Locally Differentially Private Frequency Estimation via Joint Randomized Response

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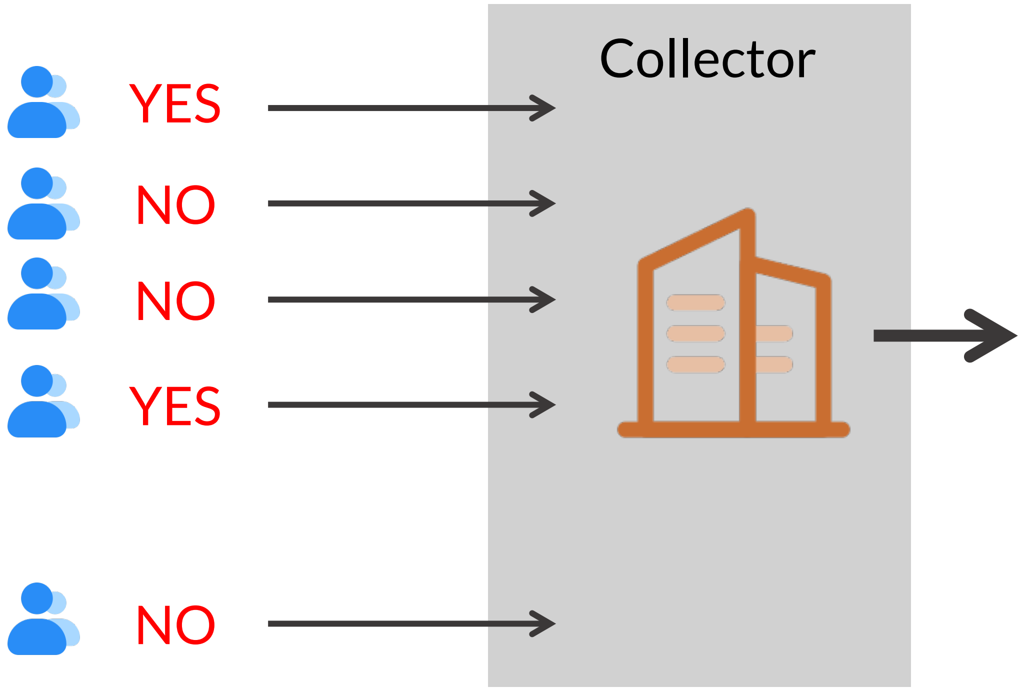
Frequency Estimation

- Social scientists: How many people engage in tax evasion?

  - ask one person if they had evaded tax

  - the person answers YES or NO



Frequency = # of YES

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- People tend to lie for such sensitive/embarrassing question

