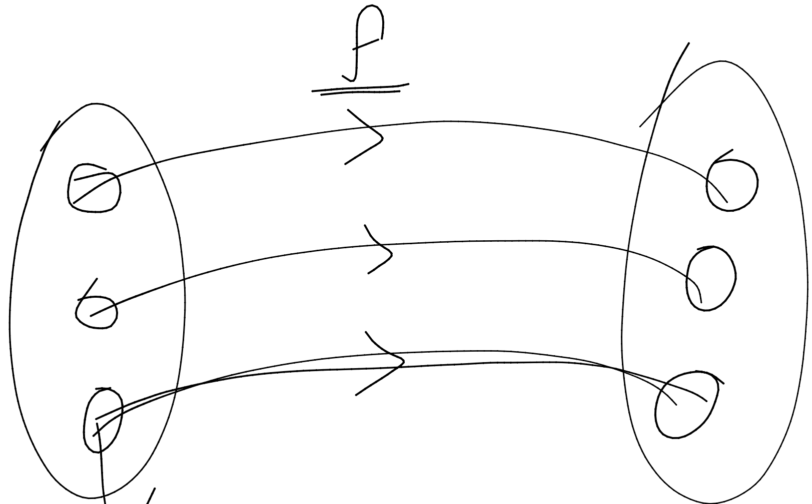
Functions →

Tuesday, 29 December 2020 8:58 AM

(1) Relation - Rule



Bowls Balls

Domain Co-domain(Ra

Inputs Outputs

$\boxed{f(x) = x}$ $f : R \longrightarrow$

$y(= f(x)) = x$

(domain)

(Input)

$R \rightarrow$ Set of Real N

ange)

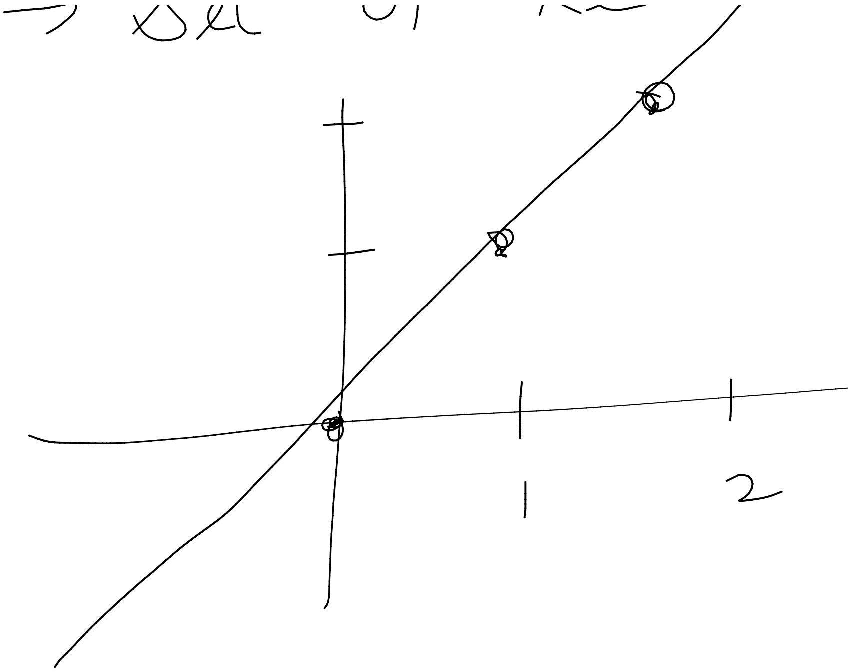
$\longrightarrow \mathbb{R}$

(Co-domain)

(output)

os. $(-\infty, \infty)$

$\mathbb{R} \to$ set of ...



