CSE1003-Digital Logic Design

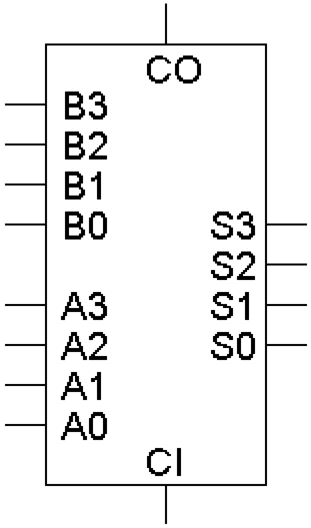
Module:7	ARITHMETIC LOGIC UNIT	9 hours
Bus Organization - ALU - Design of ALU - Status Register - Design of Shifter - Processor Unit - Design of specific Arithmetic Circuits Accumulator - Design of Accumulator.		

Arithmetic-logic units

- An arithmetic-logic unit, or ALU, performs many different arithmetic and logic operations. The ALU is the "heart" of a processor—you could say that everything else in the CPU is there to support the ALU.
- Here's the plan:
  - We'll show an arithmetic unit first, by building off ideas from the adder-subtractor circuit.
  - Then we'll talk about logic operations a bit, and build a logic unit.
  - Finally, we put these pieces together using multiplexers.
- We use some examples from the textbook, but things are re-labeled and treated a little differently.

The four-bit adder

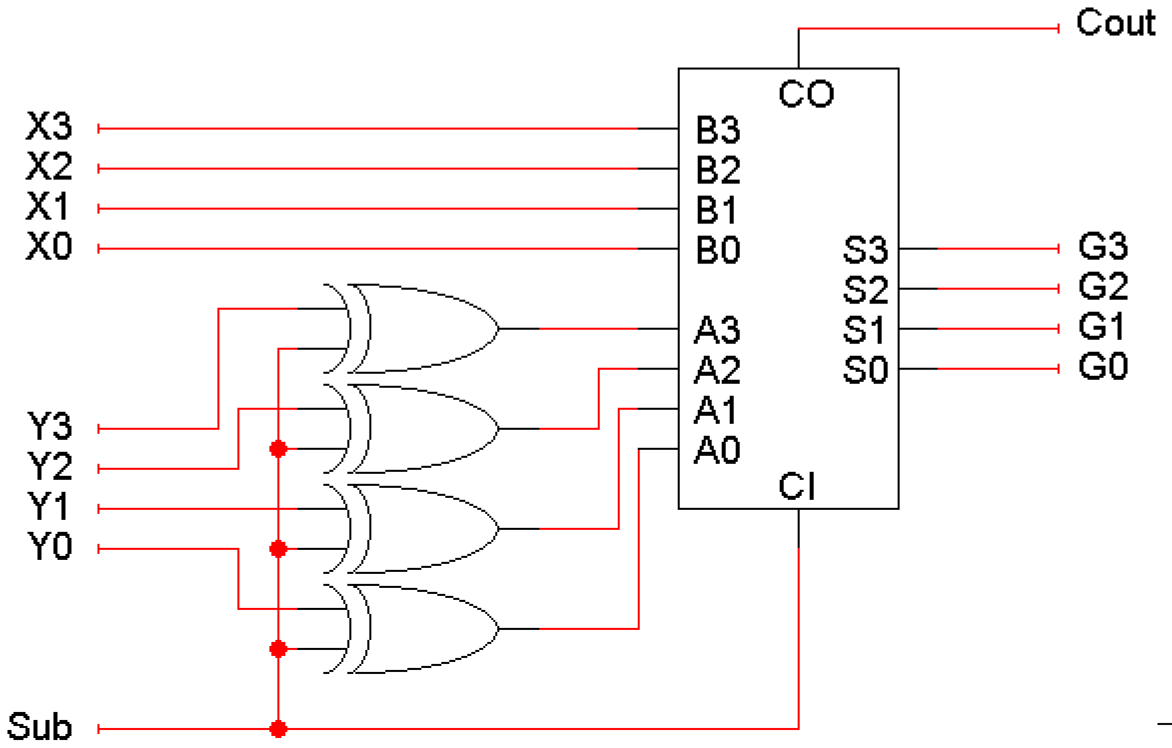
- The basic four-bit adder *always* computes S = A + B + CI.



