TABLE 1-3 ASCII Code

	ASCII Code								ASCII Code								ASCII Code						
Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0	‘	1	1	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1	a	1	1	0	0	0	0	1
“	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0	b	1	1	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1	c	1	1	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0	d	1	1	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1	e	1	1	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0	f	1	1	0	0	1	1	0
’	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1	g	1	1	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0	h	1	1	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1	i	1	1	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0	j	1	1	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1	k	1	1	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0	l	1	1	0	1	1	0	0
−	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1	m	1	1	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0	n	1	1	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1	o	1	1	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0	p	1	1	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1	q	1	1	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0	r	1	1	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1	s	1	1	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0	t	1	1	1	0	1	0	0
5	0	1	1	0	1	0	1	U	1	0	1	0	1	0	1	u	1	1	1	0	1	0	1
6	0	1	1	0	1	1	0	V	1	0	1	0	1	1	0	v	1	1	1	0	1	1	0
7	0	1	1	0	1	1	1	W	1	0	1	0	1	1	1	w	1	1	1	0	1	1	1
8	0	1	1	1	0	0	0	X	1	0	1	1	0	0	0	x	1	1	1	1	0	0	0
9	0	1	1	1	0	0	1	Y	1	0	1	1	0	0	1	y	1	1	1	1	0	0	1
:	0	1	1	1	0	1	0	Z	1	0	1	1	0	1	0	z	1	1	1	1	0	1	0
;	0	1	1	1	0	1	1	[	1	0	1	1	0	1	1	{	1	1	1	1	0	1	1
<	0	1	1	1	1	0	0	\	1	0	1	1	1	0	0	|	1	1	1	1	1	0	0
=	0	1	1	1	1	0	1	]	1	0	1	1	1	0	1	}	1	1	1	1	1	0	1
>	0	1	1	1	1	1	0	^	1	0	1	1	1	1	0	~	1	1	1	1	1	1	0
?	0	1	1	1	1	1	1	—	1	0	1	1	1	1	1	delete	1	1	1	1	1	1	1

Problems

1.1 Convert to hexadecimal and then to binary:
(a) 757.25_{10} (b) 123.17_{10} (c) 356.89_{10} (d) 1063.5_{10}

1.2 Convert to octal. Convert to hexadecimal. Then convert both of your answers to decimal, and verify that they are the same.
(a) 111010110001.011_2 (b) 10110011101.11_2

1.3 Convert to base 6: $3BA.25_{14}$ (do all of the arithmetic in decimal).

1.4 (a) Convert to hexadecimal: 1457.11_{10}. Round to two digits past the hexadecimal point.
(b) Convert your answer to binary, and then to octal.
(c) Devise a scheme for converting hexadecimal directly to base 4 and convert your answer to base 4.
(d) Convert to decimal: $DEC.A_{16}$.

1.5 Add, subtract, and multiply in binary:
(a) 1111 and 1010 (b) 110110 and 11101 (c) 100100 and 10110

1.6 Subtract in binary. Place a 1 over each column from which it was necessary to borrow.
(a) $11110100 - 1000111$ (b) $1110110 - 111101$ (c) $10110010 - 111101$

1.7 Add the following numbers in binary using 2's complement to represent negative numbers. Use a word length of 6 bits (including sign) and indicate if an overflow occurs.
(a) $21 + 11$ (b) $(-14) + (-32)$ (c) $(-25) + 18$
(d) $(-12) + 13$ (e) $(-11) + (-21)$
Repeat (a), (c), (d), and (e) using 1's complement to represent negative numbers.

1.8 A computer has a word length of 8 bits (including sign). If 2's complement is used to represent negative numbers, what range of integers can be stored in the computer? If 1's complement is used? (Express your answers in decimal.)

1.9 Construct a table for 7-3-2-1 weighted code and write 3659 using this code.

1.10 Convert to hexadecimal and then to binary.
(a) 1305.375_{10} (b) 111.33_{10} (c) 301.12_{10} (d) 1644.875_{10}

1.11 Convert to octal. Convert to hexadecimal. Then convert both of your answers to decimal, and verify that they are the same.
(a) 101111010100.101_2 (b) 100001101111.01_2

1.12 (a) Convert to base 3: 375.54_8 (do all of the arithmetic in decimal).
(b) Convert to base 4: 384.74_{10}.
(c) Convert to base 9: $A52.A4_{11}$ (do all of the arithmetic in decimal).

1.13 Convert to hexadecimal and then to binary: 544.1_9.

1.14 Convert the decimal number 97.7_{10} into a number with exactly the same value represented in the following bases. The exact value requires an infinite repeating part in the fractional part of the number. Show the steps of your derivation.
(a) binary (b) octal (c) hexadecimal (d) base 3 (e) base 5

1.15 Devise a scheme for converting base 3 numbers directly to base 9. Use your method to convert the following number to base 9: 1110212.20211_3

1.16 Convert the following decimal numbers to octal and then to binary:
(a) $2983^{63}/_{64}$ (b) 93.70 (c) $1900^{31}/_{32}$ (d) 109.30

1.17 Add, subtract, and multiply in binary:
(a) 1111 and 1001 (b) 1101001 and 110110 (c) 110010 and 11101

1.18 Subtract in binary. Place a 1 over each column from which it was necessary to borrow.
(a) 10100100 − 01110011 (b) 10010011 − 01011001
(c) 11110011 − 10011110

1.19 Divide in binary:
(a) 11101001 ÷ 101 (b) 110000001 ÷ 1110 (c) 1110010 ÷ 1001
Check your answers by multiplying out in binary and adding the remainder.

1.20 Divide in binary:
(a) 10001101 ÷ 110 (b) 110000011 ÷ 1011 (c) 1110100 ÷ 1010

1.21 Assume three digits are used to represent positive integers and also assume the following operations are correct. Determine the base of the numbers. Did any of the additions overflow?
(a) 654 + 013 = 000
(b) 024 + 043 + 013 + 033 = 223
(c) 024 + 043 + 013 + 033 = 201

1.22 What is the lowest number of bits (digits) required in the binary number approximately equal to the decimal number 0.6117_{10} so that the binary number has the same or better precision?

1.23 Convert $0.363636\ldots_{10}$ to its exact equivalent base 8 number.

1.24 (a) Verify that a number in base b can be converted to base b^3 by partitioning the digits of the base b number into groups of three consecutive digits starting at the radix point and proceeding both left and right and converting each group into a base b^3 digit. (*Hint:* Represent the base b number using the power series expansion.)
(b) Verify that a number in base b^3 can be converted to base b by expanding each digit of the base b^3 number into three consecutive digits starting at the radix point and proceeding both left and right.

1.25 Construct a table for 4-3-2-1 weighted code and write 9154 using this code.

1.26 Is it possible to construct a 5-3-1-1 weighted code? A 6-4-1-1 weighted code? Justify your answers.

1.27 Is it possible to construct a 5-4-1-1 weighted code? A 6-3-2-1 weighte code? Justify your answers.

1.28 Construct a 6-2-2-1 weighted code for decimal digits. What number does 1100 0011 represent in this code?

1.29 Construct a 5-2-2-1 weighted code for decimal digits. What numbers does 1110 0110 represent in this code?

1.30 Construct a 7-3-2-1 code for base 12 digits. Write B4A9 using this code.

1.31 (a) It is possible to have negative weights in a weighted code for the decimal digits, e.g., 8, 4, −2, and −1 can be used. Construct a table for this weighted code.
(b) If d is a decimal digit in this code, how can the code for $9 - d$ be obtained?

1.32 Convert to hexadecimal, and then give the ASCII code for the resulting hexadecimal number (including the code for the hexadecimal point):
(a) 222.22_{10} (b) 183.81_{10}

1.33 Repeat 1.7 for the following numbers:
(a) $(-10) + (-11)$ (b) $(-10) + (-6)$ (c) $(-8) + (-11)$
(d) $11 + 9$ (e) $(-11) + (-4)$

1.34 Because $A - B = A + (-B)$, the subtraction of signed numbers can be accomplished by adding the complement. Subtract each of the following pairs of 5-bit binary numbers by adding the complement of the subtrahend to the minuend. Indicate when an overflow occurs. Assume that negative numbers are represented in 1's complement. Then repeat using 2's complement.

(a)	(b)	(c)	(d)	(e)
01001	11010	10110	11011	11100
−11010	−11001	−01101	−00111	−10101

1.35 Work Problem 1.34 for the following pairs of numbers:

(a)	(b)	(c)	(d)
11010	01011	10001	10101
−10100	−11000	−01010	−11010

1.36 (a) A = 101010 and B = 011101 are 1's complement numbers. Perform the following operations and indicate whether overflow occurs.
(i) A + B (ii) A − B
(b) Repeat Part (a) assuming the numbers are 2's complement numbers.

1.37 (a) Assume the integers below are 1's complement integers. Find the 1's complement of each number, and give the decimal values of the original number and of its complement.
(i) 0000000 (ii) 1111111 (iii) 00110011 (iv) 1000000
(b) Repeat, assuming the numbers are 2's complement numbers and finding the 2's complement of them.

The inverse of $F = A'B + AB'$ is

$$F' = (A'B + AB')' = (A'B)'(AB')' = (A + B')(A' + B)$$
$$= AA' + AB + B'A' + BB' = A'B' + AB$$

We will verify that this result is correct by constructing a truth table for F and F':

A B	A'B	AB'	F = A'B + AB'	A'B'	AB	F' = A'B' + AB
0 0	0	0	0	1	0	1
0 1	1	0	1	0	0	0
1 0	0	1	1	0	0	0
1 1	0	0	0	0	1	1

In the table, note that for every combination of values of A and B for which $F = 0$, $F' = 1$; and whenever $F = 1$, $F' = 0$.

Given a Boolean expression, the *dual* is formed by replacing AND with OR, OR with AND, 0 with 1, and 1 with 0. Variables and complements are left unchanged. The dual of AND is OR and the dual of OR is AND:

$$(XYZ\ldots)^D = X + Y + Z + \ldots \qquad (X + Y + Z + \ldots)^D = XYZ\ldots \qquad \text{(2-26)}$$

The dual of an expression may be found by complementing the entire expression and then complementing each individual variable. For example, to find the dual of $AB' + C$,

$$(AB' + C)' = (AB')'C' = (A' + B)C', \qquad \text{so} \qquad (AB' + C)^D = (A + B')C$$

The laws and theorems of Boolean algebra on page 55 are listed in dual pairs. For example, Theorem 11 is $(X + Y')Y = XY$ and its dual is $XY' + Y = X + Y$ (Theorem 11D).

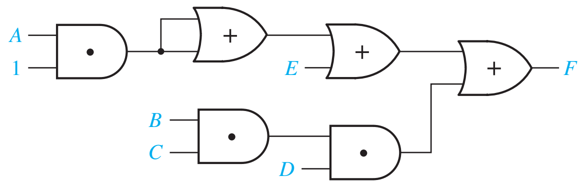
Problems

2.1 Prove the following theorems algebraically:
(a) $X(X' + Y) = XY$ (b) $X + XY = X$
(c) $XY + XY' = X$ (d) $(A + B)(A + B') = A$

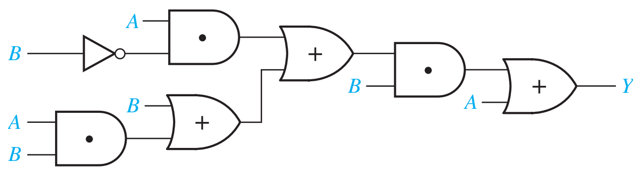
2.2 Illustrate the following theorems using circuits of switches:
(a) $X + XY = X$ (b) $X + YZ = (X + Y)(X + Z)$
In each case, explain why the circuits are equivalent.

2.3 Simplify each of the following expressions by applying *one* of the theorems. State the theorem used (see page 55).
(a) $X'Y'Z + (X'Y'Z)'$ (b) $(AB' + CD)(B'E + CD)$
(c) $ACF + AC'F$ (d) $A(C + D'B) + A'$
(e) $(A'B + C + D)(A'B + D)$ (f) $(A + BC) + (DE + F)(A + BC)'$

2.4 For each of the following circuits, find the output and design a simpler circuit having the same output. (*Hint:* Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go.)



(a)



(b)

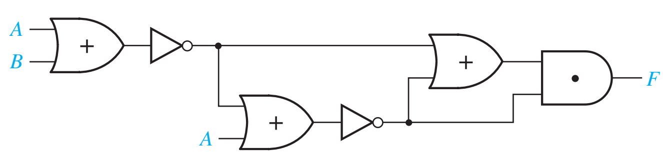
2.5 Multiply out and simplify to obtain a sum of products:
(a) $(A + B)(C + B)(D' + B)(ACD' + E)$
(b) $(A' + B + C')(A' + C' + D)(B' + D')$

2.6 Factor each of the following expressions to obtain a product of sums:
(a) $AB + C'D'$ (b) $WX + WY'X + ZYX$
(c) $A'BC + EF + DEF'$ (d) $XYZ + W'Z + XQ'Z$
(e) $ACD' + C'D' + A'C$ (f) $A + BC + DE$
(The answer to (f) should be the product of four terms, each a sum of three variables.)

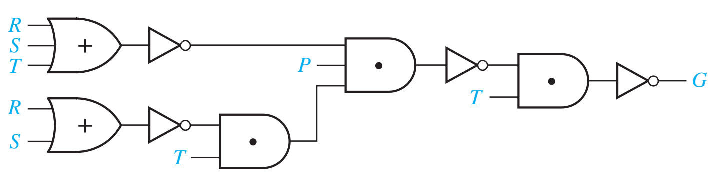
2.7 Draw a circuit that uses only one AND gate and one OR gate to realize each of the following functions:
(a) $(A + B + C + D)(A + B + C + E)(A + B + C + F)$
(b) $WXYZ + VXYZ + UXYZ$

2.8 Simplify the following expressions to a minimum sum of products.
(a) $[(AB)' + C'D]'$ (b) $[A + B(C' + D)]'$ (c) $((A + B')C)'(A + B)(C + A)'$

2.9 Find F and G and simplify:



(a)



(b)

2.10 Illustrate the following equations using circuits of switches:

(a) $XY + XY' = X$ (b) $(X + Y')Y = XY$

(c) $X + X'ZY = X + YZ$ (d) $(A + B)C + (A + B)C' = A + B$

(e) $(X + Y)(X + Z) = X + YZ$ (f) $X(X + Y) = X$

2.11 Simplify each of the following expressions by applying *one* of the theorems. State the theorem used.

(a) $(A' + B' + C)(A' + B' + C)'$ (b) $AB(C' + D) + B(C' + D)$

(c) $AB + (C' + D)(AB)'$ (d) $(A'BF + CD')(A'BF + CEG)$

(e) $[AB' + (C + D)' + E'F](C + D)$ (f) $A'(B + C)(D'E + F)' + (D'E + F)$

2.12 Simplify each of the following expressions by applying *one* of the theorems. State the theorem used.

(a) $(X + Y'Z) + (X + Y'Z)'$

(b) $[W + X'(Y + Z)][W' + X'(Y + Z)]$

(c) $(V'W + UX)'(UX + Y + Z + V'W)$

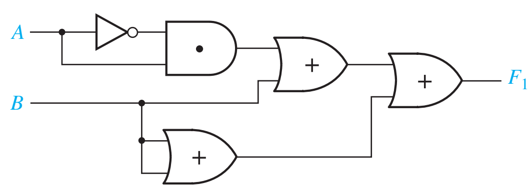
(d) $(UV' + W'X)(UV' + W'X + Y'Z)$

(e) $(W' + X)(Y + Z') + (W' + X)'(Y + Z')$

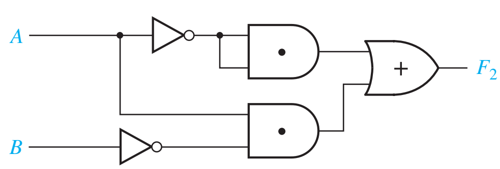
(f) $(V' + U + W)[(W + X) + Y + UZ'] + [(W + X) + UZ' + Y]$

2.13 For each of the following circuits, find the output and design a simpler circuit that has the same output. (*Hint:* Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go).

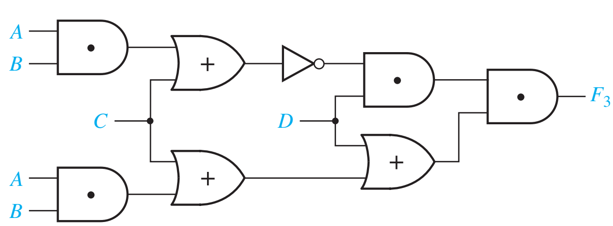
(a)



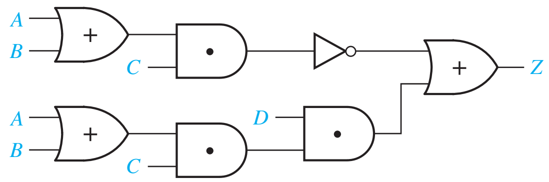
(b)



(c)



(d)

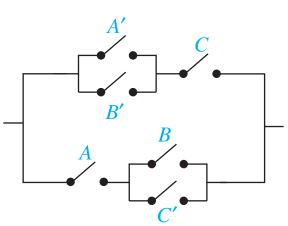


2.14 Draw a circuit that uses only one AND gate and one OR gate to realize each of the following functions:
(a) $ABCF + ACEF + ACDF$
(b) $(V + W + Y + Z)(U + W + Y + Z)(W + X + Y + Z)$

2.15 Use *only* DeMorgan's relationships and Involution to find the complements of the following functions:
(a) $f(A, B, C, D) = [A + (BCD)'][(AD)' + B(C' + A)]$
(b) $f(A, B, C, D) = AB'C + (A' + B + D)(ABD' + B')$

2.16 Using *just* the definition of the dual of a Boolean algebra expression, find the duals of the following expressions:
(a) $f(A, B, C, D) = [A + (BCD)'][(AD)' + B(C' + A)]$
(b) $f(A, B, C, D) = AB'C + (A' + B + D)(ABD' + B')$

2.17 For the following switching circuit, find the logic function expression describing the circuit by the three methods indicated, simplify each expression, and show they are equal.
(a) subdividing it into series and parallel connections of subcircuits until single switches are obtained
(b) finding all paths through the circuit (sometimes called *tie sets*), forming an AND term for each path and ORing the AND terms together
(c) finding all ways of breaking all paths through the circuit (sometimes called *cut sets*), forming an OR term for each cut set and ANDing the OR terms together.

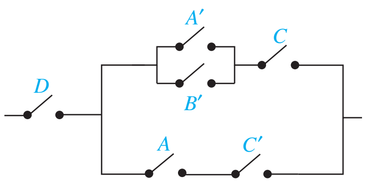


2.18 For each of the following Boolean (or switching) algebra expressions, indicate which, if any, of the following terms describe the expression: product term, sum-of-products, sum term, and product-of-sums. (More than one may apply.)
(a) $X'Y$
(b) $XY' + YZ$
(c) $(X' + Y)(WX + Z)$
(d) $X + Z$
(e) $(X' + Y)(W + Z)(X + Y' + Z')$

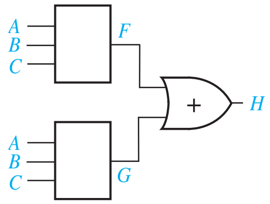
2.19 Construct a gate circuit using AND, OR, and NOT gates that corresponds one to one with the following switching algebra expression. Assume that inputs are available only in uncomplemented form. (Do not change the expression.)

$$(WX' + Y)[(W + Z)' + XYZ')]$$

2.20 For the following switch circuit:
(a) derive the switching algebra expression that corresponds one to one with the switch circuit.
(b) derive an equivalent switch circuit with a structure consisting of a parallel connection of groups of switches connected in series. (Use 9 switches.)
(c) derive an equivalent switch circuit with a structure consisting of a series connection of groups of switches connected in parallel. (Use 6 switches.)



2.21 In the following circuit, $F = (A' + B)C$. Give a truth table for G so that H is as specified in its truth table. If G can be either 0 or 1 for some input combination, leave its value unspecified.



A	B	C	H
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

2.22 Factor each of the following expressions to obtain a product of sums:
(a) $A'B' + A'CD + A'DE'$ (b) $H'I' + JK$
(c) $A'BC + A'B'C + CD'$ (d) $A'B' + (CD' + E)$
(e) $A'B'C + B'CD' + EF'$ (f) $WX'Y + W'X' + W'Y'$

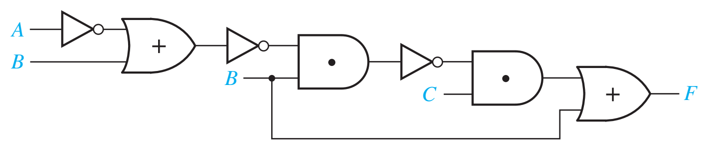
2.23 Factor each of the following expressions to obtain a product of sums:
(a) $W + U'YV$ (b) $TW + UY' + V$
(c) $A'B'C + B'CD' + B'E'$ (d) $ABC + ADE' + ABF'$

2.24 Simplify the following expressions to a minimum sum of products. Only individual variables should be complemented.
(a) $[(XY')' + (X' + Y)'Z]$ (b) $(X + (Y'(Z + W)')')'$
(c) $[(A' + B')' + (A'B'C)' + C'D]'$ (d) $(A + B)CD + (A + B)'$

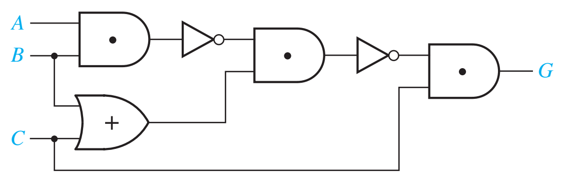
2.25 For each of the following functions find a sum-of-products expression for F'.
(a) $F(P, Q, R, S) = (R' + PQ)S$
(b) $F(W, X, Y, Z) = X + YZ(W + X')$
(c) $F(A, B, C, D) = A' + B' + ACD$

2.26 Find F, G, and H, and simplify:

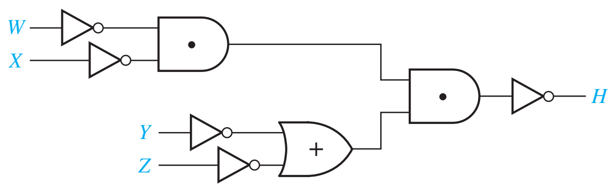
(a)



(b)



(c)



2.27 Draw a circuit that uses two OR gates and two AND gates to realize the following function:

$$F = (V + W + X)(V + X + Y)(V + Z)$$

2.28 Draw a circuit to realize the function:

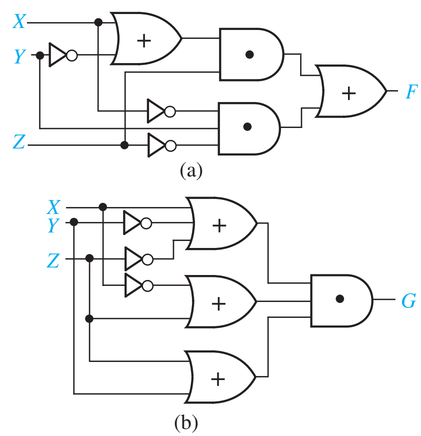
$$F = ABC + A'BC + AB'C + ABC'$$

(a) using one OR gate and three AND gates. The AND gates should have two inputs.
(b) using two OR gates and two AND gates. All of the gates should have two inputs.

2.29 Prove the following equations using truth tables:
(a) $(X + Y)(X' + Z) = XZ + X'Y$
(b) $(X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z)$
(c) $XY + YZ + X'Z = XY + X'Z$

(d) $(A + C)(AB + C') = AB + AC'$
(e) $W'XY + WZ = (W' + Z)(W + XY)$
(*Note:* Parts (a), (b), and (c) are theorems that will be introduced in Unit 3.)

2.30 Show that the following two gate circuits realize the same function.



(a) Can any term be eliminated from this expression by the direct application of the consensus theorem?
(b) If not, add a redundant sum term using the consensus theorem, and use this redundant term to eliminate one of the other terms.
(c) Finally, reduce your expression to a product of three sum terms.

Answer

(a) No
(b) Add $B + C' + G$ (consensus of $A + C' + G$ and $A' + B + G$).
Use $X(X + Y) = X$, where $X = B + C' + G$, to eliminate $B + C' + F + G$.
(c) Now eliminate $B + C' + G$ by consensus. The final answer is

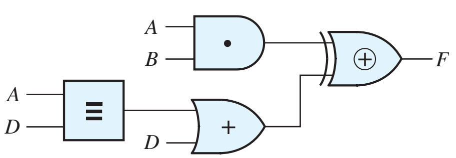
$$Z = (A + C' + G)(A + E + G)(A' + B + G)$$

Problems

3.6 In each case, multiply out to obtain a sum of products: (Simplify where possible.)
(a) $(W + X' + Z')(W' + Y')(W' + X + Z')(W + X')(W + Y + Z)$
(b) $(A + B + C + D)(A' + B' + C + D')(A' + C)(A + D)(B + C + D)$

3.7 Factor to obtain a product of sums. (Simplify where possible.)
(a) $BCD + C'D' + B'C'D + CD$
(b) $A'C'D' + ABD' + A'CD + B'D$

3.8 Write an expression for F and simplify.



3.9 Is the following distributive law valid? $A \oplus BC = (A \oplus B)(A \oplus C)$ Prove your answer.

3.10 (a) Reduce to a minimum sum of products (three terms):
$(X + W)(Y \oplus Z) + XW'$
(b) Reduce to a minimum sum of products (four terms):
$(A \oplus BC) + BD + ACD$
(c) Reduce to a minimum product of sums (three terms):
$(A' + C' + D')(A' + B + C')(A + B + D)(A + C + D)$

3.11 Simplify algebraically to a minimum sum of products (five terms):
$(A + B' + C + E')(A + B' + D' + E)(B' + C' + D' + E')$

3.12 Prove algebraically that the following equation is valid:
$A'CD'E + A'B'D' + ABCE + ABD = A'B'D' + ABD + BCD'E$

3.13 Simplify each of the following expressions:
(a) $KLMN' + K'L'MN + MN'$
(b) $KL'M' + MN' + LM'N'$
(c) $(K + L')(K' + L' + N)(L' + M + N')$
(d) $(K' + L + M' + N)(K' + M' + N + R)(K' + M' + N + R')KM$

3.14 Factor to obtain a product of sums:
(a) $K'L'M + KM'N + KLM + LM'N'$ (four terms)
(b) $KL + K'L' + L'M'N' + LMN'$ (four terms)
(c) $KL + K'L'M + L'M'N + LM'N'$ (four terms)
(d) $K'M'N + KL'N' + K'MN' + LN$ (four terms)
(e) $WXY + WX'Y + WYZ + XYZ'$ (three terms)

3.15 Multiply out to obtain a sum of products:
(a) $(K' + M' + N)(K' + M)(L + M' + N')(K' + L + M)(M + N)$ (three terms)
(b) $(K' + L' + M')(K + M + N')(K + L)(K' + N)(K' + M + N)$
(c) $(K' + L' + M)(K + N')(K' + L + N')(K + L)(K + M + N')$
(d) $(K + L + M)(K' + L' + N')(K' + L' + M')(K + L + N)$
(e) $(K + L + M)(K + M + N)(K' + L' + M')(K' + M' + N')$

3.16 Eliminate the exclusive-OR, and then factor to obtain a minimum product of sums:
(a) $(KL \oplus M) + M'N'$
(b) $M'(K \oplus N') + MN + K'N$

3.17 Algebraically prove identities involving the equivalence (exclusive-NOR) operation:
(a) $x \equiv 0 = x'$
(b) $x \equiv 1 = x$
(c) $x \equiv x = 1$
(d) $x \equiv x' = 0$
(e) $x \equiv y = y \equiv x$
(f) $(x \equiv y) \equiv z = x \equiv (y \equiv z)$
(g) $(x \equiv y)' = x' \equiv y = x \equiv y'$

3.18 Algebraically prove identities involving the exclusive-OR operation:
(a) $x \oplus 0 = x$
(b) $x \oplus 1 = x'$
(c) $x \oplus x = 0$
(d) $x \oplus x' = 1$
(e) $x \oplus y = y \oplus x$
(f) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
(g) $(x \oplus y)' = x' \oplus y = x \oplus y'$

3.19 Algebraically prove the following identities:

(a) $x + y = x \oplus y \oplus xy$

(b) $x + y = x \equiv y \equiv xy$

3.20 Algebraically prove or disprove the following distributive identities:

(a) $x(y \oplus z) = xy \oplus xz$

(b) $x + (y \oplus z) = (x + y) \oplus (x + z)$

(c) $x(y \equiv z) = xy \equiv xz$

(d) $x + (y \equiv z) = (x + y) \equiv (x + z)$

3.21 Simplify each of the following expressions using only the consensus theorem (or its dual):

(a) $BC'D' + ABC' + AC'D + AB'D + A'BD'$ (reduce to three terms)

(b) $W'Y' + WYZ + XY'Z + WX'Y$ (reduce to three terms)

(c) $(B + C + D)(A + B + C)(A' + C + D)(B' + C' + D')$

(d) $W'XY + WXZ + WY'Z + W'Z'$

(e) $A'BC' + BC'D' + A'CD + B'CD + A'BD$

(f) $(A + B + C)(B + C' + D)(A + B + D)(A' + B' + D')$

3.22 Factor $Z = ABC + DE + ACF + AD' + AB'E'$ and simplify it to the form $(X + X)(X + X)(X + X + X + X)$ (where each X represents a literal). Now express Z as a minimum sum of products in the form:

$$XX + XX + XX + XX$$

3.23 Repeat Problem 3.22 for $F = A'B + AC + BC'D' + BEF + BDF$.

3.24 Factor to obtain a product of four terms and then reduce to three terms by applying the consensus theorem: $X'Y'Z' + XYZ$

3.25 Simplify each of the following expressions:

(a) $xy + x'yz' + yz$

(b) $(xy' + z)(x + y')z$

(c) $xy' + z + (x' + y)z'$

(d) $a'd(b' + c) + a'd'(b + c') + (b' + c)(b + c')$

(e) $w'x' + x'y' + yz + w'z'$

(f) $A'BCD + A'BC'D + B'EF + CDE'G + A'DEF + A'B'EF$ (reduce to a sum of three terms)

(g) $[(a' + d' + b'c)(b + d + ac')]' + b'c'd' + a'c'd$ (reduce to three terms)

3.26 Simplify to a sum of three terms:

(a) $A'C'D' + AC' + BCD + A'CD' + A'BC + AB'C'$

(b) $A'B'C' + ABD + A'C + A'CD' + AC'D + AB'C'$

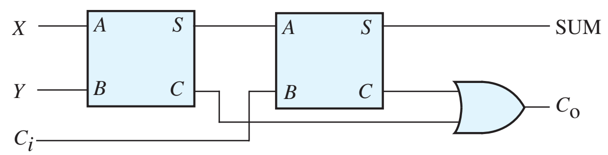
3.27 Reduce to a minimum sum of products:

$$F = WXY' + (W'Y' \equiv X) + (Y \oplus WZ).$$

3.28 Determine which of the following equations are always valid (give an algebraic proof):

(a) $a'b + b'c + c'a = ab' + bc' + ca'$
(b) $(a + b)(b + c)(c + a) = (a' + b')(b' + c')(c' + a')$
(c) $abc + ab'c' + b'cd + bc'd + ad = abc + ab'c' + b'cd + bc'd$
(d) $xy' + x'z + yz' = x'y + xz' + y'z$
(e) $(x + y)(y + z)(x + z) = (x' + y')(y' + z')(x' + z')$
(f) $abc' + ab'c + b'c'd + bcd = ab'c + abc' + ad + bcd + b'c'd$

3.29 The following circuit is implemented using two half-adder circuits. The expressions for the half-adder outputs are $S = A \oplus B$ where $\oplus$ represents the exclusive-OR function, and $C = AB$. Derive simplified sum-of-products expressions for the circuit outputs SUM and C_o. Give the truth table for the outputs.



3.30 The output of a majority circuit is 1 if a majority (more than half) of its inputs are equal to 1, and the output is 0 otherwise. Construct a truth table for a three-input majority circuit and derive a simplified sum-of-products expression for its output.

3.31 Prove algebraically:

(a) $(X' + Y')(X \equiv Z) + (X + Y)(X \oplus Z) = (X \oplus Y) + Z'$
(b) $(W' + X + Y')(W + X' + Y)(W + Y' + Z) = X'Y' + WX + XYZ + W'YZ$
(c) $ABC + A'C'D' + A'BD' + ACD = (A' + C)(A + D')(B + C' + D)$

3.32 Which of the following statements are always true? Justify your answers.

(a) If $A + B = C$, then $AD' + BD' = CD'$
(b) If $A'B + A'C = A'D$, then $B + C = D$
(c) If $A + B = C$, then $A + B + D = C + D$
(d) If $A + B + C = C + D$, then $A + B = D$

3.33 Find all possible terms that could be added to each expression using the consensus theorem. Then reduce to a minimum sum of products.

(a) $A'C' + BC + AB' + A'BD + B'C'D' + ACD'$
(b) $A'C'D' + BC'D + AB'C' + A'BC$

3.34 Simplify the following expression to a sum of two terms and then factor the result to obtain a product of sums: $abd'f' + b'cegh' + abd'f + acd'e + b'ce$

3.35 Multiply out the following expression and simplify to obtain a sum-of-products expression with three terms: $(a + c)(b' + d)(a + c' + d')(b' + c' + d')$

3.36 Factor and simplify to obtain a product-of-sums expression with four terms: $abc' + d'e + ace + b'c'd'$

3.37 (a) Show that $x \oplus y = (x \equiv y)'$
(b) Realize $a'b'c' + a'bc + ab'c + abc'$ using only two-input equivalence gates.

