

STS1201	Introduction to Problem Solving	L	T	P	J	C
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Pre-requisite	None	Syllabus version				
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Course Objectives:

- To enhance the logical reasoning skills of the students and improve the problem-solving abilities
- To strengthen the ability to solve quantitative aptitude problems
- To enrich the verbal ability of the students for academic purpose

Expected Course Outcome:

- Students will be introduced to basic concepts of Quantitative Aptitude, Logical reasoning and Verbal ability
- Students will be able to read and demonstrate good comprehension of text in areas of the student's interest
- Students will be able to demonstrate the ability to resolve problems that occur in their field.

STUDENT LEARNING OUTCOME(SLO):	5, 9, 10, 12 and 16
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Module:1	Lessons on excellence	2hours
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Skill introspection, Skill acquisition, consistent practice

Module:2	Logical Reasoning	18 hours
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Thinking Skill

- Problem Solving
- Critical Thinking
- Lateral Thinking

Taught through thought-provoking word and rebus puzzles, and word-link builder questions

Coding & decoding, Series, Analogy, Odd man out and Visual reasoning

- Coding and Decoding
- Series
- Analogy
- Odd Man Out
- Visual Reasoning

Sudoku puzzles

Solving introductory to moderate level sudoku puzzles to boost logical thinking and comfort with numbers

Attention to detail

Picture and word driven Qs to develop attention to detail as a skill

Module:3	Quantitative Aptitude	14 hours
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Speed Maths

- Addition and Subtraction of bigger numbers
- Square and square roots
- Cubes and cube roots
- Vedic maths techniques
- Multiplication Shortcuts
- Multiplication of 3 and higher digit numbers
- Simplifications
- Comparing fractions
- Shortcuts to find HCF and LCM
- Divisibility tests shortcuts

Algebra and functions

Module:4	Recruitment Essentials	5hours
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Looking at an engineering career through the prism of an effective resume

- Importance of a resume - the footprint of a person's career achievements
- How a resume looks like?
- An effective resume vs. a poor resume: what skills you must build starting today and how?

Impression Management

Getting it right for the interview:

- Grooming, dressing
- Body Language and other non-verbal signs
- Displaying the right behaviour

Module:5	Verbal Ability	6hours
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Grammar challenge

A practice paper with sentence based and passage-based questions on grammar discussed. Topics covered in questions are Nouns and Pronouns, Verbs, Subject-Verb Agreement, Pronoun-Antecedent Agreement, Punctuations

Verbal reasoning

	Total Lecture hours:	45 hours
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Mode of Evaluation: FAT, Assignments, 3 Assessments with Term End FAT (Computer Based Test)

Text Book(s):

1. FACE, Aptipedia Aptitude Encyclopedia, 2016, 1stEdition, Wiley Publications, Delhi.
2. ETHNUS, Aptimithra, 2013, 1stEdition, McGraw-Hill Education Pvt.Ltd.
3. **SMART, PlaceMentor, 2018, 1st Edition, Oxford University Press.**
4. R S Aggarwal, Quantitative Aptitude For Competitive Examinations, 2017, 3rd Edition,

S. Chand Publishing, Delhi.
Reference Book(s): Arun Sharma, Quantitative Aptitude, 2016, 7 th Edition, McGraw Hill Education Pvt. Ltd.
Recommended by Board of Studies
Approved by Academic Council

SKILL INTROSPECTION

What is knowledge?

- According to Webster's Dictionary, knowledge is "the fact or condition of knowing something with familiarity gained through experience or association". It is the information, facts, principles, skills and understanding, etc. that is acquired through education and experience.
- In practice, though, there are many possible, equally plausible definitions of knowledge, a frequently used definition of knowledge is "the ideas or understandings which an entity possesses that are used to take effective action to achieve the entity's goal(s).

What is skill?

- An ability and capacity acquired through deliberate, systematic, and sustained effort to smoothly and adaptively carryout complex activities or job functions involving ideas (cognitive skills), things (technical skills), and/or people (interpersonal skills).
- A skill set is a particular category of skills or abilities necessary to perform a job.

Knowledge vs. Skill

- Knowledge is **information acquired** through sensory input:
 - a. Reading
 - b. watching
 - c. listening
 - d. touching
- The concept of knowledge refers to **familiarity** with factual information and **theoretical concepts**.
- Knowledge can be **transferred** from one person to another or it can be self -acquired through **observation and study**.



Knowledge vs. Skill

- Skills, however, refer to the **ability to apply knowledge** to specific situations.
- Skills are developed through **practice**, through a combination of sensory input and output.
- As an example, social skills are developed through interaction with people by observing, listening, and speaking with them.
- **Trial and error** is probably the best way to achieve skills mastery.
- To make it simple,
 - Knowledge is theoretical
 - Skills are practical.

Knowledge is “what to do?”
Skill is “how to do?”

Can you explain the picture???



Knowledge vs. Skill

- **If recipe is the knowledge, then cooking is the skill.**
- To prepare any dish the ingredients and the procedure is essential, but, knowing what to do won't bring the results but, the doing i.e., the cooking will bring the results.



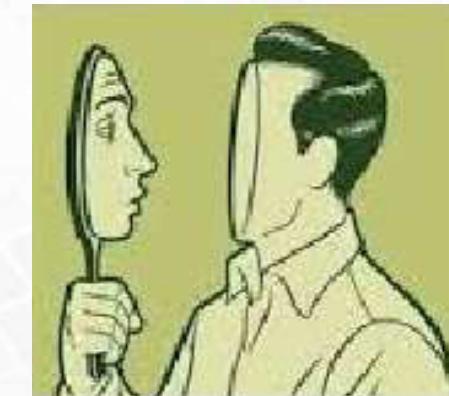
Knowledge vs. Skill

- Similarly, if Rules and Procedures would be the Knowledge, Driving would be the Skill.



What is introspection?

- It is our ability to **look at ourselves** from a detached view point so that we are able to see the flaws in our **thinking, acting and learning**.
- Our gut instinct is always active and **acts as a compass** against which we compare our actions.
- However, we sometimes fail to take that meek '**inner voice**' into consideration. Therefore we need to look within from time to time to rectify the flaws in our inner system.
- Introspection helps an individual to identify his own skill set.



Different types of skills

What's skill?

- A **skill** is the **ability to carry out a task** with pre-determined results often within a given amount of time, energy, or both.
- **Types of skills:**
 - Labour skills
 - Life skills
 - Soft skills
 - Hard skills

Three Types of Skills Classification

Skill Type	Description
Transferable/Functional	<ul style="list-style-type: none">• Actions taken to perform a task, transferable to different work functions and industries• Based on ability and aptitude• Expressed in verbs• Examples:<ul style="list-style-type: none">◦ Organize◦ Promote◦ Analyze◦ Write
Personal Traits/Attitudes	<ul style="list-style-type: none">• Traits or personality characteristics that contribute to performing work• Developed in childhood and through life experience• Expressed in adjectives• Examples:<ul style="list-style-type: none">◦ Patient◦ Diplomatic◦ Results-oriented◦ Independent
Knowledge-based	<ul style="list-style-type: none">• Knowledge of specific subjects, procedures, and information necessary to perform particular tasks• Acquired through education, training, and on-the-job experience• Expressed in nouns• Examples:<ul style="list-style-type: none">◦ Personnel Administration◦ Contract Management◦ Accounting

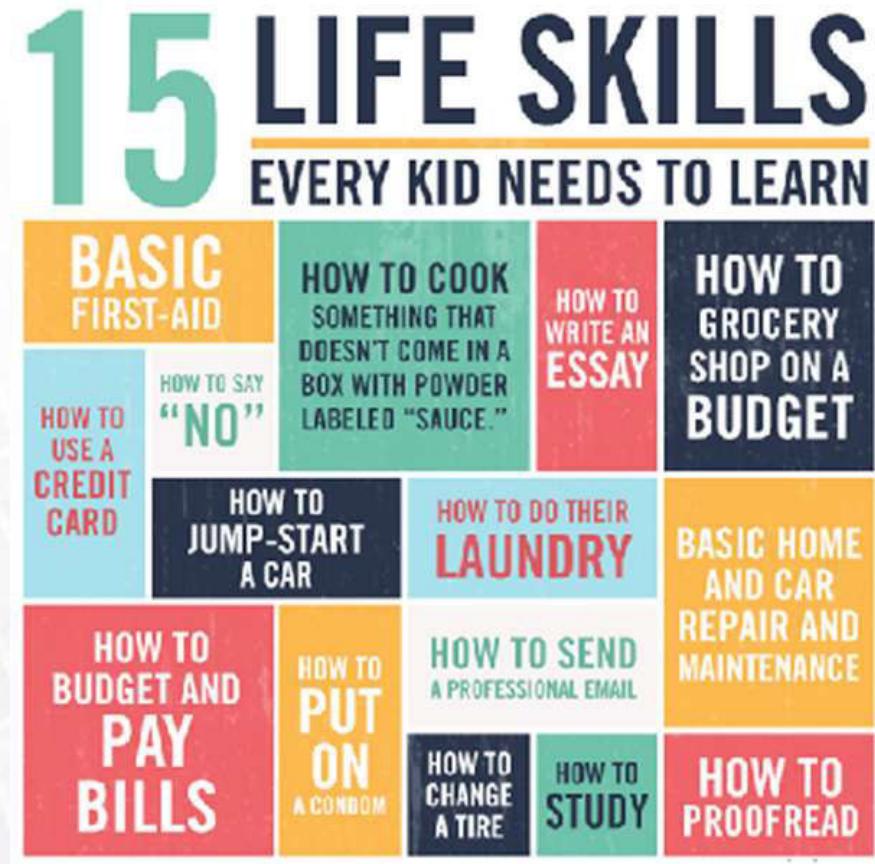
Labour Skills

- These skills are directed towards the practical working itself.
Example: Role of Electrician, Mechanic etc.



Life skills

- They are a set of human skills, acquired via learning or direct experience which are used to handle problems and questions commonly encountered in daily human life.
- The subject varies greatly depending on societal norms and community expectations.



Soft Skills

- Soft skills are a combination of interpersonal people skills, social skills, communication skills, character traits, career attributes and emotional intelligence quotient (EQ) among others.



Various soft skills

- **Empathy** -the ability to understand and share the feelings of another.
- **Creativity** -the use of imagination or original ideas to create something; inventiveness.
- **Communication**-the imparting or exchanging of information by speaking, writing, or using some other medium.
- **Time Management**- the ability to use one's time effectively or productively, especially at work.
- **Motivation** -Internal and external factors that stimulate desire and energy in people to be continually interested and committed to a job, role or subject, or to make an effort to attain a goal.
- **Negotiation**- discussion aimed at reaching an agreement.

Various soft skills

- **Personality Development-** Personality development is the development of the organized pattern of behaviors and attitudes that makes a person distinctive. Personality development occurs by the ongoing interaction of temperament , character, and environment.
- **Coaching and Mentoring-** The focus is on concrete issues, such as managing more effectively, speaking more articulately, and learning how to think strategically. This requires a content expert (coach) who is capable of teaching the coachee how to develop these skills.
- Mentoring, to be successful, requires time in which both partners can learn about one another and build a climate of trust that creates an environment in which the mentoree can feel secure in sharing the real issues that impact his or her success. Successful mentoring relationships last nine months to a year.

Hard Skills

- **Hard skills** are any skills relating to a specific task or situation. These skills are easily quantifiable unlike soft skills which are related to one's personality. Usually these include technical experience and qualifications.



Hard Skills Vs Soft Skills:

SKILLS: Goal-directed, well-organised behaviours acquired through practice and performed with economy of effort

HARD SKILLS

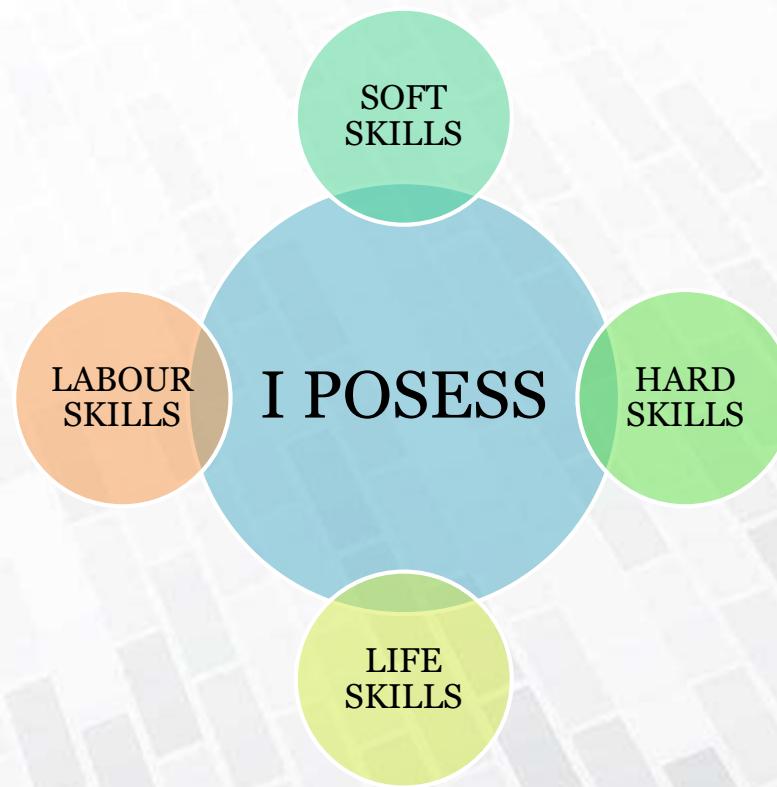
- Rule-based
- Technological/scientific
- Industrial/mechanical
- Tools/techniques
- Specialised
- Procedural/methodical
- Replicable
- Predictable
- Tangible

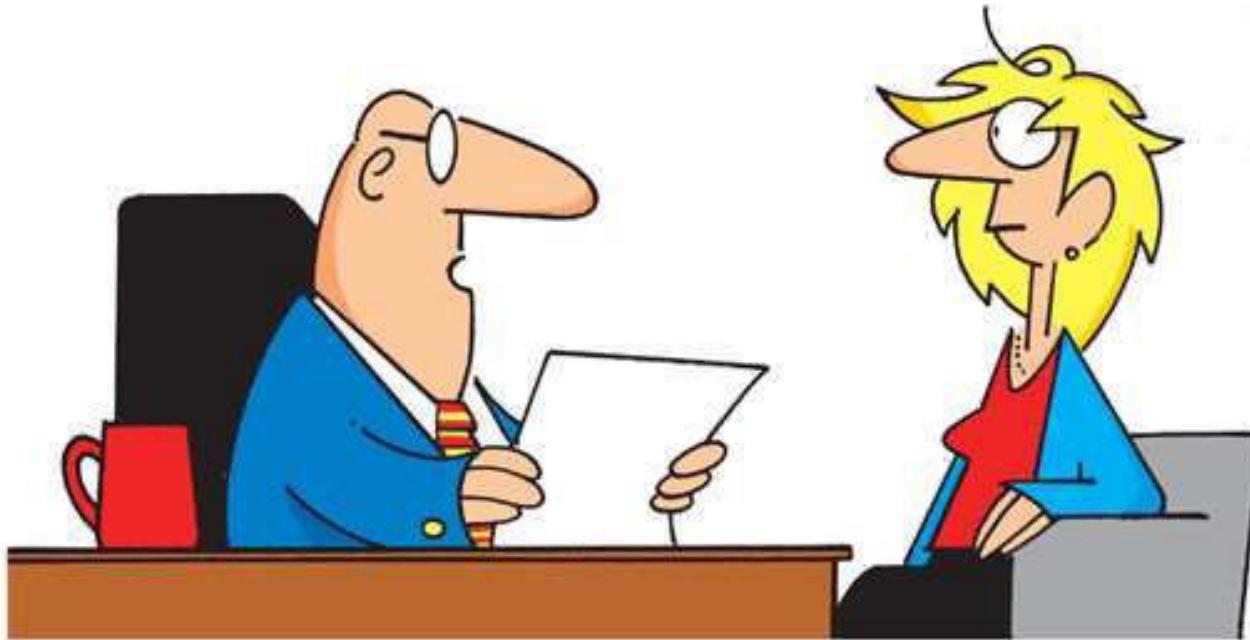
SOFT SKILLS

- Experience-based
- People-related
- Attitudinal
- Behavioural
- Non-domain-specific
- General
- Trans-situational
- Non-technical
- Intangible

Activity... When I Introspect myself?

- Students should identify their skills by analysing themselves and fill the bubble with theirs skills under each category through self-analysis.





“Any other people skills, besides 400 Facebook friends?”

Thank You...

SKILL ACQUISITION

SKILL ACQUISITION

- Skill Acquisition is the science that underpins movement learning and execution and is more commonly termed motor learning and control - (Williams & Ford, 2009)

The 4 Stages of Learning Anything



Various stages for learning

STAGE 1-UNCONSCIOUS INCOMPETENCE:

- It is the stage where an individual doesn't even know what he doesn't know. It means he is not aware of its mere existence.

STAGE 2- CONSCIOUS INCOMPETENCE:

- It's the next stage where an individual knew that he doesn't know something, which means he needs to learn something.

Various stages for learning

STAGE 3- CONSCIOUS COMPETENCE:

- He knows how to do and what to do and he does something consciously. It's like being in the beginner mode of a process.

STAGE 4- UNCONSCIOUS COMPETENCE:

- It's the final stage where one does something effortlessly. It's like being an expert in a process, still has some space to improve further.

ILLUSTRATION:

Let's take an example of playing a pc game which requires a combination of keyboard and mouse controls. We will see how the skill required to play this particular game is acquired through all the above phases.

- **STAGE 1:** An individual is unaware of the game's existence
- **STAGE 2:** An individual is aware of the game but he doesn't know to play. And he learns to play.
- **STAGE 3:** He plays consciously with much effort to gain control over both the keyboard and the mouse.
- **STAGE 4:** Finally he plays without much effort as he has gained control over the keys and mouse.

THE IMPORTANCE OF SKILL ACQUISITION

- **Self-employment** - A skilled man can be a self-employed man.
- **Diverse job opportunities** - Those who have many skills stand the chance of gaining job from many establishments.
- **Employment generation** - There will be a lot of jobs generated for the citizens of every country if the citizens are well equipped with skills.

THE IMPORTANCE OF SKILL ACQUISITION

- **Effective function** - Organizations that employ skilful workers to assist in their organizational duties lose nothing at all because there will be always effective functions performed by the employee.
- This is because the employee has acquired necessary skills needed for him to perform the work as desired by the organization.

THE IMPORTANCE OF SKILL ACQUISITION

- **Crime reduction-** skill acquisition reduces the crime rate in many nations. People begin to think on many illegal or unsocial activities they will do to make money when they do not have anything to call job of their own. But, an individual with acquired skill works and makes money from his skill set.

10,000 hours rule



- We are very well aware of this saying. But do we know the number of times we need to practise a skill in order to get it perfect?
- Well, a crazy attempt was made to figure out that mystery number and a psychologist named Malcolm Gladwell came up with an interesting theory called '10,000 hours' rule.

What does it say?

- Malcolm states that it takes about 10,000 hours of practice to become an expert at any skill, from violin to basketball to Halo.
- It was a powerful idea, based on several studies, and put some evidence behind the "practice makes perfect" argument for any skill.
- Here, the goal is not to work continuously, to finish the 10k hour quota, but to stick on to a systematic practice with an objective of improving performance every time.

What does it say?

- It isn't just about 10,000 hours of doing the activity; it's 10,000 hours of "deliberate practice."
- According to the paper, "deliberate practice is a **highly structured activity**, the explicit goal of which is to **improve performance**."
- **Deliberate practice** consists of specific training activities, drills, and exercises designed to stretch the individual's skills and thereby provide growth.

THE CONVERSE

- Though Malcolm supported his ideas with numerous experiments, it was never enough for some critics, one important critic being David Epstein.
- David Epstein in his new book '*The Sports Gene*', thoroughly disproved the theory.
- Practice is important, of course, but when it concerns physical activities, some people hold unfair advantage compared to others owing to their better physique, inherited in their genes.
- For example, Jamaicans dominate sprinting, Kenyans excel at long distance track, and tall people are much more likely to make it to the NBA, according to the book.
- Epstein also notes that the world's best in high jump, darts, and track don't need nearly 10,000 hours of practice. It's in the genes, he argues.

TO PRACTISE OR NOT TO PRACTISE?

- Gladwell came with the defence for his theory, where he states, "Epstein has written a wonderful book. But I wonder if, in his zeal to stake out a provocative claim on this one matter, he has built himself a straw man."
- Because, Gladwell only applied the 10,000-hour rule to cognitively demanding activities that needed significant thought, unlike those runners and dart-throwers.
- Gladwell has defended the 10,000-hour rule, arguing that **the rule applies to cognitively demanding tasks such as playing chess or the violin and not to domains that rely on mostly physical attributes**, such as the runners and long-jumpers cited by most critics.

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HARD SKILLS

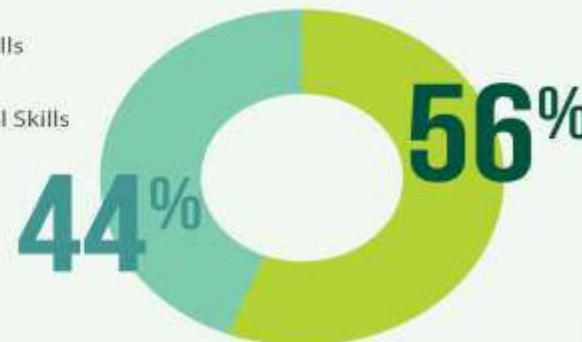
VS.

SOFT SKILLS

SO, WHAT SKILLS MATTER MOST TO RECRUITERS??

HR Pros value interpersonal skills over technical expertise.

- Technical Skills (Hard Skills)
- Interpersonal Skills (Soft Skills)

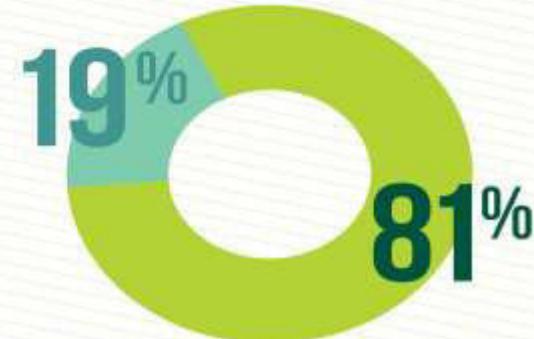


There are two types of skills required to do a job well – **hard skills** and **soft skills**.

HARD SKILLS:
Adobe Photoshop,
Microsoft Office, C++,
Certification ...

SOFT SKILLS:
Communication Skills,
Team Work, Problem Solving, Leadership Abilities...

And even fewer **Job Seekers** think their technical skills matter.



Thank You...

RECRUITMENT ESSENTIALS



Session - 1

Looking at an engineering career through the prism of an effective resume



résumé

UK /'rez.ju:.meɪ/ US /'rez.ə.meɪ/

- a short statement of the important details of something:
- (US / UK- CV) - a short written description of your education, qualifications, previous jobs, and sometimes also your personal interests, that you send to an employer when you are trying to get a job:
- A résumé is a document used and created by a person to present their background, skills, and accomplishments. Résumés can be used for a variety of reasons, but most often they are used to secure new employment.
- A typical résumé contains a "summary" of relevant job experience and education.

What to include in a résumé

Personal Details

- Include are your full name and contact information – this is usually both your phone number and email address.
- Take a pass on unnecessary information that includes your religion or nationality and **definitely not your bank account details.**

What to include in a résumé

Career Objective or Summary

- If you're a recent university leaver without much professional experience, begin your résumé or CV with a career objective in a short sentence or two.
- If you've gained experience in the workforce, a career objective is less necessary, however you may want to replace it with a career summary, describing your professional profile in a short sentence or two.

What to include in a résumé

Education

- List your most recent educational experiences first. Include your qualifications, institutions you studied at, graduation dates and other specializations. Mention any special awards and other educational achievements.

Work Experience

- If you are a fresher, Internships and Volunteer work can be mentioned here.

What to include in a résumé

Additional Information

- You may like to create headings such as ‘Skills’, ‘Strengths’ or ‘Interests’ and list information that would be relevant to the job you’re applying for.
- Information that illustrates your proficiency in languages and computer programs should be included here.

What to include in a résumé

- Keep your résumé short and concise to make a good impression in a quick glance.
- Make it clear and straightforward. Use simple text in one modern, standard font that is easy to read, and that everyone can understand.
- The design and layout of your résumé or CV should be neat and easy to read. Use only one or two easy to read fonts and include headers, bullet points and paragraphs.
- Avoid using cluttered or complicated layouts with headers, footers, tables or other items that may not look right especially, when viewed on different computers with varying software versions.



What To Exclude From Your Résumé?

- Personal details such your religion, age or marital status
- Every skill you have, especially when it **isn't relevant** to the job you're currently applying for
- Salary expectations

Résumé writing

- Résumé writing is a skill that often needs more mastering.
- Whether you've got years of experience under your belt or are newly joining the workforce, your chance of landing an interview is much more likely if you have a killer résumé to show off your education, skills and experience.

Résumés: The Good And The Bad

- Brief is best.
- Make sure you include specific skills that are relevant to the job you're applying for.
- **Avoid** writing in first or third person.
- Keep to the employer's submission requirements.
- The smallest of details can make the biggest of differences.
- Hone in on those little details that separate a good résumé from a bad one.

Résumés: The Good And The Bad

SARAH JOHNS
Mobile: 0401 111 111 | Email: sarahjohns@hotmail.com.au
Melbourne, VICTORIA, 3004

KEY SKILLS

- Financial administration
- Advanced in Microsoft Excel
- Exceptional attention to detail

PERSONAL SUMMARY

A talented financial administration professional with over two years industry experience, an exceptional eye for detail and a drive to further my career within the financial sector.

PROFESSIONAL EXPERIENCE

Administration Assistant
Westing Finance | August 2017 – Current

As the Administration Assistant I am responsible for providing support to managers and employees, assisting in daily office needs including management of Westing Finance's general administrative activities.

Key responsibilities

- Assisted the financial operations team with administrative tasks including; coordinating the full function of accounts payable and accounts receivable with high volume processing daily.
- Coordinating supplier queries, month-end duties, and credit card expenses were also among other day-to-day duties

EDUCATION & TRAINING

2013 – Current

Certificate IV in Business Administration | Finance & Payroll Course, Australian Institute of Finance

2004 – 2005

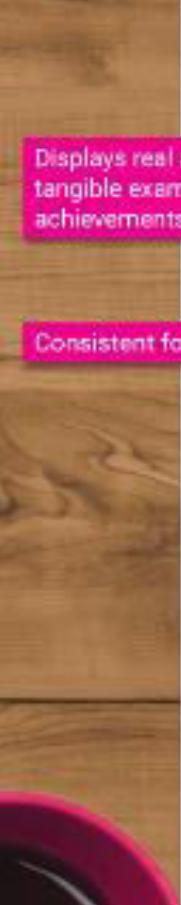
National Certificate in Business Administration & Commuting, CPIT

1998 – 2003

Burley South Secondary School

Professional email address
Only include location information that will help make your resume searchable for potential employers
Succinct and active personal summary explaining career objective and value to the organisation
Clearly highlights core skills
Lists most recent and relevant position first to showcase skills and knowledge
Correctly displays relevant training, institution and completion year
Provides a good summary of core responsibilities showcasing experience

Résumés: The Good And The Bad



Displays real and tangible examples of achievements in the role

Consistent formatting

Achievements

- Implemented a number of general administrative systems and Excel spreadsheets, removing repetitive manual tasks from the financial advisors. I also learned how to manage accounting documents and assist with portfolios.

Receptionist
Lindwood Pharmaceuticals |
January 2016 – July 2017

Key responsibilities

- Main responsibilities included welcoming and assisting visitors, as well as answering the switchboard. I also coordinated mail and courier deliveries.

Achievements

- Simplified filing and portfolio management systems, which contributed to increased organisational productivity.

REFERENCES

Available upon request

Résumés: The Good And The Bad

- Think of a résumé as “self-advertisement” that sums up on one page.
- Your résumé is one of the most important pieces of your job application.
- It gives the hiring manager an overview of the qualifications you have for the job for which you’re applying.

Résumé and a cover letter

- You should also familiarize yourself with the difference between a résumé and a cover letter:
- A résumé is typically sent with a cover letter, which is a document that provides additional information on your skills and experience in letter form.
- A résumé is a concise, often bulleted summary, while a cover letter highlights and expands on certain traits or accomplishments that would be unique or ideal assets for the particular job.

An effective résumé

- An effective résumé lays out a summary of qualifications that will push the hiring manager or employer to move forward and invite you to interview for the position.
- As well as details on skills, education, and work history, résumés can also have optional sections, such as an objective, summary statement, skills, or career highlights.
- Those sections can be added after you've compiled all the factual information you need to list on your résumé.

An effective résumé

- You can also include awards or accolades, volunteer or community experience, post-grad coursework, and skills, as well as your college education, which can move to the bottom of your résumé once you get your first job after college.

Focus on Your Achievements

- When writing the descriptions for the jobs / internships you've held/ done, focus on what you accomplished in each position rather than what you did.
- Listing quantifiable achievements in a numerical manner (eg.: reduced expenses by 10%) will help your résumé stand out.
- Be sure to match those accomplishments to the criteria the employer is seeking in the job posting.

What You Exclude From Your Resume'

- What You Exclude Is Just As Important As What You Include.
- There are some things that don't belong on a résumé for a job.
- For example, if you took a job and only stayed there for a month or so, you wouldn't want to include that position.
- If you've been out of college for more than two years, it's generally best to remove any internships you've had.
- However, this is a case where you'll want to use your common sense. If you went to college for marketing and had a marketing internship in your senior year, then worked as a steward for the next several years, you would ideally need to include only your marketing internship.
- Ultimately, you want to try to strike a balance between including experience that is both timely and relevant.

Choose a Résumé Style

There are several basic types of résumés used to apply for job openings.

- Your options include:
 - Chronological - The most common résumé type, in which you list your work experience in reverse chronological order, from the most to the least recent.
 - Functional – A functional résumé focuses on your skills and abilities rather than your work history.
 - Combination – This type of résumé lists your skills and experience before your employment history.

Which Résumé Type is Right for You?

- Which résumé type should you use for your job search? That depends on what you're trying to accomplish.
- The goal of any résumé is to show a hiring manager the applicant's strengths, skills, and experience in as short a time as possible.



According to one study, recruiters spend as little as **six seconds** reviewing a résumé before moving on to the next, so it's in your best interests to put your finest qualities and accomplishments in a prominent position on the page.

Start Compiling Your Résumé

- Once you've decided on a résumé type, it's time to start writing your résumé.
- Aim to tailor your résumé to the job you are applying to.
- It's a bad idea to send the same exact résumé to multiple openings.
- Your goal should be to write your résumé with both robots and humans in mind.
- Many organizations use **Applicant Tracking Systems [ATS]** to sort and vet résumés.
- Good keywords will refer to the job description in the posting as well.

Format Your Résumé

- Format your résumé according to typical professional standards.
 - Consistent spacing throughout, and evenly sized margins on all
- Always safe to stick with traditional formatting:
 - white page, black text, readable font
- Even if you are only sending in copies digitally, it is a good idea to print your résumé to ensure it looks professional.
- Ask someone else to give it a final review to be sure it's perfect before you click send or upload to apply for a job.

Résumé

Think of Your Résumé as a Living Document

- In the short term, you should tweak your résumé based on each job you apply to. For example, if one position you're applying to seems to weigh a certain responsibility or focus over another, you should be sure your résumé conveys your expertise in this area.
- At the same time, you should be updating your résumé adding any new skills you've learned, courses you've taken or awards you won.

8 Design Ideas for Making Your Résumé Pop

- If you want to make a great impression, you have to understand your audience. So take a moment to put yourself in the shoes of the hiring manager you wish to impress.
- Now picture the desk in front of you and the towering piles of résumés.
- How can one eager and able applicant stand out in a sea of competitors? It's a difficult task, but not impossible.

8 Design Ideas for Making Your Résumé Pop

- While you may think that visual appeal is not so critical when it comes to résumé writing, think again.
- You should seize every opportunity to catch the eye of the hiring manager.
- A sharp-looking document shows that you have the ability to present yourself with professionalism and style.
- If you are applying for design-related jobs or positions requiring a flair for visuals, an attractive résumé is even more important.

8 Design Ideas for Making Your Résumé Pop

1. Get it Together

- Make sure your content is compelling first, then worry about the design and layout. While appearance can set you apart, it's hard to impress if you don't have substance to go with that style.

2. Use a Template

- If you're not a designer and don't have time to spend on formatting, the fastest way to a good-looking résumé may be through a template. You'll find some decent free résumé templates online.

8 Design Ideas for Making Your Résumé Pop

3. Find a Great Font

- Your selection of font options is endless, free, and available all over the web. If you can't pick a favorite, your best bet is to stick with simple, thin fonts.
- Steer clear of fussy or dated-looking fonts at all costs (Comic Sans is not your friend).
- When it comes to font size, don't go any smaller than 9 points. You want to remain easily readable and maintain a professional look.

8 Design Ideas for Making Your Résumé Pop

4. Contact Information

- Your name and contact information should appear prominently at the top of the page without exception.
- Think of it as the personal letterhead at the top of your professional résumé.
- It should stand out, make a good first impression and gain credibility that will bring the reader in to the rest of your document.

8 Design Ideas for Making Your Résumé Pop

5. Make It Skimmable

- Remember that many hiring managers will first view your résumé on the computer screen.
- People read differently on a computer screen than on paper.
- They skim and scan instead of reading every word.
- That means it's critical to make sure that your résumé is skimmable — and it should only take you a few minutes to make a dramatic difference in readability.

8 Design Ideas for Making Your Résumé Pop

5. Make It Skimmable

- First, clearly mark your headings. Each heading should be bolded or italicized.
- Let the headings have some white space around them as well. A cramped résumé is no one's friend — and not very easy on the eyes.
- Keep your paragraphs and bullet point entries short.
- In general, avoid setting type in all caps. Because the letters start to look the same, words in all caps are harder to read.

8 Design Ideas for Making Your Résumé Pop

6. Color is OK!

- If wielded with a light hand, color can add a needed point of interest that will draw in the reader.
- A black and white résumé is the norm, but adding one touch of color is not unheard of.
- Consider adding a different color monogram, or making your headings a dark blue instead of black.
- Keep in mind that if you're sending the résumé electronically, the recipient may not print it out in color despite your intentions.

8 Design Ideas for Making Your Résumé Pop

7. Don't Get Too Artsy

- Even in the case of an artist or designer's résumé, one should resist the urge to use any sort of clip art, confusing lines or borders, or fancy bullets.
- Your aim is certainly to stand out, but fancy extras only serve to confuse.
- A thin, simple border line under your name and address can serve as a nice divider, and further serve your aim to make the résumé an easy read.
- There is a fine line between appropriately stylistic and cluttered/overdone.

8 Design Ideas for Making Your Résumé Pop

8. Link Them to Work Samples

- If you really want to WOW! them with your creativity, check out and set up an online portfolio.
- Use a link or QR code on your résumé to direct hiring managers to the online portfolio.
- It may be smart to link to an online portfolio to show potential employers your best work. Just make sure that the link is active.
- And don't forget that it needs to be well maintained and true to your résumé!
- Let your talent speak for you by showing off your best work. Do that by adding social media handles. Include your LinkedIn address, Twitter handle, or Instagram.



End of Session - 1

Thank You...

Recruitment Essentials

Session - 2



REVIEW
RESUME

focus activities person career skills work
guide application tips job seeker life list dates goals
education cover letter writing

RESUME

summary job search
vision interview company experience
history position offer network employers
advancement references sell salary
objective past

The purpose of a résumé

Your résumé is a marketing tool. It needs to demonstrate:

- That you are employable
- How you meet the job and the organisation's requirements
- That you have the right qualifications and education
- That you have the right experience and skills
- That you have the right level of professionalism for the job

How should I order my résumé?

Generally it's always good to present the information on your résumé in this order:

- Contact details
- Opening statement
- List of key skills
- List of technical/software skills
- Personal attributes/career overview
- Educational qualifications
- Employment history/volunteering/work placements

Do I need to change my résumé for each application?

- You need to tailor your résumé to every job application so that it responds to the specific requirements of the job you're applying for.
- You might not need to change much, but you do need to make sure your opening statement, your key skills and your personal attributes all respond to the needs of the role, based on the job ad (if there was one) and the research you've done into the job.

What NOT to put on your résumé

- Private Information
- Typos or Factual Errors
- Images And Graphics
- Information In Tables
- PDF Versions of Your Résumé

Targeted Résumés

- Résumé Reaches The Recruiter's Table Much Before Than You Do.
- Hence it is very much important to have a structured and concise résumé to make the first impression work for you.
 - Résumé to tell about you
 - To convince that you are the one
 - To sell your skills
 - To grab an interview
 - To brand yourself
 - Quick but lasting first impression
 - Summarize your career aspiration

The Ultimate 2019 Engineering Résumé



Different Skills And Industry Specialties

- Engineers have a very broad reach comprised of many different skills and industry specialties.
- You also need a variety of soft skills – such as great communication and emotional intelligence – along with the technical ones that all engineers will take as a given.
- Whether you're writing a Mechanical, Electrical, Civil, Software, or Chemical Engineering résumé, or a graduate engineer's résumé – here's what you need to know:

1. How to present your contact information
2. How to write a strong résumé summary
3. A quick word on engineering qualifications and licenses
4. The hard and soft skills of leading engineers
5. Highlighting your achievements as an engineer

How to Write a Strong Engineering Résumé Summary

Good Engineering Summary Résumé Example

Experienced, organized Mechanical Engineer with a background in project management as well as training in Electrical Engineering. In possession of strong communication and leadership skills.

Wrong

I am a Mechanical Engineer seeking employment with Millson, Inc. because I want to reach my dreams of becoming a Mechanical Engineer. I have a Master's in Engineering and I was a TA in some engineering classes, so I know what I'm doing!



The Hard and Soft Skills of Leading Engineers

- Engineering is a highly technical job – but soft skills are equally important.
- No matter what kind of engineer you are, you're going to communicate with and lead others.
- Your skills will relate to your own unique experience, but here are some hard and soft skills you could use to craft a memorable résumé.

Hard Skills for an Engineering Résumé

- Math
- Mechanical skills
- Project management
- Troubleshooting
- Prototyping
- Workflow development
- Computer skills
- Knowledge of manufacturing processes
- Knowledge of applicable laws

Soft Skills for an Engineering Résumé

- Creativity
- Problem-solving skills
- Teamwork and collaboration
- Listening skills
- Communication skills
- Ability to work under pressure
- Leadership
- Analytical thinking
- Prioritization
- Time management

Highlighting Your Achievements as an Engineer

- Skills are a necessity – but they're often shared with many other candidates. Now it's time to make yourself stand out.

Good Examples of Achievements for an Engineering Résumé

- Spearheaded the Smartphone Manufacturing initiative
- Performed Computer-Aided Design for a set of various types of cooking knives
- Completed a six-month internship with xxxxxxxxx
- Implemented Kanban system for project management

Wrong

- Made blueprints
- Designed a smartphone
- Led people
- Saved money



5 Skills Hiring Managers Look for in Engineering

- Technical Skills
- Communication Skills
- Interpersonal Skills
- Problem Solving and Critical Thinking
- Enthusiasm, Commitment and Motivation

Technical skills take first rank, as expected for jobs in the engineering field. The others, however, fall into the category of “soft skills.”

Soft Skills in the Modern Engineering Workplace

- The modern engineering workplace requires more than strictly technical skills.
- Recruiters are often looking for a combination of technical ability and transferable or soft skills.
- “Technical skills alone are not enough to ensure a successful engineering career, as engineers need to be able to function as a member of a team, think critically, and have a strong work ethic.
- It’s these soft skills that will differentiate candidates from one another.

Engineering Résumé Examples: Two Summaries

Look at these very different mechanical engineer résumé examples:

- **WRONG :**
- Mechanical engineer with nine years experience. Skilled in leadership, collaboration, lean manufacturing, and manufacturing design layout.
- **RIGHT :**
- Energetic mechanical engineer with 9+ years experience. Seeking to use proven process and production design skills to advance manufacturing excellence at Boeing. Increased production efficiency by 35% at Raytheon IADC. Decreased costs 28% and cut lead times by 22%. Lowered defects 32% department-wide. Spearheaded the team that received the Shingo Prize in 2017.

Engineering Résumé Examples: Two Summaries

- **WRONG :**
- Engineer with BS in mechanical engineering. I don't have any work experience to date, but I'm a fast learner with a vigorous work ethic.
- **RIGHT :**
- Enthusiastic engineering college graduate with freelance experience. Completed several client projects, including small consulting and research jobs. Made a watch movement in CAD with precise dimensions. Delivered each job under budget and before deadline.

Two Entry Level Engineer Résumé Samples

WRONG

- ***Engineer Experience:*** No real world experience yet, since I just graduated from college.
- ***Other Experience***
- *Clerk, Ridley's Supermarkets*
- *Bartender, Silver Street Tavern*

RIGHT

- ***Freelance Engineer 2008 - 2009***
- *Sourced a small motor gear box for a customer's specifications.*
- *Assisted with design of a small custom microscope optics system.*
- *Performed CAD design for a new sports visor idea.*
- *Consistently delivered to customer needs at 20%+ under budget.*



Two Engineering Résumé Examples

WRONG

- ***BS in Mechanical Engineering 2004 - 2008***
- ***GPA 3.6***
- ***Passed all curriculum classes in engineering.***
- But let's add some relevant details.
-

RIGHT

- ***BS in Mechanical Engineering 2004 - 2008***
- ***Followed my passion in Product Design and Manufacturing coursework.***
- ***Excelled in Mechanics and Materials.***
- ***Pursued an independent study program in Machine Layout Design.***
- ***Wrote a weekly column on Lean Manufacturing concepts in The Tech.***
-

Add things like:

- Conferences you've been to
- Articles you've written
- Courses
- Software
- Additional activities
- Volunteer work
- Professional associations

How To Do A Résumé Cover Letter:

- Make your engineering cover letter personal. Start with the recruiter's name.
- Make it relevant. If you like that the company is more concerned with "right" than "fast," then say so in your cover letter. Then link your cover letter to their needs. Mention a couple of high points from your résumé that match the job.
- End your cover letter with a call to action. Say something simple like, "I'd welcome the chance to talk with you."
- Cover letters matter. All the more relevant to entry level engineers.

Five skills to give your engineering CV the edge

Ask the following questions of your engineering CV to really make recruiters take notice.

1. Does my CV show I am a team player?
2. As an engineer do I really need to show creativity?
3. Is my eye for detail strong enough?
4. Do I show enough of my personality?
5. Can I demonstrate any data and analytical skills?

Writing An Engineer Résumé Summary Statement

Summary Statement

- A résumé summary statement is similar to an objective statement in that it is a quick way for a job seeker to catch a hiring manager's attention by summarizing critical information at the top of your résumé in an easy to read format.

How To Write A Great Résumé Summary Statement

- The goal is to get your statement down to four to six bullets (give or take a couple) distilled down into two or three laser focused sentences.
- The first thing you want to do is go back and look at the job you're applying for and determine your target audience.
 - Who are they looking for?
 - What are your top selling points?
 - What critical problems did you identify in the job posting and how are you positioned to solve them?
 - How does your summary align with the company job requirements?
 - What are your career highlights and key strengths?
 - Do you have additional certifications or achievements that set you apart?

How to Format an Engineer Résumé Summary Statement

- When writing a résumé summary statement, begin with the information that is most relevant to the specific job you want.
- Keep your summary statement between four and six lines, and, if you need more room to highlight your skills and accomplishments, create an areas of expertise table underneath it.
- The most important aspect of writing a résumé summary statement for an engineering job is focus.
- Be as specific as possible when discussing experiences that match the job duties of the engineering job you are applying for

Examples of an Engineer Résumé Summary Statement

1. Proven software engineer with extensive firewall and virus research experience. Excellent technical and communication skills. Dedicated to improving network security and eliminating vulnerabilities. Organized and analytical. Detail- and goal-oriented professional.
2. Team-focused environmental engineer with a passion for developing new methods of reducing carbon manufacturing. More than 10 years of experience in the industrial manufacturing industry, including project development and planning. Knowledge of environmental law.
3. Dedicated mechanical engineering student with internship experience in mechanical engineering. Strong interpersonal and communication skills. Leadership and integrity. Automotive focus.

End of Session - 2

Thank You...



PROBLEM SOLVING SKILLS

THE GAME OF THE SIX GLASSES

The challenge: rearrange glasses so that the glasses alternate full and empty. However, you must do this by moving only one glass.



Solution

Hold the second full glass and pour its contents into the fifth cup (counting from left to right) and replace the glass in place 2.



What is the problem?

- A doubtful or difficult matter requiring a solution.

- Something hard to understand or accomplish or deal with.

Problem types:

Different Types of Problems Different Types of Solution

Kind of Problem	Nature of Appropriate Problem-Solving Activity
Problem with unknown cause	Finding the cause
Problem with known or irrelevant cause	Generating ideas that could fix the problem
Decision between solutions with certain outcomes	Deciding on one best solution
Decision between solutions with uncertain outcomes	Deciding which solution has the highest probability of success
A jumbled list	Determining the priority order

All problems have two features in common: goals and barriers.

- Goals

Problems involve setting out to achieve some objective or desired state of affairs and can include avoiding a situation or event .

- Barriers

If there were no barriers in the way of achieving a goal, then there would be no problem. Problem solving involves overcoming the barriers or obstacles that prevent the immediate achievement of goals.

What is the problem solving process?

The process of identifying the problem, prioritizing, selecting alternatives for a solution and evaluating the outcomes.

Problem solving tools:

- Brain storming:
- Brainstorming is used to generate a large number of creative ideas when problem solving and achieving objectives. It can even be used for decision making.
- **Brainstorming Techniques Guidelines**
- 1. Brainstorming take from a few minutes to a few hours. For big problems or projects it may be done several times and over days, weeks or months.

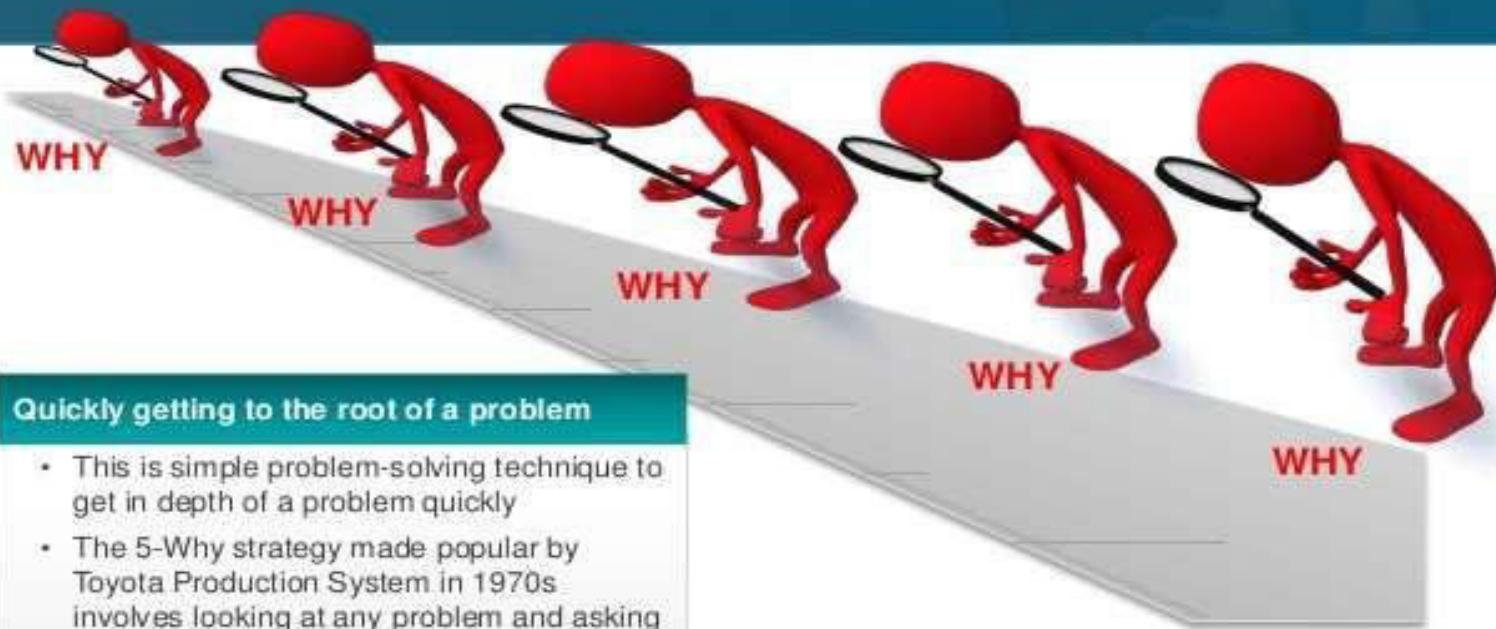
Cont. Brain storming :

- 2. Come up with as many ideas as possible. The more the better.
- 3. Don't judge any of your ideas at this time.
- Later you can use the [affinity diagram](#). to sort out your ideas.
- It is important to note that one of the fastest way to “kill” brainstorming is to judge ideas too quickly. Criticism puts a choke hold on allowing the [Right Brain Left Brain Crossover](#) to flourish

5 WHYS

- The 5 Why's is a simple problem-solving technique that helps you to get to the root of a problem quickly.
- The technique was originally developed by Sakichi Toyoda.
- The 5 Why's was used within the Toyota Motor Corporation during the evolution of its manufacturing methodologies and became very popular in the 1970s by the Toyota Production System.

5 Whys



Quickly getting to the root of a problem

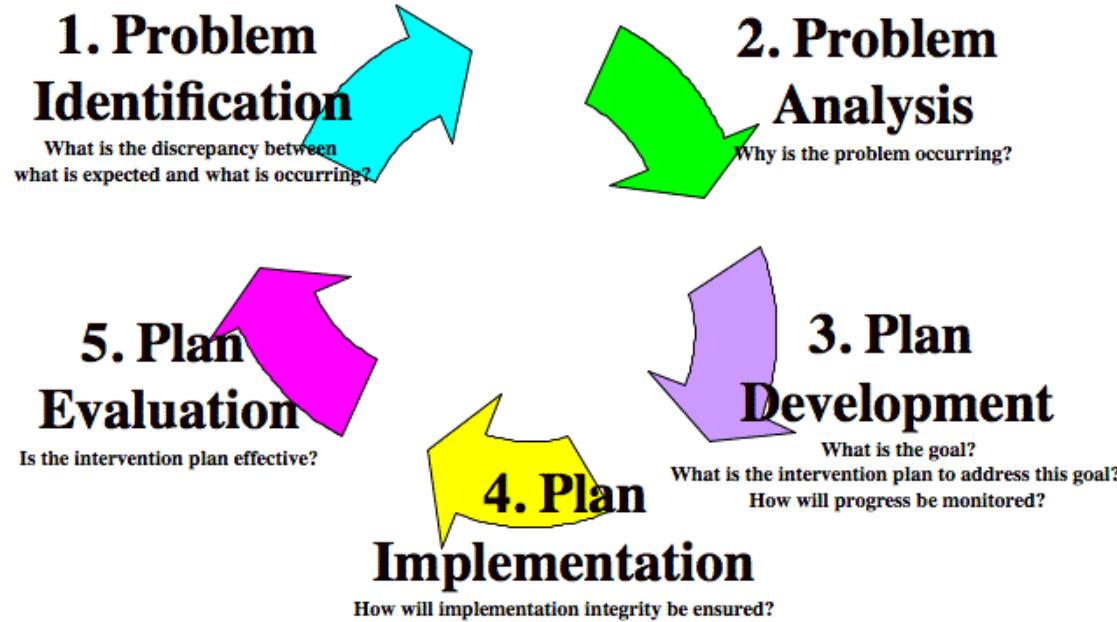
- This is simple problem-solving technique to get in depth of a problem quickly
- The 5-Why strategy made popular by Toyota Production System in 1970s involves looking at any problem and asking Why? and What caused the problem?
- Advantages of 5 Whys include:
 - It quickly identifies the root cause of the problem
 - It is simple and easy to learn and apply

IS – IS NOT

- “IS – IS NOT” is a problem solving tool that explains the rational process for finding the possible root cause of the problem.
- helps user to avoid jumping to a false cause.
- At the end of the IS – IS NOT exercise user gets a confirmed true cause .

Steps of problem solving

Problem-Solving



1- Define the problem

- Clearly state the problem

- Detect the circumstances lead to the incidence of the problem.

- Write a clear definition of the problem and the barriers encountered.

2- Find possible solution (s)

- Analyze the problem so you fully understand it and then develop ideas which will achieve your goal

- Develop several ideas to solve the problem to increase your chances of finding the best solution to the problem.

3- Choose the best solution

- This stage is a process of decision-making based on your comparing the possible outcomes of your alternative solutions; this is the “DO” stage

3- Choose the best solution

- This stage involves:
 - Identifying all parts of the solution
 - Eliminating solutions that do not meet certain criteria
 - Evaluating the solutions against the desired outcomes
 - Assessing the risks associated with the “best” solution
 - Making a decision to implement the solution

4- Implement the best solution(s)

- This stage involves accepting and carrying out the chosen course of action.
- Implementation means acting on the chosen solution.



WisdomTimes.com

5- Evaluate the best solution(s)

- Successful problem-solving involves looking at the outcome of the solution and making the necessary changes in the earlier stages if necessary in order to reach the identified goal.

Why people fail to solve problems effectively

- The following is a list of some of the reasons why people fail to find effective solutions include
 1. not being methodical
 2. lack of commitment to solving the problem
 3. misinterpreting the problem
 4. Lack of knowledge of the techniques and processes involved in problem solving

Why people fail to solve problems effectively

- 5- inability to use the techniques effectively
- 6 using a method inappropriate to the particular problem
- 7 insufficient or inaccurate information
- 8 inability to combine analytical and creative thinking
- 9 failure to ensure effective implementation

The magic factor for any problem solving is your ATTITUDE

The magic of your attitude

Vadim Kotelnikov 

* Problems

If you treat problems
as problems,
they are
problems



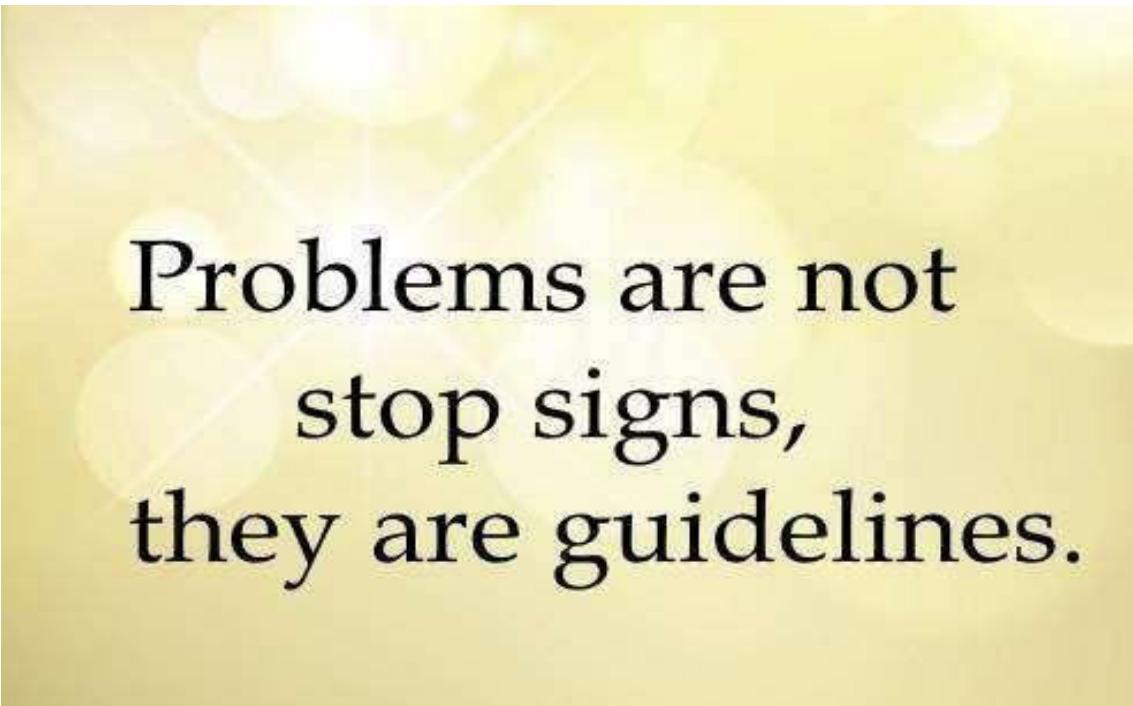
If you treat problems
as opportunities,
they are
opportunities



Don't talk about a problem, talk about an opportunity!

Take home message:

- Problems can also be opportunities they allow you to see things differently and to do things in a different way perhaps to make a fresh start.



Problems are not
stop signs,
they are guidelines.



Thank
you!

LOGICAL THINKING

HSEM1BTECHSTANDARD0719

THINKING SKILL

SESSION – 1

PROBLEM SOLVING

1. A man works on the 10-th floor and takes the elevator down to ground level at the end of the day. Yet every morning, he only takes the elevator to the 7th floor, even in a hurry. But he goes all the way to the 10-th floor when others are in the elevator with him or it is a rainy day.
Why?
2. There are two gates, one to hell and the other to heaven. Two gatekeepers, one for each gate. One of them always speaks the truth and the other always lies but you don't know which one guards which gate. You are allowed only one question and you need to find out the gate to heaven.
What is the question?
3. There is a basket full of hats. 3 of them are white and 2 of them are black. There are 3 men Tom, Tim, and Jim. They each take a hat out of the basket and put it on their heads without seeing the hat they selected or the hats the other men selected. The men arrange themselves so Tom can see Tim and Jim's hats, Tim can see Jim's hat, and Jim can't see anyone's hat.
Tom is asked what colour his hat is and he says he doesn't know.
Tim is asked the same question, and he also doesn't know.
Finally, Jim is asked the question, and he does know.
What colour is his hat?
4. A shell is tied to the side of a boat.
The shell hangs 3 metres above water level.
The water rises 2 cm in an hour.
How much time will it take before the water touches the shell?
5. This problem is also called Jelly Beans problem. This is the most commonly asked interview puzzle.
You have 3 jars that are all mislabelled. One jar contains Apple, another contains Oranges and the third jar contains a mixture of both Apple and Oranges.
You are allowed to pick as many fruits as you want from each jar to fix the labels on the jars. What is the minimum number of fruits that you have to pick and from which jars to correctly label them?



6. Bridge Riddle

A crew consists of four scientists are out of their laboratory, they are chased by demons and they will be dead if they are caught. The crew consists of a senior professor in his 80's, his son who is also a scientist whose age is 45 and they have two assistants, a young girl and young boy. The demons were chasing them and after so many hurdles they reached a bridge in the dark night of 25 feet and the weather was so bad, there were no stars and they have only one lantern to cross the bridge, without lantern they cannot get into the bridge and the bridge can carry only two persons at a time. Senior professor will take 10 minutes to cross the bridge, his son will take 5 minutes to cross the bridge, young girl will take 2 minutes and the young boy will take 1 minute to cross the bridge. The entire crew has to cross the bridge in 17 minutes and remember that the bridge can carry only 2 persons at a time and they can go only with the help of lantern. The demons will reach them in 18 minutes. Help them to escape from demons.

7. James Bond

James Bond was threatened to death by terrorists so he secured himself in a hotel in the outskirts of the city, he was allocated to Room no 308 in 25th floor of the building. One fine morning, while he was in his morning robes, sipping his lemon flavoured tea, he heard a knocking sound. He was so cautious and he reached the door and opened it. He found a young lady standing in front of him, she was elegant in her black skirt, golden colour heel, snake skinned clutch and in her glossy red lipstick. Before he questions something to her, she herself said him "Ouch sorry!! I thought it is my room and disturbed you" and she stepped back and started on her way. Suddenly the girl stopped and felt the mouth of the gun on her back neck and she was trapped by none other than Mr. Bond. Why should James Bond do this? Analyse and solve the problem.

8. Find the passcode

Assume that you are trapped in a house and the door has a passcode made in the combination of 4 whole numbers from 1 to 100, two numbers are revealed and your duty is to find other two numbers to get out of the house, if not you will die in another 2 hours. The riddle to find the number is given in the wall next to you. This riddle portrays a conversation between two persons, Jerome and Faizal.

Jerome: I know the product of two numbers but I cannot tell what the numbers are.

Faizal: I know you couldn't say that, the same way I know the sum of the two numbers but I cannot reveal the numbers.

Find your passcode and get freed.

LOGICAL THINKING

9. A wants to send a secret message to his friend B in the mail.

But C (A's Friend), who A don't trust, has access to all A's mail. So A put his message in a box with a lock. But A is not allowed to send a key!

How can A send his message through securely?

10. Murder Mystery: The Case of Fake FIR

One murder happens in a village and a police inspector asked two constables to reach the spot and take the FIR...

As it was night and too far from the police station, the two constables didn't go there and made up a fake FIR... After reading the report, the inspector said that you both are suspended for making a fake report without reaching the spot...Question is...How did the inspector find that it's a fake FIR and they didn't reach there?

The FIR is written as...

When we reached the spot, the door was open and one man aged about 40 -45 was found dead in a chair, one bulb is on in the room, fan is also switched on, one table is there in front of the dead body and on the table there were these things: one open bottle of poison, one half filled drinking water bottle, one pen, one newspaper was opened as pages 9-10, one table top calendar opened as date of June 20, one 5 rupee coin, one notebook. There was one bed also in the room... Seems like the person committed suicide.

11. Three men in a cafe order a meal the total cost of which is \$15. They each contribute \$5. The waiter takes the money to the chef who recognizes the three as friends and asks the waiter to return \$5 to the men. The waiter is not only poor at mathematics but dishonest and instead of going to the trouble of splitting the \$5 between the three he simply gives them \$1 each and pockets the remaining \$2 for himself. Now, each of the men effectively paid \$4, the total paid is therefore \$12. Add the \$2 in the waiters pocket and this comes to \$14....Where has the other \$1 gone from the original \$15?

12. You are mixing cement and the recipe calls for five gallons of water. You have a garden hose giving you all the water you need. The problem is that you only have a four gallon bucket and a seven gallon bucket and neither has graduation marks. Find a method to measure five gallons.

13. A pot contains 75 white beans and 150 black ones. Next to the pot is a large pile of black beans.

A somewhat demented cook removes the beans from the pot, one at a time, according to the following strange rule: He removes two beans from the pot at random. If at least one of the beans is black, he places it on the bean-pile and

drops the other bean, no matter what color, back in the pot. If both beans are white, on the other hand, he discards both of them and removes one black bean from the pile and drops it in the pot.

At each turn of this procedure, the pot has one less bean in it. Eventually, just one bean is left in the pot. What color is it?

14. A man is the owner of a winery who recently passed away. In his will, he left 21 barrels (seven of which are filled with wine, seven of which are half full, and seven of which are empty) to his three sons. However, the wine and barrels must be split, so that each son has the same number of full barrels, the same number of half-full barrels, and the same number of empty barrels. Note that there are no measuring devices handy. How can the barrels and wine be evenly divided?

15. 5 Sailors and a monkey landed on an island in the southern Pacific Ocean. They found a big pile of coconuts. Since they were so tired and it was getting late, they all went to sleep instead.

Allan woke up at night. He wanted to get his share right then. He divided the coconuts evenly into 5 piles, but one coconut was left. He hid one pile for himself and gave the extra one to the monkey. He went back to sleep.

Brad woke up (without knowing what had happened) after Allan. He divided the rest of the coconuts evenly into 5 piles, but one coconut was left. He hid one pile for himself and gave the extra one to the monkey. He went back to sleep.

Charlie woke up (without knowing what had happened) after Brad. He divided the rest of the coconuts evenly into 5 piles, but one coconut was left. He hid one pile for himself and gave the extra one to the monkey. He went back to sleep.

David woke up (without knowing what had happened) after Charlie. He divided the rest of the coconuts evenly into 5 piles, but one coconut was left. He hid one pile for himself and gave the extra one to the monkey. He went back to sleep.

Earl woke up (without knowing what had happened) after David. He divided the rest of the coconuts evenly into 5 piles, but one coconut was left. He hid one pile for himself and gave the extra one to the monkey. He went back to sleep.

When they all awoke in the morning, they determined to divide the rest of the coconuts evenly into 5 piles. Each sailor got a pile. Again one coconut was left, and they gave it to the monkey. Everybody got one pile. How many coconuts did each person get?

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LOGICAL THINKING – SOLUTION

HSEM1BTECHSTANDARD0719

THINKING SKILL

SESSION - 1

PROBLEM SOLVING

1. He is too short to reach the “10” button. But when others are with him, he can ask them to push the button for 10th floor. On a rainy day, he has an umbrella, so he can press the button using that.

2. Ask any guard “What would the other guard say if I ask which way is to the hell?”

And whatever answer he give is the way to the heaven.

If you end up asking the question to the truthful one, he will speak the truth and he knows that other guard is going to lie so he will show the way to the heaven.

If you end up asking the question to the liar, he will lie about the other and the answer will be the way to the heaven.

3. The hat is white. If Tom doesn't know his hat color then the other two men's hats cannot be both black otherwise he would know his is white. When Tim doesn't know his hat color either, that means Jim's hat could not be black otherwise Tim would have to know his hat was white to fulfill the information discovered through Tom's answer.

4. Never, with rising water level boat also rises.

Never – the boat floats on water.

5. Let's take a scenario. Suppose you pick from jar labelled as Apple and Oranges and you got Apple from it. That means that jar should be Apple as it is incorrectly labelled. So it has to be Apple jar.

Now the jar labelled Oranges has to be Mixed as it cannot be the Oranges jar as they are wrongly labelled and the jar labelled Apple has to be Oranges.

Similar scenario applies if it's a Oranges taken out from the jar labelled as Apple and Oranges. So you need to pick just one fruit from the jar labelled as Apple and Oranges to correctly label the jars.

ALTERNATIVE SOLUTION:

We know that every jar has wrong label on it, now we know that “apple and orange” label jar is either apple or orange, so we will pick one fruit let suppose this fruit is an apple so the label of this jar should be “apple”, so now “apple and orange” jar is now —> “apple” jar then jar which labeled orange is “apple and orange” because we know that this jar has wrong label and we have already find the “apple” jar. so now orange jar is “apple and orange” jar and obviously “apple” jar is “apple and orange” jar.

6. Set your timer to 17 minutes. Young boy and young girl will go to the other side of bridge with the lantern first and time taken by them will be 2 minutes, the boy will return in one minute, with the lantern to senior scientist and his son, now the remaining time would be 14 minutes and will handle the lantern to the scientist's son, now the senior scientist and his son will cross the bridge together in 10 minutes, remaining time would be 4 minutes now. The young girl goes back to the boy in two minutes with the lantern and they crossed the bridge together in 2 minutes and they are safe now. In rest of one minute, they disconnected the bridge so that the demons could not reach them.

7. The young lady said that she thought it was her room but she knocked the door, this made Mr. Bond to doubt on her. No person will knock their own room door.

8. He can't reveal the number because if the product of two numbers is 20 it can have two possibilities of 4*5 and 10*2. Only the product of two prime numbers cannot have possibilities, say for example 3 is a prime number and 1*3 can only give 3 and we have only 25 prime numbers. Sum of all prime numbers are almost same except the sum of 13 and 4 which is 17 and is unique. So the passcode is 13 and 4.

9. Send the box with the lock to B.

B can't open it, but can put another lock on the box.

B sends this box with the 2 locks back to A, A unlock his lock and send it back to B again.

So there is just B's lock on the box and B can now open it.

10. Newspaper are opened in even - odd sequence & not vice versa. In the FIR it was mentioned as 9-10.

11. The payments should equal the receipts. It does not make sense to add what was paid by the men (\$12) to what was received from that payment by the waiter (\$2)

Although the initial bill was \$15 dollars, one of the five dollar notes gets changed into five ones. The total the three men ultimately paid is \$12, as they get three ones back. So from the \$12 the men paid, the owner receives \$10 and the waiter receives the \$2 difference. $\$15 - \$3 = \$10 + \2 .

12. Pour the four gallon bucket filled with water into the empty seven gallon bucket. Fill the four gallon bucket up again and poor as much as you can into the seven gallon bucket until the seven gallon bucket is fill. Now there is one gallon left in the four gallon bucket. Empty the seven gallon bucket and transfer the one gallon of water into the seven gallon bucket. Fill the four gallon bucket one more time, then pour the four gallons into the seven gallon bucket making which already has one gallon in it, making a total of five gallons.

LOGICAL THINKING – SOLUTION

13. White. The cook only ever removes the white beans two at a time, and there are an odd number of them. When the cook gets to the last white bean, and picks it up along with a black bean, the white one always goes back into the pot.
14. Two half-full barrels are dumped into one of the empty barrels. Two more half-full barrels are dumped into another one of the empty barrels. This results in nine full barrels, three half-full barrels, and nine empty barrels. Each son gets three full barrels, one half-full barrel, and three empty barrels.
15. The smallest number of coconuts there could have been in the original pile is 3121.

Time	Starting File =	Monkey + Share +	New File
1	3121	=	1 + 624 + 2496
2	2496	=	1 + 499 + 1996
3	1996	=	1 + 399 + 1596
4	1596	=	1 + 319 + 1276
5	1276	=	1 + 255 + 1020
6	1020	=	0 + 204 + 816

The 3,121 coconuts would be divided as follows:

- | | |
|--------------------------|--------------|
| the monkey would get | 5 coconuts |
| the first man would get | 828 coconuts |
| the second man would get | 703 coconuts |
| the third man would get | 603 coconuts |
| the fourth man would get | 523 coconuts |
| the fifth man would get | 59 coconuts |

THINKING SKILL

SESSION - 2

REBUS PUZZLES

Can you solve these Rebus Puzzles?

1.		2.	
	FALLING ASLEEP		FOR INSTANCE
3.		4.	
	HIJACKING		NO ONE UNDERSTANDS

5.		6.	
	BACK TO SQUARE ONE		I ATE SOME PIE AND IT WAS DELICIOUS (sqrt(-1) = i, 2^3 = 8, sum, pi = Pi)
7.		8.	
	A FRIEND IN NEED		OVERSEAS TRAVEL
9.		10.	
	LOOK BOTH WAYS BEFORE YOU CROSS THE STREET		SCRAMBLE EGGS
11.		12.	
	Every cloud has a silver ring. ALTUS, CUMULUS and CIRRUS are the names of clouds and AG is the chemical symbol for silver		Beaten black and blue Bruised, either physically or emotionally.
13.		14.	
	FAT CHANCE		A CUT ABOVE THE REST

Critical Thinking

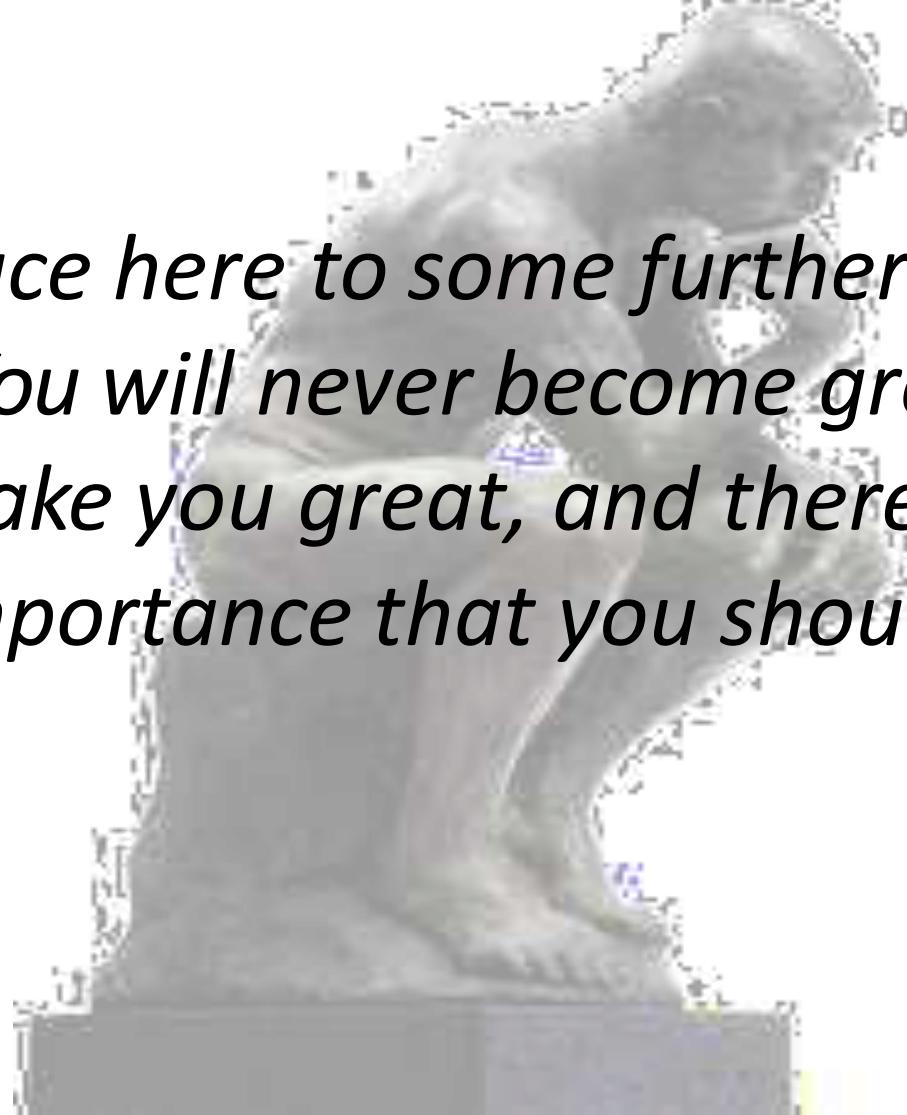
An Introduction

CRITICAL THINKING

Through this presentation you will learn:

- What is thinking?
- Types of thinking?
- What is Critical Thinking?
- Critical Thinking Standards?
- Benefits of Critical Thinking
- Barriers to Critical Thinking
- Characteristic of Critical Thinker

Do You Agree?



“GIVE place here to some further consideration of thought. You will never become great until your own thoughts make you great, and therefore it is of the first importance that you should THINK.”

What is thinking?

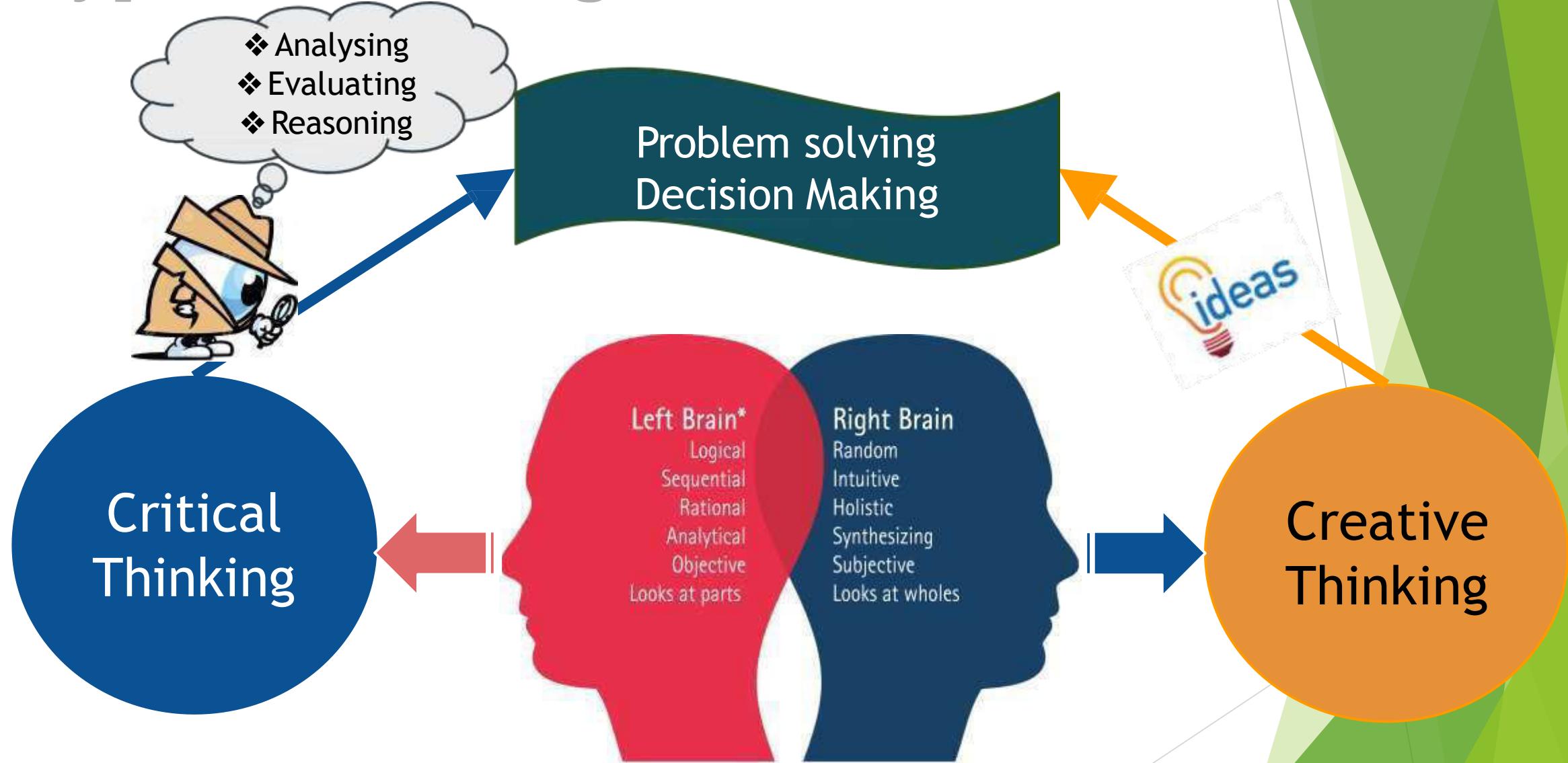
Why is the **sky** blue?
Is time travel **possible**?
Why doesn't **she** like me?
Why doesn't **he** like me?

As you start asking questions and seek answers, you are in fact **THINKING**.



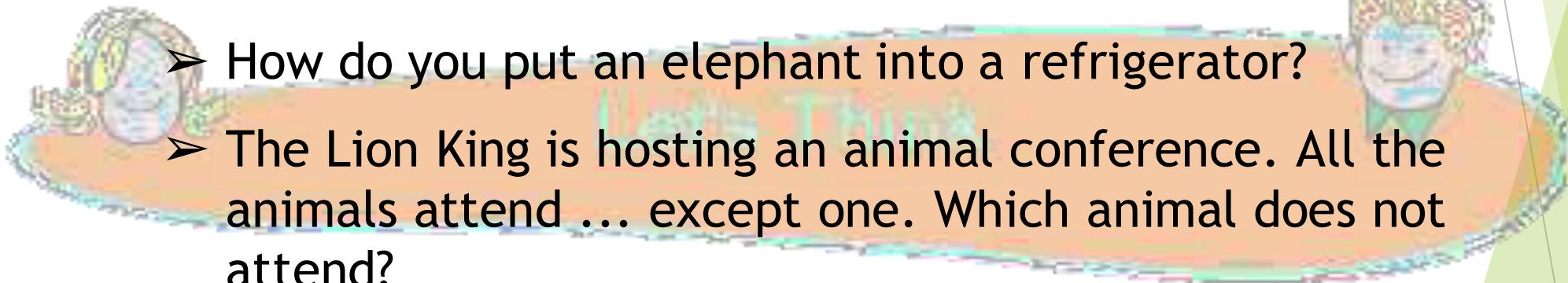
In other words:- Thinking is purposeful, organised process that we use to make sense of the world.

2. Types of thinking?



2.1 Lets think!!!

This is an old quiz (hope you remember the ans) developed by Anderson Consulting Worldwide. Answer each question in order.

- 
- How do you put a giraffe into a refrigerator?
 - How do you put an elephant into a refrigerator?
 - The Lion King is hosting an animal conference. All the animals attend ... except one. Which animal does not attend?
 - There is a river you must cross but it is used by crocodiles, and you do not have a boat. How do you manage it?

The correct answers are given in the last slide

3. What is Critical Thinking?

Example of not thinking critically



3.1 What is Critical Thinking?

Gather and
assess
Information
in a **logical**
balanced
and
reflective
way to
reach
conclusions



justified by reasoned **argument** based on available **evidence**

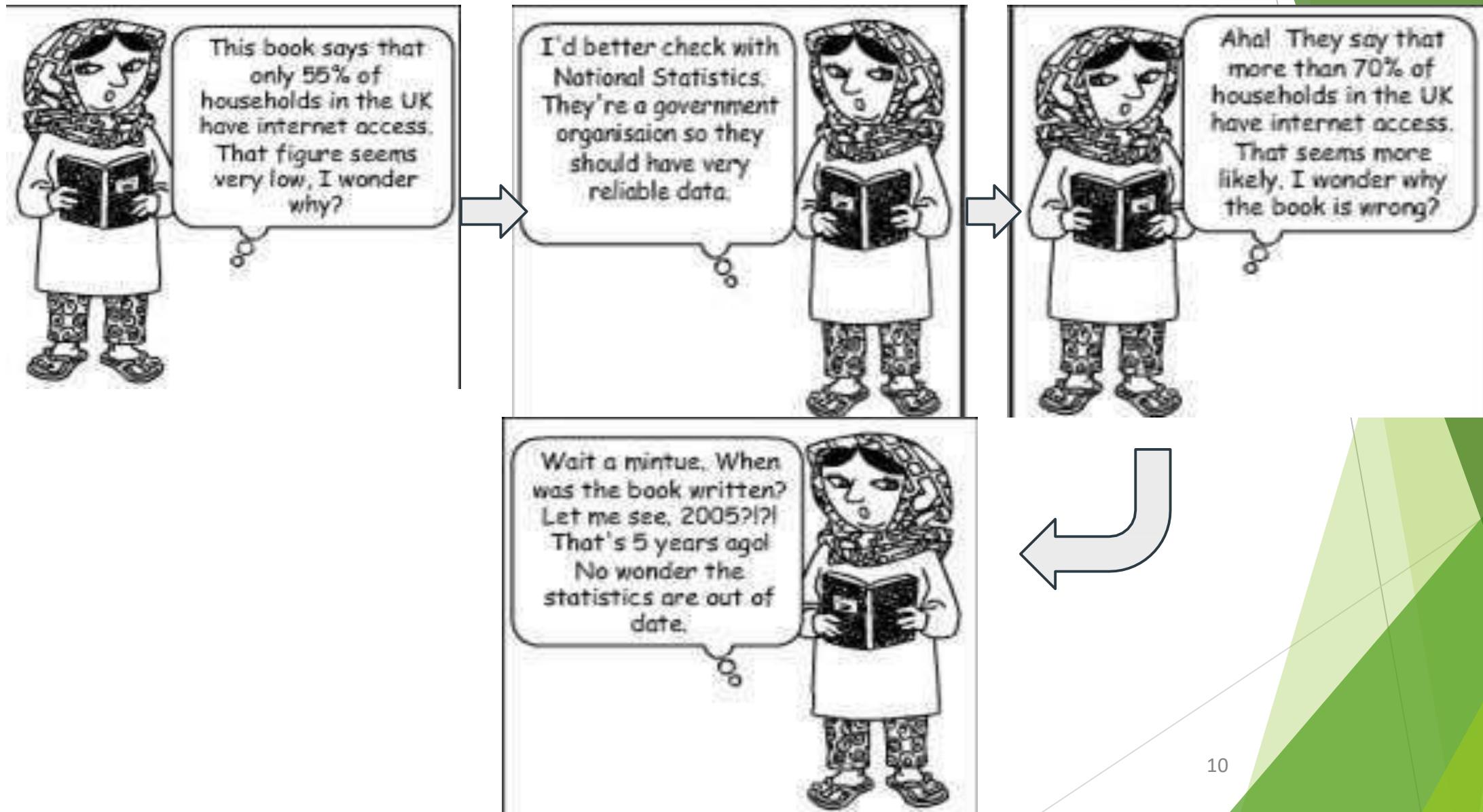
3.2 What is Critical Thinking?

