

Example 1

Compute the coefficient of correlation between X and Y , using the following data:

$X:$	1	3	5	7	8	10
$Y:$	8	12	15	17	18	20

x_i	y_i	x_i^2	y_i^2	$x_i y_i$
1	8	1	64	8
3	12	9	144	36
5	15	25	225	75
7	17	49	289	119
8	18	64	324	144
10	20	100	400	200
34	90	248	1446	582

Thus

$$n = 6$$

$$\Sigma x_i = 34, \Sigma y_i = 90$$

$$\Sigma x_i^2 = 248, \Sigma y_i^2 = 1446$$

$$\Sigma x_i y_i = 582$$

$$r_{XY} = \frac{n \Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{\{n \Sigma x^2 - (\Sigma x)^2\} \{n \Sigma y^2 - (\Sigma y)^2\}}}$$

$$= \frac{6 \times 582 - 34 \times 90}{\sqrt{\{6 \times 248 - (34)^2\} \{6 \times 1446 - (90)^2\}}}$$

$$= \frac{432}{\sqrt{332 \times 576}} = 0.9879$$

Example 2

Compute the coefficients of correlation between X and Y using the following data:

X : 65 67 66 71 67 70 68 69
 Y : 67 68 68 70 64 67 72 70

We effect change of origin in respect of both X and Y . The new origins are chosen at or near the average of extreme values. Thus we take $\frac{65+71}{2} = 68$ as the new origin for X and $\frac{64+72}{2} = 68$ as the new origin for Y . viz., we put $u_i = (x_i - 68)$ and $v_i = y_i - 68$ and find r_{UV} .

$X = x_i$	$Y = y_i$	$u_i = x_i - 68$	$v_i = y_i - 68$	u_i^2	v_i^2	$u_i v_i$
65	67	-3	-1	9	1	3
67	68	-1	0	1	0	0
66	68	-2	0	4	0	0
71	70	3	2	9	4	6
67	64	-1	-4	1	16	4
70	67	2	-1	4	1	-2
68	72	0	4	0	16	0
69	70	1	2	1	1	2
	Total	-1	2	29	39	13

$$r_{XY} = r_{UV} = \frac{n \sum uv - \sum u \cdot \sum v}{\sqrt{\{n \sum u^2 - (\sum u)^2\} \{n \sum v^2 - (\sum v)^2\}}}$$

$$= \frac{8 \times 13 - (-1) \times 2}{\sqrt{(8 \times 29 - 1)(8 \times 39 - 4)}} = \frac{106}{\sqrt{231 \times 308}} = 0.3974$$

Example 3

Find the coefficient of correlation between X and Y using the following data:

X : 5 10 15 20 25
 Y : 16 19 23 26 30

As the values of X are in arithmetic progression, we make the change of origin and scale, by choosing the middle most value 15 as the new origin and the common difference 5 as the new scale.

$$\rho_{XY} = r_{UV} = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 26}{10 \times 99} = 0.8424$$

Example 12

Ten competitors in a beauty contest were ranked by three judges as follows:

	Competitors									
Judges	1	2	3	4	5	6	7	8	9	10
A:	6	5	3	10	2	4	9	7	8	1
B:	5	8	4	7	10	2	1	6	9	3
C:	4	9	8	1	2	3	10	5	7	6

Discuss which pair of judges have the nearest approach to common taste of beauty.

Rank by A (U)	Rank by B (V)	Rank by C (W)	$d_1 =$ $U - V$	$d_2 =$ $V - W$	$d_3 =$ $U - W$	d_1^2	d_2^2	d_3^2
6	5	4	1	1	2	1	1	4
5	8	9	-3	-1	-4	9	1	16
3	4	8	-1	-4	-5	1	16	25
10	7	1	3	6	9	9	36	81
2	10	2	-8	8	0	64	64	0
4	2	3	2	-1	1	4	1	1
9	1	10	8	-9	-1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	-1	2	1	1	4	1
1	3	6	-2	-3	-5	4	9	25
Total:						157	214	158

$$r_{UV} = 1 - \frac{6 \sum d_1^2}{n(n^2 - 1)} = 1 - \frac{6 \times 157}{10 \times 99} = 0.0485$$

$$r_{VW} = 1 - \frac{6 \sum d_2^2}{n(n^2 - 1)} = 1 - \frac{6 \times 214}{10 \times 99} = -0.2970$$

$$r_{UW} = 1 - \frac{6 \sum d_3^2}{n(n^2 - 1)} = 1 - \frac{6 \times 158}{10 \times 99} = 0.0424$$

Since r_{UV} is maximum, the judges A and B may be considered to have common taste of beauty to some extent compared to other pairs of judges.

19. Find the coefficient of correlation between X and Y from the following data:

X : 10 14 18 22 26 30
 Y : 18 12 24 6 30 36

20. Calculate the coefficient of correlation between X and Y , by finding variances only, from the following data:

X : 21 23 30 54 57 58 72 78 87 90
 Y : 60 71 72 83 110 84 100 92 113 135

21. Calculate r_{XY} from the following data, where X represents production (in crore tons) and Y represents exports (in crore tons), using only the variances.

X : 55 56 58 59 60 60 62
 Y : 35 38 38 39 44 43 44

22. The following table gives the frequency of scores obtained by 65 students in a general knowledge test according to age groups. Measure the degree of linear relationship between age and general knowledge:

Test scores	Age in years			
	19	20	21	22
225	4	4	2	1
275	3	5	4	2
325	2	6	8	5
375	1	4	6	8

23. Compute the value of r_{XY} between X , the ages of husbands and Y the ages of wives from the following data:

X Y	15-25	25-35	35-45	45-55	55-65	65-75	Total
15-25	1	1	—	—	—	—	2
25-35	2	12	1	—	—	—	15
35-45	—	4	10	1	—	—	15
45-55	—	—	3	6	1	—	10
55-65	—	—	—	2	4	2	8
65-75	—	—	—	—	1	2	3
Total	3	17	14	9	6	4	53

24. Find the rank correlation coefficient between the ranks of the variable X and Y :

X : 10 15 12 17 13 16 24 14 22
 Y : 30 42 45 46 33 34 40 35 39

25. The competitors in a musical contest were ranked by the three judges A , B , C in the following order:

Rank by A : 1 6 5 10 3 2 4 9 7 8
 Rank by B : 3 5 8 4 7 10 2 1 6 9
 Rank by C : 6 4 9 8 1 2 3 10 5 7