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SURVEY ARTICLE

A Systematic Review of the Whale Optimization Algorithm: Theoretical Foundation, Improvements, and Hybridizations

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Abstract

Despite the simplicity of the whale optimization algorithm (WOA) and its success in solving some optimization problems, it faces many issues. Thus, WOA has attracted scholars' attention, and researchers frequently prefer to employ and improve it to address real-world application optimization problems. As a result, many WOA variations have been developed, usually using two main approaches improvement and hybridization. However, no comprehensive study critically reviews and analyzes WOA and its variants to find effective techniques and algorithms and develop more successful variants. Therefore, in this paper, first, the WOA is critically analyzed, then the last 5 years' developments of WOA are systematically reviewed. To do this, a new adapted PRISMA methodology is introduced to select eligible papers, including three main stages: identification, evaluation, and reporting. The evaluation stage was improved using three screening steps and strict inclusion criteria to select a reasonable number of eligible papers. Ultimately, 59 improved WOA and 57 hybrid WOA variants published by reputable publishers, including Springer, Elsevier, and IEEE, were selected as eligible papers. Effective techniques for improving and successful algorithms for hybridizing eligible WOA variants are described. The eligible WOA are reviewed in continuous, binary, single-objective, and multi/many-objective categories. The distribution of eligible WOA variants regarding their publisher, journal, application, and authors' country was visualized. It is also concluded that most papers in this area lack a comprehensive comparison with previous WOA variants and are usually compared only with other algorithms. Finally, some future directions are suggested.

1 Introduction

Metaheuristic algorithms successfully solve NP-hard problems in an acceptable response time by minimizing or maximizing any objective function [1–4]. These algorithms can be categorized into three primary categories [5]: evolutionary [6], physics-based [7, 8], and swarm intelligence [9–11]; however, there are others. Evolutionary algorithms use mechanisms such as reproduction, mutation, crossover, and selection inspired by the Darwinian evolutionary concepts to evolve the population during a predefined optimization process [12]. Like Darwin's survival, the best individual is selected and reproduced based on fitness value. Genetic algorithm (GA) [13], differential evolution (DE) [14], genetic programming (GP) [15], and biogeography-based optimizer (BBO) [16] are some flagships in this category. Concepts such as gravity, electromagnetic force, and equilibrium are employed in developing physics-based metaheuristic algorithms. Simulated annealing (SA) [17], gravitational search algorithm (GSA) [18], optics-inspired optimization (OIO) [19], thermal exchange optimization (TEO) [20], atom search optimization (ASO) [21], and quantum-based avian navigation optimizer algorithm (QANA) [22] are high cited pioneer algorithms in this category.

Swarm intelligence (SI) algorithms have modeled the behavior of groups of animals or insects working together and reacting to their surroundings. Groups of ants, flocks of birds, herds of animals, aggregations of bacteria, and schools

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of fish are examples of organisms that cooperate. The key benefits of SI algorithms are their adaptability, ease of use, and reliability [23]. Highly-cited pioneer and recently proposed SI algorithms include ant colony optimization (ACO) [24], particle swarm optimization (PSO) [25], artificial bee colony (ABC) [26], cuckoo optimization algorithm (COA) [27], krill herd (KH) [28], grey wolf optimization [29], crow search algorithm (CSA) [30], whale optimization algorithm (WOA) [31], sailfish optimizer (SFO) [32], horse herd optimization algorithm [33], starling murmuration optimizer (SMO) [34], gorilla troops optimizer (GTO) [35], and mountain gazelle optimizer (MGO) [36]. SI algorithms are more applicable than the other two categories since they have similar structures, simplicity, adaptability, robustness, high-speed convergence, and few parameters [37–39]. Therefore, many SI algorithms have been developed to solve real-world continuous [40–43] and binary [44–48] optimization problems in different applications, such as the medical [49, 50] and engineering [51–54] fields. However, SI algorithms suffer from inferior search strategies [55], which leads to premature convergence [56], an unbalancing issue [57], local optima trapping [58], and low population diversity [59–62].

In 2016, Mirjalili and Lewis [31] proposed the WOA algorithm inspired by the stunning hunting behavior of humpback whales in the ocean. It then drew the interest of numerous researchers as an SI optimization algorithm capable of resolving real-world optimization challenges. Figure 1 shows the number of WOA's citations per year, including remarkable growth from 2016 with 37 citations to the end of March 2023 with 7410 citations. This exponential growth in the number of WOA citations shows the popularity and impact of this algorithm for solving various optimization problems.

Simple but powerful search mechanisms for finding the optimal solution with high speed are the main attractions of WOA. However, like other swarm intelligence algorithms, WOA faces challenges such as falling into the local optima, premature convergence, and low population diversity. Thus, as shown in Fig. 2, many WOA variants and applications

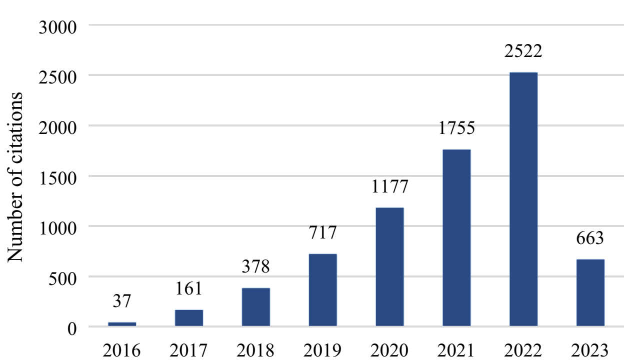


Fig. 1 The number of WOA's citations per year

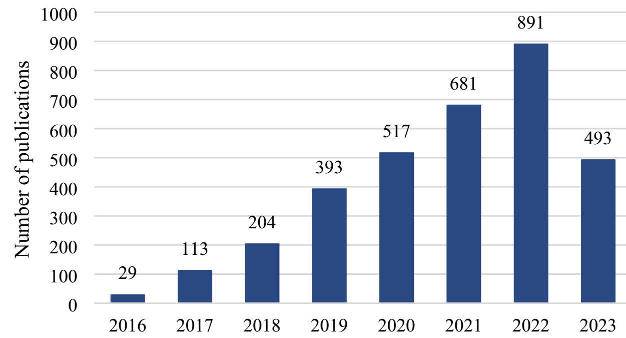


Fig. 2 Annual growth of WOA papers publishing

have been published since its introduction. These WOA variants are usually developed using improvement and hybridization approaches to enhance the performance of WOA in solving a wide variety of problems.

This high statistic of developments and applications of WOA has led to several review studies in which the importance of WOA, its variants, and applications have been discussed [63–66]. Reviewing the variants of such a popular algorithm has many advantages for guiding future research. However, although previous review studies have covered research conducted until 2020, no comprehensive systematic review of WOA variants has been done since then, which is the leading motivation of this study. Thus, this systematic review studies the WOA variants published from 2019 to the end of March 2023. Since many WOA variants have been published in this duration, more screening steps are needed to include high-quality papers in the final set. In this study, therefore, we introduce an adapted PRISMA methodology with more screening steps to find a reasonable number of the most reputable papers for full-text reading.

The introduced methodology consists of three main stages: identification, evaluation, and reporting. At first, related keywords and alternative synonyms are defined in the identification stage to extract all WOA documents from 2019 to the end of March 2023. Using these keywords for searching the Google Scholar database, this stage identified 2983 related documents. Then, in the evaluation stage, three screening steps were designed using the Rayyan tool [67] to evaluate the identified documents and find eligible papers for full-text reading. In the first screening, duplicate papers and those published by non-academic journals were removed to form a set of candidate papers. The second screening read the title and abstract of candidate papers to exclude those out of the study scope. In the third screening, the full text of the candidate papers was read, and their references list was also investigated to find eligible papers using the inclusion criteria. Finally, in the reporting stage, 116 eligible papers collected from Rayyan as the final set were first classified by six main and other publishers. Then, they were reviewed for qualitative synthesis into two main categories, improved and hybrid WOA, with 59 and 57 eligible papers, respectively. In summary, the contributions presented in this systematic review are introduced as follows:

- A complete description of WOA and its mathematical modeling to give readers a basic understanding of how this algorithm works to solve optimization problems.
- Critical analysis of search strategies' functionality to show the effectiveness of WOA in solving simple and complex optimization problems considering convergence speed, balancing exploration and exploitation, and preserving diversity.
- Introducing an adapted PRISMA methodology for further screening steps to find a reasonable number of the most valid research conducted on WOA developments.
- Describing different approaches for improving, hybridizing, binarization, and multi/many-objective WOA.
- Reviewing and analyzing outstanding WOA variants papers to find effective techniques and methods for developing future variants.
- Introducing some future research directions for developing WOA and its applications.

This systematic review is structured in the following manner. WOA and its essential components and concepts are presented in Sect. 2. In Sect. 3, a critical analysis of WOA is presented. Section 4 briefly reviews the previous surveys on WOA. In Sect. 5, an adapted PRISMA methodology is introduced. Section 6 describes the concepts used in improving and hybridization approaches for developing WOA variants. Improved and hybrid WOA variants are reviewed in Sects. 7 and 8. The steps of this systematic review are discussed through the introduced methodology, and then the findings are visualized in Sect. 9. Finally, the conclusion is presented in Sect. 10.

2 Whale Optimization Algorithm (WOA)

In 2016, Mirjalili and Lewis proposed the WOA algorithm, which has become one of the widely-used population-based metaheuristic algorithms to address global optimization problems in diverse fields [31]. This algorithm takes its inspiration from the natural hunting behavior of humpback whales. Humpback whales hunt by targeting groups of krill or small fish at the water's surface. They create distinctive bubbles along a spiral to surround and capture their prey, as shown in Fig. 3. The whales dive down and swim up to the water's surface by forming spiral bubbles around the prey. The WOA simulates the behavior of whales utilizing three strategies: encircling the prey, searching for prey (exploration phase), and spiral bubble-net attacking (exploitation phase). The position

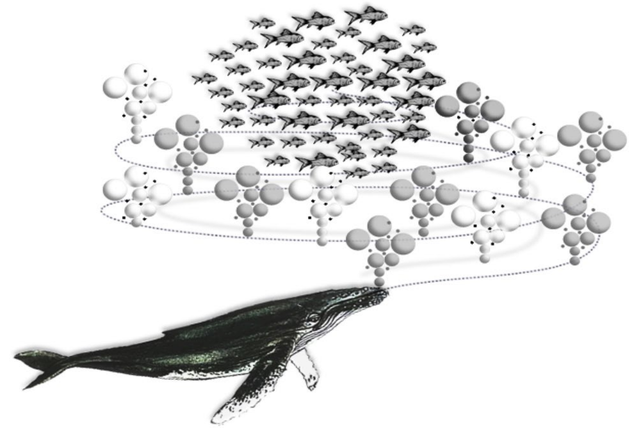


Fig. 3 The schematic of the spiral bubble-net attacking strategy

of the ith whale at iteration t is denoted by $X_i^t = \left(x_{i,1}^t, x_{i,2}^t, \ldots, x_{i,D}^t\right)$, where $i = 1, 2, \ldots, N$ and N and D are the whale population and the dimensions of the problem, respectively. In the following subsections, the strategies of WOA is mathematically presented.

2.1 Encircling Prey Strategy

Whales can find and surround their prey. In WOA, whales view the optimal solution as either the target prey or near it within the search area. During prey encirclement, other whales try to approach the best agent and update their position using Eq. (1). In this equation, t is the current iteration, X_i^t is the position of the ith whale for the current iteration, X^{*t} represents the position vector of the best-obtained solution thus far that is updated in each iteration if there is a better solution.

$$X_i^{t+1} = X^{*t} - A.D \tag{1}$$

$$D = \left|C \times X^{*t} - X_i^t\right| \tag{2}$$

where D denotes the distance between the prey X^{*t} and the whale X_i^t calculated by Eq. (2) where symbols |.| indicate the absolute value, and A and C denote coefficient vectors that are calculated using Eqs. (3) and (4) [31].

$$A = 2 \times a \times r - a \tag{3}$$

$$C = 2 \times r \tag{4}$$

$$a = 2 - t \times \left(\frac{2}{MaxIter}\right) \tag{5}$$

In Eqs. (3) and (4), the parameter r is a random value within the range of [0, 1], and a in Eq. (3) is decreased linearly during iterations from 2 to 0 according to Eq. (5). The parameters t

and *Maxiter* in Eq. (5) denotes the current iteration and the total number of iterations, respectively. The parameter a gradually narrows the whales to the encircling scope.

2.2 Spiral Bubble-Net Attacking Strategy

Humpback whales use a bubble net to corner their prey by spiraling toward them. This strategy is mathematically modeled by two methods shrinking encircling and spiral updating position.

2.2.1 Shrinking Encircling Method

This behavior is formulated by reducing the value of the convergence variable a in Eq. (3). Also, the alternate range of A fluctuation is decreased linearly using parameter a from 2 to 0 through iterations. In other words, A is a random value in the interval $[-a, a]$.

2.2.2 Spiral Updating Position Method

In this method, first, the distance between the best-obtained solution thus far X^{*t} and whale X_i^t is computed using Eq. (6). Then, moving in a spiral pattern from the current position towards an optimal solution is defined using Eq. (7). In these equations, b denotes a constant parameter for determining the logarithmic spiral shape, and l represents a random number within the range $[-1, 1]$.

$$D' = \left|X^{*t} - X_i^t\right| \tag{6}$$

$$X_i^{t+1} = D' \times e^{bl} \times \cos(2\pi l) + X^{*t} \tag{7}$$

where b is a constant parameter for determining the logarithmic spiral shape, and l is a random number within the range $[-1, 1]$.

In WOA, the humpback whale circles its prey in a contracted pattern and spirals around it as it swims. Whale chooses one of the methods shrinking encircling or spiral model with a probability of 50%, to modify the position of the whales during the optimization process. The mathematical model is defined using Eq. (8), where p is a random number in [0, 1].

$$X_i^{t+1} = \begin{cases} X^{*t} - A \times D \; if \; p < 0.5 \\ D' \times e^{bl} \times \cos(2\pi l) + X^{*t} if \; p \geq 0.5 \end{cases} \tag{8}$$

2.3 Searching for Prey Strategy

In this strategy, whales explore the problem space to find unvisited zones and increase population diversity. A randomly selected search agent updates the position of each whale. The parameter A is used to move the search agent away from a chosen randomly humpback whale. Exploration is performed using Eq. (9) to avoid getting trapped in a local minimum [31].

$$X_i^{t+1} = X_{rand} - A \times D \tag{9}$$

$$D = \left|C \times X_{rand} - X_i^t\right| \tag{10}$$

where X_{rand} is a random position vector in the search space selected from the available whales in the population, and A and C are computed using Eqs. (3) and (4).

The flowchart of WOA is depicted in Fig. 4, where N whales are randomly distributed in the search space, followed by computing their associate objective function value. Next, the control parameters' initial values are updated, and the optimization begins with the predefined iterations. Then, at each iteration, the value of parameter p is investigated. Whales update their position using the spiral updating position method defined by Eq. (7) when $p \geq 0.5$. While $p < 0.5$, whales update their position using the encircling prey strategy defined in Eq. (1) if $|A| < 1$, and the searching for prey strategy defined in Eq. (9) when $|A| \geq 1$. Then, the feasibility and fitness values of the newly obtained positions are computed. After that, the best solution is updated. Finally, WOA is terminated by the satisfying of maximum iterations.

3 Critical Analysis of WOA

Although WOA is simple, many studies show it is also powerful for solving many optimization problems. Due to its convenient and easy-to-understand search mechanisms, many WOA developments have been proposed using different approaches to solve challenging, real-world optimization problems. Improving three search strategies of WOA, encircling the prey, searching for prey, and spiral bubble-net attacking, has a high impact on its performance, which is the motivation of most developments. Therefore, this section critically analyzes the search strategies' functionality and shows the success rate of WOA facing different problems. Hence, experimental tests are performed to critically analyze WOA's behaviors in solving simple and complex optimization problems. The CEC 2018 test suite [68] is conducted due to various challenging unimodal, multimodal, hybrid, and composition test functions. In this analysis, the following objectives are considered.

- Evaluating the WOA's performance in terms of solution quality in solving unimodal, multimodal, hybrid, and composition test functions.

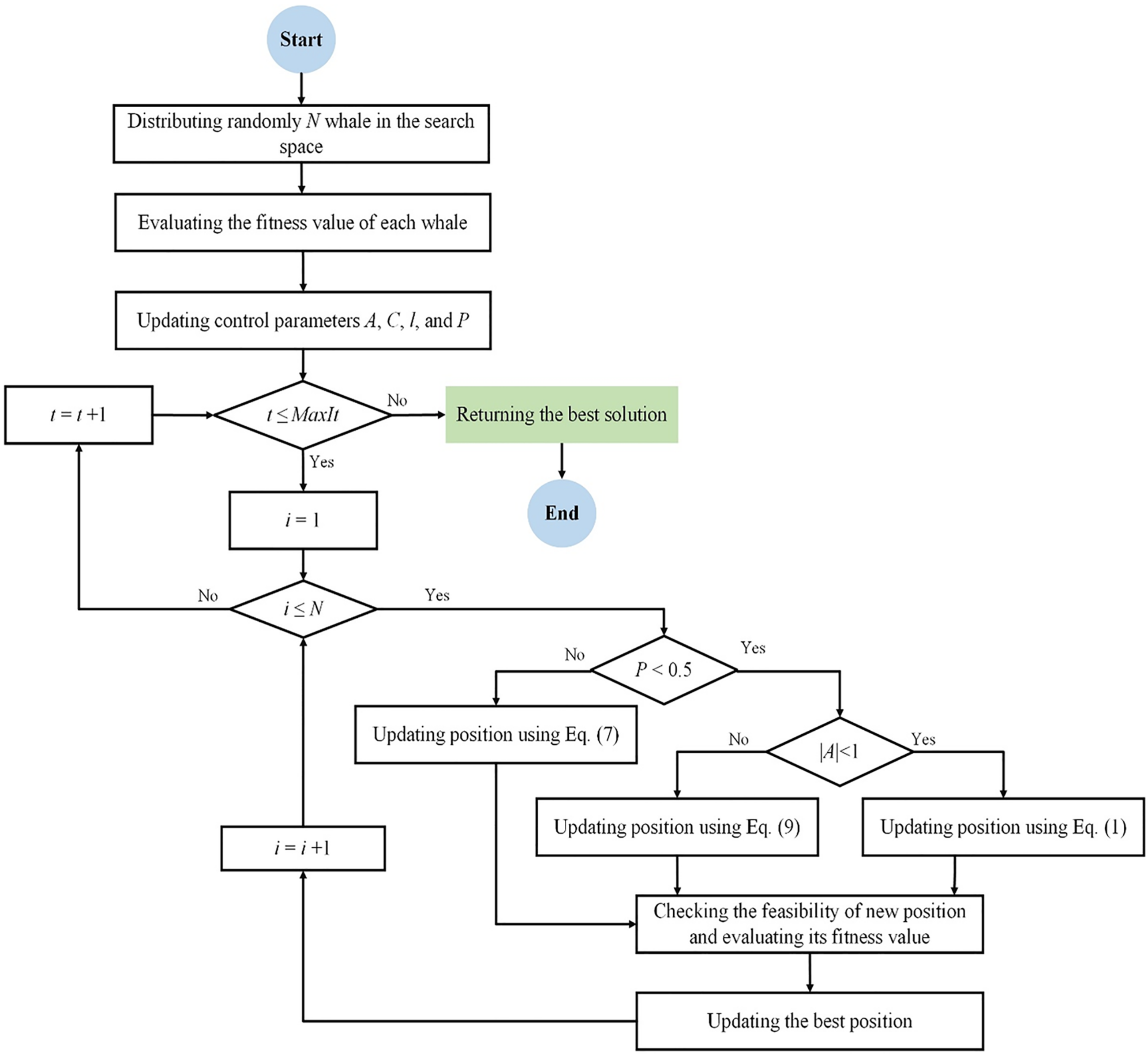


Fig. 4 The flowchart of WOA

– Analyzing the WOA's balancing behavior in switching search strategies during optimization.
– Analyzing the WOA's convergence behavior toward the promising areas.
– Analyzing the WOA's population diversity in discovering the unvisited areas and bypassing the local optima.

All experiments utilized an Intel Core i5-3770 3.4 GHz CPU and 4.00 GB of RAM. The population size of whales (N) was set to 200, and the iteration limit was $10{,}000 \times Dimension/N$, in which the whale population evolves during 30 independent runs. The value of initial WOA control parameters was set the same as the original paper, and the dimension (D) was set to 50. Ultimately, the results are reported and visualized in four groups regarding performance, balance, convergence, and population diversity to achieve the objectives considered.

3.1 Performance Evaluation

The performance of WOA in solution quality is reported in Table 1. This Table reports the results, including three statistical metrics, average (Avg), minimum (Min), and standard deviation (Std), based on values of the fitness $\left(F^{WOA}\right)$ and values of the fitness error $\left(\Delta F^{WOA}\right)$.

