Vellore Institute of Technology
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Planck's hypothesis

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Why Quantum Physics?

"Classical Physics":

- developed in 15th to 20th century;
- provides very successful description of "every day, ordinary objects"
  - motion of trains, cars, bullets,….
  - orbit of moon, planets
  - how an engine works,..
- subfields: mechanics, thermodynamics, electrodynamics,

"Quantum Physics":

- developed early 20th century, in response to shortcomings of classical physics in describing certain phenomena (blackbody radiation, photoelectric effect, emission and absorption spectra…)
- describes "small" objects (e.g. atoms and their constituents)

Some key events/observations that led to the development of quantum mechanics…

- Black body radiation spectrum (Planck, 1901)
- Photoelectric effect (Einstein, 1905)
- Model of the atom (Rutherford, 1911)
- Quantum Theory of Spectra (Bohr, 1913)
- Compton effect (Compton, 1922)

- Exclusion Principle (Pauli, 1922)
- Matter Waves (de Broglie 1925)
- Experimental test of matter waves (Davisson and Germer, 1927)

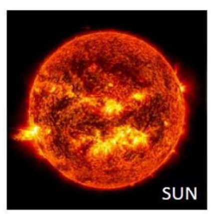
Thermal radiation

- Hot filament glows.
- Classical physics can't explain the observed wavelength distribution of EM radiation from such a hot object.
- This problem is historically the problem that leads to the rise of quantum physics during the turn of 20^{th} century

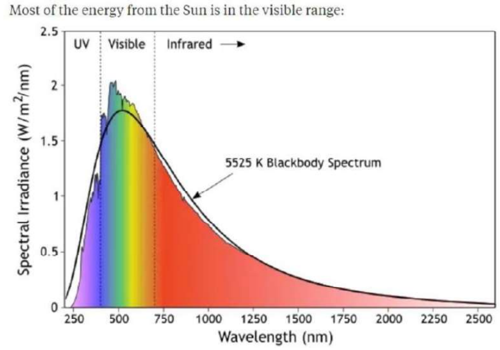
Black body radiation:

- **Black body** is an object that absorbs all the radiation incident upon it at all frequencies.
- Consider a piece of hot metal-It glows from red to yellow to white as it gets hotter and hotter.
- In fact it emits several other frequencies which the human eye cannot detect.

- When an object is at a constant temperature (thermal equilibrium with its surroundings), it absorbs and radiates energy in the same rate.
- Every object above 0K absorbs and radiates energy.

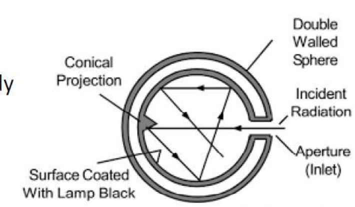


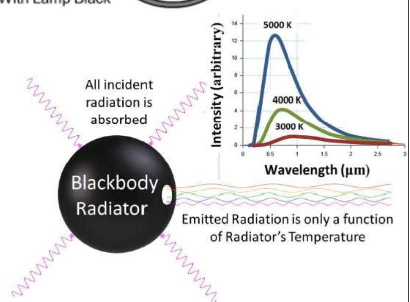
Most of the energy from the Sun is in the visible range:



Perfect Black body:

- In order to find the spectral distribution of radiation from a solid object at a constant temperature, the concept of perfect black body was introduced.
- A perfect black body completely absorbs radiation of all frequencies incident on it and emits all the radiation at a constant temperature.
- Since a perfect black body does not exist in nature, an approximation to it is made in the laboratory.
- Consider a Hollow, spherical, double walled cavity with a tiny hole on its surface leading into the cavity.
- Any radiation falling upon the hole enters into the cavity and gets trapped. It undergoes multiple reflections back and forth, until it gets absorbed.
- By using a heat source the body is maintained in constant temperature.
- The heated the walls of black body cavity radiates all frequencies that it absorbed.
- The black body spectrum indicates that the spectral intensities are higher at elevated temperatures.
- *Spectral distribution of energy depends only on the temperature of the body and not on its shape or elemental constituents.*



