Faculty of Engineering
Computer Department
Communications (ELC 325B) – Spring 2023

Assignment 3

Digital Communication

Team Members

Num	Full Name in ARABIC	SEC	BN	Code
1	عبدالرحمن حمدي احمد مخيمر	1	36	9202833
2	زياد طارق خيري علي	1	28	9202588
3	عبدالرحمن نعمان لقمان ابراهيم	2	2	9202851

Submitted to

Eng. Mohamed Khaled

Table of contents:

List of Figures

1. Part One

1.1 Gram-Schmidt Orthogonalization

- ✓ Gram-Schmidt Orthogonalization is a method for taking a set of vectors and finding a new set of orthogonal vectors that span the same space.
- ✓ The process involves taking the first vector in the set and normalizing it to create the first orthogonal vector.
- ✓ Then, for each subsequent vector, we subtract the projection of that vector onto the previous orthogonal vectors, creating a new vector that is orthogonal to all the previous vectors.
- ✓ We normalize this new vector to create the next orthogonal vector, and repeat this process until we have created a set of orthogonal vectors that span the same space.
- ✓ The resulting set of orthogonal vectors can be useful for a variety of applications, including solving systems of linear equations, finding eigenvalues and eigenvectors, and performing least-squares approximations.

NUMBER OF SAMPLES = 100

Figure 1 Φ1 VS time after using the GM_Bases function

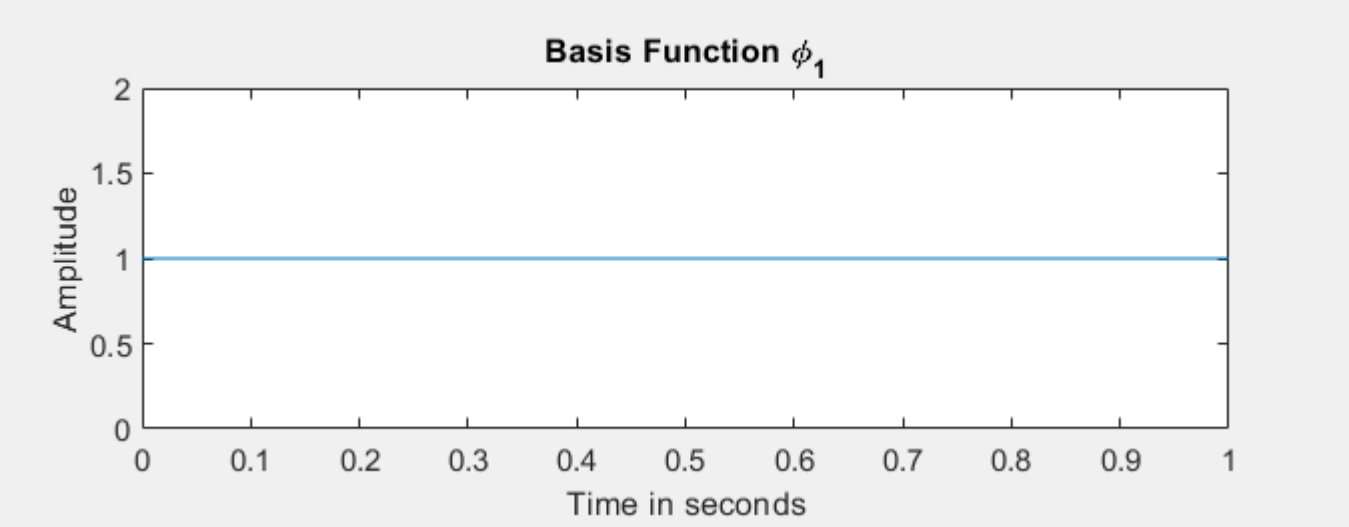
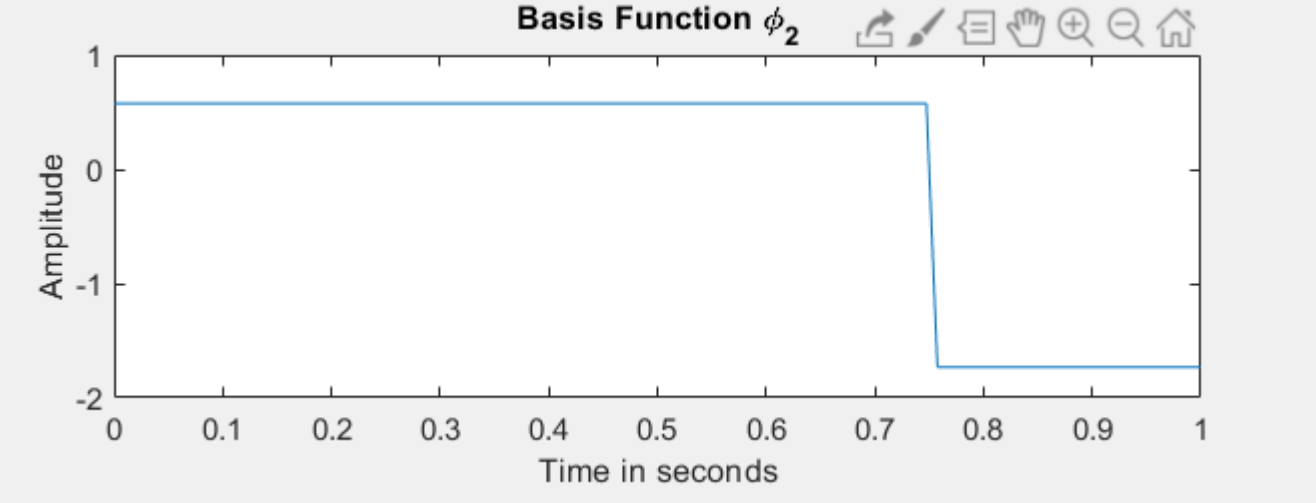


