CSE1003
Digital Logic and Design

Module 1 Introduction
Lecture 1

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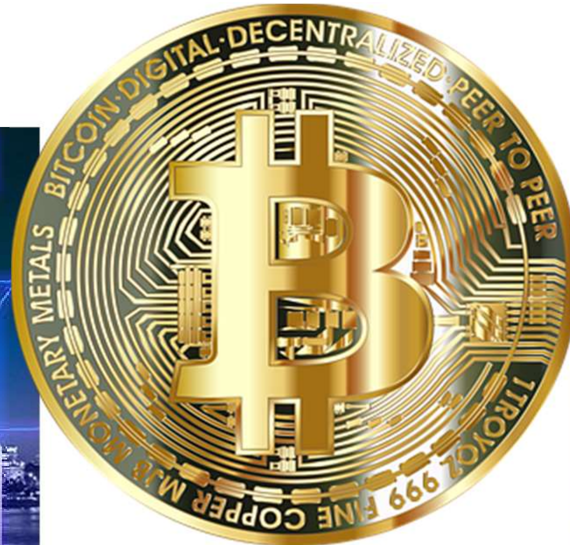
Module 1 Introduction 3 hours

- Number System
- Base Conversion
- Binary Codes
- Complements (Binary and Decimal)

Digital Systems

- Digital age and information age
- Digital computers
  - general purposes
  - many scientific, industrial and commercial applications
- Digital systems
  - telephone switching exchanges
  - digital camera
  - electronic calculators, PDA's
  - digital TV
- Discrete information-processing systems
  - manipulate discrete elements of information


BITCOIN·DIGITAL·DECENTRALIZED·PEER TO PEER
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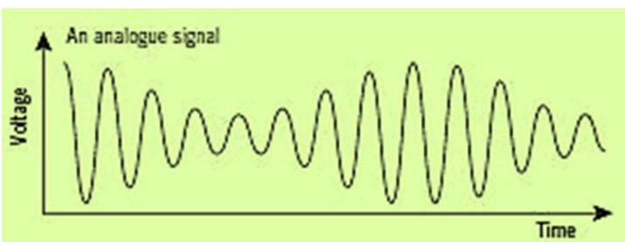
Signals

$$V(t) = \begin{cases} 1 & 0 < t < t_1 \\ = 0 & t_1 < t < t_2 \end{cases}.$$

Analog signal

- Is a continuous signal
- Any voltage level is possible at any time
- Explicit formula
  - E.g V(t)=f(t, parameters)
- Graphical representation of the signal



$v(t)$

$$v(t) = L \frac{di(t)}{dt}$$

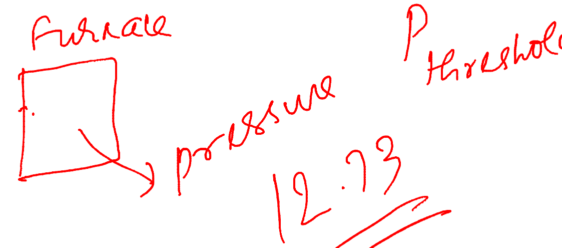
Digital signal

- Is a discrete time signal
- Discrete number of voltage levels are possible at specified time
- conveyed by the **on /off** *states* of pulses in a pulse train
- **ON** or the **off** state is a *bit,* and the time interval for the **on** or **off** state is called a *bit interval.*
- Algorithm
  - Set of conditions and operations
- V(t) can be 1 or 0

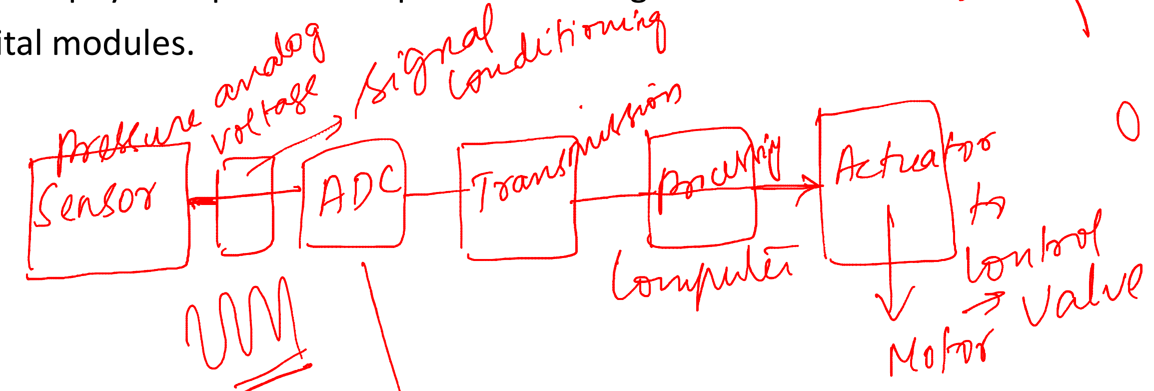
Signal

- An information variable represented by physical quantity
- For digital systems, the variable takes on discrete values
  - Two level, or binary values are the most prevalent values
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - words On and Off
- Binary values are represented by values or ranges of values of physical quantities

Digital System



- Digital systems contain devices that process the physical quantities represented in digital form.
- A digital system is an interconnection of digital modules.
- Digital techniques and systems -advantages
  - easier to design
  - have higher accuracy
  - easy programmability
  - noise immunity
  - easier storage of data
  - ease of fabrication in integrated circuit form, leading to availability of more complex functions in a smaller size.



