Supplementary Material: Robust Two-Layer Partition Clustering of Sparse Multivariate Functional Data

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This Supplementary Material provides the results of Scenarios 1-6 in the Simulation Studies. First, we present one simulation of all the above scenarios without the outlier and sparseness corruption. Next, we display the performance of clustering for Scenarios 1-6. Then, we show the performance of outlier detection for Scenarios 1-6.

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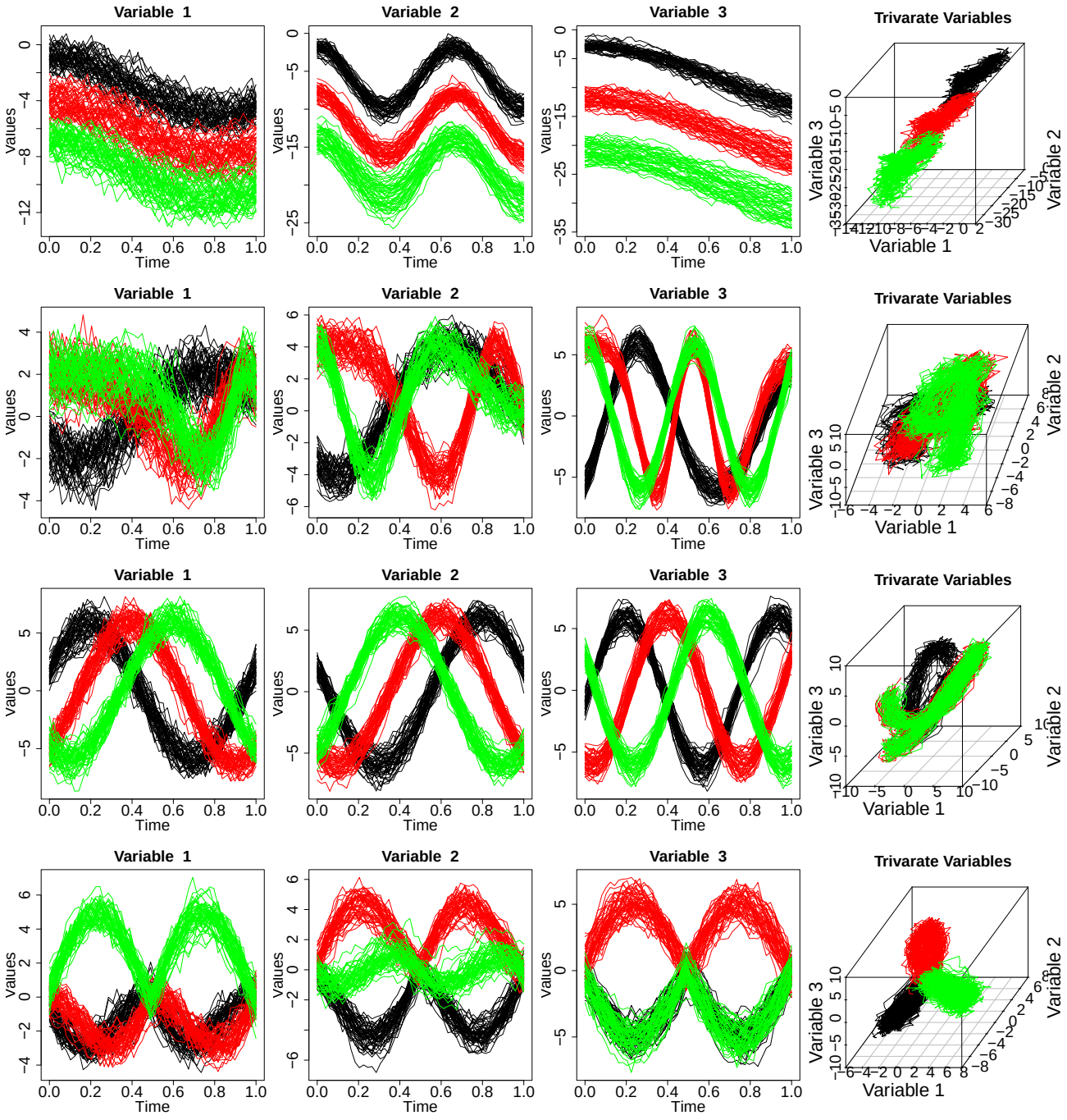
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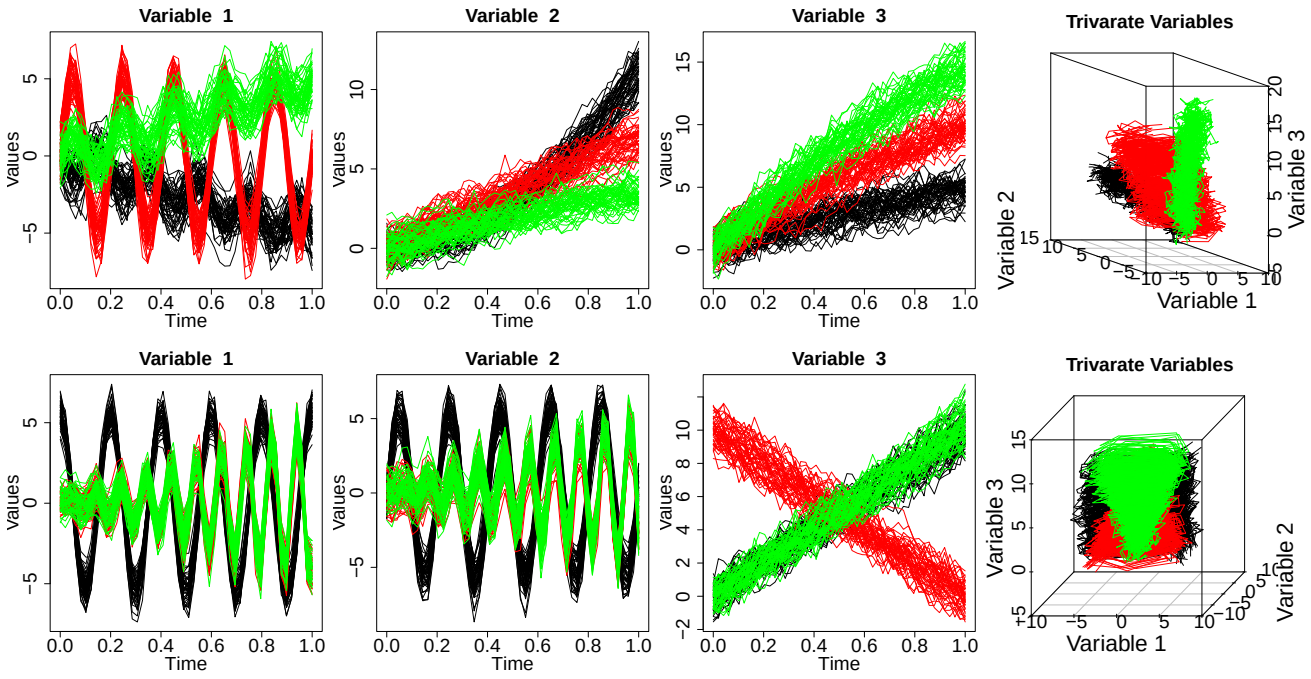

Figure S1: Rows from top to down represent samples from Scenario 1-6, respectively. Three clusters are represented in black, red, and green.

