19BCB0025
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MAT 2001 ⇒ DA - 2

Statistics for Engineers

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1. It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected.

(a) Using the binomial distribution, what is the probability that exactly 5 wells have the impurity, assuming that the conjecture is correct?

(b) What is the probability that more than 5 wells are impure?

Ans. $P(\text{well with impurity}) = 30\% = \frac{30}{100} = \frac{3}{10} \Rightarrow q = 1-p = \frac{7}{10}$

no. of wells to be tested, $n = 10$

(a) using binomial distribution,

P(exactly 5 wells have impurity, assuming conjecture is correct)

$$= P(x=5) = {}^{10}C_5 \left(\frac{3}{10}\right)^5\left(\frac{7}{10}\right)^5 = \sum_{x=0}^{5} b\left(x;10,\frac{3}{10}\right) - \sum_{x=0}^{4} b\left(x;10,\frac{3}{10}\right)$$

$$= 0.1029 \qquad = 0.9527 - 0.8497$$

Ans.

(b) $P(\text{more than 5 wells are impure}) = P(x > 5)$

$$= 1 - P(x \le 5) = 1 - 0.9527 = 0.0473$$

$$= 1 - \sum_{x=0}^{5} b\left(x;10,\frac{3}{10}\right) = 1 - \left({}^{10}C_5\left(\frac{3}{10}\right)^5\left(\frac{7}{10}\right)^5 + {}^{10}C_4\left(\frac{3}{10}\right)^4\left(\frac{7}{10}\right)^6\right.$$

$\cancel{= 1 - 0.1029 = 0.8971}$

$$\left. + {}^{10}C_3\left(\frac{3}{10}\right)^3\left(\frac{7}{10}\right)^7 + {}^{10}C_2\left(\frac{3}{10}\right)^2\left(\frac{7}{10}\right)^8 + {}^{10}C_1\left(\frac{3}{10}\right)^1\left(\frac{7}{10}\right)^9 + {}^{10}C_0\left(\frac{7}{10}\right)^{10}\right)$$

$$= 1 - \left(\sum_{x=0}^{5} {}^{10}C_x \left(\frac{3}{10}\right)^x\left(\frac{7}{10}\right)^{10-x}\right)$$

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2. In testing a certain kind of truck tire over rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that
(a) from 3 to 6 have blowouts;
(b) fewer than 4 have blowouts;
(c) more than 5 have blowouts;

Ans.

$p = \frac{25}{100} = \frac{1}{4} = P\left(\text{trucks having blowouts}\right) = 0.25$

$q = 1 - p = \frac{3}{4} = 0.75$

$n = 15$ (given)

(a) $P(\text{trucks from 3 to 6 have blowouts}) = P(3 \leq x \leq 6)$

$= \sum_{x=0}^{6} b\left(x; 15, \frac{1}{4}\right) - \sum_{x=0}^{3} b\left(x; 15, \frac{1}{4}\right)$

$= 0.9434 - 0.4613 = \underline{0.4821}$

(b) $P(\text{trucks fewer than 4 have blowouts})$

$= P(x < 4) = P(x \leq 3)$

$= \sum_{x=0}^{3} b\left(x; 15, \frac{1}{4}\right) = \underline{0.4613}$

(c) $P(\text{trucks more than 5 have blowouts})$

$= 1 - P(x \leq 5) = 1 - \sum_{x=0}^{5} \left(x; 15, \frac{1}{4}\right) = 1 - 0.8516$

$= \underline{0.1484}$ Ans

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3. Number of customers arriving per hour at a certain automobile service facility is assumed to have a Poisson distribution with mean, $\lambda = 7$.

(a) Compute the probability that more than 10 customers will arrive in a 2-hour period.

(b) What is the mean number of arrivals during a 2-hour period?

Sol:- Length of interval, $\tau = 2$; $\lambda = 7$ (given)

$\Rightarrow \lambda$ gets replaced by $\lambda\tau$ (i.e., 2λ) $(=14)$

$$\lambda \longrightarrow 2\lambda$$

$$P(X = x) = \frac{e^{-14}(14)^x}{x!} = \frac{e^{-2\lambda}(2\lambda)^x}{x!}$$

(a) $P(x > 10) = P(x \geq 11) = 1 - P(x \leq 11)$

$$= 1 - \sum_{x=0}^{11} \frac{e^{-14}(14)^x}{x!} = 1 - 0.2600 = \underline{0.7400}$$

(b) Mean number of arrivals during a 2-hour period

$= 2\lambda = 2(7) = \underline{14}$ Ans.