Critical thinking is general term given to a wide range of cognitive and intellectual skills needed to:

- Effectively identify, analyze and evaluate arguments.
- Discover and overcome personal prejudices and biases.
- Formulate and present convincing reasons in support of conclusions.
- Make reasonable, intelligent decisions about what to believe and what to do.

Note: **Critical thinking is a skill so fortunately for us we can enhance it through practice.**

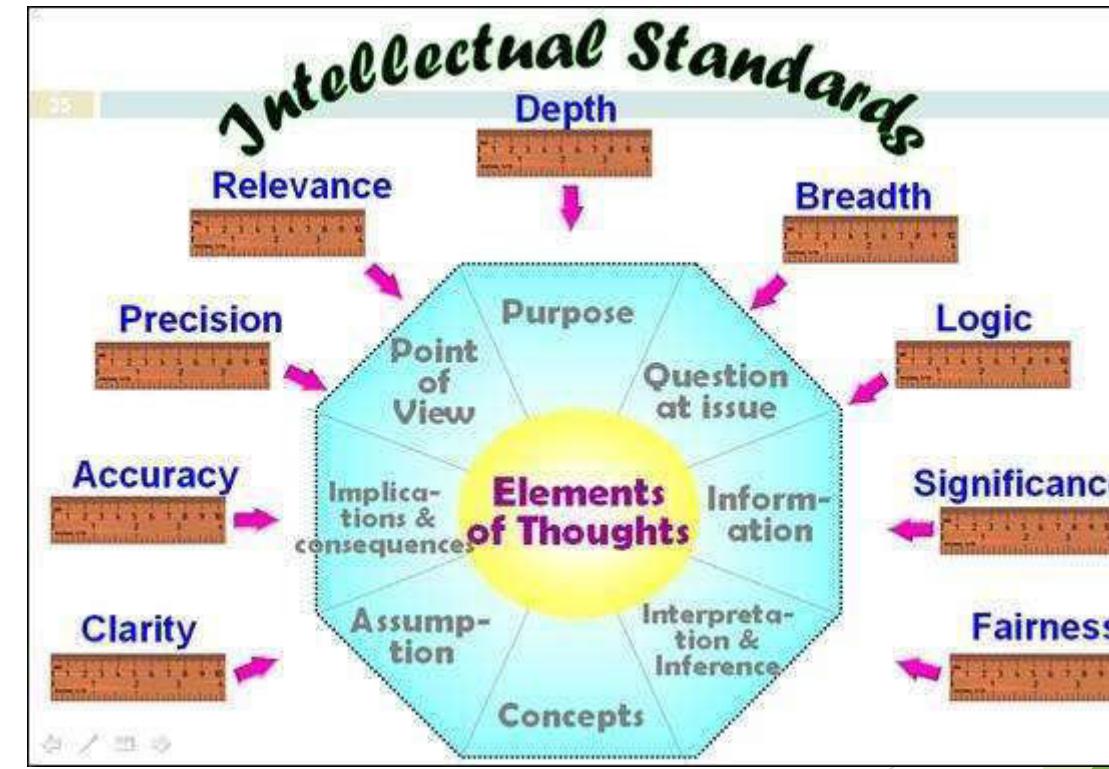
3.3 What is Critical Thinking?



4. Critical Thinking Standards?

Universal intellectual standards are standards which must be applied to thinking. To think critically requires having command of these standards

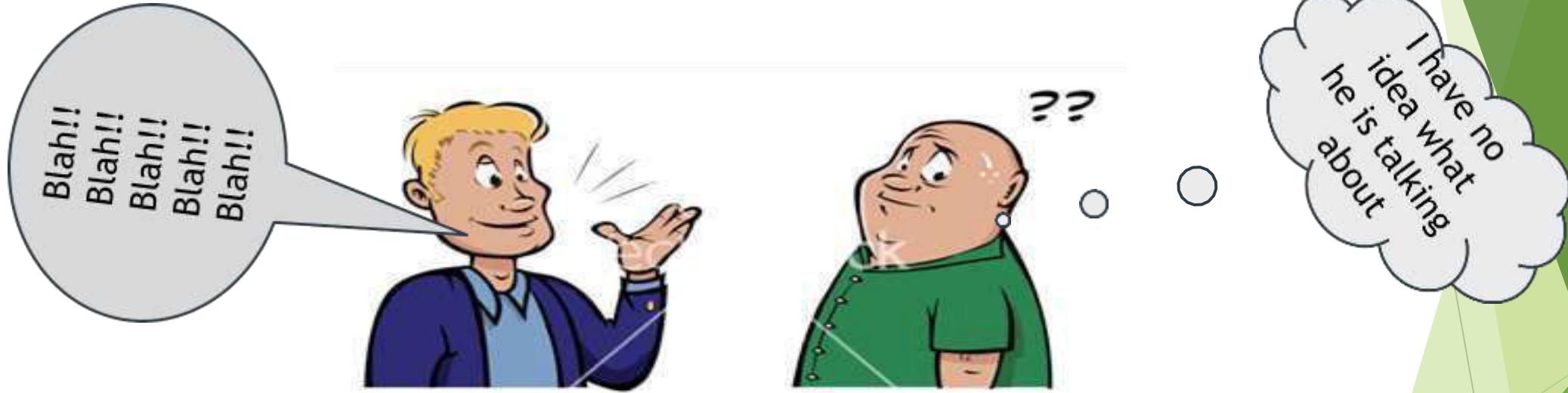
- CLARITY
- ACCURACY
- PRECISION
- RELEVANCE
- DEPTH
- BREADTH
- LOGIC
- FAIRNESS



4.1 Clarity

07.CRITICAL_THINKING_19-Jan-2021

- Could you elaborate further on that point?
- Could you express that point in another way?
- Could you give me an illustration?



Clarity is the gateway standard. If a statement is unclear, we cannot determine whether it is accurate or relevant. In fact, we cannot tell anything about it because we don't yet know what it is saying.

4.2 Accuracy

THIS DOG
WEIGHS
MORE THAN
300
POUNDS!!!



A statement can be clear but not accurate

4.3 Precision

07.CRITICAL_THINKING_19-Jan-2021

- Could you give more details?
- Could you be more specific?



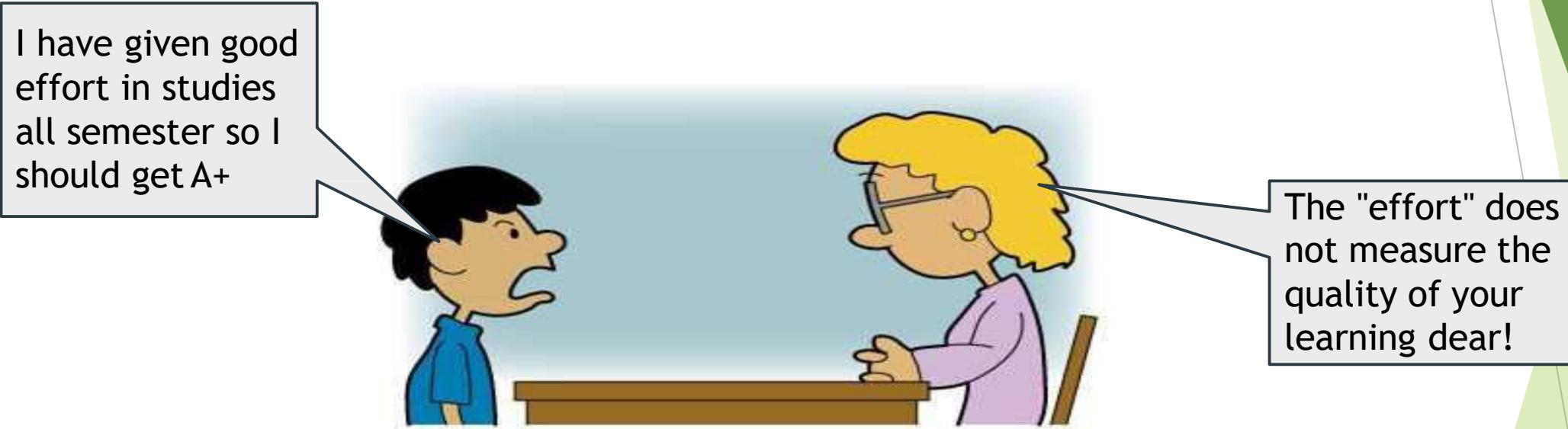
Let it go
man!!

A statement can be both clear and accurate, but not precise.

4.4 Relevance

07.CRITICAL THINKING_19-Jan-2021

- How is that connected to the question?
- How does that bear on the issue?

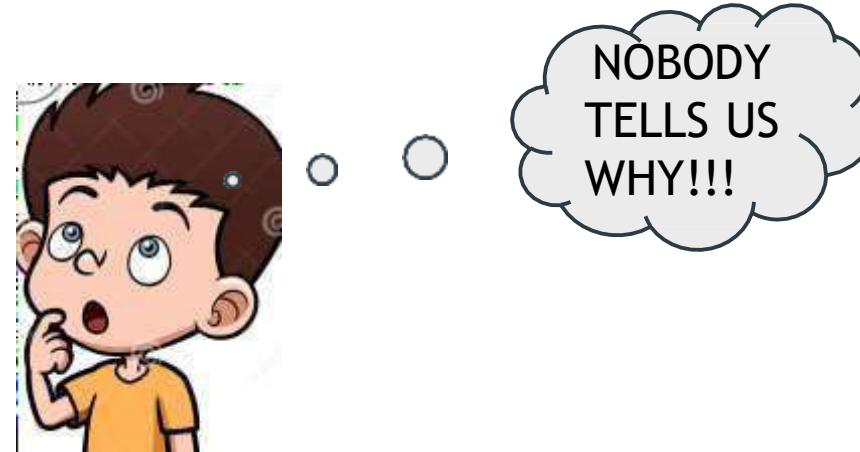


A statement can be clear, accurate and precise, but not relevant to the question at issue.

4.5 Depth

07.CRITICAL THINKING 19-Jan-2021

- How does your answer address the complexities in the question?
- How are you taking into account the problems in the question?
- Is that dealing with the most significant factors?

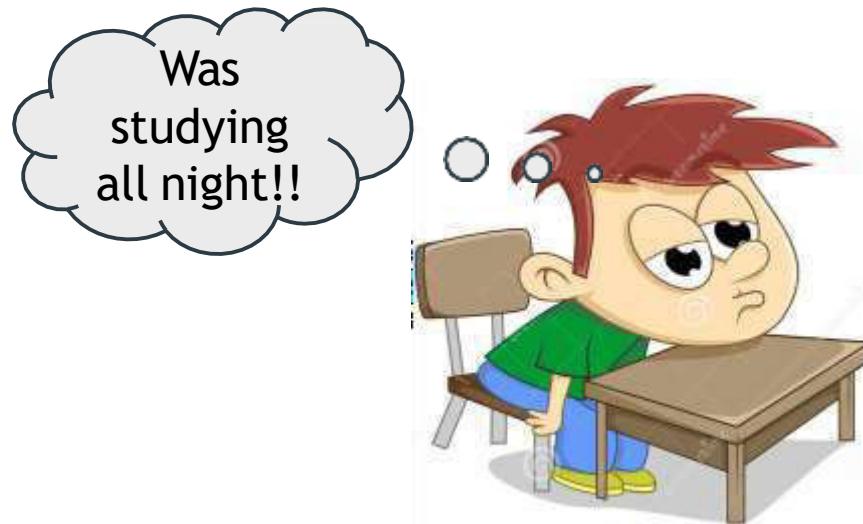


A statement can be clear, accurate, precise and relevant, but superficial.

4.6 Breadth

07.CRITICAL THINKING_19-Jan-2021

- Do we need to consider another point of view?
- Is there another way to look at this question?
- What would this look like from the point of view of . . .?



You got zero marks for “participation”, coz you didn’t participate in the class discussion at all.

A line of reasoning may be clear, accurate, precise, relevant and deep, but still ignore another side of the argument.

4.7 Logic

07.CRITICAL_THINKING_19-Jan-2021

- Does this really make sense?
- How does this follow from what I said before?
- Does this contradict a previous statement?

Superman can see through anything!!

Superman can see through walls!!

Superman can see through you!!



When the combination of thoughts are mutually supporting and make sense in combination, the thinking is "logical."

4.8 Fairness

- Do I have a vested interest in this issue?
- Am I sympathetically representing the viewpoints of others?
- Am I open minded?

"Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid."

-Albert Einstein

We do not naturally consider the rights and needs of others on the same plane with their own rights and needs, but to be a good thinker you should apply “Fairness” in your thinking.

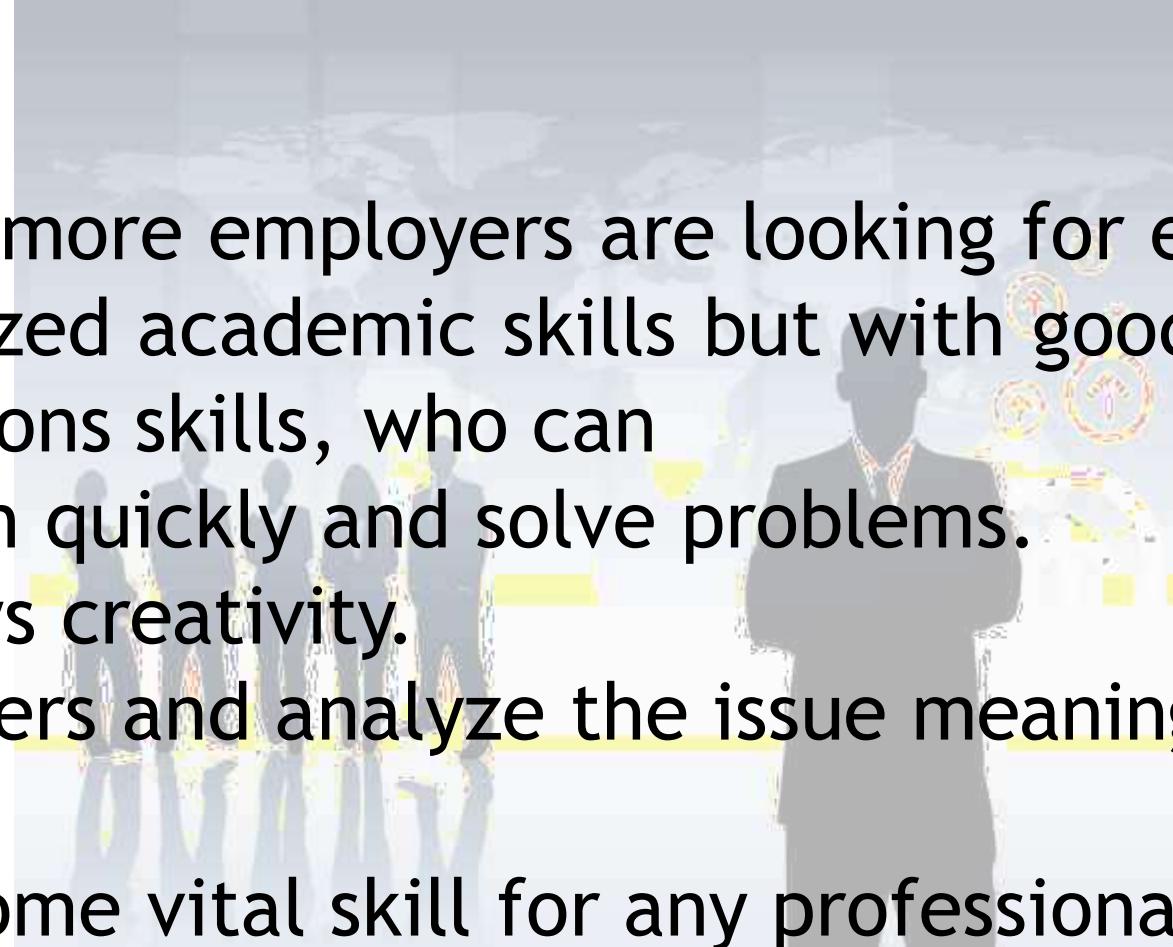
5. Benefits of Critical Thinking

At Work

As more and more employers are looking for employees not with specialized academic skills but with good thinking and communications skills, who can

- Learn quickly and solve problems.
- Shows creativity.
- Gathers and analyze the issue meaningfully.

This has become vital skill for any professional



5.1 Benefits of Critical Thinking

Daily life

- Helps us avoid making foolish decision.
- Helps us become a good citizen capable of making good decisions on important social, political and economic issues.
- Helps us in developing good thinking skill capable of examining our own assumptions and dogmas.



6. Barriers to Critical Thinking

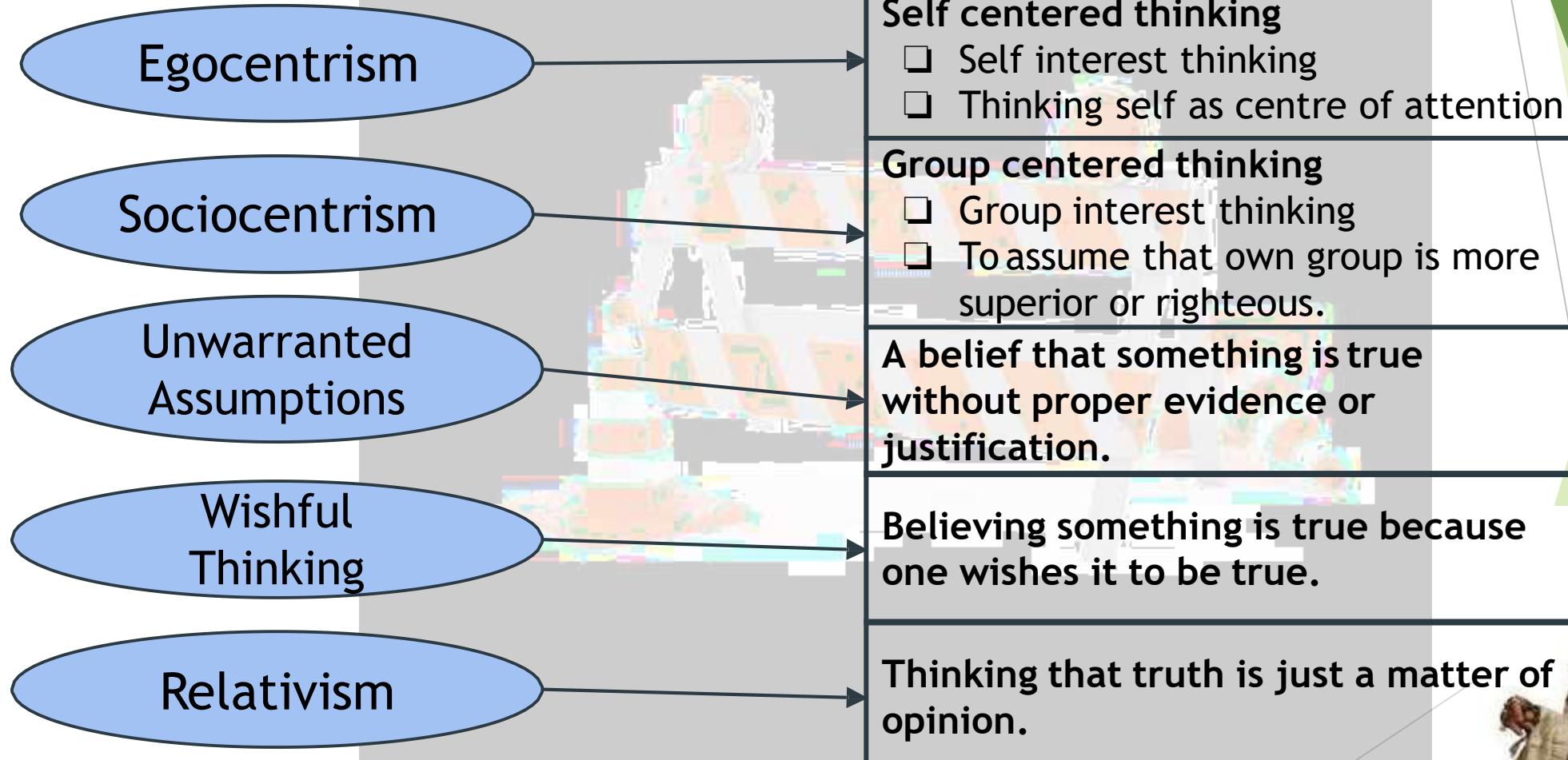
You might ask,

- If the critical thinking is so important, then why is uncritical thinking so common?
- Why is that so many people including the highly educated and intellectual people find it difficult to think critically?

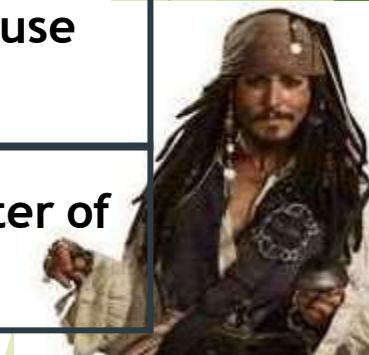


6.1 Barriers to Critical Thinking

5 most powerful **barriers** to critical thinking.



The problem is not the problem. The problem is your attitude towards the problem.

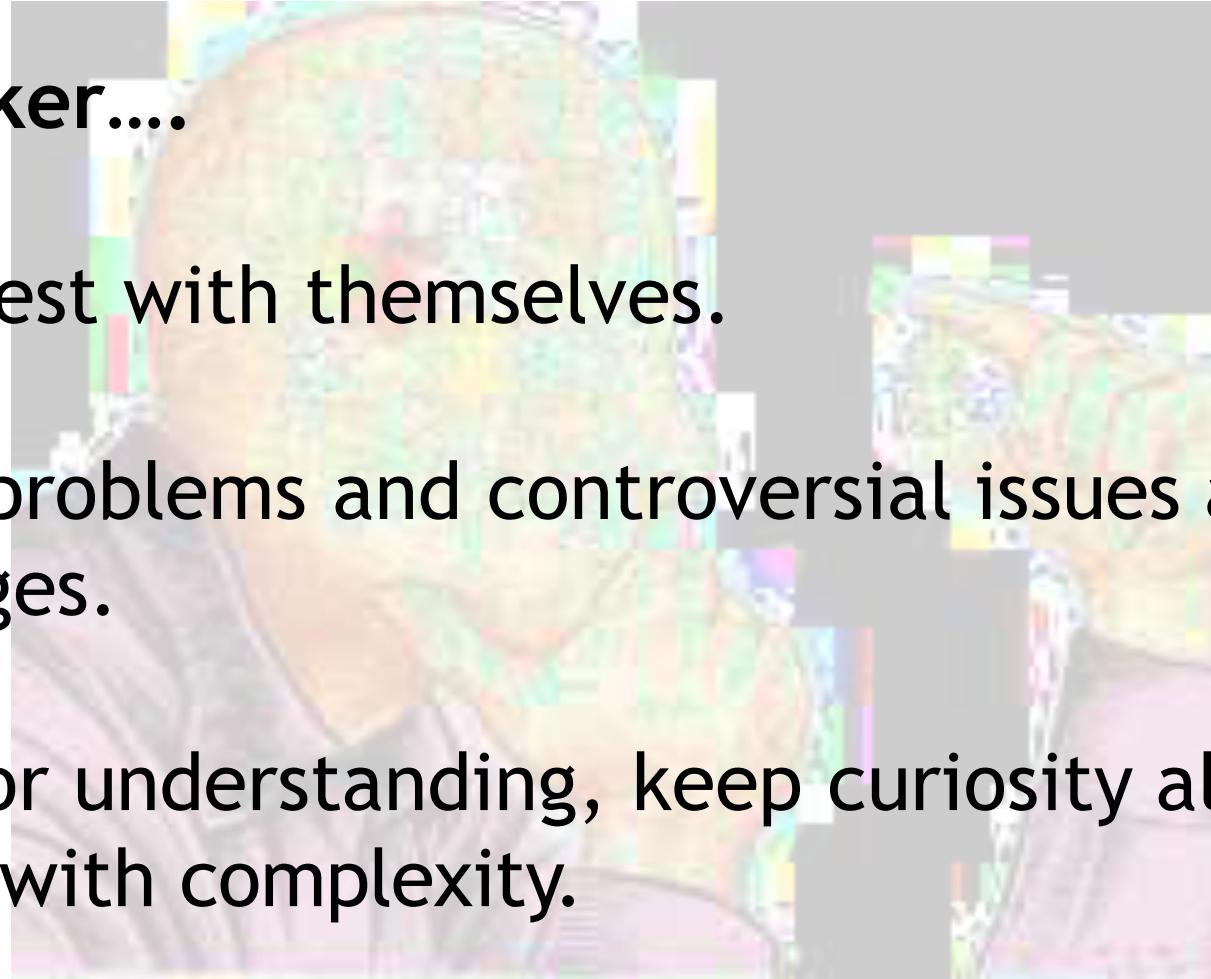


7. Characteristic of Critical Thinker

07.CRITICAL THINKING 19-Jan-2021

Critical Thinker....

- Are honest with themselves.
- Regard problems and controversial issues as exciting challenges.
- Strive for understanding, keep curiosity alive remain patient with complexity.



7. Characteristic of Critical Thinker

07.CRITICAL THINKING 19-Jan-2021

Critical Thinker....

- Base judgments on evidence rather than personal preferences.
- Are interested in other people's ideas and so are willing to read and listen attentively.
- Practice fair mindedness, and seek a balance view.
- Practice restraint, controlling their feelings rather than being controlled by them.

Answers for quiz in slide 6

How do you put a giraffe into your refrigerator?

Open the refrigerator, put in the giraffe, and close the door.

This question tests whether you tend to do simple things in an overly complicated way.

How do you put an elephant into your refrigerator?

Did you say, Open the refrigerator, put in the elephant, and close the refrigerator?

Wrong answer.

Correct answer: Open the refrigerator, take out the giraffe, put in the elephant and close the door.

This tests your ability to think through the repercussions of your previous actions.

Answers for quiz in slide 6

- The Lion King is hosting an animal conference. All the animals attend - except one. Which animal does not attend?**

Correct Answer: The elephant. The elephant is in the refrigerator. You just put him in there. This tests your memory. Okay, even if you did not answer the first three questions correctly, you still have one more chance to show your true abilities.

- There is a river you must cross but it is used by crocodiles, and you do not have a boat. How do you manage it?**

Correct Answer: You jump into the river and swim across. Have you not been listening? All the crocodiles are attending the animal conference.

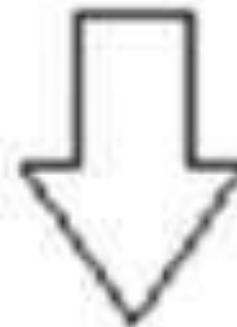
This tests whether you learn quickly from your mistakes.

THANKS

Critical Thinking Skill

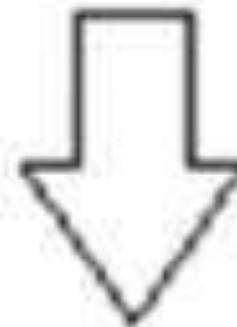
Rebus Puzzles

ASLEEP



ASLEEP

ASLEEP



ASLEEP

Falling asleep

sta4nce

stance

For Instance

10 J Q K A

10 J Q K A

HIJACKING

STANDS

0_23456789

STANDS

NO ONE UNDERSTANDS

0_23456789

I



S

1

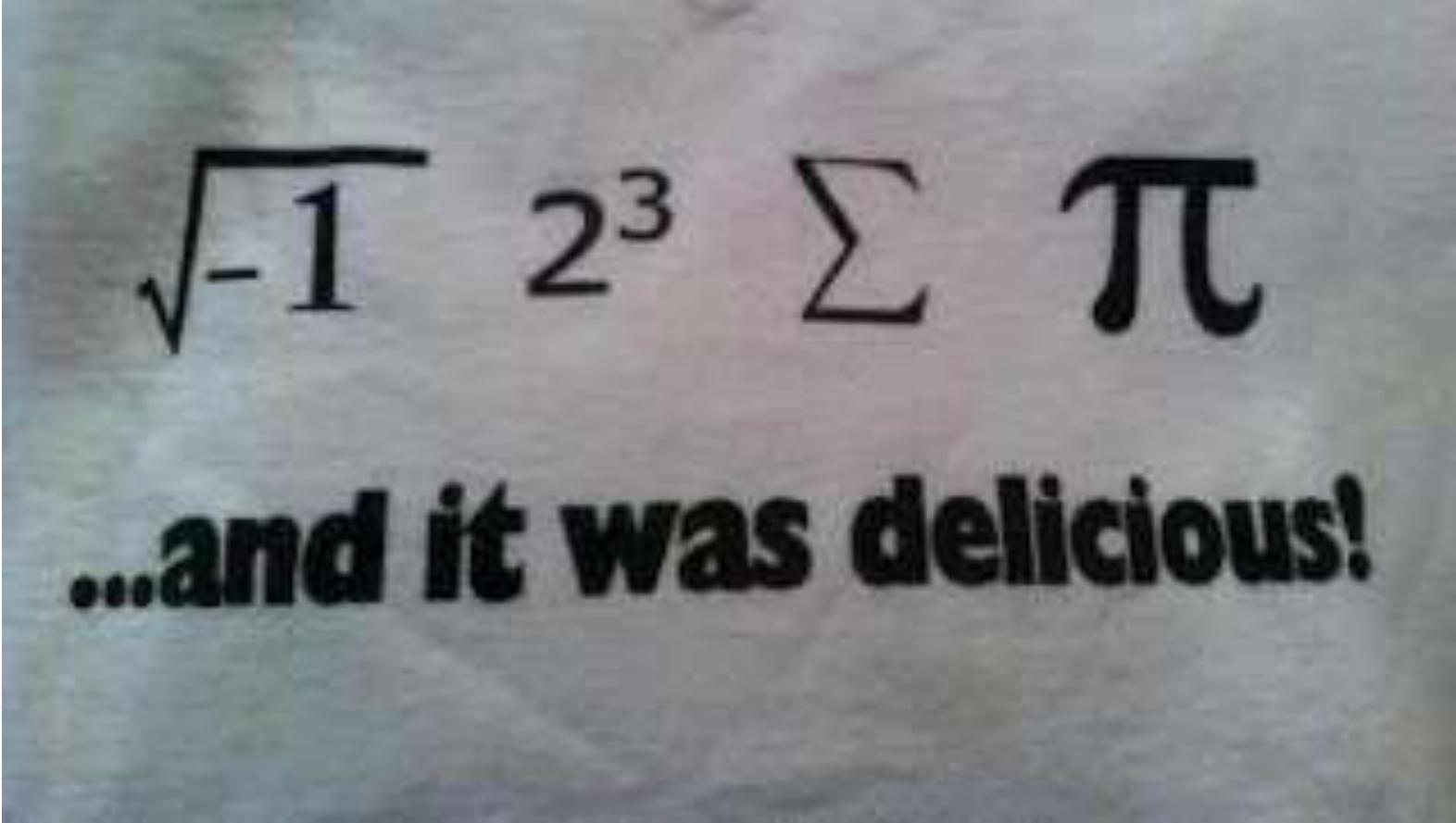


2

BACK TO SQUARE ONE

$$\sqrt{-1} \quad 2^3 \quad \sum \quad \pi$$

...and it was delicious!



$\sqrt{-1}$ 2^3 Σ π

...and it was delicious!

**I ATE SOME PIE AND IT WAS DELICIOUS
($\sqrt{-1} = i$, $2^3 = 8$, $\Sigma = \text{Sum}$, $\pi = \text{Pi}$)**

NEAFRIENDED

NEAFRIENDED

A FRIEND IN NEED

TRAVEL
—
CCCCCCCC

TRAVEL

CCCCCCCC

OVERSEAS TRAVEL

LOOK
XOOX

SiT
you
TTEEEF

LOOK
LOOK

S
T
R
Y
O
U
E
E
T

LOOK BOTH WAYS BEFORE YOU CROSS THE STREET

SGEG

SGEG

SCRAMBLE EGGS

Ag	Ag	Ag	Ag
Ag	Altus		Ag
Ag	Ag	Ag	Ag

Ag	Ag	Ag	Ag
Ag	Cumulus		Ag
Ag	Ag	Ag	Ag

Ag	Ag	Ag	Ag
Ag	Cirrus		Ag
Ag	Ag	Ag	Ag

Ag	Ag	Ag	Ag
Ag	Altus	Ag	
Ag	Ag	Ag	Ag

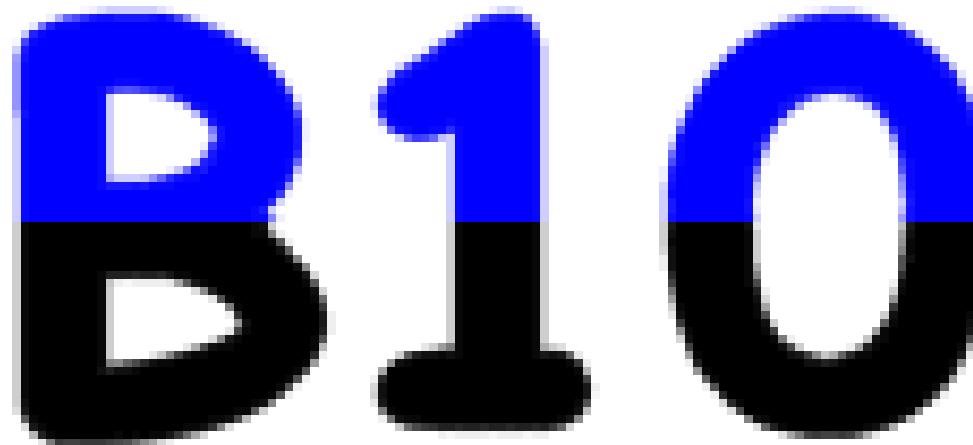
Ag	Ag	Ag	Ag
Ag	Cumulus	Ag	
Ag	Ag	Ag	Ag

Ag	Ag	Ag	Ag
Ag	Cirrus	Ag	
Ag	Ag	Ag	Ag

ANSWER: Every cloud has a silver ring.

ALTUS, CUMULUS and CIRRUS are the names of clouds and **AG** is the chemical symbol for silver

100



Beaten black and blue

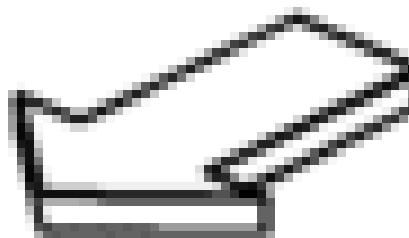
Bruised, either physically or emotionally.

CHANCE

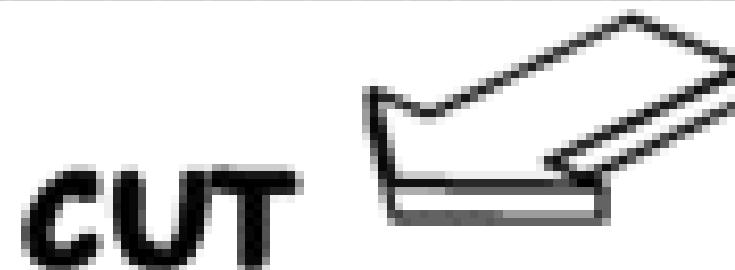
CHANCE

FAT CHANCE

CUT



CUT CUT CUT CUT
CUT CUT CUT CUT



CUT CUT CUT CUT
CUT CUT CUT CUT

A CUT ABOVE THE REST

**WHAT
WHAT
WHAT**

**WHAT
WHAT
WHAT**

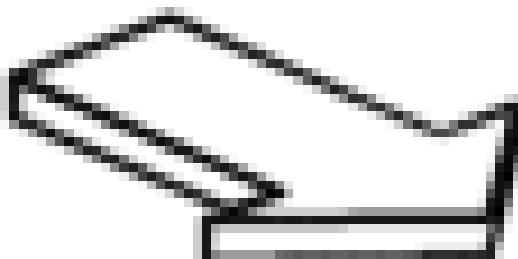
ALL OVER

WHAT
WHAT
WHAT
WHAT
ALL OVER

WHAT
WHAT
WHAT
WHAT

What's black and white and read all over.

WORLD
WORLD
WORLD





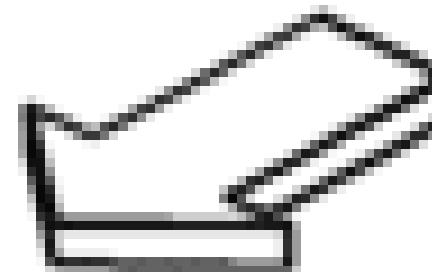
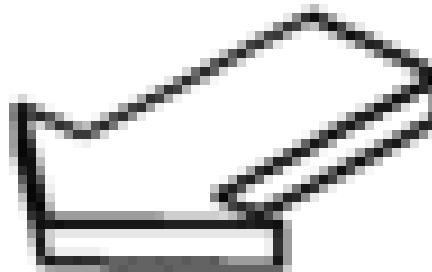
THIRD WORLD

The developing countries of Asia, Africa, and Latin America.

FEET

TEETH

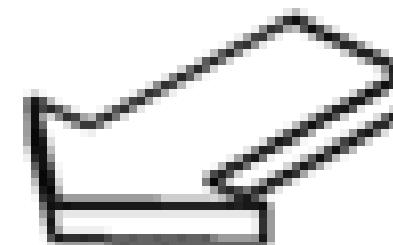
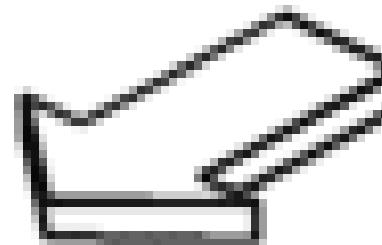
TEETH



FEET

TEET

TEET



Two left feet

STUPID**STUPID****STUPID****STUPID**

STUPID STUPID



STUPID STUPID

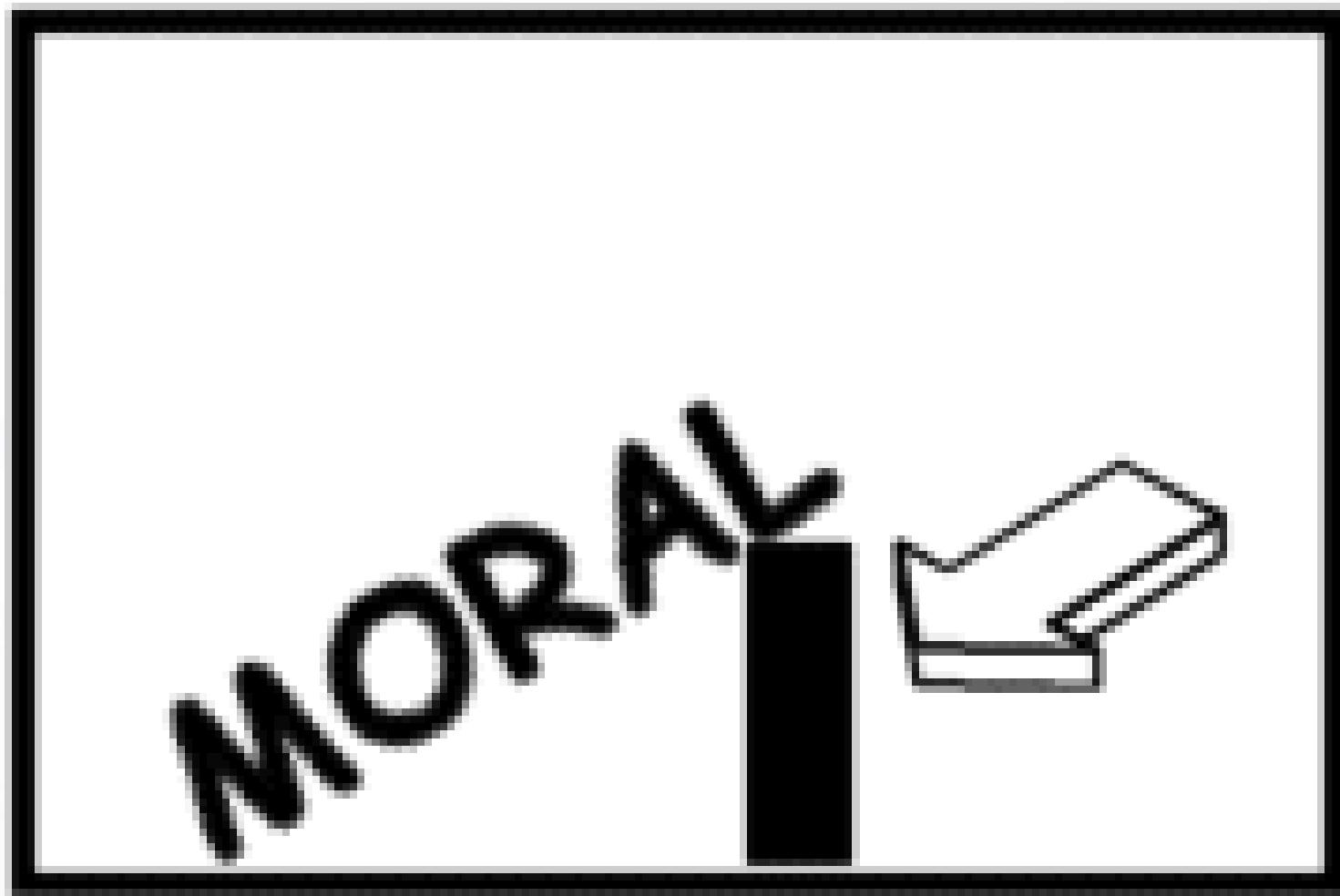
Down-right stupid!

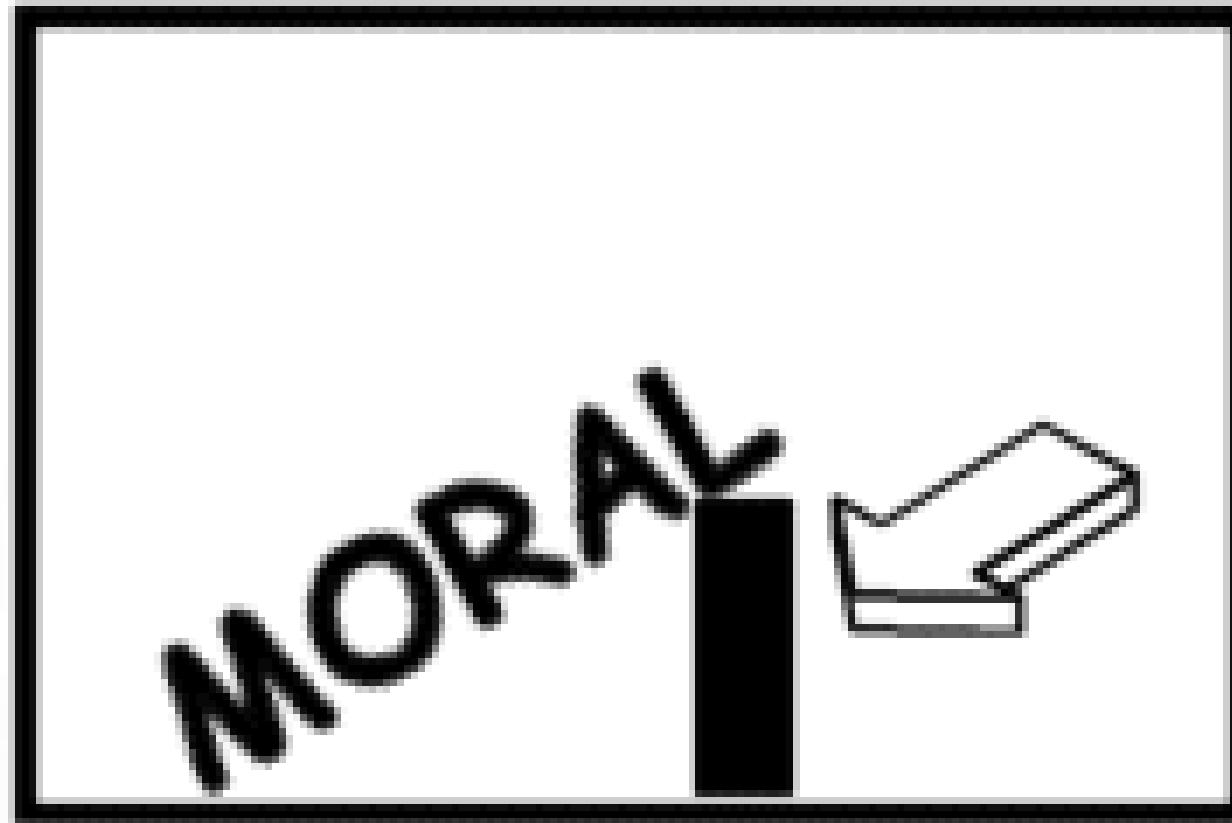
to emphasize unpleasant or bad qualities or behaviour

B
R
B
BREED
D
E

B
R
BREED
E
D

Cross breed





Moral support

HOROBOD

HOROBOD

Robin Hood

FINGER

FINGER

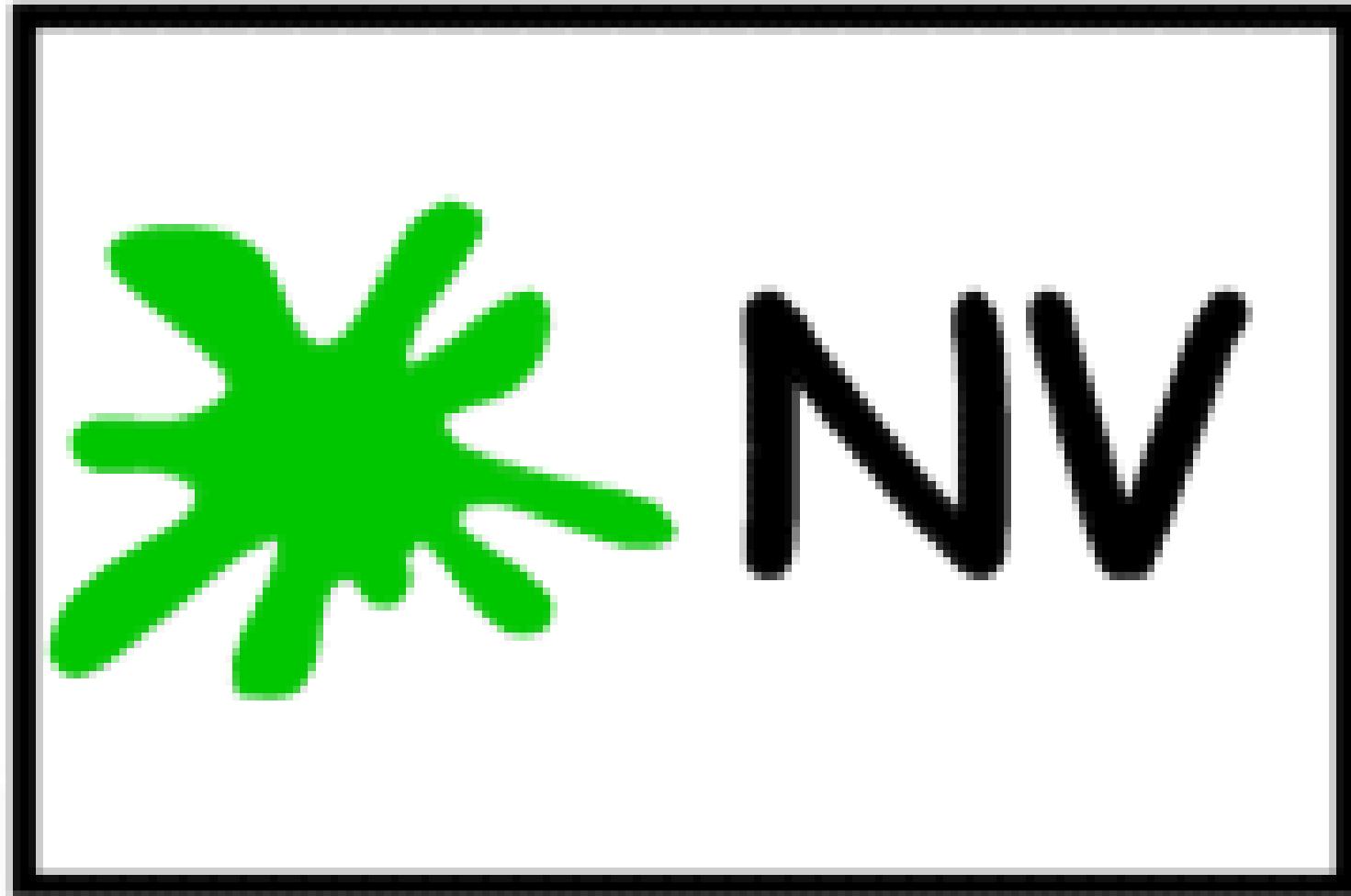
FINGER

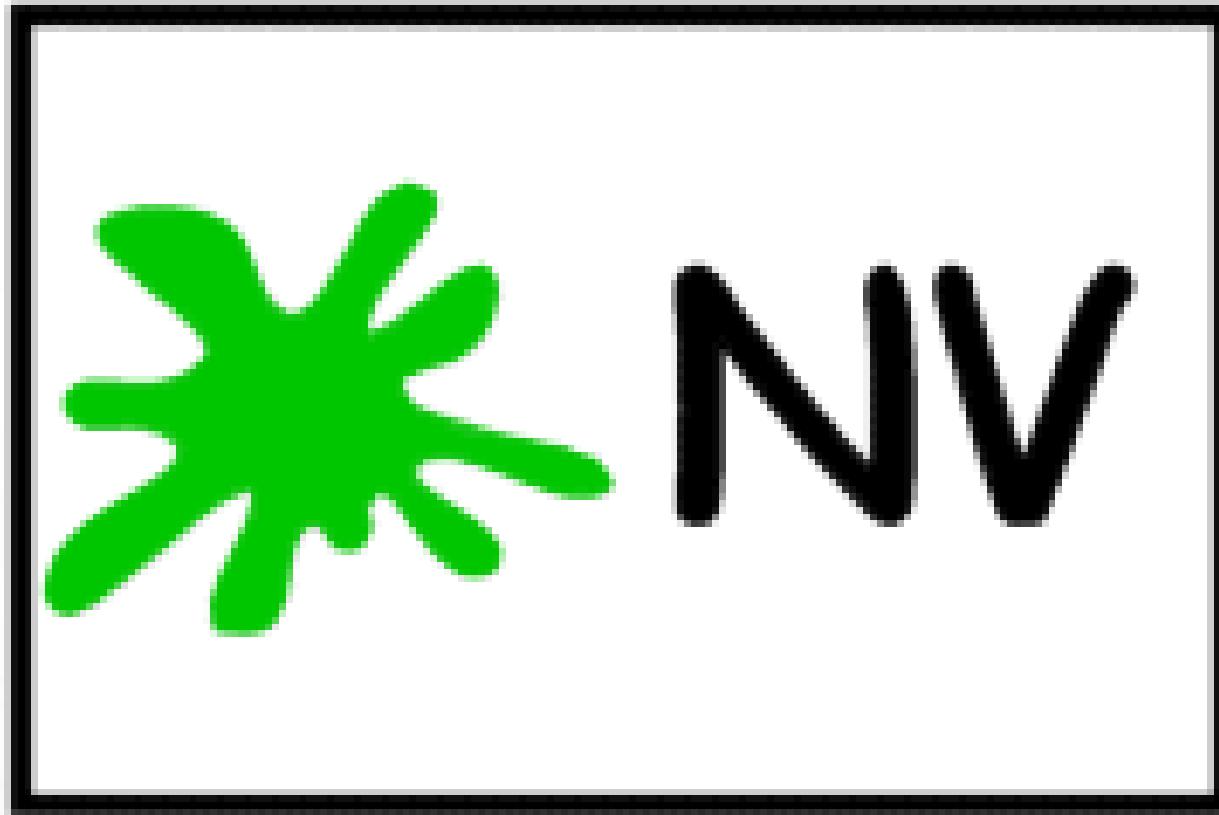
FINGER

FINGER

FINGER

Green fingers





Green with envy

MY LIFE

MY LIFE

For once in my life

M

E

A

L

M**E****A****L****Square Meal**

A B C D E F G H I J K L M
N O P Q R S T V W X Y Z

A B C D E F G H I J K L M
N O P Q R S T V W X Y Z

Missing You (U)

VA DERS

VA DERS

Space invaders

1,2,3,...,38,
39,40 LIFE

1,2,3,...,38,
39,40 LIFE

Life begins at 40!

Corporate

Corporate

Corporate downsizing

WINEEEE

WINEEEEE

Win with ease

Thank You...

Lateral Thinking

What is Lateral Thinking?

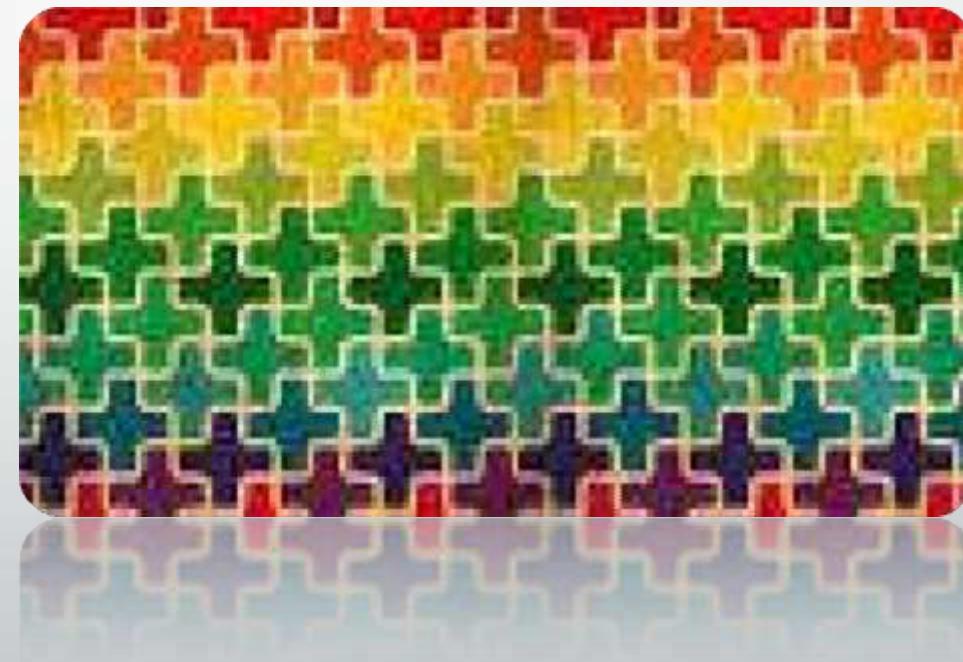
- The thinking process which involves deliberate search for alternatives
- Lateral thinking seeks the solution to problems through unorthodox methods which are generally ignored by logical thinking

How is it different?

- Our mind is an efficient self organizing information system. It uses patterns to be efficient. The normal way of thinking is therefore called vertical or logical thinking as it follows a series of definite patterns or logics to carry out an execution. Lateral Thinking doesn't follow these pre-sorted patterns.

Disadvantages of Patterns

- Expectancy
- Familiarity
- Lack of choice
- Blocked by openness
- Assumptions



Birth of the concept (Lateral Thinking)

- Edward De Bono invented the word in 1967
- He was a PhD in Psychology and served in higher academic positions in Cambridge, Oxford and Harvard University.

“The 6 Thinking Hats”

- THINKING HAT- What thinking is needed? What have we done so far? What do we do next?
- FEELINGS HAT- What do I feel about this? How do I react to this? Analyzing motions and Intuition.
- CREATIVE HAT- New ideas. What is the way to work a problem out? Different approaches.
- INFORMATION HAT- What is the truth or facts? What information do we have?
- BENEFITS HAT- What are the good points? Why is this worth doing? What positive features can be noted?
- JUDGEMENT HAT- What is wrong with this? Will it work? What are the weaknesses?

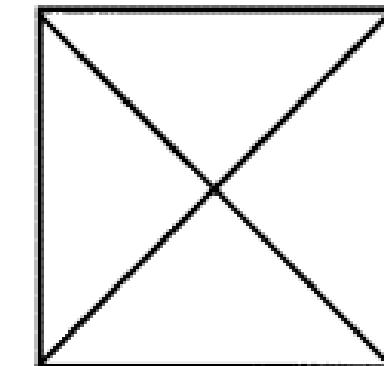
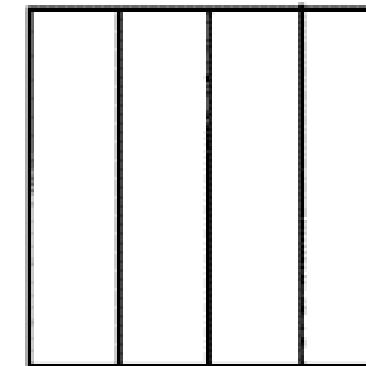
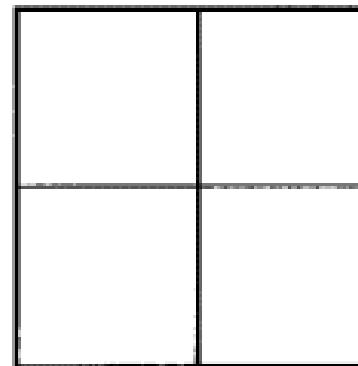
How does Lateral Thinking work?

- “You cannot dig a different hole by digging the same hole deeper”
- It is for changing the given perceptions and concepts.
- It is the technique of problem solving by approaching problems indirectly at diverse angles instead of concentrating at one particular approach.

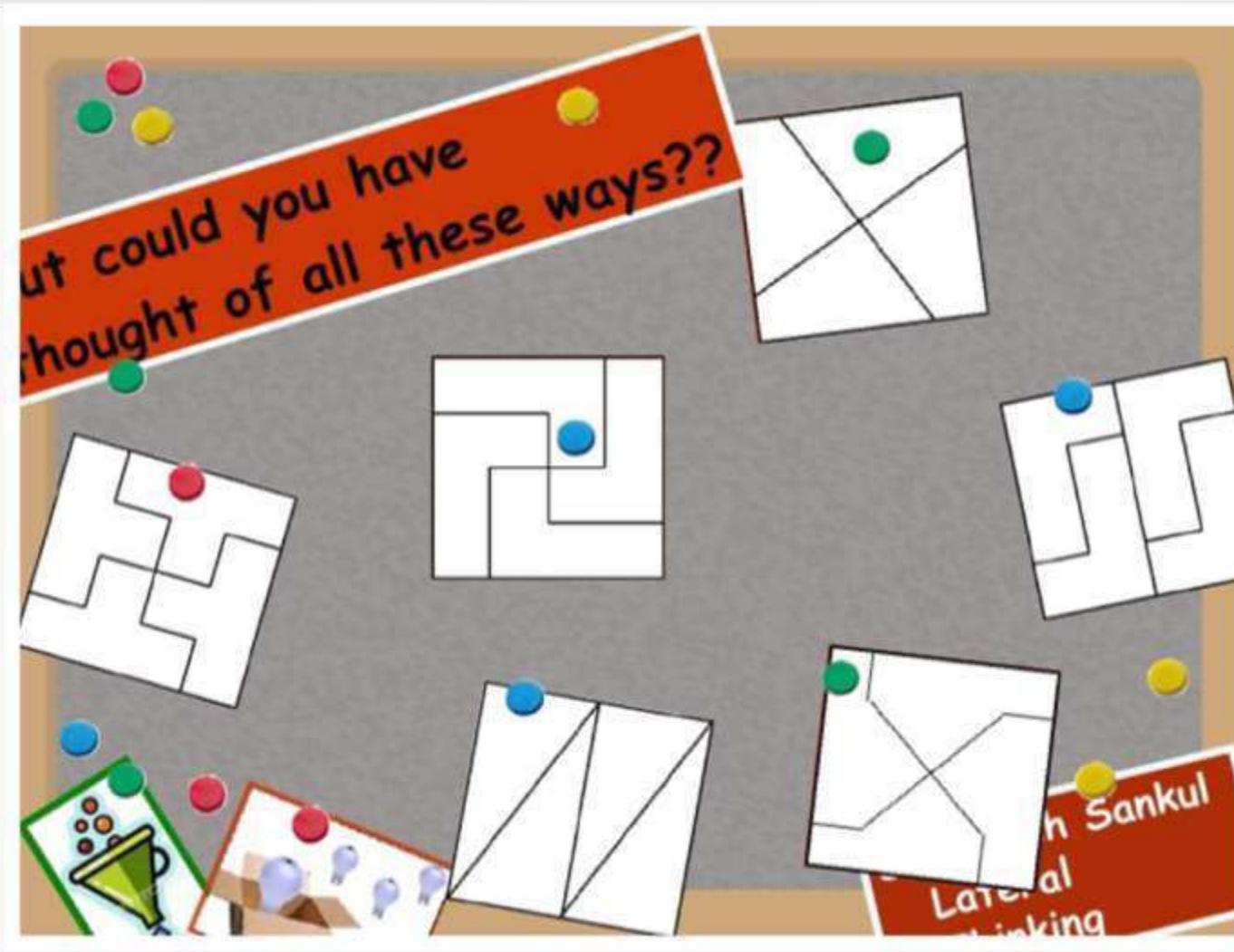
Example of Lateral Thinking

- Take a square. And try dividing it into equal four parts.

Maybe you got this....

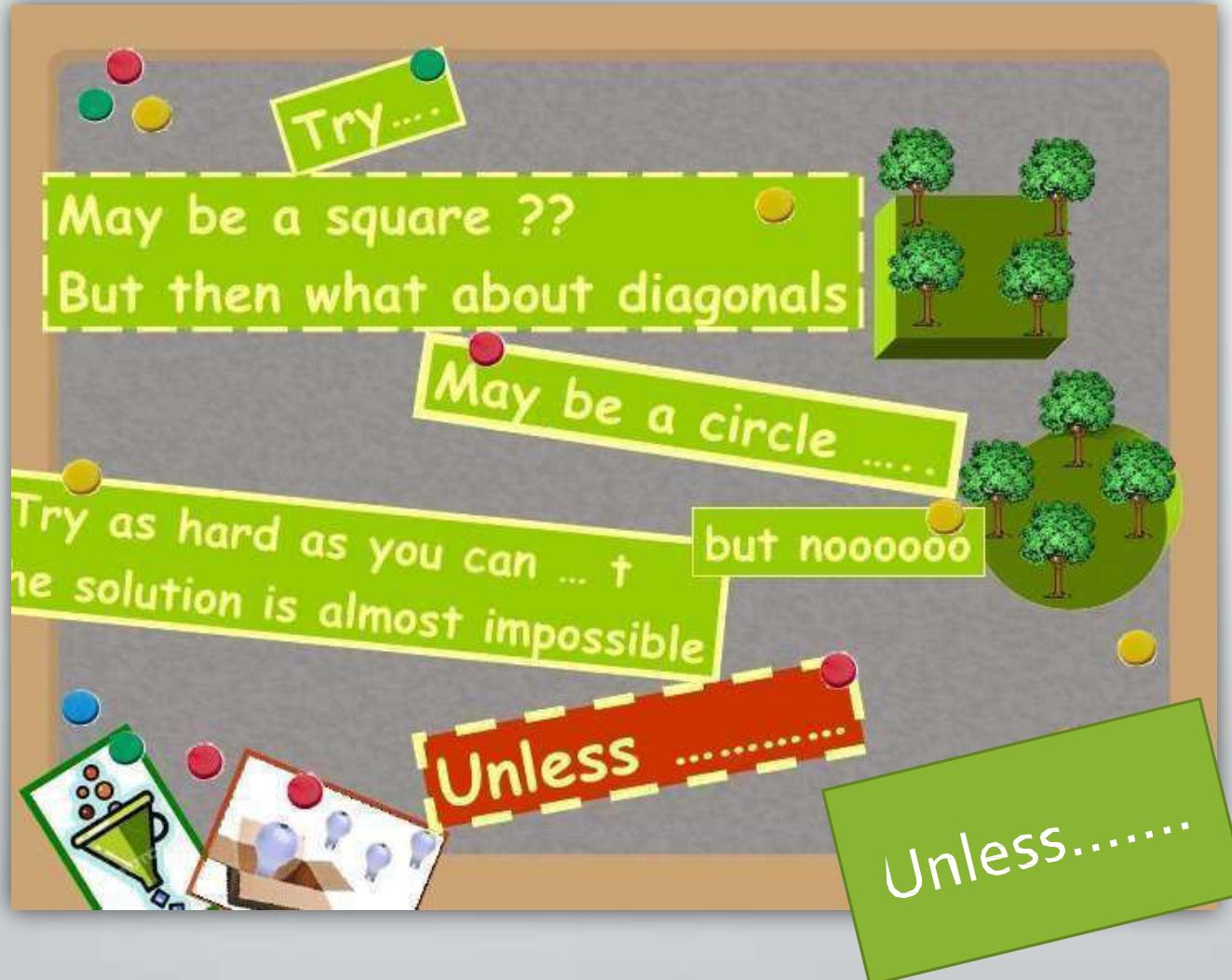


But did you think of these ways?



Another example of Lateral Thinking

- Plant four plants in such a way that they are equidistant from each other.



You think Laterally and break the assumption
of planting all trees on the same level



Difference in height....

Teaser No. 1

- There are six eggs in a basket. Six people take one egg each. How can it be that one egg is still in the basket?

The Answer is...

The last person takes
the basket along with
the last egg.



Teaser no. 2

- Acting on an anonymous phone call, the police raid a house to arrest a suspected murderer. They don't know what he looks like, but they know his name is JOHN. Inside they find a carpenter, a lorry driver, a car mechanic and a fireman playing cards. They immediately arrest the fireman without any interrogation. How did they know that they've got their man?

The Answer is...

- He was the only man. All the others were women.



CONCLUSION

- Thus, it can be concluded that the whole purpose of lateral thinking is to allow us to cut across patterns and to find new ideas. As with all creativity, the ideas must be valuable and logical in hindsight.





Thinking Skill Session - 3 Lateral Thinking

Guess the answers:

1. A girl who was just learning to drive went down a one-way street in the wrong direction, but didn't break the law.

How come?

ANS: She was walking.

Guess the answers:

2. How can you throw a ball as hard as you can and have it come back to you, even if it doesn't hit anything, there is nothing attached to it, and no one else catches or throws it?

ANS: Throw the ball straight up in the air.

Guess the answers:

3. Two students are sitting on opposite sides of the same desk. There is nothing in between them but the desk.

Why can't they see each other?

ANS: The two students have their backs to each other.

Guess the answers:

4. There are only two T's in Timothy Tuttle.

True or false?

ANS: True. There are only two T's (upper case). There are also three t's (lower case).

Guess the answers:

5. How much dirt is in a round hole that is 9 feet deep with a diameter of 3 feet?

(Hint: You don't have to do any math to get the answer.
Just use your head!)

ANS: None. You make a hole by digging out the dirt, so the hole is empty.

Guess the answers:

6. Once a dog named Nelly lived on a farm. There were three other dogs on the farm. Their names were Blackie, Whitey, and Brownie.

What do you think the fourth dog's name was?

ANS: Nelly. (If there are only four dogs on the farm, the fourth one must be Nelly!)

Guess the answers:

7. All of Jenny's pets are dogs except one. All of her pets are cats except one.

How many cats and dogs does Jenny have?

ANS: Jenny has one cat and one dog.

Guess the answers:

8. One day, two mothers and two daughters went shopping for shoes. Their shopping spree was successful – each bought a pair of shoes, and all together, they had three pairs.

How is this possible?

ANS: Only three people went shopping: a grandmother, a mother, and a daughter — but remember that the mother was the grandmother's daughter!

Guess the answers:

9. A window cleaner is cleaning the windows on the 25th floor of a skyscraper, when he slips and falls. He is not wearing a safety harness and nothing slows his fall, yet he suffered no injuries.

Explain.

ANS: He was cleaning the inside of the windows.

Guess the answers:

10. Name three consecutive days in English without using the words Tuesday, Thursday, or Saturday?

ANS: Yesterday, today, and tomorrow.

Break out of your pattern

PUZZLE 1

What is unusual about the following words?

- BOUGH
- COUGH
- DOUGH
- ROUGH
- THROUGH

ANS: They all end in O-U-G-H, but none of them rhyme

Break out of your pattern

PUZZLE 2

Which of these two sentences is correct.

- "The yolk of the egg are white."

Or

- "The yolk of the egg is white."?

ANS: Neither. The yolk of the egg is yellow.

Break out of your pattern

PUZZLE 3

- There is a word in the English language in which the first two letters signify a male, the first three signify a female, the first four signify a great man, and the whole word, a great woman. What is the word?

ANS: Heroine: HE, HER, HERO, HEROINE

Break out of your pattern

PUZZLE 4

Which one of the words displayed below does not belong to the others, and why?

- FATHER
- AUNT
- SISTER
- COUSIN
- MOTHER
- UNCLE

ANS: Cousin does not belong, since all the others refer to a specific sex.

Break out of your pattern

PUZZLE 5

Name one eight letter word that has kst in the middle, in the beginning, and at the end.

- ANS: The eight letter word is INKSTAND. In is at the beginning, kst is in the middle, and nd is at the end.

Break out of your pattern

PUZZLE 6

A cowboy rode into town on Friday, spent one night there, and left on Friday. How do you account for this?

ANS: His horse was named Friday.

Break out of your pattern

PUZZLE 7

This is a most unusual paragraph. How quickly can you find out what is so unusual about it? It looks so ordinary you'd think nothing is wrong with it - and in fact, nothing is wrong with it. It is unusual though. Why? Study it, think about it, and you may find out. Try to do it without coaching. If you work at it for a bit it will dawn on you. So jump to it and try your skill at figuring it out. Good luck - don't blow your cool!

ANS: The letter e (the most common letter in our language), does not appear in the paragraph.

Guess the answers:

PUZZLE 8

A man in a restaurant complained to the waiter that there was a fly in his cup of coffee. The waiter took the cup away and promised to bring a fresh cup of coffee. He returned a few moments later. The man tasted the coffee and complained that this was his original cup of coffee with the fly removed. He was correct, but how did he know?

ANS: He had sweetened the original cup of coffee with sugar. He therefore knew when he tasted the coffee that it was the same cup.

Guess the answers:

PUZZLE 9

A black man dressed all in black, wearing a black mask, stands at a crossroads in a totally black-painted town. All of the streetlights in town are broken. There is no moon. A black-painted car without headlights drives straight toward him, but turns in time and doesn't hit him. How did the driver know to swerve?

ANS: It was day time

Guess the answers: Puzzle 10

Many years ago in a small Indian village, a farmer had the misfortune of owing a large sum of money to a village moneylender. The moneylender, who was old and ugly, fancied the farmer's beautiful daughter. So he proposed a bargain. He said he would forgo the farmer's debt if he could marry his daughter. Both the farmer and his daughter were horrified by the proposal. So the cunning money-lender suggested that they let providence decide the matter. He told them that he would put a black pebble and a white pebble into an empty money bag. Then the girl would have to pick one pebble from the bag.

- If she picked the black pebble, she would become his wife and her father's debt would be forgiven.
- If she picked the white pebble she need not marry him and her father's debt would still be forgiven.
- If she refused to pick a pebble, her father would be thrown into jail.

They were standing on a pebble strewn path in the farmer's field. As they talked, the moneylender picked up two pebbles., the sharp-eyed girl noticed that he had picked up two black pebbles and put them into the bag. He then asked the girl to pick a pebble from the bag.

- Now, imagine that you were standing in the field. What would you have done if you were the girl?

ANS: She chooses and clumsily drops the pebble. She says luckily they can just look at the one left in the sack to see which one she picked.

Thank You...

LOGICAL THINKING

PUZZLE 10

Many years ago in a small Indian village, a farmer had the misfortune of owing a large sum of money to a village moneylender. The moneylender, who was old and ugly, fancied the farmer's beautiful daughter. So he proposed a bargain. He said he would forgo the farmer's debt if he could marry his daughter. Both the farmer and his daughter were horrified by the proposal. So the cunning money-lender suggested that they let providence decide the matter. He told them that he would put a black pebble and a white pebble into an empty money bag. Then the girl would have to pick one pebble from the bag.

If she picked the black pebble, she would become his wife and her father's debt would be forgiven.

If she picked the white pebble she need not marry him and her father's debt would still be forgiven.

If she refused to pick a pebble, her father would be thrown into jail.

They were standing on a pebble strewn path in the farmer's field. As they talked, the moneylender picked up two pebbles, the sharp-eyed girl noticed that he had picked up two black pebbles and put them into the bag. He then asked the girl to pick a pebble from the bag.

Now, imagine that you were standing in the field. What would you have done if you were the girl?

LOGICAL REASONING

SESSION - 4

॥ CODING & DECODING – I

A 'Code' is a system of conveying a message through signals. It is a method of sending a message between sender and the receiver in such a way that only the sender and the receiver can know its meaning. However 'Coding' is done according a certain pattern in the mind of the sender. Therefore, its meaning can be deciphered by a third person. Only if he carefully studies the pattern. This process is called 'Decoding'. This capability is important in many fields of application.

APPROACH

- Observe alphabets or numbers given in the code keenly.
- Find the sequence it follows whether it is ascending or descending.
- Detect the rule in which the alphabets/numbers/words follow.
- Fill the appropriate letter/number/word in the blank given.

There are many types of coding:

1. LETTER CODING

In such questions, code values are assigned to a word in terms of the alphabets. There are following types of letter coding.

Type 1

In this category, one word is coded according to a particular pattern and the candidate is asked to give the code letters for another word following the same pattern of coding.

Type 2

Sometimes a full sentence may be coded and the candidates are asked to code a word bearing in mind that code. It is possible that the code may nor follow the method of going forward or backward in the alphabetic sequence. Simply a letter of the original sentence may be given a different letter as a code. While solving puzzle, you have to keep that method in mind.

2. LETTER AND NUMERICAL CODING

In these questions either numerical values are assigned to a word or alphabetical code values are assigned to the numbers. The candidate is required to analyze the code according to the directions.

There are several ways of CODING letters of English alphabets, some consequently used are as follows-

(1) English alphabets Position from left to right:-

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

(2) English alphabets position from Right to left:-

Z	Y	X	W	V	U	T	S	R	Q	P	O	N
1	2	3	4	5	6	7	8	9	10	11	12	13

M	L	K	J	I	H	G	F	E	D	C	A	B
14	15	16	17	18	19	20	21	22	23	24	25	26

(3) Series of opposite English Alphabets:

A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

By using 'EJOTY', CFILORUX and DHLPTX formulae, we can easily remember the position of letters of English Alphabet.

3 6 9 12 15 18 21 24	4 8 12 16 20 24	5 10 15 20 25
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	↓ ↓ ↓ ↓ ↓ ↓	↓ ↓ ↓ ↓ ↓
C F I L O R U X	D H L P T X	E J O T Y

LOGICAL THINKING

Example 1: If MADRAS is coded as NBESBT. How is BOMBAY coded in that code?

Solution:

We are given that MADRAS is coded as NBESBT. In which the word MADRAS is moved one step forward to obtain the corresponding letter of the code that is M is coded as N, A is coded as B and so on. Therefore,



Implies



Likewise code may be moved one step backward also.

Example 2: If in a language APPLE is coded as 25563, RUNG is coded as 7148. Then find the code for PURPLE in that language.

Solution:

Since we are given that Apple is coded as 25563, Rung is coded as 7148. Therefore,

A P P L E	R U N G

2 5 5 6 3 7 1 4 8

Hence P is coded as 5, U is coded as 1, R is coded as 7, L is coded as 6, and E is coded as 3

Thus the code for PURPLE is 517563.

3. SUBSTITUTION

In this section an object names are substituted with different object names. We should carefully trace the substitution and answer given question.

Example 3: 'Reds' are 'blues', 'blues' are 'whites', 'whites' are 'yellows', 'yellows' are 'oranges', 'oranges' are 'pinks', then what is the colour of the sky?

Solution:

We need to find the colour of the sky and we know that the colour of sky is blue but we are given that blues are whites. So the colour of sky is white.

Example 4: If cloud is called white, white is called rain, rain is called green is called air, air is called blue and blue is called water, where will the birds fly?

- (a) Air (b) Cloud (c) Blue (d) Rain

Ans: [c]

Solution:

Clearly, the birds fly in the 'air' and 'air' is called 'blue'. So, the birds fly in the 'blue'.

- HSEM1BTECHSTANDARD0719**
- In a certain code, APPLE is written as BPPLD and CEAT is written as DEAS, then how will be HEART written in that code?
 (a) GEARS (b) HEARS (c) HEAST
 (d) TREETH (e) None of these
 - In a certain code, CAT is written as SATC and DEAR is written as QEARD. How would SING be written in that code?
 (a) BGINS (b) SGNIS (c) SIGNS
 (d) GNISS (e) None of these
 - In a certain code language, STRAIN is written as UWTDKQ. How will be PLACES written in that code?
 (a) ROCFGV (b) RNDFGV
 (c) RODEGV (d) ROCFHV
 (e) None of these
 - In a certain code language, ANTICIPATION is written as ICITNANOITAP. How will be PRODUCTIVITY written in that code?
 (a) CUDORPYTIVIT (b) CUDORPYTIUIT
 (c) CUDOPRYTIVIT (d) CUDORPTYIYIT
 (e) None of these
 - In a certain code language, the word FILAMENT is written as VPGOHKNC. How will be the word COINJUGAL written in that code?
 (a) NCIWLPQE (b) NCIWEQPL
 (c) NCWILPQE (d) NCIEWPQL
 (e) None of these

Directions for Q6 and Q7: In each question below is given a group of digits followed by four combinations of letters/symbols as options (a), (b), (c) and (d). You have to find out which of the combinations correctly represents the group of digits based on the coding system and the conditions given below and mark the number of that combination as your answer. If none of the combinations correctly represents the group of digits, mark (e) i.e., 'None of these' as your answer.

Digit	5	1	3	4	9	6	8	2	7
Letters/ symbols	P	A	K	%	R	@	D	©	R

Conditions

- If the first digit is odd and the last digit is even, the codes for the first and last digits are to be reversed.
 - If both the first and the last digits are even, both are to be coded as *.
 - If both the first and the last digits are odd, both are to be coded as \$.
6. 215349
 (a) RAPK%© (b) *APK%* (c) \$APK%\$
 (d) ©PAK%R (e) None of these

LOGICAL THINKING

7. 671254
 (a) @MA@P% (b) \$MA@P\$ (c) *MA@P*
 (d) %MA@P© (e) None of these

Directions for Q8 to Q10: Study the following information carefully and answer the questions given below it.

Digit in the numbers are to be coded as follows:

Digit	7	3	5	0	2	1	6	4	9	8
Code	N	H	L	T	F	D	R	Q	G	P

Following conditions are to be observed

- (a) If the first digit is even and the last digit is odd, they are to be coded as \$ and @ respectively.
 (b) If the first digit is odd and the last digit is even, they are to be coded as # and £ respectively.
 (c) If 0 is preceded as well as followed by an odd digit, then 0 is to be coded as †.
 (d) If 0 is preceded as well as followed by an even digit, then 0 is to be coded as ‡.
 (e) 0 is not considered as either even or odd.
8. What will be the code for 7620486?
 (a) #RF↓QP£ (b) £RF↓QP# (c) #RFTQP£
 (d) £RF↑QP# (e) None of these
9. \$QRL↑H@ could be the code for which of the following numbers?
 (a) 8465032 (b) 8456037 (c) 8455022
 (d) 6475031 (e) None of these
10. QLP↓RNT is the code for which of the following numbers?
 (a) 658070 (b) 4780650 (c) Data inadequate
 (d) None of these
11. If in a certain code, COVET is written as FRYHW, which word would be written as SHDUO?
 (a) QUAKE (b) REPAY (c) STINK
 (d) PEARL (e) TIEVP
12. If in a certain language, TRIANGLE is coded as SQHZMFKD, which word would be coded as DWZLOKD?
 (a) EXAMPLE (b) FIGMENT
 (c) DISMISS (d) DISJOIN
 (e) None of these
13. If ELCSUM is the code for MUSCLE, which word has the code LATIPAC?
 (a) CONFESS (b) CONDUCE
 (c) CAPITAL (d) CAPRICE
 (e) None of these

14. If in a certain language, ITNIETAM is the code for INTIMATE, which word has the code TREVNIETARBI?
 (a) INVRETIBRATE (b) INVERTIBARTE
 (c) INVERTIBRETA (d) INVERTIBRATE
 (e) INVERITBARTE
15. If in a certain language, DIUGNAL is the code for LANGUID, which word would be coded as ELKAHS?
 (a) SHINGLE (b) SHERBET
 (c) SHACKLE (d) SHOCKLE
 (e) None of these
16. If in a certain language DISPEL is coded as IDPSLE, how is EFFECT coded in that language?
 (a) FEEFTC (b) CTFEEF (c) EFFETC
 (d) ECTEFF (e) EEFFCT
17. In a certain language HUNTER is coded as UHNTR, how is MANAGE coded in that code?
 (a) MAANGE (b) MNAAEG
 (c) AMNAEG (d) EGNAAM
 (e) NMAAGE
18. If RAMAYANA is coded as AMARANAY, how is TULSIDAS written?
 (a) SLUTSADI (b) UTSLIDSA
 (c) SADISLUT (d) SADITULS
 (e) IDASTULS
19. If CANOE is coded as IFRRG, how is MUSIC written in that code?
 (a) NWVNI (b) MWVMH (c) NTULB
 (d) QYWMG (e) SZWLE
20. If TABLE is coded as GZYOV, how is JUICE coded?
 (a) OZLFJ (b) QFRXV (c) HOFAD
 (d) QZHMT (e) EPQIL

LOGICAL REASONING

SESSION – 5

☺ CODING & DECODING – II

21. If FOUGHT is coded as EQRKZ, how is MALE coded?
 (a) LCII (b) NZMD (c) KCMI
 (d) NBIF (e) LBID
22. If BATCH is coded as ABSDG, how is FORSAKE coded in that code?
 (a) ABDGS (b) EPQTZLD (c) EQPZLTD
 (d) GDSBA (e) None of these

LOGICAL THINKING

HSEM1BTECHSTANDARD0719

Directions for Q39 to Q40: Each of the questions below consists of a question and two statements numbered I and II are given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and

Give answer

- (a) if the data in statement I alone is sufficient to answer the question, while the data in Statement II alone is not sufficient to answer the question.

(b) if the data in statement II alone is sufficient to answer the question, while data in Statement I, alone is not sufficient to answer the question.

(c) if the data in Statement I alone or in Statement II alone is sufficient to answer the question.

(d) if the data in both the Statements I and II are not sufficient to answer the question.

(e) if the data in both the Statements I and II together are necessary to answer the question.

39. What does 'ja' mean in the code language?

Statements:

 - I. 'ja pa na' means 'go home now' in that code language.
 - II. 'na da ta' means 'come back home' in that code language.

LOGICAL THINKING – SOLUTION

HSEM1BTECHSTANDARD0719

PUZZLE 4

Cousin does not belong, since all the others refer to a specific sex.

PUZZLE 5

The eight letter word is INKSTAND. In is at the beginning, kst is in the middle, and is at the end.

PUZZLE 6

His horse was named Friday.

PUZZLE 7

The letter e (the most common letter in our language), does not appear in the paragraph.

PUZZLE 8

He had sweetened the original cup of coffee with sugar. He therefore knew when he tasted the coffee that it was the same cup.

PUZZLE 9

It was day time

PUZZLE 10

She chooses and clumsily drops the pebble. She says luckily they can just look at the one left in the sack to see which one she picked.

