Worked Example 5(A)

Example 1

Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls, (ii) at least 1 boy, (iii) at most 2 girls and (iv) children of both sexes. Assume equal probabilities for boys and girls.

Considering each child as a trial, $n = 4$. Assuming that birth of a boy is a success, $p = \frac{1}{2}$ and $q = \frac{1}{2}$. Let X denote the number of successes (boys).

(i) $P(2 \text{ boys and } 2 \text{ girls}) = P(X = 2)$

$$= 4C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{4-2}$$

$$= 6 \times \left(\frac{1}{2}\right)^4 = \frac{3}{8}$$

$\therefore$ No. of families having 2 boys and 2 girls

$$= N \cdot (P(X = 2) \text{ (where } N \text{ is the total no. of families considered)}$$

$$= 800 \times \frac{3}{8}$$

$$= 300.$$

(ii) $P(\text{at least 1 boy}) = P(X \geq 1)$

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 1 - P(X = 0)$$

$$= 1 - 4C_0 \cdot \left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^4$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

$\therefore$ No. of families having at least 1 boy

$$= 800 \times \frac{15}{16} = 750.$$

(iii) $P(\text{at most 2 girls}) = P(\text{exactly 0 girl, 1 girl or 2 girls})$

$$= P(X = 4, X = 3 \text{ or } X = 2)$$

$$= 1 - \{P(X = 0) + P(X = 1)\}$$

$$= 1 - \left\{4C_0 \cdot \left(\frac{1}{2}\right)^4 + 4C_1 \cdot \left(\frac{1}{2}\right)^4\right\}$$

$$= \frac{11}{16}$$

$\therefore$ No. of families having at most 2 girls

$$= 800 \times \frac{11}{16} = 550.$$

(iv) P(children of both sexes)

$$= 1 - P(\text{children of the same sex})$$
$$= 1 - \{P(\text{all are boys}) + P(\text{all are girls})\}$$
$$= 1 - \{P(X = 4) + P(X = 0)\}$$
$$= 1 - \left\{ 4C_4 \cdot \left(\frac{1}{2}\right)^4 + 4C_0 \cdot \left(\frac{1}{2}\right)^4 \right\}$$
$$= 1 - \frac{1}{8} = \frac{7}{8}$$

$\therefore$ No. of families having children of both sexes

$$= 800 \times \frac{7}{8} = 700.$$

Example 2

An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets?

Let the probability of getting an even number with the unfair dice be p.

Let X denote the number of even numbers obtained in 5 trials (throws).

Given: $P(X = 3) = 2 \times P(X = 2)$

i.e., $5C_3\, p^3 q^2 = 2 \times 5C_2\, p^2 q^3$

i.e., $p = 2q = 2(1 - p)$

$\therefore$ $3p = 2$ or $p = \frac{2}{3}$ and $q = \frac{1}{3}$

Now P(getting no even number)

$$= P(X = 0)$$
$$= 5C_0 \cdot p^0 \cdot q^5 = \left(\frac{1}{3}\right)^5 = \frac{1}{243}$$

$\therefore$ Number of sets having no success (even number) out of N sets $= N \times P(X = 0)$

$\therefore$ Required number of sets $= 2500 \times \frac{1}{243}$

$$= 10, \text{ nearly}$$

Two dice are thrown 120 times. Find the average number of times in which the number on the first dice exceeds the number on the second dice.

The number on the first dice exceeds that on the second die, in the following combinations:

(2, 1); (3, 1), (3, 2); (4, 1), (4, 2), (4, 3); (5, 1), (5, 2), (5, 3); (5, 4); (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),

where the numbers in the parentheses represent the numbers in the first and second dice respectively.

$\therefore \quad P(\text{success}) = P(\text{no. in the first dice exceeds the no. in the second dice})$

$$= \frac{15}{36} = \frac{5}{12}$$

This probability remains the same in all the throws that are independent.

If X is the no. of successes, then X follows a binomial distribution with parameters n (= 120) and $p\left(= \frac{5}{12}\right)$.

$$\therefore \quad E(X) = np = 120 \times \frac{5}{12} = 50$$

Example 7

Fit a binomial distribution for the following data:

x:	0	1	2	3	4	5	6	Total
f:	5	18	28	12	7	6	4	80

Fitting a binomial distribution means assuming that the given distribution is approximately binomial and hence finding the probability mass function and then finding the theoretical frequencies.

To find the binomial frequency distribution $N(q + p)^n$, which fits the given data, we require N, n and p. We assume N = total frequency = 80 and n = no. of trials = 6 from the given data.

To find p, we compute the mean of the given frequency distribution and equate it to np (mean of the binomial distribution).

x:	0	1	2	3	4	5	6	Total
f:	5	18	28	12	7	6	4	80
fx:	0	18	56	36	28	30	24	192

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{192}{80} = 2.4$$

i.e., $np = 2.4$ or $6p = 2.4$

$\therefore$ $p = 0.4$ and $q = 0.6$

If the given distribution is nearly binomial, the theoretical frequencies are given by the successive terms in the expansion of $80(0.6 + 0.4)^6$. Thus we get,

x:	0	1	2	3	4	5	6
Theoretical f:	3.73	14.93	24.88	22.12	11.06	2.95	0.33

Converting these values into whole numbers consistent with the condition that the total frequency is 80, the corresponding binomial frequency distribution is as follows:

x:	0	1	2	3	4	5	6	Total
f:	4	15	25	22	11	3	0	80

Example 8

The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month

(a) without a breakdown,

(b) with only one breakdown and

(c) with atleast one breakdown.

Let X denote the number of breakdowns of the computer in a month.

X follows a Poisson distribution with mean (parameter) $\lambda = 1.8$.

$$\therefore P\{X = r\} = \frac{e^{-\lambda} \cdot \lambda^r}{r!} = \frac{e^{-1.8} \cdot (1.8)^r}{r!}$$

(a) $P(X = 0) = e^{-1.8} = 0.1653$

(b) $P(X = 1) = e^{-1.8}(1.8) = 0.2975$

(c) $P(X \geq 1) = 1 - P(X = 0) = 0.8347$

Example 9

It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets using (i) binomial distribution and (ii) Poisson approximation to binomial distribution.