Example 1

A completely randomised design experiment with 10 plots and 3 treatments gave the following results:

Plot No. :	1	2	3	4	5	6	7	8	9	10
Treatment :	A	B	C	A	C	C	A	B	A	B
Yield :	5	4	3	7	5	1	3	4	1	7

Analyse the results for treatment effects.

Rearranging the data according to the treatments, we have the following table:

Treatment	*Yield from plots* (x_{ij})				T_i	T_i^2	n_i	$\frac{T_i^2}{n_i}$
A	5	7	3	1	16	256	4	64
B	4	4	7	–	15	225	3	75
C	3	5	1	–	9	81	3	27
				Total	$T = 40$	–	$N = 10$	166

$$\sum\sum x_{ij}^2 = (25 + 49 + 9 + 1) + (16 + 16 + 49) + (9 + 25 + 1)$$

$$= 84 + 81 + 35 = 200$$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 200 - \frac{40^2}{10} = 200 - 160 = 40$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 166 - 160 = 6$$

$\therefore$ $$Q_2 = Q - Q_1 = 40 - 6 = 34$$

ANOVA table

S.V.	*S.S.*	*d.f.*	*M.S.*	F_0
Between classes (treatments)	$Q_1 = 6$	$h - 1 = 2$	3.0	$\frac{4.86}{3.0}$
Within classes	$Q_2 = 34$	$N - h = 7$	4.86	$= 1.62$
Total	$Q = 40$	$N - 1 = 9$	–	–

From the F-table, $F_{5\%}$ $(v_1 = 7, v_2 = 2) = 19.35$
We note that $F_0 < F_{5\%}$
Let H_0 : The treatments do not differ significantly.
$\therefore$ The null hypothesis is accepted.
i.e., the treatments are not significantly different.

Example 2

The following table shows the lives in hours of four brands of electric lamps:
Brand
A : 1610, 1610, 1650, 1680, 1700, 1720, 1800
B : 1580, 1640, 1640, 1700, 1750
C : 1460, 1550, 1600, 1620, 1640, 1660, 1740, 1820
D : 1510, 1520, 1530, 1570, 1600, 1680

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps.

We subtract 1640 (= the average of the extreme values) from the given values and work out with the new values of x_{ij}

Brand	*Lives of lamps* (x_{ij})								*Ti*	n_i	$\frac{T_i^2}{n_i}$
A	– 30	– 30	10	40	60	80	160	–	290	7	12014
B	– 60	0	0	60	110	–	–	–	110	5	2420
C	– 180	– 90	– 40	– 20	0	20	100	180	– 30	8	113
D	– 130	– 120	– 110	– 70	– 40	40	–	–	– 430	6	30817
								Total	– 60	26	45364

$$\sum\sum x_{ij}^2 = (900 + 900 + 100 + 1600 + 3600 + 6400 + 25600)$$
$$+ (3600 + 0 + 0 + 3600 + 12100)$$
$$+ (32400 + 8100 + 1600 + 400 + 0 + 400 + 10000 + 32400)$$
$$+ (16900 + 14400 + 12100 + 4900 + 1600 + 1600)$$
$$= 39100 + 19300 + 85300 + 51500 = 195200$$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 1{,}95{,}200 - 138 = 1{,}95{,}062$$

$$Q_1 = \sum\frac{T_i^2}{n^i} - \frac{T^2}{n} = 45{,}364 - 138 = 45{,}226$$

$$Q_2 = Q - Q_1 = 1{,}95{,}062 - 45{,}226 = 1{,}49{,}836$$

ANOVA table

S.V.	*S.S.*	*d.f.*	*M.S.*	F_0
Between brands	$Q_1 = 45{,}226$	$h - 1 = 3$	15,075	$\frac{15{,}075}{6{,}811}$
Within brands	$Q_2 = 1{,}49{,}836$	$N - h = 22$	6,811	$= 2.21$
Total	$Q = 1{,}95{,}062$	$N - 1 = 25$	–	–

From the F-tables, $F_{5\%}$ $(v_1 = 3, v_2 = 22) = 3.06$

$$F_0 < F_{5\%}$$

Hence the null hypothesis H_0, namely, the means of the lives of the four brands are homogeneous, is accepted viz., the lives of the four brands of lamps do not differ significantly.

Note *We could have used a change of scale also. viz., we could have made the change*

$$\text{New } x_{ij} = \frac{\text{old } x_{ij} - 1640}{10}$$

and simplified the numerical work still further

Example 3

A car rental agency, which uses 5 different brands of tyres in the process of deciding the brand of tyre to purchase as standard equipment for its fleet, finds that each of 5 tyres of each brand last the following number of kilometres (in thousands):

Tyre brands

A	*B*	*C*	*D*	*E*
36	46	35	45	41
37	39	42	36	39
42	35	37	39	37
38	37	43	35	35
47	43	38	32	38

Test the hypothesis that the five tyre brands have almost the same average life.

We shift the origin to 40 and work out with the new values of x_{ij}.

Tyre brand	x_{ij}					T_i	n_i	$\frac{T_i^2}{n_i}$	$\sum_{j=1}^{5} x_{ij}^2$
A	−4	−3	2	−2	7	0	5	0	82
B	6	−1	−5	−3	3	0	5	0	80
C	−5	2	−3	3	−2	−5	5	5	51
D	5	−4	−1	−5	−8	−13	5	33.8	131
E	1	−1	−3	−5	−2	−10	5	20	40
			Total			−28	25	58.8	384

$$T = \sum_i T_i = -28; \quad \sum\sum x_{ij}^2 = \sum_i \left(\sum_j x_{ij}^2 \right) = 384$$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 384 - \frac{(-28)^2}{25} = 352.64$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 58.8 - 31.36 = 27.44$$

$$Q_2 = Q - Q_1 = 352.64 - 27.44 = 325.20$$

ANOVA table

S.V.	*S.S.*	*d.f.*	*M.S.*	F_0
Between tyre brands	$Q_1 = 27.44$	$h - 1 = 4$	6.86	$\frac{16.26}{6.86}$
Within tyre brands	$Q_2 = 325.20$	$N - h = 20$	16.26	$= 2.37$
Total	$Q = 352.64$	$N - 1 = 24$	–	–

From the F-tables, $F_{5\%}\,(v_1 = 20,\ v_2 = 4) = 5.80$

$$F_0 < F_{5\%}$$