Trajectory tracking Control of Differential Drive Mobile Robot based on improved Kinematics Controller algorithm

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***Abstract*—An important issue in wheeled mobile robot motion control is that most of robot controllers are designed relying solely on the kinematics of robots. However, when highspeed motion or heavyduty transportation are required for mobile robots, the description of the robot kinematics model is not comprehensive, and it is necessary to consider the robot dynamics model. Based on the work of reference [16], this paper makes an improvement on the kinematic controller by replacing the hyperbolic tangent function tanh(x) by the reference function sin(arctan(x)). This function ensures that the kinematic controller can improve the tracking accuracy of the robot and the distance error will be significantly reduced. Simultaneous position tracking, line speed and angular velocity tracking on the X and Y axes are achieved. Finally, the results of simulation experiments in three cases show that the tracking precision in this paper is obviously superior to the reference [16], and it has stronger robustness and better anti-jamming performance.**

***Keywords*—kinematic controller; simulation; stronger robustness; better anti-jamming performan;**

I. INTRODUCTION

Differential drive unicycle mobile robot is the most common configuration in mobile robots. It usually has two independently driven wheels and one or more unpowered wheels at the rear as a balanced structure [1]. Due to its simple configuration and good dynamics, differentially driven mobile robots have been widely used in ground cleaning [2], industrial load transport [3], subsea detection [4], surveillance [5], automatic wheelchairs [6] and many other fields.

An important issue considering the differential driving of mobile robots is that their motion controller design is mostly based on kinematic models [7]-[9]. The main reason is that dynamic models are more complex than kinematic models and mobile robots usually only use the low speed of the motor to control the loop [10]-[11].

However, when mobile robots require high-speed motion or heavy-duty transportation, the description of the robot kinematics model is not comprehensive, and it is necessary to consider the dynamics model. In [12], a robust adaptive controller by neural network is provide for perturbed and non-analog dynamic models. In addition, the literature [13] proposed a logic-based adaptive fuzzy controller whose dynamic model includes actuator dynamics.

In order to reduce the dynamic parameter variation of the robot, an adaptive controller is proposed in [14]. Using the dynamic model proposed in [15] and splitting it into two parts, the dynamic controller can be designed independently. In [16], which combines the adaptive controller of [14] and the dynamic model in [15], a dynamic model adaptive model of differential-driven mobile robot based on velocity is proposed. Based on the literature [16], this paper proposes a motion controller based on global stability, which enables the robot to track quickly and has stronger stability and robustness. At the same time, combined with the dynamic model of the literature [16] and the adaptive compensation controller of the model nature, the results of three simulation experiments are discussed. Finally, the improved method of this paper is compared with the reference [16], which greatly improves the tracking accuracy and reduces the distance error. Position tracking in the X and Y axis directions is realized, and linear speed and angular velocity tracking are also realized.

II. DYNAMIC MODELS

The dynamic model of the unicycle mobile robot in [17] is proposed. In Fig. 1, a type of differential drive unicycle mobile robot with parameters and variables is described. u and w are the linear velocity and angular velocity of a robot, G is the mass center, h is the key point of this study, ψ is the orientation of robot, a is the distance between h and the midpoint of the virtual axis connecting the drive wheels (point B), b is the distance from points G to B, and d is the distance from the center point of a independent drive wheel to the center point of another independent drive wheel.

