

16. Ans: [c]

$$\text{Let } \frac{0.009}{x} = 0.01;$$

$$\text{Then } x = \frac{0.009}{0.01} = \frac{0.9}{1} = 0.9$$

17. Ans: [d]

$$\text{Given expression} = \frac{(0.3333)}{(0.2222)} \times \frac{(0.1667)(0.8333)}{(0.6667)(0.1250)}$$

$$= \frac{3333}{2222} \times \frac{\frac{1}{6} \times \frac{5}{6}}{\frac{2}{3} \times \frac{125}{1000}}$$

$$= \left(\frac{3}{2} \times \frac{1}{6} \times \frac{5}{6} \times \frac{3}{2} \times 8 \right)$$

$$= \frac{5}{2}$$

$$= 2.50$$

18. Ans: [b]

$$4 \times 162 = 648. \text{ Sum of decimal places} = 6.$$

$$\text{So, } 0.04 \times 0.0162 = 0.000648 = 6.48 \times 10^{-4}$$

19. Ans: [b]

$$\text{Given Expression} = \frac{(a^2 - b^2)}{(a + b)(a - b)} = \frac{(a^2 - b^2)}{(a^2 - b^2)} = 1.$$

20. Ans: [a]

$$\frac{144}{0.144} = \frac{14.4}{x}$$

$$\Rightarrow \frac{144 \times 1000}{144} = \frac{14.4}{x}$$

$$\Rightarrow x = \frac{14.4}{1000} = 0.0144$$

SESSION - 10

ALGEBRA - I

1. Ans: [c]

$$7a + 8b = 53$$

$$9a + 5b = 47 \quad \dots (ii)$$

$$(i) \times 5 \rightarrow 35a + 40b = 265 \quad \dots (iii)$$

$$(ii) \times 8 \rightarrow 72a + 40b = 376 \quad \dots (iv)$$

$$(iv) - (iii) \rightarrow 37a = 111$$

$$\Rightarrow a = \frac{111}{37} = 3$$

Substituting for a in (i),

$$21 + 8b = 53 \Rightarrow b = \frac{32}{8} = 4$$

\therefore The solution is (3, 4)

2. Ans: [b]

Let the present age be x years.

$$\text{Then, } 7(x + 7) + 3(x - 3) = 12x$$

$$\Rightarrow 7x + 49 + 3x - 9 = 12x$$

$$\Rightarrow 2x = 40 \Rightarrow x = 20 \text{ years}$$

$$\therefore \text{Age after 3 years} = 20 + 3 = 23 \text{ years}$$

3. Ans: [a]

Let the initial number of chickens be x.

$$\frac{x \times 30}{1} = \frac{(x - 10) \times 150}{3}$$

$$\Rightarrow 90x = 150x - 1500$$

$$\Rightarrow 60x = 1500$$

$$\Rightarrow x = \frac{1500}{60} = 25$$

So, the initial number of chickens = 25

4. Ans: [c]

Let the cost of each chocolate be Rs.x and each biscuit be Rs.y and each lolly-pops be Rs.z.

$$\text{Then, } 4x + 6y + 12z = 36$$

$$\Rightarrow 2x + 3y + 6z = 18 \quad \dots (i)$$

$$3x + 15y + 9z = 48$$

$$\Rightarrow x + 5y + 3z = 16 \quad \dots (ii)$$

$$(ii) \times 2 - 1 \Rightarrow 7y = 14 \Rightarrow y = 2$$

\therefore The cost of 1 biscuit = Rs.2

5. Ans: [b]

Let tree II grow x feet after 1 year.

$$\therefore \left(\frac{3x}{7} + x \right) \times 3 = 3$$

$$\Rightarrow \frac{10x}{7} = 1 \Rightarrow x = \frac{7}{10} \text{ ft}$$

Tree II takes $\frac{7}{10}$ years to grow 7 ft.

