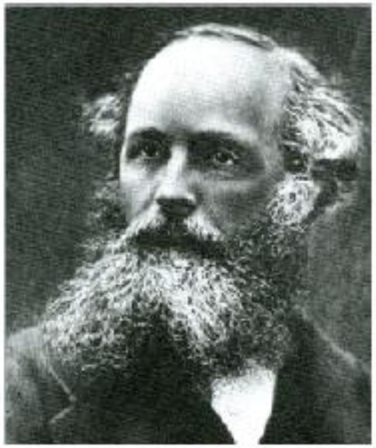
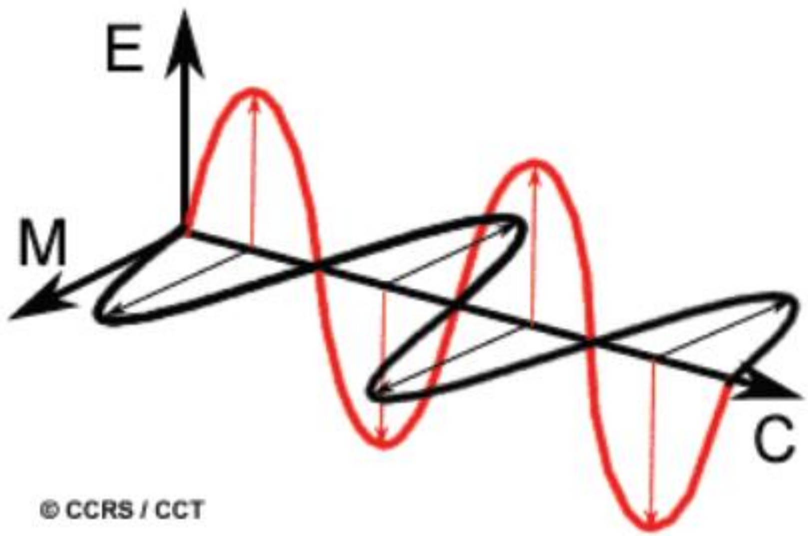
Electromagnetic radiation: wave model



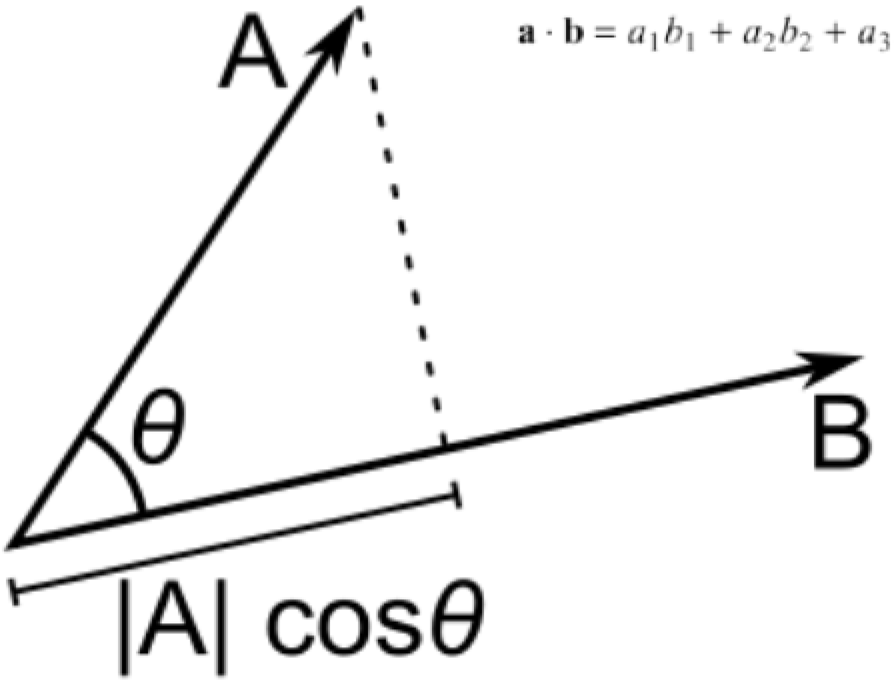
- James Clerk Maxwell (1831-1879) – Scottish mathematician and physicist
- Wave model of EM energy
  - Unified existing laws of electricity and magnetism (Newton, Faraday, Kelvin, Ampère)
  - Oscillating electric field produces a magnetic field (and vice versa) – propagates an EM wave
  - Can be described by 4 differential equations
  - Derived speed of EM wave in a vacuum

