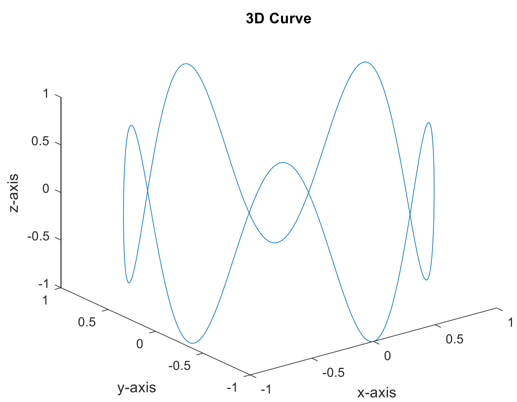
Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 3–A

Plotting 3D curves and surfaces, Taylor series of function of two variables

Commands	**Description**
`plot3(x,y,z)`	displays a three-dimensional plot of a set of data points
`comet3(x,y,z)`	displays a comet graph of the curve through the points [x(i),y(i),z(i)]
`ezplot3(funx,funy,funz)`	plots the spatial curve funx(t), funy(t), and funz(t) over the default domain 0 < t < 2π.
`ezplot3(fx,fy,fz,[tmin,tmax])`	plots the curve fx(t), fy(t), and fz(t) over the domain tmin < t < tmax
`[X,Y] = meshgrid(xgv,ygv)`	[X,Y] = meshgrid(xgv,ygv) replicates the grid vectors xgv and ygv to produce a full grid. This grid is represented by the output coordinate arrays X and Y. The output coordinate arrays X and Y contain copies of the grid vectors xgv and ygv respectively. The sizes of the output arrays are determined by the length of the grid vectors. For grid vectors xgv and ygv of length M and N respectively, X and Y will have N rows and M columns.
`surf(X,Y,Z)`	Uses Z for the colour data and surface height. X and Y are vectors or matrices defining the x and y components of a surface. If X and Y are vectors, length(X) = n andlength(Y) = m, where [m,n] = size(Z). In this case, the vertices of the surface faces are (X(*j*), Y(*i*), Z(*i*,*j*)) triples. To create X and Y matrices for arbitrary domains, use the meshgrid function.
`ezsurf(fun)`	creates a graph of fun(x,y) using the surf function. fun is plotted over the default domain: -2π < x < 2π, -2π < y < 2π.
`taylor(f,[x,y],[a,b],'order',n)`	To find the taylor series expansion of order n for the function $f(x,y)$ about the point (a,b).

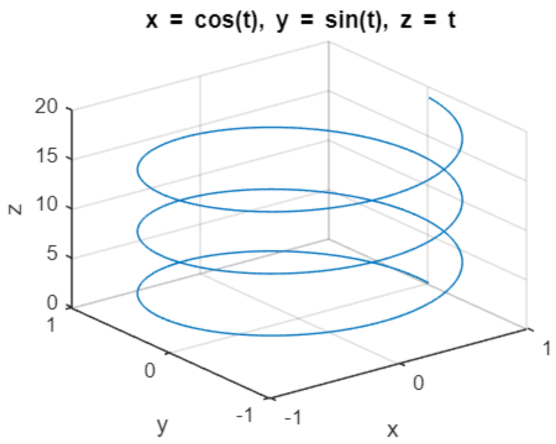
Example.1 The plotting of 3D parametric curve $x=\cos(t)$, $y=\sin(t)$, $z=\sin(5t)$ using `comet3` and `plot3` functions is given in the following code:

```
clear
clc
t=linspace(0,2*pi,500);
x=cos(t);
y=sin(t);
z=sin(5*t);
comet3(x,y,z);
plot3(x,y,z);
xlabel('x-axis');
ylabel('y-axis');
zlabel('z-axis');
title('3D Curve');
```



