07_homework_svm

December 10, 2017

1 Programming assignment 7: SVM

```
In [89]: import numpy as np
         import matplotlib.pyplot as plt
         %matplotlib inline

         from sklearn.datasets import make_blobs

         from cvxopt import matrix, solvers
```

1.1 Your task

In this sheet we will implement a simple binary SVM classifier.

We will use CVXOPT http://cvxopt.org/ - a Python library for convex optimization. If you use Anaconda, you can install it using

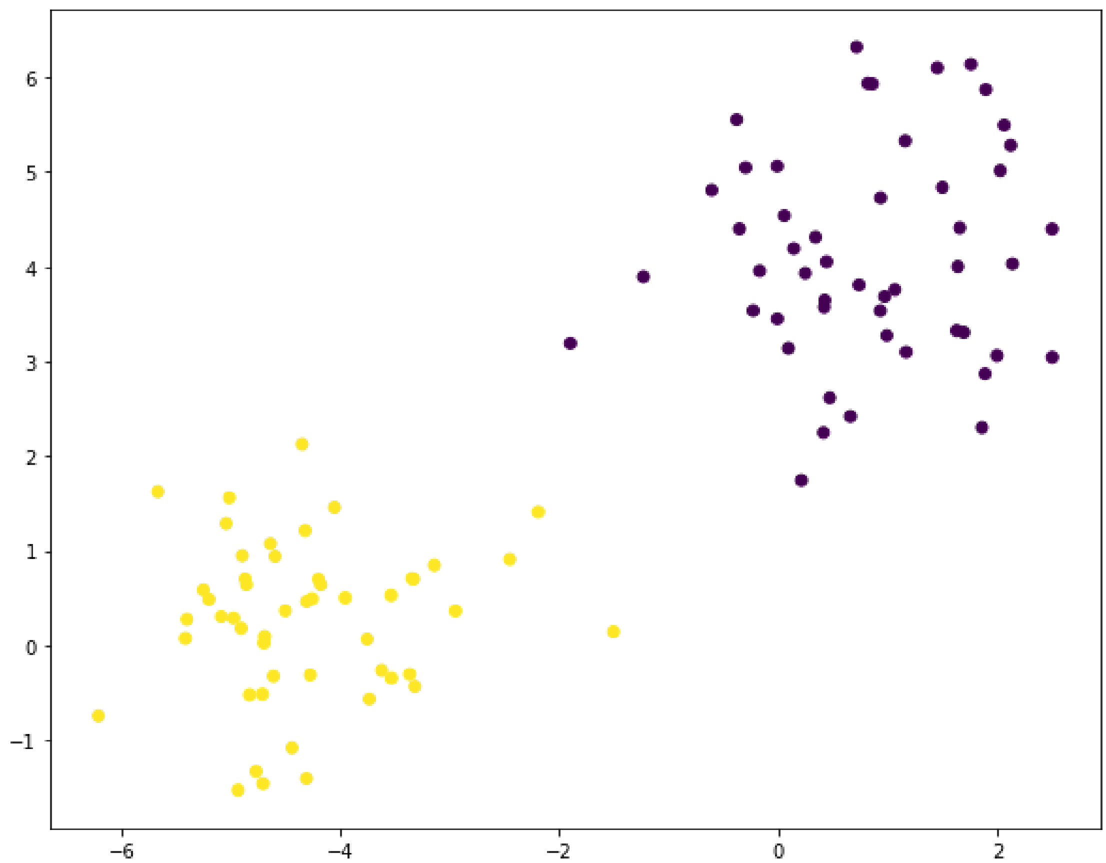
```
conda install cvxopt
```

As usual, your task is to fill out the missing code, run the notebook, convert it to PDF and attach it you your HW solution.

1.2 Generate and visualize the data

```
In [90]: N = 100  # number of samples
         D = 2  # number of dimensions
         C = 2  # number of classes
         seed = 3  # for reproducible experiments

         X, y = make_blobs(n_samples=N, n_features=D, centers=2, random_state=seed)
         y[y == 0] = -1  # it is more convenient to have {-1, 1} as class labels (instead of {
         y = y.astype(np.float)
         plt.figure(figsize=[10, 8])
         plt.scatter(X[:, 0], X[:, 1], c=y)
         plt.show()
```



1.3 Task 1: Solving the SVM dual problem

Remember, that the SVM dual problem can be formulated as a Quadratic programming (QP) problem. We will solve it using a QP solver from the `CVXOPT` library.

