

Analysing the Accuracy of CMC Predictions using Deep Learning

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Abstract

This paper presents a novel approach to predicting critical micelle concentrations (CMCs) using graph neural networks (GNNs) augmented with Gaussian processes (GPs). The proposed model uses learned latent space representations of molecules to predict CMCs and estimate uncertainties. The performance of the model on a dataset containing nonionic, cationic, anionic and zwitterionic molecules is compared against a linear model that works with extended-connectivity fingerprints (ECFPs). The GNN-based model performs slightly better than the linear ECFP model, when there is enough well-balanced training data, and achieves predictive accuracy that is comparable to published models that were evaluated on a smaller range of surfactant chemistries. We illustrate the applicability domain of our model using a molecular cartogram to visualise the latent space, which helps identify molecules for which predictions are likely to be erroneous. The proposed approach can provide valuable insights into the molecular properties that influence CMCs.

Introduction

The critical micelle concentration (CMC) of a surfactant defines the concentration above which the surfactant monomers self-assemble in solution to form micelles. The formation of micelles affects many interfacial phenomena.¹ However, predicting the CMC for an arbitrary chemical structure remains challenging.

Perhaps the most well-established predictor for CMC, X_{cmc} , is the Stauff-Klevens relationship, first published in 1953.² It formalised the observation that CMC decreases exponentially with an increase in the number of carbons in the hydrocarbon tail, n_c :

$$\log X_{cmc} = A - Bn_c, \quad B > 0 \tag{1}$$

where A and B are empirical constants that depend on the temperature and the homologous series, i.e. the headgroup. The model is simple and easily interpretable: to reduce CMC, it is sufficient to extend the surfactant's hydrocarbon tail thus defining an easy-to-apply qualitative heuristic. Its drawback as a predictive model is its very limited applicability domain; each set of parameters is only applicable to surfactants with a specific headgroup and a linear carbon tail.

There has been a wealth of investigations into making more general models for CMC prediction; here, a brief review of some diverse and promising approaches for predicting CMCs of aqueous, single-surfactant systems will be given.

Puvvada and Blankschtein³ derived a phenomenological model for studying aqueous nonionic surfactant systems that enabled prediction of CMC and other properties across a range of temperatures. They developed a model for the free energy of micellisation, from which the CMC can be calculated. Their model was parameterised by three properties of the surfactant molecule:

- The tail length, defined as the number of carbon atoms.
- The average cross-sectional area of the headgroup, which controls the steric contribu-

tion to the free energy. This must be estimated.

- The Tolman length of the tail, which approximates the thickness of an ‘interaction region’ around the tail.⁴ This must also be estimated.

The authors developed a method to estimate parameters for linear, nonionic, polyoxyethylene alcohol surfactants. The model attained a root-mean-squared error (RMSE) of approximately $0.14 \log \mu\text{M}$ for the group C_{10}E_i , where $i \in (3, 6)$, and $0.21 \log \mu\text{M}$ for the group C_{12}E_j , where $j \in (3, 8)$. However, the error is much larger for other systems, like C_8E_6 . The authors ascribed this inaccuracy to their prediction that the micelles in these systems would not exhibit much anisotropic growth, i.e. that they are mostly spherical; their model is better suited for cylindrical or disk-like (bilayer) micelles.

The connection this model established between a small set of physically meaningful properties that can be estimated and emergent properties of surfactants is extremely useful, especially because it does not explicitly require fitting to any experimental data. However, the procedures described for estimating the Tolman length are only applicable to linear hydrocarbon chains, not branched nor heterogeneous tail groups. Estimating the average cross-sectional area of the head group may also not be trivial.

Greater generalisability and better accuracy can be realised by considering the process of aggregation by identifying the behaviour of individual atoms, or small groups thereof. Simulation approaches can model the interaction of these units with each other and derive the potential energy of a configuration.^{5–7} For example, molecular dynamics (MD) simulations treat individual atoms, in the all-atomistic (AA) approach, or groups of atoms, in the coarse-grained (CG) approach, as particles in a box that exert force on each other. This allows the particles’ movement to be simulated by integrating the equations of motion.

Jorge⁸ used two approaches to simulate the self-assembly of *n*-decyltrimethylammonium bromide: an AA approach, and a united-atom approach, which groups hydrogens with their respective connecting atom. They then estimated the CMC by considering the concentration of ‘free’ surfactants, i.e. surfactants that were not in micelles, which they defined as an

aggregate containing five or more surfactants. The united-atom approach yielded a CMC with an error of $0.23 \log \mu\text{M}$. The author asserted that the simulation size, i.e. the number of particles, was too small to obtain a more accurate value of CMC. However, Jusufi and Panagiotopoulos⁹ criticised the free surfactant concentration approach for modelling CMC of ionic surfactants in general. They note that free surfactant concentration above the CMC is highly dependent on the choice of overall surfactant concentration, especially for ionic surfactants, which necessitates careful extrapolation in order to accurately determine CMCs.

The computational cost from increasing the system size can be mitigated using coarse-graining, which groups atoms into beads. This also makes simulating longer time scales accessible.¹⁰ Vishnyakov et al.¹¹ used dissipative particle dynamics (DPD) to model the CMCs of C_8E_8 , dodecyldimethylamineoxide (DDAO) and *N*-decanoyl-*N*-methyl-D-glucamide (MEGA-10). They proposed a methodology for obtaining parameters to describe the bead interactions based on known infinite dilution activity coefficients, γ_∞ . Their calculated CMC for C_8E_8 had an error relative to the average experimental value equal to $0.07 \log \mu\text{M}$, for DDAO it was $0.06 \log \mu\text{M}$, and for MEGA-10 it was $0.06 \log \mu\text{M}$.

Although this approach can produce extremely accurate results, it is limited to compounds that can be parameterised by a known γ_∞ . Furthermore, the authors do not discuss parameterising ionic surfactants, for which more complex interactions must be considered. Despite the reduced cost relative to AA-MD models of the same system scales, CG-MD approaches are still one of the more expensive approaches to CMC prediction. Nevertheless, MD simulations give very deep insight into the processes occurring at a molecular level; their results yield a precise description of the arrangement of molecules in a system, their shape and aggregation number.

Another approach is the conductor-like screening model (COSMO), which decomposes the problem by treating a molecule as a cavity with a charged surface in a solvent that acts as a dielectric continuum.¹² The cavity’s surface is described by the solvent-accessible surface

of the molecule. The geometry of this surface combined with a segment-wise description of its polarising charges can be mapped using density functional theory (DFT).

COSMO for realistic solvation (COSMO-RS) adapts the model for more complex types of solvent.¹³ Solvents only act like a dielectric continuum when they are capable of perfectly screening the COSMO-surface of a solute, i.e. every point on the molecule’s COSMO surface is matched by a point of opposite polarity due to the configuration of the solvent molecules. COSMO-RS uses statistical mechanics to determine the probability distribution describing how surface charge densities align between two molecules. This allows chemical potentials to be determined.

Turchi et al.¹⁴ used COSMO-RS to predict CMCs by treating a micelle as a separate phase and then considering the two-phase equilibrium between the micelle and an aqueous phase containing free surfactants. They modelled the micellar ‘phase’ using two strategies. Their first strategy was to treat the micellar phase as being equivalent to a bulk, homogeneous phase of surfactant. The CMC could then be determined by the equilibrium surfactant concentration in the aqueous phase. The authors argued that this approximation is more valid as the difference in polarity between the head and tail of a surfactant is reduced, which was the case for the majority of nonionic surfactants they considered.

Their second strategy was to consider the micellar phase as a bulk, homogeneous phase of an oil, whose chemistry was analogous to that of the surfactant’s tail group. They then implemented an iterative procedure to calculate the interfacial tension (IFT) between the oil and aqueous surfactant phases at different concentrations of surfactant. The concentration at which the IFT is zero yields the CMC prediction. The premise of this approach is that the oil phase is representative of a micelle’s interior, which is particularly true as the interactions between head and tail groups become more unfavourable. This is the case for highly polar head groups: primarily for ionic surfactants.

The authors recommended applying both strategies and using the lower result as the CMC prediction. They attained an RMSE of $0.81 \log \mu\text{M}$ on a dataset of 24 surfactants, containing

a mix of ionic, nonionic and zwitterionic surfactants. Excluding the two worst predictions gives a much more favourable RMSE of $0.55 \log \mu\text{M}$. It is notable that the technique can be applied across all classes of surfactants. Furthermore, the authors were able to determine which of the nonionic surfactants had a low propensity to form micelles by analysing the predicted IFT between the water and surfactant phases. If the predicted IFT was large, this implied that the free energy of micellisation would be large and therefore that the surfactants were less likely to micellise. This is a testament to the model’s interpretability.

Other approaches have extended COSMO-RS to explicitly account for the internal structure of micelles, such as COSMOmic, which treats a micelle as being made of concentric layers that each have their own surface charge profiles.¹⁵ To compute these charge profiles, the layer’s composition with respect to individual atoms of the surfactant must be known. The combination of layer-wise atomic distributions and the COSMO-surface associated with each atom gives the layer’s surface charge density profile. COSMOmic considers how the COSMO-surface of a surfactant would intersect with the layers’ surface charge profiles when it is randomly positioned and oriented within the micelle, in order to determine the partition coefficient of inserting a surfactant molecule into the micelle.

Jakobtorweihen et al.¹⁶ calculated CMCs using COSMOmic by first performing MD simulations to attain the layer-wise atomic distributions. The authors then predicted the CMCs of several polyoxyethylene alcohols by determining the partition coefficients of inserting the respective surfactant monomer into a micelle.

Although it was found that the choice of conformer used to describe the micellar layer-wise charge density profiles has a negligible effect on the results,¹⁷ the conformer used for the partitioning surfactant is important. With the best choice of conformer, the authors achieved an RMSE of $\sim 0.36 \log \mu\text{M}$ on predictions for C_iE_6 surfactants, where $i \in \{6, 8, 10, 12, 14, 16\}$. However, when considering surfactants with fixed tail length but varying head group size, C_{10}E_j , $j \in 4, 6, 8$, the authors could not identify a consistent conformer selection that would not yield significant outliers.

COSMOplex is a recent extension of COSMOmic that removes the need to perform and initial MD simulation to determine the micellar structure.¹⁸ Instead, it optimises the micellar structure using a self-consistent approach, which iteratively yields new estimates for the layer-wise charge distributions. The authors predicted the CMCs of 10 nonionic surfactants with varied head and tail group chemistries, achieving an RMSE of 0.86 log μM ; however, they noted that these results are preliminary and stand to be improved.

Although COSMO techniques are promising, Herbert¹⁹ note that many modern extensions are only available in the proprietary software package COSMOTHERM.²⁰

Another approach to predict CMCs is to use an equation of state. The reduction in complexity can be achieved using a semi-empirical method that is parameterised by fitting to experimental data.

Li et al.²¹ applied a segment-based UNIQUAC model (s-UNIQUAC) and a SAFT equation of state to predict CMCs of linear polyoxyethylene alcohols by first deriving expressions for the activity coefficient of a surfactant in water. In the s-UNIQUAC model, a segment-based local-composition model was used, and the fugacity could then be approximated using the fitted interaction energies between the segments and water. The segments used were C_2H_4 and $\text{C}_2\text{H}_4\text{O}$. In the SAFT approach, the surfactant was treated as a chain of soft-sphere segments in order to first derive the free energy of the solution and, from that, the fugacity.

