Absolute Extrema F(x,y)

Monday, 15 March 2021 12:02 PM

f must be a closed and bound.

$\{f(x) \mid x \in [a,b]\}$

Local Extrema

(i) Find C.P

(ii) evaluate your function at CP

Global Extrema

(i) Find all CP

(ii) Evaluate function at all CP

(iii) Evaluate function at endpoints (Boundary points) $[a,b]$

(iv) Max (all local max) ↳ Absolute Max.

Min (all local min) ↳ Absolute min.

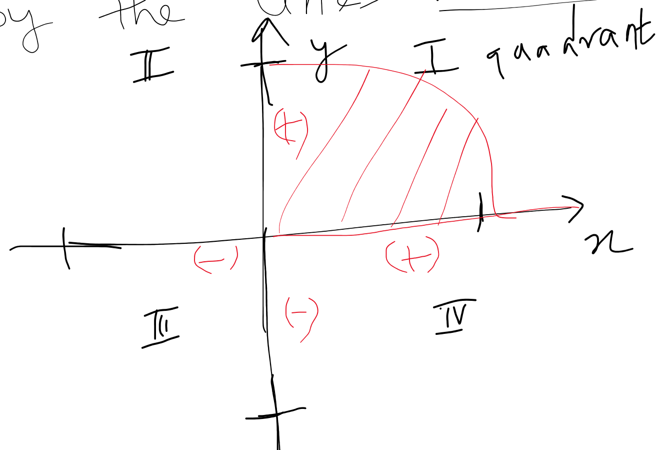
(v) Local maximum / Local minimum can be a Absolute max / Absolute min.

Problem:

Find absolute maximum and absolute minimum values of

$$\boxed{f(x,y) = 2 + 2x + 2y - x^2 - y^2}$$

on the triangular region in the 1st quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.



Step 1: Find critical points of $f(x,y)$:

$f_x = 2 - 2x$

$0 = 2 - 2x$

$\boxed{x = 1}$

$f_y = 2 - 2y$

$0 = 2 - 2y$

$\boxed{y = 1}$

∴ The C.P is $(1,1)$.

Evaluate your function at $(1,1)$.

$f(1,1) = 2 + 2(1) + 2(1) - (1)^2 - (1)^2$

$= 2 + 2 + 2 - 2$

$= 4.$

Step 2: Find Boundary points.

$y = 9 - x$

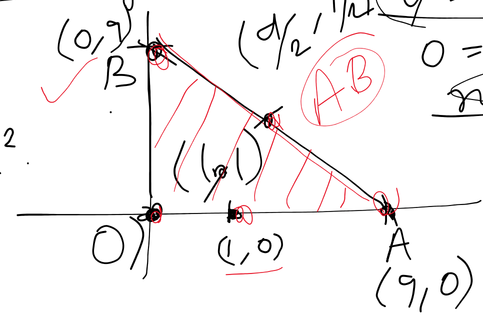
$0 = 9 - x$

$x = 9$

$(9, 0)$

On x-axis

$f(x, 0) = 2 + 2x - x^2.$



Evaluate f at OA

$$f(x,0) = 2 + 2x - x^2.$$

$$f_x(x,0) = 2 - 2x$$

$$0 = 2 - 2x$$

$$2x = 2$$

$\boxed{x = 1}$ ✓ Critical point.

Step 3:

C.P $f(1,0) = 3$

end points $f(0,0) = 2$

$f(9,0) = -61.$

Similarly: $f(0,y) = 2 + 2y - y^2$

$$f_y(0,y) = 2 - 2y$$

$\boxed{y = 1}$ C.P

$$f(0,1) = 3$$

$$f(0,0) = 2$$

$$f(0,9) = -61$$

... endpoints based on

$f(0,9) =$

We have to find endpoints based on segment AB.

$\boxed{y = 9 - x}$

$$f(x,y) = 2 + 2x + 2y - x^2 - y^2$$

$$f(x, 9-x) = 2 + 2x + 2(9-x) - x^2 - (9-x)^2$$

$$= 2 + \cancel{2x} + 18 - \cancel{2x} - x^2 - (81 + x^2 - 18x)$$

$$= 20 - x^2 - 81 - x^2 + 18x$$

$$f(x, 9-x) = 18x - 61 - 2x^2$$

find c.p

$$f_x(x, 9-x) = 18 - 4x$$

$$0 = 18 - 4x$$

$$\boxed{x = 9/2}$$

Suppose $\boxed{x = 9/2} \Rightarrow y = 9 - x$

$$y = 9 - 9/2$$

$$\boxed{y = 9/2}$$

c.p is $(9/2, 9/2)$

$$f(9/2, 9/2) = -41/2$$

$f(1,1) = 4$
$f(0,0) = 2$
$f(9,0) = -61$
$f(0,9) = -61$
$f(1,0) = 3$
$f(0,1) = 3$
$f(9/2, 9/2) = -41/2$

Absolute max $= 4$ at $(1,1)$.

Absolute min $= -61$ at $(0,9)$ a

nd $(9,0)$