Real Analysis Theorems

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1 Limsup and Liminf

Corollary 1.0.1. *If* $\lim \left|\frac{s_{n+1}}{s_n}\right|$ *exists* [*and equals* L], *then* $\lim |s_n|^{1/n}$ *exists* [*and equals* L].

2 Power Series

Given power series $\sum_{n=0}^{\infty} a_n x^n$

Theorem 2.1. *Given any* (a_n), *one of the following holds true:*

1. *The power series converges for all* $x \in \mathbb{R}$
2. *The power series converges only for* $x = 0$
3. *The power series converges for all* x *in some bounded interval centered at* 0; *the intervalmay be open, half-open or closed.*

Theorem 2.2. *Let*

$$\beta = \limsup |a_n|^{1/n} \qquad \textit{and} \qquad R = \frac{1}{\beta}$$

Then

1. *The power series converges for* $|x| < R$
2. *The power series diverges for* $|x| > R$

Also notice that $\lim \left|\frac{a_{n+1}}{a_n}\right| = \beta$, *therefore most of the time we will use* $\lim \left|\frac{a_{n+1}}{a_n}\right|$ *as it's easier to compute thatn* β.