Monday, 22 February 2021 11:58 AM

$f(x) = x^2 \rightarrow$ Single-variable.

$x \rightarrow$ Domain

$f(x) \rightarrow$ Range.

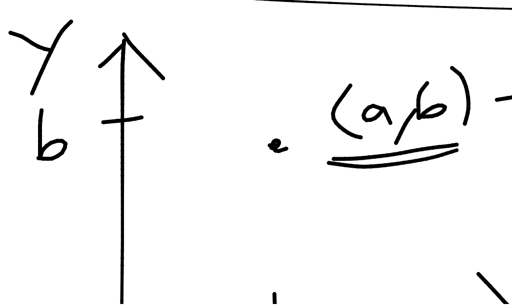
$f(x, y) = x^2 + y^2$

$$\boxed{x^2 + y^2 = r^2}$$

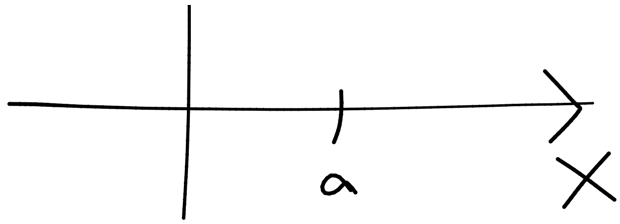
$$x^2 + y^2 = 1$$

$f(x, y, z \, \ldots \ldots \, n)$

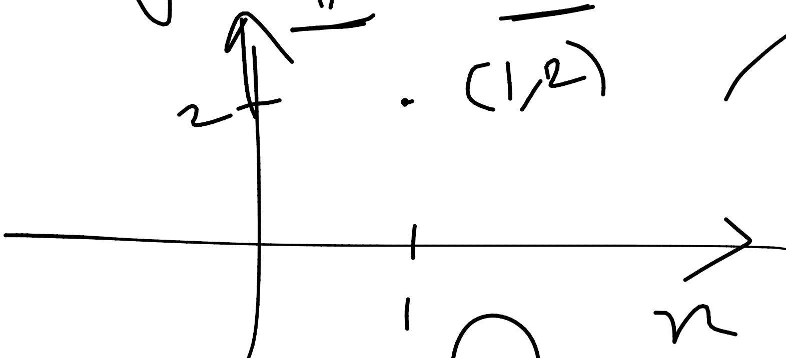
$f(x_1, x_2 \, \ldots \ldots \, x_n) \longrightarrow$ function of n-variables.

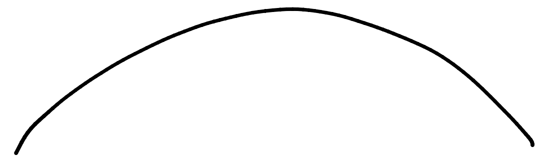


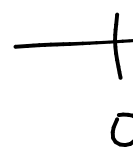
$(a,b) \rightarrow$ ordered pair.



$f : \mathbb{R}^2 \to \mathbb{R}$







$\mathbb{R}^2$

$\mathbb{R}^2 \quad \mathbb{R}^2 \quad \mathbb{R}$

$f(x, y) = 2(x + y)$

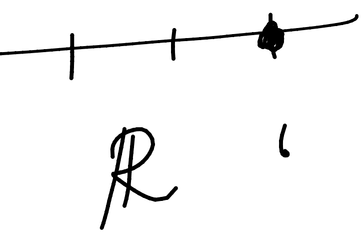
$f(1, 2) = 2(1 + 2)$

$= 2(3)$

$= 6$

$\mathbb{R}^{2\times 1}$ (or) $\boxed{\mathbb{R}^2 \to \begin{bmatrix} x \\ y \end{bmatrix} = (x, y)}$

$\mathbb{R}^{2\times 2}$



$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

$$f(3,2) = \frac{\sqrt{3+2+1}}{3-1}$$

$$\boxed{\underset{R^2}{\underline{f(3,2)}} = \frac{\sqrt{6}}{2}_{R}}$$

R^3

$f(x,y,$

② find domain

$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

$R^2 \Rightarrow ($

Denominator $x-1$

$\Rightarrow)($

$$= \frac{\sqrt{1+1+1}}{0}$$

$$= \frac{\sqrt{3}}{0} \times$$

$2) \rightarrow x+y+2$

$-\infty, \infty)$

$x = 1$

$(1, 1)$

Remove $x=1$.

$x+$

Namer $\sqrt{x+y+1}$

---

② $f(x,y) = x\ln(y^2-x)$

$y^2-x>0$ $\quad x<$

$f(x,y)=K$

$f(x,y,z)=K$

$f(x,y) = 6-3x$

for the values of

$K=-6,0,6,12$

$y+1\geq 0$

y^2

$-2y$

2

$f(x, y) = K$

$6 - 3x - 2y = K$

$3x + 2y + (K - 6) = 0$

Sub $K = -6$

(1) $3x + 2y - 12 =$

(2) $3x + 2y - 6 =$

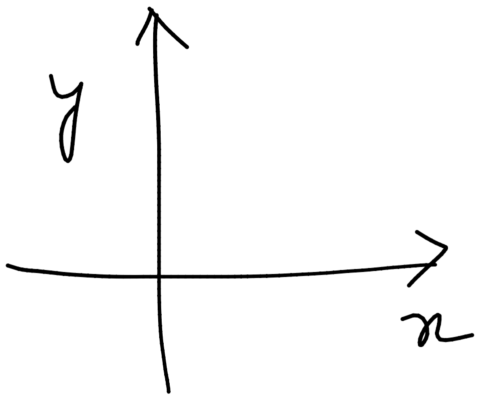
0

$=0$

$=0 \quad (K=0)$

Limits of MVC

Tuesday, 23 February 2021 9:02 AM



$f: \quad R^2 \longrightarrow R$

TO find Range:

$f(x,y) = \frac{1}{x^2} + \frac{1}{y^2}$

Domain: $D : \{ \, x, y \mid x \neq 0$ a
$y \neq 0$

$f(5,5) = \frac{1}{25} + \frac{1}{25}$

$= \frac{2}{25}$ //

$\mathbb{R}$

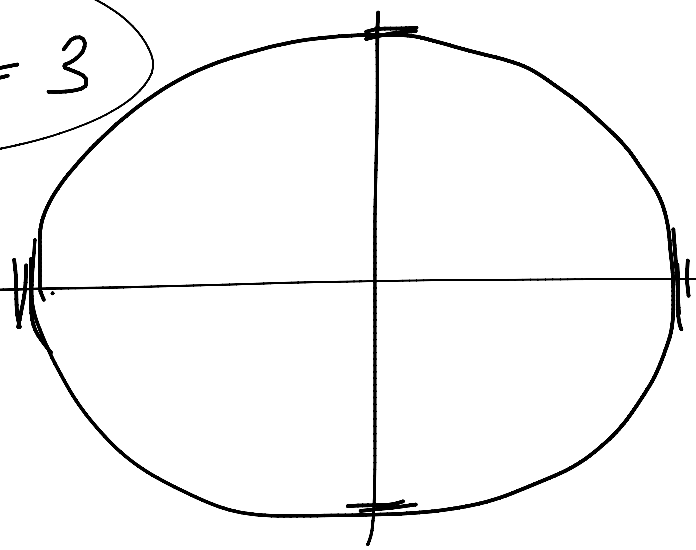
$f(-5,+5) = \frac{2}{25}$

$f(-5,-5) = \frac{2}{25}$

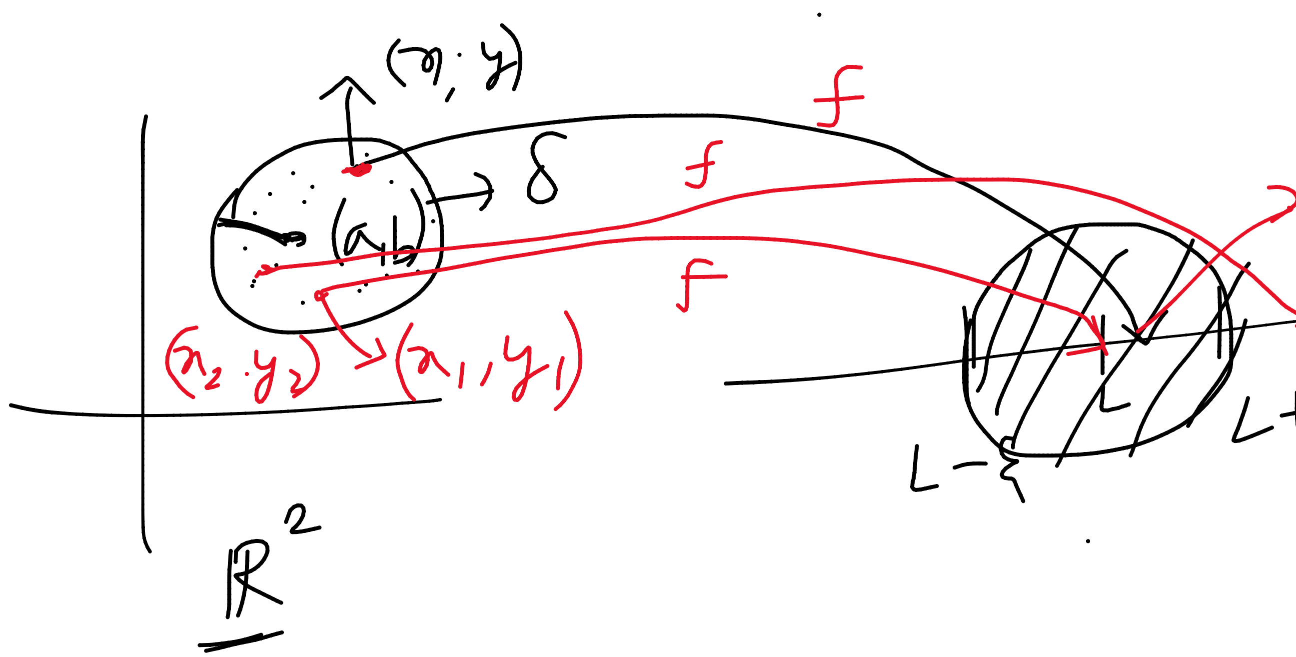
$R : \{ f(x,y) \mid R - \{0\}$

$R : \{ f(x, y) \mid R_+ - \{0\} \}$

$x^2 + y^2 = 3$



$x^2 + y^2 - 3 = 0$



$\varepsilon \rightarrow$ You may choose.

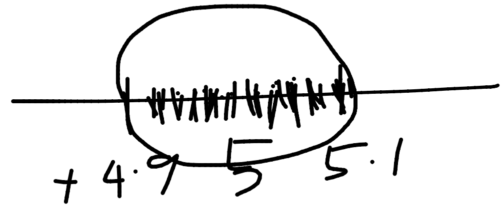
$\varepsilon = 0.1 \qquad L = 5$

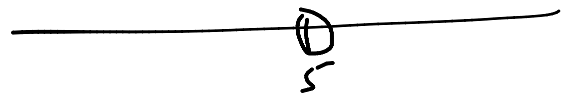
$f(x, y)$

$+\xi$

$\varepsilon = 0.1, \quad L = 5$

$\varepsilon = 0.00001$





$|x - a| < \delta \Rightarrow |f(x) - f(a)| <$

$|f(x,y) - L| < \varepsilon$

$|\sin(x)| \leq 1$

$-1 \leq \sin(x) \leq 1$

$|f(x,y) - L| < \varepsilon$

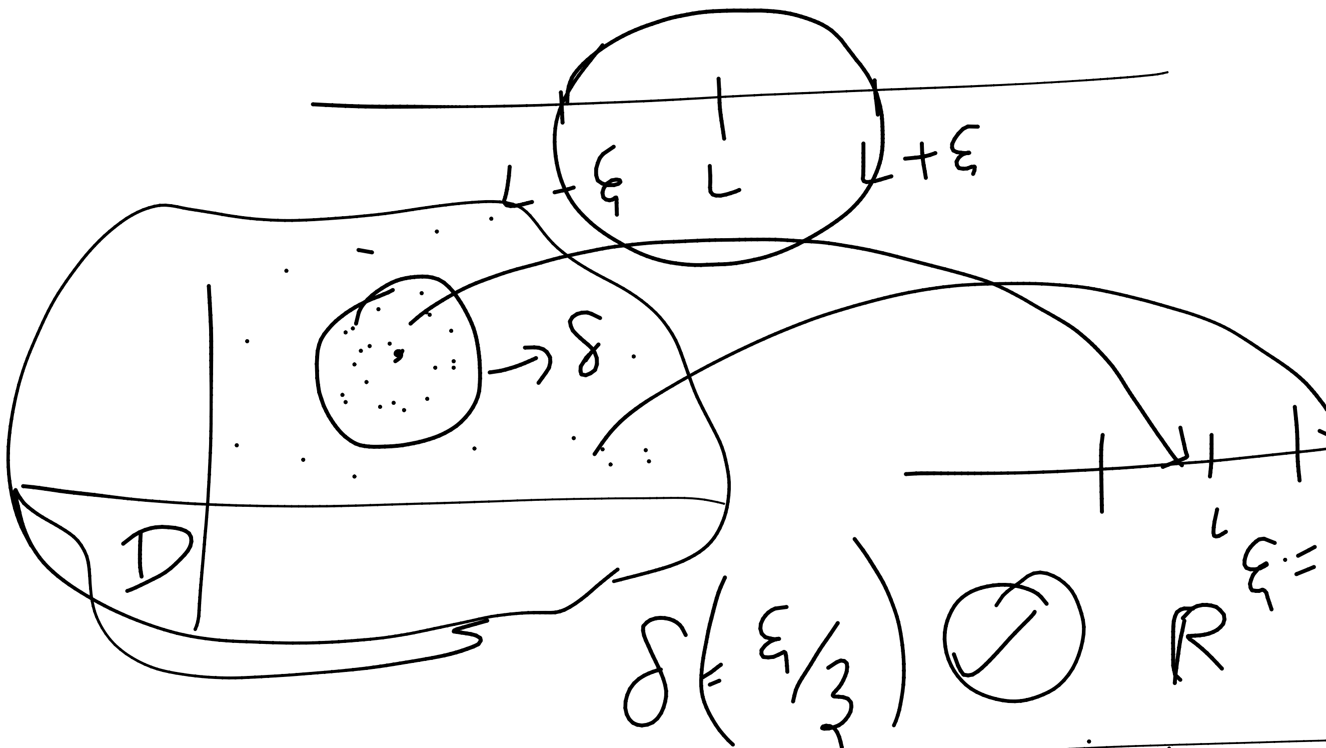
$-\varepsilon < f(x,y) - L < \varepsilon$