$\mathbb{R}^+$, $\mathbb{Q}$, $\mathbb{Z}$

$y (= f(x)) = x$

x	0	1
$f(x)$	0	1

$\{(0,0), (1,1), (2,2), (3, \ldots$

One-One

$f(x) = x$ | $f(x) = x$

3

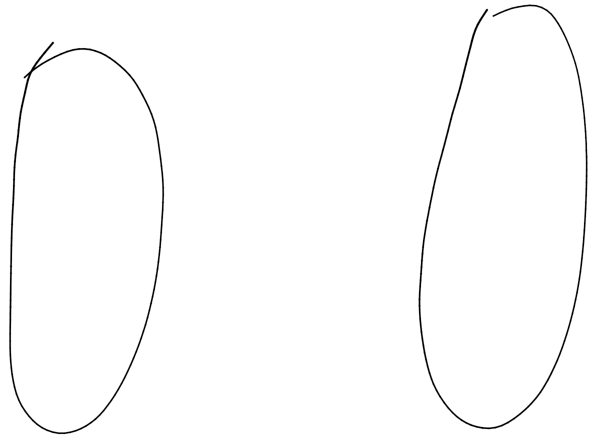
2	3
2	3

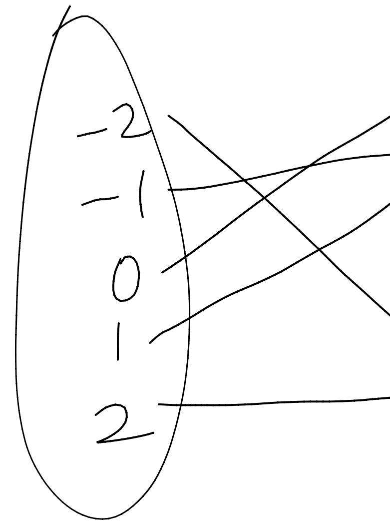
$\boxed{3)}$

$2^2 \; [R^+] \to \quad f: \mathbb{R} \to \mathbb{R}$

$f(x) = x$

1





One-one

O

Pre-image

O

not o

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nto

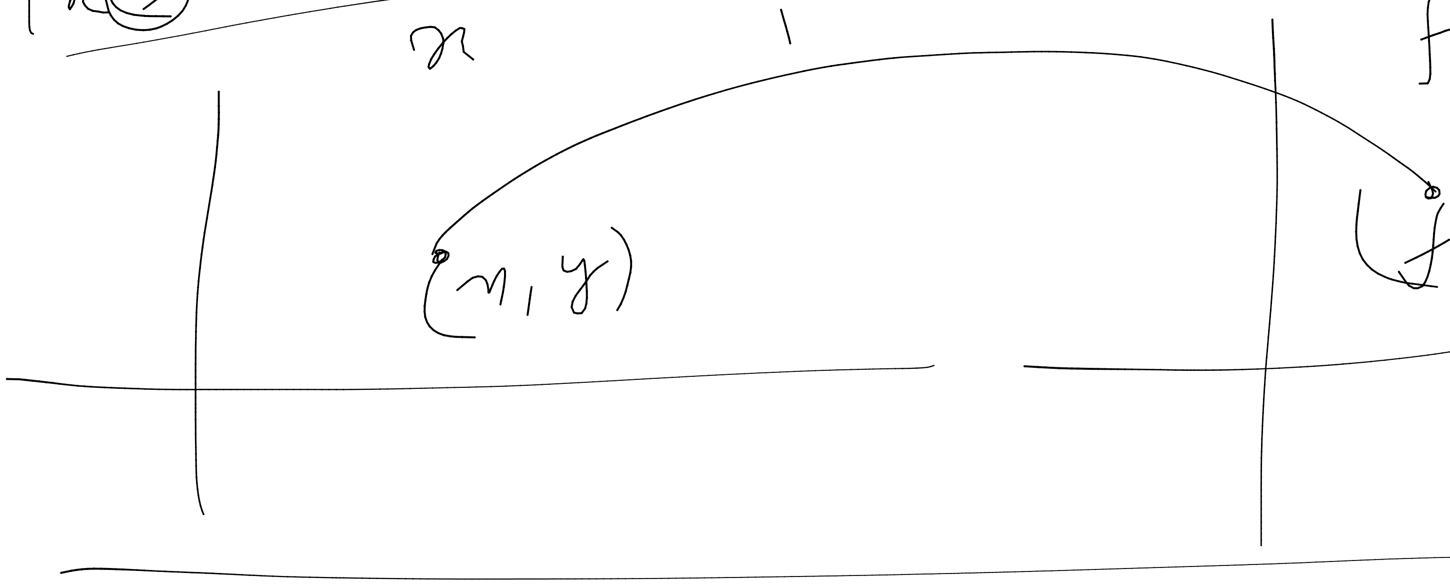
ne-One

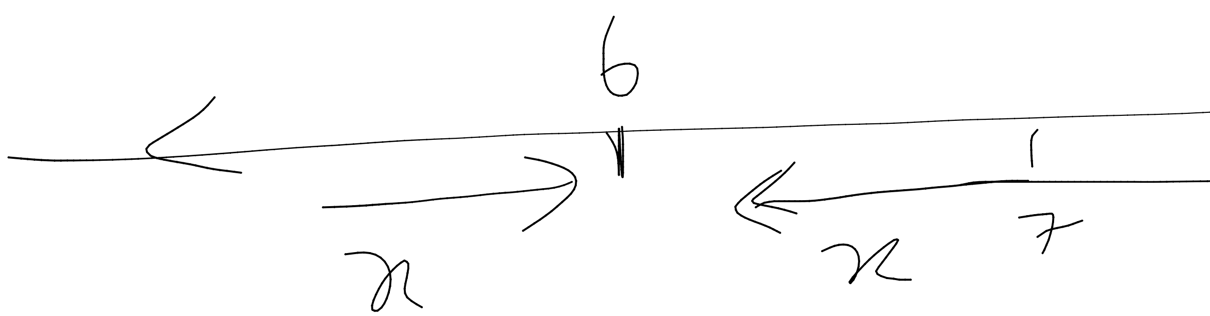
Limits and continuity

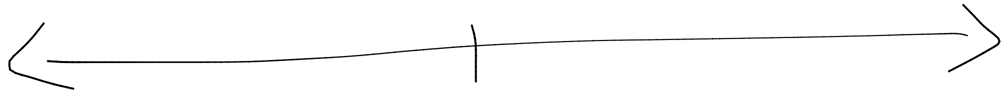
Thursday, 31 December 2020 10:02 AM

$$\boxed{f(x) = x}$$

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$







$(-x)$

$(x), f(c)$

8

$f(x)$

$f(5) =$

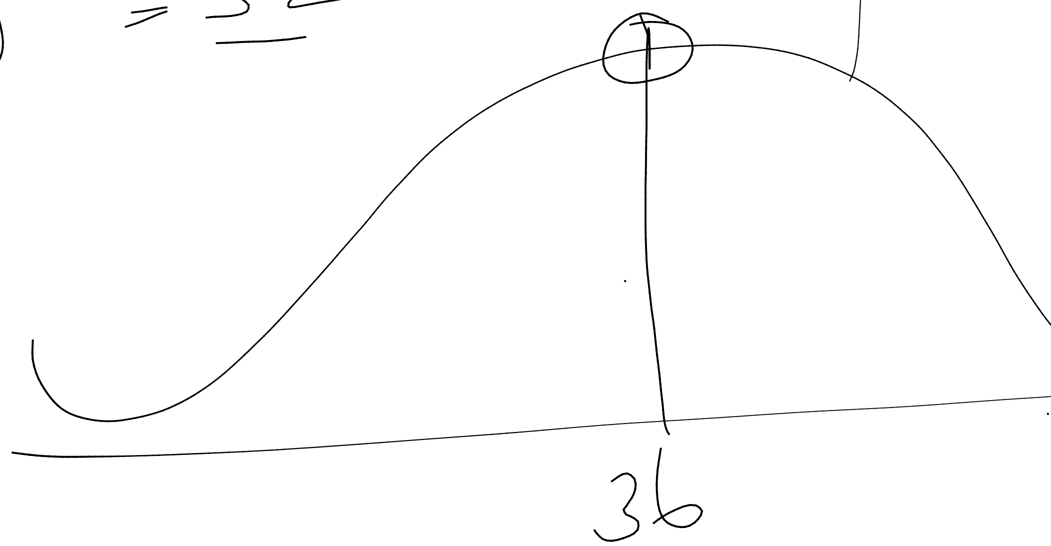
$f(6) = 36$

$f(7) = \_\_$

$f(8) = 32$

$f(w)12$

$f(5)=12$

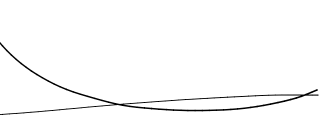


$$\lim_{x \to 6} A = \lim_{w \to 6} (12w -$$

$= 36$

$-w - w^2$

$(5) - 25$



$- w^2)$

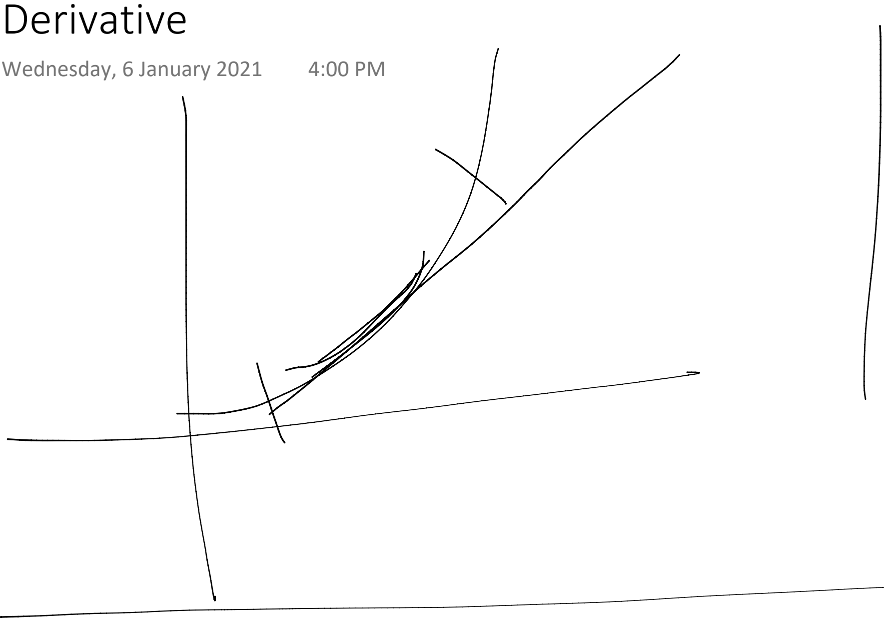
$$\lim_{x \to 5} A = 35$$

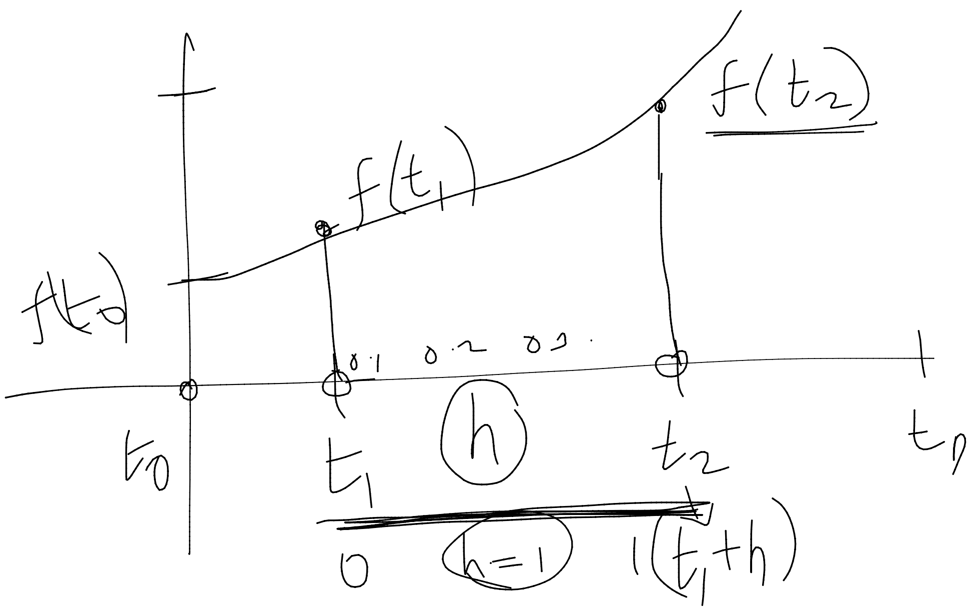
$4.5 \quad 4.6 \rightarrow 5 \leftarrow 5.75 \quad 5.9 \quad 6$

25

Derivative

Wednesday, 6 January 2021 4:00 PM





Derivative $y = f(x) = x^2 - 3$

To Find $f'(x)$

I Know

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim \frac{(x+h)^2 - 3 - (x^2 - 3)}{1}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3 - (x^2 - 3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-3} - \cancel{x^2} \cancel{+3}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} 2x + h \to 0$$

$$\boxed{f'(x) = 2x}$$

FODT

Wednesday, 20 January 2021 8:05 AM

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

$(-\infty, \infty)$ – Increasing (or) decreasing

<u>Step 1</u> :– $f'(x) = 12x^3 - 12x^2 - 24x$

$$= 12x(x^2 - x - 2)$$

$$= 12x(x^2 - 2x + x - 2)$$

$$= 12x(x^2 + x - 2x - 2)$$

$$= 12x(x(x+1) - 2(x+1))$$

$$\boxed{f'(x) = 12x(x-2)(x+1)}$$

<u>Step 2</u> : $f'(x) = 0$ [Assume]

$$12x(x-2)(x+1) = 0$$

$$12x = 0 \quad (\text{or}) \quad (x-2) = 0$$

$$(5x)(x+1) = 0$$

$12x = 0$ (or) $(x-2) = 0$

$x = 0$ (or) $x = 2$ (or

$f'(-1) = 12(-1)(-1-2$

$\boxed{f'(-1) = 0}$ verified

Step 3: critical points; 0

Step 4:

Interval	$12x$	$x+1$
$\boxed{x < -1}$	$-$	$-$
	$-$	$+$

$x = -1$

$2)(-1+1)$

$0, -1, 2$

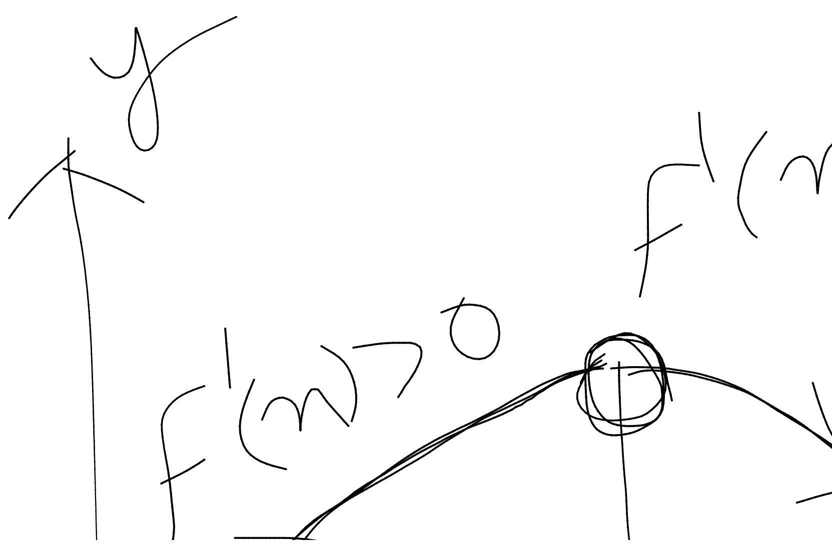
$12x(x+1)(x-2)$

$x-2$	Sign $f'(x)$	Remarks
$-$	$-$	$f'(x) < 0$ decreasing on $(-\infty$
$-$	$+$	$f'(x) > 0$

$\boxed{(0, -1)}$

$(1, 0)$

$-1 < x < 0$	$-$	$+$
$0 < x < 2$	$+$	$+$
$x > 2$	$+$	$+$



- + | $f'(x) > 0$

Increasing on $(-$

- - | $f'(x) < 0$

decreasing on $($

+ + | $f'(x) > 0$

Increasing on $($

$x) = 0$

$f'(x) < 0$

$f'(x) < 0$

$1, 0)$

$0, 2)$

$2, \infty)$

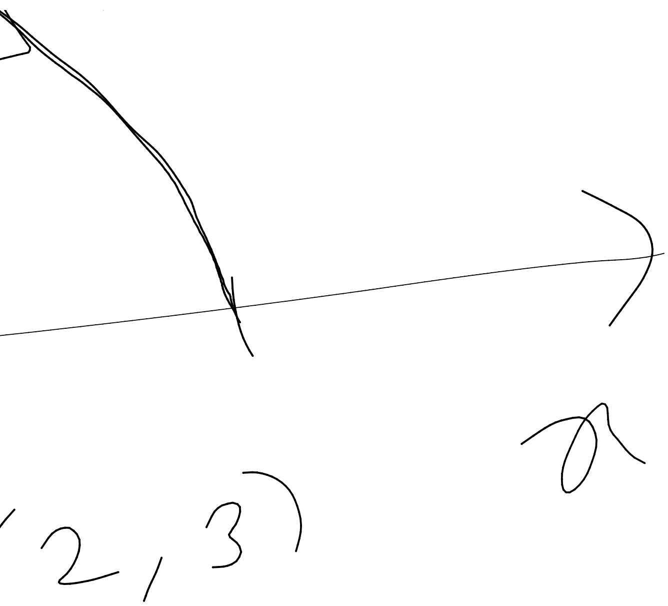
$\nearrow \; f'(x) > 0$

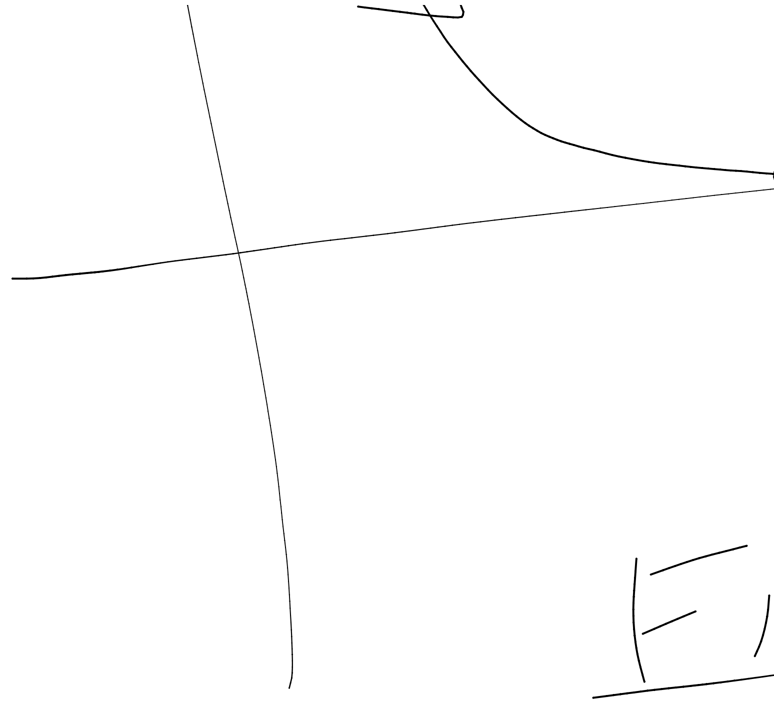
(1, 2)

Fig

From

From

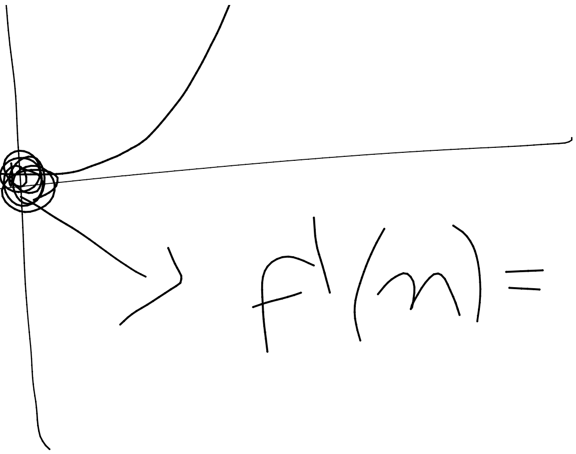




Fi

Fig 1 Sign of f

Fig 2 Sign of f



$f'(x) = 0$

g 2

(x) changes from +

local max

(x) from − to

local minimu

to —

nimum.

um.