Cheng and Chen²² compared the performance of these models on a larger dataset alongside three other models. Two of these were segment-based models: the polymer-NRTL model²¹ and a UNIFAC model.²³ The authors also employed their own modified Aranovich and Donohue (m-AD) model. The m-AD model calculates the CMC as a mole fraction, x_S^L , approximating it as the reciprocal of the limiting value of the surfactant’s activity coefficient in an aqueous solution, $\gamma_S^{L,\infty}$:

$$x_S^L = \frac{1}{\gamma_S^{L,\infty}} \tag{2}$$

The m-AD model considers the exchange equilibrium on a three-dimensional lattice of infinitely separated solvent and solute molecules in order to determine $\gamma_S^{L,\infty}$. Notably, the m-AD model is not a segment-based model; instead, the authors fitted an interchange energy, Δ , separately for each molecule. Correlations were examined between Δ and the readily calculated Kier-Hall zero-order index (KH0) of the tail groups, which indicates and the total molecular energy of the surfactant.

Where data from the literature was available, the predictive performance of the equation of state and the m-AD models on the molecular series C_nE_6 , C_nE_8 , C_nE_9 , $C_{10}E_n$ and $C_{12}E_n$ were compared, and the resulting RMSEs are summarised in Table 1. The models all have a reasonably good accuracy, but the SAFT model is particularly good.

Table 1: Comparison of the RMSEs of selected models on polyoxyethylene alcohols’ CMCs. Data from Cheng and Chen²².

Model	RMSE (log μ M)
p-NRTL	0.18
s-UNIQUAC	0.14
SAFT	0.06
UNIFAC	0.14
m-AD	0.11

Segment-based semi-empirical methods are very promising for predicting CMCs within a class of surfactants. Their major drawback is that they are only applicable to molecules that can be decomposed into segments that have trained parameters. In addition, they must respect the limitations of the theories they are based upon.

Finally, purely empirical methods have a very heavy reliance on data abundance. Empirical methods offer a way of making predictions even when a unified theory is lacking or computationally too demanding. However, without an underlying theory, the limitations of the model are not well defined and it is possible for the model to ‘learn’ trends that contradict scientific intuition.

Empirical QSPR methods require validation to determine their reliability and applicabil-

ity domain;²⁴⁻²⁶ the performance metrics during optimisation are not a reliable indicator of generality, or the performance on new molecules. This is often achieved by partitioning the available data into *training* and *test* subsets; the former is used for optimising the model’s parameters, the latter is ‘hidden’ from the model until training is complete, and the prediction metrics on the test data indicate how the model can be expected to perform in general. The test set should span the chemical space in which the model is intended to be applied.²⁶ At a high level, this means that all classes of surfactant that are within the applicability domain of the model should be represented in both the test and train data subsets.

Empirical QSPR models are general enough to be applicable to a diverse range of compounds; they can be used to design novel molecules with target properties^{27,28} and are interpretable,²⁹ meaning that they can be analysed to obtain chemical insights.

Every empirical QSPR method is characterised by two features: the choice of molecular descriptors, which are the numerical features that make up the model inputs, and the functional form of the approximator that maps the descriptors to the property prediction. This functional form includes the parameters that are optimised based upon the training data.

Mattei et al.³⁰ extended the Marrero and Gani group-contribution method³¹ to predict the CMCs of 150 nonionic surfactants. In this approach, each segment is treated as an independent *group* that contributes to the prediction in an additive, linear fashion.

The descriptors are the number of each group present in a molecule. In the original method,³¹ different ‘orders’ of groups were identified; the first-order groups are forbidden from overlapping with one another and they are formulated so that any molecule of interest can be described using these groups exclusively. Higher order groups distinguish polyfunctional molecules and isomers.³¹

Mattei et al.³⁰ introduced third-order groups to improve their model’s accuracy by analysing the molecules with the highest prediction errors after training an initial model with the first- and second-order groups from a prior work.³¹ This is an example of *feature selection*, whereby the set of descriptors is expanded or contracted to adapt to the prob-

lem.^{32,33}

The authors randomly selected 30 compounds as a test dataset, achieving a RMSE of $0.13 \log \mu\text{M}$. The model is remarkably accurate and boasts high interpretability: the fitted contributions of each group describe their effect on the CMC quantitatively, and the existence of higher order polyfunctional groups with large contributions implies that their constituent functional groups have a significant interaction with each other that affects the CMC. However, it may be difficult to determine whether a new molecule is within the applicability domain of the model, particularly because positional isomers are not necessarily distinguished from each other using the group representation.

Recently, an approach based on graph neural networks (GNNs) has produced highly accurate predictions whilst being applicable to nonionic, cationic, anionic and zwitterionic surfactants.³⁴ Neural networks have many trainable parameters and a complex functional form. This ensures their versatility as universal approximators but makes them highly susceptible to overfitting.³⁵ Neural networks potentially boast the largest applicability domain (for a single set of trained parameters) of any model discussed previously.

GNN approaches, like the one of Qin et al.³⁴, operate on molecular graphs, which are characterised by atomic nodes whose edges represent bonds. Each operation on this graph considers just the local environment of an atom, i.e. the atoms that can be reached by traversing a single bond, but by stacking these operations in sequence, the size of the environment that is considered increases. In this sense the model is similar to a group contribution approach, although the groups are determined by walking r steps along bonds from every atom in the molecule, where r is equal to the number of subsequent graph operations, so that every group overlaps. Furthermore, each ‘contribution’ is non-linear, and the number of contributions is always equal to the number of atoms in a molecule. A more detailed discussion of GNNs will be given in the Method.

Here, we build upon this previously published GNN model³⁴ in two ways: we apply a hyperparameter search algorithm to further optimise the model’s architecture and improve

its accuracy, and we implement an *uncertainty quantification* technique that yields confidence intervals alongside CMC predictions. This improved model is compared against an adaptation of the group contribution approach that determines the entire set of groups to consider using a feature selection routine. In addition, a separate dataset is introduced so as to perform external validation and to probe the limits of the models’ applicability domain, as well as to analyse the uncertainty quantification. Using the outcome of our approaches, we interpret both models, and demonstrate a technique that allows one to visualise chemical space through the ‘eyes’ of the trained GNN. We show that this technique facilitates a better understanding of the applicability domain, which is essential to assess the accuracy of CMC predictions via neural networks.

Method

Two datasets were used for training and testing:

The Qin dataset is a dataset of 202 surfactants curated by Qin et al.³⁴. To the authors’ knowledge, it is currently the largest public dataset of CMCs for several classes of surfactant collected at standard conditions in an aqueous environment between 20 °C to 25 °C. In this work, the data was further subdivided into two tasks: Qin-All, the entirety of the dataset, and Qin-Nonionics, which contains only the nonionic surfactants.

The Qin data were split into training and test subsets; the training data were used to fit the models, whilst test data were ‘locked away’. The performance metrics on the test data were used for evaluation of the models derived here. For some models, the training data were further split into optimisation and validation subsets; the optimisation data were used when calculating the loss function during model fitting. The validation data were used for on-the-fly evaluation of model performance during training, to determine how many iterations of the optimisation routine to use.

To provide a consistent benchmark of model performance with the previous work,³⁴ the same train/test data splits were used as Qin et al.³⁴. This will be referred to as the benchmarking test.

After training these models, a sensitivity analysis was carried out. The data for each task were split using a repeated, stratified k -fold cross-validation. Each molecule was assigned a class based upon its molecular fingerprint, which will be described below, in the section on Extended connectivity fingerprints. The data were then split into k folds of roughly equal size, and containing approximately the same percentage of molecules of each class, for $k \in (2, 5)$. For each fold, a model was trained using $k - 1$ folds and evaluated on the remaining fold. This was repeated thrice for $k = 2$ and twice for $k = 3$, each time with different randomisation, and there were no repetitions for $k = 4$ and $k = 5$, so that there were a similar number of models trained for each value of k .

The National Institute of Science and Technology (NIST) dataset contains 43 unique surfactant systems and their aqueous CMCs, extracted from the work of Mukerjee and Mysels³⁶. The original document compiles CMC measurements for each system across several temperatures and with various additives. Each measurement was given a quality rating depending on the authors’ assessment of the experimental method; measurements with a sufficiently good rating were categorised as ‘suggested’ measurements. The dataset used for this work was developed by taking the mean of the suggested CMC measurements between 20 °C to 25 °C, with no additives. The surfactant systems that contained elements that were not present in the dataset (Mn, Cs and Mg) were then pruned. The dataset is provided in the Supplementary Information.

The NIST data were used exclusively for external validation. Several of the surfactant systems in the dataset are not expected to be within the applicability domain of the models derived here. This includes surfactants with counterions and combinations of

functional groups that are not present in the training data. These data are included to test the robustness of the uncertainty quantification approach.

The number of each type of surfactant in the train and test subsets of the data are shown in Table 2. Only the models trained on the Qin-All dataset were evaluated on the NIST dataset, as it contained ionic compounds.

Table 2: The number of each type of surfactant contained in the train/test subsets of the CMC datasets. Missing entries mean no samples of that class.

Data subset		Number of			
Task	Train/test	Nonionics	Anionics	Cationics	Zwitterionics
Qin-All	Train	110	30	31	9
	Test	12	4	4	2
Qin-Nonionics	Train	110			
	Test	12			
NIST	Train				
	Test	12	23	6	2

The QSPR pipeline needs molecular descriptors and a functional form. The design processes for these are called *feature engineering* and *model selection*, respectively.

Feature engineering

In this work, two types of molecular descriptors are employed: extended connectivity fingerprints (ECFPs)³⁷ and molecular graphs.

Extended connectivity fingerprints (ECFPs)

In the ECFP approach, the molecule is split into atomic environments up to a given radius, r : each environment is centred on an atom and extends r steps along connecting bonds. The set of all environments in the training data up to radius r is extracted. The resulting feature vector, \vec{c} for a molecule has elements

$$c_m = \text{Count}(\mathcal{E}_m), \quad (3)$$

where \mathcal{E}_m represents the m^{th} atomic environment.

A change in headgroup composition is reflected in a change in subgraph counts. Provided the new subgraph exists in the training data, the model can adjust its prediction accordingly. Branch points in a carbon chain are distinguished from main-chain groups, as they terminate in a CH group rather than CH₂.

ECFPs are similar to a segment-based approach; however, unlike segments or groups, subgraphs can overlap. Whilst a group contribution approach requires that a canonical ‘priority’ of the groups be defined prior to featurising molecules, by using ECFPs, the manual identification of important groups and their priorities are skipped; feature importance determination is delegated to the model.

Much like a group contribution approach, these fingerprints do not necessarily distinguish between all positional or chain isomers, particularly with smaller values of r , nor are stereoisomers treated differently.

A potential disadvantage with this approach, compared to group contributions, is that the number of unique atomic environments is potentially very large relative to the size of the data available, which poses a risk of overfitting. Furthermore, larger environments necessarily envelop smaller ones, which means that there is some duplicate information in the representation: the presence of a (CH₂)₃ environment implies the presence of three CH₂ environments, so that there is multicollinearity. This redundancy can impede model fitting and interpretation. These issues can be ameliorated using a process of *feature selection*, which will be discussed below.

