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1). What are the allotropes of carbon and how do they differ from each other?

↳ Allotropes are the substances which have same ~~chemical~~ atom (and hence ~~chemical~~ composition same) but have different physical and chemical properties.

Carbon has very high number of allotropes. The most common are:-
(i) Diamond
(ii) Graphite
(iii) C_{60} Buckyball
(iv) Carbon NanoTubes
(v) C_{540} Fullerites.
(vi) C_{70}
(vii) Lonsdaleite
(viii) Amorphous Carbon and many more...

The different allotropic forms of carbon differ in number of covalent bond and arrangement of atoms. Hence, following major differences are observed.

(a) Conductivity.

⇒ The allotropes like graphite, Carbon NanoTube, etc. are highly conductive due to the presence of free electrons in each carbon of graphene whereas the other allotropes like diamond and amorphous carbon are insulators.

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(ii) Strength

⇒ Diamond - which is an carbon allotrope is the hardest substance ever known. Also, tensile strength of CNTs are 100s time greater than steel of similar dimension. However, the allotropes like graphite are not so hard and some amorphous carbons also exist.

(iii) Chemical Properties.

⇒ A carbon atom in diamond has no valence electron forming a tetrahedral structure whereas CNT and graphite has one - valence electron which allows conduction and increases reactivity.

(iv) Elasticity.

⇒ CNTs don't have weak spot found in other materials making them flexible & elastic whereas other allotropes like diamond have extremely limited elastic deformability

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2) How does the chiral vector help to identify various categories of CNT?

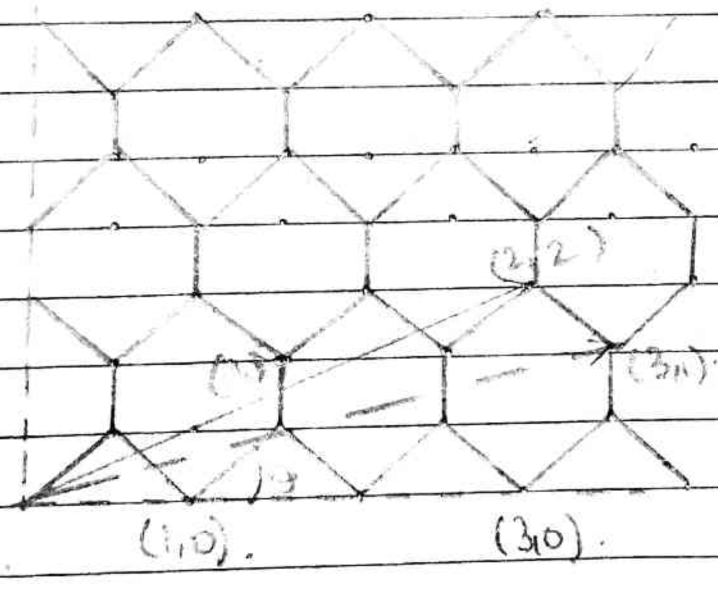
↳ The chiral vector is a vector connecting the centers of two hexagons. It helps to identify the structure of nanotubes and classify them accordingly.

For a carbon-nano-tube (CNT), chiral vector is represented as.

$$\vec{C_n} = n\vec{a_1} + m\vec{a_2}$$

where, a_1 & a_2 are defined on nearest bond length $a_{cc} = 0.144nm$

As shown in figure,



The structure of CNT depends on chiral angle (θ), defined as angle betn chiral vector and Zig-zag direction.

$$\text{i.e. } \theta = \tan^{-1}\left(\frac{\sqrt{3}m}{m+2n}\right).$$

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If $\theta = 0^\circ$ and m=0, then CNT will have zig-zag pattern.
If $0 < \theta < 30$ and $n \neq m$, then CNT will have chiral structure.
If $0 < \theta < 3^\circ$ and n=m, then CNT will have arm chair structure.

3). Write the unique properties of CNT.
↳ The properties ~~of~~ unique to CNT are:-
(i) CNTs have very high electrical and thermal conductivity
(ii) CNTs have very high tensile strength.
(iii) CNTs are flexible, tubular and highly elastic.
(iv) CNTs have a low thermal expansion coefficient.
(v) CNTs have very high tensile strength.

4). Write a short note on industrial applications of Nanotechnology.
↳ Nanotechnology is impacting the field of consumer goods, several products that incorporate nanomaterials are already in a variety of items; many of which people donot even realize contain nanoparticles, product with novel functions ranging from easy-clean to scratch-resistant. Some examples are like car bumps are made lighter, clothing is more stain repellant, sunscreen is more radiation resistant, synthetic bones are stronger, cell phone screens are stronger and light weight, sports ball are made more durable, etc.. Using nanotechnology in mid term modern textiles will become 'smart' through embedded "wearable electronics", such novel products also have a promising potential and numerous potential applications in the heavy industry. Nanotechnology is predicted to be the major driver of tech. and business in this century and holds the promise of higher performance materials, intelligent systems and new production methods with significant impact for all aspects of society.

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5). Differentiate the different types of coherence.

↳ There are basically 2 types of coherence which are differentiated :-

(i) Temporal Coherence

⇒ A beam is said to possess temporal coherence if the phase distance between any two points is constant.

i.e. f be a function at time 't' then $f(t) = f(t + \phi)$.

where ϕ = phase difference

(ii) Spatial Coherence.