- But by changing what *goes* into the adder inputs A, B and CI, we can change the adder output S.
- This is also what we did to build the combined adder-subtractor circuit.

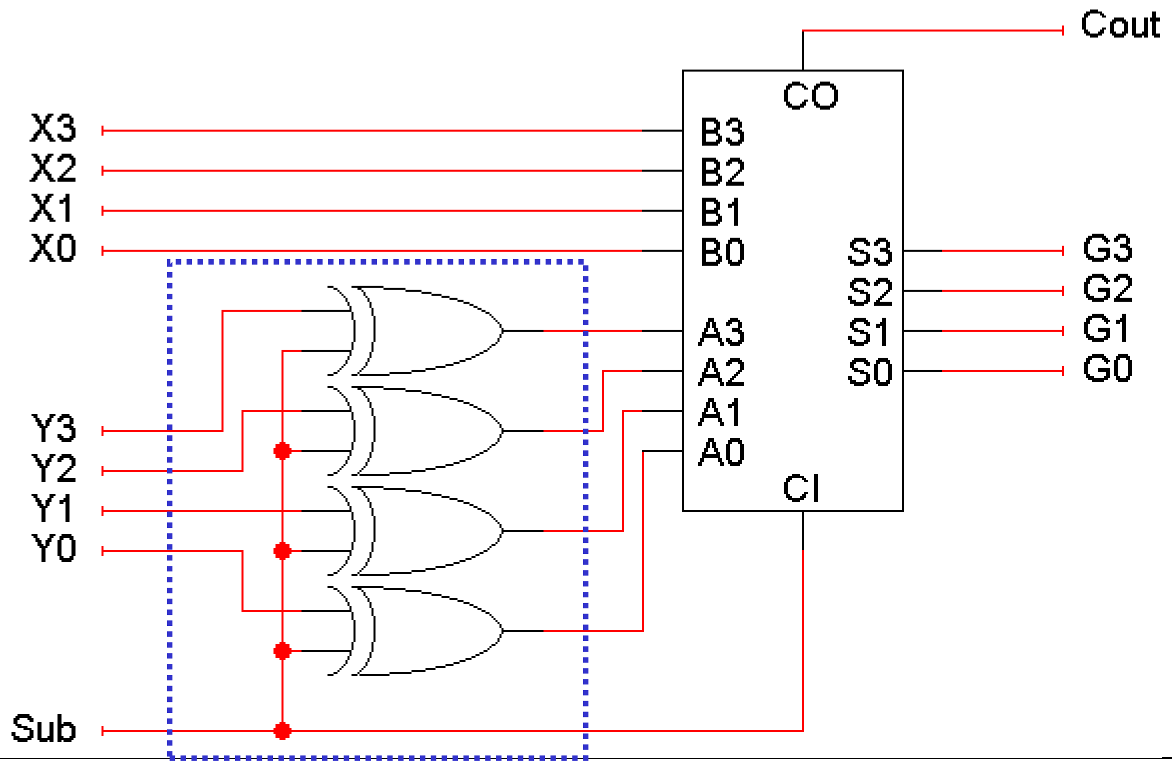
It's the adder-subtractor again!

- Here the signal Sub and some XOR gates alter the adder inputs.
  - When Sub = 0, the adder inputs A, B, CI are Y, X, 0, so the adder produces G = X + Y + 0, or just X + Y.
  - When Sub = 1, the adder inputs are Y', X and 1, so the adder output is G = X + Y' + 1, or the two's complement operation X - Y.