ECFPs are commonly employed for determining molecular similarity through the use of the Tanimoto similarity metric.^{38–40} The Tanimoto similarity is a function of two binary fingerprints, so the count-based ECFPs are first converted by $b_i = \min(1, c_i)$. That is, if an atomic environment is present in a molecule, it is assigned a one; otherwise, it is assigned

zero. The Tanimoto similarity between molecules A and B is then

$$S_{AB} = \frac{\vec{b}_A \cdot \vec{b}_B}{\vec{b}_A \cdot \vec{b}_A + \vec{b}_B \cdot \vec{b}_B - \vec{b}_A \cdot \vec{b}_B}. \quad (4)$$

In the original work by Tanimoto³⁸, a ‘distance coefficient’ is defined based on the logarithm of the similarity of two points; this is not a true distance metric as it does not satisfy the triangle inequality. Instead, the Jaccard distance, $d_{AB} = 1 - S_{AB}$, was employed here to perform clustering of the molecules of the Qin dataset. The pairwise distances between each feature-selected molecular fingerprint in the Qin dataset were computed and the OPTICS clustering algorithm was used to assign a class to each of them.⁴¹ The algorithm was parameterised with a minimum cluster size of 4 molecules. Outlying molecules, which did not fit into any other class, were assigned an ‘outliers’ class for the sake of the stratified splitting procedure.

The clusters must be assigned before determining the train/test splits for each fold during the sensitivity analysis, so in order to provide a canonical cluster assignment, the fingerprints resulting from the benchmarking test on the Qin-All task were employed.

Molecular graphs

A molecular graph describes the entire topology of a molecule. It is a popular choice for cheminformatics as well as visualisation of molecular structure. Each atom is considered a *node* and each bond an *edge*. Rather than having a single feature vector to describe the molecule as a whole, each atom is assigned its own feature vector, \vec{v}_i , based on properties such as its element, hybridisation state, charge, etc. The same set of atomic features was used here as in the work by Qin et al.³⁴. It is for this reason that the molecules in the NIST data containing elements that were not in the training data were excluded; the atomic representation includes a fixed-length one-hot encoding of the element number, and the new elements cannot be encoded without modifying the model after training. These feature

vectors are concatenated into a node feature matrix, \mathbf{V} . The graph’s structure is then defined by a binary adjacency matrix, \mathbf{A} :

$$\mathbf{A}_{ij} = \begin{cases} 1 & \text{if } i \text{ bonded to } j, \text{ or } i = j \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Molecular graphs are natural representations to visualise; see Figure 1. This exact description of the molecule’s topology enables an atomistic machine learning approach.

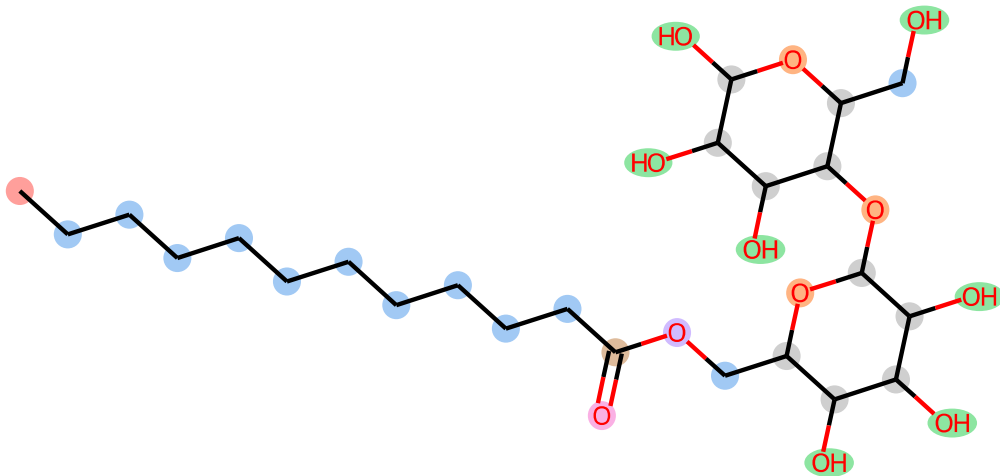


Figure 1: A molecular graph of 6-*O*-dodecanoyl-maltose. Atoms are highlighted based on their feature vectors, \vec{v}_i , so that equal feature vectors have the same colour.

Model selection

ECFP model

Based on the prior knowledge encoded in Equation 1, it is assumed that certain atomic environments have a linear relationship to $\log X_{cmc}$. It therefore seems justified to apply a linear model to the ECFP fingerprints described in Equation 3:

$$\log X_{cmc} = \vec{w} \cdot \vec{c} + b, \quad (6)$$

where \vec{w} is a trained weights vector, the elements of which correspond to the contribution of an atomic environment to the CMC, and b is an intercept (or *bias* term).

However, the issues of the large feature vector size and multicollinearity must be addressed. To that end, a process of *feature selection* was applied, whereby a subset of the atomic environments were selected for use in the model. There are several approaches to feature selection;³² here, we chose *regularisation* due to the ease of implementation a.

In this approach, we include a term in the loss function that depends on the norm of \vec{w} . The two types of constraints considered are ℓ_1 and ℓ_2 regularisation, which correspond to the inclusion of ℓ_1 and ℓ_2 norms, respectively. Concretely, the ElasticNet⁴² loss function was employed:

$$\min_{\vec{w}} \frac{1}{2n_{\text{samples}}} \left\| \mathbf{C}\vec{w} + \vec{b} - \vec{y} \right\|_2^2 + \alpha\rho \|\vec{w}\|_1 + \frac{\alpha(1-\rho)}{2} \|\vec{w}\|_2^2, \quad (7)$$

where n_{samples} is the number of training samples; \vec{y} are the training data’s true values of $\log X_{\text{cmc}}$; \vec{b} is a vector with elements all equal to b ; and α and ρ are user-defined hyperparameters describing the degree of regularisation ($\alpha \geq 0$), and the proportions of the regularisation terms ($0 < \rho < 1$), respectively. \mathbf{C} are the standardised training data feature vectors, $\{\vec{c}_n' \mid 1 \leq n \leq n_{\text{samples}}\}$, stacked row-wise into a matrix.

Standardising the environment counts ensures that they have zero mean and unit variance:

$$c_m' = \frac{c_m - u_m}{s_m}, \quad (8)$$

where u_m and s_m are the mean and standard deviation of the number of \mathcal{E}_m in each molecule in the training data. This standardisation ensures that the regularisation term is not dominated by environments with high variance and that it accounts for common and uncommon environments alike.

By imposing the ℓ_1 penalty, the model is biased towards learning a *sparse* weight vector: many of its elements will be negligible. The corresponding features can be removed from

the representation. Meanwhile, the ℓ_2 penalty means that a ‘grouping’ effect is achieved, ensuring that highly correlated groups are assigned similar weights, rather than discarding some of them. It also ensures that the upper bound on the number of selected groups is equal to the total number of groups; the model is not constrained to select a smaller subset if there are no redundant groups. Both of these are potential issues when using only an ℓ_1 norm in the loss function.^{42,43}

To determine the best values for α and ρ , 5-fold cross-validation of the training data was used. This was applied for a range of α and ρ combinations. The combination that achieved the lowest average mean-squared-error was used to train a model using the entirety of the training data. The hyperparameter search space is defined in the Supplementary Information.

The features with non-negligible fitted weights from ElasticNet were then selected for use in the final linear model. This model, ridge regression, uses just ℓ_2 regularisation so that all of the weights are non-negligible, but still addresses the issue of multicollinearity:⁴⁴

$$\min_{\vec{w}} \left\| \mathbf{C}\vec{w} + \vec{b} - \vec{y} \right\|_2^2 + \alpha \|\vec{w}\|_2^2. \quad (9)$$

A similar cross-validation method to the one described above was used to determine the best α parameter, but using leave-one-out cross-validation, whereby $k = n_{\text{samples}} - 1$. Because only one hyperparameter needs to be determined, there are far fewer trials per fold and therefore a greater number of folds can be used.

It was empirically observed that the combination of ElasticNet feature selection and a final regression with the simpler ridge regression model yielded better results, likely due to using a larger number of folds when determining the best value for α . Both models were implemented using scikit-learn.⁴⁵

Molecular graph model

The basic topology of the graph neural network (GNN) used in this work was identical to the one used by Qin et al.³⁴. The first step of the model consists of a stack of graph network layers, which mutate the node features in a molecular graph based on those of bonded atoms. These layers employ the graph convolution network (GCN) architecture introduced by Kipf and Welling⁴⁶. Layer l computes a new node feature matrix, $\mathbf{V}^{(l)}$, based on the adjacency matrix, \mathbf{A} :

$$\mathbf{V}^{(l)} = \mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}} \mathbf{V}^{(l-1)} \mathbf{W}^{(l)} + \mathbf{b}^{(l)}. \quad (10)$$

Here, $\mathbf{W}^{(l)}$ and $\mathbf{b}^{(l)}$ are the weights and biases, respectively, of layer l . We have also introduced the degree matrix,

$$D_{ii} = \sum_j A_{ij}, \quad (11)$$

so that the term $\mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}}$ normalises the adjacency matrix based on the degree of each atom.

$\mathbf{V}^{(1)}$, therefore, encodes not only information about the atom itself but its bonded neighbours. This information is used in the subsequent graph convolution so that $\mathbf{V}^{(2)}$ encodes information about the 2nd order neighbourhood, *et cetera*. The number of graphs layers, L , therefore dictates the ‘radius’ around each atom that is considered in computing the final feature vector, analogous to creating an ECFP, except that the i^{th} atomic environment is characterised by a continuous, *latent* vector, $\vec{v}_i^{(L)}$.

The next step is a pooling layer, which converts the graph to a single *latent representation vector*, $\vec{v}^{(p)}$, losing the explicit topological information. Several choices of pooling function were trialled:

Mean pooling was employed by Qin et al.³⁴. It computes the average over all atoms’ latent feature vectors.

Sum pooling computes the sum of all atoms’ latent feature vectors. This is the most

analogous to the ECFPs in that the contribution of an atomic environment scales linearly with the number of times it occurs in the molecule.

Gated attention pooling applies an *attention* mechanism to decide which environments are relevant to the prediction:⁴⁷

$$\vec{v}^{(p)} = \sum_i^N \sigma \left(\mathbf{W}_1 \vec{v}_i^{(L)} + \vec{b}_1 \right) \odot \left(\mathbf{W}_2 \vec{v}_i^{(L)} + \vec{b}_2 \right), \quad (12)$$

where \mathbf{W}_1 and \mathbf{W}_2 are trained weights and \vec{b}_1 and \vec{b}_2 are biases, σ is the sigmoid activation function, N is the number of atoms in the molecule, and \odot represents element-wise multiplication.

Attention sum pooling is a simpler variation of the above. By using a softmax function, it performs a weighted average of the atomic environments’ contributions:

$$\mathbf{X} = \text{softmax}(\mathbf{V}^{(L)} \vec{w}), \quad (13)$$

$$\vec{v}^{(p)} = \sum_i^N \mathbf{X}_i \cdot \vec{v}_i^{(L)}, \quad (14)$$

where \vec{w} are trained weights.