⇒ A beam is said to possess spatial coherence if the phase difference betn two curves of a plane at time 't' is perpendicular to the direction of propagation.

Temporal Coherence	Spatial Coherence.
(i) It is related with time, degree of monochromaticity.	(i) It is related with position, size of source.
(ii) It is also known as longitudinal coherence.	(ii) It is also known as transverse coherence.
(iii) It's measured using Michelson's Interferometer.	(iii) It's measured using Young's Double Slit Experiment

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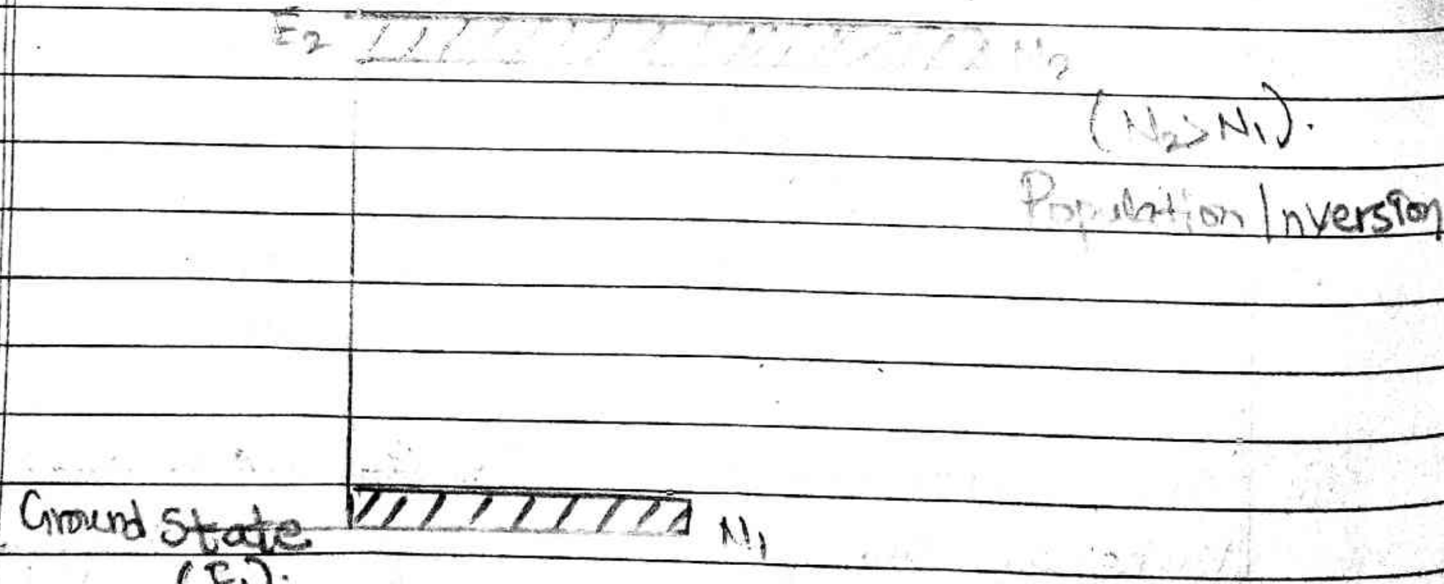
6). Why is population inversion needed for lasing action?

↳ Population inversion is the state of achieving more number of atoms in higher energy state than in ground state. It is achieved synthetically with the help of pumping.

When the population inversion is achieved, the percentage of stimulated emission increases as more atoms are in higher energy state.

So, with higher probability of stimulated emission, more photons are emitted causing the amplification of light required for laser.



N_1 — No. of atoms in ground state.
N_2 — No. of atoms in E_2 (excited).

Fig:- Population Inversion.

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7) Why are four level lasers better than three level lasers?

→ Four level lasers have two meta-stable state whereas the 3-level lasers have only one metastable state. Hence, in a four level laser the population inversion is achieved with ease as it is due to the transition from one meta-stable state to the other unlike in three level laser where the transition is from metastable state to ground state.

In a four level laser, it takes more time for atoms to reach ground state, so four level lasers require less intensive pumping and can operate in continuous mode.
That's why four level laser is better than 3-level lasers.