Laplace Transforms

Monday, 1 February 2021 12:00 PM

Linearity :—

$$\boxed{\begin{array}{l} f(x) = \sin x^{①} \text{ — } Ⓐ \\ f(x) = 2x^{①} \rightarrow Ⓑ \end{array}}$$

$$f(x+y) = f(x) + f(y)$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$a+b$

$$f(x) = \sin x$$

$$f(y) = \sin y$$

$$\sin(x+y) \neq \sin x + \sin y$$

$$f(x) = 2x$$

$$f(y) = 2y$$

$$f(x+y) = 2(x+y)$$

$$= 2x + 2y$$

$$\boxed{f(x+y) = f(x) + f(y)}$$

$f(y) = 2y$ | $\boxed{f(x+y) = f(x)+f(y)}$

I can write

$2x+2y = 2(x+y)$

$\boxed{f(x)+f(y) = f(x+y)}$

$\Rightarrow$ Try: $\underline{x^2} \Rightarrow f(x+y) = (x+y)^2$

$f(x+y) \neq x^2+y^2+2xy$

$= f(x)+f(y)+$

Laplace Transform

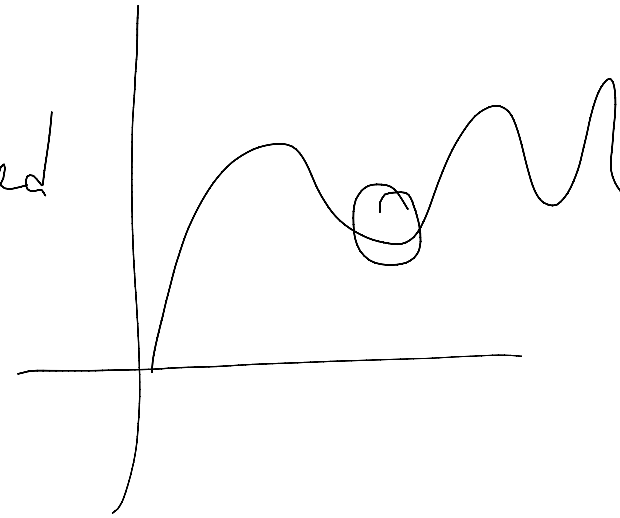
$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) \cdot dt$$

Basic Laplace Transforms

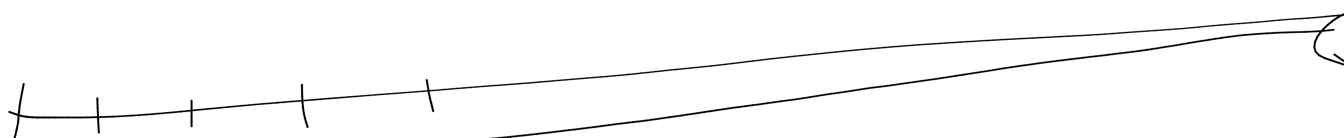
Monday, 8 February 2021 12:22 PM

$f(x) = \frac{1}{x}$ defined

on $[-\infty, \infty]$



Domain -



$-\infty$ 0.0008 1 2

→ I can take any values for x (or) my inputs

-1 -0.5 0 0.5 1

$[-1, 1]$

$(-1, 1)$

$f : \mathbb{R} \to \mathbb{R}$

$[-\infty, \infty] \to [1, 3]$

$$\mathcal{L}[f(t) + g(t)] = \mathcal{L}[f(t)] + \mathcal{L}[g(t)]$$

$\int^{\infty}$ $\quad$ ||

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) \cdot dt$$

$$\mathcal{L}[1] = \int_0^\infty e^{-st} \cdot 1 \, dt$$

$$= \int_0^\infty e^{-st} \cdot dt$$

By Calculus: $\int e^{1t} \cdot dt = \frac{e^{1t}}{1}$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

In general $\boxed{\int_a^b f(t) \, dt = f(b) - f(a)}$

$$= \left[\frac{e^{-s\times\infty}}{-s}\right] - \left[\frac{e^{-s\times 0}}{-s}\right]$$

$$= \left[-\frac{1}{se^\infty} + \frac{e^0}{s}\right]$$

$$= \left[-\frac{1}{se^{\infty}} + \frac{1}{s} \right]$$

$$= 0 + \frac{1}{s} \qquad \because e^{0} = 1$$

$$\boxed{\mathcal{L}[1] = \frac{1}{s}}$$

Try: (i) e^{at}; (ii) $\sin at$ (iii) $\cos a$

Hint(s): Given: $f(t) = \sin(at)$

Formula: $\mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} \cdot f(t) \cdot d$

$$\mathcal{L}[\sin at] = \int_0^{\infty} e^{-st} \sin a$$

$$\int u \cdot dv [$$

Refer Integration by parts i

3

t.

lt

dt

method]

n school books

Problems on Laplace Transform

Tuesday, 9 February 2021 9:10 AM

$f(t) = e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t$ ✓

$\mathcal{L}[f(t)]$

$\mathcal{L}[\underbrace{e^{2t}}_{f_1} + \underbrace{4t^3}_{f_2} - \underbrace{2\sin 3t}_{f_3} + \underbrace{3\cos 3t}_{f_4}]$

By Linearity $\left[\begin{array}{l} f(x+y) = f(x) + f(y) \\ \mathcal{L}(f(t) + g(t)) = \mathcal{L}[f(t)] + \mathcal{L}[g(t \end{array}\right.$

$\mathcal{L}[f_1(t) + f_2(t) + f_3(t) + f_4(t)] = \mathcal{L}[f_1(t)] + \mathcal{L}[f$

$= \mathcal{L}[e^{2t}] + \mathcal{L}[4t^3]$

$\mathcal{L}[e^{at}] = \frac{1}{s-a}$

Property: $\mathcal{L}$

$$\boxed{\ldots_2(t)] + \mathcal{L}[f_3(t)] + \mathcal{L}[f_4(t)]}$$

$$+\mathcal{L}[-2\sin 3t] + \mathcal{L}[3\cos 3t]$$

$$\mathcal{L}[e^{at} \cdot f(t)] = \boxed{F(s-a)}$$

$$\mathcal{L}[f(t)] = F(s)$$

$$\mathcal{L}[e^{at} f(t)] = F(s)|$$

En: e^{bt}

e^{bt}

Step 1:

Step 2:

$$\mathcal{L}[e^{at} f(t)] = F(s)\Big|_{s \to s-a}$$

$\sin t$

$f(t)$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2}$$

$$\mathcal{L}[\sin t] = \frac{1}{s^2+1}$$

$a=1$

$$= F(s)$$

$$\mathcal{L}[e^{bt} \sin t] = F(s)\Big|_{s \to s-b}$$

$$= \frac{1}{(s-b)^2+1}$$

$\mathcal{L}[3\cos 3t]$

From given:

$$= \mathcal{L}\left[e^{2t}\right] + \mathcal{L}\left[4t^3\right] \ldots$$

$$= \frac{1}{s-2} + \underbrace{4\mathcal{L}\left[t^3\right]}_{f_2} - 2 \ldots$$

Take f_2

$$4\mathcal{L}\left[t^3\right] = \frac{3!}{s^{3+1}}$$

$n = 3$

$$= \frac{3 \times 2 \times 1}{s^4}$$

$$\mathcal{L}\left(t^3\right) = \frac{6}{s^4}$$

$$+\mathcal{L}[-2\sin 3t]+\mathcal{L}[3\cos 3t]$$

$$\underbrace{\mathcal{L}[\sin 3t]}_{f_3}+\underbrace{3\mathcal{L}[\cos 3t]}_{f_4}$$

$$\mathcal{L}[t^n]=\frac{n!}{s^{n+1}}$$

$$4\mathcal{L}(t^3) = \frac{6\times 4}{s^4}$$

$$\boxed{4\mathcal{L}(t^3) = \frac{24}{s^4}}$$

Take f_3

$$\mathcal{L}(\sin 3t) = \frac{3}{s^2+9}$$

$a=3$

$$-2\mathcal{L}(\sin 3t) = -\frac{2\times 3}{s^2+9}$$

$$= -\frac{6}{s^2+9}$$

Take f_4

$$\mathcal{L}(\quad \cos t) = 3\left[\frac{s}{s^2+1}\right.$$

$$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$$

$$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$$

$$3\mathcal{L}(\cos 3t) = 3\left[\frac{\quad}{s^2+\quad}\right.$$

$$\boxed{3\mathcal{L}(\cos 3t) = \frac{3s}{s^2+9}}$$

$$\mathcal{L}(t \;\ldots) = \frac{1}{s-2} + \frac{\quad}{\quad}$$

$$(e^{2t})^2 \Rightarrow (e^{2t}\ldots$$

$$e^{a}$$

$$= e^{2t+\ldots}$$

$$(e^{2t})^2 = e^{4t}$$

$9]$ $\mathcal{L}(\cos at)$ $s^2 + a^2$

$\frac{24}{s^4} - \frac{6}{s^2+9} + \frac{3s}{s^2+9}$

$)\cdot(e^{2t})$

$\cdot e^{b} = e^{a+b}$

$2t$

$\mathcal{L}[e^{4t} = \frac{1}{s-4}]$

$$e^{2i} = e$$

$$\sin(a+b) = \underline{\sin}$$

$$\sin(a-b) = \underline{\sin}$$

$$\sin(a+b) + \sin$$

$$\sin a \cos$$

$$\sin at \cdot \cos bt$$

$a \cos b + \cos a \sin b$

$a \cos b - \cos a \sin b$

$(a-b) = 2 \sin a \cos b$

$b = \frac{1}{2} \left[\sin(a+b) + \sin(a-b) \right]$

$= \frac{1}{2} \left[\sin(at+bt) + \sin(at-bt) \right]$

$= \frac{1}{2} \left[\sin(a+b)t + \sin(a-b)t \right]$

$\frac{1}{2} \left[\mathcal{L} \left[\sin(a+b)t \right] + \mathcal{L} \left[\sin(a-b)t \right] \right.$

$a-b$

Property:

$\boxed{t}$

$\mathcal{L}\left[t^n\right.$

$\mathcal{L}\left(t^{2}\sin\right.$

$$\boxed{\frac{1}{2}\left(\frac{a+b}{s^2+(a+b)^2}\right)} + \frac{1}{2}\left[\frac{a-b}{s^2+(a-b)^2}\right]$$

$f(t)$

$f(t)$

$$[\,f(t)\,] = (-1)^n \cdot \frac{d^n F(s)}{ds^n}$$

$$= (-1)\, F^{(n)}(s)$$

$at) =$

Step 1: $\mathcal{L}(s$

r

Formula: $\mathcal{L}[$

$F'(s) = \frac{d}{ds}($

$$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2} = F(s)$$

$$n = 2 \quad \mathcal{L}[t^2]$$

$$\mathcal{L}[t^2 f(t)] = (-1)^2 F^{(2)}(s)$$

$$= (-1)^2 F''(s)$$

$$= \frac{d^2}{ds^2}(F(s))$$

$$= \frac{d}{ds}\left[F'(s)\right]$$

$$\left[\frac{a}{s^2+a^2}\right]$$

$$1 \cdot (s^2+a^2)$$

$F'(s) = \frac{d}{ds}$

$= \frac{(s}{}$

$=$

$\boxed{F'(s) = }$

$\mathcal{L}[t \cdot \sin at$

$$s^2+a^2)$$

$$\frac{(s^2+a^2)\, d/ds(a) - a \cdot d/ds(s^2+a^2)}{(s^2+a^2)^2}$$

$$\frac{0 - a \cdot 2s}{(s^2+a^2)^2}$$

$$\boxed{-\frac{2as}{(s^2+a^2)^2}}$$

$$] \Rightarrow (-1)^1 \cdot F'(s)$$

$$= (-1)^1 \cdot -\frac{2as}{(s^2+a^2)^2}$$

$$= \frac{2as}{\quad} \;//$$

$$\mathcal{L}\left[t \cdot \cos at\right.$$

$$\mathcal{L}\left[t\ e^{at}\right]$$

$$= \frac{2as}{(s^2+a^2)^2}$$

$$t] = \mathcal{L}[t^2 \cdot \sin at]$$

Second-Shifting Theorem and Unit Step

Thursday, 11 February 2021 10:01 AM

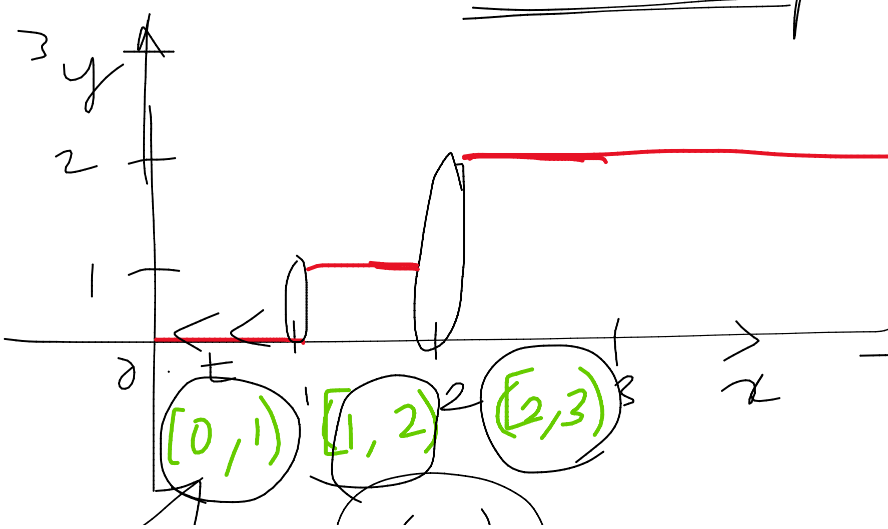
$$\int_0^\infty e^{-st} \boxed{\sin \omega(t)}\, dt$$

