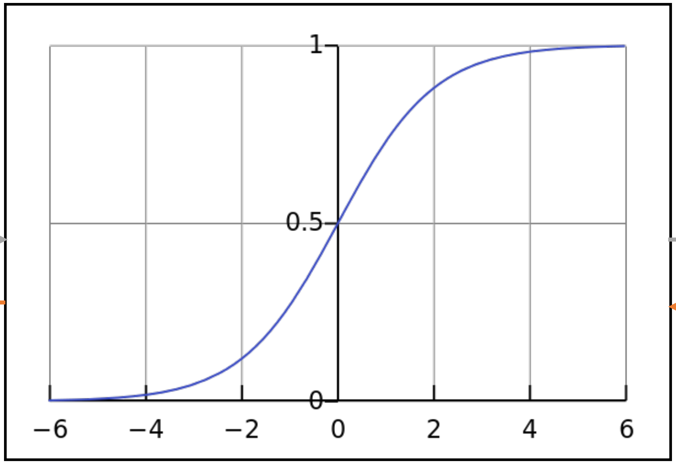
# MML minor #5

Свёрточные нейронные сети

# Sigmoid активация




$$x$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial \sigma}\frac{\partial \sigma}{\partial x}$$

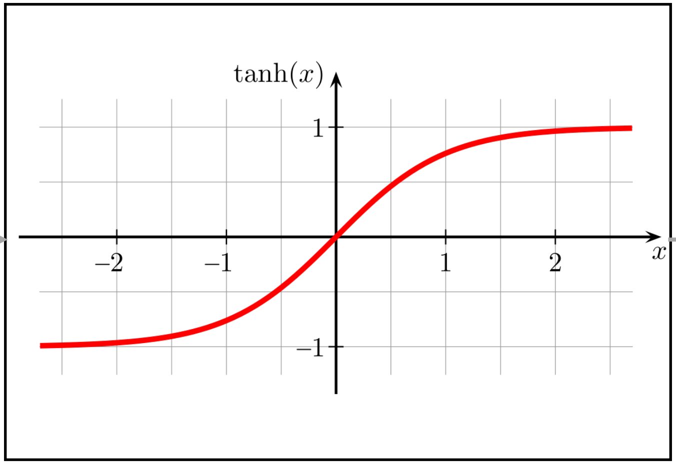
$$\frac{\partial L}{\partial \sigma}$$

$$\frac{\partial \sigma}{\partial x} = \sigma(x)(1 - \sigma(x))$$

- Нейроны с сигмоидой могут насыщаться и приводить к **угасающим градиентам**.
- Не центрированы в нуле.
- $e^x$ дорого вычислять.

# Tanh активация

$x$

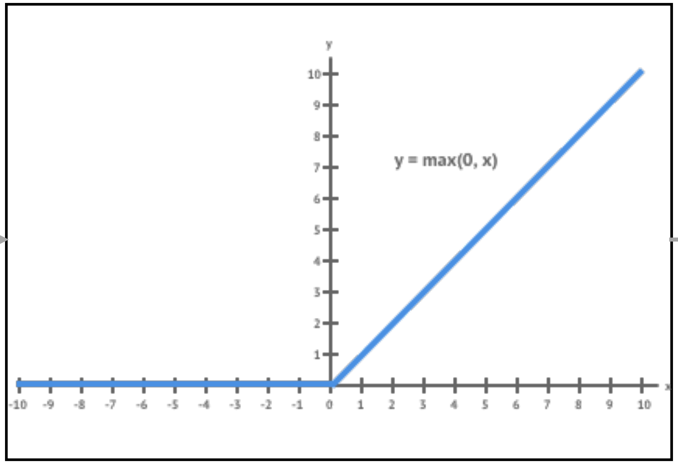



$$\tanh(x) = \frac{2}{1 + e^{-2x}} - 1$$

- Центрирован в нуле.
- Но все еще как сигмоида.

# ReLU активация

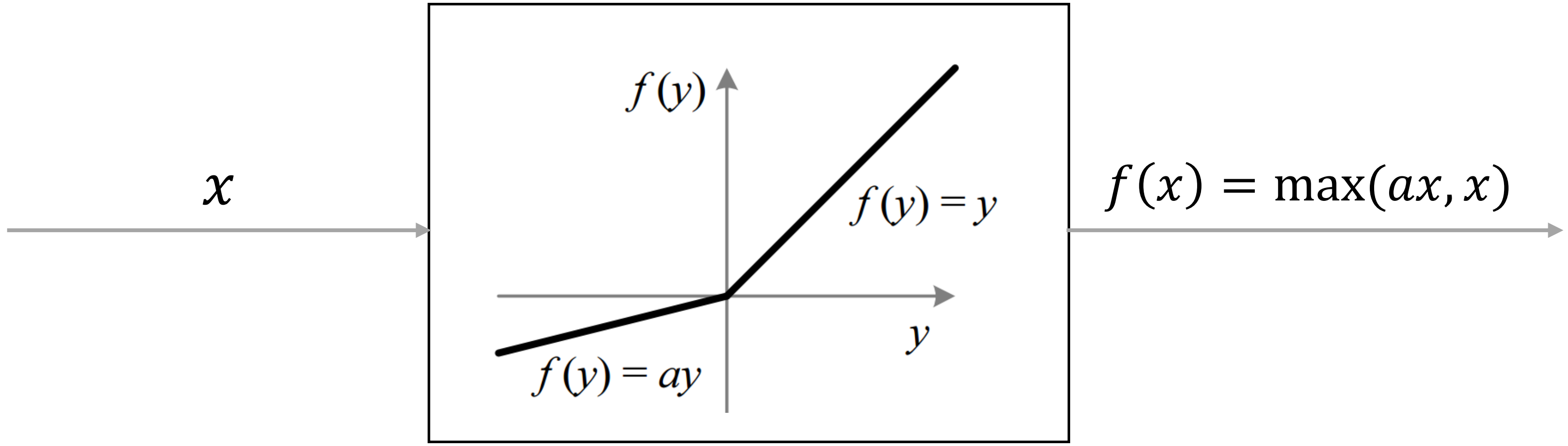
$x$




$f(x) = \max(0, x)$

- Быстро считается.
- Градиенты не угасают при $x > 0$.
- На практике ускоряет сходимость!
- Не центрирован в нуле.
- Могут умереть: если не было активации - не будет обновления!

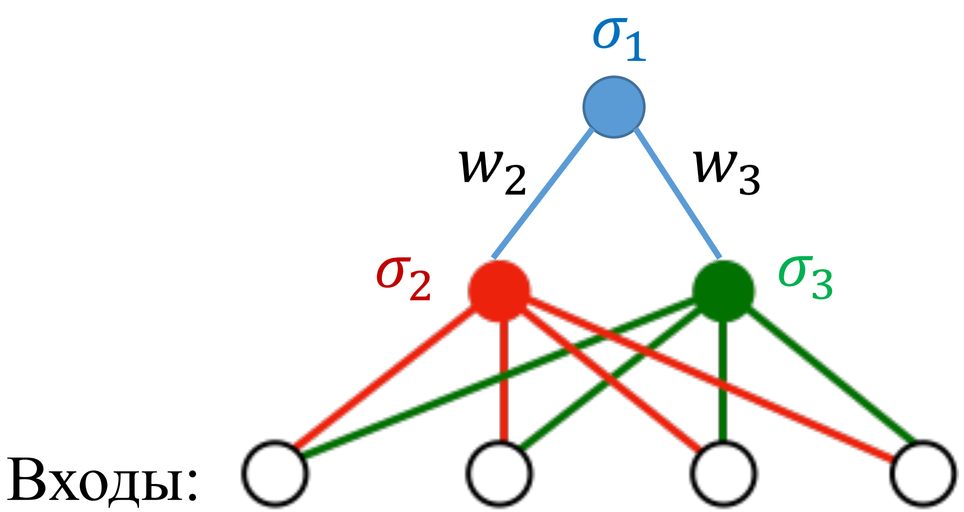
# Leaky ReLU активация




- Всегда будут обновления!
- $a \neq 1$

# Инициализация весов

Давайте начнем с нулей?




$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \sigma_1}\sigma_1(1-\sigma_1)\sigma_2$$

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial \sigma_1}\sigma_1(1-\sigma_1)\sigma_3$$

$\sigma_2$ и $\sigma_3$ обновляются одинаково!

- Нужно сломать симметрию!
- Может случайным шумом?
- Но насколько большим? $0.03 \cdot \mathcal{N}(0,1)$?

# Инициализация весов

- Линейные модели любят когда входы нормализованы.
- Нейрон это линейная комбинация входов + активация.
- Выход нейрона будет использован следующими слоями.

# Weights initializations

- Let's look at the neuron output before activation: $\sum_{i=1}^{n} x_i w_i$.

- If $E(x_i) = E(w_i) = 0$ and we generate weights independently from inputs, then $E(\sum_{i=1}^{n} x_i w_i) = 0$.

- But variance can grow with consecutive layers.

- Empirically this hurts convergence for deep networks!

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i.i.d. $w_i$ and mostly uncorrelated $x_i$

$$= \sum_{i=1}^{n} Var(x_i w_i) =$$

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independent factors $w_i$ and $x_i$

$$= \sum_{i=1}^{n} \begin{pmatrix} [E(x_i)]^2 Var(w_i) \\ +[E(w_i)]^2 Var(x_i) \\ +Var(x_i) Var(w_i) \end{pmatrix} =$$

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$$= \sum_{i=1}^{n} Var(x_i) Var(w_i) = Var(x)[n\, Var(w)]$$

We want this to be 1

# Weights initializations

- Let's use the fact that $Var(aw) = a^2 Var(w)$.

- For $[n\, Var(aw)]$ to be 1
  we need to multiply $\mathcal{N}(0,1)$ weights ($Var(w) = 1$)
  by $a = 1/\sqrt{n}$.

- Xavier initialization (Glorot et al.)
  multiplies weights by $\sqrt{2}/\sqrt{n_{in} + n_{out}}$.

- Initialization for ReLU neurons (He et al.)
  uses multiplication by $\sqrt{2}/\sqrt{n_{in}}$.

# Batch normalization

- We know how to initialize our network to constrain variance.
- But what if it grows during backpropagation?
- Batch normalization controls mean and variance of outputs **before activations**.

