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Phase and Group Velocity of EM waves

Tools Required:-

https://demonstrations.wolfram.com/GroupandPhaseVelocity/.

Objective:-

- To understand the nature of EM waves travelling in a medium with the help of Phase and Group Velocities.

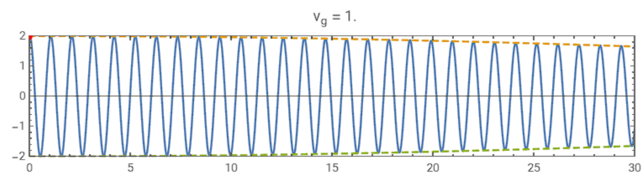
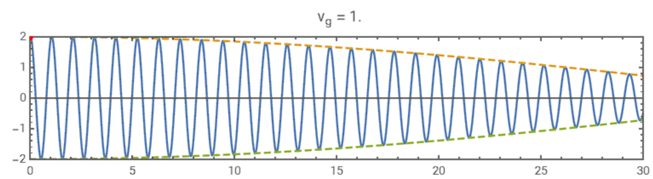
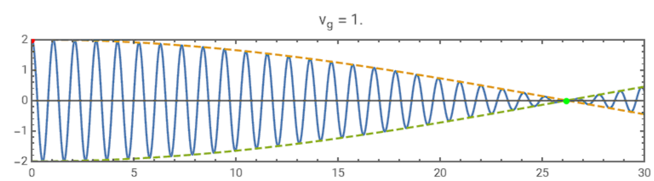
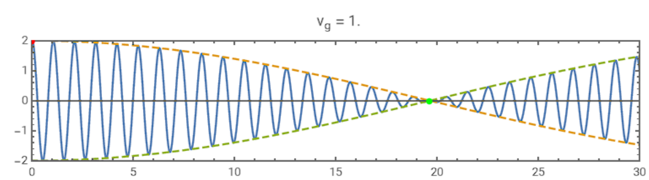
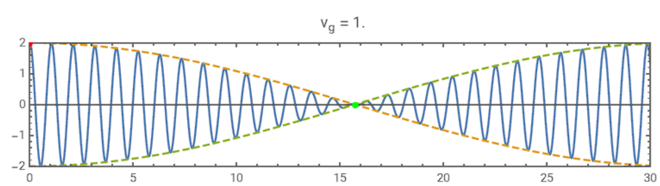
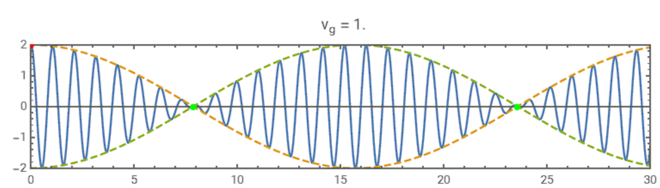
Theory:-

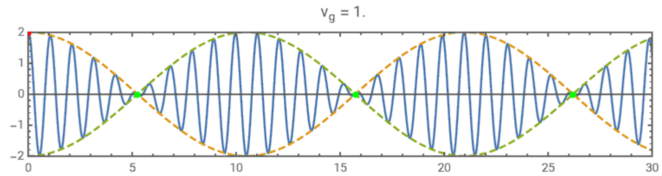
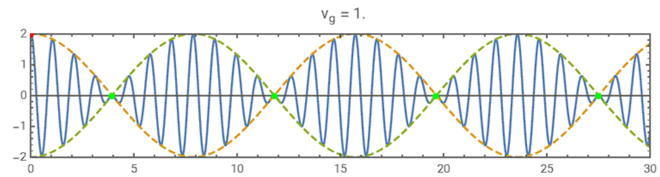
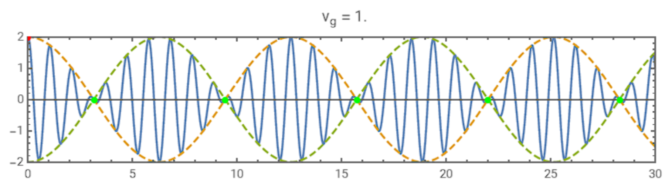
- Any real signal consisting of travelling waves of many different frequencies, which travel together as a group, at a speed that will always be less than or equal to the speed of light in vacuum. To gain some insight into what may happen when a real signal travels through a dispersive medium, we consider adding two waves of equal amplitude. When two waves travelling with amplitudes $f_1(z,t) = \cos(k_1 z - \omega_1 t)$ and $f_2(z,t) = \cos(k_2 z - \omega_2 t)$ are added, we get

