4.10 MEDIAN

Median ***is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or in descending order of their magnitudes. It divides the arranged series in two equal parts. Median is a position average, whereas the arithmetic mean is a calculated average.*** When a series consists of an even number of terms, median is the arithmetic mean of the two central items. It is generally denoted by M or M_d.

4.11 CALCULATION OF MEDIAN

4.11.1 When the Data is Ungrouped

Arrange the n values of the given variable in ascending (or descending) order of magnitudes.

Case I. When n is odd. In this case $\frac{n+1}{2}$ th term is the median,

Median: $$M_d \text{ or } M = \frac{n+1}{2} \text{ th term.}$$

Case II. When n is even. In this case, there are two middle terms $(n/2)$th and $(n/2 + 1)$th. **The median is the average of these two terms,** *i.e.*,

Median: $$M_d \text{ or } M = \frac{(n/2)\text{th term} + [(n/2)+1]\text{th term}}{2}$$

Example 25. *The number of runs scored by 11 players of a cricket team of a school are*

5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27.

Find the median.

Solution. Let us arrange the values in **ascending order**

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52. ... (*i*)

$\therefore$ **Median:** $M = \left(\frac{n+1}{2}\right)$**th value** $= \left(\frac{11+1}{2}\right)$th value $=$ 6th value.

Now, the **6th value in the data is 27.**

$\therefore$ **Median = 27 runs.**

Example 26. *The weights (in kilogram) of 15 students are as follows:*

31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30.

Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.

Solution. Weights of 15 students in **ascending order is**

27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 44, 45.

Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ **item** $= \frac{15+1}{2}$ **= 8th item = 35 kg.**

When 44 is replaced by 46 and 27 by 25, then the new data in ascending order is

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 46, 45.

New median = 8th item = 35 kg.

Example 27. *Find the median of the following items:*

6, 10, 4, 3, 9, 11, 22, 18.

Solution. Let us arrange the items in ascending order.

3, 4, 6, 9, 10, 11, 18, 22.

In this data, the number of items is $n = 8$, which is even.

$\therefore$ **Median: M = average of** $\left\{\left(\frac{n}{2}\right)\text{th and } \left(\frac{n}{2}+1\right)\text{th}\right\}$ **terms**

$= \text{Average of } \left(\frac{8}{2}\right)\text{th and } \left(\frac{8}{2}+1\right)\text{th terms}$

$=$ average of 4th and 5th terms

$= \frac{9+10}{2} = \frac{19}{2} =$ **9.5.**

Example 28. *The following table represents the marks obtained by a batch of 12 students in certain class tests in Statistics and Physics.*

Sr. No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Marks (Statistics)	*53*	*54*	*32*	*30*	*60*	*46*	*28*	*25*	*48*	*72*	*33*	*65*
Marks (Physics)	*55*	*41*	*48*	*49*	*27*	*25*	*23*	*20*	*28*	*60*	*43*	*67*

Indicate in which subject is the level of achievement higher?

Solution. ***The level of achievement is higher in that subject for which the median marks are more.***

Let us arrange the marks in the two subjects in ascending order,

Sr. No.	1	2	3	4	5	6	7	8	9	10	11	12
Marks (Statistics)	25	28	30	32	33	46	48	53	54	60	65	72
Marks (Physics)	20	23	25	27	28	41	43	48	49	55	60	67

Since the number of students is 12, the marks of the middle-most student are the mean of the marks of 6th and 7th students.

$$\therefore \quad \text{Median marks in Statistics} = \frac{46 + 48}{2} = 47.$$

$$\text{Median marks in Physics} = \frac{41 + 43}{2} = 42.$$

Here the median marks in Statistics are greater than the median marks in Physics, therefore, the level of achievement of the students is higher in Statistics.

4.11.2 Calculation of Median for Grouped Data

CASE I. When the series is discrete.

In this case, the values of the variable are arranged in ascending or descending order of magnitudes. A table is prepared showing the corresponding frequencies and cumulative frequencies.

$$\text{Median: } M = \left(\frac{n+1}{2}\right)\text{th value}$$

If x is variable which takes the values $x_1, x_2, x_3, \ldots\ldots, x_n$ with respective frequencies $f_1, f_2, f_3, \ldots\ldots f_n$; then the median of the given data is calculated by the following.

