Heisenberg Uncertainty Principle

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Bragg’s law

The phases of the beams coincide when the incident angle equals reflecting angle. The rays of the incident beam are in phase and parallel upto point z, which is the point at which top beam strikes the top layer. The second beam passes to next layer and is scattered by B. The second beam travels extra distance AB + BC. This extra distance is an integral multiple of the wavelength.

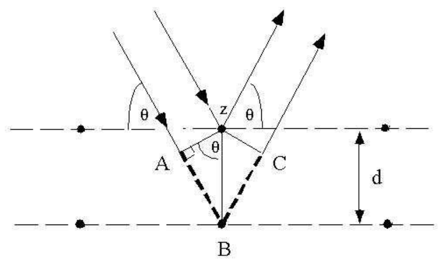
$n\lambda$ = AB+BC.

But AB=BC

$n\lambda$ = 2AB.......................................(1)

d is the hypotenuse of the right triangle Abz. Ab is opposite to angle θ

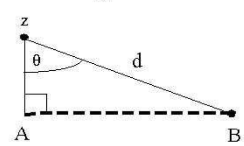
AB = d sinθ....................................(2)



Substitute equation (2) in equation (1)

$n\lambda = 2d \sin\theta$

This is equation for Bragg's law



Bragg Equation

According to Bragg Equation:

$n\lambda = 2d \sin\Theta$

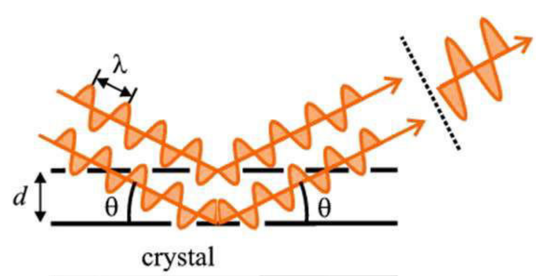
The **law** states that when the x-ray is incident onto a crystal surface, its angle of incidence, θ, will reflect back with a same angle of scattering, θ. And, when the path difference, d is equal to a whole number, n, of wavelength, a constructive interference will occur.

Therefore, according to the derivation of Bragg’s Law:

- The equation explains why the faces of crystals reflect X-ray beams at particular angles of incidence (Θ, λ).
- The variable *d* indicates the distance between the atomic layers, and the variable Lambda specifies the wavelength of the incident X-ray beam.
- n as an integer.

This observation illustrates the X-ray wave interface, which is called X-ray diffraction (XRD) and proof for the atomic structure of crystals.

If electrons act like waves, we should be able to apply Bragg’s Law to the diffraction of electrons.



Heisenberg Uncertainty Principle

The uncertainty principle says that both the position and momentum of a particle cannot be determined at the same time and accurately. The result of position and momentum is at all times greater than h/4π. The formula for Heisenberg Uncertainty principle is articulated as,

The Uncertainty Principle Equation

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

For example, the location and speed of a moving car can be determined at the same time, with minimum error. But, in microscopic particles, it will not be possible to fix the position and measure the velocity/momentum of the particle simultaneously.

Where

h is the Planck's constant ($6.62607004 \times 10^{-34}$ m^2 kg / s)
Δp is the uncertainty in momentum
Δx is the uncertainty in position

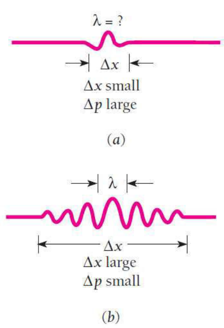


Figure 3.12 (*a*) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (*b*) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

The relationship between the distance Δx and the wave-number spread Δk depends upon the shape of the wave group

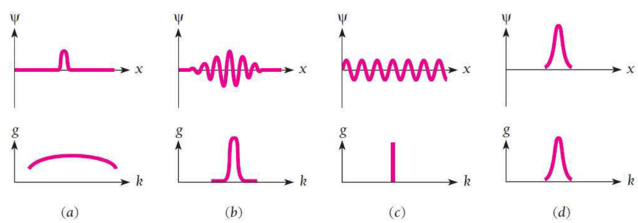


Figure 3.14 The wave functions and Fourier transforms for (*a*) a pulse, (*b*) a wave group, (*c*) a wave train, and (*d*) a Gaussian distribution. A brief disturbance needs a broader range of frequencies to describe it than a disturbance of greater duration. The Fourier transform of a Gaussian function is also a Gaussian function.

wave groups in general do not have Gaussian forms, it is more realistic to express the relationship between Δx and Δk as

$$\Delta x\, \Delta k \geq \frac{1}{2} \quad \text{.......................................(1)}$$

The de Broglie wavelength of a particle of momentum p is $\lambda = h/p$ and the corresponding wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

In terms of wave number the particle's momentum is therefore

$$p = \frac{hk}{2\pi}$$

Hence an uncertainty Δk in the wave number of the de Broglie waves associated with the particle results in an uncertainty Δp in the particle's momentum according to the formula

$$\Delta p = \frac{h\, \Delta k}{2\pi}$$

Since $\Delta x\, \Delta k \geq \frac{1}{2}$, $\Delta k \geq 1/(2\Delta x)$ and

Uncertainty principle

$$\Delta x\, \Delta p \geq \frac{h}{4\pi}$$

This equation states that the product of the uncertainty Δx in the position of an object at some instant and the uncertainty Δp in its momentum component in the x direction at the same instant is equal to or greater than $h/4\pi$.

Notes

- Quantum mechanics is the discipline of measurements on the minuscule scale. That measurements are in macro and micro-physics can lead to very diverse consequences. Heisenberg uncertainty principle or uncertainty principle is a vital concept in Quantum mechanics.

- Heisenberg's Uncertainty principle is applicable to energy and time. These uncertainties always exist in conjugate pairs such as momentum/position and energy/time.

Particle properties of wave: Matter Waves

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Matter Waves

The wave nature associated with the material particle is known as matter waves.

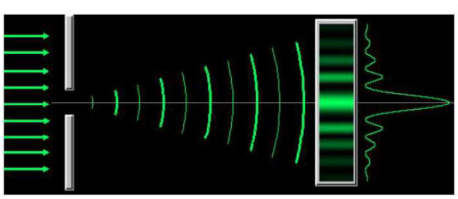
- You know about the Compton Effect, it proofs the particle nature of wave.
- Can a particle have also wave nature?
- Nature like symmetry!

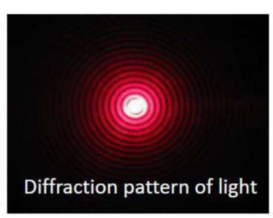
Macroscopic object

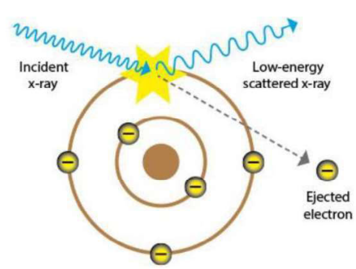
Microscopic object

Dual nature of EM radiation: Wave and particle nature

- Electromagnetic radiation behaves like waves and propagates according to Maxwell's Equations.
- Wave like nature of EM radiation was confirmed in experiments like interference and diffraction.
- Particle like nature of EM radiation was confirmed by Compton scattering and Photoelectric effect.







Compton Scattering

Particle Properties	Wave Properties
Mass (m)	Frequency (ν)
Velocity (v)	Wavelength (λ)
Momentum (p)	Amplitude (A)
Energy (E)	Intensity (I)

De Broglie's Hypothesis

- Louis Victor de Broglie in 1923 in his doctoral dissertation postulated that "because photons have wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties".
- This was a radical idea with no experimental confirmation at that time.
- According to de Broglie, electrons had a dual particle–wave nature.
- Accompanying every electron was a wave (not an electromagnetic wave!), which guided, or "piloted," the electron through space.
- This gave him a Nobel Prize in Physics in 1929 (just one year after Sir CV Raman got Nobel).

De Broglie's Wavelength

$$\lambda = \frac{h}{p}$$

Relativity

$$E = mc^2 = \sqrt{\underbrace{p^2c^2}_{\text{Kinetic energy term}} + \underbrace{m_0^2c^4}_{\text{Rest mass energy term}}}$$

rest mass = 0

Momentum of a photon

$$p = \frac{E}{c}$$

$$\frac{h}{\lambda} = \frac{E}{c}$$

Wavelength-energy relation

Photoelectric effect $E = hf = \dfrac{hc}{\lambda}$

$$\lambda = \frac{h}{p}$$

for photon

The de Broglie Hypothesis

$$\lambda \overset{?}{=} \frac{h}{p} = \frac{h}{mv}$$

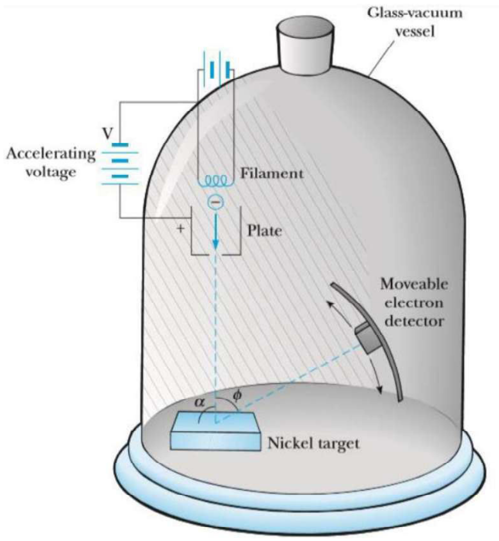
for electron?

Davisson-Germer Experiment

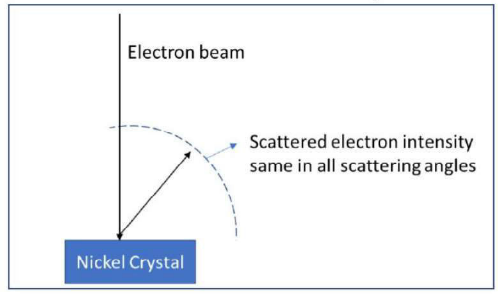
- Davisson and Germer demonstrated the wave nature of electron by their experiment.
- They set out to study the surface of metals, in particular, nickel. They intended to do that by scattering electrons from the surface of nickel and observing the scattered electron.
- A collimated beam of electron is produced is using the electron gun and made to incident on a target of nickel crystal. The electrons are scattered in all direction by the atomic planes of nickel crystal.
- The intensity of the scattered electrons is measured by an electron detector. This detector can be moved in a circular direction and the scattered electron intensity is studied at different angles.

Davisson-Germer Experiment: proves the existence of matter waves

Wave nature of electrons



Classical Theory



Davisson-Germer Experiment

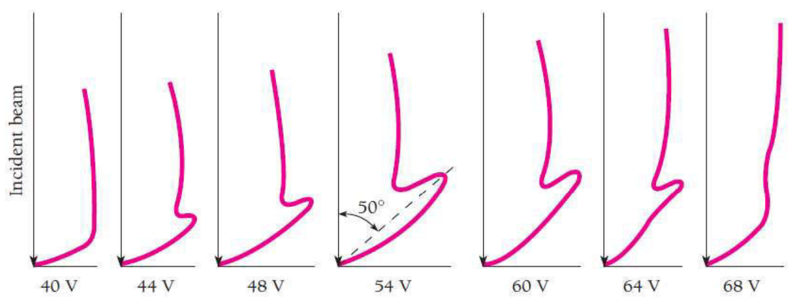
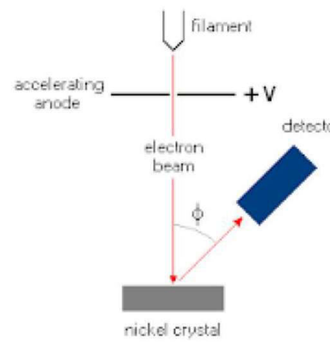
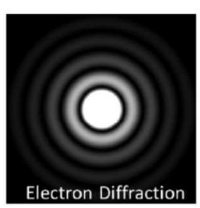


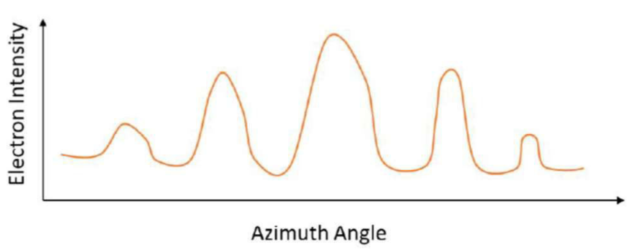
Figure 3.7 Results of the Davisson-Germer experiment, showing how the number of scattered electrons varied with the angle between the incoming beam and the crystal surface. The Bragg planes of atoms in the crystal were not parallel to the crystal surface, so the angles of incidence and scattering relative to one family of these planes were both 65° (see Fig. 3.8).



Davisson-Germer Experiment

Experimental Observation





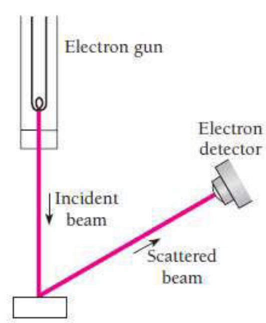


Figure 3.6 The Davisson-Germer experiment.

Observations of Davisson Germer experiment:

From this experiment, we can derive the below observations:

- We obtained the variation of the intensity (I) of the scattered electrons by changing the angle of scattering θ.
- By changing the accelerating potential difference, the accelerated voltage was varied from 44V to 68V.
- With the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle θ = 50º, we could see a strong peak in the intensity.
- This peak was the result of constructive interference of the electrons scattered from different layers of the regularly spaced atoms of the crystals.
- With the help of electron diffraction, the wavelength of matter waves was calculated to be 0.165 nm.

Using Bragg's diffraction Law:

$$2d\sin\theta = n\lambda$$
$$n = 1$$
$$\theta = 65^{o}$$
$$d = 0.91A^{\circ}$$
$$\Rightarrow \lambda = 1.66A^{\circ}$$

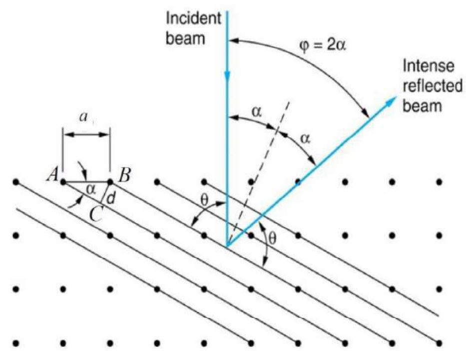


Figure (3) Bragg reflection

$$\sin\alpha = \frac{BC}{AB} = \frac{d}{a}$$
$$a = 2.15A^{\circ}$$
$$\alpha = \frac{\phi}{2} = \frac{50^{\circ}}{2} = 25^{\circ}$$
$$\therefore d = 0.91A^{\circ}$$

Using de Broglie's wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$
$$m = 9.1\times10^{-31}kg$$
$$e = 1.6\times10^{-19}C$$
$$V = 54V$$
$$\Rightarrow \lambda = 1.67A^{\circ}$$

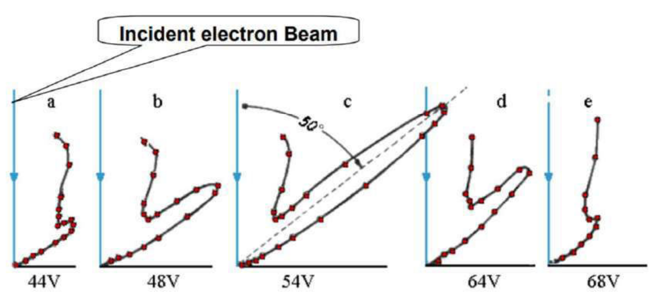


Figure (2) Diffraction of electron beam at different applied voltage

Wavelength of the electron determined using de Broglie's wavelength and Bragg's diffraction law agree well: This proves the existence of matter waves

Properties of Matter waves

1. Only generated by moving particle. If the velocity v → ∞, the wavelength will be zero and vice versa if v = 0, λ → ∞ is the wave becomes indeterminate. This shows that matter waves are generated by the motion of particles.
2. Independent of charge of the particle. Matter waves are produced by the charged or uncharged particles.
3. The velocity of the matter wave depends on the velocity of a matter particle i.e., it is not a constant, while the velocity of the EM wave is a constant.
4. The lighter the particle, the grater is the wavelength associated with it, because $\lambda \propto 1/m$.
5. Larger the velocity of the particle, smaller is the wavelength associated with the particle, because $\lambda \propto 1/v$.
6. The velocity of a matter wave is greater than the velocity of light.

Matter Waves/DeBroglie waves: $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\lambda = 10^{-34}$m

Tiny wavelength-can be ignored

Submicron Objects

e^-

$\lambda = 10^{-10}$m= 1Å

Comparable with atomic length scale

Electrons are considered as both waves and particles

What is the wave nature of electrons??

In general, whenever the de Broglie wavelength of an object is in in the range of its size, the wave nature of the object is detectable

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Planck's hypothesis

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Why Quantum Physics?

"**Classical Physics**":
- developed in 15th to 20th century;
- provides very successful description of "every day, ordinary objects"
  - motion of trains, cars, bullets,….
  - orbit of moon, planets
  - how an engine works,..
- subfields: mechanics, thermodynamics, electrodynamics,

"**Quantum Physics**":
- developed early 20th century, in response to shortcomings of classical physics in describing certain phenomena (blackbody radiation, photoelectric effect, emission and absorption spectra…)
- describes "small" objects (e.g. atoms and their constituents)

Some key events/observations that led to the development of quantum mechanics…

- Black body radiation spectrum (Planck, 1901)
- Photoelectric effect (Einstein, 1905)
- Model of the atom (Rutherford, 1911)
- Quantum Theory of Spectra (Bohr, 1913)
- Compton effect (Compton, 1922)

- Exclusion Principle (Pauli, 1922)
- Matter Waves (de Broglie 1925)
- Experimental test of matter waves (Davisson and Germer, 1927)

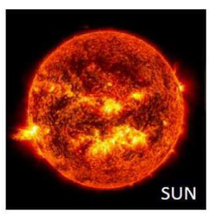
Thermal radiation

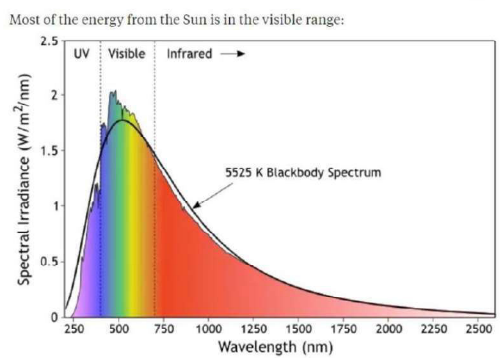
- ❑Hot filament glows.
- ❑Classical physics can't explain the observed wavelength distribution of EM radiation from such a hot object.
- ❑This problem is historically the problem that leads to the rise of quantum physics during the turn of 20th century

Black body radiation:

- **Black body** is an object that absorbs all the radiation incident upon it at all frequencies.
- Consider a piece of hot metal-It glows from red to yellow to white as it gets hotter and hotter.
- In fact it emits several other frequencies which the human eye cannot detect.

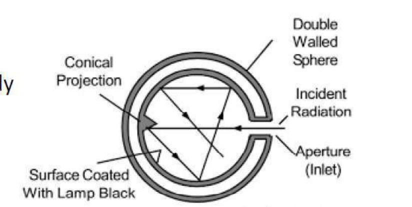
- When an object is at a constant temperature (thermal equilibrium with its surroundings), it absorbs and radiates energy in the same rate.
- Every object above 0K absorbs and radiates energy.

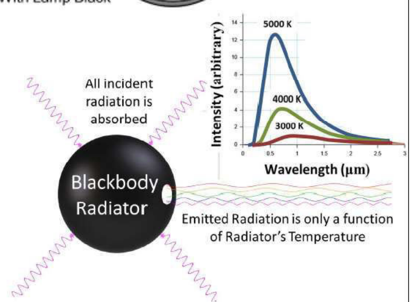




Perfect Black body:

- In order to find the spectral distribution of radiation from a solid object at a constant temperature, the concept of perfect black body was introduced.
- A perfect black body completely absorbs radiation of all frequencies incident on it and emits all the radiation at a constant temperature.
- Since a perfect black body does not exist in nature, an approximation to it is made in the laboratory.
- Consider a Hollow, spherical, double walled cavity with a tiny hole on its surface leading into the cavity.
- Any radiation falling upon the hole enters into the cavity and gets trapped. It undergoes multiple reflections back and forth, until it gets absorbed.
- By using a heat source the body is maintained in constant temperature.
- The heated the walls of black body cavity radiates all frequencies that it absorbed.
- The black body spectrum indicates that the spectral intensities are higher at elevated temperatures.
- *Spectral distribution of energy depends only on the temperature of the body and not on its shape or elemental constituents.*





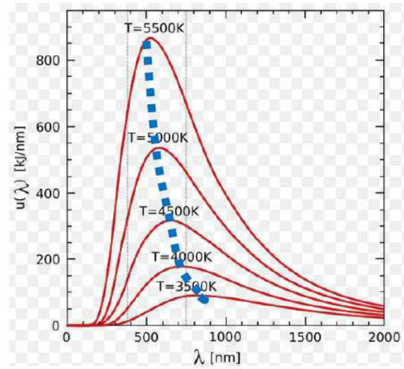
plank_s

Wein's displacement law:

The peak in the black body spectrum shifts progressively towards shorter wavelengths as the temperature increases.

$$\lambda_{max} = \frac{b}{T}$$

Where, $b = 2.898 \times 10^{-3}$ m.K



Wien's approximation: The equation does accurately describe the short wavelength (high frequency) spectrum of thermal emission from objects, but it fails to accurately fit the experimental data for long wavelengths (low frequency) emission

The blackbody radiation curve for different temperatures peaks at a wavelength is inversely proportional to the temperature.

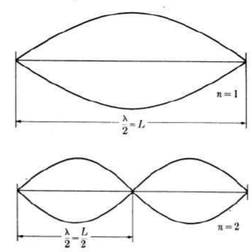
Rayleigh Jean's or classical approach:

- A cavity is at absolute temperature T.
- The walls of the cavity are perfect reflectors and radiation consist of standing electromagnetic waves.
- The condition for standing waves in such a cavity is that the path length from wall to wall must be whole number of half-wavelengths. So that the node occurs at each reflecting surface.
- The average energy per standing wave; $\bar{\epsilon} = k_B T \longrightarrow$ 1
- The density of standing waves in the cavity;

$$G(\nu)\, d\nu = \frac{8\pi}{c^3}\nu^2 d\nu \longrightarrow 2$$

- Total energy per unit volume in the cavity within the frequency interval from ν and ν + d ν is: $u(\nu)d\nu = \bar{\epsilon}\, G(\nu)d\nu \longrightarrow$ 3

$$u(\nu)\, d\nu = \frac{8\pi k_B T}{c^3}\nu^2 d\nu$$



Standing waves which can be fitted between two perfectly reflecting walls forms a standing wave.

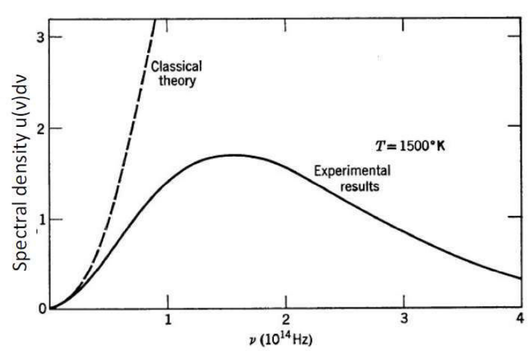
The Ultraviolet Catastrophe

Theory & experiment disagree wildly

Rayleigh Jean's law

$$u(\nu)\, d\nu = \frac{8\pi k_B T}{c^3}\nu^2 d\nu$$

- As the frequency increases (or wavelength decreases) the spectral intensity of the black body radiation should increase indefinitely .
- Rayleigh-Jean's law hold good only for the black body curve at lower frequencies (or higher wavelengths). But fails to explain high frequency (wavelength) distribution towards the UV region.

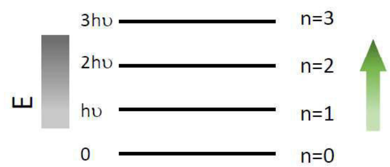


Planck's Hypothesis:

Planck made two bold and controversial assumptions concerning the nature of the oscillators in the cavity walls:

1. The energy of an oscillator can have only certain discrete values.

2. The oscillators emit or absorb energy when making a transition from one quantum state to another. The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation. If the transition is from one state to a lower adjacent state.



Planck's formulation:

- The oscillators in the cavity wall were limited to energies of E=nhν, where n= 1, 2….
- Maxwell-Boltzmann distribution law to find the number of oscillators; $\bar{\epsilon} = \frac{h\nu}{e^{h\nu/k_B T}-1}$
- The density of standing waves in the cavity; $G(\nu)\, d\nu = \frac{8\pi}{c^3}\nu^2 d\nu$
- Total energy per unit volume in the cavity within the frequency interval from ν and ν + d ν is: $u(\nu)d\nu = \bar{\epsilon}\, G(\nu)d\nu$

$$u(\nu)\, d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_B T}-1}d\nu$$

$$u(\nu)\, d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_B T}-1}d\nu$$

At high frequencies $h\upsilon >> kT$ and $e^{h\upsilon/kT} \rightarrow \infty$

NO MORE ULTRAVIOLET CATASTROPHE

Which means that $U(\nu)d\nu \longrightarrow 0$ as observed.

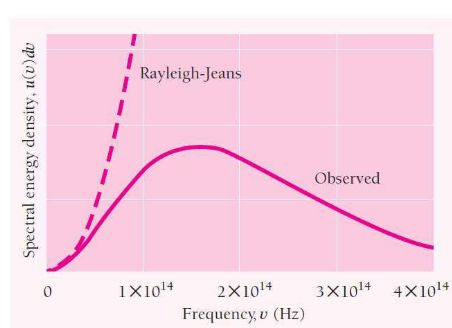
At low frequencies

$$\frac{h\upsilon}{kT} << 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots.$$

If x is small $e^x = 1 + x$

$$\frac{1}{e^{h\upsilon/kT}-1} = \frac{1}{1+\frac{h\upsilon}{kT}-1} = \frac{kT}{h\upsilon}$$



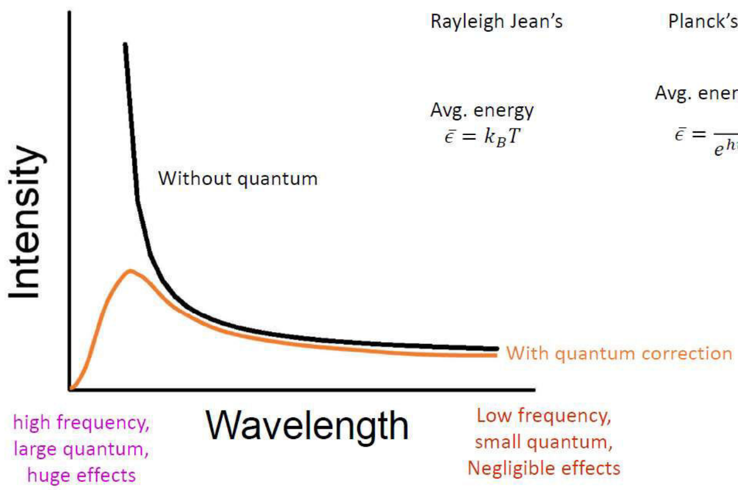
At low frequencies Planck's formula becomes

$$U(v)dv = \frac{8\pi h v^3}{c^3} dv(\frac{kT}{h\upsilon})$$

$$= \frac{8\pi k T v^2}{c^3} dv$$

which is Rayleigh Jeans formula.

How quanta overcomes the UV catastrophe



Black-body Radiation Laws:

1. Rayleigh Jean's law $\quad U(\lambda) = \frac{8\pi c k_B T}{\lambda^4}$
2. Planck's law $\quad U(\lambda) = \frac{8\pi h c^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$
3. Wein's displacement law $\quad \lambda_{peak} = \frac{b}{T}, b = 2.898\ 10^{-3} m.K$
4. Stefan-Boltzmann law $\quad j = \sigma T^4, \sigma = 5.67\ 10^{-8} W m^{-2}\ K^{-4}$

Planck radiation formulas

Frequency:

$$U(\nu) = \frac{8\pi\nu^3}{c^2}\left(\frac{1}{e^{\frac{h\nu}{k_B T}} - 1}\right)$$

Wavelength:

$$U(\lambda) = \frac{8\pi hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$

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Schrodinger Wave Equation

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Schrodinger Wave Equation

- Schrodinger wave equation describes the behaviour of a particle in a field of force or the change of a physical quantity over time. Erwin Schrödinger who developed the equation was even awarded the Nobel Prize in 1933.

- Schrodinger wave equation is a mathematical expression describing the energy and position of the electron in space and time, taking into account the matter wave nature of the electron inside an atom.

The wave Equation (Schrodinger's equation)- time dependent

Fundamental equation of Quantum Mechanics
(like second law motion of Newtonian mechanics F=ma)
Is a wave equation in the variable ψ

For standing wave equation in classical

$$y = A\cos(\omega t - kx) \qquad 1$$

Let us consider the wave equivalent of a Free Particle in a straight path at constant speed

This wave is described by general solution

$$y = A\cos(\omega t - kx) - iA\sin(\omega t - kx) \qquad 2$$

(If undamped, monochromatic harmonic wave in + x direction)
2 can be written in the form

$$y = Ae^{-i(\omega t - kx)}$$

Only real part of (2) has significance in the case of waves in a stretched string. 'y' means displacement , imaginary is discarded as irrelevant.

In quantum mechanics the wave function 'ψ' corresponds to the wave variable 'y' of wave motion in general.

However, ψ - is not measureable quantity and may therefore be complex

Wave equation

$$\Psi(x,t) = e^{i(kx-\omega t)}$$

Schrodinger time dependent wave equation

Wave equation

$$\Psi(x,t) = e^{i(kx-\omega t)}$$

From de Broglie

$$\lambda = \frac{h}{p} \Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

From Planck's

$$E = h\nu$$

$$E = h\frac{\omega}{2\pi} = \hbar\omega \qquad E/\hbar = \omega$$

$$\Psi(x,t) = e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)}$$

$$\psi = A\,Sin\frac{2\pi}{\lambda}(vt-x)$$

$$= A\,Sin\left(\underbrace{\frac{2\pi}{\lambda}v}_{f}t - \underbrace{\frac{2\pi}{\lambda}}_{k}x\right)$$

$$= A\,Sin(\omega t - kx)$$

$$\psi(x,t) = e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \qquad \frac{\partial\psi}{\partial x} = i\frac{p\psi}{\hbar}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(i\frac{p}{\hbar}\psi\right) = i\frac{p}{\hbar}\frac{\partial\psi}{\partial x} = \left(i\frac{p}{\hbar}\right)^2\psi = -\frac{p^2}{\hbar^2}\psi$$

$$p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{\partial\psi}{\partial t} = -i\frac{E}{\hbar}\psi \Rightarrow E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t} \Rightarrow E\psi = i\hbar\frac{\partial\psi}{\partial t}$$

Total Energy,

$$E = \frac{p^2}{2m} + V(x) \qquad E\psi = \frac{p^2}{2m}\psi + V(x)\psi$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi(x)}$$

Time dependent Schrödinger

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V\psi(x,y,z)}$$

In three dimension

Time independent Schrödinger wave equation

ψ(x, y, z, t) be the wave function for de Broglie waves .
The differential equation of wave given as

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)-\frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2}=0$$

The solution of differential equation in terms of time as below:

$$\psi(x, y, z, t)=\psi_0(x, y, z,)\, e^{-i\omega t}$$

Differentiating twice w.r.t. time t $\quad \frac{\partial^2\psi}{\partial t^2}=-\omega^2\psi_0 e^{-i\omega t}$ Or $\frac{\partial^2\psi}{\partial t^2}=-\omega^2\psi$

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{\omega^2}{u^2}\psi=0 \qquad \omega=2\pi\nu=2\pi\frac{u}{\lambda}$$

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{4\pi^2}{\lambda^2}\psi=0$$

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{4\pi^2}{\lambda^2}\psi=0$$

From de Broglie relation $\lambda=\frac{h}{p}$

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{4\pi^2p^2}{h^2}\psi=0$$

Total energy = Kinetic energy + Potential energy $\quad E=\frac{p^2}{2m}+V(x, y, z)$

p^2 = 2m (E-V)

$$\boxed{\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{8\pi^2m\,(E-V)}{h^2}\psi=0}$$

This is time independent Schrödinger wave equation

$$\boxed{\nabla^2\psi+\frac{2m}{\hbar^2}(E-V)\psi=0} \quad \textit{where } \nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \textit{ and } \hbar=\frac{h}{2\pi}$$

wave fn

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Wave Function (ψ)

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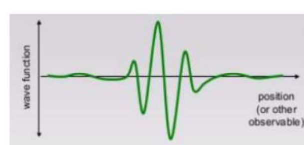
What is Wave Function?

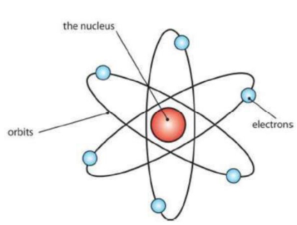
In quantum physics, a wave function is a mathematical description of a quantum state of a particle as a function of momentum, time, position, and spin. The symbol used for a wave function is a Greek letter called psi, **Ψ**.