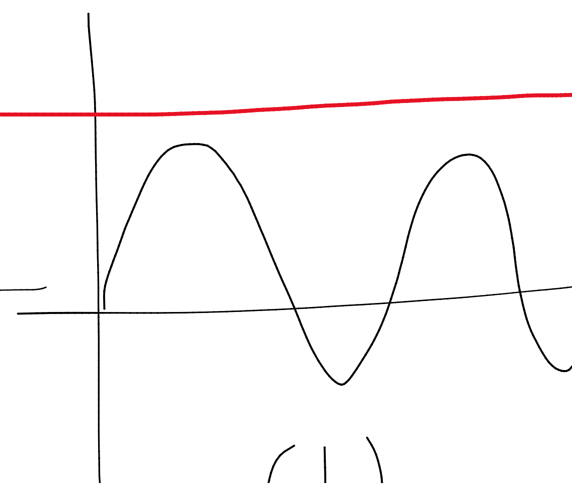
$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}[e^{at}] = \int_0^\infty e^{-st} e^{at}\, dt$$

$\sin\sqrt{t}$ $\sin(t^{1/2})$

Unit-Step function





$[0,1)\quad [1,2)$

(a)

$y = f(x) - x \in \boxed{[0,3]}$

$= 0$

$(0.1, 0)$

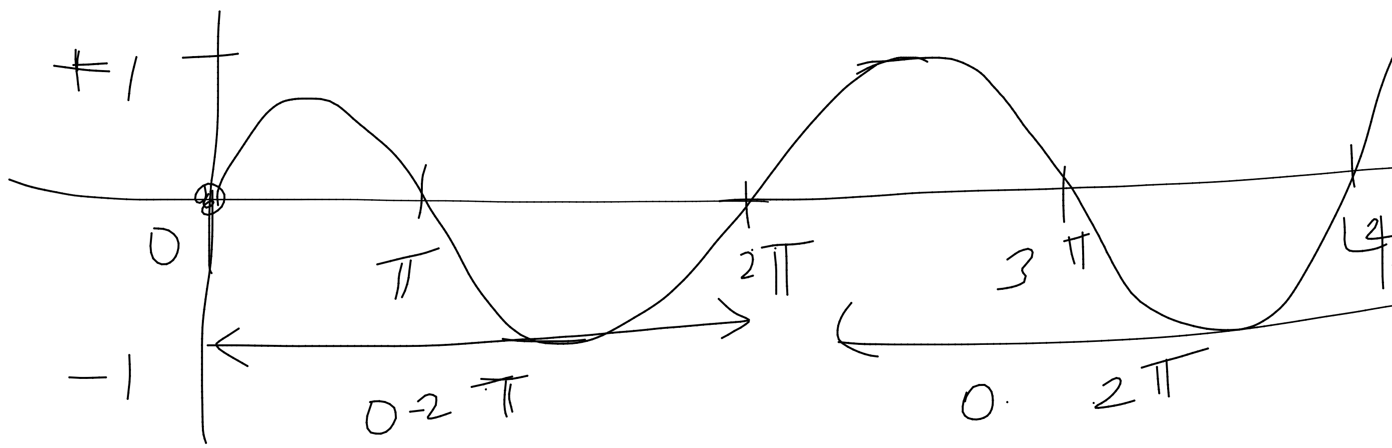
$(0.9, 0)$

(b)

$t \in [0,1)$

$$f(t) = \begin{cases} 0 & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 2 & t \geq 2 \end{cases}$$

$$\boxed{f(t+T) = f(t)}$$



$$f(t+2\pi) = f(t) \Rightarrow$$

Second-shifting theorem

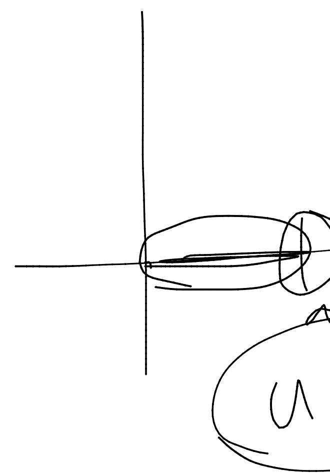
① Assume $f(t)$ is Laplace transf

$$\mathcal{L}[f(t)] = F(s) \text{ exists}$$

② $$\tilde{f} = f(t-a)\,u(t-a)$$

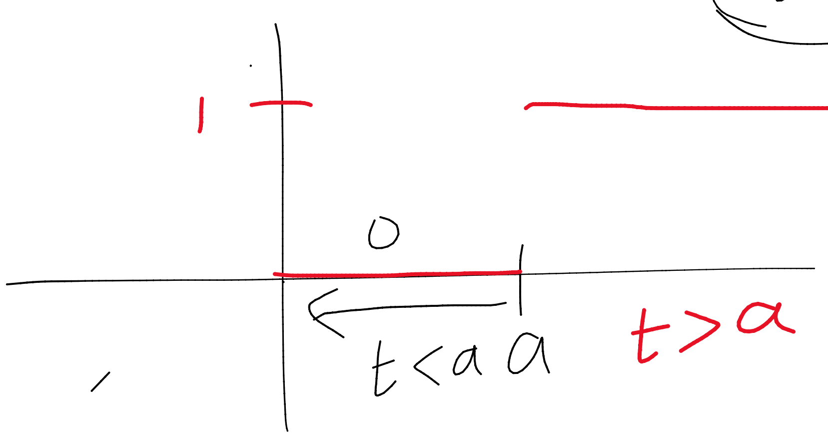
$$= \begin{cases} 0 & \text{if } t<a \\ f(t-a) & \text{if } t>a \end{cases}$$

Suppose $\underline{t=a} \Rightarrow \frac{1}{2}$.



$u(t)$

$u(t-0)$



$\{\ 0 \quad t<a$

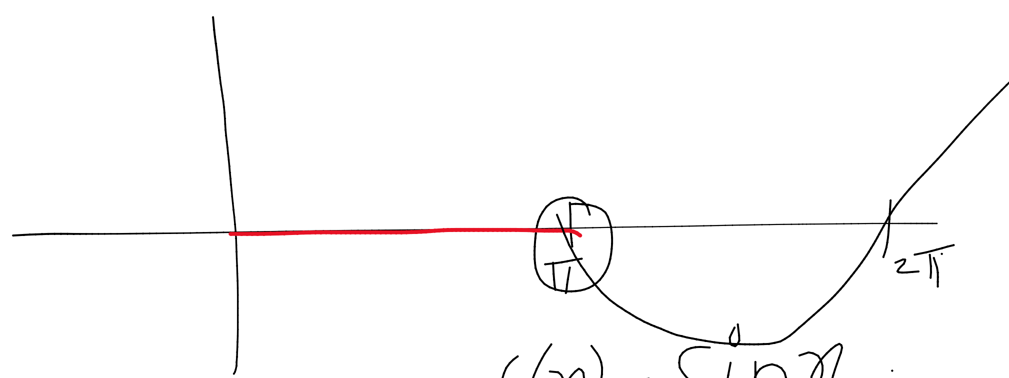
rpable

$(t-a)$

$t = a$

$$f(t) = \begin{cases} 0 & t < a \\ 1 & t > a \end{cases}$$

$$\mathcal{L}\left[f(t-a)\,u(t-a)\right] = e^{-as} \cdot F(s)$$



$f(x) = \sin x$.

<u>Problem:</u>

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$$

$f_1(t) = t \; ; \quad 0 \le t < 1$

$f_2(t) = 1 \; ; \quad t \ge 1$

$$\frac{t = a}{u(t) = 1}$$

$f(0) = 0 \ (0,0)$

$f(0.1) = (0.1, 0.1)$

$$f_2(t) = 1 \quad ; \quad t \geq 1$$

$$= \mathcal{L}\{f(t)[u(t-a) - u(t-b)$$

$$\mathcal{L}[f(t-a)\, u(t-a)] =$$

From given: Eqn. 1

$$= f_1(t)[u(t-0) - u(t-$$

$$= t\,[u(t) - u(t-1)$$

$\Big] \checkmark$

$e^{-as} F(s)$

$\Big] + f_2 \Big[u(t-1) \Big]$

$\Big] + 1 \cdot \Big[u(t-1) \Big]$

Heaviside Function

Thursday, 11 February 2021 10:33 AM

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$$

Heaviside function:

$$\mathcal{L}[f(t)] = \mathcal{L}\left[\underline{f_1(t)}[u(t-0) - u(t-1)] + f_2[u(t-1)]\right]$$

$$= \mathcal{L}\left[t[u(\underline{t}) - u(t-1)] + 1 \cdot u(t-1)\right]$$

$$= \mathcal{L}\left[\underline{t\, u(t)} - \underline{t\, u(t-1) + u(t-1)}\right]$$

Use Formula:

$$\mathcal{L}[f(t-a)\, \underline{u(t-a)}] = \boxed{e^{-as} \cdot F(s)}$$

$$\mathcal{L}[f(t)]$$

$$= \mathcal{L}\left[(t-0)\, u(t-0) - u(t-1)(t-1)\right]$$

$$= \mathcal{L}\left[(t-\underline{0})\, u(t-\underline{0})\right] - \mathcal{L}\left((t-\underline{1})(u(t-\underline{1}))\right)$$

$$= e^{-\underline{0} \cdot s}\, \frac{1!}{s^{1+1}} - e^{-\underline{1} \cdot s} \cdot \frac{1!}{s^{1+1}}$$

$$[\quad] = \frac{1}{\;} - \frac{e^{-s}}{\;}$$

$$\mathcal{L}\Big[\underset{\text{Heavy side}}{\underline{f(t)}}\Big] = \frac{1}{s^2} - \frac{e^{-s}}{s^2}$$

---

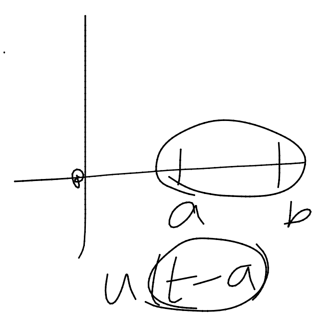
(2) $$f(t) = \begin{cases} 1 & 0 \le t < 2 \\ t^2 & 2 \le t \end{cases}$$

$$\underline{\mathcal{L}\big[f(t-a)\,u(t-a)\big] = e^{-as}F(s)}$$

$$= \mathcal{L}\big[\,1\cdot[u(t-0) - u(t-2)]$$

$$f(t) = \begin{cases} f_1 & \underline{a} \le t < \underline{b} \\ f_2 & b \le t < c \\ f_3 & t \ge c \end{cases}$$

$$\left.\begin{aligned} &f_1\cdot(u(t-a) - u(t-b)) \\ &f_2\,(u(t-b) - u(t-c)) \\ &f_3\,(u(t-c)) \end{aligned}\right\}$$



$$f(t) = f_1\,(u(t-a) - u(t-b)) + f_2((u(t-b) - u(t-c)) + f_3\,(u(t-c))$$

$$\qquad\qquad A \qquad\qquad\qquad\qquad B \qquad\qquad\qquad C$$

$$f(\ldots) = \mathcal{L}[A + B + C]$$

$$\mathcal{L}[f(t)] = \mathcal{L}[A + B + C]$$

$$\mathcal{L}[f(t-a)\,u(t-a)] = e^{-as}F(s)$$

$$f(t) = \begin{cases} 1 & 0 \le t < 2 \\ t^2 & t \ge 2 \end{cases}$$

$$1 \cdot [u(t-0) - u(t-2)] + t^2[u(t-2)]$$

$$u(t-0) - u(t-2) + t^2 u(t-2)$$

$$1 + \underline{u(t-2)}\,[t^2 - 1]$$

$$\underline{f(t-2)}$$

$$t^2 - 1 = t^2 - 1 - 4t + 4t - 4 + 4$$

$$= (t-2)^2 - 1 + 4t - 4$$

$$= (t-2)^2 - 1 +$$

Problems-Laplace Transform

Monday, 15 February 2021 12:03 PM

Laplace

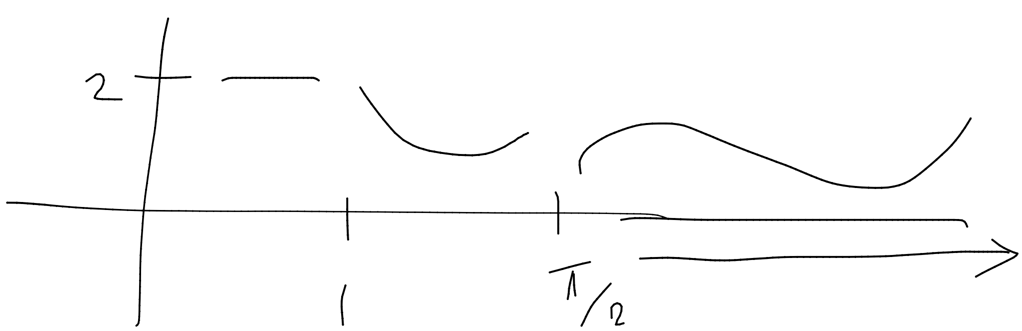
$$f(t) = \begin{cases} 2 & \text{if } 0<t<1 \\ \frac{t^2}{2} & \text{if } 1<t<\pi/2 \\ \cos t & \text{if } t>\pi/2 \end{cases}$$

