DH1

PARAJULI
20BDS0405

DATE:
PAGE NO.

1). Monochromatic X-rays of wavelength 0.7078 Å are scattered by carbon. The scattered X-rays are at an angle of 90° with the incident direction. What is the wavelength of scattered rays?

Soln: Initial wavelength $(\lambda) = 0.7078 \text{ Å}$
$= 7.078 \times 10^{-11} \text{ m}$

mass of scattering body $(m) = 12 \times 1.67 \times 10^{-27} = 2.04 \times 10^{-26}$ kg.
Planck's Constant $(h) = 6.626 \times 10^{-34}$ Js.
Speed of light $(c) = 299792458$ m/s $\approx 2.998 \times 10^{8}$ m/s
Scattering angle $(\theta) = 90°$

Then, Wavelength of Scattered Rays $(\lambda') = \frac{h}{mc}(1 - \cos\theta) + \lambda$

$$= \frac{6.626 \times 10^{-34} (1 - \cos 90)}{2.04 \times 10^{-26} \times 2.998 \times 10^{8}} + 7.08 \times 10^{-11}$$

$$= 2.43 \times 10^{-12} + 7.08 \times 10^{-11}$$

$$= (0.7078 + 0.0243) \text{ Å}$$

$$= 0.7321 \text{ Å}$$

2) Calculate the wavelength of X-rays scattered at 180° from a carbon block if the frequency of incident rays is $1.8 \times 10^{18}\ s^{-1}$.

Sol:

~~Wavelength~~ Frequency of Incident rays $(f) = 1.8 \times 10^{18}$ Hz

Wavelength of Incident rays $(\lambda) = \frac{c}{f} = 1.6656 \times 10^{-10}$ m

Scattering Angle $(\theta) = 180^\circ$

Mass of electron $(m_e) = 9.1 \times 10^{-31}$ kg

$c = 2.998 \times 10^{8}$ m/s

$h = 6.626 \times 10^{-34}$ Js

Now, Wavelength of Scattered rays $\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$

$$= 1.6656\ \mathring{A} + \frac{6.626 \times 10^{-34} \times (1+1)}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$$= 1.6656\ \mathring{A} + 0.04857\ \mathring{A}$$

$$= 1.7142\ \mathring{A}$$

M1

BIMAL PARAJULI
(20BDS0405)

DATE: / /
PAGE NO.

Q) X-ray radiation of wavelength 1.12 Å is scattered from a carbon target. Calculate (i) the wavelength of X-ray scattered at 90° wrt original direction.
(ii) Energy of scattering electron after collision.

Sol

Initial wavelength of Incident X-ray $(\lambda) = 1.12\,\text{Å} = 1.12\times10^{-10}\,m$

Angle of Scattering $(\theta) = 90°$

Planck's Constant $(h) = 6.626\times10^{-34}\,Js$

Speed of light $(c) = 2.998\times10^{8}\,m/s$

Mass of scattering electron $(m_e) = 9.1\times10^{-31}\,kg$

Then,

(i) Wavelength of Scattered light $(\lambda') = \lambda + \frac{h}{m_e c}(1-\cos\theta)$

$$= 1.12\,\text{Å} + \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times2.998\times10^{8}}(1-\cos 90°)$$

$$= 1.12\,\text{Å} + 0.00243\,\text{Å}$$

$$= 1.1443\,\text{Å}$$

(ii) After collision, Since conservation of energy is followed in elastic collision,

Gain in KE of e^- = loss in energy of photon

$$= hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right)$$

$$= 6.626\times10^{-34}\times\frac{3\times10^{8}}{10^{-10}}\times\left(\frac{1}{1.12} - \frac{1}{1.1443}\right)$$

$$= 3.766\times10^{-17}\,J$$

For electron,

rest mass, $m_e = 9.1\times10^{-31}\,kg$

~~Kinetic Energy (E) = $\frac{p^2}{2m}$ => $p = \sqrt{2mE}$~~

~~=> Linear momentum (p) = $\sqrt{2\times9.1\times10^{-31}\times3.766\times10^{-17}}$~~
~~= 8.258×10^{-24}~~

Total energy = KE + Rest mass energy

$$= 3.766\times10^{-17} + m_e c^2$$

$$= 3.766\times10^{-17} + 9.1\times10^{-31}\times(2.998\times10^{8})^2$$

$$= 3.766\times10^{-17} + 8.179\times10^{-17}$$

= 8109.776 ×10^-17 J ... 511.422 KeV

4) An Incident photon of wavelength 0.03Å recoils at 60° after collision with a free electron. Find the energy of recoiling electron.

