Software Requirements Specification for : Double Pendulum

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Contents

Revision History

Date	Version	Notes
Oct.02	1.0	Initial Draft
Nov.07	2.0	Minor Modifications

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

Symbol	**Description**	**SI Name**
kg	mass	kilogram
s	time	second
°	angle	degree
m	length	metre
N	force	newton

Table 1: Table of Units

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	unit	description
x_1	m	horizontal position of the top pendulum
x_2	m	horizontal position of the bottom pendulum
y_1	m	vertical position of the top pendulum
y_2	m	vertical position of the bottom pendulum
$x_1{}'$	m/s	horizontal velocity of the top pendulum
$x_2{}'$	m/s	horizontal velocity of the bottom pendulum
$y_1{}'$	m/s	vertical velocity of the top pendulum
$y_2{}'$	m/s	vertical velocity of the bottom pendulum
$x_1{}''$	m/s^2	horizontal acceleration of the top pendulum
$x_2{}''$	m/s^2	horizontal acceleration of the bottom pendulum
$y_1{}''$	m/s^2	vertical acceleration of the top pendulum
$y_1{}''$	m/s^2	vertical acceleration of the bottom pendulum
θ_1	$^\circ$	angle of the top pendulum(0 = vertical downwards, counter-clockwise is positive)
θ_2	$^\circ$	angle of the bottom pendulum(0 = vertical downwards, counter-clockwise is positive)
$\theta_1{}'$	$^\circ$/s	angular velocity of the top rod
$\theta_2{}'$	$^\circ$/s	angular velocity of the bottom rod
$\theta_1{}''$	$^\circ$/s^2	angular acceleration of the top rod
$\theta_2{}''$	$^\circ$/s^2	angular acceleration of the bottom rod
L_1	m	length of the top rod
L_2	m	length of the bottom rod
T_1	N	tension in the top rod
T_2	N	tension in the bottom rod
g	m/s^2	acceleration due to gravity
t	s	time of the motion

Table 2: Table of Symbols

1.3 Abbreviations and Acronyms

symbol	description
2D	Two-Dimensional
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
TM	Theoretical Model

Table 3: Abbreviations and Acronyms

2 Introduction

A double pendulum consists of two pendulums attached end to end, its moving curve is highly sensitive to initial conditions. Therefore, it is useful to have a program to simulate the motion of double pendulum to exhibit the chaotic characteristics of it. The program documented here is called Double Pendulum.

The following section provides an overview of Software Requirements Specification(SRS) for Double Pendulum. This section explains the purpose of this document, the scope of the requirements, the characteristics of intended reader, and the organization of document.

This document is based on the template introduced in Smith (2006), Smith and Lai (2005), and Smith et al. (2017).

2.1 Purpose of Document

The purpose of the document is to provide detailed requirements of the Double Pendulum project for people who are interested in the motion of double pendulum, and to provide planning for the design stage.

2.2 Scope of Requirements

The scope of the requirements includes the analysis of a two-dimentional(2D) double pendulum motion problem with various initial conditions.

2.3 Characteristics of Intended Reader

The intended readers are expected to have an understanding of advanced Physics and Calculus, with knowledge of ordinary differential equations.

2.4 Organization of Document

The following document are composed of general system description, specific description, solution characteristics specification, functional and nonfunctional requirements, likely and unlikely changes, traceability matrices and graphs, and values of auxiliary constant.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristicsand lists the system constraints.

3.1 System Context

Figure 1 shows the system context of the software. The program can be viewed abstractly following the design pattern of Inputs → Calculations → Outputs.The user provides inputs, the system does the calculations, and then provides the outputs to the user.

The responsibilities of each of the entities are listed below:

- User Responsibilities:
  - Ensure the inputs are valid
- Double Pendulum Responsibilities:
  - Detect data type mismatch, such as a string of characters instead of a floating point number
  - Do the corresponding calculations
  - Generate the graphs of angles of the rods
  - Display the graphs to users

3.2 User Characteristics

The end user should have an understanding of high school Physics and Calculus, with knowledge of ordinary differential equations.

