35. The only value of x satisfying the equation $6\sqrt{\frac{x}{4+x}}-2\sqrt{\frac{4+x}{x}}=11,\ x\in R$ is

(a) $\frac{4}{35}$ (b) $\frac{16}{3}$ (c) $-\frac{4}{35}$ (d) $-\frac{16}{3}$

36. The number of real values of x satisfying the equation $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$ is

(a) 1 (b) 2 (c) 3 (d) 4

37. Let α,β be the roots of $x^2-x+p=0$ and γ,δ be the roots of $x^2-4x+q=0$. If $\alpha,\beta,\gamma,\delta$ are in G.P., find the values of p and q.

(a) $-2,-32$ (b) $-2,3$
(c) $-6,3$ (d) $-6,-32$

38. If one root of the quadratic equation $ax^2+bx+c=0$ is three times the other, find the relationship between a, b and c.

(a) $3b^2=16ac$ (b) $b^2=4ac$
(c) $(a+c)^2=4b$ (d) $\frac{a^2+c^2}{ac}=\frac{b}{2}$

39. If the roots of the equation $\left(a^2+b^2\right)x^2-2b(a+c)x+\left(b^2+c^2\right)=0$ are equal, then a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) Cannot be determined

40. For what value of c, the quadratic equation $x^2-(c+6)x+2(2c-1)=0$ has the sum of the roots as half of their product?

(a) 5 (b) – 4 (c) 7 (d) 3

SESSION – 12

FUNCTIONS –I

Properties of Functions:

Definition of a Function: A function is a rule or formula that associates each element in the set *X* (an input) to exactly one and only one element in the set *Y* (the output). Different elements in *X* can have the same output, and not every element in *Y* has to be an output.

Definition of the Domain of a Function: The set of all possible inputs of a function is defined as the **domain**. The domain of a real-valued function defined by a formula for y in terms of x will be the set of all x input-values that result in a real y output-value unless the domain of the function is further restricted.

Definition of the Range of a Function: The set of all possible outputs of a function is defined as the **range**. The range of a real-valued function defined by a formula for y in terms of x will be the set of all y output-values that result from the x input-values in the domain.

Function Notation: Given that f(x) is given by some formula containing x, f(B) will be the same formula with each x replaced by B.

Linear Function Definition: If a function may be written in the form f(x) = mx + b where x is the independent variable and m and b are constants, then f(x) represents a linear function. The constant m is defined as the slope and the point (0, b) represents the y-intercept. An equation in this form is known to be in *Slope-Intercept Form*.

Linear Function Slope Definition: Given that f(x) = mx + b, then m is defined as the slope where: $m=\frac{y_2-y_1}{x_2-x_1}$ for any two points (x_1, y_1) and (x_2, y_2) on the line. Graphically, the slope represents the change in y with respect to x on the graph of the line.

Linear Functions of Parallel Lines: If two linear functions are given by f(x) = m_1x + b and g(x) = m_2x + b_1 and m_1 = m_2, then the graphs of f(x) and g(x) will consist of two lines that are parallel to each other.

Linear Functions of Perpendicular Lines: If two linear functions are given by f(x) = m_1x + b and g(x) = m_2x + b, and m_1 = –1/m_2, then the graphs of f(x) and g(x) will consist of two lines that are perpendicular to each other.

Graphs of Even Functions: Given a function f(x), if f(*c*) = f(–*c*) for all *c* in the domain, then f(x) is an *even* function and its graph will have symmetry with respect to the y-axis.

Graphs of Odd Functions: Given a function f(x), if f(*c*) = – f(– *c*) for all *c* in the domain, then f(x) is called an *odd* function and its graph will have symmetry with respect to the origin. Symmetry with respect to the origin implies that a 180 degree rotation of the graph about (0,0) results in an identical graph.

Functions Shifted Left: Given a function f(x) and its graph and a value of c>0, the graph of f(x + c) will be a shift of the graph of f(x) left by "c" units. This is known as the *Left Shift Function Rule*.

Functions Shifted Right: Given a function f(x) and its graph and a value of c>0, the graph of f(x – c) will be a shift of the graph of f(x) right by "c" units. This is known as the *Right Shift Function Rule*.

