ASEN 5070

Statistical Orbit Determination I

Fall 2012

Professor Jeffrey S. Parker

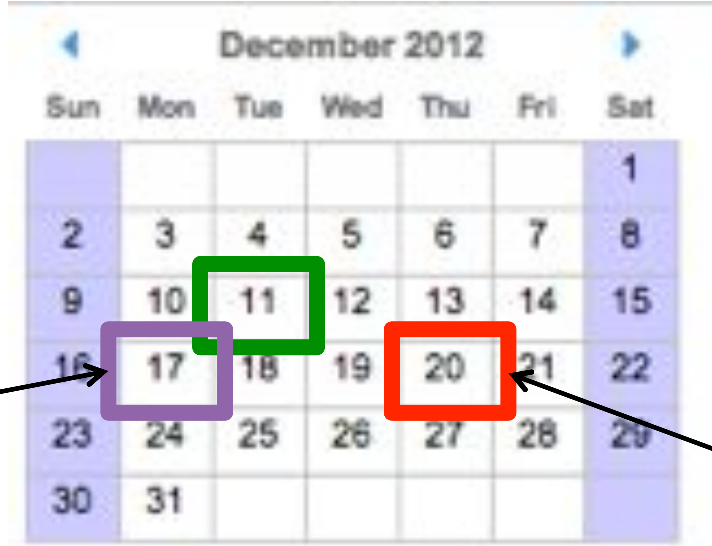
Professor George H. Born

Lecture 27: Final Lecture!

Announcements

- Today's the final lecture!
- Check your grades (for those graded anyway), especially quizzes.
- Exam 3 out today – due Monday at midnight unless you get permission otherwise.
- Thursday work day in this room.
- Everything else due Dec 20. Let me know if this is a problem!

Take-Home
Exam Due



Final Project
Due
All HW Due

Review Topics

- The Art of Stat OD
- Probability and Statistics
- The Process

- Then we'll conclude with a final visit of spaceflight operations in practice.

The Art of Stat OD

- Set up your scenario
  - What are you estimating?
  - Set up $\mathbf{X}(t)$ $\dot{\mathbf{X}}(t)$
  - What observations do you have?
  - Set up $\mathbf{Y}(t)$ $\mathbf{G}(t)$ $R(t)$ or $W(t)$
  - Do you have any *a priori* information?

    $\bar{\mathbf{x}}(t_0)$ $\bar{P}(t_0)$
  - Any consider parameters?

The Art of Stat OD

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- Set up your scenario
  - What are the dynamics?

$$\dot{\mathbf{X}}(t) = \mathbf{F}(\mathbf{X}(t), t) \qquad \dot{\Phi}(t, t_0) = A\Phi(t, t_0)$$

  - What are the observation state equations?

$$\mathbf{y} = H\mathbf{x} + \epsilon \qquad \tilde{H}(t)$$

The Art of Stat OD

- Select a Filter
  - Least Squares
  - Weighted Least Squares
  - Minimum-Variance with *a priori*
  - P-Norm Filters

- Select an algorithm
  - Batch
  - Conventional Kalman
  - Extended Kalman
  - Others

The Art of Stat OD

- Make adjustments to account for numerical issues
  - Joseph formulation to help the P matrix
  - Square root processes to keep P positive definite
    - Potter, Cholesky, Givens, Householder
    - Also improves the conditioning of any matrices
    - Square root free algorithms to speed up numerical computations
  - Process noise compensation to avoid filter saturation
    - State Noise Compensation
    - Dynamical Model Compensation

Analysis

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- After processing your observations, observe the results.
  - Is everything observable?
  - Did the covariance collapse? Diverge?
  - Do the post-fit residuals appear Gaussian?
    - If not, perhaps you need to adjust the tuning parameters or the dynamical model.

- How do the results compare with other solutions?
  - Try out all sorts of set-ups, arcs, etc, to see if everything is consistent.
  - Can you safely reduce the amount of tracking in the future?
  - Are you meeting your scientific/engineering objectives?

Review Topics

- ~~The Art of Stat OD~~
- Probability and Statistics
- The Process

- Then we'll conclude with a final visit of spaceflight operations in practice.

Probability and Statistics

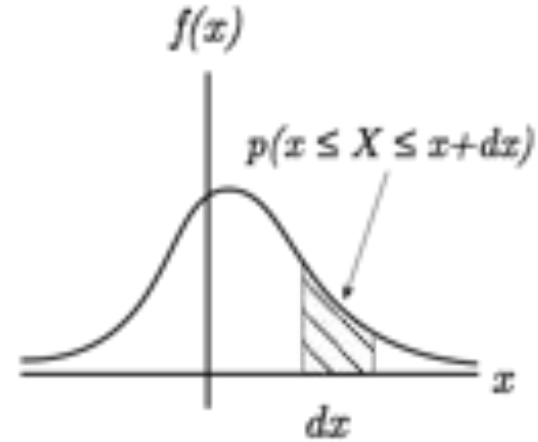
- Why do we care?

- We need to know something about the expected value and distribution of an error!
- Given:
  - An uncertain state (hopefully with a corresponding covariance)
  - Noisy observations
  - Errors in dynamical model
- We'd like to know what the "expected value" and "variance-covariance" of our state estimation errors are!

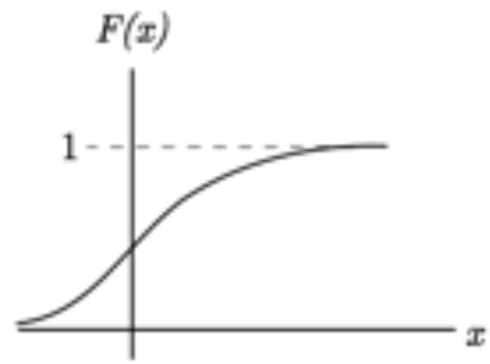
Probability and Statistics

- What is a probability density function?

- What is a cumulative distribution function?



Density function of a continuous random variable



Distribution function of a continuous random variable

Probability and Statistics

- What is a conditional probability function?

$$p\,(B/A) = \frac{p\,(A\,B)}{p\,(A)}$$

- Why do we care?
  - Bayesian statistics
  - We want to be able to compute the best estimate of our state in the presence of our observations!

Expected Value and Variance

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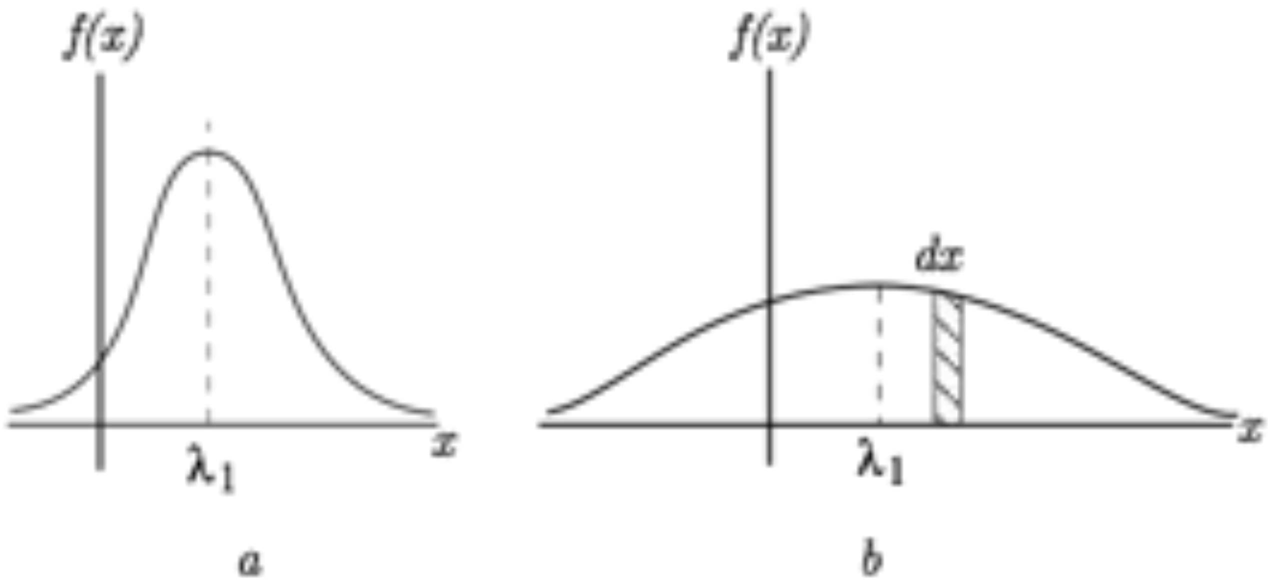
- Given a probability density function, we'd like to be able to compute E[x] and $E[(x-xb)(x-xb)^T]$



The Gaussian or Normal Density Function

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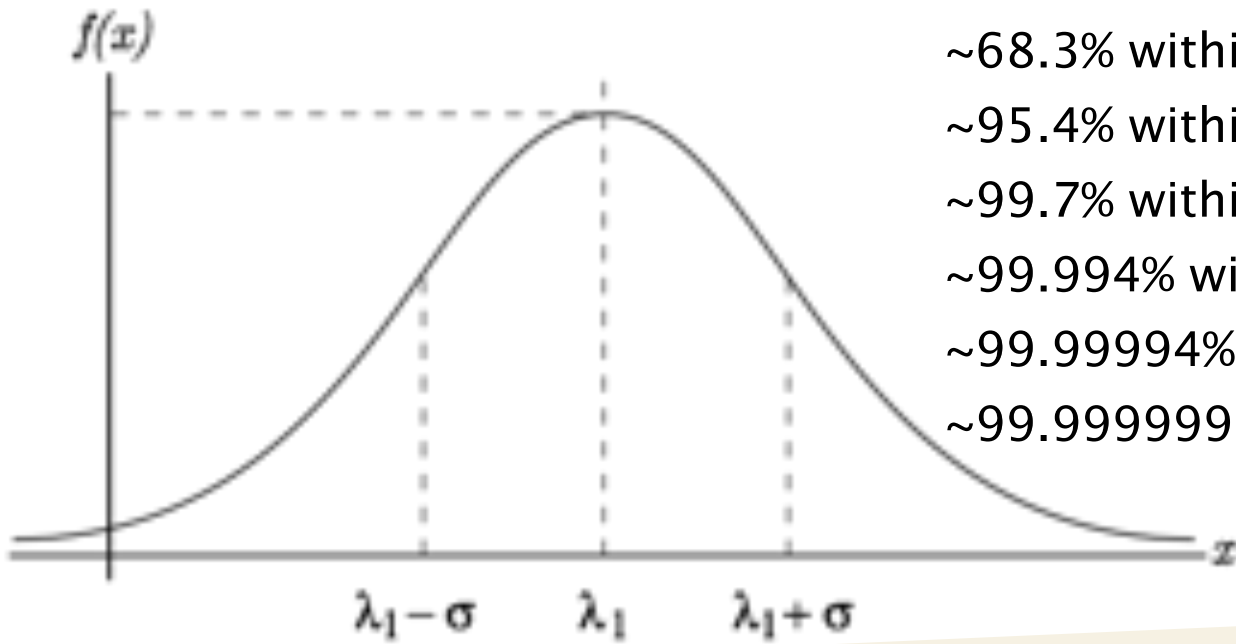
One of the most important density functions is the Gaussian.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\lambda_1}{\sigma}\right)^2\right] \quad -\infty < x < \infty\,. \tag{A.8.7}$$

- The Gaussian density function is depicted graphically as



~68.3% within 1 sigma of mean

~95.4% within 2 sigmas of mean

~99.7% within 3 sigmas of mean

~99.994% within 4

~99.99994% within 5

~99.9999998% within 6

Variance-Covariance Matrix

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$$P = E[(\mathbf{x}-\bar{\mathbf{x}})(\mathbf{x}-\bar{\mathbf{x}})^T]$$

The variance-covariance matrix for an n-dimensional random vector, X, can be written as

$$P = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & & \ddots & \vdots \\ \rho_{1n}\sigma_1\sigma_n & \rho_{2n}\sigma_2\sigma_n & \cdots & \sigma_n^2 \end{bmatrix} \qquad \text{(A.14.3)}$$

where ρ_{ij} is a measure of the degree of linear correlation between X_i and X_j.

Review Topics

- ~~The Art of Stat OD~~
- ~~Probability and Statistics~~
- The Process

- Then we'll conclude with a final visit of spaceflight operations in practice.

Fitting the data

- How do we best fit the data?

- A good solution, and one easy to code up, is the least-squares solution

$$\text{Minimize } J = \frac{1}{2}\epsilon^T \epsilon$$

$$\frac{\partial J}{\partial X} = 0$$

$$\frac{\partial^2 J}{\partial^2 X} \text{ is positive definite}$$

Fitting the data

- How do we best fit the data?

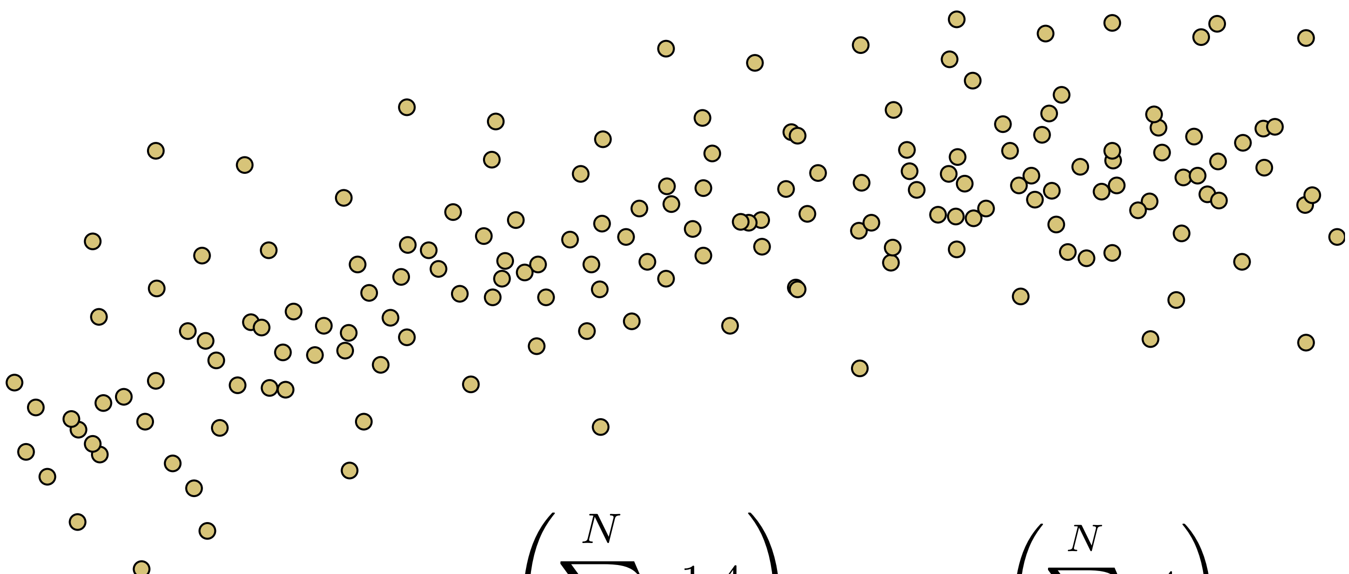
Residuals = ϵ = O−C

$$\min\left(\sum_{i=1}^{N}\epsilon_i\right)?$$ No

$$\min\left(\sum_{i=1}^{N}\epsilon_i\right)^2?$$ No

$$\min\left(\sum_{i=1}^{N}|\epsilon_i|\right)?$$ Not bad

$$\min\left(\sum_{i=1}^{N}\epsilon_i^2\right)?$$



$$\min\left(\sum_{i=1}^{N}\epsilon_i^{1.4}\right)? \qquad \min\left(\sum_{i=1}^{N}\epsilon_i^{4}\right)?$$

State Deviation and Linearization

- Linearization
- Introduce the state deviation vector

$$\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}^*(t)$$

- If the reference/nominal trajectory is close to the truth trajectory, then a linear approximation is reasonable.

State Transition Matrix

- The state transition matrix maps a deviation in the state from one epoch to another.

$$\mathbf{x}(t_2) = \Phi(t_2, t_1)\mathbf{x}(t_1)$$

$$\Phi(t_2, t_1) = \Phi(t_1, t_2)^{-1}$$

- It is constructed via numerical integration, in parallel with the trajectory itself.

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0)$$

$$\Phi(t_0, t_0) = I$$

Measurement Mapping Matrix

$$\mathbf{y}(t) = \tilde{H}(t)\mathbf{x}(t) + \epsilon(t)$$

- The Mapping Matrix

$$\tilde{H}(t) = \frac{\partial \boldsymbol{h}(t, \mathbf{X}(t))}{\partial \mathbf{X}(t)}$$

$$\tilde{H}(t) = \begin{bmatrix} \frac{\partial \rho}{\partial x} & \frac{\partial \rho}{\partial y} & \cdots & \frac{\partial \rho}{\partial \dot{z}} \\ \frac{\partial \dot{\rho}}{\partial x} & \frac{\partial \dot{\rho}}{\partial y} & \cdots & \frac{\partial \dot{\rho}}{\partial \dot{z}} \end{bmatrix}$$

Mapping an observation

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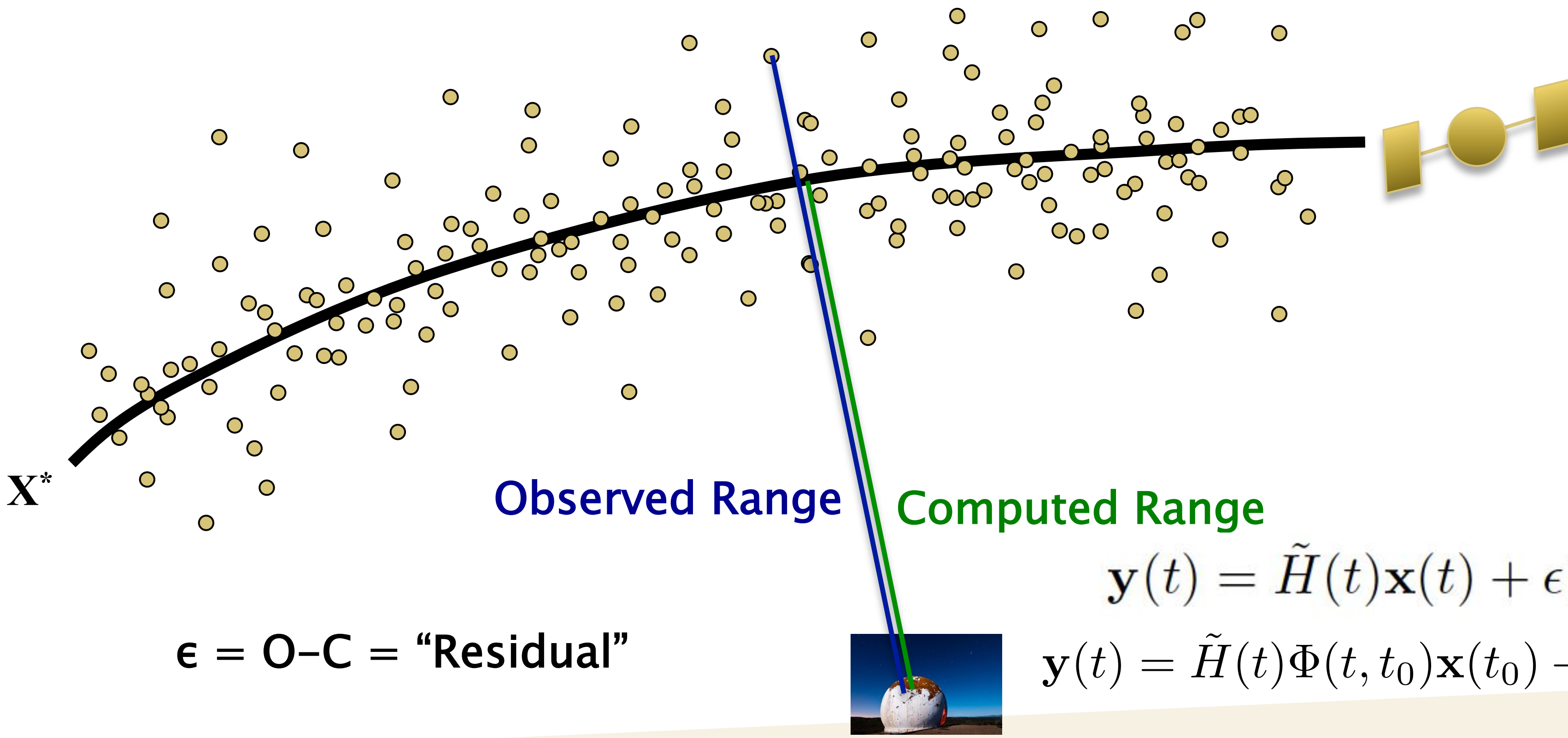
- After these mapping matrices are defined, we can relate an observation to the satellite's state at any epoch!



$$\mathbf{y}(t) = \tilde{H}(t)\mathbf{x}(t) + \epsilon(t)$$

ϵ = O–C = "Residual"

$$\mathbf{y}(t) = \tilde{H}(t)\Phi(t,t_0)\mathbf{x}(t_0) + \epsilon(t)$$

Different ways to get a *best estimate*

- Least Squares

$$\hat{\mathbf{x}}_k = \left(H^T H\right)^{-1} H^T \mathbf{y}$$

- Weighted Least Squares

$$\hat{\mathbf{x}}_k = (H^T W H)^{-1} H^T W \mathbf{y}$$

- Least Squares with *a priori*

$$\hat{\mathbf{x}}_k = \left(H^T W H + \bar{W}_k\right)^{-1} \left(H^T W \mathbf{y} + \bar{W}_k \bar{\mathbf{x}}_k\right)$$

- Min Variance

$$\hat{\mathbf{x}}_k = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \mathbf{y}$$

