); {"mk-binary-clause", true}; {"mk-bool-var", true};
("mk-clause", true); {"mk-ternary-clause", true}; {"num-allocs", true};
("probing-assigned", true); {"propagations", true}; {"restarts", true};
("rlimit-count", true); {"subsumed", true}; {"subsumption-resolution", true};
("ternary-propagations", true); {"time", false}; {"total-time", false}]]

In [65]: [|| for i in 4..36 -> (largeTaskSt.[i].Name, (smallTaskSt.[i].Rows.AsReal |> Seq.distinct |> Seq.length) = 1) ||]

Out[65]: [{"Return value", true}; {"CPUTime", false}; {"WCTime", false};
("Memory", false); {"binary_propagations", true}; {"conflicts", true};
("decisions", true); {"del_clause", true};
("dyn-subsumption-resolution", true); {"elim-blocked-clauses", true};
("elim-bool-vars", true); {"elim-clauses", true}; {"elim-literals", true};
("eliminated-applications", true); {"eliminated-vars", true};
("gc-clause", true); {"max-memory", true}; {"memory", true};
("minimized-lits", true); {"mk-binary-clause", true}; {"mk-bool-var", true};
("mk-clause", true); {"mk-ternary-clause", true}; {"num-allocs", true};
("probing-assigned", true); {"propagations", true}; {"restarts", true};
("rlimit-count", true); {"subsumed", true}; {"subsumption-resolution", true};
("ternary-propagations", true); {"time", false}; {"total-time", false}]]


```

Tests appear to be deterministic in everything, except running time, which is expected.

Minimizing Uncertainty of Performance Coefficient

Here we research relative standard deviation of median results within equal-sized group of runs depending on size of the group and typical running time of the test. The goal is to find a test, median result of a group of which will produce the least relative standard deviation within a reasonable time frame.

To put it simpler, we want a test that we will run for, say, 3 minutes, take the median result, and that result should have as less uncertainty as possible.

```
In [66]: let tableLots = Table.Load ("lots.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let lotsByFileFull = [|| for i in 0..4 -> tableLots |> Table.Filter ["File #"] (fun (x:float) -> x = float i) ||]

let lotsByFile = [|| for i in 0..4 -> [for j in 0..9 -> Column.Create (j.ToString(), (lotsByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns ||]

In [67]: [|| for i in 0..4 -> lotsByFileFull.[i].["CPUTime"].Rows.AsReal |> qsummary ||]

Out[67]: [{min = 1.953125;
lb95 = 2.03125;
lb68 = 2.1875;
median = 2.296875;
ub68 = 2.46875;
ub95 = 2.671875;
max = 3.03125;}; {min = 5.046875;
lb95 = 5.1875;
lb68 = 5.578125;
median = 5.84375;
ub68 = 6.21875;
ub95 = 6.671875;
max = 7.171875;}; {min = 268.15625;
lb95 = 273.7397135;
lb68 = 293.484375;
median = 307.265625;
ub68 = 324.2535417;
ub95 = 332.96875;
max = 347.890625;}; {min = 56.59375;
lb95 = 57.734375;
lb68 = 62.30291667;
median = 64.609375;
ub68 = 69.0;
ub95 = 72.125;
max = 75.109375;};

{min = 12.40625;
lb95 = 12.67721354;
lb68 = 13.640625;
median = 14.1875;
ub68 = 15.265625;
ub95 = 16.4375;
max = 17.234375;}||]
```

We wanted tests with average running time of 2, 5, 10, 20, and 30 seconds, but benchmarks that were completed in such times during last experient on the cluster suddenly produced different times in batch. Obviously, something was different, e.g. z3 parameters; also use of a single core was enforced in batch, while in cluster z3 had access to multiple cores.

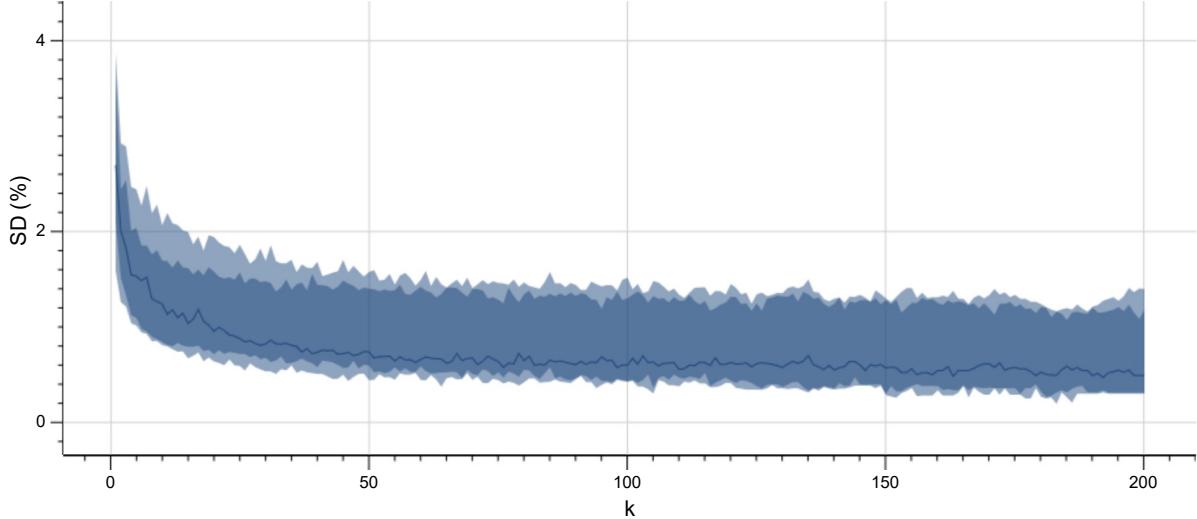
Anyway, we've got 3 good sets of data: 1st with average time of 2.2 sec; 2nd - 5.8 sec; 5th - 15 sec.