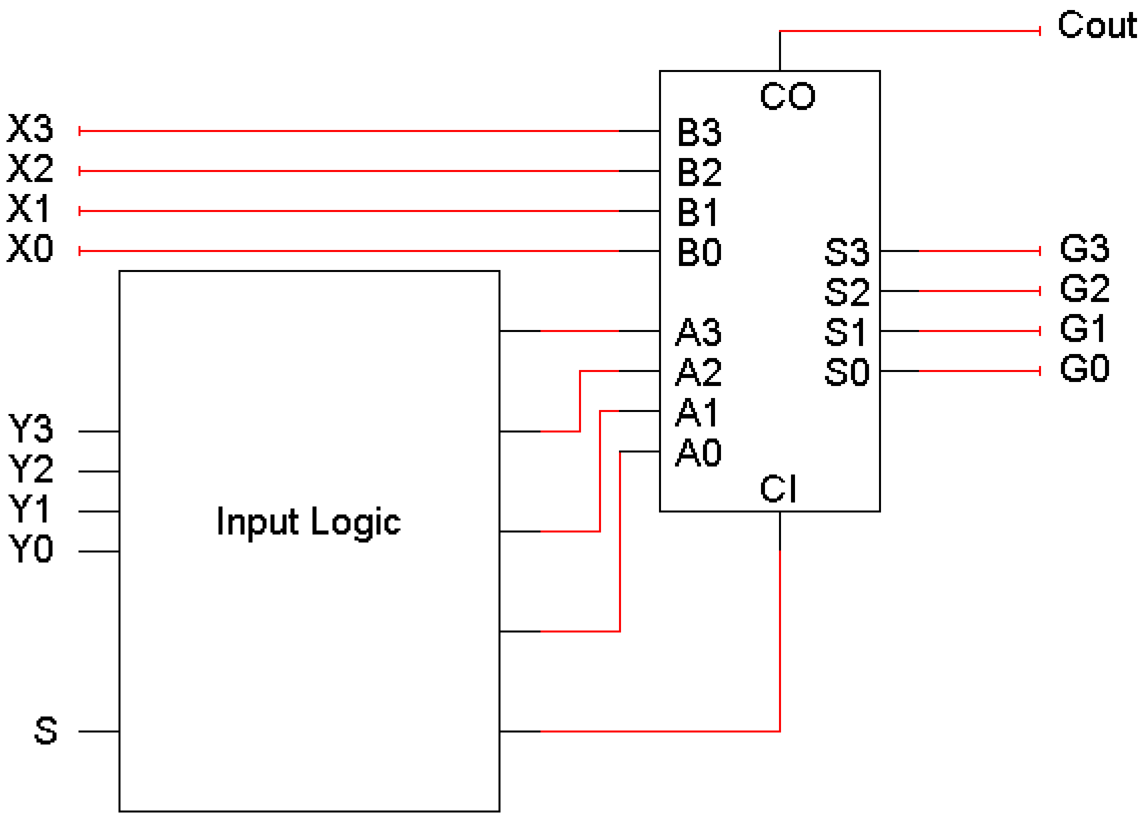
The multi-talented adder

- So we have one adder performing two separate functions.
- "Sub" acts like a function select input which determines whether the circuit performs addition or subtraction.
- Circuit-wise, all "Sub" does is modify the adder's inputs A and CI.



Modifying the adder inputs

- By following the same approach, we can use an adder to compute *other* functions as well.
- We just have to figure out which functions we want, and then put the right circuitry into the "Input Logic" box .



Some more possible functions

- We already saw how to set adder inputs A, B and CI to compute either X + Y or X - Y.
- How can we produce the increment function G = X + 1?

  One way: Set A = 0000, B = X, and CI = 1

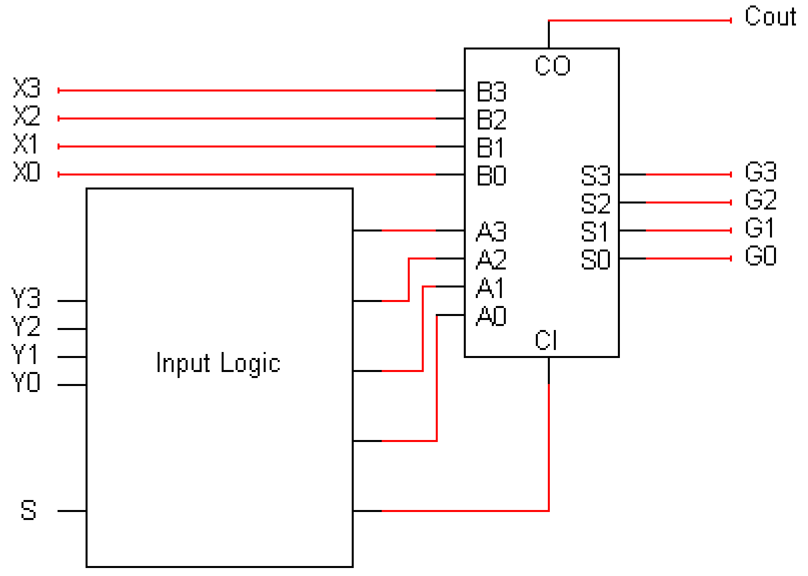
- How about decrement: G = X - 1?

