The time to repair a power generator is best described by its *pdf*

$$m(t) = \frac{t^2}{333}, \quad 1 \le t \le 10 \text{ hours}$$

(a) Find the probability that a repair will be completed in 6 hours.
(b) What is the MTTR?
(c) Find the repair rate.

(a) $P(T < 6) = P(1 \le T < 6)$, where T is the time to repair

$$= \int_1^6 m(t)\, dt$$

$$= \int_1^6 \frac{t^2}{333}\, dt = \left(\frac{t^3}{999}\right)_1^6 = 0.2152$$

(b) $$\text{MTTR} = \int_0^\infty t\,m(t)\, dt = \int_1^{10} \frac{t^3}{333}\, dt = \left(\frac{t^4}{4 \times 333}\right)_1^{10}$$

$$= 7.5 \text{ hours}$$

(c) $$\text{Repair rate} = \mu(t) = \frac{m(t)}{1 - M(t)}$$

$$= \frac{t^2 / 333}{\int_t^{10} \frac{t^2}{333}\, dt} = \frac{t^2 / 333}{\frac{1}{999}(10^3 - t^3)}$$

$$= \frac{3t^2}{1000 - t^3} \text{ per hour.}$$

A new computer has a constant failure rate of 0.02 per day (assuming continuous use) and a constant repair rate of 0.1 per day.

(a) Compute the interval availability for the first 30 days and the steady-state availability.

(b) Determine the steady-state availability if a standby unit is purchased. Assume no failures in standby.

(c) If both units are active, what is the steady-state availability?

$\lambda = 0.02/\text{day}$; $\mu = 0.1/\text{day}$.

(a)
$$A_I(T) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 \times T}\{1 - e^{-(\lambda+\mu)T}\}$$

$$\therefore\ A_I(30) = \frac{0.1}{0.12} + \frac{0.02}{(0.12)^2 \times 30}\{1 - e^{-0.12\times 30}\}$$

$$= 0.8784$$

$$A(\infty) = \frac{\mu}{\lambda+\mu} = \frac{0.1}{0.12} = 0.8333$$

(b) For the standby redundant system,

$$A_s(\infty) = \frac{\lambda\mu + \mu^2}{\mu^2 + \lambda\mu + \lambda^2} = \frac{0.002 + 0.01}{0.01 + 0.002 + 0.0004}$$

$$= 0.9677$$

For the active redundant system,

$$A_s(\infty) = 1 - \{1 - A(\infty)\}^2$$

$$= 1 - \{1 - 0.8333\}^2$$

$$= 0.9722.$$

The distribution of the time to failure of a component is Weibull with $\beta = 2.4$ and $\theta = 400$ hours and the repair distribution is lognormal with $t_M = 4.8$ hours and $s = 1.2$. Find the steady-state availability.

For the Weibull failure distribution,

$$\text{Mean} = \text{MTTF} = \theta\,\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$= 400\,\Gamma\left(1 + \frac{1}{2.4}\right)$$

$$= 400 \times \Gamma(1.42)$$

$$= 400 \times 0.88636$$

$$= 354.5 \text{ hours}$$

For the lognormal repair distribution,

$$\text{Mean} = \text{MTTR} = t_M \exp\left(s^2/2\right)$$

$$= 4.8 \times \exp\left\{(1.2)^2/2\right\}$$

$$= 9.86 \text{ hours}$$

$$A(\infty) = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

$$= \frac{345.5}{354.5 + 9.86} = 0.9729$$

Reliability testing has indicated that a voltage inverter has a 6 month reliability of 0.87 without repair facility. If repair facility is made available with an MTTR of 2.2 months, compute the availability over the 6-month period. (Assume constant failure and repair rates)

For constant failure rate λ, reliability is given by $R(t) = e^{-\lambda t}$.

As $\qquad R(6) = 0.87, \; e^{-6\lambda} = 0.87$

$\therefore \qquad \lambda = 0.0232/\text{month}$

$$\text{MTTR} = \frac{1}{\mu} = 2.2 \;\therefore\; \mu = 0.4545/\text{month}$$

Interval availability over $(0, T)$ is given by

$$A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T} \{1 - e^{-(\lambda + \mu)T}\}$$

$$A(6) = \frac{0.4545}{0.4777} + \frac{0.0232}{(0.4777)^2 \times 6} \{1 - e^{-0.4777 \times 6}\}$$

$$= 0.967.$$