Infinite Square-Well Potential

- The simplest such system is that of a particle trapped in a box with infinitely hard walls that the particle cannot penetrate. This potential is called an infinite square well and is given by

$$V(x) = \begin{cases} \infty & x \le 0, x \ge L \\ 0 & 0 < x < L \end{cases}$$

- Clearly the wave function must be zero where the potential is infinite.

- Where the potential is zero inside the box, the Schrödinger wave equation becomes $\dfrac{d^2\psi}{dx^2} = -\dfrac{2mE}{\hbar^2}\psi = -k^2\psi$ where $k = \sqrt{2mE/\hbar^2}$.

- The general solution is $\psi(x) = A\sin kx + B\cos kx$.

Quantization

- Boundary conditions of the potential dictate that the wave function must be zero at x = 0 and x = L. This yields valid solutions for integer values of n such that $kL = n\pi$.
- The wave function is now $\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right)$
- We normalize the wave function

$$\int_{-\infty}^{\infty} \psi_n^*(x)\psi_n(x)\,dx = 1 \qquad A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

- The normalized wave function becomes

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

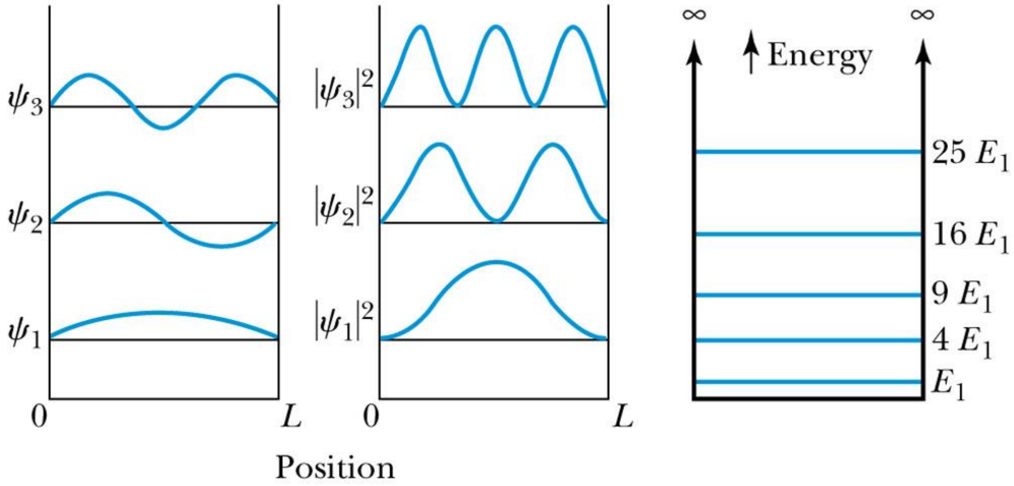
- These functions are identical to those obtained for a vibrating string with fixed ends.

