4. $$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{\frac{420}{800} - \frac{1}{2}}{\sqrt{\frac{\frac{1}{2} \times \frac{1}{2}}{800}}} = 1.414$$

5. Cal. value of $Z = 1.414 <$ Tab. value of $Z_\alpha = 1.645$

Accept H_0

Note:-

95% Confidence limits for P are given by

$$\left(p - 1.96 \times \sqrt{\frac{pq}{n}},\ p + 1.96 \times \sqrt{\frac{pq}{n}}\right)$$

prob.

Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Test the manufacturer's claim at 5% LOS. Based on the particular day's production, find also the 95% Confidence limits for the percentage of top quality product.

Soln.:-

H_0: 20% of the product manufactured is of top quality.

1. H_0: $P = \frac{20}{100} = \frac{1}{5}$, p = proportion of top quality products in the sample $= \frac{50}{400} = \frac{1}{8}$

2. H_1: $P \neq \frac{1}{5}$

3. Let LOS be 5%,

$Z_\alpha = 1.96$

Scanned by CamScanner

4. $Z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}} = \dfrac{1/8 - 1/5}{\sqrt{\dfrac{1/5 \times 4/5}{400}}} = -3.75$.

5. Cal. value of $|Z| = 3.75 >$ Tab. value of $Z_\alpha = 1.96$.

Reject H_0.

95% Confidence limits.

$$p - \sqrt{\frac{pq}{n}} \times 1.96 \le P \le p + \sqrt{\frac{pq}{n}} \times 1.96$$

$$0.125 - \sqrt{\frac{\frac{1}{8} \times \frac{7}{8}}{400}} \times 1.96 \le P \le 0.125 + \sqrt{\frac{1/8 \times \frac{7}{8}}{400}} \times 1.96$$

$$0.093 \le P \le 0.157$$

95% confidence limits for the percentage of top quality product are 9.3 and 15.7

Test of significance of the difference between two sample proportions.

Let p_1 and p_2 be the proportions of successes in two large samples of size n_1 and n_2 respectively drawn from the same population or from two population with the same proportion p.

Test statistics $Z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Scanned by CamScanner

If P is not known, an unbiased estimate of P based on the both samples, given by

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, \quad Q = 1 - P.$$

Prob-1 :

In a large city A, 20% of a random sample of 900 school boys had a slight physical defect and in another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

Soln: Given that $p_1 = 0.2$, $p_2 = 0.185$, $n_1 = 900$, $n_2 = 1600$

1. $H_0 : P_1 = P_2$
2. $H_1 : P_1 \neq P_2$ (Two tailed test is used)
3. Let the LOS be 5%, $Z_\alpha = 1.96$
4. Test statistics $Z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Here $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \dfrac{180 + 296}{900 + 1600} = 0.1904$

$$Z = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096 \left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.92$$

5. Cal. value of $Z = 0.92 < Z_\alpha = 1.96$

Accept H_0.

Scanned by CamScanner

prob.-2

Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in excise duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in excise duty.

Soln:- Let p_1 and p_2 be the proportions of the consumers before and after the increase in excise duty respectively.

$$p_1 = \frac{800}{1000} = \frac{4}{5} \quad \text{and} \quad p_2 = \frac{800}{1200} = \frac{2}{3}$$

$n_1 = 1000, \quad n_2 = 1200$

1. $H_0 : p_1 = p_2$
2. $H_1 : p_1 > p_2$ (one tailed test is used)
3. Let LOS be 1%, $Z_\alpha = 2.33$
4. $$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad \text{where } P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{800 + 800}{2200}$$

$$= 0.7273$$

$$Q = 1 - P = 0.2727$$

$$= \frac{0.8 - 0.67}{\sqrt{0.7273 \times 0.2727\left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$Z = 6.82$$

5. Cal. value of $z = 6.82 > Z_\alpha = 2.33$

Reject H_0

Scanned by CamScanner