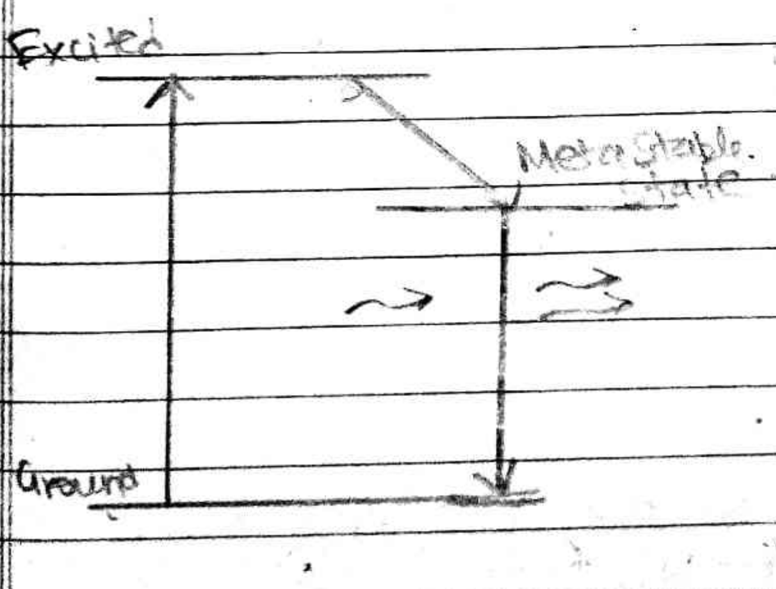

Fig:- 3 level laser working

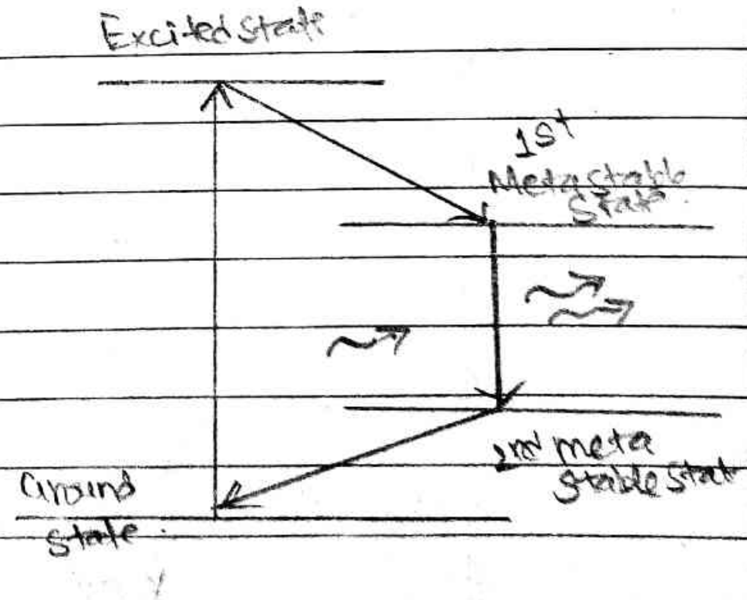

Fig:- 4 level laser working

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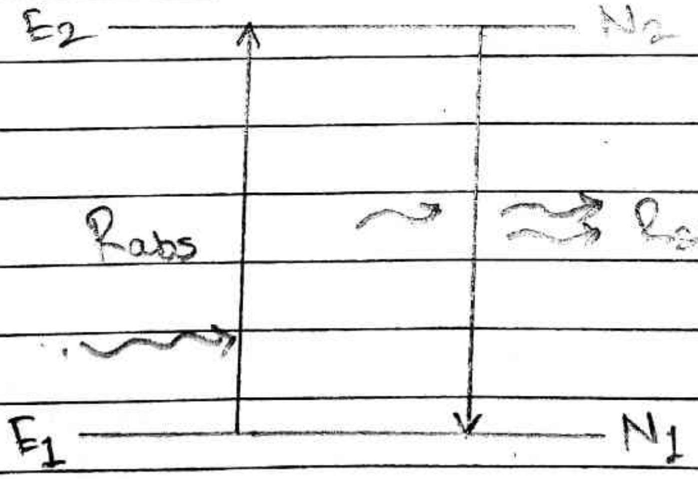
(8). Prove that the rate of stimulated absorption is same as the rate of stimulated emission.

↳ In thermal equilibrium at temperature T, with energy density Q and radiation frequency n, let N_1 and N_2 be the number of atoms present in energy state 1 and 2 at any instant.

let 'B' be the Einstein's coefficient for stimulation emission and 'A' be coefficient of stimulated absorption

Under thermal equilibrium, no. of transitions from $E_2 \rightarrow E_1$ should be equal to the no. of transitions from $E_1 \rightarrow E_2$ as shown:-



Now, the rate of stimulated absorption is:-

$$R_{abs} = B_{12} N_{12} \cdot Q \text{ ——— (i)}$$

rate of spontaneous emission.

$$R_{sp} = A_{21} N_{21} \text{ ——— (ii)}$$

rate of stimulated emission,

$$R_{st} = B_{21} \cdot N_{21} \cdot Q \text{ ——— (iii)}$$

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At equilibrium,

$R_{abs} = R_{sp} + R_{st}$.

$\Rightarrow B_{12} N_{12} \rho = A_{21} N_{21} + B_{21} N_{21} \rho$.

$\Rightarrow \rho \left(B_{12} \cdot N_{12} - B_{21} \cdot N_{21} \right) = A_{21} \cdot N_{21}$.

$$\Rightarrow \rho = \frac{A_{21} \cdot N_{21}}{B_{21} \cdot N_{21} \left(\frac{B_{12} N_{12}}{B_{21} \cdot N_{21}} - 1 \right)}$$

for conveniene, we write $N_{21} = N_2$ and $N_{12} = N_1$.