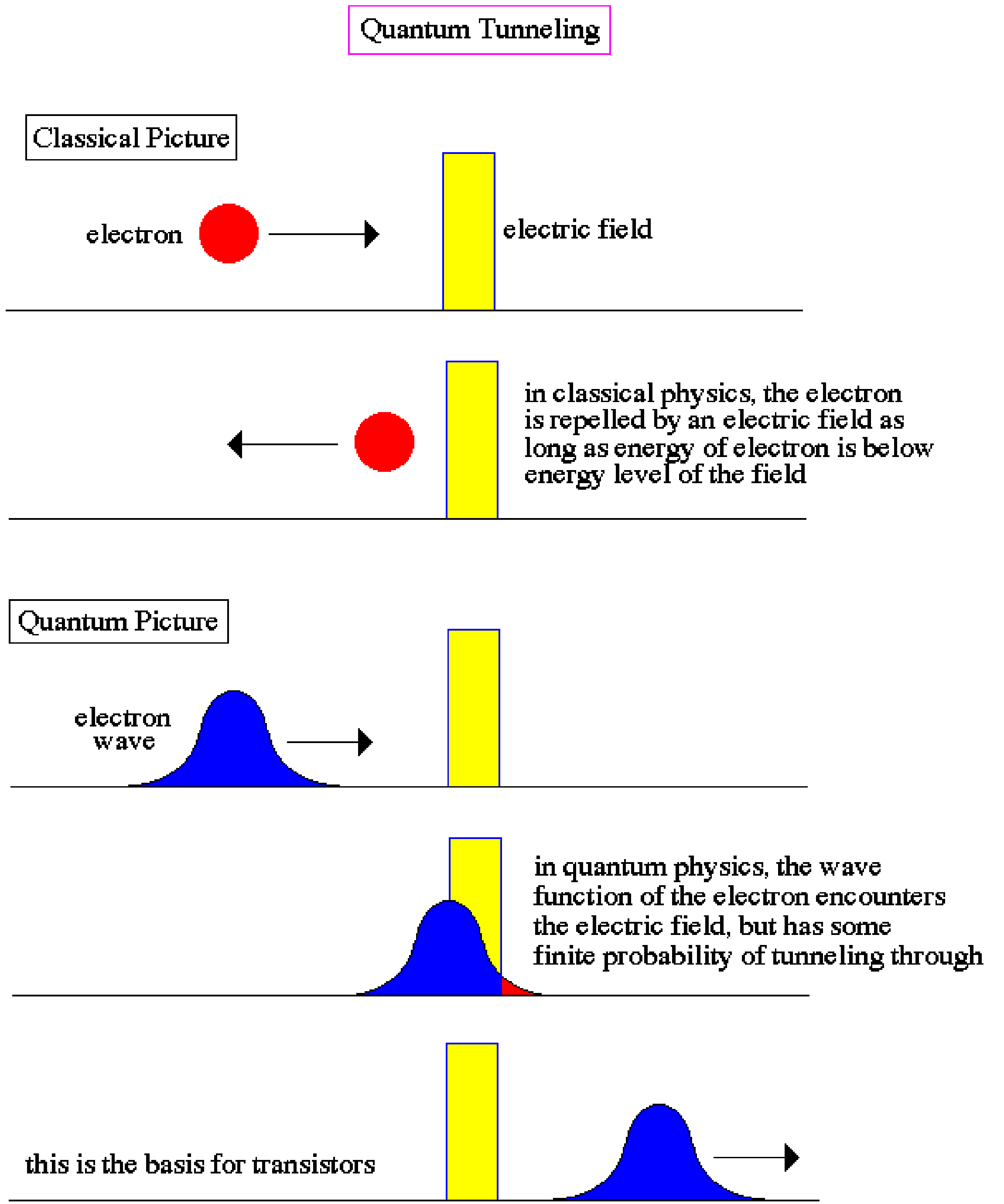
Quantized Energy

- The quantized wave number now becomes $k_n = \frac{n\pi}{L} = \sqrt{\frac{2mE_n}{\hbar^2}}$
- Solving for the energy yields

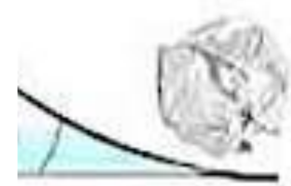
$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} \qquad (n = 1, 2, 3, \ldots)$$

- Note that the energy depends on the integer values of *n*. Hence the energy is quantized and nonzero.
- The special case of *n* = 0 is called the ground state energy. $E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$

