Compton Scattering

- Compton Effect
- Compton Theory (Derivation)
- Compton Experimental Verification

Scattering of X-rays

Two Kinds:

❑Coherent scattering or classical scattering or **Thomson scattering**
❑Incoherent scattering or **Compton scattering**

Coherent scattering:

❑X rays are scattered without any **change in wavelength.**
❑Obeys classical **electromagnetic theory**

Compton scattering: (inelastic sctattering)

❑Scattered beam consists of **two wavelengths.**

❑One is having same wavelength as the **incident beam**

❑The other is having a slightly longer wavelength called **modified beam.**

Compton Effect

A beam of monochromatic **radiation** (x-rays, γ-rays) of high frequency fall on **a low atomic no. substance**(carbon, graphite),the beam is scattered into two components.

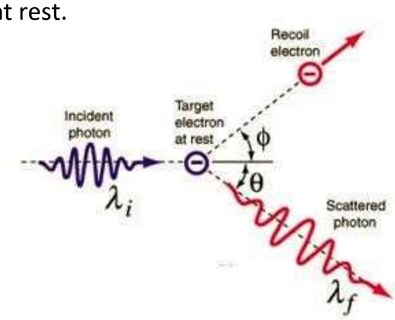
- **Modified radiation** – having lower frequency or larger wavelength
- **Un Modified radiation** – having same frequency or wavelength

This change in wavelength of the scattered X-ray is known as the Compton shift.

This effect is called **Compton Effect**.

Theory of Compton Scattering

- Compton treated this scattering as the interaction between X-ray and the matter as a particle collision between X-ray photon and loosely bound electron in the matter.
- Consider an X-ray photon of frequency ν striking an electron at rest.
- This Photon is scattered through an angle θ to x-axis.
- Let the frequency of the scattered photon be ν'.
- During the collision the photon gives energy to the electron.
- This electron moves with a velocity (v) at an angle ϕ to x-axis.



Energy Conservation:

<u>Total Energy before collision:</u>

Energy of the incident photon = $h\nu$

Energy of the electron at rest = m_0c^2

where 'm_0' is the rest mass of electron and 'c' the velocity of light.

Therefore total energy before collision = $h\nu + m_0c^2$

<u>Total energy after collision:</u>

Energy of the scattered photon = $h\nu'$

Energy of the Recoil electron = $\sqrt{p^2c^2 + m_0^2c^4}$

Therefore total energy after collision = $h\nu' + \sqrt{p^2c^2 + m_0^2c^4}$

<u>Conservation of Energy:</u>

Total energy before collision = Total energy after collision

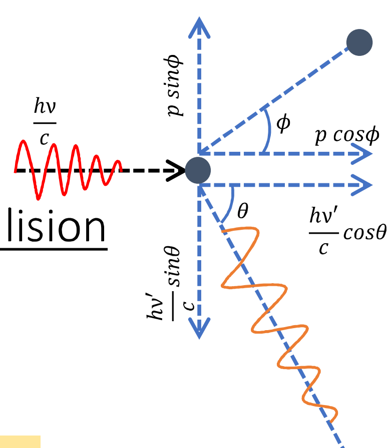
Momentum Conversation:

<u>Total momentum along x-axis before collision</u>

Momentum of incident photon along x axis = $\frac{h\nu}{c}$

Momentum of electron at rest along x axis = 0

Total momentum before collision along x axis = $\frac{h\nu}{c}$



<u>Total momentum along x-axis after collision</u>

The momentum is resolved along x axis and y axis.

momentum of scattered photon along x-axis = $\frac{h\nu'}{c}cos\theta$

momentum of recoil electron along x-axis = $p\,cos\phi$

Total momentum after collision along x-axis = $\frac{h\nu'}{c}cos\theta + p\,cos\phi$

Applying the law of conservation of momentum

Momentum before collision = momentum after collision

$$\frac{hv}{c} = \frac{hv'}{c}cos\theta + p\,cos\phi$$

$$\frac{hv}{c} - \frac{hv'}{c}cos\theta = p\,cos\phi$$

$$hv - hv'cos\theta = pc\,cos\phi$$

Total momentum along y-axis before collision

Initial momentum of photon along y axis =0
Initial momentum of electron along y axis = 0
Total momentum before collision along y axis= 0

Total momentum along y-axis after collision

momentum of scattered photon along y axis = $\frac{hv'}{c}sin\theta$
momentum of recoil electron along y axis = $p\sin\phi$

Total momentum after collision along y axis = $p\sin\phi - \frac{hv'}{c}sin\theta$

Momentum before collision = momentum after collision

$$0 = p\sin\phi - \frac{hv'}{c}sin\theta$$

$$pc\sin\phi = hv'\,sin\theta$$

From Energy conservation:

$$hv + m_0c^2 = hv' + \sqrt{p^2c^2 + m_0^2c^4} \quad \Rightarrow (hv - hv' + m_0c^2)^2 = p^2c^2 + m_0^2c^4$$

$$(hv - hv')^2 + m_0^2c^4 + 2(hv - hv')m_0c^2 = p^2c^2 + m_0^2c^4$$

$$p^2c^2 = (hv)^2 + (hv')^2 - 2(hv)(hv') + 2(hv - hv')m_0c^2 \ldots\ldots\ldots\ldots\ldots\ldots 1$$