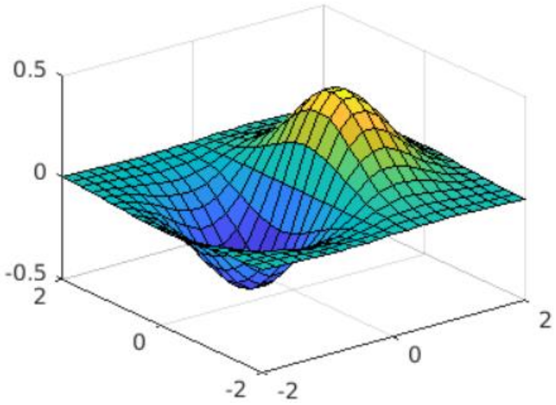
Example.2 The following code plots the helix defined by the parametric equations $x = \cos t$, $y = \sin t$, $z = t$.

```
clear
clc
syms t
x=cos(t);
y=sin(t);
z=t;
ezplot3(x,y,z,[0,6*pi])
```



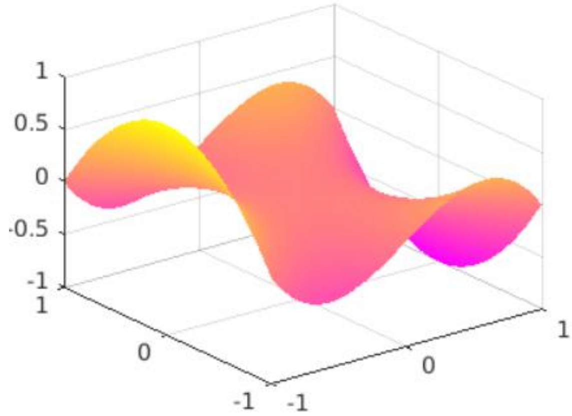
Example.3
The following code plots the surface $f(x,y) = e^{-x^2-y^2}$, $-2 \le x, y \le 2$.

```
clear
clc
[x,y]=meshgrid(-2:.2:2);
f=x.*exp(-x.^2 - y.^2);
surf(x,y,f)
```



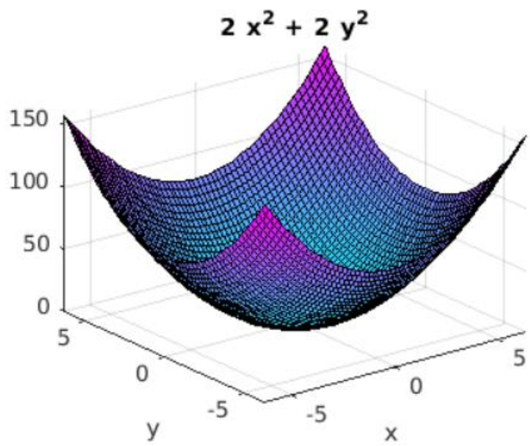
Example.4 In the following code the function $z = xy^2 - x^3$ is plotted in the interval $-1 \le x, y \le 1$

```
clear
clc
x=-1:.05:1;
y=-1:.05:1;
[x,y]=meshgrid(x,y);
z=x.*y.^2-x.^3
surf(x,y,z);
colormap spring
shading interp
```



Example.5 In the following code we used `ezsurf` to plot the function $f(x,y) = 2(x^2 + y^2)$

```
clear
clc
syms x y
f=2*(x^2+y^2)
ezsurf(f)
colormap cool
```



Taylor Series for a two variable functions:

Let $f(x,y)$ be a function of two variables x and y then the Taylor series expansion of f about the point (a,b) is given by

$$f(x,y) = f(a,b) + \left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \cdots$$

Example 6. In the following code, Taylor series of the function $f(x,y) = e^x\, cosy$ is evaluated about the origin. Also the function along with its Taylor series is plotted in the corresponding figure.

