# PH 712 Probability and Statistical Inference

Part II: Mutiple Random Variables

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## Joint distribution of multiple random variables

- The joint cdf of $n$ RVs $X_1,\dots,X_n$:

$$F_{X_1,\dots,X_n}(x_1,\dots,x_n)=\Pr(X_1\le x_1,\dots,X_n\le x_n),\quad (x_1,\dots,x_n)\in\mathbb{R}^n$$

- If $X_1,\dots,X_n$ are ALL discrete
  - Joint pmf:

$$p_{X_1,\dots,X_n}(x_1,\dots,x_n)=\Pr(X_1=x_1,\dots,X_n=x_n),\quad (x_1,\dots,x_n)\in\mathbb{R}^n$$

  - $\mathrm{supp}(X_1,\dots,X_n)=\{(x_1,\dots,x_p)\in\mathbb{R}^n : p_{X_1,\dots,X_n}(x_1,\dots,x_n)>0\}$
  - Marginal pmf of $X_i$:

$$p_{X_i}(x_i)=\sum_{x_1,\dots,x_{i-1},x_{i+1},\dots,x_n} p_{X_1,\dots,X_n}(x_1,\dots,x_n)$$

- If $X_1,\dots,X_n$ are ALL continuous
  - Joint pdf:

$$f_{X_1,\dots,X_n}(x_1,\dots,x_n)=(\partial^n/\partial x_1\cdots\partial x_n)F_{X_1,\dots,X_n}(x_1,\dots,x_n),\quad (x_1,\dots,x_n)\in\mathbb{R}^n$$

  - $\mathrm{supp}(X_1,\dots,X_n)=\{(x_1,\dots,x_n)\in\mathbb{R}^n : f_{X_1,\dots,X_n}(x_1,\dots,x_n)>0\}$
  - Marginal pdf of $X_1$:

$$f_{X_i}(x_i)=\int_{\mathbb{R}^{n-1}} f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}x_1\cdots\mathrm{d}x_{i-1}\mathrm{d}x_{i+1}\cdots\mathrm{d}x_n$$

## (Mutual) independence

- RVs $X_1,\dots,X_n$ are (mutually) independent $\Leftrightarrow$

$$F_{X_1,\dots,X_n}(x_1,\dots,x_n)=\prod_{i=1}^n F_{X_i}(x_i)$$

  - Joint pmf $p_{X_1,\dots,X_n}(x_1,\dots,x_n)=\prod_{i=1}^n p_{X_i}(x_i)$ (for discrete $X_1,\dots,X_n$)
  - Joint pdf $f_{X_1,\dots,X_n}(x_1,\dots,x_n)=\prod_{i=1}^n f_{X_i}(x_i)$ (for continuous $X_1,\dots,X_n$)