# Batch normalization

- Let's normalize $h_i$ – neuron output before activation:

$$h_i = \gamma_i \boxed{\frac{h_i - \mu_i}{\sqrt{\sigma_i^2}}} + \beta_i$$

$\rightarrow$ 0 mean, unit variance

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- Where do $\mu_i$ and $\sigma_i^2$ come from? We can estimate them having a **current training batch**!
- During testing we will use an exponential moving average over train batches:

$$0 < \alpha < 1 \qquad \begin{aligned} \mu_i &= \alpha \cdot \mathbf{mean_{batch}} + (1 - \alpha) \cdot \mu_i \\ \sigma_i^2 &= \alpha \cdot \mathbf{variance_{batch}} + (1 - \alpha) \cdot \sigma_i^2 \end{aligned}$$

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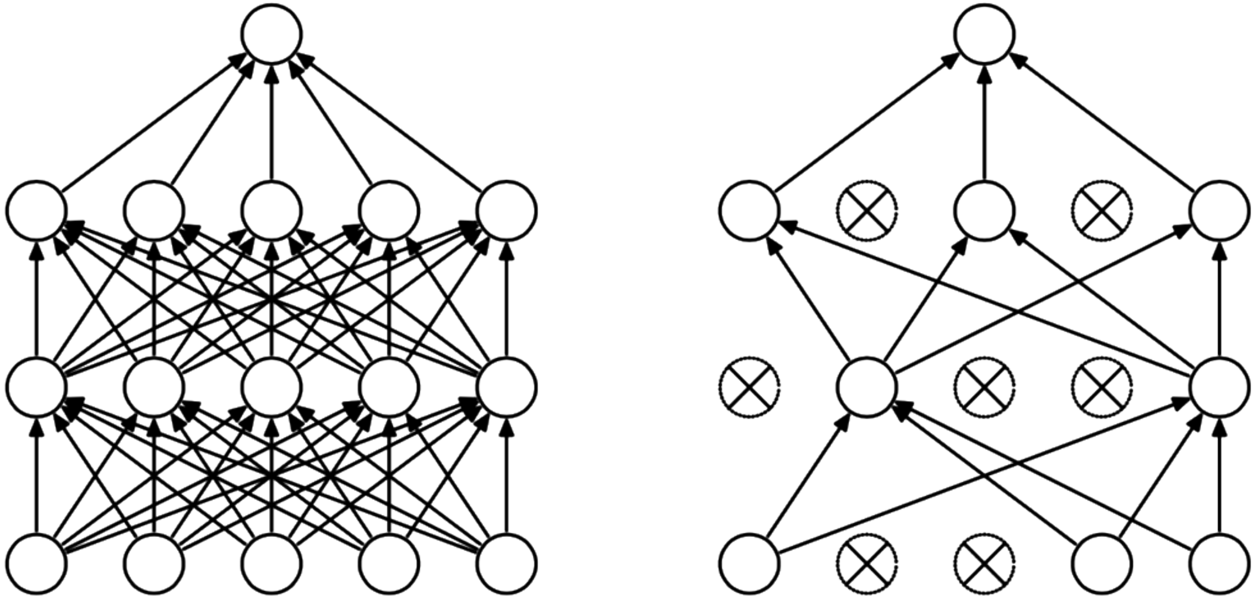
- What about $\gamma_i$ and $\beta_i$? Normalization is a differentiable operation and we can apply **backpropagation**!

# Dropout

- Regularization technique to reduce overfitting.
- We keep neurons active (non-zero) with probability $p$.
- This way we sample the network during training and change only a subset of its parameters on every iteration.



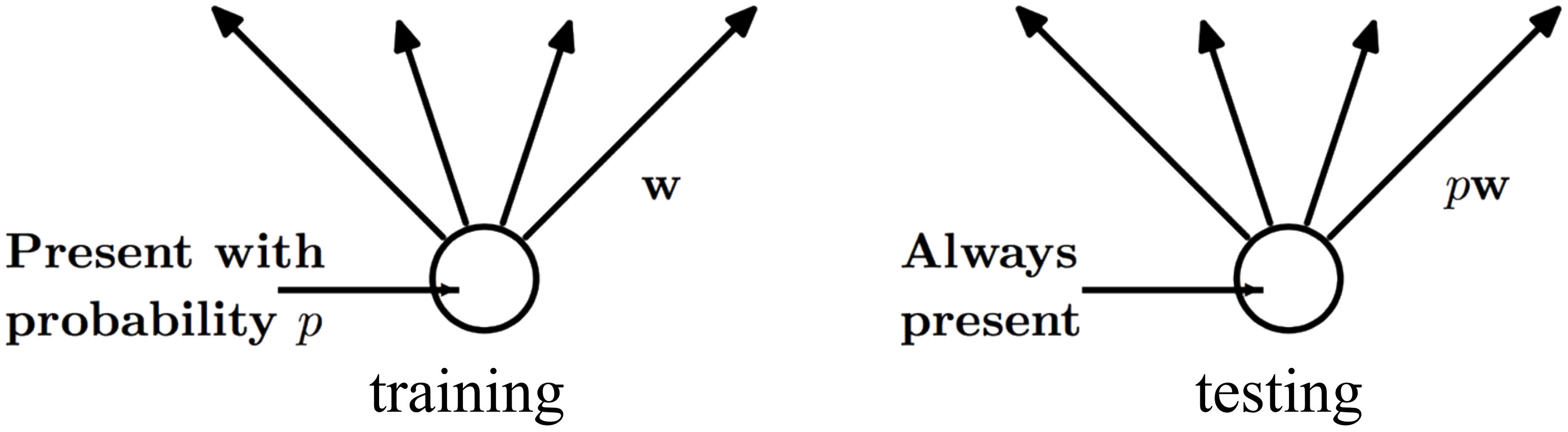
(a) Standard Neural Net (b) After applying dropout.

http://www.cs.toronto.edu/~rsalakhu/papers/srivastava14a.pdf

# Dropout

- During testing all neurons are present but their outputs are multiplied by $p$ to maintain the scale of inputs:

Expected input weight: $p \cdot w + (1 - p) \cdot 0$

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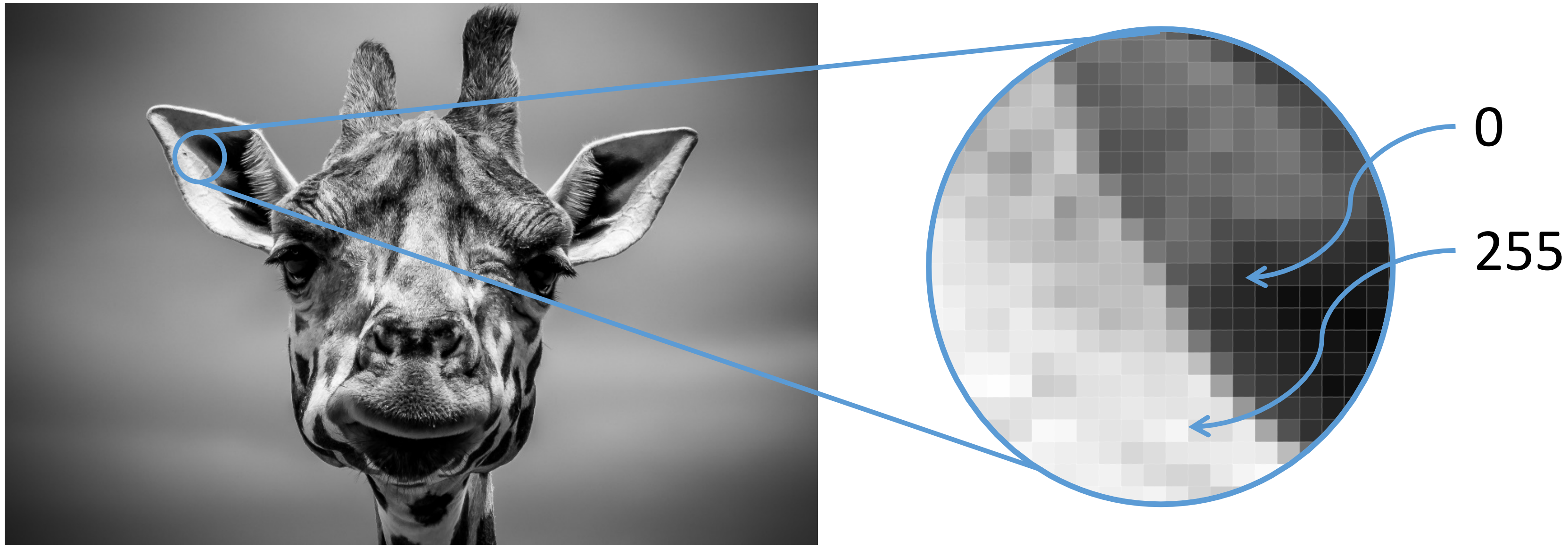
- The authors of dropout say it's similar to having an ensemble of exponentially large number of smaller networks.

# Digital representation of an image

- Grayscale image is a matrix of pixels (**pic**ture **el**ements)
- Dimensions of this matrix are called image resolution (e.g. 300 x 300)
- Each pixel stores its brightness (or **intensity**) ranging from 0 to 255, 0 intensity corresponds to black color:




- Color images store pixel intensities for 3 channels: red, green and blue

# Image as a neural network input

- Normalize input pixels: $x_{norm} = \frac{x}{255} - 0.5$

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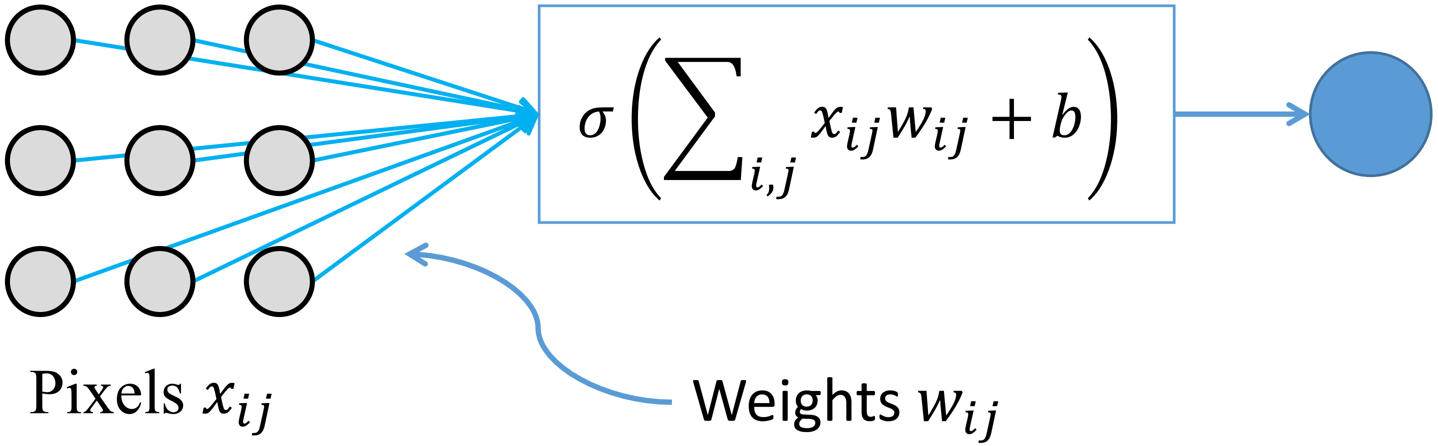
- Maybe MLP will work?



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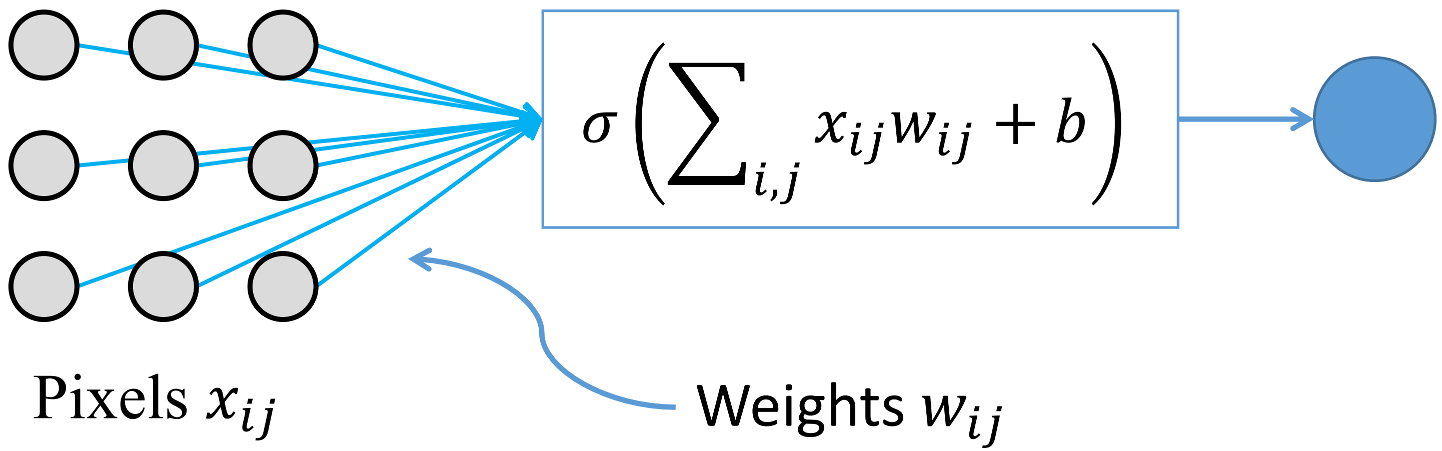
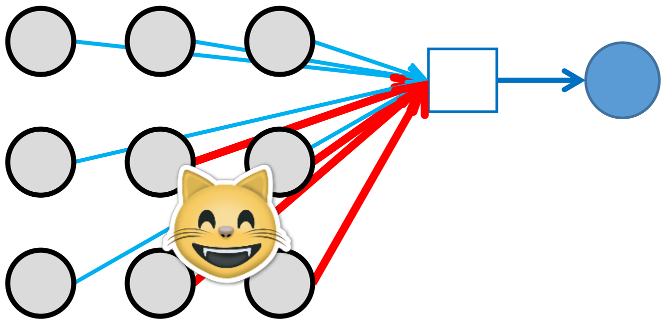



- Actually, no!

