

# **MODERN PHYSICS**

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## PREFACE

It is pleasure to present the revised edition of Modern Physics.

With great pleasure we first thank our beloved **Chancellor, Vice Presidents, Vice-chancellor, Pro Vice-chancellors, Registrar, Director (SAS)** for their kind encouragement and enthusiasm showered on us to bring out this book in a successful manner. We pay high regards to our-loving colleagues, for their untiring efforts and help at each and every step in completing this text. We express our sincere thanks to **M/s. Anuradha Publications**, who have accommodated us in every possible way to bring out this book to final shape.

Study of physics as a subject is given special attention at every stage of learning because it gives knowledge about many scientific facts and principles which are very useful to us. Keeping this idea in the mind the present book has been written in full conformity with the recent syllabus prescribed by the **VIT University** for first semester B.Tech and M.S. (S.E.) Modern Physics course.

The subject matter of each chapter of the book has been written in a simple language and in a very precise and lucid form so that the student can understand it easily without any difficulty. At the end of each chapter number of solved problems, exercise problems are given to enhance the mathematical abilities of the students which are given in accordance with University pattern. As an added advantage four model question papers are given at the end of the book to improve the confidence of the students to appear the University exams.

Experiments are arranged according to the new syllabus of first year B.Tech. Degree course. The procedure to perform every experiment is written in steps so that one can proceed very easily. Diagrams are self explanatory. Viva voce questions after each experiment covers the complete details about the experiment and provide necessary information about the principles involved. The details of few topics not mentioned in the syllabus have been put in a box.

We hope that the present edition will serve the purpose of the students.

**Authors**

# **Syllabus**

## **MODERN PHYSICS**

### **1: Quantum Physics**

**(13 Lectures)**

Black body radiation - Limitations of Classical theory - Basic idea of quantization- Planck's radiation formula - Compton effect, Experimental verification- Dual nature of electromagnetic radiation- de Broglie waves -Davison-Germer Experiment - Heisenberg uncertainty principle - Wave function and its properties -Schrödinger equation (time independent and dependent ) - particle in a 1D box-Eigen values and eigen function- Quantum mechanical tunneling (derivation) - Scanning tunneling microscope

### **2: Nano Physics**

**(05 Lectures)**

Quantum confinement: Introduction to Nanomaterials- Moore's Law - properties of nanomaterials - Quantum well - Wire - Dot - carbon nanotube; Applications of nanotechnology in sensors.

### **3: Laser Physics**

**(12 Lectures)**

Laser characteristics- Spatial and temporal coherence - Laser Principle - Einstein's coefficients - significance - population inversion - two level, three level, four level systems - laser threshold condition - Components of laser - modes (transverse and longitudinal)- He-Ne - CO<sub>2</sub> laser - Nd:YAG - Excimer laser - dye laser- Applications of lasers- Compact disc- writing and reading - Blue ray discs- Holography - recording and reconstruction.

### **4: Electromagnetic Wave Propagation**

**(06 Lectures)**

Maxwell's equations (Qualitative) - Wave equation (derivation) - EM waves- Phase velocity - Group velocity - Group index- wave guide theory- rectangular wave guide (TE and TM modes)

### **5: Fiber Optics**

**(09 Lectures)**

Light propagation through fibers (TEM mode) -Acceptance angle - numerical aperture - types of fibers - step index, graded index - single mode, multimode - attenuation - dispersion- intermodal - intramodal - application of fiber optics in communication - LED - Laser diode - Detector - pn - pin photodiode - endoscope.

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# **CHAPTER - 1**

## **QUANTUM PHYSICS**

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### **Introduction**

The end of the nineteenth century witnessed a remarkable and exciting period when number of experimental results were obtained about physical phenomena occurring in nature. Most of these results could be explained by Newtonian mechanics more popularly known as classical mechanics. Even the motion of heavenly bodies could be explained by classical mechanics.

However, when applied to electrons and nucleus, the classical mechanics gave only approximate results. Then it was found that concepts of classical mechanics are not applicable to microscopic world of atoms, electrons, protons and quarks etc., the ultimate constituents of matter. Likewise the quantization of energy is an idea entirely new to classical mechanics. The interaction of radiation with matter where photons are involved is another new concept to be incorporated in classical mechanics.

Such and many other disagreements required that a new formalism has to be developed which could be applied to bodies of atomic and nuclear dimensions. Quantum mechanics was found to be the formalism which revolutionized the thinking of physicists dealing with microscopic bodies.

First quarter of the twentieth century was an epoch making era. The founders of Quantum mechanics were mainly Max Planck, Albert Einstein, Louis de Broglie, Erwin Schrodinger, Werner Heisenberg, Dirac and Max Born. These and many others, during the period 1900 - 1930, developed Quantum Mechanics.



With the passage of time, at more and more experimental data were made available concerning the atomic and subatomic particles, elegant formalism of quantum mechanics was developed. It is now believed that classical mechanics is an approximate version of quantum mechanics.

Modern Physics essentially means the physics of the twentieth century and thereafter. Undoubtedly, the physics of the future will still refine our understanding of nature and world around us. We should be fortunate enough to be alive in this marvelous time of Modern Physics.

## 1.1 Black body Radiation

Radiation is emitted by every substance as a result of the vibration of its particles and the character of radiation depends on the nature and temperature of the substance. The ability of a body to radiate is closely related to its ability to absorb radiation as at any constant temperature, the body will be in thermal equilibrium with its surroundings and must be absorbing radiation at the same rate as it limits.

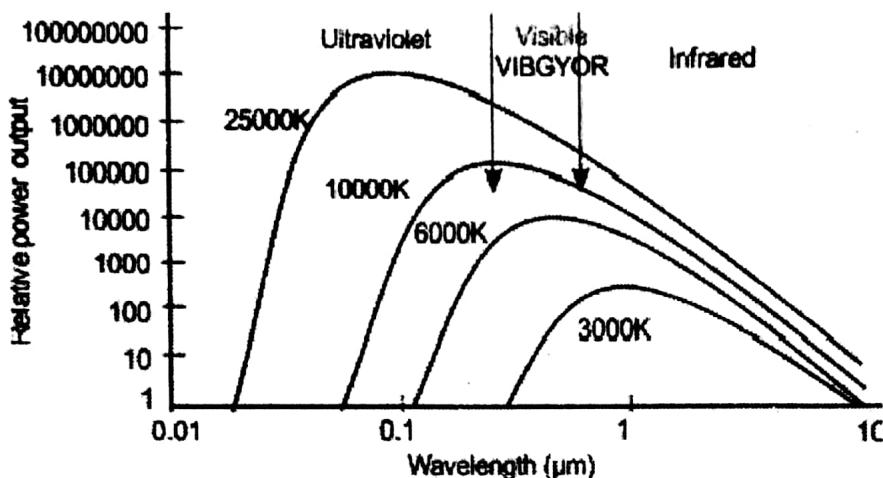
Different solids emit radiations at different rates at the same temperature; the rate is maximum when the solid is perfectly black i.e. behaves as a black body. **A black body can absorb all the radiations that falls on it.**

For practical purposes an isothermal cavity with a small aperture through which radiations from outside may be admitted and absorbed completely due to repeated reflections inside the enclosure is considered a black body. Radiations emerging from the small hole of such a hollow enclosure is, therefore, called 'black body radiations'.

The intensity of radiations i.e. total radiations emitted per unit surface area, from a black body depends only on temperature ( $T$ ) and is independent of the nature of the solid.

### 1.1.1 Spectral distribution of energy in the Black body Radiation

The radiation emitted by a black body is not confined to a single wavelength but is spread over a wide spectrum of wavelength. The experimentally observed dependence of the energy density on wavelength ( $\lambda$ ) at different temperatures is shown in figure 1.1. A close investigation reveals the following important facts:



**Figure 1.1:** Experimental dependence of the energy density on wavelength and temperature

- i) At a given temperature, the energy is not uniformly distributed in the radiation spectrum.
- ii) At a given temperature the density of radiant energy increases with increase in wavelength and becomes maximum at a particular wavelength ( $\lambda_m$ ). With further increase in wavelength, the density of radiant energy decreases.
- iii) An increase in temperature increases the energy of all spectral components. The short wavelength cutoff advance towards the origin as the temperature increases and the peak of the curve shifts to shorter wavelength, i.e. ( $\lambda_m$ ), (the wavelength for which energy emitted is maximum) decreases.

The shift of the peak of the curve was found to obey the following empirical relationship, commonly called Wien displacement law

$$\lambda_m T = \text{constant} \quad \dots (1.1)$$

With the help of classical thermodynamics, Wien has further shown that the amount of energy contained in the spectral region included within the wavelength  $\lambda$  and  $\lambda + d\lambda$  emitted by a black body is given by

$$E_\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda \quad \dots (1.2)$$

where A and B are constants.

Wien law works well only for short wavelengths. There are considerable deviations from Wien's law at long wavelength and high temperature. From equation (1.2), one obtains finite energy even for  $T = \infty$  (infinite). Rayleigh argued that it is unlikely that energy ( $E$ ) should be finite for  $T = \infty$ .

Later, Rayleigh and Jeans adopted a more rigorous method based on classical physics and came up with the following

$$E_\lambda d\lambda = C\lambda^{-4} T e^{-\lambda/T}$$

where  $C$  is another constant.

This Rayleigh-Jeans law describes the experimental curve quite well in the long wavelength region but the energy density  $E_\lambda$  increases rapidly as  $\lambda$  decreases and approaches infinity for shorter wavelengths. This failure, for explaining black body radiation, was such a crushing blow to classical physics that it is historically referred as the ultraviolet catastrophe. It led Max Planck to discover that only if light emissions is a quantum phenomenon can the correct formula for the energy density of black body radiation be obtained.

The failure of Rayleigh-Jeans law to explain the spectral energy curves of black body radiation indicated that the fundamental assumptions of classical physics required suitable modification.

### 1.1.2 Planck's Law of Radiation

Being well aware of the shortcomings of both the Rayleigh-Jeans and Wien Radiation Laws, Planck in October 1900, proposed remarkably well with the observed energy curves of the black body radiation. The formula could be written as

$$\rho(v) dv = \frac{av^3}{\exp\left(\frac{bv}{T}\right) - 1} \quad .. (1.3)$$

where  $a$  and  $b$  are numerical constants. This formula had no theoretical background and was obtained empirically by the trial and error method to explain the observed results. However, a few weeks later, i.e. in December 1900 Planck announced the theoretical derivation of his radiation law (1.3). For this, he proposed that a black body chamber was filled not only with radiation but also with molecules of a perfect gas, and dipole oscillators of molecular dimensions constituting the black body absorbed energy from the radiation

and transferred it partly or wholly to the gas molecules when the latter collided with them. This was a round about way of explaining the emission and absorption of radiation by the black body but there was no other way as the mechanism of emission and absorption of radiation by atoms and molecules by direct exchange was not known at that time. Planck calculated that the number of oscillators per unit volume was  $8\pi\nu^2/C^3$  so that the radiation density was given by

$$\rho(\nu) = \frac{8\pi\nu^2}{C^3} E(\nu) \quad .. (1.4)$$

where  $E(\nu)$  is the mean energy of the oscillator emitting radiation of frequency  $\nu$ . Now, if  $E(\nu)$  were to be determined by equipartition law, i.e.  $E(\nu) = k_B T$ , then equation (1.4) would lead to the Rayleigh-Jeans law, Planck gave up the hypothesis of continuous emission of radiation by oscillators and assumed that they emitted energy only when they acquired certain minimum energy  $E$  or an integral multiple of it,  $nE$ . That is to say an oscillator having an amount of energy  $nE$  could emit only after it had absorbed the amount  $(n+1)E$ , and after emitting the amount of energy  $E$  it reverted back to the previous state of energy  $nE$ . Thus the radiation of energy  $E$  could be emitted by only those oscillators which had energy  $E$ ,  $2E$ ,  $3E$ , ...,  $nE$ . Further  $E$  was directly proportional to  $\nu$ , i.e.

$$E = h\nu \quad .. (1.5)$$

where the constant of proportionality,  $h$ , was called the Planck's constant. The above idea can be expressed in the form of following revolutionary postulates which have become the foundation of quantum theory of radiation;

- i) The amount of energy emitted or absorbed by an oscillator is proportional to its frequency. Calling the constant of proportionality  $h$ , one can write for the change in oscillator energy

$$\Delta E = h\nu$$

- ii) An oscillator cannot have an arbitrary energy but must occupy one of a discrete set of energy states given by

$$E_n = nh\nu$$

where  $n$  is an integer or zero. It was assumed that the ground state corresponds to the zero energy state. The value of  $h = 6.62619 \times 10^{-34}$  Js. Planck's constant ( $h$ ) is a universal constant which plays important role in all quantum phenomena.

The previous picture of a continuum of oscillator states is now replaced by a discrete set of "quantised" states. Furthermore, the amount of energy emitted or absorbed is also quantised, since each quantum must correspond to the energy difference between two states of a given oscillator. Each quantum of electromagnetic energy is called a photon. Using Planck's quantum hypothesis, we now derive the famous Planck's radiation formula.

Let  $N$  be the total number of Planck's oscillator and  $E_T$  is their total energy, then the average energy per oscillator  $\bar{E}$ , is given by

$$\bar{E} = \frac{E_T}{N} \quad \dots (1.6)$$

Let  $N_0, N_1, N_2, \dots, N_n, \dots$  etc. be the number of oscillators having energies  $0, E, 2E, 3E, \dots, nE$  etc. respectively, then

$$N = N_0 + N_1 + N_2 + \dots + N_n + \dots = \sum_{n=0}^{\infty} N_n \quad \dots (1.7)$$

and  $E_T = 0 + EN_1 + 2EN_2 + \dots + nEN_n + \dots = \sum_{n=0}^{\infty} nEN_n \quad \dots (1.8)$

According to Maxwell-Boltzmann distribution function, the number of resonators or oscillators having energy  $nE$  will be

$$N_n = N_0 \exp\left(-\frac{nE}{kT}\right) \quad \dots (1.9)$$

Obviously, the higher energy states are thus less likely to be populated, and as

Putting  $y = \exp\left(-\frac{E}{kT}\right)$

$$N_n = \sum_{n=0}^{\infty} N_0 y^n$$

$$= N_0(1+y+y^2+\dots) = N_0 \left( \frac{1}{1-y} \right) \quad .. (1.10)$$

Similarly

$$\begin{aligned} E_T &= EN_0 \sum_{n=0}^{\infty} ny^n \\ &= EN_0(y+2y^2+3y^3+\dots) \\ &= EN_0 y(1+2y+3y^2+\dots) \\ &= \frac{EN_0 y}{(1-y)^2} \end{aligned} \quad .. (1.11)$$

Hence

$$\begin{aligned} \bar{E} &= \frac{E_T}{N} = \frac{EN_0 y}{(1-y)^2} \times \frac{(1-y)}{N_0} \\ &= \frac{Ey}{1-y} = \frac{E}{y^{-1}-1} \times \frac{E}{\exp^{E/kT}-1} \end{aligned} \quad .. (1.12)$$

Since  $E = hv$  and  $c = v\lambda$ , we can write  $E = hc/\lambda$

Equation 1.12 becomes

$$\bar{E} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \quad .. (1.13)$$

The number of oscillators per unit volume in the wavelength range  $\lambda$  and  $\lambda + d\lambda = 8\pi\lambda^{-4} d\lambda$

Hence the energy density of radiation between wavelength range  $\lambda$  and  $\lambda + d\lambda = (\text{Average density of an oscillator} \times \text{Number of oscillators per unit volume})$

$$\begin{aligned} E_\lambda d\lambda &= \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \times 8\pi\lambda^{-4} d\lambda \\ &= \frac{8\pi h c \lambda^{-5}}{e^{hc/\lambda kT} - 1} d\lambda \end{aligned} \quad .. (1.14)$$

In terms of frequency the energy density of radiation is

$$E_v dv = \frac{8\pi h v^3}{c^3 (e^{hv/kT} - 1)} dv \quad .. (1.15)$$

This is the **Planck's radiation formula**.

When  $v$  is large,  $\frac{hv}{kT} \gg 1$  and  $e^{\frac{hv}{kT}} - 1 = e^{\frac{hv}{kT}}$  and hence the Planck's radiation formula reduces to **Wien's law**.

When  $v$  is small,  $\frac{hv}{kT} \ll 1$  and  $e^{\frac{hv}{kT}} - 1 = \frac{hv}{kT}$  and hence the Planck's radiation formula reduces to **Rayleigh Jeans law**.

Obviously, the Planck's theory of radiation incorporates all that is valid from the older theories as special cases. It thus serves an excellent example of a conceptual advance which opened exciting new frontiers while still preserving much of the older physics.

The most important aspect of Planck's hypothesis was the new idea of discontinuous emission of radiation. According to classical physics, a system can absorb or emit any amount of energy whereas in Planck's hypothesis energy emitted by a black body is restricted by the relation  $E = hv$ . To this definite amount of energy,  $h\nu$ , Planck gave the name "quantum". Thus, while in classical physics energy is continuous variable, in quantum physics it is "quantised".

Let us now reflect for a moment on the significance of Planck's postulate. It states that the energy of a harmonic oscillator expressed in terms of its frequency of vibration, may on the average occur in units no smaller than ( $E = nhv$  where  $n = 0, 1, 2, \dots$ ). It does not say what is the energy of the oscillator at any instant, nor does it say how the value of  $h$  is to be calculated.

- i) Planck's postulate is empirical, i.e. it contains a parameter  $h$  whose value cannot be calculated except through the comparison of some expression in which it appears with the appropriate experimental results. There is no theory of Planck's constant. The only physical reason we can give for its existence is that it is invaluable for the elucidation of microscopic phenomena.

- ii) The postulate is statistical in that it refers only to the time averaged behaviour of the microscopic oscillator. Nothing is said about energy except that it varies directly with the frequency if evaluated over period of time long compared with  $v^{-1}$ .
- iii) The postulate does not suggest that a flaw exists in the Maxwell's theory of electromagnetism. It states only what energy an oscillator must have on the average if it is to remain in thermodynamic equilibrium with a radiation field. We should not conclude from this the latter must necessarily exhibit discrete energies when interacting with matter. We have verified the quantum hypothesis only for the emission of radiation by a heated solid!

## 1.2 Dual Nature of Electromagnetic Radiation

It is well known that the phenomena such as interference, diffraction and polarisation can be explained only on the basis of wave nature of light. But this wave picture of light could not explain the phenomena such as photoelectric effect, compton effect, etc.,

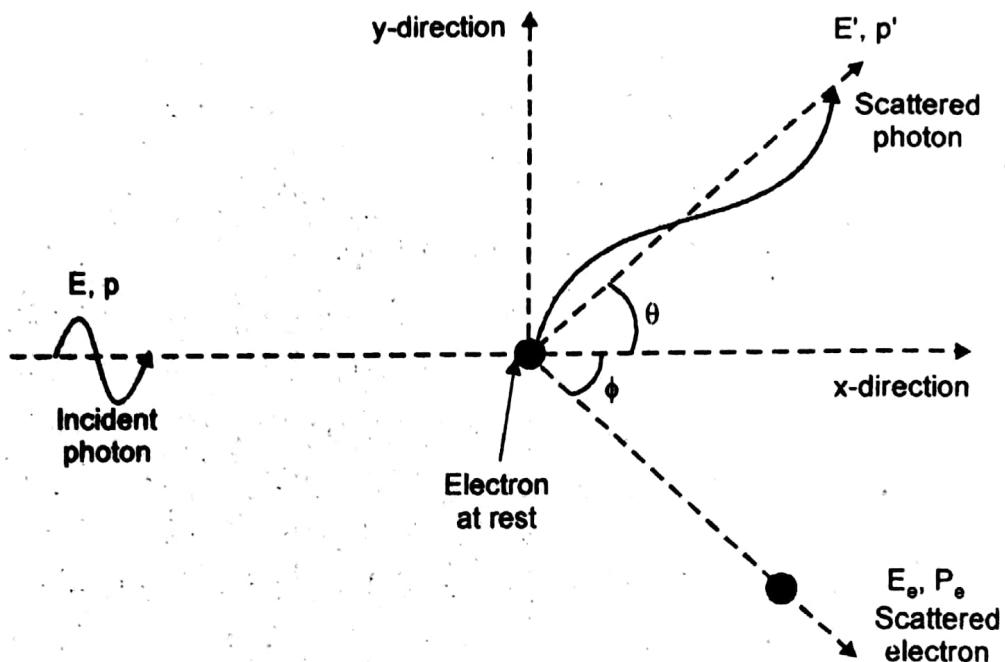
Max Planck, in 1900, put forth a new and revolutionary theory called the quantum theory of light. Planck proposed that emission and absorption of energy does not take place continuously but at discrete intervals and in discrete bundles or packets of energy. These packets of energy are known as quanta which we call photons. The energy of each photon is equal to  $hv$ , where  $h$  is the Planck's constant and  $v$  is the frequency of the associated radiation. Hence, light, or in general, any electromagnetic radiation has dual nature, meaning both wave and particle nature.

## 1.3 The Compton Effect

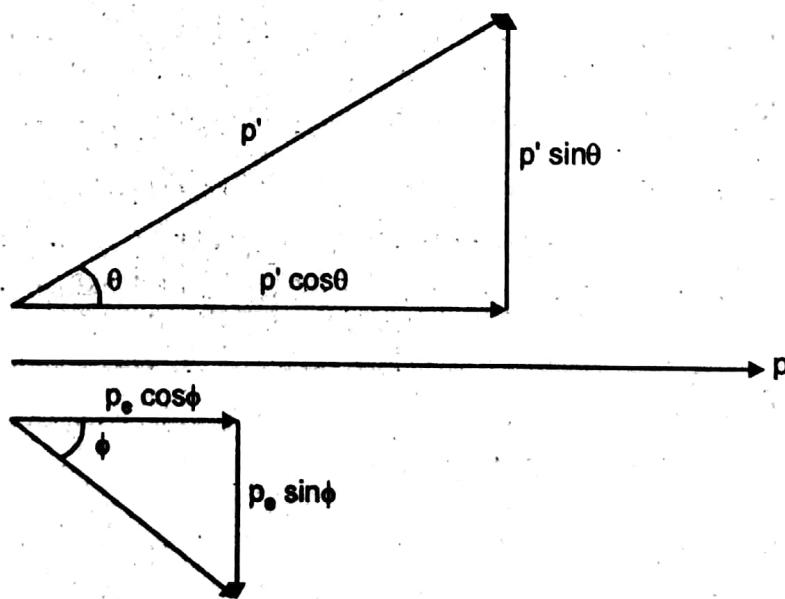
One of the ways for radiation to interact with atoms is by means of Compton effect discovered by Prof. A.H. Compton in 1923. It involves essentially scattering between a high frequency photon such as an X-ray photon and a loosely bound, nearly free electron which is assumed to be at rest.

In this scattering, the photon delivers a part of its energy to the electron and goes with a reduced energy. Obviously the scattered photon will have an increase in its wavelength (Compton scattering or incoherent scattering). This is totally contradictory to the wave picture in which the scattered radiation is less energetic than the incident radiation but has the same wavelength (Thomson scattering or coherent scattering).

The collision between the photon and the electron can be analysed in the same way as billiard - ball collisions are analysed in mechanics.



**Figure 1.2: (a)** The geometry of Compton scattering



**Figure 1.2: (b)** Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Let us assume that initially the photon has energy 'E' given by,

$$E = h\nu = \frac{hc}{\lambda} \quad \dots (1.16)$$

and the linear momentum

$$p = \frac{E}{c} \quad \dots (1.17)$$

Let  $m_e c^2$  be the rest mass energy of the electron. Let  $E'$  and  $p'$  represent energy and momentum of the scattered photon, respectively. This photon moves in a direction making an angle ' $\theta$ ' with its initial direction. Let ' $E_e$ ' and ' $p_e$ ' represent the total energy and momentum of the electron after scattering. The scattered electron moves in a direction at an angle,  $\phi$ , with respect to the initial photon.

The law of conservation of energy and the law of conservation of momentum do apply for the collision between the photon and the electron.

Applying law of conservation of energy, we have,

$$\begin{aligned} E_{\text{initial}} &= E_{\text{final}} \\ \therefore E + m_e c^2 &= E' + E_e \end{aligned} \quad \dots (1.18)$$

Momentum, unlike energy, is a vector quantity that has direction as well as magnitude and hence it should be conserved in each of two mutually perpendicular directions.

Applying the law of conservation of momentum along x-direction,

$$\begin{aligned} \text{we have, } (p_x)_{\text{initial}} &= (p_x)_{\text{final}} \\ p + 0 &= p' \cos \theta + p_e \cos \phi \\ p &= p' \cos \theta + p_e \cos \phi \end{aligned} \quad \dots (1.19)$$

$$\begin{aligned} \text{Similarly, } (p_y)_{\text{initial}} &= (p_y)_{\text{final}} \\ 0 + 0 &= p' \sin \theta - p_e \sin \phi \\ \therefore p_e \sin \phi &= p' \sin \theta \end{aligned} \quad \dots (1.20)$$

It is possible to eliminate any two of the four unknowns namely  $\theta$ ,  $\phi$ ,  $E'$  and  $E_e$  by solving the equations simultaneously. If we choose to measure the energy and the direction of the scattered photon, we eliminate  $E_e$  and  $\phi$ . The angle  $\phi$  is eliminated by combining the momentum equations.

Taking (1.19) and (1.20),

$$\begin{aligned} p_e \cos \phi &= p - p' \cos \theta \\ p_e \sin \phi &= p' \sin \theta \end{aligned}$$

By squaring and adding, we get,

$$p_e^2 = p^2 + p'^2 - 2pp' \cos \theta \quad \dots (1.21)$$

We know from the theory of relativity,

$$E_e^2 = c^2 p_e^2 + m_e^2 c^4 \quad \dots (1.22)$$

Using (1.18) and (1.21) in (1.22),

$$(E + m_e c^2 - E')^2 = c^2(p^2 + p'^2 - 2pp' \cos \theta) + m_e^2 c^4$$

$$E^2 + m_e^2 c^4 + E'^2 + 2Em_e c^2 - 2EE' - 2E'm_e c^2 = c^2 p^2 + c^2 p'^2 - 2(pc)(p'c) \cos \theta + m_e^2 c^4$$

$$E^2 + E'^2 - 2EE' + 2m_e c^2(E - E') + m_e^2 c^4 = E^2 + E'^2 - 2EE' \cos \theta + m_e^2 c^4$$

On simplifying, we get

$$2m_e c^2(E - E') = 2EE'(1 - \cos \theta)$$

$$\frac{E - E'}{EE'} = \frac{(1 - \cos \theta)}{m_e c^2}$$

$$\left( \frac{1}{E'} - \frac{1}{E} \right) = \frac{1}{m_e c^2}(1 - \cos \theta) \quad \dots (1.23)$$

This equation is often called Compton scattering formula. This equation can also be expressed in terms of change in wavelength as,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos \theta) \quad \dots (1.24)$$

$$\text{Since } E = \frac{hc}{\lambda} \text{ and } E' = \frac{hc}{\lambda'}$$

In equation (1.24),  $\lambda$  is the wavelength of the incident photon and  $\lambda'$  is the wavelength of the scattered photon. The quantity  $\frac{h}{m_e c}$  is called Compton wavelength,  $\lambda_c$ , of the electron and has a value of 2.426 pm.

Since the quantity on the right hand side in equation (1.23) as well as (1.24) is never negative, we understand that  $E'$  is always lesser than  $E$  and  $\lambda'$  is always greater than  $\lambda$ . The change in wavelength ranges from 0 to twice the Compton wavelength ( $2\lambda_c$ ) for the angles between  $0^\circ$  and  $180^\circ$ .

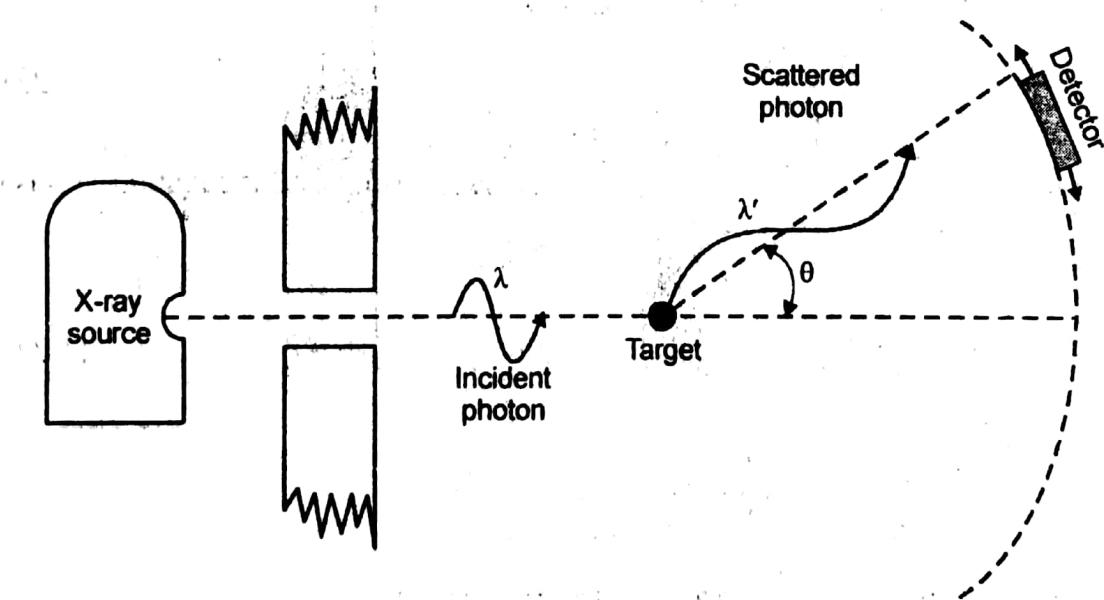
Very often the difference  $(\lambda' - \lambda)$  is referred to as Compton shift ( $\Delta\lambda$ ). Hence equation (1.24) can be written as,

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta) \quad \dots (1.25)$$

$$\therefore \Delta\lambda = \lambda_c (1 - \cos \theta) \quad \dots (1.26)$$

### 1.3.1 Experimental Demonstration of Compton Effect

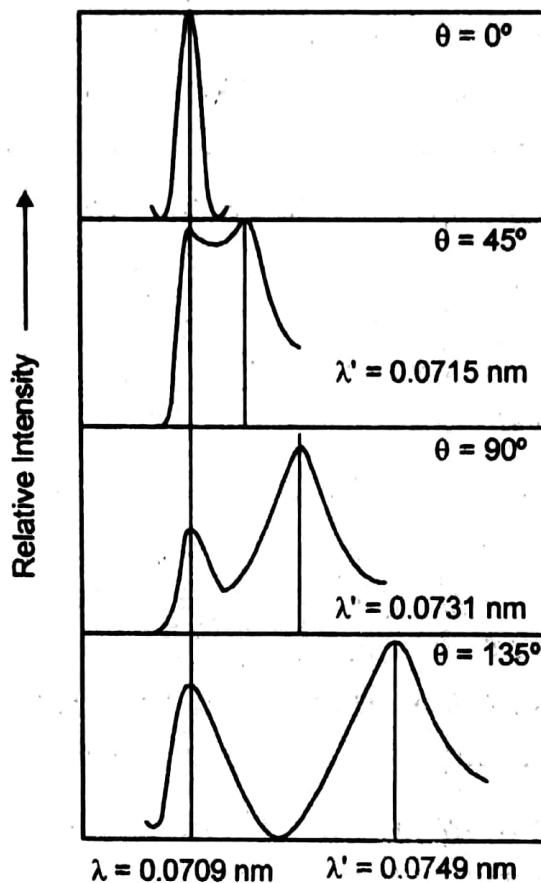
Arthur Compton, in 1923, demonstrated this scattering phenomenon with a simple experimental setup shown below:



**Figure 1.3: (a) Schematic diagram of Compton Scattering apparatus**

A beam of X-rays falls on the target of carbon atoms. A detector, capable of moving in an arc, measured the energy of the scattered X-rays at various angles,  $\theta$ .

The fig 1.3(b) shows the Compton's original results. For each angle other than  $0^\circ$ , two peaks appear. The wavelength of the first peak matches with the original wavelength.



**Figure 1.3: (b)** Compton's original results for X-ray Scattering

This peak corresponds to the photon that gets scattered from the tightly bound electrons of the atoms and hence the photon does not lose energy at all. However, there is another peak whose wavelength varies strongly with the angle as predicted by Compton formula.

Compton repeated the experiment with gamma rays and found no change in the compton shift. Again this is explicit from eqn (1.25) that the Compton shift does not depend on the incident wavelength.

Hence Compton effect constitutes a very strong evidence for the particle nature of electromagnetic radiation.

## 1.4 Matter Waves

The photoelectric effect and Compton effect have been successfully explained by the quantum theory of radiation. According to this theory the electromagnetic radiation possesses quantum nature, i.e., the energy of radiation is in the form of packets or photons. But the interference and diffraction phenomena of radiation are explained by the wave nature of radiation. Thus radiation has dual nature. Bohr stated in his complementarity

principle that for the complete description of radiation it must have dual nature that of waves as well as particles and neither the radiation nor the particle show both particle and wave aspects simultaneously.

In 1923 – 24 Louis de-Broglie proposed a hypothesis that wave particle duality is a general property and matter must also exhibit both particle and wave properties. Shortly after the pronouncement of de Broglie hypothesis Davisson and Germer in 1927 and G. P. Thomson in 1928 demonstrated the diffraction of electrons by crystals and thus confirm de Broglie hypothesis.

On the basis of the concept of dual nature of light Louis de Broglie in 1924 gave the hypothesis that particles in motion should also have wave characteristics associated with them just like the radiant energy in the form of waves has particle properties associated with it. This hypothesis is called de Broglie hypothesis. The waves associated with material particles are called matter waves. The matter waves are probability waves and are also called **de Broglie waves**. Thus both radiation and material particles have dual nature.

In addition to the above hypothesis de Broglie also gave a concept that the fundamental laws of nature must be applicable to radiation and material particles in the same way.

#### 1.4.1 Wavelength of Matter Waves

According to quantum theory of radiation the energy of a photon is

$$E = hv \quad \dots (1.27)$$

where  $h$  is Planck's constant and  $v$  is frequency of radiation. If the effective mass of a photon is  $m$ , then by Einstein's mass-energy relation.

$$E = mc^2 \quad \dots (1.28)$$

where  $c$  is velocity of light in vacuum.

From equations (1.27) and (1.28),

$$mc^2 = hv \quad \text{or} \quad mc = h\left(\frac{v}{c}\right)$$

If the momentum of the photon is  $p$ , then

$$p = mc = h\frac{v}{c}$$

But  $c = v\lambda$

$$\therefore p = h \left( \frac{v}{v\lambda} \right) = \frac{h}{\lambda}$$

$$(or) \lambda = \frac{h}{p} \quad \dots (1.29)$$

Equation (1.29) gives the wavelength of the waves associated with photon. This relation shows that the wave nature, wavelength  $\lambda$ , associated with the photon is related to the particle nature, momentum  $p$ , i.e.,  $\lambda$  is inversely proportional to  $p$  i.e.,

$$\lambda \propto \frac{1}{p}$$

The wavelength of matter waves associated with the material particles in motion can also be determined with the help of the relation derived for photons. Suppose a material particle of mass  $M$  is moving with a velocity  $v$ . Its momentum will be

$$p = Mv \quad \dots (1.30)$$

If the wavelength of matter waves associated with it is  $\lambda$ , then from equation (1.29)

$$\lambda = \frac{h}{p} = \frac{h}{Mv} \quad \dots (1.31)$$

Ordinarily the matter waves are not observed i.e., the wave property of material particles is not revealed because the magnitude of such wavelengths is so small that it is beyond the measuring capability of available instruments. This fact can be easily understood with the help of the following example.

Let the mass  $M$  of a moving particle be 1 kg and its velocity be 1 km/s =  $10^3$  m/s. The wavelength of matter waves associated with it will be

$$\begin{aligned} \lambda &= \frac{h}{Mv} = \frac{6.62 \times 10^{-34}}{1 \times 10^3} \\ &= 6.62 \times 10^{-37} \text{ m} \end{aligned}$$

The measuring capability of best instruments available today is of the order of  $10^{-15}$  m. Hence the value of  $\lambda$  for bodies which we ordinarily come across, is much less than our measuring ability. So these matter waves associated with material particles are not observed and we observe only their particle nature.

For microscopic moving material particles like electrons, protons, neutrons etc., the wavelength of matter waves associated with them can be measured because the wavelengths of these waves are of the order of  $10^{-10}$  m. For example for a beam of electrons having a speed c/5, the wavelength of matter waves will be

$$\lambda = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.6 \times 10^8} = 1.2 \times 10^{-11} \text{ m}$$

$$= 0.12 \text{ Å}$$

This wavelength is of the order of wavelength of X-rays which can be determined by studying their diffraction from crystals.

#### 1.4.2 Properties of matter waves

- i) Matter waves are not electromagnetic waves.
- ii) The wavelength associated with matter waves gets shorter when the mass of the particle becomes larger.
- iii) Matter waves can travel faster than the velocity of light.
- iv) The velocity of matter wave is not constant as that of electro magnetic radiation.
- v) The function of the wave is to pilot or to guide the matter particles.

#### 1.4.3 Wavelength of Matter Waves Associated with Different Particles

##### i) General relation for the wavelength of matter waves associated with moving particles

Suppose a particle of mass m is moving with a velocity v and has kinetic energy E.

$$\therefore E = \frac{1}{2} mv^2$$

$$\text{So that } v = \sqrt{\frac{2E}{m}}$$

The momentum of the particle

$$p = mv = m \sqrt{\frac{2E}{m}}$$

$$= \sqrt{2mE}$$

Using de-Broglie relation the wavelength of matter waves associated with the particle will be

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \quad \dots (1.32)$$

Thus for particles of same mass  $\lambda \propto \frac{1}{\sqrt{E}}$  and for particles of same energy  $\lambda \propto \frac{1}{\sqrt{m}}$ .

### ii) Wavelength of matter waves associated with accelerated electrons

Electrons can be accelerated by electric field. Let m be mass of electron and e be its charge. Its velocity becomes v then it is accelerated from rest by a potential difference of V volts. The energy gained by the electron,

$$E = \frac{1}{2} mv^2 = eV$$

so that the momentum

$$p = mv = \sqrt{2meV}$$

If the de-Broglie wavelength of electrons is  $\lambda_e$  then

$$\lambda_e = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} \quad \dots (1.33)$$

$$\text{Now } h = 6.62 \times 10^{-34} \text{ J-s,}$$

$$m = 9.1 \times 10^{-31} \text{ kg and}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore \lambda_e = \frac{6.626 \times 10^{-34}}{\left[ 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \right]^{1/2} \sqrt{V}}$$

$$= \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ m}$$

$$= \frac{12.27}{\sqrt{V}} \text{ Å} \quad \dots (1.34)$$

Thus

$$\lambda_e \propto \frac{1}{\sqrt{V}}$$

As an example, if electrons are accelerated by 100 volts, then

$$\begin{aligned}\lambda_e &= \frac{12.27}{\sqrt{100}} \text{ \AA} \\ &= \frac{12.27}{10} \text{ \AA} \\ &= 1.227 \text{ \AA}\end{aligned}$$

If  $V = 10^4$  volts, then

$$\begin{aligned}\lambda_e &= \frac{12.27}{\sqrt{10^4}} \text{ \AA} \\ &= \frac{12.27}{100} \text{ \AA} \\ &= 0.123 \text{ \AA}\end{aligned}$$

### iii) Wavelength of other accelerated charged particles

We now consider charged particles (as protons,  $\alpha$ -particles, ionised atoms etc) of mass  $m$  and charge  $q$  accelerated from rest by a potential difference of  $V$  volts so that their velocity becomes  $v$ . The energy of these particles will be

$$E = \frac{1}{2} mv^2 = qV$$

so that their momentum

$$p = mv = \sqrt{2mqV}$$

If the de-Broglie wavelength of matter waves associated with these particles is  $\lambda$ , then by de-Broglie relation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} \quad \dots (1.35)$$

### a) Accelerated protons

If protons are accelerated by a potential difference of  $V$  volts, then for protons.

$$m = m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$q = q_p = 1.6 \times 10^{-19} \text{ C}$$

$$\begin{aligned}\therefore \lambda_p &= \frac{h}{[2m_p q_p V]^{1/2}} \\ &= \frac{6.62 \times 10^{-34}}{[2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times V]^{1/2}} \\ &= \frac{0.286}{\sqrt{V}} \times 10^{-10} \text{ m} \\ &= \frac{0.286}{\sqrt{V}} \text{ Å}\end{aligned}$$

### b) Accelerated $\alpha$ -particles

$\alpha$ -particles are doubly ionised helium atoms. For  $\alpha$ -particles

$$m_\alpha = 4m_p = 4 \times 1.67 \times 10^{-27} \text{ kg} = 6.68 \times 10^{-27} \text{ kg}$$

$$q_\alpha = 2q_p = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19} \text{ C}$$

$$\begin{aligned}\therefore \lambda_\alpha &= \frac{h}{[2m_\alpha q_\alpha V]^{1/2}} \\ &= \frac{h}{[2 \times 4m_p \times 2q_p \times V]^{1/2}} \\ &= \frac{h}{[2m_\alpha q_\alpha V]^{1/2}} \\ &= \frac{\lambda_p}{2\sqrt{2}} = \frac{6.626 \times 10^{-34}}{[2 \times 6.68 \times 10^{-27} \times 3.2 \times 10^{-19} \times V]^{1/2}}\end{aligned}$$

$$\begin{aligned}
 &= \frac{0.1012 \times 10^{-10}}{\sqrt{V}} \text{ m} \\
 &= \frac{0.1012}{\sqrt{V}} \text{ Å} \quad \dots (1.36)
 \end{aligned}$$

### c) Accelerated deuterons

Deuteron is an ionised deuterium atom ( $\text{H}_2^+$ ).

For deuterons

$$\begin{aligned}
 m_d &= 2m_p = 2 \times 1.67 \times 10^{-27} \text{ kg} = 3.34 \times 10^{-27} \text{ kg} \\
 q_d &= q_p = 1.6 \times 10^{-19} \text{ C} \\
 \therefore \lambda_d &= \frac{h}{[2m_d q_d V]^{1/2}} \\
 &= \frac{h}{[2 \times m_p \times q_p \times V]^{1/2}} \\
 &= \frac{1}{\sqrt{2}} \frac{h}{[2m_a q_a V]^{1/2}} \\
 &= \frac{\lambda_p}{\sqrt{2}} = \frac{0.202}{\sqrt{V}} \text{ Å} \quad \dots (1.37)
 \end{aligned}$$

Because of the smallness of  $h$ , only for particles of atomic or nuclear size will have the wave behavior to be observable.

### d) Wave number

It is the number of waves present in one unit of length. It is equal to the inverse of the wavelength. It is represented by  $\bar{v}$  i.e.,  $\bar{v} = \frac{1}{\lambda}$ .

## 1.5 Davisson and Germer Experiment

Davisson and Germer's accidental discovery of the diffraction of electrons was the first direct evidence confirming de Broglie's hypothesis that particles can have wave properties as well. The experimental arrangement used by Davisson and Germer is shown in Fig. 1.4. It consists of an electron gun which comprises a tungsten filament F heated by

a low tension battery. The electron emitted by the filament are accelerated in an electric field of known potential difference from a high tension battery. The electrons are collimated to a fine beam by allowing them to pass through a system of suitable slits. This entire arrangement to produce a fine beam of electrons accelerated to a desired velocity is called as electron gun.

The collimated beam of electrons is made to strike a nickel target capable of rotating about an axis perpendicular to the plane of the paper. The electrons are scattered in all directions by the atoms of nickel crystal. The intensity of the electron beam scattered in a given direction is measured by allowing it to enter in a Faraday cylinder called the collector, which can be rotated about the same axis as the target. The collector is connected to a galvanometer whose deflection is proportional to the intensity of the beam entering the collector. The whole apparatus is enclosed in an evacuated chamber.

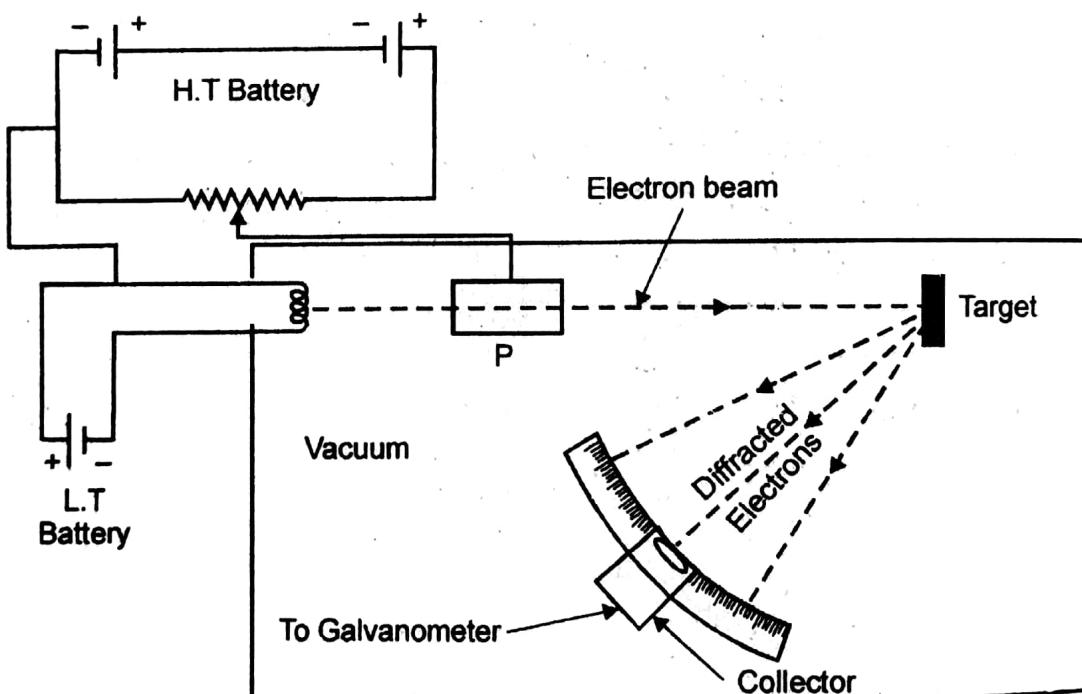
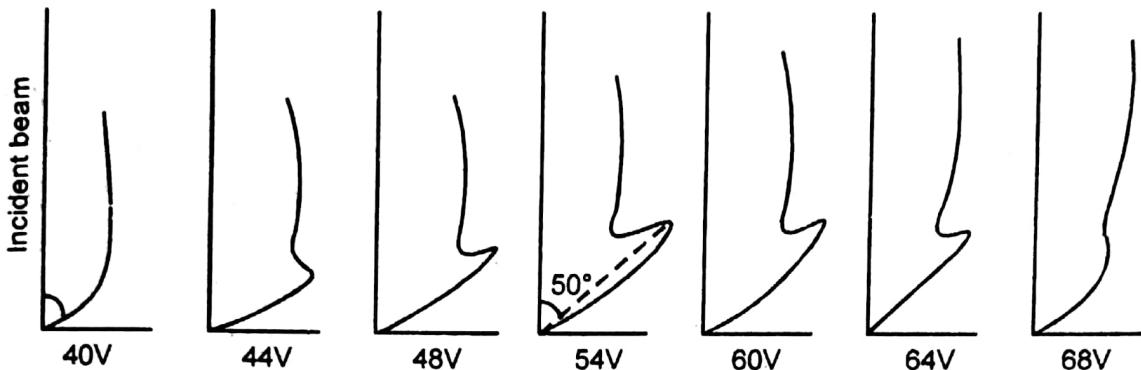


Figure 1.4: Davisson and Germer experiment

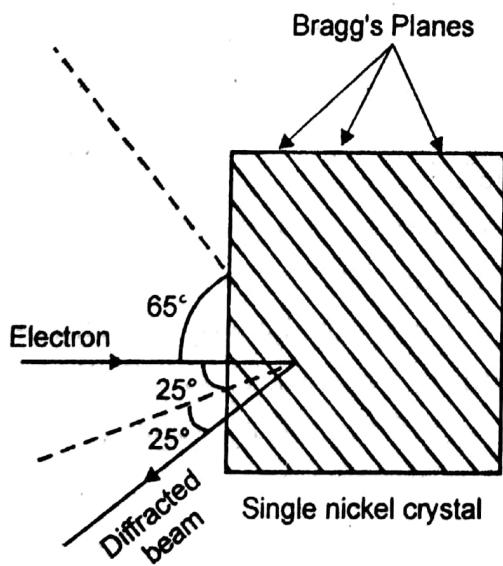
In one set of observations, the (111) face of the target nickel crystal was presented to the incident beam of electrons. The potential difference between the filament F and the electron gun was adjusted at 40 volts and then gradually increased in small steps. For each value of potential difference, polar graphs were drawn between angle  $\theta$  (the angle between the incident beam and the beam entering the collector, known as *Colatitude*) and the collector current. In these graphs, the length of the radius vector corresponding to a certain angle was drawn directly proportional to the collector current. For the potential difference

of 40 volts, a smooth curve was obtained. As the potential difference was gradually increased, a little bump began to appear on the curve which increased to a maximum for  $V = 54$  volts and  $\theta = 50^\circ$  (Fig. 1.5).



**Figure 1.5:** Polar graphs between angle  $\theta$  and intensity of scattered beam in Davisson-Germer experiment.

Beyond 54 volts, the bump again decreased to become insignificant at about 68 volts. The pronounced maxima observed corresponding to accelerating potential of 54 volts at an angle  $50^\circ$  with the electron beam provided a convincing proof that electrons were associated with waves which after scattering from the regularly spaced atoms of the nickel crystal, gave rise to a constructive interference. In the above case when the collector current is maximum at scattering (colatitude) angle of  $50^\circ$  corresponding to accelerating potential of 54 volts, the incident and the scattered beam evidently makes an angle of  $65^\circ$  with the family of Bragg's planes as shown in Fig. 1.5.



**Figure 1.6:** The strong diffracted beam at  $50^\circ$  and the Bragg's angle at  $65^\circ$

X-rays diffraction studies reveal that the spacing of planes in this family is  $0.91\text{ \AA}$ . Taking  $n = 1$ ,  $\theta = 65^\circ$ , one can calculate the wavelength of the incident wave by applying Bragg's law  $2d \sin \theta = n\lambda$ .

We have

$$2 \times 0.91 \times \sin 65^\circ = 11$$

or

$$\lambda = 2 \times 0.91 \times 0.9063 = 1.662\text{ \AA}$$

Now applying de-Broglie's relation for matter waves, we have

$$\lambda = \frac{h}{mv} = \frac{12.27}{\sqrt{V}}$$

$$\text{Since } V = 54 \text{ volts, } \lambda = \frac{12.27}{\sqrt{54}} = 1.673\text{ \AA}$$

Obviously, there is an excellent agreement between the two results. Thus Davisson and Germer's experiment may be considered to be a verification of the de-Broglie hypothesis of matter waves.

The analysis of the Davisson Germer experiment is actually less straight forward, since the energy of an electron increases when it enters a crystal by an amount equal to the work function of the surface. Hence the electron speeds in the experiment were greater inside the crystal and the corresponding de-Broglie wavelength shorter than the corresponding values outside. An additional complication arises from interference between waves diffracted by different families of Bragg planes, which restricts the occurrence of maxima to certain combinations of electron energy and angle of incidence rather than merely to any combination they obey the Bragg equation.

## 1.6 Uncertainty Principle

We have seen that the material particles like electrons, protons, neutrons etc. while in motion behave like a group of waves. They also behave as particles sometimes, as in the case of motion of these in electric and magnetic fields. Then a question arises if matter has dual characteristic: wave-particle duality, is it possible to know the exact position of a particle like a moving electron in space at some given instant. The answer to this question was given by the principle of indeterminacy or uncertainty principle stated by Werner Heisenberg in 1927.

The principle states that the position of a particle in a given direction and its momentum in the same direction cannot be measured simultaneously with unlimited accuracy. If,

however, an experiment is designed for the exact determination of one of them, the determination of the other will become completely uncertain, and vice versa. If  $\Delta_x$  is the uncertainty in position and  $\Delta p_x$  is uncertainty in momentum (in x-direction) then

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2} \quad \dots (1.38)$$

From the above equation it is clear that if a particle momentum is known accurately ( $\Delta p_x = 0$ )  $\Delta x$  is infinite so that our knowledge about its position is zero. The particle is represented by a wave of constant amplitude extending over all space.  $\Delta x$  and  $\Delta p_x$  cannot be zero simultaneously. Hence it is impossible to know both the exact position and exact momentum of a particle at the same time.