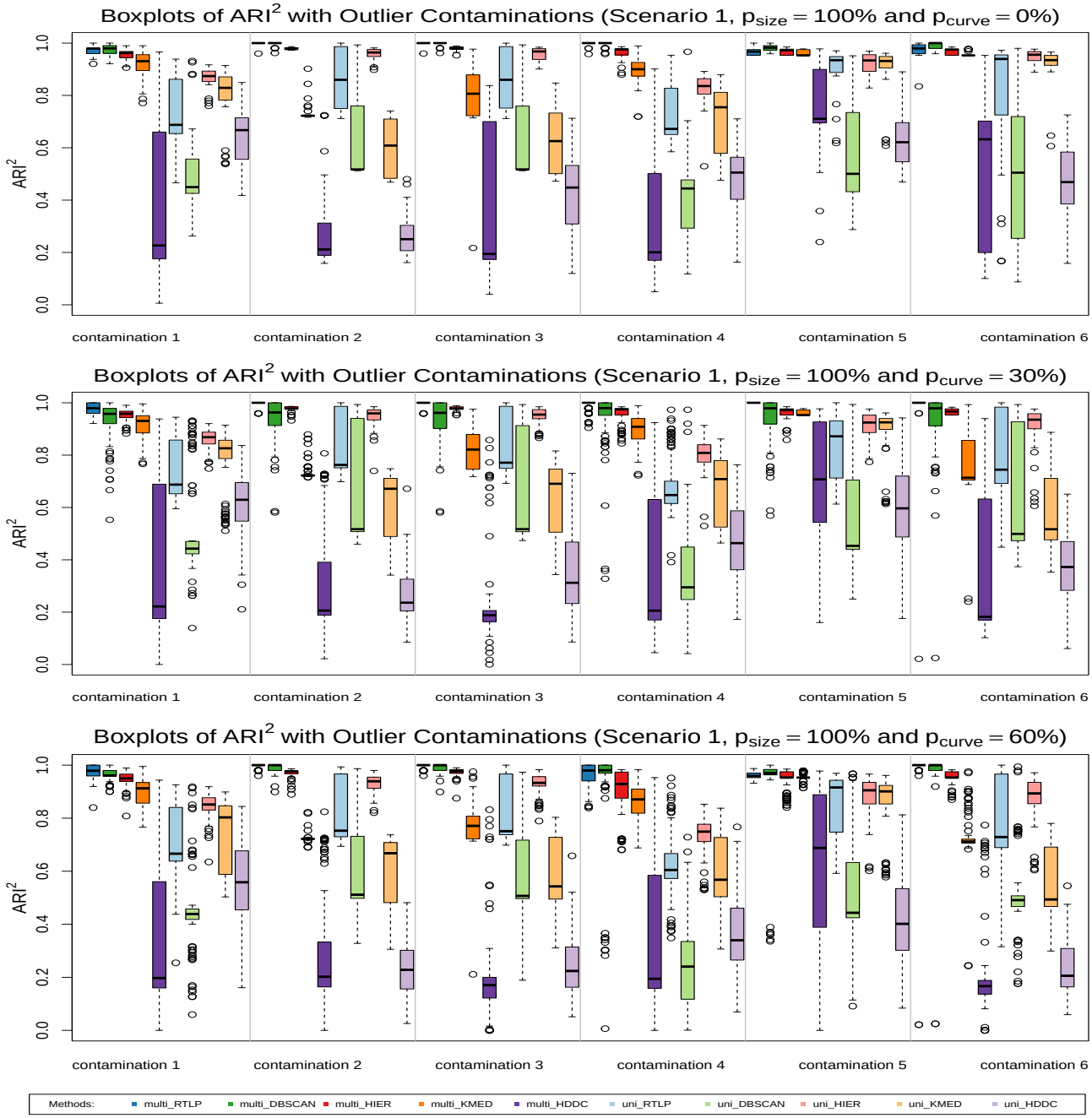

Figure S2: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 1 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

