Spectral Graph Theory – Electric Flow

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March 15, 2019

1 Introduction

In this notes, we are going to discuss the interestingproperties when turning a graph into a network of resistors.

1.1 Electrical Laws

First recall some E&M Laws:

$$I = \frac{U}{R} \qquad \text{Ohm's law}$$
$$E = I^2 R \qquad \text{Energy formula}$$
$$|I_{v,in}| = |I_{v,out}| \qquad \text{conservation of flow}$$

Note that the last law only holds for nodes that are not source or sink.

1.2 Formation on Graph

We will write a matrix formation of the problem.

- Let $G(V, E)$ be an undirected graph with $|V| = n, |E| = m$.
- Let $v \in \mathbb{R}^n$ be the vector representing the potentials of vertices.
- Edges represent the resistors, and $\forall e(u, v) \in E$, we restrict the representation (u, v) that $u < v$.
- Let $f \in \mathbb{R}^m$ representing the flow of all edges, where $f(a, b)$ represents the flow from a to b with $a < b$. Since $f(a, b)$ is directed, we have $f(a, b) = -f(b, a)$