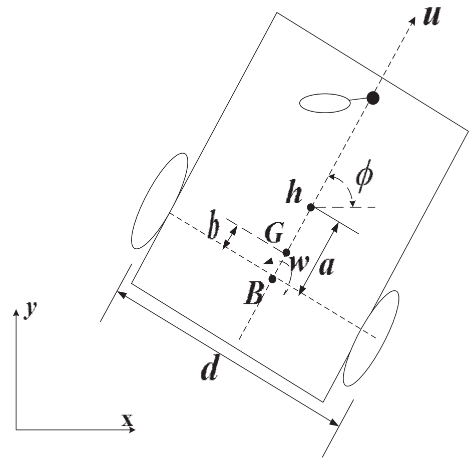


Fig. 1 The unicycle mobile robot model

In this model, $\boldsymbol{\theta} = [\theta_1, ..., \theta_6]^T$ is the physical parameters vector and $\boldsymbol{\delta} = [\delta_x \quad \delta_y \quad 0 \quad \delta_u \quad \delta_w]^T$ is the parametric uncertainties vector of unicycle mobile robot. The dynamic model of this mobile robot is as follows [8]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} u\cos\phi - aw\sin\phi \\ u\sin\phi + aw\cos\phi \\ w \\ \frac{\theta_3}{\theta_1} w^2 - \frac{\theta_4}{\theta_1} u \\ -\frac{\theta_5}{\theta_2} uw - \frac{\theta_6}{\theta_2} w \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} u_{ref} \\ w_{ref} \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ 0 \\ \delta_u \\ \delta_w \end{bmatrix} \tag{1}$$

There are some robot physical parameters in $\boldsymbol{\theta}$, such as robot mass m, inertia robot moment I_z at G, the robot motors electrical resistance R_a, the robot motors electromotive constant k_b, the robot motors torque constant k_a, the friction coefficient B_e, the wheel moment of inertia I_e, and the wheels radius r. If there are PD controllers in the robot servos to control the velocities of each motor, with positive proportional gain k_{PT}, k_{PR} and non-negative derivative gains k_{DT}, k_{DR}. If there are the same characteristics and neglectable inductances in the motors associated to both driven wheels, too. Here are some equations as follows

$$\theta_1 = \left[\frac{R_a}{k_a}(mr^2 + 2I_e) + 2rk_{DT}\right]\frac{1}{(2rk_{PT})}[s]$$

$$\theta_2 = \left[\frac{R_a}{k_a}(I_e d^2 + 2r^2(I_z + mb^2)) + 2rdk_{DR}\right] \times \frac{1}{(2rdk_{PR})}[s]$$

$$\theta_3 = \frac{R_a}{k_a}\frac{mbr}{2k_{PT}}[sm/rad^2]$$

$$\theta_4 = \frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)\frac{1}{rk_{PT}} + 1$$

$$\theta_5 = \frac{R_a}{k_a}\frac{mbr}{dk_{PR}}[s/m]$$

$$\theta_6 = \frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)\frac{d}{2rk_{PR}} + 1$$

It can be known $\theta_i > 0$, $i = 1, 2, 4, 6$. The parameters θ_3 and θ_5 can be negative and zero if and only if the centroid G coincides exactly with point B, i.e. $b = 0$.

The model (1) divides it into kinematics and dynamics. The kinematic model is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & -a\sin\phi \\ \sin\phi & a\cos\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ 0 \end{bmatrix} \tag{2}$$

The dynamic model is

$$\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1} w^2 - \frac{\theta_4}{\theta_1} u \\ -\frac{\theta_5}{\theta_2} uw - \frac{\theta_6}{\theta_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} u_{ref} \\ w_{ref} \end{bmatrix} + \begin{bmatrix} \delta_u \\ \delta_w \end{bmatrix} \tag{3}$$

Finally, we reference [16] Eq. 3 can be written as

$$\boldsymbol{\Delta} + \mathbf{H}\dot{\mathbf{v}}' + \mathbf{C}(\mathbf{v}')\mathbf{v}' + \mathbf{F}(\mathbf{v}')\mathbf{v}' = \mathbf{v_r} \tag{4}$$

where $\mathbf{v_r} = [u_{ref} \quad w_{ref}]^T$ is the vector of reference velocities, $\mathbf{v}' = \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u \\ w \end{bmatrix}$ is the vector of modified velocities, and

$$\mathbf{H} = \begin{bmatrix} \theta_1 / i & 0 \\ 0 & \theta_2 \end{bmatrix}, \ \mathbf{F}(\mathbf{v}') = \begin{bmatrix} \theta_4 / i & 0 \\ 0 & \theta_6 + (\theta_5 / i - \theta_3) iu \end{bmatrix}$$

$$\mathbf{C}(\mathbf{v}') = \begin{bmatrix} 0 & -\theta_3 w \\ \theta_3 w & 0 \end{bmatrix}, \ \boldsymbol{\Delta} = \begin{bmatrix} -\theta_1 & 0 \\ 0 & -\theta_2 \end{bmatrix}\begin{bmatrix} \delta_u \\ \delta_w \end{bmatrix}$$

III. CONTROLLER DESIGN

Fig. 2 depicts the control structure of the system, where **K**, **D**, and **R** respectively represent robot kinematic controller, adaptive dynamic compensation controller, robot controlled object.

