# Chain Rule

Thursday, 4 March 2021 10:00 AM

$W = f(x)$ | $\underline{f(x,y)}$

$\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$.

$\frac{d^2 x}{dt^2} =$ | $\frac{\partial^2 f}{\partial x^2}$ & $\frac{\partial^2 f}{\partial y^2}$.

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$y = f(x)$

$\boxed{\frac{dy}{dx} = f'(x)}$

$\boxed{\frac{d^2 y}{dx^2} = f''(x)}$

$z = f(x,y)$

$\frac{\partial z}{\partial x} = f'(x,y)$

$\frac{\partial z}{\partial y} = f'(x,y)$.

$\frac{\partial^2 z}{\partial y^2}$ (or) $\frac{\partial^2 z}{\partial x^2}$.

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## chain-rule:

$\underline{W} = f(x)$, & $\underline{x} = g(t)$ ↓ True Id

$\boxed{W = f(g(\underline{t}))}$

$\frac{dW}{dt} = \frac{dW}{dx} \cdot \frac{dx}{dt}$. | $x$ - Intermediate.

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$W = \underline{x}$ & $\underline{x = \sin t}$.

$\frac{dW}{dt} = \frac{dW}{dx} \cdot \frac{dx}{dt}$

$= 1 \cdot \cos t$

$= \cos t$

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Function of two variables:

$W = F(\underline{x}, y)$ | $x = G(t)$, $y = H(t)$

$W = F(G(t), H(t))$

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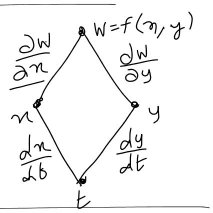
$$\boxed{\frac{dW}{dt} = \frac{\partial W}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial W}{\partial y} \cdot \frac{dy}{dt}}$$

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Problem: Given $\{ \underline{W = xy}$. & $x = \cos t$, $y = \sin t$.

Find $\frac{dW}{dt}$. | $W = F(x,y)$, $x = G(t)$, $y = H(t)$

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$\frac{dW}{dt} = \frac{\partial W}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial W}{\partial y} \cdot \frac{dy}{dt}$ — (1)

$W = xy$ | $\underline{x = \cos t}$ | $\underline{y = \sin t}$

$\frac{\partial W}{\partial x} = y$ | $\frac{dx}{dt} = -\sin t$

$\frac{\partial W}{\partial y} = x$ | $\frac{dy}{dt} = \cos t$

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$\frac{dW}{dt} = y \cdot (-\sin t) + x \cdot \cos t$

$$\frac{dw}{dt} = y\cdot(-\sin t) + x\cdot\cos t$$

$$\frac{dw}{dt} = -\sin t\cdot y + \cos t\cdot x$$

$$= -\sin t\,\sin t + \cos t\cdot\cos t$$

$$= -\sin^2 t + \cos^2 t$$

$$\boxed{\frac{dw}{dt} = \cos 2t} \quad (2)$$

Find $\left.\frac{dw}{dt}\right|_{t=\pi/2}$

$$\frac{dw}{dt} = \cos \cancel{2}\left(\frac{\pi}{\cancel{2}}\right)$$

$$= \cos\pi$$

$$= -1$$

Problem 2

2 - Independent $(r,s)$

3 - Intermediate $(x,y,z)$

$w = F(x,y,z)$ ✓

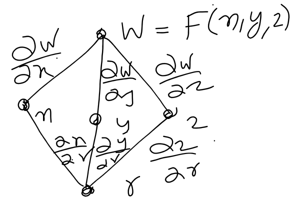
$x = G(r,s)$ ; $y = H(r,s)$; $z = I(r,s)$

$\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ (To find)

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r} \quad (1)$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s} \quad (2)$$

Problem 1




$$W = x + 2y + z^2$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$\frac{\partial w}{\partial x} = 1$

$\frac{\partial w}{\partial y} = 2$

$\frac{\partial w}{\partial z} = 2z$

$x = r/s$, $y = r^2 + \ln s$; $z = 2r$

$\frac{\partial x}{\partial r} = 1/s$; $\frac{\partial y}{\partial r} = 2r$; $\frac{\partial z}{\partial r} = 2$

$\frac{\partial x}{\partial s} = -r/s^2$ | $\frac{\partial y}{\partial s} = 1/s$ | $\frac{\partial z}{\partial s} = 0$

$F(G(t))$ $\quad F(G(r,s), H(r,s), I(r,s))$

Question: Given $w = x + 2y + z^2$ and

$x = r/s$, $y = r^2 + \ln s$

$z = 2r$

**Question:** Given $w = x + 2y + z^2$ and
$x = r/s$, $y = r^2 + \ln s$,
$z = 2r$.

**To Find:** $\dfrac{\partial w}{\partial r}$ & $\dfrac{\partial w}{\partial s}$.

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## Implicit Formula

**Suppose:** $F(x, y) \to$ differentiable

$\boxed{F(x, y) = 0} \to \dfrac{dy}{dx} = \dfrac{-F_x}{F_y}$ & $F_y \neq 0$

$F(x, y, z)$ & $z = f(x, y)$.

$\boxed{F(x, y, z) = 0}$ then $\dfrac{\partial z}{\partial x} = \dfrac{-F_x}{F_z}$

$\dfrac{\partial z}{\partial y} = \dfrac{-F_y}{F_z}$

$F_z \neq 0$