```
In [68]: let lotsByFileBGsd i k = blurGroupsT k lotsByFile.[i] |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary
```

```
In [69]: let plotLotssd i k = [Plot.line(LineX.Values ([| for j in 1..k -> float j |]),
LineY.UncertainValues ([| for j in 1..k -> lotsByFileBGsd i j |]) |> toQuantiles),
titles = Titles.line("k", "SD (%)")] |> Chart.ofList
```

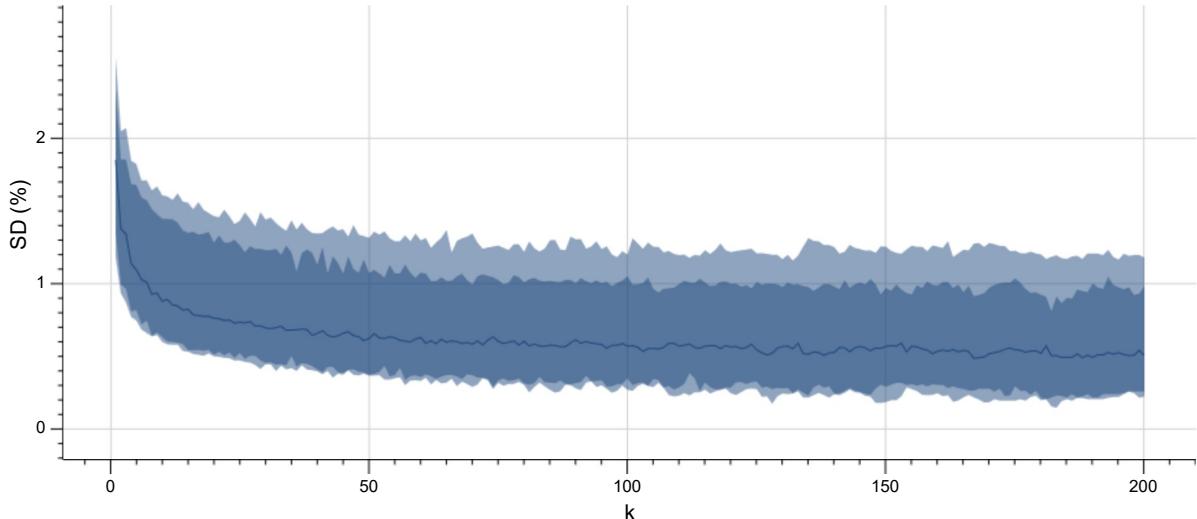
```
In [70]: plotLotssd 0 200
```

```
Out[70]:
```



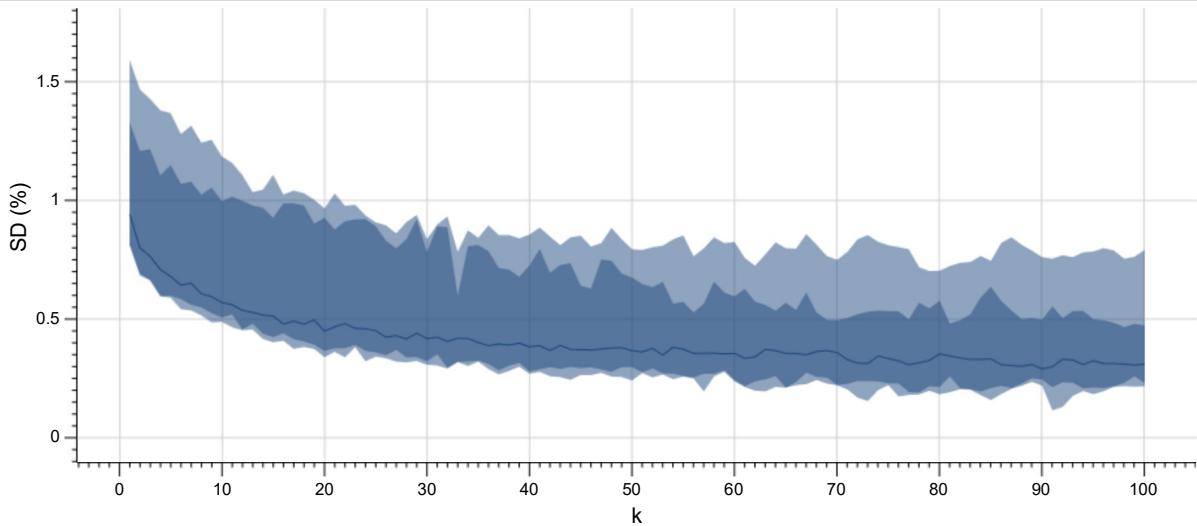
```
In [71]: plotLotssd 1 200
```

```
Out[71]:
```



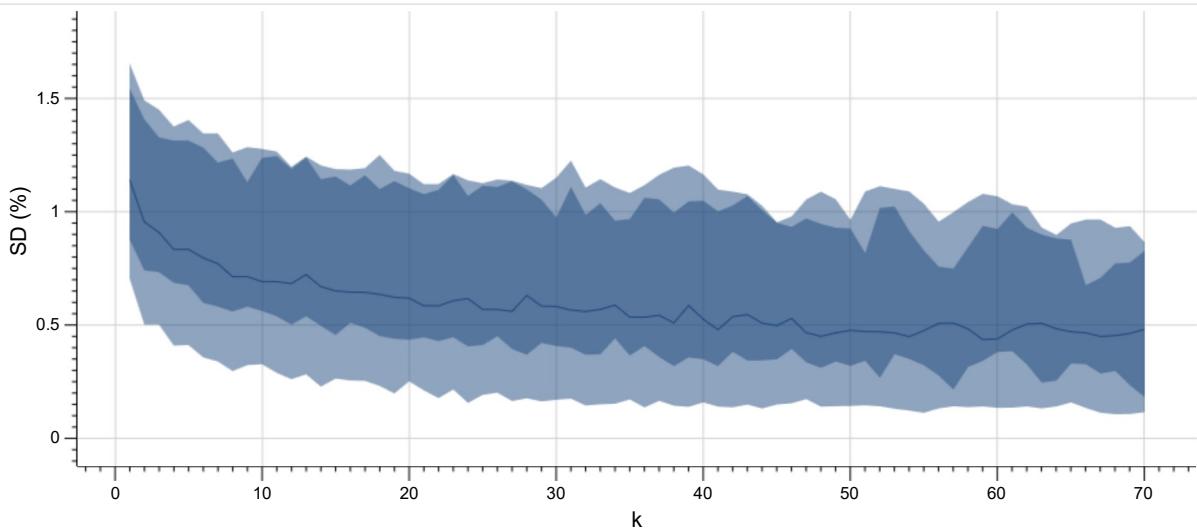
```
In [72]: plotLotssd 2 100
```