  A = 1111 (-1), B = X, CI = 0

- How about transfer: G = X?
  (This can be useful.)

  A = 0000, B = X, CI = 0

  This is almost the same as the increment function!



The role of CI

- The transfer and increment operations have the same A and B inputs, and differ only in the CI input.
- In general we can get additional functions (not all of them useful) by using both CI = 0 and CI = 1.
- Another example:
  - Two's-complement subtraction is obtained by setting $A = Y'$, $B = X$, and $CI = 1$, so $G = X + Y' + 1$.
  - If we keep $A = Y'$ and $B = X$, but set CI to 0, we get $G = X + Y'$. This turns out to be a ones' complement subtraction operation.

Table of arithmetic functions

- Here are some of the different possible arithmetic operations.
- We'll need some way to specify which function we're interested in, so we've randomly assigned a selection code to each operation.

S_2	S_1	S_0	Arithmetic operation	
0	0	0	X	(transfer)
0	0	1	X + 1	(increment)
0	1	0	X + Y	(add)
0	1	1	X + Y + 1	
1	0	0	X + Y'	(1C subtraction)
1	0	1	X + Y' + 1	(2C subtraction)
1	1	0	X – 1	(decrement)
1	1	1	X	(transfer)

Mapping the table to an adder

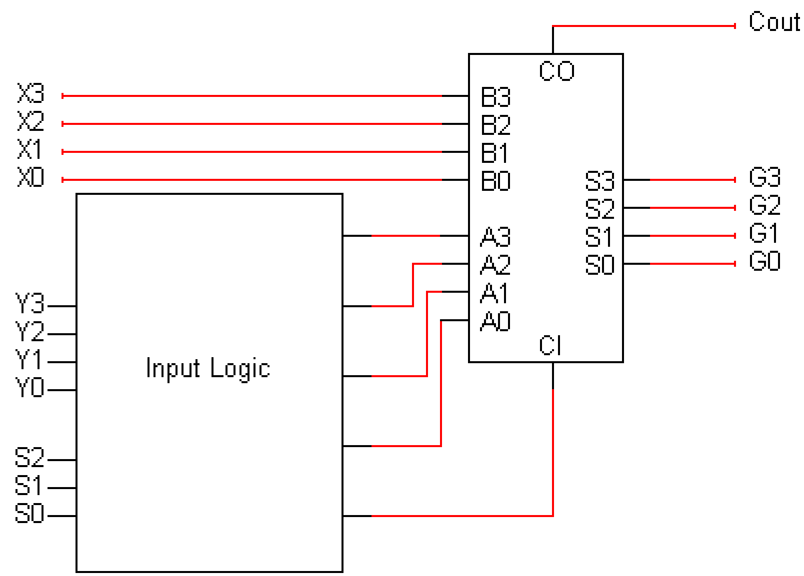
- This second table shows what the adder's inputs should be for each of our eight desired arithmetic operations.
  - Adder input CI is always the same as selection code bit S_0.
  - B is always set to X.
  - A depends only on S_2 and S_1.
- These equations depend on both the desired operations and the assignment of selection codes.

Selection code S_2	S_1	S_0	Desired arithmetic operation G (A + B + CI)		Required adder inputs A	B	CI
0	0	0	X	(transfer)	0000	X	0
0	0	1	X + 1	(increment)	0000	X	1
0	1	0	X + Y	(add)	Y	X	0
0	1	1	X + Y + 1		Y	X	1
1	0	0	X + Y'	(1C subtraction)	Y'	X	0
1	0	1	X + Y' + 1	(2C subtraction)	Y'	X	1
1	1	0	X – 1	(decrement)	1111	X	0
1	1	1	X	(transfer)	1111	X	1

Building the input logic

- All we need to do is compute the adder input *A*, given the arithmetic unit input *Y* and the function select code *S* (actually just S_2 and S_1).
- Here is an abbreviated truth table:

S_2	S_1	*A*
0	0	0000
0	1	Y
1	0	Y'
1	1	1111



- We want to pick one of these four possible values for *A*, depending on S_2 and S_1.