Table 1 WOA's performance in solving different test functions with a dimension of 50

Func	F^{Opt}	F^{WOA}			ΔF^{WOA}		
		Avg	Min	Std	Avg	Min	Std
F_1	100	1.14312E+07	2.99684E+06	1.01515E+07	1.14311E+07	2.99674E+06	1.01515E+07
F_3	300	6.15696E+04	2.59497E+04	2.98327E+04	6.12696E+04	2.56497E+04	2.98327E+04
F_4	400	6.93219E+02	6.04106E+02	4.66656E+01	2.93219E+02	2.04106E+02	4.66656E+01
F_5	500	9.45817E+02	8.31517E+02	8.93466E+01	4.45817E+02	3.31517E+02	8.93466E+01
F_6	600	6.73330E+02	6.58638E+02	7.87309E+00	7.33304E+01	5.86378E+01	7.87309E+00
F_7	700	1.65916E+03	1.48912E+03	9.02769E+01	9.59162E+02	7.89123E+02	9.02769E+01
F_8	800	1.20048E+03	1.05248E+03	8.28118E+01	4.00480E+02	2.52485E+02	8.28118E+01
F_9	900	2.24365E+04	1.32839E+04	6.20076E+03	2.15365E+04	1.23839E+04	6.20076E+03
F_{10}	1000	9.67707E+03	7.82292E+03	1.17859E+03	8.67707E+03	6.82292E+03	1.17859E+03
F_{11}	1100	1.57153E+03	1.32946E+03	9.74084E+01	4.71526E+02	2.29460E+02	9.74084E+01
F_{12}	1200	2.06771E+08	9.63731E+07	8.71823E+07	2.06770E+08	9.63719E+07	8.71823E+07
F_{13}	1300	2.08608E+05	5.71934E+04	2.18598E+05	2.07308E+05	5.58934E+04	2.18598E+05
F_{14}	1400	6.48443E+05	5.21723E+04	6.69234E+05	6.47043E+05	5.07723E+04	6.69234E+05
F_{15}	1500	7.28509E+04	1.86130E+04	4.70356E+04	7.13509E+04	1.71130E+04	4.70356E+04
F_{16}	1600	4.69187E+03	3.24505E+03	5.98508E+02	3.09187E+03	1.64505E+03	5.98508E+02
F_{17}	1700	3.91678E+03	3.12100E+03	3.75740E+02	2.21678E+03	1.42100E+03	3.75740E+02
F_{18}	1800	4.17163E+06	7.25756E+05	3.12159E+06	4.16983E+06	7.23956E+05	3.12159E+06
F_{19}	1900	2.23865E+06	1.76039E+05	1.78235E+06	2.23675E+06	1.74139E+05	1.78235E+06
F_{20}	2000	3.53152E+03	3.06860E+03	3.15794E+02	1.53152E+03	1.06860E+03	3.15794E+02
F_{21}	2100	2.85094E+03	2.67660E+03	1.06994E+02	7.50939E+02	5.76598E+02	1.06994E+02
F_{22}	2200	1.13426E+04	8.67005E+03	1.39984E+03	9.14257E+03	6.47005E+03	1.39984E+03
F_{23}	2300	3.56529E+03	3.18687E+03	1.86438E+02	1.26529E+03	8.86867E+02	1.86438E+02
F_{24}	2400	3.61483E+03	3.40589E+03	1.25782E+02	1.21483E+03	1.00589E+03	1.25782E+02
F_{25}	2500	3.14888E+03	3.07397E+03	4.28249E+01	6.48885E+02	5.73968E+02	4.28249E+01
F_{26}	2600	1.29247E+04	9.69253E+03	1.51558E+03	1.03247E+04	7.09253E+03	1.51558E+03
F_{27}	2700	4.33444E+03	3.69969E+03	4.12815E+02	1.63444E+03	9.99685E+02	4.12815E+02
F_{28}	2800	3.43850E+03	3.35494E+03	6.06162E+01	6.38501E+02	5.54937E+02	6.06162E+01
F_{29}	2900	7.25033E+03	6.04028E+03	7.07226E+02	4.35033E+03	3.14028E+03	7.07226E+02
F_{30}	3000	8.71255E+07	2.98283E+07	3.37379E+07	8.71225E+07	2.98253E+07	3.37379E+07

$$\Delta F^{WOA} = F^{WOA} - F^{Opt} \tag{11}$$

The different problems in the CEC 2018 test suite can assess the ability of WOA to exploit, explore, avoid local optima, and balance. F_1 and F_3 are unimodal test functions with one local optimum, which can benchmark the exploitation ability of WOA. The results obtained to solve unimodal test functions show that the WOA cannot properly approximate or exploit the promising area. Test functions (F_4-F_{10}) are multimodal with many local optima suitable for assessing the exploration ability. The WOA's results for multimodal functions show that whales cannot efficiently explore the entire solution space and locate the optimal solution.

Moreover, the hybrid test functions (F_{11}-F_{20}) and composition test functions (F_{21}-F_{30}) are more complex and challenging. However, the reported results from these test functions show that the WOA can find low-quality solutions for these problems. Accordingly, the WOA suffers from poor search strategies, resulting in low exploitation, exploration, and local optima trapping.

3.2 Balancing Behavior Analysis

A satisfactory balance between exploration and exploitation is crucial for achieving success when solving an optimization problem. Exploration emphasizes the ability of search strategies to find a wide range of solutions, and exploitation refers to the power of search agents to improve the existing ones or find better solutions. Therefore, the balancing analysis is quantitatively evaluated to compute the percentage of whales switching between exploration and exploitation during iterations. The balancing analysis is calculated using Eqs. (12–14) [69]. Equation (12) calculates the distance between the whale population, where $x_d\ (t)$ is the dth dimension of all whales, and x_{id} is the position of the ith whale. Then, the percentage of exploration and exploitation are calculated using Eqs. (13) and

Table 2 The overall percentage of exploration and exploitation

Percentage	Unimodal test functions		Multimodal test functions							
	F_1	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	
Exploration	17.21	12.67	13.79	14.20	18.19	6.94	13.02	11.38	15.85	
Exploitation	82.79	87.33	86.21	85.80	81.81	93.06	86.98	88.62	84.15	
Percentage	Hybrid test functions									
	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}
Exploration	13.31	15.89	18.80	15.22	17.46	13.99	15.62	17.01	16.40	13.83
Exploitation	86.69	84.11	81.20	84.78	82.54	86.01	84.38	82.99	83.60	86.17
Percentage	Composition test functions									
	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}	F_{26}	F_{27}	F_{28}	F_{29}	F_{30}
Exploration	15.49	17.79	18.02	20.28	15.96	15.49	20.60	17.89	15.63	18.87
Exploitation	84.51	82.21	81.98	79.72	84.04	84.51	79.40	82.11	84.37	81.13

(14), where $Diversity_{max}$ is the maximum diversity obtained by Eq. (12) in entire iterations [69].

$$Diversity\,(t) = \frac{1}{D}\sum_{d=1}^{D}\frac{1}{N}\sum_{i=1}^{N}\left|median\{x_d(t)\} - x_{id}(t)\right| \tag{12}$$

$$Exploration\,(\%) = \frac{Diversity(t)}{Diversity_{max}} \times 100 \tag{13}$$

$$Exploitation\,(\%) = \frac{\left|Diversity(t) - Diversity_{max}\right|}{Diversity_{max}} \times 100 \tag{14}$$

As reported in Table 2, the WOA archives a low percentage of exploration and cannot effectively strike a balance between search strategies to solve almost test functions. Also, it has the same behavior for different test functions. Moreover, Fig. 5 shows the similar behavior of WOA on some test functions, including unimodal (F_1),

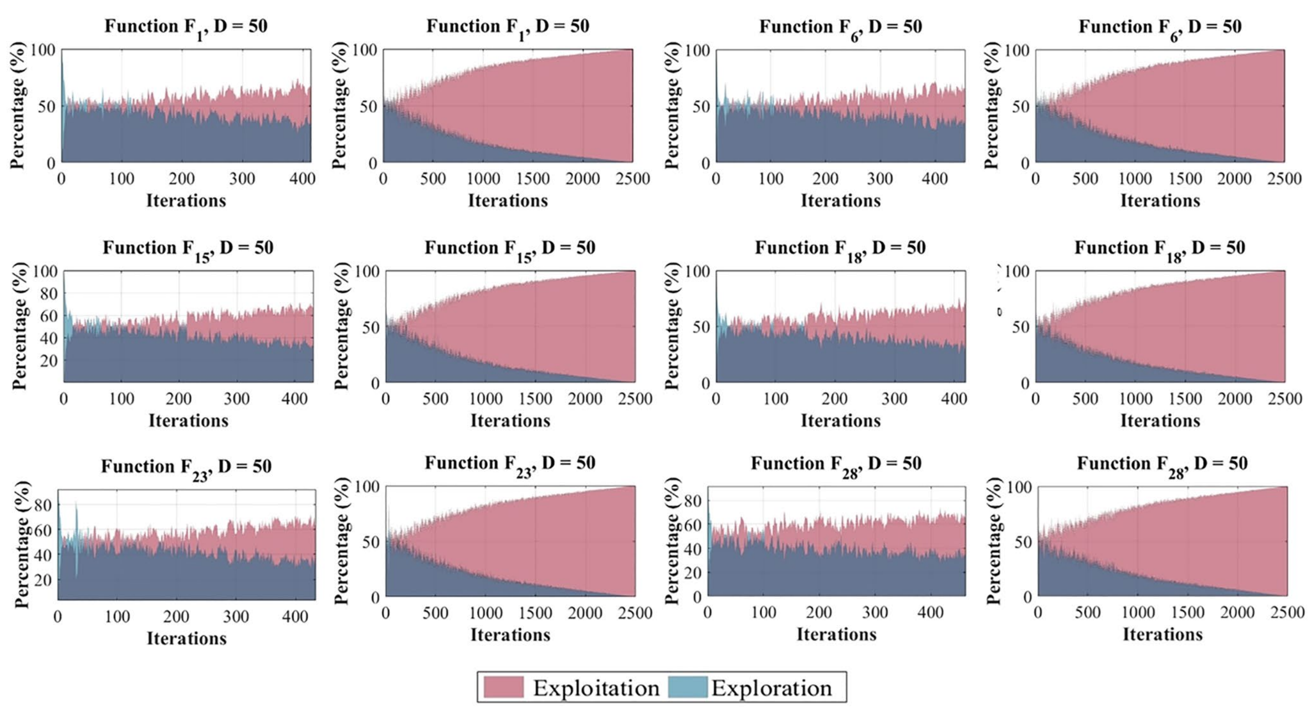


Fig. 5 WOA's balancing behavior

multimodal (F_6), hybrid (F_{15} and F_{18}), and composition (F_{23} and F_{28}).

3.3 Convergence Behavior Analysis

In this experiment analysis, the ability and convergence behavior of WOA for enhancing the candidate solution throughout the iterations is evaluated and visualized in Fig. 6. As can be seen in this figure, in the initial iterations, the fitness value of WOA significantly decreases to converge towards a position quickly. Eventually, convergence fluctuation gradually decreases in the last iterations when it reaches the promising area. The WOA's performance in Table 1 shows that it wrongly estimates the promising region or prematurely converges on this position by losing its diversity.

3.4 Population Diversity Analysis

Maintaining population diversity during optimization is the most fundamental challenge in optimizers' performance, and a low population diversity among search agents increases premature convergence to local optima trapping. Therefore, computing and visualizing the diversity of the WOA population can discover the strengths and shortcomings of the WOA during the optimization process. In this analysis, the population diversity was measured by a moment of inertia (I_c) [70] denoted in Eq. (15).

$$I_c = \sum_{j=1}^{D} \sum_{i=1}^{N} \left(x_{ji} - C_j\right)^2 \tag{15}$$

$$C_j = \frac{1}{N} \sum_{i=1}^{N} x_{ji} \tag{16}$$

where x_{ji} is the jth dimension of ith whale, C_j is computed using Eq. (16), and N is the number of whales or solutions.

As shown in Fig. 7, the WOA follows a linearly decreasing behavior for maintaining population diversity. The performance of WOA on all test functions shows similar behavior. Then, the whales maintain the variety until half of the iterations are completed and gradually decrease their distance to converge to a point. The analysis of this behavior and the performance analysis reported in Table 1 indicate that the WOA lost its diversity, resulting in the inability to discover more promising areas and improve the quality of its solution.

4 Prior Surveys in WOA and its Variants

WOA is one of the simple metaheuristic algorithms; however, the statistical information reported in the introduction shows that it is still influential in solving various optimization problems. Since many WOA variants are developed using different approaches. Therefore, a critical review of WOA variants is necessary to guide better its future developments, which is the motivation of the following review and survey studies. Soleimanian et al. [63] presented a survey of WOA and its applications by reviewing studies conducted between 2016 and 2018, summarized hybridization works, and offered varieties. They also investigated WOA's usage

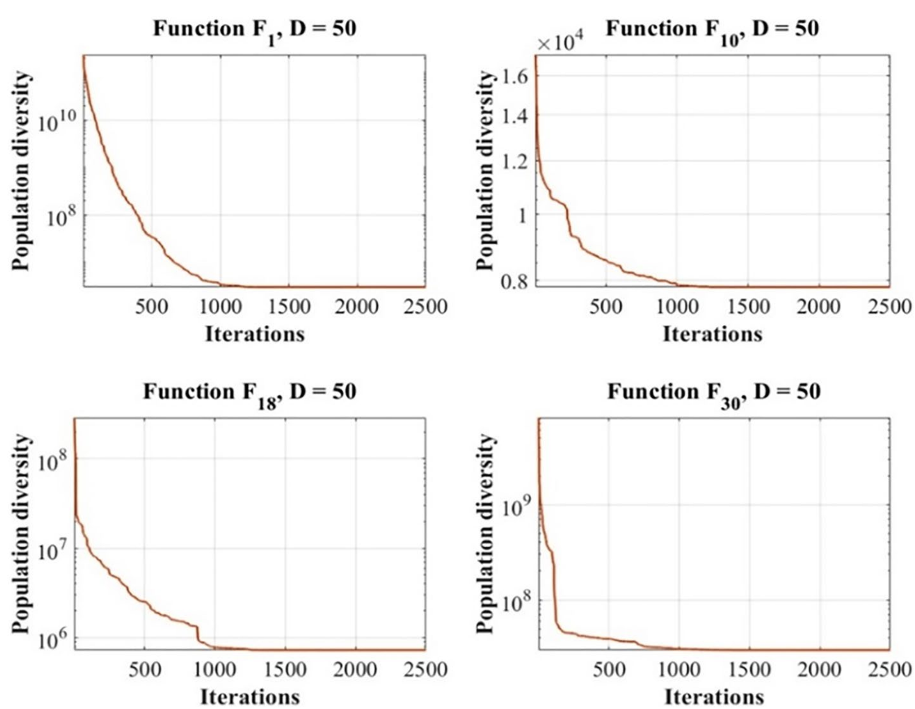


Fig. 6 WOA's convergence behavior

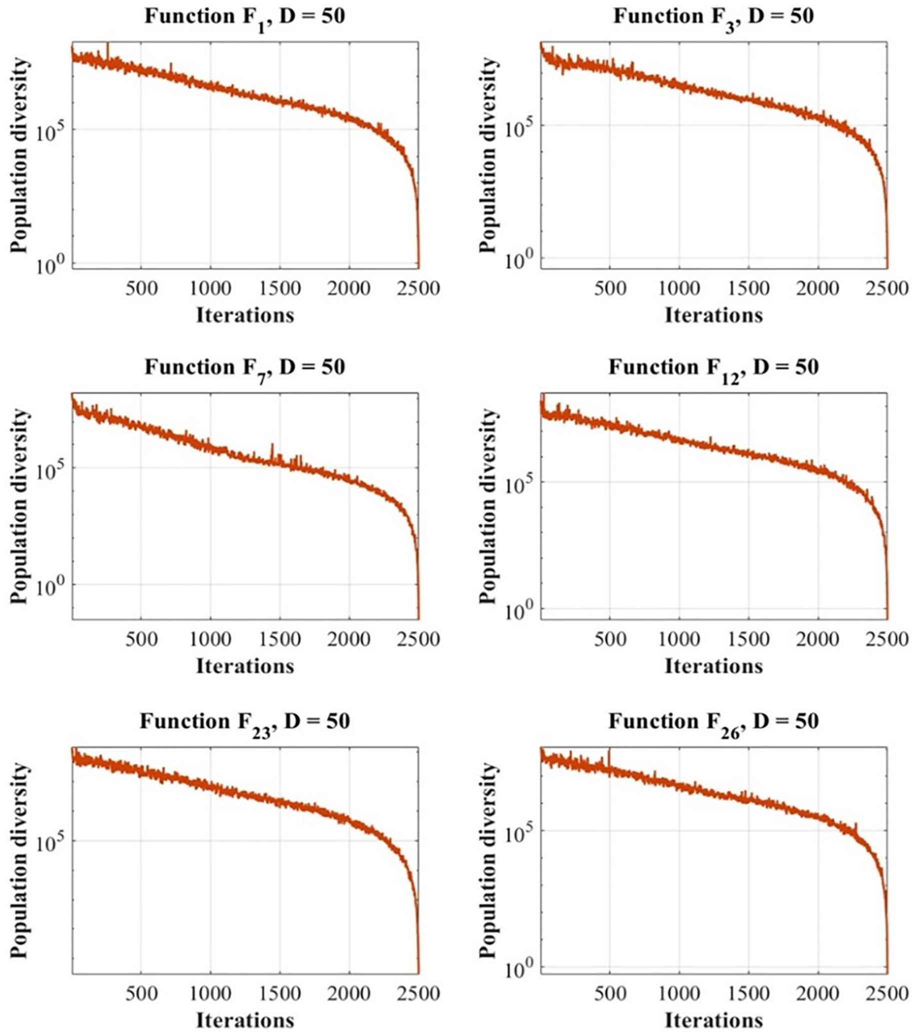


Fig. 7 WOA's population diversity

in various engineering issues, including classification, clustering, robot routing, task scheduling, networks, image processing, and other engineering applications. Their analysis results show that WOA has been primarily applied in solving engineering optimization problems. Mohammed et al. [64] presented a short systematic and meta-analysis survey on WOA regarding the algorithmic foundations, features, restrictions, improvements, combinations, and applications. First, they reported the WOA variants in two groups of modifications and hybridizations from 2016 to 2018. In addition, they proposed the WOA-BAT algorithm that uses the exploration phase of the bat algorithm and the exploitation phase of WOA. Although the WOA-BAT algorithm obtained the best performance than some canonical algorithms in solving continuous optimization problems, its superiority over the WOA variants is not investigated.

Rana et al. [65] reviewed the studies conducted on the applications, modifications, and hybridization of the WOA algorithm in the years 2016 to 2020. They selected 82 papers to review from 939 papers extracted from their search. Based on their analysis, WOA-based techniques have been used in five fields and 17 different subfields in engineering applications. The findings of this survey demonstrate that 61% of papers are modified WOA using various techniques, and 27% are hybridized with other algorithms. Moreover, 12% of WOA multi-objective variants are developed from 2016 to 2020. Although this review paper motivates the researchers to propose more novel WOA-based algorithms, introducing the new variant of WOA should cover the weak points of canonical WOA and its existing variants. In 2020, Mirjalili et al. [66] reviewed and analyzed WOA and presented brief literature on WOA, including algorithm applications, in various fields. This paper investigates the homogeneous and heterogeneous hybrid variants of WOA and explains some real-world applications of WOA. They also evaluated the efficiency of WOA on several test functions and a photonic crystal filter practical example. The findings of this study show that the WOA algorithm benefits from a good balance between two search strategies to solve this real-world problem.

In summary, most prior surveys lack a critical analysis of the WOA algorithm, its weaknesses and behaviors, an adapted methodology for extracting the eligible papers, and an introduction to effective techniques and algorithms to develop more successful variants. In addition, many WOA variants have been introduced in the last five years; therefore, an updated systematic review of this algorithm is needed. Hence, the scope of this systematic review is considered from 2019. This study analyzes the WOA algorithm critically to highlight its weakness and behaviors. The reported results of this analysis can provide informative knowledge for further developing the WOA by knowing its strengths and weaknesses. Then, an adapted methodology is introduced to extract eligible papers of WOA variants. The eligible papers are reviewed in two main categories, improved and hybrid WOA variants. The common techniques used in improved WOA and the effective algorithms applied in hybrid WOA are highlighted and explained, which can guide future developments for metaheuristic algorithms.

5 The Adapted Methodology

This section introduces an adapted methodology for our systematic review to filter the number of related documents efficiently since Fig. 2 alerts us to numerous developments in WOA. Thus, as shown in Fig. 8, we adapted the PRISMA methodology [71] to select a reasonable number of the most reputable papers for full-text reading. The introduced methodology consists of three main stages as follows.

5.1 Identification Stage

In this stage, related keywords and alternative synonyms are defined as target search problems to search and extract documents from different databases. The defined keywords include "WOA", "whale algorithm", "whale optimization", "whale optimiser", "whale optimizer", "whale optimization algorithm", and "whale optimisation algorithm". Then, the defined keywords were searched in the Google Scholar database restricted from 2019 to the end of March 2023, and 2983 documents were identified in this stage.

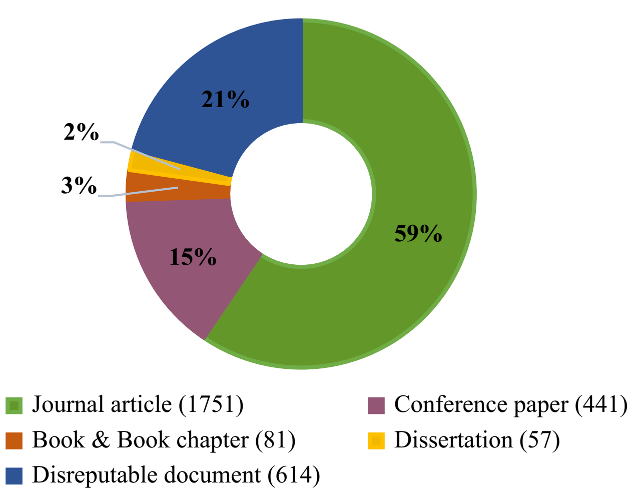


Fig. 8 The percentage of different WOA documents from 2019 to the end of March 2023

5.2 Evaluation Stage

This stage uses three screening steps to evaluate the identified documents and select eligible papers. In the screening steps, the Rayyan tool [67] is used for citation sharing and allowing comparison of decisions to include or exclude. Duplicate papers and those papers published by non-academic journals were removed in the first screening. There were 39 duplicated papers extracted from the Scopus and WOS databases. The remaining 2944 papers were checked for non-academic and disreputable journals to remove, and as shown in Fig. 9, 1751 papers published by different journals were selected for the next screening.

In the second screening, the title and abstract of candidate papers were investigated, 1096 papers were then excluded respecting the study scope, and 655 papers remained. The third screening has two steps; first, the full text of the candidate papers is read to select eligible papers using the inclusion criteria. The inclusion criteria considered for this study are listed as follows.

- C_1: Papers published from 2019 to the end of March 2023.
- C_2: Papers written in the English language.
- C_3: Non-conference papers and Non-short communications.
- C_4: Papers published in reputable journals.
- C_5: Papers focused on WOA as the main topic.
- C_6: Papers focused on developing WOA using improvement or hybridizing approaches.

Although some of these criteria were checked to reduce the number of candidate papers in the previous stage, they were carefully rechecked in this step by reading their full texts. This step selected 111 eligible papers and then increased the count to 116 by studying their references in the second step of the third screening to find more eligible papers based on inclusion criteria.