The uncertainty should not be regarded as some sort of errors in the measurement of imperfection of physical apparatus but it only emphasises the fact that momentum and position of a particle being actually non-existent in the same state.

### **The Uncertainty relation for Time and Energy**

In addition to the uncertainty relation between position co-ordinate and corresponding momentum of a moving particle in the same direction, there is a relation between other two conjugate quantities-Energy and Time. If  $\Delta E$  is the uncertainty in energy of a particle and  $\Delta t$  is the uncertainty in time, then

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \quad \dots (1.39)$$

Thus if the time of an atomic event is determined more accurately, the determination of the energy change of the atomic system is less accurate and vice versa.

The smallness of the Planck's constant  $\hbar$  makes the uncertainty principle of interest primarily in connection with the systems of atomic size.

The relation can be obtained from the following consideration.

If a particle moving with a velocity  $v$  be located anywhere within a distance,  $\Delta x$  its description in time is uncertain by

$$\Delta t \approx \frac{\Delta x}{v}$$

The kinetic energy of the particle

$$E = \frac{1}{2}mv^2$$

or  $\Delta E = mv\Delta v$

$$= vm\Delta v = v\Delta p$$

$$\approx v \frac{\hbar}{2\Delta x} \quad \text{since } \Delta p = \frac{\hbar}{2\Delta x}$$

but  $\frac{v}{\Delta x} \approx \frac{1}{\Delta t}$

$$\therefore \Delta E \approx \frac{\hbar}{2\Delta t}$$

$$\therefore \Delta E \Delta t \geq \frac{\hbar}{2}$$

Thus the energy of a particle cannot be measured accurately unless we take infinite time to measure it.

### 1.6.1 Applications of Uncertainty Principle

#### i) Non-existence of electrons inside a nucleus

The radii of typical nuclei are about  $10^{-14}$  m. For an electron to be confined within a nucleus, the uncertainty in its position should be nearly equal to  $10^{-14}$ . The corresponding uncertainty in the momentum is

$$\Delta p \approx \frac{\hbar}{\Delta x} \approx \frac{6.6 \times 10^{-34}}{10^{-14}}$$

$$\approx 6.6 \times 10^{-20} \text{ kg.m/s}$$

Therefore, the momentum of the electron must be at least comparable in magnitude with this uncertainty. An electron whose momentum is  $6.6 \times 10^{-20}$  kg. m/s has a kinetic energy ( $\approx 10^{-9}$  Joule) many times greater than its rest energy  $m_0 c^2$  ( $\approx 10^{-13}$  Joule) and we may accordingly use

$$E^2 = c^2 p^2 + m_0^2 c^4$$

Neglecting  $m_0^2 c^4$

$$E^2 \approx c^2 p^2$$

$$E \approx cp$$

$$E \approx cp \approx 6.6 \times 10^{-20} \times 3 \times 10^8$$

$$E \approx 2 \times 10^{-11} \text{ Joule}$$

$$\approx \frac{2 \times 10^{-11}}{1.6 \times 10^{-19}} \text{ eV} \approx 125 \text{ MeV}$$

Thus the kinetic energy of an electron must be of the order of 125 MeV if it has to stay inside the nucleus. The experiments indicate that the electrons associate with unstable atoms never have more than a fraction of this energy. Therefore, we conclude that electrons cannot exist within the nuclei.

### ii) Ground state energy of Hydrogen atom

The radius of hydrogen atom is of the order of  $5 \times 10^{-11} \text{ m}$ . The maximum value of uncertainty in the position of the electron is of the order  $5 \times 10^{-11} \text{ m}$ . Hence

$$\Delta x \approx 5 \times 10^{-11} \text{ m}$$

The corresponding uncertainty in the momentum is

$$\begin{aligned} \Delta p &\approx \frac{h}{\Delta x} \approx \frac{6.6 \times 10^{-34}}{5 \times 10^{-11}} \\ &\approx 1.3 \times 10^{-23} \end{aligned}$$

The velocity of such an electron will be non-relativistic and its kinetic energy

$$\begin{aligned} KE &\approx \frac{p}{2m} \approx \frac{(1.3 \times 10^{-23})^2}{2 \times 9.1 \times 10^{-31}} \\ &\approx 9.3 \times 10^{-17} \text{ Joule} \approx 5.8 \times 10^2 \text{ eV} \end{aligned}$$

which is a plausible figure.

### iii) Ground state energy of Harmonic Oscillator

The bottom of the potential energy curve for a simple harmonic oscillator classically demands that the lowest energy of the oscillator should be zero. But actually it has been

found that the lowest energy or the zero point energy is finite. It is  $\frac{1}{2}hv$ ; where  $v$  is the frequency of the oscillator. This non-zero (zero point energy) value of the lowest energy of the oscillator can be explained on the basis of uncertainty principle.

In the lowest level (ground state) the oscillator will have very small amplitude and the uncertainty in the position of the oscillator will be of the order of amplitude, i.e.,  $\Delta x \approx x_0$ ; where  $x_0$  is the amplitude in the ground state. The corresponding change in the momentum  $p_0$  will also be uncertain to the extent

$$\Delta p \approx p_0$$

The energy of the classical oscillator in the ground state is

$$E_0 = \frac{1}{2}mv^2$$

$$= \frac{1}{2}mx_0^2\omega^2 \quad \text{where } \omega \text{ is angular frequency}$$

$$E_0 = \frac{1}{2}m x_0 \omega x_0 \omega$$

$$= \frac{1}{2}x_0 \omega mv$$

$$= \frac{1}{2}x_0 \omega p_0$$

$$\approx \frac{1}{2}\omega \Delta x \Delta p \approx \frac{1}{2}\omega \hbar$$

Using

$$\hbar = \frac{h}{2\pi}$$

For the uncertainty principle

$$\Delta x \Delta p \geq \hbar$$

$$E_0 \approx \frac{1}{2} \omega \frac{\hbar}{2\pi}$$

$$= \frac{1}{2} 2\pi v \frac{\hbar}{2\pi} = \frac{1}{2} \hbar v$$

which is the zero point energy of a harmonic oscillator.

#### iv) Natural width of Spectral line

The energy time uncertainty relationship demands that if an electron stays in an excited state and comes to a lower energy state, then there is a limit on time for which the electron can stay in a state. If otherwise, there is an uncertainty in time  $\Delta t$  which governs the average length of time, the electron is in a certain state. This is the life time of the state. Thus the uncertainty in the energy  $\Delta E$  is such that

$$\Delta E \sim \frac{\hbar}{\Delta t}$$

The energy of the state cannot be known precisely, but it has a certain width  $\Delta E$ , which is inherent in the energy states (except the ground state).  $\Delta E$  is, hence, natural width of the energy state. The photon emitted during transition of electron from an excited state to the lower state has a spread in frequency rather than a single frequency ( $\Delta E = \hbar \Delta v$ ).

If  $\tau$  is life time (reciprocal of the probability per unit time for the transition). Then

$$\Delta t \sim \tau$$

and for most of the excited states, (the average life time)

$$\tau \sim 10^{-8} \text{ s}$$

i.e. atom stays in an excited state for a time of the order of  $10^{-8}$  s.

The natural width of the state is

$$\Delta E \sim \frac{\hbar}{\Delta t}$$

$$\sim \frac{h}{\tau}$$

$$\sim \frac{6.6 \times 10^{-34}}{10^{-8}}$$

$$\sim 6.6 \times 10^{-26} \text{ Joule}$$

$$(1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

or  $\Delta E \sim 4 \times 10^{-7} \text{ eV}$

Since  $h \Delta v \approx \Delta E$

$\therefore$  Width of spectral line,

$$\Delta v \sim \frac{4 \times 10^{-7}}{h}$$

$$\sim 10^{-8} \text{ s}^{-1}$$

Note: The uncertainty relations have been found to be

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

where  $\hbar = \frac{h}{2\pi}$

Similar relations will hold good for y and z directions also

$$\Delta y \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z \Delta p_z \geq \frac{\hbar}{2}$$

and  $\Delta E \Delta t \geq \frac{\hbar}{2}$

## 1.7 Wave Function

We are already familiar with sound waves, where pressure varies as they propagate in a medium. For water waves it is the height of the water above water surface which varies. In light it is the electric and magnetic field components which vary as light travels. Then question arises for matter waves. As the matter waves travel, what is the quantity that vary? Also how these waves of the variable quantity describe the matter field particle. To answer these questions, we make use of our knowledge of standing waves i.e. waves confined to a certain region, like waves produced by vibratory strings fixed at two ends, vibrating air columns in a tube, electromagnetic radiation inside a cavity. For such waves the amplitude of each point in space is fixed. Larger is the amplitude, more intense is the wave.

A similar situation exists inside the atom. The electron is situated within a distance of around  $10^{-8} - 10^{-10}$  m from a nucleus and is always confined within this distance. The matter wave associated with it may be defined in terms of standing waves which have varying amplitudes at different points in the region and practically zero outside the region. The varying quantity characterising the matter field or matter waves is denoted by  $\psi(x)$  at a point  $x$  along  $x$ -direction. It is known as wave function or wave amplitude.

Since the intensity of a wave is given by the square of the amplitude, as such the intensity of a matter wave can also be defined by the square of its amplitude, i.e., by  $|\psi(x)|^2$ . We have defined the intensity by the square of modulus of  $\psi(x)$ , because  $\psi(x)$  can be a complex function and  $|\psi(x)|^2 = \psi^*(x)\psi(x)$  where  $\psi^*(x)$  is complex conjugate of  $\psi(x)$ . If, however,  $\psi(x)$  is real then  $\psi(x) = \psi^*(x)$ .

The matter wave describes the dynamics of a particle as such wherever  $|\psi(x)|^2$  is large, we may say, that the chance of finding the particle in that region is large. It can then be said that the probability of finding a particle described by  $\psi(x)$ , in the interval  $dx$  about the point  $x$  is  $|\psi(x)|^2 dx$ . Thus  $|\psi(x)|^2$  denotes the probability per unit length.

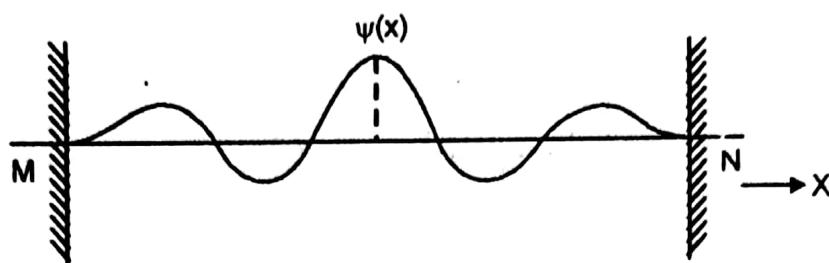
It may be noted that when  $\psi(x)$  is complex, say  $\psi(x) = A + iB$   $i = \sqrt{-1}$ , and  $A$  and  $B$  are real functions. Thus its complex conjugate ( $i$  is changed to  $-i$ ) is

$$\psi^*(x) = A - iB$$

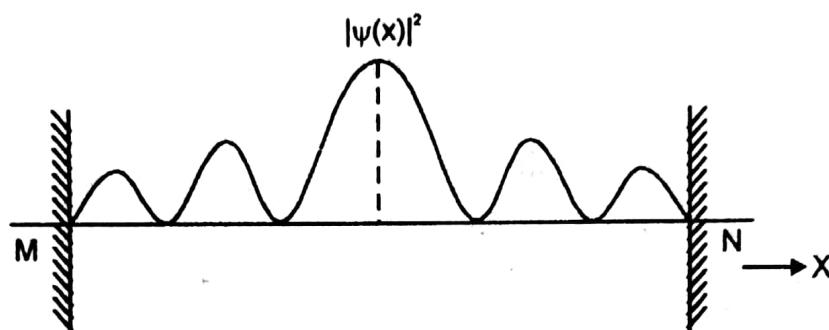
and the square of modulus of  $\psi(x)$

$$|\psi(x)|^2 = \psi^*\psi = (A - iB)(A + iB) = A^2 + B^2 \quad \dots (1.40)$$

Thus  $\psi^*\psi$  is always a positive real quantity.



**Figure 1.7: (a)** Wave function  $\psi(x)$



**Figure 1.7: (b)** Probability distribution  $|\psi(x)|^2$

In figure 1.7 the wave function  $\psi(x)$  and the probability distribution  $|\psi(x)|^2$  between a region M and N is shown.

It is thus clear that  $\psi(x)$  itself has no direct physical meaning. The value of  $\psi(x)$  defining a moving particle in space at a certain item is related to the likelihood of finding the particle there at that time. But a wave has both positive and negative amplitude and if  $\psi(x)$  is to give the likelihood or probability, then negative probability is meaningless, hence probability is defined in terms of  $|\psi(x)|^2$  which is always real, positive. Secondly, the particle to be defined by  $|\psi(x)|^2$  either exists or does not exist that is the total probability is either zero or unity, i.e., if

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 0 \quad \dots (1.41)$$

particle does not exist, and if

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \quad \dots (1.42)$$

the particle exists somewhere in space at all times.

Instead of considering in one direction, if we consider the motion in space, the wave function depends on  $x, y, z$ .

Wave function is  $\psi(x, y, z)$  and the probability of finding the particle in a volume element  $dx dy dz$ , is

$$P = |\psi(x, y, z)|^2 dx dy dz$$

or  $P = |\psi(x, y, z)|^2 dV$

where  $dV$  is the volume element. Thus

$$P = |\psi(x, y, z)|^2$$

is defined as the probability density (probability per unit volume).  $P$  gives the probability of finding the particle, described by  $\psi$ , in the volume element  $dV$  around the point given by co-ordinates  $x, y, z$ .

### 1.7.1 Physical significance of the wave function and its interpretation

In the beginning of the chapter, we have already assigned a name to the wave function that it is the amplitude of the matter wave which describes a particle having certain momentum. But the amplitude of wave can be both positive and negative, and negative amplitude (probability) cannot give description of a particle as such  $\psi(r, t)$  or the wave function, itself cannot have any direct physical interpretation.

We can only say that the wave function  $\psi(r, t)$  is a varying quantity characterising the matter waves about the point  $\vec{r}$  at time  $t$ . This is analogous to the variation of pressure in the case of sound waves in air columns and variation of electric and magnetic field components as light wave travels. However, to extract physical interpretation a clue is provided by the way in which the wave and photon theories of light describe the intensity distribution in a diffraction pattern. The intensity is proportional to the square of wave amplitude in the wave description and it is proportional to flux of photons in the particle description. Max Born suggested that this correspondence between amplitude and particle flux allows one to interpret the wave function in the following way. The quantity

$$P_{\text{total}} = |\psi(r, t)|^2 d^3r$$

is a measure of probability of finding a particle described by the wave function  $\psi(\hat{r}, t)$  in the volume element  $d^3r$  at time  $t$  around the position  $\vec{r}$ . The wave amplitude is thus a

probability amplitude, the square of which yields  $P_{\text{total}}$ . Once the particle is detected, the probability must remain constant with time, i.e. no fluctuations in  $|\psi(r, t)|^2$  with time. To have this condition we have to take  $\psi$  as a complex quantity (as done earlier).  $\psi(r, t) = A + iB$ ,  $\psi^*(r, t) = A - iB$ ,  $|\psi|^2 = \psi^* \psi = A^2 + B^2$   $A$  &  $B$ , being real functions.

The probability concept for  $\psi(r, t)$  is intimately connected with the fundamental principle that the notion of strict determinancy is no longer valid.

In figure (1.7) we have shown a graphical representation of  $\psi$  and  $|\psi|^2$  for one dimensional wave function. A similar behaviour will be exhibited by a wave function  $\psi(r, t)$  and the corresponding  $|\psi(r, t)|^2$ .

Since the particle has to exist somewhere in space at all times, as such

$$\int_{-\infty}^{\infty} |\psi(\vec{r}, t)|^2 d^3 r = 1 \quad \dots (1.43)$$

or

$$\int_{-\infty}^{\infty} P d^3 r = 1$$

where  $P = \psi^*(r, t) \psi(r, t) = |\psi(r, t)|^2 \quad \dots (1.44)$

$P$  is known as probability density (Probability per unit volume).

A wave function  $\psi(r, t)$  that obeys equation (1.43) is said to be normalised. It is a condition for normalization. Besides being normalisable, the wave function should satisfy some boundary conditions in order to have consistency between mathematical solutions and physical requirements.

The wave function should be an acceptable wave function and its space derivatives should be single valued, continuous and finite everywhere.

Thus we have for the wave function:

1.  $\psi$  defines the dynamical state of a system.

2.  $\psi$  is a complex quantity

3.  $\psi, \frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$  are finite, continuous everywhere and zero at large distances.

4. Normalisation:  $\int_{\text{All space}} \psi^* \psi d^3 r = 1$

### 1.7.2 Schrodinger's equation

We expect the wave function to describe the dynamical state of a system (or a particle). Since  $\psi$  is a wave amplitude (probability amplitude) and we have seen that  $\psi$  is expected to have a role similar to standing waves. It is, therefore, required to have wave equation for the wave function  $\psi$ . It was the genius of Erwin Schrodinger who utilised this idea and proposed an equation, known after him, as Schrodinger's wave equation. This equation is as fundamental in Quantum mechanics as Newton's equation of motion  $\vec{F} = d\vec{P}/dt$  in classical mechanics and Maxwell's equation in electromagnetism.

Such an equation must have two basic properties: Firstly it must be linear so that any combination of a given solution also be a solution of the equation. (Possibility of formation of wave packets). And secondly, the coefficients of the equation must involve only fundamental constants like  $h$ , mass and charge of the particle and not the parameters (internal) concerning a particular kind of motion of the particle (like energy and momentum or  $k$  and  $\omega$ ). This follows from the first condition. In the presence of these internal parameters otherwise, a superimposed solution belonging to different equations cannot be a solution of the equation. Since the differential equations are easier to handle it is worth-while to try it first and turns out that the requirements can be met by a different equation.

A particle moving along positive  $x$ -direction with momentum  $p$  and energy  $E$  at any time  $t$  would be expected to have a wave function defined by any one of the following form

$$\cos(kx - \omega t), \sin(kx - \omega t), e^{i(kx - \omega t)}, e^{-i(kx - \omega t)} \dots (1.45)$$

or some suitable linear combination of them.

### 1.7.3 Schrodinger time independent wave equation

Let us consider a particle of mass  $m$ , moving with a velocity  $v$ . Let  $\psi$  be the wave function of the particle along  $x$ ,  $y$  and  $z$  coordinate axes at any time  $t$ . The de-Broglie wavelength associated with it is given by  $\lambda = \frac{h}{mv}$ .

The classical differential equation of a progressive wave, moving with a wave velocity  $v$  can be written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \dots (1.46)$$

The solution for equation is assumed to be

$$\psi = \psi_0 e^{-i\omega t} \dots (1.47)$$

where  $\psi_0$  is the amplitude of the wave at the point  $(x, y, z)$ . It is a function of position.  
Differentiating equation (1.32) with respect to 't' twice, we get

$$\begin{aligned}\frac{\partial \psi}{\partial z} &= \psi_0 e^{-i\omega t} \times (-i\omega) \\ &= -i\omega \psi\end{aligned}\dots (1.48)$$

and  $\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$  ... (1.49)

Substituting equation (1.34) in (1.31), we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} (-\omega^2 \psi)$$

Introducing the Laplacian operator  $\left[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right]$  in the above equation

it becomes,

$$\nabla^2 \psi = -\frac{\omega^2}{v^2} \psi \dots (1.50)$$

But  $\omega = 2\pi v = 2\pi (v / \lambda)$   $[\because v = \lambda / \lambda]$

or  $\frac{\omega}{v} = \frac{2\pi}{\lambda}$  ... (1.51)

Substituting equation (1.51) in (1.50), we get

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \dots (1.52)$$

On introducing the wave nature of the particle (from de-Broglie's wave equation we know that,  $\lambda = \frac{h}{mv}$ ) in equation (1.52), it becomes,

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \dots (1.53)$$

If  $E$  is the total energy of the particle,  $V$  the potential energy and  $\frac{1}{2}mv^2$  the kinetic energy; then,

$$E = PE + KE$$

$$E = V + \frac{1}{2}mv^2$$

or  $E - V = \frac{1}{2}mv^2$

$$\therefore 2(E - V) = mv^2$$

Multiplying by  $m$  on both sides in the above equation we get,

$$2m(E - V) = m^2v^2 \quad \dots (1.54)$$

Substituting equation (1.54) in (1.53), we get

$$\nabla^2\psi + \frac{8\pi^2m}{h^2}(E - V)\psi = 0 \quad \dots (1.55)$$

This equation is known as Schrodinger's time independent wave equation.

For a one-dimensional motion, the Schrodinger equation takes the form

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - V)\psi = 0 \quad \dots (1.56)$$

Introducing the  $\hbar = h/2\pi$  value in equation (1.55) we get,

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad \dots (1.57)$$

### Special case

For a free particle  $V = 0$  (P.E. is zero) therefore Schrodinger wave equation becomes

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \quad \dots (1.58)$$

### 1.7.4 Schrodinger time dependent wave equation

By eliminating E in the Schrodinger time independent wave equation, we can easily get the Schrodinger time dependent wave equation.

Differentiating equation (1.47) with respect to t, we get

$$\begin{aligned}
 \frac{\partial \psi}{\partial z} &= -i\omega \psi_0 e^{i\omega t} \\
 &= -(2\pi\nu) \psi_0 e^{i\omega t} \quad (\because \omega = 2\pi\nu) \\
 &= -2\pi i \nu \psi \\
 &= -i2\pi \frac{E}{h} \psi \quad [\because E = h\nu \text{ or } \nu = E/h] \\
 &= -i \frac{E}{\hbar} \psi
 \end{aligned}$$

or       $E\psi = i\hbar \frac{\partial \psi}{\partial t}$  ... (1.59)

Substituting equation (1.59) in Schrodinger time independent wave equation (1.57), we get

$$\begin{aligned}
 \nabla^2 \psi + \frac{2m}{\hbar^2} \left[ i\hbar \frac{\partial \psi}{\partial t} - V\psi \right] &= 0 \\
 \text{or} \quad \nabla^2 \psi &= -\frac{2m}{\hbar^2} \left[ i\hbar \frac{\partial \psi}{\partial t} - V\psi \right]
 \end{aligned}$$

Multiplying LHS and RHS by  $\frac{-\hbar^2}{2m}$  and rearranging,

$$\text{or} \quad \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots (1.60)$$

This equation (1.50) is known as Schrodinger time dependent wave equation. Equation (1.50) can be written as

$$\left( \frac{-\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots (1.61)$$

$$\text{or} \quad H\psi = E\psi$$

where  $H = \left( \frac{-\hbar^2}{2m} \nabla^2 + V \right)$  is called the Hamiltonian operator, ... (1.62)

and  $E = i\hbar \frac{\partial}{\partial t}$  is called the energy operator. ... (1.63)

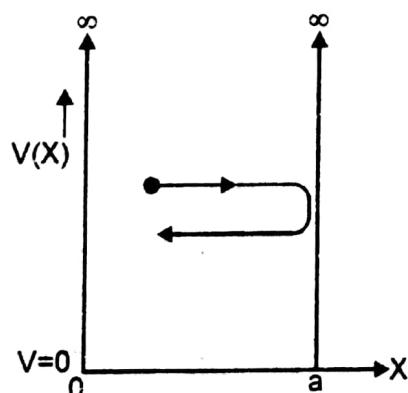
### 1.7.5 Free Particle in a one-dimensional box

Let us consider the motion of a particle (in a box) along x-direction within a region  $x = 0$  to  $x = a$ . Its motion is restricted by rigid walls of infinite height at  $x = 0$  and  $x = a$ , as shown in figure 1.8. The particle does not lose energy when it collides with such walls. Its total energy  $E$  remains constant.

The potential energy within the region  $0 < x < a$  is constant, say  $V(x) = 0$  in the region  $0 < x < a$ , where the particle is free to move. But the potential energy increases sharply to infinity on both sides of the walls at  $x = 0$  and  $x = a$ .

Since the particle cannot have infinite energy, as such it cannot exist outside the box. At the walls the particle experiences strong forces which reverses its motion. Such a box is termed as *a rectangular potential box or rectangular potential well of infinite depth and width a*.

In the region  $x < 0$  and  $x > a$ , the particle cannot exist, hence the wave function  $\psi(x)$  is zero for  $x \leq 0$  and  $x \geq a$ . Within the box, where we have taken  $V(x) = 0$  the particle moves in a force free region and the time independent Schrodinger's equation for a free particle ( $V=0$ ) is then



**Figure 1.8:** An one dimensional potential box of width a and rigid walls of infinite height

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0$$

$$\frac{d^2 \psi}{dx^2} + k^2 \psi = 0 \quad \dots(1.64)$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

[Since  $\psi$  is a function of  $x$  only, we have taken total derivative instead of partial derivative in equation (1.64)].

Equation (1.64) has the solution

$$\psi(x) = A \sin kx + B \cos kx \quad \dots(1.65)$$

$A$  and  $B$  are constants to be determined from the boundary conditions.

Since  $\psi = 0$  for  $x = 0$  from the first boundary condition,

Equation (1.65) becomes

$$\begin{aligned} 0 &= A \times 0 + B \times 1 && [\because \sin 0 = 0 \text{ & } \cos 0 = 1] \\ &= 0 + B \\ \Rightarrow B &= 0 \\ \therefore \psi(x) &= A \sin kx \end{aligned} \quad \dots(1.66)$$

To find  $A$ , apply the second boundary condition,

$$\psi = 0 \text{ for } x = a$$

$\therefore$  Equation (1.66) becomes

$$0 = A \sin ka$$

or  $ka = n\pi$ ,  $n = 1, 2, 3, \dots$

$$k = \frac{n\pi}{a} \quad \dots(1.67)$$

because  $\sin \pi, \sin 2\pi, \sin 3\pi, \dots$  are all zero.

Since  $k = \sqrt{\frac{2mE}{\hbar^2}}$ , and also  $k = \frac{n\pi}{a}$ , we find that the energy of the particle cannot have any arbitrary value, it can have only certain discrete values as determined by equation (1.68) and those are the energy eigen values.

The Energy eigen values of a free particle in a one dimensional box are:

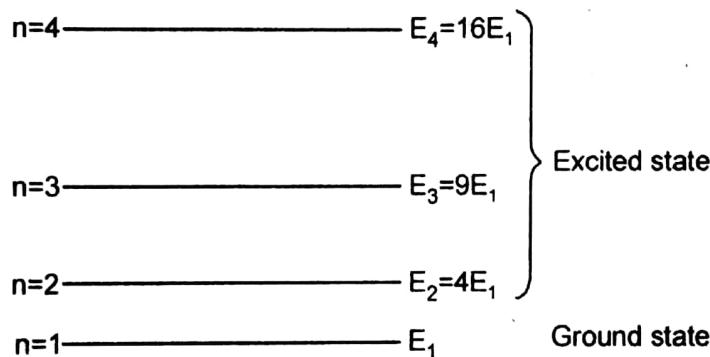
$$k^2 a^2 = \frac{2mE_n a^2}{\hbar^2} = n^2 \pi^2$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}, \quad \text{where } n = 1, 2, 3, \dots \quad \dots (1.68)$$

$n = 0$  is excluded because then  $E_0 = 0$  which is not true quantum mechanically.  $\psi$  becomes zero for  $n = 0$ , which is not permissible physically.

Thus the particle will have energy values given by equation (1.68). Its energy is quantised. If however the particle is outside the box, it is a free particle and will have continuous energy spectrum.

The discrete energy spectrum of the particle inside the box is shown in figure 1.9.



**Figure 1.9:** Energy levels for a particle inside an infinitely deep one dimensional potential box

From equation (1.68)

for  $n = 1, E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$

$$n = 2, E_2 = 4 \frac{\pi^2 \hbar^2}{2ma^2} = 4E_1$$

$$n = 3, E_3 = 9 \frac{\pi^2 \hbar^2}{2ma^2} = 9E_1$$

14:  $n = 4, E_4 = 16 \frac{\pi^2 \hbar^2}{2ma^2} = 16E_1$  and so on.

Such a situation of discrete values for the energy (eigen values) is not the peculiarity of the problem. It always happens whenever the Schrodinger's equation is solved for particle confined (due to potential) to a certain region of space then the particle will have only certain discrete energies  $E_1, E_2, E_3, \dots, E_n$  and the lowest or minimum energy is  $E_1$  and not zero as one would get ( $E = 0$ ) for the motion of particle described by classical mechanics.

The exclusion of  $E = 0$  for the particle in the potential box, and the finite value of the minimum energy

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$$

is a consequence of uncertainty principle. If  $E = 0$ , velocity  $v = 0$  and the de Broglie wave length  $\frac{h}{mv}$  would then be infinite. But for a trapped particle, to have infinite wave length is meaningless, as such the particles must have at least some kinetic energy and not zero. The uncertainty in the position of the particle is  $\Delta x \sim a$ . The particle is moving between the walls of the box with a momentum  $p$ , the uncertainty in the momentum is then  $\Delta p \approx 2p$ . The uncertainty principle demands that

$$\Delta x \Delta p \geq h \text{ (approximately)}$$

$$a \cdot 2p \geq h \quad \therefore \left( \hbar = \frac{h}{2\pi} \right)$$

$$p = \frac{h}{2a} = \frac{\hbar\pi}{a}$$

$$\frac{p^2}{2m} \geq \frac{\hbar^2\pi^2}{2ma^2}$$

or

$$E \geq E_1 \quad \dots (1.69)$$

Hence the lowest energy can never be zero. It is finite and is known as zero point energy. The zero point energy for the particle in one dimensional box of infinite depth and width 'a' is

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$$

**Eigen functions:**

The wave function of a particle in a box with energy  $E_n$  is

$$\psi_n = A \sin kx = A \sin \sqrt{\frac{2mE_n}{\hbar^2}} x \quad \dots (1.70)$$

From equation (1.67) gives

$$\psi_n = A \sin \frac{n\pi x}{a} \quad \dots (1.71)$$

Equation (1.71) gives the eigen function  $\psi_n$  corresponding to eigen values  $E_n$ . These eigen functions satisfy the necessary boundary conditions for a wave function. The normalisation condition in this case is

$$\int_{-\infty}^{\infty} |\psi_n|^2 dx = \int_{-\infty}^{\infty} |\psi_n|^2 dx = 1 \quad \dots (1.72)$$

Since the particle is confined within  $x = 0$  to  $x = a$

$$\int_0^a |\psi_n|^2 = \int_0^a A^2 \sin^2 \left( \frac{n\pi x}{a} \right) dx = 1$$

But  $\int_0^a \sin^2 \left( \frac{n\pi x}{a} \right) dx = \frac{a}{2}$

$$\therefore \int_0^a |\psi_n|^2 dx = A^2 \frac{a}{2} = 1$$

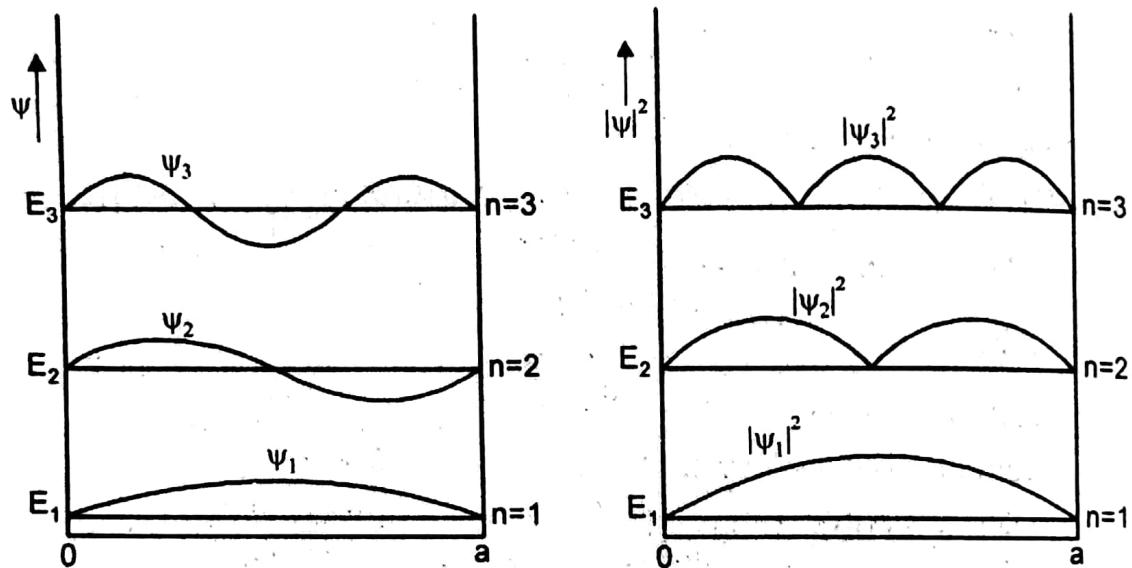
or  $A^2 = \frac{a}{2}$

$$\therefore A = \sqrt{\frac{2}{a}} \quad \text{is the normalisation constant.}$$

The normalised eigen functions of the particle are

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi}{a} x \right), \text{ where, } n = 1, 2, 3, \dots \quad \dots (1.73)$$

The wave functions  $\psi_1, \psi_2, \psi_3, \dots$  for first three energy levels corresponding to  $n = 1, n = 2, n = 3$  are shown in figure 1.10(a). In figure 1.10(b), the probability densities  $|\psi_1|^2, |\psi_2|^2$  and  $|\psi_3|^2$  are shown. Since  $\psi_n$  may be negative as well as positive  $|\psi_n|^2$  is always positive, and  $\psi_n$  being normalised, the value of  $|\psi_n|^2$  at a given  $x$  is equal to the probability density  $P$ , of finding the particle at the point. In every case  $|\psi_n|^2$  is zero at  $x = 0$  and  $x = a$ .



**Figure 1.10:** (a) First three wave functions for a particle in a one dimensional box.  
 (b) Probability density function  $|\psi|^2$  for first three states.

The eigen functions have another significant property. They are orthogonal that is

$$\int_{\text{all space}} \psi_n * \psi_m dx = 0, \quad n \neq m \quad \dots (1.74)$$

Since in the present problem, the wave functions are real and the limits for the region are  $x = 0$  to  $x = a$ .

$$\begin{aligned} \int_0^a \psi_n * \psi_m dx &= \frac{2}{a} \int_0^a \sin \frac{n\pi}{a} x \sin \frac{m\pi}{a} x dx \\ &= \frac{1}{a} \int_0^a \left[ \cos \frac{(n-m)\pi}{a} x - \cos \frac{(n+m)\pi}{a} x \right] dx = 0 \end{aligned} \quad \dots (1.75)$$

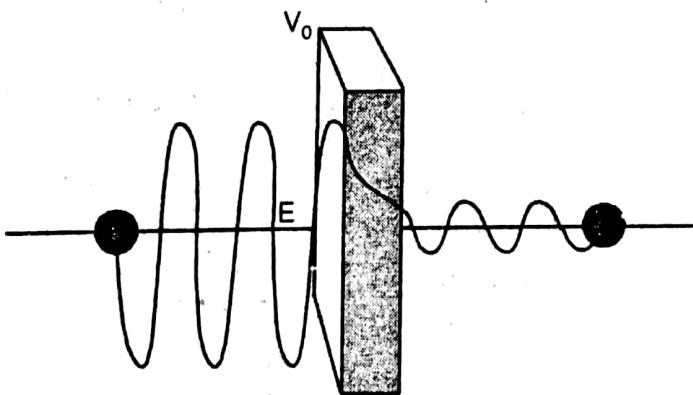
From the nature of wave function  $\psi$ , it is clear that at the walls of the box there will always be nodes ( $\psi = 0$ ) and within the range of the potential the number of nodes is equal to  $(n - 1)$ . The linear combination of the solution reflects superposition of waves giving rise to stationary waves or stationary states of the system. This means that the particle oscillates back and forth rapidly in the potential well. According to quantum probability amplitude, the probability of finding the particle at certain points corresponding to zeros of  $|\psi|^2$  becomes vanishingly small (nodes). Classically however, the probability of finding the particle is constant for all position of the particle which does not agree with low energy states defined quantum mechanically. It is only when  $n$  becomes very large, there occurs large number of rapid oscillations of the quantum probability amplitude and its average then is in agreement with the classical value.

## 1.8 Application of Quantum Mechanics

### 1.8.1 Tunnel Effect

Classically a particle with energy  $E < V_0$  (barrier height) cannot penetrate a barrier and it must rebound from the barrier. The quantum mechanical penetration of particle through the potential barrier even when the energy is less than the height of the barrier is called tunnel effect. This effect provides explanation for the following phenomena:

1. The emission of alpha particles from a radioactive element
2. The electrical breakdown of insulators
3. The switching action of a tunnel diode
4. The field emission of electrons from a cold metallic surface



**Figure 1.11:** Tunnel effect

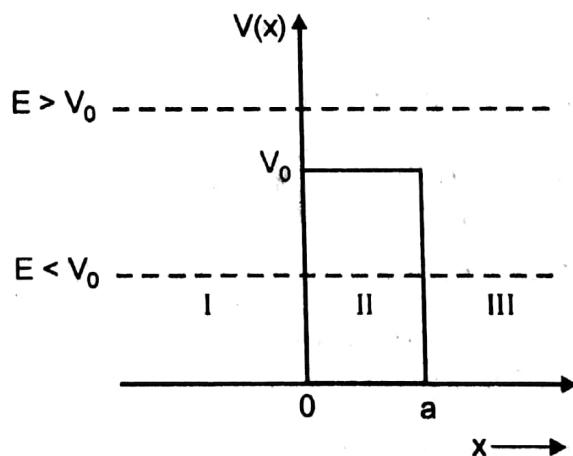
In classical mechanics, the particle is completely reflected by the potential barrier. Potential barrier is the energy inserted in order to go against the passage of electron as shown in figure.

The study of molecules of a few Angstrom size has become possible because of this new instrument STM. Traditional microscopy or imaging techniques, which employ lenses to focus light or other radiation, was diffraction limited.

Because of diffraction effect, it is therefore not possible to get a resolution better than half – wavelength of the radiation used. The new devices like STM, AFM are fundamentally different. These are not microscopes in the conventional sense and do not use radiations to image the object. These are force sensors. These scanning probe microscopes creates an image of the surface by measuring the force between the atoms on the surface and a fine, pointed tip which acts as the force sensor of the scanning probe microscope.

### 1.8.2 Barrier penetration

Let us consider the motion of a particle of mass  $m$  and total energy  $E$  in a rectangular potential barrier shown in Fig. (1.12). The potential is zero every where except over a region from  $x = 0$  to  $x = a$  where it has a constant value  $V_0$ . Such type of potential field is called potential barrier and can be represented as



**Figure 1.12:** Barrier penetration

$$V(x) = 0 \quad \text{when } x < 0 \text{ (Region I)}$$

$$V(x) = V_0 \quad \text{when } 0 < x < a \text{ (Region II)}$$

$$V(x) = 0 \quad \text{when } x > a \text{ (Region III)}$$

where  $V_0$  is a positive constant.

.. (1.76)

In quantum mechanics the motion of a particle in one dimensional time independent potential field is described by one dimensional time independent Schrodinger equation as

$$\frac{d^2 \psi(x)}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)] \psi(x) = 0 \quad .. (1.77)$$

In a region  $x < 0$  and  $x > a$ ,  $V(x) = 0$ , the Schrodinger equation can be written as

$$\left. \begin{aligned} \frac{d^2 \psi_1}{dx^2} + k^2(x) = 0 & \quad (\text{Region I}) \\ \text{and} \quad \frac{d^2 \psi_{III}}{dx^2} + k^2 \psi_{III} = 0 & \quad (\text{Region III}) \end{aligned} \right\} \quad .. (1.78)$$

where  $k = \sqrt{2mE/\hbar^2}$  and  $\psi_1$  and  $\psi_{III}$  are the wave functions in regions I and III respectively.

The steady state solutions of these equations are

$$\psi_1(x) = A e^{ikx} + B e^{-ikx}; (x < 0) \quad .. (1.79a)$$

$$\psi_{III}(x) = C e^{ikx} + D e^{-ikx}; (x > 0) \quad .. (1.79b)$$

Where A, B, C and D are arbitrary constants whose values are determined by the boundary conditions. We are assuming here that the particle encounters the barrier from left side. Thus in the region -I the incidence wave (amplitude A) propagating along  $+x$ -direction and the reflected wave (amplitude B) propagating along  $-x$ -direction both are present, but in region - III transmitted wave (Amplitude C) can propagate only in  $+x$ -direction because there is no possibility of reflected wave (amplitude D) in the region -III. Thus in equation (1.79b) D should be equal to zero. Then

$$\psi_1(x) = A e^{ikx} + B e^{-ikx}; \quad x < 0 \quad .. (1.80)$$

$$\psi_{III}(x) = C e^{ikx} \quad x > a \quad .. (1.81)$$

In region -II ( $0 < x < a$ ),  $V(x) = V_0$ . The solution of Schrodinger equation will depend on whether (i)  $E < V_0$  or (ii)  $E > V_0$ . In both cases, the solution of Schrodinger equation will be different. Therefore both cases will be discussed separately.

(i) When  $E < V_0$ , the Schrodinger equation in the region  $0 < x < a$  can be written as

$$\frac{d^2 \psi_{II}}{dx^2} - \frac{2m}{\hbar^2} (V_0 - E) \psi_{II} = 0$$

or

$$\frac{d^2 \psi_{II}}{dx^2} - \alpha^2 \psi_{II} = 0 \quad .. (1.82)$$

where  $\alpha = \sqrt{2m(V_0 - E)/\hbar^2}$  is a real constant and  $\psi_{II}$  is a wave function in a region  $0 < x < a$ . The solution of this equation will be as

$$\psi_{II} = F e^{\alpha x} + G e^{-\alpha x} \quad .. (1.83)$$

Now the boundary conditions require the continuity of  $\psi(x)$  and  $d\psi(x)/dx$ .

Apply boundary conditions at  $x = 0$ ,

$$\psi_I = \psi_{II}$$

and

$$\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$$

$$\text{at } x = 0$$

using equations (1.80) and (1.83)

$$A + B = F + G \quad .. (1.84)$$

$$ik(A - B) = \alpha(F - G) \quad .. (1.85)$$

Similarly at  $x = a$ ,

$$\psi_{II} = \psi_{III}$$

$$\frac{d\psi_{II}}{dx} = \frac{d\psi_{III}}{dx}$$

$$\text{at } x = a$$

Now using equations (1.81) and (1.83),

$$C e^{ika} = F e^{\alpha a} + G e^{-\alpha a} \quad .. (1.86)$$

$$ikC e^{ika} = \alpha(F e^{\alpha a} - G e^{-\alpha a}) \quad .. (1.87)$$

Now the equations (1.84), (1.85), (1.86) and (1.87) are to be solved to get  $B/A$  and  $C/A$ , since they are used in finding out the reflection and transmission coefficients of the particle.

From equations (1.84) and (1.85),

$$2A = F \left( 1 - \frac{i\alpha}{k} \right) + G \left( 1 + \frac{i\alpha}{k} \right) \quad .. (1.88)$$

$$\text{and } 2B = F \left( 1 + \frac{i\alpha}{k} \right) + G \left( 1 - \frac{i\alpha}{k} \right) \quad .. (1.89)$$

$$\text{or } \frac{B}{A} = \frac{(k + i\alpha) F + G (k - i\alpha)}{(k - i\alpha) F + G (k + i\alpha)} \quad .. (1.90)$$

Now dividing equation (1.87) by equation (1.86)

$$ik = \frac{\alpha (Fe^{\alpha a} - Ge^{-\alpha a})}{(Fe^{\alpha a} + Ge^{-\alpha a})}$$

$$\text{on solving, } G = - \frac{(k + i\alpha)}{(k - i\alpha)} e^{2\alpha a} F \quad .. (1.91)$$

using equation (1.91) in equation (1.90),

$$\begin{aligned} \frac{B}{A} &= \frac{(k^2 + \alpha^2)(e^{2\alpha a} - 1)}{(k^2 - \alpha^2)(e^{2\alpha a} - 1) + 2i\alpha k (e^{2\alpha a} + 1)} \\ &= \frac{(k^2 + \alpha^2) \sinh(\alpha a)}{(k^2 - \alpha^2) \sinh(\alpha a) + 2i\alpha k \cosh(\alpha a)} \quad .. (1.92) \end{aligned}$$

where  $\sinh x = (e^x - e^{-x})/2$  and  $\cosh x = (e^x + e^{-x})/2$

Similarly dividing equation (1.86) by equation (1.88), we get

$$\frac{C}{A} = \frac{2k (Fe^{\alpha a} + Ge^{-\alpha a}) e^{-ika}}{F (k - i\alpha) + G (k + i\alpha)}$$

Using equation (1.91) in this equation,

$$\frac{C}{A} = \frac{4ik\alpha e^{-\alpha a} e^{-ika}}{(k^2 - \alpha^2) \sinh(\alpha a) + 2i\alpha k \cosh(\alpha a)} \quad .. (1.93)$$

### Reflection coefficient

$$\therefore \text{Reflection coefficient } R = \left| \frac{B}{A} \right|^2 = \left( \frac{B}{A} \right) \left( \frac{B}{A} \right)$$

Using  $\cosh^2(\alpha a) = 1 + \sinh^2(\alpha a)$ .

$$k^2 = \frac{2mE}{\hbar^2} \cdot \alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

in equation (1.92),

$$R = \frac{(k^2 + \alpha^2)^2 \sinh^2 \alpha a}{(k^2 + \alpha^2)^2 \sinh^2 (\alpha a) + 4\alpha^2 k^2} \quad .. (1.94)$$

or

$$R = \left( 1 + \frac{4E(V_0 - E)}{V_0^2 \sinh^2 (\alpha a)} \right)^{-1} \quad .. (1.95)$$

### Transmission coefficient

$$T = \left| \frac{C}{A} \right|^2 = \left( \frac{C}{A} \right) \left( \frac{C}{A} \right)$$

Again using  $\cosh^2(\alpha a) = 1 + \sinh^2(\alpha a)$ .

$$k^2 = \frac{2mE}{\hbar^2} \cdot \alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

in equation (1.93)

$$T = \frac{4k^2 \alpha^2}{(k^2 + \alpha^2) \sinh^2 (\alpha a) + 4k^2 \alpha^2} \quad .. (1.96)$$

or

$$T = \left( 1 + \frac{V_0^2 \sinh^2 (\alpha a)}{4E(V_0 - E)} \right)^{-1} \quad .. (1.97)$$

It can be easily verified that

$$R + T = 1 \quad .. (1.98)$$

A very important result can be drawn from this discussion, that when the energy  $E$  of incident particle is less than the height of potential barrier  $V_0$  ( $E < V_0$ ), there is always a finite probability of transmission through the potential barrier. Classically a particle with energy  $E < V_0$  cannot penetrate a barrier and it must rebound from the barrier. The quantum barrier even when the energy is less than the height of the barrier is called tunnel effect. This effect provides an explanation for the  $\alpha$ -decay (i.e., emission of  $\alpha$ -particle from radioactive nucleus) and other several atomic and nuclear phenomena.

From equation (1.95) and (1.97) we find that when  $\lim E \rightarrow 0$ ,  $T \rightarrow 0$  and  $R \rightarrow 1$ . Thus when the energy of the particle is almost zero, the probability of transmission is negligible.

Similarly for  $E \rightarrow V_0$ ,  $\alpha a \rightarrow 0$  and  $\sinh \alpha a \rightarrow \alpha a$

The transmission coefficient

$$\underset{(\lim E \rightarrow V_0)}{T} = \left( 1 + \frac{m V_0 a^2}{2 \hbar^2} \right)^{-1} \quad .. (1.99)$$

In this equation  $m V_0 / a^2 / 2 \hbar^2$  is a dimensionless quantity and is regarded as a measure of opacity of potential barrier. As the width  $a$  or the height  $V_0$  of potential barrier or both increases, the transmission of particle decreases.

If  $\alpha a \gg 1$ , then

$$\sinh (\alpha a) \approx \frac{1}{2} e^{\alpha a}$$

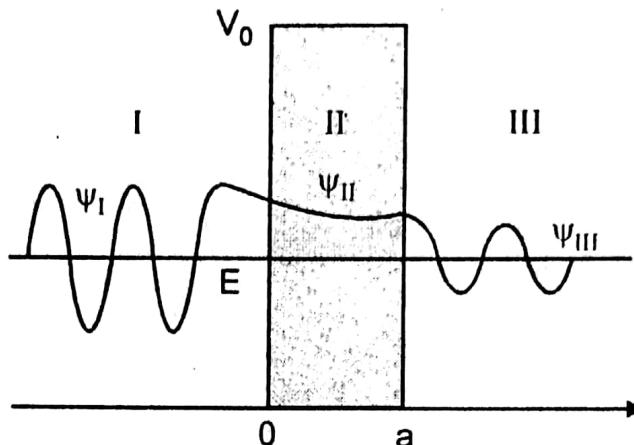
The transmission coefficient

$$T \approx \left( 1 + \frac{V_0^2 e^{\alpha a}}{16 E_0 (V_0 - E)} \right)^{-1}$$

$$\text{or } T \approx \frac{16 E (V_0 - E)}{V_0^2} e^{-2\alpha a} \quad .. (1.100)$$

As  $\alpha a$  increases,  $T$  decreases.

In case of  $E < V_0$ , the wave function in region I, II and III are shown in Fig.1.13.



**Figure 1.13:** Particle tunneling

The amplitude of wave function in region -I decreases exponentially in the region -II, even then there is some probability of transmission of wave through a potential barrier of finite width in region - III. This effect is called tunnel effect.

### (ii) When $E > V_0$

The Schrodinger equations [Equations (1.79a and 1.79b)] and their solutions [equations (1.80) and (1.81)] remain same in region I and III but in region -II ( $0 < x < a$ ), the Schrodinger equation becomes

$$\frac{d^2 \psi_{II}}{dx^2} + \frac{2m(E - V_0)}{\hbar^2} \psi_{II} = 0$$

$$\text{or } \frac{d^2 \psi_{II}}{dx^2} + k_1^2 \psi_{II} = 0$$

$$\text{Where } k_1 = \sqrt{2m(E - V_0) / \hbar^2}$$

The solution of equation in region II now,

$$k_1 = \sqrt{2m(E - V_0) / \hbar^2}$$

$$\psi_{II} = F e^{ik_1 x} + G e^{-ik_1 x}$$

On comparing this equations with equations (1.83) one finds that  $\alpha = ik_1$

Using this relation in equations (1.94) and (1.96) we get.

The reflection coefficient

$$R = \frac{(k^2 + k_1^2)^2 \sinh(k_1 a)}{(k^2 - k_1^2) \sin^2(k_1 a) + 4k^2 k_1^2}$$

or

$$R = \left( 1 + \frac{4E(E - V_0)}{V_0^2 \sin^2(k_1 a)} \right)^{-1}$$

The transmission coefficient

$$T = \frac{4k^2 k_1^2}{(k^2 + k_1^2)^2 \sin^2(k_1 a) + 4k^2 k_1^2}$$

or

$$T = \left( 1 + \frac{V_0^2 \sin^2(k_1 a)}{4E(E - V_0)} \right)^{-1}$$

Again we find that  $R + T = 1$

The most important conclusion is that the transmission probability of waves in region III is less than one even the energy E of the particle is greater than the height  $V_0$  of the potential barrier. But classically the transmission coefficient must be one for this case. As E increases with  $E > V_0$ , the transmission coefficient oscillates such that  $R + T = 1$  and for large values of E it approaches unity.

### 1.8.3 Scanning Tunneling Microscope

STM scans an electrical probe over a surface to be imaged to detect a weak electric current flowing between the tip and the surface. In 1986, Gerd Binnig and Heinrich Rohrer were awarded Nobel Prize for developing STM. It allows one to visualise regions of varying electron density and infer the position of individual atoms on the surface of a lattice. STM obtains images of conductive surfaces at an atomic scale  $10^{-10}$  m. It employs principle of electron tunneling. The quantum mechanical equations assign a finite non zero probability for an electron to move from region to another, even if it does not have sufficient energy to overcome the barrier between two regions.

STM consists of a tiny wire whose tip is fine almost to a single atom ( $\sim 2$  nm) width. This atomic tip, acting as probe, is held over the surface of specimen at a distance of about 1 nm above the surface. The tip is so close to the surface that electron clouds, of the atom at tip of the probe and of the nearest atom on the surface of specimen, overlap. When the

probe is given a potential, electrons from the atom of the specimen tunnel through the gap to reach the probe and generate a weak tunneling current. The probe is then scanned over the surface of specimen in raster pattern. The variation in tunneling current gives an image of the surface atoms of the specimen. The scanning is done in anyone of the two modes: constant current mode or constant height mode.

