The Laplace Transform of a function $f(t)$ is defined as

$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)dt$, provided the integral exists.

MATLAB Syntax used:

Command	Purpose
x=input('prompt')	Displays the text in prompt and waits for the user to input a value and press the Enter key. The user can enter expressions and can use variables in the workspace.
laplace(f)	To find the Laplace transform of a scalar symbol f with default independent variable t. The default return is a function of s.
laplace(f,w)	Returns the Laplace transform of f in symbol w instead of the default s.
laplace(f,x,w)	Assumes f as a function of the symbolic variable x and returns the Laplace transform as a function of w.
ilaplace(F)	To find the inverse Laplace transform of the scalar symbolic object F with default independent variable s. The default return is a function of t.
ilaplace(F,x)	Returns the inverse Laplace transform of the function F as a function of x instead of the default t.
ilaplace(F,w,x)	Assumes F as a function of the symbolic variable w and returns the inverse Laplace transform of F as a function of x.
heaviside(t-a)	To input the heaviside's unit step function H(t−a).
dirac(t-a)	To input the dirac delta function δ(t−a).

Example 1. The following MATLAB code finds the Laplace transform of a function $f(t)$

```
clear all
clc
syms t
f=input('Enter the function of t: ');
F=laplace(f);
disp('Laplace transform of f(t) = ');
disp(F);
```

Input:

```
Enter the function of t: t^2
```

Output:

```
Laplace transform of f(t) =
2/s^3
```

Example 2. The following MATLAB code finds the Laplace transform of $f(t)$ in terms of w.

```
clear all
clc
syms t w
f=input('Enter the function of t: ');
F=laplace(f,w);
disp('Laplace transform of f(t) = ');
disp(F);
```

Input:

```
Enter the function of t: sin(t)
```

Output:

```
Laplace transform of f(t) =
1/(w^2 + 1)
```

Example 3. The following MATLAB code finds the Laplace transform of x^3e^{-3x} in terms of w.

```
clear all
clc
syms x w
f=input('Enter the function of x: ');
F=laplace(f,x,w);
disp('Laplace transform of f(t) = ');
disp(F);
```

Input:

```
Enter the function of x: x^3*exp(-3*x)
```

Output:

```
Laplace transform of f(t) =
6/(w + 3)^4
```

Example 4: The following MATLAB code computes the Laplace Transform of

$$f(t)=\begin{cases} t^2, 0<t<2 \\ t-1, 2<t<3 \\ 7, t>3 \end{cases}$$

```
clear all
clc
syms t
f=input('Enter the function of t: ');
F=laplace(f);
F=simplify(F);
disp('Laplace transform of f(t) = ');
disp(F);
```

Input:

```
Enter the function of t: t^2*(heaviside(t)-heaviside(t-2))+
(t-1)*(heaviside(t-2)-heaviside(t-3))+7*heaviside(t-3)
```

Output:

```
Laplace transform of f(t) =
-(exp(-3*s)*(s-2*exp(3*s)+2*exp(s)+3*s^2*exp(s)+3*s*exp(s)-
5*s^2))/s^3
```

Example 5. The following MATLAB code computes the inverse Laplace transform of $F(s)$.

```
clear all
clc
syms s
F=input('Enter the function of s: ');
f=ilaplace(F);
disp('f(t) = ');
disp(f);
```

Input:

```
Enter the function of s: 6/(s^3+2*s^2-s-2)
```

Ouput:

```
f(t)=
2*exp(-2*t)-3*exp(-t)+exp(t)
```

Example 6. Write MATLAB commands to find the following:

(i) $L[\delta(t)]$ (ii) $L[\delta(t-a)]$ (iii) $L[\delta(t-a)\sin(t)]$

Solution:

(i)

```
syms t
F=laplace(dirac(t))
```

Output:

```
F =
1
```

(ii)

```
syms t a
F=laplace(dirac(t-a))
```

Output:

```
F =
piecewise(a < 0, 0, 0 <= a, exp(-a*s))
```

(iii)

```
syms t a
F=laplace(dirac(t-a)*sin(t))
```

Output:

```
F =
piecewise(a<0,0,0<=a,exp(-a*s)*sin(a))
```

Example 7. Write MATLAB commands to find (i) $L^{-1}\left[\frac{s}{s-a}\right]$ (ii) $L^{-1}\left[\frac{se^{-s}+ae^{-2s}}{s^2+a^2}\right]$

(i)

```
syms s a
f=ilaplace(s/(s-a))
```

Output

```
f=
dirac(t) + a*exp(a*t)
```

(ii)

```
syms s a
f= ilaplace((s*exp(-s)+a*exp(-2*s))/(s^2+a^2))
```

Output

```
f =
heaviside(t-1)*cos(a*(t-1))+sin(a*(t-2))*heaviside(t-2)
```

Exercise.

1. Find the Laplace transforms of the following functions:

(i) $f(t) = 1 + 2\sqrt{t} + \dfrac{3}{\sqrt{t}}$

(ii) $f(t) = \begin{cases} \sin t & ; \quad 0 \le t \le \pi \\ 0 & ; \quad \pi \le t \le 2\pi \end{cases}$

(iii) $f(t) = \sin^3 t$

(iv) $f(t) = \sin 2t \sin 3t$

(v) $f(t) = e^{-t} \sin^2 t$

(vi) $f(t) = \dfrac{\cos 2t - \cos 3t}{t}$

2. Find the inverse Laplace transforms of the following functions:

(i) $F(s) = \dfrac{6}{s^2 + 2s - 8}$

(ii) $F(s) = \dfrac{4s + 5}{(s-1)^2 (s+2)}$

(iii) $F(s) = \dfrac{s^2 + 2s - 4}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$

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