Primitive gate-based input logic

- We could build this circuit using primitive gates.
- If we want to use K-maps for simplification, then we should first expand out the abbreviated truth table.
  - The Y that appears in the output column (A) is actually an input.
  - We make that explicit in the table on the right.
- Remember A and Y are each 4 bits long!

S_2	S_1	A
0	0	0000
0	1	Y
1	0	Y'
1	1	1111

S_2	S_1	Y_i	A_i
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Primitive gate implementation

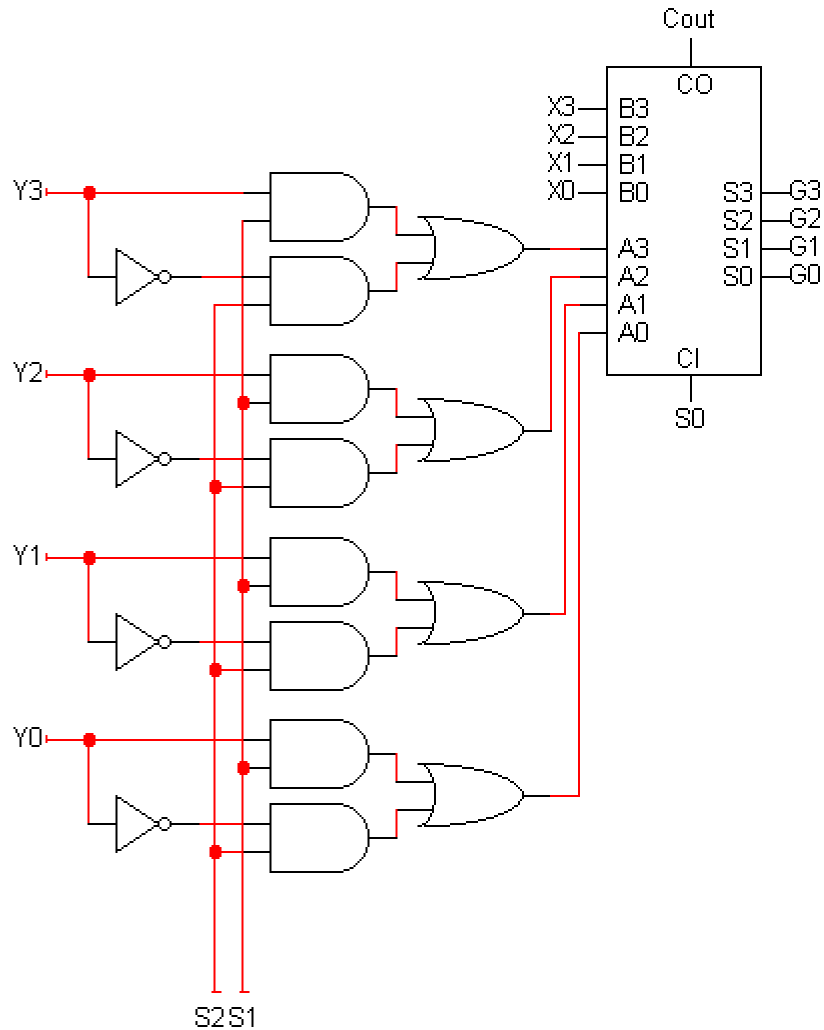
- From the truth table, we can find an MSP:

			S_1	S_1
	0	0	1	0
S_2	1	0	1	1
		Y_i	Y_i	

$$A_i = S_2 Y_i' + S_1 Y_i$$

- Again, we have to repeat this once for each bit Y3-Y0, connecting to the adder inputs A3-A0.

- This completes our arithmetic unit.



Bitwise operations

- Most computers also support logical operations like AND, OR and NOT, but extended to multi-bit words instead of just single bits.
- To apply a logical operation to two words X and Y, apply the operation on each pair of bits X_i and Y_i:

	1 0 1 1		1 0 1 1
AND	1 1 1 0	OR	1 1 1 0
	1 0 1 0		1 1 1 1

	1 0 1 1
XOR	1 1 1 0
	0 1 0 1

- We've already seen this informally in two's-complement arithmetic, when we talked about "complementing" all the bits in a number.