Functions Shifted Up: Given a function f(x) and its graph and a value of c >0, the graph of f(x) + c will be a shift of the graph of f(x) up by "c" units. This is known as the *Vertical Shift up Function Rule.*

Functions Shifted Down: Given a function f(x) and its graph and a value of c > 0, the graph of f(x) – c will be a shift of the graph of f(x) down by "c" units. This is known as the *Vertical Shift down Function Rule*.

Function Reflected Across X-axis Given a function f(x) and its graph, the graph of g(x) = – f(x) will be a reflection of the graph of f(x) across the x-axis. This is known as the *X-axis Reflection Function Rule*.

Function Reflected Across Y-axis Given a function f(x) and its graph, the graph of g(x) = f(– x) will be a reflection of the graph of f(x) across the x-axis. This is known as the *Y-axis Reflection Function Rule*.

Function Vertically Stretched Or Shrunk Given a function f(x) and its graph and a value of $c > 0$, the graph of $g(x) = c \bullet f(x)$ will be a vertical stretch of the graph of f(x). This means that all y-values of g(x) will be equal to *c* times the respective y-values of f(x). This is known as the *Vertical Stretch Function Rule*.

Definition of a Polynomial Function If f(x) may be written in the form $a_1x^n + a_2x^{n-1} + a_3x^{n-2} + ... + a_n$, then f(x) is a polynomial function of degree n where $a_1, a_2, ... a_n$ are real coefficients. Linear functions are 1st degree polynomials and quadratic functions are 2nd degree polynomials.

Graphs of Polynomials Given a function f(x) is a polynomial, it's x-intercepts will be located at the x-values x = c such that f(c) = 0. Other solution points on the graph will be located between each two x-intercepts.

Standard Form of Quadratic Functions: Quadratic functions of the form $f(x) = ax^2 + bx + c$ may always be rewritten in the form $y = a(x - h)^2 + k$. Function shift rules may then be applied to state that the graph will be a vertical stretch of $y = x^2$ and will be shifted right, left, up, or down according to the values of h and k.

Graphs of Quadratic Functions in Form $f(x) = ax^2 + bx + c$: Given $f(x) = ax^2 + bx + c$, the graph will be a shift of $g(x) = ax^2$ (meaning it has the same shape), and will have a vertex at $x = - b/2a$, $y = f(- b/2a)$.

Property of The Vertex of a Quadratic Function: The vertex of $f(x) = ax^2 + bx + c$ will be the lowest point of the graph if a > 0 and will be the highest point of the graph if a < 0. The vertex represents the minimum value of the function for a > 0 and represents the maximum value of the function if a < 0.

Function Operations: Given two functions f(x) and g(x), the operations (f + g)(x), (f – g)(x), (fg)(x), and (f/g)(x) are defined in the following way:

- $(f + g)(x) = f(x) + g(x)$ and is sometimes denoted $f + g$
- $(f - g)(x) = f(x) - g(x)$ and is sometimes denoted $f - g$
- $(fg)(x) = f(x) \bullet g(x)$ and is sometimes denoted fg
- $(f/g)(x) = f(x)/g(x)$ provided $g(x) \neq 0$. This is sometimes denoted f/g

Function Composition: Given two functions f(x) and g(x), the function composition (fog)(x), is defined in the following way:

$(f \, o \, g)(x) = f[g(x)]$ and is sometimes denoted as $f \, o \, g$

In essence, composition implies that you input the entire formula of the second function in for each x-value of the the formula in the first function, assuming x is the variable used.

Definition of Inverse Functions: Given two functions f(x) and g(x), if (f o g)(x) = x and (g o f)(x) = x, then f(x) is the inverse of g(x) and g(x) is the inverse of f(x). Each of these functions reverses the operations of the other function in reverse order. In that sense, the inverse of f(x) will consist of the identical formula with x and y interchanged - the solution for y results in "reversing" all operations on x and thus results in the formula for the inverse function.

We denote the inverse of f(x) as $f^{-1}(x)$ and we denote the inverse of g(x) as $g^{-1}(x)$.

Domain and Range of Functions That Are Inverses of Each Other: Given two functions f(x) and g(x) are inverses of each other, then

The domain of f(x) will consist of the same interval as the range of g(x).

The range of f(x) will consist of the same interval as the domain of g(x).

