Semidefinite Programming

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1 Semidefinite Programming

Definition 1. *A symmetric $n \times n$ matrix A is PSD if $x^T Ax \geq 0 \forall x$*

Theorem 1.1. *The following are equivalent:*

1. $x^T Ax \geq 0 \forall x$
2. $A = \sum_{i=0}^{n} \lambda_i v_i v_i^T$, *where* $\lambda_i \in \mathbb{R}^+$ *and* v_i *are orthonormal.*
3. $A = B^T B$ *for some* B

Proof. 1. 1) $\implies$ 2):

2. 2) $\implies$ 3):

3. 3) $\implies$ 1):

□

3) implies that if A is SDP, $A_{ij} =< b_i, b_j >$ SDP: $\min_{X \in \mathbb{R}^{n \times n}} < C, X >= \sum C_{ij} X_{i,j} = Tr(CX)$