↳ solution,

Initial wavelength $(\lambda) = 0.03 Å = 3\times10^{-12} m$

Recoiling Angle $(\theta) = 60°$

Rest Mass of free electron $(m_e) = 9.1\times10^{-31}$ kg

~~Velocity~~ $c = 2.998\times10^8 m/s$ $h = 6.626\times10^{-34}$ Js

Then,

Final wavelength $(\lambda') = ?$

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$= 3\times10^{-12} + 2.43\times10^{-12}\times(1-\cos 60)$$

$$= 4.215\times10^{-12}$$

Here, For elastic collision, Energy is conserved.

KE gained by e^- = Energy lost by photon

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{6.626\times10^{-34}\times2.998\times10^8}{10^{-10}}\left(\frac{1}{0.03} - \frac{1}{0.0...}\right)$$

$$= 1.8\times10^{-14} J$$

$$= 112.503 keV$$

Rest mass energy of electron = $m_e c^2$

$$= 9.1\times10^{-31}\times(2.998\times10^8)^2$$

$$= 8.18\times10^{-14} J$$

$$= 511.193 keV$$

Total Energy = Kinetic Energy of motion + Rest mass energy

$$= 112.503 keV + 511.193 keV$$

$$= 623.696 keV$$

$$= 9.98\times10^{-14} Joule$$

BIMAL PARAJULI (20BDS6405)

DATE: / /

PAGE NO.

Q. If an X-ray photon of wavelength $0.5\,\mathring{A}$ makes a compton collision with a free electron and scattered at 90°. Find the energy of recoiling electron.

Initial wavelength $(\lambda) = 0.5\,\mathring{A} = 5\times10^{-11}\,m$

Scattering Angle $(\theta) = 90°$

~~Rest mass~~ $m_e = 9.1\times10^{-31}\,kg$, $c = 2.998\times10^{8}\,m/s$ $h = 6.626\times10^{-34}\,Js$

Then,

$$\text{Final wavelength } (\lambda') = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$= 5\times10^{-11}\,m + \frac{6.626\times10^{-34}(1-\cos 90)}{9.1\times10^{-31}\times2.998\times10^{8}}$$

$$= 0.5\,\mathring{A} + 0.0243\,\mathring{A}$$

$$= 0.5243\,\mathring{A}$$

Then, we know that the collision betn e^- and photon is perfectly elastic. Thus, total energy is conserved mechanically.

i.e. KE gained by e^- = Loss in energy of photon.

$$\Rightarrow KE - 0 = h\nu' - h\nu = \frac{hc}{\lambda'} - \frac{hc}{\lambda}$$

$$\Rightarrow KE = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = \frac{6.626\times10^{-34}\times2.998\times10^{8}}{10^{-10}}\left(\frac{1}{0.5} - \frac{1}{0.5243}\right)$$

$$= 1.986\times10^{-15}\times0.092695$$

$$= 1.841\times10^{-16}\,J$$

$$= 1150.57\,eV$$

$$= 1.15\,keV$$

From Einstein's Mass-Energy Equivalence,

Rest mass Energy of electron $(\cancel{E_0}) = m_0c^2$

$$= 9.1\times10^{-31}\times(2.998\times10^{8})^2$$

$$= 8.18\times10^{-14}\,J$$

$$= 511.193\,keV$$

Thus, Total Energy = KE of electron + Rest mass energy of e^-

$$= 1.15\,keV + 511.193\,keV$$

$$= 512.34\,keV$$

6) In a compton's experiment, the wavelength of X-ray scattered at an angle of 45° is 0.022 Å. Calculate the wavelength of incident X-ray.