One-To-One Requirement For f(x) To Have an Inverse Function: Given a function f(x), it will only have an inverse if and only if each y-value in it's range corresponds to only 1 x-value in it's specified domain. When this is the case that each y is obtained from only 1 x-value, we say *f(x) is a one-to-one function.*

Note that a graphical way to determine that f(x) is not one-to-one is to show that a horizontal line passes through more than 1 point. This is often referred to as the **Horizontal Line Test**.

1. Let f(x) = max(2x + 1, 3 – 4x), where x is any real number. Then the minimum possible value of f (x) is:
   (a) 1/3 (b) 1/2 (c) 2/3 (d) 5/3

2. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then, at x = 0, f(x) is:
   (a) maximized whenever a > 0, b > 0
   (b) maximized whenever a > 0, b < 0
   (c) minimized whenever a > 0, b > 0
   (d) minimized whenever a > 0, b < 0

3. For the function f(x) = 2x – 1, g(x) = 5 – x, and $h(x) = x^2 + x + 1$, find range of x for which $\min\{f(x^2), h(x)\} < 3$.
   (a) $-2 < x < \sqrt{2}$ (b) $-\sqrt{2} < x < \sqrt{2}$
   (c) $-2 < x < 2$ (d) $-\sqrt{2} < x < 2$

HSEM1BTECHSTANDARD0719

4. The function f(x) = |x – 2| + |2.5 – x| + |3.6 – x|, where x is a real number, attains a minimum at:
(a) x = 2.3 (b) x = 2.5
(c) x = 2.7 (d) None of these

5. Find the minimum value of f(x) = |3x – 2| + |2x – 3|.
(a) 5/6 (b) 5/3 (c) 5/2 (d) None of these

6. Find the minimum value of f(x) = max(k – x, |x| + k).
(a) k – 1 (b) k (c) 2k (d) None of these

7. Let f(x) = ax² + bx + c, where a, b and c are certain constants and a ≠ 0. it is know that f(5) = – 3f(2) and that 3 is the root of f(x) = 0. What is the other root of f(x) = 0?
(a) – 7 (b) – 4
(c) 2 (d) Cannot be determined

8. If f(x) = x³ – 4x + p and if f(0) and f(1) are of opposite sign, then which of the following is necessarily true?
(a) – 1 < p < 2 (b) 0 < p < 3
(c) – 2 < p < 1 (d) – 3 < p < 0

9. The domain of y = $\frac{1}{\sqrt{|x|-x}}$ is
(a) (0, ∝) (b) (∝, ∝) (c) (– ∝, 0) (d) (1, ∝)

10. If f(x) = log ($\frac{1+x}{1-x}$), then
(a) f(x) is even (b) $f(x_1).f(x_2) = f(x_1+x_2)$
(c) $\frac{f(x_1)}{f(x_2)} = f(x_1 - x_2)$ (d) f(x) is odd

11. What is the minimum and maximum value of $\frac{2x}{x^2+1}$ respectively?
(a) – 1, 1 (b) – 2, 1 (c) – $\frac{1}{3}$, 0 (d) None of these

12. Let f(x) = max (2x + 1, 3 – 4x), where x is any real number. Then, the minimum possible value of f(x) is:
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$

13. Minimum value of f(x) = |3 – x| + |2 + x| + |5 – x|, will be:
(a) 0 (b) 7 (c) 8 (d) 10

14. A function f(x) is defined as follows:
(i) f(1) = 1
(ii) f(2x) = 4 f(x) + 6
(iii) f(x + 2) = f(x) + 12x + 12
then calculate f(6).
(a) 106 (b) 96 (c) 86 (d) 76

15. Let f(x) = |x – 2| + |2.5 – x| + |3.6 – x|, where x is a real number, attains a minimum at
(a) x = 2.3 (b) x = 2.5
(c) x = 2.7 (d) None of these

16. Find for what value of a is: f(n) = (a – 2)n + 3a – 4 an even function?
(a) – 2 (b) 2 (c) 3 (d) 4

17. Let g(x) = max (5 – x, x + 2). The smallest possible value of g(x) is?
(a) 4.0 (b) 4.5 (c) 1.5 (d) None of these

18. Find the maximum value of the functions 1/(x² – 3x + 2)?
(a) 11/4 (b) 1/4 (c) 0 (d) None of these

19. Let g(x) be a function such that g(x + 1) + g(x – 1) = g(x) for every real x. Then, for what value of p is the relation g(x + p) = g(x) necessarily true for every real x ?
(a) 5 (b) 3 (c) 2 (d) 6