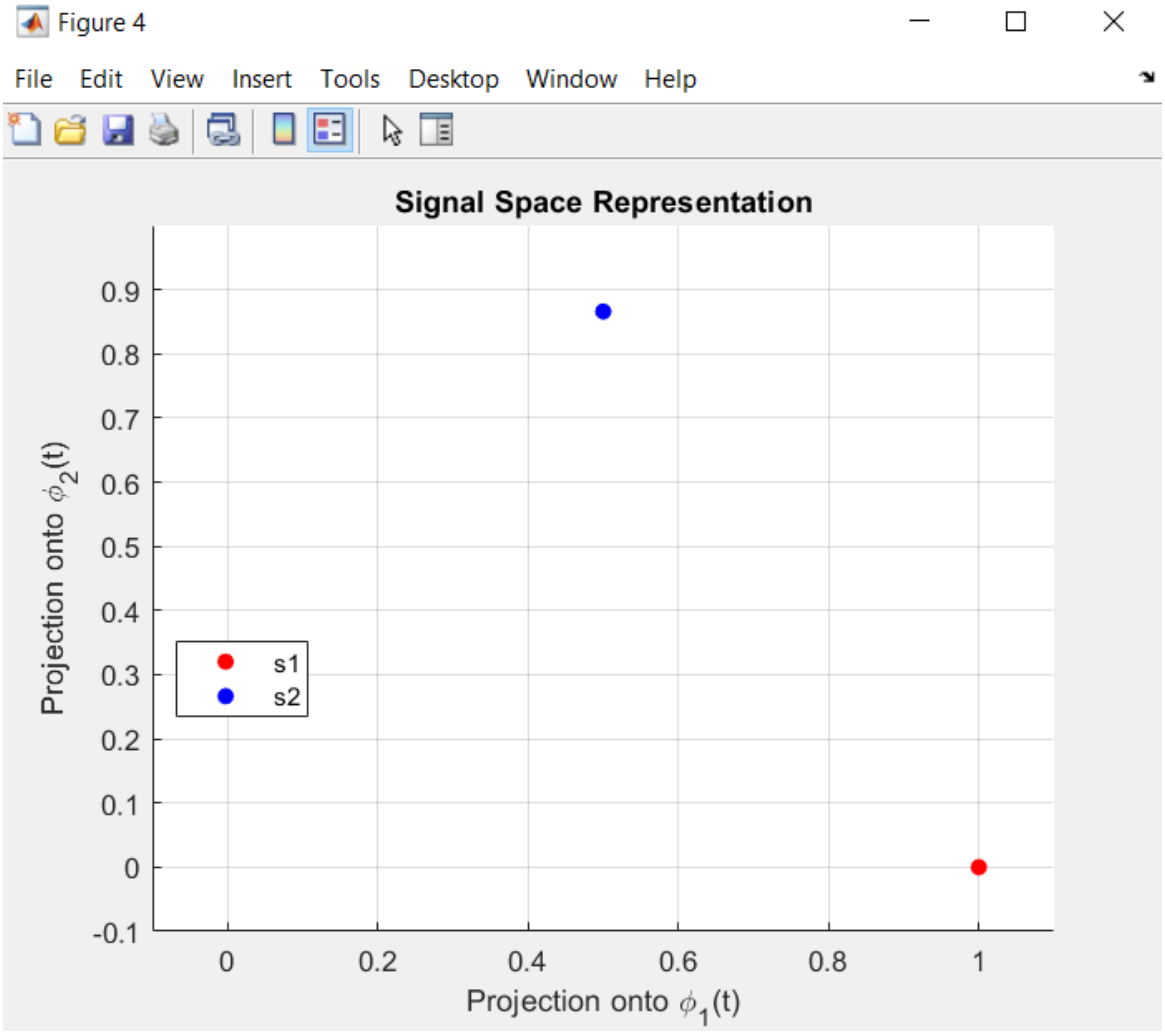
Figure 2 Φ2 VS time after using the GM_Bases function

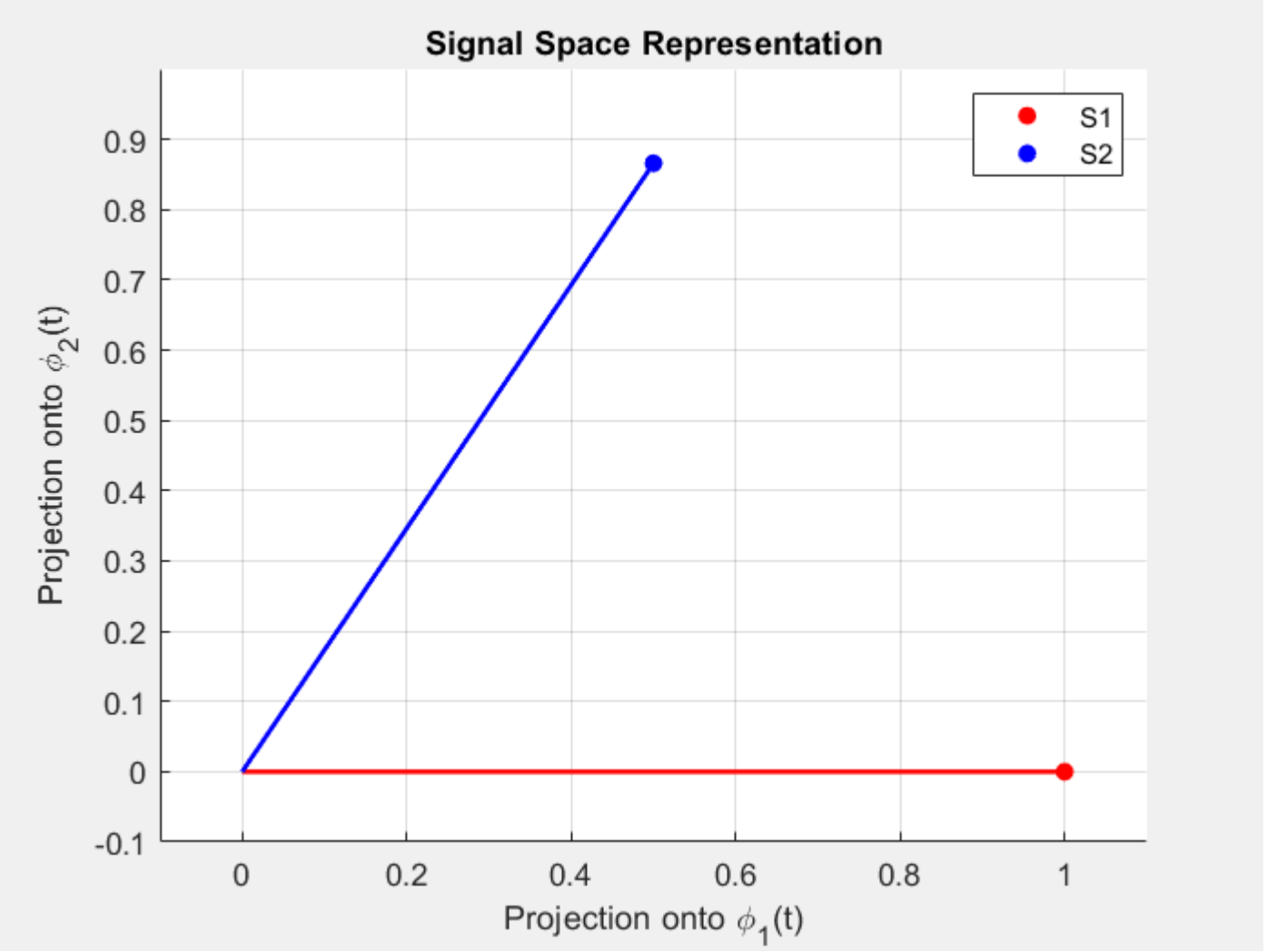


1.2 Signal Space Representation

Here we represent the signals using the base functions.