5.3 Reporting stage

In this stage, the 116 eligible papers collected from Rayyan as the final set are first classified by six main and other publishers. Then, they are reviewed for the qualitative synthesis

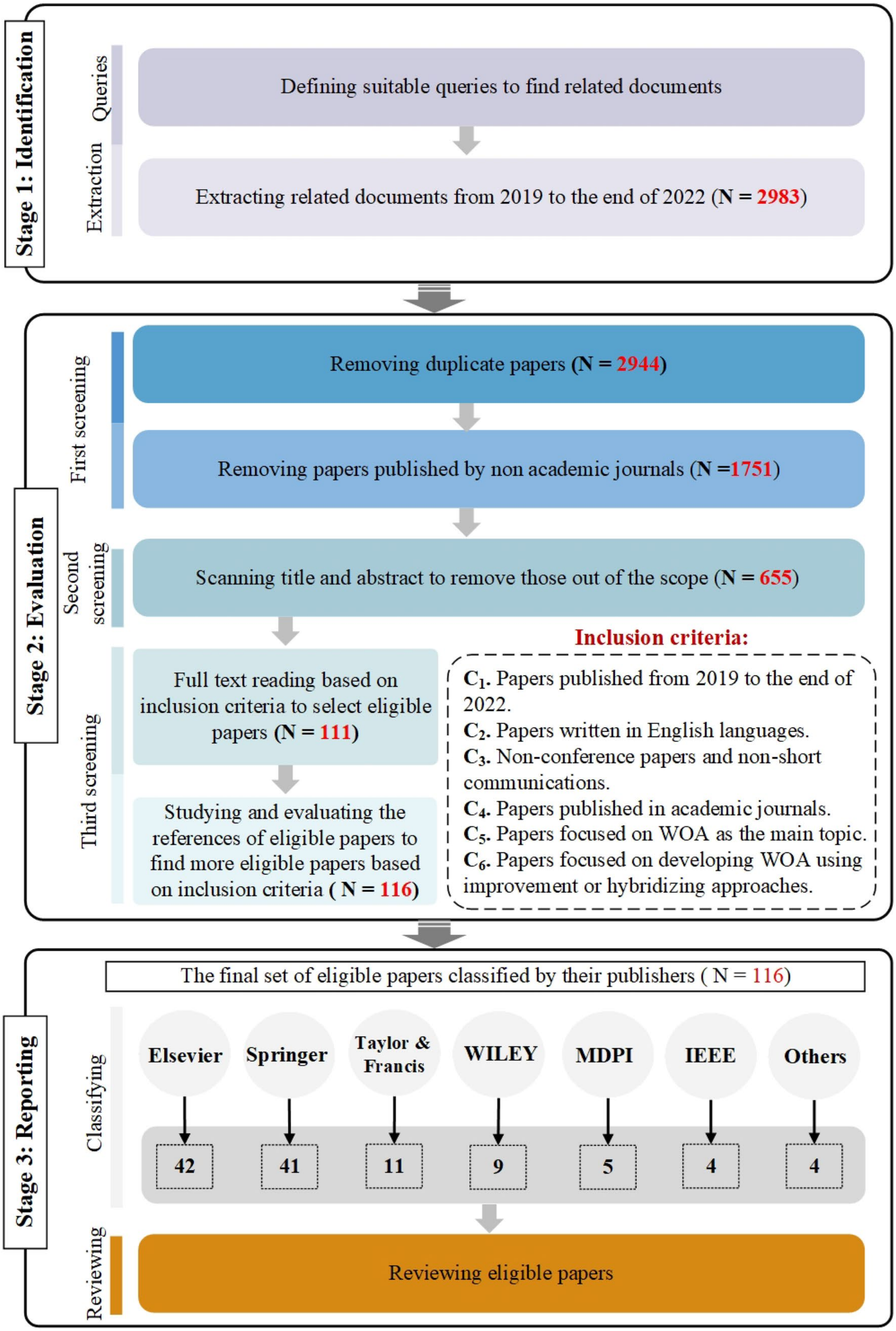


Fig. 9 The methodology of the systematic review

into two main categories, improved and hybrid WOA, with 59 and 57 eligible papers, respectively.

6 Different Approaches to Developing WOA

The WOA algorithm, like other metaheuristic algorithms, has drawbacks and limitations that make it less accurate and perform satisfactorily in some optimization problems. Experimental analysis and critical reviews [63, 64, 72] show that WOA suffers from poor search strategies [55], which leads to premature convergence [56], an unbalancing issue [57], stagnation far from local optima [58], and low population diversity [59]. Nevertheless, as shown in Fig. 10, WOA variants are used to tackle different problems in various applications, including computer and information technology (IT), electrical engineering, industrial and mechanical engineering, civil engineering and energy, medical and health, and others.

The WOA variants are mainly developed using two main approaches, improvement, and hybridization, to solve such problems. On the other hand, WOA solves continuous and single-objective problems that cannot be used for binary or multi/many-objective problems. Thus, many approaches have been proposed to develop functional binary and multi/many-objective variants of WOA. Therefore, this section explains different approaches for improving, hybridizing, binarization, and multi/many-objective WOA.

6.1 Improvement Approach

In the improvement approach, common techniques such as Lévy flight, chaotic map, opposition-based learning, mutation strategies, crossover operators, and quantum-based are usually used to improve the WOA search strategies. The common techniques used in this approach are briefly described as follows.

6.1.1 Lévy Flight Technique

Lévy flight is a statistical description of random motion in which the step lengths follow a Lévy stable distribution with heavy-tailed. The Lévy distribution is defined in Eq. (17) [73], where parameter μ is the shift parameter or location, and $\gamma > 0$ controls the scale of distribution.

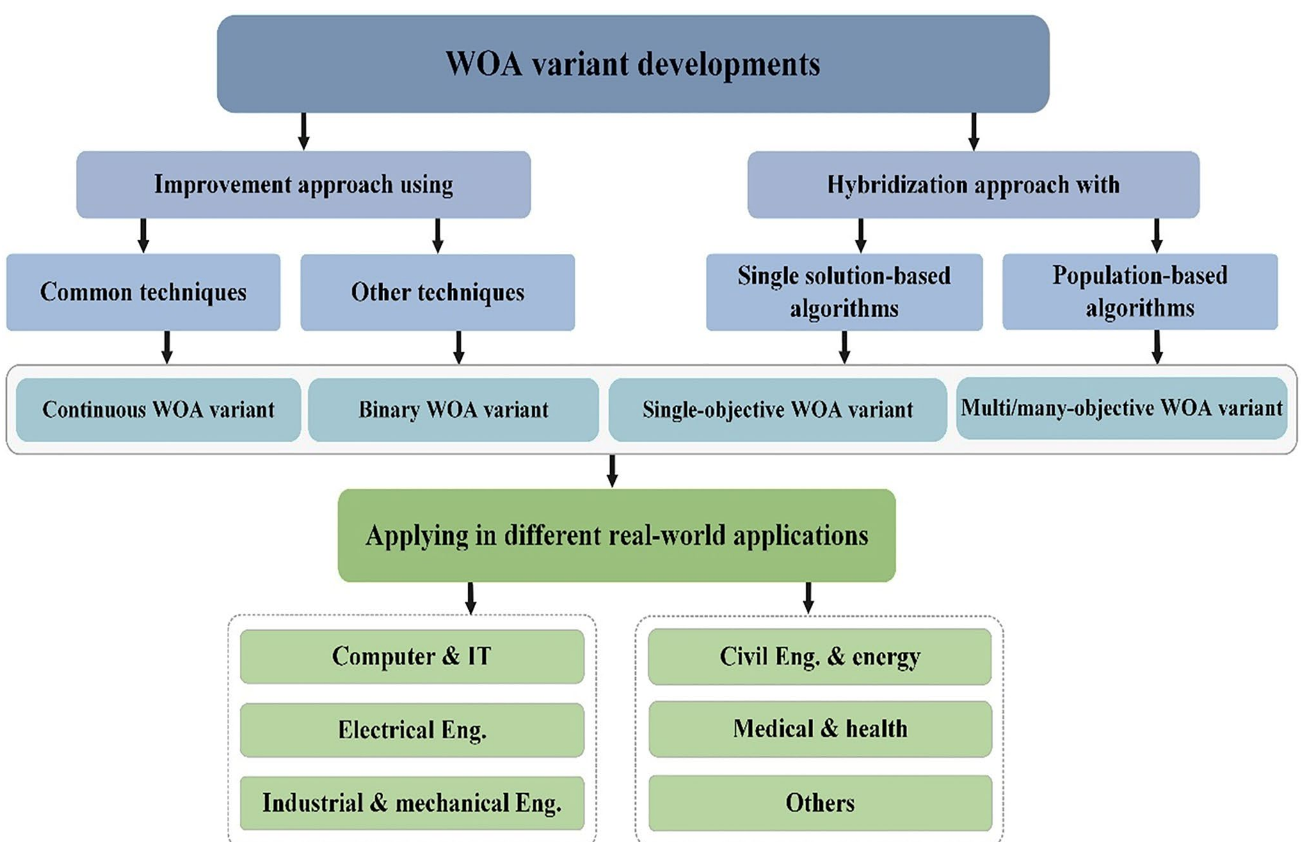


Fig. 10 The classification of WOA variant developments

$$L(S,\gamma,\mu)=\begin{cases}\sqrt{\frac{\gamma}{2\pi}}\exp\left[-\frac{\gamma}{2(s-\mu)}\right]\frac{1}{(s-\mu)^{\frac{3}{2}}} & 0<\mu<s<\infty\\ 0 & s\le 0\end{cases} \tag{17}$$

Typically, Lévy distribution is described by Fourier transform $F(k)=exp\left[-\alpha|k|^{\beta}\right]$, such that $0<\beta\le 2$ and scale factor α is within interval [− 1,1]. When $\beta = 1$, the distribution corresponds to the Cauchy probability distribution, and $\beta = 2$ corresponds to the Gaussian distribution [74]. Figure 11 shows the movement trajectory of Lévy flight for 1000 steps in 2-dimensional space with β values 1 and 2. The generation of random numbers with Lévy flight consists of two steps: choosing a random direction and generating the size of each step that follows the Lévy distribution [75], which is defined in Eq. (18) [73], where u and v are normal distributions, and Γ denotes standard Gamma function.

$$s = random \times Lévy(\beta) \sim 0.01\frac{u}{|v|^{1/\beta}}, u=\left(0,\sigma_u^2\right), v=\left(0,\sigma_v^2\right) \tag{18}$$

$$\sigma_u=\left\{\frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma(1+\beta/2)\beta 2^{(\beta-1)/2}}\right\}^{1/\beta}, \sigma_v=1$$

Lévy flight has been increasingly applied as an effective technique than the Brownian motion in metaheuristics optimization algorithms to incorporate step sizes that are randomly distributed. The Lévy flight technique can intensify local search by reducing the step size to discover new solutions near the best solution. Also, it can boost global searchability by increasing the step size. Most WOA variants improved using the Lévy flight technique show better exploration and exploitation abilities.

6.1.2 Chaotic Map Technique

A chaotic map is defined in mathematics as a random and complex phenomenon with a definite nature in the interior [76]. Chaotic systems are susceptible to their initial conditions, so a slight change in such systems' initial conditions will cause many future changes. The butterfly effect in chaos theory refers to this phenomenon [77]. Metaheuristic algorithms mainly apply randomness to generate diversification into the search process and escape from local optima. A recent metaheuristics research trend shows that replacing traditional pseudo-random number generators with chaotic maps can achieve more successful solutions to solve complex optimization problems. Thus, to boost the WOA algorithm, researchers use various chaotic maps such as logistic, sine, Chebyshev, circle, iterative, skew tent map, gauss/mouse, sinusoidal, and singer. These chaotic maps are used for generating the initial population, producing values in control parameters, and updating the position of search agents [59, 78]. The mathematical formulas of chaotic maps used in the WOA variants are reported in Table 3, where x_i can be defined based on the current position, random distribution, or other methods. Chaotic map techniques can enrich population diversity, prevent premature convergence, and speed up convergence [79]. Figure 12 shows the visualization of chaotic maps used in the WOA variants in 100 iterations.

6.1.3 Opposition-Based Learning (OBL) Technique

Since the first introduction of OBL [80], it has been used to improve many metaheuristic algorithms such as ABC [81], PSO [82], and CSA [83]. The adversarial relationship between entities inspires OBL, and its main idea is to simultaneously evaluate the current candidate solution and its opposite to find a better solution. The OBL technique is defined as follows. Suppose P $(x_1, \ldots, x_i, \ldots, x_D)$ be a candidate solution in D-dimensional space, where each real number x_i restricted between predefined interval $\left[a_i, b_i\right]$ and, $i = 1, 2, \ldots, D$. The opposite point of candidate solution P is denoted by P' $(x'_1, \ldots, x'_i, \ldots, x'_D)$ and each opposite point x'_i is computed using Eq. (19) [84].

$$x'_i = a_i + b_i - x_i \tag{19}$$

The fitness values of candidate solution P and its opposite are computed to select a point with the best fitness value and continue with the fitter one. The improved WOA

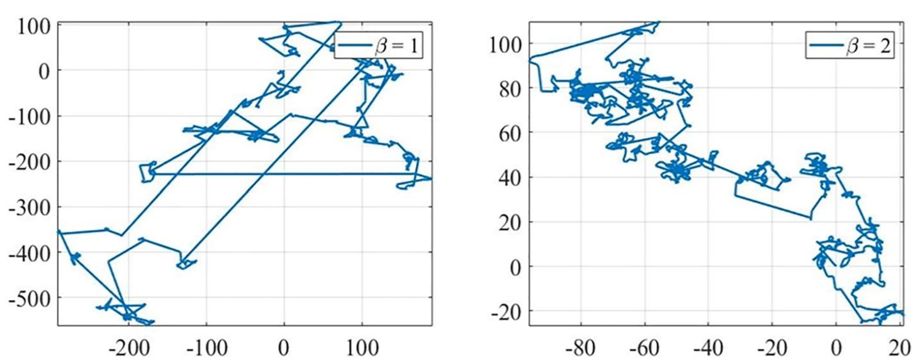


Fig. 11 The movement trajectory of Lévy flight technique for 1000 steps in 2-dimensional space

Table 3 The formulas of chaotic map used in the WOA variants

Chaotic map method	Formulas
Logistic map	$x_{i+1} = ax_i(1 - x_i), a = 4$
Sine map	$x_{i+1} = \frac{a}{4}\sin(\pi x_i), a = 4$
Chebyshev map	$x_{i+1} = Cos(iCos^{-1}(x_i))$
Circle map	$x_{i+1} = mod\left(x_i + b - \left(\frac{a}{2\pi}\right)\sin(2\pi x_i), 1\right)$, a = 0.5, b = 0.2
Iterative map	$x_{i+1} = \sin\left(\frac{a\pi}{x_i}\right), a = 0.7$
Skew tent map	$x_{i+1} = \begin{cases} x_i/\varphi & 0 \le x_i < \varphi \\ (1 - x_i)/(1 - \varphi) & \varphi \le x_i \le 1 \end{cases}, \quad \varphi \in (0,1) and x_i \in [0,1]$
Gauss/mouse map	$x_{i+1} = \begin{cases} 1 & x_i = 0 \\ \frac{1}{mod(x_i,1)} & Otherwise \end{cases}$
Sinusoidal map	$x_{i+1} = \mathrm{a}x_i^2\sin(\pi x_i), a = 2.3$
Singer map	$x_{i+1} = \mu \times (7.86x_i - 23.31x_i^2 + 28.75x_i^3 - 13.302875x_i^4), \mu = 1.07$

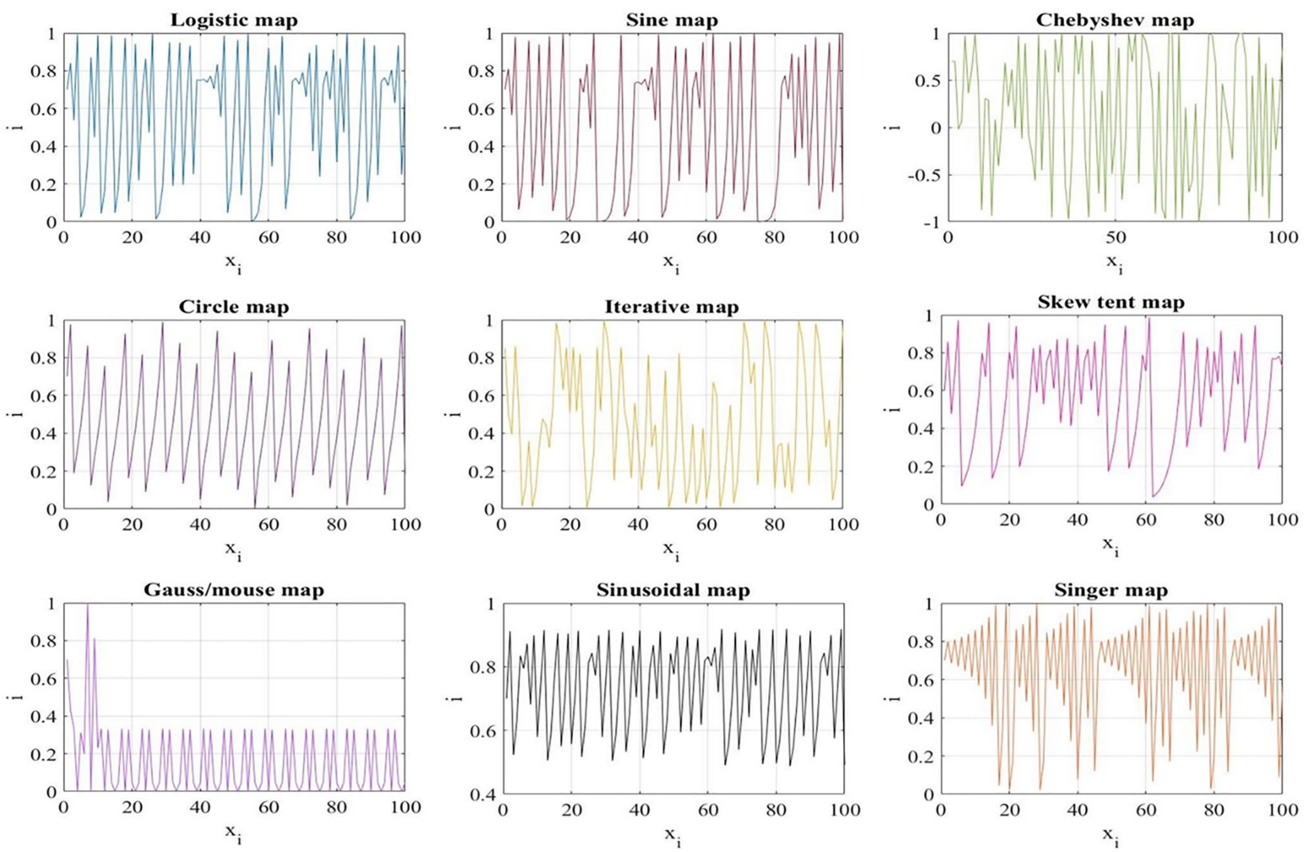


Fig. 12 The visualization of chaotic maps used in the WOA variants

algorithms using OBL show that this technique can arm the population diversity and exploration ability [85–87]. Figure 13 illustrates the OBL technique using a candidate solution P and its opposite in one and two-dimensional spaces [88].

6.1.4 Mutation Strategies and Crossover Operators

Mutation strategies and crossover operators are recognized as essential elements in evolutionary algorithms that could be adapted and used in other optimization algorithms [89]. Many

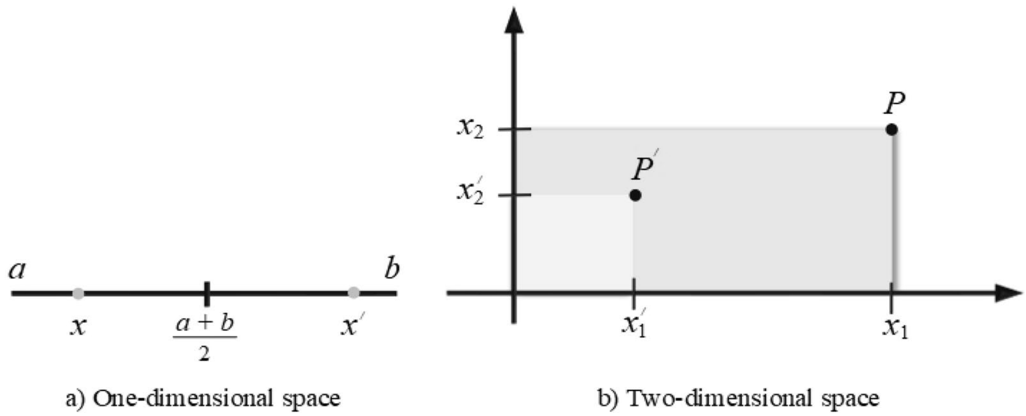


Fig. 13 The illustration of the OBL technique [88]

mutation strategies and crossover operators are proposed to increase the performance of these algorithms in handling diverse challenges. Properly selecting these elements may significantly impact the efficiency of algorithms [90].

Mutation strategies are carried out to enable random perturbations in the population. Various mutation strategies are proposed, primarily defined as DE/*x*/*y*/*z* [91], where DE stands for differential evolution, and *x* denotes a base vector to be perturbed. The symbol *y* is the number of difference vectors considered for perturbation *x*, and *z* indicates the crossover operator. The formulas of the most frequent mutation strategies are listed in Table 4, where $V_{i,g}$ is the mutant vector for the *i*th individual, $X_{i,g}$ is the position of the *i*th individual, and indices r_1, r_2, r_3, r_4, and r_5 are mutually selected different from index *i* [90, 91]. $X_{best,g}$ is the vector that obtained the best fitness value in the whole population at *g*th generation, and *F* is a positive scaling factor between interval (0, 1).

Each mutation strategy can provide different abilities. Mutation strategies can promote exploration and exploitation abilities. Mutation strategies DE/rand/1 and DE/rand/2 promote a high exploration ability by providing more perturbation. Mutation strategies DE/best/1, DE/best/2, DE/current-to-best/1, and DE/rand-to-best/1 intensify a high exploitation ability by searching around the current best individual or solution [91].

