Functional Dependence

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$u = f_1(x, y)$
$v = f_2(x, y)$

u & v are connected by some relation $f(u,v) = 0$

$f \rightarrow$ differentiable then we can say that u & v are functionally dependent.

$$J(u,v) \text{ (or) } \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

then u & v are functionally dependent

$$J(u,v,w) \text{ (or) } \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = 0$$

Problem:

Given: $u = x+y-z$; $v = x-y+z$

$w = x^2+y^2+z^2-2yz$.

P.T (i) functions $u, v,$ and w are functionally dependent.

(ii) Find the relationship between them.

Soln:

We know. $J(u,v,w) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$

Remark:

if $J(u,v,w) = 0$ then functionally dependent.

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$$J(u,v,w) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 2x & 2y-2z & 2z-2y \end{vmatrix} \quad 3 = -2$$

$$= 1\left[-(2z-2y) - (2y-2z)\right] + \cdots$$

$$= 0$$

(ii) Find relatanship between u, v & w.

$w = x^2 + y^2 + z^2 - 2yz \to (A)$

$u = x + y - z; \quad v = x - y + z$

$u + v = 2x \; (1); \quad u - v = 2y - 2z \; (2)$

$(u+v)^2 + (u-v)^2 = (2x)^2 + (2(y-z))^2 \to (1)^2 + (2)^2$

$\quad \text{''} \quad = 4x^2 + 4(y^2 + z^2 - 2yz)$

$\quad \text{''} \quad = 4[x^2 + y^2 + z^2 - 2yz]$

$\quad \text{''} \quad = 4w$ (from (A))

$u^2 + v^2 + \cancel{2uv} + u^2 + v^2 - \cancel{2uv} = 4w$

$2u^2 + 2v^2 = 4w$

$$\boxed{u^2 + v^2 = 2w}$$