SESSION - 4

☺ CODING & DECODING – I

1. Ans: [e]

$$\begin{array}{ll} +1 & +1 \\ A \rightarrow B \text{ and } C \rightarrow D & \\ P \rightarrow P & E \rightarrow E \\ P \rightarrow P & A \rightarrow A \end{array}$$

$$\begin{array}{ll} -1 \\ L \rightarrow L & T \rightarrow S \\ -1 \end{array}$$

$$\begin{array}{ll} E \rightarrow D \\ +1 \\ \text{Similarly, } H \rightarrow I \\ E \rightarrow E \end{array}$$

$$\begin{array}{ll} A \rightarrow A \\ R \rightarrow R \\ -1 \\ T \rightarrow S & \text{IEARS} \end{array}$$

2. Ans: [e]

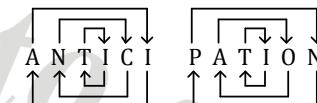
CAT becomes $(T \xrightarrow{-1} S)$ SATC and DEAR becomes $R \xrightarrow{-1}$

QEARD Similarly, SING becomes $(G \xrightarrow{-1} F)$ FINGS.

3. Ans: [a]

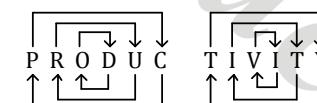
STRAIN	PLACES
$S \xrightarrow{+2} U$	$P \xrightarrow{+2} R$
$T \xrightarrow{+3} W$	$L \xrightarrow{+3} O$
$R \xrightarrow{+2} T$	$A \xrightarrow{+2} C$
$A \xrightarrow{+3} D$	$C \xrightarrow{+3} F$
$I \xrightarrow{+2} K$	$E \xrightarrow{+2} G$
$N \xrightarrow{+3} Q$	$S \xrightarrow{+3} V$

4. Ans: [a]



ICITNANOITAP

Similarly,



CUDORPYTIVIT

5. Ans: [b]

FILAMENT

Reverse order: T N E M A L $\xrightarrow{+2}$ I F
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 Code: V P G O H K M C

Similarly,

COINJUGAL

Reverse order: L A G U J N $\xrightarrow{+2}$ O C
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 Code: N C I W E Q P L

6. Ans: [e]

The first digit is even and the last digit is odd. Therefore.

2 1 5 3 4 9
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 @ A P K % R

7. Ans: [c]

Since the first and the last digits are even, both are to be coded as *. Therefore,

6 7 1 2 5 4
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 * M A © P *

8. Ans: [a]

7 6 2 0 4 8 6
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 # R F Q P E [According to conditions (b) and (d)]

LOGICAL THINKING – SOLUTION

9. Ans: [e]

\$	Q	R	L	↑	H	@
↓	↓	↓	↓	↓	↓	↓
8	4	6	5	0	3	0

10. Ans: [e]

Q	L	P	↑	R	N	T
↓	↓	↓	↓	↓	↓	↓
4	5	8	0	6	7	0

11. Ans: [d]

Each letter of the word is three steps behind the corresponding letter of the code.

∴ Required word is PEARL.

12. Ans: [a]

Each letter of the word is one step ahead of the corresponding letter of the code.

∴ Required word is EXAMPLE.

13. Ans: [c]

In the code, the letters of the word are put in the reverse order of positions.

∴ Required word is CAPITAL.

14. Ans: [d]

The letters in the first half and the last half of the code are separately reversed to obtain the word.

∴ Required word is INVERTIBRATE.

15. Ans: [c]

In the code, the letters of the word are put in the reverse order.

∴ Required word is SHACKLE.

16. Ans: [a]

In the code, every two letters of the word are reversed in order.

∴ Required code is FEEFTC.

17. Ans: [c]

In the code, the middle two letters are kept unchanged, while on either side the two letters are mutually reversed in order.

∴ Required word is AMNAEG.

18. Ans: [a]

In the code, the first four letters are reversed in arrangement and the last four letters are reversed in arrangement.

∴ Required word is SLUTSADI.

19. Ans: [e]

In the code, the first letter is the sixth alphabet, the second letter is the fifth alphabet, the third letter is the fourth alphabet and so on after the corresponding letter in the word.

∴ Required word is SZWLE.

20. Ans: [b]

If, in the word, a letter is n^{th} alphabet from the beginning, then in the code the corresponding letter is the n^{th} alphabet from the end.

LOGICAL REASONING

SESSION – 5

⦿ CODING & DECODING – II

21. Ans: [a]

In the code given, the first letter E is one place before the first letter F of the word. The second letter Q of the code is two places ahead of the second letter O of the word. The pattern of 'before' and 'ahead' thus continues.

Word →	F	O	U	G	H	T
	↓-1	↓+2	↓-3	↓+4	↓-5	↓+6
Code →	E	Q	R	K	C	Z

Accordingly,

for Word →	M	A	L	E
	↓-1	↓+2	↓-3	↓+4
Code →	L	C	I	I

22. Ans: [b]

In the code, the letters at odd places are one place before and those at even place are one place after the corresponding letters in the word.

Thus,

word →	B	A	T	C	H
	↓-1	↓+1	↓-1	↓+1	↓-1
given code →	A	B	S	D	G

Accordingly,

for Word →	F	O	R	S	A	K	E
	↓-1	↓+1	↓-1	↓+1	↓-1	↓+1	↓-1
Code →	E	P	Q	T	Z	L	D

23. Ans: [b]

The first and second letters as well as the fifth and sixth letters in the word are interchanged in the code. The other letters remain in the same positions.

Thus,

word →	H	U	M	I	D	I	T	Y
	✗	✗	↓	↓	✗	✗	↓	↓
given code →	U	H	M	I	I	D	T	Y

Accordingly,

for word →	P	O	L	I	T	I	C	S
	✗	✗	↓	↓	✗	✗	↓	↓
code →	O	P	L	I	I	T	C	S

LOGICAL THINKING – SOLUTION

24. Ans: [c]
 In the first and second statements, the common code is rps and the common word is morning. So, rps means morning.
 In the first and the third statements, the common code is ski and the common word is nice. So, ski means nice.
 Therefore, in the first statement, tri means Sunday.

25. Ans: [d]
 Tractor is used for ploughing. It is called car and hence (d).

26. Ans: [c]
 $R \rightarrow \#$ and $F \rightarrow 5$ Hence, $S \rightarrow 3$

$0 \rightarrow 4$	$I \rightarrow *$	$T \rightarrow 7$
$S \rightarrow 3$	$R \rightarrow \#$	
$O \rightarrow 4$		
$E \rightarrow \$$	$S \rightarrow 3$	$R \rightarrow \#$
	$T \rightarrow 7$	$E \rightarrow \$$

Solutions for 27 and 28

pit na sa → you are welcome

] → na → are

are very good]]

na ho pa la → they

good

ka da la → who is

they welcome good people

] → la → good

od ho pit la →

→ pit → welcome

Also, ho → they

27. Ans: [d]

people → od

28. Ans: [c]

very → pa

29. Ans: [d]

30. Ans: [d]

31. Ans: [b]

32. Ans: [a]

33. Ans: [c]

34. Ans: [b]

35. Ans: [c]

36. Ans: [c]

37. Ans: [b]

38. Ans: [a]

39. Ans: [d]

I. ja pa na → go home now]

II. na da ta → come back home] → na → home Hence,
 ja means either go or now.

40. Ans: [e]

I. → 481 → sky is blue]]

246 → sea is deep] → is → 4] → sea 6

II. → 698 → sea looks blue]. Hence, deep → 2

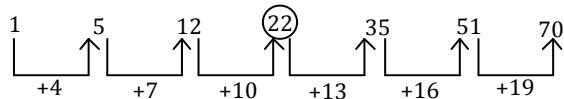
LOGICAL REASONING

SESSION - 6

NUMBER SERIES

1. Ans: [c]

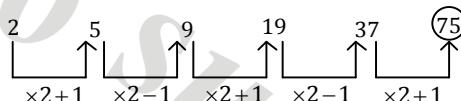
Clearly, the given series follows the following pattern.



Hence, the answer is 22 → (c).

2. Ans: [b]

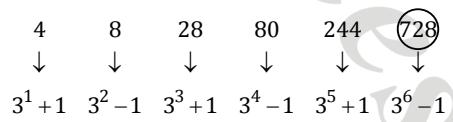
The following pattern is followed in the series.



Hence, the answer is 75 → (b).

3. Ans: [d]

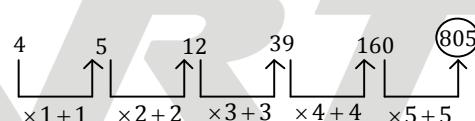
The series follows the following pattern.



Hence, the answer is 728 → (d).

4. Ans: [c]

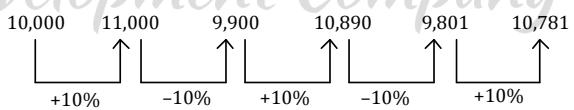
The following pattern is used in the given series.



Hence, the answer is 805 → (c).

5. Ans: [c]

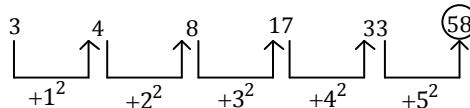
We add or subtract alternately 10% of the term to get the next term.



Hence, the answer is 10781 → (c).

6. Ans: [b]

The following pattern is used in the series.



Hence, the answer is 58 → (b).

QUANTITATIVE ABILITY

HSEM1BTECHSTANDARD0719

SESSION - 1

SPEED MATHS - I

Introduction

Learning Mathematics is a pleasure and working out sums is an interesting activity. If you don't find it interesting, that means you haven't tried to understand it. Since Maths plays an important role in the Competitive Examinations, you should have a firm determination to learn it.

The most basic things in Mathematics are the four fundamental operations – addition, subtraction, multiplication and division. All these are useful and atleast one of them are used in any type of Mathematical question.

So if we do our basic mathematical calculations faster, our valuable time is saved in each question. Speed and accuracy are very important and can be achieved only by constant practice.

Speed Maths helps you to perform the calculations faster than our traditional methods. You should also know the multiplication tables up to 20 and it is always good to practise them.

Learning the one line addition, subtraction, multiplication and division methods also is useful.

ADDITION AND SUBTRACTION

- Solve the following using one line addition method $4234 + 8238 + 646 + 5321 + 350$.

Solution:

Step 1:

Start adding the last digit from the right.

During inning total, don't exceed 10. When we exceed 10, make a tick anywhere near about our calculation and go about with the number exceeding 10.

$$4234 + 8238 + 646 + 5321 + 350 = \dots 9$$

Step 2:

Add the number of ticks with the digits in the second place.

$$4234 + 8238 + 646 + 5321 + 350 = \dots 89$$

Step 3:

Add the number of ticks with the digits in the third place.

$$4234 + 8238 + 646 + 5321 + 350 = 789$$

Step 4:

Add the number of ticks with the digits in the fourth place.

$$4234 + 8238 + 646 + 5321 + 350 = 18789$$

$$\therefore 4234 + 8238 + 646 + 5321 + 350 = 18789$$

- Simplify $412 - 83 + 70$

Solution:

Step 1:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 2$$

Step 2:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 72$$

Step 3:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 472$$

Step 4:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 8472$$

$$53921 + 6308 + 86 + 7025 + 11132 = 78472$$

$$\therefore 53921 + 6308 + 86 + 7025 + 11132 = 78472$$

- Simplify $412 - 83 + 70$

Solution:

Step 1:

To get the unit's digit required, add and subtract the digits in the unit's places according to the sign attached.

$$\text{Here } 2 - 3 + 0 = -1$$

So write as

$$412 - 83 + 70 = \dots (-1)$$

Step 2:

$$\text{Similarly we get } 1 - 8 + 7 = 0$$

$$\therefore 412 - 83 + 70 = (0) (-1)$$

Step 3:

$$412 - 83 + 70 = 4(0) (-1)$$

Step 4:

Replace the above temporary figures by real figures.

To replace (-1) by a +ve digit, borrow from digits in tens.

As the digit is 0, borrow from hundreds

$$\begin{array}{ccc} (-1) & (10)(-1) & (10) \\ (4) & (0) & (-1) \\ 3 & 9 & 9 \end{array}$$

$$\therefore 412 - 83 + 70 = 399$$

- Simplify $89978 - 12345 - 36218$

Solution:

$$\begin{array}{ccccc} \text{Step 1:} & (4) & (1) & (4) & (2) \\ \text{Step 2:} & 4 & 1 & 4 & 1 \\ & 3 & 9 & 9 & 9 \end{array} \quad (-5)$$

$$\therefore 89978 - 12345 - 36218 = 41415$$

- Simplify $28369 + 38962 - 9873$

Solution:

$$\begin{array}{ccccc} \text{Step 1:} & (5) & (7) & (4) & (5) \\ \text{Step 2:} & 5 & 7 & 4 & 5 \end{array} \quad (8)$$

$$\therefore 28369 + 38962 - 9873 = 57458$$

6. Simplify $10789 + 3946 - 2310 - 1223$

Solution:

Whenever we get a value more than 10 on adding and subtracting the digits, we will put the unit's digit and carry over the ten's digit and add it to the positive value.

$$\begin{array}{r} +1 +1 +1 \\ 10 \ 7 \ 8 \ 9 + 3946 - 2310 - 1223 \\ \hline \end{array}$$

$$= (1) (1) (2) (0) (2) = 11202$$

$$\therefore 10789 + 3946 - 2310 - 1223 = 11202$$

7. Simplify $765.819 - 89.003 + 12.038 - 86.89$

Solution:

First the number of digits after the decimal have to be equated.

$$765.819 - 89.003 + 12.038 - 86.89$$

$$\begin{array}{r} -1 -1 -1 -1 +1 \\ = 7 \ 6 \ 5 . 8 \ 1 9 - 89.003 + 12.038 - 86.890 \\ \hline \end{array}$$

$$= 7 (-9) (-8) .0 (-4) (4)$$

$$= 601.964$$

$$\therefore 765.819 - 89.003 + 12.038 - 86.890 = 601.964$$

8. Simplify $792.02 + 101.32 - 306.76$

Solution:

$$792.02 + 101.32 - 306.76$$

$$= (5) (9) (-3) (-4) (-2)$$

$$= 5 \ 8 \ 6 . 5 \ 8$$

$$\therefore 792.02 + 101.32 - 306.78 = 586.58$$

9. Simplify $1 + .1 + .01 + .001$

Solution:

$$1.000 + 0.100 + 0.010 + 0.001 = 1.111$$

10. Simplify $892.7 - 573.07 - 95.007$

Solution:

$$892.700 - 573.070 - 95.007$$

$$= 3 (-7) (-6) (7) (-7) (-7)$$

$$2 \ 2 \ 4 \ 6 \ 2 \ 3$$

$$\therefore 892.700 - 573.070 - 95.007 = 224.623$$

SPLIT AND MERGE

1. Simplify 5358×101

Solution:

Here distributive property $a(b + c) = ab + ac$ is made use of

$$5358 \times 101 = 5358 \times (100 + 1)$$

$$= 535800 + 5358 = 541158$$

2. Simplify 3897×999

Solution:

$$3897 \times 999 = 3897 \times (1000 - 1)$$

$$= 3897000 - 3897 = 3893103$$

3. Simplify 72519×9999

Solution:

$$\begin{aligned} 72519 \times 9999 &= 72519 \times (10000 - 1) \\ &= 725190000 - 72519 = 725117481 \end{aligned}$$

4. Simplify 1397×1397

Solution:

$$1397 \times 1397 = (1400 - 3) (1400 - 3)$$

Here we make use of the formula

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned} \therefore 1397 \times 1397 &= (1400)^2 + (3^2) - 6 \times 1400 \\ &= 1960000 + 9 - 8400 = 1960009 - 8400 \\ &= 1951609 \end{aligned}$$

5. Simplify 12345679×72

Solution:

$$\begin{aligned} 12345679 \times 72 &= (12345679) \times (70 + 2) \\ &= 864197530 + 24691358 = 888888888 \end{aligned}$$

6. Simplify 839478×625

Solution:

$$\begin{aligned} 839478 \times 625 &= 839478 \times \left(\frac{10}{2}\right)^4 \\ &= \frac{839478 \times 10^4}{24} = \frac{8394780000}{16} = 524673750 \end{aligned}$$

$$(a + b)(a - b) = a^2 - b^2$$

1. Evaluate $658^2 - 358^2$

Solution:

$$\begin{aligned} 658^2 - 358^2 &= (658 + 358)(658 - 358) \\ &= 1016 \times 300 = 304800 \end{aligned}$$

2. Evaluate $9717^2 - 283^2$

Solution:

$$\begin{aligned} 9717^2 - 283^2 &= (9717 + 283)(9717 - 283) \\ &= 10000 \times 9434 = 94340000 \end{aligned}$$

3. Evaluate $\frac{476^2 - 424^2}{119^2 - 106^2}$

Solution:

$$\begin{aligned} \frac{476^2 - 424^2}{119^2 - 106^2} &= \frac{(476 + 424)(476 - 424)}{(119 + 106)(119 - 106)} \\ &= \frac{900 \times 52}{225 \times 13} = \frac{900}{225} \times \frac{52}{13} = 4 \times 4 = 16 \end{aligned}$$

4. Evaluate $625^2 - 575^2$

Solution:

$$\begin{aligned} 625^2 - 575^2 &= (625 + 575)(625 - 575) \\ &= 1200 \times 50 = 60000 \end{aligned}$$

QUANTITATIVE ABILITY

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5. Evaluate $\frac{(0.783)^2 - (0.217)^2}{0.566}$

Solution:

$$\begin{aligned} \frac{(0.783)^2 - (0.217)^2}{0.566} &= \frac{(0.783 + 0.217)(0.783 - 0.217)}{0.566} \\ &= \frac{1 \times 0.566}{0.566} = 1 \end{aligned}$$

6. If $64^2 - 36^2 = 20 \times x$ find x.

Solution:

$$\begin{aligned} 64^2 - 36^2 &= 20x \\ \Rightarrow (64 + 36)(64 - 36) &= 20x \\ \Rightarrow 2800 &= 20x \\ \Rightarrow x &= 140 \end{aligned}$$

MULTIPLICATION BY 5 AND 25

To multiply by 5 follow the following 2 steps

- (i) Multiply by 10
- (ii) Divide by 2

To multiply by 25 follow the following 2 steps.

- (i) Multiply by 100
- (ii) Divided by 4

1. Multiply 257892 by 5

Solution:

$$\begin{aligned} 257892 \times 10 &= 2578920 \\ 2578920 \div 2 &= 1289460 \\ \therefore 257892 \times 5 &= 1289460 \end{aligned}$$

2. Multiply 984670 by 5

Solution:

$$\begin{aligned} 984670 \times 10 &= 9846700 \\ 9846700 \div 2 &= 4923350 \\ \therefore 984670 \times 5 &= 4923350 \end{aligned}$$

3. Multiply 12569025 by 25

Solution:

$$\begin{aligned} 12569025 \times 100 &= 1256902500 \\ 1256902500 \div 4 &= 314225625 \\ \therefore 12569025 \times 25 &= 314225625 \end{aligned}$$

4. $857609845 \times 25 = ?$

Solution:

$$\begin{aligned} 857609845 \times 100 &= 85760984500 \\ 85760984500 \div 4 &= 21440246125 \\ \therefore 857609845 \times 25 &= 21440246125 \end{aligned}$$

5. Simplify 7543.572×5

Solution:

$$7543.572 \times 10 = 75435.72$$

75435.75 $\div 2 = 37717.86$

$\therefore 7543.572 \times 2 = 37717.86$

6. $257942.652 \times 25 = ?$

Solution:

$$257942.652 \times 100 = 25794265.2$$

$$25794265.2 \div 4 = 6448566.3$$

$$\therefore 257942.652 \times 25 = 6448566.3$$

MULTIPLICATION WITH 11 TO 13

(1) Multiplication by 11

Step 1: The last digit of the number is put down as the right hand figure of the answer.

Step 2: Each successive digit of the number is added to its neighbour at the right.

1. Simplify 5892×11

Solution:

Step 1: Put down the last figure 5892 as the right hand figure of the answer $\frac{5892 \times 11}{2}$

Step 2: $9 + 2 = 11$ (Put 1 below the line and carry over 1)

$$\begin{array}{r} 5892 \times 11 \\ \hline 12 \end{array}$$

Step 3: $\frac{5892 \times 11}{812}$ ($8 + 9 + 1 = 18$, put 8 below and carry over 1)

Step 4: $\frac{5892 \times 11}{4812}$ ($5 + 8 + 1 = 14$, put 4 below and carry over 1)

Step 5: $\frac{5892 \times 11}{64812}$ ($5 + 1 = 6$, put 6 as the left hand figure)

$$\therefore 5892 \times 11 = 64812$$

2. Evaluate 23145×11

Solution:

Steps: $\frac{23145 \times 11}{5} (5 \times 1 = 5)$

$$\begin{array}{r} 23145 \times 11 \\ \hline 95 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 4595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 54595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 254595 \end{array}$$

$$\therefore 23145 \times 11 = 254595$$

QUANTITATIVE ABILITY

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3. Evaluate 89067×11

Solution:

$$\text{Steps: } 7 \times 1 = 7$$

$$6 + 7 = 13 \text{ (write 3 and carry over 1)}$$

$$0 + 6 + 1 = 7$$

$$9 + 0 = 9$$

$$8 + 9 = 17 \text{ (write 7 and carry over 1)}$$

$$0 + 8 + 1 = 9$$

$$\therefore 89067 \times 11 = 979737$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 266400084 \end{array} \quad (0 \times 2 + 2 = 2)$$

$$\therefore 22200007 \times 12 = 266400084$$

MULTIPLICATION BY 13

To multiply the number by 13

Step 1: Multiply the right hand figure by 3.

Step 2: Table each digit in turn and add to the right neighbour.

1. Simplify 9483×13

Solution:

Step 1: $\frac{9483 \times 13}{9}$ (Treble the right hand figure and write it down)

Step 2: $\frac{9483 \times 13}{79}$ ($8 \times 3 + 3 = 27$, write down 7 and carry over 2)

Step 3: $\frac{9483 \times 13}{279}$ ($4 \times 3 + 8 + 2 = 22$, write down 2 and carry over 2)

Step 4: $\frac{9483 \times 13}{3279}$ ($9 \times 3 + 4 + 2 = 33$, write down 3 and carry over 3)

Step 5: $\frac{9483 \times 13}{13279}$ ($0 \times 3 + 9 + 3 = 12$, write down 12)

$$\therefore 9483 \times 13 = 1,23,279$$

2. Simplify 456789×13

Solution:

Step 1: $\frac{456789 \times 13}{7}$ ($9 \times 3 = 27$, write 7, carry over 2)

$\frac{456789 \times 13}{57}$ ($2 + 24 + 9 = 35$, write 5, carry over 3)

$\frac{456789 \times 13}{257}$ ($3 + 21 + 8 = 32$, write 2, carry over 3)

$\frac{456789 \times 13}{8257}$ ($3 + 18 + 7 = 28$, write 8, carry over 2)

$\frac{456789 \times 13}{38257}$ ($2 + 15 + 6 = 23$, write 3, carryover 2)

$\frac{456789 \times 13}{938257}$ ($2 + 12 + 5 = 19$, write 9, carry over 1)

$\frac{456789 \times 13}{5938257}$ ($1 + 4 = 5$, write 5)

$$\therefore 456789 \times 13 = 5938257$$

MULTIPLICATION: NUMBERS CLOSER TO 100

For example to find 103×104 , we make use of 2 steps.

MULTIPLICATION BY 12

To multiply the number by 12,

Step 1: Double the right hand figure of the number

Step 2: Double each digit in turn and add to the right hand neighbour.

1. Evaluate 5324×12

Solution:

$$\begin{array}{r} 5324 \times 12 \\ \hline 8 \end{array} \quad (\text{Double the right hand figure } 4 \times 2 = 8 \text{ and}$$

write as the right hand figure of the answer)

$$\begin{array}{r} 5324 \times 12 \\ \hline 88 \end{array} \quad (2 \times 2 + 4 = 8, \text{ write down 8})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 888 \end{array} \quad (3 \times 2 + 2 = 8, \text{ write down 8})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 3888 \end{array} \quad (5 \times 2 + 3 = 13, \text{ write down 3, carry over 1})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 63888 \end{array} \quad (0 \times 2 + 5 + 1 = 6, \text{ write down 6})$$

$$\therefore 5324 \times 12 = 63888$$

2. Evaluate 22200007×12

Solution:

$$\begin{array}{r} 22200007 \times 12 \\ \hline 4 \end{array} \quad (7 \times 2 = 14, \text{ write down 4, carry over 1})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 84 \end{array} \quad (1 + 0 + 7 = 8, \text{ write down 8})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 0084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 00084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 400084 \end{array} \quad (2 \times 2 + 0 = 4, \text{ write down 4})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 6400084 \end{array} \quad (2 \times 2 + 2 = 6, \text{ write down 6})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 66400084 \end{array} \quad (2 \times 2 + 2 = 6, \text{ write down 6})$$

QUANTITATIVE ABILITY

(i) Multiply the right side digits $3 \times 4 = 12$

$$\begin{aligned} \text{(ii)} \quad 103 + 4 &= 104 + 3 = 107 \\ \therefore 103 \times 104 &= 10712 \end{aligned}$$

1. Simplify 107×102

Solution:

$$\begin{aligned} 107 + 2 &= 102 + 7 = 109 \\ 7 \times 2 &= 14 \\ \therefore 107 \times 102 &= 10914 \end{aligned}$$

2. Simplify 109×105

Solution:

$$\begin{aligned} 109 + 5 &= 105 + 9 = 114 \\ 9 \times 5 &= 45 \\ \therefore 109 \times 105 &= 11445 \end{aligned}$$

3. Simplify 98×86

Solution:

$$\begin{aligned} 98 &= 100 - 2, 86 = 100 - 14 \\ 98 - 2 &\\ 86 - 14 &\\ 98 - 14 &= 86 - 2 = 84 \\ 2 \times 14 &= 28 \\ \therefore 98 \times 86 &= 8428 \end{aligned}$$

4. Simplify 98×95

Solution:

$$\begin{aligned} 98 &= 100 - 2, 95 = 100 - 5 \\ 98 - 2 &\\ 95 - 5 &\\ 98 - 5 &= 95 - 2 = 93 \\ 2 \times 5 &= 10 \\ \therefore 98 \times 95 &= 9310 \end{aligned}$$

5. Simplify $112 \times 107 + 93 \times 96$

Solution:

$$\begin{aligned} 112 &= 100 + 12, 107 = 100 + 7 \\ 112 + 7 &= 119, 107 + 12 = 119 \\ 12 \times 7 &= 84 \\ \therefore 112 \times 107 &= 11984 \\ 93 &= 100 - 7, 96 = 100 - 4 \\ 93 - 4 &= 89, 96 - 7 = 89 \\ 97 \times 4 &= 28 \\ \therefore 93 \times 96 &= 8928 \\ \therefore 112 \times 107 + 93 \times 96 &= 11984 + 8928 = 20912 \end{aligned}$$

Solution:

Here the vertically and crosswise formula is made use of.
There are 3 steps.

Step 1: Multiply vertically on the right $7 \times 8 = 56$

Write 6 as the last digit and carry over 5

Step 2: Multiply crosswise and add with the carry over $1 \times 8 + 1 \times 7 + 5 = 20$

Write 0 as the middle digit and carry over 2.

Step 3: Multiply vertically on the left and add with the carry over $1 \times 1 + 2 = 3$

Write this as the first digit.

$$\begin{array}{r} 1 \quad | \quad 7 \\ \times \quad | \\ \hline 1 \quad 8 \end{array}$$

$$\therefore 17 \times 18 = 306$$

2. Find the product 87×92

Solution:

$$\begin{array}{r} 8 \quad | \quad 7 \\ \times \quad | \\ \hline 9 \quad 2 \end{array}$$

$$\begin{array}{r} 8004 \\ \hline \end{array}$$

$7 \times 2 = 14 \rightarrow 4$ is the last digit

$8 \times 2 + 9 \times 7 + 1 = 80 \rightarrow 0$ is the middle digit.

$8 \times 9 + 8 = 80 \rightarrow 80$ gives the first 2 digits.

$$\therefore 87 \times 92 = 8004$$

3. Simplify 61×31

Solution:

$$\begin{array}{r} 6 \quad | \quad 1 \\ \times \quad | \\ \hline 3 \quad 1 \end{array}$$

$1 \times 1 = 1 \rightarrow 1$ is the last digit.

$6 \times 1 + 3 \times 1 = 9 \rightarrow 9$ is the middle digit.

$6 \times 3 = 18 \rightarrow 18$ gives the first 2 digits

$$\therefore 61 \times 31 = 1891$$

MULTIPLICATION: TWO DIGIT NUMBERS

1. Simplify 17×18

4. Simplify 33×97

Solution:

$$\begin{array}{r}
 3 & 3 \\
 \times & \\
 9 & 7 \\
 \hline
 3201
 \end{array}$$

$3 \times 7 = 21 \rightarrow 1$ is the last digit and carry over 2.
 $3 \times 7 + 3 \times 9 + 2 = 50 \rightarrow 0$ is the middle digit carry over 5
 $3 \times 9 + 5 = 32$
 $\therefore 33 \times 97 = 3201$

5. Simplify 81×89

Solution:

$$\begin{array}{r}
 8 & 1 \\
 \times & \\
 8 & 9 \\
 \hline
 7209
 \end{array}$$

$1 \times 9 = 9 \rightarrow 9$ is the last digit
 $8 \times 9 + 8 \times 1 = 80 \rightarrow 0$ is the middle digit
 $8 \times 8 + 8 = 72 \rightarrow 72$ is the first 2 digits
 $\therefore 81 \times 89 = 7201$

SESSION - 2

SPEED MATHS - II

MULTIPLICATION: 2 DIGIT NUMBER WITH 3 DIGIT NUMBER

1. Solve 325×17

Solution:

$$\begin{array}{r}
 325 \\
 \times 17 \\
 \hline
 5 \quad \text{5} \times 7 = 35, \text{ write down } 5, \text{ carry over } 3 \\
 25 \quad \text{2} \times 7 + 5 \times 1 + 3 = 22 \text{ write down } 2, \text{ carry over } 2 \\
 525 \quad \text{3} \times 7 + 2 \times 1 + 2 = 25 \text{ write down } 5,
 \end{array}$$

$$\begin{array}{r}
 325 \\
 \times 17 \\
 \hline
 5525 \quad \text{3} \times (1) + 2 = 3 + 2 = 5 \text{ write down } 5
 \end{array}$$

2. Solve 675×78

$$\therefore 325 \times 17 = 5525$$

Solution:

$$\begin{array}{r}
 675 \\
 \times 78 \\
 \hline
 0 \quad 5 \times 8 = 40, \text{ write down } 0 \text{ carry over } 4
 \end{array}$$

$$\begin{array}{r}
 675 \\
 \times 78 \\
 \hline
 50 \quad 7 \times 8 + 5 \times 7 + 4 = 95 \text{ write down } 5, \text{ carry over } 9
 \end{array}$$

$$\begin{array}{r}
 675 \\
 \times 78 \\
 \hline
 650 \quad 6 \times 8 + 7 \times 7 + 9 = 106 \text{ write down } 6, \text{ carry over } 10
 \end{array}$$

$$\begin{array}{r}
 675 \\
 \times 78 \\
 \hline
 52650 \quad 6 \times 7 + 10 = 52 \text{ write down } 52
 \end{array}$$

$$\therefore 675 \times 78 = 52650$$

3. Simplify 95×847

Solution:

$$\begin{array}{r}
 95 \\
 \times 847 \\
 \hline
 5 \quad 5 \times 7 = 35, \text{ write down } 5, \text{ carry over } 3
 \end{array}$$

$$\begin{array}{r}
 95 \\
 \times 847 \\
 \hline
 65 \quad 9 \times 7 + 5 \times 4 + 3 = 86 \text{ write down } 6, \text{ carry over } 8
 \end{array}$$

$$\begin{array}{r}
 95 \\
 \times 847 \\
 \hline
 465 \quad 8 \times 5 + 9 \times 4 + 8 = 84 \text{ write down } 4, \text{ carry over } 9
 \end{array}$$

$$\begin{array}{r}
 95 \\
 \times 847 \\
 \hline
 80465 \quad 8 \times 9 + 8 = 80 \text{ write down } 90
 \end{array}$$

$$\therefore 95 \times 847 = 80465$$

4. Simplify 275×64

Solution:

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 0 \end{array}$$

Step 1: $5 \times 4 = 20$, write down 0 carry over 2

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 0 \ 0 \end{array}$$

Step 2: $7 \times 4 + 6 \times 5 + 2 = 60$ write down 0, carry over 6

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 6 \ 0 \ 0 \end{array}$$

Step 3: $2 \times 4 + 7 \times 6 + 6 = 56$ write down 6, carry over 5

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 17 \ 6 \ 0 \ 0 \end{array}$$

Step 4: $2 \times 6 + 5 = 17$ write down 17

5. Solve
- $251 \times 27 + 362 \times 34$

Solution:

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \end{array}$$

Step 1: $1 \times 7 = 7$, write down 7

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \ 7 \end{array}$$

Step 2: $5 \times 7 + 2 \times 1 = 37$, write down 7 carry over 3

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \ 7 \ 7 \end{array}$$

Step 3: $2 \times 7 + 5 \times 2 + 3 = 27$ write down 7, carry over 2

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 6 \ 7 \ 7 \ 7 \end{array}$$

Step 4: $2 \times 2 + 2 = 6$ write down 6

$$\therefore 251 \times 27 = 6777$$

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 8 \end{array}$$

Step 1: $2 \times 4 = 8$, write down 8

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 0 \ 8 \end{array}$$

Step 2: $6 \times 4 + 2 \times 3 = 30$ write down 0, carry over 3

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 3 \ 0 \ 8 \end{array}$$

Step 3: $3 \times 4 + 6 \times 3 + 3 = 33$ write down 3, carry over 3

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 12 \ 3 \ 0 \ 8 \end{array}$$

Step 4: $3 \times 3 + 3 = 12$ write down 12

$$\therefore 362 \times 34 = 12308$$

$$\text{So, } 251 \times 27 = 362 \times 34 = 6777 + 12308 = 19085$$

MULTIPLICATION: 3 DIGIT NUMBERS

1. Simplify
- 321×132

Solution:

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 2 \end{array}$$

Step 1: $1 \times 2 = 2$

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 7 \ 2 \end{array}$$

Step 2: $2 \times 2 + 3 \times 1 = 7$

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 3 \ 7 \ 2 \end{array}$$

Step 3: $3 \times 2 + 1 \times 1 + 2 \times 3 = 13$ (write down 3, carry over 1)

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 2 \ 3 \ 7 \ 2 \end{array}$$

Step 4: $3 \times 3 + 1 \times 2 + 1 = 12$ (write down 2, carry over 1)

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 4 \ 2 \ 3 \ 7 \ 2 \end{array}$$

Step 5: $3 \times 1 + 1 = 4$

$$\therefore 321 \times 132 = 42372$$

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2. Simplify 275×526

Solution:

$$\begin{array}{r} 275 \\ \downarrow \\ 526 \\ \hline 0 \end{array}$$

3

$$\begin{array}{r} 275 \\ \swarrow \searrow \\ 526 \\ \hline 50 \end{array}$$

carry over 5

$$\begin{array}{r} 275 \\ \swarrow \searrow \\ 526 \\ \hline 650 \end{array}$$

down 6, carry over 5

$$\begin{array}{r} 275 \\ \swarrow \searrow \\ 526 \\ \hline 4650 \end{array}$$

carry over 4

$$\begin{array}{r} 275 \\ \swarrow \searrow \\ 526 \\ \hline 144650 \end{array}$$

$\therefore 275 \times 526 = 144650$

3. Simplify 336×678

Solution:

$$\begin{array}{r} 336 \\ \downarrow \\ 678 \\ \hline 8 \end{array}$$

4

$$\begin{array}{r} 336 \\ \swarrow \searrow \\ 678 \\ \hline 08 \end{array}$$

carry over 7

Step 3:

$$\begin{array}{r} 336 \\ \swarrow \searrow \\ 678 \\ \hline 808 \end{array}$$

$3 \times 8 + 6 \times 6 + 3 \times 7 + 7 = 88$ write down 8, carry over 8

Step 4:

$$\begin{array}{r} 336 \\ \swarrow \searrow \\ 678 \\ \hline 7808 \end{array}$$

$3 \times 7 + 6 \times 3 + 8 = 47$ write down 7, carry over 4

Step 5:

$$\begin{array}{r} 336 \\ \downarrow \\ 678 \\ \hline 227808 \end{array}$$

$3 \times 6 + 4 = 22$, write down 22

$\therefore 336 \times 678 = 227808$

4. Simplify 569×952

Solution:

Step 1:

$$\begin{array}{r} 569 \\ \downarrow \\ 952 \\ \hline 8 \end{array}$$

$9 \times 2 = 18$, write down 8, carry over 1

Step 2:

$$\begin{array}{r} 569 \\ \swarrow \searrow \\ 952 \\ \hline 88 \end{array}$$

carry over 5

$6 \times 2 + 9 \times 5 + 1 = 58$ write down 8,

Step 3:

$$\begin{array}{r} 569 \\ \swarrow \searrow \\ 952 \\ \hline 688 \end{array}$$

$5 \times 2 + 9 \times 9 + 6 \times 5 + 5 = 126$ write down 6, carry over 12

Step 4:

$$\begin{array}{r} 569 \\ \swarrow \searrow \\ 952 \\ \hline 1688 \end{array}$$

1, carry over 9

$5 \times 5 + 6 \times 9 + 12 = 91$ write down 9

Step 5:

$$\begin{array}{r} 569 \\ \downarrow \\ 952 \\ \hline 541688 \end{array}$$

$5 \times 9 + 9 = 54$ write down 54

$\therefore 569 \times 952 = 541688$

5. Simplify $102 \times 304 + 207 \times 121$

Solution:

$$102 \times 304$$

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 8 \end{array}$$

Step 1: $2 \times 4 = 8$, write down 8

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 08 \end{array}$$

Step 2: $0 \times 4 + 2 \times 0 = 0$, write down 0

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 008 \end{array}$$

Step 3: $1 \times 4 + 2 \times 3 + 0 \times 0 = 10$ write down 0, carry over 1

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 1008 \end{array}$$

Step 4: $1 \times 0 + 3 \times 0 + 1 = 1$, write down 1

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 31008 \end{array}$$

Step 5: $1 \times 3 = 3$, write down 3

$$\therefore 102 \times 304 = 31008$$

$$207 \times 121$$

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 7 \end{array}$$

Step 1: $7 \times 1 = 7$, write down 7

Step 2: $0 \times 1 + 7 \times 2 = 14$, write down 4 carry over 1

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 047 \end{array}$$

Step 3: $2 \times 1 + 7 \times 1 + 0 \times 2 + 1 = 10$ write down 0 carry over 1

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 5047 \end{array}$$

Step 4: $2 \times 2 + 1 \times 0 + 1 = 5$ write down 5

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 25047 \end{array}$$

Step 5: $2 \times 1 = 2$

$$\therefore 207 \times 121 = 25047$$

$$102 \times 304 + 207 \times 121$$

$$= 31008 + 25077 = 56055$$

MULTIPLICATION - COMPLEMENTARY NUMBERS

Numbers having the same digits except the right most digits whose sum is 10 are called complementary pairs.

For example: 83, 87; 114, 116, 342, 348 are complementary pairs.

To multiply complementary numbers, there are 2 steps. If the numbers are 94 and 96 multiply the right most digits $4 \times 6 = 24$.

Multiply the first number by the number one up $9 \times 10 = 90$
 $\therefore 94 \times 96 = 9024$

1. Simplify 32×38

Solution:

$$3 \times 4 = 12$$

$$2 \times 8 = 16$$

$$\therefore 32 \times 38 = 1216$$

2. Simplify 87×83

Solution:

$$8 \times 9 = 72$$

$$7 \times 3 = 21$$

$$\therefore 87 \times 83 = 7221$$

3. Simplify 126×124

Solution:

$$12 \times 13 = 156$$

$$6 \times 4 = 24$$

$$\therefore 126 \times 124 = 15624$$

4. Simplify 243×247

Solution:

$$24 \times 25 = 600$$

$$3 \times 7 = 21$$

$$\therefore 243 \times 247 = 60021$$

5. Simplify $92 \times 98 + 67 \times 63$

Solution:

$$92 \times 98 = 9016$$

$$\begin{array}{r} 9 \times 10 = 90 \\ 2 \times 8 = 16 \end{array}$$

$$67 \times 63 = 4221$$

$$\begin{array}{r} 6 \times 7 = 42 \\ 7 \times 3 = 21 \end{array}$$

$$\therefore 92 \times 98 + 67 \times 63$$

$$= 9016 + 4221 = 13237$$

DIVISION – ONE LINE METHOD

1. Divide 38982 by 73

Solution:

73 is the divisor. We put down only the first digit 7 in the divisor column and put the other digit 3 on the top of it. As one digit (3) has been put on the top, we allot one place at the right end of the dividend to the remainder position of the answer and mark it off from the digits by a vertical line.

3	3	8	9	8	2
7		3	3	3	1
	5	3	4	0	

Step 1: Since 3 is less than 7, 38 is the first dividend.
Divide 38 by 7, Q = 5, R = 3.

Write down 5 as the first digit of the quotient and prefix the remainder (3) before 9 of the dividend.

Step 2: Now our dividend is 39. From this the product of the indexed (3) and the first digit of the quotient (5) or $3 \times 5 = 15$ is to be deducted.

The net dividend is $39 - 15 = 24$.

Dividing 24 by 7, Q = 3, R = 3.

Write down 3 as the second digit of the quotient and prefix the remainder 3 before 8 of the dividend.

Step 3: Now, the dividend is 38. From this the product 3×3 is to be subtracted and the net dividend is $38 - 9 = 29$.

Dividing 29 by 7, Q = 4 and R = 1.

Write down 4 as the third digit of the quotient and prefix 1 before 2.

Step 4: Now, the dividend is 12. From this the product $3 \times 4 = 12$ is to be subtracted and we obtain 0 as the remainder.

$$\therefore 38982 \div 73 = 534$$

2. Divide 16384 by 128

Solution:

Take 12 as the first digit and put 8 at the top of it.

Also, allot one place at the right end of the dividend to the remainder position of the answer.

8	1	6	3	8	4
12		4	11		6
	1	2	8		

Step 1: Divide 16 by 12, Q = 1, R = 4

Step 2: $43 - 8 \times 1 = 35$ is the next dividend

Divide 35 by 12, Q = 2, R = 11

Step 3: $118 - 8 \times 2 = 118 - 16 = 102$ is the next dividend.

Divide 102 by 12, Q = 8, R = 6

Step 4: $64 - 8 \times 8 = 64 - 64 = 0$ which is the remainder

$$\text{So } 16384 \div 128 = 128$$

3. Divide 601325 by 76

Solution:

Take 7 as the first digit and put 6 at the top of it.

Also allot one place at the right end of the dividend to the remainder position of the answer.

6	6	0	1	3	2
7		11	6	2	500
	7	9	1	2	.171

Step 1: Divide 60 by 7, Q = 8, R = 4

If we put 8 down, then $41 - 6 \times 8 = 41 - 48$ becomes negative which is absurd.

$$\therefore Q = 7, R = 11.$$

Step 2: $111 - 6 \times 7 = 111 - 42 = 69$ is the next dividend.

Dividing 69 by 7, Q = 9, R = 6

Step 3: $63 - 6 \times 9 = 63 - 54 = 9$ is the next dividend.

Dividing 9 by 7, Q = 1, R = 2

Step 4: $22 - 6 \times 1 = 16$ is the next dividend.

Dividing 16 by 7, Q = 2, R = 2

Step 5: $25 - 6 \times 2 = 25 - 12 = 13$ is the remainder.

If it is to be converted as a decimal, add zeros after 5 and proceed as above.

Dividing 13 by 7, Q = 1, R = 6

$$60 - 6 \times 1 = 54$$

Dividing 54 by 7, Q = 7, R = 5

$$50 - 6 \times 7 = 50 - 42 = 8$$

Dividing 8 by 7, Q = 1, R = 1

The process can be continued for any number of digits after the decimal.

$$\therefore 60132 \div 76 = 7912.171$$

QUANTITATIVE ABILITY

4. Divide 710.014 by 39 (to 4 places of decimals)

Solution:

9	7	1	0	0	1	4	0
3	4		8	2	2	6	7
1	8		2	0	5	4	

Take 3 as the first digit and put 9 at the top of it. Also allot one place before the decimal point at the right end of the dividend and also write the digits after the decimal point.

Step 1: Divide 7 by 3, Q = 2, R = 1

But if Q = 2, the next dividend will become negative. So, Q = 1, R = 4

Step 2: $41 - 9 \times 1 = 41 - 9 = 32$

Dividing 32 by 3, Q = 8, R = 8

$$80 - 9 \times 8 = 80 - 72 = 8$$

Dividing 8 by 3, Q = 2, R = 2

$$20 - 9 \times 2 = 20 - 18 = 2$$

Dividing 2 by 3, Q = 0, R = 2

$$21 - 9 \times 0 = 21 - 0 = 21$$

Dividing 21 by 3, Q = 5, R = 6

$$64 - 9 \times 5 = 64 - 45 = 19$$

Dividing 19 by 3, Q = 4, R = 7

$$\therefore 710.014 \div 39 = 18.2054$$

5. Divide 64932 by 99

Solution:

Take 9 as the first digit and put 9 at the top of it. Also allot one place at the right end of the dividend to the remainder position of the answer.

9	6	4	9	3	2	0	0	0
9	10	10	13	15	15	15	15	
6	5	5	8	7	8	7		

64 is the first dividend.

Divide 64 by 9, Q = 6, R = 10

$$109 - 6 \times 9 = 109 - 54 = 55$$

Divide 55 by 9, Q = 5, R = 10

$$103 - 9 \times 5 = 103 - 45 = 58$$

Divide 58 by 9, Q = 5, R = 13

$$132 - 45 = 87$$

Divide 87 by 9, Q = 8, R = 15

$$150 - 9 \times 8 = 150 - 72 = 78$$

Divide 78 by 9, Q = 7, R = 15

$$150 - 9 \times 7 = 150 - 63 = 87$$

Divide 87 by 9, Q = 8, R = 15

$$150 - 9 \times 8 = 150 - 72 = 78$$

Divide 78 by 9, Q = 7, R = 15

The process can be continued.

$$\therefore 64932 \div 99 = 655.8787$$

PERCENTAGE CALCULATIONS

To calculate the product of 2 numbers, percentage is used.

1. Find the value of 43×78 using percentage.

Solution:

Consider 43% of 78

$$43\% \text{ of } 78 = 10\% \text{ of } 78 + 1\% \text{ of } 78 + 1\% \text{ of } 78 + 1\% \text{ of } 78$$

$$= 7.8 + 7.8 + 7.8 + .78 + .78 + .78 = 33.54$$

43% of 78 = $.43 \times 78$ and the digits for 43×78 will be the same. Now, put back the decimal point

$$\therefore 43 \times 78 = 3354$$

2. Find the product 324×82 using percentage

Solution:

Consider 324% of 82 = 3.24×82

$$= 3 \times 82 + 8.2 + 8.2 + .82 + .82 + .82 + .82$$

$$= 246 + 16 + 3.68 = 265.68$$

$$\therefore 324 \times 82 = 26568$$

3. Find the product 252×151 using percentage

Solution:

Consider 252% of 151

$$252\% \text{ of } 151 = 2.52 \times 151$$

$$= 2 \times 151 + 15.1 + 15.1 + 15.1 + 15.1 + 15.1 + 1.51 + 1.51$$

$$= 302 + 75.5 + 3.02 = 380.52$$

$$\therefore 252 \times 151 = 38052$$

4. Find the percentage value of the ratio $\frac{53}{81}$

Solution:

$$\frac{53}{81} = \frac{40.5 + 8.1 + 4.4}{81}$$

$$= \frac{40.5}{81} + \frac{8.1}{81} + \frac{4.4}{81}$$

$$= 50\% + 10\% + \frac{4.4}{81}$$

$$= 60\% + \frac{4.05}{81} + \frac{0.35}{81}$$

$$= 65\% + 0.43\% = 65.43\%$$

5. Compare $\frac{173}{212}$ with $\frac{181}{241}$ using percentage.

Solution:

$$\begin{aligned}\frac{173}{212} &= \frac{106}{212} + \frac{53}{212} + \frac{10.6}{212} + \frac{3.4}{212} \\ &= 50\% + 25\% + 5\% + \frac{3.4}{212} \\ &= 80\% + 1.6\% = 81.6\end{aligned}$$

$$\begin{aligned}\frac{181}{241} &= \frac{120.5}{241} + \frac{60.25}{241} + \frac{.25}{241} \\ &= 50\% + 25\% + .001\% = 75.001\%\end{aligned}$$

$$81.6 > 75.001\%$$

$$\Rightarrow \frac{173}{212} > \frac{181}{241}$$

SQUARES

1. Find the square of 207

Solution:

$$207^2$$

- (1) $7^2 = 49$, write down 9 as the last digit and carry over 4.
- (2) $2 \times 0 \times 7 + 4 = 4$, write it down in the next position.
- (3) $2 \times 2 \times 7 + 0^2 = 28$, write down 8 in the third position and carry over 2.
- (4) $2 \times 0 \times 2 + 2 = 2$ write down 2 in the fourth place.
- (5) $2^2 = 4$ write down 4

$$\therefore 207^2 = 42849$$

2. Find the square of 897

Solution:

- (1) $7^2 = 49$, write down 9 in the last place and carry over 4.
- (2) $2 \times 9 \times 7 + 4 = 130$, write down 0 in the next place and carry over 13.
- (3) $2 \times 8 \times 7 + 9^2 + 3 = 206$, write down 6 in the next place and carry over 20.
- (4) $2 \times 8 \times 9 + 20 = 164$, write down 4 and carry over 16.
- (5) $8^2 + 16 = 80$, write it down.

$$\therefore (897)^2 = 804609$$

3. Find the square of 8432.

Solution:

- (1) $2^2 = 4$, write down 4 as the last digit.
- (2) $2 \times 3 \times 2 = 12$, write down 2 as the next digit and carry over 1.

- (3) $2 \times 4 \times 2 + 3^2 + 1 = 26$, write down 6 as the next digit and carry over 2.

- (4) $2 \times 8 \times 2 + 2 \times 4 \times 3 + 2 = 58$, write down 8 as the next digit and carry over 5.

- (5) $2 \times 8 \times 3 + 4^2 + 5 = 69$, write down 9 as the next digit and carry over 6.

- (6) $2 \times 8 \times 4 + 6 = 70$, write down 0 as the next digit and carry over 7.

- (7) $8^2 + 7 = 71$, write down 71.

$$\therefore (8432)^2 = 71098624$$

SQUARES OF NUMBERS FROM 51 TO 59

$$(51)^2 = 25 + 1 / 1^2 = 26 / 01 = 2601$$

$$(52)^2 = 25 + 2 / 2^2 = 27 / 04 = 2704$$

$$(53)^2 = 25 + / 3^2 = 28 / 09 = 2809$$

$$(54)^2 = 25 + 4 / 4^2 = 29 / 16 = 2916$$

$$(55)^2 = 25 + 5 / 5^2 = 30 / 25 = 3025$$

$$(59)^2 = 25 + 9 / 9^2 = 34 / 81 = 3481$$

$$1. \quad 57^2 = 25 + 7 / 7^2 = 32 / 49 = 3249$$

SQUARES OF NUMBERS WITH UNIT DIGIT AS 5

$$1. \quad 15^2 = 1 \times (1+1) / 5^2 = 2 / 25 = 225$$

$$2. \quad 35^2 = 3 \times (3+1) / 5^2 = 12 / 25 = 1225$$

$$3. \quad 85^2 = 8 \times 9 / 25 = 7225$$

$$4. \quad 225^2 = 22 \times 23 / 25 = 506 / 25 = 50625$$

SQUARES OF NUMBERS NEARER TO 10^x , $x \in \mathbb{N}$

Here the algebraic formula is used.

$$x^2 = (x^2 - y^2) + y^2 = (x+y)(x-y) + y^2$$

$$1. \quad 98^2 = (98+2)(98-2) + 2^2$$

$$= 9600 + 4 = 9604$$

$$2. \quad (104)^2 = (104+4)(104-4) + 4^2$$

$$= 10800 + 16 = 10816$$

$$3. \quad (1007)^2 = (1007+7)(1007-7) + 7^2$$

$$= 1014000 + 49 = 1014049$$

PRACTICE EXERCISE

1. Find the square of 307.
2. Find the squares of numbers 61 - 69.
3. Find the square of numbers 45 and 235.
4. Find the squares of numbers 96 and 106.

QUANTITATIVE ABILITY

SESSION – 3

SPEED MATHS – III

SQUARE ROOTS

If $y = x^2$, then $x = \sqrt{y}$ is called the square root of y .

Square roots of numbers can be found out

(i) by factorisation method (ii) by division method

1. Evaluate $\sqrt{6084}$ by factorisation method

Solution:

$$6084 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

$$= 2^2 \times 3^2 \times 13^2$$

$$\therefore \sqrt{6084} = \sqrt{2^2 \times 3^2 \times 13^2}$$

$$= 2 \times 3 \times 13 = 78$$

$$\therefore \sqrt{6084} = 78$$

$$\begin{array}{r}
 2 \sqrt{6084} \\
 2 \quad \boxed{3042} \\
 3 \quad \boxed{1521} \\
 3 \quad \boxed{507} \\
 13 \quad \boxed{169} \\
 \end{array}$$

2. Find the square root of 53824 by division method.

Solution:

(i) Mark off all the digits in pairs starting from the unit's digit.

(ii) $2^2 = 4$, remainder 1, bring down the next pair. The dividend is 138.

(iii) Double 2 and put 3 as the unit's digit. The divisor is 43.

(iv) Bring down 24, continue the process.

$$\therefore \sqrt{53824} = 232$$

$$\begin{array}{r}
 2 \sqrt{53824} \\
 2 \quad \boxed{5, 3, 8, 24} \\
 4 \quad \boxed{138} \\
 43 \quad \boxed{129} \\
 462 \quad \boxed{924} \\
 \end{array}$$

3. Evaluate $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\sqrt{248 + \sqrt{51 + \sqrt{169}}} = \sqrt{248 + \sqrt{51 + 13}}$$

$$= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$$

$$\therefore \sqrt{248 + \sqrt{51 + \sqrt{169}}} = 16$$

4. Evaluate $\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}}$

Solution:

$$\begin{aligned}
 \sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}} &= \sqrt{5 \times 5 \times 900} = \sqrt{5 \times 5 \times 30 \times 30} \\
 &= 5 \times 30 = 150
 \end{aligned}$$

5. Find the square root of $\sqrt{175.2976}$

Solution:

$$\begin{array}{r}
 13.24 \\
 1 \quad \boxed{175.2976} \\
 1 \\
 \hline
 075 \\
 69 \\
 \hline
 629 \\
 524 \\
 \hline
 10576 \\
 10576 \\
 \hline
 0
 \end{array}$$

(i) Mark off all the digits in pairs starting from either side of the decimal point.

$$\therefore \sqrt{175.2976} = 13.24$$

6. Find the value of $\sqrt{\frac{0.289}{0.00121}}$

Solution:

$$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$$

7. Find the greatest number of 4 digits which is a perfect square.

Solution:

The greatest number of 4 digits is 9999

$$\begin{array}{r}
 99 \\
 9 \quad \boxed{9999} \\
 81 \\
 \hline
 189 \\
 189 \\
 \hline
 1701 \\
 1701 \\
 \hline
 198
 \end{array}$$

Since 198 is the remainder, $9999 - 198$ will be a perfect square.

$$9999 - 198 = 9801$$

\therefore The greatest number of 4 digits which is a perfect square is 9801.

8. Find the smallest number to be subtracted from 549162 to make it a perfect square.

Solution:

$$\begin{array}{r}
 7 \quad \boxed{549162} \quad (741) \\
 49 \\
 \hline
 144 \quad \boxed{591} \\
 576 \\
 \hline
 1481 \quad \boxed{1562} \\
 1481 \\
 \hline
 81
 \end{array}$$

$\therefore 549162 - 81$ will become a perfect square.

\therefore The smallest number to be subtracted is 81.

9. Simplify $\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001}$

Solution:

$$\begin{aligned}\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} \\ = \sqrt{.0025 \times 2.25 \times .0001} \\ = \sqrt{\frac{25}{10000} \times \frac{225}{100} \times \frac{1}{10000}} \\ = \frac{\sqrt{25 \times 225}}{\sqrt{10000000000}} \\ = \frac{5 \times 15}{100000} = \frac{75}{100000} = 0.00075\end{aligned}$$

10. Find the smallest number by which 5808 should be multiplied to make it a perfect square.

Solution:

$$\begin{aligned}5808 &= 2 \times 2 \times 2 \times 2 \times 11 \times 11 \times 13 \\ &= 2^4 \times 11^2 \times 3\end{aligned}$$

∴ The required smallest number is 3.

$$\begin{array}{r} 5808 \\ 2 \boxed{2} \\ 2 \boxed{2904} \\ 2 \boxed{1452} \\ 2 \boxed{726} \\ 3 \boxed{363} \\ 11 \boxed{121} \\ 11 \end{array}$$

PRACTICE EXERCISE:

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

2. Evaluate: $\sqrt{\frac{0.289}{0.00121}}$

3. Find the greatest number of 4 digits which is a perfect square.

CUBES

Cubes of 2 digit numbers

To calculate the cube value of a 2 digit number.

Step 1: Put down the cube of the ten's digit in a row of 4 figures. Find the ratio between the digits of the given number. So, the other 3 digits in the row are in the geometrical ratio in the exact proportion.

Step 2: Put down under the second and third digits, 2 times of the second and the third digit. Then add up the 2 rows.

1. Calculate 12^3

Solution:

The ten's digit of the number is 1

So we write the first digit as $1^3 = 1$

The ratio between the digits is 1:2, the next digits will be double the previous one.

So the first row is 1 2 4 8

The second ad the third digits are 2 and 4.

So write down 4 and 8 below 2 and 4 and then add.

$$\begin{array}{r} 1 \ 2 \ 4 \ 8 \\ \quad 4 \ 8 \\ \hline 1 \ 7 \ 2 \ 8 \end{array}$$

$$\therefore 12^3 = 1728$$

2. Calculate 21^3

Solution:

$$\begin{array}{r} 8 \ 4 \ 2 \ 1 \\ \quad 8 \ 4 \\ \hline 9 \ 2 \ 6 \ 1 \end{array}$$

$$\left[8 = 2^3, 8 \div 4 = 2, 4 \div 2 = 1, 2 \div 1 = 2 \right]$$

$$\left[4 \times 2 = 8, 2 \times 2 = 4 \right]$$

$$\therefore 21^3 = 9261$$

3. Calculate 34^3

Solution:

$$\begin{array}{r} 27 \ 36 \ 48 \ 64 \\ \quad 72 \ 96 \\ \hline 39 \ 12^3 \ 150 \ 6^4 \end{array}$$

$$\left[3^3 = 27 \text{ and the ratio is } 3 : 4 \text{ and so the next number is } \frac{4}{3} \text{ or the previous } \right]$$

$$\therefore 34^3 = 39304$$

4. Calculate 93^3

Solution:

$$\begin{array}{r} 729 \ 243 \ 81 \ 27 \\ \quad 486 \ 162 \\ \hline 804 \ 753 \ 245 \ 27 \end{array}$$

$$\left[9^3 = 729, 9 : 3 = 3 : 1 \right]$$

∴ Each term is obtained by dividing by 3

$$\therefore 93^3 = 804357$$

5. Calculate 77^3

Solution:

$$\begin{array}{r} 343 \ 343 \ 343 \ 343 \\ \quad 686 \ 686 \\ \hline 456 \ 1135 \ 106^3 \ 34^3 \end{array}$$

$$\left[7^3 = 343, \text{ ratio } 1 : 1 \right]$$

$$\therefore 77^3 = 456533$$

CUBE ROOTS

If $y = x^3$, then x is called the cube root of y and is written as

$$x = \sqrt[3]{y}$$

1. Find the cube root of 2744.

Solution:

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$= 2^3 \times 7^3$$

$$\therefore \sqrt[3]{2744} = 2 \times 7$$

$$= 14$$

$$\begin{array}{r} 2 | 2744 \\ 2 | 1372 \\ 2 | 686 \\ 7 | 343 \\ 7 | 49 \\ \hline & 7 \end{array}$$

2. Find the cube root of 0.000512

Solution:

$$\sqrt[3]{0.000512} = \sqrt[3]{\frac{512}{1000000}} = \frac{\sqrt[3]{512}}{100}$$

$$\sqrt[3]{512} = \sqrt{2^9}$$

$$\therefore \sqrt[3]{512} = 2^3 = 8$$

$$\therefore \sqrt[3]{0.000512} = \frac{8}{100} = 0.08$$

$$\begin{array}{r} 2 | 512 \\ 2 | 256 \\ 2 | 128 \\ 2 | 64 \\ 2 | 32 \\ 2 | 16 \\ 2 | 8 \\ 2 | 4 \\ \hline & 2 \end{array}$$

3. Evaluate $\sqrt[3]{4 \frac{12}{125}}$

Solution:

$$\sqrt[3]{4 \frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \frac{\sqrt[3]{512}}{\sqrt[3]{125}} = \frac{8}{5} = 1 \frac{3}{5^2}$$

4. Find the smallest number by which 3600 be divided to make it a perfect cube.

Solution:

$$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$= 2^4 \times 3^2 \times 5^2$$

\therefore 3600 should be divided by $2 \times 3^2 \times 5^2$ to make it a perfect cube 8.

$$\therefore \text{The required number is } 2 \times 9 \times 25 = 450$$

$$\begin{array}{r} 2 | 3600 \\ 2 | 1800 \\ 2 | 900 \\ 2 | 450 \\ 5 | 225 \\ 3 | 45 \\ 3 | 15 \\ \hline & 5 \end{array}$$

5. Find the largest 5 digit number which is a perfect cube.

Solution:

The largest 5 digit cube is $46^3 = 97336$

PRACTICE EXERCISE - II

- Calculate 23^3 .
- Calculate 97^3 .
- Find the cube root of 5832.
- Find the cube root of 0.000216.
- Find the smallest number by which 33275 be divided to make it a perfect cube.

SIMPLIFICATION

Introduction

While performing simplification greatest care has to be taken to avoid mistakes. Developing ability to calculate is one of the major thrust areas. Developing the calculation speed is very important. Addition perhaps is the critical skill to develop the calculations. Try to practise adding 2 digit numbers to improve the calculation speed. Make sure that you are through with addition before attempting the process of subtraction. The multiplication methods followed in speed maths is useful for fast calculations.

For simplification BODMAS rule depicts the correct sequence in which the operations are to be executed so as to find out the value of a given expression.

B – Bracket, O – Of, D – Division, M – Multiplication, A – Addition, S – Subtraction.

When you simplify an expression, first the brackets have to be removed and then the other operations in the given order.

1. Simplify $100 + 20 \times 55$

Solution:

$$\begin{aligned} 100 + 20 \times 55 &= 100 + 1100 \\ &= 1200 \end{aligned}$$

2. Simplify $50040 \div 139 - 60$

Solution:

$$\begin{aligned} &= \frac{50040}{139} - 60 \\ &= 360 - 60 = 300 \end{aligned}$$

$$\begin{array}{r} 9 | 5 0 0 4 | 0 \\ 13 | \quad 115 \\ \hline & 3 6 0 \end{array}$$

3. Simplify $\frac{480 \times 15 - 12 \times 20 + 7 \times 60}{140 \times 8 + 2 \times 55}$

Solution:

$$\begin{aligned} &\frac{480 \times 15 - 12 \times 20 + 7 \times 60}{140 \times 8 + 2 \times 55} = \frac{7200 - 240 + 420}{1120 + 110} \\ &= \frac{6960 + 420}{1230} = \frac{7380}{130} = 6 \end{aligned}$$

4. Simplify $\frac{3}{5} \text{ of } \frac{4}{7} \text{ of } \frac{5}{9} \text{ of } \frac{21}{24} \text{ of } 504$.

Solution:

$$\begin{aligned} &\frac{3}{5} \text{ of } \frac{4}{7} \text{ of } \frac{5}{9} \text{ of } \frac{21}{24} \text{ of } 504 \\ &= \frac{3}{5} \times \frac{4}{7} \times \frac{5}{9} \times \frac{21}{24} \times 504 \\ &= 84 \end{aligned}$$

5. Simplify $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3}$

Solution:

$$\begin{aligned} & \frac{9}{2} \times \frac{13}{3} - \frac{25}{3} \times \frac{3}{17} \\ &= \frac{39}{2} - \frac{25}{17} = \frac{663 - 50}{34} = \frac{613}{34} = 18\frac{1}{34} \end{aligned}$$

6. Simplify $0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004)$

Solution:

$$\begin{aligned} & 0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004) \\ &= 0.00008 \times \frac{0.0072}{0.000048} \\ &= \frac{0.8 \times 0.0072}{0.48} = \frac{8}{10} \times \frac{72}{10000} \times \frac{100}{48} = \frac{12}{1000} = 0.012 \end{aligned}$$

7. If $3 - [1.6 - \{3.2 - (3.2 + 2.25 \div x)\}] = 0.65$, find the value of x.

Solution:

$$\begin{aligned} & 3 - [1.6 - \{3.2 - (3.2 + 2.25 \div x)\}] = 0.65 \\ & \Rightarrow 3 - \left[1.6 - \left\{ 3.2 - \left(3.2 + \frac{2.25}{x} \right) \right\} \right] = 0.65 \\ & \Rightarrow 3 - \left[1.6 - \left(3.2 - 3.2 - \frac{2.25}{x} \right) \right] = 0.65 \\ & \Rightarrow 3 - \left[1.6 + \frac{2.25}{x} \right] = 0.65 \\ & \Rightarrow 3 - 1.6 - \frac{2.25}{x} = 0.65 \\ & \Rightarrow 1.4 - \frac{2.25}{x} = 0.65 \\ & \Rightarrow \frac{2.25}{x} = 1.4 - 0.65 = 0.75 \\ & \Rightarrow x = \frac{2.25}{0.75} = \frac{225}{75} = 3 \end{aligned}$$

8. Simplify $\frac{2 + \frac{1}{4}}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4}}}}$

Solution:

$$\begin{aligned} & \frac{2 + \frac{1}{4}}{\frac{3}{5}} = \frac{2 + \frac{1}{19}}{\frac{5}{1}} = \frac{2 + \frac{5}{19}}{2 + \frac{1}{3 + \frac{1}{4}}} \\ &= \frac{2 + \frac{5}{19}}{2 + \frac{1}{2 + \frac{1}{3 + \frac{5}{4}}}} = \frac{2 + \frac{5}{19}}{2 + \frac{1}{2 + \frac{1}{3 + \frac{5}{4}}}} \\ &= \frac{2 + \frac{5}{19}}{2 + \frac{1}{2 + \frac{1}{2 + \frac{5}{19}}}} = 1 \end{aligned}$$

9. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$, where x, y, z are natural numbers,

find x, y, z.

Solution:

$$\begin{aligned} \frac{37}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \\ \Rightarrow \frac{37}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \\ \Rightarrow 2 + \frac{11}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \end{aligned}$$

$$\Rightarrow \frac{11}{13} = \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{13}{11} = 1 + \frac{2}{11}$$

$$\Rightarrow x = 1, \frac{1}{y + \frac{1}{z}} = \frac{2}{11} \Rightarrow x = 1, y + \frac{1}{z} = \frac{11}{2}$$

$$\Rightarrow x = 1, y + \frac{1}{z} = 5 + \frac{1}{2} \Rightarrow x = 1, y = 5, z = 2$$

 $\therefore x, y, z$ are 1, 5, 2.

10. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find $y - x$.

Solution:

$$\begin{cases} 4x + 5y = 83 \\ \frac{3x}{2y} = \frac{21}{22} \end{cases}$$

$$\Rightarrow 66x = 42y \Rightarrow y = \frac{66}{42}x = \frac{11}{7}x$$

$$4x + 5 \times \frac{11}{7}x = 83$$

$$\Rightarrow 28x + 55x = 581$$

$$\Rightarrow 83x = 581 \Rightarrow x = 7$$

$$4x + 5y = 83 \Rightarrow 4 \times 7 + 5y = 83 \Rightarrow 5y = 55 \Rightarrow y = 11$$

$$\therefore y - x = 11 - 7 = 4$$

11. Find the sum

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$$

Solution:

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$$

QUANTITATIVE ABILITY

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$$\begin{aligned}
 &= \frac{1}{2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \frac{1}{6.7} \\
 &\quad + \frac{1}{7.8} + \frac{1}{8.9} + \frac{1}{9.10} + \frac{1}{10.11} + \frac{1}{11.12} \\
 &= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) \\
 &\quad + \left(\frac{1}{7} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{11}\right) + \left(\frac{1}{11} - \frac{1}{12}\right) \\
 &= \frac{1}{2} + \frac{1}{2} - \frac{1}{12} = 1 - \frac{1}{12} = \frac{11}{12}
 \end{aligned}$$

12. Simplify $\frac{(856+167)^2 + (856-167)^2}{856 \times 856 + 167 \times 167}$

Solution:

$$\begin{aligned}
 &\frac{(856+167)^2 + (856-167)^2}{856^2 + 167^2} \\
 &= \frac{2[856^2 + 167^2]}{856^2 + 167^2} \\
 &\quad [\text{use the formula } (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)] \\
 &= 2
 \end{aligned}$$

13. Simplify $\frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \div \frac{\frac{3}{2} - 2\frac{1}{2}}{\frac{4}{3} - 3\frac{1}{3}}$

Solution:

$$\begin{aligned}
 &\frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \div \frac{\frac{3}{2} - 2\frac{1}{2}}{\frac{4}{3} - 3\frac{1}{3}} \\
 &= \frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \times \frac{\left(\frac{4}{3}\right) - \left(\frac{3}{3}\right)}{\left(\frac{3}{2}\right) - \left(\frac{2}{2}\right)} \\
 &= \frac{\frac{3}{2} + 2\frac{1}{2}}{\frac{4}{3} + 3\frac{1}{3}} \\
 &\quad [\text{using the formula } a^2 - b^2 = (a-b)(a+b)]
 \end{aligned}$$

$$\frac{\frac{11}{4} + \frac{5}{3}}{\frac{19}{4} + \frac{10}{3}} = \frac{\frac{22+15}{6}}{\frac{57+40}{12}} = \frac{37}{6} \times \frac{12}{97} = \frac{74}{97}$$

14. Simplify $\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2}$

Solution:

$$\begin{aligned}
 &\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2} = \frac{\left[\left(\frac{3}{5}\right) - \left(\frac{2}{5}\right)\right] \left[\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \frac{3}{5} \times \frac{2}{5}\right]}{\left[\left(\frac{3}{5}\right) - \left(\frac{2}{5}\right)\right] \left[\left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)\right]} \\
 &= \frac{\frac{9}{25} + \frac{4}{25} + \frac{6}{25}}{\frac{3}{5} + \frac{2}{5}} = \frac{\frac{19}{25}}{\frac{5}{5}} = \frac{19}{25}
 \end{aligned}$$

PRACTICE EXERCISE

1. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$ where x, y, z are natural numbers find x, y, z.

2. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find y - x.

3. Simplify $\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2}$

4. Find the sum $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \dots + \frac{1}{132}$

SESSION - 4

SPEED MATHS - IV

PRACTICE EXERCISE

- Add 707325, 192382, 58009, 564943 and 656.
(a) 1523315 (b) 331969 (c) 1623225 (d) 3129621
- Simplify: $5124 - 829 + 731 - 435$
(a) 4732 (b) 4591 (c) 3921 (d) 3865
- Solve: 5732×1002
(a) 5743464 (b) 5742464 (c) 5843464 (d) 5925461
- Evaluate: $8492^2 - 508^2$
(a) 70856000 (b) 7284600 (c) 71856000 (d) 70856000
- Multiply 84726 by 5.
(a) 435630 (b) 432650 (c) 423630 (d) 413630
- Multiply 625 by 25.
(a) 15625 (b) 30625 (c) 15725 (d) 120625

QUANTITATIVE ABILITY – SOLUTION

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SESSION – 2

SPEED MATHS - II

PRACTICE EXERCISE

1. Find the square of 307.

Solution:

$$307^2$$

(1) $7^2 = 49$, write down 9 as the last digit and carry over 4.

(2) $3 \times 0 \times 7 + 4 = 4$, write it down in the next position.

(3) $2 \times 3 \times 7 + 0^2 = 42$, write down 2 in the third position and carry over 4.

(4) $3 \times 0 \times 3 + 4 = 4$ write down 4 in the fourth place.

(5) $3^2 = 9$ write down 9

$$\therefore 307^2 = 94249$$

2. Find the squares of numbers 61 – 69.

Solution:

$$(61)^2 = 25 + 11 / 11^2 = 36 / 121 = 3721$$

$$(62)^2 = 25 + 12 / 12^2 = 37 / 144 = 3844$$

$$(63)^2 = 25 + 13 / 13^2 = 38 / 169 = 3969$$

$$(64)^2 = 25 + 14 / 14^2 = 39 / 196 = 4096$$

$$(65)^2 = 25 + 15 / 15^2 = 40 / 225 = 4225$$

$$(66)^2 = 25 + 16 / 16^2 = 41 / 256 = 4356$$

$$(67)^2 = 25 + 17 / 17^2 = 42 / 289 = 4489$$

$$(68)^2 = 25 + 18 / 18^2 = 43 / 324 = 4624$$

$$(69)^2 = 25 + 19 / 19^2 = 44 / 361 = 4761$$

3. Find the square of numbers 45 and 235.

Solution:

$$45^2 = 4 \times (4+1) / 5^2 = 20 / 25 = 2025$$

$$235^2 = 23 \times 24 / 25 = 552 / 25 = 55225$$

4. Find the squares of numbers 96 and 106.

Solution:

$$96^2 = (96+4)(96-4) + 4^2 \\ = 9200 + 16 = 9216$$

$$(106)^2 = (106+6)(106-6) + 6^2 \\ = 11200 + 36 = 11236$$

SESSION – 3

SPEED MATHS - III

PRACTICE EXERCISE – I

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\sqrt{248 + \sqrt{51 + \sqrt{169}}} = \sqrt{248 + \sqrt{51 + 13}}$$

$$= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$$

$$\therefore \sqrt{248 + \sqrt{51 + \sqrt{169}}} = 16$$

2. Evaluate: $\sqrt{\frac{0.289}{0.00121}}$

Solution:

$$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$$

Find the greatest number of 4 digits which is a perfect square.

Solution:

The greatest number of 4 digits is 9999

$$\begin{array}{r}
 99 \\
 9 \overline{)9999} \\
 81 \\
 \hline
 189 \\
 189 \\
 \hline
 1701 \\
 \hline
 198
 \end{array}$$

Since 198 is the remainder, $9999 - 198$ will be a perfect square.

$$9999 - 198 = 9801$$

\therefore The greatest number of 4 digits which is a perfect square is 9801.

PRACTICE EXERCISE – II

1. Calculate 23^3 .

Solution:

The ten's digit of the given number is 2.

So, we write the first number in the first row as 8.

The ratio between the digits is 2:3 and so the next three numbers should also be in the same ratio with the previous one.

If the next number is a two digit number, the ten's digit is written as carryover below the previous number.

In the second row, twice the second and third numbers are written below them in the same manner.

	8	2	8	7
	1	1	2	
		4	6	
	2	3		
1	2	1	6	7

QUANTITATIVE ABILITY – SOLUTION

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2. Calculate 97^3 .

Solution:

$$\begin{array}{r}
 729 & 567 & 441 & 343 \\
 1134 & 882 & & \\
 \hline
 729 & 1701 & 1323 & 343
 \end{array}$$

$$\therefore 97^3 = 912673$$

3. Find the cube root of 5832.

Solution:

$$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 2^3 \times 3^3 \times 3^3$$

$$\therefore \sqrt[3]{5832} = 2 \times 3 \times 3$$

$$= 18$$

4. Find the cube root of 0.000216.

Solution:

$$\sqrt[3]{0.000216} = \sqrt[3]{\frac{216}{1000000}} = \frac{\sqrt[3]{216}}{100}$$

$$\sqrt[3]{216} = \sqrt{2^3 \times 3^3}$$

$$\therefore \sqrt[3]{216} = 2 \times 3 = 6$$

$$\therefore \sqrt[3]{0.000216} = \frac{6}{100} = 0.06$$

$$\begin{array}{r}
 2 \boxed{216} \\
 2 \boxed{108} \\
 2 \boxed{54} \\
 3 \boxed{27} \\
 3 \boxed{9} \\
 3
 \end{array}$$

5. Find the smallest number by which 33275 be divided to make it a perfect cube.

Solution:

$$33275 = 5 \times 5 \times 11 \times 11 \times 11$$

$$= 5^2 \times 11^3$$

\therefore 33275 should be divided by 5^2 to make it a perfect cube.

$$\begin{array}{r}
 5 \boxed{33275} \\
 5 \boxed{6655} \\
 11 \boxed{1331} \\
 11 \boxed{121} \\
 11
 \end{array}$$

\therefore The required number is $5^2 = 25$

SIMPLIFICATION

PRACTICE EXERCISE

1. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$ where x, y, z are natural numbers

find x, y, z.

Solution:

$$\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow \frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow 2 + \frac{11}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow \frac{11}{13} = \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{13}{11} = 1 + \frac{2}{11}$$

$$\Rightarrow x = 1, \frac{1}{y + \frac{1}{z}} = \frac{2}{11} \Rightarrow x = 1, y + \frac{1}{z} = \frac{11}{2}$$

$$\Rightarrow x = 1, y + \frac{1}{z} = 5 + \frac{1}{2} \Rightarrow x = 1, y = 5, z = 2$$

$\therefore x, y, z$ are 1, 5, 2.

2. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find $y - x$.

Solution:

$$\left. \begin{array}{l} 4x + 5y = 83 \\ \frac{3x}{2y} = \frac{21}{22} \end{array} \right\}$$

$$\Rightarrow 66x = 42y \Rightarrow y = \frac{66}{42}x = \frac{11}{7}x$$

$$4x + 5 \times \frac{11}{7}x = 83$$

$$\Rightarrow 28x + 55x = 581$$

$$\Rightarrow 83x = 581 \Rightarrow x = 7$$

$$4x + 5y = 83 \Rightarrow 4 \times 7 + 5y = 83 \Rightarrow 5y = 55 \Rightarrow y = 11$$

$$\therefore y - x = 11 - 7 = 4$$

3. Find the sum $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$

$$\begin{aligned}
 &= \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} \\
 &\quad + \frac{1}{8 \cdot 9} + \frac{1}{9 \cdot 10} + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} \\
 &= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{6} \right) \\
 &\quad + \left(\frac{1}{6} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{10} \right) + \left(\frac{1}{10} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{12} \right)
 \end{aligned}$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{12} = 1 - \frac{1}{12} = \frac{11}{12}$$

SESSION - 2

SPEED MATHS - II

PRACTICE EXERCISE

1. Find the square of 307.

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$$307^2$$

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Solution:

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SESSION - 3

SPEED MATHS - III

PRACTICE EXERCISE - I

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\sqrt{248 + \sqrt{51 + \sqrt{169}}} = \sqrt{248 + \sqrt{51 + 13}}$$

$$= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$$

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2.

$$\text{Evaluate: } \sqrt{\frac{0.289}{0.00121}}$$

Solution:

$$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$$

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The greatest number of 4 digits is 9999

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 9 \overline{)9999} \\
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 198
 \end{array}$$

Since 198 is the remainder, $9999 - 198$ will be a perfect square.

$$9999 - 198 = 9801$$

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2. Calculate 97^3 .

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$$\begin{array}{r} 729 & 567 & 441 & 343 \\ & 1134 & 882 & \\ \hline 729 & 1701 & 1323 & 343 \end{array}$$

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Solution:

$$\sqrt[3]{0.000216} = \sqrt[3]{\frac{216}{1000000}} = \frac{\sqrt[3]{216}}{100}$$

$$\sqrt[3]{216} = \sqrt{2^3 \times 3^3}$$

$$\therefore \sqrt[3]{216} = 2 \times 3 = 6$$

$$\therefore \sqrt[3]{0.000216} = \frac{6}{100} = 0.06$$

$$\begin{array}{r} 2 | 216 \\ 2 | 108 \\ 2 | 54 \\ 3 | 27 \\ 3 | 9 \\ \hline 3 \end{array}$$

5. Find the smallest number by which 33275 be divided to make it a perfect cube.

Solution:

$$33275 = 5 \times 5 \times 11 \times 11 \times 11 \\ = 5^2 \times 11^3$$

\therefore 33275 should be divided by 5^2 to make it a perfect cube.

\therefore The required number is $5^2 = 25$

$$\begin{array}{r} 5 | 33275 \\ 5 | 6655 \\ 11 | 1331 \\ 11 | 121 \\ \hline 11 \end{array}$$

$$\Rightarrow \frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow 2 + \frac{11}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow \frac{11}{13} = \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{13}{11} = 1 + \frac{2}{11}$$

$$\Rightarrow x = 1, \frac{1}{y + \frac{1}{z}} = \frac{2}{11} \Rightarrow x = 1, y + \frac{1}{z} = \frac{11}{2}$$

$$\Rightarrow x = 1, y + \frac{1}{z} = 5 + \frac{1}{2} \Rightarrow x = 1, y = 5, z = 2$$

$\therefore x, y, z$ are 1, 5, 2.

2. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find $y - x$.

Solution:

$$\left. \begin{array}{l} 4x + 5y = 83 \\ \frac{3x}{2y} = \frac{21}{22} \end{array} \right\}$$

$$\Rightarrow 66x = 42y \Rightarrow y = \frac{66}{42}x = \frac{11}{7}x$$

$$4x + 5 \times \frac{11}{7}x = 83$$

$$\Rightarrow 28x + 55x = 581$$

$$\Rightarrow 83x = 581 \Rightarrow x = 7$$

$$4x + 5y = 83 \Rightarrow 4 \times 7 + 5y = 83 \Rightarrow 5y = 55 \Rightarrow y = 11$$

$$\therefore y - x = 11 - 7 = 4$$

3. Find the sum $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$

$$\begin{aligned} &= \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} \\ &\quad + \frac{1}{8 \cdot 9} + \frac{1}{9 \cdot 10} + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} \\ &= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{6} \right) \\ &\quad + \left(\frac{1}{6} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{10} \right) + \left(\frac{1}{10} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{12} \right) \end{aligned}$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{12} = 1 - \frac{1}{12} = \frac{11}{12}$$

SIMPLIFICATION

PRACTICE EXERCISE

1. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$ where x, y, z are natural numbers

find x, y, z.

Solution:

$$\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

10. Multiply 95 with 83.

Ans: [b]

$$95 = 100 - 5$$

$$83 = 100 - 17$$

$$95 - 17 = 83 - 5 = 78$$

$$17 \times 5 = 85$$

$$\therefore 83 \times 95 = 7885$$

11. Find the value of 147×143 .

Ans: [a]

$$\begin{array}{r} 147 \\ \downarrow \\ 143 \\ \hline 1 \end{array}$$

Step 1: 2 carry

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 21 \end{array}$$

Step 2: $28 + 12 + 2 = 42$ carry '4'

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 021 \end{array}$$

Step 3: In 30 place '0' 3 as carry

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 1021 \end{array}$$

Step 4: $4+4+3=11$ 1 as carry

$$\begin{array}{r} 147 \\ \downarrow \\ 143 \\ \hline 11021 \end{array}$$

Step 5: $3 \times 1 + 1 = 4$

$$\therefore 147 \times 143 = 11021$$

12. Find the value of 671×359 .

Ans: [d]

$$\begin{array}{r} 671 \\ \downarrow \\ 359 \\ \hline 9 \end{array}$$

Step 1:

$$\begin{array}{r} 671 \\ \times 359 \\ \hline 89 \end{array}$$

Step 2: $63 + 5 = 68$ 6 carry

$$\begin{array}{r} 671 \\ \times 359 \\ \hline 0889 \end{array}$$

Step 3: $= 30 + 21 + 9 = 60$ 6 as carry

$$\begin{array}{r} 671 \\ \downarrow \\ 359 \\ \hline 240889 \end{array}$$

Step 4: $18 + 6 = 24$

$$\therefore 671 \times 359 = 240889$$

13. Multiply 678 by 37.

Ans: [a]

$$\begin{array}{r} 678 \\ \times 37 \\ \hline 25086 \end{array}$$

14. Divide 45625 by 73.

Ans: [b]

$$45625 \div 73$$

$$\begin{array}{r} 45625 \\ 73 \quad 9125 \\ \swarrow \quad \searrow \\ 5 \quad 1825 \\ \swarrow \quad \searrow \\ 5 \quad 365 \\ \swarrow \quad \searrow \\ 5 \quad 73 \end{array}$$

$$\frac{5^4 \times 73}{73} = 5^4 = 625$$

15. Divide $718.589 \div 96$ to 3 places of decimals.

Ans: [a]

16. Find the value of 92×67 using percentage.

Ans: [b]

$$\begin{array}{r} 92 \\ (x) \\ \times 67 \\ \hline 6164 \end{array}$$

17. Find the largest number of 3 digits which is a perfect square.

Ans: [a]

Largest 3 digit number is 999.