By using a wave function, the probability of finding an electron within the matter-wave can be explained. This can be obtained by including an imaginary number that is squared to get a real number solution resulting in the position of an electron.

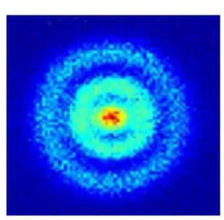
Wave function: $\psi(x,t)$

Electron
- Particle — Can be easily located in space
- Wave — Wave function — Probability of finding the electron in space

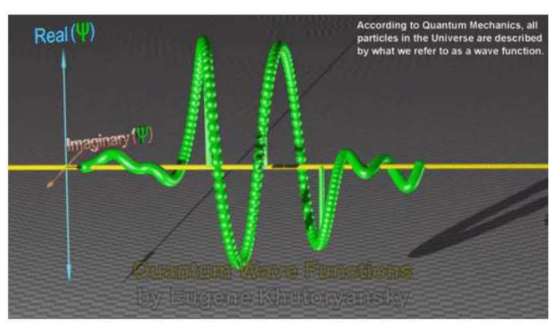




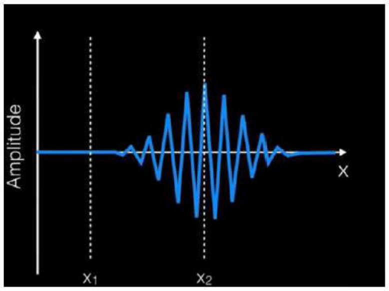
Schematic: Circular orbits



Actual: Hydrogen atom: Static wavefunction

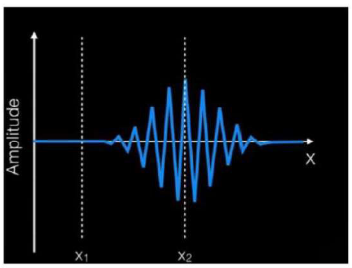


Wave function: $\psi(x,t)$

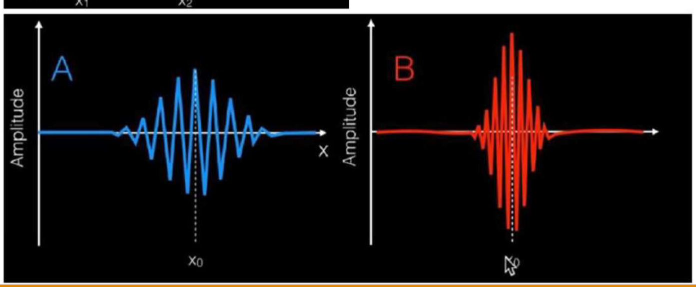


- Probability is maximum at x_2 since the amplitude of the wavefunction maximum.
- Provides all the information about the particle like position, energy and momentum at a given time.

Wave function: $\psi(x,t)$



- Maximum probability=Highest amplitude



Properties of Wave function

- $\Psi(x,t)$ is complex. It can be written in the form
$$\Psi(x,t) = A(x,t) + i\,B(x,t)$$
where A and B are real functions.

- Complex conjugate of Ψ is defined as
$$\Psi^* = A - iB$$
- $|\Psi|^2 = \Psi * \Psi = A^2 + B^2$
Therefore $|\Psi|^2 = \Psi * \Psi$is always positive and real.

- While Ψ itself has no physical interpretation, $|\Psi|^2$ evaluated at a particular place at a particular time equals to the probability of finding the body there at that time.

- Normalization: $\int_{-\infty}^{+\infty} |\Psi|^2 dV = 1$, If a wavefunction is not normalized, we can make it by dividing it with a normalization constant.

Properties of Wave function: $\psi(x,t)$

- Ψ describes the possibility of finding the particle at (x,y,z) at time t.
- Ψ is a complex quantity. Probability is always positive real quantity. $\int_{x_2}^{x_1} |\psi|^2 dx$
- Ψ must be Finite everywhere
- Ψ must be Single-valued
- Ψ must be Continuous
- Ψ must be Normalizable $\int_{-\infty}^{+\infty} |\psi|^2 dV = 1$
- By itself Ψ has no physical significance. Square of absolute magnitude give probability density.

Wave function $$\Psi = A + iB$$

where A and B are real functions. The complex conjugate Ψ^* of Ψ is

Complex conjugate $$\Psi^* = A - iB$$

and so $$|\Psi|^2 = \Psi^*\Psi = A^2 - i^2B^2 = A^2 + B^2$$

since $i^2 = -1$. Hence $|\Psi|^2 = \Psi^*\Psi$ is always a positive real quantity, as required.

Normalization

Even before we consider the actual calculation of Ψ, we can establish certain requirements it must always fulfill. For one thing, since $|\Psi|^2$ is proportional to the probability density P of finding the body described by Ψ, the integral of $|\Psi|^2$ over all space must be finite—the body is *somewhere*, after all. If

$$\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 0$$

the particle does not exist, and the integral obviously cannot be ∞ and still mean anything. Furthermore, $|\Psi|^2$ cannot be negative or complex because of the way it is defined. The only possibility left is that the integral be a finite quantity if Ψ is to describe properly a real body.

wave fn

It is usually convenient to have $|\Psi|^2$ be *equal* to the probability density P of finding the particle described by Ψ, rather than merely be proportional to P. If $|\Psi|^2$ is to

Normalization
$$\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 1 \tag{5.1}$$

since if the particle exists somewhere at all times,

$$\int_{-\infty}^{\infty} P \, dV = 1$$

A wave function that obeys Eq. (5.1) is said to be **normalized.** Every acceptable wave function can be normalized by multiplying it by an appropriate constant; we shall shortly see how this is done.

The Wavefunction

- $|\psi|^2 \, dx$ corresponds to a physically meaningful quantity -
  - the probability of finding the particle near x
- $\left|\psi^* \dfrac{d\psi}{dx}\right| dx$ is related to the momentum probability density -
  - the probability of finding a particle with a particular momentum

PHYSICALLY MEANINGFUL STATES MUST HAVE THE FOLLOWING PROPERTIES:

ψ(x) must be single-valued, and finite
(finite to avoid infinite probability density)

ψ(x) must be continuous, with finite dψ/dx
(because dψ/dx is related to the momentum density)

In regions with finite potential, dψ/dx must be continuous
(with finite $d^2\psi/dx^2$, to avoid infinite energies)

There is usually no significance to the overall *sign* of ψ(x) (it goes away when we take the absolute square)
(In fact, ψ(x,t) is usually complex !)

Compton Scattering

- Compton Effect
- Compton Theory (Derivation)
- Compton Experimental Verification

Scattering of X-rays

Two Kinds:

❑Coherent scattering or classical scattering or **Thomson scattering**
❑Incoherent scattering or **Compton scattering**

Coherent scattering:

❑X rays are scattered without any **change in wavelength.**
❑Obeys classical **electromagnetic theory**

Compton scattering: (inelastic sctattering)

❑Scattered beam consists of **two wavelengths.**

❑One is having same wavelength as the **incident beam**

❑The other is having a slightly longer wavelength called **modified beam.**

Compton Effect

A beam of monochromatic **radiation** (x-rays, γ-rays) of high frequency fall on **a low atomic no. substance**(carbon, graphite),the beam is scattered into two components.

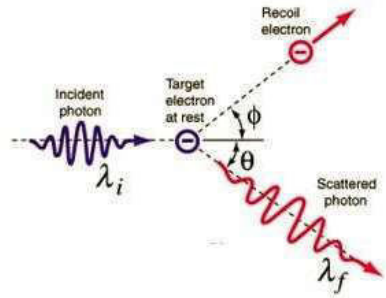
- **Modified radiation** – having lower frequency or larger wavelength
- **Un Modified radiation** – having same frequency or wavelength

This change in wavelength of the scattered X-ray is known as the Compton shift.

This effect is called **Compton Effect**.

Theory of Compton Scattering

- Compton treated this scattering as the interaction between X-ray and the matter as a particle collision between X-ray photon and loosely bound electron in the matter.
- Consider an X-ray photon of frequency ν striking an electron at rest.
- This Photon is scattered through an angle θ to x-axis.
- Let the frequency of the scattered photon be ν'.
- During the collision the photon gives energy to the electron.
- This electron moves with a velocity (v) at an angle ϕ to x-axis.



Energy Conservation:

<u>Total Energy before collision:</u>

Energy of the incident photon = $h\nu$

Energy of the electron at rest = m_0c^2

where 'm_0' is the rest mass of electron and 'c' the velocity of light.

Therefore total energy before collision = $h\nu + m_0c^2$

<u>Total energy after collision:</u>

Energy of the scattered photon = $h\nu'$

Energy of the Recoil electron = $\sqrt{p^2c^2 + m_0^2c^4}$

Therefore total energy after collision = $h\nu' + \sqrt{p^2c^2 + m_0^2c^4}$

<u>Conservation of Energy:</u>

Total energy before collision = Total energy after collision

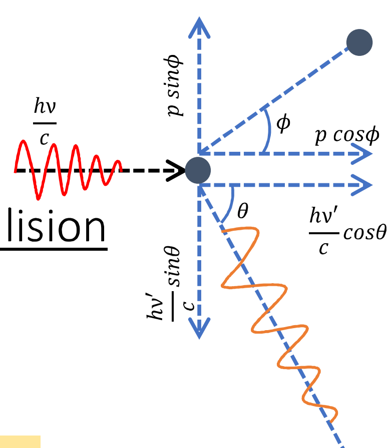
Momentum Conversation:

<u>Total momentum along x-axis before collision</u>

Momentum of incident photon along x axis = $\frac{h\nu}{c}$

Momentum of electron at rest along x axis = 0

Total momentum before collision along x axis = $\frac{h\nu}{c}$



<u>Total momentum along x-axis after collision</u>

The momentum is resolved along x axis and y axis.

momentum of scattered photon along x-axis = $\frac{h\nu'}{c}cos\theta$

momentum of recoil electron along x-axis = $p\ cos\phi$

Total momentum after collision along x-axis = $\frac{h\nu'}{c}cos\theta + p\ cos\phi$

Applying the law of conservation of momentum

Momentum before collision = momentum after collision

$$\frac{h\nu}{c} = \frac{h\nu'}{c}cos\theta + p\,cos\phi$$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}cos\theta = p\,cos\phi$$

$$h\nu - h\nu' cos\theta = pc\,cos\phi$$

Total momentum along y-axis before collision

Initial momentum of photon along y axis =0
Initial momentum of electron along y axis = 0
Total momentum before collision along y axis= 0

Total momentum along y-axis after collision

momentum of scattered photon along y axis = $\frac{h\nu'}{c}sin\theta$
momentum of recoil electron along y axis = $p\sin\phi$

Total momentum after collision along y axis = $p\sin\phi - \frac{h\nu'}{c}sin\theta$

Momentum before collision = momentum after collision

$$0 = p\sin\phi - \frac{h\nu'}{c}sin\theta$$

$$pc\sin\phi = h\nu'\,sin\theta$$

From Energy conservation:

$$h\nu + m_0c^2 = h\nu' + \sqrt{p^2c^2 + m_0^2c^4} \qquad \Rightarrow (h\nu - h\nu' + m_0c^2)^2 = p^2c^2 + m_0^2c^4$$

$$(h\nu - h\nu')^2 + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2 = p^2c^2 + m_0^2c^4$$

$$p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') + 2(h\nu - h\nu')m_0c^2 \ldots\ldots\ldots\ldots\ldots\ldots 1$$

From Momentum conservation:

$$p^2c^2cos^2\phi + p^2c^2sin^2\phi = (h\nu - h\nu' cos\theta)^2 + h\nu' sin^2\theta$$

$$p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')cos\theta \ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equation 1 and 2:

$$\cancel{(h\nu)^2 + (h\nu')^2} - 2(h\nu)(h\nu')cos\theta = \cancel{(h\nu)^2 + (h\nu')^2} - 2(h\nu)(h\nu') + 2(h\nu - h\nu')m_0c^2$$

$$-2(h\nu)(h\nu')cos\theta = -2(h\nu)(h\nu') + 2(h\nu - h\nu')m_0c^2$$

$$2(h\nu)(h\nu')(1 - cos\theta) = 2(h\nu - h\nu')m_0c^2$$

$$h\nu\nu' \,(1 - cos\theta) = (\nu - \nu')m_0c^2$$

$$\frac{(\nu - \nu')}{\nu\nu'} = \frac{h}{m_0c^2}(1 - cos\theta) \Rightarrow \left(\frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'}\right) = \frac{h}{m_0c^2}(1 - cos\theta)$$

$$\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_0c^2}(1 - cos\theta) \Rightarrow \left(\frac{c}{\nu'} - \frac{c}{\nu}\right) = \frac{h}{m_0c}(1 - cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1 - cos\theta)$$

$$\lambda' - \lambda = \lambda_c\,(1 - cos\theta)$$

Where, Compton wavelength $\lambda_c = \frac{h}{m_0c} = 0.0243\ \text{Å}$

Therefore the change in wavelength is given by; $d\lambda = \frac{h}{m_0c}(1 - cos\theta)$

- ❑ The change in wavelength **dλ does not depend** on the
  - (i) wavelength of the **incident photon**
  - (ii) Nature of the **scattering material**
- ❑ The change in wavelength **dλ depends** only on the **scattering angle.**

Case (1) When **θ=0** then,

$$d\lambda = \frac{h}{m_0c}(1 - 1);\, d\lambda = 0$$

Case (2) When θ=**90°** then,

$$d\lambda = \frac{h}{m_0c}(1 - 0);\, d\lambda = \frac{h}{m_0c}$$

Substituting the values for h,m_0 and c

$$d\lambda = \frac{h}{m_0c} = 0.0243\ \text{Å}$$

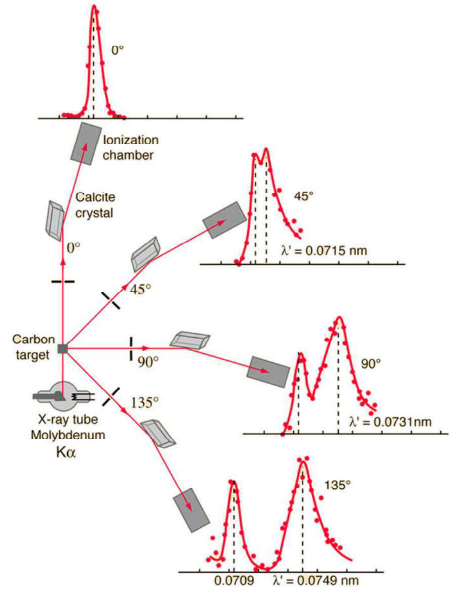
Case(3) When θ**=180°** then,

$$d\lambda = \frac{h}{m_0c}(1 - (-1));\, d\lambda = \frac{2h}{m_0c} = \ 0.0486\ \text{Å}$$

The change in wavelength is maximum at 180^0

Experimental verification

1. A beam of mono chromatic X ray beam is allowed to fall on the **scattering material.**
2. The scattered beam is received by a **Bragg spectrometer**.
3. The **intensity of the scattered beam** is measured for various angles of scattering.
4. A graph is plotted between the **intensity and the wavelength.**



Problems:

1. X-ray of wavelength 1.4 Å are scattered from a block of carbon. What will be the wave length of scattered X-rays at (i) 180° (ii) 90° (iii) 0°. At what scattering angle the maximum Compton shift is observed?

2. Explain the Compton scattering with a schematic. How does Compton shift vary with the photon scattering angle. In Compton experiment how much would be the maximum Compton shift, if electron is replaced with a neutron (mass of neutron is 2000 times that of electron.)

3. X-rays are scattered by Na crystal. Compare the wavelength of X-rays and the Compton wavelength of Na atom. What is change in wavelength you observe in this Compton scattering? [M_{Na}=3.82×10^{-26} Kg]

4. A photon of energy 3keV collides with an electron initially at rest. If photon emerges at an angle of 60^{o}, calculate kinetic energy of recoiling electron in electron volts.

5. A photon carries 2.00 x10^{-14} J of energy. It undergoes Compton scattering in a block of carbon. What is the largest fractional change in energy the photon can undergo as a result?

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Particle in a 1-D box

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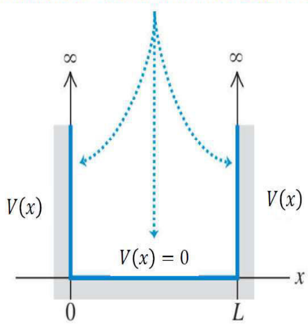
VIT Vellore

Particle in One–Dimensional Box

$$V(x) = 0; 0 < x < L,$$
$$= \infty; x \leq 0 \ and \ x \geq L$$

The potential energy V is zero in the interval 0<x<L and is infinite else where.



Schrodinger equation will reduce to:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial^2 x^2} = E\psi(x); 0 \leq x \leq L$$

Solution:

$$\psi(x) = A\cos\frac{\sqrt{2mE}}{\hbar} + B\sin\frac{\sqrt{2mE}}{\hbar}$$

Boundary conditions:

$$\psi(0) = 0 \Rightarrow A = 0$$

$$\psi(L) = 0 \Rightarrow \sin\frac{\sqrt{2mE}}{\hbar}L = 0$$

- Classical Physics: The particle can exist anywhere in the box and follow a path in accordance to Newton's Laws.
- Quantum Physics: The particle is expressed by a wave function and there are certain areas more likely to contain the particle within the box.

$$\sin\frac{\sqrt{2mE}}{\hbar}L = 0 \Rightarrow \frac{\sqrt{2mE}}{\hbar} = n\pi \Rightarrow \boxed{En = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

Eigen value

Wave Function:

$$\psi_n(x) = B\sin\frac{\sqrt{2mE_n}}{\hbar} = B\sin\frac{n\pi}{L}$$

Wave function must be normalizable

$$\int_{-\infty}^{+\infty}|\psi(x)|^2 dx = 1 \Rightarrow \int_{-\infty}^{+\infty} B^2\sin^2(\frac{n\pi}{L}x)dx = 1 \Rightarrow B = \sqrt{\frac{2}{L}}$$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x}$$

Wave Function for a Particle in One-Dimensional Box

- Energy is quantized
- Solving for the energy yields

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} \qquad (n = 1, 2, 3, \ldots)$$

- Note that the energy depends on the integer values of *n*. Hence the energy is quantized and nonzero.

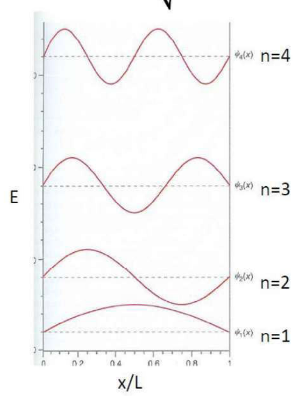
$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

Quantized Energy

$$\psi_n(x) = \sqrt{\frac{2}{L}} sin \frac{n\pi}{L}$$

Applying the Born Interpretation

$$|\psi_n(x)|^2 = \frac{2}{L}\left(\sin\frac{n\pi x}{L}\right)^2$$

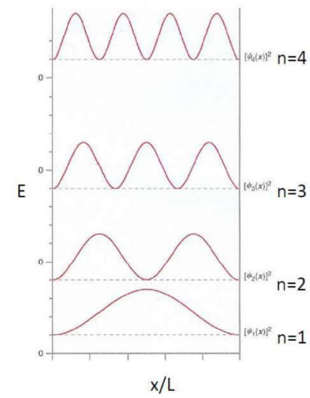


- The quantized wave number now becomes
- Solving for the energy yields

$$E_n = n^2 \frac{\pi^2 h^2}{2mL^2} \qquad (n = 1, 2, 3, \ldots)$$

- Note that the energy depends on the integer values of *n*. Hence the energy is quantized and nonzero.
- The special case of *n* = 1 is called the ground state energy.

$$E_1 = \frac{\pi^2 h^2}{2mL^2}$$



Quantized Energy

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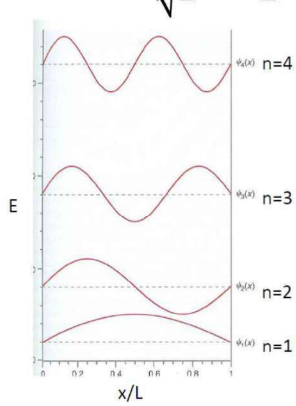
Applying the Born Interpretation

$$|\psi_n(x)|^2 = \frac{2}{L}\left(\sin\frac{n\pi x}{L}\right)^2$$



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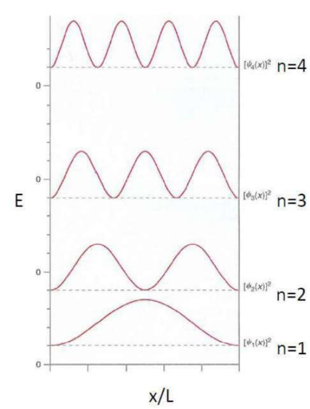

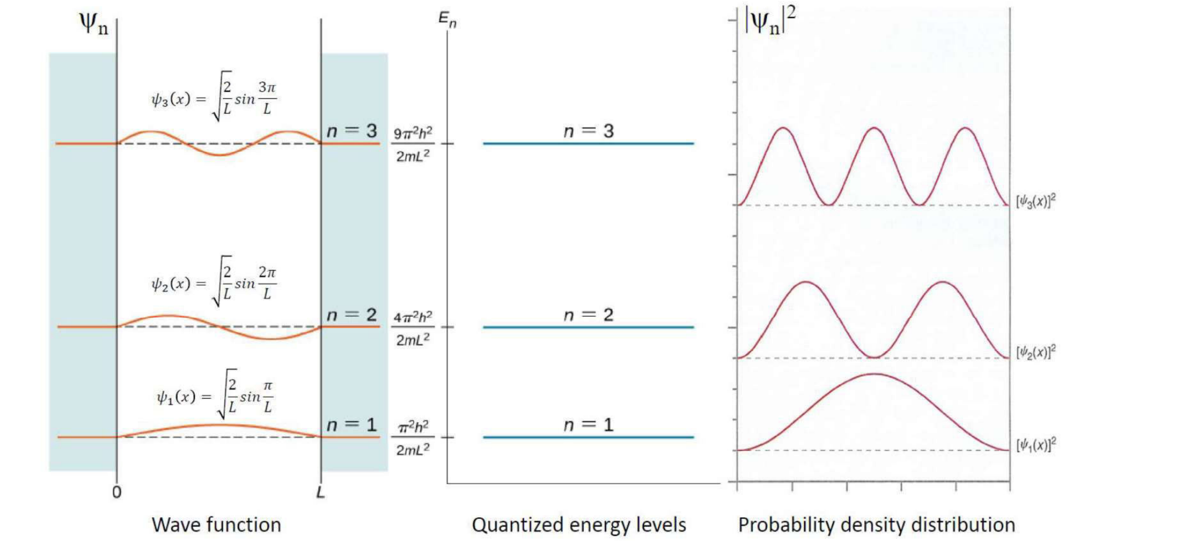

Ψ_n
$\psi_3(x) = \sqrt{\frac{2}{L}} sin \frac{3\pi}{L}$
$\psi_2(x) = \sqrt{\frac{2}{L}} sin \frac{2\pi}{L}$
$\psi_1(x) = \sqrt{\frac{2}{L}} sin \frac{\pi}{L}$
$n = 3$
$n = 2$
$n = 1$
$\frac{9\pi^2\hbar^2}{2mL^2}$
$\frac{4\pi^2\hbar^2}{2mL^2}$
$\frac{\pi^2\hbar^2}{2mL^2}$
0
L
E_n
$|\Psi_n|^2$
$[\psi_3(x)]^2$
$[\psi_2(x)]^2$
$[\psi_1(x)]^2$
Wave function
Quantized energy levels
Probability density distribution

PARTICLE IN 3D CUBICAL BOX

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Apply 1D box energy and wave function in 3D as

$$E_x = \frac{n_x^2 h^2}{8mL_1^2} \qquad E_y = \frac{n_y^2 h^2}{8mL_2^2} \qquad E_z = \frac{n_z^2 h^2}{8mL_3^2}$$

$$En = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \psi_n(x) = \sqrt{\frac{2}{L}} sin\frac{n\pi}{L}x$$

Wave function

$$X(x) = \sqrt{\frac{2}{L_1}} \sin\left(\frac{n_x \pi x}{L_1}\right)$$

$$Y(y) = \sqrt{\frac{2}{L_2}} \sin\left(\frac{n_y \pi y}{L_2}\right)$$

$$Z(z) = \sqrt{\frac{2}{L_3}} \sin\left(\frac{n_z \pi z}{L_3}\right)$$

The wave function 3D as

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

$$\sqrt{\frac{2}{L_1}} \sin\left(\frac{n_x \pi x}{L_1}\right) \sqrt{\frac{2}{L_2}} \sin\left(\frac{n_y \pi y}{L_2}\right) \sqrt{\frac{2}{L_3}} \sin\left(\frac{n_z \pi z}{L_3}\right)$$

$$\boxed{\psi(x,y,z) = \sqrt{\frac{8}{L_1 L_2 L_3}} \sin\left(\frac{n_x \pi x}{L_1}\right) \sin\left(\frac{n_y \pi y}{L_2}\right) \sin\left(\frac{n_z \pi z}{L_3}\right)}$$

This is the total wave function of free particle in 3D box

The Energy of particle 3D as

$$E = E_x + E_y + E_z$$

$$E = \frac{n_x^2 h^2}{8mL_1^2} + \frac{n_y^2 h^2}{8mL_2^2} + \frac{n_z^2 h^2}{8mL_3^2}$$

$$\boxed{E = \frac{h^2}{8m}\left[\frac{n_x^2}{L_1^2} + \frac{n_y^2}{L_2^2} + \frac{n_z^2}{L_3^2}\right]}$$

This is the total energy of free particle in 3D box

Degeneracy of energy level of particle in 3D box

Distinct energy level posses same energy

$$E = \frac{h^2}{8m}\left[\frac{n_x^2}{L_1^2} + \frac{n_y^2}{L_2^2} + \frac{n_z^2}{L_3^2}\right]$$

total energy of free particle in 3D box

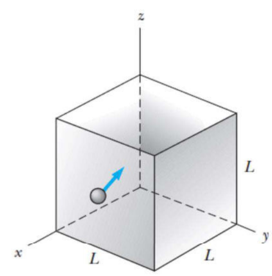

Figure 8.1 A particle confined to move in a cubic box of sides L. Inside the box $U = 0$. The potential energy is infinite at the walls and outside the box.

For cube L_1=L_2=L_3

$$E_{n_x, n_y, n_z} = \frac{h^2}{8mL^2}\left[n_x^2 + n_y^2 + n_z^2\right]$$

SCANNING TUNNELING EFFECT

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Scanning Tunneling Microscopy (STM)

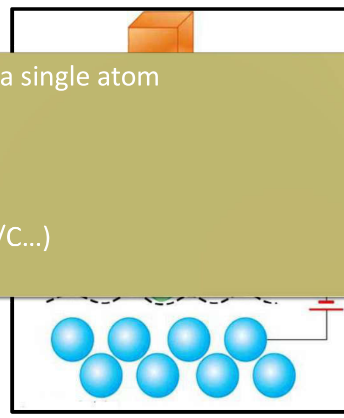
Uses the principle of Quantum mechanical tunneling. Invented in 1981 by Gerd Binnig and Heinrich Rohrer at IBM Zurich. Awarded the Nobel prize in Physics after 5 years

- An electrically conducting probe with a very

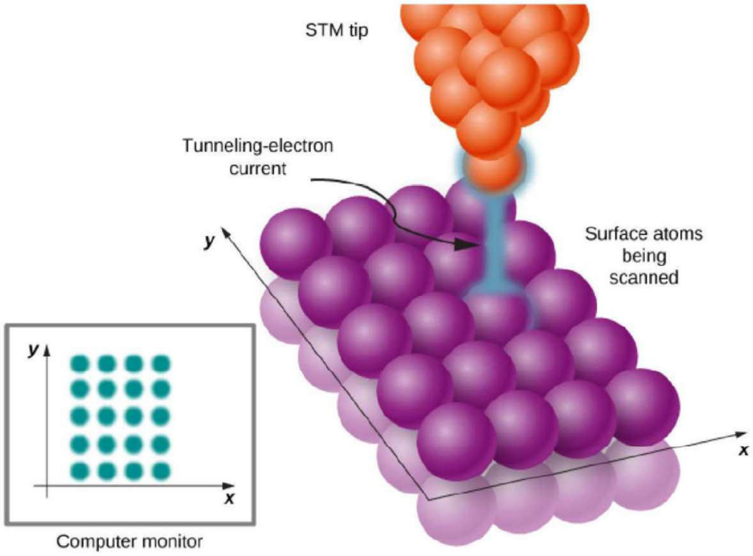
STM tip: Atomically sharp needle and terminates in a single atom

- ✓ Pure metals (W, Au)
- ✓ Alloys (Pt-Rh, Pt-Ir)
- ✓ Chemically modified conductor (W/S, Pt-Rh/S, W/C…)

- The tip and the surface are the two walls of the "potential well".

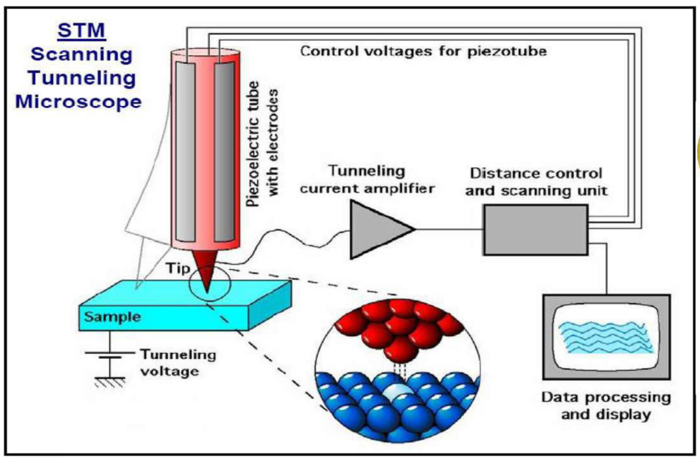


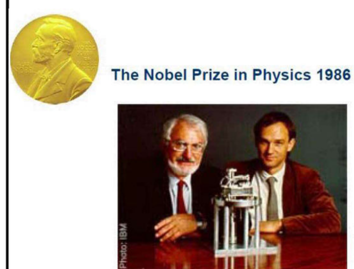
- When the tip is brought about 1 nm of the sample, electrons from the sample begin to tunnel through the 1 nm gap into the conducting tip or vice versa.
- The resulting tunneling current varies with tip to sample spacing and is the signal used to create an STM image.
- For tunneling to occur, both the sample and tip must be conductors or semiconductors. Hence, STMs cannot be used for imaging insulating materials.
- The Angstrom level distance control between the tip and sample surface is required which is acquired by piezoelectric motors.
- A piezoelectric is a crystal that creates potential differences when mechanical stresses are imposed on it or vice versa.
- The piezoelectric motors can be used in a feedback system to adjust the motion of the tip in all the 3 dimensions.



In STM, a surface at a constant potential is being scanned by a narrow tip moving along the surface. When the STM tip moves close to surface atoms, electrons can tunnel from the surface to the tip. This tunneling-electron current is continually monitored while the tip is in motion. The amount of current at location (*x*,*y*) gives information about the elevation of the tip above the surface at this location. In this way, a detailed topographical map of the surface is created and displayed on a computer monitor.

UHV





Nobel Laureates Heinrich Rohrer and Gerd Binnig

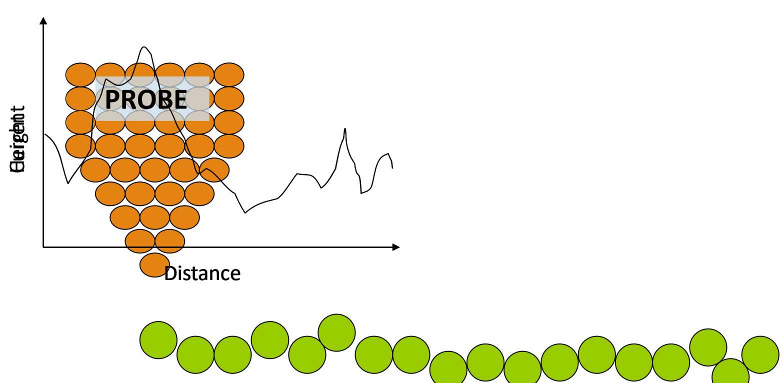
- ❖ Raster the tip across the surface and using the current as a feedback signal.
- ❖ The tip-surface separation is controlled to be constant by keeping the tunneling current at a constant value. (Constant Height Mode)
- ❖ The voltage necessary to keep the tip at a constant separation is used to produce a computer image of the surface.

Probing a surface using STM

STM can be used either in the Constant Current mode or in the Constant Height mode

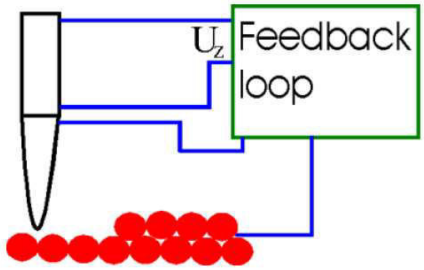
Or we could vary the height in search for constant current.

