14. ' 10. 0.957.

7.10 SPEARMAN'S RANK CORRELATION COEFFICIENT

The **coefficient of rank correlation** is based on the various values of the variates and is denoted by R. *It is applied to the problems in which data cannot be measured quantitatively but qualitative assessment is possible such as beauty, honesty etc.* In this case the best individual is given rank number 1, next rank 2 and so on. The **coefficient of rank correlation** is given by the formula :

$$R = 1 - \frac{6 \Sigma D^2}{n(n^2 - 1)},$$

where D^2 is the square of the difference of corresponding ranks, and n is the number of pairs of observations.

7.10-1. When the Ranks are Given.

WORKING RULE

Step I. Calculate the difference of ranks of x from the corresponding ranks of y and write it under the column headed by D.

Step II. Square the difference D and write it under the column headed by D^2.

Step III. Apply the formula

$$R = 1 - \frac{6 \Sigma D^2}{n(n^2 - 1)},$$

where n is the total number of pairs of observations.

Example 19. *Two judges in a beauty competition rank the 12 entries as fallows.*

X :	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Y :	*12*	*9*	*6*	*10*	*3*	*5*	*4*	*7*	*8*	*2*	*11*	*1*

Calculate the rank correlation coefficient between X and Y.

Example 27. *Quotation of index numbers of equity share prices of certain joint stock company and prices of preference share are given below:*

Years :	*1961*	*1962*	*1963*	*1964*	*1965*	*1966*	*1967*
Equity :	*97.5*	*99.4*	*90.6*	*96.2*	*95.1*	*98.4*	*97.1*
Preference :	*75.1*	*75.9*	*77.1*	*78.2*	*79.0*	*74.8*	*70.1*

Use the method of rank correlation to determine the relationship between equity share and preference share prices.

Solution. Here first we assign the ranks in the descending orders.

Table : Computation of Rank Correlation Coeficient

X	Y	*Rank of X* R_1	*Rank of Y* R_2	$d = R_1 - R_2$	d^2
97.5	75.1	4	5	– 1	1
99.4	75.9	1	4	– 3	9
98.6	77.1	2	3	– 1	1
96.2	78.2	6	2	4	16
95.1	79.0	7	1	6	36
98.4	74.0	3	6	– 3	9
97.1	70.2	5	7	– 2	4
				$\Sigma d = 0$	$\Sigma d^2 = 76$

Here $n = 7$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 76}{7(49-1)} = 1 - \frac{456}{336} = 1 - 1.357 = -\mathbf{0.357}$$

Example 28. *Ten competitors in a beauty contest are ranked by three judges in the following orders:*

1st Judge :	*1*	*6*	*5*	*10*	*3*	*2*	*4*	*9*	*7*	*8*
2nd Judge :	*3*	*5*	*8*	*4*	*7*	*10*	*2*	*1*	*6*	*9*
3rd Judge :	*6*	*4*	*9*	*8*	*1*	*2*	*3*	*10*	*5*	*7*

Use the correlation coefficient to determine which pair of judges has the nearest approach to common taste in beauty.

Solution. Let R_1, R_2, R_3, respectively be the ranks given by first, second and third judge. Let r_{ij} be the rank correlation coefficient between the rank given by ith and jth judges, $i \neq j$, $i = 1, 2, 3, j = 1, 2, 3$. Let $D_{ij} = R_i - R_j$ be the difference of ranks of an individual given by ith and jth judge.

Table : Computation of Rank Correlation Coeficient

R_1	R_2	R_3	$D_{12} = R_1 - R_2$	$D_{13} = R_1 - R_3$	$D_{23} = R_2 - R_3$	D_{12}^2	D_{13}^2	D_{23}^2
1	3	6	– 2	– 5	– 3	4	25	9
6	5	4	1	2	1	1	4	1
5	8	9	– 3	– 4	– 1	9	16	1
10	4	8	6	2	– 4	36	4	16
3	7	1	– 4	2	6	16	4	36
2	10	2	– 8	0	8	64	0	64
4	2	3	2	1	– 1	4	1	1
9	1	10	8	– 1	9	64	1	81
7	6	5	1	2	1	1	4	1
8	9	7	– 1	1	2	1	1	4
			$\Sigma D_{12} = 0$	$\Sigma D_{13} = 0$	$\Sigma D_{23} = 0$	$\Sigma D_{12}^2 = 200$	$\Sigma D_{13}^2 = 60$	$\Sigma D_{23}^2 = 214$

Here $n = 10$

$$\therefore \quad R_{12} = 1 - \frac{6\Sigma D_{12}^2}{n(n^2-1)} = 1 - \frac{6 \times 200}{10 \times 99} = -\frac{7}{33} = -0.2121.$$

$$R_{13} = 1 - \frac{6\Sigma D_{13}^2}{n(n^2-1)} = 1 - \frac{6 \times 60}{10 \times 99} = \frac{7}{11} = 0.6363.$$

$$R_{23} = 1 - \frac{6\Sigma D_{23}^2}{n(n^2-1)} = 1 - \frac{6 \times 214}{10 \times 99} = \frac{-49}{165} = -0.2970.$$

Since R_{13} is maximum, so the pair of first and third judge has the nearest approach to the common taste of beauty.