

PH 712 Probability and Statistical Inference

Part III: Mutiple Random Variables

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Characterizing the joint distribution of multiple random variables

- The joint cdf of X_1, \dots, X_n : $F_{X_1, \dots, X_n}(x_1, \dots, x_n) = \Pr(X_1 \leq x_1, \dots, X_n \leq x_n), (x_1, \dots, x_n) \in \mathbb{R}^n$
- Discrete X_1, \dots, X_n
 - Joint pmf $p_{X_1, \dots, X_n}(x_1, \dots, x_n) = \Pr(X_1 = x_1, \dots, X_n = x_n), (x_1, \dots, x_n) \in \mathbb{R}^n$
 - $\text{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_p) \in \mathbb{R}^n : p_{X_1, \dots, X_n}(x_1, \dots, x_n) > 0\}$
 - Marginal pmf of X_i :

$$p_{X_i}(x_i) = \sum_{x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n} p_{X_1, \dots, X_n}(x_1, \dots, x_n)$$

- Continuous X_1, \dots, X_n
 - Joint pdf $f_{X_1, \dots, X_n}(x_1, \dots, x_n) = (\partial^p / \partial x_1 \cdots \partial x_n) F_{X_1, \dots, X_n}(x_1, \dots, x_n), (x_1, \dots, x_n) \in \mathbb{R}^n$
 - $\text{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_n) \in \mathbb{R}^n : f_{X_1, \dots, X_n}(x_1, \dots, x_n) > 0\}$
 - Marginal pdf of X_1 :

$$f_{X_i}(x_i) = \int_{\mathbb{R}^{n-1}} f_{\mathbf{X}}(\mathbf{x}) dx_1 \cdots dx_{i-1} dx_{i+1} \cdots dx_n$$

(Mutual) independence

- RVs X_1, \dots, X_n are (mutually) independent \Leftrightarrow

$$F_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n F_{X_i}(x_i)$$

- Joint pmf $p_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n p_{X_i}(x_i)$ (for discrete X_1, \dots, X_n)
 - Joint pdf $f_{X_1, \dots, X_n}(x_1, \dots, x_n) = \prod_{i=1}^n f_{X_i}(x_i)$ (for continuous X_1, \dots, X_n)
- (Optional) X_1, \dots, X_p are (mutually) independent $\Rightarrow M_{X_1, \dots, X_n}(t_1, \dots, t_n) = \prod_{i=1}^n M_{X_i}(t_i)$