Engineering Physics

Introduction to Modern Physics

NOTES

Topics:

1. Planck's concept (hypothesis)
2. Compton Effect
3. Particle properties of wave: Matter Waves,Davisson Germer Experiment, Heisenberg Uncertainty Principle, Wave function
4. Schrodinger equation (time dependent & independent).

Engineering Physics
(PHY 1701)

Module 1: Introduction to Modern Physics
Planck's concept (hypothesis), Crompton - Effect, Particle properties of wave: Matter waves, Davisson Germer Experiment, Heisenberg Uncertainity principle, Wave function, and Schrodinger equation (time dependent & independent).

1. Planck's concept (hypothesis)

Que. Write down Plank's black body radiation formulae, draw the energy density Vs frequency plot. Explain how plank's formula resolved the issue of ultraviolet catastrophe [CAT-1 VIT Feb 2017]

or

Que. Draw the spectrum of black body radiation. Explain the ultraviolet catstrope through proper diagram [CAT-1 VIT win sem 2016-17]

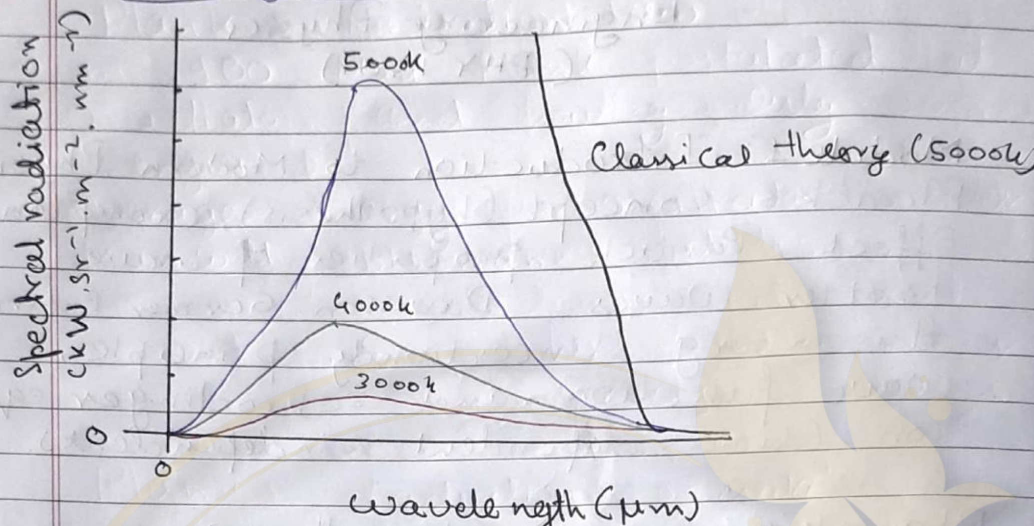
or

Que. Write down the considerations of Rayleigh and Jeans in deriving Rayleigh Jeans formula for explaining black body radiation spectrum. Discuss the shortcomings of their formula [CAT-1 VIT Fall sem 2017]

Solⁿ Black body

A black body is defined as a body which can absorb all the radiations incident on it. It then radiates energy depending on its equilibrium - temperature. The wavelength of black body radiation is independent of its shape

Blackbody radiation:



Observation - 1

As Temperature T of body increases, Intensity of radiation from body increases

- Stefan - Boltzman law:

$$I(T) = e\sigma T^4$$

e - emissivity (1 for ideal blackbody, 0.9 for practical cases)

σ - Stefan - Boltzman Constant, $\sigma = 5.67\times10^{-8}\ Wm^{-2}k^{-4}$

Observation - 2

Higher the temperature, lower is the wavelength of the most intense part of the spectrum

- Wien's displacement law

$$\lambda_{max} T = \text{Constant} = 2.898\times10^{-3}\ mK$$

Explanations for Blackbody Spectrum

~~1. We~~

1. Wien exponential law

- $u(\nu, T)$, Energy density $= A\nu^3 e^{-B\nu/T}$

where A & B are Constants

→ This explanation has good agreement at lower wavelength (higher frequency)
→ when measured to higher wavelength upto 60μm, experimental data was not fitting

2. Rayleigh Jeans law

Rayleigh Jeans assumed Blackbody to be Radiation filled cavity.