3.3 System Constraints

There is no system constraints for this project.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

A system is needed to efficiently and correctly predict the motion of a double pendulum.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

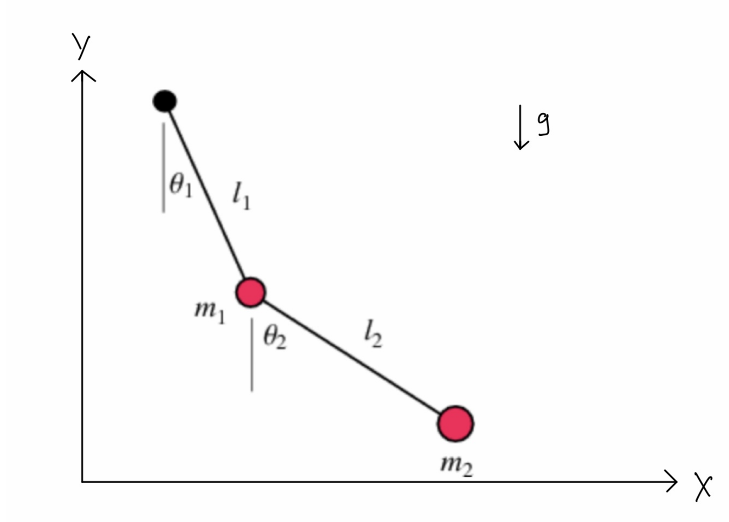

Figure 1: The physics system

- Gravity: The force that attacts one physical body with mass to another.
- Cartesian coordinate system: A coordinate system that specifies each point uniquely in a plane by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length.

4.1.2 Physical System Description

The physics system of Double Pendulum, as shown in (Section The physics system), includes the following elements:

PS1: The top mass (with initial angle θ_1)

PS2: The bottom mass (with initial angle θ_2)

PS3: The top rod

PS4: The bottom rod

4.1.3 Goal Statement

Given the masses, lengths of rods, initial angles of masses, and the gravitational constant, the goal statements are:

GS1: Calculate the motion of the two masses.

4.2 Solution Characteristics Specification

The instance models that govern Double Pendulum are presented in section Instance Models. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refines the scope by providing more detail.

A1: The motion of Double Pendulum is two-dimensional(2D).

A2: A Cartesian coordinate system is used.

A3: The direction of the y-axis is directly opposite to gravity.

A4: The double pendulum is attached to the origin.

A5: The motion is small enough that the curvature of the Earth can be neglected.

A6: Time starts at zero.

A7: Air drag is neglected.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that Double Pendulum is based on.

Number	T1
Label	**Newton's laws of motion**
Equation	$F = ma$
Description	F is the force(N)) m is the mass(kg) a is the acceleration($\frac{m}{s^2}$)
Source	https://www.grc.nasa.gov/www/k-12/airplane/newton.html
Ref. By	IM1, IM2

Number	T2
Label	**Velocity**
Equation	$$v = \frac{dx}{dt}$$
Description	v is the velocity($\frac{m}{s}$) t is the time(s) x is the position(m)
Source	http://wearcam.org/absement/Derivatives_of_displacement.htm
Ref. By	DD5, DD6, DD7, DD8

Number	T3
Label	**Acceleration**
Equation	$$a = \frac{dv}{dt}$$
Description	a is the acceleration($\frac{m}{s^2}$) t is the time(s) v is the velocity($\frac{m}{s}$)
Source	http://wearcam.org/absement/Derivatives_of_displacement.htm
Ref. By	DD9, DD10, DD11, DD12

4.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Number	DD1
Label	**x-component of initial positions of the top pendulum**
Symbol	x_1
SI Units	m
Equation	$$x_1 = L_1 sin\theta_1$$
Description	x_1 is the horizontal position of the top pendulum(m) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod($^\circ$)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html Neumann
Ref. By	DD3, DD5

