Area between the curves

If f and g are continuous with $f(x) \geq g(x)$ for $x \in [a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral

$$A = \int_a^b [f(x) - g(x)]dx.$$

Also, if a region's bounding curves f and g are described by functions of y, where f denotes the right hand curve and g denotes the left hand curve, $f(y) - g(y)$ being non negative, then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from $y = c$ to d is the integral

$$A = \int_c^d [f(y) - g(y)]dy.$$

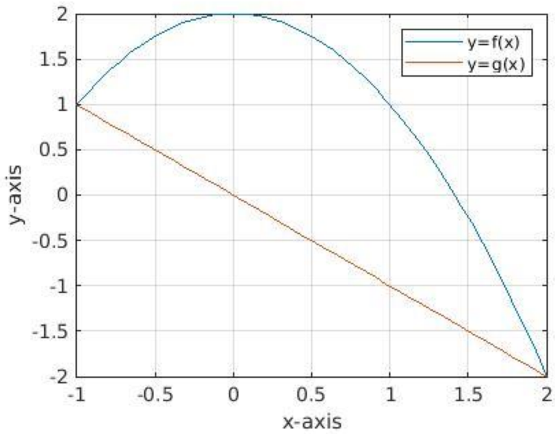
Example 1.

The area bounded by the curves $y = 2 - x^2$ and the line $y = -x$, from $x = -1$ to 2 is given by the following code:

```
clear
clc
syms x
f(x)=2-x^2; % Upper curve
g(x)=-x;    % Lower curve
I=[-1,2]; % Interval of Integration
a=I(1); b=I(2);
A=int(f(x)-g(x),a,b); % Finding the area by integration
disp('Area bounded by the curves f(x) and g(x) is:' );
disp(A);
fplot(f(x),[a,b]);grid on;hold on; %Plotting the upper curve
fplot(g(x),[a,b]);hold off         %Plotting the lower curve
xlabel('x-axis');ylabel('y-axis');
legend('y=f(x)','y=g(x)');
```

Output

```
Area bounded by the curves f(x) and g(x) is:
9/2
```



Volume of solid of revolution – Disc method

The solid figure formed by revolving a plane curve about an axis is called Solid of revolution.

If the solid is formed by revolving the curve $y = f(x)$ about a line $y = c$ (parallel to the $x-$axis), then the volume of the solid formed is given by $V = \int_a^b \pi[y-c]^2\, dx$.

Note: For $c = 0$, the axis of revolution will be the $x-$axis itself.

Example 2.

The volume of the solid generated by the revolving the curve $y = \sqrt{x}$ about the line $y = 1$ from $x = 1$ to $x = 4$ is given by the following code:

MATLAB Syntax used:

The code below consists of two parts.

The first part evaluates the volume of the solid generated by revolving the curve $y = f(x)$ about the line $y = yr$ (axis of revolution) for $x \in (a,b)$.

In the second part we give the visualization of the solid of revolution in the 3-D space.

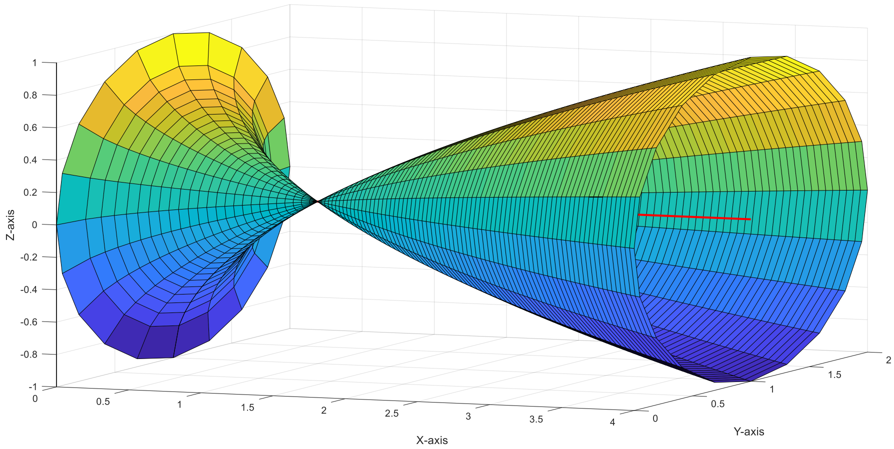
Some of the MATLAB commands used in the code are explained here.

`int(f(x),a,b)`	evaluates the integration of $f(x)$ between the limits a and b
`fx=matlabFunction(f)`	This converts Symbolic Function f to Anonymous Function *fx*
`[X,Y,Z]=cylinder(r);`	This returns the *x*-, *y*-, and *z*-coordinates of a cylinder using *r* to define a profile curve. cylinder treats each element in `r` as a radius at equally spaced heights along the unit height of the cylinder. The cylinder has 20 equally spaced points around its circumference.
`surf(X,Y,Z)`	This creates a three-dimensional surface plot. The function plots the values in matrix Z as heights above a grid in the *x*-*y* plane defined by X and Y.

```
%Evaluation of Volume of solid of revolution
clear all
clc
syms x
f(x)=sqrt(x);          % Given function
yr=1;                  % Axis of revolution y=yr
I=[0,4];               % Interval of integration
a=I(1);b=I(2);
vol=pi*int((f(x)-yr)^2,a,b);
disp('Volume of solid of revolution is: ');
disp(vol);
% Visualization if solid of revolution
fx=matlabFunction(f);
xv = linspace(a,b,101); % Creates 101 points from a to b
[X,Y,Z] = cylinder(fx(xv)-yr);
Z = a+Z.*(b-a);     % Extending the default unit height of the
cylinder profile to the interval of integration.
surf(Z,Y+yr,X)    % Plotting the solid of revolution about y=yr
hold on;
plot([a b],[yr yr],'-r','LineWidth',2); % Plotting the line y=yr
view(22,11);       % 3-D graph viewpoint specification
xlabel('X-axis');ylabel('Y-axis');zlabel('Z-axis');
```

Output

```
Volume of solid of revolution is:
(4*pi)/3
```



Exercise:

1. Find the area of the region bounded by the curve $y = x^2 - 2x$ and the line $y = x$.
2. To find the area of the region bounded by the curves $y^2 = x$, $y = x - 2$ in the first quadrant.
3. Find the area of the region bounded by the curves $x = y^3$ and $x = y^2$.
4. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the curve $y = \dfrac{4}{x^2 + 4}$, the $x-$axis, and the lines $x = 0$ and $x = 2$.

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