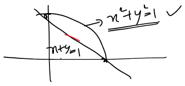
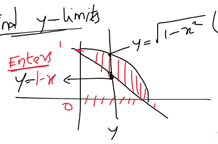
How to Find Limits of Integration



Ex.

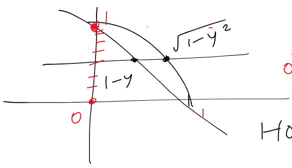
[Vertical Cross-Section]

Find y-limits



$$\int_{y=1-x}^{y=\sqrt{1-x^2}} f(x,y)\,dy\,dx$$

Find x-limits:



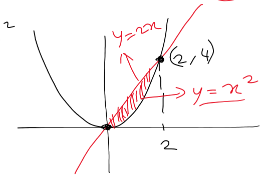
$$\int_0^1\int_{x=1-y}^{x=\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

Horizontal Cross-Section.

Sketch the region of integration for the integral.

$$\int_0^2\int_{y=x^2}^{2x=y} (4x+2)\,dy\,dx \Rightarrow \int_{y_1}^{y_2}\int_{x=h_1(y)}^{x=h_2(y)} (4x+2)\,dx\,dy$$

Sketching:



$x^2=y$

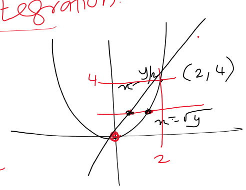
$\boxed{x=\pm\sqrt{y}}$ ✓ ignore $-\sqrt{y}$.

Change the order of integration:

$y = 0$ to 4

$0 \le y \le 4$

$y/2 \le x \le \sqrt{y}$



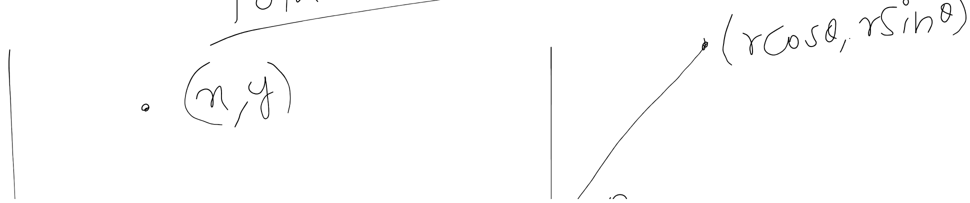
$$\boxed{\int_0^4\int_{y/2}^{\sqrt{y}} (4x+2)\,dx\,dy} \longrightarrow 2$$

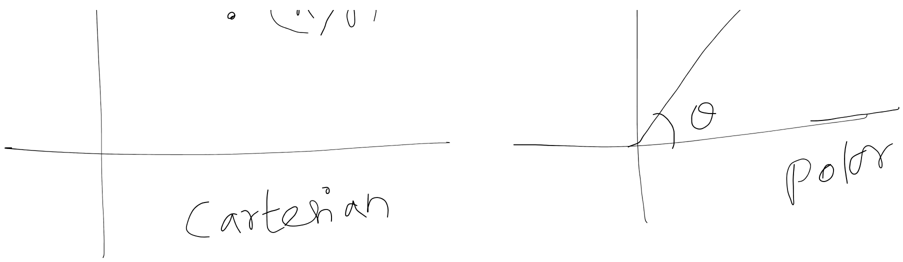
Ex:

$$\int_0^1\int_{y=2}^{y=4-2x} dy\,dx \rightarrow \iint dx\,dy$$

$$\int_0^1\int_{x=y}^{x=\sqrt{y}} dx\,dy \rightarrow \iint dy\,dx$$

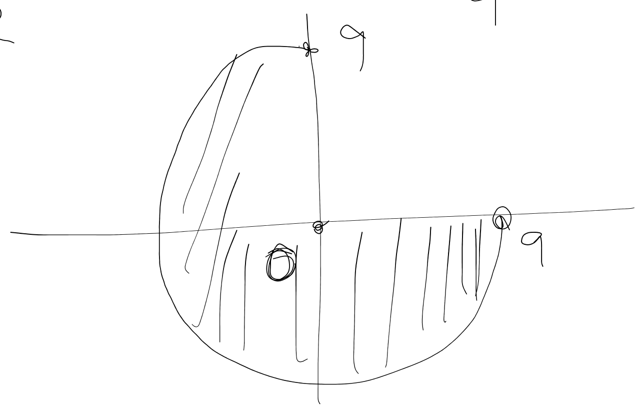
Polar co-ordinates:





polar form.