Bitwise operations in programming

- Languages like C, C++ Java and HDLs provide bitwise logical operations:

  & (AND) | (OR) ^ (XOR) ~ (NOT)

- These operations treat each integer as a bunch of individual bits:

  13 & 25 = 9 because 01101 & 11001 = 01001

- They are *not* the same as the operators &&, || and !, which treat each integer as a single logical value (0 is false, everything else is true):

  13 && 25 = 1 because true && true = true

- Bitwise operators are often used in programs to set a bunch of Boolean options, or flags, with one argument.
- Easy to represent sets of fixed universe size with bits:
  - 1: is member, 0 not a member. Unions: OR, Intersections: AND

Bitwise operations in networking

- IP addresses are actually 32-bit binary numbers, and bitwise operations can be used to find network information.
- For example, you can bitwise-AND an address 192.168.10.43 with a "subnet mask" to find the "network address," or which network the machine is connected to.

```
  192.168. 10. 43 = 11000000.10101000.00001010.00101011
& 255.255.255.224 = 11111111.11111111.11111111.11100000
-------------------------------------------------------
  192.168. 10. 32 = 11000000.10101000.00001010.00100000
```

- You can use bitwise-OR to generate a "broadcast address," for sending data to all machines on the local network.

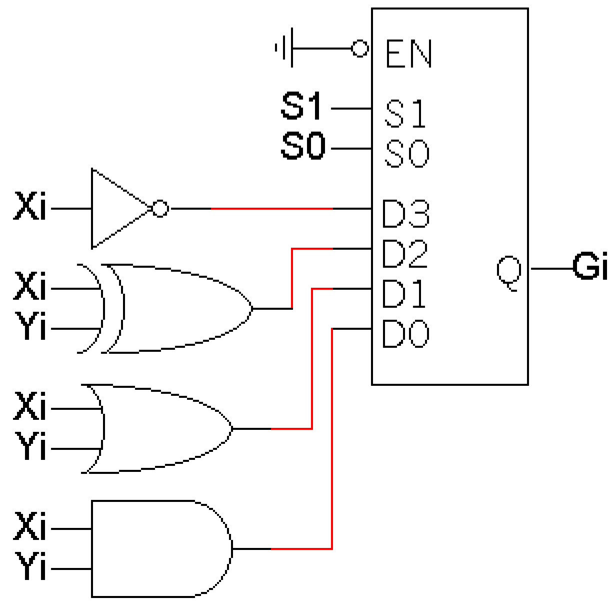
```
  192.168. 10. 43 = 11000000.10101000.00001010.00101011
|   0.  0.  0. 31 = 00000000.00000000.00000000.00011111
-------------------------------------------------------
  192.168. 10. 63 = 11000000.10101000.00001010.00111111
```

Defining a logic unit

- *A logic unit* supports different logical functions on two multi-bit inputs X and Y, producing an output G.
- This abbreviated table shows four possible functions and assigns a selection code S to each.

