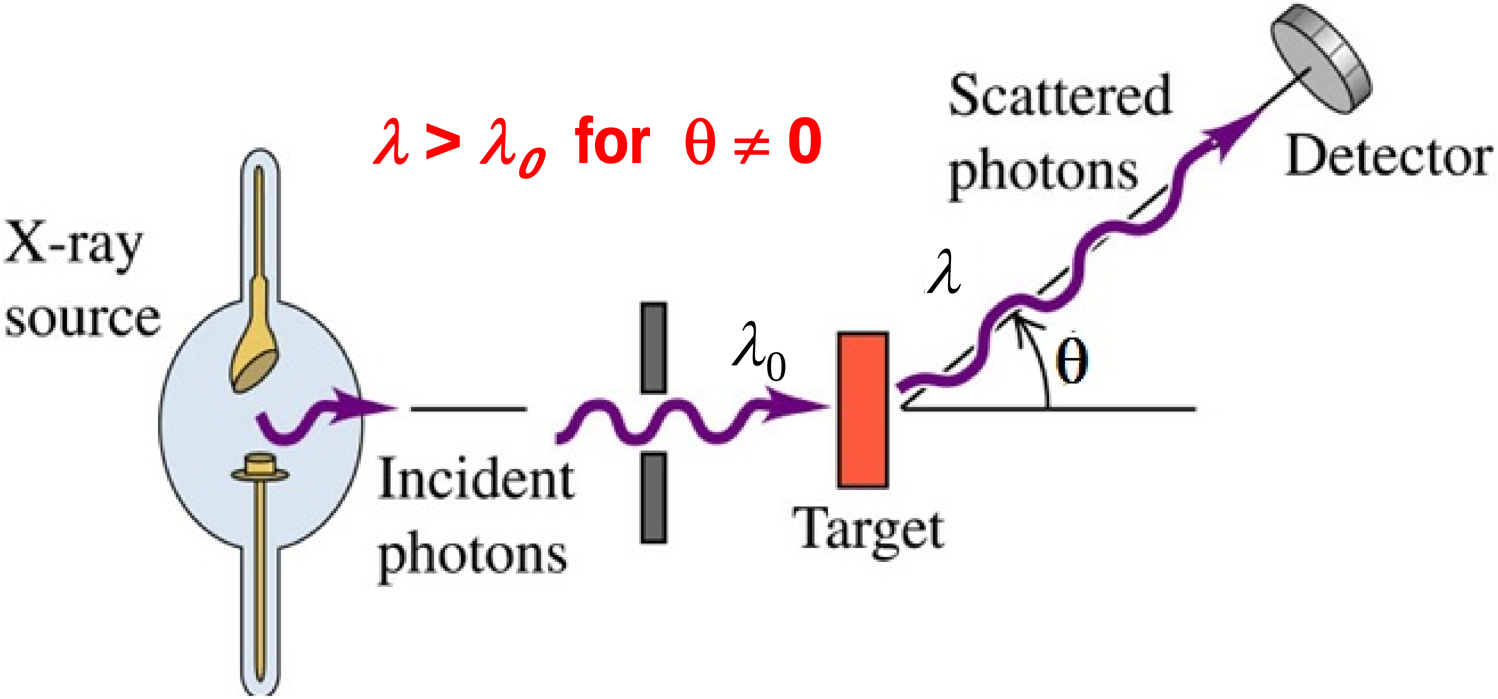
