

Problem: $x=0, y=0$

Expand: $e^x \log(1+y)$ at $(0,0)$ using

Taylor series of order 3.

$$f(x,y) \approx f(0,0) + \frac{1}{1!} [f_x(0,0)x + f_y(0,0)y] \\ + \frac{1}{2!} [f_{xx}(0,0)x^2 + 2f_{xy}(0,0)xy + f_{yy}(0,0)y^2] \\ + \frac{1}{3!} [f_{xxx}(0,0)x^3 + 3f_{xxy}(0,0)x^2y + 3f_{xyy}(0,0)xy^2 + f_{yyy}(0,0)y^3]$$

$$f_x(x,y) = \frac{\partial}{\partial x} [f(x,y)] \\ = \frac{\partial}{\partial x} [e^x \log(1+y)] \\ = e^x \log(1+y)$$

$$f_x(0,0) = e^{(0)} \log(1+0) \\ = 1 \cdot 0 \\ = 0 //$$

$$f_{xxy} = \frac{\partial}{\partial x} \left[\frac{\partial}{\partial x} \left[\frac{\partial}{\partial y} f(x,y) \right] \right]$$

$$\begin{aligned}
 &= \frac{\partial}{\partial x} \left[\frac{\partial}{\partial x} \left[\frac{\partial}{\partial y} (e^x \log(1+y)) \right] \right] \\
 &= \frac{\partial}{\partial x} \left[\frac{\partial}{\partial x} \left[e^x \cdot \frac{1}{1+y} \right] \right] \\
 &= \frac{\partial}{\partial x} \left(e^x \cdot \frac{1}{1+y} \right)
 \end{aligned}$$

$$f_{xxy}(x,y) = e^x \cdot \frac{1}{1+y}$$

$$\begin{aligned}
 f_{xxy}(0,0) &= e^{(0)} \cdot \frac{1}{1+0} \\
 &= 1 \cdot 1 \\
 &= \underline{\underline{1}}
 \end{aligned}$$