```
Out[72]:
```



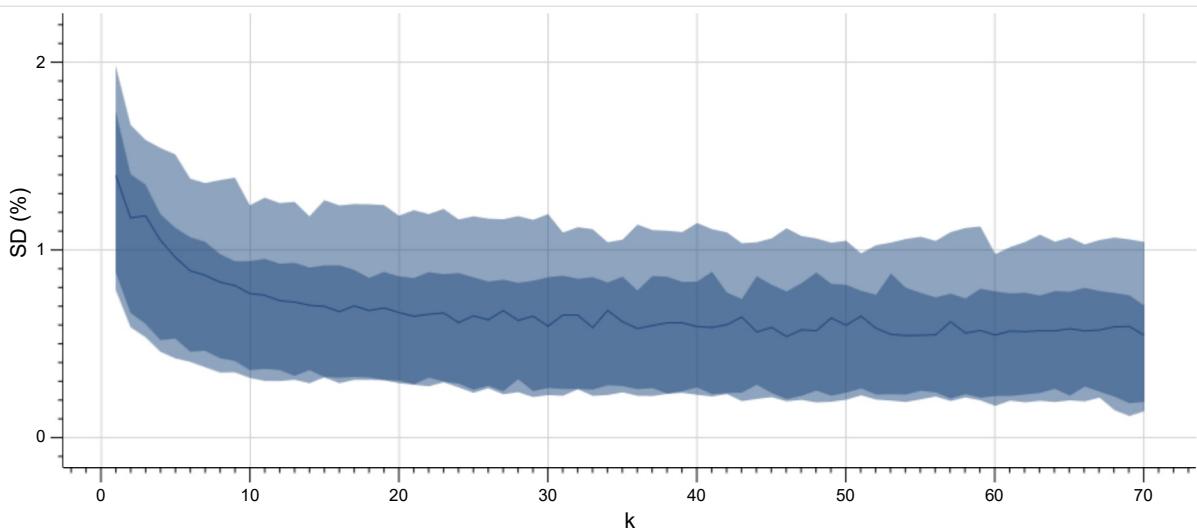
```
In [73]: plotLotssd 3 70
```

```
Out[73]:
```



```
In [74]: plotLotssd 4 70
```

```
Out[74]:
```



So,

- 50+ runs of 2.3 sec problem give us 0.7-1.6% of uncertainty within 130 sec
- 50+ runs of 5 sec problem give us 0.4-1.3% of uncertainty within 250 sec
- 10+ runs of 15 sec problem give us 0.3-1.3% of uncertainty within 150 sec

Among the listed, the last one appears most pleasant. We need to experiment more, obviously. Also, it must be mentioned, that longer (1 min and 5 min) problems do not produce any better results within 10 minutes time frame. In fact, to reduce uncertainty to values less than 1% we need to run these experiments for more than an hour.

Averaging Results for Short-Running Experiments to Decrease Uncertainty

As we've seen above, shot-running experiments have a rather high (compared to the same of long-running ones) uncertainty, which can be decreased by repeating the experiment multiple times and picking the median value.

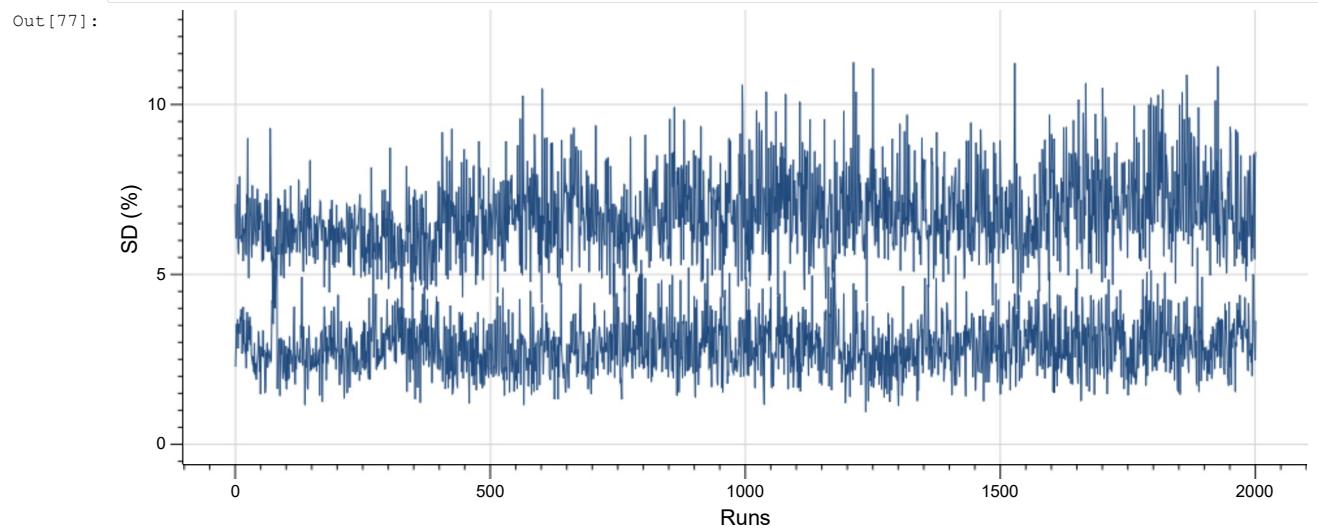
Here we're going to try to do that with the data obtained on previous experiment: we'll compute performance coefficient based on 10 runs of 15 sec task, then we'll apply them to median results of groups of 2.3 sec experiment runs, and we'll watch, how will it affect the uncertainty of end result.