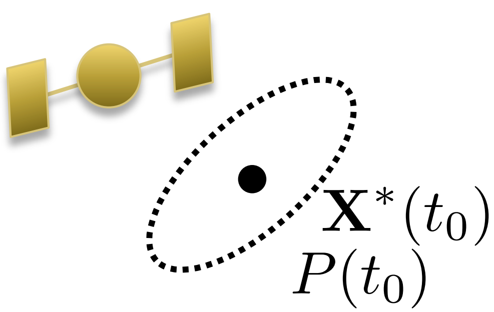
- Min Variance with *a priori*

$$\hat{\mathbf{x}}_k = \left(\tilde{H}_k^T R^{-1} \tilde{H}_k + \bar{P}_k^{-1}\right)^{-1} \left(\tilde{H}_k^T R^{-1} \mathbf{y}_k + \bar{P}_k^{-1} \bar{\mathbf{x}}_k\right)$$

Review of the Stat OD Process

- Setup.
  - Given: an initial state $\mathbf{X}^*(t_0)$
  - Optional: an initial covariance $P(t_0)$

$$\mathbf{X}^*(t_0) = [\ x_0,\ y_0,\ z_0,\ \dot{x}_0,\ \dot{y}_0,\ \dot{z}_0,\ c_1,\ c_2\ ]^T$$



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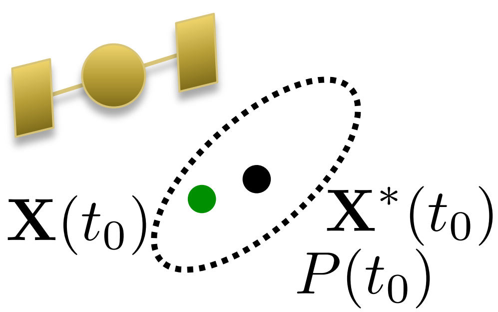
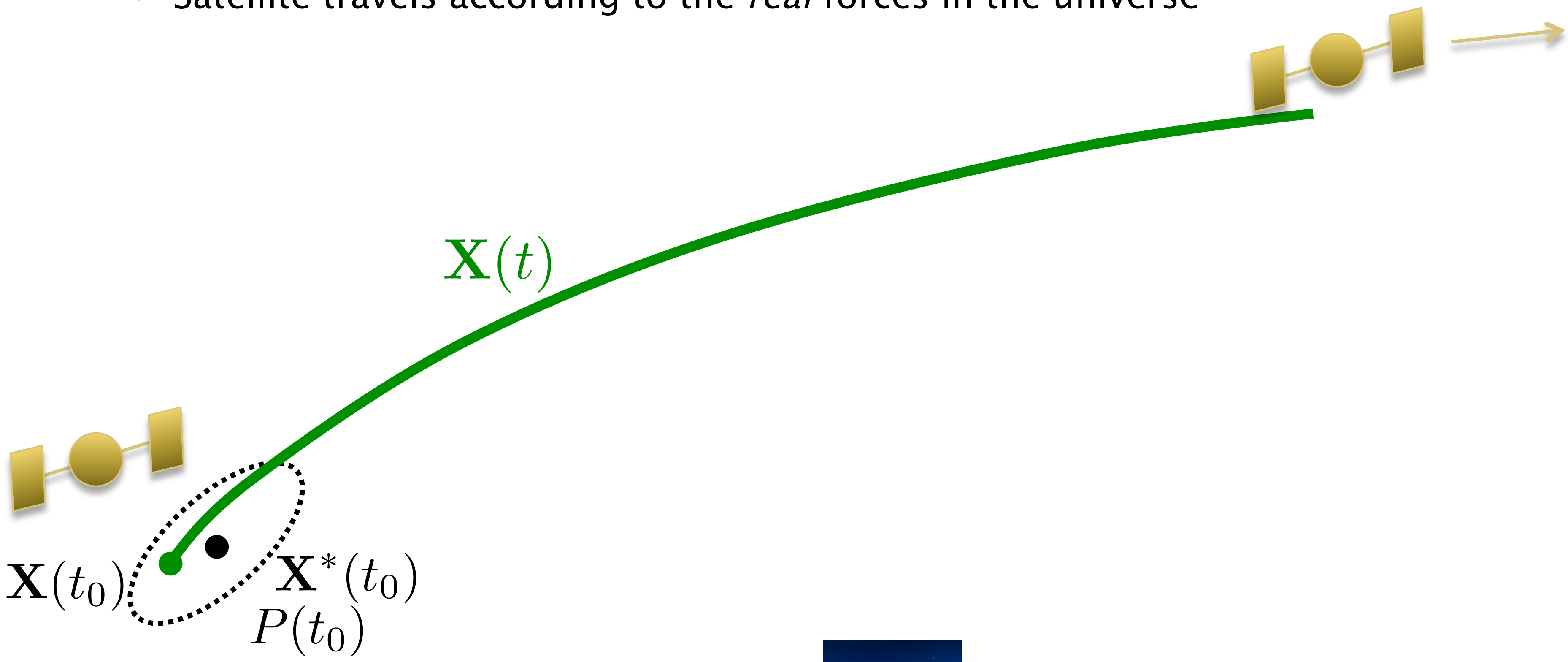
  - The satellite will *not* be there, but will (hopefully) be nearby
    - True state = $\mathbf{X}(t_0)$



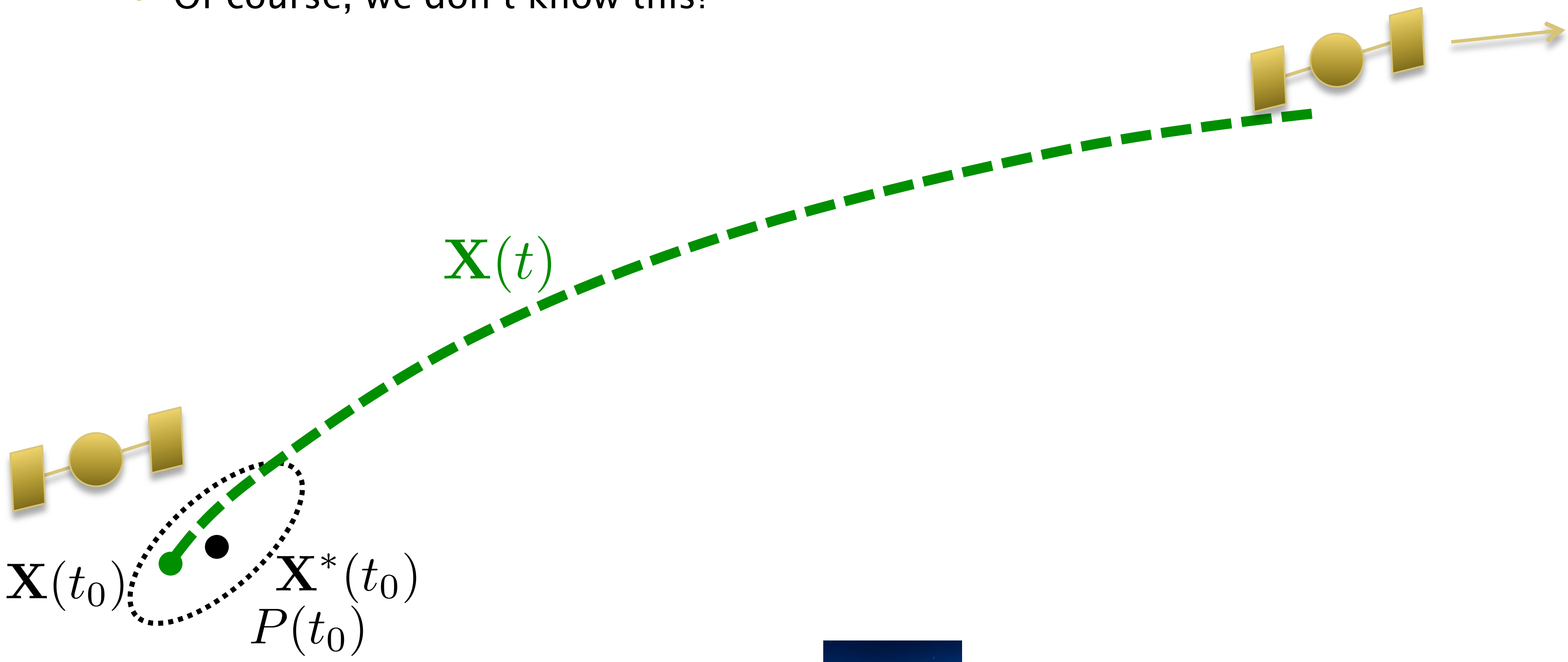
Review of the Stat OD Process

- What really happens
  - Satellite travels according to the *real* forces in the universe



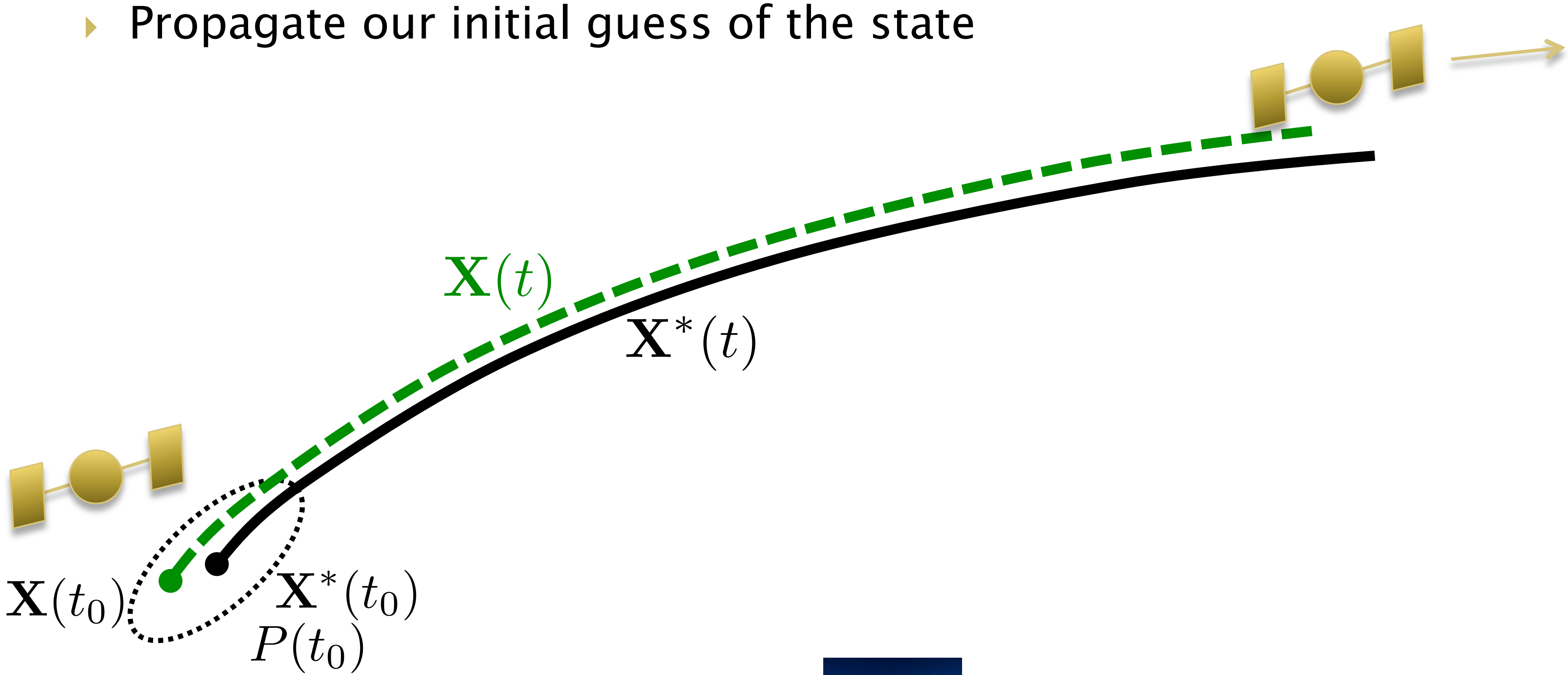
Review of the Stat OD Process

- What really happens
  - Of course, we don't know this!



Review of the Stat OD Process

- Model reality as best as possible
- Propagate our initial guess of the state



Review of the Stat OD Process

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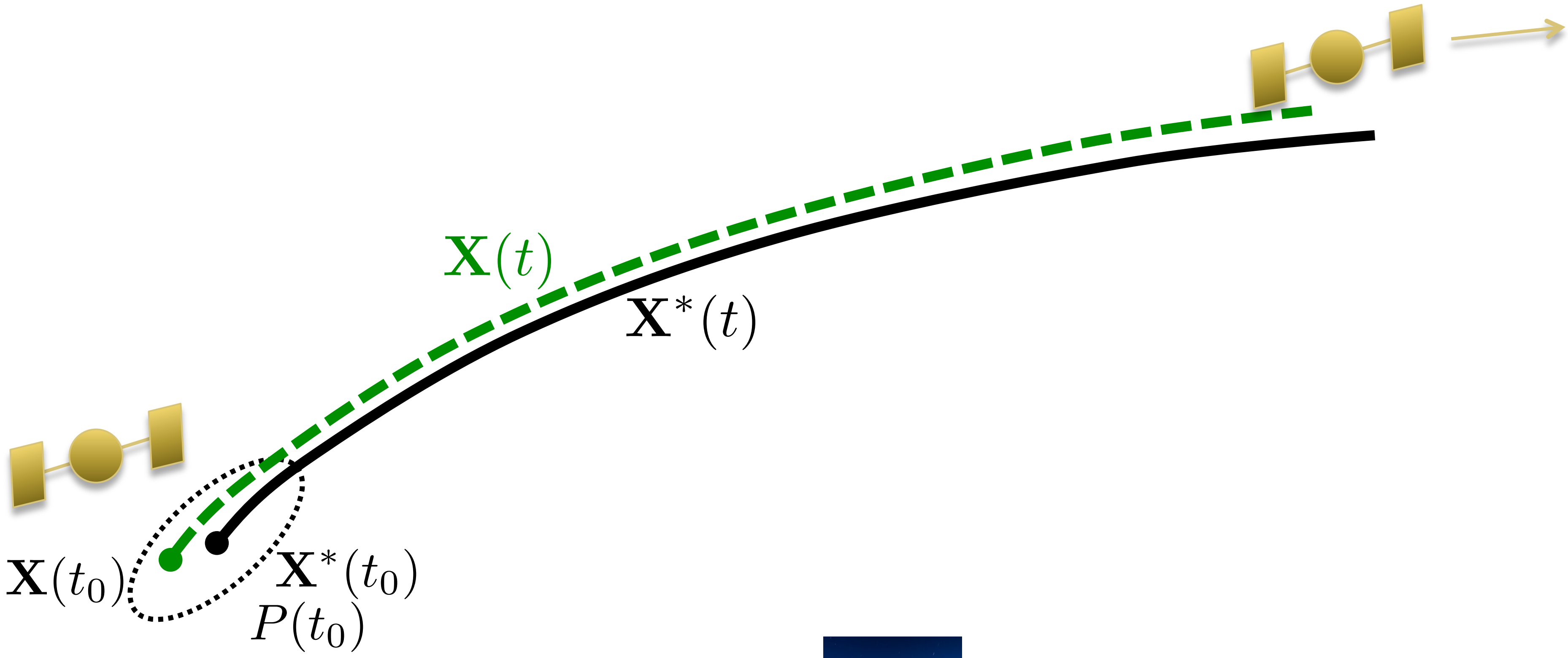
- Goal: Determine how to modify $\mathbf{X}^*(t)$ to match $\mathbf{X}(t)$



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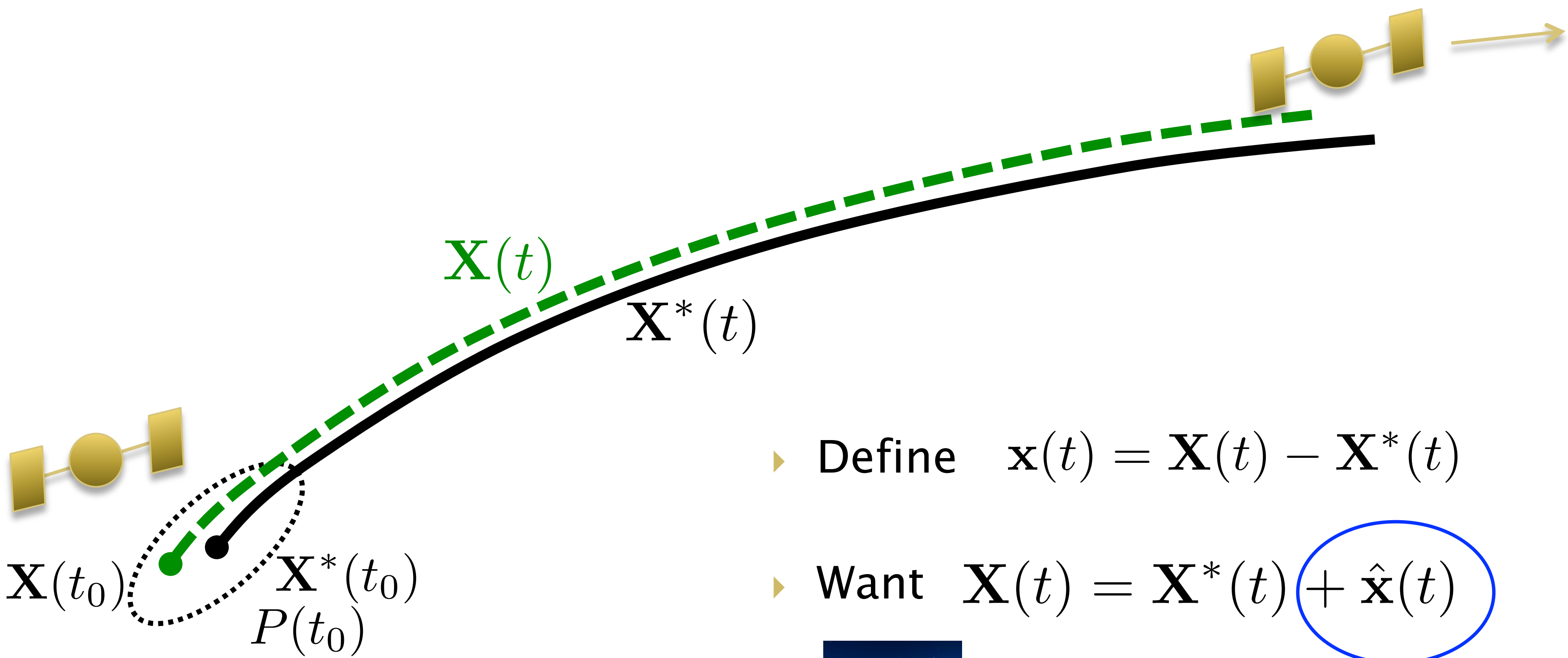

- Define $\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}^*(t)$
- Want $\mathbf{X}(t) = \mathbf{X}^*(t) + \hat{\mathbf{x}}(t)$

Review of the Stat OD Process

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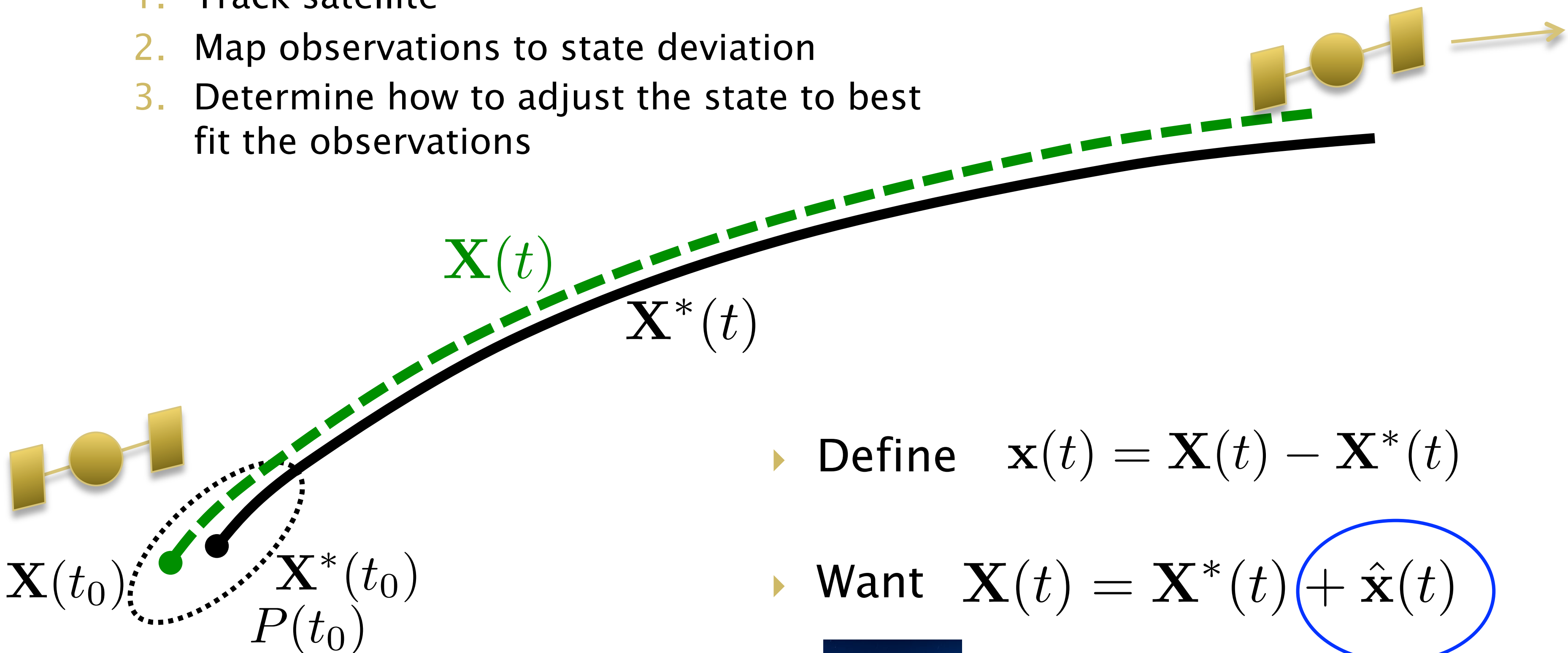
- Process:
  1. Track satellite
  2. Map observations to state deviation
  3. Determine how to adjust the state to best fit the observations