Using Second-Shifting theorem:

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}[f(t)] = \int_0^1 e^{-st} f_1(t)\,dt + \int_1^{\pi/2} e^{-st} f_2(t) + \int_{\pi/2}^{\infty}$$

$f_1(t) = 2$, $f_2(t) = \frac{t^2}{2}$, $f_3(t) = \mathrm{Co}$



∞

$e^{-st} \cdot f_1(t)\, dt$

st

$t, > a$

Second Shifting theorem:

$$\boxed{L\left[f(t-a)\,u(t-a)\right] = e^{-as}F(s)}$$

Given:

$$L[f(t)] = 2\left[H(t-a) - H(t-b\right.$$

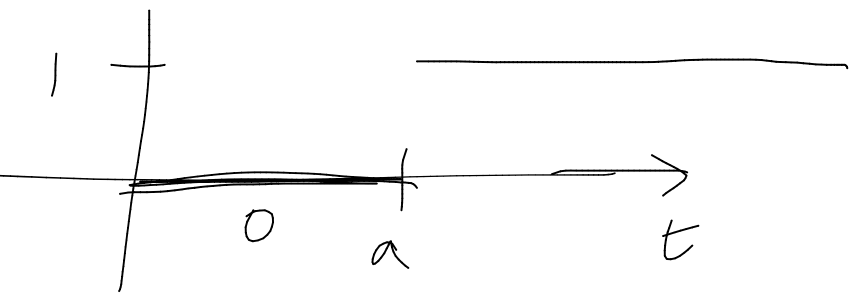
$$f(t) = \begin{cases} 2 \\ t^2/2 \\ \cos t \end{cases}$$

a

b

$$f(t) = \begin{cases} 2 \\ 2 \end{cases}$$

$$u(t-a) = \begin{cases} 1 & t > a \\ 0 & 0 < t < a \end{cases}$$



$$\Big] + \frac{t^2}{2}\Big[H(t-b) - H(t-c)\Big] +$$

$$\cos t \,\Big[H(t-c)\Big] \longrightarrow 1$$

$$\left.\begin{array}{l} < t < b \\ < t < c \\ t > c \end{array}\right\}$$

$$0 < t < 1$$

$$< t < \pi/$$

$$f(t) = \begin{cases} - \\ t^2/2 \\ \cos t \end{cases}$$

$a = 0, \quad b = 1,$

$f(t) = 2\,[H(t-0)$

A

Take A

$\mathcal{L}\,[2H(t-0) - H($

$1 < t < \pi/2$

$t > \pi/2$

$\boxed{= \pi/2}$

$- H(t-1)] + \frac{t^2}{2}\underbrace{\left[H(t-1) - H(t-\pi/2)\right]}_{B}$

$+ \cos t \underbrace{\left[H(t-\pi/2)\right]}_{C}$

$t-1)]$ (by linearity)

$f(t) H(t-a)$

$$= 2\,\mathcal{L}\left[H(t-0)\right] - 2\,\mathcal{L}$$

$$= 2\,e^{-0\cdot s}\,\mathcal{L}[1] - 2\,e^{-}$$

$$= 2\cdot \frac{1}{s} - 2\,\frac{e^{-s}}{s}$$

$$\boxed{\text{For } (A) = \frac{2}{s}\left[1 - e^{-s}\right]}$$

Take (B)

$$\mathcal{L}\left[\frac{t^2}{2}\left(H(t-1) - H(t-\pi/2)\right)\right.$$

$$\mathcal{L}\left(\frac{t^2}{2}\left(H(t-1)\right)\right) - \frac{1}{2}\mathcal{L}\left[t^2\right.$$

$\mathcal{L}\left[H(t-1)\right]$ $\qquad$ $(f(t))H(t-a)$

$s \cdot 1 \cdot \mathcal{L}[1]$ $\qquad$ $\mathcal{L}[1] = \frac{1}{s}$

$\rightarrow 2$

$H(t-\pi/2)]$

$$\frac{1}{2} \mathcal{L}\left[t^2 \left(H(t-1)\right)\right] - \frac{1}{2}\mathcal{L}\left[\right.$$

$f(t)$ $H(t-1)$

Objective to form $f($

$$f(t) \cdot H(t-1) = t^2 \; H($$

$$= t^2 H(t-$$

$$= \left(t^2 - 2t\right.$$

$$= \left((t-1)^2\right.$$

$$= \left((t-1)^2\right.$$

$$= \left((t-1)\right.$$

$t-a)\,H(t-a)$

$(t-1)$

$(t-1)^2 = t^2 - 2t + 1$

$1)$

$+ 1 + 2t - 1)\,H(t-1)$

$^2 + 2t - 1)\,H(t-1)$

$+ 2t - 2 + 1)\,H(t-1)$

$^2 + 2(t-1) + 1)\,H(t-1)$

$^2 \ldots) + 2(t-1)\,H(t-1) + 1\,H(t$

$$t^2 \cdot H(t-1) = (t-1$$

$$\mathcal{L}\left[t^2 \cdot H(t-1)\right] = \mathcal{L}\left[(t-\right.$$

$$= e^{-}$$

$$= e$$

$$\boxed{f(t-\underline{a})\, H}$$

$$-\frac{1}{2}\mathcal{L}\left[t^2 H\left(t-\frac{\pi}{2}\right)\right.$$

$)^2 H(t-1) + 2(t-1) H(t-1) + \dots$

$] + 2\mathcal{L}[(t-1) H(t-1)] + \dots$

$-1)^2 H(t-1)]$

$e^{-1 \cdot s} \mathcal{L}[t^2] + 2 \cdot e^{-1 \cdot s} \mathcal{L}(t) + e \dots$

$$\boxed{(t-a) = e^{-as} \mathcal{L}[f(t)] = e^{-as} \cdot F(s)}$$

$\mathcal{L}$

$(t - \pi/2)^2 =$

$=$

$$\mathcal{L}\left[H(t-1)\right]$$

$$e^{-1.s}\,\mathcal{L}[1]$$

$$\mathcal{L}\left[t^n\right] = \frac{n!}{s^{n+1}}$$

$$t^2 - 2\cdot t\,\pi/2 + \pi^2/4 \quad — \quad ($$

$$t^2 - 2t\,\pi/2 + \pi^2/4 + {}^2$$

ii.

$$t\,\pi/2 - \pi^2/4$$

$$\mathcal{L}\left(t^2 H\left(t - \pi/2\right)\right) =$$

$=$

$=$

$\boxed{t^2}$

$= \mathcal{L}\left\{\left(t - \frac{\pi}{2}\right)^2 \cdot H(t -\right.$

$$\left(t-\pi/2\right)^2 + 2t\,\pi/2$$

$$-\left(t-\pi/2\right)^2 + 2t\,\pi/2$$

$$= \left(t-\pi/2\right)^2 + 2\,\pi/2$$

$$-\pi/2\big)\Big\} + \frac{2\pi}{2}\int\Big[\big(t$$

$$2 \quad \int\Big[$$

$$\pi^2/4$$

$$- 2\pi^2/4 + \pi^2/4$$

$$\boxed{\left(t - \pi/2\right) + \pi^2/4}$$

$$\left.\left(t - \pi/2\right) H\left(t - \pi/2\right)\right]$$

$$H\left(t - \pi/2\right)\right]$$

3 2

a

$$= e^{-\frac{\pi}{2} \cdot s} \cdot \frac{2}{s^3} + \pi$$

$\underline{\underline{\cos t}} \cdot H(t - \pi$

$$+ \frac{\pi^2}{4} \mathcal{L} \Big[$$

$$\cdot\ e^{-\frac{\pi}{2} \cdot s} \cdot \frac{1}{s^2} + \frac{\pi^2}{4}$$

$$\left(\frac{\pi}{2}\right)\Big] - \cdot \ \cos($$

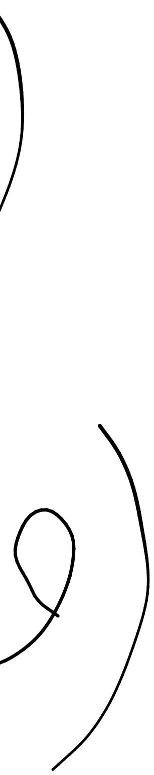
$$\frac{\pi}{})\Big] \qquad e^{0}$$

$H(t - \pi/2)]$

$\frac{1}{s}$

$\theta = \sin(\pi/_2 - \theta)$

$(-\theta) = -\sin($



$2\Big[-$

-2

$-e$

$--e$

$-\sin^{\circ}\left(t-\frac{\pi}{2}\right) H($

$\left[\sin^{\circ}\left(t-\frac{\pi}{2}\right) t\right.$

$e^{-\frac{\pi}{2}\cdot s} \mathcal{L}\left[\sin^{\circ}(t\right.$

$e^{-\frac{\pi}{2}\cdot s} \frac{1}{\;\;\;\;\;\;^{2}}$

$(t - \pi/2)]$ $\delta \, \mathrm{sin}$

$+ ((t - \pi/2))]$

$]$ $\mathcal{L}[\sin a$

Here I

$n(-\theta) =$

$$t = \frac{a}{s^2+a^2}$$

$$\boxed{a = 1}$$

$= -$

$- \mathcal{L}\left[\cos\right.$

$$\frac{\quad}{s^2+1^2}$$

$$t \cdot H\left(t-\pi/2\right)\Big]$$

Unit-Impu

$$= -\frac{e^{-\pi/2 \cdot s}}{s^2+1}$$

lr:

$2(s(t-$

$$-a)) = e^{-as}$$

$$\delta(t-a) = \Bigg\{$$

$$\infty$$

$$\int (t-a)\, dt$$

0 $t \neq a$

∞ $t = a$

$= 1$

$-\infty$

$\mathcal{L}(1) =$

$\mathcal{L}(t^2)$

$d'(t-c)$

$$\frac{1}{s} - \qquad \mathcal{L}^{-1}$$

$$= \frac{2!}{s^2+1} \qquad \mathcal{L}^{-1}$$

$$\left(\frac{1}{s}\right) = 1$$

$$\left(\frac{2}{s^3}\right) = t^2.$$

$\int^{2}$

$$\frac{1}{+2s-1} = \frac{1}{(s-1}$$

$$= \mathcal{L}^{-1} \left\{ \frac{}{(s} \right.$$

$$\frac{\quad}{\quad)^2}$$

$$\frac{1}{-1)^2}$$

$$\mathcal{L}[e^{at}] =$$

$$\frac{1}{(s-a)}$$

Inverse Laplace Transfrom

Monday, 1 February 2021 12:03 PM

$$\boxed{\begin{aligned} a_1 x + b_1 y + c_1 z &= 0 \\ a_2 x + b_2 y + c_2 z &= 0 \\ a_3 x + c_3 z &= 0 \end{aligned}}$$

$$\frac{dx}{dt} = ax(t) + by(t)$$

$$\frac{dy}{dt} = cx(t) + dy(t)$$

Lap

Alg. Eq

D.E

$f(t) \longrightarrow F(s)$

a le
rans form

T

$$f(t) \xrightarrow[L.T]{} F(s)$$

$\downarrow$ Inverse L

$$f(t)$$

$$\int \underbrace{\boxed{t \cdot e^t}}_{u \cdot dv} dt$$

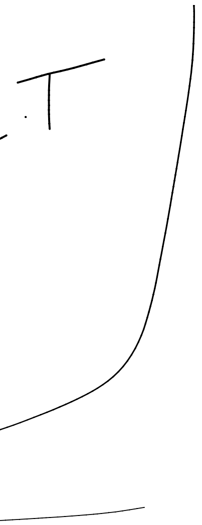
$u = t, \quad dv = e^t \cdot dt$

$du = dt \quad v = e^t.$

$$= uv - \int v \cdot du$$

$$= \int_{-\infty}^{\infty} \to \text{Indefinite } t$$

$$\int^3$$


T

$\left|\frac{J}{2}\right|$

<u>Given</u>: $f(t) = 1$

<u>To Find</u>: $\mathcal{L}(f(t))$ (or) $F(s)$

Formula: $\left.\begin{matrix} F(s) \\ \mathcal{L}(f(t)) \end{matrix}\right\} = \int_0^\infty e^{-st} f(t)\, dt$