Electromagnetic radiation



- EM wave is:
- Electric field (E) perpendicular to magnetic field (M)
- Travels at velocity, c (3×10^8 ms^{-1}, in a vacuum)

Dot (scalar) product



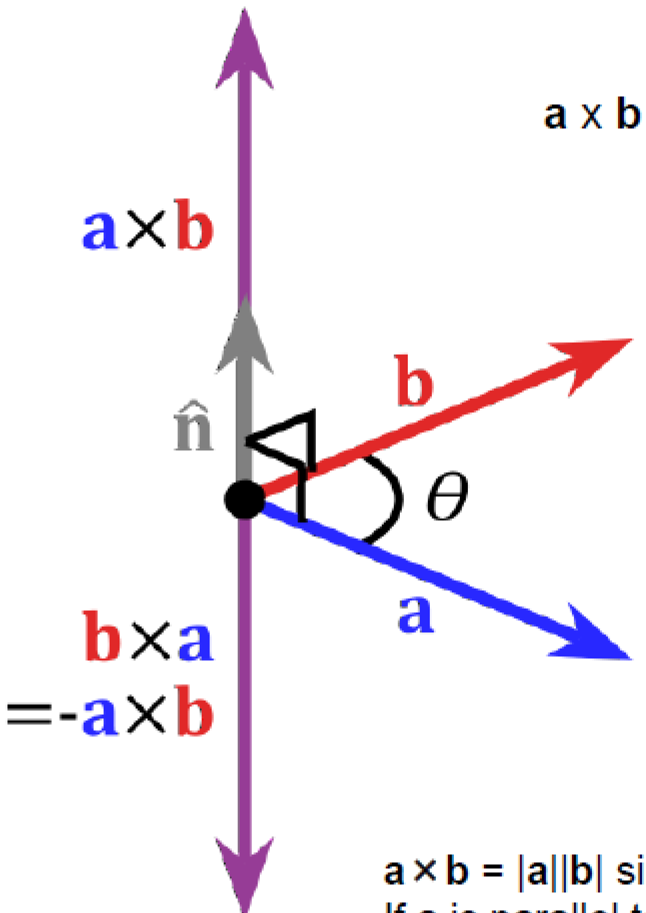
$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

A•**B** = |**A**||**B**|cos θ

If **A** is perpendicular to **B**, the dot product of **A** and **B** is zero

Cross (vector) product

$\mathbf{a} \times \mathbf{b} = [(a_2b_3 - a_3b_2), (a_3b_1 - a_1b_3), (a_1b_2 - a_2b_1)]$



$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin \theta \, \mathbf{n}$

If **a** is parallel to **b**, the cross product of **a** and **b** is zero

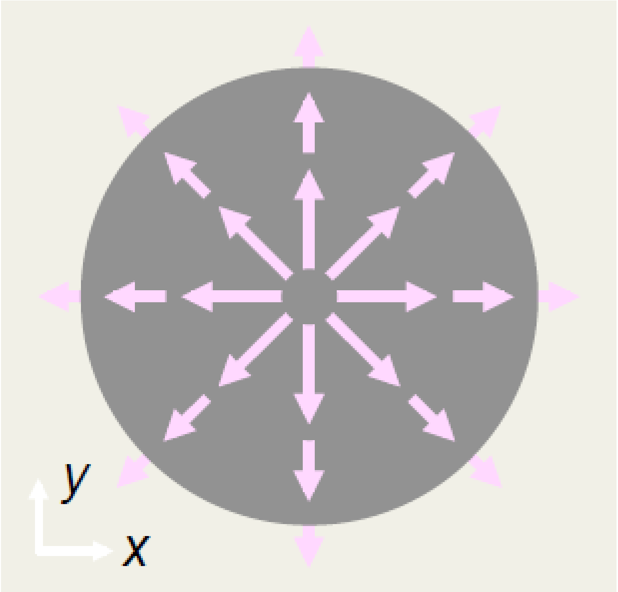
Div, Grad, Curl

The Divergence of a vector function (scalar):

$$\nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

The Divergence is nonzero if there are sources or sinks.