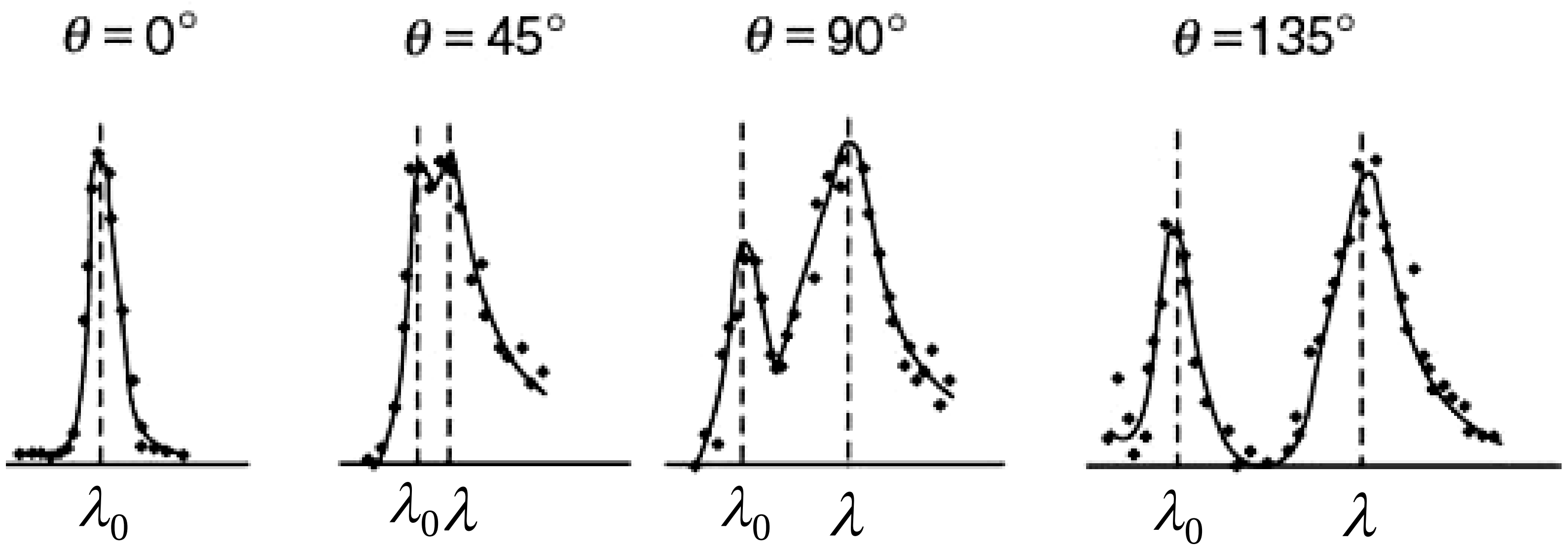
Compton Effect