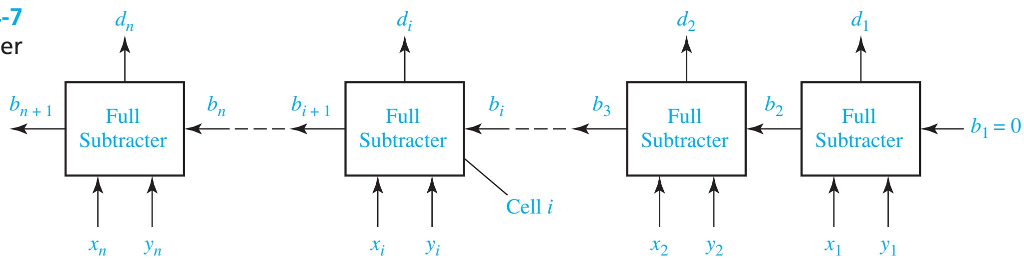


FIGURE 4-7
Parallel Subtracter

TABLE 4-6
Truth Table for Binary Full Subtracter

x_i y_i b_i	b_{i+1} d_i
0 0 0	0 0
0 0 1	1 1
0 1 0	1 1
0 1 1	1 0
1 0 0	0 1
1 0 1	0 0
1 1 0	0 0
1 1 1	1 1

y_i are subtracted from x_i to form the difference d_i, and a borrow signal ($b_{i+1} = 1$) is generated if it is necessary to borrow from the next column.

Table 4-6 gives the truth table for a binary full subtracter. Consider the following case, where $x_i = 0$, $y_i = 1$ and $b_i = 1$:

	Column i Before Borrow	Column i After Borrow	
x_i	0	10	
$-b_i$	-1	-1	
$-y_i$	-1	-1	
d_i		0	($b_{i+1} = 1$)

Note that in column i, we cannot immediately subtract y_i and b_i from x_i. Hence, we must borrow from column $i + 1$. Borrowing 1 from column $i + 1$ is equivalent to setting b_{i+1} to 1 and adding 10 (2_{10}) to x_i. We then have $d_i = 10 - 1 - 1 = 0$. Verify that Table 4-6 is correct for the other input combinations and use it to work out several examples of binary subtraction.

Problems

4.1 Represent each of the following sentences by a Boolean equation.
(a) The company safe should be unlocked only when Mr. Jones is in the office or Mr. Evans is in the office, and only when the company is open for business, and only when the security guard is present.

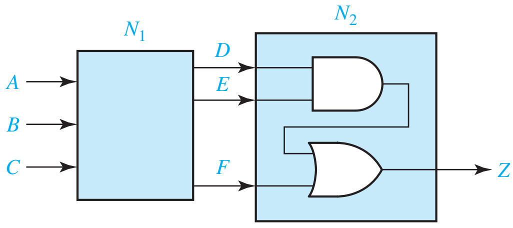
(b) You should wear your overshoes if you are outside in a heavy rain and you are wearing your new suede shoes, or if your mother tells you to.
(c) You should laugh at a joke if it is funny, it is in good taste, and it is not offensive to others, or if it is told in class by your professor (regardless of whether it is funny and in good taste) and it is not offensive to others.
(d) The elevator door should open if the elevator is stopped, it is level with the floor, and the timer has not expired, or if the elevator is stopped, it is level with the floor, and a button is pressed.

4.2 A flow rate sensing device used on a liquid transport pipeline functions as follows. The device provides a 5-bit output where all five bits are zero if the flow rate is less than 10 gallons per minute. The first bit is 1 if the flow rate is at least 10 gallons per minute; the first and second bits are 1 if the flow rate is at least 20 gallons per minute; the first, second, and third bits are 1 if the flow rate is at least 30 gallons per minute; and so on. The five bits, represented by the logical variables A, B, C, D, and E, are used as inputs to a device that provides two outputs Y and Z.
(a) Write an equation for the output Y if we want Y to be 1 iff the flow rate is less than 30 gallons per minute.
(b) Write an equation for the output Z if we want Z to be 1 iff the flow rate is at least 20 gallons per minute but less than 50 gallons per minute.

4.3 Given $F_1 = \Sigma\, m(0, 4, 5, 6)$ and $F_2 = \Sigma\, m(0, 3, 6, 7)$ find the minterm expression for $F_1 + F_2$. State a general rule for finding the expression for $F_1 + F_2$ given the minterm expansions for F_1 and F_2. Prove your answer by using the general form of the minterm expansion.

4.4 (a) How many switching functions of two variables (x and y) are there?
(b) Give each function in truth table form and in reduced algebraic form.

4.5 A combinational circuit is divided into two subcircuits N_1 and N_2 as shown. The truth table for N_1 is given. Assume that the input combinations $ABC = 110$ and $ABC = 010$ never occur. Change as many of the values of D, E, and F to don't-cares as you can without changing the value of the output Z.



A	B	C	D	E	F
0	0	0	1	1	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	0	1	0
1	1	1	0	0	0

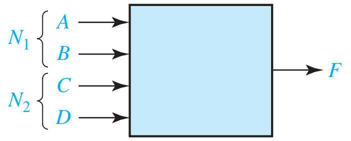
4.6 Work (a) and (b) with the following truth table:

A	B	C	F	G
0	0	0	1	0
0	0	1	X	1
0	1	0	0	X
0	1	1	0	1
1	0	0	0	0
1	0	1	X	1
1	1	0	1	X
1	1	1	1	1

(a) Find the simplest expression for F, and specify the values of the don't-cares that lead to this expression.
(b) Repeat (a) for G. (*Hint:* Can you make G the same as one of the inputs by properly choosing the values for the don't-care?)

4.7 Each of three coins has two sides, heads and tails. Represent the heads or tails status of each coin by a logical variable (A for the first coin, B for the second coin, and C for the third) where the logical variable is 1 for heads and 0 for tails. Write a logic function $F(A, B, C)$ which is 1 iff exactly one of the coins is heads after a toss of the coins. Express F
(a) as a minterm expansion.
(b) as a maxterm expansion.

4.8 A switching circuit has four inputs as shown. A and B represent the first and second bits of a binary number N_1. C and D represent the first and second bits of a binary number N_2. The output is to be 1 only if the product $N_1 \times N_2$ is less than or equal to 2.
(a) Find the minterm expansion for F.
(b) Find the maxterm expansion for F.
Express your answers in both decimal notation and algebraic form.



4.9 Given: $F(a, b, c) = abc' + b'$.
(a) Express F as a minterm expansion. (Use m-notation.)
(b) Express F as a maxterm expansion. (Use M-notation.)
(c) Express F' as a minterm expansion. (Use m-notation.)
(d) Express F' as a maxterm expansion. (Use M-notation.)

4.10 Work Problem 4.9 using:
$F(a, b, c, d) = (a + b + d)(a' + c)(a' + b' + c')(a + b + c' + d')$

4.11 (a) Implement a full subtracter using a minimum number of gates.
(b) Compare the logic equations for the full adder and full subtracter. What is the relation between s_i and d_i? Between c_{i+1} and b_{i+1}?

4.12 Design a circuit which will perform the following function on three 4-bit numbers:

$$(X_3X_2X_1X_0 + Y_3Y_2Y_1Y_0) - Z_3Z_2Z_1Z_0$$

It will give a result $S_3S_2S_1S_0$, a carry, and a borrow. Use eight full adders and any other type of gates. Assume that negative numbers are represented in 2's complement.

4.13 A combinational logic circuit has four inputs (A, B, C, and D) and one output Z. The output is 1 iff the input has three consecutive 0's or three consecutive 1's. For example, if $A = 1, B = 0, C = 0$, and $D = 0$, then $Z = 1$, but if $A = 0, B = 1, C = 0$, and $D = 0$, then $Z = 0$. Design the circuit using one four-input OR gate and four three-input AND gates.

4.14 Design a combinational logic circuit which has one output Z and a 4-bit input $ABCD$ representing a binary number. Z should be 1 iff the input is at least 5, but is no greater than 11. Use one OR gate (three inputs) and three AND gates (with no more than three inputs each).

4.15 A logic circuit realizing the function f has four inputs A, B, C, and D. The three inputs A, B, and C are the binary representation of the digits 0 through 7 with A being the most-significant bit. The input D is an odd-parity bit, i.e., the value of D is such that A, B, C, and D always contain an odd number of 1's. (For example, the digit 1 is represented by $ABC = 001$ and $D = 0$, and the digit 3 is represented by $ABCD = 0111$.) The function f has value 1 if the input digit is a prime number. (A number is prime if it is divisible only by itself and 1; 1 is considered to be prime and 0 is not.)
(a) List the minterms and don't-care minterms of f in algebraic form.
(b) List the maxterms and don't-care maxterms of f in algebraic form.

4.16 A priority encoder circuit has four inputs, x_3, x_2, x_1, and x_0. The circuit has three outputs: z, y_1, and y_0. If one of the inputs is 1, z is 1 and y_1 and y_0 represent a 2-bit, binary number whose value equals the index of the highest numbered input that is 1. For example, if x_2 is 1 and x_3 is 0, then the outputs are $z = 1$ and $y_1 = 1$ and $y_0 = 0$. If all inputs are 0, $z = 0$ and y_1 and y_0 are don't-cares.
(a) List in decimal form the minterms and don't-care minterms of each output.
(b) List in decimal form the maxterms and don't-care maxterms of each output.

4.17 The 9's complement of a decimal digit d (0 to 9) is defined to be $9 - d$. A logic circuit produces the 9's complement of an input digit where the input and output digits are represented in BCD. Label the inputs A, B, C, and D, and label the outputs W, X, Y and Z.

(a) Determine the minterms and don't-care minterms for each of the outputs.

(b) Determine the maxterms and don't-care maxterms for each of the outputs.

4.18 Repeat Problem 4.17 for the case where the input and output digits are represented using the 4-2-2-1 weighted code. (If only one weight of 2 is required for decimal digits less than 5, select the rightmost 2. In addition, select the codes so that $W = A'$, $X = B'$, $Y = C'$, and $Z = D'$. (There are two possible codes with these restrictions.)

4.19 Each of the following sentences has two possible interpretations depending on whether the AND or OR is done first. Write an equation for each interpretation.

(a) The buzzer will sound if the key is in the ignition switch, and the car door is open, or the seat belts are not fastened.

(b) You will gain weight if you eat too much, or you do not exercise enough, and your metabolism rate is too low.

(c) The speaker will be damaged if the volume is set too high, and loud music is played, or the stereo is too powerful.

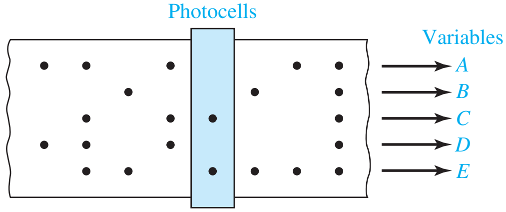
(d) The roads will be very slippery if it snows, or it rains, and there is oil on the road.

4.20 A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A, B, and C are 1 if there is a key inserted into lock 1, 2, or 3, respectively. Write an equation for the variable Z which is 1 iff the door should open.

4.21 A paper tape reader used as an input device to a computer has five rows of holes as shown. A hole punched in the tape indicates a logic 1, and no hole indicates a logic 0. As each hole pattern passes under the photocells, the pattern is translated into logic signals on lines A, B, C, D, and E. All patterns of holes indicate a valid character with two exceptions. A pattern consisting of none of the possible holes punched is not used because it is impossible to distinguish between this pattern and the unpunched space between patterns. An incorrect pattern punched on the tape is erased by punching all five holes in that position. Therefore, a valid character punched on the tape will have at least one hole but will not have all five holes punched.

(a) Write an equation for a variable Z which is 1 iff a valid character is being read.

(b) Write an equation for a variable Y which is 1 iff the hole pattern being read has holes punched only in rows C and E.



4.22 A computer interface to a line printer has seven data lines that control the movement of the paper and the print head and determine which character to print. The data lines are labeled A, B, C, D, E, F, and G, and each represents a binary 0 or 1. When the data lines are interpreted as a 7-bit binary number with line A being the most significant bit, the data lines can represent the numbers 0 to 127_{10}. The number 13_{10} is the command to return the print head to the beginning of a line, the number 10_{10} means to advance the paper by one line, and the numbers 32_{10} to 127_{10} represent printing characters.

(a) Write an equation for the variable X which is 1 iff the data lines indicate a command to return the print head to the beginning of the line.

(b) Write an equation for the variable Y which is 1 iff there is an advance paper command on the data lines.

(c) Write an equation for the variable Z which is 1 iff the data lines indicate a printable character. (*Hint:* Consider the binary representations of the numbers 0–31 and 32–127 and write the equation for Z with only two terms.)

4.23 Given $F_1 = \Pi\, M(0, 4, 5, 6)$ and $F_2 = \Pi\, M(0, 4, 7)$, find the maxterm expansion for F_1F_2. State a general rule for finding the maxterm expansion of F_1F_2 given the maxterm expansions of F_1 and F_2.
Prove your answer by using the general form of the maxterm expansion.

4.24 Given $F_1 = \Pi\, M(0, 4, 5, 6)$ and $F_2 = \Pi\, M(0, 4, 7)$, find the maxterm expansion for $F_1 + F_2$.
State a general rule for finding the maxterm expansion of $F_1 + F_2$, given the maxterm expansions of F_1 and F_2.
Prove your answer by using the general form of the maxterm expansion.

4.25 Four chairs are placed in a row:

$$\boxed{A}\ \boxed{B}\ \boxed{C}\ \boxed{D}$$

Each chair may be occupied (1) or empty (0). Give the minterm and maxterm expansion for each logic function described.

(a) $F(A, B, C, D)$ is 1 iff there are no adjacent empty chairs.

(b) $G(A, B, C, D)$ is 1 iff the chairs on the ends are both empty.

(c) $H(A, B, C, D)$ is 1 iff at least three chairs are full.

(d) $J(A, B, C, D)$ is 1 iff there are more people sitting in the left two chairs than in the right two chairs.

4.26 Four chairs (A, B, C, and D) are placed in a circle: A next to B, B next to C, C next to D, and D next to A. Each chair may be occupied (1) or empty (0). Give the minterm and maxterm expansion for each of the following logic functions:

(a) $F(A, B, C, D)$ is 1 iff there are no adjacent empty chairs.

(b) $G(A, B, C, D)$ is 1 iff there are at least three adjacent empty chairs.

(c) $H(A, B, C, D)$ is 1 iff at least three chairs are full.
(d) $J(A, B, C, D)$ is 1 iff there are more people sitting in chairs A and B than chairs C and D.

4.27 Given $f(a, b, c) = a(b + c')$.
(a) Express f as a minterm expansion (use m-notation).
(b) Express f as maxterm expansion (use M-notation).
(c) Express f' as a minterm expansion (use m-notation).
(d) Express f' as a maxterm expansion (use M-notation).

4.28 Work Problem 4.27 using $f(a, b, c, d) = acd + bd' + a'c'd + ab'cd + a'b'cd'$.

4.29 Find both the minterm expansion and maxterm expansion for the following functions, using *algebraic manipulations*:
(a) $f(A, B, C, D) = AB + A'CD$
(b) $f(A, B, C, D) = (A + B + D')(A' + C)(C + D)$

4.30 Given $F'(A, B, C, D) = \Sigma\, m(0, 1, 2, 6, 7, 13, 15)$.
(a) Find the minterm expansion for F (both decimal and algebraic form).
(b) Find the maxterm expansion for F (both decimal and algebraic form).

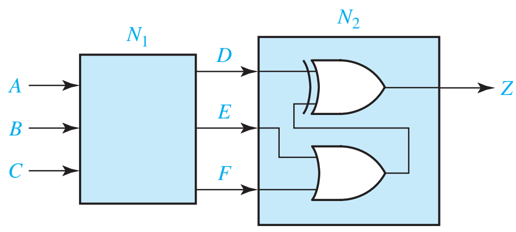
4.31 Repeat Problem 4.30 for $F'(A, B, C, D) = \Sigma\, m(1, 2, 5, 6, 10, 15)$.

4.32 Work parts (a) through (d) with the given truth table.

A	B	C	F_1	F_2	F_3	F_4
0	0	0	1	1	0	1
0	0	1	X	0	0	0
0	1	0	0	1	X	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	X	0	1	0
1	1	0	0	X	X	X
1	1	1	1	X	1	X

(a) Find the simplest expression for F_1, and specify the values for the don't-cares that lead to this expression.
(b) Repeat for F_2.
(c) Repeat for F_3.
(d) Repeat for F_4.

4.33 Work Problem 4.5 using the following circuits and truth table. Assume that the input combinations of $ABC = 011$ and $ABC = 110$ will never occur.



A	B	C	D	E	F
0	0	0	1	1	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	0	0	1
1	1	0	0	0	1
1	1	1	1	0	1

4.34 Work Problem 4.7 for the following logic functions:
(a) $G_1(A, B, C)$ is 1 iff all the coins landed on the same side (heads or tails).
(b) $G_2(A, B, C)$ is 1 iff the second coin landed on the same side as the first coin.

4.35 A combinational circuit has four inputs (A, B, C, D) and three outputs (X, Y, Z). XYZ represents a binary number whose value equals the number of 1's at the input. For example if $ABCD = 1011$, $XYZ = 011$.
(a) Find the minterm expansions for X, Y, and Z.
(b) Find the maxterm expansions for Y and Z.

4.36 A combinational circuit has four inputs (A, B, C, D) and four outputs (W, X, Y, Z). $WXYZ$ represents an excess-3 coded number whose value equals the number of 1's at the input. For example, if $ABCD = 1101$, $WXYZ = 0110$.
(a) Find the minterm expansions for X, Y, and Z.
(b) Find the maxterm expansions for Y and Z.

4.37 A combinational circuit has four inputs (A, B, C, D), which represent a binary-coded-decimal digit. The circuit has two groups of four outputs—S, T, U, V, and W, X, Y, Z. Each group represents a BCD digit. The output digits represent a decimal number which is five times the input number. For example, if $ABCD = 0111$, the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs.
(a) Construct the truth table.
(b) Write down the minimum expressions for the outputs by inspection of the truth table. (*Hint:* Try to match output columns in the table with input columns.)

4.38 Work Problem 4.37 where the BCD outputs represent a decimal number that is 1 more than four times the input number. For example, if $ABCD = 0011$, the outputs are 0001 0011.

4.39 Design a circuit which will add a 4-bit binary number to a 5-bit binary number. Use five full adders. Assume negative numbers are represented in 2's complement. (*Hint:* How do you make a 4-bit binary number into a 5-bit binary number, without making a negative number positive or a positive number negative? Try writing down the representation for −3 as a 3-bit 2's complement number, a 4-bit 2's complement number, and a 5-bit 2's complement number. Recall that one way to find the 2's complement of a binary number is to complement *all* bits to the left of the first 1.)

4.40 A half adder is a circuit that adds two bits to give a sum and a carry. Give the truth table for a half adder, and design the circuit using only two gates. Then design a circuit which will find the 2's complement of a 4-bit binary number. Use four half adders and any additional gates. (*Hint:* Recall that one way to find the 2's complement of a binary number is to complement *all* bits, and then add 1.)

4.41 (a) Write the switching function $f(x, y) = x + y$ as a sum of minterms and as a product of maxterms.

(b) Consider the Boolean algebra of four elements $\{0, 1, a, b\}$ specified by the following operation tables and the Boolean function $f(x, y) = ax + by$ where a and b are two of the elements in the Boolean algebra. Write $f(x, y)$ in a sum-of-minterms form.

(c) Write the Boolean function of part (b) in a product-of-maxterms form.

(d) Give a table of combinations for the Boolean function of Part (b). (*Note*: The table of combinations has 16 rows, not just 4.)

(e) Which four rows of the table of combinations completely specify the function of Part (b)? Verify your answer.

	′
0	1
1	0
a	*b*
b	*a*

+	0	1	*a*	*b*
0	0	1	*a*	*b*
1	1	1	1	1
a	*a*	1	*a*	1
b	*b*	1	1	*b*

•	0	1	*a*	*b*
0	0	0	0	0
1	0	1	*a*	*b*
a	0	*a*	*a*	0
b	0	*b*	0	*b*

4.42 (a) If m_1 and m_2 are minterms of n variables, prove that $m_1 + m_2 = m_1 \oplus m_2$.

(b) Prove that any switching function can be written as the exclusive-OR sum of products where each product does not contain a complemented literal. [*Hint*: Start with the function written as a sum of minterms and use Part (a).]

Write down two different minimum sum-of-products expressions for f.

$f =$ ______________________

$f =$ ______________________

Answer:

$$f = a'd'e' + ace + a'ce' + bde' + \begin{Bmatrix} abc \\ or \\ bce' \end{Bmatrix} + \begin{Bmatrix} b'c'de + a'c'de \\ b'c'de + a'bc'd \\ ab'de + a'c'de \end{Bmatrix}$$

Problems

5.3 Find the minimum sum of products for each function using a Karnaugh map.
(a) $f_1(a, b, c) = m_0 + m_2 + m_5 + m_6$ (b) $f_2(d, e, f) = \Sigma\, m(0,1,2,4)$
(c) $f_3(r, s, t) = rt' + r's' + r's$ (d) $f_4(x, y, z) = M_0 \bullet M_5$

5.4 (a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.)

$$F(A,B,C,D) = BD' + B'CD + ABC + ABC'D + B'D'$$

(b) Find the minimum sum of products.
(c) Find the minimum product of sums.

5.5 A switching circuit has two control inputs (C_1 and C_2), two data inputs (X_1 and X_2), and one output (Z). The circuit performs one of the logic operations AND, OR, EQU (equivalence), or XOR (exclusive OR) on the two data inputs. The function performed depends on the control inputs:

C_1	C_2	Function Performed by Circuit
0	0	OR
0	1	XOR
1	0	AND
1	1	EQU

(a) Derive a truth table for Z.
(b) Use a Karnaugh map to find a minimum AND-OR gate circuit to realize Z.

5.6 Find the minimum sum-of-products expression for each function. Underline the essential prime implicants in your answer and tell which minterm makes each one essential.
(a) $f(a, b, c, d) = \Sigma\, m(0, 1, 3, 5, 6, 7, 11, 12, 14)$
(b) $f(a, b, c, d) = \Pi\, M(1, 9, 11, 12, 14)$
(c) $f(a, b, c, d) = \Pi\, M(5, 7, 13, 14, 15) \bullet \Pi\, D(1, 2, 3, 9)$

5.7 Find the minimum sum-of-products expression for each function.
(a) $f(a, b, c, d) = \Sigma\, m(0, 2, 3, 4, 7, 8, 14)$
(b) $f(a, b, c, d) = \Sigma\, m(1, 2, 4, 15) + \Sigma\, d(0, 3, 14)$
(c) $f(a, b, c, d) = \Pi\, M(1, 2, 3, 4, 9, 15)$
(d) $f(a, b, c, d) = \Pi\, M(0, 2, 4, 6, 8) \bullet \Pi\, D(1, 12, 9, 15)$

5.8 Find the minimum sum of products and the minimum product of sums for each function:
(a) $f(a, b, c, d) = \Pi\, M(0, 1, 6, 8, 11, 12) \bullet \Pi\, D(3, 7, 14, 15)$
(b) $f(a, b, c, d) = \Sigma\, m(1, 3, 4, 11) + \Sigma\, d(2, 7, 8, 12, 14, 15)$

5.9 Find the minimum sum of products and the minimum product of sums for each function:
(a) $F(A, B, C, D, E) = \Sigma\, m(0, 1, 2, 6, 7, 9, 10, 15, 16, 18, 20, 21, 27, 30) + \Sigma\, d(3, 4, 11, 12, 19)$
(b) $F(A, B, C, D, E) = \Pi\, M(0, 3, 6, 9, 11, 19, 20, 24, 25, 26, 27, 28, 29, 30) \bullet \Pi\, D(1, 2, 12, 13)$

5.10 $F(a, b, c, d, e) = \Sigma\, m(0, 3, 4, 5, 6, 7, 8, 12, 13, 14, 16, 21, 23, 24, 29, 31)$
(a) Find the essential prime implicants using a Karnaugh map, and indicate why each one of the chosen prime implicants is essential (there are four essential prime implicants).
(b) Find all of the prime implicants by using the Karnaugh map. (There are nine in all.)

5.11 Find a minimum product-of-sums solution for f. Underline the essential prime implicants.

$$f(a, b, c, d, e) = \Sigma\, m(2, 4, 5, 6, 7, 8, 10, 12, 14, 16, 19, 27, 28, 29, 31) + \Sigma\, d(1, 30)$$

5.12 Given $F = AB'D' + A'B + A'C + CD$.
(a) Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebric notation).
(b) Use a Karnaugh map to find the minimum sum-of-products form for F'.
(c) Find the minimum product of sums for F.

5.13 Find the minimum sum of products for the given expression. Then, make minterm 5 a don't-care term and verify that the minimum sum of products is unchanged. Now, start again with the original expression and find each minterm which could *individually* be made a don't-care without changing the minimum sum of products.

$$F(A, B, C, D) = A'C' + B'C + ACD' + BC'D$$

5.14 Find the minimum sum-of-products expressions for each of these functions.
(a) $f_1(A, B, C) = m_1 + m_2 + m_5 + m_7$
(b) $f_2(d, e, f) = \Sigma\, m(1, 5, 6, 7)$
(c) $f_3(r, s, t) = rs' + r's' + st'$
(d) $f_4(a, b, c) = m_0 + m_2 + m_3 + m_7$
(e) $f_5(n, p, q) = \Sigma\, m(1, 3, 4, 5)$
(f) $f_6(x, y, z) = M_1 M_7$

5.15 Find the minimum product-of-sums expression for each of the functions in Problem 5.14.

5.16 Find the minimum sum of products for each of these functions.

(a) $f_1(A, B, C) = m_1 + m_3 + m_4 + m_6$ (b) $f_2(d, e, f) = \Sigma\, m(1, 4, 5, 7)$
(c) $f_3(r, s, t) = r't' + rs' + rs$ (d) $f_1(a, b, c) = m_3 + m_4 + m_6 + m_7$
(e) $f_2(n, p, q) = \Sigma\, m(2, 3, 5, 7)$ (f) $f_4(x, y, z) = M_3M_6$

5.17 (a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.)

$$F(A,B,C,D) = A'B' + CD' + ABC + A'B'CD' + ABCD'$$

(b) Find the minimum sum of products.
(c) Find the minimum product of sums.

5.18 Work Problem 5.17 for the following:

$$f(A,B,C,D) = A'B' + A'B'C' + A'BD' + AC'D + A'BD + AB'CD'$$

5.19 A switching circuit has two control inputs (C_1 and C_2), two data inputs (X_1 and X_2), and one output (Z). The circuit performs logic operations on the two data inputs, as shown in this table:

C_1	C_2	Function Performed by Circuit
0	0	X_1X_2
0	1	$X_1 \oplus X_2$
1	0	$X_1' + X_2$
1	1	$X_1 \equiv X_2$

(a) Derive a truth table for Z.
(b) Use a Karnaugh map to find a minimum OR-AND gate circuit to realize Z.

5.20 Use Karnaugh maps to find all possible minimum sum-of-products expressions for each function.

(a) $F(a, b, c) = \Pi\, M(3, 4)$
(b) $g(d, e, f) = \Sigma\, m(1, 4, 6) + \Sigma\, d(0, 2, 7)$
(c) $F(p, q, r) = (p + q' + r)(p' + q + r')$
(d) $F(s, t, u) = \Sigma\, m(1, 2, 3) + \Sigma\, d(0, 5, 7)$
(e) $f(a, b, c) = \Pi\, M(2, 3, 4)$
(f) $G(D, E, F) = \Sigma\, m(1, 6) + \Sigma\, d(0, 3, 5)$

5.21 Simplify the following expression first by using a map and then by using Boolean algebra. Use the map as a guide to determine which theorems to apply to which terms for the algebraic simplification.

$$F = a'b'c' + a'c'd + bcd + abc + ab'$$

5.22 Find all prime implicants and all minimum sum-of-products expressions for each of the following functions.
(a) $f(A,B,C,D) = \Sigma\, m(4, 11, 12, 13, 14) + \Sigma\, d(5, 6, 7, 8, 9, 10)$
(b) $f(A,B,C,D) = \Sigma\, m(3, 11, 12, 13, 14) + \Sigma\, d(5, 6, 7, 8, 9, 10)$
(c) $f(A,B,C,D) = \Sigma\, m(1, 2, 4, 13, 14) + \Sigma\, d(5, 6, 7, 8, 9, 10)$
(d) $f(A,B,C,D) = \Sigma\, m(4, 15) + \Sigma\, d(5, 6, 7, 8, 9, 10)$
(e) $f(A,B,C,D) = \Sigma\, m(3, 4, 11, 15) + \Sigma\, d(5, 6, 7, 8, 9, 10)$
(f) $f(A,B,C,D) = \Sigma\, m(4) + \Sigma\, d(5, 6, 7, 8, 9, 10, 11, 12, 13, 14)$
(g) $f(A,B,C,D) = \Sigma\, m(4, 15) + \Sigma\, d(0, 1, 2, 5, 6, 7, 8, 9, 10)$

5.23 For each function in Problem 5.22, find all minimum product-of-sums expressions.

5.24 Find the minimum sum-of-products expression for
(a) $\Sigma\, m(0, 2, 3, 5, 6, 7, 11, 12, 13)$
(b) $\Sigma\, m(2, 4, 8) + \Sigma\, d(0, 3, 7)$
(c) $\Sigma\, m(1, 5, 6, 7, 13) + \Sigma\, d(4, 8)$
(d) $f(w, x, y, z) = \Sigma\, m(0, 3, 5, 7, 8, 9, 10, 12, 13) + \Sigma\, d(1, 6, 11, 14)$
(e) $\Pi\, M(0, 1, 2, 5, 7, 9, 11) \bullet \Pi\, D(4, 10, 13)$

5.25 Work Problem 5.24 for the following:
(a) $f(a, b, c, d) = \Sigma\, m(1, 3, 4, 5, 7, 9, 13, 15)$
(b) $f(a, b, c, d) = \Pi\, M(0, 3, 5, 8, 11)$
(c) $f(a, b, c, d) = \Sigma\, m(0, 2, 6, 9, 13, 14) + \Sigma\, d(3, 8, 10)$
(d) $f(a, b, c, d) = \Pi\, M(0, 2, 6, 7, 9, 12, 13) \bullet \Pi\, D(1, 3, 5)$

5.26 Find the minimum product of sums for the following. Underline the essential prime implicants in your answer.
(a) $\Pi\, M(0, 2, 4, 5, 6, 9, 14) \bullet \Pi\, D(10, 11)$
(b) $\Sigma\, m(1, 3, 8, 9, 15) + \Sigma\, d(6, 7, 12)$

5.27 Find a minimum sum-of-products and a minimum product-of-sums expression for each function:
(a) $f(A, B, C, D) = \Pi\, M(0, 2, 10, 11, 12, 14, 15) \bullet \Pi\, D(5, 7)$
(b) $f(w, x, y, z) = \Sigma\, m(0, 3, 5, 7, 8, 9, 10, 12, 13) + \Sigma\, d(1, 6, 11, 14)$

5.28 A logic *circuit* realizes the function $F(a, b, c, d) = a'b' + a'cd + ac'd + ab'd'$. Assuming that $a = c$ never occurs when $b = d = 1$, find a simplified expression for F.

5.29 Given $F = AB'D' + A'B + A'C + CD$.
(a) Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebric notation).

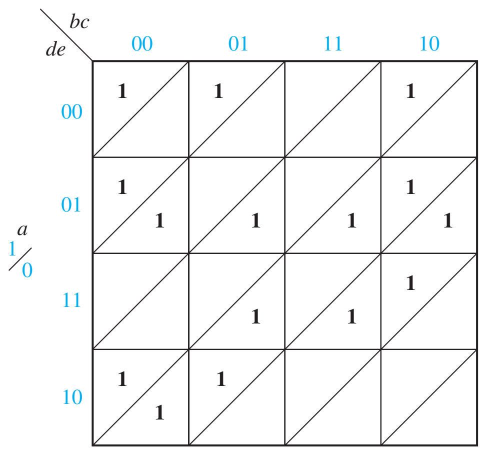
(b) Use a Karnaugh map to find the minimum sum-of-products form for F'.
(c) Find the minimum product of sums for F.

5.30 Assuming that the inputs $ABCD = 0101$, $ABCD = 1001$, $ABCD = 1011$ never occur, find a simplified expression for

$$F = A'BC'D + A'B'D + A'CD + ABD + ABC$$

5.31 Find all of the prime implicants for each of the functions plotted on page 150.

5.32 Find all of the prime implicants for each of the plotted functions:



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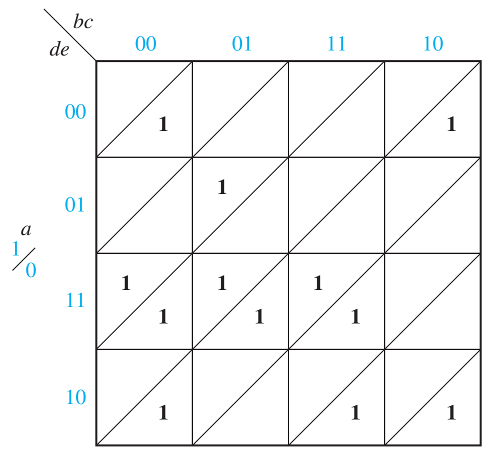

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5.33 Given that $f(a, b, c, d, e) = \Sigma\, m(6, 7, 9, 11, 12, 13, 16, 17, 18, 20, 21, 23, 25, 28)$, using a Karnaugh map,
(a) Find the essential prime implicants (three).
(b) Find the minimum sum of products (7 terms).
(c) Find all of the prime implicants (twelve).

5.34 A logic circuit realizing the function f has four inputs a, b, c, d. The three inputs a, b, and c are the binary representation of the digits 0 through 7 with a being the most significant bit. The input d is an odd-parity bit; that is, the value of d is such that a, b, c, and d always contains an odd number of 1's. (For example, the digit 1 is represented by $abc = 001$ and $d = 0$, and the digit 3 is represented by $abcd = 0111$.) The function f has value 1 if the input digit is a prime number. (A number is prime if it is divisible only by itself and 1; 1 is considered to be prime, and 0 is not.)
(a) Draw a Karnaugh map for f.
(b) Find all prime implicants of f.
(c) Find all minimum sum of products for f.
(d) Find all prime implicants of f'.
(e) Find all minimum product of sums for f.

5.35 The decimal digits 0 though 9 are represented using five bits A, B, C, D, and E. The bits A, B, C, and D are the BCD representation of the decimal digit, and bit E is a parity bit that makes the five bits have odd parity. The function $F(A, B, C, D, E)$ has value 1 if the decimal digit represented by A, B, C, D, and E is divisible by either 3 or 4. (Zero is divisible by 3 and 4.)

(a) Draw a Karnaugh map for f.
(b) Find all prime implicants of f. (Prime implicants containing only don't-cares need not be included.)
(c) Find all minimum sum of products for f.
(d) Find all prime implicants of f'.
(e) Find all minimum product of sums for f.

