Reg. No:- 20BDS0405 Name:- Bimal Parajuli

Course:- MAT 1011 (Calculus for Engineers).

Max Marks: 10

Slot:- G1&TG1

Class No:- VL2020210505038

Due date:- 20/03/2021

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1). Verify mean value theorem for $f(x) = 2x^2 - 3x + 1$, $x \in [0,2]$.

Sol^n. For $f(x) = 2x^2 - 3x + 1$.

The function is a polynomial. Hence, it is continuous on $\mathbb{R}$.
Thus, continuous on $[0,2]$.

$f'(x) = 4x - 3$

Also, it is differentiable all over $\mathbb{R}$. Thus, over $(0,2)$, it's differentiable.

Take $a=0$, $b=2$,

At $x=0$, $f(x) = 2\times0^2 - 3\times0 + 1 = 1$

At $x=2$, $f(x) = 2\times2^2 - 3\times2 + 1 = 3$

Thus, according to Mean Value Theorem, theorem, there should be at least one c such that $c \in [0,2]$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$\Rightarrow 4c - 3 = \frac{3-1}{2} = 1$$

$$\Rightarrow 4c - 3 = 1$$

$$\Rightarrow 4c = 4$$

$\Rightarrow c = 1$ which lies in interval $[0,2]$.

Hence, mean value theorem is verified for $f(x) = 2x^2 - 3x + 1$ on $[0,2]$

Given $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$, find where the function 'f' is increasing and decreasing. Also provide local maximum and minimum values.

Sol^n:

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x$

For critical points, $f'(x) = 0$

$\Rightarrow 12x(x^2 - x - 2) = 0$

$= 12x(x^2 - 2x + x - 2) = 0$

$= 12x(x+1)(x-2) = 0$

$\Rightarrow$ Critical points are at $x = 0, 2, -1$.

$\therefore$ The intervals formed by critical points are $(-\infty, -1)$, $(1, 0)$, $(0, 2)$, $(2, \infty)$.

Intervals \ Sign of	x	$(x+1)$	$(x-2)$	$x(x+1)(x-2)$
$(-\infty, -1)$	$-ve$	$-ve$	$-ve$	$-ve$
$(-1, 0)$	$-ve$	$+ve$	$-ve$	$+ve$
$(0, 2)$	$+ve$	$+ve$	$-ve$	$-ve$
$(2, \infty)$	$+ve$	$+ve$	$+ve$	$+ve$

Thus, $f'(x) > 0$ in $(-1, 0) \cup (2, \infty)$.

$f'(x) < 0$ in $(-\infty, -1) \cup (1, 2)$.

Thus, the function is increasing in the interval $(-1, 0)$ and $(2, \infty)$.

while decreasing in $(-\infty, -1)$ and $(0, 2)$.

Evaluating at critical points,

$f''(x) = 36x^2 - 24x - 24 = 12(3x^2 - 2x - 2)$

At $x = -1$, $f''(x) = 12\{3\times(-1)^2 - 2\times(-1) - 2\} = 12(3 + 2 - 2) = 36 > 0$ (minima)

At $x = 0$, $f''(x) = 12(3\times 0^2 - 2\times 0 - 2) = 12(0 - 0 - 2) = -24 < 0$ (maxima).

At $x = 2$, $f''(x) = 12\times(3\times 2^2 - 2\times 2 - 2) = 12(12 - 4 - 2) = 72 > 0$ (minima).

$\therefore$ At $x = 0$, $f(x) = 3\times 0^4 - 4\times 0^3 - 12\times 0^2 + 5 = 5$ $\therefore$ Local Maxima $= (0, 5)$.

At $x = 2$, $f(x) = 3\times 2^4 - 4\times 2^3 - 12\times 2^2 + 5 = -27$ $\therefore$ Local minima $= (2, -27)$.

At $x = -1$, $f(x) = 3\times(-1)^4 - 4\times(-1)^3 - 12\times(-1)^2 + 5 = 0$ $\therefore$ Local minima $= (-1, 0)$.

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Sketch the graph of $f(x) = 1-9x-6x^2-x^3$ by hand and include the coordinates of any local, absolute extreme points and inflection points.

Sol.

$f(x) = 1-9x-6x^2-x^3$

$f'(x) = -9-12x-3x^2 = -3(x^2+4x+3)$

$f''(x) = -12-6x = -6(x+2)$.

For critical points, take $f'(x)=0 \Rightarrow x^2+4x+3=0$

$\Rightarrow (x+3)(x+1)=0$

$\Rightarrow x = -1, -3$

At $x=-1$, $f''(x) = -6(2-1) = -6<0$ (Local maxima).

