

Software Requirements Specification for : Double Pendulum

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Revision History

Date	Version	Notes
Oct.02	1.0	Initial Draft
Nov.07	2.0	Minor Modifications

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d’Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

Symbol	Description	SI Name
kg	mass	kilogram
s	time	second
°	angle	degree
m	length	metre
N	force	newton

Table 1: Table of Units

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	unit	description
x_1	m	horizontal position of the top pendulum
x_2	m	horizontal position of the bottom pendulum
y_1	m	vertical position of the top pendulum
y_2	m	vertical position of the bottom pendulum
x_1'	m/s	horizontal velocity of the top pendulum
x_2'	m/s	horizontal velocity of the bottom pendulum
y_1'	m/s	vertical velocity of the top pendulum
y_2'	m/s	vertical velocity of the bottom pendulum
x_1''	m/s ²	horizontal acceleration of the top pendulum
x_2''	m/s ²	horizontal acceleration of the bottom pendulum
y_1''	m/s ²	vertical acceleration of the top pendulum
y_2''	m/s ²	vertical acceleration of the bottom pendulum
θ_1	°	angle of the top pendulum(0 = vertical downwards, counter-clockwise is positive)
θ_2	°	angle of the bottom pendulum(0 = vertical downwards, counter-clockwise is positive)
θ_1'	°/s	angular velocity of the top rod
θ_2'	°/s	angular velocity of the bottom rod
θ_1''	°/s ²	angular acceleration of the top rod
θ_2''	°/s ²	angular acceleration of the bottom rod
m_1	kg	mass of the top mass
m_2	kg	mass of the bottom mass
L_1	m	length of the top rod
L_2	m	length of the bottom rod
T_1	N	tension in the top rod
T_2	N	tension in the bottom rod
g	m/s ²	acceleration due to gravity
t	s	time of the motion

Table 2: Table of Symbols

1.3 Abbreviations and Acronyms

symbol	description
2D	Two-Dimensional
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
TM	Theoretical Model

Table 3: Abbreviations and Acronyms

2 Introduction

A double pendulum consists of two pendulums attached end to end, its moving curve is highly sensitive to initial conditions. Therefore, it is useful to have a program to simulate the motion of double pendulum to exhibit the chaotic characteristics of it. The program documented here is called Double Pendulum.

The following section provides an overview of Software Requirements Specification(SRS) for Double Pendulum. This section explains the purpose of this document, the scope of the requirements, the characteristics of intended reader, and the organization of document.

This document is based on the template introduced in [Smith \(2006\)](#), [Smith and Lai \(2005\)](#), and [Smith et al. \(2017\)](#).

2.1 Purpose of Document

The purpose of the document is to provide detailed requirements of the Double Pendulum project for people who are interested in the motion of double pendulum, and to provide planning for the design stage.

2.2 Scope of Requirements

The scope of the requirements includes the analysis of a two-dimensional(2D) double pendulum motion problem with various initial conditions.

2.3 Characteristics of Intended Reader

The intended readers are expected to have an understanding of advanced Physics and Calculus, with knowledge of ordinary differential equations.

2.4 Organization of Document

The following document are composed of general system description, specific description, solution characteristics specification, functional and nonfunctional requirements, likely and unlikely changes, traceability matrices and graphs, and values of auxiliary constant.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1 System Context

Figure 1 shows the system context of the software. The program can be viewed abstractly following the design pattern of Inputs \rightarrow Calculations \rightarrow Outputs. The user provides inputs, the system does the calculations, and then provides the outputs to the user.

The responsibilities of each of the entities are listed below:

- User Responsibilities:
 - Ensure the inputs are valid
- Double Pendulum Responsibilities:
 - Detect data type mismatch, such as a string of characters instead of a floating point number
 - Do the corresponding calculations
 - Generate the graphs of angles of the rods
 - Display the graphs to users

3.2 User Characteristics

The end user should have an understanding of high school Physics and Calculus, with knowledge of ordinary differential equations.