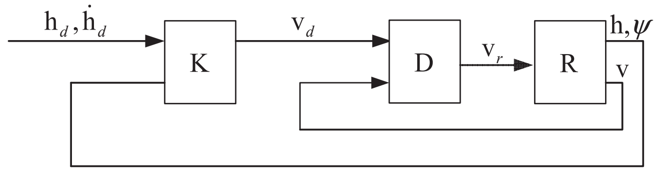


Fig. 2 The structure of control system

It is shown that the inputs of Kinematic controller are the desired position $\mathbf{h_d} = [x_d \quad y_d]^T$, velocity $\dot{\mathbf{h}}_\mathbf{d}$ from the references trajectory, $\mathbf{h} = [x \quad y]^T$ represent the actual position of the robot, and orientation ψ, Calculate the desired robot velocities $\mathbf{v_d} = [u_d \quad w_d]^T$ as its output by the kinematic controller. The output of Kinematic controller and the actual velocities $\mathbf{v} = [u \quad w]^T$ of the robot are the inputs of the adaptive dynamic compensation controller.

A. *The Kinematic Controller*

- Design

The robot's kinematic model (2) without disturbance is as follows

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}\cos\phi & -a\sin\phi\\ \sin\phi & a\cos\phi\\ 0 & 1\end{bmatrix}\begin{bmatrix}u\\ w\end{bmatrix}$$

Considering output are the points needed $\mathbf{h}=\begin{bmatrix}x & y\end{bmatrix}^T$. Then

$$\dot{\mathbf{h}}=\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}=\begin{bmatrix}\cos\phi & -a\sin\phi\\ \sin\phi & a\cos\phi\end{bmatrix}\begin{bmatrix}u\\ w\end{bmatrix}=\mathrm{A}\begin{bmatrix}u\\ w\end{bmatrix} \tag{5}$$

Whose inverse is

$$A^{-1}=\begin{bmatrix}\cos\phi & \sin\phi\\ -\frac{1}{a}\sin\phi & \frac{1}{a}\cos\phi\end{bmatrix}$$

We have

$$\begin{bmatrix}u\\ w\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi\\ -\frac{1}{a}\sin\phi & \frac{1}{a}\cos\phi\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} \tag{6}$$

and the kinematics control law of the mobile robot is given as follows

$$\begin{bmatrix}u_d\\ w_d\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi\\ -\frac{1}{a}\sin\phi & \frac{1}{a}\cos\phi\end{bmatrix}\begin{bmatrix}\dot{x}_d+l_x\sin(\arctan(l_x\tilde{x}))\\ \dot{y}_d+l_y\sin(\arctan(l_y\tilde{y}))\end{bmatrix} \tag{7}$$

Here, $\tilde{\mathbf{h}}=\begin{bmatrix}\tilde{x} & \tilde{y}\end{bmatrix}^T$ is the distance error variable obtained by $\mathbf{h_d}-\mathbf{h}$; l_x, $l_y\in\Re$ are saturation constants; and $a>0$.

- Stability analysis

Suppose equation (3) and equation (4) are equal, which means that the dynamics of mobile robots are negligible. Therefore, the following closed-loop equation for velocity error can be obtained.

$$\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{y}}\end{bmatrix}+\begin{bmatrix}l_x & 0\\ 0 & l_y\end{bmatrix}\begin{bmatrix}\sin(\arctan(l_x\tilde{x}))\\ \sin(\arctan(l_y\tilde{y}))\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \tag{8}$$

Eq. 8 can be written as

$$\dot{\tilde{\mathbf{h}}}=-\begin{bmatrix}l_x\sin(\arctan(l_x\tilde{x})) & l_y\sin(\arctan(l_y\tilde{y}))\end{bmatrix}^T$$

Consider the following Lyapunov function

$$V=\frac{1}{2}\tilde{\mathbf{h}}^T\tilde{\mathbf{h}}>0$$

is given, whose derivative

$$\begin{aligned}\dot{V}&=\tilde{\mathbf{h}}^T\dot{\tilde{\mathbf{h}}}\\ &=-l_x\tilde{x}\sin(\arctan(l_x\tilde{x}))-l_y\tilde{y}\sin(\arctan(l_y\tilde{y}))<0\end{aligned}$$

is negative definite.