At $x=-3$, $f''(x) = -6(-3+2) = 6>0$ (Local minima).

For Maxima, $x=-1$, $f(x) = 1-9\times(-1)-6\times(-1)^2-(-1)^3$

$= 1+9-6+1$

$= 5$

Minima, $x=-3$, $f(x) = 1-9(-3)-6\times(-3)^2-(-3)^3$

$= 1+27-54+27$

$= 1$

In interval $(-\infty,-3)$, taking -5 as testing point, $f'(x) = -3((-5)^2+4\times(-5)+3)$

$= -3\times(25-20+3)$

$= -24 < 0$ ∴ function is decreasing

In interval $(-3,-1)$ taking -2 as testing point, $f'(x) = -3((-2)^2+4\times(-2)+3)$

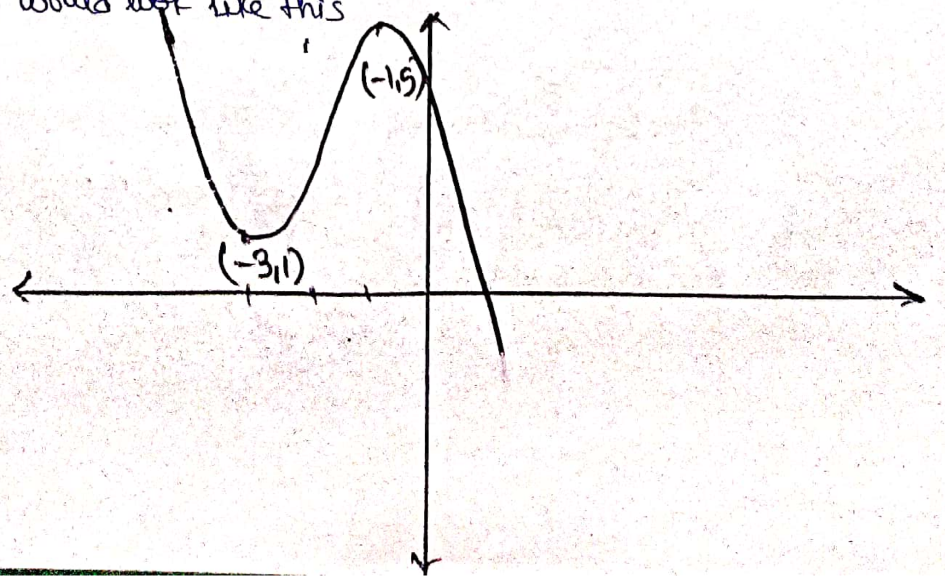
$= -3\times(4-8+3)$

$= 3>0$ ∴ function is increasing.

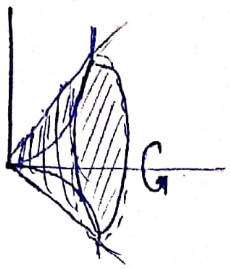
In interval $(-1,\infty)$, taking 0 as testing point $f'(x) = -3(0^2+4\times0+3)$

$= -3<0$ ∴ function is decreasing

Thus the graph would look like this



A cup like object is made by rotating the area between $y=2x^2$ and $y=x+1$ with $x\geq 0$ around x-axis. Find the volume of material needed to make cup (Units cm)



Given equations of curves:-

$$y=2x^2 \quad \text{——— (i)}$$
$$y=x+1 \quad \text{——— (ii)}$$

Solving (i) and (ii),

$$2x^2=x+1$$
$$\Rightarrow 2x^2-x-1=0$$
$$\Rightarrow 2x^2-2x+x-1=0$$
$$2x(x-1)+1(x-1)=0$$
$$\Rightarrow (2x+1)(x-1)=0$$
$$\Rightarrow x=1, -1/2$$

Since $x\geq 0$, negative value is neglected.

Thus, Volume of material needed $= \pi\left[\int_0^1 (x+1)^2 dx - \int_0^1 (2x^2)^2 dx\right]$

$$= \pi\left[\int_0^1 (x+1)^2-(2x^2)^2 dx\right]$$
$$= \pi\left[\int_0^1 (x^2+2x+1-4x^4)dx\right]$$
$$= \pi\left[\frac{x^3}{3}+x^2+x-\frac{4}{5}x^5\right]_0^1$$
$$= \pi\cdot\left(\frac{1}{3}+1+1-\frac{4}{5}\right)$$
$$= \pi\cdot\frac{23}{15}$$
$$= \frac{23\pi}{15} \text{ cubic cm.}$$