The general form of a QP is

$$\min_{\mathbf{x}} \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x}$$

$$\text{subject to} \quad \mathbf{G}\mathbf{x} \preceq \mathbf{h}$$

$$\text{and} \quad \mathbf{A}\mathbf{x} = \mathbf{b}$$

where $\preceq$ denotes "elementwise less than or equal to".

Your task is to formulate the SVM dual problems as a QP and solve it using `CVXOPT`, i.e. specify the matrices $\mathbf{P}, \mathbf{G}, \mathbf{A}$ and vectors $\mathbf{q}, \mathbf{h}, \mathbf{b}$.

```
In [138]: def solve_dual_svm(X, y):
              """Solve the dual formulation of the SVM problem.

              Parameters
```

```
    ----------
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).

    Returns
    -------
    alphas : array, shape [N]
        Solution of the dual problem.
    """
    # TODO
    # These variables have to be of type cvxopt.matrix
    n = X.shape[0]

    P = matrix(np.outer(y,y) * X.dot(X.T))
    q = matrix(-1.0, (n,1))
    G = matrix(-np.eye(n))
    h = matrix(0.0, (n,1))
    A = matrix(y[:,np.newaxis].T)
    b = matrix(0.0)
    solvers.options['show_progress'] = False
    solution = solvers.qp(P, q, G, h, A, b)
    alphas = np.array(solution['x'])
    return alphas
```

1.4 Task 2: Recovering the weights and the bias

```
In [141]: def compute_weights_and_bias(alpha, X, y):
              """Recover the weights w and the bias b using the dual solution alpha.

              Parameters
              ----------
              alpha : array, shape [N]
                  Solution of the dual problem.
              X : array, shape [N, D]
                  Input features.
              y : array, shape [N]
                  Binary class labels (in {-1, 1} format).

              Returns
              -------
              w : array, shape [D]
                  Weight vector.
              b : float
                  Bias term.
              """
              w = X.T.dot((alpha*y[:,np.newaxis]))
```

```
    b = y[np.argmax(alpha)]-w.T.dot(X[np.argmax(alpha),:])
    return w, b
```

1.5 Visualize the result (nothing to do here)

```
In [99]: def plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b):
             """Plot the data as a scatter plot together with the separating hyperplane.

             Parameters
             ----------
             X : array, shape [N, D]
                 Input features.
             y : array, shape [N]
                 Binary class labels (in {-1, 1} format).
             alpha : array, shape [N]
                 Solution of the dual problem.
             w : array, shape [D]
                 Weight vector.
             b : float
                 Bias term.
             """
             plt.figure(figsize=[10, 8])
             # Plot the hyperplane
             slope = -w[0] / w[1]
             intercept = -b / w[1]
             x = np.linspace(X[:, 0].min(), X[:, 0].max())
             plt.plot(x, x * slope + intercept, 'k-', label='decision boundary')
             # Plot all the datapoints
             plt.scatter(X[:, 0], X[:, 1], c=y)
             # Mark the support vectors
             support_vecs = (alpha > 1e-4).reshape(-1)
             plt.scatter(X[support_vecs, 0], X[support_vecs, 1], c=y[support_vecs],
                         s=250, marker='*', label='support vectors')
             plt.xlabel('$x_1$')
             plt.ylabel('$x_2$')
             plt.legend(loc='upper left')
```

The reference solution is

```
w = array([[-0.69192638],
           [-1.00973312]])

b = 0.907667782
```

Indices of the support vectors are

```
[38, 47, 92]
```

```
In [142]: alpha = solve_dual_svm(X, y)
          w, b = compute_weights_and_bias(alpha, X, y)
          print("w: {}".format(w))
          print("b: {}".format(b))
          plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b)
          plt.show()
```

```
w: [[-0.69192638]
 [-1.00973312]]
b: [ 0.90766774]
```