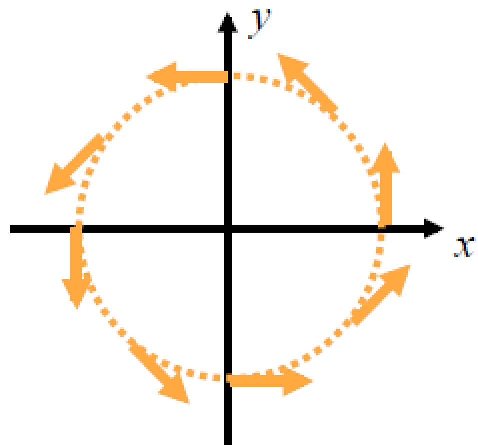
A 2D source with a large divergence:

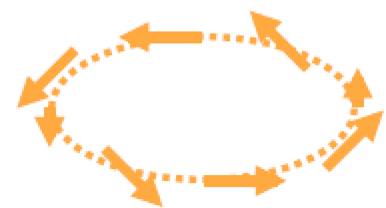


Div, Grad, Curl

The Curl of a vector function $\vec{f}$:

$$\vec{\nabla}\times\vec{f} \equiv \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{dz},\ \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{dx},\ \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{dy} \right)$$





Functions that tend to curl around have large curls.

Div, Grad, Curl

The Laplacian of a scalar function :

$$\nabla^2 f \equiv \vec{\nabla} \cdot \vec{\nabla} f = \vec{\nabla} \cdot \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

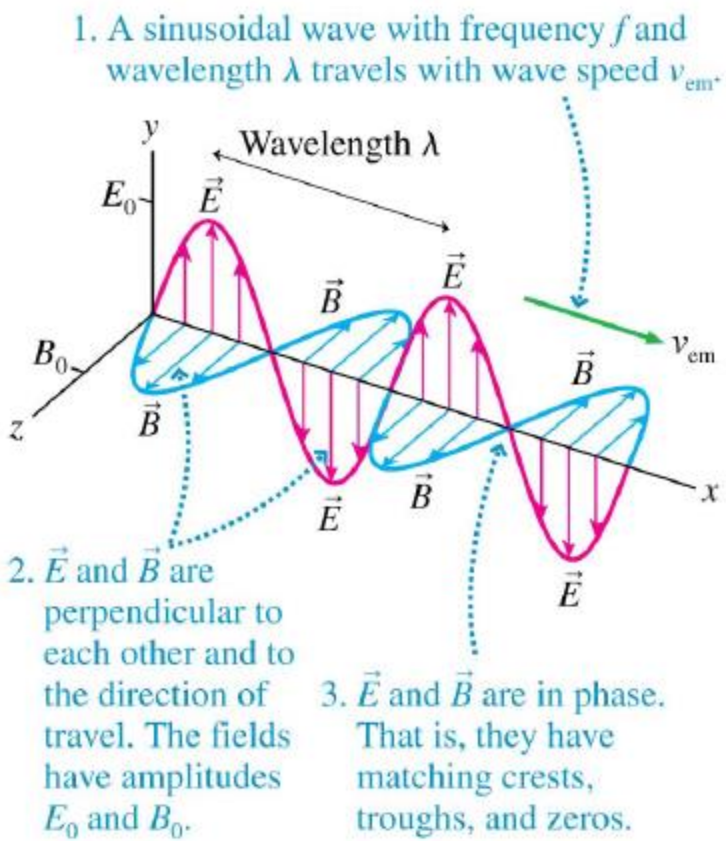
The Laplacian of a vector function is the same, but for each component of *f*:

$$\nabla^2 \vec{f} = \left(\frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_x}{\partial y^2} + \frac{\partial^2 f_x}{\partial z^2}, \frac{\partial^2 f_y}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_y}{\partial z^2}, \frac{\partial^2 f_z}{\partial x^2} + \frac{\partial^2 f_z}{\partial y^2} + \frac{\partial^2 f_z}{\partial z^2} \right)$$

The Laplacian tells us the curvature of a vector function.