We could either probe at a constant height and look at current variations as we pass.



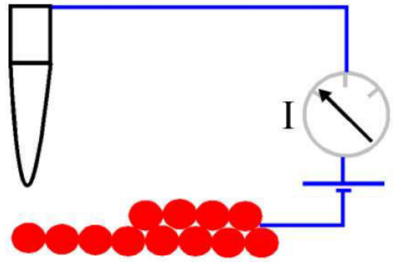
STM working procedure

Constant current mode



- ❑ ***Image the surface with constant tunnel current and variable height***
- ❑ ***Feed back loop help to maintain constant current***
- ❑ ***Surface (height) structure can detect***

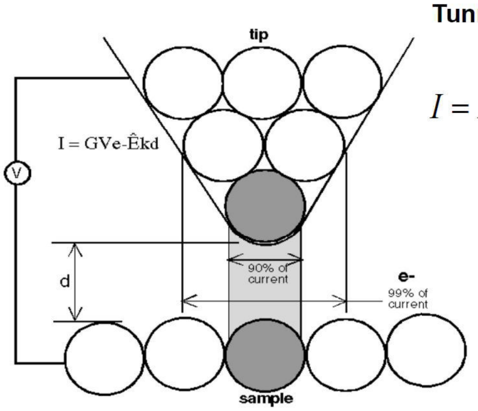
Constant height mode



- ❑ ***Image the surface with constant height and variable tunnel current***
- ❑ ***Electron density on the surface can detect***

STM working Principle

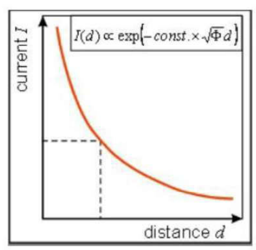
STM works on the principle of quantum mechanical tunneling



Tunneling current

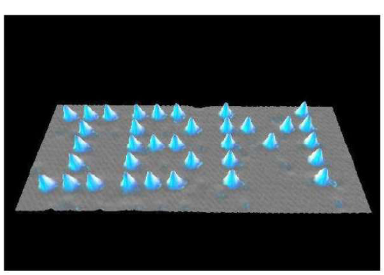
$$I = A \times eV \times e^{-2\sqrt{\frac{2m\Phi}{\hbar^2}}d}$$

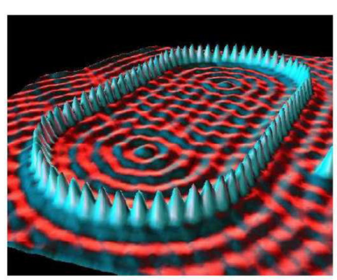
A is constant
e is electron charge
V is voltage
m is mass of electron
Φ is work function of metal tip
d is distance between tip and sample



Playing with Atoms

- At this distance the coulomb force between the tip and an atom of the sample is actually enough to move the atom.
- This has allowed physicists to create images and structures on the atomic level.





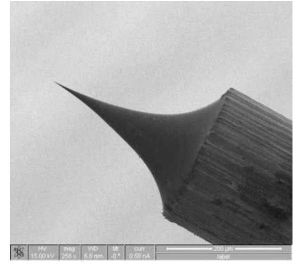
Images courtesy of IBM

Advantages

- No damage to the sample
- Vertical resolution superior to SEM
- Spectroscopy of individual atoms
- Relatively Low Cost

Disadvantages

- Samples limited to conductors and semiconductors
- Limited Biological Applications: AFM
- Generally a difficult technique to perform



It is very important that the tip of the probe be a single atom.

Tungsten is commonly used as Electro-chemical etching techniques can create very sharp tips like the one above.

Figures of Merit

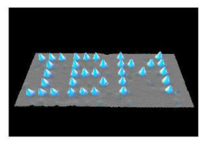
- Maximum Vertical Resolution: .1 Å
- Maximum Lateral Resolution: 1 Å
- Maximum Field of View: 100 μm

STM Applications

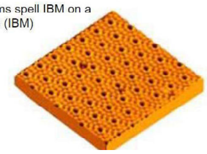
Widely used in nanotechnology

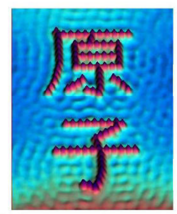
- ❖ Image the surface structure
- ❖ Estimate surface roughness
- ❖ 3D images of the surface
- ❖ Locate the defect on the surface of crystal
- ❖ Understand electric structure of materials

STM allows manipulation of individual atoms (1989)



Xenon atoms spell IBM on a nickel plate (IBM)





Iron atoms spell "Atom" on copper in Kanji characters. (IBM)

TUNNELING EFFECT

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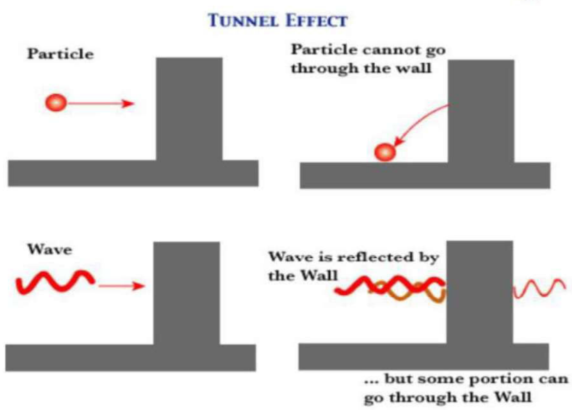
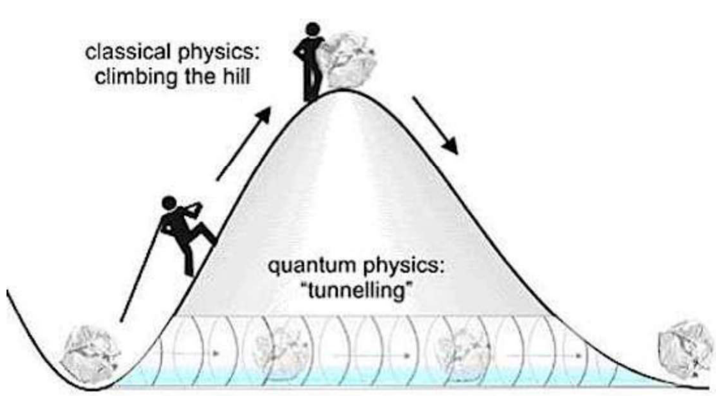
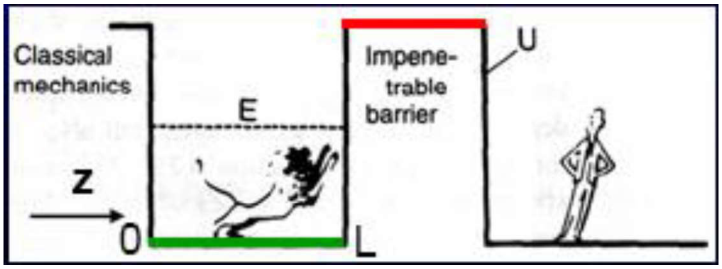
Quantum Mechanical Tunneling Effect

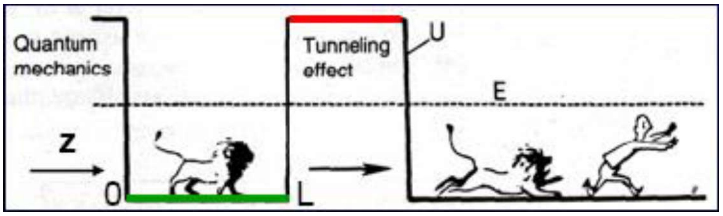


The quantum particle exhibits wave-like nature, it can reflect and transmit (tunnel) through the potential barrier

Quantum Tunneling







DEFINITION

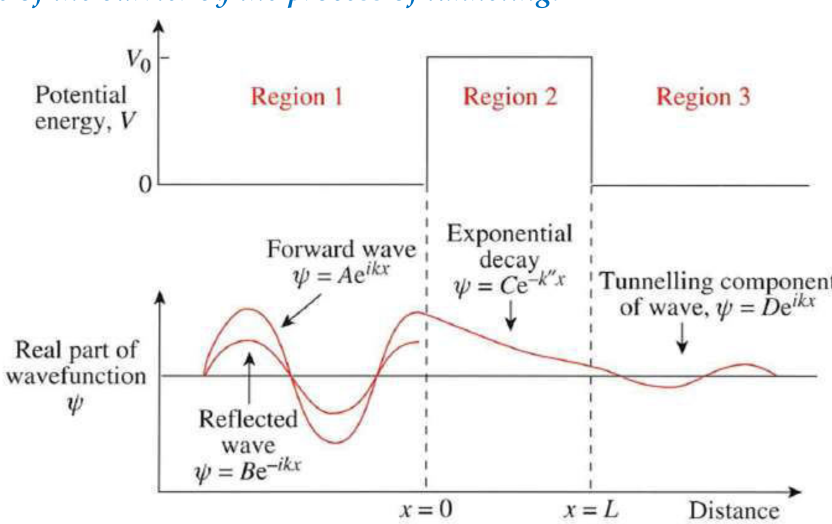
*Another phenomenon explained by the particle-wave is **tunneling**, which occurs when a particle actually passes through a seemingly impenetrable barrier. When a particle hits a barrier, it either has enough energy to break through or it doesn't and bounces back. But with a wave, part of it can pass through while part of it is reflected, making it possible for the particle to appear on the other side.*

***Tunnel effect:** when a particle is able to cross a potential barrier even when its energy is less than the barrier height, then this phenomenon is called "tunnel effect'. It is purely quantum mechanical phenomenon, never realizable classically.*

Quantum tunneling or tunneling is the quantum mechanical phenomenon where a wavefunction can propagate through a potential barrier.

The emission of α-particles from atomic nuclei is an example of tunnel effect

In quantum mechanics there is finite probability that the particle will appear on the other side of the barrier by the process of tunneling.



One dimensional Schrodinger equation to be solved for this problem is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}(E-U)\psi=0$$

Depending upon +ve or –ve of (E-U), we will have different solutions

REGION –I

U(x) = 0 for -∞ > x > 0

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}E\psi=0$$

Solution

$$\psi_I = Ae^{ikx}+Be^{-ikx}$$

Incident wave Reflected wave

A, B and k are constant where $k=\frac{\sqrt{2mE}}{\hbar}$

Since the probability of the particle tunneling through the barrier is very small, the reflected wave is going to be almost as strong as incident wave

REGION –II U(X) = U_0 FOR 0 < X < L

$$\frac{\partial^2\psi_{II}}{\partial x^2}+\frac{2m}{\hbar^2}(E-U_0)\psi_{II}=0$$

$$\frac{\partial^2\psi_{II}}{\partial x^2}-\frac{2m}{\hbar^2}(U_0-E)\psi_{II}=0 \longrightarrow \because U_0 \rangle E$$

$$\frac{\partial^2\psi_{II}}{\partial x^2}-k_2^{\,2}\psi_{II}=0 \longrightarrow where \qquad k_2=\frac{\sqrt{2m(U_0-E)}}{\hbar} \longrightarrow (1)$$

$$\psi_{II}=Ce^{-k_2x}+De^{+k_2x}$$

There is no 'i' this means that 'ψII' would not be an oscillating wave

But simply an exponential decay

So $\psi_{II}=Ce^{-k_2x}$

REGION –III U(X) = 0 FOR L< X < ∞

$$\frac{\partial^2\psi_{III}}{\partial x^2}+\frac{2m}{\hbar^2}E\psi_{III}=0$$

Solution

$$\psi_{III}=Fe^{ikx}$$

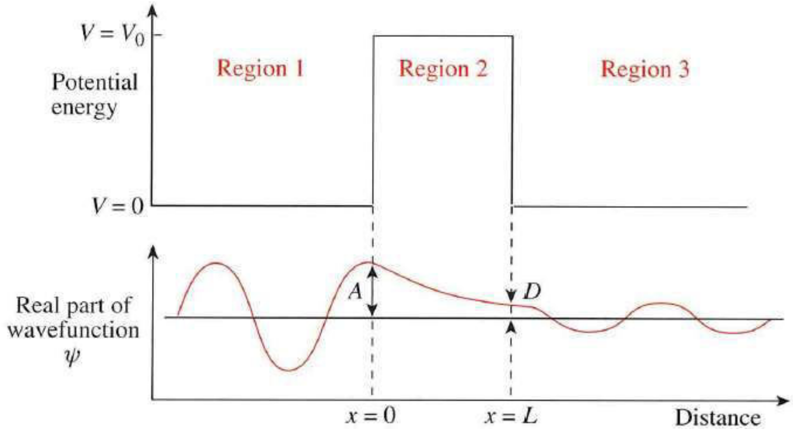
Forwarded way

F is a constant where $k=\frac{\sqrt{2mE}}{\hbar}$

The wave eq. has the Same form of eq. in The region –I

One difference is The only particle moving Away from the barrier

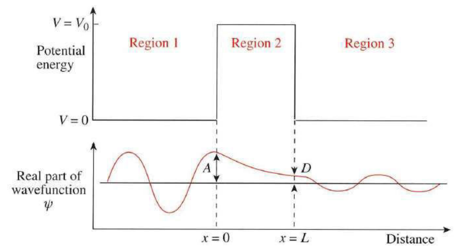
No particle approaching the barrier on Other side



Total wave function = joining together the separate wave functions from 3 regions

This should be continuous in either 'ψ' and '$d\psi/dx$' at the boundaries

∴ '$\psi_1 = \psi_2$' and '$d\psi_1/dx = d\psi_2/dx$'



- **similarly**
- **∴ '$\psi_3 = \psi_2$' and '$d\psi_3/dx = d\psi_2/dx$'**
- **From approximation**
- **The reflected component of ψ_1, e^{-ikx} can be ignored**

$$\psi_I = Ae^{ikx}$$

- **And boundary x=0** $\psi_I = \psi_{II} \longrightarrow Ae^{ik.0} = Ce^{-k.0}$

$$A=C \longrightarrow \textbf{(2)}$$

- **matching the magnitude not gradient of ψ_1 and ψ_2**
- **At boundary x=L**

$$\psi_{II} = \psi_{III} \longrightarrow Ce^{-k_2.L} = Fe^{ikL} \longrightarrow \textbf{(3)}$$

$$F = Ae^{-L(k_2+ik)} \longrightarrow \textbf{(4)}$$

- **The probability of the particles that leave the barrier after tunneling is proportional to**

$$\psi_{III}{}^*\psi_{III} = F^*e^{-ikx}Fe^{ikx}$$

Particles that arrive in front of the barrier is

$$= A^*e^{-ikx}Ae^{ikx}$$

The tunneling probability P is the ratio between the above two

$$P = \frac{F^*e^{-ikx}Fe^{ikx}}{A^*e^{-ikx}Ae^{ikx}} = \frac{F^*F}{A^*A}$$

$$P = \frac{A^*e^{-L(k_2+ik)}Ae^{-L(k_2-ik)}}{A^*A} = e^{-2Lk_2}$$

$$P = e^{-\frac{2L\sqrt{2m(U_0-E)}}{\hbar}}$$

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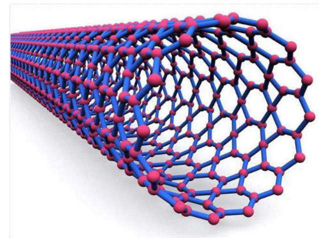
(Deemed to be University under section 3 of UGC Act, 1956)

CARBON NANOTUBES (CNT): SYNTHESIS & APPLICATIONS



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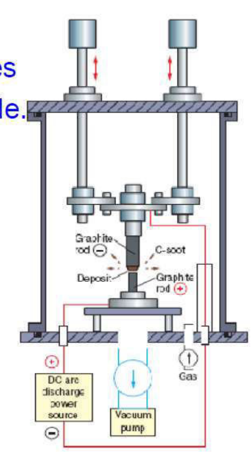
Synthesis of Carbon NanoTube

- Most widely used methodologies
  1. Arc-discharge method
  2. Laser ablation method
  3. Thermal synthesis
     - 3.1 Chemical vapour deposition
     - 3.2 Flame synthesis
  4. Plasma Enhanced CVD synthesis

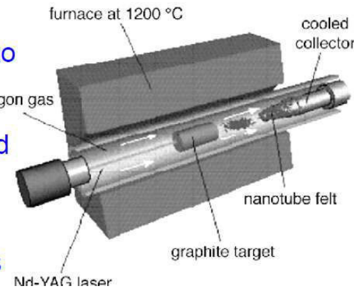
Arc-discharge Method

- Evaporation of graphite rod by applying a DC arc voltage in an inert gas (He).
- The evaporated anode generates fullerenes in the form of soot deposited on the cathode.
- The deposited product contains CNTs.
- MWCNTs are produced for pure graphite.
- Graphite rod with metal catalysts (Fe, Co) produced SWNTs.
- Methane or Hydrogen gas environments were found to be more effective for higher yield and crystallinity of MWNTs.



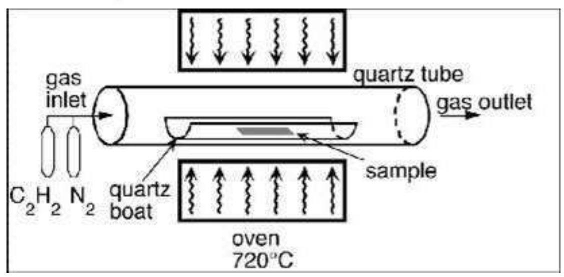
Laser Ablation Method

- High power laser was focused onto a carbon target (graphite) maintained at high temperature (1200°C) inside a (quartz) tubular furnace to vaporize carbon from the graphite target.
- An Inert gas (Argon at 500 torr) environment was maintained inside the tube, which carries the vapours from high temperature chamber into a cooled collector.
- A metal (1.2% Co / Ni) doped graphite (98.8%) source is used to produce SWNT.
- Other parameters for quantity and quality of CNTs are Type and amount of catalyst, Laser power, Temperature, Pressure, Inert gas



Chemical Vapour Deposition Method

Hydrocarbon + Fe/Co/Ni catalyst $\xrightarrow{550\text{-}750^{\circ}C}$ CNT



Steps:

- Dissociation of hydrocarbon.
- Dissolution and saturation of C atoms in metal nanoparticle.
- Precipitation of Carbon.

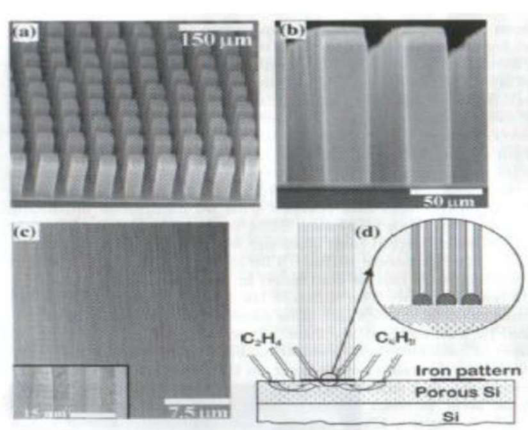
Choice of catalyst material?

Base Growth Mode or Tip Growth Mode?

- Metal support interactions

Controlled Growth by CVD

Methane + Porous Si + Fe pattern $\xrightarrow{CVD}$



a) SEM image of aligned nanotubes.
a) SEM image of side view of towers. Self-alignment due to Van der Walls interaction.
a) High magnification SEM image showing aligned nanotubes.
d) Growth Process: Base growth mode.

cnt s n a

APPLICATIONS

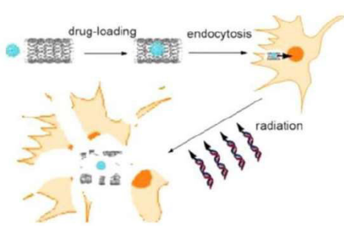
Carbon Nanotube can be used for a wide range of new and existing applications:

- Conductive plastics
- Flat-panel displays
- Gas storage
- Antifouling paint
- Structural composite materials
- Micro- and nano-electronics
- Radar-absorbing coating
- Technical textiles
- Ultra-capacitors
- Atomic Force Microscope (AFM) tips
- Batteries with improved lifetime
- Biosensors for harmful gases
- Extra strong fibers

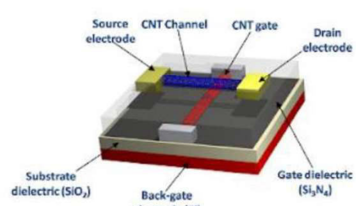
Bionics

Crisis 2.0 inspired body armor for soldiers –made of light weight high strength carbon nanotube composites

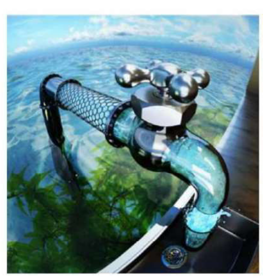
Bionic arm-Artificial muscles by contracting carbon nanoribbons



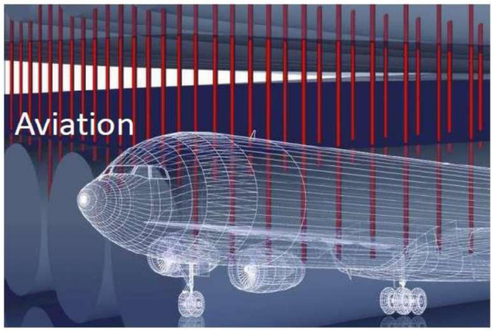
Using CNTs as carrier agents in Hyperthermia for cancer therapy



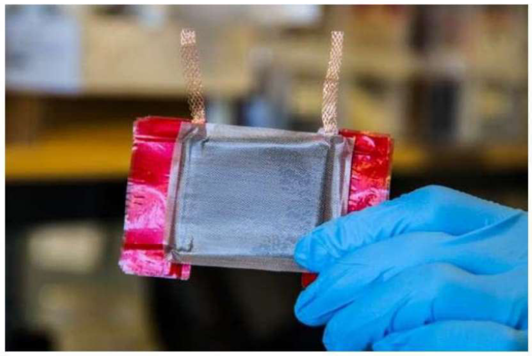
CNT based nano transistor



Removal of salt from seawater using carbon nanotube filters



advanced composite materials such as carbon fiber reinforced plastic—extremely light, durable materials that reduce the overall weight of the plane by as much as 20 percent compared to aluminum-bodied planes. Such lightweight airframes translate directly to fuel savings, which is a major point in advanced composites' favor.



Energy storage-battery based on CNT

Electronics

electronic packaging to meet electrostatic discharges (ESD) and high cleanliness and also to avoid Overheating

- IC trays and Wafer Carriers
- IC test sockets

Automotive

electrically conductive additives for automotive fuel system line components requiring **electrical conductivity**.

Thermoplastic exterior parts, such as fenders, mirror housings, and door handles

Aeronautic

flame retardant protection of fuel tanks and exhaust parts.

Construction

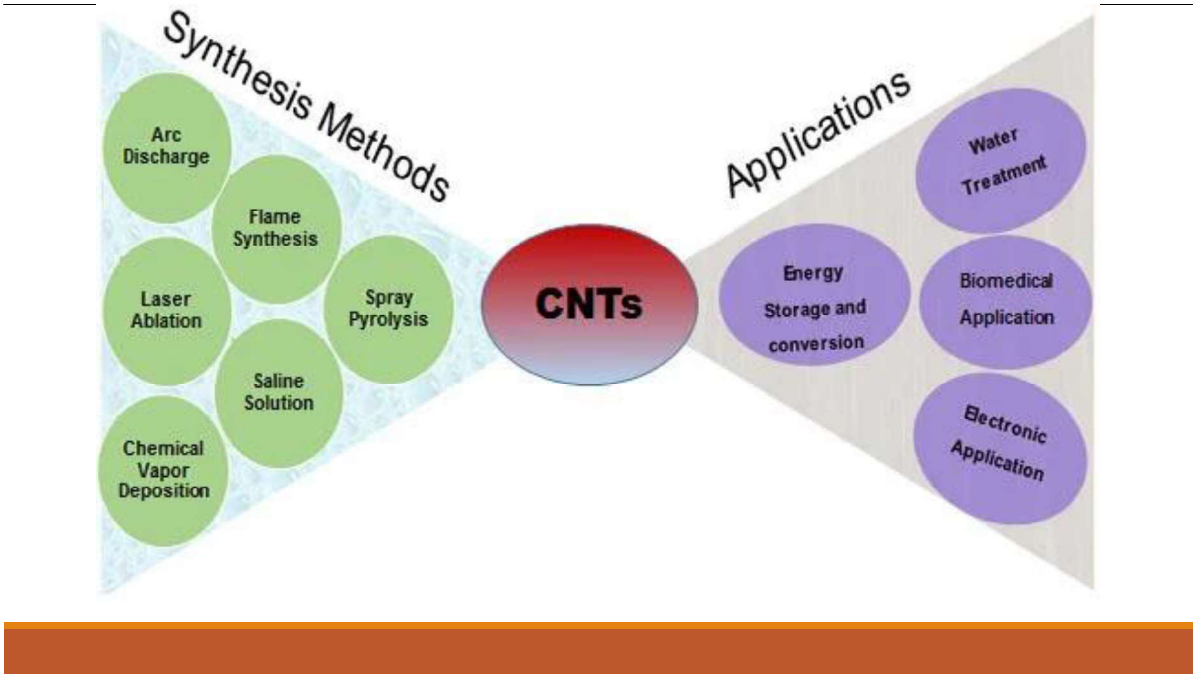
CNT provides protection for construction substrates, including **metal, concrete**, **wood**, **plaster** and **fiberboard**.

an array of **flexible** and **cost-efficient** solutions:

- Excellent thermal barrier even at low coating thicknesses
- Fast curing solutions and shortened curing time for off-site coating
- creating more design flexibility
- Better scratch resistance when handling
- No use of solvents or water
- Very low smoke density and toxicity

Sports

bike frames, hockey sticks, tennis rackets,
golf shafts, and skis.


Synthesis Methods
Arc Discharge
Flame Synthesis
Laser Ablation
Spray Pyrolysis
Saline Solution
Chemical Vapor Deposition
CNTs
Applications
Water Treatment
Energy Storage and conversion
Biomedical Application
Electronic Application

cnt s n a

CARBON NANOTUBES (CNT)

Dr Rajeshkumar Mohanraman
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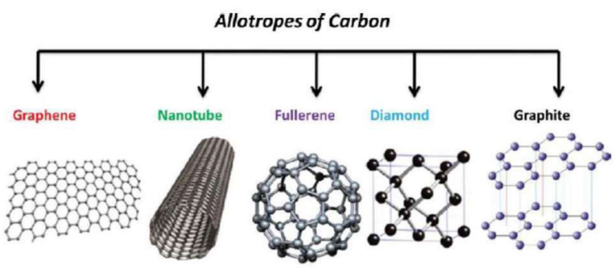
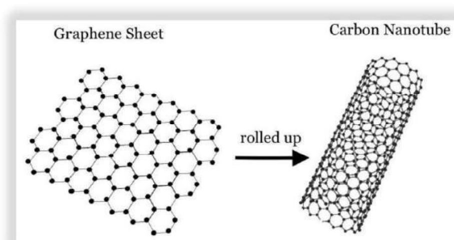
What are Carbon Nano Tubes (CNTs) ?

- CNTs are allotropes of carbon with cylindrical nanostructure.
- The structure of CNT is formed by a layer of carbon atom that are bonded together in hexagonal (honeycomb) mesh.
- This one atom thick layer of carbon is called Graphene.
- Graphene is wrapped in the shape of a cylinder to form CNTs.
- Tube diameter has dimensions in nanometer-2D quantum confinement-Quantum wire.

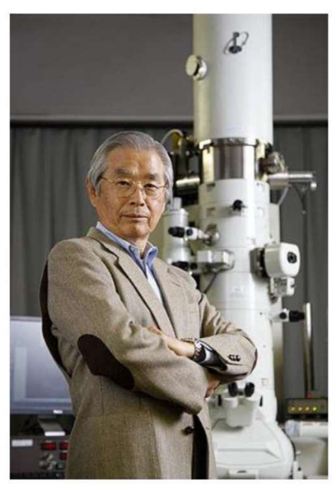




Discovery of CNTs

1991:Sumio Iijima - NEC Laboratory in Tsukuba- used high-resolution transmission electron microscopy to observe carbon nanotubes.

(carbon soot of graphite electrodes during an arc discharge, that was intended to produce fullerenes)

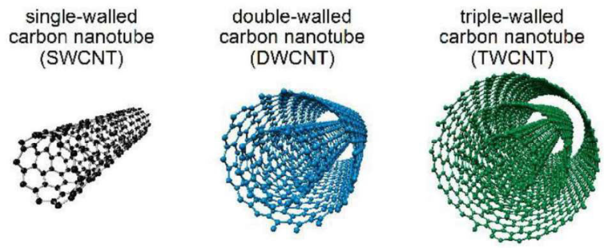


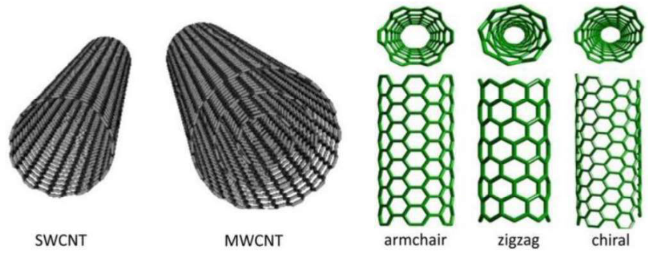
Types of Carbon nanotubes

Based on their shape and size. CNTs are classified as single walled carbon nanotube (SWCNT) and Multiwalled carbon nanotube (MWCNT)

1. SWCNT-single outer wall of graphene rolled up
2. MWCNTs-Multiple CNTs one inside the other.

Further classification of CNTs based on their Chirality. Angle of twist in the rolled up carbon layer.





Properties of CNTs

- The cylindrical carbon molecules have interesting properties that make them potentially useful in many applications in nanotechnology, electronics, optics , fields of materials science, potential uses in architectural fields.

- Exhibit extraordinary strength and unique electrical properties, and are efficient conductors of heat.

Strength Properties

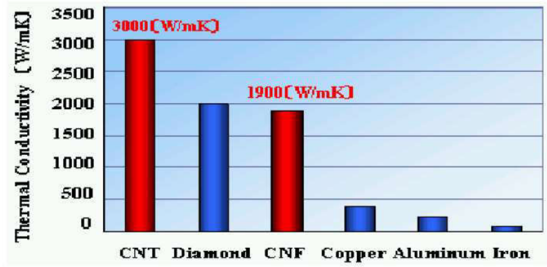
Carbon nanotubes have the strongest tensile strength of any material known. **It also has the highest modulus of elasticity.**

Electrical Properties

- If the nanotube structure is armchair, then the electrical properties are metallic.
- If the nanotube structure is chiral then the electrical properties can be either semiconducting with a very small band gap, otherwise the nanotube is a moderate semiconductor.
- In theory, metallic nanotubes can carry an electrical current density of 4×109 A/cm^2 which is more than 1,000 times greater than metals such as copper.

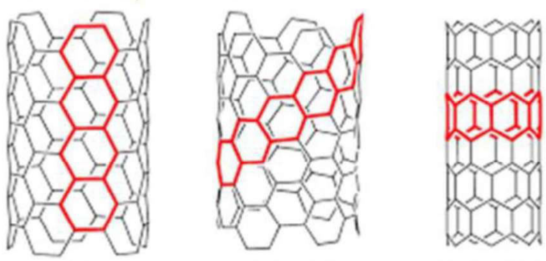
Thermal conductivity of CNTs

- Excellent thermal conductivity along the length of the tube but not along lateral direction
- Thermal conductivity for CNT, K=3500 W/mK. This value is 100 times greater than aluminum.



Electronic property of CNTs

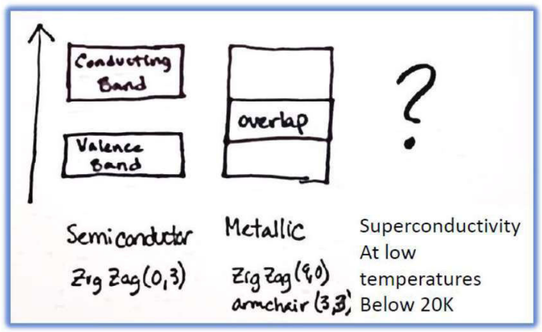
- Electronic properties are determined by the shape and structure of CNTs.
- Nanotubes are mostly metallic or semiconductors based on their chirality.
- Chirality changes band gap and hence the electrical conductivity of SWCNT



[5,5] CNT
Armchair
Metallic

[7,5] CNT
Chiral
Semiconducting

[7,0] CNT
Zigzag
Semiconducting



Chirality	Band gap Eg (eV)	
Arm chair (3,3)	0.73eV	Metal
Zig zag (0,3)	2.72eV	Semiconductor
Chiral (2,3)	0.93eV	Semiconductor

Properties of Carbon Allotropes

Allotrope	Hardness	Tensile strength	Conducts heat	Conducts electricity
Coal	+	+	+	no
Graphite	++	++	+++++	+++++
Diamond	+++++	Not known	+++	no
Buckyballs	+++++	++++	+	+
Carbon Nanotubes	++++++	+++++	+++++	++++++

Structure of Carbon Nanotubes

- **Single-wall carbon nanotube (SWNT)**
- **Multiwall Nanotubes (MWNT)**

Single-wall nanotubes (SWNT)

Tubes of graphite that are normally capped at the ends and a single cylindrical wall. The structure of a SWNT can be visualized as a layer of graphite, a single atom thick, called graphene, which is rolled into a seamless cylinder.