So we find out square root of 999 by long division method and check whether it is a perfect square or root.

$$\begin{array}{r} 31 \\ \sqrt{999} \\ 3 \quad 99 \\ +3 \quad 9 \\ \hline 61 \quad 99 \\ \quad \quad 61 \\ \hline 38 \end{array}$$

So, $999 - 38 = 961$

961 is a perfect square.

18. Find the value of $\sqrt{0.0025} \times \sqrt{2.25} \times \sqrt{0.0001}$.

Ans: [d]

$$0.05 \times 1.5 \times 0.01$$

$$0.00075$$

None of these

Speed Maths

Square & Square root

1. Find the square of 207.

Solution:

$$207^2$$

- (1) $7^2 = 49$, write down 9 as the last digit and carry over 4.
 - (2) $2 \times 0 \times 7 + 4 = 4$, write it down in the next position.
 - (3) $2 \times 2 \times 7 + 0^2 = 28$, write down 8 in the third position and carry over 2.
 - (4) $2 \times 0 \times 2 + 2 = 2$ write down 2 in the fourth place.
 - (5) $2^2 = 4$ write down 4
- $\therefore 207^2 = 42849$

2. Find the square of 897.

Solution:

(1) $7^2 = 49$, write down 9 in the last place and carry over 4.

(2) $2 \times 9 \times 7 + 4 = 130$, write down 0 in the next place and carry over 13.

(3) $2 \times 8 \times 7 + 9^2 + 3 = 206$, write down 6 in the next place and carry over 20.

(4) $2 \times 8 \times 9 + 20 = 164$, write down 4 and carry over 16.

(5) $8^2 + 16 = 80$, write it down.

$$\therefore (897)^2 = 804609$$

3. Find the square of 8432.

Solution:

(1) $2^2 = 4$, write down 4 as the last digit.

(2) $2 \times 3 \times 2 = 12$, write down 2 as the next digit and carry over 1.

(3) $2 \times 4 \times 2 + 3^2 + 1 = 26$, write down 6 as the next digit and carry over 2.

(4) $2 \times 8 \times 2 + 2 \times 4 \times 3 + 2 = 58$, write down 8 as the next digit and carry over 5.

(5) $2 \times 8 \times 3 + 4^2 + 5 = 69$, write down 9 as the next digit and carry over 6.

(6) $2 \times 8 \times 4 + 6 = 70$, write down 0 as the next digit and carry over 7.

(7) $8^2 + 7 = 71$, write down 71.

$$\therefore (8432)^2 = 71098624$$

$$(51)^2 = 25 + 1 / 1^2 = 26 / 01 = 2601$$

$$(52)^2 = 25 + 2 / 2^2 = 27 / 04 = 2704$$

$$(53)^2 = 25 + 3 / 3^2 = 28 / 09 = 2809$$

$$(54)^2 = 25 + 4 / 4^2 = 29 / 16 = 2916$$

$$(55)^2 = 25 + 5 / 5^2 = 30 / 25 = 3025$$

$$(59)^2 = 25 + 9 / 9^2 = 34 / 81 = 3481$$

1. $57^2 = 25 + 7 / 7^2 = 32 / 49 = 3249$

$$1. \quad 15^2 = 1 \times (1+1)/5^2 = 2/25 = 225$$

$$2. \quad 35^2 = 3 \times (3+1)/5^2 = 12/25 = 1225$$

$$3. \quad 85^2 = 8 \times 9/25 = 7225$$

$$4. \quad 225^2 = 22 \times 23/25 = 506/25 = 50625$$

Here the algebraic formula is used.

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

1. $98^2 = (98 + 2)(98 - 2) + 2^2$
 $= 9600 + 4 = 9604$

2. $(104)^2 = (104 + 4)(104 - 4) + 4^2$
 $= 10800 + 16 = 10816$

3. $(1007)^2 = (1007 + 7)(1007 - 7) + 7^2$
 $= 1014000 + 49 = 1014049$

Exercise

1. Find the square of 307.
 2. Find the squares of numbers 61 – 69.
 3. Find the square of numbers 45 and 235.
 4. Find the squares of numbers 96 and 106.

Square root

If $y = x^2$, then $x = \sqrt{y}$ is called the square root of y .

Square roots of numbers can be found out

(i) by factorisation method (ii) by division method

- Evaluate $\sqrt{6084}$ by factorisation method.

Solution:

$$6084 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

$$= 2^2 \times 3^2 \times 13^2$$

$$\therefore \sqrt{6084} = \sqrt{2^2 \times 3^2 \times 13^2} \mid$$

$$= 2 \times 3 \times 13 = 78$$

$$\therefore \sqrt{6084} = 78$$

A long division diagram for finding the square root of 6084. The number 6084 is at the top. A vertical line to the left of the first two digits (60) is labeled '2'. A horizontal line below 60 is labeled '60'. Another vertical line to the left of the next two digits (84) is labeled '2'. A horizontal line below 84 is labeled '30'. A third vertical line to the left of the next two digits (42) is labeled '3'. A horizontal line below 42 is labeled '15'. A fourth vertical line to the left of the next two digits (21) is labeled '3'. A horizontal line below 21 is labeled '5'. A fifth vertical line to the left of the next two digits (13) is labeled '13'. A horizontal line below 13 is labeled '16'. A final horizontal line below 16 is labeled '13'.

2. Find the square root of 53824 by division method.

Solution:

- (i) Mark off all the digits in pairs starting from the unit's digit.
- (ii) $2^2 = 4$, remainder 1, bring down the next pair. The dividend is 138.
- (iii) Double 2 and put 3 as the unit's digit. The divisor is 43.
- (iv) Bring down 24, continue the process.

$$\therefore \sqrt{53824} = 232$$

$$\begin{array}{r} 232 \\ \hline 5, 3, 8, 24 \\ 4 \\ \hline 43 \\ 138 \\ 129 \\ \hline 462 \\ 924 \\ 924 \\ \hline 0 \end{array}$$

3. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\begin{aligned}\sqrt{248 + \sqrt{51 + \sqrt{169}}} &= \sqrt{248 + \sqrt{51 + 13}} \\&= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16 \\&\therefore \sqrt{248 + \sqrt{51 + \sqrt{169}}} = 16\end{aligned}$$

4. Evaluate:

$$\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}}$$

Solution:

$$\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}} = \sqrt{5 \times 5 \times 900} = \sqrt{5 \times 5 \times 30 \times 30} = 5 \times 30 = 150$$

Exercise

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

2. Evaluate: $\sqrt{\frac{0.289}{0.00121}}$

3. Find the greatest number of 4 digits which is a perfect square.

Speed Maths

Cubes & Cube roots

CUBES

Cubes of 2 digit numbers

To calculate the cube value of a 2 digit number.

Step 1: Put down the cube of the ten's digit in a row of 4 figures. Find the ratio between the digits of the given number. So, the other 3 digits in the row are in the geometrical ratio in the exact proportion.

Step 2: Put down under the second and third digits, 2 times of the second and the third digit. Then add up the 2 rows.

1. Calculate 12^3 .

Solution:

The ten's digit of the number is 1

So we write the first digit as $1^3 = 1$

The ratio between the digits is 1:2, the next digits will be double the previous one.

So the first row is 1 2 4 8

The second ad the third digits are 2 and 4.

So write down 4 and 8 below 2 and 4 and then add.

1 2 4 8

$$\begin{array}{r} 4 \ 8 \\ \hline 1 \ 7 \ 2 \ 8 \end{array}$$

$$\therefore 12^3 = 1728$$

2. Calculate 21^3 .

Solution:

8 4 2 1

$$\begin{array}{r} 8 & 4 \\ \underline{-} & 8 & 4 \\ 9 & 2 & 6 & 1 \end{array} \quad [8 = 2^3, 8 \div 4 = 2, 4 \div 2 = 1, 2 \div 1 = 2]$$

$$\therefore 21^3 = 9261 \quad [4 \times 2 = 8, 2 \times 2 = 4]$$

3. Calculate 34^3 .

Solution:

27 36 48 64

72 96

39 123 150 64

$$\therefore 34^3 = 39304$$

4. Calculate 93^3 .

Solution:

$$\begin{array}{r}
 729 & 243 & 81 & 27 \\
 & 486 & 162 \\
 \hline
 804 & 753 & 245 & 27
 \end{array}$$

$$9^3 = 729, 9 : 3 = 3 : 1$$

∴ Each term is obtained by dividing by 3

$$\therefore 93^3 = 804357$$

5. Calculate 77^3 .

Solution:

$$\begin{array}{r}
 343 & 343 & 343 & 343 \\
 + & 686 & 686 \\
 \hline
 456 & 1135 & 1063 & 343
 \end{array}$$

$$7^3 = 343, \text{ ratio } 1 : 1$$

$$\therefore 77^3 = 456533$$

Cube roots

If $y = x^3$, then x is called the cube root of y and is written as $x = \sqrt[3]{y}$

- Find the cube root of 2744.

Solution:

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$= 2^3 \times 7^3$$

$$\therefore \sqrt[3]{2744} = 2 \times 7$$

$$= 14$$

A handwritten diagram showing the long division method for finding the cube root of 2744. The process is as follows:

- 2744 is divided by 2, resulting in 1372.
- 1372 is divided by 2, resulting in 686.
- 686 is divided by 2, resulting in 343.
- 343 is divided by 7, resulting in 49.
- 49 is divided by 7, resulting in 7.

The quotient, 14, is written below the line.

2. Find the cube root of 0.000512.

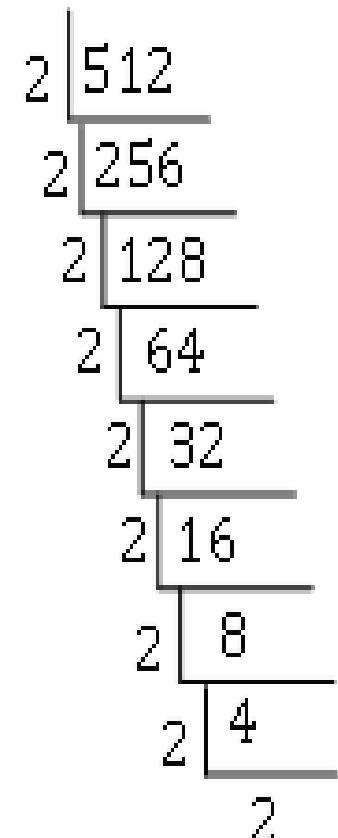
Solution:

$$\sqrt[3]{0.000512} = \sqrt[3]{\frac{512}{1000000}} = \frac{\sqrt[3]{512}}{100}$$

$$\sqrt[3]{512} = \sqrt{2^9}$$

$$\therefore \sqrt[3]{512} = 2^3 = 8$$

$$\therefore \sqrt[3]{0.000512} = \frac{8}{100} = 0.08$$



3. Evaluate $\sqrt[3]{4\frac{12}{125}}$.

Solution:

$$\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \frac{\sqrt[3]{512}}{\sqrt[3]{125}} = \frac{8}{5} = 1\frac{3}{5^2}$$

4. Find the smallest number by which 3600 be divided to make it a perfect cube.

Solution:

$$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$= 2^4 \times 3^2 \times 5^2 |$$

∴ 3600 should be divided by $2 \times 3^2 \times 5^2$ to make it a perfect cube 8.

∴ The required number is $2 \times 9 \times 25 = 450$

2	3600
2	1800
2	900
2	450
5	225
3	45
3	15
	5

CUBES OF 2 DIGIT NUMBERS

Exercise:

1. Calculate 23^3 .

2. Calculate 97^3 .

3. Calculate 103^3 .

4. Calculate 111^3 .

CUBE ROOTS

Exercise:

3. Find the cube root of 5832.

4. Find the cube root of 0.000216.

5. Find the smallest number by which 33275 be divided to make it a perfect square.

QUANTITATIVE ABILITY

HSEM1BTECHSTANDARD0719

SESSION - 1

SPEED MATHS - I

Introduction

Learning Mathematics is a pleasure and working out sums is an interesting activity. If you don't find it interesting, that means you haven't tried to understand it. Since Maths plays an important role in the Competitive Examinations, you should have a firm determination to learn it.

The most basic things in Mathematics are the four fundamental operations – addition, subtraction, multiplication and division. All these are useful and atleast one of them are used in any type of Mathematical question.

So if we do our basic mathematical calculations faster, our valuable time is saved in each question. Speed and accuracy are very important and can be achieved only by constant practice.

Speed Maths helps you to perform the calculations faster than our traditional methods. You should also know the multiplication tables up to 20 and it is always good to practise them.

Learning the one line addition, subtraction, multiplication and division methods also is useful.

ADDITION AND SUBTRACTION

- Solve the following using one line addition method $4234 + 8238 + 646 + 5321 + 350$.

Solution:

Step 1:

Start adding the last digit from the right.

During inning total, don't exceed 10. When we exceed 10, make a tick anywhere near about our calculation and go about with the number exceeding 10.

$$4234 + 8238 + 646 + 5321 + 350 = \dots 9$$

Step 2:

Add the number of ticks with the digits in the second place.

$$4234 + 8238 + 646 + 5321 + 350 = \dots 89$$

Step 3:

Add the number of ticks with the digits in the third place.

$$4234 + 8238 + 646 + 5321 + 350 = 789$$

Step 4:

Add the number of ticks with the digits in the fourth place.

$$4234 + 8238 + 646 + 5321 + 350 = 18789$$

$$\therefore 4234 + 8238 + 646 + 5321 + 350 = 18789$$

- Simplify $412 - 83 + 70$

Solution:

Step 1:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 2$$

Step 2:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 72$$

Step 3:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 472$$

Step 4:

$$53921 + 6308 + 86 + 7025 + 11132 = \dots 8472$$

$$53921 + 6308 + 86 + 7025 + 11132 = 78472$$

$$\therefore 53921 + 6308 + 86 + 7025 + 11132 = 78472$$

- Simplify $412 - 83 + 70$

Solution:

Step 1:

To get the unit's digit required, add and subtract the digits in the unit's places according to the sign attached.

$$Here 2 - 3 + 0 = -1$$

So write as

$$412 - 83 + 70 = \dots (-1)$$

Step 2:

Similarly we get $1 - 8 + 7 = 0$

$$\therefore 412 - 83 + 70 = (0) (-1)$$

Step 3:

$$412 - 83 + 70 = 4(0) (-1)$$

Step 4:

Replace the above temporary figures by real figures.

To replace (-1) by a +ve digit, borrow from digits in tens.

As the digit is 0, borrow from hundreds

$$(1) \qquad \qquad (10)(-1) \qquad \qquad (10)$$

$$(4) \qquad \qquad (0) \qquad \qquad (-1)$$

$$3 \qquad \qquad 9 \qquad \qquad 9$$

$$\therefore 412 - 83 + 70 = 399$$

- Simplify $89978 - 12345 - 36218$

Solution:

$$Step 1: \quad (4) \qquad (1) \qquad (4) \qquad (2) \qquad (-5)$$

$$Step 2: \quad 4 \qquad 1 \qquad 4 \qquad 1 \qquad 5$$

$$\therefore 89978 - 12345 - 36218 = 41415$$

- Simplify $28369 + 38962 - 9873$

Solution:

$$Step 1: \quad (5) \quad (7) \quad (4) \quad (5) \quad (8)$$

$$Step 2: \quad 5 \quad 7 \quad 4 \quad 5 \quad 8$$

$$\therefore 28369 + 38962 - 9873 = 57458$$

QUANTITATIVE ABILITY

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6. Simplify $10789 + 3946 - 2310 - 1223$

Solution:

Whenever we get a value more than 10 on adding and subtracting the digits, we will put the unit's digit and carry over the ten's digit and add it to the positive value.

$$\begin{array}{r} \begin{smallmatrix} +1 & +1 & +1 \\ 10 & 7 & 8 & 9 \end{smallmatrix} \\ + 3946 - 2310 - 1223 \end{array}$$

$$= (1) (1) (2) (0) (2) = 11202$$

$$\therefore 10789 + 3946 - 2310 - 1223 = 11202$$

7. Simplify $765.819 - 89.003 + 12.038 - 86.89$

Solution:

First the number of digits after the decimal have to be equated.

$$765.819 - 89.003 + 12.038 - 86.89$$

$$\begin{array}{r} \begin{smallmatrix} -1 & -1 & -1 & +1 \\ 7 & 6 & 5 & . & 8 & 1 & 9 \end{smallmatrix} \\ - 89.003 + 12.038 - 86.89 \end{array}$$

$$= 7 (-9) (-8) .0 (-4) (4)$$

$$= 601.964$$

$$\therefore 765.819 - 89.003 + 12.038 - 86.89 = 601.964$$

8. Simplify $792.02 + 101.32 - 306.76$

Solution:

$$792.02 + 101.32 - 306.76$$

$$= (5) (9) (-3) (-4) (-2)$$

$$= 5 \quad 8 \quad 6 \quad . \quad 5 \quad 8$$

$$\therefore 792.02 + 101.32 - 306.78 = 586.58$$

9. Simplify $1 + .1 + .01 + .001$

Solution:

$$1.000 + 0.100 + 0.010 + 0.001 = 1.111$$

10. Simplify $892.7 - 573.07 - 95.007$

Solution:

$$892.700 - 573.070 - 95.007$$

$$= 3 (-7) (-6) (7) (-7) (-7)$$

$$\begin{array}{r} \begin{smallmatrix} 2 & 2 & 4 & 6 & 2 & 3 \end{smallmatrix} \\ \dots 892.700 - 573.070 - 95.007 = 224.623 \end{array}$$

SPLIT AND MERGE

1. Simplify 5358×101

Solution:

Here distributive property $a(b + c) = ab + ac$ is made use of

$$5358 \times 101 = 5358 \times (100 + 1)$$

$$= 535800 + 5358 = 541158$$

2. Simplify 3897×999

Solution:

$$3897 \times 999 = 3897 \times (1000 - 1)$$

$$= 3897000 - 3897 = 3893103$$

3. Simplify 72519×9999

Solution:

$$\begin{aligned} 72519 \times 9999 &= 72519 \times (10000 - 1) \\ &= 725190000 - 72519 = 725117481 \end{aligned}$$

4. Simplify 1397×1397

Solution:

$$1397 \times 1397 = (1400 - 3) (1400 - 3)$$

Here we make use of the formula

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned} \therefore 1397 \times 1397 &= (1400)^2 + (3^2) - 6 \times 1400 \\ &= 1960000 + 9 - 8400 = 1960009 - 8400 \\ &= 1951609 \end{aligned}$$

5. Simplify 12345679×72

Solution:

$$\begin{aligned} 12345679 \times 72 &= (12345679) \times (70 + 2) \\ &= 864197530 + 24691358 = 888888888 \end{aligned}$$

6. Simplify 839478×625

Solution:

$$\begin{aligned} 839478 \times 625 &= 839478 \times \left(\frac{10}{2}\right)^4 \\ &= \frac{839478 \times 10^4}{24} = \frac{8394780000}{16} = 524673750 \end{aligned}$$

$$(a + b)(a - b) = a^2 - b^2$$

1. Evaluate $658^2 - 358^2$

Solution:

$$\begin{aligned} 658^2 - 358^2 &= (658 + 358)(658 - 358) \\ &= 1016 \times 300 = 304800 \end{aligned}$$

2. Evaluate $9717^2 - 283^2$

Solution:

$$\begin{aligned} 9717^2 - 283^2 &= (9717 + 283)(9717 - 283) \\ &= 10000 \times 9434 = 94340000 \end{aligned}$$

3. Evaluate $\frac{476^2 - 424^2}{119^2 - 106^2}$

Solution:

$$\begin{aligned} \frac{476^2 - 424^2}{119^2 - 106^2} &= \frac{(476 + 424)(476 - 424)}{(119 + 106)(119 - 106)} \\ &= \frac{900 \times 52}{225 \times 13} = \frac{900}{225} \times \frac{52}{13} = 4 \times 4 = 16 \end{aligned}$$

4. Evaluate $625^2 - 575^2$

Solution:

$$\begin{aligned} 625^2 - 575^2 &= (625 + 575)(625 - 575) \\ &= 1200 \times 50 = 60000 \end{aligned}$$

QUANTITATIVE ABILITY

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5. Evaluate $\frac{(0.783)^2 - (0.217)^2}{0.566}$

Solution:

$$\begin{aligned} \frac{(0.783)^2 - (0.217)^2}{0.566} &= \frac{(0.783 + 0.217)(0.783 - 0.217)}{0.566} \\ &= \frac{1 \times 0.566}{0.566} = 1 \end{aligned}$$

6. If $64^2 - 36^2 = 20 \times x$ find x.

Solution:

$$\begin{aligned} 64^2 - 36^2 &= 20x \\ \Rightarrow (64 + 36)(64 - 36) &= 20x \\ \Rightarrow 2800 &= 20x \\ \Rightarrow x &= 140 \end{aligned}$$

MULTIPLICATION BY 5 AND 25

To multiply by 5 follow the following 2 steps

- (i) Multiply by 10
- (ii) Divide by 2

To multiply by 25 follow the following 2 steps.

- (i) Multiply by 100
- (ii) Divided by 4

1. Multiply 257892 by 5

Solution:

$$\begin{aligned} 257892 \times 10 &= 2578920 \\ 2578920 \div 2 &= 1289460 \\ \therefore 257892 \times 5 &= 1289460 \end{aligned}$$

2. Multiply 984670 by 5

Solution:

$$\begin{aligned} 984670 \times 10 &= 9846700 \\ 9846700 \div 2 &= 4923350 \\ \therefore 984670 \times 5 &= 4923350 \end{aligned}$$

3. Multiply 12569025 by 25

Solution:

$$\begin{aligned} 12569025 \times 100 &= 1256902500 \\ 1256902500 \div 4 &= 314225625 \\ \therefore 12569025 \times 25 &= 314225625 \end{aligned}$$

4. $857609845 \times 25 = ?$

Solution:

$$\begin{aligned} 857609845 \times 100 &= 85760984500 \\ 85760984500 \div 4 &= 21440246125 \\ \therefore 857609845 \times 25 &= 21440246125 \end{aligned}$$

5. Simplify 7543.572×5

Solution:

$$7543.572 \times 10 = 75435.72$$

75435.75 $\div 2 = 37717.86$

$\therefore 7543.572 \times 2 = 37717.86$

6. $257942.652 \times 25 = ?$

Solution:

$$257942.652 \times 100 = 25794265.2$$

$$25794265.2 \div 4 = 6448566.3$$

$$\therefore 257942.652 \times 25 = 6448566.3$$

MULTIPLICATION WITH 11 TO 13

(1) Multiplication by 11

Step 1: The last digit of the number is put down as the right hand figure of the answer.

Step 2: Each successive digit of the number is added to its neighbour at the right.

1. Simplify 5892×11

Solution:

Step 1: Put down the last figure 5892 as the right hand figure of the answer $\frac{5892 \times 11}{2}$

Step 2: $9 + 2 = 11$ (Put 1 below the line and carry over 1)

$$\begin{array}{r} 5892 \times 11 \\ \hline 12 \end{array}$$

Step 3: $\frac{5892 \times 11}{812}$ ($8 + 9 + 1 = 18$, put 8 below and carry over 1)

Step 4: $\frac{5892 \times 11}{4812}$ ($5 + 8 + 1 = 14$, put 4 below and carry over 1)

Step 5: $\frac{5892 \times 11}{64812}$ ($5 + 1 = 6$, put 6 as the left hand figure)

$$\therefore 5892 \times 11 = 64812$$

2. Evaluate 23145×11

Solution:

Steps: $\frac{23145 \times 11}{5} (5 \times 1 = 5)$

$$\begin{array}{r} 23145 \times 11 \\ \hline 95 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 4595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 54595 \end{array}$$

$$\begin{array}{r} 23145 \times 11 \\ \hline 254595 \end{array}$$

$$\therefore 23145 \times 11 = 254595$$

QUANTITATIVE ABILITY

HSEM1BTECHSTANDARD0719

3. Evaluate 89067×11

Solution:

$$\text{Steps: } 7 \times 1 = 7$$

$$6 + 7 = 13 \text{ (write 3 and carry over 1)}$$

$$0 + 6 + 1 = 7$$

$$9 + 0 = 9$$

$$8 + 9 = 17 \text{ (write 7 and carry over 1)}$$

$$0 + 8 + 1 = 9$$

$$\therefore 89067 \times 11 = 979737$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 266400084 \end{array} \quad (0 \times 2 + 2 = 2)$$

$$\therefore 22200007 \times 12 = 266400084$$

MULTIPLICATION BY 13

To multiply the number by 13

Step 1: Multiply the right hand figure by 3.

Step 2: Table each digit in turn and add to the right neighbour.

1. Simplify 9483×13

Solution:

Step 1: $\frac{9483 \times 13}{9}$ (Treble the right hand figure and write it down)

Step 2: $\frac{9483 \times 13}{79}$ ($8 \times 3 + 3 = 27$, write down 7 and carry over 2)

Step 3: $\frac{9483 \times 13}{279}$ ($4 \times 3 + 8 + 2 = 22$, write down 2 and carry over 2)

Step 4: $\frac{9483 \times 13}{3279}$ ($9 \times 3 + 4 + 2 = 33$, write down 3 and carry over 3)

Step 5: $\frac{9483 \times 13}{13279}$ ($0 \times 3 + 9 + 3 = 12$, write down 12)

$$\therefore 9483 \times 13 = 1,23,279$$

2. Simplify 456789×13

Solution:

Step 1: $\frac{456789 \times 13}{7}$ ($9 \times 3 = 27$, write 7, carry over 2)

$\frac{456789 \times 13}{57}$ ($2 + 24 + 9 = 35$, write 5, carry over 3)

$\frac{456789 \times 13}{257}$ ($3 + 21 + 8 = 32$, write 2, carry over 3)

$\frac{456789 \times 13}{8257}$ ($3 + 18 + 7 = 28$, write 8, carry over 2)

$\frac{456789 \times 13}{38257}$ ($2 + 15 + 6 = 23$, write 3, carryover 2)

$\frac{456789 \times 13}{938257}$ ($2 + 12 + 5 = 19$, write 9, carry over 1)

$\frac{456789 \times 13}{5938257}$ ($1 + 4 = 5$, write 5)

$$\therefore 456789 \times 13 = 5938257$$

MULTIPLICATION: NUMBERS CLOSER TO 100

For example to find 103×104 , we make use of 2 steps.

MULTIPLICATION BY 12

To multiply the number by 12,

Step 1: Double the right hand figure of the number

Step 2: Double each digit in turn and add to the right hand neighbour.

1. Evaluate 5324×12

Solution:

$$\begin{array}{r} 5324 \times 12 \\ \hline 8 \end{array} \quad (\text{Double the right hand figure } 4 \times 2 = 8 \text{ and}$$

write as the right hand figure of the answer)

$$\begin{array}{r} 5324 \times 12 \\ \hline 88 \end{array} \quad (2 \times 2 + 4 = 8, \text{ write down 8})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 888 \end{array} \quad (3 \times 2 + 2 = 8, \text{ write down 8})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 3888 \end{array} \quad (5 \times 2 + 3 = 13, \text{ write down 3, carry over 1})$$

$$\begin{array}{r} 5324 \times 12 \\ \hline 63888 \end{array} \quad (0 \times 2 + 5 + 1 = 6, \text{ write down 6})$$

$$\therefore 5324 \times 12 = 63888$$

2. Evaluate 22200007×12

Solution:

$$\begin{array}{r} 22200007 \times 12 \\ \hline 4 \end{array} \quad (7 \times 2 = 14, \text{ write down 4, carry over 1})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 84 \end{array} \quad (1 + 0 + 7 = 8, \text{ write down 8})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 0084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 00084 \end{array} \quad (0 + 0 = 0, \text{ write down 0})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 400084 \end{array} \quad (2 \times 2 + 0 = 4, \text{ write down 4})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 6400084 \end{array} \quad (2 \times 2 + 2 = 6, \text{ write down 6})$$

$$\begin{array}{r} 22200007 \times 12 \\ \hline 66400084 \end{array} \quad (2 \times 2 + 2 = 6, \text{ write down 6})$$

QUANTITATIVE ABILITY

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- (i) Multiply the right side digits $3 \times 4 = 12$
 (ii) $103 + 4 = 104 + 3 = 107$
 $\therefore 103 \times 104 = 10712$

1. Simplify 107×102

Solution:

$$\begin{aligned} 107 + 2 &= 102 + 7 = 109 \\ 7 \times 2 &= 14 \\ \therefore 107 \times 102 &= 10914 \end{aligned}$$

2. Simplify 109×105

Solution:

$$\begin{aligned} 109 + 5 &= 105 + 9 = 114 \\ 9 \times 5 &= 45 \\ \therefore 109 \times 105 &= 11445 \end{aligned}$$

3. Simplify 98×86

Solution:

$$\begin{aligned} 98 &= 100 - 2, 86 = 100 - 14 \\ 98 - 2 & \\ 86 - 14 & \\ 98 - 14 &= 86 - 2 = 84 \\ 2 \times 14 &= 28 \\ \therefore 98 \times 86 &= 8428 \end{aligned}$$

4. Simplify 98×95

Solution:

$$\begin{aligned} 98 &= 100 - 2, 95 = 100 - 5 \\ 98 - 2 & \\ 95 - 5 & \\ 98 - 5 &= 95 - 2 = 93 \\ 2 \times 5 &= 10 \\ \therefore 98 \times 95 &= 9310 \end{aligned}$$

5. Simplify $112 \times 107 + 93 \times 96$

Solution:

$$\begin{aligned} 112 &= 100 + 12, 107 = 100 + 7 \\ 112 + 7 &= 119, 107 + 12 = 119 \\ 12 \times 7 &= 84 \\ \therefore 112 \times 107 &= 11984 \\ 93 &= 100 - 7, 96 = 100 - 4 \\ 93 - 4 &= 89, 96 - 7 = 89 \\ 97 \times 4 &= 28 \\ \therefore 93 \times 96 &= 8928 \\ \therefore 112 \times 107 + 93 \times 96 & \\ &= 11984 + 8928 = 20912 \end{aligned}$$

MULTIPLICATION: TWO DIGIT NUMBERS

1. Simplify 17×18

Solution:

Here the vertically and crosswise formula is made use of.
 There are 3 steps.

Step 1: Multiply vertically on the right $7 \times 8 = 56$

Write 6 as the last digit and carry over 5

Step 2: Multiply crosswise and add with the carry over
 $1 \times 8 + 1 \times 7 + 5 = 20$

Write 0 as the middle digit and carry over 2.

Step 3: Multiply vertically on the left and add with the carry over $1 \times 1 + 2 = 3$

Write this as the first digit.

$$\begin{array}{r} 1 & 7 \\ \times & 8 \\ \hline 1 & 8 \end{array}$$

$$\therefore 17 \times 18 = 306$$

2. Find the product 87×92

Solution:

$$\begin{array}{r} 8 & 7 \\ \times & 9 \\ \hline 9 & 2 \end{array}$$

$$\underline{\quad\quad\quad}$$

$$8\ 0\ 4$$

$7 \times 2 = 14 \rightarrow 4$ is the last digit

$8 \times 2 + 9 \times 7 + 1 = 80 \rightarrow 0$ is the middle digit.

$8 \times 9 + 8 = 80 \rightarrow 80$ gives the first 2 digits.

$$\therefore 87 \times 92 = 8004$$

3. Simplify 61×31

Solution:

$$\begin{array}{r} 6 & 1 \\ \times & 3 \\ \hline 3 & 1 \end{array}$$

$$\underline{\quad\quad\quad}$$

$$1\ 8\ 9\ 1$$

$1 \times 1 = 1 \rightarrow 1$ is the last digit.

$6 \times 1 + 3 \times 1 = 9 \rightarrow 9$ is the middle digit.

$6 \times 3 = 18 \rightarrow 18$ gives the first 2 digits

$$\therefore 61 \times 31 = 1891$$

4. Simplify 33×97

Solution:

QUANTITATIVE ABILITY

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$$\begin{array}{r}
 3 & 3 \\
 \times & \diagdown \\
 9 & 7 \\
 \hline
 3 & 2 & 0 & 1
 \end{array}$$

$3 \times 7 = 21 \rightarrow 1$ is the last digit and carry over 2.

$3 \times 7 + 3 \times 9 + 2 = 50 \rightarrow 0$ is the middle digit carry over 5

$$3 \times 9 + 5 = 32$$

$$\therefore 33 \times 97 = 3201$$

5. Simplify 81×89

Solution:

$$\begin{array}{r}
 8 & 1 \\
 \times & \diagdown \\
 8 & 9 \\
 \hline
 7 & 2 & 0 & 9
 \end{array}$$

$1 \times 9 = 9 \rightarrow 9$ is the last digit

$8 \times 9 + 8 \times 1 = 80 \rightarrow 0$ is the middle digit

$8 \times 8 + 8 = 72 \rightarrow 72$ is the first 2 digits

$$\therefore 81 \times 89 = 7209$$

SESSION - 2

⑥ SPEED MATHS - II

MULTIPLICATION: 2 DIGIT NUMBER WITH 3 DIGIT NUMBER

1. Solve 325×17

Solution:

$$\begin{array}{r}
 3 & 2 & 5 \\
 \times & \diagdown \\
 1 & 7 \\
 \hline
 5
 \end{array}$$

Step 1: $5 \times 7 = 35$, write down 5, carry over 3

$$\begin{array}{r}
 3 & 2 & 5 \\
 \times & \diagdown \\
 1 & 7 \\
 \hline
 2 & 5
 \end{array}$$

Step 2: $2 \times 7 + 5 \times 1 + 3 = 22$ write down 2, carry over 2

$$\begin{array}{r}
 3 & 2 & 5 \\
 \times & \diagdown \\
 1 & 7 \\
 \hline
 5 & 2 & 5
 \end{array}$$

Step 3: $3 \times 7 + 2 \times 1 + 2 = 25$ write down 5, carry over 2

$$\begin{array}{r}
 3 & 2 & 5 \\
 \times & \diagdown \\
 1 & 7 \\
 \hline
 5 & 5 & 2 & 5
 \end{array}$$

Step 4: $3 \times (1) + 2 = 3 + 2 = 5$ write down 5

$$\therefore 325 \times 17 = 5525$$

2. Solve 675×78

Solution:

$$\begin{array}{r}
 6 & 7 & 5 \\
 \times & \diagdown \\
 7 & 8 \\
 \hline
 0
 \end{array}$$

Step 1: $5 \times 8 = 40$, write down 0 carry over 4

$$\begin{array}{r}
 6 & 7 & 5 \\
 \times & \diagdown \\
 7 & 8 \\
 \hline
 5 & 0
 \end{array}$$

Step 2: $7 \times 8 + 5 \times 7 + 4 = 95$ write down 5, carry over 9

$$\begin{array}{r}
 6 & 7 & 5 \\
 \times & \diagdown \\
 7 & 8 \\
 \hline
 6 & 5 & 0
 \end{array}$$

Step 3: $6 \times 8 + 7 \times 7 + 9 = 106$ write down 6, carry over 10

$$\begin{array}{r}
 6 & 7 & 5 \\
 \times & \diagdown \\
 7 & 8 \\
 \hline
 5 & 2 & 6 & 5 & 0
 \end{array}$$

Step 4: $6 \times 7 + 10 = 52$ write down 52

$$\therefore 675 \times 78 = 52650$$

3. Simplify 95×847

Solution:

$$\begin{array}{r}
 9 & 5 \\
 \times & \diagdown \\
 8 & 4 & 7 \\
 \hline
 5
 \end{array}$$

Step 1: $5 \times 7 = 35$, write down 5, carry over 3

$$\begin{array}{r}
 9 & 5 \\
 \times & \diagdown \\
 8 & 4 & 7 \\
 \hline
 6 & 5
 \end{array}$$

Step 2: $9 \times 7 + 5 \times 4 + 3 = 86$ write down 6, carry over 8

$$\begin{array}{r}
 9 & 5 \\
 \times & \diagdown \\
 8 & 4 & 7 \\
 \hline
 4 & 6 & 5
 \end{array}$$

Step 3: $8 \times 5 + 9 \times 4 + 8 = 84$ write down 4, carry over 9

$$\begin{array}{r}
 9 & 5 \\
 \times & \diagdown \\
 8 & 4 & 7 \\
 \hline
 8 & 0 & 4 & 6 & 5
 \end{array}$$

Step 4: $8 \times 9 + 8 = 80$ write down 90

$$\therefore 95 \times 847 = 80465$$

4. Simplify 275×64

Solution:

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 0 \end{array}$$

Step 1: $5 \times 4 = 20$, write down 0 carry over 2

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 0 \ 0 \end{array}$$

Step 2: $7 \times 4 + 6 \times 5 + 2 = 60$ write down 0, carry over 6

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 6 \ 0 \ 0 \end{array}$$

Step 3: $2 \times 4 + 7 \times 6 + 6 = 56$ write down 6, carry over 5

$$\begin{array}{r} 2 \ 7 \ 5 \\ \times \ 6 \ 4 \\ \hline 17 \ 6 \ 0 \ 0 \end{array}$$

Step 4: $2 \times 6 + 5 = 17$ write down 17

5. Solve $251 \times 27 + 362 \times 34$

Solution:

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \end{array}$$

Step 1: $1 \times 7 = 7$, write down 7

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \ 7 \end{array}$$

Step 2: $5 \times 7 + 2 \times 1 = 37$, write down 7 carry over 3

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 7 \ 7 \ 7 \end{array}$$

Step 3: $2 \times 7 + 5 \times 2 + 3 = 27$ write down 7, carry over 2

$$\begin{array}{r} 2 \ 5 \ 1 \\ \times \ 2 \ 7 \\ \hline 6 \ 7 \ 7 \ 7 \end{array}$$

Step 4: $2 \times 2 + 2 = 6$ write down 6

$\therefore 251 \times 27 = 6777$

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 8 \end{array}$$

Step 1: $2 \times 4 = 8$, write down 8

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 0 \ 8 \end{array}$$

Step 2: $6 \times 4 + 2 \times 3 = 30$ write down 0, carry over 3

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 3 \ 0 \ 8 \end{array}$$

Step 3: $3 \times 4 + 6 \times 3 + 3 = 33$ write down 3, carry over 3

$$\begin{array}{r} 3 \ 6 \ 2 \\ \times \ 3 \ 4 \\ \hline 12 \ 3 \ 0 \ 8 \end{array}$$

Step 4: $3 \times 3 + 3 = 12$ write down 12

$\therefore 362 \times 34 = 12308$

So, $251 \times 27 + 362 \times 34 = 6777 + 12308 = 19085$

MULTIPLICATION: 3 DIGIT NUMBERS

1. Simplify 321×132

Solution:

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 2 \end{array}$$

Step 1: $1 \times 2 = 2$

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 7 \ 2 \end{array}$$

Step 2: $2 \times 2 + 3 \times 1 = 7$

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 3 \ 7 \ 2 \end{array}$$

Step 3: $3 \times 2 + 1 \times 1 + 2 \times 3 = 13$ (write down 3, carry over 1)

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 2 \ 3 \ 7 \ 2 \end{array}$$

Step 4: $3 \times 3 + 1 \times 2 + 1 = 12$ (write down 2, carry over 1)

$$\begin{array}{r} 3 \ 2 \ 1 \\ \times \ 1 \ 3 \ 2 \\ \hline 4 \ 2 \ 3 \ 7 \ 2 \end{array}$$

Step 5: $3 \times 1 + 1 = 4$

$\therefore 321 \times 132 = 42372$

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2. Simplify 275×526

Solution:

$$\begin{array}{r} 275 \\ \times 526 \\ \hline 0 \end{array}$$

Step 1: $5 \times 6 = 30$, write down 0, carry over

$$\begin{array}{r} 275 \\ \times 526 \\ \hline 50 \end{array}$$

Step 2: $7 \times 6 + 5 \times 2 + 3 = 55$, write down 5, carry over 5

$$\begin{array}{r} 275 \\ \times 526 \\ \hline 650 \end{array}$$

Step 3: $2 \times 6 + 5 \times 5 + 7 \times 2 + 5 = 56$ write down 6, carry over 5

$$\begin{array}{r} 275 \\ \times 526 \\ \hline 4650 \end{array}$$

Step 4: $2 \times 2 + 7 \times 5 + 5 = 44$, write down 4, carry over 4

$$\begin{array}{r} 275 \\ \times 526 \\ \hline 144650 \end{array}$$

Step 5: $2 \times 5 + 4 = 14$, write down 14

3. Simplify 336×678

Solution:

$$\begin{array}{r} 336 \\ \times 678 \\ \hline 8 \end{array}$$

Step 1: $6 \times 8 = 48$, write down 8, carry over

$$\begin{array}{r} 336 \\ \times 678 \\ \hline 08 \end{array}$$

Step 2: $3 \times 8 + 6 \times 7 + 4 = 70$ write down 0, carry over 7

$$\begin{array}{r} 336 \\ \times 678 \\ \hline 808 \end{array}$$

Step 3: $3 \times 8 + 6 \times 6 + 3 \times 7 + 7 = 88$ write down 8, carry over 8

$$\begin{array}{r} 336 \\ \times 678 \\ \hline 7808 \end{array}$$

Step 4: $3 \times 7 + 6 \times 3 + 8 = 47$ write down 7, carry over 4

$$\begin{array}{r} 336 \\ \times 678 \\ \hline 227808 \end{array}$$

Step 5: $3 \times 6 + 4 = 22$, write down 22

4. Simplify 569×952

Solution:

$$\begin{array}{r} 569 \\ \times 952 \\ \hline 8 \end{array}$$

Step 1: $9 \times 2 = 18$, write down 8, carry over 1

$$\begin{array}{r} 569 \\ \times 952 \\ \hline 88 \end{array}$$

Step 2: $6 \times 2 + 9 \times 5 + 1 = 58$ write down 8, carry over 5

$$\begin{array}{r} 569 \\ \times 952 \\ \hline 688 \end{array}$$

Step 3: $5 \times 2 + 9 \times 9 + 6 \times 5 + 5 = 126$ write down 6, carry over 12

$$\begin{array}{r} 569 \\ \times 952 \\ \hline 1688 \end{array}$$

Step 4: $5 \times 5 + 6 \times 9 + 12 = 91$ write down 1, carry over 9

$$\begin{array}{r} 569 \\ \times 952 \\ \hline 541688 \end{array}$$

Step 5: $5 \times 9 + 9 = 54$ write down 54

$$\therefore 569 \times 952 = 541688$$

5. Simplify $102 \times 304 + 207 \times 121$

Solution:

$$102 \times 304$$

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 8 \end{array}$$

Step 1: $2 \times 4 = 8$, write down 8

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 08 \end{array}$$

Step 2: $0 \times 4 + 2 \times 0 = 0$, write down 0

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 008 \end{array}$$

Step 3: $1 \times 4 + 2 \times 3 + 0 \times 0 = 10$ write down 0, carry over 1

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 1008 \end{array}$$

Step 4: $1 \times 0 + 3 \times 0 + 1 = 1$, write down 1

$$\begin{array}{r} 102 \\ \times 304 \\ \hline 31008 \end{array}$$

Step 5: $1 \times 3 = 3$, write down 3

$$\therefore 102 \times 304 = 31008$$

$$207 \times 121$$

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 7 \end{array}$$

Step 1: $7 \times 1 = 7$, write down 7

Step 2: $0 \times 1 + 7 \times 2 = 14$, write down 4 carry over 1

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 047 \end{array}$$

Step 3: $2 \times 1 + 7 \times 1 + 0 \times 2 + 1 = 10$ write down 0 carry over 1

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 5047 \end{array}$$

Step 4: $2 \times 2 + 1 \times 0 + 1 = 5$ write down 5

$$\begin{array}{r} 207 \\ \times 121 \\ \hline 25047 \end{array}$$

Step 5: $2 \times 1 = 2$

$$\therefore 207 \times 121 = 25047$$

$$102 \times 304 + 207 \times 121$$

$$= 31008 + 25077 = 56055$$

MULTIPLICATION - COMPLEMENTARY NUMBERS

Numbers having the same digits except the right most digits whose sum is 10 are called complementary pairs.

For example: 83, 87; 114, 116, 342, 348 are complementary pairs.

To multiply complementary numbers, there are 2 steps. If the numbers are 94 and 96 multiply the right most digits $4 \times 6 = 24$.

Multiply the first number by the number one up $9 \times 10 = 90$
 $\therefore 94 \times 96 = 9024$

1. Simplify 32×38

Solution:

$$3 \times 4 = 12$$

$$2 \times 8 = 16$$

$$\therefore 32 \times 38 = 1216$$

2. Simplify 87×83

Solution:

$$8 \times 9 = 72$$

$$7 \times 3 = 21$$

$$\therefore 87 \times 83 = 7221$$

3. Simplify 126×124

Solution:

$$12 \times 13 = 156$$

$$6 \times 4 = 24$$

$$\therefore 126 \times 124 = 15624$$

4. Simplify 243×247

Solution:

$$24 \times 25 = 600$$

$$3 \times 7 = 21$$

$$\therefore 243 \times 247 = 60021$$

QUANTITATIVE ABILITY

5. Simplify $92 \times 98 + 67 \times 63$

Solution:

$$92 \times 98 = 9016$$

$$\begin{array}{r} 9 \times 10 = 90 \\ 2 \times 8 = 16 \end{array}$$

$$67 \times 63 = 4221$$

$$\begin{array}{r} 6 \times 7 = 42 \\ 7 \times 3 = 21 \end{array}$$

$$\therefore 92 \times 98 + 67 \times 63$$

$$= 9016 + 4221 = 13237$$

DIVISION – ONE LINE METHOD

1. Divide 38982 by 73

Solution:

73 is the divisor. We put down only the first digit 7 in the divisor column and put the other digit 3 on the top of it. As one digit (3) has been put on the top, we allot one place at the right end of the dividend to the remainder position of the answer and mark it off from the digits by a vertical line.

3					2
	3	8	9	8	
7	3	3	3	1	
	5	3	4	0	

Step 1: Since 3 is less than 7, 38 is the first dividend.
Divide 38 by 7, Q = 5, R = 3.

Write down 5 as the first digit of the quotient and prefix the remainder (3) before 9 of the dividend.

Step 2: Now our dividend is 39. From this the product of the indexed (3) and the first digit of the quotient (5) or $3 \times 5 = 15$ is to be deducted.

The net dividend is $39 - 15 = 24$.

Dividing 24 by 7, Q = 3, R = 3.

Write down 3 as the second digit of the quotient and prefix the remainder 3 before 8 of the dividend.

Step 3: Now, the dividend is 38. From this the product 3×3 is to be subtracted and the net dividend is $38 - 9 = 29$.

Dividing 29 by 7, Q = 4 and R = 1.

Write down 4 as the third digit of the quotient and prefix 1 before 2.

Step 4: Now, the dividend is 12. From this the product $3 \times 4 = 12$ is to be subtracted and we obtain 0 as the remainder.

$$\therefore 38982 \div 73 = 534$$

2. Divide 16384 by 128

Solution:

Take 12 as the first digit and put 8 at the top of it.

Also, allot one place at the right end of the dividend to the remainder position of the answer.

8					4
	1	6	3	8	
12	4	11			6
	1	2	8		

Step 1: Divide 16 by 12, Q = 1, R = 4

Step 2: $43 - 8 \times 1 = 35$ is the next dividend

Divide 35 by 12, Q = 2, R = 11

Step 3: $118 - 8 \times 2 = 118 - 16 = 102$ is the next dividend.

Divide 102 by 12, Q = 8, R = 6

Step 4: $64 - 8 \times 8 = 64 - 64 = 0$ which is the remainder

$$\text{So } 16384 \div 128 = 128$$

3. Divide 601325 by 76

Solution:

Take 7 as the first digit and put 6 at the top of it.

Also allot one place at the right end of the dividend to the remainder position of the answer.

6					500
	6	0	1	3	
7	11	6	2		265
	7	9	1	2	.171

Step 1: Divide 60 by 7, Q = 8, R = 4

If we put 8 down, then $41 - 6 \times 8 = 41 - 48$ becomes negative which is absurd.

$$\therefore Q = 7, R = 11.$$

Step 2: $111 - 6 \times 7 = 111 - 42 = 69$ is the next dividend.

Dividing 69 by 7, Q = 9, R = 6

Step 3: $63 - 6 \times 9 = 63 - 54 = 9$ is the next dividend.

Dividing 9 by 7, Q = 1, R = 2

Step 4: $22 - 6 \times 1 = 16$ is the next dividend.

Dividing 16 by 7, Q = 2, R = 2

Step 5: $25 - 6 \times 2 = 25 - 12 = 13$ is the remainder.

If it is to be converted as a decimal, add zeros after 5 and proceed as above.

Dividing 13 by 7, Q = 1, R = 6

$$60 - 6 \times 1 = 54$$

Dividing 54 by 7, Q = 7, R = 5

$$50 - 6 \times 7 = 50 - 42 = 8$$

Dividing 8 by 7, Q = 1, R = 1

The process can be continued for any number of digits after the decimal.

$$\therefore 60132 \div 76 = 7912.171$$

4. Divide 710.014 by 39 (to 4 places of decimals)

Solution:

9	7	1	0	0	1	4	0
3	4		8	2	2	6	7
1	8		2	0	5	4	

Take 3 as the first digit and put 9 at the top of it. Also allot one place before the decimal point at the right end of the dividend and also write the digits after the decimal point.

Step 1: Divide 7 by 3, Q = 2, R = 1

But if Q = 2, the next dividend will become negative. So, Q = 1, R = 4

Step 2: $41 - 9 \times 1 = 41 - 9 = 32$

Dividing 32 by 3, Q = 8, R = 8

$$80 - 9 \times 8 = 80 - 72 = 8$$

Dividing 8 by 3, Q = 2, R = 2

$$20 - 9 \times 2 = 20 - 18 = 2$$

Dividing 2 by 3, Q = 0, R = 2

$$21 - 9 \times 0 = 21 - 0 = 21$$

Dividing 21 by 3, Q = 5, R = 6

$$64 - 9 \times 5 = 64 - 45 = 19$$

Dividing 19 by 3, Q = 4, R = 7

$$\therefore 710.014 \div 39 = 18.2054$$

5. Divide 64932 by 99

Solution:

Take 9 as the first digit and put 9 at the top of it. Also allot one place at the right end of the dividend to the remainder position of the answer.

9	6	4	9	3	2	0	0	0	
9	10	10	13	15	15	15	15	15	
6	5	5	8	7	8	7			

64 is the first dividend.

Divide 64 by 9, Q = 6, R = 10

$$109 - 6 \times 9 = 109 - 54 = 55$$

Divide 55 by 9, Q = 5, R = 10

$$103 - 9 \times 5 = 103 - 45 = 58$$

Divide 58 by 9, Q = 5, R = 13

$$132 - 45 = 87$$

Divide 87 by 9, Q = 8, R = 15

$$150 - 9 \times 8 = 150 - 72 = 78$$

Divide 78 by 9, Q = 7, R = 15

$$150 - 9 \times 7 = 150 - 63 = 87$$

Divide 87 by 9, Q = 8, R = 15

$$150 - 9 \times 8 = 150 - 72 = 78$$

Divide 78 by 9, Q = 7, R = 15

The process can be continued.

$$\therefore 64932 \div 99 = 655.8787$$

PERCENTAGE CALCULATIONS

To calculate the product of 2 numbers, percentage is used.

1. Find the value of 43×78 using percentage.

Solution:

Consider 43% of 78

$$\begin{aligned} 43\% \text{ of } 78 &= 10\% \text{ of } 78 + 10\% \text{ of } 78 + 10\% \text{ of } 78 + 10\% \text{ of } \\ &78 + 1\% \text{ of } 78 + 1\% \text{ of } 78 + 1\% \text{ of } 78 \\ &= 7.8 + 7.8 + 7.8 + .78 + .78 = 33.54 \end{aligned}$$

43% of 78 = .43 × 78 and the digits for 43×78 will be the same. Now, put back the decimal point

$$\therefore 43 \times 78 = 3354$$

2. Find the product 324×82 using percentage

Solution:

Consider 324% of 82 = 3.24×82

$$\begin{aligned} &= 3 \times 82 + 8.2 + 8.2 + .82 + .82 + .82 + .82 \\ &= 246 + 16 + 3.68 = 265.68 \\ &\therefore 324 \times 82 = 26568 \end{aligned}$$

3. Find the product 252×151 using percentage

Solution:

Consider 252% of 151

$$252\% \text{ of } 151 = 2.52 \times 151$$

$$\begin{aligned} &= 2 \times 151 + 15.1 + 15.1 + 15.1 + 15.1 + 15.1 + 1.51 + 1.51 \\ &= 302 + 75.5 + 3.02 = 380.52 \end{aligned}$$

$$\therefore 252 \times 151 = 38052$$

4. Find the percentage value of the ratio $\frac{53}{81}$

Solution:

$$\frac{53}{81} = \frac{40.5 + 8.1 + 4.4}{81}$$

$$= \frac{40.5}{81} + \frac{8.1}{81} + \frac{4.4}{81}$$

$$= 50\% + 10\% + \frac{4.4}{81}$$

$$= 60\% + \frac{4.05}{81} + \frac{0.35}{81}$$

$$= 65\% + 0.43\% = 65.43\%$$

5. Compare $\frac{173}{212}$ with $\frac{181}{241}$ using percentage.

Solution:

$$\begin{aligned}\frac{173}{212} &= \frac{106}{212} + \frac{53}{212} + \frac{10.6}{212} + \frac{3.4}{212} \\ &= 50\% + 25\% + 5\% + \frac{3.4}{212} \\ &= 80\% + 1.6\% = 81.6\end{aligned}$$

$$\begin{aligned}\frac{181}{241} &= \frac{120.5}{241} + \frac{60.25}{241} + \frac{.25}{241} \\ &= 50\% + 25\% + .001\% = 75.001\%\end{aligned}$$

$$81.6 > 75.001\%$$

$$\Rightarrow \frac{173}{212} > \frac{181}{241}$$

SQUARES

1. Find the square of 207

Solution:

$$207^2$$

- (1) $7^2 = 49$, write down 9 as the last digit and carry over 4.
- (2) $2 \times 0 \times 7 + 4 = 4$, write it down in the next position.
- (3) $2 \times 2 \times 7 + 0^2 = 28$, write down 8 in the third position and carry over 2.
- (4) $2 \times 0 \times 2 + 2 = 2$ write down 2 in the fourth place.
- (5) $2^2 = 4$ write down 4

$$\therefore 207^2 = 42849$$

2. Find the square of 897

Solution:

- (1) $7^2 = 49$, write down 9 in the last place and carry over 4.
- (2) $2 \times 9 \times 7 + 4 = 130$, write down 0 in the next place and carry over 13.
- (3) $2 \times 8 \times 7 + 9^2 + 3 = 206$, write down 6 in the next place and carry over 20.
- (4) $2 \times 8 \times 9 + 20 = 164$, write down 4 and carry over 16.
- (5) $8^2 + 16 = 80$, write it down.

$$\therefore (897)^2 = 804609$$

3. Find the square of 8432.

Solution:

- (1) $2^2 = 4$, write down 4 as the last digit.
- (2) $2 \times 3 \times 2 = 12$, write down 2 as the next digit and carry over 1.

- (3) $2 \times 4 \times 2 + 3^2 + 1 = 26$, write down 6 as the next digit and carry over 2.

- (4) $2 \times 8 \times 2 + 2 \times 4 \times 3 + 2 = 58$, write down 8 as the next digit and carry over 5.

- (5) $2 \times 8 \times 3 + 4^2 + 5 = 69$, write down 9 as the next digit and carry over 6.

- (6) $2 \times 8 \times 4 + 6 = 70$, write down 0 as the next digit and carry over 7.

- (7) $8^2 + 7 = 71$, write down 71.

$$\therefore (8432)^2 = 71098624$$

SQUARES OF NUMBERS FROM 51 TO 59

$$(51)^2 = 25 + 1 / 1^2 = 26 / 01 = 2601$$

$$(52)^2 = 25 + 2 / 2^2 = 27 / 04 = 2704$$

$$(53)^2 = 25 + / 3^2 = 28 / 09 = 2809$$

$$(54)^2 = 25 + 4 / 4^2 = 29 / 16 = 2916$$

$$(55)^2 = 25 + 5 / 5^2 = 30 / 25 = 3025$$

$$(59)^2 = 25 + 9 / 9^2 = 34 / 81 = 3481$$

$$1. \quad 57^2 = 25 + 7 / 7^2 = 32 / 49 = 3249$$

SQUARES OF NUMBERS WITH UNIT DIGIT AS 5

$$1. \quad 15^2 = 1 \times (1+1) / 5^2 = 2 / 25 = 225$$

$$2. \quad 35^2 = 3 \times (3+1) / 5^2 = 12 / 25 = 1225$$

$$3. \quad 85^2 = 8 \times 9 / 25 = 7225$$

$$4. \quad 225^2 = 22 \times 23 / 25 = 506 / 25 = 50625$$

SQUARES OF NUMBERS NEARER TO 10^x , $x \in \mathbb{N}$

Here the algebraic formula is used.

$$x^2 = (x^2 - y^2) + y^2 = (x+y)(x-y) + y^2$$

$$1. \quad 98^2 = (98+2)(98-2) + 2^2$$

$$= 9600 + 4 = 9604$$

$$2. \quad (104)^2 = (104+4)(104-4) + 4^2$$

$$= 10800 + 16 = 10816$$

$$3. \quad (1007)^2 = (1007+7)(1007-7) + 7^2$$

$$= 1014000 + 49 = 1014049$$

PRACTICE EXERCISE

1. Find the square of 307.
2. Find the squares of numbers 61 - 69.
3. Find the square of numbers 45 and 235.
4. Find the squares of numbers 96 and 106.

SESSION - 3

SPEED MATHS - III

SQUARE ROOTS

If $y = x^2$, then $x = \sqrt{y}$ is called the square root of y .

Square roots of numbers can be found out

(i) by factorisation method (ii) by division method

1. Evaluate $\sqrt{6084}$ by factorisation method

Solution:

$$6084 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

$$= 2^2 \times 3^2 \times 13^2$$

$$\therefore \sqrt{6084} = \sqrt{2^2 \times 3^2 \times 13^2}$$

$$= 2 \times 3 \times 13 = 78$$

$$\therefore \sqrt{6084} = 78$$

$$\begin{array}{r} 2 | 6084 \\ 2 | 3042 \\ 3 | 1521 \\ 3 | 507 \\ 13 | 169 \\ \hline & 13 \end{array}$$

2. Find the square root of 53824 by division method.

Solution:

(i) Mark off all the digits in pairs starting from the unit's digit.

(ii) $2^2 = 4$, remainder 1, bring down the next pair. The dividend is 138.

(iii) Double 2 and put 3 as the unit's digit. The divisor is 43.

(iv) Bring down 24, continue the process.

$$\therefore \sqrt{53824} = 232$$

$$\begin{array}{r} 232 \\ 2 | 5, 3, 8, 24 \\ 4 | 138 \\ 129 \\ \hline 462 | 924 \\ \hline 924 \\ 0 \end{array}$$

3. Evaluate $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\sqrt{248 + \sqrt{51 + \sqrt{169}}} = \sqrt{248 + \sqrt{51 + 13}}$$

$$= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$$

$$\therefore \sqrt{248 + \sqrt{51 + \sqrt{169}}} = 16$$

4. Evaluate $\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}}$

Solution:

$$\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}} = \sqrt{5 \times 5 \times 900} = \sqrt{5 \times 5 \times 30 \times 30}$$

$$= 5 \times 30 = 150$$

5. Find the square root of $\sqrt{175.2976}$

Solution:

$$\begin{array}{r} 13.24 \\ 1 | 1,75.2976 \\ 1 | \\ 23 | 075 \\ 69 \\ \hline 262 | 629 \\ 524 \\ \hline 2644 | 10576 \\ 10576 \\ \hline 0 \end{array}$$

(i) Mark off all the digits in pairs starting from either side of the decimal point.

$$\therefore \sqrt{175.2976} = 13.24$$

6. Find the value of $\sqrt{\frac{0.289}{0.00121}}$

Solution:

$$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$$

7. Find the greatest number of 4 digits which is a perfect square.

Solution:

The greatest number of 4 digits is 9999

$$\begin{array}{r} 99 \\ 9 | 9999 \\ 81 \\ \hline 189 | 1899 \\ 1701 \\ \hline 198 \end{array}$$

Since 198 is the remainder, $9999 - 198$ will be a perfect square.

$$9999 - 198 = 9801$$

\therefore The greatest number of 4 digits which is a perfect square is 9801.

8. Find the smallest number to be subtracted from 549162 to make it a perfect square.

Solution:

$$\begin{array}{r} 7 | 549162 (741) \\ 49 \\ \hline 144 | 591 \\ 576 \\ \hline 1481 | 1562 \\ 1481 \\ \hline 81 \end{array}$$

$\therefore 549162 - 81$ will become a perfect square.

\therefore The smallest number to be subtracted is 81.

9. Simplify $\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001}$

Solution:

$$\begin{aligned}\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} \\ = \sqrt{.0025 \times 2.25 \times .0001} \\ = \sqrt{\frac{25}{10000} \times \frac{225}{100} \times \frac{1}{10000}} \\ = \frac{\sqrt{25 \times 225}}{\sqrt{10000000000}} \\ = \frac{5 \times 15}{100000} = \frac{75}{100000} = 0.00075\end{aligned}$$

10. Find the smallest number by which 5808 should be multiplied to make it a perfect square.

Solution:

$$\begin{aligned}5808 &= 2 \times 2 \times 2 \times 2 \times 11 \times 11 \times 13 \\ &= 2^4 \times 11^2 \times 3\end{aligned}$$

∴ The required smallest number is 3.

$$\begin{array}{r} 5808 \\ 2 \boxed{2} \\ 2 \boxed{2904} \\ 2 \boxed{1452} \\ 2 \boxed{726} \\ 3 \boxed{363} \\ 11 \boxed{121} \\ 11 \end{array}$$

PRACTICE EXERCISE:

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

2. Evaluate: $\sqrt{\frac{0.289}{0.00121}}$

3. Find the greatest number of 4 digits which is a perfect square.

CUBES

Cubes of 2 digit numbers

To calculate the cube value of a 2 digit number.

Step 1: Put down the cube of the ten's digit in a row of 4 figures. Find the ratio between the digits of the given number. So, the other 3 digits in the row are in the geometrical ratio in the exact proportion.

Step 2: Put down under the second and third digits, 2 times of the second and the third digit. Then add up the 2 rows.

1. Calculate 12^3

Solution:

The ten's digit of the number is 1

So we write the first digit as $1^3 = 1$

The ratio between the digits is 1:2, the next digits will be double the previous one.

So the first row is 1 2 4 8

The second ad the third digits are 2 and 4.

So write down 4 and 8 below 2 and 4 and then add.

$$\begin{array}{r} 1 \ 2 \ 4 \ 8 \\ \quad 4 \ 8 \\ \hline 1 \ 7 \ 2 \ 8 \end{array}$$

∴ $12^3 = 1728$

2. Calculate 21^3

Solution:

$$\begin{array}{r} 8 \ 4 \ 2 \ 1 \\ \quad 8 \ 4 \\ \hline 9 \ 2 \ 6 \ 1 \end{array}$$

$[8 = 2^3, 8 \div 4 = 2, 4 \div 2 = 1, 2 \div 1 = 2]$

$[4 \times 2 = 8, 2 \times 2 = 4]$

$\therefore 21^3 = 9261$

3. Calculate 34^3

Solution:

$$\begin{array}{r} 27 \ 36 \ 48 \ 64 \\ \quad 72 \ 96 \\ \hline 39 \ 12^3 \ 150 \ 6^4 \end{array}$$

$[3^3 = 27 \text{ and the ratio is } 3 : 4 \text{ and so the next number is } \frac{4}{3} \text{ or the previous}]$

$\therefore 34^3 = 39304$

4. Calculate 93^3

Solution:

$$\begin{array}{r} 729 \ 243 \ 81 \ 27 \\ \quad 486 \ 162 \\ \hline 804 \ 753 \ 245 \ 27 \end{array}$$

$[9^3 = 729, 9 : 3 = 3 : 1]$

∴ Each term is obtained by dividing by 3

$\therefore 93^3 = 804357$

5. Calculate 77^3

Solution:

$$\begin{array}{r} 343 \ 343 \ 343 \ 343 \\ \quad 686 \ 686 \\ \hline 456 \ 1135 \ 106^3 \ 34^3 \end{array}$$

$[7^3 = 343, \text{ ratio } 1 : 1]$

$\therefore 77^3 = 456533$

CUBE ROOTS

If $y = x^3$, then x is called the cube root of y and is written as

$$x = \sqrt[3]{y}$$

1. Find the cube root of 2744.

Solution:

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$= 2^3 \times 7^3$$

$$\therefore \sqrt[3]{2744} = 2 \times 7$$

$$= 14$$

$$\begin{array}{r} 2 | 2744 \\ 2 | 1372 \\ 2 | 686 \\ 7 | 343 \\ 7 | 49 \\ \hline & 7 \end{array}$$

2. Find the cube root of 0.000512

Solution:

$$\sqrt[3]{0.000512} = \sqrt[3]{\frac{512}{1000000}} = \frac{\sqrt[3]{512}}{100}$$

$$\sqrt[3]{512} = \sqrt{2^9}$$

$$\therefore \sqrt[3]{512} = 2^3 = 8$$

$$\therefore \sqrt[3]{0.000512} = \frac{8}{100} = 0.08$$

$$\begin{array}{r} 2 | 512 \\ 2 | 256 \\ 2 | 128 \\ 2 | 64 \\ 2 | 32 \\ 2 | 16 \\ 2 | 8 \\ 2 | 4 \\ \hline & 2 \end{array}$$

3. Evaluate $\sqrt[3]{4 \frac{12}{125}}$

Solution:

$$\sqrt[3]{4 \frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \frac{\sqrt[3]{512}}{\sqrt[3]{125}} = \frac{8}{5} = 1 \frac{3}{5^2}$$

4. Find the smallest number by which 3600 be divided to make it a perfect cube.

Solution:

$$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$= 2^4 \times 3^2 \times 5^2$$

\therefore 3600 should be divided by $2 \times 3^2 \times 5^2$ to make it a perfect cube 8.

$$\therefore \text{The required number is } 2 \times 9 \times 25 = 450$$

$$\begin{array}{r} 2 | 3600 \\ 2 | 1800 \\ 2 | 900 \\ 2 | 450 \\ 5 | 225 \\ 3 | 45 \\ 3 | 15 \\ \hline & 5 \end{array}$$

5. Find the largest 5 digit number which is a perfect cube.

Solution:

The largest 5 digit cube is $46^3 = 97336$

PRACTICE EXERCISE - II

- Calculate 23^3 .
- Calculate 97^3 .
- Find the cube root of 5832.
- Find the cube root of 0.000216.
- Find the smallest number by which 33275 be divided to make it a perfect cube.

SIMPLIFICATION

Introduction

While performing simplification greatest care has to be taken to avoid mistakes. Developing ability to calculate is one of the major thrust areas. Developing the calculation speed is very important. Addition perhaps is the critical skill to develop the calculations. Try to practise adding 2 digit numbers to improve the calculation speed. Make sure that you are through with addition before attempting the process of subtraction. The multiplication methods followed in speed maths is useful for fast calculations.

For simplification BODMAS rule depicts the correct sequence in which the operations are to be executed so as to find out the value of a given expression.

B – Bracket, O – Of, D – Division, M – Multiplication, A – Addition, S – Subtraction.

When you simplify an expression, first the brackets have to be removed and then the other operations in the given order.

1. Simplify $100 + 20 \times 55$

Solution:

$$\begin{aligned} 100 + 20 \times 55 &= 100 + 1100 \\ &= 1200 \end{aligned}$$

2. Simplify $50040 \div 139 - 60$

Solution:

$$\begin{aligned} &= \frac{50040}{139} - 60 \\ &= 360 - 60 = 300 \end{aligned}$$

$$\begin{array}{r} 9 | 5 0 0 4 | 0 \\ 13 | \quad 115 \\ \hline & 3 6 0 \end{array}$$

3. Simplify $\frac{480 \times 15 - 12 \times 20 + 7 \times 60}{140 \times 8 + 2 \times 55}$

Solution:

$$\begin{aligned} &\frac{480 \times 15 - 12 \times 20 + 7 \times 60}{140 \times 8 + 2 \times 55} = \frac{7200 - 240 + 420}{1120 + 110} \\ &= \frac{6960 + 420}{1230} = \frac{7380}{130} = 6 \end{aligned}$$

4. Simplify $\frac{3}{5} \text{ of } \frac{4}{7} \text{ of } \frac{5}{9} \text{ of } \frac{21}{24} \text{ of } 504$.

Solution:

$$\begin{aligned} &\frac{3}{5} \text{ of } \frac{4}{7} \text{ of } \frac{5}{9} \text{ of } \frac{21}{24} \text{ of } 504 \\ &= \frac{3}{5} \times \frac{4}{7} \times \frac{5}{9} \times \frac{21}{24} \times 504 \\ &= 84 \end{aligned}$$

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5. Simplify $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3}$

Solution:

$$\begin{aligned} & \frac{9}{2} \times \frac{13}{3} - \frac{25}{3} \times \frac{3}{17} \\ &= \frac{39}{2} - \frac{25}{17} = \frac{663 - 50}{34} = \frac{613}{34} = 18\frac{1}{34} \end{aligned}$$

6. Simplify $0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004)$

Solution:

$$\begin{aligned} & 0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004) \\ &= 0.00008 \times \frac{0.0072}{0.000048} \\ &= \frac{0.8 \times 0.0072}{0.48} = \frac{8}{10} \times \frac{72}{10000} \times \frac{100}{48} = \frac{12}{1000} = 0.012 \end{aligned}$$

7. If $3 - [1.6 - \{3.2 - (3.2 + 2.25 \div x)\}] = 0.65$, find the value of x.

Solution:

$$\begin{aligned} & 3 - [1.6 - \{3.2 - (3.2 + 2.25 \div x)\}] = 0.65 \\ & \Rightarrow 3 - \left[1.6 - \left\{ 3.2 - \left(3.2 + \frac{2.25}{x} \right) \right\} \right] = 0.65 \\ & \Rightarrow 3 - \left[1.6 - \left(3.2 - 3.2 - \frac{2.25}{x} \right) \right] = 0.65 \\ & \Rightarrow 3 - \left[1.6 + \frac{2.25}{x} \right] = 0.65 \\ & \Rightarrow 3 - 1.6 - \frac{2.25}{x} = 0.65 \\ & \Rightarrow 1.4 - \frac{2.25}{x} = 0.65 \\ & \Rightarrow \frac{2.25}{x} = 1.4 - 0.65 = 0.75 \\ & \Rightarrow x = \frac{2.25}{0.75} = \frac{225}{75} = 3 \end{aligned}$$

8. Simplify $\frac{2 + \frac{1}{4}}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4}}}}$

Solution:

$$\begin{aligned} & \frac{2 + \frac{1}{4}}{\frac{3}{5}} = \frac{2 + \frac{1}{19}}{\frac{5}{1}} = \frac{2 + \frac{5}{19}}{\frac{1}{1}} \\ & \frac{2 + \frac{1}{3 + \frac{1}{4}}}{2 + \frac{1}{3 + \frac{1}{5}}} = \frac{2 + \frac{1}{1 + \frac{1}{4}}}{2 + \frac{1}{3 + \frac{1}{4}}} \\ & = \frac{2 + \frac{5}{19}}{2 + \frac{1}{19}} = \frac{2 + \frac{5}{19}}{2 + \frac{5}{19}} = 1 \end{aligned}$$

9. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$, where x, y, z are natural numbers,

find x, y, z.

Solution:

$$\begin{aligned} \frac{37}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \\ \Rightarrow \frac{37}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \\ \Rightarrow 2 + \frac{11}{13} &= 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \end{aligned}$$

$$\Rightarrow \frac{11}{13} = \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{13}{11} = 1 + \frac{2}{11}$$

$$\Rightarrow x = 1, \frac{1}{y + \frac{1}{z}} = \frac{2}{11} \Rightarrow x = 1, y + \frac{1}{z} = \frac{11}{2}$$

$$\Rightarrow x = 1, y + \frac{1}{z} = 5 + \frac{1}{2} \Rightarrow x = 1, y = 5, z = 2$$

∴ x, y, z are 1, 5, 2.

10. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find y - x.

Solution:

$$\left. \begin{array}{l} 4x + 5y = 83 \\ \frac{3x}{2y} = \frac{21}{22} \end{array} \right\}$$

$$\Rightarrow 66x = 42y \Rightarrow y = \frac{66}{42}x = \frac{11}{7}x$$

$$4x + 5 \times \frac{11}{7}x = 83$$

$$\Rightarrow 28x + 55x = 581$$

$$\Rightarrow 83x = 581 \Rightarrow x = 7$$

$$4x + 5y = 83 \Rightarrow 4 \times 7 + 5y = 83 \Rightarrow 5y = 55 \Rightarrow y = 11$$

$$\therefore y - x = 11 - 7 = 4$$

11. Find the sum

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$$

Solution:

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$$

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$$\begin{aligned}
 &= \frac{1}{2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \frac{1}{6.7} \\
 &\quad + \frac{1}{7.8} + \frac{1}{8.9} + \frac{1}{9.10} + \frac{1}{10.11} + \frac{1}{11.12} \\
 &= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) \\
 &\quad + \left(\frac{1}{7} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{11}\right) + \left(\frac{1}{11} - \frac{1}{12}\right) \\
 &= \frac{1}{2} + \frac{1}{2} - \frac{1}{12} = 1 - \frac{1}{12} = \frac{11}{12}
 \end{aligned}$$

12. Simplify $\frac{(856+167)^2 + (856-167)^2}{856 \times 856 + 167 \times 167}$

Solution:

$$\begin{aligned}
 &\frac{(856+167)^2 + (856-167)^2}{856^2 + 167^2} \\
 &= \frac{2[856^2 + 167^2]}{856^2 + 167^2} \\
 &\quad [\text{use the formula } (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)] \\
 &= 2
 \end{aligned}$$

13. Simplify $\frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \div \frac{\frac{3}{2} - 2\frac{1}{2}}{\frac{4}{3} - 3\frac{1}{3}}$

Solution:

$$\begin{aligned}
 &\frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \div \frac{\frac{3}{2} - 2\frac{1}{2}}{\frac{4}{3} - 3\frac{1}{3}} \\
 &= \frac{\left(\frac{3}{2}\right)^2 - \left(\frac{2}{2}\right)^2}{\left(\frac{4}{3}\right)^2 - \left(\frac{3}{3}\right)^2} \times \frac{\left(\frac{4}{3}\right) - \left(\frac{3}{3}\right)}{\left(\frac{3}{2}\right) - \left(\frac{2}{2}\right)} \\
 &= \frac{\frac{3}{2} + 2\frac{1}{2}}{\frac{4}{3} + 3\frac{1}{3}} \\
 &\quad [\text{using the formula } a^2 - b^2 = (a-b)(a+b)]
 \end{aligned}$$

$$\frac{\frac{11}{3} + \frac{5}{2}}{\frac{19}{4} + \frac{10}{3}} = \frac{\frac{22+15}{6}}{\frac{57+40}{12}} = \frac{37}{6} \times \frac{12}{97} = \frac{74}{97}$$

14. Simplify $\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2}$

Solution:

$$\begin{aligned}
 &\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2} = \frac{\left[\left(\frac{3}{5}\right) - \left(\frac{2}{5}\right)\right] \left[\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \frac{3}{5} \times \frac{2}{5}\right]}{\left[\left(\frac{3}{5}\right) - \left(\frac{2}{5}\right)\right] \left[\left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)\right]} \\
 &= \frac{\frac{9}{25} + \frac{4}{25} + \frac{6}{25}}{\frac{3}{5} + \frac{2}{5}} = \frac{\frac{19}{25}}{\frac{5}{5}} = \frac{19}{25}
 \end{aligned}$$

PRACTICE EXERCISE

1. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$ where x, y, z are natural numbers
find x, y, z.

2. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find y - x.

3. Simplify $\frac{\left(\frac{3}{5}\right)^3 - \left(\frac{2}{5}\right)^3}{\left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2}$

4. Find the sum $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \dots + \frac{1}{132}$

SESSION - 4

SPEED MATHS - IV

PRACTICE EXERCISE

1. Add 707325, 192382, 58009, 564943 and 656.
(a) 1523315 (b) 331969 (c) 1623225 (d) 3129621
2. Simplify: $5124 - 829 + 731 - 435$
(a) 4732 (b) 4591 (c) 3921 (d) 3865
3. Solve: 5732×1002
(a) 5743464 (b) 5742464 (c) 5843464 (d) 5925461
4. Evaluate: $8492^2 - 508^2$
(a) 70856000 (b) 7284600 (c) 71856000 (d) 70856000
5. Multiply 84726 by 5.
(a) 435630 (b) 432650 (c) 423630 (d) 413630
6. Multiply 625 by 25.
(a) 15625 (b) 30625 (c) 15725 (d) 120625

SESSION - 2

SPEED MATHS - II

PRACTICE EXERCISE

1. Find the square of 307.

Solution:

$$307^2$$

(1) $7^2 = 49$, write down 9 as the last digit and carry over 4.

(2) $3 \times 0 \times 7 + 4 = 4$, write it down in the next position.

(3) $2 \times 3 \times 7 + 0^2 = 42$, write down 2 in the third position and carry over 4.

(4) $3 \times 0 \times 3 + 4 = 4$ write down 4 in the fourth place.

(5) $3^2 = 9$ write down 9

$$\therefore 307^2 = 94249$$

2. Find the squares of numbers 61 – 69.

Solution:

$$(61)^2 = 25 + 11 / 11^2 = 36 / 121 = 3721$$

$$(62)^2 = 25 + 12 / 12^2 = 37 / 144 = 3844$$

$$(63)^2 = 25 + 13 / 13^2 = 38 / 169 = 3969$$

$$(64)^2 = 25 + 14 / 14^2 = 39 / 196 = 4096$$

$$(65)^2 = 25 + 15 / 15^2 = 40 / 225 = 4225$$

$$(66)^2 = 25 + 16 / 16^2 = 41 / 256 = 4356$$

$$(67)^2 = 25 + 17 / 17^2 = 42 / 289 = 4489$$

$$(68)^2 = 25 + 18 / 18^2 = 43 / 324 = 4624$$

$$(69)^2 = 25 + 19 / 19^2 = 44 / 361 = 4761$$

3. Find the square of numbers 45 and 235.

Solution:

$$45^2 = 4 \times (4+1) / 5^2 = 20 / 25 = 2025$$

$$235^2 = 23 \times 24 / 25 = 552 / 25 = 55225$$

4. Find the squares of numbers 96 and 106.

Solution:

$$96^2 = (96+4)(96-4) + 4^2 \\ = 9200 + 16 = 9216$$

$$(106)^2 = (106+6)(106-6) + 6^2 \\ = 11200 + 36 = 11236$$

SESSION - 3

SPEED MATHS - III

PRACTICE EXERCISE - I

1. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

Solution:

$$\sqrt{248 + \sqrt{51 + \sqrt{169}}} = \sqrt{248 + \sqrt{51 + 13}}$$

$$= \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$$

$$\therefore \sqrt{248 + \sqrt{51 + \sqrt{169}}} = 16$$

- 2.

$$\text{Evaluate: } \sqrt{\frac{0.289}{0.00121}}$$

Solution:

$$\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$$

Find the greatest number of 4 digits which is a perfect square.

Solution:

The greatest number of 4 digits is 9999

$$\begin{array}{r} 99 \\ 9 \overline{)9999} \\ 81 \\ \hline 189 \\ 189 \\ \hline 1701 \\ 1701 \\ \hline 198 \end{array}$$

Since 198 is the remainder, $9999 - 198$ will be a perfect square.

$$9999 - 198 = 9801$$

\therefore The greatest number of 4 digits which is a perfect square is 9801.

PRACTICE EXERCISE - II

1. Calculate 23^3 .

Solution:

The ten's digit of the given number is 2.

So, we write the first number in the first row as 8.

The ratio between the digits is 2:3 and so the next three numbers should also be in the same ratio with the previous one.

If the next number is a two digit number, the ten's digit is written as carryover below the previous number.

In the second row, twice the second and third numbers are written below them in the same manner.

	8	2	8	7
	1	1	2	
		4	6	
	2	3		
1	2	1	6	7

2. Calculate 97^3 .

Solution:

$$\begin{array}{r} 729 & 567 & 441 & 343 \\ & 1134 & 882 & \\ \hline 729 & 1701 & 1323 & 343 \end{array}$$

$$\therefore 97^3 = 912673$$

3. Find the cube root of 5832.

Solution:

$$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 2^3 \times 3^3 \times 3^3$$

$$\therefore \sqrt[3]{5832} = 2 \times 3 \times 3 \\ = 18$$

4. Find the cube root of 0.000216.

Solution:

$$\sqrt[3]{0.000216} = \sqrt[3]{\frac{216}{1000000}} = \frac{\sqrt[3]{216}}{100}$$

$$\sqrt[3]{216} = \sqrt{2^3 \times 3^3}$$

$$\therefore \sqrt[3]{216} = 2 \times 3 = 6$$

$$\therefore \sqrt[3]{0.000216} = \frac{6}{100} = 0.06$$

$$\begin{array}{r} 2 | 216 \\ 2 | 108 \\ 2 | 54 \\ 3 | 27 \\ 3 | 9 \\ \hline 3 \end{array}$$

5. Find the smallest number by which 33275 be divided to make it a perfect cube.

Solution:

$$33275 = 5 \times 5 \times 11 \times 11 \times 11$$

$$= 5^2 \times 11^3$$

\therefore 33275 should be divided by 5^2 to make it a perfect cube.

$$\begin{array}{r} 5 | 33275 \\ 5 | 6655 \\ 11 | 1331 \\ 11 | 121 \\ \hline 11 \end{array}$$

\therefore The required number is $5^2 = 25$

SIMPLIFICATION

PRACTICE EXERCISE

1. If $\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$ where x, y, z are natural numbers find x, y, z.

Solution:

$$\frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow \frac{37}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \\ \Rightarrow 2 + \frac{11}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow 2 + \frac{11}{13} = 2 + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}$$

$$\Rightarrow \frac{11}{13} = \frac{1}{x + \frac{1}{y + \frac{1}{z}}} \Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{13}{11} = 1 + \frac{2}{11}$$

$$\Rightarrow x = 1, \frac{1}{y + \frac{1}{z}} = \frac{2}{11} \Rightarrow x = 1, y + \frac{1}{z} = \frac{11}{2}$$

$$\Rightarrow x = 1, y + \frac{1}{z} = 5 + \frac{1}{2} \Rightarrow x = 1, y = 5, z = 2$$

$\therefore x, y, z$ are 1, 5, 2.

2. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, find y - x.