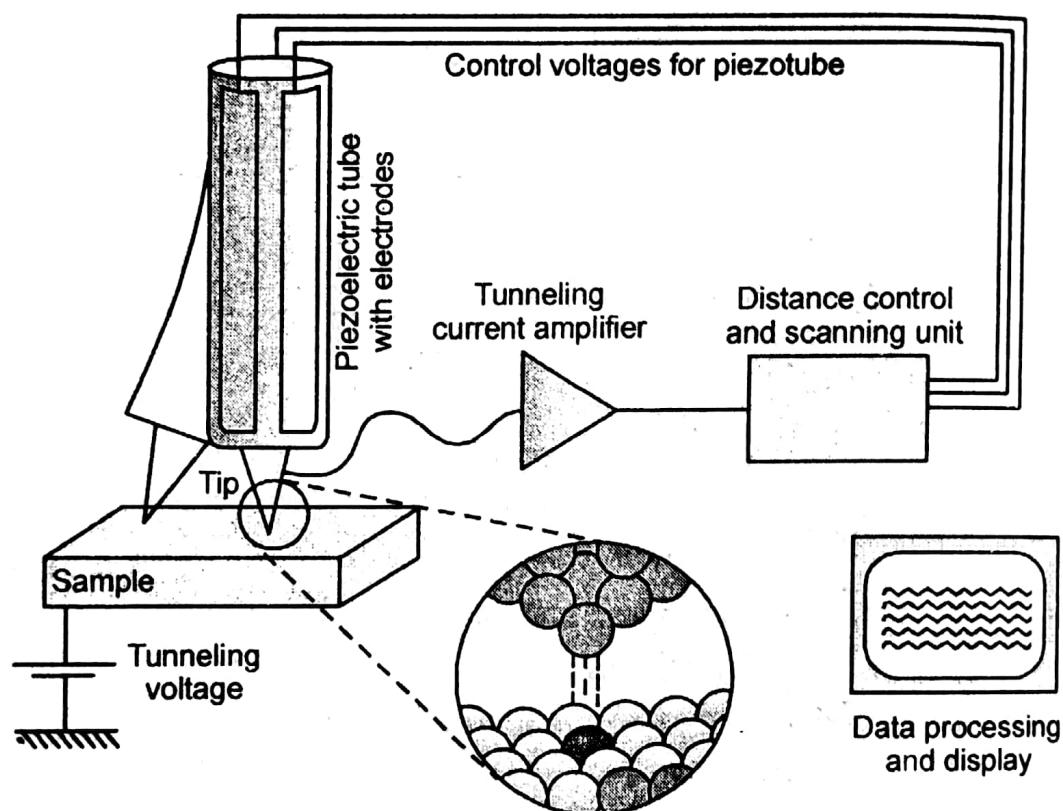


Figure 1.14: (a) Scanning Tunneling Microscope

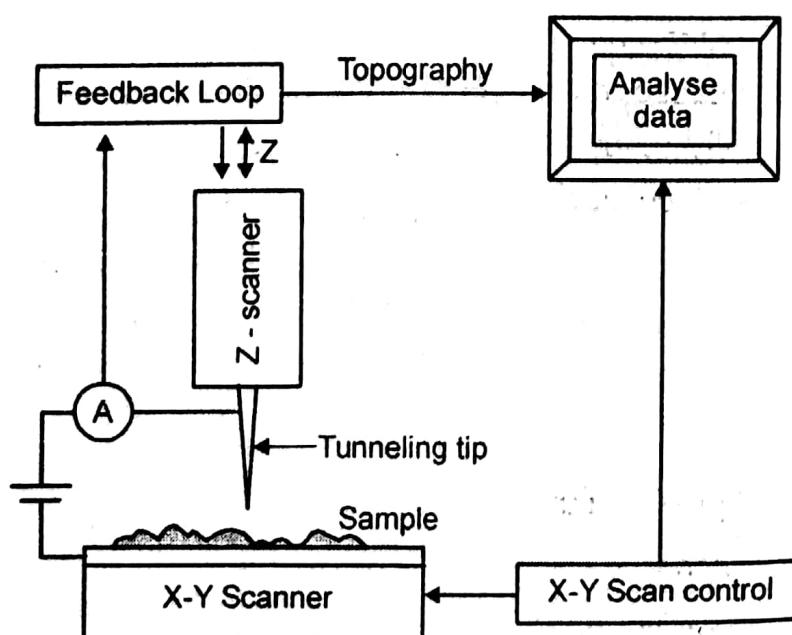
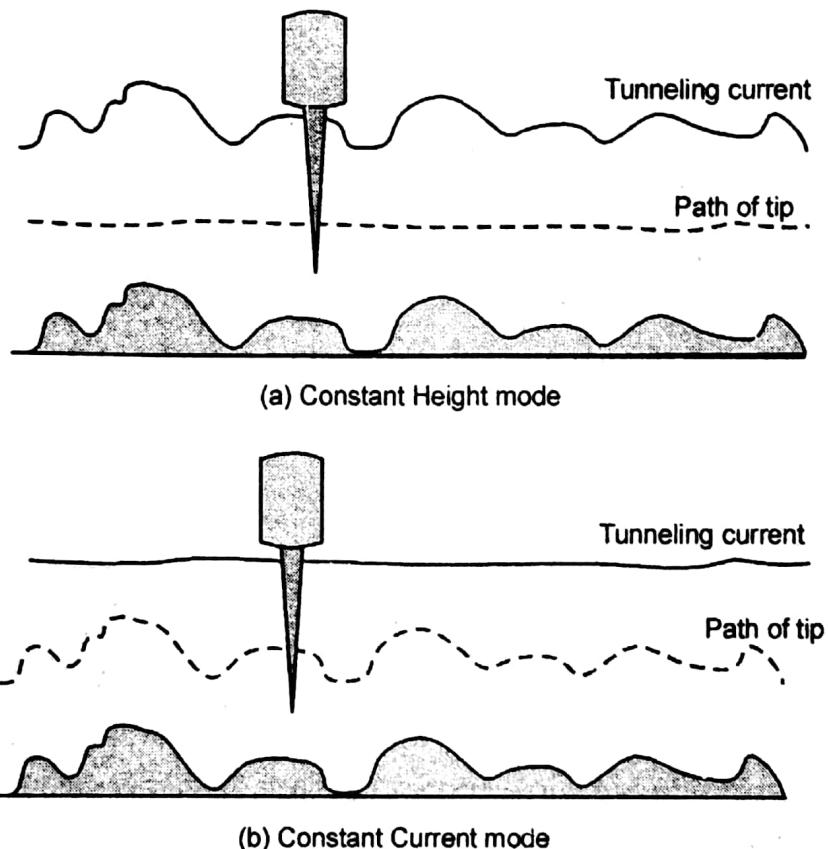


Figure 1.14: (b) Block diagram of STM

If the probe scans the surface at a constant height, the tunneling current fluctuates as the tip passes over different atoms. Depending upon the nature of atom (and associated electrons cloud) the tunneling current varies and the image recorded gives the atom by atom variation over the specimen. This is *constant height mode*. (Figure 1.15 (a))



**Figure 1.15:** Two different modes of STM

In *constant current mode* (Figure 1.15 (b)), as the tunneling current changes feedback electronics changes the height of the probe above the specimen surface to get constant current. Generally the tunneling current changes exponentially with distance between probe and specimen surface. For a change of gap by about 0.1 nm current changes by a factor of 10. For the same reason, it is a disadvantage if the surface is contaminated. Another limitation is that the specimen must have a conducting surface for the tunneling current to flow.

### Uses

It provides a 3D profile of the surface which helps in

- i) surface roughness.
- ii) observing surface defects.
- iii) determining the size and conformation of the atoms on the surface.

## Solved Problems

- 1. X-rays of wavelength 0.240 nm are Compton scattered and the scattered beam is observed at an angle  $60^\circ$  relative to the incident beam. Find the wavelength of the scattered X-rays.**

### Solution

$$\text{We know, } \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

$$\text{But } \frac{h}{m_e c} = 2.426 \text{ pm;}$$

$$\lambda = 0.240 \text{ nm}, \theta = 60^\circ$$

$$\therefore \lambda' = \lambda + 2.426 \text{ pm} (1 - \cos \theta)$$

$$\therefore \lambda' = 0.240 \text{ nm} + 2.426 \text{ pm} (1 - \cos 60^\circ)$$

$$= 0.240 \text{ nm} + 0.0012 \text{ nm}$$

$$\lambda' = 0.2412 \text{ nm}$$

- 2. A beam of X-rays is scattered by a target. At  $45^\circ$  from the beam direction the scattered X-rays have a wavelength of 2.2 pm. What is the wavelength of X-rays in the direct beam?**

*Given data:*

$$\theta = 45^\circ, \lambda' = 2.2 \text{ pm}; \lambda = ?$$

### Solution

$$\lambda' - \lambda = \lambda_c (1 - \cos \theta)$$

$$\lambda = \lambda' - \lambda_c (1 - \cos \theta)$$

Plugging the values,

$$\lambda = 2.2 \text{ pm} - 2.426 \text{ pm} (1 - \cos 45^\circ)$$

$$\lambda = 1.489 \text{ pm}$$

- 3. At what scattering angle will incident 100 keV X-rays leave a target with an energy of 90 keV?**

*Given data:*

$$E = 100 \text{ keV}; E' = 90 \text{ keV}$$

**Solution**

$$\left( \frac{1}{E'} - \frac{1}{E} \right) = \frac{1}{m_e c^2} (1 - \cos \theta)$$

Plugging the values in

$$m_e c^2 \left[ \frac{1}{E'} - \frac{1}{E} \right] = (1 - \cos \theta)$$

We get,  $\cos \theta = 0.4428$

$$\theta \approx 64^\circ$$

- 4. An X-ray photon of wavelength 10 pm is scattered through  $110^\circ$  by an electron. What is the kinetic energy of the scattered electron?**

*Given data:*

$$\lambda = 10 \text{ pm}; \theta = 110^\circ$$

*Formula:*

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

By plugging the values, we have

$$\Delta\lambda = \lambda' - \lambda = 2.426 \text{ pm} \times (1 - \cos 110^\circ)$$

$$\Delta\lambda = \lambda' - \lambda = 3.256 \text{ pm}$$

∴ The scattered wavelength,

$$\begin{aligned} \lambda' &= \lambda + \Delta\lambda = 10 \text{ pm} + 3.256 \text{ pm} \\ &= 13.26 \text{ pm} \end{aligned}$$

∴ The kinetic energy of the electron is,

$$K_e = E - E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

Plugging for  $h$ ,  $c$ ,  $\lambda$  and  $\lambda'$ , we have  $K_e = 30.5 \text{ keV}$ .

5. In an experiment of Compton scattering, the incident radiation has wavelength, 2 Å. Calculate the energy of recoil electron which scatters radiation through  $60^\circ$ .

**Solution**

Change in wavelength in Compton scattering

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

$$\lambda = 2 \times 10^{-10} \text{ m and } \lambda_c = 2.426 \text{ pm}$$

$$\text{when } \theta = 60^\circ$$

$$\lambda' = 2 \times 10^{-10} \text{ m} + 2.426 \times 10^{-12} \text{ m} (1 - \cos 60^\circ)$$

$$\lambda' = 2.012 \times 10^{-10} \text{ m}$$

Hence the energy of recoil electron which scatters radiation through  $60^\circ$  is given

$$E = h\nu - h\nu' = hc \left[ \frac{1}{\lambda} - \frac{1}{\lambda'} \right]$$

$$\begin{aligned} & 6.626 \times 10^{-34} \text{ Js} \times \\ &= 3 \times 10^8 \text{ ms}^{-1} \left[ \frac{1}{2 \times 10^{-10} \text{ m}} - \frac{1}{2.012 \times 10^{-10} \text{ m}} \right] \end{aligned}$$

$$E = 5.928 \times 10^{-18} \text{ J}$$

or

$$E = 37 \text{ eV}$$

6. Calculate the de-Broglie wave length of: (i) 1 MeV proton; (ii) 25 eV electron;

**Solution**

i)  $\lambda$  of proton

$$= \frac{28.7 \times 10^{-10}}{\sqrt{E}} \text{ m } [\because E = 10^6 \text{ eV}]$$

$$= \frac{28.7 \times 10^{-10}}{\sqrt{10^6}} \text{ m } = \frac{28.7}{10^3} \text{ Å}$$

$$= 2.87 \times 10^{-2} \text{ Å}$$

$$\text{ii) } \lambda \text{ of electron} = \frac{12.2}{\sqrt{10^6}} \text{ Å}$$

$$= \frac{12.2}{5} = 2.45 \text{ Å}$$

7. The position and momentum of 2 KeV electrons are measured simultaneously. If its position is located to within 1 Å, what is the percentage of uncertainty in its momentum?

### Solution

The energy of the electron

$$\begin{aligned} E &= 2 \text{ KeV} \\ &= 2 \times 10^3 \times 1.6 \times 10^{-19} \text{ Joule} \end{aligned}$$

The uncertainty in the location of its position

$$\Delta x = 1 \text{ Å} = 10^{-10} \text{ m}$$

The uncertainty in the momentum

$$\begin{aligned} \Delta p &= \frac{h}{2\pi} \cdot \frac{1}{\Delta x} = \frac{6.6 \times 10^{-34}}{2 \times 3.14} \times \frac{1}{10^{-10}} \\ &= 1.1 \times 10^{-24} \text{ kg-m/sec} \end{aligned}$$

But the momentum  $p = \sqrt{2mE}$

$$\begin{aligned} &= \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 10^3 \times 1.6 \times 10^{-19}} \\ &= 2.4 \times 10^{-23} \text{ kg.m sec}^{-1} \end{aligned}$$

$\therefore$  The percentage of uncertainty in its momentum

$$\begin{aligned} \frac{\Delta p}{p} \times 100 &= \frac{1.1 \times 10^{-24}}{2.4 \times 10^{-23}} \times 100 \\ &= 4.67 \% \end{aligned}$$

8. The life time of an energy state is  $10^{-8}$  sec., calculate the uncertainty in the frequency of the photon emitted during the transition (de-excitation of the atom).

**Solution**

The uncertainty in photon energy

$$\Delta E \sim \frac{h}{\Delta t}$$

The uncertainty in the time of de-excitation is of the order of the life time of the state  
 $\Delta t \approx 10^{-8}$  sec.

$$\therefore \Delta E = \frac{h}{10^{-8}} = \frac{1.054 \times 10^{-34}}{10^{-8}}$$

$$= 1.1 \times 10^{-26} \text{ J}$$

$$\therefore \Delta v = \frac{\Delta E}{h} = \frac{1.1 \times 10^{-26}}{6.6 \times 10^{-34}}$$

$$= 1.6 \times 10^7 \text{ Hz}$$

9. If the uncertainty in the location of a particle is equal to its de Broglie wavelength, what is the uncertainty with velocity?

**Solution**

Given data:  $\Delta x = 1$

Formula  $\Delta x \Delta p = h$

$$\Delta x \cdot m \Delta v = h$$

$$\Delta v = \frac{h}{m \lambda}$$

$$= \frac{h}{m \times \cancel{h} / mv} = v$$

$$s \Delta v = v$$

**10. A nucleon is confined to a nucleus of diameter  $5 \times 10^{-4}$  m. Calculate the minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon.**

### Solution

Given data:  $(\Delta x)_{\max} = 5 \times 10^{-4}$  m

Formula:  $(\Delta p)_{\min} (\Delta x)_{\max} = h$

$$\begin{aligned} (\Delta p)_{\min} &= \frac{h}{(\Delta x)_{\max}} = \frac{6.62 \times 10^{-34}}{5 \times 10^{-4}} \\ &= 1.33 \times 10^{-20} \text{ kg-m/sec} \end{aligned}$$

p cannot be less than  $(\Delta P)_{\min}$

$$p_{\min} = (\Delta p)_{\min}$$

$$\begin{aligned} E &= \frac{p^2}{2m} = \frac{(1.33 \times 10^{-20})^2}{2 \times 1.675 \times 10^{-27}} \\ &= \frac{0.529 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 0.33 \text{ MeV} \end{aligned}$$

**11. Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1 Å, given mass of the electron  $9.11 \times 10^{-31}$  kg m and  $h = 6.63 \times 10^{-34}$  Js.**

### Solution

Given data:  $a = 1 \text{ \AA} = 10^{-10} \text{ m}$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

Formula:  $E = \frac{h^2}{8ma^2}$

$$E = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (10^{-10})^2} \text{ J}$$

$$= 9.1 \times 10^{-19} \text{ J}$$

$$= \frac{9.1 \times 10^{-19}}{1.602 \times 10^{-19}} \text{ eV}$$

$$= 5.68 \text{ eV}$$

- 12. An electron is bound by a potential which closely approaches an infinite square well of width  $2.5 \times 10^{-10} \text{ m}$ . Calculate the lowest three permissible quantum energies the electron can have.**

### Solution

Given data:  $a = 2.5 \times 10^{-10} \text{ m}$

Formula:  $E_n = \frac{n^2 h^2}{8ma^2}$

$$E_1 = \frac{(6.63 \times 10^{-34})}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2} = 9.63 \times 10^{-19} \text{ J}$$

$$E_1 = 6 \text{ eV}$$

$$E_2 = 24 \text{ eV} \quad \text{and} \quad E_3 = 54 \text{ eV}$$

- 13. A particle moving in one-dimensional potential box (of infinite height) of width 25 Å. Calculate the probability of finding the particle within an interval of 5 Å at the centres of the box when it is in its state of least energy.**

### Solution

Given data:  $a = 25 \text{ \AA} = 25 \times 10^{-10} \text{ m}$

$$\Delta x = 5 \text{ \AA} = 5 \times 10^{-10} \text{ m}$$

Formula: The wave function of particle enclosed with an infinite well is given by

$$\psi(x) = \sqrt{\frac{2}{a}} \sin n \frac{\pi x}{a}$$

when the particle is in the least energy state  $n = 1$ ,

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$$

At the centre of the box  $x = a/2$ . The probability of finding the particle in the unit interval at the centre of the box is

$$\begin{aligned} |\psi(x)|^2 &= \left[ \sqrt{\frac{2}{a}} \sin \frac{\pi(a/2)}{a} \right]^2 \\ &= \frac{2}{a} \sin^2 \frac{\pi}{2} \\ &= \frac{2}{a} \end{aligned}$$

The probability  $p$  in the interval  $\Delta x$  is

$$\begin{aligned} p &= |\psi(x)|^2 \Delta x \\ &= \frac{2}{a} \Delta x \\ &= \frac{2 \times 5 \times 10^{10}}{25 \times 10^{-10}} \\ &= 0.4 \end{aligned}$$

- 14. Find the K.E. of a neutron which has a wavelength of  $3\text{\AA}$ . At what angle will such a neutron undergo first order Bragg reflection from a calcite crystal for which the grating space is  $3.039\text{\AA}$ ?**

### Solution

We know that

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad E = \frac{h^2}{2m\lambda^2}$$

$$\begin{aligned} \therefore E &= \frac{(6.6 \times 10^{-34})^2}{2 \times (1.6 \times 10^{-27})(3 \times 10^{-10})^2} \\ &= 1.45 \times 10^{-21} \text{ J} \end{aligned}$$

Further,

$$2d \sin \theta = n\lambda$$

or

$$\sin \theta = \frac{n\lambda}{2d}$$

$$= \frac{1 \times (3 \times 10^{-10})}{2 \times (3.039 \times 10^{-10})}$$

$$= 0.4936$$

$$\theta = \sin^{-1}(0.4936) = 29^\circ 33'$$

**15. Consider an electron whose total energy is 5 eV approaching a barrier whose height is 6 eV and width is 7 Å. Find out de Broglie wave length of incident electron and probability of transmission through the barrier. (Mass of electron =  $9.1 \times 10^{-31}$  kg, Planck's constant =  $6.6 \times 10^{-34}$  Js)**

**Solution**

$$\text{Energy of incident electron } E = 5 \text{ eV}$$

$$\therefore \text{Accelerating voltage of incident electron} = 5 \text{ volt}$$

de Broglie wave length of an electron

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

$$= \frac{12.27}{\sqrt{5}}$$

$$= 5.48 \text{ Å}$$

$$\text{Given: Height of potential barrier } V_0 = 6 \text{ eV}$$

$$\text{Width of potential barrier } a = 7 \text{ Å}$$

$$= 7 \times 10^{-10} \text{ m}$$

$$\therefore \text{Since Planck's constant } \hbar = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{2 \times 3.14}$$

$$= 1.054 \times 10^{-34} \text{ Js}$$

The transmission coefficient

$$T = \frac{16E(V_0 - E)}{V_0^2} e^{-2\alpha a}$$

Where

$$\begin{aligned}\alpha a &= \left[ \frac{2m}{\hbar^2} (V_0 - E) \right]^{1/2} a \\ &= \left[ \frac{2 \times 6.1 \times 10^{-31} \times (6-5) \times 1.6 \times 10^{-19}}{1.054 \times 10^{-34} \times 1.054 \times 10^{-34}} \right]^{1/2} \times 7 \times 10^{-10} \\ &= 5.12 \times 10^9 \times 7 \times 10^{-10} = 3.584 \\ T &= \frac{16 \times 5 \times (6-5)}{6 \times 6} \exp[-2 \times 3.584] \\ &= 2.2 \exp(-7.17) \\ &= 1.69 \times 10^{-3}\end{aligned}$$

**16.** A beam of electrons is incident on a potential barrier of height 5 eV and width 0.2 mm. What should be the energy of the electron so that half of them are able to penetrate through the barrier?

**Solution**

Given

$$V_0 = 5 \text{ eV}$$

$$a = 0.2 \text{ nm} = 2 \times 10^{-10} \text{ m}$$

Since  $V_0 > E$ , the transmission probability

$$T = \frac{16E(V_0 - E)}{V_0^2} e^{-2\alpha a}$$

$$T = T_0 e^{-2\alpha a}$$

where  $T_0$  is the maximum transmission probability

$$\text{Given: } T = \frac{T_0}{2}$$

$$\chi = \frac{T_0}{2} e^{-2\alpha a}$$

or

$$2\alpha a = \log 2$$

but

$$a = \sqrt{2m(V_0 - E)/\hbar^2}$$

$$(V_0 - E) = \frac{\hbar^2 (\log_e 2)^2}{8ma^2}$$

$$= \frac{(0.693)^2 \times (1.054 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.83 \times 10^{-20} \text{ J}$$

$$= \frac{1.83 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 0.11 \text{ eV}$$

$$E = V_0 - 0.11$$

$$= 5 - 0.11$$

$$= 4.890 \text{ eV}$$

### Points to Remember

1. Black body is an object which absorbs all the electromagnetic radiation incident on it and hence appears black. The absorption power of a black body is 1 for all frequencies.
2. Planck arrived at his black body formula by making two assumptions. (i) The energy of a charged oscillator of frequency  $\nu$  is limited to discrete values  $n\hbar$  and (ii) during light emission or absorption the change in energy of an oscillator is  $\hbar$ .
3. Classical theory cannot be applied to atomic phenomenon e.g., motion of an electron in an atom, stability of the atom or nucleus, spectrum of hydrogen atom.
4. According to Thomson Scattering, there is no change in wavelength of the scattered light.
5. According to Compton scattering, the scattered radiation has larger wavelength.
6. Compton scattering is an experimental evidence for the particle nature of electromagnetic radiation.

7. The unmodified peak in the compton's scattering results is due to the interaction between the photon and the electron which is very strongly attached to the nucleus.
8. Compton shift does not depend on the incident wavelength.
9. Exchange of energy between electromagnetic radiation and matter is only in discrete packets called **photons**. Because the photon travels at the speed of light, it must have zero mass, otherwise its momentum and energy would be infinite. Similarly, a photon's rest energy also be zero.
10. The wave nature associated with a moving particle is known **matter wave or de Broglie wave**. The following reasons inspired Louis de Broglie to propose the concept of matter waves: (i) The universe is symmetrical. (ii) The radiation possesses dual nature. If radiation possesses dual nature, the matter will also possess dual nature.
11. Davisson and Germer experiment: Accelerated electrons are diffracted by crystals as X-rays. By Bragg's law,  $2ds\sin\theta = n\lambda$ . Using a Nickel crystal electrons accelerated by 54 V produce the first diffraction maximum at  $\theta = 50^\circ$ .
12. According to uncertainty principle the position and momentum, or energy and time, of an electron cannot be measured simultaneously with unlimited precision.
13. The variable quantity characterizing de Broglie waves associated with moving particle is called the **wave function or state function** and is denoted by  $\psi$ .
14. The equation that describes the wave nature of a particle in mathematical form is known as **Schrodinger wave equation**.  $-i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi - V\psi$  is known as **Schrodinger three dimensional time dependent wave equation**, where  $\nabla^2$  is a Laplacian operator.
15.  $\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V)\psi = 0$  is known as three dimensional **time independent Schrodinger wave equation**. For a free particle, the potential energy,  $V = 0$ .
16. Applications of Schrodinger wave equation:
  - i) Particle in a box (like electrons in metals)
  - ii) Particles in a potential well (radioactive disintegration)
  - iii) Hydrogen atom
  - iv) Rotating molecule.

17. Energy of a free particle in a box,  $E_n = \frac{n\hbar^2}{2ma^2} = \frac{n\pi^2\hbar^2}{8ma^2}$
18. The application of quantum concepts does not lead to any observable effects in macro world but assumes significance in the micro world. The underlying reason for this difference is that the magnitude of *Planck's constant is extremely small*.
19. The quantum mechanical penetration of particle through the potential barrier even when the energy is less than the height of the barrier is called **tunnel effect**.
20. Transmission coefficient is defined as the ratio of probability flux transmitted through the barrier to the probability flux incident upon the barrier. Reflection coefficient is defined as the ratio of probability flux reflected from the barrier to the probability flux incident upon the barrier.
21. The probability of particles being transmitted or reflected is determined by the ratio of the appropriate  $\Psi^*\Psi$ ; the probability of the wave being either transmitted or reflected has to be unity.
22. In STM, the tunneling current is an exponential function of distance. Based on quantum mechanics, the tunneling current ( $I_t$ ) is,  $I_t = e^{-kd}$  where  $d$  is the distance between tip and sample surface.

### Review Questions

1. Why does the sun apparently act as a black body?
2. Why did scientists choose to study black body radiation from something as complicated as a hollow container rather than the radiation from something simple like a thin solid cylinder?
3. Deduce Stefan's law from Planck's radiation law.
4. Deduce Wien's displacement law from Planck's radiation formula.
5. Show that the Planck's radiation law agrees with the Rayleigh Jeans formula for large wavelengths.
6. A source operating at a frequency of 100 MHz radiates a power of 100 kW. Calculate the number of quanta of energy emitted per second. [Ans:  $15 \times 10^{29} \text{ s}^{-1}$ ]
7. Explain the Planck's quantum hypothesis.
8. Derive Planck's radiation formula using Maxwell distribution function.

9. Why do we say that light has both wave and particle properties.
10. Describe briefly the failure of the classical theory to explain the black body radiation
11. Differentiate between Compton scattering and Thomson scattering.
12. What is Compton effect? How does it support the particle nature of light?
13. Discuss Compton scattering and obtain an expression for the Compton shift.
14. Explain why compton effect is not observed easily for low frequency electromagnetic radiation?
15. How do you account for the unmodified peak in the Compton's results?
16. If Compton effect had been observed initially for gamma rays, would the Compton shift have varied? Defend your answer.
17. Describe the Davisson and Germer experiment. How does it confirm the evidence of matter waves?
18. Explain Heisenberg's principle of uncertainty. Discuss its validity by two examples.
19. Write down the energy-time uncertainty relations. Hence explain the natural width of spectral lines.
20. Use momentum position uncertainty relation to explain
  - i) non-existence of electrons inside the nucleus
  - ii) the ground state energy of hydrogen atom
  - iii) non-zero value of the energy of harmonic oscillator in its ground state.
21. What are the reasons that prompted de Broglie to propose the concept of matter waves?
22. Differentiate between  $\psi$  and  $|\psi|^2$ .
23. Obtain Schrodinger time independent wave equation.
24. Obtain Schrodinger time dependent wave equation.
25. State Schrodinger's time independent equation. Hence, obtain expression for eigen function of particle in one dimensional potential well of length  $a$ .
26. Show that the energy of an electron in one dimensional box varied as the square of natural numbers.

27. Write the Schrodinger equation for the particle in a box and solve it to obtain the eigen values and eigen functions.
28. Prove that the eigen functions of a particle moving in one dimensional box are orthogonal.
29. Find the energy eigen values and eigen functions for a particle enclosed in a one dimensional box.
30. Set up the time dependent Schrodinger equation for a non-relativistic particle and hence obtain the time independent equation.
31. Which of the following are eigen functions of the operator  $d^2/dx^2$ ? Give the eigen values where appropriate (i)  $\sin x$  (ii)  $\cos x$  (iii)  $e^x$  (iv)  $e^{-x}$  (v)  $\sin^2 x$ .
32. What is potential barrier? Calculate the reflection and transmission coefficients of rectangular potential barrier for  $E > V_0$ . Also prove that transmission coefficient will be one for certain values of energy.
33. Which is more effective to prevent tunneling, the barrier height or the barrier width?
34. Can a particle be observed while it is tunneling through the barrier? What would be its wavelength, momentum and kinetic energy while it tunnels through the barrier?
35. Calculate the probability for the penetration of a barrier of height 3 eV and width 10 Å by an electron of kinetic energy 2.5 eV.
36. A beam of electrons with energy 10 eV is incident on a potential barrier of height 20 eV and width 5 Å. Calculate the transmission probability of electrons. Given mass of an electron  $9.1 \times 10^{-31}$  kg and  $\hbar = 10^{-34}$  Js. [Ans:  $1.66 \times 10^{-7}$ ]
37. A furnace has walls of temperature  $1600^\circ\text{C}$ . What is the wavelength of maximum intensity emitted when a small hole is made up? [Hint: Wiens's displacement law,  $\lambda_{\max} T = 2.898 \times 10^{-3}$  m K;  $\lambda_{\max} = 1550$  nm]
38. What is tunneling? Explain with a neat sketch the principle, construction and working of a STM.
39. An X-ray photon of frequency  $1.5 \times 10^{19}$  Hz suffers Compton scattering. The frequency of the scattered radiation is  $1.2 \times 10^{19}$  Hz. Calculate the kinetic energy of the recoil electron. [Ans: 12.4 keV]

40. An X-ray photon of initial frequency  $3 \times 10^{19}$  Hz collides with an electron in the carbon block. It is scattered at  $90^\circ$ . Calculate the new frequency. [Ans:  $2.4 \times 10^{19}$  Hz]
41. An X-ray photon of wavelength  $0.10\text{\AA}$  is scattered by a carbon block. Calculate
- The wavelength of photon scattered at  $60^\circ$ .
  - The energy and velocity of the recoil electron.
  - The recoil angle [Ans: (i)  $0.112\text{\AA}$ ; (ii)  $12.5\text{ keV}$ , (iii)  $\theta \approx 60^\circ$ ]
42. A photon collides with a free proton, initially at rest. The photon is deflected by  $90^\circ$ . What is its change of wavelength? (Given, mass of the proton,  $m_p = 1.673 \times 10^{-27}\text{ kg}$ ) [Ans:  $1.32 \times 10^{-5}\text{\AA}$ ]
43. A photon of energy  $1.02\text{ MeV}$  is scattered through  $90^\circ$  by a free electron. Calculate the energy of the photon and electron after interaction. [Ans:  $0.51\text{ MeV}$ ,  $0.51\text{ MeV}$ ]
44. Compute the de Broglie wavelength of the following:
- A 1 tonne car travelling at  $100\text{ m/s}$ .
  - A  $10\text{ g}$  bullet travelling at  $500\text{ m/s}$ .
  - A smoke particle of mass  $10^{-9}\text{ g}$  moving at  $1\text{ cm/s}$ .
  - An electron with a kinetic energy of  $1\text{ eV}$ .
  - An electron with a kinetic energy of  $100\text{ MeV}$ .
- [Ans:  $6.6 \times 10^{-39}\text{ m}$ ;  $1.3 \times 10^{-34}\text{ m}$ ;  $6.6 \times 10^{-20}\text{ m}$ ;  $1.2\text{ nm}$ ;  $12\text{ fm}$ ]
45. Calculate the energy of a proton whose de-Broglie wavelength is  $0.5\text{\AA}$ . [Ans:  $0.33\text{ eV}$ ]
46. A rifle bullet of mass  $20\text{ g}$  is moving with a velocity of  $300\text{ m/s}$ . Calculate the de-Broglie wave length associated with it. [Ans:  $4.1 \times 10^{-34}\text{ m}$ ]
47. An electron is accelerated through a potential difference  $V$  volts. Prove that its de-Broglie wave length is given by:
- $$\lambda = \frac{2.2}{\sqrt{V}}\text{\AA}$$
48. Find the kinetic energy of a neutron whose wavelength is  $10^{-14}\text{ m}$ . [Ans:  $8.2\text{ MeV}$ ]
49. A nucleon is confined in a nucleus of radius  $5 \times 10^{-15}\text{ m}$ . Calculate the minimum uncertainty in the momentum of the nucleon. [Ans:  $1.326 \times 10^{-19}\text{ kg-m/s}$ ]

50. The position and momentum of 1 keV electron are determined simultaneously. If its position is located with in  $1\text{ \AA}$ , what is the percentage of uncertainty in its momentum?  
[Ans: 6.2%]
51. The time period of a radar vibration is  $0.25\text{ }\mu\text{s}$ . What is the uncertainty in the energy of the photon?  
[Ans:  $2.6 \times 10^{-9}\text{ eV}$ ]
52. A measurement establishes the position of a proton with an accuracy of  $\pm 10^{-11}\text{ m}$ . Find the uncertainty in the proton's position one second later. Assume proton's speed  $v \ll c$  (Mass of proton =  $1.67 \times 10^{-27}\text{ kg}$ )  
[Ans: 3.15 km]

# **CHAPTER - 2**

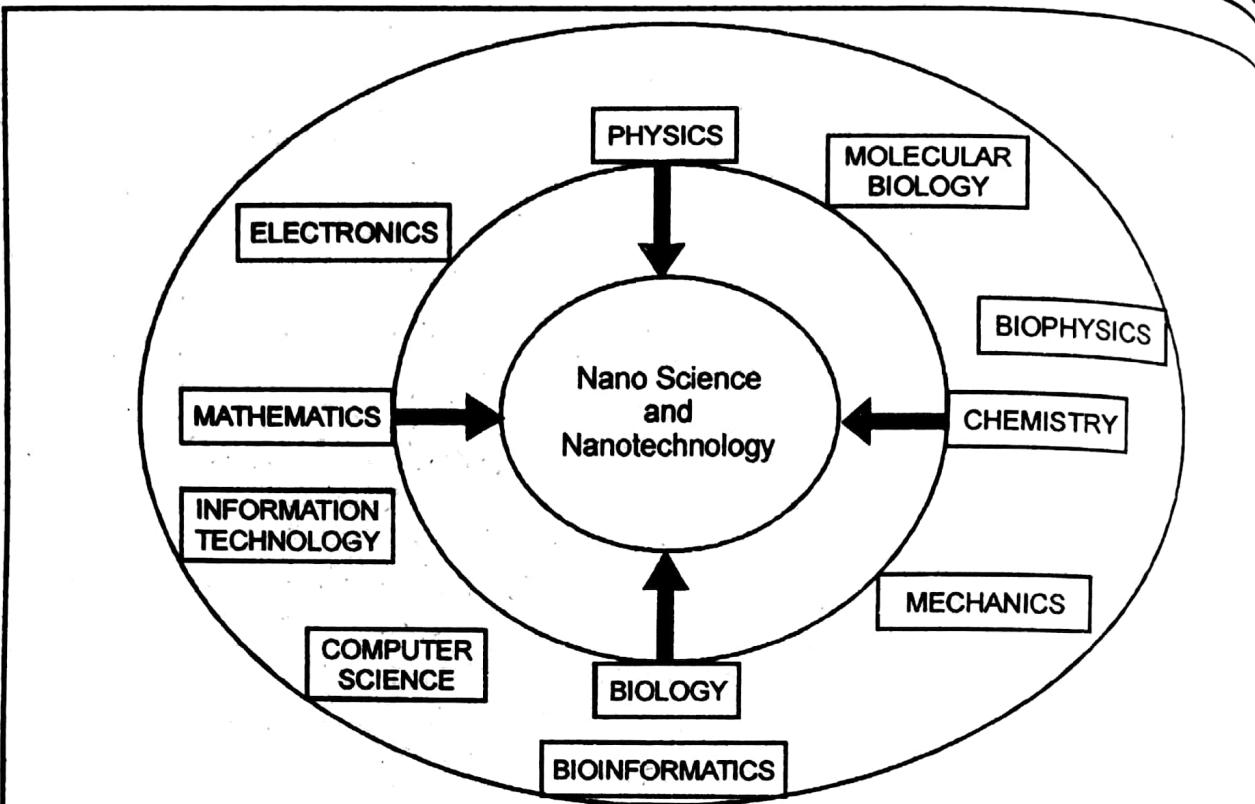
## **NANO PHYSICS**

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### **2.1 Introduction**

Manufactured materials are made from atoms. The properties of those materials depend on how these atoms are arranged. If one rearranges the atoms in coal, he gets diamond and if one arranges the atoms in sand he gets computer chips. Theoretical analysis makes it clear that one can do a lot more. One of the basic principles about matter is positional control. The control may be either macroscopic or microscopic level but the results could be much lighter, stronger and more precise. A new technology creates a new science called *nanotechnology* which is about rearranging atoms whichever way we want. Nanotechnology is an umbrella term that covers many areas of research dealing with objects that are measured in nanometers. It is a sub classification of technology in physics, chemistry, biology and engineering fields [Figure 2.1]. *Nanoscience* is the study of the fundamental principles of atoms, molecules and structures with at least one dimension roughly between 1 and 100 nanometers. The fundamental goal of nanoscience or nanotechnology is to understand the evolution with decreasing crystal size of molecular properties from bulk properties.

Nanoscience and nanotechnologies incorporate exciting areas of research and development at the interface between physics, chemistry, biology, computer science, bioinformatics, molecular and Information Technology. Figure 2.1 depicts the convergence of nanotechnology from all existing various boundary areas. In this, convergence refers to the multiple ways in which nanotechnologies will combine in the future with other developments in new technology (reflecting its genuinely interdisciplinary nature). Convergence probably presents some of the biggest uncertainties, with respect to what is genuinely plausible and when new technologies might actually come into use.



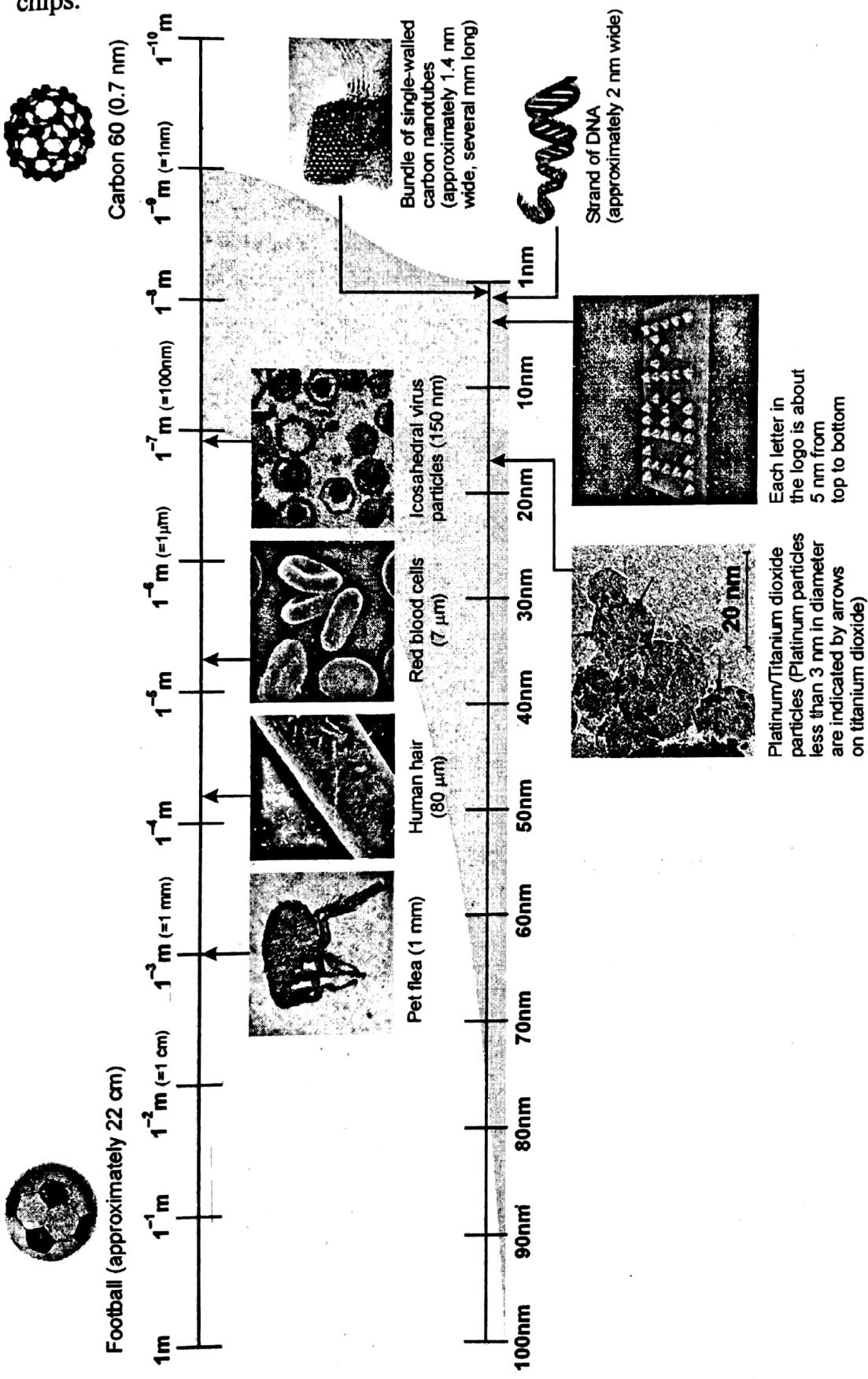
**Figure 2.1:** Convergence towards nanoscience

Nanoparticles have existed for billions of years: Geckos hang upside down on the ceiling by nanotech effect; on each tow they have millions of tiny hairs, and each hair has a minute force which holds on to the ceiling, Terpene: hydrocarbons in the essential oils and resins of trees (turpentine) and volcanic ash contains nanoparticles. A human hair is approximately 80,000nm wide, and a red blood cell approximately 7000nm wide.

Figure 2.2 shows the nanometre in context. Atoms are below a nanometre in size, whereas many molecules, including some proteins, range from a nanometre upwards. The conceptual underpinnings of nanotechnologies were first laid out in 1959 by the physicist *Richard Feynman*, in his lecture ‘There’s plenty of room at the bottom’. Feynman explored the possibility of manipulating material at the scale of individual atoms and molecules, imagining the whole of the Encyclopaedia Britannica written on the head of a pin and foreseeing the increasing ability to examine and control matter at the nanoscale.

The term ‘nanotechnology’ was not used until 1974, when *Norio Taniguchi*, a researcher at the University of Tokyo, Japan used it to refer to the ability to engineer materials precisely at the nanometre level. The primary driving force for miniaturization at that time came from the electronics industry, which aimed to develop tools to create smaller (and therefore faster and more complex) electronic devices on silicon

chips.



**Figure 5.2:** Length scale showing the nanometre in context. The length scale at the top ranges from  $1\text{ m}$  to  $10^{-10}\text{ m}$ , and illustrates the size of a football compared to a carbon 60 ( $\text{C}_{60}$ ) molecule, also known as a buckyball. For comparison the world is approximately one hundred million times larger than a football, which is in turn one hundred million times larger than a buckyball. The section from  $10^{-7}\text{ m}$  (100nm) to  $10^{-9}\text{ m}$  (1nm) is expanded below. The lengthscale of interest for nanoscience and nanotechnologies is from 100nm down to the atomic scale - approximately 0.2nm.

## 2.2 Nanoscale

**Nanotechnology** is a field of applied science that throws light on the design, synthesis, characterization and the application of materials and devices on the nanoscale. A **nanoscale** is an object having nanometer size (1-100 nm range).

A nanometer (nm) is one-billionth ( $10^{-9}$ ) of a meter or 10 Å, so particles having a radius of about 1000 Å can be considered and is derived from the Greek word for dwarf, 'nano'.

A nanometer is about the width of six bonded carbon atoms and our fingernails grow approximately one nm per second. Nanoparticles are generally considered to be a number of atoms or molecules bonded together with a radius of 100nm. Nanomaterials, are grown either by way of a *top-down* approach (a bulk material is reduced in size to nanoscale pattern) or a *bottom-up* approach (grown atom by atom).

Table 2.1 gives a variation between molecules, nanoparticles and bulk materials. A cluster of 1 nm radius has approximately 25 atoms, but most of the atoms are on the surface of the cluster.

**Table 2.1:** Distinction between molecules, nanoparticles and bulk

No. of atoms	Radius (nm)	Type
1 10	1	Molecules
$10^2$ $10^6$	10	Nanoparticles
$10^7$ onwards	< 100	Bulk

**Nanoscience** is the study of phenomena and manipulation of materials at atomic, molecular and macromolecular scales, where properties differ significantly from those at a larger scale.

**Nanotechnologies** are the design, characterisation, production and application of structures, devices and systems by controlling shape and size at nanometer scale.

### 2.3 Nanophase materials

In a nanostructure, electrons are confined in the nanoscale dimension(s), but are free to move in other dimension(s). Nanophase materials or simply *nano materials* are the nanostructured materials having a characteristic length less than 100 nm. These are recently developed new materials having so many new properties. These have a three dimensional structure with a domain size smaller than 100 nm. These are characterized by a large number of grain boundary interfaces in which local atomic arrangements are different from those of the crystal lattice. A cluster of nanoparticles contains less than  $10^4$  molecules or atoms corresponding to a diameter of only a few nanometers.

Thus one can conclude that particle's size in nano materials is about 1 nm. With these small sized particles one can get the different properties (electronic, optical, electrical, magnetic, chemical and mechanical) which are different from the bulk materials. Having a size between the molecular and bulk solid state structures, the nano particles have hybrid properties. They have nonlinear optical and magnetic properties.

The change in geometry continue to small sizes containing only a few atoms, at which point all traces of the bulk arrangement will be lost. It is clear that at these sizes the clusters are no longer fragments of the bulk structure. Therefore the metal aggregates cannot be simply treated as minute elements of the block of a metal, as implied by the term divided metals. Because the conduction band present in the bulk metal will be absent and instead there would be discrete states at the band edge. Electrons will therefore undergo quantum confinement in very small metal particles, showing quantum dots, further a high proportion of the atoms in small metal nanoparticles will be present at the surface. There is remarkable increase in the surface to bulk ration, with decrease in size. For instance, nanoparticles of 10 nm diameter would have approximately 10% of surface atoms, while in 1nm size-particle there will be 100% surface atoms.

Starting from the bulk, the first effect of reducing particle size is to create more surface sites. This changes the surface pressure and results in a change in the inter-particle spacing. The change in the inter-particle spacing and the larger surface to volume ratio in particles has concomitant effect on the material properties. Variations in surface free energy change the chemical potential. This affects the thermodynamical properties of which the simplest example is melting point of Au particle as a function of size decreases. The variation in inter-particle spacing and geometry also result in a variation in electronic properties with size. For small particles, the electronic states

are not continuous but discrete, due to confinement of the electron wave function. As the size decreases, the electronic bands become narrower and the delocalized electronic states are transformed to more localized molecular bonds. The electronic property accessible for experimental measurements is the ionization potential. The ionization potentials at smaller sizes are higher than the bulk work function and showed considerable fluctuations as a function of size. Photo electronic spectroscopy is used to probe the changes in electronic energy levels with size for free cluster. A negative charge created on the clusters by exposing them to low energy electronic energy levels with size for free cluster. The anionic clusters are then crossed with a fixed frequency laser beam and the resulting photo detached electrons are analysed. Therefore, the spectra are finger prints of the ground and excited states of neutral cluster. They provide information on vertical and adiabatic electron affinities and the low lying electronic states.

Further, the large surface to volume ratio and the variations in geometry and electronic structures have a strong effect on catalytic properties. The reactivity of small clusters found to vary by order of magnitude, when the cluster size is changed by only few atoms. Another classic example is that storage ability of hydrogen is metals. Most metals do not absorb hydrogen, and even among those that do, hydrogen is absorbed dissociative on surfaces with hydrogen to metal ratio of one. This limit can be significantly enhanced in small sizes. It is found that small positively charged clusters of Pd, Pt and Ni generated in molecular beams containing two to sixty atoms, can absorb up to eight hydrogen atoms per metal atom. The number of absorbed hydrogen atoms decreases with increasing cluster size and approaches one for clusters containing around 60 atoms. This shows that small particles may be useful for hydrogen storage devices. The fundamental challenges in nanostructure materials are

- Ability to control the scale (size) of the system
- Ability to obtain the required composition; not only the average composition but also the details such as defects, concentration gradients etc.
- Understand the influence of the size of building blocks in nano structured materials as well as the influence of microstructure on the physical, chemical and mechanical properties of the material.

A wide range of synthetic procedures are used in preparation of nanomaterials such as

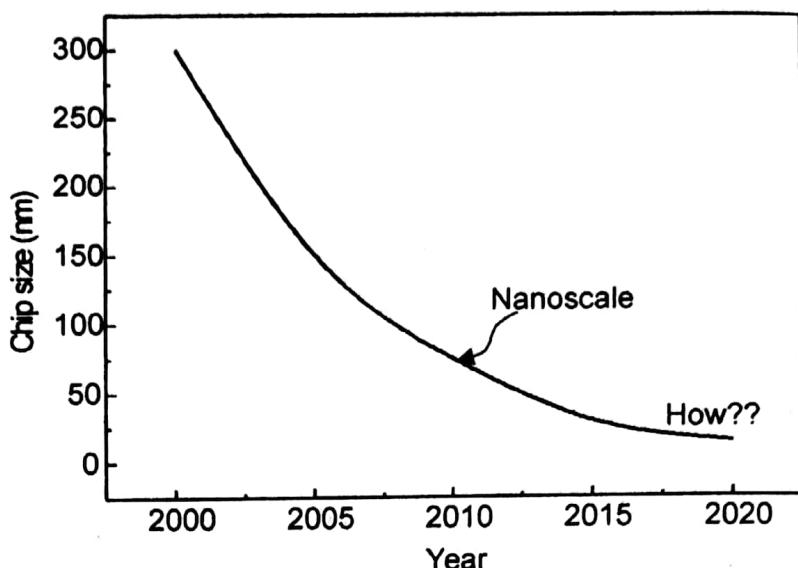
- Gas phase evaporation method
- Reduction of metal ions
- Decomposition of organometallic compounds and cluster complexes
- Metal atom vapour synthesis
- Metal vapour deposition into polymer
- Encapsulation in inert gas matrices
- Sol-Gel processing: single-phase materials, multiphase materials, hybrids

**Scanning Tunneling Microscope (STM)** is a technology evolved to allow the manipulation of individual atoms and molecules. What makes nanostructures unique and interesting? For bulk materials (e.g., a Cu wire, a cup of water), their intrinsic physical properties, such as density, conductivity and chemical reactivity, are independent of their sizes. For example, if a one-meter Cu wire is cut into a few pieces; those intrinsic properties of the shorter wires remain the same as in the original wire. If the dividing process is repeated again and again, this invariance cannot be kept indefinitely. Certainly, we know that the properties are changed greatly when the wire is divided into individual Cu atoms (even more at the level of electrons, protons and neutrons). Significant property changes often start when we get down to the nanoscales.

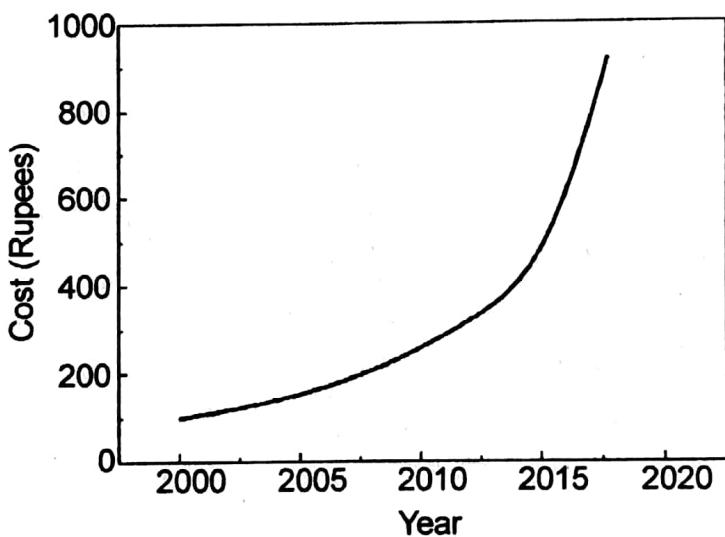
## 2.4 Moore's law

*Gordon Moore*, one of the founders of the Intel Corporation, came up with two empirical laws to explain the amazing development in integrated circuit electronics. Moore's first law (called Moore's law) says that the amount of space required to install a transistor on a chip shrinks by roughly half every 18 months. This means that the space that could hold one transistor 15 years ago can hold approximately 1000 transistors today. Figure 2.3 shows Moore's law in a graphical way.