Figure 3 Signal Space representation of signals s1,s2




Signal Space Representation
S1
S2
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
Projection onto $\phi_2(t)$
0
0.2
0.4
0.6
0.8
1
Projection onto $\phi_1(t)$

1.3 Signal Space Representation with adding AWGN

-the expected real points will be solid and the received will be hollow

Case 1: $\mathbf{10}\ log(E/\sigma^2)\ =\ \mathbf{10}\ dB$

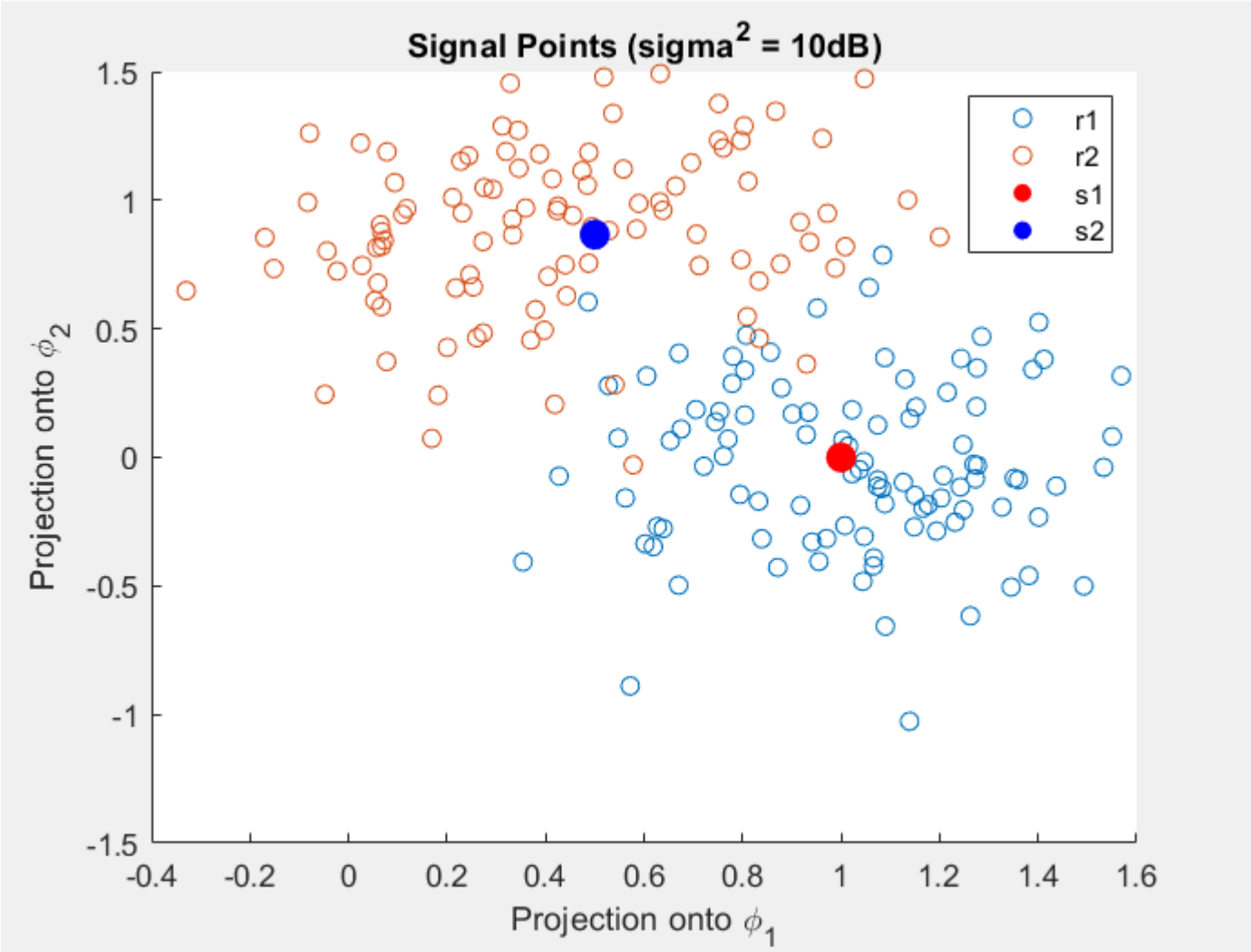


Figure 4 Signal Space representation of signals s1,s2 with E/σ¬2 =10dB

Case 2: $\mathbf{10}\ log(E/\sigma^2)\ =\ \mathbf{0}\ dB$

