Hence '$H_0$ : the five tyre brands have almost the same average life) is accepted viz., the five tyre brands do not differ significantly in their lives.

## Example 4

In order to determine whether there is significant difference in the durability of 3 makes of computers, samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are as follows :

*Makes*

| A | B | C |
|---|---|---|
| 5 | 8 | 7 |
| 6 | 10 | 3 |
| 8 | 11 | 5 |
| 9 | 12 | 4 |
| 7 | 4 | 1 |

In view of the above data, what conclusion can you draw?

| *Make* | $x_{ij}$ | | | | | $T_i$ | $n_i$ | $T_i^2 / n_i$ | $\sum_j x_{ij}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| A | 5 | 6 | 8 | 9 | 7 | 35 | 5 | 245 | 255 |
| B | 8 | 10 | 11 | 12 | 4 | 45 | 5 | 405 | 445 |
| C | 7 | 3 | 5 | 4 | 1 | 20 | 5 | 80 | 100 |
| | | | | | *Total* | 100 | 15 | 730 | 800 |

$$T = \sum T_i = 100 \; ; \sum\sum x_{ij}^2 = 800; \; N = \sum n_i = 15$$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 800 - \frac{100^2}{15} = 133.33$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 730 - 666.67 = 63.33$$

$$Q_2 = Q - Q_1 = 70$$

***ANOVA table***

| *S.V.* | *S.S.* | *d.f.* | *M.S.* | $F_0$ |
|---|---|---|---|---|
| Between makes | $Q_1 = 63.33$ | $h - 1 = 2$ | 31.67 | $\frac{31.67}{5.83}$ |
| Within makes | $Q_2 = 70$ | $N - h = 12$ | 5.83 | $= 5.43$ |
| *Total* | $Q = 133.33$ | $N - 1 = 14$ | – | – |

From the $F$-tables, $F_{5\%}$ $(v_1 = 2, v_2 = 12) = 3.88$

$$F_0 > F_{5\%}$$

Hence the null hypothesis ($H_0$: the 3 makes of computers do not differ in the durability) is rejected.
viz., there is significant difference in the durability of the 3 makes of computers.

## Example 5

Three varieties of a crop are tested in a randomised block design with four replications, the layout being as given below: The yields are given in kilograms. Analyse for significance

| | | | |
|---|---|---|---|
| C48 | A51 | B52 | A49 |
| A47 | B49 | C52 | C51 |
| B49 | C53 | A49 | B50 |

Rewriting the data such that the rows represent the blocks and the columns represent the varieties of the crop (as assumed in the discussion of analysis of variance for two factors of classification), we have the following table:

*Crops*

| *Blocks* | *A* | *B* | *C* |
|---|---|---|---|
| 1 | 47 | 49 | 48 |
| 2 | 51 | 49 | 53 |
| 3 | 49 | 52 | 52 |
| 4 | 49 | 50 | 51 |

We shift the origin to 50 and work out with the new values of $x_{ij}$.

***Crops***

| *Blocks* | *A* | *B* | *C* | $T_i$ | $T_i^2 / k$ | $\sum_j x_{ij}^2$ |
|---|---|---|---|---|---|---|
| 1 | − 3 | − 1 | − 2 | − 6 | 36/3 = 12 | 14 |
| 2 | 1 | − 1 | 3 | 3 | 9/3 = 3 | 11 |
| 3 | − 1 | 2 | 2 | 3 | 9/3 = 3 | 9 |
| 4 | − 1 | 0 | 1 | 0 | 0/3 = 0 | 2 |
| $T_j$ | − 4 | 0 | 4 | $T = 0$ | $\sum \frac{T_i^2}{k} = 18$ | 36 |
| $T_j^2 / h$ | $\frac{16}{4} = 4$ | $\frac{0}{4} = 0$ | $\frac{16}{4} = 4$ | $\sum \frac{T_i^2}{h} = 8$ | | |
| $\sum_i x_{ij}^2$ | 12 | 6 | 18 | 36 | | |

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 36 - \frac{0^2}{12} = 36$$

$$Q_1 = \frac{1}{k}\sum T_i^2 - \frac{T^2}{N} = 18 - 0 = 18$$

$$Q_2 = \frac{1}{h}\sum T_j^2 - \frac{T^2}{N} = 8 - 0 = 8$$

$$Q_3 = Q - Q_1 - Q_2 = 36 - 18 - 8 = 10$$

***ANOVA table***

| *S.V.* | *S.S.* | *d.f.* | *M.S.* | $F_0$ |
|---|---|---|---|---|
| Between rows (blocks) | $Q_1 = 18$ | $h - 1 = 3$ | 6 | $\frac{6}{1.67} = 3.6$ |
| Between columns (crops) | $Q_2 = 8$ | $k - 1 = 2$ | 4 | $\frac{4}{1.67} = 2.4$ |
| Residual | $Q_3 = 10$ | $(h-1)(k-1) = 6$ | 1.67 | – |
| *Total* | $Q = 36$ | $hk - 1 = 11$ | – | – |

From $F$-tables, $F_{5\%}$ $(v_1 = 3, v_2 = 6) = 4.76$ and $F_{5\%}$ $(v_1 = 2, v_2 = 6) = 5.14$
Considering the difference between rows, we see that $F_0$ $(= 3.6) < F_{5\%}$ $(= 4.76)$
Hence the difference between the rows is not significant. ($H_0$ is accepted) viz., the blocks do not differ significantly with respect to the yield.

Considering the difference between columns, we see that $F_0$ $(= 2.4) < F_{5\%}$ $(= 5.14)$

Hence the difference between the columns is not significant. ($H_0$ is accepted) viz., the varieties of crop do not differ significantly with respect to the yield.

**Example 6**