Relation between Einstein's A and B Coefficient

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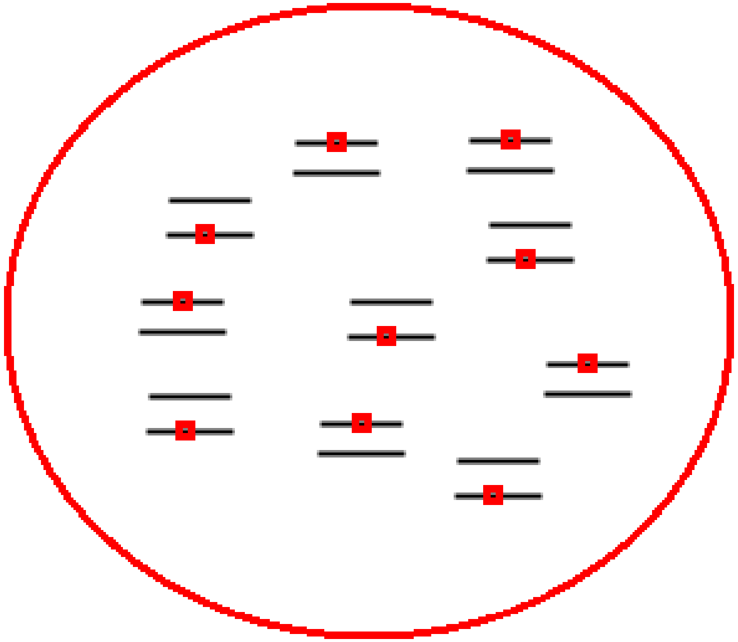
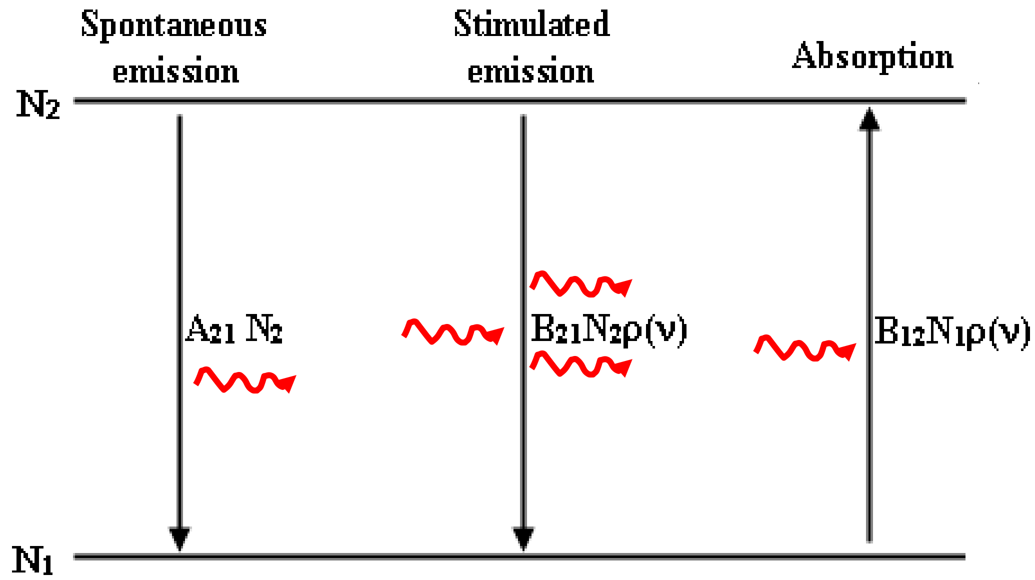
- In thermal equilibrium at temperature T, with radiation frequency n and energy density Q. Let N_1 and N_2 be the number of atoms in energy states 1and 2 respectively at any instant. The number of atoms in state 1 absorb a photon and give rise to absorption per unit time
- Under thermal equilibrium transitions from E_1 to E_2 should be equal to transitions from E_2 to E_1
- No. of atoms absorbing photons per unit volume = the number of atoms emitting photons per second

- Einstein's A and B coefficients





- Under thermal equilibrium transitions from E_1 to E_2 should be equal to transitions from E_2 to E_1

No. of atoms absorbing photons per unit volume = the number of atoms emitting photons per second

$$B_{12}\rho(\nu)N_1 = A_{21}N_2 + B_{21}\rho(\nu)N_2$$

$$B_{12}\rho(\nu)N_1 - B_{21}\rho(\nu)N_2 = A_{21}N_2$$

$$\rho(\nu)[B_{12}N_1 – B_{21}N_2] = A_{21}N_2$$

$$\rho(\nu) = A_{21}N_2 / [B_{12}N_1 – B_{21}N_2]$$

Taking $B_{21}N_2$ in the denominator as common

$$\rho(\nu) = A_{21}N_2 / B_{21}N_2 \{ 1 / [(B_{12}N_1 / B_{21}N_2) – 1] \}$$

$$\rho(\nu) = (A_{21} / B_{21}) \{ 1 / [(B_{12}N_1 / B_{21}N_2) – 1]\}$$

It was proved thermodynamically that $B_{12} = B_{21}$

Then

$$\rho(\nu) = (A_{21} / B_{21}) \{ 1 / [(N_1 / N_2) – 1]\}$$

Distribution of atoms under equilibrium among diff. states is given by Boltzmann's law according to which

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} \qquad \frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} \qquad \frac{N_2}{N_1} = e^{-(h\upsilon)/kT}$$

Therefore

$$\rho(\nu) = (A_{21}/B_{21}) [1 / (e^{h\nu/kT} - 1)]$$

The above equation is the energy density of photon of frequency ν in equilibrium with atoms in energy states 1 and 2 at temperature T

comparing with Plank's radiation formula

$$\rho(\nu) = (8\pi h\nu^3/c^3) [1 / (e^{h\nu/kT} - 1)]$$

We get

$$A_{21}/B_{21} = 8\pi h\nu^3/c^3$$

The above equation gives the relation between Einstein's A and B coefficients

Significance

- $B_{12} = B_{21}$, The probability of spontaneous emission is same as that of induced absorption. This means that if these two processes will occur at equal rates, so that no population inversion can be attained in a two-level system.

- The ratio of spontaneous emission and stimulated emission is proportional to v^3. This implies that the probability of spontaneous emission dominates over induced emission more and more as the energy difference between the two states increases.