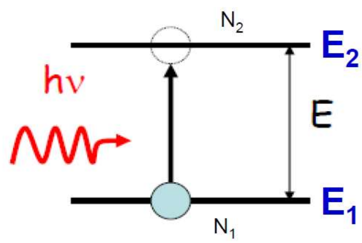
EINSTEIN COEFFICIENTS

Dr Rajeshkumar Mohanraman
Assistant Professor Grade 1
School of Advanced Sciences
VIT Vellore

Absorption or Stimulated absorption



$E_2 – E_1 = h\nu$

The atoms present in the lower energy states absorb the incident photon and get excited to the higher energy level. This process is said to be absorption of light.

N_1 and N_2 are the no. of atoms present in E_1 and E_2

E_1 = Ground state
E_2 = Excited State
E = hν(Photon Energy)

Energy levels

The rate of absorption (R_{12}) depends on no. of atoms N_1 present in E_1 & spectral energy density (ρ_ν) of the incident photon.

$R_{12} \propto \rho_\nu$
$\propto N_1$

$\boxed{R_{12} = B_{12}\, \rho_\nu\, N_1}$

Where B_{12} known as coefficient for absorption or Einstein's coefficient.

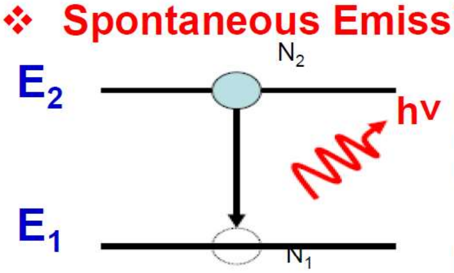
Emission of Light

The atom in the excited state exists in that state only for a short period of time. The average time spent by an atom in the excited state (in the order of 10^{-8} s) is known as life time of an atom.

In some energy levels the atoms exists for a longer time (10^{-3} s). These energy levels are called metastable state. After spending a short period of time in the excited state, it automatically returns to lower energy state by emitting the excess of energy possessed by it. The process is said to be emission of light. They are two types

- **Spontaneous Emission**

Spontaneous emission was postulated by Bohr.



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E = hν(Photon Energy)

Energy levels

$E_2 – E_1 = h\nu$

N_1 and N_2 are the no. of atoms present in E_1 and E_2

Consider an atom in the excited state. It will exist in that state only for a short period of time. After that it will return to lower energy state. If the atom lying in upper energy level returns to lower energy level without any external inducement, than the emission is said to be **spontaneous emission**.

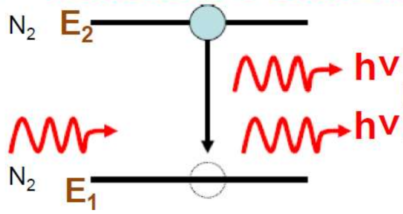
The rate of spontaneous emission R_{21} (spot) can be written as

$$R_{21}\text{ (spot)} \propto N_2$$

$\boxed{R_{21}\text{ (spot)} = A_{21} N_2}$ Where A_{21} known as spontaneous emission coefficient or Einstein's coefficient.

❖ Stimulated Emission

Stimulated emission was postulated by Einstein.



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E = hv(Photon Energy)

Energy levels

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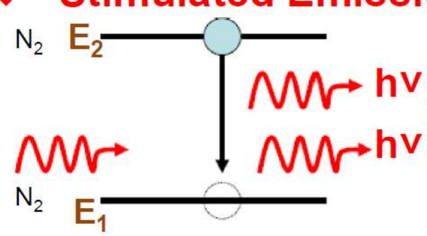

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Energy levels

$$E_2 - E_1 = h\nu$$

Consider an atom in the metastable state (in the order of 10^{-3} s). At that time, if an external source of radiation energy **hν** is incident on the system, then the atom is stimulated to emit a radiation of energy **hν** and hence it returns to the ground state. This phenomenon is called **stimulated emission.**

The rate of stimulated emission R_{21} (sti) can be written as

$$R_{21}\text{ (sti)} \propto \rho_\nu$$
$$\propto N_2$$

$$\boxed{R_{21}\text{ (Sti)} = B_{21}\,\rho_\nu\,N_2}$$

Where B_{12} known as coefficient for Stimulated emisson or Einstein's coefficient. ρ_ν is te energy density of the incident radiation.

Einstein obtained a mathematical expression for the existence of two different kinds of processes,
(1) Spontaneous emission
(2) Stimulated emission
Consider all atoms are in thermal equilibrium at T. Radiation of freq. ν & energy density ρ_ν.
N1 & N2 are atoms or populations in E1 & E2 respectively.
In equilibrium, absorption rates & emission rates must be same.

Rate of absorption = Rate of emission

$$B_{12}\,\rho_\nu\,N_1 = A_{21}\,N_2 + B_{21}\,\rho_\nu\,N_2$$

$$A_{21}\,N_2 = \rho_\nu\,[B_{12}N_1 - B_{21}N_2]$$

$$\text{So, } \rho_\nu = [A_{21}\,N_2 / (B_{12}\,N_1 - B_{21}\,N_2)] \text{ ---------(1)}$$

$$\rho_\nu = A_{21}/B_{21} / [B_{12}N_1/B_{21}N_2 - 1] \text{ ....(2)}$$

Boltzmann distribution law,

$$N_1 = N_0 e^{-E_1/kT}$$
$$N_2 = N_0 e^{-E_2/kT} \quad \text{..... (3)}$$

$$\frac{N_1}{N_2} = e^{(E_2-E_1)/kT} \quad \text{..... (4)}$$

$$E_2 - E_1 = h\nu \text{ ....(5)}$$

$$N1/N2 = e^{h\nu/KT} \text{ ....(6)}$$

$$\rho_\nu = \frac{\frac{A_{21}}{B_{21}}}{[\frac{B_{12}}{B_{21}} e^{h\nu/kT} - 1]} \quad \text{............(7)}$$

According to Planck's radiation formula,

$$\rho_\nu = \frac{8\pi h\,\nu^3}{c^3}\left(\frac{1}{[e^{h\nu/kT} - 1]}\right) \quad \text{......8}$$

If the expressions of equation 7 and 8 to be identical, we must have

$$B_{12} = B_{21} \;\&\; A_{21}/B_{21} = \frac{8\pi h\,\nu^3}{c^3}$$

Einstein coefficients A and B

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}$$

Significance

- **The stimulated emission rate per atom is the same as the absorption rate per atom**
- **The ratio between spontaneous emission and stimulated emission is proportional to v^3**
- **The probability of spontaneous emission rapidly increases with the energy difference between the two states**

Significance of Einstein's coefficients

- Coefficients A_{21}, B_{21} and B_{12} are interrelated and can be calculated if one is known
- Stimulated emission and absorption coefficients are equal at least for non-degenerate energy states
- Since B_{21}/A_{21} is proportional to reciprocal of cube of n, higher the frequency smaller the B_{21}