Trigonometria

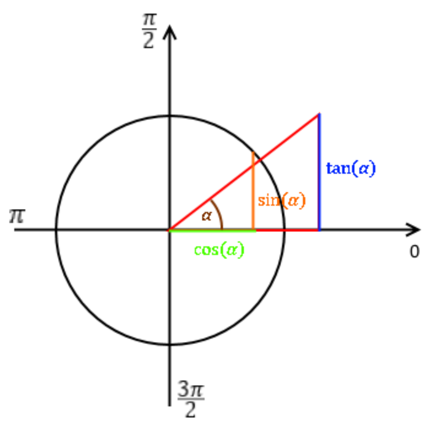
Tabela trigonométrica

α	$\frac{\pi}{6}$ ou $30°$	$\frac{\pi}{4}$ ou $45°$	$\frac{\pi}{3}$ ou $60°$
sin(α)	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos(α)	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan(α)	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Conversões básicas (Graus ↔ Radianos)

Graus	$360°$	$180°$	$90°$	$60°$	$45°$	$30°$
Radianos	2π	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

Variação das funções trigonométricas

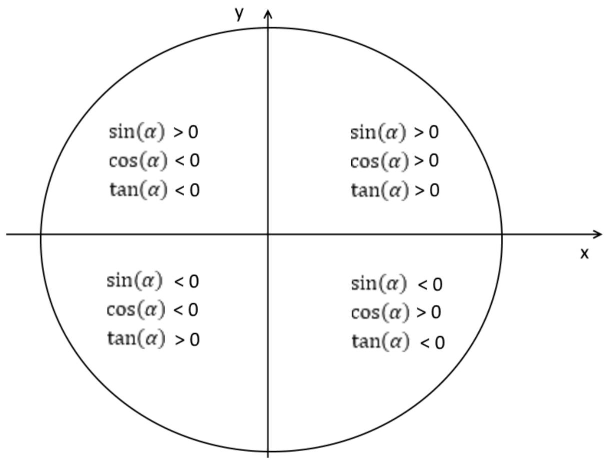


$$-1 \leq \sin(\alpha) \leq 1$$

$$-1 \leq \cos(\alpha) \leq 1$$

$$-\infty \leq \tan(\alpha) \leq +\infty$$

$\tan\left(\frac{\pi}{2}\right)$ e $\tan\left(\frac{3\pi}{2}\right)$ não estão definidos.



$\boldsymbol{\alpha°}$	$\mathbf{sin}(\boldsymbol{\alpha})$	$\mathbf{cos}(\boldsymbol{\alpha})$	$\mathbf{tan}(\boldsymbol{\alpha})$	$\boldsymbol{\alpha\ rads}$
0°	0	1	0	$0\ rads$
90°	1	0	Não definido	$\frac{\pi}{2}\ rads$
180°	0	-1	0	$\pi\ rads$
270°	-1	0	Não definido	$\frac{3\pi}{2}\ rads$

Razões trigonométricas de α e $-\alpha$

1. $\cos(-\alpha) = \cos(\alpha)$
2. $\sin(-\alpha) = \sin(\alpha)$
3. $\tan(-\alpha) = -\tan(\alpha)$

4. $\cos(\pi - \alpha) = -\cos(\alpha)$
5. $\sin(\pi - \alpha) = \sin(\alpha)$
6. $\tan(\pi - \alpha) = -\tan(\alpha)$

7. $\cos(\pi + \alpha) = -\cos(\alpha)$
8. $\sin(\pi + \alpha) = -\sin(\alpha)$
9. $\tan(\pi + \alpha) = \tan(\alpha)$

10. $\sin(90° - \alpha) = \cos(\alpha)$
11. $\cos(90° - \alpha) = \sin(\alpha)$
12. $\tan(90° - \alpha) = \frac{1}{\tan(\alpha)}$

13. $\sin(90° + \alpha) = \cos(\alpha)$
14. $\cos(90° + \alpha) = -\sin(\alpha)$
15. $\tan(90° + \alpha) = -\frac{1}{\tan(\alpha)}$

16. $\sin(60°) = \cos(30°)$
17. $\cos(60°) = \sin(30°)$
18. $\tan(60°) = \frac{1}{\tan(30°)}$

19. $\sin(270° - \alpha) = -\cos(\alpha)$
20. $\cos(270° - \alpha) = -\sin(\alpha)$
21. $\tan(270° - \alpha) = \frac{1}{\tan(\alpha)}$

