Solution:

$$E(X) = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$$

or

$$E(X) = \frac{1}{3} \times 0 + \frac{1}{2} \times 1 + 0 \times 2 + 3 \times \frac{1}{6} = 1.$$

$$E(X^2) = p_1x_1^2 + p_2x_2^2 + p_3x_3^2 + p_4x_4^2$$

or

$$E(X^2) = \frac{1}{3} \times 0 + \frac{1}{2} \times 1 + 0 \times 4 + \frac{1}{6} \times 9 = 2.$$

$$E[X - E(X)]^2 = E(X^2) - [E(X)]^2 = 4 - 1 = 3.$$

Now,

$$E(Y) = E(2X + 1) = 2E(X) + 1 = 2 \times 1 + 1 = 3 = \bar{Y}.$$

$$\text{Var}(Y) = E(Y - \bar{Y})^2 = E(2X + 1 - 3)^2 = E(2X - 2)^2 = 4E(X - 1)^2$$

$$= 4\left[(0-1)^2 \times \frac{1}{3} + (1-1)^2 \times \frac{1}{2} + (2-1)^2 \times 0 + (3-1)^2 \times \frac{1}{6}\right]$$

$$= 4\left[\frac{1}{3} + 0 + 0 + \frac{4}{6}\right] = 4.$$

Hence, V (Y) = 4 and E [X – E (X)]² = 3.

Example 9. *The probability that there is atleast one error in an account statement prepared by A is 0.2 and for B and C they are 0.25 and 0.4 respectively. A, B and C prepared 10, 16 and 20 statements respectively. Find the expected number of correct statements in all.*

Solution: Let p_1, p_2, p_3 respectively denote the probability of the events that there is no error in the account prepared by A, B and C. Then

p_1 = **1 – (probability of at least one error in the account statement prepared by A)**
= 1 – 0.02 = 0.8.

Similarly, $p_2 = 1 - 0.25 = \mathbf{0.75}$, and $p_3 = 1 - 0.4 = \mathbf{0.6}$

Also, $x_1 = 10$, $x_2 = 16$, $x_3 = 20$.

$$E(X) = p_1x_1 + p_2x_2 + p_3x_3 + \ldots\ldots + p_nx_n$$

$$E(X) = (0.8 \times 10) + (0.75 \times 16) + (0.6 \times 20) = 8 + 12 + 12 = \mathbf{32}.$$

Hence, the expected number of all correct statements would be 32.

Example 11. *The p.m.f. f (x) of a discrete random variable X is given by*

X	1	2	3
f (x)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Find E (x) and V (x).

Solution: Expectation : $E(X) = x_1 \cdot f(x_1) + x_2 \cdot f(x_2) + x_3 \cdot f(x_3)$

$$= 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 3 \times \frac{1}{6} = \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = \frac{10}{6} = \mathbf{1.67.}$$

Variance: $\quad Var(X) = E[(X - Ex)^2]$

$$= [x_1 - E(X)]^2 \cdot f(x_1) + [x^2 - E(X)]^2 \cdot f(x^2) + [x^3 - E(X)]^2 \cdot f(x_3)$$

$$= (1 - 1.67)^2 \times \frac{1}{2} + (2 - 1.67)^2 \times \frac{1}{3} + (3 - 1.67)^2 \times \frac{1}{6}$$

$$= 0.4489 \times \frac{1}{2} + 0.1089 \times \frac{1}{3} + 1.7689 \times \frac{1}{6}$$

$$= 0.22445 + 0.0363 + 0.2948 = \mathbf{0.55555.}$$

Example 12. *Let X be a random variable assuming values x_1, x_2 and x_3. Then the function f defined by $f(x_i) = P(X = x_i)$ is given by*

X :	– 3	6	9
$P(X = x_i)$:	1/6	1/2	1/3

Find E (X), E (X^2) and E $(2X + 1)^2$.

Solution:

TABLE: Computation of E (X)

x	*p*	*px*	*px²*
– 3	1/6	– 1/2	9/2
6	1/2	3	12
9	1/3	3	27
Total		$\Sigma px = 5.5$	$\Sigma px^2 = 43.5$

$\therefore$ **E (X) = Σ px = 5.5** **and** **E (X^2) = 43.5.**

Also, **E (2X + 1) = 2*E* (*X*) + 1 = 2 × 5.5 + 1 = 12.**

Example 21. *Find the expected value and variance of the following probability distribution:*

x :	-10	-20	30	75	80
$p(x)$:	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{20}$

Solution:

Expected Value: $E(x) = x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \cdot p(x_3) + x_4 \cdot p(x_4) + x_5 \cdot p(x_5)$

$$= \left(-10\times\frac{1}{5}\right)+\left(-20\times\frac{3}{20}\right)+\left(30\times\frac{1}{2}\right)+\left(75\times\frac{1}{10}\right)+\left(80\times\frac{1}{20}\right)$$

$$= -2-3+15+7.5+4 = 26.5-5 = \mathbf{21.5.}$$

Variance: $V(x) = E[(x-E(x))^2]$

$$= [x_1 - E(x)]^2 \cdot p(x_1) + [x_2 - E(x)]^2 \cdot p(x_2) + [x_3 - E(x)]^2 \cdot p(x_3) + [x_4 - E(x)]^2 \cdot p(x_4) + [x_5 - E(x)]^2 \cdot p(x_5)$$

$$= (-10-21.5)^2 \times \frac{1}{5} + (-20-21.5)^2 \times \frac{3}{20} + (30-21.5)^2 \times \frac{1}{2} + (75-21.5)^2 \times \frac{1}{10} + (80-21.5)^2 \times \frac{1}{20}$$

$$= (-31.5)^2 \times \frac{1}{5} + (-41.5)^2 \times \frac{3}{20} + (8.5)^2 \times \frac{1}{2} + (53.5)^2 \times \frac{1}{10} + (58.5)^2 \times \frac{1}{20}$$

$$= 992.25 \times \frac{1}{5} + 1722.25 \times \frac{3}{20} + 72.25 \times \frac{1}{2} + 2862.2 \times \frac{1}{10} + 3422.2 \times \frac{1}{20}$$

$$= 198.45 + 258.34 + 36.12 + 286.22 + 171.11 = \mathbf{950.24.}$$

Example 22. *If three coins are tossed, find the expectation and variance of the number of heads.*