```
In [75]: //computing coefficients
let typicalTime = 14.1875
let perfCoefs = blurGroupsT 10 (lotsByFile.[4]) |> Seq.map (fun c -> typicalTime / c.Rows.AsReal.[0]) |> Array.ofSeq
perfCoefs

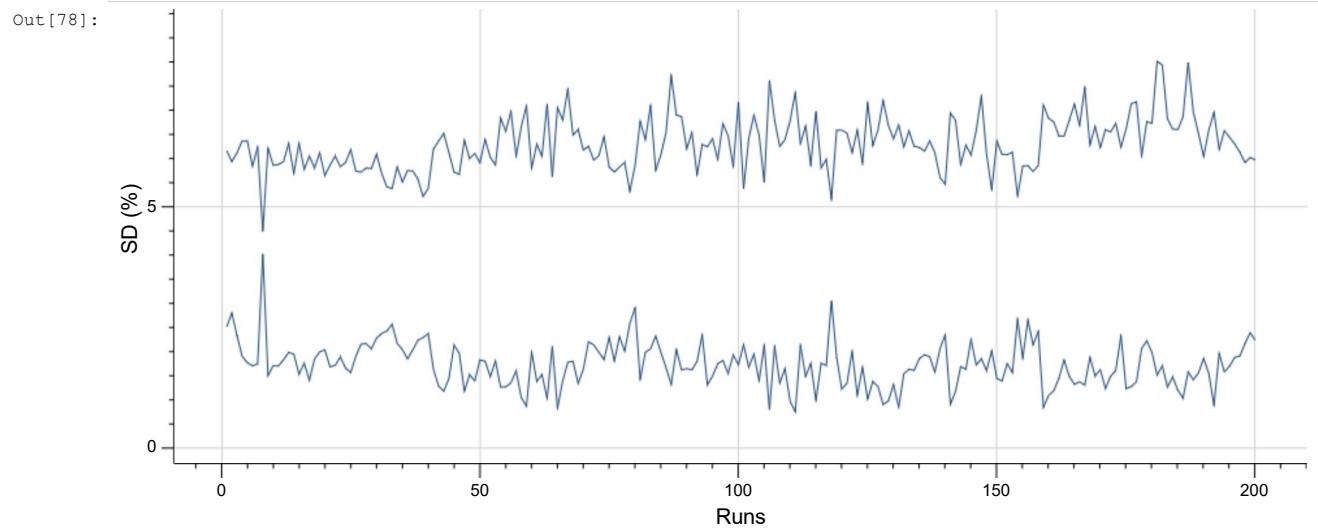
Out[75]: [|0.9222955815; 1.044879171; 0.8590350047; 1.03652968; 1.006651885; 1.002207506;
0.9848156182; 1.115479115; 1.003314917; 0.9956140351|]
```

```
In [76]: let times2 = lotsByFile.[0]
let times2grouped k = blurGroupsT k times2
let times2groupedAdj k = times2grouped k |> Seq.mapi (fun i c -> Column.Create (c.Name, c.Rows.AsReal |> Seq.map
(fun v -> v * perfCoefs.[i])) ) |> List.ofSeq |> Table.OfColumns
```

```
In [77]: [ (drawRowsUncertaintiesP <| times2groupedAdj 1).Plots.[0]; (drawRowsUncertaintiesP <| times2grouped 1).Plots.[0]
] |> Chart.ofList
```

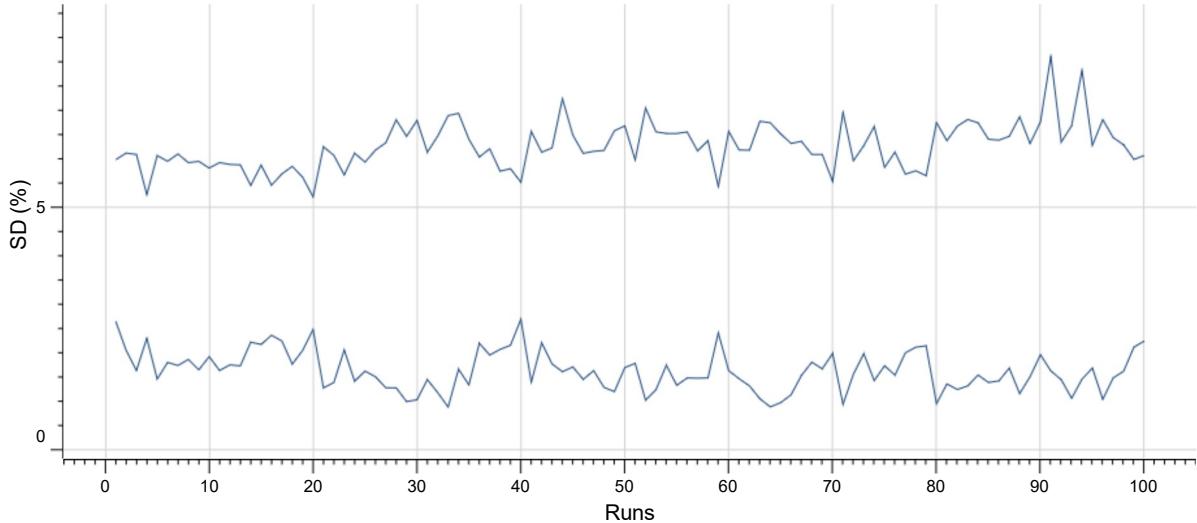