↳ Solution,

Wavelength of Scattered X-rays (λ') = $0.022\,\text{Å} = 2.2\times10^{-12}\,m$

Scattering Angle (θ) = $45°$

Rest Mass of electron (m_e) = $9.1\times10^{-31}\,kg$

$h = 6.626\times10^{-34}\,Js$ $\qquad c = 2.998\times10^{8}\,m/s$

Then,

Wavelength of incident X-rays (λ) = $\lambda' - \frac{h}{m_e c}(1-\cos\theta)$

$$= 2.2\times10^{-12} - \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times2.998\times10^{8}}(1-\cos45°)$$

$$= 2.2\times10^{-12} - 2.43\times10^{-12}\times(1-\cos45)$$

$$= 2.2\times10^{-12} - 0.712\times10^{-12}$$

$$= 1.488\times10^{-12}\,m$$

$$= 0.01488\,\text{Å}$$

7) X-ray of wavelength 0.24 nm are compton scattered and scattered beam is observed at 60° relative to the incident beam. Find the wavelength of scattered rays.

↳ Solution.

Wavelength of Incident X-rays (λ) = $0.24\,nm = 2.4\times10^{-10}\,m$

Scattering Angle (θ) = $60°$

Rest mass of electron (m_e) = $9.1\times10^{-31}\,kg$

$h = 6.626\times10^{-34}\,Js$ $\qquad c = 2.998\times10^{8}\,m/s$

Now,

Wavelength of Scattered X-rays (λ') = $\lambda + \frac{h}{m_e c}(1-\cos\theta)$

$$= 2.4\times10^{-10} + \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times2.998\times10^{8}}(1-\cos60)$$

$$= 2.4\times10^{-10} + 0.0243\times10^{-10}\times(1-\tfrac{1}{2})$$

$$= 2.412\times10^{-10}\,m$$

(8) In an experiment of compton scattering, the incident radiation has wavelength 2Å. Calculate the energy of recoil electron which scatters the radiation through 60°.

↳ Solution,

Wavelength of incident radiation $(\lambda) = 2Å = 2\times10^{-10}$ m

Scattering Angle $(\theta) = 60°$

Rest mass of electron $(m_e) = 9.1\times10^{-31}$ kg

$h = 6.626\times10^{-34}$ Js $\quad c = 2.998\times10^{8}$ m/s

Then,

Final wavelength after scattering $(\lambda') = \lambda + \frac{h}{m_e c}(1-\cos\theta)$

$$= 2\times10^{-10} + \frac{6.626\times10^{-34}(1-\cos60°)}{9.1\times10^{-31}\times2.998\times10^{8}}$$

$$= 2Å + 0.243\times0.5Å$$

$$= 2.1215Å$$

Now, The collision betn photon and e^- is supposed to be perfectly elastic. so, by using principle of energy conservation,

Change in KE of e^- = loss in energy of photon

ie. $KE - 0 = h\nu - h\nu'$

$$\Rightarrow KE = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) = \frac{6.626\times10^{-34}\times2.998\times10^{8}}{10^{-10}}\left(\frac{1}{2}-\frac{1}{2.1215}\right)$$

$$= 5.688\times10^{-17} \text{ Joule}$$

$$= 355.52 \text{ eV}$$

$$= 0.3555 \text{ KeV}$$

Rest mass Energy of e^- $= m_e c^2 = 9.1\times10^{-31}\times(2.998\times10^{8})^2$

$$= 8.18\times10^{-14} \text{ Joule}$$

$$= 511.192 \text{ keV}$$

Then,

Total Energy = KE + Rest mass energy

$$= (511.19 + 0.3555) \text{ keV}$$

$$= 511.545 \text{ KeV}$$

$$= 8.18\times10^{-14} \text{ J}$$

DA1

DATE: / /
PAGE NO.

Uncertainty Principle:-

1) The life time of an energy state is 10^{-8}s. Calculate the uncertainty in the frequency of photon emitted during transition (de-excitation).

Soln:

Life time $(\Delta T) = 10^{-8}$s

Uncertainty in frequency $(\Delta \nu) = ?$

We know that, from heisenberg's Uncertainty principle,

$$\Delta E \cdot \Delta T \geq \frac{h}{4\pi}$$

$$\Rightarrow (h \cdot \Delta \nu) \cdot \Delta T \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta \nu \geq \frac{\cancel{h}}{4\pi \cancel{h} \cdot \Delta T}$$

$$\Rightarrow \Delta \nu \geq \frac{1}{4\pi T}$$

$$\Rightarrow \Delta \nu \geq 7957747.155 \text{Hz}$$

$$\approx 7.96 \times 10^{6} \text{ Hz}$$

Thus, the minimum uncertainty in frequency of photon is 7.96 MHz.

