Example 17. *Find the standard deviation of the following data:*

48, 43, 65, 57, 31, 60, 37, 48, 59, 78.

Solution. Let us prepare the following table in order to calculate the value of *S.D.* by assuming $A = 50$.

Value (x)	$d = x - A = (x - 50)$	d^2
48	−2	4
43	−7	49
65	15	225
57	7	49
31	−19	361
60	10	100
37	−13	169
48	−2	4
59	9	81
78	28	784
$n = 10$	$\Sigma d = 26$	$\Sigma d^2 = 1826$

Here
$$\bar{x} = A + \frac{\Sigma d}{n} = 50 + \frac{26}{10} = \mathbf{52.6},$$

which is a fraction. Let us apply the short-cut formula in order to calculate S.D.

$$\therefore \text{ S.D.} = \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{1826}{10} - \left(\frac{26}{10}\right)^2} = \sqrt{182.60 - 6.76} = \sqrt{175.84} = \mathbf{13.26.}$$

5.7 CALCULATION OF STANDARD DEVIATION – DISCRETE SERIES OR GROUPED DATA

The standard deviation of a discrete series or grouped data can be calculated by any one of the following three methods.

(a) ***Actual Mean Method or Direct Method***

(b) ***Assumed Mean Method or Short-cut Method***

(c) ***Step Deviation Method***

5.7.1 Actual Mean Method or Direct Method

The standard deviation for the discrete series is given by the formula:

$$\sigma = \sqrt{\frac{\Sigma f (x - \bar{x})^2}{n}},$$

where $\bar{x}$ is the arithmetic mean, x is the size of item, f is the corresponding frequency and $n = \Sigma f$.

However, in practice, this method is rarely used because if the arithmetic mean is a **fraction,** the calculations take a lot of time and are cumbersome.

5.7.2 Assumed Mean Method or Short-cut Method

In this method we use the following formula to calculate the standard deviation σ:

$$\sigma = \sqrt{\frac{\Sigma f d^2}{n} - \left(\frac{\Sigma f d}{n}\right)^2},$$

where **A = is the assumed mean, $d = x - A$, and $n = \Sigma f$.**

WORKING RULE

STEP I Take any item of the given series as assumed mean A.

STEP II Take the deviations of the items from the assumed mean A and denote it by d.

STEP III Multiply the deviations by the respective frequency and denote it by fd. Obtain the total Σfd.

STEP IV Calculate d^2, where d's are obtained in **step II**.

STEP V Multiply the squared deviations by respective frequencies to get Σfd^2.

STEP VI Find the value of $\sigma^2 = \frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2$

STEP VII Take the square root of σ^2 obtained in **step VI** to get the value of **standard deviation σ.**

The above method is illustrated by the following example.

Example 19. *Find the standard deviation from the following data:*

Size of the item :	*10*	*11*	*12*	*13*	*14*	*15*	*16*
Frequency :	2	7	*11*	*15*	*10*	*4*	*1*

Also find the coefficient of variation.

Solution.

Table: Computation of Standard Deviation

Size of the item: (x)	*Frequency:* (f)	$d = x - A$, $A = 13$	fd	d^2	fd^2
10	2	– 3	– 6	9	18
11	7	– 2	– 14	4	28
12	11	– 1	– 11	1	11
13	15	0	0	0	0
14	10	1	10	1	10
15	4	2	8	4	16
16	1	3	3	9	9
Total	$n = \Sigma f = 50$		$\Sigma fd = -10$		$\Sigma fd^2 = 92$

Now **Mean :** $\bar{x} = A + \frac{\Sigma fd}{n} = 13 + \frac{(-10)}{50} = \mathbf{12.8.}$

Here $\bar{x}$ **= 12.8, is a fraction.**

$$\therefore \quad \text{S.D.} = \sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = \sqrt{\frac{92}{50} - \left(\frac{-10}{50}\right)^2} = \sqrt{1.84 - 0.04} = \sqrt{1.80} = \mathbf{1.342.}$$

$\therefore$ **Now the coefficient of variation** $= \frac{\sigma}{\bar{x}} \times 100 = \frac{1.342}{12.8} \times 100 = \mathbf{10.4.}$

5.7.3 Step Deviation Method

In this method we divide the deviations by a common class interval and use the following formula for computing standard deviation:

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} \times i,$$

where i = common class interval, $d = \frac{x - A}{i}$, A is assumed mean, f is the respective frequency.

WORKING RULE

STEP I Find the mid-values or mid-points of the various classes and denote it by m.

STEP II Take any one of the values of m's as the assumed mean A (Generally, the middle value is taken as A).

STEP III Take the deviations of the mid-points from the assumed mean A and divide it by class interval or common factor i. Denote it by d (*or* d').

STEP IV Multiply the respective frequencies f with the corresponding deviation d and obtain Σfd.

STEP V Square the deviations d and multiply it with their respective frequencies. Obtain Σfd^2.

STEP VI Substitute the values of Σfd, Σfd^2, i in the formula:

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} \times i, \text{ where } n = \Sigma f$$

to get **the desired standard deviation σ.**

Example 21. *Find the standard deviation for the following distribution:*

Marks	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*70 – 80*
No. of Students	*5*	*12*	*15*	*20*	*10*	*4*	*2*

Solution. Let us prepare the following table in order to calculate the standard deviation, by assuming $A = 45$.

Table: Computation of Standard Deviation

Marks (Class interval)	*No. of Students* (f)	*Mid-value* (m)	$d = \frac{m-45}{10}$	fd	fd^2
10 – 20	5	15	– 3	– 15	45
20 – 30	12	25	– 2	– 24	48
30 – 40	15	35	– 1	– 15	15
40 – 50	20	45	0	0	0
50 – 60	10	55	1	10	10
60 – 70	4	65	2	8	16
70 – 80	2	75	3	6	18
Total	$\mathbf{\Sigma f = n = 68}$			$\mathbf{\Sigma fd = -30}$	$\mathbf{\Sigma fd^2 = 152}$

$$\therefore\ \sigma = i \times \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = 10 \times \sqrt{\frac{152}{68} - \left(\frac{-30}{68}\right)^2} = 10 \times \sqrt{(2.2352 - 0.1946)}$$

$$= 10 \times \sqrt{2.0406} = \mathbf{14.3\ Approx.}$$

Example 22. *Find the standard deviation by the step deviation method for the following data:*

Class-Interval:	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*
Frequency:	*6*	*14*	*10*	*8*	*1*	*3*	*8*

Solution. Let the assumed mean be $A = 35$, $d = \frac{x - A}{c} = \frac{x - 35}{10}$, where $i = 10$