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Review of the Stat OD Process

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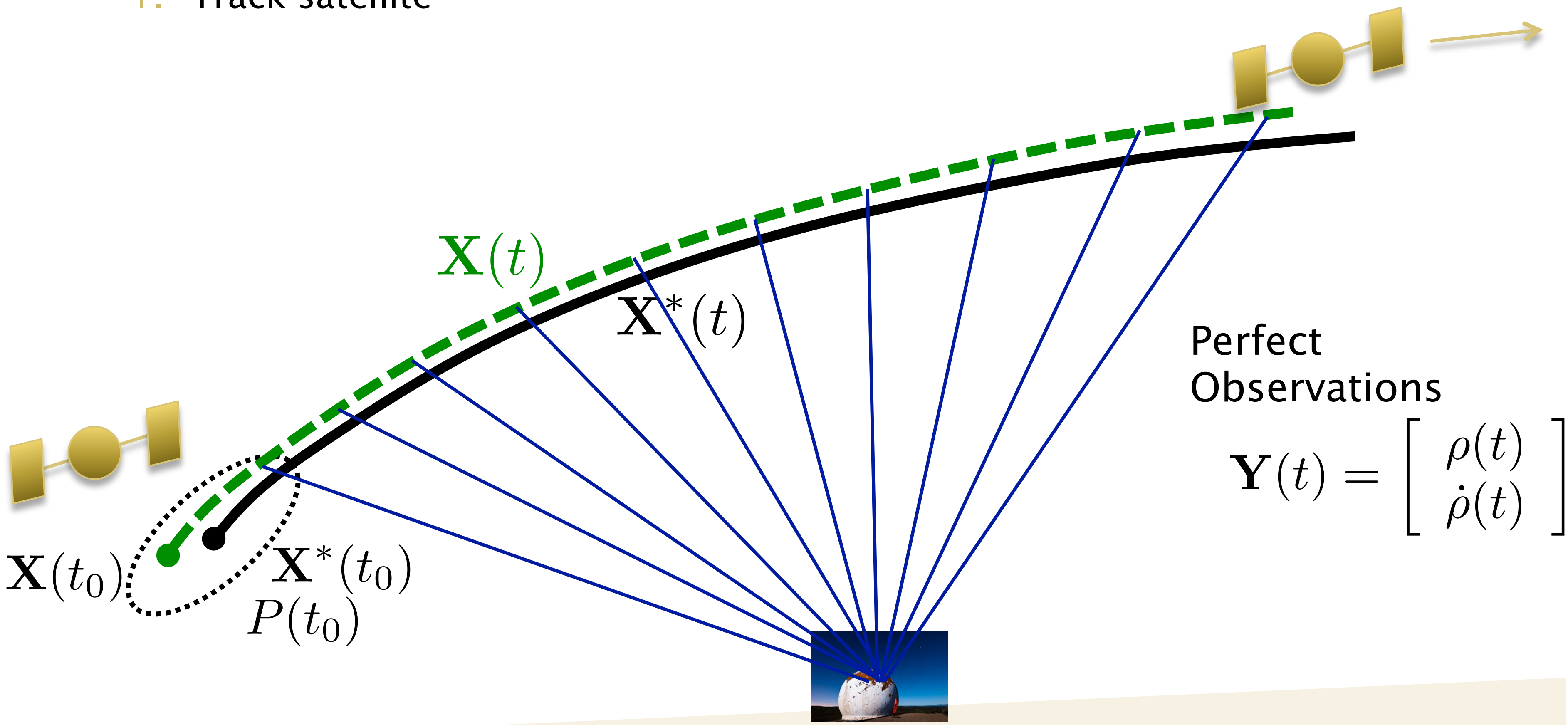

Review of the Stat OD Process

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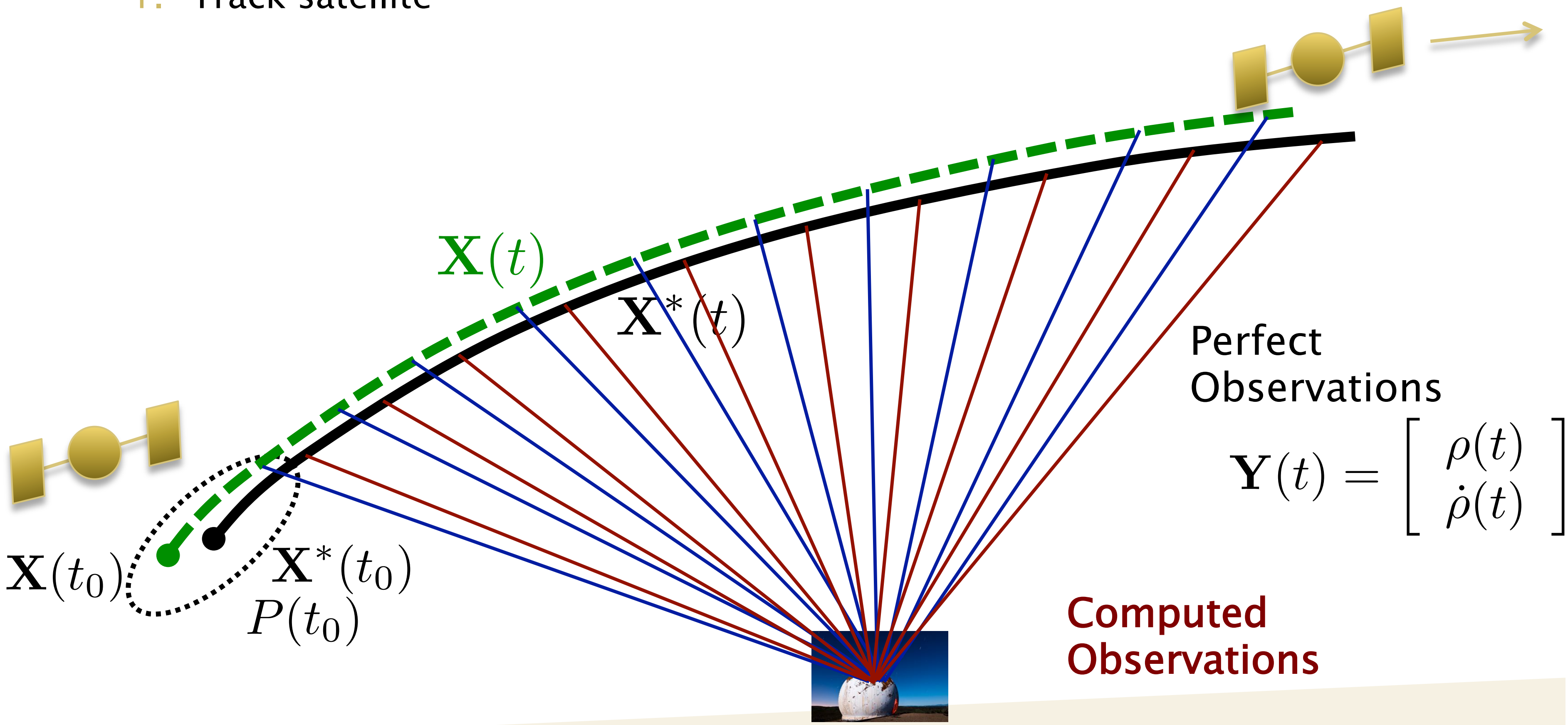

Review of the Stat OD Process

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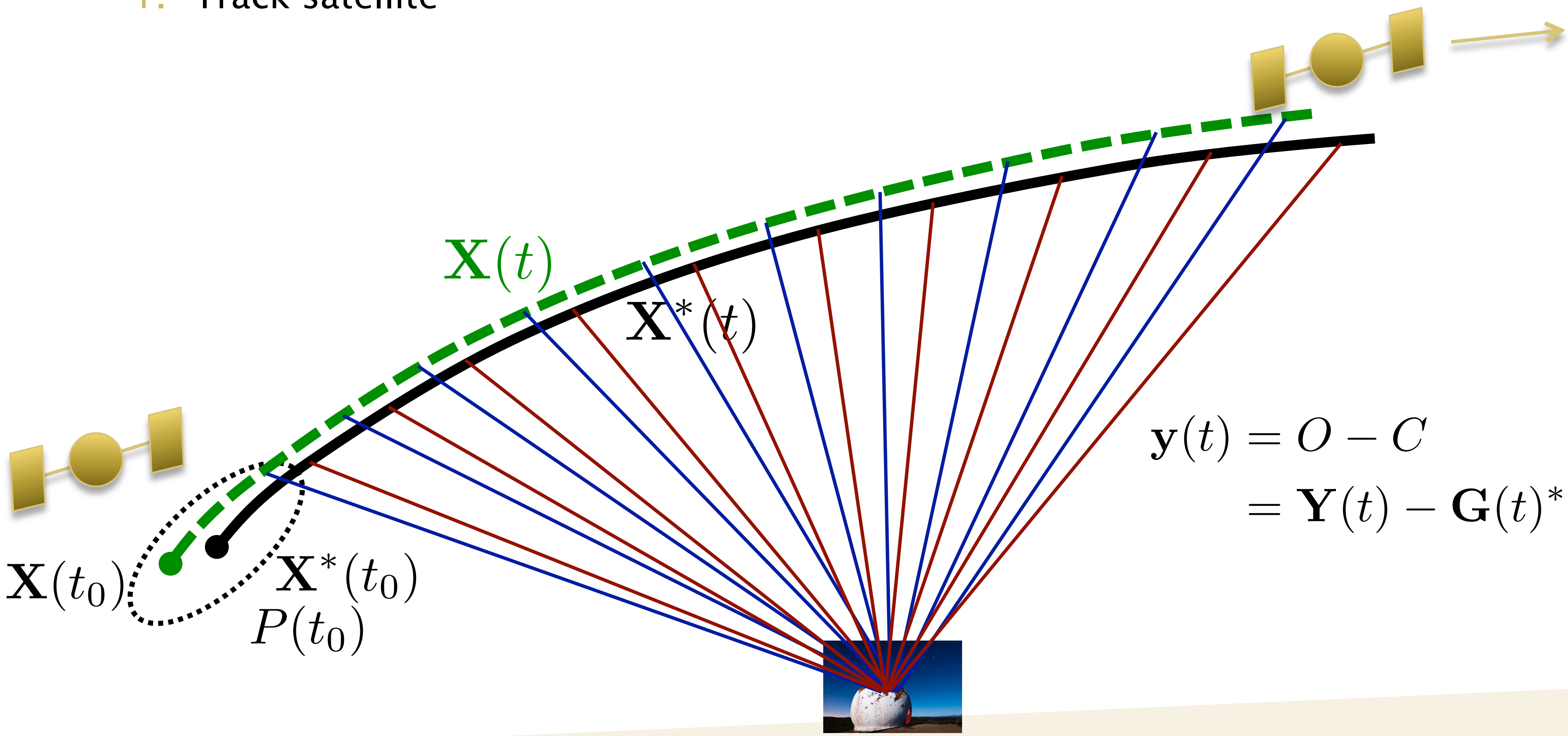
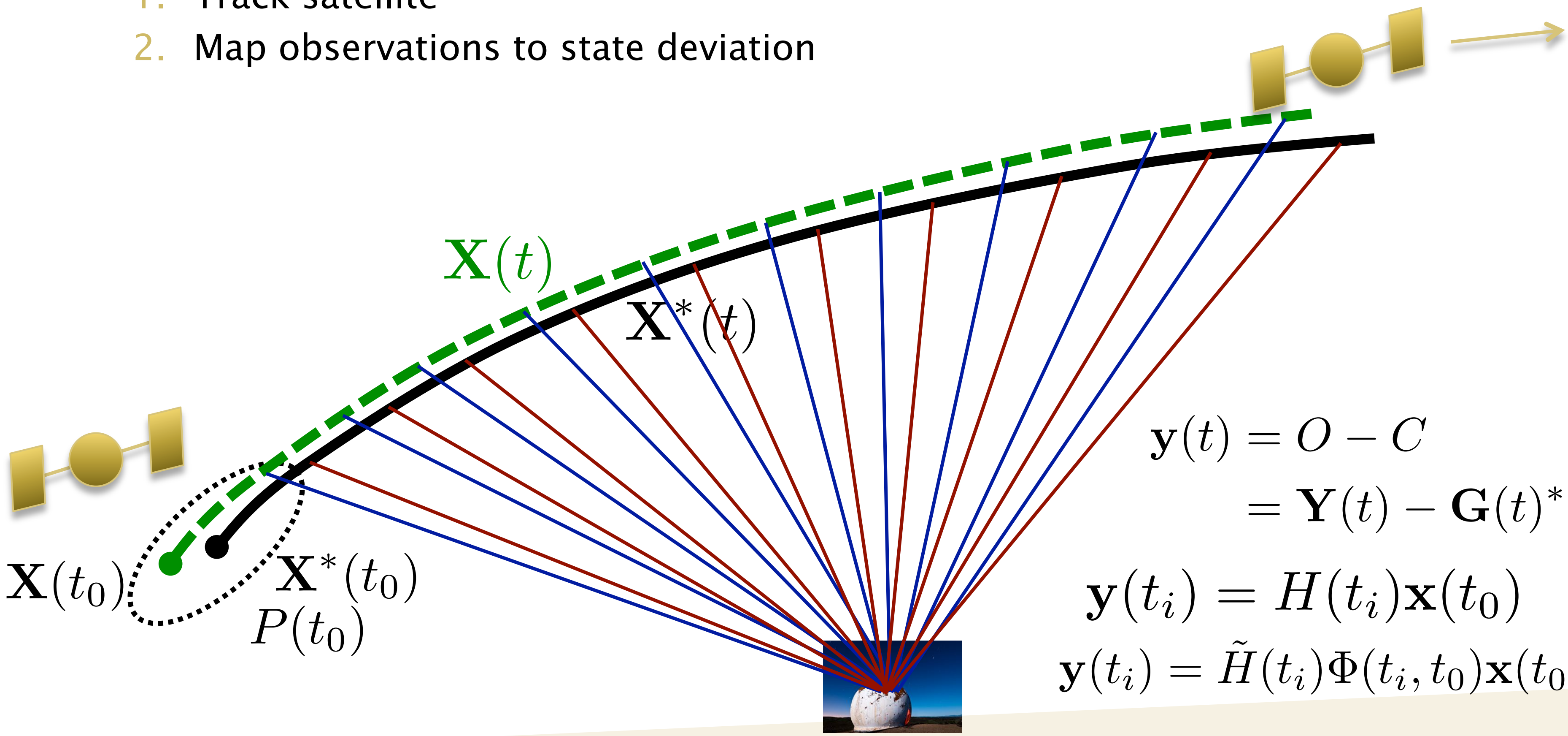


$$
\begin{aligned}
\mathbf{y}(t) &= O - C \\
&= \mathbf{Y}(t) - \mathbf{G}(t)^*
\end{aligned}
$$

Review of the Stat OD Process

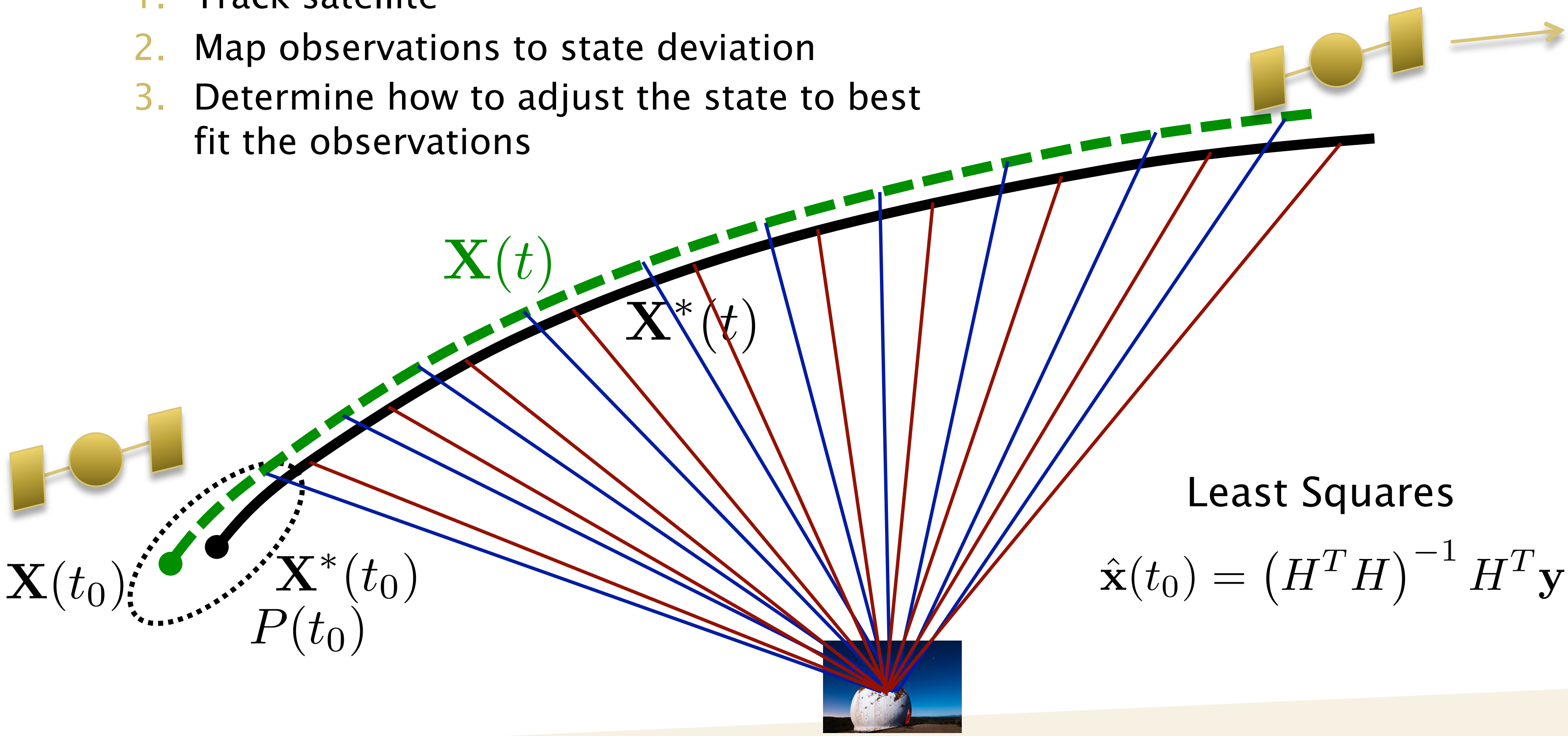
- Process:
  1. Track satellite
  2. Map observations to state deviation



$$\mathbf{y}(t) = O - C$$
$$= \mathbf{Y}(t) - \mathbf{G}(t)^*$$
$$\mathbf{y}(t_i) = H(t_i)\mathbf{x}(t_0)$$
$$\mathbf{y}(t_i) = \tilde{H}(t_i)\Phi(t_i, t_0)\mathbf{x}(t_0)$$

Review of the Stat OD Process

- Process:
  1. Track satellite
  2. Map observations to state deviation
  3. Determine how to adjust the state to best fit the observations



Least Squares

$$\hat{\mathbf{x}}(t_0) = \left(H^T H\right)^{-1} H^T \mathbf{y}$$

Review of the Stat OD Process

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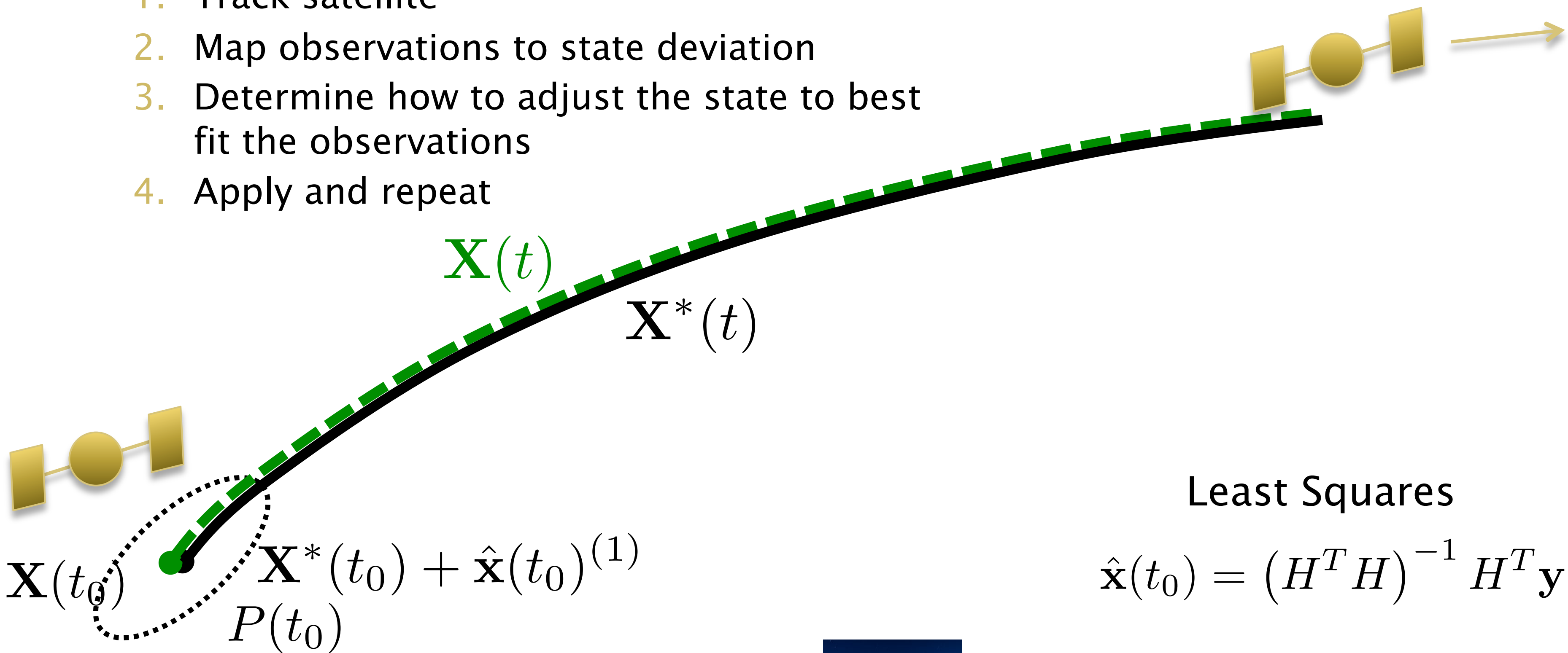
- Process:
  1. Track satellite
  2. Map observations to state deviation
  3. Determine how to adjust the state to best fit the observations
  4. Apply and repeat