```
In [78]: [ (drawRowsUncertaintiesP <| times2groupedAdj 10).Plots.[0]; (drawRowsUncertaintiesP <| times2grouped 10).Plots.[0]
] |> Chart.ofList
```



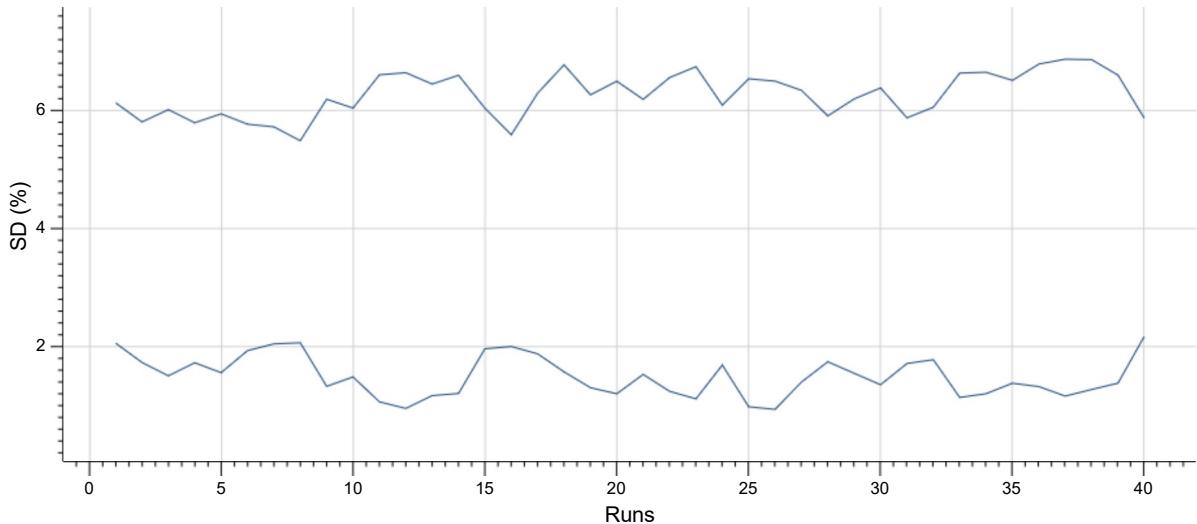
```
In [79]: [ (drawRowsUncertaintiesP <| times2groupedAdj 20).Plots.[0]; (drawRowsUncertaintiesP <| times2grouped 20).Plots.[0] ] |> Chart.ofList
```

Out[79]:



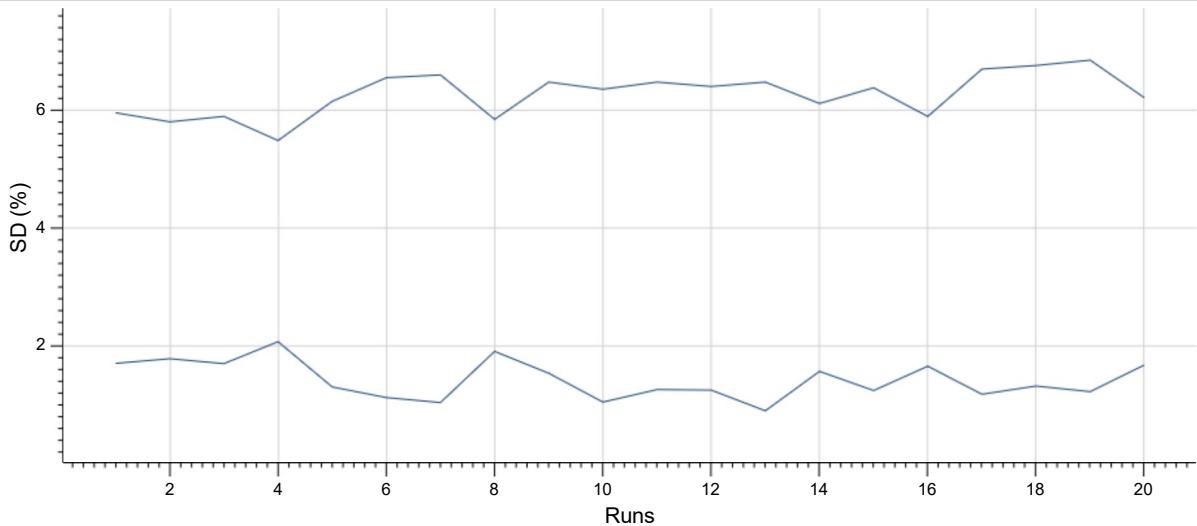
```
In [80]: [ (drawRowsUncertaintiesP <| times2groupedAdj 50).Plots.[0]; (drawRowsUncertaintiesP <| times2grouped 50).Plots.[0] ] |> Chart.ofList
```

Out[80]:



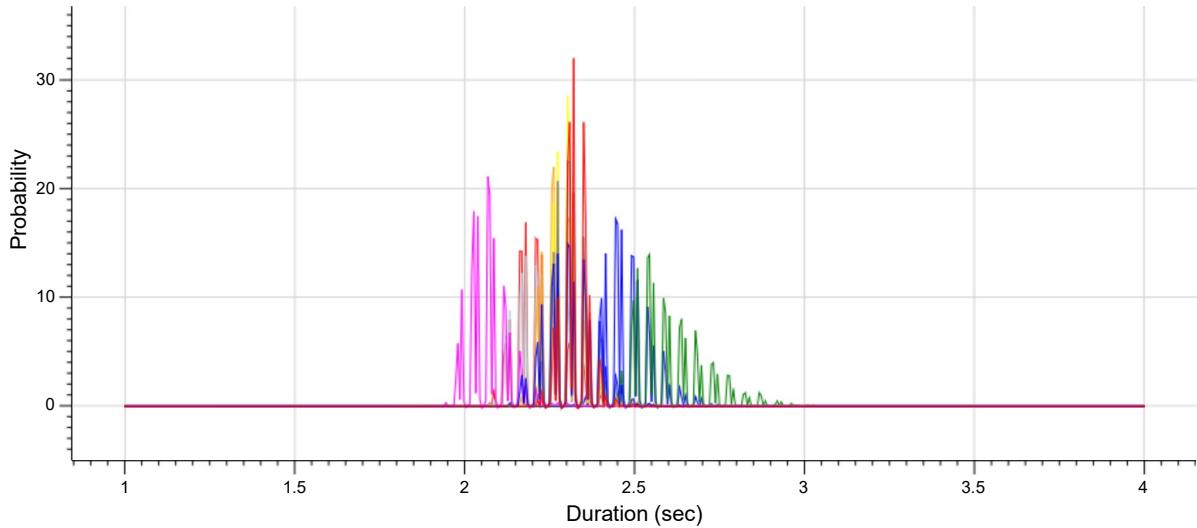
```
In [81]: [ (drawRowsUncertaintiesP <| times2groupedAdj 100).Plots.[0]; (drawRowsUncertaintiesP <| times2grouped 100).Plots.[0] ] |> Chart.ofList
```

Out[81]:



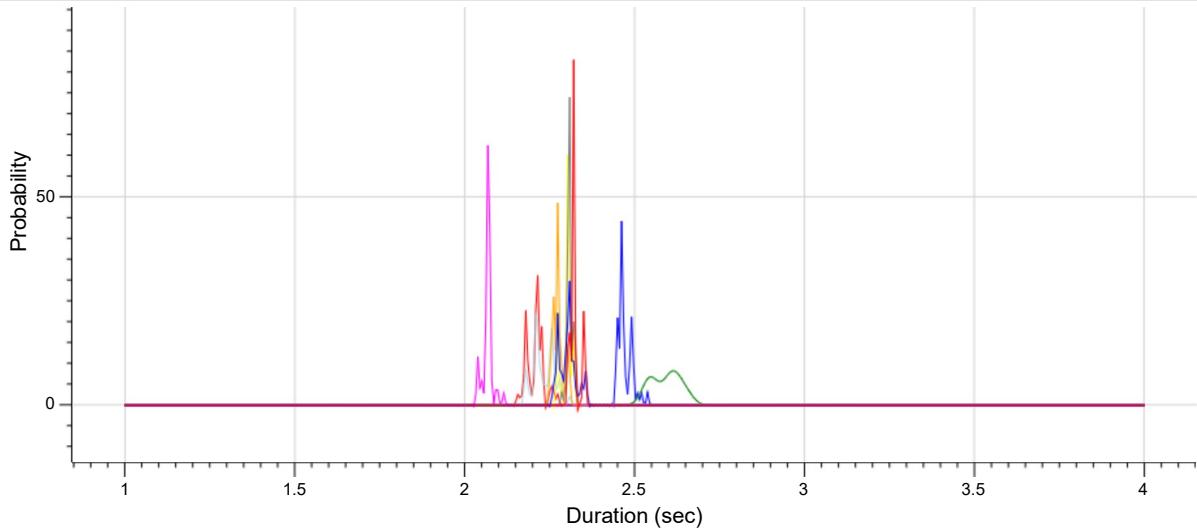
```
In [82]: drawPdfColumnsRange (times2grouped 1) 1.0 4.0
```

