The reliability of a turbine blade is given by $R(t)=\left(1-\frac{t}{t_0}\right)^2$, $0\le t\le t_0$, where t_0 is the maximum life of the blade.

(a) Show that the blades are experiencing wear out.

(b) Compute MTTF as a function of the maximum life.

(c) If the maximum life is 2000 operating hours, what is the design life for a reliability of 0.90?

(a) $$R(t)=\left(1-\frac{t}{t_0}\right)^2, \quad 0\le t\le t_0$$

Now $$\lambda(t)=-\frac{R'(t)}{R(t)}$$

$$=-\left(1-\frac{t}{t_0}\right)^{-2}\left\{-\frac{2}{t_0}\left(1-\frac{t}{t_0}\right)\right\}$$

$$=\frac{2}{t_0-t}$$

$$\lambda'(t) = \frac{2}{(t_0 - t)^2} > 0 \text{ and so } \lambda(t) \text{ is an increasing function of } t.$$

When the failure rate increases with time, it indicates that the blades are experiencing wear out.

(b) $$\text{MTTF} = \int_0^\infty R(t)\,dt = \int_0^{t_0} \left(1 - \frac{t}{t_0}\right)^2 dt$$

$$= \left[-\frac{t_0}{3}\left(1 - \frac{t}{t_0}\right)^3\right]_0^{t_0} = \frac{t_0}{3}$$

(c) When $t_0 = 2000$, $R(t_D) = 0.90$

i.e., $$\left(1 - \frac{t_D}{2000}\right)^2 = 0.90$$

$\therefore$ $$1 - \frac{t_D}{2000} = 0.9487$$

$\therefore$ $$t_D = 102.63 \text{ hours.}$$

Given that $R(t) = e^{-\sqrt{0.001t}}$, $t \geq 0$

(a) Compute the reliability for a 50 hours mission.

(b) Show that the hazard rate is decreasing.

(c) Given a 10-hour wear-in period, compute the reliability for a 50-hour mission.

(d) What is the design life for a reliability of 0.95, given a 10-hour wear-in period?

(a) $$R(t) = e^{-\sqrt{0.001t}}, \quad t \geq 0$$

$$\therefore \quad R(50) = e^{-\sqrt{0.001 \times 50}} = 0.9512$$

(b) $$\lambda(t) = \frac{-R'(t)}{R(t)} = -e^{\sqrt{0.001t}} \times e^{-\sqrt{0.001t}} \times -\sqrt{0.001} \times \frac{1}{2\sqrt{t}}$$

$$= \frac{\sqrt{0.001}}{2\sqrt{t}},$$ which is a decreasing function of t.

(c) $$R(t/T_0) = \frac{R(t+T_0)}{R(T_0)}$$

$$\therefore \quad R(50/10) = \frac{R(60)}{R(10)} = \frac{e^{-\sqrt{0.001 \times 60}}}{e^{-\sqrt{0.001 \times 10}}} = 0.8651$$

(d) $R(t_D/10) = 0.95$

i.e., $$\frac{R(t_D + 10)}{R(10)} = 0.95$$

i.e., $$e^{-\sqrt{0.001\times(t_D+10)}} = 0.95 \times e^{-\sqrt{0.001\times 10}}$$

$\therefore$ $$\sqrt{0.001\times(t_D+10)} = 0.15129$$

$\therefore$ $$t_D = 12.89 \text{ hours.}$$

A one-year guarantee is given based on the assumption that no more than 10% of the items will be returned. Assuming an exponential distribution, what is the maximum failure rate that can be tolerated?

If T is the time to failure of the item,

then $P(T \geq 1) \geq 0.9$ ($\because$ no more than 10% will be returned)

i.e., $R(1) \geq 0.9$

i.e., $\int_{1}^{\infty} \lambda e^{-\lambda t}\, dt \geq 0.9$ ($\because$ T follows an exponential distribution)

i.e., $(-e^{-\lambda t})_{1}^{\infty} \geq 0.9$

i.e., $e^{-\lambda} \geq 0.9$

$\therefore$ $-\lambda \geq -0.1054$

$\therefore$ $\lambda \leq 0.1054$/year