Q) Find extrema of $f(x,y) = xy$ constrained by ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

so $f(x,y) = xy$

$$g(x,y) = \frac{x^2}{8} + \frac{y^2}{2} - 1$$

By Lagrange's multiplier,

$$\frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x}, \quad \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y}$$

$$\Rightarrow y = \lambda \frac{x}{4} \quad - ①, \qquad x = \lambda y \quad - ②$$

Case 2 $\lambda = \pm 2.$

So, $x = \pm 2y.$ and $\cancel{y = \pm \frac{2x}{2}}$.

Let $y = \pm 1, \quad x = \pm 2.$

$f(-2,-1) = xy = 2$

$f(2,-1) = -2$] minimum

$f(-2,1) = -2$

$f(2,1) = 2.$

maximum.

Q) Find the maximum and minimum distance of $(3,4,12)$ subjected to sphere $x^2+y^2+z^2=4.$

Q) Find point on the plane $x+2y+3z=13$ closest to point $(1,1,1).$

Q) Find the farthest distance from point $(1,1,1)$ connected to the sphere $x^2+y^2+z^2=4.$

Problem :-

Q) $$\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

$$= \int_0^{\pi}\int_0^{\pi} \sin\phi \left[\frac{\rho^3}{3}\right]_0^{2\sin\phi} d\phi \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi}\int_0^{\pi} \sin\phi \; 8\sin^3\phi \, d\phi \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi}\int_0^{\pi} 2.(4\sin^3\phi) \, d\phi \, d\theta.$$

$$= \frac{2}{3}\int_0^{\pi}\int_0^{\pi} (3\sin\phi - \sin 3\phi) \, d\phi \, d\theta.$$

$$= \frac{2}{3}\int_0^{\pi} \left[-3\cos\phi + \frac{\cos 3\phi}{3}\right]_0^{\pi} d\theta$$

$$= \frac{2}{3}\int_0^{\pi} \left(-3(-1) - \frac{1}{3}\right.$$

3) Use spherical coordinate to evaluate

i) $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx.$

$= \frac{64}{9}\pi.$

~~us~~

given function
$F(x,y,z) = z^2\sqrt{x^2+y^2+z^2}$

is bounded by $z=0$ and $z=\sqrt{4-x^2-y^2}$

So, using spherical coordinates,

$\Rightarrow \rho\cos\phi = \sqrt{4-\rho^2\sin^2\phi\cos^2\theta-\rho^2\sin^2\phi\sin}$

$\Rightarrow \rho\cos\phi = \sqrt{4-\rho^2\sin^2\phi}.$

$\Rightarrow \rho^2\cos^2\phi + \rho^2\sin^2\phi = 4$

$\rho^2 = 4$

$\rho = -2, 2$

-2 is neglected so, $\rho = 2.$

$$= \frac{32}{9} \cancel{d\theta} \int_0^{2\pi} d\theta = \frac{32}{9} \theta\Big|_0^{2\pi}$$

$$= \frac{64\pi}{9} \text{ units}^3$$

1) $$I = \int_0^6 \int_0^4 x \, dx \, dy$$

$$= \frac{1}{2}\int_0^6 x^2\Big|_0^4 dy = \int_0^6 \frac{1}{2} \times (16-0)$$

$$= 8 \int_0^6 dy = 8 \times 6 = 48 \text{ units}^2$$

2) $$I = \int_0^{\sqrt{3}} \int_0^{x} dy \, dx = \int_0^{\sqrt{3}} y\Big|_0^x dx$$

$$= \int_0^{\sqrt{3}} x \, dx = \frac{1}{2} x^2\Big|_0^{\sqrt{3}} = \frac{3}{2} \text{ units}^2$$

