ANOVA table

S.V.	S.S.	d.f.	M.S.	F_0
Between rows (workers)	$Q_1 = 161.5$	$h - 1 = 4$	40.375	$\dfrac{40.375}{6.142} = 6.57$
Between Cols. (machines)	$Q_2 = 338.8$	$k - 1 = 3$	112.933	$\dfrac{112.933}{6.142} = 18.39$
Residual	$Q_3 = 73.7$	$(h - 1)(k - 1) = 12$	6.142	–
Total	$Q = 574$	$hk - 1 = 19$	–	–

From the F-tables, $F_{5\%}(v_1 = 4, v_2 = 12) = 3.26$

and $F_{5\%}(v_1 = 3, v_2 = 12) = 3.49$

With respect to the rows, $F_0(= 6.57) > F_{5\%}(= 3.26)$

With respect to the columns, $F_0(= 18.39) > F_{5\%}(= 3.49)$

    Hence the 5 workers differ significantly and the 4 machine types also differ significantly with respect to mean productivity.

Example 8

Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover. The results are as follows (recovery time in days)

Doctor	Treatment 1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Discuss the difference between (a) doctors and (b) treatments.

We subtracted 15 from the given values and work out with the new values of x_{ij}.

Doctor	Treatment 1	2	3	4	T_i	$\dfrac{T_i^2}{k}$	$\sum_j x_{ij}^2$
A	−5	−1	4	5	3	2.25	67
B	−4	0	2	6	4	4.00	56
C	−6	−3	1	4	−4	4.00	62
D	−7	−2	2	5	−2	1.00	82
T_j	−22	−6	9	20	$T = 1$	$\sum \dfrac{T_j^2}{k} = 11.25$	267
T_j^2 / h	121	9	20.25	100	$\sum T_j^2 / h = 250.25$		
$\sum_i x_{ij}^2$	126	14	25	102	267		

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 267 - \frac{1}{16} = 266.94$$

$$Q_1 = \sum \frac{T_i^2}{k} - \frac{T^2}{N} = 11.25 - 0.0625 = 11.19$$

$$Q_2 = \sum \frac{T_j^2}{h} - \frac{T^2}{N} = 250.25 - 0.0625 - 250.19$$

$$Q_3 = Q - Q_1 - Q_2 = 266.94 - 261.38 = 5.56$$

ANOVA table

S.V.	S.S.	d.f.	M.S.	F_0
Between rows (doctors)	$Q_1 = 11.19$	$h - 1 = 3$	3.73	$\frac{3.73}{0.62} = 6.02$
Between cols. (treatments)	$Q_2 = 250.19$	$k - 1 = 3$	83.40	$\frac{83.40}{0.62} = 134.52$
Residual	$Q_3 = 5.56$	$(h - 1)(k - 1) = 9$	0.62	–
Total	$Q = 266.94$	$hk - 1 = 15$	–	–

From the F-tables, $F_{5\%}$ ($v_1 = 3$, $v_2 = 9$) $= 3.86$

Since $F_0 > F_{5\%}$ with respect to rows and columns, the difference between the doctors is significant and that between the treatments is highly significant.

34. In order to compare three burners B_1, B_2 and B_3, one observation is made on each burner on each of four successive days. The data are tabulated below:

	B_1	B_2	B_3
Day 1	21	23	24
Day 2	18	17	23
Day 3	18	21	20
Day 4	17	20	22

Perform an analysis of variance on these data and find whether the difference between (i) the days and (ii) the burners significant at 5% LOS.

35. A company appoints 4 salesman *A*, *B*, *C* and *D* and observes their sales in 3 seasons summer, winter and monsoon. The figures (in lakhs of Rs) are given in the following table :

	Salesmen			
Season	*A*	*B*	*C*	*D*
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

Carry out an analysis of variance.

36. The following data represent the numbers of units of production per day turned out by 4 different workers using 5 different types of machines:

	Machine type				
Worker	*A*	*B*	*C*	*D*	*E*
1	4	5	3	7	6
2	6	8	6	5	4
3	7	6.	7	8	8
4	3	5	4	8	2

On the basis of this information, can it be concluded that (i) the mean productivity is the same for different machines (ii) the workers do not differ with regard to productivity?

37. The number of automobiles arriving at 4 tall gates were recorded for a 2 hours time period (10 A.M. to 12 noon) for each of six working days. The data are as follows :

Day	Gate 1	Gate 2	Gate 3	Gate 4
Mon	200	228	212	301
Tues	208	230	215	305
Wed	225	240	228	288
Thur	223	242	224	212
Fri	228	210	235	215
Sat	220	208	245	200