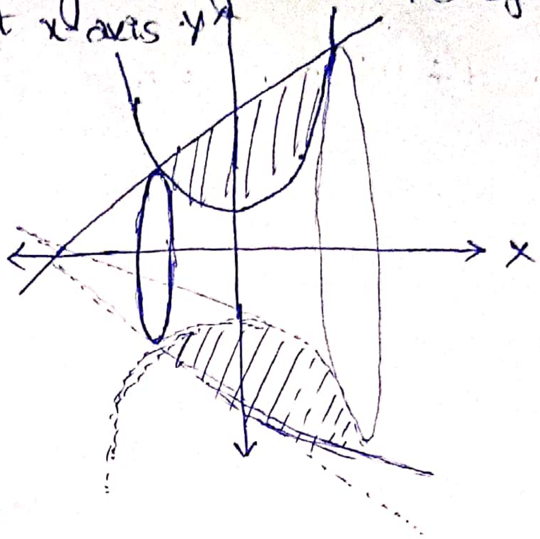
20BDS0405 BIMAL PARAJULI (5)

Find the volume of solids generated by revolving the regions bounded by the lines and curves $y=x+3$ and $y=x^2+1$ about x axis

Sol:

The given equations are:-

$$y = x+3 \quad \text{——— (i)}$$
$$y = x^2+1 \quad \text{——— (ii)}$$



Solving (i) and (ii),

$$x+3 = x^2+1$$
$$\Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow (x-2)(x+1) = 0.$$
$$\Rightarrow x = 2, -1.$$

Volume of solid enclosed by them is:-

$$V = \pi\left[\int_{-1}^{2}\left[(x+3)^2 - (x^2+1)^2\right] dx\right]$$
$$= \pi\left[\int_{-1}^{2}(x^2+6x+9 - x^4 - 2x^2 - 1)\,dx\right]$$
$$= \pi\left[\int_{-1}^{2}(8 - x^4 - x^2 + 6x)\,dx\right]$$
$$= \pi\left[8x - \frac{x^5}{5} - \frac{x^3}{3} + 3x^2\right]_{-1}^{2}$$
$$= \pi\left[\left(16 - \frac{32}{5} - \frac{8}{3} + 12\right) - \left(-8 + \frac{1}{5} + \frac{1}{3} + 3\right)\right]$$
$$= \pi\left[\frac{28\times15 - 32\times3 - 8\times5 + 5\times15 - 1\times3 - 1\times5}{15}\right]$$
$$= \frac{\pi\times351}{15} = \frac{117\pi}{15} \text{ cubic units.}$$

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Find the following:-

(a). $\mathcal{L}[e^{2t} t^2]$.

Sol. We have $\mathcal{L}t^n = \frac{\Gamma(n+1)}{s^{n+1}}$

So, $\mathcal{L}t^2 = \frac{\Gamma 3}{s^3} = \frac{2!}{s^3} = f(s)$ let

Using 1st shifting theorem,

$\mathcal{L}(e^{2t}t^2) = f(s-2)$

$= \frac{2!}{(s-2)^3}$

$= \frac{2!}{(s-2)^3}$

(b). $\mathcal{L}[e^{2t}\cos 2t]$

Sol. We have $\mathcal{L}\cos at = \frac{s}{s^2+a^2}$

So, $\mathcal{L}\cos 2t = \frac{s}{s^2+4} = f(s)$ let

Using 1st shifting theorem,

$\mathcal{L}(e^{2t}\cos 2t) = f(s-2)$

$= \frac{(s-2)}{(s-2)^2+4}$

$= \frac{s-2}{s^2-4s+8}$

(c). $\mathcal{L}^{-1}\left[\frac{e^{2s}}{s^2}\right]$

Sol. $\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = \frac{t^{2-1}}{(2-1)!}$

$= t$

Using 2nd shifting theorem,

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\,h(t-a)$.

Here, a = -2,

$\mathcal{L}^{-1}\left(e^{2s}\cdot\frac{1}{s^2}\right) = f(t+2)\,h(t+2)$

$= (t+2)H(t+2)$.

Graph the function $f(t) = 3[h(t-1) - h(t-4)]$ for $t \geq 0$ and $h(t)$ is a heaviside function. Also find the $L[f(t)]$.

Soln

Given that:- $f(t) = 3(h(t-1) - h(t-4))$

$$= 3 \begin{cases} 0 \text{ for } t < 1 \\ 1 \text{ for } 1 < t < 4 \\ 0 \text{ for } 4 < t \end{cases}$$

$$= \begin{cases} 0 \text{ for } t < 1 \text{ \& } 4 < t \\ 3 \text{ for } 1 < t < 4 \end{cases}$$

Following would be the graph for it.

