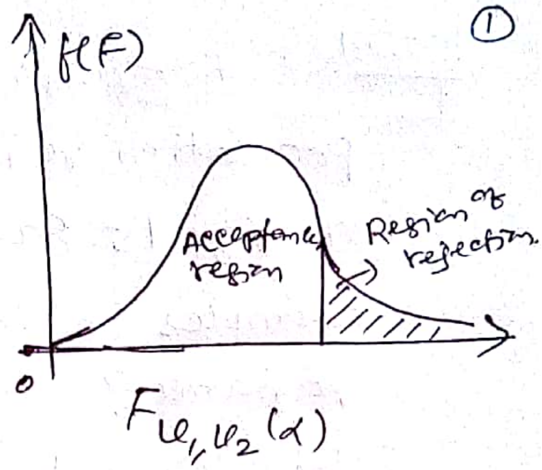
F - Distribution

The probability curve of the F-distribution is



F-test is used to test the significance of the difference between population variances σ_1^2 and σ_2^2.

We shall estimate σ_1^2 and σ_2^2 based on sample variances s_1^2 and s_2^2

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} \quad \text{with d.f } \nu_1 = n_1 - 1$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} \quad \text{with d.f } \nu_2 = n_2 - 1.$$

$$F = \frac{\sigma_1^2}{\sigma_2^2} \text{ or } \frac{\sigma_2^2}{\sigma_1^2} \quad \text{ie } F = \frac{\text{Greater variance}}{\text{Smaller variance}}$$

If $F > F_{(\nu_1,\nu_2)}(\alpha)$, then the difference between σ_1^2 and σ_2^2 is significant at LOS 'α'.

Note 1: 1. F test is not a two tailed test and is always a right tailed test, since F cannot be negative.

2. To test whether two samples comes from the same normal population, first we should test the equality of variances by F-test and then apply t-test for the significance of the difference of two sample means.

Prob.-1 : A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

Soln:- Given that $n_1 = 13$, $n_2 = 15$, $\sigma_1^2 = 3.0$, $\sigma_2^2 = 2.5$,

$\nu_1 = n_1 - 1 = 12$, $\nu_2 = n_2 - 1 = 14$

1. $H_0 : \sigma_1^2 = \sigma_2^2$

2. $H_1 : \sigma_1^2 \neq \sigma_2^2$

3. Let the LOS be 5%, Tabulated value of

$F_{0.05}(\nu_1, \nu_2) = 2.53$ by F-table.

4. $F = \dfrac{\text{Greater variance}}{\text{Smaller variance}} = \dfrac{\sigma_1^2}{\sigma_2^2} = \dfrac{3}{2.5} = 1.2$

5. Calculated value of $F = 1.2 <$ Tab. value of

$F_{0.05}(12, 14) = 2.53$

Accept H_0.

ie Two samples could have come from two normal populations with the same variance.

Prob.2 : Time taken by workers in performing a job are given below

Method I : 20 16 26 27 23 22

Method II : 27 33 42 35 32 34 38

Test whether there is any significant difference between the variances of time distribution.

Soln :- Let us calculate the variances of the samples.

x_1	$d_1 = x_1 - 22$	d_1^2	x_2	$d_2 = x_2 - 34$	d_2^2
20	-2	4	27	-7	49
16	-6	36	33	-1	1
26	4	16	42	8	64
27	5	25	35	1	1
23	1	1	32	-2	4
22	0	0	34	0	0
			38	4	16
134	2	82	241	3	135

$$s_1^2 = \frac{1}{n_1}\Sigma d_1^2 - \left(\frac{\Sigma d_1}{n_1}\right)^2 = \frac{82}{6} - \left(\frac{2}{6}\right)^2 = 13.23$$

$$s_2^2 = \frac{1}{n_2}\Sigma d_2^2 - \left(\frac{\Sigma d_2}{n_2}\right)^2 = \frac{135}{7} - \left(\frac{3}{7}\right)^2 = 19.11$$

$$n_1 = 6, \; n_2 = 7, \; s_1^2 = 13.23, \; s_2^2 = 19.11$$

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{6 \times 13.23}{5} = 15.88 \; ; \; \nu_1 = 5$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{7 \times 19.11}{6} = 22.30 \; ; \; \nu_2 = 6$$

1. $H_0 : \sigma_1^2 = \sigma_2^2$
2. $H_1 : \sigma_1^2 \neq \sigma_2^2$
3. Let the LOS be 5%, $F_{0.05}(6,5) = 4.28$
4. $F = \frac{\sigma_2^2}{\sigma_1^2} = \frac{22.30}{15.88} = 1.40$
5. Cal. value of $F = 1.40$ < Tab. value of $F_{0.05}(6,4) = 4.28$

Accept H_0

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Prob. 3: Two random samples gave the following data.

	Size	mean	variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population.

Soln:- Given that $n_1 = 8$, $n_2 = 11$, $s_1^2 = 1.2$, $s_2^2 = 2.5$

$\bar{x}_1 = 9.6$, $\bar{x}_2 = 16.5$

I. F-test - to test population variances

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 1.37, \quad \sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 2.7$$

$$\nu_1 = n_1 - 1 = 7, \quad \nu_2 = 11 - 1 = 10$$

1. $H_0: \sigma_1^2 = \sigma_2^2$; 3. Let the LOS be 5%.
2. $H_1: \sigma_1^2 \neq \sigma_2^2$; Tab. value of $F_{0.05}(10,7) = 3.64$ by F-table

4. $F = \frac{\sigma_2^2}{\sigma_1^2} = \frac{2.7}{1.37} = 2.007$

5. Cal. value of F < Tab. value of $F_{0.05}(10,7) = 3.64$

Accept H_0.

II: t-test - to test the equality of means.

1. $H_0: \bar{x}_1 = \bar{x}_2$; 2. $H_1: \bar{x}_1 \neq \bar{x}_2$;
3. Let LOS be 5%, $t_{0.05}(\nu = 17) = 2.11$ from t-table

4. $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -10.05$$

5. Cal. value of $|t| = 10.05$ > Tab. value of $t_{0.05} = 2.11$

Reject H_0.

The two samples could not have been drawn from the same normal population.