Solution: Let X denote the number of heads obtained in a random throw of 3 coins. Then X is a random variable with the following probability distribution.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Now, $E(X) = \Sigma x \cdot p(x) = \left(0\times\frac{1}{8}\right)+\left(1\times\frac{3}{8}\right)+\left(2\times\frac{3}{8}\right)+\left(3\times\frac{1}{8}\right) = \frac{3}{2}$.

$E(X^2) = \Sigma x^2 \cdot p(x) = \left(0^2\times\frac{1}{8}\right)+\left(1^2\times\frac{3}{8}\right)+\left(2^2\times\frac{3}{8}\right)+\left(3^2\times\frac{1}{8}\right) = 3$.

$\therefore$ **Variance** $(X) = E(X^2) - [E(X)]^2 = 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4} = \frac{3}{4}$.

Hence

Example 31. *Calculate the expected value of X, the sum of the scores when two dice are rolled.*

Solution: Let the random variable X denote the sum of the scores obtained on a pair of dice when thrown. Then X takes the following values with corresponding probabilities as

x_i :	2	3	4	5	6	7	8	9	10	11	12
p_i :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\therefore \quad E(X) = \Sigma x_i p_i = 2\times\frac{1}{36}+3\times\frac{2}{36}+4\times\frac{3}{36}+5\;\frac{4}{36}+6\times\frac{5}{36}+7\times\frac{6}{36}$$

$$+8\times\frac{5}{36}+9\times\frac{4}{36}+10\times\frac{3}{36}+11\times\frac{2}{36}+12\times\frac{1}{36}$$

$$= \frac{1}{36}[2+6+12+20+30+42+40+36+30+22+12] = \frac{252}{36} = 7.$$

Example 32. *A random variable has the following probability distribution:*

x	:	4	5	6	8
P	:	0.1	0.3	0.4	0.2

Find $E[X - E(X)]^2$.

Solution.

	p	$X \times p$	$X - E(X)$	$2[X - E(X)]^2$	$[X - E(X)]^2 \times p$
[illegible]	0.1	0.4	− 1.9	3.61	0.361
[illegible]	0.3	1.5	− 0.9	0.81	0.243
[illegible]	0.4	2.4	0.1	0.01	0.004
[illegible]	0.2	1.6	2.1	4.41	0.882
	Total			**8.84**	**1.49**

$$E(X) = \Sigma\ X \times p = 5.9$$

$$\therefore \quad E[X - E(X)]^2 = 1.49$$

Example 33. *The probability distribution of a random variable X is given as:*

$$X = -2,\ 3,\ 1; \qquad P(X) = 1/3,\ 1/2,\ 1/6.$$

Find $E(2X + 5)$ and $E(X^2)$.

Solution.

$$E(X) = p_1X_1 + p_2X_3 + p_3X_3 = \frac{-2}{3} + \frac{3}{2} + \frac{1}{6} = \frac{6}{6} = 1$$

$$E(2X + 5) = 2[E(X)] + 5 = 2 \times 1 + 5 = 7$$

$$E(X^2) = p_1X_1^2 + p_2X_2^2 + p_3X_3^2 = \frac{1}{3} \times 4 + \frac{1}{2} \times 9 + \frac{1}{6} \times 1 = 6.$$

Example 34. *Define mathematical expectation. For the following probability distribution, calculate the mean and the variance of the random variable x.*

x :	8	12	16	20	24
$P(x)$:	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

Solution. Mathematical expectation of X is defined as

$$E(X) = p_1x_1 + p_2x_2 + \dots + p_nX_n = \text{Mean}$$

$$\therefore \text{ Mean} = E(X) = 8\left(\frac{1}{8}\right) + 12\left(\frac{1}{6}\right) + 16\left(\frac{3}{8}\right) + 20\left(\frac{1}{4}\right) + 24\left(\frac{1}{12}\right) = 1 + 2 + 6 + 5 + 2 = 16$$

$$\text{Variance} = E\left(X - \bar{X}\right)^2$$

$$= (8-16)^2\left(\frac{1}{8}\right) + (12-16)^2\left(\frac{1}{6}\right) + (16-16)^2\left(\frac{3}{8}\right) + (20-16)^2\left(\frac{1}{4}\right) + (24-16)^2\left(\frac{1}{12}\right) = 20.$$

Example 35. *A random variable X has the following probability distribution:*

Value of X:	− 2	− 1	0	1	2	3
$P(X = x)$:	0.1	K	0.2	$2K$	0.3	K

(i) Find the value of K.

(ii) Find the expected value and variance of X

Solution. In the case of probability distribution of a random variable, we know **that sum of the probability is one, *i.e.*, $\Sigma\, p(x) = 1$.**

$$\Rightarrow 0.1 + K + 0.2 + 2K + 0.3 + K = 1 \quad \Rightarrow \quad 4K + 0.6 = 1 \Rightarrow K = 0.1.$$