Plotting

Q: Plot beta prior distributions $\mathrm{beta}(\theta|2,2)$, Bernoulli likelihoods $\mathrm{Ber}(\mathrm{HHTHH}|\theta)$, and their corresponding posterior distributions $p(\theta|\mathrm{HHTHH})$.

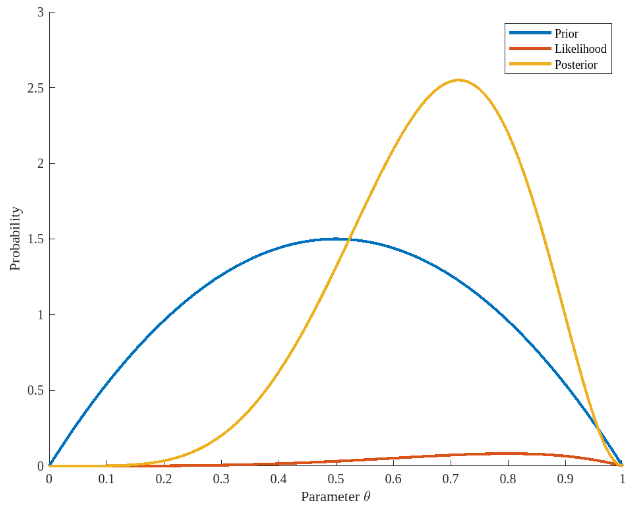
For Bernoulli likelihood, here it is

$$\mathrm{Ber}(\mathrm{HHTHH}|\theta) = \theta^4(1-\theta)^1.$$

Then the posterior distribution is

$$\begin{aligned} p(\theta|\mathrm{HHTHH}) &= \frac{\theta^4(1-\theta)^1}{p(\mathrm{HHTHH})} \times \mathrm{beta}(\theta \mid 2,2) \\ &= \frac{\mathrm{B}(2,2)}{\mathrm{B}(4+2,1+2)} \times \theta^4(1-\theta)^1 \times \mathrm{beta}(\theta \mid 2,2) \\ &= \frac{\theta^5(1-\theta)^2}{\mathrm{B}(6,3)} = \mathrm{beta}(\theta \mid 6,3). \end{aligned}$$

The three plots are shown below:



Posterior Prediction

Q: Compute the posterior predictive $p(\mathrm{H}|\mathrm{HHTHH})$.

$$\begin{aligned} p(\mathrm{H}|\mathrm{HHTHH},\alpha,\beta) &= \int_0^1 p(\mathrm{H}|\theta)p(\theta|\mathrm{HHTHH})\mathrm{d}\theta \\ &= \int_0^1 \theta\, \mathrm{beta}(\theta \mid 6,3)\mathrm{d}\theta \\ &= \mathrm{E}_\theta[\mathrm{beta}(\theta \mid 6,3)] \\ &= \frac{6}{6+3} = \frac{2}{3}. \end{aligned}$$

Model Comparison

Q: Compare simple and complex models by computing the marginal likelihoods.

Marginal likelihood of "HHTH" in model M_1 is:

$$p(\mathrm{HHTH}|M_1) = (\frac{1}{2})^3 \frac{1}{2} = \frac{1}{16}.$$

Marginal likelihood of "HHTH" in model M_2 is:

$$\begin{aligned} p(\mathrm{HHTH}|M_2) &= \int_0^1 \theta^3(1-\theta)\mathrm{beta}(\theta \mid 2,2)\ \mathrm{d}\theta \\ &= \frac{\mathrm{B}(5,3)}{\mathrm{B}(2,2)} = \frac{2}{35}. \end{aligned}$$

So the Bayes factor is $35/32 > 1$. Then model M_1 is better.