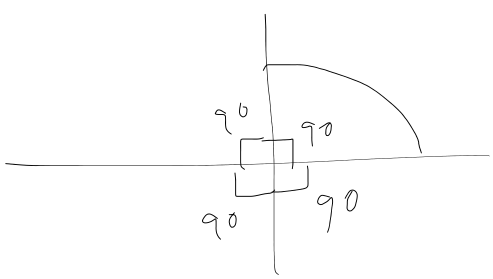
$$\iint_R f(x,y)\, dy\, dx = \iint_G f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

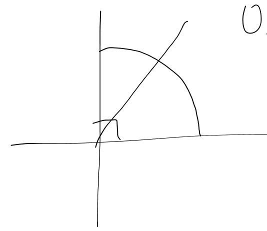


$r = 9$

$0 \leq r \leq 9$

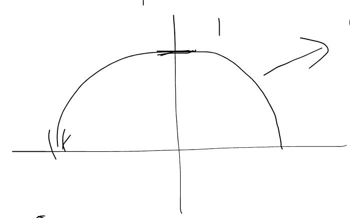
$\leq \theta \leq$





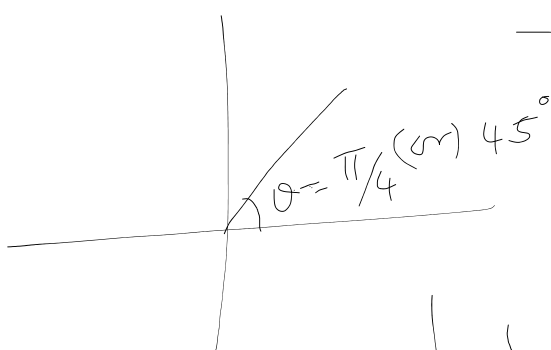
$0 \leq \theta \leq 90$

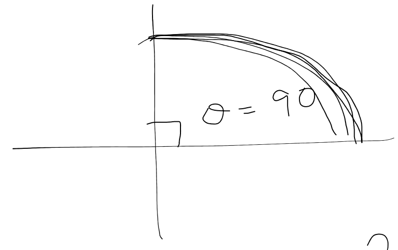
$\pi/2$

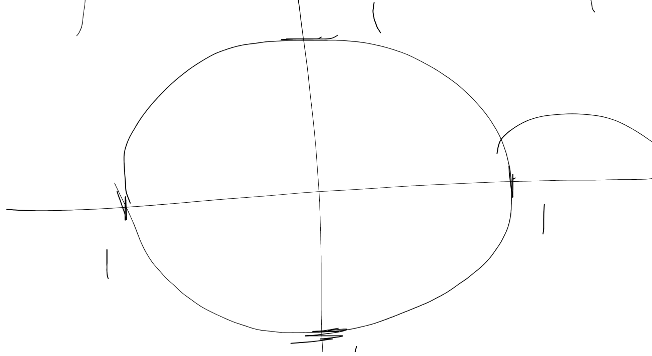


$0 \leq r \leq 1$

$0 \leq \theta \leq \pi$



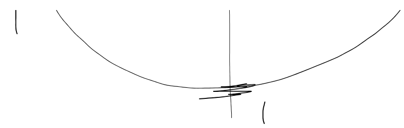




$x^2 + y^2 = 1$

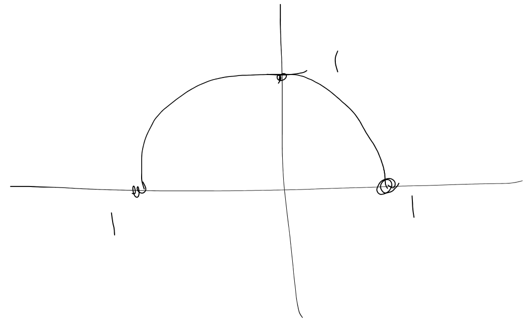
$x^2 + y^2 = r^2$

$0 \leq \theta \leq 2\pi$



$x^2+y^2=$

$0 \le \theta \le \pi$



Change into polar form & Evaluate

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx \longrightarrow \text{polar form}$$