4. The average number of customers who arrive at a counter of a certain bank per minute is 2. Using poisson distribution, find the probability that during a given minute, there are no ~~more~~ customers appear?

Sol:- Given, mean = avg. no. of customers per minute = 2

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P(during a given minute, there are no customers appear)
$= P(x=0)$; x = no. of customers

$$P(X=x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!} = \frac{e^{-2}\, 2^x}{x!}$$

$$P(x=0) = \frac{e^{-2}\cdot 2^0}{0!} = \underline{0.1353}$$

Ans.

5. Heights of 1000 students are normally distributed with a mean of 174.5 cm and a s.d. of 6.9 cm. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights
(a) less than 160 cm?
(b) between 171.5 and 182 cm inclusive?
(c) equal to 175 cm?
(d) greater than or equal to 188 cm?

Sol:- mean, $\mu = 174.5$
s.d., $\sigma = 6.9$

$$z = \frac{x-\mu}{\sigma} = \frac{x-174.5}{6.9}$$

x = height of student

(a) $P(x<160) = P\left(z < \frac{160-174.5}{6.9}\right) = P(z< -2.101)$
$= P(z<-2.10)$
$= 0.0179$

no. of students with $P(x<160)$
$= 1000 \times 0.0179 \approx \underline{18 \text{ students}}$

(b) $P(171.5 \leq x \leq 182) = P\left(\frac{171.5-174.5}{6.9} \leq z \leq \frac{182-174.5}{6.9}\right)$

$= P(-0.435 \leq z \leq 1.087) = P(-0.44 \leq z \leq 1.09)$
$= P(z \leq 1.09) - P(z \leq -0.435)$
$= 0.8621 - 0.3300 = \underline{0.5321}$

no. of students
$= 1000 \times 0.5321 = \underline{532 \text{ students}}$

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(c) $P(x=175) = P\left(z = \frac{175-174.5}{6.9}\right) = P(z = 0.07)$

$= P(x<175) - P(x<174)$

$= P(z<0.07) - P(z<-0.07)$

$= 0.5279 - 0.4721$

$= 0.0558$

$\Rightarrow$ no. of students with height 175 cm $= 1000 \times 0.0558$

$\approx$ 55 students

(d) $P(x \geq 188) = 1 - P(x<188)$

$= 1 - P\left(z < \frac{188-174.5}{6.9}\right) = 1 - P(z<1.96)$

$= 1 - 0.9750 = 0.025$

$\Rightarrow$ no. of students with height ≥ 188 cm $= 1000 \times 0.025$

$=$ 25 students

Ans

7. The weights of a large number of miniature poodles are approximately normally distributed with a mean of 8 kg and a s.d. of 0.9 kg. If measurements are recorded to the nearest tenth of a kilogram, find the fraction of these poodles with weights :

(a) over 9.5 kg ;
(b) of atmost 8.6 kg ;
(c) between 7.3 and 9.1 kg inclusive;

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Sol: mean, $\mu = 8$ $\quad z = \frac{x-\mu}{\sigma} = \frac{x-8}{0.9}$; x = weight

s.d., $\sigma = 0.9$

(a) $P(x > 9.5) = 1 - P(x < 9.5)$

$= 1 - P\left(z < \frac{9.5-8}{0.9}\right) = 1 - P(z < 1.67)$

$= \underline{0.9525}$

(b) $P(x \leq 8.6) = P\left(z < \frac{8.6-8}{0.9}\right) = P(z \leq 0.67)$

$= \underline{0.7486}$

(c) $P(7.3 \leq x \leq 9.1) = P\left(\frac{7.3-8}{0.9} \leq z \leq \frac{9.1-8}{0.9}\right)$

$= P(-0.78 \leq z \leq 1.22)$

$= P(z \leq 1.22) - P(z \leq -0.78)$

$= 0.8888 - 0.2177$

$= \underline{0.6711}$

Ans

8. The life in years of a certain type of electrical switches has an exponential distribution with an average life 2. If 100 of these switches are installed in different systems, what is the probability that atmost 30 fail during the first year.