- The real world, however, is analogue.
- Analogue variables are digitized at the input with the help of an analogue-to-digital converter block and reconverted back to analogue form at the output using a digital-to-analogue converter block.
- **To understand the operation of each digital module, it is necessary to have a basic knowledge of digital circuits and their logical function.**

Number Systems

Decimal → 10 base
Binary → 2
Octal → 8
Hexadecimal → 16

- Representation of numbers
- Radix: "base", the primitive unit for group of numbers, e.g. for decimal arithmetic radix=10 ("base" 10)
- For every system, we need arithmetic operations (addition, subtraction, multiplication)
- Also, conversion from one base to the other

Introduction to Number Systems

Positional Notation

- A positive number N can be written as:

$$N = (a_{n-1}a_{n-2}a_{n-3}\ \ldots .\ a_1a_0.a_{-1}a_{-2}\ \ldots .\ a_{-m})_r$$

MSD LSD

. = radix point
r = radix or base of number system
n = number of integer digits to the left of radix point
m = number of fractional digits to the right of radix point
a_i = integer digit i 0 to n−1
a_j = fractional digit j → −1 to −m
a_{n-1} = most significant digit
a_{-m} = least significant digit

1325
$a_3\ a_2\ a_1\ a_0$

$1325_{10} = 1\times10^3 + 3\times10^2 + 2\times10^1 + 5\times10^0$
$1000 + 300 + 20 + 5 = 1325$

Polynomial Notation

- A positive number N can also be written as:

$$N = \sum_{i=-m}^{n-1} a_i r^i$$

where a_i is a coefficient between 0 to 9, and i denotes the weight $(=10^i)$ of a_i

Common Number Systems

Decimal Number System

- Radix or Base 10
- Digits: 0,1,2,3,4,5,6,7,8,9
- Example: 1045_{10}
- First 17 positive integers
- 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16

Octal Number System

- Radix or Base 8
- Digits: 0,1,2,3,4,5,6,7
- Example: 137.21_8
- First 17 positive integers
- 0,1,2,3,4,5,6,7, 10,11,12,13,14,15,16,17,20

Binary Number System

- Radix or Base 2
- Digits: 0, 1
- Example: 1010110_2
- First 17 positive integers
- 0,1,10,11,100,101,110,111, 1000,1001,1010,1011,1100 ,1101,1110,1111,10000

17

000
001
010
011
100
101
110

Hexadecimal Number System

- Radix or Base 16
- Digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D, E,F
- Example: $EF56_{16}$
- First 17 positive integers
- 0,1,2,3,4,5,6,7,8,9,A,B,C,D, E,F,10

Summary

Dec	Bin	Hex	Octal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7

Dec	Bin	Hex	Octal
8	1000	8	10
9	1001	9	11
10	1010	A	12
11	1011	B	13
12	1100	C	14
13	1101	D	15
14	1110	E	16
15	1111	F	17

Binary Arithmetic

Binary Addition

- Single Bit Addition Table

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 10 Note “carry”

carry sum

$$
\begin{array}{r}
\scriptstyle 1\ 1\ 1\ \ \ \\
1\,1\,0\,1 \\
+\,0\,1\,1\,1 \\
\hline
1\,0\,1\,0\,0
\end{array}
$$

Binary Subtraction

- Single Bit Subtraction Table

0 - 0 = 0

1 - 0 = 1

1 - 1 = 0

0 - 1 = 1 with a “borrow”

$$
\begin{array}{r}
\scriptstyle 0\ \ 0\ \ \ \ \ \ \ \\
\cancel{1}\,\cancel{1}\,0\,1 \\
-\,0\,1\,1\,1 \\
\hline
0\,1\,1\,0
\end{array}
$$

Binary Arithmetic

Binary Multiplication

- Single Bit Multiplication Table

0 x 0 = 0

0 x 1 = 0

1 x 0 = 0

1 x 1 = 1

```
     1 1 0 1
   x 0 1 1 1
   ---------
     1 1 0 1
   1 1 0 1
 1 1 0 1
0 0 0 0
-------------
1 0 1 1 0 1 1
```

Binary Division

- Single Bit Division Table

0 / 0 = N/A

0 / 1 = 0

1 / 0 = N/A

1 / 1 = 1

110 ÷ 11

```
     10
11)110
    11
   ---
   000
```

Conversion Methods - Series substitution

Expand number in original base using

$$N = \sum_{i=-m}^{n-1} a_i r^i$$

$$N = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} +_{.} \dots . + a_1 r^1 + a_0 + \cdots + a_{-m} r^{-m}$$

Most Significant Bit (MSB)

Least Significant Bit (MSB)

convert from

Binary to Decimal

$(1001.0101)_2 = 9.3125_{10}$

integer

fractional

$1001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$2^0 = 1$

$= 8 + 0 + 0 + 1 = 9$

$.0101 = 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} = 0 + 0.25 + 0 + 0.0625 = 0.3125$