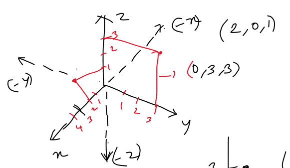
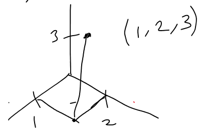
Triple Integrals

Thursday, 25 March 2021 10:02 AM





(1, 1,)

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Triple - Integral

$F(x, y, z)$ - function

$$V = \iint_D f(x, y)\, dA$$

$$V = \iiint_D F(x, y, z)\, dV$$

$$\iiint_D F(x,y,z)\, dz\, dx\, dy = \int_{y_1}^{y_2} \int_{x=g_1(y)}^{x=g_2(y)} \int_{z=h_1(x,y)}^{z=h_2(x,y)} F(x,y,z)\, dz\, dx\, dy$$

$$\iiint_D F(x,y,z)\, dz\, dy\, dx = \int_{x_1}^{x_1} \int_{y=g_1(x)}^{y=g_2(x)} \int_{z=h_1(x,y)}^{z=h_2(x,y)} F(x,y,z)\, dz\, dy\, dx$$

Problem:

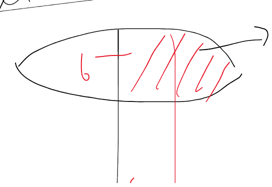
Find the volume of Region D, enclosed by the surfaces $z = x^2 + 3y^2$, $z = 8 - x^2 - y^2$, & $\boxed{F(x,y,z) = 1}$

Sol:

$$V = \iiint_D dz\, dy\, dx$$

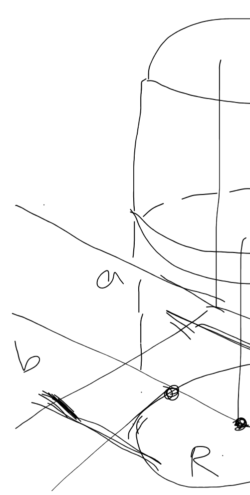
To find limits of z, y, x

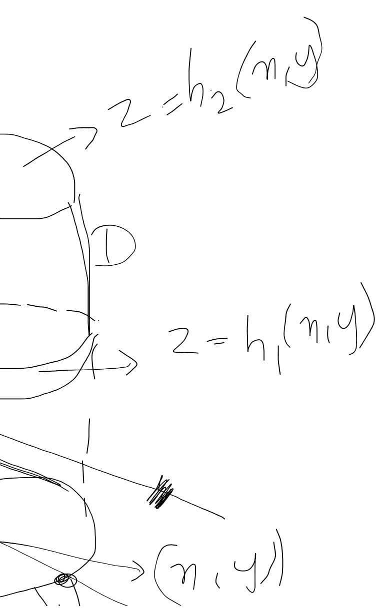
Let us find limits of z

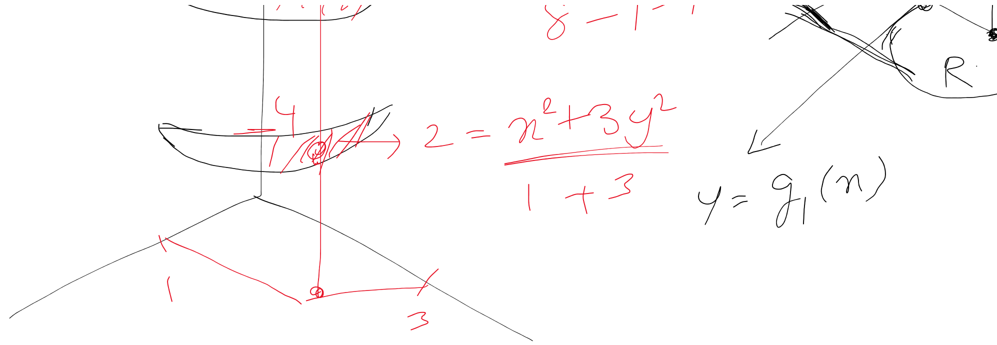


$\rightarrow z = 8 - x^2 - y^2$

$8 - 1 - 1$




z = h2(x, y)
D
z = h1(x, y)
(x, y)



$z = 8 - x^2 - y^2$

$z = x^2 + 3y^2$

$8 - x^2 - y^2$

$8 = x^2 +$

$8 = 2x$

$4 = x^2 +$

$2y^2 = 4$

$y = \pm$

y bound below by =

above by =

x-limits: put $y = 0$

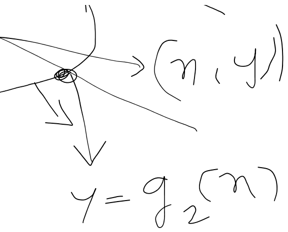
$x^2 = 4 + 0$

$x = \pm 2$

lower limit is $-$

upper limit is $+\sqrt{\frac{4-x^2}{2}}$

$\int^{2} \int^{+\sqrt{\frac{4-x^2}{2}}}$



$= x^2 + 3y^2$

$3y^2 + x^2 + y^2$

${}^2 + 4y^2$

$\boxed{-2y^2}$ ✓

$- x^2$ ✓

$\sqrt{\frac{4-x^2}{2}}$

$- \sqrt{\frac{4-x^2}{2}}$

$+ \sqrt{\frac{4-x^2}{2}}$

2

-2

$\int^{8-x^2-y^2} dz\, dy\, dx$

$$\int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}$$

$$V = \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\quad}}$$

$$= \int_{-2}^{2}$$

$$=$$

$$=$$

$$=$$

$$\int dz\,dy\,dx$$

$$x^2+3y^2$$

$$\frac{4-x^2}{2}$$

$$[z]_{x^2+3y^2}^{8-x^2-y^2}\,dy\,dx$$

$$\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} 8-x^2-y^2-x^2-3y^2\,dy\,dx$$

$$\int_{-2}^{2}\boxed{\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} 8-2x^2-4y^2\,dy}\,dx$$

$$\int_{-2}^{2}\left(8y-2x^2y-\tfrac{4}{3}y^3\right)_{-\sqrt{\frac{4-x^2}{2}}}^{+\sqrt{\frac{4-x^2}{2}}}\,dx$$

$$\int_{-2}^{2} 8\sqrt{\left(\frac{4-x^2}{2}\right)}-2x^2\sqrt{\left(\frac{4-x^2}{2}\right)}-\tfrac{4}{3}\left(\sqrt{\frac{4-x^2}{2}}\right)^3$$

$$\frac{4\sqrt{2}}{}\int^{2}\left(4-x^2\right)^{3/2}\,dx$$

x

=

2

$$\frac{4\sqrt{2}}{3}\int_{-2}^{2}(4-x^2)^{3/2}\,dx \qquad x$$

Cylindrical co-ordinates

$$\iiint f(x,y,z)\,dV = \iiint_G f(r\cos\theta, r\sin\theta, z)\,dz$$

$x = r\cos\theta,\ y = r$

$z = z \quad \theta = ta$

r - radius

Problem

z - limits (bounded above & below by Surfaces)

r - limits $(r = f(\theta))$

θ - limits

Problem

Using Triple integration in Cylindrical - … of region G

$r\,dr\,d\theta$.

$\sin\theta$

$n^{-1}(y/n)$.

y

Using Triple integration [illegible] region G
Co-ordinates to find vol. of region G
bounded by the hemisphere $z = \sqrt{2}$
below by xy-plane $(z=0)$ and
laterally by the cylinder $x^2+y^2=9$

$\sqrt{5-x^2-y^2}$