Moore's second law predicts that the cost of building a chip manufacturing plant doubles with every other chip generation or roughly every 36 months. Microprocessors and memory circuits follow Moore's law.



(a) Moore's first law



(b) Moore's second law

**Figure 2.3**

## 2.5 Semiconducting nanoparticles

Because of their role in quantum dots, nanoparticles made of the elements, which are normal constituents of semiconductors, have been the subject of much study, with particular emphasis on their electronic properties. The title of this section, "semiconducting nanoparticles", is somewhat misleading. Nanoparticles made of cadmium, germanium, or silicon are not themselves semiconductors. A nanoparticle of Si<sub>n</sub> can be made by laser evaporation of a Si substrate in the region of a helium gas pulse. The beam of neutral clusters is photolyzed by a UV laser producing ionized clusters whose mass to charge ratio is then measured in a mass spectrometer. The

most striking property of nanoparticles made of semiconducting elements is the pronounced changes in their optical properties compared to those of the bulk material. There is a significant shift in the optical absorption spectra toward the blue (shorter wavelength) as the particle size is reduced.

In a bulk semiconductor a bound electron-hole pair, called an *exciton*, can be produced by a photon having an energy greater than that of the band gap of the material. The band gap is the energy separation between the top filled energy level of the valence band and the nearest unfilled level in the conduction band above it. The photon excites an electron from the filled band to the unfilled band above. The result is a hole in the otherwise filled valence band, which corresponds to an electron with an effective positive charge. Because of the Coulomb attraction between the positive hole and the negative electron, a bound pair, called an *exciton*, is formed that can move through the lattice. The separation between the hole and the electron is many lattice parameters. The existence of the exciton has a strong influence on the electronic properties of the semiconductor and its optical absorption. The exciton can be modeled as a hydrogen-like atom and has energy levels with relative spacings analogous to the energy levels of the hydrogen atom but with lower actual energies. Light-induced transitions between these hydrogen-like energy levels produce a series of optical absorptions that can be labeled by the principal quantum numbers of the hydrogen energy levels. We are particularly interested in what happens when the size of the nanoparticle becomes smaller than or comparable to the radius of the orbit of the electron-hole pair. There are two situations, called the *weak-confinement* and the *strong-confinement regimes*. In the weak regime the particle radius is larger than the radius of the electron-hole pair, but the range of motion of the exciton is limited, which causes a blue shift of the absorption spectrum. When the radius of the particle is smaller than the orbital radius of the electron-hole pair, the motion of the electron and the hole become independent, and the exciton does not exist. The hole and the electron have their own set of energy levels.

## 2.6 Nanocomposites

Nanocomposites are special type of materials originating from suitable combination of two or more such nanoparticles by some appropriate techniques, resulting in materials having unique physical properties. Novel properties of Nanocomposites can be derived from the successful combination of the features of parent constituents into a single material. These properties obtained by nanomaterials are consequences of their finite smaller size, dimensionality, organization and interaction with each other as well as

the surrounding environment. These properties include enhanced quantum electronic and optical nonlinearities as a consequence of the spatial, quantum and dielectric confinement of electrons, holes and excitons with respect to the nanoscale of the materials. Nanosize metal particles pose a number of unsolved problems and require new approaches for their synthesis, stabilization and characterization.

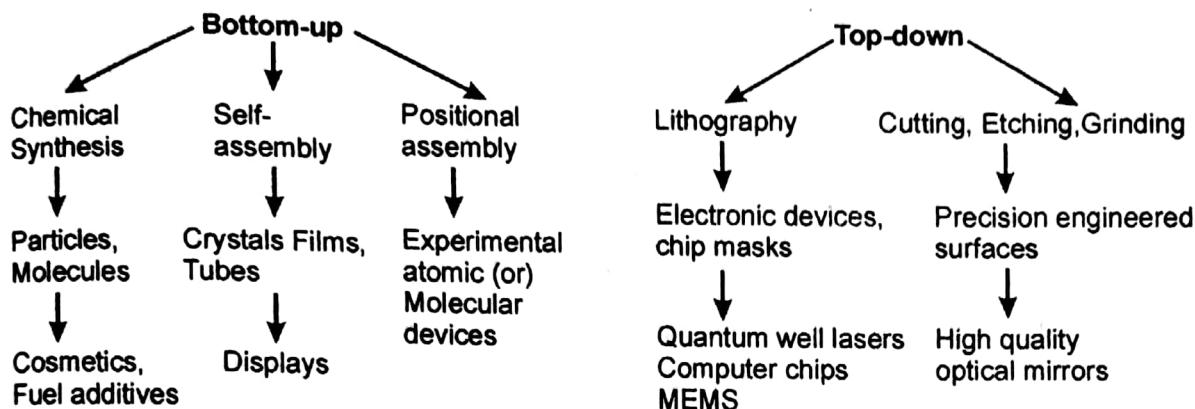
Different approaches that are commonly used to attain nanocomposites are:

- Polymeric nanocomposites
- Organic –inorganic systems
  - One dimensional inorganic structures
  - 3D inorganic structures
  - Layered structures
- Inorganic – inorganic systems
  - Nanocomposites in stable colloidal form
  - Nanocomposites with improved physical properties

### 2.6.1 Synthesis of Nanoparticles

If you take a sheet of paper and tear it in half and tear it pieces in half and so on, you will soon find that it becomes increasingly difficult to produce smaller pieces. Similarly, it is quite difficult if not impossible to produce nanomaterials by breaking down larger particles. How then can we go about producing nano sized particles? Just as a wall be built up by piling bricks one over the other, particles may be created by assembling together their building blocks which may be atoms or molecules. This can be done much more easily than trying to break down a large particle into a nano particle. Nano particles are synthesized by bringing together atoms or molecules which are initially apart to form a larger cluster of atoms. This is similar to how crystals grow in a solution. However, things are not that simple and these particles quickly grow to be too large. It is thus important to arrest or limit the growth of these particles when they have grown to the desired size. There are a wide variety of techniques that are capable of creating nanostructures with various degrees of quality, speed and cost. These manufacturing approaches fall under two categories: '*bottom-up*', and '*top-down*'. In recent years the limits of each approach, in terms of feature size and quality that can be achieved, have started to converge. A diagram illustrating some of the

types of materials and products that these two approaches are used for is shown below in figure 2.4.



**Figure 2.4:** The use of bottom-up and top-down techniques in manufacturing

## 2.7 Properties of nanomaterials

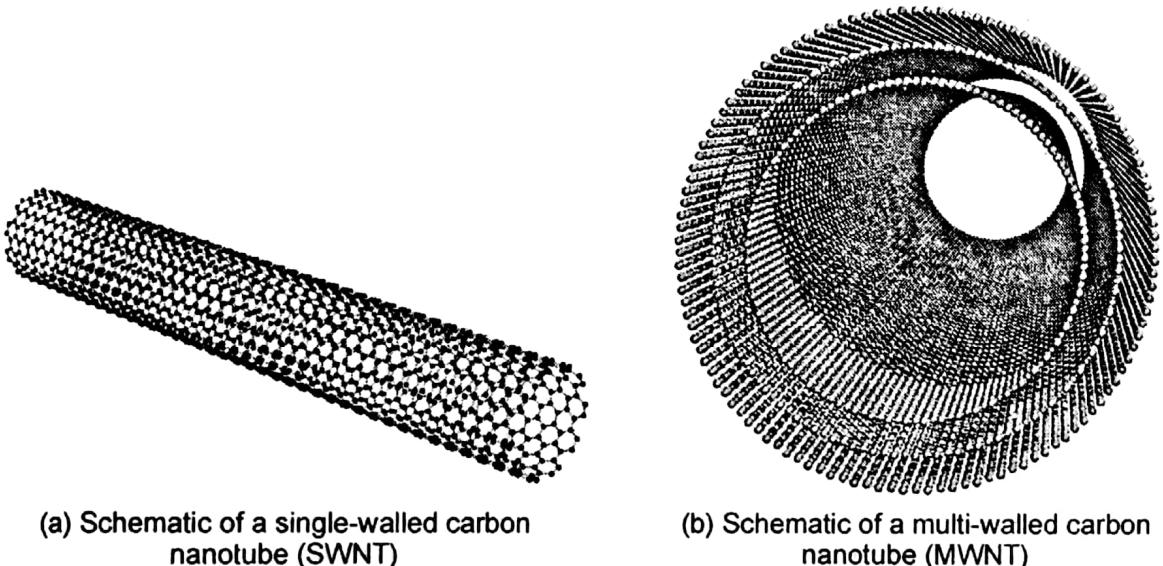
Two principal factors cause the properties of nanomaterials to differ significantly from other materials: increased relative surface area, and quantum effects. These factors can change or enhance properties such as reactivity, strength and electrical characteristics. As a particle decreases in size, a greater proportion of atoms are found at the surface compared to those inside. For example, a particle of size 30 nm has 5% of its atoms on its surface, at 10 nm 20% of its atoms, and at 3 nm 50% of its atoms. Thus nanoparticles have a much greater surface area per unit mass compared with larger particles. As growth and catalytic chemical reactions occur at surfaces, this means that a given mass of material in nanoparticulate form will be much more reactive than the same mass of material made up of larger particles.

What makes nanoparticles very interesting and endows them with their unique properties is that their size is smaller than critical lengths that characterize many physical phenomena. Generally, the physical properties of nanomaterials can be characterized by some critical length, a thermal diffusion length or a scattering length, for example the electrical conductivity of a conductor is strongly determined by the distance that electron travels between collisions with the vibrating atoms or the impurities of the solid. This distance is called mean free path or the scattering length. If the sizes of particles are less than this characteristic length, it is possible that new physics may occur. Perhaps a working definition of a nanoparticle is an aggregate of atoms between one and 100 nm viewed as a sub division of a bulk material, and of dimension less than the characteristic length of some phenomena. The interesting properties of nanoparticles are mainly due to their reduction in size.

- i) As the size is reduced from the bulk the electronic bands in metals become narrower and the value of ionization potential is raised.
- ii) When we have metals in nanophase, the metals absorb hydrogen. This property can be used for hydrogen storage in metals.
- iii) The thermodynamic properties are drastically different for nanoparticles. For example in the case of gold in nanophase condition, its melting point decreases from 1200 K to 800 K when the particle size decreases from 300 Å to 20 Å.
- iv) The properties of nano particles of magnetic materials are changed so that they are behaving as other kind of magnetic materials. For example Fe, Co and Ni are ferromagnetic in the bulk state. But they are changed into superparamagnetic to the nanophase. Similarly Na, K and Rb are paramagnetic in the bulk state. But they are changed into ferromagnetic in the nanophase. Further the magnetic nanomaterials are used to nonvolatile magnetic storages and permanent magnets.
- v) Regarding mechanical properties, they are inferior with respect to elastic strength. But the plastic behaviour is increased. In some nanophase materials, it is found that there is decrease in hardness with decreasing grain size when the grain size is less than 10 nm. But for many nanocrystalline pure metals (10 nm) the hardness is about 2 – 7 times greater than that of large grained ( $> 1 \mu\text{m}$ ) metal.
- vi) These have high solid state phase transition pressure, lower Debye temperature and high self diffusion coefficient, high catalytic activity and lower ferroelectric phase transition temperature.

**Nanotube** is a one dimensional fullerene with a cylindrical tube. **Carbon nanotubes** (CNTs) are molecular scale tubes of graphite carbon or allotropes of carbon with a nanostructure that can have a length-to-diameter ratio greater than  $10^6$  and were discovered in 1991 by Sumio Iijima. These cylindrical carbon molecules have novel properties that make them potentially useful in many applications in nanotechnology, electronics, optics and other fields of materials science, as well as extensive use in architectural fields. They exhibit extraordinary strength and unique electrical properties, and are efficient conductors of heat. Inorganic nanotubes have also been synthesized.

Nanotubes are members of the *fullerene* structural family, which also includes the spherical buckyballs. The cylindrical nanotube usually has at least one end capped with a hemisphere of the buckyball structure. Their name is derived from their size, since the diameter of a nanotube is in the order of a few nanometers (approximately 1/50,000th of the width of a human hair), while they can be up to several millimeters in length (as of 2008). Nanotubes are categorized as *single-walled nanotubes* (one tube) and *multi-walled nanotubes* (several concentric tubes). Both of these are typically a few nanometres in diameter and several micrometres ( $10^{-6}\text{ m}$ ) to centimetres long.



**Figure 2.5**

Carbon nanotubes are the strongest and stiffest materials on earth, in terms of tensile strength and elastic modulus respectively. Bulk carbon nanotubes have already been used as composite fibers in polymers to improve the mechanical, thermal and electrical properties of the bulk product. Carbon nanotubes have also been successfully used in the construction of handlebars for mountain bikes. The handlebar, created by Easton, which has won awards for its incredible strength and light weight.

Carbon nanotubes have also been proposed as a possible gene delivery vehicle and for use in combination with radiofrequency fields to destroy cancer cells. Carbon nanotubes are said to have the strength of diamond, and research is being made into weaving them into clothes to create stab-proof and bulletproof clothing. The nanotubes would effectively stop the bullet from penetrating the body but the force and velocity of the bullet would be likely to cause broken bones and internal bleeding.

## 2.8 Quantum well

When layered in particular ways, doped silica can trap conduction electrons in a membrane so thin that, from one face to the other, their behavior as tiny quantum wave packets takes precedence over their behavior as particles. This structure is called a quantum well. From there, confining the electrons along a second dimension produces a quantum wire, and finally, with three dimensions, a quantum dot."

Here electrons are confined in one dimension (1D), free in other 2D. It can be realized by sandwiching a narrow-bandgap semiconductor layer between the wide-gap ones. A quantum well is often called a 3D electronic system.

## 2.9 Quantum wires

A strip of conducting material about 10 nanometers or less in width and thickness that displays quantum-mechanical effects like universal conductance fluctuations called quantum wires. It is another form of quantum dot, but unlike the three-dimension "dot," a quantum wire is confined only in two dimensions - that is it has "length," and allows the electrons to propagate in a "particle-like" fashion. Constructed typically on a semiconductor base, and (among other things) used to produce very intense laser beams, switchable up to multi-gigahertz per second.

Real quantum wires include polymer chains, nanowires and nanotubes. A single walled nanotube can have a diameter of 2 mm and a length of 100  $\mu\text{m}$ , form a narrow dimensional structure called a *nanowire*.

## 2.10 Quantum dots

Quantum dots are semi-conducting crystals of nanometre dimensions. In other words **nanocrystals** (NCs), nanometer-size crystals of cadmium-based semiconductors are also referred to as quantum dots (QDs). Due to their incredibly small size (ranging from  $<1.5$  to  $>8$  nanometers), these usually non-fluorescing compounds develop intense, long-lasting colors excitable by UV and visible light LEDs, lasers, etc. The colors produced are a function of the particle size (blue to red and infrared, depending on the diameter of the nanocrystals) with the smallest NCs fluorescing in the blue and green. The fluorescence is due to a phenomenon called *quantum confinement*. Quantum confinement is the confinement of electrons in the nanoscale dimensions result in quantization of energy and momentum, and reduced dimensionality of electronic states.

When electrons and holes (exciton pairs) generated by photons are confined within a space, or quantum box, smaller than the Bohr exciton radius (i.e., the normal, off-the-shelf, preferred, bulk material electron-to-hole distance of an exciton pair), the semiconductor's band gap widens and its fluorescence shifts towards the blue. The smaller that box, the bluer the fluorescence. Conversely, the looser the confinement, the bigger the quantum box, the redder the fluorescence.

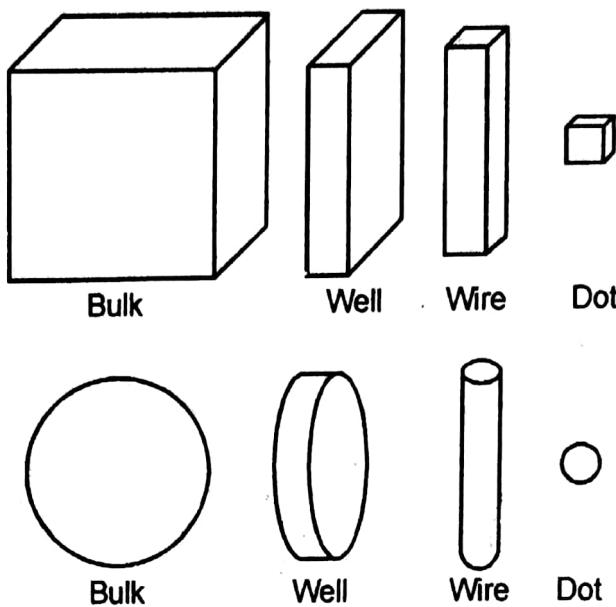
Eventually, one enters the infrared and the invisible. The fluorescence is generated when electron and hole recombine. The restricted confinement requires additional energy and this causes the wavelength shift.

Quantum dots are in fact very small physical cavities in which a finite number of particles, for example electrons, are trapped. These dots are commonly engineered by the current silicon technology. The physics behind quantum dot is very simple — quantum mechanics QM which is the most fundamental (and well accepted) theory currently known to mankind. QM governs the dynamics of everything in the microscopic world. Quantum dot is a physical (approximately) realization of the textbook example of the so-called particle-in-a-box which illustrates the quantization of energy when a particle is placed inside a (sufficiently small) box.

It has become recognized in recent years that the electronic and optical properties of metals and semiconductors change dramatically as the particulates of the materials are reduced to approach the nanometer size range of dimensions. At such size levels, the physical dimensions of the material may have a critical effect on the electronic or optical behaviour of the material. The restriction of the electronic wave function to smaller and smaller regions of space within a particle of material (the resonance cavity) is referred to as "quantum confinement".

In short, semiconductor structures in the nanometer size range, exhibiting the characteristics of quantum confinement, are commonly referred to as "quantum dots" when the confinement is in three dimensions, "quantum wires" when the confinement is in two dimensions, and "quantum wells" when the confinement is in one dimension.

Films of NCs in polymers or coatings on indium tin oxide (ITO) glass can produce electroluminescent devices. NCs are being used in solar cell research. Some manufacturers are talking about adding these crystals to fabrics for brighteners and intense colorants.



**Figure 2.6:** Generation of rectangular and curvilinear nanostructures

Despite the existence of ongoing work in the area of quantum dots for over a decade, the field remains one of the most interesting and cutting-edge areas of science.

## 2.11 Nanolithography

The word “lithography” originally referred to making objects from stones. A *lithograph* is an image (usually on paper) that is produced by carving a pattern on the stone, inking the stone, and then pushing the inked stone onto the paper.

Many types of small-scale lithography operate in very much this way. Indeed, the common methods used to make current computer chips normally use optical (or x-ray) lithography, in which a master mask is made using chemical methods and light passes through that mask to produce the actual chip structures. It works just like a silk screen for a T-shirt.

Nanoscale lithography really can't use visible light because the wavelength of visible light is at least 400 nanometers, so structures smaller than that are difficult to make directly using it. This is one of the reasons that continuing Moore's law into the nanoscale will require entirely new preparation methods.

Despite this, there are several techniques for doing small-scale lithography. One of the most straight forward and elegant is *micro-imprint lithography*, largely developed by George Whitesides and his research group at Harvard. This method works in the same way as the rubber stamps that are still found in post offices. A pattern is inscribed onto a rubber surface (in this case actually a rubber-like silicon/oxygen polymer), and that rubber surface is then coated with molecular ink. The ink

can then be stamped out onto a surface: this is paper in the post office, but it could be a metal, polymer, oxide, or any other surface in small-scale stamps. Small-scale stamping is more complex, but it is very inexpensive and can be used to make numerous copies. Originally, the stamps worked at the larger micron (1000-nanometer) scale, but recent improvements are bringing it to the nanoscale.

The technique of nanolithography is the patterning a thin film where the line resolution is below 100 nm. There are several primary techniques that are used for patterning in the nanoscale regime.

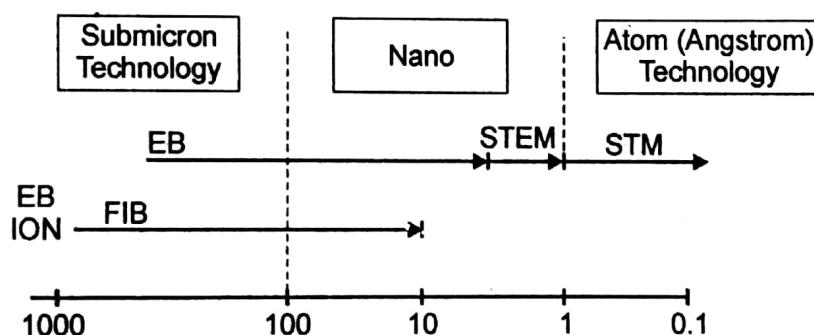
Electron beam lithography

X-ray lithography

Scanning probe microscope lithography

Nano imprint lithography

Dip pen nanolithography



**Figure 2.7: Focused Ion Beam (FIB)**

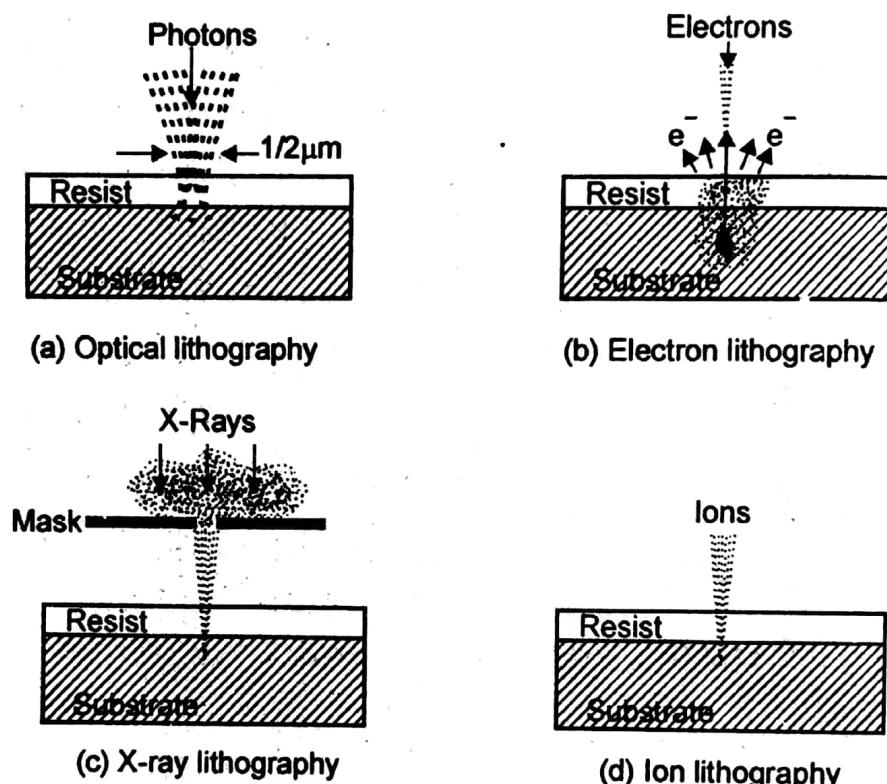
Lithography involves the transfer of a pattern to a photosensitive material by selective exposure to a radiation source such as light. A photo sensitive material is the one that experiences a change in its physical properties when exposed to a radiation source. If we selectively expose a photosensitive material to radiation by masking some of the radiation, the pattern of the radiation is transferred to the material exposed.

### 2.11.1 Electron beam Lithography

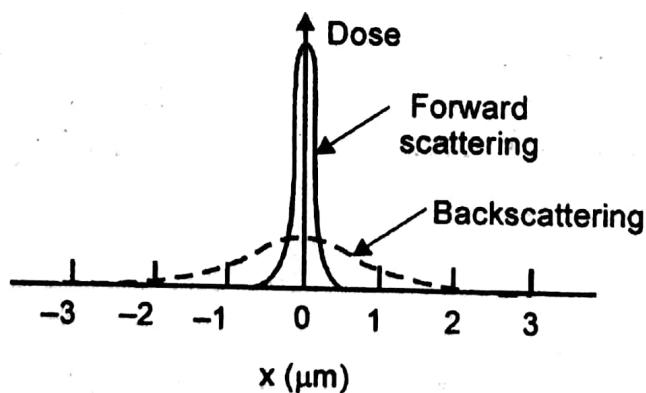
Although optical lithography is most common, there exist other lithographic techniques, as depicted in figure 2.8. Electron lithography offers high resolution because of the small wavelength of electrons ( $\leq 0.1$  nm for 10–50 keV electrons). The resolution of an electron lithographic system is not limited by diffraction, but rather by electron scattering in the resist (Figure 2.8 & Figure 2.9) and by the various aberrations of the electron optics.

The advantages of electron lithography are:

1. Generation of micron and submicron resist geometries
2. Highly automated and precisely controlled operation
3. Greater depth of focus
4. Direct patterning without a mask



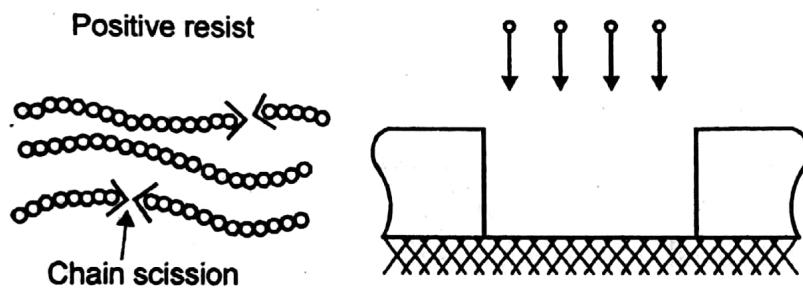
**Figure 2.8:** Types of lithographic methods



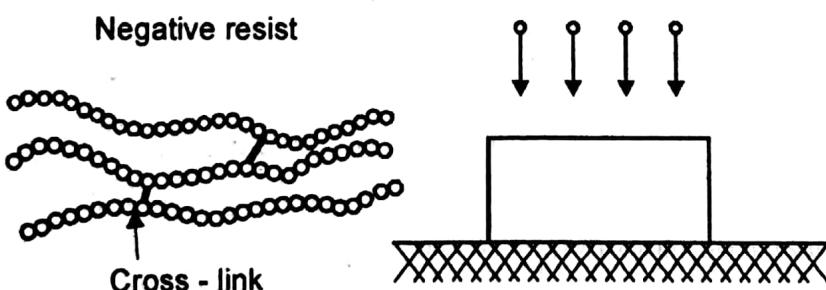
**Figure 2.9:** Dose distribution for forward scattering and backscattering at the resist - substrate interface

The biggest disadvantage of electron lithography is its low throughput (approximately 5 wafers / hour at less than  $0.1\text{ }\mu\text{m}$  resolution). Therefore, electron lithography is primarily used in the production of photomasks and in situations that require small number of custom circuits.

There are two basic ways to scan an electron beam. In raster scanning, the patterns are written by an electron beam that moves through a regular pattern. The beam scans sequentially over the entire area and is blanked off where no exposure is required. On the contrary, in vector scanning, the electron beam is directed only to the requested pattern features and hops from features to features. Time is therefore saved in a vector scan system.



(a) Schematic of the chemical reaction of a positive resist



(b) Schematic of the chemical reaction of a negative resist used in electron beam lithography

**Figure 2.10**

Electron resists are polymers. For a positive electron resist, the polymer-electron interaction causes chain scission, that is, broken chemical bonds (Figure 2.10(a)). The irradiated areas can be dissolved in a developer solution that attacks low-molecular-weight material. Common positive electron resists are poly (methyl methacrylate), abbreviated PMMA, and poly (butene-1 sulfone), abbreviated PBS. Positive electron resists typically have resolution of  $0.1\text{ }\mu\text{m}$  or better. When electrons

impact a negative electron resist, polymer linking is induced (Figure 2.10 (b)). Poly(glycidyl methacrylate-co-ethyl-acrylate), abbreviated COP, is a common negative electron resist. Like a negative photoresist, COP swells during developing, and resolution is limited to about 1  $\mu\text{m}$ .

Electron beam lithography is usually done with SEM and scanned at a controlled rate over the surface of some type of resist material. The scan rate is adjusted to deliver dose to a selected area of the resist. The  $e^-$  beam interacts with the material sufficiently to remove the resist material.

### 2.11.2 X-ray Lithography

X-ray lithography employs a shadow printing method similar to optical proximity printing. The x-ray wavelength (0.4 to 5 nm) is much shorter than that of UV light (200 to 400 nm). Hence, diffraction effects are reduced and higher resolution can be attained. For instance, for an x-ray wavelength of 0.5 nm and a gap of 40  $\mu\text{m}$ ,  $l_m$  is equal to 0.2  $\mu\text{m}$ . X-ray lithography has a higher throughput when compared to e-beam lithography because parallel exposure can be adopted. However, on account of the finite size of the x-ray source and the finite mask-to-wafer gap, a penumbral effect results which degrades the resolution at the edge of a feature. As shown in Figure 2.11, the penumbral blur, on the edge of the resist image is given by:

$$\delta = ag / L \quad \dots(2.1)$$

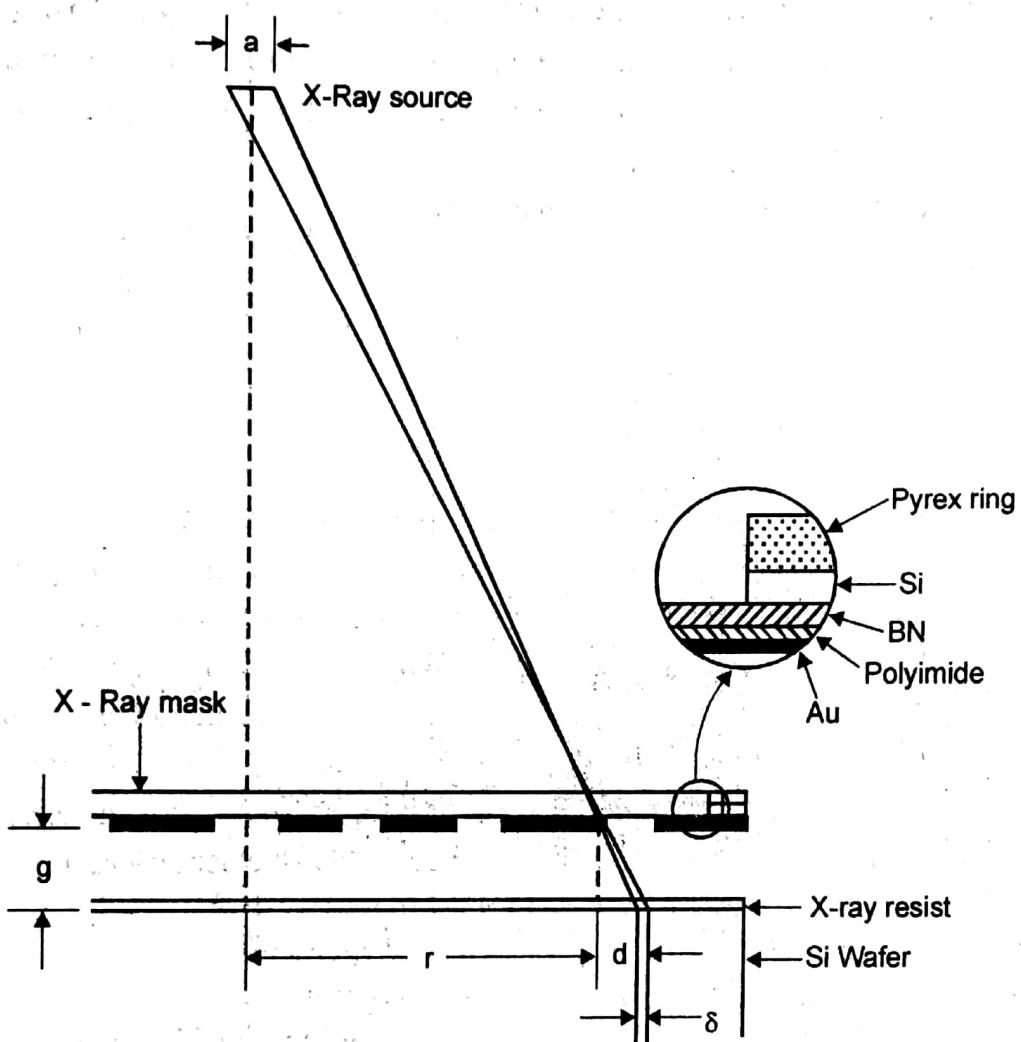
where  $a$  is the diameter of the x-ray source,  $g$  is the gap spacing, and  $L$  is the distance from the source to the x-ray mask. If  $a = 3 \text{ mm}$ ,  $g = 40 \mu\text{m}$ , and  $L = 50 \text{ cm}$ ,  $3$  is on the order of 0.2  $\mu\text{m}$ .

An additional geometric effect is the lateral magnification error due to the finite mask-to-wafer gap and the non-vertical incidence of the x-ray beam. The projected images of the mask are shifted laterally by an amount  $d$ , called runout:

$$\delta = rg / L \quad \dots(2.2)$$

where  $r$  denotes the radial distance from the center of the wafer. For a 125-mm wafer, the runout error can be as large as 5  $\mu\text{m}$  for  $g = 40 \mu\text{m}$  and  $L = 50 \text{ cm}$ . This runout error must be compensated for during the mask making process.

Electron beam resists can be used in x-ray lithography because when an x-ray photon impinges on the specimen, electron emission results. One of the most attractive x-ray resist is DCOPA (dichloropropyl acrylate and glycidyl methacrylate-co-ethyl acrylate), as it has a relatively low threshold ( $\sim 10 \text{ mJ/cm}^2$ ).



**Figure 2.11:** Geometric effects in X-ray lithography.

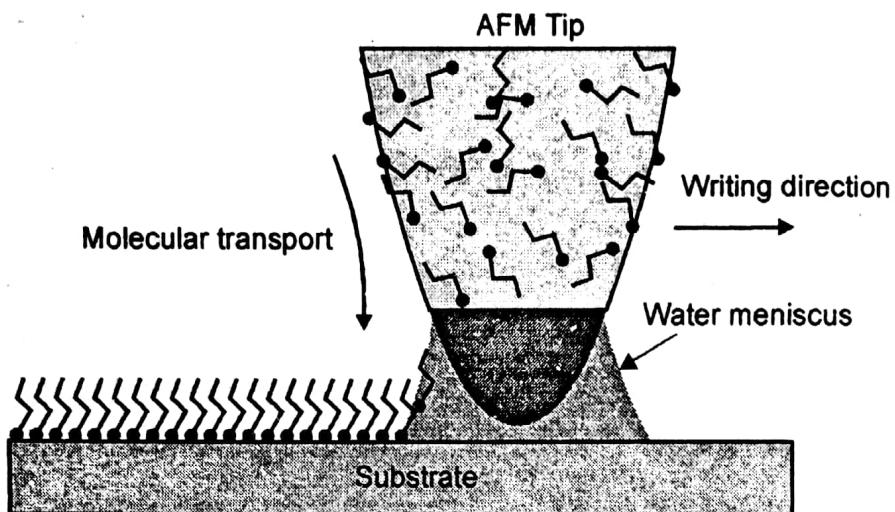
Insert shows the X-ray mask structure

Here the mask is made of an x-ray transparent material with a pattern of high Z material either etched or deposited on it. The mask is held in proximity to the wafer and exposed to x-rays. The mask is the limiting factor in x-ray lithography. The resolution of the pattern is depending on the variations in the mask. Another concern in the pattern accuracy is the accuracy of the pattern created on the mask.

### 2.11.3 Dip Pen Nanolithography

One way to construct arbitrary structures on surfaces is to write them in exactly the same way that we write ink lines using a fountain pen. To make such lines at the nanoscale, it is necessary to have a nano-pen. Fortunately, AFM tips are ideal nano-pens. *Dip pen nanolithography* (DPN) is named after the old-fashioned dip pen that was used in schoolrooms in the 19th century. The principle of DPN is shown in

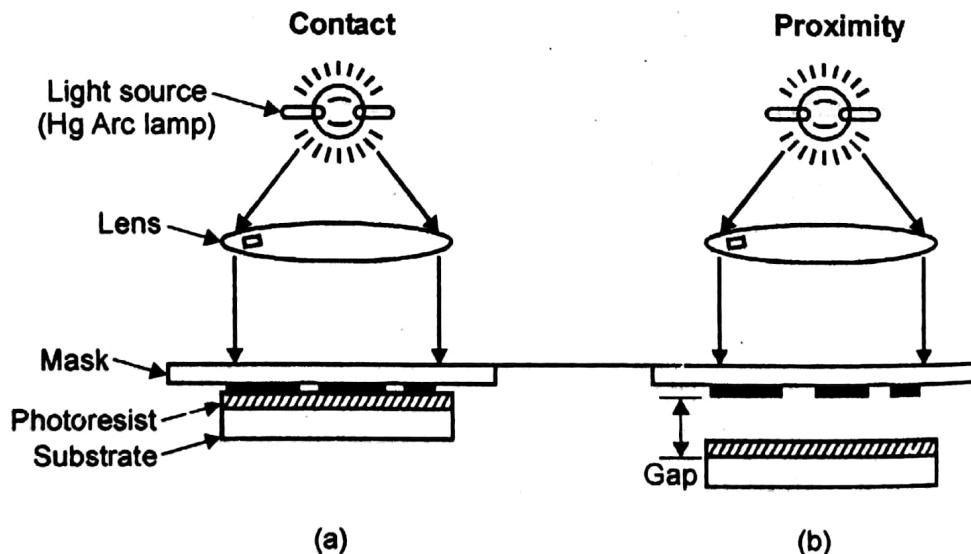
figure 2.12, and the excerpt from Feyman's speech in figure is one DPN-assembled structure. In DPN, a reservoir of "ink" (atoms or molecules) is stored on the top of the scanning probe tip, which is manipulated across the surface, leaving lines and patterns behind.



**Figure 2.12:** Dip pen lithography process - the wiggly lines are molecular "ink"

#### 2.11.4 Optical lithography

The vast majority of lithographic equipment for IC fabrication is optical equipment using ultraviolet light ( $\lambda \approx 0.2 \mu\text{m}$  to  $0.4 \mu\text{m}$ ) or deep ultraviolet light. There are basically two optical exposure methods: shadow printing and projection printing.



(a) Contact printing, (b) Proximity printing.

**Figure 2.13**

In shadow printing, the mask and wafer may be in direct contact, as in contact printing, or in close proximity, as in proximity printing (Figure 2.13). Contact printing

yields very high resolution ( $\sim 1 \mu\text{m}$ ), but suffers from major drawback caused by dust particles or silicon specks accidentally embedded into the mask, thereby causing permanent damage to the mask and defects in the wafers. Proximity printing is not as prone to particle damage. However, the small gap between the mask and wafer (typically  $10 \mu\text{m}$  to  $50 \mu\text{m}$ ) introduces optical diffraction at the feature edges on the photomasks and the resolution is typically degraded to the  $2$  to  $5 \mu\text{m}$  regime.

The minimum line-width that can be printed,  $l_m$ , in shadow printing is roughly given by

$$l_m = (\lambda g)^{1/2} \quad \dots(2.1)$$

where  $\lambda$  is the wavelength of the exposure radiation and  $g$  is the gap between the mask and the wafer and includes the thickness of the resist. For typical values of  $X$  ( $\sim 0.4 \mu\text{m}$ ) and  $g$  ( $\sim 50 \mu\text{m}$ ),  $l_m$  is on the order of  $4.5 \mu\text{m}$ . Equation (1) imparts that the minimum linewidth can be improved by reducing the wavelength  $\lambda$  (that is, going to deep UV spectral region) or the gap  $g$ .

In order to circumvent problems associated with shadow printing, projection printing exposure tools have been developed to project an image of the mask patterns onto a resist-coated wafer many centimeters away from the mask. The small image area is scanned or stepped over the wafer to cover the entire surface. Figure 2.14 depicts the various ways to project and scan the image. The resolution of a projection system is given by

$$l_m = \lambda / NA \quad \dots(2.2)$$

where  $\lambda$  is the wavelength of the exposure radiation and  $NA$  is the numerical aperture given by

$$NA = \bar{n} \sin \theta \quad \dots(2.3)$$

where  $\bar{n}$  denotes the refractive index of the imaging medium ( $\bar{n} = 1$  in air) and  $\theta$  is the half angle of the cone of light converging to a point image at the wafer as shown in Figure 2.14. The depth of focus,  $\Delta z$ , can be expressed as

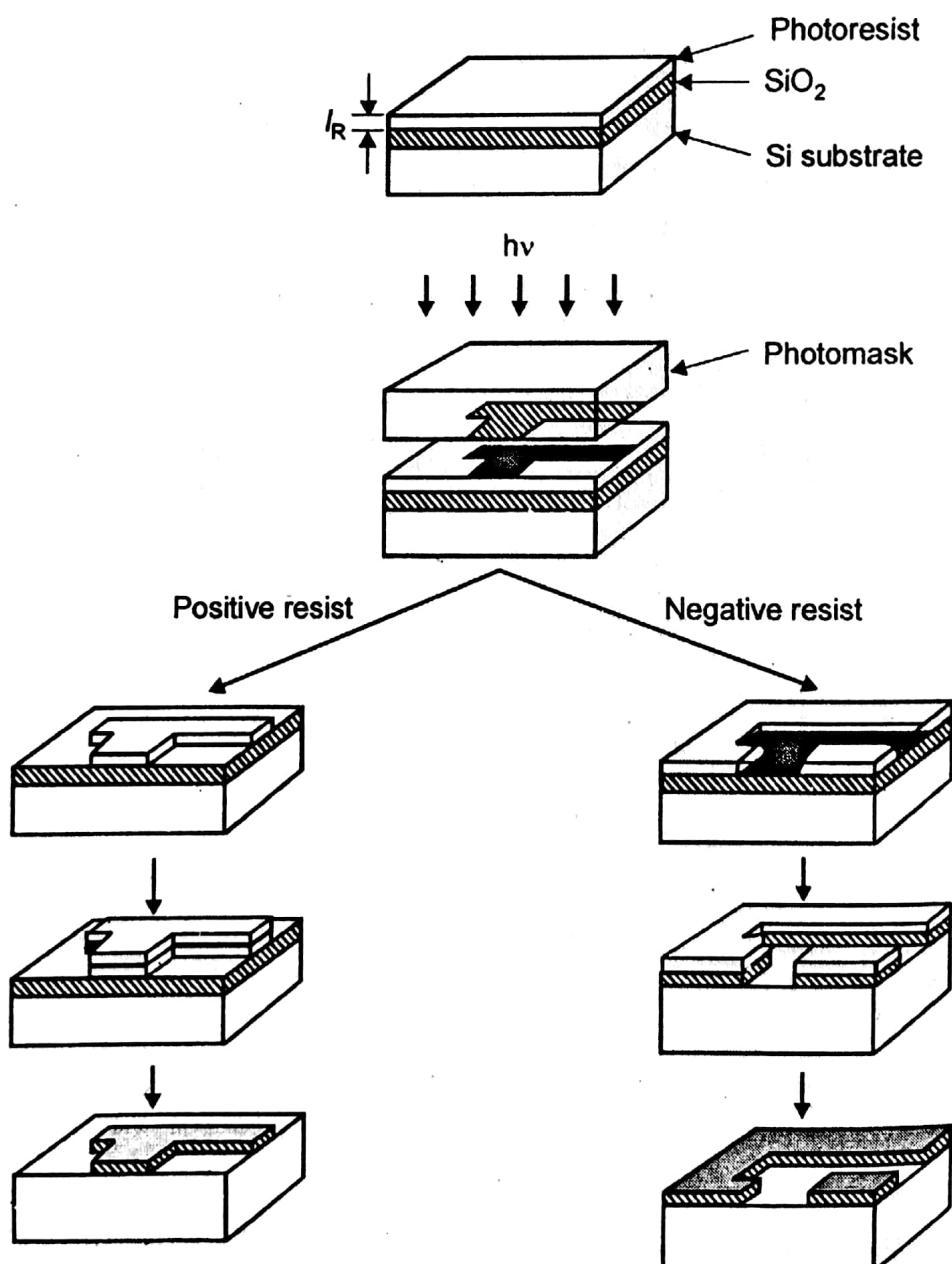
$$\Delta z = \pm \frac{l_m}{2 \tan \theta} \cong \pm \frac{l_m}{2 \sin \theta} = \pm \frac{\bar{n} \lambda}{[2(NA)^2]} \quad \dots(2.4)$$

Resolution can be enhanced by reducing  $\lambda$  and this explains the trend towards shorter wavelength in optical lithography. Typically, scanning projection systems are

capable of about  $1 \mu\text{m}$  resolution, while step-and-repeat projection systems can accomplish deep sub-micrometer spatial resolution demanded by modern integrated circuits, especially when coupled with deep UV radiation sources.

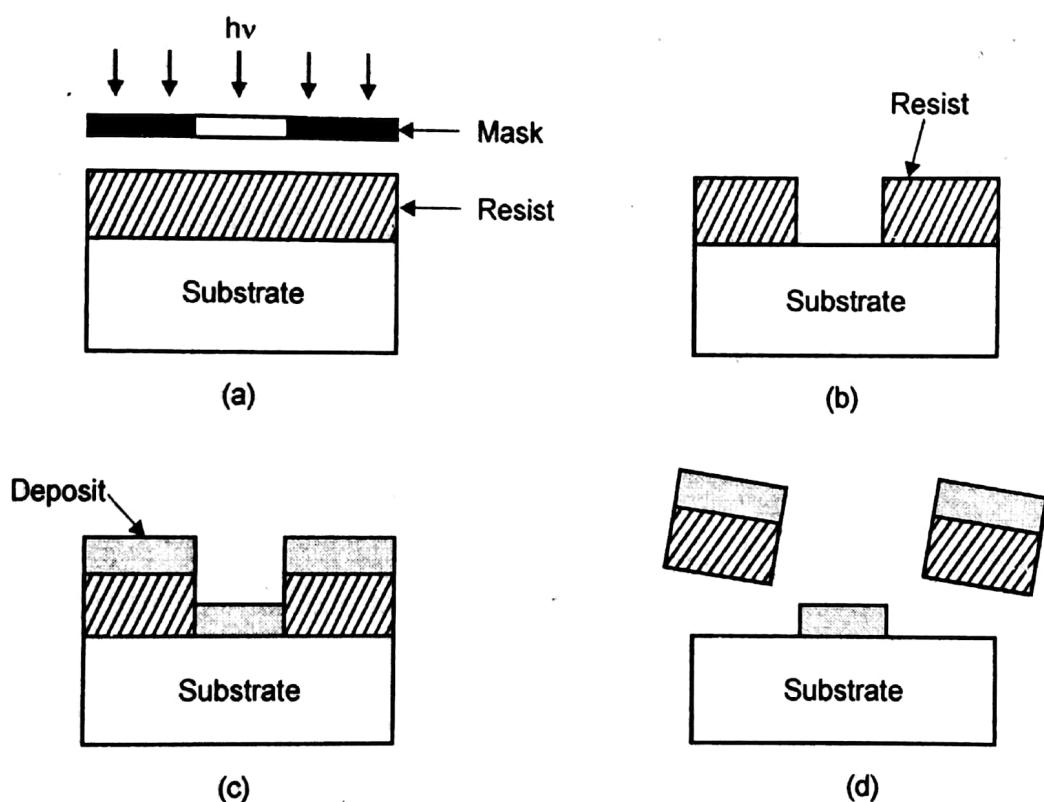
### Clean Room

An IC fabrication facility requires a clean room, particularly in lithography areas. Dust particles settling on semiconductor wafers and lithographic masks can cause defects in the devices. In a clean room, the total number of dust particles per unit volume must be tightly controlled along with other parameters, such as temperature, humidity, pressure and so on.



**Figure 2.14:** Optical lithographic transfer process

Figure 2.14 illustrates the steps of transferring IC patterns from a mask to a wafer. The wafer is placed in a clean room that typically is illuminated with yellow light as photoresists are not sensitive to wavelengths greater than  $0.5 \mu\text{m}$ . The wafer is held on a vacuum spindle, and approximately  $1 \text{ cm}^3$  of liquid resist is applied to the center of the wafer. The wafer is then spun for about 30 seconds. The thickness of the resulting resist film,  $l_R$ , is directly proportional to its viscosity as well as the percent solid content indigenous to the resist, and varies inversely with the spin speed. For spin speeds in the range of 1000 to 10000 rpm, film thicknesses of the order of 0.5 to 1  $\mu\text{m}$  can be accomplished.



**Figure 2.15:** Lift-off process for pattern transfer

The wafer is then given a pre-exposure bake ( $80^\circ\text{C}$  to  $100^\circ\text{C}$ ) to remove solvent and improve adhesion. The wafer is aligned with respect to the mask in an optical lithographic system prior to exposure to UV or deep UV light.

For a positive photoresist, the exposed portions are dissolved in the developer solution. The wafer is then rinsed, dried, and then put in an ambient that etches the exposed insulating layer but does not attack the resist. Finally, the resist is stripped, leaving behind an insulator image (or pattern) that is the same as the opaque image on the mask.

For a negative photoresist, the exposed area becomes insoluble, and the final insulator pattern is the reverse of the opaque image on the mask.

The insulator image can be employed as a mask for subsequent processing. For instance, ion implantation can be performed to dope the exposed regions selectively. Figure 2.15 illustrates the lift-off technique. This method suffices if the film thickness is smaller than that of the photoresist.

### Photoresist

A photoresist is a radiation-sensitive compound. For positive resists, the exposed region becomes more soluble and this more readily removed in the developing process.

The net result is that the patterns formed in the positive resist are the same as those on the mask. For negative resists, the exposed regions become less soluble, and the patterns engraved are the reverse of the mask patterns.

A positive photoresist consists of three constituents: a photoresistive compound, a base resin, and an organic solvent. Prior to exposure, the photosensitive compound is insoluble in the developer solution. After irradiation, the photosensitive compound in the exposed pattern areas absorbs energy, changes its chemical structure, and transforms into a more soluble species. Upon developing the exposed areas are expunged.

Negative photoresists are polymers combined with a photosensitive compound. Following exposure, the photosensitive compound absorbs the radiation energy and converts it into chemical energy to initiate a chain reaction, thereby causing crosslinking of the polymer molecules.

The cross-linked polymer has a higher molecular weight and becomes insoluble in the developer solution. After development, the unexposed portions are removed. One major drawback of a negative photoresist is that the resist absorbs developer solvent and swells, thus limiting the resolution of a negative photoresist.

## 2.12 Applications of Nanotechnology

Nanoscale materials have been used for many decades in several applications including mass-market consumer products. Well known is a glass for windows which is coated with titanium oxide nanoparticles that react to sunlight as break down dirt. When water hits the glass, it spreads evenly over the glass surface instead of forming droplets and runs off rapidly, taking the dirt with it. Nanomaterials are highly used in vehicle industry to reinforce certain properties of vehicle bumpers and to improve the adhesive properties of paints.

There are two types of applications. The structural applications are based on the mechanical properties of the nano structured or nanophase materials. These are used to produce super plastic ceramics and extremely hard metals.

Functional applications are based on the transformation of external signals such as the filtering of the incident light, the change of electrical resistance in different gas concentrations and luminescent behaviour when electrically activated.

Other applications are:

- Sunglasses using protective and antireflective ultrathin polymer coatings.
- Scratch resistant coatings that are transparent, ultrathin, easy maintenance, well suited for daily use and reasonably priced.
- Windproof and waterproof cloths preventing wrinkling or staining and guarding against electrostatic discharges.
- Sunscreens and cosmetics based on nanotech are widely used because of their purity and cleanliness. For antiwrinkle creams, a polymer capsule is used to transport active agents like vitamins.
- Sports equipments like tennis rackets with carbon nanotubes have improved torsion and flex resistance. Long lasting tennis balls are made up of nanocoated inner core clay which increases the lifetime of conventional balls.
- In televisions, the electrons in a field effect display are fixed through vacuum at a layer of phosphorescent glass covered with pixels.
- Automobile fuel lines made with carbon nanotubes that inhibit static electricity and reduce the risk of explosions.
- A nano-improved golf ball that increases the efficiency of the transfer of energy from the club to the ball and reduces off-axis rotation for greater control.
- “Intelligent” nanocoatings for windows that reflect solar heat in the summer and transmit in winter.
- Cutting tools made of nanocrystalline materials, such as tungsten carbide, tantalum carbide and titanium carbide, are more wear and erosion-resistant, and last longer than their conventional (large-grained) counterparts. They are finding applications in the drills used to bore holes in circuit boards.

