

STAT 3690 Lecture 01

zhiyanggeezhou.github.io

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Contact

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Timeline

- Lectures
 - Mon/Wed/Fri 9:30–10:20 via Zoom (tentatively)
- Office Hour
 - (Instructor) Wed/Fri 10:20–11:20 via Zoom (tentatively)
 - (Marker) TBA
- Exam
 - Midterm: (tentatively) Mar. 7, 2022
 - Final project: TBD

Grading

- Assignments (20%)
 - Scanned/photographed and submitted to Crowdmark
 - Attaching both outputs and source codes if R is used in computation
 - Including necessary interpretation
 - Organized in a clear and readable way
 - Accepting NO late submission
- Midterm (30%)
 - Take-home
 - Open-book
 - Time-sensitive
- Final project (50%)
 - Individual report with an analysis of recent dataset(s)
 - To be detailed later

Materials

- Reading list (recommended but not required)
 - R. A. Johnson & D. W. Wichern. (2007). *Applied Multivariate Statistical Analysis*, 5/6th Ed. London: Pearson Education.

- * Textbook, abbr. J&W
 - * 2HR print reserve in the Sciences and Technology Library
- A. C. Rencher & W. F. Christensen. (2012). *Methods of Multivariate Analysis*, 3rd Ed. Hoboken: Wiley.
 - * Electronically accessible via library
- D. Salsburg (2001). *The lady tasting tea: how statistics revolutionized science in the twentieth century*. New York: WH Freeman.
- Lecture notes and beyond
 - zhiyanggeezhou.github.io
 - UM Learn

Outline

- Topics to be covered
 - Multivariate normal distribution
 - Inference on a mean vector
 - Comparisons of several multivariate means
 - Multivariate linear regression
 - Principal component analysis
 - Factor analysis
 - Canonical correlation analysis
 - and so forth

R basics

- Installation
 - download and install BASE *R* from <https://cran.r-project.org>
 - download and install *Rstudio* from <https://www.rstudio.com>
 - download and install packages via *Rstudio*
- Working directory
 - When you ask *R* to open a certain file, it will look in the working directory for this file.
 - When you tell *R* to save a data file or figure, it will save it in the working directory.

```
getwd()
mainDir <- "c:/"
subDir <- "stat3690Lec01"
dir.create(file.path(mainDir, subDir), showWarnings = FALSE)
setwd(file.path(mainDir, subDir))
```

- Packages
 - installation: `install.packages()`
 - loading: `library()`

```
install.packages('nlme')
library(nlme)
```

- Help manual: `help()`, `?`, google, stackoverflow, etc.

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- *R* is free but not cheap
 - Open-source
 - Citing packages
 - NO quality control
 - Requiring statistical sophistication
 - Time-consuming to become a master

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- References for *R*
 - M. L. Rizzo (2019) Statistical Computing with R, 2nd Ed. (forthcoming)
 - O. Jones, R. Maillardet, A. Robinson (2014) Introduction to Scientific Programming and Simulation Using R, 2nd Ed.
 -
 - Courses online
 - <https://www.pluralsight.com/search?q=R>
 -
 - Data types: let `str()` or `class()` tell you
 - numbers (integer, real, or complex)
 - characters (“abc”)
 - logical (TRUE or FALSE)
 - date & time
 - factor (commonly encountered in this course)
 - NA (different from Inf, “ ’”, 0, NaN etc.)

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- Data structures: let `str()` or `class()` tell you
 - vector: an ordered collection of the same data type
 - matrix: two-dimensional collection of the same data type
 - array: more than two dimensional collection of the same data type
 - data frame: collection of vectors of same length but of arbitrary data types
 - list: collection of arbitrary objects

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- Data input and output
 - create
 - * vector: `c()`, `seq()`, `rep()`
 - * matrix: `matrix()`, `cbind()`, `rbind()`
 - * data frame
 - output: `write.table()`, `write.csv()`, `write.xlsx()`
 - import: `read.table()`, `read.csv()`, `read.xlsx()`
 - * header: whether or not assume variable names in first row
 - * `stringsAsFactors`: whether or not convert character string to factors
 - `scan()`: a more general way to input data
 - `save.image()` and `load()`: save and reload workspace
 - `source()`: run R script