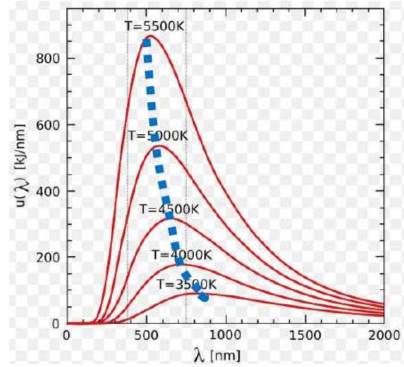


Wein's displacement law:

The peak in the black body spectrum shifts progressively towards shorter wavelengths as the temperature increases.

$$\lambda_{max} = \frac{b}{T}$$

Where, $b = 2.898 \times 10^{-3}$ m.K



Wien's approximation: The equation does accurately describe the short wavelength (high frequency) spectrum of thermal emission from objects, but it fails to accurately fit the experimental data for long wavelengths (low frequency) emission

The blackbody radiation curve for different temperatures peaks at a wavelength is inversely proportional to the temperature.

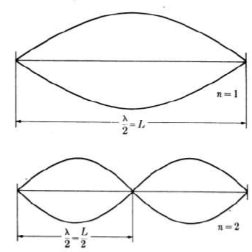
Rayleigh Jean's or classical approach:

- A cavity is at absolute temperature T.
- The walls of the cavity are perfect reflectors and radiation consist of standing electromagnetic waves.
- The condition for standing waves in such a cavity is that the path length from wall to wall must be whole number of half-wavelengths. So that the node occurs at each reflecting surface.
- The average energy per standing wave; $\bar{\epsilon} = k_B T$ → 1
- The density of standing waves in the cavity;

$$G(\text{v})\, d\text{v} = \frac{8\pi}{c^3}\text{v}^2 d\text{v} \quad \rightarrow 2$$

- Total energy per unit volume in the cavity within the frequency interval from v and v + d v is: $u(\text{v})d\text{v} = \bar{\epsilon}\, G(\text{v})d\text{v}$ → 3

$$u(\text{v})\, d\text{v} = \frac{8\pi k_B T}{c^3}\text{v}^2 d\text{v}$$



Standing waves which can be fitted between two perfectly reflecting walls forms a standing wave.

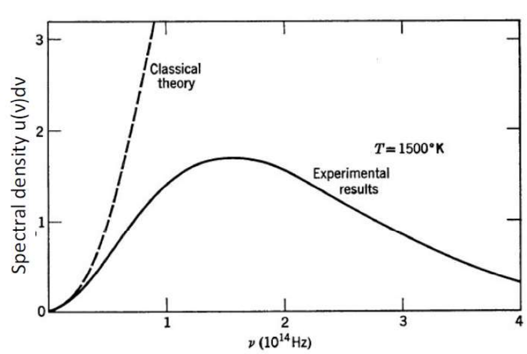
The Ultraviolet Catastrophe

Theory & experiment disagree wildly

Rayleigh Jean's law

$$u(\text{v})\, d\text{v} = \frac{8\pi k_B T}{c^3}\text{v}^2 d\text{v}$$

- As the frequency increases (or wavelength decreases) the spectral intensity of the black body radiation should increase indefinitely .
- Rayleigh-Jean's law hold good only for the black body curve at lower frequencies (or higher wavelengths). But fails to explain high frequency (wavelength) distribution towards the UV region.

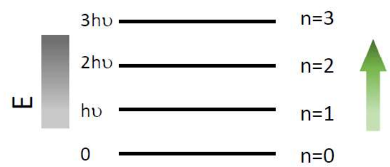


Planck's Hypothesis:

Planck made two bold and controversial assumptions concerning the nature of the oscillators in the cavity walls:

1. The energy of an oscillator can have only certain discrete values.

2. The oscillators emit or absorb energy when making a transition from one quantum state to another. The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation. If the transition is from one state to a lower adjacent state.