$$f_1(z,t) + f_2(z,t) = \cos(k_1 z + \omega_1 t) + \cos(k_2 z - \omega_2 t)$$
$$= 2\cos\left(\frac{\Delta k \cdot z - \Delta\omega t}{2}\right) \cdot \cos\left(\frac{\bar{k} z - \bar{\omega} t}{2}\right)$$

where, $\Delta k = k_1 - k_2$, $\Delta\omega = \omega_1 - \omega_2$, $\bar{k} = k_1 + k_2$, $\bar{\omega} = \omega_1 + \omega_2$

The result is a fast oscillating wave that travels with a phase velocity $v_p = \frac{\bar{\omega}}{\bar{k}}$ and it's amplitude being modulated by $2\cos\left(\frac{\Delta k z - \Delta\omega t}{2}\right)$ in space and time. This modulated wave moves at the group velocity given by $v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k}$

S.No	Δw	Δk	Wave pattern of the resultant waves	V_g
1	0.02	0.02		1
2	0.04	0.04		1
3	0.06	0.06		1
4	0.08	0.08		1
5	0.1	0.1		1
6	0.2	0.2		1

7	0.3	0.3		1
8	0.4	0.4		1
9	0.5	0.5		1

<u>Table: Observation of wave pattern on various differences in frequency and wavelength</u>

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Inferences:-

1). Are the wave patterns for various values of $\Delta\omega$ and Δk same? If not, why?

↳ The wave patterns are different for various values of $\Delta\omega$ and Δk. For a given group velocity, (1 in above case), increasing $\Delta\omega$ implies increase in difference in frequency of two waves which means - for a match of crest and trough the waves have to travel shorter distance than before. eventually forming more wavepackets in a given distance.

Mathematically, $F(z,t) = 2\cos\left(\frac{\Delta k}{2}z - \frac{\Delta\omega}{2}t\right)\cos(\bar{k}z - \bar{\omega}t)$.

If $\Delta\omega$ and Δk both increase, the argument of amplitude part will increase with time and distance, as a result the length of an envelope will shorten.

Hence, the wave patterns are not same for various values of $\Delta\omega$ and Δk.

2). Comment on phase velocity (V_p) of the waves for increased values of $\Delta\omega$ and Δk.

↳ In case of a dispersive medium, phase velocity changes with respect to k and $V_p = \frac{\omega}{k}$.

In a non-dispersive medium ($\mu = 1$). V_p will be constant for all wavelengths. Thus, phase velocity doesnot depend upon change in $\Delta\omega$ and Δk.

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3) Why do we see V_p and V_g being the same.

↳ We have formula for group velocity and phase velocity:-

$$V_p = V_g - k\, dV_p/dk$$

Since, V_p is constant for non-dispersive medium ·ie. $\Delta V_p = 0$;

ie. $\frac{dV}{dk} = 0$

$$\boxed{\therefore V_p = V_g}$$

Hence, for non-dispersive medium - i.e. vacuum, they are equal

4) Draw a typical dispersion curve (ω-k curve) for $V_p = V_g$ and $V_p \neq V_g$ cases.

↳ Since, ω is a function of k,

For a non-dispersive medium (ie. $\mu = 1$), $\omega \propto k$ and (ω-constant) is proportional to k for dispersive medium. In both cases, V_p is the proportionality constant.

Case I: When $V_g = V_p$,

This case is possible in non-dispersive medium. The relation between ω and k is:- $\omega = ak$; $a = V_p$.

