4. *It may be a value which does not correspond to actual value.*
5. *It cannot be obtained by inspection.*

Uses:

1. *It is used in those cases where it is necessary to average ratios which express rate of change.*
2. *It is also used for the construction of index numbers.*

Example 47. *Compute the Geometric Mean for the following data:*

10, 110, 120, 50, 52, 80, 37, 60.

Solution. Let us prepare the following table:

Size of item (x)	*log x*	*Value of log x*
10	log 10	1.000
110	log 110	2.0414
120	log 120	2.0792
50	log 50	1.6990
52	log 52	1.7160
80	log 80	1.9031
37	log 37	1.5682
60	log 60	1.7782
n = 8		**Σ log x = 13.7851**

Now $$\log G = \frac{1}{n} \Sigma \log x = \frac{13.7851}{8} = \mathbf{1.723.}$$

∴ $$G = \text{antilog } 1.723 = \mathbf{52.84.}$$

Example 48. *Calculate the geometric mean for the following data:*

x :	*12*	*13*	*14*	*15*	*16*	*17*
f :	*5*	*4*	*4*	*3*	*2*	*1*

Solution. Let us prepare the following table in order to calculate the geometric mean for the given data:

x	*log x*	*f*	*f × log x*
12	1.0792	5	5.3960
13	1.1139	4	4.4556
14	1.1461	4	4.5844
15	1.1761	3	3.5283
16	1.2041	2	2.4092
17	1.2304	1	1.2304
		n = Σf = 19	**Σf log x = 21.6029**

Here $$n = 19$$

∴ $$\log G = \frac{\Sigma f \log x}{n} = \frac{21.6029}{19} = 1.137.$$

∴ $$G = \text{Antilog } (1.137) = \mathbf{13.71.}$$

Example 49. *Find the geometric mean for the following data:*

Marks :	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*
No. of students :	*4*	*8*	*10*	*6*	*7*

Solution. Let us prepare the following table in order to calculate the geometric mean for the given data:

Marks	*Mid-value* (x)	*Frequency* (f)	*log x*	$f \times$ *log x*
0 – 10	5	4	0.6990	2.7960
10 – 20	15	8	1.1761	9.4088
20 – 30	25	10	1.3979	13.9790
30 – 40	35	6	1.5441	9.2646
40 – 50	45	7	1.6532	11.5724
	Total	$\boldsymbol{\Sigma f = n = 35}$		$\boldsymbol{\Sigma f \log x = 47.0208}$

Now $$\log G = \frac{\Sigma f \log x}{n} = \frac{47.0208}{35} = 1.3435.$$

$$\therefore \quad G = \text{antilog}(1.3435) = \mathbf{22.055 \text{ marks.}}$$

Example 50. *Find the geometric mean of the following distribution:*

Marks	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*
No. of students	*5*	*8*	*3*	*4*

Solution. Let us prepare the following table in order to calculate geometric mean for the given data:

Marks	*Mid–value* (x)	*Frequency* (f)	*log x*	$f \times$ *log x*
0 – 10	5	5	0.6990	3.495
10 – 20	15	8	1.1761	9.4088
20 – 30	25	3	1.3979	4.1937
30 - 40	35	4	1.5441	6.1764
		$\boldsymbol{n = 20}$		$\boldsymbol{\Sigma f \log x = 23.2739}$

$$\therefore \quad \log G = \frac{\Sigma f \log x}{n} = \frac{23.2739}{20} = 1.1637$$

$$\Rightarrow \quad \log G = \mathbf{1.1636.}$$

$$\therefore \quad G = \text{Antilog}(1.1637) = \mathbf{14.578 \text{ marks.}}$$

4.22 HARMONIC MEAN

Definition. The harmonic mean of n items $x_1, x_2, x_3, \ldots\ldots, x_n$ is defined as;

$$\textbf{Harmonic Mean} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots\ldots + \frac{1}{x_n}}$$

For example, the harmonic mean of 2, 4 and 5 is $= \dfrac{3}{\frac{1}{2} + \frac{1}{4} + \frac{1}{5}} = \dfrac{60}{19} = \mathbf{3.16.}$

Harmonic Mean of Frequency Distribution Data. Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be n items which occur with frequencies $f_1, f_2, f_3, \ldots\ldots, f_n$ respectively. Then their Harmonic Mean is given by:

$$\text{Harmonic Mean} = \frac{f_1 + f_2 + f_3 + \ldots\ldots + f_n}{\left(\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + \ldots\ldots + \frac{f_n}{x_n}\right)} = \frac{\Sigma f_i}{\Sigma f_i \times \frac{1}{x_i}}$$

4.23 MERITS, DEMERITS AND USES OF HARMONIC MEAN

Merits:

1. *It is easy to calculate.*
2. *It is rigidly defined.*
3. *It gives largest weight to the smallest items and can be used whenever so desired.*
4. *It is a useful average when we deal with* ***average of rates.***

Demerits:

It cannot be located by inspection.