- A washing machine that inhibits bacterial growth on the cloths it washes. Samsung's Silver Care technology harnesses the antimicrobial properties of silver releasing 100 quadrillion silver ions into your cold-water wash. Silver is toxic to aquatic life, so *Samsung* has been notified by the EPA that their washing machine may be regulated as a pesticide. They make a refrigerator, too.
- Hydrogen storage devices.
- Very thin permanent magnets with high energy products used for high density magnetic recording.
- Nanomaterials have very high magneto resistance. This property is also used in the magnetic storage devices.
- Make alloy from immiscible materials and high TC superconductors.
- Quantum wells, quantum dots and quantum wires have quantum confinement are produced from semiconductor nanomaterials which are acting as computer storage (memory) materials with high density.
- Nanomaterials have a large volume function of grain boundaries or large ratio between surface area and volume. This property is used to get improved mechanical properties like higher hardness in ceramics.

The nanomaterials find wide applications in fields such as coating & films, ceramics, adhesives, surface modifiers, packaging, fire retardants, pharmaceuticals, cosmetics, resins and glasses, magnetic materials, catalysts etc.

Some typical properties and possibilities of future applications of nanomaterials are tabulated below:

**Table 2.2:** Properties and applications of nanomaterials

<i>Sl.No.</i>	<i>Properties</i>	<i>Applications</i>
1.	Single magnetic domain	Magnetic recording, data encryption
2.	Small mean free path of electrons electrons in a solid	Special conductors
3.	High and selective optical absorption of Metal particles	Colour filters, solar absorbers, photo voltaics, Photographic materials, molecular filters
4.	Uniform mixture of different kinds of superfine particles	R & D of new materials
5.	Grain size too small for stable dislocation	High strength and hardness of metallic materials
6.	Surface interface—Large specific surface area	Catalysis, sensors
7.	Large surface area, small heat capacity	Heat exchange materials, combustion catalysis
8.	Optical devices based on transparent, rigid silica host	Tunable laser, semiconductors, non-linear optics, pick-up lens, photochromic
9.	Photovoltaic	Power cables, superconductors
10.	Cluster coating and metallization	Special resistors, temperature sensors

### 2.12.1 Aerospace components

Due to the risks involved in flying, aircraft manufacturers strive to make the aerospace components stronger, tougher, and last longer. One of the key properties required of the aircraft components is the fatigue strength, which decreases with the component's age. By making the components out of stronger materials, the life of the aircraft is greatly increased. The fatigue strength increases with a reduction in the grain

size of the material. Nanomaterials provide such a significant reduction in the grain size over conventional materials that the fatigue life is increased by an average of 200–300%.

Furthermore, components made of nanomaterials are stronger and can operate at higher temperatures; aircrafts can fly faster and more efficiently (for the same amount of aviation fuel). In spacecrafts, elevated-temperature strength of the material is crucial because the components (such as rocket engines, thrusters, and vectoring nozzles) operate at much higher temperatures than aircrafts and higher speeds. Nanomaterials are perfect candidates for spacecraft applications, as well.

### 2.12.2 Sensors

Ceramics are hard, brittle and difficult to machine. However, with a reduction in grain size to the nanoscale, ceramic ductility can be increased. Zirconia, normally a hard, brittle ceramic, has even been rendered superplastic (for example, able to be deformed up to 300% of its original length).

Nanocrystalline ceramics, such as silicon nitride and silicon carbide, have been used in such automotive applications as high-strength springs, ball bearings and valve lifters, because they can be easily formed and machined, as well as exhibiting excellent chemical and high-temperature properties.

They are also used as components in high-temperature furnaces. Nanocrystalline ceramics can be pressed into complex net shapes and sintered at significantly lower temperatures than conventional ceramics.

In their widest sense, nanotechnologies have been used by industries for decades (semiconductors), and in some cases considerably longer (chemicals). However, developments over the past 20 years in the tools used to characterise materials, have led to an increased understanding of the behaviour and properties of matter at very small size scales.

Increased knowledge of the relationship between the structure and properties of nanomaterials has enabled the production of materials and devices with higher performance and increased functionality. This progress has taken place steadily over several years; so, at least so far, the influence of nanotechnologies on industry can be described as evolutionary rather than revolutionary. This is also evident in the current production rates of nanoparticles and nanomaterials which, although increasing, are negligible compared with bulk chemicals and materials.

### 2.12.3 Medicine

Applications in the field of medicine are especially promising. Areas such as disease diagnosis, drug delivery and molecular imaging are being intensively researched.

The small size of nanoparticles endows them with properties that can be very useful in oncology, particularly in imaging. Quantum dots (nanoparticles with quantum confinement properties, such as size-tunable light emission), when used in conjunction with MRI (magnetic resonance imaging), can produce exceptional images of tumor sites. These nanoparticles are much brighter than organic dyes and only need one light source for excitation. This means that the use of fluorescent quantum dots could produce a higher contrast image and at a lower cost than today's organic dyes used as contrast media.

Another nanoproperty, high surface area to volume ratio, allows many functional groups to be attached to a nanoparticle, which can seek out and bind to certain tumor cells. Additionally, the small size of nanoparticles (10 to 100 nanometers), allows them to preferentially accumulate at tumor sites (because tumors lack an effective lymphatic drainage system). A very exciting research question is how to make these imaging nanoparticles do more things for cancer. For instance, is it possible to manufacture multifunctional nanoparticles that would detect, image, and then proceed to treat a tumor? This question is under vigorous investigation; the answer to which could shape the future of cancer treatment. A promising new cancer treatment that may one day replace radiation and chemotherapy is edging closer to human trials. Kanzius RF therapy attaches microscopic nanoparticles to cancer cells and then the RF (radio frequency) transmitter apparatus exposes the nanoparticles to the radio frequency signal they heat up, destroying the cancer cells, but don't damage healthy cells nearby.

- ❖ Target therapies
- ❖ Image single cells
- ❖ Develop biosensors to: Monitor therapy within a cell, Screen for disease markers, Create biomaterials and engineered tissues.

With the growing side effects of recently introduced drug antibiotics and the developing resistance of germs to these drugs, the interest in use of silver has recently been rekindled. The colloidal silver solutions are far more effective than silver nitrate

and require far less quantities of Silver (less than 0.25% of the Silver in Silver Nitrate used before). Silver is non toxic to the human body and has been used in preparation of food items in India for hundreds of years e.g. Silver foil on sweets. During the past few years, the technology for making Nano sized particles of Silver has developed to a great extent and the broad spectrum antibiotic properties of silver have thereby been enhanced to cover in more effective manner all kinds of gram positive and gram negative bacteria. Nano silver particles are also highly effective against fungi and viruses of all types including hepatitis, herpes and HIV virus, to name a few.

The drug, a compound known as TAK-779, was originally found to bind to a specific location on human T-cells, which blocks the HIV virus' entry to the body's immune system. Unfortunately, the portion of the drug's molecule that made binding possible had unpleasant side effects. When that portion of the molecule- an ammonium salt-was removed, the drug lost its binding ability.

That's when Melander and colleagues from the University of North Carolina at Chapel Hill and the University of Colorado at Boulder turned to gold as the answer. The element is non-reactive in the human body, and would be the perfect "scaffold" to attach molecules of the drug to in the absence of the ammonium salt, holding the drug molecules together and concentrating their effect.

"The idea is that by attaching these individual molecules of the drug with a weak binding ability to the gold nanoparticle, you can magnify their ability to bind," Melander says.

The researchers' theory proved correct. They started with a modified version of TAK-779, which didn't include the harmful ammonium salt. After testing, they found that attaching 12 molecules of the modified drug (SDC-1721) to one nanoparticle of gold restored the drug's ability to prevent HIV infection in primary cultured patient cells. When only one molecule of the drug was attached to the gold nanoparticle, the compound was unable to prevent HIV infection, indicating that the multivalency of the drug was important for its activity.

Current medical implants, such as orthopaedic implants and heart valves, are made of titanium and stainless steel alloys, primarily because they are biocompatible. Unfortunately, in some cases these metal alloys may wear out within the lifetime of the patient. Nanocrystalline zirconium oxide (zirconia) is hard, wearresistant, bio-corrosion resistant and bio-compatible. It therefore presents an attractive alternative material for

implants. It and other nanoceramics can also be made as strong, light aerogels by sol-gel techniques. Nanocrystalline silicon carbide is a candidate material for artificial heart valves primarily because of its low weight, high strength and inertness.

In future, *Nanonets* – networks of carbon nanotubes enable numerous basic electronic functions at low cost. The durable nature of nanonets make them suitable devices, like electronic paper, flexible touch screens, solar cells, sensors.

## 2.13 Disadvantages and Risks of Nanotechnology

Initial costs will be high and products will be exclusive; eventually prices fall. Manufacturers aren't required to disclose the presence of nano-materials in their labeling.

Some nanoparticles are extremely combustible so they could spontaneously burst into flames. Some nanomaterials seem to linger in the environment. Nanoparticles can enter the body and its vital organs [through the skin and nasal passage], including the brain, much more easily than can larger particles. There seems to be a significant danger that public acceptance of a whole range of beneficial applications of nanotechnologies, particularly in the environmental domain, might be threatened by too close an association with military applications.

In conclusion, nanotechnologies will have an impact across many branches of science and technology and can be expected to influence a range of areas of human endeavour.

## Points to Remember

1. Richard Feynman - "There's plenty of room at the bottom" 1959, 29<sup>th</sup> Dec (Annual meeting of American physical society at the California institute of technology).
2. Gordon Moore, Head of research at Fairchild Semiconductors (Founder of Intel) suggested in 1965 issue of Electronics that "The number of transistors that could be crammed into each new generation of chips would remain roughly the same."
3. Two laws of Moore

### *Moore's 1<sup>st</sup> Law*

The amount of space required to install a transistor on a chip shrinks by roughly half in every 18 months.

### *Moore's 2<sup>nd</sup> Law*

The cost of building a chip manufacturing plant doubles with every other chip generation in every 36 months.

4. Future of Moore's Law – Semiconductor industry crisis
  - 1) Overheating – solved by Nanotechnology
  - 2) Chip manufacturing – solved by Nanotechnology
  - 3) The material crisis – solved by Nanotechnology
  - 4) Quantum effects – solved by Nanotechnology

– Small object that behaves as a whole units in terms of transport properties  
(diameter range 100 nm to 2500 nm for fine particles for nanoparticles 1 to 100 nm)
5. Surface area more than bulk materials and surface/volume high.  
(E.g.) Cu bulk rod 50 nm scale can be bent but same rod made up of nanoparticles can't be bent since they don't exhibit malleability and ductility.
6. *Quantum dots – Electrons confined in all three dimensions*  
Quantum computing, biological analysis

$$\text{Band gap} \propto \frac{1}{\text{size of dot}}$$

*Quantum wires – Electrons confined in 2 dimensions*

High electrical conductivity, light weight, small diameter, low chemical reactivity and high tensile strength.

*Quantum wells – Electrons confined in one dimension*

Sharper density of states used in diode lasers.

7. Nanomanipulation means to carry out precise and controlled manipulation of atoms, molecules and nanostructures on a surface.
8. Lithography means the process of transferring a pattern into a reactive polymer film, termed as resist which will subsequently be used to replicate that pattern into a underlying thin film (or) substrate.

## Review Questions

### Short Answer Questions

1. What is nano? Why do we care about nano? How it is going to revolutionize the world?
2. What is nanoscale?
3. Define nanostructured material. Classify nanostructured materials in detail.
4. What is nanoscience?
5. What are nanophase materials?
6. What are nanomaterials?
7. What is the difference between nanoscience and nanotechnology? Does science of technology change?
8. Why nanotechnology now? Why we did not hear about it in the past?
9. Name few nanomaterials that are found in nature.
10. Why objects in the nanoscale cannot be seen by visible light? How do we see them?
11. What are the characteristic properties of objects in the nanoscale?
12. Why does nanotechnology have to be used responsibly?
13. Are there nanoobjects in our body? Name a few.
14. What are the likely impacts of nanotechnology?
15. State Moore's I law and II law? How these laws are related with nanotechnology?
16. What is nanomachine?
17. Give some present and future applications of nanomaterial.
18. How can nanotechnology be used for industrial development?
19. Contrast nanotechnology from other emerging technologies.
20. What could go wrong during a nanomedical procedure?
21. What products are there already on the market which contains nanoparticles or carbon nanotubes?
22. List few applications of carbon nanotubes.

23. What is the necessity of quantum well in nanoscience?
24. Define quantum wire with a sketch.
25. Define quantum dots.
26. How do we manipulate objects in nano dimension?
27. What do you mean by positive and negative resist?
28. What are the types of lithography? Explain optical lithography with a neat sketch.
29. What are nanopen and nanopencil?
30. List out the disadvantages and risks found in using nanomaterials.

### Descriptive Questions

31. Give an account on nanomaterials and their applications.
32. What are nanomaterials and explain their properties?
33. Discuss about quantum well, wire and dot with their significance.
34. How do we study the properties of nanoobjects?
35. Discuss the principle and applications of nanolithography.
36. Illustrate the role of nanomaterials in sensors.
37. Explain the usage of nanomaterials in medicine.
38. Summarize the landmarks in nanotechnology.
39. What are carbon nanotubes? Mention its properties and applications.
40. Enumerate the details of top-down and bottom-up approaches in nanotechnology. Give examples.
41. What are the types of CNT? Highlight the properties of CNT.
42. Write a short note on nanofabrication. Explain electron beam lithography with a neat sketch.
43. Describe the principle and working of an optical lithographic technique with necessary diagram.
44. Compare and contrast nanotechnology from other emerging technologies. With neat sketch explain the function of dip pen lithography.

# CHAPTER - 3

## LASER PHYSICS

### Introduction

Laser, perhaps one of the most exciting discoveries of the twentieth century, is an acronym for [light amplification by stimulated emission of radiation.] The first successful operation of a laser was done by Maiman in 1960 using a ruby crystal. [Laser light, like light from any other ordinary source, is emitted when atoms make a transition from quantum state of higher energy to a state of lower energy. However, it has unique properties not found in the light from ordinary sources.] Let us now discuss its basic principle, working of some laser systems, and its important applications.

### 3.1 Characteristics of Laser

The enormous growth of laser technology has stimulated a broad range of scientific and engineering applications that exploit some of the unique properties of laser light. These properties are derived from the distinctive way laser light is produced in contrast to the generation of ordinary light. Light from a laser differs from light from conventional sources in number of ways. The most striking features of a laser beam are

#### i) Monochromaticity

[If light coming from a source has only one frequency of oscillation, the light is said to be monochromatic and the source, a monochromatic source. In practice it is not possible to produce light with only one frequency.]

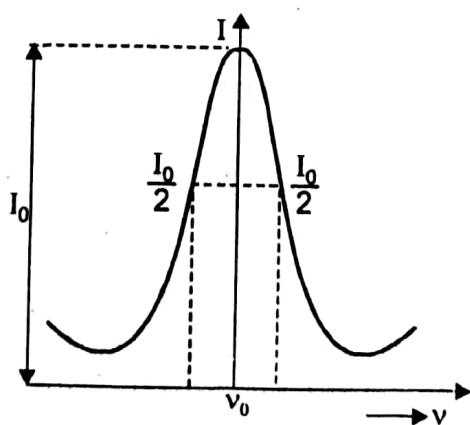


Figure 3.1: (a) Line Width diagram

[Light coming out of any source consists of band of frequencies closely spaced around a central frequency ' $\nu_0$ '. The band of frequencies,  $\Delta\nu$ , is called the linewidth or bandwidth. The light from conventional sources has large linewidths of the order of  $10^{10}$  Hz or more. On the other hand, light from lasers is more monochromatic having linewidths to 100 Hz.]

### ii) Higher degree of coherence

[Light waves are said to be coherent if they are in phase with each other.] For example if they maintain crest-to crest and trough-to-trough correspondence.

[Two things are necessary for light waves to be coherent. First they must start with the same phase at the same position. Secondly their wavelengths must be the same or they will drift out of phase because the crests of the higher frequency will arrive ahead of the crests of the lower frequency wave.]

[The light that emerges from a conventional light source is a jumble of short waves which combine with each other in a random manner. On the other hand the light from a laser is a resultant of a large number of identical photons which are in phase and therefore exhibits a high degree of coherence.]

### iii) High intensity

The power output of a laser may vary from a few milliwatts to a few kilowatts. But this energy is concentrated in a beam of very small cross section. The intensity of a laser beam is approximately given by

$$I = \left(\frac{10}{\lambda}\right)^2 P \quad \dots(3.1)$$

where  $P$  is the power radiated by the laser.

In the case of 1mW He-Ne laser

$$\lambda = 6328 \times 10^{-10} \text{ m}$$

$$I = \frac{100 \times 10^{-10} \text{ W}}{(6328 \times 10^{-10})^2 \text{ m}^2} = 2.5 \times 10^{11} \text{ W/m}^2$$

To obtain light of same intensity from a tungsten bulb, it would have to be raised to a temperature of  $4.6 \times 10^6$  K. The normal operating temperature of the bulb is about 2000 K.

#### iv) Directionality

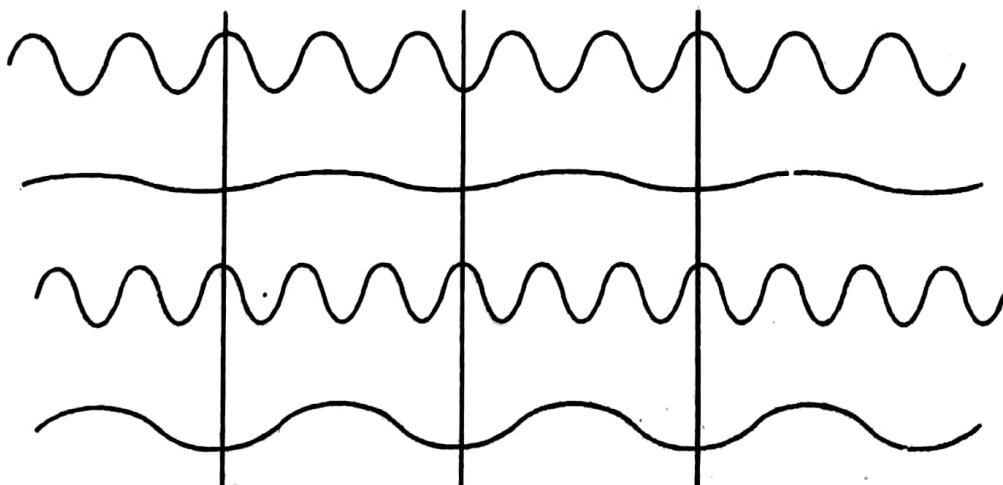
[Conventional light sources emit light in all directions. But light from laser diverges very little. Light beam can travel as a parallel beam upto a distance of  $d^2/\lambda$  where  $d$  is the diameter of the aperture through which the light is passing and  $\lambda$  is the wavelength of the light used. After travelling the distance  $d^2/\lambda$ , the light beam spreads radially. In ordinary light beam the angular spread is given by  $\Delta\theta = \lambda/d$ . For a laser beam the angular spread is 1mm per 1m, but for an ordinary source of light the angular spread is 1m per 1m. This shows the directionality of laser beam.] A 10-cm wide laser beam has a diameter of only 5 km after reaching the moon which is at an approximate distance of 3,85,000 km.

#### 3.1.1 Coherence

[Conventional light sources produce incoherent light.] This means that the light that emerges from a conventional light source is mixture of waves at various frequencies that reinforce or cancel each other in a random fashion. Figure 3.1(b) depicts this situation. Obviously, the wavefront thus produced varies from point to point and changes from time to time. [The wave from a laser is called almost coherent because it is an orderly wave of one frequency where the whole beam is spatially in phase. There are, thus, two independent facets of this coherence: namely, temporal coherence and spatial coherence.]

#### 3.1.2 Temporal Coherence

This type of coherence refers to the correlation between the radiation field at a point and the radiation field at the same point at a later time, i.e. the relation between  $E(x, y, z, t_1)$  and  $E(x, y, z, t_2)$ , where these represent the radiation field at the point  $(x, y, z)$  at times  $t_1$  and  $t_2$  respectively.



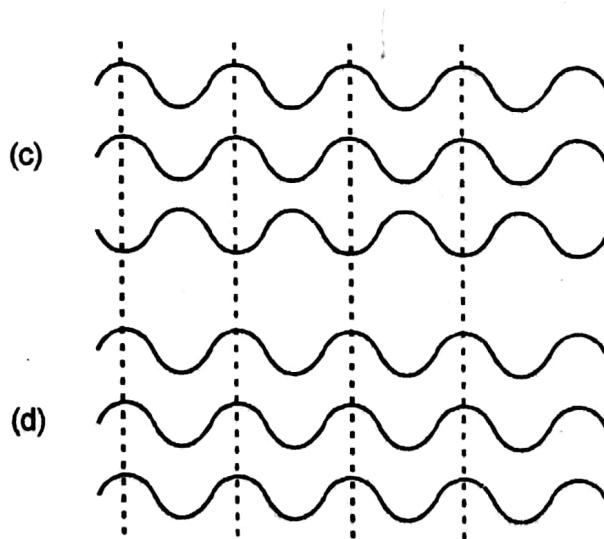
**Figure 3.1: (b) Incoherent waves. There is no relationship between one wave and another**

If the phase difference between the two radiation fields is constant during the period normally covered by observations ( $\sim$  few microseconds), the wave is said to have temporal coherence. If the phase difference between two radiation fields changes many times and in an irregular way during the shortest period of observation, the wave is said to be incoherent.

Time coherence comes about when each cycle of the wave takes exactly the same time to pass a point in space. This really means that the frequency of the wave is not varying and is given the name **monochromatic**.

### 3.1.3 Spatial Coherence

Two fields at two different points on a wavefront of a given electromagnetic wave are said to be space coherent if they preserve a constant phase difference over any time, i.e., space coherency requires that the waves not only are of the same frequency, but that they are in phase in space. Figure 3.1 shows these conditions. In figure 3.1(c) the waves are monochromatic (time coherent) but are not in sequence in space. This can occur when the source of the wave is physically broad rather than a point source. Figure 3.1(d) shows a spatially and time-coherent wave. The whole wavefront is in step and each cycle take the same length of time. The very nature of laser mechanism produces this coherent signal.



**Figure 3.1:** (c) Time coherent, space incoherent,  
(d) Time, space coherent

Spatial coherence is possible even the two beams are individually time incoherent, as long as any phase change in one of the beams is accompanied by a simultaneous equal phase change in the other beam. With the ordinary light sources, this is only possible if the two beams have been derived from the same part of the source.

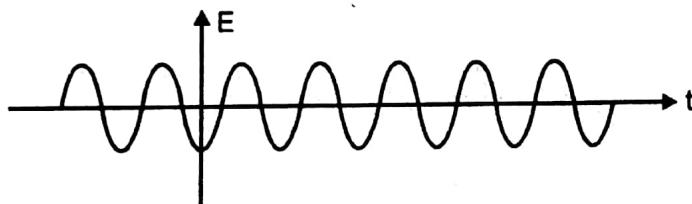
Time incoherence is characteristic of single beam of light, whereas space incoherence concerns the relationship between two separate beams of light. Two beams of light produced in different parts of light source will have been emitted by different groups of atoms. Each beam will be time-incoherent and will suffer random phase changes, as a result of which the phase difference between the two beams will also suffer rapid and random changes. Two such light beams are said to be space incoherent.

### Coherence Time and Coherence Length

Let us consider that the radiation field  $E$ , from a light source is an ideal sinusoidal function of time  $t$ , at any given position can be written as

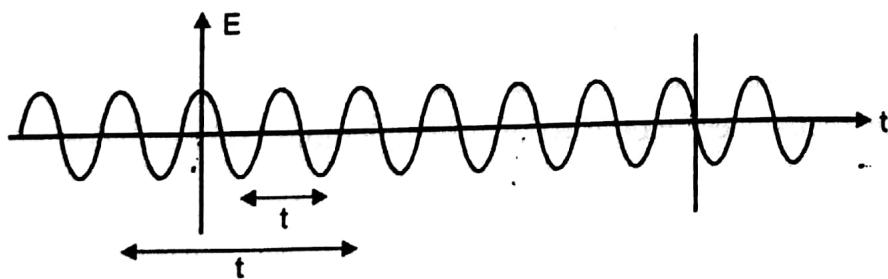
$$E = a \cos(\omega_0 t + \phi) \quad \dots (3.1.a)$$

Here  $a$  is the magnitude of the field,  $\omega_0$  the angular frequency and  $\phi$  the phase. Figure 3.1 (e) depicts the field given by (3.1.a)



**Figure 3.1: (e)**

However, no emitted light produces a perfect sinusoidal variation of field  $E$  with time  $t$ , and for any actual light source the magnitude  $a$  and phase  $\phi$  will vary with time. One can represent, a more realistic picture of the radiation from an emitting source of light as shown in figure 3.1 (e).



**Figure 3.1: (f)**

Figure 3.1 (f) represents the average time duration for which an ideal sinusoidal emission occurs.  $\tau$  is called **coherence time or interval of coherence**.

Let  $T$  denote the period of the oscillation, where  $= \frac{\omega_0}{2\pi}$ . The spatial dimension,  $L$ , for which the light may be considered to be a perfect sinusoidal is given by

$$L \approx CT \approx \frac{\tau}{T} \lambda \quad .. (3.1.b)$$

Here  $C$  is the velocity of light and  $L$  is called the **coherence length**. After time  $\tau$ , there is no correlation between the phases of the waves.

In terms of purity  $Q$  of the spectral line ( $Q \approx \lambda / \Delta\lambda$  where  $\Delta\lambda$  half width of the spectral line), we have

$$C_t \approx L \approx \lambda Q \quad .. (3.1.c)$$

### 3.2 Einstein's Prediction

When we see light from any source, we actually see electrons jumping from excited states to lower states. This type of emission of light which occurs on its own is known as spontaneous emission and is responsible for the light coming from candles, electric bulbs, fire, sun etc.

Einstein predicted in 1917, there must be a second emission process to establish thermal equilibrium. For example, if we illuminate a material with light of suitable frequency, the atoms in it absorb light and go to higher energy state.

The excited atoms tend to return randomly to the lower energy state. As the ground state population is very large, more and more atoms are excited under the action of incident light and it is likely that a stage may be reached where all atoms are excited. This violates thermal equilibrium condition. Therefore Einstein suggested that there could be an additional emission mechanism, by which the excited atoms can make downward transitions. He predicted that the photons in the light field induce the excited atoms to fall to lower energy state and give up their excess energy in the form of photons. He called this type of second emission as stimulated emission.

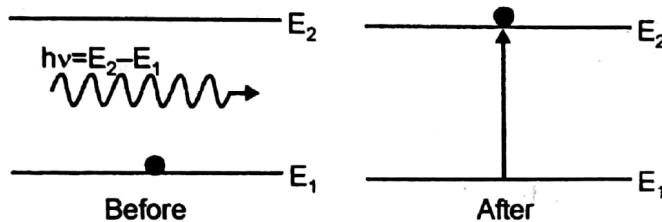
### 3.3 The Three Fundamental Processes

Let us consider a medium consisting of identical atoms capable of being excited from the energy level 1 to the energy level 2 by absorption of photons. Let the levels be denoted by  $E_1$  and  $E_2$  and their populations be  $N_1$  and  $N_2$ , respectively. Let the atoms be in thermal equilibrium. In the equilibrium condition the number of atomic-upward transitions

must be equal to the number of downward transitions. Thus no net photons are generated or lost. However, when the atoms are subjected to an external light of frequency, 'v', the following three processes occur in the medium.

### i) Absorption

An atom in the ground state  $E_1$  absorbs an incident photon and makes a transition to the excited state  $E_2$ . This transition is known as induced or stimulated absorption or simply absorption.



**Figure 3.2:** Absorption process

Corresponding to each absorption transition, one photon disappears from the incident field and one atom adds to the population at the excited energy level  $E_2$ . This process may be represented as



\* → atom is in the excited state

The number of absorption transitions occurring in the material at any instant will be proportional to the population in the lower level and the number of photons per unit volume in the incident beam. The rate of absorption may be expressed as

$$R_{\text{abs}} = B_{12} \rho(v) N_1 \quad \dots(3.2)$$

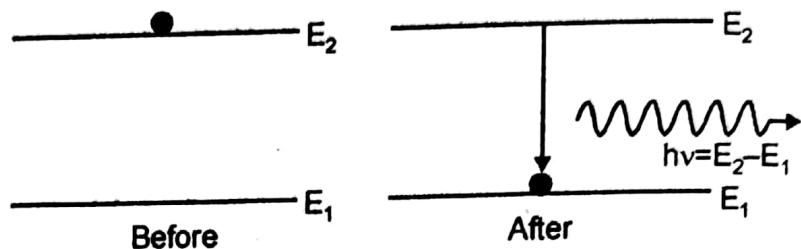
$\rho(v)$  - energy density of incident light

$B_{12}$  - Einstein coefficient for absorption

At thermal equilibrium, the population in the lower energy state is far larger than that in the higher energy state. Therefore as light propagates through the medium, it gets absorbed.

### ii) Spontaneous emission

An excited atom can stay at the excited level for an average life time ( $10^{-8}$  s). If this atom is not stimulated by any other agent during its short lifetime, the excited atom undergoes a transition to the lower level on its own. During the transition it gives up the excess energy in the form of photon. This process in which an excited atom emits a photon without any external agent is known as spontaneous emission.



**Figure 3.3:** Spontaneous emission process

This process is represented as



The number of photons generated will be proportional to the population of the excited level only. The rate of spontaneous emission may be expressed as

$$R_{SP} = A_{21} N_2 \quad \dots (3.3)$$

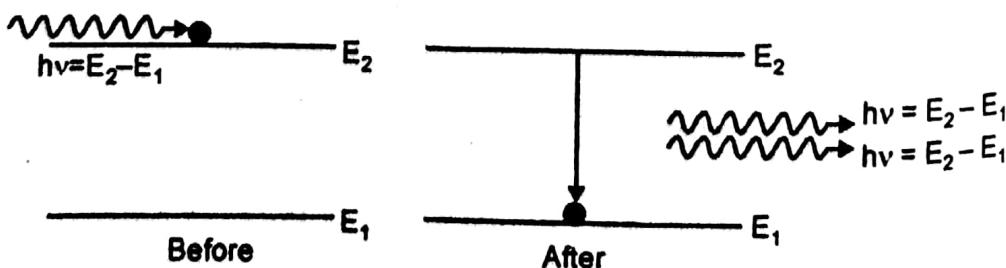
$A_{21}$  - Einstein coefficient for spontaneous emission.

Thus the process of spontaneous emission is independent of energy density.

### iii) Stimulated emission

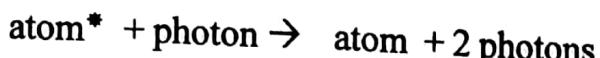
An atom in the excited state need not wait for spontaneous emission to occur. If a photon with appropriate energy ( $h\nu = E_2 - E_1$ ) interacts with the excited atom, it can trigger the atom to undergo transition to the lower level and to emit another photon.

The process of emission of photons by an excited atom through a forced transition occurring under the influence of an external agent is called induced or stimulated emission.



**Figure 3.4:** Stimulated emission process

The process may be represented as

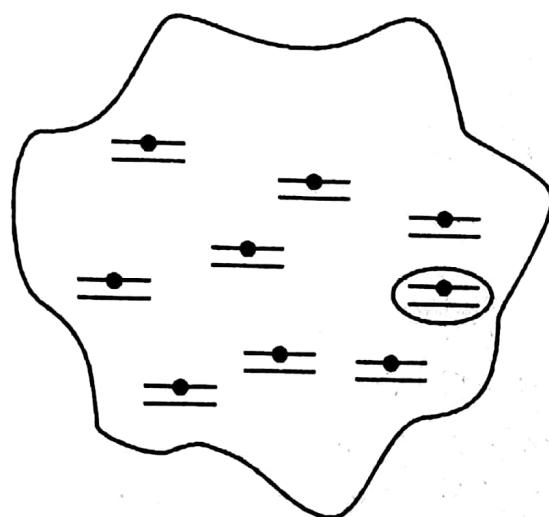


The rate of stimulated emission of photons is given by

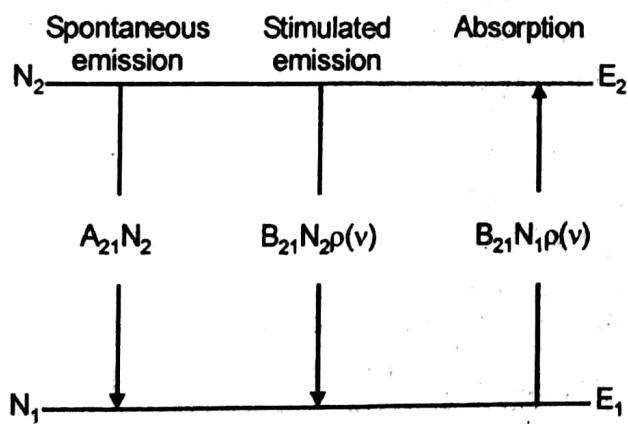
$$R_{st} = B_{21}\rho(v) N_2 \quad \dots(3.4)$$

$B_{21}$  - Einstein coefficient for stimulated emission

### 3.4 Einstein's A and B Co-efficients



**Figure 3.5:** A blackbody at a temperature 'T' emits radiation that interacts with the atoms in the blackbody



**Figure 3.6:** Radiative processes that affect the number of atoms at energy  $E_1$  and  $E_2$ .

The two emission processes remove atoms from level  $E_2$  add them to level  $E_1$ .

The absorption process involves transition from  $E_1$  to  $E_2$ .

Einstein's proof for the existence of stimulated emission grew out of his desire to understand the basic mechanism involved in the interaction between electromagnetic radiation and matter. Let us assume matter (a collection of atoms) is in thermodynamic equilibrium with black body radiation field. The atoms and the resonant radiation are contained in an enclosure at some temperature T and interact with one another. The figure 3.5 shows a simplified picture of two level atoms and radiation bound inside of an arbitrary unit volume. If thermodynamic equilibrium exists, the number of atoms  $N_2$  at energy level  $E_2$ , the number of atoms  $N_1$  at energy level  $E_1$ , and the number of photons in the radiation field will all remain constant. This condition requires that the number of transitions from  $E_2$  to  $E_1$  must be equal to the number of transitions from  $E_1$  to  $E_2$ . Thus

$$\text{The number of atoms absorbing photons per second per unit volume} = \text{The number of atoms emitting photons per second per unit volume}$$

$$\text{The number of atoms absorbing photons per second per unit volume} = B_{12} \rho(v) N_1$$

$$\text{The number of atoms emitting photons per second per unit volume} = A_{21} N_2 + B_{21} \rho(v) N_2$$

In equilibrium condition, the number of transitions from  $E_2$  to  $E_1$  must be equal to the number of transitions from  $E_1$  to  $E_2$ . Thus

$$B_{12} \rho(v) N_1 = A_{21} N_2 + B_{21} \rho(v) N_2$$

$$\rho(v) [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$$

$$\rho(v) = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

$$\rho(v) = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\frac{N_1}{N_2} \frac{B_{12}}{B_{21}} - 1} \right]$$

Einstein proved thermodynamically that the probability of absorption is equal to the probability of stimulated emission.

$$\text{i.e. } B_{12} = B_{21} \quad \dots(3.5)$$

Then we have

$$\rho = \frac{A_{21}}{B_{21}} \frac{1}{\frac{N_1}{N_2} - 1}$$

The equilibrium distribution of atoms among different energy states is given by Boltzmann's law according to which

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}}$$

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

$$= e^{-(hv/kT)}$$

$$\therefore \rho = \frac{A_{21}}{B_{21}} \left[ \frac{1}{e^{hv/kT} - 1} \right] \quad \dots (3.6)$$

This is the formula for the energy density of photon of frequency  $v$  in equilibrium with atoms in energy states 1 and 2, at temperature T.

Comparing it with Planck's radiation formula

$$\rho(v) = \frac{8\pi hv^3}{c^3} \left[ \frac{1}{e^{hv/kT} - 1} \right] \quad \dots (3.7)$$

From equations 3.6 and 3.7 we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi hv^3}{c^3} \quad \dots (3.8)$$

Equation 3.5 and 3.8 are called Einstein's relations and it gives the relationship between Einstein's A and B coefficients.

### Significance of Einstein's Coefficients

- Einstein coefficients  $A_{21}$ ,  $B_{21}$  and  $B_{12}$  are all interrelated. If one is known, by measurement or calculation, all are known.

2. The stimulated emission coefficient  $B_{21}$  and the absorption coefficient  $B_{12}$ , are equal, at least for the case of non degenerate energy states. The rates  $R_{st} = N_2 B_{21} \rho(v)$  and  $R_{sp} = N_1 B_{12} \rho(v)$  differ depending upon the population densities  $N_2$  and  $N_1$ . If  $N_2$  is greater than  $N_1$  and a radiation field interacts with the atoms, stimulated emission exceeds absorption and photons will be added to the field.

If  $N_1$  is greater than  $N_2$ , absorption exceeds stimulated emission and photons will be removed from the field.

$N_2 > N_1$  leads to increase in  $\rho(v)$  and hence, amplification.

$N_1 > N_2$  leads to decrease in  $\rho(v)$  and hence, attenuation.

For laser to operate, it is necessary that  $N_2 > N_1$ . This is the condition of population inversion. Without population inversion laser action cannot occur.

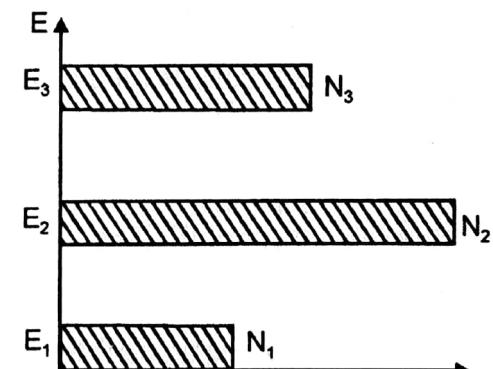
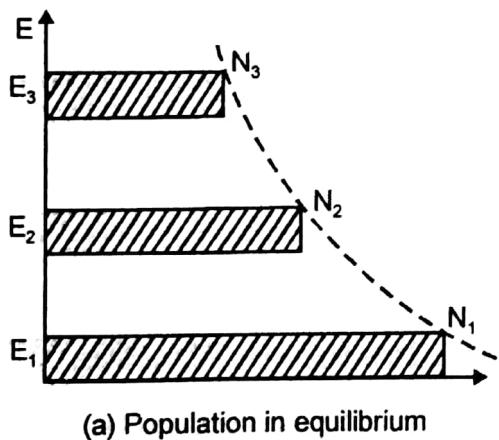
3. Since  $B_{21}/A_{21}$  is proportional to the reciprocal of the cube of the frequency  $v$ , the higher the frequency (shorter the wavelength) the smaller  $B_{21}$  becomes in comparison with  $A_{21}$ . Since  $B_{21}$  is related to stimulated emission and  $A_{21}$  is related to spontaneous emission it would seem that lasers of short wavelength radiation would be more difficult to build and operate.
4. Although the relations  $A_{21}$ ,  $B_{21}$  and  $B_{12}$  were derived based on the condition of thermal equilibrium, they are valid and hold under any condition. The laser while operating is hardly an enclosure in thermodynamic equilibrium. Yet A and B coefficient relationships hold good because they are characteristic of the atom, are equally valid whether the atom is in the intense radiation field of a laser cavity or in a hot furnace that can be treated as a blackbody in thermodynamic equilibrium. So, two important ideas emerge from a review of Einstein's study of the interaction of electromagnetic radiation with matter which are useful for the successful operation of laser.
- i) Stimulated emission, that leads to light amplification.
  - ii) Population inversion of atoms in energy levels must be achieved if the stimulated emission process producing coherent photons is to outrival the absorption process removing photons.

### 3.5 Population Inversion

When an atomic system is in equilibrium, absorption and spontaneous emission take place side by side. But, because  $N_2 < N_1$ , absorption dominates. However laser operation requires obtaining stimulated emission exclusively. To achieve a high percentage of stimulated emission, a majority of the atoms should be at the higher energy level than at the lower level. The non equilibrium state in which the population  $N_2$  of the upper energy level exceeds the population  $N_1$  of the lower energy level is known as population inversion.

Consider a system that has three energy states  $E_1$ ,  $E_2$  and  $E_3$ . With the system in equilibrium, the uppermost level  $E_3$  is populated least and lowest level  $E_1$  is populated most.

The dotted curve shown in Figure 3.7(a) represents a normal Boltzmann distribution. Since the population in three states is such that  $N_3 < N_2 < N_1$ , the system absorbs photons rather than emitting photons. However if the system is supplied with external energy such that  $N_2$  exceeds  $N_1$ , we say that the system has reached population inversion.



**Figure 3.7**

Under the population inversion condition, the stimulated emission can produce a cascade of light.

### 3.6 Pumping

In order to realize and maintain the state of population inversion, it is necessary that atoms must be continuously promoted from the lower level to the higher level. So, energy is supplied by some means to the laser medium to raise atoms from the lower level to the excited level, thus maintaining population at the excited level at a value greater than that of the lower level.

The process by which atoms are raised from the lower level to the upper level is called pumping.

### **Methods of Pumping**

There are several ways of pumping a laser and producing the population inversion necessary for stimulated emission to occur. The most commonly used methods are discussed here.

#### **a) Optical Pumping**

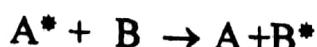
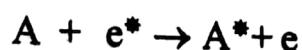
In optical pumping, a light source is used to supply luminous energy. This energy comes in the form of short flashes of light. For example in Ruby laser a light source such as helical xenon flash lamp is used to illuminate the laser medium and the photons of appropriate frequency excite the atoms to an uppermost level. Optical pumping is suitable for any laser medium which is transparent to pump light. Example: Ruby laser, Nd:YAG laser.

#### **b) Electrical Discharge**

This method is preferred in gases. In the case of gas lasers a high voltage ( $\text{KV m}^{-1}$ ) causes the electrons emitted by the cathode to be accelerated towards the anode. Some of these electrons will collide on the atoms of the active medium, ionize the medium and raise it to the excited state, thus producing the population inversion needed. Example:  $\text{CO}_2$  laser.

#### **c) Inelastic atom-atom collisions**

In this type of pumping excitation by electric discharge still provides the initial excitation which raises one type of atoms to their excited state. These atoms then collide inelastically with another type of atoms and it is these latter atoms that provide the population inversion needed for laser emission.



Example: He - Ne laser.

#### **d) Direct Conversion**

In the case of semiconductor lasers it is not the atoms that are excited. It is the current carriers namely electrons and holes which are excited and a population inversion is achieved in the junction region. The electrons recombine with holes in the junction regions producing laser light. Thus in semiconductor lasers, a direct conversion of electrical energy into light energy takes place.

### 3.6.1 Active Medium

An active medium is a medium, which when excited, reaches the state of population inversion, and eventually causes light amplification. The active medium may be solid, a liquid or a gas.

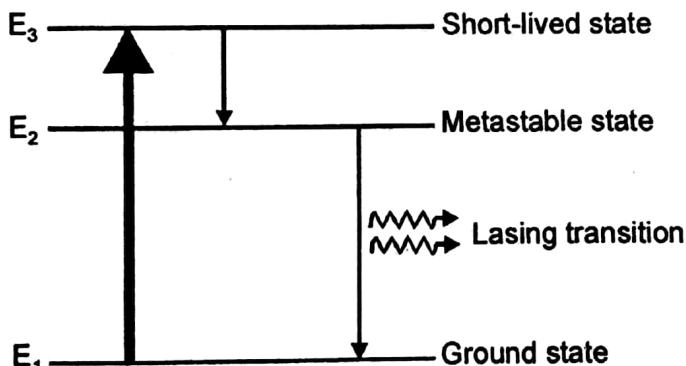
### 3.6.2 Two Level Pumping Scheme

It seems that the simple and straight forward method to achieve population inversion is to pump and maintain excess of atoms into the excited state by applying intense radiation. But basically a two level pumping scheme is not suitable for attaining population inversion.

This is due to two basic reasons.

1. It is difficult to keep a collection of atoms in their excited states until they are stimulated to emit photon.
2. The atoms that happen to be in their ground state will undergo absorption and will thus remove photons from the beam as it builds up.

### 3.6.3 Three Level Pumping Scheme



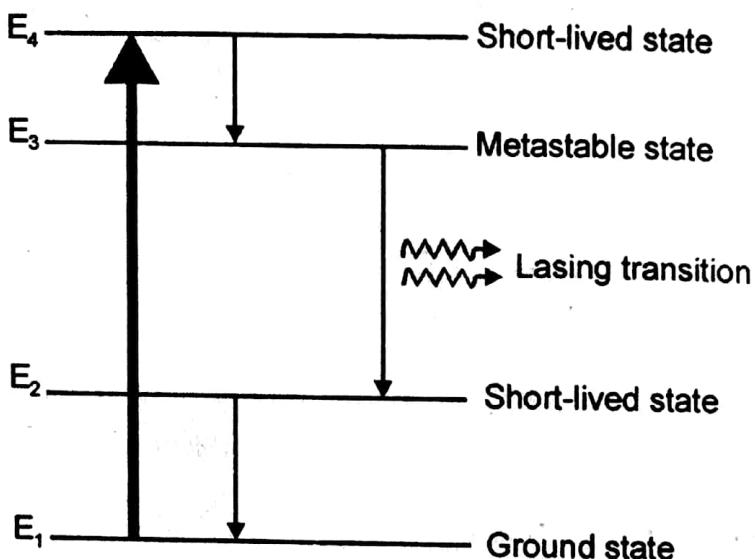
**Figure 3.8:(a)** A three-level laser

In the three level pumping scheme the atoms originally in the ground state are pumped into the excited state by some external source of energy (an electric pulse or a flash of light). The excited atoms decay by spontaneous emission very rapidly into a lower excited state which is a metastable state.

Atoms stay in metastable state for about  $10^{-6}$  to  $10^{-3}$  s. Therefore it is possible for a large number of atoms to accumulate at a metastable level. In the metastable state population can exceed the population of a lower level and lead to the state of population inversion. If the metastable state does not exist, there could be no population inversion, no

stimulated emission and hence no laser operation. This system solves the first problem arose with the two level laser i.e., placing the collection of atoms in their excited states. But it does not solve the second problem i.e. any atom in the ground state will absorb the lasing transition and remove photons from the beam. Three level scheme requires high pump powers and can produce the light only in pulses.

### 3.6.4 Four Level Pumping Scheme

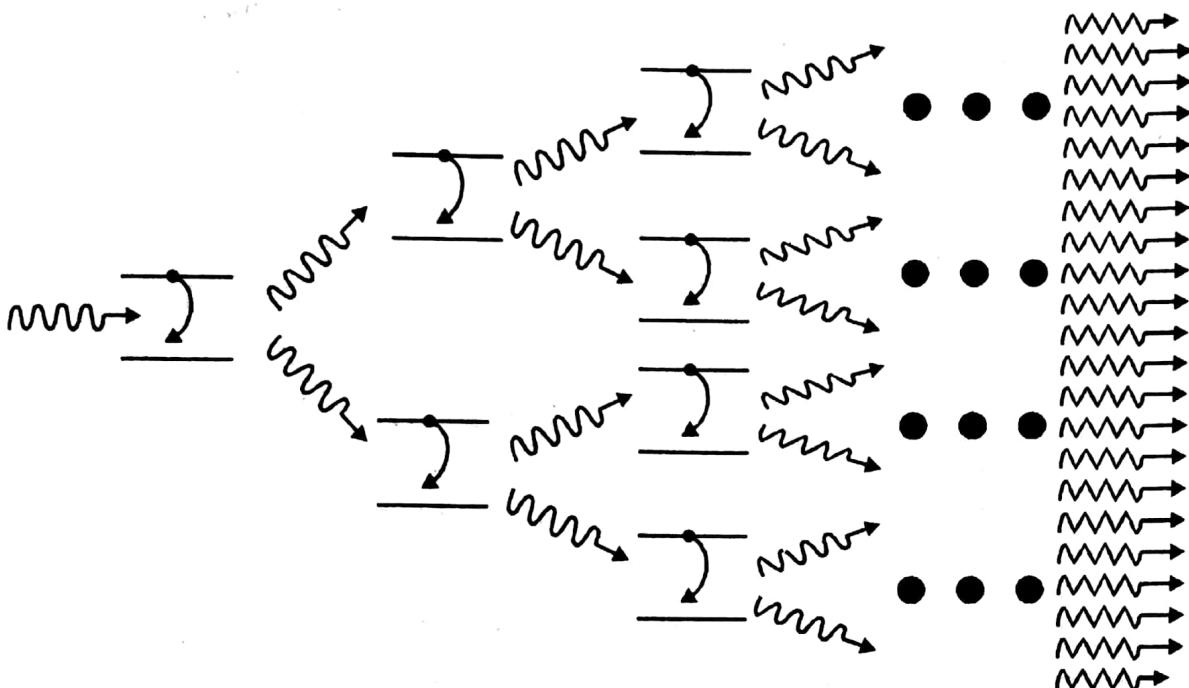


**Figure 3.8:(b) Four-level laser**

Four level laser relieves the difficulty. The ground state atoms are pumped to the excited state and decay rapidly to the metastable state as in the three level laser. The lasing transition proceeds from the metastable state ( $E_3$ ) to  $E_2$ , a short lived state from where the atoms decay rapidly to the ground state. The atoms in the ground state cannot absorb at the energy of the lasing transition. Four level laser requires less pumping energy than three level laser and can operate in a continuous mode.

### Principle of Laser

The outstanding feature of this process is the multiplication of photons. For one photon hitting an excited atom, there are two photons emerging. The two photons are in phase and travel along the same direction. These two photons stimulate two excited atoms in their path and produce four photons which are in phase and travel along the same direction. These four photons can in turn stimulate four excited atoms and generate eight photons and so on.



**Figure 3.9:** Buildup of intense beam in a laser. Each emitted photon interacts with an excited atom and produce two photons. Multiplication of stimulated photons.

### Components required for laser action

A laser requires three components for operation

1. Energy source
2. Active medium
3. Optical cavity

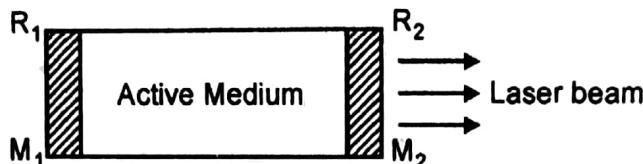
The energy source that will raise the system to an excited state. Next there is an active medium that, when excited, achieves population inversion and subsequently lasers. Third in most cases there is an optical cavity that provides the feedback necessary for laser oscillation. The optical cavity is formed by two mirrors facing each other. One of the mirrors is coated to 100% reflectance, the other mirror is partially transparent to let some of the radiation pass through.

### 3.7 Gain Coefficient - Threshold Gain Coefficient

The gain of a laser depends on several factors. Foremost among them is the separation of energy levels that produce laser transition. If the two levels are too apart, the gain is higher because the laser transition energy is a larger fraction of the pump transition energy. If the two levels are closer together, the gain is lower.

To ensure laser oscillations the population inversion and hence the gain within the medium must be large enough to overcome various losses, including the loss of energy in the laser output, which may exist in the system. Hence there is a minimum or threshold gain coefficient which is required to initiate and sustain laser oscillations.

Let us consider a laser medium of length  $L$  enclosed between the two mirrors  $M_1$  and  $M_2$ , which have their reflectances  $R_1$  and  $R_2$ , respectively.



**Figure 3.10:** Laser medium

In any laser oscillator initially a few photons would be emitted spontaneously and these photons start the stimulated emission process. Let photons due to stimulated emission constitute initial irradiance represented by  $I_0$ .

According to exponential law, whenever a beam passes through a medium, there will be some absorption of power in that medium. Hence the irradiance of the light when it comes out, decreases. Here the initial irradiance after traversing the distance ' $L$ ' is

$$I_1 = I_0 \exp(-\alpha L) \quad \dots(3.9)$$

where  $\alpha$  is absorption coefficient. Since the exponential law is concerned with the situation where the system is in thermal equilibrium,  $N_1$  will be greater than  $N_2$  and hence absorption dominates. Hence the exponent ( $\alpha L$ ) in the above equation is negative and absorption coefficient  $\alpha$  is positive.

But laser oscillator incorporates pumping energy. Hence population inversion will be maintained.

$$\therefore N_2 > N_1$$

Hence, here amplification dominates and therefore, exponent is positive and absorption coefficient ( $\alpha$ ) is negative. Indeed laser emission may be considered negative absorption.