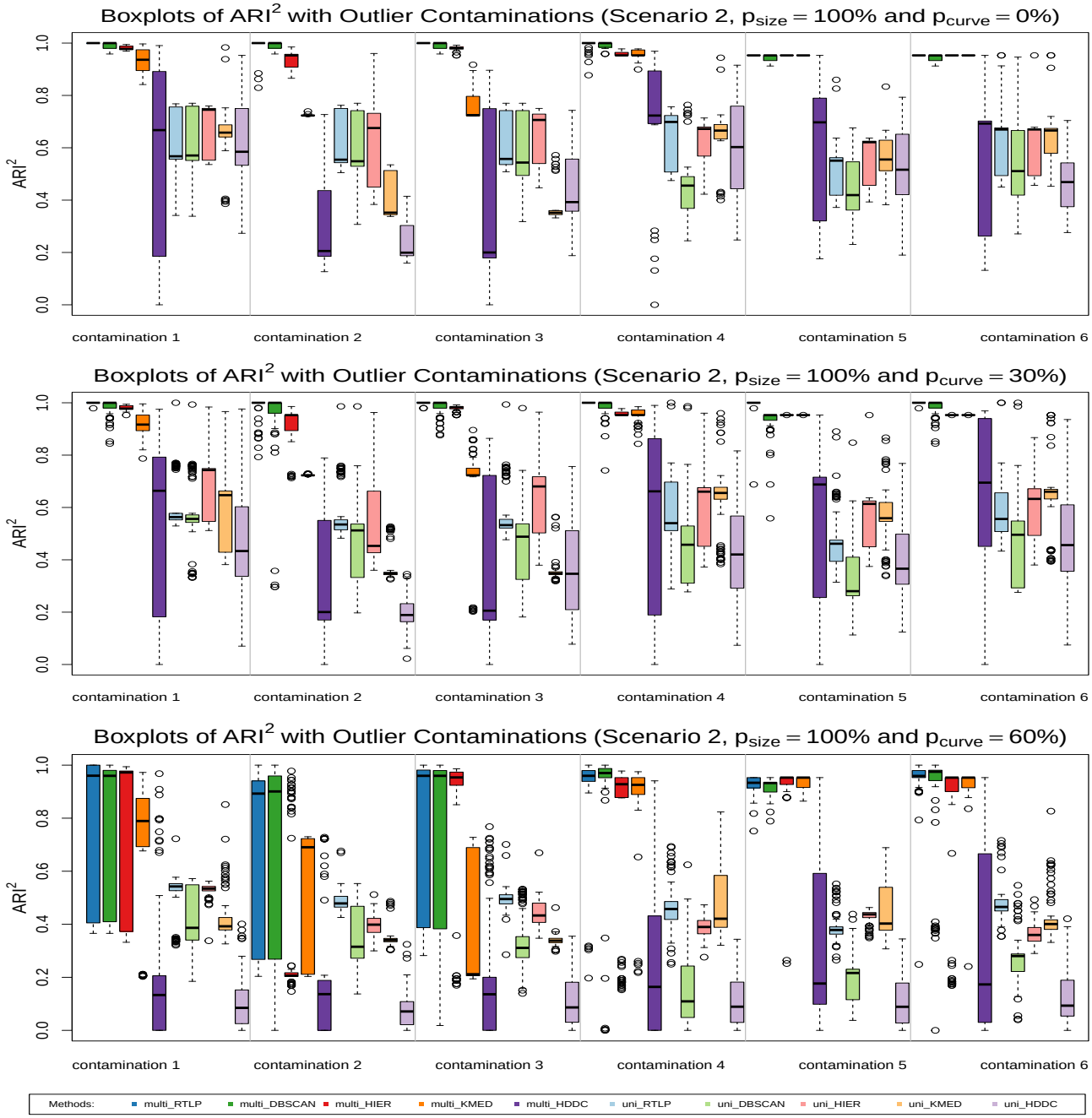

Figure S3: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 2 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

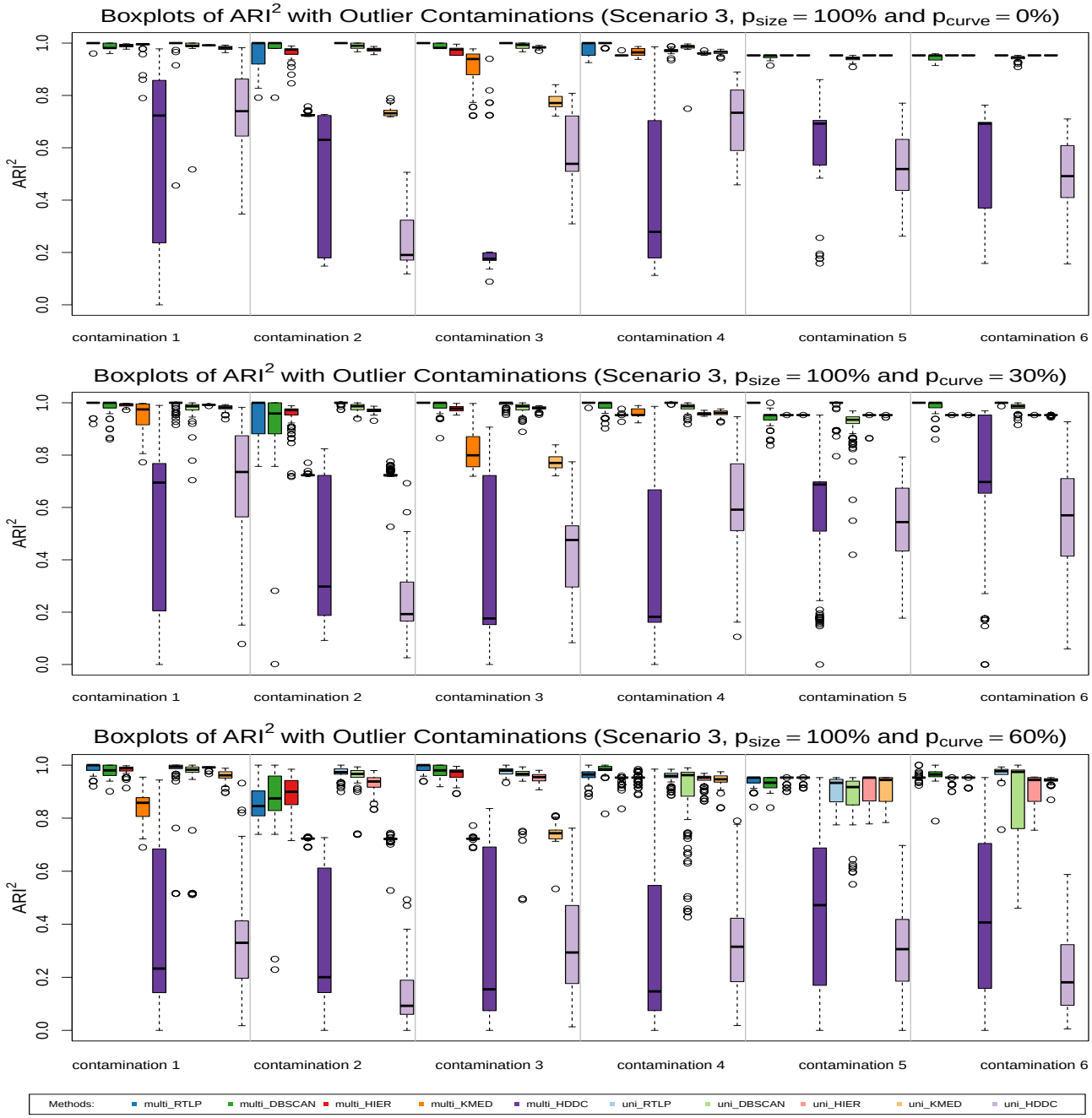

Figure S4: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 3 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