Uses:

1. *The harmonic mean is especially useful in averaging time rates, in finding the average price per unit when the data gives the amount of commodity for a given price and in the development of index numbers.*
2. *It is used when rates are expressed as x per y, where x is constant.*

It is illustrated by the following example:

Example 51. *Compute the harmonic mean for the following data:*

Marks obtained	20	21	22	23	24	25
No. of students	4	2	7	1	3	1

Solution. Let us compute the following table:

x	$\frac{1}{x}$	f	$f \times \frac{1}{x}$
20	0.05000	4	0.2000
21	0.04762	2	0.09524
22	0.04545	7	0.31815
23	0.04348	1	0.04348
24	0.04167	3	0.12501
25	0.04000	1	0.04000
	Total	**18**	$\Sigma f \times \frac{1}{x} = \mathbf{0.82188}$

Now $\quad \Sigma f \times \frac{1}{x} = 0.82188$ and $n = 18$.

$\therefore \quad$ **Harmonic Mean** $= \frac{18}{0.82188} = \mathbf{21.9.}$

4.25 CHOICE OF AN AVERAGE FOR DECISION MAKING

We have studied the various kinds of averages such as: **Arithmetic mean, Median, Mode, Geometric mean and Harmonic mean.** It is important to know **when and how to use which average?** Thus averages cannot be used indiscriminately. A judicial selection of averages for sound statistical analysis depends upon the following factors:

(*i*) *The nature of the variable involved.*

(*ii*) *The purpose of analysis.*

(*iii*) *The system of classification adopted.*

(*iv*) *The quality, nature and availability of data.*

(*v*) *The study of average for further statistical computation required for the enquiry in mind.*

We give below the suitability of some of the averages.

Arithmetic Mean. It is, generally, used in business. Whenever, we talk of average cost of production or sale or average wages, we use arithmetic mean. It is also used for further statistical calculations such as **standard deviation.** Arithmetic mean is not recommended while dealing with frequency distribution with extreme observations or open end classes.

Median. It is to be used for finding the average when the data is qualitative, *i.e.*, for finding average of intelligence, honesty, beauty etc., median is the only average to be used. It is a **positional average** as it divides the entire series in two equal parts, 50% of actual values will be below and 50% will be above it. It is suitable when there are **open extreme classes** or where there are **extreme**

values. It is commonly used for **average wages of worker** as it would avoid the influence of a few very high or very low wage rates.

Mode. It is a **positional average**. It is to be used while dealing with **open end classes.** It is particularly used in **business,** when the businessman is not interested in the magnitude but only in the most common or fashionable value.

Geometric and Harmonic Means. Geometric mean and Harmonic mean are known as **ratio averages** as they are most appropriate where the data comprise **rates, ratios or percentages instead of actual quantities.** *Geometric mean is to be used while dealing with rates and ratios. Harmonic mean is to be used in compiling special types of average rates or ratios, where time factor is variable and the act being performed, e.g., distance is constant.*

4.26 COMPARISON AMONG MEAN, MEDIAN AND MODE

	Mean	Median	Mode
Average	It is a *calculated average.*	It is a *positional average.*	It is a *positional average.*
Calculation	It is based on all the observations.	It is the middle most value which divides the series into two equal parts.	It is the value around which the items of the series tend to concentrate densely.
Treatment	It is capable of mathematical treatments.	It is not capable of mathematical treatments.	It is not capable of mathematical treatments.
Items	It involves all the items for calculation.	It does not consider all the items.	Does not consider all the items.
Array	It does not require arraying.	Arraying of the values of the items in the series is essential.	Arraying of the values of the items in the series is essential.
Extreme values	It is affected by extreme and abnormal values of the items in the series.	It is not affected by the extreme values.	It is not affected by the extreme values.
Result	There is only one mean.	There is only one median.	In a series there may be one mode or more than one mode or no mode.
Reliability	Most reliable measure.	Less reliable.	Less reliable.
Use	It is simple and widely used in statistical treatment and interpretation.	Not popular and is used only in appropriate cases.	Not popular and is used only in appropriate cases.