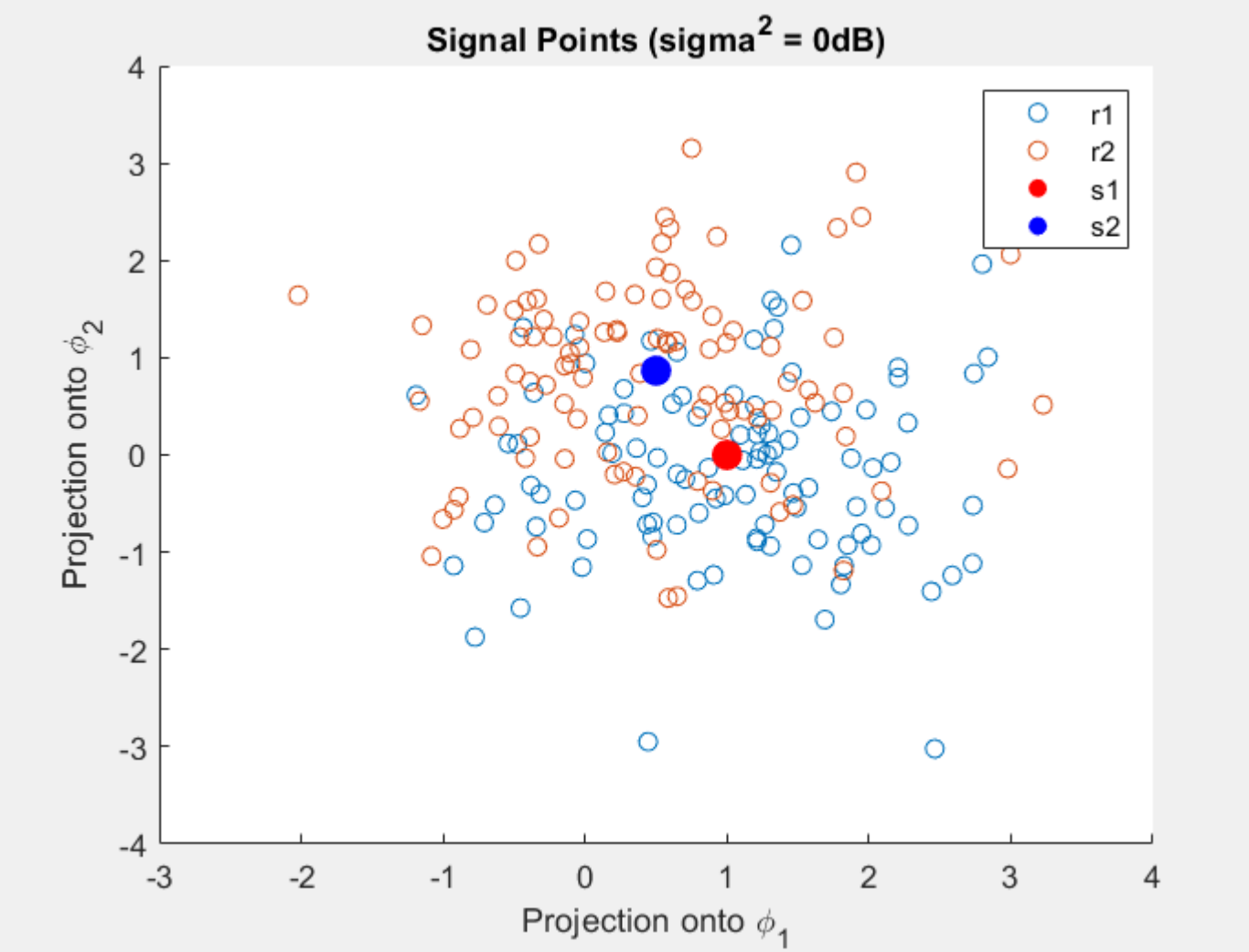


Figure 5 Signal Space representation of signals s1,s2 with E/σ¬2 =0dB

Case 3: $\mathbf{10\ log(E/\sigma^2)\ =\ -5\ dB}$

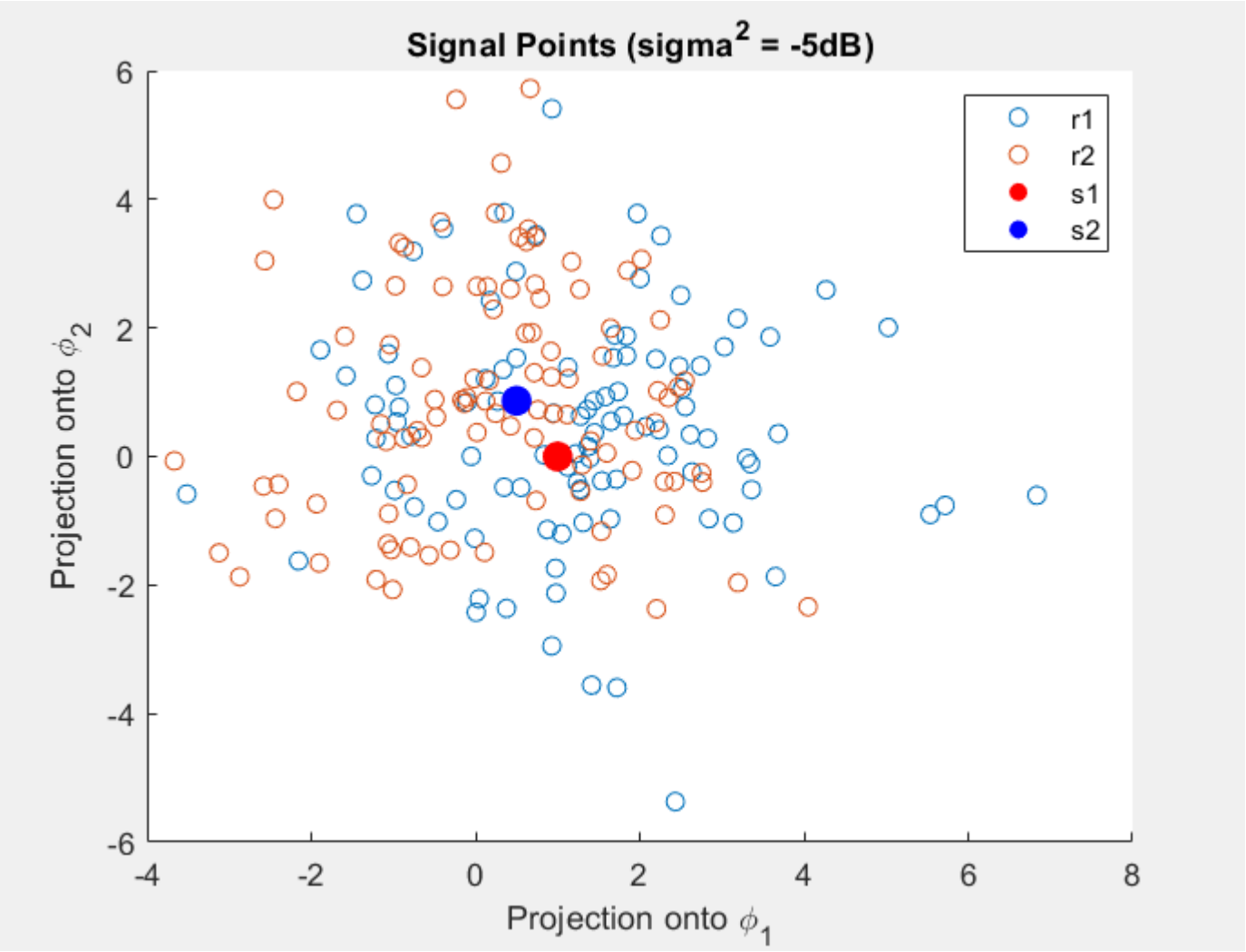


Figure 6 Signal Space representation of signals s1,s2 with E/σ¬2 =-5dB

1.4 Noise Effect on Signal Space

The noise affects the signal space representation by introducing variability and spreading out the signal points. As the noise variance σ^2 increases, the effect of noise becomes more prominent and the signal points become more scattered.