2) The nucleon is confined in a nucleus of radius 5×10^{-15} m. Calculate the minimum uncertainty in momentum of electron.

Soln:

Maximum uncertainty in distance $(\Delta x) = 5 \times 10^{-15}$ m

Uncertainty in momentum $(\Delta p) = ?$

From Heisenberg's Uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\Rightarrow \Delta p \geq \frac{6.626 \times 10^{-34}}{4\pi \times 5 \times 10^{-15}}$$

$$\Rightarrow \Delta p \geq 1.054 \times 10^{-20} \text{ kg ms}^{-1}$$

$\therefore$ The minimum uncertainty in momentum is 1.054×10^{-20} kg ms^{-1}.

(11) The position and momentum of 1 keV electron are determined simultaneously. If it's position is located within 1 Å. What is the percentage of uncertainty in it's momentum.

↳ Sol:- Mass of e^- $(m_e) = 9.1 \times 10^{-31}$ kg

Energy of e^- $(E) = 1 \text{ keV} = 1 \times 1.6 \times 10^{-16}$ J $= 1.6 \times 10^{-16}$ J

Momentum of e^- $(p) = ?$

We know, $E = \frac{1}{2}mv^2$

$$\Rightarrow E = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}}$$

$$= 1.7 \times 10^{-23} \text{ kgms}^{-1}.$$

Then, uncertainty in position $(\Delta x) = 1\text{Å} = 10^{-10}$ m.

By using Heisenberg's Uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\Rightarrow \Delta p \geq \frac{6.626 \times 10^{-34}}{4\pi \times 10^{-10}}$$

$$\Rightarrow \Delta p \geq 5.273 \times 10^{-25} \text{ kgms}^{-1}.$$

Thus, minimum uncertainty in momentum $= 5.273 \times 10^{-25}$ kgms^{-1}.

Then,

% Uncertainty of momentum $= \frac{\Delta p}{p} \times 100\%$

$$= \frac{5.27 \times 10^{-25}}{1.7 \times 10^{-23}} \times 100\%$$

$$= 3.102\%.$$

Scanned with CamScanner

Q) The time period of a radar vibration is 0.25 μs. What is the uncertainty in energy of photon?

Solution.

Time period of radar vibration (ΔT) = 0.25μs = 2.5×10^{-7} s

Uncertainty in Energy (ΔE) = ?

Then, From Heisenberg's Uncertainty principle,

$$\Delta E \cdot \Delta T \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta E \geq \frac{h}{4\pi \Delta T}$$

$$\Rightarrow \Delta E \geq \frac{6.626 \times 10^{-34}}{4\pi \times 2.5 \times 10^{-7}}$$

$$\Rightarrow \Delta E \geq 2.11 \times 10^{-28} \text{ Joule}$$

∴ The minimum uncertainty in Energy of photon is 2.11×10^{-28} Joule.

13). Determine the energy values of an electron confined in a box of width 1Å.

↳ Sol:– Mass of e^- $(m) = 9.1 \times 10^{-31}$ kg.

Width of potential well / box $(L) = 1\text{Å} = 10^{-10}$ m

$h = 6.626 \times 10^{-34}$ Js

Then, Ground state energy $(E_1) = \frac{1^2 \cdot \pi^2 \times h^2}{2 \cdot m \cdot L^2 \cdot 4\pi^2} = \frac{\pi^2 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2 \times \pi^2}$

$= 6.03 \times 10^{-18}$ J

$= 37.692$ eV.

First excited state $(E_2) = \frac{2^2 \cdot \pi^2 \times h^2}{2 \cdot m L^2 \cdot 4\pi^2} = 4 \times E_1$

$= 4 \times 6.03 \times 10^{-18}$

$= 2.41 \times 10^{-17}$ J

$= 150.768$ eV.

Second Excited State $(E_3) = \frac{3^2 \cdot \pi^2 h^2}{2mL^2 4\pi^2} = 9 \times E_1$

$= 9 \times 6.03 \times 10^{-18}$

$= 5.42 \times 10^{-17}$ J

$= 339.228$ eV.

