

PH 712 Probability and Statistical Inference

Part II: Mutiple Random Variables

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Joint distribution of multiple random variables

- The joint cdf of n RVs X_1, \dots, X_n :

$$F_{X_1, \dots, X_n}(x_1, \dots, x_n) = \Pr(X_1 \leq x_1, \dots, X_n \leq x_n), \quad (x_1, \dots, x_n) \in \mathbb{R}^n$$

- If X_1, \dots, X_n are ALL discrete

- Joint pmf:

$$p_{X_1, \dots, X_n}(x_1, \dots, x_n) = \Pr(X_1 = x_1, \dots, X_n = x_n), \quad (x_1, \dots, x_n) \in \mathbb{R}^n$$

- $\text{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_p) \in \mathbb{R}^n : p_{X_1, \dots, X_n}(x_1, \dots, x_n) > 0\}$
- Marginal pmf of X_i :

$$p_{X_i}(x_i) = \sum_{x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n} p_{X_1, \dots, X_n}(x_1, \dots, x_n)$$

- If X_1, \dots, X_n are ALL continuous

- Joint pdf:

$$f_{X_1, \dots, X_n}(x_1, \dots, x_n) = (\partial^n / \partial x_1 \cdots \partial x_n) F_{X_1, \dots, X_n}(x_1, \dots, x_n), \quad (x_1, \dots, x_n) \in \mathbb{R}^n$$

- $\text{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_n) \in \mathbb{R}^n : f_{X_1, \dots, X_n}(x_1, \dots, x_n) > 0\}$
- Marginal pdf of X_1 :

$$f_{X_i}(x_i) = \int_{\mathbb{R}^{n-1}} f_{\mathbf{X}}(\mathbf{x}) dx_1 \cdots dx_{i-1} dx_{i+1} \cdots dx_n$$

(Mutual) independence

- RVs X_1, \dots, X_n are (mutually) independent \Leftrightarrow

$$F_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n F_{X_i}(x_i)$$

- Joint pmf $p_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n p_{X_i}(x_i)$ (for discrete X_1, \dots, X_n)
- Joint pdf $f_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n f_{X_i}(x_i)$ (for continuous X_1, \dots, X_n)

Example Lec2.1

- X_1, X_2 are iid Bernoulli RVs with pmf $p_{X_i}(0) = 1 - p$ and $p_{X_i}(1) = p$, $i = 1, 2$. Write the joint pmf of (X_1, X_2) .

- Let (X_1, X_2) be iid Poisson RVs with pmf $p_{X_i}(k) = \frac{e^{-3} \cdot 3^k}{k!}$, $k = 0, 1, 2, \dots$, $i = 1, 2$. Write the joint pmf of (X_1, X_2) .

- X_1, X_2 are iid uniform RVs with pdf $f_{X_i}(x) = \mathbf{1}_{[0,1]}(x)$, $i = 1, 2$. Write the joint pdf of (X_1, X_2) .

- Let (X_1, X_2) be iid exponential RVs with pdf $f_{X_i}(x) = 2e^{-2x} \cdot \mathbf{1}_{(0,\infty)}(x)$, $i = 1, 2$. Write the joint pdf of (X_1, X_2) .