\therefore Time required = 10 years

6. Ans: [d]

$$x^2 - 7x + 12 = 0$$

$\dots (i)$ Sum of the roots = 7, product of the roots = 12

$$\Rightarrow \alpha + \beta = 7, \alpha\beta = 12$$

$$\text{Sum of the reciprocals of the roots} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{7}{12}$$

$$\text{Product of the reciprocals of the roots} = \frac{1}{\alpha\beta} = \frac{1}{12}$$

$$\therefore \text{The required equation is } x^2 - \frac{7}{12}x + \frac{1}{12} = 0$$

$$\Rightarrow 12x^2 - 7x + 1 = 0$$

7. Ans: [b]

$$\sqrt{4x+9} - \sqrt{11x+1} - \sqrt{7x+4} = 0$$

$$\Rightarrow \sqrt{4x+9} - \sqrt{7x+4} = \sqrt{11x+1}$$

$$\Rightarrow (4x+9) + (7x+4) - 2\sqrt{(4x+9)(7x+4)} = 11x+1$$

$$\Rightarrow 2\sqrt{(4x+9)(7x+4)} = 12$$

$$\Rightarrow (4x+9)(7x+4) = 36$$

$$\Rightarrow 28x^2 + 79x + 36 = 36 \Rightarrow 28x^2 + 79x = 0$$

$$\Rightarrow x = 0 \text{ or } -\frac{79}{28}$$

As $x \geq -\frac{1}{11}$, $x = -\frac{79}{28}$ is not a root.

\therefore The solution is $x = 0$

There is 1 solution

8. Ans: [d]

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$$

If the equation has real roots,

$$\cos^2 p - 4\sin p(\cos p - 1) \geq 0$$

$$\Rightarrow \cos^2 p - 4\sin p \cos p + 4\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$$

$$(\cos p - 2\sin p)^2 \text{ is always } \geq 0$$

For $1 - \sin p$ to be non-negative, $\sin p \leq 1$

This is possible in the interval $(0, \pi)$

9. Ans: [b]

$$5x^2 + 4x + p(p-2) = 0$$

The roots are real if discriminant ≥ 0

$$\Rightarrow 16 - 20p(p-2) \geq 0$$

$$\Rightarrow 4 - 5p(p-2) \geq 0$$

$$\Rightarrow p(p-2) \leq \frac{4}{5}$$

The roots will be of opposite sign if $\frac{p(p-2)}{5} < 0$

$$\Rightarrow p(p-2) < 0 \Rightarrow p < 0 \text{ and } p > 2$$

or

$$p > 0 \text{ and } p < 2$$

$$\Rightarrow 0 < p < 2 \text{ or } (0, 2)$$

10. Ans: [c]

Since α, β are the roots of the equation

$$(x-a)(x-b) = c \text{ or } x^2 - (a+b)x + ab - c = 0$$

$$\alpha + \beta = a + b,$$

$$\alpha\beta = ab - c$$

Since $a + b = \alpha + \beta$ and $ab = \alpha\beta + c$, a, b are the roots of

$$x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$$

$$\Rightarrow (x - \alpha)(x - \beta) + c = 0$$

\therefore The required roots are a and b .

11. Ans: [a]

p and q are the roots of $x^2 - 2x + A = 0$

$$\Rightarrow p + q = 2, pq = A$$

r and s are the roots of $x^2 - 18x + B = 0$

$$\Rightarrow r + s = 18, rs = B$$

Since p, q, r, s are in A.P.,

Let $p = a - 3d, q = a - d, r = a + d, s = a + 3d$

As $p < q < r < s, d > 0$

$$2 = p + q = 2a - 4d$$

$$18 = r + s = 2a + 4d$$

Solving, we get $a = 5, d = 2$

$$p = -1, q = 3, r = 7 \text{ and } s = 11$$

$$A = pq = -3$$

$$B = rs = 77$$

$\therefore (A, B)$ is $(-3, 77)$

12. Ans: [c]

Let the fraction be x .

$$x + \frac{1}{x} = \frac{85}{18}$$

$$\Rightarrow x^2 + 1 = \frac{85}{18}x \Rightarrow 18x^2 - 85x + 18 = 0$$

$$\Rightarrow (9x - 2)(2x - 9) = 0$$

$$\Rightarrow x = \frac{2}{9} \text{ or } \frac{9}{2}$$

$$\text{But } x \neq \frac{9}{2}$$

\therefore The fraction is $\frac{2}{9}$

13. Ans: [a]

α, β are the roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

$$\text{But } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow (\alpha\beta)^2(\alpha + \beta) = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow \frac{c^2}{a^2} \left(-\frac{b}{a} \right) = \frac{b^2}{a^2} - 2\frac{c}{a}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow 2a^2c = ab^2 + bc^2$$