Number of standing wave per unit volume or Density of Standing wave is

$$G(\nu)d\nu = \frac{8\pi \nu^2 d\nu}{c^3}$$

According to classic mechanics, average energy at temperature T is 3/2 KT

Here we assume one dimensional cavity

wave can move ~~into~~ in two direction and 2 degrees of freedom.
Average energy in this case is KT

Rayleigh Jean Formula

$$u(\nu)\, d\nu = \frac{8\pi \nu^2 KT d\nu}{c^3}$$

Drawback of Rayleigh Jean

As per Rayleigh Jean
$\nu \to \infty$, $u(\nu)d\nu \to \infty$
but in reality $\nu \to \infty$, $u(\nu)d\nu \to 0$
Also it says that total energy density from $\nu = 0$ to $\nu = \infty$ is ∞ at all temperature

$\boxed{\text{This discrepancy is termed "Ultra Violet Catastrophe"}}$

Plank's Radiation law

- In 1900, Max Plank postulated that the oscillators and having only discrete energies
- Average energy per oscillator considering Maxwell Boltzmann statistics,

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

Plank's radiation Formula

$$u(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{c^3} \times \frac{1}{e^{h\nu/kT} - 1}$$

plank's radiation formula at limiting cases

Case 1: Higher frequency or lower wavelength,

$$\frac{h\nu}{kT} >> 1$$

$$\frac{1}{e^{h\nu/kT} - 1} = e^{-h\nu/kT}$$

or

$$u(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{c^3} e^{-h\nu/kT}$$

Which is Wien's exponential law,

$$\left(u(\nu, T) = A\nu^3 e^{-B\nu/T} \right)$$

Case 2 : low frequency or High wavelength

$$\frac{h\nu}{kT} << 1$$

• By Taylor series expansion,

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots$$

$$\frac{1}{e^{\frac{h\nu}{kT}} - 1} = \frac{1}{1 + \frac{h\nu}{kT} + \left(\frac{h\nu}{kT}\right)^2 \times \frac{1}{2} + \cdots - 1}$$

On further approximation

$$\frac{1}{e^{h\nu/kT} - 1} = \frac{kT}{h\nu} \quad \text{and}$$

$$u(\nu)d\nu = \frac{8\pi \nu^2 kT d\nu}{c^3}$$

Which is Rayleigh Jean Formula

2. Compton Effect

Que. Compton effect was explained by assuming elastic collision of a photon and free electron. Draw a neat diagram to depict the Collision event and write down the moment balance and energy balance equation
[5 marks, CAT-1 VIT Win Sem 2016-17]

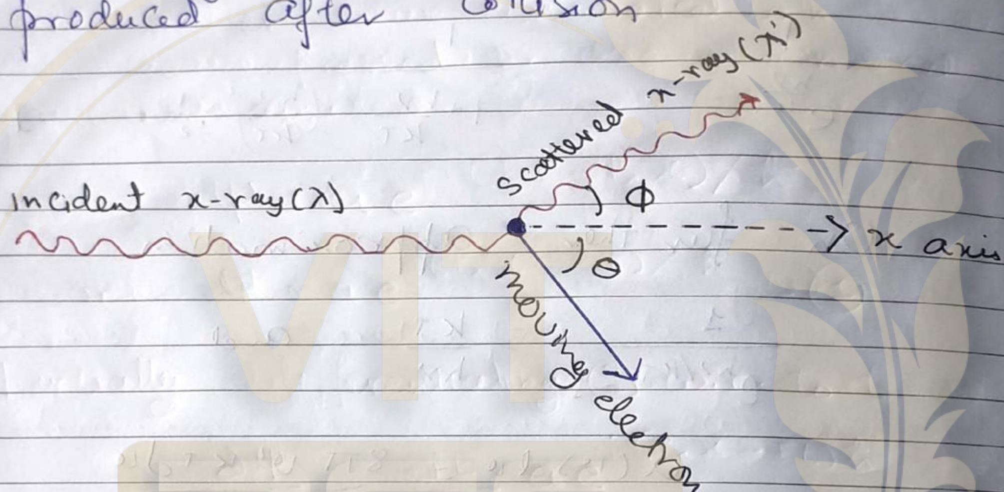
or

Que. Show that the Compton shift is independent of wavelength of incident photon
[5 marks, CAT-1 VIT Win Sem 2016-17]

Solⁿ

Compton Effect

The Compton Effect was an experiment Conducted by Arthur H. Compton in 1923, where Compton directed electromagnetic waves with short wavelength (X-rays) at electrons and found that incident waves had slightly shorter wavelength than that the waves produced after Collision