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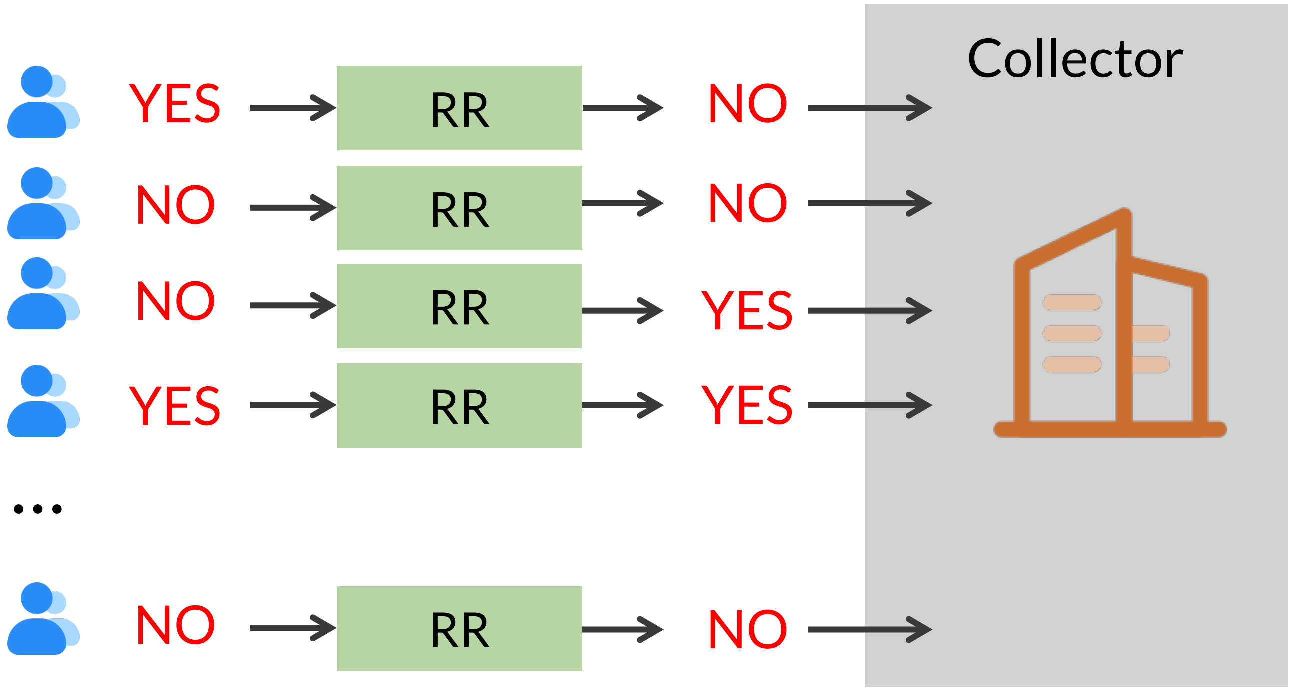

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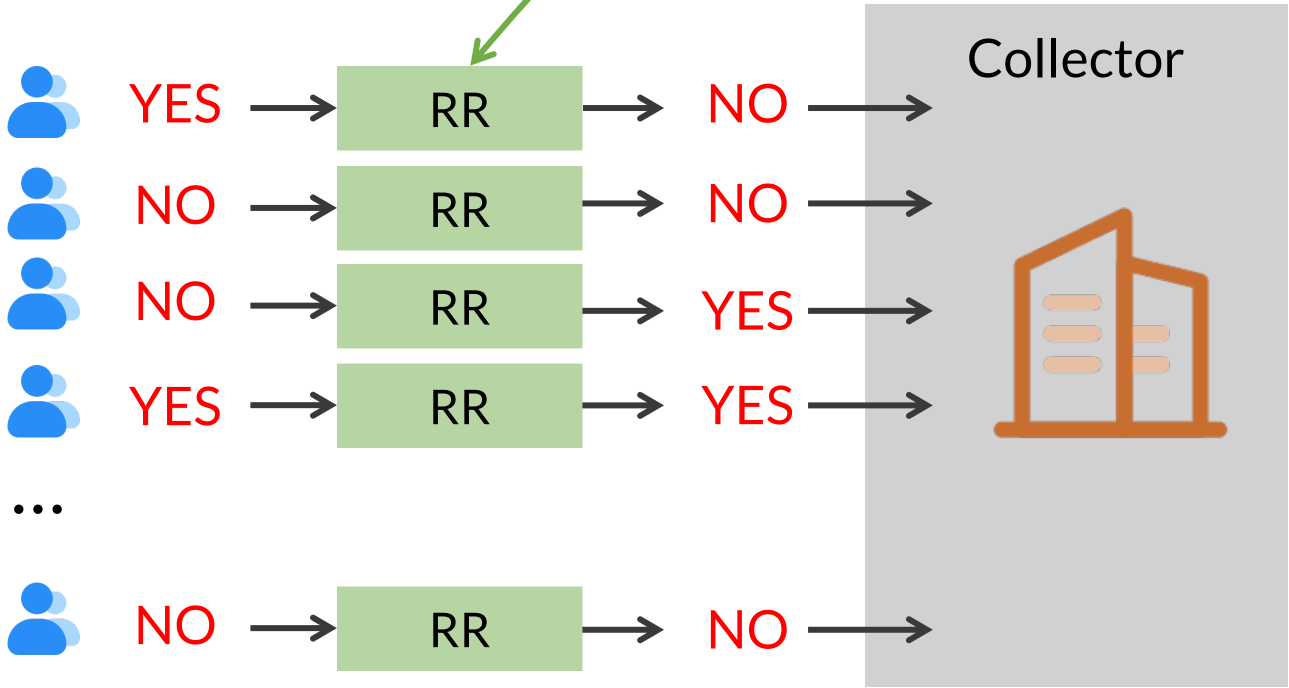
RR:
answer truth with probability p