After training the model, $\vec{v}^{(p)}$ effectively acts as a machine-learned representation of the molecule that captures only the information about its topology and composition that is useful for predicting the CMC. Finding an optimised representation is a feature of neural networks that happens implicitly during training, called *representation learning*.⁴⁸ The final step is a readout neural network: a multi-layer perceptron which acts as a nonlinear approximator to map this latent representation vector to the CMC property prediction. Each layer in this neural network, called a ‘dense’ layer, outputs a new vector, $\vec{v}^{(l)}$:

$$\vec{v}^{(l)} = \mathbf{W}^{(l)} \vec{v}^{(l-1)} + \vec{b}^{(l)}, \quad (15)$$

The full network’s architecture is illustrated in Figure 2. The model was implemented using the open-source library Spektral⁴⁹ and optimised using an Adam optimiser⁵⁰ to minimise the mean squared error.

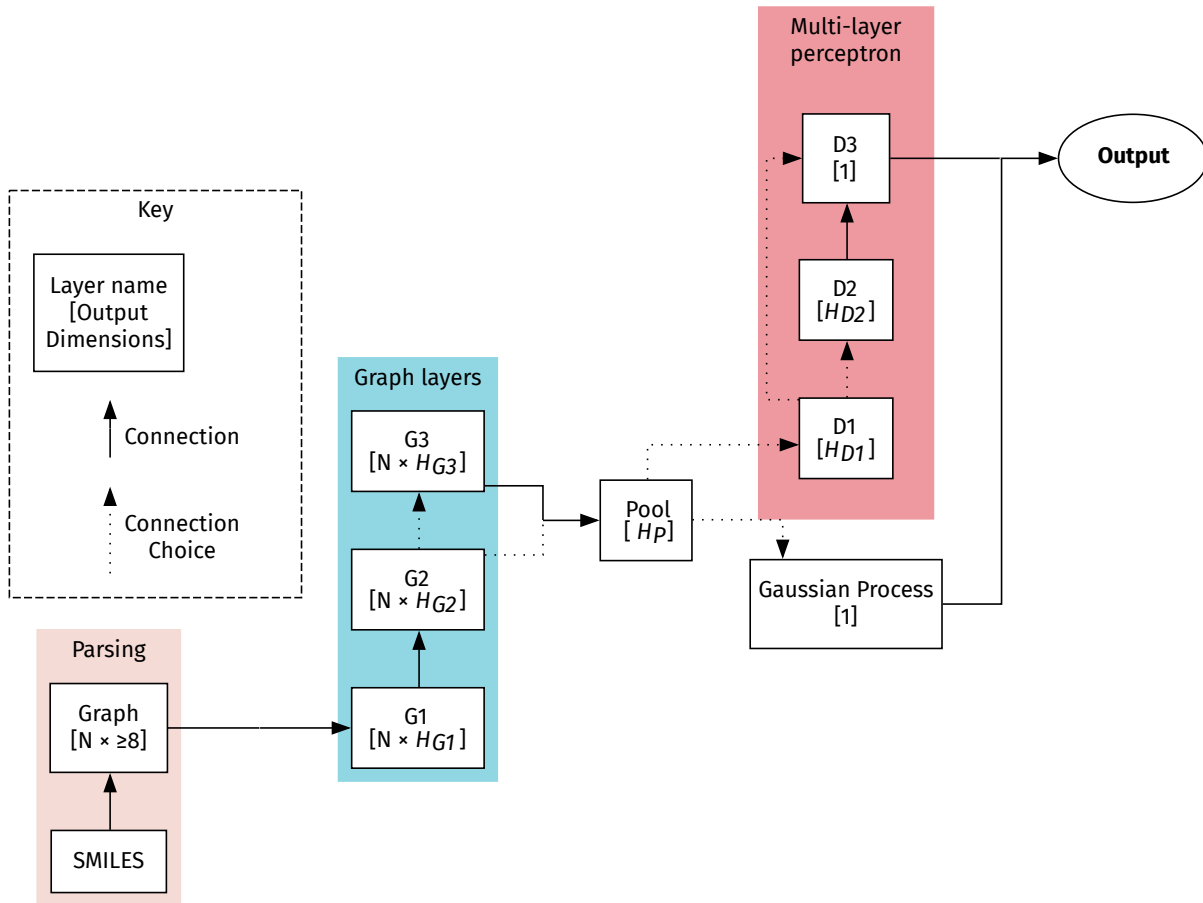


Figure 2: Schematic of the neural network architecture. Here, N represents the number of constituent atoms/ions in the input molecule and H represents a hyperparameter. The size of the pooling layer output, H_P , is only independent in the case of a gated attention-pooling layer. Otherwise, it is equal to the number of columns of the graph layer that feeds into it (H_{G2} or H_{G3}).

A neural network’s topology describes the types of layers used, i.e. their functional form and the connection between them. Layers that are parameterised by a weight matrix, \mathbf{W} , may have different ‘sizes’, meaning that the dimensionality of their output is arbitrary and can be adjusted by changing the dimensions of \mathbf{W} . The graph layers, dense layers and the gated attention pool all have this property. These sizes, the type of pooling layer and the

number of each graph and dense layer, are all hyperparameters that can be adjusted prior to training. To determine the best combination of hyperparameters for predicting CMCs, an automated searching procedure was employed.

Optimising GNN hyperparameters

The Hyperband approach,⁵¹ implemented in Keras Tuner,⁵² was used to select a good combination of hyperparameters for the model. Hyperband provides a way to efficiently evaluate the performance of a large search space of hyperparameter configurations. The algorithm assesses several combinations of hyperparameters, initially allocating only a small number of resources to each trial. The hyperparameters for the trials with the best performance are then allocated more resources, whilst the remainder is discarded. A reduction factor of 3 was chosen, meaning that 2/3 of the trials were discarded after each iteration. This procedure iterates until the best configuration is found.

The algorithm can be executed multiple times if resources are available to obtain a more reliable result; the training procedure is stochastic, and therefore the performance of two trials with the same hyperparameters may be different. In this case, a single run was performed. The training data was partitioned into an optimisation subset and a validation subset in a ratio of 9:1. The trials were fit to the optimisation subset and evaluated based on the RMSE of their predictions on the validation subset. The best hyperparameters determined on the benchmark tasks were then used during the sensitivity analysis.

Adding uncertainty with a Gaussian process

To improve the model’s reliability, a *surrogate* model was employed that could yield uncertainty estimates alongside CMC predictions. The approach is based on the Convolution-Fed Gaussian Process of Tran et al.⁵³. The model first computes the latent representation vector, $\vec{v}^{(p)}$, of an input molecule using a trained GNN. $\vec{v}^{(p)}$ is then standardised, similar to

Equation 8, but in this case the standardisation applies across each latent feature, n :

$$v_n^{(p)'} = \frac{v_n^{(p)} - u_n}{s_n}. \quad (16)$$

Again, u_n and s_n were determined from the training molecules’ latent representations.

The standardised latent representation vectors of the training data serve as index points for a Gaussian process (GP); see Figure 2. The GP’s predicted mean and standard deviation define a predicted normal distribution of a molecule’s CMC, $\log X_{cmc} \sim \mathcal{N}(\mu, \sigma)$.

In this work, the GPs were defined using a Matérn kernel with parameter $1/2$, and a fixed noise variance of 1×10^{-5} . Furthermore, the multi-layer perceptron component of the GNN used to calculate $\vec{v}^{(p)}$ was employed as the GP’s mean function. The kernel parameters were optimised with an Adam optimiser.⁵⁰ The same optimisation/validation splitting was used as for the GNN hyperparameter search and training was stopped after 1000 iterations without improvement in the validation predictions’ RMSE, or after a total of 5000 iterations. The software implementation was based on GPFlow.⁵⁴

Visualising the latent space

In order to better understand the model’s interpretation of chemical space, we exploit the GP’s kernel, optimised during training, to plot the molecules in 2D space in a way that respects the model’s perception of their ‘similarity’. This approach is inspired by Isayev et al.⁵⁵, who compared the fingerprints of several inorganic compounds to develop so-called materials catrograms. By employing the machine-learned latent representations of molecules, our cartogram reflects molecular similarities learned by the model itself.

Having trained the GP, the learned kernel function was computed between every pair of molecules in the combined data, NIST and Qin. These kernel values were then normalised within the range $[0, 1]$. Each molecule was assigned a node in a graph and these were connected by edges. Each edge was given a weight equal to the normalised kernel value

between the two nodes it connected.

This was the starting condition for computing a *force-directed graph layout*. The nodes are initialised with a set of 2D coordinates that are uniformly distributed and they are then moved according to forces acting upon them; primarily an attractive force that acts along the edges. In addition, there are pairwise repulsive forces acting amongst the nodes that serves to ensure that the equilibrium distance between two nodes is non-zero. In this work, the Force Atlas 2 algorithm was employed.^{56,57}

Results

ECFP feature selection

The number of atomic environments remaining after each stage of the feature selection process is reported in Table 3. Notably, the ratio of the number of features to the size of the training dataset is similar for both tasks ($\sim 74\%$) and so is the ratio of the initial number of features to the number of selected features (31% to 33%). The number of features is large compared to many of the empirical models described above, but not compared to the number of parameters for the GNN, which is significantly larger. Because this model also aims to cover a large part of chemical space, a large number of parameters is to be expected.

Table 3: The number of atomic environments at each stage of the ECFP feature selection process.

Task	Number of training data atomic environments		
	Initially	Found in multiple molecules	With non-negligible weight
Qin-Nonionics	260	201	81
Qin-All	410	302	134

The OPTICS clustering routine on the Qin-All dataset using these fingerprints resulted in 24 classes and 31 outlying molecules. The resulting classifications for each molecule are available in the Supplementary information.

Hyperband tuning

725 trials were conducted for each of the Qin training datasets. The best hyperparameters discovered on each set are described in Table 4.

Table 4: The best hyperparameters discovered during searching. The H values refer to the dimensions of the corresponding layer, see Figure 2. Values for H_{G3} and H_{D2} have been omitted where the layers were not included in the model, and the values of H_P were only independent for the gated attention pool, so that they are omitted here as well.

Hyperparameter	Best value for	
	Qin-Nonionics	Qin-All
# Graph layers	2	3
H_{G1}	320	64
H_{G2}	256	64
H_{G3}	—	128
Pooling layer	Mean pool	Sum pool
H_P	—	—
# Dense layers	2	2
H_{D1}	128	256
H_{D2}	—	—

Benchmark model performance

The performances of all the trained models on the benchmark tasks are reported in Table 5. All of the models developed here outperformed those of Qin et al.³⁴. For every task, the most accurate model was either the GNN or the combined GNN with GP (GNN/GP). The linear model’s performance is surprisingly good, considering its relative simplicity, faster optimisation and the far smaller number of parameters it constitutes.

As expected, the performance of all models on the NIST data is significantly worse than on the test data. This supports the hypothesis that the NIST data molecules are outside of the applicability domain of the models developed here, but it does not exclude the possibility that the models are instead overfitted. Therefore, a more detailed analysis of the applicability domain will be provided below.