$f(x,y) - L + L < \varepsilon +$

$< \xi$

$$-\varepsilon + L < f(x,y) - L + L < \varepsilon +$$

$$L - \varepsilon < f(x,y) < L + \varepsilon$$



$$\lim_{(x,y)\to(0,0)} = \frac{xy}{x^2+y^2}$$

steps: $y = 0 \Rightarrow \lim_{(x,y)\to(0,0)} =$

$\lim \quad =$

L

$\frac{N}{3}$

$\frac{0}{1} = 0$

$\frac{0}{} = 0$

$\cancel{x=0} \Rightarrow \lim\limits_{(x,y)\to(0,0)} =$

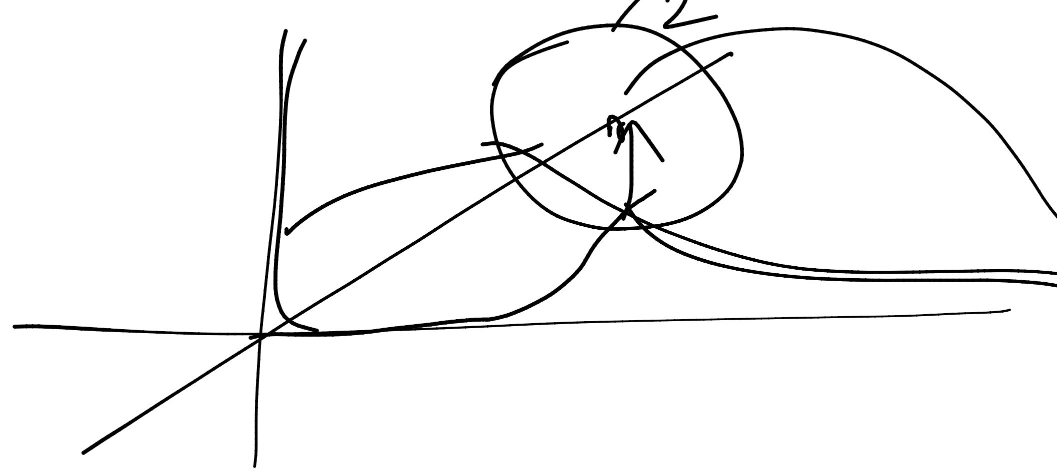
$\boxed{\cancel{y=mx}} \Rightarrow \lim\limits_{(x,y)\to(0,0)} \Big[$

$= \lim\limits_{(x,y)\to(0,0)} \Big[$

$= \lim\limits_{(x,y)\to(0,0)} \Big[$

$m=1 \qquad = \lim\limits_{(x,y)\to(0,0)} \Big[$

$= 1/2$ ✓



$\frac{0}{1} = 0$

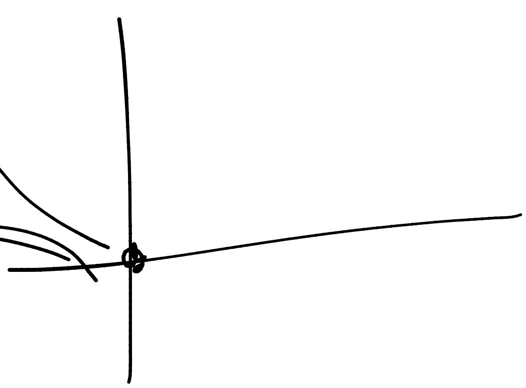
$\left. \frac{(mx)}{x^2+(mx)^2} \right]$

$\left. \frac{mx^2}{x^2+m^2x^2} \right]$

$\left. \frac{m}{1+m^2} \right]$

$\left. \frac{1}{1+1} \right) = \frac{1}{2}$

$\boxed{= L_2 \neq L_3}$



path

$\lim_{x \to 0} f(x) \to L_1$ on $\underline{C_1}$

$\lim_{x \to 0} f(x) \to L_2$ on C_2

Suppose $L_1 = L_2$ (Lim

Suppose $L_1 \neq L_2$ (Lim

---

(1) $C_1 : y = 0$	$\lim_{(x,y) \to (0,0)}$
(2) $C_2 : x = 0$	
(3) $\underline{y} = mx$	$\underline{C_1}, C_2$ -
(4) $y = x^2$	

C_3 $\underline{y = \underline{m}}$

$y =$

$\lim_{(x,y) \to (0,0)}$

$=$

path

path

it exists)

it does not exist)