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Review of the Stat OD Process

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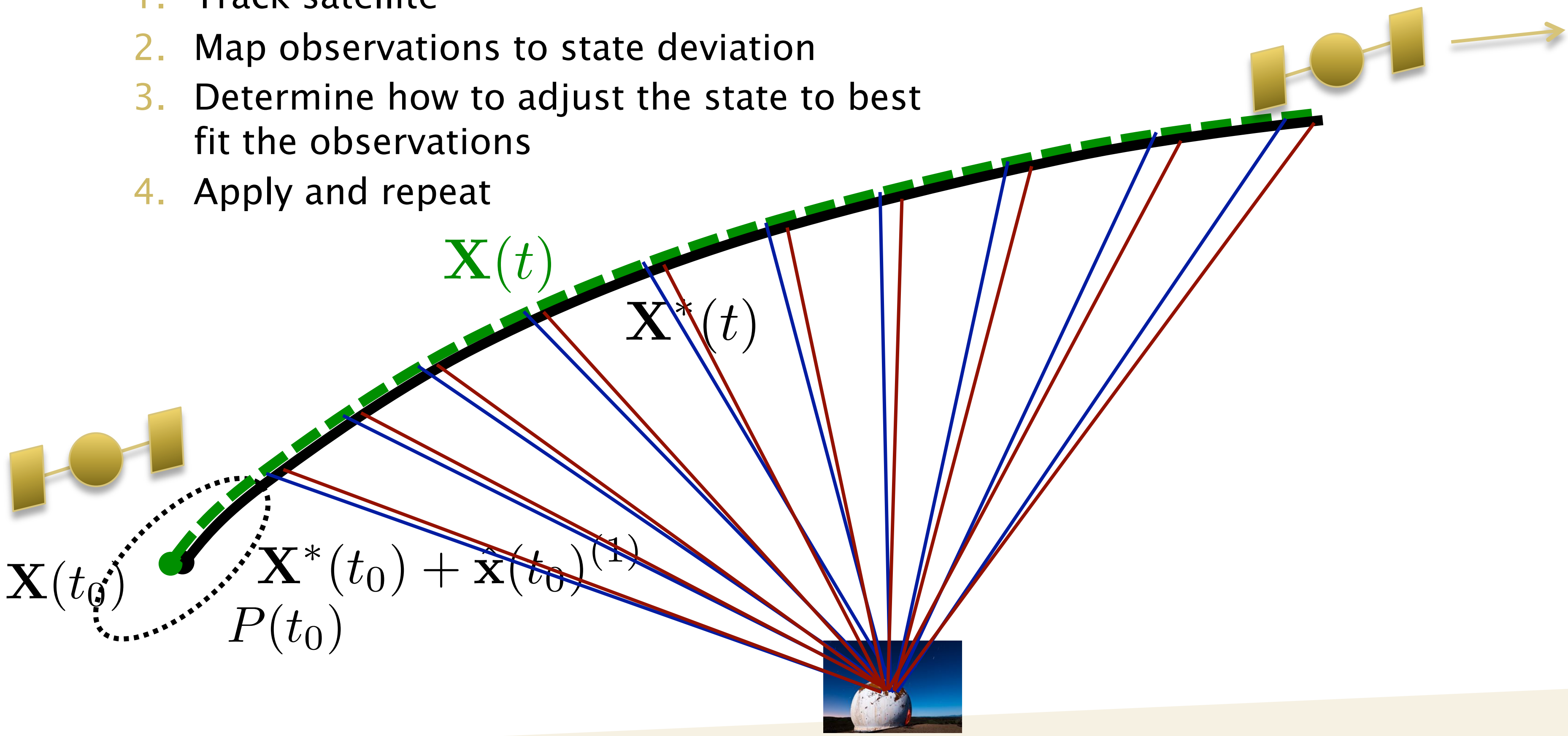

Review of the Stat OD Process

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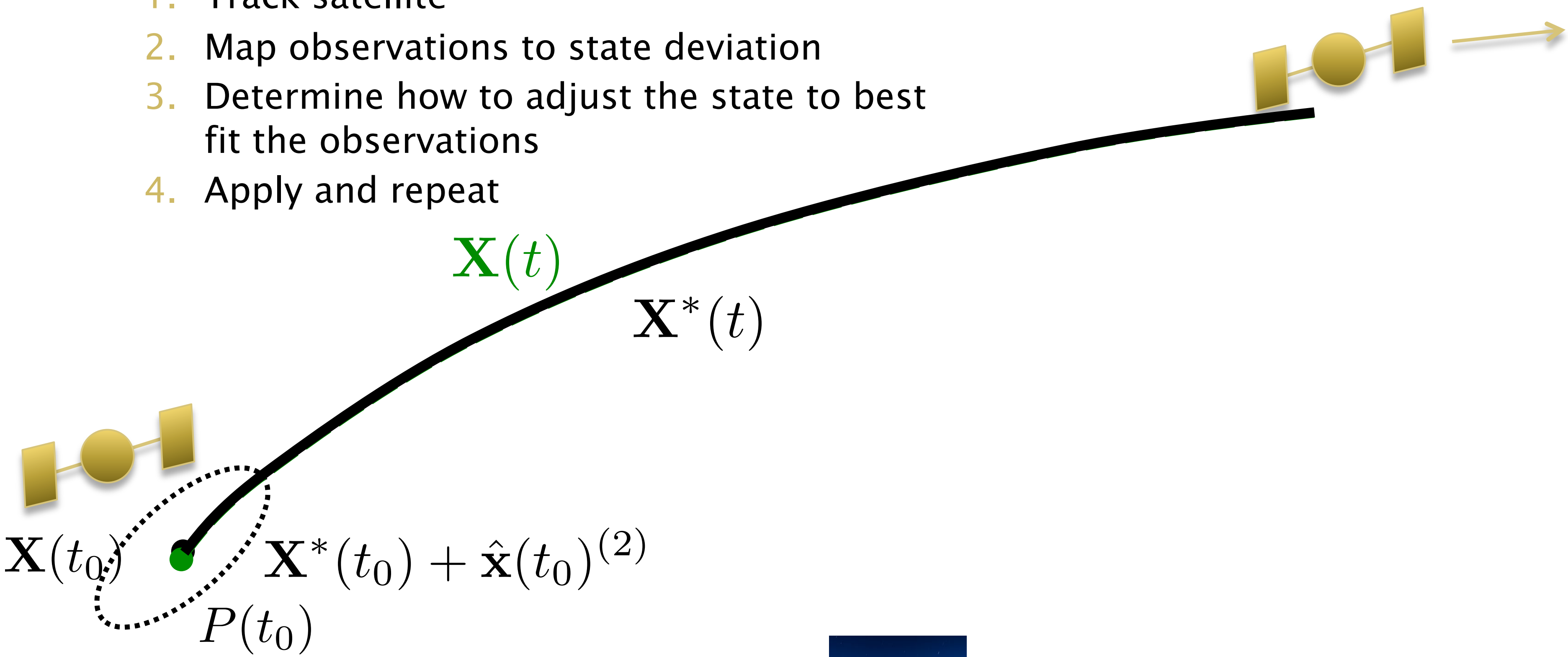

Review of the Stat OD Process

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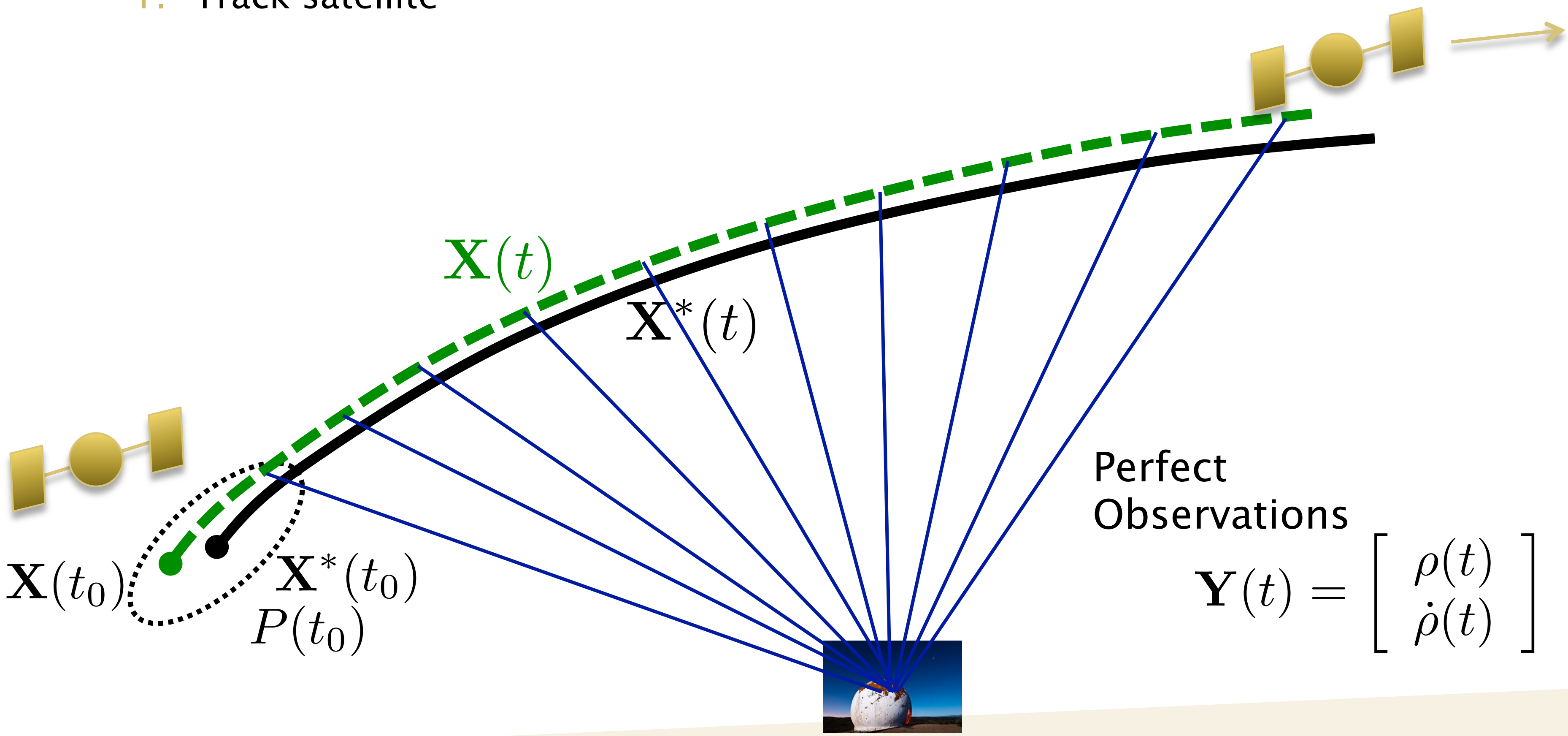
- Process:
  1. Track satellite



Review of the Stat OD Process

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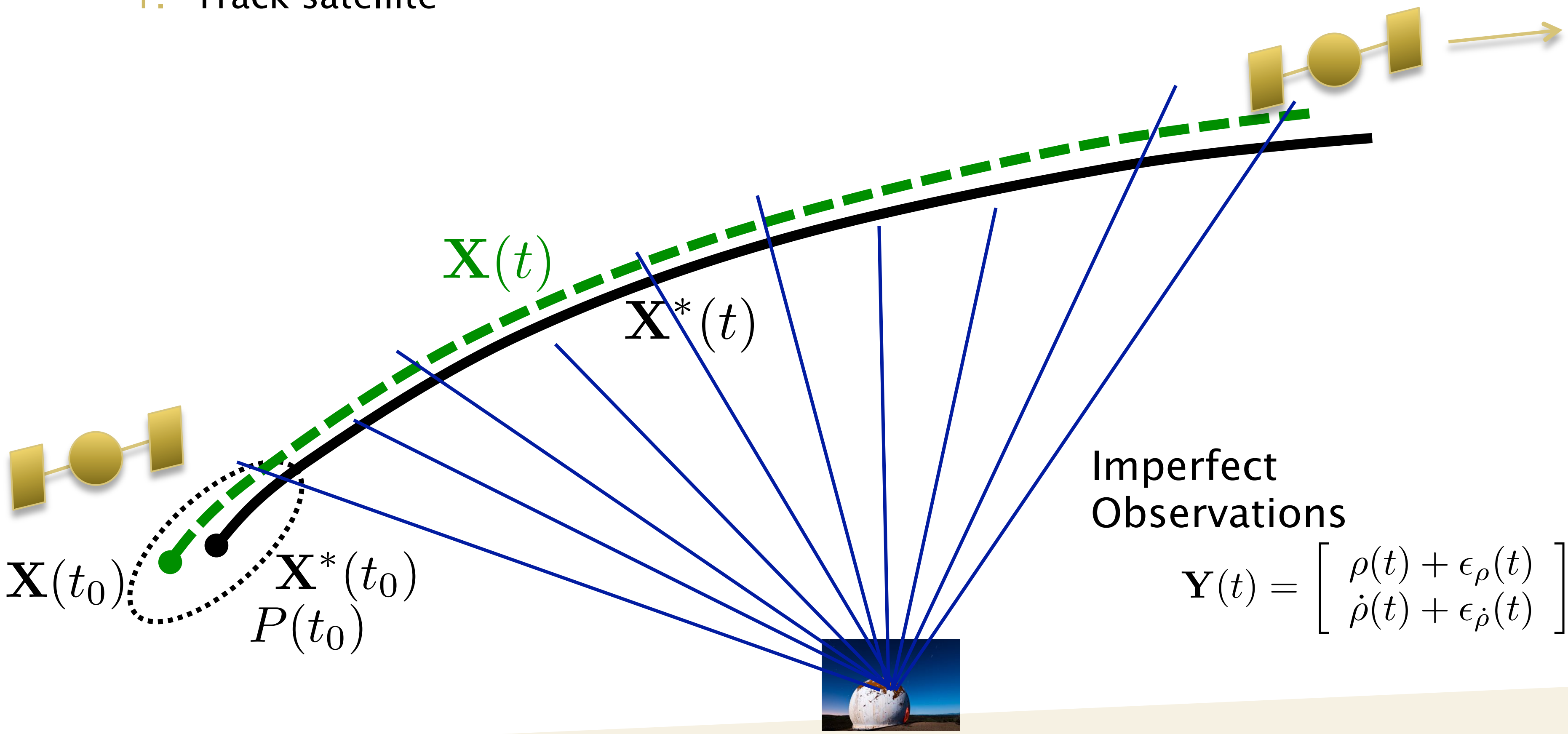

Review of the Stat OD Process

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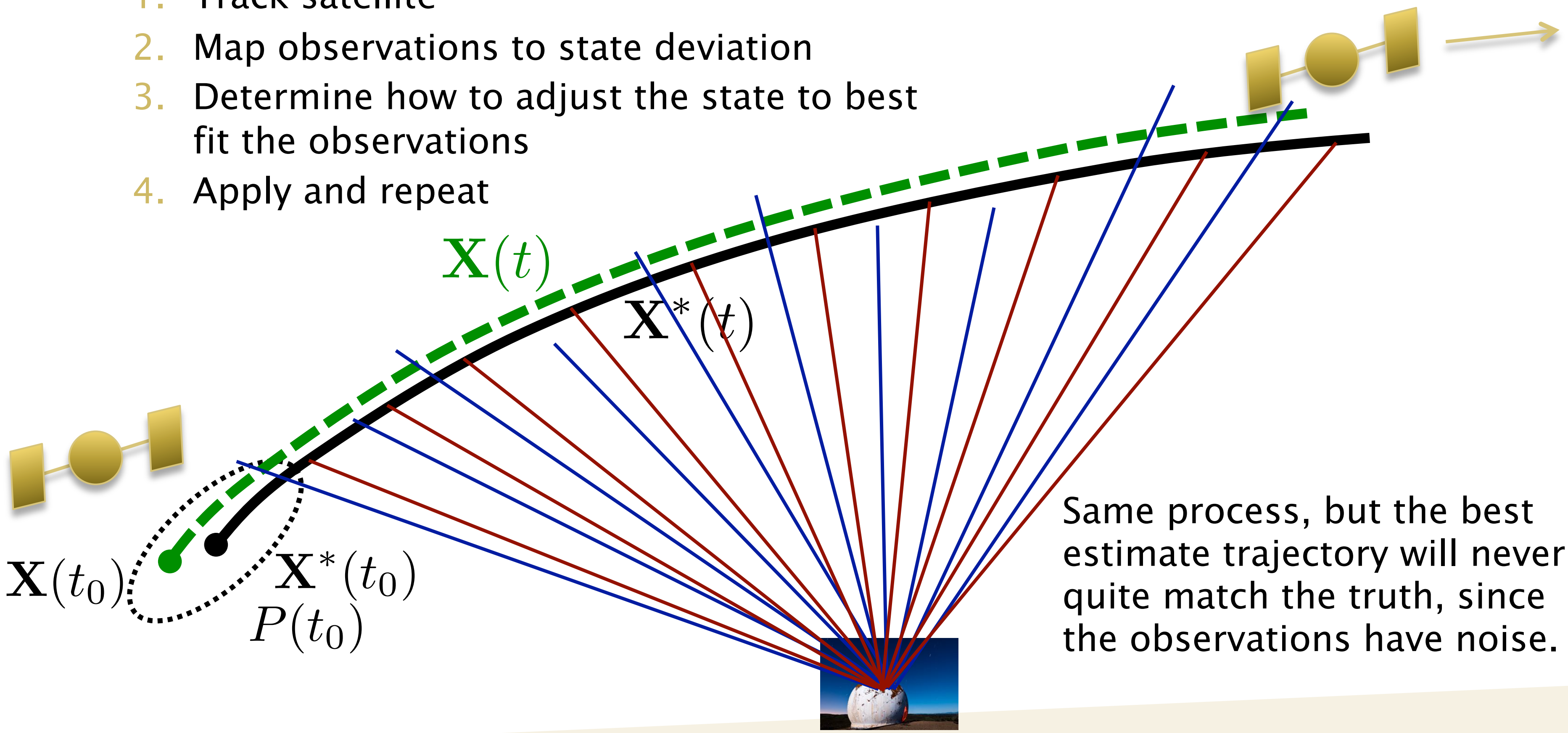

Same process, but the best estimate trajectory will never quite match the truth, since the observations have noise.