$$\Rightarrow ab^2, ca^2, bc^2 \text{ are in A.P..}$$

14. Ans: [d]

Since α is the root of $a^2x^2 + bx + c = 0$

$$a^2\alpha^2 + b\alpha + c = 0$$

Since β is the root of $a^2x^2 - bx - c = 0$

$$a^2\beta^2 - b\beta + c = 0$$

Let $f(x) = a^2x^2 + 2bx + 2c$

$$f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$$

$$= 2(a^2\alpha^2 + b\alpha + c) - a^2\alpha^2$$

$$= 2 \times 0 - a^2\alpha^2 = -a^2\alpha^2 < 0$$

$$f(\beta) = a^2\beta^2 + 2b\beta + 2c$$

$$= 3a^2\beta^2 - 2(\alpha^2\beta^2 - b\beta - c)$$

$$= 3a^2\beta^2 - 0$$

$$= 3a^2\beta^2 > 0$$

\therefore In the interval (α, β) , $f(x)$ becomes 0 atleast once

Hence $\alpha < \gamma < \beta$

15. Ans: [b]

$ax^2 + bx + c = 0$ has roots α & β .

$$\therefore \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 - bx(x-1) + c(x-1)^2 = 0$$

$$\Rightarrow (a-b+c)x^2 + (b-2c)x + c = 0$$

$$\text{Disc} = (b-2c)^2 - 4c(a-b+c)$$

$$= b^2 - 4bc + 4c^2 - 4ac + 4bc - 4c^2$$

$$= b^2 - 4ac$$

If A and B are the roots of the equation

$$A = \frac{-(b-2c) + \sqrt{b^2 - 4ac}}{2(a-b+c)}, B = \frac{-(b-2c) - \sqrt{b^2 - 4ac}}{2(a-b+c)}$$

$$A = \frac{-\frac{b}{2a} + \frac{c}{a} + \frac{\sqrt{b^2 - 4ac}}{2a}}{1 - \frac{b}{a} + \frac{c}{a}}, B = \frac{-\frac{b}{2a} + \frac{c}{a} - \frac{\sqrt{b^2 - 4ac}}{2a}}{1 - \frac{b}{a} + \frac{c}{a}}$$

$$A = \frac{\alpha + \alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{\alpha(1+\beta)}{(1+\alpha)(1+\beta)} = \frac{\alpha}{1+\alpha}$$

$$B = \frac{\beta + \alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{\beta(1+\alpha)}{(1+\alpha)(1+\beta)} = \frac{\beta}{1+\beta}$$

16. Ans: [c]

$$2\sqrt{5} - 1 > \sqrt{3} \Rightarrow \tan^{-1}(2\sqrt{5} - 1) > \tan^{-1}\sqrt{3} = \frac{\pi}{3} > 1$$

$$\therefore A = \tan^{-1}(2\sqrt{5} - 1) > 1$$

Let the other root be B

$$\text{Then } AB = 1 \Rightarrow B = \frac{1}{A} < 1$$

17. Ans: [b]

p and q are the roots of $x^2 + px + q = 0$

$$\Rightarrow pq = q, p + q = -p$$

$$\Rightarrow q(p-1) = 0 \Rightarrow q = 0 \text{ or } p = 1$$

If $q = 0$, we get $p = 0$

If $p = 1$, we get $q = -p = -1$

Thus $p = 1$ or 0

18. Ans: [c]

$$ax^2 + 2bx + c = 0$$

Since a, b, c are in G.P., $b^2 = ac \Rightarrow b = \sqrt{ac}$

The equation can be written as

$$ax^2 + 2\sqrt{ac}x + c = 0$$

$$\Rightarrow (\sqrt{ax} + \sqrt{c})^2 = 0$$

$$\Rightarrow x = -\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$$

Also $ax^2 + 2bx + c = 0$ has equal roots.

So the two given equations have a common root if $-\sqrt{\frac{c}{a}}$ is

a root of $dx^2 + 2ex + f = 0$

$$\Rightarrow d\left(-\sqrt{\frac{c}{a}}\right) + 2e\sqrt{\frac{c}{a}} + f = 0$$

$$\Rightarrow \frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \quad [\because b = \sqrt{ac}]$$

$$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$$

$$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{a}{d}, \frac{b}{e}, \frac{c}{f} \text{ are in H.P.}$$