Specifically, when the noise variance σ^2 is small, the signal points tend to cluster closely around the ideal signal representation. As the noise variance increases, the signal points become more spread out and exhibit greater dispersion. This dispersion is due to the random nature of the noise, which introduces variations in the observed signal values.

The effect of noise on the signal space representation can be observed in the scatter plots. As the noise level (variance) increases, the signal points become more spread out, leading to a larger variation in the signal space representation. This can be seen as an increase in the dispersion of the signal points in the scatter plots. The noise affects the signal space representation by introducing uncertainty and causing deviations from the ideal signal projections onto the basis functions. Therefore, increasing the noise level (increasing σ 2) generally results in a larger effect of noise on the signal space representation.

2. Appendix A: Codes for Part One:

A.1 Code for Gram-Schmidt Orthogonalization

```
function [phi1, phi2] = GM_Bases(s1, s2)
    % Calculate the first basis function (phi1)
    phi1 = s1 / norm(s1);

    % Check if s2 is linearly independent from s1
    if dot(s2, phi1) ~= 0
        % Calculate the second basis function (phi2)
        phi2 = s2 - dot(s2, phi1) * phi1;
        phi2 = phi2 / norm(phi2);
    else
        % s2 is linearly dependent on s1, so phi2 is a zero vector
        phi2 = zeros(size(s2));
    end
    phi1 = phi1 * sqrt(length(s1));
    phi2 = phi2 * sqrt(length(s2));
end
```

A.2 Code for Signal Space representation

```
function [v1, v2] = signal_space(s, phi1, phi2)
    % Get the length of each basis function
    phi1_len = length(phi1);
    phi2_len = length(phi2);

    % Set new phi1 after dividing by the square root of their lengths
    phi1 = phi1 / sqrt(phi1_len);
    phi2 = phi2 / sqrt(phi2_len);

    % Calculating the dot product between the signal and phi just
like the
    % lecture
    dotProduct_1 = dot(s, phi1);
```

```
    dotProduct_2 = dot(s, phi2);

    % Calculate the projections (correlations) of s over phi1 and
phi2
    % Then divide by the square root of each phi
    v1 = dotProduct_1 / sqrt(phi1_len);
    v2 = dotProduct_2 / sqrt(phi2_len);
end
```

A.3 Code for plotting the bases functions

```
% Get phi1 & phi2 from the GM_Bases function
[phi1, phi2] = GM_Bases(s1, s2);

% Plot phi1
figure;
subplot(2,1,1);
plot(T, phi1);
title('Basis Function \phi_1');
xlabel('Time in seconds');
ylabel('Amplitude');

% Plot phi2
subplot(2,1,2);
plot(T, phi2);
title('Basis Function \phi_2');
xlabel('Time in seconds');
ylabel('Amplitude');
```

A.4 Code for plotting the Signal space Representations

```
% Calculate the signal space representation using signal_space
function
% For s1(t)
[s1_v1, s1_v2] = signal_space(s1, phi1, phi2);
% For s2(t)
[s2_v1, s2_v2] = signal_space(s2, phi1, phi2);

% Plot the signal space representation we just obtained
figure;
scatter(s1_v1, s1_v2, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
hold on;
scatter(s2_v1, s2_v2, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
title('Signal Space Representation');
xlabel('Projection onto \phi_1(t)');
ylabel('Projection onto \phi_2(t)');
legend('Location', 'best');
axis([-0.1 1.1 -0.1 1]);
grid on;

% Line Plot
figure;
scatter(s1_v1, s1_v2, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
hold on;
scatter(s2_v1, s2_v2, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
plot([0, s1_v1], [0, s1_v2], 'r', 'LineWidth', 1.5);
plot([0, s2_v1], [0, s2_v2], 'b', 'LineWidth', 1.5);
title('Signal Space Representation');
xlabel('Projection onto \phi_1(t)');
ylabel('Projection onto \phi_2(t)');
legend('S1', 'S2');
axis([-0.1 1.1 -0.1 1]);
grid on;
```

A.5 Code for effect of noise on the Signal space Representations

```
% Random Samples
% Set the random number generator seed for reproducibility
rng(0);

% Number of samples
numberOfSamples = 100;

% Variance values (dB) for different noise levels
values = [-5, 0, 10];

% Set the length of values
L = length(values);

% Get size of s1 & s2
sz_s1 = size(s1);
sz_s2 = size(s2);

% Get length of s1 & s2
len_s1 = length(s1);
len_s2 = length(s2);

for i = 1:L
    r1V1 = [];
    r1V2 = [];
    r2V1 = [];
    r2V2 = [];
    for sample = 1:numberOfSamples
        % Convert dB to linear scale
        val = 10^(values(i)/10);

        sq_s1 = s1.^2;
        sq_s2 = s2.^2;

        energy1 = sum(sq_s1);
        energy2 = sum(sq_s2);

        sigma1 = sqrt(energy1 / val);
```

```
        sigma2 = sqrt(energy2 / val);

        % Add noise to the original signal
        w1 = sigma1 * randn(sz_s1);
        w2 = sigma2 * randn(sz_s2);

        r1 = w1 + s1;
        r2 = w2 + s2;

        % Calculate the signal points of the input using signal_space
function
        [r1_v1, r1_v2] = signal_space(r1, phi1, phi2);
        [r2_v1, r2_v2] = signal_space(r2, phi1, phi2);

        % Append to lists
        r1V1(end + 1) = r1_v1;
        r1V2(end + 1) = r1_v2;
        r2V1(end + 1) = r2_v1;
        r2V2(end + 1) = r2_v2;
    end
    disp(values(i));
    % Plot the signal points of generated samples of r1 & r2
    figure;
    % Plot r1 samples
    scatter(r1V1, r1V2, 'DisplayName', 'r1');
    hold on;
    % Plot r2 samples
    scatter(r2V1, r2V2, 'DisplayName', 'r2');
    hold on
    % Plot the input signal s1(t)
    scatter(s1_v1, s1_v2, 100, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
    hold on
    % Plot the input signal s2(t)
    scatter(s2_v1, s2_v2, 100, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
    % Title/xLabel/yLabel
    title(sprintf('Signal Points (sigma^2 = %ddB)', values(i)));
    xlabel('Projection onto \phi_1');
    ylabel('Projection onto \phi_2');
```