The system is globally asymptotically stable, so we have the position errors $\tilde{x}(t)\to 0$ and $\tilde{y}(t)\to 0$ as $t\to\infty$.

B. The adaptive dynamic compensation controller

In order to improve the tracking accuracy and robustness of mobile robot trajectory, an adaptive dynamic compensation controller is proposed in this part. Its inputs are the output of kinematic controller $\mathbf{v_d}$ and actual robot velocities $\mathbf{v}$ in Fig. 2. Define the corrected velocity vector $\mathbf{v'_d}$ as

$\mathbf{v'_d}=\begin{bmatrix}u'_d\\ w_d\end{bmatrix}=\begin{bmatrix}i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_d\\ w_d\end{bmatrix}$, where $i=1rad^2/s$ is to ensure unit consistency, and the velocity errors is $\tilde{\mathbf{v}}'=\mathbf{v'_d}-\mathbf{v}'$.

For the robot dynamics controller design, its equation (4) is rewritten into a linear parameterized combination.

$$\mathbf{v_r}=\boldsymbol{G'\theta}=\begin{bmatrix}\dot{u} & 0 & -w^2 & u & 0 & 0\\ 0 & \dot{w} & 0 & 0 & uw & w\end{bmatrix}\boldsymbol{\theta} \tag{9}$$

The parameter uncertainty vector is negligible, so considering the parameter uncertainty, the following control law can be obtained

$$\mathbf{v_r}=\hat{\mathbf{H}}(\dot{\mathbf{v}}'_\mathbf{d}+\mathbf{T}(\tilde{\mathbf{v}}'))+\hat{\mathbf{C}}\mathbf{v'_d}+\hat{\mathbf{F}}\mathbf{v'_d} \tag{10}$$

Where $\hat{\mathbf{H}}$, $\hat{\mathbf{C}}$, and $\hat{\mathbf{F}}$ are estimates of $\mathbf{H}$, $\mathbf{C}$, and $\mathbf{F}$, respectively, $\mathbf{T}(\tilde{\mathbf{v}}')=\begin{bmatrix}l_u & 0\\ 0 & l_w\end{bmatrix}\begin{bmatrix}\sin(\arctan(l_u i\tilde{u}))\\ \sin(\arctan(l_w\tilde{w}))\end{bmatrix}$, l_u, $l_w\in\Re$ are saturation constants, and $\tilde{w}=w_d-w$, $\tilde{u}=u_d-u$ are the current velocity errors. The term $\mathbf{T}(\tilde{\mathbf{v}}')$ avoid that the orders which robot receives are too big. The reader should refer to [16] for more details about the dynamic compensation controller.

When the dynamic parameters are not correctly identified, so an adaptive parameter update law is designed by a linear parametric combination as follows

$$\mathbf{v_r}=\mathbf{G}\hat{\boldsymbol{\theta}}=\begin{bmatrix}\sigma_1 & 0 & -w_dw & u_d & 0 & 0\\ 0 & \sigma_2 & (iu_dw-iuw_d) & 0 & uw_d & w_d\end{bmatrix}\hat{\boldsymbol{\theta}} \tag{11}$$

Where $\hat{\boldsymbol{\theta}}$ is the parameter estimate vector, $\sigma_1=\dot{u}_d+l_u\sin(\arctan(l_u\tilde{u}))$, $\sigma_2=\dot{w}_d+l_w\sin(\arctan(l_w\tilde{w}))$.

