Mean value theorem:

Suppose that the function $y = f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point in (a,b), then there is at least one number c in (a,b) so that

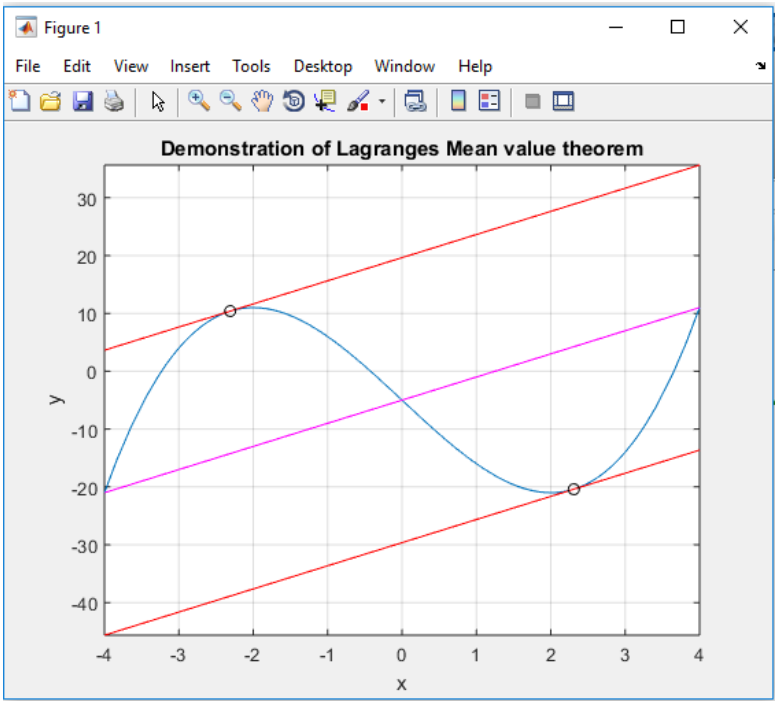
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The code given below illustrates the verification of Lagrange's theorem for the function $f(x) = x^3 - 12x - 5$ on the interval $[-4,4]$.

```
clear
clc
syms x y
f(x)=x^3-12*x-5;I=[-4,4]; % Input the function and interval
a=I(1);b=I(2);
Df=diff(f,x);
m=(f(b)-f(a))/(b-a); %Slope of Secant Line
c=solve(Df==m, x);
c=c(a<c&c<b);
disp('Values of c lying in the interval I are');
disp(double(c));
T=f(c)+m*(x-c); %Tangents at x=c
disp('The Tangent Lines at c are');
disp(vpa(y==T,4));
figure
fplot(f,I); grid on; hold on;
fplot(T, I, 'r');      %Tangent Lines
plot(c, double(f(c)), 'ko');
plot(I, double(f(I)), 'm');    %Secant Line
xlabel('x'); ylabel('y');
title('Demonstration of Lagranges Mean value theorem');
```

Output:

```
Values of c lying in the interval I are
   -2.3094
    2.3094
The Tangent Lines at c are
 y == 4.0*x + 19.63
 y == 4.0*x - 29.63
```



Exercise:

1. Using MATLAB find the tangent to the curves $y=\sqrt{x}$ at $x=4$ and show graphically.
2. Using MATLAB find the tangent to the curves $y=-\sin(x/2)$ at the origin and and show graphically.
3. Verify Rolle's theorem for the function $(x+2)^3(x-3)^4$ in the interval $[-2,3]$. Plot the curve along with the secant joining the end points and the tangents at points which satisfy Rolle's theorem.
4. Verify Lagrange's mean value theorem for the function $f(x)=x+e^{3x}$ in the interval $[0,1]$. Plot the curve along with the secant joining the end points and the tangents at points which satisfy Lagrange's mean value theorem.