Modified Booth's Algorithm

- Guarantees that the maximum number of summands that must be added is n/2 for n-bit operands.
- Bit pair recoding technique
- Observe the following:
  - The pair (+1, -1) is equivalent to the pair (0, +1)
  - (+1, 0) is equivalent to (0, +2)
  - (-1,
- Exam

$$\bullet \quad \bullet \quad \bullet \quad x_7 \quad x_6 \quad x_5 \quad x_4 \quad x_3 \quad x_2 \quad x_1 \quad x_0 \quad (x_{-1})$$

$$y_7\, y_6 \qquad y_5\, y_4 \qquad y_3\, y_2 \qquad y_1\, y_0$$

Modified Booth's Algorithm

$$\begin{array}{r}
101101 \\
\times\ \ \text{-}1\ \ 1 \\
\hline
1101101 \\
010011 \\
\hline
\cancel{1}0010011 \\
\hline
\end{array}
\qquad
\begin{array}{r}
101101 \\
\times\ \ \ \ \text{-}1 \\
\hline
010011 \\
\\
\hline
0010011 \\
\hline
\end{array}$$

$$\begin{array}{r}
101101 \\
\times\ \ 1\ \ \text{-}1 \\
\hline
0010011 \\
101101 \\
\hline
1101101 \\
\hline
\end{array}
\qquad
\begin{array}{r}
101101 \\
\times\ \ \ \ 1 \\
\hline
101101 \\
\\
\hline
1101101 \\
\hline
\end{array}$$

Modified Booth's Algorithm

$$\begin{array}{r}
1\,0\,1\,1\,0\,1 \\
\times \quad 1 \quad 0 \\
\hline
0\,0\,0\,0\,0\,0\,0 \\
1\,0\,1\,1\,0\,1 \\
\hline
1\,0\,1\,1\,0\,1\,0 \\
\hline
\end{array}
\qquad
\begin{array}{r}
1\,0\,1\,1\,0\,1 \\
\times \qquad +2 \\
\hline
1\,0\,1\,1\,0\,1\,0 \\
\\
\hline
1\,0\,1\,1\,0\,1\,0 \\
\hline
\end{array}$$

$$\begin{array}{r}
1\,0\,1\,1\,0\,1 \\
\times \; -1 \quad 0 \\
\hline
0\,0\,0\,0\,0\,0\,0 \\
0\,1\,0\,0\,1\,1 \\
\hline
0\,1\,0\,0\,1\,1\,0 \\
\hline
\end{array}
\qquad
\begin{array}{r}
1\,0\,1\,1\,0\,1 \\
\times \qquad -2 \\
\hline
0\,1\,0\,0\,1\,1\,0 \\
\\
\hline
0\,1\,0\,0\,1\,1\,0 \\
\hline
\end{array}$$

Table of Multiplicand and Selection decisions

Multiplier Bit-Pair		Multiplier bit on the right i – 1	Booths Represenation		Multiplicand selected at position i
i + 1	i				
0	0	0	0	0	0 x M
0	0	1	0	1	+ 1 x M
0	1	0	1	-1	+1 x M
0	1	1	1	0	+ 2 x M
1	0	0	-1	0	- 2 x M
1	0	1	-1	1	- 1 x M
1	1	0	0	-1	-1 x M
1	1	1	0	0	0 x M

Select Line (encoding)	Partial Products (operation)
000	add 0
001	add multiplicand
010	add mulitplicand
011	add 2*multiplicand
100	subtract 2*multiplicand
101	subtract multiplicand
110	subtract multiplicand
111	subtract 0

Multiplication requiring only n/2 summands

	0	1	1	0	1	(+13)
×	1	1	0	1	0	(−6)

					0	1	1	0	1	
					0	−1	+1	−1	0	
0	0	0	0	0	0	0	0	0	0	
1	1	1	1	1	0	0	1	1		
0	0	0	0	1	1	0	1			
1	1	1	0	0	1	1				
0	0	0	0	0	0					
1	1	1	0	1	1	0	0	1	0	(−78)

					0	1	1	0	1
					0		−1		−2
1	1	1	1	1	0	0	1	1	0
1	1	1	1	0	0	1	1		
0	0	0	0	0	0				
1	1	1	0	1	1	0	0	1	0

Modified Booth's Multiplication - Example

Example: -9 × -13 = 117

M = 110111, $\overline{M} + 1$ = 001001

Comment	A	Q	Q_{-1}	SC
	000000	110011	0	3
Subtract M	001001			
	001001			
Ashr	000100	111001	1	
Ashr	000010	011100	1	2
Add M	110111			
	111001			
Ashr	111100	101110	0	
Ashr	111110	010111	0	1
Subtract M	001001			
	000111			
Ashr	000011	101011	1	
Ashr	000001	110101	1	0

Modified Booth's Multiplication - Example

Example: 13x9 = 117

M =001101, $\overline{M}$ + 1 = 110011

Comment	A	Q	Q_{-1}	SC
	000000	001001	0	3
Add M	001101			
	001101			
Ashr	000110	100100	1	
Ashr	000011	**01**0010	0	2
Add shifted 2's	100110			
	101001			
Ashr	110100	101001	0	
Ashr	111010	010100	1	1
Add M	001101			
	000111			
Ashr	000011	101010	0	
Ashr	000001	110101	0	0