5.36 Rework Problem 5.35 assuming the decimal digits are represented in excess-3 rather than BCD.

5.37 The function $F(A, B, C, D, E) = \Sigma\, m(1, 7, 8, 13, 16, 19) + \Sigma\, d(0, 3, 5, 6, 9, 10, 12, 15, 17, 18, 20, 23, 24, 27, 29, 30)$.

(a) Draw a Karnaugh map for f.
(b) Find all prime implicants of f. (Prime implicants containing only don't-cares need not be included.)
(c) Find all minimum sum of products for f.
(d) Find all prime implicants of f'.
(e) Find all minimum product of sums for f.

5.38 $F(a, b, c, d, e) = \Sigma\, m(0, 1, 4, 5, 9, 10, 11, 12, 14, 18, 20, 21, 22, 25, 26, 28)$

(a) Find the essential prime implicants using a Karnaugh map, and indicate why each one of the chosen prime implicants is essential (there are four essential prime implicants).
(b) Find all of the prime implicants by using the Karnaugh map (there are 13 in all).

5.39 Find the minimum sum-of-products expression for F. Underline the essential prime implicants in this expression.

(a) $f(a, b, c, d, e) = \Sigma\, m(0, 1, 3, 4, 6, 7, 8, 10, 11, 15, 16, 18, 19, 24, 25, 28, 29, 31) + \Sigma\, d(5, 9, 30)$
(b) $f(a, b, c, d, e) = \Sigma\, m(1, 3, 5, 8, 9, 15, 16, 20, 21, 23, 27, 28, 31)$

5.40 Work Problem 5.39 with

$$F(A, B, C, D, E) = \Pi\, M(2, 3, 4, 8, 9, 10, 14, 15, 16, 18, 19, 20, 23, 24, 30, 31)$$

5.41 Find the minimum sum-of-products expression for F. Underline the essential prime implicants in your expression.

$$F(A, B, C, D, E) = \Sigma\, m(0, 2, 3, 5, 8, 11, 13, 20, 25, 26, 30) + \Sigma\, d(6, 7, 9, 24)$$

5.42 $F(V, W, X, Y, Z) = \Pi\, M(0, 3, 5, 6, 7, 8, 11, 13, 14, 15, 18, 20, 22, 24) \bullet \Pi\, D(1, 2, 16, 17)$

(a) Find a minimum sum-of-products expression for F. Underline the essential prime implicants.

(b) Find a minimum product-of-sums expression for F. Underline the essential prime implicants.

5.43 Find the minimum product of sums for

(a) $F(a, b, c, d, e) = \Sigma\, m(1, 2, 3, 4, 5, 6, 25, 26, 27, 28, 29, 30, 31)$

(b) $F(a, b, c, d, e) = \Sigma\, m(1, 5, 12, 13, 14, 16, 17, 21, 23, 24, 30, 31) + \Sigma\, d(0, 2, 3, 4)$

5.44 Find a minimum product-of-sums expression for each of the following functions:

(a) $F(v, w, x, y, z) = \Sigma\, m(4, 5, 8, 9, 12, 13, 18, 20, 21, 22, 25, 28, 30, 31)$

(b) $F(a, b, c, d, e) = \Pi\, M(2, 4, 5, 6, 8, 10, 12, 13, 16, 17, 18, 22, 23, 24) \cdot \Pi\, D(0, 11, 30, 31)$

5.45 Find the minimum sum of products for each function. Then, make the specified minterm a don't-care and verify that the minimum sum of products is unchanged. Now, start again with the original expression and find each minterm which could individually be made a don't-care, without changing the minimum sum of products.

(a) $F(A, B, C, D) = A'C' + A'B' + ACD' + BC'D$, minterm 2

(b) $F(A, B, C, D) = A'BD + AC'D + AB' + BCD + A'C'D$, minterm 7

5.46 $F(V, W, X, Y, Z) = \Pi\, M(0, 3, 6, 9, 11, 19, 20, 24, 25, 26, 27, 28, 29, 30) \cdot \Pi\, D(1, 2, 12, 13)$

(a) Find two minimum sum-of-products expressions for F.

(b) Underline the essential prime implicants in your answer and tell why each one is essential.

When a student builds the circuit in a lab, he finds that when $A = B = C = D = 1$, the output F has the wrong value, and that the gate outputs are as shown in Figure 8-13. The reason for the incorrect value of F can be determined as follows:

1. The output of gate 7 (F) is wrong, but this wrong output is consistent with the inputs to gate 7, that is, $1 + 0 = 1$. Therefore, one of the inputs to gate 7 must be wrong.

2. In order for gate 7 to have the correct output ($F = 0$), both inputs must be 0. Therefore, the output of gate 5 is wrong. However, the output of gate 5 is consistent with its inputs because $1 \cdot 1 \cdot 1 = 1$. Therefore, one of the inputs to gate 5 must be wrong.

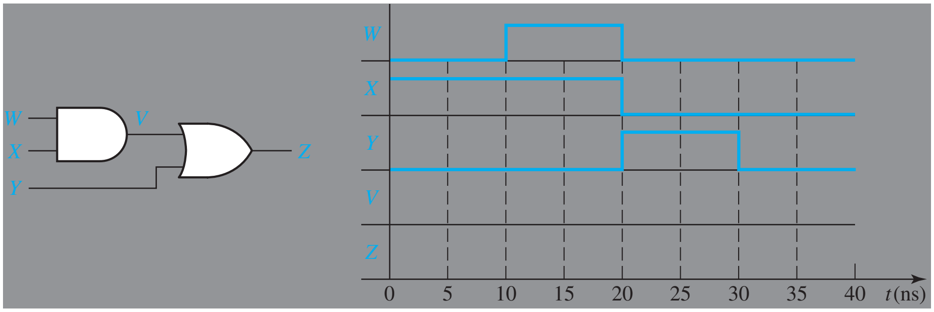
3. Either the output of gate 3 is wrong, or the A or B input to gate 5 is wrong. Because $C'D + CD' = 0$, the output of gate 3 is wrong.

4. The output of gate 3 is not consistent with the outputs of gates 1 and 2 because $0 + 0 \neq 1$. Therefore, either one of the inputs to gate 3 is connected wrong, gate 3 is defective, or one of the input connections to gate 3 is defective.

This example illustrates how to troubleshoot a logic circuit by starting at the output gate and working back until the wrong connection or defective gate is located.

Problems

8.1 Complete the timing diagram for the given circuit. Assume that both gates have a propagation delay of 5 ns.



8.2 Consider the following logic function.

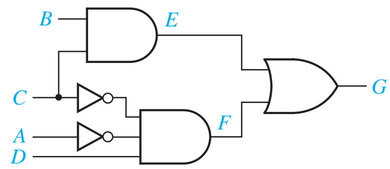
$$F(A, B, C, D) = \Sigma\, m(0, 4, 5, 10, 11, 13, 14, 15)$$

(a) Find two different minimum circuits which implement F using AND and OR gates. Identify two hazards in each circuit.

(b) Find an AND-OR circuit for F which has no hazards.

(c) Find an OR-AND circuit for F which has no hazards.

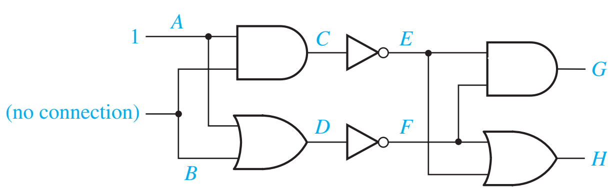
8.3 For the following circuit:



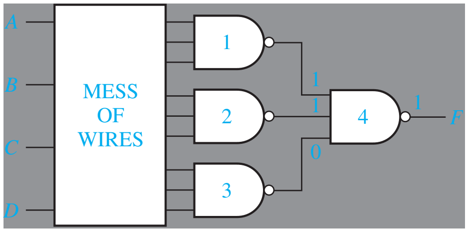
(a) Assume that the inverters have a delay of 1 ns and the other gates have a delay of 2 ns. Initially $A = 0$ and $B = C = D = 1$, and C changes to 0 at time = 2 ns. Draw a timing diagram and identify the transient that occurs.

(b) Modify the circuit to eliminate the hazard.

8.4 Using four-valued logic, find A, B, C, D, E, F, G, and H.

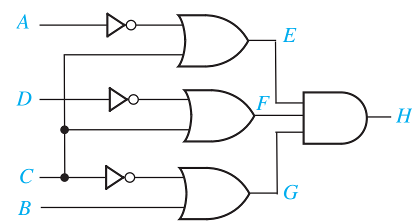


8.5 The circuit below was designed to implement the logic equation $F = AB'D + BC'D' + BCD$, but it is not working properly. The input wires to gates 1, 2, and 3 are so tightly packed, it would take you a while to trace them all back to see whether the inputs are correct. It would be nice to only have to trace whichever one is incorrectly wired. When $A = B = 0$ and $C = D = 1$, the inputs and outputs of gate 4 are as shown. Is gate 4 working properly? If so, which of the other gates either is connected incorrectly or is malfunctioning?



8.6 (a) Assume the inverters have a delay of 1 ns and the other gates have a delay of 2 ns. Initially $A = B = 0$ and $C = D = 1$; C changes to 0 at time 2 ns. Draw a timing diagram showing the glitch corresponding to the hazard.

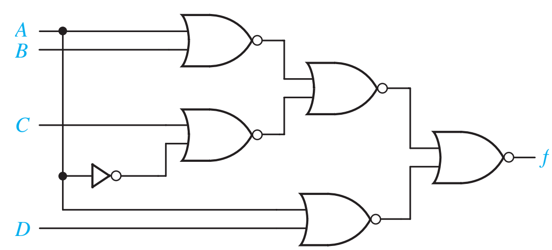
(b) Modify the circuit so that it is hazard free. (Leave the circuit as a two-level, OR-AND circuit.)



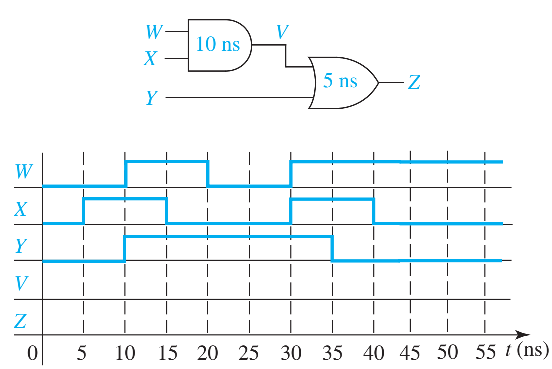
8.7 A two-level, NOR-NOR circuit implements the function
$f(a, b, c, d) = (a + d')(b' + c + d)(a' + c' + d')(b' + c' + d)$.
(a) Find all hazards in the circuit.
(b) Redesign the circuit as a two-level, NOR-NOR circuit free of all hazards and using a minimum number of gates.

8.8 $F(A, B, C, D) = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 9, 13, 15)$
(a) Find three different minimum AND-OR circuits that implement F. Identify two hazards in each circuit. Then find an AND-OR circuit for *F* that has no hazards.
(b) There are two minimum OR-AND circuits for *F*; each has one hazard. Identify the hazard in each circuit, and then find an OR-AND circuit for *F* that has no hazards.

8.9 Consider the following three-level NOR circuit:
(a) Find all hazards in this circuit.
(b) Redesign the circuit as a three-level NOR circuit that is free of all hazards.



8.10 Draw the timing diagram for *V* and *Z* for the circuit. Assume that the AND gate has a delay of 10 ns and the OR gate has a delay of 5 ns.

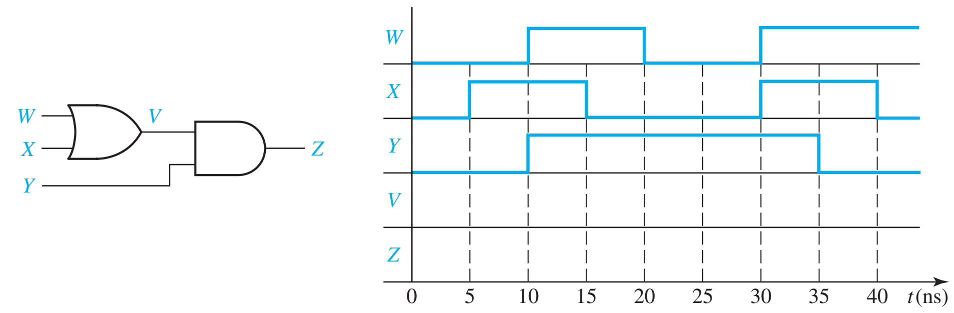


8.11 Consider the three-level circuit corresponding to the expression f(A, B, C, D) = (A + B)(B′C′ + BD′).

(a) Find all hazards in this circuit.

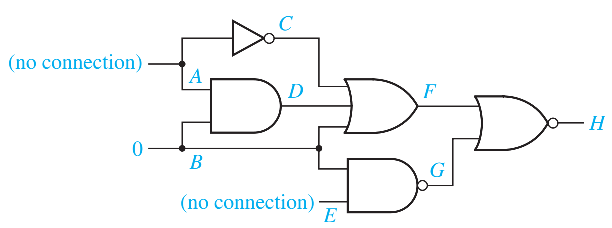
(b) Redesign the circuit as a three-level NOR circuit that is free of all hazards.

8.12 Complete the timing diagram for the given circuit. Assume that both gates have a propagation delay of 5 ns.

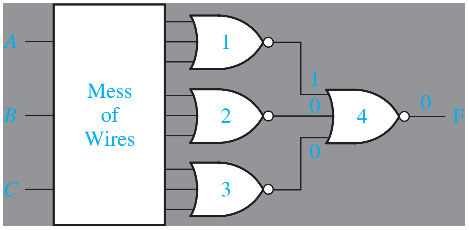


8.13 Implement the logic function from Figure 8.10(b) as a minimum sum of products. Find the static hazards and tell what minterms they are between. Implement the same logic function as a sum of products without any hazards.

8.14 Using four-valued logic, find A, B, C, D, E, F, G, and H.



8.15 The following circuit was designed to implement the logic equation $F = (A + B' + C')(A' + B + C')(A' + B' + C)$, but it is not working properly. The input wires to gates 1, 2, and 3 are so tightly packed, it would take you a while to trace them all back to see whether the inputs are correct. It would be nice to only have to trace whichever one is incorrectly wired. When $A = B = C = 1$, the inputs and outputs of gate 4 are as shown. Is gate 4 working properly? If so, which of the other gates either is connected incorrectly or is malfunctioning?



8.16 Consider the following logic function.

$$F(A, B, C, D) = \Sigma\, m(0, 2, 5, 6, 7, 8, 9, 12, 13, 15)$$

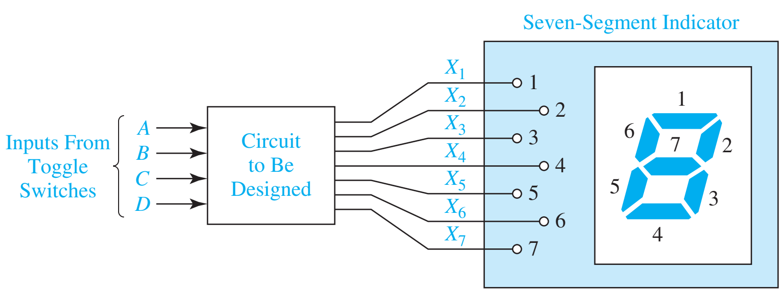
(a) Find two different minimum AND-OR circuits which implement F. Identify two hazards in each circuit. Then find an AND-OR circuit for F that has no hazards.
(b) The minimum OR-AND circuit for F has one hazard. Identify it, and then find an OR-AND circuit for F that has no hazards.

Design Problems

Seven-Segment Indicator

Several of the problems involve the design of a circuit to drive a seven-segment indicator (see Figure 8-14). The seven-segment indicator can be used to display any one of the decimal digits 0 through 9. For example, "1" is displayed by lighting segments 2 and 3, "2" by lighting segments 1, 2, 7, 5, and 4, and "8" by lighting all seven segments. A segment is lighted when a logic 1 is applied to the corresponding input on the display module.

FIGURE 8-14 Circuit Driving Seven-Segment Module



8.A Design an 8-4-2-1 BCD code converter to drive a seven-segment indicator. The four inputs to the converter circuit (A, B, C, and D in Figure 8-14) represent an 8-4-2-1 binary-coded-decimal digit. Assume that only input combinations representing the digits 0 through 9 can occur as inputs, so that the combinations 1010 through 1111 are don't-cares. Design your circuit using only two-, three-, and four-input NAND gates and inverters. Try to minimize the number of gates required. The variables A, B, C, and D will be available from toggle switches.

Use  (not ) for 6. Use  (not ) for 9.

Any solution that uses 18 or fewer gates and inverters (not counting the four inverters for the inputs) is acceptable.

8.B Design an excess-3 code converter to drive a seven-segment indicator. The four inputs to the converter circuit (A, B, C, and D in Figure 8-14) represent an excess-3

Problems

9.1 (a) Show how two 2-to-1 multiplexers (with no added gates) could be connected to form a 3-to-1 MUX. Input selection should be as follows:
If $AB = 00$, select I_0
If $AB = 01$, select I_1
If $AB = 1-$ (B is a don't-care), select I_2

(b) Show how two 4-to-1 and one 2-to-1 multiplexers could be connected to form an 8-to-1 MUX with three control inputs.

(c) Show how four 2-to-1 and one 4-to-1 multiplexers could be connected to form an 8-to-1 MUX with three control inputs.

9.2 Design a circuit which will either subtract X from Y or Y from X, depending on the value of A. If $A = 1$, the output should be $X - Y$, and if $A = 0$, the output should be $Y - X$. Use a 4-bit subtracter and two 4-bit 2-to-1 multiplexers (with bus inputs and outputs as in Figure 9-5).

9.3 Repeat 9.2 using a 4-bit subtracter, four 4-bit three-state buffers (with bus inputs and outputs), and one inverter.

9.4 Realize a full adder using a 3-to-8 line decoder (as in Figure 9-13) and
(a) two OR gates.
(b) two NOR gates.

9.5 Derive the logic equations for a 4-to-2 priority encoder. Refer to your table in the Study Guide, Part 4(b).

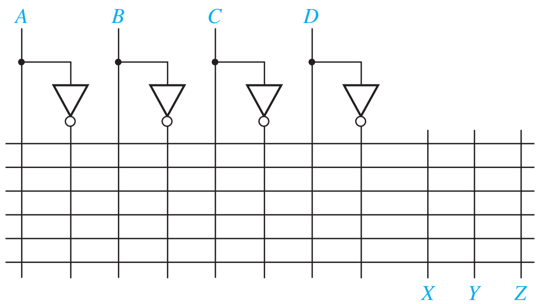
9.6 Design a circuit equivalent to Figure 9-11 using a 4-to-1 MUX (with bus inputs as in Figure 9-5). Use a 4-to-2 line priority encoder to generate the control signals.

9.7 An adder for Gray-coded-decimal digits (see Table 1-2) is to be designed using a ROM. The adder should add two Gray-coded digits and give the Gray-coded sum and a carry. For example, $1011 + 1010 = 0010$ with a carry of 1 ($7 + 6 = 13$). Draw a block diagram showing the required ROM inputs and outputs. What size ROM is required? Indicate how the truth table for the ROM would be specified by giving some typical rows.

9.8 The following PLA will be used to implement the following equations:

$$X = AB'D + A'C' + BC + C'D'$$
$$Y = A'C' + AC + C'D'$$
$$Z = CD + A'C' + AB'D$$

(a) Indicate the connections that will be made to program the PLA to implement these equations.



(b) Specify the truth table for a ROM which realizes these same equations.

9.9 Show how to implement a full subtracter using a PAL. See Figure 9-29.

9.10 (a) If the ROM in the hexadecimal to ASCII code converter of Figure 9-22 is replaced with a PAL, give the internal connection diagram.
(b) If the same ROM is replaced with a PLA, give the PLA table.

9.11 (a) Sometimes the programmable MUX (1) in Figure 9-31 helps us to save AND gates. Consider the case in which $F = c'd' + bc' + a'c$. If programmable MUX (1) is not set to invert F (i.e., $G = F$), how many AND gates are needed? If the MUX is set to invert F (i.e., $G = F'$), how many AND gates are needed?
(b) Repeat (a) for $F = a'b' + c'd'$.

9.12 (a) Implement a 3-variable function generator using a PAL with inputs a, b, c, and 1 (use the input inverter to get 0 also). Give the internal connection diagram. Leave the connections to 0 and 1 disconnected, so that any 3-variable function can be implemented by connecting only 0 and 1.
(b) Now connect 0 and 1 so that the function generator implements the sum function for a full adder. See Figure 9-34.

9.13 Expand the following function about the variable b.
$F = ab'cde' + bc'd'e + a'cd'e + ac'de'$

9.14 (a) Implement the following function using only 2-to-1 MUXes:
$R = ab'h' + bch' + eg'h + fgh$.
(b) Repeat using only tri-state buffers.

9.15 Show how to make a 4-to-1 MUX, using an 8-to-1 MUX.

9.16 Implement a 32-to-1 multiplexer using two 16-to-1 multiplexers and a 2-to-1 multiplexer in two ways: (a) Connect the most significant select line to the 2-to-1 multiplexer, and (b) connect the least significant select line to the 2-to-1 multiplexer.

9.17 2-to-1 multiplexers with an active high output and active high enable are to be used in the following implementations:

(a) Show how to implement a 4-to-1 multiplexer with an active high output and no enable using two of the 2-to-1 MUXes and a minimum number of additional gates.

(b) Repeat part (a) for a 4-to-1 multiplexer with an active low output.

(c) Repeat part (b) assuming the output of the 2-to-1 MUX is 1 (rather than 0) when the enable is 0.

9.18 Realize a BCD to excess-3 code converter using a 4-to-10 decoder with active low outputs and a minimum number of gates.

9.19 Use a 4-to-1 multiplexer and a minimum number of external gates to realize the function $F(w, x, y, z) = \Sigma\, m(3, 4, 5, 7, 10, 14) + \Sigma\, d(1, 6, 15)$.
The inputs are only available uncomplemented.

9.20 Realize the function $f(a, b, c, d, e) = \Sigma\, m(6, 7, 9, 11, 12, 13, 16, 17, 18, 20, 21, 23, 25, 28)$ using a 16-to-1 MUX with control inputs b, c, d, and e. Each data input should be 0, 1, a, or a'. Hint: Start with a minterm expansion of F and combine minterms to eliminate a and a' where possible.

9.21 Implement a full adder

(a) using two 8-to-1 MUXes. Connect X, Y, and C_{in} to the control inputs of the MUXes and connect 1 or 0 to each data input.

(b) using two 4-to-1 MUXes and one inverter. Connect X and Y to the control inputs of the MUXes, and connect 1's, 0's, C_{in}, or C_{in}' to each data input.

(c) again using two 4-to-1 MUXes, but this time connect C_{in} and Y to the control inputs of the MUXes, and connect 1's, 0's, X, or X' to each data input. Note that in this fashion, any N-variable logic function may be implemented using a $2^{(N-1)}$-to-1 MUX.

9.22 Repeat Problem 9.21 for a full subtracter, except use B_{in} instead of C_{in}.

9.23 Make a circuit which gives the absolute value of a 4-bit binary number. Use four full adders, four multiplexers, and four inverters. Assume negative numbers are represented in 2's complement. Recall that one way to find the 2's complement of a binary number is to invert all of the bits and then add 1.

9.24 Show how to make a 4-to-1 MUX using four three-state buffers and a decoder.

9.25 Show how to make an 8-to-1 MUX using two 4-to-1 MUXes, two three-state buffers, and one inverter.

9.26 Realize a full subtracter using a 3-to-8 line decoder with inverting outputs and

(a) two NAND gates.

(b) two AND gates.

9.27 Show how to make the 8-to-3 priority encoder of Figure 9-16 using two 4-to-2 priority encoders and any additional necessary gates.

9.28 Design an adder for excess-3 decimal digits (see Table 1-2) using a ROM. Add two excess-3 digits and give the excess-3 sum and a carry. For example, 1010 + 1001 = 0110 with a carry of 1 (7 + 6 = 13). Draw a block diagram showing the required ROM inputs and outputs. What size ROM is required? Indicate how the truth table for the ROM would be specified by giving some typical rows.

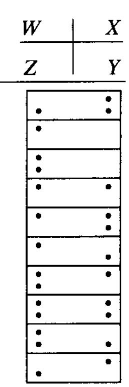
9.29 A circuit has four inputs *RSTU* and four outputs *VWYZ*. *RSTU* represents a binary-coded-decimal digit. *VW* represents the quotient and *YZ* the remainder when *RSTU* is divided by 3 (*VW* and *YZ* represent 2-bit binary numbers). Assume that invalid inputs do not occur. Realize the circuit using
(a) a ROM.
(b) a minimum two-level NAND-gate circuit.
(c) a PLA (specify the PLA table).

9.30 Repeat Problem 9.29 if the inputs *RSTU* represent a decimal digit in Gray code (see Table 1-2).

9.31 (a) Find a minimum two-level NOR gate circuit to realize F_1 and F_2. Use as many common gates as possible.
$F_1(a, b, c, d) = \Sigma\, m(1, 2, 4, 5, 6, 8, 10, 12, 14)$
$F_2(a, b, c, d) = \Sigma\, m(2, 4, 6, 8, 10, 11, 12, 14, 15)$
(b) Realize F_1 and F_2 using a PLA. Give the PLA table and internal connection diagram for the PLA.

9.32 Braille is a system which allows a blind person to read alphanumerics by feeling a pattern of raised dots. Design a circuit that converts BCD to Braille. The table shows the correspondence between BCD and Braille.
(a) Use a multiple-output NAND-gate circuit.

A	*B*	*C*	*D*
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1



(b) Use a PLA. Give the PLA table.
(c) Specify the connection pattern for the PLA.

9.33 (a) Implement your solution to Problem 7.10 using a PLA. Specify the PLA table and draw the internal connection diagram for the PLA using dots to indicate the presence of switching elements.
(b) Repeat (a) for Problem 7.41.
(c) Repeat (a) for Problem 7.43.

9.34 Show how to make an 8-to-1 MUX using a PAL. Assume that PAL has 14 inputs and six outputs and assume that each output OR gate may have up to four AND terms as inputs, as in Figure 9-29. (*Hint*: Wire some outputs of the PAL around to the inputs, external to the PAL. Some PALs allow this inside the PAL to save inputs.)

9.35 Work Problem 9.34 but make the 8-to-3 priority encoder of Figure 9-16 instead of a MUX.

9.36 The function $F = CD'E + CDE + A'D'E + A'B'DE' + BCD$ is to be implemented in an FPGA which uses 3-variable lookup tables.
(a) Expand F about the variables A and B
(b) Expand F about the variables B and C.
(c) Expand F about the variables A and C.
(d) Any 5-variable function can be implemented using four 3-variable lookup tables and a 4-to-1 MUX, but this time we are lucky. Use your preceding answers to implement F using only three 3-variable lookup tables and a 4-to-1 MUX. Give the truth tables for the lookup tables.

9.37 Work Problem 9.36 for $F = B'D'E' + AB'C + C'DE' + A'BC'D$.

9.38 Implement a 4-to-1 MUX using a CLB of the type shown in Figure 9-33. Specify the function realized by each function generator.

9.39 Realize the function $f(A, B, C, D) = A'C' + A'B'D' + ACD + A'BD$.
(a) Use a single 8-to-1 multiplexer with an active low enable and an active high output. Use A, C, and D as the select inputs where A is the most significant and D is the least significant.
(b) Repeat Part (a) assuming the multiplexer enable is active high and output is active low.
(c) Use a single 4-to-1 multiplexer with an active low enable and an active high output and a minimum of additional gates. Show the function expansion both algebraically and on a Karnaugh map.

9.40 Repeat Problem 9.39 for the function
$f(A, B, C, D, E) = A'C'E' + A'B'D'E' + ACDE' + A'BDE'$.

9.41 $F(a, b, c, d) = a' + ac'd' + b'cd' + ad$.

(a) Using Shannon's expansion theorem, expand F about the variable d.

(b) Use the expansion in Part (a) to realize the function using two 4-variable LUTs and a 2-to-1 MUX. Specify the LUT inputs.

(c) Give the truth table for each LUT.

9.42 Repeat 9.41 for $F(a, b, c, d) = cd' + ad' + a'b'cd + bc'$.

9.43 Repeat 9.41 for $F(a, b, c, d) = bd + bc' + ac'd + a'd'$.

II. SOLUTIONS TO HOMEWORK PROBLEMS

Unit 1 Problem Solutions

1.1 (a) 757.25_{10}

$$\begin{array}{rl} 16\,\underline{|\,757} & \\ 16\,\underline{|\,47} & r5 \\ 16\,\underline{|\,2} & r15=F_{16} \\ 0 & r2 \end{array} \qquad \begin{array}{r} 0.25 \\ \underline{\quad 16} \\ (4).00 \end{array}$$

$\therefore\ 757.25_{10} = 2F5.40_{16}$

$= \underset{2}{\underline{0010}}\ \underset{F}{\underline{1111}}\ \underset{5}{\underline{0101}}.\underset{4}{\underline{0100}}\ \underset{0}{\underline{0000}}{}_2$

1.1 (c) 356.89_{10}

$$\begin{array}{rl} 16\,\underline{|\,356} & \\ 16\,\underline{|\,22} & r4 \\ 16\,\underline{|\,1} & r6 \\ 0 & r1 \end{array} \qquad \begin{array}{r} 0.89 \\ \underline{\quad 16} \\ (14).24 \\ \underline{\quad 16} \\ (3).84 \\ \underline{\quad 16} \\ (13).44 \\ \underline{\quad 16} \\ (7).04 \end{array}$$

$\therefore\ 356.89_{10} = 164.E3_{16}$

$= \underset{1}{\underline{0001}}\ \underset{6}{\underline{0110}}\ \underset{4}{\underline{0100}}.\underset{E}{\underline{1110}}\ \underset{3}{\underline{0011}}{}_2$

1.2 (a) $EB1.6_{16} = E \times 16^2 + B \times 16^1 + 1 \times 16^0 + 6 \times 16^{-1}$

$= 14 \times 256 + 11 \times 16 + 1 + 6/16 = 3761.375_{10}$

$\underset{E}{\underline{1110}}\ \underset{B}{\underline{1011}}\ \underset{1}{\underline{0001}}.\underset{6}{\underline{011(0)}}{}_2$

$7261.3_8 = 7 \times 8^3 + 2 \times 8^2 + 6 \times 8^1 + 1 + 3 \times 8^{-1}$

$= 7 \times 512 + 2 \times 64 + 6 \times 8 + 1 + 3/8 = 3761.375_{10}$

$\underset{7}{\underline{111}}\ \underset{2}{\underline{010}}\ \underset{6}{\underline{110}}\ \underset{1}{\underline{001}}.\underset{3}{\underline{011}}{}_8$

1.3 $3BA.25_{14} = 3 \times 14^2 + 11 \times 14^1 + 10 \times 14^0 + 2 \times 14^{-1} + 5 \times 14^{-2}$

$= 588 + 154 + 10 + 0.1684 = 752.1684_{10}$

$$\begin{array}{rl} 6\,\underline{|\,752} & \\ 6\,\underline{|\,125} & r2 \\ 6\,\underline{|\,20} & r5 \\ 6\,\underline{|\,3} & r2 \\ 0 & r3 \end{array} \qquad \begin{array}{r} 0.1684 \\ \underline{\qquad 6} \\ (1).0104 \\ \underline{\qquad 6} \\ (0).0624 \\ \underline{\qquad 6} \\ (0).3744 \\ \underline{\qquad 6} \\ (2).2464 \\ \underline{\qquad 6} \\ (1).4784 \end{array}$$

$\therefore\ 3BA.25_{14} = 752.1684_{10} = 3252.1002_6$

1.1 (b) 123.17_{10}

$$\begin{array}{rl} 16\,\underline{|\,123} & \\ 16\,\underline{|\,7} & r11 \\ 0 & r7 \end{array} \qquad \begin{array}{r} 0.17 \\ \underline{\quad 16} \\ (2).72 \\ \underline{\quad 16} \\ (11).52 \\ \underline{\quad 16} \\ (8).32 \end{array}$$

$\therefore 123.17_{10} = 7B.2B_{16}$

$= \underset{7}{\underline{0111}}\ \underset{B}{\underline{1011}}.\underset{2}{\underline{0010}}\ \underset{B}{\underline{1011}}{}_2$

1.1 (d) 1063.5_{10}

$$\begin{array}{rl} 16\,\underline{|\,1063} & \\ 16\,\underline{|\,66} & r7 \\ 16\,\underline{|\,4} & r2 \\ 0 & r4 \end{array} \qquad \begin{array}{r} 0.5 \\ \underline{\quad 16} \\ (8).00 \end{array}$$

$\therefore 1063.5_{10} = 427.8_{16}$

$= \underset{4}{\underline{0100}}\ \underset{2}{\underline{0010}}\ \underset{7}{\underline{0111}}.\underset{8}{\underline{1000}}{}_2$

1.2 (b) $59D.C_{16} = 5 \times 16^2 + 9 \times 16^1 + D \times 16^0 + C \times 16^{-1}$

$= 5 \times 256 + 9 \times 16 + 13 + 12/16 = 1437.75_{10}$

$\underset{5}{\underline{0101}}\ \underset{9}{\underline{1001}}\ \underset{D}{\underline{1101}}.\underset{C}{\underline{1100}}{}_{16}$

$2635.6_8 = 2 \times 8^3 + 6 \times 8^2 + 3 \times 8^1 + 5 \times 8^0 + 6 \times 8^{-1}$

$= 2 \times 512 + 6 \times 64 + 3 \times 8 + 5 + 6/8 = 1437.75_{10}$

$\underset{2}{\underline{010}}\ \underset{6}{\underline{110}}\ \underset{3}{\underline{011}}\ \underset{5}{\underline{101}}.\underset{6}{\underline{110}}{}_8$

1.4 (b) 1457.11_{10}

$$\begin{array}{rl} 16\,\underline{|\,1457} & \\ 16\,\underline{|\,91} & r1 \\ 16\,\underline{|\,5} & r11=B_{16} \\ 0 & r5 \end{array} \qquad \begin{array}{r} 0.11 \\ \underline{\quad 16} \\ (1).76 \\ \underline{\quad 16} \\ (12).16 \end{array}$$