Review of the Stat OD Process

- Process:
  1. Track satellite
  2. Map observations to state deviation
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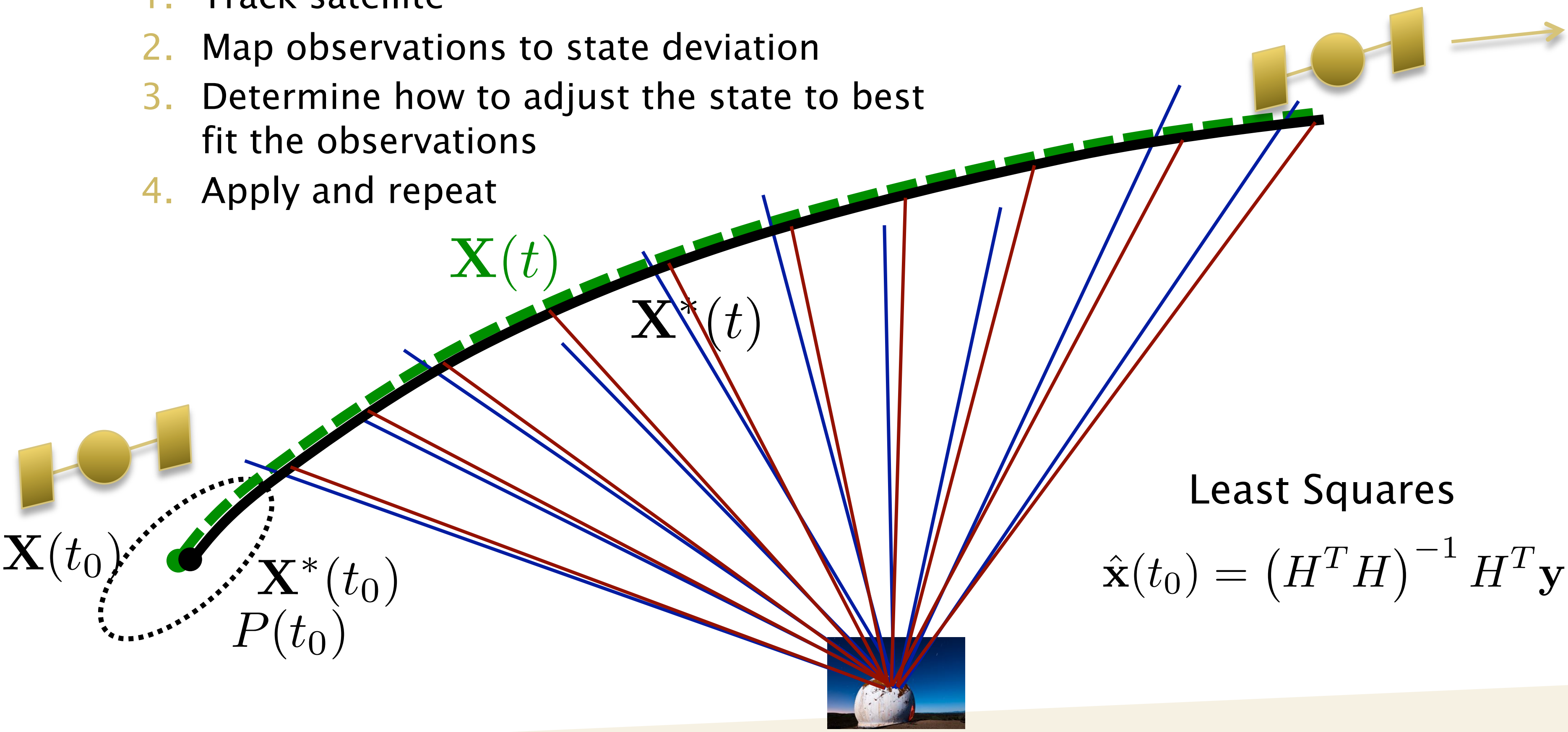
  4. Apply and repeat



Least Squares

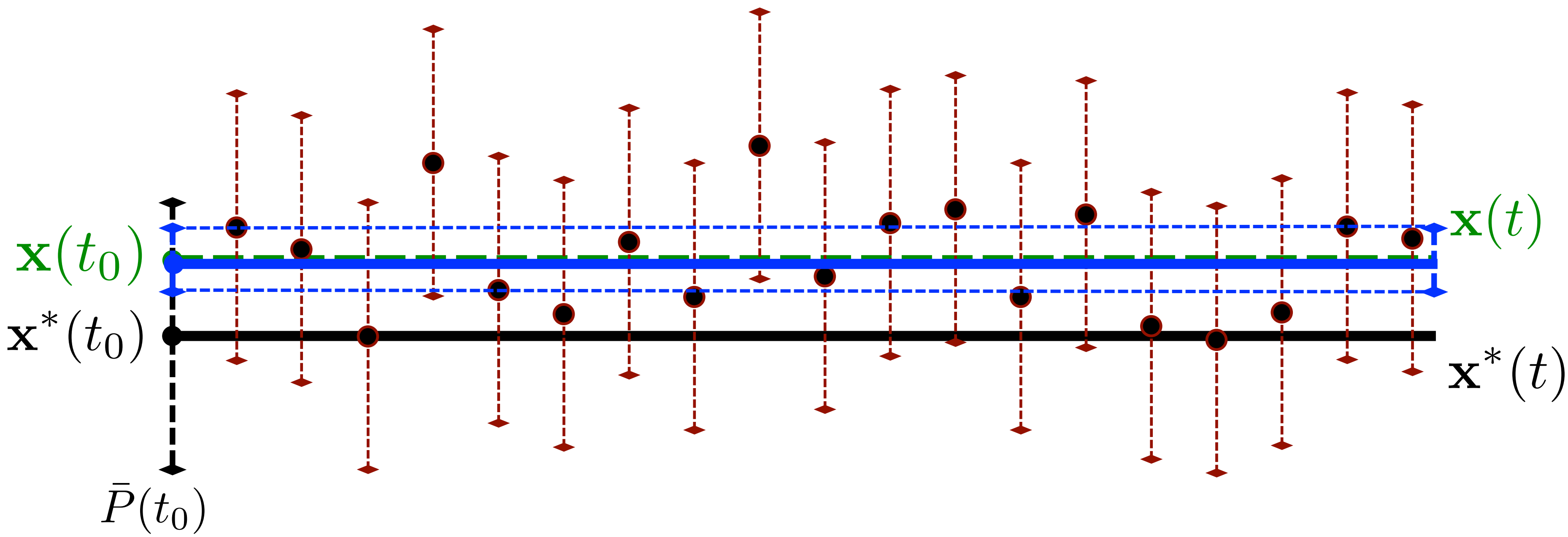
$$\hat{\mathbf{x}}(t_0) = \left(H^T H\right)^{-1} H^T \mathbf{y}$$

Filter Options

- Batch
  - Using any of the Least-Squares derivations
- Sequential
  - CKF
  - EKF
  - UKF (others)

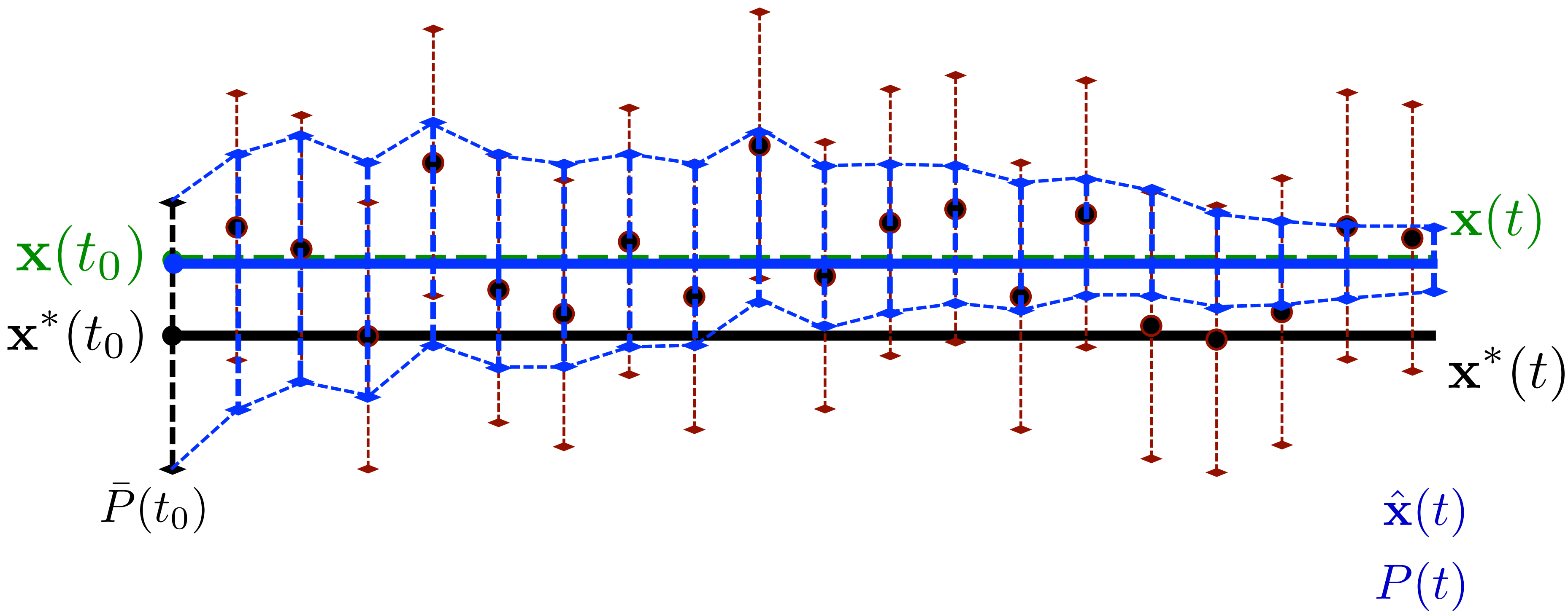
Stat OD Conceptualization

- Least Squares (Batch)



Stat OD Conceptualization

- Conventional Kalman



Stat OD Conceptualization

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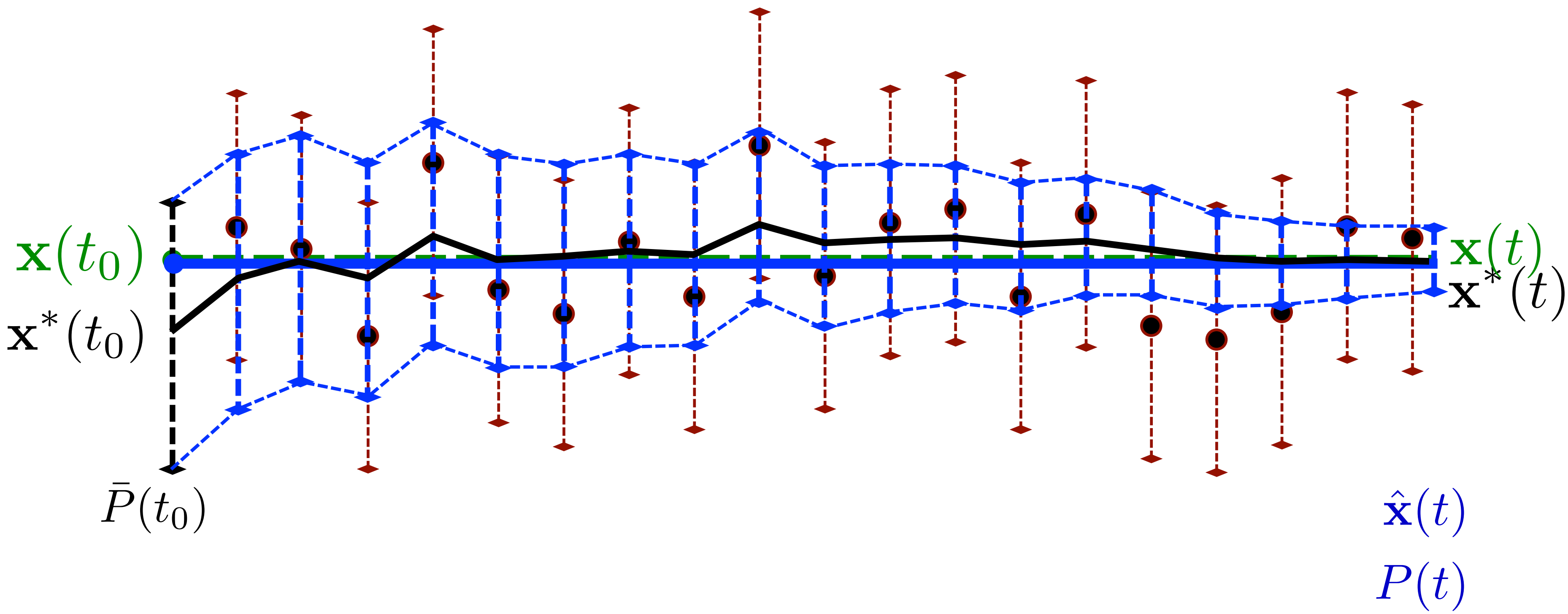
- EKF



Numerical Issues

- As we’ve seen, computers aren’t perfect and Stat OD is a very sensitive subject!

Joseph Formulation

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- Replace

$$P_k = \left(I - K_k \tilde{H}_k^T\right) \bar{P}_k$$

with

$$P_k = \left(I - K_k \tilde{H}_k^T\right) \bar{P}_k \left(I - K_k \tilde{H}_k^T\right)^T + K_k R_k K_k^T$$

- This formulation will always retain a symmetric matrix, but it may still lose positive definiteness.

Square Root Filter Algorithms

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- Define *W*, the square root of P:

$$WW^T = P$$

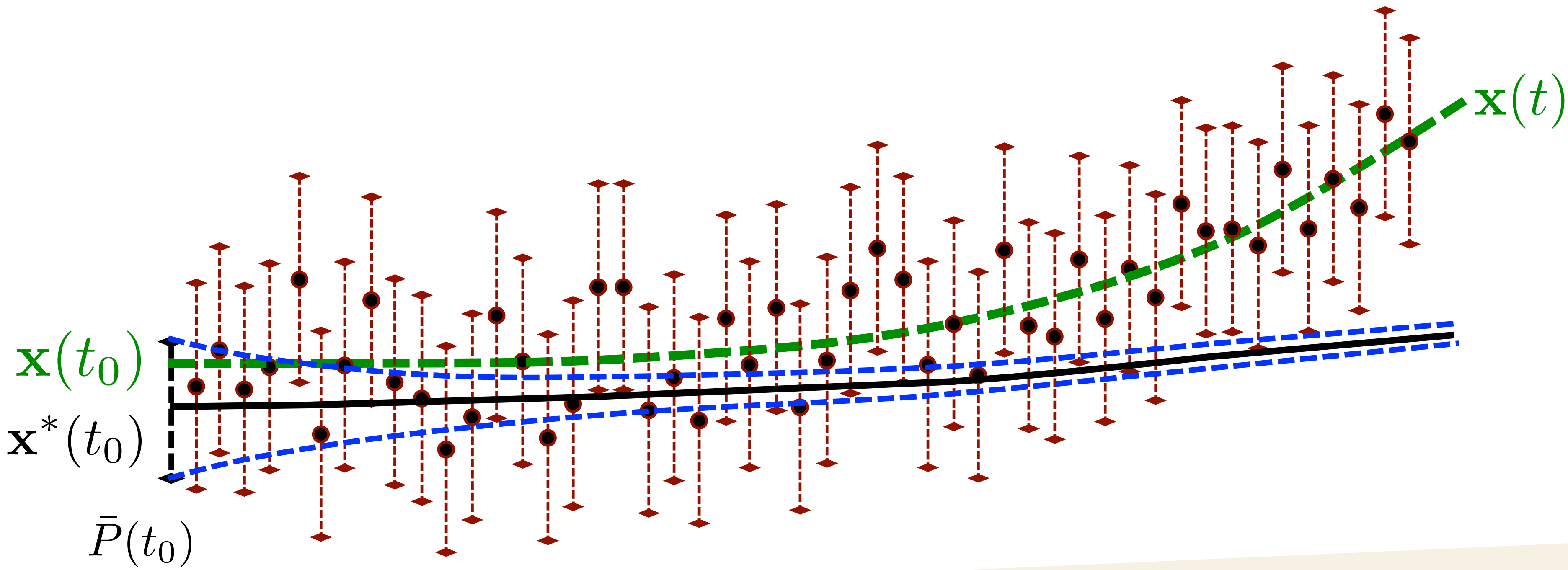
- Observe that if we have *W*, then computing *P* in this manner will always result in a symmetric PD matrix.

- Perform measurement updates and time updates on *W* rather than *P*.
  - Often process one observation at a time (rather than a vector of observations).

- *W* may be constructed via Givens, Householder, Cholesky, or other methods.

Filter Saturation

- Applying the sequential algorithm to a large amount of data will cause the covariance to shrink down too far.



Filter Saturation

- We need a way to add expected process noise to our filter.

- Many ways to do that:
  - State Noise Compensation
    - Constant noise
    - Piecewise constant noise
    - Correlated noise
    - White noise
  - Dynamic Model Compensation
    - 1^{st} order linear stochastic noise
    - More complex model compensations

Process Noise Time Update

- New time update:
- Old: $\bar{P}_k = \Phi(t_k, t_{k-1}) P_{k-1} \Phi^T(t_k, t_{k-1})$
- New Time Update: $\bar{P}_k = \Phi_{k,k-1} P_{k-1} \Phi^T_{k,k-1} + Q_\eta(t_k)$
- General: $\bar{P}_k = \Phi_{k,k-1} P_{k-1} \Phi^T_{k,k-1} + \Gamma_{k,k-1} Q_k \Gamma^T_{k,k-1}$

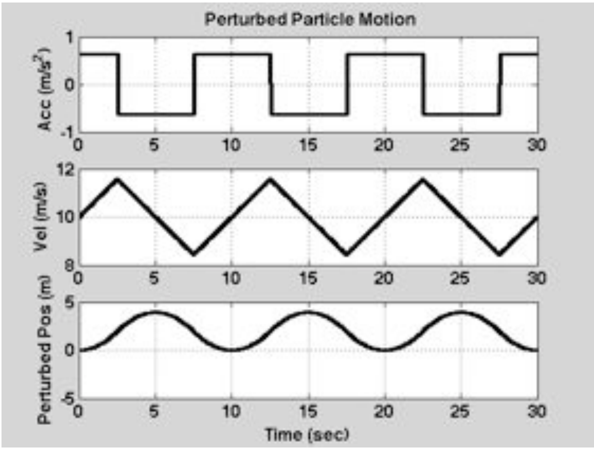
Process Noise

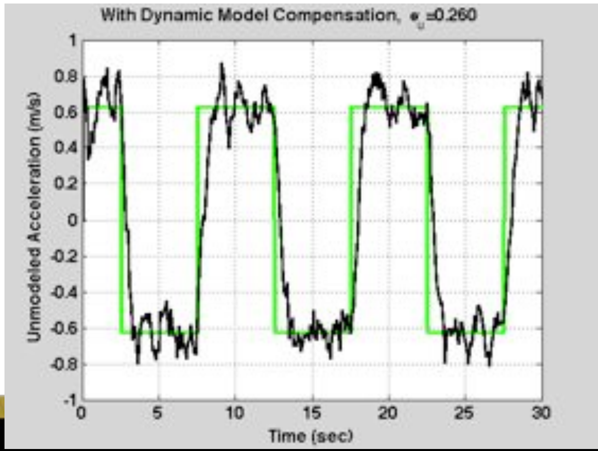
- State Noise Compensation usually compensates for unmodeled dynamics by assuming it is some white noise process.

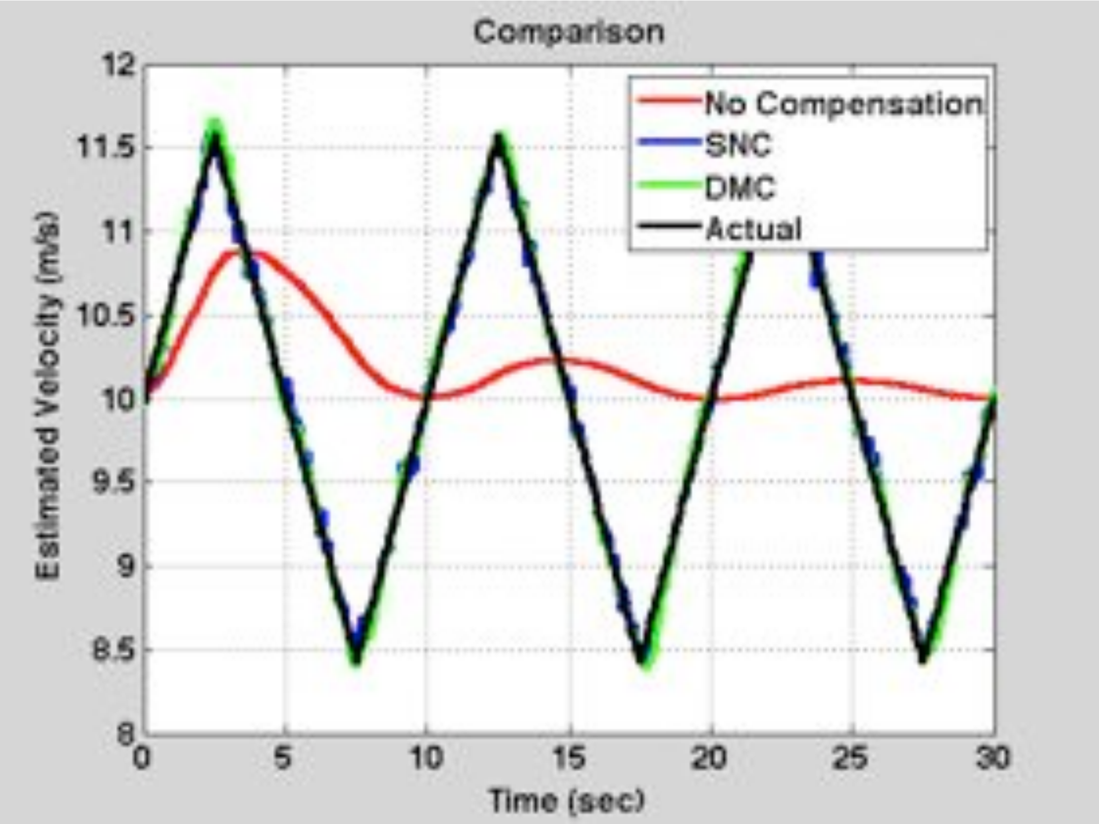
- Dynamic Model Compensation also estimates what the unmodeled acceleration is.
  - Adds a term or two to the state.

