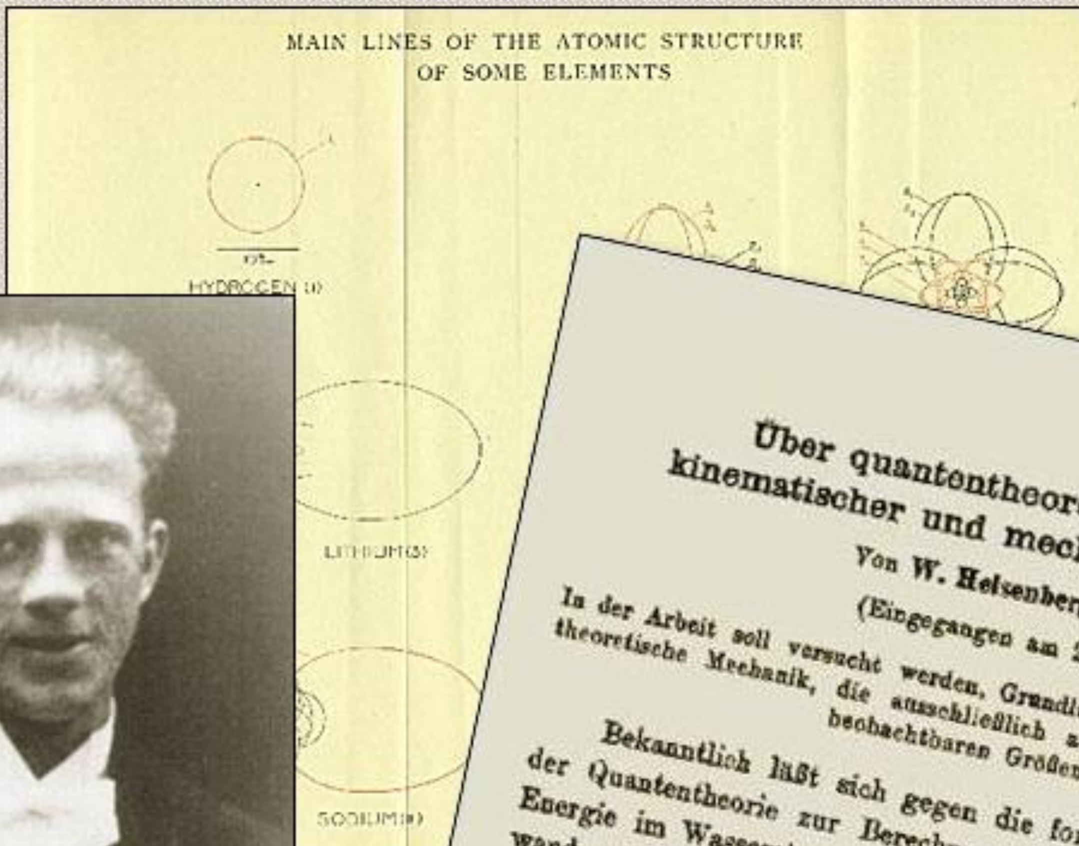
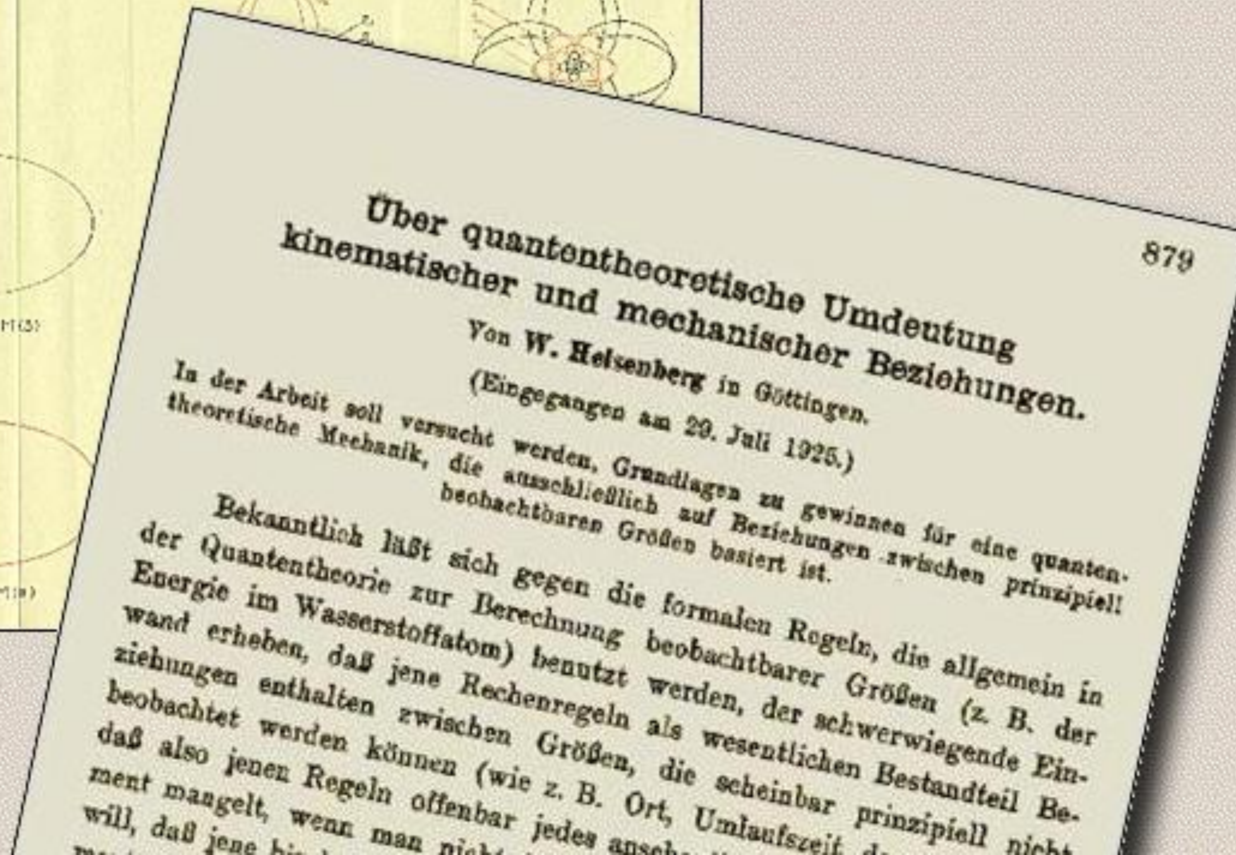
W. Heisenberg