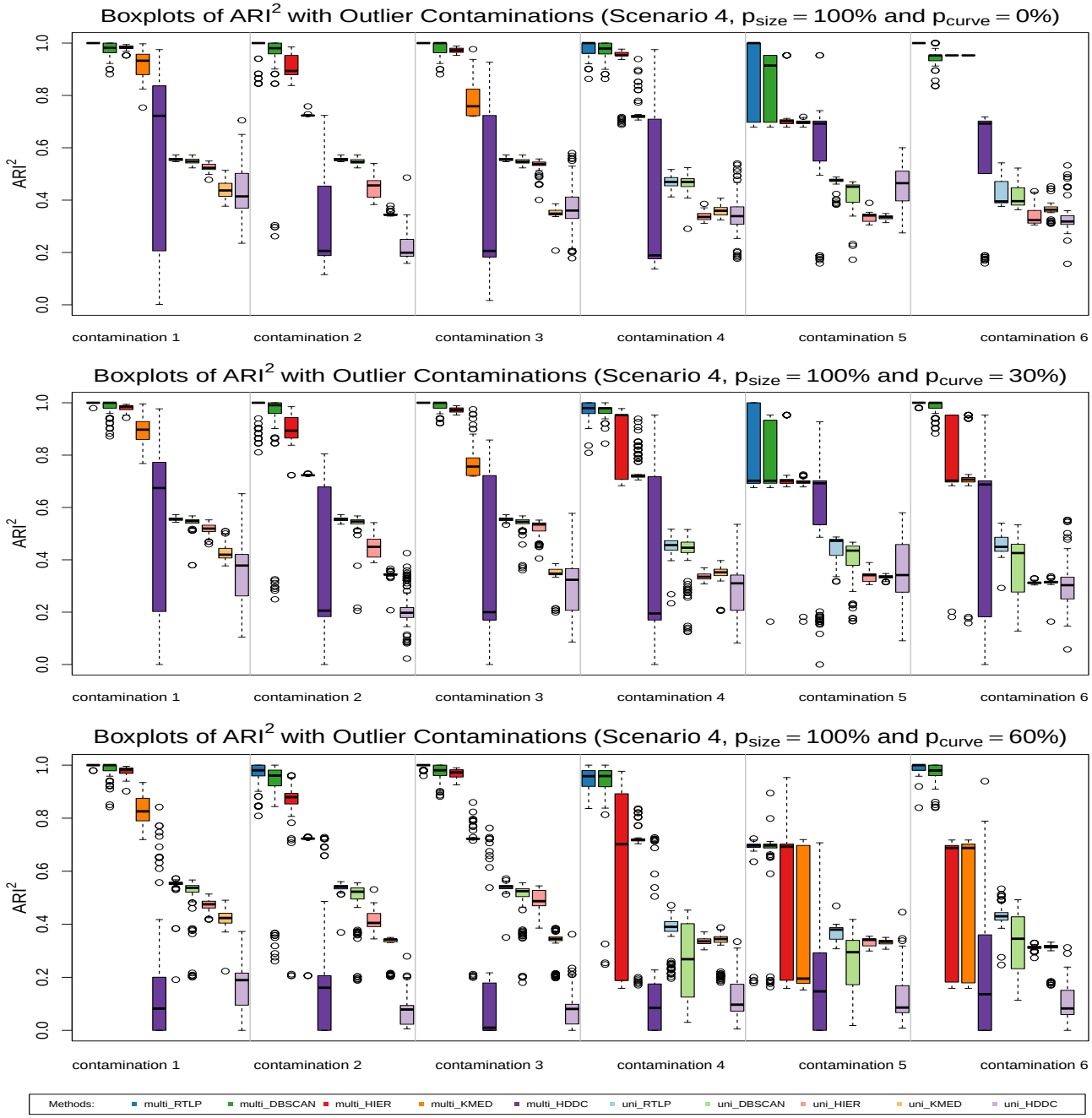

Figure S5: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 4 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

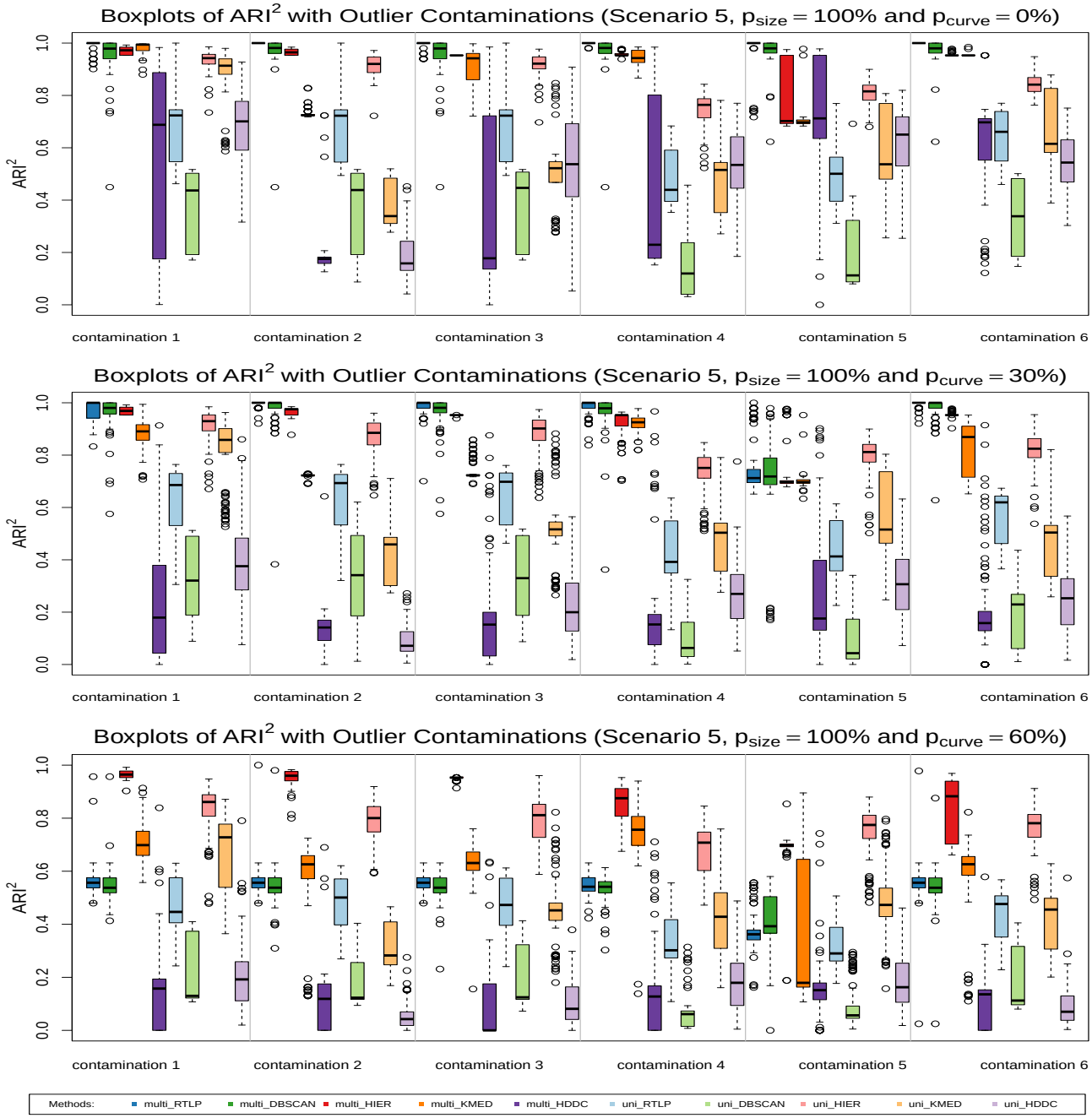

Figure S6: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 5 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