Example SNC / DMC

- Square Wave unmodeled acceleration







Tuning the Filter

- To tune the filter:
- Practice on many simulated runs.
- Aim to tune the filter to add the least amount of inflation to the P-matrix while reducing the residuals to near-noise.
- Check the trace of the P-matrix. Is it too large?
- Do you know anything about the uncertainty in the model? Perhaps you can orient the Q matrix to accommodate for expected uncertainties (drag, SRP, maneuver errors, etc)

Smoothing

- Estimate using process noise

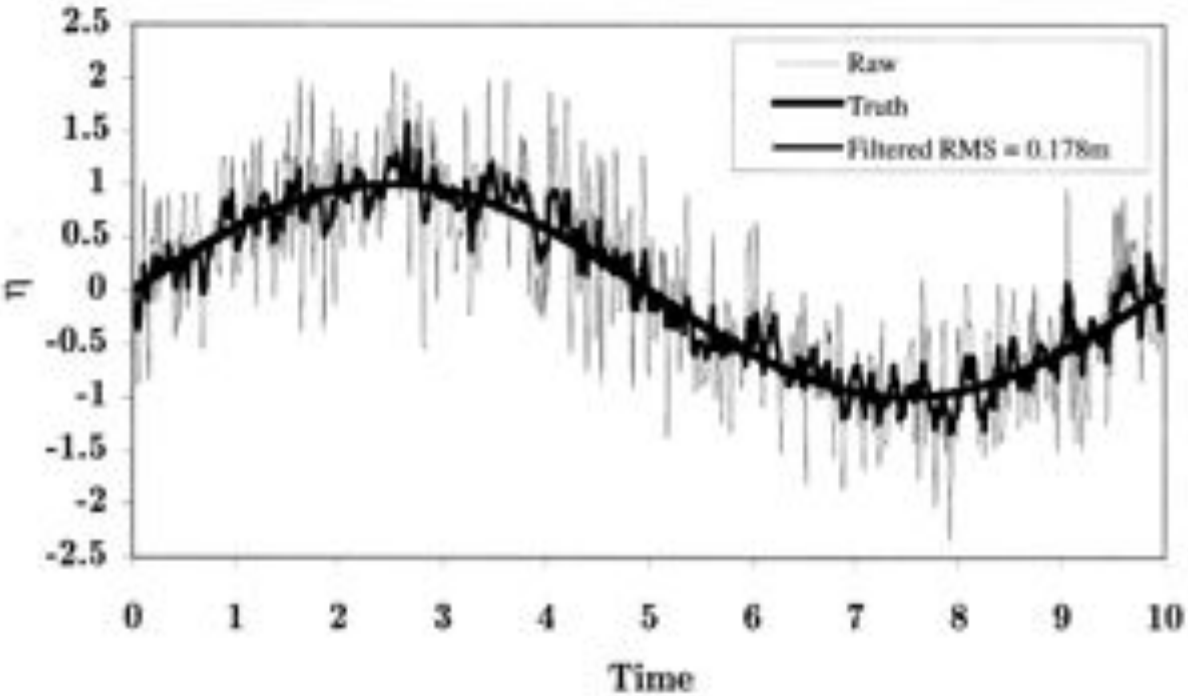


Figure 4.19.1: Process noise/sine wave recovery showing truth, raw data (truth plus noise) and the filtered solution. $\eta_0 = 0$, $\sigma = 2.49$, $\beta = .045$.

Smoothing

- Smoothed estimate

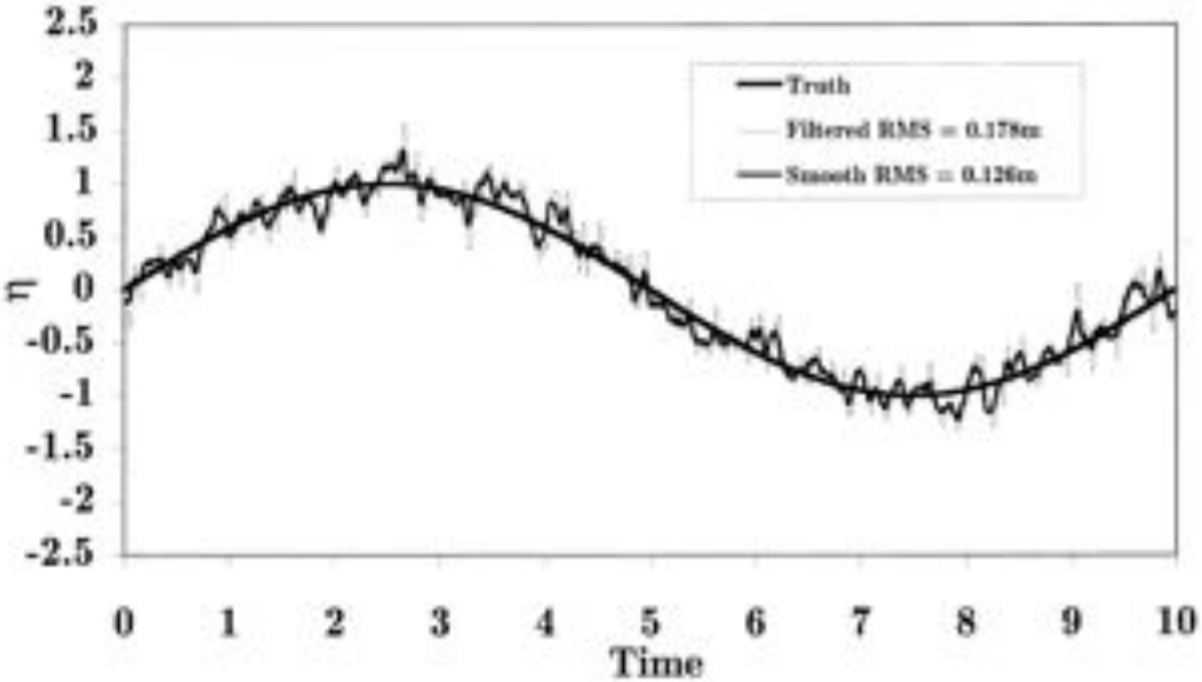


Figure 4.19.2: Process noise/sine wave recovery showing the truth, the filtered, and the smoothed solution. $\bar{\eta}_0 = 0$, $\sigma = 2.49$, $\beta = .045$.

Using Information

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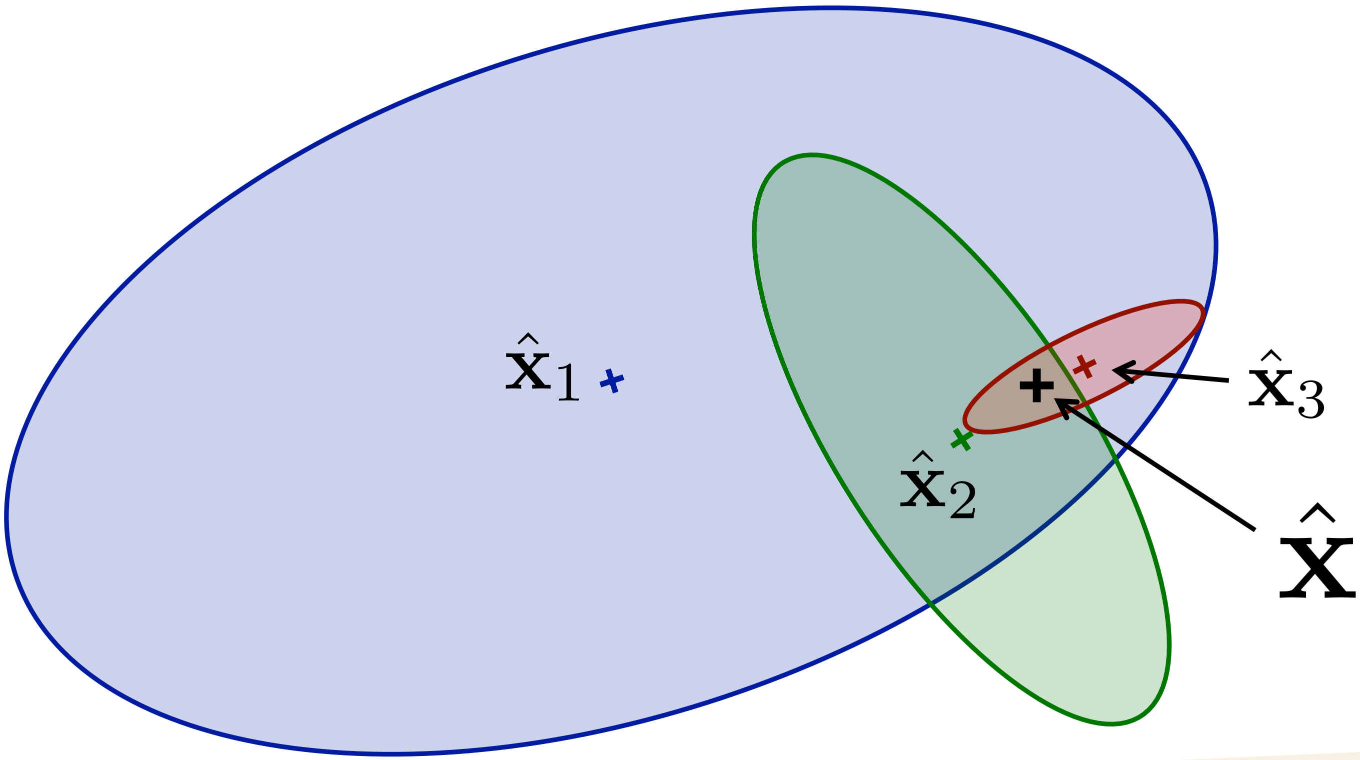
- We may be interested in comparing solutions that were generated using a variety of sources.
  - Not all of these solutions will be independent!

- If you have multiple solutions that *are* independent and uncorrelated, then we can combine them to make a *best* estimate.

$$\hat{\mathbf{x}} = \left(\sum_{i=1}^{n} P_i^{-1} \right)^{-1} \sum_{i=1}^{n} P_i^{-1} \hat{\mathbf{x}}_i$$

Combining Estimates

- Where would the optimal estimate lie given these estimates and associated errors covariances?



Covariance → Probability Ellipsoid

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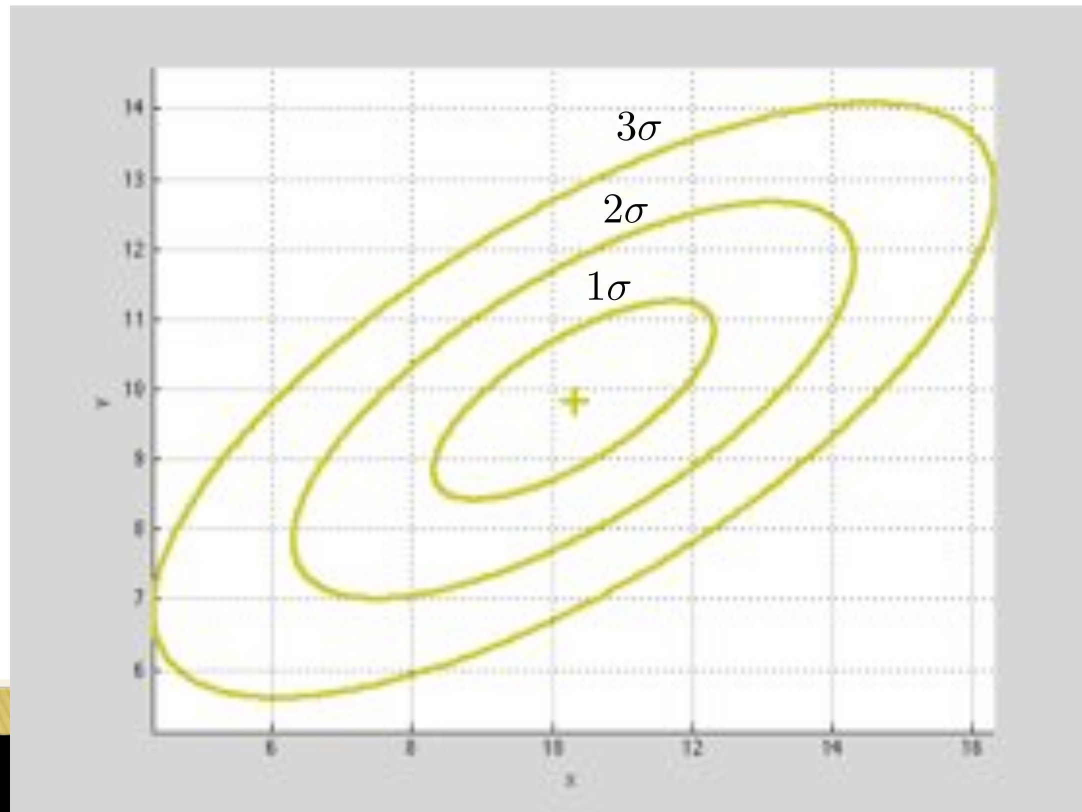
- Demonstrated how to convert a covariance matrix into a probability ellipsoid.
  - Plot it as an ellipse onto your design space
  - E.g., B-Plane

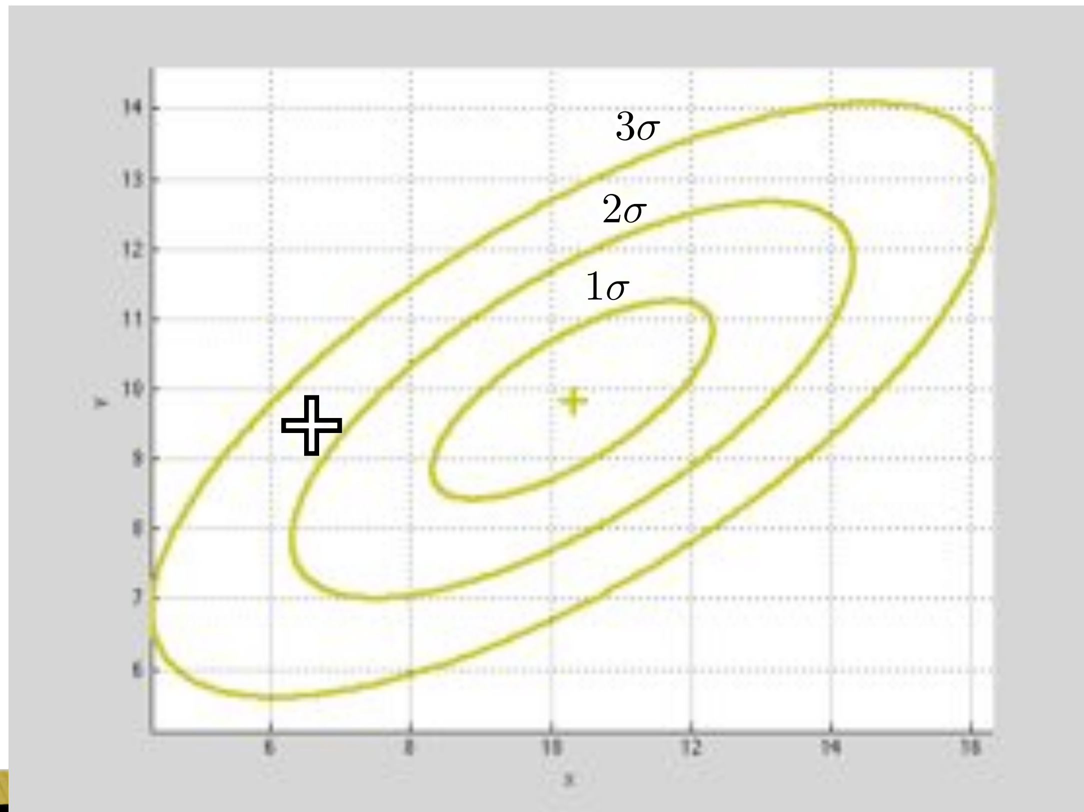
- Example:

$$P = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$



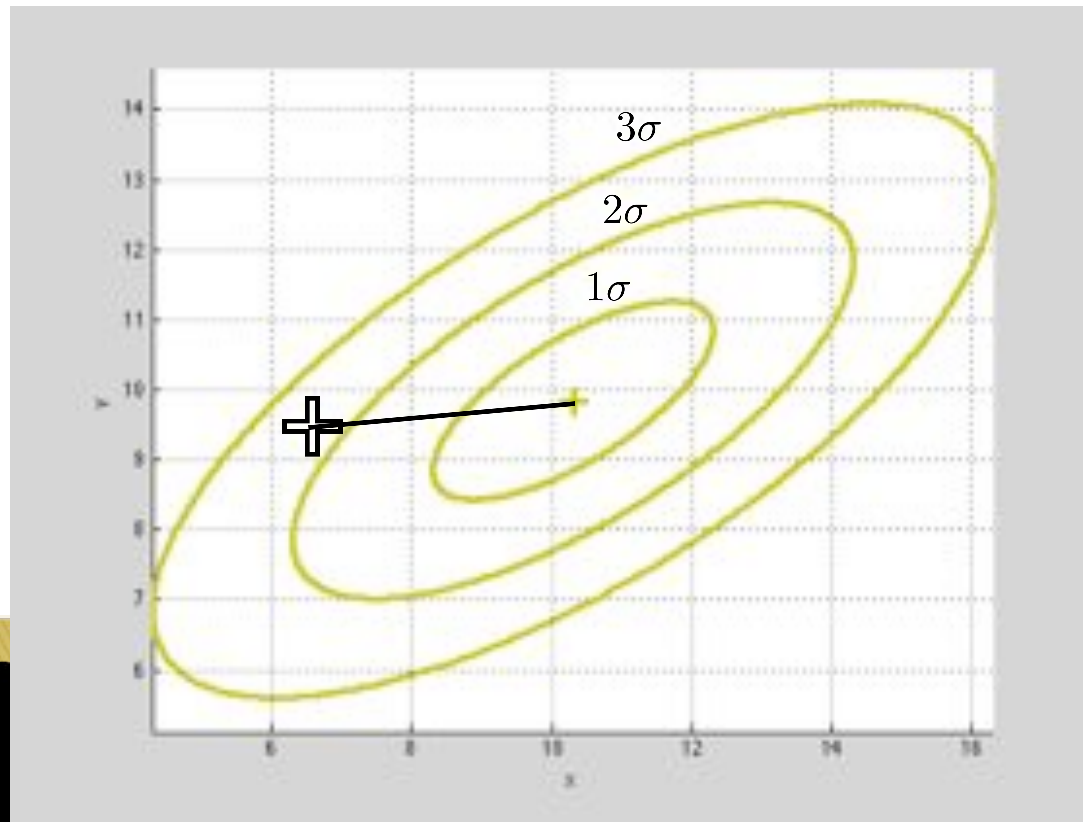
Covariance → Probability Ellipsoid

- Question: how many sigmas is this example point away from the mean?



Covariance → Probability Ellipsoid

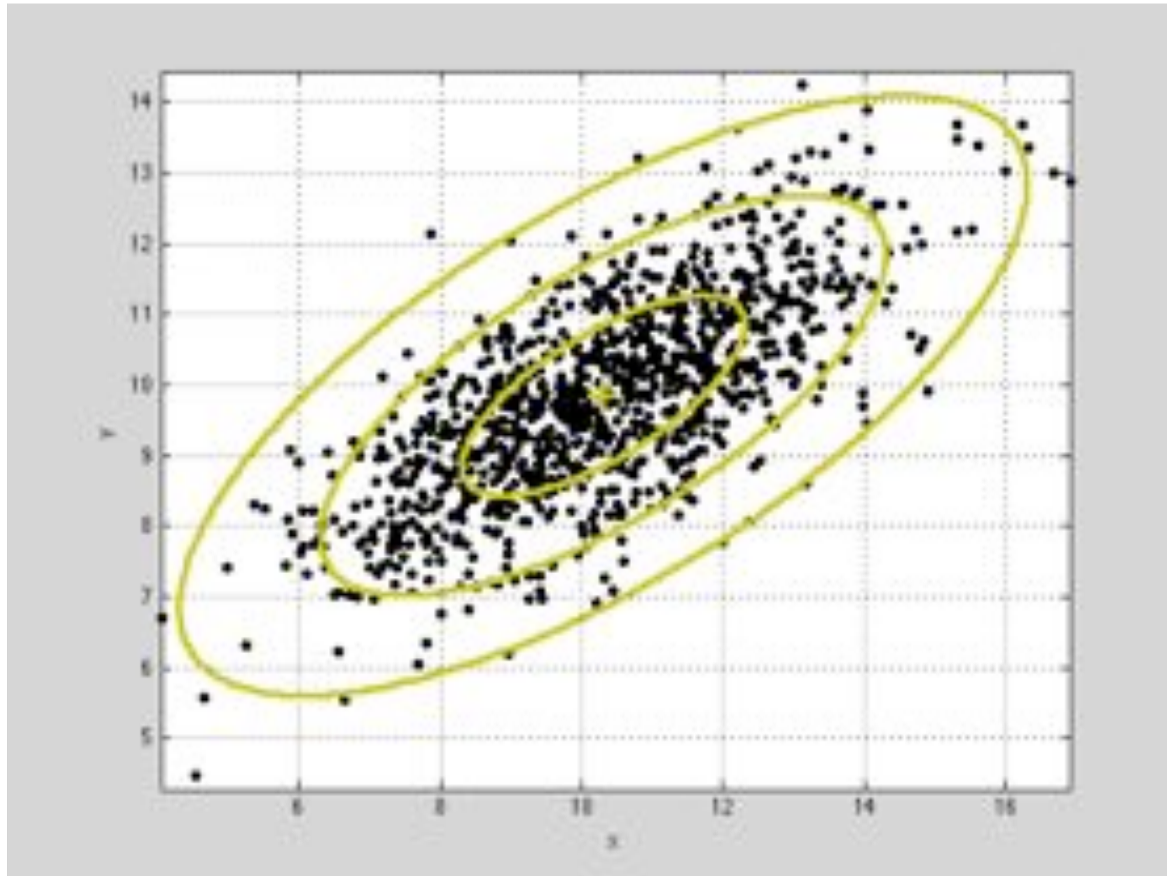
- Answer: ~2.4
- Rotate the point into the principal axes, then scale its new x' and y' values by the semi-axes and compute the distance to the mean.
- Nsigma = sqrt($(x'/a)^2 + (y'/b)^2$)



Monte Carlo

- Use Monte Carlo to sample a covariance.

  - S = chol(P)
  - for i=1:1000
    - e = randn(n,1)
    - x = S'*e
  - end



Spaceflight Operations

- I wanted to finish Stat OD by talking about Spaceflight operations in practice.
- Interplanetary and Earth orbiting

Flight Operations

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- Good practices in spaceflight navigation
  - Practice! Tune your filter on all kinds of data. Be ready for a wide variety of data.
  - Perform lots of solutions and compare them.
    - Solutions over different arcs of time, long and short
    - Solutions with different data types
    - Solutions with different parameters, consider parameters, process noise, etc.
  - Present the information in a manager-friendly format, but understand the details.
    - B-Plane, error ellipses, covariance, residuals
  - Look at that covariance in different ways.
  - Don't neglect the numerical details.