# Why not MLP?

- Let's say we want to train a "cat detector"



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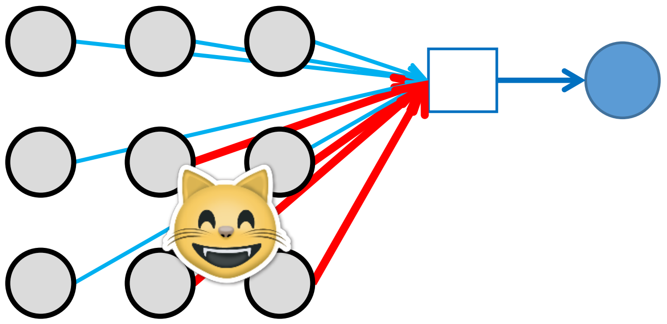

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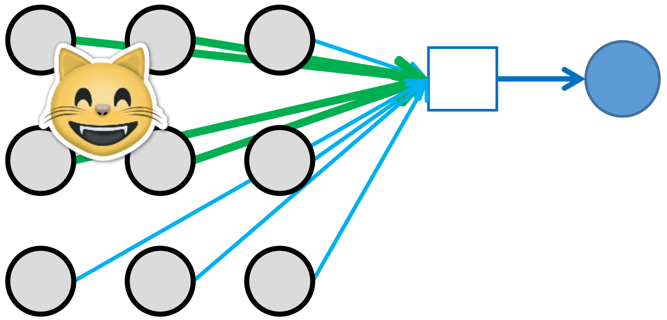

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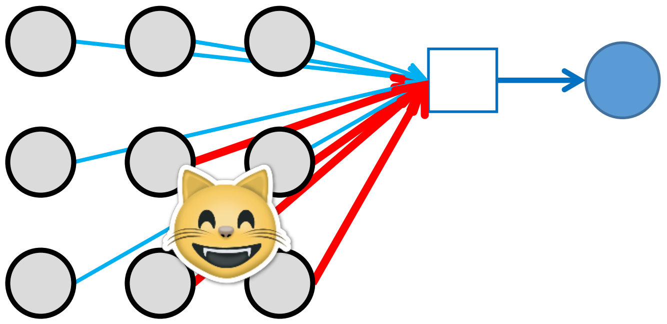

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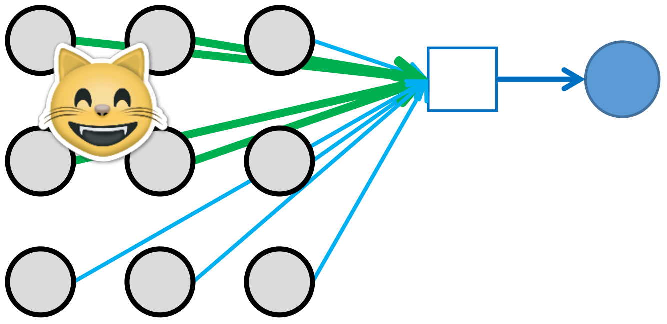
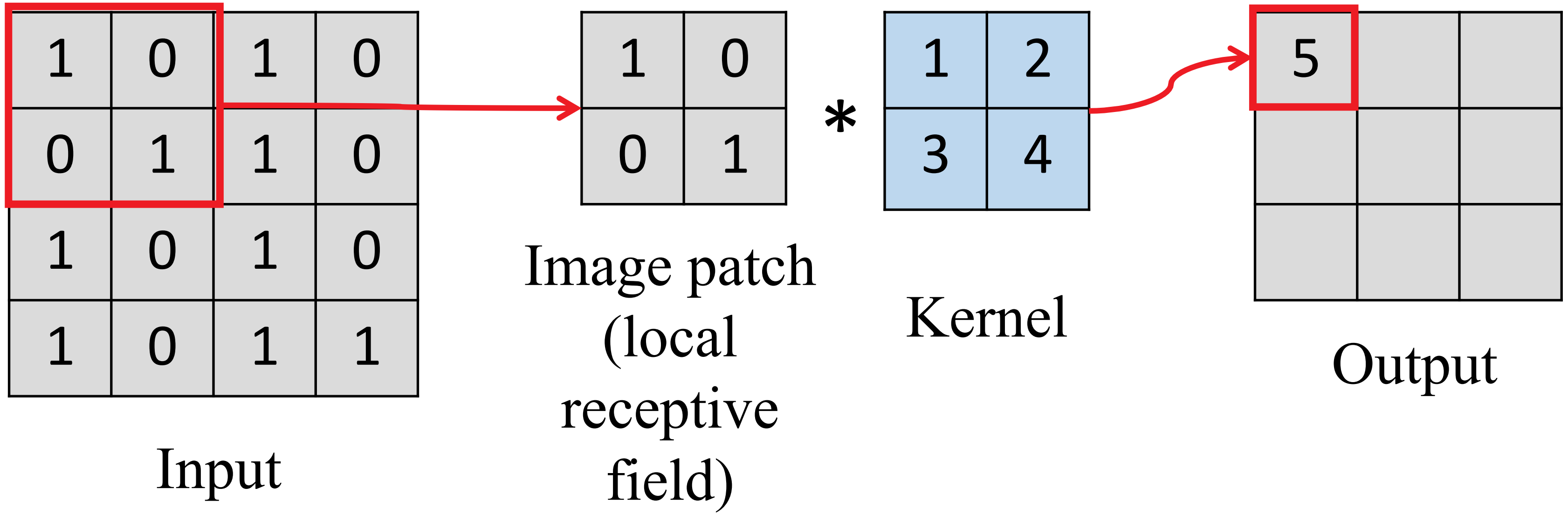


On this training image green weights $w_{ij}$ will change…

- We learn the same “cat features” in different areas and don’t fully utilize the training set!
- What if cats in the test set appear in different places?

# Convolutions will help!

Convolution is a dot product of a **kernel** (or filter)
and a patch of an image (**local receptive field**) of the same size



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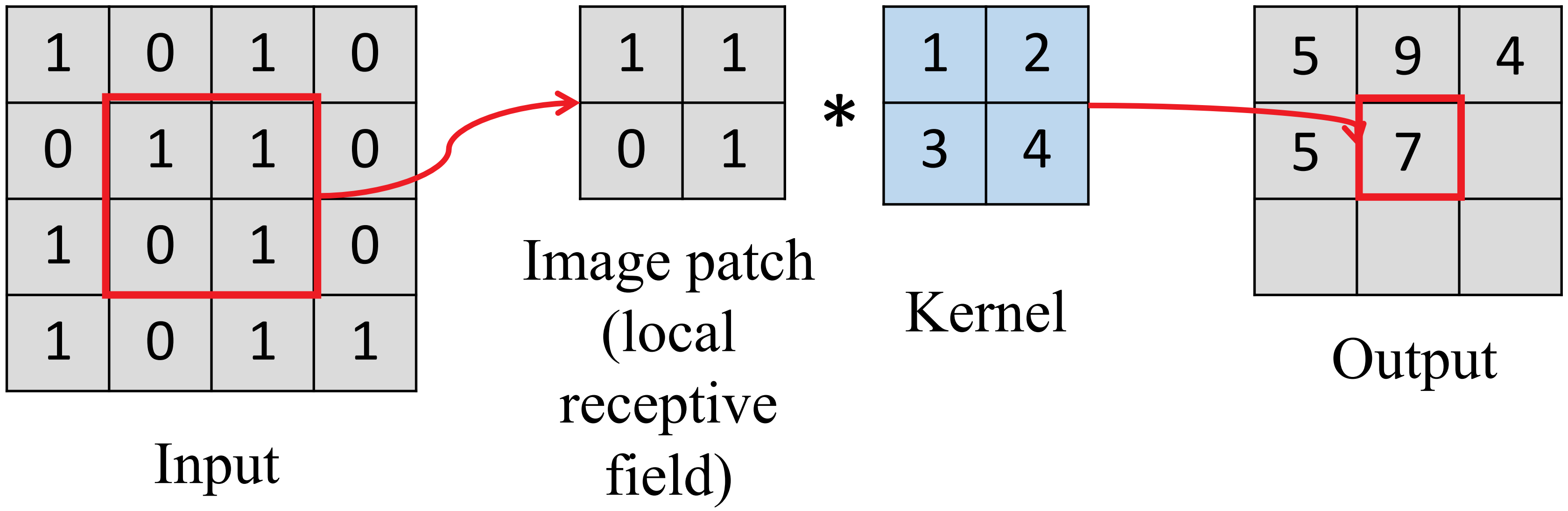