3) $$I = \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2) \, dx \, dy$$

$$= \int_0^1 \left[\frac{x^3}{3} + y^2 x \right]_0^{x=\sqrt{1-y^2}} dy$$

$$= \int_0^1 \left(\frac{(1-y^2)^3}{3} + y^2\sqrt{1-y^2} \right) dy$$

Q) Using triple integrals in cylindrical coordinates to find Vol of region G bounded by hemisphere $z = \sqrt{25-x^2-y^2}$ below by XY plane and laterally by cylinder $x^2+y^2=9$.

upper curve   low

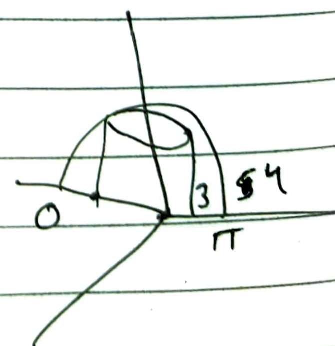
$z = \sqrt{25-x^2-y^2}$ & $z=0$

put $z=0$ in upper & $x = r\cos\theta$, $y = r\sin\theta$.

$25 - r^2\cos^2\theta - r^2\sin^2\theta = 0$

$25 = r^2 \cdot 1$

$r = -5, 5.$



Given cylinder,

$x^2+y^2=9$, $x = r\cos\theta$, $y = r\sin\theta$

$r^2 \cdot 1 = 9$

$r = -3, 3.$

$\sqrt{a^2-x^2}$

$\frac{\pi}{2}$

Now z bounds is. $z=0$ and

using beta

$$\int_0^1 (1-u)\, du = \int_0^1 u^0 \cdot (1-u)^1\, du$$

So can be written as.

$$\beta(1,2) = \frac{\Gamma(1)\,\Gamma(2)}{\Gamma(1+2)}$$

So. $\iint_R x\, y\, dy\, dx$

$$= \frac{a^2 b^2}{4} \times \beta(1,2) = \frac{a^2 b^2}{4} \times \frac{\cancel{0} \times 1!}{2!}$$

$$= \frac{a^2 b^2}{8}$$

Q) Using Beta - Gamma function.

$\iint_R \sqrt{xy}\, dx\, dy$ bounded by the lines

$x = 0,\ y = 0,\ x + y = 1$.

Vector diffrentiation of a function $f(x, y)$

is $D_v f(x, y) = \nabla f(x, y) \times \hat{v}$

where $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$

Q) Find DD of ~~f~~ function $2xy + z^2$ at $(1, -1, 3)$ in the direction of $i + 2j + 2k$.

Q) Find DD of $xyz^2 + xz$ at $(1, 1, 1)$ in direction of normal to surface $3xy^2 + y = z$ at $(0, 1, 1)$.

$\vec{v} = \cancel{i+j}\ \frac{1}{\sqrt{2}} \langle 1,1 \rangle$, find $\vec{v}$.

Decreases most rapidly at

$$\vec{v} = -\frac{1}{\sqrt{2}} \langle 1, 1 \rangle = \frac{1}{\sqrt{2}} \langle -1, -1 \rangle.$$

$$\vec{v} = \frac{1}{\sqrt{2}} (-i-j)$$

Q) Given $f(x,y) = x^2 + xy + y^2$ at $(-1,1)$, find the

i) find direction when f is rapidly increasing

ii) " " " " " " decreasing

$$\nabla f = \langle 2x+y,\ x+2y \rangle$$

$$|\nabla f| = \sqrt{(2x+y)^2 + (x+2y)^2}$$

$$|\nabla f|_{(-1,1)} = \sqrt{(-2+1)^2 + (-1+2)^2} = \sqrt{1+1}$$

$$= \sqrt{2}.$$

i) So the direction ~~is~~ where f is inc.

$$\vec{v} = \frac{1}{\sqrt{2}} \nabla f_{(-1,1)} = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$$

ii) f is decreasin

$$\hat{v} = -\frac{1}{\sqrt{2}} \nabla f_{(-1,1)} = -\frac{1}{\sqrt{2}} \langle -1, 1 \rangle.$$