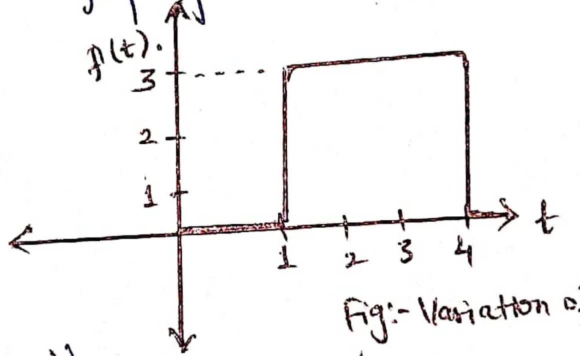

Fig:- Variation of $f(t)$ vs. t.

$$L[f(t)] = L(3(h(t-1) - h(t-4)))$$

$$= L[3 \cdot h(t-1)] - L[3 \cdot h(t-4)]$$

$$= e^{-s} L(3) - e^{-4s} L(3)$$

$$= \frac{e^{-s} \cdot 3}{s} - \frac{e^{-4s} \cdot 3}{s}$$

$$= \frac{3}{s}(e^{-s} - e^{-4s})$$

$$\therefore L[f(t)] = \frac{3}{s}(e^{-s} - e^{-4s})$$

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Use the partial decomposition method to find.

a). $\mathcal{L}^{-1}\left[\frac{12}{(s-3)(s+1)}\right]$

$\Rightarrow$ Sol:

$$\frac{12}{(s-3)(s+1)} = \frac{A}{(s-3)} + \frac{B}{(s+1)}$$

$$\Rightarrow \frac{12}{(s-3)(s+1)} = \frac{A(s+1)+B(s-3)}{(s-3)(s+1)}$$

$$\Rightarrow 12 = A(s+1) + B(s-3)$$

Putting $s=-1$ in above eq^n, $B=-3$

Putting $s=3$ in above eq^n, $A=3$

$$\therefore \mathcal{L}^{-1}\left[\frac{12}{(s-3)(s+1)}\right] = \mathcal{L}^{-1}\left[\frac{3}{(s-3)} - \frac{3}{(s+1)}\right]$$

$$= 3\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) - 3\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) \quad \text{[Using Linearity Property]}.$$

$$= 3e^{3t}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - 3e^{-t}.\mathcal{L}^{-1}\left(\frac{1}{s}\right) \quad \text{(using inverse first shifting property)}$$

$$= 3e^{3t} - 3e^{-t}$$

$$= 3(e^{3t} - e^{-t}).$$

$$\therefore \mathcal{L}^{-1}\left(\frac{12}{(s-3)(s-2)}\right) = 3(e^{3t} - e^{-t})$$

(b) $\mathcal{L}^{-1}\left(\frac{24e^{-5s}}{s^2-9}\right)$

Sol: We know, $\mathcal{L}^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{24}{s^2-9}\right) = 8\cdot\mathcal{L}^{-1}\left(\frac{3}{s^2-9}\right) = 8\cdot\sinh 3t$$

Now, from 2nd shifting theorem, $\mathcal{L}^{-1}\left(e^{-as}f(s)\right) = f(t-a)\cdot h(t-a)$.

Thus, $\mathcal{L}^{-1}\left(e^{-5s}\cdot\frac{24}{s^2-9}\right) = 8.\sinh 3(t-5).H(t-5)$

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Graph the function $f(t) = t[h(t-1) - h(t-3)]$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, find $L[f(t)]$.

Sol:- Given that,

$$f(t) = t \cdot [h(t-1) - h(t-3)]$$

$$= t \cdot \begin{cases} 0 & \text{for } t<1 \\ 1 & \text{for } 1<t<3 \\ 0 & \text{for } t>3 \end{cases}$$

$$= \begin{cases} 0 & \text{for } 0<t<1 \;\&\; t>3 \\ t & \text{for } 1<t<3 \end{cases}$$

The graph would be as follows:-

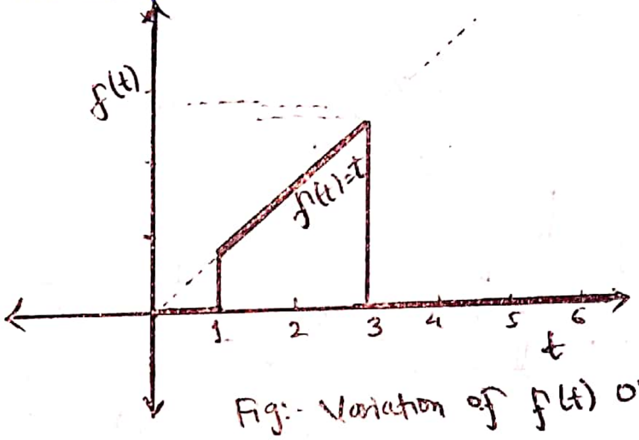

Fig:- Variation of f(t) over time.