19. Ans: [b]

$$x^2 + px + 1 = (x-a)(x-b)$$

$$x^2 + qx + 1 = (x-c)(x-d)$$

$$(a-c)(b-c)(a+d)(b+d)$$

$$= (c-a)(c-b)(-a-d)(-b-d)$$

$$= (c^2 + pc + 1)[(-d)^2 - pd + 1]$$

$$= (c^2 + pc + 1)(d^2 - pd + 1)$$

$$\text{But, both } c^2 + pc + 1 = 0, d^2 - pd + 1 = 0$$

$$\therefore (a-c)(b-c)(a+d)(b+d)$$

$$= (pc - qc)(-qd - pd) \quad [\because c^2 + 1 = -pc, d^2 + 1 = pd]$$

$$= cd(p-q)(-q-p)$$

$$= cd(q-p)(p+q)$$

$$= cd(q^2 - p^2) = q^2 - p^2 \quad [\because cd = 1]$$

20. Ans: [d]

$$[a^2 - 5a + 3]x^2 + (3a - 1)x + 2 = 0$$

Let α and 2α be the roots

$$\therefore (a^2 - 5a + 3)\alpha^2 + (3a - 1)\alpha + 2 = 0 \quad \dots (i)$$

$$(a^2 - 5a + 3)(4\alpha^2) + (3a - 1)(2\alpha) + 2 = 0 \quad \dots (ii)$$

$$(i) \times 4 - (ii) \rightarrow (3a - 1)2\alpha + 6 = 0$$

$$\Rightarrow \alpha = \frac{-3}{3a - 1}$$

Putting this value in (i)

$$(a^2 - 5a + 3)9 - (3a - 1)^2(3) + 2(3a - 1)^2 = 0$$

$$\Rightarrow 9a^2 - 45a + 27 - 9a^2 + 18a - 3 + 18a^2 - 12a + 2 = 0$$

$$\Rightarrow -39a + 26 = 0$$

$$\Rightarrow a = \frac{2}{3}$$

SESSION - 11

ALGEBRA - II

21. Ans: [c]

$$5x + 9y + 17z = a$$

$$2x + 3y + 8z = c$$

$$4x + 8y + 12z = b$$

$$4a - 3b - 4c = 4(5x + 9y + 17z)$$

$$-3(4x + 8y + 12z)$$

$$-4(2x + 3y + 8z)$$

$$= 20x + 36y + 68z$$

$$-12x - 24y - 36z$$

$$-8x - 12y - 32z = 0$$

$$\Rightarrow 4a - 3b - 4c = 0$$

22. Ans: [a]

$$ax^2 + bx + c$$

$$\text{when } x = 0, c = 4$$

$$f(-1) = 4 \Rightarrow a - b + c = 4$$

$$f(-2) = 6 \Rightarrow 4a - 2b + c = 6$$

$$\Rightarrow \begin{cases} a - b = 0 \\ 4a - 2b = 2 \end{cases}$$

$$\Rightarrow a = 1, b = 1$$

$$\therefore a, b, c \text{ are } 1, 1, 4.$$

23. Ans: [d]

$$3x + 2y + z = 17$$

... (i),

$$2x + 4y + 6z = 38$$

... (ii)

$$2 \times (i) + (ii) \rightarrow 8x + 8y + 8z = 34 + 38 = 72$$

$$\Rightarrow x + y + z = 9$$

24. Ans: [b]

$x = 1$ is a root of the quadratic equations

$$ax^2 + ax + 3 = 0 \text{ and } x^2 + x + b = 0$$

$$\therefore a + a + 3 = 0 \text{ and } 1 + 1 + b = 0$$

$$\Rightarrow a = -\frac{3}{2} \text{ and } b = -2$$

$$\Rightarrow ab = \left(-\frac{3}{2}\right)(-2) = 3$$

$$\Rightarrow ab = 3$$

25. Ans: [d]

Subtract 3 from both the sides of the given equation,

$$\frac{x-a}{b+c} - 1 + \frac{x-b}{c+a} - 1 + \frac{x-c}{a+b} - 1 = 0$$

$$(x-a-b-c) \left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) = 0$$

$$\text{By the given data, } \left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) \neq 0$$

$$\Rightarrow x - a - b - c = 0 \text{ or } x = a + b + c$$

26. Ans: [a]

$$\frac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0 \Rightarrow x^2 - 7x + 12 > 0$$

$$\Rightarrow (x-4)(x-3) > 0$$

$$\Rightarrow x - 4 > 0, x - 3 > 0 \text{ or}$$

$$x - 4 < 0, x - 3 < 0$$

$$\Rightarrow x > 4 \text{ and } x > 3 \text{ or}$$

$$x < 4 \text{ and } x < 3$$

$$\Rightarrow x > 4 \text{ or } x < 3$$

27. Ans: [c]

$$\text{Let } x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\Rightarrow 2y^2 - 4 - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow (2y - 5)(y - 2) = 0$$