The crossover operator produces offspring $U_{j,i,g}$ by combining every individual with the corresponding mutant vector $V_{j,i,g}$ to extend their exploration ability in a much wider area of the problem space [89]. The most used crossover operators are binomial and exponential crossovers. The binomial crossover operator defined in Eq. (20), where NP is the total number of population, D is the total number of dimensions, $rnd_d(0,1)$ is a uniform random number for *d*th dimension, CR is a predefined crossover parameter between interval (0, 1), and parameter j_{rand} is an integer value that is randomly selected in the range of dimensions.

$$U_{j,i,g} = \begin{cases} V_{j,i,g} & if\ rnd_d(0,1) \le Cr\ or\ j = j_{rand}, \\ X_{j,i,g} & otherwise \\ i = 1, 2, \ldots, NP\ j = 1, 2, \ldots, D \end{cases} \tag{20}$$

6.1.5 Quantum-Based Technique

Quantum approximate optimization algorithms [92–94] have performed quantum computing concepts such as qubit [95], quantum rotation gate [96], and quantum encoding [97] for developing search strategies and control parameters. In quantum computation, the fundamental information element is called a qubit or a quantum bit. A qubit is defined using

Table 4 Some well-known mutation strategies [90, 91]

Name	Formula
DE/rand/1	$V_{i,g} = X_{r1,g} + F\left(X_{r2,g} - X_{r3,g}\right)$
DE/best/1	$V_{i,g} = X_{best,g} + F\left(X_{r1,g} - X_{r2,g}\right)$
DE/current-to-best/1	$V_{i,g} = X_{i,g} + F\left(X_{best,g} - X_{i,g}\right) + F\left(X_{r1,g} - X_{r2,g}\right)$
DE/best/2	$V_{i,g} = X_{best,g} + F\left(X_{r1,g} - X_{r2,g}\right) + F\left(X_{r3,g} - X_{r4,g}\right)$
DE/rand/2	$V_{i,g} = X_{r1,g} + F\left(X_{r2,g} - X_{r3,g}\right) + F\left(X_{r4,g} - X_{r5,g}\right)$
DE/rand-to-best/1	$V_{i,g} = X_{i,g} + F\left(X_{best,g} - X_{r1,g}\right) + F\left(X_{r2,g} - X_{r3,g}\right)$

Eq. (21) with two states of 0 or 1, where α and β are probability amplitudes. A quantum qubit can exist in state 0, state 1, or a combination of the two.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{21}$$

A quantum qubit found in the state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ with probability $|\alpha|^2$ and found in the state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with probability $|\beta|^2$. As shown in Eq. (22), the probability of the system being detected in either state $|0\rangle$ or $|1\rangle$ is equal to 1 [98].

$$|\alpha|^2 + |\beta|^2 = 1 \tag{22}$$

WOA quantum-based variants used such concepts to boost their performance. The structure of an N-qubit individual is represented in Eq. (23).

$$\begin{pmatrix} \alpha_1 & \dots & \alpha_i & \dots & \alpha_N \\ \beta_1 & \dots & \beta_i & \dots & \beta_N \end{pmatrix}, \quad i = 1, 2, \dots, N \tag{23}$$

A quantum rotation gate is a commonly used operator for updating the values of α and β denoted using Eq. (24) [98]. The qubit's probability amplitudes after rotation are represented by $\propto'_i$ and β'_i. Also, θ_i denotes the rotation angle of the ith qubit. For simplicity, Fig. 14 visualizes a qubit state vector updating in a Bloch sphere [98].

$$\begin{pmatrix} \alpha'_i \\ \beta'_i \end{pmatrix} = R(\theta_i) \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} \tag{24}$$

6.2 Hybridization Approach

Based on the No-free-lunch theorem [99], no optimization algorithm is superior to all others for solving different problems. Each optimization algorithm, even the most successful existing one, usually suffers from search strategy weaknesses which is the main reason for developing the huge number of optimization algorithms. Instead of proposing new algorithms, hybridizing existing algorithms' search strategies can be a shortcut to developing an effective search strategy to cope with different properties of complex problems. However, hybridization can enhance performance significantly if the participant algorithms are analyzed very well, and the reasons behind hybridization are to tackle their weaknesses.

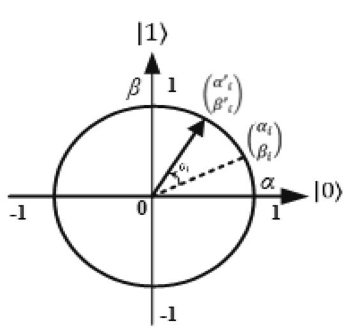


Fig. 14 Bloch sphere for updating qubit state vector

Hybridization methods are divided into two main categories: low-level and high-level [100]. Low-level hybridization algorithms perform the functional composition of a single optimization method by replacing a given function of a metaheuristic with another one [100]. High-level hybrid algorithms are self-contained, and there is no direct association with the internal mechanisms of a metaheuristic. There are different frameworks to hybrid some algorithms and form a meaningful search strategy by incorporating some search strategies, which can add up the advantages of each other and simultaneously increase performance [94, 100, 101]. However, real-world optimization problems have many complex properties, such as complexity, non-linear constraints, discrete search spaces, and high dimensionality. Therefore, various search strategies with the necessary properties are needed to develop an effective hybridization, usually found in two primary algorithms: ones that rely on a single solution and those that focus on a population. Accordingly, there have been developed many WOA variants by hybridizing with different optimization algorithms from these categories [102]. The results show that hybridizing WOA with algorithms from these two categories can significantly enhance the canonical WOA's performance [103–105].

6.2.1 Hybridizing with Single Solution-Based Algorithms

These algorithms perform a local search process by manipulating a single solution to improve this solution in its neighborhood [106]. Single solution-based algorithms focus on exploitation and are recognized as suitable candidates for hybridizing with WOA that suffer from poor exploitation ability. There are a few hybridizing between WOA with single solution-based algorithms such as tabu search (TS) [107] and hill climbing (HC) [108].

6.2.2 Hybridizing with Population-Based Algorithms

Population-based metaheuristic algorithms employ search agents to explore a problem space and capture the promising areas in the earlier stage of the optimization process. Then, they eventually search the vicinity of the discovered regions to achieve a near-optimal solution using local search strategies. Population-based metaheuristic algorithms benefit effective initialization mechanisms, meaningful search strategies, striking a trade-off between exploration and exploitation, and fine-tuning control parameters to achieve a near-optimal solution without trapping in local optima and losing the population diversity. Therefore, a wide range of population-based metaheuristic algorithms is suitable for hybridizing with WOA, especially in three main categories: evolutionary, physics-based, and swarm intelligence algorithms.

6.2.3 Hybridizing with Evolutionary Algorithms

Evolutionary algorithms are efficient bio-inspired algorithms inspired by biological evolution, such as mutation, selection, reproduction, and recombination [109]. Genetic algorithm [110], differential evolution [111], and evolutionary strategy are the prominent evolutionary optimization algorithms to capture global solutions in solving complex optimization problems. Evolutionary algorithms can approximate the optimal region early in the evolutionary process and maintain the population diversity in the final iterations using properties such as mutation, crossover, selection, self-adaptation, and external archives. These properties caused many hybrid WOA variants to be proposed, such as [103, 112, 113] since the introduction of WOA.

6.2.4 Hybridizing with Physics-Based Algorithms

These algorithms apply mathematical rules, laws of motion, and physical concepts such as quantum theory [22], optic [19], and Newton's gravitational law [18] to design effective search strategies which can effectively approximate the promising areas. To improve the search strategies of WOA, researchers hybridized it with physics-based algorithms such as the sine cosine algorithm (SCA) [102] and thermal exchange optimization (TEO) [114].

6.2.5 Hybridizing with Swarm Intelligence (SI) Algorithms

SI algorithms mimic the collective and simple behavior of social organisms such as insects, birds, and animals to develop efficient optimizers. Because of its straightforward design with limited control parameters and an information-sharing mechanism, SI algorithms are recognized as widespread and most-used optimizers for hybridizing with other algorithms. Accordingly, many hybrid WOA variants have been developed by hybridizing with SI algorithms such as particle swarm optimization (PSO) [115], artificial bee colony algorithm (ABC) [116], grey wolf optimizer (GWO) [117], firefly algorithm (FA) [118], slime mould algorithm (SMA) [119], dragonfly algorithm (DA) [120], seagull optimization algorithm (SOA) [121], woodpecker mating algorithm (WMA) [105], and salp swarm algorithm (SSA) [122].

6.3 Binarization Approach

Like most metaheuristic algorithms, WOA has been fundamentally developed for continuous search spaces; however, many complex real-world optimization problems with discrete search spaces exist [123]. Hence, the binarization approach maps continuous search space to a binary using binarization techniques [124]. In the binarization approach, the continuous value of each dimension is mapped to either 1 or 0 [125]. Thus, in the binary problem defined in Eq. (25), the objective function should be minimized (or maximized), where x_j is a decision variable, and D is the problem's dimension [126].

$$\begin{aligned} &min/max\ F(x_1, x_2, \ldots, x_j, \ldots, x_D), \\ &\forall x_j \in \{0, 1\} and\, j = 1, 2, \ldots, D. \end{aligned} \tag{25}$$

The performance of binary optimization algorithms strongly depends on their searchability and binarization techniques. The binarization techniques applied in binary WOA variants are transfer functions, binary operators, and variable threshold methods [127]. The most used transfer functions involved in binarizing WOA are the S-shaped transfer function [128], the V-shaped transfer function [129], and the U-shaped transfer function [55]. The sigmoid or S-shaped transfer function variants can enhance the performance of the binary WOA variants [130, 131] by providing a higher bit-flipping probability with a smaller absolute value, encouraging a better exploration ability in the early stages. Also, a considerable absolute value can achieve better exploitation ability in the final stages of the optimization process [131, 132]. The variants of the V-shaped transfer function can alleviate the weak points of sigmoid transfer function variants to solve binary optimization problems [133]. The semantics of different transfer functions are shown in Fig. 15.

6.4 Single and Multi/Many-Objective Approach

The main goal for solving any optimization problem is to locate the best acceptable solution, given the constraints and requirements of the problem [134]. For each problem, an objective function is established to compute the fitness values of solutions. Defining the suitable objective function for each problem is one of the essential steps in optimization [135, 136]. Depending on the problem's nature and the objective function, optimization problems are generally categorized into three main categories: single-objective, multi-objective, and many-objective.

Single-objective problems have only one objective function defined, and the objective is to find a single solution representing the global optimum in the problem space [137]. In

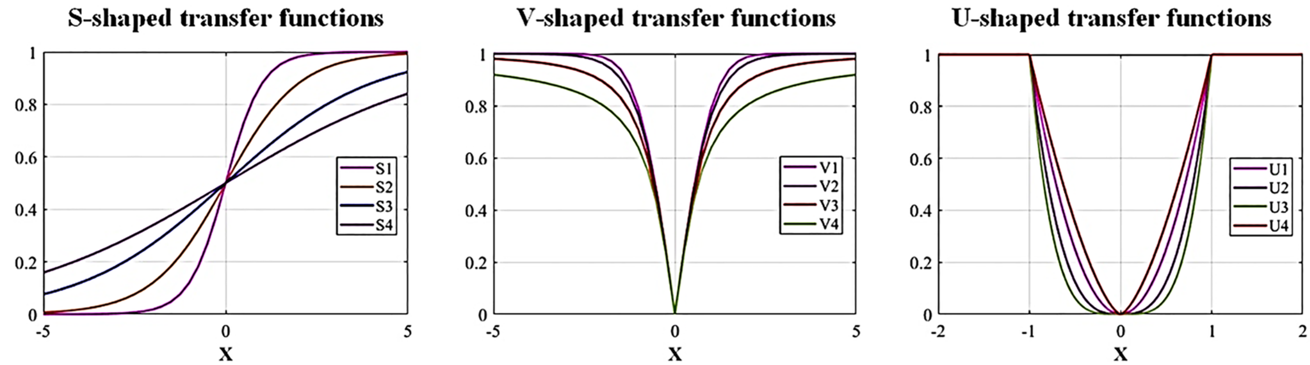


Fig. 15 The semantics of transfer functions used in binary WOA variants

general, a single-objective optimization problem is defined using Eq. (26), where $F(x)$ denotes a minimization (or maximization) objective function and X is decision variables.

$$min/max\ F(x)x \in R, \quad j = 1, 2, \dots, D \tag{26}$$

In solving many optimization problems, considering a single objective function cannot show all the characteristics of the desired system. Hence, it is necessary to use multi-objective optimization algorithms (MOO) to describe multiple aspects of the problem, represented by multiple objective functions [138, 139]. The MOO problems involve two or less than five conflicting objectives that must be optimized simultaneously [140]. In optimization problems, multi-objective algorithms are not responsive when the number of desired objectives increases. Many-objective algorithms are proposed [141], the same as multi-objective with more than four objectives [142]. A multi/many-objective optimization problem is mathematically defined using Eq. (27) [143] to deal with M objectives of equal importance, and N denotes the number of solutions. g and h show the inequality and equality constraints, respectively. An unconstrained multi-objective optimization problem is defined when $p = q = 0$. Each variable x_i is restricted between lower bound x_i^L, and upper bound x_i^U.

$$F : X \rightarrow R^M \tag{27}$$

$$min/max\ F_1(x), F_2(x), F_3(x), \dots, F_M(x)$$

$$\text{Subject to } g_j(x) \geq 0, \quad j = 1, 2, \dots, J$$

$$h_k(x) = 0, \quad k = 1, 2, \dots, K$$

$$x_i^L \leq x_i \leq x_i^U, \quad \text{i} = 1,\ 2,\ \dots\ N$$

Each object should be minimized or maximized and set X denotes the feasible set of decision vectors representing parameters for the values selected to satisfy constraints. Maximizing an objective function is the same as minimizing its negative or inverse. Pareto and scalarization are common methods to solve MOO problems that do not require complicated mathematical equations. The Pareto method uses a continuously evolving algorithm to produce a dominant and a non-dominant solution. The main aim of solving multi-objective problems is to find an optimal solution set of objective functions called the "Pareto Front" [140, 144]. A point $x.^\star$ in the search space (X) is called Pareto optimal if there is no other point like $x \in X$, as shown in Eq. (28) for at least one function F_i [140]

$$\forall\, i\, F(x) \leq F\left(x^\star\right) \text{and } F_i(x) \leq F_i\left(x^\star\right) \tag{28}$$

Meanwhile, the scalarization method transforms multiple objectives into a single solution using weights to form a scalar objective function [145]. There are three scalarization weights: equal weights, rank-order centroid weights, rank-sum weights, and weighted sum. Weighted sum scalarization, defined in Eq. (29) [146], is a frequently used method that transforms a multi-objective into a single-objective optimization problem, where W_m is the predefined weight for mth objective. One of the most popular multi-objective problems solved using the scalarization method is a feature selection problem in which objectives are linearly combined with predefined weights and turned into a single-objective problem [147]. Many multi-objective optimization problems have been solved by WOA multi-objective algorithm [148–150].

$$\text{Maximize/Minimize } F(x) = \sum_{m=1}^{M} W_m f_m(x) \tag{29}$$

7 Improved WOA Variants

Using the adapted methodology introduced in this study, including the three screening steps and strict inclusion criteria, 116 papers were collected to review in two categories, improved and hybrid WOA variants. This section reviews improved WOA variants, including 59 papers that were improved using either the common techniques of Lévy flight, chaotic map, opposition-based learning, mutation, and crossover, or quantum-based or other techniques. Figure 16 shows the usage percentage of these techniques in improved WOA variants in which an improved paper could use more than one common or different technique. For example, mutation & crossover and chaotic map are the most common techniques for improving WOA, with 16%; moreover, the usage percentage of other techniques, such as adaptive parameter strategies, is 39%. The improved WOA variants are described in the following section as two main groups, continuous and binary.

7.1 Improved Continuous WOA Variants

Most (49 of 59) improved WOA variants were developed to solve continuous problems. This section reviews them in two groups, single-objective and multi/many-objective.

7.1.1 Improved Continuous Single-Objective WOA Variants

This subsection reviews continuous WOA variants that have used common techniques, including Lévy flight, chaotic map, opposition-based learning, mutation strategies, crossover operators, or other techniques to improve WOA for solving continuous single-objective problems.

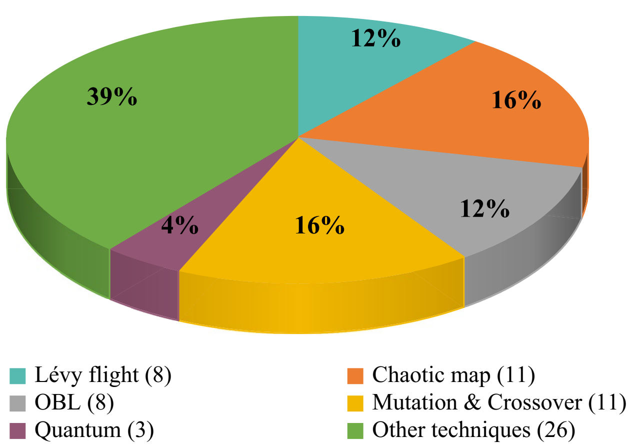


Fig. 16 Percentage of using common and other techniques in improved WOA variants

7.1.1.1 Using Common Techniques The improved continuous single-objective WOA variants that have used at least one common technique are reviewed and summarized in Table 5. Chen et al. [151] proposed a balanced whale optimization algorithm (BWOA) using Lévy flight and chaotic local search strategy to alleviate weaknesses of slow convergence rate and getting stuck in the local optimum. BWOA uses the Lévy flight technique to update the position of search agents and a chaotic local search strategy to exploit the vicinity of the best solution. Although the experimental results reveal that BWOA is an effective and efficient helper tool for complex optimization models and scenarios, the applicability of the BWOA algorithm must be investigated for practical cases, especially in engineering optimization control. Abdel-Basset et al. [152] introduced an enhanced Lévy-based whale optimization algorithm for efficient virtual machine placement in cloud computing environments, focused on maximizing bandwidth utilization. The Lévy flight technique adjusts the whale movement with a more diffuse distribution, enhancing the exploration ability to find more suitable solutions. The experimental results show that this algorithm could minimize the number of physical machines running. Still, its robustness must be assessed in cloud computing applications such as multi-objective VM placement, dynamic migration, and cloud scheduling.

The DEWCO [153] introduced a hyper-heuristic WOA to enhance the exploration ability and local optima avoidance of WOA within two stages. In the first stage, the differential evolution algorithm finds the optimal configuration from the chaotic map, OBL technique, and a portion of the population. In the second stage, this optimal configuration is applied to improve the performance of WOA for solving the optimization problem. The experimental results show that the DEWCO can find the optimal solutions on test functions better than the other compared algorithms. However, further experimental evaluations are needed to assess the applicability of DEWCO. Singh [154] developed the Laplacian whale optimization algorithm (LXWOA), which performed better than WOA and gradient algorithms in solving classical benchmark functions. LXWOA performs the Laplace crossover operator in the best and randomly selected whale to avoid immature convergence and stagnation problems. Zhang et al. [155] introduced the Gaussian mutation and differential evolution operators to boost the performance of WOA to locate electric vehicle charging stations with service capacity. Results show that this algorithm effectively finds planning problems and reduces social costs. To accurately predict the water resources demand in the Shaanxi province of China, another improved WOA based on social learning and a wavelet mutation strategy named SMWOA [156] was proposed. SMWOA uses an adaptive social learning strategy and wavelet jumps to estimate optimal model parameters. SMWOA can strike a better balance between

Table 5 Improved continuous single-objective WOA variants that use at least one common technique

Author and reference	Year	Common techniques	Other techniques	Evaluation benchmarks	Applications
Chen et al. [151]	2019	Lévy flight and chaotic map	–	–	Industrial and mechanical Eng
Abdel-Basset et al. [152]	2019	Lévy flight	–	–	Computer and IT
Abd Elaziz et al. [153]	2019	Chaotic map and opposition-based learning	Population ratio	CEC 2005 test functions	–
Singh [154]	2019	Crossover operators	–	CEC test functions	–
Zhang et al. [155]	2019	Lévy flight and mutation strategy	–	CEC test functions	Electronic Eng
Guo et al. [156]	2020	Mutation strategy	Social learning	CEC test functions 2017	Civil Eng. and energy
Abdel-Basset et al. [157]	2020	Mutation strategy	Local search strategy	–	Computer and IT
Deepa et al. [158]	2021	Lévy flight	–	–	Computer and IT
Nadimi-Shahraki et al. [159]	2021	Lévy flight	Brownian motion	–	Electronic Eng
Huang et al. [160]	2021	Lévy flight	–	CEC test functions	Civil Eng. and energy
Gao et al. [59]	2021	Chaotic map	Non-linear strategy and the inverse incomplete function	CEC test functions	Computer and IT
Priyanga et al. [78]	2021	Chaotic map	–	–	Medical and health
Saravanan et al. [85]	2021	Chaotic map	–	–	Computer and IT
Fan et al. [86]	2021	Chaotic map and opposition-based learning	Adaptive inertia weight and joint search mechanism	CEC test functions	–
Liu et al. [56]	2022	Lévy flight	Adaptive weights and judgment mechanism	CEC test functions	–
Deng et al. [161]	2022	Chaotic map and opposition-based learning	Adaptive coefficients	CEC test functions	–
Sun et al. [87]	2022	Chaotic map	Inertia weight strategy and feedback strategy	CEC test functions	–
Huang et al. [162]	2022	Chaotic map	Segment control parameters and Adaptive weights, an adaptive learning factor, and a Cauchy perturbation strategy	–	Electronic Eng
Cao et al. [57]	2022	Opposition-based learning	Adaptive inertia weight	CEC test functions 2017	Industrial and mechanical Eng
Gao et al. [163]	2022	Lévy flight	Adaptive factors	–	Electronic Eng
Li et al. [164]	2023	DE/rand/1 mutation strategy and Lévy flight mechanism	Elite reverse learning strategy and nonlinear convergence factor	CEC test functions 2017	–
Wang et al. [165]	2023	Mutation strategy	Adaptive adjustment method, enhanced bubble-net attack, roulette selection operator, and improved removing similarity operation	CEC test functions 2017	Computer and IT

exploitation and exploration and increase the algorithm's ability to escape from local optima.

Abdel-Basset et al. [157] proposed a modified hybrid whale optimization algorithm (MHWA) for the scheduling problem in multimedia data objects. The solutions' quality is improved by repeatedly applying mutation and reverse block insertion operations. Experimental results show that MHWA can find the best makespan and reduce the average lateness of jobs. The MHWA was not evaluated for other practical problems. Deepa et al. [158] improved WOA using Lévy flight (LWOA) to find where nodes are deployed in wireless sensor networks. The Lévy flight technique can improve the WOA's exploration ability and prevent falling into the local optimum. In addition, LWOA dramatically enhances the coverage of nodes, leading to an overall improvement in network performance. Nadimi-Shahraki et al. [159] proposed an effective whale optimization algorithm to solve the optimal power flow (EWOA-OPF) problem. EWOA-OPF uses the Lévy motion to modify the prey encirclement, and the prey search strategy is boosted using the Brownian movement. The results show that EWOA-OPF can improve the exploitability and maintain an appropriate balance between the search strategies. The EWOA-OPF algorithm can find promising solutions for large-dimensional OPF problems with more control parameters.