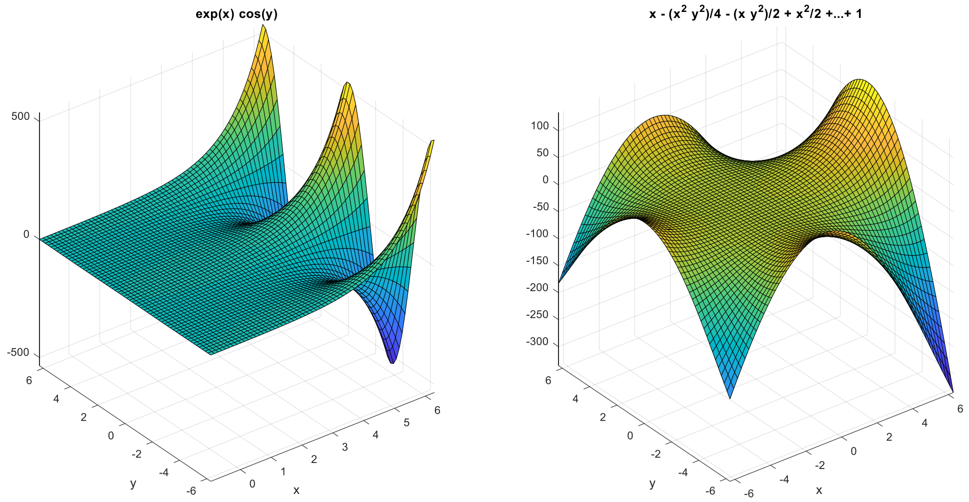
```
clear
clc
close all
syms x y
f=input('Enter the function f(x,y): ');
I=input('Enter the point[a,b] at which Taylor series is sought: ');
a=I(1);b=I(2);
n=input('Enter the order of series:');
tayser=taylor(f,[x,y],[a,b],'order',n)
subplot(1,2,1);
ezsurf(f); %Function plot
subplot(1,2,2);
ezsurf(tayser); % Taylors series of f
```

Input

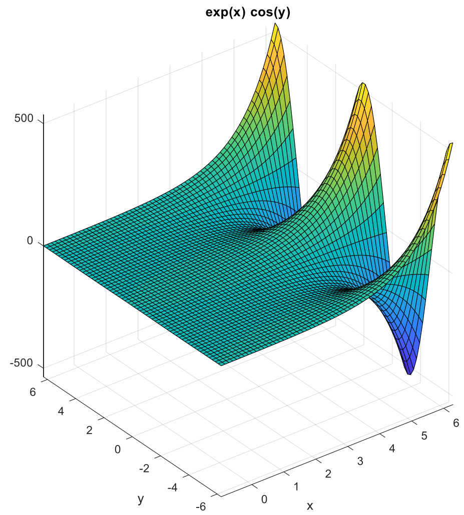
```
Enter the function f(x,y): exp(x)*cos(y)
Enter the point [a,b] around which Taylor series is sought: [0 0]
Enter the order of series:5
```

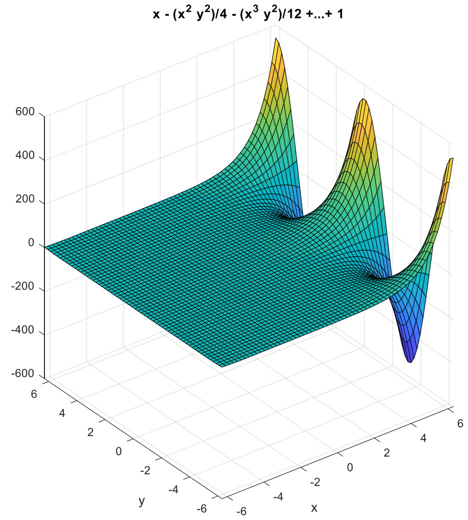
Output

```
tayser =
x^4/24 + x^3/6 - (x^2*y^2)/4 + x^2/2 - (x*y^2)/2 + x + y^4/24 - y^2/2 + 1
```



Note: The above program when executed with higher order (`n=20`), produces a better approximation than with order 5, the same is shown in the figure below.





Exercise:

1. Draw the surface of the function $f(x,y)=e^x+e^y$ using ezsurf.
2. Draw the 3-D plot for the function $f(t)=(t,\ t^2,\ t^3)$, where $0 \le t \le 100$.
3. Using 'surf' plot the surface $f(x,y) = x(x^2 + y^2)$.
4. Expand $f(x,y) = e^x \ln(1+y)$ in terms of x and y upto the terms of 3rd degree using Taylor series.
5. Expand e^{xy} in Taylor series the neighbourhood of $(1,1)$.