Johnson-Lindenstrauss, Compress Sensing

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February 14, 2019

1 Johnson-Lindenstrauss (JL)

1.1 Formal Theorem

For a set of n points $x_1, \ldots, x_n \in \mathbb{R}^d$, then for a random $k = \frac{c \log n}{\varepsilon^2}$ dimensional subspace, we project these points to this subspace.

JL states that with probability $1 - \frac{1}{n^{c-2}}$, we have:

$$(1-\varepsilon)\sqrt{\frac{k}{d}}\,|x_i - x_j| \leq |y_i - y_j| \leq (1+\varepsilon)\sqrt{\frac{k}{d}}\,|x_i - x_j|$$

Furthermore, y_i is the projection of x_i on to the subspace.

1.2 Intuitive Interpretation

$|x_i - x_j|$ is the original distance between two points, and similar for $|y_i - y_j|$. Therefore the inequality is stating that after projecting to the subspace of dimension k, with high probability that the pariwise distances are scaled by $\sqrt{\frac{k}{d}}$ within factor of $1 \pm \varepsilon$