

$$\begin{aligned}
S_i^x &= \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, S_i^y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, S_i^z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
S_i^x S_j^x &= \frac{1}{4} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, S_i^y S_j^y = \frac{1}{4} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, S_i^z S_j^z = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
H_{ij} &= -J_x S_i^x S_j^x - J_y S_i^y S_j^y - J_z S_i^z S_j^z = \begin{pmatrix} -\frac{J_z}{4} & 0 & 0 & -\frac{J_x - J_y}{4} \\ 0 & \frac{J_z}{4} & -\frac{J_x + J_y}{4} & 0 \\ 0 & -\frac{J_x + J_y}{4} & \frac{J_z}{4} & 0 \\ -\frac{J_x - J_y}{4} & 0 & 0 & -\frac{J_z}{4} \end{pmatrix} \\
C - H_{ij} &= \begin{pmatrix} C + \frac{J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & C - \frac{J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & C - \frac{J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & C + \frac{J_z}{4} \end{pmatrix}
\end{aligned}$$

$\begin{matrix} S_t \\ S_{t+\delta t} \end{matrix}$	$ \downarrow\downarrow\rangle$	$ \downarrow\uparrow\rangle$	$ \uparrow\downarrow\rangle$	$ \uparrow\uparrow\rangle$
$\langle\downarrow\downarrow $				
$\langle\downarrow\uparrow $				
$\langle\uparrow\downarrow $				
$\langle\uparrow\uparrow $				

$$\begin{array}{c} \text{Diagram 1} \\ \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \end{array}$$

$$\begin{array}{c} \text{Diagram 1} \\ \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \end{array}$$

$$\begin{array}{c} \text{Diagram 1} \\ \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right) \end{array}$$

consider the case $J_x \geq J_y \geq 0$, when

$$J_z \geq J_x, C = \frac{J_z}{4}, C - H_{ij} = \begin{pmatrix} \frac{J_z}{2} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & 0 & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & 0 & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_z}{2} \end{pmatrix} =$$

$$\begin{array}{c} \text{Diagram 1} \\ \frac{J_x + J_y}{4} \text{Diagram 1} + \frac{J_x - J_y}{4} \text{Diagram 2} + \frac{J_z - J_x}{2} \text{Diagram 3} \\ 0 \leq J_z \leq J_x, C = \frac{J_x}{4}, C - H_{ij} = \begin{pmatrix} \frac{J_x + J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & \frac{J_x - J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & \frac{J_x - J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_x + J_z}{4} \end{pmatrix} = \end{array}$$

$$\begin{array}{c} \text{Diagram 1} \\ \frac{J_y + J_z}{4} \text{Diagram 1} + \frac{J_x - J_y}{4} \text{Diagram 2} + \frac{J_x - J_z}{4} \text{Diagram 3} \\ -J_x \leq J_z \leq 0, C = \frac{J_x}{4}, C - H_{ij} = \begin{pmatrix} \frac{J_x + J_z}{4} & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & \frac{J_x - J_z}{4} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & \frac{J_x - J_z}{4} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & \frac{J_x + J_z}{4} \end{pmatrix} = \end{array}$$

$$\begin{aligned}
& \frac{J_x + J_z}{4} \left[\text{diagram 1} \right] + \frac{J_x - J_y}{4} \left[\text{diagram 2} \right] + \frac{J_y - J_z}{4} \left[\text{diagram 3} \right] \\
J_z \leq -J_x, \quad C = -\frac{J_z}{4}, \quad C - H_{ij} = & \begin{pmatrix} 0 & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & -\frac{J_z}{2} & \frac{J_x + J_y}{4} & 0 \\ 0 & \frac{J_x + J_y}{4} & -\frac{J_z}{2} & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & 0 \end{pmatrix} = \\
& \frac{J_x - J_y}{4} \left[\text{diagram 4} \right] + \frac{J_x + J_y}{4} \left[\text{diagram 5} \right] + \frac{-J_z - J_x}{2} \left[\text{diagram 6} \right]
\end{aligned}$$