And so on ___

$\therefore E_1 = 37.692$ eV

$E_2 = 150.768$ eV

$E_3 = 339.228$ eV

$mc^2 \quad \frac{M}{\sqrt{1-\frac{v^2}{c^2}}} \cdot v = x$

$\frac{cm}{\sqrt{c^2-v^2}} \cdot v = x$

$\frac{cm}{x} = \frac{\sqrt{c^2-v^2}}{v}$

(4): Find the lowest energy level and momentum of e^- in one dimensional potential well of width $1\mathring{A}$.

Soln.

Mass of e^- $(m) = 9.1 \times 10^{-31}$ kg

Width of potential well $(L) = 1\mathring{A} = 10^{-10}$ m

$h = 6.626 \times 10^{-34}$ Js

Then, Lowest energy is in the ground state of e^-.

Taking $n=1$, $E_n = \frac{n^2h^2}{2mL^2 \cdot 4} = \frac{1 \cdot (6.626\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2\times4}$

$= 6.03\times10^{-18}$ Joule.

$= 37.69$ eV.

Also, In ~~ground state, the lowest~~ the potential well, for minimum momentum, momentum is equal to the uncertainty in momentum. For uncertainty in momentum to be lowest, uncertainty in position must be maximum according to the uncertainty principle.

$\therefore \Delta x = L = 10^{-10}$ m

Minimum Momentum $(p) = \Delta p = \frac{h}{4\pi\Delta x} = \frac{6.626\times10^{-34}}{4\pi\times10^{-10}}$

$= 5.27\times10^{-25}$ kgms^{-1}.

$\therefore$ Lowest Energy $(E) = 6.03\times10^{-18}$ J

Lowest Momentum $(p) = 5.27\times10^{-25}$ kgms^{-1}.

15) A particle is moving in one-dimensional potential box (of infinite height) of width 50Å. Calculate the probability of finding the particle within an interval of 10Å at the center of box when it is in state of least energy.

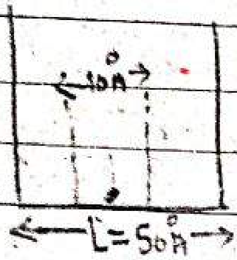
↳ sol.

Width of potential box (L) $= 50\text{Å} = 5\times10^{-9}$ m

Then,

After normalizing the wave function,

$$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$



At lowest energy state, we take $n=1$,

$$\therefore \psi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right).$$

Since it's real, it's conjugate will be same.

$$\psi^* = \psi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right).$$

Then, Probability of finding it from $\frac{2L}{5}$ to $\frac{3L}{5}$ is:-

$$\text{Probability} = \int_{2L/5}^{3L/5} \psi^*\psi\, dx$$

$$= \int \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)\cdot\sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{2}{L}\int \sin^2\frac{\pi x}{L}$$

$$= \frac{2}{L}\int \frac{1-\cos\frac{2\pi x}{L}}{2}$$

$$= \frac{1}{L}\int 1-\cos\frac{2\pi x}{L}$$

$$= \frac{1}{L}\left(x - \left(\sin\frac{2\pi x}{L}\right)\cdot\frac{L}{2\pi}\right)_{2L/5}^{3L/5}$$

$$= \frac{1}{L}\times\left(\frac{3L}{5} - \frac{2L}{5}\right) - \frac{1}{L}\times\frac{L}{2\pi}\left(\sin\frac{2\pi}{L}\frac{3L}{5} - \sin\frac{2\pi}{L}\frac{2L}{5}\right)$$

$$= \left(\frac{3}{5}-\frac{2}{5}\right) - \frac{1}{2\pi}\times\left(\sin\frac{6\pi}{5} - \sin\frac{4\pi}{5}\right)$$

$$= 0.2 + 0.187 = 0.387$$

$\therefore$ The probability is 0.387

16). Find the lowest energy of e^- confined to move in a cubical box of length $0.5\ \mathring{A}$.

↳ sol

Mass of e^- $(m_e) = 9.1 \times 10^{-31}$ kg.

Dimensions of box are: $a = b = c = 0.5\ \mathring{A} = 5 \times 10^{-11}$ m.

$$\text{Energy of particle } (E) = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

Taking $n_x = n_y = n_z = 1$,

$$E = \frac{h^2 \times 3}{8m \cdot a^2}$$

$$= 7.25 \times 10^{-17} \text{ Joule.}$$

$$= 452.3 \text{ eV.}$$

17. ~~Ques:~~ Whether electrons are present inside atomic nuclei or not? Prove using Heisenberg Uncertainty Principle.