Now from Conservation of Momentum

Initial momentum along x-axis = Final momentum along x-axis

$$(h\nu/c) = (h\nu'/c)\cos\phi + p\cos\theta \quad -(1)$$

Initial momentum along y-axis = Final momentum along y-axis

$$0 = (h\nu'/c)\sin\phi - p\sin\theta \quad -(2)$$

Multiplying with C and re-arranging,

$$pc\cos\theta = h\nu - h\nu'\cos\phi \quad -(3)$$

$$pc\sin\theta = h\nu'\sin\phi \quad -(4)$$

Squaring and adding (3) & (4)

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \quad \text{--- (5)}$$

Now considering energy

Total energy of a moving particle, $E = E_0 + KE$
$= mc^2 + KE$

but, $E = \sqrt{p^2c^2 + (mc^2)^2}$ $\left\{ E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \right.$

$$\Rightarrow (mc^2 + KE)^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow p^2c^2 = KE^2 + 2mc^2KE$$

but $KE = h\nu - h\nu'$

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu') \quad \text{--- (6)}$$

Comparing (5) & (6)

$$\cancel{(h\nu)^2} - 2(h\nu)(h\nu')\cos\phi + \cancel{(h\nu')^2} = \cancel{(h\nu)^2} - 2(h\nu)(h\nu') + \cancel{(h\nu')^2} + 2mc^2(h\nu - h\nu')$$

$$\Rightarrow 2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1-\cos\phi)$$

Substituting $\nu = c/\lambda$
and $\nu' = c/\lambda'$

$$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda}\cdot\frac{1}{\lambda'}(1-\cos\phi)$$

$$\Rightarrow \boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)}$$

Compton Shift

1) Monochromatic X-Rays of wavelength 0.7078Å are scattered by carbon. The X-rays scattered at an angle of 90° with the direction of incident beam are observed. What is the wavelength of the scattered X-Rays?

A) Formula: $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

Given: $\lambda = 0.7078\,Å$ (λ: wavelength of incident X-Ray)

$\theta = 90°$ (θ: Scattering angle)

Therefore,

$$\lambda' = (0.7078\,Å) + \left(\frac{h}{m_e c}\right)(1-\cos 90°)$$

$$= 0.7078\,Å + (0.02426\,Å)\cdot(1)$$

$$= 0.7324\,Å$$

2) Calculate the wavelength of X-Rays scattered at 180° from a carbon block if the frequency of the incident rays is $1.8 \times 10^{18}\,s^{-1}$.

A) Formula: $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

Given: $\theta = 180°$ (scattering angle); $\nu = 1.8 \times 10^{18}\,s^{-1}$ (frequency of incident rays)

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{1.8 \times 10^{18}} = 1.667\,Å$$

Therefore,

$$\lambda' = 1.667\,Å + (0.02426\,Å) \times (1-\cos 180°)$$

$$= 1.667\,Å + (0.02426\,Å) \times 2$$

$$= 1.71552\,Å$$

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3) X-Radiation of wavelength 1.12 Å is scattered from a carbon target. Calculate (i) the wavelength of X-ray scattered at an angle of 90° with respect to the original direction and (ii) the energy of scattering electron after collision.

A)

Formula: $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

Given: $\lambda = 1.12$ Å (wavelength of incident X-ray)

(i) Given $\theta = 90°$

$$\lambda' = 1.12 + (0.02426 \text{ Å}) \cdot (1-\cos 90°)$$
$$= (1.12 + 0.02426) \text{ Å}$$
$$= 1.1446 \text{ Å}$$

(ii) Using energy conservation,

$$E_0 + E_e = E_e' + E_e$$
$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e$$

$$\frac{6.626\times10^{-34}\times3\times10^{8}}{1.12\times10^{-10}} + (9.1\times10^{-31}\times9\times10^{16}) = \frac{6.626\times10^{-34}\times3\times10^{8}}{1.1446\times10^{-10}} + E_e$$

$$E_e = 1.775\times10^{-15} + 8.19\times10^{-14} - 1.737\times10^{-15}$$

$$\boxed{E_e = 8.194\times10^{-14} \text{ J}}$$

4) An incident photon of wavelength ~~3 Å~~ 0.03 Å recoils at an angle of 60° after collision with a free electron. Find the energy of the recoiling electron.

