10_homework_dim_reduction

January 14, 2018

1 Programming assignment 10: Dimensionality Reduction

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt

        %matplotlib inline
```

1.1 PCA Task

Given the data in the matrix X your tasks is to: * Calculate the covariance matrix Σ. * Calculate eigenvalues and eigenvectors of Σ. * Plot the original data X and the eigenvectors to a single diagram. What do you observe? Which eigenvector corresponds to the smallest eigenvalue? * Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new subspace. * Transform all vectors in X in this new subspace by expressing all vectors in X in this new basis.

1.1.1 The given data X

```
In [2]: X = np.array([(-3,-2),(-2,-1),(-1,0),(0,1),
                      (1,2),(2,3),(-2,-2),(-1,-1),
                      (0,0),(1,1),(2,2), (-2,-3),
                      (-1,-2),(0,-1),(1,0), (2,1),(3,2)])
```

1.1.2 Task 1: Calculate the covariance matrix Σ

```
In [3]: def get_covariance(X):
            """Calculates the covariance matrix of the input data.

            Parameters
            ----------
            X : array, shape [N, D]
                Data matrix.

            Returns
            -------
            Sigma : array, shape [D, D]
                Covariance matrix
```

```
    """
    xmean = np.sum(X, axis=0)/X.shape[0]
    return X.T.dot(X) - np.outer(xmean,xmean)
```

1.1.3 Task 2: Calculate eigenvalues and eigenvectors of Σ.

```
In [4]: def get_eigen(S):
            """Calculates the eigenvalues and eigenvectors of the input matrix.

            Parameters
            ----------
            S : array, shape [D, D]
                Square symmetric positive definite matrix.

            Returns
            -------
            L : array, shape [D]
                Eigenvalues of S
            U : array, shape [D, D]
                Eigenvectors of S

            """
            L, U = np.linalg.eig(S)
            return L, U.T
```

1.1.4 Task 3: Plot the original data X and the eigenvectors to a single diagram.

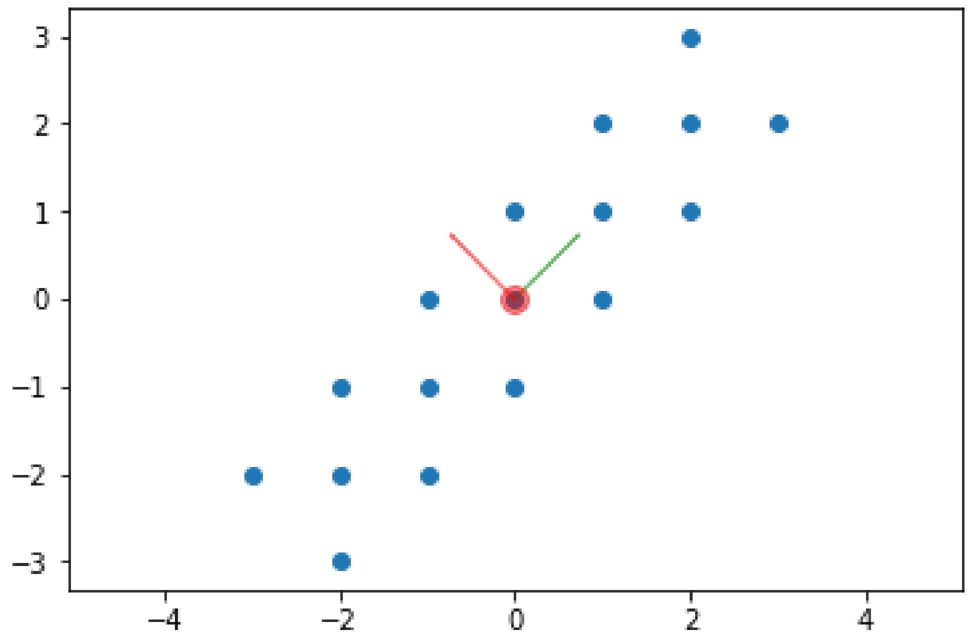
```
In [5]: # plot the original data
        plt.scatter(X[:, 0], X[:, 1])

        # plot the mean of the data
        mean_d1, mean_d2 = X.mean(0)
        plt.plot(mean_d1, mean_d2, 'o', markersize=10, color='red', alpha=0.5)