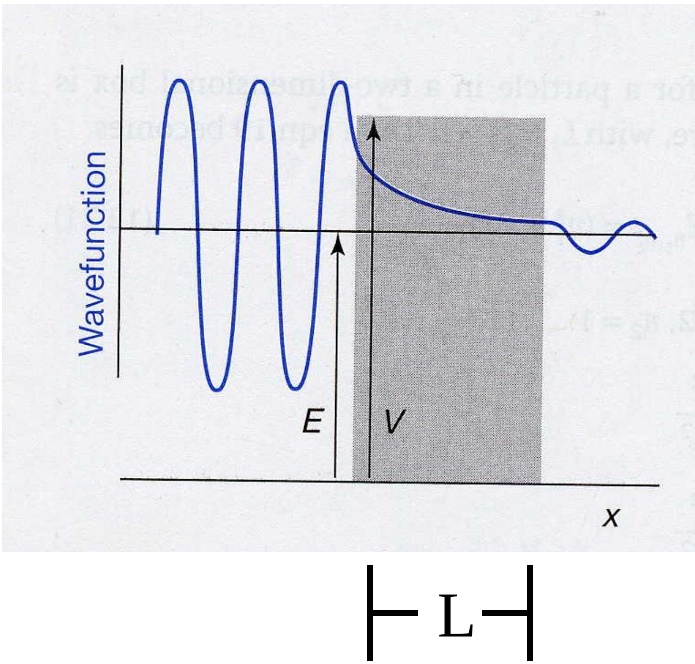


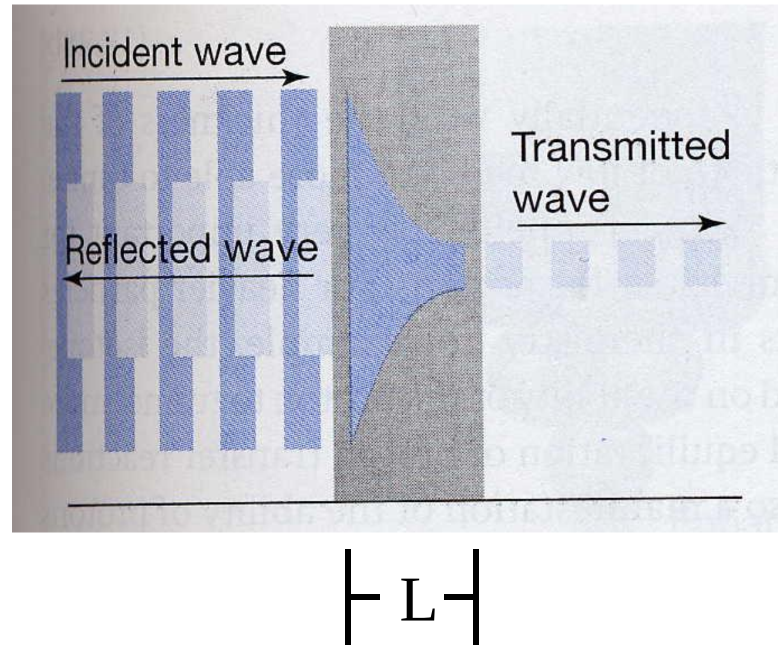


Quantum Tunneling
Classical Picture
electron
electric field
in classical physics, the electron is repelled by an electric field as long as energy of electron is below energy level of the field
Quantum Picture
electron wave
in quantum physics, the wave function of the electron encounters the electric field, but has some finite probability of tunneling through
this is the basis for transistors