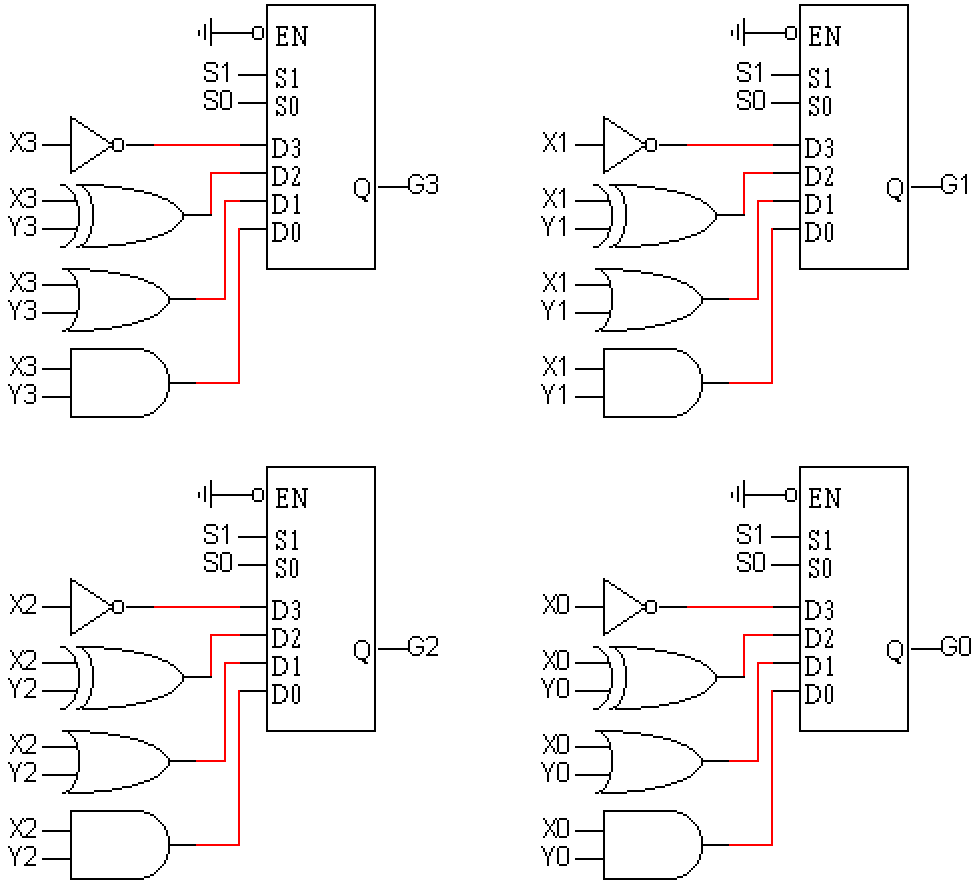
S_1	S_0	Output
0	0	$G_i = X_i Y_i$
0	1	$G_i = X_i + Y_i$
1	0	$G_i = X_i \oplus Y_i$
1	1	$G_i = X_i'$

- We'll just use multiplexers and some primitive gates to implement this.
- Again, we need one multiplexer for *each bit* of X and Y.



Our simple logic unit

- Inputs:
  - X (4 bits)
  - Y (4 bits)
  - S (2 bits)
- Outputs:
  - G (4 bits)



Combining the arithmetic and logic units

- Now we have two pieces of the puzzle:
  - An arithmetic unit that can compute eight functions on 4-bit inputs.
  - A logic unit that can perform four functions on 4-bit inputs.

- We can combine these together into a single circuit, an arithmetic-logic unit (ALU).