$\therefore\ 1457.11_{10} = 5B1.1C_{16}$

$$5B1.1C_{16} = \overset{5}{\overbrace{0101}}\overset{B}{\overbrace{1011}}\overset{1}{\overbrace{0001}}.\overset{1}{\overbrace{0001}}\overset{C}{\overbrace{1100}}{}_2 = 2661.070_8$$

$$(\underset{2}{\underbrace{010}}\,\underset{6}{\underbrace{110}}\,\underset{6}{\underbrace{110}}\,\underset{1}{\underbrace{001}}.\underset{0}{\underbrace{000}}\,\underset{7}{\underbrace{111}}\,\underset{0}{\underbrace{00}})$$

1.4 (c) $5B1.1C_{16} = \underset{5}{\underline{11}}\ \underset{B}{\underline{23}}\ \underset{1}{\underline{01}}.\underset{1}{\underline{01}}\ \underset{C}{\underline{30}}{}_4$

1.4 (d) $DEC.A_{16} = D \times 16^2 + E \times 16^1 + C \times 16^0 + A \times 16^{-1}$

$= 3328 + 224 + 12 + 0.625 = 3564.625_{10}$

Unit 1 Solutions

1.5 (a)

$$\begin{array}{r} {}^{1\,1\,1}\\ 1111\\ +\underline{1010}\\ 11001 \end{array}\ \text{(Add)} \qquad \begin{array}{r} 1111\\ -\underline{1010}\\ 0101 \end{array}\ \text{(Sub)} \qquad \begin{array}{r} 1111\\ \underline{\times 1010}\\ 0000\\ \underline{1111}\\ 11110\\ \underline{0000}\\ 011110\\ \underline{1111}\\ 10010110 \end{array}\ \text{(Multiply)}$$

1.5 (b, c) *See FLD p. 692 for solutions.*

1.6, 1.7, 1.8 *See FLD p. 692 for solutions.*

1.10 (a) 1305.375_{10}

$$\begin{array}{rl} 16\,\underline{|\,1305} & \\ 16\,\underline{|\,81} & r9\\ 5 & r1 \end{array} \qquad \begin{array}{r} 0.375\\ \underline{16}\\ (6).000 \end{array}$$

$\therefore\ 1305.375_{10} = 519.600_{16}$
$= \underset{5}{\underline{0101}}\ \underset{1}{\underline{0001}}\ \underset{9}{\underline{1001}}.\underset{6}{\underline{0110}}\ \underset{0}{\underline{0000}}\ \underset{0}{\underline{0000}}{}_2$

1.10 (b) 11.33_{10}

$$\begin{array}{rl} 16\,\underline{|\,111} & \\ 6 & r15 = F_{16} \end{array} \qquad \begin{array}{r} 0.33\\ \underline{16}\\ (5).28\\ \underline{16}\\ (4).48 \end{array}$$

$\therefore\ 111.33_{10} = 6F.54_{16}$
$= \underset{6}{\underline{0110}}\ \underset{F}{\underline{1111}}.\underset{5}{\underline{0101}}\ \underset{4}{\underline{0100}}{}_2$

1.10 (c) 301.12_{10}

$$\begin{array}{rl} 16\,\underline{|\,301} & \\ 16\,\underline{|\,18} & r13\\ 1 & r2 \end{array} \qquad \begin{array}{r} 0.12\\ \underline{16}\\ (1).92\\ \underline{16}\\ (14).72 \end{array}$$

$\therefore\ 301.12_{10} = 12D.1E_{16}$
$= \underset{1}{\underline{0001}}\ \underset{2}{\underline{0010}}\ \underset{D}{\underline{1101}}.\underset{1}{\underline{0001}}\ \underset{E}{\underline{1110}}{}_2$

1.10 (d) 1644.875_{10}

$$\begin{array}{rl} 16\,\underline{|\,1644} & \\ 16\,\underline{|\,102} & r12\\ 6 & r6 \end{array} \qquad \begin{array}{r} 0.875\\ \underline{16}\\ (14).000 \end{array}$$

$\therefore\ 1644.875_{10} = 66C.E00_{16}$
$= \underset{6}{\underline{0110}}\ \underset{6}{\underline{0110}}\ \underset{C}{\underline{1100}}.\underset{E}{\underline{1110}}\ \underset{0}{\underline{0000}}\ \underset{0}{\underline{0000}}{}_2$

1.11 (a) $101\ 111\ 010\ 100.101\ _2 = 5724.5_8$
$= 5 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 4 \times 8^0 + 5 \times 8^{-1}$
$= 5 \times 512 + 7 \times 64 + 2 \times 8 + 4 + 5/8$
$= 3028.625_{10}$

$1011\ 1101\ 0100.1010_2 = BD4.A_{16}$
$B \times 16^2 + D \times 16^1 + 4 \times 16^0 + A \times 16^{-1}$
$11 \times 256 + 13 \times 16 + 4 + 10/16$
$= 3028.625_{10}$

1.11 (b) $100\ 001\ 101\ 111.010_2 = 4157.2_8$
$= 4 \times 8^3 + 1 \times 8^2\ \ 5 \times 8^1 + 7 \times 8^0 + 2 \times 8^{-1}$
$= 4 \times 512 + 1 \times 64 + 5 \times 8 + 7 + 2/8$
$= 2159.25_{10}$

$1000\ 0110\ 1111.0100_2 = 86F.4_{16}$
$= 8 \times 16^2 + 6 \times 16^1 + F \times 16^0 \times 4 \times 16^{-1}$
$= 8 \times 256 + 6 \times 16 + 15 + 4/16$
$= 2159.25_{10}$

1.12 (a) $375.54_8 = 3 \times 64 + 7 \times 8 + 5 + 5/8 + 4/64$
$= 253.6875_{10}$

$$\begin{array}{rl} 3\,\underline{|\,253} & \\ 3\,\underline{|\,84} & r1\\ 3\,\underline{|\,28} & r0\\ 3\,\underline{|\,9} & r1\\ 3\,\underline{|\,3} & r0\\ 3\,\underline{|\,1} & r0\\ 0 & r1 \end{array} \qquad \begin{array}{r} 0.69\\ \underline{3}\\ (2).07\\ \underline{3}\\ (0).21\\ 3\\ (0).63\\ 3\\ (1).89 \end{array}$$

$\therefore\ 375.54_8 = 100101.2001_3$

1.12 (b) 384.74_{10}

$$\begin{array}{rl} 4\,\underline{|\,384} & \\ 4\,\underline{|\,96} & r0\\ 4\,\underline{|\,24} & r0\\ 4\,\underline{|\,6} & r0\\ 4\,\underline{|\,1} & r2\\ 0 & r1 \end{array} \qquad \begin{array}{r} 0.74\\ \underline{4}\\ (2).96\\ \underline{4}\\ (3).84\\ 4\\ (3).36 \end{array}$$

$\therefore\ 384.74_{10} = 12000.233113_4...$

1.12 (c) $A52.A4_{11} = 10 \times 121 + 5 \times 11 + 2 + 10/11 + 4/121$
$= 1267.94_{10}$

9 | 1267		0.94
9 | 140	r7	9
9 | 15	r5	(8).46
9 | 1	r6	9
0	r1	(4).14

$\therefore A52.A4_{11} = 1267.94_{10} = 1657.8427_9...$

1.13 $544.1_9 = 5 \times 9^2 + 4 \times 9^1 + 4 \times 9^0 + 1 \times 9^{-1}$
$= 5 \times 81 + 4 \times 9 + 4 + 1/9$
$= 445\ 1/9_{10}$

16 | 445		1/9
16 | 27	r13	16
16 | 1	r11	(1)7/9
0	r1	16
		(12)4/9
		16
		(7)1/9

$\therefore 544.1_9 = 1BD.1C7_{16}$
$= 1\ 1011\ 1101.0001\ 1100\ 0111_2...$

1.14 (a), (b), (c) (c)

16 | 97		.7
16 | 6	r1	16
0	r6	(11).2
		16
		(3).2

$\therefore 97.7_{10} = 61.B3333...._{16}$
(a) $61.B3333.._{16}$
$= 110\ 0001.1011\ 0011\ 0011\ 0011\ 0011..._2$
(b) $1\ 100\ 001.101\ 100\ 110\ 011\ 001\ 100\ 11..._2$
$= 141.5\ 4631\ 4631...._8$

1.14 (d)

3 | 97		.7
3 | 32	r1	3
3 | 10	r2	(2).1
3 | 3	r1	3
3 | 1	r0	(0).3
0	r1	3
		(0).9
		3
		(2).7

$\therefore 97.7_{10} = 10121.2002...._3$

1.14 (e)

5 | 97		.7
5 | 19	r2	5
5 | 3	r4	(3).5
0	r3	5
		(2).5

$\therefore 97.7_{10} = 342.322.._5$

1.15 1110212.20211_3
$01\ 11\ 02\ 12.20\ 21\ 10 = 1425.673_9$

Base 3	Base 9
00	0
01	1
02	2
10	3
11	4
12	5
20	6
21	7
22	8

1.16 (a) $2983\ 63/64_{10} =$

8 | 2983		0.984
8 | 372	r7	8
8 | 46	r4	(7).872
9 | 5	r6	8
0	r5	(6).976

$\therefore 2983\ 63/64_{10} = 5647.76_8$ (or 5647.77_8)
$= 101\ 110\ 100\ 111.111\ 110_2$
(or $101\ 110\ 100\ 111.111\ 111_2$)

1.16 (b) 93.70_{10}

8 | 93		0.70
8 | 11	r5	8
8 | 1	r3	(5).60
0	r1	8
		(4).80

$\therefore 93.70_{10} = 135.54_8 = 001\ 011\ 101.101\ 100_2$

1.16 (c) $1900\ 31/32_{10}$

```
8 | 1900              0.969
8 | 273    r4             8
8 | 29     r5        (7).752
9 | 3      r5             8
    0      r3        (6).016
```

$\therefore\ 1900\ 31/32_{10} = 3554.76_{8}$
$= 011\ 101\ 101\ 100.111\ 110_{2}$

1.16 (d) 109.30_{10}

```
8 | 109              0.30
8 | 13    r5            8
8 | 1     r5        (2).40
    0     r1            8
                    (3).20
```

$\therefore\ 109.30_{10} = 155.23_{8}$
$= 001\ 101\ 101.010\ 011_{2}$

1.17 (a)

```
 111          111
 1111 (Add)   1111 (Subtract)
 1001         1001
11000         0110

       1111 (Multiply)
       1001
       1111
      0000
      01111
     0000
     001111
    1111
   10000111
```

1.17 (b)

```
1                   11 11
1101001(Add)        1101001 (Sub)
 110110              110110
10011111             110011

          1101001   (Mult)
           110110
          0000000
         1101001
         11010010
        1101001
        1001110110
       0000000
       1001110110
      1101001
     100100000110
     1101001
    1011000100110
```

1.17(c)

```
1                   111  1
110010   (Add)      110010   (Sub)
  11101              11101
1001111              10101

      110010  (Mult)
       11101
      110010
     000000
     0110010
    110010
    11111010
   110010
  1010001010
  110010
 10110101010
```

1.18

```
     1  1   1                1   1
(a)  10100100         (b)   10010011
     01110011               01011001
      0110001               00111010

        11
(c)  11110011
     10011110
     01010101
```

1.19(a)

```
          101110  Quotient
101   )11101001
        101
         1001
          101
          1000
           101
            110
            101
             11  Remainder
```

1.19(b)

```
              11011  Quotient
1110   )110000001
         1110
         10100
          1110
           11000
            1110
            10101
             1110
              111 Remainder
```

1.19(c)

```
           1100   Quotient
1001 )1110010
      1001
       1010
       1001
          110 Remainder
```

1.20(a)

```
         10111   Quotient
110 )10001101
      110
       1011
        110
        1010
         110
          1001
           110
            11 Remainder
```

1.20(b)

```
           100011   Quotient
1011 )110000011
       1011
          10001
           1011
            1101
            1011
              10 Remainder
```

1.20(c)

```
              1011    Quotient
1010    )1110100
          1010
           10010
            1010
            10000
             1010
              110 Remainder
```

1.21 (a) 4 + 3 is 10 in base 7, i.e., the sum digit is 0 with a carry of 1 to the next column. 1 + 5 + 4 is 10 in base 7. 1 + 6 + 0 is 10 in base 7. This overflows since the correct sum is 1000_7.

(b) 4 + 3 + 3 + 3 = 13 in base 10 and 23 in base 5. Try base 10. 1 + 2 + 4 + 1 + 3 = 11 in base 10 so base 10 does not produce a sum digit of 2. Try base 5. 2 + 2 + 4 + 1 + 3 = 22 in base 5 so base 5 works.

(c) 4 + 3 + 3 + 3 = 31 in base 4, 21 in base 6, and 11 in base 12. Try base 12. 1 + 2 + 4 + 1 + 3 = B in base 12 so base 12 does not work. Try base 4. 3 + 2 + 4 + 1 + 3 = 31 in base 4 so base 4 does not work. Try base 6. 2 + 2 + 4 + 1 + 3 = 20 so base 6 is correct.

1.22 If the binary number has n bits (to the right of the radix point), then its precision is $(1/2^{n+1})$. So to have the same precision, n must satisfy

$(1/2^{n+1}) < (1/2)(1/10^4)$ or $n > 4/(\log 2) = 13.28$ so n must be 14.

1.23

$.363636....$
$= (36/10^2)(1 + 1/10^2 + 1/10^4 + 1/10^6 + \ldots)$
$= (36/10^2)[1/(1 - 1/10^2)] = (36/10^2)[10^2/99]$
$= 36/99 = 4/11$

8(4/11) = 2 + 10/11
8(10/11) = 7 + 3/11
8(3/11) = 2 + 2/11
8(2/11) = 1 + 5/11
8(5/11) =3 + 7/11
8(7/11) = 5+ 1/11
8(1/11) = 0 + 8/11
8(8/11) = 5 + 9/11
8(9/11) = 6 + 6/11
8(6/11) = 4 + 4/11
8(4/11) = 2 + 10/11
Repeats: .27213505642…….

1.24 (a) Expand the base b number into a power series

$N = d_{3k-1}b^{3k-1} + d_{3k-2}b^{3k-2} + d_{3k-3}b^{3k-3} + \ldots + d_5b^5 + d_4b^4 + d_3b^3 + d_2b^2 + d_1b^1 + d_0b^0 + d_{-1}b^{-1} + d_{-2}b^{-2} + d_{-3}b^{-3} + \ldots + d_{-3m+2}b^{-3m+2} + d_{-3m+1}b^{-3m+1} + d_{-3m}b^{-3m}$ where each d_i has a value from 0 to (b-1). (Note that 0's can be appended to the number so that it has a multiple of 3 digits to the left and right of the radix point.) Factor b^3 from each group of 3 consecutive digits of the number to obtain

$N = (d_{3k-1}b^2 + d_{3k-2}b^1 + d_{3k-3}b^0)(b^3)^{(k-1)} + \ldots + (d_5b^2 + d_4b^1 + d_3b^0)(b^3)^1 + (d_2b^2 + d_1b^1 + d_0b^0)(b^3)^0 + (d_{-1}b^2 + d_{-2}b^1 + d_{-3}b^0)(b^3)^{-1} + \ldots + (d_{-3m+2}b^2 + d_{-3m+1}b^1 + d_{-3m}b^0)(b^3)^{-m}$

Each $(d_{3i-1}b^2 + d_{3i-2}b^1 + d_{3i-3}b^0)$ has a value from 0 to $[(b-1)b^2 + (b-1)b^1 + (b-1)b^0]$

$= (b-1)(b^2 + b^1 + b^0) = (b^3-1)$

so it is a valid digit in a base b^3 number. Consequently, the last expression is the power series expansion for a base b^3 number.

1.24 (b) Expand the base b^3 number into a power series

$N = d_k(b^3)^k + d_{k-1}(b^3)^{k-1} + \ldots + d_1(b^3)^1 + d_0(b^3)^0 + d_{-1}(b^3)^{-1} + \ldots + d_{-m}(b^3)^{-m}$

where each d_i has a value from 0 to $(b^3 -1)$. Consequently, d_i can be represented as a base b number in the form

$(e_{3i-1}b^2 + e_{3i-2}b^1 + e_{3i-3}b^0)$

Where each e_j has a value from 0 to (b-1). Substituting these expressions for the d_i produces a power series expansion for a base b number.

Unit 1 Solutions

1.25

	4 3 2 1
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 1 0 0
4	1 0 0 0
5	1 0 0 1
6	1 0 1 0
7	1 1 0 0
8	1 1 0 1
9	1 1 1 0

9154 = 1110 0001 1001 1000

1.26 5-3-1-1 is possible, but 6-4-1-1 is not, because there is no way to represent 3 or 9.

	5 3 1 1	Alternate Solutions:
0	0 0 0 0	
1	0 0 0 1	(0010)
2	0 0 1 1	
3	0 1 0 0	
4	0 1 0 1	(0110)
5	1 0 0 0	
6	1 0 0 1	(1010)
7	1 0 1 1	
8	1 1 0 0	
9	1 1 0 1	(1110)

1.27 5-4-1-1 is not possible, because there is no way to represent 3 or 8. 6-3-2-1 is possible:

	6 3 2 1
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 1 0 0
4	0 1 0 1
5	0 1 1 0
6	1 0 0 0
7	1 0 0 1
8	1 0 1 0
9	1 1 0 0

1.28

	6 2 2 1	Alternate Solutions:
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	(0100)
3	0 0 1 1	(0101)
4	0 1 1 0	
5	0 1 1 1	
6	1 0 0 0	
7	1 0 0 1	
8	1 0 1 0	(1100)
9	1 0 1 1	(1101)

1100 0011 = 83

1.29

	5 2 2 1	Alternate Solutions:
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	(0100)
3	0 0 1 1	(0101)
4	0 1 1 0	
5	1 0 0 0	
6	1 0 0 1	
7	1 0 1 0	(1100)
8	1 0 1 1	(1101)
9	1 1 1 0	

1110 0110 = 94

1.30

	7 3 2 1	Alternate Solutions:
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	
3	0 1 0 0	(0011)
4	0 1 0 1	
5	0 1 1 0	
6	0 1 1 1	
7	1 0 0 0	
8	1 0 0 1	
9	1 0 1 0	
A	1 1 0 0	(1011)
B	1 1 0 1	

B4A9 = 1101 0101 1100 1010
Alt.: = " " 1011 "

1.31 (a)

	8 4 -2 -1
0	0 0 0 0
1	0 1 1 1
2	0 1 1 0
3	0 1 0 1
4	0 1 0 0
5	1 0 1 1
6	1 0 1 0
7	1 0 0 1
8	1 0 0 0
9	1 1 1 1

(b)
The 9's complement of a decimal number represented with this weighted code can be obtained by replacing 0's with 1's and 1's with 0's (bit-by-bit complement).

1.32 (a) 222.22_{10}

```
16 | 222            0.22
16 | 13   r14       × 16
      0   r13      (3).52
                   × 16
                   (8).32
```

$\therefore\ 222.22_{10} = \text{DE.38}_{16}$

= 1000100 1000101 0101110 0110011 0111000
(D E . 3 8)

1.32 (b) 183.81_{10}

```
16 | 183            0.81
16 | 11   r7        × 16
      0   r11     (12).96
                    × 16
                  (15).36
```

$\therefore\ 183.81_{10} = \text{B7.CF}_{16}$

= 1000010 0110111 0101110 1000011 1000110
(B 7 . C F)

1.33 (a)

```
In 2's complement        In 1's complement
 (–10) + (–11)            (–10) + (–11)
  110110                    110101
  110101                    110100
(1)101011 (–21)          (1)101001
                                 1
                            101010 (–21)
```

1.33 (b)

```
In 2's complement        In 1's complement
  (–10) + (–6)            (–10) + (–6)
  110110                    110101
  111010                    111001
(1)110000 (–16)          (1)101110
                                 1
                            101111 (–16)
```

1.33 (c)

```
In 2's complement        In 1's complement
  (–8) + (–11)             (–8) + (–11)
  111000                    110111
  110101                    110100
(1)101101 (–19)          (1)101011
                                 1
                            101100 (–19)
```

1.33(d)

```
In 2's complement        In 1's complement
  11 + 9                    11 + 9
  001011                    001011
  001001                    001001
  010100 (20)               010100 (20)
```

1.33 (e)

```
In 2's complement        In 1's complement
  (–11) + (–4)             (–11) + (–4)
  110101                    110100
  111100                    111011
(1)110001 (–15)          (1)101111
                                 1
                            110000 (–15)
```

1.34 (a) 01001-11010

```
In 2's complement        In 1's complement
     01001                    01001
   + 00110                  + 00101
     01111                    01110
```

1.34 (b)

```
In 2's complement        In 1's complement
     11010                    11010
   + 00111                  + 00110
  (1)00001                 (1)00000
                                  1
                              00001
```

1.34 (c)

```
In 2's complement        In 1's complement
     10110                    10110
   + 10011                  + 10010
  (1)01001                 (1)01000
  overflow                        1
                              01001
                           overflow
```

1.34 (d)

```
In 2's complement        In 1's complement
     11011                    11011
    +11001                   +11000
  (1)10100                 (1)10011
                                  1
                              10100
```

1.34 (e)

```
In 2's complement        In 1's complement
     11100                    11100
   + 01011                  + 01010
  (1)00111                 (1)00110
                                  1
                              00111
```

1.35 (a)

In 2's complement	In 1's complement
11010	11010
+ 01100	+ 01011
(1)00110	(1)00101
	1
	00110

1.35 (b)

In 2's complement	In 1's complement
01011	01011
+ 01000	+ 00111
10011	10010

1.35 (c)

In 2's complement	In 1's complement
10001	10001
+ 10110	+ 10101
(1)00111	(1)00110
overflow	1
	00111
	overflow

1.35 (d)

In 2's complement	In 1's complement
10101	10101
+ 00110	+ 00101
11011	11010

1.36

(a)

add	subt
101010	101010
+ 011101	- 011101
(1)000111	001101
1	overflow
001000	

(b)

add	subt
101010	101010
+ 011101	- 011101
(1)000111	001101
	overflow

1.37

(a)

	complement
i) 00000000 (0)	11111111 (-0)
ii) 11111110 (-1)	00000001 (1)
iii) 00110011 (51)	11001100 (-51)
iv) 10000000 (-127)	01111111 (127)

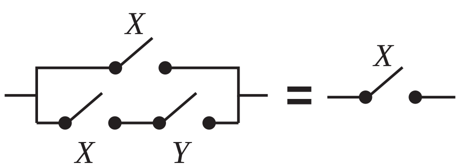
(b)

i) 00000000 (0)	00000000 (0)
ii) 11111110 (-2)	00000010 (2)
iii) 00110011 (51)	11001101 (-51)
iv) 10000000 (-128)	10000000 (-128)

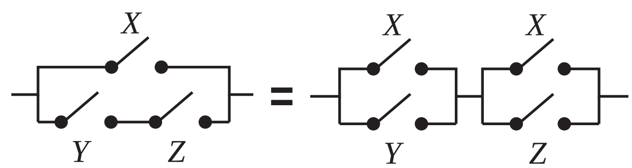
Unit 2 Problem Solutions

2.1 *See FLD p. 693 for solution.*

2.2 (a) In both cases, if X = 0, the transmission is 0, and if X = 1, the transmission is 1.



2.2 (b) In both cases, if X = 0, the transmission is YZ, and if X = 1, the transmission is 1.



2.3 *Answer is in FLD p. 693*

2.4 (a) $F = [(A{\cdot}1) + (A{\cdot}1)] + E + BCD = A + E + BCD$

2.4 (b) $Y = (AB' + (AB + B))\,B + A = (AB' + B)\,B + A$
$= (A + B)\,B + A = AB + B + A = A + B$

2.5 (a) $(A + B)\,(C + B)\,(D' + B)\,(ACD' + E)$
$= (AC + B)\,(D' + B)\,(ACD' + E)$ By Th. 8D
$= (ACD' + B)\,(ACD' + E)$ By Th. 8D
$= ACD' + BE$ By Th. 8D

2.5 (b) $(A' + B + C')\,(A' + C' + D)\,(B' + D')$
$= (A' + C' + BD)\,(B' + D')$
{By Th. 8D with $X = A' + C'$}
$= A'B' + B'C' + B'BD + A'D' + C'D' + BDD'$
$= A'B' + A'D' + C'B' + C'D'$

2.6 (a) $AB + C'D' = (AB + C')\,(AB + D')$
$= (A + C')\,(B + C')\,(A + D')\,(B + D')$

2.6 (b) $WX + WY'X + ZYX = X(W + WY' + ZY)$
$= X(W + ZY)$ {By Th. 10}
$= X(W + Z)\,(W + Y)$

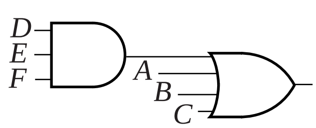
2.6 (c) $A'BC + EF + DEF' = A'BC + E(F + DF')$
$= A'BC + E(F + D) = (A'BC + E)\,(A'BC + F + D)$
$= (A' + E)\,(B + E)\,(C + E)\,(A' + F + D)$
$(B + F + D)\,(C + F + D)$

2.6 (d) $XYZ + W'Z + XQ'Z = Z(XY + W' + XQ')$
$= Z[W' + X(Y + Q')]$
$= Z(W' + X)\,(W' + Y + Q')$ By Th. 8D

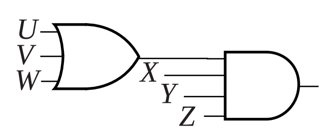
2.6 (e) $ACD' + C'D' + A'C = D'\,(AC + C') + A'C$
$= D'\,(A + C') + A'C$ By Th. 11D
$= (D' + A'C)\,(A + C' + A'C)$
$= (D' + A')\,(D' + C)\,(A + C' + A')$ By Th. 11D
$= (A' + D')\,(C + D')$

2.6 (f) $A + BC + DE$
$= (A + BC + D)(A + BC + E)$
$= (A + B + D)(A + C + D)(A + B + E)(A + C + E)$

2.7 (a) $(\underline{A + B + C} + D)\,(\underline{A + B + C} + E)\,(\underline{A + B + C} + F)$
$= \underline{A + B + C} + DEF$
Apply second distributive law (Th. 8D) twice



2.7 (b) $W\underline{XYZ} + V\underline{XYZ} + U\underline{XYZ} = \underline{XYZ}\,(W + V + U)$
By first distributive law (Th. 8)



2.8 (a) $[(AB)' + C'D]' = AB(C'D)' = AB(C + D')$
$= ABC + ABD'$

2.8 (b) $[A + B\,(C' + D)]' = A'(B(C' + D))'$
$= A'(B' + (C' + D)') = A'(B' + CD')$
$= A'B' + A'CD'$

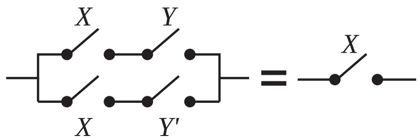
2.8 (c) $((A + B')\,C)'\,(A + B)\,(C + A)'$
$= (A'B + C')\,(A + B)C'A' = (A'B + C')A'BC'$
$= A'BC'$

2.9 (a) $F = [(A + B)' + (A + (A + B)')']\,(A + (A + B)')'$
$= (A + (A + B)')'$
By Th. 10D with $X=(A+(A+B)')' = A'(A + B) = A'B$

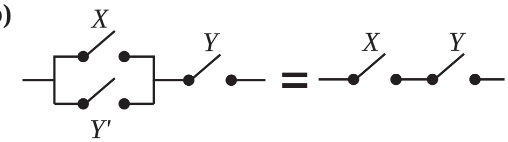
2.9 (b) $G = \{[(R + S + T)'\,PT(R + S)']'\,T\}'$
$= (R + S + T)'\,PT(R + S)' + T'$
$= T' + (R'S'T')\,P(R'S')T = T' + PR'S'T'T = T'$

Unit 2 Solutions

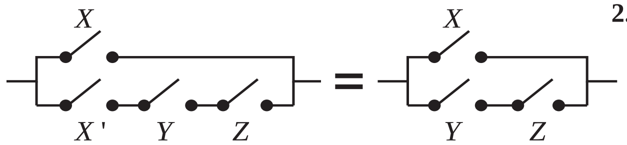
2.10 (a)



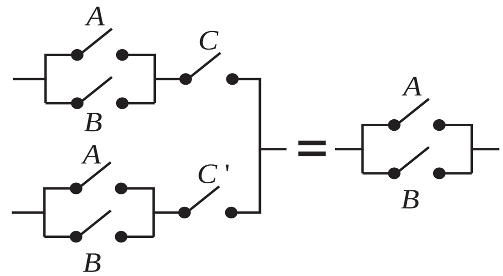
2.10 (b)



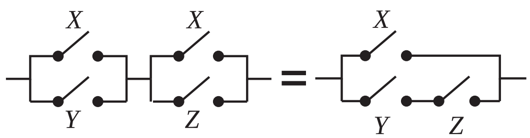
2.10 (c)



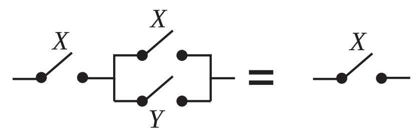
2.10 (d)



2.10 (e)



2.10 (f)



2.11 (a) $(A' + B' + C)(A' + B' + C)' = 0$ By Th. 5D

2.11 (b) $AB(C' + D) + B(C' + D) = B(C' + D)$ By Th. 10

2.11 (c) $AB + (C' + D)(AB)' = AB + C' + D$
By Th. 11D

2.11 (d) $(A'BF + CD')(A'BF + CEG) = A'BF + CD'EG$
By Th. 8D

2.11 (e) $[AB' + (C + D)' + E'F](C + D)$
$= AB'(C + D) + E'F(C + D)$ By Th. 8

2.11 (f) $A'(B + C)(D'E + F)' + (D'E + F)$
$= A'(B + C) + D'E + F$ By Th. 11D

2.12 (a) $(X + Y'Z) + (X + Y'Z)' = 1$ By Th. 5

2.12 (b) $[W + X'(Y + Z)][W' + X'(Y + Z)] = X'(Y + Z)$
By Th. 9D

2.12 (c) $(V'W + UX)'(UX + Y + Z + V'W) = (V'W + UX)'$
$(Y + Z)$ By Th. 11

2.12 (d) $(UV' + W'X)(UV' + W'X + Y'Z) = UV' + W'X$
By Th. 10D

2.12 (e) $(W' + X)(Y + Z') + (W' + X)'(Y + Z')$
$= (Y + Z')$ By Th. 9

2.12 (f) $(V' + U + W)[(W + X) + Y + UZ'] + [(W + X) +$
$UZ' + Y] = (W + X) + UZ' + Y$ By Th. 10

2.13 (a) $F_1 = A'A + B + (B + B) = 0 + B + B = B$

2.13 (b) $F_2 = A'A' + AB' = A' + AB' = A' + B'$

2.13 (c) $F_3 = [(AB + C)'D][(AB + C) + D]$
$= (AB + C)'D\,(AB + C) + (AB + C)'D$
$= (AB + C)'D$ By Th. 5D & Th. 2D

2.13 (d) $Z = [(A + B)C]' + (A + B)CD = [(A + B)C]' + D$
By Th. 11D with $Y = [(A + B)C]'$
$= A'B' + C' + D'$

2.14 (a) $ACF(B + E + D)$

2.14 (b) $W + Y + Z + VUX$

2.15 (a) $f' = \{[A + (BCD)'][(AD)' + B(C' + A)]\}'$
$= [A + (BCD)']' + [(AD)' + B(C' + A)]'$
$= A'(BCD)'' + (AD)''[B(C' + A)]'$
$= A'BCD + AD[B' + (C' + A)']$
$= A'BCD + AD[B' + C''A']$
$= A'BCD + AD[B' + CA']$

2.15(b) $f' = [AB'C + (A' + B + D)(ABD' + B')]'$
$= (AB'C)'[(A' + B + D)(ABD' + B')]'$
$= (A' + B'' + C')[(A' + B + D)' + (ABD')'B'']$
$= (A' + B + C')[A''B'D' + (A' + B' + D'')B]$
$= (A' + B + C')[AB'D' + (A' + B' + D)B]$

2.16 (a) $f^D = [A + (BCD)'][(AD)' + B(C' + A)]^D$
$= [A(B + C + D)'] + [(A + D)'(B + C'A)]$

2.16 (b) $f^D = [AB'C + (A' + B + D)(ABD' + B')]^D$
$= (A + B' + C)[A'BD + (A + B + D')B']$

2.17 (a) $f = [(A' + B)C] + [A(B + C')]$
$= A'C + B'C + AB + AC'$
$= A'C + B'C + AB + AC' + BC$
$= A'C + C + AB + AC' = C + AB + A = C + A$

2.17 (b) $f = A'C + B'C + AB + AC' = A + C$

2.17 (c) $f = (A' + B' + A)(A + C)(A' + B' + C' + B)$
$(B + C + C') = (A + C)$

2.18 (a) product term, sum-of-products, product-of-sums)

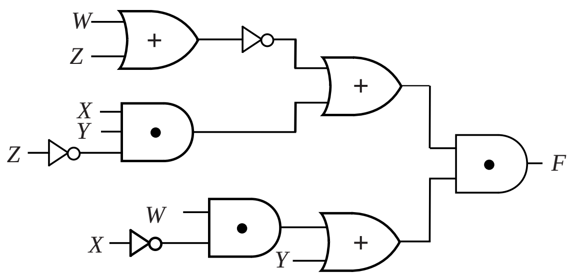
2.18 (b) sum-of-products

2.18 (c) none apply

2.18 (d) sum term, sum-of-products, product-of-sums

2.18 (e) product-of-sums

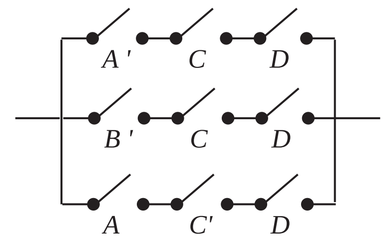
2.19



2.20 (a) $F = D[(A' + B')C + AC']$

2.20 (b) $F = D[(A' + B')C + AC']$

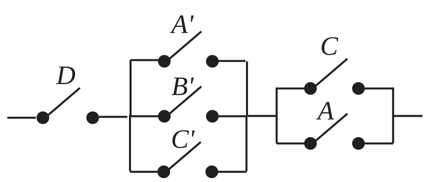
$= A'CD + B'CD + AC'D$



2.20 (c) $F = D[(A' + B')C + AC']$

$= D(A' + B' + AC')(C + AC')$

$= D(A' + B' + C')(C + A)$



2.21

A	B	C	H	F	G
0	0	0	0	0	0
0	0	1	1	1	x
0	1	0	1	0	1
0	1	1	1	1	x
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	1	1	x

