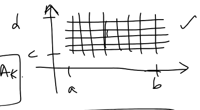
Fubinis Theorem

Monday, 22 March 2021 12:00 PM

Riemann Sum

$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta A_k$



$$\boxed{\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k, y_k) \cdot \Delta A_k}$$

Can be defined through

$$\boxed{= \iint_R f(x,y) \cdot dy\, dx}$$

$\Updownarrow$ equal

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k, y_k) \cdot \Delta A_k$$

$$\text{Volume} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k, y_k)\Delta A_k = \iint_R f(x,y)\, dy\, dx$$

Fubini's Theorem:-

$$\iint_R f(x,y) \cdot \underline{dy}\, dx = \iint_R f(x,y)\, \underline{dx \cdot dy}$$

Theorem:

If $f(x,y)$ - Continuous throughout the rectangular region

$$R: \begin{cases} a \le x \le b \\ c \le y \le d \end{cases} \quad \text{then}$$

$$\iint_R f(x,y) \cdot dA = \int_a^b \boxed{\int_c^d f(x,y) \cdot dy} \cdot dx$$

$$= \int_c^d \boxed{\int_a^b f(x,y) \cdot dx} \cdot dy$$

Problem:

Calculate the volume under the plane

$$z = 4 - x - y \text{ over } R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 1 \end{cases} \text{ in}$$

xy plane

Soln $\int^{x=2} \underline{A(x)} \cdot dx$

$= \int_{y=0}^{y=1} A(y) \cdot dy \to ①$

Soln $\int_{x=0} \underline{\underline{A(x)}} \, dx$

$$A(x) = \int_{y=0}^{y=1} 4 - x - y \, dy$$

$$\int_{x=0}^{x=2} \left[\int_{y=0}^{y=1} \underline{4 - x - y \, dy} \right] dx$$

$$\int_{x=0}^{x=2} \left[4y - xy - y^2/2 \right]_0^1 dx$$

$$\int_{x=0}^{x=2} \left[\left(\underline{4(1)} - x(1) - (1)^2/2 \right) - \left(\frac{4(0) - x(0) - \frac{0}{2}}{0} \right) \right] dx$$

$$= \int_{x=0}^{x=2} \left[4 - x - 1/2 - 0 \right] dx$$

$$= \left[4x - \frac{x^2}{2} - \frac{x}{2} \right]_0^2$$

$$= \left[8 - 4/2 - 2/2 \right]$$

$$= \left[8 - 2 - 1 \right]$$

$$= 5 //$$

$$y = 0$$

$$= \int_{x=0}^{x=2} 4 - x - y \, dx \quad (2)$$

$$= \int_0^1 \left[\int_0^2 4 - x - \ldots \right.$$

$$= 5 //$$

$$\iint_R f(x,y) \, dA = \int_a^b \int_c^d f(x,y) \, \underline{dy} \, dx \qquad a \le x \le b$$

$$= \int^d \int^b f(x,y) \, dx \, dy \qquad c \le y \le d$$

$$y \cdot dx \Big) \cdot dy$$

R

$$= \int_c^d \int_a^b f(x,y)\, dx\, dy$$

Note: $\int_a^b \int_c^d \underline{f(x,y)}\, dy\, dx = \int_a^b g(x)\, dx$

Problem:

Calculate vol. of a regi[on]
by surface $z = 2\sin$[…]
and bounded below by

region
$R: \{(x,y) \in \mathbb{R}^2 / 0 \leq x$[…]
$0 \leq$[…]

$$\text{Vol. of Region} = \int_a^b \int_c^d f(x,y$$[…]

$$= \int_a^b \int_c^d 2\sin x$$[…]

$$= 2\int^b \sin x\, dx$$

$$\ast \int_c^d h(x)\, dx$$

on bounded

$nx \cdot \cos y$

the rectangular

$$\le \pi/2$$

$$y \le \pi/4$$

$) \, dy \, dx$

$\cos y \, dy \, dx$

$$\int_c^d \cos y \cdot dy \checkmark$$

$$= 2\int_0^{\pi/2} \underline{\sin x \, dx} \int_0^{\pi}$$

$$= 2(-\cos x)_0^{\pi/2} \; (S$$

$$= 2\left(-\cos\left(\pi/2\right)+\cos(0)\right)\left(\sin\right.$$

$$= 2(-0+1)\;\left(\frac{1}{\sqrt{2}} - 0\right.$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2} \,//$$

<u>Alternatively</u> $\int_0^{\pi/2} \boxed{\int_0^{\pi/}}$

$$= \int_0^{\pi/2} \underline{2\sin x} \int_0$$

$$= \sqrt{2} \,//$$

<u>H.W</u>

$$\iint$$

$/4$ $\cos y \cdot dy$

$iny)_0^{\pi/4}$

$(\pi/4) - \sin(0))$

$)$

$/4$ $2\sin x \cdot \underline{\cos y \cdot dy} \cdot dx$

$\pi/4$ $\cos y \cdot dy \cdot dx$

$\underline{w}$

$, \ln | D \cdot \{ x \in [-\pi, 0]$

① $\iint_R y \sin(x$

Ans: 4

② $\iint_R \frac{x y^3}{x^2+1} \, dA$

③ $\iint_R x y e^{xy}$

Fubini

Evalu: $\iint_R$

where R —

by the x —

lin

$+y) \, dA \;\Big|\; R: \begin{cases} x \in [-\pi, 0] \\ y \in [0, \pi] \end{cases}$