$\frac{3x^2y}{x^2+y^2}$

$\rightarrow \quad L_1 = L_2 = 0$

mx

$\frac{3x^2 m x}{x^2+m^2x^2}$

$\frac{3m x^3}{x^2+m^2x^2}$

$y = x^2$

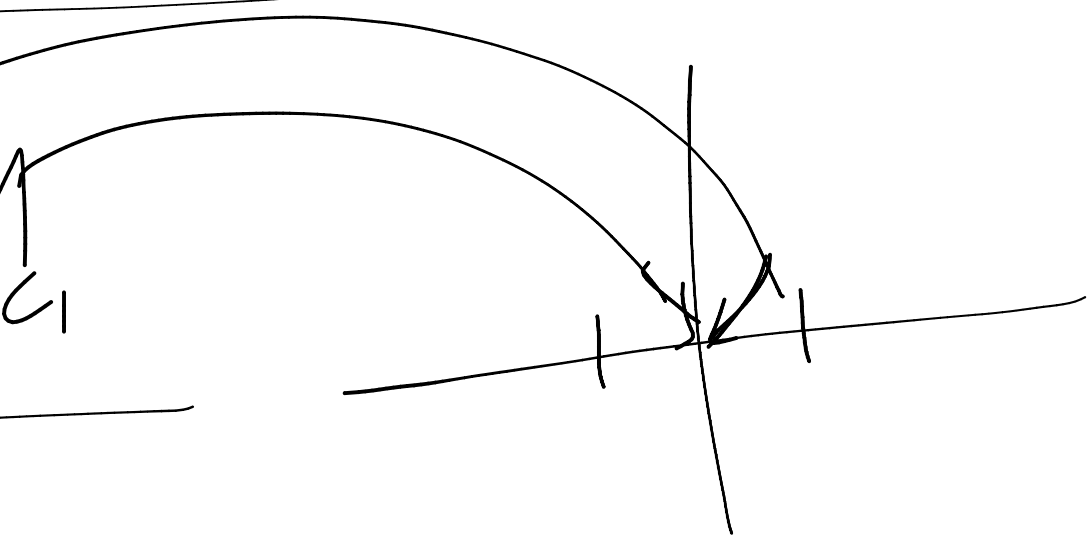
$| f(x, y)$

lim

$$= \frac{3m\,x}{x^2(1+m^2)}$$

$$= \frac{3m\,x}{1+m^2}$$

$$\Rightarrow 0$$



$$| < \delta$$

$$-L| < \varepsilon$$

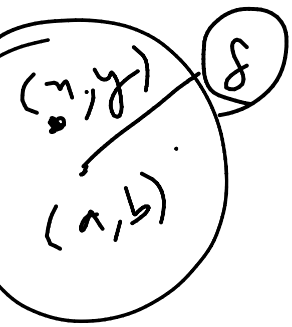
$$\Rightarrow |3x^2y - 0| < \varepsilon.$$

$$\lim_{(x,y)\to(0,0)}$$

(a, b) -

=

$$\Rightarrow \left| \frac{3x^2y}{x^2+y^2} - 0 \right| < \varepsilon.$$



$$\Rightarrow \sqrt{(x-a)^2+(y-b)^2}$$

$$0 < \sqrt{(x-a)^2+(y-b)^2} < \delta.$$

$(0, 0)$

$$0 < \sqrt{x^2+y^2} < \delta.$$

$$\Rightarrow \left| \frac{3x^2y}{x^2+y^2} - 0 \right| < \varepsilon.$$

$$= \frac{3x^2|y|}{x^2+y^2}$$

$(y^2 \geq 0)$

$\Rightarrow$ x

x

x^2

$x^2 + y$

$\leq x^2 + y^2 \qquad (y^2 \geq 0)$

$$\boxed{\frac{x^2}{x^2+y^2} \leq 1}$$

$$= \frac{3x^2|y|}{x^2+y^2}$$

$$\leq 3 \cdot 1 \cdot |y|$$

$$\leq 3 \cdot \sqrt{y^2}$$

$$\leq 3\sqrt{x^2+y^2}$$

$$< 3\delta$$

PDE

Tuesday, 2 March 2021 9:16 AM

$$f(x,y) = \underline{x^2} + \underline{3xy} + y - 1$$

$$f_x \text{ (or) } \frac{\partial f}{\partial x} = 2x + 3y \quad \text{— (1)}$$

$$f_y \text{ (or) } \frac{\partial f}{\partial y} = 3x + 1 \quad \text{— (2)}$$

$$f_x(4, -5) = 2(4) + 3(-5)$$
$$= -7$$

$$f_y(4, -5) = 13$$

$$f(x,y) = \frac{2y}{y + \cos x}$$

$$f_x = \left[\frac{(y + \cos x)\overset{0}{\frac{\partial}{\partial x}(2y)} - 2y\frac{\partial}{\partial x} \cdots}{(y + \cos x)^2} \right.$$

$$0 - 2y(-\sin x)$$

$$\frac{\ldots n\left(\underset{0}{\underline{y}}+\cos x\right)}{\quad}$$

$$\left[\ \frac{u}{v}=\frac{vu'-v'u}{v^2}\ \right]$$

$$= \frac{0 - 2y(-\sin x)}{(y+\cos x)^2}$$

$$\boxed{f_x = \frac{2y\sin x}{(y+\cos x)^2}}$$

$$f_y = \frac{2\cos x}{(y+\cos x)^2} \text{ //.}$$

3

$$yz - \ln z = x + y$$

$$\boxed{z = f(x,y)}$$

$$\boxed{\frac{\partial z}{\partial x} \to \text{Find}}$$

Take: $\underline{yz - \ln z =}$

y .

$x + y \quad — \quad 3$

$= \quad \partial \; (x) + \partial \; (y)$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}(\ln z$$

$$y\frac{\partial z}{\partial x} - \frac{1}{2}\,\frac{\partial}{\partial}$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{2}\right)$$

$$\frac{\partial z}{\partial x} = \frac{}{y}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{}{z}}$$

$$\frac{\partial z}{\partial y} =$$

Se

$$\left.z\right) = \frac{\partial}{\partial x}(x) + \cancel{\frac{\partial}{\partial x}(y)}$$

$$\frac{\partial z}{\partial x} = 1 + 0 \qquad \left[\; z = f(x,y) \right]$$

$$) = 1$$

$$\frac{1}{y - 1/2}$$

$$\boxed{\frac{2}{y-1}}$$

[Try !!!]

second order derivative

$$\frac{d^{1}x}{dt^{1}} = \text{Order}$$

$$\frac{d^{2}x}{dt^{2}} = \text{orde}$$

$$\frac{d^{n}x}{dt^{n}} = O$$

$$a_n \frac{d^n x}{dt^n} + a_{n-1}$$

n^{th}

first order

r is 1

er is 2

rder is n

$$\frac{d^{n-1}x}{dt^{n}} + \cdots + a_1\frac{dx}{dt} + a_0 .$$

- order.

ODE

∂f .

f_x, (or)

Second order

f_{xx} (

f_{yy} (

f_{xy}

$\boxed{f_{yx} =}$

$\therefore \boxed{f_{xy} =}$

$$\frac{\partial f}{\partial x}.$$

PDF

(or) $\frac{\partial^2 f}{\partial x^2}$.

(or) $\frac{\partial^2 f}{\partial y^2}$.