Planck's formulation:

- The oscillators in the cavity wall were limited to energies of E=nhv, where n= 1, 2....
- Maxwell-Boltzmann distribution law to find the number of oscillators; $\bar{\epsilon} = \frac{h\nu}{e^{h\nu/k_B T} - 1}$
- The density of standing waves in the cavity; $G(\nu)\, d\nu = \frac{8\pi}{c^3}\nu^2 d\nu$
- Total energy per unit volume in the cavity within the frequency interval from ν and ν + d ν is: $u(\nu)d\nu = \bar{\epsilon}\, G(\nu)d\nu$

$$u(\nu)\, d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_B T} - 1}d\nu$$

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At high frequencies $h\upsilon >> kT$ and $e^{h\upsilon/kT} \rightarrow \infty$

NO MORE ULTRAVIOLET CATASTROPHE

Which means that $U(\nu)d\nu \longrightarrow 0$ as observed.

At low frequencies

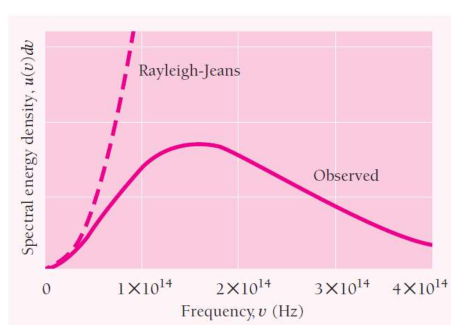
$$\frac{h\upsilon}{kT} << 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + ...$$

If x is small

$$e^x = 1 + x$$

$$\frac{1}{e^{h\upsilon/kT} - 1} = \frac{1}{1 + \frac{h\upsilon}{kT} - 1} = \frac{kT}{h\upsilon}$$



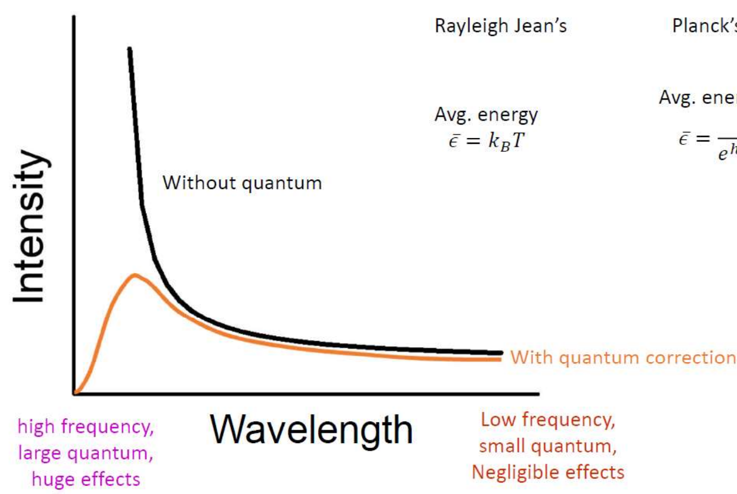
At low frequencies Planck's formula becomes

$$U(v)dv = \frac{8\pi h v^3}{c^3} dv(\frac{kT}{h\upsilon})$$

$$= \frac{8\pi kTv^2}{c^3} dv$$

which is Rayleigh Jeans formula.

How quanta overcomes the UV catastrophe



Rayleigh Jean's

Avg. energy
$\bar{\epsilon} = k_B T$

Planck's

Avg. energy
$\bar{\epsilon} = \frac{h\upsilon}{e^{h\upsilon/k_B T} - 1}$

Black-body Radiation Laws:

1. Rayleigh Jean's law $$U(\lambda) = \frac{8\pi c k_B T}{\lambda^4}$$
2. Planck's law $$U(\lambda) = \frac{8\pi hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$
3. Wein's displacement law $$\lambda_{peak} = \frac{b}{T}, b = 2.898\ 10^{-3} m.K$$
4. Stefan-Boltzmann law $$j = \sigma T^4, \sigma = 5.67\ 10^{-8} W m^{-2}\ K^{-4}$$

Planck radiation formulas

Frequency:

$$U(\nu) = \frac{8\pi\nu^3}{c^2}\left(\frac{1}{e^{\frac{h\nu}{k_B T}} - 1}\right)$$

Wavelength:

$$U(\lambda) = \frac{8\pi hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$