```
Out[82]:
```



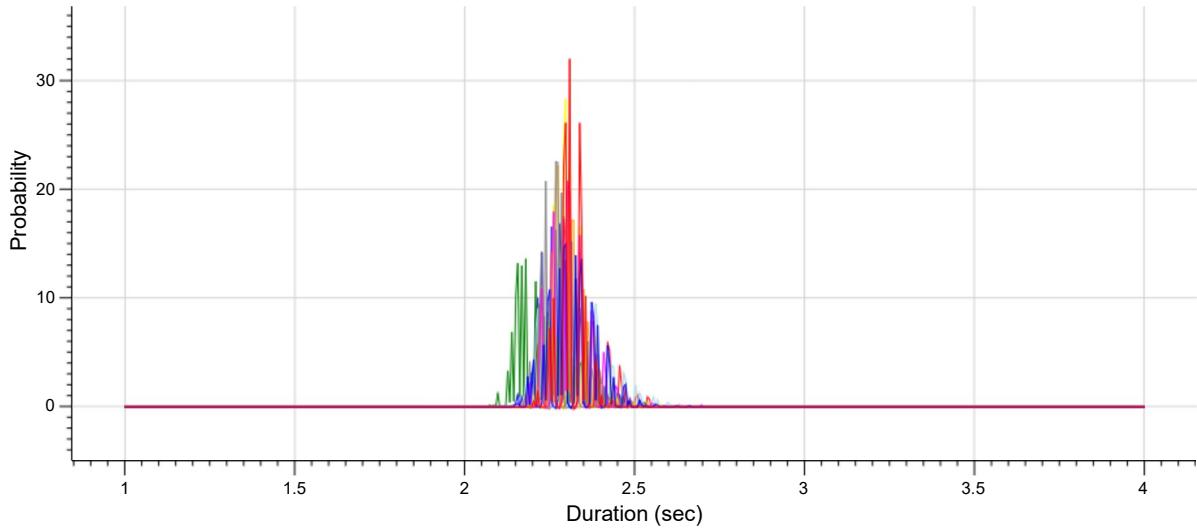
```
In [83]: drawPdfColumnsRange (times2grouped 50) 1.0 4.0
```

```
Out[83]:
```



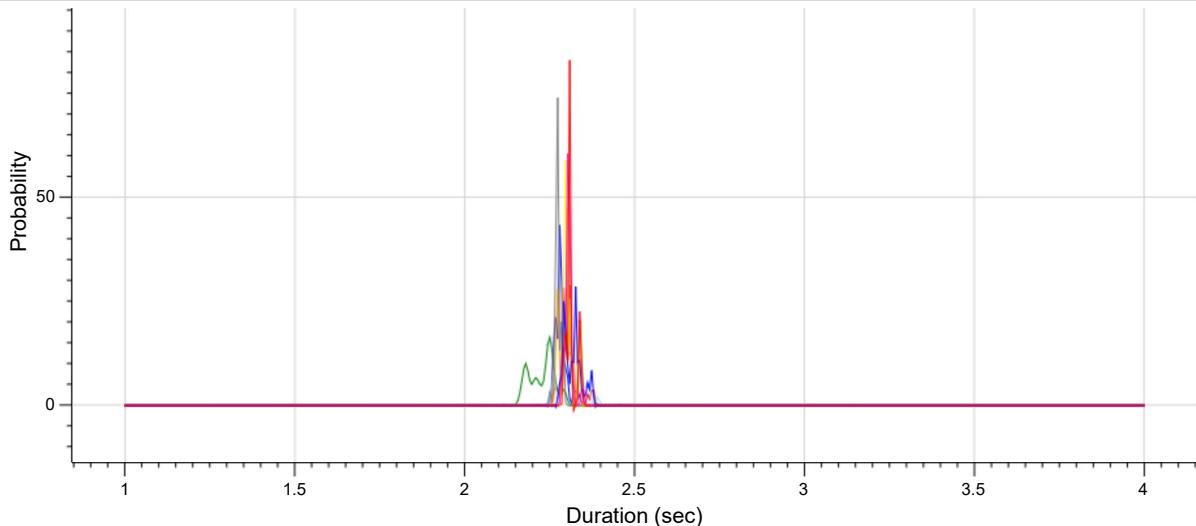
```
In [84]: drawPdfColumnsRange (times2groupedAdj 1) 1.0 4.0
```

```
Out[84]:
```



```
In [85]: drawPdfColumnsRange (times2groupedAdj 50) 1.0 4.0
```

Out[85]:



Finding the Best Test Problem

```
In [86]: let table15s = Table.Load ("15s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t15sByFileFull = [| for i in 0..7 -> table15s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t15sByFile = [| for i in 0..7 -> [| for j in 0..9 -> Column.Create (j.ToString(), (t15sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) | |> Table.OfColumns |]
```

```
In [87]: let tableBGsd table k = blurGroupST k table |> Seq.map (fun c -> uncertaintyP c.Rows.AsReal) |> qsummary