Lecture 13

and Uncertainty





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Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

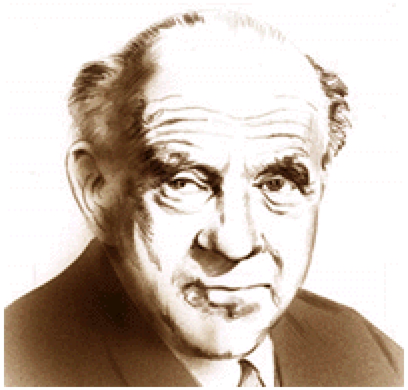
(Eingegangen am 29. Juli 1925.)

In der Arbeit soll versucht werden, Grundlagen zu gewinnen für eine quantentheoretische Mechanik, die ausschließlich auf Beziehungen zwischen prinzipiell beobachtbaren Größen basiert ist.

Bekanntlich läßt sich gegen die formalen Regeln, die allgemein in der Quantentheorie zur Berechnung beobachtbarer Größen (z. B. der Energie im Wasserstoffatom) benutzt werden, der schwerwiegende Einwand erheben, daß jene Rechenregeln als wesentlichen Bestandteil Beziehungen enthalten zwischen Größen, die scheinbar prinzipiell nicht beobachtet werden können (wie z. B. Ort, Umlaufszeit, d… daß also jenen Regeln offenbar jedes ansch… ment mangelt, wenn man nicht… will, daß jene…

Heisenberg realised that ...

- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way
- This introduces an unavoidable uncertainty into the result
- One can never measure all the properties exactly



Werner Heisenberg (1901-1976)

Heisenberg's Uncertainty Principle

uncertainty in momentum

$$\Delta x \, \Delta p \geq \frac{h}{4\pi}$$

uncertainty in position

The more accurately you know the position (i.e., the smaller Δx is) , the less accurately you know the momentum (i.e., the larger Δp is); and vice versa