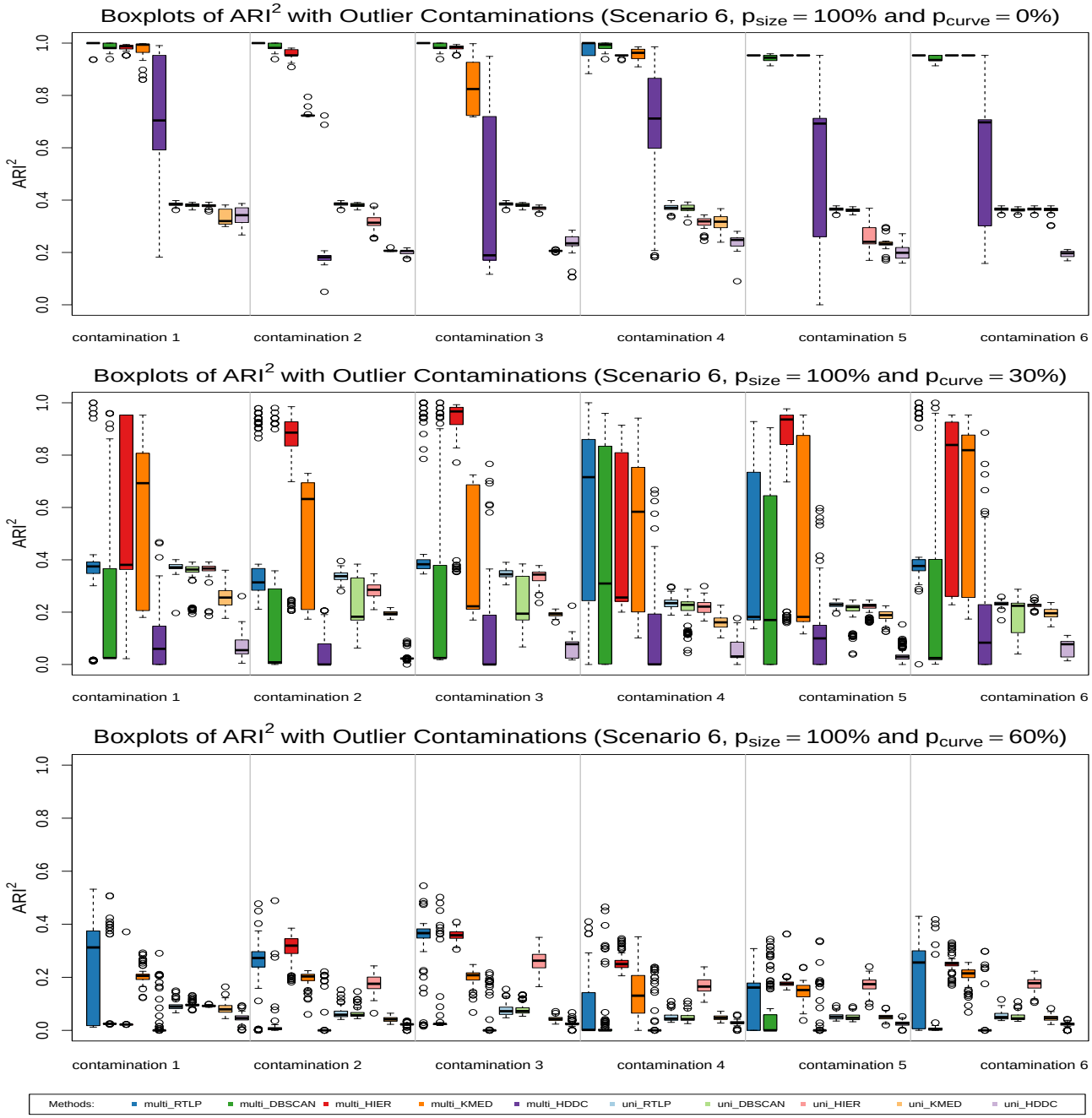

Figure S7: Panels from the top to bottom display the boxplots of ARI^2 in Scenario 6 under $p_{curve} = 0, 30\%$ and 60%. Ten methods are compared in all settings with six contaminations. Here, $K = 3$ and 100 simulation replicates. The methods from left to right are the multivariate and average marginal univariate versions of RTLP, DBSCAN, agglomerative hierarchical, K-medoids, and funHDDC methods.

Table S1: Correct outlier detection percentage p_c (%) and false outlier detection percentage p_f (%) for multivariate and univariate RTLP and DBSCAN with different outlier contaminations for Scenarios 4 and 5. Higher p_c and lower p_f values for each setting given the contamination and p_{curve} are indicated in bold. The proportion of outliers is 10%, and p_{size} is 100% for all settings. Simulations were conducted with 100 replicates.

(a) Scenario 1

p_{curve} \ Methods		Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	92.0 (6.8)	**0.1** (0.3)	99.5 (1.8)	**0.1** (0.2)	**99.7** (1.3)	**0.1** (0.2)
	multi_DBSCAN	**92.4** (6.2)	1.7 (2.3)	**99.7** (1.5)	2.1 (2.9)	**99.7** (1.3)	2.1 (2.9)
	uni_RTLP	63.6 (9.6)	0.2 (0.3)	96.0 (3.2)	0.2 (0.3)	96.1 (3.0)	0.2 (0.4)
	uni_DBSCAN	65.0 (9.1)	0.5 (0.5)	96.2 (3.0)	0.7 (0.9)	96.1 (3.0)	0.7 (0.9)
30%	multi_RTLP	92.3 (6.3)	**0.0** (0.0)	99.8 (1.1)	**0.0** (0.1)	99.8 (1.1)	**0.0** (0.1)
	multi_DBSCAN	**92.4** (6.2)	2.0 (3.5)	**100.0** (0.0)	2.5 (3.7)	**100.0** (0.0)	2.5 (3.7)
	uni_RTLP	65.9 (8.5)	0.2 (0.5)	100.0 (0.3)	0.3 (0.5)	99.5 (1.0)	0.2 (0.5)
	uni_DBSCAN	66.3 (8.3)	0.5 (0.8)	100.0 (0.3)	0.9 (1.0)	99.5 (1.0)	0.8 (1.0)
60%	multi_RTLP	92.0 (6.8)	**0.1** (0.3)	**99.5** (1.8)	**0.1** (0.2)	**99.7** (1.3)	**0.1** (0.2)
	multi_DBSCAN	**92.5** (6.2)	1.1 (2.7)	**99.5** (1.8)	1.3 (2.9)	**99.7** (1.3)	1.4 (2.9)
	uni_RTLP	65.5 (8.6)	0.2 (0.3)	96.0 (3.2)	0.2 (0.3)	96.1 (3.0)	0.3 (0.5)
	uni_DBSCAN	66.3 (8.2)	0.6 (0.6)	96.2 (3.0)	0.9 (1.1)	96.1 (3.0)	0.9 (0.9)

p_{curve} \ Methods		Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	99.2 (2.9)	**0.0** (0.4)	98.4 (11.4)	**0.1** (0.5)	94.1 (12.1)	0.4 (1.2)
	multi_DBSCAN	**99.3** (2.5)	2.0 (2.9)	**100.0** (0.0)	2.4 (3.1)	**100.0** (0.0)	3.0 (3.7)
	uni_RTLP	62.5 (14.3)	0.1 (0.4)	76.3 (14.2)	0.2 (0.5)	84.3 (15.7)	**0.2** (0.3)
	uni_DBSCAN	41.7 (12.3)	0.1 (0.4)	99.1 (14.0)	0.3 (0.8)	90.3 (15.8)	0.3 (0.9)
30%	multi_RTLP	97.3 (6.4)	**0.0** (0.1)	**100.0** (0.0)	**0.0** (0.0)	99.8 (1.1)	**0.0** (0.1)
	multi_DBSCAN	**97.8** (5.0)	1.4 (2.9)	**100.0** (0.0)	2.6 (4.1)	**100.0** (0.0)	2.4 (3.7)
	uni_RTLP	55.4 (13.9)	0.1 (0.4)	76.6 (14.2)	0.2 (0.4)	91.8 (9.4)	0.1 (0.2)
	uni_DBSCAN	63.8 (11.8)	0.2 (0.4)	79.6 (15.0)	0.7 (0.8)	92.7 (8.1)	0.9 (1.0)
60%	multi_RTLP	89.4 (13.5)	**0.0** (0.2)	99.0 (10.0)	**0.1** (0.2)	99.7 (1.3)	**0.1** (0.2)
	multi_DBSCAN	**93.2** (11.4)	0.5 (1.3)	**99.8** (1.0)	1.1 (2.7)	**99.9** (0.7)	1.1 (2.7)
	uni_RTLP	40.2 (16.4)	0.2 (0.5)	67.2 (18.3)	0.3 (0.4)	89.7 (8.9)	0.2 (0.3)
	uni_DBSCAN	49.5 (14.5)	0.1 (0.4)	72.9 (16.5)	0.6 (0.7)	90.4 (8.0)	0.7 (0.8)