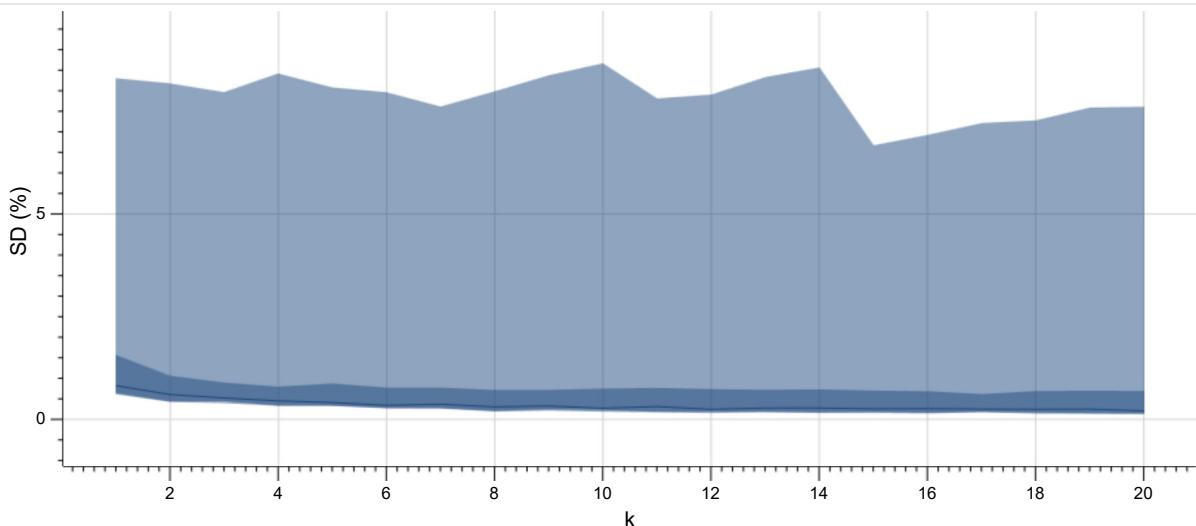
let plotsd table k = [Plot.line(LineX.Values ( [| for j in 1..k -> float j |]), LineY.UncertainValues ( [| for j in 1..k -> tableBGsd table j |] |> toQuantiles), titles = Titles.line("k", "SD (%))")] |> Chart.ofList
```

```
In [88]: let table35s = Table.Load ("35s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t35s = [| for j in 0..9 -> Column.Create (j.ToString(), (table35s |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) | |> Table.OfColumns
```

```
In [89]: plotsd t35s 20
```

Out[89]:



```
In [90]: let table25s = Table.Load ("25s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t25s = [| for j in 0..9 -> Column.Create (j.ToString(), (table25s |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) | |> Table.OfColumns
```

```
In [91]: plotsd t25s 20
```

Out[91]:

SD (%)

k

```
In [92]: let table5s = Table.Load ("5s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t5sByFileFull = [| for i in 0..12 -> table5s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t5sByFile = [| for i in 0..12 -> [for j in 0..9 -> Column.Create (j.ToString(), (t5sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns |]
```

```
In [93]: let table10s = Table.Load ("10s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t10sByFileFull = [| for i in 0..5 -> table10s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t10sByFile = [| for i in 0..5 -> [for j in 0..9 -> Column.Create (j.ToString(), (t10sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns |]
```

```
In [94]: let table7s = Table.Load ("7s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t7sByFileFull = [| for i in 0..19 -> table7s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t7sByFile = [| for i in 0..19 -> [for j in 0..9 -> Column.Create (j.ToString(), (t7sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns |]
```

```
In [95]: let table12s = Table.Load ("12s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t12sByFileFull = [| for i in 0..7 -> table12s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t12sByFile = [| for i in 0..7 -> [for j in 0..4 -> Column.Create (j.ToString(), (t12sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns |]
```

```
In [96]: let table20s = Table.Load ("20s.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })

let t20sByFileFull = [| for i in 0..4 -> table20s |> Table.Filter ["File #"] (fun (x:float) -> x = float i) |]

let t20sByFile = [| for i in 0..4 -> [for j in 0..4 -> Column.Create (j.ToString(), (t5sByFileFull.[i] |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns |]
```

```
In [97]: (tableBGsd t7sByFile.[3] 60).ub95
```

Out[97]: 1.542811847

```
In [98]: let ub95 (tables: Table array) (groupSize: int) = tables |> Array.map (fun t -> (tableBGsd t groupSize).ub95)

let minInd arr = arr |> Array.fold (fun (acc, ind, i) elem -> if elem < acc then (elem, i+1, i+1) else (acc, ind, i+1)) (arr.[0] + 1.0, -1, -1)

let minub95Ind tables groupSize = ub95 tables groupSize |> minInd
```

```
In [99]: minub95Ind t5sByFile 60
```

Out[99]: (0.5778548224, 1, 12)

```
In [100]: minub95Ind t7sByFile 60
```

Out[100]: (0.6824209025, 17, 19)

```
In [101]: minub95Ind t10sByFile 60
```

Out[101]: (0.8074975797, 3, 5)

```
In [102]: minub95Ind t12sByFile 45
```

```
Out[102]: (0.9086729278, 0, 7)
```

```
In [103]: minub95Ind t15sByFile 40
```

```
Out[103]: (0.3834899836, 7, 7)
```

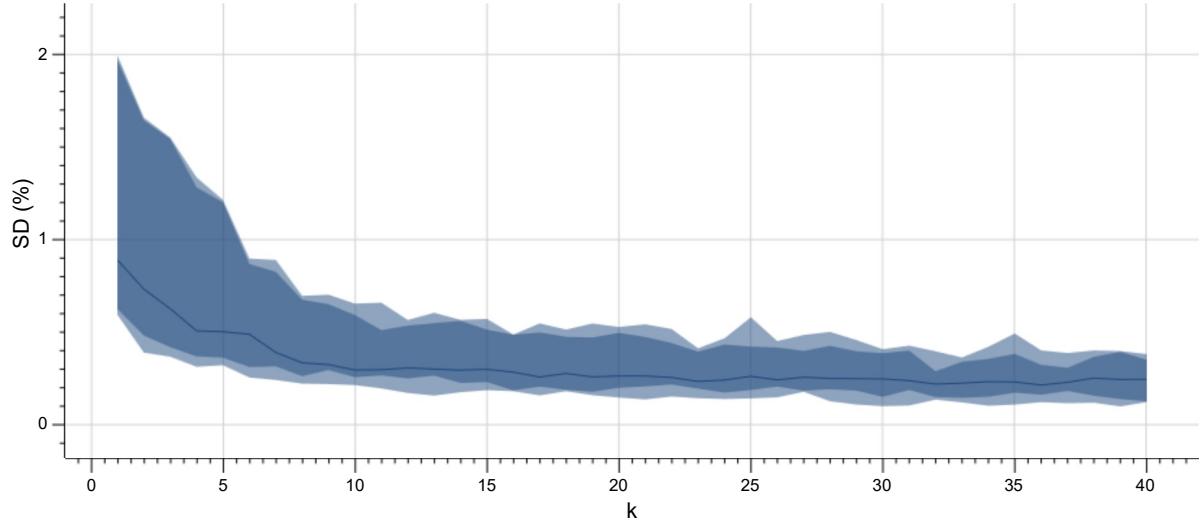
```
In [104]: minub95Ind t20sByFile 30
```

