

PH 712 Probability and Statistical Inference

Part X: Confidence Set/Interval

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Confidence set (CB Sec 9.2.1 & 9.3.1)

- Called a confidence interval (CI) If the set is an interval
- True (but unknown) value of parameter θ , say θ_0
- $(1 - \alpha) \times 100\%$ confidence set, say $C(X_1, \dots, X_n)$: $C(X_1, \dots, X_n)$ covers θ_0 with probability AT LEAST $(1 - \alpha) \times 100\%$, i.e., $\Pr\{\theta_0 \in C(X_1, \dots, X_n)\} \geq (1 - \alpha) \times 100\%$
 - $C(X_1, \dots, X_n)$ is a set defined on sample X_1, \dots, X_n and hence is randomized, while θ_0 is fixed
 - Coverage probability: $(1 - \alpha) \times 100\%$

Construction of a confidence set by inverting a level α test

- (CB Thm 9.2.2) Implementation
 1. For each $\theta^* \in \Theta$, find the rejection region, say $R(\theta^*)$, of a level α test of hypotheses $H_0 : \theta = \theta^*$ vs. $H_1 : \theta \neq \theta^*$
 2. $C(x_1, \dots, x_n) = \{\theta : (x_1, \dots, x_n) \in \text{supp}(X_1, \dots, X_n)/R(\theta)\}$,
 - $\text{supp}(X_1, \dots, X_n)/R(\theta)$: the complementary set of $R(\theta)$.
- $(1 - \alpha) \times 100\%$ confidence set $C(X_1, \dots, X_n)$ does not cover $\theta_0 \Leftrightarrow$ reject $H_0 : \theta = \theta_0$ (vs. $H_1 : \theta \neq \theta_0$) at level α
- Special cases:
 - $(1 - \alpha) \times 100\%$ asymptotic LRT confidence set for θ : $\{\theta : -2(\ell(\theta) - \ell(\hat{\theta}_{\text{ML}})) < \chi_{1,1-\alpha}^2\}$
 - $(1 - \alpha) \times 100\%$ Wald confidence set for θ : $\{\theta : |\hat{\theta}_{\text{ML}} - \theta|/\sqrt{\widehat{\text{var}}(\hat{\theta}_{\text{ML}})} < \Phi_{1-\alpha/2}^{-1}\}$
 - $(1 - \alpha) \times 100\%$ score confidence set for θ : $\{\theta : |\ell'(\theta)|/\sqrt{I_n(\theta)} < \Phi_{1-\alpha/2}^{-1}\}$