Hence in a laser oscillator, irradiance of the beam will grow exponentially in accordance with the equation.

$$I_1 = I_0 \exp(kL) \quad \dots(3.10)$$

where  $k$  is called small signal gain coefficient ( $k = -\alpha$ )

But to sustain laser oscillations, the gain coefficient must be atleast large enough to overcome the losses in the system.

The sources of loss include the following:

1. Transmission at the mirrors, that is the useful output (one of the mirrors is usually made as reflective as possible while the other, the output mirror, may have a reflectance of 90 %)
2. Absorption and scattering by the mirrors.
3. Diffraction around the boundary of the mirrors.
4. Absorption in the laser medium due to transitions other than the desired one.
5. Scattering at optical inhomogeneity in the laser medium.

To simplify matters let us include those losses, other than the useful output and mirror losses, in a single effective volume loss coefficient  $\gamma$ . This reduces the effective gain coefficient to  $(k - \gamma)$

Equation (3.10) becomes

$$I_1 = I_0 \exp [ (k - \gamma) L ] \quad \dots (3.11)$$

After reflection at  $M_2$ , the beam irradiance will be  $I_0 R_2 \exp [ (k - \gamma) L ]$

and when the beam comes to the end, it will have grown to  $I_0 R_2 \exp [ 2(k - \gamma) L ]$ .

After reflection at  $M_1$ , the irradiance will be  $I_0 R_1 R_2 \exp [ 2(k - \gamma) L ]$ .

We can determine the threshold gain coefficient required from the condition that the round trip gain,  $G$ , in the irradiance of the beam must be atleast unity.

If  $G$  were less than this the oscillations would die out. Whereas if  $G$  were greater than unity oscillations would grow.

$$\text{The Round trip gain } G = \frac{\text{Final irradiance}}{\text{Initial irradiance}}$$

$$\therefore G = \frac{I_0 R_1 R_2 \exp [ 2(k - \gamma) L ]}{I_0}$$

$$G = R_1 R_2 \exp [ 2(k - \gamma) L ] \quad \dots (3.12)$$

Thus the threshold condition for laser oscillation is

$$R_1 R_2 \exp [2(k_{th} - \gamma)L] = 1$$

$$\ln R_1 R_2 2(k_{th} - \gamma)L = \ln 1$$

$$2k_{th}L = 2\gamma L + \ln \left( \frac{1}{R_1 R_2} \right)$$

$$K_{th} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \quad \dots (3.13)$$

The first term in the above equation represents the volume losses, while the second is the loss in the form of the useful output.

Thus the condition for steady state laser operation is that gain equals the sum of losses. In lasers designed for continuous output the gain becomes constant at the threshold value. This is because if the round trip gain were less than or greater than unity the cavity energy would correspondingly increase or decrease. It is only when G has been equal to unity for a while that the cavity energy (and hence laser output) settles down to a steady state value. This phenomenon is referred to as gain saturation.

### 3.8 Different kinds of Lasers

Solid state lasers, Gas lasers, Liquid lasers, Dye Lasers and Semiconductor lasers are the important kinds of lasers.

Solid state lasers : Ruby laser, Nd : YAG laser

Gas lasers : He-Ne laser, CO<sub>2</sub> laser, Argon-ion laser, Copper-vapour laser

Liquid lasers : Se OCl<sub>2</sub> laser, Europium chelate laser

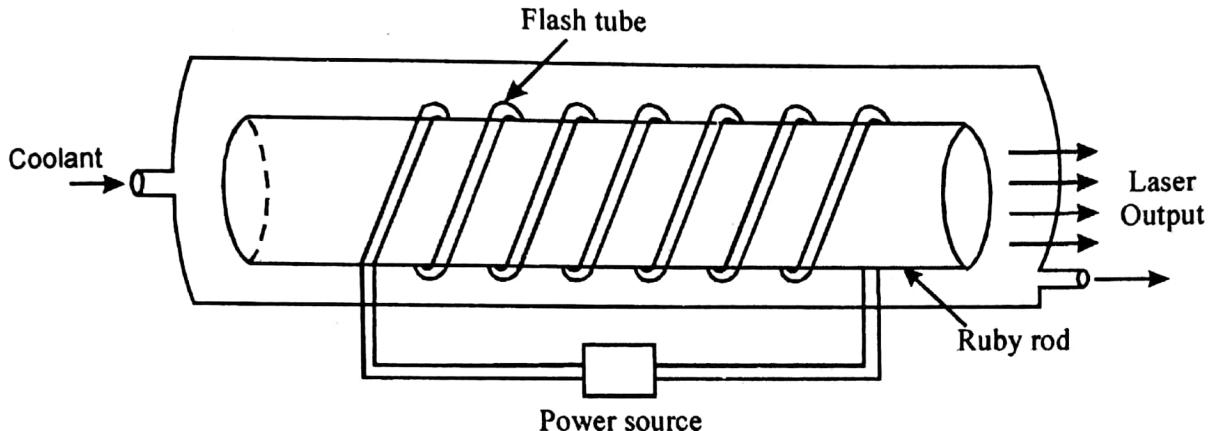
Dye lasers : Rhodamine 6G laser, Coumarin dye laser

Semiconductor lasers : Ga As laser, In P laser.

### 3.9 Ruby Laser

Ruby laser is a three level solid state laser and was constructed by Maiman. It is a pulsed laser having very high power of hundreds of Mega watt in a single pulse with

10 nanosecond duration. It is used for various industrial applications like surface hardening, cladding of various industrial products, etc. Recently erbium ( $\text{Er}^{3+}$ ) doped ruby lasers are available and have higher merits than ordinary chromium ( $\text{Cr}^{3+}$ ) doped ruby laser.



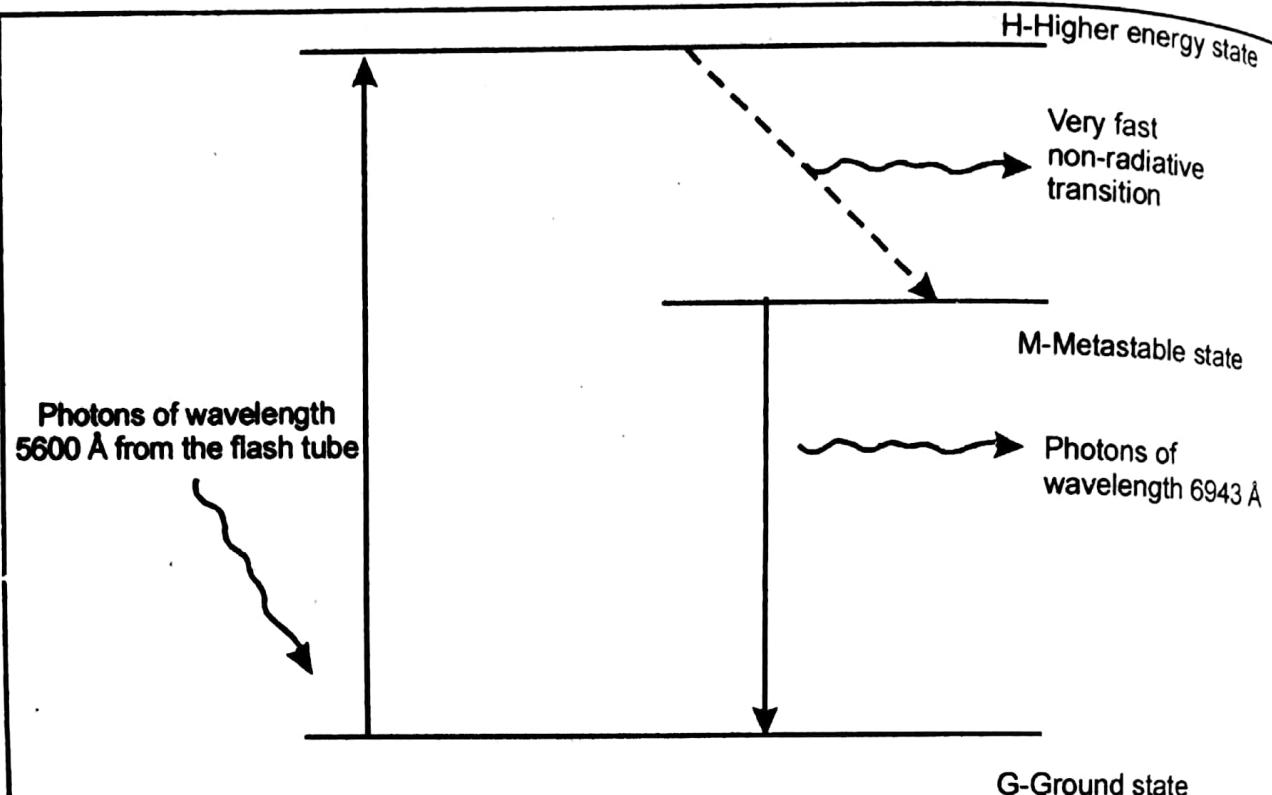
**Figure 3.11:** Ruby laser

### Construction

In ruby laser the active element is pure aluminium oxide ( $\text{Al}_2\text{O}_3$ ) with 0.05% of chromium and it is shaped into a cylinder with parallel transparent sides and reflectors at both ends. Among the two end reflectors, one is a perfect reflector and other is a partial reflector. The diameter of the active element is about 0.5 cm and its length is about few centimetres. Pumping source is a flash lamp. Chromium atoms are particularly responsive to light having a wavelength 5600 Å. Most flash lamps like Xenon flash lamps are able to supply energy in this wavelength range. Flash lamp tube is spirally wound over the curved surface of the ruby rod and is connected to a power supply as shown in figure 3.11. The emitted wavelength of radiation is 6943 Å.

### Working

- Once chromium atoms have been excited to an upper energy level 'H' by absorbing light photons of wavelength 5600 Å from the flash lamp they require two steps to return to their ground state 'G'. First step is from 'H' state to metastable state 'M' which is a shorter jump and energy emitted in this transition is passed to the crystal lattice as heat. The energy is not radiated in the form of photons and this transition is called "*radiationless transition*" (Figure 3.12).



**Figure 3.12: Three levels in Ruby Laser “Transition”**

- The chromium atoms returned to M level can remain in this state for several milliseconds. The accumulation of excited atoms at M level increases the population at M level and then transition occurs from M to G level emitting out some photons by spontaneous emission initially in a random manner.
- Due to continuous working of flash lamp, the chromium atoms are continuously raised to higher energy state ‘H’ and then to M level.
- At a particular stage, population of excited chromium atoms is more at M than at G. Hence there is **population inversion**. The emitted photons of wavelength 6943 Å stimulate or induce the chromium atoms at level M to undergo transition. This results in stimulated emission of other identical photons and a cascade begins.
- The photons travelling parallel to the axis of the ruby rod are used for stimulation while the photons travelling in other directions will pass out from the ruby rod. In the mean time, the photons undergo multiple reflections from the mirrors placed at the ends of the ruby rod and the intensity of the laser radiation grows to a higher value and subsequently, comes out through the partial reflector and it serves as output laser beam.
- The emitted photon and stimulating photon are in phase and have same frequency and are travelling in the same direction. Thus the laser beam has directionality along with spatial and temporal coherence.

[Note: Metastable state 'M' consists of two closed sublevels. There is another laser transition between one of these closed sub-levels and ground state giving an emission line at a wavelength 6927 Å. But this transition probability is very small].

### 3.10 Neodymium Laser

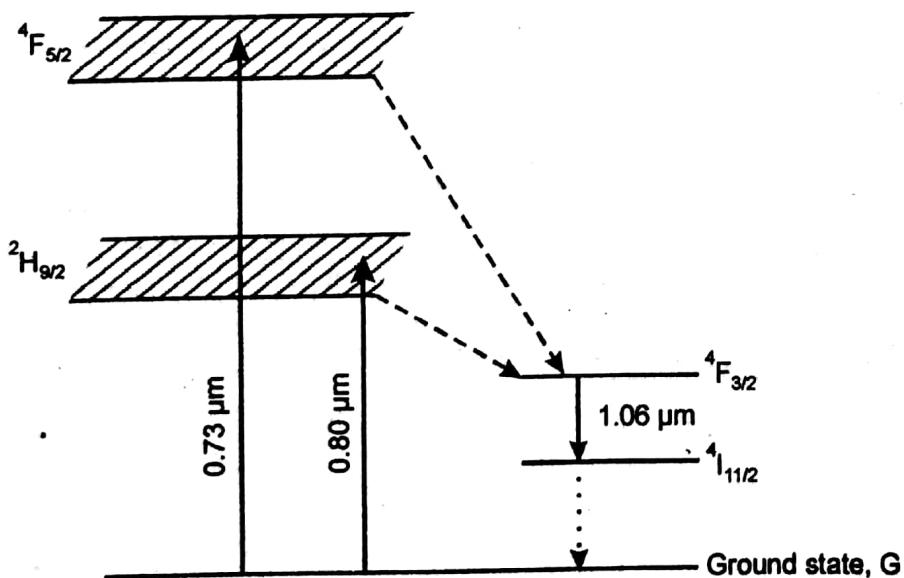
This is a rare earth laser system. There are two types of Neodymium lasers

- i) Nd : YAG laser
- ii) Nd : glass laser

These are four level solid state lasers with high power pulses having shorter pulse width and high repetition rate. These can give continuous power output also. Among these, Nd : YAG lasers are widely used in the industries for cutting, welding, drilling and surface hardening of the industrial products. In medicine it is used to treat gastrointestinal bleeding and to do intracular eye surgeries. Further Nd glass lasers are used to produce nuclear fusion process.

#### Construction of Nd : YAG laser

The construction of Nd : YAG laser is similar to Ruby laser.



**Figure 3.13:** Energy levels of  $\text{Nd}^{3+}$  YAG laser

In Yttrium Aluminium Garnet (YAG -  $\text{Y}_3\text{Al}_5\text{O}_{12}$ ) crystal which acts as the active element, yttrium ions ( $\text{Y}^{3+}$ ) are replaced by Neodymium ions ( $\text{Nd}^{3+}$ ). Neodymium atoms are pumped by a krypton flash lamp, which has krypton at high pressure or a xenon lamp, which has xenon at moderate pressures. Nd : YAG is shaped into a cylinder with parallel transparent sides and reflectors at both ends.

### Action (Figure 3.13)

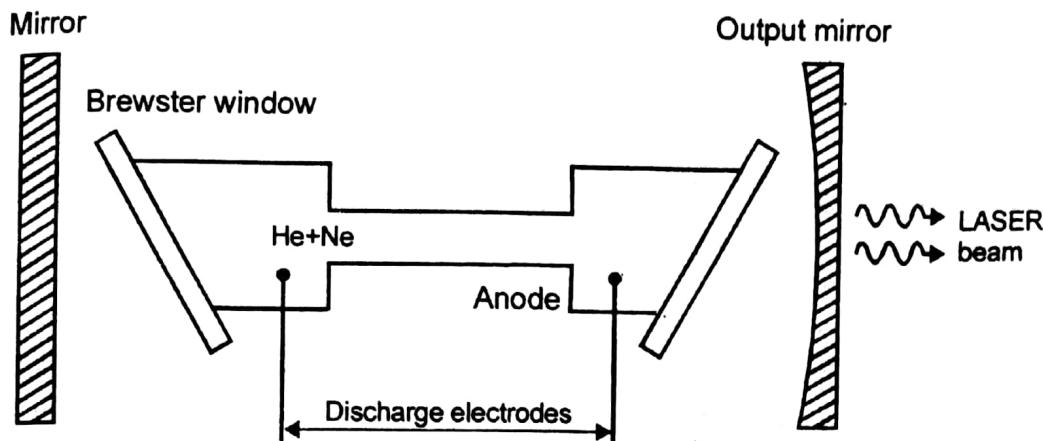
1. The Neodymium atoms are pumped by a krypton flash lamp, from the ground state 'G' to the higher energy levels  $^4\text{F}_{5/2}$  and  $^2\text{H}_{9/2}$  by absorbing radiations of wavelengths  $0.73 \mu\text{m}$  and  $0.80 \mu\text{m}$  respectively.
2. From these energy levels the atoms are transferred to  $^4\text{F}_{3/2}$  which is a metastable state by radiationless transitions.
3. Population inversion takes place with respect  $^4\text{F}_{3/2}$  which is virtually empty at room temperature.
4. Stimulated emission takes place between  $^4\text{F}_{3/2}$  and  $^4\text{I}_{11/2}$  giving a laser radiation of wavelength,  $1.06 \mu\text{m}$ .
5. The energy  $^4\text{I}_{11/2}$  is about  $0.25 \text{ eV}$  above the ground state and the transition of atoms from  $^4\text{I}_{11/2}$  to ground state takes place by non-radiative processes and is quite fast.

### 3.11 He-Ne Laser

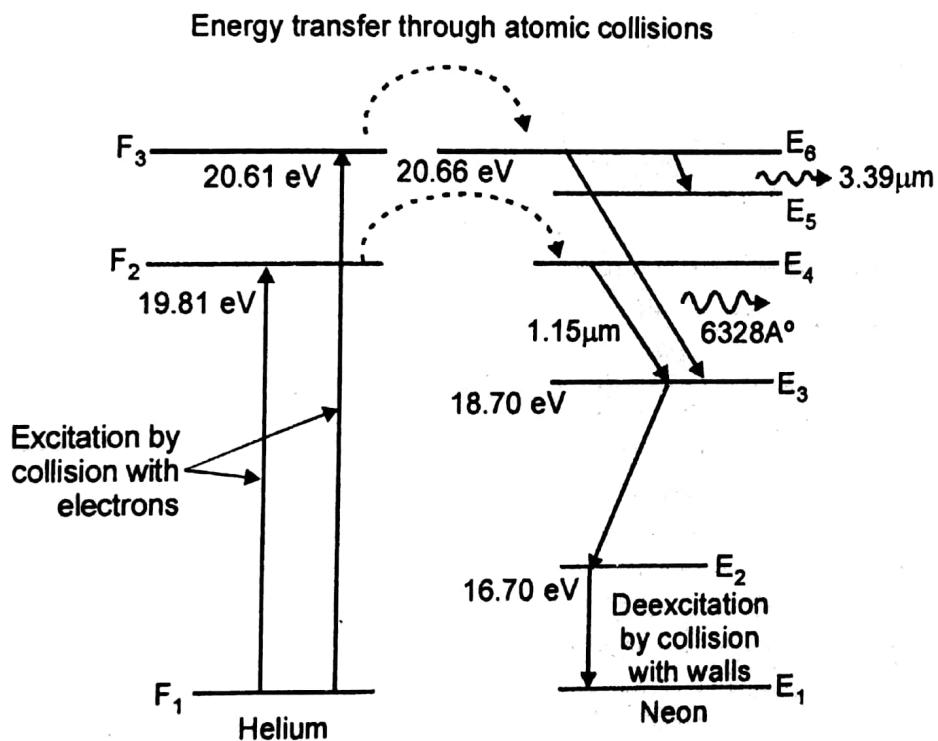
The first successful demonstration of a gas laser was done by Ali Javan in 1961. Helium - Neon is an atomic laser which employs four - level pumping scheme. The active medium is a mixture of 10 parts of Helium and one part of Neon. Neon atoms are active centres and have the energy levels suitable for laser transitions while helium atoms help efficient excitation of neon atoms.

The mixture is kept at a pressure of 10 torr in a pyrex tube of 25 cm in length and 2 mm in diameter. Since the cavity windows are kept outside the tube, Brewster windows may be used at the ends of the tube to minimize reflection losses. This will also result in a plane polarized output.

Voltages of a few kilovolts applied across the tube to initiate a gas discharge.



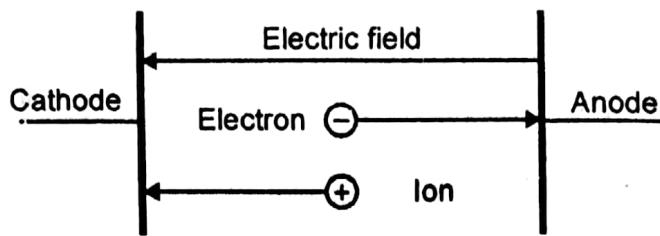
**Figure 3.14:** Schematic diagram of He-Ne laser



**Figure 3.15:** Energy level diagram of He-Ne laser

### Working

When the power is switched on, a high voltage of 10 kV is applied across the gas. It is sufficient to ionize the gas. The electrons and ions produced in the process of discharge are accelerated towards the anode and cathode respectively.



Since electrons have a smaller mass, they acquire a higher velocity. They transfer their kinetic energy to helium atoms through inelastic atom-atom collisions. Helium atoms are much more readily excited by electrons impact because they are fairly light. Thus the initial excitation affects only the helium atoms with the result that these atoms are excited to the levels  $F_2$  and  $F_3$ , which lie at 19.81 eV and 20.61 eV respectively above the ground state. These levels are metastable levels and the excited helium atoms cannot return to the ground state through spontaneous emission. Here radiative transition is not possible. The excited helium atom can return to the normal state by transferring their energy to neon atoms through collision. Such an energy transfer can take place when two colliding atoms have identical energy states. It is called resonant transfer of energy. The neon energy levels namely  $E_6$  and  $E_4$  nearly coincide with  $F_3$  and  $F_2$  levels of helium atom. Therefore the resonant transfer of energy can occur readily.

The additional 0.05 eV energy required is provided by the kinetic energy of helium atoms. When a helium atom in the metastable state collides with a Neon atom in the ground state, the neon atom is excited to  $E_4$  or  $E_6$  level. Now the helium atom drops back to the ground state. This is the pumping mechanism of He-Ne laser. Thus the role of the helium atoms is to excite neon atoms and cause population inversion.

Occasionally a neon atom in the  $E_6$  and  $E_4$  level emit a photon parallel to the axis of the tube. This photon will cause stimulated emission by other atoms and a beam of coherent radiation will eventually build up traveling along the tube axis.

In reality the neon energy levels  $E_6$ ,  $E_5$ ,  $E_4$ ,  $E_3$  and  $E_2$  are not single but a group of lines. Consequently several laser transitions are possible. However the following three are main laser transitions:

- i)  $E_6 \rightarrow E_3$  : This transition generates a laser beam of red colour at 6328 Å.
- ii)  $E_4 \rightarrow E_3$  : This produces IR beam at a wavelength of 1.15 μm.
- iii)  $E_6 \rightarrow E_5$  : It generates light in far IR region at 3.39 μm.

The level  $E_2$  is a metastable state. The neon atoms tend to accumulate at this level if they are not somehow removed from the level.  $E_2 \rightarrow E_1$  transition can be induced by collisions with the walls of the discharge tube.

### Applications of He - Ne laser

- a) Laser Printing
- b) Bar-code reading
- c) Interferometry

## 3.12 CO<sub>2</sub> Laser

The CO<sub>2</sub> laser is the most important laser of its class and in terms of technological applications it unquestionably ranks first. It exhibits both high efficiency (upto 40%) and high power output. CO<sub>2</sub> laser is an example of molecular gas laser and it was designed by CKN Patel.

### Principle

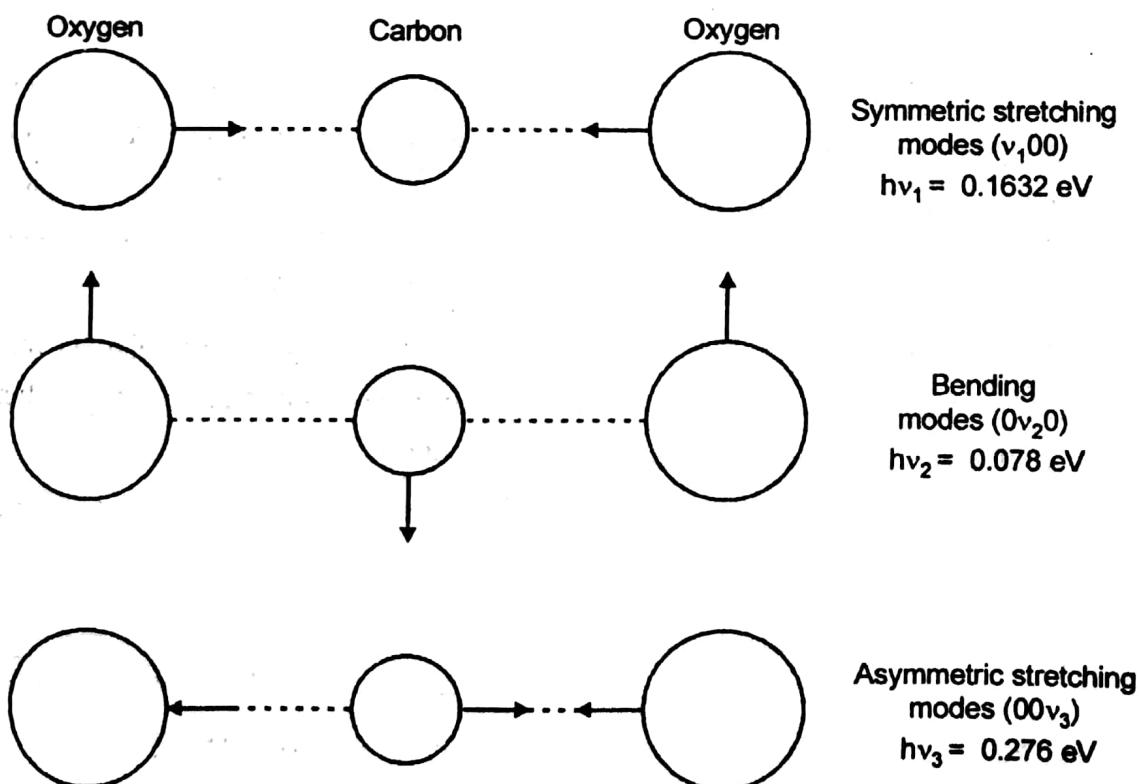


Figure 3.16: Vibrational modes of CO<sub>2</sub> molecule

In the case of isolated atom the electron energy levels are discrete and narrow. On the other hand energy spectrum of molecules is complex. Each electron level is associated with a number of equispaced vibrational levels and each vibrational level in turn has a number of rotational levels.

The  $\text{CO}_2$  molecule is basically a linear molecule consisting of two oxygen atoms and a carbon atom between them. It can undergo three fundamental vibrational oscillations which are called vibrational modes. They are symmetric stretching mode, bending mode, asymmetric stretching mode.

In the symmetric stretching mode, the oxygen atoms oscillate along the axis of the molecule by simultaneously departing and approaching the carbon atom in between.

In the bending mode, all the three atoms of the molecule undergo vibrational oscillations by moving perpendicularly to the molecular axis.

In the asymmetric stretching mode, the oxygen atoms also move along the axis of the molecule but both in one direction relative to the carbon atom itself moving to the other side.

We denote the frequencies of the symmetric stretching, bending and asymmetric stretching modes by  $v_1$ ,  $v_2$  and  $v_3$  respectively. The energy contents of these vibrations are

$$\hbar v_1 = 0.163 \text{ eV}, \hbar v_2 = 0.078 \text{ eV}, \hbar v_3 = 0.276 \text{ eV}$$

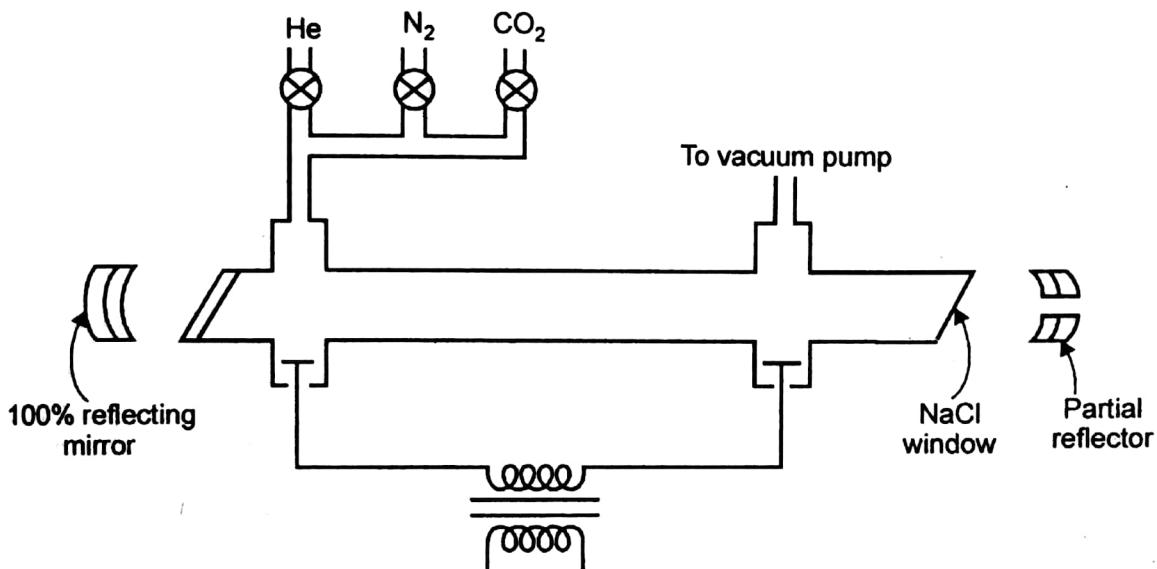
According to quantum theory, the energy of oscillation of a molecule in any one mode can have only discrete values, which are all integral multiples of some fundamental value. At any one time, a  $\text{CO}_2$  molecule can vibrate in a linear combination of the three fundamental modes.

The energy state of the molecule can be represented by three numbers (ijk). These numbers represent the amount of energy or the number of energy quanta associated with each mode. For example the number (030) means that the molecule in this energy state is in pure bending mode with three units of energy. Each vibrational state is associated with rotational states corresponding to rotation of the molecule about its centre of mass. The separations between vibrational rotational states are usually much smaller on the energy scale than separation between electronic states.

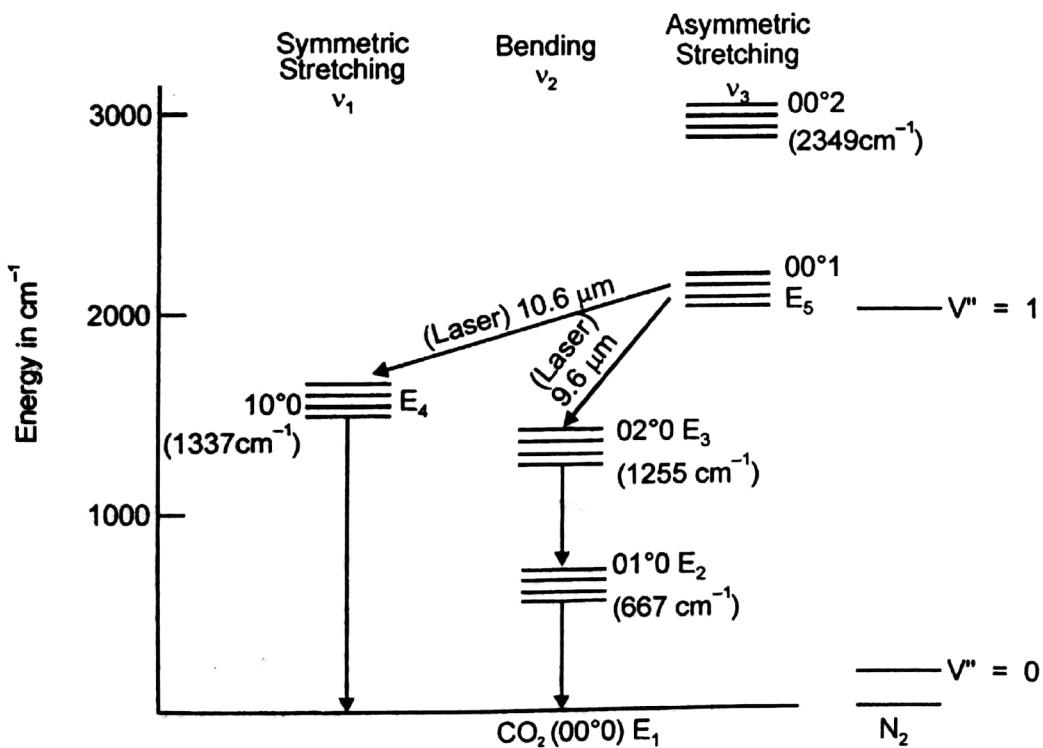
### **Construction**

The schematic diagram of a typical  $\text{CO}_2$  laser is shown in figure. The discharge tube is 3.5 cm in diameter and 5m in length and discharge is produced by d.c. excitation. Sodium chloride Brewster windows are used at the end. The confocal silicon mirrors are

coated with aluminium, forming the resonant cavity. The discharge tube is filled with a mixture of carbondioxide, nitrogen and helium gases in 1:4:5 proportions with a pressure of 0.33 torr, 1.2 torr and 7 torr respectively. A high dc voltage causes an electric discharge to pass through the tube. The discharge breaks down  $\text{CO}_2$  molecules to form oxygen and carbon mono oxide. Therefore a small amount of water vapour is added to the gaseous mixture which regenerates  $\text{CO}_2$ .



**Figure 3.17:(a)** Schematic diagram of  $\text{CO}_2$  laser



**Figure 3.17:(b)** Energy level of  $\text{CO}_2$  laser

### Working

In the CO<sub>2</sub> laser nitrogen plays the same role as that of Helium in a He-Ne laser. The nitrogen molecules go to the excited state by collision with electrons.

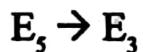
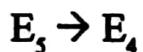


The excited nitrogen atom transfers its energy to the CO<sub>2</sub> molecule and the CO<sub>2</sub> molecule gets excited.



The lowest vibrational level of nitrogen has nearly as much energy as the symmetric stretching mode of CO<sub>2</sub> molecule and so the excited nitrogen molecules readily transfer energy to the CO<sub>2</sub> molecules in resonant collisions.

As a result, CO<sub>2</sub> molecules are excited to (001) E<sub>5</sub> level. The (100) E<sub>4</sub> level in CO<sub>2</sub> has a lower energy and cannot be populated. So, population inversion is established between (001) E<sub>5</sub> level and (100) E<sub>4</sub> level. Simultaneously in the same manner population inversion is created between (001) E<sub>5</sub> level and (020) E<sub>3</sub> level.



Lasing transitions produce IR radiation at wavelength 10.6 μm and 9.6 μm respectively. E<sub>5</sub> and E<sub>4</sub> levels are metastable states. The CO<sub>2</sub> molecules at these levels fall to the lower bending level E<sub>3</sub> through inelastic collisions with helium atoms.

Helium has a high thermal conductivity and acts as a coolant by transporting waste heat to the tube walls and by assisting in the deactivation of the lower level.

### Applications of CO<sub>2</sub> laser

CO<sub>2</sub> laser is mainly useful in the material processing such as drilling, cutting, welding, alloying etc. In medical science it is used for cauterising and cutting.

### 3.13 Excimer Laser

They also belong to a family of gas lasers that produce nanosecond long powerful pulses at UV region (308, 248 or 193 nm) of electromagnetic spectrum. These lasers use mixture of gases. The interest in excimer lasers principally of heavy noble gases (Xe, Kr, Ar) and the halogens (F, Cl, Br, I) is due to the relatively efficient production of their excited state by electron beam collisions and the fact that their emission wavelengths lie in

the ultraviolet ( $0.2 < \lambda < 0.4 \mu\text{m}$ ) region of the spectrum, a region not covered well by other types of lasers. The important excimer laser gases are krypton fluoride, xenon fluoride, argon fluoride and xenon chloride. It can generate billion watt of power pulses.

### 3.14 Dye Laser

Dye lasers, as the name suggests, use liquid organic dyes. Liquid dye laser beam covers a wide range of wavelengths and thus have the great advantage of being tunable. The user can select a fine tuned wavelength as required. Liquid dye lasers can emit laser beams from 250 nm (UV) through the whole visible spectrum to 1800 nm (IR). In the liquid dye laser, the dye is the active medium.

It is usually dissolved in a liquid solvent such as alcohol or ethylene glycol (antifreeze). The source of energy for a liquid dye laser is usually a flash lamp or another laser like argon or a krypton ion laser. An example of a liquid dye laser is a rhodamine 6G laser. The emitted light from the rhodamine 6G laser is having a broad range of frequencies from 570 to 665 nm which includes the orange colour light of 590 nm.

This laser is tunable using optical lenses and prisms to select the desired wavelength. This capability is of great value in certain types of spectroscopic investigations in and near visible wavelengths. Liquid dye lasers are used for investigation process in chemical reactions.

### 3.15 Free Electron Laser

The conventional microwave sources produce waves at wavelength  $\lambda \geq 1 \text{ cm}$  and the different types of lasers operate at infrared and optical wavelengths. Hence, there remained a big void of sources in the range  $1 \text{ cm} \geq \lambda \geq 30 \text{ mm}$ .

Free electron laser emerged as versatile device of megawatt power that filled the entire void and proved to be suitable for satellite communications, precision radar and plasma heating in thermonuclear fusion device. FEL also produces tunable radiation at infrared, optical wavelengths and even much shorter wavelengths upto X-rays.

### 3.16 Chemical Laser

Some lasers are stimulated by chemical reactions instead of an outside source of energy. Chemical lasers most often utilize gases as active medium and the end products of the reaction are excited energy states that are capable of emitting photons. Some chemical lasers can produce pulses of energy as enormous as 200 gigawatts. ( $\sim 10^{12}$  i.e., Billion watt).

Among the most important chemical lasers are systems that utilize hydrogen fluoride (HF) and deuterium fluoride (DF) as the active laser molecule.

These devices offer several attractive features, including both CW and pulsed operation; fairly large output powers; shorter IR lasing wavelengths ( $3\text{-}4 \mu\text{m}$ ). Chemical lasers are becoming increasingly important as a research tool and probably will be available in the commercial market in the near future.

### 3.17 Laser Welding

In the basic welding process two metals are placed in contact and the region round the contact is heated until the materials are melt and fused together. Enough heat must be supplied to cause melting of a sufficient volume of material but not enough to give rise to significant amounts of vaporization, otherwise weak porous welds are produced.

Laser welding however has a number of advantages:

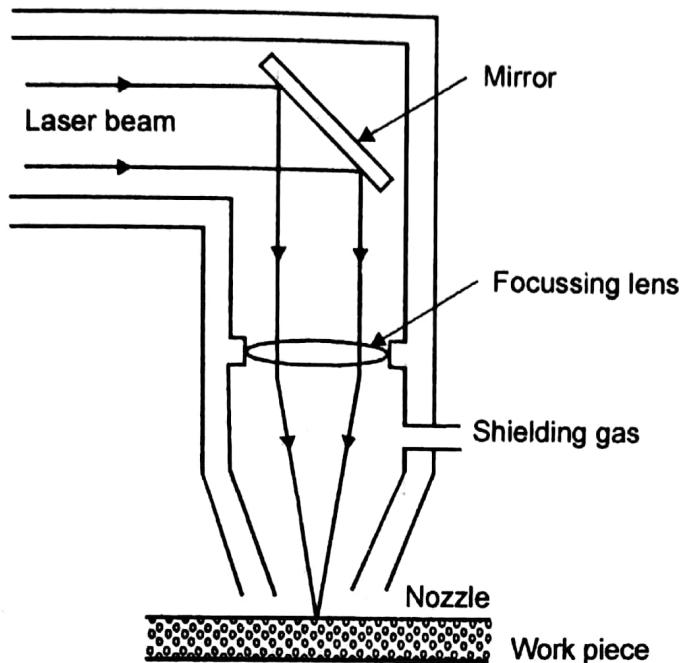
- i) There is no physical contact with external components
- ii) The heating is very localised
- iii) Dissimilar metals can be welded
- iv) Welding can be carried out in a controlled atmosphere with the work piece sealed if necessary within optically transparent materials
- v) Possibility to weld in inaccessible regions
- vi) Heat affected area is small

Welding is normally carried out using a shielding gas. This is an inert gas, usually argon or helium, which is applied to the welding area via a nozzle.

The main purpose of shielding gas is to cover the weld area and eliminate oxidation. It also helps to remove any metal vapour that may be formed.

One typical example of laser welding is the welding of curved contours under the body of automobiles.

The automobile is made to move while computer controlled beam deflectors control the laser beam which performs the welding. A  $\text{CO}_2$  laser can also weld sheets of films of plastic materials.



**Figure 3.18:** Laser welding

### 3.18 Laser Drilling

Hole perforating by the laser relies on the intense evaporation of the material heated by powerful light pulses of  $10^{-4}$  to  $10^{-3}$  sec duration. Use of series of short pulses minimises the energy diffused laterally into the workpiece and assists in controlling the size and shape of the hole.

Laser drilling offers a number of advantages over traditional tools:

- i) Non contact process and does not require a physical drill bit.
- ii) Problem of wear and broken drills does not arise.
- iii) As there is no physical contact between drill and workpiece, the process becomes faster
- iv) Extremely high precision in any direction.
- v) Drilling of hard material is possible.

For example, a hole of 4.7 mm in diameter and 2 mm deep in a diamond die can be made in about 15 minutes. Using a mechanical process this would take about 24 hours.

Lasers have advantage where high speeds and small holes in deformable materials are required, for example perforation in plastic, nozzles and nylon buttons, baby bottle nipple, aircraft engine turbine blade etc.

### 3.19 Laser Cutting

In cutting, the aim is to vaporize the material as quickly as possible and to produce a narrow heat affected zone as possible with minimum distortion of workpiece. Compared to other cutting techniques, lasers offer the following advantages.

- i) Minimal amount of mechanical distortion and thermal damage introduced in the material being cut.
- ii) The process does not introduce any contamination.
- iii) Possibility of cutting in two and even three dimensions according to complicated profile.
- iv) Easy automation of the process and high production rates.

Laser cutting is mainly useful in paper, cloth, plywood, glass, ceramics, sheetmetal industries. It is also useful in aerospace industries, specifically in the fabrication of spacecraft.

### 3.20 Optical Disk Systems

In recent years optical disks have been used increasingly for educational programs, entertainment and general audio-visual communications. In the field of data storage direct optical recording systems are becoming popular as computer peripherals, where the combination of very high information capacity and rapid random access makes optical disks an attractive alternative to other forms of computer memory store.

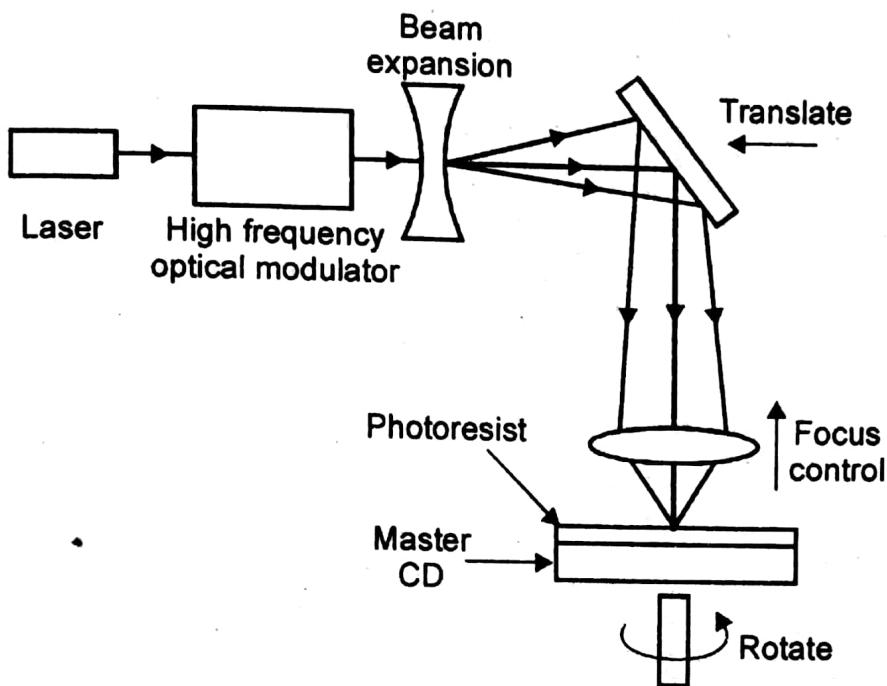
The high information capacity, long shelf life and long storage life are leading to applications in archival storage. Apart from high storage density, is the absence of physical contact between the reading head and the information storage medium, which prevents wear.

#### Recording Medium

The information is recorded along a continuous spiral track which contains micro-pits on a plastic material (polymethyl methacrylate) with a metallic coating.

The diameter of the CD is 120 mm and it is divided into tracks. The track is divided into blocks. The information tracks are located between 50 and 116 mm. The total length of the track is about 5 kilometers.

### Recording and readout from optical disk



**Figure 3.19:** Schematic diagram of laser beam recorder

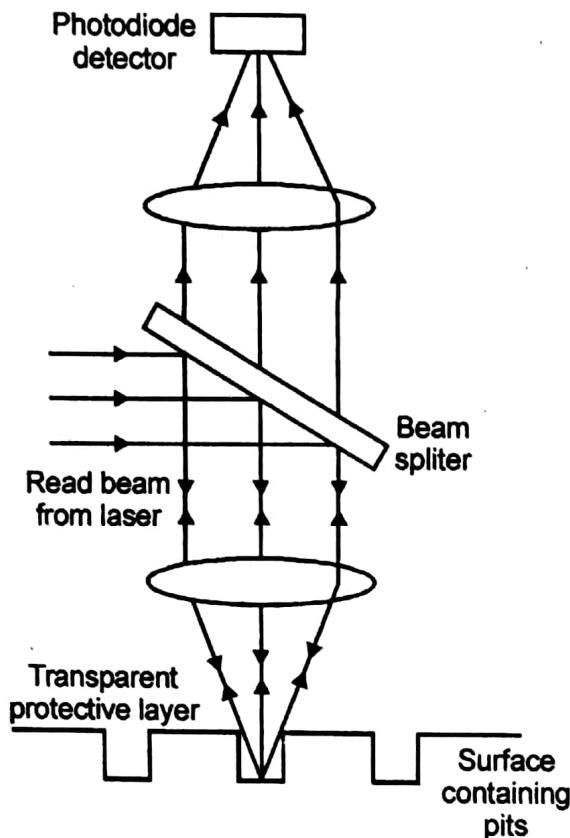
Recording digital information on a CD is done by pointing a laser (high power) radiation on a photo-sensitive coating which changes its properties when exposed to specific wavelength of light.

The information loaded in binary form (0 or 1) is transferred to a modulator, which is a device that controls the light output of the laser. This laser radiation creates the pits on the photoresist and the result is a master CD.

The master CD is used in a process of Electro-plating of photopolymer replication to stamp hundreds of CD per hour with hot pressing.

In the production of the master CD, the goal is to make the pits as small as possible, in order to have the maximum available density of information storage.

Since diffraction of light dictates the limit to the size of the minimum laser spot size, the best way to increase the density of information storage is to use shorter wavelength.



**Figure 3.20:** Readout from an optical disk

Reading information from the CD is done by illuminating it with a focussed beam from a diode laser. The diode laser radiation is focussed by an optical system into a small spot on the surface of the CD.

The radiation is reflected back to a detector from both the upper metallic surface and the bottom of the pit. The CD spins under the light beam and the laser beam scans the tracks. When the beam is reflected from the upper metallic surface or the bottom of the pit, there is only one reflected wave to the detector.

When the laser beam is reflected from a transition between the upper and lower metallic surfaces, two waves (with a phase difference of  $\pi$ ) are reflected to the detector. These two waves interfere destructively, (cancel each other) and the detector reads no reflected signal.

When the detector sees the transition between the pit and the upper surface, the detector reads one. Otherwise the reflection is from metallic surface and there is no transition (no destructive interference) so the reading is zero.

### Types of optical disks

- a) Read only optical disk.
- b) WORM (Write Once, Read Many times)
- c) CD - WR (writing many times to the CD)

Lastly DVD (Digital Versatile Disk) that can store upto 4.7 GB. This DVD can store information on both sides. So the total amount of information is 9.4 GB.

## 3.21 Holography

Using lasers, we can get three dimensional lensless photography called holography. Using interference techniques we can take hologram which is analogous to negative of the photographic film. In one hologram we can store so much information and if a hologram is broken, then each piece will act an individual hologram.

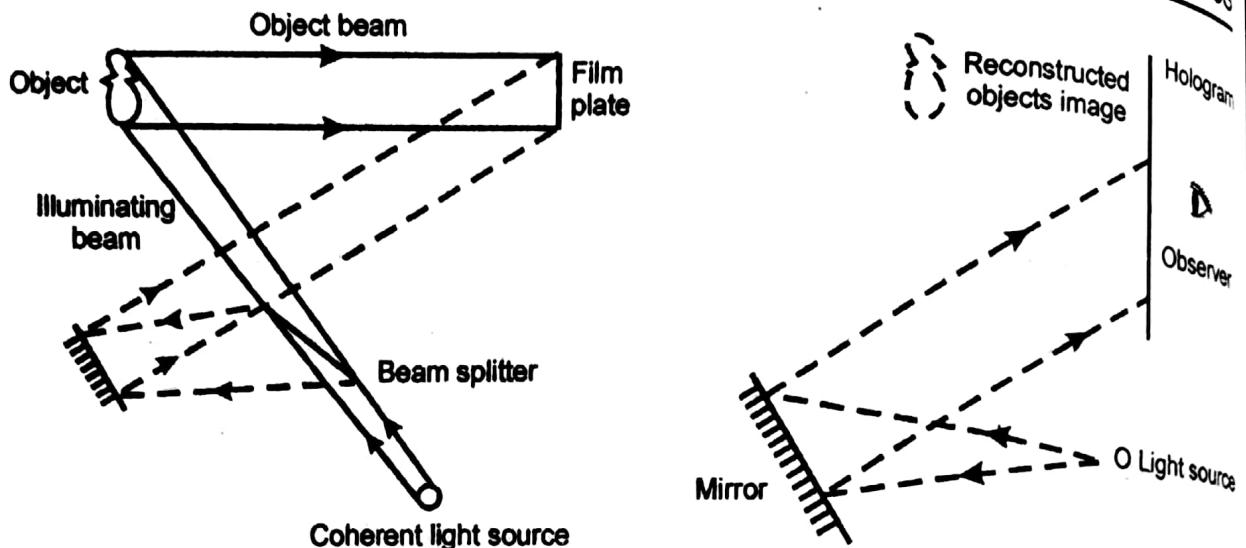
In conventional photography, only the distribution of the amplitude is recorded in two dimensional projection of an object onto the plane of the photograph. We cannot get complete information of the object because the phase distribution of that object is lost during recording. But in holography along with the amplitude, phase is also recorded by interference pattern.

### Recording and reconstruction of a hologram

#### A) Recording

The process of making a hologram is called recording or construction and the method of displaying the hologram is termed as reconstruction. Hologram records the interference pattern formed by the superposition of two coherent light waves; one scattered by the object (the object or signal wave) and the other by auxiliary (the reference wave).

Figure 3.21(a) shows a simple arrangement for recording holograms. A low power helium-neon laser is used as the light source because it emits coherent light. The light beam is spread and then divided into two beams by a beam splitter. One beam of light is called the illuminating beam and it illuminates the object to be photographed. The object, in turn, reflects part of the illuminating beam onto the film such as Kodak or AGFA. This beam is called the object beam. The second beam of light leaving the beam splitter is called the reference beam. It is redirected to the film by front surface mirror.

**a) Recording of a hologram****b) Reconstruction of a holographic image****Figure 3.21: Holographic recording and reconstruction**

On the film the reference and object beams interfere with each other and the interference pattern is recorded on the film as microscopic swirls. When it is developed the hologram appears transparent.

### B) Reconstruction

In reconstruction the object is recreated by directing a beam of light at the film as shown in figure 3.21(b). The reconstruction beam need not be a laser light. We can use ordinary sodium light. By moving the hologram through the light until the beam is striking the hologram at approximately the same angle as the reference beam we can see the three dimensional virtual image of the object.

### Uses of holography

- i) A hologram is a very reliable method of data storage. The most valuable information can be stored in holographic form.
- ii) Holographic non-destructive testing can be used to identify stresses in a pipe fitting, the stress points on a wheel and the vibration pattern of a guitar. It enables us to determine the degree and nature of deformation of the observed surface and to study the vibrational objects.
- iii) Holography plays a very important role in optical signal processing.
- iv) It can also be used for character recognition and for identification of finger prints, etc.

- v) Using holography, one can produce diffraction gratings. Defects arised as a result of the periodic errors in the ruling engine while making conventional gratings are completely avoided in the holographic gratings formed by interference between two coherent beams. The bright and dark fringes in the grating are acting as transparent space and opaque rulings of a conventional grating.
- vi) Holography is also empoyed in the production of photographic masks used to produce microelectronic circuits.