$$\Rightarrow y = \frac{5}{2}, 2$$

$$\text{when } x + \frac{1}{x} = \frac{5}{2}, \text{ we get } x = 2 \text{ or } \frac{1}{2}$$

$$\text{when } x + \frac{1}{x} = 2, \text{ we get } x = 1$$

$$\therefore \text{There are 3 values for } x = 2, \frac{1}{2}, 1$$

28. Ans: [b]

$$2, 3 \text{ are the roots of } 2x^3 + mx^2 - 13x + n = 0$$

$$\therefore 16 + 4m - 26 + n = 0, 54 + 9m - 39 + n = 0$$

$$\Rightarrow 4m + n = 10, 9m + n = -15$$

Solving for m and n, we get

$$5m = -25 \Rightarrow m = -5$$

$$\text{and } n = 10 - 4m = 10 + 20 = 30$$

$$\therefore m = -5, n = 30$$

29. Ans: [a]

Let the number of chocolates in the plate be x.

$$\therefore \frac{1}{3}x + \frac{1}{3} \times \frac{2}{3}x + \frac{1}{3} \times \left[x - \left(\frac{1}{3} + \frac{2}{9}\right)x\right] + 8 = x$$

$$\Rightarrow \frac{1}{3}x + \frac{2}{9}x + \frac{4}{27}x + 8 = x$$

$$\Rightarrow \frac{19}{27}x + 8 = x \Rightarrow \frac{8x}{27} = 8 \Rightarrow x = 27$$

\therefore The number of chocolates in the plate = 27.

30. Ans: [b]

$$\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{(x-2)}{(x+4)}$$

$$\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{(x-2)}{(x+4)}$$

$$(x^3 + x^2 - 22x - 40) - (x^3 + x^2 - 24x + 36) = 0$$

$$2x - 76 = 0$$

i.e. $(x - 38) = 0$, has only one root.

31. Ans: [a]

Let the number of pens, pencils and erasers be x, y, z respectively.

$$x + y + z = 100 \quad \dots (i)$$

$$5x + y + \frac{z}{20} = 100$$

$$\Rightarrow 100x + 20y + z = 2000 \quad \dots (ii)$$

$$(ii) - (i) \rightarrow 99x + 19y = 1900 \quad \dots (iii)$$

Since x and y are positive integers, $x = 19$ and $y = 1$

\therefore The number of pencils = 1

32. Ans: [c]

Let the present age be x years.

$$x + 8 = 3(x - 4)$$

$$\Rightarrow 2x = 20 \Rightarrow x = 10 \text{ years}$$

\therefore Age will be 24 years after 14 years from now.

33. Ans: [a]

Let the length of each step be x inches.

$$18x = 16(x + 2) \Rightarrow 2x = 32 \Rightarrow x = 16 \text{ inches}$$

Distance between the home and the lake

$$= 18 \times 16 \text{ inches}$$

$$= \frac{18 \times 16}{12} \text{ feet}$$

$$= 24 \text{ feet}$$

34. Ans: [a]

$$f(x) = x^2 + 2x - 5$$

$$g(x) = 5x + 30$$

$$g[f(x)] = 5(x^2 + 2x - 5) + 30$$

$$= 5x^2 + 10x + 5$$

$$\therefore 5x^2 + 10x + 5 = 0$$

$$\Rightarrow x^2 + 2x + 1 = 0$$

$$\Rightarrow (x + 1)^2 = 0$$

$$\Rightarrow x = -1, -1$$

35. Ans: [d]

$$6\sqrt{\frac{x}{x+4}} - 2\sqrt{\frac{x+4}{x}} = 11$$

$$\text{Let } \sqrt{\frac{x}{x+4}} = y$$

$$\therefore 6y - \frac{2}{y} = 11$$

$$\Rightarrow 6y^2 - 11y - 2 = 0$$

$$\Rightarrow y = -\frac{1}{6} \text{ or } y = 2$$

But y cannot be negative

So, $y = 2$

$$\sqrt{\frac{x}{x+4}} = 2 \Rightarrow \frac{x}{x+4} = 4$$

$$\Rightarrow x = 4x + 16$$

$$\Rightarrow 3x = -16$$

$$\Rightarrow x = -\frac{16}{3}$$

36. Ans: [c]

$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

Put $x + \frac{1}{x} = y$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = y^2 - 2$$

$$\therefore 2(y^2 - 2) - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow (2y - 5)(y - 2) = 0$$

$$\Rightarrow y = \frac{5}{2} \text{ or } 2$$

When $y = \frac{5}{2}$, $x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2 \text{ or } \frac{1}{2}$