$$\mathrm{RR}(x) = \begin{cases} x & \text{w.p. } p \\ \neg x & \text{w.p. } 1-p \end{cases}$$



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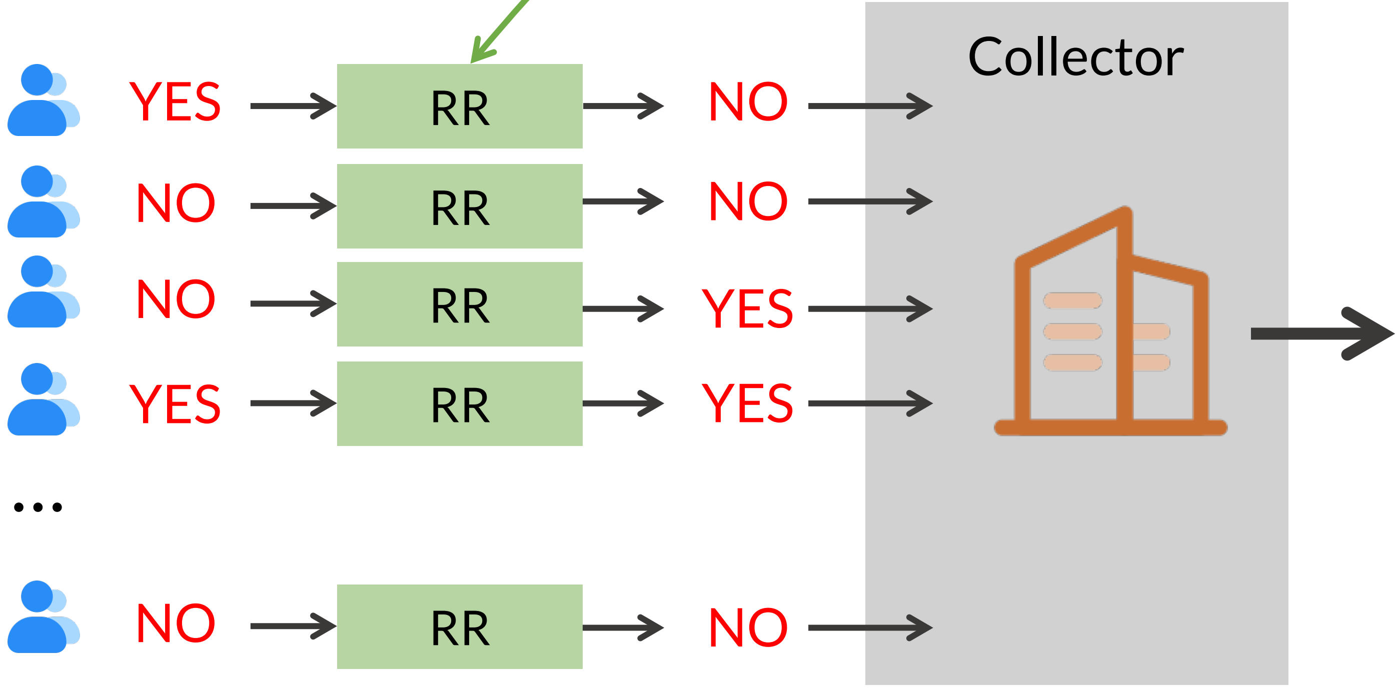

