Spectral Graph Theory

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1 Introduction

1.1 Adjacency Matrix Representation of A Graph

We represent a Graph G with a square adjacency matrix A, where entry

$$a_{ij} = \begin{cases} 1 & \text{if (i, j) is an edge in } G \\ 0 & \text{otherwise} \end{cases}$$

Evidently, the matrix is symmetric, or $a_{ij} = a_{ji}$. Therefore by spectral theorem, we have a orthonormal basis of eigenvectors, with real eigenvalues associated.

2 Counting Paths with Adjacency Matrix

Theorem L. et G be a graph on labeled vertices, let A be its adjacency matrix, and let k be a positive integer. Then $A_{i,j}^k$ is equal to the number of walks from i to j that are of length k .