(b) Scenario 2

p_{curve} \ Methods		Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**100.0** (0.0)	**0.0** (0.0)	97.0 (12.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.7 (1.2)	**97.1** (11.4)	0.8 (1.3)	**100.0** (0.0)	0.8 (1.3)
	uni_RTLP	**100.0** (0.0)	**0.0** (0.0)	93.8 (3.6)	**0.0** (0.0)	91.7 (6.4)	**0.0** (0.0)
	uni_DBSCAN	**100.0** (0.0)	0.4 (0.4)	93.3 (3.9)	0.3 (0.4)	91.8 (5.9)	0.2 (0.3)
30%	multi_RTLP	**100.0** (0.0)	**0.0** (0.1)	96.0 (13.4)	**0.0** (0.1)	**99.9** (0.7)	**0.0** (0.1)
	multi_DBSCAN	**100.0** (0.0)	0.6 (1.1)	**96.1** (13.0)	0.6 (0.9)	**99.9** (0.7)	0.6 (1.0)
	uni_RTLP	**100.0** (0.0)	0.1 (0.1)	89.5 (6.2)	**0.0** (0.1)	89.1 (6.2)	0.0 (0.2)
	uni_DBSCAN	**100.0** (0.0)	0.4 (0.5)	89.0 (6.5)	0.2 (0.4)	89.2 (5.9)	0.2 (0.3)
60%	multi_RTLP	**100.0** (0.0)	0.8 (1.3)	63.6 (29.1)	0.4 (0.8)	95.2 (8.4)	0.6 (1.1)
	multi_DBSCAN	**100.0** (0.0)	1.4 (2.1)	62.5 (30.6)	0.6 (1.2)	**95.4** (8.0)	1.0 (1.7)
	uni_RTLP	99.9 (0.7)	**0.5** (0.6)	**75.6** (10.3)	**0.4** (0.5)	78.2 (8.6)	**0.5** (0.6)
	uni_DBSCAN	**100.0** (0.0)	1.4 (2.1)	75.3 (10.4)	0.7 (0.8)	78.9 (7.8)	0.6 (0.7)

p_{curve} \ Methods		Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**100.0** (0.0)	**0.0** (0.0)	**100.0** (1.4)	**0.0** (0.4)	**100.0** (0.7)	**0.0** (0.5)
	multi_DBSCAN	**100.0** (0.0)	0.7 (1.2)	0.0 (0.0)	0.9 (1.3)	2.0 (14.3)	0.9 (1.3)
	uni_RTLP	90.2 (9.9)	0.2 (0.4)	54.8 (20.5)	0.1 (0.3)	87.1 (16.4)	0.1 (0.3)
	uni_DBSCAN	98.3 (8.8)	0.2 (0.4)	94.7 (4.8)	0.2 (0.5)	93.8 (9.2)	0.1 (0.4)
30%	multi_RTLP	**100.0** (0.0)	**0.0** (0.0)	**99.0** (10.0)	**0.0** (0.1)	**100.0** (0.0)	**0.0** (0.1)
	multi_DBSCAN	**100.0** (0.0)	0.6 (1.3)	0.0 (0.0)	0.7 (1.9)	**100.0** (0.0)	0.6 (1.1)
	uni_RTLP	87.8 (9.8)	0.2 (0.4)	55.4 (17.2)	0.2 (0.5)	88.3 (15.8)	0.2 (0.5)
	uni_DBSCAN	83.2 (8.1)	0.3 (0.4)	0.0 (0.0)	0.4 (0.6)	72.6 (12.1)	0.4 (0.6)
60%	multi_RTLP	**97.7** (9.3)	**0.9** (1.3)	**92.0** (27.3)	**0.8** (1.3)	**98.6** (10.1)	0.8 (1.3)
	multi_DBSCAN	96.0 (14.7)	1.5 (1.8)	0.0 (0.0)	1.4 (2.1)	96.7 (16.5)	1.4 (2.0)
	uni_RTLP	77.5 (13.8)	**0.9** (1.3)	54.5 (16.4)	0.7 (0.9)	77.9 (15.8)	**0.8** (0.9)
	uni_DBSCAN	74.8 (12.2)	1.1 (2.2)	0.0 (0.0)	1.1 (2.5)	70.9 (12.3)	1.0 (1.0)

(c) Scenario 3

p_{curve}	Methods	Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	99.9 (1.0)	**0.0** (0.1)	86.9 (20.5)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	1.0 (1.7)	86.9 (20.5)	0.5 (1.1)	**100.0** (0.0)	0.9 (1.7)
	uni_RTLP	97.2 (7.3)	**0.0** (0.1)	**100.0** (0.0)	**0.0** (0.0)	99.8 (1.6)	**0.0** (0.0)
	uni_DBSCAN	98.7 (4.4)	0.7 (0.5)	**100.0** (0.0)	0.7 (0.5)	99.8 (1.6)	0.7 (0.5)
30%	multi_RTLP	99.5 (2.7)	**0.0** (0.1)	78.6 (25.2)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.5 (1.0)	78.6 (25.3)	0.2 (0.5)	**100.0** (0.0)	0.4 (0.8)
	uni_RTLP	98.4 (4.8)	**0.0** (0.1)	**99.7** (1.4)	**0.0** (0.0)	98.8 (2.8)	**0.0** (0.0)
	uni_DBSCAN	99.6 (2.5)	0.8 (1.1)	**99.7** (1.4)	0.6 (0.5)	98.8 (2.9)	0.6 (0.7)
60%	multi_RTLP	99.7 (2.0)	0.3 (0.5)	53.4 (27.7)	**0.1** (0.4)	**99.1** (2.6)	0.3 (0.6)
	multi_DBSCAN	**100.0** (0.0)	0.9 (1.2)	53.4 (27.7)	0.2 (0.5)	**99.1** (2.6)	0.9 (1.0)
	uni_RTLP	99.4 (2.1)	**0.2** (0.3)	91.9 (6.5)	0.2 (0.2)	93.8 (4.4)	**0.2** (0.3)
	uni_DBSCAN	**100.0** (0.0)	0.8 (0.7)	**92.2** (6.5)	0.6 (0.6)	94.2 (4.2)	0.7 (0.7)