Huang et al. [160] proposed the LWOA algorithm using the Lévy flight technique to improve the convergence speed of WOA for structural damage identification applications. LWOA outperforms other algorithms in terms of computational efficiency and accuracy. It can precisely locate damage in various structures and enhance the speed of damage detection in beam and frame structures. Other real-world applications were not considered when evaluating LWOA. Gao et al. [59] improved WOA using the chaotic map technique and a non-linear strategy to cope with local optima trapping and low convergence speed (STNWOA). In the population initialization phase, the skew tent chaotic map increases the population diversity. Moreover, the incomplete inverse function Γ is used to dynamically adjust the convergence coefficient and strike a balance between search strategies. The experimental results show that the STNWOA algorithm attains the best convergence rate and optimization accuracy with more computational cost.

Priyanga et al. [78] proposed a combined framework for predicting heart disease using electronic health records, which consists of a recurrent neural network-logistic chaos-based whale optimization. They used chaos theory in encryption techniques to develop secure image encryption methods. The comparative analysis results reveal the LCBWO algorithm's substantial effectiveness as it provided accurate predictions. However, further experimental analysis is needed to examine the framework's reliability in different datasets and especially analyze risk levels associated with heart disease. Saravanan et al. [85] proposed a coefficient-improved whale optimization algorithm (CI-WOA) by hybridizing WOA with two chaotic maps, piece-wise linear and logistic, for image encryption. The results indicate that the CI-WOA can find a better solution than other comparative algorithms by maximizing the information entropy between the cipher image and the original. However, experiments were limited only to image encryption. Fan et al. [86] introduced an enhanced WOA algorithm combining chaotic mapping and joint search (JSWOA) to tackle high-dimensional global optimization challenges. JSWOA uses a chaotic tent map and adaptive inertial weighting to preserve the population diversity and improve the convergence speed, respectively. In addition, JSWOA updates the position of whales using the opposition-based learning mechanism to enhance the whale population's diversity. The results indicate that the JSWOA algorithm outperforms WOA and rival algorithms regarding solution precision and convergence speed for high-dimensional global optimization, as shown by experimental results.

The EGE-WOA algorithm [56] improves the convergence behavior of WOA based on convergent dual adaptive weights and enhances global discovery efficiency based on judging the predation status mechanism. in addition, the EGE-WOA performs the Lévy flight technique to improve global searchability. The experimental results show that the EGE-WOA can enhance the WOA's efficiency in global exploration and improve its convergence behavior. Furthermore, the effects on some constrained real cases show that the EGE-WOA outperforms other algorithms considering the mean and standard deviation. Deng et al. [161] introduced the improved whale optimization algorithm (IWOA) to address the limitations of the canonical WOA and better solve global optimization problems. IWOA initializes the population using the chaotic map technique and then merges the scent of the black widow algorithm with the opposition-based teaching approach to enhance the convergence speed and the exploration ability of the canonical WOA. Simulation results show that IWOA performs better than other comparative algorithms regarding convergence speed, stability, accuracy, and global performance.

Sun et al. [87] proposed a modified WOA based on a multi-strategy named MSWOA to improve the convergence speed and stagnation issues. MSWOA uses a tent map to boost the population distribution in the initialization phase, iteration-based update strategies to balance search strategies, and an optimal feedback strategy to strengthen the global search ability. The results indicate that MSWOA outperforms the other algorithms in the convergence factor and stability. The performance of MSWOA needs to be assessed in real-world applications. Huang et al. [162] proposed an improved whale optimization algorithm to adapt the parameter-adaptive proportional-integral-differential (PID). The optimal PID parameters are obtained using the IWOA algorithm. The IWOA algorithm uses the chaotic map in the initialization phase of the optimization process to improve the convergence speed and alleviate the tendency to fall into the local optima. Also, it uses segment control parameters and adaptive weights to improve the global search ability in early iterations and the convergence speed in late iterations. Moreover, IWOA is eqipped with an adaptive learning factor and Cauchy perturbation strategy to control the variability of each individual's learning ability and prevent local optima trapping. Results show that the IWOA-PID algorithm can accurately retrieve the boundary heat flux variation in real time with proper resistance and self-adaptability.

Cao et al. [57] proposed an enhanced whale optimization algorithm (EWOA) using improved dynamic opposite learning (IDOL) and adaptive inertia weight strategy to overcome WOA issues. They use the adaptive inertia weight strategy to modify the location of the prey to prevent it from getting stuck in a suboptimal solution. The EWOA balanced the ability to explore and exploit to handle global optimization and has better results than other algorithms. However, some potential research work still exists to apply IDOL to different metaheuristic algorithms that lack proper balancing. Gao et al. [163] introduced a photovoltaic output power prediction model using the hybridization of an improved whale optimization algorithm (IMWOA) and a support vector machine (SVM). IMWOA optimizes the kernel function parameter and penalty coefficient values in SVM for solving short-term photovoltaic power prediction. The findings demonstrate that IMWOA outperforms other algorithms regarding photovoltaic output power prediction accuracy. Li et al. [164] proposed the modified WOA (MWOA) algorithm with a multi-strategy mechanism to solve numerical optimization problems. The MWOA algorithm uses the elite reverse learning strategy, nonlinear convergence factor, DE/rand/1 mutation strategy, and Lévy flight disturbance strategy to improve convergence speed and balance between search strategies and avoid local optima trapping. Wang et al. [165] proposed a whale optimization algorithm with combined mutation and removing similarity (CRWOA) to solve global optimization and multilevel thresholding image segmentation. The CRWOA algorithm uses the adaptive adjustment method in the encircling prey stage and an enhanced bubble-net attack update formula to improve the local search ability. Moreover, it uses a roulette selection operator to randomly select whales and strengthen the possibility of generating outstanding whales in the random search stage. Furthermore, a combined mutation operator based on fitness value balances the exploration and exploitation abilities and maintains the population diversity using the improved removing similarity operation.

7.1.1.2 Using Other Techniques In this subsection, the improved continuous single-objective WOA variants that have used one or more techniques except common techniques are reviewed and outlined in Table 6. Sun et al. [58] introduced an improvement of WOA using a quadratic interpolation operator (QIWOA) to enhance the performance of the WOA algorithm in solving high-dimensional opti-

Table 6 Improved continuous single-objective WOA variants that use other techniques

Author and reference	Year	Improvement techniques	Evaluation benchmarks	Application
Sun et al. [58]	2019	Quadratic interpolation process	CEC test functions	–
Zhang and Liu [166]	2019	Lamarckian learning	CEC test functions	–
Azizi et al. [167]	2019	Fuzzy logic controller	–	Industrial and mechanical Eng
Bozorgi et al. [168]	2019	Re-initialization and adaptive parameter strategies	CEC test functions	–
Qiao et al. [169]	2019	Adaptive change strategy and introducing jump behavior	CEC test functions	Industrial and mechanical Eng
Khadanga et al. [170]	2020	Correction factor	CEC test functions	Electronic Eng
Qiao et al.[171]	2020	Adaptive search-surround mechanism and jumping behavior	–	Industrial and mechanical Eng
Jiang et al. [172]	2020	Multi-population strategy	CEC test functions	Industrial and mechanical Eng
Heidari et al. [173]	2020	β-hill climbing and Associative learning approaches	CEC test functions 2017	–
Sun et al. [174]	2021	Dividing population	CEC test functions	–
Anitha et al. [175]	2021	Multi-level thresholding	–	Computer and IT
Chakraborty et al. [176]	2021	Selection parameter and a non-linear weight vector	CEC test functions	–
Paul et al. [177]	2022	Correction factors	–	Electronic Eng
Chakraborty et al. [178]	2022	Random solution selection process and cooperative hunting strategy	CEC test functions 2017	Computer and IT
Mohammadbeigi et al. [179]	2022	Sequential quadratic programming	–	Civil Eng. and energy
Shen et al. [180]	2022	Multi-population evolution	CEC test functions 2019	Industrial and mechanical Eng
Mohanty et al. [181]	2022	Sine function	CEC test functions	Electronic Eng
Liu et al. [182]	2023	Global random position, information exchange mechanisms, and a random disturbance factor	–	Computer and IT
Chakraborty et al. [183]	2023	Local elite method and inertia weight	CEC test functions 2019	Computer and IT

mization problems. Zhang and Liu [166] developed WOA by proposing the WOALam algorithm based on Lamarckian learning for global optimization problems. WOALam initializes the population based on the excellent point sets theory and uses the upper confidence limit algorithm to measure the search agents' development potential. Then WOALam, based on Lamarck's evolutionary theory, selects search agents with more significant development potential to perform an advanced local search. The results proved that WOALam could balance exploration and exploitation abilities and have a better convergence rate and accuracy than the other algorithms.

Azizi et al. [167] introduced an upgraded whale optimization algorithm (UWOA) for fuzzy logic-based vibration control as an active vibration control algorithm in a nonlinear steel structure. UWOA adjusts the parameters of the fuzzy controller system using the continuous-time concept. The UWOA algorithm can provide competitive results based on different earthquake records. However, UWOA's assessment was limited to only one problem, and other possible tests were not evaluated. To improve the efficiency of WOA, Bozorgi et al. [168] developed an improved whale optimization algorithm (IWOA) by combining the exploration capability of differential evolution with the exploitation capability of WOA. In addition, IWOA utilizes re-initialization and adaptive parameter techniques to regulate its search process, resulting in improved final solutions and faster convergence than other algorithms. The robustness of IWOA can be tested for multi-objective optimization and training neural networks. Qiao et al. [169] introduced a predictive model that merges an enhanced whale swarm algorithm with a relevance vector machine for precise short-term forecasting of natural gas load. It assists model implementation using empirical mode decomposition, approximate entropy, and the C–C method. The findings demonstrate that this prediction model can escape from local optima. Moreover, it has superior optimization performance and convergence speed than other models, but its computational time is relatively long.

Khadanga et al. [170] proposed the MWOA algorithm using a new search strategy to improve WOA run time and solution quality for the load frequency controller design of a two-area power system composed of a photovoltaic grid and thermal generator. The results of the MWOA algorithm show that load frequency control is more efficiently managed using the MWOA-based proportional–integral–derivative with filter (MWOA-based PIDF) compared to the traditional proportional–integral–derivative controller. However, experiments were limited to this problem, and other practical cases were not evaluated. Qiao et al. [171] proposed an enhanced WOA (IWOA) using a Volterra adaptive filter, chaotic characteristics recognition, and phase space reconstruction to dynamically determine the model input and accurately predict short-term natural gas consumption. Since IWOA demonstrates more effective results as a prediction model than advanced models, IWOA robustness should be assessed in other fields.

Jiang et al. [172] proposed the whale army optimization algorithm (WAROA) to solve global optimization and constraint engineering problems. WAROA divides the whale population into two search and deep prey units. The relationship between these units determines the most astute and intelligent members as commanders to guide the deep prey group to search for the underlying local optimum. In addition, WAROA adjusts the parameters in both the exploitation and exploration phases to improve its performance. The results show that the WAROA algorithm improves convergence with lower computational complexity than other algorithms. The findings prove the superiority of WOROA in solving large-dimensional problems. However, the scalability of WOROA and its application to other real-world problems has not been investigated. Heidari et al. [173] proposed the BMWOA algorithm using the local search engine β-hill climbing algorithm and associative learning approaches to boost the exploitation capability of the canonical WOA. The experimental results on multi-dimensional problems show the superiority of the BMWOA compared to other competitive algorithms. The BMWOA needs further enhancement regarding exploration and exploitation.

Sun et al. [174] presented an improved multi-population WOA (MIWOA) to enhance the performance of WOA in solving high-dimensional optimization problems. MIWOA applies multi-population exploitation and exploration processes to divide the population into better and worse subpopulations. The better subpopulations improve exploitation ability, and the worse subpopulations improve exploration ability. Then MIWOA uses the current optimal whale and weighted center to enhance learning and the interpolation method to improve exploitation by searching near the current optimum solution. Also, MIWOA balances the exploitation and exploration abilities using a control parameter. Selecting threshold values for multi-level thresholding is very important in color image segmentation. Anitha et al. [175] presented a modified whale optimization algorithm (MWOA) to optimize the threshold selection for a multi-level color image thresholding problem. MWOA uses Otsu's and Kapur's functions as a fitness function. In addition, MWOA controls the whales' position by adapting the cosine function and regulating search agents' movements using the correction factors in position updates. MWOA yields better results than other competitive algorithms regarding image quality, feature conservation, and convergence rate. However, MWOA was evaluated only with this problem; other practical tests were not considered.

Chakraborty et al. [176] enhanced the performance of the canonical whale optimization algorithm using a unique selection parameter, co-efficient vectors, and a non-linear weight vector to solve large-scale optimization problems. A unique selection parameter can adequately balance global and local search strategies. Co-efficient vectors increase the searchability, and a non-linear weight vector can boost the exhaustive search nearby the best solution. Test results show better performance of this algorithm than other comparative algorithms on higher-dimensional problems. To reduce congestion in the transmission system, Paul et al. [177] proposed an improved WOA algorithm for optimal rescheduling of generators. The MWOA algorithm uses two correction factors to enhance the balance between exploration and exploitation abilities and prevent premature convergence. The results show that the MWOA algorithm significantly reduces the compression cost and improves system voltage compared to competing algorithms. Chakraborty et al. [178] proposed an improved WOA method (ImWOA) to solve numerical optimization and real-world applications. ImWOA employs a randomized selection method to enhance exploration during the prey search phase and a collaborative hunting approach to enhance exploitation efficiency in the standard WOA. Comparison of ImWOA results with competing algorithms confirms its diversity and convergence analysis superiority. However, high complexity and unbalancing between local and global search are drawbacks of ImWOA. Mohammadbeigi et al. [179] enhanced the whale optimization algorithm using sequential quadratic programming to improve the exploration ability of WOA. The WOA-SQP algorithm solves an optimal chiller loading problem in energy conservation. Experimental results of the WOA-SQP algorithm on several case studies show it is more efficient than other algorithms.

Shen et al. [180] proposed an improved WOA based on multi-population evolution (MEWOA) to overcome slow convergence speed and falling into local optima issues. The MEWOA classified individuals into three equal-sized sub-populations according to their fitness, and different movement strategies apply for each sub-population to increase the ability of exploration and exploitation abilities. The results show the superiority of the MEWOA in comparison with the other algorithms. Mohanty et al. [181] presented a sine-adapted improved whale optimization algorithm (SiWOA) to solve benchmark functions and the distributed power system (DPS) problem. SiWOA incorporates the sine function with WOA for parameter selection and engages scaling factors to WOA to balance local and global search strategies during optimization. Results demonstrate that the SiWOA could best solve the DPS problem. Liu et al. [182] proposed an improved whale optimization algorithm to obtain the preliminary edge of unmanned aerial vehicle-captured color images. This method applies the global random position and information exchange mechanisms into the random walk foraging formula of the canonical WOA. Meanwhile, a random disturbance factor is used in the predator–prey mechanism of the spiral bubble net. Chakraborty et al. [183] introduced elite-based WOA (EBWOA) to solve engineering-design and cloud task scheduling problems. The EBWOA algorithm uses only the encircling prey and bubble-net attack phases. Moreover, the EBWOA algorithm applies the local elite method to encircling the prey phase to improve exploitation ability and an inertia weight in all phases to scour the region.

7.1.2 Improved Continuous Multi/many-Objective WOA Variants

This subsection reviews improved continuous WOA variants that have used different techniques for improving WOA to solve multi/many-objective problems; these are summarized in Table 7.

Wang et al. [148] proposed an opposition-based multi-objective whale optimization algorithm (MOWOA) to solve a data clustering problem and a hydropower plant scheduling problem in China. MOWOA uses opposition-based learning, global grid ranking, and a suitable external archive strategy to increase convergence speed and population diversity. MOWOA performs well for benchmark problems but has not been validated in real-world engineering problems. Sulaiman et al. [184] proposed the improved whale optimization algorithm (IWOA) to solve the plate-fin heat exchanger's thermal economic multi-objective optimization problems. In addition, IWOA converges to the required results quickly. IWOA can solve problems with complex objectives and highly non-linear constraints. Hou et al. [97] proposed a method for designing gas turbine controllers using a multi-objective economic model predictive control and a quantum simultaneous whale optimization algorithm. Their findings demonstrate the superiority of the proposed approach in terms of improved financial performance, accuracy, speed, and stability. However, this study was evaluated only by solving the power generation process.

Li et al. [149] introduced a multi-objective WOA algorithm that uses a multi-leader guiding system. This algorithm divides search agents into two groups—leaders and regular—using a grid system and multiple leaders to guide the population in searching the solution space. The algorithm then employs differential evolution and whale optimization to generate offspring for the leaders and regular groups. Additionally, an opposition-based learning mechanism enhances the population distribution during the early stage. The obtained results prove the excellent performance of this algorithm in real-world applications. Abdel-Basset et al. [185] proposed a hybrid optimization method that combines the whale optimization algorithm with the

Table 7 Improved continuous multi/many-objective WOA variants

Author and Ref	Year	Common techniques	Other techniques	Evaluation benchmarks	Application
Wang et al. [148]	2019	Opposition-based learning	Global grid ranking opposition	–	Industrial and mechanical Eng
Sulaiman et al. [184]	2019	–	Initialization strategy	–	Industrial and mechanical Eng
Hou et al. [97]	2020	Quantum-based	Quantum coding and simultaneous search	–	Industrial and mechanical Eng
Li et al. [149]	2021	Opposition-based learning	Multi-leader guiding		Bi-objective and tri-objective, and unconstrained problems
Abdel-Basset et al. [185]	2021	–	Nelder-mead method and Pareto archive evolution strategy	DTLZ, CEC 2009, and GLT	–
Yankai et al. [186]	2021	–	Pareto-based optimal solution	–	Electronic Eng
Kotary et al. [187]	2021	–	Leader selection method	–	Computer and IT
Paul et al. [188]	2023	Chaotic-oppositional learning	–	–	Industrial and mechanical Eng

Nelder–Mead simplex method to tackle multi-objective problems. Integrating the Nelder–Mead simplex and Pareto archive evolution strategy helps maintain diversity in the population and hastens convergence. The experiments' results showed that this algorithm outperforms other existing multi-objective algorithms. However, it has not yet been applied to real-world scenarios.

Hence, Yankai et al. [186] introduced a multi-objective mathematical model (MOHFSP-DRP) that considers dynamic device reconfiguration and devices' adjustable processing modes to solve a Chinese company's digital hot-rolling workshop problem. Although the proposed IMOWOA performs well compared to NSGA-II and SPEA2 in solving the MOHFSP-DRP, there are still limitations for devices that adopt consistent processing modes. Kotary et al. [187] proposed a many-objective whale optimization algorithm to handle robust distributed clustering in wireless sensor network (WSN) applications. First, a swarm-based many-objective whale optimization (MaOWOA) is introduced to reference the point-based leader selection method in updating the solutions. The findings of this algorithm show that the MaOWOA algorithm could give a better convergence rate and maintain diversity on many-objective DTLZ test problems. Then, the distributed many-objective clustering using WOA (DMaOWOA) is introduced to conduct robust distributed clustering in WSN applications. DMaOWOA uses a weight-based method to distinguish and remove the outliers and a diffusion method for distributed clustering. The findings on a synthetic dataset and three real-world WSN datasets demonstrate the algorithm's superiority over others. Paul et al. [188] introduced a chaotic-oppositional learning WOA (COWOA) algorithm to solve wind and solar-based multi-objective hydro-thermal scheduling problems. The COWOA algorithm handles the cost function's nonlinearity using two test systems. The first test system consists of four hydro and three thermal units, and the second test system consists of one wind unit and one solar unit with four hydro and one thermal generating units.

7.2 Improved Binary WOA Variants

The techniques introduced to alleviate the weakness of canonical WOA to solve different binary optimization problems are reviewed in this subsection. Table 8 displays improved binary WOA variants.

Tubishat et al. [189] proposed an improved WOA algorithm, IWOA, to select useful features in sentiment analysis for the Arabic language. IWOA uses elite opposition-based learning in the initialization phase and mutation, crossover, and selection operators at the end of each optimization phase. In addition, IWOA uses Information Gain as a filter-based feature selection technique for feature ranking. The results show that the IWOA algorithm has better efficiency than other algorithms regarding the accuracy and number of selected features. Li and He [190] proposed a wrapper-based feature selection approach using a modified whale optimization algorithm and ideal point method to identify critical quality characteristics in unbalanced production data. In this approach, a modified fast, non-dominated sorting method is employed to maximize the geometric mean and minimize the size of the feature subset. Experimental results show the effectiveness and efficiency of this algorithm compared to standard multi-objective optimization methods.

Agrawal et al. [96] proposed a feature selection method based on an improved quantum-based WOA algorithm called QWOA. QWOA uses the quantum representation for

Table 8 Improved binary WOA variants

Author and reference	Year	Single/multi-objective	Improving techniques	Binarization techniques	Datasets and benchmarks
Tubishat et al. [189]	2019	Single-objective	OBL, mutation, and crossover	Gain information	Arabic benchmark dataset
Li et al. [190]	2020	Multi-objective	Mutation strategy	Threshold method	Unbalanced production datasets
Agrawal et al. [96]	2020	Single-objective	Quantum technique, mutation strategy, and crossover operator	Binary operator	High dimensional datasets
Yusof et al. [191]	2022	Single-objective	Logistic-Tent chaotic map	Transfer function	drug dataset
Zhang et al. [192]	2022	Multi-objective	Crossover operator	Threshold method	Two-sided disassembly line
Kaur et al. [128]	2022	Single-objective	Quantum technique	Transfer function	Wizard-of-Oz dataset
Nadimi-Shahraki et al. [55]	2022	Single-objective	Pooling mechanism and multi-search strategies	Transfer function	Medical diseases detection and CEC test function 2018
Toloueiashtian et al. [193]	2022	Single-objective	Exploration and spiral attack operators	Binary operator	Computer and IT
Xing et al. [194]	2022	Single-objective	OBL and Gaussian barebone mechanism	-	CEC test functions 2014 and 2020, and Medical diseases
Zhang et al. [195]	2023	Single-objective	Random contraction and the Rosenbrock method	Transfer function	COVID-19 disease

the position of search agents and the quantum rotation gate operator to balance exploration and exploitation search strategies. In addition, QWOA applied mutation and crossover operators to improve the mechanisms of the shrinking and spiral movement of the search agents. Experiments demonstrate the QWOA algorithm's superior efficiency compared to other widely known algorithms. However, the QWOA is only effective for feature selection or binary unconstrained optimization problems. Yusof et al. [191] introduced a chaotic time-varying binary whale optimization algorithm ($CBWOA_{TV}$) to optimize the feature selection process in amphetamine-type stimulants (ATS) and non-ATS drugs classification. The $CBWOA_{TV}$ algorithm applies a non-linear time-varying modified sigmoid transfer function and a hybrid Logistic-Tent chaotic map to establish an appropriate balance between exploration and exploitation abilities. The results show that the CBWOATV algorithm effectively prevents local optima trapping.