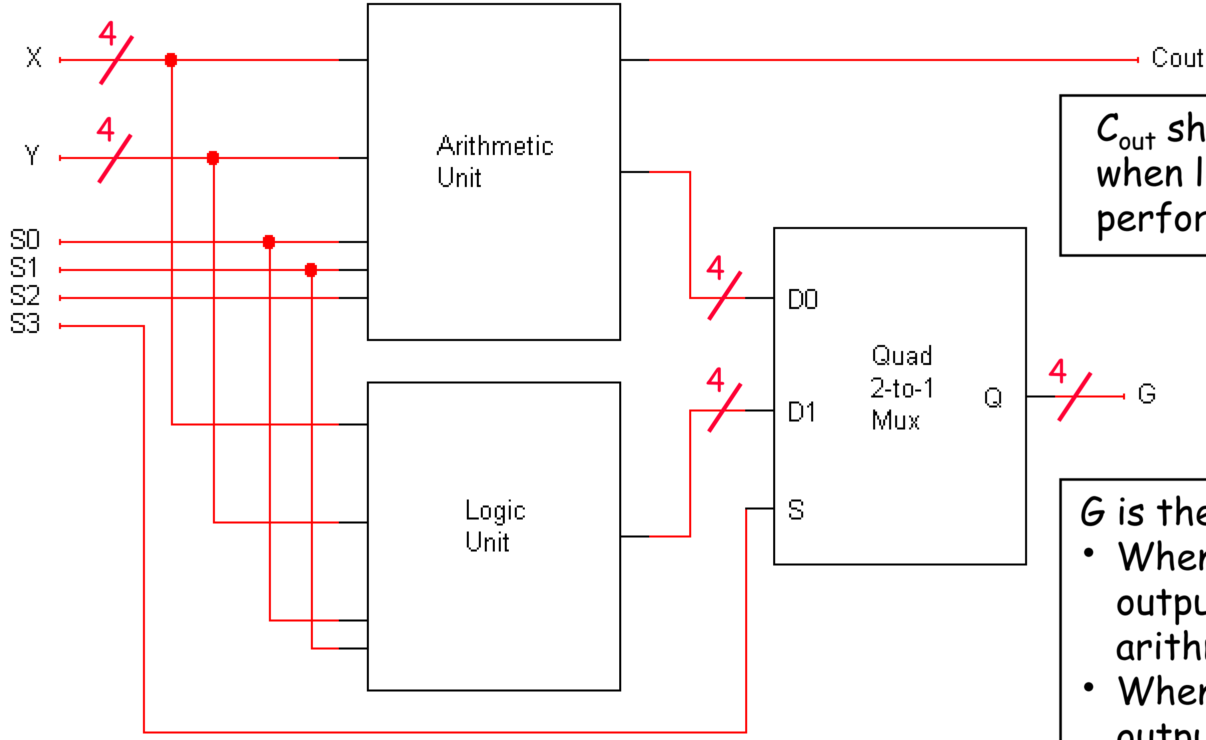
Our ALU function table

- This table shows a sample function table for an ALU.
- All of the arithmetic operations have S_3=0, and all of the logical operations have S_3=1.
- These are the same functions we saw when we built our arithmetic and logic units a few minutes ago.
- Since our ALU only has 4 logical operations, we don't need S_2. The operation done by the logic unit depends only on S_1 and S_0.

S_3	S_2	S_1	S_0	Operation
0	0	0	0	$G = X$
0	0	0	1	$G = X + 1$
0	0	1	0	$G = X + Y$
0	0	1	1	$G = X + Y + 1$
0	1	0	0	$G = X + Y'$
0	1	0	1	$G = X + Y' + 1$
0	1	1	0	$G = X - 1$
0	1	1	1	$G = X$
1	x	0	0	$G = X$ and Y
1	x	0	1	$G = X$ or Y
1	x	1	0	$G = X \oplus Y$
1	x	1	1	$G = X'$

A complete ALU circuit

The / and 4 on a line indicate that it's actually *four* lines.



C_{out} should be ignored when logic operations are performed (when S3=1).

G is the final ALU output.

- When S3 = 0, the final output comes from the arithmetic unit.
- When S3 = 1, the output comes from the logic unit.

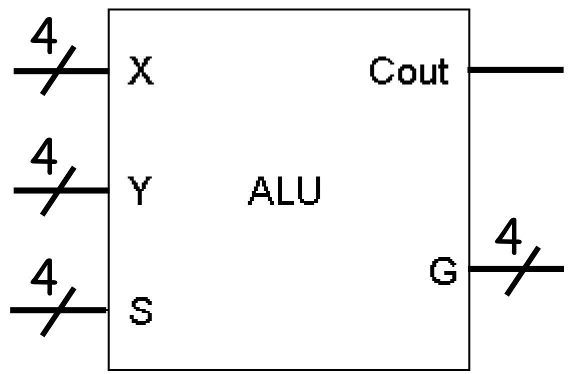
The arithmetic and logic units share the select inputs S1 and S0, but only the arithmetic unit uses S2.

Comments on the multiplexer

- *Both* the arithmetic unit and the logic unit are “active” and produce outputs.
  - The mux determines whether the final result comes from the arithmetic or logic unit.
  - The output of the other one is effectively ignored.
- Our hardware scheme may seem like wasted effort, but it’s not really.
  - “Deactivating” one or the other wouldn’t save that much time.
  - We have to build hardware for both units anyway, so we might as well run them together.
- This is a very common use of multiplexers in logic design.

The completed ALU

- This ALU is a good example of hierarchical design.
  - With the 12 inputs, the truth table would have had $2^{12} = 4096$ lines. That's an awful lot of paper.
  - Instead, we were able to use components that we've seen before to construct the entire circuit from a couple of easy-to-understand components.
- As always, we encapsulate the complete circuit in a "black box" so we can reuse it in fancier circuits.



ALU summary

- We looked at:
  - Building adders hierarchically, starting with one-bit full adders.
  - Representations of negative numbers to simplify subtraction.
  - Using adders to implement a variety of arithmetic functions.
  - Logic functions applied to multi-bit quantities.
  - Combining all of these operations into one unit, the ALU.
- Where are we now?
  - We started at the very bottom, with primitive gates, and now we can understand a small but critical part of a CPU.
  - This all built upon our knowledge of Boolean algebra, Karnaugh maps, multiplexers, circuit analysis and design, and data representations.