

**Department of Mathematics**  
**School of Advanced Sciences**  
**MAT 1011 – Calculus for Engineers (MATLAB)**  
**Experiment 5–B**  
**Line integral and work done**

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**Aim:**

To write MATLAB codes to find the work done by a force  $\vec{F}$  and visualize the force field with the path.

**Mathematical form:**

Let the given function be  $\vec{F} = F_1(x, y, z)\hat{i} + F_2(x, y, z)\hat{j} + F_3(x, y, z)\hat{k}$ , where  $(x, y, z)$  given in parametric form  $\vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$ ,  $a < t < b$ . Then

$$\int_a^b \vec{F} \cdot d\vec{r} = \int_a^b \left( \vec{F}(r(t)) \cdot \frac{d\vec{r}}{dt} \right) dt$$

**Example 1.** Finding the line integral  $\int_C \vec{F} \cdot d\vec{r}$  along the given curve  $C$  given by  $x(t) = t + \sin(\pi t/2)$ ,

$y(t) = t + \cos(\pi t/2)$ ,  $0 \leq t \leq 1$ , where  $\vec{F} = xy^2\hat{i} + x^2y\hat{j}$ .

```
clc
clear all
syms x y t
f=input('Enter the components of 2D vector function [u,v] ');
r=input('Enter x,y in parametric form');
I=input('Enter the limits of integration for t in the form [a,b]');
a=I(1);b=I(2);
dr=diff(r,t);
F=subs(f,{x,y},r);
Fdr=sum(F.*dr);
I=int(Fdr,t,a,b)
P(x,y)=f(1);Q(x,y)=f(2);
x1=linspace(-2*pi,2*pi,10); y1=x1;
[X,Y] = meshgrid(x1,y1);
U=P(X,Y); V=Q(X,Y);
quiver(X,Y,U,V,1.5)
hold on
t1=linspace(0,2*pi);
x=subs(r(1),t1);y=subs(r(2),t1);
plot(x,y,'r')
```

**Input**

Enter the components of 2D vector function [u,v]:

[x\*y^2 x^2\*y]

Enter x(t) and y(t) in parametric form:

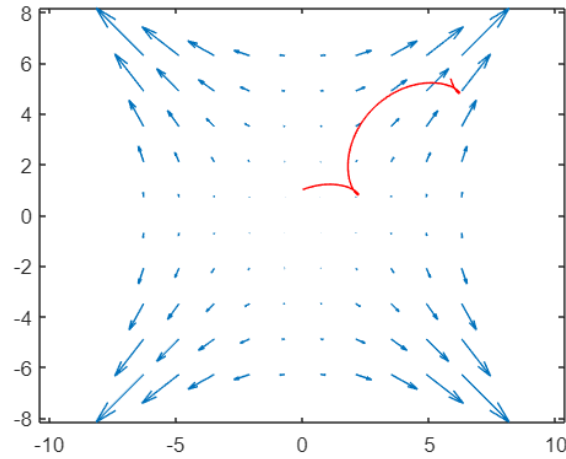
[t+sin((pi\*t)/2) t+cos((pi\*t)/2)]

Enter the limits of integration for t in the form [a,b]:

[0,1]

## Output

I =  
2



**Example 2.** Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  along the given curve  $C$  given by  $r: [t, t^2, t^3]$   $0 \leq t \leq 1$ , where

$$\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}.$$

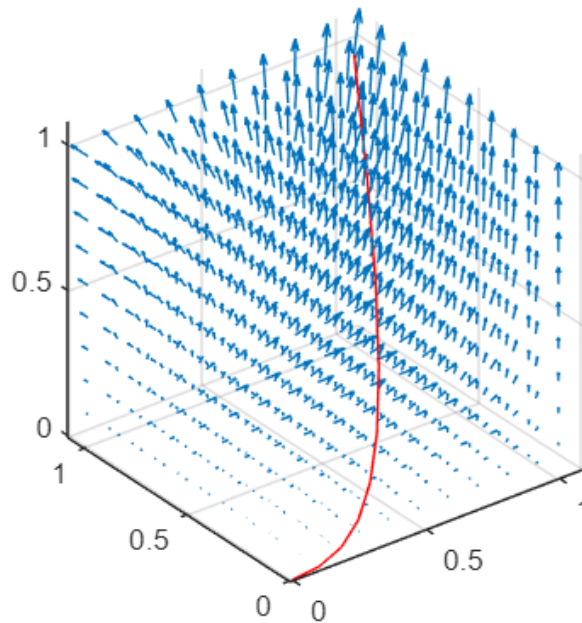
```
clc
clear all
syms x y z t
f=input('Enter the components of 3D vector function [u,v,w] ');
r=input('Enter x,y,z in parametric form');
I=input('Enter the limits of integration for t in the form [a,b]');
a=I(1);b=I(2);
dr=diff(r,t);
F=subs(f,{x,y,z},r);
Fdr=sum(F.*dr);
I=int(Fdr,t,a,b)
P(x,y,z)=f(1);Q(x,y,z)=f(2); R(x,y,z)=f(3);
x1=linspace(0,1,10); y1=x1; z1=x1;
[X,Y,Z] = meshgrid(x1,y1,z1);
U=P(X,Y,Z); V=Q(X,Y,Z); W=R(X,Y,Z);
quiver3(X,Y,Z,U,V,W,1.5)
hold on
t1=linspace(0,1,10);
x=subs(r(1),t1);y=subs(r(2),t1);z=subs(r(3),t1);
plot3(x,y,z,'r')
```

## Input

```
Enter the components of 2D vector function [u,v,w]
[x*y y*z z*x]
Enter r in parametric form [x(t) y(t) z(t)]
[t t^2 t^3]
Enter the limits of integration for t in the form [a,b]
[0,1]
```

## Output

27/28



**Exercise:**

- 1) Find the work done for the force  $\vec{F}(x,y,z) = yz\vec{i} + xz\vec{j} + (xy+2z)\vec{k}$  along the line segment from  $(1,0,-2)$  to  $(4,6,3)$ .
- 2) ~~Find the work done for the force  $\vec{F}(x,y,z) = x^2\vec{i} + y^2\vec{j}$  along the arc of the parabola  $y = 2x^2$  from  $(-1,2)$  to  $(2,8)$ .~~

Find the work done for the force. ---  
 $(0,0,0)$  to  $(4,6,3)$