Zhang et al. [192] presented an improved WOA-based discrete multi-objective model for creating a two-sided disassembly line using two different crossover operators with four main objectives: many paired stations, idle index, demand index, and risk index. The crossover operators simulate the whales' predatory behavior to improve the solution quality. Also, a disturbance factor is applied to reduce the probability of local optima trapping. The experimental results verify the algorithm's superiority compared to other algorithms. Kaur et al. [128] proposed a quantum-based whale optimization algorithm (QWOA) for detecting depression on the publicly available Distress Analysis Interview Corpus Wizard-of-Oz (DAICWOZ) dataset. QWOA uses a two-phase speech-based depression detection system to select an effective feature subset of relevant and non-redundant speech features. The results prove that the performance of the QWOA outperforms traditional wrapper-based evolutionary methods significantly. However, experiments were limited to speech-based depression, and other practical cases were not evaluated.

Nadimi-Shahraki [55] proposed the enhanced whale optimization algorithm (E-WOA) for solving continuous problems and its binary version (BE-WOA) to select an effective medical feature selection with COVID-19 as the case study. E-WOA boosts population diversity using a pooling mechanism, strikes a better balance using preferential selection, enriches the exploration ability using a migration strategy, and improves the exploitation ability using an enriched encircling prey strategy. In a comprehensive comparison with other previous WOA variants, E-WOA is superior. The experimental results also show that the BE-WOA is better than comparative optimization algorithms in selecting effective features. Toloueiashtian et al. [193] proposed an improved WOA algorithm to solve the point coverage problem in network applications. This algorithm discovers the best solution for whales using discovery operations, spiral attacks, and bubble network attacks. The experimental results demonstrate that this algorithm improves the lifespan of the coverage area, outperforming compared algorithms in most cases.

Xing et al. [194] introduced the QGBWOA algorithm to boost the searchability of canonical WOA for solving the feature selection problem and COVID-19 image segmentation. QGBWOA uses quasi-opposition-based learning to enrich local searchability and the Gaussian barebone mechanism to maintain population diversity.

The performance of the QGBWOA was assessed to solve test functions CEC 2014 and CEC 2020, the feature selection problem, and the multi-threshold image segmentation applications. Results show that the QGBWOA could improve the convergence speed and local and global search capabilities of WOA; however, it suffers from high time complexity. Zhang et al. [195] introduced the RRWOA algorithm by improving the whale optimization using random contraction (RC) and the Rosenbrock method. The RRWOA algorithm uses the k-nearest neighbor classifier to detect COVID-19 disease. In addition, the RRWOA algorithm uses the RC strategy to enhance the algorithm's exploration ability and the RM strategy to improve the quality of solutions.

8 Hybrid Variants of WOA

As explained in subSect. 6.2, WOA is hybridized with two categories of single-solution-based and population-based algorithms, including evolutionary, physics-based, and swarm, to cope with its inherent weaknesses. As explained in Sect. 5, 57 of the 116 collected papers for this systematic review belong to hybrid WOA variants. Figure 17 shows the percentage of different categories of metaheuristic algorithms that have hybridized with WOA. In hybridizing with WOA, the swarm intelligence algorithms from the population-based category have the most significant percentage, 58%. The evolutionary algorithms are the second with 19%; single solution-based algorithms are the third with 11%, and physics-based algorithms are the fourth with 9%. Finally, algorithms inspired by other concepts in nature are hybridized with WOA. These total 3% because

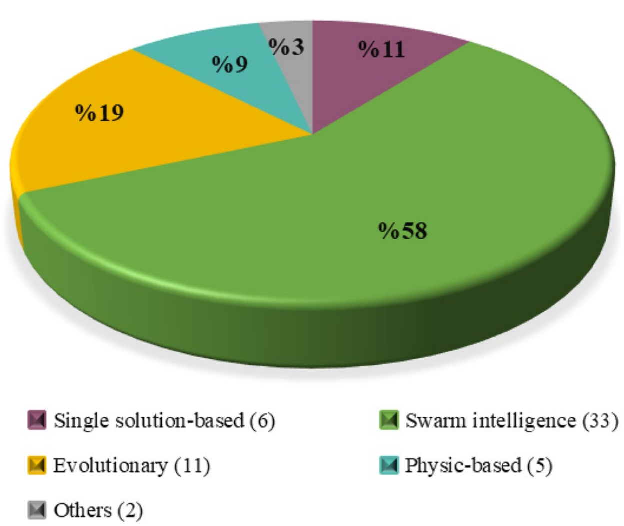


Fig. 17 Percentage of different categories of metaheuristic algorithms hybridized with WOA

the algorithms in this category mostly cannot incorporate WOA's search strategies to tackle weak points. In the following subsections, the hybrid variants of WOA are reviewed in two main groups, continuous and binary, and are summarized in Tables 9, 10, 11.

8.1 Hybrid Continuous WOA Variants

The review of hybrid WOA variant papers from 2019 to the end of March 2023 showed that most of them are developed for continuous applications, equaling 49 of 57 papers. This section investigates the hybrid continuous WOA variants in two main groups, single-objective and multi-objective.

8.1.1 Hybrid Continuous Single-Objective WOA Variants

As explained in subSect. 6.2, WOA is hybridized with two main categories of population-based and single-solution-based algorithms to alleviate its weaknesses in solving various problems. The hybrid WOA variants that were hybridized with at least one population-based or single-solution-based algorithm are reviewed and summarized in Table 9.

8.1.1.1 Hybridizing with Population-Based Algorithms Reading and reviewing the WOA variant papers show that they are mostly hybridized with one or more algorithms from three main categories, evolutionary, physics-based, and swarm intelligence.

Evolutionary algorithms have prominent abilities, such as effective global search strategies, self-adaptive mechanisms, and archiving methods [196, 197], encouraging researchers to alleviate the weak WOA points using such abilities. However, most WOA variants claim that the canonical WOA suffers from a meaningful search strategy to solve real-world optimization problems, which causes them to encounter issues such as premature convergence, local optima trapping, and population diversity loss. In the following, the hybridization algorithms presented to overcome WOA weaknesses in solving optimization problems, such as managing surface water resources, unit commitment scheduling, load sharing for parallel compressors, and global optimization, are reviewed in detail. Mohammadi et al. [112] proposed a hybrid whale-genetic algorithm (HWGA) to optimally exploit reservoirs in managing surface water resources. The HWGA algorithm randomly generates the initial population for WOA and GA to start their optimization process in parallel. Then, two obtained populations merge to form the best population for the next iteration. Results in the optimal operation of continuous-time four-reservoir benchmark system and ten-reservoir benchmark system show that the HWGA can boost the low convergence rate of GA and poor precision of WOA.

Table 9 Hybrid continuous single-objective WOA variants

Author and Ref	Year	Hybridized with	Benefit of hybridization	Problem/Application
Mohammadi et al. [112]	2019	GA	Improving exploration and exploitation abilities	CEC test functions
Mahalingam et al. [198]	2019	GWO and GA	Improving the convergence speed and escaping from local optima	Computer and IT
Laskar et al. [199]	2019	PSO	Improving exploration and exploitation abilities	CEC test functions and industrial and mechanical Eng
Korashy et al. [200]	2019	GWO	Improving exploitation ability	Electronic Eng
Prabhakar et al. [122]	2019	SSA	Improving convergence speed	CEC test functions and Computer and IT
Singh et al. [201]	2020	DE	Balancing between exploration and exploitation abilities	Electronic Eng
Li et al. [103]	2020	DE	Balancing between exploration and exploitation abilities	Civil Eng. and energy
Abo-Elyousr et al. [202]	2020	PSO	Improving exploitation ability	Electronic Eng
Rathore et al. [203]	2020	GWO	Balancing between exploration and exploitation abilities	Computer and IT
Mohammed et al. [204]	2020	GWO	Escaping from the local optima	Industrial and mechanical Eng
Abdel-Basset et al. [119]	2020	SMA	Balancing between exploration and exploitation abilities	Medical and health
Chakraborty et al. [205]	2021	DE	Improving exploration and exploitation abilities, and convergence	CEC test functions 2019 and industrial and mechanical Eng
Lee et al. [114]	2021	GA, and TEO	Balancing between the exploration and exploitation abilities and escaping from the local optima	CEC test functions 2017
Çimen et al. [206]	2021	FA	Improving the solution quality	Electronic Eng
Nasrollahzadeh et al. [115]	2021	PSO	Balancing between exploration and exploitation abilities	Computer and IT
Asghari et al. [207]	2021	PSO	Avoiding local optima	CEC test functions and engineering problems
Zhang and Wen [208]	2021	GWO	Improving global search strategy	CEC test functions
Asghari et al. [143]	2021	GWO	Improving local optima and slow convergence rate	CEC test functions
Che et al. [121]	2021	SOA	Balancing between exploration and exploitation abilities	CEC test functions/ Industrial and mechanical Eng
Tawhid et al. [209]	2021	FPA	Decreasing the execution time and the complexity of WOA	CEC test functions 2014
Li et al. [210]	2022	GA	Improving the performance of WOA	Industrial and mechanical Eng
Tan et al. [211]	2022	EO	Increasing the solution quality and robustness	CEC test functions 2017, 2019, and 2020
Seyyedabbasi [212]	2022	SCA	Improving exploitation ability	CEC test functions and Industrial and mechanical Eng
Lakshmi et al. [104]	2022	SCA	Improving the exploration ability	CEC test functions and Computer and IT
Dey et al. [102]	2022	SCA	Increases the exploration capability	Electronic Eng
Zhang et al. [105]	2022	WMA	Improving convergence speed	CEC test functions
Tang et al. [213]	2022	ABC	Accelerating the convergence rate and escaping from the local optimum	CEC test functions 2019 and engineering mathematical models
Saminathan et al. [214]	2022	FOA	Improving exploitation ability	Computer and IT
Talasila et al. [120]	2022	DA	Improving the solution quality	Computer and IT
Jindal and Singh [117]	2022	GWO	Improving exploitation ability	Computer and IT
Obadina et al. [215]	2022	GWO	Improving exploration ability	Computer and IT
Braik et al. [216]	2022	CSA	Improving the performance of WOA to solve a real-world problem	Computer and IT
Fan et al. [217]	2022	SSA and LOBL	Improve the exploitation and exploration abilities and population diversity	CEC test functions and Industrial and mechanical Eng
Fan et al. [218]	2022	SSA	Improving the performance of WOA	Civil Eng. and energy

Table 9 (continued)

Author and Ref	Year	Hybridized with	Benefit of hybridization	Problem/Application
Zhang et al. [219]	2022	SSA	Improving the exploration ability and boosting local optima escaping	CEC test functions and Industrial and mechanical Eng
Ponmalar et al. [223]	2022	Tabu	Improving the solution quality	Computer and IT
Ghany et al. [224]	2022	Tabu and EA	Boosting population diversity	Computer and IT
Xu et al. [225]	2023	SA	Balancing between the exploration and exploitation abilities and escaping from the local optima	Computer and IT
Hsu et al. [220]	2023	PSO	Improving the solution quality	Industrial and mechanical Eng
Uzer et al. [221]	2023	PSO	Escaping from the local optima and Balances between the exploration and exploitation abilities	CEC test functions and engineering problems
Wang et al. [222]	2023	DE	Generating a better initial population, maintaining the population diversity, and exploration ability	Electronic Eng

Table 10 Hybrid continuous multi-objective WOA variants

Author and Ref	Year	Hybridized with	Benefit of hybridization	Problem/Application
Bhandakkar et al. [226]	2020	SMA	Balancing between exploration and exploitation abilities	Electronic Eng
Dewi et al. [232]	2020	TS	Escaping from the local optimum	Civil Eng. and energy
Reddy et al. [227]	2021	GWO	Improving the solution quality	Computer and IT
Rana et al. [228]	2022	DE	Improving population diversity and the local search ability	Computer and IT
Wang et al. [229]	2022	GA	Improving the solution quality and execution times	Industrial and mechanical Eng
Siahroodi et al. [230]	2022	GWO, and DE	Improving the convergence speed	Electronic Eng
Oladepo et al. [231]	2022	PSO	Improving exploration ability	Electronic Eng

Table 11 Hybrid binary WOA variants

Author and Ref	Year	Hybridized with	Benefit of hybridization	Problem/Application
Nagarajan et al. [108]	2019	Hill climbing	Improving the solution quality	Computer and IT
Vijh et al. [233]	2020	PSO	Improving the convergence speed	Medical and health
Monica et al. [118]	2021	FA	Escaping from the local optima	Medical and health
Mohammadzadeh et al. [234]	2021	FPA	Improving the convergence speed	Computer and IT
Stephan et al. [116]	2021	ABC	Improving the exploration ability	Medical and health
Hussain et al. [235]	2022	ABC	Improving the convergence speed	Computer and IT
Alwajih et al. [236]	2022	HHO	Improving the solution quality	Computer and IT
Mafarja et al. [237]	2023	GWO, HHO	Improving the exploration ability	Computer and IT
Uzer et al. [238]	2023	WOA, PSO	Improving the solution quality	Industrial and mechanical Eng

Some WOA variants focus on improving exploitation search strategy and balancing the canonical WOA by hybridizing it with SI algorithms based on terrestrial animal behavior. They mostly used mechanisms such as searching for prey, information sharing, herd leadership, and attacking prey to combine with WOA search strategies. Mahalingam and Subramoniam [198] proposed the optimal weighted centroid (OWC) technique that uses the WOA algorithm with the fusion of the grey wolf optimization (GWO) for efficient moving object analysis. Then a modified kernel fuzzy C means algorithm using an optimally weighted centroid is introduced to find the best centroid for performing clustering and achieving optimal background separation. This technique can avoid local optima trapping and increase the convergence rate, improving the statement sensitivity level. Laskar et al. [199] suggested a hybrid whale-particle swarm optimization (HWPSO) algorithm to tackle electronic design optimization problems. HWPSO introduces the forced technique to encourage exploration ability and the capping technique to improve the search mechanism during exploitation. The results show that the HWPSO can reduce the stagnation effect and improve convergence speed in a conventional PSO using the exploration strategy of humpback whales.

Korashy et al. [200] proposed the HWGO algorithm by hybridizing WOA with a grey wolf optimizer (GWO) to boost the performance and reliability of WOA in solving the coordination problem. HWGO can improve the local search ability of WOA using the leadership hierarchy of GWO. Prabhakar et al. [122] proposed the SSWOA algorithm by hybridizing salp swarm optimization (SSA) and WOA for the conformal antenna array application. The SSWOA algorithm uses the SSA algorithm to guide the evolution process and the WOA algorithm as an assistant. SSWOA updates the position search agents using SSA and then evolves the best positions using WOA to enhance the optimum searchability. The results show that the hybridization of SSA and WOA can significantly improve the convergence speed and accuracy to solve antenna array pattern synthesis. Singh et al. [201] proposed the WODEGA algorithm by hybridizing WOA, differential evolution, and the genetic algorithm for solving the unit commitment scheduling problem. WODEGA reduces the operation cost within three phases. In the first phase, WODEGA computes the priority of generators and commits the generators to be ready for taking the load. The WODEGA hybridizes the genetic algorithm and differential evolution in the second phase. In the last phase, WODEGA applies the post-hybridization mechanism using WOA to reduce the operation cost. Results show that WODEGA is promising in reducing operating costs for power companies.

In solving the optimization of load sharing for parallel compressors problem, Li et al. [103] show that WOA suffers from poor exploration ability while the differential evolution (DE) algorithm benefits from this ability. Hence, they developed the DWOA algorithm by hybridizing WOA and DE. DWOA used a DE algorithm in the second half of the evolutionary process and crossed the mutant vectors of DE and WOA when $|A| < 1$ and $\rho < 0.5$ to improve exploration ability and escape from local optima. As a result, DWOA can improve comprehensive compressor station optimization with a 2% achievement compared to competitors.

Abo-Elyousr et al. [202] developed a hybrid PSO-WOA to generate a robust load frequency controller with a mass-less inertia photovoltaics system. PSO-WOA benefits from the exploitation ability of the conventional particle swarm optimization algorithm and the exploration ability of the WOA to optimize the scaling factors of the proportional-integral controllers. Rathore et al. [203] boosted the exploitation and exploration abilities of canonical WOA by hybridizing it with a grey wolf optimization algorithm (GWO). In the first iteration, the hybrid GWO algorithm randomly distributed search agents in the search space to start an optimization process. Then each algorithm evolves its population independently. At the end of each iteration, the search agents share experiences, and the best-obtained solutions are considered for the rest of the iterations. Results show that the hybrid WGWO algorithm can select efficient cluster heads regarding the delay, packet delivery ratio, energy consumption, network lifetime, and throughput.

To overcome the poor exploitation ability of the WOA algorithm and escape from local optima, Mohammed et al. [204] proposed the WOAGWO algorithm by combining WOA with grey wolf optimization. WOAGWO adds the hunting mechanism of GWO in the WOA exploitation phase to strengthen it and introduce a technique to improve the exploration phase of the WOA after each iteration. The WOAGWO algorithm could effectively boost the search strategy of WOA by benefiting from the high exploitation ability of GWO for solving global numerical optimization and pressure vessel design problems. Abdel-Basset et al. [119] proposed a hybrid slime mould algorithm (SMA) with the WOA named HSMA_WOA to tackle the image segmentation problem. The WOA is utilized initially during optimization until a designated iteration has been accomplished. HSMA_WOA uses the WOA algorithm to explore the search space when the iteration counter reaches a predefined value. Then HSMA_WOA applies SMA to exploit in the vicinity of the best-so-far solution if the distance between the fitness value of the current solution and the best solution is more than a specific value; otherwise, HSMA_WOA explores another region to find the best-so-far solution. HSMA_WOA uses the exploration ability of SMA to re-initialize the solutions located in a predefined probability of the search space. Results show that the HSMA_WOA algorithm can attain the maximum of Kapur's entropy using the high exploration ability of WOA at the start of the optimization process and the high exploitation ability of SMA around the best-so-far solution. The biggest challenge of this algorithm is determining the correct control parameter values.

Since WOA suffers from poor search strategy, it cannot perform appropriately in terms of solution quality to solve unimodal, multi-modal, and hybrid test functions. Hence, Chakraborty et al. [205] proposed the m-SDWOA algorithm to boost the performance of WOA to solve the IEEE CEC 2019 benchmark function, which consists of various test functions with different properties such as unimodal, multi-modal, and hybrid. The m-SDWOA algorithm hybridizes WOA with the modified mutualism and commensalism segment strategies of the symbiotic organisms search algorithm and mutation strategy from the differential algorithm. m-SDWOA applies selection parameter γ to arrange balances between the exploration and exploitation phases effectively. m-SDWOA can improve the exploration ability of canonical WOA using the modified mutualism phase and DE/rand/1/bin mutation strategy and enhance exploitation ability using the commensalism phase, encircling prey, and bubble-net attacking methods. To improve the global search strategy of WOA, Lee et al. [114] proposed the GWOA-TEO algorithm by hybridizing WOA with GA and thermal exchange optimization algorithm. The GWOA-TEO algorithm improves the exploration ability of WOA using GA and the exploitation ability using TEO. Moreover, GWOA-TEO is equipped with a memory that saves previously successful solutions, a combination technique using this memory, and a method to update positions. Therefore, GWOA-TEO could perform excellently in solving global optimization problems using such mechanisms.

Çimen et al. [206] optimized the parameters of the predictive control model by hybridizing the firefly algorithm and WOA. The FA-WOA algorithm uses the fireflies' attractiveness and the whales' hunting characteristics. Also, the FA-WOA algorithm introduces variables related to the attractiveness of fireflies. Results show that the FA-WOA algorithm can effectively optimize the predictive control parameters. Motion sensors are essential elements of environmental automation, and determining the best quantity and placement of motion sensors are among the primary difficulties in this field. Hence, Nasrollahzadeh et al. [115] merged the advantages of WOA and PSO and proposed an improved metaheuristic algorithm to find optimized motion sensor placement in smart homes and intelligent environments. A hybrid WOA-PSO algorithm could solve some of the drawbacks of PSO and WOA in handling high-dimensional issues with intricate objective functions. Asghari et al. [207] proposed the CWP algorithm by hybridizing WOA with chaotic maps and particle swarm optimization and the MCWP algorithm as a multi-swarm version of the CWP to solve global and engineering optimization problems. The CWP algorithm uses chaotic maps to initialize the populations of WOA and PSO and then starts the optimization process using two separate populations. The CWP algorithm uses the chaotic maps and the quasi-opposition-based learning (QOBL) technique to re-initialize the populations that have not improved for five consecutive iterations. The MCWP algorithm divides the population into several smaller sub-swarms to explore the search space. MCWP uses a roulette wheel selection operator to select the search agents and exploit promising areas of the search space. The results show that the MCWP could significantly increase convergence speed toward the global optimum and better avoid local optima trapping than some canonical optimization algorithms. However, CWP and MCWP suffer from the lack of impact analysis on each search strategy. Also, time and computational complexities are the most critical issues for the CWP algorithm.

