Semidefinite Programming

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1 Semidefinite Programming

Definition 1. *A symmetric $n \times n$ matrix A is PSD if $x^T A x \geq 0 \forall x$*

Theorem 1.1. *The following are equivalent:*

1. *$x^T A x \geq 0 \forall x$*

2. *$A = \sum_{i=0}^{n} \lambda_i v_i v_i^T$, where $\lambda_i \in \mathbb{R}^+$ and v_i are orthonormal.*

3. *$A = B^T B$ for some B*

Proof.   1. 1) $\implies$ 2):

2. 2) $\implies$ 3):

3. 3) $\implies$ 1):

$\square$

3) implies that if A is SDP, $A_{ij} = <b_i, b_j>$ SDP: $\min\limits_{X \in \mathbb{R}^{n \times n}}$