Typically have a diameter of **close to 1 nm**. The tube length, however, can be **many thousands of times longer**.

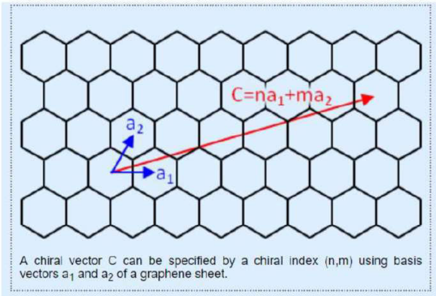
They can be twisted, flattened, and bent into small circles or around sharp bends without breaking.

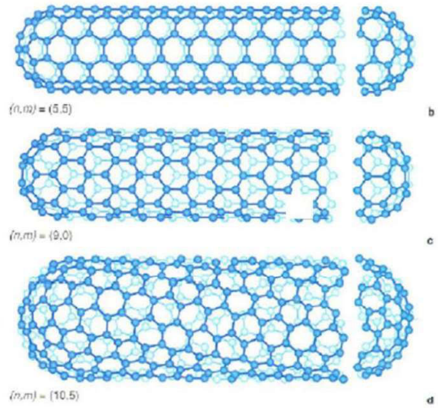
Unique electronic and mechanical properties which can be used in numerous applications (field-emission displays, nanocomposite materials, nanosensors, and logic elements). Expected as next generation of miniaturized electronics.

Chirality of single-walled carbon nanotubes

- A sheet made of carbon atoms arranged in a honeycomb lattice is called a graphene sheet.
- Single-walled carbon nanotubes are made by rolling up such a graphene sheet into a tube with a diameter of few nanometers.
- In order to form a seamless tube, you would need to take two of the hexagons and overlap them.
- A vector connecting the centers of the two hexagons is called the chiral vector, and it determines the structure of a single-walled carbon nanotube.
- Chiral vector C can be written as C = n a1 + m a2 where a1 and a2 are basis vectors of the graphene lattice. The pair of integers (n,m) is called the chiral index or just chirality. This implies that the structure of a single-walled carbon nanotube is completely determined by chirality.
- What's really interesting about single walled carbon nanotubes is that their electronic structure can become either semiconducting or metallic depending on this chirality, and the bandgap energy also depends on chirality.
- Unfortunately, currently we do not have control over chirality when we synthesize carbon nanotubes. Nevertheless, measurements on individual nanotubes can clarify the different characteristics that depend on the chirality.

Chirality of single-walled carbon nanotubes





Chiral angle is θ

Armchair (n,m) = (5,5)
$\theta = 30°$

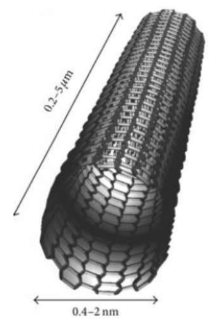
Zig Zag (n,m) = (9,0)
$\theta = 0°$

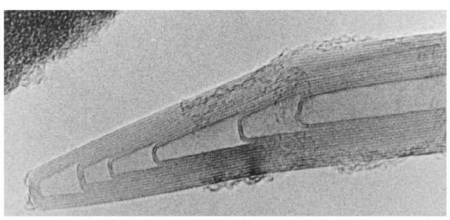
Chiral (n,m) = (10,5)
$0° < \theta < 30°$

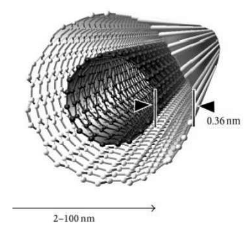
Two types of CNTs

- Single walled (SWNT): A single-atom thick graphite (graphene) sheet rolled into cylinder and capped with fullerene hemisphere.

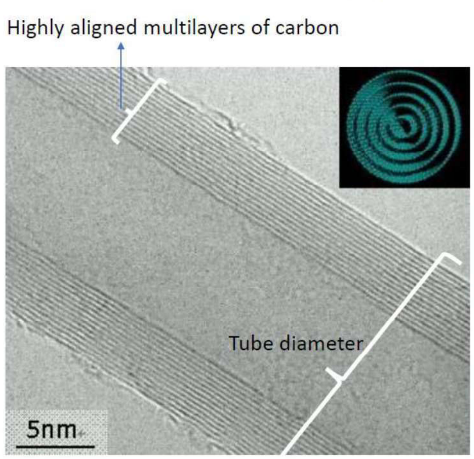
- Multi-walled (MWNT): Multiple rolled layers (concentric tubes) of graphene.



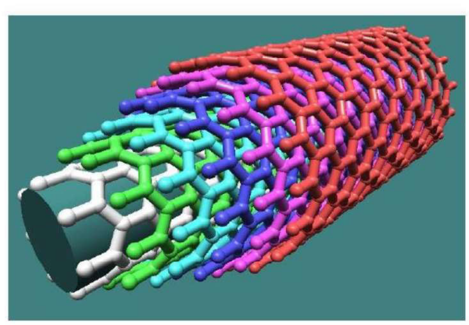




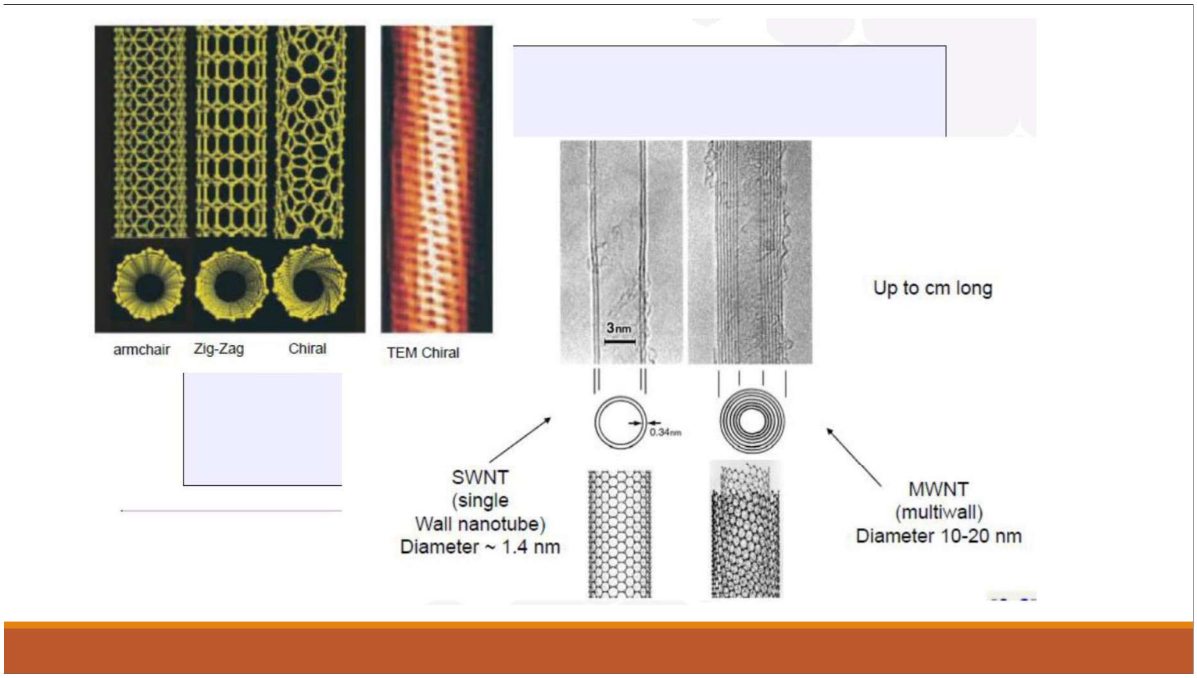
Multiwall Nanotubes



TEM image of multi walled carbon nanotube (MWCNT)



cnt



armchair
Zig-Zag
Chiral
TEM Chiral
3nm
0.34nm
Up to cm long
SWNT
(single
Wall nanotube)
Diameter ~ 1.4 nm
MWNT
(multiwall)
Diameter 10-20 nm

INTRODUCTION TO NANOMATERIALS

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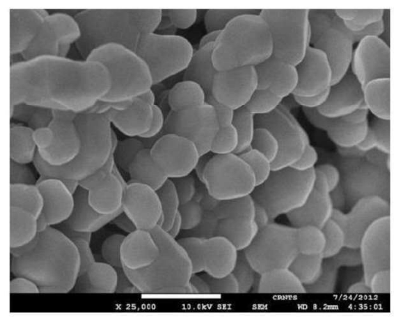
Physical Properties of nanomaterials

- Nano-sized materials exhibit some remarkable properties that may be different from the physical properties of bulk materials.
- Some are known but a lot more to be discovered
- The origin of these unique properties may be attributed to
  - Large fraction of surface atoms
  - Large surface energy
  - Quantum confinement
  - Reduced imperfections



- ➢ Mechanical
- ➢ Electrical
- ➢ Optical
- ➢ Thermal
- ➢ Magnetic

Types of Physical properties :

– Optical (e.g. color, transparency)

– Electrical (e.g. conductivity)

– Mechanical (e.g. hardness, melting point)

– Magnetic (e.g. Super Paramagnetism)

Optical properties in Nanomaterials

➢Nanomaterials have attracted much interest due to their novel optical properties, which differ remarkably from bulk crystals.

➢Applications based on optical properties of nanomaterials include optical detector, sensor, imaging, display, solar cell, photocatalysis, photoelectrochemistry and biomedicine.

➢ The optical properties of nanomaterials depend on parameters such as size, shape, surface characteristics, and other variables including doping and interaction with the surrounding environment.

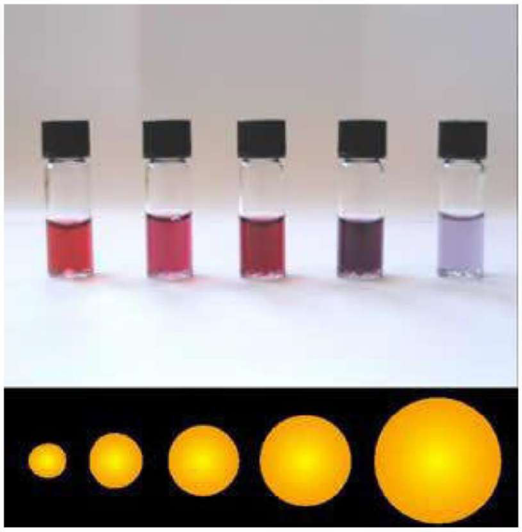
➢ Shape can have dramatic influence on optical properties of metal nanostructures.

✓Bulk Gold appears Yellow in color.

✓Nanosized Gold appears Red in color.

✓The particles are so small that electrons are not free to move about as in bulk gold.

✓Because this movement is restricted, the particles react differently with light.

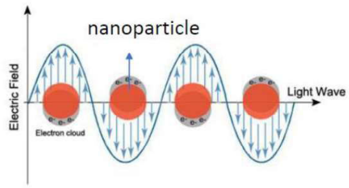


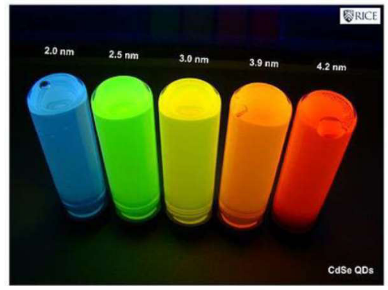
Optical properties of nanomaterials can be significantly different from bulk crystals. E.g. The optical absorption peak of a semiconductor nanoparticle shifts to short wavelength, due to an increased band gap. The colour of metallic nanoparticles may change with their sizes due to surface plasmon resonance.

Bulk gold	small particle of gold
shine as a metal;	no metallic, don't shine
Chemically not reactive (make jewel)	Reactive

Optical Property

- Optical properties of nanomaterials can be significantly different from bulk crystals. E.g. The optical absorption peak of a semiconductor nanoparticle shifts to short wavelength, due to an increased band gap.
- The wavelength of light absorbed increases as a function of increasing nanoparticle size.
- The colour of metallic nanoparticles may change with their sizes due to **surface plasmon resonance.** Eg. Bulk gold appears yellow while gold nanoparticles appear red below 20nm.
- **Incident light frequency is absorbed based on plasmon resonance frequency which in turn depends on particle size.**





Emission colors of CdSe nanoparticles of different sizes. Smaller particles emit blue light because the exciton energy increases as the size decreases

Electrical Properties

The properties like conductivity or resistivity are come under category of electrical properties. These properties are observed to change at nanoscale level like optical properties.

✓The electronic structure of Nano materials is different from its bulk material.

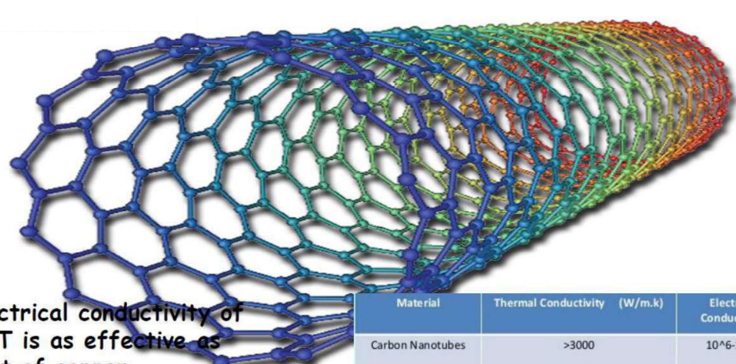
✓The density of the energy states in the conduction band changes.

✓When the energy spacing between two energy levels is more than K_BT **, energy gap is created.**

✓Nano clusters of different sizes will have different electronic structures and different energy level separations.

✓The Ionization potential at Nano sizes are higher than that for the bulk materials.

Electrical conductivity decreases with a reduced dimension due to increased surface scattering. However, electrical conductivity of nanomaterials could also be enhanced appreciably, due to the better ordering in microstructure



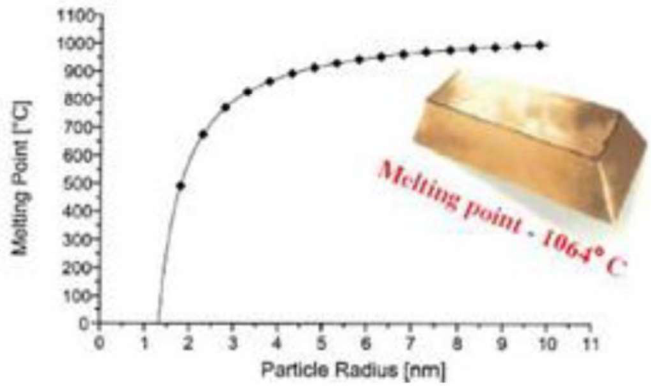
Electrical conductivity of CNT is as effective as that of copper

Very high current carrying capacity

Material	Thermal Conductivity (W/m.k)	Electrical Conductivity
Carbon Nanotubes	>3000	10^6-10^7
Copper	400	6*10^7
Carbon Fiber – Pitch	1000	2-8.5*10^6
Carbon Fiber – PAN	8-105	6.5-14*10^6

Thermal properties

Nanomaterials may have a significantly lower melting point or phase transition temperature and appreciably reduced lattice constants, due to a huge fraction of surface atoms in the total amount of atoms



Mechanical Properties

Mechanical properties of nanomaterials are one or two orders of magnitude higher than that of single crystals in the bulk form. The enhancement in mechanical strength is simply due to the reduced probability of imperfection .

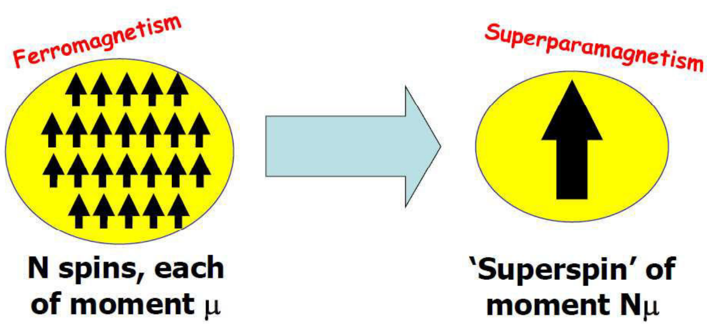
Magnetic Properties

✓Nano-structured materials are distinctly different from that of bulk materials.
✓Ferromagnetism disappears and transfers to super-paramagnetism in the nanometer scale due to the huge surface energy.
✓Surface atoms are not only different to bulk atoms, but they can also be modified by interaction with other chemical species .
✓The Magnetic Moment of Nano particles is found to be very less when compared them with its bulk size.
✓It should be possible that non-ferromagnetic bulk materials exhibit ferromagnetic-like behavior when prepared in nano range.

Changes in properties

Magnetic properties

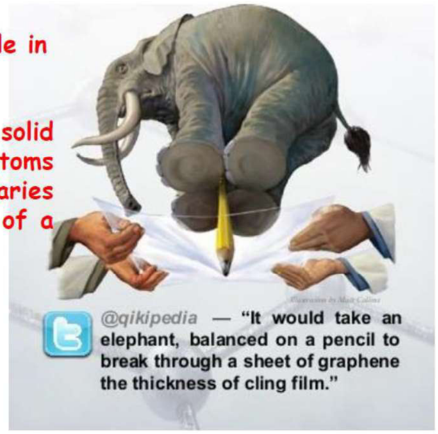
Magnetic properties of nanostructured materials are distinctively different from that of bulk materials. Ferromagnetism of bulk materials disappears and transfers to superparamagnetism in the nanometer scale due to the huge surface energy.



Mechanical properties of nanomaterials may reach the theoretical strength, which are **one or two** orders of magnitude **higher than** that of single crystals in the bulk form. The enhancement in mechanical strength is due to the **reduced probability of defects**.

Grain boundaries play a significant role in the materials properties

As the grain size d of the solid decreases, the proportion of atoms located at or near grain boundaries relative to those within the interior of a crystalline grain, scales as 1/d.



Examples of Size effect

	Property	Influence of size reduction on Property
Due to Surface to Volume Ratio	Structural	Decrease or Increase of Lattice parameters, Structure Transformation
	Mechanical	Enhancement of hardness, strength, ductility Increase in wear resistance
	Thermal	Decrease of melting point and Phase transition temp.
Due to Quantum Confinement	Electronic	Increase in band gap
	Optical	Increase of absorption in UV range Nonlinear optical properties

INTRODUCTION TO NANOMATERIALS

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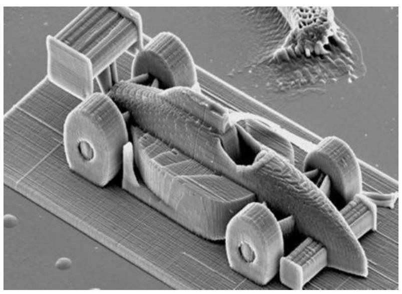
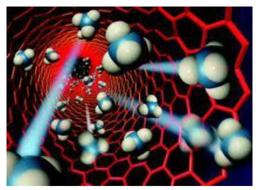
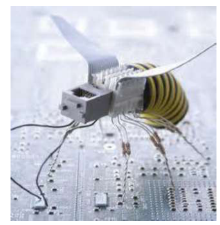
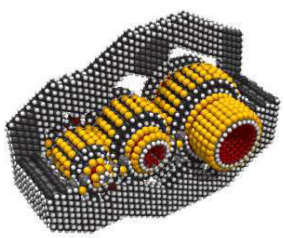
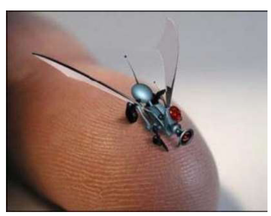
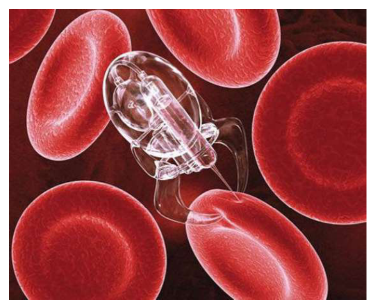




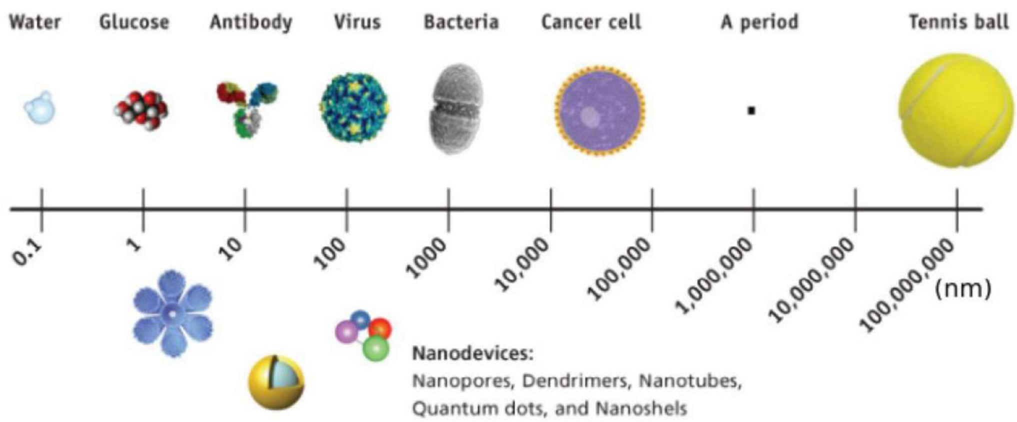
NANO TECHNOLOGY

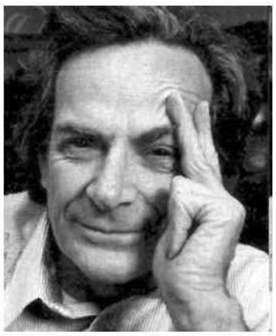
What is Nano?

- ❖Nano refers to the scale of nanometers.
- ❖A nanometer is one billionth of a meter
- ❖This is the scale of molecules, proteins, and other nano-objects that are the topics of this course.
- ❖The Nano-scale involves the range from approximately 100 nm to 1 nm.

Finger print of objects



ORIGIN OF NANOSCIENCE



Richard P. Feynman

"There is a plenty of room at the bottom"

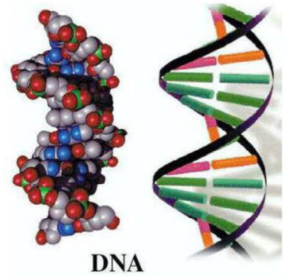
(Lecture in 1959 at the annual meeting of the American Physical Society)

Huge information can be stored at the tip of a ball pin. Molecules and atoms can be manipulated to store information

Professor Norio Taniguchi was the first person to use the term 'nanotechnology' in 1974.

"'Nano-technology' mainly consists of the processing of, separation, consolidation, and deformation of materials by one atom or by one molecule."

Examples of Nano-scale Structures



DNA
~2.5 nm diameter

Carbon nanotube
~2 nm in diameter

Nano Science

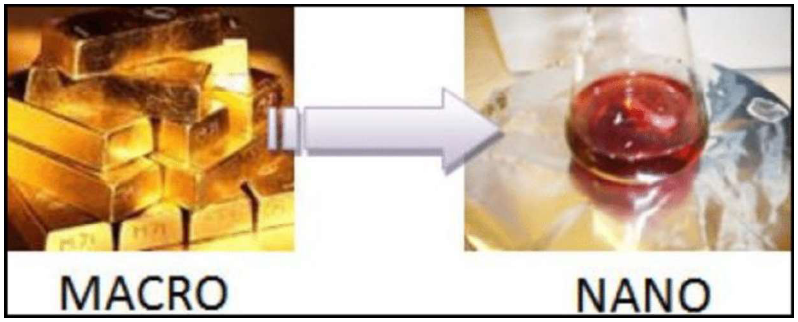
Nano Science can be defined as the study of phenomena and manipulation of materials at Atomic, Molecular and Macromolecular scales where properties differ significantly from those at a larger scale.

Nano Science is the study and understanding of properties of Nano Particles.

What is Nano-technology??

Nanotechnology can be defined as the design, characterization, production and application of structures devices and systems by controlling shape and size at a Nano meter Scale.

Professor Norio Taniguchi was the first person to use the term 'nanotechnology' in 1974.



What is Nano material??

Nano Materials could be defined as the materials with at least one of its dimensions in the range of 1-100 nm.

Why are nanomaterials so special?

The properties of Nano Materials are very much different from those at a larger scale.

Physical properties change due to :

1. Large Surface Area to Volume Ratio
(a) large fraction of surface atoms
(b) large surface energy
2. Quantum Confinement

These factors can charge or enhance properties such as reactivity, strength and electrical characteristics.

Increase in surface area to volume ratio:

The ratio of surface area to volume ratio is large for nano materials.

Example1:To understand this let us consider a spherical material of radius'r'. Then its surface area to volume ratio is 3/r. Due to decrease of r, the ratio increases predominantly.

Spherical material

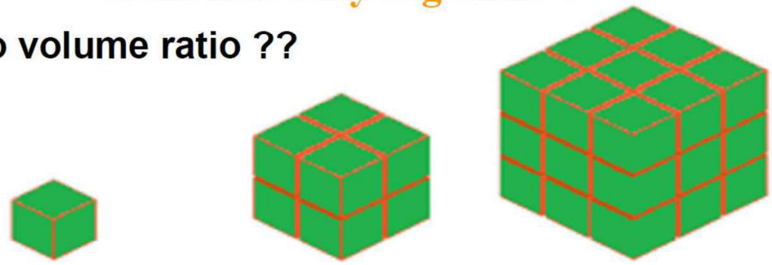
For a sphere of radius 'r', we have

$$\frac{\text{Surface area of the sphere}}{\text{Volume of the sphere}} = \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{3}{r}$$

This shows that by reducing the size (or radius) of a spherical object, the spherical area to volume ratio increases.

Does Size Play Big Role??

Surface to volume ratio ??



1 cm	2 cm	3 cm
SA = 6 cm^2	SA = 24 cm^2	SA = 54 cm^2
Vol = 1 cm^3	Vol = 8 cm^3	Vol = 27 cm^3
SA:Vol = 6:1	SA:Vol = 3:1	SA:Vol = 2:1

Quantum Confinement

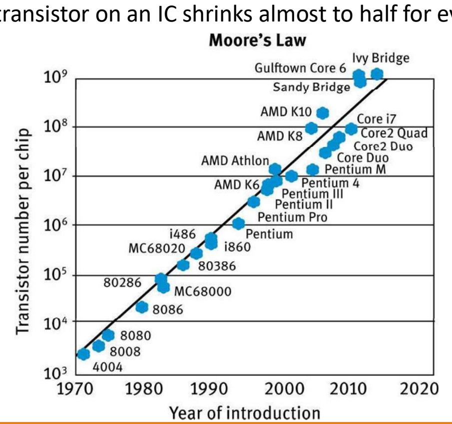
In Nano Crystals, the Electronic energy levels are not continuous as in the bulk but are discrete (finite density of states), because of the confinement of the electronic wave function to the physical dimensions of the particles. This phenomenon is called Quantum confinement and therefore Nano Crystals are also referred to as quantum dots (QDs).

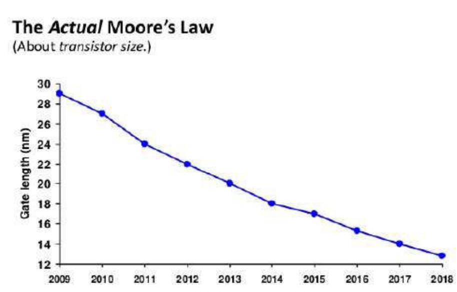
Moore's law

Mr. Gordon Moore observed that the size of transistor reduces linearly over a time. It is empirical law.

Definition

The no. of transistors on a specific area (cm^{-2}) of an IC (integrated circuit) doubles every 18 months. Or the size of transistor on an IC shrinks almost to half for every 18 months.

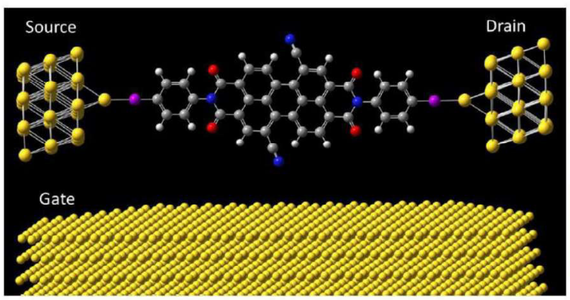




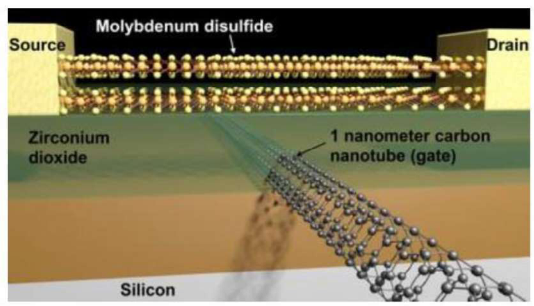
Deviation from Moore's law

Now, the size of a transistor reaches close to critical dimensions and hence Moore's law may deviates from linear behavior.

Smaller Transistors on the way….



Molecular Transistor



Nanotube as gate material

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TYPES OF NANOMATERIALS

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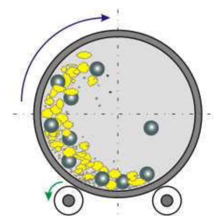
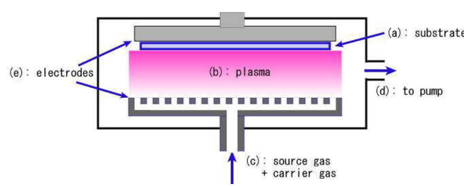
Methods of preparing nanomaterials

Physical and Chemical methods

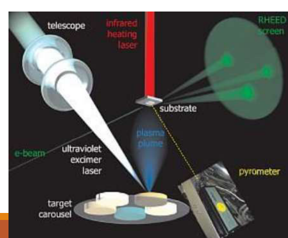
Physical Method -
Grind materials Using ball milling



Evaporate materials to gas phase and solidify



Laser



Chemical methods (most powerful)

React a metal salt with an alcohol or
some other reducing agent

Metal compound along with a reagent
in a boiling solvent (sealed vessel)

Under this condition,
many kinds of nano-particles are formed

types

Hydrothermal and solvothermal methods

Water used as a boiling solvent - Hydrothermal

Organic boiling solvent like a hydrocarbon - solvothermal

For example, if we take a metal acetate and heat in a boiling hydrocarbon, we get metal or metal oxide nanoparticles

Today we have reached a level where can make nanomaterial of any compound in any shape we desired

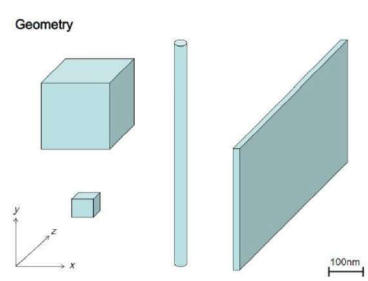
Nanostructured materials

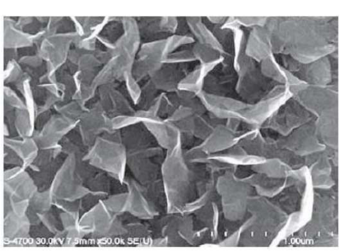
- **3D: Bulk material**
- **2D: Quantum well, Thin films**
- **1D: Quantum wires, nanowires**
- **0D: Quantum dots**

Quantum Confinement

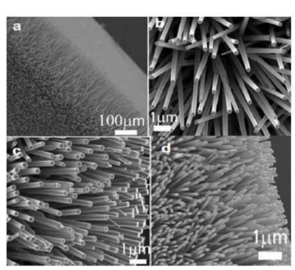
Quantum Confinement is the spatial confinement of electron-hole pairs (excitons) in one or more dimensions within a material.

1D confinement: Quantum Wells
2D confinement: Quantum Wire
3D confinement: Quantum Dot

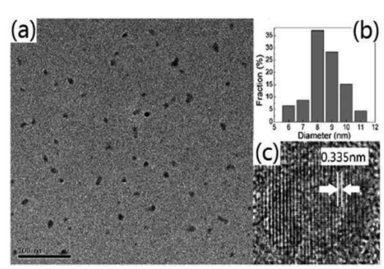




2D-Graphene Nanosheet



1D- ZnO Nanowire



0D- Carbon quantum dots

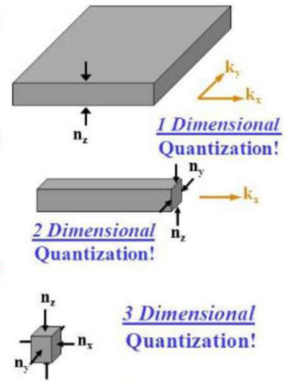
types

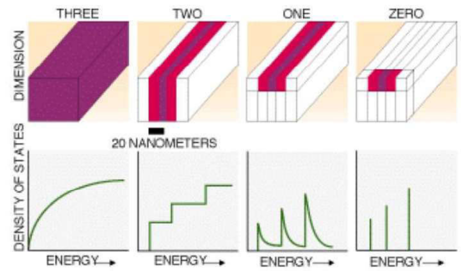
Quantum Confinement in Nanostructures: Overview

Electrons *Confined in 1 Direction*:
Quantum Wells (thin films):
⇒ Electrons can easily move in *2 Dimensions*!