So,

$$\rho = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12} N_1}{B_{21} N_2} - 1 \right)} \quad \text{--- (IV)}$$

We know the planck's ~~radiation~~ law gives:

$$\rho = \frac{8 \pi h \nu^3}{c^3} \times \frac{1}{\left(e^{h\nu/KT} - 1 \right)} \quad \text{--- (V)}$$

The distribution of atoms is given by Boltzmann's ~~constant~~ law, according to which,

$$\frac{N_2}{N_1} = \frac{e^{-E_2/KT}}{e^{-E_1/KT}}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{(E_2 - E_1)/KT}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{\Delta E/KT} \quad (\because \Delta E = h\nu).$$

$$\Rightarrow \frac{N_1}{N_2} = e^{h\nu/KT} \quad \text{--- (VI)}$$

so, the eqn (IV) becomes,

$$\rho = \frac{A_{21}}{B_{21}} \times \left(\frac{1}{\frac{B_{12}}{B_{21}} e^{h\nu/KT} - 1} \right) \quad \text{--- (VII)}$$

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Comparing (VI) and (VII), we get,

$$\frac{B_{12}}{B_{21}} = 1.$$

$$\therefore B_{12} = B_{21}$$

and $$\frac{A_{12}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

Since, $B_{12} = B_{21}$, eqn (i) and (ii) becomes,

$$R_{abs} = B_{12} N_1 Q$$

and $$R_{st} = B_{12} N_2 Q.$$

$\therefore$ R_{abs} will be equal to R_{st} only if $N_1 = N_2$.

Hence, it's proved that the rate of stimulated emission will be equal to rate of stimulated absorption only if $N_1 = N_2$.

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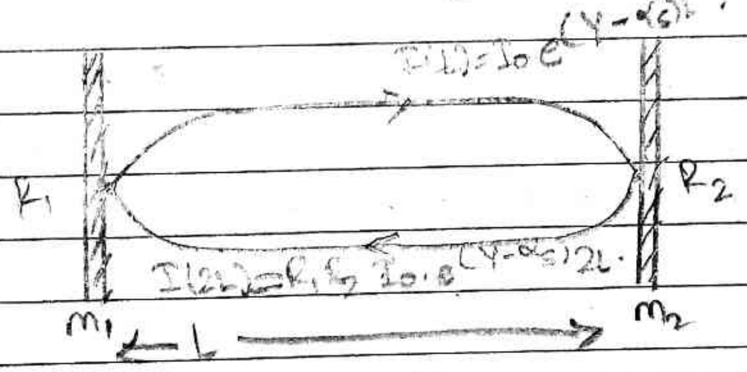
(5). Derive the expression for threshold gain of laser.

↳ laser threshold is the lowest excitation energy at which the laser's output is dominated by stimulated emission.

In a laser, active medium acts as an amplifier of waves and when laser light bounces back and forth in the optical resonator as shown, it suffers some loss.



let two mirrors M_1 and m_2 with reflectivity R_1 and R_2 separated by distance 'L' have empty space filled by active medium. let the initial intensity of light be I_0 travelling from m_1 to m_2, the beam intensity increases from:

$$I(L) = I_0 \cdot e^{(\gamma-\alpha_s)L}.$$

where, γ = gain of laser.

α_s = loss due to scattering, diffraction and absorption in medium.

after reflection at m_2 intensity of beam will be.

$$I(L) = R_2 I_0 \cdot e^{(\gamma-\alpha_s)L}.$$

after round trip beam intensity will be.

$$I(2L) = R_1 R_2 \cdot I_0 \cdot e^{(\gamma-\alpha_s)2L}.$$

And the amplification obtained during the round trip :-

$$G = \frac{I(2L)}{I_0} = R_1 R_2 e^{(\gamma-\alpha_s)2L} \quad \text{—— (1)}.$$

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here, $R_1 R_2$ represents the loses at mirror.

Now, for lasing action to occur, $G \geq 1$.

$$\Rightarrow R_1 R_2 e^{(\gamma - \alpha_s) 2L} \geq 1.$$

$$\Rightarrow e^{(\gamma - \alpha_s) 2L} \geq \frac{1}{R_1 R_2}$$

$$\Rightarrow (\gamma - \alpha_s) 2L \geq \ln\left(\frac{1}{R_1 R_2}\right).$$

$$\Rightarrow \gamma \geq \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right). \quad \text{--- (2)}$$

Here, γ = threshold gain.

As pumping is increased, value of γ^{th} also increases called threshold gain and laser starts oscillating.

Hence, the value of threshold gain is:-

$$\gamma^{th} = \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

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(10). Describe the construction and working of a He-Ne laser with suitable diagram.

↳ He-Ne is a 4 level laser which can operate in continuous mode.

Constnection :-

It cosists of a quartz tube of length 35cm and diameter 1cm which is filled with He & Ne in ratio 5:1 to 20:1 at one end perfect reflecton is fixed and another partial reflection.