3.3 System Constraints

There is no system constraints for this project.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

A system is needed to efficiently and correctly predict the motion of a double pendulum.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

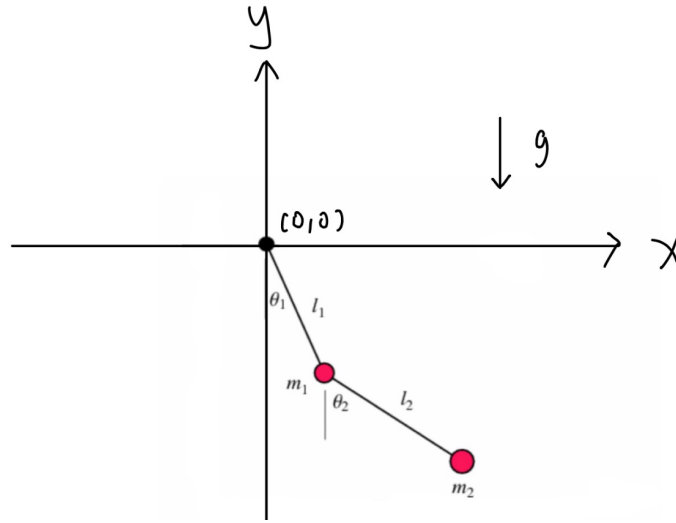


Figure 1: The physics system

- Gravity: The force that attracts one physical body with mass to another.
- Cartesian coordinate system: A coordinate system that specifies each point uniquely in a plane by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length.

4.1.2 Physical System Description

The physics system of Double Pendulum, as shown in (Section [The physics system](#)), includes the following elements:

PS1: The top mass (with initial angle θ_1)

PS2: The bottom mass (with initial angle θ_2)

PS3: The top rod

PS4: The bottom rod

4.1.3 Goal Statement

Given the masses, lengths of rods, initial angles of masses, and the gravitational constant, the goal statements are:

GS1: Calculate the motion of the two masses.

4.2 Solution Characteristics Specification

The instance models that govern Double Pendulum are presented in section **Instance Models**. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refines the scope by providing more detail.

A1: The motion of Double Pendulum is two-dimensional(2D).

A2: A Cartesian coordinate system is used.

A3: The Cartesian coordinate system is right-handed, where the positive x and y axes point right and up.

A4: The direction of the y-axis is directly opposite to gravity.

A5: The double pendulum is attached to the origin.

A6: The motion is small enough that the curvature of the Earth can be neglected.

A7: Time starts at zero.

A8: Air drag is neglected.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that Double Pendulum is based on.

Number	T1
Label	Newton's laws of motion
Equation	$F = ma$
Description	F is the force(N) m is the mass(kg) a is the acceleration($\frac{m}{s^2}$)
Source	https://www.grc.nasa.gov/www/k-12/airplane/newton.html
Ref. By	GD9, GD10

Number	T2
Label	Velocity
Equation	$v = \frac{dx}{dt}$
Description	<p>v is the velocity($\frac{m}{s}$)</p> <p>t is the time(s)</p> <p>x is the position(m)</p>
Source	http://wearcam.org/absement/Derivatives_of_displacement.htm
Ref. By	GD1, GD2, GD3, GD4
Number	T3
Label	Acceleration
Equation	$a = \frac{dv}{dt}$
Description	<p>a is the acceleration($\frac{m}{s^2}$)</p> <p>t is the time(s)</p> <p>v is the velocity($\frac{m}{s}$)</p>
Source	http://wearcam.org/absement/Derivatives_of_displacement.htm
Ref. By	GD5, GD6, GD7, GD8

4.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Number	GD1
Label	x-component of velocity of the top pendulum
Symbol	x_1'
SI Units	m/s
Equation	$x_1' = \theta_1' L_1 \cos \theta_1$
Description	x_1' is the horizontal velocity of the top pendulum(m/s) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod($^\circ$) θ_1' is the angular velocity of the top rod($^\circ$ /s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD5

Detailed derivation of x-component of velocity1:

According to T2:

$$x_1' = \frac{dx_1}{dt}$$

Substitute DD1 in the equation above:

$$x_1' = \frac{d(L_1 \sin \theta_1)}{dt}$$

Apply the chain rule, we get the x-component of velocity of the top rod:

$$x_1' = \theta_1' L_1 \cos \theta_1$$

Number	GD2
Label	y-component of velocity of the top mass
Symbol	y_1'
SI Units	m/s
Equation	$y_1' = \theta_1' L_1 \sin \theta_1$
Description	y_1' is the vertical velocity of the top pendulum(m/s) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod($^\circ$) θ_1' is the angular velocity of the top rod($^\circ$ /s)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD6

Detailed derivation of y-component of velocity1:

According to T2:

$$y_1' = \frac{dy_1}{dt}$$

Substitute DD2 in the equation above:

$$y_1' = \frac{-d(L_1 \cos \theta_1)}{dt}$$

Apply the chain rule, we get the y-component of velocity of the top rod:

$$y_1' = \theta_1' L_1 \sin \theta_1$$

Number	GD3
Label	x-component of velocity of the bottom pendulum
Symbol	x_2'
SI Units	m/s
Equation	$x_2' = x_1' + \theta_2' L_2 \cos \theta_2$
Description	x_2' is the horizontal velocity of the bottom pendulum(m/s) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod($^\circ$) θ_2' is the angular velocity of the bottom rod($^\circ$ /s) x_1' is the horizontal velocity of the top pendulum(m/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD7

Detailed derivation of x-component of velocity2:

According to T2:

$$x_2' = \frac{dx_2}{dt}$$

Substitute DD3 in the equation above:

$$x_2' = \frac{d(x_1 + L_2 \sin \theta_2)}{dt}$$

Apply the chain rule, we get the x-component of velocity of the bottom rod:

$$x_2' = x_1' + \theta_2' L_2 \cos \theta_2$$

Number	GD4
Label	y-component of velocity of the bottom pendulum
Symbol	y_2'
SI Units	m/s
Equation	$y_2' = y_1' + \theta_2' L_2 \sin \theta_2$
Description	y_2' is the vertical velocity of the bottom pendulum(m/s) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod($^\circ$) θ_2' is the angular velocity of the bottom rod($^\circ$ /s) y_1' is the vertical velocity of the top pendulum(m/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD8

Detailed derivation of y-component of velocity2:

According to T2:

$$y_2' = \frac{dy_2}{dt}$$

Substitute DD4 in the equation above:

$$y_2' = \frac{-d(y_1 - L_2 \cos \theta_2)}{dt}$$

Apply the chain rule, we get the y-component of velocity of the top rod:

$$y_2' = y_1' + \theta_2' L_2 \sin \theta_2$$

Number	GD5
Label	x-component of acceleration of the top pendulum
Symbol	x_1''
SI Units	m/s ²
Equation	$x_1'' = -\theta_1'^2 L_1 \sin\theta_1 + \theta_1'' L_1 \cos\theta_1$
Description	x_1'' is the horizontal acceleration of the top pendulum(°/s ²) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) θ_1' is the angular velocity of the top rod(°/s) θ_1'' is the angular acceleration of the top rod(°/s ²)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD7, GD9

Detailed derivation of x-component of acceleration1:

According to T3:

$$x_1'' = \frac{dx_1'}{dt}$$

Substitute DD1 in the equation above:

$$x_1'' = \frac{d(\theta_1' L_1 \cos\theta_1)}{dt}$$

Apply the chain rule, we get the x-component of acceleration of the top rod:

$$x_1'' = -\theta_1'^2 L_1 \sin\theta_1 + \theta_1'' L_1 \cos\theta_1$$

Number	GD6
Label	y-component of acceleration of the top pendulum
Symbol	y_1''
SI Units	m/s ²
Equation	$y_1'' = \theta_1'^2 L_1 \cos\theta_1 + \theta_1'' L_1 \sin\theta_1$
Description	y_1'' is the vertical acceleration of the top pendulum(°/s ²) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) θ_1' is the angular velocity of the top rod(°/s) θ_1'' is the angular acceleration of the top rod(°/s ²)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD8, GD9

Detailed derivation of y-component of acceleration1:

According to T3:

$$y_1'' = \frac{dy_1'}{dt}$$

Substitute DD2 in the equation above:

$$y_1'' = \frac{d(\theta_1' L_1 \sin\theta_1)}{dt}$$

Apply the chain rule, we get the y-component of acceleration of the top rod:

$$y_1'' = \theta_1'^2 L_1 \cos\theta_1 + \theta_1'' L_1 \sin\theta_1$$

Number	GD7
Label	x-component of acceleration of the bottom pendulum
Symbol	x_2''
SI Units	m/s ²
Equation	$x_2'' = x_1'' - \theta_2'^2 L_2 \sin\theta_2 + \theta_2'' L_2 \cos\theta_2$
Description	x_2'' is the horizontal acceleration of the bottom pendulum(°/s ²) L_2 is the length of the bottom rod(m) θ_2 is the angle of the bottom rod(°) θ_2' is the angular velocity of the bottom rod(°/s) θ_2'' is the angular acceleration of the bottom rod(°/s ²) x_1'' is the horizontal acceleration of the top pendulum(°/s ²)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD10