Number	DD2
Label	**y-component of initial positions of the top pendulum**
Symbol	y_1
SI Units	m
Equation	$$y_1 = -L_1 cos\theta_2$$
Description	y_1 is the vertical position of the top pendulum(m) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod($^\circ$)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD4, DD8

Number	DD3
Label	**x-component of initial positions of the bottom mass**
Symbol	x_2
SI Units	m
Equation	$$x_2 = x_1 + L_2 sin\theta_2$$
Description	x_2 is the horizontal position of the bottom pendulum(m) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod(°) x_1 is the horizontal position of the top pendulum(m)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD7

Number	DD4
Label	**y-component of initial positions of the bottom pendulum**
Symbol	y_2
SI Units	m
Equation	$$y_2 = y_1 - L_2 cos\theta_2$$
Description	y_2 is the vertical position of the bottom pendulum(m) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod(°) y_1 is the vertical position of the top pendulum(m)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD8

Number	DD5
Label	**x-component of velocity of the top pendulum**
Symbol	${x_1}'$
SI Units	m/s
Equation	$${x_1}' = {\theta_1}' L_1 cos\theta_1$$
Description	${x_1}'$ is the horizontal velocity of the top pendulum(m/s) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) ${\theta_1}'$ is the angular velocity of the top rod(°/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD9

Number	DD6
Label	**y-component of velocity of the top mass**
Symbol	${y_1}'$
SI Units	m/s
Equation	$${y_1}' = {\theta_1}' L_1 sin\theta_1$$
Description	${y_1}'$ is the vertical velocity of the top pendulum(m/s) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) ${\theta_1}'$ is the angular velocity of the top rod(°/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD10

Number	DD7
Label	**x-component of velocity of the bottom pendulum**
Symbol	x_2'
SI Units	m/s
Equation	$$x_2' = x_1' + \theta_2' L_2 cos\theta_2$$
Description	x_2' is the horizontal velocity of the bottom pendulum(m/s) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod(°) θ_2' is the angular velocity of the bottom rod(°/s) x_1' is the horizontal velocity of the top pendulum(m/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD11

Number	DD8
Label	**y-component of velocity of the bottom pendulum**
Symbol	y_2'
SI Units	m/s
Equation	$$y_2' = y_1' + \theta_2' L_2 sin\theta_2$$
Description	y_2' is the vertical velocity of the bottom pendulum(m/s) L_2 is the length of the bottom rod(m) θ_2 is the angle of the botom rod(°) θ_2' is the angular velocity of the bottom rod(°/s) y_1' is the vertical velocity of the top pendulum(m/s)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD12

Number	DD9
Label	**x-component of acceleration of the top pendulum**
Symbol	x_1”
SI Units	m/s^2
Equation	$$x_1'' = -\theta_1'^2 L_1 sin\theta_1 + \theta_1'' L_1 cos\theta_1$$
Description	x_1” is the horizontal acceleration of the top pendulum(°/s^2) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) θ_1' is the angular velocity of the top rod(°/s) θ_1'' is the angular acceleration of the top rod(°/s^2)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD11, IM1

Number	DD10
Label	**y-component of acceleration of the top pendulum**
Symbol	y_1”
SI Units	m/s^2
Equation	$$y_1'' = \theta_1'^2 L_1 cos\theta_1 + \theta_1'' L_1 sin\theta_1$$
Description	y_1” is the vertical acceleration of the top pendulum(°/s^2) L_1 is the length of the top rod(m) θ_1 is the angle of the top rod(°) θ_1' is the angular velocity of the top rod(°/s) θ_1'' is the angular acceleration of the top rod(°/s^2)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	DD12, IM1