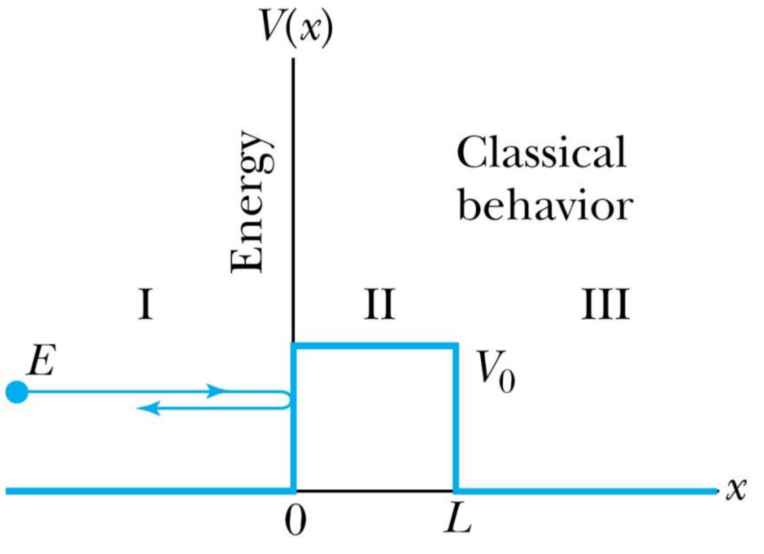
Quantum Mechanical Tunneling





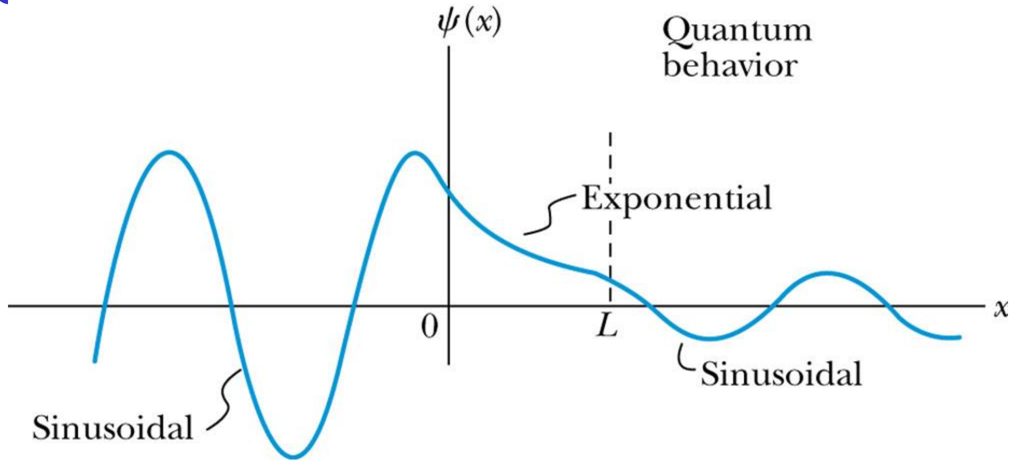
- Quantum mechanics allows a small particle, such as an electron, to overcome a potential barrier larger than its kinetic energy.
- Tunneling is possible because of the wave-like properties of matter.

Tunneling



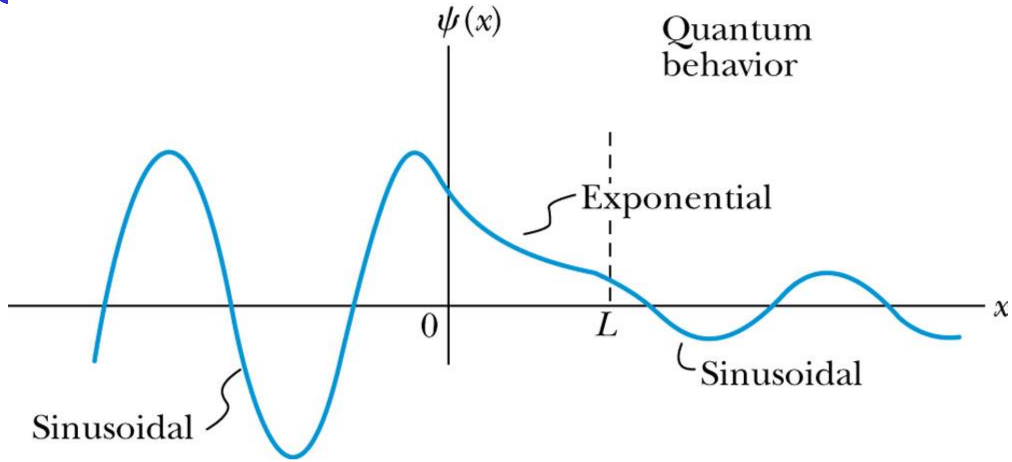
- Consider the potential energy function in which the potential energy of the system is zero everywhere except L.
- Square barrier – U is the barrier height
- If E>U, classically the particle reflected back.
- Classically if the particle exist in region II, its KE would be negative
- Classically region II and III are forbidden to the particle

Tunneling



- Acc to QM, all the regions are accessible regardless of its energy
- Schrodinger eqn has valid solutions for I and III which are sinusoidal, region II solution is exponential
- Applying the boundary conditions, the wave functions in the three regions must join smoothly at the boundaries
- Thus mathematically it is satisfied