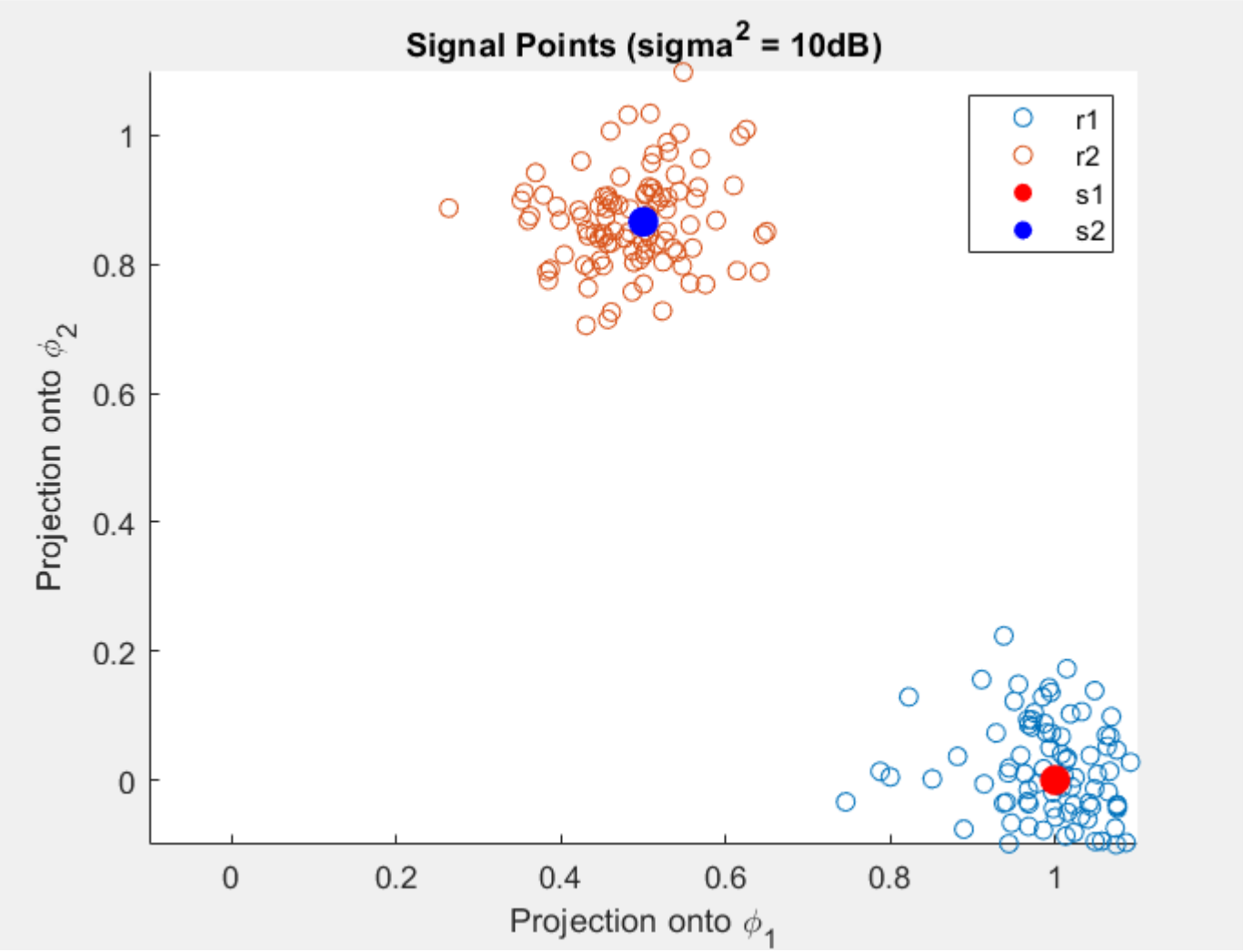
```
    legend('r1', 'r2', 's1', 's2');
    %axis([-4 5 -1 2]);
end
```

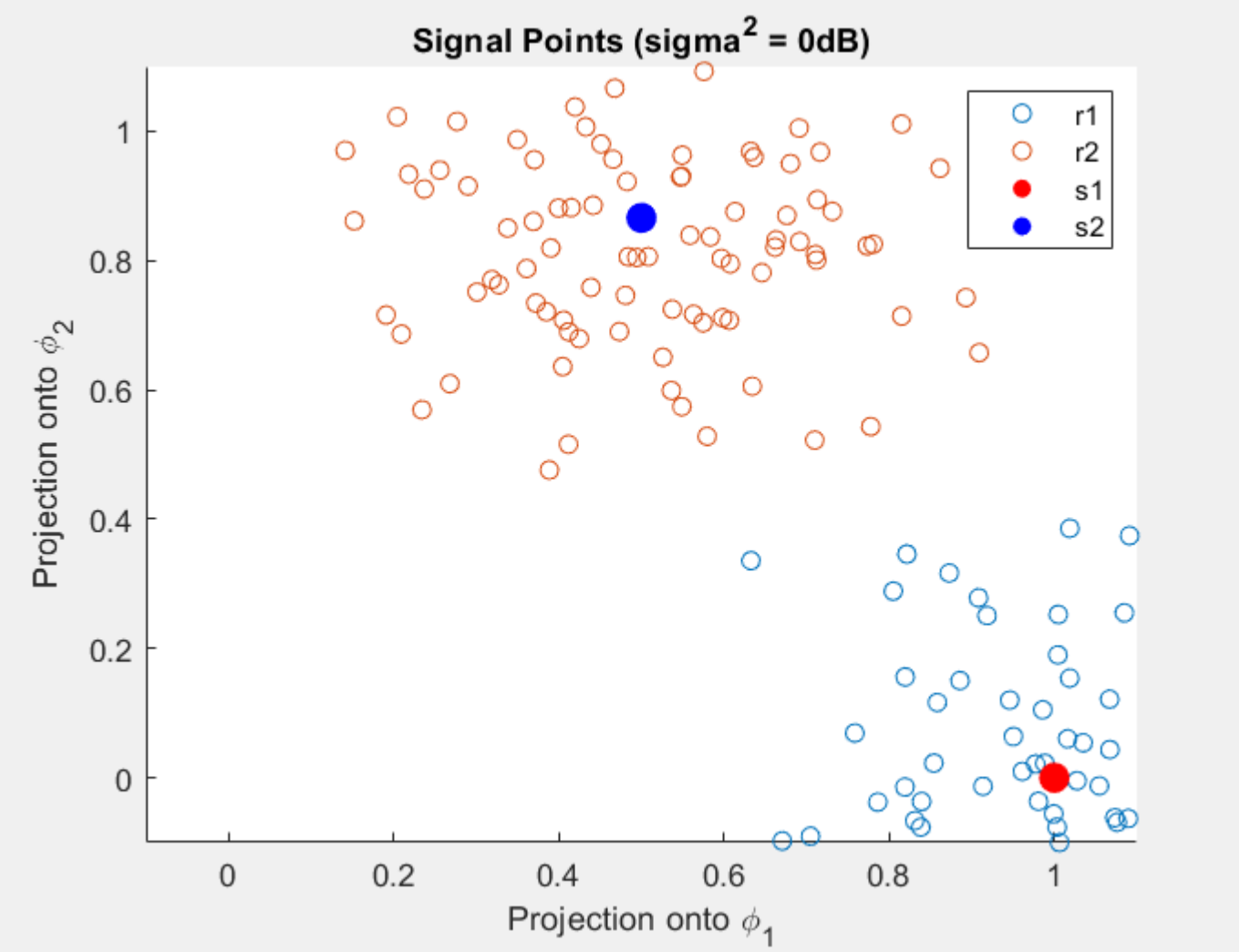
IMPORTANT NOTE:

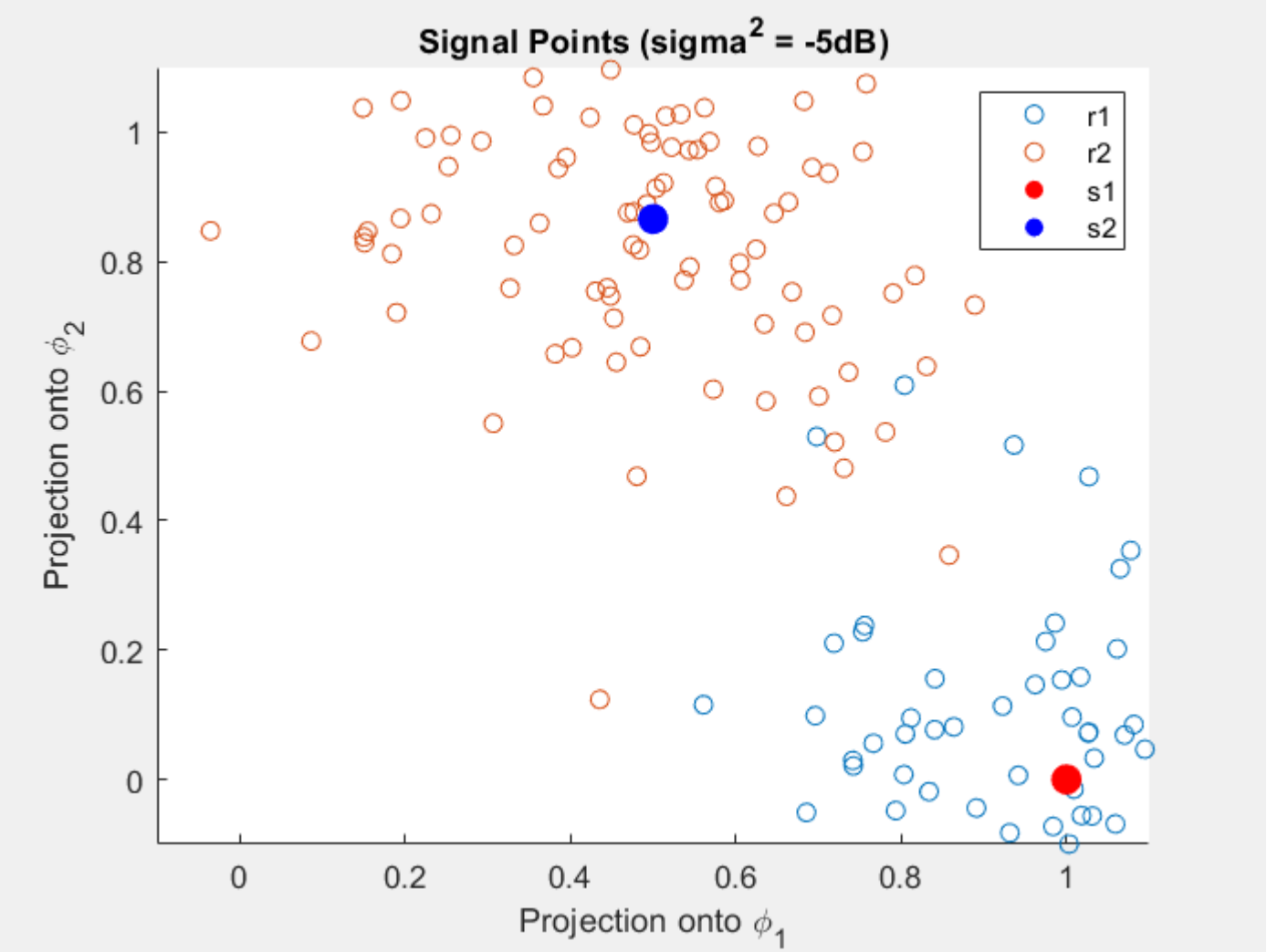
Manually calculating the noise from the equation SNR = E / sigma^2.. We already have the linear SNR value and the **energy** is calculated by summing the square of the signal components, we can easily get sigma. Then, to calculate the noise, it'll be the sigma multiplied by the random number from 1 to the size of the signal. This noise is then added on to the original signal to obtain r1 and r2. This way gave us the above plots which we submitted.

The above approach was the most logical one but didn't give us the same graph shapes in the assignment video.