When $y = 2$, $x + \frac{1}{x} = 2 \Rightarrow x = 1$

Hence $x = 2, \frac{1}{2}, 1$

\therefore The number of real values = 3

37. Ans: [a]

α, β are the roots of $x^2 - x + p = 0$

$$\Rightarrow \alpha + \beta = 1, \alpha\beta = p$$

γ, δ are the roots of $x^2 - 4x + q = 0$

$$\Rightarrow \gamma + \delta = 4, \gamma\delta = q$$

Let r be the common ratio of the G.P..

Then $\frac{\gamma + \delta}{\alpha + \beta} = 4 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

When $r = 2$

We get $\alpha(1 + r) = 1 \Rightarrow \alpha = \frac{1}{1 + r} = \frac{1}{3}$

$p = \alpha\beta = \alpha \times \alpha r = \alpha^2 r = \frac{2}{9}$ which is not an integer

When $r = -2$,

$$\alpha(1 + r) = 1 \Rightarrow \alpha = -1$$

$p = \alpha^2 r = -2$

$$q = r\delta = (\alpha r^2)(\alpha r^3) = \alpha^2 r^5$$

$$= (-2)^5 = -32$$

\therefore The values of p and q are $-2, -32$.

38. Ans: [a]

$$ax^2 + bx + c = 0$$

Let the roots be α and 3α .

$$\therefore 3\alpha + \alpha = -\frac{b}{a} \Rightarrow 4\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{4a}$$

$$3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow 3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a} \Rightarrow \frac{3b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow \frac{3b^2}{16a} = c$$

$$\Rightarrow 3b^2 = 16ac$$

39. Ans: [b]

$$(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$$

Since the roots are equal, the discriminant is equal to 0.

$$4b^2(a + c)^2 - 4(a^2 + b^2)(b^2 + c^2) = 0$$

$$\Rightarrow b^2a^2 + 2b^2ac + b^2c^2 - a^2b^2 - b^4 - a^2c^2 - b^2c^2 = 0$$

$$\Rightarrow 2b^2ac - b^4 - a^2c^2 = 0$$

$$\Rightarrow b^4 + a^2c^2 - 2b^2ac = 0$$

$$\Rightarrow (b^2 - ac)^2 = 0 \Rightarrow b^2 = ac$$

$$\Rightarrow a, b, c \text{ are in G.P.}$$

40. Ans: [c]

$$x^2 - (c + 6)x + 2(2c - 1) = 0$$

Sum of the roots = $c + 6$

Product of the roots = $2(2c - 1)$

$$c + 6 = \frac{1}{2} \times 2(2c - 1)$$

$$\Rightarrow c + 6 = 2c - 1$$

$$\Rightarrow c = 7$$

SESSION - 12

FUNCTIONS - I

1. Ans: [d]

$F(x) = \max(2x + 1, 3 - 4x)$ is minimum when $2x + 1 = 3 - 4x$

$$\text{i.e. } 6x = 2$$

$$x = 2/6 = 1/3$$

therefore minimum possible value of $f(x)$ is

$$(2x + 1) \text{ at } x = 1/3 = 2 \times 1/3 + 1 = 5/3$$

$$\text{or } (3 - 4x) \text{ at } x = 1/3 = 3 - 4 \times 1/3 = 5/3$$

2. Ans: [d]

$$F(x) = ax^2 - b|x|$$

$$ax^2 > 0 \text{ for } a > 0 \text{ and}$$

$$(-b|x|) > 0 \text{ for } b < 0$$

$$F(x) = ax^2 - b|x| > 0 \text{ for } x \neq 0$$

$$F(0) = ax^2 - b|x| = 0 \text{ for } x = 0$$

If $x = 0$ $f(x)$ is minimised whenever, $a > 0, b < 0$.

3. Ans: [a]

$$\text{Min } \{f(x/2), h(x)\} < 3$$

$$f(x/2) < 3 \text{ or } h(x) < 3$$

$$2x^2 - 1 < 3 \text{ or } x^2 + x + 1 < 3$$