Number	DD11
Label	**x-component of acceleration of the bottom pendulum**
Symbol	x_2"
SI Units	m/s^2
Equation	$$x_2'' = x_1'' - {\theta_2'}^2 L_2 sin\theta_2 + \theta_2'' L_2 cos\theta_2$$
Description	x_2" is the horizontal acceleration of the bottom pendulum($^\circ$/s^2) L_2 is the length of the bottom rod(m) θ_2 is the angle of the bottom rod($^\circ$) θ_2' is the angular velocity of the bottom rod($^\circ$/s) θ_2'' is the angular acceleration of the bottom rod($^\circ$/s^2) x_1" is the horizontal acceleration of the top pendulum($^\circ$/s^2)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	IM2

Number	DD12
Label	**y-component of acceleration of the bottom pendulum**
Symbol	y_2"
SI Units	m/s^2
Equation	$$y_2'' = y_1'' + {\theta_2'}^2 L_2 cos\theta_2 + \theta_2'' L_2 sin\theta_2$$
Description	y_2" is the vertical acceleration of the bottom pendulum($^\circ$/s^2) L_2 is the length of the bottom rod(m) θ_2 is the angle of the bottom rod($^\circ$) θ_2' is the angular velocity of the bottom rod($^\circ$/s) θ_2'' is the angular acceleration of the bottom rod($^\circ$/s^2) y_1" is the vertical acceleration of the top pendulum($^\circ$/s^2)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	IM2

4.2.5 Instance Models

This section transforms the problem defined in section Specific System Description into one which is expressed in mathematical terms. It uses concrete symbols defined in section Data Definitions to replace the abstract symbols in the models identified in section Theoretical Models and section General Definitions.

Number	IM1
Label	**calOfForceOnTopPendulum**
Input	T_1, T_2, θ_1, θ_2,m_1, g
Output	m_1x_1'', m_1y_1''
Input Constraints	θ_1,$\theta_2 \neq 0$
Output Constraints	None
Equation	$$m_1x_1'' = -T_1 sin\theta_1 + T_2 sin\theta_2$$ $$m_1y_1'' = T_1 cos\theta_1 - T_2 cos\theta_2 - m_1 g$$
Description	m_1x_1'' is the horizontal net force on the top pendulum(N) m_1y_1'' is the veritical net force on the top pendulum(N)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	T1

Number	IM2
Label	**calOfForceOnBottomPendulum**
Input	T_2, θ_2,m_2, g
Output	$m_2 x_2''$, $m_2 y_2''$
Input Constraints	$\theta_2 \neq 0$
Output Constraints	None
Equation	$$m_2 x_2'' = -T_2 sin\theta_2$$ $$m_2 y_2'' = T_2 cos\theta_2 - m_2 g$$
Description	$m_2 x_2''$ is the horizontal net force on the bottom pendulum(N) $m_2 y_2''$ is the veritical net force on the bottom pendulum(N)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	T1

Number	IM3
Label	**calOfAngularAccelerationOfTopPendulum**
Input	m_1,m_2,θ_1,θ_2,L_1,L_2,g
Output	$\theta_1{}''$, $\theta_2{}''$
Input Constraints	$\theta_1,\theta_2 \neq 0$
Output Constraints	None
Equation	$$\theta_1{}'' = \frac{-g(2m_1+m_2)sin\theta_1 - m_2 g sin(\theta_1 - 2\theta_2) - 2sin(\theta_1-\theta_2)m_2({\theta_2{}'}^2 L_2 + {\theta_2{}'}^2 L_1 cos(\theta_1 - \theta}{L_1(2m_1+m_2-m_2cos(2\theta_1-2\theta_2))}$$ $$\theta_2{}'' = \frac{2sin(\theta_1-\theta_2)(\theta_1{}' L_1(m1+m2) + g(m_1+m_2)cos(\theta_1) + (\theta_2{}')^2 L_2 m_2 cos(\theta_1 - \theta_2}{L_2(2m_1+m_2-m_2cos(2\theta_1-2\theta_2))}$$
Description	$m_2 x_2{}''$ is the horizontal net force on the bottom pendulum(N) $m_2 y_2{}''$ is the veritical net force on the bottom pendulum(N)
Sources	https://www.myphysicslab.com/pendulum/double-pendulum-en.html
Ref. By	IM1, DD9, DD10, DD11, DD12