Solution:

$$\left. \begin{array}{l} 4x + 5y = 83 \\ \frac{3x}{2y} = \frac{21}{22} \end{array} \right\}$$

$$\Rightarrow 66x = 42y \Rightarrow y = \frac{66}{42}x = \frac{11}{7}x$$

$$4x + 5 \times \frac{11}{7}x = 83$$

$$\Rightarrow 28x + 55x = 581$$

$$\Rightarrow 83x = 581 \Rightarrow x = 7$$

$$4x + 5y = 83 \Rightarrow 4 \times 7 + 5y = 83 \Rightarrow 5y = 55 \Rightarrow y = 11$$

$$\therefore y - x = 11 - 7 = 4$$

3. Find the sum $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}$

$$\begin{aligned} &= \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} \\ &\quad + \frac{1}{8 \cdot 9} + \frac{1}{9 \cdot 10} + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} \\ &= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{6} \right) \\ &\quad + \left(\frac{1}{6} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{10} \right) + \left(\frac{1}{10} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{12} \right) \end{aligned}$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{12} = 1 - \frac{1}{12} = \frac{11}{12}$$

10. Multiply 95 with 83.

Ans: [b]

$$95 = 100 - 5$$

$$83 = 100 - 17$$

$$95 - 17 = 83 - 5 = 78$$

$$17 \times 5 = 85$$

$$\therefore 83 \times 95 = 7885$$

11. Find the value of 147×143 .

Ans: [a]

$$\begin{array}{r} 147 \\ \downarrow \\ 143 \\ \hline 1 \end{array}$$

Step 1: 2 carry

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 21 \end{array}$$

Step 2: $28 + 12 + 2 = 42$ carry '4'

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 021 \end{array}$$

Step 3: In 30 place '0' 3 as carry

$$\begin{array}{r} 147 \\ \times 143 \\ \hline 1021 \end{array}$$

Step 4: $4+4+3=11$ 1 as carry

$$\begin{array}{r} 147 \\ \downarrow \\ 143 \\ \hline 11021 \end{array}$$

Step 5: $3 \times 1 + 1 = 4$

$$\therefore 147 \times 143 = 11021$$

12. Find the value of 671×359 .

Ans: [d]

$$\begin{array}{r} 671 \\ \downarrow \\ 359 \\ \hline 9 \end{array}$$

Step 1:

$$\begin{array}{r} 671 \\ \times 359 \\ \hline 89 \end{array}$$

Step 2: $63 + 5 = 68$ 6 carry

$$\begin{array}{r} 671 \\ \times 359 \\ \hline 0889 \end{array}$$

Step 3: $= 30 + 21 + 9 = 60$ 6 as carry

$$\begin{array}{r} 671 \\ \downarrow \\ 359 \\ \hline 240889 \end{array}$$

Step 4: $18 + 6 = 24$

$$\therefore 671 \times 359 = 240889$$

13. Multiply 678 by 37.

Ans: [a]

$$\begin{array}{r} 678 \\ \times 37 \\ \hline 25086 \end{array}$$

14. Divide 45625 by 73.

Ans: [b]

$$45625 \div 73$$

$$\begin{array}{r} 45625 \\ 5 \swarrow 9125 \\ 5 \swarrow 1825 \\ 5 \swarrow 365 \\ 5 \swarrow 73 \end{array}$$

$$\frac{5^4 \times 73}{73} = 5^4 = 625$$

15. Divide $718.589 \div 96$ to 3 places of decimals.

Ans: [a]

16. Find the value of 92×67 using percentage.

Ans: [b]

$$\begin{array}{r} 92 \\ (x) \\ \times 67 \\ \hline 6164 \end{array}$$

17. Find the largest number of 3 digits which is a perfect square.

Ans: [a]

Largest 3 digit number is 999.

So we find out square root of 999 by long division method and check whether it is a perfect square or root.

$$\begin{array}{r} 31 \\ \sqrt{999} \\ 3 \quad | \quad 999 \\ +3 \quad \quad 9 \\ \hline 61 \quad | \quad 99 \\ \quad \quad 61 \\ \hline \quad \quad 38 \end{array}$$

So, $999 - 38 = 961$

961 is a perfect square.

18. Find the value of $\sqrt{0.0025} \times \sqrt{2.25} \times \sqrt{0.0001}$.

Ans: [d]

$$0.05 \times 1.5 \times 0.01$$

$$0.00075$$

None of these

RECRUITMENT ESSENTIALS



RECRUITMENT ESSENTIALS

SESSION - 3

IMPRESSION MANAGEMENT



WHAT IS IMPRESSION?

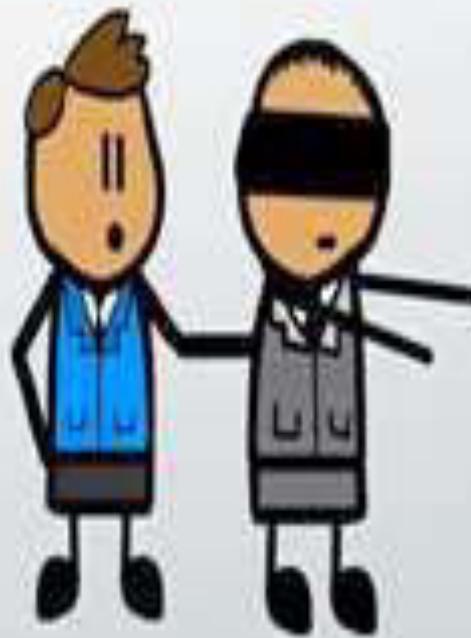
An idea, feeling, or opinion about something or someone, esp. one formed without conscious thought or on the basis of little evidence. An effect produced on someone.



WHAT IS FIRST IMPRESSION

- **IMPRESSION** - an opinion or feeling that you have about someone or something you have seen but do not know very well
- **FIRST IMPRESSION** - opinions that you form immediately, before thinking thoroughly
- Probably, it will take less than 5 seconds to judge
- Not very fair
- More stereotype

Impression Management



A person influencing another person's interpretation of a person, place or thing by controlling the information they receive.

What Is Impression Management?

- Impression management is the effort to influence other people's perceptions.
- The theory goes on to explain that we try to make the perception consistent with our goals.
- Many of us can identify with the desire (and resulting actions) to be seen in a certain way or cause someone or something we care about to be seen a certain way.
- Sometimes it's conscious and sometimes it isn't, but when we pay attention, we may find several perceptions we are striving to get from others.

What Is Impression Management?



What my friends think I do



What my mom thinks I do



What society thinks I do

What Is Impression Management?

- An example of impression management theory in play is in sports such as soccer.

At an important game, a player would want to showcase himself / herself in the best light possible, because there are college recruiters watching.

These people would have the flashiest pair of cleats and try and perform their best to show off their skills.

Their main goal may be to impress the college recruiters in a way that maximizes their chances of being chosen for a college team rather than winning the game.

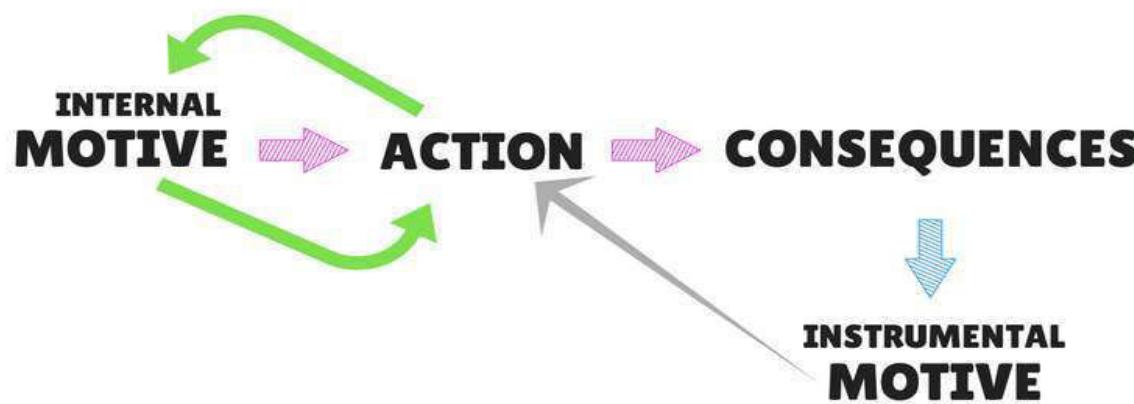
Why We Do It?

- There are two main motives for trying to manage the impressions of others:
 - the instrumental and
 - the expressive.



The instrumental motive

- The **instrumental** motive includes the desire for increased self-esteem. The fundamental meaning of instrumental motivation is the gaining of rewards.
- So, when we try to manage perceptions to get something back from another person, we are motivated by instrumental purposes.



The instrumental motive

- For example,

If a sales representative shares with her customer that she uses this particular bar of soap and goes on to explain how it has helped her skin, her reason for encouraging this sale might be her desire for a raise.

- Besides seeking a raise, a person could be looking for acceptance, respect, more friends, etc.



The expressive motive

- Have you ever felt like you wanted to redefine yourself or how you think you have been seen by others? Maybe when you were growing up, your parents wanted you to dress, speak or act a certain way, and you wanted to show them they couldn't define you. Whether or not we have felt this way, many people become aware of how they are viewed and want to change that view of them.
- An **expressive** motive comes down to wanting to be in charge of one's personal behavior and identity. It can come from a response to social norms, expectations or restrictions, and it seeks to show others something different.

The expressive motive



Importance of Impression Management:

- Making a good impression on others and managing by way of maintaining good deeds and actions must be regarded as one of the most important approaches to life.
- It not only aids in leaving a lasting impression on others but it also helps in attaining desired results successfully.
- Impression management has considerable implications in all the aspects of life in which you have a need to give others your overall impression as good.

Importance of Impression Management:

- Managing impression is essential for an individual to live a good life, as it enables one to find the effective ways of self-improvement.
- It helps one in leading in the direction of self-improvement positively, for it develops self-motivation by enhancing the awareness of self and developing self-concept.



What matters most is how you see yourself

- Many a time or at some point in time of life, one may be disappointed due to being perceived differently from exactly how he should be.
- It is owing to the fact that one is unable to make a favorable impression on others.
- There may be several factors influencing the perception of others.

WHAT MATTERS MOST
IS HOW YOU SEE YOURSELF.



What matters most is how you see yourself

- Hence it is important to take those factors, contributing to the formation of wrong perception of others, into consideration and focus on such aspects that may help in getting a good impression about him.



- In this manner, not only one will have managed to create a favorable and indelible impression but also to clear spurious impression.

Types of impression management

The two important types of impression management are:

- **Constructive-** Helps in the formation of self-identity.

Ex.: dressing, appearance, make up, hair style, etc...



- **Strategic-** Helps in the attainment of some interpersonal goals. For Example: hand shaking, communication, body language, etc.

Two Types of Impression Management



Constructive -- helps in the formation of self identity

Strategic -- helps in the attainment of some interpersonal goal

How to use Impression Management effectively?

There are several ways by which people can use Impression Management effectively.

- People tend to talk about their personal characteristics portraying them in a manner so as to make themselves look superior amongst all.
- People tend to give excuses or try to justify their acts so as to make them acceptable to everyone.
- People try to portray their deceptive nature of being courteous just to show how down to earth they are and how readily they accept their mistakes.

How to use Impression Management effectively?

- If somebody has achieved something high and of great importance then people tend to take credit of that worthy act by telling everybody how they played a vital role by offering a helping hand to accomplish the task.
- Cajoling is one way of making yourself likeable by everybody as it's a typical human behavior of people who like being flattered.
- Returning favors of the good behavior that is showed to you can create miracles for people who do the same.

This is “Good Tit for Tat’ method.

Why do people adopt the technique of Impression Management?

- *There are basically two main reasons that drive people to adopt the art Impression Management*
 - ✓ Effectiveness
 - ✓ Conflict Avoidance



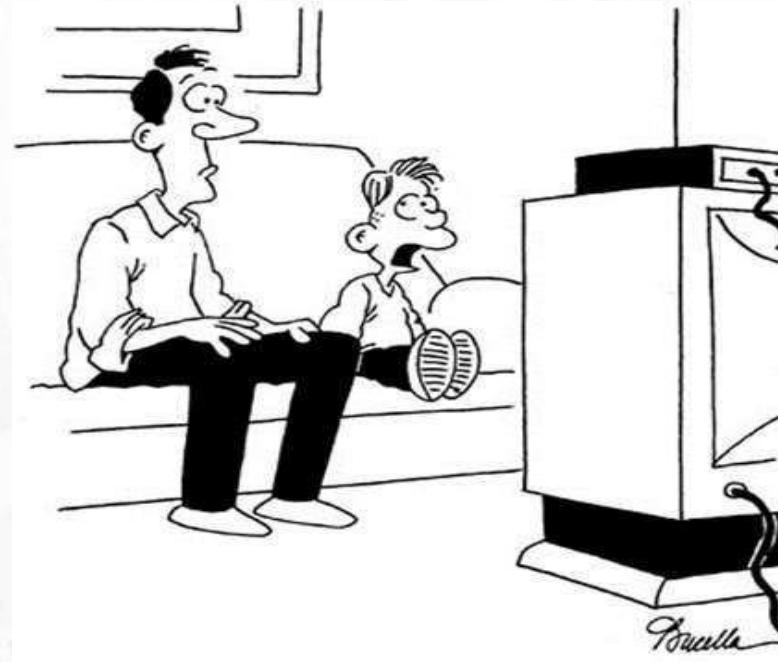
Effectiveness

- Many people feel that by managing their impression they can get by or charm any person and make them work according to their wishes
- E.g. A boy / girl coming out of a Mercedes Benz wearing branded clothes to pick up his girl / boy friend, is trying to impress the friend.



Conflict avoidance

- The best way to avoid conflict is to act sweet with soft voice.
- People who follow the art of Impression Management never try to win an argument. They believe in, “Do not try too hard to win an argument, you may lose a friend”.



“You can watch cartoons with me Dad, as long as you don't ask me to explain them to you.”

Techniques in Impression Management

Impression Management Tactics

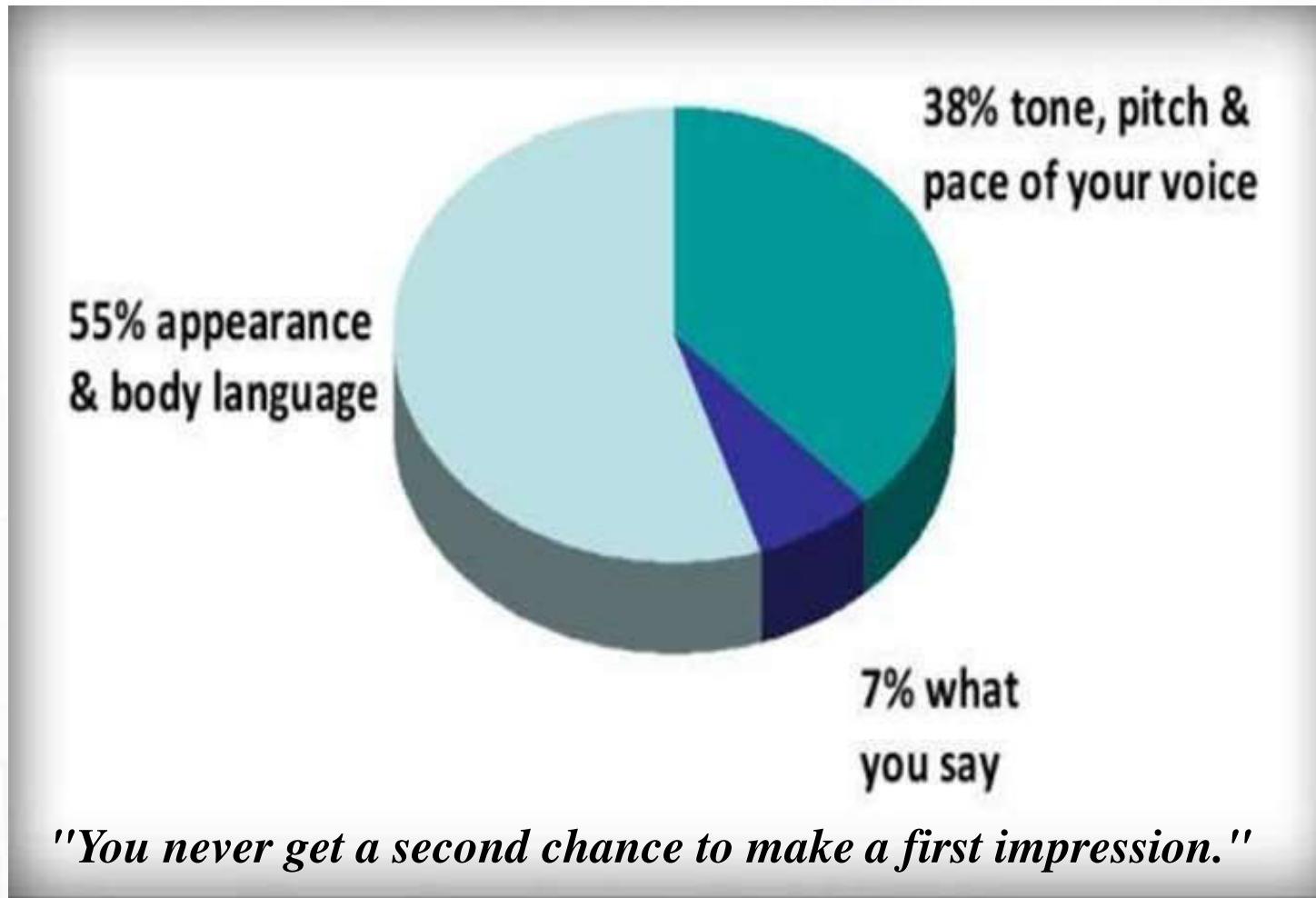
Behavioral Matching	The target of perception matches his or her behavior to that of the perceiver.	A subordinate tries to imitate her boss's behavior by being modest and soft-spoken because her boss is modest and soft-spoken.
Self-Promotion	The target tries to present herself or himself in as positive a light as possible.	A worker reminds his boss about his past accomplishments and associates with co-workers who are evaluated highly.
Conforming to Situational Norms	The target follows agreed-upon rules for behavior in the organization.	A worker stays late every night even if she has completed all of her assignments because staying late is one of the norms of her organization.
Appreciating or Flattering Others	The target compliments the perceiver. This tactic works best when flattery is not extreme and when it involves a dimension important to the perceiver.	A coworker compliments a manager on his excellent handling of a troublesome employee.
Being Consistent	The target's beliefs and behaviors are consistent. There is agreement between the target's verbal and nonverbal behaviors.	A subordinate delivering a message to his boss looks the boss straight in the eye and has a sincere expression on his face.

A person's first
impression will
influence their overall
opinion of you



—YOU—
NEVER
get a second
CHANCE
TO MAKE A
—Great
—**FIRST**—
IMPRESSION

How to make a good Impression



BODY LANGUAGE

Body Language

Say What You Mean!

Using Body Language
to Send a Message



People take in information through five senses

But not all of them are equal:

IT'S WHAT YOU DON'T SAY THAT COUNTS!



Body Language - an element of Communication skills

- It's a kind of non verbal communication we make – both conscious and unconscious.
- It includes facial expressions, body movements, and gestures.
- It is very effective, maybe even so more than speech.
- Remember the saying, "Actions speak louder than words".

Body language

- Body language, and more technically the study of body language, is also known as kinesics (pronounced 'kineesicks'), which is derived from the Greek word kinesis, meaning motion.
- The way you communicate through body language can determine your success in everything from relationships to your career. Paying closer attention to the messages you send through body language can help you succeed.
- Body language goes both ways:
 - Your own body language reveals your feelings and meanings to others.
 - Other people's body language reveals their feelings and meanings to you.

Body language

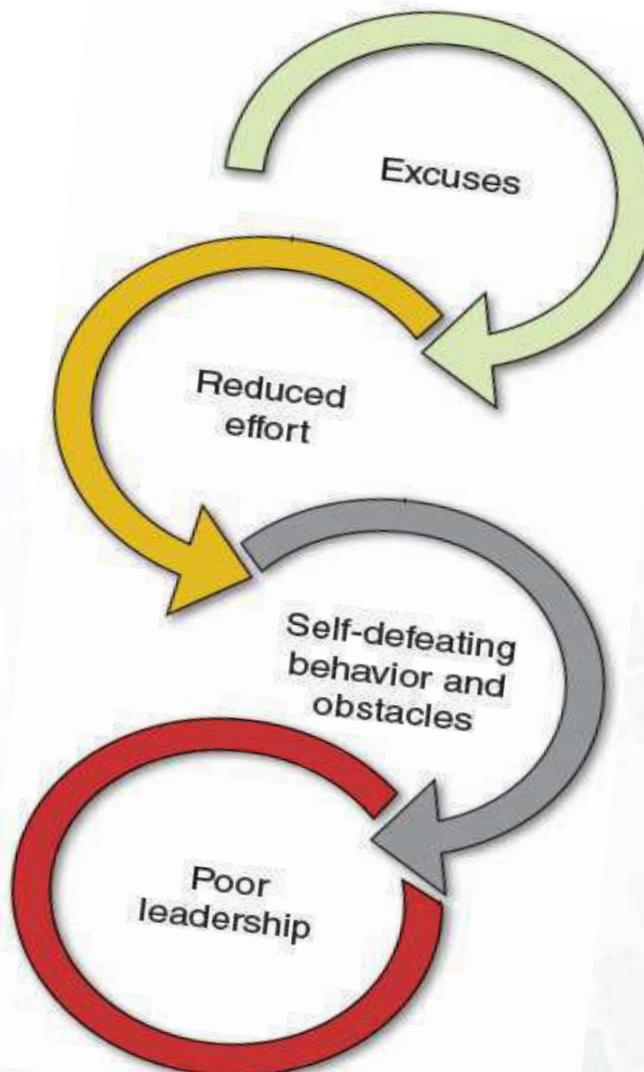
- Body language is not just about how we hold and move our bodies.
- Body language potentially (although not always, depending on the definition you choose to apply) encompasses:
 - how we position our bodies
 - our closeness to and the space between us and other people (proxemics), and how this changes
 - our facial expressions
 - our eyes especially and how our eyes move and focus, etc
 - how we touch ourselves and others
 - how our bodies connect with other non-bodily things, for instance, pens, cigarettes, spectacles and clothing
 - our breathing, and other less noticeable physical effects, for example our heartbeat and perspiration

Components or types of body language

- Facial expressions
- Body movements and posture
- Gestures
- Eye contact
- Touch
- Space
- Tone or voice

First Impression- Body Language

Positive Signals:	Negative Signals:
Upright posture	Slouching
Good eye contact	Avoiding eye contact
Own Comfort level	Fidgeting
Proper gesture	Touching face/ hair
Stillness	Defensive gestures
Active listening	Frowning



Everyone has a few habits they could improve but you may not even recognize that you have some of these habits.

Nonverbal Signals

Evaluating nonverbal signals

Eye contact	Is eye contact being made? If so, is it overly intense or just right?
Facial expression	What is their face showing? Is it masklike and unexpressive, or emotionally present and filled with interest?
Tone of voice	Does their voice project warmth, confidence, and interest, or is it strained and blocked?
Posture and gesture	Are their bodies relaxed or stiff and immobile? Are shoulders tense and raised, or slightly sloped?
Touch	Is there any physical contact? Is it appropriate to the situation? Does it make you feel uncomfortable?
Intensity	Do they seem flat, cool, and disinterested, or over-the-top and melodramatic?
Timing and pace	Is there an easy flow of information back and forth? Do nonverbal responses come too quickly or too slowly?
Sounds	Do you hear sounds that indicate caring or concern?

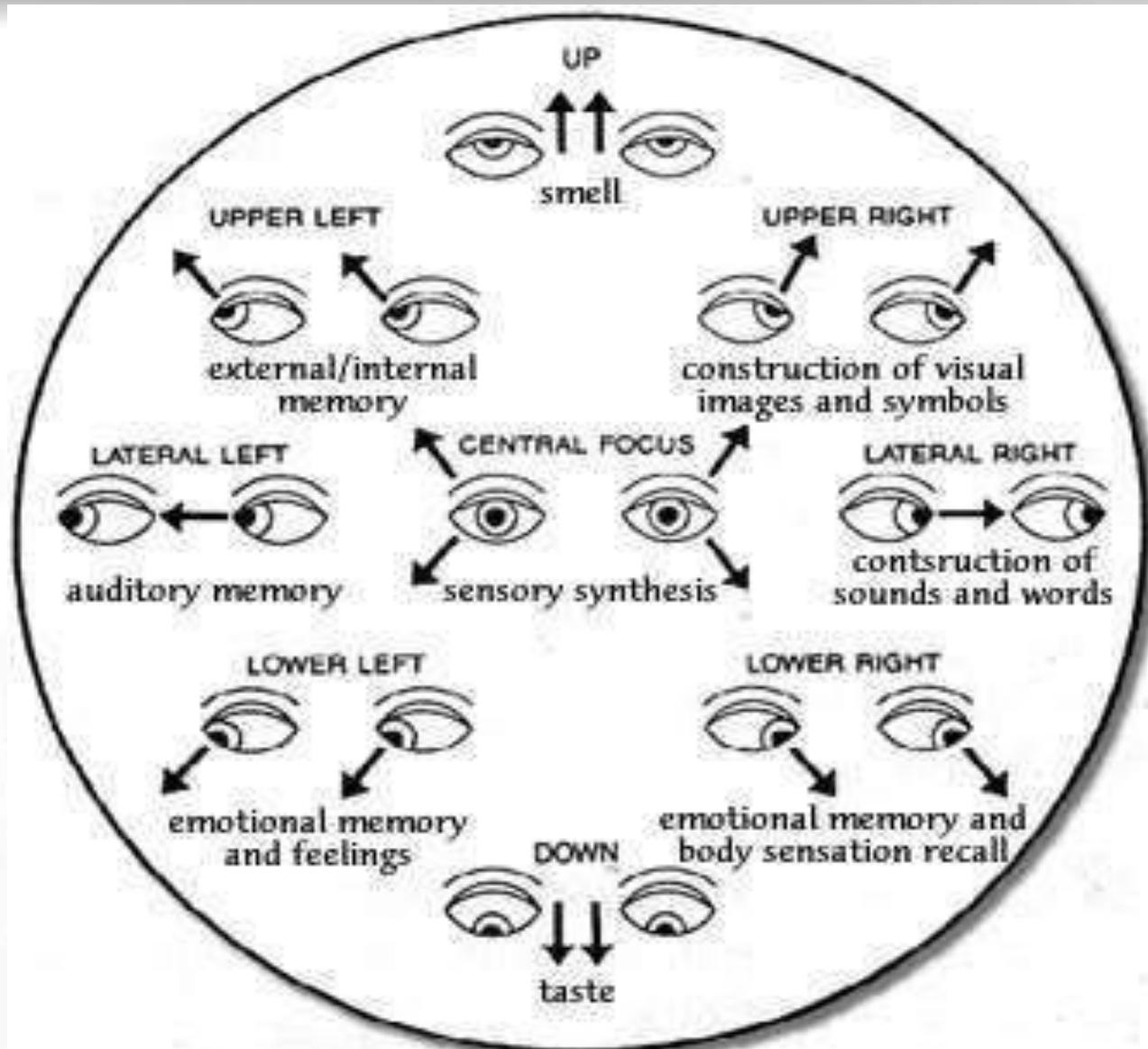
Nonverbal Communication Cues

Nonverbal communication cues can play five roles:

- **Repetition:** they can repeat the message the person is making verbally.
- **Contradiction:** they can contradict a message the individual is trying to convey.
- **Substitution:** they can substitute for a verbal message. For example, a person's eyes can often convey a far more vivid message than words do.
- **Complementing:** they may add to or complement a verbal message. A boss who pats a person on the back in addition to giving praise can increase the impact of the message.
- **Accenting:** they may accent or underline a verbal message. Pounding the table, for example, can underline a message.

Eyes

- Our eyes are a very significant aspect of the non-verbal signals we send to others.
- To a lesser or greater extent we all 'read' people's eyes without knowing how or why, and this ability seems to be inborn.
- For example we know if we have eye contact with someone at an almost unbelievable distance. Far too far away to be able to see the detail of a person's eyes - 30-40 metres away or more sometimes - we know when there is eye contact. This is an absolutely awesome capability when you think about it.
- Incredibly also, we can see whether another person's eyes are focused on us or not, and we can detect easily the differences between a 'glazed over' blank stare, a piercing look, a moistening eye long before tears come, and an awkward or secret glance.

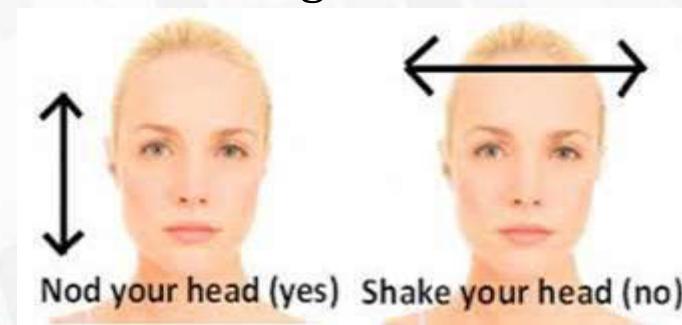


Mouth

- The mouth is associated with very many body language signals, which is not surprising given its functions - obviously speech.
- The mouth is a tremendously flexible and expressive part of the body too, performing a central role in facial expressions.
- Smiling is a big part of facial body language. As a general rule real smiles are symmetrical and produce creases around the eyes and mouth, whereas fake smiles, for whatever reason, tend to be mouth-only gestures.

Head

- The head is very significant in body language. The head is used a lot in directional (likes and dislikes) body language, and in defensive (self-protection) body language too.
- A person's head, due to a very flexible neck structure, can turn, jut forward, withdraw, tilt sideways, forwards, backwards. All of these movements have meanings, which given some thought about other signals can be understood. The head - when our hands interact with it - is therefore dynamic and busy in communicating all sorts of messages - consciously and unconsciously.





Defensive

Arrogance

Annoyed



Are you being defensive? - I don't think so!



Irritated

Happy

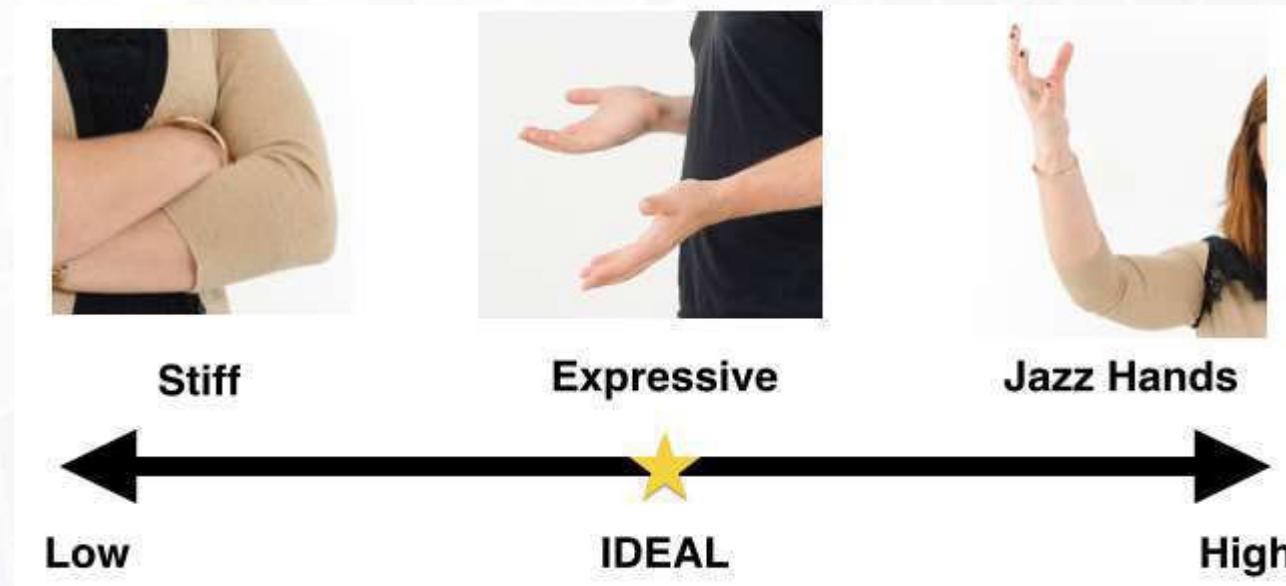
Confused

Worried



Arms

- Arms act as defensive barriers when across the body, and conversely indicate feelings of openness and security when in open positions, especially combined with open palms. Arms are quite reliable indicators of mood and feeling, especially when interpreted with other body language.



"High Power" body language (top row)
vs.
"Low Power" body language (bottom row)

(Images courtesy of Amy Cuddy, Harvard University)



Hands

- Body language involving hands is extensive.
- They are extremely expressive and flexible tools, so it is natural for hands to be used a lot in signalling consciously - as with emphasizing gestures - or unconsciously - as in a wide range of unintentional movements which indicate otherwise hidden feelings and thoughts.

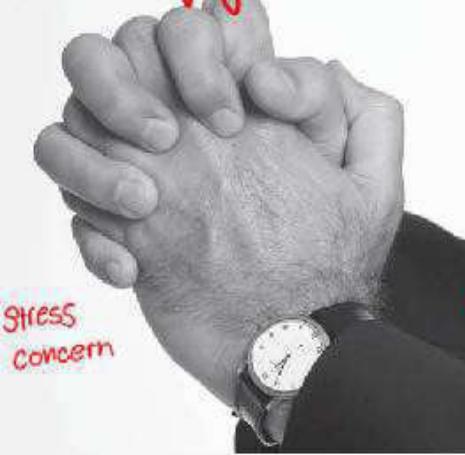


Hands

Hands body language is used for various purposes, notably:

- Emphasis : pointing, jabbing, and chopping actions, etc
- illustration : shaping, mimicking actions or sizing things in the air etc
- specific conscious signals : the thumbs-up, the Victory-sign, greeting people and waving goodbye etc.
- and more interestingly in unconscious signals - including interaction with items like pens and cigarettes and other parts of the body - indicating feelings such as doubt, deceit, pressure, openness, expectation, etc.

Hand-wringing



Steeping of Hands



positive



negative



Self-preening



dismissiveness

Fingertips placed apart on surface



Finger pointing



Have a good handshake

- A good handshake is an important part of making a great first impression. A well timed, firm but not aggressive handshake says you are confident, social and professional. A weak, finger-tip or overly-macho handshake can give the impression that you lack confidence or basic social skills.

The 5 S's of a Perfect Handshake

S**MILE**

S**TRAIGHT**

S**LIDE**

S**STRENGTH**

S**HAKE**



Different Kinds of Handshakes

The Hand Hug



Cold and Clammy



Dominator



The Knuckle Crusher



Different Kinds of Handshakes

The Finger Squeeze



The Polite Pinch



The Dead Fish



A Good Handshake

How to give a good handshake...

Hold your hand vertically in an L shape



Then wrap your fingers around



Legs and feet

- Legs and feet body language is more difficult to control consciously or fake than some body language of arms and hands and face. Legs and feet can therefore provide good clues to feelings and moods, if you know the signs.
- Also consider that when people sit for half-an-hour or more they tend to change their leg positions, which can include leg crossing purely for comfort reasons. Again allow for this when interpreting signals.
- Leg signals tend to be supported by corresponding arms signals, for example crossed arms and crossed legs, which aside from comfort reasons generally indicate detachment, disinterest, rejection or insecurity, etc

Keywords – Non Verbal

Nonverbal communication cues can play five roles:

- **Accept** : Accept the message the person is saying verbally – by nodding
- **Contradiction**: We can contradict a message the individual is trying to convey – using hand and head gestures
- **Substitution**: they can substitute for a verbal message. For example, a person's eyes can often convey a far more vivid message than words
- **Complementing**: they may add to or complement a verbal message. A boss who pats a person on the back in addition to giving praise can increase the impact of the message
- **Accenting**: they may accent or underline a verbal message - Pounding the table, for example, can underline a message.

How nonverbal communication can go wrong

- What you communicate through your body language and nonverbal signals affects how others see you, how well they like and respect you, and whether or not they trust you.
- Unfortunately, many people send confusing or negative nonverbal signals without even knowing it. When this happens, both connection and trust are damaged.



Evaluating nonverbal signals

- **Eye contact** – Is eye contact being made? If so, is it overly intense or just right?
- **Facial expression** – What is their face showing? Is it masklike and inexpressive, or emotionally present and filled with interest?
- **Tone of voice** – Does the person's voice project warmth, confidence, and interest, or is it strained and blocked?
- **Posture and gesture** – Is their body relaxed or stiff and immobile? Are shoulders tense and raised, or relaxed?
- **Touch** – Is there any physical contact? Is it appropriate to the situation? Does it make you feel uncomfortable?
- **Intensity** – Does the person seem flat, cool, and disinterested, or over-the-top and melodramatic?
- **Timing and place** – Is there an easy flow of information back and forth? Do nonverbal responses come too quickly or too slowly?
- **Sounds** – Do you hear sounds that indicate caring or concern?



End of Session - 3

Thank You...

RECRUITMENT ESSENTIALS

SESSION - 4

IMPRESSION MANAGEMENT



Grooming and Etiquette

What is Etiquette??

Webster's II New College Dictionary defines Etiquette as:

The forms and practices prescribed by social convention or by authority.

Does How Etiquette Benefit You?

- Differentiates you from others in a competitive job market
- Enables you to be confident in a variety of settings with a variety of people
- Honors commitment to excellence and quality
- Modifies distracting behaviors and develops admired conduct

“Be one step ahead, practice the social skills necessary to help you make a great first impression and stand out in a competitive job market”.

-Kenneth-

Critical Etiquette Topics to Consider

- Etiquette Basics
- Professional Appearance
- Office Etiquette
- Dining and Table Manners

Etiquette Basics

(Creating a positive image)

- ***Behaviour:***

- Exhibit a positive attitude and pleasant demeanor
- Use a firm handshake
- Maintain good eye contact
- Appropriate introductions – introduce someone by their title and last name (Ms. Mrs. Mr. Dr. Swami), unless otherwise specified
- Rise when you are introducing someone or you are being introduced
- Nonverbal communication is important
- Show common respect and consideration for others

- Do you have these? If your answer is yes, good for you.
- But if your answer is no, then you know it is time to take this training seriously or learn them yourself

DRESSING, APPEARANCE AND GROOMING

- Have you ever judged a person based solely on their dressing? While you may think you haven't, you probably have.
- Before you even hear them speak or know anything about them, you probably make certain assumptions about a person based on their dressing.
- Often, we expect certain people to dress a certain way before we learn anything about their credentials.

DRESSING, APPEARANCE AND GROOMING

- If you have ever gone on a job interview, you have probably put much thought into what you wore.
- Most people try to pick out one of their nicest outfits to wear, so they make a good first impression before the employer ever asks them any questions.
- First impressions are very important and dressing is one of the biggest factors in first impressions.
- We all know that our clothing is important, especially in the work setting, but what may be unfamiliar is a clear sense of what our clothes are communicating — not only to others, but to ourselves.

Types of design elements

- Here are three distinct types of design elements, along with their “innate messages”:
 - **Light color:** fresh, open, louder, flexible
 - **Dark color:** calm, sophisticated, soothing, precise
 - **Bright color:** energetic, cheerful, stimulating, responsive
-
- **Straight lines:** formal, crisp, strong, persuasive
- **Curved lines:** casual, graceful, romantic, approachable
-
- **Firm fabrics:** stable, official, grounded, classic
- **Pliable fabrics:** casual, agreeable, relaxed, temporary

Equally important is well fitting and appropriate fitting clothes. When buttons pucker, you draw attention to your body size exceeding your clothes allotment. Alternatively, properly fitting clothes can hide a few extra pounds

Good Grooming vs. Bad Grooming

27.5%

think that it shows their interest in the position

24.8%

think that it shows the candidate has good character

20.2%

think that it is an important job requirement

60.2%

think that it's important to appear fit, healthy, clean, happy, energetic, confident and well dressed

48.5%

spend a moderate amount of their income on grooming and clothes

75.8%

make a considerable effort in looking after their appearance



51.9%

say that tattoos and piercings are not allowed in their office

77.3%

think that you risk being negatively judged on your personal grooming and clothes

46.4%

are not aware of the company dress code and found out about it later

TIP

Three basic rules for making the right impression at work:

1. Presentation does count
2. A casual office environment doesn't mean that you have to dress carelessly
3. Dress as you want to be seen; a person who takes their job seriously, is professional, and upward-bound."

Professional Appearance

- ***Grooming is fundamental***
 - Hair clean and styled appropriately
 - Clean nails, skin and teeth
 - Many professionals wear make-up
(depends on field)
 - Check fragrance and clothing care

Professional Appearance

- ***Wardrobe***
 - Professional Business Wardrobe
 - For women: skirted/pant suit, blouse or dress shirt, sari,salwar-kameez, clean and well maintained dress shoes (generally closed-toe shoe)
 - For men: suit, dress shirt, tie (well maintained dress shoes)
 - Outerwear
 - Appropriate for women/men: Trench coat, umbrella

Professional Work Attire



Professional Appearance

Business Casual Wardrobe

- For women: dress pants, shirt, blouse, sari well maintained dress or casual shoe (no tennis shoes, flip flops, etc.)
- For men: slacks/khaki pants, polo shirt, or other collared dress shirt, well maintained dress or casual shoes (no tennis shoes, flip flops, etc.)

****NOTHING SLOPPY****

Casual Work Attire



Photo: Lands End / www.landsend.com

Common Business Etiquette

Professional Etiquette :

Meeting and Greeting



Handshake: Offer entire hand, web-to-web, shake lightly and release

Know whom to introduce first Junior to senior Fellow worker to client

Eliminate slang / jargon from your vocabulary

Always on time, always organized, always ready

How To Introduce Yourself



- Stand up
- Look the person in the eye
- Extend your hand for a firm web-to-web handshake. Avoid:
 - Bone-crushing handshakes
 - “Dead fish” handshakes
 - Grabbing someone’s fingers
- Say your name and something about yourself

“Hello, I’m John Smith. I work in Process Improvement over in Polymers Division.”

Greetings



- Respond to a 'How do you do', with another 'How do YOU do', not 'Fine'. And to a 'Thank you', with 'You're welcome', not 'Mention Not'.
- At work, make it a habit to greet pleasantly everyone you see for the first time in the day, particularly subordinate staff. Do not go on an ego trip and wait for others, including juniors, to greet you first. Initiate the greeting. Respond to all greetings, especially from juniors, with a smile and pleasant eye contact, and perhaps a brief word.
- Depending on the culture of the company, use Sir, Mr. or first names. If you are a new employee, even when you see others address senior officers and women by the first name, wait till you are invited to do so, or ask how they would like to be addressed by you. Do not use pet names at work.

Business Introductions



- In business, introductions are based on power and hierarchy.
- Simply, persons of lesser authority are introduced to persons of greater authority.
- Gender plays no role in business etiquette; nor does it affect the order of introductions.
- For example, you would say, "Mr./Ms. Greater Authority, I would like to introduce Mr./Ms. Lesser Authority." However, the person holding the highest rank may not be Mr./Ms. Greater Authority. A client, for instance, always takes precedence over anyone in your organization, as does an elected official. Here are examples of pecking order:

Handshakes



- The more senior and older executives and visitors put their hand out first to shake hands.
- Peers make a spontaneous simultaneous move.
- Both men and women must keep the grip firm, neither crushing nor limp.
- Do not hold the hand too long.
- The more important person signals the release by a slight lessening of the grip. During the handshake, smile and maintain pleasant eye contact.
- In the office, both men and women always rise to shake hands. Ideally, in the office, walk out from behind your desk to shake hands.
- In a social setting, the woman puts her hand out first. She may also remain seated, as may a very old man, when shaking hands with a much younger or equal age person. At a cocktail party, put down, or hold discreetly in the left hand, your drink and cigarette to shake hands. Resume smoking only after the guest has taken a seat or has moved on.
- In traditional cultures, if a non-smoking older lady or man converses with you, it is best to put out your cigarette

Exchanging visiting cards



- Worldwide today, the Japanese style of exchanging calling cards is in practice:
- Hold your card out, so that the writing faces the person you want to present your card to. Hold the card, using the thumbs and index fingers of both hands to grip it at the left and right tops. Bow as you extend the card towards the other person. The more junior bows deeper than the senior does. The receiver takes the card with the same thumb and index finger 'pincer movement'.
- When you receive a card, thank the presenter, but do not put it away immediately in your pocket or wallet. Do not fold or mutilate it. Do not write on it. Do not use it to fan yourself, clean your fingernail or scratch your nose with it. Hold it with respect, look at it with interest, and ask for any clarifications or correct pronunciations. Finally, place it in a proper cardholder.
- If you are in a meeting with up to five or six people, place all their cards in front of you, in the order in which they are seated. As they talk, look at the card to connect the name with the face.

Protocols

- Always use last names with customers unless they are about your age and rank
- Don't keep customers waiting
- Escort clients out
- When someone of higher rank or from outside the organization enters, everyone in the office stands
- Junior employees stand until seniors sit



10 Common Etiquette Mistakes

- Eating with your mouth open
- Grooming in public
- Being a gym slob
- Talking solely about yourself
- Treating service staff poorly
- Not giving up your seat
- Being late
- Arguing over a check
- Not removing yourself to take a cell phone call
- Failing to introduce people

Dining Etiquette

The Importance of Dining Appropriately

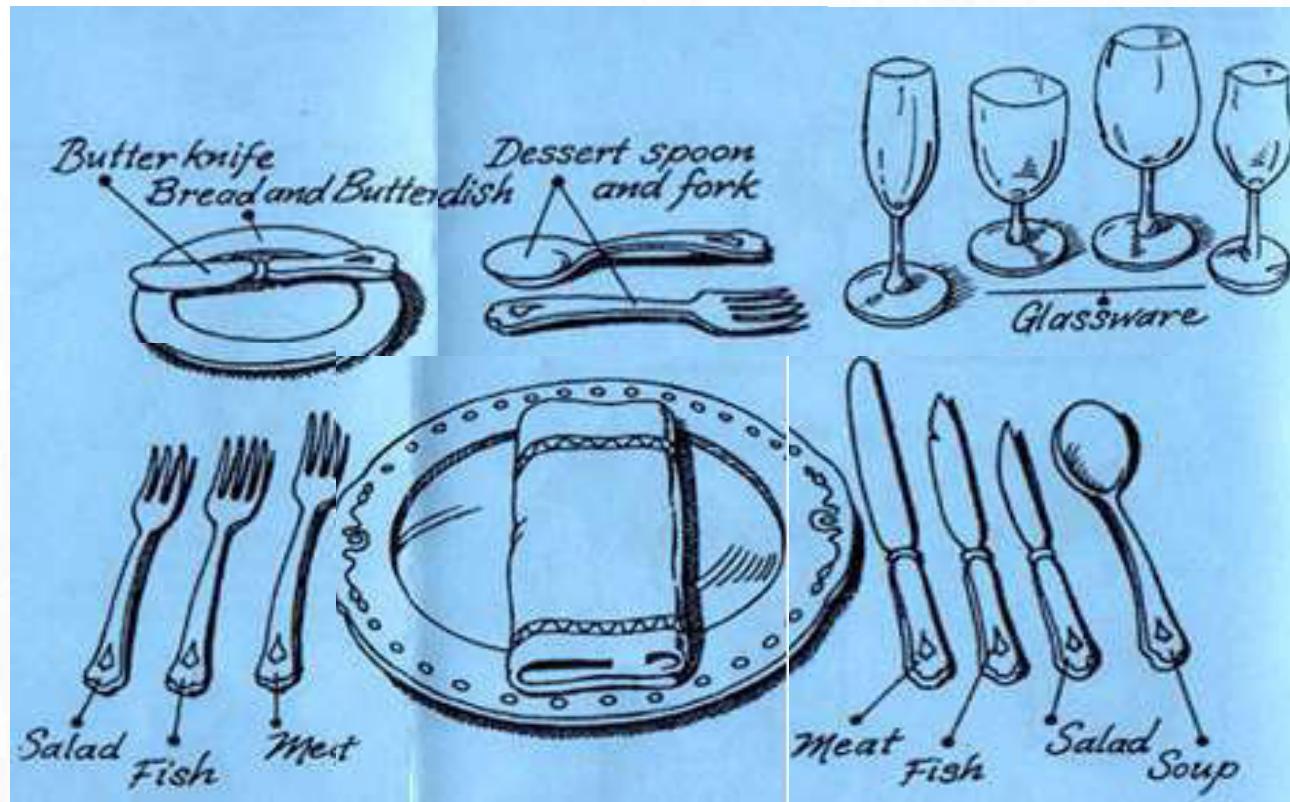
80% of second interviews involve a business meal. Therefore Table Manners is an integral part etiquettes.

Professional Dining Basics

Style Dining

- Arrive on time
- Wait to sit until host/hostess indicated the seating arrangement
- Put napkin in lap before drinking or eating
- Order easy to eat food
- Don't order the most expensive items on the menu
- Wait until everyone has been served before you begin to eat
- Bring food to your mouth - NOT your head to your plate
- Salt/Pepper pass together
- Generally pass food to the right
- Rest utensils on plate while talking
- Do not talk with your mouth full

Individual Place Setting



Professional Dining Basics

- Proper posture is important
- Table manners please!!
 - (No gum, no elbows on the table)
- **“Please and Thank You”**
- Turn your cell phone off
- Be responsible for keeping up and positively contributing to the conversation
- Small Talk is appropriate – topics such as :
 - Books, sports, food, theater, travel, current events etc.
 - Follow employer's lead

- Serve guests of honor, woman first, then male, then counterclockwise around the table. Serve host then hostess last.
- Small party of 6 or less, wait to eat until hostess begins. At a large party, hostess urges everyone to begin as they are served
- Place knife and fork on plate after using, knife with sharp edge facing in and fork with tines up (American) or down (Continental) and placed so they will not fall off the plate.

- Never place used silverware on the table or leave it in a cup or small bowl.
- A used soup spoon is left in a large soup plate or on the plate under the soup bowl.
- A used coffee spoon is placed on the saucer beneath the handle of the cup.
- Unused silver is left on the table.

- When you are finished, place the fork and knife parallel to each other, so they lie either across horizontally the center of the plate or are on the diagonal, with the handles pointing to the right.
- Dessert silverware – Place so the spoon can be picked up with the right hand and the fork so it can be picked up with the left hand.
- The napkin – as soon as you are seated, remove the napkin from your place setting, unfold it, and place it in your lap.

- If your napkin falls on the floor during a formal meal, signal a waiter that you need a fresh one.
- When you leave the table at the end of a meal, place your napkin loosely folded next to your plate. You may place it in the napkin ring if one is present. Do not place your napkin in your empty plate.



- When eating soup or dessert that has liquid, it is acceptable to tip the bowl when necessary, but tip it away from you.
- Test liquids before eating by testing a small amount with a spoon to see if it is too hot. Never blow on food to cool it.

- Do not automatically add salt or pepper before tasting. If you need to add to suit your taste, do it unobtrusively.
- When asked to pass the salt or pepper, pick up both the salt and pepper and place them on the table within reach of the person next to you who will do the same, and so on, until they reach the person who asked for them. They are not passed hand-to-hand.

- To remove inedible items from the mouth, it should go out the same way it went in.
- Olive pits can be delicately dropped onto an open palm before putting them onto your plate.
- A piece of bone discovered in a bit of chicken should be returned to the plate by way of the fork.
- Fish is an exception to the rule. It is fine to remove the tiny bones with your fingers, since they would be difficult to drop from your mouth onto the fork.

- Big seeds should be removed with a spoon.
- An extremely fatty piece of meat that you simply can't bring yourself to swallow, it will be necessary to surreptitiously spit it into your napkin, so that you can keep it out of sight.
- Just like your mother told you, Keep your elbows off the table!

- If you are someone's guest at a restaurant, ask the person what he/she recommends. By doing this, you will learn the price range guidelines and have an idea of what to order.
- Usually order an item in the mid price range. Keep in mind that the person who typically initiates the meal will pay.
- Don't order appetizers or dessert unless your host does. It is inappropriate for your meal to cost more than your host's meal.
- When ordering, avoid foods that are difficult to eat gracefully.
- Be prepared. If there is a purpose to the luncheon or dinner meeting, make it clear when extending or accepting an invitation. Bring writing materials.

- If you're dining at someone's home and aren't sure what to do, follow the actions of the host or hostess as a guide.
- Cut food into small pieces for eating. If you try to eat large pieces, you may have difficulty chewing and might choke.
- People from different countries and cultures have table manners that may be different from yours. Respect and accept people with other customs.



End of Session - 4

Thank You...



RECRUITMENT ESSENTIALS

SESSION - 5

IMPRESSION MANAGEMENT



Activities...

Match the following

Non - Verbal Communication	Interpretation
Facial Expressions	
Frown	Embarrassment
Smile	Anger
Raised eyebrows	Displeasure, unhappiness
Narrowed eyes	Friendliness, happiness
Blushing	Disbelief, amazement

Match the following

Non - Verbal Communication	Interpretation
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Frown	Embarrassment
Smile	Anger
Raised eyebrows	Displeasure, unhappiness
Narrowed eyes	Friendliness, happiness
Blushing	Disbelief, amazement

Match the following

Hand/Arm Gestures	
Pointing finger	Disbelief, puzzlement, uncertainty
Folded arms	Open to suggestions, relaxed
Arms at side	Not open to change, preparing to speak
Hands uplifted outward	Authority, displeasure, lecturing

Match the following

Hand/Arm Gestures	
Pointing finger	Disbelief, puzzlement, uncertainty
Folded arms	Open to suggestions, relaxed
Arms at side	Not open to change, preparing to speak
Hands uplifted outward	Authority, displeasure, lecturing

Match the following

Body Postures	
Fidgeting, doodling	Nervousness
Hands on hips	Listening, great concern
Shrugging shoulders	Boredom, lack of interest
Squared stance or shoulders	Boredom
Biting lip, shifting, jingling money	Anger, defensiveness
Sitting on the edge of a chair	Indifference
Slouching in chair	Problem-solving, concern, listening

Match the following

Body Postures	
Fidgeting, doodling	Nervousness
Hands on hips	Listening, great concern
Shrugging shoulders	Boredom, lack of interest
Squared stance or shoulders	Boredom
Biting lip, shifting, jingling money	Anger, defensiveness
Sitting on the edge of a chair	Indifference
Slouching in chair	Problem-solving, concern, listening

Match the following

Proxemics (Physical Space)	
From physical contact to 18 inches	Social space
From 18 inches to 4 feet	Public space
From 4 feet to 8 feet	High status
From 8 feet outward	Intimate space
Prestigious titles	Personal space

Match the following

Proxemics (Physical Space)	
From physical contact to 18 inches	Social space
From 18 inches to 4 feet	Public space
From 4 feet to 8 feet	High status
From 8 feet outward	Intimate space
Prestigious titles	Personal space

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

1. What expression is shown on Carlton's face?



- (a) Worried (b) Panicked (c) Excited (d) Scared

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

2. Which word best describes Maria's expression?



- (a) Ecstatic (b) Furious (c) Mischievous (d) Angry

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

3. Michael is shown here as ...



- (a) Angry (b) Thoughtful (c) Enthusiastic (d) Relieved

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

4. Patrick's expression is ...



- (a) Happy (b) Concerned (c) Disturbed (d) Anguished

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

5. Maria's expression could be described as ...



- (a) Excited (b) Intrigued (c) Disgusted (d) Fascinated

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

6. Eric expression could be described as ...



- (a) Happy (b) Excited (c) Overjoyed (d) Disbelieving

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

7. Janice expression is:



- (a) Excited (b) Worried (c) Surprised (d) Perplexed

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

8. Carlton face clearly shows the emotion of:



- (a) Confusion
- (b) Sadness and hurt
- (c) Excitement and anticipation
- (d) Love and desire

Facial Expression quiz

A quiz to test how well you can read facial expressions using facial expressions of characters.

9. Dexter felt a strong emotion, when two fighting boys fell through the glass door – what emotion is it:



- (a) Shocked (b) Passionate (c) Angry (d) Curious

Facial Expression quiz

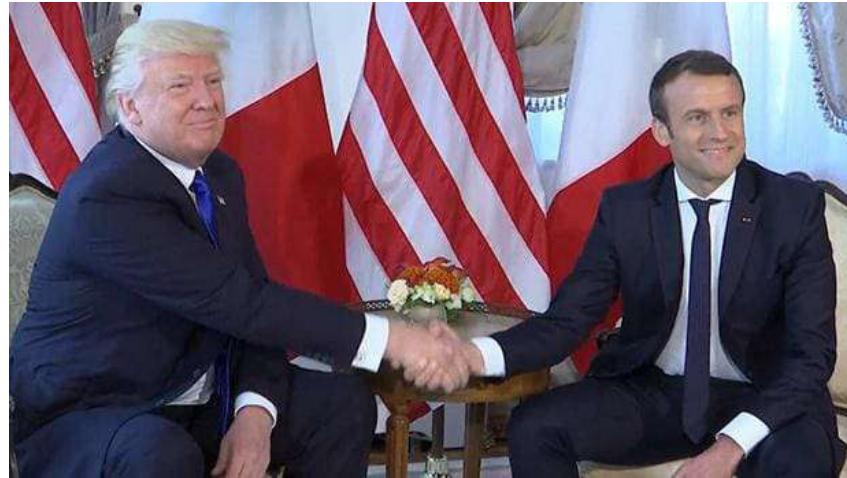
A quiz to test how well you can read facial expressions using facial expressions of characters.

10. Kat's facial expression in below pic shows her attitude of

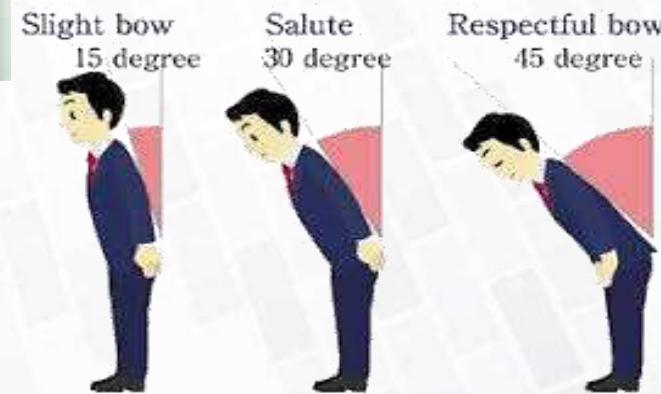


- (a) Contempt (b) Delight (c) Joy (d) Excitement

Activity: Compare the images and make a study on Donald Trump's body language.



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



What do these gestures mean?...



Guess the body language



1. (a) Upset (b) Telling lie (c) Tears of joy (d) Surprised

Guess the body language



2. (a) Excitement (b) Frustration (c) Headache (d) Shouting

Guess the body language



3. (a) Happy (b) Depressed (c) Tired (d) Bored

Guess the body language



4. (a) Confused (b) Thinking
(c) Brainstorming (d) None of these

Guess the body language



5. (a) Amazed (b) Thinking (c) Meditating (d) Sorrow

Guess the body language



6. (a) Tired (b) Sleeping (c) Upset (d) Depressed

Guess the body language



7. (a) Pleasing (b) Explaining (c) Anger (d) Conversing

Guess the body language



8. (a) Listening (b) Staring (c) Joking (d) None of these

Guess the body language



9. (a) Argument (b) Conversation (c) Casual talk (d) Fight

Guess the body language



10. (a) Boredom (b) Interested (c) Tired (d) Sad

End of Session - 5

Thank You...

SESSION - 1

PARTS OF SPEECH

There are eight parts of speech in the English language: noun, pronoun, verb, adjective, adverb, preposition, conjunction, and interjection. The part of speech indicates how the word functions in meaning as well as grammatically within the sentence. An individual word can function as more than one part of speech when used in different circumstances.

NOUN

A noun is the name of a person, place, thing, or idea.

- *man... College... house... happiness*

Proper nouns always start with a capital letter; common nouns do not. Nouns can be singular or plural, concrete or abstract. Nouns show possession by adding 's'.

Nouns can function in different roles within a sentence; for example, a noun can be a subject, direct object, indirect object, subject complement, or object of a preposition.

- *The young girl brought me a very long letter from the teacher, and then she quickly disappeared.*

Some more Examples

The **highlighted** words in the following sentences are all nouns:

- Late last **year** our **neighbours** bought a **goat**.
- **Portia White** was an **opera singer**.
- The **bus inspector** looked at all the **passengers' passes**.
- According to **Plutarch**, the **library** at **Alexandria** was destroyed in 48 B.C.
- **Philosophy** is of little **comfort** to the **starving**.

Noun Plurals

Most nouns change their form to indicate number by adding "-s" or "-es", as illustrated in the following sentences:

- When Matthew was small he rarely told the **truth** if he thought he was going to be punished.
- Many people do not believe that **truths** are self-evident.
- As they walked through the silent house, they were startled by an unexpected **echo**.
- I like to shout into the quarry and listen to the **echoes** that return.
- He tripped over a **box** left carelessly in the hallway.
- Since we are moving, we will need many **boxes**.

There are other nouns which form the plural by changing the last letter before adding "s". Some words ending in "f" form the plural by deleting "f" and adding "ves," and words ending in "y" form the plural by deleting the "y" and adding "ies," as in the following pairs of sentences:

- The harbour at Marble Mountain has one **wharf**.
- There are several **wharves** in Halifax Harbour.
- Warsaw is their favourite **city** because it reminds them of their courtship.
- The vacation my grandparents won includes trips to twelve European **cities**.
- The children circled around the headmaster and shouted, "Are you a **mouse** or a man?"
- The audience was shocked when all five men admitted that they were afraid of **mice**.

Other nouns form the plural irregularly. If English is your first language, you probably know most of these already: when in doubt, consult a good dictionary.

Possessive Nouns

In the possessive case, a noun or pronoun changes its form to show that it owns or is closely related to something else. Usually, nouns become possessive by adding a combination of an apostrophe and the letter "s."

You can form the possessive case of a singular noun that does not end in "s" by adding an apostrophe and "s," as in the following sentences:

- The red suitcase is **Cassandra's**.
- The only luggage that was lost was the **prime minister's**.
- The exhausted recruits were woken before dawn by the **drill sergeant's** screams.
- The **miner's** face was covered in coal dust.

You can form the possessive case of a singular noun that ends in "s" by adding an apostrophe alone or by adding an apostrophe and "s," as in the following examples:

- The **bus's** seats are very uncomfortable.
- The **bus'** seats are very uncomfortable.
- The film crew accidentally crushed the **platypus's** eggs.
- The film crew accidentally crushed the **platypus'** eggs.
- **Felicia Hemans's** poetry was once more popular than Lord Byron's.
- **Felicia Hemans'** poetry was once more popular than Lord Byron's.

You can form the possessive case of a plural noun that does not end in "s" by adding an apostrophe and a "s," as in the following examples:

- The **children's** mittens were scattered on the floor of the porch.
- The **sheep's** pen was mucked out every day.
- Since we have a complex appeal process, a **jury's** verdict is not always final.
- The **men's** hockey team will be playing as soon as the **women's** team is finished.
- The hunter followed the **moose's** trail all morning but lost it in the afternoon.
- You can form the possessive case of a plural noun that *does* end in "s" by adding an apostrophe:
- The concert was interrupted by the **dogs'** barking, the **ducks'** quacking, and the **babies'** squalling.
- The **janitors'** room is downstairs and to the left.
- My uncle spent many hours trying to locate the **squirrels'** nest.

Using Possessive Nouns

When you read the following sentences, you will notice that a noun in the possessive case frequently functions as an adjective modifying another noun:

- The **miner's** face was covered in coal dust.

Here the possessive noun "miner's" is used to modify the noun "face" and together with the article "the," they make up the noun phrase that is the sentence's subject.

- The concert was interrupted by the **dogs'** barking, the **ducks'** quacking, and the **babies'** squalling.

In this sentence, each possessive noun modifies a gerund. The possessive noun "dogs'" modifies "barking," "ducks'" modifies "quacking," and "babies'" modifies "squalling."

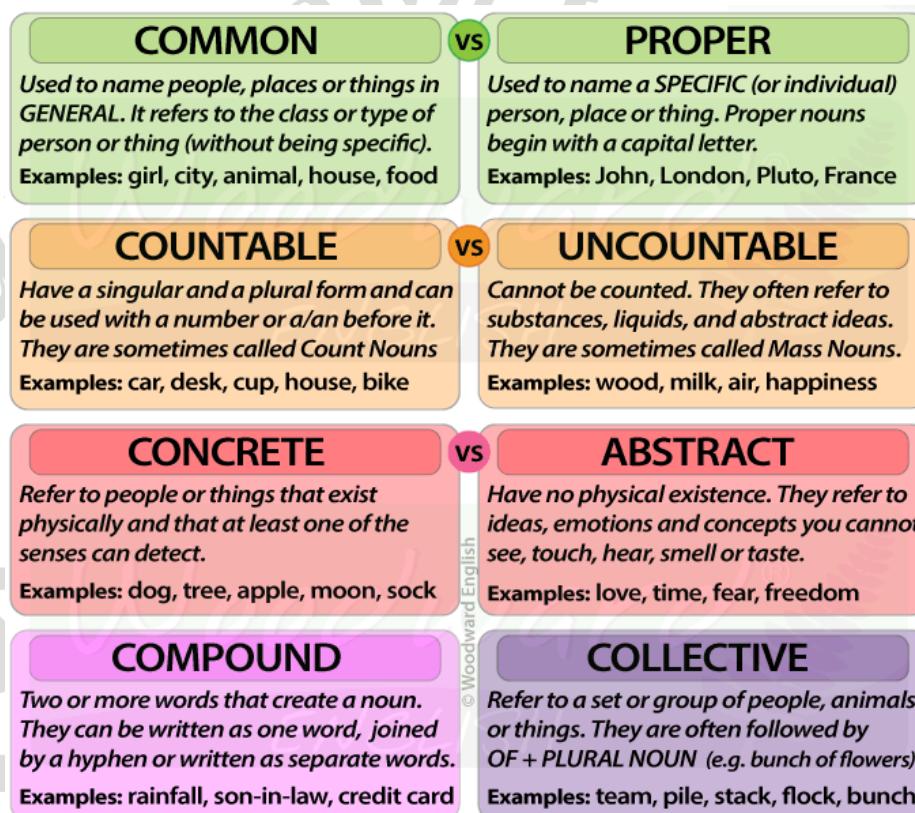
- The film crew accidentally crushed the **platypus's** eggs.

In this example the possessive noun "platypus's" modifies the noun "eggs" and the noun phrase "the platypus's eggs" is the direct object of the verb "crushed."

- My uncle spent many hours trying to locate the **squirrels'** nest.

In this sentence the possessive noun "squirrels'" is used to modify the noun "nest" and the noun phrase "the squirrels' nest" is the object of the infinitive phrase "to locate."

Types of Nouns



There are many different types of nouns. As you know, you capitalise some nouns, such as "Canada" or "Louise," and do not capitalise others, such as "badger" or "tree" (unless they appear at the beginning of a sentence). In fact, grammarians have developed a whole series of noun types, including the proper noun, the common noun, the concrete noun, the abstract noun, the countable noun (also called the count noun), the non-countable noun (also called the mass noun), and the collective noun. You should note that a noun will belong to more than one type: it will be proper or common, abstract or concrete, and countable or non-countable or collective.

Proper Nouns

You always write a **proper noun** with a capital letter, since the noun represents the name of a specific person, place, or thing. The names of days of the week, months, historical documents, institutions, organisations, religions, their holy texts and their adherents are proper nouns. A proper noun is the opposite of a common noun

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In each of the following sentences, the proper nouns are **highlighted**:

- The **Maroons** were transported from **Jamaica** and forced to build the fortifications in **Halifax**.
- Many people dread **Monday** mornings.
- **Beltane** is celebrated on the first of **May**.
- **Abraham** appears in the **Talmud** and in the **Koran**.
- Last year, I had a **Baptist**, a **Buddhist**, and a **Gardnerian Witch** as roommates.

Common Nouns

A **common noun** is a noun referring to a person, place, or thing in a general sense -- usually, you should write it with a capital letter only when it begins a sentence. A common noun is the opposite of a proper noun.

In each of the following sentences, the common nouns are **highlighted**:

- According to the **sign**, the nearest **town** is 60 **miles** away.
- All the **gardens** in the **neighbourhood** were invaded by **beetles** this **summer**.
- I don't understand why some **people** insist on having six different **kinds** of **mustard** in their **cupboards**.
- The road **crew** was startled by the **sight** of three large **moose** crossing the **road**.
- Many child-care **workers** are underpaid.

Sometimes you will make proper nouns out of common nouns, as in the following examples:

- The tenants in the **Garnet Apartments** are appealing the large and sudden increase in their rent.
- The meals in the Bouncing **Bean Restaurant** are less expensive than meals in ordinary restaurants.
- Many witches refer to the Renaissance as the Burning **Times**.
- The **Diary of Anne Frank** is often a child's first introduction to the history of the **Holocaust**.

Concrete Nouns

A **concrete noun** is a noun which names anything (or anyone) that you can perceive through your physical senses: touch, sight, taste, hearing, or smell. A concrete noun is the opposite of a abstract noun.

The **highlighted** words in the following sentences are all concrete nouns:

- The **judge** handed the **files** to the **clerk**.
- Whenever they take the **dog** to the **beach**, it spends hours chasing **waves**.
- The real estate **agent** urged the **couple** to buy the second **house** because it had new **shingles**.
- As the **car** drove past the **park**, the **thump** of a disco **tune** overwhelmed the string **quartet's** rendition of a **minuet**.
- The **book binder** replaced the flimsy paper **cover** with a sturdy, cloth-covered **board**.

Abstract Nouns

An **abstract noun** is a noun which names anything which you can *not* perceive through your five physical senses, and is the opposite of a concrete noun. The **highlighted** words in the following sentences are all abstract nouns:

- Buying the fire extinguisher was an **afterthought**.
- Tillie is amused by people who are nostalgic about **childhood**.
- **Justice** often seems to slip out of our grasp.
- Some scientists believe that **schizophrenia** is transmitted genetically.

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Countable Nouns

A **countable noun** (or **count noun**) is a noun with both a singular and a plural form, and it names anything (or anyone) that you can *count*. You can make a countable noun plural and attach it to a plural verb in a sentence. Countable nouns are the opposite of non-countable nouns and collective nouns.

In each of the following sentences, the **highlighted** words are countable nouns:

- We painted the **table** red and the **chairs** blue.
- Since he inherited his **aunt's library**, Jerome spends every **weekend** indexing his **books**.
- Miriam found six silver **dollars** in the **toe** of a **sock**.
- The oak **tree** lost three **branches** in the **hurricane**.
- Over the **course** of twenty-seven **years**, Martha Ballad delivered just over eight hundred **babies**.

Non-Countable Nouns

A **non-countable noun** (or **mass noun**) is a noun which does not have a plural form, and which refers to something that you could (or would) not usually count. A non-countable noun always takes a singular verb in a sentence. Non-countable nouns are similar to collective nouns, and are the opposite of countable nouns.

The **highlighted** words in the following sentences are non-countable nouns:

- Joseph Priestly discovered **oxygen**.
- The word "oxygen" cannot normally be made plural.
- **Oxygen** is essential to human life.
 - Since "oxygen" is a non-countable noun, it takes the singular verb "is" rather than the plural verb "are."
- We decided to sell the **furniture** rather than take it with us when we moved.
 - You cannot make the noun "furniture" plural.
- The **furniture** is heaped in the middle of the room.
 - Since "furniture" is a non-countable noun, it takes a singular verb, "is heaped."
- The crew spread the **gravel** over the roadbed.
 - You cannot make the non-countable noun "gravel" plural.
- **Gravel** is more expensive than I thought.
 - Since "gravel" is a non-countable noun, it takes the singular verb form "is."

Collective Nouns

A **collective noun** is a noun naming a group of things, animals, or persons. You could count the individual members of the group, but you usually think of the group as a whole is generally as one unit. You need to be able to recognise collective nouns in order to maintain subject-verb agreement. A collective noun is similar to a non-countable noun, and is roughly the opposite of a countable noun.

In each of the following sentences, the **highlighted** word is a collective noun:

- The **flock** of geese spends most of its time in the pasture.
- The collective noun "flock" takes the singular verb "spends."
- The **jury** is dining on take-out chicken tonight.
 - In this example the collective noun "jury" is the subject of the singular compound verb "is dining."
- The steering **committee** meets every Wednesday afternoon.
 - Here the collective noun "committee" takes a singular verb, "meets."

- The **class** was startled by the bursting light bulb.
 - In this sentence the word "class" is a collective noun and takes the singular compound verb "was startled."

Exercise 1

Identify the nouns in the following passage.

- While we were traveling through the countryside we passed a small village. We spotted a postman delivering letters while a greengrocer, dressed in a bottle green shirt was selling vegetables.

Ans:

- | | | | |
|---------------|-----------|--------------|-----------|
| • Countryside | • Letters | • Vegetables | • Village |
| • Greengrocer | • Postman | • Shirt | |

Explanation: Noun is a part of speech (or word class) that is used to name or identify a person, place, thing, quality, or action.

SESSION – 2

❖ PRONOUN

A pronoun is a word used in place of a noun.

- She... we... they... it

A pronoun is usually substituted for a specific noun, which is called its antecedent. In the sentence above, the antecedent for the pronoun she is the girl. Pronouns are further defined by type: personal pronouns refer to specific persons or things; possessive pronouns indicate ownership; reflexive pronouns are used to emphasize another noun or pronoun; relative pronouns introduce a subordinate clause; and demonstrative pronouns identify, point to, or refer to nouns.

- The young girl brought **me** a very long letter from the teacher, and then **she** quickly disappeared.

Personal Pronouns

A **personal pronoun** refers to a specific person or thing and changes its form to indicate person, number, gender, and case.

Subjective Personal Pronouns

A **subjective personal pronoun** indicates that the pronoun is acting as the subject of the sentence. The subjective personal pronouns are "I," "you," "she," "he," "it," "we," "you," "they."

In the following sentences, each of the **highlighted** words is a subjective personal pronoun and acts as the subject of the sentence:

- I was glad to find the bus pass in the bottom of the green knapsack.
- You are surely the strangest child I have ever met.
- He stole the money from a shop keeper.
- When she was a young woman, she earned her living as a coal miner.
- After many years, they returned to their homeland.
- We will meet at the library at 3:30 p.m.
- It is on the counter.
- Are you the delegates from Malaga watch?

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Objective Personal Pronouns

An **objective personal pronoun** indicates that the pronoun is acting as an object of a verb, compound verb, preposition, or infinitive phrase. The objective personal pronouns are: "me," "you," "her," "him," "it," "us," "you," and "them."

In the following sentences, each of the **highlighted** words is an objective personal pronoun:

- Seamus stole the selkie's skin and forced **her** to live with **him**.
 - The objective personal pronoun "her" is the direct object of the verb "forced" and the objective personal pronoun "him" is the object of the preposition "with."
- After reading the pamphlet, Judy threw **it** into the garbage can.
 - The pronoun "it" is the direct object of the verb "threw."
- The agitated assistant stood up and faced the angry delegates and said, "Our leader will address **you** in five minutes."
 - In this sentence, the pronoun "you" is the direct object of the verb "address."
- Deborah and Roberta will meet **us** at the newest café in the market.
 - Here the objective personal pronoun "us" is the direct object of the compound verb "will meet."
- Give the list to **me**.
 - Here the objective personal pronoun "me" is the object of the preposition "to."
- I'm not sure that my contact will talk to **you**.
 - Similarly in this example, the objective personal pronoun "you" is the object of the preposition "to."
- Christopher was surprised to see **her** at the drag races.
 - Here the objective personal pronoun "her" is the object of the infinitive phrase "to see."

Possessive Personal Pronouns

A **possessive pronoun** indicates that the pronoun is acting as a marker of possession and defines who owns a particular object or person. The **possessive personal pronouns** are "mine," "yours," "hers," "his," "its," "ours," and "theirs." Note that possessive personal pronouns are very similar to possessive adjectives like "my," "her," and "their."

In each of the following sentences, the **highlighted** word is a possessive personal pronoun:

- The smallest gift is **mine**.
 - Here the possessive pronoun "mine" functions as a subject complement.
- This is **yours**.
 - Here too the possessive pronoun "yours" functions as a subject complement.
- His is on the kitchen counter.
 - In this example, the possessive pronoun "his" acts as the subject of the sentence.
- Theirs will be delivered tomorrow.
 - In this sentence, the possessive pronoun "theirs" is the subject of the sentence.
- Ours is the green one on the corner.
 - Here too the possessive pronoun "ours" function as the subject of the sentence.

Demonstrative Pronouns

A **demonstrative pronoun** points to and identifies a noun or a pronoun. "This" and "these" refer to things that are nearby either in space or in time, while "that" and "those" refer to things that are farther away in space or time.

The demonstrative pronouns are "this," "that," "these," and "those." "This" and "that" are used to refer to singular nouns or noun phrases and "these" and "those" are used to refer to plural nouns and noun phrases. Note that the demonstrative pronouns are identical to demonstrative adjectives, though, obviously, you use them differently. It is also important to note that "that" can also be used as a relative pronoun.

In the following sentences, each of the **highlighted** words is a demonstrative pronoun:

- **This** must not continue.
 - Here "this" is used as the subject of the compound verb "must not continue."
- **This** is puny; **that** is the tree I want.
 - In this example "this" is used as subject and refers to something close to the speaker. The demonstrative pronoun "that" is also a subject but refers to something farther away from the speaker.
- Three customers wanted **these**.
 - Here "these" is the direct object of the verb "wanted."

Interrogative Pronouns

An **interrogative pronoun** is used to ask questions. The interrogative pronouns are "who," "whom," "which," "what" and the compounds formed with the suffix "ever" ("whoever," "whomever," "whichever," and "whatever"). Note that either "which" or "what" can also be used as an interrogative adjective, and that "who," "whom," or "which" can also be used as a relative pronoun.

You will find "who," "whom," and occasionally "which" used to refer to people, and "which" and "what" used to refer to things and to animals.

"Who" acts as the subject of a verb, while "whom" acts as the object of a verb, preposition, or a verbal.

The **highlighted** word in each of the following sentences is an interrogative pronoun:

- **Which** wants to see the dentist first?
 - "Which" is the subject of the sentence.
- **Who** wrote the novel Rockbound?
 - Similarly "who" is the subject of the sentence.
- **Whom** do you think we should invite?
 - In this sentence, "whom" is the object of the verb "invite."
- To **whom** do you wish to speak?
 - Here the interrogative pronoun "whom" is the object of the preposition "to."
- **Who** will meet the delegates at the train station?
 - In this sentence, the interrogative pronoun "who" is the subject of the compound verb "will meet."
- To **whom** did you give the paper?
 - In this example the interrogative pronoun "whom" is the object of the preposition "to."
- **What** did she say?
 - Here the interrogative pronoun "what" is the direct object of the verb "say."

Relative Pronouns

You can use a **relative pronoun** is used to link one phrase or clause to another phrase or clause. The relative pronouns are "who," "whom," "that," and "which." The compounds "whoever," "whomever," and "whichever" are also relative pronouns.

You can use the relative pronouns "who" and "whoever" to refer to the subject of a clause or sentence, and "whom" and "whomever" to refer to the objects of a verb, a verbal or a preposition.

In each of the following sentences, the **highlighted** word is a relative pronoun.

- You may invite **whomever** you like to the party.
 - The relative pronoun "whomever" is the direct object of the compound verb "may invite."
- The candidate **who** wins the greatest popular vote is not always elected.

- In this sentence, the relative pronoun is the subject of the verb "wins" and introduces the subordinate clause "who wins the greatest popular vote." This subordinate clause acts as an adjective modifying "candidate."
- In a time of crisis, the manager asks the workers **whom** she believes to be the most efficient to arrive an hour earlier than usual.
 - In this sentence "whom" is the direct object of the verb "believes" and introduces the subordinate clause "whom she believes to be the most efficient". This subordinate clause modifies the noun "workers."
- **Whoever** broke the window will have to replace it.
 - Here "whoever" functions as the subject of the verb "broke."
- The crate **which** was left in the corridor has now been moved into the storage closet.
 - In this example "which" acts as the subject of the compound verb "was left" and introduces the subordinate clause "which was left in the corridor." The subordinate clause acts as an adjective modifying the noun "crate."
- I will read **whichever** manuscript arrives first.
 - Here "whichever" modifies the noun "manuscript" and introduces the subordinate clause "whichever manuscript arrives first." The subordinate clause functions as the direct object of the compound verb "will read."

Indefinite Pronouns

An **indefinite pronoun** is a pronoun referring to an identifiable but not specified person or thing. An indefinite pronoun conveys the idea of all, any, none, or some.

The most common indefinite pronouns are "all," "another," "any," "anybody," "anyone," "anything," "each," "everybody," "everyone," "everything," "few," "many," "nobody," "none," "one," "several," "some," "somebody," and "someone." Note that some indefinite pronouns can also be used as indefinite adjectives.

The **highlighted** words in the following sentences are indefinite pronouns:

- **Many** were invited to the lunch but only twelve showed up.
 - Here "many" acts as the subject of the compound verb "were invited."
- The office had been searched and **everything** was thrown onto the floor.
 - In this example, "everything" acts as a subject of the compound verb "was thrown."
- We donated **everything** we found in the attic to the woman's shelter garage sale.
 - In this sentence, "everything" is the direct object of the verb "donated."
- Although they looked everywhere for extra copies of the magazine, they found **none**.
 - Here too the indefinite pronoun functions as a direct object: "none" is the direct object of "found."
- Make sure you give **everyone** a copy of the amended bylaws.
 - In this example, "everyone" is the indirect object of the verb "give" -- the direct object is the noun phrase "a copy of the amended bylaws."
- Give a registration package to **each**.
 - Here "each" is the object of the preposition "to."

Reflexive Pronouns

You can use a **reflexive pronoun** to refer back to the subject of the clause or sentence.

The reflexive pronouns are "myself," "yourself," "herself," "himself," "itself," "ourselves," "yourselves," and "themselves." Note each of these can also act as an intensive pronoun.

Each of the **highlighted** words in the following sentences is a reflexive pronoun:

- Diabetics give **themselves** insulin shots several times a day.
- The Dean often does the photocopying **herself** so that the secretaries can do more important work.
- After the party, I asked **myself** why I had faxed invitations to everyone in my office building.
- Richard usually remembered to send a copy of his e-mail to **himself**.
- Although the landlord promised to paint the apartment, we ended up doing it **ourselves**.

Intensive Pronouns

An **intensive pronoun** is a pronoun used to emphasise its antecedent. Intensive pronouns are identical in form to reflexive pronouns.

The **highlighted** words in the following sentences are intensive pronouns:

- I **myself** believe that aliens should abduct my sister.
- The Prime Minister **himself** said that he would lower taxes.
- They **themselves** promised to come to the party even though they had a final exam at the same time.

Exercise 2

Replace the nouns with a suitable pronoun.

1. Paul is studying while Sita is singing.
2. Students are smart.
3. I can trust Sita.
4. The table is made by John.

Exercise 3

Fill in the blanks with suitable pronouns.

1. I blame _____ for the deed.
2. Look at the sun. How bright _____ is!
3. I own the car. It is _____.
4. _____ is there waiting?
5. The pen _____ I bought yesterday was lost.

Exercise 4

Combine the pairs of sentences using the given pronoun.

1. John came late. You wanted to meet him. (whom)

.....

2. The necklace was lost. It was made of gold. (which)

.....

Exercise 2**Replace the nouns with a suitable pronoun.**

A pronoun is a part of speech which stands for noun. 'He' for masculine gender 'She' for feminine gender and 'It' for objects.

1. 'Paul' should be replaced by 'He' and 'Sita' should be replaced by 'she'.

Explanation: Since Paul is a male it is replaced by 'He' and Sita is a female so it is replaced by 'She'.

2. Students must be replaced by THEY (3rd person plural)

Explanation: 'THEY' is a pronoun which is used to refer two or more people or things of 3rd person previously mentioned or easily identified.