Zhang and Wen [208] proposed the HWOAG algorithm by hybridizing WOA with gathering strategies. HWOAG uses an individual-based updating strategy instead of a dimension-based one to increase suitability for high-dimensional applications. Then, HWOAG proposes the OWOA algorithm by embedding a random opposition learning strategy into the individual-based WOA and proposes the OWOAG algorithm by integrating GWO into OWOA. OWOAG employs techniques such as altering the max parameter, refining the current population, and introducing random variations during the initial search phase to establish stronger exploration searchability. OWOAG tries to enrich exploitation searchability in the second search stage by generating new solutions based on the historical best solutions and adding the global-best spiral operator in the spiral updating stage. Results show that the HWOAG algorithm could increase the searchability of WOA on high-dimensional problems and clustering datasets. Asghari et al. [143] proposed a chaotic grey wolf optimization algorithm and WOA (CGWW) for solving continuous optimization problems. The CGWW algorithm uses the chaotic map technique to initialize the population and set the movement parameters, the roulette wheel selection method to choose the search agents with better scores, and multi-swarm characteristics to establish a more diverse population. Although the CGWW can solve continuous optimization problems effectively, results in composite test functions show that it suffers from inadequate balancing between search strategies.

Che et al. [121] developed WSOA by hybridizing WOA with a seagull optimization algorithm (SOA) to solve global optimization problems. WSOA merges WOA's concentric reduction approach with SOA's spiraling assault strategy to enhance solution quality. WSOA also uses the Lévy flight strategy to balance between search strategies and avoid premature convergence. Furthermore WOA can be hybridized with other natural phenomena, such as flower pollination, to improve its exploitation ability. Tawhid and Ibrahim [209] proposed the WOFPA algorithm based on the flower pollination algorithm (FPA) and WOA to resolve non-linear systems and optimization problems without constraints. The WOFPA algorithm introduces three steps: a spiral model, prey search or random search, and local pollination. In addition, WOFPA uses the local pollination approach from FPA to rely on selecting random agents and boost exploitation ability. The results show that WOFPA can alleviate disadvantages of the WOA and FPA algorithms by timely converging search agents to the promising area. Li et al. [210] proposed SHM-C&G&A as a hybrid approach for forecasting ship motion using a convolutional neural network (CNN), gated recurrent unit (GRU), attention mechanism (AM), and a hybrid genetic cloud whale optimization algorithm (GCWOA). GCWOA optimizes the hyperparameters of the SHM-C&G&A model. Results expose that the SHM-C&G&A model exhibits better nonlinear characteristics than the other models considered in this paper. Furthermore, the

GCWOA improved the performance of CNN in the forecasting process.

Tan et al. [211] introduced the equilibrium whale optimization algorithm (EWOA) by hybridizing WOA with an equilibrium optimizer (EO) to solve benchmark problems CEC 2017, CEC 2019, and CEC 2020. EWOA combines the encirclement and net-bubble approaches using EO's weight balancing technique and also has memory optimization, multiple top solutions, and adjusted coefficient adjustment to enhance solution quality. Results show that EWOA benefits from low execution time, low computational complexity, an appropriate trade-off between global and local searches, efficient search capability, and superb clustering properties. Seyyedabbasi [212] proposed the WOASCALF algorithm, hybridized with the sine cosine algorithm (SCA) and Lévy flight strategy, to boost WOA in solving global optimization problems. WOASCALF uses the hybridization of the WOA, SCA, and Lévy flight to update search agents' positions. The results show that the WOASCALF benefits SCA in terms of exploitation ability and profits of Lévy flight strategy in terms of population diversity by providing small and large jumps in different directions of the search space. Lakshmi et al. [104] proposed WOA-SCA by hybridization of SCA and WOA to solve the facial emotion recognition problem. Results show that WOA-SCA can improve the exploration strategy of the WOA effectively and obtain average emotion recognition accuracy equal to 98%. Dey et al. [102] proposed WOA-SCA to boost the exploration ability of canonical WOA and increase its applicability for solving the electricity market prices problem. WOA-SCA benefits from the exploitation ability of WOA and the exploration ability of the sine cosine algorithm. Results show that the WOA-SCA could reduce the generation cost of a microgrid system and yield better quality solutions with the least computational time.

The main idea behind the hybridizing WOA with SI algorithm based on bird behaviors is using bird survival behaviors, including nesting, mating, predator defense, feeding, and social interaction, to overcome WOA weaknesses. HWMWOA [105] combines WOA with the woodpecker mating algorithm (WMA) for solving global optimization problems and data classification applications. HWMWOA uses a modified position update mechanism of WMA, a self-regulation Cauchy mutation operator, and an arithmetic spiral movement with a novel search guide pattern to enhance the searchability of WOA. The results prove that the HWMWOA can overcome the poor population diversity and premature convergence of WOA. Tang et al. [213] proposed a hybrid WOA with an artificial bee colony (ACWOA) to solve global optimization problems. The ACWOA adds non-linear convergence factors and adaptive inertia weight coefficients to the movement phase of the whales to accelerate the convergence rate and chaotic map technique to the population initialization phase to avoid local optima trapping. Results of the ACWOA show that it can reduce the weaknesses of WOA and achieve the best solution quality for solving global optimization problems.

Saminathan and Thangavel [214] hybridized the fruit fly optimization algorithm (FOA) and WOA (FOA-WOA) to solve energy efficiency problems in a mobile ad-hoc network application. Results show that FAO-WOA can perform better than ant colony optimization and genetic algorithm regarding packet delivery ratio, delay, and energy consumption. Moreover, FAO-WOA can reduce the occasional network topology change when the nodes move randomly in any direction with limited battery life. Talasila [120] hybridized WOA with a dragonfly algorithm to optimize weights of the generative adversarial network (GAN) for text-to-image synthesis applications. At first, cross-modal feature grouping performs text-to-image encoding. Next, the bidirectional long short-term memory model transforms text embeddings into textual feature vectors. Then, the optimal weights of GAN are determined using the dragon customized whale optimization (DC-WO) algorithm. Results prove that the hybrid algorithm can effectively optimize the weights of GAN and achieve the best solution for the text-to-image synthesis application. Jindal et al. [117] hybridized GWO and WOA to determine malicious transactions in the role- and non-role-based access control supervised databases. The hybrid algorithm detects outsider threats by extracting data dependency rules from the database logs using a frequent sequential pattern mining named CM-SPADE. It detects insider threats by assigning previous user activities and the modified metaheuristic clustering algorithm. Lastly, malicious transactions are recognized by matching the user role profile and comparing adherence to the transaction pattern.

Obadina et al. [215] proposed a grey-box modeling approach and fuzzy logic control by hybridizing GWO and WOA for real-time trajectory control. The white box model forms using the Euler–Lagrange formulation, then re-tunes the parameters using GWO-WOA to form a grey-box model. This approach could improve the trajectory-tracking performance of leader–follower robot in terms of time delay, steady-state error, and mean squared error. Braik et al. [216] proposed the HWOA algorithm by hybridizing WOA and the chameleon swarm algorithm (CSA) to adaptively determine the optimal parameter values of the incomplete beta function in the image contrast enhancement application. HWOA uses bilateral gamma correction to generate better contrast and brightness while preserving edge detail. The results showed that HWOA could improve the performance of canonical WOA and surpass some of the optimization algorithms dedicated to this comparison in terms of average peak signal-to-noise ratio, average structural similarity index, and average values of entropy results.

Fan et al. [217] introduced ESSAWOA, which combines WOA, the salp swarm algorithm (SSA), and the lens opposition-based learning strategy (LOBL). ESSAWOA has three steps. First, ESSAWOA updates positions using SSA's exploitation and leader mechanism before using WOA. Secondly, ESSAWOA applies SSA's nonlinear parameter to the WOA for encircling prey and bubble-net attacking. Third, ESSAWOA uses the LOBL strategy to increase population diversity. The results show that the ESSAWOA can significantly improve exploitation and exploration abilities and benefits of better population diversity. Fan et al. [218] introduced a two-stage structural damage detection strategy based on variational mode decomposition (VMD), the fast, independent component analysis (FastICA), and an enhanced whale optimization algorithm integrated with the salp swarm algorithm (ESSAWOA). First, VMD and FastICA decompose and process the structure's initial response signals to distinguish the preliminary damage time. Then, the ESSAWOA algorithm identifies the structural parameters at different periods to determine the damage's location and extent. The results show that the structural damage detection strategy could effectively detect the damage's location, time, and extent. Zhang et al. [219] proposed the ISSWOA algorithm by hybridizing the sparrow search algorithm (SSA) and WOA for solving global optimization and engineering problems. The ISSWOA algorithm enhances the exploration search strategy using the WOA's improved spiral update mechanism. As introduced by the Levy flight, the guard mechanism of SSA enhances the capability to evade local optima. As a result, ISSWOA can obtain the best results to solve test functions and complex engineering problems.

Hsu et al. [220] hybridized WOA with particle swarm optimization for scheduling a dual-command storage/retrieval (S/R) machine. This paper first defines a mixed integer linear programming for the dual-command block sequencing problem of the S/R machine to minimize the total operational time. Then, a framework consisting of three hybrid approaches, Hybrid1 (WOA + PSO), Hybrid2 (WOA + PSO), and Hybrid3 (WOA + PSO), is proposed through which the Hybrid3 (WOA + PSO) outperforms the others. Uzer et al. [221] proposed five hybrid variants: WOAWWPSO, WOAFWPSO, WOALFWWPSO, WOALFFWPSO, and WOALFVWPSO by combining WOA and particle swarm optimization (PSO). WOALFWWPSO, WOALFFWPSO, and WOALFVWPSO use the Lévy flight algorithm with different combinations. The results show that WOALFVWPSO is a superior variant for solving mathematical optimization problems. Wang et al. [222] introduced a hybrid whale optimization algorithm (HWOA) to optimize the stacking sequence of arbitrary quadrilateral composite plates. The HWOA algorithm uses an adversarial learning strategy to generate a better initial population. Moreover, this algorithm uses the mutation operator and opposition-based learning (OBL) strategy to maintain the diversity of the evolutionary process and the nonlinear convergence factor to improve the global exploration ability of HWOA when solving discrete optimization problems.

8.1.1.2 Hybridizing with Single-Solution-Based Algorithms Most WOA variants claim that the canonical WOA suffers from a poor exploitation search strategy and can't exploit its neighborhood effectively. Hence, the canonical WOA must effectively guide a local search out of local optima. Therefore, the simplest and most effective solution is to intensify the exploitation search strategy using single-solution-based algorithms. Ponmalar et al. [223] optimized convolutional neural network architecture using the hybridization of WOA and tabu search optimization algorithm for intrusion detection in big data. Results demonstrate that the WOA can offer an effective architecture with high solution quality by enriching its local search. The findings of this study indicate that this hybridization could guide WOA out of local optima and avoid the search process getting stuck at a strict local minimum. Ghany et al. [224] proposed the WOATS algorithm by combing WOA with tabu search (TS) to solve data clustering applications. WOATS changes the position of humpback whales according to their original locations rather than the prey location and stores multiple best solutions about solution space using the elite memory components of TS. WOATS also applies the crossover operator of evolutionary computing methods to boost population diversity. The results show that the WOATS could bypass the local optima and increase the exploration ability in high dimensional clustering biological data sets. Xu et al. [225] proposed a hybrid improved-whale-optimization–simulated-annealing algorithm (IWOA-SA) for trajectory planning of quadruped robots. The IWOA-SA algorithm uses the simulated annealing mechanism to alleviate the local optima trapping and adaptive weights to balance the exploration and exploitation abilities in the canonical WOA. Moreover, the convergence ability of the IWOA-SA algorithm to the global optimal is proved using Markov chains of stochastic process theory.

8.1.2 Hybrid Continuous Multi-Objective WOA Variants

Multi-objective optimization issues involve two or more conflicting objectives by nature. Finding an optimal solution that satisfies all conflicting objectives is challenging for optimization algorithms. In this situation, a set of feasible solutions are considered a final solution. As mentioned in subSect. 6.4, WOA variants solve multi-objective optimization problems such as security-aware clustering, green vehicle routing, power flow controller, and harmonic power markets using two methods, Pareto and scalarization, which are reviewed as follows. The hybrid WOA variants to solve multi-objective optimization problems are presented in Table 10.

Bhandakkar and Mathew [226] proposed the ISMA algorithm by hybridizing WOA with a slime mould algorithm (SMA) for optimal allocation in a hybrid power flow controller (HPFC). ISMA uses the position updating strategy of the WOA to enhance the search strategy of SMA. The HPFC formulation is a multi-objective optimization problem in which the ISMA algorithm minimizes real power loss and system generation cost. The ISMA was applied to determine the optimal allocation of HPFC to enhance the voltage stability of the power system. In the optimization algorithm, maximal load parameters and bus voltage magnitude profiles were used to estimate the performance of the power system. The results show that the ISMA algorithm improves the maximum power loss by determining the suitable location for a combined power flow controller. In their study, Dewi and Utama [205] introduced a hybrid whale optimization algorithm to reduce the cost associated with fuel consumption, carbon emission, and vehicle utilization in the green vehicle routing problem. HWOA combines WOA with the tabu search algorithm and local search procedure. In HWOA, 10% of whales use the tabu search algorithm to improve the exploitation ability of canonical WOA. Furthermore, the local search procedure improves the solution quality by swapping and flipping operations. The swap operation randomly selects two positions and then swaps them, and the flip operation reverses the order in which jobs are chosen randomly. Since the findings show that the HWOA algorithm can effectively address the green vehicle routing problem; however, it has a considerable computation time due to having a local search procedure at the end of each iteration when the WOA finds the solution.

Reddy et al. [227] proposed a cluster-based routing model by hybridizing the grey wolf algorithm and WOA for security-aware clustering in wireless sensor network applications. Security-aware clustering problem is defined as a multi-objective function by different constraints such as distance, delay, security, and energy. The results show that this hybridization could overcome the weakness of the WOA algorithm and achieve appropriate effectiveness concerning live node analysis, throughput, and normalized network energy. Rana et al. [228] introduced M-WODE by hybridizing WOA and the differential evolution (DE) algorithm as a hybrid multi-objective to solve virtual machine scheduling in a dynamic cloud environment. M-WODE uses the DE algorithm to replace the randomly generated solution produced using the WOA search strategy to guarantee population diversity and strengthen the local search ability. To escape local optima trapping, M-WODE applies the Pareto front made using the WOA. The experimental results showed that the M-WODE could perform best on makespan and cost trade-offs. Wang et al. [229] introduced WOA-GA by combining the whale optimization algorithm and the Genetic algorithm (GA) to find the optimal time-jerk trajectory for industrial robots. For this problem, two objectives, the integral of the squared jerk along the entire trajectory and the total execution time, are considered in which WOA-GA tries to obtain the whole of the optimal trajectory by minimizing these objectives. The numerical results show that the WOA-GA creates paths with smooth, curved lines and minimal durations.

Siahroodi et al. [230] proposed a multi-objective framework by hybridizing three algorithms, WOA, the grey wolf optimization, and differential evolution, for solving the harmonic power market problem, including plug-in electric vehicles as harmonic compensators. The harmonic power market problem is a multi-objective problem in which the Pareto method finds the optimal set. Oladepo et al. [231] proposed a hybrid particle swarm optimization and whale optimization algorithm for optimizing small grid-connected photovoltaic/hydropower in the power system problem. This problem is a multi-objective issue, where the hybrid approach aims to reduce feeder voltage fluctuation and power loss. This algorithm hybridizes the exploration ability of the particle swarm optimization with the exploitation ability of WOA to enhance the speed of convergence and efficiency.

8.2 Hybrid Binary WOA Variants

This subsection presents the hybrid WOA variants proposed to solve binary optimization problems such as feature selection, lung tumor detection, missing value imputation, and wireless sensor networks.

The existence of missing values in electronic health records strongly reduces the performance of classifiers and results in inaccurate inferences. Since many imputing methods are proposed to address missing data in high-dimensional datasets, feature weighting is an effective solution to mitigate the adverse effects of irrelevant features in the imputation process and improve the accuracy of classifiers. Hence, Nagarajan et al. [108] proposed the LAHCAWOA approach for feature weighting by hybridizing whale optimization and late acceptance hill-climbing algorithm (LAHCAWOA) to boost classification performance in electronic health records applications. Although the results demonstrate that the LAHCAWOA approach based on the nearest neighbor classifier can improve the performance, the LAHCAWOA approach's computational time when solving large high-dimensional datasets is high. Vijh et al. [233] proposed the WOA_APSO algorithm by hybridizing the whale optimization algorithm (WOA) and adaptive particle swarm optimization (APSO) for automatic lung tumor detection. The WOA_APSO algorithm selects the optimized features subset using embedding linear discriminant analysis. Although the WOA_APSO algorithm could achieve promising results for 3-dimensional medical imaging, the effectiveness of the WOA_APSO algorithm needs to be assessed using different medical imaging modalities.

Monica et al. [118] proposed the FOW algorithm by hybridizing binary WOA and the firefly algorithm (FA) to select optimal clinical treatment and diagnosis based on gait features. The FOW algorithm merges fireflies' attraction strategy and the exploration phases of WOA. The reported results show that the FOW algorithm can effectively solve the trapping problems of FA and WOA. The FOW algorithm increases the classifier models' performance by choosing the best three features: step length, stride length, and swing ratio, independent of the right or left side. Mohammadzadeh et al. [234] proposed the HWOAFPA algorithm by hybridizing binary WOA, the flower pollination algorithm (FPA), and the opposition-based learning (OBL) algorithm for solving feature selection problems. HWOAFPA starts the optimization process using the WOA algorithm and then updates the whale's position using the OBL algorithm to intensify the search process in the opposite direction. Then HWOAFPA performs the FPA to enhance accuracy and convergence rate. The HWOAFPA algorithm uses the scalarization binary method to solve a multi-objective optimization problem. Stephan et al. [116] proposed the HAW algorithm by combining the employee bee phase of an artificial bee colony and the bubble net attacking method of WOA to address the feature selection problem and parameter optimization in the artificial neural network. HAW applies the exploitation strategy of humpback whales in the employee bee attacking phase to find better food source positions. HAW introduces the mutative initialization phase as the explorative phase to boost exploration ability. HAW achieves optimal accuracy, simplicity, and computation time using backpropagation learning in various breast cancer datasets. Based on the reported results, the HAW has not been tested for other medical and high-dimensional datasets.

Hussain et al. [235] proposed the WOA-ABC algorithm as an intrusion detection system by hybridizing WOA and the artificial bee colony (ABC) algorithm to detect malicious behaviors in wireless sensor networks. The WOA-ABC algorithm optimizes the architecture of a convolution neural network by benefiting from ABC's global search ability and rapid convergence speed. The results show that the WOA-ABC algorithm could detect malicious behaviors in wireless sensor networks. Alwajih et al. [236] proposed the BWOAHHO memetic technique by hybridizing binary hybrid WOA with harris hawks optimization (HHO) for solving feature selection problems. BWOAHHO uses the transfer function as a binarization method to transfer continuous values to binary ones. Results show that BWOAHHO can select the most effective features with high classification performance in the 18 datasets using a wrapper k-nearest neighbor classifier. Mafarja et al. [237] introduced the SBEWOA algorithm as a machine-learning framework for software fault prediction. The SBEWOA algorithm first uses pre-processing and re-sampling methods on seven classifiers K-nearest neighbors (KNN), Naive Bayes (NB), linear discriminant analysis (LDA), linear regression (LR), decision tree (DT), support vector machine (SVM), and random forest (RF). Due to the RF classifier outperforming all other classifiers, it improves using the binary whale optimization algorithm (BWOA). The BWOA uses the exploration strategies applied in the grey wolf optimizer (GWO) and harris hawks optimization (HHO) algorithms to eliminate irrelevant and redundant features. Uzer et al. [238] proposed the BWPLFS as a wrapper feature selection method hybridized with a WOA, particle swarm optimization, and the Lévy Flight method.

9 Discussion

Since many WOA developments have been published from 2019 to the end of March 2023, an adapted PRISMA was introduced to screen these developments carefully and select a reasonable number of eligible papers for review. Three main stages, identification, evaluation, and reporting, were considered in the adapted methodology. The identification stage found and extracted 2983 papers by searching defined keywords in the Google Scholar database. Then more screening steps and strict inclusion criteria were considered in the evaluation stage of the introduced methodology to find a reasonable number of eligible papers for full-test reading and reviewing. Three different screening steps considered in the evaluation stage limit the scope of this systematic review to only WOA variants published in reputable journals from 2019 to the end of March 2023. The evaluation stage found 116 eligible papers as the final set. The first screening selected 1751 papers from 2944 extracted papers by removing 39 duplicate papers and 1193 papers published by non-academic journals and conferences. Figure 18 shows the distribution of the WOA papers selected by the first screening for reputable publishers, including Springer, IEEE, Elsevier, MDPI, Wiley Online Library, and Taylor and Francis. Among the 1751 papers selected in this screening, Springer and Elsevier publishers published the highest percentage of WOA developments, with 24%.

Figure 19 shows the reputable journals that have published more than 10 WOA papers selected by the first screening of the evaluation stage. The IEEE Access journal has published the maximum number of WOA papers, with 76.