↳ Ans: No,

Electrons are not ~~possible to be~~ present inside the nucleus. It is because if the electrons were present inside the nucleus, their energy would be way higher than experimentally observed.

- During radioactive decay, β-particles are emitted ~~when~~ from Nucleus whose energy is of the order of 2-3 MeV which is way less than the minimum energy that would be needed for e^- if it were to be present in nucleus.

Mathematically, we can use Heisenberg's Uncertainty principle to prove it. Since, the dimensions of nucleus is ~~the~~ of the order of 10^{-14} m, If the e^- were present inside nucleus, it's uncertainty ~~should~~ in position should be less or equal to 10^{-14} m. i.e. $\Delta x \leq 10^{-14}$ m.

$$\therefore \Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\Rightarrow \Delta p \geq \frac{h}{4\pi \times 10^{-14}}$$

$$\Rightarrow \Delta p \geq 5.27 \times 10^{-20} \ kg\,ms^{-1}$$

Thus, the momentum of e^- should be at least 5.27×10^{-20} kg ms^{-1}.

Then, Energy of e^- $E = \sqrt{m_0^2c^4 + p^2c^2}$

$$= \sqrt{(9.1\times10^{-31})^2 \times (3\times10^8)^4 + (5.27\times10^{-20})^2 (3\times10^8)^2}$$

$$= \sqrt{2.5 \times 10^{-22}}$$

$$= 1.58 \times 10^{-11} \ J$$

$$= 98.81 \ MeV$$

But e^- observed experimentally during β-decay is seen to have 2-3 MeV only. Thus, e^- can't be present inside nucleus.

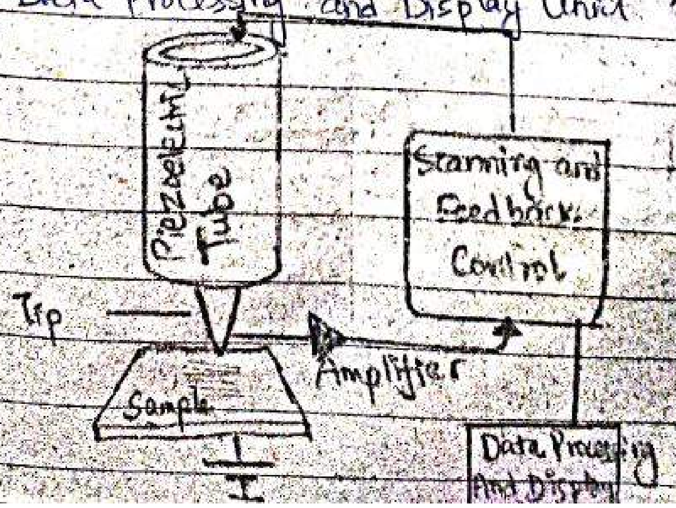
(18) Explain the working parts of Scanning Tunnelling Microscope in detail (2 page + 2 fig.)

Scanning Tunneling Microscope is a device that is based on quantum tunneling principle and used for imaging the surfaces at atomic level. It senses the surface by using an extremely fine pointed tip that can distinguish surface characteristics smaller than 0.1nm. Thus, the individual images can be routinely examined ~~and~~ imaged and manipulated.

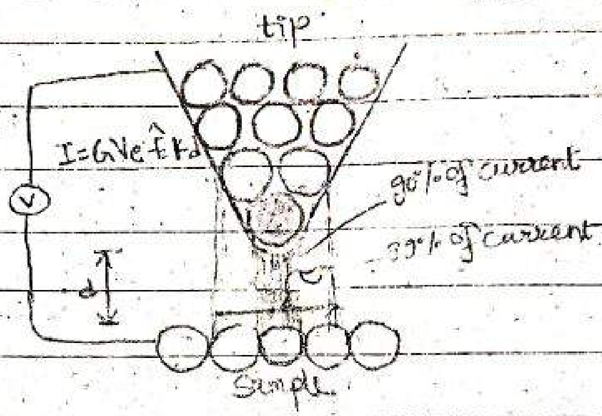
It's based on quantum tunneling. When the tip is brought closed to the surface to be examined, a bias voltage applied between the two allows e^- to tunnel through the space between them. The resulting current is a function of tip position, applied voltage, and Local Density Of State of the sample. For tunneling to take place, both sample and the tip should be either ~~semi~~ conductor or semiconductor. Thus, STM can't image insulating materials.