estimated frequency

$$= \frac{\#\text{ of YES} - \# \times q}{p-q}$$

Unbiased:
expectation = truth

Privacy: RR Satisfies LDP

- A mechanism $\mathcal{M}$ satisfies LDP if

For any truth x_1, x_2,
and randomized answer y:

$$\max \frac{\Pr[\mathcal{M}(x_1) = y]}{\Pr[\mathcal{M}(x_2) = y]} \leq e^{\varepsilon}$$

Distinguishability of x_1 (YES) and x_2 (NO)
from y (randomized answer)

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Distinguishability of x_1 (YES) and x_2 (NO)
from y (randomized answer)

- **quantifiable hardness** to distinguish x_1 (YES) and x_2 (NO) from the randomized answer y

- against inference from data collectors or adversaries

Utility: RR’s Variance

- Randomization reduces data utility

$$\mathrm{Var}\left[\frac{\#\text{ of }\text{YES} - \# \times q}{p-q}\right] = \frac{\mathrm{Var}[\#\text{ of YES}]}{(p-q)^2} = \frac{npq}{(p-q)^2}$$

- summation of variance from n **independent** randomization

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- larger $p \in (0.5, 1]$ → lower variance → larger privacy parameter ε

↑ data utility  ↓ privacy

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- Q: Can **correlated** (joint) randomization improve this privacy-utility tradeoff?

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- **Example:** 2-person ($x_1 = \text{YES}$ and $x_2 = \text{YES}$) with $p = 0.8$ ($\text{P}[T = 1] = 0.8$)

RR: Joint distribution

	$T_1 = 1$	$T_1 = 0$
$T_2 = 1$	0.64 (=0.8× 0.8)	0.16 (=0.2× 0.8)
$T_2 = 0$	0.16 (=0.8× 0.2)	0.04 (=0.2× 0.2)

Truthfulness of x_1

Truthfulness of x_2

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$\mathrm{P}[T_1 = 1] = 0.6 + 0.2 = 0.8$

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$\mathbf{P}[T_1 = 0 \cap T_2 = 0] = 0$

$\mathbf{P}[T_1 = 0] \cdot \mathbf{P}[T_2 = 0]$=0.04

NOT independent T_1 and T_2

Utility: JRR's Variance

- Same estimator as RR

$$\text{Expectation:}\quad \mathrm{E}[\#\text{ of YES}] = \sum_{i=1}^{\#} \mathrm{P}[y_i = \text{YES}] = n_{\text{YES}} \cdot p + (\# - n_{\text{YES}}) \cdot q$$

$$\hat{n}_{\text{YES}} = \boxed{\frac{\#\text{ of YES} - 2q}{p-q}} \text{ is unbiased}$$

Identical to RR

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- Variance: (# = 2, $p = 0.8$)

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JRR

# of YES	0	1	2
Probability	0	$0.2+0.2$	0.6

$\mathrm{Var}[\#\text{ of YES}] = E[(X-\mu)^2] = 0.24$

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$\text{Var}[\#\text{ of YES}] = E[(X-\mu)^2] \approx 0.32$

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JRR **Better utility**

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Privacy: NOT as Simple as RR

- If any person can be an adversary

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T_1: I am an adversary

When I report untruthfully ($T_1 = 0$),
My partner will report truthfully ($T_2 = 1$)

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- Correlation results in privacy leakage (2 slides later)

General JRR

- Correlated randomization with 2 persons x_{2i-1} and x_{2i}

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	$T_{2i-1} = 1$	$T_{2i-1} = 0$
$T_{2i} = 1$	$p^2 + \rho pq$	$(1-\rho)pq$
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$\rho \in [-1,1]$: correlated coefficient