Using formula,

$$\lambda' - (0.03\times10^{-10}) = \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}.(1-\cos60^\circ)$$

$$\lambda' - (0.03\times10^{-10}) = (0.02426\times10^{-10}).(1-\tfrac{1}{2})$$

$$\lambda' = \frac{(0.02426\times10^{-10})}{2} + 0.03\times10^{-10}$$

$$= \cancel{0.02713\,\mathring{A}} = 0.04213\,\mathring{A}$$

Using energy conservation,

$$E_0 + E_e = E_e' + E$$

$[\text{where } E_e = m_e c^2]$

$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e'$$

$$\frac{6.626\times10^{-34}\times3\times10^{8}}{0.03\times10^{-10}} + (9.1\times10^{-31}\times9\times10^{16}) = E_e' + \frac{6.626\times10^{-34}\times3\times10^{8}}{0.04213\times10^{-10}}$$

$$6.626\times10^{-14} + 8.19\times10^{-14} = E_e' + 4.72\times10^{-14}$$

$$E_e' = 10.096\times10^{-14}\ J$$

Therefore, the energy of the recoiling electron is ~~130~~ 10.096×10^{-14} J.

If an X-ray photon of wavelength 0.5 Å makes a Compton collision with a free electron in carbon and is scattered at 90°, find the energy of the recoiling electron.

A) Formula: $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

where λ: wavelength of incident X-Ray
θ: Scattered angle

Given: $\lambda = 0.5 \times 10^{-10} m$
$\theta = 90°$

$\lambda' = 0.5 \times 10^{-10} + (0.02426 Å) \cdot (1-\cos 90°)$
$= (0.5 + 0.02426) \times 10^{-10}$
$= 0.52426 Å$

Using energy conversation,
$E_0 + E_e = E_e' + E$

So,
$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e$$

$$\frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{0.5 \times 10^{-10}} + (9.1 \times 10^{-31} \times 9 \times 10^{16}) - \left(\frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{0.52426 \times 10^{-10}}\right) = E_e'$$

$3.98 \times 10^{-15} + 8.19 \times 10^{-14} - 3.79 \times 10^{-15} = E_e'$

$E_e' = 8.209 \times 10^{-14} J$

Therefore, the energy of the recoiling electron is $8.209 \times 10^{-14} J$

6) In a Compton experiment the wavelength of X-ray radiation scattered at an angle of 45° is 0.022 Å. Calculate the wavelength of incident X-rays.

A) Formula:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Given: λ' (Wavelength of scattered X-Ray) : ~~45°~~ 0.022Å

θ (Scattered angle) : 45°

So,

$$\lambda = \lambda' - \frac{h}{m_e c}(1-\cos\theta)$$

$$= (0.022\times10^{-10}) - (0.02426\times10^{-10})\cdot\left(1-\frac{1}{\sqrt{2}}\right)$$

$$= 0.022\times10^{-10} - 7.1\times10^{-13}$$

$$= 0.0149\,\text{Å}$$

Therefore, the wavelength of the incident X-Ray is 0.0149 Å

7) X-Ray of wavelength 0.240nm are Compton scattered and the scattered beam is observed at an angle ~~of~~ 60° relative to the incident beam. Find the wavelength of the scattered X-Rays.

A) Formula:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Given: λ (wavelength of incident X-Ray) : 0.24×10^{-9}m

θ (Scattering angle) : 60°

$$= 0.24\times10^{-9} + (0.02426\times10^{-10})\cdot(1-\tfrac{1}{2})$$

$$= 0.24\times10^{-9} + 1.213\times10^{-12}$$

$$= 2.412\ \text{Å}$$

Therefore, the wavelength of the scattered X-Rays is 2.412 Å

8) In an experiment of Compton scattering, the incident radiation has wavelength 2 Å. Calculate the energy of recoil electron which scatters through 60°.

A)

Formula: $\lambda' - \lambda = \dfrac{h}{m_e c}(1-\cos\theta)$

Given: λ (wavelength of incident radiation) : 2×10^{-10} m

θ (Scattering angle) : 60°

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$= 2\times10^{-10} + (0.02426\times10^{-10})(1-\tfrac{1}{2})$$

$$= 2\times10^{-10} + 0.01213\times10^{-10}$$

$$= 2.01213\ \text{Å}$$

Using energy conversation,

$$E_0 + E_e = E_e' + E$$

So,

$$\frac{hc}{\lambda} + m_e c^2 = E_e' + \frac{hc}{\lambda'}$$

$$\frac{6.626\times10^{-34}\times3\times10^{8}}{2\times10^{-10}} + (9.1\times10^{-31}\times9\times10^{16}) - \frac{6.626\times10^{-34}\times3\times10^{8}}{2.01213\times10^{-10}} = E_e'$$

Kunal Kulkarni

$8.289\times10^{-14} - 9.879\times10^{-16} = Ee'$

$Ee' = 8.190\times10^{-14}$ J

Therefore, the energy of the scattering electron is 8.190×10^{-14} J.