The most important parts of a typical STM are:-

* Piezoelectric Tube
* Scanning and Feed back Control Unit.
* Tunneling Tip.
* Tunneling Current Amplifier.
* Data Processing and Display Unit.



Working:- In STM, Bias Voltage is applied betn sharp conductive tip and a conductive sample. The sample is approached to few angstroms from tip, the tunneling occurs that indicates the proximity of the tip to sample with very high accuracy.



It operates in following 2 modes:-

① Constant Current mode.

↳ In this mode, the STM uses feedback to keep the tunneling current constant by moving and adjusting the height of scaner at each measurement point. Eg:- If the system detects increase in tunneling current, it adjusts the voltage applied to z-axis scanner to increase the distance between tip and sample. In constant current mode, the motion of scaner constitutes the data set. If the system keeps the current constant to within few per cent, tip-to-sample distance will be constant to within a few hundredths of an angstrom. Although this method is very slow, it can measure irregular surfaces with high precision.

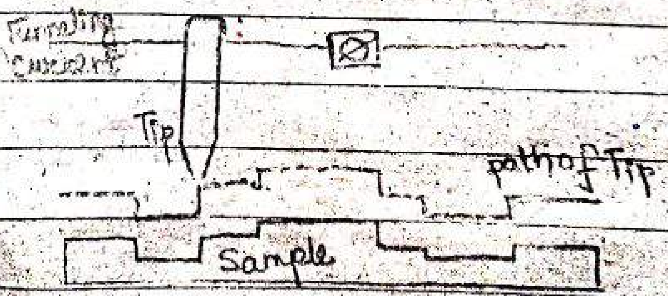


Fig:- Constant Current Mode

(2) Constant height mode.

↳ In this mode, the tip travels in a horizontal plane above the sample at a constant height and the tunneling current through its tip varies depending on the topography and the local surface electronic properties of the sample. The tunneling current measured at each location on the sample surface constitute the data set,

$$I_t = e^{-kd}$$

Thus, the variation in distance of tip from sample surface causes exponential variation in current which is recorded as data set and analysed. This method is faster than constant current mode. because the system doesn't have to use feedback and move scanner up & down

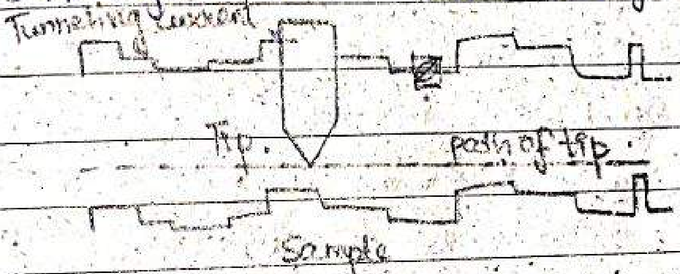


Fig:- Constant height mode.

1Q). Write briefly the underlying principle used in Davison-Germer experiment to verify wave nature of e^- experimentally. (2 pages).

↳ ~~Solution~~

Davison-Germer experiment used the principle of diffraction to experimentally verify the wave nature of e^-.

In Davison-Germer experiment, they assumed that if e^- showed wave property, it should undergo diffraction like X-ray gets diffracted in von-laue experiment.

In order to test the De-Broglie's hypothesis that matter behaved like waves, Davisson and Germer set up an experiment very similar to what might be used to look at the interference pattern from X-ray scattering from a crystal surface. The basic idea is that the planar nature of crystal structure provides scattering surface at regular intervals; thus waves that scatter from one surface can constructively or destructively interfere from waves that scatter from the next crystal plane deeper into the crystal.