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- Parenthesis in *R*
 - parenthesis `()` to enclose inputs for functions
 - square brackets `[]`, `[[]]` for indexing
 - braces `{ }` to enclose forloop or statements such as if or ifelse

```
# Create numeric vectors
v1 = c(1,2,3); v1
v2 = seq(4,6,by=0.5); v2
v3 = c(v1,v2); v3
v4 = rep(pi,5); v4
v5 = rep(v1,2); v5
v6 = rep(v1,each=2); v6
```

```

# Create Character vector
v7 <- c("one", "two", "three"); v7
# Select specific elements
v1[c(1,3)]
v7[2]

# Create matrices
m1 = matrix(-1:4, nrow=2); m1
m2 = matrix(-1:4, nrow=2, byrow=TRUE); m2
m3 = cbind(m1,m2); m3
(m4 = cbind(m1,m2))

# Create a data frame
e <- c(1,2,3,4)
f <- c("red", "white", "black", NA)
g <- c(TRUE,TRUE,TRUE,FALSE)
mydata <- data.frame(e,f,g)
names(mydata) <- c("ID", "Color", "Passed") # name variable
mydata

# Output
write.csv(mydata, file='mydata.csv', row.names=F)

# Import
(simple = read.csv('mydata.csv', header=TRUE, stringsAsFactors=TRUE))
class(simple)
class(simple[[1]])
class(simple[[2]])
class(simple[[3]])
(simple = read.csv('mydata.csv', header=FALSE, stringsAsFactors=FALSE))
class(simple[[3]])

# EXERCISE
# Create a matrix with 2 rows and 6 columns such that it contains the numbers 1,4,7,...,34.
# Make sure the numbers are increasing row-wise; ie, 4 should be in the second column.
# Use the seq() function to generate the numbers. Do NOT type them out by hand!

# ANSWER
matrix(seq(from=1, to=34, by=3), nrow=2)

```

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- Elementary arithmetic operators
 - +, -, *, /, ^
 - log, exp, sin, cos, tan, sqrt
 - FALSE and TRUE becoming 0 and 1, respectively
 - sum(), mean(), median(), min(), max(), var(), sd(), summary()
 - Matrix calculation
 - element-wise multiplication: A * B
 - matrix multiplication: A %*% B
 - singular value decomposition: eigen(A)
 - Loops: for() and while()
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- Probabilities
 - normal distribution: dnorm(), pnorm(), qnorm(), rnorm()

- uniform distribution: `dunif()`, `punif()`, `qunif()`, `runif()`
- multivariate normal distribution: `dmvnorm()`, `rmvnorm()`

```
# Generate two datasets
set.seed(100)
x = rnorm(250, mean=0, sd=1)
y = runif(250, -3, 3)
```

- Basic plots
 - strip chart, histogram, box plot, scatter plot
 - Package `ggplot2` (RECOMMENDED)
-

```
# Strip chart
stripchart(x)

# Histogram
hist(x)

# Box plot
boxplot(x)

# Side-by-side box plot
xy = data.frame(normal=x, uniform=y)
boxplot(xy)

# Scatter Plot with fitted line
plot(x, y, xlab="x", ylab = "y", main = "scatter plot between x and y")
abline(lm(y~x))
```

```
# EXERCISE
# Play with a data set called "Gasoline" included in the package "nlme".
# 1. How many variables are contained in this data set? What are they?
# 2. Generate a histogram of yield and calculate the five number summary for it.
#    What is the shape of the histogram?
# 3. Generate side-by-side boxplots,
#    comparing the temperature at which all the gasoline is vaporized (endpoint) to sample.
#    Does it seem that the temperatures at which all the gasoline is vaporized differ by sample?
# 4. Generate a plot that illustrates the relationship between yield and endpoint.
#    Describe the relationship between these two variables.
# 5. What if the plot created in Q4 were separated by sample?
#    Generate a plot of yield v.s. endpoint, separated by sample.
```

```
# ANSWER
attach(nlme::Gasoline)
# 1. Six variables: yield, endpoint, sample, API, vapor, ASTM
# 2.
summary(yield)
hist(yield, nclass=50)
# 3.
boxplot(endpoint ~ Sample)
anova(lm(endpoint ~ Sample))
```

```
# 4.
plot(x=endpoint, y=yield, xlab="endpoint", ylab = "yield",
     main = "scatter plot between endpoint and yield")
abline(lm(yield~endpoint))
# 5.
par(mfrow=c(2,5))
for (i in 1:10){
  plot(x=endpoint[Sample==i], y=yield[Sample==i], xlab='', ylab='', main=paste('Sample=', i))
  abline(lm(yield[Sample==i]~endpoint[Sample==i]))
}
# Do not forget to detach the dataset after using it.
detach(nlme::Gasoline)
```