### Defense

1. Laser beams are used to guide the missiles. The laser guided missiles hit the target with a very high accuracy.
2. Laser weapons are used to disable the enemy weapon and to destroy it.
3. LIDAR is an acronym for LIght Detection And Ranging. It uses the laser beam in RADAR to detect the direction, velocity, and position of an object precisely. The light produced by a pulse laser is made to return after reflection from the moving object. The time delay of the laser beam for the round-trip is used to find the range of the target. Using the Doppler shift principle, the velocity of the moving object is determined.

### Chemistry

Laser beam is used to accelerate a chemical reaction. It is also used to produce a new compound by simply breaking an atomic bond.

### Photography

Laser beams are used to produce hologram, a lensless photography. The image is recorded by means of interference of lights.

### Optical data storage

Since laser beam can be focused into a small spot using a convex lens, it is used to store more information in the optical data storage medium. It is used to write and read in the compact discs (CDs).

## Solved Problems

1. The He-Ne system is capable of lasing at several different IR wavelengths, the prominent one being  $3.3913 \mu\text{m}$ . Determine the energy difference (in eV) between upper and lower levels for this wavelength.

*Given data:*

Wavelength,  $\lambda = 3.3913 \mu\text{m}$

**Solution**

$$\begin{aligned} E &= \frac{12400 \text{ (eV)}}{\lambda (\text{\AA})} = \frac{12400}{33913} \text{ (eV)} \\ &= 0.37 \text{ eV} \end{aligned}$$

2. Find the ratio of population of the two states in He-Ne laser that produces light of wavelength  $6328 \text{\AA}$  at  $27^\circ \text{C}$ .

*Given data:*

Wavelength,  $\lambda = 6328 \text{\AA}$

Temperature,  $T = 27^\circ \text{C} = 300 \text{ K}$

**Solution**

$$E = \frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

$$E_2 - E_1 = \frac{12400}{6328} \text{ eV} = 1.96 \text{ eV}$$

$$\begin{aligned} \therefore \frac{N_2}{N_1} &= \exp \left[ \frac{-1.96 \text{ eV}}{(8.61 \times 10^{-5} \text{ eV/K}) 300 \text{ K}} \right] \\ &= e^{-75.88} = 1.1 \times 10^{-33} \end{aligned}$$

3. The CO<sub>2</sub> laser is one of the most powerful lasers. The energy difference between the two levels is 0.117 eV. Determine the frequency and wavelength of radiation.

*Given data*

Energy interval,  $\lambda = 0.117 \text{ eV}$

**Solution**

$$\lambda = \frac{12400}{E(\text{eV})} = \frac{12400}{0.117}$$

$$\lambda = 105983 \text{ Å} = 10.5 \mu\text{m.}$$

$$\gamma = \frac{c}{\lambda} = \frac{3 \times 10^8}{10.5 \times 10^6} = 2.9 \times 10^{13} \text{ Hz}$$

4. A He-Ne laser produces an output power of 5 mW. If it emits light of wavelength 632.8 nm, calculate the number of photons emitted by the laser in one second.

*Given data:*

Output power, P = 5 mW

Wavelength,  $\lambda = 632.8 \text{ nm}$

**Solution**

$$\begin{aligned} \text{Energy of one photon, } h\nu &= \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}} \\ &= 3.141 \times 10^{-19} \text{ J} \\ &= 1.96 \text{ eV} \end{aligned}$$

$$\begin{aligned} \text{Number of photons emitted} &= \frac{\text{output power}}{\text{energy of one photon}} = \frac{5 \times 10^{-3}}{3.141 \times 10^{-19}} \\ &= 1.591 \times 10^{16} \text{ photons per second} \end{aligned}$$

The number of photons emitted per second by a He-Ne laser is  $1.591 \times 10^{16}$  photons per second.

5. A transition between the energy level  $E_2$  and  $E_1$  produces a light of wavelength 632.8 nm, calculate the energy of the emitted photons.

*Given data:*

$$\text{Wavelength, } \lambda = 632.8 \text{ nm}$$

**Solution**

$$\text{Energy of the emitted photon, } E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 3.141 \times 10^{-19} \text{ J} = 1.96 \text{ eV}$$

$$\text{The energy of the photon} = 1.96 \text{ eV}$$

6. A system has three energy levels  $E_1$ ,  $E_2$  and  $E_3$ . The energy levels  $E_1$  and  $E_2$  are at 0 eV and 1.4 eV respectively. If the lasing action takes place from the energy level  $E_3$  to  $E_2$ , and emits a light of wavelength 1.15  $\mu\text{m}$ , find the value of  $E_3$ .

*Given data:*

$$\text{The value of first energy level, } E_1 = 0 \text{ eV}$$

$$\text{Value of second energy level, } E_2 = 1.4 \text{ eV}$$

$$\text{Wavelength, } \lambda = 1.15 \mu\text{m}$$

**Solution**

$$\text{Energy of the emitted photon, } E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.15 \times 10^{-6}}$$

$$= 1.728 \times 10^{-19} \text{ J}$$

$$= 1.079 \text{ eV}$$

$$\begin{aligned}
 \text{The energy value of } E_3 &= E_2 + h\nu \\
 &= 1.4 \text{ eV} + 1.079 \text{ eV} \\
 &= 2.479 \text{ eV}
 \end{aligned}$$

$$\text{The energy value of } E_3 = 2.479 \text{ eV}$$

- 7. A laser transition takes place from an energy level at 3.2 eV to another level at 1.6 eV. Calculate the wavelength of the laser beam emitted.**

*Given data:*

$$\text{The value of higher energy level } E_1 = 3.2 \text{ eV}$$

$$\text{The value of lower energy level } E_2 = 1.6 \text{ eV}$$

**Solution**

$$\begin{aligned}
 \text{Energy difference, } E_2 - E_1 &= 3.2 - 1.6 \\
 &= 1.6 \text{ eV}
 \end{aligned}$$

$$\begin{aligned}
 \text{Wavelength, } \lambda &= \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 1.6 \times 10^{-6}} \\
 &= 7.7648 \times 10^{-7} \text{ m}
 \end{aligned}$$

$$\text{The wavelength of the photon, } \lambda = 7.7648 \times 10^{-7} \text{ m}$$

- 8. The band gap of GaAs is 1.42 eV. What is the wavelength of the laser beam emitted by a GaAs diode laser?**

*Given data:*

$$\text{Band gap of GaAs} = 1.42 \text{ eV}$$

**Solution**

Wavelength of laser emitted by GaAs,

$$\begin{aligned}
 \lambda &= \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.42 \times 1.6 \times 10^{-6}} \\
 &= 8.749 \times 10^{-7} \text{ m}
 \end{aligned}$$

$$\text{The wavelength of the laser emitted by GaAs, } \lambda = 8.749 \times 10^{-7} \text{ m}$$

9. Calculate the relative population of the energy levels  $N_1$  and  $N_2$  at 300 K,  $\lambda = 500 \text{ nm}$ .

*Given data:*

Temperature,  $T = 300 \text{ K}$

Wavelength,  $\lambda = 500 \text{ nm}$

**Solution**

From Maxwell and Boltzmann law, the relative population is given by

$$\frac{N_1}{N_2} = \frac{\exp\left(-\frac{E_1}{kT}\right)}{\exp\left(-\frac{E_2}{kT}\right)} = \exp\left(-\frac{E_1 - E_2}{kT}\right) = \exp\left(\frac{hv}{kT}\right)$$

Substituting the values of  $T$  and  $\lambda$ , we get

$$\begin{aligned} \frac{N_1}{N_2} &= \exp\left(\frac{hv}{kT}\right) = \exp\left(\frac{hc}{\lambda kT}\right) \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9} \times 1.38 \times 10^{-23} \times 300} = \exp(96.029) \\ &= 5.068 \times 10^{41} \end{aligned}$$

The relative population between  $N_1$  and  $N_2$  is  $5.068 \times 10^{41}$ .

10. Examine the possibility of stimulated emission at 300 K, and  $\lambda = 600 \text{ nm}$ .

*Given data:*

Temperature,  $T = 300 \text{ K}$

Wavelength,  $\lambda = 600 \text{ nm}$

**Solution**

The ratio between the stimulated emissions to spontaneous emission is given by

$$\frac{\text{stimulated emission}}{\text{spontaneous emission}} = \frac{1}{\exp\left(\frac{hv}{kT}\right) - 1} = \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Substituting the values of  $T$  and  $\lambda$ , we get

$$\begin{aligned}\frac{\text{stimulated emission}}{\text{spontaneous emission}} &= \frac{1}{\exp\left(\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9} \times 1.38 \times 10^{-23} \times 300}\right) - 1} \\ &= \frac{1}{\exp(80.024) - 1} \\ &= 1.762 \times 10^{-35}\end{aligned}$$

The ratio between stimulated emission and spontaneous emission is  $1.762 \times 10^{-35}$ . Therefore, the stimulated emission is not possible in this condition.

- 11. Calculate the efficiency of a He-Ne laser, if it produces an output power of 5 mW and if it is operated with a current of 10 mA at 3 kV.**

*Given data:*

$$\text{Output power, } P = 5 \text{ mW} = 5 \times 10^{-3} \text{ W}$$

$$\text{Current, } I = 10 \text{ mA} = 10 \times 10^{-3} \text{ A}$$

$$\text{Voltage, } V = 5 \text{ kV} = 3 \times 10^3 \text{ V}$$

**Solution**

$$\begin{aligned}\text{Efficiency} &= \frac{\text{output power}}{\text{input power}} \times 100 \% \\ &= \frac{5 \times 10^{-3}}{10 \times 10^{-3} \times 3 \times 10^3} \times 100 \% = 0.016667 \% \end{aligned}$$

$$\text{The efficiency of the laser} = 0.016667 \%$$

- 12. A laser beam emits an output power of 1 mW. If it is focused as a spot having a diameter of 1 μm, calculate the intensity of the laser beam.**

*Given data:*

$$\text{Output power, } P = 1 \text{ mW} = 1 \times 10^{-3} \text{ W}$$

$$\text{Diameter} = 1 \mu\text{m}$$

$$\text{Radius, } r = 0.5 \mu\text{m} = 0.5 \times 10^{-6} \text{ m}$$

**Solution**

$$\text{Intensity of laser} = \frac{\text{power}}{\text{area of cross section}} = \frac{1 \times 10^{-3}}{\pi(0.5 \times 10^{-6})^2}$$

$$= 1.273 \times 10^9 \text{ W m}^{-2}$$

$$\text{The intensity of the laser} = 1.273 \times 10^9 \text{ W m}^{-2}$$

- 13. A laser beam of wavelength 632.8 nm is made to fall on a wall that lies at a distance of 5 m and if it produces a spot having a diameter of 1 mm, calculate the angular spread and the divergence of the laser beam.**

*Given data:*

$$\text{Wavelength, } \lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$$

$$\text{Distance, } D = 5 \text{ m}$$

$$\text{Diameter, } d = 1 \text{ mm}$$

**Solution**

$$\text{Angular spread, } \Delta\theta = \frac{\lambda}{d} = \frac{632.8 \times 10^{-9}}{1 \times 10^{-3}}$$

$$= 6.328 \times 10^{-4} \text{ radian}$$

$$\text{Radius of the spread, } r = (D \times \Delta\theta) = \left(D \frac{\lambda}{d}\right)$$

$$= 5 \times 6.328 \times 10^{-4}$$

$$= 3.164 \text{ mm}$$

$$\text{Area of the spread} = \pi(D \times \Delta\theta)^2 = \pi \left(D \frac{\lambda}{d}\right)^2$$

$$= \pi(5 \times 6.328 \times 10^{-4})^2$$

$$= 3.145 \times 10^{-5} \text{ m}^2$$

$$\text{Angular spread} = 6.328 \times 10^{-4} \text{ radian}$$

$$\text{Area of the spread} = 3.145 \times 10^{-5} \text{ m}^2$$

14. The coherence length for sodium D<sub>1</sub> line is 2.5 cm. Determine (a) the spectral width of the line,  $\Delta\lambda$ , (b) the purity factor, Q, (c) the coherence time  $\tau$ . Take wavelength of light 6000 Å.

### Solution

$$a) \Delta\lambda \approx \frac{\lambda'}{2L} \approx \frac{36 \times 10^{-10}}{5} \text{ cm} \approx 7 \text{ Å}$$

$$b) Q \approx \frac{\lambda'}{\Delta\lambda} \approx \frac{6 \times 10^{-5}}{7 \times 10^{-8}} \approx 10^3$$

$$c) \tau \approx \frac{L}{C} = \frac{2.5}{3 \times 10^{10}} \text{ s} \approx 0.8 \times 10^{-10} \text{ s}.$$

### Points to Remember

- Laser stands for Light Amplification by Stimulated Emission of Radiation.
- Situation when  $N_2 > N_1$  is population inversion.
- The process of achieving  $N_2 > N_1$  is pumping.
- Ruby laser produces pulsed output.
- He-Ne laser produces continuous output.
- Helium acts as coolant in CO<sub>2</sub> laser.
- Information is recorded using high power laser and data is read out using low power laser.
- The role of He in a He-Ne laser is to produce population inversion with Ne atoms.
- The role of N<sub>2</sub> in a CO<sub>2</sub> laser is to produce population inversion with CO<sub>2</sub> molecules.
- Hologram is constructed by means of interference of light between the reference beam and the light reflected from the object.

### Formulae

$$1. E = \frac{hc}{\lambda} = \frac{12400 \text{ (eV)}}{\lambda \left( \frac{\text{Å}}{\text{A}} \right)}$$

2. Number of photons in each pulse =  $\frac{\text{Energy}}{h\nu} = \frac{\text{Energy}}{hc} \lambda$
3. Number of photons emitted per minute =  $\frac{\text{Total energy emitted per minute}}{\text{energy of 1 photon}}$
4. Ratio of population of two states,  $\frac{N_2}{N_1} = e^{-\left(\frac{E_2 - E_1}{kT}\right)} = e^{-\left(\frac{h\nu}{kT}\right)}$
5. Intensity =  $\frac{\text{Power}}{\text{Area}}$
6. Angular spread,  $d\theta = \frac{\text{Wavelength}}{\text{Aperture diameter}}$
7. Area of spread =  $\pi (\text{distance} \times d\theta)^2$
8. Efficiency,  $\eta = \frac{\text{optical output}}{\text{electrical input}} \times 100 \%$

## Review Questions

### Short Answer Questions

1. What is absorption of light?
2. What is meant by emission of light?
3. What is stimulated emission? What is spontaneous emission?
4. How laser is different from conventional light sources?
5. Write the mathematical expression for the stimulated and spontaneous emissions and explain the terms.
6. Distinguish between spontaneous and stimulated emission.
7. What is meant by the population inversion?
8. What is optical pumping?
9. What is electron excitation?
10. What is inelastic atom–atom collision?
11. What is a resonator?
12. Mention about the different types of resonators used for laser production.

13. Mention the distinct properties of laser.
14. What is monochromaticity?
15. What is intensity of laser?
16. What is meant by directionality of laser?
17. What is coherence?
18. Mention the different types of lasers.
19. What are the advantages and drawbacks of ruby laser?
20. What is a solid state laser? Give an example.
21. What are the advantages and drawbacks of He-Ne laser?
22. Distinguish between a ruby laser and a He-Ne laser.
23. Mention the different types of vibration in CO<sub>2</sub> laser.
24. What are the advantages and drawbacks of CO<sub>2</sub> laser.
25. What are the advantages and drawbacks of a Nd:YAG laser.
26. What are the advantages of laser welding over other welding processes?

### **Descriptive Questions**

27. Discuss the fundamental process involved in laser action. Derive Einstein's A and B coefficients and mention its significance.
28. What do you mean by pumping? Explain the different types of pumping system in laser.
29. What are the losses that occur in laser media? Derive an expression for the threshold gain coefficient.
30. With a neat sketch, explain the construction and working of a He-Ne laser.
31. What are the fundamental modes of vibration of CO<sub>2</sub> molecule? Explain the working of CO<sub>2</sub> laser.
32. Explain how information can be recorded and readout in an optical storage system.
33. If the laser action occurs by the transition from an excited state to the ground state, E<sub>1</sub> = 0, and if it produces light of 693 nm wavelength, what is the energy level of the excited state? [Ans. 1.78 eV]

34. Transition occurs between a metastable state  $E_3$  and an energy state  $E_2$ , just above the ground state. If emission is at  $1.1 \mu\text{m}$  and if  $E_2 = 0.4 \times 10^{-19} \text{ J}$ , how much energy is contained in the  $E_3$  state? [Ans. 1.37 eV]
35. A relative population, or Boltzmann ratio, of  $1/e$  is often considered representative of the ratio of populations in two energy states at room temperature,  $T = 300 \text{ K}$ . As an example, determine the wavelength of the radiation emitted at that temperature.  
[Ans.  $48 \times 10^{-6} \text{ m}$ ]
36. The length of a laser tube is 150 mm and the gain factor of the laser material is  $0.0005/\text{cm}$ . If one of the cavity mirror reflects 100% light that is incident on it, what is the required reflectance of the other cavity mirror? [Ans. 0.98]
37. A lamp is operating at a temperature of  $727^\circ\text{C}$  and emitting radiation of wavelength of  $0.5 \mu\text{m}$ . Calculate the ratio between stimulated and spontaneous emission rate.  
[Ans.  $3.17 \times 10^{-13}$ ]
38. Explain the characteristics and applications of Nd-YAG laser.
39. Explain the principle of excimer laser.
40. Write short notes on dye laser.
41. Discuss the principle involved in chemical laser. How does it differ from other lasers?

# **CHAPTER - 4**

## **ELECTROMAGNETIC WAVE PROPAGATION**

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### **4.1 Maxwell's Equations**

Time varying magnetic fields give rise to electric field and vice versa. It would be nice to speak of electromagnetic fields rather than electric and magnetic fields. The behaviour of time dependent electromagnetic fields is described by a set of equations known as Maxwell's equations. These equations are mathematical expressions of certain experimental results.

There are four fundamental equations of electromagnetism known as Maxwell's equations which may be written in differential form as

- i) Gauss's law for electrostatics  $\nabla \cdot D = \rho$
- ii) Gauss's law for magnetic field  $\nabla \cdot B = 0$
- iii) Faraday's law for electromagnetic induction  $\nabla \times E = - \frac{\partial B}{\partial t}$
- iv) Ampere's law for magneto motive force  $\nabla \times E = J + \frac{\partial D}{\partial t}$

where  $D$  is the electric displacement vector in  $C/m^2$

$\rho$  is the charge density in  $C/m^3$

$B$  is the magnetic induction in  $Wb/m^2$

$E$  is the electric field intensity in  $V/m$

$H$  is the magnetic field intensity in  $A/m$

and  $J$  is the current density.

Each of Maxwell's equations represents a generalization of certain experimental observations: Equation (1) represents the Gauss's law in electrostatics which in turn derives from Coulomb's law. Equation (2) represents Gauss's law in magnetostatics which is usually said to represent the fact that isolated magnetic poles do not exist in our physical world. Equation (3) represents differential form of Faraday's law of electromagnetic induction and the last equation represents Maxwell's modification of Ampere's law to include time varying fields.

## 4.2 Electromagnetic Wave Equation

The application of Maxwell's equations is the prediction of existence of electromagnetic wave. Electromagnetic wave equation can be obtained from Maxwell's equations.

The Maxwell's equation from Faraday's law is given by,

$$\begin{aligned}\nabla \times E &= -\frac{\partial B}{\partial t} \\ &= -\mu \frac{\partial H}{\partial t}\end{aligned}$$

Take curl on both sides,

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad \dots (4.1)$$

But Maxwell's equation from Ampere's law is

$$\begin{aligned}\nabla \times H &= J + \frac{\partial B}{\partial t} \\ &= \sigma E + \epsilon \frac{\partial E}{\partial t}\end{aligned}$$

Differentiating

$$\begin{aligned}\nabla \times \frac{\partial H}{\partial t} &= \frac{\partial \nabla \times H}{\partial t} \\ &= \frac{\partial}{\partial t} \left( \sigma E + \epsilon \frac{\partial E}{\partial t} \right) \\ \nabla \times \frac{\partial H}{\partial t} &= \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \quad \dots (4.2)\end{aligned}$$

Substituting the equation (4.2) in equation (4.1)

$$\begin{aligned}\nabla \times \nabla \times E &= -\mu \left[ \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \right] \\ &= -\mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \dots (4.3)\end{aligned}$$

But according to the identity

$$\nabla \times \nabla \times E = \nabla (\nabla \cdot E) - \nabla^2 E \quad \dots (4.4)$$

But

$$\nabla \cdot E = \frac{1}{\epsilon} \nabla \cdot D$$

Since there is not net charge within the conductor, the charge density  $\rho = 0$ .

$$\nabla \cdot D = 0$$

$$\nabla \cdot E = 0$$

Then equation (4.4) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \quad \dots (4.5)$$

Comparing the equations (4.3) and (4.5)

$$\nabla^2 E = -\mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

$\nabla^2 E - \mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$

... (4.6)

This is the wave equation for electric field E.

The wave equation for Magnetic field H is obtained in a similar manner as follows.

The Maxwell's equation from Ampere's law is given by,

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Take curl on both sides,

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \epsilon \nabla \times \frac{\partial E}{\partial t} \quad \dots (4.7)$$

But Maxwell's equation from Faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Differentiating,  $\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial H^2}{\partial t^2}$

Substituting the values of  $\nabla \times E$  and  $\nabla \times \frac{\partial E}{\partial t}$  in equation (4.7)

$$\nabla \times \nabla \times H = -\mu \sigma \frac{\partial H}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2} \quad \dots (4.8)$$

But the identity is

$$\nabla \times \nabla \times H = \nabla (\nabla \cdot H) - \nabla^2 H$$

But

$$\nabla \cdot B = \mu \nabla \cdot H = 0$$

Then,

$$\nabla \times \nabla \times H = \nabla^2 H \quad \dots (4.9)$$

Comparing the equations (4.8) and (4.9)

$$\nabla^2 H = -\mu \sigma \frac{\partial H}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H - \mu \sigma \frac{\partial H}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2} = 0$$

... (4.10)

This is the wave equation for magnetic field H.

### 4.3 Wave Equation for Free Space

For free space (dielectric medium) the conductivity of the medium is zero. (i.e.,  $\sigma = 0$ ) and there is no charge containing in it (i.e.,  $\rho = 0$ ). The electromagnetic wave equations for free space can be obtained from Maxwell's equations.

$$\nabla^2 E - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

... (4.11)

This is the wave equation for free space in terms of Electric field.

The wave equation for free space in terms of magnetic field H is obtained in a similar manner as follows.

$$\nabla^2 H - \mu \epsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots (4.12)$$

For free space  $\mu_r = 1$  and  $\epsilon_r = 1$  (air)

Then the wave equation becomes

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{or} \quad \nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

#### 4.4 Phase velocity and group velocity

The speed with which the individual component wave of the wave packet moves is called the phase velocity  $v_p$ , which is defined by

$$v_p = \frac{2\pi v}{2\pi/\lambda} = v\lambda$$

For a particle,

$$E = hv = mc^2 \quad \text{and} \quad \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$v_p = \frac{mc^2}{h} = \frac{h}{mv} = \frac{c^2}{v}$$

For photon  $v = c$  and therefore  $v_p = c$  as expected. But for a material particle,  $v$  is always less than  $c$  and hence  $v_p$  is always greater than  $c$ , that is  $v < c$  and  $v_p > c$ .

Group velocity  $v_g$  is the velocity with which the envelope of the wave packet moves. It is not difficult to show that

$$V_g = \frac{d\omega}{dk}$$

Using the relations  $E = \hbar\omega$  and  $p = k\hbar$

$$v_g = \frac{d\omega}{dk} = \left( \frac{d\omega}{dE} \right) \left( \frac{dE}{dp} \right) \left( \frac{dp}{dk} \right) = \frac{1}{\hbar} \left( \frac{dE}{dp} \right) (\hbar) = \frac{dE}{dp}$$

For a classical free particle,  $E = p^2/2m$ . Therefore,

$$v_g = \frac{dE}{dp} = \frac{p}{m} = v$$

which is the velocity of the particle.

For a relativistic particle,  $E^2 = c^2 p^2 + m_0^2 c^4$  and therefore

$$v_g = \frac{dE}{dp} = \frac{c^2 p}{E} = \frac{c^2 mv}{mc^2} = v$$

That is, the velocity of a particle is always equal to the group velocity of the corresponding wave packet. In other words, the particle and the associated wave packet move together.

## 4.5 Rectangular Waveguides

Waveguides usually take the form of rectangular or circular cylinders because of its simpler forms in use and less expensive to manufacture. A hollow conducting metallic tube of uniform cross-section (rectangular or circular) is used for propagating electromagnetic waves, waves that are guided along the surface (walls) of the tube is called waveguide. Propagation of waveguide can be considered as a phenomenon in which waves are reflected from wall to wall and hence pass down the waveguide in a zig-zag fashion. The resultant wave travels longitudinally down along the waveguide.

In order to determine the electromagnetic field configuration within the guide, Maxwell's equations are solved subject to the appropriate boundary conditions at the walls of the guide.

## 4.6 Guided Waves

The electromagnetic waves that are guided along or over conducting or dielectric surface are called guided waves.

### 4.6.1 Transverse Electric (TE) Waves

Transverse Electric (TE) waves are waves in which the electric field strength is entirely transverse. It has a magnetic field strength ( $H_z$ ) in the direction of propagation and no component of electric field  $E_z$  in the same direction ( $E_z = 0$ ).

### 4.6.2 Transverse Magnetic (TM) Waves

Transverse magnetic (TM) waves are waves in which the magnetic field strength  $H$  is entirely transverse. It has an electric field strength  $E_z$  in the direction of propagation and no component of magnetic field  $H_z$  in the same direction ( $H_z = 0$ ).

### 4.6.3 Transverse Electro-magnetic (TEM) waves

It is a special type transverse magnetic wave in which electric field  $E$  along the direction of propagation is also zero. The transverse electromagnetic (TEM) waves are waves in which both electric and magnetic fields are transverse entirely but have no component of  $E_z$  and  $H_z$ . It is referred to as principal waves.

### Solved Problems

1. Wave on the surface of water travel with a phase velocity  $V_p = \sqrt{g\lambda/2\pi}$ , where  $g$  is the acceleration due to gravity. Show that the group velocity of a wave packet comprised of these waves is  $v_p/2$ .

**Solution:**

$$\text{Phase velocity, } v_p = \sqrt{g\lambda/2\pi} = \sqrt{\frac{g}{k}}$$

By definition  $v_p = \omega/k$  and hence,

$$\frac{\omega}{k} = \sqrt{\frac{g}{k}} \quad \text{or} \quad \omega = \sqrt{gk}$$

$$\text{Therefore, group velocity } v_g = \frac{d\omega}{dk} = \frac{1}{2} \sqrt{\frac{g}{k}} = \frac{v_p}{2}$$

### Points to Remember

1. In free space the electromagnetic waves travel with the speed of light.
2. The electromagnetic field vectors  $E$  and  $H$  are mutually perpendicular and they are also perpendicular to the direction of propagation of electromagnetic waves. Thereby indicating the electromagnetic waves are transverse in nature.
3. If a region consists of two or more wave trains, then the physical velocity of propagation of waves is called the group velocity. The velocity of individual wave is called phase velocity.

## Review Questions

1. What is the nature of electromagnetic wave?
2. Write Maxwell's equations.
3. What do you mean by displacement current? Is it due to free charge carriers?
4. Define phase velocity and group velocity.
5. Derive the wave equation for magnetic field.
6. Obtain the wave equation for electric field using Maxwell's equations.
7. Distinguish between TE and TM mode.

# **CHAPTER - 5**

## **FIBER OPTICS**

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### **5.1 Introduction**

Fiber optics is a branch of optics that deals with the study of propagation of light through dielectric waveguides e.g., optical fibers. The transmission medium in fiber-optic communications system is an optical fiber. An optical fiber is a transparent flexible filament that guides light from a transmitter to a receiver. A fiber-optic communications system is the most preferred telecommunications system for several reasons that will be discussed in what follows. An optical fiber forms the heart of the fiber communications system that possesses a lot of advantages compared to copper or co-axial or even satellite communications.

### **5.2 Advantages of Fiber-Optic Communications**

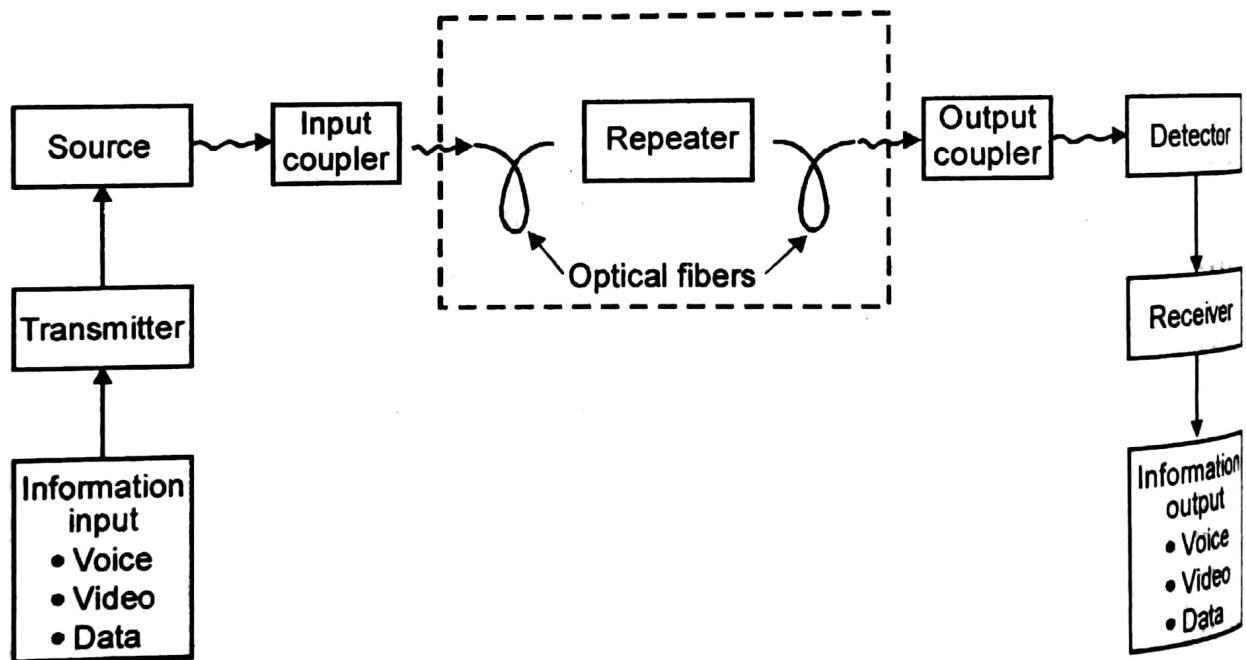
- The bandwidth, which is the information carrying capacity, of optical communication system is very large. Theoretically a bandwidth of the order of 50 Tb/s is possible.
- Owing to the fact that photons carry the information in optical fibers, the speed at which the data are transmitted is quite high.
- The attenuation is very less (of the order of 0.15 dB/km).
- Fibers are light in weight and small in size.
- Digital communication is possible.
- Since optical fibers are made of dielectric material, the communication does not suffer from either electromagnetic interference (EMI) or radio-frequency interference (RFI).

- It forms a very secure communication since it is not possible to tap the signal without sender's knowledge.
- A fiber presents no spark or fire hazard, so it cannot cause explosions or fires as a faulty copper cable can.
- Optical communication is not at all affected by weather conditions.

However, optical fiber communication is a wired communication. Apart from this cutting and jointing of fibers require a greater skill. But the advantages undoubtedly outweigh the drawbacks and hence fiber optic communication continues to be in the forefront among the other communication technologies of the telecommunication industry.

### 5.3 A Fiber - Optic Communication System

The block diagram represents the generalized configuration of a fiber - optic communication system.

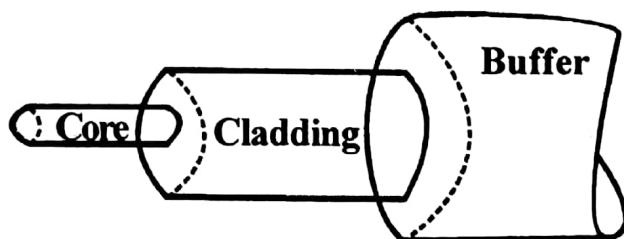


**Figure 5.1:** Generalized configuration of a fiber-optic communication system

The information input may be either voice or video or data. Therefore an input transducer is required for converting the non-electrical input into an electrical input. The transmitter (or the modulator, as it is often called) converts the electric signal into the proper form and impresses this signal onto the electromagnetic wave generated by an optoelectronic source. The light from the source is coupled to the fiber by a coupler. The light is transmitted through the fiber by the phenomenon of total internal reflection. As the

optical signals propagate through the fiber, they get attenuated due to scattering, absorption and bending of the fiber. Hence the regeneration of signal is necessary. A repeater just does this. The conventional electronic repeaters are being replaced, nowadays, by optical amplifiers. There is another coupler at the output which directs the light to the detector. The detector converts light into electricity back. The receiver section filters out undesired frequencies and amplifies the photocurrent. Finally, the information must be presented in a form that can be interpreted by a human observer. Suitable output transducers are required for achieving this transformation.

#### 5.4 Structure of an Optical Fiber



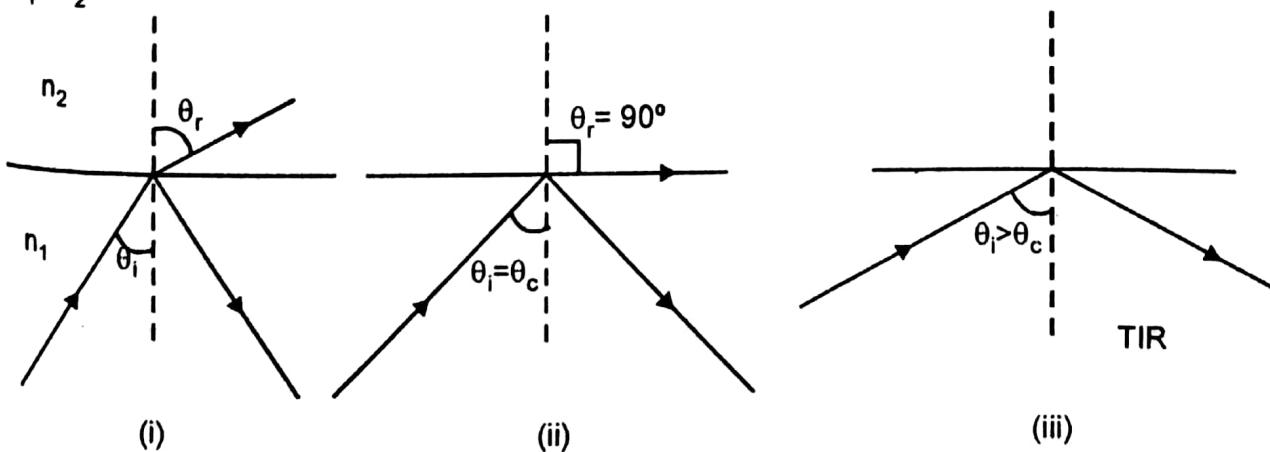
**Figure 5.2:** Schematic of a single-fiber structure

An optical fiber is a thin, transparent, flexible strand that consists of a core surrounded by cladding. The core and cladding of an optical fiber are made from the same material (a type of glass called silica) and they differ only in their refractive indices. The refractive index of the core ( $n_1$ ) will be slightly greater than that of the cladding ( $n_2$ ). Normally an elastic plastic buffer encapsulates the fiber.

#### 5.5 Light Propagation through Fibers

The basic principle behind the transmission of light through the optical fibers is ‘total internal reflection’ which is described below:

$$n_1 > n_2$$



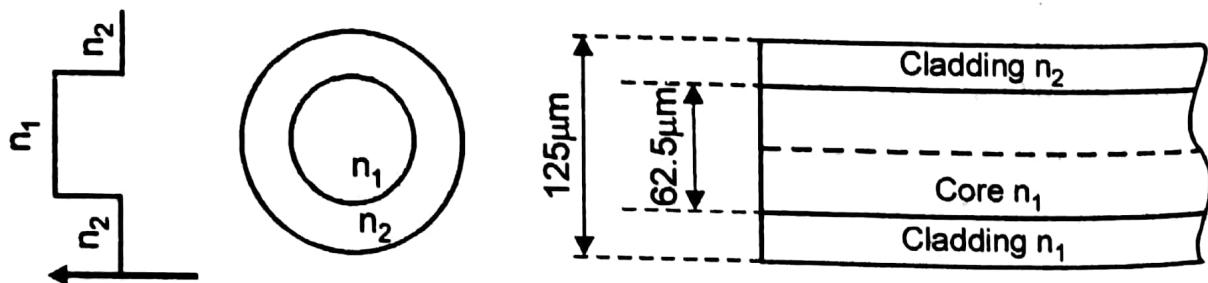
**Figure 5.3:** Representation of critical incident angle and total internal reflection

Consider two media of refractive indices  $n_1$  and  $n_2$ , separated by the boundary. Let  $n_1 > n_2$  and from the diagram, it is clear that light passes from the denser to the rarer medium. In the first diagram, the ray makes an angle of incidence ( $\theta_i$ ) and gets refracted through an angle ( $\theta_r$ ). Here the refracted ray bends away from the normal as the light travels from the denser to the rarer medium.

As we increase the angle of incidence, the angle of refraction also increases. The most important position is shown in the figure 5.3 (ii) where the angle of incidence,  $\theta_i$ , reaches the critical value,  $\theta_c$ . At this stage no light penetrates into the second medium. The incident angle at which the angle of refraction equals  $90^\circ$  is called the *critical incident angle*,  $\theta_c$ . If we continue to increase the angle of incidence so that  $\theta_i > \theta_c$ , all light will be reflected back into the incident medium, as in Figure 5.3 (iii). This phenomenon is called *total internal reflection* (TIR) since all the light is reflected back to the medium of incidence. TIR is what keeps light inside an optical fiber.

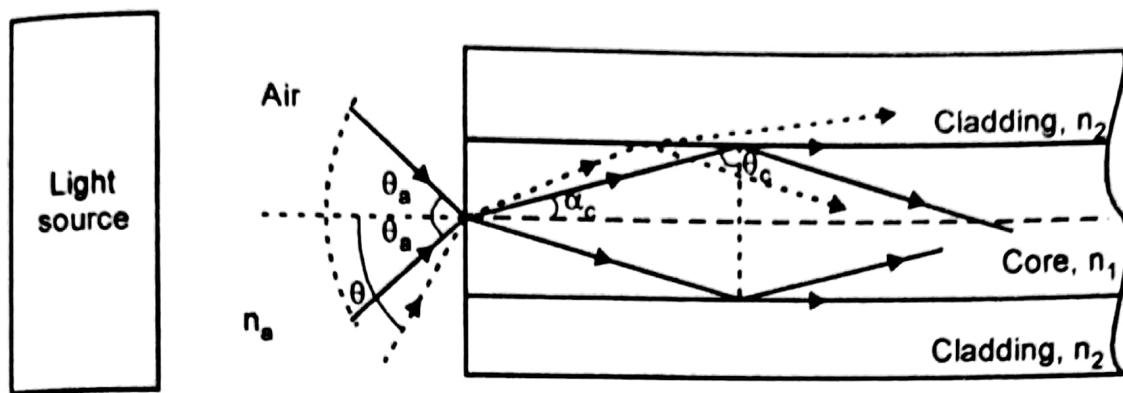
## 5.6 Conditions for Light Propagation in Optical Fibers

As has been discussed already, an optical fiber has an inner core of higher refractive index,  $n_1$ , surrounded by an outer core of relatively lower refractive index  $n_2$ . Hence the refractive index of the core is uniform throughout the core and decreases abruptly at core-cladding boundary. This fiber is called a *step-index fiber* as the refractive index profile shows that the refractive index decreases step wise. This is clearly shown in the Figure 5.4.



**Figure 5.4:** Basic structure of a step-index fiber

Since the refractive index of the core ( $n_1$ ) is greater than that of the cladding ( $n_2$ ), the light is totally internally reflected provided the angle of incidence is greater than the *critical incident angle* ( $\theta_c$ ). This is the angle the beam makes with the line perpendicular to the optical boundary between the core and cladding.



**Figure 5.5:** Launching light into an optical fiber

From the Figure 5.5, it is clear that to save light inside a strand of fiber, we need to have it strike the core-cladding boundary at the critical incident angle,  $\theta_c$ , or above it, in order to provide total internal reflection.

Applying Snell's law at the core - cladding boundary, we have,

$$\begin{aligned} n_1 \sin \theta_c &= n_2 \sin 90^\circ \\ \sin \theta_c &= (n_2/n_1) \\ \theta_c &= \sin^{-1}(n_2/n_1) \end{aligned} \quad \dots (5.1)$$

At this point, it is necessary to emphasize the fact that there is so called *critical propagation angle*,  $\alpha_c$ , that represents the requirement to save the light within the core. It is the maximum angle which the ray can make with the fiber axis while entering into the fiber so that light is confined within the core. Any angle lesser than  $\alpha_c$  is allowed. However, when the propagation angle is greater than the critical propagation angle, the light not only undergoes reflection but also refraction. Hence, to save light inside an optical fiber, it is necessary to direct rays at this critical propagation angle,  $\alpha_c$ , or even at a lesser angle.

From the Figure 5.5, we can get an expression for  $\alpha_c$  as follows:

From the figure,

$$\begin{aligned} \theta_c &= 90^\circ - \alpha_c \\ \sin \theta_c &= \sin(90^\circ - \alpha_c) = \cos \alpha_c \end{aligned}$$

But from (5.1),  $\sin \theta_c = n_2/n_1$

$$\therefore \cos \alpha_c = n_2/n_1$$

From this relation, we can write,

$$\begin{aligned}\sin \alpha_c &= \sqrt{1 - \cos^2 \alpha_c} = \sqrt{1 - (n_2/n_1)^2} \\ \therefore \quad \sin \alpha_c &= \sqrt{1 - (n_2/n_1)^2} \\ \alpha_c &= \sin^{-1} \sqrt{1 - (n_2/n_1)^2} \quad \dots (5.2)\end{aligned}$$

Obviously, the next task is to direct the beam from outside so that it does fall at or below the critical propagation angle. At the gap-fiber interface, the beam at an angle,  $\theta_a$ , is the incident beam and the beam at an angle,  $\alpha_c$ , is the launched one, which is the refracted beam with respect to gap-core interface.

Applying Snell's law at gap-fiber interface, we have,

$$n_a \sin \theta_a = n_1 \sin \alpha_c$$

For air,  $n_a \approx 1$ ,

$$\therefore \quad \sin \theta_a = n_1 \sin \alpha_c$$

Using (5.2), we have,

$$\sin \theta_a = \sqrt{n_1^2 - n_2^2} \quad \dots (5.3)$$

From the figure, it is clear that the angle,  $\theta_a$ , is a spatial angle. Light will be saved inside the fiber if it comes from a light source bounded by the cone,  $2\theta_a$ . The angle,  $\theta_a$ , is called conical half angle. Generally the angle,  $\theta_a$ , is referred to as *acceptance angle*.

In conclusion, to save light inside a fiber, all rays must propagate at critical angle,  $\alpha_c$ , or less. In order for us to maintain the light inside the fiber at this angle, we have to direct it from outside the fiber at an angle,  $\theta_a$ , or less. In Figure 5.5, a ray represented as dotted line making an angle,  $\theta$ , with the fiber axis (exceeding the acceptance angle) is found to exceed subsequently,  $\alpha_c$ , and hence undergoes both reflection and refraction.

Despite having discussed many angles that form the conditions for achieving TIR, in practice, fiber-optic communications technology does not operate with any angles but, integrates all these factors into one characteristic called *numerical aperture* (NA). It is expressed as,

$$NA = \sin \theta_a \quad \dots (5.4)$$

However the following formula is the most often used one:

$$NA = \sqrt{n_1^2 - n_2^2} \quad \dots\dots (5.5)$$

From these relations, one can conclude that fiber-optic communications technology makes use of numerical aperture, NA, which describes the ability of an optical fiber to gather light from a source and then the ability to preserve, or save, this light inside the fiber because of total internal reflection.

Numerical aperture characterises the ability of the fiber to gather light from a source. It is more convenient to define a parameter called *relative index*,  $\Delta$ , whose variation could get us fibers of different NA values. It is given as,

$$\Delta = \left( \frac{n_1 - n_2}{n} \right)$$

where  $n$  is the average refractive index of the core and cladding.

$$\therefore \Delta = \frac{(n_1 - n_2)(n_1 + n_2)}{n(n_1 + n_2)}$$

$$\Delta = \frac{n_1^2 - n_2^2}{n_1(n_1 + n_2)} \quad [ \because n = n_1 = n_2 ]$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$n_1^2 - n_2^2 = n_1^2 2\Delta$$

$$\sqrt{n_1^2 - n_2^2} = n_1 \sqrt{2\Delta}$$

$$\therefore NA = n_1 \sqrt{2\Delta} \quad \dots\dots (5.6)$$

The present day fiber-optic communications system suffers from the following three major impediments:

1. Dispersion
2. Attenuation
3. Nonlinear effects.

Of these three aspects, we shall be focussing only on the first aspect. However the reasons for attenuation have already been mentioned in this chapter. Nonlinear effects arise in optical fibers whenever optical power through fibers becomes very large. Unfortunately nonlinear effects are highly detrimental in fiber communications and efforts to cope with them are being attempted.

## 5.7 Dispersion

Dispersion is the spreading of a light pulse as it travels down the length of an optical fiber. Dispersion ultimately limits the information-carrying capacity of a fiber. In fiber-optic communications, there are two major types of dispersions, namely,

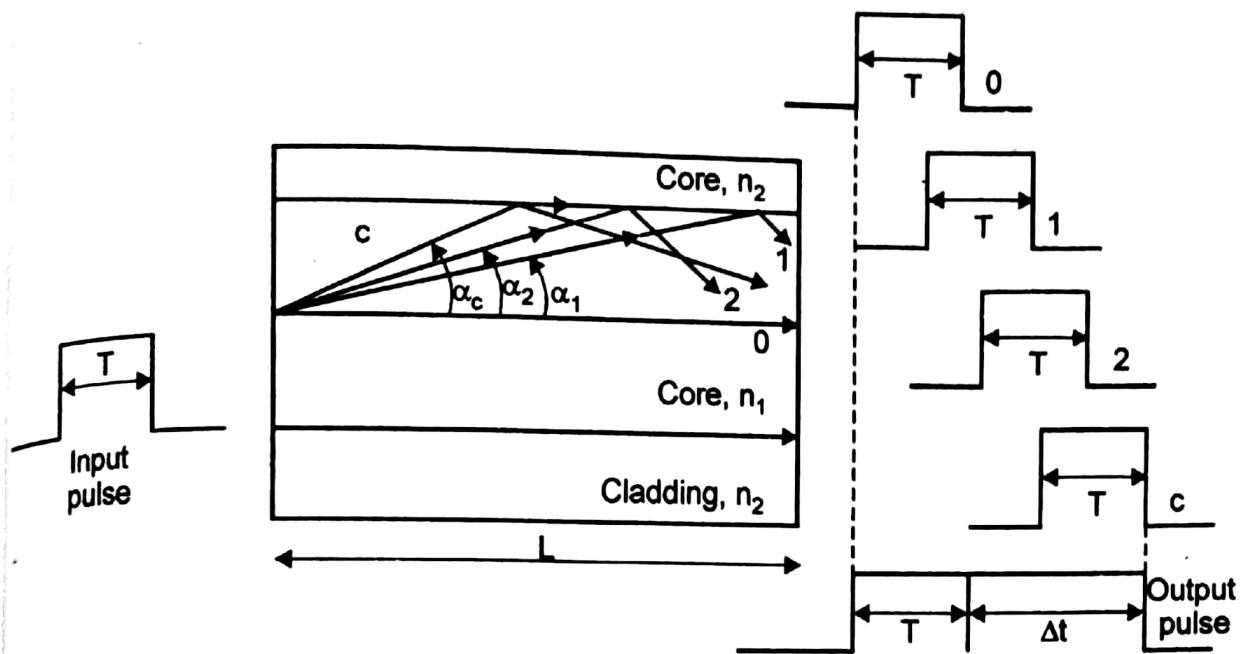
1. Intermodal dispersion and
2. Intramodal dispersion

### 5.7.1 Intermodal Dispersion

When an optical pulse is launched into a fiber, the optical power in the pulse is distributed over all of the modes of the fiber. The propagation of light within the fiber can be described in terms of a set of guided electromagnetic waves called the ‘modes’. Roughly, we can say that a mode is the path taken by the light rays while entering into the fiber. Hence different modes will travel with different propagation angles, the range being zero (corresponding to the fundamental mode) to the critical value,  $\alpha_c$  (corresponding to the critical mode). These modes take different routes but travel with the same velocity and hence, at the end of the fiber, they arrive at different timings. This ultimately causes the pulse widening. This is called *intermodal dispersion* or simply *modal dispersion* since it is the dispersion that arises within the different modes of a single pulse.

#### Measuring Intermodal Dispersion

Consider a beam propagating inside a fiber, taking into account the mode concept, as in Figure 5.6. For digital transmission, a light pulse represents logic 1, and no light pulse represents logic 0. Such light pulses, radiated by light source, enter a fiber, where each pulse breaks down into a set of small pulses carried by an individual mode. At the fiber output, individual pulses recombine and, since they are overlapping, the receiver sees one long light pulse whose rising edge is from the fundamental mode and whose falling edge is from the critical mode. This explanation is depicted in Figure 5.6, where four modes are shown as an example.



**Figure 5.6:** Modes in an optical fiber

To ascertain why these individual pulses arrive at the receiver end at different times, let us do a simple calculation. A zero-order mode travelling along the fiber axis needs time,

$$t_0 = L/v$$

to reach the receiver end. Here,  $L$  is the link length and  $v \left( = \frac{c}{n_1} \right)$  is the light velocity within the core having refractive index  $n_1$ . 'c' is the speed of light in vacuum. The highest order mode propagating at the critical angle needs time,

$$t_c = \frac{L}{(v \cos \alpha_c)}$$

to complete the path. Therefore the pulse widening due to intermodal dispersion is,

$$\Delta t_{SI} = t_c - t_0 = \frac{L}{v \cos \alpha_c} - \frac{L}{v}$$

$$\Delta t_{SI} = \frac{L}{v} \left[ \frac{1}{\cos \alpha_c} - 1 \right]$$

$$= \frac{L}{v} \left[ \frac{n_1}{n_2} - 1 \right] \quad \left[ \because \cos \alpha_c = \frac{n_2}{n_1} \right]$$

$$= \frac{Ln_1}{c} \left[ \frac{n_1 - n_2}{n_2} \right] \quad \left[ \because v = \frac{c}{n_1} \right]$$

Since  $n_2 \approx n$ , we can rewrite this equation as,

$$\Delta t_{SI} = \frac{Ln_1}{c} \left[ \frac{n_1 - n_2}{n} \right]$$

$$\Delta t_{SI} = \left( \frac{Ln_1}{c} \right) \Delta \quad \dots (5.7)$$

Where ' $\Delta$ ' is the relative refractive index. Equation (5.7) represents the pulse widening stemming from intermodal dispersion in the case of a step index fiber. Hence the intermodal dispersion ultimately restricts the bit rate of the optical communication system.

The first solution to the modal dispersion came with the realization of so called *graded-index fiber* and the physics behind reducing intermodal dispersion using graded-index fiber is as follows:

It should be recalled at this stage that intermodal dispersion arises in a step index fiber owing to the fact that different beams travel at the same velocity, but over different distances and hence arrive at the receiver end at different times.

A clever modification in the refractive index profile has considerably reduced the intermodal dispersion. Such a fiber is called *graded-index fiber*. In this fiber, refractive index is maximum (say,  $n_1$ ) at the axis of the core and decreases radially until it meets the core-cladding boundary. Hence the mode propagating along the centerline of this fiber - the shortest distance - travels at the lowest speed because it meets the highest refractive index. The mode traveling closer to the fiber cladding - the longer distance - propagates at the higher speed because it meets a lower refractive index. These are clearly shown in the Figure 5.7. Hence the fractions of an input pulse delivered by the different modes arrive at the receiver end more or less simultaneously. Therefore, intermodal dispersion will be relatively less and bit rate can be increased. It has been estimated that a graded-index fiber has a modal dispersion  $\Delta/8$  times less than that of a step-index fiber. However graded index fibers are relatively expensive when compared to step-index fibers because of the requirement of complex index profile.

Though a graded-index fiber exhibits relatively less intermodal dispersion, this dispersion is not totally absent. Hence a complete solution for the intermodal dispersion was thought of and subsequently realized too. Such a fiber is called *single-mode fiber*.