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JRR – Privacy Model in This Paper

- Use random grouping to form 2-person groups for correlated randomization

  - secure shuffling by multi-party computing (MPC)

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  - the adversary cannot control randomness, but can infer their partner's

$$\mathrm{P}\left[\begin{matrix}\text{the adversary}\\ \text{knows id}\end{matrix}\right] = \frac{m}{n-1}$$

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$$\mathrm{P}[\mathrm{JRR}(x_2) = 1 \mid \underbrace{\mathrm{JRR}(x_1) = 0}_{\text{Adversary knows}}] = \frac{(1-\rho)pq}{q} = \underbrace{(1-\rho)p}_{\text{Higher confidence}}$$

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Adversary's confidence

$$\frac{m}{n-1} \cdot (1-\rho)p$$

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$$P[\text{the adversary}\ ] = \frac{m}{n-1}$$

m: # of adversaries

Privacy affected by

m	# adversaries
n	# of persons
ρ	Correlated coefficient

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Adversary knows

Higher confidence

Adversary's confidence

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JRR – Formal Privacy & Utility

Theorem. Assume there is a set of data contributors $\mathcal{T}_m$ whose reporting truthfulness is known to the adversary. For any data contributor i, the JRR mechanism satisfies:

$$\frac{\Pr[\ \mathrm{JRR}(x_i)\ |\mathcal{T}_m]}{\Pr\left[\ \mathrm{JRR}\left(x_i'\right)\ |\mathcal{T}_m\right]} \leq e^{\varepsilon},\quad \text{where}\quad \varepsilon = \ln \frac{mp_{\max} + (n-m-1)p}{mp_{\min} + (n-m-1)q}.$$

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$m = |\mathcal{T}_m|$:
# of adversaries

$p_{\max} = \max\{(1-\rho)p, p+\rho q\}$:
confidence of adversaries
inferring a specific value

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Theorem. The variance of JRR's estimator $\hat{\boldsymbol{n}}_{\boldsymbol{v}}$ is

$$\mathrm{Var}[\hat{\boldsymbol{n}}_{\boldsymbol{v}}] = \frac{pq}{(p-q)^2} \cdot \left(n + \frac{\rho\left((2n_{\mathrm{YES}} - n)^2 - n\right)}{n-1}\right).$$

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Affected by # of original values