Electrons *Confined in 2 Directions*:
Quantum Wires:
⇒ Electrons can easily move in *1 Dimension*!

Electrons *Confined in 3 Directions*:
Quantum Dots:
⇒ Electrons can easily move in *0 Dimensions*!





- Density of states (dn/dE) is a function of dimensionality.
- Disceretness increases for lower dimensions of particles.

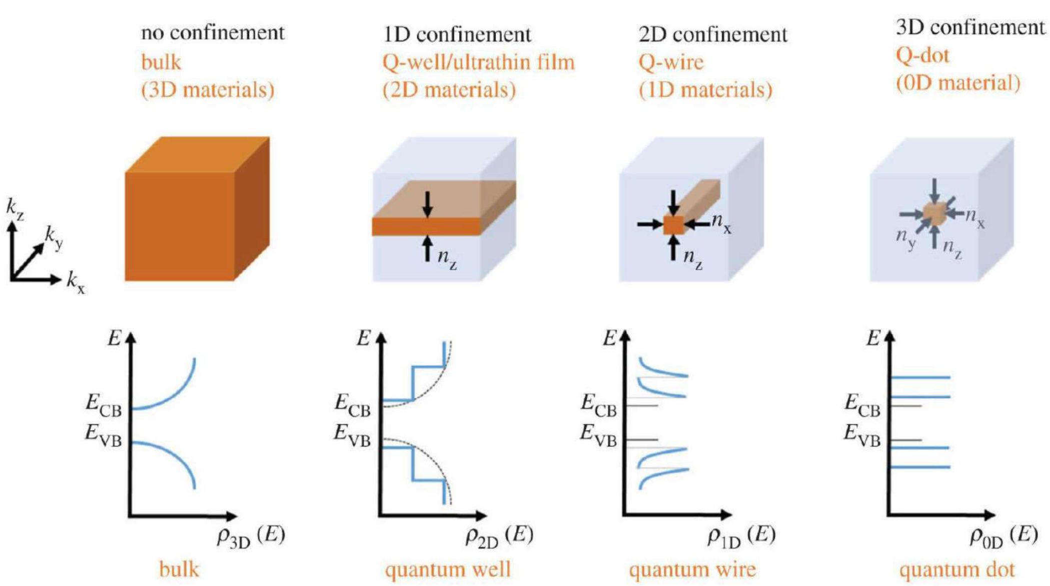
Dimensionality

Zero-dimensional (quantum dots) in which the movement of electrons is confined in all three dimensions.

one-dimensional (quantum wires) in which the electrons can only move freely in the X-direction.

Two-dimensional (quantum wells) in which case the free electron can move in the X-Y plane.

Three dimensional (nanostructured material built of nanoparticles as building blocks) in which the free electron can move in the X, Y and Z directions.



types

Quantum well

•It is a two-dimensional system

•The electron can move in two directions and restricted in one direction.

Quantum Wire

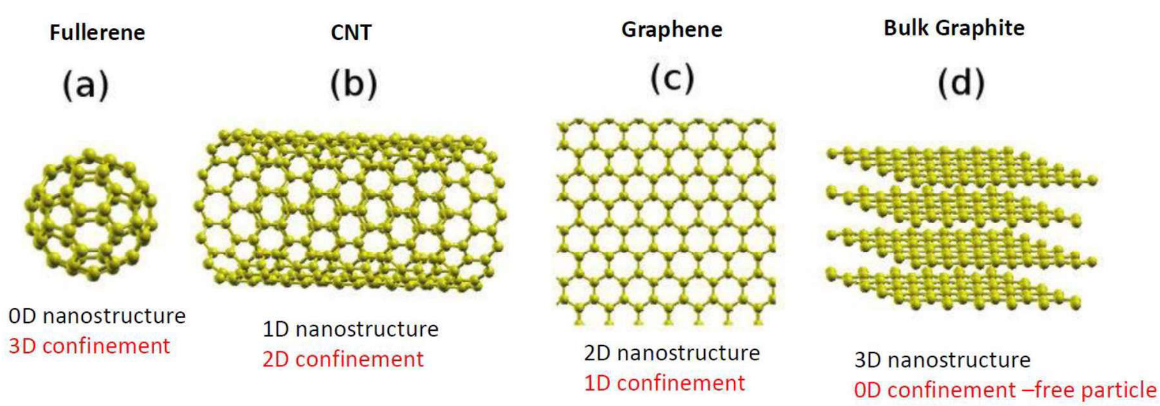
•It is a one-dimensional system

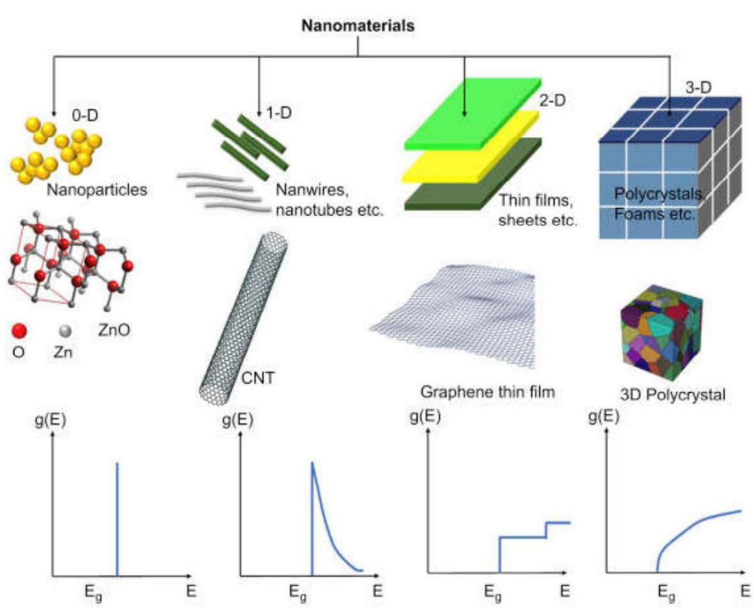
•The electron can move in one direction and restricted in two directions.

Quantum dot

•It is a zero-dimensional system

•The electron movement was restricted in entire three dimensions



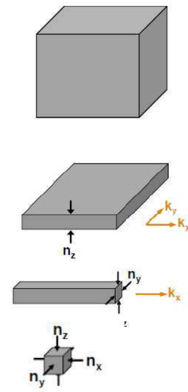


types

Quantum confinement is more prominent in semiconductors because they have an energy gap in their electronic band structure.

Metals do not have a bandgap, so quantum size effects are less prevalent. Quantum confinement is only observed at dimensions below 2 nm.

STRUCTURE	SPATIAL DIMENSION	CONFINEMENT DIMENSION
Bulk	3	0
Surface/ Film (Quantum Well)	2	1
Nanotubes, -wires (Quantum wire)	1	2
Nano-particles, clusters (Quantum dots)	0	3



EINSTEIN COEFFICIENTS

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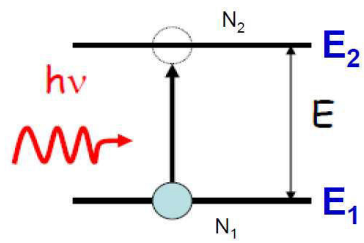
Absorption or Stimulated absorption



$E_2 - E_1 = h\nu$

The atoms present in the lower energy states absorb the incident photon and get excited to the higher energy level. This process is said to be absorption of light.

N_1 and N_2 are the no. of atoms present in E_1 and E_2

E_1 = Ground state
E_2 = Excited State
E = hν(Photon Energy)

Energy levels

The rate of absorption (R_{12}) depends on no. of atoms N_1 present in E_1 & spectral energy density (ρ_ν) of the incident photon.

$R_{12} \propto \rho_\nu$
$\propto N_1$

$\boxed{R_{12} = B_{12}\, \rho_\nu\, N_1}$

Where B_{12} known as coefficient for absorption or Einstein's coefficient.

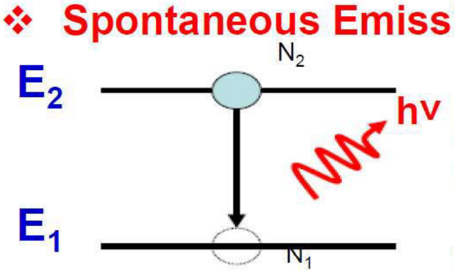
Emission of Light

The atom in the excited state exists in that state only for a short period of time. The average time spent by an atom in the excited state (in the order of 10^{-8} s) is known as life time of an atom.

In some energy levels the atoms exists for a longer time (10^{-3} s). These energy levels are called metastable state. After spending a short period of time in the excited state, it automatically returns to lower energy state by emitting the excess of energy possessed by it. The process is said to be emission of light. They are two types

- **Spontaneous Emission**

Spontaneous emission was postulated by Bohr.



E_1 = Ground state
E_2 = Excited State
E = hν(Photon Energy)

Energy levels

$E_2 - E_1 = h\nu$

N_1 and N_2 are the no. of atoms present in E_1 and E_2

Consider an atom in the excited state. It will exist in that state only for a short period of time. After that it will return to lower energy state. If the atom lying in upper energy level returns to lower energy level without any external inducement, than the emission is said to be **spontaneous emission**.

The rate of spontaneous emission R_{21} (spot) can be written as

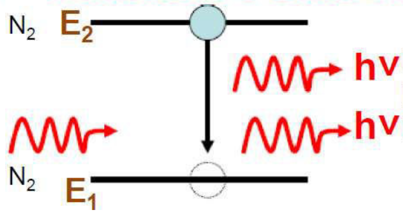
$$R_{21}\text{ (spot)} \ \alpha \ N_2$$

$$\boxed{R_{21}\text{ (spot)} = A_{21} N_2}$$

Where A_{21} known as spontaneous emission coefficient or Einstein's coefficient.

❖ Stimulated Emission

Stimulated emission was postulated by Einstein.



E_1 = Ground state
E_2 = Excited State
E = hv(Photon Energy)

Energy levels

$$E_2 - E_1 = h\nu$$

Consider an atom in the excited state. It will exist in that state only for a short period of time. After that it will return to lower energy state. If the atom lying in upper energy level returns to lower energy level without any external inducement, than the emission is said to be **spontaneous emission**.

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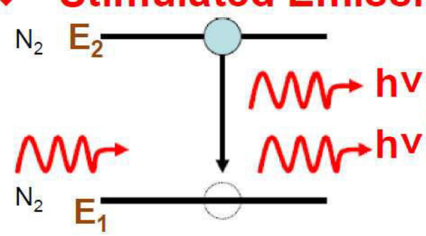

E_1 = Ground state
E_2 = Excited State
E = hv(Photon Energy)

Energy levels

$$E_2 - E_1 = h\nu$$

Consider an atom in the metastable state (in the order of 10^{-3} s). At that time, if an external source of radiation energy **hν** is incident on the system, then the atom is stimulated to emit a radiation of energy **hν** and hence it returns to the ground state. This phenomenon is called **stimulated emission.**

The rate of stimulated emission R_{21} (sti) can be written as

$$R_{21}\text{ (sti)} \ \alpha \ \rho_\nu$$
$$\alpha \ N_2$$

$$\boxed{R_{21}\text{ (Sti)} = B_{21} \rho_\nu N_2}$$

Where B_{12} known as coefficient for Stimulated emisson or Einstein's coefficient. ρ_ν is te energy density of the incident radiation.

Einstein obtained a mathematical expression for the existence of two different kinds of processes,
(1) Spontaneous emission
(2) Stimulated emission
Consider all atoms are in thermal equilibrium at T. Radiation of freq. ν & energy density ρ_ν.
N1 & N2 are atoms or populations in E1 & E2 respectively.
In equilibrium, absorption rates & emission rates must be same.

Rate of absorption = Rate of emission

$$B_{12}\, \rho_\nu\, N_1 = A_{21}\, N_2 + B_{21}\, \rho_\nu\, N_2$$

$$A_{21}\, N_2 = \rho_\nu\, [B_{12} N_1 - B_{21} N_2]$$

$$\text{So, } \rho_\nu = [A_{21}\, N_2 / (B_{12}\, N_1 - B_{21}\, N_2)] \quad \text{---------(1)}$$

$$\rho_\nu = A_{21}/B_{21} / [B_{12}N_1/B_{21}N_2 - 1] \;\ldots\ldots\text{(2)}$$

Boltzmann distribution law,

$$N_1 = N_0 e^{-E_1/kT}$$
$$N_2 = N_0 e^{-E_2/kT} \quad \ldots\ldots \text{(3)}$$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT} \quad \ldots\ldots \text{(4)} \qquad E_2 - E_1 = h\nu \;\ldots\text{(5)}$$

$$N1/N2 = e^{h\nu/KT} \;\ldots\ldots\ldots\ldots\text{(6)}$$

$$\rho_\nu = \frac{\dfrac{A_{21}}{B_{21}}}{\left[\dfrac{B_{12}}{B_{21}} e^{h\nu/kT} - 1\right]} \quad \ldots\ldots\ldots\ldots\text{(7)}$$

According to Planck's radiation formula,

$$\rho_\nu = \frac{8\pi h \nu^3}{c^3}\left(\frac{1}{[e^{h\nu/kT} - 1]}\right) \quad \ldots\ldots 8$$

If the expressions of equation 7 and 8 to be identical, we must have

$$B_{12} = B_{21} \;\&\; A_{21}/B_{21} = \frac{8\pi h \nu^3}{c^3}$$

Einstein coefficients A and B

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}$$

Significance

- The stimulated emission rate per atom is the same as the absorption rate per atom
- The ratio between spontaneous emission and stimulated emission is proportional to v^3
- The probability of spontaneous emission rapidly increases with the energy difference between the two states

Significance of Einstein's coefficients

- Coefficients A_{21}, B_{21} and B_{12} are interrelated and can be calculated if one is known
- Stimulated emission and absorption coefficients are equal at least for non-degenerate energy states
- Since B_{21}/A_{21} is proportional to reciprocal of cube of n, higher the frequency smaller the B_{21}

GAIN COEFFICIENT

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Main Components

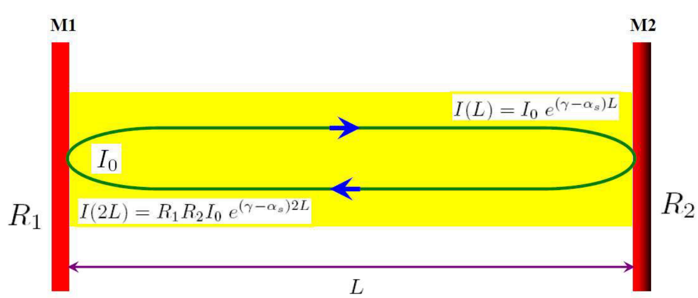
Three main components of **ANY** lasers are

(i) **The active medium**
(ii) **The pumping source**
(iii) **The optical resonator**

- The active medium acts as an **amplifier** for light waves
- For amplification, **medium** should be in a state of **Population inversion**
- Population inversion – **metastable levels** – lifetime is bit longer as compared to excited state
- The active medium placed inside an optical resonator – acts as an **oscillator**
- A pair of mirrors + active medium - optical resonators

Threshold condition for LASING action

- **Output of the active medium bouncing back and forth in the optical resonator.**
- **During amplification it suffers various losses**



The losses occur mainly due to

1. Transmission at the output mirror
2. Scattering, diffraction and absorption of light with in the active medium

- Proper build up of laser oscillation: The amplification between two constructive reflections of light from rear end mirror must balance the losses.
- Assume that the active medium of the laser fills the space between the mirrors M1 and M2.
- The reflectivity of both mirrors are R1 and R2
- Mirrors be separated by a distance L
- Let the intensity of light beam at M1 be I_0
- Traveling from M1 to M2, the beam intensity increases from

$$I(L) = I_0 \, e^{(\gamma - \alpha_s)L}$$

Where γ is the gain of the laser medium

α_s losses due to scattering, diffraction and absorption in the medium

After reflection at mirror M2, the beam intensity will be

$$I(L) = R_2 I_0 \, e^{(\gamma - \alpha_s)L}$$

After complete the round trip the final intensity will be

$$I(2L) = R_1 R_2 I_0 \, e^{(\gamma - \alpha_s)2L}$$

The amplification obtained during the round trip

$$G = \frac{I(2L)}{I_0} = R_1 R_2 \, e^{(\gamma - \alpha_s)2L}$$

The product R_1R_2 represents the losses at the mirror

The losses are balanced by gain, when the amplification factor

$$G \geq 1$$

It requires that

$$R_1 R_2 \, e^{(\gamma - \alpha_s)2L} \geq 1$$

$$e^{(\gamma - \alpha_s)2L} \geq \frac{1}{R_1 R_2}$$

$$(\gamma - \alpha_s)2L \geq ln\left(\frac{1}{R_1 R_2}\right)$$

$$\gamma - \alpha_s \geq \frac{1}{2L} ln \left(\frac{1}{R_1 R_2} \right)$$

$$\gamma \geq \alpha_s + \frac{1}{2L} ln \left(\frac{1}{R_1 R_2} \right)$$

The above equation is the condition for the lasing action

This equation is determined the threshold value of pumping energy for lasing action

As the pump power is slowly increased, a value of γ_{th} called threshold value is reached and the laser starts oscillations

The threshold value γ_{th} is given by

$$\gamma_{th} = \alpha_s + \frac{1}{2L} ln \left(\frac{1}{R_1 R_2} \right)$$

Gain threshold - γ_{th}

1. Gain required to just balance the total losses in a gain medium. The loss in gain medium may be due to three factors:
2. Absorption of photons within the gain medium
3. Scattering of photons within the gain medium
4. Transmission of photons through the reflecting mirrors
5. Lasing action begins just above the gain threshold -γ_{th}>α (loss co-eff).
6. For efficient lasing action $\frac{1}{2L}\ln(R_1R_2) \longrightarrow 0$
7. Which means the mirrors should be highly reflecting (such that the product of their reflectances $R_1 x R_2 \approx 1$) or the optical cavity length should be infinite.
8. The losses (α) due to absorption and scattering in the gain medium cannot be controlled so for perfectly reflecting mirrors and long optical cavity, γ_{th}>α

Laser threshold condition:
- Gain balances the losses within medium
- Stimulated emission begins to dominate spontaneous emission
- Laser action begin with exponential amplification of photons due to sustained oscillations

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HELIUM-NEON LASER

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Main Components

Three main components of **ANY** lasers are

(i) The active medium
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(iii) The optical resonator

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- A pair of mirrors + active medium - optical resonators

Helium – Neon laser

❖ First continuous laser developed by Ali Javan, W. Bennutt and D. Herriot in 1961.

❖ Operation wavelength is 632.8 nm (red portion of visible spectrum).

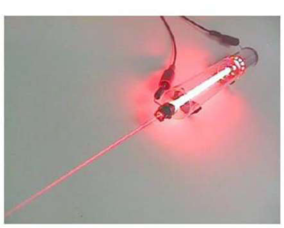
❖ 4-level laser scheme.

❖ More directional and monochromatic than solid state lasers.

❖ Output is moderate compared with solid state lasers.

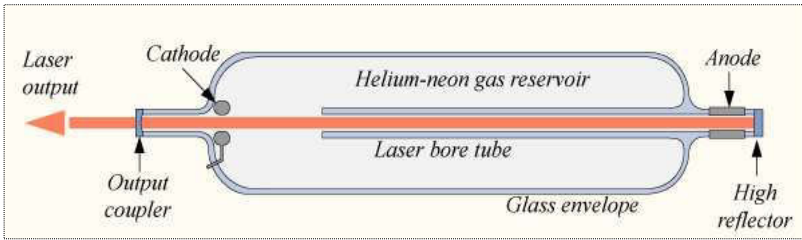
❖ Active medium is a mixture of He and Ne gas in 10:1 ratio (atomic percentage).

❖ Ne atoms are active centres for lasing action. He only helps in efficient excitation of Ne atoms.


He-Ne laser – without protective jacket

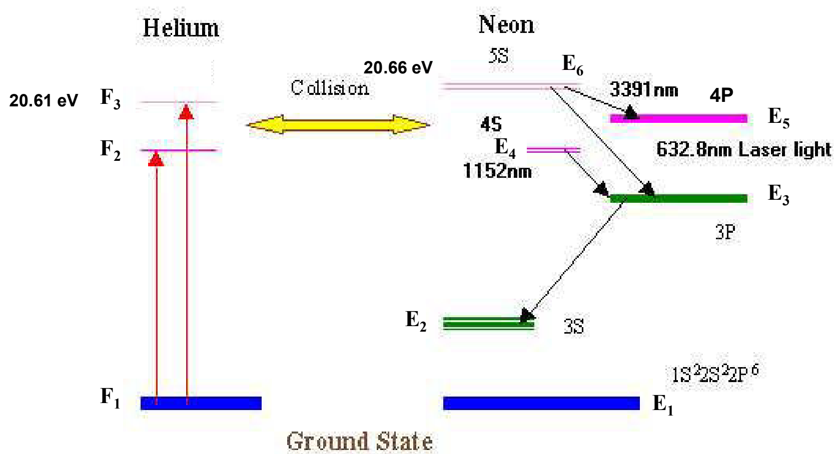
Completely protected commercial He-Ne laser

Construction of He-Ne laser



- ✓ Set up consists of a **discharge tube of length 80 cm and bore diameter of 1.5 cm**.
- ✓ **Gain medium of the laser is a mixture of He and Ne** as the name suggests in ~10:1 ratio. It is contained at **low pressure** (an average 50 Pa/cm of cavity length) in a glass envelope.
- ✓ The pumping is provided electrically by creating an discharge. The **electrical discharge is created by applying ~ 1 KV** through an anode and cathode present at each end of the glass tube. The typical current value ranges from 5 – 100 mA for continuous mode operation.

He-Ne Energy level diagram



$$He + e_1 \rightarrow He^* + e_2 \qquad He^* + Ne \rightarrow He + Ne^*$$

- ✓ When voltage is applied to the electrodes it **ionizes the gas, the electrons and ions** thus produced are accelerated towards anode and cathode respectively.
- ✓ Electrons acquire higher velocity due to its smaller mass when compared to the others. They transfer **K.E to He atoms through inelastic collision**.
- ✓ **He atoms are readily excited by electrons impact** because of its fairly light mass.
- ✓ Thus He atoms are **excited to F_2 and F_3 states which lie at 19.81 and 20.61 eV** respectively.
- ✓ These are **metastable states** and these atoms cannot return to ground state readily by spontaneous emission.

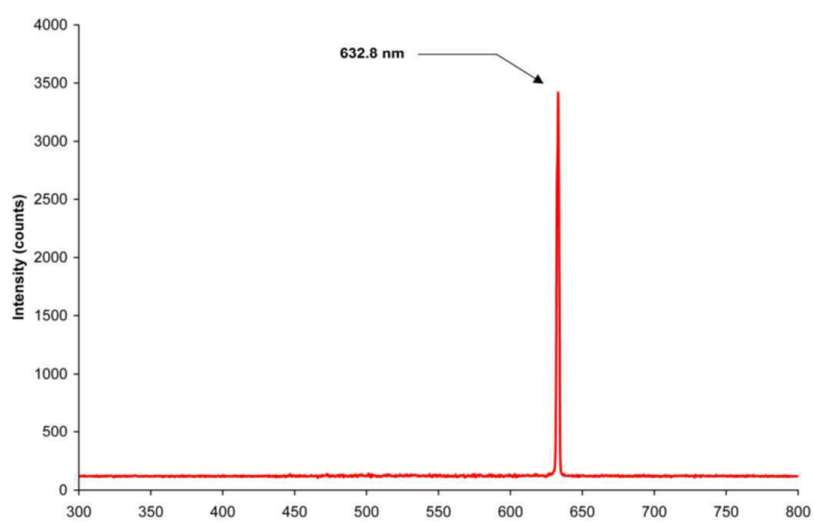
- ✓ These atoms **return to ground state by transferring energy to Ne atom** in the state which has identical energy. Such transfer is called **resonant transfer of energy**. (The direct excitation of Ne atoms is in-efficient compared to He)
- ✓ **Neon energy levels E_6 and E_4 nearly coincide with F_3 and F_2 of Helium,** so resonant transfer can occur.
- ✓ **The additional energy 0.05 eV is provided by the K.E of the He** atom.
- ✓ This **energy exchange process occurs with high probability only because of the accidental near equality of the two excitation energies** of the two levels in these atoms. Thus, the purpose of population inversion is fulfilled.

- ❖ Ne atoms in the E_6 level and E_4 level emit a photon parallel to the axis of the tube.
- ❖ This photon travels through the gas mixture parallel to the axis of tube, it is **reflected back and forth by the mirror ends until it stimulates an excited Ne atom and causes it to emit a photon with the stimulating photon**.
- ❖ In reality neon energy levels E_6, E_5, E_4, E_3, E_2 are not single but a group of lines. Consequently several laser transitions are possible.

- ❖ Three main laser transitions are

1. **E_6 to E_3 – generates laser beam of red colour at 6328Å**
2. **E_4 to E_3 – IR beam at wavelength of 1.15 μm**
3. **E_6 to E_5 – light in Far IR region at 3.39 μm**

He-Ne Laser Spectrum



Applications of He-Ne laser

- The Narrow red beam of He-Ne laser is used in supermarkets to read bar codes.
- The He- Ne Laser is used in Holography in producing the 3D images of objects.
- He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

He-Ne lasers uses transitions among the various excited electronic states of an atom.

Advantages	Disadvantages
Emits laser light in the visible portion of the spectrum	Low efficiency
High stability	Low gain
Low cost	Output power is small
Operates without damage at higher temperatures	

CAUTION

Helium-neon lasers are common in the introductory physics laboratories, but they can still be dangerous! According to Garmire, an unfocused 1-mW HeNe laser has a brightness equal to sunshine on a clear day (0.1 watt/cm^2) and is just as dangerous to stare at directly.

LC

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LASER: CHARACTERISTICS

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Reference Books:

- **Modern Physics**
  Arthur Beiser
- **Understanding Lasers**
  Jeff Hecht
- **Lasers fundamentals and applications**
  K. Thyagarajan, A. Ghatak

What is LASER?

Light Amplification by Stimulated Emission of Radiation

- Radiation: means electromagnetic radiation.
- Stimulated Emission: the way lasers produce light.
- Amplification: increase the amount of light emitted.
- Light: type of electromagnetic radiation produced.

Applications of LASER

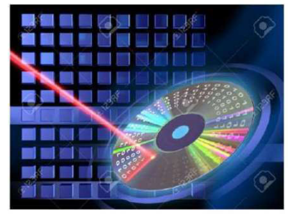
Surveying

Laser Eye surgery

Welding and Cutting

Laser Communication

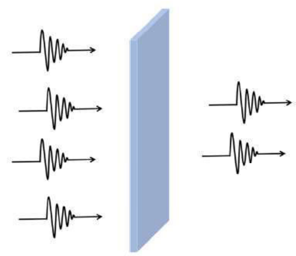
Barcode scanners



CD Read/Write

What happens when light interact with matter:

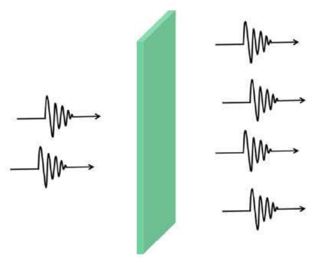
When light passes through materials it is usually absorbed.



In certain circumstances light may be amplified.

This was called "negative absorption".

It is the basis of laser action.

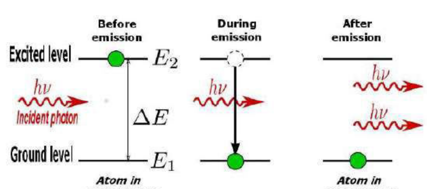


Radiation and its interaction with matter

When radiation interacts with matter the following processes may occur:

- Stimulated Absorption
- Spontaneous Emission
- Stimulated Emission

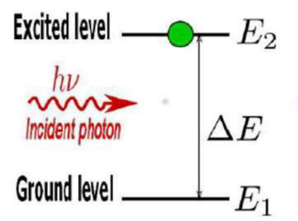
- let us consider two energy levels E1 and E2 some atom or molecule of a given material.
- The lowest energy level E1 is called the ground state and the higher energy levels are excited states. Here E1 < E2.
- The two levels in this discussion could be any two out of the infinite set of levels possessed by the atom.



$$E_2 - E_1 = \Delta E = h\nu$$

Stimulated Absorption

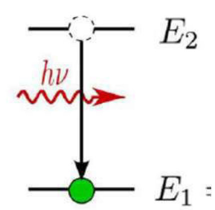
- Let us now assume that the atom is initially lying in ground state E1.
- The atom will remain in this level unless some external stimulus is applied to it.
- Let us assume, that an electromagnetic radiation of frequency ν is incident on the material.
- If the energy difference *E*2 -*E*1 is equal to the incident photon energy hν, then the atom to undergo transition from lower energy level to the excited state.
- This is the *stimulated absorption* process.



$$E_2 - E_1 = \Delta E = h\nu$$

Spontaneous Emission

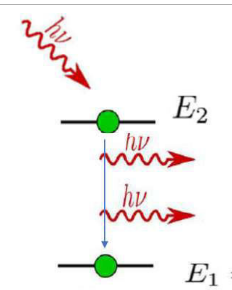
- Let us now assume that the atom is initially in level 2. Since *E*2 > *E*1, the atom will tend to decay to level 1.
- During this transition the atom must release the corresponding energy difference, *E*2-*E*1.
- Often this energy is released as electromagnetic radiation and this process is called *Spontaneous Emission*.
- Spontaneous emission is therefore characterized by the emission of a photon of energy *hv=E*2-*E*1.



$$E_2 - E_1 = \Delta E = h\nu$$

Stimulated Emission

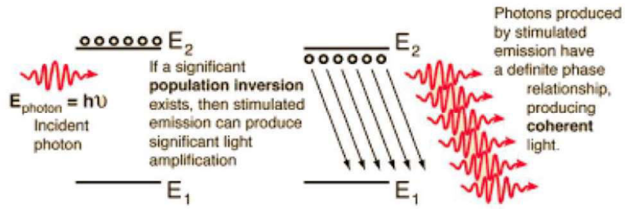
- Let us assume that the atom is found initially in level 2.
- If electromagnetic wave of frequency ν is incident on the material.
- The e.m wave has the same frequency as the atomic frequency, there is a finite probability that this wave will force the atom to undergo the transition from level 2 to level 1.
- In this case the energy difference *E*2 -*E*1 is delivered in the form of an e.m. wave + incident radiation.
- This phenomenon where radiation is emitted upon stimulation of an excited atom to transit from Level 2 to level one by the emission of two photons is called *Stimulated emission*.

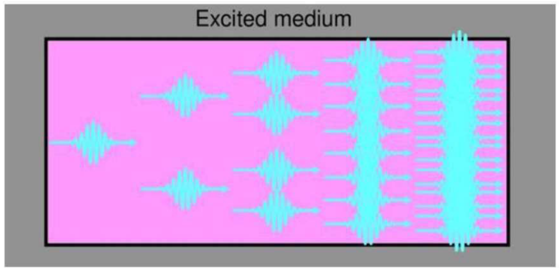


$$E_2 - E_1 = \Delta E = h\nu$$

Spontaneous Emission	Stimulated Emission
Radiative decay process from level 2 to level 1 - occurs on its own after lifetime.	Occurs due to external stimulus –radiation of energy ΔE=E2-E1=hν
Phases of emitted photons is different for different atoms	Incident e.m. wave and the emitted wave of any atom are in phase and along the same direction
Incoherent and broad bandwidth of emitted light	Coherent and narrow bandwidth of emitted light

Stimulated Emission leads to a chain reaction and laser emission



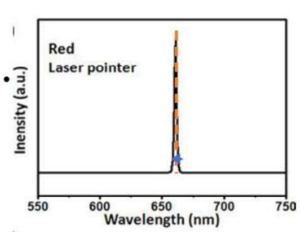


Properties of laser beam

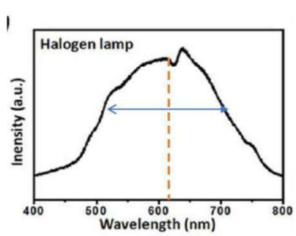
1. Monochromaticity
2. Coherence
3. Directionality
4. Intensity and Brightness

Monochromaticity:

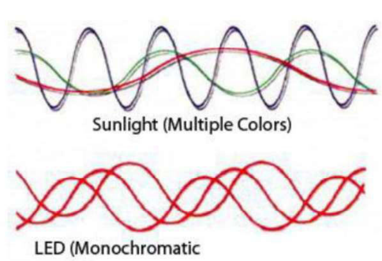
- Light from a conventional broadband source consists of a range of frequencies/wavelength. This range of frequencies is called bandwidth.
- Laser light has narrow bandwidth. Thus, resembling a single frequency source called Monochromatic source.
- Monochromaticity in lasers may arise due to two reasons:
    - Only the e.m. waves of a given frequency are amplified
    - The two mirror arrangement in the resonant cavity. Oscillations can occur only at the resonant frequencies of the cavity



Narrow bandwidth Laser source



Broad Bandwidth conventional light source



Coherence:

- Coherence is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.
- When two waves are in Phase with each other, they are said to be coherent.
- Ordinary light source is incoherent and the wave front differs at every point.
- Laser light is generated as a long continuous wave train as compared to conventional light source. Due to High coherence it results in extremely high power.

