$\dots$ have $k = -2.37$.

Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

: The z values corresponding to $x_1 = 45$ and $x_2 = 62$ are

$$z_1 = \frac{45 - 50}{10} = -0.5, \quad \text{and} \quad z_2 = \frac{62 - 50}{10} = 1.2.$$

Therefore,

$$P(45 < X < 62) = P(-0.5 < Z < 1.2).$$

The $P(-0.5 < Z < 1.2)$ is shown by the area of the shaded region of Figure 6.11. This area may be found by subtracting the area to the left of the ordinate $z = -0.5$ from the entire area to the left of $z = 1.2$. Using Table A.3, we have

$$\begin{aligned} P(45 < X < 62) &= P(-0.5 < Z < 1.2) = P(Z < 1.2) - P(Z < -0.5) \\ &= 0.8849 - 0.3085 = 0.5764. \end{aligned}$$

Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

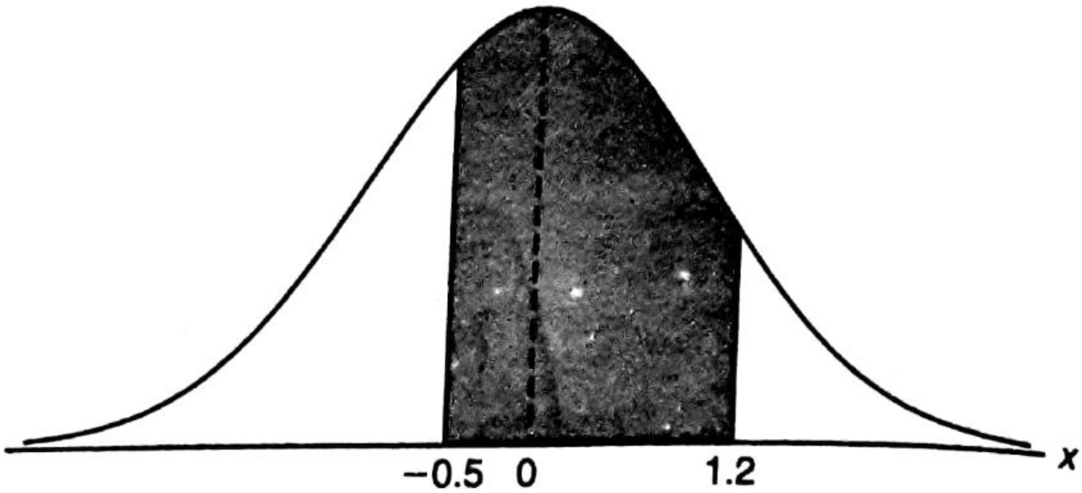

Figure 6.11: Area for Example 6.4.

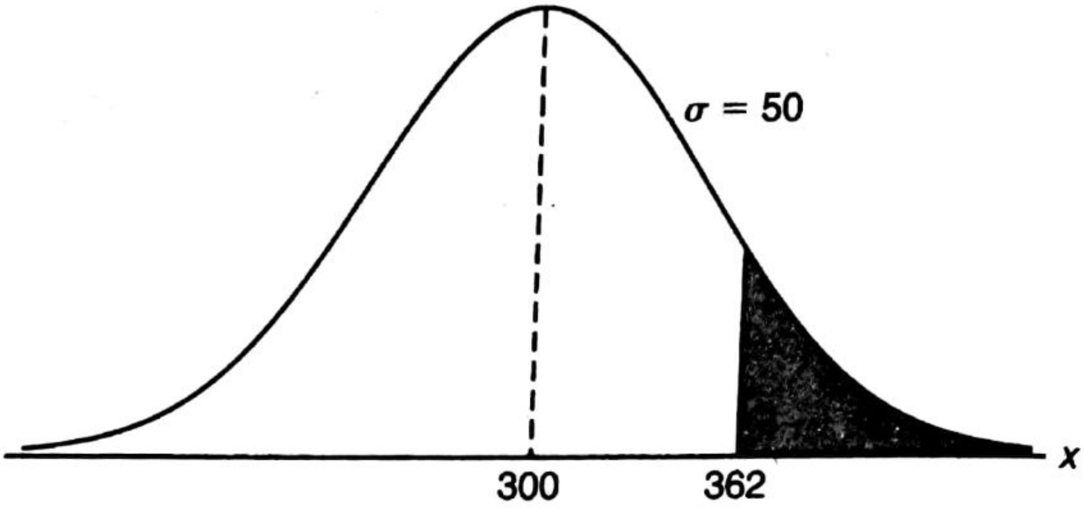

Figure 6.12: Area for Example 6.5.

