TUNNELING EFFECT

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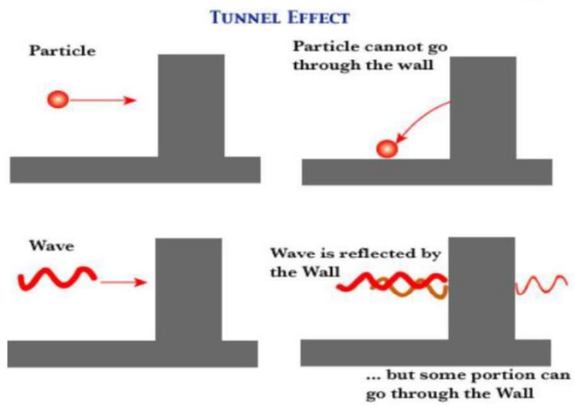
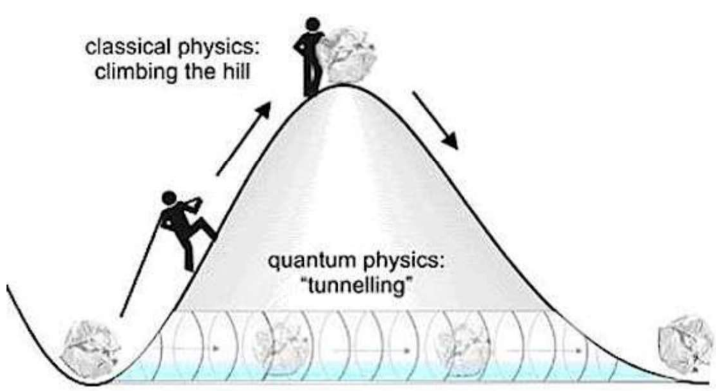
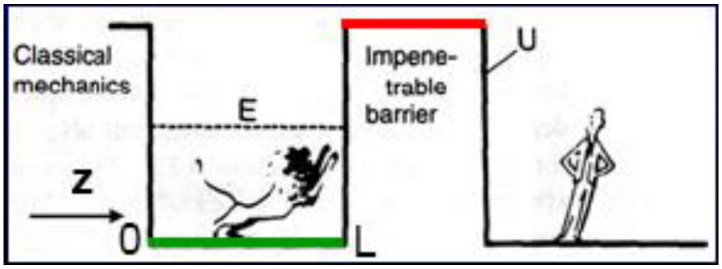
Quantum Mechanical Tunneling Effect

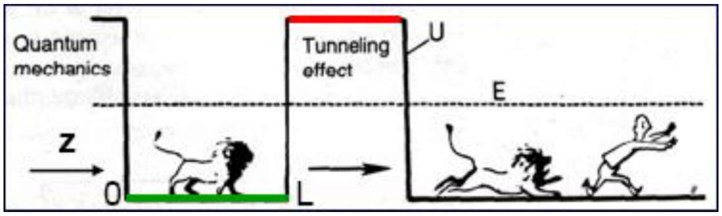


The quantum particle exhibits wave-like nature, it can reflect and transmit (tunnel) through the potential barrier

Quantum Tunneling







DEFINITION

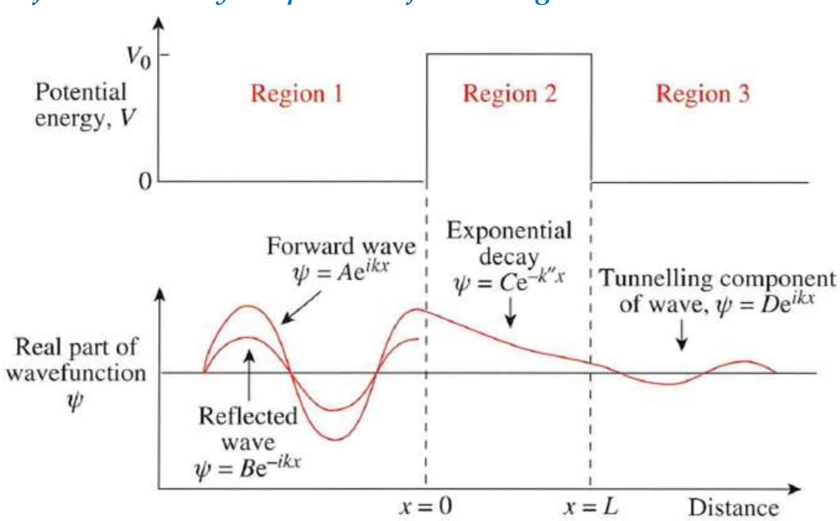
*Another phenomenon explained by the particle-wave is **tunneling**, which occurs when a particle actually passes through a seemingly impenetrable barrier. When a particle hits a barrier, it either has enough energy to break through or it doesn't and bounces back. But with a wave, part of it can pass through while part of it is reflected, making it possible for the particle to appear on the other side.*

Tunnel effect: *when a particle is able to cross a potential barrier even when its energy is less than the barrier height, then this phenomenon is called "tunnel effect'. It is purely quantum mechanical phenomenon, never realizable classically.*

Quantum tunneling or tunneling is the quantum mechanical phenomenon where a wavefunction can propagate through a potential barrier.

The emission of α-particles from atomic nuclei is an example of tunnel effect

In quantum mechanics there is finite probability that the particle will appear on the other side of the barrier by the process of tunneling.



