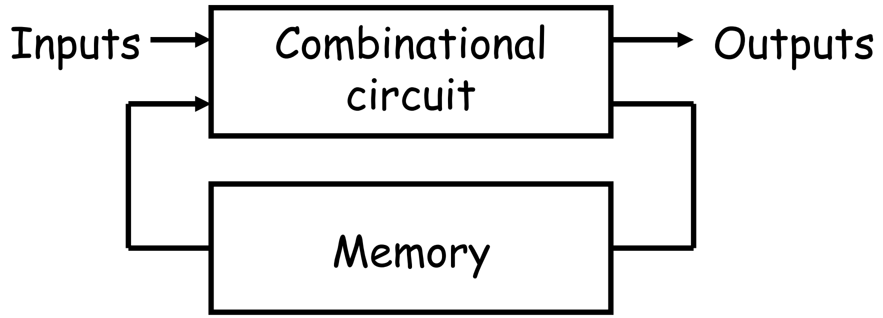
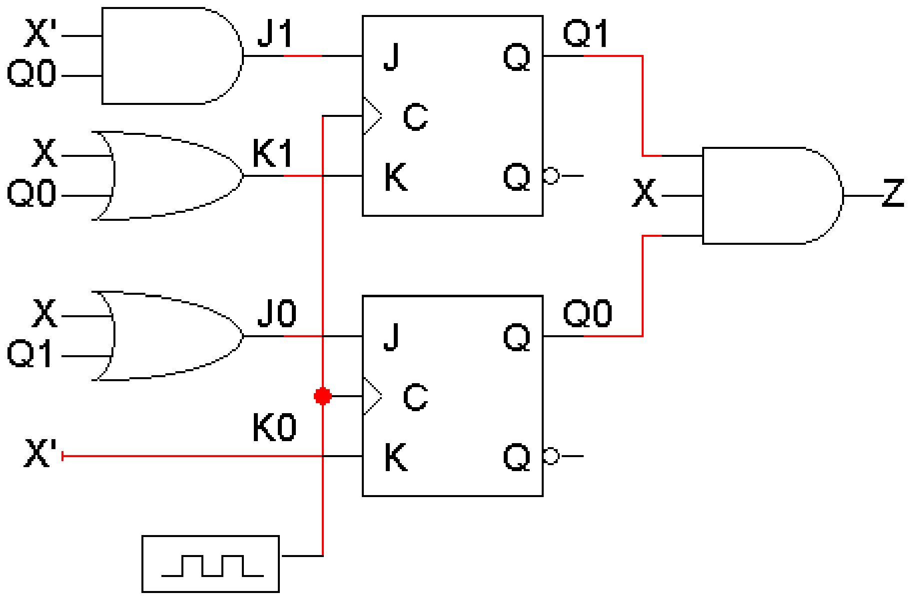
Sequential Circuit Analysis

Mealy or Moore???

- The output of a sequential circuit can be expressed in two different ways:
  - Moore model: Outputs= f(present state)
  - Mealy model: Outputs: f(present state, inputs)



An example sequential circuit

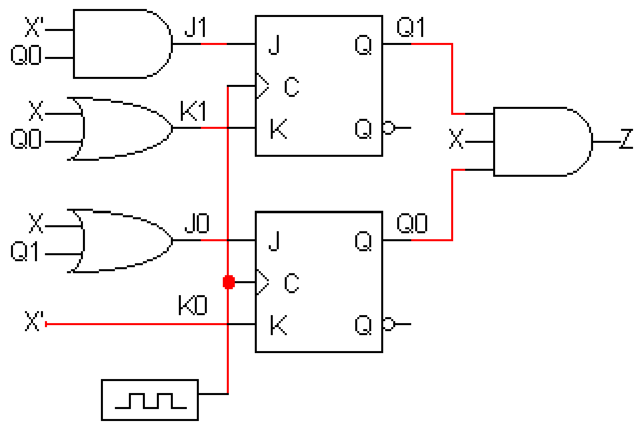


- A sequential circuit with two JK flip-flops
- State or memory: Q1Q0
- One input: X; One output: Z

How do you describe a sequential circuit?