As the name indicates, this fiber sustains only one mode of propagation (fundamental mode). This is done by reducing the core diameter and relative refractive index. Since there is only one mode of propagation, intermodal dispersion is totally absent in the case of single-mode fibers. But it is the most expensive fiber to manufacture and most difficult to maintain, largely because of the difficulty in maintaining an accurate core size. Indeed the core size of a single mode fiber may vary from 4 to 11  $\mu\text{m}$ .

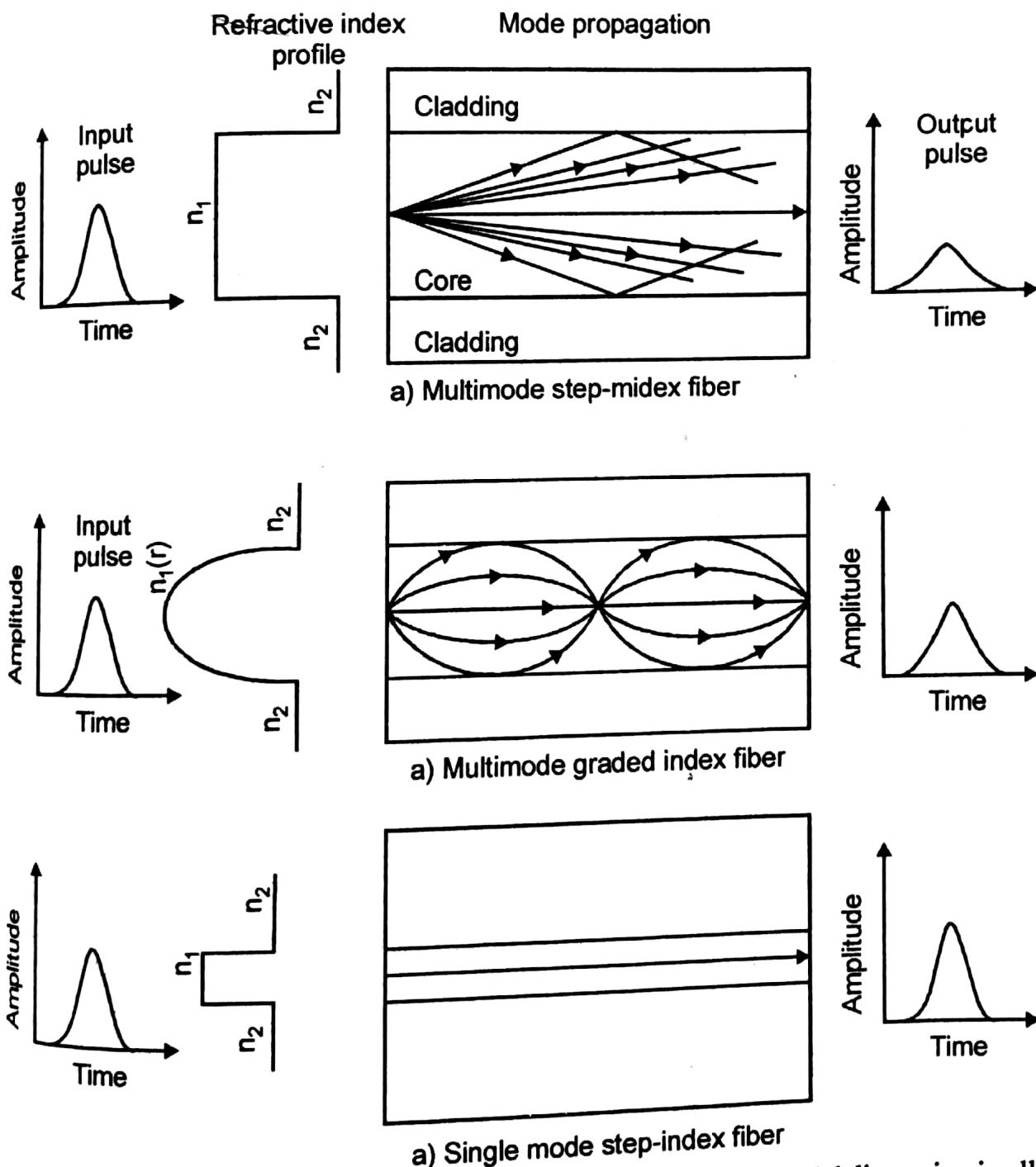


Figure 5.7: Schematic diagram for pulse broadening due to intermodal dispersion in all the three types of fibers

While step-index and graded index fibers are used for short and intermediate-distance networks, single mode fibers are the most popular type of link, particularly for long-distance communications.

From the Figure 5.7, it is clear that a multimode step-index fiber, which is generally called a step-index fiber, suffers from maximum intermodal dispersion. It is seen that a multimodal graded-index fiber, which is generally called a graded-index fiber, suffers from relatively less amount of intermodal dispersion. It is only the single mode step-index fiber (which is popularly called just a single-mode fiber) that does not suffer from intermodal dispersion at all.

However single-mode fibers do suffer from chromatic dispersion which is discussed below.

### 5.7.2 Intramodal Dispersion

As the name indicates, it is the dispersion that arises within a mode of the pulse. It is otherwise called chromatic dispersion. This dispersion is further classified as follows:

- i) Material dispersion
- ii) Waveguide dispersion

#### i) Material Dispersion

As no optical source is perfectly monochromatic, each pulse of light has several spectral components. These spectral components experience different indices inside the fiber and travel with different speeds. Hence they arrive at the end of the fiber with different timings. This dispersion is called material dispersion since it results from the refractive index variation of the material of the fiber with the wavelength of light propagating through the fiber. Material dispersion plays a major role in limiting the bandwidth of a single mode fiber.

#### ii) Waveguide Dispersion

It is caused by the fact that light is guided by a structure - here, an optical fiber. This dispersion, though exists in multimode fibers, is seen prominently only with single mode fibers. After entering a single-mode fiber, an information - carrying light pulse is distributed between the core and the cladding. Its major portion travels within the core and the rest within the cladding. Both the portions propagate at different velocities since the core and cladding have different indices. The pulse will spread simply because light is confined within a structure having different refractive indices - the core - cladding combination of the fiber. This dispersion is called *waveguide dispersion*. However, waveguide dispersion in a single-mode fiber is relatively small compared with material dispersion. It should be

noted that multimode fibers also suffer from chromatic dispersion. However modal dispersion is the major factor limiting multimode fiber bandwidth.

## 5.8 Meridional and Skew beams

All beams (modes) propagating within an optical fiber are divided into two categories: *meridional beams* and *skew beams*. Meridional beams are those that intersect the centerline of the fiber; skew beams propagate without intersecting the fiber's central axis. Skew rays are not confined to a single plane, but instead tend to follow a helical-type path along the fiber. These rays are more difficult to track as they travel along the fiber, since they do not lie in a single plane. Although skew rays constitute a major portion of the total number of guided rays, their analysis is not necessary to obtain a general picture of rays propagating in a fiber. Hence, it is sufficient to consider the meridional rays for all practical purposes.

In all the discussions of this chapter, only meridional rays have been considered

## 5.9 Types of Fibers

Optical fibers are classified based on the

- i) material with which they are made
- ii) refractive index profile and
- iii) number of modes employed

Based on the material used, we have following types of fibers.

- a) Low loss fiber
- b) Medium loss fiber
- c) Higher loss fiber

In low and medium loss fibers the core material is generally glass and is surrounded by either a glass or a plastic cladding. In the case of higher loss fiber, both core and cladding are made of plastic.

Variations in the material composition of the core have given rise to two commonly used fibers, namely, step-index fiber and graded index fiber.

In the case of a step-index fiber, the refractive index of the core is uniform throughout and undergoes an abrupt change at the cladding boundary. In graded index fibers, the core refractive index is made to vary as a function of the radial distance from the centre of the fiber.

Both the step and the graded index fibers can be further divided into single mode and multimode classes. As the name implies, a single mode fiber sustains only one mode of propagation, whereas multimode fibers sustain many hundreds of modes.

Multimode fibers offer several advantages compared with single-mode fibers. The larger core radii of multimode fibers make it easier to launch optical power into the fiber and facilitate the connecting of similar fibers. Another major advantage is that light can be launched into a multimode fiber using a light-emitting diode (LED) source, whereas single-mode fibers must generally be excited with laser diodes. However, as seen, multimode fibers suffer from intermodal dispersion. Hence, the most preferred fiber for communication purposes is single-mode fiber and 80% of the fibers laid all over the world belong to this category.

## 5.10 Applications of Fiber Optics in Communication

Owing to the several advantages of fiber-optic communications that were discussed in the beginning of this chapter, telecommunication industries consider optical communication as their primary technology. Several novel fibers designed especially to tackle the problem of dispersion have made a lot of improvements in communications technology.

We are living in an information society, where the efficient transfer of information is highly relevant to our well-being. Fiber-optic systems form the very means of such information transfer and have a very important role, directly or indirectly, in the development of almost every sphere of life.

The following are the areas in which fiber-optic technology has made a considerable impact:

- 1) Voice communication
  - Inter-office
  - Intercity
  - Intercontinental links
- 2) Video communication
  - TV Broadcast
  - Remote monitoring
  - Videophones
  - Cable television (CATV)
  - Wired city
- 3) Data transfer
  - Inter-office data link
  - Computers
  - Local area networks
  - Satellite ground stations
- 4) Internet
  - E-mail
  - Video conferencing

The greatest advantage with optical fibers is their adaptability to any technological revisions. In recent times, there have been a lot of improvements in fiber-optic communication systems, and the following technologies are main players for the same :

- Wavelength Division Multiplexing (WDM)
- Optical amplifiers
- Optical solitons

WDM represents the technology of sending several signals through a single fiber, ultimately increasing the information carrying capacity of a single fiber.

Optical amplifiers have the advantage of amplifying weak signal in the light domain itself. The most popular optical amplifier as on today is the erbium-doped fiber amplifier (EDFA). It is an ordinary fiber doped with erbium ions with a suitable pump laser. This has replaced the conventional electronic repeaters.

An optical soliton is a laser pulse realized in single-mode fibers by a careful balancing effects of dispersion and nonlinearity. It has the ability to travel several thousands of km through an optical fiber without dispersion and is more likely to be the candidate for transferring information through fiber links in future.

In recent years, fiber optics has found major application in sensor technology due to the inherent advantages of optical fibers.

## 5.11 Source

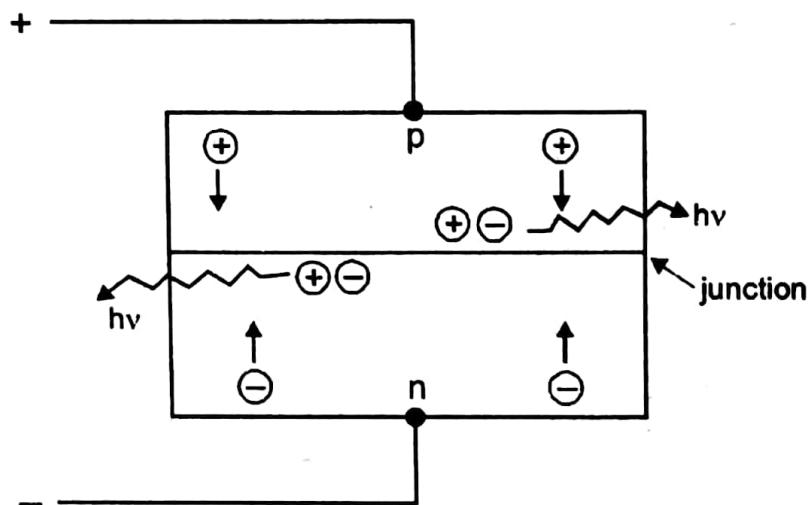
### 5.11.1 Light emitting diode (LED)

LEDs have been around for more than 35 years. They have found application in nearly every consumer-electronic device: TV sets, VCRs, telephones, car electronics, and many others. They are used in fiber-optic communications, mostly because of their small size and long life.

To embark on the discussion of an LED, a basic knowledge of semiconductor physics is a must. A semiconductor is a material that does not conduct electricity at normal temperature but starts conducting when the temperature is increased. It has a lower valence band (VB) which will be normally filled with electrons and a upper conduction band (CB) which will be empty. There is so called ‘energy gap’ that separates these two bands. To increase the conductivity of this intrinsic semiconductor, doping is done. By suitable doping, we can get both p-type and n-type material. When these materials are fused, they form a p-n junction diode.

A light-emitting diode is a semiconductor diode made by the creation of a junction with n-type and p-type materials. When the diode is forward biased, electrons and holes enter the depletion layer and recombine.

Unlike the case of a regular diode, these recombinations produce light. The recombination in the case of regular diodes is called non-radiative and in the case of LEDs, the recombination is called radiative.

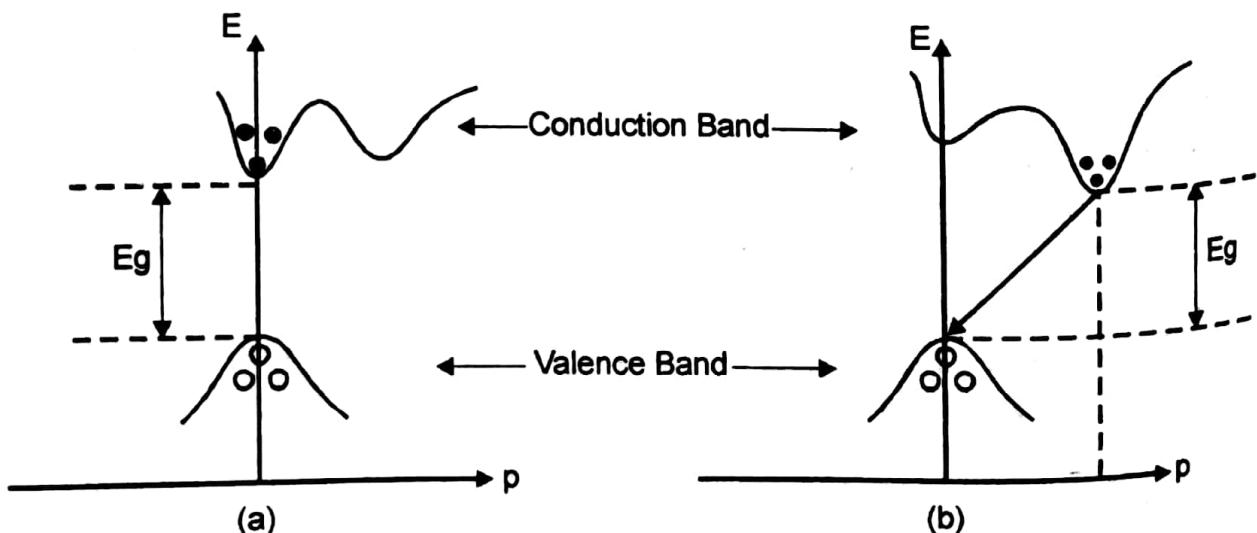


**Figure 5.8:** Basic idea of an LED

There are two types of semiconducting materials, namely,

- direct bandgap semiconductors
- indirect bandgap semiconductors.

The energy-momentum diagrams for these two types of materials are shown in figure 5.9.



**Figure 5.9:** Energy Vs momentum for (a) direct and (b) indirect bandgap semiconductors

In direct bandgap materials, the energy corresponding to the bottom of the CB and that corresponding to the top of the VB have almost the same values of the crystal momentum.

Thus there is a high probability of the direct recombination of electrons and holes, giving rise to the emission of photons. The materials in this category include GaAs, GaSb, InAs, etc.

In indirect bandgap materials, the energy corresponding to the bottom of the CB has excess crystal momentum as compared to that corresponding to the top of the VB.

Here, the electron-hole radiative recombination requires the simultaneous emission of a right photon as well as a right phonon (crystal lattice vibration) in order to conserve the momentum. The probability of such a transition is, therefore, low. The materials in this category include Ge, Si, etc.

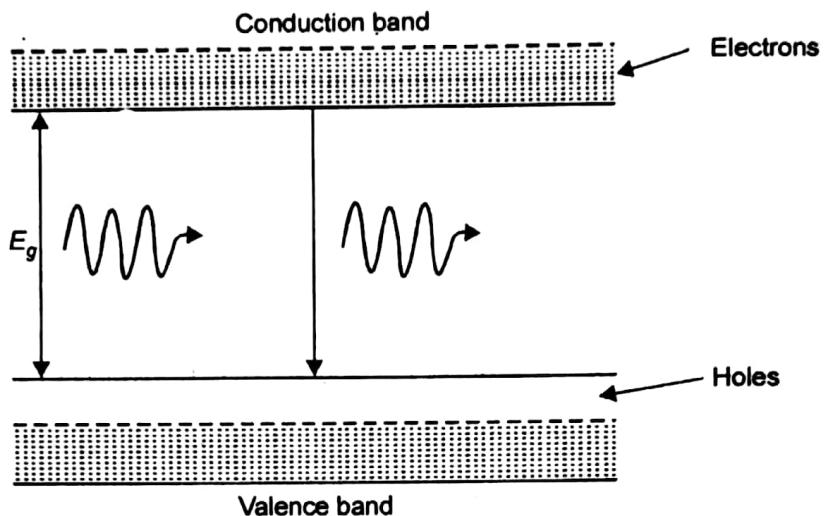
### 5.11.2 Laser diode

The semiconductor laser is one of the most important types of lasers in today's fiber-optic communication systems owing to several advantages such as the capability of direct modulation in the GHz region, small size and low cost, direct pumping with conventional electronic circuitry and compatibility with optical fibers.

The basic mechanism responsible for light emission from a semiconductor is the recombination of electrons and holes at p-n junction when a current is passed through the diode. As in any other laser systems, there are three possible interaction processes: (a) an electron in the valence band (VB) can absorb the incident radiation and be excited to the conduction band (CB) leading to the generation of electron hole pair; (b) an electron can make a spontaneous transition in which it combines with a hole, i.e., it makes a transition from the CB to VB, and during the process, it emits radiation; (c) stimulated emission may occur in which the incident radiation stimulates an electron in the CB to make a transition to the VB, and during the process, it emits radiation. If now by some mechanism, a large density of electrons is created at the bottom of the CB and simultaneously in the same region a large density of holes is produced at the top of the VB as in Figure 5.10, then such a semiconductor can amplify optical radiation at a frequency which corresponds to energy slightly greater than the bandgap energy.

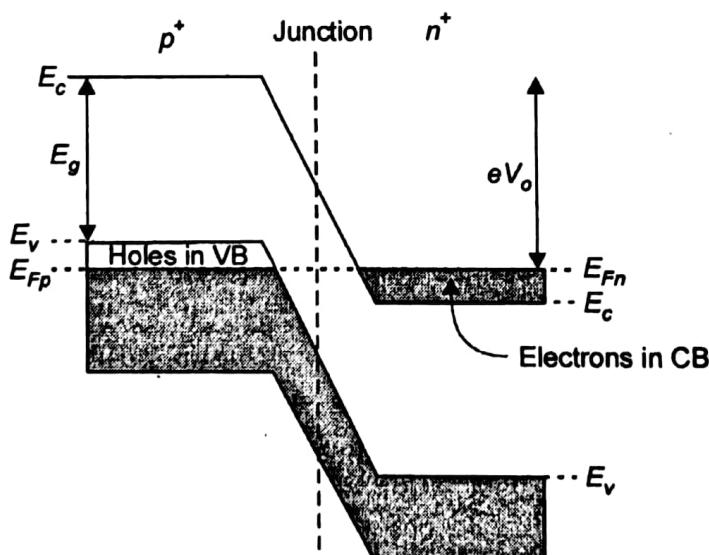
Thus, when a current is passed through a p-n junction under forward bias, the injected electrons and holes will increase the density of electrons in the CB and holes in the VB at a region close to the junction. Further, at some value of current, the stimulated emission

rate will exceed the absorption rate and amplification will begin. As the current is further increased, at some threshold value of current, the amplification will overcome the losses in the cavity and the laser will begin to emit coherent radiation.



**Figure 5.10:** Light amplification mechanism in a p-n junction diode

Consider a degenerately doped direct bandgap semiconductor p-n junction whose band diagram is shown in the Figure 5.11.

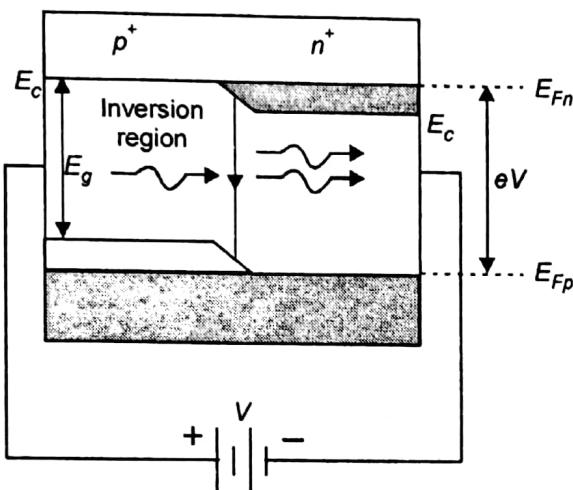


**Figure 5.11:** The energy band diagram of a degenerately doped p-n junction with no bias

By degenerate doping we mean that the Fermi level  $E_{Fp}$  in the p-side is in the valence band (VB) and the  $E_{Fn}$  in the n-side is in the conduction band (CB). All energy levels up to the Fermi level can be taken to be occupied by electrons as in Figure 5.11. In the absence of an applied voltage, the Fermi level is continuous across the diode,  $E_{Fp} = E_{Fn}$ . The depletion region or the space charge layer (SCL) in such a p-n junction is very narrow.

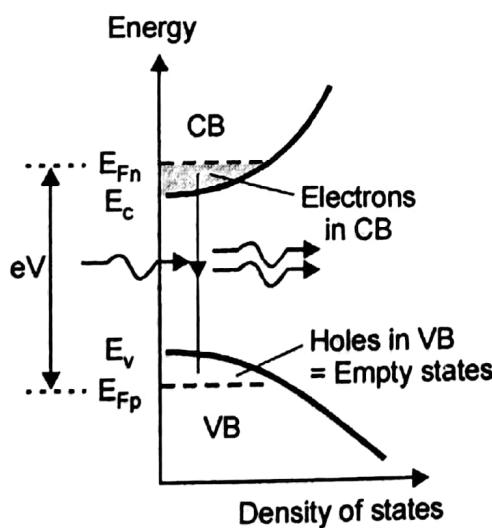
There is a built-in voltage,  $V_0$ , which gives rise to a potential energy barrier  $eV_0$  that prevents the electrons in the CB of  $n^+$ -side diffusing into the CB of the  $p^+$ . There is a similar barrier stopping hole diffusion from the  $p^+$ -side to  $n^+$ -side.

Suppose this junction is forward biased by a voltage  $V$  greater than the bandgap voltage;  $eV > E_g$  as shown in Figure 5.12.



**Figure 5.12:** Band diagram with a sufficiently large forward bias

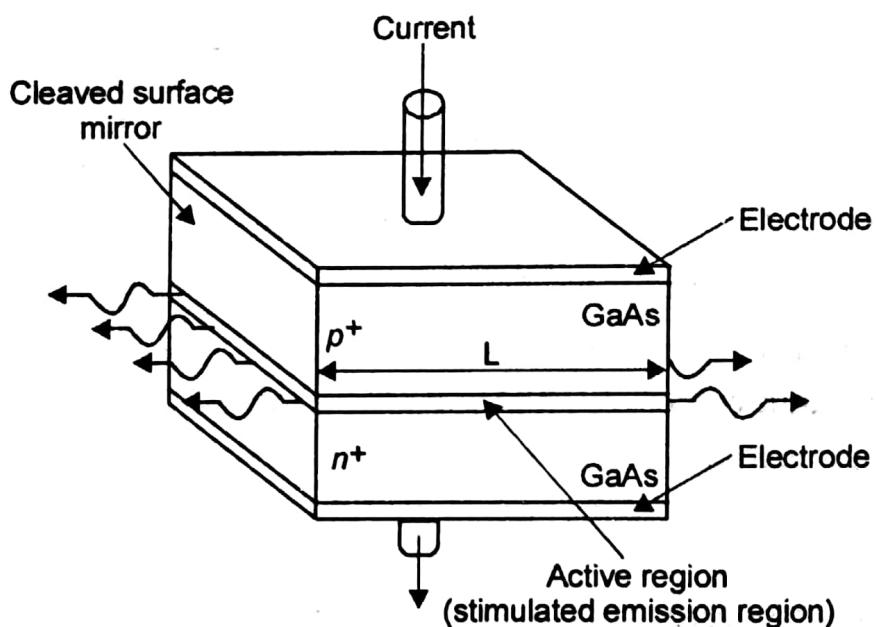
The separation between  $E_{Fn}$  and  $E_{Fp}$  is now the applied potential energy or  $eV$ . The applied voltage diminishes the built-in potential barrier to almost zero, which means that the electrons flow into the SCL and flow over to the  $p^+$ -side to constitute the diode current. There is a similar reduction in the potential barrier for holes from  $p^+$ -side to  $n^+$ -side. The final result is that electrons from  $n^+$ -side and holes from  $p^+$ -side flow into the SCL, and this SCL region is no longer depleted as apparent in the Figure 5.12.



**Figure 5.13:** The density of states and the energy distribution of electrons and holes in the CB and VB respectively under forward bias such that  $E_{Fn} - E_{Fp} > E_g$

If we draw the energy band diagram with  $E_{Fn} - E_{Fp} = eV > E_g$  this conclusion is apparent. In this region, there are more electrons in the CB at energies near  $E_c$  than electrons in the VB near  $E_v$ , as illustrated by density of states diagram for the junction region in Figure 5.13. In other words, there is a population inversion between energies near  $E_c$  and those near  $E_v$  around the junction.

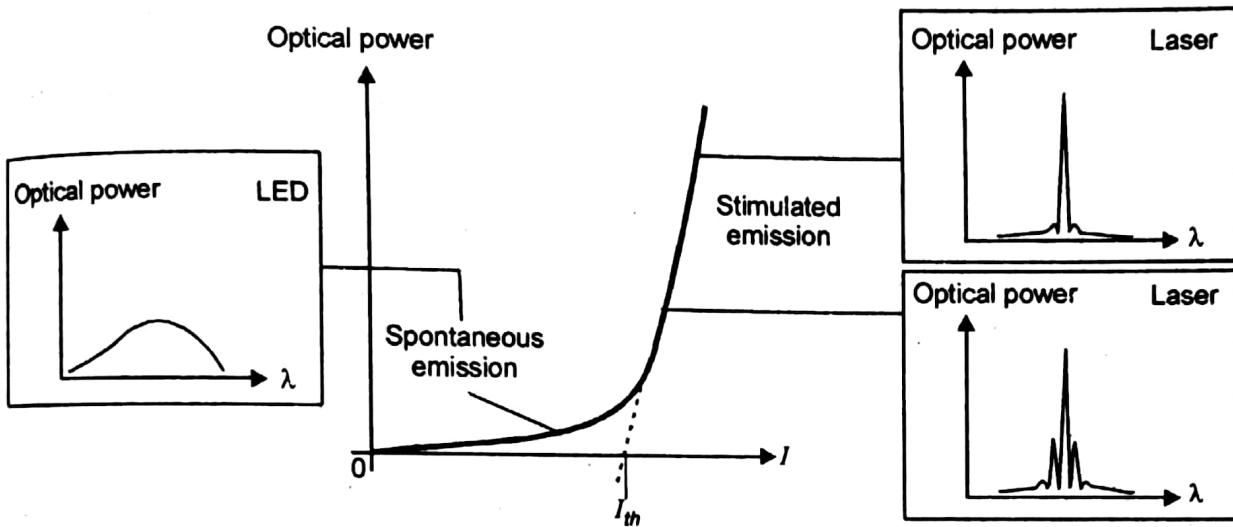
This population inversion region is a layer along the junction and is called inversion layer or the active region. An incoming photon with an energy of  $(E_c - E_v)$  cannot excite an electron from  $E_v$  to  $E_c$  as there are almost none near  $E_v$ . It can, however, stimulate an electron to fall down from  $E_c$  to  $E_v$  as shown in Figure 5.14. In other words, the incoming photon stimulates direct recombination.



**Figure 5.14:** Structure of a homojunction diode laser

A schematic structure for a homojunction laser is shown in Figure 5.14. Laser action can only be sustained within the narrow region called active region. Optical feedback is usually obtained by using the reflections from carefully cleaved end faces of the semiconductor. Although the amount of reflection is relatively small, it is often sufficient for lasing to take place. The amount of population inversion, and hence gain, is determined by the current flowing. At low current any population inversion achieved is offset by the losses present and lasing does not occur. Any radiation generated is due to spontaneous emission (as in an LED) which increases linearly with drive current. Beyond a critical current (the *threshold current*,  $I_h$ ), however, lasing commences and radiative output then increases very rapidly with increasing current, as shown in Figure 5.15.

One big difference between semiconductor lasers and most other lasers is their exceedingly small size. This severely limits the total power output, since at high powers the optical flux can cause mechanical damage to the facets and bring about failure of the laser.



**Figure 5.15:** Output optical power Vs. diode current ( $I$ ) characteristics and the corresponding output spectrum of a laser diode

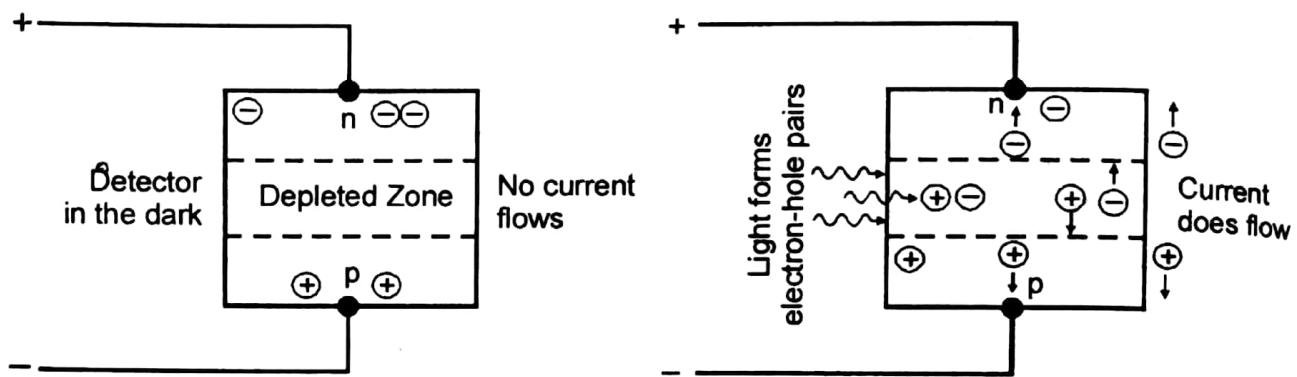
If high powers are required, then arrays of single lasers are used, which may be physically separate or fabricated on the same substrate.

## 5.12 Detector

### 5.12.1 p-n photodiode

The detectors used in fiber-optic communications are semiconductor photodiodes or photodetectors, which get their name from their ability to detect light. The simplest semiconductor detectors are solar cells, where incident light energy raises valence band electrons to the conduction band, generating an electric voltage. Unfortunately, such photovoltaic detectors are slow and insensitive.

Photodiodes are much faster and more sensitive if electrically reverse-biased as shown in Figure 5.16. The reverse bias draws current-carrying electrons and holes out of the junction region, creating a depleted region, which stops current from passing through the diode. Light of a suitable wavelength can create electron-hole pairs in this region by raising an electron from the valence band to the conduction band, leaving a hole behind. The bias voltage causes these current carriers to drift quickly away from the junction region, so a current flows proportional to the light illuminating the detector. The detector that has been discussed is called a p-n photodiode as it is formed by fusing a p type and n-type semiconductor.



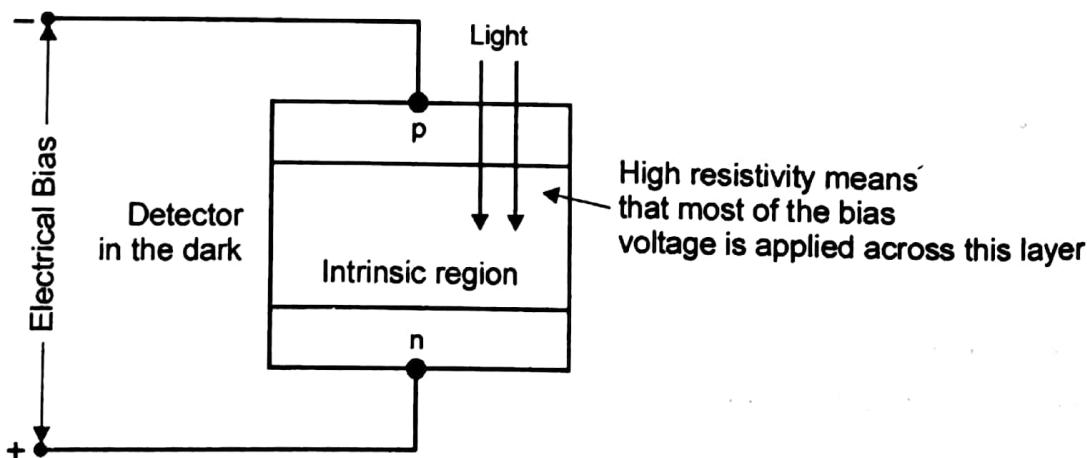
**Figure 5.16:** A p-n photodiode

There are several advantages of reverse biasing and the the following two are very important :

- As soon as the incident photon creates an electron-hole pair, this voltage helps to separate these free charges carriers and quickly removes them from the depletion region, thus generating photocurrent.
- The other advantage of reverse biasing is its ability to eliminate what is called ‘dark current’. Because any free charge carriers that are occasionally created without light will be swept away by the reverse-bias voltage.

### 5.12.2 p-i-n photodiode

The major feature of this positive-intrinsic-negative photodiode is that it consists of a thick, lightly doped intrinsic layer sandwiched between thin p and n regions.



**Figure 5.17:** A simple p-i-n Photodiode

The first and major feature of a p-i-n photodiode is that its intrinsic layer is its depletion layer where the absorption of photons occurs. Since the intrinsic layer is naturally thick,

most of the incident photons enter this layer and generate electron-hole pairs. This action results in the high quantum efficiency of this device.

Since the intrinsic layer contains almost no free charge carriers, the electric field across this layer is large. The result is the efficient separation of electrons and holes generated by the incident photons. In addition, this field decreases the dark current by sweeping away all thermally generated charge carriers. Another advantage of a p-i-n photodiode is that reverse-biasing voltage can be small because the thickness of the depletion region is controlled by the thickness of the intrinsic layer, not by the reverse voltage.

A p-i-n photodiode is the most commonly employed light detector in today's fiber-optic communications systems because of its ease in fabrication, high reliability, low noise, low voltage and relatively high bandwidth.

### 5.12.3 Responsivity (R) and Quantum efficiency ( $\eta$ )

The input for a photodiode is light power (P). The output is current, which is usually called photocurrent ( $I_p$ ) because it is caused by light. It follows from the principle of operation that the more photons that strike the active area of a photodiode, the more charge carriers will be created and hence greater will be the photocurrent.

$$\therefore I_p \propto P$$

$$\therefore I_p = RP \quad \dots (5.8)$$

where 'R' is a constant, called *responsivity*, measured in A/W.

We know photocurrent is the number of electrons ( $N_e$ ) times the electron charge (e) per unit time.

$$\therefore I_p = (e.N_e)/t \quad \dots (5.9)$$

where  $N_e$  represents the number of electrons generated and e, the electronic charge. On the other hand, light power is light energy per unit of time, where light energy is equal to the energy of a photon ( $E_p$ ) times the number of photons ( $N_p$ ).

$$\therefore P = (N_p E_p)/t \quad \dots (5.10)$$

Substituting  $E_p = \left(\frac{hc}{\lambda}\right)$  and dividing  $I_p$  by P, we have,

$$R = \frac{I_p}{P} = \left( \frac{N_e}{N_p} \right) \left( \frac{e\lambda}{hc} \right) \quad \dots (5.11)$$

The ratio of the number of produced electrons,  $N_e$ , to the number of falling photons,  $N_p$ , shows how efficiently the semiconductor material converts light into current. This ratio is called the *quantum efficiency*,  $\eta$ , of a photodiode.

$$\therefore \eta = \left( \frac{N_e}{N_p} \right) \quad \dots (5.12)$$

The quantum efficiency of a regular photodiode ranges from 50% to almost 100%.

$$\therefore R = \eta \left( \frac{e\lambda}{hc} \right) \quad \dots (5.13)$$

This is the equation connecting responsivity and quantum efficiency of a photodiode.

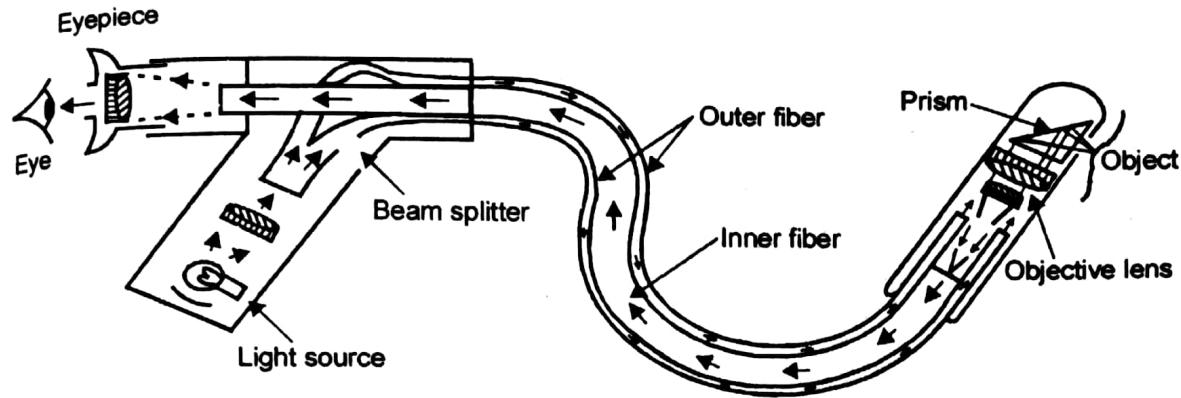
### 5.13 Endoscope

The term ‘endo’ means ‘inside’ and ‘scope’ means ‘device to look or see’. Hence an endoscope is a device to see or look inside.

Bundles of free fibers whose ends are bound together (e.g., with epoxy), ground, and polished form flexible light guides. When the fibers are carefully arranged so that their terminations occupy the same relative positions in both of the bound ends of the bundle, it is said to be a coherent bundle. Such an arrangement is capable of transmitting images and is consequently known as a flexible image carrier.

Coherent bundles are frequently fashioned by winding fibers on a drum to make ribbons, which are then carefully layered. When one end of such a device is placed face down flat on an illuminated surface, a point-by-point image of whatever is beneath it will appear at the other end. These bundles can be tipped off with a small lens, so that they need not be in contact with the object under examination.

Nowadays it is common to use fiber-optic instruments to poke into all sorts of unlikely places, from nuclear reactor cores and jet engines to stomachs and reproductive organs. When a device is used to examine internal body cavities, it is called an endoscope or a fiberscope. This category includes bronchoscopes, colonoscopes, gastroscopes, and so forth, all of which are generally less than about 200 cm in length. Figure 5.18 represents the structure of an endoscope.



**Figure 5.18: Endoscope**

Normally, in an endoscope, there will be a coherent bundle encapsulated within a fiber through which light will be sent to illuminate the region of examination. The reflected light from the region will be collected by the coherent bundle of fibers. A light collecting optical arrangement and an image viewing optical arrangement are placed at the respective fiber ends as shown in the diagram. However, for a wider field of view and better image quality, a telescope system is added in the internal part of the endoscope.

Endoscopes and fiber-optic sensors are the very important applications of optical fibers apart from their use in communications.

### Solved problems

1. In an optical fiber, the core refractive index is 1.4513 and the cladding refractive index is 1.4468. What is
  - i) Critical incident angle?
  - ii) Critical propagation angle ?
  - iii) Acceptance angle ?
  - iv) The numerical aperture ?

### Solution

Given data :  $n_1 = 1.4513$ ;  $n_2 = 1.4468$

$$\sin \theta_c = \frac{n_2}{n_1} \Rightarrow \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

$$\theta_c = \sin^{-1} \left( \frac{1.4468}{1.4513} \right)$$

$\theta_c = 85.49^\circ$

$$\text{We know, } \cos \alpha_c = \frac{n_2}{n_1} = \frac{1.4468}{1.4513}$$

$$\therefore \alpha_c = \cos^{-1} \left( \frac{1.4468}{1.4513} \right) = 4.51^\circ$$

$$\boxed{\alpha_c = 4.51^\circ}$$

$$\begin{aligned} \text{NA} &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{(1.4513)^3 - (1.4468)^2} \end{aligned}$$

$$\boxed{\text{NA} = 0.114}$$

$$\begin{aligned} \text{NA} &= \sin \theta_a \quad \therefore \theta_a = \sin^{-1}(\text{NA}) \\ &\theta_a = \sin^{-1}(0.114) \end{aligned}$$

$$\boxed{\theta_a = 6.55^\circ}$$

- 2. Calculate the numerical aperture of an optical fiber whose core index is 1.48 and relative index is 0.02.**

### Solution

$$\text{We know, } \text{NA} = n_1 \sqrt{2\Delta}$$

$$\text{Given data: } n_1 = 1.48 \text{ and } \Delta = 0.02$$

$$\therefore \text{NA} = 1.48 \sqrt{2 \times 0.02}$$

$$\text{NA} = 0.296$$

- 3. Calculate the numerical aperture of a step-index fiber having  $n_1 = 1.48$  and  $n_2 = 1.46$ . What is the maximum entrance angle for this fiber if the outer medium is air with  $n_a = 1.00$**

### Solution

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\text{Given data: } n_1 = 1.48; n_2 = 1.46; n, \text{ refractive index of air} \approx 1.00$$

$$NA = \sqrt{(1.48)^2 - (1.46)^2} = 0.2425 \Rightarrow NA = 0.2425$$

But  $NA = n_a \sin \theta_a \Rightarrow \theta_a = \sin^{-1}(NA)$   $[\because n_a \approx 1]$   
 $\theta_a = \sin^{-1}(0.2425) = 14^\circ$   
 $\theta_a = 14^\circ$

4. A Si p-i-n photodiode has a quantum efficiency of 0.7 at a wavelength of 0.85 μm. Calculate its responsivity.

Solution

Given data:  $\eta = 0.7$ ;  $\lambda = 0.85 \mu\text{m}$

$$R = ?$$

We Know,  $R = \eta \left( \frac{e\lambda}{hc} \right)$

$$\therefore R = 0.7 \times \frac{1.602 \times 10^{-19} \text{C} \times 0.85 \times 10^{-6} \text{m}}{6.626 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{ms}^{-1}}$$

$$R = 0.48 \text{A/W}$$

5. A particular photodetector has a responsivity of 0.6A/W for a light of wavelength 1.3μm. Calculate its quantum efficiency.

Solution

Given data:  $R = 0.6 \text{A/W}$ ;  $\lambda = 1.3 \mu\text{m}$

$$\eta = ?$$

We Know,  $R = \eta \left( \frac{e\lambda}{hc} \right)$

$$\therefore \eta = R \left( \frac{hc}{e\lambda} \right)$$

$$\eta = \frac{0.6 \text{A/W} \times 6.626 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{ms}^{-1}}{1.602 \times 10^{-19} \text{C} \times 1.3 \times 10^{-6} \text{m}}$$

$$\eta = 0.573$$

6. If a step-index fiber has a core of refractive index 1.5 and a cladding of refractive index 1.48, calculate, assuming that the fiber is kept in air, the  
 (a) NA of the fiber, (b) angles  $\theta_a$ ,  $\alpha_c$  and  $\theta_c$ .

### Solution

*Given data:*

$$n_1 = 1.5; n_2 = 1.48; n_a = 1$$

$$a) \text{ NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.5)^2 - (1.48)^2} = 0.244$$

$$b) \text{ NA} = n_a \sin \theta_a = 1 \times \sin \theta_a$$

$$\theta_a = \sin^{-1} \text{NA} = \sin^{-1}(0.244)$$

$$\theta_a = 14.13^\circ$$

$$\text{Also } n_a \sin \theta_a = n_1 \sin \alpha_c$$

$$\text{or } 0.244 = 1.5 \sin \alpha_c$$

$$\alpha_c = \sin^{-1} \left( \frac{0.244}{1.5} \right) = 9.36^\circ$$

Further

$$\sin \theta_c = \left( \frac{n_2}{n_1} \right) = \left( \frac{1.48}{1.5} \right)$$

$$\theta_c = \sin^{-1} \left( \frac{1.48}{1.5} \right) = 80.63^\circ$$

7. A p-n photodiode has a quantum efficiency of 70% for photons of energy  $1.52 \times 10^{-19} \text{ J}$ . Calculate, (a) the wavelength at which the diode is operating and (b) the optical power required to achieve a photocurrent of  $3 \mu\text{A}$  when the wavelength of the incident photons is that calculated in part (a).

### Solution

- (a) The energy of the photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{1.52 \times 10^{-19} \text{ J}} \\ = 1.3 \mu\text{m}$$

$$(b) R = \frac{\eta e}{hv} = \frac{0.70 \times 1.602 \times 10^{-19} \text{ C}}{1.52 \times 10^{-19} \text{ J}} \\ = 0.738 \text{ AW}^{-1}$$

Since  $R = \frac{I_p}{P}$

$$P = \frac{I_p}{R} = \frac{3 \times 10^{-6} \text{ A}}{0.738 \text{ AW}^{-1}} = 4.07 \mu\text{W}$$

8. A pin photodiode, on an average, generates one electron-hole pair per two incident photons at a wavelength of  $0.85 \mu\text{m}$ . Assuming all the photo-generated electrons are collected, calculate (a) the quantum efficiency of the diode, (b) the maximum possible bandgap energy (in eV) of the semiconductor, assuming the incident wavelength to be a long-wave length cut-off; and (c) the mean output photocurrent when the incident optical power is  $10 \mu\text{W}$ .

**Solution:**

$$a) \eta = \frac{1}{2} = 0.5 \text{ (i.e.,) } 50\%$$

$$b) E_g = \frac{hc}{\lambda_c} = \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{0.85 \times 10^{-6} \text{ m}} \\ = 2.34 \times 10^{-19} \text{ J} = 1.46 \text{ eV}$$

$$c) I_p = RP = \left( \frac{\eta e}{hv} \right) P \\ = \frac{0.5 \times 1.602 \times 10^{-19} \text{ C}}{2.34 \times 10^{-19} \text{ J}} \times 10 \times 10^{-6} \text{ W} = 3.42 \mu\text{A}$$

9. A step index fiber has higher core index and lower cladding index of 1.5 and 1.45, respectively. Calculate (a) critical incident angle of the fiber, (b) the corresponding acceptance angle of the fiber in air and (c) NA of the fiber.

### Solution

*Given data:*

$$n_1 = 1.5; \quad n_2 = 1.45$$

$$\text{a)} \quad \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1.45}{1.50} \right) = 75.17^\circ$$

$$\text{b)} \quad n_a \sin \theta_a = n_1 \sin \alpha_c \\ \therefore \theta_c = 75^\circ, \alpha_c = 90^\circ - 75.17^\circ = 14.83^\circ$$

$$\therefore \sin \theta_a = (1.5) \sin (14.83) = 0.384 \quad [\because n_a \approx 1]$$

$$\therefore \theta_a = \sin^{-1}(0.384) = 22.58^\circ$$

$$\text{c)} \quad \text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.5)^2 - (1.45)^2} = 0.3841$$

10. When  $2.5 \times 10^{12}$  photons generated by a laser source of wavelength  $0.85 \mu\text{m}$  are incident on a photodiode,  $1.5 \times 10^{12}$  electrons on an average are collected at the output terminal. Calculate the quantum efficiency and the responsivity of the photodiode at the above wavelength.

### Solution

Quantum efficiency,

$$\eta = \frac{\text{No. of electrons generated}}{\text{No. of incident photons}} = \frac{1.5 \times 10^{12}}{2.5 \times 10^{12}} = 0.6$$

$$\text{Responsivity, } R = \eta \left( \frac{e\lambda}{hc} \right)$$

$$R = \frac{0.6 \times 1.602 \times 10^{-19} \text{C} \times 0.85 \times 10^{-6} \text{m}}{6.626 \times 10^{-34} \text{J s} \times 3 \times 10^8 \text{ ms}^{-1}} = 0.4110 \text{ AW}^{-1}$$

## Points to Remember

- Light is guided through optical fibers by total internal reflection if it enters the fiber within an acceptance angle, measured directly or as the numerical aperture.
- The core of an optical fiber must have a higher refractive index than the cladding surrounding it.
- An optical fiber that supports only the fundamental mode is called a single-mode fiber.
- Intermodal dispersion is maximum in the case of step index fibers, minimum in the case of graded index fibers and totally absent in single-mode fibers.
- Single mode fibers suffer from chromatic dispersion only.
- Both inter and intramodal dispersions arise in multimode fibers.
- Meridional beams intersect the centerline of the fiber.
- Skew beams don't intersect the centerline of the fiber.
- LEDs are realized using direct bandgap semiconductors.
- Detectors always need to be reverse-biased.
- Responsivity is measured in A/W.
- Quantum efficiency of a photodiode is the ratio of the number of produced electrons to the total number of falling photons.

## Review Questions

### Short Answer Questions

1. Mention a few advantages of fiber-optic communications.
2. Draw the basic structure of an optical fiber and name its parts.
3. What are the conditions that are to be met to achieve total internal reflection?
4. What is critical incident angle?
5. Define critical propagation angle.
6. What do you mean by the acceptance angle?
7. Define numerical aperture of an optical fiber.
8. Differentiate between a step-index and a graded-index fiber.
9. Differentiate between a multimode fiber and a single-mode fiber.
10. What is intermodal dispersion?
11. What is the reason for the origin of the material dispersion ?
12. What is waveguide dispersion?
13. Why is intermodal dispersion less in a graded-index fiber when compared to a step-index fiber?
14. Does a single-mode fiber suffer from intermodal dispersion? Defend your answer.
15. Differentiate between a meridional ray and a skew ray.
16. Why is light emission not possible with indirect bandgap semiconductors?
17. What is the role of a detector in a fiber-optic system?
18. What is a 'dark current'?
19. Mention any two advantages of reverse biasing in the case of a photodiode.
20. What is the main advantage of a p-i-n photodiode over a p-n photodiode?
21. Define responsivity of a photodiode.
22. Define quantum efficiency of a photodiode.
23. What is an endoscope?

24. A silica fiber with a core diameter large enough to be considered by ray theory analysis has a core refractive index 1.50 and a cladding refractive index 1.47.

Determine

- the critical angle at the core-cladding interface
- the NA of the fiber
- the acceptance angle in air for the fiber.

$$[\theta_c = 78.5^\circ; \text{NA} = 0.30; \theta_a = 17.5^\circ]$$

25. A step-index fiber has an acceptance angle of  $20^\circ$  in air and a relative refractive index difference of 3%. Estimate the NA and the critical angle at the core-cladding interface.

$$[\text{NA} = 0.34, \theta_c = 76^\circ]$$

26. Photons of wavelength  $0.90 \mu\text{m}$  are incident on a p-n photodiode at a rate of  $5 \times 10^{10}/\text{s}$  and, on an average, the electrons are collected at the terminals of the diode at the rate of  $2 \times 10^{10}/\text{s}$ .

Calculate

- the quantum efficiency and
- the responsivity of the diode at this wavelength.

$$[\eta = 0.4; R = 0.29 \text{ A/W}]$$

27. Calculate the responsivity of an ideal p-n photodiode at  $0.85 \mu\text{m}$ .  $[R = 0.685 \text{ A/W}]$

### Descriptive Questions

- Explain, with the block diagram, the operation of a typical fiber-optic communication system. List the advantages of fiber-optic communications.
- Describe the conditions for the light propagation through optical fibers and hence explain the following terms:
  - Critical incident angle
  - Critical propagation angle
  - Acceptance angle
  - Numerical aperture
- Draw the structure of an optical fiber and explain its parts. Explain, in detail, the different types of optical fibers with suitable diagrams.

31. What is dispersion? Calculate the pulse spreading due to intermodal dispersion in the case of a step-index fiber.
32. Describe the two dispersions that appear under the title of intramodal dispersion. Explain why single mode fibers are the most preferred one for long-distance communications.
33. Explain the construction and the operation of a light emitting diode. Describe about both the direct and indirect bandgap semiconductors.
34. Describe the construction and working of a semiconductor laser. Mention its merit and demerits.
35. Explain about a p-i-n photodiode with a suitable diagram. What are its advantages compared with a simple p-n photodiode? Arrive at the relationship connecting responsivity and quantum efficiency of a photodiode.
36. Explain the operation of an endoscope with a neat sketch.