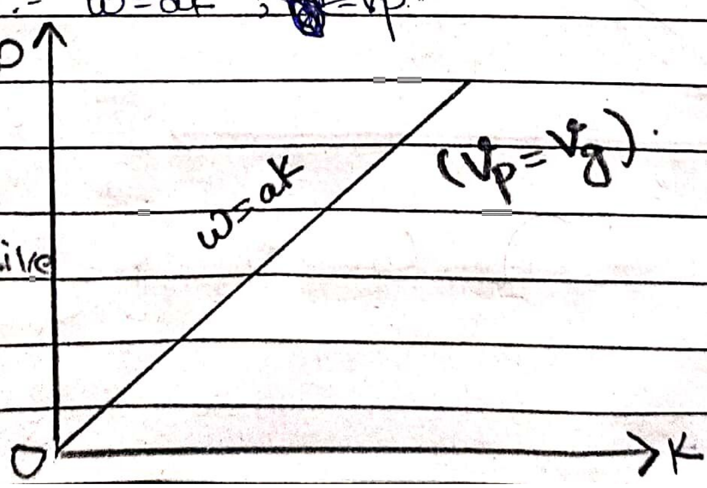

Fig:- ω-k curve for non-dispersive medium.

Case-II : $V_p \neq V_g$:

~~$V_p > V_g$~~. This case is possible in dispersive medium.
The relation between ω and K is :-

(a): For $V_g > V_p$:

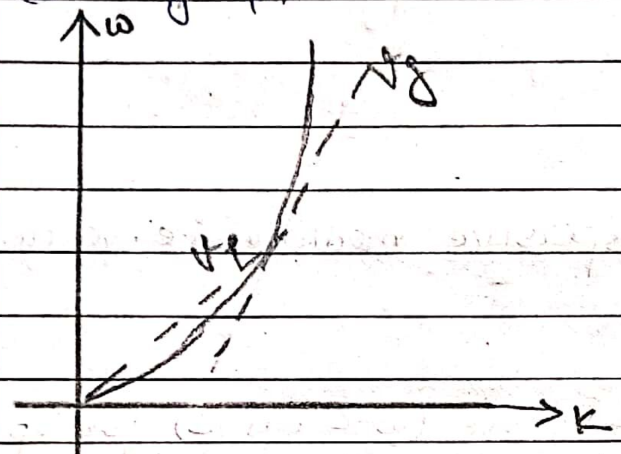

Fig:- Dispersion Curve for $V_g > V_p$

(b): For $V_g < V_p$.

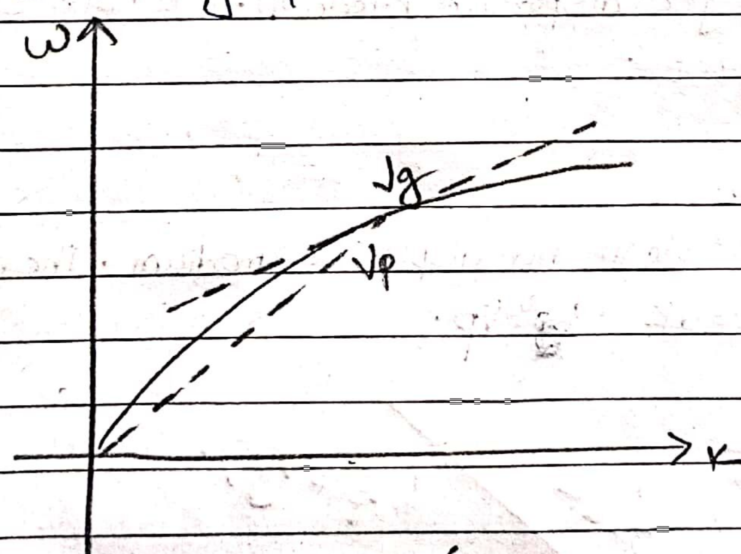

Fig:- Dispersion Curve for $V_p > V_g$.