2.22 (a) $A'B' + A'CD + A'DE'$

$= A'(B' + CD + DE')$

$= A'[B' + D(C + E')]$

$= A'(B' + D)(B' + C + E')$

2.22 (b) $H'I' + JK$

$= (H'I' + J)(H'I' + K)$

$= (H' + J)(I' + J)(H' + K)(I' + K)$

2.22 (c) $A'BC + AB'C + CD'$

$= C(A'B + AB' + D')$

$= C[(A + B)(A' + B') + D']$

$= C(A + B + D')(A' + B' + D')$

2.22 (d) $A'B' + (CD' + E) = A'B' + (C + E)(D' + E)$

$= (A'B' + C + E)(A'B' + D' + E)$

$= (A' + C + E)(B' + C + E)$

$(A' + D' + E)(B' + D' + E)$

2.22 (e) $A'B'C + B'CD' + EF' = A'B'C + B'CD' + EF'$

$= B'C(A' + D') + EF'$

$= (B'C + EF')(A' + D' + EF')$

$= (B' + E)(B' + F')(C + E)(C + F')$

$(A' + D' + E)(A' + D' + F')$

2.22 (f) $WX'Y + W'X' + W'Y' = X'(WY + W') + W'Y'$

$= X'(W' + Y) + W'Y'$

$= (X' + W')(X' + Y')(W' + Y + W')(W' + Y + Y')$

$= (X' + W')(X' + Y')(W' + Y)$

2.23 (a) $W + U'YV = (W + U')(W + Y)(W + V)$

2.23 (b) $TW + UY' + V$

$= (T+U+Z)(T+Y'+V)(W+U+V)(W+Y'+V)$

2.23 (c) $A'B'C + B'CD' + B'E' = B'(A'C + CD' + E')$

$= B'[E' + C(A' + D')]$

$= B'(E' + C)(E' + A' + D')$

2.23 (d) $ABC + ADE' + ABF' = A(BC + DE' + BF')$

$= A[DE' + B(C + F')]$

$= A(DE' + B)(DE' + C + F')$

$= A(B + D)(B + E')(C + F' + D)(C + F' + E')$

Unit 2 Solutions

2.24 (a) $[(XY')' + (X' + Y)'Z] = X' + Y + (X' + Y)'Z$
$= X' + Y' + Z$ By Th. 11D with $Y = (X' + Y)$

2.24 (b) $(X + (Y'(Z + W)')')' = X'Y'(Z + W)' = X'Y'Z'W'$

2.24 (c) $[(A' + B')' + (A'B'C)' + C'D]'$
$= (A' + B')A'B'C(C + D') = A'B'C$

2.24 (d) $(A + B)\,CD + (A + B)' = CD + (A + B)'$
{By Th. 11D with $Y = (A + B)'$}
$= CD + A'B'$

2.25 (a) $F(P, Q, R, S)' = [(R' + PQ)S]' = R(P' + Q') + S'$
$= RP' + RQ' + S'$

2.25 (b) $F(W, X, Y, Z)' = [X + YZ(W + X')]'$
$= [X + X'YZ + WYZ]'$
$= [X + YZ + WYZ]' = [X + YZ]'$
$= X'Y' + X'Z'$

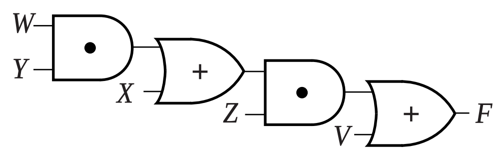
2.25 (c) $F(A, B, C, D)' = [A' + B' + ACD]'$
$= [A' + B' + CD]' = AB(C' + D')$

2.26 (a) $F = [(A' + B)'B]'C + B = [A' + B + B']C + B$
$= C + B$

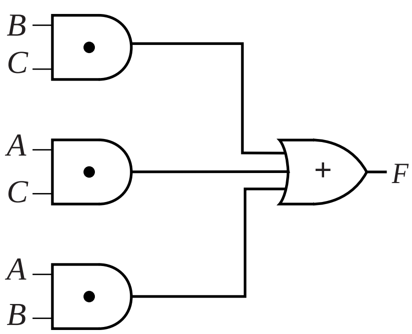
2.26 (b) $G = [(AB)'(B + C)]'C = (AB + B'C')C = ABC$

2.26 (c) $H = [W'X'(Y' + Z')]' = W + X + YZ$

2.27 $F = (\underline{V + X} + W)\,(\underline{V + X} + Y)\,(V + Z)$
$= (V + X + WY)(V + Z) = V + Z\,(X + WY)$
By Th. 8D with $X = V$

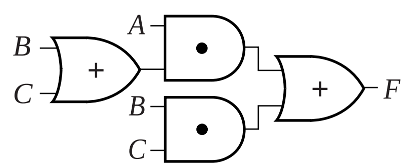


2.28 (a) $F = ABC + A'BC + AB'C + ABC'$
$= BC + AB'C + ABC'$ (By Th. 9)
$= C\,(B + AB') + ABC' = C\,(A + B) + ABC'$
(By Th. 11D)
$= AC + BC + ABC' = AC + B\,(C + AC')$
$= AC + B\,(A + C) = AC + AB + BC$



2.28 (b) Beginning with the answer to (a):

$F = A\,(B + C) + BC$



Alternate solutions:

$F = AB + C(A + B)$

$F = AC + B(A + C)$

2.29 (a)

$X\ Y\ Z$	$X+Y$	$X'+Z$	$(X+Y)(X'+Z)$	XZ	$X'Y$	$XZ+X'Y$
0 0 0	0	1	0	0	0	0
0 0 1	0	1	0	0	0	0
0 1 0	1	1	1	0	1	1
0 1 1	1	1	1	0	1	1
1 0 0	1	0	0	0	0	0
1 0 1	1	1	1	1	0	1
1 1 0	1	0	0	0	0	0
1 1 1	1	1	1	1	0	1

2.29 (b)

$X\ Y\ Z$	$X+Y$	$Y+Z$	$X'+Z$	$(X+Y)(Y+Z)(X'+Z)$	$(X+Y)(X'+Z)$
0 0 0	0	0	1	0	0
0 0 1	0	1	1	0	0
0 1 0	1	1	1	1	1
0 1 1	1	1	1	1	1
1 0 0	1	0	0	0	0
1 0 1	1	1	1	1	1
1 1 0	1	1	0	0	0
1 1 1	1	1	1	1	1

2-29 (c)

$X\ Y\ Z$	XY	YZ	$X'Z$	$XY+YZ+X'Z$	$XY+X'Z$
0 0 0	0	0	0	0	0
0 0 1	0	0	1	1	1
0 1 0	0	0	0	0	0
0 1 1	0	1	1	1	1
1 0 0	0	0	0	0	0
1 0 1	0	0	0	0	0
1 1 0	1	0	0	1	1
1 1 1	1	1	0	1	1

2.29 (d)

$A\ B\ C$	$A+C$	$AB+C'$	$(A+C)(AB+C')$	AB	AC'	$AB+AC'$
0 0 0	0	1	0	0	0	0
0 0 1	1	0	0	0	0	0
0 1 0	0	1	0	0	0	0
0 1 1	1	0	0	0	0	0
1 0 0	1	1	1	0	1	1
1 0 1	1	0	0	0	0	0
1 1 0	1	1	1	1	1	1
1 1 1	1	1	1	1	0	1

2.29 (e)

$W\ X\ Y\ Z$	$W'XY$	WZ	$W'XY+WZ$	$W'+Z$	$W+XY$	$(W'+Z)(W+XY)$
0 0 0 0	0	0	0	1	0	0
0 0 0 1	0	0	0	1	0	0
0 0 1 0	0	0	0	1	0	0
0 0 1 1	0	0	0	1	0	0
0 1 0 0	0	0	0	1	0	0
0 1 0 1	0	0	0	1	0	0
0 1 1 0	1	0	1	1	1	1
0 1 1 1	1	0	1	1	1	1
1 0 0 0	0	0	0	0	1	0
1 0 0 1	0	1	1	1	1	1
1 0 1 0	0	0	0	0	1	0
1 0 1 1	0	1	1	1	1	1
1 1 0 0	0	0	0	0	1	0
1 1 0 1	0	1	1	1	1	1
1 1 1 0	0	0	0	0	1	0
1 1 1 1	0	1	1	1	1	1

2.30

$F = (X+Y')Z + X'YZ'$ (from the circuit)

$= (X+Y'+ X'YZ')(Z+X'YZ')$ (distributive law)

$= (X+Y'+X')(X+Y'+Y)(X+Y'+Z')(Z+X')(Z+Y)(Z+Z')$ (distributive law)

$= (1+Y')(X+1)(X+Y'+Z')(Z+X')(Z+Y)(1)$ (complementation laws)

$= (1)(1)(X+Y'+Z')(Z+X')(Z+Y)(1)$ (0 and 1 operations)

$= (X+Y'+Z')(Z+X')(Z+Y)$ (0 and 1 operations)

$G = (X +Y' + Z')(X' + Z)(Y + Z)$ (from the circuit)

Unit 2 Solutions

Unit 3 Problem Solutions

3.6 (a) $(W + X' + Z')(W' + Y')(W' + X + Z')(W + X')(W + Y + Z)$

$= (W + X')(W' + Y')(W' + X + Z')(W + Y + Z)$

$= (W + X')[W' + Y'(X + Z')](W + Y + Z)$

$= [W + X'(Y + Z)][W' + Y'(X + Z')] = WY'(X + Z') + W'X'(Y + Z)$ {Using $(X + Y)(X' + Z) = X'Y + XZ$ with $X=W$}
$= WY'X + WY'Z' + W'X'Y + W'X'Z$

3.6 (b) $(A + B + C + D)(A' + B' + C + D')(A' + C)(A + D)(B + C + D)$

$= (B + C + D)(A' + C)(A + D) = (B + C + D)(A'D + AC)$ {Using $(X + Y)(X' + Z) = X'Y + XZ$ with $X = A$}
$= \cancel{A'DB} + \cancel{A'DC} + A'D + \cancel{ABC} + AC + \cancel{ACD} = A'D + AC$

3.7 (a) $BCD + C'D' + B'C'D + CD$

$= CD + C'(D' + B'D) = (C' + D)[C + (D' + B'D)]$ {Using $(X + Y)(X' + Z) = X'Y + XZ$ with $X=C$}
$= (C' + D)[C + (D' + B')(D' + D)] = (C' + D)(C + D' + B')$

3.7 (b) $A'C'D' + ABD' + A'CD + B'D$

$= D'(A'C' + AB) + D(A'C + B')$
$= D'[(A' + B)(A + C')] + D[(B' + A')(B' + C)]$ {Using $XY + X'Z = (X' + Y)(X + Z)$ twice inside the brackets}
$= [D + (A' + B)(A + C')][D' + (B' + A')(B' + C)]$ {Using $XY + X'Z = (X' + Y)(X + Z)$ with $X = D$}
$= (D + A' + B)(D + A + C')(D' + B' + A')(D' + B' + C)$ {Using the Distributive Law}

3.8
$F = AB \oplus [(A \equiv D) + D] = AB \oplus (\cancel{AD} + A'D' + D) = AB \oplus (A'D' + D) = AB \oplus (A' + D)$
$= (AB)'(A' + D) + AB(A' + D)' = (A' + B')(A' + D) + AB(AD')$
$= A' + B'D + ABD'$ {Using $(X + Y)(X + Z) = X + YZ$} $= A' + BD' + B'D$ {Using $X + X'Y = X + Y$}

3.9 $A \oplus BC = (A \oplus B)(A \oplus C)$ is not a valid distributive law. PROOF: Let $A = 1$, $B = 1$, $C = 0$.
LHS: $A \oplus BC = 1 \oplus 1 \cdot 0 = 1 \oplus 0 = 1$. RHS: $(A \oplus B)(A \oplus C) = (1 \oplus 1)(1 \oplus 0) = 0 \cdot 1 = 0$.

3.10 (a) $(X + W)(Y \oplus Z) + XW'$
$= (X + W)(YZ' + Y'Z) + XW'$

$= XYZ' + XY'Z + WYZ' + WY'Z + XW'$

Using Consensus Theorem
$WYZ' + WY'Z + XW'$

3.10 (b) $(A \oplus BC) + BD + ACD = A'BC + A(BC)' + BD + ACD$
$= A'BC + A(B' + C') + BD + ACD$
$= A'BC + AB' + AC' + BD + ACD$

$= A'BC + AB' + AC' + AD + BD + \cancel{ACD}$
(Add consensus term AD, eliminate ACD)
$= A'BC + AB' + AC' + BD$
(Remove consensus term AD)

3.10 (c) $(A' + C' + D')(A' + B + C')(A + B + D)(A + C + D)$

$= (A' + C' + D')(B + C' + D)\cancel{(A' + B + C')}\cancel{(A + B + D)}(A + C + D)$ Add consensus term

$= (A' + B + C')(A + B + D)$
$= (A' + C' + D')(B + C' + D)(A + C + D)$ Removing consensus terms

Unit 3 Solutions

3.11 $(\underline{A + B'} + C + E')(\underline{A + B'} + D' + E)(B' + C' + D' + E') = [A + B' + (C + \underline{E'})(D' + \underline{E})](B' + C' + D' + E')$

$= (A + \underline{B'} + D'E' + CE)(\underline{B'} + C' + D' + E') = B' + (A + D'E' + CE)(C' + D' + E')$

CD' {Add consensus term}

$= B' + AC' + AD' + AE' + C'D'E' + D'E' + \underline{D'E'} + \underline{CD'E} = B' + \underline{AC'} + AD' + AE' + \underline{CD'} + \cancel{CD'E} + D'E'$

$= B' + AC' + AE' + CD' + D'E'$

3.12 $\underline{A'CD'E} + A'B'D' + \underline{ABCE} + ABD = A'B'D' + ABD + BCD'E$

Proof: *LHS:* $A'CD'E + \underline{BCD'E} + \underline{A'B'D'} + ABCE + \underline{ABD}$ Add consensus term to left-hand side and use it to eliminate two consensus terms

$= BCD'E + A'B'D' + ABD$ This yields the right-hand side.

$\therefore$ LHS = RHS

3.13 (a) $KLMN' + K'L'MN + \underline{MN'} = K'L'MN + MN' = M(K'L'N + N') = M(N' + K'L')$ {Th. 11C with $Y = N'$} $= MN' + K'L'M$

3.13 (b) $KL'M' + M\underline{N'} + LM'\underline{N'} = KL'M' + N'(\underline{M} + L\underline{M'}) = KL'M' + N'(M + L) = KL'M' + MN' + LN'$

3.13 (c) $(K + \underline{L'})(K' + \underline{L'} + N)(\underline{L'} + M + N') = L' + \underline{K}(\underline{K'} + N)(M + N') = L' + K\underline{N}(M + \underline{N'}) = L' + KMN$

3.13 (d) $(\underline{K'} + L + \underline{M'} + N)(\underline{K' + M'} + N + E)(\underline{K' + M'} + N + E')\,KM$

$= [K' + M' + (L + N)(\underline{N + R})(\underline{N + R'})]\,KM$ {Th. 8N twice with $X = K' + M'$} $= [K' + M' + (\cancel{L + N})N]\,KM$

$= [\underline{K'} + \underline{M'} + N]\,\underline{KM} = KMN$

3.14 (a) $K'L'\underline{M} + K\underline{M'}N + KL\underline{M} + L\underline{M'}N' = M'(KN + LN') + M(K'L' + KL)$

$= M'[(K + N')(L + N)] + M[(K' + L)(K + L')]$ {Th. 14 twice with $X = N$ and $X = L$}

$= [M + (K + N')(L + N)][M' + (K' + L)(K + L')]$ {Th. 14 with $X = M$}

$= (M + K + N')(M + L + N)(M' + K' + L)(M' + K + L')$ {Distributive Law}

3.14 (b) $K\underline{L} + K'\underline{L'} + \underline{L'}M'N' + \underline{L}MN' = L'(K' + M'N') + L(K + MN')$

$= (L + K' + M'N')(L' + K + MN')$ {Th. 14 with $X = L$}

$= (L + K' + M')(L + K' + N')(L' + K + M)(L' + K + N')$

3.14 (c) $K\underline{L} + K'\underline{L'}M + \underline{L'}M'N + \underline{L}M'N' = L'[K'\underline{M} + \underline{M'}N] + L[\underline{K + M'N'}] = \underline{L'}[(M + N)(M' + K')] + \underline{L}[(K + M')(K + N')]$

$= [L + (M + N)(M' + K')][L' + (K + M')(K + N')] = (L + M + N)(L + M' + K')(L' + K + M')(L' + K + N')$

3.14 (d) $K'M'\underline{N} + KL'\underline{N'} + K'M\underline{N'} + L\underline{N} = N(\underline{K'M' + L}) + N'(\underline{K}L' + \underline{K'}M) = \underline{N}(L + K')(L + M') + \underline{N'}(L' + K')(K + M)$

$= [N' + (L + K')(L + M')][N + (L' + K')(K + M)] = (N' + L + K')(N' + L + M')(N + L' + K')(N + K + M)$

3.14 (e) $\underline{W}X\underline{Y} + \underline{W}X'\underline{Y} + \underline{W}\underline{Y}Z + XYZ' = WY(\underline{X} + \underline{X'} + Z) + XYZ' = W\underline{Y} + X\underline{Y}Z' = Y(W + XZ') = Y(W + X)(W + Z')$

3.15 (a) $(\underline{K' + M' + N})(\underline{K' + M})(\underline{L + M' + N'})(\cancel{K' + L + M})(\underline{M + N})$

$= (M' + NL + K'N')(M + K'N) = M(LN + K'N') + (M'K'N)$ {Using $XY + X'Z = (X + Z)(X' + Y)$ with $X = M$}

$= MLN + MK'N' + M'K'N$

3.15 (b) $(\underline{K' + L' + M'})(\underline{K + M + N'})(\underline{K + L})(\underline{K' + N})(\cancel{K' + M + N})$

$= [K' + N(L' + M')][K + L(M + N')] = KN(L' + M') + K'L(M + N') = KNL' + KNM' + K'LM + K'LN'$

3.15 (c) $(\underline{K' + L' + M})(\underline{K + N'})(\underline{K' + L + N'})(\underline{K + L})(\cancel{K + M + N'})$

$= [K' + (L' + M)(L + N')](K + LN') = (K' + LM + L'N')(K + LN')$ {Th. 14 with $X = L$}

$= K(LM + L'N') + K'LN'$ {By Th. 14 with $X = K$}

$= KLM + KL'N' + K'LN'$

3.15 (d) $(K + L + M)(K' + L' + N')(K' + L' + M')(K + L + N) = (K + L + MN)(K' + L' + M'N')$
$= K(L' + M'N') + K'(L + MN)$ {Th. 14 with $X = K$} $= KL' + KM'N' + K'L + K'MN$

3.15 (e) $(K + L + M)(K + M + N)(K' + L' + M')(K' + M' + N') = (K + M + LN)(K' + M' + L'N')$
$= K(M' + L'N') + K'(M + LN) = KM' + KL'N' + K'M + K'LN$
Alt. soln's: $KM' + K'M + L'MN' + LM'N$ (or) $KM' + K'M + K'LN + L'MN'$ (or) $KM' + K'M + KL'N' + LM'N$

3.16 (a) $(\underline{KL \oplus M}) + M'N' = \underline{(KL)'}M + KLM' + M'N' = (K' + L')M + KL\underline{M'} + \underline{M'}N' = \underline{M}(K' + L') + \underline{M'}(KL + N')$
$= (M' + K' + L')(M + N' + KL) = (M' + K' + L')(M + N' + K)(M + N' + L)$

3.16 (b) $M'(\underline{K \oplus N'}) + MN + K'N = M'[K'N' + KN] + MN + K'N = K'M'N' + KM'\underline{N} + M\underline{N} + K'\underline{N}$
$= K'M'N' + N(M + \underline{K}M' + \underline{K'})$
$= K'M'N' + N(\underline{M} + K' + \underline{M'}) = K'M'\underline{N'} + \underline{N} = N + K'M' = (K' + N)(M' + N)$

3.17
(a) $x \equiv 0 = x(0) + x'(0)' = x'$
(b) $x \equiv 1 = x(1) + x'(1)' = x$
(c) $x \equiv x = x(x) + x'(x)' = x + x' = 1$
(d) $x \equiv x' = x(x') + x'(x')' = 0$
(e) $x \equiv y = xy + x'y' = yx + y'x' = y \equiv x$
(f) $(x \equiv y) \equiv z = (xy + x'y') \equiv z = (xy + x'y')z + (xy' + x'y)z' = xyz + x'y'z + xy'z' + x'yz'$
$= x(yz + y'z') + x'(y'z + yz') = x(yz + y'z') + x'(yz + yz)' = x \equiv (yz + y'z') = x \equiv (y \equiv z)$
(g) $(x \equiv y)' = (xy + x'y')' = (x' + y')(x + y) = x'y + xy' = x' \equiv y = xy' + x'y = x \equiv y'$

3.18
(a) $x \oplus 0 = x(0)' + x'(0) = x$
(b) $x \oplus 1 = x(1)' + x'(1)' = x'$
(c) $x \oplus x = x(x)' + x'(x) = 0$
(d) $x \oplus x' = x(x')' + x'(x') = x + x' = 1$
(e) $x \oplus y = xy' + x'y = y'x + yx' = y \oplus x$
(f) $(x \oplus y) \oplus z = (xy' + x'y) \oplus z = (xy' + x'y)z' + (xy' + x'y)'z = xy'z' + x'yz' + xyz + x'y'z$
$= x(yz + y'z') + x'(yz' + y'z) = x(yz' + yz')' + x'(yz' + y'z) = x \oplus (yz + y'z') = x \oplus (y \oplus z)$
(g) $(x \oplus y)' = (xy' + x'y)' = (x' + y)(x + y') = x'y' + xy = x' \oplus y = xy + x'y' = x \oplus y'$

3.19
(a) $x \oplus y \oplus xy = x \oplus [y(xy)' + y'(xy)] = x \oplus [yx'] = x(yx')' + x'(yx') = x(y'+x) + x'y = x + x'y = x + y$
(b) $x \equiv y \equiv xy = (xy + x'y') \equiv xy = (xy + x'y')xy + (xy + x'y')'(xy)' = xy + (xy' + x'y)(x' + y') = xy + x'y + xy'$
$= x + y$

3.20
(a) $xy \oplus xz = xy(x' + z') + (x' + y')xz = xyz' + xyz = x(yz' + yz') = x(y \oplus z)$
(b) For $y = 1$, the left hand side is $x + z'$ but the right hand side is $x'z'$ which are not equal.
(c) For $y = 0$, the left hand side is xz' but the right hand side is $x' + z'$ which are not equal.
(d) $(x + y) \equiv (x + z) = (x + y)(x + z) + (x + y)'(x + z)' = x + yz + (x'y')(x'z') = x + yz + x'y'z' = x + yz + y'z'$
$= x + (y \equiv z)$

3.21 (a) $BC'D' + \underline{ABC'} + \cancel{AC'D} + \underline{AB'D} + A'BD' = \cancel{BC'D'} + \underline{ABC'} + AB'D + \underline{A'BD'} = ABC' + AB'D + A'BD'$

3.21 (b) $W'Y' + \underline{WYZ} + \underline{XY'Z} + WX'Y + WXZ = \underline{W'Y'} + WYZ + \cancel{XY'Z} + WX'Y + \underline{WXZ} = W'Y' + WYZ + \underline{WX'Y} + \underline{WXZ}$
$= W'Y' + WX'Y + WXZ$

3.21 (c) $(\cancel{B + C + D})(\underline{A + B + C})(\underline{A' + C + D})(B' + C' + D') = (A + B + C)(A' + C + D)(B' + C' + D')$

3.21 (d) $\underline{W'XY} + \underline{WXZ} + WY'Z + W'Z' = \cancel{W'XY} + \cancel{WXZ} + \underline{WY'Z} + \underline{W'Z'} + \underline{XYZ} = WY'Z + W'Z' + XYZ$
XYZ (add consensus term)

Unit 3 Solutions

3.21 (e) $\cancel{A'BC'} + BC'D' + \cancel{A'CD} + B'CD + A'BD = BC'D' + B'CD + A'BD$

3.21 (f) $(A + B + C)(B + C' + D)\cancel{(A + B + D)}(A' + B' + D') = (A + B + C)(B + C' + D)(A' + B' + D')$

3.22 $Z = ABC + DE + ACF + AD' + AB'E' = A(BC + CF + D' + B'E') + DE$

$= (A + DE)(DE + BC + CF + D' + B'E')$ {By Th. 8D with $X = DE$}

$= (A + D)(A + E)(BC + CF + D' + E + B'E')$

$= (A + D)(A + E)(D' + E + B' + BC + CF)$ {Since $E + B'E' = E + B'$}

$= (A + D)(A + E)(D' + E + B' + C + CF)$ {Since $B' + BC = B' + C$}

$= (A + D)(A + E)(D' + E + B' + C)$ {Since $C + CF = C$}

$= (A + DE)(D' + E + B' + C)$

$= AD' + AE + AB' + AC + DE + DEB' + DEC$ {eliminate consensus term AE; use $X + XY = X$ where $X = DE$}

$= AD' + AB' + AC + DE$

3.23 $F = A'B + AC + BC'D' + BEF + BDF = (A + B)(A' + C) + B(C'D' + EF + DF)$

$= [(A + B)(A' + C) + B][(A + B)(A' + C) + C'D' + EF + DF]$

$= (A + B)(A' + C + B)(A + B + C'D' + EF + DF)(A' + C + C'D' + EF + DF)$

$B + C$ $\qquad$ $C + D'$

$= (A + B)\cancel{(A' + C + B)}(C + B)\cancel{(A + B + C'D' + EF + DF)}(A' + C + D' + EF + DF)$

$= (A + B)(B + C)(A' + C + D' + FE + DF) = (A + B)(B + C)(A' + C + D' + F + FE)$

$= (A + B)(B + C)(A' + C + D' + F)$

$= (B + AC)(A' + C + D' + F)$

$= (A'B + BC + BD' + BF + AC + ACD' + ACF = A'B + BD' + BF + AC$

use consensus, $X + XY = X$ where $X = AC$

3.24 $X'Y'Z' + XYZ = (X + Y'Z')(X' + YZ) = (X + Y')(X + Z')(X' + Y)(X' + Z)(Y + Z')$

$= (X + Y')(X + Z')(X' + Y)(X' + Z)(Y + Z') = (X + Y')(X + Z')(X' + Z)(Y + Z')$

$= (X + Y')(X' + Z)(Y + Z')$

Alt.: $(X' + Y)(Y' + Z)(X + Z')$ by adding $(Y' + Z)$ as consensus in 3rd step

3.25 (a) $xy + x'yz' + yz = y(x + x'z') + yz = xy + yz' + yz$

$= xy + y = y$

Alternate Solution: $xy + x'yz' + yz = y(x + x'z' + z)$

$= y(x + z' + z) = y(x + 1) = y$

3.25 (b) $(xy' + z)(x + y')z = (xy' + xz + y'z)z$

$= xy'z + xz + y'z = xz + y'z$

Alternate Solution: $(xy' + z)(x+y')z = z(x + y')$

$= zx + zy'$

3.25 (c) $xy' + z + (x' + y)z'$

$= x'y + (x' + y)$ {By Th. 11D with $Y = z$}

$= xy' + x' + y = x + x' + y = 1 + y = 1$

Alt.: $xy' + z + (x' + y)z' = (xy' + z) + (xy' + z)' = 1$

3.25 (d) $a'd(b' + c) + a'd'(b + c') + (b' + c)(b + c')$

$= a'b'd + a'cd + a'bd' + a'c'd' + b'c' + bc$

$= a'b'd + a'bd' + b'c' + bc'$

Other Solutions: $b'c' + bc + a'c'd' + a'b'd$

$b'c' + bc + a'c'd' + a'cd$

$b'c' + bc + a'bd' + a'cd$

3.25 (e) $w'x' + x'y' + yz + w'z' + x'z$ Add redundant term

$= w'x' + x'y' + yz + w'z' + x'z$

$= x'y' + yz + w'z' + x'z$ Remove redundant term

$= x'y' + yz + w'z'$

3.25 (f) $A'BCD + A'BC'D + B'EF + CDE'G + A'DEF + A'B'EF$

$= A'BD + B'EF + CDE'G + A'DEF$ (consensus)

$= A'BD + B'EF + CDE'G$

3.25 (g) $[(a' + d' + b'c)(b + d + ac')]' + b'c'd' + a'c'd$

$= ad(b + c') + b'd'(a' + c) + b'c'd' + a'c'd$

$= abd + ac'd + a'b'd' + b'cd' + b'c'd' + a'c'd$

$c'd \qquad b'd'$

$= abd + a'b'd' + b'd' + c'd = abd + b'd' + c'd$

3.26 (a) $A'C'D' + AC' + BCD + A'CD' + A'BC + AB'C'$

$= A'D' + AC' + BCD + A'BC$ consensus

$= A'D' + AC' + BCD$

3.26 (b) $A'B'C' + ABD + A'C + \cancel{A'CD'} + AC'D + AB'C'$

$= B'C' + ABD + A'C + AC'D$

$= B'C' + ABD + A'C$

3.27 $WXY' + (W'Y' \equiv X) + (Y \oplus WZ)$

$= WXY' + W'Y'X + (W'Y')'X' + Y(WZ)' + Y'WZ$

$= WXY' + W'XY' + (W + Y)X' + Y(W' + Z') + Y'WZ$

$= XY' + WX' + X'Y + W'Y + YZ' + WY'Z + WY'$

$= XY' + WX' + \cancel{X'Y} + W'Y + YZ' + \cancel{WY'Z} + WY'$

$= XY' + WX' + W'Y + YZ' + WY'$

$= XY' + WX' + W'Y + YZ'$

Alternate Solutions: $F = W'Y + WX' + WZ' + XY'$

$F = YZ' + W'X + XY' + WY'$

$F = W'X + X'Y + XZ' + WY'$

$F = W'X + XY' + WZ' + WY'$

3.28 (a) VALID: $a'b + b'c + c'a$

$= a'b(c + c') + (a + a')b'c + (b + b')ac'$

$= a'bc + a'bc' + ab'c + a'b'c + abc' + ab'c'$

$= a'c + \qquad bc' + \qquad ab'$

Alternate Solution: $a'b + b'c + c'a$

Add all consensus terms: ab', bc', ca'

$\therefore$ We get $= a'b + b'c + c'a + ab' + bc' + ca'$

$= ab' + bc' + ca'$

3.28 (b) NOT VALID. Counterexample: a = 0, b = 1, c = 0. LHS = 0, RHS = 1. $\therefore$ This equation is *not* always valid.

In fact, the two sides of the equation are complements: $[(a + b)(b + c)(c + a)]'$

$= [(b + ac)(a + c)]' = [ab + ac + bc]'$

$= (a' + b')(a' + c')(b' + c')$

3.28 (c) VALID. Starting with the right side, add consensus terms

$RHS = abc + ab'c' + b'cd + bc'd + acd + ac'd$

$= abc + ab'c' + b'cd + bc'd + acd + ac'd$

$= abc + ab'c' + b'cd + bc'd + ad = LHS$

3.28 (d) VALID: LHS $= xy' + x'z + yz'$

consensus terms: $y'z, xz', x'y$

$= xy' + x'z + yz' + y'z + xz' + x'y$

$= y'z + xz' + x'y =$ RHS

3.28 (e) NOT VALID. Counterexample: x = 0, y = 1, z = 0, then LHS = 0, RHS = 1. $\therefore$ This equation is *not* always valid. In fact, the two sides of the equations are complements.

LHS $= (x + y)(y + z)(x + z)$

$= [(x + y)' + (y + z)' + (x + z)']'$

$= (x'y' + y'z' + x'z')' = [x'(y' + z') + y'z']'$

$= [(x' + y'z')(y' + z' + y'z')]'$

$= [(x' + y')(x' + z')(y' + z')]'$

$\neq (x' + y')(y' + z')(x' + z')$

3.28 (f) VALID: LHS $= abc' + ab'c + b'c'd + bcd$

consensus terms: $ab'd, \quad abd$

$= abc' + ab'c + b'c'd + bcd + ab'd + abd$

ad

$abc' + ab'c + ad + bcd + b'c'd =$ RHS

Unit 3 Solutions

3.29 $SUM = (X \oplus Y) \oplus C_i = (XY' + X'Y) \oplus C_i$
$= (XY' + X'Y)C_i' + (XY' + X'Y)'C_i$
$= XY'C_i' + X'YC_i' + X'Y'C_i + XYC_i$

$C_o = (X \oplus Y)C_i + XY$
$= XY'C_i + X'YC_i + XY$
$= XC_i + YC_i + XY$

X	Y	C_i	SUM	C_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

3.30

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$F = AB + AC + BC$

3.31 (a) VALID:
LHS $= (X' + Y')(X \oplus Z) + (X + Y)(X \oplus Z)$
$= (X' + Y')(X'Z' + XZ) + (X + Y)(X'Z + XZ')$
$= \underline{X'Z'} + \cancel{X'YZ'} + \underline{XY'Z} + \underline{X'YZ} + \underline{XZ'} + \cancel{XYZ'}$

$= \underline{X'Z'} + (\underline{XY' + X'Y})Z + \underline{XZ'}$
$= \underline{Z'} + \underline{Z}(X \oplus Y) = Z' + (X \oplus Y) =$ RHS

3.31 (b) LHS $= (W' + X + Y')(\underline{W} + X' + Y)(\underline{W} + Y' + Z) = (W' + X + Y')(W + (X' + \underline{Y})(\underline{Y'} + Z))$
$= (\underline{W'} + X + Y')(\underline{W} + (X'Y' + YZ)) = (W'(X'Y' + YZ) + W(X + Y')) = \underline{W'X'Y'} + \underline{W'YZ} + \underline{WX} + \underline{WY'}$

consensus terms: $X'Y'$ XYZ

$= W'X'Y' + W'YZ + WX + WY' + XYZ + X'Y' = W'X'Y' + W'X'Z + W'YZ + XYZ + \underline{WX} + WY' + \underline{X'Y'}$

$= \cancel{W'X'Z} + \underline{W'YZ} + XYZ + WX + \underline{X'Y'} = W'YZ + XYZ + WX + X'Y'$

3.31 (c) LHS $= \underline{A}B\underline{C} + \underline{A}'C'\underline{D'} + \underline{A}'B\underline{D'} + \underline{AC}D = \underline{A}C(B + D) + \underline{A}'D'(B + C') = (A + D'(B + C'))(A' + C(B + D))$
$= (A + D')(\underline{A + B + C'})(A' + C)(\underline{A' + B + D}) = (A + D')(A + B + C')(\underline{A' + C})(A' + B + D)(\underline{B + C' + D})$

consensus: $B + C' + D$

$= (\underline{A + D'})(A + B + C')(A' + C)(\underline{B + C' + D}) = (A + D')(A' + C)(B + C' + D) =$ RHS

3.32 (a) VALID. $[A + B = C] \Rightarrow [D'(A + B) = D'(C)]$
$[A + B = C] \Rightarrow [AD' + BD' = CD']$

3.32 (b) NOT VALID. Counterexample: $A = 1, B = C = 0$
and $D = 1$ then LHS $= (0)(0) + (0)(0) = 0$
RHS $= (0)(1) = 0 =$ LHS
but $B + C = 0 + 0 = 0$; $D = 1 \neq B + C$
$\therefore$ The statement is false.