$\Big|\; R: \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 2 \end{cases} = \Big\}$

${}^{2} \; dA \;\Big|\; R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 1 \end{cases} = \Big\}$

's Strong theorem

$f(x,y) \, dA$

rectangular region bounded

axis, line $y = x$ &

ne $x = 1$

$$\iint_R f(x,y)\,dA = \int_a^b$$

$$\int_c^d$$

Eval: $\iint_R \frac{\sin x}{x}$

bounded by

$$\int_0^1 \int_{y=0}^{y=x} \frac{\sin}{}$$

$$= \int_0^1$$

$$= \int_0^1$$

$$= \int^1 ($$

$$\int^{y=h_2(x)}_{y=h_1(x)} f(x,y)\, dy\, dx$$

$$\int^{x=g_2(x)}_{x=g_1(y)} f(x,y)\, dx\, dy$$

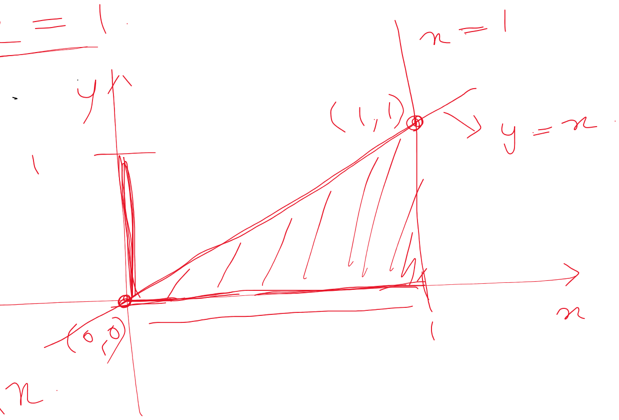
Problem

where R is a triangle

x - axis, line $y = x$ and

$y = 0$

line $x = 1$



①

$$\frac{\sin x}{x}\, dy\, dx$$

$$\int^{y=x}_{y=0} \frac{\sin x}{x}\, dy\, dx$$

$$\left[\frac{\sin x}{x} \cdot y\right]_0^{y=x} dx$$

$$\left(\frac{\sin x}{x} \cdot x - 0\right) dx$$

Evaluate:

Sketch

$$\int_a^b \int_{y=h_1(}^{y=h}$$

$$\int_c^d$$

$\frac{\sin}{x} \cdot x - 0)$

$\int^1 \sin x \, dx$

$(-\cos x)\Big|_0^1$

$= -\cos(1) + \cos(0)$

$= -0.54\ldots + 1$

$= 0.46\ldots$ //

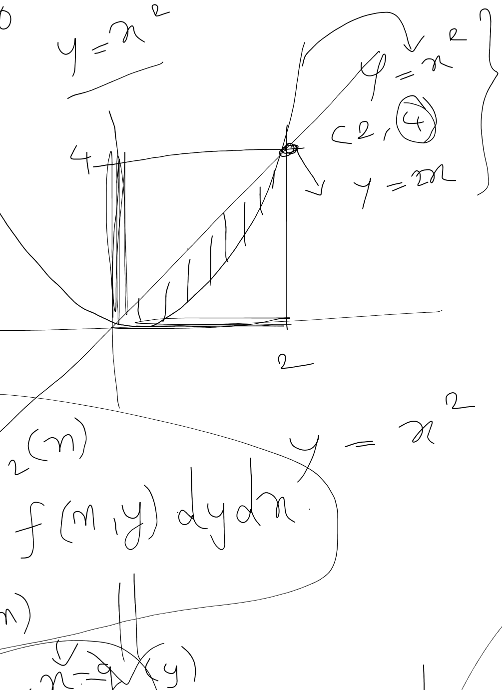
Problem 2

$\int_0^2 \int^{2x} 4x + 2 \; dy \, dx$ (Ans 8)

$y = x^2$

$2x = x^2$

$\boxed{\begin{matrix} x = \pm\sqrt{y} \\ x = y/2 \end{matrix}}$



$x = \pm\sqrt{y}$

$y = x^2$ into

$x = \pm\sqrt{y}$

$\int \int^{g_2(x)} f(x, y) \, dy \, dx$

$\int_{x = g_1(y)}^{x = g_2(y)} f(x, y) \, dx \, dy$

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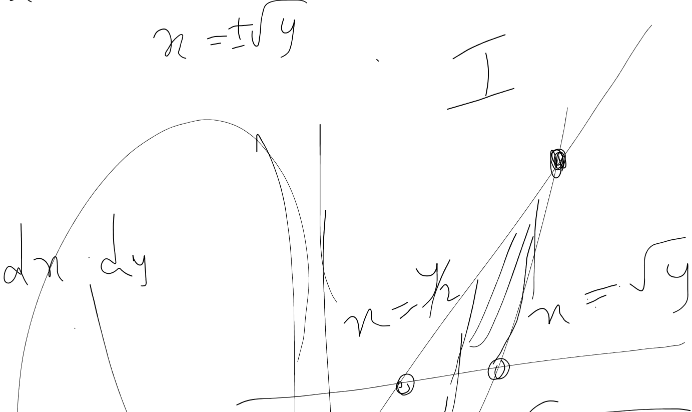

$x = g_1(y)$

$$\int_{x=y/2}^{x=\sqrt{y}} f(x,y)\,dx\,dy$$