CSE1003
Digital Logic and Design

Module 2
BOOLEAN ALGEBRA L3

Dr. S. Hemamalini
Professor
School of Electrical Engineering
VIT Chennai

Module 2
BOOLEAN ALGEBRA 8 hrs

Boolean algebra

- Properties of Boolean algebra
- Boolean functions
- Canonical and Standard forms
- Logic gates - Universal gates
- Karnaugh map - Don't care conditions
- Tabulation Method

CANONICAL AND STANDARD FORMS

- An arbitrary logic function can be expressed in the following forms.

(*i*) Sum of the Products (SOP)

(*ii*) Product of the Sums (POS)

- ***Product Term*****:** In Boolean algebra, the logical product of several variables on which a function depends is considered to be a product term. In other words, the AND function is referred to as a product term or standard product.
- ***Sum Term*****:** The logical sum of several variables on which a function depends is considered to be a sum term. An OR function is referred to as a sum term.

$f(A, B, C) = A + B + C$

CANONICAL AND STANDARD FORMS

- ***Sum of Products* (SOP):** The logical sum of two or more logical product terms is referred to as a sum of products expression. It is basically an OR operation on AND operated variables. This form is also called the Disjunctive Normal Form (DNF).

Y = AB + BC + AC

Y = A'B + BC + AC'

- In a sum-of-products form, a single overbar cannot extend over more than one variable, although more than one variable in a term can have an overbar.
- For example, $\bar{A}\,B\bar{C}$ can be a term in sum-of-products expression but not $\overline{ABC}$.

$Y = A'B + B'A$

Product of Sums (POS)

- The logical product of two or more logical sum terms is called a product of sums expression.
- It is an AND operation on OR operated variables.
- This form is also called the Conjunctive Normal Form (CNF).

Y = (A + B + C)(A + B′ + C)(A + B + C′)
Y = (A + B + C)(A′ + B′ + C′)

- In the product-of-sums (POS) expression, a single overbar cannot extend over more than one variable, although more than one variable in a term can have an overbar.
- For example, a product-of-sums (POS) expression can have the term $\bar{A} + B + \bar{C}$ but not $\overline{A + B + C}$.

CANONICAL AND STANDARD FORMS

Standard form: The standard form of the Boolean function is when it is expressed in sum of the products or product of the sums fashion.

Standard or Canonical Sum-of-products (SOP) Form

- If each term in the sum of products form contains all the variables (literals), then the expression is known as standard sum of products form or canonical sum of products form.
- In this form each product term contains all the variables of the function either in complemented or uncomplemented form.

$$f(A,B,C,D) = AB\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}D$$

$\cancel{f(A,B) = AB + B}$

- **MINTERM**
- Each of the product terms in the standard SOP form is called a minterm i.e., a product term which contains all the variables of the function either in complemented or uncomplemented form is called a minterm.

MINTERM

$2^2 = 4$

1. An n variable function can have in all 2^n minterms. A minterm is equal to 1 for only one combination of variables.

$f(A, B) = \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB = 1$

For example the term $A\bar{B}C\bar{D}$ is equal to 1 only when $A = 1, B = 0, C = 1$ and $D = 0$.

2. The sum of the minterms whose value is equal to 1 is the standard sum of products form of the function.

3. The minterms are often denoted as m_0 , m_1, m_2,..., where the subscripts are the decimal equivalent of the binary number of the minterms.

4. For minterms, the binary words are formed by representing each non-complemented variable by a 1 and each complemented variable by a 0.

For example, for the minterm ~~$A\bar{B}\bar{C}D$~~ binary number is 1001 and decimal equivalent is 9. Hence it is represented as $m9$

$A\bar{B}\bar{C}D$

$n = 3 \quad 2^3 = 8$

Minterms for 3 variables and their designation

0 – Complem
1 – Uncompl

A	B	C	Minterm	Designation
0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0
0	0	1	$\overline{A}\,\overline{B}C$	m_1
0	1	0	$\overline{A}B\overline{C}$	m_2
0	1	1	$\overline{A}BC$	m_3
1	0	0	$A\overline{B}\,\overline{C}$	m_4
1	0	1	$A\overline{B}C$	m_5
1	1	0	$AB\overline{C}$	m_6
1	1	1	ABC	m_7

Σ Notation

Σ notation is used to represent sum-of-products Boolean expressions.