To calculate $L[f(t)]$,

$$L[f(t)] = L[t \cdot h(t-1) - t \cdot h(t-3)]$$

$$= L[t \cdot h(t-1)] - L[t \cdot h(t-3)]$$

$$= e^{-s} L(t+1) - e^{-3s} L(t+3) \quad \text{(using 2nd shifting property)}.$$

$$= e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right) - e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s}\right)$$

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Use the Convolution Theorem to find:-

(a) $L^{-1}\left\{\frac{1}{(s^2+4)(s-1)}\right\}$

Sol:

$$\frac{1}{(s^2+4)(s-1)} = G(s) * F(s) \ (\text{let}).$$

where $G(s) = \frac{1}{(s^2+4)}$, $F(s) = \frac{1}{(s-1)}$

Now, $g(t) = L^{-1}(G(s))$
$= L^{-1}\left(\frac{1}{s^2+4}\right)$
$= \frac{\sin 2t}{2}$

Similarly, $f(t) = L^{-1}(F(s))$
$= L^{-1}\left(\frac{1}{s-1}\right)$
$= e^{1t} = e^{t}$

The convolution of $g(t)$ & $f(t)$ would be :-

$$(g*f)(t) = \int_0^t \frac{1}{2}\sin 2(t-\tau)\cdot e^{\tau} d\tau = I \text{ (suppose)}.$$

Then, $I = \int_0^t \frac{1}{2}\sin 2(t-\tau)\cdot e^{\tau} d\tau$

$$= \frac{1}{2}\left(\left[\sin 2(t-\tau)\, e^{\tau}\right]_0^t - \int_0^t \cos 2(t-\tau)\cdot(-2)\cdot e^{\tau} d\tau\right)$$

$$= \left[\frac{1}{2}\sin 2(t-\tau)\cdot e^{\tau}\right]_0^t - \left[\cos 2(t-\tau)\cdot e^{\tau}\right]_0^t - 4\int_0^t \frac{1}{2}\sin 2(t-\tau)e^{\tau} d\tau.$$

$\Rightarrow (I+4I) = -\frac{1}{2}\sin 2t + e^{t} - \cos 2t$

$\Rightarrow 5I = -\frac{1}{2}\sin 2t + e^{t} - \cos 2t.$

$\Rightarrow I = -\frac{1}{10}\sin 2t + \frac{e^t}{5} - \frac{\cos 2t}{5}$

$$\therefore L^{-1}\left(\frac{1}{(s^2+4)(s-1)}\right) = -\frac{1}{10}\sin 2t + \frac{e^t}{5} - \frac{\cos 2t}{5}$$

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(B). $L^{-1}\left[\frac{1}{s^2(s+6)}\right]$.

Sol^n:

$$L^{-1}\left[\frac{1}{s^2(s+6)}\right] = ?$$

Let $G(s) = \frac{1}{s^2}$, $I(s) = \frac{1}{s+6}$

$$g(t) = L^{-1}[G(s)] = L^{-1}\left(\frac{1}{s^2}\right) = t$$

$$i(t) = L^{-1}[H(s)] = L^{-1}\left(\frac{1}{s+6}\right) = e^{-6t}L^{-1}\left(\frac{1}{s}\right) = e^{-6t}.$$

Then, the convolution of $g(t)$ and $i(t)$ is:-

$$(g*i)(t) = \int_0^t (t-\tau).e^{-6\tau}d\tau$$

$$= \int_0^t t.e^{-6\tau}d\tau - \int_0^t \tau.e^{-6\tau}d\tau$$

$$= t.\left[\frac{e^{-6\tau}}{-6}\right]_0^t - \left(\left[\tau\frac{e^{-6\tau}}{-6}\right]_0^t - \int_0^t \frac{e^{-6\tau}}{-6}d\tau\right)$$

$$= \cancel{\frac{t.e^{-6t}}{-6}} + \frac{t}{6} + \cancel{\frac{te^{-6t}}{6}} + \left[\frac{e^{-6\tau}}{36}\right]_0^t$$

$$= \frac{t}{6} + \frac{e^{-6t}}{36} - \frac{1}{36}$$

Thus, $L^{-1}\left(\frac{1}{s^2(s+6)}\right) = \frac{t}{6} + \frac{e^{-6t}}{36} - \frac{1}{36}$