## **Plotting**

Q: Plot beta prior distributions  $beta(\theta|2,2)$ , Bernoulli likelihoods  $Ber(HHTHH|\theta)$ , and their corresponding posterior distributions  $p(\theta|HHTHH)$ .

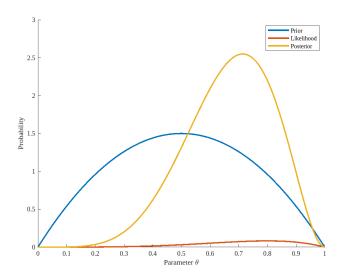
For Bernoulli likelihood, here it is

$$Ber(HHTHH|\theta) = \theta^4(1-\theta)^1.$$

Then the posterior distribution is

$$egin{aligned} p( heta|\mathrm{HHTHH}) &= rac{ heta^4(1- heta)^1}{p(\mathrm{HHTHH})} imes \mathrm{beta}( heta \mid 2,2) \ &= rac{\mathrm{B}(2,2)}{\mathrm{B}(4+2,1+2)} imes heta^4(1- heta)^1 imes \mathrm{beta}( heta \mid 2,2) \ &= rac{ heta^5(1- heta)^2}{\mathrm{B}(6,3)} = \mathrm{beta}( heta \mid 6,3). \end{aligned}$$

The three plots are shown below:



## **Posterior Prediction**

Q: Compute the posterior predictive p(H|HHTHH).

$$egin{aligned} p(\mathrm{H}|\mathrm{HHTHH},lpha,eta) &= \int_0^1 p(\mathrm{H}| heta)p( heta|\mathrm{HHTHH})\mathrm{d} heta \ &= \int_0^1 heta \, \mathrm{beta}( heta \, | \, 6, 3)\mathrm{d} heta \ &= \mathrm{E}_ heta(\mathrm{beta}( heta \, | \, 6, 3)) \ &= rac{4+2}{4+2+1+2} = rac{2}{3}. \end{aligned}$$

## **Model Comparison**

Q: Compare simple and complex models by computing the marginal likelihoods.

Marginal likelihood of "HHTH" in model  $M_1$  is:

$$p(\text{HHTH}|M_1) = (\frac{1}{2})^3 \frac{1}{2} = \frac{1}{16}.$$

Marginal likelihood of "HHTH" in model  ${\cal M}_2$  is:

$$egin{align} p( ext{HHTH}|M_2) &= \int_0^1 heta^3 (1- heta) ext{beta}( heta \mid 2,2) \; ext{d} heta \ &= rac{ ext{B}(5,3)}{ ext{B}(2,2)} = rac{2}{35}. \end{split}$$

So the Bayes factor is 35/32>1. Then model  $M_1$  is better.