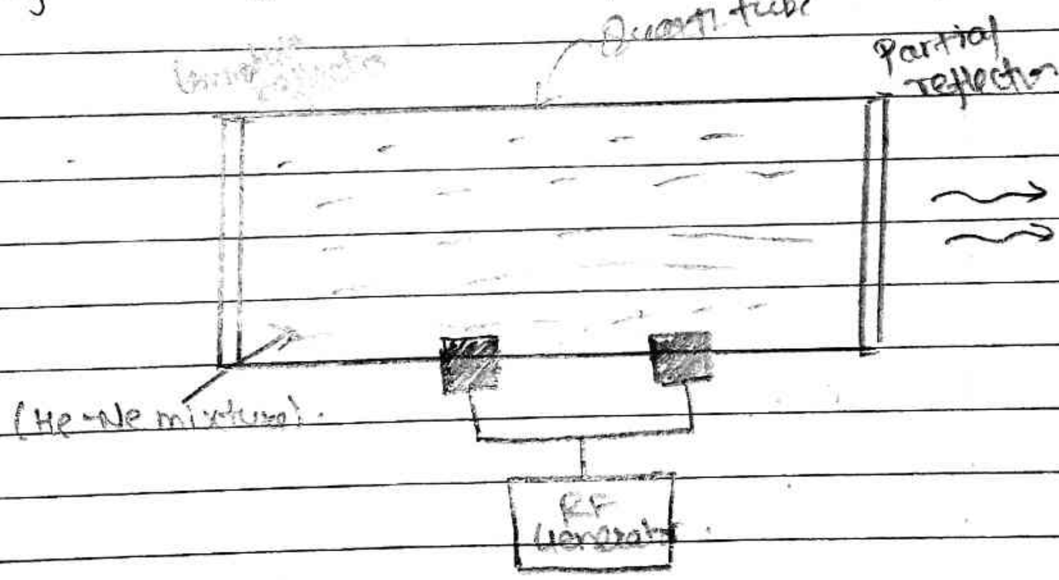

Fig:- Construction of He-Ne laser.

Working.

(1) When voltage is supplied. He atoms jump to excited state due to the Inelastic atom collision.

(2) Some excited level of He is close to that of Ne - So, there occurs transfer of energy betn He & Ne atoms. So, He returns to ground state and Ne atoms get excited.

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3). So, population inversion is created in Ne atoms which helps in production of lasing action. Thus, monochromatic laser (red) is produced

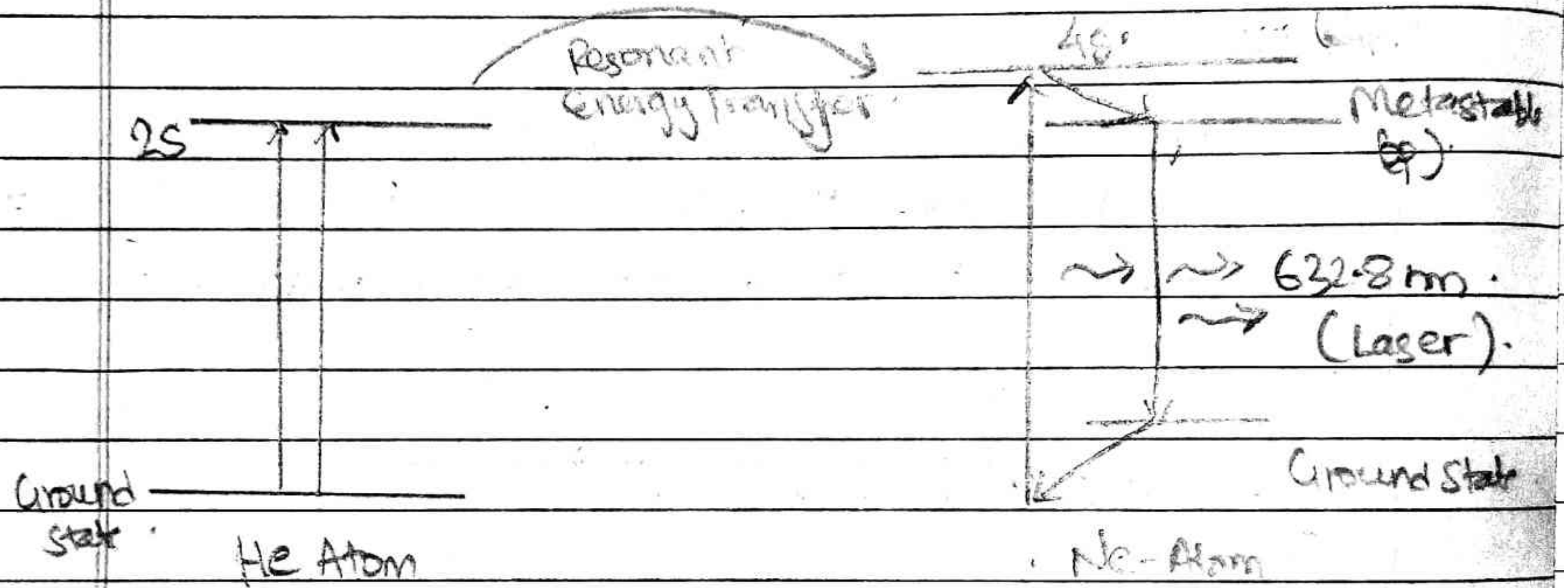

Fig:- Working of He-Ne laser.

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u). Describe the construction and working of Nd YAG laser with diagram.

↳ Nd-YAG laser is 4 level laser with solid state medium. (Nd- YAG crystal).

Construction:-

(i) It consists of ends made of elliptical cylindrical reflector, polished with silver to produce resonance for lasing.

(ii) Krypton lamp is used as pumping device (optical pumping).