3.32 (c) VALID. $[A + B = C] \Rightarrow [(A + B) + D = (C) + D]$
$[A + B = C] \Rightarrow [A + B + D = C + D]$

3.32 (d) NOT VALID. Counterexample: $C = 1, A = B = 0$
and $D = 1$ then LHS $= 0 + 0 + 1 = 1$
RHS $= 1 + 1 = 1 =$ LHS
but $A + B = 0 + 0 = 0 \neq D$
$\therefore$ The statement is false.

3.33 (a) $A'C' + BC + AB' + A'BD + B'C'D' + ACD'$
Consensus terms: (1) $B'C'$ using $A'C' + AB'$
(2) $A'B$ using $A'C' + BC$ (3) AC using $AB' + BC$
(4) $AB'D'$ using $B'C'D' + ACD'$
Using 1, 2, 3: $A'C' + BC + AB' + \cancel{A'BD} + \cancel{B'C'D'}$
$+ \cancel{ACD'} + B'C' + A'B + AC = A'C' + BC + AB'$
(Using the consensus theorem to remove the added terms since the terms that generated them are still present.)

3.33 (b) $A'C'D' + BC'D + AB'C' + A'BC$
Consensus terms:
(1) $A'BC'$ using $A'C'D' + BC'D$
(2) $AC'D$ using $AB'C' + BC'D$
(3) $B'C'D'$ using $A'C'D' + AB'C'$
(4) $A'BD'$ using $A'C'D' + A'BC$
(5) $A'BD$ using $BC'D + A'BC$
Using 1: $A'C'D' + BC'D + AB'C' + \cancel{A'BC} + A'B$, which is the minimum solution.

3.34 $abd'f' + b'cegh' + abd'f + acd'e + b'ce$
$= (abd'f' + abd'f) + (b'cegh' + b'ce) + acd'e$
$= abd' + b'ce + acd'e$
$= abd' + b'ce$ (consensus)
$= (b + ce)(b' + ad')$
$= (b + c)(b + e)(b' + a)(b' + d')$

3.35 $(a + c)(b' + d)(a + c' + d')(b' + c' + d')$
$= (a + cd')(b' + c'd)$
$= ab' + ac'd + b'cd'$

3.36 $abc' + d'e + ace + b'c'd'$
$= (d' + abc' + ace + b'c'd')(e + abc' + ace + b'c'd')$
$= (d' + abc' + ace)(e + abc' + b'c'd')$
$= [d' + a(bc' + ce)][e + c'(ab + b'd')]$
$= [d' + a(b + c)(c' + e)][e + c'(a + b')(b + d')]$
$= (d' + a)(d' + b + c)(d' + c' + e)(e + c')$
$(e + a + b')(e + b + d')$
$= (d' + a)(d' + b + c)(e + c')$
$(e + a + b')(e + b + d')$
$= (d' + a)(d' + b + c)(e + c')(e + a + b')$
(consensus)

3.37 (a) $(x \equiv y)' = (xy + x'y')' = (x' + y')(x + y)$
$= x'y + xy' = x \oplus y$
(b) $a'b'c' + a'bc + ab'c + abc'$
$= a'(b'c' + bc) + a(b'c + bc')$
$= a'(b \equiv c) + a\,(b \equiv c)'$
$= a' \equiv (b \equiv c)$

Unit 4 Problem Solutions

4.1 *See FLD p. 695 for solution.*

4.2

A B C D E		*y*	*z*
0 0 0 0 0	(less than 10 gpm)	+	
1 0 0 0 0	(at least 10 gpm)	+	
1 1 0 0 0	(at least 20 gpm)	+	+
1 1 1 0 0	(at least 30 gpm)		+
1 1 1 1 0	(at least 40 gpm)		+
1 1 1 1 1	(at least 50 gpm)		

4.2 (a) $Y = A'B'C'D'E' + AB'C'D'E' + ABC'D'E'$

4.2 (b) $Z = ABC'D'E' + ABCD'E' + ABCDE'$

4.3 $F_1 = \sum m(0, 4, 5, 6)$; $F_2 = \sum m(0, 3, 4, 6, 7)$; $F_1 + F_2 = \sum m(0, 3, 4, 5, 6, 7)$

General rule: $F_1 + F_2$ is the sum of all minterms that are present in either F_1 or F_2.

Proof: Let $F_1 = \sum_{i=0}^{2^n-1} a_i m_i$; $F_2 = \sum_{j=0}^{2^n-1} b_j m_j$; $F_1 + F_2 = \sum_{i=0}^{2^n-1} a_i m_i + \sum_{j=0}^{2^n-1} b_j m_j = a_0 m_0 + a_1 m_1 + a_2 m_2 + \ldots$

$+ b_0 m_0 + b_1 m_1 + b_2 m_2 + \ldots = (a_0 + b_0)\, m_0 + (a_1 + b_1)\, m_1 + (a_2 + b_2)\, m_2 + \ldots = \sum_{i=0}^{2^n-1} (a_i + b_i)\, m_i$

4.4 (a) $2^{2^n} = 2^{2^2} = 2^4 = 16$

4.4 (b)

$x\ y$	z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}
0 0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0 1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1 0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1 1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
	0	$x'y'$	$x'y$	x'	xy'	y'	$x'y+xy'$	$x'+y'$	xy	$x'y'+xy$	y	$x'+y$	x	$x+y'$	$x+y$	1

4.5

A B C	*D E F*	*Z*
0 0 0	1 1 X[3]	1
0 0 1	X[2] X[2] 1	1
0 1 0	X[1] X[1] X[1]	X
0 1 1	X[2] X[2] 1	1
1 0 0	X[4] 0 0	0
1 0 1	X[2] X[2] 1	1
1 1 0	X[1] X[1] X[1]	X
1 1 1	X[4] 0 0	0

Alternate Solutions

A B C	*D E F*	*Z*
0 1 1	1 1 X[3]	1
1 1 1	0 X[4] 0	0

[1] These truth table entries were made don't cares because *ABC* = 110 and *ABC* = 010 can never occur

[2] These truth table entries were made don't cares because when *F* is 1, the output *Z* of the OR gate will be 1 regardless of its other input. So changing *D* and *E* cannot affect *Z*.

[3] These truth table entries were made don't cares because when *D* and *E* are both 1, the output *Z* of the OR gate will be 1 regardless of the value of *F*.

[4] These truth table entries were made don't cares because when one input of the AND gate is 0, the output will be 0 regardless of the value of its other input.

4.6 (a) Of the four possible combinations of d_1 & d_5, $d_1 = 1$ and $d_5 = 0$ gives the best solution:
$F = A'B'C' + A'B'C + ABC' + ABC = A'B' + AB$

4.6 (b) By inspection, $G = C$ when both don’t cares are set to 0.

Unit 4 Solutions

4.7 (a) Exactly one variable not complemented: $F = A'B'C + A'BC' + AB'C' = \sum m(1, 2, 4)$

4.7 (b) Remaining terms are maxterms:
$F = \prod M(0, 3, 5, 6, 7) = (A + B + C)(A + B' + C')(A' + B + C')(A' + B' + C)(A' + B' + C')$

4.8

$A\,B\,C\,D$		F
0 0 0 0	$0 \times 0 = 0 \le 2$	1
0 0 0 1	$0 \times 1 = 0 \le 2$	1
0 0 1 0	$0 \times 2 = 0 \le 2$	1
0 0 1 1	$0 \times 3 = 0 \le 2$	1
0 1 0 0	$1 \times 0 = 0 \le 2$	1
0 1 0 1	$1 \times 1 = 1 \le 2$	1
0 1 1 0	$1 \times 2 = 2 \le 2$	1
0 1 1 1	$1 \times 3 = 3 > 2$	0
1 0 0 0	$2 \times 0 = 0 \le 2$	1
1 0 0 1	$2 \times 1 = 2 \le 2$	1
1 0 1 0	$2 \times 2 = 4 > 2$	0
1 0 1 1	$2 \times 3 = 6 > 2$	0
1 1 0 0	$3 \times 0 = 0 \le 2$	1
1 1 0 1	$3 \times 1 = 3 > 2$	0
1 1 1 0	$3 \times 2 = 6 > 2$	0
1 1 1 1	$3 \times 3 = 9 > 2$	0

4.8 (a) $F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 6, 8, 9, 12)$
Refer to FLD p. 695 for full term expansion

4.8 (b) $F(A, B, C, D) = \prod M(7, 10, 11, 13, 14, 15)$
Refer to FLD p. 695 for full term expansion

4.9 (a) $F = abc' + b'(a + a')(c + c') = abc' + ab'c + ab'c' + a'b'c + a'b'c'$; $F = \sum m(0, 1, 4, 5, 6)$

4.9 (b) Remaining terms are maxterms: $F = \prod M(2, 3, 7)$

4.9 (c) Maxterms of F are minterms of F':
$F' = \sum m(2, 3, 7)$

4.9 (d) Minterms of F are maxterms of F':
$F' = \prod M(0, 1, 4, 5, 6)$

4.10

$$F(a, b, c, d) = (a + b + d)(a' + c)(a' + b' + c')(a + b + c' + d')$$

$$= (a + b + c + d)(a + b + c' + d)(a' + c + bb' + dd')(a' + b' + c' + d)(a' + b' + c' + d')(a + b + c' + d')$$

$$= (a + b + c + d)(a + b + c' + d)(a' + b + c + d)(a' + b + c + d')(a' + b' + c + d)(a' + b' + c + d')$$
$$(a' + b' + c' + d)(a' + b' + c' + d')(a + b + c' + d')$$

4.10 (a) $F = \sum m(1, 4, 5, 6, 7, 10, 11)$

4.10 (b) $F = \prod M(0, 2, 3, 8, 9, 12, 13, 14, 15)$

4.10 (c) $F' = \sum m(0, 2, 3, 8, 9, 12, 13, 14, 15)$

4.10 (d) $F' = \prod M(1, 4, 5, 6, 7, 10, 11)$

4.11 (a) difference, $d_i = x_i \oplus y_i \oplus b_i$; $b_{i+1} = b_i x_i' + x_i' y_i + b_i y_i$

$x_i\,y_i\,b_i$	b_{i+1}	d_i
0 0 0	0	0
0 0 1	1	1
0 1 0	1	1
0 1 1	1	0
1 0 0	0	1
1 0 1	0	0
1 1 0	0	0
1 1 1	1	1

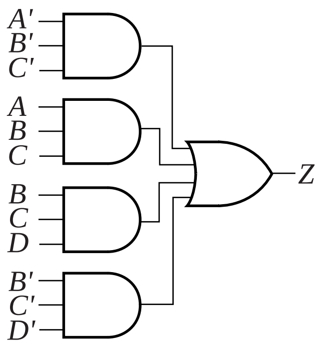
4.11 (b) $d_i = s_i$; b_{i+1} is the same as c_{i+1} with x_i replaced by x_i'

4.12 *See FLD p. 696 for solution.*

4.13

A B C D	Z
0 0 0 0	1
0 0 0 1	1
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	0
1 0 1 0	0
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	1
1 1 1 1	1

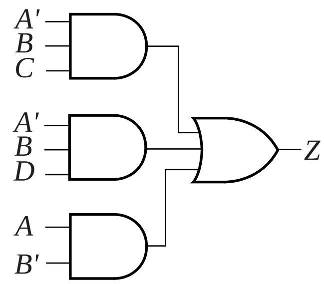
$Z = A'B'C'D' + A'B'C'D + AB'C'D' + ABCD' + ABCD + A'BCD$
$= A'B'C' + ABC + AB'C'D' + A'BCD$
$= A'B'C' + ABC + AB'C'D' + A'BCD + \underline{BCD} + \underline{B'C'D'}$
(Added consensus terms)
$\therefore Z = A'B'C' + ABC + BCD + B'C'D'$



4.14

A B C D	Z
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	1
0 1 1 0	1
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	1
1 0 1 1	1
1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
1 1 1 1	0

$Z = A'BC'D + A'BCD' + A'BCD + AB'C'D' + AB'C'D + AB'CD' + AB'CD$
$= A'BD + AB'C' + AB'C + A'BCD'$
$= AB' + A'BD + A'BCD' + \underline{A'BC}$
(Added consensus terms)
$\therefore Z = AB' + A'BD + A'BC$



4.15 (a) Prime digits are 1, 3, 5, and 7 represented as 0010, 0111, 1011 and 1110. The minterms are $A'B'CD'$, $A'BCD$, $AB'CD$ and $ABCD'$. The don't care minterms are $A'B'C'D'$, $A'B'CD$, $A'BC'D$, $A'BCD'$, $AB'C'D$, $AB'CD'$, $ABC'D'$ and $ABCD$.

(b) Nonprime digits are 0, 2, 4, and 6 represented as 0001, 0100, 1000 and 1101. The maxterms are $A + B + C + D'$, $A + B' + C + D$, $A' + B + C + D$ and $A' + B' + C + D'$. The don't care maxterms are $A + B + C + D$, $A + B + C' + D'$, $A + B' + C + D'$, $A + B' + C' + D$, $A' + B + C + D'$, $A' + B + C' + D$, $A' + B' + C + D$ and $A' + B' + C' + D'$.

4.16 Truth Table

$x_3x_2x_1x_0$	$z\,y_1y_0$
0 0 0 0	0 x x
0 0 0 1	1 0 0
0 0 1 0	1 0 1
0 0 1 1	1 0 1
0 1 0 0	1 1 0
0 1 0 1	1 1 0
0 1 1 0	1 1 0
0 1 1 1	1 1 0
1 0 0 0	1 1 1
1 0 0 1	1 1 1
1 0 1 0	1 1 1
1 0 1 1	1 1 1
1 1 0 0	1 1 1
1 1 0 1	1 1 1
1 1 1 0	1 1 1
1 1 1 1	1 1 1

(a)
minterms of z: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
minterms of y_1: 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
don't care minterm: 0
minterms of y_0: 2, 3, 8, 9, 10, 11, 12, 13, 14, 15
don't care minterm: 0

(b)
maxterms of z: 0
maxterms of y_1: 1, 2, 3
don't care maxterm: 0
maxterms of y_0: 1, 4, 5, 6, 7
don't care maxterm: 0

4.17 Truth Table

A B C D	W X Y Z
0 0 0 0	1 0 0 1
0 0 0 1	1 0 0 0
0 0 1 0	0 1 1 1
0 0 1 1	0 1 1 0
0 1 0 0	0 1 0 1
0 1 0 1	0 1 0 0
0 1 1 0	0 0 1 1
0 1 1 1	0 0 1 0
1 0 0 0	0 0 0 1
1 0 0 1	0 0 0 0
1 0 1 0	x x x x
1 0 1 1	x x x x
1 1 0 0	x x x x
1 1 0 1	x x x x
1 1 1 0	x x x x
1 1 1 1	x x x x

(a)
minterms of w: 0, 1
minterms of x: 2, 3, 4, 5
minterms of y: 2, 3, 6, 7
minterms of z: 0, 2, 4, 6, 8
don't care minterms: 10, 11, 12, 13, 14, 15

(b)
maxterms of w: 2, 3, 4, 5, 6, 7, 8, 9
maxterms of x: 0, 1, 6, 7, 8, 9
maxterms of y: 0, 1, 4, 5, 8, 9
maxterms of z: 1, 3, 5, 7, 9
don't care maxterms: 10, 11, 12, 13, 14, 15

4.18 (a), (b) Truth Table

$A\,B\,C\,D$	$W X\,Y\,Z$
0 0 0 0	1 1 1 1
0 0 0 1	1 1 1 0
0 0 1 0	1 1 0 1
0 0 1 1	1 1 0 0
0 1 0 0	x x x x
0 1 0 1	x x x x
0 1 1 0	1 0 0 1
0 1 1 1	x x x x
1 0 0 0	x x x x
1 0 0 1	0 1 1 0
1 0 1 0	x x x x
1 0 1 1	x x x x
1 1 0 0	0 0 1 1
1 1 0 1	0 0 1 0
1 1 1 0	0 0 0 1
1 1 1 1	0 0 0 0

(a)
minterms of W: 0, 1, 2, 3, 6
minterms of X: 0, 1, 2, 3, 9
minterms of Y: 0, 1, 9, 12, 13
minterms of Z: 0, 2, 6, 12,14
don't care minterms: 4, 5, 7, 8, 10, 11

(b)
maxterms of W: 9, 12, 13, 14, 15
maxterms of X: 6, 12, 13, 14, 15
maxterms of Y: 2, 3, 6, 14, 15
maxterms of Z: 1, 3, 9, 13, 15
don't care maxterms: 4, 5, 7, 8, 10, 11

4.18 (a), (b) Alternative Truth Table

$A\,B\,C\,D$	$W X\,Y\,Z$
0 0 0 0	1 1 1 1
0 0 0 1	1 1 1 0
0 0 1 0	1 1 0 1
0 0 1 1	1 1 0 0
0 1 0 0	x x x x
0 1 0 1	x x x x
0 1 1 0	x x x x
0 1 1 1	1 0 0 0
1 0 0 0	0 1 1 1
1 0 0 1	x x x x
1 0 1 0	x x x x
1 0 1 1	x x x x
1 1 0 0	0 0 1 1
1 1 0 1	0 0 1 0
1 1 1 0	0 0 0 1
1 1 1 1	0 0 0 0

(a)
minterms of W: 0, 1, 2, 3, 7
minterms of X: 0, 1, 2, 3, 8
minterms of Y: 0, 1, 9, 12, 13
minterms of Z: 0, 2, 8, 12, 14
don't care minterms: 4, 5, 6, 9, 10, 11

(b)
maxterms of W: 8, 12, 13, 14, 15
maxterms of X: 7, 12, 13, 14,15
maxterms of Y: 2, 3, 7, 14, 15
maxterms of Z: 1, 3, 7, 13, 15
don't care maxterms: 4, 5, 6, 9, 10, 11

4.19 (a) The $\underbrace{\text{buzzer will sound}}_{B}$ if $\underbrace{\text{the key is in the ignition switch}}_{K}$ and $\underbrace{\text{the car door is open}}_{D}$, or $\underbrace{\text{the seat belts are not fastened}}_{S'}$.
∴ The two possible interpretations are: $B = KD + S'$ and $B = K(D + S')$

4.19 (b) You $\underbrace{\text{will gain weight}}_{W}$ if you $\underbrace{\text{eat too much}}_{F}$, or you do $\underbrace{\text{not exercise enough}}_{E'}$ and $\underbrace{\text{your metabolism rate is too low}}_{M}$.
∴ The two possible interpretations are: $W = (F + E')\,M$ and $W = F + E'M$

4.19 (c) The $\underbrace{\text{speaker will be damaged}}_{D}$ if the $\underbrace{\text{volume is set too high}}_{V}$ and $\underbrace{\text{loud music is played}}_{M}$ or the $\underbrace{\text{stereo is too powerful}}_{S}$.
∴ The two possible interpretations are: $D = VM + S$ and $D = V\,(M + S)$

4.19 (d) The $\underbrace{\text{roads will be very slippery}}_{V}$ if $\underbrace{\text{it snows}}_{S}$ or $\underbrace{\text{it rains}}_{R}$ and $\underbrace{\text{there is oil on the road}}_{O}$.
∴ The two possible interpretations are: $V = (S + R)\,O$ and $V = S + RO$

4.20 $Z = AB + AC + BC$

4.21 $Z = (ABCDE + A'B'C'D'E')'$; $Y = A'B'CD'E$

4.22 (a) $13_{10} = D_{16} = 0001101$; ∴ $X = A'B'C'DEF'G$

4.22 (b) $10_{10} = 0001010$; ∴ $Y = A'B'C'DE'FG'$

4.22 (c) $0_{10} = 0000000_2$; $64_{10} = 1000000_2$; $31_{10} = 0011111_2$; $127_{10} = 1111111_2$; $32_{10} = 0100000_2$; ∴ $Z = (A'B')' = A + B$

4.23 $F_1F_2 = \prod M(0, 4, 5, 6, 7)$. General rule: F_1F_2 is the product of all maxterms that are present in either F_1 or F_2.
Proof:

Let $F_1 = \prod (a_i + M_i)$; $F_2 = \prod (b_j + M_j)$; $F_1F_2 = \prod (a_i + M_i) \prod (b_j + M_j)$

$= (a_0 + M_0)(b_0 + M_0)(a_1 + M_1)(b_1 + M_1)(a_2 + M_2)(b_2 + M_2) \ldots = (a_0b_0 + M_0)(a_1b_1 + M_1)(a_2b_2 + M_2) \ldots$

$= \prod (a_ib_i + M_i)$

Maxterm M_i is present in F_1F_2 iff $a_ib_i = 0$, i.e., if either $a_i = 0$ or $b_i = 0$. Maxterm M_i is present in F_1 iff $a_i = 0$. Maxterm M_i is present in F_2 iff $b_i = 0$. Therefore, maxterm M_i is present in F_1F_2 iff it is present in F_1 or F_2.

4.24 $F_1 + F_2 = \prod M(0, 4)$. General rule: $F_1 + F_2$ is the product of all maxterms that are present in both F_1 and F_2.
Proof:

Let $F_1 = \sum_{i=0}^{2^n-1} (a_i m_i)$; $F_2 = \sum_{i=0}^{2^n-1} (b_j m_j)$; $F_1 + F_2 = \sum_{i=0}^{2^n-1} (a_i m_i) + \sum_{i=0}^{2^n-1} (b_j m_j)$

$= a_0 m_0 + b_0 m_0 + a_1 m_1 + b_1 m_1 + a_2 m_2 + b_2 m_2) \ldots = (a_0 + b_0)m_0 + (a_1 + b_1)m_1 + (a_2 + b_2)m_2 + \ldots$

$= \sum_{i=0}^{2^n-1} (a_i + b_i)m_i$

Minterm m_i is present in $F_1 + F_2$ iff $a_i + b_i = 1$, i.e., if either $a_i = 1$ or $b_i = 1$ so maxterm M_i is present in $F_1 + F_2$ if $a_i = 0$ and $b_i = 0$. Therefore, maxterm M_i is present in $F_1 + F_2$ iff it is present in both F_1 and F_2.

4.25

A B C D	*F G H J*
0 0 0 0	0 1 0 0
0 0 0 1	0 0 0 0
0 0 1 0	0 1 0 0
0 0 1 1	0 0 0 0
0 1 0 0	0 1 0 1
0 1 0 1	1 0 0 0
0 1 1 0	1 1 0 0
0 1 1 1	1 0 1 0
1 0 0 0	0 0 0 1
1 0 0 1	0 0 0 0
1 0 1 0	1 0 0 0
1 0 1 1	1 0 1 0
1 1 0 0	0 0 0 1
1 1 0 1	1 0 1 1
1 1 1 0	1 0 1 1
1 1 1 1	1 0 1 0

(a) $F(A, B, C, D) = \sum m(5, 6, 7, 10, 11, 13, 14, 15) = \prod M(0, 1, 2, 3, 4, 8, 9, 12)$

(b) G (A, B, C, D) $= \sum m(0, 2, 4, 6) = \prod M(1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

(c) $H(A, B, C, D) = \sum m(7, 11, 13, 14, 15) = \prod M(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12)$

(d) $J(A, B, C, D) = \sum m(4, 8, 12, 13, 14) = \prod M(0, 1, 2, 3, 5, 6, 7, 9, 10, 11, 15)$

4.26

A B C D	*F G H J*
0 0 0 0	0 1 0 0
0 0 0 1	0 1 0 0
0 0 1 0	0 1 0 0
0 0 1 1	0 0 0 0
0 1 0 0	0 1 0 1
0 1 0 1	1 0 0 0
0 1 1 0	0 0 0 0
0 1 1 1	1 0 1 0
1 0 0 0	0 1 0 1
1 0 0 1	0 0 0 0
1 0 1 0	1 0 0 0
1 0 1 1	1 0 1 0
1 1 0 0	0 0 0 1
1 1 0 1	1 0 1 1
1 1 1 0	1 0 1 1
1 1 1 1	1 0 1 0

(a) $F(A, B, C, D) = \sum m(5, 7, 10, 11, 13, 14, 15) = \prod M(0, 1, 2, 3, 4, 6, 8, 9, 12)$

(b) G (A, B, C, D) $= \sum m(0, 1, 2, 4, 8) = \prod M(3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15)$

(c) $H(A, B, C, D) = \sum m(7, 11, 13, 14, 15) = \prod M(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12)$

(d) $J(A, B, C, D) = \sum m(4, 8, 12, 13, 14) = \prod M(0, 1, 2, 3, 5, 6, 7, 9, 10, 11, 15)$

4.27 You can also work this problem using a truth table, as in problem 4.28.

$f(a, b, c) = a(b + c') = ab + ac' = ab(c + c') + a(b + b')c' = \underset{m_7}{\underline{abc}} + \underset{m_6}{\underline{abc'}} + \underset{m_6}{\underline{abc'}} + \underset{m_4}{\underline{ab'c'}}$

$f = \sum m(4, 6, 7)$ $\quad f = \prod M(0, 1, 2, 3, 5)$

$f' = \sum m(0, 1, 2, 3, 5)$ $\quad f' = \prod M(4, 6, 7)$

4.28

a b c d	*f*
0 0 0 0	0
0 0 0 1	1
0 0 1 0	1
0 0 1 1	0
0 1 0 0	1
0 1 0 1	1
0 1 1 0	1
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	1
1 1 0 0	1
1 1 0 1	0
1 1 1 0	1
1 1 1 1	1

(a) $f = \sum m(1, 2, 4, 5, 6, 11, 12, 14, 15)$

(b) $f = \prod M(0, 3, 7, 8, 9, 10, 13)$

(c) $f' = \sum m(0, 3, 7, 8, 9, 10, 13)$

(d) $f' = \prod M(1, 2, 4, 5, 6, 11, 12, 14, 15)$

You can also work this problem algebraically, as in problem 4.27.

4.29 (a) $f(A,B,C,D) = AB + A'CD = ABC'D' + ABC'D$
$+ ABCD' + ABCD + A'B'CD + A'BCD$
$= (A+A'CD)(B+A'CD) = (A+C)(A+D)(A'+B)$
$(B+C)(B+D)$
$f(A,B,C,D) = (A+B'+C+D')(A+B'+C+D)$
$(A+B+C+D')(A+B+C+D)(A+B'+C'+D)$
$(A+B'+C+D)(A+B+C'+D)(A+B+C+D)$
$(A'+B+C'+D')(A'+B+C'+D)(A'+B+C+D')$
$(A'+B+C+D)(A'+B+C+D')(A'+B+C+D)$
$(A+B+C+D')(A+B+C+D)(A'+B+C'+D)$
$(A'+B+C+D)(A+B+C'+D)(A+B+C+D)$
$= (A+B'+C+D')(A+B+C+D')(A+B'+C'+D)$
$(A+B'+C+D)(A+B+C'+D)\ (A+B+C+D)$
$(A'+B+C'+D')(A'+B+C'+D)(A'+B+C+D')$
$(A'+B+C+D)$
Note: Consensus could have been applied twice to write $f = (A+C)(A+D)(A'+B)$ *and save some work.*

4.29 (b) $f(A,B,C,D) = (A+B+D')(A'+C)(C+D)$
$= (A+B+D')(A'D+C) = AC+A'BD+BC+CD'$
$= AC(B+B')(D+D')+A'BD(C+C')$
$+BC(A+A')(D+D')+(A+A')(B+B')CD'$
$= ABCD+ABCD'+AB'CD+AB'CD'+A'BCD$
$+A'BC'D+ABCD+ABCD'+ABCD+A'BCD'$
$+ABCD'+AB'CD'+A'BCD'+A'B'CD'$
$= ABCD+ABCD'+AB'CD+AB'CD'+A'BCD$
$+A'BC'D+A'BCD'+A'B'CD'$
$f(A,B,C,D) = (A+B+CC'+D')(A'+BB'+C+DD')$
$(AA'+BB'+C+D)$
$= (A+B+C+D')(A+B+C'+D')(A'+B+C+D)$
$(A'+B+C+D')(A'+B'+C+D)(A'+B'+C+D')$
$(A+B+C+D)\ (A+B'+C+D)(A'+B+C+D)$
$(A'+B'+C+D)$
$= (A+B+C+D')(A+B+C'+D')(A'+B+C+D)$
$(A'+B+C+D')(A'+B'+C+D)(A'+B'+C+D')$
$(A+B+C+D)(A+B'+C+D)$

4.30 (a) $F(A, B, C, D) = \sum m(3, 4, 5, 8, 9, 10, 11, 12, 14)$
$F = A'B'CD + A'BC'D' + A'BC'D + AB'C'D' +$
$AB'C'D + AB'CD' + AB'CD + ABC'D' + ABCD'$

4.30 (b) $F(\mathrm{A, B, C, D}) = \prod M(0, 1, 2, 6, 7, 13, 15)$
$F = (A + B + C + D)(A + B + C + D')$
$(A + B + C' + D)(A + B' + C' + D)$
$(A + B' + C' + D')(A' + B' + C + D')$
$(A' + B' + C' + D')$

4.31 (a) $F(A, B, C, D) = \sum m(0, 3, 4, 7, 8, 9, 11, 12, 13, 14) = \underset{m_0}{\underline{A'B'C'D'}} + \underset{m_3}{\underline{A'B'CD}} + \underset{m_4}{\underline{A'BC'D'}} + \underset{m_7}{\underline{A'BCD}} + \underset{m_8}{\underline{AB'C'D'}} + \underset{m_9}{\underline{AB'C'D}}$
$+ \underset{m_{11}}{\underline{AB'CD}} + \underset{m_{12}}{\underline{ABC'D'}} + \underset{m_{13}}{\underline{ABC'D}} + \underset{m_{14}}{\underline{ABCD'}}$

4.31 (b) $F(A, B, C, D) = \prod M(1, 2, 5, 6, 10, 15) = (\underset{M_1}{\underline{A + B + C + D'}})\ (\underset{M_2}{\underline{A + B + C' + D}})\ (\underset{M_5}{\underline{A + B' + C + D'}})\ (\underset{M_6}{\underline{A + B' + C' + D}})$
$(\underset{M_{10}}{\underline{A' + B + C' + D}})\ (\underset{M_{15}}{\underline{A' + B' + C' + D'}})$

4.32 (a) If don't cares are changed to (1, 1), respectively,
$F_1 = A'B'C' + ABC + A'B'C + AB'C$
$= A'B' + AC$

4.32 (b) If don't cares are changed to (1, 0), respectively
$F_2 = A'B'C' + A'BC' + AB'C' + ABC' = C'$

4.32 (c) If don't cares are changed to (1, 1), respectively
$F_3 = (A + B + C)\ (A + B + C') = A + B$

4.32 (d) If don't cares are changed to (0, 1), respectively
$F_4 = A'B'C' + A'BC + AB'C' + ABC$
$= B'C' + BC$

4.33

A B C	*D E F*	*Z*
0 0 0	1 1 X[2]	0
0 0 1	0 1 X[2]	1
0 1 0	0 X[2] 1	1
0 1 1	X[1] X[1] X[1]	X
1 0 0	0 1 X[2]	1
1 0 1	0 X[2] 1	1
1 1 0	X[1] X[1] X[1]	X
1 1 1	1 X[2] 1	0

[1] These truth table entries were made don't cares because *ABC* = 110 and *ABC* = 011 can never occur.
[2] These truth table entries were made don't cares because when one input of the OR gate is 1, the output will be 1 regardless of the value of its other input.

