5.5 STANDARD DEVIATION

Standard deviation is the most important and commonly used measure of dispersion. *It measures the absolute dispersion or variability of a distribution. A small standard deviation means a high degree of uniformity in the observations as well as homogeneity of the series.* It is extremely useful in judging the representativeness of the mean.

Definition. ***Standard deviation is the positive square root of the average of squared deviations taken from arithmetic mean.*** It is, generally, denoted by the Greek alphabet σ or by S.D. or s.d. Let x be a random variate which takes on n values, *viz.*, $x_1, x_2, x_3, \ldots\ldots, x_n$, then the standard deviation of these n observations is given by.

Standard Deviation: $$\sigma = \sqrt{\Sigma\frac{(x-\bar{x})^2}{n}}, \text{ where}$$

$$\bar{x} = \frac{\Sigma x}{n} \text{ is the mean of these observations}$$

ALTERNATIVELY

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

Also $$\sigma = \sqrt{\frac{\Sigma(x-x)^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} \quad \text{or} \quad \sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} \quad \left[\because \bar{x} = \frac{\Sigma x}{n}\right]$$

But the formula $$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2},$$

is used when the items are very small. On the other hand the relevance of this method is particularly useful when computers are used where by the values, even when they are of high magnitude, can be used directly for calculating σ.

Standard deviation is also known as **Root Mean Square Deviation.**

Example 16. *Find the standard deviation of 3, 4, 5, 6.*

Solution. Here $n = 4$, $\Sigma x = 3 + 4 + 5 + 6 = 18$.

$$\Sigma x^2 = 3^2 + 4^2 + 5^2 + 6^2 = 9 + 16 + 25 + 36 = 86.$$

$$\therefore \quad \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{86}{4} - \left(\frac{18}{4}\right)^2}$$

$$= \sqrt{21.5 - (4.5)^2} = \sqrt{21.5 - 20.25} = \sqrt{1.25} = \mathbf{1.12} \text{ nearly.}$$

5.5.3 Merits, Demerits and Uses of Standard Deviation

Merits:

1. *It is based on all the observations.*
2. *It is rigidly defined.*
3. *It has a greater mathematical significance and is capable of further mathematical treatments.*
4. *It represents the true measurement of dispersion of a series.*
5. *It is least affected by fluctuation of sampling.*
6. *It is not reliable and dependable measure of dispersion.*
7. *It is extremely useful in correlation etc.*

Demerits:

1. *It is difficult to compute unlike other measures of dispersion.*
2. *It is not simple to understand.*
3. *It gives more weightage to extreme values.*
4. *It consumes much time and labour while computing it.*

Uses:

1. *It is widely used in biological studies.*
2. *It is used in fitting a normal curve to a frequency distribution.*
3. *It is most widely used as a measure of dispersion.*

5.6 CALCULATION OF STANDARD DEVIATION – INDIVIDUAL OBSERVATIONS

When the data under consideration consists of individual observations, the standard deviation may be computed by any of the following two methods:

(*a*) **By taking deviations of the items from the actual mean.**

(*b*) **By taking deviations of the items from an assumed mean.**

5.6.1 When the Deviations are Taken from the Actual Arithmetic Mean.

This method is known as **Direct Method.**

Direct Method

In case of simple series, the standard deviation can be obtained by the formula:

$$\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}} \quad \text{or} \quad \sigma = \sqrt{\frac{\Sigma d^2}{n}}, \text{ where } d = x_i - \bar{x}$$

and x_i = value of the variable or observation, $\bar{x}$ = arithmetic mean, and

n = total number of observations.

WORKING RULE

STEP I Calculate the arithmetic mean $\bar{x}$.

STEP II Take the deviations of the items from the mean, *i.e.*, calculate $d = x_i - \bar{x}$

STEP III Take the sum of the square of all these deviations, *i.e.*, find $\Sigma d^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$

STEP IV Find the mean of the squared deviations obtained in **step III**, *i.e.*, $\frac{\Sigma d^2}{n}$, where n is the total number of observations. It is known as **variance.**

STEP V Take the under root of variance to get the desired standard deviation.

The above method is explained by the following example.

Example 17. *Find the standard deviation of 16, 13, 17, 22.*

Solution. Here A.M. $= \bar{x} = \frac{16+13+17+22}{4} = \frac{68}{4} = \mathbf{17.}$

Let us prepare the following table in order to calculate the standard deviation.

(x)	$d = x - \bar{x} = x - 17$	$d^2 = (x - \bar{x})^2$
16	-1	1
13	-4	16
17	0	0
22	5	25
		$\Sigma d^2 = 42$

Now $$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \frac{\Sigma d^2}{n} = \sqrt{\frac{42}{4}} = \mathbf{3.24.}$$

5.6.2 When the Deviations are Taken from the Assumed Mean.

This method is also known as **Short-cut Method.**

Short-cut Method

This method is applied to calculate standard deviation, when **the mean of the data comes out to be a fraction.** In that case, it is very difficult and tedious to find the deviations of all observations from the **actual mean** by the above method. The formula used is:

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2},$$

where $d = x - A$, A = assumed mean, n = total number of observations.

WORKING RULE

STEP I Take any arbitrary number as the assumed mean **A**.

STEP II Take the deviations from the assumed mean and denote it by d, *i.e.*, $d = x - A$. Take the total of these deviations, *i.e.*, obtain Σd.

STEP III Square these deviations and find Σd^2.

STEP IV Calculate $\frac{\Sigma d}{n}, \left(\frac{\Sigma d}{n}\right)^2$ and $\frac{\Sigma d^2}{n}$, where n is the total number of obser-vations.

STEP V Find $\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2$. Take its square root to get the **standard deviation** of the given data.