Missions that require navigation

- All of them.

Spacecraft missions

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- All spacecraft require OD
  - For communication

- Most spacecraft require OD for other reasons
  - For science
  - For mission planning
  - For maneuver design

The State Parameters

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- Typical spacecraft solutions include the following parameters:
  - Position of the spacecraft
  - Velocity of the spacecraft
  - Attitude of the spacecraft
    - At discrete moments, or a time history
  - Maneuver components
    - As complicated as needed to model the maneuvers well, including:
    - Right ascension and declination of thrust vector
    - Pointing time-series
    - Thrust time-series
    - Mass-flow time-series
  - Small forces
    - ACS, desats, venting, etc.
  - Solar Pressure: bias, stochastic
  - Range biases to each ground station, per pass
  - Gravity field of target body, as applicable
  - Miscellaneous parameters that describe any modeled acceleration on the spacecraft.

Only non-spacecraft parameters in a typical state!

Tracking Data

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- Tracking data may include many types of data – and often *should* include many types of data:
  - Ground observations:
    - Doppler
    - Range
    - 1-Way, 2-Way, 3-Way
    - Angles when very near the Earth
    - Delta-DOR when further from Earth
  - Relative to other spacecraft, vehicles, bodies
    - GPS
    - Autonav
    - LiAISON
    - Formation Flying
  - Spacecraft measurements:
    - Accelerations, including drag-free corrections, thrust, etc.
    - Measured mass-flow
    - Attitude measurements

Radial vs. Angular Measurements

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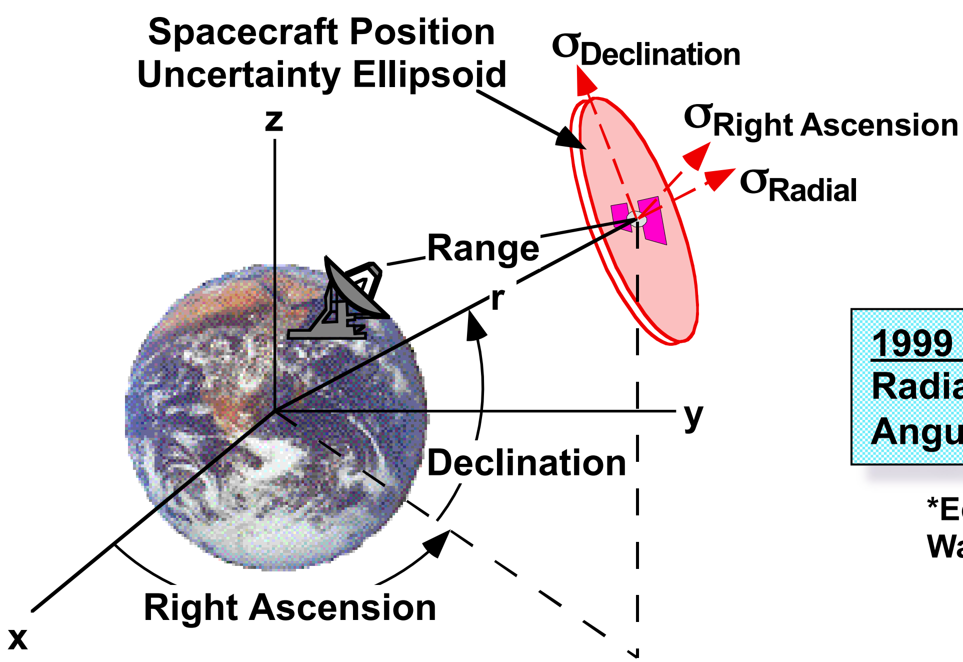
- For most interplanetary missions, S/C position uncertainty is much smaller in Earth-spacecraft (“radial”) direction than in any angular (“plane-of-sky”) direction



1999 Capability	Position	Velocity
Radial Error	2 m	0.1 mm/s
Angular Error (at 1 AU)	3 km*	0.1 m/s

*Equivalent to angle subtended by quarter atop Washington Monument as viewed from Chicago

Different Solutions

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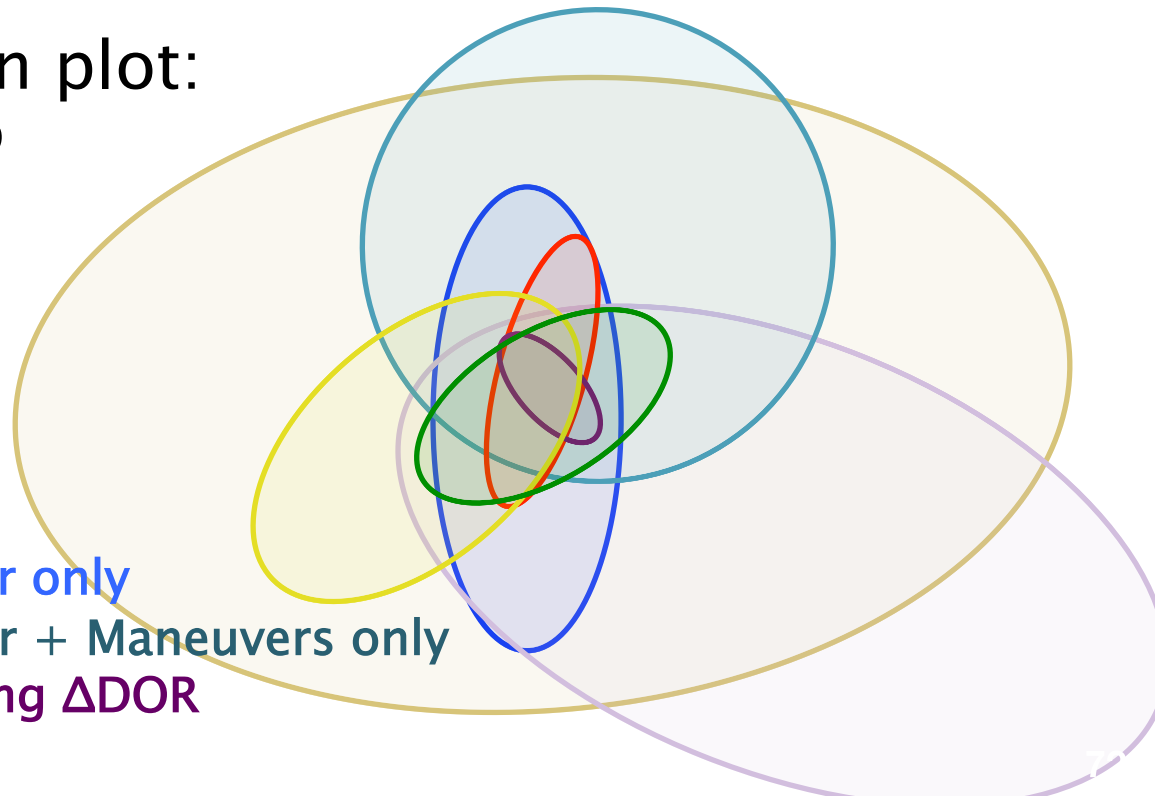
- Solutions are generated using numerous combinations of data types.
- This works to prevent another "unit-conversion" problem, as well as measurement corruption issues such as issues with singularities.
- A good navigation plot:
  - All ellipses overlap



Spacecraft missions

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- We presented a few practical examples
  - Mars Odyssey
  - Cassini navigating through a Titan flyby
  - Chandrayaan-1
  - Of course plenty of GRAIL references

Mars Odyssey Interplanetary Trajectory

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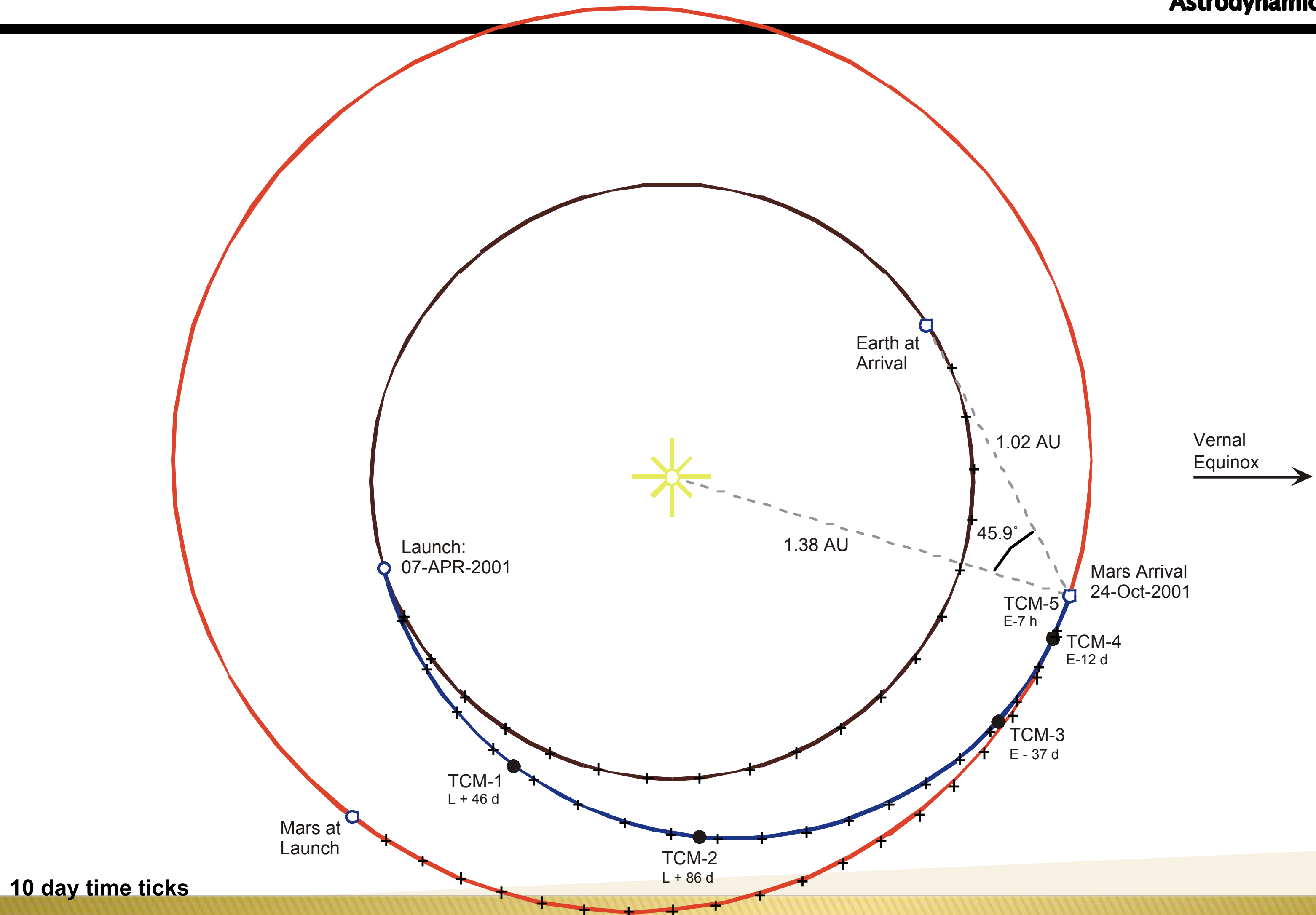

The B-plane

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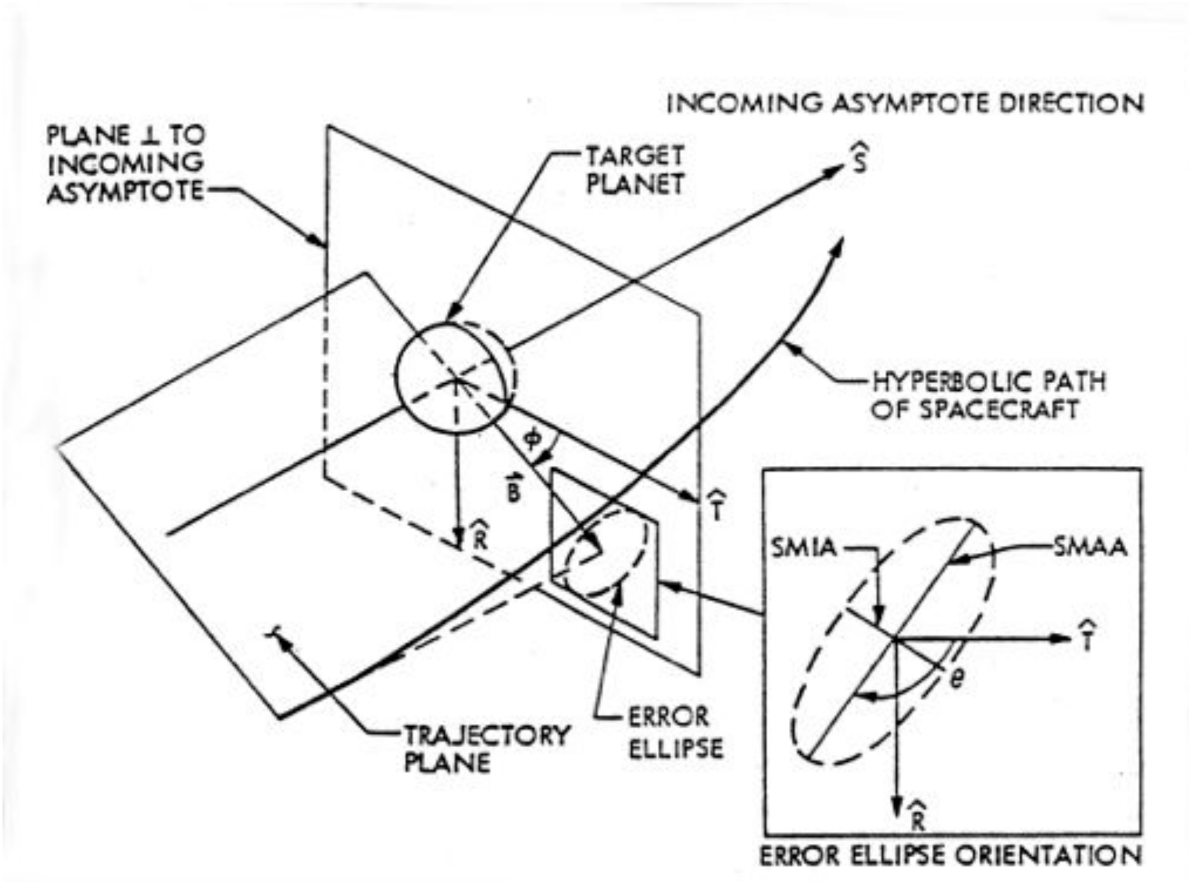

Mars Odyssey Navigation

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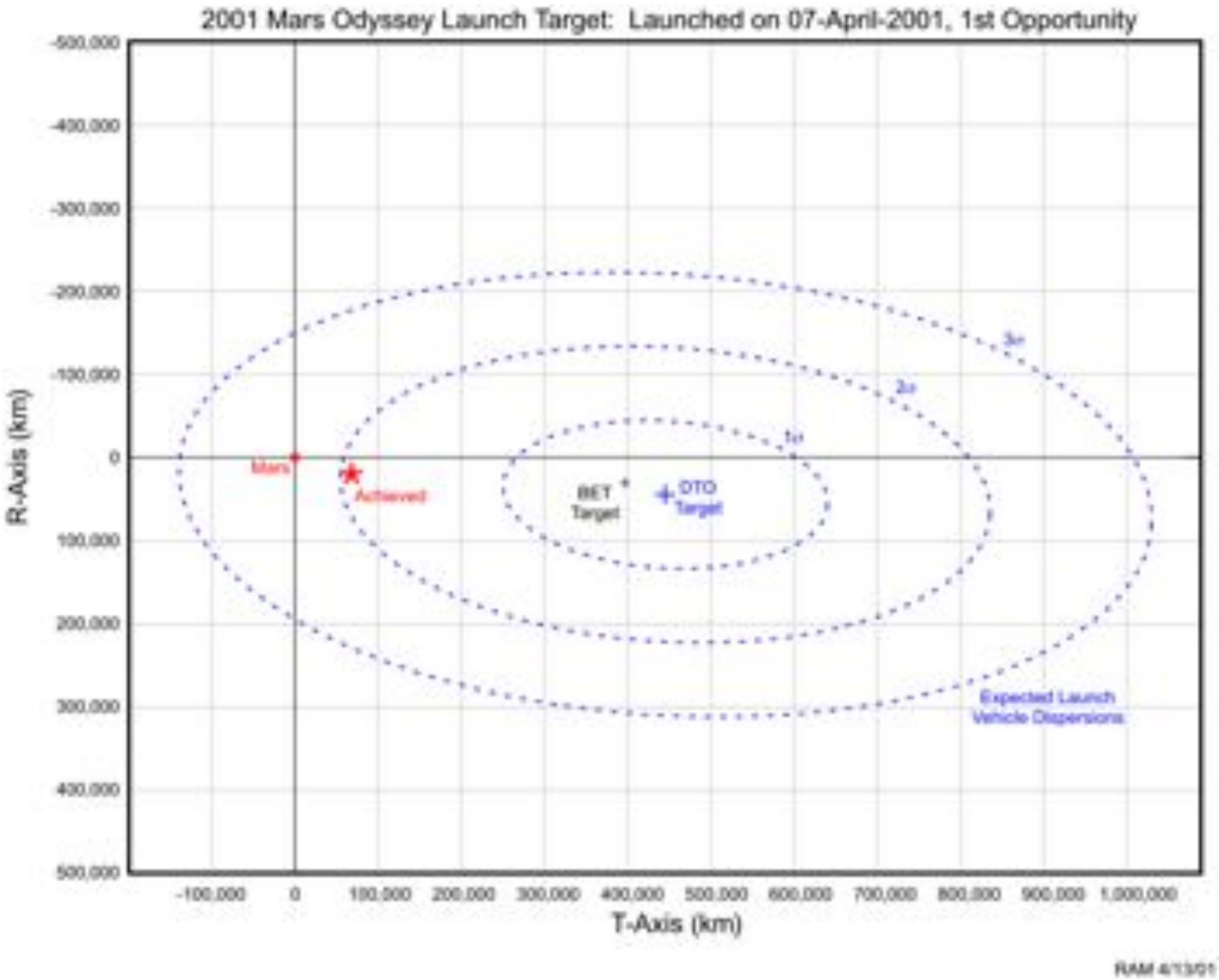
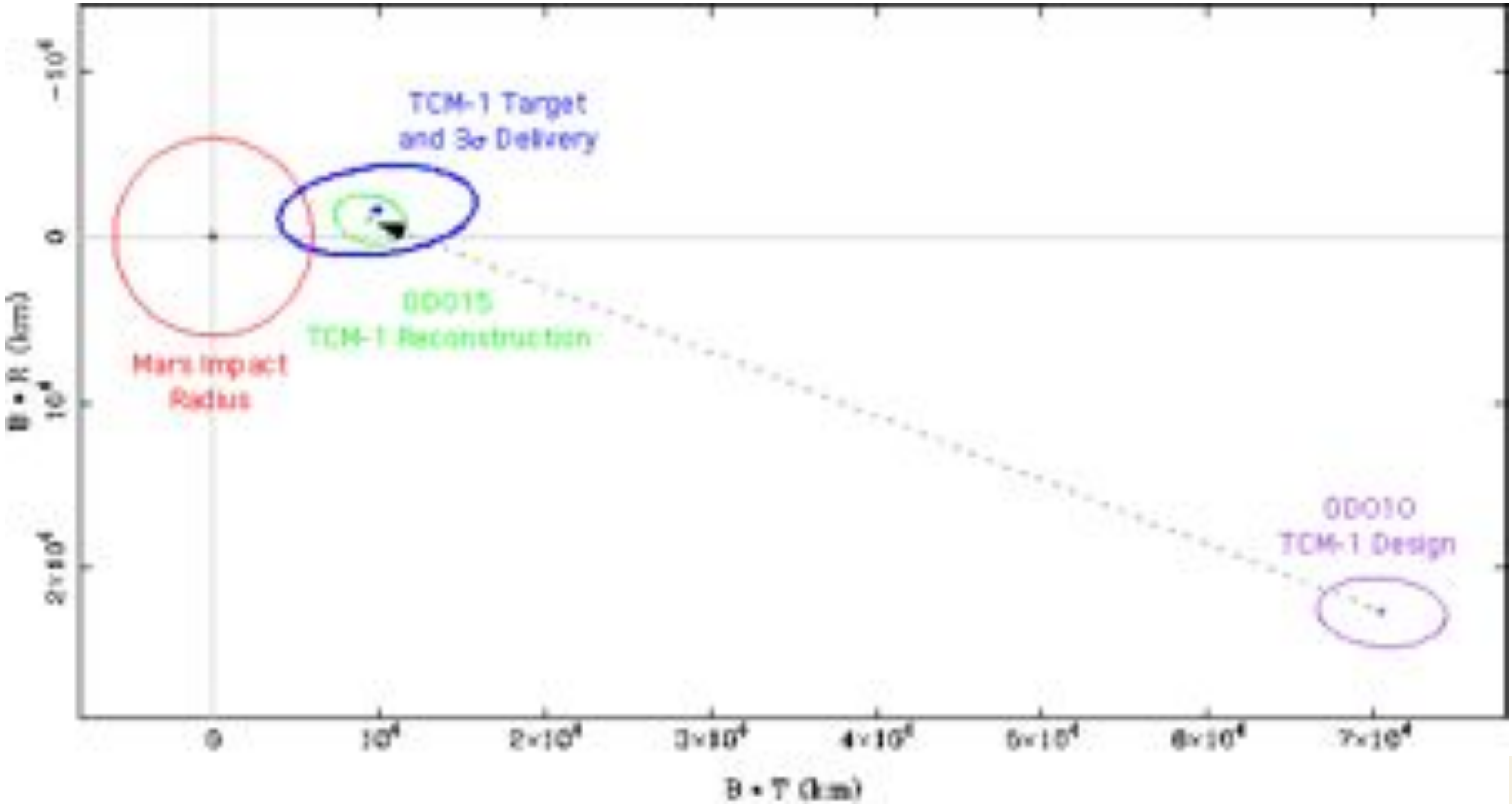