Detailed derivation of x-component of acceleration2:

According to T3:

$$x_2'' = \frac{dx_2'}{dt}$$

Substitute DD3 in the equation above:

$$x_2'' = \frac{d(x_1' + \theta_2' L_2 \cos\theta_2)}{dt}$$

Apply the chain rule, we get the x-component of acceleration of the bottom rod:

$$x_2'' = x_1'' - \theta_2'^2 L_2 \sin\theta_2 + \theta_2'' L_2 \cos\theta_2$$

Number	GD8
Label	y-component of acceleration of the bottom pendulum
Symbol	y_2''
SI Units	m/s ²
Equation	$y_2'' = y_1'' + \theta_2'^2 L_2 \cos\theta_2 + \theta_2'' L_2 \sin\theta_2$
Description	y_2'' is the vertical acceleration of the bottom pendulum(°/s ²) L_2 is the length of the bottom rod(m) θ_2 is the angle of the bottom rod(°) θ_2' is the angular velocity of the bottom rod(°/s) θ_2'' is the angular acceleration of the bottom rod(°/s ²) y_1'' is the vertical acceleration of the top pendulum(°/s ²)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD10

Detailed derivation of y-component of acceleration2:

According to T3:

$$y_2'' = \frac{dy_2'}{dt}$$

Substitute DD4 in the equation above:

$$y_1'' = \frac{d(y_1' + \theta_2' L_2 \sin\theta_2)}{dt}$$

Apply the chain rule, we get the y-component of acceleration of the bottom rod:

$$y_2'' = y_1'' + \theta_2'^2 L_2 \cos\theta_2 + \theta_2'' L_2 \sin\theta_2$$

Number	GD9
Label	calOffForceOnTopPendulum
Input	$T_1, T_2, \theta_1, \theta_2, m_1, g$
Output	$m_1 x_1'', m_1 y_1''$
Input Constraints	$\theta_1, \theta_2 \neq 0$
Output Constraints	None
Equation	$m_1 x_1'' = -T_1 \sin \theta_1 + T_2 \sin \theta_2$ $m_1 y_1'' = T_1 \cos \theta_1 - T_2 \cos \theta_2 - m_1 g$
Description	$m_1 x_1''$ is the horizontal net force on the top pendulum(N) $m_1 y_1''$ is the vertical net force on the top pendulum(N)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	T1

Detailed derivation of force on top mass:

We treat the two pendulum masses as point particles. Begin by drawing the free body diagram for the top mass and writing an expression for the net force acting on it.

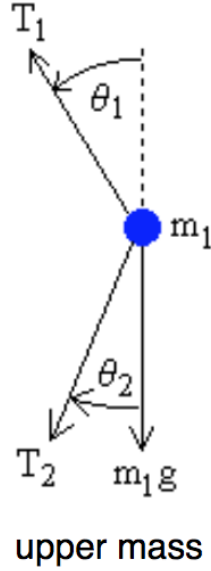


Figure 2: The free body diagram of the top mass

Define these variables:

T = tension in the rod

m = mass of pendulum

g = gravitational constant

The forces on the top mass are upper mass are the tension in the upper rod T_1 , the tension in the lower rod T_2 , and the gravity $-m_1 g$. Using Newton's law $F = ma$, the net force on the top mass in x axis is:

$$m_1 x_1'' = -T_1 \sin \theta_1 + T_2 \sin \theta_2 \quad (1)$$

and the net force on the top mass in y axis is:

$$m_1 y_1'' = T_1 \cos \theta_1 - T_2 \cos \theta_2 - m_1 g \quad (2)$$

Number	GD10
Label	calOffForceOnBottomPendulum
Input	T_2, θ_2, m_2, g
Output	m_2x_2'', m_2y_2''
Input Constraints	$\theta_2 \neq 0$
Output Constraints	None
Equation	$m_2x_2'' = -T_2\sin\theta_2$ $m_2y_2'' = T_2\cos\theta_2 - m_2g$
Description	m_2x_2'' is the horizontal net force on the bottom pendulum(N) m_2y_2'' is the vertical net force on the bottom pendulum(N)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	T1

Detailed derivation of force on bottom mass:

For the bottom pendulum, its free body diagram is:

The forces on the bottom pendulum are the tension in the bottom rod T_2 , and the gravity $-m_2g$. So the net force on the bottom mass in x-axis is:

$$m_2x_2'' = -T_2\sin\theta_2 \quad (3)$$

and the net force on the bottom mass in y-axis is:

$$m_2y_2'' = T_2\cos\theta_2 - m_2g \quad (4)$$

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

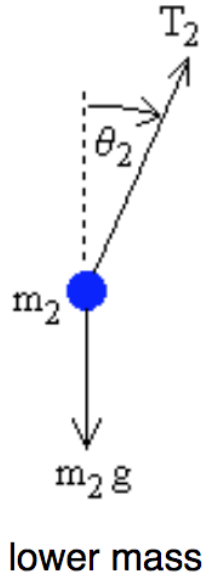


Figure 3: The free body diagram of the bottom mass

Number	DD1
Label	x-component of initial positions of the top pendulum
Symbol	x_1
SI Units	m
Equation	$x_1 = L_1 \sin \theta_1$
Description	x_1 is the horizontal position of the top pendulum(m) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod($^\circ$)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html Neumann
Ref. By	DD3, GD1

Number	DD2
Label	y-component of initial positions of the top pendulum
Symbol	y_1
SI Units	m
Equation	$y_1 = -L_1 \cos \theta_2$
Description	y_1 is the vertical position of the top pendulum(m) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD4, GD4
Number	DD3
Label	x-component of initial positions of the bottom mass
Symbol	x_2
SI Units	m
Equation	$x_2 = x_1 + L_2 \sin \theta_2$
Description	x_2 is the horizontal position of the bottom pendulum(m) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod(°) x_1 is the horizontal position of the top pendulum(m)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD3

Number	DD4
Label	y-component of initial positions of the bottom pendulum
Symbol	y_2
SI Units	m
Equation	$y_2 = y_1 - L_2 \cos \theta_2$
Description	y_2 is the vertical position of the bottom pendulum(m) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod($^\circ$) y_1 is the vertical position of the top pendulum(m)
Sources	https://www.mypysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD4

4.2.5 Instance Models

This section transforms the problem defined in section **Specific System Description** into one which is expressed in mathematical terms. It uses concrete symbols defined in section **Data Definitions** to replace the abstract symbols in the models identified in section **Theoretical Models** and section **General Definitions**.

Number	IM1
Label	AngularAccelerationToFind $\theta_1''(t), \theta_2''(t)$
Input	$m_1, m_2, \theta_1, \theta_2, L_1, L_2, g$
Output	<p>$\theta_1''(t), \theta_2''(t)$, such that</p> $\theta_1'' = \frac{f - g}{h}$ <p>where</p> $f = -g(2m_1 + m_2)\sin\theta_1 - m_2g\sin(\theta_1 - 2\theta_2)^2L_1\cos(d))$ $g = 2(\sin(\theta_1 - \theta_2)m_2(\theta_2'^2L_2 + \theta_2'^2L_1\cos(d))$ $h = L_1(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2))$ <p>where</p> $d = \theta_1 - \theta_2$ <p>, and</p> $\theta_2'' = \frac{2\sin(d)(\theta_1'L_1(m_1 + m_2) + g(m_1 + m_2)\cos(\theta_1) + (\theta_2')^2L_2m_2\cos(d))}{L_2(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2))}$ <p>where</p> $d = \theta_1 - \theta_2$ <p>$\theta_1''(0) = \theta_1$ and $\theta_2''(0) = \theta_2$ from user inputs</p>
Input Constraints	$\theta_1 \neq 0$ or $\theta_2 \neq 0$
Output Constraints	None
Description	<p>m_2x_2'' is the horizontal net force on the bottom pendulum(N)</p> <p>m_2y_2'' is the vertical net force on the bottom pendulum(N)</p>
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	GD9, GD5, GD6, GD7, GD8

Detailed derivation of calculation of angular acceleration of pendulums:

By solving equations 3 and 4 for $T_2 \sin \theta_2$ and $T_2 \cos \theta_2$ and then substituting into equations 1 and 2, we get:

$$m_1 x_1'' = -T_1 \sin \theta_1 - m_2 x_2'' \quad (5)$$

$$m_1 y_1'' = T_1 \cos \theta_1 - m_2 y_2'' - m_2 g - m_1 g \quad (6)$$

Multiply equation 5 by $\cos \theta_1$ and equation 6 by $\sin \theta_1$ and rearrange to get:

$$T_1 \sin \theta_1 \cos \theta_1 = -\cos \theta_1 (m_1 x_1'' + m_2 x_2'') \quad (7)$$

$$T_1 \sin \theta_1 \cos \theta_1 = \sin \theta_1 (m_1 y_1'' + m_2 y_2'' + m_2 g + m_1 g) \quad (8)$$