The second screening checked 1751 selected papers by reading their title and abstract and removed 1096 papers, including only the usage of the canonical WOA. Thus, the second screening step reduced the number of candidate

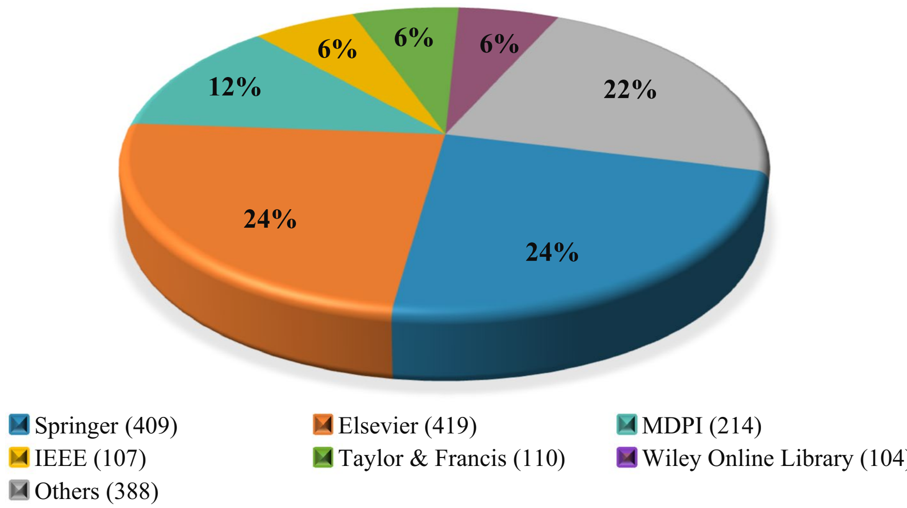


Fig. 18 Distribution of the WOA papers selected by the first screening in terms of their publishers

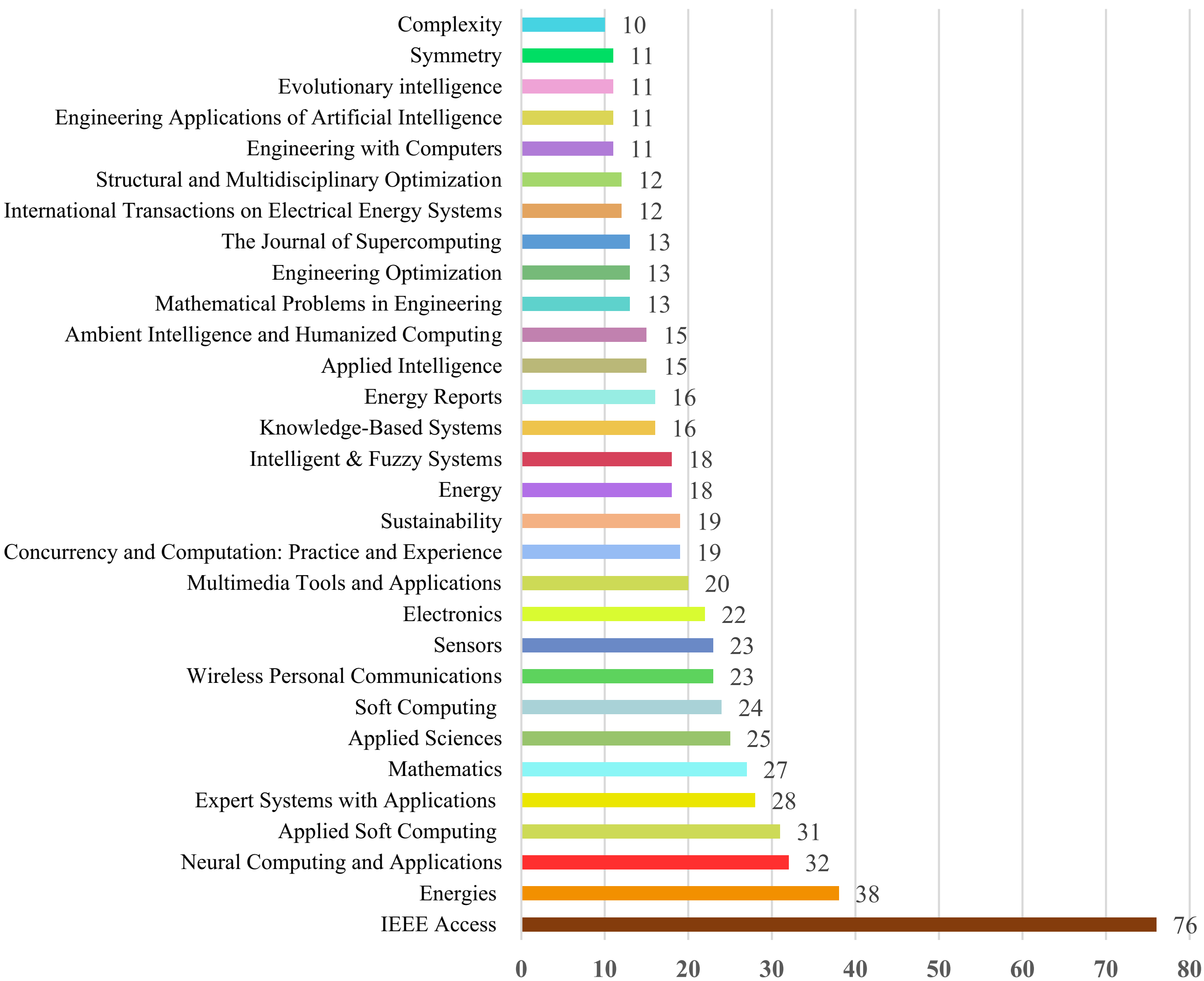


Fig. 19 Journals that have published more than ten WOA papers selected by the first screening

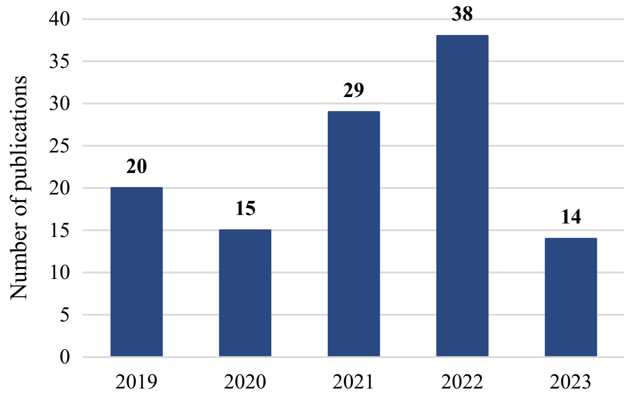


Fig. 20 The distribution of eligible WOA papers from 2019 to the end of March 2023

papers to 655 for full-text reading. The third screening formed the final set of eligible papers by verifying 111 papers for full-text reading from these 655 candidate papers, according to the inclusion criteria considered in the methodology. Additionally, the second step of the third screening found another five eligible papers by investigating the references list of 111 papers in the final set. Finally, 116 eligible papers were classified into six main and other publishers in the reporting stage. Figure 20 shows the distribution of the eligible papers published from 2019 to the end of March 2023, with a maximum of 38 in 2022. Then, the eligible papers in the final set were reviewed, which led us to consider two main approaches improved and hybrid WOA. The eligible papers were then classified as 59 for improved and 57 for hybrid variants.

The distribution of the eligible papers is visualized in terms of their publisher, journal, application, and authors' country. For example, as shown in Fig. 21, Elsevier and Springer publishers have published the maximum number of eligible papers on improved and hybrid WOA.

Figures 22 and 23 show the journals that have published eligible papers on improved and hybrid WOA. These figures show that the Applied Soft Computing journal has published the maximum number of eligible papers, with eight. Then, Neural Computing and Applications journal is ranked second, publishing six WOA eligible papers. The third rank belongs to the Soft Computing journal, with five papers.

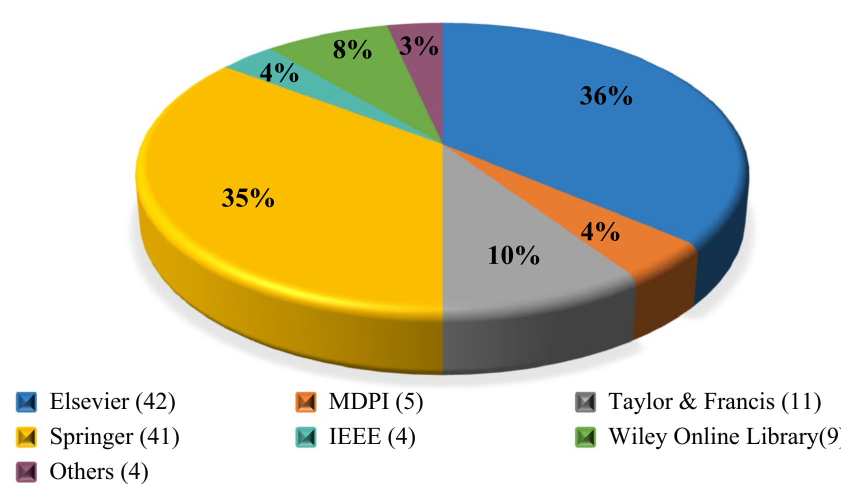
Figure 24 shows the distribution of eligible papers in terms of application, including computer and IT, electrical engineering, industrial and mechanical engineering, civil engineering and energy, medical and health, and others. As shown in this figure, computers and IT have the highest percentage of using WOA variants, with 29%. The second rank belongs to Industrial and mechanical engineering with 21%. The remaining eligible papers are 17% in electrical engineering, 9% in medical and health, and 5% in civil engineering and energy. 19% of papers were in other applications showing a vast domain of WOA variants.

Figure 25 shows the distribution of the corresponding authors' countries of the WOA eligible papers. Chinese researchers have published the maximum number of WOA variants with 39 papers. India and Iran are the second and third-ranked countries that have developed WOA variants to solve optimization problems.

10 Conclusion and Future Directions

Many WOA developments have been published from 2019 to the end of March 2023, and there is still no comprehensive systematic review of WOA variants. Therefore, this systematic review was necessary to guide WOA and other optimizers' future developments. In this study, we first used the

Fig. 21 The distribution of WOA eligible papers in terms of publisher



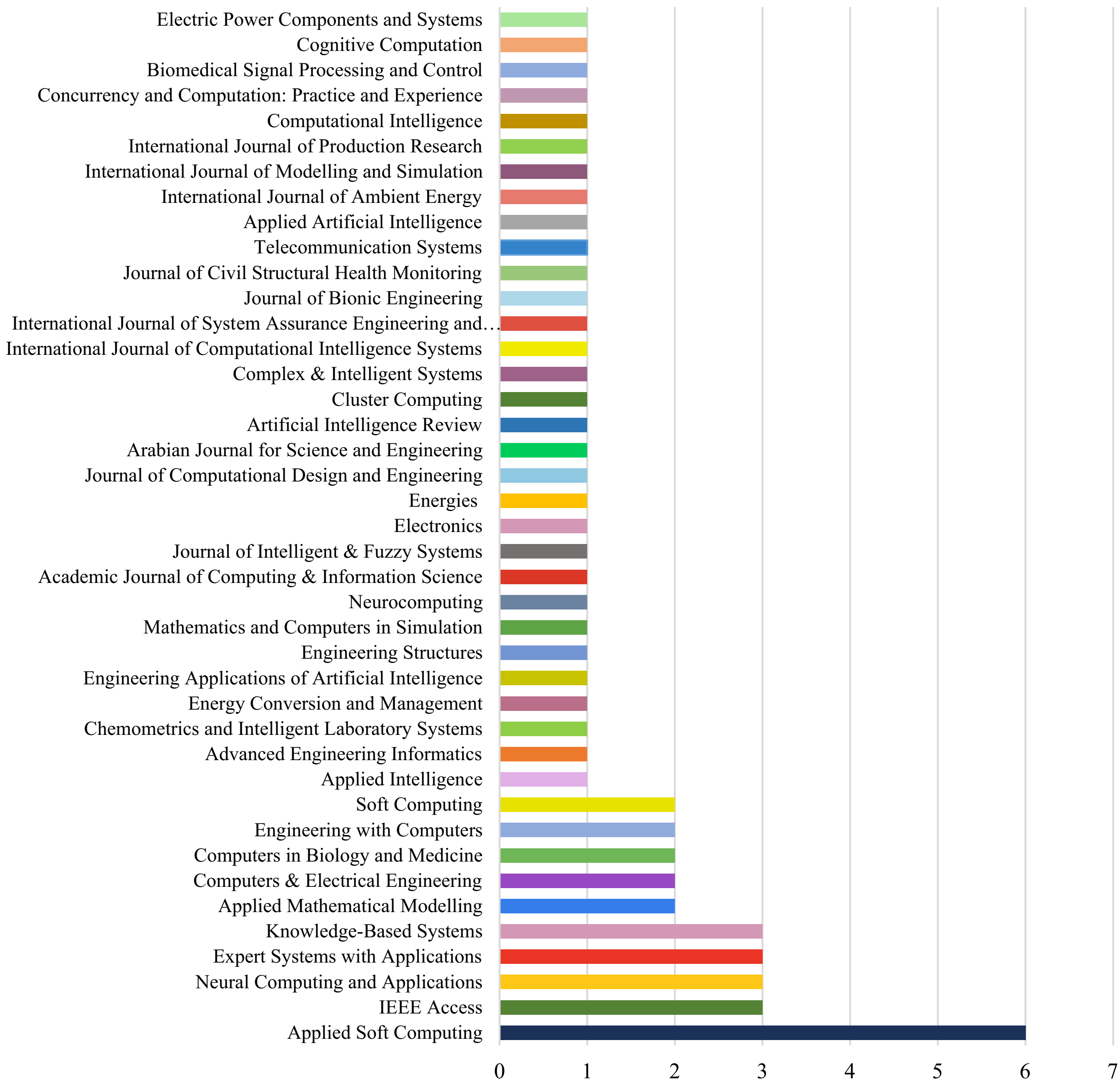


Fig. 22 The distribution of eligible papers of improved WOA in terms of journal

PRISMA methodology [71], by which 2983 related papers were extracted by defining suitable keywords and queries in the identification stage. However, using the second stage of the PRISMA, the required reduction was not obtainable, and many papers remained for full-text reading. Thus, to strictly screen the vast number of WOA developments and reach a reasonable number of reputable articles, an adapted PRISMA methodology consisting of three main stages, identification, evaluation, and reporting, was introduced. Its evaluation stage was armed by more screening steps and strict inclusion criteria, through which the extracted papers were reduced to 116 eligible papers for reviewing and reporting. Our strict inclusion criteria limited the extracted papers to only those published by reputable journals in the scope of the study, including improving and hybrid WOA variants. Then, in the reporting stage, the eligible papers were carefully studied and reviewed through two main approaches, improving and hybrid WOA variants. The WOA variants papers were also classified into continuous, binary, single-objective, and multi/many-objective to be thoroughly reviewed along with their results.

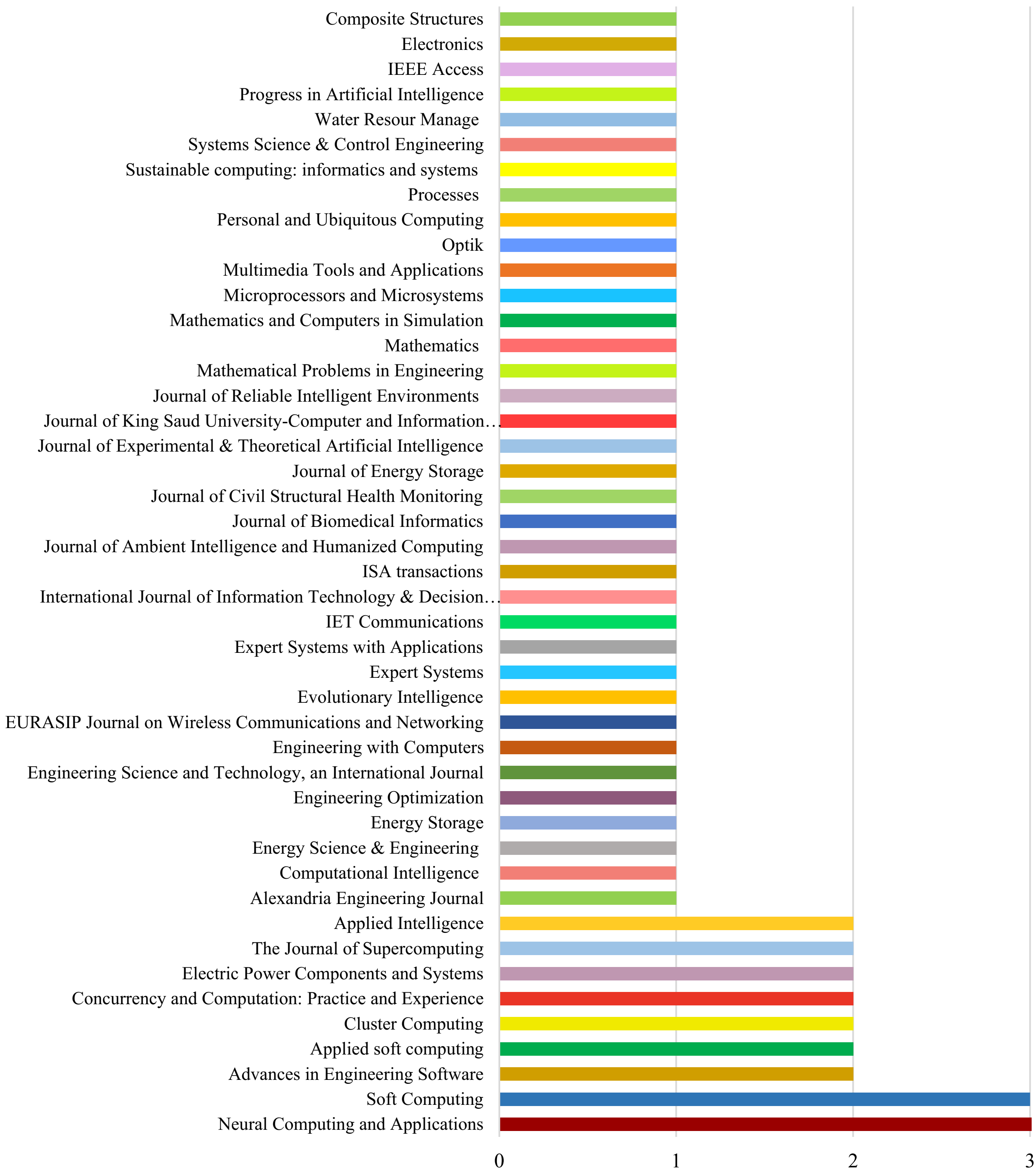


Fig. 23 The distribution of hybrid WOA eligible papers in terms of journal

In Sect. 6, different approaches and techniques were described by which most WOA variants were developed. Reviewing the WOA variants in Sects. 7 and 8 showed that the Lévy flight, chaotic map, opposition-based learning, quantum-based techniques, mutation strategies, and crossover operators are valuable in improving WOA. Additionally, WOA can be effectively hybridized with other population-based algorithms in the evolutionary, physics-based, and mostly swarm intelligence categories. Although reviewing the eligible papers shows that both improved and hybrid approaches successfully

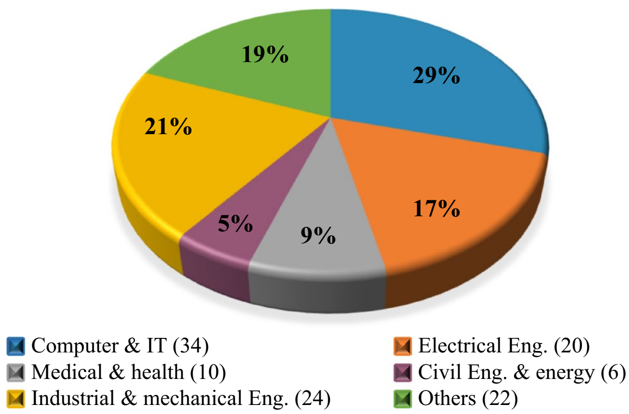


Fig. 24 The distribution of WOA eligible papers in terms of application

tackle one or more WOA weaknesses such as local optima trapping, low population diversity, and imbalance between exploration and exploitation, the improved WOA variants have more benefits because of using the required technique in their improvement intently. Most WOA variants are developed for solving continuous single-objective problems; however, binary problems, such as feature selection, were effectively solved by binary WOA variants. This study also showed that WOA successfully solves real optimization problems in various applications, including computer and IT, electrical engineering, industrial and mechanical engineering, civil engineering and energy, and medical and health. It is also concluded that most WOA variant papers lack a comprehensive comparison with previous WOA variants and are usually compared with other algorithms.

There is still space for enhancing the performance of WOA by tuning its parameters, hybridizing with other algorithms, and improving using different techniques. Thus, some possible future directions for WOA development can be as follows.

- Critical analysis studies to comprehensively determine the strength and weaknesses of WOA in solving different optimization problems through which future WOA variants can be well-designed.
- A sensitive analysis to fine-tune the WOA parameters since they significantly impact the WOA's performance.
- Hybridizing WOA with other metaheuristic algorithms, especially those that can effectively incorporate with WOA.
- A systematic review of the usage of WOA and analysis of its performance in different applications since there are many papers for only using the canonical WOA.

In future directions, the WOA developments that have been applied to solve real-world engineering optimization problems should be systematically reviewed. Moreover, a critical analysis of those WOA variants can overcome the WOA weaknesses to find their main benefits and use them in developing other algorithms, and applying the effective techniques introduced in this study to improve different metaheuristic algorithms.

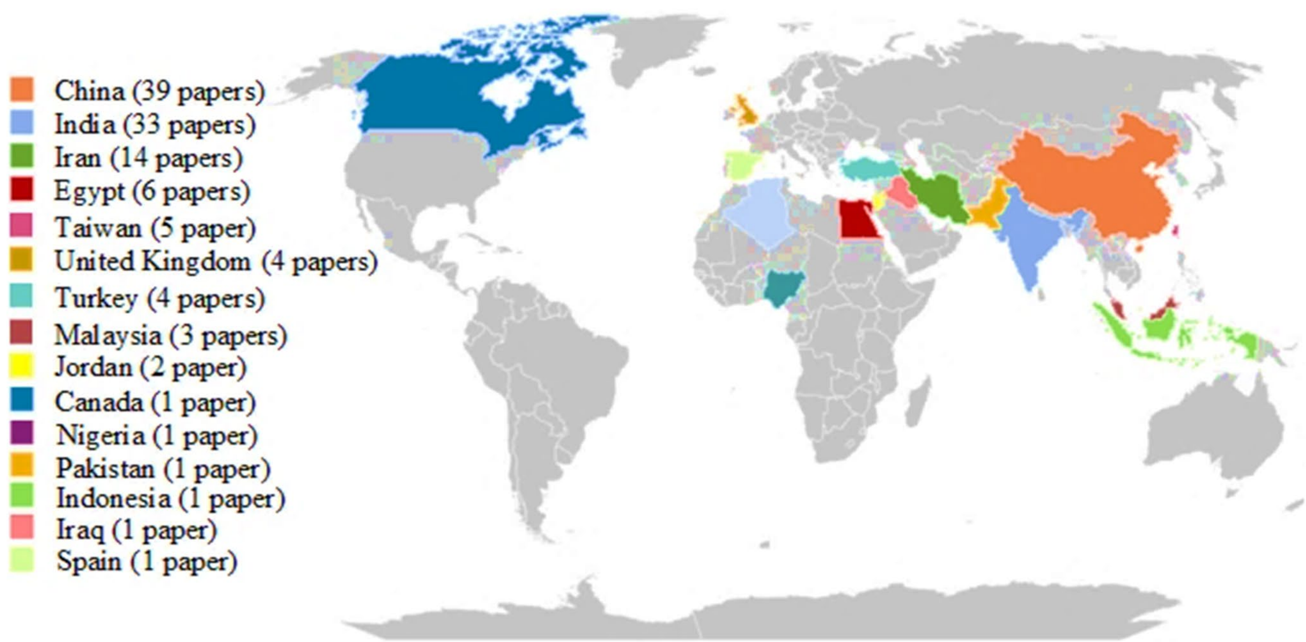


Fig. 25 The distribution of corresponding authors' country of the WOA eligible papers

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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