CS 224N: Assignment 5 (2021)

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March 19, 2021

1 Attention exploration (21 points)

(a) Copying in attention

$$k_j^T q \gg k_i^T q, i \neq j$$

(b) An average of two

$$q = t(k_a + k_b), t \gg 0$$

(c) Drawbacks of single-headed attention

i.

$$q = t(u_a + u_b), t \gg 0$$

ii.

we got $k_a \sim \mathcal{N}\left(\mu_a, \alpha I + \frac{1}{2}(\mu_a \mu_a^\top)\right)$, and for vanishingly small α: $k_a \approx \varepsilon_a \mu_a$, $\varepsilon_a \sim \mathcal{N}(1, \frac{1}{2})$, when $q = t(u_a + u_b), t \gg 0$:

$$k_i^T q \approx 0 \text{ for } i \notin \{a, b\}$$
$$k_a^T q \approx \varepsilon_a t$$
$$k_b^T q \approx \varepsilon_b t$$

then:

$$\begin{aligned} c &\approx \frac{\exp(\varepsilon_a t)}{\exp(\varepsilon_a t) + \exp(\varepsilon_b t)} v_a + \frac{\exp(\varepsilon_b t)}{\exp(\varepsilon_a t) + \exp(\varepsilon_b t)} v_b \\ &= \frac{1}{\exp((\varepsilon_b - \varepsilon_a)t) + 1} v_a + \frac{1}{\exp((\varepsilon_a - \varepsilon_b)t) + 1} v_b \end{aligned}$$

since $\varepsilon_a, \varepsilon_b \sim \mathcal{N}(1, \frac{1}{2})$, when $\varepsilon_a > \varepsilon_b$, c will be closer to v_a, vice versa. (ie. c will be closer to those with larger $\|k\|$)

(d) Benefits of multi-headed attention

i.

$$q_a = t_1 \mu_a, t_1 \gg 0$$
$$q_b = t_2 \mu_b, t_2 \gg 0$$

ii.

$$k_a^T q = \varepsilon_a t_1$$
$$k_b^T q = \varepsilon_b t_2$$

then:

$$c_1 \approx v_a, c_2 \approx v_b$$
$$c = \frac{1}{2}(c_1 + c_2) \approx \frac{1}{2}(v_a + v_b)$$

(e) Key-Query-Value self-attention in neural networks

i.

$$c_2 \approx u_a$$

It's impossible for c_2 to approximate u_b by adding either u_d or u_c to x_2. Say, if we add u_d, α_{21} increases, which means the weight of x_1 increases, but u_d and u_b will increase equally in c_2, that's why c_2 can never be approximated to u_b.

ii.

$$\begin{aligned} V &= u_b u_b^T \odot \frac{1}{\|u_b\|_2^2} - u_c u_c^T \odot \frac{1}{\|u_c\|_2^2} \\ &= \left(u_b u_b^T - u_c u_c^T\right) \odot \frac{1}{\beta^2} \\ K &= I \\ Q &= u_d u_a^T \odot \frac{1}{\|u_a\|_2^2} + u_c u_d^T \odot \frac{1}{\|u_d\|_2^2} \\ &= \left(u_d u_a^T + u_c u_d^T\right) \odot \frac{1}{\beta^2} \end{aligned}$$

Proof:

$$v_1 = u_b, v_2 = 0, v_3 = u_b - u_c$$
$$q_1 = u_c, q_2 = u_d, q_3 = 0$$
$$k_i = x_i, i \in \{1, 2, 3\}$$

so,

$$\alpha_1 \approx [0, 0, 1], \alpha_2 \approx [1, 0, 0]$$
$$c_1 \approx v_3 = u_b - u_c, c_2 \approx v_1 = u_b$$

2 Pretrained Transformer models and knowledge access (35 points)

(a)

3 Considerations in pretrained knowledge (5 points)