General

limits θ   r-limits

$$\int\int \boxed{f(r\cos\theta, r\sin\theta)\, r\, d}$$

limits — θ   r-limits

Sol: $y=0$ & $y=\sqrt{1-x^2}$

$y^2 = 1-x^2$

$\boxed{x^2+y^2=1}$

r-limits is $0 \le r \le 1$

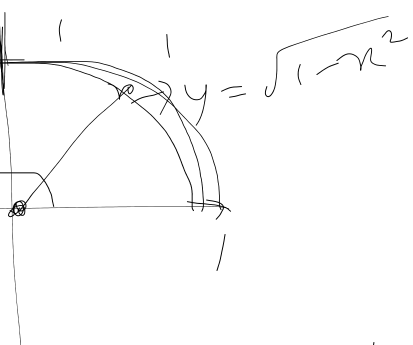
$0 \le \theta \le \pi$

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx = \int_{0}^{\pi}\boxed{\int_{0}^{1} r\,dr}\,d\theta$$

$$= \int^{\pi}\left(\frac{r^2}{2}\right)_0^1 d\theta$$

ral form

$r\,dr\,d\theta$



$0 \leq \theta \leq \pi$

$0 \leq r \leq 1$

$$= \int_0 \left(\frac{}{2}\right)_0 d\theta$$

$$= \frac{1}{2}\int_0^{\pi} d\theta = \frac{1}{2}\left[\theta\right]_0$$

$$= \frac{1}{2}(\pi - 0)$$

$$= \pi/2 \;//$$

$$\int_0^1 \int_1^{\sqrt{1-y^2}} \frac{x^2}{1}$$

$$x \qquad \int_{-1}^{1} \int_{y=h_1(x)}^{y=h_2(x)} f$$

$$\int_{-1}^{1} \int^{x=\sqrt{1-y}} \qquad x$$

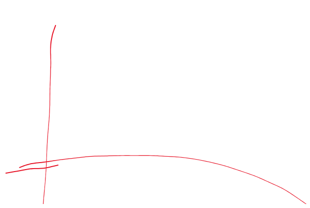
$$-1 \qquad x = 1$$

$+y^2 \, dx \, dy$

$(x, y) \, dy \, dx$

y^2

$x^2+y^2 \, dx \, dy$

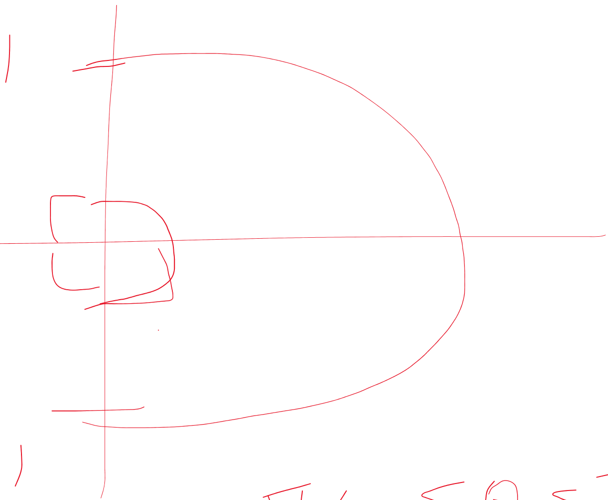


Ex:

$$\int_0^6 \int_0^{4} \cdots$$

$$\int_0^{\sqrt{3}} \int_1^{\sqrt{1\cdots}} \cdots$$

$$\int_0^1 \int_0 \cdots$$



$$-\pi/2 \leq \theta \leq \pi/2$$

$x \, dx \, dy$

x

$dy \, dx$

$-y^2 \,(x^2+y^2) \, dx \, dy$