One dimensional Schrodinger equation to be solved for this problem is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}(E-U)\psi=0$$

Depending upon +ve or –ve of (E-U), we will have different solutions

REGION –I

U(x) = 0 for -∞ > x > 0

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}E\psi=0$$

Solution

$$\psi_I = Ae^{ikx}+Be^{-ikx}$$

Incident wave Reflected wave

A, B and k are constant where $k=\frac{\sqrt{2mE}}{\hbar}$

Since the probability of the particle tunneling through the barrier is very small, the reflected wave is going to be almost as strong as incident wave

REGION –II U(X) = U_0 FOR 0 < X < L

$$\frac{\partial^2\psi_{II}}{\partial x^2}+\frac{2m}{\hbar^2}(E-U_0)\psi_{II}=0$$

$$\frac{\partial^2\psi_{II}}{\partial x^2}-\frac{2m}{\hbar^2}(U_0-E)\psi_{II}=0 \longrightarrow \because U_0 \rangle E$$

$$\frac{\partial^2\psi_{II}}{\partial x^2}-k_2{}^2\psi_{II}=0 \longrightarrow where \quad k_2=\frac{\sqrt{2m(U_0-E)}}{\hbar} \longrightarrow (1)$$

$$\psi_{II}=Ce^{-k_2x}+De^{+k_2x}$$

There is no 'i' this means that 'ψII' would not be an oscillating wave

But simply an exponential decay

So $\psi_{II}=Ce^{-k_2x}$

REGION –III U(X) = 0 FOR L< X < ∞

$$\frac{\partial^2\psi_{III}}{\partial x^2}+\frac{2m}{\hbar^2}E\psi_{III}=0$$

The wave eq. has the Same form of eq. in The region –I

One difference is The only particle moving Away from the barrier

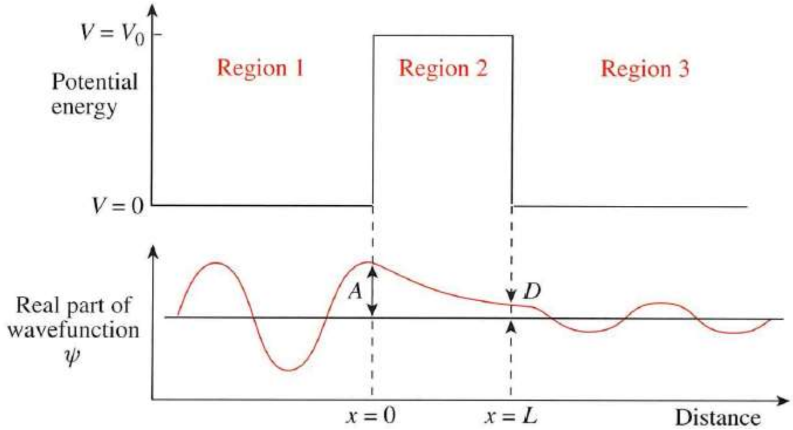
No particle approaching the barrier on Other side

Solution

$$\psi_{III}=Fe^{ikx}$$

Forwarded way

F is a constant where $k=\frac{\sqrt{2mE}}{\hbar}$



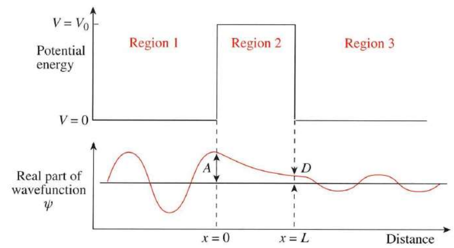
Total wave function = joining together the separate wave functions from 3 regions

This should be continuous in either 'ψ' and 'dψ/dx' at the boundaries

∴ '$\psi_1 = \psi_2$' and '$d\psi_1/dx = d\psi_2/dx$'

- **similarly**
- **∴ '$\psi_3 = \psi_2$' and '$d\psi_3/dx = d\psi_2/dx$'**
- **From approximation**
- **The reflected component of ψ_1, e^{-ikx} can be ignored**

$$\psi_I = Ae^{ikx}$$



- **And boundary x=0** $\quad \psi_I = \psi_{II} \longrightarrow Ae^{ik.0} = Ce^{-k.0}$

$$A = C \longrightarrow \textbf{(2)}$$

- **matching the magnitude not gradient of ψ_1 and ψ_2**
- **At boundary x=L**

$$\psi_{II} = \psi_{III} \longrightarrow Ce^{-k_2.L} = Fe^{ikL} \longrightarrow \textbf{(3)}$$

$$F = Ae^{-L(k_2+ik)} \longrightarrow \textbf{(4)}$$

- **The probability of the particles that leave the barrier after tunneling is proportional to**

$\psi_{III}{}^*\psi_{III}$ $\quad = F^*e^{-ikx}Fe^{ikx}$

Particles that arrive in front of the barrier is

$$= A^*e^{-ikx}Ae^{ikx}$$

The tunneling probability P is the ratio between the above two

$$P = \frac{F^*e^{-ikx}Fe^{ikx}}{A^*e^{-ikx}Ae^{ikx}} = \frac{F^*F}{A^*A}$$

$$P = \frac{A^*e^{-L(k_2+ik)}Ae^{-L(k_2-ik)}}{A^*A} = e^{-2Lk_2}$$

$$P = e^{-\frac{2L\sqrt{2m(U_0-E)}}{\hbar}}$$