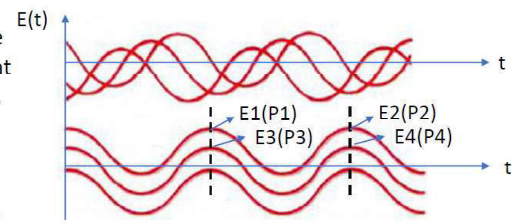
LED (Monochromatic and Non-coherent)

Laser (Monochromatic and Coherent)

- There are two types of Coherence:
  1. Spatial Coherence
  2. Temporal Coherence

Spatial Coherence:

- When two identical light waves separated by a distance are travelling in the same direction and have constant phases at different times, then they are said to be Spatially coherent.
- Spatial coherence is inversely proportional to the divergence or directionality of laser beam.
- Spatial coherence in lasers allows them to be focused to a tight spot and also allows a laser beam to stay narrow over great distances



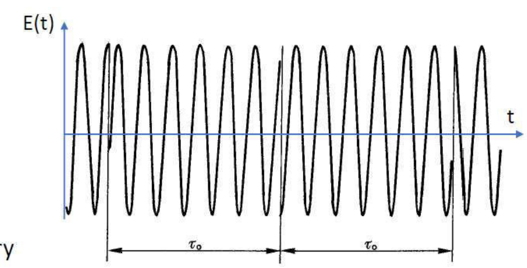
Variation of Electric field with time

- To define spatial coherence, let us consider two points *P*1 and *P*2 that, at time *t* =0, lie on the same wave-front of some given e.m. wave.
- *Let E*1 (t) and E2 (t) be the electric fields at P1 and P1 at time t=0.
- By definition, the difference between the phases of the two field at time *t* =0 is zero. Now, if this difference remains zero at any time *t* > 0, we will say that there is *perfect spatial coherence*.

$\Delta p' = (p_1 - p_2)$ is same as $\Delta p'' = (p_3 - p_4)$ At t=0 and t>0

Temporal Coherence:

- When a light wave travelling in a direction has constant phase at different time periods then it is said to have temporal coherence.
- Temporal coherence is directly proportional to the Monochromaticity of laser beam.
- Temporal coherence allows them to emit light with a very narrow spectrum



Let us consider the electric field of the e.m. wave at a given point P, at times t and t +τ_0.
If, for a given time delay τ_0, the phase difference between the two field (E1(t) and E2(t)) remains the same for any time t, we will say that there is a temporal coherence over a time τ_0.

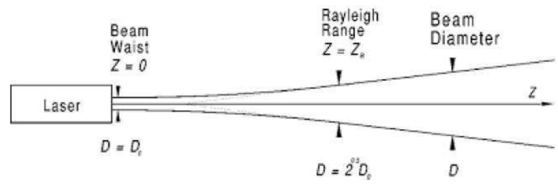
Note: The two concepts of temporal and spatial coherence are indeed independent of each other. In fact, examples can be given of a wave having perfect spatial coherence but only limited temporal coherence (or vice versa).

Directionality:

Laser light can be focused to small dimensions making it highly directional.
Directionality is measured in terms of beam divergence.
Laser beam has less divergence or spreading and remains as parallel set of rays to a certain range called Rayleigh range.

Rayleigh criterion $\theta = \frac{1.22\lambda}{D}$

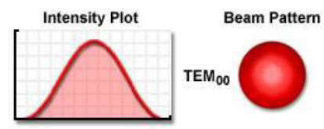
Beam divergence measured form Rayleigh criterion.
Θ-angle of divergence
λ-wavelength of laser light
D- diameter of laser's aperture.



Intensity:

Power per unit area is the measure of Intensity.
High intensity due to directionality.

$$I = \frac{P}{\pi r^2}$$

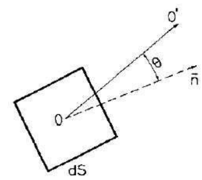


Brightness:

Brightness of a given source of light is defined as the power emitted per unit surface area per unit solid angle.

Consider dS-Elemental surface at point O, dP-power emitted by the elemental surface dS into a solid angle dΩ.

$$dP = B \cos\theta \, dS \, d\Omega$$



$\vec{n} = normal\ vector\ to\ dS$
Θ=angle between OO' and $\vec{n}$
cos Θ=projection of OO' on $\vec{n}$

Laser beam of Power P, with a circular cross-section of aperture diameter D and divergence $\theta = \frac{\beta\lambda}{D}$ β-Numerical Co-efficient=1.22

Since θ is small, cos θ=1

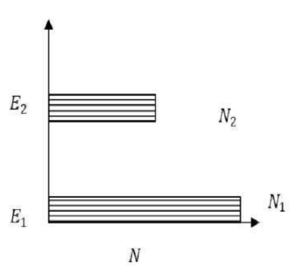
Area of circular laser beam spot $= \frac{\pi D^2}{4}$

Emission solid Angle, $d\Omega = \pi\theta^2$

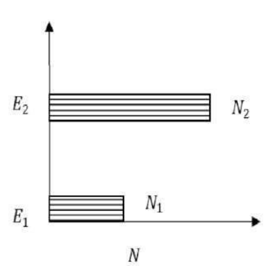
$$\therefore B = \left(\frac{2}{\pi\beta\lambda}\right)^2 P$$ → Brightness of the Laser Beam

Population inversion

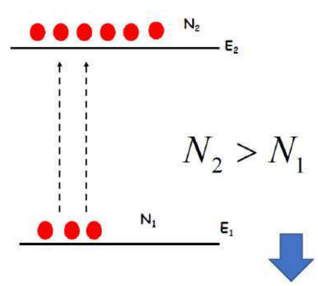
The number of atoms present in the excited (or higher) state is greater than the number of atoms present in the ground state (or lower) state is called population inversion.



At normal conditions $N_1 > N_2$



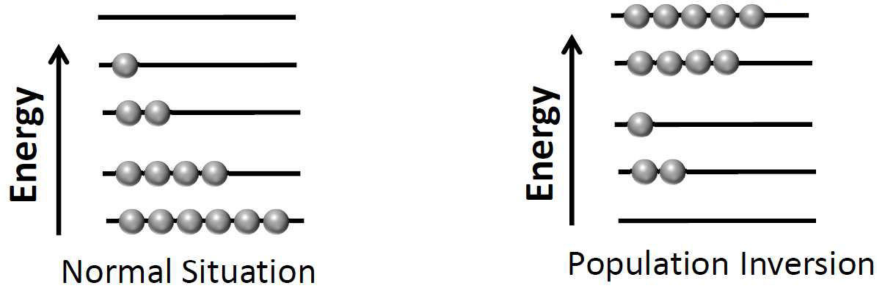
After population inversion is achieved $N_2 > N_1$



Necessary condition for laser action to take place

Population Inversion

In normal situation all things wants to be lowest energy state.



For stimulated emission to dominate, the majority of atoms must be in an excited state, so spontaneously emitted photons are more likely to stimulate emission than to be absorbed by atoms in the ground state. This is called a population inversion.

Nd:YAG Laser

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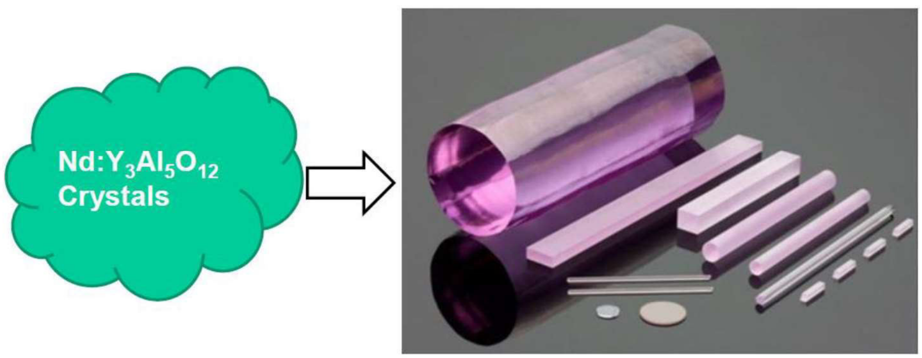
Nd:YAG laser **Solid state laser**

Neodymium-doped Yttrium Aluminum Garnet

Chemical composition Nd:$Y_3Al_5O_{12}$



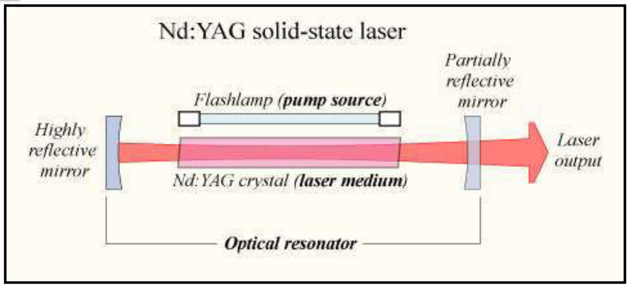
Called as "doped Insulator laser"

Nd:YAG laser

- Ruby ($Cr:Al_2O_3$) was the 1st solid state medium laser developed by T.H. Maiman in 1960. 4-level laser. Pure material acts as host and the dopant acts as guest material responsible for lasing action.

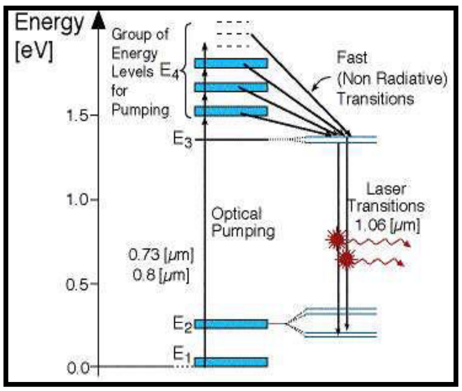
- Neodymium doped Yttrium Aluminum Garnate ($Y_3Al_5O_{13}$).
- It is the most popular type of solid state laser.
- Here, Y^{+3} ions in YAG crystal are partially replaced by Nd^{+3} ions.
- The crystal atoms do not participate in the lasing action but serve as a host lattice in which the active centers (Nd^{+3}) reside.

Construction



- Nd: YAG laser is made up of **elliptical cylindrical reflector.**
- One end is fixed with focus **krypton lamp acting as pumping device**.
- Other focus is silvered – flash from the krypton lamp after reflection concentrate at YAG rod placed at the other end.
- Ends of the laser rod is polished with silver to achieve the resonance mechanism of lasing action

Working of Nd:YAG laser



- **Optical pumping excites** the Nd^{+3} ions from the ground state energy E_1 to higher energy level E_4 and above by absorbing radiations of wavelengths **800 nm and 730 nm** respectively.
- The transition from **higher energy levels to E_3 is a non-radiative** transition.
- **E_3 is a metastable state** and upon continuous excitation, **population inversion of Nd^{+3} ions is achieved between the metastable state E_3 and E_2.**

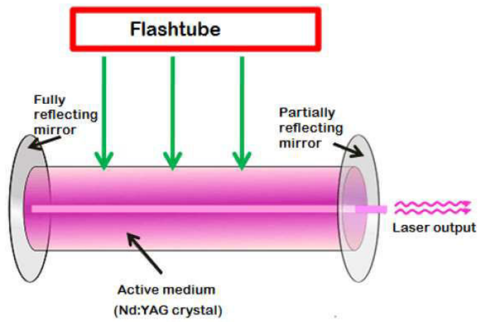
- Any of the spontaneously emitted photon will make the excited Nd^{+3} ions to undergo a transition between E_3 to E_2 state producing coherent stimulated photons.
- As a result the **transition $E_3 \rightarrow E_2$ yields a coherent laser beam of wavelength 1064 nm.**
- The Nd^{+3} ions then make a transition **$E_2 \rightarrow E_1$ which is a non-radiative transition**.

Applications:

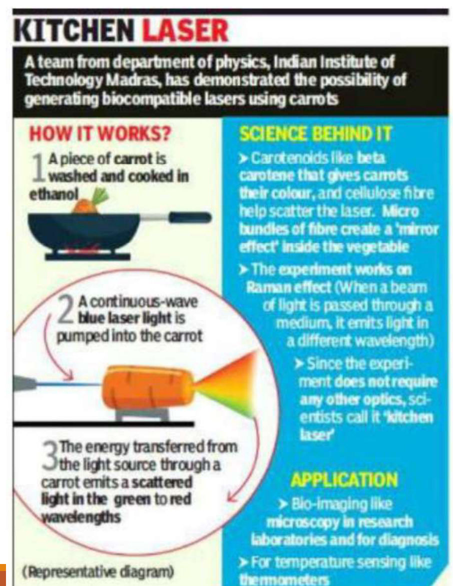
1. Widely used in material processing such as drilling, cutting, etching, welding, surface hardening etc.
2. In military for range finding and target destination.
3. In medical field for cateract surgery, gall bladder surgery anointd to treat gastrointestinal bleeding.

Nd:YAG laser Principle

The Nd ion has many energy levels and due to optical pumping these ions raised to excited levels. During the transition from metastable state E_3 to lower energy state E_2, the laser beam of wavelength 1064nm emitted.



In A First , IIT Madras Researchers Generate Lasers From Carrots



Uses of Lasers

- ❑Due to their properties of coherence, high intensity and high monochromaticity, lasers prove to be useful in almost all fields of the society.
- ❑In general, they are widely used for scientific, military, medical and industrial purposes.

Scientific Uses of Lasers

- Spectroscopy
- Heat treatment
- Weather
- Lunar laser ranging
- Photochemistry
- Laser scanner
- Laser cooling
- Nuclear fusion
- Microscopy

Military Uses of Lasers

- Directly as an energy weapon
- Defensive countermeasures
- Disorientation
- Guidance
- Firearms

Industrial Uses of Lasers

- Lasers are used in a vast variety of areas in the industry, but the most important applications of lasers include cutting, welding and drilling.

- Other uses include laser pointers, engraving, OLED display manufacturing, 3D laser scanners, etc.

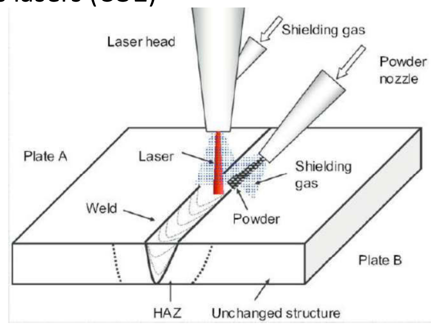
Welding

❖Two metals are placed in contact, and the area around the point heated until the metals are fused together.

❖Type of laser used: solid state (Nd:YAG and ruby) and gas lasers (CO2)

❖Advantages of using lasers:

- ❑No physical contact
- ❑Heating is localised
- ❑Dissimilar metals can be weld
- ❑Controlled Atmosphere
- ❑Can be used in inaccessible regions



❖Examples: Curved contours of automobiles. CO2 laser can also be used to weld plastic films.

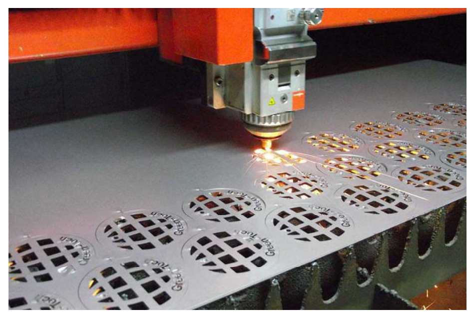
Cutting

❖The aim is to vaporise quickly and produce narrow heat affected zone with minimum distortion

❖Types of laser used: CO2, Nd:YAG

❖Advantages of using lasers:

- ❑ Minimal mechanical distortion and thermal damage
- ❑ No contamination
- ❑ Complicated profiles
- ❑ Easy automation



❖Examples: Paper, cloth, plywood, glass, ceramics, aerospace industries

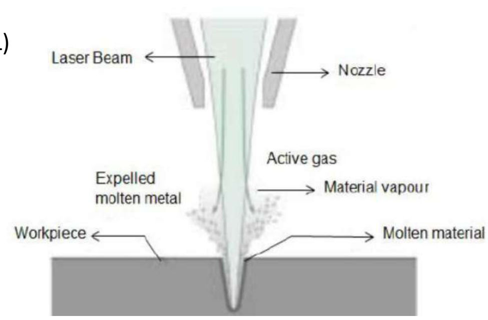
Drilling

❖ Creating thru-holes, by repeated pulses of focused laser energy. Each pulse lasts for 0.0001 - 0.001 seconds.

❖ Types of laser used: CO2, Nd:YAG, Copper Vapour Laser (CVL)

❖ Advantages of using lasers:

- ❑ Non-contact, so physical drill bit needed
- ❑ High precision
- ❑ Faster process
- ❑ Drilling hard materials is possible



❖ Examples: baby bottle nipples, aircraft engine turbine blade, nozzles

Hertz Experiment of Electromagnetic Waves

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GROUP INDEX

Refractive index is defined as

$$n = \frac{Velocity\ of\ light_{vacuum}}{Velocity\ of\ light_{medium}}$$

$$n = \frac{c}{V_p} \qquad V_p = \frac{c}{n} = \frac{\omega}{k}$$

$$ck = n\omega$$

$$c\frac{d}{d\omega}(k) = \frac{d}{d\omega}(n\omega)$$

$$\frac{dk}{d\omega} = \frac{1}{c}\frac{d}{d\omega}(n\omega) \longrightarrow \textbf{(1)}$$

$$Group\ velocity, V_g = \frac{d\omega}{dk}$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} \longrightarrow \textbf{(2)}$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{1}{c}\frac{d}{d\omega}(n\omega)$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{1}{c}\left[n\frac{d\omega}{d\omega} + \omega\frac{dn}{d\omega}\right]$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{1}{c}\left[n + \omega\frac{dn}{d\omega}\right]$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{1}{c}\left[n + \omega\frac{dn}{d\omega}\right]$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{\left[n + \omega\frac{dn}{d\omega}\right]}{c} = \frac{n_g}{c}$$

$$n_g = n + \omega\frac{dn}{d\omega} \longrightarrow \textbf{(3)}$$

In terms of wavelength Group Index is written as

$$\omega\frac{dn}{d\omega} = \frac{2\pi c}{\lambda}\frac{dn}{d\left(\frac{2\pi c}{\lambda}\right)}$$

$$= \frac{\cancel{(2\pi c)}}{\lambda}\frac{dn}{\cancel{(2\pi c)}d\left(\lambda^{-1}\right)} = \frac{1}{\lambda}\frac{dn}{d\left(\lambda^{-1}\right)}$$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\omega\frac{dn}{d\omega} = \frac{1}{\lambda}\frac{dn}{(-)\lambda^{-2}d\lambda}$$

$$\omega\frac{dn}{d\omega} = -\lambda\frac{dn}{d\lambda}$$

$$n_g = n - \lambda\frac{dn}{d\lambda} \longrightarrow \textbf{(4)}$$

Hertz Experiment

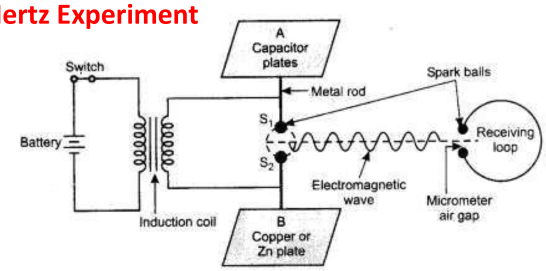
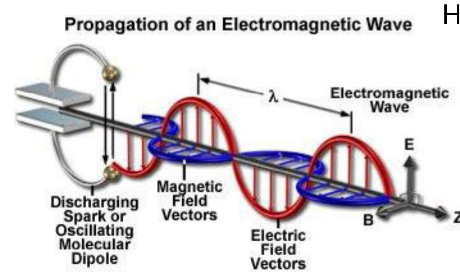


Fig : Sketch of the apparatus used by Hertz for producing and detecting radiowaves



- The existence of electromagnetic wave was confirmed experimentally by Hertz in 1888. This experiment is based on the fact that an oscillating electric charge radiates electromagnetic waves. The energy of these waves is due to kinetic energy of the oscillating charge.

- The experiment arrangement is as shown in figure. It consists of two metal plates A and B placed at a distance of 60 cm from each other. The metal plates are connected to two polished metal spheres S_1 and S_2 by means of metal rods. Using an induction coil a high potential difference is applied across the small gap between the spheres.

- Due to high potential difference across S_1 and S_2, the air in small gap between the spheres get ionized and provides a path for the discharge of the plates. A spark is produced between S_1 and S_2 and electromagnetic waves of high frequency radiated.

- Hertz was able to produce electromagnetic waves of frequency about 5×10^7 Hz. Here the plates A and B acts an a capacitor having small capacitance value C and connecting wire provide low inductance L. Thus the plates and the rods (with spheres) constitute an LC combination. The high frequency oscillation of charges between the plates is given by

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

MAXWELL'S EQUATION

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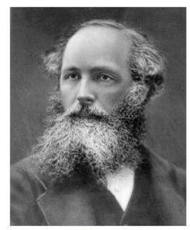
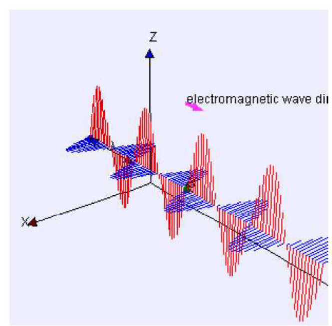
Maxwell's Equations

- The equations are named after the physicist and mathematician James Clerk Maxwell, who between 1861 and 1862 published an early form of the equations that included the Lorentz force law.
- Maxwell's field equations are a set of partial differential equations that describe how electric and magnetic fields are generated by charges, currents, and changing fields.
- One important consequence of these equations is that they demonstrate how fluctuating electric and magnetic fields propagate at the speed of light.
- This eventually led to the identification of light waves as electromagnetic in nature.

Electromagnetic theory of light


James Clerk Maxwell



Maxwell's Equations [instantaneous, differential form]

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{(Faraday's law)}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \text{(Ampere's law)}$$

$$\nabla \cdot \boldsymbol{D} = \rho \qquad \text{(Gauss's law)}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \text{(Gauss's law for magnetic fields)}$$

Electromagnetic Fields

$\vec{E}$: Electric Field Intensity (V/m)

$\vec{D}$: Electric Flux Density (C/m^2)

$\vec{H}$: Magentic Field Intensity (A/m)

$\vec{B}$: Magnetic Flux Density $(T) or (Wb/m^2)$

$\vec{J}$: Current Density (A/m^2)

V : Electric Potential (V)

ρ_V : Volume Charge Density (C/m^3)

Permeability of free space- $\mu_0 = 4\pi \times 10^{-7} (N/A^2)$

Permittivity of free space- $\varepsilon_0 = 8.85 \times 10^{-12} (C^2/Nm^2)$

Physical Significance of Maxwell Equation

Gauss's Law of Electrostatic	$\oint_S \boldsymbol{D} \cdot \boldsymbol{ds} = \oint_V \rho_v \, dv = Q$	Electric flux through a closed surface is proportional to the charged enclosed
Gauss's Law of magneto static	$\oint_S \boldsymbol{B} \cdot \boldsymbol{ds} = 0$	The total magnetic flux through a closed surface is zero
Faraday's law of Induction	$\oint_L \boldsymbol{E} \cdot \boldsymbol{dl} = -\frac{\partial}{\partial t} \iint_S \boldsymbol{B} \cdot \boldsymbol{ds}$	Changing magnetic flux produces an electric field
Maxwell's modified Ampere's Circuital Law	$\oint_L \boldsymbol{H} \cdot \boldsymbol{dl} = \iint_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot \boldsymbol{ds}$	Electric current and changing electric flux produces a magnetic field

4.2 Electromagnetic Wave Equation

The application of Maxwell's equations is the prediction of existence of electromagnetic wave. Electromagnetic wave equation can be obtained from Maxwell's equations.

The Maxwell's equation from Faraday's law is given by,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$H = \frac{B}{\mu}$ → *Magnetic field strength*

$$= -\mu \frac{\partial H}{\partial t}$$

Take curl on both sides,

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \qquad \text{... (4.1)}$$

But Maxwell's equation from Ampere's law is

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}}$$

$D = \varepsilon E$ → *Electric Displacement vector*

J = σ **E** of Ohm's law

$$= \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Differentiating

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{... (4.2)}$$

Substituting the equation (4.2) in equation (4.1)

$$\nabla \times \nabla \times E = -\mu\left[\sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2}\right]$$

$$= -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{... (4.3)}$$

But according to the identity

$$\nabla \times \nabla \times E = \nabla(\nabla \bullet E) - \nabla^2 E \quad \text{... (4.4)}$$

But

$$\nabla \bullet E = \frac{1}{\varepsilon} \nabla \bullet D$$

Since there is not net charge within the conductor, the charge density $\rho = 0$.

$$\nabla \bullet D = 0$$

$$\nabla \bullet E = 0$$

Then equation (4.4) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \quad \text{... (4.5)}$$

$$\nabla \times \nabla \times E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{... (4.3)}$$

Comparing the equations (4.3) and (4.5)

$$\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0} \quad \text{... (4.6)}$$

This is the wave equation for electric field E.

The wave equation for Magnetic field H is obtained in a similar manner as follows.

The Maxwell's equation from Ampere's law is given by,

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Take curl on both sides,

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \varepsilon \nabla \times \frac{\partial E}{\partial t} \quad \text{... (4.7)}$$

But Maxwell's equation from Faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Differentiating,

$$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial H^2}{\partial t^2}$$

Substituting the values of $\nabla \times E$ and $\nabla \times \frac{\partial E}{\partial t}$ in equation (4.7)

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \quad \text{... (4.8)}$$

But the identity is

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H$$

But $\nabla \cdot B = \mu \nabla \cdot H = 0$

Then, $\nabla \times \nabla \times H = \nabla^2 H \quad \ldots (4.9)$

Comparing the equations (4.8) and (4.9)

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \quad \ldots (4.8)$$

$$\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0} \quad \ldots (4.10)$$

This is the wave equation for magnetic field H.

4.3 Wave Equation for Free Space

For free space (dielectric medium) the conductivity of the medium is zero. (i.e., $\sigma = 0$) and there is no charge containing in it (i.e, $\rho = 0$). The electromagnetic wave equations for free space can be obtained from Maxwell's equations.

$$\boxed{\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0} \quad \ldots (4.11)$$

This is the wave equation for free space interms of Electric field.

The wave equation for free space in terms of magnetic field H is obtained in a similar manner as follows.

$$\boxed{\nabla^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0} \quad \ldots (4.12)$$

For free space $\mu_r = 1$ and $\varepsilon_r = 1$ (air)

Then the wave equation becomes

$$\boxed{\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0} \quad \text{or} \quad \boxed{\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0}$$

Module:6 Propagation of EM waves in Optical fibers

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$$NA = \sqrt{n_1^2 - n_2^2} \longrightarrow (1)$$

$$\Delta = \frac{n_1 - n_2}{n_1} \longrightarrow (2)$$

From eq (2)

$$\Delta n_1 = n_1 - n_2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1+n_2)(n_1-n_2)}$$

$$NA = \sqrt{(n_1+n_2)\,\Delta n_1}$$

here n_1 is nearer to n_2

$$NA = \sqrt{2 n_1 \Delta n_1} = \sqrt{2 \Delta n_1^2}$$

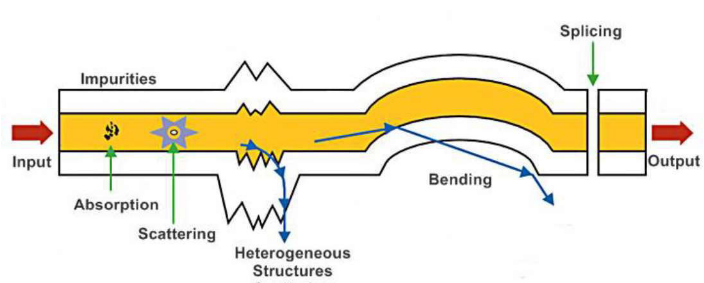
$$\boxed{NA = n_1 \sqrt{2\Delta}}$$

Losses in Optical fiber

Optical fibers have negligible losses when light traveling inside a core

The transmission through an optical fiber is limited by

- **Attenuation**
- **Dispersion.**



- When light travels along the fibre, there is a loss of optical power, which is called attenuation.

Definition:

Attenuation: Ratio of optical input power (P_i) to the optical output power (P_o)

Optical Input power: The power transmitted into the fibre from an optical source

Optical output power: The power received at the fibre end

Attenuation is the rate at which the signal light decreases in intensity. For this reason, glass fiber (which has a low attenuation) is used for long-distance fiber optic cables; plastic fiber has a higher attenuation and, hence, shorter range.

Attenuation

Fiber Attenuation

This relation defines the signal attenuation or absorption coefficient in terms of length L of the fibre:

$$\boxed{\alpha = \frac{10}{L}\log_{10}\frac{P_i}{P_0}}$$

Length **L** of the fibre is expressed in **kilometers**

Here, the unit of Attenuation is decibels/kilometer i.e. **dB/km.**

The main causes of attenuation in optical fibre are:

(a)Absorption

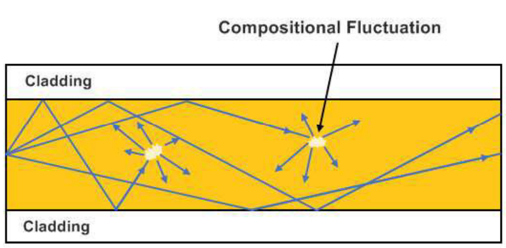
(b)Scattering

(c)Bending losses

Each mechanism of loss is influenced by the properties of fibre Material and fibre structure.

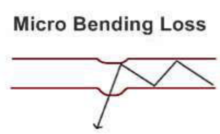
Intrinsic Optical Fiber Losses

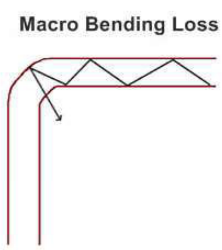
Scattering losses in optical fiber are due to microscopic variations in the material density, compositional fluctuations, structural inhomogeneities and manufacturing defects.



Extrinsic Optical Fiber Losses

Bending is the common problem that can cause optical fiber losses generated by improper fiber optic handling. There are two basic types. One is micro bending, and the other one is macro bending (shown in the picture below). Macro bending refers to a large bend in the fiber (with more than a 2mm radius).





Dispersion

- ❑ Dispersion is the broadening of actual time-width of the pulse due to material properties and imperfections.
- ❑ As pulse travels down the fiber, dispersion causes pulse spreading. This limits the distance travelled by the pulse and the bit rate of data on optical fiber.
- ❑ Dispersion mechanisms cause broadening of the transmitted light pulses as they travel along the fiber.

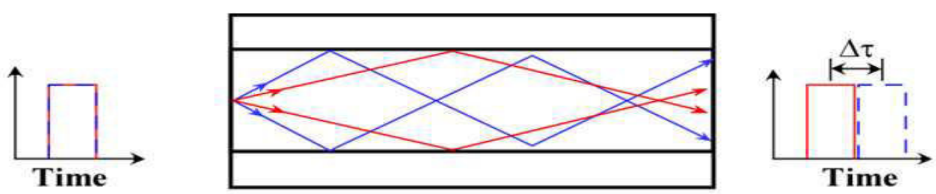
Signal broadening in a fiber

There are two major types of dispersion in fiber-optics

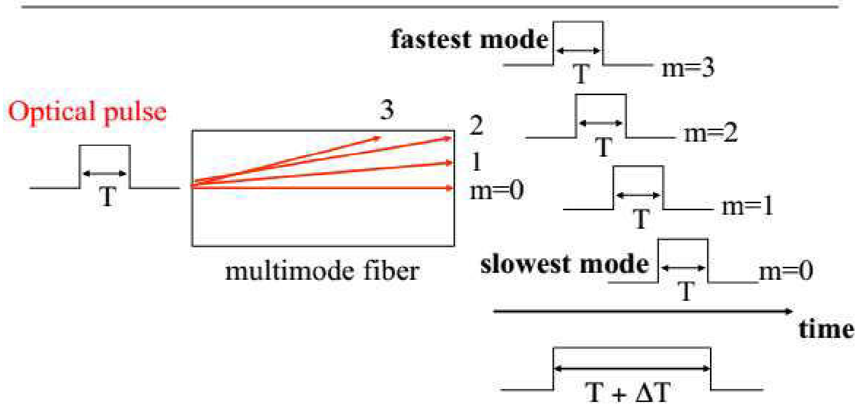
- ❑**Intermodal**
- ❑**Intramodal**

Intermodal

- ❖Dispersion caused by multipath propagation of light energy is referred to as intermodal dispersion.
- ❖Different modes will travel with different propagation angles, hence these modes takes different routes but travel with the same velocity, but at the end of fiber they come at different timings.
- ❖This causes pulse widening
- ❖Signal degradation occurs due to different values of group delay for each individual mode at a single frequency



Modal dispersion results in pulse broadening



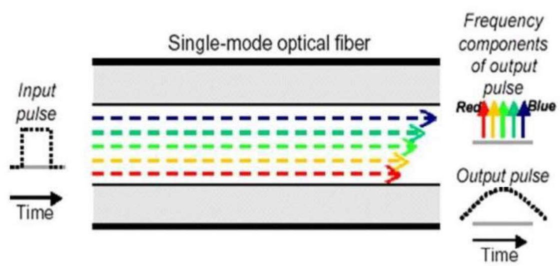
modal dispersion: different modes arrive at the receiver with different delays => pulse broadening

Intramodal dispersion

- Pulse broadening within a single mode is called as intramodal dispersion or chromatic dispersion.
- Since this phenomenon is wavelength dependent and group velocity is a function of wavelength, it is also called as group velocity dispersion (GVD).
- Two types: 1) **Material dispersion** 2) **Waveguide dispersion**

Material dispersion

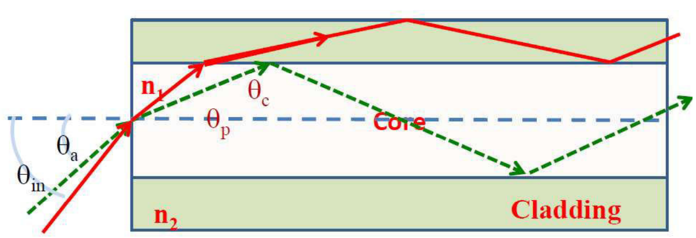
- It is the pulse spreading due to the dispersive properties of material.
- It arises from variation of refractive index of the core material as a function of wavelength.
- Material dispersion is a property of glass as a material and will always exist irrespective of the structure of the fiber.
- Called material dispersion since it results from the refractive index variation of the material of the fiber with the wavelength of light propagating through it.