Implications

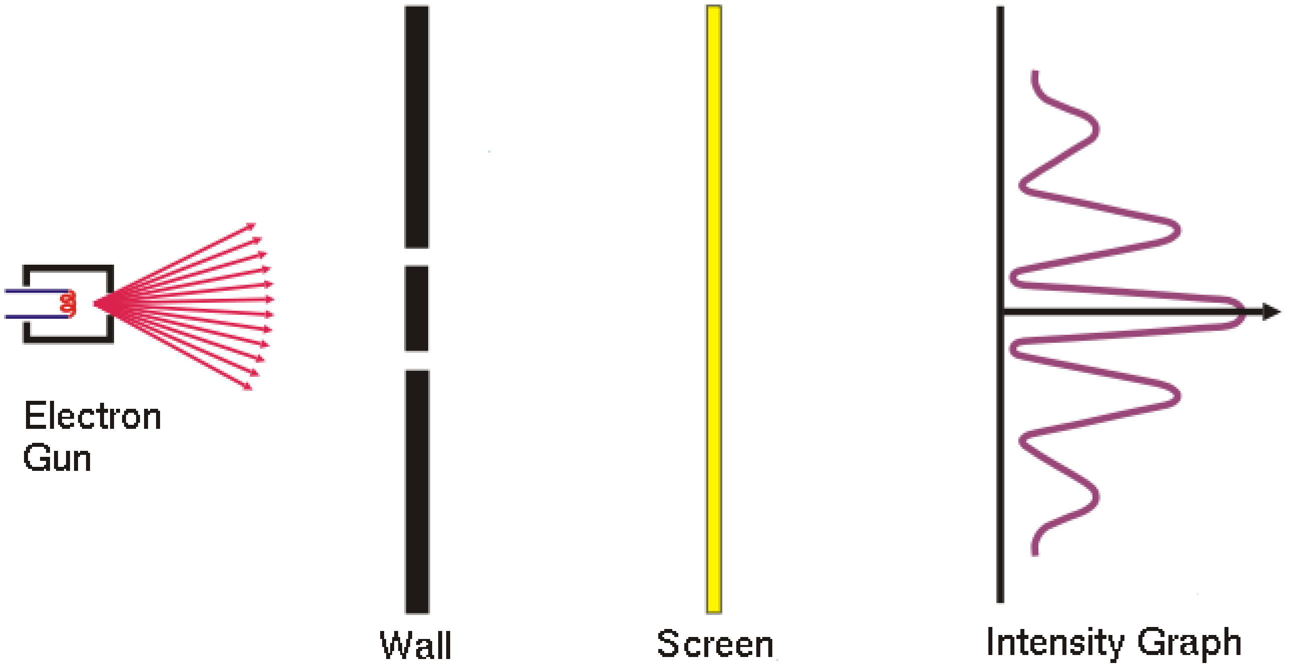
- It is impossible to know *both* the position and momentum exactly, i.e., $\Delta x=0$ and $\Delta p=0$
- These uncertainties are inherent in the physical world and have nothing to do with the skill of the observer
- Because h is so small, these uncertainties are not observable in normal everyday situations

Heisenberg's Uncertainty Principle involving energy and time

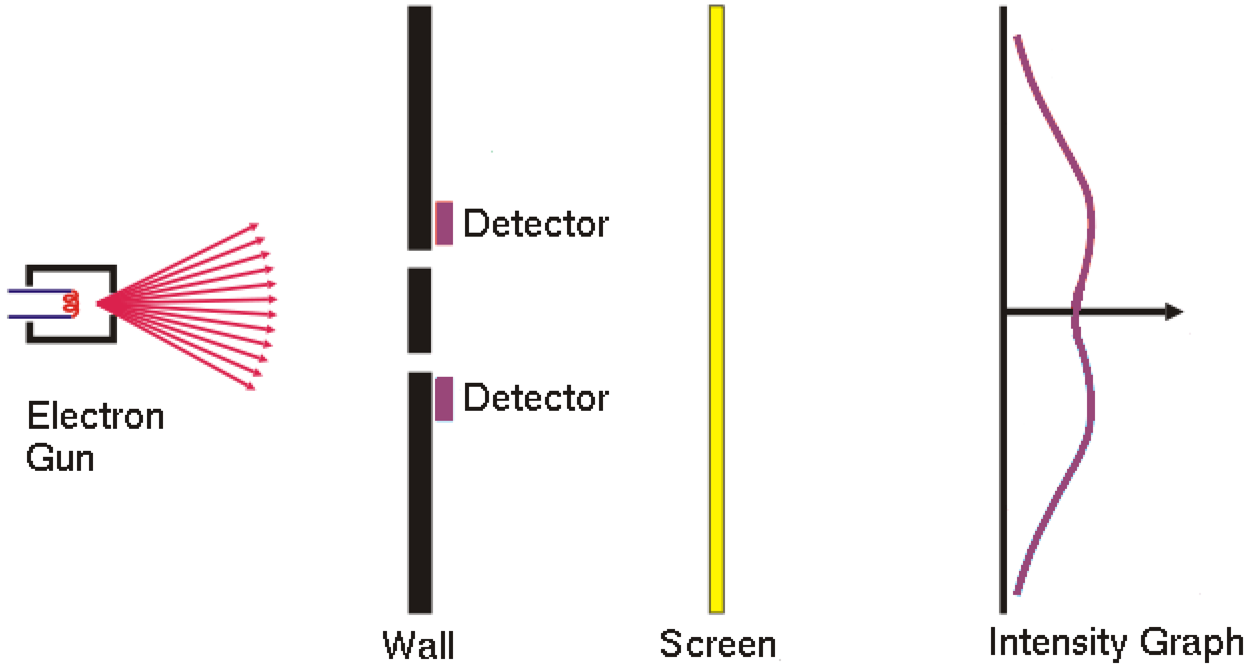
$$\triangle E \, \triangle t \geq \frac{h}{4\pi}$$