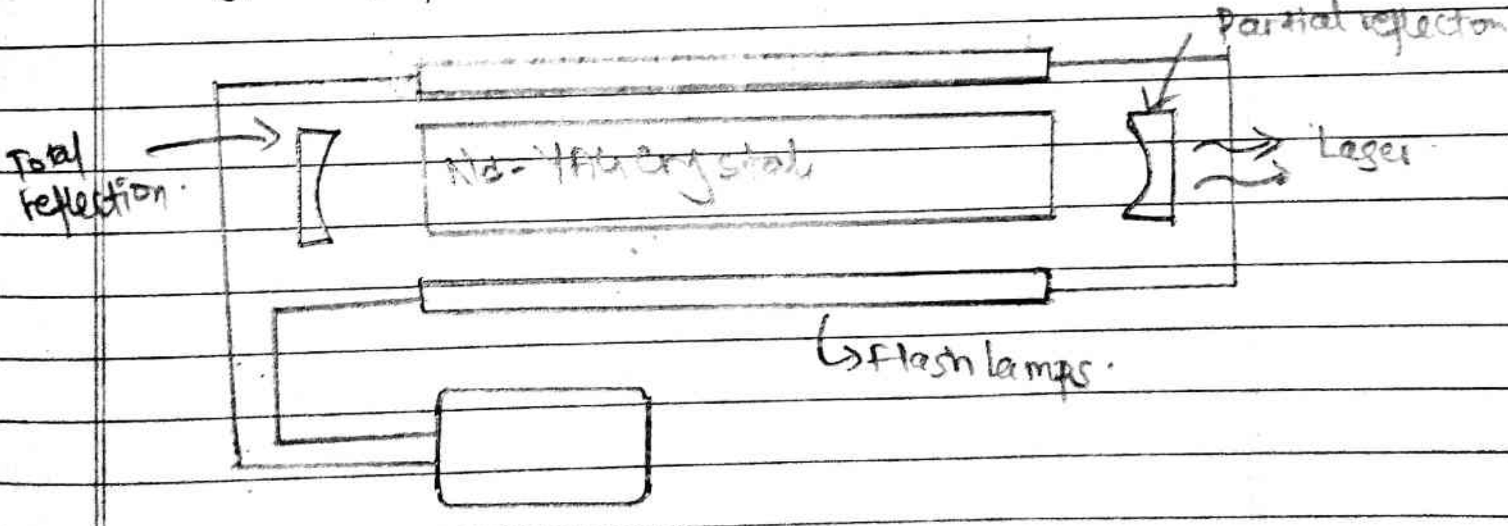

Fig:- Construction of Nd-YAG laser.

Working:-

(i) When light is flashed to the Nd-YAG crystal, Nd atoms get excited.

(ii). Nd-atoms first get excited to unstable level but very quickly return to meta-stable stable. E_3.

(iii). Population inversion is established betn E_2 and E_3 and laser is produced by 4 level action.

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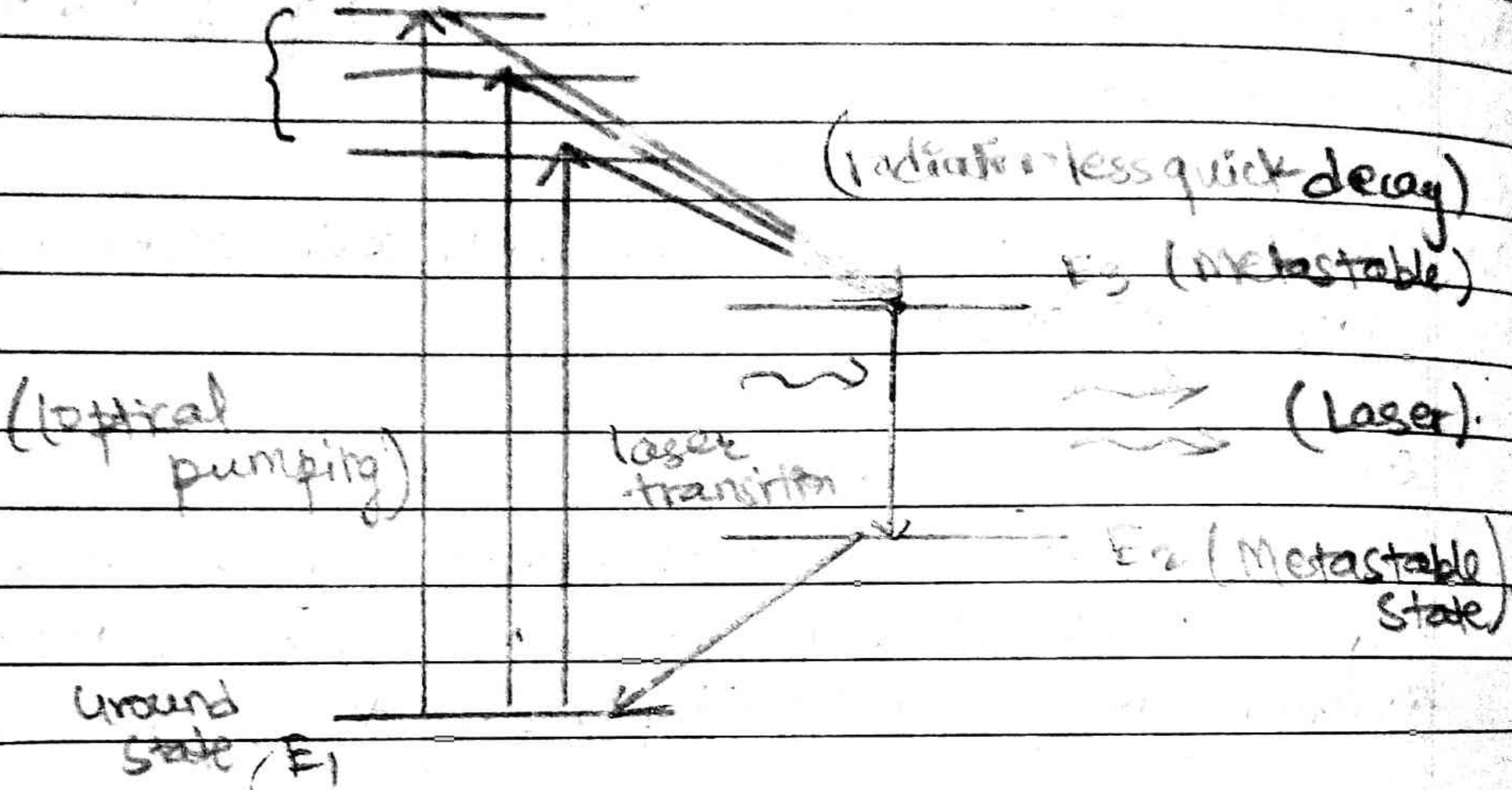