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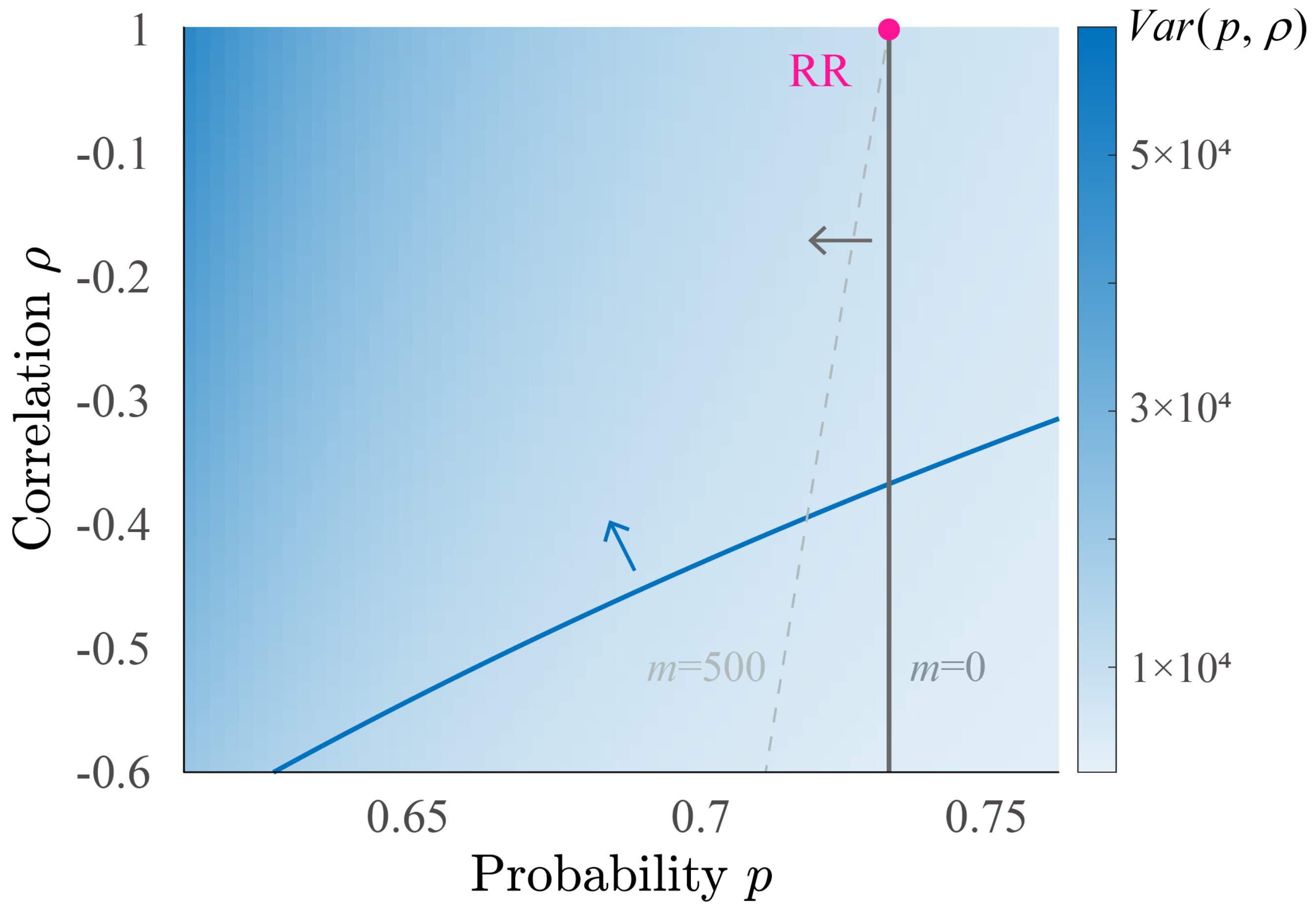
Independent randomization (RR)

Correlated randomization

JRR – Variance Heatmap

- Effect of $\rho, p,$ and m on variance (when $\varepsilon = 1, n = 10^4$, and $n_{\text{Yes}} = 200$)