# Convolutions have been used for a while

Kernel

$$*\begin{bmatrix}-1 & -1 & -1\\ -1 & 8 & -1\\ -1 & -1 & -1\end{bmatrix} =$$

Edge detection

Original image

Sums up to 0 (black color) when the patch is a solid fill

# Convolutions have been used for a while

Kernel

*

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | 8 | -1 |
| -1 | -1 | -1 |

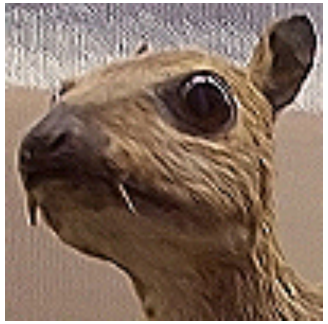
=

Edge detection

*

| | | |
|---|---|---|
| 0 | -1 | 0 |
| -1 | 5 | -1 |
| 0 | -1 | 0 |

=



Sharpening

Original image

Doesn’t change an image for solid fills

Adds a little intensity on the edges

# Convolutions have been used for a while

Kernel

Original image

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Edge detection

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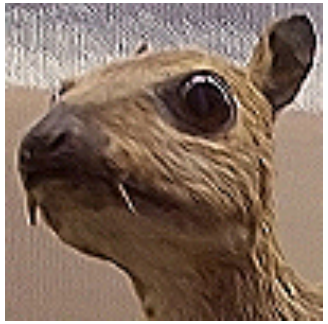
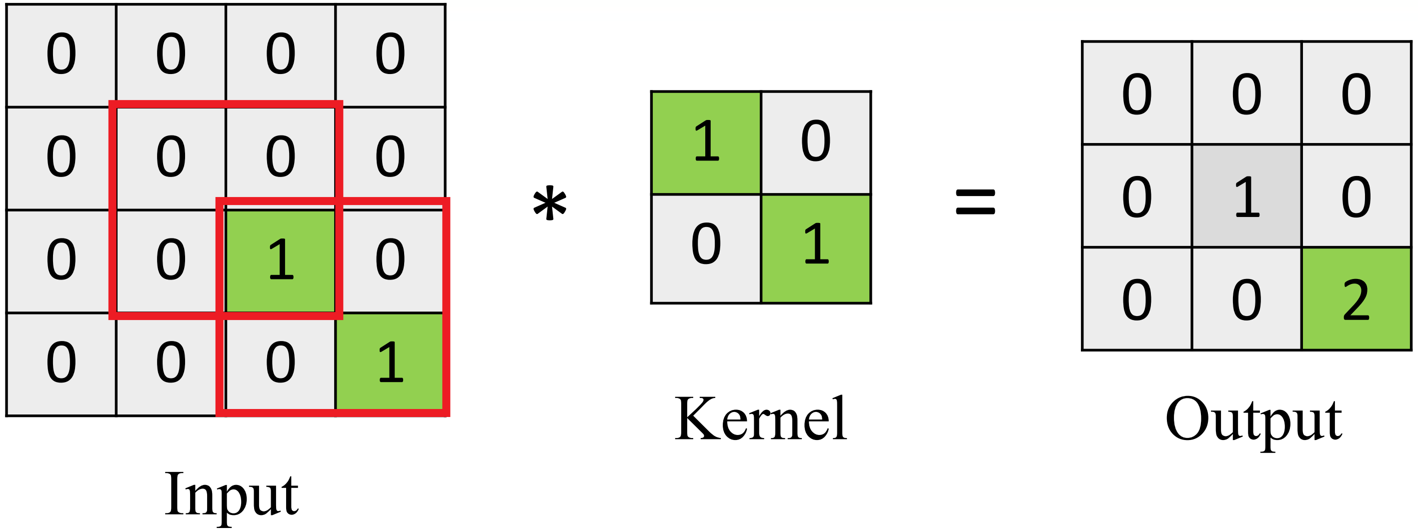


Sharpening

$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Blurring

# Convolution is similar to correlation



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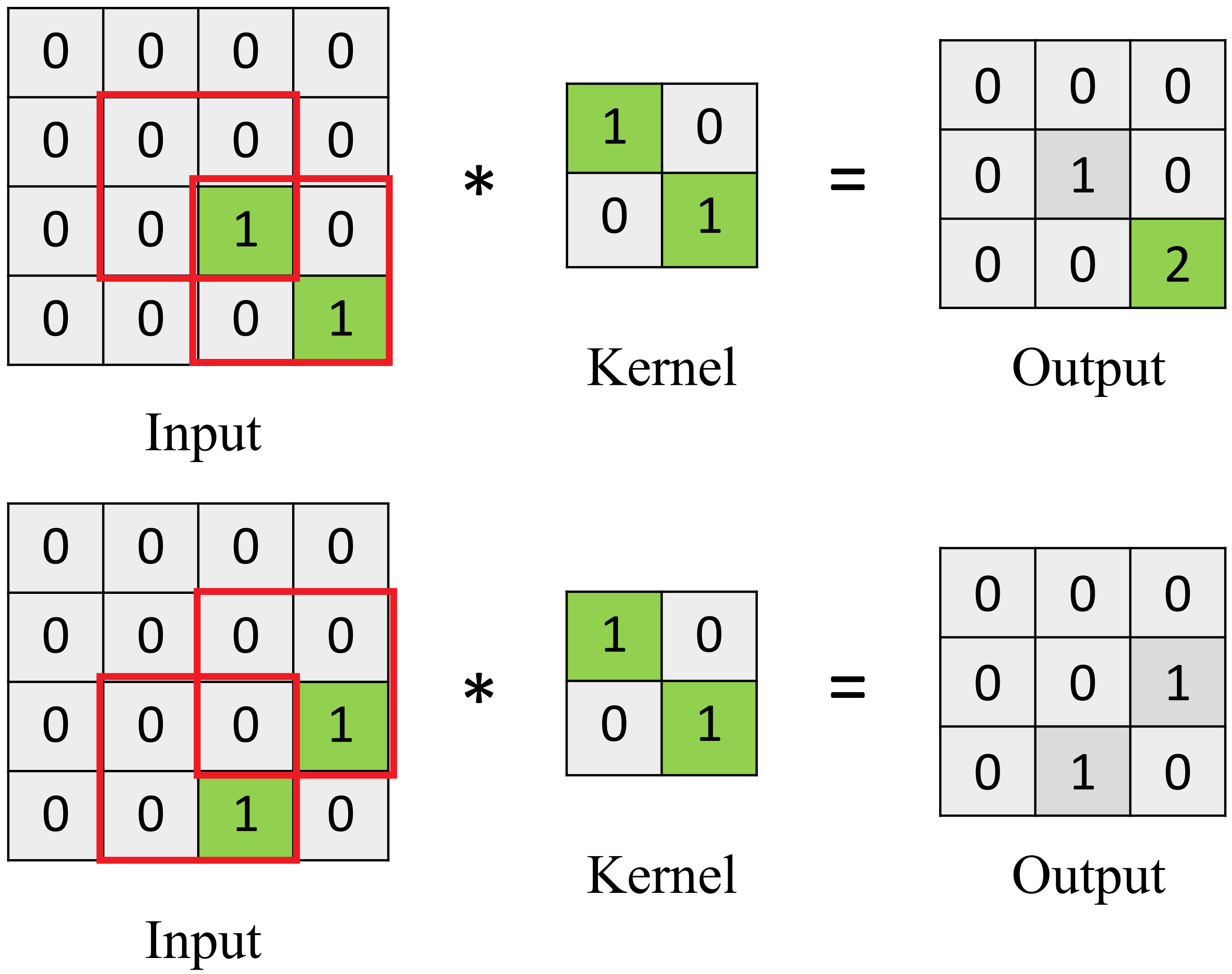

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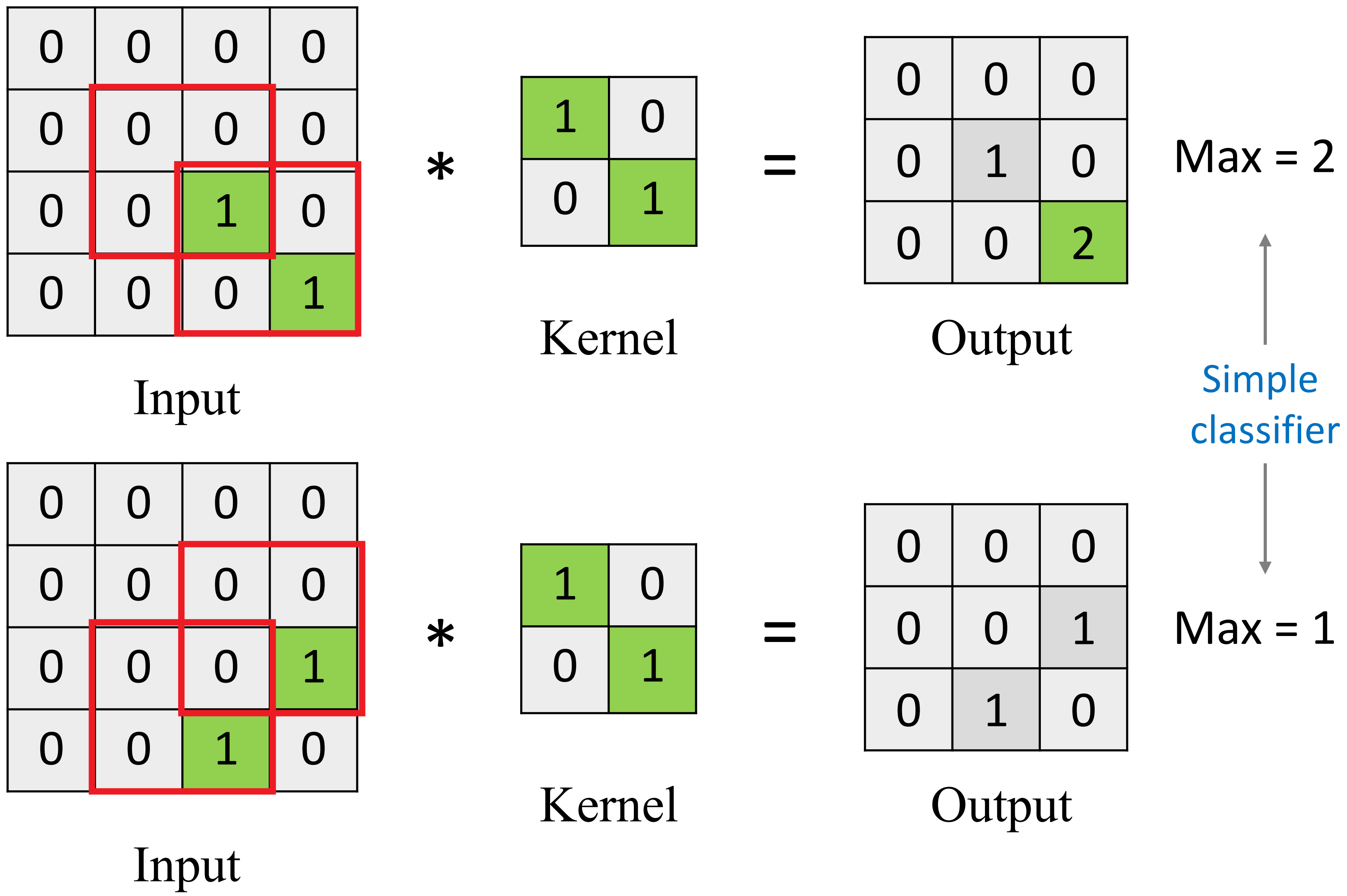