CD causes the shorter λ to travel faster than the longer λ.

Waveguide dispersion

- It occurs because a single mode fiber confines only about 80% of the optical power to the core.
- Dispersion thus arises since the 20% light propagating in the cladding travels faster than light confined to the core.
- The amount of waveguide dispersion depends on the structure of the fiber and can be varied by altering the parameters such as NA, core radius etc.



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Module:6 Propagation of EM waves in Optical fibers

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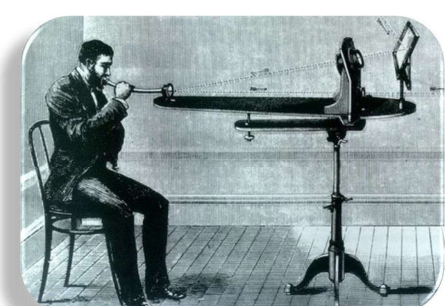
Module-6 Propagation of Electromagnetic waves in optical fibers

- Optical fiber
- Light propagation in optical fiber
- Acceptance angle and numerical aperture Type of fibers
- Attenuation and dispersion

1880 – Alexander Graham Bell

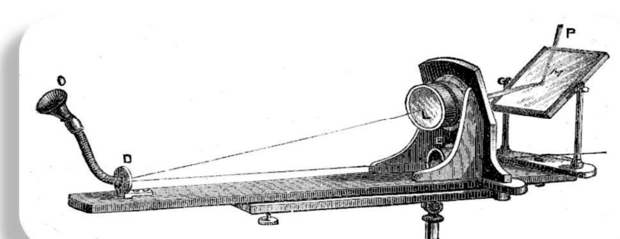


first practical application of transmitting information by bending light

Photophone - telecommunications device that allows transmission of speech on a beam of light

small mirror with light that vibrated to the sound

receiver that would in turn focus light on and off of crystalline selenium cells which would change their resistance
The first wireless data transmission in the world occurred from two rooftops over 700ft apart using this technology.



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Advantages of Optical Fibre

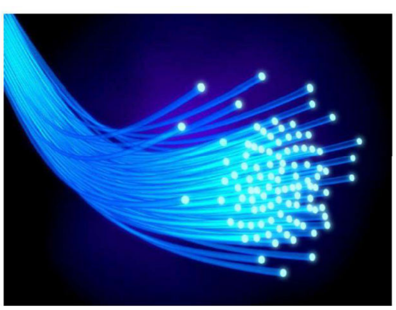
- Thinner, smaller in size
- Flexible, so can be bend
- Less Expensive
- Higher Carrying Capacity
- Less Signal Degradation & Digital Signals
- Light Signals
- Non-Flammable, non-conductive, non- radiative, non-inductive
- Light Weight
- No short circuit as in metals
- No need to ground, so no voltage problem

Areas of Application

- **Telecommunications**
- **Local Area Networks**
- **Cable TV**
- **CCTV (Closed-circuit television)**
- **Optical Fiber Sensors**

Optical fiber

Optical fiber is a transparent, flexible cylindrical dielectric waveguide made of low loss materials (silica glass). It has a central core embedded in an outer cladding of slightly low refractive index



Structure of an optical Fiber

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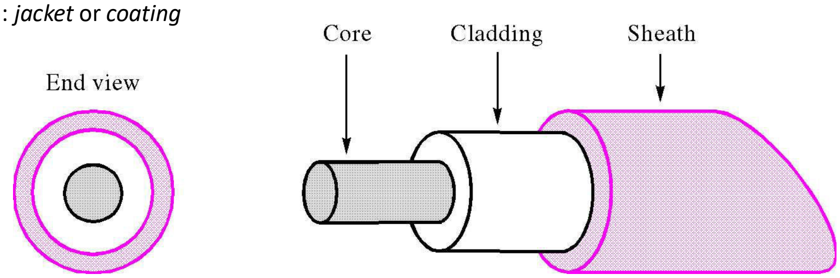
An optical fiber consists of 3 distinct parts:

1) Core : silica glass, SiO_2
2) Cladding: glass or plastic
3) Sheath: *jacket* or *coating*



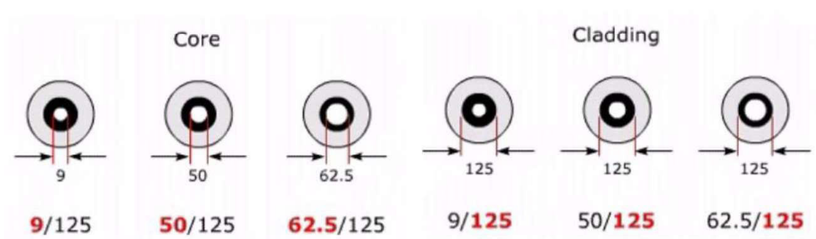
n_1 is refractive index of core
n_2 is refractive index of cladding

$n_1 > n_2$

Structure of Optical fiber

- The core and cladding act as an optical wave-guide.
- Core - it is a transmission area of fiber - typical core diameters range from 50 to 500 µm
- Cladding - it surrounds the core and *has a different index of refraction* less than the core.
- The size of the fibers are specified by the outer diameters of the core and cladding.
- A 50/150 fiber means that the core diameter is 50 µm while the outside dimension of both the core plus cladding together is 150 µm.
- The core and cladding are surrounded by the sheath.

What do the fiber terms 9/125, 50/125 and 62.5/125 (micron)?



Remember: A **micron** (short for micrometer) is one-millionth of a meter

Typically n(cladding) < n(core)

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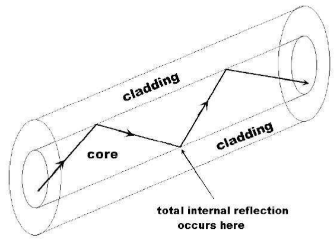
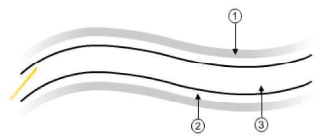
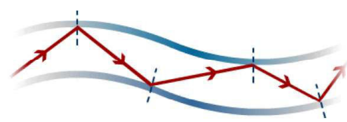
Working principle of Optical fiber

Optical fiber works under the principle of total internal reflection





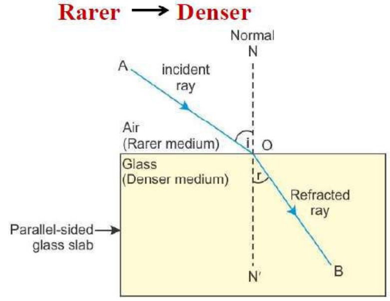


Optics basics

Rarer medium : medium with less refractive index (n_1)
Denser medium : medium with high refractive index (n_2) **($n_2 > n_1$)**

***i* is angle of incident and *r* is angle of refraction**

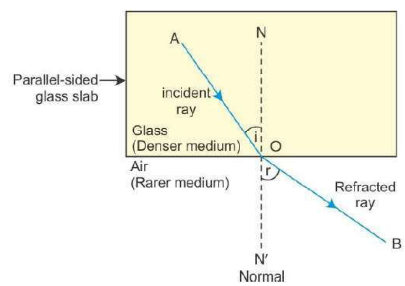
Rarer → Denser



The refracted ray bends towards the normal

(i > r)

Denser → Rarer

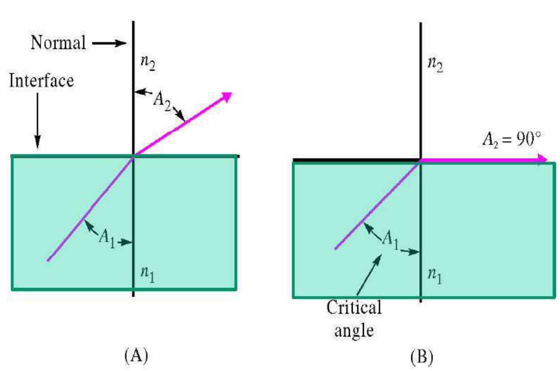


The refracted ray bends away from the normal

(i < r)

Critical angle

When a light ray passes from denser medium to a rarer medium, then the critical angle defined as the angle of incidence that will produce a 90° angle of refraction.



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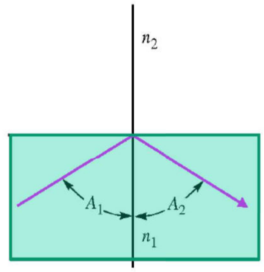
Total Internal Reflection

The angle of incidence exceeds the critical angle, then the incident ray is reflected in the same medium and this phenomenon is called total internal reflection.



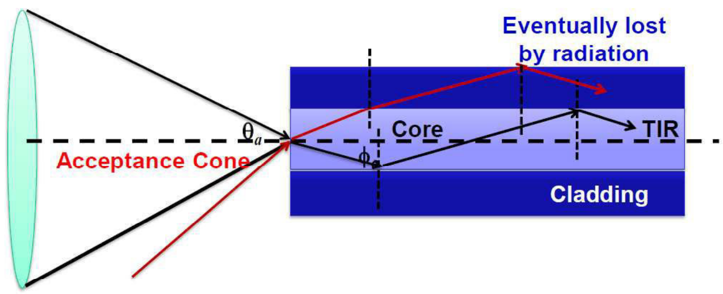
(C)

Condition for internal reflection

- **The ray of light should be traverse from denser to rare medium.**
- **The incident angle should be more than the Critical angle.**

ACCEPTANCE ANGLE

The maximum angle of incident with respect to the fiber core axis at the core-cladding interface, which allow to transmit the light in the fiber by total internal reflection.



Acceptance cone : Multimode optical fiber will only propagates light that enters the fiber with in a certain cone known as acceptance cone of the fiber .

Half angle of the cone is the acceptance angle

- Any rays which are incident on into the fiber core at an angle greater than θ_a will be transmitted to the core-cladding interface at an angle less than ϕ_c and will not be totally internally reflected.
- Thus, for rays to be transmitted by total internal reflection within the fiber core they must be incident on the fiber core within an acceptance cone defined by the conical half angle θ_a.
- Hence, θ_a is the maximum angle to the axis at which light may enter the fiber in order to be propagated and is often referred to as the acceptance angle for the fiber.

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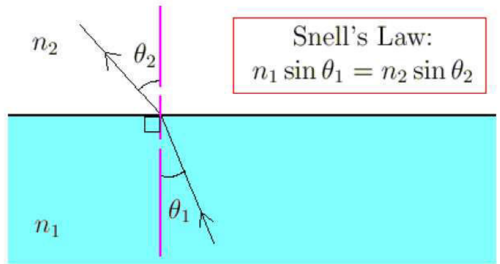
Snell's law



θ_1 is incident angle in medium 1 and θ_2 is refracted angle in medium 2

The ratio of the sine angle of incident rays in medium 1 to the refracted rays in medium 2 is equal to the ratio of refractive index of medium 2 to that of medium 1

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}} \qquad \sin\theta_1 n_1 = \sin\theta_2 n_2$$

References

- Djafar K. Mynbaev and Lowell L.Scheiner, Fiber Optic Communication Technology, 2011, Pearson.
- Ajoy Ghatak and K. Thyagarajan, Introduction to Fiber Optics, 2010, Cambridge University Press.

Module:6 Propagation of EM waves in Optical fibers

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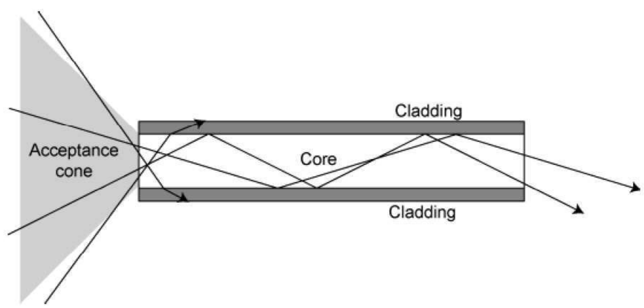
Acceptance cone



- The maximum angle, represented in three-dimensional view as a cone, at which an optical fiber will accept incident light. Within that cone, as defined by those angles, a light source can inject an optical signal into the fiber core and the signal will remain in the core, reflecting off of the interface between the core and cladding.

- Incident rays which fall within the acceptance cone of the fiber are transmitted, whereas those which fall outside of the acceptance cone are lost in the cladding.

Acceptance cone

- The imaginary light cone with twice the acceptance angle as the vertex angle, is known as the acceptance cone.

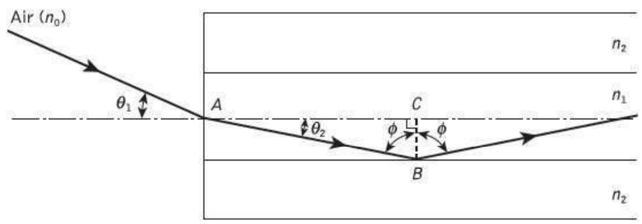
- The maximum angle, represented in three-dimensional view as a **cone**, at which an **optical fiber** will accept incident light. ... At a more severe angle, i.e., outside the **cone**, the signal will penetrate the interface and enter, and perhaps be lost in, the cladding.

- a light **ray** must **enter** the core with an **angle less than** a particular **angle** called the **acceptance angle of** the **fibre**. **A ray which enters the fiber with an angle** greater **than the acceptance angle** will be lost in the cladding.

Numerical Aperture (NA) :

Numerical aperture (NA) of the fiber is the light collecting efficiency of the fiber and is a measure of the amount of light rays can be accepted by the fiber.

Relation between acceptance angle and refractive index is achieved through **Numerical Aperture**



Derivation for Numerical aperture:

Apply Snell's law at point A

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \text{ ------------------------------} \rightarrow \textbf{(1)}$$

Apply Snell's law at point B

$$\frac{\sin\theta_c}{\sin 90} = \frac{n_2}{n_1}$$

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}} \text{ ------------------------------} \rightarrow \textbf{(2)}$$

From equation 1

In the triangle ABC, $\theta_r = \frac{\pi}{2} - \phi$

$$\sin\theta_1 = n_1 \sin\left(\frac{\pi}{2} - \theta\right) = n_1 \cos\theta$$

$$\boxed{\sin\theta_i = \frac{n_1}{n_0}\cos\theta_c}$$

$$\sin\theta_i = \frac{n_1}{n_0}\sqrt{1-\sin^2\theta_c}$$

From equation 2

$$\sin\theta_i = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

$$\sin\theta_i = \sqrt{n_1^2 - n_2^2}$$

Since the refractive index of air is 1, then $n_o = 1$

Where $\sqrt{n_1^2 - n_2^2}$ is called numerical aperture (N.A) of the fiber

$$N.A = \sin\theta_i = \sqrt{n_1^2 - n_2^2}$$

Fractional refractive index or relative refractive index

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$N.A = n_1\sqrt{2\Delta}$$

Acceptance angle

$$\theta_i = \sin^{-1}(N.A)$$

$$\theta_i = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

Numerical Problems

Activity 1: The refractive indices for core and cladding for a step index fibre are 1.52 and 1.41 respectively. Calculate (1) Critical angle (2) Numerical Aperture (3) The maximum incidence angle

Hints: Given Here $= \mu_1 = \mu_{core} = 1.52$, and $\mu_2 = \mu_{clad} = 1.41$

Critical angle $\theta_c = \sin^{-1}(\mu_2 / \mu_1)$ **Ans = 68.06⁰**

Numerical Aperture $\sqrt{\mu_1{}^2 - \mu_2{}^2}$ **Ans = 0.568**

Maximum incidence angle $(\theta_0) = \sin^{-1}\left(\sqrt{\mu_1{}^2 - \mu_2{}^2}\right)$

Ans $\theta_0 = 34.6^0$

Solved Problem (1) : A fiber has the following characteristics: $n_1 = 1.35$ (core index) and $\Delta = 2\%$. Find the N.A and the acceptance angle.

n1 = 1.35 ; $\Delta = 2\% = 0.02$

W.K.T
$= 1.35 \times (2 \times 0.02)^{1/2} = 0.27$
$\theta_a = \sin^{-1}(N.A) = \sin^{-1}(0.27) = 15.66^\circ$

Solved Problem (2): A silica optical fiber has a core refractive index of 1.50 and a cladding refractive index of 1.47. Determine (i) the critical angle at the core – cladding interface, (ii) the N.A for the fiber and (iii) the acceptance angle for the fiber.

The critical angle $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.47}{1.5}\right) = 78.5^\circ$

The numerical aperture $NA = (n_1^2 - n_2^2)^{\frac{1}{2}}$

$(1.50^2 - 1.47^2)^{1/2} = 0.30$

The acceptance angle $= 2\theta_a = 2\sin^{-1}(N.A) = 2\sin^{-1}(0.30) = 34.9^\circ$

Critical angle = 78.5º ; N.A = 0.30 ; Acceptance angle = 34.9º

Activity 1: Calculate the numerical aperture for the optical fiber whose core refractive index is 1.52 and cladding has 1.43;

Activity 2: Calculate the acceptance angle for the fiber of NA = 0.39

Types of Optical fiber

Dependents of the refractive index, the optical fiber classified as two types

- **Step index fiber**
- **Graded index fiber**

Step index fiber

A step-index **fiber has a central core with a *uniform* refractive index. The core is surrounded by an outside cladding with a uniform refractive index less than that of the central core.**

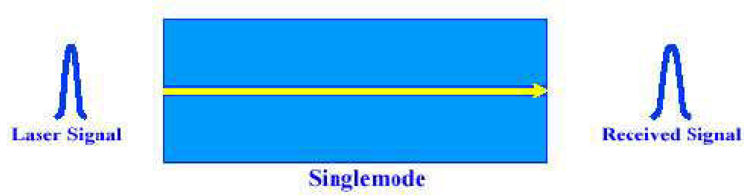
Single-Mode Step-Index Fiber

It has a central core that is sufficiently small so that there is essentially only one path that light may take as it propagates down the cable.

The refractive index of the cladding is slightly less than that of the central core and is uniform throughout the cladding.

Consequently, all light rays follow approximately the same path down the cable and take approximately the same amount of time to travel the length of the cable.

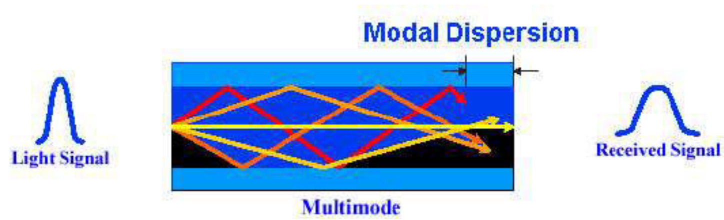
Used for very high speed, large bandwidth and long-distance transmission.



Multi-mode Step-Index Fiber

- The light rays that strike the core/cladding interface at an angle greater than the critical angle are propagated down the core in a zigzag fashion, continuously reflecting off the interface boundary.
- There are many paths that a light ray may follow as it propagates down the fiber. As a result, all light rays do not follow the same path and hence do not take the same amount of time to travel the length of the fiber.

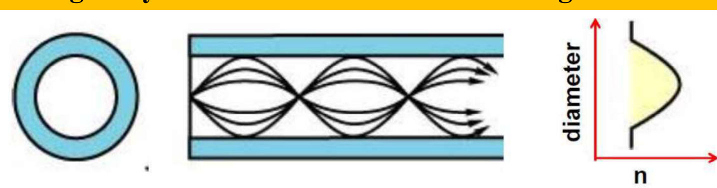
Best designed for short transmission distances and is suited for use in LAN systems and video surveillance



Graded-Index Fibers

The refractive index of core gradually changes, while the refractive index of cladding remains constant

In graded index fibers, the core center has high refractive index and it is radially decreases on moving away from the center to core-cladding interface



Graded-index fibers are in general multimode fibers

Large core diameter (> 50 µm) Example: 62.5/125

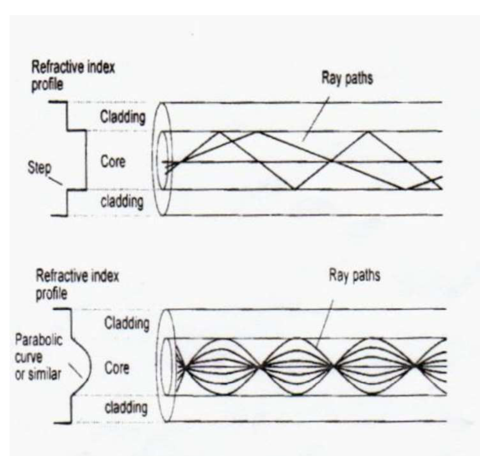
Two main types of cables

Step Index Fibre

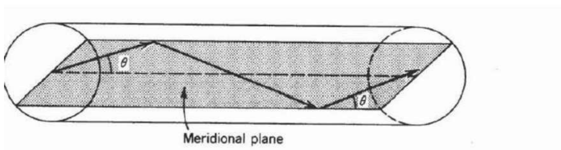
This cable has a specific index of refraction for the core and the cladding. It causes deformations due to the various paths lengths of the light ray. This is called modal distortion. It is the cheapest type of cabling. Within the cladding and the core, the refractive index is constant.

Graded Index Fibre

In graded index fibre, rays of light follow sinusoidal paths. Although the paths are different lengths, they all reach the end of the fibre at the same time. Multimode dispersion is eliminated and pulse spreading is reduced. Graded Index fibre can hold the same amount of energy as multimode fibre. The disadvantage is that this takes place at only one wavelength.

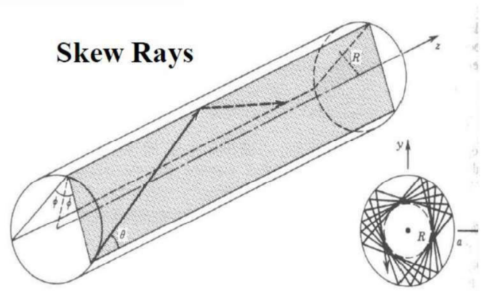


Step Index	Graded Index
Refractive index is single step	Refractive index is gradually decreasing from center to wards interface of core and cladding
Propagation is in the form of meridional rays and it passes through the fiber axis	Propagation is in the form of Skew rays and it will not cross the fiber axis
Path of propagation is in zig-zag	Path of propagation is in Helical
Lower bandwidth	Higher bandwidth
Distortion is more (high angle ray arrive later than low angle ray, hence signal pulse is broadened)	Distortion is less due to self focusing effect (different paths in different speeds, at outer it has faster than inner, hence reach at same time)
No. of modes propagation $N_{step} = 4.9(d*NA/\lambda)^2$	No. of modes propagation $N_{graded} = N_{step}/2$



Meridional Rays

Skew Rays



A **meridional ray** is a **ray** that passes through the axis of an optical fiber. A skew **ray** is a **ray** that travels in a non-planar zig-zag path and never crosses the axis of an optical fiber.

V-number (fiber parameter)

The number of propagation modes in step-index optical fiber can be determined from v-number or a normalized frequency (horizontal wave number)

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a n_1}{\lambda}\sqrt{2\Delta}$$

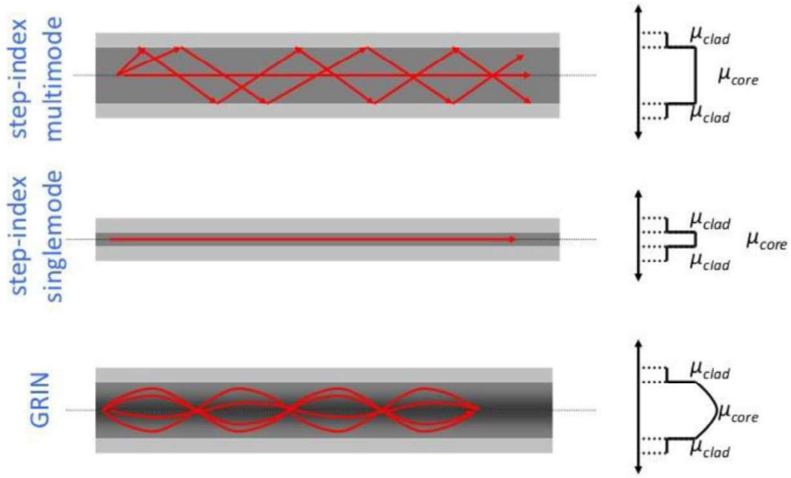
Where a is core radius and n_1 is core refractive index

If V <2.405, single mode optical fiber only one mode can propagates through the fiber

If V >> 2.405, it is multimode optical fiber

Number of modes can be
$M=V^2/2$ (Step index fiber)
$M=V^2/4$ (Graded index fiber)

Types of Fibers



Parameters of Optical fibres

Acceptance angle =

$$\sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\theta_0 = \sin^{-1}\left(\sqrt{\mu_1^2 - \mu_2^2}\right)$$

Acceptance Cone = $2\theta_0$

Numerical Aperture (NA) = $\sin\theta_0$ = $\sqrt{\mu_1^2 - \mu_2^2}$

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Module:7 Endoscopy

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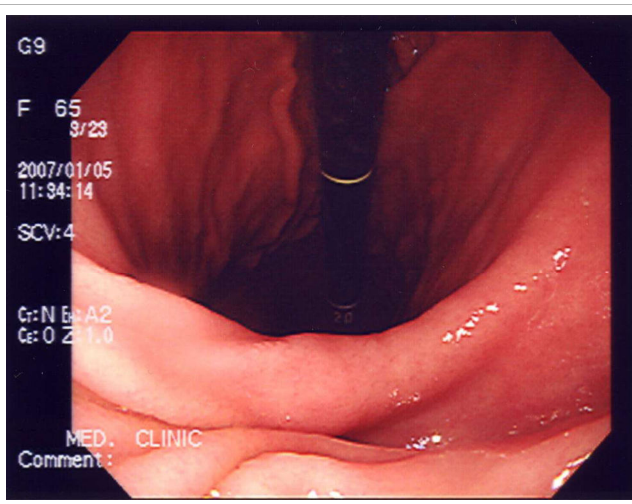
Applications of Optical fiber

Endoscopy

- Endoscopy Greek Word "Endo" means "Inside"
  "Skopeein " means "To See"

- Endoscopy, is the examination of internal body cavities using a specialized medical instrument called an endoscope.

- Physicians use endoscopy to diagnose, monitor, and surgically treat various medical problems.



The endoscope gives visual evidence of the problem, such as ulceration or inflammation

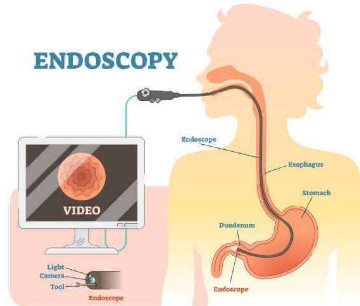
It can be used to collect a sample of tissue; remove problematic tissue

It is used to take photograph of the hollow internal organs

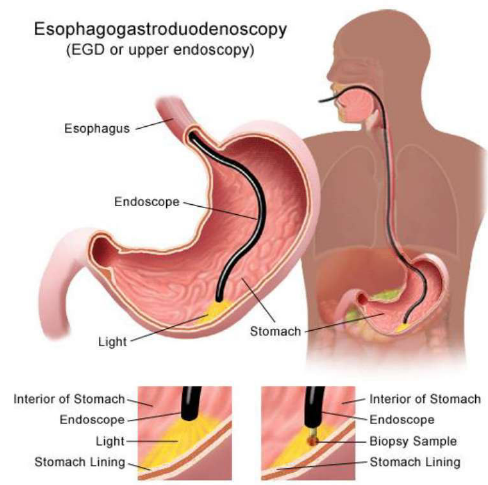
Endoscopes and fiber optic sensors are the very important applications of optical fibers apart from their use in communications

An endoscope can consist of:

- An endoscope is a slender, flexible tube equipped with lenses and a light source. Illumination is done by the help of a number of optical fibres.
- These optical fibers are arranged such that their terminations occupy the same relative positions in both the bound ends of the bundle. (coherent bundle)
- Reflected light rays are collected by CCD (Charge coupled device) and electrical signals are produced, which are fed to the video monitor to get image.
- Through one channel of endoscope water and air is conducted to wash and dry the surgical site.



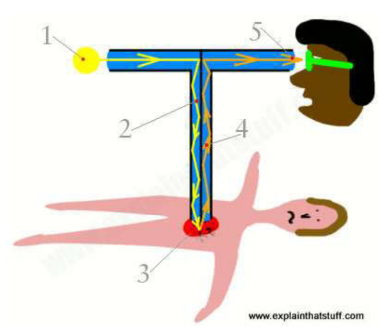
- The endoscope also has a channel through which surgeons can manipulate tiny instruments, such as forceps, surgical scissors, and suction devices.
- A variety of instruments can be fitted to the endoscope for different purposes.
- A surgeon introduces the endoscope into the body either through a body opening, such as the mouth or the anus, or through a small incision in the skin.

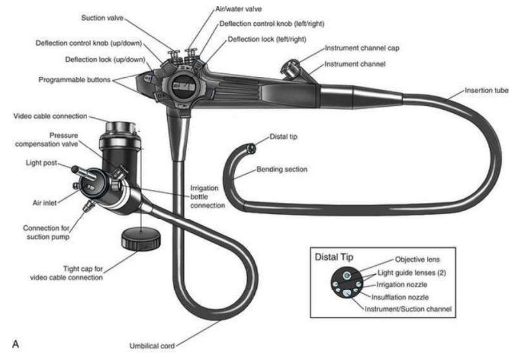


How do endoscopes work?

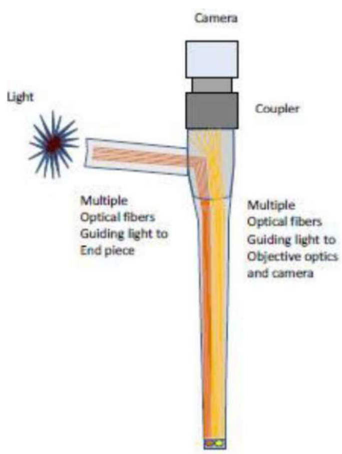
Here's how endoscopy works:

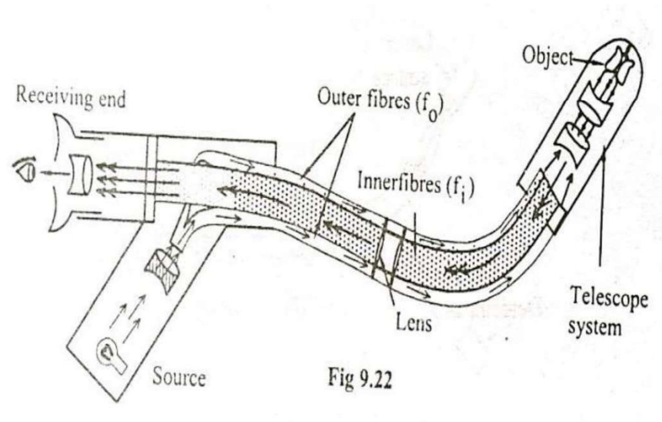
1. One of the two main endoscope cables carries light from a bright lamp in the operating room into the body, illuminating the cavity where the endoscope has been inserted.
2. The light bounces along the walls of the cable into the patient's body cavity.
3. The diseased or injured part of the patient's body is illuminated by the light shining in.
4. Light reflected off the body part travels back up a separate fiber-optic cable, bouncing off the glass walls as it goes.
5. The light shines into the physician's eyepiece so he or she can see what's happening inside the patient's body. Sometimes the fiber-optic cable is directed into a video camera (which displays what's happening on a television monitor) or a CCD (which can capture images like a digital camera or feed them into a computer for various kinds of image enhancement).





Endoscope system





Why Do I Need an Endoscopy?