        # calculate the covariance matrix
        Sigma = get_covariance(X)
        # calculate the eigenvector and eigenvalues of Sigma
        L, U = get_eigen(Sigma)

        plt.axis('equal')
        plt.arrow(mean_d1, mean_d2, U[0, 0], U[0, 1], width=0.01, color='green', alpha=0.5)
        plt.arrow(mean_d1, mean_d2, U[1, 0], U[1, 1], width=0.01, color='red', alpha=0.5)

Out[5]: <matplotlib.patches.FancyArrow at 0x7f9b8ce11da0>
```



What do you observe in the above plot? Which eigenvector corresponds to the smallest eigenvalue?

Write your answer here:

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

1.1.5 Task 4: Transform the data

Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new subspace. Transform all vectors in X in this new subspace by expressing all vectors in X in this new basis.

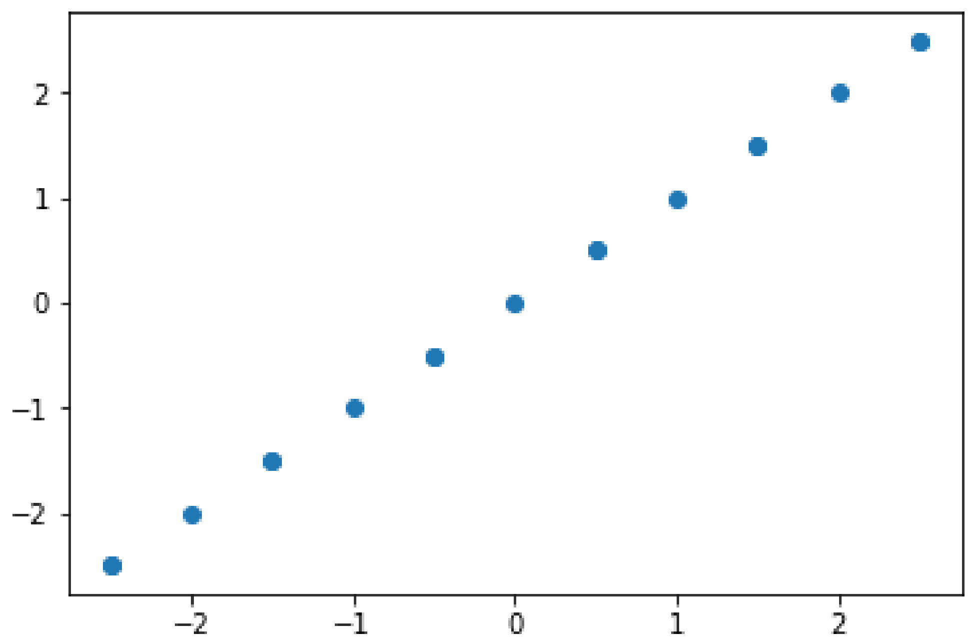
```
In [6]: def transform(X, U, L):
            """Transforms the data in the new subspace spanned by the eigenvector correspondin
            Parameters
            ----------
            X : array, shape [N, D]
                Data matrix.
            L : array, shape [D]
                Eigenvalues of Sigma_X
            U : array, shape [D, D]
                Eigenvectors of Sigma_X

            Returns
            -------
            X_t : array, shape [N, 1]
                Transformed data
```

```
        """
        Xtilde = X - np.sum(X, axis=0)/X.shape[0]
        g = U[np.argmax(L), :]
        return Xtilde.dot(g)
```

In [7]:
```
X_t = transform(X, U, L)
```

In [8]:
```
# Transform the reduced points back to the original space to validate the results
v = U[np.argmax(L), :]
Xnew = np.array([y * v for y in X_t])
plt.scatter(Xnew[:,0], Xnew[:,1])
plt.show()
```



1.2 Task SVD

1.2.1 Task 5: Given the matrix M find its SVD decomposition $M = U \cdot \Sigma \cdot V$ and reduce it to one dimension using the approach described in the lecture.

In [9]:
```
M = np.array([[1, 2], [6, 3],[0, 2]])
```

In [10]:
```
def reduce_to_one_dimension(M):
    """Reduces the input matrix to one dimension using its SVD decomposition.
    Parameters
    ----------
    M : array, shape [N, D]
        Input matrix.
```

```
        Returns
        -------
        M_t: array, shape [N, 1]
            Reduce matrix.
        """
        U,S,V = np.linalg.svd(M)
        return M.dot((V.T)[0,:])
```

In [11]:
```
M_t = reduce_to_one_dimension(M)
print(M_t)
```

```
[-1.90211303 -6.68109819 -1.05146222]
```