Sol: mean $= E(x) = \frac{1}{\lambda} = 2 \Rightarrow \underline{\lambda = 0.5}$;

$n = 100$ = total no. of switches;

~~x = no. of switches to be considered~~

~~$P(X \leq 30)$~~

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x = no. of years that the electrical switches survived successfully
= life of electrical switches

$P(x<1)$ = probability that 1 switch couldn't last for 1 year

$$= \int_0^1 f(x)\,dx$$

$$f(x) = \lambda \cdot e^{-\lambda x} = \frac{1}{2} e^{-0.5x}$$

$$\therefore P(x<1) = \int_0^1 \frac{1}{2} e^{-\frac{x}{2}}\,dx = 0.393$$

Let p represent number of switches ~~working~~ having life <1 year, then using Binomial distribution,

$$P(p \le 30) = \sum_0^{30} b(x;100,0.393) = \sum_{x=0}^{30} {}^{100}C_x (0.393)^x (1-0.393)^{100-x}$$

$\begin{pmatrix} n=100 \\ p=0.393 \end{pmatrix}$

$= 0.0342$

Ans.

Q 9. In a certain city, daily consumption of electric power is a random variable X having a gamma distribution with mean 6 and variance 12.
Find (i) α and β
(ii) P(on a given day, daily consumption will exceed 12 mkw/hr).

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Sol:- $f(x;\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, \quad x>0$

for gamma distribution, mean $= \alpha\beta = 6$
variance $= \alpha\beta^2 = 6\beta = 12$

$\Rightarrow \boxed{\begin{matrix}\beta = 2 \\ \alpha = 3\end{matrix}}$

$f(x;3,2) = \frac{1}{2^3\,\Gamma(3)} x^2 e^{-x/2}$

$= \frac{1}{8\times 2!} x^2 e^{-x/2} = \frac{1}{16} x^2 \cdot e^{-x/2}$

$P(x>12) = \int_{12}^{\infty} f(x)\,dx = \int_{12}^{\infty} \frac{1}{16} x^2 e^{-x/2}\,dx$

$= \underline{0.062}$

Ans.

11. If the service life, in hours, of a semiconductor is a random variable having a Weibull distribution with parameters $\alpha = 0.0025$ and $\beta = 0.5 = \frac{1}{2}$

(i) How long can such a semiconductor be expected to last?

(ii) P(such a semiconductor will still be in operating condition after 4000 hours)

Sol:- (i) Mean,

$\mu = \alpha^{-1/\beta}\,\Gamma\left(1+\frac{1}{\beta}\right) = (0.025)^{-1/0.5}\,\Gamma(1+2)$

$= 1600 \times 2! = \underline{3200}$ hours

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(ii) $P(x > 4000) = 1 - P(x \leq 4000)$ [Here, x = no. of hours for semiconductor to be operational]

$= 1 - e^{-\alpha x^{\beta}}$

$= 1 - e^{-0.025(4000)^{0.5}}$

$= \underline{0.794}$

10. In a binomial research study, it was determined that the survival time, in weeks, of an animal subjected to a certain exposure of gamma radiation has a gamma distribution with $\alpha = 5$ and $\beta = 10$.

(a) What is the mean survival time of a randomly selected animal of the type used in the experiment?

(b) What is the probability that the animal survives more than 30 weeks?

Sol:- $\alpha = 5$

$\beta = 10$

(a) mean, $= \alpha\beta = \underline{50 \text{ weeks}}$

(b) x = survival time of a randomly selected animal

$P(x > 30) = 1 - P(x \leq 30)$

$$= 1 - \int_0^{30} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \, dx$$

$$= 1 - \int_0^{30} \frac{1}{10^5 \Gamma(5)} x^4 e^{-x/10} \, dx$$

$= 1 - 0.185 = \underline{0.815}$ Ans.

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12 Random samples drawn from two places gave the following data relating to the heights of adult males.

	Place A	Place B
Mean height (inches)	$68.50 = \bar{x}_1$	$68.58 = \bar{x}_2$
s.d.	$2.5 = \sigma_1$	$3.0 = \sigma_2$
sample size	$1200 = n_1$	$1500 = n_2$

Test at 5% level of significance that the mean height is the same for adults in the two places.