$= \frac{\partial^2 f}{\partial x \cdot \partial y}$

$\boxed{\frac{\partial^2 f}{\partial y \cdot \partial x}.} \Rightarrow \underline{\frac{\partial^2 f}{\partial x \cdot \partial y}} \Rightarrow \underline{\underline{\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)}}$

$\boxed{f_{yx}.}$

x

$f(x, y) = \underline{x \, \cos}$

To find

$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial}$

Start: with $\frac{\partial f}{\partial x}, \frac{\partial}{\partial}$

$\frac{\partial f(x, y)}{\partial x} = \frac{\cos}{}$

$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial}$

$\boxed{\frac{\partial^2 f}{\partial x^2} = ye}$

(2) $\frac{\partial f}{} = x(-\sin y)$

$\ldots os\, y + ye^x$

$\ldots y \,,\quad \frac{\partial^2 f}{\partial y^2} \,,\quad \frac{\partial^2 f}{\partial y \cdot \partial x} \,.$

$\frac{\ldots f}{\ldots y} \,.$

$\ldots os\, y + y \cdot e^x$

$\frac{\ldots}{\ldots x}\left(\underbrace{\cos y}_{0} + \underline{ye^x} \right)$

$\boxed{\ldots e^x}$

$\ldots + e^x$

(2) $\frac{\partial f}{\partial y} = x(-\sin y)$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left[-\right.$$

$$= -x \cdot c$$

$$\boxed{\frac{\partial^2 f}{\partial y^2} = -x\, co}$$

(3) $\frac{\partial^2 f}{\partial x \cdot \partial y}$ (or) $\frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}\right.$

$$= \frac{\partial}{\partial x}\left[-x\sin y +\right.$$

$$\boxed{\frac{\partial^2 f}{\partial x \cdot \partial y} = -\sin y + e^{x}}$$

$$\frac{\partial^2 f}{} \quad \frac{\partial}{}\left[\frac{\partial f}{}\right]$$

$+ c$

$x \sin y + \underbrace{e^x}_{0}]$

$\cos y$

$\boxed{sy}$

$= f_{xy}$

$e^x]$

4

$$f_{xyz} = f_{yzx}$$

$$\Updownarrow$$

$$f_{xyz} = f_{zyx}.$$

(4) $$\frac{\partial^2 f}{\partial y \cdot \partial x} = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}\right] =$$

$$= \frac{\partial}{\partial y}\left[\cos y +\right.$$

$$\boxed{\frac{\partial^2 f}{\partial y \cdot \partial x} = -\sin y + e}$$

It can be seen that

eqn. (4) = eq

$$\boxed{\frac{\partial^2 f}{\partial x \cdot \partial y} = \frac{\partial^2 f}{\partial y \cdot \partial x}}$$

(5) $f(x, y, z) = 1 - 2$

To find f => f_{yx}

f_{yx}.

$y e^{x}]$

x.

$-$ 5

n. 5

$xy^2z + x^2y$.

$= f_{xy}$.

To find $f_{yxyz} \Rightarrow f_{yx}$

Start: $f_y = -4xyz +$

$$\frac{\partial}{\partial x}(f_y) = -4yz + 2$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f_y}{\partial x}\right) = -4z +$$

$$\frac{\partial}{\partial z}\left[\frac{\partial^2 f_y}{\partial y \cdot \partial x}\right] = -4$$

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right.\right.\right.$$

Thomas

$= 1\,\underline{xy}$

x^2

$2x$

0

$f(x, y, z))))$

14.3

f_{xyz}

$\int \int \int \; dx$

$dy \quad dz$

Chain Rule

Thursday, 4 March 2021 10:00 AM

$W = f(x)$.

$\dfrac{d^2x}{dt^2} =$

$f(x, y)$

$\dfrac{\partial f}{\partial x}$ & $\dfrac{\partial f}{\partial y}$.

$\dfrac{\partial^2 f}{\partial x^2}$ & $\dfrac{\partial^2 f}{\partial y^2}$.

$y = f(x)$

$\boxed{\dfrac{dy}{dx} = f'(x)}$

$\boxed{\dfrac{d^2y}{dx^2} = f''(x)}$

$z = f(x, y)$

$\dfrac{\partial z}{\partial x} = f'(x, y)$

$\dfrac{\partial z}{\partial y} = f'(x, y)$

$\dfrac{\partial^2 z}{\partial y^2}$ (or) $\dfrac{\partial^2 z}{\partial x^2}$

Chain-rule:

$W = f(x)$, & $x = g(t)$

↓ True;

True
Id

$$\boxed{w = f(g(t))}$$

x - Interme

$$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt}$$

$w = x$ & $x = \sin t$

$$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt}$$

$$= 1 \cdot \cos t$$

$$= \cos t$$

Function of two variables:

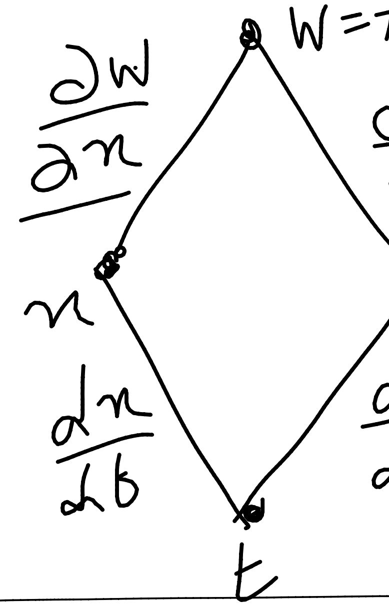
$$w = F(x, y) \quad \Big| \quad \begin{aligned} x &= G(t) \\ y &= H(t) \end{aligned}$$

$$w = F(G(t), H(t))$$

diale.

$$W = F(G(t), H(t))$$

$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}}$$



Problem:

Given $\begin{cases} W = xy & \& \ x = \cos \\ & \ \ y = \sin \end{cases}$

Find $\dfrac{dw}{dt}$.

$W = F(x, y$

$x = G(t)$

$y = H(t)$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{d}{d}$$

$W = xy$

$\underline{x = \cos t}$ $\quad$ $\underline{y =}$

$\dfrac{\partial w}{\partial x} = y$

$\dfrac{dx}{dt} = -$

$f(x, y)$

$\frac{\partial w}{\partial y}$

y

$\frac{dy}{dt}$

---

t

t.

$)$

$\frac{y}{t}$ — 1

$\sin t$

$\sin t$

$$\frac{\partial w}{\partial x} = y$$

$$\frac{\partial w}{\partial y} = x$$

$$\frac{dy}{dt} = c$$

$$\frac{dw}{dt} = y \cdot (-\sin t) + x \cdot \cos t$$

$$\frac{dw}{dt} = -\sin t \cdot y + \cos t \cdot$$

$$= -\sin t \sin t + \cos t$$

$$= -\sin^2 t + \cos^2 t$$

$$\boxed{\frac{dw}{dt} = \cos 2t} \quad (2)$$

Find $\left.\frac{dw}{dt}\right|_{t=\pi/2}$.

$$\frac{dw}{dt} = \cos \cancel{2}\left(\frac{\pi}{\cancel{2}}\right)$$

$$\cos \pi$$

cost

---

x.