# Convolution is translation equivariant

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

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Max = 2

Didn't change

$$\underset{\text{Input}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

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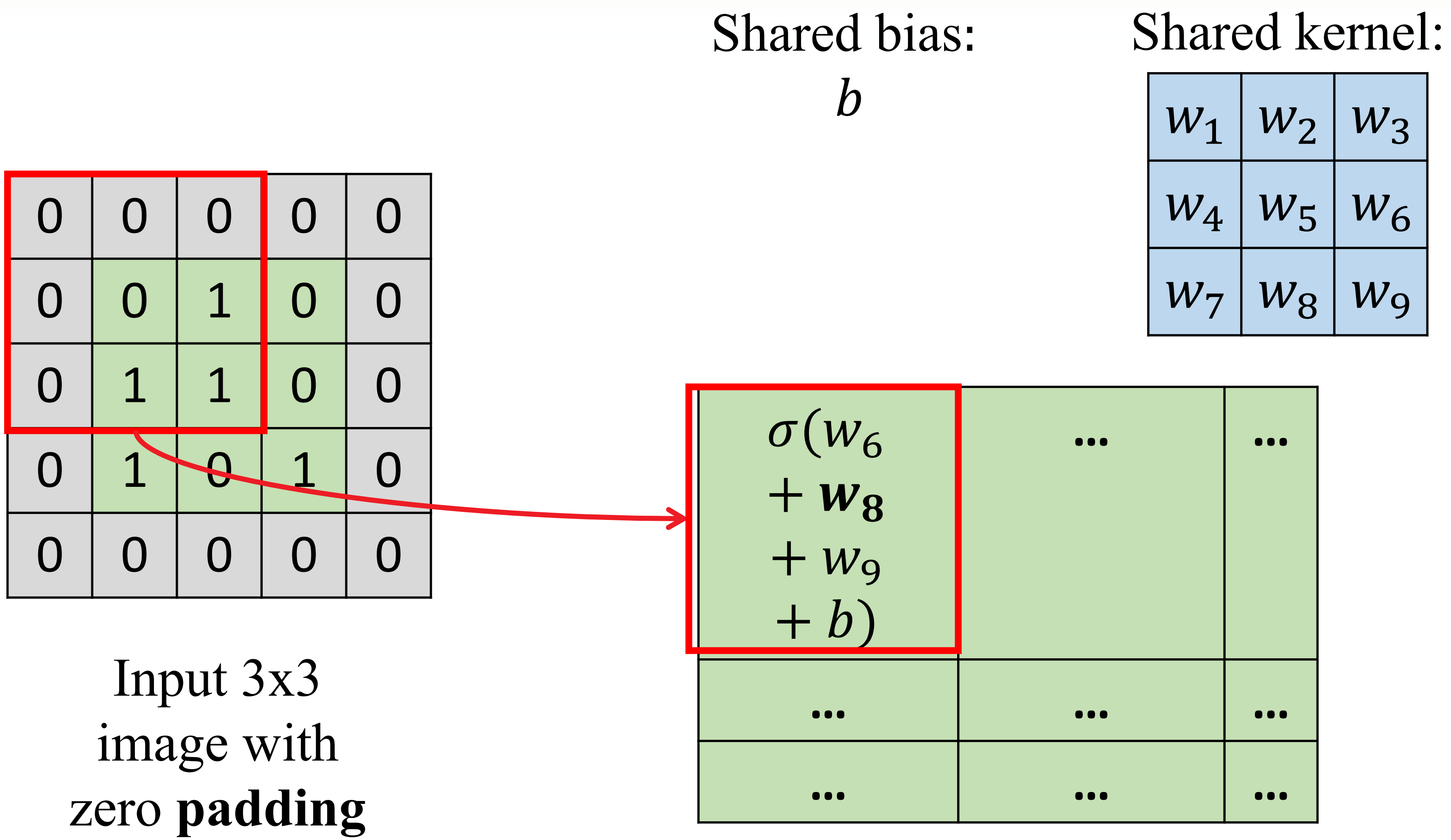
# Convolutional layer in neural network




Input 3x3 image with zero **padding** (grey area)

9 output neurons (**feature map**) with only 10 parameters

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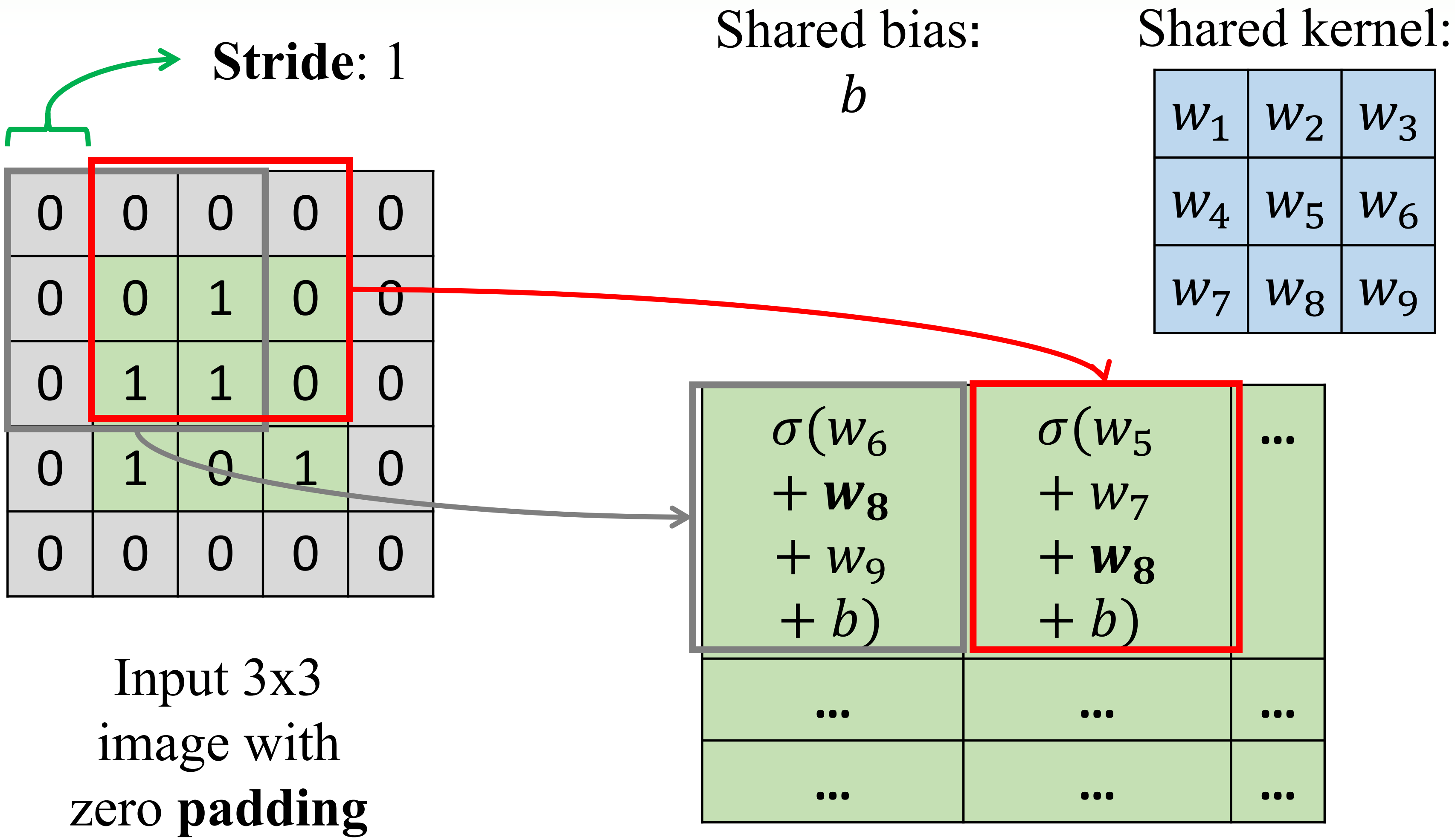


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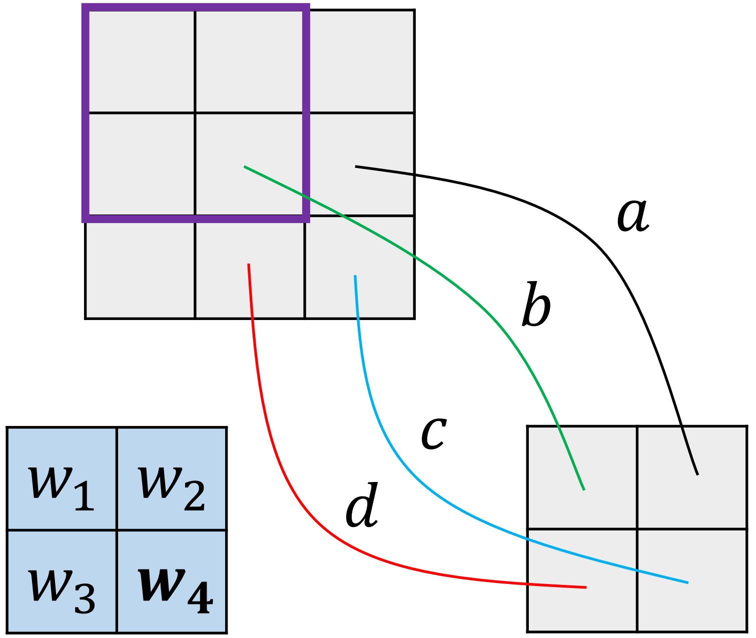
# Backpropagation for CNN

Gradients are first calculated as if the kernel weights were not shared:



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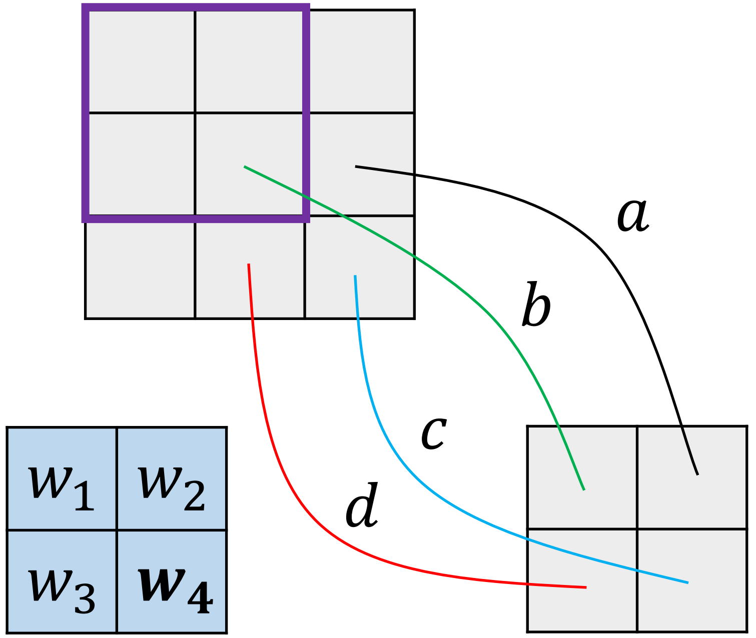


$$a = a - \gamma \frac{\partial L}{\partial a} \qquad b = b - \gamma \frac{\partial L}{\partial b}$$

$$c = c - \gamma \frac{\partial L}{\partial c} \qquad d = d - \gamma \frac{\partial L}{\partial d}$$

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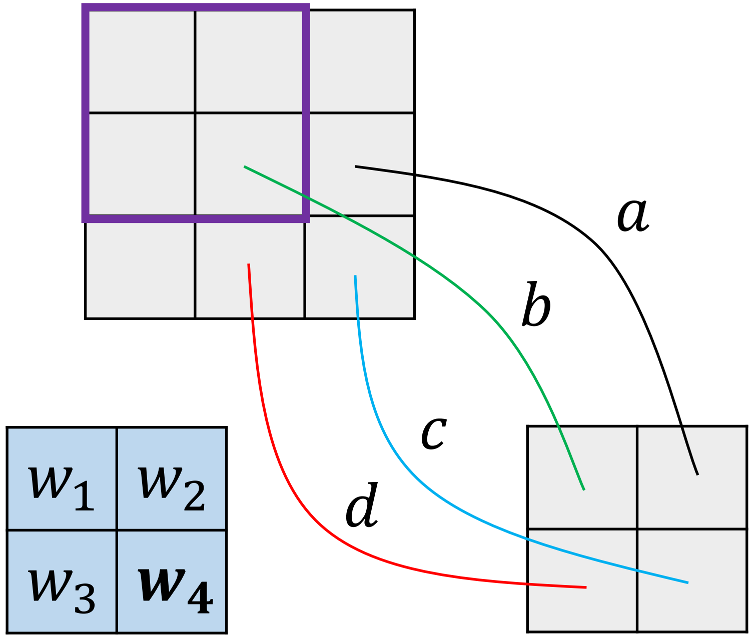


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$$w_4 = w_4 - \gamma \left( \frac{\partial L}{\partial a} + \frac{\partial L}{\partial b} + \frac{\partial L}{\partial c} + \frac{\partial L}{\partial d} \right)$$

Gradients of the same shared weight are summed up!