***n*:** The normal probability distribution showing the desired area is shown by Figure 6.12. To find the $P(X > 362)$, we need to evaluate the area under the normal curve to the right of $x = 362$. This can be done by transforming $x = 362$ to the corresponding z value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find that

$$z = \frac{362 - 300}{50} = 1.24.$$

Hence

$$P(X > 362) = P(Z > 1.24) = 1 - P(Z < 1.24) = 1 - 0.8925 = 0.1075.$$

According to Chebyshev's theorem, the probability that a random variable

Example 6.7: A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

***Solution*:** First construct a diagram such as Figure 6.14, showing the given distribution of battery lives and the desired area. To find the $P(X < 2.3)$, we need to evaluate the area under the normal curve to the left of 2.3. This is accomplished by finding the area to the left of the corresponding z value. Hence we find that

$$z = \frac{2.3 - 3}{0.5} = -1.4,$$

and then using Table A.3 we have

$$P(X < 2.3) = P(Z < -1.4) = 0.0808.$$

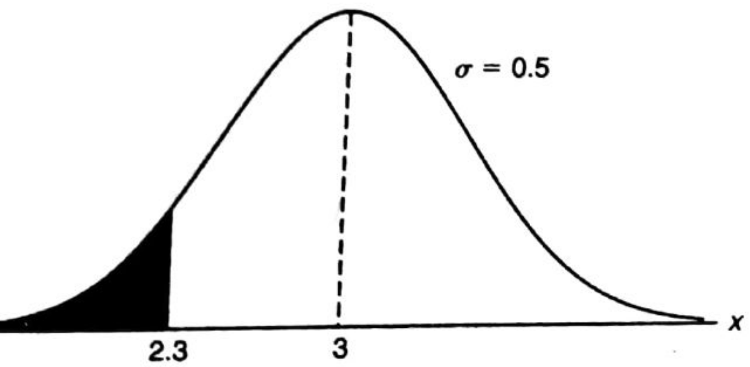

Figure 6.14: Area for Example 6.7.

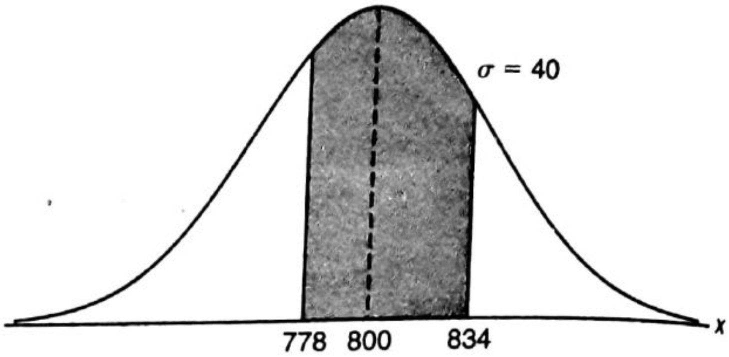

Figure 6.15: Area for Example 6.8.

Example 6.8: An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

***Solution*:** The distribution of light bulbs is illustrated by Figure 6.15. The z values corresponding to $x_1 = 778$ and $x_2 = 834$ are

$$z_1 = \frac{778 - 800}{40} = -0.55, \quad \text{and} \quad z_2 = \frac{834 - 800}{40} = 0.85.$$

Hence

$$P(778 < X < 834) = P(-0.55 < Z < 0.85) = P(Z < 0.85) - P(Z < -0.55)$$
$$= 0.8023 - 0.2912 = 0.5111.$$

Example 6.9: In an industrial process the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On the average, how many manufactured ball bearings will be scrapped?

Solution: The distribution of diameters is illustrated by Figure 6.16. The values corresponding to the specification limits are $x_1 = 2.99$ and $x_2 = 3.01$. The corresponding z values are

$$z_1 = \frac{2.99 - 3.0}{0.005} = -2.0, \quad \text{and} \quad z_2 = \frac{3.01 - 3.0}{0.005} = +2.0.$$

Hence

$$P(2.99 < X < 3.01) = P(-2.0 < Z < 2.0).$$

From Table A.3, $P(Z < -2.0) = 0.0228$. Due to symmetry of the normal distribution, we find that

$$P(Z < -2.0) + P(Z > 2.0) = 2(0.0228) = 0.0456.$$

As a result, it is anticipated that on the average, 4.56% of manufactured ball bearings will be scrapped.

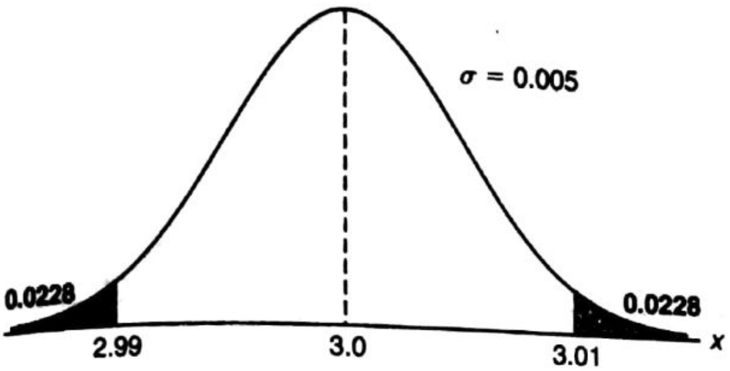

Figure 6.16: Area for Example 6.9.