20. A function f(x) satisfies f(1) = 3600 and f(1) + f(2) +……f(n) = n²f(n), for all positive integers n > 1. What is the value of f(9)?
(a) 200 (b) 100 (c) 120 (d) 80

SESSION – 13

FUNCTIONS – II

1. If $f(x)=ax^2+bx+1,\ f(1)=4,\ f(-2)=1,$ find f(x).
(a) x^2-2x+1 (b) x^2-3x+1
(c) x^2+2x+1 (d) None of these

2. Find the domain of $f(x)=\sqrt{x}$ where f is a real function.
(a) $(-\infty,\infty)$ (b) $(0,\infty)$
(c) $(0,-\infty)$ (d) None of these

3. Find the range of $f(x)=\sqrt{16-x^2}$.
(a) (0, 4) (b) [– 4, 4] (c) (– 4, 0) (d) [0, 4]

4. Which of the following is an even function?
(a) $|x|-x$ (b) x^2+x^3 (c) $e^{3x}+e^{-3x}$ (d) $\frac{|x^2|}{3x}$

5. Let $f(x)=e^{2x}$ & $g(x)=\log x$, find $fog(2)$.
(a) 2 (b) 4 (c) 0 (d) e^4

6. Which of the following two functions have same domain?
$$f(x)=\frac{x^2+1}{x};\quad g(x)=|x|+1;\ h(x)=x^2+2x$$
(a) f and g (b) g and h
(c) f and h (d) None of these

7. Find the domain of the function $y=5e^{\sqrt{x^2-1}}\log(x-1)$.
(a) $(-\infty,\infty)$ (b) $R-(-1,1)$
(c) $(1,\infty)$ (d) $(-\infty,-1)$

8. How many onto functions can be defined from the set $A=\{1,2,3,4\}$ to $B=\{p,q,r\}$?

(a) 81 (b) 36 (c) 64 (d) 45

9. $n(A)=a, n(B)=b$ and $n(C)=c$. We can define a function that is 1 – 1 but not onto from A to B, a function that is onto but not 1 – 1 from B to C, and a function that is 1 – 1 but not onto from C to A. Arrange a, b, c in ascending order.

(a) a < b < c (b) b < c < a
(c) c < a < b (d) c < b < a

10. Find the range of $f(x)=\frac{x^2+6x+6}{x^2+6x+12}$.

(a) $[3,\infty]$ (b) $[-1,1]$
(c) $[1,1]$ (d) None of these

11. Find f(f(3)), if $f(x)=x^3-2x^2+x+1$.

(a) 31 (b) 1873 (c) 13 (d) 169

12. Find the domain of $f(x)=\frac{x}{2-x}$

(a) R – {2} (b) R
(c) (–2, 2) (d) None of these

13. Find the domain of $f(x)=\sqrt{x^2-25}$.

(a) $(-5,5)$ (b) $[0,5]$
(c) $R-(-5,5)$ (d) None of these

14. Let f be the exponential function and g be the logarithmic function, find fg(1)

(a) 1 (b) ∞
(c) 0 (d) None of these

15. Find the domain of the function $f(x)=\frac{1}{\sqrt{x^2-3x}}$.

(a) (0, 3) (b) $R-[0,3]$
(c) R (d) None of these

16. If $f(x)=x+2$, $g(x)=\frac{1}{x}$ and $h(x)=x^2$ then find $fogoh(3)$.

(a) $2\frac{1}{9}$ (b) $\frac{9}{19}$
(c) $\frac{1}{9}$ (d) None of these

17. The domain of the function $f(x)=\frac{|x+3|}{x+3}$ is

(a) R (b) $R-\{3\}$
(c) $R-\{-3\}$ (d) $R-(-3,3)$

18. If f is an even function and g is an odd function, then the function fog is _____

(a) an even (b) odd
(c) neither even nor odd (d) periodic function

19. Which of the following functions from z (set of integers) to z are bijections?

(a) $f(x)=x+5$ (b) $f(x)=x^5$
(c) $f(x)=3x+2$ (d) $f(x)=x^2+x+1$

20. If $f(x)=\sqrt{3-x}$ and $g(x)=\sqrt{1-x}$, then find the domain of $fog(x)$.

(a) $(-\infty,3)$ (b) $[3,\infty]$
(c) $(1,-3)$ (d) None of these