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- 300x300 input, 300x300 output, 5x5 kernel – **26** parameters in convolutional layer and $\mathbf{8.1\times10^9}$ parameters in fully connected layer (each output is a perceptron)

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- 300x300 input, 300x300 output, 5x5 kernel – **26** parameters in convolutional layer and $\mathbf{8.1{\times}10^{9}}$ parameters in fully connected layer (each output is a perceptron)

- Convolutional layer can be viewed as a special case of a fully connected layer when all the weights outside
  the **local receptive field** of each neuron equal 0
  and kernel parameters are shared between neurons

# A color image input

- Let's say we have a color image as an input, which is $W\times H\times C_{in}$ **tensor** (multidimensional array), where
- W – is an image width,
- $H$ – is an image height,
- $C_{in}$ – is a number of input channels (e.g. 3 RGB channels).

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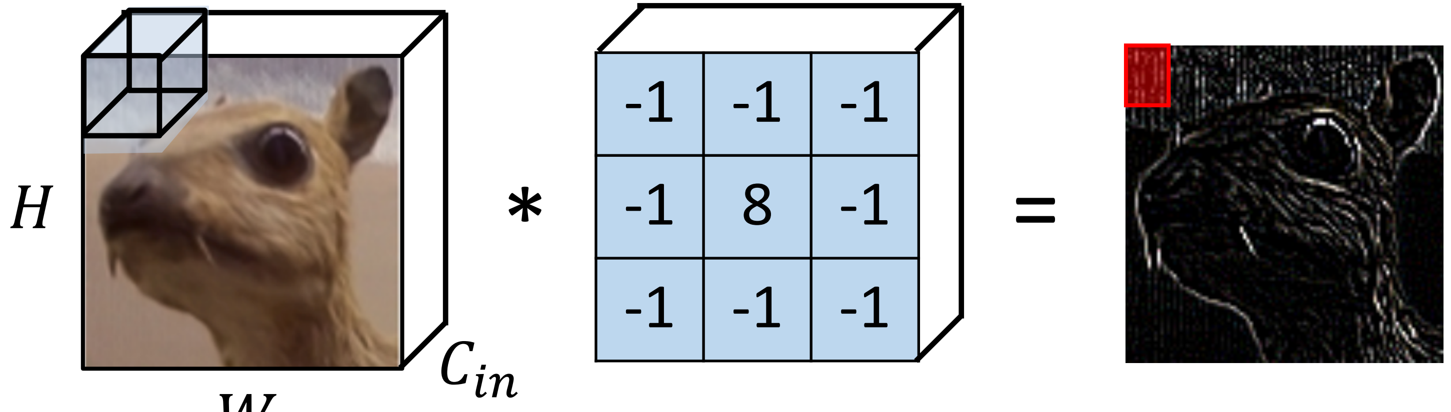


kernel of size $W_k \times H_k \times C_{in}$

feature map

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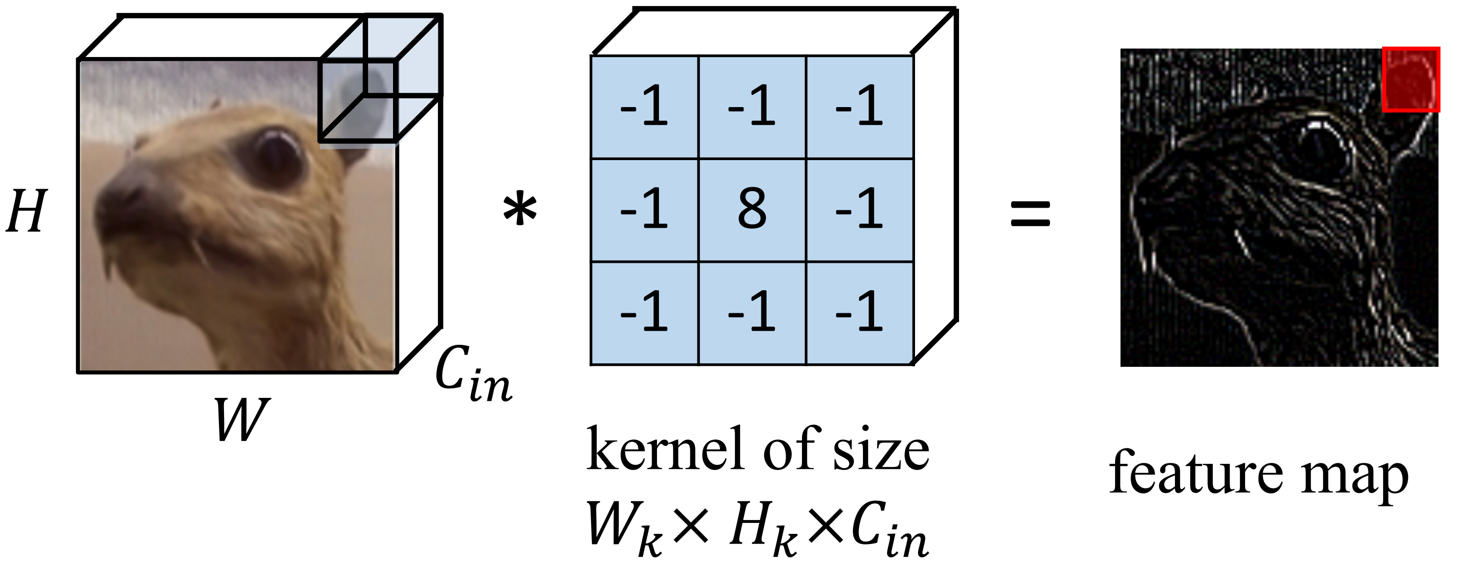
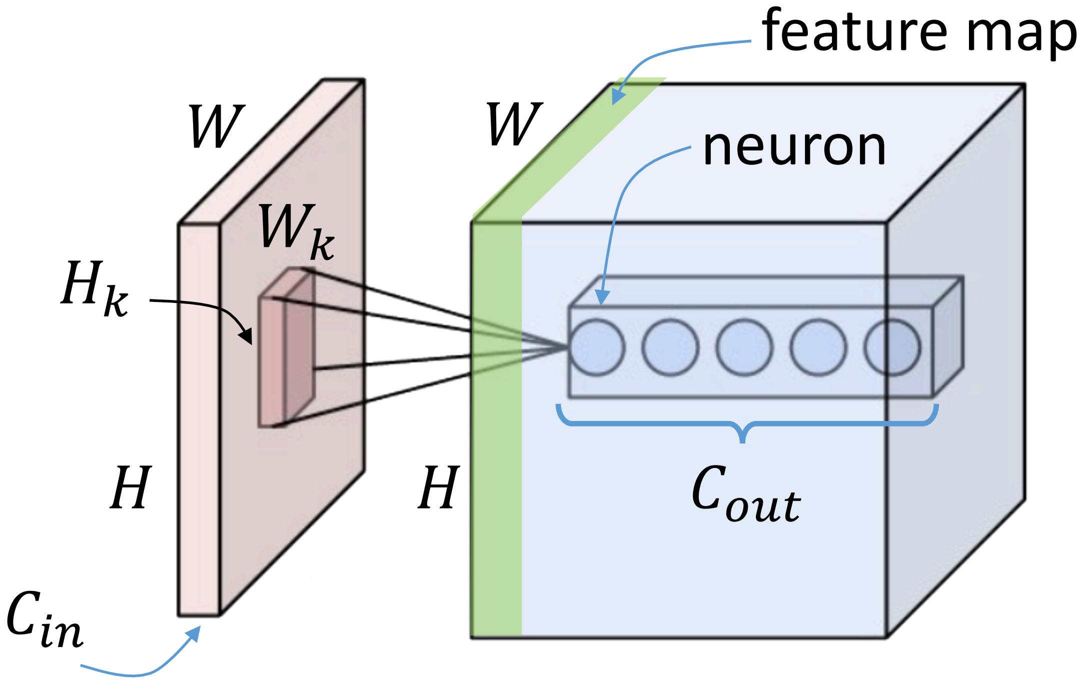


# One kernel is not enough!

- We want to train $C_{out}$ kernels of size $W_k \times H_k \times C_{in}$.
- Having a stride of 1 and enough zero padding we can have $W \times H \times C_{out}$ output neurons.



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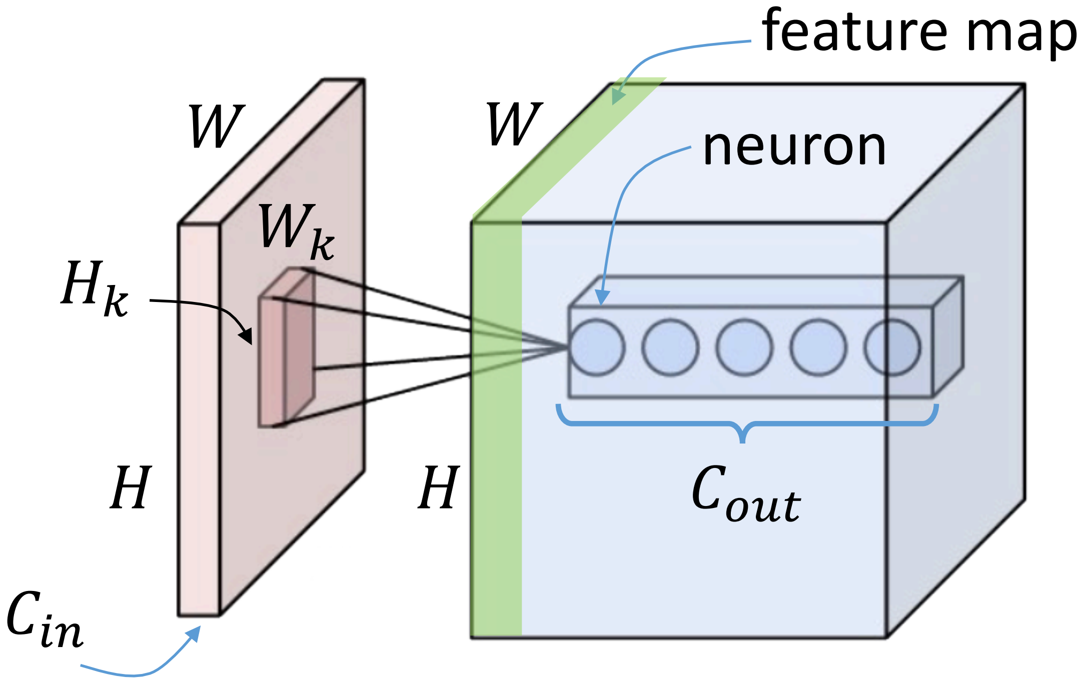


- Using $(W_k * H_k * C_{in} + 1) * C_{out}$ parameters.

# One convolutional layer is not enough!

- Let's say neurons of the $1^{st}$ convolutional layer look at the patches of the image of size 3x3.
- What if an object of interest is bigger than that?
- We need a $2^{nd}$ convolutional layer on top of the $1^{st}$!