Exemplos:

a. $\sin(240°) = \sin(270° - 30°) = -\cos(30°)$
b. $\cos(210°) = \cos(270° - 60°) = -\sin(60°)$
c. $\tan(225°) = \tan(270° - 45°) = \frac{1}{\tan(45°)}$

22. $\sin(270° + \alpha) = -\cos(\alpha)$
23. $\cos(270° + \alpha) = \sin(\alpha)$
24. $\tan(270° + \alpha) = -\frac{1}{\tan(\alpha)}$

Exemplos:

a. $\sin(300°) = \sin(270° + 30°) = -\cos(30°)$
b. $\cos(330°) = \cos(270° + 60°) = \sin(60°)$
c. $\tan(315°) = \tan(270° + 45°) = \frac{1}{\tan(45°)}$

Função seno

Domínio	$D = \mathbb{R}$
Contradomínio	$D' = [-1; 1]$
Zeros	$k\pi, k \in \mathbb{Z}$
Máximo	1
Mínimo	-1
Maximizante	$\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$
Minimizante	$-\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$
Função seno é ímpar: $f(x) = -f(x), \forall_x \in D_f$	

Função cosseno

Domínio	$D = \mathbb{R}$
Contradomínio	$D' = [-1; 1]$
Zeros	$\frac{\pi}{2} + k\pi, k \in \mathbb{Z}$
Máximo	1
Mínimo	-1
Maximizante	$2k\pi, k \in \mathbb{Z}$
Minimizante	$\pi + 2k\pi, k \in \mathbb{Z}$
Função cosseno é par: $f(x) = f(-x), \forall_x \in D_f$	

Função tangente

Domínio	$D = \mathbb{R} \backslash \{x \in \mathbb{R}: x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$ ou $D = \{x \in \mathbb{R}: x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$
Contradomínio	$D' =]-\infty; +\infty[$
Zeros	$k\pi, k \in \mathbb{Z}$
$\boldsymbol{f(-x) = -f(x)}$	

Equações trigonométricas

$\sin(x) = \sin(\alpha) \Leftrightarrow x = \alpha + 2k\pi \vee x = (\pi - \alpha) + 2k\pi, k \in \mathbb{Z}$

$\cos(x) = \cos(\alpha) \Leftrightarrow x = \alpha + 2k\pi \vee x = -\alpha + 2k\pi, k \in \mathbb{Z}$

$\tan(x) = \tan(\alpha) \Leftrightarrow x = \alpha + k\pi, k \in \mathbb{Z}$

Exemplos:

a) $\sqrt{12}\sin\left(\frac{x}{5}\right) = -3 \Leftrightarrow \sin\left(\frac{x}{5}\right) = \frac{-3}{\sqrt{12}} \Leftrightarrow \sin\left(\frac{x}{5}\right) = \frac{-3\sqrt{12}}{12} \Leftrightarrow \sin\left(\frac{x}{5}\right) = \frac{-3*2\sqrt{3}}{12} \Leftrightarrow \sin\left(\frac{x}{5}\right) = \frac{-\sqrt{3}}{2} \Leftrightarrow \frac{x}{5} = -\frac{\pi}{3} + 2k\pi \vee \frac{x}{5} = \left(\pi + \frac{\pi}{3}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow x = -\frac{5\pi}{3} + 10k\pi \vee \frac{x}{5} = \frac{4\pi}{3} + 2k\pi \Leftrightarrow x = -\frac{5\pi}{3} + 10k\pi \vee x = \frac{20\pi}{3} + 10k\pi$

b) $2\cos(3x) = -1 \Leftrightarrow \cos(3x) = -\frac{1}{2} \Leftrightarrow 3x = \left(\pi - \frac{\pi}{3}\right) + 2k\pi \vee 3x = \left(\pi + \frac{\pi}{3}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow 3x = \frac{2\pi}{3} + 2k\pi \vee 3x = \frac{4\pi}{3} + 2k\pi \Leftrightarrow x = \frac{2\pi}{9} + 2k\pi \vee x = \frac{4\pi}{9} + 2k\pi$

c) $12\tan\left(\frac{x}{2}\right) = \sqrt{48}, em\ [-2\pi; 5\pi] \Leftrightarrow \tan\left(\frac{x}{2}\right) = \frac{\sqrt{48}}{12} \Leftrightarrow \tan\left(\frac{x}{2}\right) = \frac{4\sqrt{3}}{12} \Leftrightarrow \tan\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3} \Leftrightarrow \frac{x}{2} = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{3} + 2k\pi$

