Real Analysis Theorems

Zhiwei Zhang

April 8, 2019

1 Limsup and Liminf

Corollary 1.0.1. *If* $\lim \left|\frac{s_{n+1}}{s_n}\right|$ *exists [and equals* L*], then* $\lim |s_n|^{1/n}$ *exists [and equals* L*].*

2 Power Series

Given power series $\sum_{n=0}^{\infty} a_n x^n$

Theorem 2.1. *Given any* (a_n)*, one of the following holds true:*

1. *The power series converges for all* $x \in \mathbb{R}$
2. *The power series converges only for* $x = 0$
3. *The power series converges for all* x *in some bounded interval centered at* 0*; the intervalmay be open, half-open or closed.*

Theorem 2.2. *Let*

$$\beta = \limsup |a_n|^{1/n} \qquad \text{and} \qquad R = \frac{1}{\beta}$$

Then

1. *The power series converges for* $|x| < R$
2. *The power series diverges for* $|x| > R$

Also notice that $\lim \left|\frac{a_{n+1}}{a_n}\right| = \beta$*, therefore most of the time we will use* $\lim \left|\frac{a_{n+1}}{a_n}\right|$ *as it's easier to compute thatn* β*.*

3 More on Uniform Convergence

Theorem 3.1. *Let* (f_n) *be a sequence of continuous functions on* $[a, b]$*, and suppose* $f_n \to f$ *uniformly on* $[a, b]$*. Then*

$$\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$$

Definition 3.2. *A sequence (f_n) of functions defined on a set $S \subseteq \mathbb{R}$ is uniformly Cauchy on S if*

$$\begin{gathered}\textit{for each } \epsilon > 0 \textit{ there exists a number } N \textit{ such that} \\ |f_n(x) - f_m(x)| < \epsilon \textit{ for all } x \in S \textit{ and all } m, n > N\end{gathered}$$

Theorem 3.3. *Let (f_n) be a sequence of functions defined and uniformly Cauchy on a set $S \subseteq \mathbb{R}$. Then there exists a function f on S such that $f_n \to f$ uniformly on S.*