From Momentum conservation:

$$p^2c^2cos^2\phi + p^2c^2sin^2\phi = (hv - hv'cos\theta)^2 + hv'sin^2\theta$$

$$p^2c^2 = (hv)^2 + (hv')^2 - 2(hv)(hv')cos\theta \ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equation 1 and 2:

$$\cancel{(hv)^2 + (hv')^2} - 2(hv)(hv')cos\theta = \cancel{(hv)^2 + (hv')^2} - 2(hv)(hv') + 2(hv - hv')m_0c^2$$

$$-2(hv)(hv')cos\theta = -2(hv)(hv') + 2(hv - hv')m_0c^2$$

$$2(hv)(hv')(1 - cos\theta) = 2(hv - hv')m_0c^2$$

$$hvv'\ (1 - cos\theta) = (v - v')m_0c^2$$

$$\frac{(v - v')}{vv'} = \frac{h}{m_0c^2}(1 - cos\theta) \Rightarrow \left(\frac{v}{vv'} - \frac{v'}{vv'}\right) = \frac{h}{m_0c^2}(1 - cos\theta)$$

$$\left(\frac{1}{v'} - \frac{1}{v}\right) = \frac{h}{m_0c^2}(1 - cos\theta) \Rightarrow \left(\frac{c}{v'} - \frac{c}{v}\right) = \frac{h}{m_0c}(1 - cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - cos\theta)}$$

$$\boxed{\lambda' - \lambda = \lambda_c\ (1 - cos\theta)}$$

Where, Compton wavelength $\boxed{\lambda_c = \frac{h}{m_0c} = 0.0243\ \text{Å}}$

Therefore the change in wavelength is given by; $d\lambda = \frac{h}{m_0c}(1 - cos\theta)$

- The change in wavelength **dλ does not depend** on the
  - (i) wavelength of the **incident photon**
  - (ii) Nature of the **scattering material**
- The change in wavelength **dλ depends** only on the **scattering angle.**

Case (1) When **θ=0** then,

$$d\lambda = \frac{h}{m_0c}(1 - 1);\ d\lambda = 0$$

Case (2) When **θ=90°** then,

$$d\lambda = \frac{h}{m_0c}(1 - 0);\ d\lambda = \frac{h}{m_0c}$$

Substituting the values for h,m_0 and c

$$d\lambda = \frac{h}{m_0c} = 0.0243\ \text{Å}$$

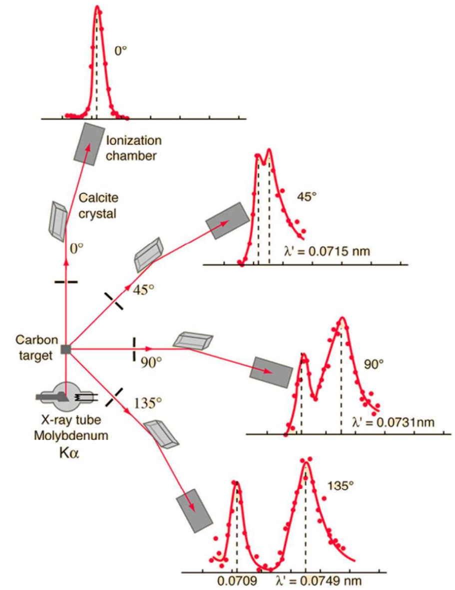
Case(3) When **θ=180°** then,

$$d\lambda = \frac{h}{m_0c}(1 - (-1));\ d\lambda = \frac{2h}{m_0c} = \ 0.0486\ \text{Å}$$

The change in wavelength is maximum at 180^0

Experimental verification

1. A beam of mono chromatic X ray beam is allowed to fall on the **scattering material.**
2. The scattered beam is received by a **Bragg spectrometer**.
3. The **intensity of the scattered beam** is measured for various angles of scattering.
4. A graph is plotted between the **intensity and the wavelength.**



Problems:

1. X-ray of wavelength 1.4 Å are scattered from a block of carbon. What will be the wave length of scattered X-rays at (i) 180° (ii) 90° (iii) 0°. At what scattering angle the maximum Compton shift is observed?

2. Explain the Compton scattering with a schematic. How does Compton shift vary with the photon scattering angle. In Compton experiment how much would be the maximum Compton shift, if electron is replaced with a neutron (mass of neutron is 2000 times that of electron.)

3. X-rays are scattered by Na crystal. Compare the wavelength of X-rays and the Compton wavelength of Na atom. What is change in wavelength you observe in this Compton scattering? [$M_{Na}=3.82\times10^{-26}$ Kg]

4. A photon of energy 3keV collides with an electron initially at rest. If photon emerges at an angle of 60^o, calculate kinetic energy of recoiling electron in electron volts.

5. A photon carries $2.00\ \text{x}10^{-14}$ J of energy. It undergoes Compton scattering in a block of carbon. What is the largest fractional change in energy the photon can undergo as a result?