$k = 0: x = \frac{\pi}{3}$

$k = 1: x = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

$k = 2: x = \frac{\pi}{3} + 2 * 2 * \pi = \frac{\pi}{3} + 4\pi = \frac{13\pi}{3}$

$k = -1: x = \frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}$

$k = -2: x = \frac{\pi}{3} + 2 * (-2) * \pi = \frac{\pi}{3} - 4\pi = -\frac{11\pi}{3}$

$x \in \{-\frac{5\pi}{3}; \frac{\pi}{3}; \frac{7\pi}{3}; \frac{13\pi}{3}\}$

Fórmulas trigonométricas

$(\sin(\alpha))^2 + (\cos(\alpha))^2 = 1$

$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$

$(\tan(\alpha))^2 + 1 = \frac{1}{(\cos(\alpha))^2}$

$1 + \frac{1}{(\tan(\alpha))^2} = \frac{1}{(\sin(\alpha))^2}$

$\sin(a + b) = \sin(a) * \cos(b) + \sin(b) * \cos(a)$

$\cos(a + b) = \cos(a) * \cos(b) - \sin(a) * \sin(b)$

$\tan(a + b) = \frac{\tan(a) - \tan(b)}{1 - \tan(a) * \tan(b)}$

$\sin(a - b) = \sin(a) * \sin(b) - \sin(b) * \cos(a)$

$\cos(a - b) = \cos(\mathrm{a}) * \cos(\mathrm{b}) + \sin(\mathrm{a}) * \sin(b)$

$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a) * \tan(b)}$

$\sin(2a) = 2 \sin(a) * \cos(a)$

$\cos(2a) = (\cos(a))^2 - (\sin(a))^2$

$\tan(2a) = \frac{2 \tan(a)}{1 - (\tan(a))^2}$

Função periódica

Diz-se que f é uma função periódica de período T, se e só se: $f(x + \mathrm{T}) = f(x), \forall_x \in D_f$.

Família de funções (cálculo do Período)

Funções do tipo y = c +dsin(ax+b) e y = c dcos(ax +b)

- $D = \mathbb{R}$
- $D'(por\ enquadramentos)$
- $\mathrm{T} = \frac{2\pi}{|a|}$

Para mostrar que T é período: $f(x + \mathrm{T}) = \mathrm{f(x)}, \forall_x \in D_f$.

Funções do tipo y = c + d tan(ax+b)

$D = \{x \in \mathbb{R}: ax + b \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$

- $D' = \mathbb{R}$
- $\mathrm{T} = \frac{\pi}{|a|}$

Limites em trigonometria

Não existem

- $\lim_{x \to \pm\infty} (\cos x)$
- $\lim_{x \to \pm\infty} (\sin x)$
- $\lim_{x \to \pm\infty} (\tan x)$

Limites infinitos

- $\lim_{x \to \frac{\pi}{2}^-} (\tan x) = +\infty$
- $\lim_{x \to \frac{\pi}{2}^+} (\tan x) = -\infty$
- $\lim_{x \to \frac{3\pi}{2}^-} (\tan x) = +\infty$
- $\lim_{x \to \frac{3\pi}{2}^+} (\tan x) = -\infty$

Limites associados a y = sin(x) / x

- $\lim_{x \to -\infty} \left(\frac{\sin x}{x}\right) = 0$
- $\lim_{x \to +\infty} \left(\frac{\sin x}{x}\right) = 0$
- $\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$

Demonstração:

$$-1 \leq \sin x \leq 1 \Leftrightarrow -\frac{1}{x} \leq \sin x \leq \frac{1}{x} \Leftrightarrow 0 \leq \sin x \leq 0$$

Assim, $\lim_{x \to \pm\infty} \left(\frac{\sin x}{x}\right) = 0.$

Regras de derivação das funções trigonométricas

- $(\sin u)' = u' * \cos u$
- $(\cos u)' = -u' * \sin u$
- $(\tan u)' = \frac{u'}{\cos^2 u}$
- $(\tan u)' = u' * (1 + \tan^2 u)$