Doctors will often recommend endoscopy to evaluate:

- ❑ Stomach pain
- ❑ Ulcers, gastritis, or difficulty swallowing
- ❑ Digestive tract bleeding
- ❑ Changes in bowel habits (chronic constipation or diarrhea)
- ❑ Polyps or growths in the colon

In addition, your doctor may use an endoscope to take a biopsy (removal of tissue) to look for the presence of disease.

opto

Module:7 Optoelectronic Devices & Applications of Optical fibers

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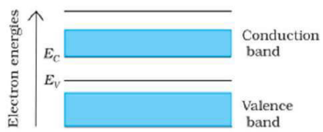
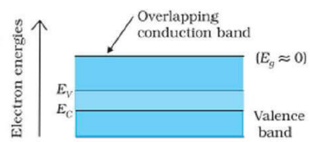
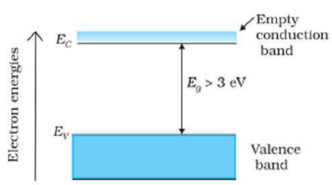


E_c → lowest energy level of the C.B. and E_v → highest energy level of the V.B. In a solid where the valence band is partially empty, electrons from the lower energy levels can move to the higher levels making conduction possible.

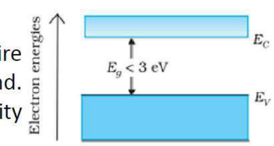
E_g → energy gap. If in a solid the conduction and valence bands overlap each other, electrons can easily move from the VB to the CB. This makes a large number of electrons available for conduction.



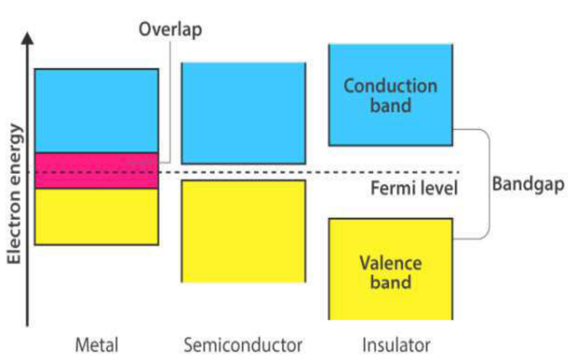


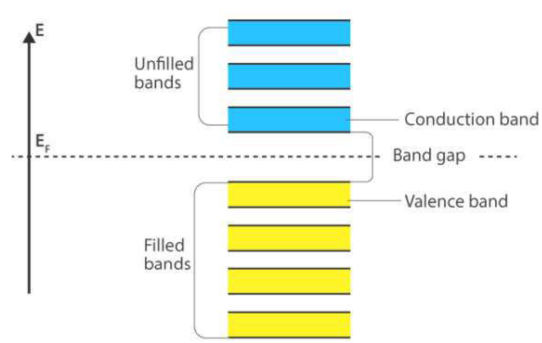
E_g is very large (>3 eV). Due to this large gap, electrons cannot be excited to move from the valence to the conduction band by thermal excitation, there are no free electrons in the conduction band and no conductivity. These are insulators.

If energy gap is small (< 3 eV). Since the gap is small, some electrons acquire enough energy even at room temperature and enter the conduction band. These electrons can move in the conduction band increasing the conductivity of the solid. These are semiconductors.

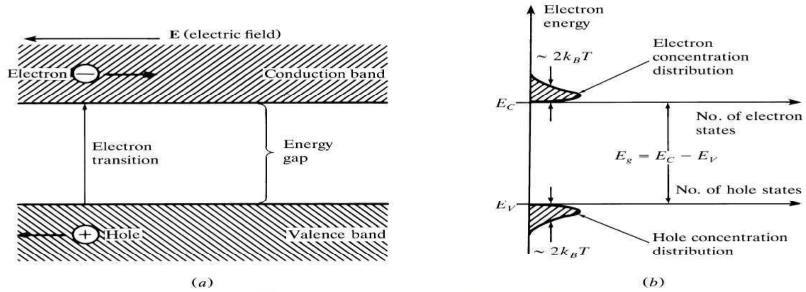


Band Theory of Solids





Basic Semiconductor Physics



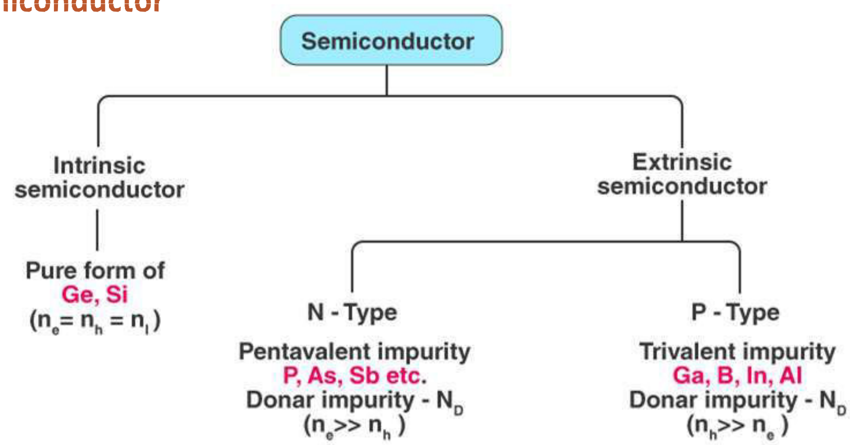
Boltzmann's constant $k_B = 1.38\times10^{-23}$ JK^{-1}

a) Energy level diagrams showing the excitation of an electron from the valence band to the conduction band. The resultant free electron can freely move under the application of electric field.

b) Equal electron & hole concentrations in an intrinsic semiconductor created by the thermal excitation of electrons across the band gap

Types of Semiconductors

- ❑ **Intrinsic Semiconductor**
- ❑ **Extrinsic Semiconductor**

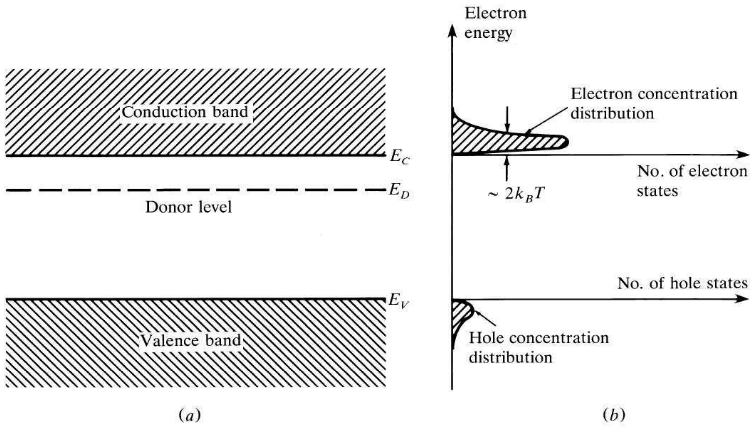


•**Intrinsic Semiconductor (Pure semiconductor):** An intrinsic semiconductor is made up a very pure form of semiconductor material. It's conductivity is solely determined by the thermally generated carriers.

• **Extrinsic Semiconductor (Doped semiconductor):** When an impurity atom is added with a pure semiconductor, then it is called extrinsic semiconductor. The process of adding impurity to a semiconductor is called doping. The added impurity is called dopant.

- ❑ **Majority carriers**: electrons in *n*-type or holes in *p*-type.
- ❑ **Minority carriers**: holes in *n*-type or electrons in *p*-type.
- ❑ The operation of semiconductor devices is essentially based on the **injection** and **extraction** of minority carriers.

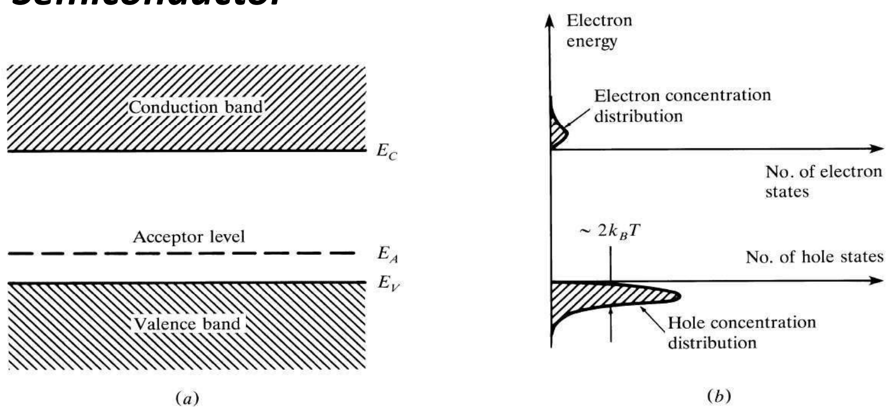
n-Type Semiconductor



Number of free electrons >> number of holes in n-type semiconductor. That is why free electrons are called majority carriers, and holes are called minority carriers in the **n-type semiconductor**.

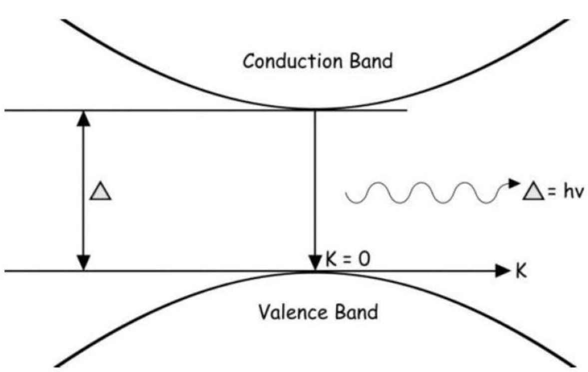
a) Donor level in an *n*-type semiconductor.
b) The ionization of donor impurities creates an increased electron concentration distribution

p-Type Semiconductor



a) Acceptor level in an *p*-type semiconductor.
b) The ionization of acceptor impurities creates an increased hole concentration distribution

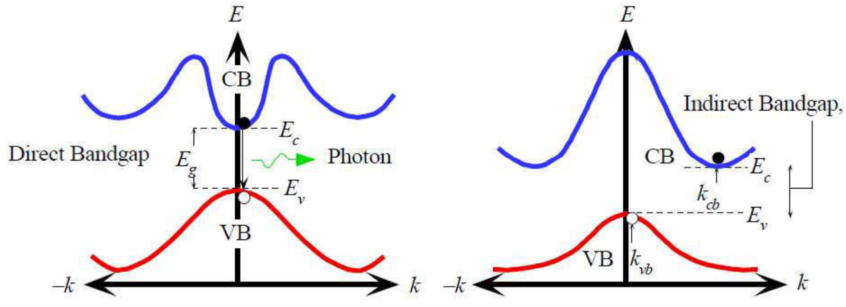
Direct Band Gap Semiconductors



- Those semiconductor materials which have a direct band gap are the ones that emit photons.
- Example of material which has direct band gap is Gallium Arsenide(GaAs).

In a direct bandgap material, the bottom of the energy level of conduction band lies directly above the topmost energy level of the valence band on the Energy vs Momentum (wave vector 'k') diagram. When electrons and hole recombine, energy E = hv corresponding to the energy gap △ (eV) is escaped in the form of light energy or photons where h is the Planck's constant and v is the frequency of light.

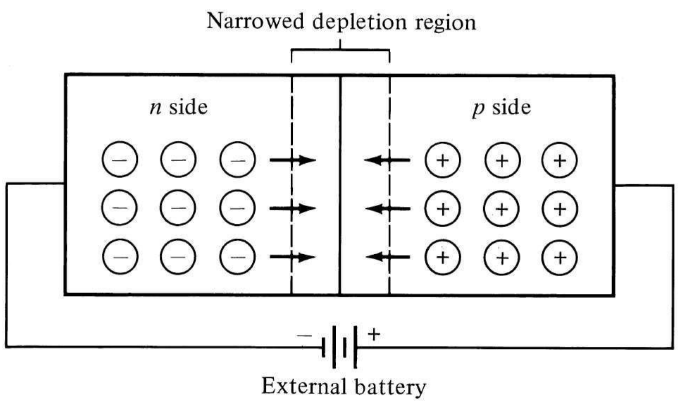
Indirect Band Gap Semiconductors



(a) GaAs (b) Si

(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor.

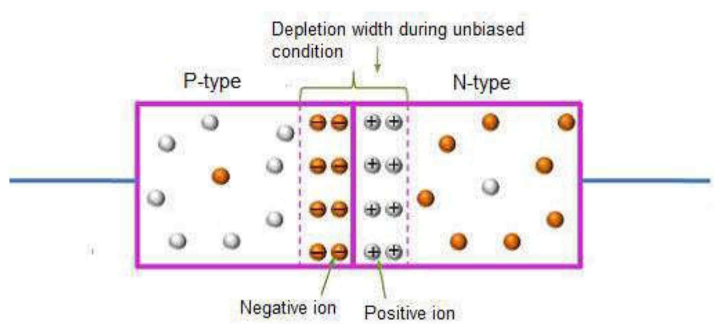
Forward-biased *PN* Junction



Lowering the barrier potential with a forward bias allows majority carriers to diffuse across the junction.

The PN Junction

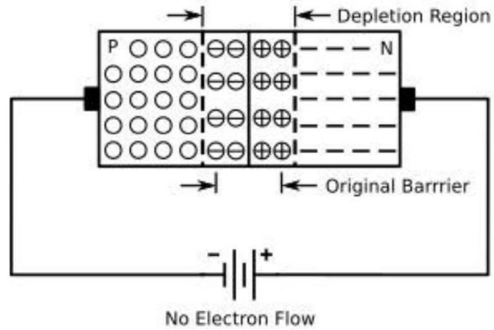
Zero Bias



- No external voltage is applied to the P-N junction.
- The electrons diffuse to the P-side and simultaneously holes diffuse towards the N side through the junction, and then combine with each other.
- Due to this an electric field is generated by these charge carriers. Electric field opposes further diffusion of charged carriers so that there is no movement in the middle region. This region is known as depletion width or space charge.

Reversed-biased *PN* Junction

Connecting the *p-type* region to the *negative* terminal of the battery and the *n-type* region to the *positive* terminal corresponds to reverse bias. It will increase the junction barrier and thereby offer a high resistance to the current flow through the junction. This type of bias is known as reverse bias



Module:7 Photodetector

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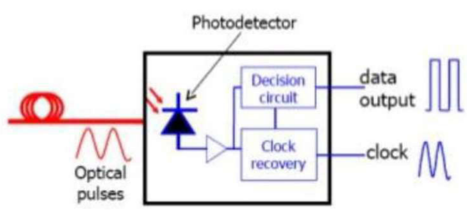
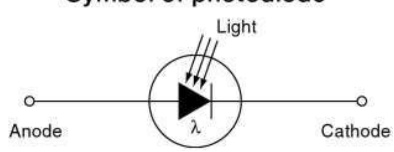
Photodetector

A photodetector is a p-n junction device which absorb light and convert optical energy into electrical energy



Symbol of photodiode



Photodiodes

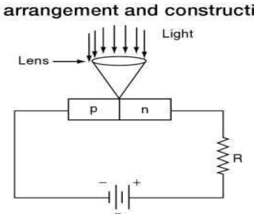
Photodetector

A photo diode is a reverse biased semiconductor p-n diode whose reverse current increases with the increase in intensity of light incident at the junction.

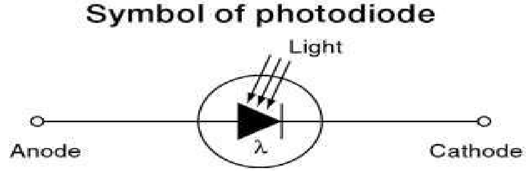
When light is incident on the p-n junction of a photo diode, the atoms at the junction absorb the energy of photons and create more free electrons and holes.

These additional carriers increase the reverse current. As the intensity of incident light on the junction increase the reverse current also increases. The lens is used to concentrate the light to the junction region.

The basic biasing arrangement and construction of a photodiode



Symbol of photodiode

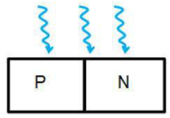


Types of Photodiode

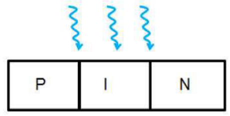
Photo detectors are made up of semiconductor materials, absorb incident photons and produce electrons/holes

Photo detector (diode) works on the reverse bias condition

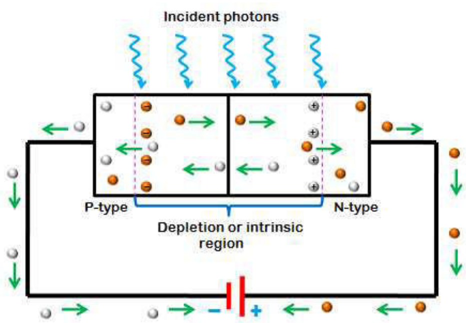
PN Photodiode (p-type and n-type)



PIN photodiode (p-type, intrinsic, n-type)



Working PN photo diode



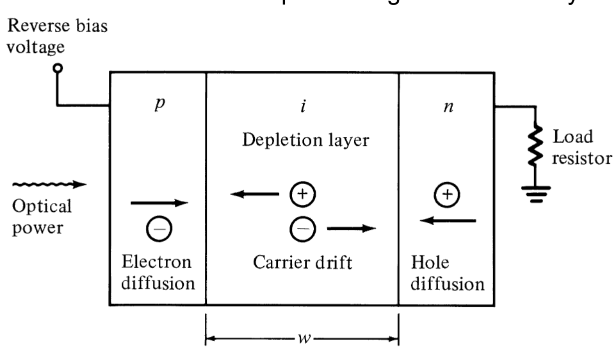
Width of the depletion region increases upon reverse bias of the p-n junction diode leading to higher quantum efficiency

The strong depletion region electric field and the external electric field increase the drift velocity of the free electrons

Because of this high drift velocity, the minority carriers generated in the depletion region will cross the p-n junction and constitute a current flow in the circuit

Working PIN photo diode

In PIN photodiode, an addition layer called intrinsic semiconductor is placed between the p-type and n-type semiconductor to increase width of depletion region and thereby **the minority carrier current.**



When light energy is applied to the PIN diode, most part of the energy is observed by the intrinsic or depletion region. As a result, a large number of electron-hole pairs are generated.

Photons entering these layer produces charge carriers, this action results in high quantum efficiency of this device.

Dark Current

when no light is applied to the reverse bias photodiode, it carries a small reverse current due to external voltage. This small electric current under the absence of light is called dark current. It is denoted by I λ.

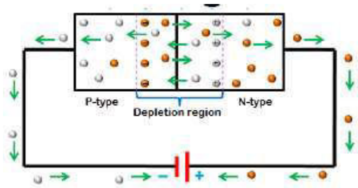
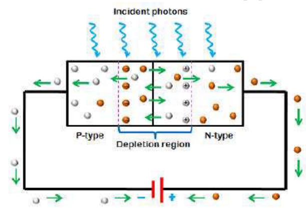


Photo Current

The electric current generated in the photodiode due to the application of light is called photocurrent.



In a photodiode, reverse current is independent of bias voltage and it is mostly depends on the light power

What are the properties of Photodiode?

- The noise produced by the photodiode is very less.
- The life span of the photodiode is very long.
- It is one of the light weighted and compact diodes.
- With respect to incident light, the linearity of the diode is good.
- Wide spectral response is expected from the photodiode.
- It can be mechanically rugged

Responsivity (R) and Quantum efficiency (η)

Responsivity is a measure of the conversion efficiency of a photo detector.

The photocurrent produced is proportional to the power of incident beam

$$I_p \propto P \text{ (input power)}$$
$$I_p = RP$$

R is a constant called responsivity, measured in Amp/Watt

$$\boxed{R = \frac{I_{photo}}{P}} \text{ --------------------------- [1]}$$

- **Responsivity**

$$\boxed{R = \frac{\text{Photocurrent (A)}}{\text{Incident Optical Power (W)}} = \frac{I_{ph}}{P_0}}$$

$$I_{photo} = \frac{N_e . e}{t}$$

$$P = \frac{N_p . E_p}{t}$$

Substitute these equation in equation 1

$$\boxed{R = \frac{N_e . e}{N_p . E_p}} \text{ ---------------------------- [2]}$$

Substitute as $E_p = \dfrac{hc}{\lambda}$ **in equation 2**

$$R = \frac{N_e . e\lambda}{N_p . hc}$$

Quantum Efficiency of PIN photodiode

- The electrical current produced by e^- and h^+ generated by incident photons are called as photocurrent (I_p). I_p is proportional to light power (P) (of suitable $h\nu$).
- -¿ $I_p \propto P$; $I_p = RP$. $\implies$ $R = \frac{I_p}{P}$, here R-responsivity of photodiode.
- $I_p = \frac{Q}{t} = \frac{N_e e}{t}$; here $\frac{N_t}{t}$-no. of e^- generated per unit time.
- Light power $P = \frac{E}{t} = \frac{N_p E_p}{t}$; here $\frac{N_p}{t}$- no. of photons incident on depletion region per unit time. E_p- average energy of incident photon.
- $$R = \frac{I_p}{P} = \frac{\frac{N_e e}{t}}{\frac{N_p E_p}{t}} = \frac{N_e}{N_p}\frac{e}{E_p} = \frac{N_e}{N_p}\left(\frac{e\lambda_p}{hc}\right)$$

$$R = \left(\frac{N_e}{N_p}\right)\left(\frac{e\lambda}{hc}\right)$$

$$\boxed{R = \eta \frac{e\lambda}{hc}}$$

Responsivity R is a measure of the conversion efficiency of a photo detector.

Where $\eta = \dfrac{N_e}{N_p}$ **is called quantum efficiency of a photodiode**

The ratio of number of electrons produced (N_e) to the number of photons falling (N_p) shows the efficiency of the semiconductor material to convert light into current.

This ratio is called as <u>quantum efficiency</u> η of a photo diode

A p-n photodiode has a quantum efficiency of 50% at a wavelength of 0.9 μm. Calculate (i) the responsivity at 0.9 μm (ii) the received optical power if the mean photocurrent is 10^{-6} A and (iii) the corresponding number of received photons per second at this wavelength.

A Si pin photodiode has an active light receiving area of diameter 0.4 mm. When radiation of wavelength 700 nm (red light) and intensity 0.1 mW cm^{-2} is incident it generates a photocurrent of 56.6 nA. What is the responsivity and QE of the photodiode at 700 nm?

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Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Module:7 Sources LED & Laser diode

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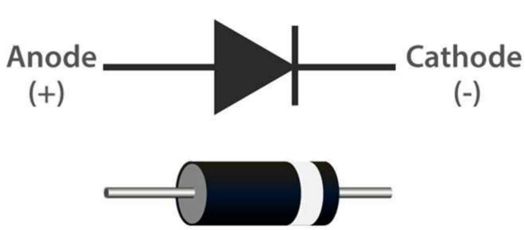
Optical source

- ❑ **LED (Light emitting diode)**
- ❑ **Laser diode**

Diode



- ❑ A semiconductor diode is a p-n junction diode. It is a two-terminal device that conducts current only in one direction.
- ❑ Diodes can be made of either of the two semiconductor materials, silicon and germanium.
- ❑ When the anode voltage is more positive than the cathode voltage, the diode is said to be forward-biased and it conducts readily with a relatively low-voltage drop.
- ❑ Likewise, when the cathode voltage is more positive than the anode, the diode is said to be reverse-biased.
- ❑ The arrow in the diode symbol represents the direction of conventional current flow when the diode conducts.

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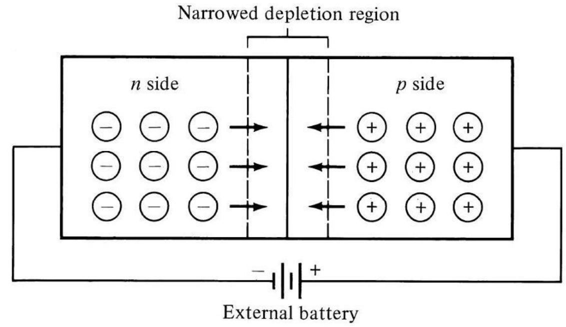
Ordinary PN junction Diode working

Ordinary diode works (produce current) in forward bias condition

Forward bias condition: Positive terminal of the battery connected to p-type semiconductor and negative terminal connected to n-type semiconductor



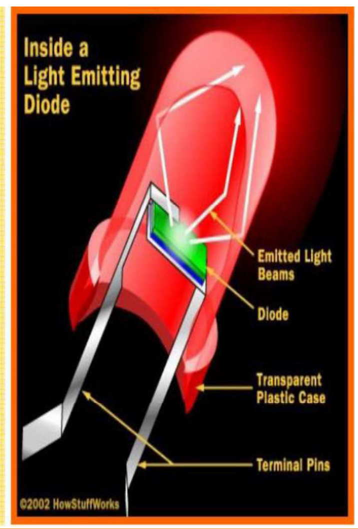
Ordinary diode is non-radiative

Do not produce light up on electron and hole transport

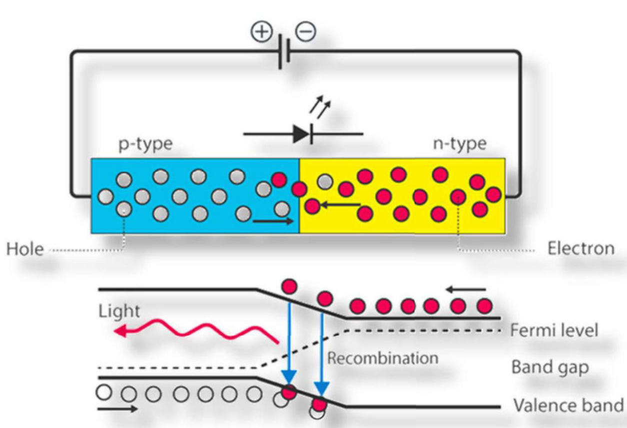
What is LED?

LED are semiconductor p-n junctions that under forward bias conditions can emit radiation by electroluminescence in the UV, visible or infrared regions of the electromagnetic spectrum. The qaunta of light energy released is approximately proportional to the band gap of the semiconductor.

- A light emitting diode (LED) is essentially a PN junction opto-semiconductor chip.
- LED's convert electrical energy into light energy.
- Emits a monochromatic (single color) light when operated in a forward biased direction.
- The quanta of light energy released is approximately proportional to the band gap energy of the semiconductor.



Working Principle of LED



- A typical LED needs a forward biased p-n junction
- When this movement of free electron and hole takes place, there is a change in the energy level as the voltage drops from the conduction band to the valance band. There is a release of energy due to the motion of the electron.
- In standard diodes, the release of energy in the manner of heat. But in LED the release of energy in the form of photons would emit light energy.
- **Electrons and Holes recombine and produce light**
- The entire process is known as **electroluminescence**, and the diodes are known as a light-emitting diode.

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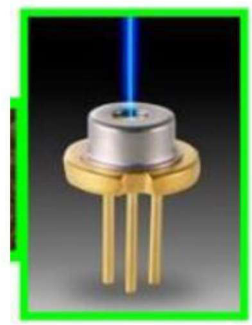
LED

Advantage	Disadvantage
Smaller in size	Power output is low
Low cost	Intensity is less than laser
Long Life	Cannot travel longer distance
Different colours	Incoherent and not in phase
Operation at low voltage	Have no directionality
Very fast response (10^{-9}s)	
Easy Intensity control	
Less scattering	

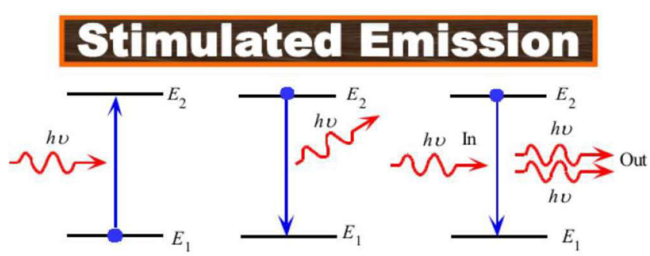
Semiconductor Diode Laser

- **A semiconductor laser is a specially fabricated pn junction device (both the p and n regions are highly doped) which emits coherent light when it is forward biased.**
- **Gallium Arsenide (GaAs) is used as a semiconductor in these lasers. It emits light in near IR region.**



Semiconductor lasers can also be made to emit light in the spectrum from UV to IR using different semiconductor materials e.g. InGaAs, AlGaAs etc.

Laser Principle



(a) Absorption (b) Spontaneous emission (c) Stimulated emission

- ❑ **To design a laser diode, the p-n junction must be <u>heavily doped</u> i.e., p and n materials must be <u>degenerately doped.</u>**
- ❑ **By degenerated doping, the Fermi level of the n-side will lie in the conduction band whereas the fermi level in the p-region will lie in the valence band.**
- ❑ **Light emission should be stimulated.**
- ❑ **Stimulated emission - an electron in a higher energy state interacts with a photon that stimulates it to return to a lower energy state, and a photon is released.**

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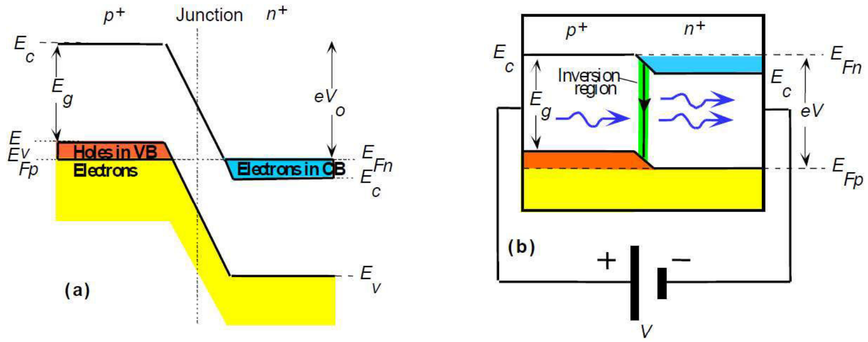
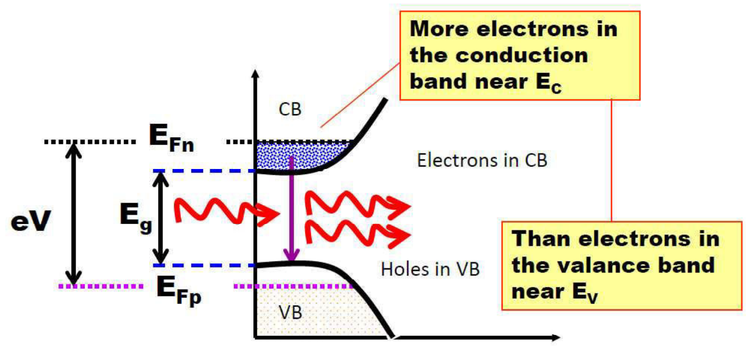
Working of Semiconductor lasers

After positive bias



- When the junction is forward biased, Electron can flow to p-region and holes can flow to n-region
- at low voltage the electron and hole recombine and cause spontaneous emission
- But when the forward voltage reaches a threshold value, the carrier concentration rises to very high value.

Population Inversion in Diode Laser



Population inversion is achieved by direct conversion method of pumping

E_{Fn}-E_{fP} = eV

eV > E_g

eV = forward bias voltage

Fwd Diode current pumping → injection pumping

❑There is therefore a **population inversion** between energies near E_C and near E_V around the junction.

❑This only achieved when degenerately doped p-n junction is forward bias with energy > E_{gap}

Laser diode

Advantage	Disadvantage
When laser diode is compared with other light-emitting devices, the operational power is less in the laser diode	These diodes are expensive when compared to other light-emitting devices.
The handling of these diodes is easy as they are small.	The light generated by these diodes adversely affect the eyes.
The light generated by these diodes is of high efficiency	

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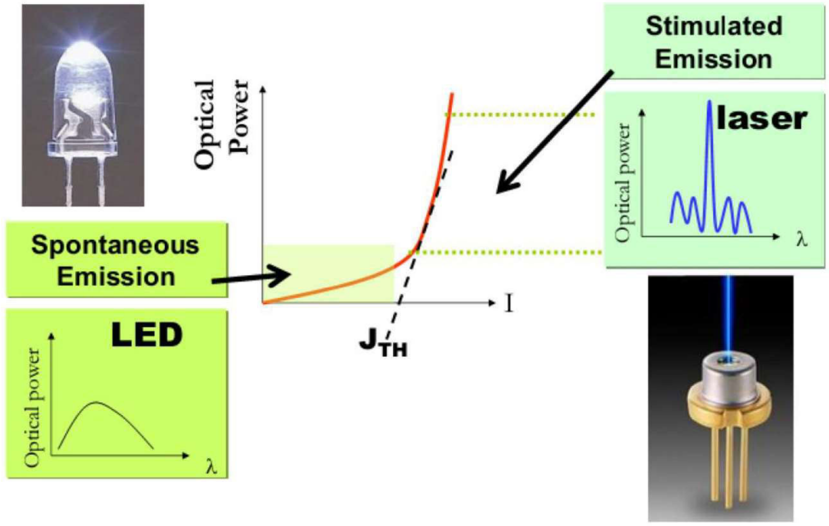
LED vs Laser diode



Comparison of a Semiconductor Diode Laser and LED

Semiconductor Diode Laser	LED
Stimulated radiation	Spontaneous radiation
narrow line width	broad spectral
coherent	incoherent
higher output power	lower output power
a threshold device	no threshold current
strong temperature dependence	weak temperature dependence
higher coupling efficiency to a fiber	lower coupling efficiency