STAT 3690 Lecture 15

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Testing for equality of covariance matrices (J&W Sec. 6.6)

- Model: m independent samples, where
  - $\mathbf{X}_{11}, \ldots, \mathbf{X}_{1n_1} \overset{\text{iid}}{\sim} MVN_p(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$
  - $\vdots$
  - $\mathbf{X}_{m1}, \ldots, \mathbf{X}_{mn_m} \overset{\text{iid}}{\sim} MVN_p(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$
- Hypotheses $H_0 : \boldsymbol{\Sigma}_1 = \cdots = \boldsymbol{\Sigma}_m$ v.s. H_1 : otherwise
- MLE of $(\boldsymbol{\mu}_1, \ldots, \boldsymbol{\mu}_m, \boldsymbol{\Sigma}_1, \ldots, \boldsymbol{\Sigma}_m)$
  - Under H_0
    - $\hat{\boldsymbol{\mu}}_i = \bar{\mathbf{X}}_i = n_i^{-1} \sum_j \mathbf{X}_{ij}$
    - $\widehat{\boldsymbol{\Sigma}}_i = (\sum_i n_i)^{-1} \mathbf{SSP}_{\text{w}} = (\sum_i n_i)^{-1} \sum_{ij} (\mathbf{X}_{ij} - \bar{\mathbf{X}}_i)(\mathbf{X}_{ij} - \bar{\mathbf{X}}_i)^\top$ for all i
  - No restriction on $\boldsymbol{\Sigma}_i$
    - $\hat{\boldsymbol{\mu}}_i = \bar{\mathbf{X}}_i = n_i^{-1} \sum_j \mathbf{X}_{ij}$
    - $\widehat{\boldsymbol{\Sigma}}_i = n_i^{-1}(n_i - 1)\mathbf{S}_i = n_i^{-1} \sum_j (\mathbf{X}_{ij} - \bar{\mathbf{X}}_i)(\mathbf{X}_{ij} - \bar{\mathbf{X}}_i)^\top$
- Likelihood ratio

$$\lambda = \prod_i \left[\frac{\det\{n_i^{-1}(n_i - 1)\mathbf{S}_i\}}{\det\{(\sum_i n_i)^{-1}(\sum_i n_i - m)\mathbf{S}_{\text{pool}}\}} \right]^{n_i/2}$$

  - $\mathbf{S}_{\text{pool}} = (\sum_i n_i - m)^{-1} \mathbf{SSP}_{\text{w}}$

- Box's M test statistic (a modification of LRT)

$$M = -2 \ln \prod_i \left(\frac{\det \mathbf{S}_i}{\det \mathbf{S}_{\text{pool}}} \right)^{(n_i - 1)/2}$$

  - Under H_0

$$(1 - u)M \approx \chi^2(p(p+1)(m-1)/2)$$

    - $u = \{\sum_i (n_i - 1)^{-1} - (\sum_i n_i - m)^{-1}\}\{6(p+1)(m-1)\}^{-1}(2p^2 + 3p - 1)$
- Rejection region at level α

$$\left\{ x_{11}, \ldots, x_{1n_1}, x_{21}, \ldots, x_{mn_m} : (1 - u)M \geq \chi^2_{1-\alpha, p(p+1)(m-1)/2} \right\}$$

- p-value

$$1 - F_{\chi^2_{1-\alpha, p(p+1)(m-1)/2}}\{(1 - u)M\}$$

- Exercise: factors in producing plastic film (continued)
  - Check the equality of covariance matrices for `RATE="Low"` and `RATE="High"`