This leads to the equation

$$\sin \theta_1 (m_1 y_1'' + m_2 y_2'' + m_2 g + m_1 g) = -\cos \theta_1 (m_1 x_1'' + m_2 x_2'') \quad (9)$$

Next, multiply equation 3 by $\cos \theta_2$ and equation 4 by $\sin \theta_2$ and rearrange to get:

$$T_2 \sin \theta_2 \cos \theta_2 = -\cos \theta_2 (m_2 x_2'') \quad (10)$$

$$T_2 \sin \theta_2 \cos \theta_2 = \sin \theta_2 (m_2 y_2'' + m_2 g) \quad (11)$$

Which leads to

$$\sin \theta_2 (m_2 y_2'' + m_2 g) = -\cos \theta_2 (m_2 x_2'') \quad (12)$$

Next we need to use a computer algebra program to solve equations 9 and 12 for θ_1'' , θ_2'' in terms of θ_1 , θ_1' , θ_2 , θ_2' .

4.2.6 Input Data Constraints

Table **Input Variables** shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
m_1	$m_1 > 0$	-	kg	10%
m_2	$m_2 > 0$	-	kg	10%
L_1	$L_1 > 0$	-	m	10%
L_2	$L_2 > 0$	-	m	10%
θ_1	-	-	°	10%
θ_2	-	-	°	10%
g	$g > 0$	-	N/kg	10%
t	$t > 0$	-	s	60%

Table 4: Input Variables

4.2.7 Properties of a Correct Solution

The correct solution should contain a list of data for both $\theta_1(t)$ and $\theta_2(t)$.

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviors that the software is expected to complete.

R1: Input the following quantities, which define the initial conditions:

symbol	unit	description
m_1	kg	mass of the top mass
m_2	kg	mass of the bottom mass
L_1	m	length of the top rod
L_1	m	length of the bottom rod
θ_1	°	initial angle of the top mass
θ_2	°	initial angle of the bottom mass

g	m/s^2	gravitational constant
t	s	time of the motion

R2: Check the entered input values to ensure that they do not exceed the data constraints mention in section [Input Data Constraints](#).

R3: Calculate and output the position of two masses over the the simulation time.

R4: Output the results to a file.

R5: Output graphs of $\theta_1(t)$ and $\theta_2(t)$ over the simulation time.

5.2 Nonfunctional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

NFR6: The outputs of the code have the properties described in section [Properties of a Correct Solution](#).

NFR7: The code is tested with complete verification and validation plan.

NFR8: The code is modularized.

NFR9: The traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, unlikely changes, and modules is completely recorded in traceability matrices in teh SRS and module guide.

NFR10: The code is able to be run in different environments.

6 Likely Changes

LC1: Air drag might be considered.(Ref. by [A8](#))

7 Unlikely Changes

LC2: The units used will not likely to be changed.

	A1	A2	A3	A4	A5	A6	A7	A8
T1								
T2								
T3								
DD1								
DD2								
DD3								
DD4								
GD1								
GD2								
GD3								
GD4								
DD5								
GD6								
GD7								
GD8								
GD9	X	X	X	X		X		X
GD10	X	X	X	X		X		X
IM1	X	X	X	X	X	X		
LC1								X
LC2								

Table 6: Traceability Matrix Showing the Connections Between Assumptions and Other Items

8 Traceability Matrices and Graphs

	T1	T2	T3	DD1	DD2	DD3	DD4	GD1	GD2	GD3	GD4	GD5	GD6	GD7	DD8	GD9	GD10	IM1	LC1	LC2
T1																X	X			
T2								X	X	X	X									
T3												X	X	X	X					
DD1						X		X												
DD2							X				X									
DD3										X										
DD4											X									
GD1												X								
GD2													X							
GD3														X						
GD4															X					
GD5														X		X				
GD6															X	X				
GD7																	X			
GD8																	X			
GD9	X																			
GD10	X																			
IM1	X											X	X	X	X					
LC1																				
LC2																				

Table 5: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1	R1	R2	R3	R4	R5
IM1			X	X		X
R1	X					
R2	X					
R3	X					
R4	X					
R5	X					

Table 7: Traceability Matrix Showing the Connections Between Requirements and Instance Models

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9 Values of Auxiliary Constants

NA.