Matrix properties

- Determinant and trace
 - Applicable only to square matrices
 - Properties for determinant
 - * $|\mathbf{A}^\top| = |\mathbf{A}|$
 - * $|\mathbf{A}^{-1}| = |\mathbf{A}|^{-1}$
 - * $|c\mathbf{A}| = c^n |\mathbf{A}|$ for $n \times n$ matrix \mathbf{A} and scalar c
 - * $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$ if \mathbf{A} and \mathbf{B} are square matrices of the identical dimension
 - * $|\mathbf{A}| = \prod_i \lambda_i$
 - Properties for trace
 - * $\text{tr}(c\mathbf{A}) = c \text{tr}(\mathbf{A})$ for scalar c
 - * $\text{tr}(\mathbf{A} + \mathbf{B}) = \text{tr}(\mathbf{A}) + \text{tr}(\mathbf{B})$ if \mathbf{A} and \mathbf{B} are square matrices of the identical dimension
 - * $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ for $m \times n$ \mathbf{A} and $n \times m$ \mathbf{B}
 - * $(\text{tr}(\mathbf{AA}^\top))^{1/2} = (\sum_{i,j} a_{ij}^2)^{1/2}$ Frobenius norm (a generalization of Euclidean norm)
 - * $\text{tr}(\mathbf{A}) = \sum_i \lambda_i$
-
- Exercise: Prove that
 1. $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ for $m \times n$ \mathbf{A} and $n \times m$ \mathbf{B} .
 2. $\text{tr}(\mathbf{A}_1 \cdots \mathbf{A}_k) = \text{tr}(\mathbf{A}_{k'+1} \cdots \mathbf{A}_k \mathbf{A}_1 \cdots \mathbf{A}_{k'})$ for $1 < k' < k$.
 3. $\text{tr}(\mathbf{A}) = \sum_i \lambda_i$.
 4. $|\mathbf{A}| = \prod_i \lambda_i$.
 - Hint: Jordan matrix decomposition: there exists a Jordan normal (or canonical) form \mathbf{J} and invertible \mathbf{U} such that $\mathbf{A} = \mathbf{U}\mathbf{J}\mathbf{U}^{-1}$ for any square \mathbf{A} .
 - Remark: $|\mathbf{A}|$ and $\text{tr}(\mathbf{A})$ can be taken as measures of the size of \mathbf{A} when \mathbf{A} is positive definite.
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- Proof:
 1. $\text{tr}(\mathbf{AB}) = \sum_i \sum_j a_{ij} b_{ji} = \sum_j \sum_i b_{ji} a_{ij} = \text{tr}(\mathbf{BA})$.
 2. Take $\mathbf{A}_1 \cdots \mathbf{A}_{k'}$ and $\mathbf{A}_{k'+1} \cdots \mathbf{A}_k$ as a whole, respectively.
 3. $\text{tr}(\mathbf{U}\mathbf{J}\mathbf{U}^{-1}) = \text{tr}(\mathbf{J}\mathbf{U}^{-1}\mathbf{U}) = \text{tr}(\mathbf{J}) = \sum_i \lambda_i$.
 4. $|\mathbf{A}| = |\mathbf{U}\mathbf{J}\mathbf{U}^{-1}| = |\mathbf{U}||\mathbf{J}||\mathbf{U}^{-1}| = |\mathbf{J}|$.