```
Out[104]: (0.6922145329, 1, 4)
```

File, appearing to be the best choice is bench_7536.smt2 (7th file in 15s set)

```
In [105]: plotsd t15sByFile.[7] 40
```

```
Out[105]:
```

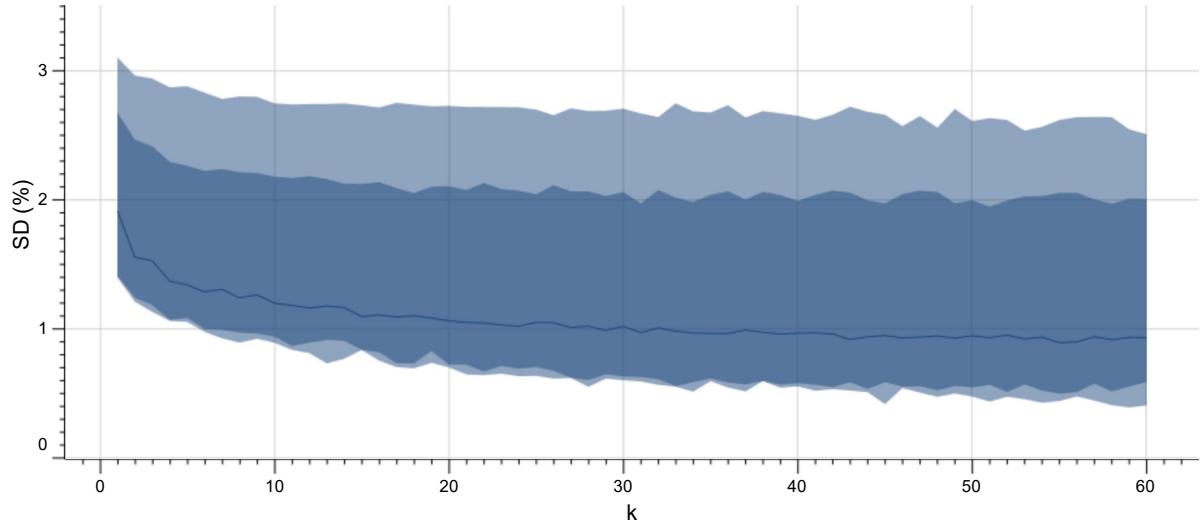


```
In [106]: let tableBest = Table.Load ("best.tsv", { Angara.Data.DelimitedFile.ReadSettings.Default with Delimiter = Angara.Data.DelimitedFile.Delimiter.Tab })
```

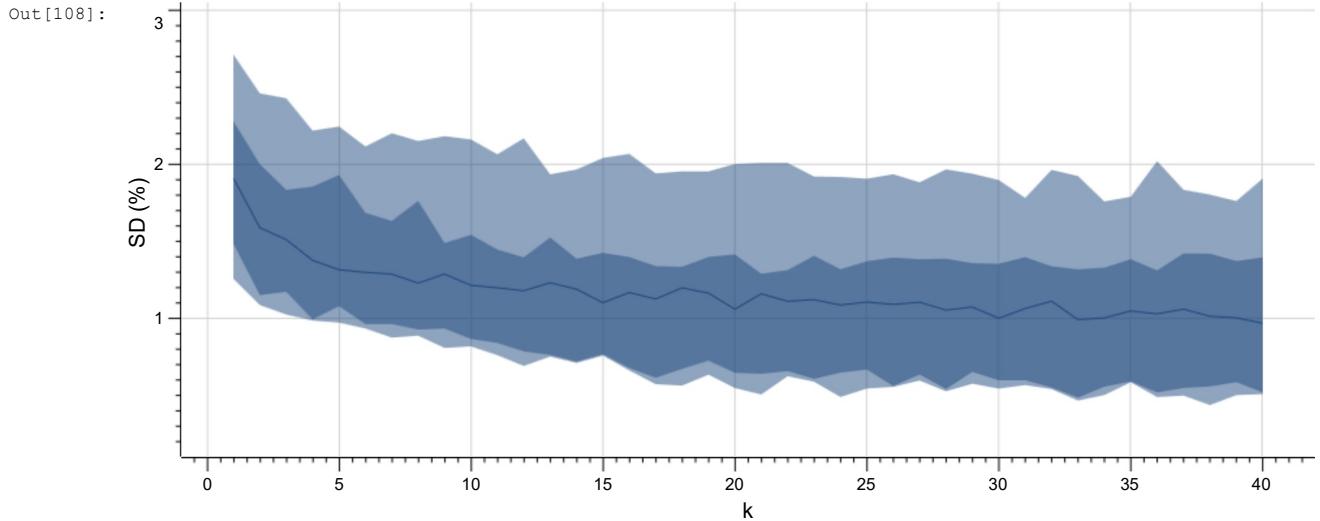
```
let tBest = [for j in 0..9 -> Column.Create (j.ToString(), (tableBest |> Table.Filter ["Task #"] (fun (x: float) -> x = float j)).["CPUTime"].Rows.AsReal) ] |> Table.OfColumns
```

```
In [107]: plotsd tBest 60
```

```
Out[107]:
```



```
In [108]: let best_cut1 = takeRows 400 tBest  
plotsd best_cut1 40
```



```
In [109]: qsummary tableBest.["CPUTime"].Rows.AsReal
```

```
Out[109]: {min = 14.140625;  
lb95 = 14.640625;  
lb68 = 15.5625;  
median = 16.34375;  
ub68 = 17.03125;  
ub95 = 17.921875;  
max = 18.828125;}
```

```
In [ ]:
```