Fig:- working of Nd-YAG laser.

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13). How do you measure the monochromaticity of a light source?

↳ Monochromaticity of a light source can be measured by analyzing the spectrum emitted by it.

Monochromaticity is the property, degree or the extent to which the light produced by a source are of same or nearly same wavelength.

There can be ~~sep~~ several methods possible to measure the monochromaticity of a light source which eventually relies on the fundamental principle of finding out whether or not the emitted waves have almost common wavelength. Some common methods are like fourier analysis of the superposed wave packets or by the measurement of temporal coherence using an interferometer which eventually correlates to monochromaticity.

Q). For a light source of frequency 10^{14} Hz, the bandwidth of gaseous and solid state laser is 500Hz and 10^{9} Hz respectively. Find which is more monochromatic, solid state or gaseous?

↳ Sol

Frequency of light source (rated)., $f = 10^{14}$ Hz.

Band Width of gaseous laser (Δf_g) = 500Hz.

Band width of Solid State laser (Δf_s) = 10^{9} Hz.

Then, Monochromaticity of gaseous laser (M_g) = $\frac{\Delta f_g}{f} = \frac{500}{10^{14}} = 5\times10^{-12}$

Monochromaticity of Solid state laser (M_s) = $\frac{\Delta f_s}{f} = \frac{10^{9}}{10^{14}} = 10^{-5}$

Here, we see, $M_g < M_s$.

- Hence Gaseous state laser is more monochromatic than solid state

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14). Explain why two level lasing is not possible. A laser lamp operates by a discharge of 1000μF capacitor bank charged to 4 KV to produce flash of 10J energy. Find the efficiency.

↳ In a simple two-level system, it is not possible to obtain a population inversion with optical pumping for lasing action because the rate of absorption becomes equal to the rate of emission. The energy being used to pump the atoms to upper laser state has equal probability of stimulating them back down and at most only achieve equal population of a two-level system. i.e. ($N_2 = N_1$) and hence population inversion ($N_2 >> N_1$) is not achieved.

Soln

For Capacitor, Capacitance (C) = 10^{-3} Farad

Potential difference (V) = 4000 Volts.

Energy stored in capacitor (E) = $\frac{1}{2}CV^2$

$= \frac{10^{-3} \times (4000)^2}{2}$

$= 8 \times 10^{3}$ Joules.

(E_{in}). Energy used by laser = Energy stored in Capacitor

= 8000 Joules.

(E_{out}). Output energy of laser = 10 Joules.

Then, Efficiency (η) = $\frac{E_{out}}{E_{in}} \times 100\%$

$= \frac{10}{8000} \times 100\%$

$= \frac{1}{8}\%$

$= 0.125\%$

Hence, Efficiency of the given laser lamp is 0.125%.

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15). An atom has two atomic levels separated by 2.26 eV energy. At what temperature is (N_u/N_l) half? ($k = 1.38 \times 10^{-23}$ J/K)

↳ Sol: $k = 1.38 \times 10^{-23}$, $T = ?$

Separation of energy levels $(\Delta E) = 2.26\,eV$
$= 2.26 \times 10^{-19} \times 1.6$ J
$= 3.616 \times 10^{-19}$ J.

~~Energy~~ No. of e^- in upper state and lower state are N_u and N_l respectively.

We know, $N_u = N_l \cdot e^{-\frac{\Delta E}{kT}}$

$$\frac{N_u}{N_l} = e^{-\frac{\Delta E}{kT}}$$

$$\Rightarrow \frac{-\Delta E}{kT} = \ln\left(\frac{N_u}{N_l}\right).$$

$$\Rightarrow T = \frac{-\Delta E}{k \ln\left(\frac{N_u}{N_l}\right)} = \frac{-3.616 \times 10^{-19}}{1.38 \times 10^{-23} \times \ln\left(\frac{1}{2}\right)}$$

$$= 37802.8\,K.$$

Hence, $\frac{N_u}{N_l}$ is half at nearly 37800K which is a relatively very high temperature, which is equivalent to that inside the sun.

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16). Find the ratio of spontaneous and stimulated emission for a two level dye laser system of frequency 5×10^{14} Hz at 2000K.