· Cost

dt

$= \cos \pi$

$= -1$

Problem: 2

2 - Independent (r, s)
3 - Intermediate $(x, y$

$w = F(x, y, z)$ ✓
$x = G(r, s)$; $y = H(r, s)$; z

$\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ (To find

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}$$

Problem

$)$

$^{2})$

$= I(r,s)$

$d)$

$$\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r} \quad \text{— } 1$$

$$\cdot\ \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s} \quad \text{— } 2$$

∂w

$W = F(x, y, z)$

∂w

Problem

$$W = x + 2y + z^2$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \ldots$$

$$\frac{\partial w}{\partial x} = 1$$

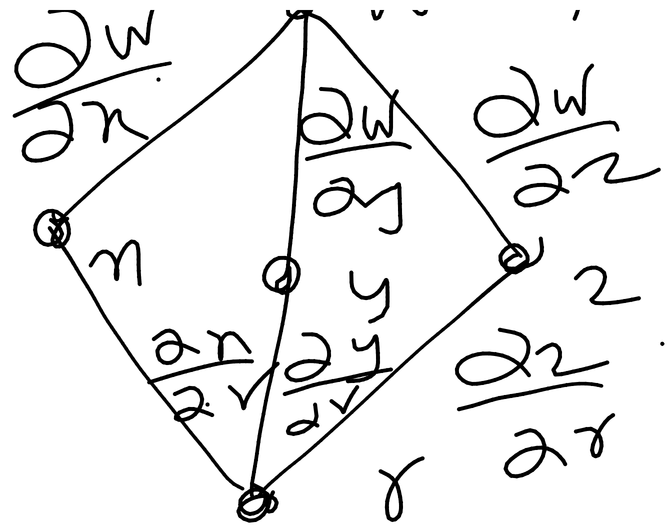
$$\frac{\partial w}{\partial y} = 2$$

$$\frac{\partial w}{\partial z} = 2z$$

$x = r \ldots$

$$\frac{\partial x}{\partial r} = \ldots$$

$$\frac{\partial x}{\partial s} \ldots$$



$$\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$+\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$/s,\ y = r^2+\ln s;\ z = 2r$

$= 1/s,\ \frac{\partial y}{\partial r} = 2r,\ \frac{\partial z}{\partial r} = 2$

$= -r/s^2,\ \frac{\partial y}{\partial s} = 1/s$

$H(r,s),\ T(r,s)$

$$\left.\frac{\partial z}{\partial s}\right| = 0$$

$F(g(t))$ $F(G(r,s)$

Question: Given W
r

To Find: $\frac{\partial}{\partial}$

, $H(r,s)$, $I(r,s)$)

$w = x + 2y + z^2$ and

$x = r/s$, $y = r^2 + \ln s$,

$z = 2r$.

$\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$.

Implicit Formula

Suppose: $F($

$F(x, y$

$F(x,$

$F(x$

Implicit formula

$(x, y) \rightarrow$ differen

$\boxed{y) = 0} \rightarrow \quad \frac{dy}{dx} =$

$y, z) \quad \& \quad z = f($

$\boxed{, y, z) = 0}$ then

tiable

$$\frac{-F_x}{F_y} \checkmark \quad \& \quad F_y \neq 0$$

$$\frac{(x, y)}{}$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} \quad \Bigg| \quad F_z$$

$\neq 0$

IF(n

, y , z

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Total Derivative

Monday, 8 March 2021 11:58 AM

Laplace eqn.

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$$

$v = f(x, y, z)$

Problem:

1) If $V=(x^2+y^2+z^2)^{-1/2}$ P.T

$$\boxed{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0}$$

Soln:

$$\frac{\partial^2 v}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right).$$

$V=(x^2+y^2+z^2)^{-1/2}$

$$\frac{\partial v}{\partial x}=-\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2-1}\cdot\cancel{2}x$$

$$=-x(x^2+y^2+z^2)^{-3/2}.$$

$$(x^2+y^2+z^2)^{-3/2}$$

/2.

$$\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) = \frac{\partial}{\partial x}\left(\underset{u}{-x}\,\underset{v}{\underline{\left(x^2+y^2+z^2\right)^{-3/2}}}\right)$$

$$= -\left[1\cdot\left(x^2+y^2+z^2\right)^{-3/2} + x\left[-\right.\right.$$

$$= -\left[\left(x^2+y^2+z^2\right)^{-3/2} - \right.$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[+3x^2\right.$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2\right.$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x\right.$$

$$\frac{\partial^2 V}{\partial x^2} = \left(x^2+y^2+z^2\right)^{-5/2}\left[2\right.$$

$$\cdots {}^{3}/\cancel{2}\left(x^2+y^2+z^2\right)^{-5/2}\cdot \cancel{2}x\Big]$$

$$\cdots 3x^2\left(x^2+y^2+z^2\right)^{-5/2}\Big]$$

$$\cdots \frac{\underline{3x}}{}$$

$$\cdots {}^{2} - \frac{\left(x^2+y^2+z^2\right)^{-3/2}}{\left(x^2+y^2+z^2\right)^{-5/2}}\Bigg]$$

$$\cdots - \left(x^2+y^2+z^2\right)^{-3/2+5/2}\Big]$$

$$\cdots x^2 - \left(x^2+y^2+z^2\right)\Big]$$

$$\cdots x^2 - y^2 - z^2\Big].$$

$$\cdots)^{-1/2}$$

∂x

$|||^{ly}$ $\dfrac{\partial^2 V}{\partial y^2}$ & $\dfrac{\partial^2 V}{\partial z^2}$ | $V =$

$$\frac{\partial^2 V}{\partial y^2} = (x^2+y^2+z^2)^{-5}$$

$$\frac{\partial^2 V}{\partial z^2} = (x^2+y^2+z^2)^{-5}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} =$$

$$\Rightarrow \quad (x^2+y^2+z^2)^{-5/2}$$

---

a function V

$(x^2+y^2+z^2)^{-1/2}$

$^{5/2}\left[2y^2-x^2-z^2\right]$

$^{5/2}\left[2z^2-y^2-x^2\right]$

0

$\left[0\right]=0.$

Satisfies the Laplace

A <u>function</u> v

equation $\left(\frac{\partial^2 v}{\partial x^2} + \right.$

is called as

<u>Harm</u>

<u>P</u>

$u = \log(x^3$

<u>P.T</u> $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right.$

satisfies "..."

