Solved Problem 2.112 (AU May/June 2006 - MA 1204)

Each of the 8 tubes of a radio set has a life length (in years) which may be considered as a random variable with Weibull distribution with parameter $\alpha = 25$, $\beta = 2$. If these tubes function independently of one another, what is the probability that no tube will have to be replaced during the first 2 month of service:

Solution

If X is a random variable of life length of each tube
Given X follow weibull distribution

$$f(x) = \alpha\, \beta\, x^{\beta-1}\, e^{-\alpha\, x^{\beta}} \qquad x > 0$$

Given $\alpha = 25,\ \beta = 2$

$$\Rightarrow f(x) = 50\, x\, e^{-25\, x^2}$$

P (Tube not replace the first 2 months)

$$= P\left(X > \frac{1}{6}\right)$$

$$\left(\because \text{ the parameter is in years } \Rightarrow 2 \text{ months} = \frac{1}{6} \text{ year}\right)$$

$$= \int_{\frac{1}{6}}^{\infty} 50\, x\, e^{-25\, x^2}\, dx$$

takes $\quad 25\, x^2 = t$

$$50\, x\, dx = dt$$

when $\quad x = \frac{1}{6}; \quad t = \frac{25}{6}$

$$x = \infty \quad t = \infty$$

$$= \int_{\frac{25}{6}}^{\infty} e^{-t}\, dt$$

$$= \left[\frac{e^{-t}}{-1}\right]_{\frac{25}{6}}^{\infty}$$

$$= -\left[0 - e^{-\frac{25}{6}}\right] = e^{-\frac{25}{6}} = 0.0155$$

Solved Problem 2.113

Let X be the service life of a semiconductor having Weibull with $\alpha = 0.025$ and $\beta = 0.5$ as parameter. Find the probability that the semiconductor will be working after 3000 hours.

Solution

X is Weibull distribution with p.d.f:

$$f(x) = \alpha\, \beta\, x^{\beta-1}\, e^{-\alpha\, x^{\beta}} \qquad x > 0$$

Given $\alpha = 0.025$; $\beta = 0.5$

$$\text{then} \qquad f(x) = (0.025)(0.5)\, x^{0.5-1}\, e^{-0.025\, x^{0.5}}$$
$$= 0.0125\, x^{-0.5}\, e^{-0.025 x^{0.5}}$$

P[semi - conductor working after 3000 hours]

$$P[X \geq 3000] = \int_{3000}^{\infty} f(x)\, dx$$
$$= \int_{3000}^{\infty} 0.0125\, x^{-0.5}\, e^{-0.025\, x^{0.5}}$$

$$\text{take} \qquad 0.025\, x^{0.5} = t$$
$$(0.025)(0.5)\, x^{-0.5}\, dx = dt$$

when $x = \infty$ $\quad t = \infty$
$x = 3000$ $\quad t = 1.369$

$$= \int_{1.369}^{\infty} e^{-t}\, dt$$
$$= \left[\frac{e^{-t}}{-1}\right]_{1.369}^{\infty}$$
$$= e^{-1.369} = 0.2543$$

Solved Problem 2.114

Let X be random variable of the failure (in minutes) of electric components with parameter $\alpha = \frac{1}{5}$, $\beta = \frac{1}{3}$. Find the

(i) expected time the product will last

(ii) the probability that the component will fail is less than 10 hours.

Solution

X is a random varible with weibull distribution.

$$\text{then} \qquad f(x) = \alpha\, \beta\, x^{\beta-1}\, e^{\alpha\, x^{\beta}}$$

Given $\alpha = \frac{1}{5}$, $\quad \beta = \frac{1}{3}$

(i) Now, the expected time the product will last is

$$
\begin{aligned}
E[X] &= \frac{1}{\alpha^{\beta}} \Gamma\left(\frac{1}{\beta}+1\right) \\
&= \frac{1}{\left(\frac{1}{5}\right)^{3}} \Gamma 4 \\
&= (0.2)^{-3} \Gamma 4 \\
&= 125 \times 3! \\
&= 125 \times 6 = 750 \text{ minutes}
\end{aligned}
$$

(ii) Probability of lasting less than 10 hours

i.e., is probability of lasting less than 600 minutes

W.K.T $\quad F(X) = P(X \leq x)$

$$
\begin{aligned}
P(X \leq 600) &= 1 - e^{-\alpha\, x^{\beta}} \\
&= 1 - e^{-\frac{1}{5}\, x^{\frac{1}{3}}} \\
&= 1 - e^{-\frac{1}{5}\,(600)} \\
&= 1 - e^{-1.687} \\
&= 1 - 0.1850 = 0.8149
\end{aligned}
$$

Let X be random variable of the life time of a certain kind of an energy back up battery (in hours) follows weibull distribution with $\alpha = 0.1$ and $\beta = 0.5$. Find

(i) The mean life time of these batteries.

(ii) The probability that the battery last more than 300 hours.

(iii) The probability that such a battery will not last 100 hours.

Solution

X is a random variable density the life time of any back up battery.

then $$f(x) = \alpha\, \beta\, x^{\beta-1}\, e^{-\alpha\, x^{\beta}} \qquad x > 0$$

Given $\alpha = 0.1$ and $\beta = 0.5$

$$\Rightarrow \qquad f(x) = (0.1)(0.5)\, x^{0.5-1}\, e^{-0.1\, x^{0.5}}$$

$$= 0.05\, x^{-0.5}\, e^{-0.1\, x^{0.5}}$$

(i) To find the mean life time

$$E[X] = \alpha^{-\frac{1}{\beta}} \left\lceil \left(\frac{1}{\beta} + 1\right)\right.$$

$$= (0.1)^{-\frac{1}{0.5}} \left\lceil \left(\frac{1}{0.5} + 1\right)\right.$$

$$= (0.1)^{-2} \lceil 3 \qquad \left[\text{put } 0.5 = \frac{1}{2}\right]$$

$$= \frac{1}{0.01}\, 2 \qquad [\because \lceil \overline{n} = n!]$$

$= 200$ hours

$$P(X > 300) = \int_{300}^{\infty} (0.05)\ x^{0.5}\ e^{-0.1\ x^{0.5}}\ dx$$

take $\quad u = 0.1\ x^{0.5}$

$$du = 0.05\ x^{-0.5}\ dx$$

When $\quad x = \infty \qquad u = \infty$

$$x = 300 \qquad u = 0.1(300)^{-0.5} = \sqrt{3}$$

$$= \int_{\sqrt{3}}^{\infty} e^{-u}\ du$$

$$= \left[\frac{e^{-u}}{-1}\right]_{\sqrt{3}}^{\infty}$$

$$= -\left[0 - e^{-\sqrt{3}}\right] = 0.177$$

$$P(X < 100) \int_{0}^{100} (0.05)\ x^{-0.5}\ e^{-0.01\ x^{0.5}}\ dx$$

take $\quad u = 0.1\ x^{0.5}$

$$= du = 0.05\ x^{-0.5}\ dx$$

$$x = 100 \qquad u = 1$$

$$x = 0 \qquad u = 0$$

$$= \int_{0}^{1} e^{-u}\ du$$

$$= -\left[e^{-1} - 1\right]$$

$$= \left[1 - e^{-1}\right] = 0.6321$$