3. 'Sita' should be replaced by 'she'.

Explanation: Since Sita is a female so it is replaced by 'She'.

4. 'John' is replaced by 'He'.

Explanation: Since John is a male it is replaced by 'He'.

Exercise 3**Fill in with suitable pronouns.**

1. Myself

Explanation: When the subject and the object refer to the same person or thing, we use reflexive pronoun. Here 'myself' is a reflexive pronoun of the subject 'I'.

2. It

Explanation: 'It' is the suitable pronoun which is used to denote 'The Sun' which is a 3rd person singular-non being..

3. Mine

Explanation: 'Mine' is a possessive pronoun that demonstrates ownerships..

4. Who

Explanation: The pronoun who, is an interrogative and relative pronoun, used chiefly to refer human beings.

5. That

Explanation: 'That' is a pronoun which is used to refer a specific thing

Exercise 4**Combine the pairs of sentences using the given pronoun.**

1. John, whom you wanted to meet, came late.

Explanation: 'Whom' is also an interrogative pronoun; it is used in place of the object of a question. And 'whom' can be used in statements, in place of the object of a clause.

2. The necklace, which was made of gold, was last.

Explanation: It is used as a function word to introduce a relative clause and to modify a noun in that clause and to refer together with that noun.

VEDIC MATHEMATICS

1. Find the square of 75

A. 2555

B. 4525

C. 3555

D. 5625

Answer: Option D

Explanation:

Multiply 5 by 5 and put 25 as your right part of answer.

Multiply 7 with the next higher digit ie $(7+1)=8$ gives

56 as the left part of the answer,

Answer is 5625

2. Find the value of 93×86

A. 2448

B. 5998

C. 7998

D. 9318

Answer: Option C

Explanation:

$$100-86=14, 100-93=7$$

$$14*7=98 \text{ (last two digits)}$$

$$93-14=86-7=79$$

Therefore, the correct answer is 7998

3. Find the square of 55

A. 4255

B. 3025

C. 6525

D. 7225

Answer: Option B

Explanation:

$$55^2 \Rightarrow (5*6)25 = 3025$$

4. Find the value of $51 * 27$

A. 1377

B. 1355

C. 3227

D. 4027

Answer: Option A

Explanation:

51

27

$$1 * 7 = 07 \text{ (write 07)}$$

$1 * 2 + 5 * 7 + 2 \text{ (carried over)} = 37$ (write 7 and 3 is carried over to the next step)

$$5 * 2 + 3 \text{ (carried over)} = 13 \text{ (write 13)}$$

Therefore, 1377 is the answer.

5. Find the square of 107

A. 11449

B. 10449

C. 13443

D. 14439

Answer: Option A

Explanation:

107 is 7 more than the base 100, 7^2 is 49.
Then the left side is $107+7=114$

Thus, square becomes 11449.

6. Addition of $39+6$

A. 45

B. 50

C. 65

D. 35

Answer: Option A

Explanation:

Here 39 is close to 40 and is less than it.

So take 1 from 6 to make 40 and then there will be 5 which should be added to give 40.

7. Round 14.851 to the nearest tenth.

A. 14.85

B. 10

C. 14.9

D. 15

Answer: Option C

Explanation:

The tenths place is the first number to the right of the decimal. The number 8 is in the tenths place. To decide whether to round up or stay the same, look at the number to the right of the tenths place. Since that number is 5 or above, round the tenths place up to 9 and drop the digits after the tenths place.

8. Round 468.235 to the nearest hundredth

- A. 500
- B. 568.235
- C. 468.24
- D. 468.2

Answer: Option C

Explanation:

The hundredths place is the second digit to the right of the decimal point (3). To decide how to round, you must look at the digit to the right of the hundredths place (5). Since this digit is 5 or greater, the hundredths place is rounded up to 4, producing the number 468.24.

9. Divide 6.8×105 by 2.0×102 . Write your answer in scientific notation.

- A. 4.8×103
- B. 4.8×102.5
- C. 3.4×103
- D. 3.4×104

Answer: Option C

Explanation:

To divide numbers written in scientific notation, divide the first numbers ($6.8 \div 2.0 = 3.4$); then divide the powers of 10, which means you subtract the exponents of 10 ($10^5 \div 10^2 = 10^3$). The answer is 3.4×10^3 .

10. The following are four times from a 400-meter race. Which is the fastest time?
- A. 10.1
 - B. 10.14
 - C. 10.2
 - D. 10.09

Answer: Option D

Explanation:

The fastest time is the smallest number. If you chose c, you chose the slowest time since it is the largest number (this person took the longest amount of time to finish the race). To compare decimals easily, make the numbers have the same number of decimal places;
 $10.09 < 10.10 < 10.14 < 10.20$.

11. How much greater is 0.0543 than 0.002?

A. 0.0343

B. 0.0072

C. 0.0523

D. 0.0563

Answer : Option C

Explanation:

To find out how much greater a number is, you need to subtract; $0.0543 - 0.002 = 0.0523$.

To subtract decimals, line the numbers up vertically so that the decimal points align. Then, subtract normally.

12. Which number falls between 5.56 and 5.81?

A. 5.54

B. 5.87

C. 5.6

D. 5.27

Answer: Option C

Explanation:

If you add a zero to the end of 5.6 to get 5.60, it is easier to see that $5.56 < 5.60 < 5.81$.

13. Which of the following decimals is the greatest number?

A. 0.064

B. 0.007

C. 0.1

D. 0.04236

Answer: Option C

Explanation:

If you add zeros to the end of each of the numbers so that each number has 5 places after the decimal point, it is easier to compare the numbers; $0.00700 < 0.04236 < 0.06400 < 0.10000$.

14. What is the smallest possible number that can be created with four decimal places using the numbers 3, 5, 6, and 8?

A. 0.8653

B. 0.3568

C. 0.6538

D. 0.5368

Answer: Option B

Explanation:

Place the smallest number in the largest place value and work your way down, putting the digits in ascending order. Thus, the answer is 0.3568.

15. Which of the following numbers is equivalent to 12.087?

- A. 12.0087
- B. 120.087
- C. 12.0870
- D. 102.087

Answer: Option C

Explanation:

Zeros can be added to the end (right) of the decimal portion of a number without changing the value of the number; 12.0870 is equivalent to 12.087—a 0 has just been added to the end of the number.

16. Which of the following numbers will yield a number larger than 23.4 when it is multiplied by 23.4?

- A. 0.999
- B. 0.0008
- C. 0.3
- D. 1.0002

Answer: Option D

Explanation:

When multiplying by a number less than 1, you get a product that is less than the number you started with. Multiplying by a number greater than 1 gives you a larger number than you started with. Therefore, multiplying by 1.0002 will yield a number larger than the one you started with

17. How will the decimal point move when 245.398 is multiplied by 100?
- A. It will move three places to the right.
 - B. It will move three places to the left.
 - C. It will move two places to the right.
 - D. It will move two places to the left.

Answer: Option C

Explanation:

It is moved two places to the right. When multiplying by multiples of 10, the decimal point is moved to the right according to the number of zeros. For example: Multiply by 10 and move the decimal one place; multiply by 1,000 and move the decimal three places

18. If 967.234 is divided by 10, how will the decimal point move?

- A. It will move one place to the right.
- B. It will move one place to the left.
- C. It will move two places to the right.
- D. It will move two places to the left.

Answer: Option B

Explanation:

It will move one place to the left. When dividing by multiples of 10, the decimal point is moved to the left according to the number of zeros. For example: Divide by 100 and move the decimal two places; divide by 1,000 and move the decimal three places.

19. A pair of pants costs \$24. The cost was reduced by 8%. What is the new cost of the pants?
- A. \$25.92
 - B. \$21.06
 - C. \$22.08
 - D. \$16.00

Answer: Option C

Explanation:

If the cost of the pants is reduced by 8%, the cost of the pants is 92% of the original cost ($100\% - 8\% = 92\%$).

To find 92% of the original cost,
 $\$24 \times 0.92 = \22.08 .

20. Peter purchased 14 new baseball cards for his collection. This increased the size of his collection by 35%. How many baseball cards does Peter now have?

A. 5

B. 54

C. 40

D. 34

Answer: Option B

Explanation:

First, find how many baseball cards Peter had originally. Use a proportion to find the original number of baseball cards; part/whole = %/100 \times 0002. The 14 baseball cards that he added to his collection is the part. The whole number of baseball cards is what we are looking for, so call it x.

The % is 35 (the percent of increase);

$$\frac{14}{x} = \frac{35}{100} \times 0002 \Rightarrow (14)(100) = 35x \Rightarrow x = 40$$

The original number of baseball cards was 40, and 14 more were added to the collection for a total of 54 cards.

21. Joey has 30 pages to read for history class tonight. He decided that he would take a break when he finished reading 70% of the pages assigned. How many pages must he read before he takes a break?
- A. 7
- B. 21
- C. 9
- D. 18

Answer: Option

Explanation:

To find 70% of 30, you must multiply 30 by the decimal equivalent of 70% (0.70); $30 \times 0.70 = 21$.

22. The Dow Jones Industrial Average fell 2% today. The Dow began the day at 8,800. What was the Dow at the end of the day after the 2% drop?
- A. 8,600
 - B. 8,976
 - C. 8,624
 - D. 8,720

Answer: Option C

Explanation:

The Dow lost 2%, so it is worth 98% of what it was worth at the beginning of the day ($100\% - 2\% = 98\%$).

To find 98% of 8,800

$$8,800 \times 0.98 = 8,624.$$

23. The population of Hamden was 350,000 in 1990. By 2000, the population had decreased to 329,000. What percent of decrease is this?
- A. 16%
 - B. 7.5%
 - C. 6%
 - D. 6.4%

Answer : Option C

Explanation:

First, find the number of residents who left Hamden by subtracting the new population from the old population:
 $350,000 - 329,000 = 21,000$.

The population decreased by 21,000.

To find what percent this is of the original population,
 $21,000 \div 350,000 = 0.06$; $0.06 = 6\%$.

24. Rebecca is 12.5% taller than Debbie. Debbie is 64 inches tall. How tall is Rebecca?
- A. 42 inches
 - B. 8 inches
 - C. 56 inches
 - D. 72 inches

Answer: Option D

Explanation:

Since Rebecca is 12.5% taller than Debbie,
She is 112.5% of Debbie's height ($100\% + 12.5\% = 112.5\%$).

To find 112.5% of Debbie's height,
 $64 \times 1.125 = 72$ inches.

25. A tent originally sold for \$260 and has been marked down to \$208. What is the percent of discount?
- A. 20%
 - B. 25%
 - C. 52%
 - D. 18%

Answer: Option A

Explanation:

Find the number of dollars off.

$$\$260 - \$208 = \$52.$$

Next, determine what percent of the original price

$$\$52 \div \$260 = 0.20; 0.20 \text{ is equivalent to } 20\%.$$

26. The football boosters club had 80 T-shirts made to sell at football games. By mid-October, they had only 12 left. What percent of the shirts had been sold?
- A. 85%
 - B. 15%
 - C. 60%
 - D. 40%

Answer: Option A

Explanation:

Determine the number of T-shirts sold;

$80 - 12 = 68$. To find what percent of the original number of shirts 68 is

$68 \div 80 = 0.85$; 0.85 is equivalent to 85%.

27. What is 19% of 26?

A. 21.06

B. 4.94

C. 19

D. 5

Answer: Option B

Explanation:

To find 19% of 26

$$26 \times 0.19 = 4.94.$$

28. 64% of the students in the school play are boys. If there are 75 students in the play, how many are boys?

- A. 64
- B. 45
- C. 27
- D. 48

Answer: Option D

Explanation:

To find 64% of 75, multiply 75 by the decimal equivalent of 64% (0.64); $75 \times 0.64 = 48$.

29. Coastal Cable had 1,440,000 customers in January of 2002. During the first half of 2002 the company launched a huge advertising campaign. By the end of 2002 they had 1,800,000 customers. What is the percent of increase?

- A. 36%
- B. 21%
- C. 20%
- D. 25%

Answer: Option D

Explanation:

Coastal Cable gained a total of 360,000 customers
 $(1,800,000 - 1,440,000 = 360,000)$.

To find out what percent of the original number of customers 360,000 represents,

$360,000 \div 1,440,000 = 0.25$; 0.25 is equivalent to 25%.

30. 450 girls were surveyed about their favorite sport, 24% said that basketball is their favorite sport, 13% said that ice hockey is their favorite sport, and 41% said that softball is their favorite sport. The remaining girls said that field hockey is their favorite sport. What percent of the girls surveyed said that field hockey is their favorite sport?

A. 37%

B. 22%

C. 78%

D. 35%

Answer : Option B

Explanation:

The percents must add to 100%;

$$24\% + 13\% + 41\% = 78\%.$$

If 78% of the girls surveyed have been accounted for, the remainder of the girls must have said that field hockey is their favorite sport.

To find the percent that said field hockey is their favorite sport, subtract 78% from 100%;

$$100\% - 78\% = 22\%;$$

22% of the girls said that field hockey is their favorite sport.

40. What number is code for 'deep'?

Statements:

- I. '481' means 'sky is blue' and '246' means 'sea is deep'.
- II. '698' means 'sea looks blue'.

LOGICAL REASONING**SESSION - 6****NUMBER SERIES**

Type 1: It is a sequence of numerical digits where each term is obtained according to certain rule or pattern.

(a) Inserting the missing number

Under this type, a series of numbers is given with one number or term missing from its place. You have to identify the pattern followed in the formation of the series and then find out the missing term accordingly.

(b) Finding the wrong term

In the questions under this type, one of the terms in the given series does not fit in with the pattern of the series and is, therefore, termed as wrong. The candidate is to find out that wrong number after identifying the specific pattern of the series.

Directions for Q1 to Q8: Find the missing number in the following series.

1. 1, 5, 12, __, 35, 51, 70
(a) 23 (b) 24 (c) 22 (d) 27
2. 2, 5, 9, 19, 37, __
(a) 73 (b) 75 (c) 83 (d) 85
3. 4, 8, 28, 80, 244, __
(a) 278 (b) 428 (c) 628 (d) 728
4. 4, 5, 12, 39, 160, __
(a) 225 (b) 695 (c) 805 (d) 790
5. 10,000, 11,000, 9900, 10896, 9801, __
(a) 10,241 (b) 10,423 (c) 10,781 (d) 10,521
6. 3, 4, 8, 17, 33, __
(a) 57 (b) 58 (c) 48 (d) 47
7. 1, 4, 27, 16, 125, 36, 343, 49, __
(a) 719 (b) 729 (c) 739 (d) 749
8. 3, 15, 5, 35, 7, 63, 9, __, 11
(a) 97 (b) 89 (c) 109 (d) 99

Directions for Q9 to Q16: Find the wrong number in the following.

9. 69, 55, 26, 13, 5
(a) 5 (b) 13 (c) 26 (d) 55
10. 4, 19, 49, 93, 154, 229
(a) 93 (b) 229 (c) 319 (d) 19
11. 80, 42, 24, 13.5, 8.75, 6.375
(a) 8.75 (b) 13.5 (c) 24 (d) 6.375
12. 64, 71, 80, 91, 104, 119, 135, 155
(a) 71 (b) 80 (c) 135 (d) 104
13. 823, 734, 645, 556, 476, 378, 289
(a) 645 (b) 476 (c) 378 (d) 289
14. 895, 870, 821, 740, 619, 445, 225
(a) 445 (b) 821 (c) 619 (d) 225
15. 19, 26, 33, 46, 59, 74, 91
(a) 26 (b) 33 (c) 46 (d) 74
16. 32, 36, 41, 61, 86, 122, 171, 235
(a) 41 (b) 61 (c) 86 (d) 122

Directions for Q17 to Q21: What is the next number in the series?

17. 2, 6, 12, 20, 30, 42, 56
(a) 60 (b) 64 (c) 72 (d) 70
18. 165, 195, 255, 285, 345
(a) 375 (b) 420 (c) 435 (d) 390
19. 5, 10, 13, 26, 29, 58, 61
(a) 122 (b) 64 (c) 125 (d) 128
20. 51975, 9450, 2100, 600, 240, 160
(a) 80 (b) 120 (c) 320 (d) 240
21. 444, 467, 513, 582, 674, 789
(a) 950 (b) 904 (c) 927 (d) 984

Directions for Q22 to Q25: Find the number in the place of the question mark.

22. 15, 25, 40, 130, (?), 2560
(a) 500 (b) 520 (c) 510 (d) 480
23. 6, 42, (?), 1260, 5040, 15120, 30240
(a) 546 (b) 424 (c) 252 (d) 328
24. 282, 286, 302, (?), 402, 502
(a) 366 (b) 318 (c) 326 (d) 338
25. 6, 4, 8, 23, (?), 385.25
(a) 84.5 (b) 73 (c) 78.5 (d) 82

LOGICAL THINKING – SOLUTION

24. Ans: [c]

In the first and second statements, the common code is rps and the common word is morning. So, rps means morning. In the first and the third statements, the common code is ski and the common word is nice. So, ski means nice. Therefore, in the first statement, tri means Sunday.

25. Ans: [d]

Tractor is used for ploughing. It is called car and hence (d).

26. Ans: [c]

$R \rightarrow \#$ and $F \rightarrow 5$ Hence, $S \rightarrow 3$

$$\begin{array}{lll}
 0 \rightarrow 4 & I \rightarrow * & T \rightarrow 7 \\
 S \rightarrow 3 & R \rightarrow \# & \\
 0 \rightarrow 4 & & \\
 E \rightarrow \$ & S \rightarrow 3 & R \rightarrow \# \\
 & T \rightarrow 7 & E \rightarrow \$
 \end{array}$$

Solutions for 27 and 28

pit na sa → you are welcome

are very good]]

good

they welcome good people

] → na → are

na ho pa la → they

ka da la → who is

] → la → good

od ho pit la →

→ pit → welcome

Also, ho → they

27. Ans: [d]

people → od

28. Ans: [c]

very → pa

29. Ans: [d]

30. Ans: [d]

31. Ans: [b]

32. Ans: [a]

33. Ans: [c]

34. Ans: [b]

35. Ans: [c]

36. Ans: [c]

37. Ans: [b]

38. Ans: [a]

39. Ans: [d]

I. ja pa na → go home now]

II. na da ta → come back home] → na → home Hence, ja means either go or now.

40. Ans: [e]

I. → 481 → sky is blue]]

246 → sea is deep] → is → 4] → sea 6

II. → 698 → sea looks blue]. Hence, deep → 2

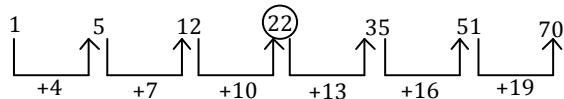
LOGICAL REASONING

SESSION - 6

NUMBER SERIES

1. Ans: [c]

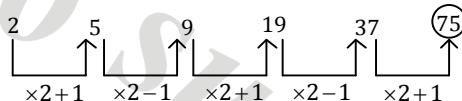
Clearly, the given series follows the following pattern.



Hence, the answer is 22 → (c).

2. Ans: [b]

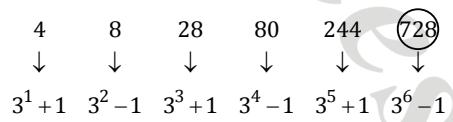
The following pattern is followed in the series.



Hence, the answer is 75 → (b).

3. Ans: [d]

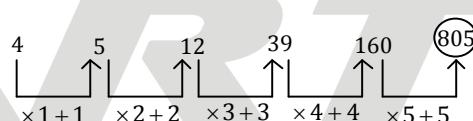
The series follows the following pattern.



Hence, the answer is 728 → (d).

4. Ans: [c]

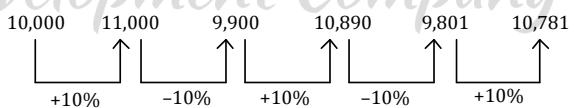
The following pattern is used in the given series.



Hence, the answer is 805 → (c).

5. Ans: [c]

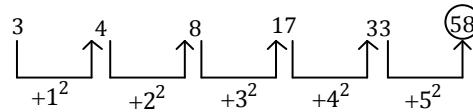
We add or subtract alternately 10% of the term to get the next term.



Hence, the answer is 10781 → (c).

6. Ans: [b]

The following pattern is used in the series.



Hence, the answer is 58 → (b).

LOGICAL THINKING – SOLUTION

7. Ans: [b]

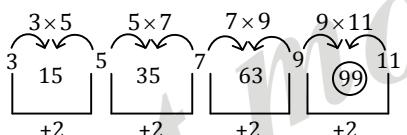
The sequence consists of two series, the one consisting of cubes of odd numbers and the other the square of even numbers.

1	27	125	343	729
↓	4	16	36	49
1^3	3^3	5^3	7^3	9^3
2^2	4^2	6^2	7^2	

The answer is 729 → (b).

8. Ans: [d]

The following pattern is followed in the series.



Hence, the answer is 99 → (d).

9. Ans: [a]

It may be seen that each term in the series is one more than the product of the digits of the preceding term. By way of explanation, the 2nd term = $6 \times 9 + 1 = 55$

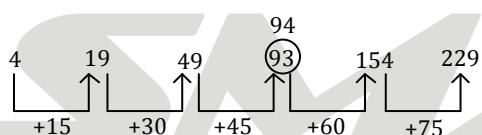
Similarly, $5 \times 5 + 1 = 26$, $2 \times 6 + 1 = 13$, $1 \times 3 + 1 = 4$

Therefore, the 5th term should be 4 and not 5.

So, 5 is the wrong term → (a).

10. Ans: [a]

The pattern in the series is as follows.

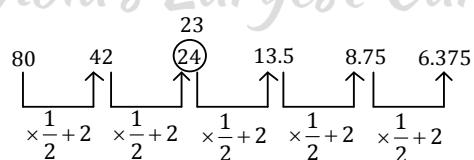


The fourth term 93 is wrong and is to be replaced by 94.

Hence, the answer is 93 → (a).

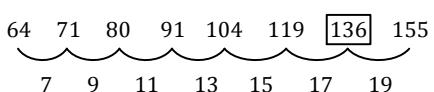
11. Ans: [c]

The pattern of the series is as follows.



Hence, the third term 24 is wrong and should be replaced by 23. Hence, the answer is 24 → (c).

12. Ans: [c]



The numbers are obtained by adding 7, 9, 11, 13, 15, 17, 19 to the previous number.

$$119 + 17 = 136, 136 + 19 = 155$$

∴ 135 is the wrong number and it should be replaced by 136.

13. Ans: [b]

823 734 645 556 476 378 289

In this series, unit's digit and ten's digit of each number is increased by 1 and the hundred's digit decreased by 1.

The series should be

823 734 645 556 467 378 289

∴ 476 is the wrong number and it should be replaced by 467.

14. Ans: [a]

895 870 821 740 619 445 225
 $\begin{array}{ccccccc} -25 & -49 & -81 & -121 & -174 & -220 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ -5^2 & -7^2 & -9^2 & -11^2 & -13^2 & -15^2 \end{array}$

Here each number is obtained by subtracting from the previous number $5^2, 7^2, 9^2, 11^2, 13^2, 15^2$ respectively.

$$619 - 13^2 = 450, 450 - 15^2 = 225$$

∴ 445 is the wrong number and it should be replaced by 450.

15. Ans: [b]

19, 26, 35, 46, 59, 74, 91
 $\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 7 & 9 & 11 & 13 & 15 & 17 & \end{array}$

The series is obtained by adding 7, 9, 11, 13, 15, 17 to the previous numbers.

$$26 + 9 = 35, 35 + 11 = 46$$

∴ 33 is the wrong number and should be replaced by 35.

16. Ans: [a]

32 36 45 61 86 122 171 235
 $\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 9 & 16 & 25 & 36 & 49 & 64 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2^2 & 3^2 & 4^2 & 5^2 & 6^2 & 7^2 & 8^2 \end{array}$

$$\therefore 36 + 9 = 45, 45 + 16 = 61$$

∴ 41 is wrong and should be replaced by 45.

17. Ans: [c]

2, 6, 12, 20, 30, 42, 56

$$2 = 1 \times 2, 6 = 2 \times 3, 12 = 3 \times 4, 20 = 4 \times 5, 30 = 5 \times 6$$

$$42 = 6 \times 7, 56 = 7 \times 8$$

∴ 8 × 9 = 72 is the next number

18. Ans: [c]
 $165, 195, 255, 285, 345$
 $11 \times 15 = 165$

$13 \times 15 = 195$

$17 \times 15 = 255$

$19 \times 15 = 285$

$23 \times 15 = 345$

The next number will be $29 \times 15 = 435$

19. Ans: [a]
 $5, 10, 13, 26, 29, 58, 61$
 $\underbrace{2}_{\times 2} \quad \underbrace{+3}_{\times 2} \quad \underbrace{+3}_{\times 2} \quad \underbrace{\times 2}_{+3}$

The numbers are alternately multiplied by 2 and added to 3.
 \therefore The next number is $61 \times 2 = 122$

20. Ans: [c]
 $51975, 9450, 2100, 600, 240, 160$
 $\underbrace{\frac{2}{11}}_{\times 2}, \underbrace{\frac{2}{9}}_{\times 2}, \underbrace{\frac{2}{7}}_{\times 2}, \underbrace{\frac{2}{5}}_{\times 2}, \underbrace{\frac{2}{3}}_{\times 2}, \underbrace{\frac{2}{1}}_{\times 2}$
 $160 \times 2 = 320$

The next number is $160 \times \frac{2}{1} = 320$

21. Ans: [c]
 $444, 467, 513, 582, 674, 789$
 $\underbrace{23}_{23}, \underbrace{46}_{23}, \underbrace{69}_{23}, \underbrace{92}_{23}, \underbrace{115}_{23}, \underbrace{138}_{23}$
 $789 + 138 = 927$

The next number is 927.

22. Ans: [c]
 510
 $15, 25, 40, 130, (?), 2560$
 $15 \times 1 + 10 = 25, 25 \times 2 - 10 = 40, 40 \times 3 + 10 = 130$
 \therefore The next number will be $130 \times 4 - 10 = 510$

$510 \times 5 + 10 = 2560$

\therefore The required number is 510.

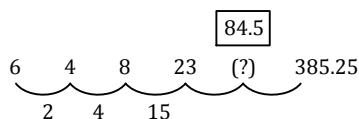
23. Ans: [c]
 252
 $6, 42, (?), 1260, 5040, 15120, 30240$
 $\underbrace{\times 7}_{\times 6}, \underbrace{\times 6}_{\times 5}, \underbrace{\times 5}_{\times 4}, \underbrace{\times 4}_{\times 3}, \underbrace{\times 3}_{\times 2}, \underbrace{\times 2}_{\times 2}$

The number is $42 \times 6 = 252$

24. Ans: [d]
 338
 $282, 286, 302, (?), 402, 502$
 $\begin{matrix} 4 & 16 & 36 & 64 & 100 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2^2 & 4^2 & 6^2 & 8^2 & 10^2 \end{matrix}$

\therefore The number is $302 + 36 = 338$

25. Ans: [a]



$$\begin{aligned} 6 \times 0.5 + 1 &= 4, 4 \times 1.5 + 2 = 8 \\ 8 \times 2.5 + 3 &= 23, 23 \times 3.5 + 4 = 84.5 \\ 84.5 \times 4.5 + 5 &= 385.25 \end{aligned}$$

\therefore The required number is 84.5.

LOGICAL REASONING

SESSION - 7

⑨ ALPHA SERIES

1. Ans: [a]

Observing closely, it may be seen that the first and the second letters of each term are moved five places forward to obtain the corresponding letters of the next term.

Accordingly the first letter of the missing term (5th term) will be five places ahead of P i.e. U, and the second letter must be five places ahead of R i.e., W.

So, the missing term is UW. Hence, the answer is (a).

2. Ans: [b]

It may be seen that the second term U is three places after R and so also all succeeding terms. So, the next term after D is 3 places after D i.e., G.

Hence the answer is G → (b).

3. Ans: [c]

$$H \xrightarrow{+1} I \xrightarrow{+2} K \xrightarrow{+3} N \xrightarrow{+4} (R)$$

The missing term is R → (c).

4. Ans: [a]

$$\text{First letter: } A \xrightarrow{+1} B \xrightarrow{+1} C \xrightarrow{+1} (D)$$

$$\text{Second letter: } I \xrightarrow{+1} J \xrightarrow{+1} K \xrightarrow{+1} (L)$$

Hence, the missing term is DL → (a).

5. Ans: [a]

First letter:

$$D \xrightarrow{+3} G \xrightarrow{+4} K \xrightarrow{+3} N \xrightarrow{+4} R \xrightarrow{+3} (U)$$

Second letter:

$$F \xrightarrow{+4} J \xrightarrow{+3} M \xrightarrow{+4} Q \xrightarrow{+3} T \xrightarrow{+4} (X)$$

Hence, the missing term is UX → (d)

6. Ans: [b]

Analysing the given terms, the pattern of the series is as follows.

LOGICAL THINKING

LOGICAL REASONING

SESSION - 7

ALPHA SERIES

Type 2: Letter Series

Here, the series includes alphabetical letters in single, pair or groups. The terms of the series form a certain pattern as regards the position of letters in the English alphabet. The candidate is to identify the pattern and accordingly find the missing term or the wrong term in the given series.

Directions for Q1 and Q2: Which term comes next in the sequence.

1. AC, FH, KM, PR, ____
(a) UW (b) VW (c) UX
(d) TV (e) None of these

2. R, U, X, A, D, ____
(a) F (b) G (c) H (d)

Directions for Q3 to Q8: In the following alphabet series, one or more terms are missing. Choose the missing term from the given alternatives.

3. H, I, K, N, ____
(a) O (b) Q (c) R (d) S

4. AI, BJ, CK, ____
(a) DL (b) DM (c) DH (d) LM

5. DF, GJ, KM, NQ, RT, ____
(a) UW (b) YZ (c) XZ (d) UX (e) YA

6. UPI, ___, ODP, MBQ, IAW
(a) RHJ (b) SHJ (c) SIJ (d) THK (e) TIJ

7. BZA, DYC, FXE, ___, JVI
(a) HUG (b) HWG (c) UHG
(d) WHG (e) None of these

8. PMT, OOS, NQR, MSQ, ____
(a) LUP (b) LVP (c) LVR (d) LWPP

Directions for Q9 to Q12: In each of the following questions, various terms of a letter series are given with one term missing as shown by (?). Choose the missing term out of the given alternatives.

9. A CD GHI ?, UVWXY
(a) LMNO (b) MNO (c) NOPQ
(d) NOP (e) MNOP

10. BL FP JT ? RB VF
(a) MW (b) NX (c) OY
(d) MX (e) NW

11. EJOT DHLP CFIL ?
 (a) BDFH (b) BHLM (c) DEIJ (d) DGKL

Directions for Q13 to Q17: Which of the following will come in place of the question mark according to English Alphabetical?

ALPHANUMERIC SERIES

Type 3: Under this type, the terms of the given series are a combination of letters and numbers and they follow a particular pattern of their own.

Type 4: This type of questions usually consist of a series of small letters of alphabet which follow a certain definite pattern. However, some letters will be missing from the series. The candidate is to identify the pattern and accordingly choose the proper sequence of the missing letters from the given alternatives.

Directions for Q1 to Q3: In the following letter-number series questions, one or more terms are missing. Choose the missing term out of the given alternatives.

1. 2B, 4C, 8E, 14H, ____
(a) 16K (b) 20I (c) 20L (d) 22L

2. A1, C3, F6, J10, O15, ____
(a) U21 (b) V21 (c) T20 (d) U20

3. P3, M8, ____ G24, D35
(a) K15 (b) J13 (c) I13 (d) J15

Directions for Q4 to Q6: What should come in the place of question mark (?) in the following alpha-numeric series?

4. KM5, IP8, GS11, EV14, ?
(a) BY17 (b) CZ17 (c) BX17 (d) CY17

5. ST39, UV43, WX47, ?
(a) YZ47 (b) YZ52 (c) YZ51 (d) YX50

18. Ans: [c]
 $165, 195, 255, 285, 345$

$$11 \times 15 = 165$$

$$13 \times 15 = 195$$

$$17 \times 15 = 255$$

$$19 \times 15 = 285$$

$$23 \times 15 = 345$$

The next number will be $29 \times 15 = 435$

19. Ans: [a]
 $5, 10, 13, 26, 29, 58, 61$

$$\underbrace{5}_{\times 2} \quad \underbrace{10}_{+3} \quad \underbrace{13}_{\times 2} \quad \underbrace{26}_{+3} \quad \underbrace{29}_{\times 2} \quad \underbrace{58}_{+3} \quad 61$$

The numbers are alternately multiplied by 2 and added to 3.
 \therefore The next number is $61 \times 2 = 122$

20. Ans: [c]
 $51975, 9450, 2100, 600, 240, 160$

$$160 \times 2 = 320$$

$$\underbrace{51975}_{\times \frac{2}{11}} \quad \underbrace{9450}_{\times \frac{2}{9}} \quad \underbrace{2100}_{\times \frac{2}{7}} \quad \underbrace{600}_{\times \frac{2}{5}} \quad \underbrace{240}_{\times \frac{2}{3}} \quad \underbrace{160}_{\times \frac{2}{1}}$$

The next number is $160 \times \frac{2}{1} = 320$

21. Ans: [c]
 $444, 467, 513, 582, 674, 789$

$$789 + 138 = 927$$

$$\underbrace{444}_{23} \quad \underbrace{467}_{23} \quad \underbrace{513}_{23} \quad \underbrace{582}_{23} \quad \underbrace{674}_{23} \quad \underbrace{789}_{23}$$

The next number is 927.

22. Ans: [c]
 510

$$15, 25, 40, 130, (?), 2560$$

$$15 \times 1 + 10 = 25, 25 \times 2 - 10 = 40, 40 \times 3 + 10 = 130$$

The next number will be $130 \times 4 - 10 = 510$

$$510 \times 5 + 10 = 2560$$

\therefore The required number is 510.

23. Ans: [c]
 252

$$\underbrace{6}_{\times 7} \quad \underbrace{42}_{\times 6} \quad \underbrace{(?)}_{\times 5} \quad \underbrace{1260}_{\times 4} \quad \underbrace{5040}_{\times 3} \quad \underbrace{15120}_{\times 2} \quad \underbrace{30240}_{\times 2}$$

The number is $42 \times 6 = 252$

24. Ans: [d]
 338

$$\begin{array}{ccccccc} 282 & 286 & 302 & (?) & 402 & 502 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 16 & 36 & 64 & 100 & \\ 2^2 & 4^2 & 6^2 & 8^2 & 10^2 & \end{array}$$

\therefore The number is $302 + 36 = 338$

25. Ans: [a]

$$\begin{array}{ccccc} 6 & 4 & 8 & 23 & (?) \\ \underbrace{2}_{\text{6}} & \underbrace{4}_{\text{4}} & \underbrace{15}_{\text{8}} & \underbrace{23}_{\text{23}} & \underbrace{?}_{\text{84.5}} \\ & & & & \end{array} \quad 385.25$$

$$6 \times 0.5 + 1 = 4, 4 \times 1.5 + 2 = 8$$

$$8 \times 2.5 + 3 = 23, 23 \times 3.5 + 4 = 84.5$$

$$84.5 \times 4.5 + 5 = 385.25$$

\therefore The required number is 84.5.

LOGICAL REASONING

SESSION - 7

⑨ ALPHA SERIES

1. Ans: [a]

Observing closely, it may be seen that the first and the second letters of each term are moved five places forward to obtain the corresponding letters of the next term.

Accordingly the first letter of the missing term (5th term) will be five places ahead of P i.e. U, and the second letter must be five places ahead of R i.e., W.

So, the missing term is UW. Hence, the answer is (a).

2. Ans: [b]

It may be seen that the second term U is three places after R and so also all succeeding terms. So, the next term after D is 3 places after D i.e., G.

Hence the answer is G \rightarrow (b).

3. Ans: [c]

$$H \xrightarrow{+1} I \xrightarrow{+2} K \xrightarrow{+3} N \xrightarrow{+4} (R)$$

The missing term is R \rightarrow (c).

4. Ans: [a]

$$\text{First letter: } A \xrightarrow{+1} B \xrightarrow{+1} C \xrightarrow{+1} (D)$$

$$\text{Second letter: } I \xrightarrow{+1} J \xrightarrow{+1} K \xrightarrow{+1} (L)$$

Hence, the missing term is DL \rightarrow (a).

5. Ans: [a]

First letter:

$$D \xrightarrow{+3} G \xrightarrow{+4} K \xrightarrow{+3} N \xrightarrow{+4} R \xrightarrow{+3} (U)$$

Second letter:

$$F \xrightarrow{+4} J \xrightarrow{+3} M \xrightarrow{+4} Q \xrightarrow{+3} T \xrightarrow{+4} (X)$$

Hence, the missing term is UX \rightarrow (d)

6. Ans: [b]

Analysing the given terms, the pattern of the series is as follows.

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$$\begin{array}{ccccccc} U & \xrightarrow{-2} & (S) & \xrightarrow{-4} & O & \xrightarrow{-2} & M & \xrightarrow{-4} & I \\ P & \xrightarrow{-8} & (H) & \xrightarrow{-4} & D & \xrightarrow{-2} & B & \xrightarrow{-1} & A \\ I & \xrightarrow{+1} & (J) & \xrightarrow{+6} & P & \xrightarrow{+1} & Q & \xrightarrow{+6} & W \end{array}$$

Hence, the missing term is SHJ → (b).

7. Ans: [b]

First letter: B $\xrightarrow{+2}$ D $\xrightarrow{+2}$ F $\xrightarrow{+2}$ (H) $\xrightarrow{+2}$ J

Second letter:

Z $\xrightarrow{-1}$ Y $\xrightarrow{-1}$ X $\xrightarrow{-1}$ (W) $\xrightarrow{-1}$ V

Third letter: A $\xrightarrow{+2}$ C $\xrightarrow{+2}$ E $\xrightarrow{+2}$ (G) $\xrightarrow{+2}$ I

Hence the missing term is HWG → (b).

8. Ans: [a]

First letter: P $\xrightarrow{-1}$ O $\xrightarrow{-1}$ N $\xrightarrow{-1}$ M $\xrightarrow{-1}$ (L)

Second letter:

M $\xrightarrow{+2}$ O $\xrightarrow{+2}$ Q $\xrightarrow{+2}$ S $\xrightarrow{+2}$ (U)

Third letter: T $\xrightarrow{-1}$ S $\xrightarrow{-1}$ R $\xrightarrow{-1}$ Q $\xrightarrow{-1}$ (P)

Hence the missing term is LUP → (a).

9. Ans: [e]

The first term consists of one letter, the second term consists of two consecutive letters and the third term consists of three consecutive letters. So, the required term will consist of four consecutive letters. Also there is a gap of one letter between the letter of the first term and first letter of the second term and a gap of two letters between the last of the second term and first letter of the third term. So, the first letter of the required term would be four steps ahead of the last letter of the third term.

Accordingly,

$$\begin{array}{c} G \quad H \quad I \\ \downarrow +4 \\ ? \longrightarrow M \quad N \quad O \quad P \end{array}$$

10. Ans: [b]

Each letter of a term of the series is four steps ahead of the corresponding letter of the preceding term.

Accordingly,

$$\begin{array}{l} 3^{\text{rd}} \text{ term} \rightarrow J \quad T \\ ? \quad \rightarrow N \quad X \end{array}$$

11. Ans: [a]

The letters of each term are moved backward one, two, three and four steps to obtain the corresponding letters of the next term.

Accordingly,

$$\begin{array}{cccccc} 3^{\text{rd}} \text{ term} \rightarrow & C & F & I & L \\ & \downarrow -1 & \downarrow -2 & \downarrow -3 & \downarrow -4 \\ 4^{\text{th}} \text{ term} (?) \rightarrow & B & D & F & H \end{array}$$

12. Ans: [e]

Each term of the series consists of two alternate letters and there is a gap of two letters between the last letter of each term and the first letter of the next term.:

Accordingly,

$$\begin{array}{l} 4^{\text{th}} \text{ term} \longrightarrow Q \quad S \\ \downarrow +3 \\ V \xrightarrow{+2} X \\ 5^{\text{th}} \text{ term} (?) \longrightarrow VX \end{array}$$

13. Ans: [b]

Each letter of the terms is moving differently, the first one by $-3, -3, -3$, the second one by $+1, +1, +1$ and the third one by $+2, +3, +4$ respectively.

$$\begin{array}{l} 3^{\text{rd}} \text{ term} \longrightarrow Q \quad H \quad G \\ \downarrow -3 \quad \downarrow +1 \quad \downarrow +4 \\ 4^{\text{th}} \text{ term} (?) \longrightarrow N \quad I \quad K \end{array}$$

14. Ans: [a]

Letters in odd positions move by -3 steps and letters in even positions move by -4 steps.

Accordingly,

$$\begin{array}{ccccccccccccc} & & & & & & & & & & & & & N & C \\ Z & S & W & O & T & K & Q & G & ? & ? & & & & & \\ \hline & -3 & & -3 & & -3 & & -3 & & -3 & & & & & \end{array}$$

15. Ans: [c]

The number of letters in the terms of the given series increases by one at each step.

The first letter of each term is two steps ahead of the last letter of the preceding term. However, each term consists of consecutive letters in order.

$$\begin{array}{ccccccccccccc} AB & DEF & HIJK & MNOPQ & STUVWX \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ +2 & +2 & +2 & +2 & +2 \end{array}$$

ALPHANUMERIC SERIES

1. Ans: [d]

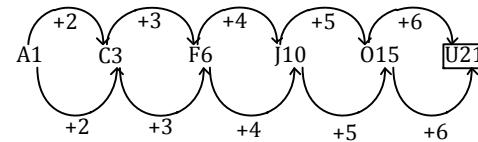
Numbers: $2 \xrightarrow{+2} 4 \xrightarrow{+4} 8 \xrightarrow{+6} 14 \xrightarrow{+8} (22)$

Letters: B $\xrightarrow{+1}$ C $\xrightarrow{+2}$ E $\xrightarrow{+3}$ H $\xrightarrow{+4}$ (L)

So, the missing term is 22L → (d).

2. Ans: [a]

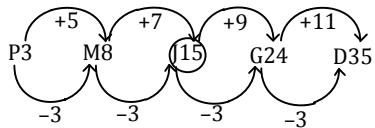
The pattern of the given series is as follows.



The missing term is, therefore, U21 → (a).

3. Ans: [d]

The pattern followed in the series is as follows.



The missing term is J15 → (d).

4. Ans: [d]

The first letter of each term is moved two steps backward and the second letter is moved three steps forward to obtain the corresponding letters of the next term. The number in each term is 3 more than that in the preceding term.

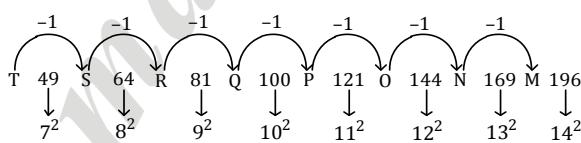
5. Ans: [c]

6. Ans: [b]

Because the letters are the same, concentrate on the number series, which is a simple 2, 3, 4, 5, 6 series, and follows each letter in order.

7. Ans: [a]

The pattern followed in the series is as follows.



The missing terms are N169 and M196 → (a).

8. Ans: [b]

The pattern followed by the letters is

First letter: G → J → M → P → S

Third letter: T → R → P → N → L

The pattern followed by the number is 4, $4 \times 2 + 1(9)$, $9 \times 2 + 2(20)$, $20 \times 2 + 3(3)$, $43 \times 2 + 4(90)$.

The second number 10 is wrong and is to be replaced by 9.

So, the second term J10R does not fit in the given series.

The correct second term is J9R.

Hence, the answer is (b).

9. Ans: [b]

The series pattern is aceg/aceg/aceg

So, the required answer is ecag → (b).

10. Ans: [a]

The series pattern is abba/abba/abba

So, the required answer is baab.

11. Ans: [b]

The pattern of the series is ab/ab/ab/ab/ab with 'ab' repeated six times.

So, the required answer is abbab → (b).

12. Ans: [a]

The pattern of the series is abc/bca/cab/abc/bca.

The letters change places in a cyclic order.

So, the required answer is abcbb → (a).

13. Ans: [a]

The pattern of the series is bhcca/ccaab/aabbcc/bbccaa.

So, the required answer is baabc → (a).

14. Ans: [a]

15. Ans: [a]

abczy/abcdxw/abcdevu

LOGICAL REASONING

SESSION - 8

Odd Man Out

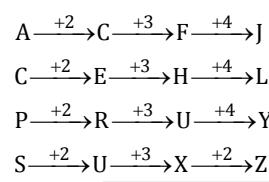
1. Ans: [c] 2. Ans: [d] 3. Ans: [c]

4. Ans: [d] 5. Ans: [c] 6. Ans: [c]

7. Ans: [d] 8. Ans: [d]

9. Ans: [d]

The pattern of the series is shown below.

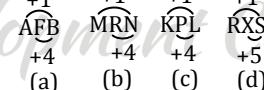


10. Ans: [b]

In all options except (b), the third and the fourth letters are reverse.

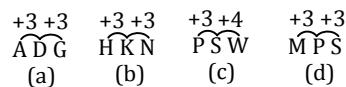
11. Ans: [d]

The pattern of the series is illustrated below,



12. Ans: [c]

The pattern if the series is illustrated below,



13. Ans: [a]

The pattern of the series is explained below,
AA will be the odd one.

Since alternatively, Rank of B is 2 so, BB

LOGICAL THINKING

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6. B₂CD, ?, BCD₄, ?, BC₆D
 (a) BC₃D, B₅CD (b) BC₂D, BC₃D
 (c) BC₃D, C₃CD (d) BC₃D, BC₅D

Directions for Q7 and Q8: In the following letter-number series questions, one or more terms are missing. Choose the missing term out of the given alternatives.

7. T49, S64, R81, Q100, P121, 0144, __, __
 (a) N169, M196 (b) N160, M190
 (c) N164, M194 (d) U36, T46
8. Find the term which does not fit into the series given below.
 G4T, J10R, M20P, P43N, S90L
 (a) G4T (b) J10R (c) M20P (d) P43N

Directions for Q9 to Q13: In each of the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

9. ac - ga - eg - ce -
 (a) dbag (b) ecag (c) deag (d) ebdg
10. a - ba - bb - ab - a
 (a) baab (b) aaba (c) abab (d) baaa
11. - - aba - - ba - ab
 (a) abbba (b) abbab (c) baabb (d) bbaba
12. - bc - ca - aba - c - ca
 (a) abcbb (b) bbbcc (c) bacba (d) abbcc
13. b - ccacca - ba - bbc - bc - a
 (a) baabc (b) abaaa (c) acbca (d) bacab

Directions for Q14 and Q15: In each of the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Select the correct alternative.

14. aa_aabb_b_aa_aabb_bb
 (a) bbbaa (b) bbbbba (c) aabbbb (d) babba
15. abc_yabc_xw abcdev_
 (a) zdu (b) dxu (c) cdu
 (d) zyw (e) None of these

2. (a) PRS (b) TVX (c) FIK (d) LME
3. (a) EGK (b) BEN (c) CGP (d) AEU
4. (a) HIKJ (b) KLNMM (c) STVU (d) RSTU
5. (a) BCEH (b) PQSV (c) CDGK (d) STVY

Directions for Q6 to Q8: From the given choices select the odd man out.

6. (a) ABDC (b) FGIH (c) KLMN (d) QRSTS
7. (a) aaa b FG (b) bbb c GH (c) hhh i MN
 (d) ddd c HI (e) fff g KL
8. (a) aaAA eeFF (b) ppQQ uuVV
 (c) IIIMM qqRR (d) rsss wwXX
 (e) ooPP ttUU

Directions for Q9 and Q30: From the given choices select the odd man out.

9. (a) ACFJ (b) CEHL (c) PRUY (d) SUXZ
10. (a) ABDC (b) MNPQ (c) PQSR (d) STVU
11. (a) AFB (b) MRN (c) KPL (d) RXS
12. (a) ADG (b) HKN (c) PSW (d) MPS
13. (a) AA (b) BB (c) DDDD (d) EEEE
14. (a) BAK (b) DCM (c) HGQ (d) MNW
15. (a) DFE (b) PRQ (c) MPN (d) SUT
16. (a) MQ2 (b) RV2 (c) PS3 (d) DM3
17. (a) DEH (b) CDI (c) KLQ (d) PRV
18. (a) PRQ (b) ACB (c) TUV (d) QSR
19. (a) FHKO (b) CEHL (c) ZBEJ (d) XZCG
20. (a) AEK (b) DFH (c) TWZ (d) MOQ
21. (a) DFK (b) PRW (c) EGL (d) TVZ
22. (a) BD2 (b) GI2 (c) NR4 (d) SV2
23. (a) ADP (b) QTX (c) HKR (d) STE
24. (a) FU (b) DW (c) CX (d) NR
25. (a) STV (b) XYA (c) KLN (d) BDE
26. (a) FJN (b) HLO (c) CGK (d) KOS
27. (a) BAD (b) FEH (c) POS (d) TSV
28. (a) BDF (b) MQT (c) HKN (d) PTX
29. (a) DEB (b) FGD (c) PQN (d) TUS
30. (a) YDA (b) AEC (c) HLJ (d) RVT

LOGICAL REASONING

SESSION - 8

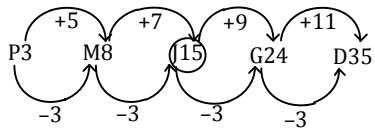
Odd Man Out

Directions for Q1 to Q5: From the given choices select the odd man out.

1. (a) KML (b) PRQ (c) NPQ (d) TVU

3. Ans: [d]

The pattern followed in the series is as follows.



The missing term is J15 → (d).

4. Ans: [d]

The first letter of each term is moved two steps backward and the second letter is moved three steps forward to obtain the corresponding letters of the next term. The number in each term is 3 more than that in the preceding term.

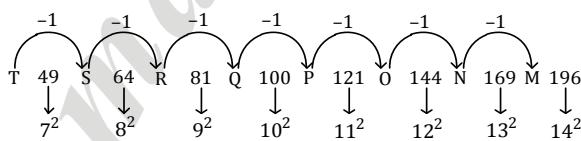
5. Ans: [c]

6. Ans: [b]

Because the letters are the same, concentrate on the number series, which is a simple 2, 3, 4, 5, 6 series, and follows each letter in order.

7. Ans: [a]

The pattern followed in the series is as follows.



The missing terms are N169 and M196 → (a).

8. Ans: [b]

The pattern followed by the letters is

First letter: G →⁺³ J →⁺³ M →⁺³ P →⁺³ S

Third letter: T →⁻² R →⁻² P →⁻² N →⁻² L

The pattern followed by the number is 4, $4 \times 2 + 1(9)$, $9 \times 2 + 2(20)$, $20 \times 2 + 3(3)$, $43 \times 2 + 4(90)$.

The second number 10 is wrong and is to be replaced by 9.

So, the second term J10R does not fit in the given series.

The correct second term is J9R.

Hence, the answer is (b).

9. Ans: [b]

The series pattern is aceg/aceg/aceg

So, the required answer is ecag → (b).

10. Ans: [a]

The series pattern is abba/abba/abba

So, the required answer is baab.

11. Ans: [b]

The pattern of the series is ab/ab/ab/ab/ab with 'ab' repeated six times.

So, the required answer is abbab → (b).

12. Ans: [a]

The pattern of the series is abc/bca/cab/abc/bca.

The letters change places in a cyclic order.

So, the required answer is abcbb → (a).

13. Ans: [a]

The pattern of the series is bhcca/ccaab/aabbcc/bbccaa.

So, the required answer is baabc → (a).

14. Ans: [a]

15. Ans: [a]

abczy/abcdxw/abcdevu

LOGICAL REASONING

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Odd Man Out

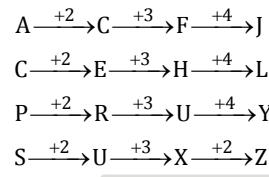
1. Ans: [c] 2. Ans: [d] 3. Ans: [c]

4. Ans: [d] 5. Ans: [c] 6. Ans: [c]

7. Ans: [d] 8. Ans: [d]

9. Ans: [d]

The pattern of the series is shown below.



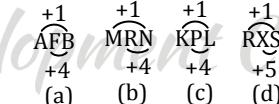
∴ SUXZ is the odd man out.

10. Ans: [b]

In all options except (b), the third and the fourth letters are reverse.

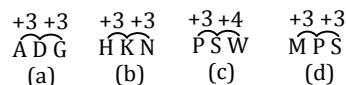
11. Ans: [d]

The pattern of the series is illustrated below,



12. Ans: [c]

The pattern if the series is illustrated below,



13. Ans: [a]

The pattern of the series is explained below,

AA will be the odd one.

Since alternatively, Rank of B is 2 so, BB

LOGICAL THINKING – SOLUTION

HSEM1BTECHSTANDARD0719

Rank of D is 4 so, DDDD

and similarly, Rank of E is 5 so, EEEEE and
since, Rank of A is 1 so, it has to Be (a)

14. Ans: [d]

The pattern of the series is illustrated below,

BAK	<u>DCM</u>	<u>HGQ</u>	<u>MNW</u>
-1	-1	-1	+1
(a)	(b)	(c)	(d)

15. Ans: [c]

The pattern of the series is illustrated below,

<u>D</u> FE	<u>P</u> RQ	<u>M</u> PN	<u>S</u> U
+1	+1	+1	+1
(a)	(b)	(c)	(d)

16. Ans: [c]

The pattern of the series is illustrated below,

<u>M</u> Q2	<u>R</u> V2	<u>P</u> S3	<u>D</u> M3
+4	+4	+3	+9
(a)	(b)	(c)	(d)

17. Ans: [d]

The pattern of the series is illustrated below,

<u>D</u> EH	<u>C</u> DI	<u>K</u> LQ	<u>P</u> RV
+1	+1	+1	+2
(a)	(b)	(c)	(d)

18. Ans: [c]

The pattern of the series is illustrated below,

<u>P</u> RQ	<u>A</u> CB	<u>T</u> UV	<u>Q</u> SR
+1	+1	+1+1	+1
(a)	(b)	(c)	(d)

19. Ans: [c]

The pattern of the series is illustrated below,

<u>F</u> HKO	<u>C</u> EHL	<u>Z</u> B ² EJ	<u>X</u> ZCG
+2+3+4	+2+3+4	+2+3+5	+2+3+4
(a)	(b)	(c)	(d)

20. Ans: [a]

All terms have common difference except for AEK.

21. Ans: [d]

The pattern of the series is shown as follows,

(a) D F K	(b) P R W	(c) E G L	(d) T V Z
1 4	1 4	1 4	1 3
GHIJ	QSTUV	FHIJK	UWXYZ

Thus, the odd one out is TVZ.

22. Ans: [d]

The pattern of the series is shown as follows,

(a) B D 2	(b) G I 2	(c) N R 4	(d) S V 2
2-4	7-9	14-18	19-22

Thus, the odd one out is SV2 because the difference between 19 and 22 is 3.

23. Ans: [d]

ADP — 1, 4, 16 { $1 + 4 + 16 = 21$ }

QTX — 17, 20, 24 { $17 + 20 + 24 = 61$ }

HKR — 8, 11, 18 { $8 + 11 + 18 = 37$ }

STE — 19, 20, 5 { $19 + 20 + 5 = 44$ }

Except "STE", other words produce odd results

24. Ans: [d]

F U → difference → 15

D W → difference → 19

C X → difference → 21

N R → difference → 4 (the difference is an even no.)

Hence, the answer is option d.

25. Ans: [d]

The pattern of the series from one letter to the other is first the succeeding letter and then the next letter to the succeeding letter.

The odd one out is BDE which does follow the series.

Hence, the answer is option d.

26. Ans: [b]

Option a, c and d has a difference of 4 positions between the successive letters. While, option b does not fall into the pattern.

Hence, the answer is option b.

27. Ans: [c]

The pattern in three of these terms is that the difference between the first letter and the second letter is 1 while the difference between the second and the third letter is 3. Option c does not fall into this pattern.

Hence, the answer is option c.

28. Ans: [b]

Except for the term MQT, other terms are having equal difference between the successive letters

Hence, the answer is option b.

29. Ans: [d]

(a) DEB - One letter between D and B as in English alphabet

(b) FGD - One letter between D and B as in English alphabet

(c) PQN - One letter between D and B as in English alphabet

(d) TUS - S and T are consecutive letters in English alphabet

Hence, the answer is option d.

30. Ans: [a]

The pattern common to three terms is that the difference between the first two terms is 4 and the last two terms is 2. YDA is the odd one out.

Hence, the answer is option a.

$$1.23 \times 17 + 427 - 52\% \text{ of } 1450 = ?^2$$

- a) 64 b) 58 c) 8 d) 16

Correct Option: C

$$23 \times 17 + 427 - 52\% \text{ of } 1450 = ?^2$$

$$?^2 = 391 + 427 - 754$$

$$?^2 = 391 + 427 - 754$$

$$?^2 = 64 = 8$$

Hence, option C is correct.

$$2. 62\% \text{ of } 16850 + 32\% \text{ of } 7345 = 52\% \text{ of } 645 + ?$$

- a) 10328 b) 12462 c) 10358 d) 12360

$$62\% \text{ of } 16850 + 32\% \text{ of } 7345 = 52\% \text{ of } 645 + ?$$

$$10447 + 2350.40 = 335.40 + ?$$

$$? = 12462$$

Hence, option B is correct.

$$3. \frac{3}{5} \text{ of } 3245 + 32\% \text{ of } 6250 - (?)^2 = 1035$$

- a) 64 b) 62 c) 57 d) 58

$$1947 + 2000 - 103 = (?)^2$$

$$3947 - 103 = (?)^2$$

$$3844 = (?)^2$$

$$? = 62$$

Hence, option B is correct.

$$4. 23568 + 33852 + 17183 - 52549 = ?$$

- a) 20084 b) 22184 c) 21084 d) 22084 e) None of these

$$74603 - 52549 = 22054$$

Hence, option E is correct.

5. $1454 + 2365 + 9710 + 3020 = ?$

- a) 20718 b) 18121 c) 16549 d) 14226

$$1454 + 2365 + 9710 + 3020 = ?$$

$$? = 1454 + 2365 + 9710 + 3020$$

$$? = 16549$$

Hence, option C is correct.

6. 67.5% of 960 + ?% of 640 = 728

- a) 12.5 b) 25 c) 12 d) 11

$$67.5\% \text{ of } 960 + ?\% \text{ of } 640 = 728$$

$$\frac{67.5}{100} \times 960 + \frac{?}{100} \times 640 = 728$$

$$648 + \frac{?}{100} \times 640 = 728$$

$$\frac{?}{100} \times 640 = 728 - 648$$

$$\frac{?}{100} \times 640 = 80$$

$$? = 80 \times \frac{100}{640}$$

$$? = 12.5$$

Hence, option A is correct.

7. $6992 \div 19 - ?\% \text{ of } 652 = -4196$

- a) 700 b) 600 c) 300 d) 800

$$6992 \div 19 - ?\% \text{ of } 652 = -4196$$

$$368 + 4196 = ?\% \text{ of } 652$$

$$?\% \text{ of } 652 = 4564$$

$$? = 4564 \div 652 \times 100$$

$$? = 700$$

8. $13 \frac{2}{3} \% \text{ of } 3300 + 25\% \text{ of } 184 = 40\% \text{ of } ?$

- a) 1242.5 b) 1361.5 c) 1124 d) 1220

$$13 \frac{2}{3} \% \text{ of } 3300 + 25\% \text{ of } 184 = 40\% \text{ of } ?$$

$$\frac{41}{300} \times 3300 + \frac{1}{4} \times 184 = 41 \times 11 + 46 = 40\% \text{ of } ?$$

$$451 + 46 = \frac{2}{5} \times ?$$

$$? = 497 \times \frac{5}{2} = 1242.5$$

Hence, option A is correct.

9.

$$8125 \div 13 \div 2 \frac{1}{2} \times 10^2 = ?$$

- a) 12500 b) 25000 c) 50000 d) 37500
 $(2^{-3} + 12.5\% \text{ of } 624) \times 4^4 = 5^3 \times ?$

$$\left(\frac{1}{8} + \frac{1}{8} \times 624\right) \times 256 = 5^3 \times ?$$

b)

$$\frac{625}{8} \times 256 = 125 \times ?$$

c)

$$? = 5 \times 32 = 160$$

Hence, option C is correct.

$$10. (13456 - 712) \div 27^2 = ? \div 3 \div 3 \div 3 \div 2$$

- a) 472 b) 236 c) 832 d) 944

$$(13456 - 712) \div 27^2 = ? \div 3 \div 3 \div 3 \div 2$$

$$\frac{12744}{27 \times 27} = \frac{?}{27 \times 2}$$

$$? = 472 \times 2 = 944$$

Hence, option D is correct.

$$11. \quad 0.16 \times 55 \times 180 + ?^2 = 12^3$$

- a) 484 b) 12 c) 22 d) 32

$$0.16 \times 55 \times 180 + ?^2 = 12^3$$

$$\frac{16}{100} \times 55 \times 180 + ?^2 = 1728$$

$$1584 + ?^2 = 1728$$

$$?^2 = 1728 - 1584 = 144 = 12^2$$

$$? = \pm 12$$

Hence, option B is correct.

$$12. \quad 1.25 \times 844 + 0.5 \times 432 + ? = 2500$$

- a) 1229 b) 1341 c) 1339 d) 1489

$$1.25 \times 844 + 0.5 \times 432 + ? = 2500$$

$$\frac{5}{4} \times 844 + \frac{1}{2} \times 432 + ? = 2500$$

$$? = 2500 - (5 \times 211) - 216$$

$$? = 2500 - 1271 = 1229$$

Hence, option A is correct.

$$13. \quad 75\% \text{ of } 12^2 + ? = 40\% \text{ of } 600$$

- a) 145 b) 132 c) 112 d) 158

$$75\% \text{ of } 12^2 + ? = 40\% \text{ of } 600$$

$$\frac{3}{4} \times 144 + ? = \frac{2}{5} \times 600$$

$$? = 240 - 108$$

$$? = 132$$

Hence, option B is correct.

$$14. \quad 366.633 + 636.36 - 666.333 - 33.366 + 3336.33 = ?$$

- a) 3639.624 b) 4532.224
 c) 3242.332 d) 4426.634

$$366.633 + 636.36 - 666.333 - 33.366 + 3336.33 = ?$$

$$\Rightarrow 3639.624 = ?$$

Hence, option A is correct.

$$15. 8200 \times 67 - 32518 = ? \times 90 + 12$$

a) 5743

b) 6587

c) 5796

d) 6425

$$8200 \times 67 - 32518 = ? \times 90 + 12$$

$$\Rightarrow 549400 - 32518 = ? \times 90 + 12$$

$$\Rightarrow 516882 = ? \times 90 + 12$$

$$\Rightarrow ? \times 90 = 516882 - 12$$

$$\Rightarrow ? = \frac{516870}{90}$$

$$\Rightarrow ? = 5743$$

Hence, option A is correct.

HSEM1BTECHSTANDARD0719

4. $M^4 + N^4 - 2$ is divisible by 16 if M & N are
 (a) 234, 528 (b) 541, 684
 (c) 384, 4495 (d) 297, 981
5. Which of the following number is divisible by 5?
 (a) 424242242423₄
 (b) 4444333221₅
 (c) 11111122222333344455₆
 (d) None of these
6. Find the remainder when $(135678)_{16}$ is divided by 5.
 (a) 0 (b) 1 (c) 2 (d) 4
7. A 60 digit number is formed by writing natural numbers from 1 as given below 12345....
 Find the remainder when this number is divided by 32.
 (a) 9 (b) 0 (c) 5 (d) 31
8. The number of positive integers not greater than 1000, which are not divisible by 3, 5 and 7 is
 (a) 457 (b) 675
 (c) 543 (d) None of these
9. The number of positive integers 'n' from 35 to 150 such that $(n-1)!$ is not divisible by n is
 (a) 67 (b) 33 (c) 23 (d) 25
10. When 10^x is divided by 13, remainder is 1. If x is a natural number less than 153 then how many values can 'x' take?
 (a) 12 (b) 10 (c) 30 (d) 25
11. On base 8, first 100 natural numbers are written. How many of these numbers are divisible by 7?
 (a) 70 (b) 21 (c) 14 (d) 28
12. Let $N = n^6 + 3n^5 - 5n^4 - 15n^3 + 4n^2 + 12n$ for any $n \in \mathbb{N}$.
 The greatest divisor of N among the number given below is
 (a) 2 (b) 6 (c) 10 (d) 30
13. Find the largest 'n' such that $n+10$ divides n^3+100 .
 (a) 890 (b) 99
 (c) n=1000 (d) None of these
14. Let $x_n = 6^n + 8^n$. Find the remainder when x_{49} is divided by 49.
 (a) 48 (b) 1 (c) 47 (d) 0
15. Let $N = n^{12} - n^8 - n^4 + 1$ where n is any odd number.
 Which one of the following can divide N?
 (a) 27 (b) 512
 (c) 1029 (d) None of these
16. Which of the following can divide $3^{123456789} + 1$?
 (a) 2 (b) 8 (c) 4 (d) 16
17. Find the number of positive divisors of 10^{9999} but not 10^{9998} .
 (a) 1999 (b) 10000^2 (c) 19999 (d) 9999^2
18. $N = 2 \times 4 \times 6 \times \dots \times (100 \text{ numbers}) - 1 \times 3 \times 5 \times \dots \times (100 \text{ numbers})$
 Which one of the following can divide N?
 (a) 1986 (b) 2300 (c) 1000 (d) 2001
19. For any positive integer n, $2^n + 9^n - 4^n - 7^n$ is divisible by
 (a) 3 (b) 10 (c) 7 (d) 3
20. Find the number of positive integer 'n' for which $n \leq 2000$ and 6 is a factor of $n^2 + 3n + 2$.
 (a) 1334 (b) 666 (c) 444 (d) 556

SESSION – 8

 NUMBER PROPERTIES HCF & LCM

1. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case, but when divided by 7 leaves no remainder.
 (a) 8004 (b) 13004 (c) 18004 (d) 18014
2. What is the greatest number which divides 852, 1065 and 1491 exactly?
 (a) 193 (b) 183 (c) 223 (d) 213
3. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.
 (a) 12 (b) 18 (c) 24 (d) 32
4. Find the side of the largest square slabs which can be paved on the floor of a room 5 m 44 cm long and 3 m 74 cm broad.
 (a) 56 cm (b) 42 cm (c) 38 cm (d) 34 cm
5. Find the greatest number of 4 digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 as remainders respectively.
 (a) 9654 (b) 9666 (c) 9664 (d) 9864
6. Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds. After what interval of time will they keep on tolling together?
 (a) 3240 secs (b) 3080 secs
 (c) 3140 secs (d) 3200 secs
7. The least perfect square number which is divisible by 3, 4, 5, 6 and 8 is
 (a) 900 (b) 1200 (c) 2500 (d) 3600

QUANTITATIVE ABILITY

8. The HCF of two numbers is 11 and their LCM is 693. If one of the numbers is 77, find the other.
 (a) 909 (b) 119 (c) 66 (d) 99
9. The sum of the HCF and LCM of two numbers is 680 and the LCM is 84 times the HCF. If one of the numbers is 56, find the other number.
 (a) 84 (b) 12 (c) 8 (d) 96
10. The ratio of two numbers is 3:4. Their HCF is 4. Find the LCM.
 (a) 12 (b) 16 (c) 24 (d) 48
11. Philip, Tom and Brad start jogging around a circular field and complete a single round in 18 secs, 22 secs and 30 secs respectively. In how much time will they meet again at the starting point?
 (a) 3 min 15 secs (b) 21 min
 (c) 16 min 30 secs (d) 12 min
12. The HCF of two numbers is 8. Which one of the following can never be their LCM?
 (a) 24 (b) 48 (c) 56 (d) 60
13. Three numbers which are co-primes to each other are such that the product of the first two is 551 and that of the last two is 1073. Find the sum of the three numbers.
 (a) 75 (b) 81 (c) 85 (d) 89
14. The sum of two numbers is 216 and their HCF is 27. Find the numbers.
 (a) 27, 189 (b) 81, 189
 (c) 108, 108 (d) 154, 162
15. HCF of 3240, 3600 and a third number is 36 and their LCM is $2^4 \times 3^5 \times 5^2 \times 7^2$. Find the third number.
 (a) $2^2 \times 3^5 \times 7^2$ (b) $2^2 \times 5^3 \times 7^2$
 (c) $2^5 \times 5^2 \times 7^2$ (d) $2^3 \times 3^3 \times 7^2$
16. The HCF and LCM of two numbers are 33 and 264. When the first number is divided by 2, the quotient is 33. Find the other number.
 (a) 66 (b) 132 (c) 198 (d) 99
17. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes?
 (a) 11 km/min (b) 13 km/min
 (c) 17 km/min (d) None of these
18. Find the greatest number of 5 digits that will give us a remainder of 5 when divided by 8 and 9 respectively.
 (a) 99931 (b) 99941
 (c) 99725 (d) None of these
19. Two equilateral triangles have the sides of length 34 and 85 respectively. Find the greatest length of the rope that can measure both of them exactly. How many such equal parts can be measured?
 (a) 17, 21 (b) 19, 18
 (c) 21, 14 (d) None of these
20. The sum of two numbers is 528 and their HCF is 33. The number of pairs of such numbers satisfying the above condition is
 (a) 6 (b) 12 (c) 8 (d) 4

SESSION - 9

NUMBER PROPERTIES FRACTIONS & DECIMALS

1. Which of the following has fractions in ascending order?
 (a) $\frac{2}{5}, \frac{3}{5}, \frac{1}{3}, \frac{4}{7}, \frac{5}{6}$ (b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{5}{6}, \frac{4}{7}$
 (c) $\frac{1}{3}, \frac{2}{5}, \frac{5}{6}, \frac{4}{7}, \frac{3}{5}$ (d) $\frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}, \frac{5}{6}$
2. Which of the following has fractions in descending order?
 (a) $\frac{5}{6}, \frac{4}{7}, \frac{2}{5}, \frac{3}{5}, \frac{1}{3}$ (b) $\frac{5}{6}, \frac{3}{5}, \frac{4}{7}, \frac{2}{5}, \frac{1}{3}$
 (c) $\frac{4}{7}, \frac{1}{3}, \frac{2}{5}, \frac{5}{6}, \frac{3}{5}$ (d) $\frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}, \frac{5}{6}$
3. Convert 0.737373... into vulgar fraction?
 (a) $\frac{73}{99}$ (b) $\frac{73}{100}$ (c) $\frac{73}{90}$ (d) $\frac{73}{900}$
4. Convert $0.\overline{67}$ into vulgar fraction.
 (a) $\frac{67}{99}$ (b) $\frac{67}{90}$ (c) $\frac{61}{90}$ (d) $\frac{61}{100}$
5. Find the correct expression for $5.\overline{46}$ in the fractional form.
 (a) $\frac{541}{100}$ (b) $\frac{541}{99}$ (c) $\frac{546}{99}$ (d) $\frac{541}{900}$
6. $0.2\overline{34} + 0.1\overline{888} = ?$
 (a) 0.42 $\overline{32}$ (b) 0.41 $\overline{32}$
 (c) 0.42 $\overline{33}$ (d) 0.4231
7. $3.\overline{23} - 2.\overline{03} + 1.\overline{55}$
 (a) 2. $\overline{75}$ (b) 2.75 (c) 2.70 (d) 2. $\overline{71}$
8. Find the product of $0.\overline{09} \times 7.\overline{3}$
 (a) 0. $\overline{67}$ (b) 0.657
 (c) 0. $\overline{6}$ (d) None of these

9. If $\frac{347.624}{0.0089} = a$, then find the value of $\frac{347624}{0.0089} = ?$
 (a) $\frac{a}{10}$ (b) $10a$ (c) $\frac{a}{1000}$ (d) $1000a$
10. Find the value of

$$\frac{(0.555 \times 0.555 - 0.555 \times 0.020 + 0.020 \times 0.020)}{(0.555 \times 0.555 \times 0.555) + (0.020 \times 0.020 \times 0.020)}$$

 (a) 1.55 (b) 1.74 (c) 2.36 (d) 5.02
11. Evaluate: $\frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61}$
 (a) 2 (b) 4 (c) 6 (d) 8
12. What decimal of an hour is a second?
 (a) 0.0025 (b) 0.0256
 (c) 0.00027 (d) 0.000126
13. The value of $\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + (0.1)^2}$ is:
 (a) 0.86 (b) 0.95 (c) 0.97 (d) 1.06
14. The value of $\frac{0.1 \times 0.1 \times 0.1 + 0.02 \times 0.02 \times 0.02}{0.2 \times 0.2 \times 0.2 + 0.04 \times 0.04 \times 0.04}$ is:
 (a) 0.0125 (b) 0.125
 (c) 0.25 (d) 0.5
15. If $2994 \div 14.5 = 172$, then $29.94 \div 1.45 = ?$
 (a) 0.172 (b) 1.72 (c) 17.2 (d) 172
16. $\frac{0.009}{?} = 0.01$
 (a) 0.0009 (b) 0.09 (c) 0.9 (d) 9
17. $\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$ is approximately equal to:
 (a) 2 (b) 2.40 (c) 2.43 (d) 2.50
18. 0.04×0.0162 is equal to:
 (a) 6.48×10^{-3} (b) 6.48×10^{-4}
 (c) 6.48×10^{-5} (d) 6.48×10^{-6}
19. $\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1}$ is equal to:
 (a) 0.5 (b) 1.0 (c) 20 (d) 22
20. If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is:
 (a) 0.0144 (b) 1.44 (c) 14.4 (d) 144

SESSION - 10

 ALGEBRA - I

Introduction

- The equation of the form $ax + b = 0$ is a linear equation in one variable x .
- The equations of the form $ax + by + c = 0$ where $a, b \neq 0$ is called the linear equation in two variables x and y .
- Simultaneous equations are a pair of equations of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. The solution of these equations is (x, y) an ordered pair.
- The set of three equations of the form $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$ is called the system of linear equations in three variables x, y, z .
- An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation. Here x is the variable and $a, b, c \in \mathbb{R}$ are the constants.
 a, b, c are called the coefficients of the equation.
- The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 The sum of the roots $= -\frac{b}{a}$, the product of the roots $= \frac{c}{a}$
- The roots of the quadratic equation depends on $b^2 - 4ac$ which is called the discriminant of the equation.
 - The equation has real and distinct roots when $b^2 - 4ac > 0$
 - The equation has real and equal roots when $b^2 - 4ac = 0$
 - The equation has complex roots when $b^2 - 4ac < 0$
 - If $b^2 - 4ac$ is a perfect square the equation has two rational roots.
- If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$. The quadratic equation whose roots are α and β is $(x - \alpha)(x - \beta) = 0$.

∴ The side of the largest square is 34 cm

$$\begin{array}{r}
 374) 544 (1 \\
 \quad\quad\quad 374 \\
 \hline
 \quad\quad\quad 170) 374 (2 \\
 \quad\quad\quad \quad\quad\quad 340 \\
 \quad\quad\quad \quad\quad\quad 34) 170 (5 \\
 \quad\quad\quad \quad\quad\quad 170 \\
 \hline
 \quad\quad\quad\quad\quad 0
 \end{array}$$

5. Ans: [a]

$$10 - 4 = 6, 15 - 9 = 6, 21 - 15 = 6, 28 - 22 = 6$$

LCM of 10, 15, 21 and 28 = 420

The greatest 4 digit number is 9999.

$$9999 \div 420 = Q + R339$$

∴ The greatest 4 digit number divisible by 420

$$\rightarrow 9999 - 339 = 9660$$

$$\therefore \text{The required number} = 9660 - 6 = 9654$$

2	10, 15, 21, 28
5	5, 15, 21, 14
3	1, 3, 21, 14
7	1, 1, 7, 14
	1, 1, 1, 2

420	9999 (23
	840
	1599
	1260
	339

6. Ans: [a]

The interval of time is the LCM of the numbers 36, 45, 72, 81 and 108.

$$\text{LCM} = 3240$$

∴ They will keep on tolling together after every interval of 3240 secs.

2	36, 45, 72, 81, 108
2	18, 45, 36, 81, 54
3	9, 45, 18, 81, 27
3	3, 15, 6, 27, 9
3	1, 5, 2, 9, 3
	1, 5, 2, 3, 1

7. Ans: [d]

$$\text{LCM of } 3, 4, 5, 6, 8 = 120$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

To make it a perfect square, it should be multiplied by $2 \times 3 \times 5 = 30$

$$\therefore \text{The required number is } 120 \times 30 \rightarrow 3600$$

2	3, 4, 5, 6, 8
2	3, 2, 5, 3, 4
3	3, 1, 5, 3, 2
	1, 1, 5, 1, 2

8. Ans: [d]

$$\text{HCF} = 11, \text{LCM} = 693, \text{One number} = 77$$

HCF \times LCM = Product of the two numbers

$$\therefore \text{Other number} = \frac{11 \times 693}{77} = 99$$

9. Ans: [d]

$$\text{HCF} + \text{LCM} = 680$$

$$\text{LCM} = 84 \times \text{HCF}$$

$$\therefore \text{HCF} + 84 \times \text{HCF} = 680$$

$$\Rightarrow 85 \times \text{HCF} = 680$$

$$\Rightarrow \text{HCF} = 8$$

$$\text{LCM} = 84 \times 8 = 672$$

$$\text{One number} = 56$$

$$\therefore \text{Other number} = \frac{672 \times 8}{56} = 96$$

10. Ans: [d]

Let the numbers be $3x$ and $4x$.

$$\therefore \text{HCF} = x = 4$$

⇒ The numbers are 12 and 16.

$$\therefore \text{LCM} = \frac{12 \times 16}{4} = 48$$

11. Ans: [c]

They will meet again at the starting point after running for the time equal to the LCM of 18 secs, 22 secs and 30 secs.

$$\text{LCM} = 990 \text{ secs} = 16 \text{ min 30 secs}$$

∴ The time is 16 min 30 secs

2	18, 30, 22
3	9, 15, 11
	3, 5, 11

12. Ans: [d]

The LCM should be a multiple of the HCF.

⇒ LCM should be a multiple of 8.

$$24 = 8 \times 3, 48 = 8 \times 6, 56 = 8 \times 7$$

But 60 is not a multiple of 8.

∴ The required number is 60.

13. Ans: [c]

Since the 3 numbers are co-prime, they contain only 1 as a common factor. Also the two products have the middle number common.

$$\text{So middle number} = \text{HCF of } 551 \text{ and } 1073 = 29$$

$$\therefore \text{First number} = 551 / 29 = 19$$

$$\text{Third number} = 1073 / 29 = 37$$

$$\therefore \text{Sum of the three numbers} = 19 + 29 + 37 = 85$$

$$\begin{array}{r}
 551) 1073 (1 \\
 \quad\quad\quad 551 \\
 \hline
 \quad\quad\quad 522) 551 (1 \\
 \quad\quad\quad \quad\quad\quad 522 \\
 \hline
 \quad\quad\quad\quad\quad 29) 522 (18 \\
 \quad\quad\quad\quad\quad \quad\quad\quad 29 \\
 \hline
 \quad\quad\quad\quad\quad\quad\quad\quad 232 \\
 \quad\quad\quad\quad\quad\quad\quad\quad 232 \\
 \hline
 \quad\quad\quad\quad\quad\quad\quad\quad 0
 \end{array}$$

14. Ans: [a]

QUANTITATIVE ABILITY – SOLUTION

Let the two numbers be $27x$ and $27y$.

$$\therefore \text{Their sum} = 27x + 27y = 27(x + y) = 216 \Rightarrow x + y = 8$$

Co-primes with sum 8 are (1, 7) and (3, 5).

\therefore The required numbers are $(27 \times 1, 27 \times 7)$ or $(27 \times 3, 27 \times 5)$.

\Rightarrow The numbers are (27, 189) or (81, 135).

Out of these, the matching choice is 27, 189.

\therefore The numbers are 27 and 189.

15. Ans: [a]

$$3240 = 2^3 \times 3^4 \times 5$$

$$3600 = 2^4 \times 3^2 \times 5^2$$

$$\text{HCF} = 36 = 2^2 \times 3^2$$

Since HCF is the product of the lowest powers of common factors, the third number must have $(2^2 \times 3^2)$ as its factor.

Since LCM is the product of the highest powers of common prime factors, the third number must have 3^5 and 7^2 as its factors.

$$\therefore \text{Third number} = 2^2 \times 3^5 \times 7^2$$

16. Ans: [b]

$$\text{HCF} = 33, \text{LCM} = 264$$

$$\text{First number} = 2 \times 33 = 66$$

$$\therefore \text{Second number} = \frac{\text{HCF} \times \text{LCM}}{\text{First number}} = \frac{33 \times 264}{66} = 132$$

17. Ans: [c]

The greatest possible rate can be obtained by finding the HCF of 51 and 85.

$$\text{HCF of } 51 \text{ and } 85 = 17$$

\therefore The required possible speed is 17 km/min

$$\begin{array}{r} 51) 85 (1 \\ \quad 51 \\ \hline \quad 34) 51 (1 \\ \quad \quad 34 \\ \hline \quad \quad 17) 34 (2 \\ \quad \quad \quad 34 \\ \hline \quad \quad \quad 0 \end{array}$$

18. Ans: [b]

$$\text{LCM of } 8 \text{ and } 9 = 72$$

The largest five digit multiple of 72 = 99936

$$\therefore \text{The required number} = 99936 + 5 = 99941$$

19. Ans: [a]

$$\text{HCF of } 34 \text{ and } 85 = 17$$

$$34 = 2 \times 17$$

\therefore The greatest length of the rope = 17

$$85 = 5 \times 17$$

$$\begin{aligned} \text{Number of equal parts to be measured} &= \frac{34 \times 3}{17} + \frac{85 \times 3}{17} \\ &= (2 \times 3) + (5 \times 3) = 6 + 15 = 21 \end{aligned}$$

20. Ans: [d]

Let the numbers be $33x$ and $33y$.

$$\therefore 33x + 33y = 33(x + y) = 528$$

$$\Rightarrow x + y = 16$$

The number of possible pairs are (1, 15), (3, 13), (5, 11), (7, 9).

\therefore The numbers with sum 528 are $(33 \times 1, 33 \times 15)$, $(33 \times 3, 33 \times 13)$, $(33 \times 5, 33 \times 11)$ and $(33 \times 7, 33 \times 9)$.

So, there are 4 such pairs.

SESSION - 9

NUMBER PROPERTIES FRACTIONS & DECIMALS

1. Ans: [d]

Option 1: 0.4, 0.6, 0.33, 0.5, 0.8 --- (wrong)

Option 2: 0.3, 0.4, 0.6, 0.8, 0.5 --- (wrong)

Option 3: 0.3, 0.4, 0.8, 0.5, 0.6 --- (wrong)

Option 4: 0.33, 0.4, 0.5, 0.6, 0.8 --- (correct)

$0.33 < 0.4 < 0.5 < 0.6 < 0.8$ ---- (Ascending order)

2. Ans: [b]

Option 1: 0.8, 0.5, 0.4, 0.6, 0.33 --- (wrong)

Option 2: 0.8, 0.6, 0.5, 0.4, 0.3 --- (correct)

Option 3: 0.5, 0.3, 0.4, 0.8, 0.6 --- (wrong)

Option 4: 0.33, 0.4, 0.5, 0.6, 0.8 --- (wrong)

$0.8 > 0.6 > 0.5 > 0.4 > 0.3$ ---- (Descending order)

3. Ans: [a]

In a decimal fraction, if there are n numbers of repeated numbers after a decimal point, then just write one repeated number in the numerator and in denominator take n number of nines equal to repeated numbers you observe after the decimal point.

0.737373... is written as $\overline{0.73}$

Numerator = 73 ---- (one repeated number)

Denominator = 99 ---- (73 is the number which is repeated)

$$\text{Vulgar fraction} = \frac{73}{99}$$

4. Ans: [c]

Numerator = (All digits after decimal point) Repeated digits only once - (Non-repetitive digit after decimal point)

Denominator = Take 9 as many times the repetitive digit, followed by zeros equal to number of non-repetitive digits.