Table 5: Benchmark task evaluation results for the models trained in this work versus those of the previous work. The best RMSE for each task is emboldened.

Model	Test RMSE (log μM)		
	Qin-Nonionics	Qin-All	NIST
Previous work ³⁴	0.23	0.30	–
ECFP	0.19	0.26	1.57
GNN	0.15	0.29	1.35
GNN/GP	1.38	0.21	1.32

Finally, it is noted that the GNN/GP model’s predictive performance on the Qin-Nonionics task was very poor. This indicates that the spacing between the molecules’ latent representation vectors, determined from the corresponding GNN, was not a good indicator of similarity with respect to CMC prediction.

Uncertainty quantification

The RMSE alone does not capture the quality of the predicted standard deviations. One metric that captures these is the negative log-likelihood (NLL) of observing the true CMCs, given the model’s predicted normal distributions:

$$\text{NLL} = - \sum_n \log p_n(\hat{y}_n), \quad (17)$$

where subscript n is the index of the data, \hat{y}_n is the true CMC value and p_n is the probability density function of the normal distribution $\mathcal{N}(\mu_n, \sigma_n)$, where μ_n and σ_n are the predicted mean and standard deviation. This metric indicates the relative performance of different models on the same data. (Note that its value scales with the size of the data.) It does not give a good indication of the quality of any individual model in isolation, however. The NLL values are included in the supplementary information for comparison against future work.

To assess the models’ quality individually, the predictions can be visualised against the true CMCs in a parity plot; see Figure 3a. Alternatively, a calibration plot can be used, which compares the cumulative distribution of the residuals against the expected distribu-

tion, given a model’s predicted standard deviations. The expected distribution stipulates what would be observed if the residuals were drawn from the distributions predicted by the corresponding model. Deviations from this distribution indicate whether the model was over- or underconfident (c.f. Tran et al.⁵³). The calibration plots are shown in Figure 3b.

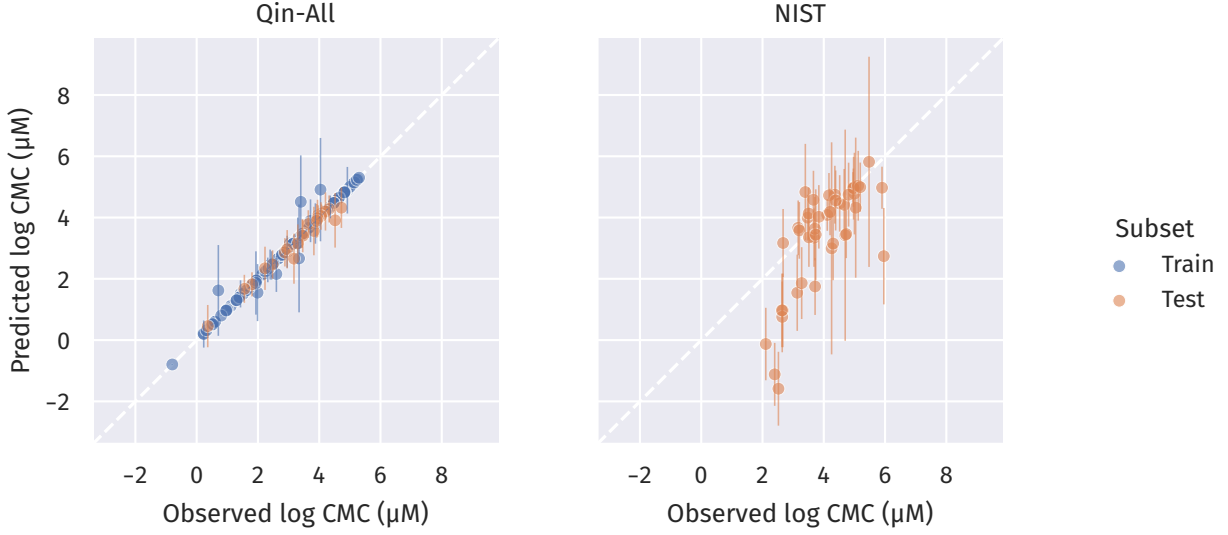
The S-shaped calibration curve for the Qin-All test data indicates that the model was underconfident in its predictions. In fact, there is a spike in the number of observed residuals that are close to the centre of the distribution. The corresponding parity plot shows that the predicted uncertainties were relatively small. The NIST data calibration curve’s asymmetry indicates its tendency to underestimate the CMCs. It shows a generally good agreement with the ideal distribution, with the greatest discrepancy above 0.8, which indicates that there are several surfactants whose CMC predictions are too low and that these predictions are overconfident, i.e. their predicted standard deviations are too small.

ECFP interpretations

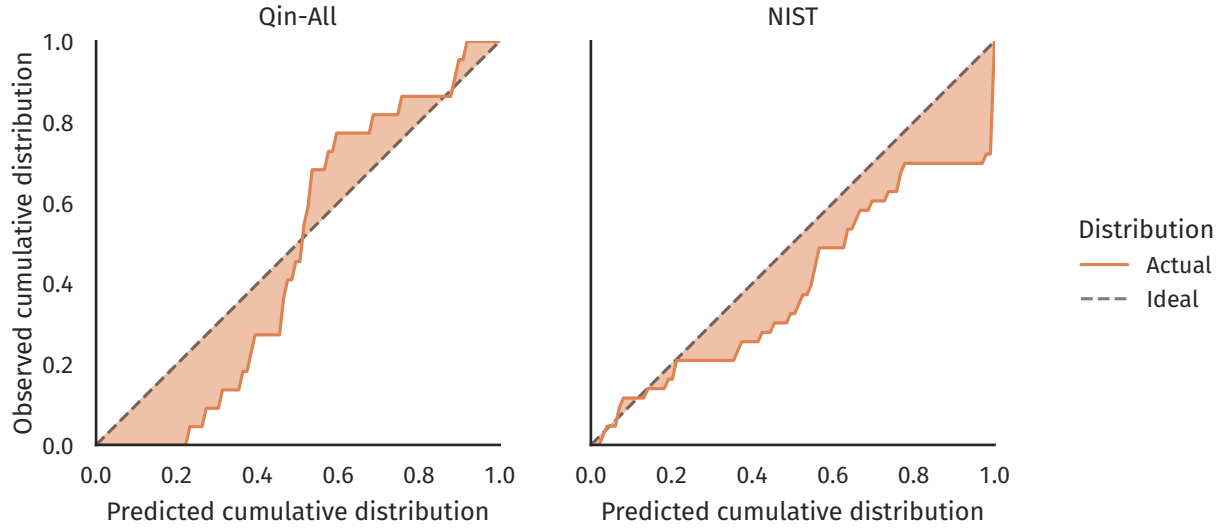
The weights of the ECFP models are coefficients corresponding to the scaled counts of the selected atomic environments. Referring to Equations 6 and 8, these coefficients indicate the change in a predicted CMC when the count of \mathcal{E}_m increases by s_m from its average, u_m . A more readily interpreted value can be achieved by rescaling the coefficient, w_m , for an environment:

$$w'_m = \frac{w_m(1 - u_m)}{s_m}, \quad (18)$$

which indicates the difference in predicted CMC between a molecule containing one \mathcal{E}_m and a molecule without any \mathcal{E}_m , but which otherwise contains exactly the same number as all the other environments. This scaled weight can be interpreted as a rough indication of the relative importance of different environments to determining CMC; ‘rough’ because it may not be physically plausible that two molecules exist that are distinguished only by the number of \mathcal{E}_m that they contain. This is particularly true of larger environments that



(a)



(b)

Figure 3: (a) Parity plots of the predicted CMCs from the GNN/GP model and 95 % confidence intervals for the Qin-All and NIST datasets and (b) corresponding calibration plots for the test data predictions. The ‘ideal’ distribution line indicates the cumulative distribution that would be obtained if residuals were drawn from the model’s predicted distributions.

envelope smaller ones. The largest scaled weights for the two ECFP models are given in Table 6.

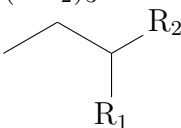
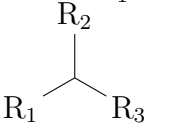
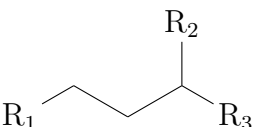
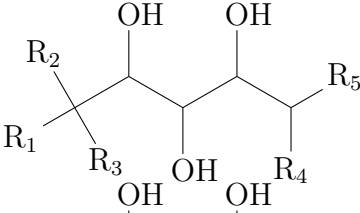
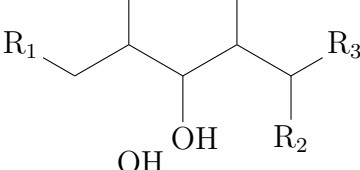
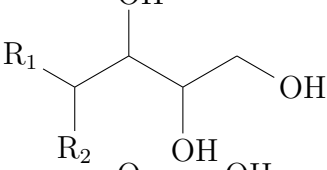
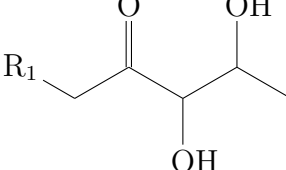
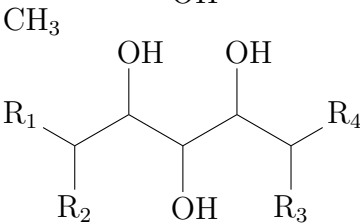
Both the Qin-Nonionic and Qin-All models agree that alkyl chain environments constitute the top two most important contributors to CMC, suggesting that tail length is the most important factor. The model trained on all surfactant classes includes two counterions in its most important environments: Cl^- and Br^- . This is to be expected; ionic surfactants typically have much larger CMCs than nonionics, and the model appears to distinguish these by their counterion. The Qin-Nonionics model identifies environments from the headgroups of sugar-based surfactants as being important. These surfactant headgroups possessed relatively complex topologies and therefore several environments; it may have been necessary for the model to use many of these environments in order to accurately distinguish between their CMCs.

Applicability domain analysis

Several of the molecules included in the NIST dataset were expected to be outside of the applicability domain, which justifies their poor prediction accuracy. The majority of the outliers’ CMCs are underpredicted, and the GNN/GP model is overconfident in their predictions. The 13 surfactants whose predictions’ residuals are greater than the 95 % confidence interval (CI) are shown in Figure 4. These constitute 3 nonionic surfactants, 1 zwitterionic surfactant and 9 ionic surfactants. Of the ionic surfactants, 5 have quaternary ammonium salts (quats) as counterions.

It is useful to examine the types of counterions in these molecules as compared to the training data. In the training data, there are no examples with a bromate, nitrate or carboxylate counterion, but there are two examples of a quat counterion. However, the quats in the training data are isotropic: tetrapropylammonium and tetramethylammonium. In contrast, those that are underpredicted are highly anisotropic and are effectively surfactants themselves; the behaviour of these compounds, in terms of CMC, can be expected to be very

Table 6: The atomic environments with the greatest importance to CMC according to the trained ECFP models.

