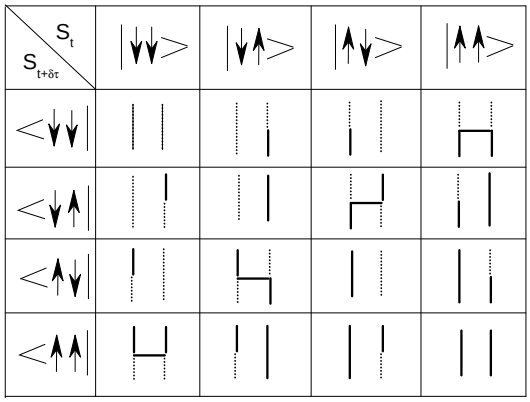
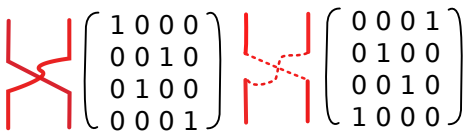
$$S_i^x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ S_i^y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ S_i^z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

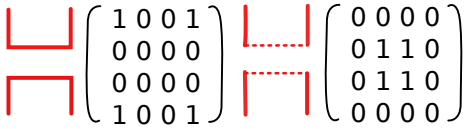
$$S_i^x S_j^x = \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},\ S_i^y S_j^y = \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},\ S_i^z S_j^z = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

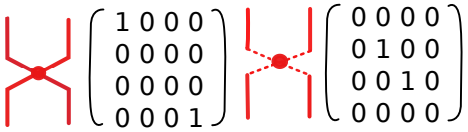
$$H_{ij} = -J_x S_i^x S_j^x - J_y S_i^y S_j^y - J_z S_i^z S_j^z = \begin{pmatrix} -\frac{J_z}{4} & 0 & 0 & -\frac{J_x - J_y}{4} \\ 0 & \frac{J_z}{4} & -\frac{J_x + J_y}{4} & 0 \\ 0 & -\frac{J_x + J_y}{4} & \frac{J_z}{4} & 0 \\ -\frac{J_x - J_y}{4} & 0 & 0 & -\frac{J_z}{4} \end{pmatrix}$$

$$C - H_{ij} = \begin{pmatrix} C + \frac{J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & C - \frac{J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & C - \frac{J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & C + \frac{J_z}{4} \end{pmatrix}$$



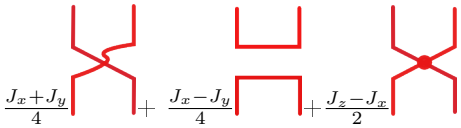




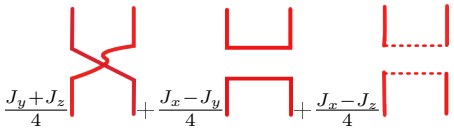


consider the case $J_x \geq J_y \geq 0$, when

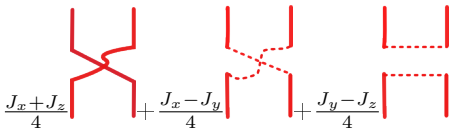
$$J_z \geq J_x,\ C = \tfrac{J_z}{4},\ C - H_{ij} = \begin{pmatrix} \frac{J_z}{2} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & 0 & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & 0 & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_z}{2} \end{pmatrix} =$$



$$0 \leq J_z \leq J_x,\ C = \tfrac{J_x}{4},\ C - H_{ij} = \begin{pmatrix} \frac{J_x + J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & \frac{J_x - J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & \frac{J_x - J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_x + J_z}{4} \end{pmatrix} =$$



$$-J_x \leq J_z \leq 0,\ C = \tfrac{Jx}{4},\ C - H_{ij} = \begin{pmatrix} \frac{J_x + J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & \frac{J_x - J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & \frac{J_x - J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_x + J_z}{4} \end{pmatrix} =$$



$$J_z \leq -J_x,\ C = -\tfrac{J_z}{4},\ C - H_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{J_x-J_y}{4} \\ 0 & -\frac{J_z}{2} & \frac{J_x+J_y}{4} & 0 \\ 0 & \frac{J_x+J_y}{4} & -\frac{J_z}{2} & 0 \\ \frac{J_x-J_y}{4} & 0 & 0 & 0 \end{pmatrix} =$$