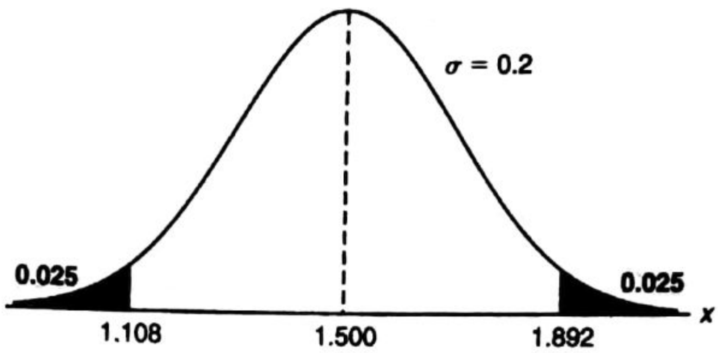

Figure 6.17: Specifications for Example 6.10.

Example 6.10: Gauges are used to reject all components where a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2. Determine the value d such that the specifications "cover" 95% of the measurements.

Solution: From Table A.3 we know that

$$P(-1.96 < Z < 1.96) = 0.95.$$

Therefore,

$$1.96 = \frac{(1.50 + d) - 1.50}{0.2},$$

from which we obtain

$$d = (0.2)(1.96) = 0.392.$$

An illustration of the specifications is shown in Figure 6.17.

Example 6.11: A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

Solution: A percentage is found by multiplying the relative frequency by 100%. Since the relative frequency for an interval is equal to the probability of falling in the interval, we must find the area to the right of $x = 43$ in Figure 6.18. This can be done by transforming $x = 43$ to the corresponding z value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find

$$z = \frac{43 - 40}{2} = 1.5.$$

Therefore,

$$P(X > 43) = P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668.$$

Hence, 6.68% of the resistors will have a resistance exceeding 43 ohms.

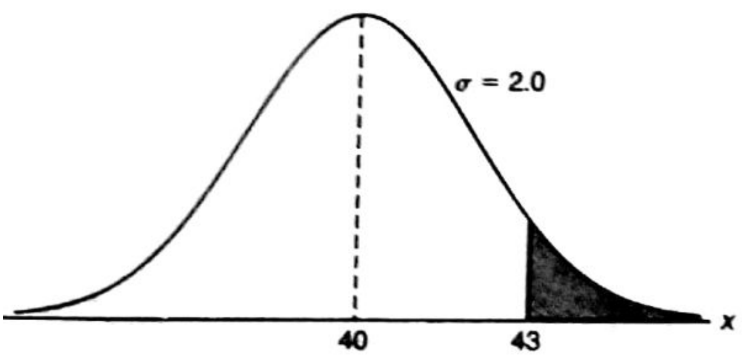

Figure 6.18: Area for Example 6.11.

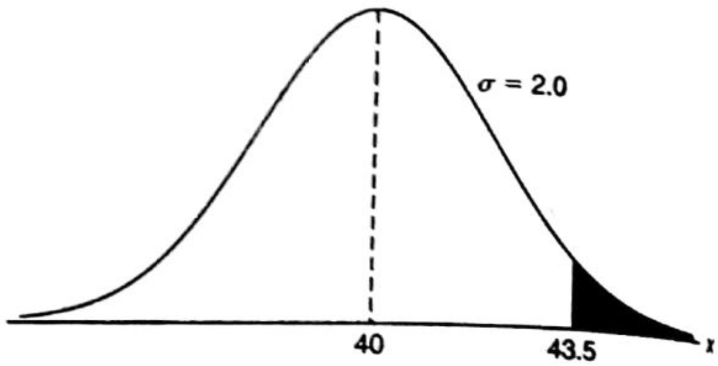

Figure 6.19: Area for Example 6.12.

Example 6.12: Find the percentage of resistances exceeding 43 ohms for Example 6.11 if resistance is measured to the nearest ohm.

Solution: This problem differs from Example 6.11 in that we now assign a measurement of 43 ohms to all resistors whose resistances are greater than 42.5 and less than 43.5. We are actually approximating a discrete distribution by means of a continuous normal distribution. The required area is the region shaded to the right of 43.5 in Figure 6.19. We now find that

$$z = \frac{43.5 - 40}{2} = 1.75.$$

Hence

$$P(X > 43.5) = P(Z > 1.75) = 1 - P(Z < 1.75) = 1 - 0.9599 = 0.0401.$$

Therefore, 4.01% of the resistances exceed 43 ohms when measured to the nearest ohm. The difference $6.68\% - 4.01\% = 2.67\%$ between this answer and that of Example 6.11 represents all those resistors having a resistance greater than 43 and less than 43.5 that are now being recorded as 43 ohms.