↳ Sol

Frequency of laser system (ν) = 5×10^{14} Hz.
Temperature (T) = 2000 K.
We have formula, $Q = \frac{A_{21}}{B_{21}.(e^{h\nu/CT}-1)}$ where A, B → Einstein's coeff.

$$\Rightarrow \frac{A_{21}}{B_{21}\cdot Q} = (e^{h\nu/CT}-1)$$

Rate of spontanous emission, $R_{spon} = A_{21}\times N_2$ —— (i)
Rate of stimulated emission, $R_{stim} = B_{21}\times N_2\cdot Q$ —— (ii)
Dividing

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17). Nd-YAG laser might have a 20ns pulse width, energy of 5mJ per pulse and operates at 25 pulses ~~repetitions~~ per ~~pulse~~ second. Find the average power of this laser. What are the advantages of pulsed laser?

↳ Sol:-

Pulse width = 20 ns

Energy per pulse (E) = 5mJ = 0.005 J

Rate of repeatitions (R) = 25 pulses per second.

ficient.

Now,

$$\text{Power of laser} = \frac{\text{Energy emitted}}{\text{Unit time}} = \frac{\text{Energy of each pulse} \times \text{No of pulse}}{\text{Unit time}}$$

$$= E \times R$$

$$= 0.005 \times 25$$

$$= 0.125 \text{ Watts}.$$

The advantages of a pulsed laser are:-

(i). It's conceptually simple and easier to setup than continuous laser.

(ii). It is versatile especially for high power laser applications like cutting, welding, etc. due to very high peak power.

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18). CO_2 laser emits light of wavelength 10.6 μm. If the output power is 10W, calculate the number of photons emitted per minute.

↳ Sol

Wavelength of light = 10.6 μm.
$= 1.06 \times 10^{-5}$ m.

Total power out = 10 W

Then, let, rate of photon emission $= \left(\frac{n}{t}\right)$.

Then, $P = \frac{n}{t} \times$ energy per photon.

$$\Rightarrow P = \frac{n}{t} \times \frac{hc}{\lambda}$$

$$\Rightarrow \frac{n}{t} = \frac{P\lambda}{hc} = \frac{10 \times 1.06 \times 10^{-5}}{6.624 \times 10^{-34} \times 3 \times 10^{8}} = 5.33 \times 10^{20} \text{ photons per second}$$

Taking $t = 60$ seconds,

No. of photons emitted $(n) = 5.33 \times 10^{20} \times 60$
$= 3.2 \times 10^{22}$ photons.

Hence, 3.2×10^{22} photons are emitted in a minute.

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19). Starting from the integral form, derive differential form of Gauss Theorem for electrostatics and explain it's physical significance.

↳ so^n:-

The integral form of Gauss Theorem in Electrostatics states that the flux of electric field out of an arbitrary closed surface is proportional to the electric charge enclosed by the surface, irrespective of how the charge is distributed.

ie. $$\oiint_S E\, dA = \phi_E \quad , \quad \phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$$

where ε_0 is permittivity of medium.

for any closed surface 'S' containing charge Q. By the divergence theorem, this equation is equivalent to:-

$$\iiint_V \nabla \cdot E\, dV = \frac{Q_{enclosed}}{\varepsilon_0}$$

for any volume 'V' containing charge Q, by the relation between charge and charge density, the equation equivalents to ::

$$\iiint_V \nabla \cdot E \cdot dV = \iiint_V \frac{\rho}{\varepsilon_0} dV$$

for any volume 'V', In order for this equation to be simultaneously true for every possible volume V, it is necessary (and sufficient) for integrands to be equal everywhere. Therefore, the equation is equivalent to:-

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

Which is the differential form of Gauss Theorem for electrostatics

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where $\vec{\nabla}$ is gradient operator.

ie. $\vec{\nabla} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$.

The physical significances of this law are:-

* It is an important law relating the charge distribution with electricfield.
* It can be used for solid of any shape like conical, spherical or irregular or others as long as the surface is closed.
* It can be used to derive further laws in electrostatics like columb's law.

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20). Derive the wave equation for electric field in free space using Maxwell's equations.

↳ Maxwell's equations for electric field and magnetic field are given as :-

$$\vec{\nabla}\cdot\vec{E} = \frac{Q}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 J + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

Putting all together shows that oscillating electrical field produces variable magnetic field and changing magnetic field produces electric field and so on.

In a free space, we can assume the current density (J) and charge (Q) to be zero.

Hence, equations above are reduced to :-

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{--- (I)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{--- (II)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{--- (III)}$$

$$\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \quad \text{--- (IV)}$$

Taking curl of equation (III),

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\vec{\nabla}\frac{\partial\vec{B}}{\partial t}$$

$$\Rightarrow \vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

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$$\Rightarrow \vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad \text{(from eq}^n \text{ 4)}.$$

$$\Rightarrow \vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{———— (V)}.$$

[μ_0 and ε_0 are constants called permittivity and permeabil

We know that,

$$\vec{\nabla} \cdot [\vec{\nabla} \times \vec{F}(x,y,z)] = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F} \quad \text{———— (VI)}.$$

So, eq^n (V) becomes,

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \left(\frac{\partial \vec{E}}{\partial t^2}\right).$$

Since, $\vec{\nabla} \cdot \vec{E} = 0$,

$$\Rightarrow -\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\therefore \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{———— (VII)}.$$

is the wave eq^n for electric field in free space.