Sol:

1) $H_0: \bar{x}_1 = \bar{x}_2$

2) $H_1: \bar{x}_1 \neq \bar{x}_2$ (two-tailed test)

3) $\alpha = 5\%$

$\Rightarrow Z_\alpha = 1.96$

4) $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{68.50 - 68.58}{\sqrt{\frac{(2.5)^2}{1200} + \frac{(3.0)^2}{1500}}} = -0.76$$

$\Rightarrow |Z| = 0.76$

5) $\because$ Calculated value of $|Z| = 0.76$ < tabulated value of $Z_\alpha = 1.96$

$\Rightarrow H_0$ accepted.

Ans.

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13. A sample of 900 items found to have a mean of 3.47 cm. Can it be reasonably regarded as a simple sample from a population mean height is 3.23 cm and s.d. = 2.31 cm?

Sol^n.- $n = 900$

$\bar{x} = 3.23$; $\mu = 3.47$

$\sigma = 2.31$

1) $H_0 : \bar{x} = \mu$
2) $H_1 : \bar{x} \neq \mu$ (two-tailed test)
3) Let $\alpha = 5\%$. $\Rightarrow$ $Z_\alpha = 1.96$
4) $$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.23 - 3.47}{2.31/\sqrt{900}} = -3.17 \Rightarrow |Z| = 3.17$$
5) Calculated value of $|Z| (= 3.17) >$ Tab. value of $Z_\alpha (= 1.96)$

$\Rightarrow H_0$ rejected

$\Rightarrow H_1$ accepted

i.e., $\bar{x} \neq \mu$

Ans.

14. In a random sample of 500 people from Maharashtra, 200 are found to be consumers of vegetable oil and in another sample of 400 persons from Gujarat, 200 are found to be consumers of vegetable oil. Test at 5% level of significance, whether the data reveal a significant difference between Maharashtra and Gujarat so far as the population of vegetable oil consumers are concerned?

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Sol^n.

Maharashtra

$n_1 = 500$

$p_1 = \frac{200}{500} = 2/5$

Gujarat

$n_2 = 400$

$p_2 = \frac{200}{400} = 1/2$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{500(2/5) + (400)\,1/2}{500 + 400} = \frac{4}{9}$$

$$q = 1 - p = 5/9$$

$\therefore$ 1) $H_0 : p_1 = p_2$

2) $H_1 : p_1 \neq p_2$ (two-tailed test)

3) $\alpha = 5\%$

$\Rightarrow z_\alpha = 1.96$

4)
$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\frac{2}{5} - \frac{1}{2}}{\sqrt{\frac{4}{9} \times \frac{5}{9}\left(\frac{1}{500} + \frac{1}{400}\right)}} = -3$$

$\Rightarrow |z| = 3$

5) Calculated value of $|z|$ $(= 3)$ $>$ Tabulated value of z_α $(= 1.96)$

$\Rightarrow H_0$ rejected

$\Rightarrow H_1$ accepted

$\Rightarrow$ data reveals a significant difference. Ans

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15. In a certain factory, there are 2 independent processes for manufacturing the same item. The mean weight in a sample of 250 items produced from 1 process is found to be 120 gms with a s.d. of 12 gms, while from another process are 124 and 14 in a sample of 400 items. Is the difference between the mean weights significant at 1% level of significance?

Sol:

$n_1 = 250$ $\quad$ $n_2 = 400$

$\bar{x}_1 = 120$ gms $\quad$ $\bar{x}_2 = 124$ gms

$\sigma_1 = 12$ gms $\quad$ $\sigma_2 = 14$ gms

1) $H_0 : \bar{x}_1 = \bar{x}_2$

2) $H_1 : \bar{x}_1 \neq \bar{x}_2$ (two-tailed test)

3) $\alpha = 1\%$

$z_\alpha = 2.58$

4)

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{120-124}{\sqrt{\frac{(12)^2}{250} + \frac{(14)^2}{400}}} = -3.874$$

$\Rightarrow |z| = 3.874$

5) $\because$ Calculated value of $|z|$ $(= 3.87)$ $>$ Tabulated value of z_α $(= 2.58)$

$\Rightarrow$ reject H_0

$\Rightarrow$ accept H_1

$\Rightarrow$ difference between mean weights of given two samples is significant.

Ans