Uncertainty Principle

9. The life time of an energy state is 10^{-8} s. Calculate the uncertainty in the frequency of the photon emitted during the transition (de-excitation of the atom)

A) Formula: $\cancel{\Delta x \Delta p_x \geq \frac{h}{4\pi}}$ $\Delta E \Delta t \geq \frac{h}{4\pi}$

Given: Δt (life time of an energy state) : 10^{-8} s

So, $\Delta E(10^{-8}) \geq \frac{6.626\times10^{-34}}{4\times3.142}$

$\Delta E \geq 5.27\times10^{-27}$

As $E = h\nu$

$\nu = \frac{\Delta E}{h} = \frac{5.27\times10^{-27}}{6.626\times10^{-34}}$

$= 7.954\times10^{6}\, s^{-1}$

Therefore, the frequency of the photon emitted during the transition is $7.954\times10^{6}\, s^{-1}$

10) A nucleon is confined in a nucleus of radius 5×10^{-15} m. Calculate the minimum uncertainty in the momentum of the nucleon.

A) Formula: $\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}$

Given: Δx (Radius of nucleus) $= 5 \times 10^{-15}$ m

$$(5 \times 10^{-15}) \cdot \Delta p_x \geq \frac{6.626 \times 10^{-34}}{4 \times 3.142}$$

$$\Delta p_x \geq 1.05 \times 10^{-20}$$

Removing the sign, the minimum uncertainty in the momentum of the nuclon is 1.05×10^{-20} kgm/s

11) The position and momentum of 1keV electron are determined simultaneously. If its position is located in 1 Å, what is the percentage of uncertainty in its momentum?

A) Formula: $\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}$, $p = \sqrt{2mE}$

Given: Δx (position of electron): 1×10^{-10} m
~~Charge on~~ Energy of electron: 1keV

$$1 \times 10^{-10} \cdot (\Delta p_x) \geq \frac{6.626 \times 10^{-34}}{4 \times 3.142}$$

$$\Delta p_x \geq 5.27 \times 10^{-25}$$

$$\text{Energy} = h\nu = 1.6 \times 10^{-16} \text{ J}$$

$$\nu = \frac{1.6 \times 10^{-16}}{6.626 \times 10^{-34}}$$

$$= 2.453 \times 10^{17} \text{ Hz}$$

Therefore,

percentage of uncertainty in momentum $= \frac{\Delta p}{p} \times 100$

$p = \sqrt{2mE}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}}$

$= 1.706 \times 10^{-23}$ kgm/s

$= \frac{5.27 \times 10^{-25}}{\cancel{5.7718 \times 10^{-25}}\ 1.706 \times 10^{-23}} \times 100$

$= \cancel{0.9727 \times 100} = 3.089\%$

$= \cancel{97.27}\%$

Therefore, the percentage of uncertainty in its momentum is 97.27%.

12) The time period of a radar vibration is 0.25μs. What is the uncertainty in the energy of the photon?

A) Formula: $\Delta E . \Delta t \geq \frac{h}{4\pi}$

Given:

Δt (time period of a radar vibration) : 0.25×10^{-6} s

So,

$$\Delta E (0.25 \times 10^{-6}) \geq \frac{6.626 \times 10^{-34}}{4 \times 3.142}$$

$$\Delta E \geq \cancel{5.27}\ 2.11 \times 10^{-28}$$

Therefore, the uncertainty in the energy of the photon is 2.11×10^{-28} J

17) Whether electrons are present in atomic nuclei or not? Prove using Heisenberg uncertainty principle.