<u>Substitute the given</u>: $f(t) = 1$

$$F(s) = \int_0^\infty e^{-st}(1)\, dt$$

$$= \int_0^\infty e^{-st}\, dt$$

$$= \left[\frac{e^{-st}}{-}\right]^{\infty}$$

$$= \left[\frac{e}{-s} \right]_0$$

$$= \left[\frac{e^{-s\infty}}{-s} \right] - \left[\frac{e^{-s \cdot 0}}{-s} \right]$$

$$= 0 + \frac{e^0}{s}$$

$$\boxed{F(s) = \frac{1}{s}}$$

$$\boxed{\mathcal{L}(1) = \frac{1}{s}}$$ Answer

---

(2) $\boxed{f(t) = e^{at}}$

$$\mathcal{L}(e^{at}) = \int_0^\infty e^{-st} \cdot e^{at} \cdot dt$$

$$= \int^\infty e^{t(a-s)} \cdot dt$$

$$e^a \cdot e^b = e^{a+b}$$

$$= \int_0^\infty e^{t(a-s)} \, dt$$

$$= \left[\frac{e^{t(a-s)}}{a-s} \right]_0^\infty$$

$$= \left[\frac{e^{-\infty(s-a)}}{-(s-a)} \right] - \left[\frac{e^{-0}}{-(\ldots} \right.$$

$$= 0 + \frac{1}{s-a}$$

$$\boxed{\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}}$$

---

③ $\boxed{f(t) = t^n}$

$$\mathcal{L}(t^n) = \int_0^\infty \underline{e^{-st} \cdot t^n} \, d\ldots$$

$$(s-a) \Big]$$

$$\overline{s-a)}$$

$$t$$

$$= \int_0^0 \underbrace{t^n}_{u} \cdot \underbrace{e^{-st} \cdot dt}_{dv}$$

$u = t^n$, $dv = e^{-st} \cdot dt$

$du = n t^{n-1} \cdot dt$ $\boxed{v = \frac{e}{-}}$

$$\Rightarrow \int t^n \cdot e^{-st} \cdot dt = \int u \cdot dv$$

$$= uv - \int v$$

$$= \left[t^n \cdot \frac{e^{-st}}{-s} \right]_0^\infty$$

$$\Rightarrow 0 + \frac{n}{s}$$

$n \ell \int t$

$$\boxed{\frac{-st}{-s}}$$

$$du$$

$$\boxed{\int_0^\infty \frac{e^{-st}}{-s}\, n t^{n-1}\, dt}$$

$$\int_0^\infty \boxed{e^{-st}\, t^{n-1}}\, dt$$

$$t^{n-1}$$

$$= \frac{n}{s} \cdot \underline{\mathcal{L}(t}$$

$$= \frac{n}{s} \cdot \frac{(n-1)}{s}$$

$$= \frac{n \cdot (n-1)(}{}$$

$$= \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(t^n) = \frac{n!}{s^{n}}$$

$$\mathcal{L}(t^2) = \frac{2}{s}$$

$$\mathcal{L}(\cos at) =$$

$$n-1)$$

$$n=n$$

$$\mathcal{L}(t^{n-2}) \qquad = t^{n-n}$$

$$(n-2) \cdots n \, \mathcal{L}(t^{0}) = (t^{0}) = 1$$

$$s^{n}$$

$$(s>0)$$

$$\boxed{\frac{\;}{\;+1}}$$

$$\frac{!}{2+1} = \frac{2}{s^{3}}$$

$$\int^{\infty} e^{-st} \cos at \cdot dt$$

$\mathcal{L}(\cos at) =$

Linerity: ① $\mathcal{L}($
② $\mathcal{L}($

$\sin(a+b) \neq \sin$

$\mathcal{L}(1) =$

$\mathcal{L}(e^{at}) =$

$\mathcal{L}(t^n)$

$\mathcal{L}(\cos a$

$\mathcal{L}(\sin a$

$\int_0 e^{-} \cos at \, dt$

$f+g) = \boxed{\mathcal{L}(f) + \mathcal{L}(g)}$

$af) = a\mathcal{L}(f)$

$n(a) + \sin(b)$

$1/s$

$= \frac{1}{s-a} = F(s)$

$= \frac{n!}{s^{n+1}}$

$t) = \frac{s}{s^2+a^2}$

$t) = \frac{a}{s^2+a^2}$

$\mathcal{L}(e^{iat}) = \frac{1}{s-ia}$

$= \frac{1}{s-ia} \times \frac{s+ia}{s+ia}$

$= \frac{s+ia}{s^2+a^2}$

$\mathcal{L}(e^{iat}) = \frac{s}{s^2+a^2} + i\frac{a}{s^2+a^2}$

$\Downarrow \qquad \Downarrow$

$\mathcal{L}(e^{iat}) = \mathcal{L}(\cos at) + i\mathcal{L}(\sin a$

at)

$2(\sin$

$[-\infty,$

$[2$

s^2+a^2

$\mathcal{L}(e^{iat}) = \mathcal{L}(\cos at) + i\mathcal{L}$

∞

n

$, 5) * [5, 10]$

f_1

Inverse LT-Convolution

Tuesday, 16 February 2021 8:56 AM

Convolution:

$f(t)$, $g(t)$ } - piecewise continuous.

Convolution of f, g is defined by

$$(f * g)(t) = \int_0^t f(\tau) \cdot g(t-\tau)\, d\tau .$$

$$(g * f)(t) = \int_0^t g(\tau) \cdot f(t-\tau) \cdot d\tau$$

$\underline{f(t) = e^{3t}}$; $g(t) = e^{7t}$.

$$(f * g)(t) = \int_0^t \underline{f(\tau)} \cdot g\underline{(t-\tau)} \cdot d\tau .$$

$$f(\tau) = e^{3\tau} . \quad g(t-\tau) = e^{7(t-\tau)} .$$

$$\underline{e^{3t} * e^{7t}} = \int_0^t \underline{e^{3\tau}} \cdot e^{7t-7\tau} \cdot d\tau$$

$$= \int_0^t e^{3\tau} \cdot \underline{e^{7t} \cdot e^{-7\tau}} \cdot d\tau \quad [e^{a+b} = e^a \cdot e^b]$$

$$= e^{7t} \int_0^t e^{3\tau} \cdot e^{-7\tau} \cdot d\tau$$

$$= e^{7t} \int_0^t e^{3\tau - 7\tau} \cdot d\tau$$

$$= e^{7t} \cdot \int_0^t e^{-4\tau} \cdot d\tau$$

$$= e^{7t} \cdot \left[\frac{e^{-4\tau}}{-4}\right]_0^t$$

$$= e^{7t} \cdot [-1/4] \left[e^{-4 \cdot t} - e^{-4 \cdot 0}\right]$$

$$= -1/4 \, e^{7t} \left[e^{-4t} - 1\right]$$

$$= -1/4 \, e^{7t - 4t} + 1/4 \, e^{7t}$$

$$= -1/4 \, e^{3t} + 1/4 \, e^{7t}$$

Properties in Inverse LT

Tuesday, 16 February 2021 9:14 AM

$$\mathcal{L}[f(t)] = F(s)$$
$$\mathcal{L}^{-1}[F(s)] = f(t).$$

$$\mathcal{L}^{-1}(1/s) = 1$$
$$\mathcal{L}^{-1}\left[\frac{1}{s^2+1^2}\right] = \sin t$$
$$\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] = \cos 2t$$
$$\mathcal{L}^{-1}\left[\frac{s}{s^2+2}\right] = \cos\sqrt{2}t$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2-2}\right] = \cosh\sqrt{2}t$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+4)^2}\right]$$

By 1st shifting property

$$\mathcal{L}[e^{at} f(t)] = F(s-a)$$
$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+4)^2}\right)$$

By partial fraction.

$$\frac{1}{(s+4)^2} = \frac{A}{s+4} + \frac{B}{(s+4)^2}$$

Inverse LT

Thursday, 18 February 2021 12:59 PM

$$\mathcal{L}\left[f(t)\cdot e^{at}\right] = F(s-a)$$

$$= F(s)\Big|_{s\to s-a}$$

$$\mathcal{L}^{-1}\left[F(s-a)\right] = f(t)\cdot e^{at}$$

$$= \frac{s-3}{(s-3)^2+9} =$$

$\left[\ \mathcal{L}[\cos at] = \right.$

$\mathcal{L}[e^{bt}\cos a$

$b = 3, \quad a = 3$

$$\mathcal{L}^{-1}\left[\frac{s-3}{(s-3)^2+9}\right] = e^{3t}$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-a)^2+1^2}\right]$$

/

$$\frac{s}{s^2+a^2}]$$

$$t] = \frac{s-b}{(s-b)^2+a^2}.$$

cos 3t.

$$\mathcal{L}[\sin bt] = \frac{b}{s^2+b^2}.$$

$$\mathcal{L}^{-1}\left[\frac{\ldots}{(s-a)^2+b^2}\right]$$

By $\underline{\underline{F.S.T}}$

$$\Rightarrow e^{+at}\,\mathcal{L}^{-1}\left[F \ldots\right.$$

$$\Rightarrow e^{at}\cdot\mathcal{L}^{-1}\left[\frac{\ldots}{s^{\ldots}}\right.$$

$$\Rightarrow e^{at}\,\mathcal{L}^{-1}\left[\ldots\right.$$

$$= e^{at}\cdot 1/b\cdot s\ldots$$

$$F(s)\Big]$$

$$\frac{1}{s^2+b^2}\Big]$$

$$\frac{b}{b}\cdot\frac{1}{s^2+b^2}\Big]$$

$$\sin bt$$

$$\left[t^n\right] = n!$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-a)^n}\right]$$

Use F. S. T

$$e^{at}\mathcal{L}^{-1}$$

$$\frac{1}{s^3} \Rightarrow$$

Second:

$$\Rightarrow \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$=$$

$$\left[\frac{n!}{s^{n+1}}\right]$$

$$\frac{1}{s^{2+1}} \Rightarrow \frac{2!}{2!}\frac{1}{s^{2+1}}$$

$$= t^2$$

$$\Rightarrow \quad e^{-as}F(s)$$

$$\mathcal{L}\left[f(t-a)\right.$$

$$\mathcal{L}\left[f(t-a)\right.$$

$$\mathcal{L}^{-1}\left[\frac{e^{-as}}{s^n}\right]$$

We can div

$$= e^{-as}$$

$\mathcal{L}[f(t-a)] = e^{-as} F(s)$

$\mathcal{L}[u(t-a)]$

---

ectly use SIT

$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right]$

Inverse LT using Partial Fractions

Thursday, 18 February 2021 1:22 PM

Partial Fractions

$$\mathcal{L}^{-1}\left[\frac{1}{(s-1)(s+2)(s+3)}\right]$$

A

from A

$$\frac{1}{(s-1)(s+2)(s+3)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$\frac{1}{\cancel{(s-1)(s+2)(s+3)}} = \frac{A(s+2)(s+3) + B(s-1)(s\ldots}{\cancel{(s-1)(s+2)\ldots}}$$

$$1 = \underline{A}(s+2)(s+3) + B(\underline{s-1})(s+\ldots$$

Objective is to find $\underline{A, B, \ldots}$

Put $s = 1$ (solve for A)