When we attempted to draw the plots the way you did in the tutorial, and using the **awgn** function we got the plots you showed us in the tutorial but there was a problem that when we change the number of samples, the scatter changes and no longer contains the same correlation. Therefore, it's not considered correct. This gave us the following plots which are exactly like yours:


Signal Points (sigma^2 = 10dB)
r1
r2
s1
s2
Projection onto ϕ_2
Projection onto ϕ_1
0
0.2
0.4
0.6
0.8
1


Signal Points (sigma^2 = 0dB)
r1
r2
s1
s2
Projection onto ϕ_2
Projection onto ϕ_1
0
0.2
0.4
0.6
0.8
1



COMPLETE CODE

```
clc;
clear all;
close all;

% Time axis
N = 100;
```

```
T = linspace(0, 1, N);

% Input signal s1(t)
s1 = ones(1, N);

% Input signal s2(t)
s2 = zeros(1, N);

% Setting from 0 to 0.75 as 1, else -1
s2(1: floor(0.75 * N)) = 1;
s2(floor(0.75 * N) + 1:end) = -1;

% Plot s1(t)
figure;
plot(T, s1);
title('Input Signal s1(t)');
xlabel('Time in seconds');
ylabel('Amplitude');

% Plot s2(t)
figure;
plot(T, s2);
title('Input Signal s2(t)');
xlabel('Time in seconds');
ylabel('Amplitude');

% Get phi1 & phi2 from the GM_Bases function
[phi1, phi2] = GM_Bases(s1, s2);

% Plot phi1
figure;
subplot(2,1,1);
plot(T, phi1);
title('Basis Function \phi_1');
xlabel('Time in seconds');
ylabel('Amplitude');

% Plot phi2
subplot(2,1,2);
plot(T, phi2);
```

```
title('Basis Function \phi_2');
xlabel('Time in seconds');
ylabel('Amplitude');

% Calculate the signal space representation using signal_space
function
% For s1(t)
[s1_v1, s1_v2] = signal_space(s1, phi1, phi2);
% For s2(t)
[s2_v1, s2_v2] = signal_space(s2, phi1, phi2);

% Plot the signal space representation we just obtained
figure;
scatter(s1_v1, s1_v2, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
hold on;
scatter(s2_v1, s2_v2, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
title('Signal Space Representation');
xlabel('Projection onto \phi_1(t)');
ylabel('Projection onto \phi_2(t)');
legend('Location', 'best');
axis([-0.1 1.1 -0.1 1]);
grid on;

% Line Plot
figure;
scatter(s1_v1, s1_v2, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
hold on;
scatter(s2_v1, s2_v2, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
plot([0, s1_v1], [0, s1_v2], 'r', 'LineWidth', 1.5);
plot([0, s2_v1], [0, s2_v2], 'b', 'LineWidth', 1.5);
title('Signal Space Representation');
xlabel('Projection onto \phi_1(t)');
ylabel('Projection onto \phi_2(t)');
legend('S1', 'S2');
axis([-0.1 1.1 -0.1 1]);
grid on;
```

```
% Random Samples
% Set the random number generator seed for reproducibility
rng(0);

% Number of samples
numberOfSamples = 100;

% Variance values (dB) for different noise levels
values = [-5, 0, 10];

% Set the length of values
L = length(values);

% Get size of s1 & s2
sz_s1 = size(s1);
sz_s2 = size(s2);

% Get length of s1 & s2
len_s1 = length(s1);
len_s2 = length(s2);

for i = 1:L
    r1V1 = [];
    r1V2 = [];
    r2V1 = [];
    r2V2 = [];
    for sample = 1:numberOfSamples
        % Convert dB to linear scale
        val = 10^(values(i)/10);

        sq_s1 = s1.^2;
        sq_s2 = s2.^2;

        energy1 = sum(sq_s1);
        energy2 = sum(sq_s2);

        sigma1 = sqrt(energy1 / val);
        sigma2 = sqrt(energy2 / val);
```

```
        % Add noise to the original signal
        w1 = sigma1 * randn(sz_s1);
        w2 = sigma2 * randn(sz_s2);

        r1 = w1 + s1;
        r2 = w2 + s2;

        % Calculate the signal points of the input using signal_space
function
        [r1_v1, r1_v2] = signal_space(r1, phi1, phi2);
        [r2_v1, r2_v2] = signal_space(r2, phi1, phi2);

        % Append to lists
        r1V1(end + 1) = r1_v1;
        r1V2(end + 1) = r1_v2;
        r2V1(end + 1) = r2_v1;
        r2V2(end + 1) = r2_v2;
    end
    disp(values(i));
    % Plot the signal points of generated samples of r1 & r2
    figure;
    % Plot r1 samples
    scatter(r1V1, r1V2, 'DisplayName', 'r1');
    hold on;
    % Plot r2 samples
    scatter(r2V1, r2V2, 'DisplayName', 'r2');
    hold on
    % Plot the input signal s1(t)
    scatter(s1_v1, s1_v2, 100, 'filled', 'MarkerFaceColor', 'red',
'DisplayName', 's1');
    hold on
    % Plot the input signal s2(t)
    scatter(s2_v1, s2_v2, 100, 'filled', 'MarkerFaceColor', 'blue',
'DisplayName', 's2');
    % Title/xLabel/yLabel
    title(sprintf('Signal Points (sigma^2 = %ddB)', values(i)));
    xlabel('Projection onto \phi_1');
    ylabel('Projection onto \phi_2');
    legend('r1', 'r2', 's1', 's2');
    %axis([-4 5 -1 2]);
```

```
end

function [phi1, phi2] = GM_Bases(s1, s2)
    % Calculate the first basis function (phi1)
    phi1 = s1 / norm(s1);

    % Check if s2 is linearly independent from s1
    if dot(s2, phi1) ~= 0
        % Calculate the second basis function (phi2)
        phi2 = s2 - dot(s2, phi1) * phi1;
        phi2 = phi2 / norm(phi2);
    else
        % s2 is linearly dependent on s1, so phi2 is a zero vector
        phi2 = zeros(size(s2));
    end
    phi1 = phi1 * sqrt(length(s1));
    phi2 = phi2 * sqrt(length(s2));
end

function [v1, v2] = signal_space(s, phi1, phi2)
    % Get the length of each basis function
    phi1_len = length(phi1);
    phi2_len = length(phi2);

    % Set new phi1 after dividing by the square root of their lengths
    phi1 = phi1 / sqrt(phi1_len);
    phi2 = phi2 / sqrt(phi2_len);

    % Calculating the dot product between the signal and phi just
like the
    % lecture
    dotProduct_1 = dot(s, phi1);
    dotProduct_2 = dot(s, phi2);

    % Calculate the projections (correlations) of s over phi1 and
phi2
    % Then divide by the square root of each phi
    v1 = dotProduct_1 / sqrt(phi1_len);
    v2 = dotProduct_2 / sqrt(phi2_len);
end
```