By selecting the updating law as

$$\dot{\hat{\boldsymbol{\theta}}}=\boldsymbol{\gamma}\mathbf{G}^\mathrm{T}\tilde{\mathbf{v}}' \tag{12}$$

However, the law of renewal of parameters (12) will cause robustness problems. Therefore, the robust correction law is proposed by introducing such the term

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\gamma}\mathbf{G}^{\mathrm{T}}\tilde{\boldsymbol{v}}' - \boldsymbol{\gamma}\boldsymbol{\Gamma}\hat{\boldsymbol{\theta}} \tag{13}$$

with diagonal matrix $\gamma > 0$, $\Gamma > 0$.

IV. EXPERIMENTAL RESULTS

In order to illustrate the effectiveness of the method, we compared it with previous methods [16] in three cases. The first case uses only kinematic models and kinematic controllers. In the second case, the dynamic controller and dynamic model are added. In the third case, an adaptive dynamics compensation controller was used in the case of the second addition of the dynamic model.

The robot control law is given by Eq. 4 and 10, and the robust adaptive update law is given by Eq. 13. Then apply the robot control structure shown in Fig. 2, and simulate the application through MATLAB/Simulink.

In all simulations, the common parameters are as follows: fixed sample time of 0.1s; saturation parameters $l_u = 1$, $l_w = 1$;

$$\gamma = diag(1.7, 1.1, 0.5, 0.3, 0.01, 0.5)$$
$$\Gamma = diag(0.0005, 0.001, 0.001, 0.00006, 0.001, 0.001)$$

The robot starts at position $(0.2, 0.0)$ m with orientation 0 degrees, and should track a 8-shaped reference trajectory starting at $(0.0, 0.0)$ m.

Case I simulation results for kinematic controller and kinematic model

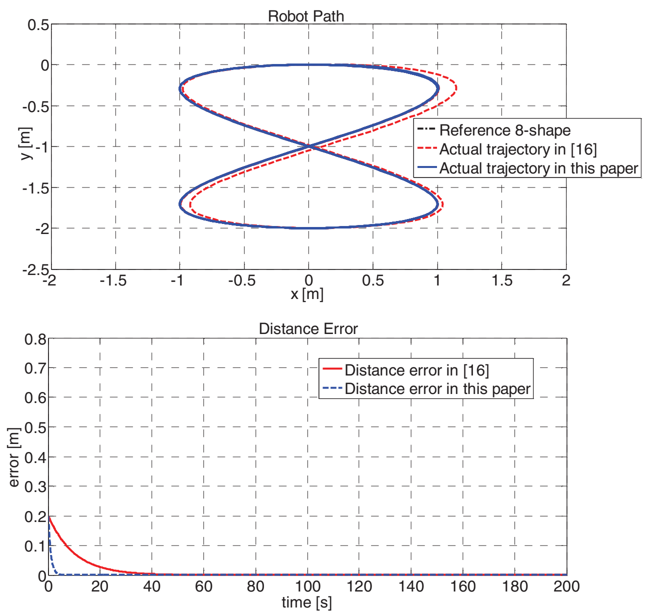
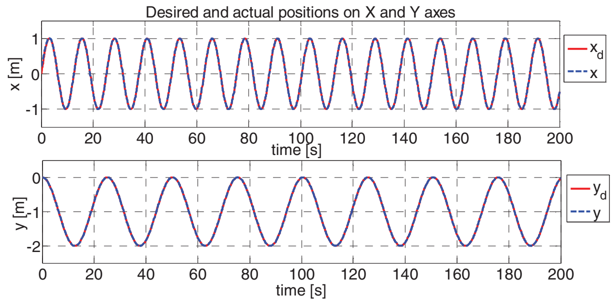
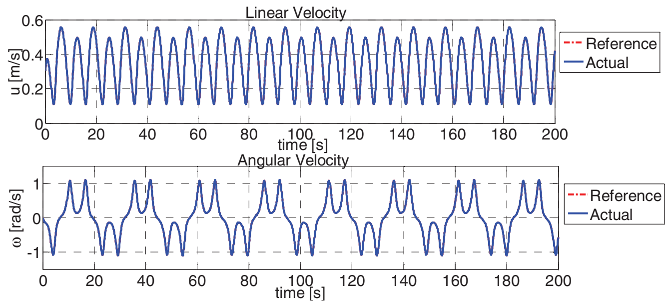