$(1+2)(1+3) + B(0)(\ldots$

$$+3) + C(s+2)(s-1)$$

$$\cancel{)(s+3)}$$

$$3) + C(s+2)(s-1) - 1$$

$$C \quad \text{of } 1$$

$$(4) + C(3)(0)$$

Put

$$1 = A(1+2)(1+3) + \frac{B(0)(}{0}$$

$$1 = A(3)(4)$$

$$\boxed{A = 1/12}$$

Put $s = -2$ (solve for B

$$1 = \underbrace{A(-2+2)(-2+3)}_{0}$$

$$1 = 0 + B(-3)(1)$$

$$1 = -3B$$

$$\boxed{B = -1/3}$$

$4) + \dfrac{\quad}{0}$

$+ B(-2-1)(-2+3) + C(0)$

$+ 0$

Put $s = -3$ (Solve for

$$1 = A(0) + B(0) + $$

$$1 = 0 + 0 + C(-1)$$

$$\boxed{C = 1/4}$$

$$\frac{1}{(s-1)(s+2)(s+3)} = \frac{1}{12}\left[\frac{1}{s-1}\right.$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-1)(s-2)(s+3)}\right] = \mathcal{L}^{-1}\left[\right.$$

c)

$c(-3+2)(-3-1)$

(-4)

$$\Big] - \frac{1}{3}\left[\frac{1}{s+2}\right] + \frac{1}{4}\left[\frac{1}{s+3}\right]$$

$$\frac{1}{12}\left[\frac{1}{(s-1)}\right] - \mathcal{L}^{-1}\left[\frac{1}{3}\,\frac{1}{s+2}\right] +$$

$$\mathcal{L}^{-1}\left[\frac{1}{\quad}\right.$$

$$\frac{1}{4}\mathcal{L}^{-1}\left[\frac{1}{s+3}\right]$$

$$\Big] + 1/4\,\mathcal{L}^{-1}\left[\frac{1}{s+3}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-1)(s+2)(s+3)}\right] =$$

$$\tfrac{1}{12}\mathcal{L}^{-1}\frac{1}{(s-1)} - \tfrac{1}{3}\mathcal{L}^{-1}\left[\frac{1}{s+2}\right.$$

$$\boxed{\begin{gathered}\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a} \\ \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}\end{gathered}}$$

$$\tfrac{1}{12}e^{t} - \tfrac{1}{3}e^{-2t} + \tfrac{1}{4}$$

$$1$$

$] + 1/4\,\sigma\,[\partial + 3]$

X

e^{-3t}

$$\frac{s-1}{(s-2)(s-3)(s-4)}$$

2

$$\mathcal{L}^{-1}\left[\frac{2s^2-6s+5}{s^3-6s^2+11s}\right]$$

$$Q(s)=s^3-6s^2+11s$$

Synthetic

to find

Division

Factor

$f(s) = s^3 - 6s + 11$

$f(-1) = (-1)^3 - 6($

$= -1 - 6$

$= \underline{\text{ }\psi\text{ }}$ to find

$\boxed{1, 2, 3, 6}$

$\boxed{\begin{matrix} -1, & 1 \\ -2, & 2 \\ -3, & 3 \\ -6, & 6 \end{matrix}}$

$(-1)^2 + 11(-1) - 6$

$-11 - 6 \neq 0$ [-1 is

not a f

tact

not

uctor]

$$f(1) = (1)^3 - 6(1$$

$$= 1 - 6 +$$

$$= 0 \,//$$

$$f(2) = (2)^3 -$$

$$= 8 -$$

$$= 30$$

$) + 11(1) - 6$

$+ 11 - 6$

$s = 1$

$(s - 1) = 0$ ✓

$- 6(2)^2 + 11(2) - 6$

$24 + 22 - 6$

-30

$= 0$

$(5-2)$

$f(3) = (3)^3 -$

$= 27 -$

$= 60 -$

$= 0$

is a factor

$-6(3)^2+11(3)-6$

$-54+33-6$

-60

factor

$$(s-3)$$

$$\frac{2s^2-6s+}{(s-1)(s-2)}$$

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-}$$

a ta Cotro

$$\boxed{\frac{5}{(s-3)} =}$$

$$\frac{}{3)} = \frac{A}{s-1} + \frac{B}{s-2}$$

$(s-2)(s-3) + B(s-1)($

$$+\frac{C}{s-3}$$

$$s-3) + C(s-2)(s-1)$$

$$2s^2 - 6s + 5$$

Put $s = 1$

$$2(1)^2 - 6(1)$$

$$2 - 6$$

$= \underline{A}(s-2)(s-3) + B(s-$

$+5 = A(-1)(-2) + B$

$+5 = +2A$

$1 = 2A$

$$\boxed{A = 1/2}$$

$(0) + c(0)$

$$\boxed{C = 5/2}$$

$$s = 3$$

$$\frac{2s^2 - 6s + \ldots}{(s-1)(s-2)(s-\ldots}$$

$$\boxed{B = -1}$$

$$s = 2$$

$$= 2\left[\frac{1}{s-1}\right.$$

$$-4e^{t} - e$$

$$+\frac{-1}{s-2}+\frac{\frac{5}{2}}{s-3}\Bigg]$$

$$e^{2t}+\frac{5}{2}e^{3t}$$

$\Rightarrow$

$= y_2 e$

$\mathcal{L}^{-1}\left[\frac{1}{s^3}\right] \Rightarrow \mathcal{L}$

$= \mathcal{L}^{-}$

$= \frac{1}{2!}\mathcal{L}^{-1}$

$$^{-1}\left[\frac{1}{s^3}+\frac{6}{s^2+4}\right]$$

$$\left(\frac{1}{s^3}\right)+6\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right)$$

$$\left(\frac{2!}{s^{2+1}}\right)+\frac{6}{2}\mathcal{L}^{-1}\left[\frac{2}{s^2+}\right.$$

$$2 \quad 3\sin 2t$$

-2^2

16

$= 1/2$

$= + 3 \cdot \sin 2i \cdot$

Problems-Convolution

Saturday, 20 February 2021 10:25 AM

$$\frac{5.5}{(s+1.5)(s-4)} \Rightarrow 5.5\left[\underbrace{\frac{1}{s+1.5}}_{F(s)} \cdot \underbrace{\frac{1}{s-4}}_{G(s)}\right]$$

$$= 5.5\left(\underbrace{e^{-1.5t}}_{f(t)} \cdot \underbrace{e^{4t}}_{g(t)}\right)$$

$$= \int_0^t f(\tau) \cdot g(t-\tau) \cdot d\tau$$

$$= \int_0^t e^{-1.5\tau} \cdot e^{4(t-\tau)} \cdot d\tau$$

$$= \int_0^t e^{-1.5\tau + 4t - 4\tau} \cdot d\tau$$

$$= e^{4t} \int_0^t e^{-5.5\tau} \cdot d\tau$$

$$= e^{4t} \cdot \left[\frac{e^{-5.5\tau}}{-5.5}\right]_0^t$$

$$= e^{4t} \cdot \left[\frac{e^{-5.5t}}{-5.5} - \frac{e^0}{-5.5}\right]$$

$$= e^{4t}\left[\frac{1}{5.5} - \frac{e^{-5.5t}}{5.5}\right]$$

$$= \frac{e^{4t}}{5.5}\left[1 - e^{-5.5t}\right]$$

$$= \frac{1}{5.5}\left[e^{4t} - e^{-1.5t}\right]$$

$$f * g(t) = \int_0^t f(s)\, g(t-s)\, ds$$

$$= \int_0^t f(\theta)\, g(t-\theta) \cdot d\theta$$

$$= \int_0^t f(r) \cdot g(t-r) \cdot dr$$

$$= \int_0^t f(\nu) \cdot g(t-\nu) \cdot d\nu$$

$$= \int_0^t f(\tau) \cdot g(t-\tau) \cdot d\tau$$

$$= 5.5\left(\frac{A}{s+1.5} + \frac{B}{s-4}\right).$$

Inverse Laplace Transform Problems-1

Saturday, 20 February 2021 9:12 AM

$$\mathcal{L}[1] = 1/s$$

$$\mathcal{L}[f(t)] = F(s)$$

$$\mathcal{L}^{-1}[F(s)] = f(t)$$

Linearity:

$$\mathcal{L}^{-1}\Big[\underbrace{\frac{1}{s-2}}_{F(s)} + \underbrace{\frac{1}{s-3}}_{F(s)} + \underbrace{\frac{1}{s-4}}_{F(s)}\Big]$$

$$\mathcal{L}^{-1}\Big[\underbrace{\frac{1}{s-2}}_{F(s)}\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-3}\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-4}\Big]$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$e^{at} = \mathcal{L}^{-1}\left(\frac{1}{s-a}\right) \qquad a = 2$$

$$\boxed{\mathcal{L}[e^{2t}] = \frac{1}{s-2}}$$

$$\mathcal{L}[e^{2t}] = \frac{1}{s-2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{2t}$$

② $$\mathcal{L}[e^{at} \cdot f(t)] = F(s)\Big|_{s \to s-a}$$

$$\Downarrow$$

$$= \mathcal{L}[f(t)]$$

1^{st} shifti property.

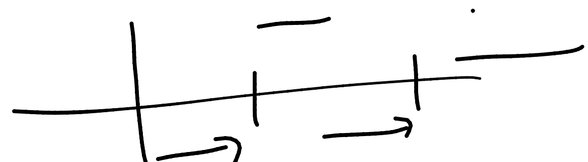
③ $$\mathcal{L}[t^n \cdot f(t)] = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}[t^2 f(t)] = (-1)^2 F''(s)$$

$$= (-1)^2 \frac{d^2}{ds^2} F(s).$$

$$\therefore \mathcal{L}^{-1}[(-1)^2 \cdot F''(s)] = t^2 \cdot f(t)$$

Second shiting:



$$\ldots = e^{-as} \cdot F(s)$$

$$\mathcal{L}\left[f(t-a)u(t-a)\right] = e^{-as} \cdot F(s)$$
$$= e^{-as}\mathcal{L}\left[f(t)\right]$$

$f(t-a) \to f(t)$

$f(t-0) = 1$

$$\mathcal{L}\left[1 \cdot u(t-a)\right] = \frac{e^{-as}}{s} \leftarrow \mathcal{L}[1]$$

$$\boxed{\mathcal{L}^{-1}\left[e^{-as} F(s)\right] = f(t-a)\,u(t-a)}$$

Problem:

$$\mathcal{L}^{-1}\left[\frac{e^{-s}}{s^2+s}\right]$$

$\mathcal{L}^{-1}\left[e^{-as} \cdot F(s)\right]$

$\boxed{a = 1}$

$\mathcal{L}^{-1}\ F(s)]$

Step: Ignore e^{-s}

$$F(s) = \frac{1}{s(s+1)}$$

$$\mathcal{L}^{-1}[F(s = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right]$$

$$= f(t).$$

$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right]$

$$\Rightarrow \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

Solve: Find A, B

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$\frac{1}{\cancel{s(s+1)}} = \frac{A(s+1) + B(s)}{\cancel{s(s+1)}}$$

$$A(s+1) + B(s) = 1 \quad \text{--- (1)}$$

Put $s = 0$ in (1) to Find A

$$A(1) \quad B(0) = 1$$

$$\boxed{A = 1}$$

Put $s = -1$ in (1) to Find B

$$A(0) + B(-1) = 1$$

$\underline{3}$.

-1 .

$-$

$-$ 1

$\underline{B}$.

$H(0) \ldots$

$$\boxed{B = -1}$$

$$\frac{1}{s(s+1)} = \frac{1}{s} + \frac{(-1)}{s+1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right] = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right] = \underline{\underline{1 - e^{-t}}}$$

$$\mathcal{L}\left[f(t-a)\, \underline{\underline{u(t-a)}}\right] = e^{-as} F(s)$$

$$\boxed{a=1} \quad \mathcal{L}^{-1}\left[F(s)\right] = \underline{\underline{1 - e^{-t}}}$$

$$\mathcal{L}^{-1}\left(e^{-s} F(s)\right) = \left(1 - e^{-(t-1)}\right)$$

$$e^{-t}$$

$$\frac{1}{s+1}\Big]$$

$$\left(f(t)\right)$$

$$u(t-1)$$

$f(t) = 1 - e^{-t}$

$\boxed{f(t-1) = 1 - e^{-(t-1)}}$ ✓

$u(t-1)$

$\mathcal{L}^{-1}\left[e^{-s} F(s)\right] = f(t-1) \cdot u$

$= \left(1 - e^{-(t-1)}\right)$

$= u(t-1) - u$

② $\mathcal{L}^{-1}\left[\dfrac{s e^{-5s}}{s^2+4}\right]$ ✓.