4.34 (a) $G_1(A, B, C) = \sum m(0, 7) = \prod M(1, 2, 3, 4, 5, 6)$

4.34 (b) $G_2(A, B, C) = \sum m(0, 1, 6, 7) = \prod M(2, 3, 4, 5)$

4.35 (a)

A B C D	1's	X Y Z
0 0 0 0	0	0 0 0
0 0 0 1	1	0 0 1
0 0 1 0	1	0 0 1
0 0 1 1	2	0 1 0
0 1 0 0	1	0 0 1
0 1 0 1	2	0 1 0
0 1 1 0	2	0 1 0
0 1 1 1	3	0 1 1
1 0 0 0	1	0 0 1
1 0 0 1	2	0 1 0
1 0 1 0	2	0 1 0
1 0 1 1	3	0 1 1
1 1 0 0	2	0 1 0
1 1 0 1	3	0 1 1
1 1 1 0	3	0 1 1
1 1 1 1	4	1 0 0

$X = ABCD$

$Y = A'B'CD + A'BC'D + A'BCD' + A'BCD + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABC'D + ABCD'$

$Z = A'B'C'D + A'B'CD' + A'BC'D' + A'BCD + AB'C'D' + AB'CD + ABC'D + ABCD'$

4.35 (b) $Y = (A + B + C + D)(A + B + C + D')(A + B + C' + D)(A + B' + C + D)(A' + B + C + D)(A' + B' + C' + D')$

$Z = (A + B + C + D)(A + B' + C + D')(A + B' + C' + D)(A' + B + C + D')(A' + B + C' + D)(A' + B' + C + D)(A' + B' + C' + D')$

4.36 (a)

A B C D	W X Y Z
0 0 0 0	0 0 1 1
0 0 0 1	0 1 0 0
0 0 1 0	0 1 0 0
0 0 1 1	0 1 0 1
0 1 0 0	0 1 0 0
0 1 0 1	0 1 0 1
0 1 1 0	0 1 0 1
0 1 1 1	0 1 1 0
1 0 0 0	0 1 0 0
1 0 0 1	0 1 0 1
1 0 1 0	0 1 0 1
1 0 1 1	0 1 1 0
1 1 0 0	0 1 0 1
1 1 0 1	0 1 1 0
1 1 1 0	0 1 1 0
1 1 1 1	0 1 1 1

$X = A'B'C'D + A'B'CD' + A'B'CD + A'BC'D' + A'BC'D + A'BCD' + A'BCD + AB'C'D' + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABC'D + ABCD' + ABCD$

$Y = A'B'C'D' + A'BCD + ABC'D + ABCD' + ABCD$

$Z = A'B'C'D' + A'B'CD + A'BC'D + A'BCD' + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABCD$

4.36 (b) $Y = (A + B + C + D')(A + B + C' + D)(A + B + C' + D')(A + B' + C + D)(A + B' + C + D')(A + B' + C' + D)(A' + B + C + D)(A' + B + C + D')(A' + B + C' + D)(A' + B + C' + D')(A' + B' + C + D)$

$Z = (A + B + C + D')(A + B + C' + D)(A + B' + C + D)(A + B' + C' + D)(A' + B + C + D)(A' + B' + C + D')(A' + B' + C' + D)$

4.37

A B C D		S T U V	W X Y Z
0 0 0 0	0 × 5 = 00	0 0 0 0	0 0 0 0
0 0 0 1	1 × 5 = 05	0 0 0 0	0 1 0 1
0 0 1 0	2 × 5 = 10	0 0 0 1	0 0 0 0
0 0 1 1	3 × 5 = 15	0 0 0 1	0 1 0 1
0 1 0 0	4 × 5 = 20	0 0 1 0	0 0 0 0
0 1 0 1	5 × 5 = 25	0 0 1 0	0 1 0 1
0 1 1 0	6 × 5 = 30	0 0 1 1	0 0 0 0
0 1 1 1	7 × 5 = 35	0 0 1 1	0 1 0 1
1 0 0 0	8 × 5 = 40	0 1 0 0	0 0 0 0
1 0 0 1	9 × 5 =45	0 1 0 0	0 1 0 1

Note: Rows 1010 through 1111 have don't care outputs.

$S = 0, T = A, U = B, V = C, W = 0, X = D, Y = 0, Z = D$

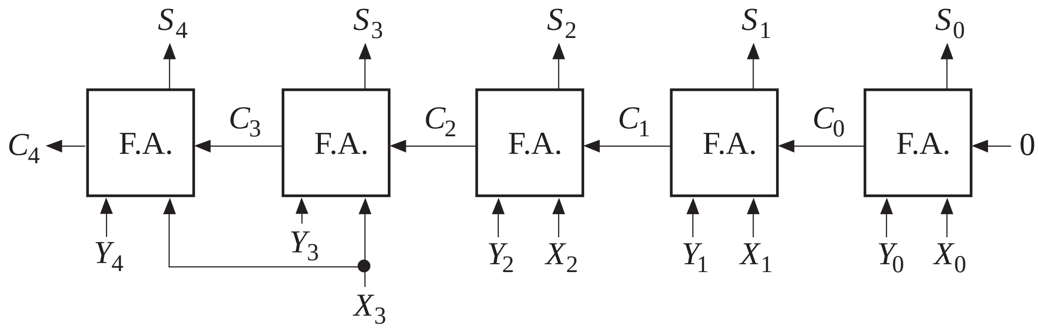
4.38

A B C D		S T U V	W X Y Z
0 0 0 0	0 × 4 + 1 = 01	0 0 0 0	0 0 0 1
0 0 0 1	1 × 4 + 1 = 05	0 0 0 0	0 1 0 1
0 0 1 0	2 × 4 + 1 = 09	0 0 0 0	1 0 0 1
0 0 1 1	3 × 4 + 1 = 13	0 0 0 1	0 0 1 1
0 1 0 0	4 × 4 + 1 = 17	0 0 0 1	0 1 1 1
0 1 0 1	5 × 4 + 1 = 21	0 0 1 0	0 0 0 1
0 1 1 0	6 × 4 + 1 = 25	0 0 1 0	0 1 0 1
0 1 1 1	7 × 4 + 1 = 29	0 0 1 0	1 0 0 1
1 0 0 0	8 × 4 + 1 = 33	0 0 1 1	0 0 1 1
1 0 0 1	9 × 4 + 1 =37	0 0 1 1	0 1 1 1

Note: Rows 1010 through 1111 have don't care outputs.

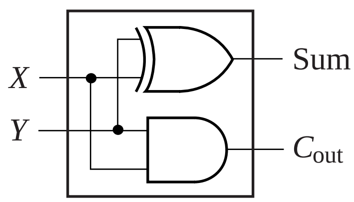
$S = 0, T = 0, U = BD + BC + A,$
$V = B'CD + BC'D' + A, W = B'CD' + BCD,$
$X = B'C'D + BD', Y = B'CD + BC'D' + A, Z = 1$

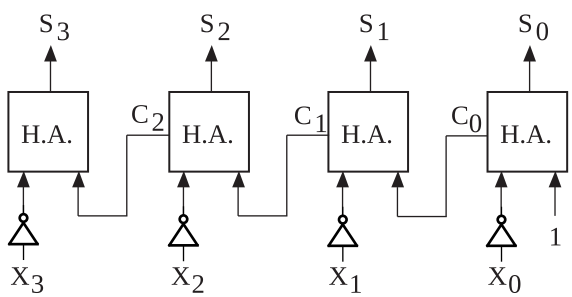
4.39 Notice that the sign bit X_3 of the 4-bit number is extended to the leftmost full adder as well.



4.40

XY	Sum	Cout
0 0	0	0
0 1	1	0
1 0	1	0
1 1	0	1





4.41 (a), (b), (c)

(a)
$f = x(y+y')+y(x+x') = xy+xy'+x'y$ (sum-of-minterms)
$f = x+y$ already in product-of-maxterms form

(b)
$f = ax+by = ax(y+y')+by(x+x')$
$= axy+axy'+bxy+bx'y = (a+b)xy+axy'+bx'y$
$= xy+axy'+bx'y$

(c)
$f' = (a'+x')(b'+y') = (b+x')(a+y')$
$= ab+ax'+by'+x'y' = ax'(y+y')+by'(x+x')+x'y'$
$= ax'y+ax'y'+by'x+by'x'+x'y'$
$= ax'y+by'x+x'y'(a+b+1) = ax'y+by'x+x'y'$ *so*
$f = (a'+x+y')(b'+x'+y)(x+y)$
$= (b+x+y')(a+x'+y)(x+y)$
Alternatively,
$f = ax+by = (a+by)(x+by) = (a+b)(a+y)(x+b)(x+y)$
$= (a+xx'+y)(b+yy'+x)(x+y)$
$= (a+x+y)(a+x'+y)(b+x+y)(b+x+y')(x+y)$
$= [(a+x+y)(b+x+y)(x+y)](a+x'+y)(b+x+y')$
$= (ab+x+y)(a+x'+y)(b+x+y')$
$= (x+y)(a+x'+y)(b+x+y')$

4.41 (d), (e)

(d)

x y	*f*	*x y*	*f*
0 0	*0*	*a 0*	*a*
0 1	*b*	*a 1*	*1*
0 a	*0*	*a a*	*a*
0 b	*b*	*a b*	*1*
1 0	*a*	*b 0*	*0*
1 1	*1*	*b 1*	*b*
1 a	*a*	*b a*	*0*
1 b	*1*	*b b*	*b*

(e)
f(x,y) is completely specified by the coefficients of the minterms in the sum of minterms expression. These coefficients are determined by the value of the function for xy = 00, 01, 10 and 11.

4.42

(a) $m_1 + m_2 = m_1(m_2' + m_2) + (m_1' + m_1)m_2$
$= m_1m_2' + m_1m_2 + m_1'm_2$
But $m_1m_2 = 0$, *so* $m_1 + m_2 = m_1m_2' + m_1'm_2$
$= m_1 \oplus m_2$.
(b) Using part (a), any function can be written as the exclusive-or sum of its minterms. However, if a product contains a complemented literal, it can be written as the exclusive-or sum of two products without a complemented literal by using
$x'p = (x \oplus 1)p = xp \oplus p$.
By repeated application of the preceding relationship, all complemented literals can be removed from the products.

Unit 5 Problem Solutions

5.3 (a)

bc \ a	0	1
00	1	
01		1
11		
10	1	1

$f_1 = a'c' + a\,b'c + b\,c'$

5.3 (b)

ef \ d	0	1
00	1	1
01	1	
11		
10	1	

$f_2 = d'e' + d'f' + e'f'$

5.3 (c)

st \ r	0	1
00	1	1
01	1	
11	1	
10	1	1

$f_3 = r' + t'$

5.3 (d)

yz \ x	0	1
00	0	1
01	1	0
11	1	1
10	1	1

$f_4 = x'z + y + x\,z'$

5.4 (a)

CD \ AB	00	01	11	10
00	1	1	1	1
01			1	
11	1		1	1
10	1	1	1	1

$F = BD' + B'CD + ABC + ABC'D + B'D'$

5.4 (b)

CD \ AB	00	01	11	10
00	1	1	1	1
01			1	
11	1		1	1
10	1	1	1	1

$F = D' + B'C + A\,B$

5.4 (c)

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	1	0	1	1
10	1	1	1	1

$F = (A + B' + D')(B + C + D')$

5.5 (a) See FLD p. 697 for solution.

5.5 (b)

X_1X_2 \ C_1C_2	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	1	0	1	1
10	1	1	0	0

$Z = C_1'X_1'X_2 + C_1'X_1X_2' + C_1C_2X_1'X_2' + C_1X_1X_2 + C_1'C_2'X_2$

Alt:

$Z = C_1'X_1'X_2 + C_1'X_1X_2' + C_1C_2X_1'X_2' + C_1X_1X_2 + C_1'C_2'X_1$

$Z = C_1'X_1'X_2 + C_1'X_1X_2' + C_1C_2X_1'X_2' + C_1X_1X_2 + C_2'X_1X_2$

5.6 (a)

cd \ ab	00	01	11	10
00	1*		1*	
01	1	1*		
11	1	1		1*
10		1	1	

$f = \underline{a'b'c'} + \underline{a'd} + \underline{b'cd} + \underline{abd'} + bcd'$

Alt: $f = \underline{a'b'c'} + \underline{a'd} + \underline{b'cd} + \underline{abd'} + a'bc$

(*) Indicates a minterm that makes the corresponding prime implicant essential.

$a'd \rightarrow m_5$; $a'b'c' \rightarrow m_0$; $b'cd \rightarrow m_{11}$; $abd' \rightarrow m_{12}$

5.6 (b)

cd \ ab	00	01	11	10
00	1	1	0	1*
01	0	1	1*	0
11	1*	1	1	0
10	1	1	0	1

$F = \underline{a'c} + \underline{b'd'} + \underline{bd} + a'd'$

Alt: $F = \underline{a'c} + \underline{b'd'} + \underline{bd} + a'b$

(*) Indicates a minterm that makes the corresponding prime implicant essential.

$bd \rightarrow m_{13}$ or m_{15}; $a'c \rightarrow m_3$; $b'd' \rightarrow m_8$ or m_{10}

Unit 5 Solutions

5.6 (c)

cd \ ab	00	01	11	10
00	1	1	1 *	1
01	X	0	0	X
11	X	0	0	1 *
10	X	1 *	0	1 *

F = a'd' + b' + c'd'

(*) Indicates a minterm that makes the corresponding prime implicant essential.

c'd' $\rightarrow m_{12}$; a'd' $\rightarrow m_6$; b' $\rightarrow m_{10}$ or m_{11}

5.7 (a)

cd \ ab	00	01	11	10
00	1	1		1
01				
11	1	1		
10	1		1	

f = a'c'd' + a'cd + b'c'd' + abcd' + a'b'd'

Alt: f = a'c'd' + a'cd + b'c'd' + abcd' + a'b'c

5.7 (b)

cd \ ab	00	01	11	10
00	X	1		
01	1			
11	X		1	
10	1		X	

f = a'b' + a'c'd' + abc

5.7 (c)

cd \ ab	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	0	1	0	1
10	0	1	1	1

f = b'c'd' + ab'c + a'bc + bc'd + ad'

5.7 (d)

CD \ AB	00	01	11	10
00	0	0	X	0
01	X	1	1	X
11	1	1	X	1
10	0	0	1	1

F = D + A C

5.8 (a)

cd \ ab	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	X	X	X	0
10	1	0	X	1

f = (c'+ d')(b'+ c')(a + b + c)(a'+ c + d)

cd \ ab	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	X	X	X	0
10	1	0	X	1

f = a'bc' + ac'd + b'cd'

5.8 (b)

cd \ ab	00	01	11	10
00	0	1	X	X
01	1	0	0	0
11	1	X	X	1
10	X	0	X	0

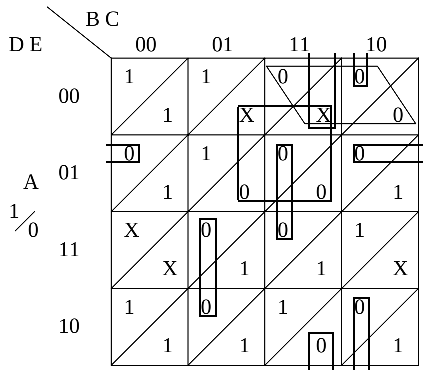
f = (a'+ c)(b'+ d')(b + d)(c'+ d)

Alt: f = (a'+ c)(b'+ d')(b + d)(b'+ c')

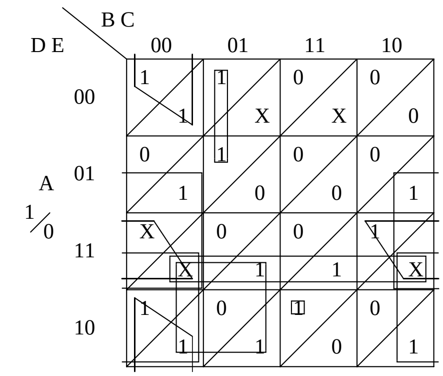
cd \ ab	00	01	11	10
00	0	1	X	X
01	1	0	0	0
11	1	X	X	1
10	X	0	X	0

f = a'b'd + bc'd' + cd

5.9 (a)



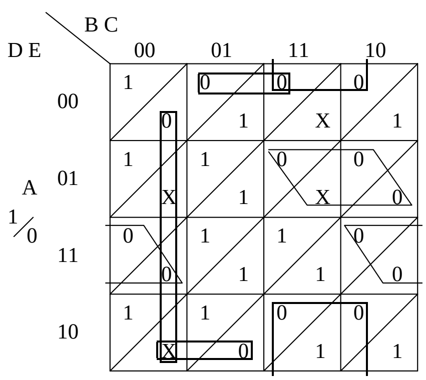
F = (A'+ B'+ C + E)(A'+ B + C'+ D')(A + B'+ C'+ E)
(B'+ D + E)(A + C'+ D)(A'+ C + D + E')(A'+ B'+ C'+ E')



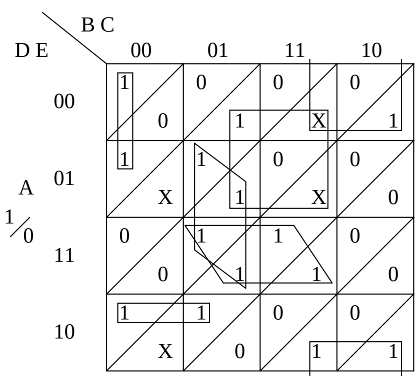
F = A'C'E + A'C'D + A'D E + A B'C D' + C'D E
+ A B C D E' + B'C'E' + A'B'D

Alt: F = A'C'E + A'C'D + A'D E + A B'C D' + C'D E
+ A B C D E' + B'C'E' + A'B'E'

5.9 (b)



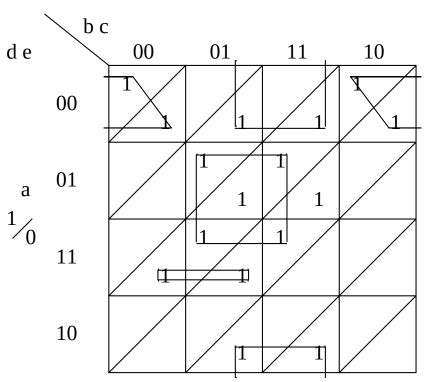
F = (A'+ B'+ E)(A'+ C'+ D + E)(C + D'+ E')
(A + B + D'+ E)(A + B + C)(B'+ D + E')



F = A'C D' + A'B E' + C D E + A B'C'D' + A B'D E' + B'C E

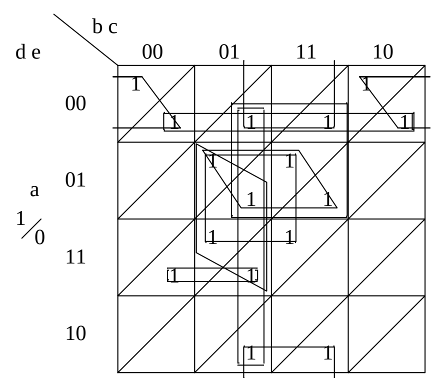
Alt:
F = A'C D' + A'B E' + C D E + A B'C'E' + A B'C D + B'D'E
F = A'C D' + A'B E' + C D E + A B'C'D' + A B'D E' + B'D'E
F = A'C D' + A'B E' + C D E + A B'C'E' + A B'D E' + B'D'E

5.10 (a)



Essential prime implicants: *c'd'e'* (m_{16}, m_{24}), *a'ce'* (m_{14}), *ace* (m_{31}), *a'b'de* (m_3)

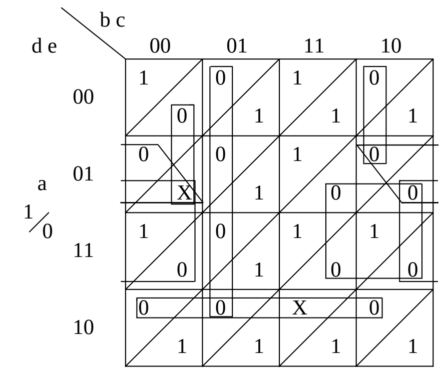
5.10 (b)



Prime implicants: *a'b'de, a'd'e', cd'e, a'ce', ace, a'b'c, b'ce, c'd'e', a'cd'*

Unit 5 Solutions

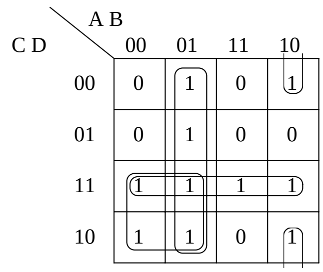
5.11



f = (a'+ b + c') (a'+ d'+ e) (a + b'+ e') (a + c + e')
(a + b + c + d) (a'+b'+ c + d) (c + d + e')

Alt: f = (a'+ b + c') (a'+ d'+ e) (a + b'+ e') (a + c + e')
(a + b + c + d) (a'+b'+ c + e) (c + d + e')

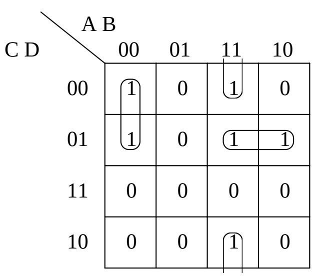
5.12 (a)



F = A B'D' + A'B + A'C + C D

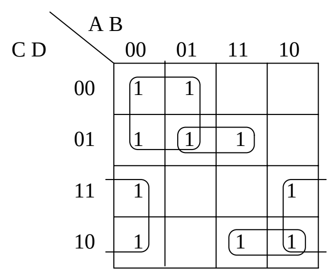
F = ∏ M(0, 1, 9, 12, 13, 14) = $(A + B + C + D)$
$(A + B + C + D')$ $(A' + B' + C + D)$
$(A' + B' + C + D')$ $(A' + B' + C' + D)$
$(A' + B + C + D')$

5.12 (b)



F' = A'B'C' + A B D' + A C'D

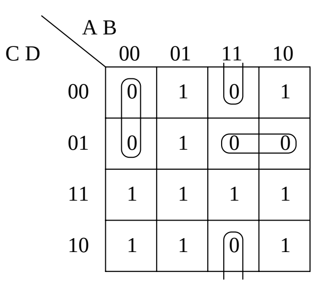
5.13



F = A C D' + B C'D + B'C + A'C'

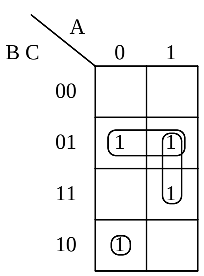
Minterms m_0, m_1, m_2, m_3, m_4, m_{10}, and m_{11} can be made don't cares, individually, without changing the given expression. However, if m_{13} or m_{14} is made a don't care, the term BC'D or the term ACD' (respectively) is not needed in the expression.

5.12 (c)



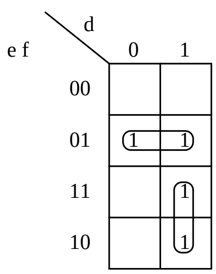
F = (A'+ B'+ D)(A + B + C)(A'+ C + D')

5.14 (a)



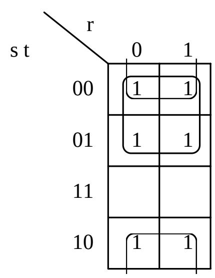
f_1 = B'C + A'BC' + AC

5.14 (b)



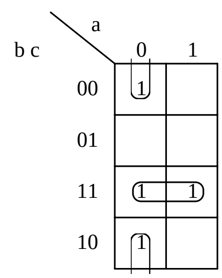
f_2 = e'f + d e

5.14 (c)



f_3 = s' + t'

5.14 (d)



f_4 = a'c' + bc

5.14 (e)

p q \ n	0	1
00		1
01	1	1
11	1	
10		

$f_5 = n'q + np'$

5.14 (f)

y z \ x	0	1
00	1	1
01		1
11	1	
10	1	1

$f_6 = z' + x'y + xy'$

5.15 (a)

B C \ A	0	1
00	0	0
01		
11	0	
10		0

$f_1 = (A'+ C)(A + B'+ C')(B + C)$

5.15 (b)

e f \ d	0	1
00	0	0
01		
11	0	
10	0	

$f_2 = (e + f)(d + e')$

5.15 (c)

s t \ r	0	1
00		
01		
11	0	0
10		

$f_3 = (s'+ t')$

5.15 (d)

b c \ a	0	1
00		0
01	0	0
11		
10		0

$f_4 = (b + c')(a'+ c)$

5.15 (e)

p q \ n	0	1
00	0	
01		
11		0
10	0	0

$f_5 = (n + q)(n'+ p')$

5.15 (f)

y z \ x	0	1
00		
01	0	
11		0
10		

$f_6 = (x + y + z')(x'+ y'+ z')$

5.16 (a)

B C \ A	0	1
00		1
01	1	
11	1	
10		1

$f_1 = A'C + AC'$

5.16 (b)

e f \ d	0	1
00		1
01	1	1
11		1
10		

$f_2 = e'f + de' + df$

5.16 (c)

s t \ r	0	1
00	1	1
01		1
11		1
10	1	1

$f_3 = t' + r$

5.16 (d)

b c \ a	0	1
00		1
01		
11	1	1
10		1

$f_1 = bc + ac'$

5.16 (e)

p q \ n	0	1
00		
01		1
11	1	1
10	1	

$f_2 = n'p + nq$

5.16 (f)

y z \ x	0	1
00	1	1
01	1	1
11	0	1
10	1	0

$f_4 = y' + x'z' + xz$

5.17 (a) & (b)

C D \ A B	00	01	11	10
00	1			
01	1			
11	1		1	
10	1	1	1	1

$F = A'B' + CD' + ABC$

5.17 (c)

CD \ AB	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	1	0
10	1	1	1	1

$F = (B' + C)(A + B' + D')(A' + C)(A' + B + D')$

5.18 (a) & (b)

CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	1	1	0	0
10	1	1	0	1

$F = A' + C'D + B'CD'$

5.18 (c)

CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	1	1	0	0
10	1	1	0	1

$F = (A' + C + D)(A' + C' + D')(A' + B' + D)$

Alt: $F = (A' + C + D)(A' + C' + D')(A' + B' + C')$

5.19 (a)

$C_1\,C_2\,X_1\,X_2$	Z
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	1
0 1 0 0	0
0 1 0 1	1
0 1 1 0	1
0 1 1 1	0
1 0 0 0	1
1 0 0 1	1
1 0 1 0	0
1 0 1 1	1
1 1 0 0	1
1 1 0 1	0
1 1 1 0	0
1 1 1 1	1

5.19 (b)

X_1X_2 \ C_1C_2	00	01	11	10
00	0	0	1	1
01	0	1	0	1
11	1	0	1	1
10	0	1	0	0

$F = (C_1 + C_2 + X_1)(C_1 + X_1 + X_2)(C_1 + C_2' + X_1' + X_2')$
$(C_1' + C_2' + X_1 + X_2')(C_1' + X_1' + X_2)\left\{\begin{matrix}(C_2 + C_2 + X_2) \\ \text{or} \\ (C_2 + X_1' + X_2)\end{matrix}\right\}$

5.20 (a)

bc \ a	0	1
00	1	
01	1	1
11		1
10	1	1

$F = a'c' + b'c + ab$ or
$= a'b' + bc' + ac$

5.20 (b)

ef \ d	0	1
00	X	1
01	1	
11		X
10	X	1

$g = d'e' + f'$

5.20 (c)

qr \ p	0	1
00	1	1
01	1	
11	1	1
10		1

$F = p'r + q'r' + pq$ or
$= p'q' + pr' + qr$

5.20 (d)

tu \ s	0	1
00	X	
01	1	X
11	1	X
10	1	

$F = s'$

5.20 (e)

bc \ a	0	1
00	1	
01	1	1
11		1
10		1

$f = a'b' + ab + b'c$ or
$= a'b' + ab + ac$

5.20 (f)

EF \ D	0	1
00	X	
01	1	X
11	X	
10		1

$G = DEF' + D'E'$
$G = DEF' + D'F$
$G = DEF' + E'F$

5.21

c d \ a b	00	01	11	10
00	1			1
01	1	1		1
11		1	1	1
10			1	1

$F = a'b'c' + a'c'd + bcd + abc + ab'$
$= (a'b'c' + ab') + a'c'd + bcd + (abc + ab')$
$= (a'c' + a)b' + (a'c'd + bcd) + a(bc + b')$
$= (c' + a)b' + (a'c'd + bcd + a'bd) + a(c + b')$
$= (b'c' + a'bd + a'c'd) + (bcd + a'bd + ac) + ab'$
$= (b'c' + ac + ab') + a'bd$
$= b'c' + ac + a'bd$

5.22 (a)

C D \ A B	00	01	11	10
00		1	1	X
01		X	1	X
11		X		1
10		X	1	X

PIs: $AB', BC', AD', BD', AC', A'B$

$f = AB' + BD' + AC'$ or
$= AB' + BC' + BD'$ or
$= AB' + BC' + AD'$

5.22 (b)

C D \ A B	00	01	11	10
00			1	X
01		X	1	X
11	1	X		1
10		X	1	X

PIs: $B'CD, AC', AD', AB', BC'D, BCD', A'CD, A'BD, A'BC$

$f = B'CD + AC' + AD'$

5.22 (c)

C D \ A B	00	01	11	10
00		1		X
01	1	X	1	X
11		X		
10	1	X	1	X

PIs: $C'D, CD', A'B, AB'C', AB'D'$

$f = C'D + CD' + A'B$

5.22 (d)

C D \ A B	00	01	11	10
00		1		X
01		X		X
11		X	1	
10		X		X

PIs: $A'B, BCD, AB'C', AB'D'$

$f = A'B + BCD$

5.22 (e)

C D \ A B	00	01	11	10
00		1		X
01		X		X
11	1	X	1	1
10		X		X

PIs: $CD, A'B, AB'$

$f = CD + A'B$

5.22 (f)

C D \ A B	00	01	11	10
00		1	X	X
01		X	X	X
11		X		X
10		X	X	X

PIs: $A'B, BC', BD', AC', AD', AB'$

$f = BD'$
$f = BC'$
$f = A'B$

5.22 (g)

C D \ A B	00	01	11	10
00	X	1		X
01	X	X		X
11		X	1	
10	X	X		X

PIs: $BCD, A'C', A'D', A'B, B'D', B'C'$

$f = BCD + A'B$ or
$f = BCD + A'D'$ or
$f = BCD + A'C'$

5.23 (a)

C D \ A B	00	01	11	10
00	0			X
01	0	X		X
11	0	X	0	
10	0	X		X

PIs: $(B' + C' + D'), (A + B), (A + D'), (A + C')$

$f = (B' + C' + D')(A + B)$

5.23 (b)

C D \ A B	00	01	11	10
00	0	0		X
01	0	X		X
11		X	0	
10	0	X		X

PIs: $(B' + C' + D'), (A + C), (A + D), (B + D), (B + C), (A + B')$

$f = (B' + C' + D')(A + D)(B + C)$ or
$= (B' + C' + D')(A + C)(B + D)$ or
$= (B' + C' + D')(A + C)(A + D)$

5.23 (c)

C D \ A B	00	01	11	10
00	0		0	X
01		X		X
11	0	X	0	0
10		X		X

PIs: $(B + C + D), (C' + D'), (A' + C + D), (A' + B), (A + B' + D'), (A + B' + C')$

$f = (B + C + D)(C' + D')(A' + C + D)$

5.23 (d)

C D \ A B	00	01	11	10
00	0		0	X
01	0	X	0	X
11	0	X		0
10	0	X	0	X

PIs: $(B), (A' + C), (A' + D), (C + D'), (C' + D), (A + D'), (B + D'), (A + C')$

$f = (B)(C + D')(A' + D)$ or
$= (B)(A' + C)(C' + D)$ or
$= (B)(A' + C)(A' + D)$

Unit 5 Solutions

5.23 (e)

C D \ A B	00	01	11	10
00	0		0	X
01	0	X	0	X
11		X		
10	0	X	0	X

PIs: (C + D'), (A'+ D), (B + D), (A' + C),
(B + C), (C' + D), (A + B' + D'), (A + B'+ C')

f = (A'+ C) (B + C) (C'+ D) or
= (C + D') (A'+ D) (B + D)

5.23 (f)

C D \ A B	00	01	11	10
00	0		X	X
01	0	X	X	X
11	0	X	0	X
10	0	X	X	X

PIs: (B), (D'), (A'), (C')

f = (B) (A') or
= (B) (C') or
= (B) (D')

5.23 (g)

C D \ A B	00	01	11	10
00	X		0	X
01	X	X	0	X
11	0	X		0
10	X	X	0	X

PIs: (B)(A'+ D)(A'+ C)(A + C')
(C'+ D)(C + D')(A + D')

f = (B)(A'+ C)(C'+ D) or
= (B)(A'+ D)(C+ D') or
= (B)(A'+ D)(A'+ C)

5.24 (a)

C D \ A B	00	01	11	10
00	1		1	
01		1	1	
11	1	1		1
10	1	1		

F = A B C' + B'C D + A'C + A'B'D' + A'B D

Alt: F = A B C' + B'C D + A'C + A'B'D' + B C'D

5.24 (b)

C D \ A B	00	01	11	10
00	X	1		1
01				
11	X	X		
10	1			

F = A'C'D' + B'C'D' + A'B'D'

Alt: F = A'C'D' + B'C'D' + A'B'C

5.24 (c)

C D \ A B	00	01	11	10
00		X		X
01	1	1	1	
11		1		
10		1		

F = A'C'D + A'B + B C'D

5.24 (d)

y z \ w x	00	01	11	10
00	1		1	1
01	X	1	1	1
11	1	1		X
10		X	X	1

f = x'y' + w'z + y'z + wz'

Alt: f = x'y' + wy' + w'z + wz'
f = x'y' + wy' + w'z + wx'

5.24 (e)

C D \ A B	00	01	11	10
00	0	X	1	1
01	0	0	X	0
11	1	0	1	0
10	0	1	1	X

F = A'B'C D + BD' + AD' + AB

5.25 (a)

c d \ a b	00	01	11	10
00		1		
01	1	1	1	1
11	1	1	1	
10				

f = a'd + a'bc' + c'd + bd

5.25 (b)

c d \ a b	00	01	11	10
00	0	1	1	0
01	1	0	1	1
11	0	1	1	0
10	1	1	1	1

f= b'c'd + cd' + bd' + bc + ab

5.25 (c)

c d \ a b	00	01	11	10
00	1			X
01			1	1
11	X			
10	1	1	1	X

f = b'd' + cd' + ac'd

5.25 (d)

c d \ a b	00	01	11	10
00	0	1	0	1
01	X	X	0	0
11	X	0	1	1
10	0	0	1	1

f = a'bc' + ab'd' + ac

5.26 (a)

CD \ AB	00	01	11	10
00	0	0		
01		0		0
11				X
10	0	0	0	X

$F = \underline{(C'+ D)}\,\underline{(A'+ B + D')}\,\underline{(A + B'+ C)}\,\underline{(A + D)}$

5.26 (b)

CD \ AB	00	01	11	10
00	0	0	X	1
01	1	0	0	1
11	1	X	1	0
10	0	X	0	0

$F = \underline{(B'+ C)}\,\underline{(A'+ B + C')}\,\underline{(A + D)}\,(C'+ D)$

Alt: $F = \underline{(B'+ C)}\,\underline{(A'+ B + C')}\,\underline{(A + D)}\,(B'+ D)$

5.27 (a)

CD \ AB	00	01	11	10
00	0	1	0	1
01	1	X	1	1
11	1	X	0	0
10	0	1	0	0

$f = (A'+ B'+ D)(A'+ C')(A + B + D)$

CD \ AB	00	01	11	10
00	0	1	0	1
01	1	X	1	1
11	1	X	0	0
10	0	1	0	0

$f = C'D + AB'C' + A'B + A'D$

5.27 (b)

yz \ wx	00	01	11	10
00	1	0	1	1
01	X	1	1	1
11	1	1	0	X
10	0	X	X	1

$f = x'y' + w'z + y'z + wz'$

Alt: $\begin{cases} f = x'y' + wy' + w'z + wz' \\ f = x'y' + wy' + w'z + wx' \end{cases}$

yz \ wx	00	01	11	10
00	1	0	1	1
01	X	1	1	1
11	1	1	0	X
10	0	X	X	1

$f = (w + x'+ z)(w + y'+ z)(w'+ y'+ z')$

Alt: $f = (w + x'+ z)(w + y'+ z)(w'+ x'+ y')$

5.28

cd \ ab	00	01	11	10
00	1			1
01	1	X	1	1
11	1	1	X	
10	1			1

$F = b'd' + a'd + c'd$

Notice that *abcd* = 0101 and 1111 never occur, so minterms 5 and 15 are don't cares.