Fig. 3 Only kinematic controllers and kinematic models, where represent robot path tracking, and represent the evolution of distance error



(a)

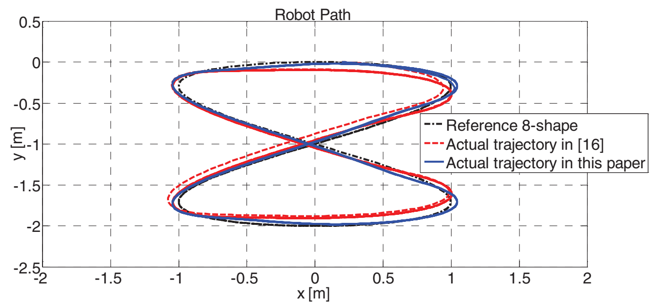


(b)

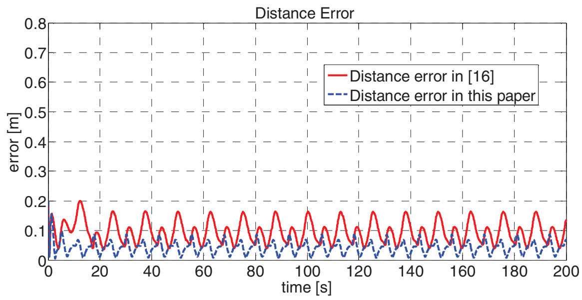
Fig. 4 Only kinematic controllers and kinematic models, where (a) represent robot positions tracking on the X and Y axes, and (b) represent the velocities tracking of the robot

The case I simulation results are shown in Fig. 3(a) and (b). The robot path tracking and distance error evolved into perfect tracking. Fig. 4(a) and (b) show the robot's X-axis and Y-axis position and velocity tracking, respectively. Since the robot dynamics model is not considered, the difference between the reference speed and the actual speed is not seen in Fig. 4b, and the distance error evolution will remain at zero. In addition, the proposed method and the reference [16], the robot path tracking speed Fast, the distance error tends to zero only about 3s.

Case II simulation results of kinematic controller, kinematic model and dynamic model

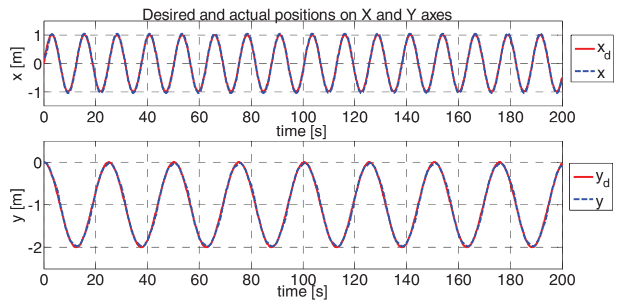


(a)

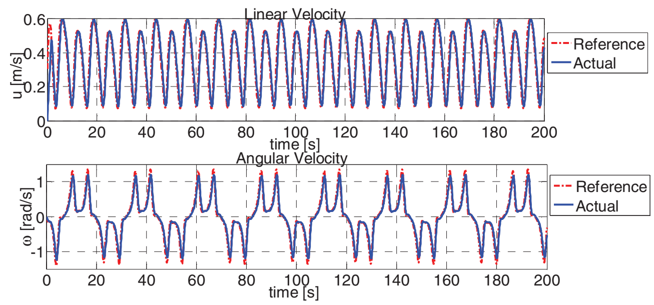


(b)

Fig. 5 Kinematic controller, kinematic model, dynamic model combination, where (a) represent robot path tracking, and (b) represent the evolution of distance error



(a)



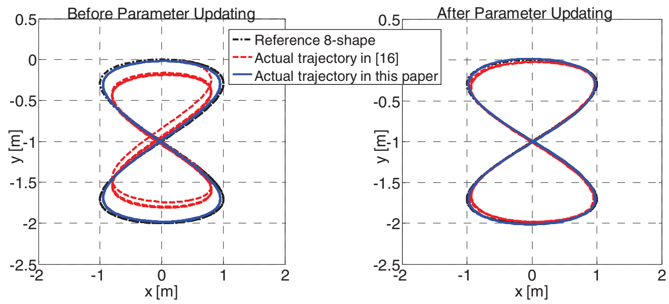
(b)