The propagation direction of a light wave

FIGURE 35.19 A sinusoidal electromagnetic wave.



$$\vec{v} = \vec{E} \times \vec{B}$$

Right-hand screw rule

Maxwell's Equations

- Four equations relating electric (**E**) and magnetic fields (**B**) – vector fields
- ε_0 is electric permittivity of free space (or vacuum permittivity - a constant) – *resistance to formation of an electric field in a vacuum*
- $\varepsilon_0 = 8.854188 \times 10^{-12}$ Farad m^{-1}
- μ_0 is magnetic permeability of free space (or magnetic constant - a constant) – *resistance to formation of a magnetic field in a vacuum*
- $\mu_0 = 1.2566 \times 10^{-6}$ T.m/A (T = Tesla; SI unit of magnetic field)

$$\nabla \bullet E = \frac{\rho}{\varepsilon_0}$$

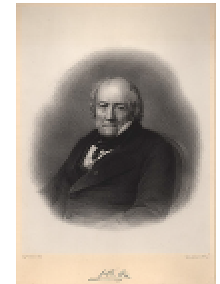
$$\nabla \bullet B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

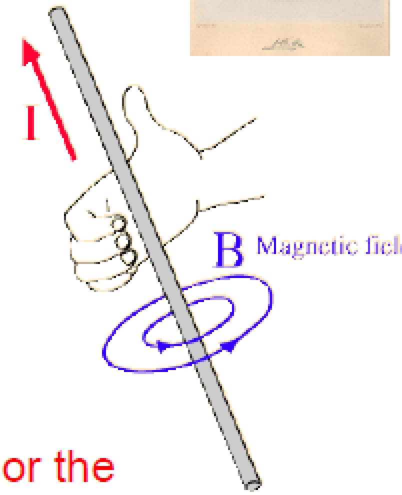
$$\nabla \times B = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

Note: $\nabla\bullet$ is 'divergence' operator and ∇x is 'curl' operator

Biot-Savart Law (1820)



- Jean-Baptiste Biot and Felix Savart (French physicist and chemist)
- The magnetic field **B** at a point a distance **R** from an infinitely long wire carrying current **I** has magnitude:



$$B = \frac{\mu_0 I}{2\pi R}$$

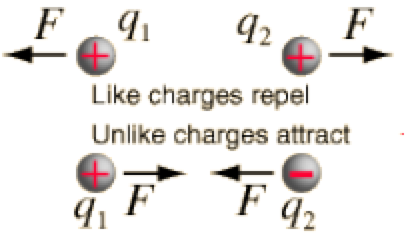
- Where μ_0 is the magnetic permeability of free space or the magnetic constant
- Constant of proportionality linking magnetic field and distance from a current
- Magnetic field strength decreases with distance from the wire
- $\mu_0 = 1.2566\times10^{-6}$ T.m/A (T = Tesla; SI unit of magnetic field)

Coulomb's Law (1783)

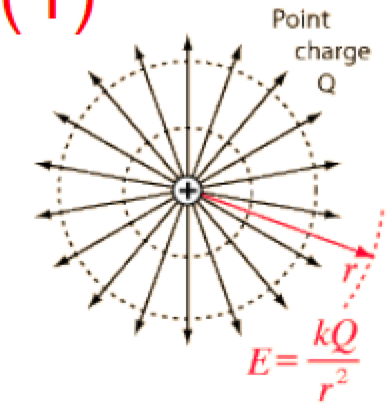
- Charles Augustin de Coulomb (French physicist)
- The magnitude of the electrostatic force (F) between two point electric charges (q_1, q_2) is given by:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

- Where ε_0 is the electric permittivity or electric constant
- Like charges repel, opposite charges attract
- $\varepsilon_0 = 8.854188 \times 10^{-12}$ Farad m^{-1}



Maxwell's Equations (1)

