Plotting

Q: Plot beta prior distributions $beta(\theta|2,2)$, Bernoulli likelihoods $Ber(HHTHH|\theta)$, and their corresponding posterior distributions $p(\theta|HHTHH)$.

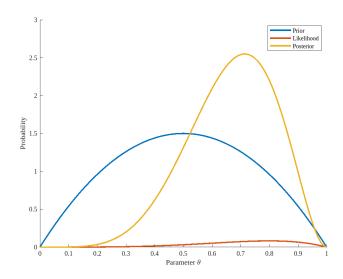
For Bernoulli likelihood, here it is

$$Ber(HHTHH|\theta) = \theta^4(1-\theta)^1.$$

Then the posterior distribution is

$$egin{aligned} p(heta|\mathrm{HHTHH}) &= rac{ heta^4(1- heta)^1}{p(\mathrm{HHTHH})} imes \mathrm{beta}(heta \mid 2,2) \ &= rac{\mathrm{B}(2,2)}{\mathrm{B}(4+2,1+2)} imes heta^4(1- heta)^1 imes \mathrm{beta}(heta \mid 2,2) \ &= rac{ heta^5(1- heta)^2}{\mathrm{B}(6,3)} = \mathrm{beta}(heta \mid 6,3). \end{aligned}$$

The three plots are shown below:



Posterior Prediction

Q: Compute the posterior predictive p(H|HHTHH).

$$egin{aligned} p(\mathbf{H}|\mathbf{HHTHH}, lpha, eta) &= \int_0^1 p(\mathbf{H}| heta) p(heta|\mathbf{HHTHH}) \mathrm{d} heta \ &= \int_0^1 heta \, \mathrm{beta}(heta \mid 6, 3) \mathrm{d} heta \ &= \mathrm{E}_ heta[\mathrm{beta}(heta \mid 6, 3)] \ &= rac{6}{6+3} = rac{2}{3}. \end{aligned}$$

Model Comparison

Q: Compare simple and complex models by computing the marginal likelihoods.

Marginal likelihood of "HHTH" in model M_1 is:

$$p(\text{HHTH}|M_1) = (\frac{1}{2})^3 \frac{1}{2} = \frac{1}{16}.$$

Marginal likelihood of "HHTH" in model ${\cal M}_2$ is:

$$egin{align} p(ext{HHTH}|M_2) &= \int_0^1 heta^3 (1- heta) ext{beta}(heta \mid 2,2) \; ext{d} heta \ &= rac{ ext{B}(5,3)}{ ext{B}(2,2)} = rac{2}{35}. \end{split}$$

So the Bayes factor is 35/32>1. Then model M_1 is better.