- A combinational circuit - Truth table, which shows how the outputs are related to the inputs
- A sequential circuit - State table, which shows inputs *and* current states on the left, and outputs *and* next states on the right
  - Need to find the next state of the FFs based on the present state and inputs
  - Need to find the output of the circuit as a function of
    - current state for a circuit of the Moore model
    - current state and inputs for a circuit of the Mealy model

State table of example circuit



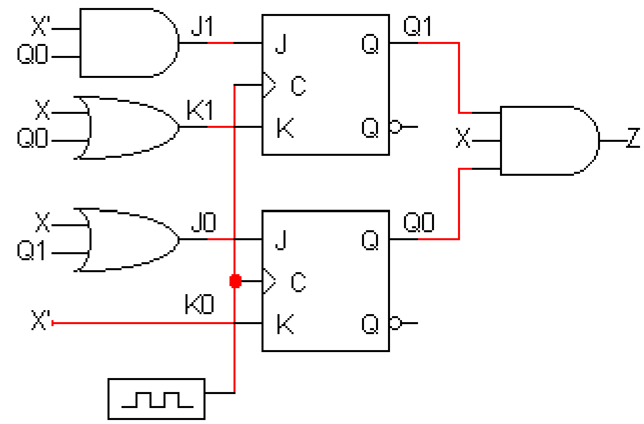
Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

The outputs are easy

- From the diagram, you can see that

$$Z = Q_1 Q_0 X$$

Mealy model circuit !!!



Present State Q_1	Q_0	Inputs X	Next State Q_1	Q_0	Outputs Z
0	0	0			0
0	0	1			0
0	1	0			0
0	1	1			0
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1

Flip-flop input equations

- Finding the next states is harder

  Step 1:
  Find Boolean expressions for the flip-flop inputs
  i.e., How do the inputs (say, J & K) to the flip-flops depend on the current state and input

  Step 2:
  Use these expressions to find the actual flip-flop input values for each possible combination of present states and inputs
  i.e., Fill in the state table (with new intermediate columns)

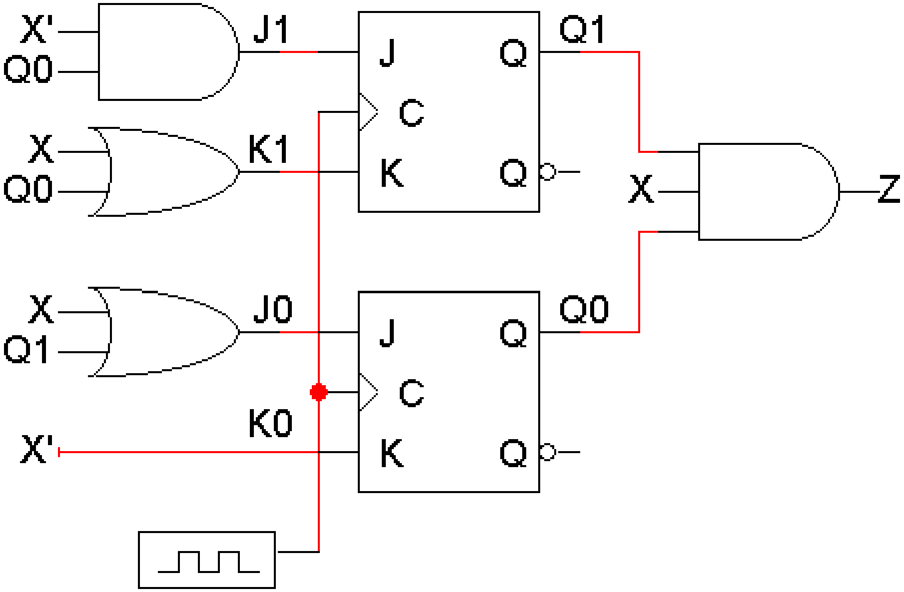
  Step 3:
  Use flip-flop characteristic tables or equations to find the next states, based on the flip-flop input values and the present states

Step 1: Flip-flop input equations

- For our example, the flip-flop input equations are:

$$J_1 = X' \, Q_0$$
$$K_1 = X + Q_0$$

$$J_0 = X + Q_1$$
$$K_0 = X'$$



Step 2: Flip-flop input values

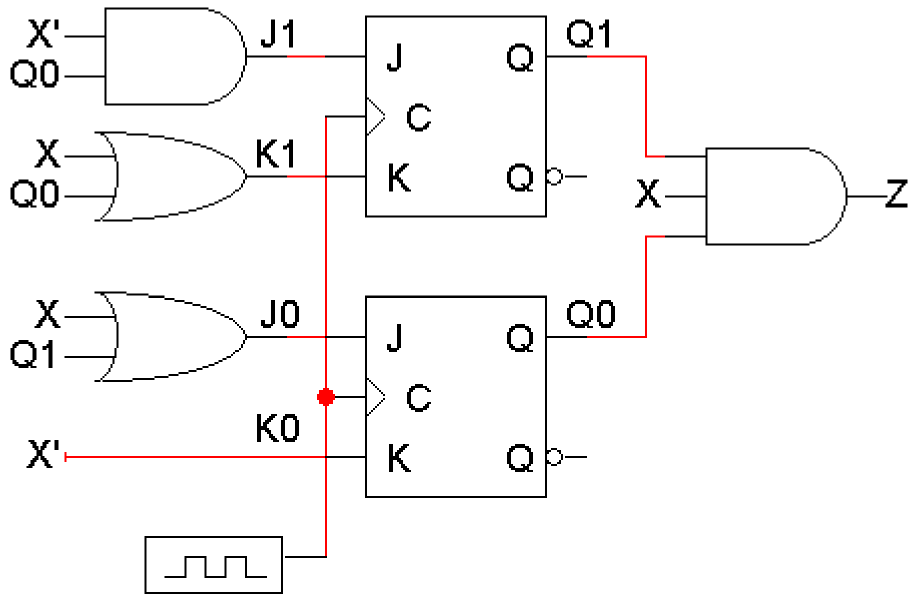
- With these equations, we can make a table showing J_1, K_1, J_0 and K_0 for the different combinations of present state Q_1Q_0 and input X

$J_1 = X' Q_0$ $\qquad$ $J_0 = X + Q_1$

$K_1 = X + Q_0$ $\qquad$ $K_0 = X'$

Present State		Inputs	Flip-flop Inputs			
Q_1	Q_0	X	J_1	K_1	J_0	K_0
0	0	0	0	0	0	1
0	0	1	0	1	1	0
0	1	0	1	1	0	1
0	1	1	0	1	1	0
1	0	0	0	0	1	1
1	0	1	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0

Step 2: Flip-flop input values



Present State		Inputs	Flip-flop Inputs			
Q_1	Q_0	X	J_1	K_1	J_0	K_0
0	0	0	0	0	0	1
0	0	1	0	1	1	0
0	1	0	1	1	0	1
0	1	1	0	1	1	0
1	0	0	0	0	1	1
1	0	1	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0

Step 3: Find the next states

- Finally, use the JK flip-flop characteristic tables or equations to find the next state of *each* flip-flop, based on its present state and inputs

- The general JK flip-flop characteristic equation is:

$$Q(t+1) = K'Q(t) + JQ'(t)$$

- In our example circuit, we have two JK flip-flops, so we have to apply this equation to *each* of them:

$$Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$$
$$Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$$

- We can also determine the next state for each input/current state combination directly from the characteristic table

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

Step 3 concluded

- The next states for Q_1 and Q_0, are calculated using these equations:

$$Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$$
$$Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$$

Present State		Inputs	FF Inputs				Next State	
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0
0	0	0	0	0	0	1		
0	0	1	0	1	1	0		
0	1	0	1	1	0	1	1	
0	1	1	0	1	1	0		
1	0	0	0	0	1	1		
1	0	1	0	1	1	0		
1	1	0	1	1	1	1		
1	1	1	0	1	1	0		

Step 3 concluded

- Using the characteristic equations:

  $Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$
  $Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$

Or the characteristic table

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	Q'(t)

Present State		Inputs	FF Inputs				Next State	
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0
$\boxed{0}$	0	0	0	0	0	1	0	
0	0	1	0	1	1	0		
0	1	0	1	1	0	1		
0	1	1	0	1	1	0		
1	0	0	0	0	1	1		
1	0	1	0	1	1	0		
$\boxed{1}$	1	0	1	1	1	1	0	
1	1	1	0	1	1	0		

Step 3 concluded

- Finally, here are the next states for Q_1 and Q_0, using these equations:

$$Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$$
$$Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$$

Present State		Inputs	FF Inputs				Next State	
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0
0	0	0	0	0	0	1	0	0
0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	1	0
0	1	1	0	1	1	0	0	1
1	0	0	0	0	1	1	1	1
1	0	1	0	1	1	0	0	1
1	1	0	1	1	1	1	0	0
1	1	1	0	1	1	0	0	1

Getting the state table columns straight

- The table starts with Present State and Inputs
  - Present State and Inputs yield FF Inputs
  - Present State and FF Inputs yield Next State, based on the flip-flop characteristic tables
  - Present State and Inputs yield Output

Present State		Inputs	FF Inputs				Next State		Outputs
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0	Z
0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	1	0	0	1	0
0	1	0	1	1	0	1	1	0	0
0	1	1	0	1	1	0	0	1	0
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0	1	0
1	1	0	1	1	1	1	0	0	0
1	1	1	0	1	1	0	0	1	1