Mars Odyssey Navigation

TCM-1 Execution Date: 23-May-01



Mars Odyssey Navigation

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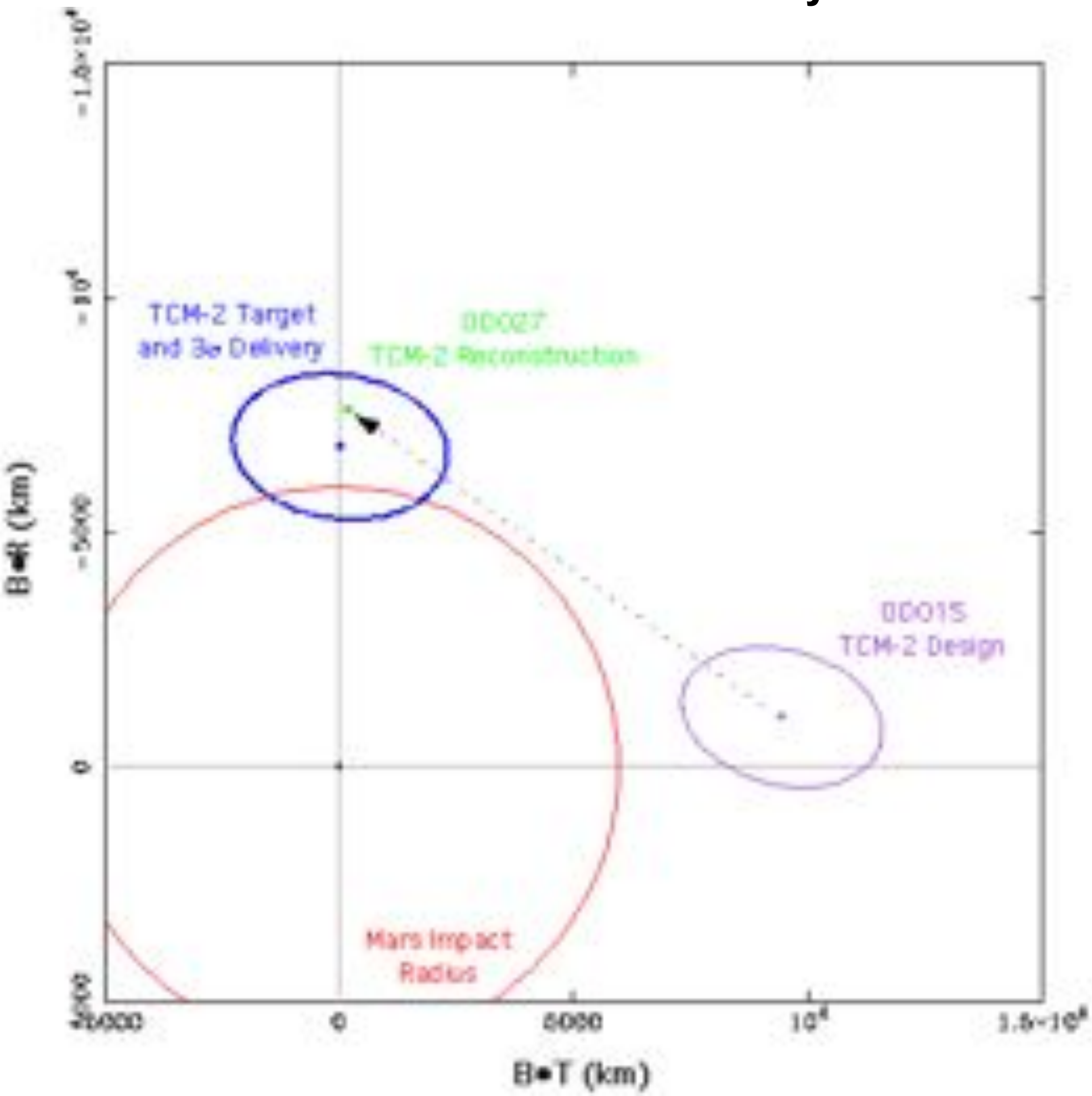
TCM-2 Execution Date: 02-July-01



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Mars Odyssey Navigation

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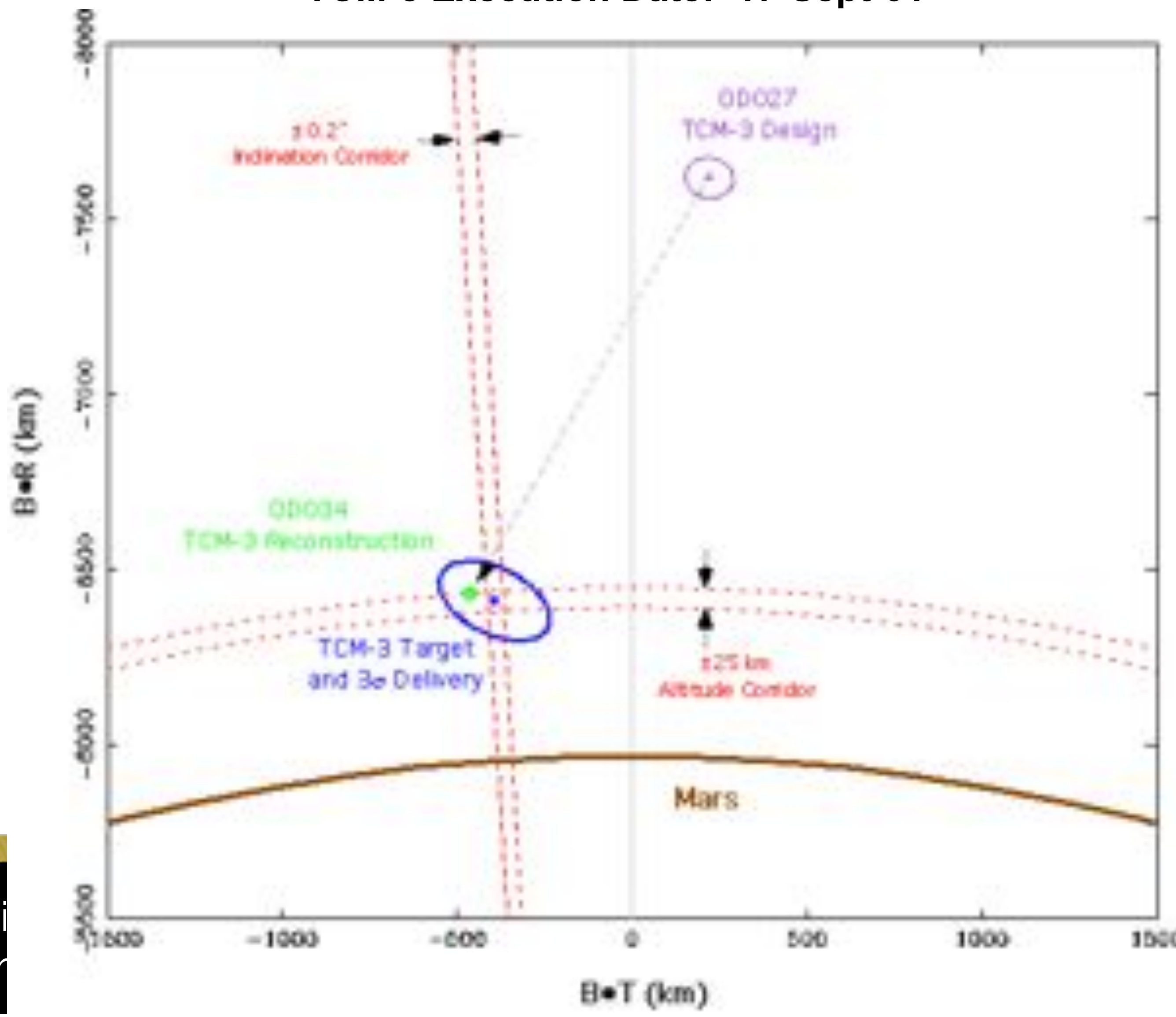
TCM-3 Execution Date: 17-Sept-01



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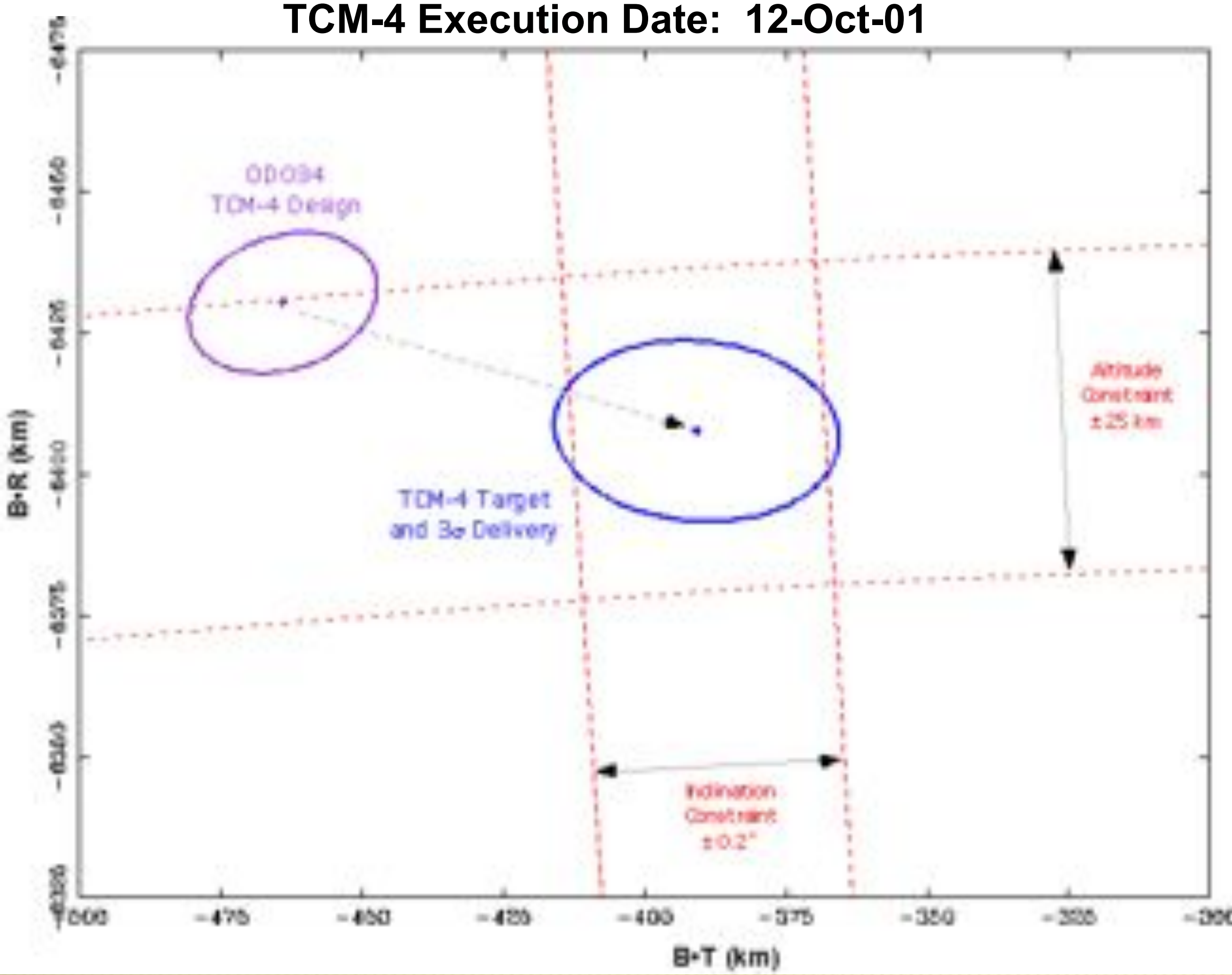
Mars Odyssey Navigation

TCM-4 Execution Date: 12-Oct-01



Target

Alt: 300 km
Inc: 93.47°

Current Estimate (OD034)

Alt: 324.1±11 km
Inc: 94.10°±0.2°

Current Miss (Est-Target)

Alt: +24 km
Inc: +0.6°

TCM-4 to Correct Miss

ΔV: 0.08 m/s

Data Type Contributions to Solution

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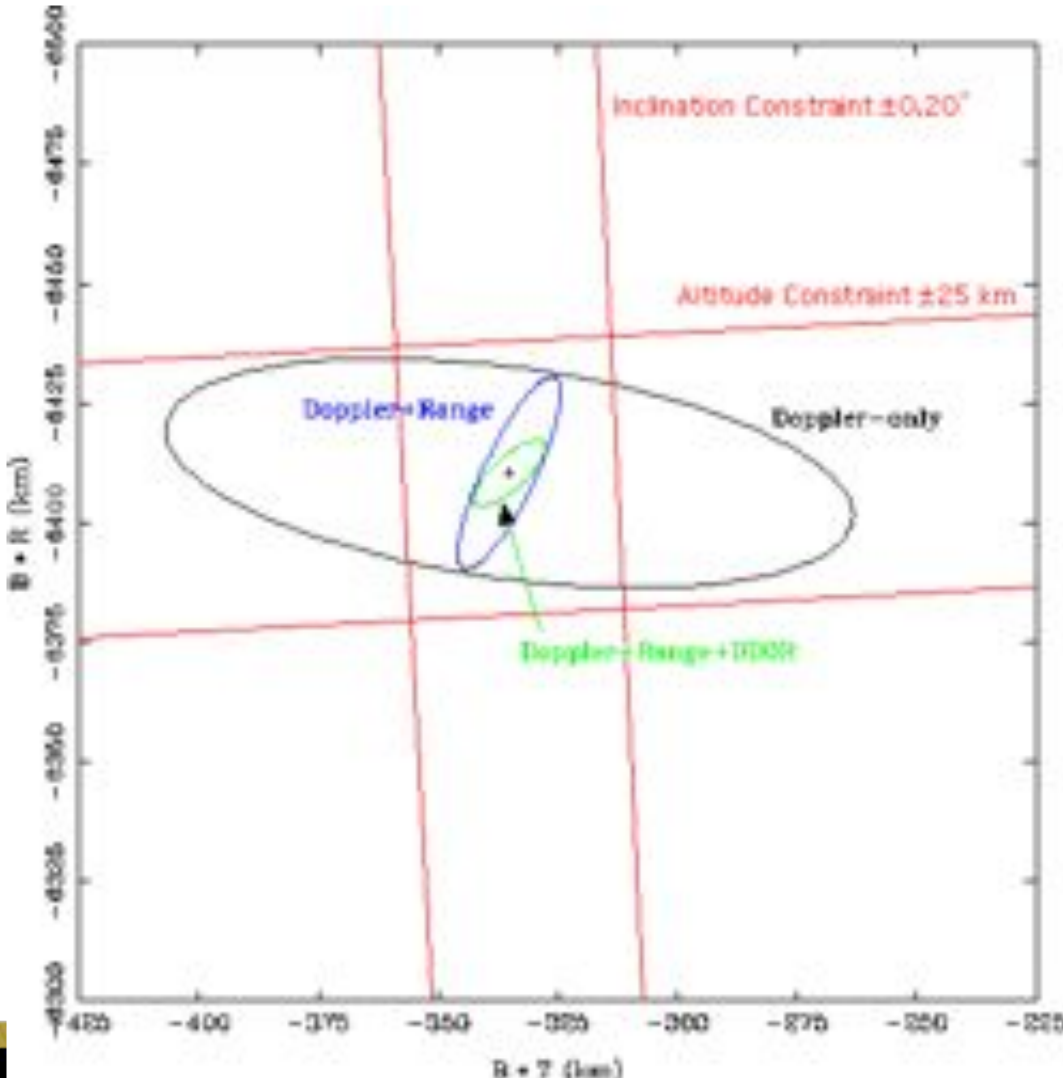

OD Knowledge
at the time of
TCM-4 Design (3σ)

Goal:
Altitude: 300 km ± 25 km
Inclination: 93.5° ± 0.2°

Achieved:
Altitude: 300.75 km
Inclination: 93.51°

Other interplanetary missions

- If ODY required 4 TCMs to achieve a good insertion, imagine what Cassini goes through.

- Cassini:
  - 2 Venus flybys
  - 1 Earth flyby
  - 1 Jupiter flyby
  - 90+ Titan flybys
  - Dozens of other targeted satellite flybys

  - Total number of OD solutions? Thousands.

Lessons Learned from CH-1

- Fast and furious can achieve success.
  - Often eats into fuel budget
  - Have to be prepared to work rapidly

- Good practice to:
  - Think through all possible outcomes.
  - Include time for contingencies.
  - Have a contingency plan at all times.

GRAIL's Navigation Plan

- GRAIL's operations have been planned to avoid conflicts between the two spacecraft.
  - Only one spacecraft is performing a maneuver on a given day.
  - A 5-day maneuver design block precedes any significant maneuver design. Contingencies can reduce this to a 3-day, 5-shift block.
    - This is at least 5 times more time than we had for CH-1!
  - Many contingency plans exist.

Earth Orbiters

- Huge difference between interplanetary and Earth orbiters
  - Short data arcs
  - Good geometry variations
  - Many potential tracking stations
  - Other data types
    - Radiometric, GPS, optical, GBORN
  - No significant light-time operational issues
  - Often tighter absolute requirements
  - Collision avoidance
  - So many Earth orbiters!

Techniques for OD Solution Accuracy Assessment

Examine P, the estimation error variance-covariance matrix

1. Plot tracking data residuals
   - Are they Gaussian?
   - What is the mean and RMS for each data type?
   - If these agree with the *a priori* data statistics, one can believe that P actually represents the estimation errors – this will probably never happen.

2. In general the correlations in P will be nearly correct but the variances will be optimistic unless process noise has been added and properly calibrated (tuned).

3. Do solution overlaps and compute statistics on them
   - Any common biases will cancel

Techniques for OD Solution Accuracy Assessment

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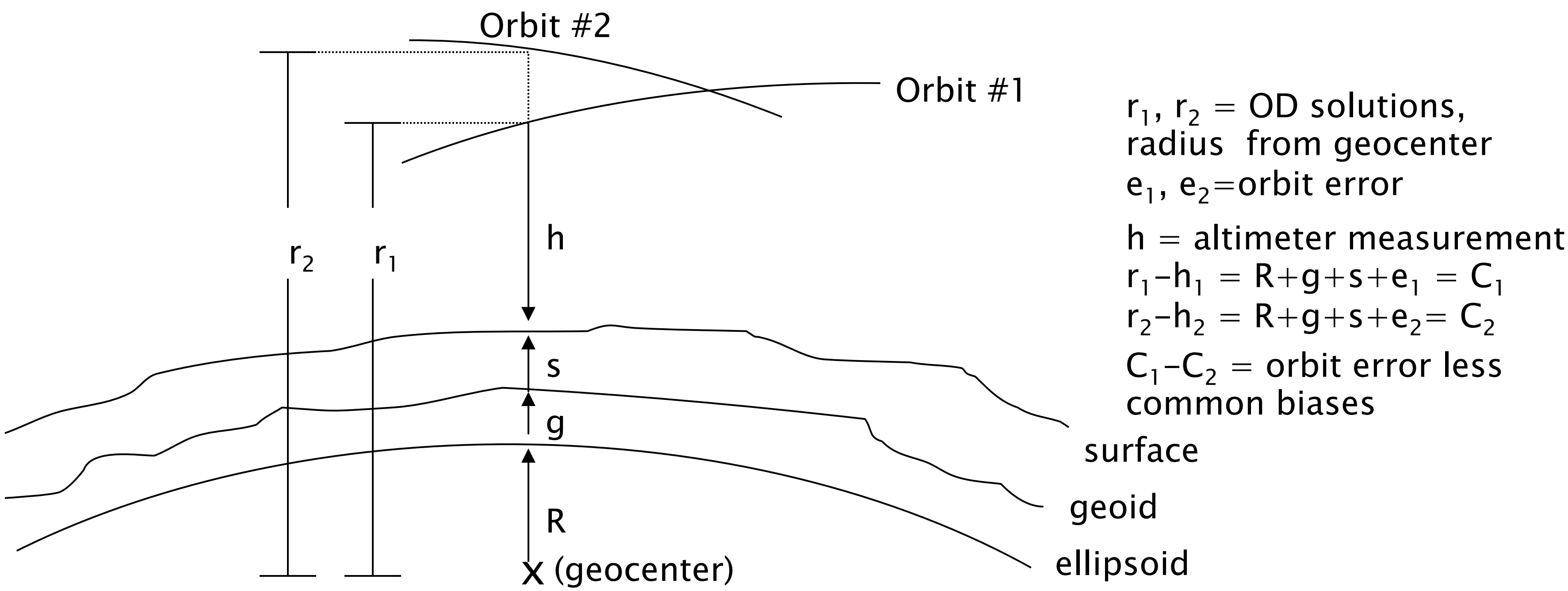
4. If the spacecraft has laser tracking compare spacecraft slant range computed from OD solutions with withheld laser range measurements. Lasers are generally accurate to 1 or 2 cm.

5. If the spacecraft carries an altimeter, examine cross over differences over the ocean or any point where the altimeter measures accurately and local surface elevation is constant.



The End

- Statistical Orbit Determination
  - The art of gracefully knowing that you don't know where your satellite is.
  - But now you can determine just how limited your uncertainty is!
  - Stat OD II will delve into more operational details, such as consider filters, unscented Kalman filters, more process noise, smoothing, and other topics to help you keep your $1B satellite flying straight.

The End

- Check your grades (for those graded anyway), especially quizzes.
- Exam 3 out today – due Monday at midnight unless you get permission otherwise.
- Thursday work day in this room.
- Everything else due Dec 20. Let me know if this is a problem!