Electromagnetic Wave equation

- EM wave eqn can be derived by using Maxwell's equation for EM wave propagation through a homogeneous, isotropic dielectric medium
- As the dielectric medium offers infinite resistance to the electric current – conductivity 's' is zero   i.e., $J = 0$

  ($J = sE$)
- In homogeneous isotropic medium, there is no volume distribution of charge, thus the charge density is Zero.

  hence $J = 0$; $r = 0$; $D = e_0 e_r E$ and $B = m_0 m_r H = mH$

- Hence Maxwell's equation for a dielectric medium becomes

$$\nabla \bullet E = 0 \longrightarrow \quad (1)$$

$$\nabla \bullet B = 0 \longrightarrow \quad (2)$$

$$\nabla x E = -\frac{\partial B}{\partial t} \longrightarrow \quad (3)$$

$$\nabla x B = \mu\varepsilon \frac{\partial E}{\partial t} \longrightarrow \quad (4)$$

Taking Curl of eqn (4), we get

$$\nabla \mathrm{x} \nabla \mathrm{x} B = \nabla \mathrm{x} \mu\varepsilon \frac{\partial E}{\partial t}$$

$$= \mu\varepsilon \left(\nabla \mathrm{x} \frac{\partial E}{\partial t} \right)$$

$$= \mu\varepsilon \frac{\partial}{\partial t} \left(\nabla \mathrm{x} E \right)$$

since $\nabla \mathrm{x} E = -\frac{\partial B}{\partial t}$

$$= \mu\varepsilon \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t} \right) = -\mu\varepsilon \frac{\partial^2 B}{\partial t^2} \longrightarrow \quad (5)$$

- We have $$\nabla \mathrm{x} \nabla \mathrm{x} B = \nabla(\nabla \bullet B) - \nabla^2 B$$

$$\nabla(0) - \nabla^2 B$$

therefore

$$\nabla \mathrm{x} \nabla \mathrm{x} B = -\nabla^2 B \longrightarrow \qquad (6)$$

Substituting eqn (6) in (5) we get

$$-\nabla^2 B = -\mu\varepsilon\left(\frac{\partial^2 B}{\partial t^2}\right)$$

or

$$\nabla^2 B = \mu\varepsilon\left(\frac{\partial^2 B}{\partial t^2}\right) \longrightarrow \qquad (7)$$

Similarly from eqn (3) we can show that

$$\nabla^2 E = \mu\varepsilon\left(\frac{\partial^2 E}{\partial t^2}\right) \longrightarrow \qquad (8)$$

Eqn (7 & 8) represents the relation between the space and time variation of magnetic field B and electric field E. –Wave equations for B and E resp.

- The above eqns are similar to general form of differential equation of wave motion – given by

$$\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \longrightarrow \qquad (9)$$

where v is the velocity of the wave and y in its amplitude

Comparing eqn (8 & 9) $\mu\varepsilon$ and $1/v^2$ has the same significance

So we find that variations of E and B are propagated in homogeneous, isotropic medium with a velocity given by

$$1/v^2 = \mu\varepsilon \text{ or } v^2 = 1/\mu\varepsilon$$

Therefore $\quad v = 1/(\mu\varepsilon)^{1/2}$

Where μ and ε are permeability and permittivity of the medium

For free space $\quad v = 1/(\mu_0\varepsilon_0)^{1/2} = 3 \times 10^8$ m/s