Fig. 6 Kinematic controller, kinematic model, dynamic model combination, where (a) represent robot positions tracking on the X and Y axes, and (b) represent the velocities tracking of the robot

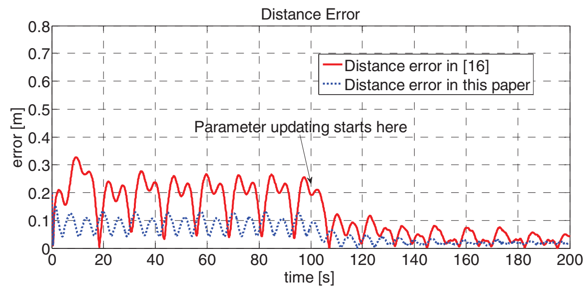
The case II simulation results, from Fig. 5(a) and (b), can be seen that the robot path generated by the design method can not be completely tracked, and the distance error will not be reduced to zero, but the oscillation is about 0.05 m compared to the reference [16], its tracking accuracy is improved, and the distance error tends to be zero. It is known from Fig. 6(a) and (b) that the actual speed of the robot is not exactly equal to the speed produced by the kinematic controller.

The case I and case II simulation results show that considering the kinematic model is very important for evaluating controller performance.

Case III simulation results of adaptive dynamic compensation

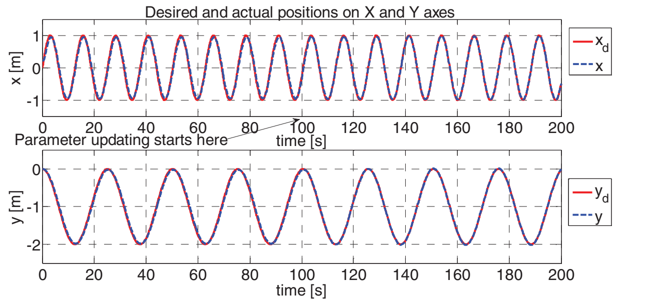


(a)

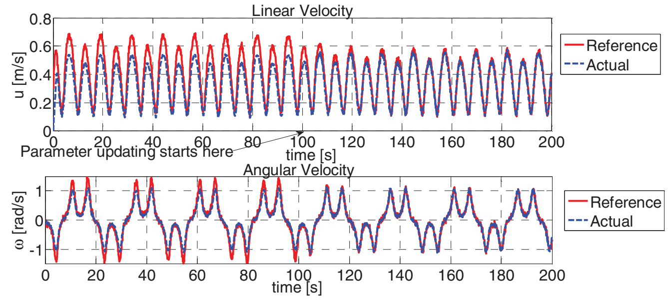


(b)

Fig. 7 In the case of adaptive dynamic compensation, (a) represent the robot path tracking before and after the parameter update law, and (b) represent the evolution process of the distance error before and after the parameter update law



(a)



(b)

Fig. 8 In the case of adaptive dynamic compensation, (a) represent the position tracking of the robot before and after the parameter update law, and (b) represent the velocities tracking of the robot before and after the parameter update law

The results of the case III simulation, from Fig. 7(a) and (b), show that the tracking accuracy of the robot path is significantly superior to the parameter update after the parameter update, and the design method of this paper is superior to the reference before and after the parameter update [16]. The distance error oscillates around 0.1m before the parameter is updated. After the parameter is updated, the distance error will be infinitely close to zero, and the parameter update time is 100 seconds. Compared with the reference [16], the distance error is significantly reduced. From Fig. 8(a) and (b), it is found that after the parameter update, the actual X and Y position tracking and speed tracking of the robot significantly improve the tracking accuracy. This indicates the importance of designing dynamic models in robot motion controllers.

V. CONCLUSION

Aiming at the kinematics model of mobile robot, this paper proposes a kinematic controller based on global asymptotic stability, which is combined with the speed-based adaptive dynamic compensation controller design. The simulation results show that the validity and superiority of the design method [16], and the designed controller has stronger robustness and better tracking precision.

Acknowledgment

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