$0.\bar{6}\bar{7}$ is a mixed recurring fraction.

$$\text{Numerator} = \frac{67 - 6}{90} = \frac{61}{90}$$

5. Ans: [b]

$$5.\overline{46} = 5 + 0.\overline{46} = 5 + \frac{46}{99} = \frac{495 + 46}{99} = \frac{541}{99}$$

Convert $0.\overline{46}$ into fraction and then add 5 to the fraction obtained.

6. Ans: [a]

$0.\overline{2343} + 0.\overline{1888}$ are mixed recurring decimal

$$0.\overline{2343} = \frac{2343 - 23}{9900} = \frac{2320}{9900}$$

$$0.\overline{1888} = \frac{1888 - 18}{9900} = \frac{1870}{9900}$$

$$0.\overline{2343} + 0.\overline{1888}$$

$$\frac{2320}{9900} + \frac{1870}{9900} = \frac{2320 + 1870}{9900} = \frac{4190}{9900} = 0.\overline{4232}$$

7. Ans: [b]

$$3.\overline{23} - 2.\overline{03} + 1.\overline{55}$$

$$\text{Step 1: } (3 + 0.\overline{23}) + (1 + 0.\overline{55}) - (2 + 0.\overline{03})$$

$$\text{Step 2: } (4 + (0.23 + 0.55)) - (2 + 0.03)$$

Step 3: Convert decimal numbers into vulgar fractions

$$0.\overline{23} = \frac{23}{99}$$

$$0.\overline{55} = \frac{55}{99}$$

$$0.\overline{03} = \frac{9}{99}$$

Step 4: Substituting the values, we get

$$= \left[4 + \left(\frac{23}{99} + \frac{55}{99} \right) \right] - \left[2 + \frac{3}{99} \right]$$

$$= \left[2 + \left(\frac{23}{99} + \frac{55}{99} - \frac{3}{99} \right) \right]$$

$$= \left[2 + \frac{75}{99} \right] = 2.\overline{75}$$

8. Ans: [c]

$$0.09 = \frac{9}{99}$$

$$7.3 = 7 + \frac{3}{9} = \frac{66}{9}$$

$$\frac{9}{99} \times \frac{66}{9} = \frac{2}{3} = \frac{6}{9}$$

$\frac{6}{9}$ can be easily converted into decimal form = $0.\overline{6}$

9. Ans: [c]

$$\text{Given: } \frac{347.624}{0.0089} = a$$

$$\text{The value of } \frac{347624}{0.0089} \div 1000 = a \div 1000 = \frac{a}{1000}$$

10. Ans: [b]

The given numerical is in the form

$$\frac{(a^2 - ab + b^2)}{(a^3 + b^3)} = \frac{(a^2 - ab + b^2)}{(a+b)(a^2 - ab + b^2)} = \frac{1}{(a+b)}$$

$$\frac{(0.555^2) - (0.555 \times 0.020) + (0.20^2)}{(0.555^3) + 0.020^3}$$

$$= \frac{1}{(0.555 + 0.020)} = 1.74$$

This type of numerical can be easily solved, if all basic formulae are known.

11. Ans: [b]

Given Expression

$$= \frac{a^2 - b^2}{a - b} = \frac{(a+b)(a-b)}{(a-b)} = (a+b) = (2.39 + 1.61) = 4.$$

12. Ans: [c]

$$\text{Required decimal} = \frac{1}{60 \times 60} = \frac{1}{3600} = 0.00027$$

13. Ans: [a]

$$\text{Given expression} = \frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + (0.96 \times 0.1) + (0.1)^2}$$

$$= \left(\frac{a^3 - b^3}{a^2 + ab + b^2} \right)$$

$$= (a - b)$$

$$= (0.96 - 0.1)$$

$$= 0.86$$

14. Ans: [b]

$$\text{Given expression} = \frac{(0.1)^3 + (0.02)^3}{2^3[(0.1)^3 + (0.02)^3]} = \frac{1}{8} = 0.125$$

15. Ans: [c]

$$\frac{29.94}{14.5} = \frac{299.4}{145}$$

$= \left(\frac{2994}{145} \times \frac{1}{10} \right)$ [Here, substitute 172 in the place of 2994/14.5]

$$= \frac{172}{10}$$

$$= 17.2$$

QUANTITATIVE ABILITY

PRACTICE EXERCISE

1. If $7a + 8b = 53$ and $9a + 5b = 47$, find the values of a and b.
 (a) (4, 5) (b) (4, 3) (c) (3, 4) (d) (5, 4)
2. If I add 7 times my age seven years from now and 3 times my age three years ago, I get 12 times my current age. How old will I be 3 years from now?
 (a) 22 years (b) 23 years (c) 24 years (d) 25 years
3. A farmer has some chickens and feed for them. The feed lasts for exactly 30 days. He sold 10 of the chickens and purchased some feed such that the entire feed with him will last for 150 days. Now, he has thrice the initial feed. Find the initial number of chickens.
 (a) 25 (b) 30 (c) 40 (d) Cannot be determined
4. The cost of 4 chocolates, 6 biscuits and 12 lolly-pops is Rs.36. The cost of 15 biscuits, 9 lolly-pops and 3 chocolates is Rs.48. Find the cost of 1 biscuit.
 (a) Rs.4 (b) Rs.3 (c) Rs.2 (d) Rs.1
5. Tree I grows at $\frac{3}{7}$ th of tree II. If both the trees together grow 3 ft for every 3 years, find the time required by tree II to grow 7 ft.
 (a) 7 years (b) 10 years (c) 8 years (d) 12 years
6. Find the quadratic equation whose roots are the reciprocals of the roots of the equation $x^2 - 7x + 12 = 0$.
 (a) $x^2 - 12x + 7 = 0$ (b) $x^2 + 12x - 7 = 0$
 (c) $12x^2 + 7x - 1 = 0$ (d) $12x^2 - 7x + 1 = 0$
7. The equation $\sqrt{4x+9} - \sqrt{11x+1} - \sqrt{7x+4} = 0$ has
 (a) no solution (b) 1 solution (c) 2 solutions (d) more than 2 solutions
8. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ has real roots if p lies in the interval
 (a) $(0, 2\pi)$ (b) $(-\pi, 0)$
 (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$
9. The set of values of p for which the roots of the equation $5x^2 + 4x + p(p-2) = 0$ are of opposite sign is
 (a) $(-\infty, 0)$ (b) $(0, 2)$
 (c) $(0, \infty)$ (d) None of these

10. If α, β are the roots of the equation $(x-a)(x-b)=c$ with $c \neq 0$, find the roots of the equation $(x-\alpha)(x-\beta)+c=0$.
 (a) a, c (b) b, c
 (c) a, b (d) a + c, b + c
11. If p and q are the roots of $x^2 - 2x + A = 0$ and r and s are the roots of $x^2 - 18x + B = 0$ and $p < q < r < s$ are in A.P., find (A, B).
 (a) (-3, 77) (b) (77, -3)
 (c) (-3, -77) (d) None of these
12. The sum of a fraction and its reciprocal equals $\frac{85}{18}$. Find the fraction.
 (a) $\frac{2}{6}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$
13. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then
 (a) ab^2, ca^2, bc^2 are in A.P.
 (b) ab^2, bc^2, ca^2 are in A.P.
 (c) ab^2, bc^2, ac^2 are in A.P.
 (d) None of these
14. Let a, b, c be $\in \mathbb{R}$ and $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies
 (a) $\gamma - \frac{1}{2}(\alpha + \beta)$ (b) $\gamma = \alpha + \frac{\beta}{2}$
 (c) $\gamma = \alpha + \beta$ (d) $\alpha < \gamma < \beta$
15. If α, β are the roots of $ax^2 + bx + c = 0$, then the roots of the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ are
 (a) $\alpha - 1, \beta - 2$ (b) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$
 (c) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$ (d) None of these
16. If for the quadratic equation $x^2 - kx + 1 = 0$, one of the roots is A such that $\tan A = 2\sqrt{5} - 1$, then the other root is
 (a) greater than 1 (b) greater than 2
 (c) less than 1 (d) None of these

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17. If p and q are the roots of the equation $x^2 + px + q = 0$, then
 (a) $p = 1$ (b) $p = 1$ or 0
 (c) $p = -2$ (d) $p = -2$ or 0
18. If a, b, c are positive real numbers which are in G.P., then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in
 (a) A.P. (b) G.P.
 (c) H.P. (d) None of these
19. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$, the value of $(a - c)(b - c)(a + d)(b + d)$ is
 (a) $p^2 - q^2$ (b) $q^2 - p^2$
 (c) $q^2 + p^2$ (d) None of these
20. Find the value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other.
 (a) $-\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$
- SESSION - 11**
- ALGEBRA - II**
21. If $5x + 9y + 17z = a$, $4x + 8y + 12z = b$ and $2x + 3y + 8z = c$ have atleast 1 solution for x, y, z and a, b, c $\neq 0$ then which of the following is true?
 (a) $4a - 3b - 3c = 0$ (b) $3a - 4b - 3c = 0$
 (c) $4a - 3b - 4c = 0$ (d) None of these
22. If the expression $ax^2 + bx + c$ is equal to 4 when $x = 0$, leaves a remainder 4 when divided by $x + 1$, and remainder 6 when divided by $x + 2$, the values of a, b, c are
 (a) 1, 1, 4 (b) 2, 2, 4
 (c) 3, 3, 4 (d) 4, 4, 4
23. If $3x + 2y + z = 17$ and $4y + 2x + 6z = 38$, then find the value of $x + y + z$.
 (a) 6 (b) 7 (c) 8 (d) 9
24. If 2 quadratic equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ have a common root, $x = 1$ find out which of the following statements is correct.
 (a) $a + b = -3.5$ (b) $ab = 3$
 (c) $\frac{a}{b} = \frac{3}{4}$ (d) $a - b = -0.5$
25. If a, b, c are natural numbers and $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$, find the value of x.
 (a) abc (b) 1 (c) 0 (d) $a + b + c$
26. Solve: $\frac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0$
 (a) $x < 3$ or $x > 4$ (b) $3 < x < 4$
 (c) $4 < x < 24$ (d) $0 < x < 3$
27. Find the number of real values of x satisfying the equation
 $2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$
 (a) 1 (b) 2 (c) 3 (d) 4
28. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are the roots of the equation $f(x) = 0$, then the values of m and n are
 (a) -5, -30 (b) -5, 30
 (c) 5, 30 (d) None of these
29. There are some chocolates in a plate. Ajay took one third of them, Vijay took one third of the remaining and finally Karan took one third of the remaining. If at the end there were 8 chocolates, find the number of chocolates initially in the plate.
 (a) 27 (b) 24 (c) 32 (d) 36
30. The equation $\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{(x-2)}{(x+4)}$ has ___ root/s.
 (a) No (b) One
 (c) Two (d) Three
31. Karen went to buy 100 items which include only pens, pencils and erasers. The cost of each pen and pencil are Rs.5 and Re.1 respectively. If the cost of 20 erasers is Re.1 and he spent Rs.100 to buy 100 items. Find the number of pencils he bought.
 (a) 1 (b) 51
 (c) 49 (d) Cannot be determined
32. Eight years hence I will be thrice as old as I was 4 years ago. After how many years will I be 24 years?
 (a) 20 (b) 18 (c) 14 (d) 12
33. Ram takes 18 steps to go to a lake from his home. While coming back, he increased the steps length by 2 inches and took only 16 steps. What is the distance between his home and the lake?
 (a) 24 feet (b) 8 feet (c) 16 feet (d) 36 feet
34. If $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$, find the roots of the quadratic equation $g[f(x)] = 0$.
 (a) -1, -1 (b) 2, -1
 (c) $-1 + \sqrt{2}, -1 - \sqrt{2}$ (d) 1, 2

16. Ans: [c]

$$\text{Let } \frac{0.009}{x} = 0.01;$$

$$\text{Then } x = \frac{0.009}{0.01} = \frac{0.9}{1} = 0.9$$

17. Ans: [d]

$$\text{Given expression} = \frac{(0.3333)}{(0.2222)} \times \frac{(0.1667)(0.8333)}{(0.6667)(0.1250)}$$

$$\begin{aligned} &= \frac{3333}{2222} \times \frac{\frac{1}{6} \times \frac{5}{6}}{\frac{2}{3} \times \frac{125}{1000}} \\ &= \left(\frac{3}{2} \times \frac{1}{6} \times \frac{5}{6} \times \frac{3}{2} \times 8 \right) \\ &= \frac{5}{2} \\ &= 2.50 \end{aligned}$$

18. Ans: [b]

$4 \times 162 = 648$. Sum of decimal places = 6.

$$\text{So, } 0.04 \times 0.0162 = 0.000648 = 6.48 \times 10^{-4}$$

19. Ans: [b]

$$\text{Given Expression} = \frac{(a^2 - b^2)}{(a+b)(a-b)} = \frac{(a^2 - b^2)}{(a^2 - b^2)} = 1.$$

20. Ans: [a]

$$\begin{aligned} \frac{144}{0.144} &= \frac{14.4}{x} \\ \Rightarrow \frac{144 \times 1000}{144} &= \frac{14.4}{x} \\ \Rightarrow x &= \frac{14.4}{1000} = 0.0144 \end{aligned}$$

2. Ans: [b]

Let the present age be x years.

$$\text{Then, } 7(x+7) + 3(x-3) = 12x$$

$$\Rightarrow 7x + 49 + 3x - 9 = 12x$$

$$\Rightarrow 2x = 40 \Rightarrow x = 20 \text{ years}$$

\therefore Age after 3 years = $20 + 3 = 23$ years

3. Ans: [a]

Let the initial number of chickens be x .

$$\frac{x \times 30}{1} = \frac{(x-10) \times 150}{3}$$

$$\Rightarrow 90x = 150x - 1500$$

$$\Rightarrow 60x = 1500$$

$$\Rightarrow x = \frac{1500}{60} = 25$$

So, the initial number of chickens = 25

4. Ans: [c]

Let the cost of each chocolate be Rs. x and each biscuit be Rs. y and each lolly-pops be Rs. z .

$$\text{Then, } 4x + 6y + 12z = 36$$

$$\Rightarrow 2x + 3y + 6z = 18 \quad \dots (\text{i})$$

$$3x + 15y + 9z = 48$$

$$\Rightarrow x + 5y + 3z = 16 \quad \dots (\text{ii})$$

$$(\text{ii}) \times 2 - 1 \Rightarrow 7y = 14 \Rightarrow y = 2$$

\therefore The cost of 1 biscuit = Rs.2

5. Ans: [b]

Let tree II grow x feet after 1 year.

$$\therefore \left(\frac{3x}{7} + x \right) \times 3 = 3$$

$$\Rightarrow \frac{10x}{7} = 1 \Rightarrow x = \frac{7}{10} \text{ ft}$$

Tree II takes $\frac{7}{10}$ years to grow 7 ft.

\therefore Time required = 10 years

6. Ans: [d]

$$x^2 - 7x + 12 = 0$$

$\dots (\text{i})$ Sum of the roots = 7, product of the roots = 12

$$\Rightarrow \alpha + \beta = 7, \alpha\beta = 12$$

$$\text{Sum of the reciprocals of the roots} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{7}{12}$$

$$\text{Product of the reciprocals of the roots} = \frac{1}{\alpha\beta} = \frac{1}{12}$$

$$\therefore \text{The required equation is } x^2 - \frac{7}{12}x + \frac{1}{12} = 0$$

1. Ans: [c]

$$7a + 8b = 53$$

$$9a + 5b = 47$$

$$(\text{i}) \times 5 \rightarrow 35a + 40b = 265$$

$$(\text{ii}) \times 8 \rightarrow 72a + 40b = 376$$

$$(\text{iv}) - (\text{iii}) \rightarrow 37a = 111$$

$$\Rightarrow a = \frac{111}{37} = 3$$

Substituting for a in (i),

$$21 + 8b = 53 \Rightarrow b = \frac{32}{8} = 4$$

\therefore The solution is (3, 4)

QUANTITATIVE ABILITY – SOLUTION

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$$\Rightarrow 12x^2 - 7x + 1 = 0$$

7. Ans: [b]

$$\begin{aligned} \sqrt{4x+9} - \sqrt{11x+1} - \sqrt{7x+4} &= 0 \\ \sqrt{4x+9} - \sqrt{7x+4} &= \sqrt{11x+1} \\ (4x+9) + (7x+4) - 2\sqrt{(4x+9)(7x+4)} &= 11x+1 \\ 2\sqrt{(4x+9)(7x+4)} &= 12 \\ (4x+9)(7x+4) &= 36 \\ 28x^2 + 79x + 36 &= 36 \quad \Rightarrow 28x^2 + 79x = 0 \\ x = 0 \text{ or } -\frac{79}{28} & \end{aligned}$$

As $x \geq -\frac{1}{11}$, $x = -\frac{79}{28}$ is not a root.

∴ The solution is $x = 0$

There is 1 solution

8. Ans: [d]

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$$

If the equation has real roots,

$$\cos^2 p - 4\sin p(\cos p - 1) \geq 0$$

$$\Rightarrow \cos^2 p - 4\sin p \cos p + 4\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$$

$$(\cos p - 2\sin p)^2 \text{ is always } \geq 0$$

For $1 - \sin p$ to be non-negative, $\sin p \leq 1$

This is possible in the interval $(0, \pi)$

9. Ans: [b]

$$5x^2 + 4x + p(p-2) = 0$$

The roots are real if discriminant ≥ 0

$$\Rightarrow 16 - 20p(p-2) \geq 0$$

$$\Rightarrow 4 - 5p(p-2) \geq 0$$

$$\Rightarrow p(p-2) \leq \frac{4}{5}$$

The roots will be of opposite sign if $\frac{p(p-2)}{5} < 0$

$$\Rightarrow p(p-2) < 0 \Rightarrow p < 0 \text{ and } p > 2$$

or

$$p > 0 \text{ and } p < 2$$

$$\Rightarrow 0 < p < 2 \text{ or } (0, 2)$$

10. Ans: [c]

Since α, β are the roots of the equation

$$(x-a)(x-b) = c \text{ or } x^2 - (a+b)x + ab - c = 0$$

$$\alpha + \beta = a + b, \quad \alpha\beta = ab - c$$

Since $a + b = \alpha + \beta$ and $ab = \alpha\beta + c$, a, b are the roots of

$$x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$$

$$\Rightarrow (x-\alpha)(x-\beta) + c = 0$$

∴ The required roots are a and b .

11. Ans: [a]

p and q are the roots of $x^2 - 2x + A = 0$

$$\Rightarrow p + q = 2, pq = A$$

r and s are the roots of $x^2 - 18x + B = 0$

$$\Rightarrow r + s = 18, rs = B$$

Since p, q, r, s are in A.P.,

Let $p = a - 3d, q = a - d, r = a + d, s = a + 3d$

As $p < q < r < s, d > 0$

$$2 = p + q = 2a - 4d$$

$$18 = r + s = 2a + 4d$$

Solving, we get $a = 5, d = 2$

$$p = -1, q = 3, r = 7 \text{ and } s = 11$$

$$A = pq = -3$$

$$B = rs = 77$$

∴ (A, B) is $(-3, 77)$

12. Ans: [c]

Let the fraction be x .

$$x + \frac{1}{x} = \frac{85}{18}$$

$$\Rightarrow x^2 + 1 = \frac{85}{18}x \Rightarrow 18x^2 - 85x + 18 = 0$$

$$\Rightarrow (9x-2)(2x-9) = 0$$

$$\Rightarrow x = \frac{2}{9} \text{ or } \frac{9}{2}$$

But $x \neq \frac{9}{2}$

∴ The fraction is $\frac{2}{9}$

13. Ans: [a]

α, β are the roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\text{But } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow (\alpha\beta)^2(\alpha + \beta) = (\alpha + \beta)^2 - 2\alpha\beta$$

QUANTITATIVE ABILITY – SOLUTION

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$$\Rightarrow \frac{c^2}{a^2} \left(-\frac{b}{a} \right) = \frac{b^2}{a^2} - 2 \frac{c}{a}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow 2a^2c = ab^2 + bc^2$$

$\Rightarrow ab^2, ca^2, bc^2$ are in A.P..

14. Ans: [d]

Since α is the root of $a^2x^2 + bx + c = 0$

$$a^2\alpha^2 + b\alpha + c = 0$$

Since β is the root of $a^2x^2 - bx - c = 0$

$$a^2\beta^2 - b\beta + c = 0$$

Let $f(x) = a^2x^2 + 2bx + 2c$

$$f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$$

$$= 2(a^2\alpha^2 + bx + c) - a^2\alpha^2$$

$$= 2 \times 0 - a^2\alpha^2 = -a^2\alpha^2 < 0$$

$$f(\beta) = a^2\beta^2 + 2b\beta + 2c$$

$$= 3\alpha^2\beta^2 - 2(\alpha^2\beta^2 - b\beta - c)$$

$$= 3\alpha^2\beta^2 - 0$$

$$= 3\alpha^2\beta^2 > 0$$

\therefore In the interval (α, β) , $f(x)$ becomes 0 atleast once

Hence $\alpha < \gamma < \beta$

15. Ans: [b]

$ax^2 + bx + c = 0$ has roots α & β .

$$\therefore \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 - bx(x-1) + c(x-1)^2 = 0$$

$$\Rightarrow (a-b+c)x^2 + (b-2c)x + c = 0$$

$$\text{Discr} = (b-2c)^2 - 4c(a-b+c)$$

$$= b^2 - 4bc + 4c^2 - 4ac + 4bc - 4c^2 \\ = b^2 - 4ac$$

If A and B are the roots of the equation

$$A = \frac{-(b-2c) + \sqrt{b^2 - 4ac}}{2(a-b+c)}, B = \frac{-(b-2c) - \sqrt{b^2 - 4ac}}{2(a-b+c)}$$

$$A = \frac{-\frac{b}{2a} + \frac{c}{a} + \frac{\sqrt{b^2 - 4ac}}{2a}}{1 - \frac{b}{a} + \frac{c}{a}}, B = \frac{-\frac{b}{2a} + \frac{c}{a} - \frac{\sqrt{b^2 - 4ac}}{2a}}{1 - \frac{b}{a} + \frac{c}{a}}$$

$$A = \frac{\alpha + \alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{\alpha(1+\beta)}{(1+\alpha)(1+\beta)} = \frac{\alpha}{1+\alpha}$$

$$B = \frac{\beta + \alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{\beta(1+\alpha)}{(1+\alpha)(1+\beta)} = \frac{\beta}{1+\beta}$$

16. Ans: [c]

$$2\sqrt{5} - 1 > \sqrt{3} \Rightarrow \tan^{-1}(2\sqrt{5} - 1) > \tan^{-1}\sqrt{3} = \frac{\pi}{3} > 1$$

$$\therefore A = \tan^{-1}(2\sqrt{5} - 1) > 1$$

Let the other root be B

$$\text{Then } AB = 1 \Rightarrow B = \frac{1}{A} < 1$$

17. Ans: [b]

p and q are the roots of $x^2 + px + q = 0$

$$\Rightarrow pq = q, p + q = -p$$

$$\Rightarrow q(p-1) = 0 \Rightarrow q = 0 \text{ or } p = 1$$

If $q = 0$, we get $p = 0$

If $p = 1$, we get $q = -p - p = -2$

Thus $p = 1$ or 0

18. Ans: [c]

$$ax^2 + 2bx + c = 0$$

Since a, b, c are in G.P., $b^2 = ac \Rightarrow b = \sqrt{ac}$

The equation can be written as

$$ax^2 + 2\sqrt{ac}x + c = 0$$

$$\Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0$$

$$\Rightarrow x = -\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$$

Also $ax^2 + 2bx + c = 0$ has equal roots.

So the two given equations have a common root if $-\sqrt{\frac{c}{a}}$ is a root of $dx^2 + 2ex + f = 0$

$$\Rightarrow d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$$

$$\Rightarrow \frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \quad [\because b = \sqrt{ac}]$$

$$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$$

$$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{a}{d}, \frac{b}{e}, \frac{c}{f} \text{ are in H.P.}$$

19. Ans: [b]

$$x^2 + px + 1 = (x-a)(x-b)$$

$$x^2 + qx + 1 = (x-c)(x-d)$$

$$(a-c)(b-c)(a+d)(b+d)$$

QUANTITATIVE ABILITY – SOLUTION

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$$= (c-a)(c-b)(-a-d)(-b-d)$$

$$= (c^2 + pc + 1) [(-d)^2 - pd + 1]$$

$$= (c^2 + pc + 1)(d^2 - pd + 1)$$

But, both $c^2 + pc + 1 = 0$, $d^2 - pd + 1 = 0$

$$\therefore (a-c)(b-c)(a+d)(b+d)$$

$$= (pc - qc)(-qd - pd) \quad [\because c^2 + 1 = -pc, d^2 + 1 = pd]$$

$$= cd(p-q)(-q-p)$$

$$= cd(q-p)(p+q)$$

$$= cd(q^2 - p^2) = q^2 - p^2 \quad [\because cd = 1]$$

20. Ans: [d]

$$[a^2 - 5a + 3]x^2 + (3a - 1)x + 2 = 0$$

Let α and 2α be the roots

$$\therefore (a^2 - 5a + 3)\alpha^2 + (3a - 1)\alpha + 2 = 0 \quad \dots (i)$$

$$(a^2 - 5a + 3)(4\alpha^2) + (3a - 1)(2\alpha) + 2 = 0 \quad \dots (ii)$$

$$(i) \times 4 - (ii) \rightarrow (3a - 1)2\alpha + 6 = 0$$

$$\Rightarrow \alpha = \frac{-3}{3a - 1}$$

Putting this value in (i)

$$(a^2 - 5a + 3)9 - (3a - 1)^2(3) + 2(3a - 1)^2 = 0$$

$$\Rightarrow 9a^2 - 45a + 27 - 9a^2 + 18a - 3 + 18a^2 - 12a + 2 = 0$$

$$\Rightarrow -39a + 26 = 0$$

$$\Rightarrow a = \frac{2}{3}$$

SESSION - 11

ALGEBRA - II

21. Ans: [c]

$$5x + 9y + 17z = a$$

$$2x + 3y + 8z = c$$

$$4x + 8y + 12z = b$$

$$4a - 3b - 4c = 4(5x + 9y + 17z)$$

$$-3(4x + 8y + 12z)$$

$$-4(2x + 3y + 8z)$$

$$= 20x + 36y + 68z$$

$$-12x - 24y - 36z$$

$$-8x - 12y - 32z = 0$$

$$\Rightarrow 4a - 3b - 4c = 0$$

22. Ans: [a]

$$ax^2 + bx + c$$

when $x = 0, c = 4$

$$f(-1) = 4 \Rightarrow a - b + c = 4 \quad \left. \right\}$$

$$f(-2) = 6 \Rightarrow 4a - 2b + c = 6 \quad \left. \right\}$$

$$\begin{cases} a - b = 0 \\ 4a - 2b = 2 \end{cases}$$

$$\Rightarrow a = 1, b = 1$$

$\therefore a, b, c$ are 1, 1, 4.

23. Ans: [d]

$$3x + 2y + z = 17 \quad \dots (i),$$

$$2x + 4y + 6z = 38 \quad \dots (ii)$$

$$2 \times (i) + (ii) \rightarrow 8x + 8y + 8z = 34 + 38 = 72$$

$$\Rightarrow x + y + z = 9$$

24. Ans: [b]

$x = 1$ is a root of the quadratic equations

$$ax^2 + ax + 3 = 0 \text{ and } x^2 + x + b = 0$$

$$\therefore a + a + 3 = 0 \text{ and } 1 + 1 + b = 0$$

$$\Rightarrow a = -\frac{3}{2} \text{ and } b = -2$$

$$\Rightarrow ab = \left(-\frac{3}{2}\right)(-2) = 3$$

$$\Rightarrow ab = 3$$

25. Ans: [d]

Subtract 3 from both the sides of the given equation,

$$\frac{x-a}{b+c} - 1 + \frac{x-b}{c+a} - 1 + \frac{x-c}{a+b} - 1 = 0$$

$$(x-a-b-c) \left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) = 0$$

By the given data, $\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) \neq 0$

$$\Rightarrow x - a - b - c = 0 \text{ or } x = a + b + c$$

26. Ans: [a]

$$\frac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0 \Rightarrow x^2 - 7x + 12 > 0$$

$$\Rightarrow (x-4)(x-3) > 0$$

$$\Rightarrow x - 4 > 0, x - 3 > 0 \text{ or}$$

$$x - 4 < 0, x - 3 < 0$$

$$\Rightarrow x > 4 \text{ and } x < 3 \text{ or}$$

$$x < 4 \text{ and } x < 3$$

$$\Rightarrow x > 4 \text{ or } x < 3$$

27. Ans: [c]

$$\text{Let } x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

QUANTITATIVE ABILITY – SOLUTION

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$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\Rightarrow 2y^2 - 4 - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow (2y - 5)(y - 2) = 0$$

$$\Rightarrow y = \frac{5}{2}, 2$$

when $x + \frac{1}{x} = \frac{5}{2}$, we get $x = 2$ or $\frac{1}{2}$

when $x + \frac{1}{x} = 2$, we get $x = 1$

\therefore There are 3 values for $x = 2, \frac{1}{2}, 1$

28. Ans: [b]

2, 3 are the roots of $2x^3 + mx^2 - 13x + n = 0$

$$\therefore 16 + 4m - 26 + n = 0, 54 + 9m - 39 + n = 0$$

$$\Rightarrow 4m + n = 10, 9m + n = -15$$

Solving for m and n, we get

$$5m = -25 \Rightarrow m = -5$$

$$\text{and } n = 10 - 4m = 10 + 20 = 30$$

$$\therefore m = -5, n = 30$$

29. Ans: [a]

Let the number of chocolates in the plate be x.

$$\therefore \frac{1}{3}x + \frac{1}{3} \times \frac{2}{3}x + \frac{1}{3} \times \left[x - \left(\frac{1}{3} + \frac{2}{9}\right)x\right] + 8 = x$$

$$\Rightarrow \frac{1}{3}x + \frac{2}{9}x + \frac{4}{27}x + 8 = x$$

$$\Rightarrow \frac{19}{27}x + 8 = x \Rightarrow \frac{8x}{27} = 8 \Rightarrow x = 27$$

\therefore The number of chocolates in the plate = 27.

30. Ans: [b]

$$\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{(x-2)}{(x+4)}$$

$$(x^3 + x^2 - 22x - 40) - (x^3 + x^2 - 24x + 36) = 0$$

$$2x - 76 = 0$$

i.e. $(x - 38) = 0$, has only one root.

31. Ans: [a]

Let the number of pens, pencils and erasers be x, y, z respectively.

$$x + y + z = 100 \quad \dots \text{(i)}$$

$$5x + y + \frac{z}{20} = 100$$

$$\Rightarrow 100x + 20y + z = 2000 \quad \dots \text{(ii)}$$

$$\text{(ii)} - \text{(i)} \rightarrow 99x + 19y = 1900 \quad \dots \text{(iii)}$$

Since x and y are positive integers, x = 19 and y = 1
 \therefore The number of pencils = 1

32. Ans: [c]

Let the present age be x years.

$$x + 8 = 3(x - 4)$$

$$\Rightarrow 2x = 20 \Rightarrow x = 10 \text{ years}$$

\therefore Age will be 24 years after 14 years from now.

33. Ans: [a]

Let the length of each step be x inches.

$$18x = 16(x + 2) \Rightarrow 2x = 32 \Rightarrow x = 16 \text{ inches}$$

Distance between the home and the lake

$$= 18 \times 16 \text{ inches}$$

$$= \frac{18 \times 16}{12} \text{ feet}$$

$$= 24 \text{ feet}$$

34. Ans: [a]

$$f(x) = x^2 + 2x - 5$$

$$g(x) = 5x + 30$$

$$g[f(x)] = 5(x^2 + 2x - 5) + 30$$

$$= 5x^2 + 10x + 5$$

$$\therefore 5x^2 + 10x + 5 = 0$$

$$\Rightarrow x^2 + 2x + 1 = 0$$

$$\Rightarrow (x + 1)^2 = 0$$

$$\Rightarrow x = -1, -1$$

35. Ans: [d]

$$6\sqrt{\frac{x}{x+4}} - 2\sqrt{\frac{x+4}{x}} = 11$$

$$\text{Let } \sqrt{\frac{x}{x+4}} = y$$

$$\therefore 6y - \frac{2}{y} = 11$$

$$\Rightarrow 6y^2 - 11y - 2 = 0$$

$$\Rightarrow y = -\frac{1}{6} \text{ or } y = 2$$

But y cannot be negative

So, y = 2

$$\sqrt{\frac{x}{x+4}} = 2 \Rightarrow \frac{x}{x+4} = 4$$

$$\Rightarrow x = 4x + 16$$

$$\Rightarrow 3x = -16$$

$$\Rightarrow x = -\frac{16}{3}$$

36. Ans: [c]

QUANTITATIVE ABILITY – SOLUTION

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$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\text{Put } x + \frac{1}{x} = y$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = y^2 - 2$$

$$\therefore 2(y^2 - 2) - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow (2y - 5)(y - 2) = 0$$

$$\Rightarrow y = \frac{5}{2} \text{ or } 2$$

$$\text{When } y = \frac{5}{2}, \quad x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2 \text{ or } \frac{1}{2}$$

$$\text{When } y = 2, \quad x + \frac{1}{x} = 2 \Rightarrow x = 1$$

$$\text{Hence } x = 2, \frac{1}{2}, 1$$

\therefore The number of real values = 3

37. Ans: [a]

α, β are the roots of $x^2 - x + p = 0$

$$\Rightarrow \alpha + \beta = 1, \alpha\beta = p$$

γ, δ are the roots of $x^2 - 4x + q = 0$

$$\Rightarrow \gamma + \delta = 4, \gamma\delta = q$$

Let r be the common ratio of the G.P..

$$\text{Then } \frac{\gamma + \delta}{\alpha + \beta} = 4 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$$

When r = 2

$$\text{We get } \alpha(1+r) = 1 \Rightarrow \alpha = \frac{1}{1+r} = \frac{1}{3}$$

$p = \alpha\beta = \alpha \times ar = \alpha^2r = \frac{2}{9}$ which is not an integer

When r = -2,

$$\alpha(1+r) = 1 \Rightarrow \alpha = -1$$

$$p = \alpha^2r = -2$$

$$q = r\delta = (\alpha r^2)(\alpha r^3) = \alpha^2 r^5$$

$$= (-2)^5 = -32$$

\therefore The values of p and q are -2, -32.

38. Ans: [a]

$$ax^2 + bx + c = 0$$

Let the roots be α and 3α .

$$\therefore 3\alpha + \alpha = -\frac{b}{a} \Rightarrow 4\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{4a}$$

$$3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a} \Rightarrow \frac{3b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow \frac{3b^2}{16a} = c$$

$$\Rightarrow 3b^2 = 16ac$$

39. Ans: [b]

$$(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$$

Since the roots are equal, the discriminant is equal to 0.

$$4b^2(a+c)^2 - 4(a^2+b^2)(b^2+c^2) = 0$$

$$\Rightarrow b^2a^2 + 2b^2ac + b^2c^2 - a^2b^2 - b^4 - a^2c^2 - b^2c^2 = 0$$

$$\Rightarrow 2b^2ac - b^4 - a^2c^2 = 0$$

$$\Rightarrow b^4 + a^2c^2 - 2b^2ac = 0$$

$$\Rightarrow (b^2 - ac)^2 = 0 \Rightarrow b^2 = ac$$

$\Rightarrow a, b, c$ are in G.P.

40. Ans: [c]

$$x^2 - (c+6)x + 2(2c-1) = 0$$

Sum of the roots = $c + 6$

Product of the roots = $2(2c-1)$

$$c + 6 = \frac{1}{2} \times 2(2c-1)$$

$$\Rightarrow c + 6 = 2c - 1$$

$$\Rightarrow c = 7$$

SESSION - 12

⑥ FUNCTIONS - I

1. Ans: [d]

$F(x) = \max(2x+1, 3-4x)$ is minimum when $2x+1 = 3-4x$

$$\text{i.e. } 6x = 2$$

$$x = 2/6 = 1/3$$

therefore minimum possible value of $f(x)$ is

$$(2x+1)(\text{at } x = 1/3) = 2 * 1/3 + 1 = 5/3$$

$$\text{or } (3-4x) \text{ at } x = 1/3 = 3 - 4 * 1/3 = 5/3$$

2. Ans: [d]

$$F(x) = ax^2 - b|x|$$

$ax^2 > 0$ for $a > 0$ and

$(-b|x|) > 0$ for $b < 0$

$$F(x) = ax^2 - b|x| > 0 \text{ for } x \neq 0$$

$$F(0) = ax^2 - b|x| = 0 \text{ for } x = 0$$

If $x = 0$ $f(x)$ is minimised whenever, $a > 0, b < 0$.

3. Ans: [a]

$$\min \{f(x), h(x)\} < 3$$

$$f(x) < 3 \text{ or } h(x) < 3$$

$$2x^2 - 1 < 3 \text{ or } x^2 + x + 1 < 3$$

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35. The only value of x satisfying the equation

$$6\sqrt{\frac{x}{4+x}} - 2\sqrt{\frac{4+x}{x}} = 11, \quad x \in \mathbb{R}$$

- (a) $\frac{4}{35}$ (b) $\frac{16}{3}$ (c) $-\frac{4}{35}$ (d) $-\frac{16}{3}$

36. The number of real values of x satisfying the equation

$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

- (a) 1 (b) 2 (c) 3 (d) 4

37. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., find the values of p and q .

- (a) $-2, -32$ (b) $-2, 3$
 (c) $-6, 3$ (d) $-6, -32$

38. If one root of the quadratic equation $ax^2 + bx + c = 0$ is three times the other, find the relationship between a, b and c .

- (a) $3b^2 = 16ac$ (b) $b^2 = 4ac$
 (c) $(a+c)^2 = 4b$ (d) $\frac{a^2 + c^2}{ac} = \frac{b}{2}$

39. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$ are equal, then a, b, c are in

- (a) A.P. (b) G.P.
 (c) H.P. (d) Cannot be determined

40. For what value of c , the quadratic equation $x^2 - (c+6)x + 2(2c-1) = 0$ has the sum of the roots as half of their product?

- (a) 5 (b) -4 (c) 7 (d) 3

SESSION - 12

FUNCTIONS - I

Properties of Functions:

Definition of a Function: A function is a rule or formula that associates each element in the set X (an input) to exactly one and only one element in the set Y (the output). Different elements in X can have the same output, and not every element in Y has to be an output.

Definition of the Domain of a Function: The set of all possible inputs of a function is defined as the **domain**. The domain of a real-valued function defined by a formula for y in terms of x will be the set of all x input-values that result in a real y output-value unless the domain of the function is further restricted.

Definition of the Range of a Function: The set of all possible outputs of a function is defined as the **range**. The range of a real-valued function defined by a formula for y in terms of x will be the set of all y output-values that result from the x input-values in the domain.

Function Notation: Given that $f(x)$ is given by some formula containing x , $f(B)$ will be the same formula with each x replaced by B .

Linear Function Definition: If a function may be written in the form $f(x) = mx + b$ where x is the independent variable and m and b are constants, then $f(x)$ represents a linear function. The constant m is defined as the slope and the point $(0, b)$ represents the y -intercept. An equation in this form is known to be in *Slope-Intercept Form*.

Linear Function Slope Definition: Given that $f(x) = mx + b$, then m is defined as the slope where: $m = \frac{y_2 - y_1}{x_2 - x_1}$ for any two points (x_1, y_1) and (x_2, y_2) on the line. Graphically, the slope represents the change in y with respect to x on the graph of the line.

Linear Functions of Parallel Lines: If two linear functions are given by $f(x) = m_1x + b$ and $g(x) = m_2x + b_1$ and $m_1 = m_2$, then the graphs of $f(x)$ and $g(x)$ will consist of two lines that are parallel to each other.

Linear Functions of Perpendicular Lines: If two linear functions are given by $f(x) = m_1x + b$ and $g(x) = m_2x + b$, and $m_1 = -1/m_2$, then the graphs of $f(x)$ and $g(x)$ will consist of two lines that are perpendicular to each other.

Graphs of Even Functions: Given a function $f(x)$, if $f(c) = f(-c)$ for all c in the domain, then $f(x)$ is an *even* function and its graph will have symmetry with respect to the y -axis.

Graphs of Odd Functions: Given a function $f(x)$, if $f(c) = -f(-c)$ for all c in the domain, then $f(x)$ is called an *odd* function and its graph will have symmetry with respect to the origin. Symmetry with respect to the origin implies that a 180 degree rotation of the graph about $(0,0)$ results in an identical graph.

Functions Shifted Left: Given a function $f(x)$ and its graph and a value of $c > 0$, the graph of $f(x + c)$ will be a shift of the graph of $f(x)$ left by " c " units. This is known as the *Left Shift Function Rule*.

Functions Shifted Right: Given a function $f(x)$ and its graph and a value of $c > 0$, the graph of $f(x - c)$ will be a shift of the graph of $f(x)$ right by " c " units. This is known as the *Right Shift Function Rule*.

Functions Shifted Up: Given a function $f(x)$ and its graph and a value of $c > 0$, the graph of $f(x) + c$ will be a shift of the graph of $f(x)$ up by " c " units. This is known as the *Vertical Shift up Function Rule*.

Functions Shifted Down: Given a function $f(x)$ and its graph and a value of $c > 0$, the graph of $f(x) - c$ will be a shift of the graph of

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graph of $f(x)$ down by "c" units. This is known as the *Vertical Shift down Function Rule*.

Function Reflected Across X-axis Given a function $f(x)$ and its graph, the graph of $g(x) = -f(x)$ will be a reflection of the graph of $f(x)$ across the x-axis. This is known as the *X-axis Reflection Function Rule*.

Function Reflected Across Y-axis Given a function $f(x)$ and its graph, the graph of $g(x) = f(-x)$ will be a reflection of the graph of $f(x)$ across the x-axis. This is known as the *Y-axis Reflection Function Rule*.

Function Vertically Stretched Or Shrunk Given a function $f(x)$ and its graph and a value of $c > 0$, the graph of $g(x) = c \bullet f(x)$ will be a vertical stretch of the graph of $f(x)$. This means that all y-values of $g(x)$ will be equal to c times the respective y-values of $f(x)$. This is known as the *Vertical Stretch Function Rule*.

Definition of a Polynomial Function If $f(x)$ may be written in the form $a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \dots + a_n$, then $f(x)$ is a polynomial function of degree n where a_1, a_2, \dots, a_n are real coefficients. Linear functions are 1st degree polynomials and quadratic functions are 2nd degree polynomials.

Graphs of Polynomials Given a function $f(x)$ is a polynomial, it's x-intercepts will be located at the x-values $x = c$ such that $f(c) = 0$. Other solution points on the graph will be located between each two x-intercepts.

Standard Form of Quadratic Functions: Quadratic functions of the form $f(x) = ax^2 + bx + c$ may always be rewritten in the form $y = a(x - h)^2 + k$. Function shift rules may then be applied to state that the graph will be a vertical stretch of $y = x^2$ and will be shifted right, left, up, or down according to the values of h and k .

Graphs of Quadratic Functions in Form $f(x) = ax^2 + bx + c$: Given $f(x) = ax^2 + bx + c$, the graph will be a shift of $g(x) = ax^2$ (meaning it has the same shape), and will have a vertex at $x = -b/2a, y = f(-b/2a)$.

Property of The Vertex of a Quadratic Function: The vertex of $f(x) = ax^2 + bx + c$ will be the lowest point of the graph if $a > 0$ and will be the highest point of the graph if $a < 0$. The vertex represents the minimum value of the function for $a > 0$ and represents the maximum value of the function if $a < 0$.

Function Operations: Given two functions $f(x)$ and $g(x)$, the operations $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$ are defined in the following way:

- $(f + g)(x) = f(x) + g(x)$ and is sometimes denoted $f + g$
- $(f - g)(x) = f(x) - g(x)$ and is sometimes denoted $f - g$
- $(fg)(x) = f(x) \bullet g(x)$ and is sometimes denoted fg
- $(f/g)(x) = f(x)/g(x)$ provided $g(x) \neq 0$. This is sometimes denoted f/g

Function Composition: Given two functions $f(x)$ and $g(x)$, the function composition $(f \circ g)(x)$, is defined in the following way:

$(f \circ g)(x) = f[g(x)]$ and is sometimes denoted as $f \circ g$

In essence, composition implies that you input the entire formula of the second function in for each x-value of the the formula in the first function, assuming x is the variable used.

Definition of Inverse Functions: Given two functions $f(x)$ and $g(x)$, if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ is the inverse of $g(x)$ and $g(x)$ is the inverse of $f(x)$. Each of these functions reverses the operations of the other function in reverse order. In that sense, the inverse of $f(x)$ will consist of the identical formula with x and y interchanged - the solution for y results in "reversing" all operations on x and thus results in the formula for the inverse function.

We denote the inverse of $f(x)$ as $f^{-1}(x)$ and we denote the inverse of $g(x)$ as $g^{-1}(x)$.

Domain and Range of Functions That Are Inverses of Each Other: Given two functions $f(x)$ and $g(x)$ are inverses of each other, then

The domain of $f(x)$ will consist of the same interval as the range of $g(x)$.

The range of $f(x)$ will consist of the same interval as the domain of $g(x)$.

One-To-One Requirement For $f(x)$ To Have an Inverse Function: Given a function $f(x)$, it will only have an inverse if and only if each y-value in it's range corresponds to only 1 x-value in it's specified domain. When this is the case that each y is obtained from only 1 x-value, we say $f(x)$ is a one-to-one function.

Note that a graphical way to determine that $f(x)$ is not one-to-one is to show that a horizontal line passes through more than 1 point. This is often referred to as the **Horizontal Line Test**.

1. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is:

(a) 1/3	(b) 1/2	(c) 2/3	(d) 5/3
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2. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then, at $x = 0$, $f(x)$ is:

(a) maximized whenever $a > 0, b > 0$	(b) maximized whenever $a > 0, b < 0$
---------------------------------------	---------------------------------------

(c) minimized whenever $a > 0, b > 0$	(d) minimized whenever $a > 0, b < 0$
---------------------------------------	---------------------------------------
3. For the function $f(x) = 2x - 1$, $g(x) = 5 - x$, and $h(x) = x^2 + x + 1$, find range of x for which $\min\{f(x^2), h(x)\} < 3$.

(a) $-2 < x < \sqrt{2}$	(b) $-\sqrt{2} < x < \sqrt{2}$
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(c) $-2 < x < 2$	(d) $-\sqrt{2} < x < 2$
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4. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at:
 (a) $x = 2.3$ (b) $x = 2.5$
 (c) $x = 2.7$ (d) None of these
5. Find the minimum value of $f(x) = |3x - 2| + |2x - 3|$.
 (a) $5/6$ (b) $5/3$ (c) $5/2$ (d) None of these
6. Find the minimum value of $f(x) = \max(k - x, |x| + k)$.
 (a) $k - 1$ (b) k (c) $2k$ (d) None of these
7. Let $f(x) = ax^2 + bx + c$, where a, b and c are certain constants and $a \neq 0$. It is known that $f(5) = -3f(2)$ and that 3 is the root of $f(x) = 0$. What is the other root of $f(x) = 0$?
 (a) -7 (b) -4
 (c) 2 (d) Cannot be determined
8. If $f(x) = x^3 - 4x + p$ and if $f(0)$ and $f(1)$ are of opposite sign, then which of the following is necessarily true?
 (a) $-1 < p < 2$ (b) $0 < p < 3$
 (c) $-2 < p < 1$ (d) $-3 < p < 0$
9. The domain of $y = \frac{1}{\sqrt{|x|-x}}$ is
 (a) $(0, \infty)$ (b) (∞, ∞) (c) $(-\infty, 0)$ (d) $(1, \infty)$
10. If $f(x) = \log(\frac{1+x}{1-x})$, then
 (a) $f(x)$ is even (b) $f(x_1)f(x_2) = f(x_1+x_2)$
 (c) $\frac{f(x_1)}{f(x_2)} = f(x_1 - x_2)$ (d) $f(x)$ is odd
11. What is the minimum and maximum value of $\frac{2x}{x^2+1}$ respectively?
 (a) $-1, 1$ (b) $-2, 1$ (c) $-\frac{1}{3}, 0$ (d) None of these
12. Let $f(x) = \max(2x+1, 3-4x)$, where x is any real number. Then, the minimum possible value of $f(x)$ is:
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$
13. Minimum value of $f(x) = |3 - x| + |2 + x| + |5 - x|$, will be:
 (a) 0 (b) 7 (c) 8 (d) 10
14. A function $f(x)$ is defined as follows:
 (i) $f(1) = 1$
 (ii) $f(2x) = 4f(x) + 6$
 (iii) $f(x+2) = f(x) + 12x + 12$
 then calculate $f(6)$.
 (a) 106 (b) 96 (c) 86 (d) 76
15. Let $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at
 (a) $x = 2.3$ (b) $x = 2.5$
 (c) $x = 2.7$ (d) None of these
16. Find for what value of a is: $f(n) = (a - 2)n + 3a - 4$ an even function?
 (a) -2 (b) 2 (c) 3 (d) 4
17. Let $g(x) = \max(5 - x, x + 2)$. The smallest possible value of $g(x)$ is?
 (a) 4.0 (b) 4.5 (c) 1.5 (d) None of these
18. Find the maximum value of the functions $1/(x^2 - 3x + 2)$?
 (a) $11/4$ (b) $1/4$ (c) 0 (d) None of these
19. Let $g(x)$ be a function such that $g(x+1) + g(x-1) = g(x)$ for every real x . Then, for what value of p is the relation $g(x+p) = g(x)$ necessarily true for every real x ?
 (a) 5 (b) 3 (c) 2 (d) 6
20. A function $f(x)$ satisfies $f(1) = 3600$ and $f(1) + f(2) + \dots + f(n) = n^2f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$?
 (a) 200 (b) 100 (c) 120 (d) 80

SESSION - 13

FUNCTIONS - II

1. If $f(x) = ax^2 + bx + 1$, $f(1) = 4$, $f(-2) = 1$, find $f(x)$.
 (a) $x^2 - 2x + 1$ (b) $x^2 - 3x + 1$
 (c) $x^2 + 2x + 1$ (d) None of these
2. Find the domain of $f(x) = \sqrt{x}$ where f is a real function.
 (a) $(-\infty, \infty)$ (b) $(0, \infty)$
 (c) $(0, -\infty)$ (d) None of these
3. Find the range of $f(x) = \sqrt{16 - x^2}$.
 (a) $(0, 4)$ (b) $[-4, 4]$ (c) $(-4, 0)$ (d) $[0, 4]$
4. Which of the following is an even function?
 (a) $|x| - x$ (b) $x^2 + x^3$ (c) $e^{3x} + e^{-3x}$ (d) $\frac{|x|^2}{3x}$
5. Let $f(x) = e^{2x}$ & $g(x) = \log x$, find $fog(2)$.
 (a) 2 (b) 4 (c) 0 (d) e^4
6. Which of the following two functions have same domain?
 $f(x) = \frac{x^2 + 1}{x}$; $g(x) = |x| + 1$; $h(x) = x^2 + 2x$
 (a) f and g (b) g and h
 (c) f and h (d) None of these
7. Find the domain of the function $y = 5e^{\sqrt{x^2-1}} \log(x-1)$.
 (a) $(-\infty, \infty)$ (b) $R - (-1, 1)$
 (c) $(1, \infty)$ (d) $(-\infty, -1)$

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8. How many onto functions can be defined from the set $A = \{1, 2, 3, 4\}$ to $B = \{p, q, r\}$?
 (a) 81 (b) 36 (c) 64 (d) 45
9. $n(A) = a, n(B) = b$ and $n(C) = c$. We can define a function that is 1 - 1 but not onto from A to B, a function that is onto but not 1 - 1 from B to C, and a function that is 1 - 1 but not onto from C to A. Arrange a, b, c in ascending order.
 (a) $a < b < c$ (b) $b < c < a$
 (c) $c < a < b$ (d) $c < b < a$
10. Find the range of $f(x) = \frac{x^2 + 6x + 6}{x^2 + 6x + 12}$.
 (a) $[3, \infty]$ (b) $[-1, 1]$
 (c) $[1, 1]$ (d) None of these
11. Find $f(f(3))$, if $f(x) = x^3 - 2x^2 + x + 1$.
 (a) 31 (b) 1873 (c) 13 (d) 169
12. Find the domain of $f(x) = \frac{x}{2-x}$
 (a) $R - \{2\}$ (b) R
 (c) $(-2, 2)$ (d) None of these
13. Find the domain of $f(x) = \sqrt{x^2 - 25}$.
 (a) $(-5, 5)$ (b) $[0, 5]$
 (c) $R - (-5, 5)$ (d) None of these
14. Let f be the exponential function and g be the logarithmic function, find $fg(1)$
 (a) 1 (b) ∞
 (c) 0 (d) None of these
15. Find the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 3x}}$.
 (a) $(0, 3)$ (b) $R - [0, 3]$
 (c) R (d) None of these
16. If $f(x) = x + 2$, $g(x) = \frac{1}{x}$ and $h(x) = x^2$ then find $fogoh(3)$.
 (a) $2\frac{1}{9}$ (b) $\frac{9}{19}$
 (c) $\frac{1}{9}$ (d) None of these
17. The domain of the function $f(x) = \frac{|x+3|}{x+3}$ is
 (a) R (b) $R - \{3\}$
 (c) $R - \{-3\}$ (d) $R - (-3, 3)$
18. If f is an even function and g is an odd function, then the function fog is _____.
 (a) an even (b) odd
 (c) neither even nor odd (d) periodic function
19. Which of the following functions from z (set of integers) to z are bijections?
 (a) $f(x) = x + 5$ (b) $f(x) = x^5$
 (c) $f(x) = 3x + 2$ (d) $f(x) = x^2 + x + 1$
20. If $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{1-x}$, then find the domain of $fog(x)$.
 (a) $(-\infty, 3)$ (b) $[3, \infty]$
 (c) $(1, -3)$ (d) None of these

QUANTITATIVE ABILITY – SOLUTION

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$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\text{Put } x + \frac{1}{x} = y$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = y^2 - 2$$

$$\therefore 2(y^2 - 2) - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow (2y - 5)(y - 2) = 0$$

$$\Rightarrow y = \frac{5}{2} \text{ or } 2$$

$$\text{When } y = \frac{5}{2}, \quad x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2 \text{ or } \frac{1}{2}$$

$$\text{When } y = 2, \quad x + \frac{1}{x} = 2 \Rightarrow x = 1$$

$$\text{Hence } x = 2, \frac{1}{2}, 1$$

\therefore The number of real values = 3

37. Ans: [a]

α, β are the roots of $x^2 - x + p = 0$

$$\Rightarrow \alpha + \beta = 1, \alpha\beta = p$$

γ, δ are the roots of $x^2 - 4x + q = 0$

$$\Rightarrow \gamma + \delta = 4, \gamma\delta = q$$

Let r be the common ratio of the G.P..

$$\text{Then } \frac{\gamma + \delta}{\alpha + \beta} = 4 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$$

When r = 2

$$\text{We get } \alpha(1+r) = 1 \Rightarrow \alpha = \frac{1}{1+r} = \frac{1}{3}$$

$p = \alpha\beta = \alpha \times ar = \alpha^2r = \frac{2}{9}$ which is not an integer

When r = -2,

$$\alpha(1+r) = 1 \Rightarrow \alpha = -1$$

$$p = \alpha^2r = -2$$

$$q = r\delta = (\alpha r^2)(\alpha r^3) = \alpha^2 r^5$$

$$= (-2)^5 = -32$$

\therefore The values of p and q are -2, -32.

38. Ans: [a]

$$ax^2 + bx + c = 0$$

Let the roots be α and 3α .

$$\therefore 3\alpha + \alpha = -\frac{b}{a} \Rightarrow 4\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{4a}$$

$$3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a} \Rightarrow \frac{3b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow \frac{3b^2}{16a} = c$$

$$\Rightarrow 3b^2 = 16ac$$

39. Ans: [b]

$$(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$$

Since the roots are equal, the discriminant is equal to 0.

$$4b^2(a+c)^2 - 4(a^2+b^2)(b^2+c^2) = 0$$

$$\Rightarrow b^2a^2 + 2b^2ac + b^2c^2 - a^2b^2 - b^4 - a^2c^2 - b^2c^2 = 0$$

$$\Rightarrow 2b^2ac - b^4 - a^2c^2 = 0$$

$$\Rightarrow b^4 + a^2c^2 - 2b^2ac = 0$$

$$\Rightarrow (b^2 - ac)^2 = 0 \Rightarrow b^2 = ac$$

$\Rightarrow a, b, c$ are in G.P.

40. Ans: [c]

$$x^2 - (c+6)x + 2(2c-1) = 0$$

Sum of the roots = $c + 6$

Product of the roots = $2(2c-1)$

$$c + 6 = \frac{1}{2} \times 2(2c-1)$$

$$\Rightarrow c + 6 = 2c - 1$$

$$\Rightarrow c = 7$$

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FUNCTIONS - I

1. Ans: [d]

$F(x) = \max(2x+1, 3-4x)$ is minimum when $2x+1 = 3-4x$

$$\text{i.e. } 6x = 2$$

$$x = 2/6 = 1/3$$

therefore minimum possible value of $f(x)$ is

$$(2x+1)(\text{at } x = 1/3) = 2 * 1/3 + 1 = 5/3$$

$$\text{or } (3-4x) \text{ at } x = 1/3 = 3 - 4 * 1/3 = 5/3$$

2. Ans: [d]

$$F(x) = ax^2 - b|x|$$

$ax^2 > 0$ for $a > 0$ and

$(-b|x|) > 0$ for $b < 0$

$$F(x) = ax^2 - b|x| > 0 \text{ for } x \neq 0$$

$$F(0) = ax^2 - b|x| = 0 \text{ for } x = 0$$

If $x = 0$ $f(x)$ is minimised whenever, $a > 0, b < 0$.

3. Ans: [a]

$$\min \{f(x), h(x)\} < 3$$

$$f(x) < 3 \text{ or } h(x) < 3$$

$$2x^2 - 1 < 3 \text{ or } x^2 + x + 1 < 3$$

QUANTITATIVE ABILITY – SOLUTION

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- $x^2 - 2 < 0$ or $x^2 + x - 2 < 0$
 $-\sqrt{2} < x < \sqrt{2}$ or $(x+2)(x-1) < 0$
 $-\sqrt{2} < x < \sqrt{2}$ or $-2 < x < 1$
- Both the above ranges for x , satisfy the inequality Min $\{f(x^2), h(x)\} < 3$
 $-2 < x < \sqrt{2}$ (as $-2 < -\sqrt{2}$ and $\sqrt{2} > 1$)
4. Ans: [b]
 $F(x) = |x - 2| + |2.5 - x| + |3.6 - x|$
 Sub. from the options, the values of x in the function, when $x = 2.3$
 $F(x) = 0.3 + 0.2 + 1.3 = 1.8$
 when $x = 2.5$
 $F(x) = 0.5 + 0 + 1.1 = 1.6$
 When $x = 2.7$
 $F(x) = 0.7 + 0.2 + 0.9 = 1.8$
 Thus for any value of x , $f(x)$ will be greater than 1.6
 $\Rightarrow f(x)$ is minimum at $x = 2.5$.
5. Ans: [b]
 $F(x) = |3x - 2| + |2x - 3|$
 $= 3|x - 2/3| + 2|x - 3/2|$
 For $x \geq 3/2$, $f(x) = 3x - 2 + 2x - 3$
 $= 5(x - 1)$
 So minimum value = $5(3/2 - 1) = 5/2$
 For $3/2 \geq x \geq 2/3$,
 $F(x) = 3x - 2 + 3 - 2x = x + 1$
 So minimum value = $2/3 + 1 = 5/3$
 For $x \leq 2/3$, $f(x) = 2 - 3x + 3 - 2x$
 $= 5(1 - x)$
 So, minimum value = $5(1 - 2/3) = 5/3$
 So, minimum value of $f(x)$ is $5/3$.
6. Ans: [b]
 Since $|x| \geq -x$ for any value of x ,
 $k - x \leq |x| + k$ for any value of x
 $f(x) = |x| + k$ for any $x \geq k$ as $|x| \geq 0$ and $f(0) = k$
 So, minimum value of $f(x)$ is k .
7. Ans: [b]
 Given $f(x) = ax^2 + bx + c$ ($a \neq 0$).
 3 is a root of $f(x)$
 $9a + 3b + c = 0$... (1)
 Also, $f(5) = -3f(2)$.
 $25a + 5b + c = -3(4a + 2b + c)$
 $= -12a - 6b - 3c$
 $37a + 11b + 4c = 0$... (2)
 From two equations $a - b = 0$ $a = b$
 Thus we get $f(x) = ax^2 + ax + c$
 Dividing $f(x)$ by $x - 3$, we get $c = -12a$
 $F(x) = ax^2 + ax - 12a$
 $F(x) = 0, -4$ is the second root.
8. Ans: [b]
 $F(x) = x^3 - 4x + p$
 $F(0) = +p$ and
 $F(1) = 1 - 4 + p = -3 + p$
 $F(0)$ and $f(1)$ are of opposite signs.
 If p is positive, $(p - 3)$ has to be negative and p has to take values less than 3 i.e. $0 < p < 3$.
9. Ans: [c]
 For D_f , $|x| - x > 0$, $|x| > x$ i.e., $x < |x|$ which is true if $x < 0$.
 $D_f = (-\infty, 0)$.
10. Ans: [d]
 Since $f(-x) = \log \frac{1-x}{1+x} = \log \left(\frac{1+x}{1-x} \right)^{-1} = -\log \frac{1+x}{1-x} = -f(x)$
 $\therefore f(x)$ is odd.
11. Ans: [a]
 Let $y = \frac{2x}{x^2+1} \implies x^2y - 2x + y = 0$
 Since x is real, discriminant $4 - 4y^2 \geq 0$
 $1 - y^2 \geq 0$
 $y^2 \leq 1$
 $|y| \leq 1$
 $-1 \leq y \leq 1$.
12. Ans: [d]
 As $f(x) = \max(2x + 1, 3 - 4x)$
 We know that $f(x)$ would be minimum at the point of intersection of these curves.
 i.e., $2x + 1 = 3 - 4x$
 i.e., $6x = 2 \implies x = \frac{1}{3}$
 Hence, minimum value of $f(x)$ is $\frac{5}{3}$.
13. Ans: [b]
 Minimum possible value of any expression inside mod is zero. So we will check for $x = 3, -2$ and $x = 5$. At $x = 3$ we will get minimum value, which is 7.
14. Ans: [a]
 Using property (iii) with $x = 1$,
 $f(3) = f(1) + 12(1) + 12 = 1 + 12 + 12 = 25$
 since $f(1) = 1$ by property (i).
 Using property (ii) with $x = 3$,
 $f(6) = 4f(3) + 6 = 4(25) + 6 = 106$
 Therefore, the value of $f(6)$ is 106.
15. Ans: [b]
 $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$ attains minimum value when any of the terms = 0.
16. Ans: [b]

QUANTITATIVE ABILITY – SOLUTION

If $a = 2$, the function is constant.

17. Ans: [d]

$$g(x) = \max(5 - x, x + 2)$$

We have to draw graph and find the point of intersection.

$$y = 5 - x$$

$$y = x + 2$$

Hence at the point of intersection of two straight lines.

Smallest of $g(x) = 3.5$

18. Ans: [d]

The denominator $x^2 - 3x + 2$ has real roots. Hence the maximum value of the function $f(x)$ will be infinity.

19. Ans: [d]

$$g(x+1) + g(x-1) = g(x)$$

$$g(x+2) + g(x) = g(x+1)$$

Adding these two equations, we get

$$g(x+2) + g(x-1) = 0$$

$$g(x+3) + g(x) = 0 \dots (1)$$

$$g(x+4) + g(x+1) = 0$$

$$g(x+5) + g(x+2) = 0$$

$$g(x+6) + g(x+3) = 0$$

$$g(x+6) - g(x) = 0 \text{ (From (1))}$$

20. Ans: [d]

$$\text{Given function} = f(1) + f(2) + f(3) + f(4) + \dots = n^2 f(n)$$

$$\text{Given } f(1) = 3600$$

For $n = 2$,

$$f(1) + f(2) = 2^2 f(2)$$

$$\text{i.e. } 2^2 f(2) - f(2) = f(1)$$

$$f(2) = f(1)/(2^2 - 1) \dots (1)$$

For $n = 3$

$$f(1) + f(2) + f(3) = 3^2 f(3)$$

put the value of $f(2)$ from (1)

$$\rightarrow f(1) + f(1)/(2^2 - 1) = 3^2 f(3) - f(3)$$

$$\rightarrow f(1) + f(1)/(2^2 - 1) = (3^2 - 1)f(3)$$

now take $f(1)$ in left side

$$\text{i.e. } f(1) = [1 + 1/(2^2 - 1)] = f(3)(3^2 - 1)$$

$$\text{i.e. } f(3) = f(1) \times 2^2 / (2^2 - 1) \times 1 / (3^2 - 1)$$

$$f(3) = 600$$

Similarly

$$f(9) = f(1) \times (2^2 \times 3^2 \times 4^2 \dots 8^2) / ((2^2 - 1)(3^2 - 1)$$

$$(4^2 - 1) \dots (9^2 - 1)$$

$$f(9) = 80$$

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FUNCTIONS - II

1. Ans: [c]

$$f(1) = a + b + 1 = 4$$

$$\therefore a + b = 3$$

$$\dots (1)$$

$$f(-2) = 4a - 2b + 1 = 1$$

$$\therefore 4a - 2b = 0 \quad \dots (2)$$

On solving (1) & (2), $a = 1$ & $b = 2$

So, the required equation is $x^2 + 2x + 1$.

2. Ans: [b]

Square root of a negative number is not a real number.

So its domain = $(0, \infty)$

Note: $\sqrt{0} = 0$ and so it is included.

3. Ans: [d]

$$\text{Let } f(x) = y$$

$$\Rightarrow \sqrt{16 - x^2} = y, \text{ or } x = \pm \sqrt{16 - y^2}$$

x is defined, when $16 - y^2 \geq 0$

$$\Rightarrow (4 + y)(4 - y) \geq 0$$

$$\text{or } (y + 4)(y - 4) \leq 0$$

$$\Rightarrow y \in [-4, 4]$$

But $f(x)$ gives only non-negative values.

$$\text{so } y \in [0, 4]$$

4. Ans: [c]

$$\text{For even function, } f(x) = f(-x)$$

Option (c) satisfies this condition.

$$f(-x) = e^{-3x} + e^{3x} = f(x)$$

5. Ans: [b]

$$\log(2) = f(\log 2) = e^{2\log 2}$$

$$= e^{\log 4} = 4$$

6. Ans: [b]

Domain of $f(x) = R - \{0\}$ (As at $x = 0$, $f(x)$ is not defined)

Domain of $g(x)$ is defined throughout R .

Domain of $h(x)$ is defined throughout R .

$g(x)$ and $h(x)$ have the same domain.

7. Ans: [c]

$$y = 5e^{\sqrt{x^2 - 1}} \log(x - 1)$$

Since, $x^2 - 1 \geq 0$ and $x - 1 > 0$

$$\Rightarrow (x+1)(x-1) \geq 0 \text{ and } x > 1$$

$$\Rightarrow x \in R - (-1, 1) \text{ and } x > 1$$

\therefore domain is $(1, \infty)$

8. Ans: [b]

Total number of functions possible = $3^4 = 81$

A's each element can be mapped to any of the 3 elements in B.

QUANTITATIVE ABILITY – SOLUTION

HSEM1BTECHSTANDARD0719

To find number of into functions,

Case i: Let A being mapped to any two elements of B.

So, number of into functions = $2^4 - 2 = 14$

(Note: Each element of A can be mapped to 2 elements of B and 2 is subtracted to eliminate 2 function which are mapped to single element of B)

So, total number of into functions with element of A mapped to 2 elements of B = $(2^4 - 2) \times 3 = 14 \times 3 = 42$

Case ii: All elements of A mapped to one element of B.

Number of such into functions = 3

So, total number of onto functions = $81 - 42 - 3 = 36$

9. Ans: [c]

Since $A \rightarrow B$ is 1 - 1, not onto, $\Rightarrow a < b$

Since $B \rightarrow C$ is onto, not 1 - 1, $\Rightarrow b > c$

Since $C \rightarrow A$ is 1 - 1, not onto, $\Rightarrow c < a$

So, $c < a < b$

10. Ans: [b]

$$\frac{x^2 + 6x + 6}{x^2 + 6x + 12} = \frac{x^2 + 6x + 12 - 6}{x^2 + 6x + 12}$$

$$= 1 - \frac{6}{x^2 + 6x + 12}$$

Range of $x^2 + 6x + 12$

$$= x^2 + 6x + 9 + 3 = (x + 3)^2 + 3$$

Range of $(x + 3)^2 + 3 = [3, \infty]$

$$\text{Range of } \frac{6}{(x + 3)^2 + 3} = [0, 2]$$

$$\begin{aligned} \text{Range of } f(x) &= 1 - [0, 2] \\ &= [-1, 1] \end{aligned}$$

11. Ans: [b]

$$f(3) = 27 - 18 + 3 + 1 = 13$$

$$f(13) = 13^3 - 2 \times 13^2 + 13 + 1 = 1873$$

12. Ans: [a]

$f(x)$ is not defined for $x = 2$.

So domain of $f(x) = R - \{2\}$

13. Ans: [c]

$f(x)$ is not defined for $x^2 - 25 < 0$

$$\Rightarrow (x+5)(x-5) < 0 \text{ (not defined)}$$

$$\Rightarrow (x+5)(x-5) \geq 0 \text{ (defined)}$$

$\Rightarrow x \leq -5 \text{ & } x \geq 5$

\Rightarrow Defined in $R - (-5, 5)$

14. Ans: [c]

$$(fg)x = f(x) \cdot g(x) = e^x \cdot \log x$$

$$\therefore (fg)(1) = e^1 \cdot \log 1 = 0$$

15. Ans: [b]

$f(x)$ is defined only when $x^2 - 3x > 0$ and $x^2 - 3x \neq 0$

or $x^2 - 3x > 0$, $x(x-3) > 0$ and $x \neq 0$ or 3

$$\Rightarrow x \notin [0, 3]$$

$$\Rightarrow x \in R - [0, 3]$$

16. Ans: [a]

$$h(3) = 9$$

$$goh(3) = g(9) = \frac{1}{9}$$

$$\text{fogoh}(3) = f\left(\frac{1}{9}\right) = \frac{1}{9} + 2 = \frac{19}{9} = 2\frac{1}{9}$$

17. Ans: [c]

$$f(x) = \frac{|x+3|}{x+3}$$

Since, $x+3 \neq 0$, domain of $f(x) = R - \{-3\}$

18. Ans: [a]

$$\text{fog}(-x) = f(-g(x)) = \text{fog}(x)$$

so fog is an even function.

19. Ans: [a]

$b - f(x) = x^5$ is not onto function as $3 \in z$ does not have a preimage in z .

$c - f(x) = 3x + 2$ is not onto function as numbers of the form $3x + 1, 3x$ do not have pre image in z .

$d - f(x) = x^2 + x + 1$ is not a 1 - 1 function as $f(-4) = f(3) = 13$ but $-4 \neq 3$

$a - f(x) = x + 5$ is a 1 - 1 and on to function.

20. Ans: [d]

$$\text{fog}(x) = f(\sqrt{1-x}) = \sqrt{3 - \sqrt{1-x}}$$

For fog to be defined, $1-x \geq 0$ and $3 - \sqrt{1-x} \geq 0$

$$1 \geq x \text{ and } x \geq -8 \Rightarrow -8 \leq x \leq 1$$

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9. Grandpa claims that Martian measles _____ (causes/cause) green and purple spots to erupt all over a person's body.
10. The dog, digging holes in the sofa cushions _____ (is/are) Peanut, Elizabeth's new puppy.
11. Even though the jury _____ (want/wants) to believe that the defendant did not feed Elvis to the Loch Ness Monster, much of the evidence points to the guilt.
12. Neither the students nor their instructor _____ (is/are) happy with the long cafeteria line for squid eyeball stew.
13. Neither of Freud's parents _____ (has/have) much intelligence when it comes to choosing spouses. Freud's father has married five times, and Mom just presented him with stepfather number three.
14. Each of these women _____ (wish/wishes) that she had read the care instructions before washing the delicate and expensive dresses.
15. Who needs a pet if you live in Florida? There already _____ (is/are) lizards that sneak in between the window screens and cockroaches that take up residence in every bathroom and kitchen.
16. Susan is one of those students who _____ (kiss/kisses) up to the instructor every chance that they get.
17. There _____ (is/are) more calories in a bowl of chocolate-broccoli breakfast cereal than you might think.

SESSION – 5

❖ PUNCTUATIONS

To fully understand basic grammar rules, you also need to look at punctuation rules.

- **Capitalization** is important. All sentences must start with a capital, or upper-case, letter. Titles of people, books, magazines, movies, specific places, etc. are capitalized. Organizations and compass points are capitalized.
- Every sentence needs a **punctuation mark** at the end of it. These include a period, exclamation mark, or question mark.
- **Colons** are used to separate a sentence from a list of items, to introduce a long, direct quote, or between two sentences (or clauses) when the second one explains the first.
- **Semicolons** can take the place of a conjunction and are placed before introductory words like "therefore" or "however." They are also used to separate a list of things if there are commas within each unit.
- There are a lot of rules for **commas**. The basic ones are that commas separate things in a series and go wherever there is a pause in the sentence. They surround the name of a person being addressed, separate the day of the month from the year in a date, and separate a town from the state.
- **Parentheses** enclose things that clarify or numbers and letters that are part of a list.
- **Apostrophes** are used in contractions to take the place of one or more letters and also to show possession. An apostrophe and "s" is added if the noun is singular and an apostrophe alone is added if the noun is plural.

VERBAL ABILITY

EXERCISE 8

Directions for Q1 to Q10: Choose the correct punctuated sentence.

1. (a) Michael Suza Emperor of Ethiopia visited Ghana in 2004
 (b) Michael Suza Emperor of ethiopia visited Ghana in 2004.
 (c) Michael Suza , Emperor of Ethiopia visited Ghana in 2004.
 (d) Michael Suza Emperor of Ethiopia visited Ghana in 2004.

Answer: [c]

2. (a) although he studied very hard for the examination he did not pass
 (b) Although, he studied very hard for the examination, he did not pass.
 (c) Although he studied very hard for the examination, he did not pass.
 (d) Although he studied very hard for the examination....he did not pass.

Answer: [c]

3. (a) i do not believe you chewe the teacher said when you say this is your own work.
 (b) I do not believe you. Chewe the teacher said. When you say this is your own work.
 (c) "I do not believe you ,Chewe," the teacher said, " when you say this is your own work."
 (d) " I do not believe you Chewe the teacher said when you say this is your own work."

Answer: [c]

4. (a) For breakfast, we had cereal, toast, peanut butter and jelly.
 (b) For breakfast we had Cereal Toast Peanut Butter and Jelly.
 (c) for breakfast we had cereal. toast. peanut butter and jelly.
 (d) For breakfast we had Cereal....toast....peanut butter and jelly....

Answer: [a]

5. (a) we moved from san juan puerto rico on december 1 2004
 (b) we moved from san juan. puerto rico, on december 1 2004,
 (c) We moved from San Juan, Puerto Rico on December 1, 2004.
 (d) We moved, from San Juan Puerto Rico on December 1..2004.

Answer: [c]

6. (a) i cant hold on much longer screamed akiko
 (b) I can't hold on much longer, screamed Akiko.
 (c) I can't hold on much longer. Screamed Akiko.
 (d) "I can't hold on much longer," screamed Akiko.

Answer: [d]

7. (a) the woman screamed to her son stop pulling the dogs tail
 (b) The woman screamed to her, " Son stop pulling the dog's tail!"
 (c) The woman screamed to her son, "Stop pulling the dogs tail."
 (d) The woman screamed to her son. stop pulling the dogs tail.

Answer: [c]

8. (a) we were sorry to see that the sign on the door said closed
 (b) We are sorry to see that the sign on the door said closed.
 (c) We are sorry to see, that the sign on the door, said "closed."
 (d) We are sorry, to see that the sign on the door, said "Closed."

Answer: [c]

9. (a) when she got there she asked the cashier would you accept a cheque
 (b) When she got there, she asked the cashier, "Would you accept a cheque?"
 (c) when she got there she asked the cashier. would you accept a cheque?
 (d) When she got there she asked the cashier, would you accept a cheque?

Answer: [b]

10. (a) ann joanne and alison were playing in anns garden
 (b) Ann, Joanne and Alison were playing in anns garden.
 (c) Ann, Joanne and Alison were playing in Anns garden.
 (d) Ann, Joanne and Alison were playing in Ann's garden.

Answer: [d]

Exercise 9

Fill in the blanks.

1. We decided _____ (**run**) through the forest.
2. I learned _____ (**ride**) the bike at the age of 5.
3. She doesn't mind _____ (**work**) the night shift.
4. The man asked me how _____ (**get**) to the airport.
5. I can't imagine Peter _____ (**go**) by bike.
6. Are you thinking of _____ (**visit**) London?
7. He agreed _____ (**buy**) a new car.
8. The teacher expected Sarah _____ (**study**) hard.
9. I look forward to _____ (**see**) you at the weekend.
10. The question is easy _____ (**answer**).

Exercise 10

Fill in the blanks.

1. A wedding involves _____ (**negotiate**) with everyone in the family.
2. He denies _____ (**steal**) the money.
3. He claims _____ (**be**) a millionaire but I don't believe him.
4. I expect _____ (**be**) there about seven.
5. Julia reported _____ (**see**) the boys to the police.

6. The teenager refused _____ (go) on holiday with his parents.
7. I understand _____ (be) late once or twice, but every day is too much!
8. That criminal deserves _____ (get) a long sentence.
9. We arranged _____ (meet) at four but at four thirty she still hadn't arrived.
10. She mentioned _____ (go) to the cinema, but I don't know what she decided to do in the end.

Exercise 9

1. We decided **to run** through the forest.
2. I learned **to ride** the bike at the age of 5.
3. She doesn't mind **working** the night shift.
4. The man asked me how **to get** to the airport.
5. I can't imagine Peter **going** by bike.
6. Are you thinking of **visiting** London?
7. He agreed **to buy** a new car.
8. The teacher expected Sarah **to study** hard.
9. I look forward to **seeing** you at the weekend.
10. The question is easy **to answer**.

Exercise 10

1. A wedding involves **negotiating** with everyone in the family.
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8. That criminal deserves **to get** a long sentence.
9. We arranged **to meet** at four but at four thirty she still hadn't arrived.
10. She mentioned **going** to the cinema, but I don't know what she decided to do in the end.

8. The bed _____ made as soon as I got up.
9. Sarah _____ ski or roller skate.
10. _____ Matthew bring coffee?

Exercise 5

- | | | | |
|----------|-------------|------------|------------|
| 1. were, | 2. has, | 3. did, | 4. didn't, |
| 5. is, | 6. doesn't, | 7. didn't, | 8. don't, |
| 9. were, | 10. hasn't | | |

Exercise 6

1. Jessica **is** taking John to the airport.
2. If he **doesn't** arrive on time, he'll have to take a later flight.
3. Unfortunately, our dinner **has been** eaten by the dog.
4. I **have** purchased a new pair of shoes to replace the ones that were lost in my luggage.
5. We hope you **don't** have an accident on your way to school.
6. She **was** baking a pie for dessert.
7. Dad **has been** working hard all day.
8. The bed **was** made as soon as I got up.
9. Sarah **doesn't** ski or roller skate.
10. **Did** Matthew bring coffee?

SESSION – 4**☺ SUBJECT-VERB AGREEMENT**

Subject verb agreement is where we agree the verb with the subject.

A sentence is made up of 2 parts, subject and verb.

Subject that tells what the sentence is about. It can be either a noun or a pronoun. It can be either singular or plural.

Verb represents the action of a sentence.

How to make the verb agree with the subject?

- 1) Identify the subject of the sentence.
- 2) Decide if the subject is singular or plural.
- 3) Lastly, decide which verb form will match with the subject.

Subject Verb Agreement



You are my best friend!



Subject Verb Disagreement



You is so mean!



RULES:

1. Verb-number agreement

Singular subjects take singular verbs and plural subjects take plural verbs.

- The **list** of to-dos **was** too long for me to handle.

The **lists** of to-dos **were** too long for me to handle.

- Even **an animal** has **its** own territory.

Even **animals** have **their** own territory.

EXCEPTION to this rule:

A plural verb is always required after 'YOU' even when it is used in singular, referring to one person. For e.g.:

- You **were** going to send over the document.

2. Rule for when verb has 's' at the end

The 's' is added after a noun indicates plural. But **an 's' after a verb indicates THIRD PERSON SINGULAR** i.e. the sentence is in third person and the subject is singular. For e.g.:

- She **goes** to the library every single day.
- They **go** to the restaurant every day for their favourite dish.

3. The verb has to agree only with the true, main subject

Not with the intervening plural object of a preposition or any other intervening plural.

- The **box** of Nestlé's chocolates **is** missing.

Here, the main subject is box, not chocolates. Hence, we use 'is' instead of 'are'.

More examples:

- His **experience** as a teacher to young kids **gives** him a lot of understanding.
- The **prices** of the new iPhone **vary** from country to country.

4. Subjects joined by 'AND' are usually plural and take plural verbs.

- His laptop **and** my ipad **were** stolen from the desk.
- Chennai **and** Kolkata **have** very hot weather.

VERBAL ABILITY

EXCEPTIONS to this rule:

- If the subject has **two singular nouns connected by AND**; and both are about the **same person / thing**, then the **verb remains singular**.
 - My best friend **and** roommate **is** going to US for a vacation.
 - Soup **and** bread **is** our Sunday breakfast.
- When **two subjects connected by AND** are preceded by '**each, every or many**', a **singular verb** is used.
 - **Every** chair, table and sofa, every single piece of furniture in the house **is** up for auction.
 - **Every** man and woman in the store **is** requested to go through the security check.

5. Rule for 'with, together with, along with, besides, as well as, including, in addition to'

Words like 'with, together with, along with, besides, as well as, including, in addition to, etc. do not affect the number of the verb. If the main subject is singular, the verb has to be singular; if the subject is plural, the verb has to be plural.

- The television, **along with** the cabinet, **is to be sold**.
- Our chief competitor, **as well as** ourselves, **is obliged** to increase the prices.
- The decoration of the room, **including** all the paintings on the walls, **is** most pleasing.

6. Rule when both singular and plural subjects are present.

If the subject is made up of both singular and plural words connected by or, nor, either – or, neither – nor, not only, but also then the verb agrees with the nearer part of the subject.

- Neither the quality nor the **prices have** changed.
- Neither the prices nor the **quality has** changed.
- Neither the salesman nor **the buyers are** in favour of the system of management.
- Neither the buyers nor the **salesman is in favour** if the system of the management.

7. Rule for neither-nor, either-or & or

If the subject consists of two singular words connected by 'or, neither- nor, either – or', the subject is singular and requires a singular verb.

- **Neither** the laptop **nor** the phone **was** in working order.
- **Either** January **or** February **is going to be** her wedding month.

8. Rule for nouns that are plural in form but singular in meaning

Nouns that are plural in form but singular in meaning such as news, measles, mumps, physics, electronics, tactics, economics and so on usually take singular verbs.

- **News is** traveling faster than ever before.
- **Physics has** fascinated my hostel mate for months.

Some nouns ending in '-ics' such as **athletics, statistic and politics** are considered singular when referring to an organized body of knowledge and plural when referring to individual facts, qualities or activities.

- **Athletics provide** good recreation. (i.e. various games)
- **Economics is** an important subject for every field of study.

9. A linking verb usually agrees with its subject, not with its compliment.

- Excessive **absences were** the reason for his failure.
- The reason of his **failure was** excessive absences.

