## Example 3

A random variable $X$ may assume 4 values with probabilities $(1 + 3x)/4$, $(1 - x)/4$, $(1 + 2x)/4$ and $(1 - 4x)/4$. Find the condition on $x$ so that these values represent the probability function of $X$?

$$P(X = x_1) = p_1 = (1 + 3x)/4;\ p_2 = (1 - x)/4;$$
$$p_3 = (1 + 2x)/4;\ p_4 = (1 - 4x)/4$$

If the given probabilities represent a probability function, each $p_i \geq 0$ and $\sum_i p_i = 1$.

In this problem, $p_1 + p_2 + p_3 + p_4 = 1$, for any $x$.

But $p_1 \geq 0$, if $x \geq -1/3$; $p_2 \geq 0$, if $x \leq 1$; $p_3 \geq 0$, if $x \geq -1/2$ and $p_4 \geq 0$, if $x \leq 1/4$.

Therefore, the values of $x$ for which a probability function is defined lie in the range $-1/3 \leq x \leq 1/4$.

## Example 4

If the random variable $X$ takes the values 1, 2, 3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$, find the probability distribution and cumulative distribution function of $X$.

Let $P(X = 3) = 30K$. Since $2P(X = 1) = 30K$, $P(X = 1) = 15K$.

Similarly $P(X = 2) = 10K$ and $P(X = 4) = 6K$.

Since $\Sigma p_i = 1$, $15K + 10K + 30K + 6K = 1$.

$$\therefore K = \frac{1}{61}$$

The probability distribution of $X$ is given in the following table:

| $X = i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p_i$ | $\frac{15}{61}$ | $\frac{10}{61}$ | $\frac{30}{61}$ | $\frac{6}{61}$ |

The cdf $F(x)$ is defined as $F(x) = P(X \le x)$. Accordingly the cdf for the above distribution is found out as follows:

When $x < 1$, $F(x) = 0$

When $1 \le x < 2$, $F(x) = P(X = 1) = \dfrac{15}{61}$

When $2 \le x < 3$, $F(x) = P(X = 1) + P(X = 2) = \dfrac{25}{61}$

When $3 \le x < 4$, $F(x) = P(X = 1) + P(X = 2) + P(X = 3) = \dfrac{55}{61}$

When $x \ge 4$, $F(x) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$.

$4 \le x$

## Example 5

A random variable X has the following probability distribution.

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$: | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
| $p(x)$: | 0.1 | $K$ | 0.2 | $2K$ | 0.3 | $3K$ |

(a) Find $K$, (b) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$, (c) find the cdf of $X$ and (d) evaluate the mean of $X$. (BU — Apr. 96)

(a) Since $\Sigma\, P(x) = 1$, $6K + 0.6 = 1$

$$\therefore \quad K = \frac{1}{15} = 0.066667$$

$\therefore$ the probability distribution becomes

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ : | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
| $p(x)$ : | 1/10 | 1/15 | 1/5 | 2/15 | 3/10 | 1/5 |

(b) $P(X < 2) \quad = P(X = -2, -1, 0 \text{ or } 1)$

$= P(X = -2) + P(X = -1) + P(X = 0) + P(X = 1)$

[since the events $(X = -2)$, $(X = -1)$ etc. are mutually exclusive]

$$= \frac{1}{10} + \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{1}{2}$$

$P(-2 < X < 2) = P(X = -1, 0 \text{ or } 1)$

$$= P(X = -1) + P(X = 0) + P(X = 1)$$

$$= \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{2}{5}$$

(c) $F(x) = 0$, when $x < -2$

$$= \frac{1}{10}, \text{ when } -2 \le x < -1$$

$$= \frac{1}{6}, \text{ when } -1 \le x < 0$$

$$= \frac{11}{30}, \text{ when } 0 \le x < 1$$

$$= \frac{1}{2}, \text{ when } 1 \le x < 2$$

$$= \frac{4}{5}, \text{ when } 2 \le x < 3$$

$$= 1, \text{ when } 3 \le x$$

$\frac{1}{10} + \frac{1}{15} + \frac{1}{5} = \frac{3+2+6}{30} = \frac{11}{30}$

$\frac{11}{30} + \frac{2}{15} = \frac{15}{30} = \frac{1}{2}$

$\frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

$\frac{4}{5} + \frac{1}{5} = 1$

(d) The mean of $X$ is defined as $E(X) = \Sigma\, xp(x)$ (refer to Chapter 4)

$$\therefore \text{Mean of } X = \left(-2 \times \frac{1}{10}\right) + \left(-1 \times \frac{1}{15}\right) + \left(0 \times \frac{1}{5}\right)$$

$$+ \left(1 \times \frac{2}{15}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{5}\right)$$

$$= -\frac{1}{5} - \frac{1}{15} + \frac{2}{15} + \frac{3}{5} + \frac{3}{5} = \frac{16}{15}$$

## Example 6

The probability function of an infinite discrete distribution is given by $P(X = j) = 1/2^j$ ($j = 1, 2, \ldots, \infty$). Verify that the total probability is 1 and find the mean and variance of the distribution. Find also $P(X$ is even), $P(X \ge 5)$ and $P(X$ is divisible by 3).

Let $P(X = j) = p_j$

$$\sum_{j=1}^{\infty} p_j = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \infty, \text{ that is a geometric series.}$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

The mean of $X$ is defined as $E(X) = \sum_{j=1}^{\infty} jp_j$ (refer to Chapter 4).

$$\therefore \quad E(X) = a + 2a^2 + 3a^3 + \ldots \infty, \text{ where } a = \frac{1}{2}$$

$$= a(1 + 2a + 3a^2 + \ldots \infty)$$