2·6. MEDIAN

Median of a distribution is the value of the variable which divides it into two equal parts. It is the value which exceeds and is exceeded by the same number of observations, *i.e.*, it is the value such that the number of observations above it is equal to the number of observations below it. The median is thus a *positional* average.

In case of ungrouped data, if the number of observations is odd then median is the middle value after the values have been arranged in ascending or descending order of magnitude. In case of even number of observations, there are two middle terms and median is obtained by taking the arithmetic mean of the middle terms. For example, the median of the values 25, 20, 15, 35, 18, *i.e.*, 15, 18, 20, 25, 35 is 20 and the median of 8, 20, 50, 25, 15, 30, *i.e.*, of 8, 15, 20, 25, 30, 50 is $\frac{1}{2}(20 + 25) = 22{\cdot}5$.

Remarks. In case of even number of observations, in fact any value lying between the two middle values can be taken as median but conventionally we take it to be the mean of the middle terms.

In case of discrete frequency distribution median is obtained by considering the cumulative frequencies. The steps for calculating median are given below :

(*i*) Find $\frac{1}{2}N$, where $N = \sum_i f_i$.

(*ii*) See the (less than) cumulative frequency (*c.f.*) just greater than $\frac{1}{2}N$.

(*iii*) The corresponding value of x is median.

Example 2·6. *Obtain the median for the following frequency distribution :*

x :	1	2	3	4	5	6	7	8	9
f :	8	10	11	16	20	25	15	9	6

Solution.

COMPUTATION OF MEDIAN

x	f	$c.f.$
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120
Total	$N = 120$	

Here $N = 120 \Rightarrow \frac{N}{2} = 60$

The cumulative frequency (*c.f.*) just greater than $\frac{1}{2}N$ is 65 and the value of x corresponding to 65 is 5. Therefore, median is 5.

2·6·1. Median for Continuous Frequency Distribution. In the case of continuous frequency distribution, the class corresponding to the *c.f.* just greater than $\frac{1}{2}N$ is called the *median class* and the value of median is obtained by the following formula :

$$Median = l + \frac{h}{f}\left(\frac{N}{2} - c\right) \qquad \dots (2·6)$$

where l is the lower limit of the median class,

f is the frequency of the median class,

h is the magnitude of the median class,

'c' is the *c.f.* of the class *preceding* the median class,

and $N = \sum f$.

Derivation of the Median Formula (2·6). Let us consider the following continuous frequency distribution, $(x_1 < x_2 < \dots < x_{n+1})$:

Class interval :	$x_1 - x_2$	$x_2 - x_3$	...	$x_k - x_{k+1}$	...	$x_n - x_{n+1}$
Frequency :	f_1	f_2	...	f_k	...	f_n

The cumulative frequency distribution is given by :

Class interval :	$x_1 - x_2$	$x_2 - x_3$	...	$x_k - x_{k+1}$	...	$x_n - x_{n+1}$
Frequency :	F_1	F_2	...	F_k	...	F_n

Example 2·7. *Find the median wage of the following distribution :*

Wages (in Rs.) :	*2,000—3,000*	*3,000—4,000*	*4,000—5,000*	*5,000—6,000*	*6,000—7,000*
No. of workers :	*3*	*5*	*20*	*10*	*5*

Solution. COMPUTATION OF MEDIAN

Wages (in Rs.)	*No. of employees*	*c.f.*
2,000—3,000	3	3
3,000—4,000	5	8
4,000—5,000	20	28
5,000—6,000	10	38
6,000—7,000	5	43

Here $\frac{1}{2}N = \frac{1}{2}(43) = 21{\cdot}5$.

Cumulative frequency just greater than 21·5 is 28 and the corresponding class is 4,000—5,000. Thus median class is 4,000—5,000.

Hence using (2·6), Median $= 4{,}000 + \dfrac{1{,}000}{20}(21{\cdot}5 - 8) = 4{,}000 + 675 = 4{,}675$.

Thus median wage is Rs. 4,675.

2·6·2. Merits and Demerits of Median

Merits	*Demerits*
1. It is rigidly defined.	1. In case of even number of observations median cannot be determined exactly. We merely estimate it by taking the mean of two middle terms.
2. It is easily understood and is easy to calculate. In some cases it can be located merely by inspection.	2. It is not based on all the observations. For example, the median of 10, 25, 50, 60 and 64 is 50. We can replace the observations 10 and 25 by any two values which are smaller than 50 and the observations 60 and 65 by any two values greater than 50, without affecting the value of median. This property is sometimes described by saying that median is *insensitive*.
3. It is not at all affected by extreme values.	3. It is not amenable to algebraic treatment.
4. It can be calculated for distributions with open-end classes.	4. As compared with mean, it is affected much by fluctuations of sampling.

Uses. (*i*) Median is the only average to be used while dealing with qualitative data which cannot be measured quantitatively but still can be arranged in ascending or descending order of magnitude, *e.g.*, to find the average intelligence or average honesty among a group of people.

(*ii*) It is to be used for determining the typical value in problems concerning wages, distribution of wealth, etc.

2·7. MODE

2·7. MODE

Let us consider the following statements :

(*i*) The average height of an Indian (male) is 5′-6″.

(*ii*) The average size of the shoes sold in a shop is 7.

(*iii*) An average student in a hostel spends Rs. 750 per month.

In all the above cases, the average referred to is mode. *Mode is the value which occurs most frequently in a set of observations and around which the other items of the set cluster densely. In other words, mode is the value of the variable which is predominant in the series.* Thus, in the case of discrete frequency distribution, mode is the value of x corresponding to maximum frequency. For example, in the following frequency distribution :

x :	1	2	3	4	5	6	7	8
f :	4	9	16	25	22	15	7	3

value of x corresponding to the maximum frequency, *viz.*, 25 is 4. Hence mode is 4.

But in any one (or more) of the following cases :

(*i*) if the maximum frequency is repeated,

(*ii*) if the maximum frequency occurs in the very beginning or at the end of the distribution, and

(*iii*) if there are irregularities in the distribution,

the value of mode is determined by the *method of grouping,* which is illustrated below by an example.

Example 2·12. *Find the mode for the following distribution :*

Class-interval :	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80
Frequency :	5	8	7	12	28	20	10	10

Solution. Here maximum frequency is 28. Thus the class 40—50 is the modal class. Using mode formula (2·7), the value of mode is given by :

$$\text{Mode} = 40 + \frac{10(28-12)}{(2 \times 28 - 12 - 20)} = 40 + 6{\cdot}666 = 46{\cdot}67 \text{ (approx.)}$$