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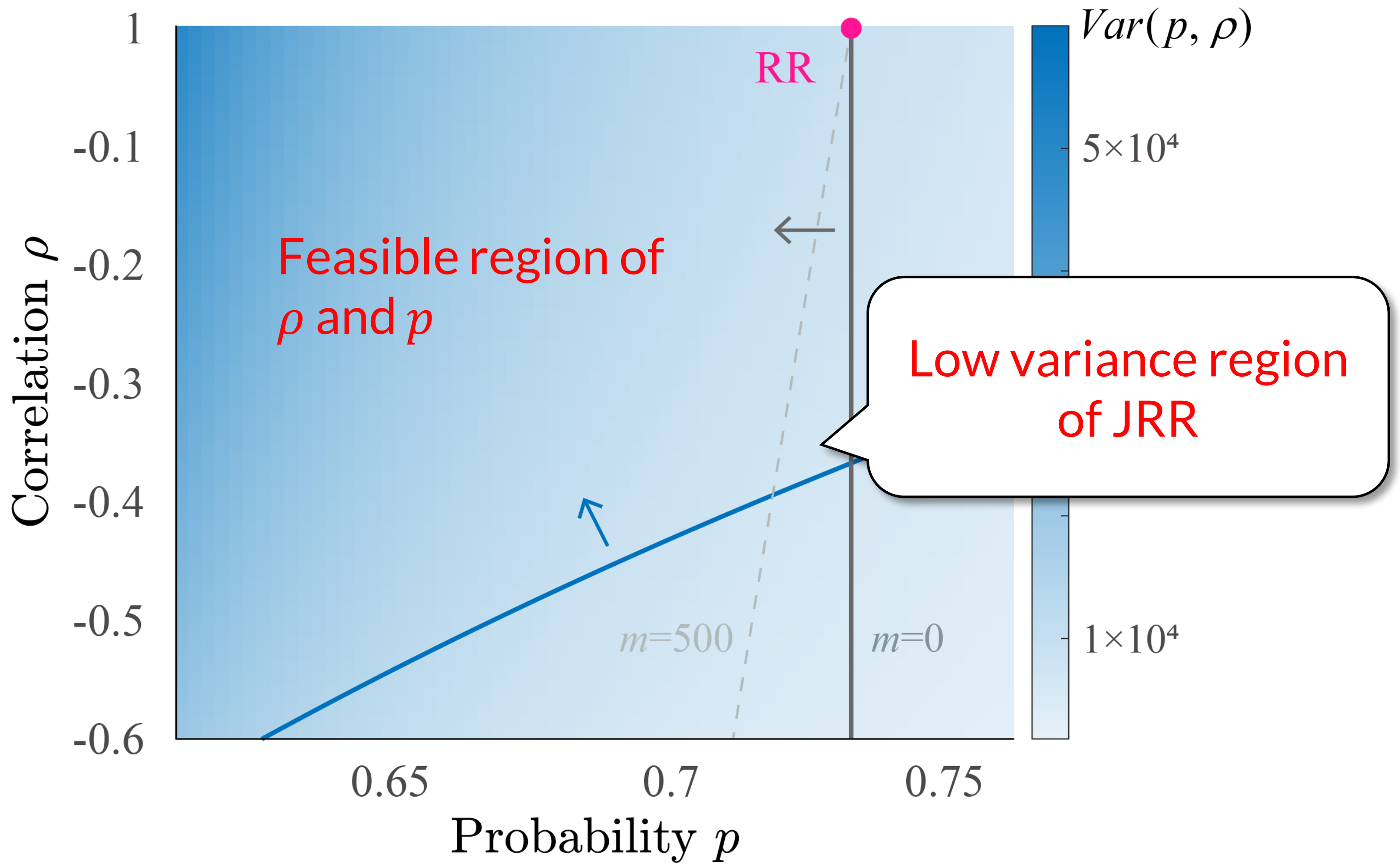

Summary

- Correlated randomization can improve the data utility of frequency estimation
- **JRR: Privacy & utility model for correlated randomization**

- What's more in the paper

  - selection the tradeoff between ρ and p

  - practical protocol design

  - prototype extensions to non-binary data and larger-size group

  - evaluations on synthetic and real-world datasets

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YES → RR → NO →
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…
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Collector

estimated frequency

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  - any person can be an adversary
  - if a group contains an adversary, the ad… (after random grouping)
  - the adversary cannot control random…

$\mathrm{P}[\text{the adversary}] = \frac{m}{n-1}$ → m: # of adversaries

Privacy affected by

m	# adversaries
n	# of persons
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Adversary knows Higher confidence

…versary's confidence

$\frac{m}{n-1} \cdot (1-\rho)p$

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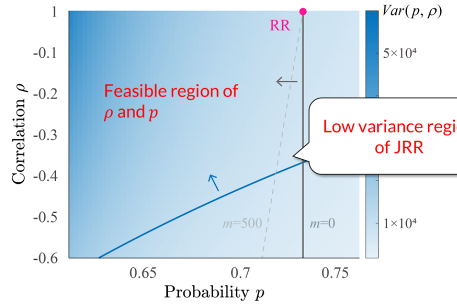

Thank you!