- The more accurately we know the energy of a body, the less accurately we know how long it possessed that energy
- The energy can be known with perfect precision ($\Delta E = 0$), only if the measurement is made over an infinite period of time ($\Delta t = \infty$)

Double-Slit Experiment:
cannot predict where electron would land

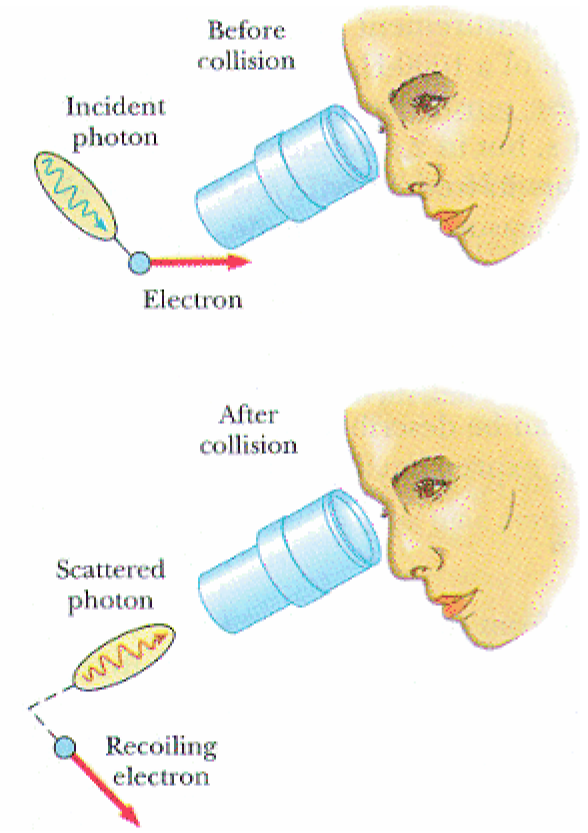


Double-Slit Experiment:
act of observation affects behaviour of electron



Measuring the position and momentum of an electron

- Shine light on electron and detect reflected light using a microscope
- Minimum uncertainty in position is given by the wavelength of the light
- So to determine the position accurately, it is necessary to use light with a short wavelength



Measuring the position and momentum of an electron (cont'd)

- By Planck's law $E = hc/\lambda$, a photon with a short wavelength has a large energy
- Thus, it would impart a large 'kick' to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength!

Fundamental Trade Off …

- Use light with short wavelength:
  - accurate measurement of position but not momentum
- Use light with long wavelength:
  - accurate measurement of momentum but not position

Example of Baseball

- A pitcher throws a 0.1-kg baseball at 40 m/s
- So momentum is 0.1 x 40 = 4 kg m/s
- Suppose the momentum is measured to an accuracy of 1 percent , i.e.,

$$\Delta p = 0.01\, p = 4 \times 10^{-2} \text{ kg m/s}$$

Example of Baseball (cont'd)

- The uncertainty in position is then

$$\triangle x \geq \frac{h}{4\pi \triangle p} = 1.3 \times 10^{-33} \text{ m}$$

- No wonder one does not observe the effects of the uncertainty principle in everyday life!

Example of Electron

- Same situation, but baseball replaced by an electron which has mass 9.11 x 10^{-31} kg

- So momentum = 3.6 x 10^{-29} kg m/s
  and its uncertainty = 3.6 x 10^{-31} kg m/s

- The uncertainty in position is then

$$\triangle x \geq \frac{h}{4\pi \triangle p} = 1.4 \times 10^{-4} \text{ m}$$