The schematic of their apparatus is as shown:-

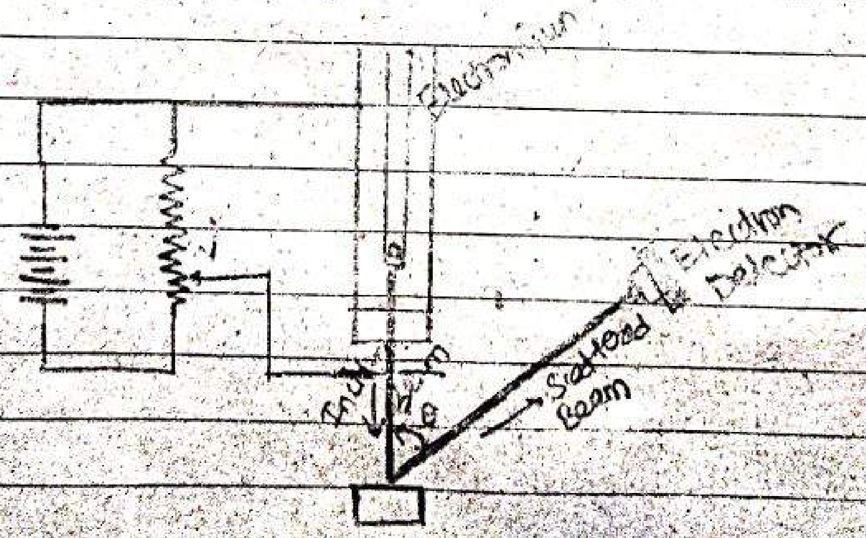
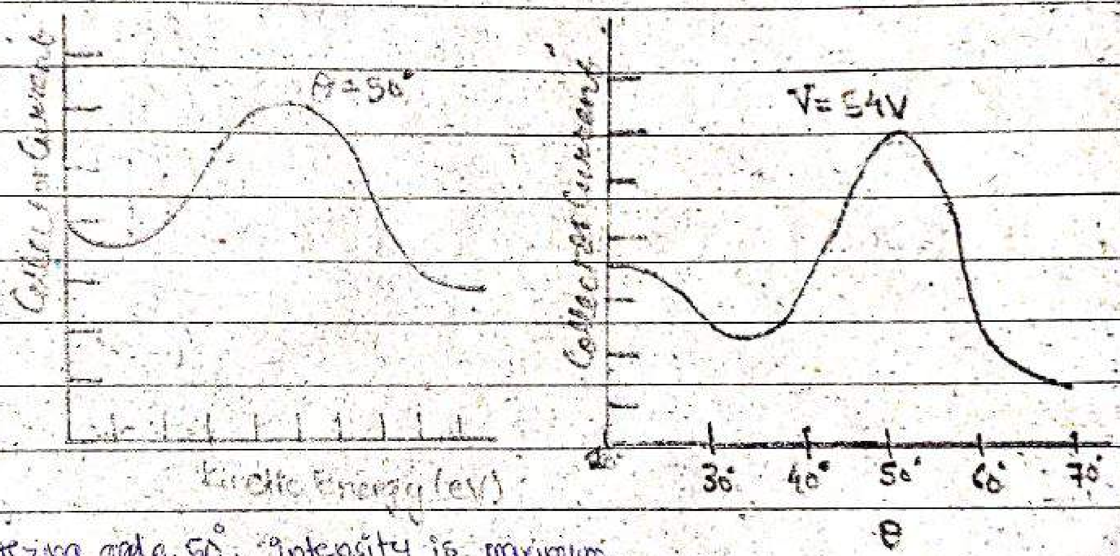


Fig:- Davison-Germer Experiment

This simple apparatus sends an e^- beam with an adjustable energy to a crystal surface and then measures the current of e^- detected at a particular scattering angle(θ). The results of an energy scan at a particular angle and an angle scan at a fixed energy are shown as follows. Both show a characteristic shape indicative of an interference pattern and consistent with the planar separation in the crystal. This was a dynamic proof of the wave nature of matter.



At scattering angle 50°, intensity is maximum.

$\therefore$ Glancing Angle(θ) $= \frac{180 - \cancel{\theta}\,50^\circ}{2} = 65^\circ$

From Bragg's law: $n\lambda = 2d\sin\theta$

For first order, $n = 1$, $\Rightarrow \lambda = 2d\sin\theta$

$= 2 \times 0.91 \times 10^{-10} \times 0.9063$

$\approx 1.65 \times 10^{-10}$ m

$= 1.65$ Å

Also, From Debroglie's hypothesis,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$\approx 1.66 \times 10^{-10}$ Å

$= 1.66$ Å

We can observe that the wavelength of matter wave from the bragg's law and de broglie's hypothesis are almost identical. Thus, Davison-Germer successfully verifies the De-Broglie's hypothesis experimentally.