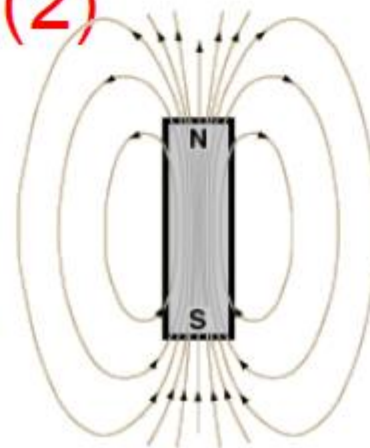


$$\nabla \bullet E = \frac{\rho}{\varepsilon_0}$$

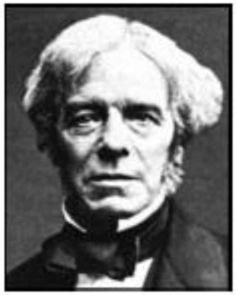
- Gauss' law for electricity: the electric flux out of any closed surface is proportional to the total charge enclosed within the surface; i.e. a charge will radiate a measurable field of influence around it.
- **E** = electric field, ρ = net charge inside, ε_0 = vacuum permittivity (constant)
- Recall: divergence of a vector field is a measure of its tendency to converge on or repel from a point.
- Direction of an electric field is the direction of the force it would exert on a positive charge placed in the field
- If a region of space has more electrons than protons, the total charge is negative, and the direction of the electric field is negative (inwards), and vice versa.

Maxwell's Equations (2)

$$\nabla \bullet B = 0$$

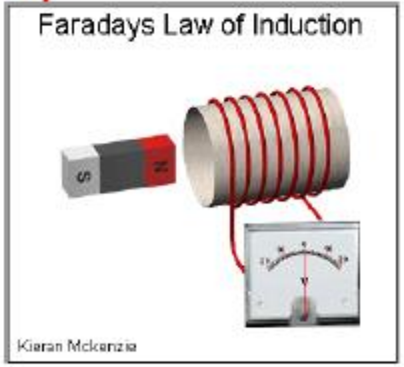


- Gauss' law for magnetism: the net magnetic flux out of any closed surface is zero (i.e. magnetic monopoles do not exist)
- **B** = magnetic field; magnetic flux = **B**A (A = area perpendicular to field **B**)
- Recall: divergence of a vector field is a measure of its tendency to converge on or repel from a point.
- Magnetic sources are dipole sources and magnetic field lines are loops – we cannot isolate N or S 'monopoles' (unlike electric sources or point charges – protons, electrons)
- Magnetic monopoles *could* exist, but have never been observed



Maxwell's Equations (3)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$



- Faraday's Law of Induction: the curl of the electric field (**E**) is equal to the negative of rate of change of the magnetic flux through the area enclosed by the loop
- **E** = electric field; **B** = magnetic field
- Recall: curl of a vector field is a vector with magnitude equal to the maximum 'circulation' at each point and oriented perpendicularly to this plane of circulation for each point.
- Magnetic field weakens ➔ curl of electric field is positive and vice versa
- Hence changing magnetic fields affect the curl ('circulation') of the electric field – basis of electric generators (moving magnet induces current in a conducting loop)

Maxwell's Equations (4)

$$\nabla \times B = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

- Ampère's Law: the curl of the magnetic field (**B**) is proportional to the electric current flowing through the loop

AND to the rate of change of the electric field. ← added by Maxwell

- **B** = magnetic field; J = current density (current per unit area); **E** = electric field
- The curl of a magnetic field is basically a measure of its strength
- First term on RHS: in the presence of an electric current (J), there is always a magnetic field around it; **B** is dependent on J (*e.g., electromagnets*)
- Second term on RHS: a changing electric field generates a magnetic field.
- Therefore, generation of a magnetic field does not require electric current, only a changing electric field. An oscillating electric field produces a variable magnetic field (as dE/dT changes)

Putting it all together….

- An oscillating electric field produces a variable magnetic field. A changing magnetic field produces an electric field….and so on.
- In ‘free space’ (vacuum) we can assume current density (J) and charge (ρ) are zero i.e. there are no electric currents or charges
- Equations become:

$$\nabla \bullet E = 0$$

$$\nabla \bullet B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$