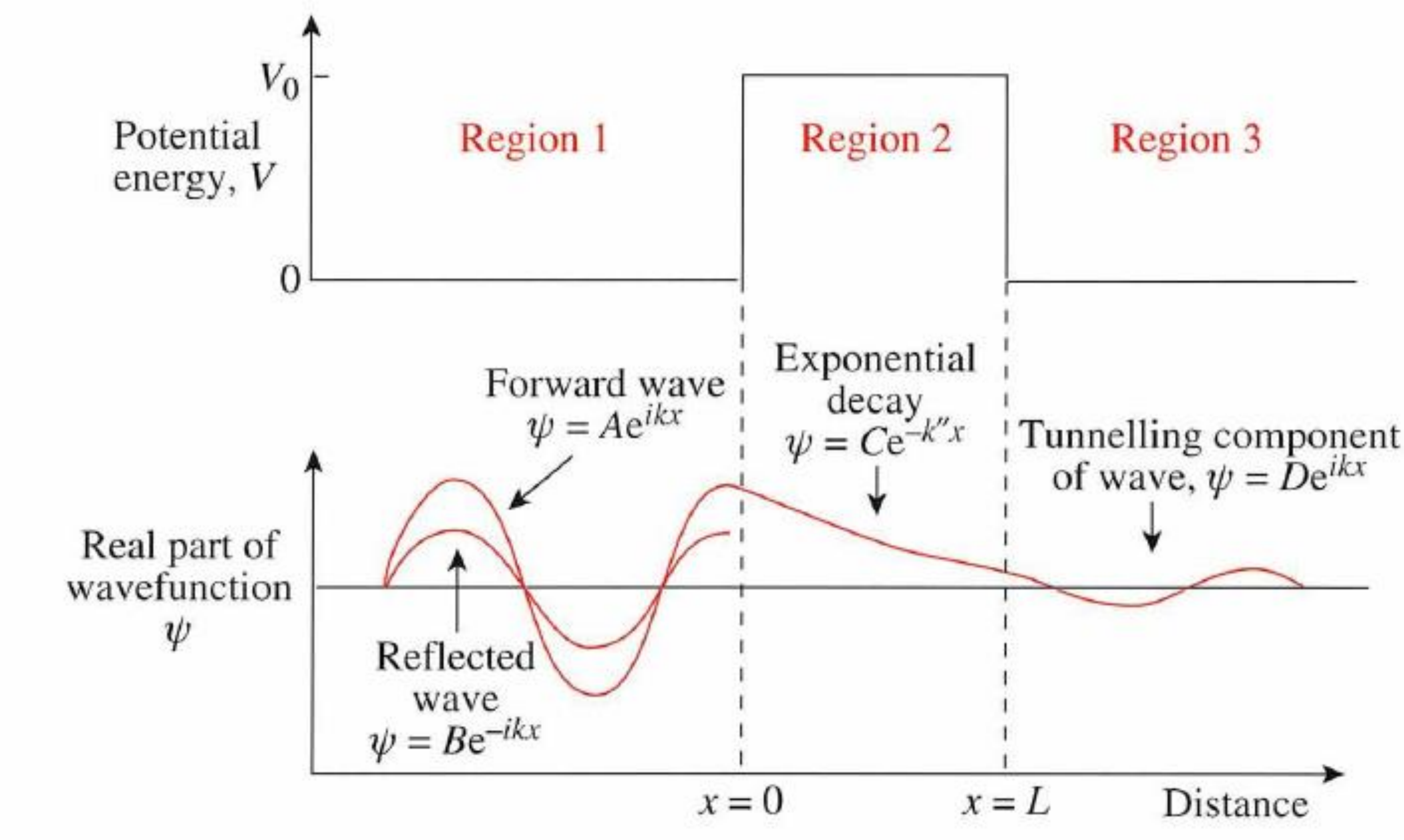
Tunneling



- Hence finding the particle beyond the barrier in region III is nonzero
- The movement of the particle to the far side of the barrier is called tunneling or barrier penetration

~~CLASSICAL~~ **QUANTUM**

Tunneling



V_0
Potential energy, V
0
Region 1
Region 2
Region 3
Forward wave
$\psi = Ae^{ikx}$
Exponential decay
$\psi = Ce^{-k''x}$
Tunnelling component of wave, $\psi = De^{ikx}$
Real part of wavefunction ψ
Reflected wave
$\psi = Be^{-ikx}$
$x = 0$
$x = L$
Distance

Transmission and Reflection coefficient

- The transmission coefficient represents the probability that the particle penetrates to the other side of the barrier
- The reflection coefficient is the probability that the particle is reflected by the barrier

$$T + R = 1$$

$$T = e^{-2CL}, \text{ where } C = \frac{\sqrt{2m(U-E)}}{\hbar}$$