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0)$$ then

harmonic function

nonic

Practice:

$$+y^3+z^3-3xyz)$$

$$\left(\ldots + \frac{\partial}{\partial z}\right)^2 \cdot u = \frac{-9}{(x+y+z)^2}$$

$$\left(\frac{\partial u}{\ldots} + \frac{\partial u}{\ldots} + \frac{\partial u}{\ldots}\right)$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial}\right.$$

$$\frac{\partial^2 z}{\partial t^2}$$

$$x = r\,($$

$$\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\Big)\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)$$

Wave- equation:

$$= c^2 \cdot \frac{\partial^2 z}{\partial x^2}$$

polar- co-ordinates

$$\cos\theta, \quad y = r\sin\theta$$

$$\theta = \tan^{-1}(y/x)$$

$$r = \sqrt{x^2+y^2}$$

Problem:

P.T $\dfrac{\partial x}{\partial \theta}$.

$\dfrac{\partial y}{\partial \theta}$

Soln' $x =$

$$x = e^{r\cos\theta} \cdot \cos(r\sin$$

$$y = e^{r\cos\theta} \cdot \sin(r\sin\theta$$

$$= -r \cdot \frac{\partial y}{\partial r} \quad (1)$$

$$= +r \cdot \frac{\partial x}{\partial r} \quad (2)$$

$$e^{r\cos\theta} \cdot \cos(r\sin\theta).$$

$n\theta)$

$)$

m

If x i

2 cm/sec

y = 1 cm.

be cha

function

neither

Statement-Proble

ncreases at the r

at the instant 2

at what rate r

nging in order that

$2xy - 3x^2y$ shall

increasing (or) dec

m:

ate of

$r = 3$cm and

must 'y'

the

be

reasing?

Rate of

$y = 1,$

$u = 2x$

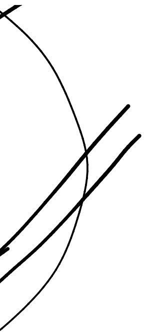
$\frac{du}{dt} = 0$

change :

$$\frac{dx}{dt} = 2$$

$x = 3$.

$y - 3x^2y \rightarrow$ neith
non



her decreasing

\+ increasing

To find

$\dfrac{d}{dt}$

Soln

$\dfrac{du}{dt}$

O

$$\boxed{\frac{dy}{dt} = ?}$$

$u = 2xy$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}$$

$$= \underline{\underline{\quad}}$$

$$) = (2y - 6xy)(2) -$$

$$y - 3x^2 y$$

$$\cdot \boxed{\frac{dy}{dt}}$$

$$+ (2x - 3x^2) \cdot \frac{dy}{dt}$$

$$\frac{dy}{dx}$$

$$=$$

$$=$$

$$= \frac{-(2y - 6xy)(2)}{2x - 3x^2}$$

$$\frac{-(2(1) - 6(3)(1))(2)}{2(3) - 3(3)^2}$$

$$= \frac{-(2 - 18)(2)}{6 - 27}$$

$$= \frac{+(-16)(2)}{}$$

Sub:

$x = 3$ | Present

$y = 1$ | value

$$\frac{dy}{dt}$$

Ex: u

$$|x^3 + y$$

$= \frac{\quad}{\times 21}$

$$\boxed{= \frac{-32}{21} \text{ cm.}}$$

Implicit di

$= x \log xy$

$^3 + 3xy = 1 \quad \boxed{f(x}$

fferentiation

$\left(u = f(x, y)\right)$

$\boxed{, y) = c}$

$x + y$

$+3xy-1$

$$\boxed{\frac{dy}{dx} = \frac{-\frac{\partial f}{\partial x}}{+\frac{\partial f}{\partial y}}}$$

$x^3 + y^3 + 3xy - 1 =$

$f(x, y) =$

$f(\ \ ) \quad x^3 + y^3 +$

$= 0$

0

$+ 3xy - 1$

t

<u>To find</u>

$\frac{du}{dt}$

$\frac{du}{dx}$

$f(x, y) = x^3 + y^3 +$

$$\boxed{\frac{du}{dx}}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial}{\partial}$$

$$= \frac{\partial u}{\partial x} \cdot \cancel{\frac{dx}{dx}} + \frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial y} \cdot \frac{dy}{dt} \cdot$$

$$\frac{\partial u}{\partial y} \cdot \frac{dy}{dx} \cdot$$

$$\boxed{\frac{du}{dx}}$$

$$\frac{dy}{dx}$$

$$= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

$$= \frac{-\frac{\partial f}{\partial x}}{+\frac{\partial f}{\partial y}} = \frac{-(3x}{3y}$$

$$x^2+y$$

$$\frac{dy}{dx}$$

$$\frac{{}^2 + 3y)}{{}^2 + 3x} = \frac{-\cancel{3}(x}{\cancel{3}(y}$$

$$\frac{x^2+y)}{y^2+x)}$$

$$\Big| \; \frac{du}{dx}$$

$$u = x \cdot ($$

$$=$$

$$\frac{y}{x} = \frac{\quad}{y^2+x}$$

$\log xy$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{d}{d}$$

$$\frac{dy}{dx} = \frac{x^2 + y}{y^2 +}$$

$$\frac{dy}{dx}$$

$$\frac{y}{x} \cdot$$

$$\sqrt{x^2 +$$


φ

$$\log x \cdot y) + x$$

$$1 + \log(xy$$

$$\cdot \frac{1}{\cancel{x}\cancel{y}} \cdot \cancel{y} + \Big(x$$

$$\Big) + \left(\frac{x}{y}\right)\left[\frac{x}{x}\right.$$

$$x \cdot \frac{1}{\cancel{x}y} \cdot \cancel{x} \cdot \frac{x+}{x+y}$$

$$\frac{{}^2+y}{+y^2}$$

$$(\ldots) = C$$

$$\frac{y}{y^2}\Bigg)$$

To find

$$= f(x, y)$$

$$\frac{du}{dx}$$

(x

$\cancel{z}$ $f(x$

$)$ $\dfrac{du}{dy}$

$y) =$

Jacobian Matrix

Wednesday, 10 March 2021 4:04 PM

Suppose: $u(x, y)$ & $v(x, y)$

$$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = J(u, v) = J(u(x,y), v(x,y))$$

$u(x, y, z)$ & $v(x, y, z)$, $w(x, y, z)$

$$J(u, v, w) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

Same:

$$\frac{\partial(u, v, w)}{\partial(x, y, z)}$$

Properties:

$$J = \frac{\partial(u, v)}{\partial(x, y)} \; ; \quad J' = \frac{\partial(x, y)}{\partial(u, v)}$$

$$J \cdot J' = \frac{\partial(u, v)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(u, v)}$$

$$J \cdot J' = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial y}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\frac{\partial u}{\partial u} = 1, \Rightarrow \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial u} + \boxed{\frac{\partial u}{\partial y}} \cdot \frac{\partial y}{\partial u}$$