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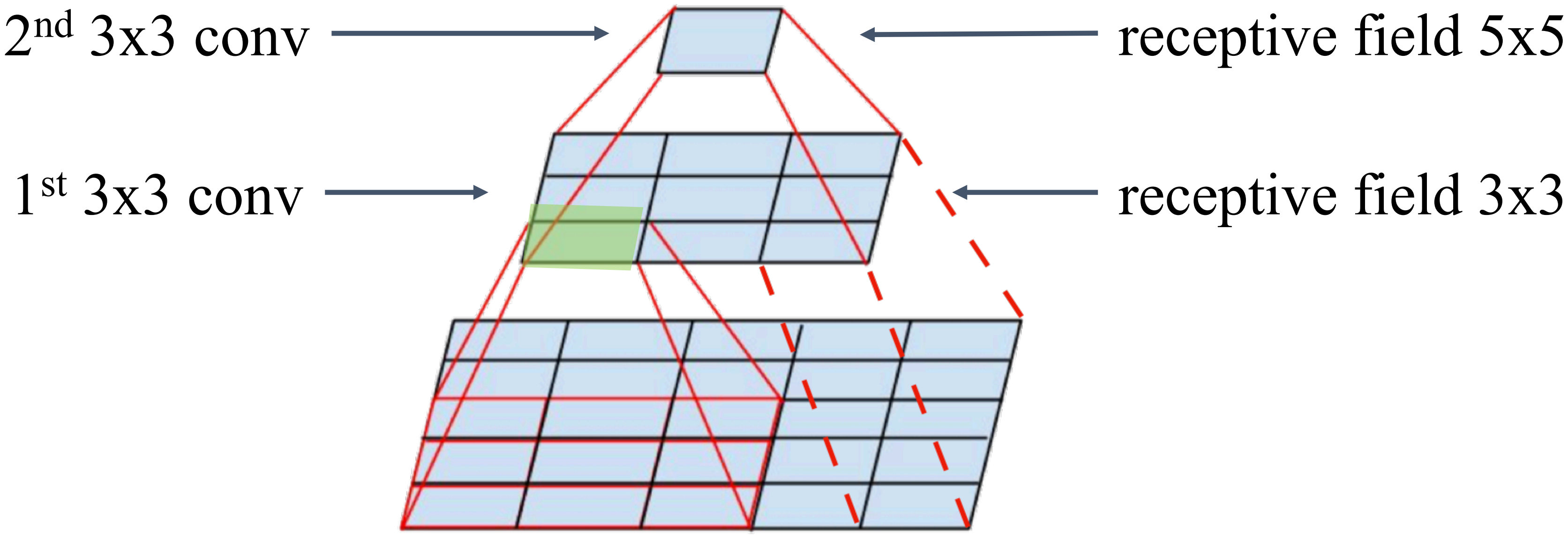
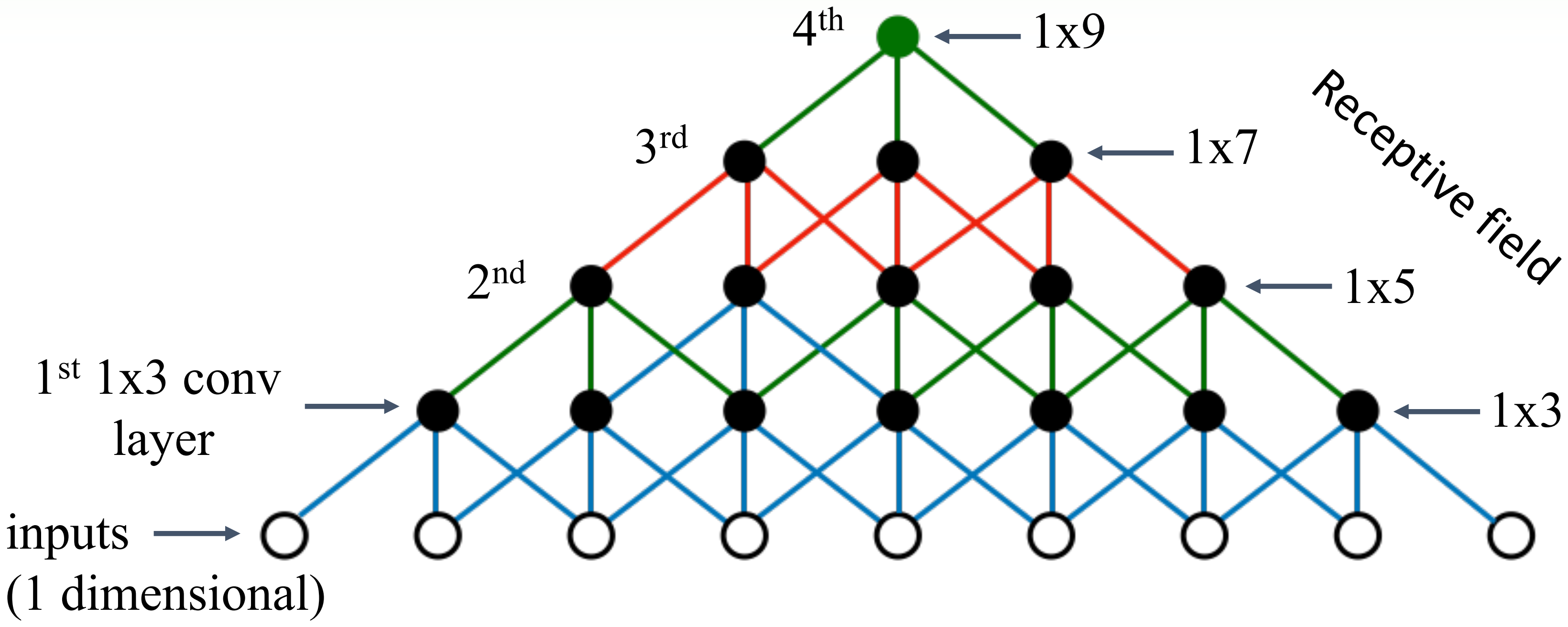


# Receptive field after N convolutional layers



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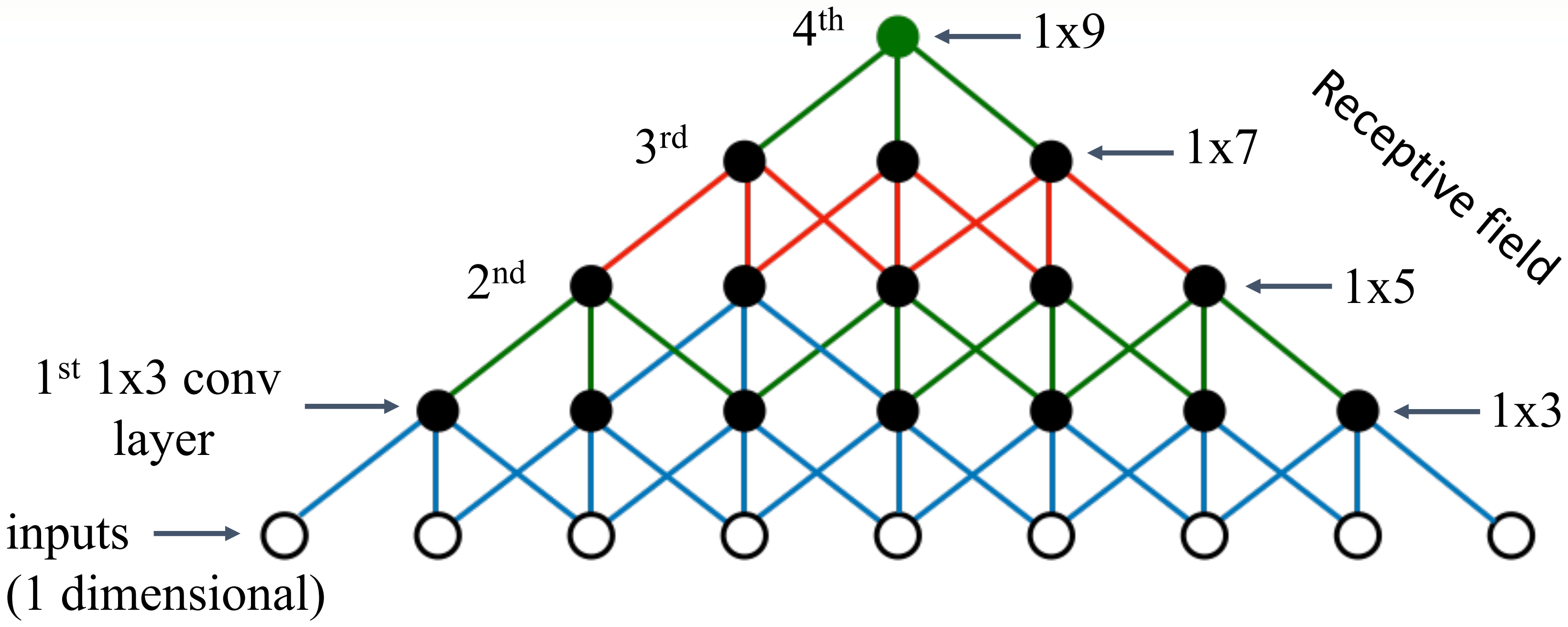
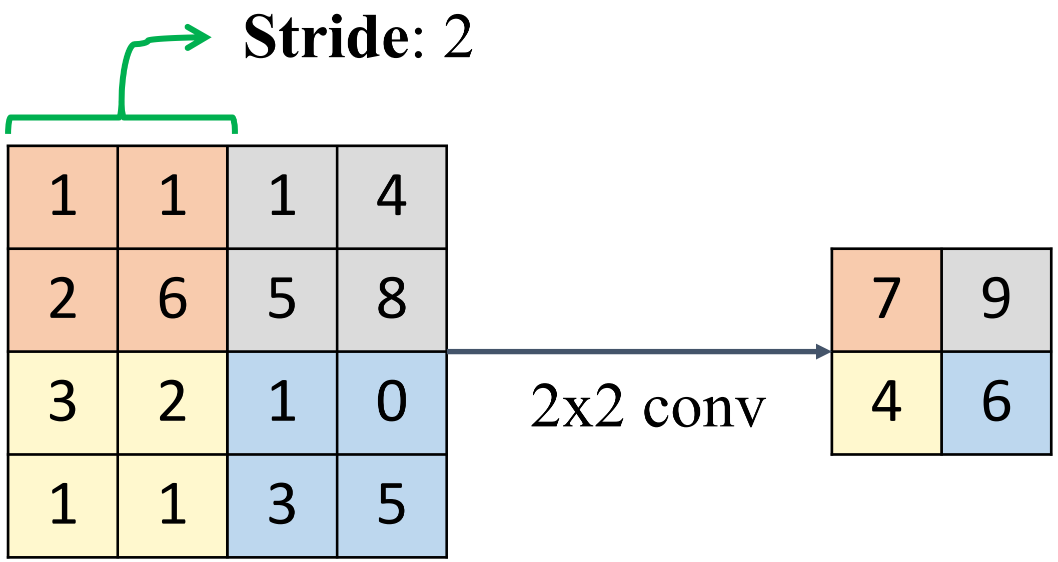



- If we stack $N$ convolutional layers with the same kernel size 3x3 the receptive field on $N$-th layer will be $\mathbf{2N+1 \times 2N+1}$.
- It looks like we need to stack a lot of convolutional layers!
  To be able to identify objects as big as the input image **300x300** we will need **150** convolutional layers!

# We need to grow receptive field faster!

- We can increase a **stride** in our convolutional layer to reduce the output dimensions!