Standard SOP form:

$$f(A,B,C) = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C$$

Another way of representing the function in canonical SOP form is by showing the sum of minterms for which the function value equals 1.

$$f(A,B,C) = m_1 + m_2 + m_3 + m_5$$

Another way of representing the function in canonical form is by listing the decimal equivalents of the minterms for which *f* = 1.

$$f(A,B,C) = \sum m(1,2,3,5)$$

Standard product of sum (POS) form

- **Maxterm:** A sum term which contains each of the n variables in either complemented or uncomplemented form is called a maxterm.

$f(A,B,C) = (\bar{A}+B+\bar{C})(A+\bar{B}+C)$

maxterm

$A - 0$

$\bar{A} - 1$

Maxterms for 3 variables and their designation

A	B	C	Maxterm	Designation
0	0	0	$A+B+C$	M_0
0	0	1	$A+B+\overline{C}$	M_1
0	1	0	$A+\overline{B}+C$	M_2
0	1	1	$A+\overline{B}+\overline{C}$	M_3
1	0	0	$\overline{A}+B+C$	M_4
1	0	1	$\overline{A}+B+\overline{C}$	M_5
1	1	0	$\overline{A}+\overline{B}+C$	M_6
1	1	1	$\overline{A}+\overline{B}+\overline{C}$	M_7

Maxterms

1. For an n-variable function, there will be at the most 2^n maxterms. A maxterm assumes the value 0 only for one combination of the variables.

For example, the term $A + \overline{B} + C + \overline{D}$ is 0 only when $A = 0, B = 1, C = 0$ and $D = 1$. For all other combinations it will be 1.

2. The product of maxterms whose value is 0 gives the standard or canonical product of sums form of the function.

3. Maxterms are often represented as M_0 , M_1, M_2,....,where the subscripts denote decimal equivalent of the binary number of the maxterms.

4. For maxterms, the binary words are formed by representing each non-complemented variable by a 0 and each complemented variable by a 1.

For example, for the maxterm $A + \overline{B} + \overline{C} + D$ binary number is 0110 and decimal equivalent is 6. Hence, it is represented as M_6 .

Standard product of sum (POS) form

- **Π Notation**

Standard POS form:

$$f(A,B,C) = (A+B+C)(\overline{A}+B+C)(\overline{A}+\overline{B}+C)(\overline{A}+\overline{B}+\overline{C})$$

Another way of representing the function in canonical POS form is by showing the product of maxterms for which the function value equals 0.

$$f(A,B,C) = M_0 \cdot M_4 \cdot M_6 \cdot M_7$$

Another way of representing the function in canonical POS form is by listing the decimal equivalents of the maxterms for which $f = 0$.

$$f(A,B,C) = \Pi(0,4,6,7)$$

CONVERTING EXPRESSIONS TO STANDARD SOP OR POS FORMS

$f(A,B) = A\ (B+B')$

Converting SOP Form to Standard SOP Form

1. Find the missing literal in each product term if any.

2. If one or more variables are missing in any term, expand that term by multiplying it with the sum of each one of the missing variables and its complement.

For example, in a three variable SOP function consider a term AB' . The third variable C is missing. So, to convert it into standard form multiply the term by $(C + C')$ and expand it as $AB'(C + C') = AB'C + AB'C'$

3. Remove repeated product terms if any.

Converting POS Form to standard POS Form

1. Find the missing literals in each sum term if any.

2. If one or more variables are missing in any sum term, expand that term by adding the products of each of the missing variable and its complement.

3. For expanding, apply rule $A + BC = (A + B)(A + C)$. For example, in a three variable POS function consider a term $A + B'$. The third variable C is missing. So, to convert it into standard form add the term (CC') and expand it as $A + B' + CC' = (A + B' + C)(A + B' + C')$.

4. Remove repeated sum terms if any.

$$f(A,B,C) = (A+B') + CC' = \underline{\underline{(A+B'+C)}}\,\underline{\underline{(A+B'+C')}}$$

Simplifying logic circuits

$$Z = ABC + A\bar{B}\cdot(\overline{\bar{A}\bar{C}}) = ABC + A\bar{B}(\bar{\bar{A}} + \bar{\bar{C}})$$

$$= ABC + A\bar{B}(A+C) = ABC + A\bar{B}A + A\bar{B}C$$

$$= ABC + A\bar{B} + A\bar{B}C = AC(B+\bar{B}) + A\bar{B}$$

$$= AC + A\bar{B} = A(C+\bar{B})$$