Qin-All		Qin-Nonionics	
Environment	Scaled weight	Environment	Scaled Weight
$(\text{CH}_2)_5$	-0.64	$(\text{CH}_2)_5$	-0.76
$(\text{CH}_2)_3$	-0.55	$(\text{CH}_2)_3$	-0.69
Cl^-	0.31		-0.29
Br^-	0.29		-0.25
	-0.27		-0.19
CH_2	-0.23		0.14
$\text{R}_1-\text{O}-\text{R}_2$	0.18		-0.12
OH	-0.17		0.09
$\text{R}_1-\text{O}-(\text{CH}_2)_2\text{OH}$	-0.14	CH_3	-0.06
$\text{CH}_2-\text{O}-(\text{CH}_2)_2\text{OH}$	-0.14		0.05

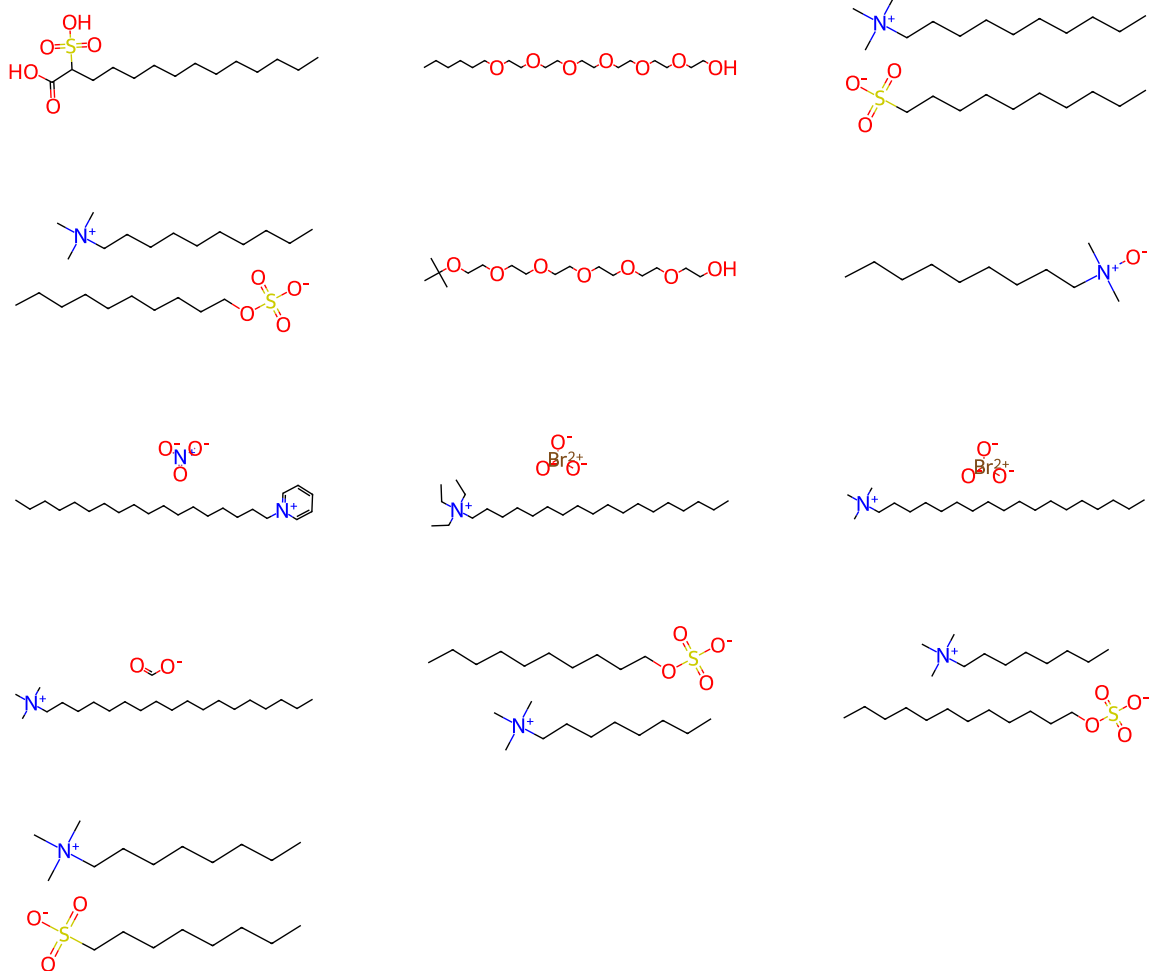


Figure 4: The 13 surfactants and counterions in the NIST dataset with residuals that were greater than the 95 % confidence interval.

different to that of the surfactants in the training data. Furthermore, the two polyoxyethylene alcohols in Figure 4 have remarkably small tail groups relative to the examples in the training data, which justifies their erroneous CMC predictions. This leaves two surfactants among the outliers that might reasonably be expected to lie within the applicability domain, constituting 4.7 % of the total NIST dataset, which is close to the 5 % that are expected to be outside the 95 % CI.

To gain insight into the relationships between molecules, we exploit the kernel function from the trained GP to create a molecular cartogram, as explained in the Method. The resulting cartogram is shown in Figure 5. The NIST molecules whose residuals are above the 95 % CI are also highlighted separately.

From the cartogram of the entire data, it is apparent that the majority of the surfactants are segregated based upon the type of counterion in the solution. This suggests that the model has learned that the counterion is an important factor for determining CMC and the weights associated with this counterion have a profound impact on the CMC prediction. However, the visualisation of the NIST molecules with CMCs above the 95 % CI indicates that the model classifies all of these molecules as being primarily similar to the nonionic or zwitterionic compounds. This is clearly erroneous for the ionic compounds: the quats, the bromates, the carboxylates and the nitrates. This suggests that the model has not learned appropriate weights for these counterions, hence it fails to properly segregate them.

Sensitivity analysis

Figure 6 shows the test set evaluations for the GNN and ECFP models using repeated, stratified k -fold cross-validation. The ECFP and GNN models show very similar performance on the Qin-All dataset, but the ECFP model outperforms the GNN model for all training ratios on the Qin-Nonionics dataset. This is indicative of the propensity of neural networks to overfit, especially on such small dataset sizes. Notably, there is also a much broader distribution of RMSEs on the Qin-Nonionics data. This suggests that both of the Qin-

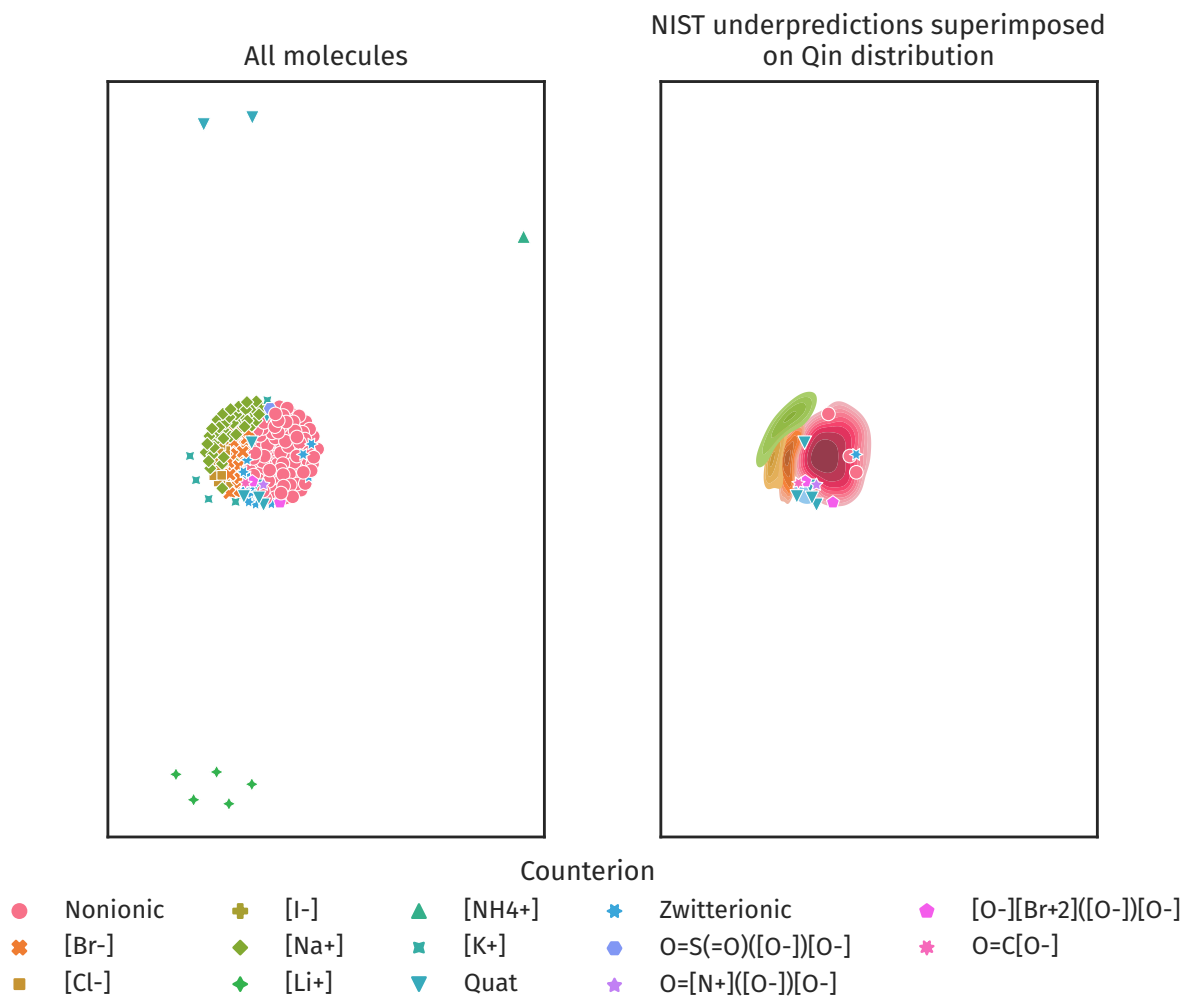


Figure 5: Cartogram of the molecules in the combined NIST and Qin datasets using a force-directed graph layout. Left: The entirety of the combined NIST and Qin datasets, where each molecule is assigned a point. Right: The NIST molecules that are overconfidently underpredicted, so that the residual is above the 95 % confidence interval, are superimposed on a kernel density estimate (KDE) plot of the Qin data. This KDE plot is an estimate of the distribution of the Qin molecules on the left, coloured by the type of counterion.

Nonionics models are much more sensitive to the specific molecules that are included in the train and test splits.

Extrapolating the trend lines indicates that the benchmark performance is much better than what would be expected. This highlights the importance of selecting an appropriate training data split, which spans the entirety of the chemical space of interest. On a dataset as small as the one used here, it is not possible to achieve this without using a high training ratio.

Discussion

The ideal molecular representation of an active chemical depends on the task at hand. Ideally, it should be compact but complete;^{58,59} ‘as simple as possible, but not simpler.’ To that end, the representation should contain enough information to distinguish between isomers with distinct properties. However, concessions can be made if we restrict the model’s domain and self-imposed limits on the type of isomers we expose the model to, both during training and in use. Representations may also include descriptions of state, such as temperature and pressure,⁶⁰ but this is redundant in cases where the training data spans a very limited range of states.

The representations employed by both the GNN and the linear models capture topological information and the performances of all of the models on in-domain data suggest that this is sufficient for the task of predicting CMC very accurately. However, both models are unable to distinguish between certain positional isomers, depending on the size of the atomic environments that they consider. In the case of ECFPs, this is dictated by the radius around each atom that is included in the fingerprint, whilst for the GNNs, this is determined by the number of consecutive graph layers.