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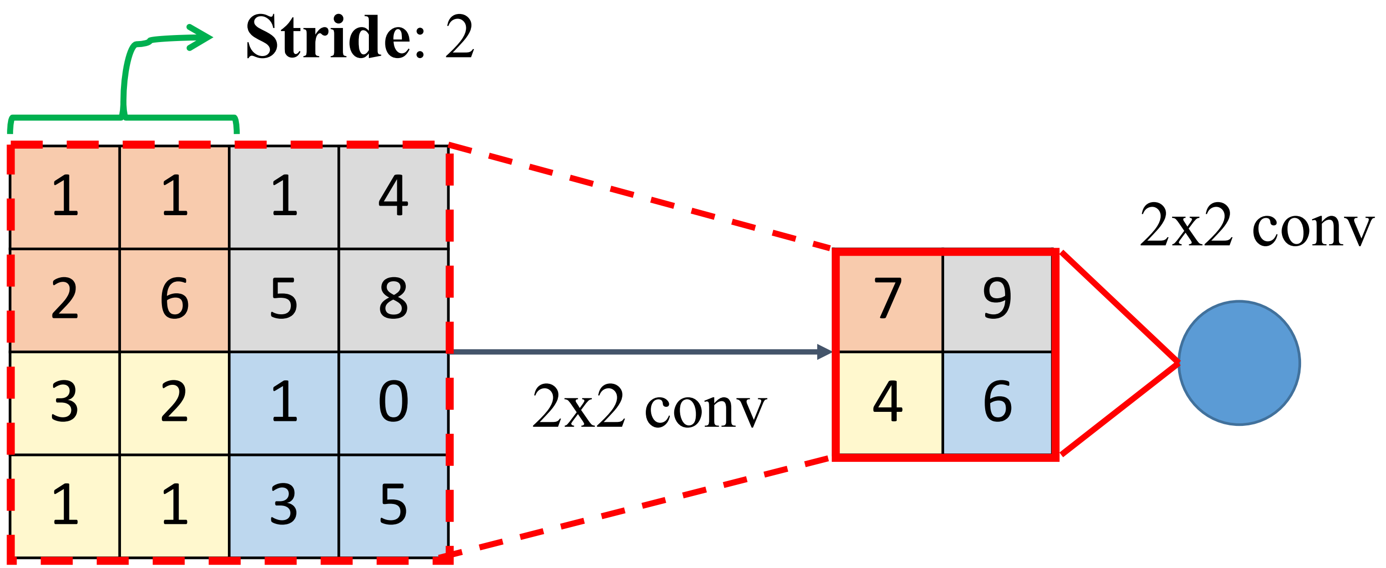



Further convolutions will effectively **double** their receptive field!

# How do we maintain translation invariance?

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

$$\underset{\text{Input}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

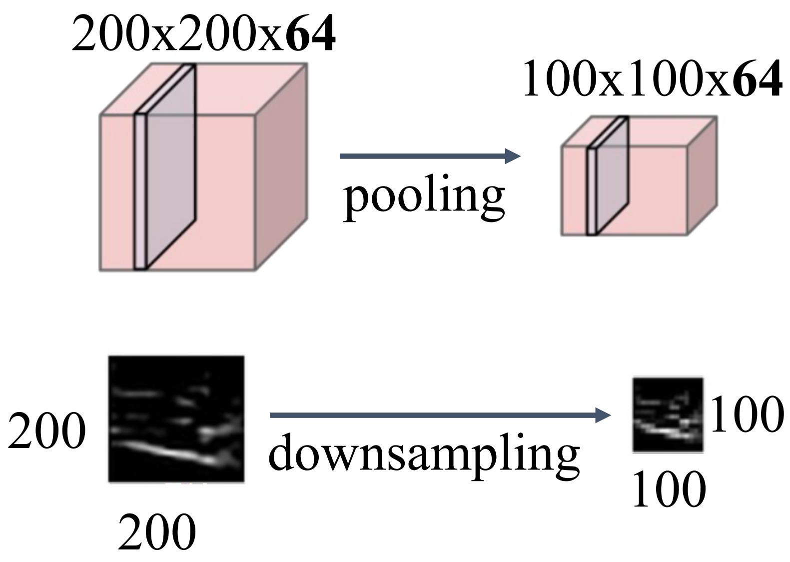
Max = 2

Didn't change

Max = 2

# Pooling layer will help!

- This layer works like a convolutional layer but doesn't have kernel, instead it calculates **maximum** or **average** of input patch values.




Single depth slice

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 4 |
| 2 | **6** | 5 | **8** |
| **3** | 2 | 1 | 0 |
| 1 | 1 | 3 | **5** |

| | |
|---|---|
| 6 | 8 |
| 3 | 5 |

2x2 **max pooling** with stride 2

# Backpropagation for max pooling layer

Strictly speaking: maximum is not a differentiable function!

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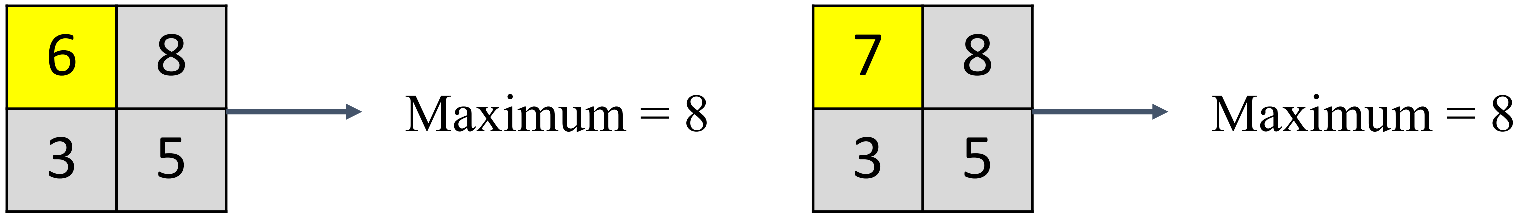


There is no gradient with respect to non maximum patch neurons,
since changing them slightly
does not affect the output.

# Backpropagation for max pooling layer

Strictly speaking: maximum is not a differentiable function!

| | |
|---|---|
| 6 | 8 |
| 3 | 5 |

$\rightarrow$ Maximum = 8

| | |
|---|---|
| 7 | 8 |
| 3 | 5 |

$\rightarrow$ Maximum = 8

There is no gradient with respect to non maximum patch neurons,
since changing them slightly
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| | |
|---|---|
| 6 | 8 |
| 3 | 5 |

$\rightarrow$ Maximum = 8

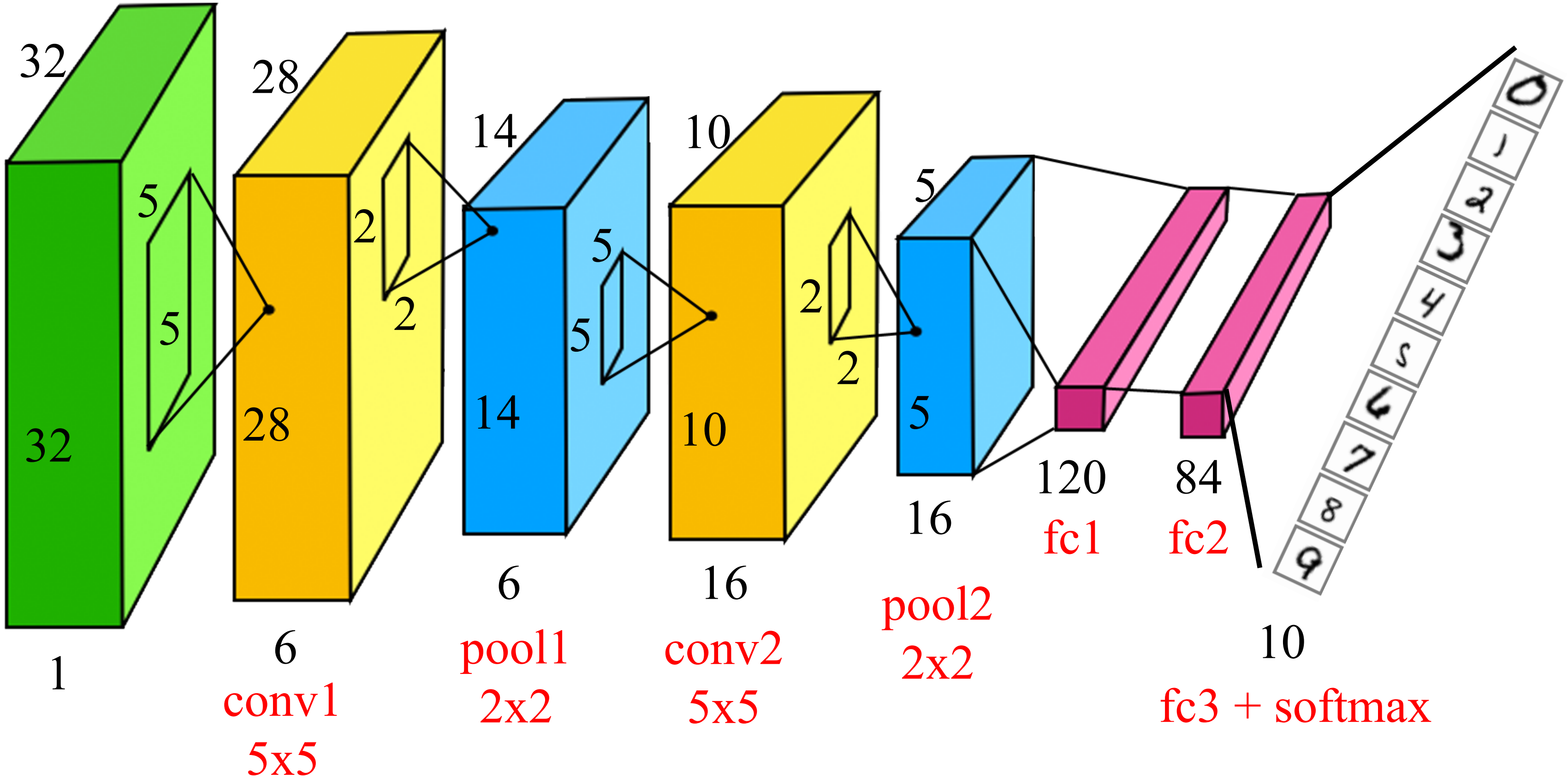
| | |
|---|---|
| 7 | 9 |
| 3 | 5 |

$\rightarrow$ Maximum = 9

For the maximum patch neuron we have a gradient of 1.

# Putting it all together into a simple CNN

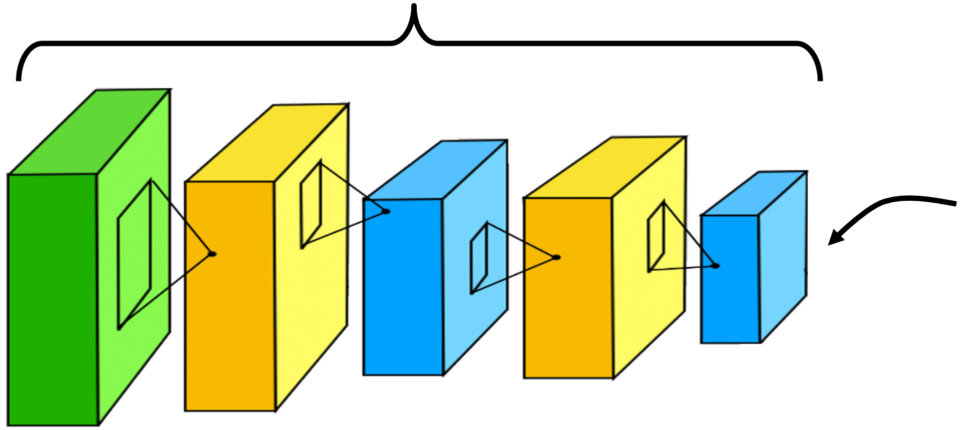
- LeNet-5 architecture (1998) for handwritten digits recognition on MNIST dataset:



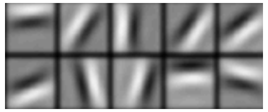

http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf

# Learning deep representations

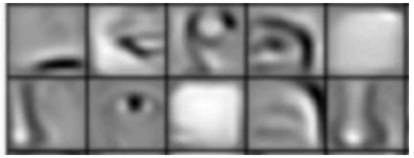
- Neurons of deep convolutional layers learn complex representations that can be used as features for classification with MLP.




Inputs that provide highest activations:



conv1



conv2

conv3

http://web.eecs.umich.edu/~honglak/icml09-ConvolutionalDeepBeliefNetworks.pdf

# Ссылки

- http://cs231n.stanford.edu/
- http://cs231n.github.io/convolutional-networks/
- https://brohrer.github.io/how_convolutional_neural_networks_work.html
- https://blog.keras.io/how-convolutional-neural-networks-see-the-world.html