p_{curve}	Methods	Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.9 (1.7)	24.5 (43.4)	0.7 (1.3)	38.8 (49.2)	0.7 (1.4)
	uni_RTLP	**100.0** (0.0)	**0.0** (0.0)	98.6 (6.7)	**0.0** (0.0)	98.0 (8.1)	0.0 (0.1)
	uni_DBSCAN	**100.0** (0.0)	0.7 (0.5)	12.9 (20.2)	0.9 (0.8)	21.1 (23.3)	0.9 (0.8)
30%	multi_RTLP	**100.0** (0.0)	0.0 (0.1)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.5 (0.8)	5.0 (21.9)	0.5 (0.9)	**100.0** (0.0)	0.4 (0.9)
	uni_RTLP	**100.0** (0.0)	**0.0** (0.0)	93.6 (14.7)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	uni_DBSCAN	**100.0** (0.0)	0.6 (0.6)	3.7 (10.5)	0.6 (0.7)	**100.0** (0.0)	0.6 (0.6)
60%	multi_RTLP	**100.0** (0.0)	0.3 (0.5)	**100.0** (0.0)	0.3 (0.5)	**100.0** (0.0)	0.3 (0.5)
	multi_DBSCAN	**100.0** (0.0)	0.8 (0.8)	0.0 (0.0)	0.8 (1.0)	**100.0** (0.0)	0.8 (1.0)
	uni_RTLP	99.7 (3.3)	**0.2** (0.3)	85.7 (17.9)	**0.1** (0.2)	99.7 (0.8)	**0.2** (0.3)
	uni_DBSCAN	99.9 (0.5)	0.8 (0.7)	1.0 (5.7)	0.7 (0.7)	99.6 (1.4)	0.7 (0.6)

(d) Scenario 4

p_{curve}	Methods	Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**100.0** (0.0)	**0.0** (0.0)	92.4 (17.1)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.8 (1.1)	92.4 (17.1)	0.6 (1.0)	**100.0** (0.0)	0.8 (1.1)
	uni_RTLP	**100.0** (0.0)	0.0 (0.1)	**99.9** (0.4)	0.0 (0.1)	**100.0** (0.0)	0.0 (0.1)
	uni_DBSCAN	**100.0** (0.0)	0.5 (0.4)	**99.9** (0.4)	0.5 (0.4)	**100.0** (0.0)	0.5 (0.4)
30%	multi_RTLP	**100.0** (0.0)	**0.0** (0.1)	93.8 (15.5)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	0.7 (1.1)	93.8 (15.5)	0.5 (0.9)	**100.0** (0.0)	0.5 (0.7)
	uni_RTLP	**100.0** (0.0)	**0.0** (0.1)	99.5 (1.2)	0.0 (0.1)	99.4 (1.3)	0.0 (0.1)
	uni_DBSCAN	**100.0** (0.0)	0.6 (0.6)	**99.5** (1.1)	0.6 (0.6)	99.4 (1.3)	0.6 (0.7)
60%	multi_RTLP	**100.0** (0.0)	**0.0** (0.1)	89.9 (14.4)	**0.0** (0.1)	99.3 (2.0)	**0.0** (0.1)
	multi_DBSCAN	**100.0** (0.0)	0.7 (1.1)	89.9 (14.5)	0.5 (1.1)	**99.4** (1.9)	0.8 (1.1)
	uni_RTLP	99.8 (1.6)	0.1 (0.2)	94.4 (3.2)	0.1 (0.2)	94.4 (3.4)	0.1 (0.2)
	uni_DBSCAN	99.9 (0.7)	1.1 (1.1)	**94.5** (3.2)	0.9 (0.9)	94.9 (3.2)	1.0 (1.0)

p_{curve}	Methods	Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**92.5** (11.3)	**0.0** (0.0)	**63.3** (48.7)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	93.3 (10.6)	0.4 (1.0)	0.0 (0.0)	0.4 (0.7)	18.4 (39.1)	0.8 (1.1)
	uni_RTLP	65.5 (11.2)	0.1 (0.1)	61.8 (11.8)	0.1 (0.1)	44.0 (16.3)	0.0 (0.1)
	uni_DBSCAN	67.9 (9.0)	0.4 (0.4)	4.1 (13.0)	0.5 (0.5)	7.8 (14.4)	0.2 (0.2)
30%	multi_RTLP	91.3 (11.5)	**0.0** (0.0)	30.0 (46.1)	**0.0** (0.1)	**99.9** (0.9)	**0.0** (0.1)
	multi_DBSCAN	**92.9** (8.6)	0.2 (0.5)	0.0 (0.0)	0.2 (0.6)	**99.9** (0.9)	0.6 (1.0)
	uni_RTLP	59.1 (12.4)	0.1 (0.2)	**56.3** (17.4)	0.1 (0.2)	62.6 (12.4)	0.1 (0.1)
	uni_DBSCAN	61.4 (9.6)	0.4 (0.6)	3.0 (9.6)	0.7 (0.7)	64.8 (12.8)	0.4 (0.4)
60%	multi_RTLP	**82.3** (14.5)	**0.0** (0.2)	0.0 (0.0)	**0.1** (0.3)	97.3 (6.8)	**0.0** (0.1)
	multi_DBSCAN	82.3 (16.1)	0.2 (0.5)	0.0 (0.0)	0.3 (0.8)	**97.1** (7.2)	0.7 (1.2)
	uni_RTLP	31.6 (13.2)	0.2 (0.3)	**22.3** (16.1)	0.3 (0.3)	54.3 (11.3)	0.2 (0.2)
	uni_DBSCAN	37.4 (11.3)	0.6 (0.7)	0.3 (3.3)	1.3 (1.5)	53.7 (9.4)	0.8 (1.3)

(e) Scenario 5

p_{curve}	Methods	Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	97.8 (5.0)	0.2 (0.5)	**100.0** (0.0)	**0.0** (0.0)	98.9 (2.5)	**0.2** (0.4)
	multi_DBSCAN	**100.0** (0.0)	2.2 (4.2)	**100.0** (0.0)	1.3 (3.6)	**100.0** (0.0)	2.2 (4.2)
	uni_RTLP	99.9 (1.6)	**0.2** (0.4)	99.3 (3.1)	0.2 (0.3)	**100.0** (0.0)	**0.2** (0.4)
	uni_DBSCAN	**100.0** (0.0)	0.4 (0.6)	99.3 (3.1)	0.4 (0.5)	**100.0** (0.0)	0.4 (0.6)
30%	multi_RTLP	96.4 (6.7)	**0.5** (0.8)	**99.9** (0.9)	**0.1** (0.4)	98.7 (2.7)	**0.4** (1.4)
	multi_DBSCAN	**100.0** (0.0)	1.6 (3.9)	**99.9** (0.9)	0.6 (1.0)	**100.0** (0.0)	1.5 (4.0)
	uni_RTLP	97.8 (4.8)	0.8 (2.0)	95.8 (5.3)	0.5 (1.4)	98.0 (2.4)	0.5 (1.4)
	uni_DBSCAN	99.6 (1.2)	1.2 (2.2)	96.1 (5.3)	0.7 (1.3)	98.2 (2.4)	0.6 (1.2)
60%	multi_RTLP	99.8 (2.0)	32.7 (4.5)	**100.0** (0.0)	33.0 (3.6)	**99.8** (1.1)	33.3 (1.2)
	multi_DBSCAN	**100.0** (0.0)	33.5 (4.1)	**100.0** (0.0)	33.8 (3.9)	**99.8** (1.1)	34.1 (2.2)
	uni_RTLP	98.5 (4.0)	**11.5** (0.9)	95.4 (3.5)	11.4 (1.0)	96.1 (3.5)	**11.3** (1.4)
	uni_DBSCAN	**100.0** (0.2)	11.6 (0.9)	95.5 (3.4)	**11.4** (0.6)	96.2 (3.5)	11.4 (1.2)