A) According to the Heisenberg uncertainty principle, the position and the velocity of an object cannot both be measured accurately, at the same time, even in theory. Mathematically, the formula of the principle is expressed as:

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Or, $$\Delta x \cdot (mv) \geq \frac{h}{4\pi}$$

As the diameter of nucleus is 1.6×10^{-15} m, if we keep this as its position and fix it then using the formula we can obtain the least possible value of the velocity of an electron, if any, is present within the nucleus. Substituting:

$$v \geq \frac{6.626 \times 10^{-34}}{4\pi \times 1.6 \times 10^{-15} \times 9.1 \times 10^{-31}}$$

$$v \geq 3.6214 \times 10^{10} \text{ m/s}$$

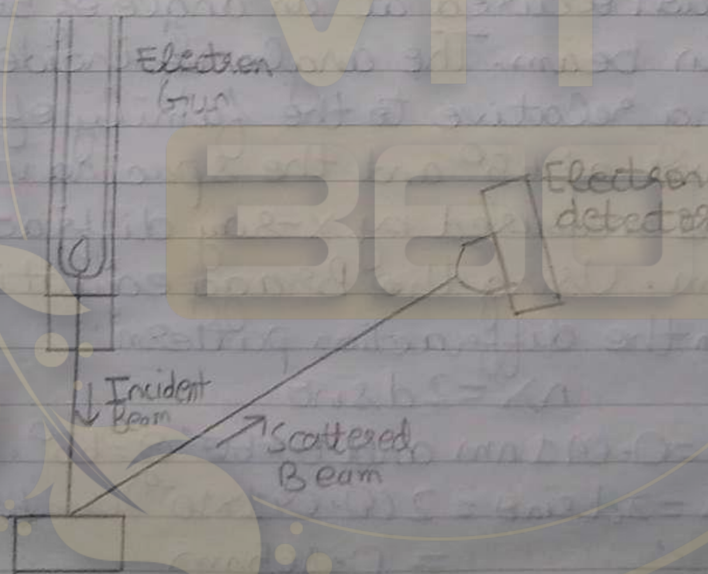
So, the least possible value of the electron would be 3.62×10^{10} m/s which exceeds the speed of light (3×10^8 m/s). Hence, ~~the~~ no electron can exist inside the nucleus as its speed exceeds the speed of light which is impossible. On the contrary, if we consider the value of atomic radius to be about 10^{-10} m, we get the speed of the electron as 5.79×10^5 m/s, which being less than the speed of light means that electrons can exist outside the nucleus.

Thus, using the Heisenberg Uncertainty principle it can clearly be proven that electrons cannot be present inside any atomic nucleus.

19) Write briefly the underlying principle used in Davison-Germer experiment to verify wave nature of electron experimentally.

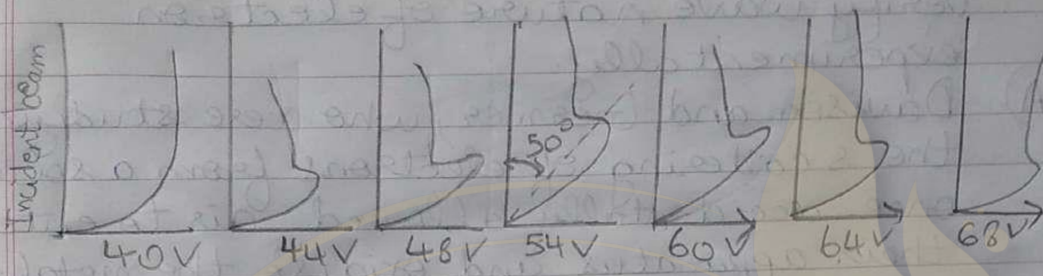
A) Davison and Germer, who were studying the scattering of electrons from a solid) once accidentally allowed air to enter in their apparatus and oxidise the metal. In a bid to reduce the oxide to pure nickel, the sample was strongly heated. Following this, the experimental results of the electron intensity with the angle changed drastically.

Experimental Setup:



After the oxide was heated, distant maxima and minima were observed (whose intensity depended on the electron energy) instead of the earlier continuous variation.

This unusual experimental observation caused De Broglie to believe that electrons were showing wave behaviour. The polar graphs after the accident are shown below:



To confirm De Broglie's hypothesis about 'De Broglie's waves', further study was done. When a beam of 54-eV electrons was directed perpendicularly ~~on~~ at the nickel target, a sharp maximum in the electron distribution occured at an angle of 50° with the origin beam. The angle of incidence and scattering relative to the family of Bragg planes was at 65° and the spacing in the planes as measured by x-ray diffraction was 0.091nm. Using the Bragg equation for maxima in the diffraction pattern:

$$n\lambda = 2d\sin\theta$$

where $d = 0.091$ nm and ~~$\theta = 65$~~ $\theta = 65°$. So for $n=1$:

$$\lambda = 2d\sin\theta = 2(0.091\times10^{-9})(\sin 65°)$$
$$= 0.165\text{nm}$$

Using de Broglie's formula, $\lambda = \frac{h}{\gamma m v}$ (where $\gamma = 1$

as the electron kinetic energy (54 eV) is negligible to mc^2 (0.51MeV)).