A different look

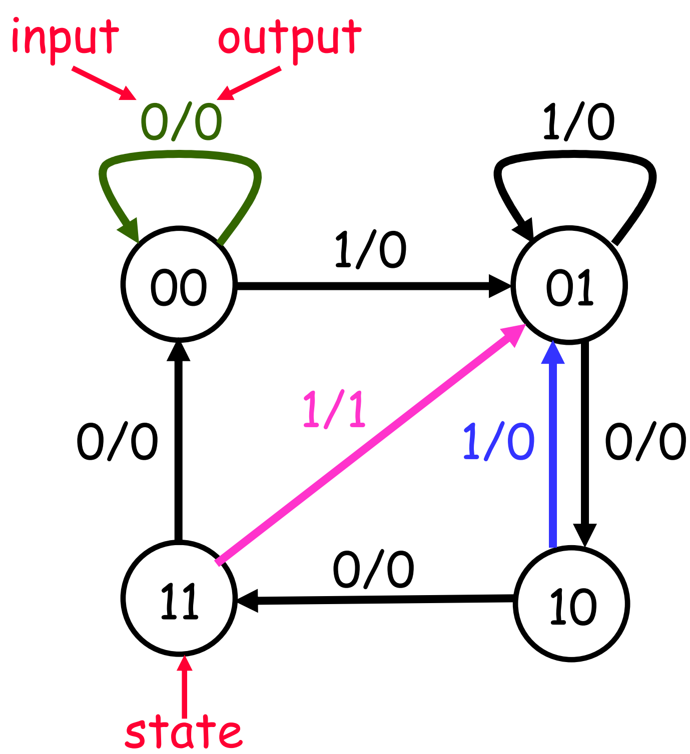
Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1

Present State		Next State				Output Z	
		Input X= 0		Input X= 1			
Q1	Q0					X= 0	X= 1
0	0	0	0	0	1	0	0
0	1	1	0	0	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	0	1	0	1

State diagrams (Mealy model)

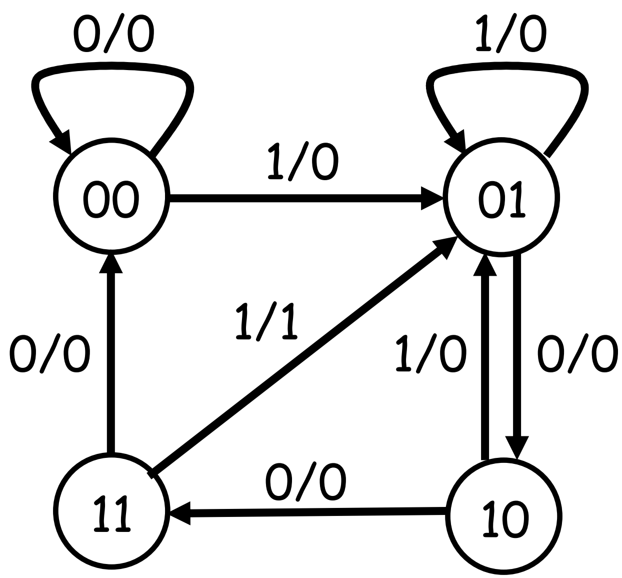
- We can also represent the state table graphically with a state diagram
- A diagram corresponding to our example state table is shown below

Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1

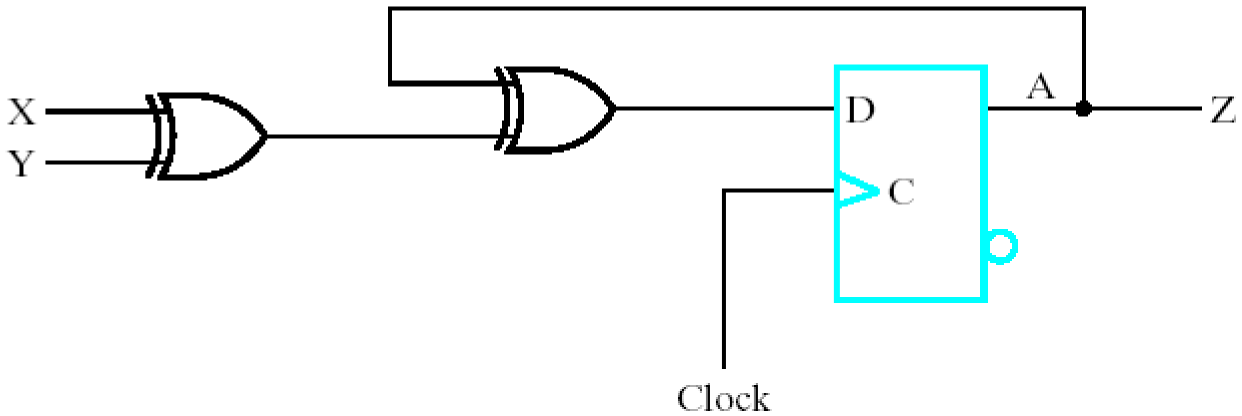


Sizes of state diagrams

- Always check the size of your state diagrams
  - If there are *n* flip-flops, there should be 2^n nodes in the diagram
  - If there are *m* inputs, then each node will have 2^m outgoing arrows
- In our example,
  - We have two flip-flops, and thus four states or nodes.
  - There is one input, so each node has two outgoing arrows.



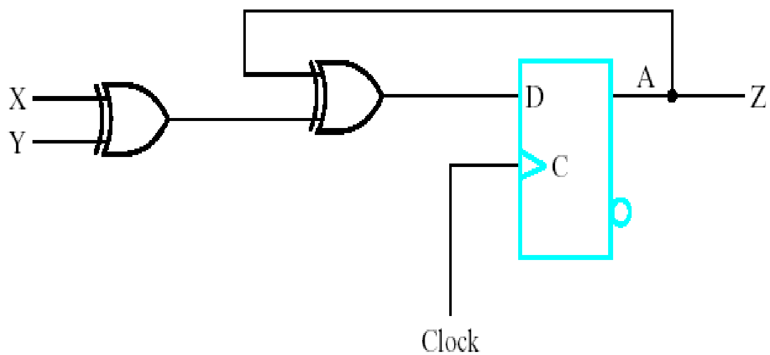
Moore type circuit



(a)

- Two inputs: X and Y; One output: Z
- One state: A
- Note that Z= A, just a function of the current state

State table (Moore)



(a)

Present state A	Inputs X	Inputs Y	Next state A	Output Z
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

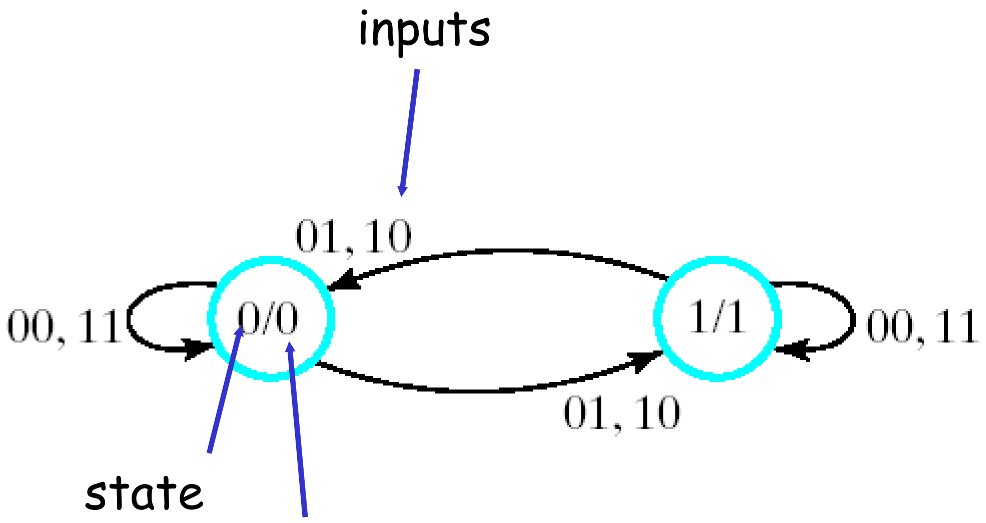
(b) State table

Present State A	Next State Inputs XY 00	01	10	11	Output Z
0	0	1	1	0	0
1	1	0	0	1	1

State diagram (Moore)

Present state	Inputs		Next state	Output
A	X	Y	A	Z
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(b) State table



(b)

Sequential circuit analysis summary

- To analyze sequential circuits, you have to:
  - Find Boolean expressions for the outputs of the circuit and the flip-flop inputs
  - Use these expressions to fill in the output and flip-flop input columns in the state table
  - Finally, use the characteristic equation or characteristic table of the flip-flop to fill in the next state columns.
- The result of sequential circuit analysis is a state table or a state diagram describing the circuit