By second-shifting:

$= u(t-5)\cos 2(t-5)$

Step 1: $e^{-5s} \Rightarrow u(t-5)$

To find $f(t-5)$

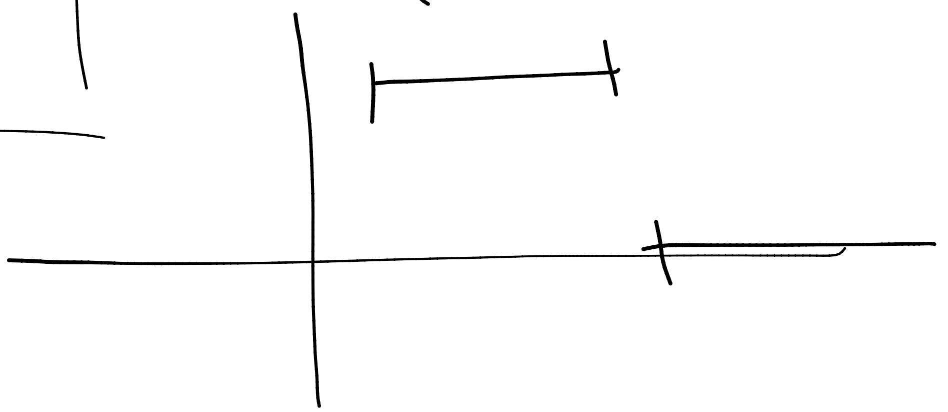
$\mathcal{L}^{-1}[$

$(t-1)$

$)(u(t-1))$

$(t-1)\cdot e^{-(t-1)}$

$$u(t-a) = \begin{cases} 1 & \\ 0 & \end{cases}$$



$=(s)]$

To find $f(t-5)$

start from: $f(t) = \mathcal{L}^{-1}[F$

$$F(s) = \frac{s}{s^2+2^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2+2^2}\right] = \cos 2t.$$

$$f(t) = \cos 2t.$$

$$f(t-5) = \cos 2(t-5)$$

$$\mathcal{L}^{-1}\left[\frac{s \cdot e^{-5t}}{s^2+4}\right] = \cos 2(t-5$$

③ $\mathcal{L}^{-1}$ [logarithmic

$\Downarrow$

$\mathcal{L}^{-1}[\tan^{-1}]$

$F(s)]$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

$$\mathcal{L}^{-1}\left[\underbrace{\frac{s}{s^2+2^2}}_{F(s)}\right] = \underbrace{\cos 2t}_{f(t)}$$

$)\,u(t-5).$

$\mathcal{L}$ | tan

$$\mathcal{L}\left[t \cdot f(t)\right] = -\frac{d}{d}$$

$$= -F$$

$$t \cdot f(t) = \mathcal{L}^{-1}$$

Problem:

$$\mathcal{L}^{-1}\left[\ln\left(\frac{s+2}{s-s}\right.\right.$$

Step 1: $F(s) = \ln\left(\frac{s+}{s}\right.$

$F(s) = \ln ((s-$

$F(s)$

$'(s)$

$\boxed{\left[-F'(s)\right]}$.

$\boxed{\dfrac{\;\;2}{\;\;}}$.

$\left(\dfrac{\;\;+2}{\;\;-5}\right)$

$+2) - \ln(s-5)$

$$F(s) = \ln (s \ldots$$

$$\boxed{t \cdot f(t) = \mathcal{L}^{-1} \ldots}$$

$$F'(s) = \frac{1}{s+2 \ldots}$$

$$= \mathcal{L}^{-1} \left[- \ldots \right.$$

$$= \mathcal{L}^{-1} \left[- \ldots \right.$$

$$\boxed{\mathcal{L}^{-1} \left[-F'(s) \right] = -e^{-2t} \ldots}$$

$$t \cdot f(t) = \mathcal{L}^{-1} \left[\ldots \right.$$

$$+2)$$

$$\boxed{\left[-\underline{F'(s)}\right]}$$

$$-\frac{1}{s-5}$$

$$\left(\frac{1}{s+2}\right)+\frac{1}{s-5}\Bigg]$$

$$\frac{1}{s+2}+\frac{1}{s-5}\Bigg)$$

$$\boxed{+e^{5t}}$$

$$\left[-F'(s)\right]$$

$$t \qquad e^{-2t}$$

$t \cdot f(t) = e^{5}$

$\boxed{f(t) = \frac{e}{}}$

$\mathcal{L}\left[t \cdot f(t)\right] = (-1)^n F$

$t \cdot f(t) = (-1)^n$

$\boxed{f(t) = \mathcal{L}^{-1}\{(-}$

$f(t) = \frac{e^{5t} -}{t}$

$t \quad - \quad e^{-2t}$

$$\boxed{\dfrac{{}^{5t} - e^{-2t}}{t}}$$

$'(s)$

$F'(s)$

$$\boxed{\dfrac{)^{n} F'(s)]}{t}}$$

e^{-2t}

$-2t$

$$\mathcal{L}[f(t)] = \frac{e^{5t} -}{t}$$

② $\mathcal{L}^{-1}\left[\tan^{-1}\left(1/s\right)\right]$

$\frac{d}{dx} \tan$

$\Rightarrow \mathcal{L}^{-1}\left[-F'(s)\right]$

$F'(s) \frac{d}{ds}$

$= \mathcal{L}^{-1}\left[-\frac{s}{s^2+1}\right]$

$$= -\frac{\cos t}{t}$$

$\mathcal{L}^{-1}\left[\tan^{-1}\left(1/s\right)\right] =$

$$e^{-2t}$$

$$\text{an}^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2+x^2}$$

$$\tan^{-1}\left(\frac{1}{s}\right) = \frac{s}{s^2+1}$$

$a = s,\quad x = 1$

$$\frac{\cos t}{t}$$

$\mathcal{L}\left[t \tan^{-1}(1/s)\right]$

Convolution Theorem:

$$\mathcal{L}\left[(f*g)(t)\right] =$$

$$[f*g](t) = \int$$

Problems:

$$\mathcal{L}^{-1}\left[\frac{1}{s^3(\,}\right.$$

$$\int_0^t f(\tau)\, g(t-\tau) \cdot d\tau$$

$$\mathcal{L}^{-1}\left[\int_0^t f(\tau) \cdot g(t-\tau)\, d\tau\right]$$

$$\left[\frac{3}{(s^2-3)}\right]$$

$f(s)$ [convolution

$$\mathcal{L}\left[(f*g)(t)\right] =$$

$$F(s) = \mathcal{L}[f(t$$

$$G(s) = \mathcal{L}[g$$

$$\mathcal{L}^{-1}\left[F(s)\right] =$$

$$\mathcal{L}^{-1}\left[G(s)\right] =$$

$$\mathcal{L}^{-1}\left[\frac{3}{s(s^2+2)}\right.$$

$F(s) * G(s)$ [Convolution]

$(t)]$

$(t)]$

$f(t)$ —

$g(t)$ —

$$\mathcal{L}^{-1}\left[\frac{\quad}{s^3(s^2-3)}\right.$$

Objective: $\boxed{F(s)\cdot G(s}$

Take: $\dfrac{3}{s^3(s^2-3)} = 3$

$= 3$

$=$

$=$

$$\left[\frac{1}{S^3} * \frac{1}{s^2-3}\right]$$

$$\left[\frac{1}{2!}\cdot\frac{2!}{s^3} * \frac{1}{\sqrt{3}}\,\frac{\sqrt{3}}{s^2-(\sqrt{3})^2}\right]$$

$F(s)$ $\qquad$ $G(s)$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{3}}\left[\frac{2!}{s^3} * \frac{\sqrt{3}}{s^2-(\sqrt{3})^2}\right]$$

$$\sqrt{3}\left[\frac{2!}{} * \frac{\sqrt{3}}{s^2-(\sqrt{3})^2}\right]$$

$$=$$

$$=$$

$$-$$

$$f(t) = t$$

$$f(\tau) = t$$

$$\frac{\sqrt{3}}{2}\left(\frac{2}{s^3} * \frac{}{s^2-(\sqrt{3})^2}\right)$$

$$\frac{\sqrt{3}}{2}\left[\mathcal{L}^{-1}\left(\frac{2!}{s^3}\right) \cdot \mathcal{L}^{-1}\left(\frac{\sqrt{3}}{s^2-(\sqrt{3})}\right.\right.$$

$$= \frac{\sqrt{3}}{2}\left(\underset{f(t)}{t^2} \cdot \underset{g(t)}{\sinh\sqrt{3}t}\right)$$

$$\int_0^t \underline{f(\tau) \cdot g(t-\tau)}$$

$$g(t) = \sinh\sqrt{3}t$$

$$g(t-\tau) = \sinh\sqrt{3}(t$$

$$\cdots{}^{2}\Big)\Bigg]$$

$$\cdots d\tau\Bigg)$$

$$\cdots - \tau)\Big)$$

$H(s) =$

$=$

Ex: 2 $\frac{}{s}$

$\mathcal{L}^{-1}\left[\frac{}{s(}\right.$

$$\int_0^t t^2 \cdot \sinh\sqrt{3}(t-$$

$$\frac{1}{s(s^2+1)} = \underbrace{\frac{1}{s}}_{F(s)} \cdot \underbrace{\frac{1}{s^2+1}}_{G(s)}$$

$$\left(\frac{1}{(s^2+1)}\right) = \mathcal{L}^{-1}\left[\frac{1}{s} \cdot \frac{1}{s^2+1}\right]$$

$$= {}^{-1}\left[\frac{1}{s}\right] \cdot \mathcal{L}^{-1}\left[\frac{1}{s^2+$$

$$1 \cdot \sin t$$

$\tau)\, d\tau$

$<$

$-1]$

$$\int_0^t f(\tau$$

1

$= 1 \cdot \sin t$

$g(t) \quad f(t)$

$) \cdot g(t-\tau) \cdot d\tau \Rightarrow \int_0^t \sin \tau$

$= \left[-\cos \tau \right.$

$= -\cos t +$

$= \underline{\underline{1 - \cos t}}$

$(f * g)(t) = \int_0^t f(\tau) \cdot$

$$(1-0)\, d\tau$$

$$]_0^t$$

$$\cos(0)$$

$$g(t-\tau) \cdot d\tau$$

2

0

$$\mathcal{L}\left[(f * g)(t)\right] = \int_0^t f(\tau$$

$$\to \mathcal{L}(f(t)) \cdot \mathcal{L}$$

$$F(s) \cdot G($$

$$\mathcal{L}(f(t)) = F(s) \quad \Big| \quad \mathcal{L}(g(t$$

$$f(t) = \mathcal{L}^{-1}(F(s)) \quad \Big| \quad g(t)$$

$$e^{-3s}$$

$$t)\cdot g(t-\tau)\,d\tau$$

$$\left[g(t)\right]$$

$$(s)$$

$$)) = G(s)$$

$$= \mathcal{L}^{-1}\left[G(s)\right]$$

12

Step 1:

Step 2

$$\frac{e^{-3s}}{(s-1)^3}$$

Ignore: $e^{-3s} \Rightarrow f(t-3)$

find $f(t) = \mathcal{L}^{-1}$

$$= \mathcal{L}^{-1}\left[\frac{1}{(s-1)^3}\right]$$

$u(t-3)$

$[F(s)]$

Step 3

$\boxed{f(}$

$f(t$

1^{st} Shifting

$$= e^t \cdot \mathcal{L}^{-1}\left[\frac{1}{s^3}\right]$$

$$= e^t \cdot \mathcal{L}^{-1}\left[\frac{2}{2 \ldots}\right.$$

$$\boxed{\ldots(t) = \frac{e^t}{2} \cdot \left[t^2\right]}$$

$$\ldots-3) = \frac{e^{(t-3)}}{\ldots}\left((t-3\ldots\right.$$

$$\frac{!}{!} \cdot \frac{1}{s^3} \Big]$$

$$)^2\Big) \cdot u(t-3)$$

Ju

ult

1

$\Rightarrow$

$$\frac{\quad}{2}$$

$$-3)$$

$$\frac{5.5}{(s+1.5)(s-4)} :$$

$$5.5\left[\frac{1}{s+1.5}\right.$$