10. Rule for nouns that do not have singular forms

Plural verbs are required for many nouns that have no singular form, such as proceeds, goods, ashes, remains, credentials, premises etc.

- The **proceeds of** the magic show **are** to be given to the fund for soldier's welfare.
- The **goods are** being dispatched today by goods train.

SUBJECT VERB AGREEMENT FOR COLLECTIVE NOUNS

What are collective nouns?

A collective noun is a word that represents a group of persons, animals or things. For e.g. audience, committee, company, council, army, police, society, board, department, cabinet etc. the following rules govern the form of verb to be used with a collective noun.

1. When a group acts as a unit, the verb should be singular.

- The **committee has** agreed to submit its report on Friday.
- The **board of directors meets** once in a month.
- The **firm is one of the most** reputed in the country.
- The **majority has** made its decision.

2. When the members of the group are thought of as acting separately, the verb should be plural.

- The **teams are arguing** over who should be the captain (individual members in the team are arguing).
 - The **committee were** not in agreement on the action to be taken.
- The **audience were** cheering and laughing, even crying.

3. Company names may be either singular or plural, according to their meaning.

The plural form emphasizes the individual personnel making up the company.

- **Mudra and corporation have** retained the goodwill of their customers.
- The **oil corporation is** located at Nariman Point, Mumbai.

VERBAL ABILITY

4. Rule for nouns expressing time, money or quantity

When nouns expressing periods of time, amounts of money or quantities are considered as a singular unit, singular verbs are used. For e.g.

- **Rs.10** seems too much for the job.
- **3 months** is too long a time to wait.
- The **number of board members** is very small.
- **That Rs.1 lakh was an** inheritance from my father.
- Yes, **5m is ample** for a suit.

5. After such expressions as 'one half of', 'two-thirds of', 'a part of', 'a majority of'

- Use a singular verb if a singular noun follows the 'of'.
 - **A part of** the office **is** closed.
 - **Two-third of** the mailing list **has been** typed.
 - A **majority of** 3500 **indicates his** popularity in the constituency.
- Use a plural verb when a plural noun follows the 'of'.
 - Part **of the walls** **are** to be painted.
 - **Two thirds of** our workers live in the suburbs.
 - The **majority of our staff members** live in villages.

6. Rules for 'The number'

The expression '**the number**' has a singular meaning and requires a singular verb, whereas the expression '**a number**' has a plural meaning and takes a plural verb.

- **The number** of board members **is** very small.
- A **number** of board members **were** absent.
- **The number** of orders **is** still to be executed **is** estimated at nearly a 100.
- A **number** of our staff **are** going on leave.

7. In sentences containing the words 'one of', the verb is chosen as follows:

- In simple form, one of or one of the, a singular verb is used.
 - **One of the** reasons for his demotion **is** his carelessness.
 - **One of the pens** **is** missing from my desk.

The sentences containing phrases 'one of those who' or 'one of the things that', a plural verb is required.

- **He is one of those** managers who **favour** increasing the staff.

Here favour agrees with those. In the phrase one of those who, those is the plural object of the preposition of. In the subordinate clause who favour, the relative pronoun who is the subject and must agree with its antecedent those.

- Mr. Verma is **one of our officers** who is accompanying me.
- He is **one of our employees** who are always alert.

However, when only precedes one of / one of those, a singular verb is used.

- Ramesh is **only one of our employees** who is always alert.
- Mr Verma is **the only one of our officers** accompanying me.

8. Rule of singular nouns and plural verbs

Certain collective nouns, those who are singular in form, are always used in the plural sense and take a plural verb. For e.g. gentry, cattle, poultry, alphabets, offspring etc.

- These **poultry** are ready for sale.
- There are **26 alphabets** in English.
- The **cattle** are grazing near the canal.

9. The always singular nouns

Certain nouns are always used in singular and followed by singular verbs. These are not used in the plural sense and do not take on plural verbs. For e.g. **hair, issue, advice, information, scenery, luggage, mischief, bread, abuse, furniture, land, business, machinery, poultry** etc. (Detailed discussion right after examples)

- Her **hair** has turned grey now.
- The **scenery of Kasauli** is beautiful.
- **Is there any information** in this regard?
- **All the machinery** is old.
- I have sold **all the furniture** that was useless.
- My **luggage** is lying at the bus stand.

COUNTABLE Vs. UNCOUNTABLE

Countable Nouns: These are the names of objects, people etc. that we can count, e.g. book, pen, apple, boy, sister, doctor, horse.

Uncountable nouns: These are the names of things, which we cannot count, e.g., milk, oil, sugar, gold, and honesty. They mainly denote substances and abstract things.

E.g. Nature (uncountable)

NOTE:

- **Countable nouns** have plural forms and can be used with **a/an**.
- Uncountable nouns** do not have plural forms and cannot be used with **a/an**.

For e.g. we say books but we do not say "milks".

- The following nouns are usually uncountable in English: **Advice, news, information, furniture, luggage work, business, weather, traffic, scenery, paper, and bread.** Most of these are **countable in Indian languages** and therefore Indian students often wrongly use them with 'a/an' and in the plural.

Example:

- He gave me **an advice**. (Incorrect) (ek se kya hogा??)
- He gave me **some advice** (correct) (or, a piece of advice)
- The **sceneries** here **are** very good. (Incorrect)
- The **scenery** here **is** very good. (Correct)

10. Each, every, either, neither

The words 'each, every, either, neither', used as pronouns or else adjectives, are always singular and require singular verbs.

- **Each of them does have** political ambitions.
- **Each employee is** responsible for clearing is desk in the evening.
- **Neither of the boys is** eligible for taking the examination.
- **Neither boy is** eligible for selection.

EXCEPTION: If a parenthetical each follows a plural noun or pronoun, the verb should be singular.

- The **members each feel their** responsibility.
- They **each have their own** problem.
- **10 each of these books** is required.

11. All, any, many

All, any, more, most, some – may be singular or plural depending on the meaning, and take verbs accordingly.

- **Some of the books** seem too old.
- **Some of the food is** not good. (food is a singular noun)
- **All the typing has** been finished. (typing is an activity. Can't be plural)
- All the reports **have** been typed.
- **Most of the goods have** been sold.
- **Most of the stock has** been sold, but more of these shirts are due. (stock is always singular)

12. The titles of books or magazines are considered singular and take singular verbs.

- The **Hindustan times** still **has** wide circulation.
- The **Shiva Trilogy is** a best seller.

VERBAL ABILITY

13. The always singular words which take singular verbs

The following words and their compounds are always singular and requires a singular verb.

Body (anybody, everybody, nobody, somebody)

Thing (anything, everything, nothing, something)

One (anyone, everyone, no one, someone)

- **Something** is wrong **with him** these days.
- **Everybody** in the office **has** tickets.
- **Everyone** is required to clear their dues.
- **Nobody knows** the trouble I have seen.
- **No one is** entitled to have his debts cancelled.

14. Rules for relative pronouns

A relative pronoun is one which establishes a relationship between two subjects (who, which, that). When it is used as a subject, it takes a singular or plural verb to accord with its antecedent i.e. if the subject is singular, use a singular verb and so on.

- Measles is among the **diseases that are curable**.
- This is only one of the **local papers that print** a weekly horoscope.

EXERCISE 7

Fill in the blanks.

1. Every pale tomato slice, wilted pickle, and brown lettuce leaf _____ (cost/costs) an extra 25 cents at Bernie's Burger Emporium.
2. Not only the Smiths but also Tonya _____ (has/have) agreed to try one of the world-famous chocolate-broccoli muffins.
3. The Smiths, along with Tonya, _____ (hope/hopes) to avoid indigestion after eating these weird muffins.
4. On the sidewalk _____ (is/are) many little lizards sunning themselves on the hot concrete.
5. Even though Antonio has many friends who love their Chevrolets and Buicks, he has always believed that General Motors _____ (makes/make) lemons.
6. My dog Floyd, together with Buster the cat, _____ (likes/like) to play with money; the cat swats crumpled bills onto the floor where the dog shreds them to pieces.
7. Even though Johnson and Johnson _____ (warm/warms) consumers not to insert Q-tips into their ears, people refuse to read directions and frequently puncture their eardrums.
8. All of my important keys _____ (is/are) now stuck in the drain pipe of my bathroom sink. Buster, my kitten, doesn't realize how much his playfulness inconveniences me.

9. Grandpa claims that Martian measles _____ (causes/cause) green and purple spots to erupt all over a person's body.
10. The dog, digging holes in the sofa cushions _____ (is/are) Peanut, Elizabeth's new puppy.
11. Even though the jury _____ (want/wants) to believe that the defendant did not feed Elvis to the Loch Ness Monster, much of the evidence points to the guilt.
12. Neither the students nor their instructor _____ (is/are) happy with the long cafeteria line for squid eyeball stew.
13. Neither of Freud's parents _____ (has/have) much intelligence when it comes to choosing spouses. Freud's father has married five times, and Mom just presented him with stepfather number three.
14. Each of these women _____ (wish/wishes) that she had read the care instructions before washing the delicate and expensive dresses.
15. Who needs a pet if you live in Florida? There already _____ (is/are) lizards that sneak in between the window screens and cockroaches that take up residence in every bathroom and kitchen.
16. Susan is one of those students who _____ (kiss/kisses) up to the instructor every chance that they get.
17. There _____ (is/are) more calories in a bowl of chocolate-broccoli breakfast cereal than you might think.

SESSION – 5

❖ PUNCTUATIONS

To fully understand basic grammar rules, you also need to look at punctuation rules.

- **Capitalization** is important. All sentences must start with a capital, or upper-case, letter. Titles of people, books, magazines, movies, specific places, etc. are capitalized. Organizations and compass points are capitalized.
- Every sentence needs a **punctuation mark** at the end of it. These include a period, exclamation mark, or question mark.
- **Colons** are used to separate a sentence from a list of items, to introduce a long, direct quote, or between two sentences (or clauses) when the second one explains the first.
- **Semicolons** can take the place of a conjunction and are placed before introductory words like "therefore" or "however." They are also used to separate a list of things if there are commas within each unit.
- There are a lot of rules for **commas**. The basic ones are that commas separate things in a series and go wherever there is a pause in the sentence. They surround the name of a person being addressed, separate the day of the month from the year in a date, and separate a town from the state.
- **Parentheses** enclose things that clarify or numbers and letters that are part of a list.
- **Apostrophes** are used in contractions to take the place of one or more letters and also to show possession. An apostrophe and "s" is added if the noun is singular and an apostrophe alone is added if the noun is plural.

9. Grandpa claims that Martian measles _____ (causes/cause) green and purple spots to erupt all over a person's body.
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16. Susan is one of those students who _____ (kiss/kisses) up to the instructor every chance that they get.
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SESSION – 5

❖ PUNCTUATIONS

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VERBAL ABILITY

EXERCISE 8

Directions for Q1 to Q10: Choose the correct punctuated sentence.

1. (a) Michael Suza Emperor of Ethiopia visited Ghana in 2004
 (b) Michael Suza Emperor of ethiopia visited Ghana in 2004.
 (c) Michael Suza , Emperor of Ethiopia visited Ghana in 2004.
 (d) Michael Suza Emperor of Ethiopia visited Ghana in 2004.

Answer: [c]

2. (a) although he studied very hard for the examination he did not pass
 (b) Although, he studied very hard for the examination, he did not pass.
 (c) Although he studied very hard for the examination, he did not pass.
 (d) Although he studied very hard for the examination....he did not pass.

Answer: [c]

3. (a) i do not believe you chewe the teacher said when you say this is your own work.
 (b) I do not believe you. Chewe the teacher said. When you say this is your own work.
 (c) "I do not believe you ,Chewe," the teacher said, " when you say this is your own work."
 (d) " I do not believe you Chewe the teacher said when you say this is your own work."

Answer: [c]

4. (a) For breakfast, we had cereal, toast, peanut butter and jelly.
 (b) For breakfast we had Cereal Toast Peanut Butter and Jelly.
 (c) for breakfast we had cereal. toast. peanut butter and jelly.
 (d) For breakfast we had Cereal....toast....peanut butter and jelly....

Answer: [a]

5. (a) we moved from san juan puerto rico on december 1 2004
 (b) we moved from san juan. puerto rico, on december 1 2004,
 (c) We moved from San Juan, Puerto Rico on December 1, 2004.
 (d) We moved, from San Juan Puerto Rico on December 1..2004.

Answer: [c]

6. (a) i cant hold on much longer screamed akiko
 (b) I can't hold on much longer, screamed Akiko.
 (c) I can't hold on much longer. Screamed Akiko.
 (d) "I can't hold on much longer," screamed Akiko.

Answer: [d]

7. (a) the woman screamed to her son stop pulling the dogs tail
 (b) The woman screamed to her, " Son stop pulling the dog's tail!"
 (c) The woman screamed to her son, "Stop pulling the dogs tail."
 (d) The woman screamed to her son. stop pulling the dogs tail.

Answer: [c]

8. (a) we were sorry to see that the sign on the door said closed
 (b) We are sorry to see that the sign on the door said closed.
 (c) We are sorry to see, that the sign on the door, said "closed."
 (d) We are sorry, to see that the sign on the door, said "Closed."

Answer: [c]

9. (a) when she got there she asked the cashier would you accept a cheque
 (b) When she got there, she asked the cashier, "Would you accept a cheque?"
 (c) when she got there she asked the cashier. would you accept a cheque?
 (d) When she got there she asked the cashier, would you accept a cheque?

Answer: [b]

10. (a) ann joanne and alison were playing in anns garden
 (b) Ann, Joanne and Alison were playing in anns garden.
 (c) Ann, Joanne and Alison were playing in Anns garden.
 (d) Ann, Joanne and Alison were playing in Ann's garden.

Answer: [d]

Exercise 9

Fill in the blanks.

1. We decided _____ (**run**) through the forest.
2. I learned _____ (**ride**) the bike at the age of 5.
3. She doesn't mind _____ (**work**) the night shift.
4. The man asked me how _____ (**get**) to the airport.
5. I can't imagine Peter _____ (**go**) by bike.
6. Are you thinking of _____ (**visit**) London?
7. He agreed _____ (**buy**) a new car.
8. The teacher expected Sarah _____ (**study**) hard.
9. I look forward to _____ (**see**) you at the weekend.
10. The question is easy _____ (**answer**).

Exercise 10

Fill in the blanks.

1. A wedding involves _____ (**negotiate**) with everyone in the family.
2. He denies _____ (**steal**) the money.
3. He claims _____ (**be**) a millionaire but I don't believe him.
4. I expect _____ (**be**) there about seven.
5. Julia reported _____ (**see**) the boys to the police.

6. The teenager refused _____ (go) on holiday with his parents.
7. I understand _____ (be) late once or twice, but every day is too much!
8. That criminal deserves _____ (get) a long sentence.
9. We arranged _____ (meet) at four but at four thirty she still hadn't arrived.
10. She mentioned _____ (go) to the cinema, but I don't know what she decided to do in the end.

Exercise 9

1. We decided **to run** through the forest.
2. I learned **to ride** the bike at the age of 5.
3. She doesn't mind **working** the night shift.
4. The man asked me how **to get** to the airport.
5. I can't imagine Peter **going** by bike.
6. Are you thinking of **visiting** London?
7. He agreed **to buy** a new car.
8. The teacher expected Sarah **to study** hard.
9. I look forward to **seeing** you at the weekend.
10. The question is easy **to answer**.

Exercise 10

1. A wedding involves **negotiating** with everyone in the family.
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10. She mentioned **going** to the cinema, but I don't know what she decided to do in the end.

VERBAL ABILITY

SESSION – 3

❖ VERBS

A verb expresses action or being.

- *jump... is... write... become*

The verb in a sentence expresses action or being. There is a main verb and sometimes one or more helping verbs. ("She can sing." *Sing* is the main verb; *can* is the modal verb.) A verb must agree with its subject in number (both are singular or both are plural). Verbs also take different forms to express tense.

- *The young girl brought me a very long letter from the teacher, and then she quickly disappeared.*
 - *The verb or compound verb is the critical element of the predicate of a sentence.*

In each of the following sentences, the verb or compound verb is **highlighted**:

- *Dracula bites his victims on the neck.*
 - *The verb "bites" describes the action Dracula takes.*
- *In early October, Giselle will plant twenty tulip bulbs.*
 - *Here the compound verb "will plant" describes an action that will take place in the future.*
- *My first teacher was Miss Crawford, but I remember the janitor Mr. Weatherbee more vividly.*
 - *In this sentence, the verb "was" (the simple past tense tense of "is") identifies a particular person and the verb "remember" describes a mental action.*
- *Karl Creelman bicycled around the world in 1899, but his diaries and his bicycle were destroyed.*
 - *In this sentence, the compound verb "were destroyed" describes an action which took place in the past.*

Different types of verbs:

1) Transitive Verb– Transitive Verb is the verb that needs object and usually followed by noun. These transitive verbs include arrest, avoid, do, enjoy, find, force, get, give, grab, hit, like, pull, report, shock, take, tell, touch, want, warn ...

Example:

- She **takes** a book.
- I **need** a chair.
- They **speak** English.

2) Intransitive Verb– Intransitive Verb is the verb which does not need object, but it needs adverbial modifier. These intransitive verbs include appear, come, fall, go, happen, matter, sleep, swim, wait...

Example:

- He **cries**.
- They **dance** well.
- She **sings** beautifully.

3) Auxiliary Verb– Auxiliary Verbs are used to form question and negative sentence, and they are usually used with main verb to form many different kinds of tenses.

Be is used to form Continuous Tense and Passive Voice.

Example:

- The dog is biting a child.
- A child is bitten by the dog.

Have/Has is used to form Perfect Tense.

Example:

- They have known me for 3 years.
- She has had dinner already.

Do/Does is used to form Question and Negative Sentence in the Present Simple when the sentence doesn't have a special verb. Moreover, we can also use them to show the emphasis sentence.

Example:

- He doesn't eat meat.
- Do you love her?
- I do live here. (Emphasis form)

4) Modal Verb- Modal Verbs are the verbs that are used to talk about ability, permission, obligation and necessity, obligation and advice, possibility, probability, request, offer, suggestion, habit and promise. Most Modal Verbs can form question and negative sentence by themselves. Some modal verbs are **can, could, shall, should, will, would, may, might, must ought to, dare, need not, used to**.

5) Gerunds- A **gerund** is a verb in its ing (present participle) form that functions as a noun that names an activity rather than a person or thing. Any action verb can be made into a gerund.

Spelling Tip for Gerunds

Verb-ing (Present Participle)

- Add ing to most verbs. Ex. play > playing, cry > crying, bark > barking
- For verbs that end in e, remove the e and add ing. Ex: slide > sliding, ride > riding
- For verbs that end in i.e, change the i.e to y and add ing. Ex: die > dying, tie > tying
- For a verb whose **last** syllable is written with a consonant-vowel-consonant and is stressed, double the last letter before adding ing. Ex: beg > begging, begin > beginning. **However:** enter > entering (last syllable is not stressed)

Examples:

Gerunds can appear at the beginning of a sentence when used as a subject:

- **Jogging** is a hobby of mine.

Gerunds can act as an object following the verb:

- Daniel quit **smoking** a year ago.

Gerunds can serve as an object after a preposition:

- I look forward to **helping** you paint the house.

6) Infinitives- An infinitive is a verb form that acts as other parts of speech in a sentence. It is formed with to + base form of the verb. Ex: to buy, to work.

Examples:

An object following the verb:

- Jim always forgets **to eat**

A subject at the beginning of a sentence:

- **To travel** around the world requires a lot of time and money.

An adverb modifying a verb:

- You promised **to buy** me a diamond ring.

An adjective modifying a noun:

- Tara has the ability **to succeed**.

Some verbs are directly followed by an infinitive:

- Do you want **to call** your family now?

Some verbs are directly followed by a noun or pronoun and then by an infinitive:

- I convinced Catherine **to become** vegetarian.

❖ ADVERB

- **An adverb modifies or describes a verb, an adjective, or another adverb.**

gently... extremely... carefully... well

An adverb describes or modifies a verb, an adjective, or another adverb, but never a noun. It usually answers the questions of when, where, how, why, under what conditions, or to what degree. Adverbs often end in -ly.

*The young girl brought me a **very** long letter from the teacher, and she **quickly** disappeared.*

In the following examples, each of the **highlighted** words is an adverb:

- The seamstress **quickly** made the mourning clothes.
 - In this sentence, the adverb "quickly" modifies the verb "made" and indicates in what manner (or how fast) the clothing was constructed.
- The midwives waited **patiently** through a long labour.
 - Similarly in this sentence, the adverb "patiently" modifies the verb "waited" and describes the manner in which the midwives waited.
- The **boldly** spoken words would return to haunt the rebel.
 - In this sentence the adverb "boldly" modifies the adjective "spoken."
- We urged him to dial the number more **expeditiously**.
 - Here the adverb "more" modifies the adverb "expeditiously."
- **Unfortunately**, the bank closed at three **today**.
 - In this example, the adverb "unfortunately" modifies the entire sentence.

VERBAL ABILITY

SESSION – 6

❖ VERBAL REASONING

Exercise 11

Directions for Q1 to Q10: Which one of the following argument (I. OR II.) represents the strongest counter (if any) to given statements? Choose options A, B, C, D or E accordingly:

A. Strengthening & Weakening arguments

1. **Statement:** Should there be a law to punish parents who get their minor children married?

Arguments:

- I. Yes, a minor girl is physiologically not prepared to conceive a baby.
 - II. No, this has been a custom prevailing since many centuries.
- | | |
|-----------------------------------|------------------------------------|
| (a) if only argument I is strong. | (b) if only argument II is strong. |
| (c) if either I or II is strong. | (d) if neither I nor II is strong. |
| (e) if both I and II are strong. | |

2. **Statement:** Should the institute conduct classes in remote villages?

Arguments:

- I. Yes, this will help those students who belong to villages and cannot visit urban areas for studies.
 - II. No, this is not an economically viable proposal, as the number of students who attend such classes cannot contribute to break-even.
- | | |
|-----------------------------------|------------------------------------|
| (a) if only argument I is strong. | (b) if only argument II is strong. |
| (c) if either I or II is strong. | (d) if neither I nor II is strong. |
| (e) if both I and II are strong. | |

3. **Statement:** Are these sanctuaries, which are meant to protect the endangered animals, necessary?

Arguments:

- I. Yes, these are necessary as it is our responsibility to conserve environment and to provide posterity with a better world to live in.
 - II. No, these are a huge burden on our receding economy.
- | | |
|-----------------------------------|------------------------------------|
| (a) if only argument I is strong. | (b) if only argument II is strong. |
| (c) if either I or II is strong. | (d) if neither I nor II is strong. |
| (e) if both I and II are strong. | |

4. **Statement:** Are there any good politicians left in this world?

Arguments:

- I. Yes, so many poor people are sustaining themselves and improving economically.
 - II. No, there is nothing in this world which is completely good or completely bad.
- | | |
|-----------------------------------|------------------------------------|
| (a) if only argument I is strong. | (b) if only argument II is strong. |
| (c) if either I or II is strong. | (d) if neither I nor II is strong. |
| (e) if both I and II are strong. | |

5. **Statement:** Should the teachers be stopped from beating the students?

Arguments:

- I. Yes, child psychologists say that beating hinders the learning process in a child.
- II. No, spare the cane and spoil the child.
- (a) if only argument I is strong.
- (b) if only argument II is strong.
- (c) if either I or II is strong.
- (d) if neither I nor II is strong.
- (e) if both I and II are strong.

6. **Statement:** Should there be an upper age limit for a person to assume the post of prime minister?

Arguments:

- I. Yes, a leader must not only be wise and experienced but also be energetic and young enough to understand the trends of the world.
- II. No, older the leader, better he is, as he can lead more effectively owing to his experience.
- (a) if only argument I is strong.
- (b) if only argument II is strong.
- (c) if either I or II is strong.
- (d) if neither I nor II is strong.
- (e) if both I and II are strong.

7. **Statement:** Should Jacobs, a hardware company, enter the software industry?

Arguments:

- I. Yes, if there is an indigenous software development wing, the Research and Development of hardware would be more effective.
- II. No, software industry is in recession.
- (a) if only argument I is strong.
- (b) if only argument II is strong.
- (c) if either I or II is strong.
- (d) if neither I nor II is strong.
- (e) if both I and II are strong.

8. **Statement:** It is wise on the part of the government to disinvest from PSUs?

Arguments:

- I. Yes, overheads of PSUs are increasing making the total cost of production higher and higher.
- II. No, the fundamental purpose of PSUs is to provide employment to the people.
- (a) if only argument I is strong.
- (b) if only argument II is strong.
- (c) if either I or II is strong.
- (d) if neither I nor II is strong.
- (e) if both I and II are strong.

9. **Statement:** Should the manufacturing of leather products be prohibited?

Arguments:

- I. Yes, killing animals for any reason other than survivals is making the environment dangerous.
- II. NO, leather products are the biggest foreign exchange earners.
- (a) if only argument I is strong.
- (b) if only argument II is strong.
- (c) if either I or II is strong.
- (d) if neither I nor II is strong.
- (e) if both I and II are strong.

10. **Statement:** Should Abacus be introduced as a part of the curriculum by schools?

Arguments:

Exercise 12

Directions for Q11 to Q20: Following questions consist of one statement followed by two assumptions. You have to decide which one of the Assumptions (if any) sounds practical that can be followed. Choose options A, B, C, D or E accordingly.

- 11. Statement:** I will go to Kishore binding centre to repair my big Oxford dictionary book.

Assumptions:

- 12. Statement:** RBI has imposed Rs.1 crore fine on Baroda bank for violating KYC norms.

Assumptions:

13. **Statement:** Asian development bank will provide 120 million dollar for India-Bangladesh cross border electricity

Assumptions:

- 14. Statement:** Union cabinet allows 100% FDI for white label ATMs

Assumptions:

15. **Statement:** FIFA banned Nepal's football Chief for 10 years.

Assumptions:

- I. After Nepal, India is FIFA's next target.
- II. India will cut its relation with Nepal in football.
- (a) Only assumption I is practical.
- (b) Only assumption II is practical.
- (c) Either assumption I or II is practical.
- (d) Neither assumption I nor II is practical.
- (e) Both the assumptions are practical.

16. **Statement:** Karnataka government banned drug Ketoprofen.

Assumptions:

- I. There were some copyright issues.
- II. This drug is very harmful for humans.
- (a) Only assumption I is practical.
- (b) Only assumption II is practical.
- (c) Either assumption I or II is practical.
- (d) Neither assumption I nor II is practical.
- (e) Both the assumptions are practical.

17. **Statement:** Usually state medium students have lesser IQs than the CBSE medium students.

Assumptions:

- I. The presence of workshops in CBSE curriculum increase students' IQ.
- II. State medium students are normally from poor background.
- (a) Only assumption I is practical.
- (b) Only assumption II is practical.
- (c) Either assumption I or II is practical.
- (d) Neither assumption I nor II is practical.
- (e) Both the assumptions are practical.

18. **Statement:** Government has relaxed FDI investment norms in defence sector.

Assumptions:

- I. When investment will be more, we can attack Pakistan.
- II. Relaxed FDI in defence sector will create havoc among the defence employees and they will look for other jobs.
- (a) Only assumption I is practical.
- (b) Only assumption II is practical.
- (c) Either assumption I or II is practical.
- (d) Neither assumption I nor II is practical.
- (e) Both the assumptions are practical.

19. **Statement:** North Korea has tested fired hydrogen bomb.

Assumptions:

- I. India should make a hydrogen bomb for its safety.
- II. United Nations should put a sharp eye on North Korea.
- (a) Only assumption I is practical.
- (b) Only assumption II is practical.
- (c) Either assumption I or II is practical.
- (d) Neither assumption I nor II is practical.
- (e) Both the assumptions are practical.

20. **Statement:** In today's economic crisis only the ideals of limited family and hard labor in the field of education can lead India towards prosperity.

Assumptions:

- I. Ideals of limited family and hard labor in the field of education are correlated with India's prosperity.
 - II. A large family faces difficulty in bearing the expenses on education.
- | | |
|---|---|
| (a) Only assumption I is practical. | (b) Only assumption II is practical. |
| (c) Either assumption I or II is practical. | (d) Neither assumption I nor II is practical. |
| (e) Both the assumptions are practical. | |

Exercise 11

1. Answer: [a]

Statement I explains a valid reason to go against such marriages because it is wrong to encourage anything that jeopardises the health of a person. Hence, (I) is a strong argument.

Statement II is not a valid argument because a custom being followed since a long time does not necessarily make it good. Only I is strong.

2. Answer: [e]

Statement I: The basic purpose of conducting classes is to help students. Hence, statement I is a strong argument as it conveys this idea.

Statement II: If the institute conducts classes with the intention of making profits, then this is a valid point to be considered. Hence, statement II is also strong.

3. Answer: [e]

Sanctuary for endangered animals becomes necessary when it improves and betters the living conditions of the world. But at the same time, if an economy is not able to provide basic necessities to its people, then it cannot think of constructing sanctuaries. Therefore, the decision is based on both the factors viz. requirement and funds supply. Hence, both I and II are strong arguments.

4. Answer: [d]

Just because some poor people are improving economically, we cannot conclude that there are good politicians. Any reason could be ascribed to such an improvement. Statement II is very vague and is not giving any reasons for the argument that it proposes. Hence, both the statements are weak arguments.

5. Answer: [a]

Anything which hinders the learning process in a child must be controlled and, if possible, be prohibited. The aim of teacher should be to make a child learn. According to argument I, beating causes hindrance to learning process of a child thus defeating the ultimate goal. It also supports its argument by referring to the findings of psychologists. Hence, I is a strong argument. Argument II states that a child will be spoiled if it is not caned. Caning is an extreme measure; there are also other softer methods to rectify a child. Moreover, the argument is not substantiated by any facts. Hence, II is a weak argument.

6. Answer: [e]

Statement I says that the prime minister must be "young enough" for some explained reasons. This implies that the prime minister must not be older than a certain age limit. This statement is strong.

Statement II: This says that there should not be an upper limit because older persons can perform better because of the experience gained. This is also strong.

VERBAL ABILITY

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7. Answer: [b]

Statement I: This statement is talking about a software wing for indigenous purposes [to support R and D], whereas the question is about entering the industry. This statement is irrelevant and hence the argument is weak.

Statement II: This is an important point to be considered before entering any industry.

8. Answer: [d]

Statement I says that total cost is increasing. But this does not mention whether production is also correspondingly increasing or not. If production is also increasing, then there would not be any problem. Hence, this statement is weak.

Statement II: We do not know whether the employees would be removed or not. It is likely that the employment is not disturbed. Hence this is also a weak argument.

9. Answer: [d]

Statement I: There are two points to be considered.

(a) Whether clothing is for survival or not?

(b) Can we use the leather of those animals which are killed for food?

Because statement I is not explaining the answers to the above questions. It is considered to be a weak argument.

Statement II: It is most likely that the reason behind the proposal is more related to morals and feelings rather than money. Foreign exchange cannot be a valid argument in this context. Hence, both are weak arguments.

10. Answer: [e]

Both the statements, if true, are valid and strong arguments.

Exercise 12

11. Answer : [e]

Here both the assumptions are practical because the shop does its job properly with low rates.

12. Answer: [b]

Bank's number of branches is not mentioned in the statement. Hence; assumption I is not practical but RBI is strict, so it imposed penalty. Therefore; only assumption II is practical.

13. Answer: [b]

We do not know whether ADB bank will pay more than World Bank or not but it is for sure that this type of initiative will help the people of both countries.

14. Answer: [a]

100% FDI means more incoming of money and thereby causing number of white level ATMs to increase further. Assumption II is however not related to given statement.

15. Answer: [d]

Here none of the assumptions follows because both are not logical at all.

16. Answer: [d]

Ban can be done on any of the issues. We cannot assume for sure.

VERBAL ABILITY

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17. Answer: [d]

We do not know whether workshops are there in state medium board or not and it is not always mandatory that all the students of state boards are from poor background only.

18. Answer: [d]

None of the given assumptions are logical.

19. Answer: [b]

Just because one country has made hydrogen bomb does not mean that another country will also do the same but instead keeping an eye upon the country is a good practice.

20. Answer: [a]

Here, if the sentence talks about prosperity of the nation then the assumption is that only the two factors mentioned can lead to prosperity. Hence, assumption I is practical. But nothing is mentioned about why larger families will be deterrent to prosperity. While it is common sense that a large family faces difficulty in bearing the expenses on education, it is not practical in the statement given.

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LOGICAL THINKING

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VISUAL REASONING - I

SESSION - 10

ABSTRACT REASONING

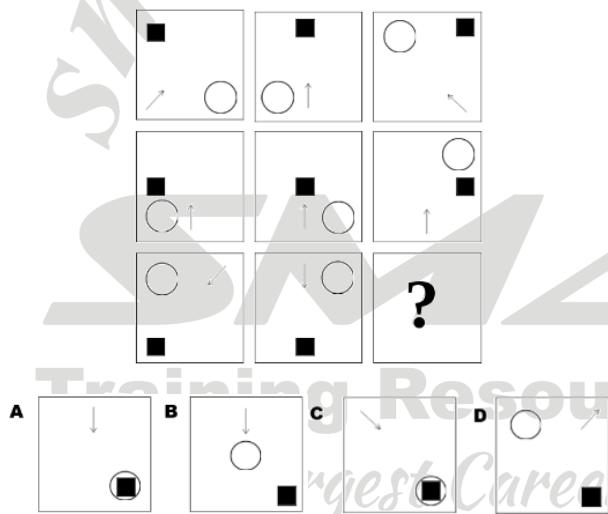
The abstract reasoning test is also called the conceptual reasoning test (also known as inductive reasoning or diagrammatic reasoning tests). It measures your lateral thinking skills, which are your ability to quickly identify patterns, logical rules and trends in new data, integrate this information, and apply it to solve problems. You also need to see the relationships between shapes and figures, identify rules and similarities, and quickly apply these to identify the answer. It measures what most people would refer to as 'street smarts' and the ability to 'think on your feet'.

Generally, abstract reasoning tests measure non-verbal abilities. Abstract reasoning tests are almost always a part of any job assessment or intelligence testing setup.

While there are many permutations of question types and formats, there are some general concepts which are common, and it can be useful to familiarise yourself with these.

Common question types in abstract reasoning tests

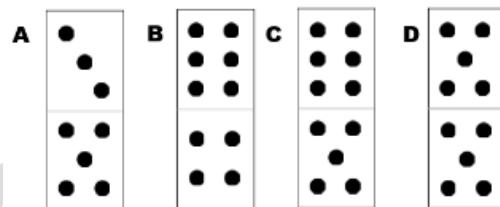
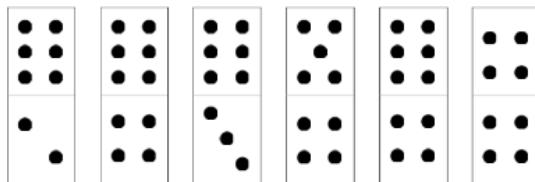
Question 1: Identify the missing square



This type of question requires you to look at the patterns in the squares and understand their relationship to one another, so as to identify the missing square.

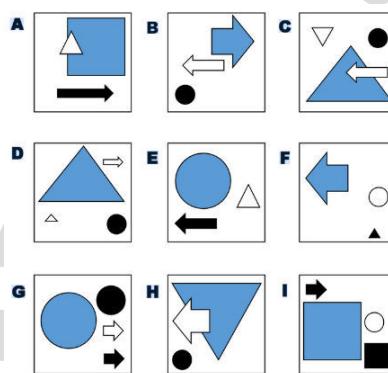
There are three rules to spot in this question. Firstly, there is a relative positional rule: the position of the black square corresponds to the position its square holds within the diagram. Secondly, there is a movement rule, in that the circle moves around the boxes in a clockwise position. Finally, the arrows in the first and third columns are reflections of one another. The correct answer is therefore C.

Question 2: Complete the sequence



This requires you to identify and understand the pattern behind the order in which the dominoes are presented. There are two patterns here. The first, third and fifth (and therefore seventh) dominoes have the rule that the number of the top is six and the number on the bottom is increasing by one each time. The second pattern includes the second, fourth and sixth dominoes, and has the rule that the number on the bottom is four and the number at the top is decreasing by one each time. The correct answer is therefore C.

Question 3: Identify the odd one out

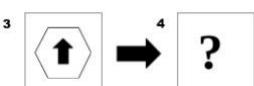
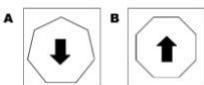
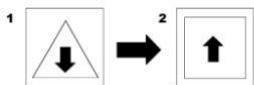


This type of question requires you to look at some data, identify the pattern or rules, and then spot which square does not meet those rules. Watch out for relative position, number of items, relationship between items, colour, shape, and orientation of shapes: there are many different variations on these rules and there may be some extraneous data in there that complicates the rules.

For example, in this question some of the squares have three items in and some have four – you need to work out whether or not that is important. In this particular case, there are two rules. The first is that the largest shape must be grey, and the second is that the bottom shape must be black. The odd one out is therefore C, as the bottom shape is stripy and not black.

LOGICAL THINKING

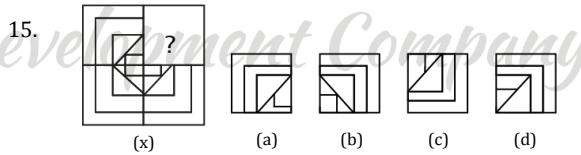
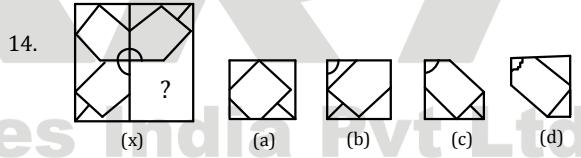
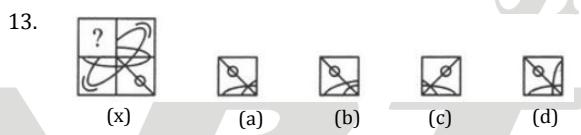
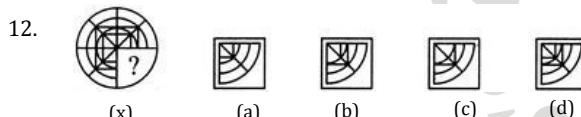
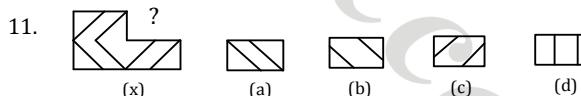
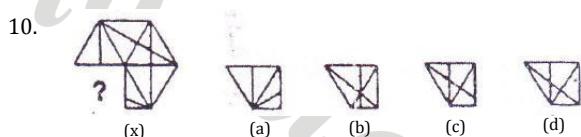
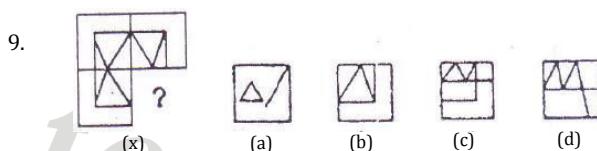
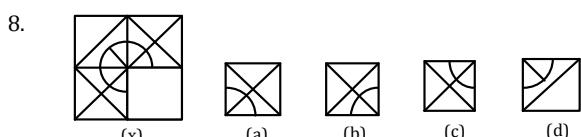
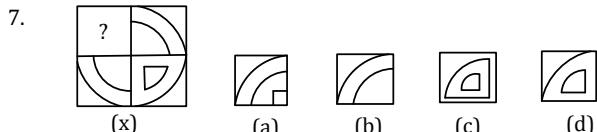
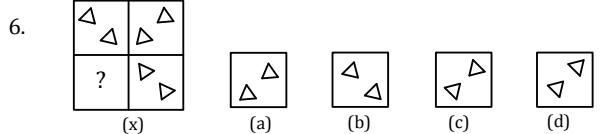
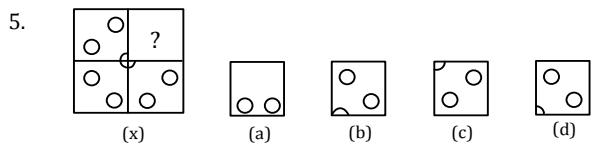
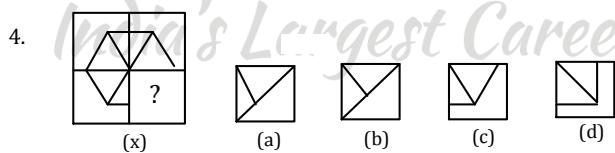
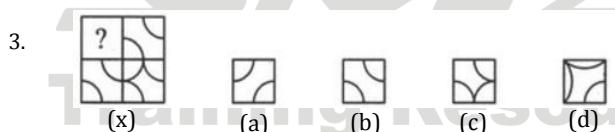
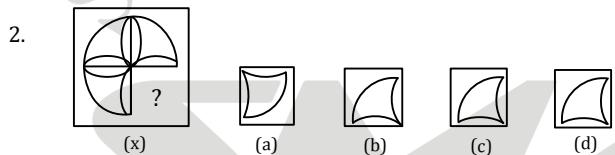
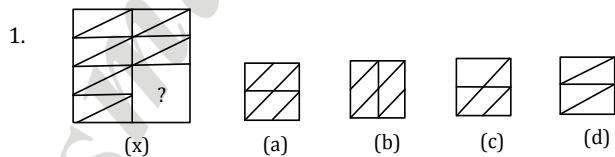
Question 4: 1 is to 2, as 3 is to 4



This type of question is all about relationships between data: being able to recognise what links two boxes together and then apply this rule to a new shape to solve the problem.

There are two rules here. First, the shape in box 2 has one more side than the shape in box 1. As the shape in box 3 has six sides, the correct shape for box 4 must have seven sides. The second rule is around the arrow and the rule is that for shapes with an even number of sides, the arrow points up. For shapes with an odd number of sides the arrow points down. The correct answer is therefore A.

Directions for Q1 to Q15: In each of the following questions, complete the missing portion of the given pattern by selecting from the given alternatives (a), (b), (c) and (d).



INPUT-TYPE DIAGRAMMATIC REASONING

Diagrammatic reasoning questions (also known as logical reasoning or inductive reasoning questions) are designed to assess your logical reasoning ability.

These tests involve rules that must be applied to a sequence of either shapes or letters. They may involve symbols that change either figures or objects. For example, a symbol may signify that triangles must be replaced with squares. Alternatively, they may involve shapes that apply rules to a sequence of letters. For

LOGICAL THINKING

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example, one shape will reverse the letters and another may signify that a letter must be dropped or added. These tests require you to visualize quickly how the sequence will change after a series of transformations.

Directions for Q16 and Q17: According to a certain code language, words in column 1 are written in capital letters and in column 2 their codes are given. The codes in column 2 are jumbled up. Decode the language and choose the correct code for the word given in each of the questions that follow.

Column 1	Column 2
CHAIN	acmvq
FRAIL	pnqal
TEAM	wjqd
DESTINY	xwtjazc
TOIL	ajgl
VARY	qbzn
NAUGHTY	iqcfvzj

16. DAILY
 (a) aqzla (b) alzqi (c) izqat (d) aqtzc
17. TEAR
 (a) nqjw (b) qwmc
 (c) jwqc (d) fjcw

Directions for Q18 and Q19: Answer the questions based on the code given below.

- AB Delete the last character
 BC Replace the third character with the next in the alphabet
 CD Insert the letter P between the third and fourth characters
 DE Exchange the first and last characters
 EF Replace the second character with the previous letter in the alphabet
 FG Replace the fifth character with the next in the alphabet
 GH Reverse the whole sequence of letters
 HI Delete the third character
18. MOZLUCK → AB + FG + CD →
 (a) MOZPLVK (b) MOZLVC
 (c) MOZPLVCK (d) MOZPLVC
19. CNPTTBM → HI + EF + DE →
 (a) MMTTBC (b) MMTTBM
 (c) MNTTBBC (d) CMTTBC

Directions for Q20 and Q21: Answer the questions based on the information given below.

- ⌚ Remove all shading
 @ Switch the top and middle shapes
 Σ Add a vertical line to the shape at the bottom of the series

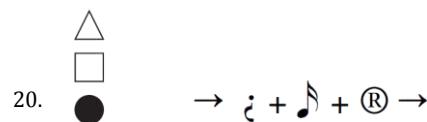
♪ Switch the top and bottom shapes

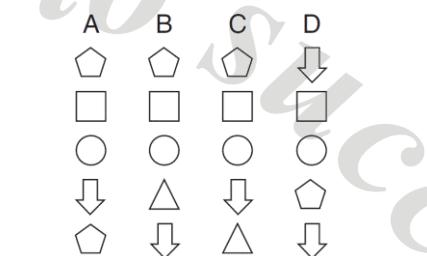
Ω Replace the top shape with ▲

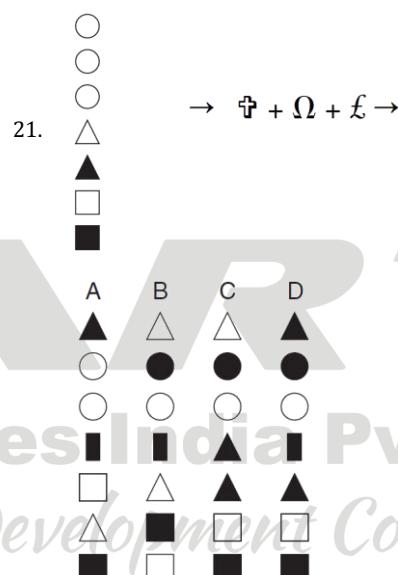
⊕ Shade the second shape

£ Replace the middle shape with ■

® Switch the fourth and last shapes

20. 



21. 

Directions for Q22 and Q23: Answer the questions based on the information given below.

□ Reverse the whole sequence

△ Drop all Ts

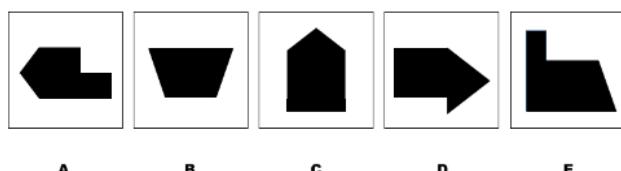
○ Add TE to the end of the sequence

↓ Exchange the first two and last two characters

☆ Add an A between the fourth and fifth characters

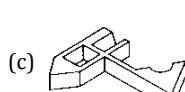
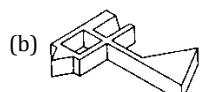
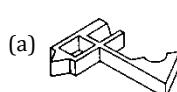
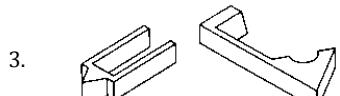
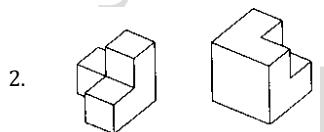
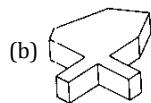
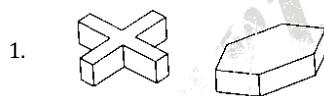
Example

Which image can be made from the three shapes shown?

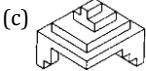
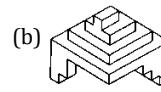
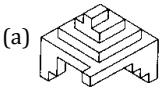
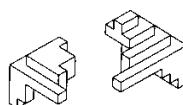


The answer is (C).

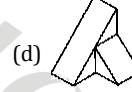
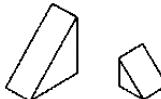
Directions for Q1 to Q15: Identify the new shape that could be constructed if the two example shapes were combined.



4.



5.



6. Find the odd one out.



(a) 2

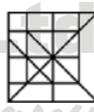
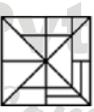
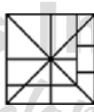
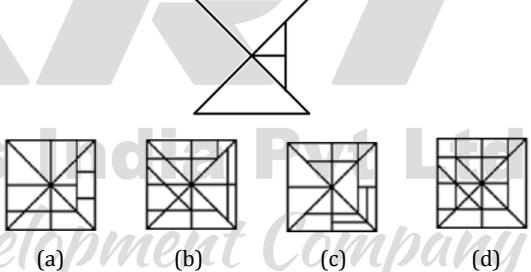
(b) 3

(c) 1

(d) 5

7.

The figure below is a part of one of the figures given as option. Find that figure.

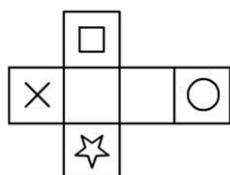


8. Find the mirror image of the figure given below.



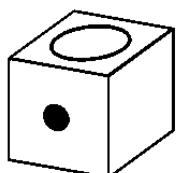
LOGICAL THINKING

9. Find the object that the pattern below can fold into.



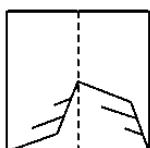
- (a)
- (b)
- (c)
- (d)

10. Find the pattern that can be folded into the object below.



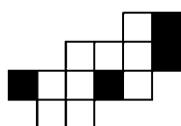
- (a)
- (b)
- (c)
- (d)

11. How would the pattern appear when the transparent sheet is folded along the dotted line?



- (a)
- (b)
- (c)
- (d)

12. Which of the five groups below can be combined to make the figure given below?



- (a)
- (b)
- (c)
- (d)

Directions for Q13 to Q15: Choose the correct figure of the three-dimensional question shape from the four suggested answer.



- (a)
- (b)
- (c)
- (d)



- (a)
- (b)
- (c)
- (d)

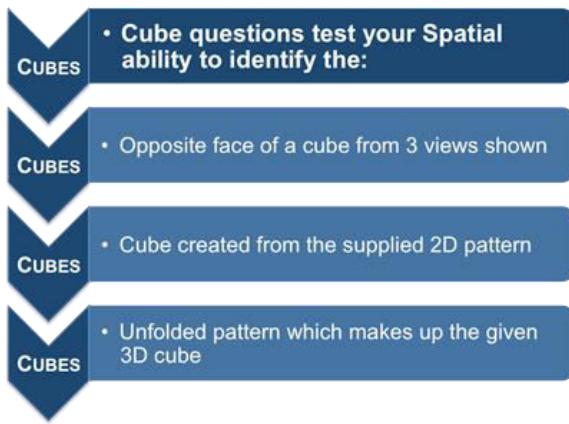


- (a)
- (b)
- (c)
- (d)

CUBES

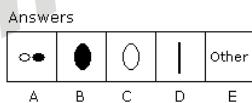
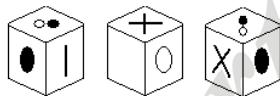
These questions show you several (usually 3) views of a 3-dimensional cube with unique symbols or markings on each face and then asks you a question about it.

LOGICAL THINKING



Example

Three views of the same cube are shown above. Which symbol is opposite the X?

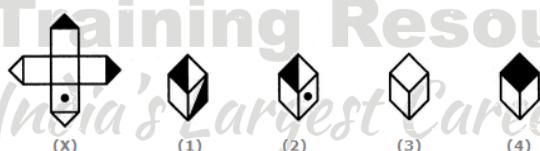


Answer: [D]

In the question above for example, you can simply use a process of elimination. If you can see a symbol on the same illustration as the 'X' then it cannot be opposite. The second and third cubes eliminate A, B and C. This leaves only D and 'other' as possibilities. D has edges shared with A and B which would be consistent with the third cube illustrated. Therefore D is correct.

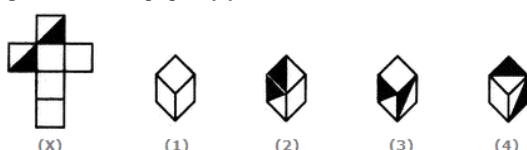
Directions for Q16 and Q17: The sheet of paper shown in the figure (X) given on the left hand side, in each problem, is folded to form a box. Choose from amongst the alternatives (1), (2), (3) and (4), the boxes that are similar to the box that will be formed.

16. Choose the box that is similar to the box formed from the given sheet of paper (X).



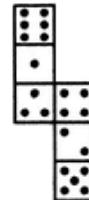
- (a) 1 and 2 only (b) 2 and 4 only
(c) 2 and 3 only (d) 1 and 4 only

17. Choose the box that is similar to the box formed from the given sheet of paper (X).



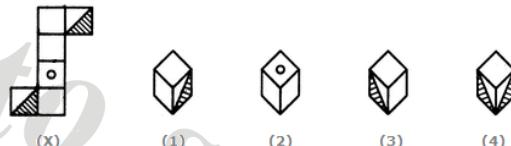
- (a) 1 and 4 only (b) 3 and 4 only
(c) 1 and 2 only (d) 2 and 3 only

18. How many dots lie opposite to the face having three dots, when the given figure is folded to form a cube?



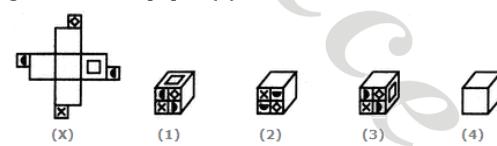
- (a) 2 (b) 4 (c) 5 (d) 6

19. Choose the box that is similar to the box formed from the given sheet of paper (X).



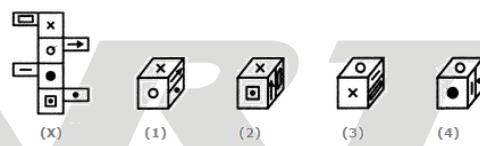
- (a) 1 and 3 only (b) 1 and 4 only
(c) 2 and 4 only (d) 3 and 4 only

20. Choose the box that is similar to the box formed from the given sheet of paper (X).



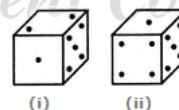
- (a) 1, 2 and 3 only (b) 2 and 3 only
(c) 1, 3 and 4 only (d) 2, 3 and 4 only

21. Choose the box that is similar to the box formed from the given sheet of paper (X).



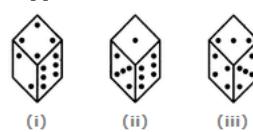
- (a) 1 and 2 only (b) 1 and 3 only
(c) 3 and 4 only (d) 1, 2, 3 and 4

22. Observe the dots on a dice (one to six dots) in the following figures. How many dots are contained on the face opposite to that containing four dots?



- (a) 2 (b) 3
(c) 6 (d) Cannot be determined

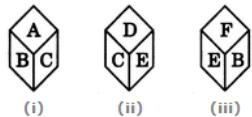
23. Three different positions of a dice are shown below. How many dots lie opposite 2 dots?



- (a) 1 (b) 3 (c) 5 (d) 6

LOGICAL THINKING

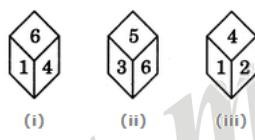
24. The six faces of a dice have been marked with alphabets A, B, C, D, E and F respectively. This dice is rolled down three times. The three positions are shown as:



Find the alphabet opposite A.

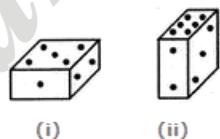
- (a) C (b) D (c) E (d) F

25. Three positions of a dice are given. Based on them find out which number is found opposite the number 2 in the given cube.



- (a) 6 (b) 5 (c) 3 (d) 1

26. Two positions of a parallelepiped are shown below. When the number 3 will be on the top side, then which number will be at the bottom?



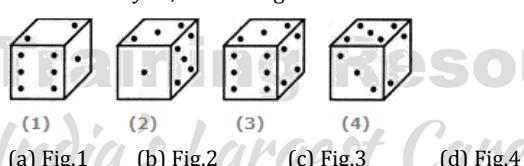
- (a) 1 (b) 4 (c) 5 (d) 6

27. A dice is numbered from 1 to 6 in different ways.

If 1 is adjacent to 2, 3 and 5, then which of the following statements is necessarily true?

- (a) 4 is adjacent to 6 (b) 2 is adjacent to 5
(c) 1 is adjacent to 6 (d) 1 is adjacent to 4

28. If the total number of dots on opposite faces of a cubical block is always 7, find the figure which is correct.



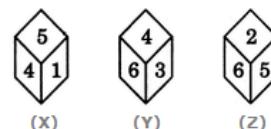
- (a) Fig.1 (b) Fig.2 (c) Fig.3 (d) Fig.4

29. A dice is numbered from 1 to 6 in different ways.

If 1 is opposite to 5 and 2 is opposite to 3, then

- (a) 4 is adjacent to 3 and 6
(b) 2 is adjacent to 4 and 6
(c) 4 is adjacent to 5 and 6
(d) 6 is adjacent to 3 and 4

30. Three different positions X, Y and Z of a dice are shown in the figures given below. Which of the hidden numbers adjacent to 5 in position X is/are common to the hidden numbers adjacent to 5 in position Z?



- (a) 1 and 4 (b) 2
(c) 6 (d) None

REASONING ABILITY

SESSION - 12

SUDOKU - I

- The objective is to fill a 9x9 grid so that each column, each row, and each of the nine 3x3 boxes (also called blocks or regions) contains the digits from 1 to 9.
- A cell is the smallest block in the game. A row, column and region consists of 9 cells and the whole game consists of 81 cells. A region has thicker lines surrounding it. This simply makes it easier to play the game.

5			1			4		
2	7	4				6		
	8		9		4			
8	1		4	6		3	2	
		2	3		1			
7	6		9	1		5	8	
		5		3		1		
		5			9	2	7	
1			2			3		

Look for the easy play first: When you first start to play a Sudoku puzzle, look for where you have the easiest opportunities to add a number. Usually this is where there is a crowded square or a row that is almost full of numbers. You can quickly use process of elimination to figure out where to place a number.

9			5			6		
	6		4			8	2	
7			2	8		3		
4	8	2	1			5	6	
	1	7		5	9		3	
3	9	5	6	2				
					5	6	8	
7								
2	4	3				7	9	

This number must be a 6

For example, if there is a square that already has numbers 1-7, you know that you only need to figure out where to put numbers 8 and 9. Look at the rows that feed into that row or square – sometimes you will be able to eliminate one number or the other, and can quickly fill in the gaps.

LOGICAL THINKING

Look for which numbers are missing: Sudoku is about placing numbers where they don't already exist – it's a logical process of elimination. If a number already exists in a row or square, then that number cannot be placed again. Your challenge is to keep thinking and looking and spotting opportunities to add numbers where they haven't already been placed.

For example, if the top row of a Sudoku puzzle already has the numbers 1, 7, 8, 5, 9 and 2, this means that the row still needs numbers 3, 4, and 6. Look in the nearby rows (within the same squares) to see if you can rule out any of those three missing numbers.

	4		2	6		9		
2	6	1			7			
1	7				5			
		3			2			
	3		5		9	8		
2					6	3		
1		9	2					
		4					5	
7	4	9	6	1				

Since this row already has a 7, 4, 9, 6, 1, and 5, you know that this number must be a 2, 3, or 8

Keep moving: Sudoku rewards the “roving eye” – if you feel stuck, don’t concentrate too hard on one part of the puzzle grid. Instead, let your eye and your mind wander to a different place on the grid where you haven’t placed any numbers yet, and see which new possibilities become apparent to you.

1.

	6	1	2	4				
2		5				1		
4				5	6			
			4	3				
		2	5					
	5			2				

2.

	4			1				
5	6		2					
2		4	3	2				
1			5		6			
			1	6				
			4		3			

3.

	1			4		
		2		6	5	
	2	3	4			
6		5		1	2	
	6	1				
3		4				

4.

7	4			9		2		
8	9			1	7		6	5
				8				
			9	5		3	2	1
	3						8	
9	8	2		3	1			
				7				
4	1		3	6			5	2
		7	2			1	3	

5.

	1	3		4	9	7	8	
9								4
5	4			7	1	9	2	
1			6				9	
6		7	9			5		
		5					6	
	6				5			8
8			1				4	
			3	8	6	1		

	7	5				4	2	6
9		2				3		
	3			5				
			8					7
3					7	1	8	
7				4		2	6	
	2		4	9		5		
8								
5	9	1	7	2	8			4

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7.

	2							
	8			1	2	4		
	6	2	8		1			
			3				6	
			6					
		9		8	3		1	
2	4		3				5	
	8	7	2				3	
	3		1	5				

3.

			8					1
			5				4	2
1	4	9					5	
7	9	1			6			
					5		7	
	3				9	2		
			6		3	9		
6				1			2	
		2		4		1		3

4.

7		1						8
	3			4			7	
			5	7				1
			2	8				
3		4				6		5
	6		4			7		
	4		3		1		2	
1			8			5		3
2			5	4				

5.

						5	8	
1		8		3			9	
	4			2			1	
3		4	6					
		2						3
	6							7
			5					
8	2	7						
			1					

❖ SUDOKU - II

1.

7	2		3	5				
		3	7	6				5
5					8	3	4	
4		2			5		3	
	3		4		2		8	
	8		1			7		4
	1	9	5					2
6				2	1	8		
			4	3		7	9	

2.

			3	1				
	6				2	9	4	
	5		6		2			
6				7				5
				6			1	
1	4	3			9	6		
8		6		3				
5				4		9		1
7				5		8	6	

LOGICAL THINKING – SOLUTION

25. Ans: [a]

From figures (i) and (ii), we conclude that the numbers 1, 4, 3 and 5 lie adjacent to the number 6. Clearly, the number 2 lies opposite 6 and conversely 6 lies opposite 2.

26. Ans: [c]

The number 2 is common to both the figures. We assume the parallelepiped in fig. (ii) to be rotated so that 2 appears at the same position as in fig. (i) i.e. on the RHS face and the numbers 6 and 3 move to the faces hidden behind the numbers 1 and 5 respectively [in fig. (i)]. Then, the combined figure will have 1 opposite 6 and 5 opposite 3. Thus, when 3 will be on the top, then 5 will appear at the bottom.

27. Ans: [a]

If 1 is adjacent to 2, 3 and 5, then either 4 or 6 lies opposite to 1. So, the numbers 4 and 6 cannot lie opposite to each other. Hence, 4 necessarily lies adjacent to 6.

28. Ans: [b]

Since the total number of dots on opposite faces is always 7, therefore, 1 dot appears opposite 6 dots, 2 dots appear opposite 5 dots and 3 dots appear opposite 4 dots.

29. Ans: [b]

If 1 is opposite to 5 and 2 is opposite to 3, then 4 definitely lies opposite to 6. Therefore, 2 cannot lie opposite to any of the two numbers - 4 or 6. Hence, 2 necessarily lies adjacent to both 4 and 6.

30. Ans: [d]

From positions X and Y we conclude that 1, 5, 6 and 3 lie adjacent to 4. Therefore, 2 must lie opposite 4. From positions Y and Z we conclude that 4, 3, 2 and 5 lie adjacent to 6. Therefore, 1 must lie opposite 6. Thus, 2 lies opposite 4, 1 lies opposite 6 and consequently 5 lies opposite 3.

Since 3 lies opposite 5 (as analysed above), it follows that 1, 4, 6 and 2 lie adjacent to 5. Out of these four numbers, the hidden numbers adjacent to 5 in position X are 6 and 2 and the hidden numbers adjacent to 5 in position Z are 1 and 4. Clearly, there is no number common.

REASONING ABILITY

SESSION – 12

❖ SUDOKU – I

1.

3	6	1	2	4	5
2	4	5	3	6	1
4	2	3	1	5	6
5	1	6	4	3	2
6	3	2	5	1	4
1	5	4	6	2	3

2.

3	4	2	6	1	5
5	6	1	2	3	4
2	5	4	3	2	1
4	1	3	5	4	6
6	3	5	1	6	2
1	2	6	4	5	3

3.

5	1	6	2	4	3
4	3	2	1	6	5
1	2	3	4	5	6
6	4	5	3	1	2
2	6	1	5	3	4
3	5	4	6	2	1

4.

7	4	1	5	9	6	2	3	8
8	9	3	2	1	7	4	6	5
2	5	6	4	8	3	1	9	7
6	7	4	9	5	8	3	2	1
1	3	5	7	4	2	6	8	9
9	8	2	6	3	1	5	7	4
3	2	9	1	7	5	8	4	6
4	1	8	3	6	9	7	5	2
5	6	7	8	2	4	9	1	3

5.

2	1	3	5	4	9	7	8	6
9	7	8	3	2	6	1	5	4
5	4	6	8	7	1	9	2	3
1	2	4	6	5	3	8	9	7
6	8	7	9	1	4	5	3	2
3	9	5	7	8	2	4	6	1
4	6	1	2	9	5	3	7	8
8	3	9	1	6	7	2	4	5
7	5	2	4	3	8	6	1	9

6.

8	7	5	9	3	1	4	2	6
9	6	2	8	7	4	3	5	1
1	3	4	6	5	2	8	7	9
2	4	6	1	8	5	9	3	7
3	5	9	2	6	7	1	8	4
7	1	8	3	4	9	2	6	5
6	2	7	4	9	3	5	1	8
4	8	3	5	1	6	7	9	2
5	9	1	7	2	8	6	4	3

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7.

1	2	5	4	7	9	6	3	8
3	8	9	5	6	1	2	4	7
4	7	6	2	8	3	1	5	9
7	9	4	1	3	2	5	8	6
8	3	1	6	5	7	4	9	2
6	5	2	9	4	8	3	7	1
2	4	7	3	9	6	8	1	5
5	1	8	7	2	4	9	6	3
9	6	3	8	1	5	7	2	4

4.

7	9	1	2	6	3	4	5	8
6	5	3	1	8	4	2	7	9
4	8	2	9	5	7	3	6	1
5	1	7	6	2	8	9	3	4
3	2	4	7	1	9	6	8	5
8	6	9	4	3	5	7	1	2
9	4	5	3	7	1	8	2	6
1	7	6	8	9	2	5	4	3
2	3	8	5	4	6	1	9	7

5.

2	6	3	7	9	1	5	8	4
1	5	8	6	3	4	7	9	2
7	4	9	8	2	5	3	1	6
9	3	1	4	6	7	8	2	5
4	7	2	9	5	8	1	6	3
5	8	6	2	1	3	9	4	7
3	1	4	5	8	6	2	7	9
8	2	7	3	4	9	6	5	1
6	9	5	1	7	2	4	3	8

6.

2	5	8	4	7	3	6	9	1
3	4	1	8	6	9	2	7	5
7	6	9	1	2	5	8	3	4
6	9	7	5	8	1	3	4	2
5	2	3	7	9	4	1	8	6
1	8	4	6	3	2	9	5	7
9	7	5	3	1	6	4	2	8
8	1	2	9	4	7	5	6	3
4	3	6	2	5	8	7	1	9

7.

6	3	7	5	1	9	2	4	8
8	2	4	7	6	3	9	1	5
9	1	5	2	8	4	7	3	6
2	4	8	6	5	1	3	7	9
7	6	9	3	2	8	1	5	4
1	5	3	9	4	7	8	6	2
5	7	2	1	9	6	4	8	3
3	8	6	4	7	2	5	9	1
4	9	1	8	3	5	6	2	7

3.

3	2	6	8	5	4	7	9	1
8	7	5	3	9	1	6	4	2
1	4	9	7	6	2	3	5	8
7	9	1	2	8	6	5	3	4
2	6	4	1	3	5	8	7	9
5	3	8	4	7	9	2	1	6
4	1	7	6	2	3	9	8	5
6	5	3	9	1	8	4	2	7
9	8	2	5	4	7	1	6	3

LOGICAL THINKING

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LOGICAL THINKING

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LOGICAL THINKING

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25. Study the following arrangement carefully and answer the questions given below:

R J 3 D # T M 9 % P E \$ K 2 H A * N 5 I * 4 6 U 7 β I V 8 F

How many such consonants are there in the above arrangement, each of which is immediately preceded by a symbol but not immediately followed by a vowel?

- (a) None (b) One (c) Two
 (d) Three (e) More than three

REASONING ABILITY

SESSION - 15

❖ ATTENTION TO DETAIL - II

Directions for Q1 to Q5: Follow the directions given below to answer the questions that follow. Your answer for each question below would be

- (A) If ALL THREE items given in the question are exactly ALIKE.
 (B) If only the FIRST and SECOND items are exactly ALIKE.
 (C) If only the FIRST and THIRD items are exactly ALIKE.
 (D) If only the SECOND and THIRD items are exactly ALIKE.
 (E) If ALL THREE items are DIFFERENT.

1. 0452-9858762, 0452-9858762, 0452-9858762
 (a) A (b) B (c) C (d) D (e) E

2. NIINININN, NIININNNNN, NIINIININN
 (a) A (b) B (c) C (d) D (e) E

3. LLMLLKLMPUU, LLMLLKLMPUU, LLMLLKLMPUU
 (a) A (b) B (c) C (d) D (e) E

4. 4665.8009291, 4665.7999291, 4665.8009291
 (a) A (b) B (c) C (d) D (e) E

5. 808088080.8080, 808008080.8080, 808088080.8080
 (a) A (b) B (c) C (d) D (e) E

Directions for Q6: Select one of the options from the given choices.

- (A) If ALL THREE items in the given question are ALIKE.
 (B) If only the FIRST and the SECOND items are exactly ALIKE.
 (C) If only the FIRST and THIRD items are exactly ALIKE.
 (D) If only the SECOND and THIRD items are exactly ALIKE.
 (E) If all THREE Items are DIFFERENT.

6. MNMNNNNMM, MNMMNMNNN, MNMNNNNMM
 (a) A (b) B (c) C (d) D (e) E

7. If * stands for /
 / stands for -
 + stands for *
 and - stands for +
 Then $9/8*7+5-10=?$
 (a) 13.3 (b) 10.8 (c) 10.7 (d) 11.4

8. If * stands for /
 / stands for -
 + stands for *
 - stands for +
 Then $9/15*9+2-9=?$
 (a) 14.7 (b) 15.3 (c) 14.1 (d) 16.2

9. If * stands for /
 / stands for -
 + stands for *
 and - stands for +
 then $36/12*4+50-8=?$
 (a) -106 (b) -189
 (c) -1909 (d) None of these

10. In the following question, the following letters indicate mathematical operations as indicated below:

A: Addition

V: Equal to

S: Subtraction

W: Greater than

M: Multiplication

X: Less than

D: Division

Out of the four alternatives given to this question, only one is correct according to the above letter symbols. Identify the correct one.

- (a) 6 S 7 A 2 M 3 W 0 D 7
 (b) 6 A 7 S 2 M 3 W 0 A 7
 (c) 6 S 7 M 2 S 3 W 0 M 7
 (d) 6 M 7 S 2 A 3 X 0 D 7

11. If * stands for -

- / stands for +
 + stands for /
 and - stands for *

Then which of the following is TRUE?

- (a) $16/8*6+90-12 = 23.2$
 (b) $8*12/6+90-12 = 7.2$
 (c) $16*6/8+16-12 = -4.1$
 (d) $12*16/6+90-12 = 8$

LOGICAL THINKING – SOLUTION

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REASONING ABILITY

SESSION - 14

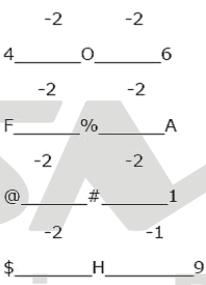
⌚ ATTENTION TO DETAIL - I

1. Ans: [c]
2. Ans: [d]
3. Ans: [d]
4. Ans: [a]
5. Ans: [d]
6. Ans: [c]

Consonant –Cosonant-Number
B -F -1
G -P -7
7. Ans: [b]

Letter-Vowels-Letter/Number
R - O - T
L - L = L
 $12 - 4 = 8^{\text{TH}}$
So the required element is %.
8. Ans: [d]

L - L = L
 $12 - 4 = 8^{\text{TH}}$
So the required element is %.
9. Ans: [d]


10. Ans: [b]

R - R = R
 $10 - 5 = 5^{\text{th}}$ from right end after deleting all the symbols is 5.

11. Ans: [b]

Remember, $R_{12} - R_4 = R = 6$
12. Ans: [a]

The numbers which are preceded by a symbol are \$2,*8.
13. Ans: [b]

2 is between B and H.
14. Ans: [c]

F 1 4 K B 5 9 # D R 2 @ 7 M G % H V T 3 8 * 1 J A

15. Ans: [b]
1 4 K B 5 9 # D R 2 @ 7 M G % H V T 3 8 * 1 J A
16. Ans: [b]
E G 4 B H 7 5 @ K 8 D N E Q Z \$ W 3 C 1 9 * L B 2 S 6
17. Ans: [b]

The letters used in the series are B Q E R D U K A P. sum of the positions of consonants according to alphabet = 2 + 17 + 18 + 4 + 11 + 16 = 68.
Similarly, sum of the vowels positions = 5 + 21 + 2 = 27.
Hence required difference = 68 - 27 = 41.
18. Ans: [e]
M K 3 \$ R E 5 F % T U J * 8 P H B N 2 1 S # A 3 7 D 4
19. Ans: [e]

Here the rule followed is : All the groups consist of three elements. Where, 1st element + 2 = 2nd element and 2nd element + 3 = 3rd element.
20. Ans: [b]

$(18 + 3) = 21^{\text{st}}$ letter from the right in the reverse series or, 21st letter from the left in the original series.
21. Ans: [d]
2 2 Z 1 3 9 4 8 Z 6 5 4 2 8 3 5 7 4 6 5 9 7 8 6 4 3 9 Z 4 6 5 2
22. Ans: [c]
F 6 Z 7 1 T 3 U X R 5 2 9 P 4 B A 7 8 D 4 6 F G H 2 P 3 Q R
23. Ans: [a]
F 6 Z 7 1 T 3 U X R 5 2 9 P 4 B A 7 8 D 4 6 F G H 2 P 3 Q R
24. Ans: [a]
W 3 + S 4 ? \$ P Q 6 R 7 . H T 5 8 # B F L § 4 * M J 9 G N 2
25. Ans: [d]

Here, we have to look for symbol-consonant arrangement:
R J 3 D # T M 9 % P E \$ K 2 H A * N 5 I * 4 6 U 7 B 1 V 8 F

REASONING ABILITY

SESSION - 15

⌚ ATTENTION TO DETAIL - II

1. Ans: [a]

ALL THREE items given in the question are exactly ALIKE.
2. Ans: [c]

Only the FIRST and THIRD items are exactly ALIKE.
3. Ans: [a]

All the Three items are exactly ALIKE.
So correct option is A.

LOGICAL THINKING – SOLUTION

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4. Ans: [c]
Only the FIRST and THIRD items are exactly ALIKE.

5. Ans: [c]
Only the FIRST and THIRD items are exactly ALIKE.

6. Ans: [c]

7. Ans: [a]

$$9-8/7*5+10$$

$$9-40/7+10$$

$$93/7 = 13.3$$

8. Ans: [a]

$$9-15/9*2+9$$

$$=18-30/9$$

$$=132/9$$

$$=14.66$$

9. Ans: [a]

$$36-12/4*50+8$$

$$=44-150$$

$$=-106$$

10. Ans: [a] 11. Ans: [a] 12. Ans: [a]

13. Ans: [b] 14. Ans: 18 15. Ans: [d]

16. Ans: [b] 17. Ans: [a] 18. Ans: [d]

19. Ans: [b] 20. Ans: [d] 21. Ans: [c]

22. Ans: [a] 23. Ans: [e] 24. Ans: [d]

25. Ans: [b] 26. Ans: [e] 27. Ans: 13

28. Ans: 13 29. Ans: 18 30. Ans: 3



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LOGICAL THINKING – SOLUTION

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18. Ans: [b]

The pattern of the series is illustrated below,

$$\begin{array}{ccc} & +4 & \\ \text{BHE} & : & \text{FLI} \\ +4 & & +4 \\ +4 & & +4 \end{array} \quad \begin{array}{ccc} & +4 & +4 \\ \text{JPM} & : & \text{NTQ} \\ & +4 & \end{array}$$

19. Ans: [c]

The pattern of the series is illustrated below,

$$\begin{array}{ccc} & +4 & \\ \text{D} & 4 & \text{T} \\ -4 & & \end{array} \quad \text{:} \quad \begin{array}{ccc} & +5 & \\ \text{L} & 5 & \text{R} \\ -5 & & \end{array} \quad \text{:} \quad \begin{array}{ccc} & +5 & \\ \text{Q} & 5 & \text{M} \\ -5 & & \end{array}$$

20. Ans: [d]

The analogy/relationship from the L.H.S. of the question can be identified as shown below

$$\begin{array}{l} 06 \rightarrow 60 \\ \swarrow \\ 120 \rightarrow 210 \end{array}$$

Letters in the 1st and 2nd positions are interchanged as shown above.

Hence, the answer is option (d).

21. Ans: [a]

The pattern of the series is illustrated below,

$$\begin{array}{cccc} \text{E} & \text{H} & \text{K} & \text{N} \\ +1 \downarrow & -1 \downarrow & +1 \downarrow & -1 \downarrow \\ \text{F} & \text{G} & \text{L} & \text{M} \end{array} \quad \text{Similarly,} \quad \begin{array}{cccc} \text{C} & \text{F} & \text{I} & \text{L} \\ +1 \downarrow & -1 \downarrow & +1 \downarrow & -1 \downarrow \\ \text{D} & \text{E} & \text{J} & \text{K} \end{array}$$

Hence, the answer is option [a].

22. Ans: [c]

The pattern of the series is illustrated below,

$$\begin{array}{cccc} \text{C} & \text{E} & \text{G} & \text{I} \\ +7 \boxed{+7} & +7 \boxed{+7} & +7 \boxed{+7} & +7 \boxed{+7} \\ \text{J} & \text{L} & \text{N} & \text{P} \end{array} \quad \begin{array}{cccc} \text{Q} & \text{S} & \text{U} & \text{W} \\ +7 \boxed{+7} & +7 \boxed{+7} & +7 \boxed{+7} & +7 \boxed{+7} \\ \text{X} & \text{Z} & \text{B} & \text{D} \end{array}$$

Hence, the answer is option [c].

23. Ans: [a]

In the pairs given, the first two letters are swapped.

Hence, the answer option is [a].

24. Ans: [c]

The relationship between the letters in the L.H.S. of the question increase by 4. So, the answer is MPNO.

Hence, the answer is option c.

25. Ans: [d]

The relationship between the terms can be identified as follows,

The difference is 2 between the 1st letters of the terms. This is same with the 2nd letters. The difference is 1 between the 3rd letters and this is same for the 4th letters as well.

Applying the same to UIOZ, we get WKPA.
Hence, the answer is option [d].

26. Ans: [a]

The relationship between the terms can be identified as follows,

The difference is 3 between the 1st letters of the terms. This is same with the 2nd and 3rd letters of the terms as well.

Applying the same to IHK, we get LKN.
Hence, the answer is option [a].

27. Ans: [c]

28. Ans: [c]

First letter increases by 2

The number decreases by 5

Third letter decreases by 2

29. Ans: [b]

30. Ans: [a]

VISUAL REASONING – I

SESSION – 10

ABSTRACT REASONING

- | | | | |
|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (c) | 4. (c) |
| 5. (d) | 6. (d) | 7. (d) | 8. (d) |
| 9. (b) | 10. (c) | 11. (a) | 12. (d) |
| 13. (a) | 14. (c) | 15. (b) | |

INPUT-TYPE DIAGRAMMATIC REASONING

16. Ans: [c]

For finding the code of the word DAILY, we know the codes for the letter A, I, L, Y are q, a, l, z respectively. But code for D is not known. So the coding of DAILY will definitely have all the four letters. q, a, l, z being the codes for A, I, L, Y. Now, option (a) is not our answer because 'a' has been repeated twice. Option (b) is not our answer because I cannot be the code for D because I is not available in the coding of DESTINY. Option (d) is also not our answer where code 'l' is missing. Hence option (c) is our answer and code for D is t, which is also present in the coding of DESTINY.

17. Ans: [a]

The codes for the letters T, E, A, R are j, w, q, n respectively.

LOGICAL THINKING – SOLUTION

HSEM1BTECHFASTRACK0719

18. Ans: [D]
The first change deletes the K, next the U is changed to V and finally a P is inserted between the Z and the L.
19. Ans: [A]
The first change deletes the P, then the N is replaced with an M and finally the C and the first M are exchanged.
20. Ans: [B] 21. Ans: [D] 22. Ans: [B] 23. Ans: [D]
24. Ans: [A] 25. Ans: [D]

VISUAL REASONING – II

SESSION – 11

SPATIAL REASONING

- | | | | |
|---------|---------|---------|---------|
| 1. (a) | 2. (a) | 3. (a) | 4. (a) |
| 5. (d) | 6. (b) | 7. (d) | 8. (c) |
| 9. (d) | 10. (b) | 11. (d) | 12. (c) |
| 13. (a) | 14. (b) | 15. (b) | |

CUBES

16. Ans: [c]
The fig. (X) is similar to the Form VI. So, when a cube is formed by folding the sheet shown in fig. (X), then  is one of the faces of the cube. However, the cube in fig. (1) has two such faces and fig. (4) has a face which is completely shaded. So, these two cubes cannot be formed. Hence, only the cubes in figures (2) and (3) can be formed.
17. Ans: [a]
The fig. (X) is similar to the Form I. So, when the sheet shown in fig. (X) is folded to form a cube then one of the two half-shaded faces lies opposite to one of the blank faces and the other half-shaded face lies opposite to another blank face. The two remaining blank faces lie opposite to each other. Thus, both the cubes shown in figures (1) and (4) can be formed when the sheet shown in fig. (X) is folded. Also, though the cubes shown in figures (2) and (3) have faces that can appear adjacent to each other but the cube formed by folding the sheet in fig. (X) cannot be rotated to form either of the two. Hence, the cubes in figures (2) and (3) cannot be formed.
18. Ans: [d]
The given figure is similar to Form V. Therefore, when this figure is folded to form a cube then the face bearing six dots will lie opposite the face bearing three dots.

19. Ans: [a]
The fig. (X) is similar to Form II. So, when the sheet shown in fig. (X) is folded to form a cube then the two half-shaded faces lie opposite to each other, the face bearing a circle lies opposite to one of the two blank faces and the

two remaining blank faces lie opposite to each other. Therefore, the cubes shown in fig. (4) which has the two half-shaded faces adjacent to each other, cannot be formed by folding the sheet shown in fig. (X). Also, the cube shown in fig. (2) has the face bearing a circle adjacent to two blank faces. This is not possible since there is one blank face opposite to the circle and one blank face opposite to the third blank face. Hence, only the cubes in figures (1) and (3) can be formed.

20. Ans: [d]
The fig. (X) is similar to the Form VII. So, when a cube is formed by folding the sheet shown in fig. (X), then  is one of the faces of the cube and this face lies opposite to a blank face. Also, a face bearing a square lies opposite to another blank face. The remaining two blank faces lie opposite to each other. Clearly, in the cube shown in fig. (1), the face consisting of the four symbols is not the same as that formed (as shown above). Hence, the cube in fig. (1) cannot be formed.
21. Ans: [b]
The fig. (X) is similar to the Form VIII. So, when a cube is formed by folding the sheet shown in fig. (X), then  and  are the two faces and these two faces lie opposite to each other. Also, the face bearing the 'x' sign lies opposite to the face bearing the black circle and the face bearing the white circle lies opposite to the face bearing the square (having a dot inside it). Now, the cubes in figures (2) and (4) consist of faces which are not formed when the sheet in fig. (X) is folded. Hence, these two cubes are not formed. Therefore, only the cubes in figures (1) and (3) are formed.
22. Ans: [a]
We shall assume the dice in fig. (ii) to be rotated so that the 5 dots appear at the same position as in fig. (i) i.e. on RHS face (i.e. on face II as per activity 1) and 1 dot appears at the same position as in fig. (i) i.e. on Front face (i.e. on face I). Then, from the, two figures, 2 dots appear on the top face (i.e. on face V) and 4 dots appear on the Bottom face (i.e. on face VI). Since, these two faces are opposite to each other, therefore, two dots are contained on the face opposite to that containing four dots.
23. Ans: [c]
From figures (ii) and (iii), we conclude that 1, 6, 3 and 4 dots lie adjacent to 5 dots. Therefore, 2 dots must lie opposite 5 dots. Conversely, 5 dots must lie opposite 2 dots.
24. Ans: [c]
From figures (ii) and (iii), we conclude that the alphabets C, D, B and F appear adjacent to the alphabet E. Therefore, the alphabet A appears opposite E. Conversely, E appears opposite A.