As, $KE = \frac{1}{2}mv^2$

YouTube/Vit 360 /vit360_

the electron momentum mv can be written as:

$$mv = \sqrt{2mKE}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 54\,eV \times 1.6 \times 10^{-19}\,J/eV}$$

$$= 4 \times 10^{-24}\ kgm/s$$

Thus, electron wavelength $(\lambda) = \frac{h}{mv}$

$$= \frac{6.626 \times 10^{-34}}{4 \times 10^{-24}} = 1.66 \times 10^{-10}\,m$$

$$= 0.166\,nm$$

As the observed wavelength was 0.165nm we can clearly verify the presence of De-Broglie waves (electron waves). As in the theoretical proof, we have treated the electron as a wave which matches with the practical observation, the Davison-Germer experiment successfully proves that electrons can show wave behaviour under specific conditions.

Schrödinger's Equation

Time Dependent Schrödinger's Equation

Consider a particle of mass m moving in positive x-direction. The potential energy of the particle is $V(x)$, momentum is P_x and the total energy of the particle E

The matter wave associated with the particle can be expressed by one-dimensional wave function as:

$$\psi = a e^{-i(Et - P_x x)/\hbar}$$

or $$\psi = a e^{i(P_x x - Et)/\hbar} \quad - (1)$$

Partially differentiating (1) w.r. to t we get

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} a e^{i(P_x x - Et)\hbar}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi \quad \text{or}$$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad - (2)$$

Similarly partially differentiating (1) w.r.to x we get

$$\frac{\partial \psi}{\partial x} = \frac{iP_x}{\hbar} a e^{i(P_x x - Et)\hbar}$$

$$\frac{\partial \psi}{\partial x} = \frac{iP_x}{\hbar} \psi$$

$$P_x \psi = -i\hbar \frac{\partial \psi}{\partial x} \quad -(3)$$

Now, according to classical Mechanics, total Energy of particle is

$$E = \frac{P_x^2}{2m} + V$$

or, $$E\psi = \left(\frac{P_x^2}{2m} + V\right)\psi$$

Substituting values of E and P_x

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}(-i\hbar)^2 \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

or, $$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

This is Time-dependent Schrödinger wave equation in one-dimensional moving particle.

Three-dimensional Time-dependent Schrödinger wave equation is written as:

$$-\frac{\hbar}{2m}\nabla^2\psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

Time-independent Schrödinger's Equation

Consider a particle of mass m moving in positive x-direction under force F(x). In the force field, the potential energy of the particle V does not depend on time t, but depends only on the position x.
let momentum of particle be P_x and total energy of particle be E.

The wave function representing the plane wave associated with the particle is

$$\psi = a e^{-i(Et - P_x x)/\hbar} \quad \text{or}$$

$$\psi = a e^{i(P_x x - Et)/\hbar} \quad -(1)$$

partially differentiating w.r.to x in (1)

$$\frac{\partial \psi}{\partial x} = \frac{i P_x}{\hbar} a e^{i(P_x x - Et)/\hbar}$$

$$\frac{\partial \psi}{\partial x} = \frac{i P_x}{\hbar} \psi \quad \text{or}$$

$$-i\hbar \frac{\partial \psi}{\partial x} = P_x \psi \quad -(2)$$

According to classical mechanics, total energy of particle is given as

$$E = \frac{P_x^2}{2m} + V$$

$$E\psi = \left(\frac{P_x^2}{2m} + V\right)\psi$$

Substituting value of P_x we get

$$E\psi = \frac{1}{2m}(-i\hbar)^2 \frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

or

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

This is the time independent Schrödinger's equation for one-dimensional motion of a particle. In case of three-dimensional it is written as

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

[Tips to study: 1) Note whatever important point your faculty asks to remember
2)If time permits read Engineering physics by Kalainathan S
3)While going through this note summarise points on a paper
4)Try to solve previous year's paper (will be uploaded around 15th October,2020)]