Wave Function (ψ)

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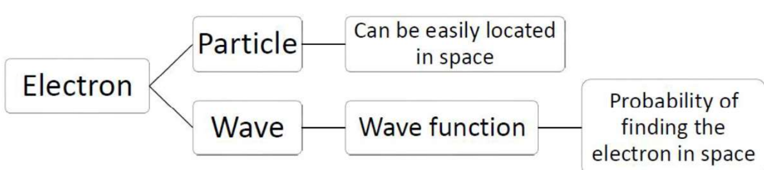
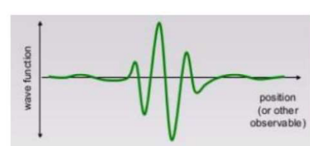
What is Wave Function?

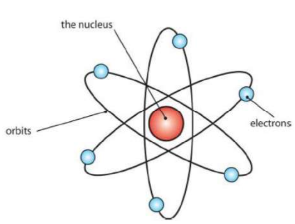
In quantum physics, a wave function is a mathematical description of a quantum state of a particle as a function of momentum, time, position, and spin. The symbol used for a wave function is a Greek letter called psi, **Ψ**.

By using a wave function, the probability of finding an electron within the matter-wave can be explained. This can be obtained by including an imaginary number that is squared to get a real number solution resulting in the position of an electron.

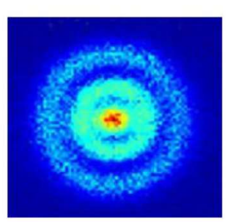
Wave function: $\psi(x,t)$



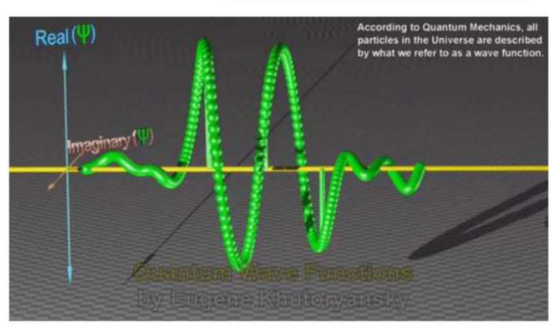




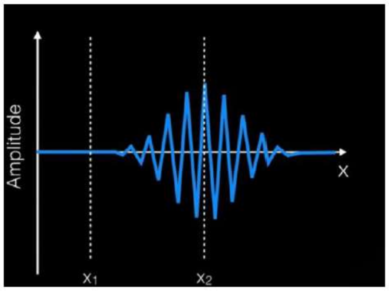
Schematic: Circular orbits



Actual: Hydrogen atom: Static wavefunction

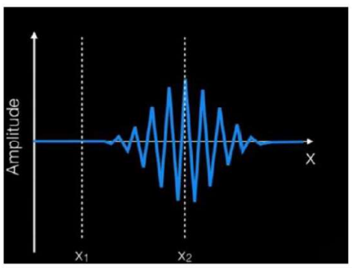


Wave function: $\psi(x,t)$

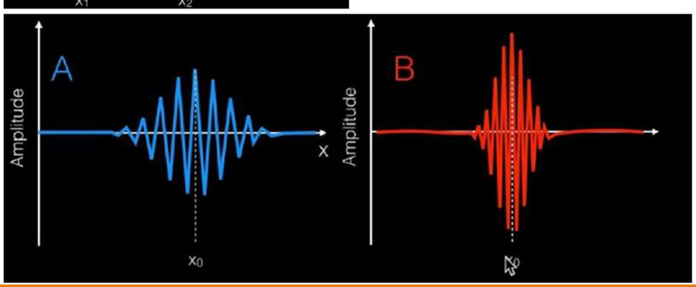


- Probability is maximum at x_2 since the amplitude of the wavefunction maximum.
- Provides all the information about the particle like position, energy and momentum at a given time.

Wave function: $\psi(x,t)$



- Maximum probability=Highest amplitude



Properties of Wave function

- $\Psi(x,t)$ is complex. It can be written in the form

$$\Psi(x,t) \;=\; A(x,t) \;+\; i\,B(x,t)$$

  where A and B are real functions.

- Complex conjugate of Ψ is defined as

$$\Psi^{*} = A - iB$$

- $|\Psi|^2 = \Psi * \Psi = A^2 + B^2$
  Therefore $|\Psi|^2 = \Psi * \Psi$is always positive and real.

- While Ψ itself has no physical interpretation, $|\Psi|^2$ evaluated at a particular place at a particular time equals to the probability of finding the body there at that time.

- Normalization: $\int_{-\infty}^{+\infty}|\Psi|^2 dV = 1$, If a wavefunction is not normalized, we can make it by dividing it with a normalization constant.

Properties of Wave function: $\psi(x,t)$

- Ψ describes the possibility of finding the particle at (x,y,z) at time t.
- Ψ is a complex quantity. Probability is always positive real quantity. $\int_{x_2}^{x_1} |\psi|^2 \, dx$
- Ψ must be Finite everywhere
- Ψ must be Single-valued
- Ψ must be Continuous
- Ψ must be Normalizable $\int_{-\infty}^{+\infty} |\psi|^2 \, dV = 1$
- By itself Ψ has no physical significance. Square of absolute magnitude give probability density.

Wave function $\Psi = A + iB$

where A and B are real functions. The complex conjugate Ψ^* of Ψ is

Complex conjugate $\Psi^* = A - iB$

and so $|\Psi|^2 = \Psi^*\Psi = A^2 - i^2B^2 = A^2 + B^2$

since $i^2 = -1$. Hence $|\Psi|^2 = \Psi^*\Psi$ is always a positive real quantity, as required.

Normalization

Even before we consider the actual calculation of Ψ, we can establish certain requirements it must always fulfill. For one thing, since $|\Psi|^2$ is proportional to the probability density P of finding the body described by Ψ, the integral of $|\Psi|^2$ over all space must be finite—the body is *somewhere*, after all. If

$$\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 0$$

the particle does not exist, and the integral obviously cannot be ∞ and still mean anything. Furthermore, $|\Psi|^2$ cannot be negative or complex because of the way it is defined. The only possibility left is that the integral be a finite quantity if Ψ is to describe properly a real body.

It is usually convenient to have $|\Psi|^2$ be *equal* to the probability density P of finding the particle described by Ψ, rather than merely be proportional to P. If $|\Psi|^2$ is to

Normalization
$$\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 1 \tag{5.1}$$

since if the particle exists somewhere at all times,

$$\int_{-\infty}^{\infty} P \, dV = 1$$

A wave function that obeys Eq. (5.1) is said to be **normalized.** Every acceptable wave function can be normalized by multiplying it by an appropriate constant; we shall shortly see how this is done.

The Wavefunction

- $|\psi|^2 \, dx$ corresponds to a physically meaningful quantity -
  - the probability of finding the particle near x
- $\left|\psi^* \frac{d\psi}{dx}\right| dx$ is related to the momentum probability density -
  - the probability of finding a particle with a particular momentum

PHYSICALLY MEANINGFUL STATES MUST HAVE THE FOLLOWING PROPERTIES:

ψ(x) must be single-valued, and finite
(finite to avoid infinite probability density)

ψ(x) must be continuous, with finite dψ/dx
(because dψ/dx is related to the momentum density)

In regions with finite potential, dψ/dx must be continuous
(with finite $d^2\psi/dx^2$, to avoid infinite energies)

There is usually no significance to the overall *sign* of ψ(x) (it goes away when we take the absolute square)
(In fact, ψ(x,t) is usually complex !)