Increasing these parameters both increases computational cost and model complexity, introducing more parameters and therefore requiring more data to optimise them appro-

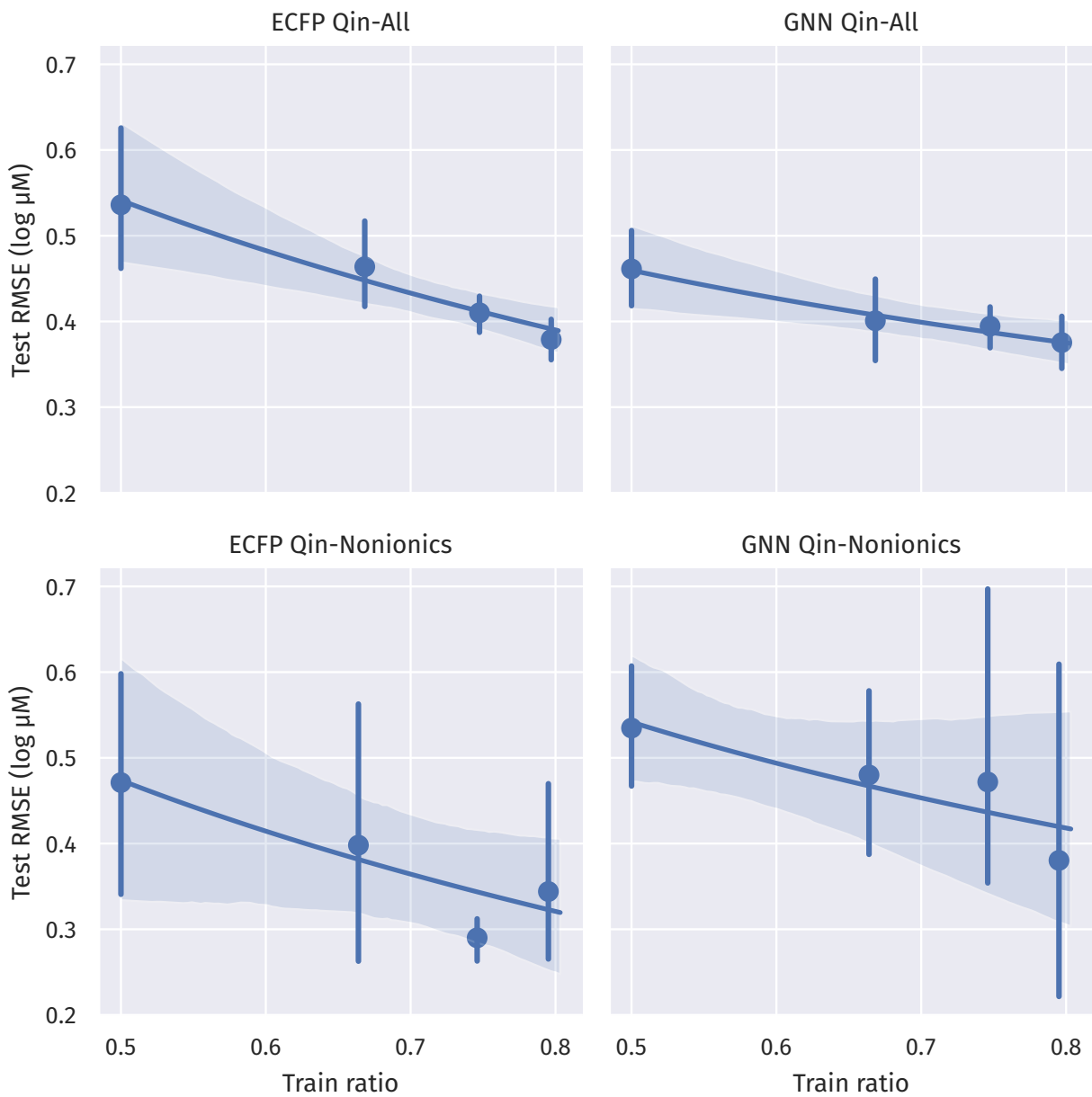


Figure 6: Test set RMSEs for the sensitivity analysis models. The average RMSE for each value of k is indicated, as well as the range of the RMSEs. A logarithmic fit to the data is shown, with a 95 % confidence interval determined by bootstrapping, using 1000 repeats.

privately. The sensitivity analysis demonstrated that this also increases the propensity for overfitting; however, the benchmarking results demonstrate that using a proper selection of training samples can yield more accurate models. In cases where there are fewer samples available, and some chemical classes are poorly represented in the training data, the simpler, linear model may be preferable.

One of the great advantages of using such a topological approach is that the contributions of each molecular fragment can be explicitly determined, as shown in the section on ECFP interpretations. In the case of the GNN, introducing a kernel to the model, via a Gaussian process operating on the GNN’s learned latent space representations, offers a quantitative measure of molecular similarity that can simultaneously be employed for adding uncertainty to the CMC predictions and visualising the chemical space of the training data. Superimposing the test data onto this cartogram highlights which molecules may be associated with erroneous predictions, based on the fact that they are clustered amongst training data molecules with very different chemistries.

Case-by-case examination of these molecules highlights the nature of these chemical differences, which can be related to properties important for micellisation. Broadly speaking, the model failed in three cases:

- Where ionic effects of the solute were not learned, as in the case of the counterions that were unique to the NIST data.
- When there was a significant difference in the strength of the hydrophobic interactions, as in the case of the surfactants with very small tail groups.
- When the counterion was itself surfactant-like, as in the case of the ammonium quaternary ions, which implies the system should better be described as a binary mixture of surfactants. Notably, the models developed here were not trained against predictions of CMC for mixed surfactant systems.

These results stress the importance of applying domain knowledge in developing and analysing

the results of deep learning models. The uncertainty quantification is unreliable when the systems’ behaviour is starkly different from what the model can be expected to learn from the training data.

Future efforts may consider incorporating another term in the loss function for the GNN that explicitly biases the model towards learning a form of $\vec{v}^{(p)}$ that captures chemical similarity based on user-defined metrics; for example, the length of the carbon tail. This approach would enable chemical knowledge to be explicitly encoded within the model and may correct for some of the aforementioned failure cases. Alternatively, a variational Gaussian process could be used, which approximates the Gaussian process using a fixed-size set of ‘pseudo-points’;⁶¹ this would enable the entire GNN/GP model to be trained at once using backpropagation and can be applied when the training data size is larger.⁶²

Conclusions

Empirical models were developed and applied to predict CMCs from two datasets of aqueous surfactants. One dataset was partitioned into training and test data (Qin-All), and a subset of the nonionic surfactants within this data was also used as a separate prediction task (Qin-Nonionics). The NIST dataset was collected from a different source and contained molecules with somewhat different chemistries than the above.

A linear model based on ECFPs demonstrated remarkably good performance, improving on a previous work³⁴ that applied a more complex GNN model, despite using a smaller number of parameters and having a much faster optimisation time. A new model was presented that improved the architecture of previous work’s GNN using a hyperparameter search algorithm, which was capable of obtaining better performances than the ECFP model on the Qin-Nonionics task and demonstrated a better ability to generalise to the NIST dataset.

Sensitivity analysis showed that the GNN models have a tendency to overfit when training

data samples do not adequately cover the chemical space of interest. When using small datasets, with only a few examples of certain surfactant classes, our analysis suggests that it may be preferable to use a simpler functional form, like the linear ECFP model.

A surrogate model was developed by feeding the latent space representation of a molecule, learned by the GNN model, to a Gaussian process. This yielded uncertainty estimates alongside CMC predictions. Although this model failed when applied to the Qin-Nonionics task, it yielded the best predictive performance of all of the models trained here for the Qin-All task, as well as providing good uncertainty estimates on the in-domain NIST test data. This approach would allow practitioners to gauge their confidence in the model’s predictions for systems within the applicability domain.

Finally, the kernel function that is learned while training the Gaussian process was employed to visualise the chemical space using a molecular cartogram. By analysing this cartogram, it was shown that chemical intuition could be employed to determine which molecules were likely poorly represented in the latent space, based on the fact that they were surrounded by molecules with different chemistries. This proves to be a useful technique for exploring the limits of a model’s applicability domain, as well as understanding why the model yields its predictions for a given molecule based on proximity to training set molecules within the cartogram.

This work demonstrates the potential of Gaussian processes to add uncertainty quantification to machine learning models with minimal overhead. There is still scope to overcome the limitations of these models with respect to small datasets, such as Qin-Nonionics, and out-of-domain molecules, which could be achieved by explicitly biasing the latent space.

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Supporting Information Available

Source code for featurisation and model training, graph neural network logs and metrics for hyperparameter optimisation and final training, and individual model predictions is available at <https://github.com/a-ws-m/CaMCaNN/>.

References

- (1) Rosen, M. J.; Kunjappu, J. T. *Surfactants and Interfacial Phenomena*; John Wiley & Sons, 2012
.
- (2) Klevens, H. B. Structure and Aggregation in Dilute Solution of Surface Active Agents. *Journal of the American Oil Chemists Society* **1953**, *30*, 74–80
.
- (3) Puvvada, S.; Blankschtein, D. Molecular-thermodynamic Approach to Predict Micellization, Phase Behavior and Phase Separation of Micellar Solutions. I. Application to Nonionic Surfactants. *The Journal of Chemical Physics* **1990**, *92*, 3710–3724
.
- (4) de Miguel, R.; Rubí, J. M. Gibbs Thermodynamics and Surface Properties at the Nanoscale. *The Journal of Chemical Physics* **2021**, *155*, 221101
.