4.2.6 Input Data Constraints

Table Input Variables shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
m_1	$m_1 > 0$	-	kg	10%
m_2	$m_2 > 0$	-	kg	10%
L_1	$L_1 > 0$	-	m	10%
L_2	$L_2 > 0$	-	m	10%
θ_1	$\theta_1 \neq 0$	-	°	10%
θ_2	$\theta_2 \neq 0$	-	°	10%
g	$g > 0$	-	N/kg	10%

Table 4: Input Variables

4.2.7 Properties of a Correct Solution

Table Output Variables shows the data constraints on the output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable.

Var	Physical Constraints
θ_1''	$\theta_1'' \neq 0$
θ_2''	$\theta_2'' \neq 0$

Table 5: Output Variables

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviors that the software is expected to complete.

R1: Input the required values in the software.

R2: Check the entered input values to ensure that they do not exceed the data constraints mention in section Input Data Constraints.

R3: Calculate the equation for the following values: θ_1(t) and θ_1(t).

R4: Output the results to a file.

R5: Output graphs of θ_1(t) and θ_1(t).

5.2 Nonfunctional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

NFR6: The outputs of the code have the properties described in section Properties of a Correct Solution.

NFR7: The code is tested with complete verification and validation plan.

NFR8: The code is modularized.

NFR9: The traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, unlikely changes, and modules is completely recorded in traceability matrices in teh SRS and module guide.

NFR10: The code is able to be run in different environments.

6 Likely Changes

LC1: Air drag might be considered.(Ref. by A7)

7 Unlikely Changes

LC2: The units used will not likely to be changed.

	A1	A2	A3	A4	A5	A6	A7
T1							
T2							
T3							
DD1							
DD2							
DD3							
DD4							
DD5							
DD6							
DD7							
DD8							
DD9							
DD10							
DD11							
DD12							
IM1	X	X	X		X		X
IM2	X	X	X		X		X
IM3	X	X	X	X	X		
LC1							X
LC2							

Table 7: Traceability Matrix Showing the Connections Between Assumptions and Other Items

8 Traceability Matrices and Graphs

	T1	T2	T3	DD1	DD2	DD3	DD4	DD5	DD6	DD7	DD8	DD9	DD10	DD11	DD12	IM1	IM2	IM3	LC1	LC2
T1																X	X			
T2								X	X	X	X									
T3												X	X	X	X					
DD1						X		X												
DD2							X				X									
DD3										X										
DD4											X									
GD5												X								
GD6													X							
GD7														X						
GD8															X					
GD9														X		X				
GD10															X	X				
GD11																	X			
GD12																	X			
IM1	X																			
IM2	X																			
IM3	X											X	X	X	X					
LC1																				
LC2																				

Table 6: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1	IM2	IM3	R1	R2	R3	R4	R5
IM1					X	X		X
IM2					X	X		X
IM3					X	X		X
R1	X	X	X					
R2	X	X	X					
R3	X	X	X					
R4	X	X	X					
R5	X	X	X					

Table 8: Traceability Matrix Showing the Connections Between Requirements and Instance Models

References

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W. Spencer Smith. Systematic development of requirements documentation for general purpose scientific computing software. In *Proceedings of the 14th IEEE International Requirements Engineering Conference, RE 2006*, pages 209–218, Minneapolis / St. Paul, Minnesota, 2006. URL http://www.ifi.unizh.ch/req/events/RE06/.

W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Ȧgerfalk, and N. Kraiem, editors, *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05*, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.

W. Spencer Smith, John McCutchan, and Jacques Carette. Commonality analysis for a family of material models. Technical Report CAS-17-01-SS, McMaster University, Department of Computing and Software, 2017.

9 Values of Auxiliary Constants

NA.