WORKING RULE

Step I. Arrange the values of the variable in ascending or descending order of magnitudes.

Step II. Find the cumulative frequency (c.f.).

Step III. Find $\frac{N}{2}$, when $N = \Sigma f_i$.

Step IV. Find the cumulative frequency just greater than $N/2$ and determine the corresponding value of the variable.

Step V. The value obtained in **Step IV** above is the required **median.**

Example 29. *Calculate median for the following data:*

No. of students	*6*	*4*	*16*	*7*	*8*	*2*
Marks	*20*	*9*	*25*	*50*	*40*	*80*

Solution. Arranging the marks in ascending order and preparing the following table:

Table: Computation of Median

Marks	*Frequency*	*Cumulative Frequency*
9	4	4
20	6	10
25	16	26
40	8	34
50	7	41
80	2	
	$n = \Sigma f = 43$	

Example 32. *The following table gives the weekly expenditure of 100 families. Find the median weekly expenditure.*

Weekly Expenditure (in Rs.)	*Number of families*
0 – 10	*14*
10 – 20	*23*
20 – 30	*27*
39 – 40	*21*
40 – 50	*15*

Solution. Let us prepare a table which gives the frequencies and cumulative frequencies.

Table: Computation of Median

Weekly Expenditure (in Rs.)	*Number of families (frequency) f*	*Cumulative frequency*
0 – 10	14	14
10 – 20	23	37
20 – 30	27	64
30 – 40	21	85
40 – 50	15	100
		Here $n = \Sigma f = 100$

$\therefore$ Median = $\left(\frac{n}{2}\right)$th value = $\left(\frac{100}{2}\right)$th value = 50th value.

Median class = 20 – 30.

Here $\frac{n}{2} = 50$, $L = 20$, $C = 37$, $i = 10$.

$\therefore$ **Median** = $L + \frac{(n/2) - C}{f} \times i = 20 + \frac{50 - 37}{27} \times 10 = 20 + \frac{13}{27} \times 10 = 20 + 4.815 =$ **24.815.**

Hence, **median = 24.815.**

Example 33. *The following table gives the marks obtained by 50 students in Economics. Find the median.*

Marks	*No. of students*	*Marks*	*No. of students*
10 – 14	*4*	*30 – 34*	*7*
15 – 19	*6*	*35 – 39*	*3*
20 – 24	*10*	*40 – 44*	*9*
25 – 29	*5*	*45 – 49*	*6*

Solution. Let us prepare the table showing the frequencies and cumulative frequencies.

Here $n = 50$, $\therefore \frac{n}{2} = 25$.

Example 35. *The scores on a reading comprehension text of 1000 students are given below:*

Scores (out of 75)	*Frequency*	*Scores (out of 75)*	*Frequency*
0 – 5	*6*	*25 – 30*	*250*
5 – 10	*12*	*30 – 35*	*185*
10 – 15	*50*	*35 – 40*	*110*
15 – 20	*120*	*40 – 45*	*32*
20 – 25	*225*	*45 – 50*	*10*

Find the median score.

Solution. **Table: Computation of Median**

Score (out of 75)	*Frequency (f)*	*Cumulative Frequency (c.f.)*
0 – 5	6	6
5 – 10	12	18
10 – 15	50	68
15 – 20	120	188
20 – 25	225	413
25 – 30	250	663
30 – 35	185	848
35 – 40	110	958
40 – 45	32	990
45 – 50	10	1000
		$n = \Sigma f = 1000$

$\therefore$ **Median** $= \left(\frac{n}{2}\right)$**th value** $= \left(\frac{1000}{2}\right)$th value = **500th value.**

Median class : 25 – 30. Here $\frac{n}{2} = 500$, $L = 25$, $C = 413$, $i = 5$.

$$\therefore \quad \text{Median} = L + \frac{n/2 - C}{f} \times i = 25 + \frac{500 - 413}{250} \times 5$$

$$= 25 + \frac{87}{50} = 25 + 1.74 = \mathbf{26.74.}$$

Example 36. *Calculate the mean and median for the following data:*

Height (in cm)	*No. of boys*	*Height (in cm)*	*No. of boys*
135 – 140	*4*	*155 – 160*	*24*
140 – 145	*9*	*160 – 165*	*10*
145 – 150	*18*	*165 – 170*	*5*
150 – 155	*28*	*170 – 175*	*2*