RELIABILITY

Reliability

Reliability may be defined as the probability that a component (or a system) will perform properly for a specified period of time 't' under a given set of operating conditions.

In other words, it is the probability that the component does not fail during the interval (0, t).

Concept of Reliability: Time to failure or Life length

If a component is put in to operation at t = 0 and if T is the time until it fails or ceases to function properly then T is called the life length or time to failure of the component.

Here $T \geq 0$ is a continuous random variable with some probability function f(t). Then the reliability function of the component at time 't' denoted by R(t) s defined as

$$R(t) = P(T > t) \text{ or } 1 - P(T \leq t)$$

$R(t) = 1 - F(t)$ where F(t) is the CDF of T, given by $F(t) = \int_0^t f(t)dt$

$$R(t) = 1 - \int_0^t f(t)dt = \int_t^\infty f(t)dt$$

$$R(t) = \int_t^\infty f(t)dt$$

$F(t) = \int_0^t f(t)dt$; Since $F(0) = 0$; $F(\infty) = 1$.

By the property of $R(0) = 1$, $R(\infty) = 0$

$$0 \leq R(t) \leq 1$$

Since $\frac{d}{dt}\big(F(t)\big) = f(t)$; $\qquad f(t) = -\frac{d}{dt}(R(t))$

The conditional probability of failure in the interval $(t, t+\Delta t)$, given that the component has survived up to time t,

$$P(t \le T \le t+\Delta t \,/\, T \ge t) = P(t \le T \le t+\Delta t) \,/\, P(T \ge t) = f(t)\,\Delta t/(1 - F(t))$$

$$= f(t)\,\Delta t/R(t)$$

Definition

Instantaneous Failure Rate/Hazard function

The conditional probability of failure per unit time is given by f(t)/R(t) (by taking $\Delta t = 1$) is called the Instantaneous failure rate or Hazard function of the component denoted by

$$\lambda(t) = f(t)/R(t)$$

By using the above result we have $f(t) = \lambda(t)e^{-\int_0^t \lambda(t)dt}$

Definition

MTTF- Mean Time To Failure

The expected value of the time to failure T is denoted by E(T) and is called the mean time to failure (MTTF)

$$\text{MTTF} = E(T) = \int_0^\infty R(t)dt$$

$$\text{Var}(T) = \sigma^2_T = \int_0^\infty t^2 f(t)dt - (MTTF)^2$$

Burn-in period (or after warranty period T_0)

Using conditional reliability, we will find the reliability of a component or system following wear in period (or burn in period) or after warranty period(T_0)

$$R(t/T_0) = P(T > T_0 + t \,/\, T > T_0) = P(T > T_0 + t)/P(T > T_0) = R(T_0 + t)/R(T_0)$$

$$= R(t/T_0) = e^{-\int_{T0}^{T0+t} \lambda(t)dt}$$