- (5) Frenkel, D.; Smit, B. *Understanding Molecular Simulation: From Algorithms to Applications*; Elsevier, 2001
.
- (6) Joshi, S. Y.; Deshmukh, S. A. A Review of Advancements in Coarse-Grained Molecular Dynamics Simulations. *Molecular Simulation* **2021**, *47*, 786–803
.
- (7) Filipe, H. A. L.; Loura, L. M. S. Molecular Dynamics Simulations: Advances and Applications. *Molecules* **2022**, *27*, 2105
.
- (8) Jorge, M. Molecular Dynamics Simulation of Self-Assembly of n-Decyltrimethylammonium Bromide Micelles. *Langmuir* **2008**, *24*, 5714–5725
.
- (9) Jusufi, A.; Panagiotopoulos, A. Z. Explicit- and Implicit-Solvent Simulations of Micellization in Surfactant Solutions. *Langmuir* **2015**, *31*, 3283–3292
.
- (10) Fitzgerald, G.; DeJoannis, J.; Meunier, M. In *Modeling, Characterization, and Production of Nanomaterials*; Tewary, V. K., Zhang, Y., Eds.; Woodhead Publishing Series in Electronic and Optical Materials; Woodhead Publishing, 2015; pp 3–53
.
- (11) Vishnyakov, A.; Lee, M.-T.; Neimark, A. V. Prediction of the Critical Micelle Concentration of Nonionic Surfactants by Dissipative Particle Dynamics Simulations. *The Journal of Physical Chemistry Letters* **2013**, *4*, 797–802
.
- (12) Klamt, A.; Schüürmann, G. COSMO: A New Approach to Dielectric Screening in Sol-

- vents with Explicit Expressions for the Screening Energy and Its Gradient. *Journal of the Chemical Society, Perkin Transactions 2* **1993**, 0, 799–805
- .
- (13) Klamt, A.; Eckert, F.; Arlt, W. COSMO-RS: An Alternative to Simulation for Calculating Thermodynamic Properties of Liquid Mixtures. *Annual Review of Chemical and Biomolecular Engineering* **2010**, 1, 101–122
- .
- (14) Turchi, M.; Karcz, A. P.; Andersson, M. P. First-Principles Prediction of Critical Micellar Concentrations for Ionic and Nonionic Surfactants. *Journal of Colloid and Interface Science* **2022**, 606, 618–627
- .
- (15) Klamt, A.; Huniar, U.; Spycher, S.; Keldenich, J. COSMOmic: A Mechanistic Approach to the Calculation of Membrane-Water Partition Coefficients and Internal Distributions within Membranes and Micelles. *The Journal of Physical Chemistry B* **2008**, 112, 12148–12157
- .
- (16) Jakobtorweihen, S.; Yordanova, D.; Smirnova, I. Predicting Critical Micelle Concentrations with Molecular Dynamics Simulations and COSMOmic. *Chemie Ingenieur Technik* **2017**, 89, 1288–1296
- .
- (17) Jakobtorweihen, S.; Ingram, T.; Smirnova, I. Combination of COSMOmic and Molecular Dynamics Simulations for the Calculation of Membrane–Water Partition Coefficients. *Journal of Computational Chemistry* **2013**, 34, 1332–1340
- .
- (18) Klamt, A.; Schwöbel, J.; Huniar, U.; Koch, L.; Terzi, S.; Gaudin, T. COSMOplex: Self-

Consistent Simulation of Self-Organizing Inhomogeneous Systems Based on COSMO-RS. *Physical Chemistry Chemical Physics* **2019**, *21*, 9225–9238

.

- (19) Herbert, J. M. Dielectric Continuum Methods for Quantum Chemistry. *WIREs Computational Molecular Science* **2021**, *11*, e1519

.

- (20) Eckert, F.; Klamt, A. Fast Solvent Screening via Quantum Chemistry: COSMO-RS Approach. *AIChE Journal* **2002**, *48*, 369–385

.

- (21) Li, X.-S.; Lu, J.-F.; Li, Y.-G.; Liu, J.-C. Studies on UNIQUAC and SAFT Equations for Nonionic Surfactant Solutions. *Fluid Phase Equilibria* **1998**, *153*, 215–229

.

- (22) Cheng, J.-S.; Chen, Y.-P. Correlation of the Critical Micelle Concentration for Aqueous Solutions of Nonionic Surfactants. *Fluid Phase Equilibria* **2005**, *232*, 37–43

.

- (23) Voutsas, E. C.; Flores, M. V.; Spiliotis, N.; Bell, G.; Halling, P. J.; Tassios, D. P. Prediction of Critical Micelle Concentrations of Nonionic Surfactants in Aqueous and Non-aqueous Solvents with UNIFAC. *Industrial & Engineering Chemistry Research* **2001**, *40*, 2362–2366

.

- (24) Veerasamy, R.; Rajak, H.; Jain, A.; Sivadasan, S.; Varghese, C. P.; Agrawal, R. K. Validation of QSAR Models-Strategies and Importance. *Int. J. Drug Des. Discov* **2011**, *3*, 511–519

.

- (25) Tropsha, A. Best Practices for QSAR Model Development, Validation, and Exploitation. *Molecular informatics* **2010**, *29*, 476–488
.
- (26) Leonard, J. T.; Roy, K. On Selection of Training and Test Sets for the Development of Predictive QSAR Models. *QSAR & Combinatorial Science* **2006**, *25*, 235–251
.
- (27) Gantzer, P.; Creton, B.; Nieto-Draghi, C. Inverse-QSPR for de Novo Design: A Review. *Molecular Informatics* **2020**, *39*, 1900087
.
- (28) Bolboaca, S. D.; Jantschi, L.; Diudea, M. V. Molecular Design and QSARs/QSPRs with Molecular Descriptors Family. *Current Computer - Aided Drug Design* **2013**, *9*, 195–205
.
- (29) Zefirov, N. S.; Palyulin, V. A. Fragmental Approach in QSPR. *Journal of Chemical Information and Computer Sciences* **2002**, *42*, 1112–1122
.
- (30) Mattei, M.; Kontogeorgis, G. M.; Gani, R. Modeling of the Critical Micelle Concentration (CMC) of Nonionic Surfactants with an Extended Group-Contribution Method. *Industrial & Engineering Chemistry Research* **2013**, *52*, 12236–12246
.
- (31) Gani, R.; Harper, P. M.; Hostrup, M. Automatic Creation of Missing Groups through Connectivity Index for Pure-Component Property Prediction. *Industrial & Engineering Chemistry Research* **2005**, *44*, 7262–7269
.

- (32) Li, J.; Cheng, K.; Wang, S.; Morstatter, F.; Trevino, R. P.; Tang, J.; Liu, H. Feature Selection: A Data Perspective. *ACM Computing Surveys* **2017**, *50*, 94:1–94:45
- .
- (33) Guyon, I.; Elisseeff, A. An Introduction to Variable and Feature Selection. *Journal of machine learning research* **2003**, *3*, 1157–1182
- .
- (34) Qin, S.; Jin, T.; Van Lehn, R. C.; Zavala, V. M. Predicting Critical Micelle Concentrations for Surfactants Using Graph Convolutional Neural Networks. *The Journal of Physical Chemistry B* **2021**, *125*, 10610–10620
- .
- (35) Bejani, M. M.; Ghatee, M. A Systematic Review on Overfitting Control in Shallow and Deep Neural Networks. *Artificial Intelligence Review* **2021**, *54*, 6391–6438
- .
- (36) Mukerjee, P.; Mysels, K. J. *Critical Micelle Concentrations of Aqueous Surfactant Systems*; 1971; pp 51–65
- .
- (37) Rogers, D.; Hahn, M. Extended-Connectivity Fingerprints. *Journal of Chemical Information and Modeling* **2010**, *50*, 742–754
- .
- (38) Tanimoto, T. T. Elementary Mathematical Theory of Classification and Prediction. **1958**,
- (39) Bajusz, D.; Rácz, A.; Héberger, K. Why Is Tanimoto Index an Appropriate Choice for Fingerprint-Based Similarity Calculations? *Journal of Cheminformatics* **2015**, *7*, 20
- .

- (40) Butina, D. Unsupervised Data Base Clustering Based on Daylight’s Fingerprint and Tanimoto Similarity: A Fast and Automated Way To Cluster Small and Large Data Sets. *Journal of Chemical Information and Computer Sciences* **1999**, *39*, 747–750
.
- (41) Ankerst, M.; Breunig, M. M.; Kriegel, H.-P.; Sander, J. OPTICS: Ordering Points to Identify the Clustering Structure. *ACM SIGMOD Record* **1999**, *28*, 49–60
.
- (42) Zou, H.; Hastie, T. Regularization and Variable Selection via the Elastic Net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* **2005**, *67*, 301–320
.
- (43) Efron, B.; Hastie, T.; Johnstone, I.; Tibshirani, R. Least Angle Regression. *The Annals of Statistics* **2004**, *32*, 407–499
.
- (44) McDonald, G. C. Ridge Regression. *WIREs Computational Statistics* **2009**, *1*, 93–100
.
- (45) Pedregosa, F. et al. Scikit-Learn: Machine Learning in Python. *Journal of Machine Learning Research* **2011**, *12*, 2825–2830
.
- (46) Kipf, T. N.; Welling, M. Semi-Supervised Classification with Graph Convolutional Networks. 2017
.
- (47) Li, Y.; Tarlow, D.; Brockschmidt, M.; Zemel, R. Gated Graph Sequence Neural Networks. 2017
.

- (48) Goodfellow, I.; Bengio, Y.; Courville, A. *Deep Learning*; MIT Press, 2016; pp 524–554
.
- (49) Grattarola, D.; Alippi, C. Graph Neural Networks in TensorFlow and Keras with Spectral. 2020
.
- (50) Kingma, D. P.; Ba, J. Adam: A Method for Stochastic Optimization. 2017
.
- (51) Li, L.; Jamieson, K.; DeSalvo, G.; Rostamizadeh, A.; Talwalkar, A. Hyperband: A Novel Bandit-Based Approach to Hyperparameter Optimization. *Journal of Machine Learning Research* **2018**, *18*, 1–52
.
- (52) Chollet, F., et al. Keras. 2015
.
- (53) Tran, K.; Neiswanger, W.; Yoon, J.; Zhang, Q.; Xing, E.; Ulissi, Z. W. Methods for Comparing Uncertainty Quantifications for Material Property Predictions. *Machine Learning: Science and Technology* **2020**, *1*, 025006
.
- (54) Matthews, A. G. d. G.; van der Wilk, M.; Nickson, T.; Fujii, Keisuke.; Boukouvalas, A.; León-Villagrà, P.; Ghahramani, Z.; Hensman, J. GPflow: A Gaussian Process Library Using TensorFlow. *Journal of Machine Learning Research* **2017**, *18*, 1–6
.
- (55) Isayev, O.; Fourches, D.; Muratov, E. N.; Oses, C.; Rasch, K.; Tropsha, A.; Curtarolo, S. Materials Cartography: Representing and Mining Materials Space Using Structural and Electronic Fingerprints. *Chemistry of Materials* **2015**, *27*, 735–743
.

- (56) Jacomy, M.; Venturini, T.; Heymann, S.; Bastian, M. ForceAtlas2, a Continuous Graph Layout Algorithm for Handy Network Visualization Designed for the Gephi Software. *PLOS ONE* **2014**, *9*, e98679
- .
- (57) Bastian, M.; Heymann, S.; Jacomy, M. Gephi: An Open Source Software for Exploring and Manipulating Networks. 2009
- .
- (58) Faber, F.; Lindmaa, A.; von Lilienfeld, O. A.; Armiento, R. Crystal Structure Representations for Machine Learning Models of Formation Energies. *International Journal of Quantum Chemistry* **2015**, *115*, 1094–1101
- .
- (59) Himanen, L.; Jäger, M. O. J.; Morooka, E. V.; Federici Canova, F.; Ranawat, Y. S.; Gao, D. Z.; Rinke, P.; Foster, A. S. Dscribe: Library of Descriptors for Machine Learning in Materials Science. *Computer Physics Communications* **2020**, *247*, 106949
- .
- (60) Chen, C.; Ye, W.; Zuo, Y.; Zheng, C.; Ong, S. P. Graph Networks as a Universal Machine Learning Framework for Molecules and Crystals. *Chemistry of Materials* **2019**, *31*, 3564–3572
- .
- (61) Hensman, J.; Fusi, N.; Lawrence, N. D. Gaussian Processes for Big Data. 2013
- .
- (62) Moriarty, A.; Morita, K.; Butler, K. T.; Walsh, A. UnlockNN: Uncertainty Quantification for Neural Network Models of Chemical Systems. *Journal of Open Source Software* **2022**, *7*, 3700
- .

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