p_{curve}	Methods	Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**99.7** (1.3)	**0.0** (0.0)	89.1 (28.7)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**99.7** (1.3)	1.2 (3.6)	**95.6** (14.4)	0.7 (2.3)	**100.0** (0.0)	1.1 (2.4)
	uni_RTLP	37.5 (14.5)	0.2 (0.5)	38.2 (19.9)	0.3 (0.4)	91.7 (14.1)	0.2 (0.4)
	uni_DBSCAN	44.3 (13.3)	0.1 (0.2)	19.3 (14.2)	0.4 (1.0)	68.7 (8.1)	0.5 (0.7)
30%	multi_RTLP	97.7 (5.4)	**0.3** (0.8)	21.0 (34.7)	**0.4** (0.7)	**100.0** (0.0)	**0.2** (0.5)
	multi_DBSCAN	**97.9** (4.7)	1.0 (1.5)	29.1 (35.4)	1.1 (1.8)	**100.0** (0.0)	0.9 (2.3)
	uni_RTLP	36.0 (11.4)	4.1 (4.9)	**31.4** (14.7)	2.2 (3.7)	66.8 (2.3)	0.6 (0.9)
	uni_DBSCAN	39.2 (10.7)	4.8 (4.8)	21.3 (12.2)	3.0 (3.9)	66.7 (0.0)	1.3 (1.7)
60%	multi_RTLP	96.3 (9.0)	33.3 (1.2)	19.2 (31.3)	32.3 (5.4)	**100.0** (0.0)	32.6 (4.8)
	multi_DBSCAN	**97.9** (4.4)	33.9 (2.2)	**40.4** (36.7)	32.4 (5.5)	**100.0** (0.0)	33.4 (4.8)
	uni_RTLP	33.8 (12.7)	11.4 (0.7)	34.0 (16.2)	11.7 (0.8)	71.3 (7.6)	**11.5** (0.7)
	uni_DBSCAN	37.9 (11.4)	**11.2** (0.5)	22.6 (12.8)	**11.6** (1.0)	76.2 (8.2)	11.5 (0.9)

(f) Scenario 6

p_{curve}	Methods	Contamination 1		Contamination 2		Contamination 3	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	98.8 (3.8)	0.1 (0.2)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**100.0** (0.0)	1.3 (1.6)	**100.0** (0.0)	1.1 (1.6)	**100.0** (0.0)	1.1 (1.6)
	uni_RTLP	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	uni_DBSCAN	**100.0** (0.0)	1.0 (0.9)	**100.0** (0.0)	1.0 (0.9)	**100.0** (0.0)	1.0 (0.9)
30%	multi_RTLP	94.5 (9.0)	0.5 (0.7)	61.8 (21.0)	**0.2** (0.5)	**99.1** (2.5)	0.4 (1.2)
	multi_DBSCAN	**100.0** (0.0)	**0.2** (0.7)	57.7 (21.2)	**0.2** (0.5)	99.0 (2.7)	0.5 (1.6)
	uni_RTLP	99.8 (1.7)	0.9 (0.7)	80.7 (8.9)	0.4 (0.5)	85.3 (8.1)	**0.4** (0.4)
	uni_DBSCAN	**100.0** (0.0)	1.5 (0.9)	**81.3** (8.6)	0.7 (0.6)	86.6 (7.2)	0.7 (0.6)
60%	multi_RTLP	91.7 (9.0)	**2.1** (4.8)	49.7 (21.3)	**1.9** (5.5)	95.7 (5.7)	2.5 (6.4)
	multi_DBSCAN	**100.0** (0.0)	3.0 (6.6)	51.5 (22.5)	3.6 (10.4)	**96.9** (4.4)	4.4 (8.8)
	uni_RTLP	75.7 (8.3)	3.1 (3.2)	55.8 (10.2)	2.6 (3.4)	67.8 (11.1)	**2.4** (3.4)
	uni_DBSCAN	99.9 (0.4)	4.8 (4.7)	**59.8** (9.9)	4.2 (5.3)	74.7 (9.4)	4.1 (5.0)

p_{curve}	Methods	Contamination 4		Contamination 5		Contamination 6	
		p_c	p_f	p_c	p_f	p_c	p_f
0%	multi_RTLP	**99.9** (1.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	multi_DBSCAN	**99.9** (1.0)	1.1 (1.6)	59.2 (49.7)	1.0 (1.4)	75.5 (43.4)	1.1 (1.4)
	uni_RTLP	95.7 (4.1)	0.0 (0.0)	**100.0** (0.0)	**0.0** (0.0)	**100.0** (0.0)	**0.0** (0.0)
	uni_DBSCAN	97.4 (2.5)	0.9 (0.9)	34.0 (17.3)	1.0 (0.8)	50.3 (21.6)	1.0 (0.8)
30%	multi_RTLP	42.3 (37.4)	1.4 (1.9)	26.7 (34.1)	3.8 (8.3)	91.1 (16.8)	**0.5** (0.7)
	multi_DBSCAN	**49.6** (30.4)	1.9 (3.3)	0.0 (0.0)	1.3 (2.4)	**91.1** (13.4)	0.7 (1.5)
	uni_RTLP	37.6 (9.5)	**0.8** (0.6)	**34.1** (14.9)	**1.3** (1.0)	80.0 (7.0)	0.7 (0.5)
	uni_DBSCAN	41.8 (6.8)	1.9 (1.2)	31.3 (8.0)	1.6 (1.0)	37.1 (6.3)	1.2 (0.9)
60%	multi_RTLP	16.6 (22.7)	14.2 (9.3)	0.0 (0.0)	**3.4** (5.9)	**100.0** (0.0)	**1.9** (2.4)
	multi_DBSCAN	18.7 (21.5)	16.7 (20.0)	0.0 (0.0)	17.3 (20.4)	50.4 (20.9)	4.3 (12.5)
	uni_RTLP	**33.1** (6.5)	14.9 (3.5)	12.9 (9.4)	15.1 (3.7)	51.1 (4.6)	9.0 (1.9)
	uni_DBSCAN	32.5 (4.8)	**9.0** (5.5)	**33.0** (3.3)	11.6 (12.4)	34.6 (5.9)	9.7 (8.2)