Compton discovered that when a beam of X-rays is scattered from a target, the wavelengths of the scattered X-rays are slightly greater than the wavelength of incident beam.

$\lambda > \lambda_0$ **for** $\theta \neq 0$



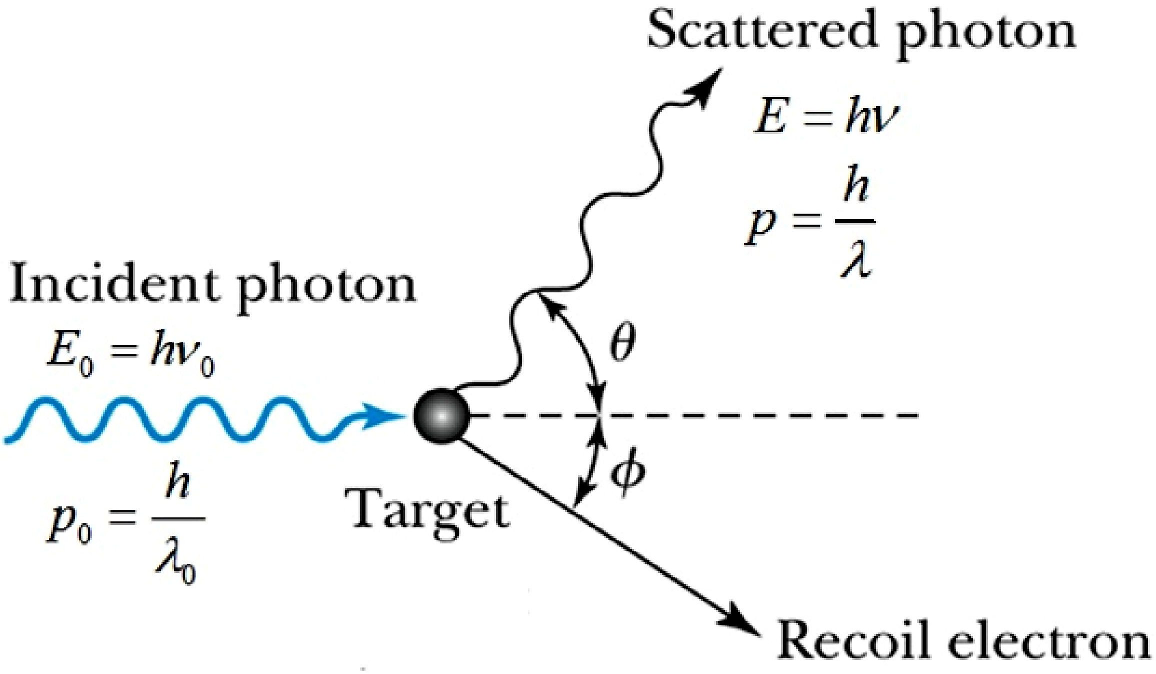


Compton experimental results for four different values of the scattering angle θ

- The peak at λ_0 is due to the scattering of the incident radiation from the tightly bound inner electrons of the atom.
- The second peak represents the radiation scattered from the loosely bound, nearly free outer electrons

Compton Shift $\Delta\lambda = \lambda - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$

- The scattering of X-rays by electrons is treated as the collision between photons and electrons.
- The electrons which are loosely bound can be assumed as almost free particles at rest.



Before Collision

Energy of incident photons $E_0 = h\nu_0$

Linear momentum of incident photons $p_0 = \dfrac{E_0}{c} = \dfrac{h\nu_0}{c}$

Relativistic relation between energy and momentum $E = \sqrt{p^2c^2 + m_0^2c^4}$

Rest mass of photon, m_0 = 0. So $E_0 = p_0 c$

Energy of electrons = $m_e c^2$

Free electron at rest… No momentum

After Collision

Energy of scattered photons $E = h\nu$

Linear momentum of scattered photons $p = \dfrac{h\nu}{c}$

Energy of electrons $E_e = \sqrt{p_e^2c^2 + m_e^2c^4}$

Linear momentum of recoiled electron = p_e

Conservation of momentum

$$\vec{p}_0 + 0 = \vec{p} + \vec{p}_e \quad \text{or} \quad \vec{p}_e = \vec{p}_0 - \vec{p}$$

$$p_e^2 = \left(\vec{p}_0 - \vec{p}\right)\cdot\left(\vec{p}_0 - \vec{p}\right) = p_0^2 + p^2 - 2\vec{p}_0 \bullet \vec{p}$$

$$\boxed{p_e^2 = p_0^2 + p^2 - 2p_0 p\cos\theta}$$

Conservation of Energy

$$E_0 + m_e c^2 = E + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$\left[\left(E_0 - E\right) + m_e c^2\right] = \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$\left[\left(E_0 - E\right) + m_e c^2\right]^2 = p_e^2 c^2 + m_e^2 c^4$$

$$\left(E_0 - E\right)^2 + 2\left(E_0 - E\right)m_e c^2 = p_e^2 c^2$$

$$(E_0 - E)^2 + 2(E_0 - E)m_e c^2 = p_0^2 c^2 + p^2 c^2 - 2p_0 p c^2 \cos\theta$$

$$= E_0^2 + E^2 - 2E_0 E \cos\theta$$

$$\cancel{(E_0 - E)^2} + 2(E_0 - E)m_e c^2 = \cancel{E_0^2 + E^2 - 2E_0 E} + 2E_0 E - 2E_0 E \cos\theta$$

$$2(E_0 - E)m_e c^2 = 2E_0 E(1 - \cos\theta)$$

$$\left(\frac{1}{E} - \frac{1}{E_0}\right) = \frac{1}{m_e c^2}(1 - \cos\theta)$$

$$\left(\frac{1}{\nu} - \frac{1}{\nu_0}\right) = \frac{h}{m_e c^2}(1 - \cos\theta)$$

$$\boxed{\lambda - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)}$$

Compton wavelength

$$\lambda_c = \frac{h}{m_e c} = 0.00243 \quad \text{nm}$$