$u(x,y)$

Property-2

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} \checkmark$$

$$u(x,y) = u(y,x)$$

$$u(x,y) = x + y \text{ (or)}$$

$$\sin(x+y) \quad \alpha$$

$$\left| \frac{\partial u}{} \quad \frac{\partial u}{} \right| \quad u = (x,y$$

$$\begin{vmatrix} \dfrac{\partial u}{\partial y} & \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} & \dfrac{\partial v}{\partial x} \end{vmatrix}$$

$u = (x, y)$

$v = (x, y)$

$u = (y, x)$

$v = (y, x)$

$$\frac{\partial(u, v)}{\partial(x, y)} \quad / \quad \frac{\partial(x, y)}{\partial(u, v)}$$

$$\Downarrow$$

$u(x, y) \qquad x(u, v)$

Problem:

$x = r\cos\theta \qquad y = r\sin\theta$

1)

1)

)

$$\frac{\partial x}{\partial r} =$$

$$\frac{\partial x}{\partial \theta} =$$

$$\frac{\partial y}{\partial r} =$$

$$\frac{\partial y}{\partial \theta}.$$

$$\frac{\partial (x,y)}{\partial (r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial} & \end{vmatrix} \checkmark$$

Problem:

$x = r\cos\theta$, $y = r\sin\theta$.

$$J = \frac{\partial (x,y)}{\partial (r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$\frac{\partial x}{} \quad \cos\theta \qquad \frac{\partial y}{} = \sin\theta$

$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r(\cos^2\theta + \sin^2\theta$$

$$= r$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = r.$$

$$\boxed{x = \underline{\rho}\cos\underline{\phi}, \quad y = \rho\sin\phi}$$

$$z = z$$

Given.

$z = z$

$$\left.\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)}\right\} \text{To find}$$

$$\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial z} \end{vmatrix}$$

$$\boxed{\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)} = \rho \cdot}$$

Problem:

$u = x^2 - y^2, \quad v = 2$

$\rightarrow$ Jacobian

answer

xy.

$r\sin\theta$.

$x = r\cos\theta\ ,\ y =$

Given

<u>To find</u> $\dfrac{\partial(u,v)}{\partial(r,\theta)}$.

<u>Formula by chain rul</u>

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v}{\partial(x,}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} =$$

$r\sin\theta$

e.

$\frac{\partial(x,y)}{\partial(r,\theta)}$.

$4r^3$. $\longrightarrow$ What are the ind dependent

$$\partial(r,\theta)$$

$$u = xy^2 ,$$

$$w = x +$$

Find $\dfrac{\partial(x,y}{\partial(u}$

Problem:

$r = x^2 + y^2 + z^2$

r

$y + z$.

$y, z)$. $\Rightarrow$ $\dfrac{1}{\partial(u, v, w)}$

$, w)$ $\partial(x, w$

$$u - (xyz) = 0$$

$$(x^2 + y^2 + z^2) = 0$$

$$w - (x + y + z) = 0$$

$$w)$$

$$z)$$

$z(u, v$

$f(x, y, z, v$

$$\boxed{\frac{dy}{dx}}$$

$v, w)$ $\quad \partial(x, y$

$u, v, w) = 0 \quad f(x$

$$= \boxed{\frac{-\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial y}}}$$

$1,2)$

$,y) = 0$

$$\frac{\partial(x, y}{\partial(u, v}$$

$$= \frac{1}{2(x-y)(y-z)(z}$$

$$\frac{y, z)}{v, w)} = (-1)^3 \cdot \frac{\partial(F}{\partial(} \over \frac{\partial(}{\partial(}$$

$-x)$ ✓

$\overline{, f_2, f_3)}$

$u, v, w)$

$\overline{f_1, f_2, f_3)}$

(x, y, z)

Functional Dependence

Saturday, 13 March 2021 2:38 PM

$$u = f_1(x, y)$$
$$v = f_2(x, y)$$

u & v are Connecte
by some relation
$f(u, v) = 0$

$f \to$ differentiable then we can
Say that u & v are functiona
dependent.

$$J(u,v) \text{ (or) } \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

then u & v are functionally

$$J(u,v,w) \text{ (or) } \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \\ \ldots \end{vmatrix}$$

d

ally

$$\Bigg| = 0$$

dependent

$$\begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \partial w & \partial w \end{vmatrix} = 0$$

$\left| \frac{\partial w}{\partial x} \right.$

Problem:

Given: $u = x+y-z$;

$w = x^2+y^2+z$

P.T (i) functions u, v

(ii) Find the

Soln:

We Know. $J(u, v,$

Remark:

if $J(u, v, w) = 0$

$\frac{\partial w}{\partial y} \quad \frac{\partial w}{\partial z} \Big|$

$v = \underline{\underline{x - y + z}}$

$\underline{{}^2 - 2yz.}$

$\underline{,\ \text{and}\ w}$ are functionally dependent.

relationship between them.

$$w) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

then functionally dependent.

$(1) \quad -1 \Big| \quad (3) = -(2)$

$$J(u, v, w) = \begin{vmatrix} 1 \\ 1 \\ 2x \end{vmatrix}$$

$$= 1\left[-(2z\right.$$

$$= 0$$

(ii) Find relatans

$w = x^2 + y^2$

$u = x + y -$

$u + v = 2x$ ①;

$(u+v)^2 + (u-v)^2$

$\parallel$

$$\begin{array}{cc} & -1 \\ 1 & \\ -1 & 1 \\ 2y-2z & 2z-2y \end{array} \qquad ③ = -②$$

$$-2y) - (2y-2z)\Big] + \cdots$$

hip between $\underline{u, v \;\&\; w}$.

$$^2 + z^2 - 2yz \longrightarrow Ⓐ$$

$$z; \quad v = x - y + z$$

$$u - v = 2y - 2z \quad —②$$

$$= (2x)^2 + \big(2(y-z)\big)^2 \to \big(①^2 + ②^2\big)$$

$$= 4x^2 + 4(y^2 + z^2 - 2yz)$$

$$\big[\,2 \ldots ^2 + z^2 - 2yz\big]$$

//

//

//

$u^2 + v^2 \cancel{+2uv} + u^2$

$2u^2 + 2v^2$

$\boxed{u^2 + v^2}$

$$= 4\left[x^2 + y^2 + z^2 - 2yz\right]$$

$$= 4W \text{ (from } A \text{)}$$

$$\ldots^2 + v^2 - \cancel{2uv} = 4W$$

$$\ldots = 4W$$

$$\boxed{\ldots^2 = 2W}$$

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