5.29 (a)

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

$F = AB'D' + A'B + A'C + CD$

$$F = \prod M(0, 1, 9, 12, 13, 14) = (A + B + C + D)(A + B + C + D')(A' + B + C + D')(A' + B' + C + D)(A' + B' + C + D')(A' + B' + C' + D)$$

5.29 (b)

CD \ AB	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	0	0	0	0
10	0	0	1	0

$F' = ABD' + A'B'C' + AC'D$

Unit 5 Solutions

5.29 (c)

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

F = (A'+ B'+ D)(A + B + C)(A'+ C + D')

5.30

CD \ AB	00	01	11	10
00				
01	1	X	1	X
11	1	1	1	X
10			1	

F = D + AB C

5.31 Prime implicants for f ': abc'e, ac'd', ab'e', a'ce, b'c'de', c'd'e, a'd'e

Prime implicants for f: a'd'e', ace, a'ce', bde', abc, bce', b'c'de, a'c'de, a'bc'd, ab'de

5.32 For F: b'c'de', a'ce, ab'e', ac'd', abc'e, c'd'e, a'd'e

For G: ab'ce, a'bcd, a'bde', cde, b'de, a'b'c'd, a'c'e'

5.33 5-variable mirror image map

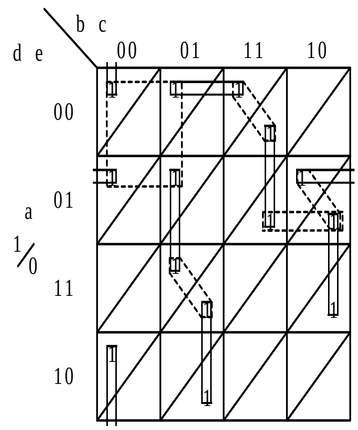
de \ abc	000	001	011	010	110	111	101	100
00			1			1	1	1
01			1	1	1		1	1
11		1		1			1	
10		1						1

Essential PIs: ab'c'e', a'bc'e, a'b'cd

f = a'b'cd + a'bc'e + ab'c'e' + a'bcd' + ab'ce + ac'd'e + acd'e'

Other PIs: ab'd', b cd'e', bc'd'e, a'bd'e, b'cde

5-variable diagonal map



5.34 (a)

cd \ ab	00	01	11	10
00	X	1	X	
01		X		X
11	X	1	X	1
10	1	X	1	X

5.34 (b) & (c)

cd \ ab	00	01	11	10
00	X	1	X	
01		X		X
11	X	1	X	1
10	1	X	1	X

PIs: bd', a'b, a'd', c, ab'd

f = b d' + c or
= a'b + c or
= a'd' + c

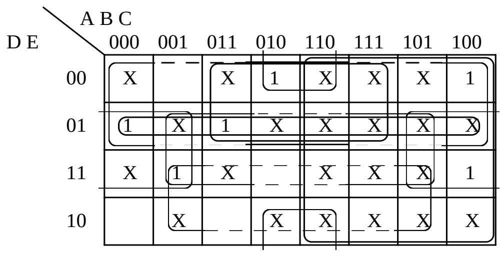
5.34 (d) & (e)

cd \ ab	00	01	11	10
00	X		X	0
01	0	X	0	X
11	X		X	
10		X		X

PIs: (c + d'), (a'+ c), (b + c), (a + b +d'), (a + b'+c'+d), (a'+b'+d'), (a'+b+d)

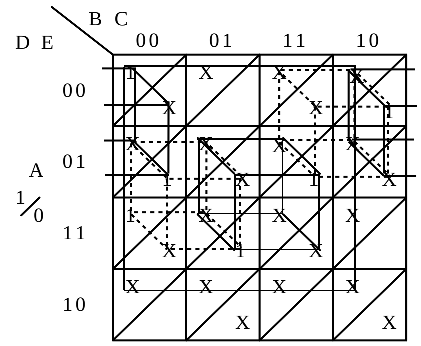
f = (c + d')(a'+ c) or
= (b + c)(c + d') or
= (b + c)(a'+ c)

5.35 (a), (b) & (c) 5-variable mirror image map

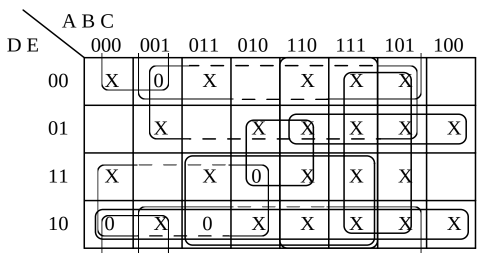


PIs: A, C'D', B'E, C E, B D', D'E, B C'E', B'C D
F= A + B'E + BD' or
= A + C'D' + CE

5-variable diagonal map

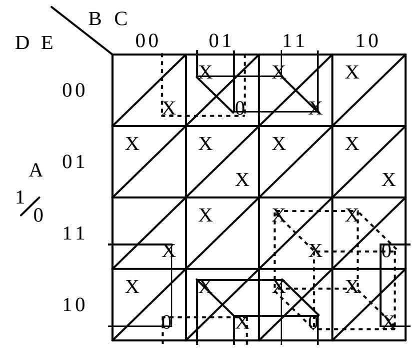


5.35 (d), & (e) 5-variable mirror image map

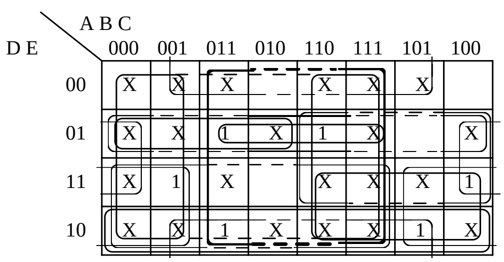


PIs: (A'+ B'), (A'+ C'), (A'+D + E'), (B'+D'), (B'+ C + E'),
(C'+ E), (D'+E), (B + C'+ D), (A + C + D'), (A + B + E)
F = (B'+D')(A + B + E) or
= (C'+ E)(A + C + D')

5-variable diagonal map

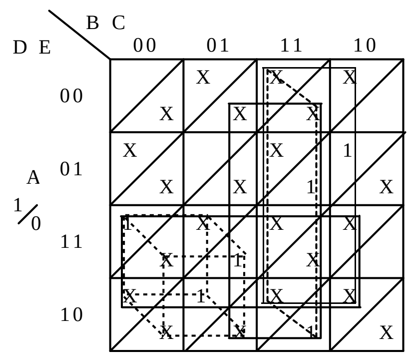


5.36 (a), (b) & (c) 5-variable mirror image map

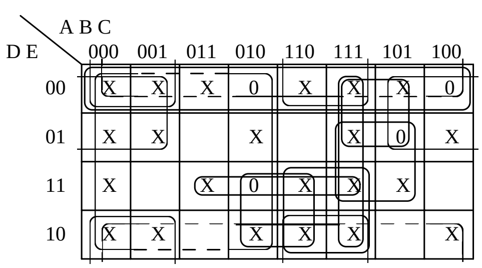


PIs: A B, A D, A C'E, B C, B D'E, C D, D E', C E',
A'C, B'D, C'D'E,A'D'E, B'C'E, A'B'
F = A'C + B'D + AB or
= B'D + AB + BC or
= A'C + AB + AD

5-variable diagonal map

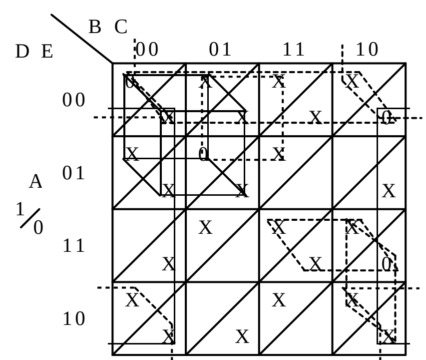


5.36 (d), (e) 5-variable mirror image map



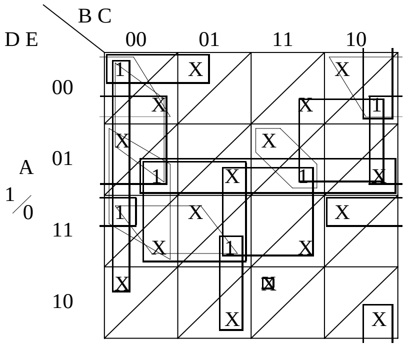
PIs: (A'+ B'+ C'), (A'+B'+D'), (A'+ B'+ E), (A'+ C'+ E'),
(A'+C'+ D), (B'+ D'+ E'), (B'+ C+ D'), (D + E),
(A + B + E), (A + C), (B + D), (C + E)
F = (A + C)(B + D)

5-variable diagonal map



Unit 5 Solutions

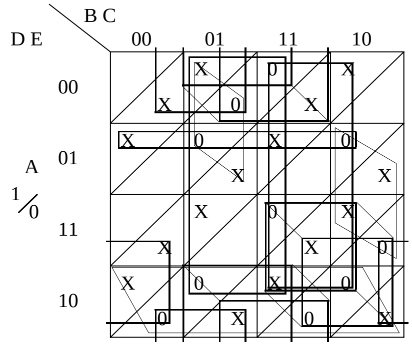
5.37 (a), (b) & (c)



PIs: A B C D E', B C D'E, A C'D E, A B'D'E', A B'C', A'B C'E', A'B D', A'B'C D, A'D'E, A'C E, B'D E, A'B'E, B'C'E, C'D'E', A'C'D', B'C'D'

F = A'B'E + A'B D' + A B'C' or
= A'C'D' + A'C E + A B'C' or
= A'D'E + B'D E + C'D'E' or
= A'B D' + B'C'D' + B'D E or
= A'C E + B'C'E + C'D'E'

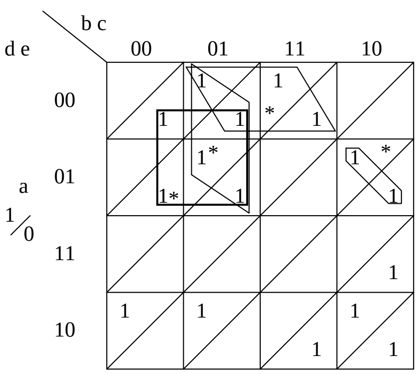
5.37 (d), & (e)



PIs: (B'+ C + E'), (C'+ E), (D'+ E), (A'+D +E'), (A'+ C'), (B'+D'), (A'+B'), (A + C+D'), (A + B + E)

F = (A'+ D + E')(C'+ E)(D'+ E)(B'+ D') or
= (A + B + E)(B'+ D')(A'+ B')(A'+ C') or
= (A + C + D')(C'+ E)(A'+ B')(A'+ C') or
= (B + C'+ D)(D'+ E)(B'+ D')(A'+ B') or
= (B'+ C + E')(C'+ E)(D'+ E)(A'+ C')

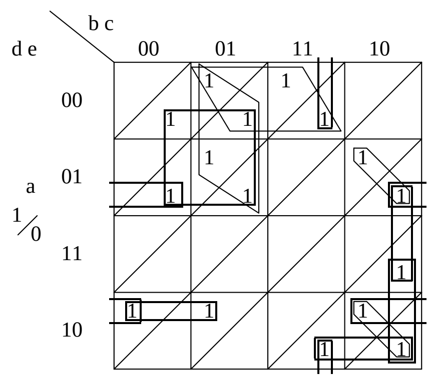
5.38 (a)



(*) Indicates a minterm that makes the corresponding prime implicant essential.

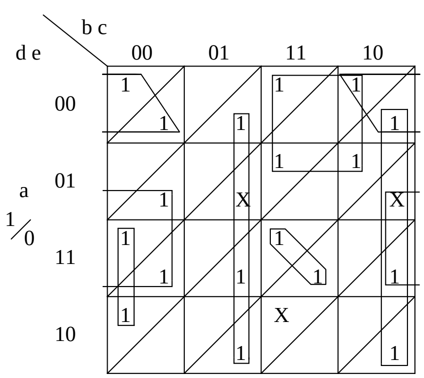
a'b'd'$\rightarrow m_{1}$; cd'e'$\rightarrow m_{28}$; bc'd'e$\rightarrow m_{25}$; b'cd'$\rightarrow m_{21}$

5.38 (b)



a'b'd', cd'e', bc'd'e, b'cd', ac'de', ab'ce', ab'de', a'c'd'e, a'bc'e, a'bc'd, bc'de', a'bde', a'bce'

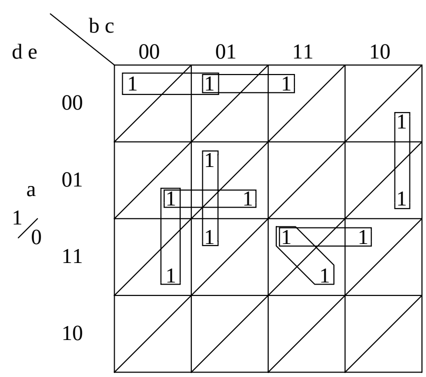
5.39 (a)



f = $\underline{a'b'c}$ + $\underline{a'bc'}$ + $\underline{ab'c'd}$ + $\underline{c'd'e'}$ + $\underline{abd'}$ + $\underline{bcde}$ + a'c'e

Alt: f = $\underline{a'b'c}$ + $\underline{a'bc'}$ + $\underline{ab'c'd}$ + $\underline{c'd'e'}$ + $\underline{abd'}$ + $\underline{bcde}$ + a'b'e

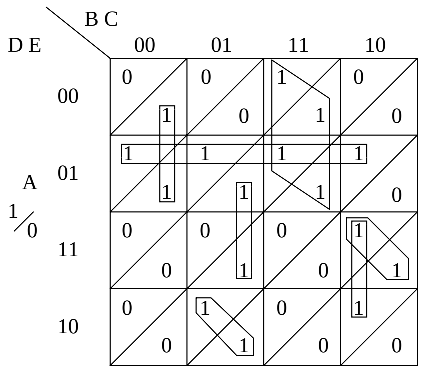
5.39 (b)



f = $\underline{a'b'c'e}$ + $\underline{a'bc'd'}$ + $\underline{bcde}$ + $\underline{ab'd'e'}$ + $\underline{abde}$ + $\underline{acd'e'}$ + a'b'd'e + ab'ce

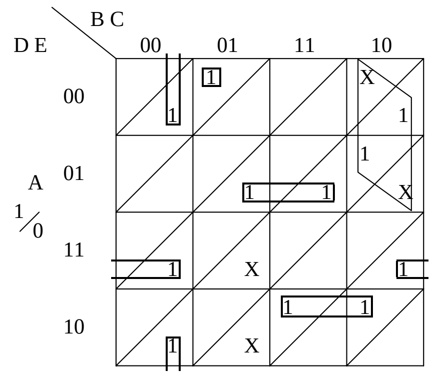
alt:
f = $\underline{a'b'c'e}$ + $\underline{a'bc'd'}$ + $\underline{bcde}$ + $\underline{ab'd'e'}$ + $\underline{abde}$ + $\underline{acd'e'}$ + b'cd'e + ab'ce
f = $\underline{a'b'c'e}$ + $\underline{a'bc'd'}$ + $\underline{bcde}$ + $\underline{ab'd'e'}$ + $\underline{abde}$ + $\underline{acd'e'}$ + b'cd'e + acde

5.40



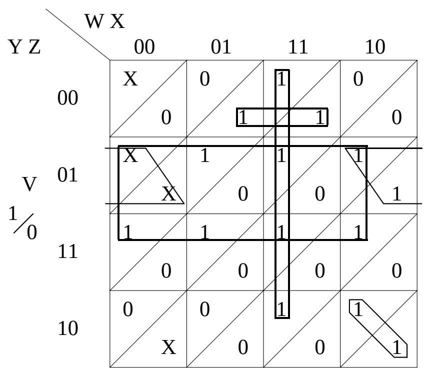
$F = \underline{A'B'C'D'} + \underline{BC'DE} + \underline{BCD'} + \underline{B'CDE'} + \underline{ABC'D}$
$+ \underline{A'B'CE} + AD'E$

5.41



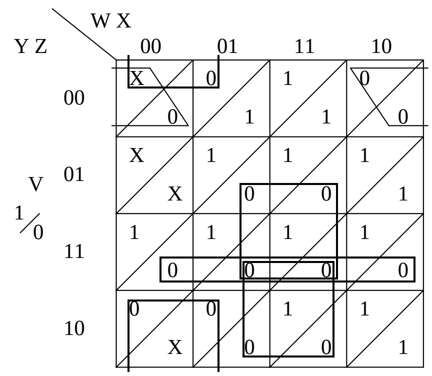
$F = \underline{AB'CD'E'} + \underline{BC'D'} + \underline{ABDE'}$
$+ A'B'C'E' + A'C'DE + A'CD'E$

5.42 (a)



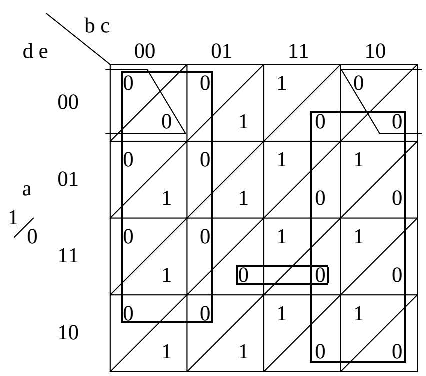
$F = \underline{V'XY'Z'} + \underline{X'Y'Z} + \underline{VZ} + WX'YZ' + VWX$

5.42 (b)



$F = \underline{(X + Y + Z)}\ \underline{(V + Y' + Z')}\ \underline{(V + X' + Z')}$
$\underline{(V + X' + Y')}\ \underline{(V' + W + Z)}$

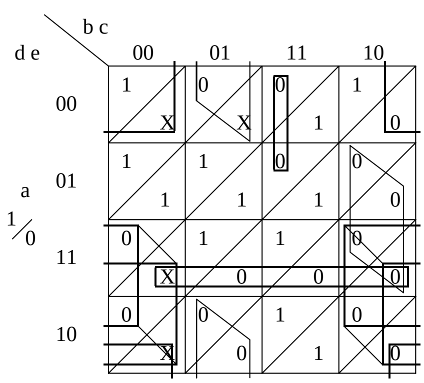
5.43 (a)



$F = (c + d + e)(a' + b)(a + b')(a + c' + d' + e')$

Alt: $F = (c + d + e)(a' + b)(a + b')(b + c' + d' + e')$

5.43 (b)

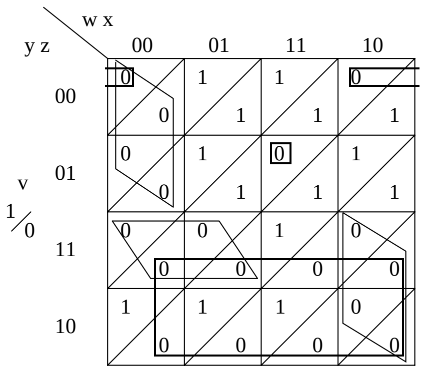


$F = (c + d')(a + d' + e')(b' + c + e')(a' + b' + c' + d)$
$(a + c + e)(b + c' + e)$

Alt: $F = (c + d')(a + d' + e')(a + b' + c)(b' + c + e')$
$(a' + b' + c' + d)(b + c' + e)$

Unit 5 Solutions

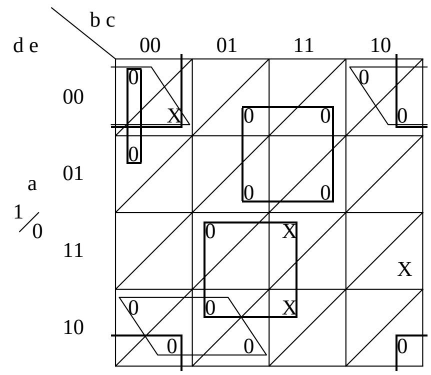
5.44 (a)



F = (v'+ w'+ x'+ y + z')(w + y'+ z')(v + y')(w + x + y)
(v'+ x + y + z) (w'+ x + y')

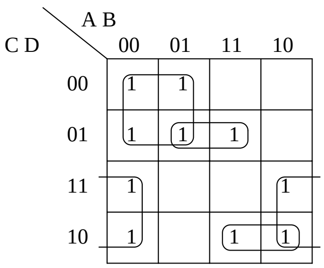
Alt:
F = (v'+ w'+ x'+ y + z')(w + y'+ z')(v + y')(w + x + y)
(v'+ w'+ x + z)(w'+ x + y')
F = (v'+ w'+ x'+ y + z')(w + y'+ z')(v + y')(w + x + y)
(v'+ w'+ x + z)(x + y'+ z')

5.44 (b)



$F = (c + d + e)(a' + c' + d')(a' + b + c + d)$
$(a + c' + d)(b + d' + e)(a + c + e)$

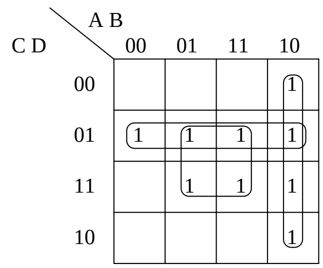
5.45 (a)



F = ACD' + BC'D + B'C + A'C'

m_4, m_{13}, or m_{14} change the minimum sum of products, removing A'C', BC'D, or ACD', respectively.

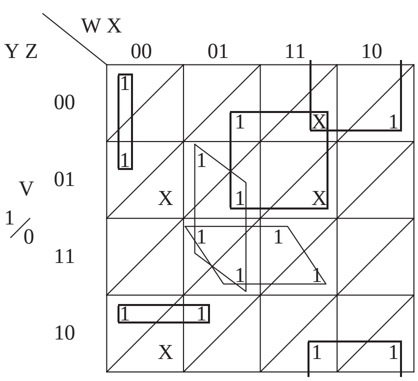
5.45 (b)



F = C'D + BD + AB'

Changing m_1 to a don't care removes *C'D* from the solution.

5.46 (a)



$F = \underbrace{V'XY'}_{m_4} + \underbrace{V'WZ'}_{m_8} + \underbrace{XYZ}_{m_{31}} + VW'X'Y' + VW'Y Z' + W'XZ$

$F = \underline{V'XY'} + \underline{V'WZ'} + \underline{XYZ} + VW'X'Z' + VW'XY + W'Y'Z$

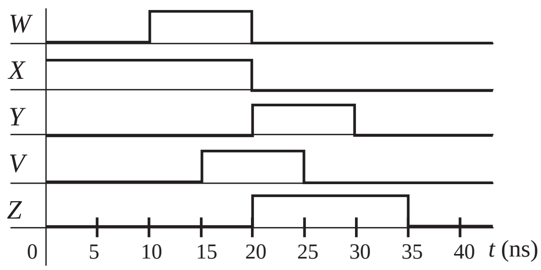
$F = \underline{V'XY'} + \underline{V'WZ'} + \underline{XYZ} + VW'X'Y' + VW'Y Z' + W'Y'Z$

$F = \underline{V'XY'} + \underline{V'WZ'} + \underline{XYZ} + VW'X'Z' + VW'Y Z' + W'Y'Z$

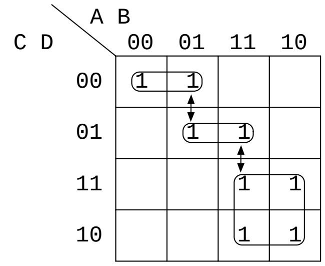
5.46 (b) $V'WZ' \rightarrow m_8$; $XYZ \rightarrow m_{31}$; $V'XY' \rightarrow m_4$

Unit 8 Problem Solutions

8.1



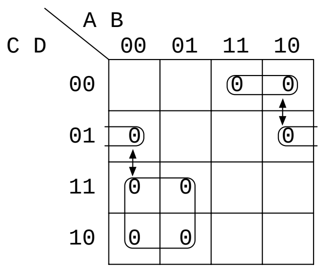
8.2 (a)



F = A'C'D' + A C + B C'D

Static 1-hazards: 1101↔1111 and 0100↔0101

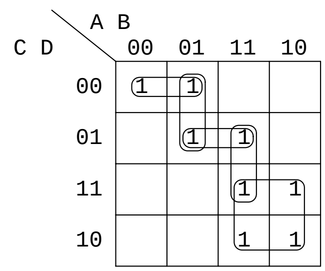
8.2 (a) (contd)



F = (A + C') (A'+ C + D) (B + C + D')

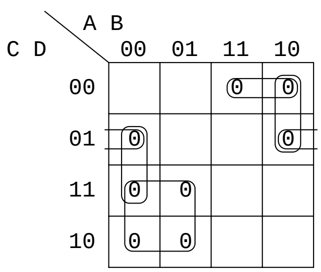
Static 0-hazards are: 0001↔0011 and 1000↔1001

8.2 (b)



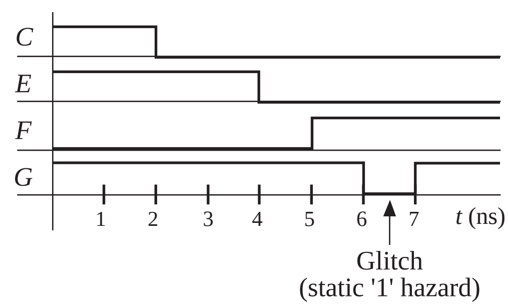
F^t = A'C'D' + A C + B C'D + <u>A'BC'</u> + <u>ABD</u>

8.2 (c)

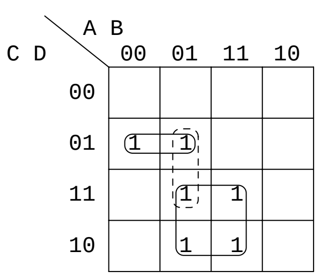


$F^t = (A + C')\,(A' + C + D)\,(B + C + D')$
$\underline{(A' + B + C)}\ \underline{(A + B + D')}$

8.3 (a)



8.3 (b) Modified circuit (to avoid hazards)



G = A'C'D + B C + A'BD

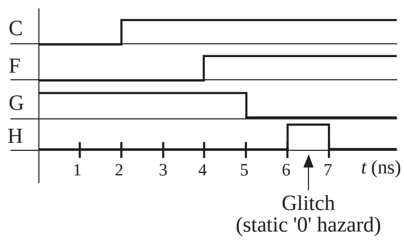
8.4 $A = 1$; $B = \mathrm{Z}$; $C = 1 \cdot \mathrm{Z} = \mathrm{X}$; $D = 1 + \mathrm{Z} = 1$;
$E = \mathrm{X}' = \mathrm{X}$; $F = 1' = 0$; $G = \mathrm{X} \cdot 0 = 0$;
$H = \mathrm{X} + 0 = \mathrm{X}$
See FLD Table 8-1, p. 231.

Unit 8 Solutions

8.5 $A = B = 0$, $C = D = 1$
So $F = AB'D + BC'D' + BCD = 0$

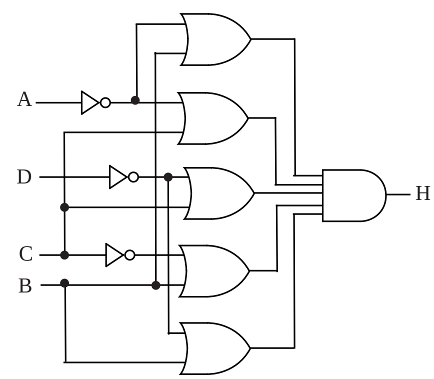
But in the figure, gate 4 outputs F = 1, indicating something is wrong. For the last NAND gate, F = 0 only when all its inputs are 1. But the output of gate 3 is 0. Therefore, gate 4 is working properly, but gate 3 is connected incorrectly or is malfunctioning.

8.6 (a)

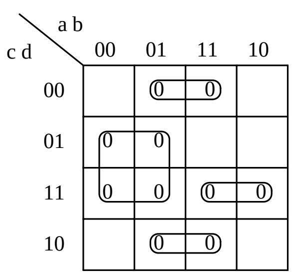


The circuit has three static 0-hazards: 0001↔0011, 1001↔1011 and 1000↔1010. Two sum terms are needed to eliminate the hazards: (A' + B) (B + D')

8.6 (b)



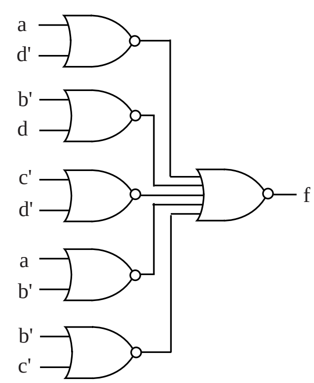
8.7 (a)



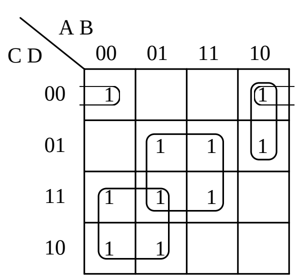
f = (a+d')(b'+c+d)(a'+c'+d')(b'+c'+d)

The static-0 hazards are 0100↔0101, 0100↔0110, 0111↔0110, 1100↔1110, 1111↔1110, 0011↔1011 and 0111↔1111.

8.7 (b) The minimal POS expression for f is f(a,b,c,d) = $(a + d')(b' + d)(c' + d')$ but $(a + b')$ and $(b' + c')$ must be added to eliminate the static-0 hazards.

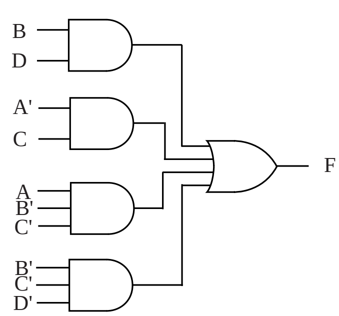


8.8 (a)

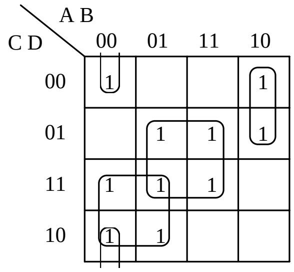


F = B D + A'C + A B'C' + B'C'D' .

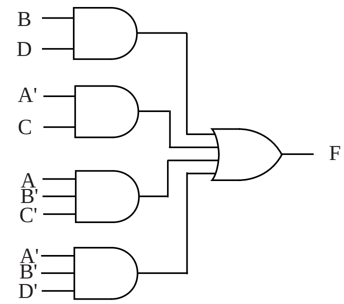
Static-1 Hazards: 0000↔0010, 1101↔1001



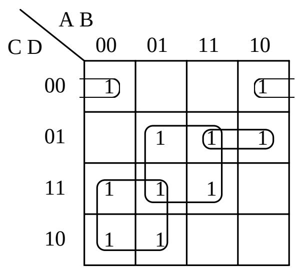
8.8 (a) contd



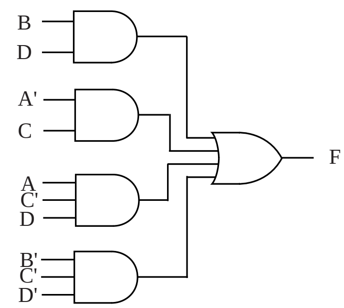
F = B D + A'C + A B'C' + A'B'D' .



Static-1 Hazards: 0000↔1000, 1101↔1001

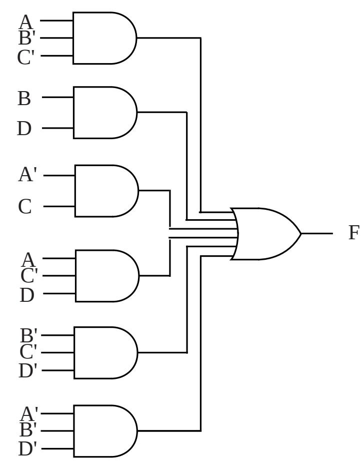


F = B D + A'C + A C'D + B'C'D' .

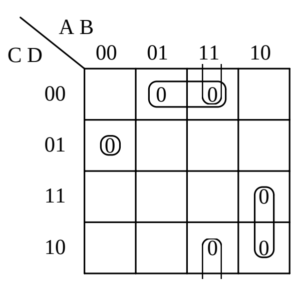


Static-1 Hazards: 0000↔0010, 1000↔1001

Hazard-free AND-OR circuit function:
f(A, B, C, D) = BD + A'C + AC'D + B'C'D' + A'B'D' + AB'C'

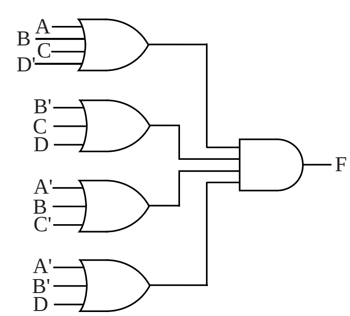


8.8 (b)



F = (A + B + C + D')(B'+ C + D)
(A'+ B + C')(A'+ B'+ D)

Static-0 Hazard: 1110↔1010

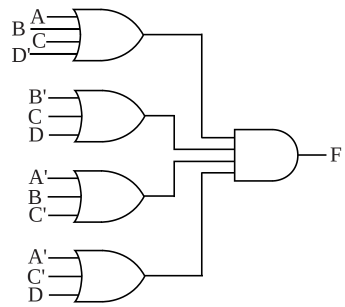


Unit 8 Solutions

8.8 (b) contd

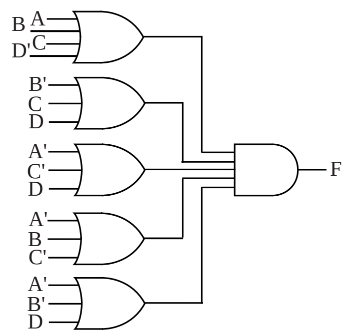
CD \ AB	00	01	11	10
00		0	0	
01	0			
11				0
10			0	0

F = (A + B + C + D')(B'+ C + D)
(A'+ B + C')(A'+ C'+ D)



Static-0 Hazard: 1100↔1110

Hazard-free OR-AND circuit function:
$f(A, B, C, D) = (A + B + C + D')(B' + C + D)$
$(A' + B + C')\ (A' + B' + D)\ (A' + C' + D)$



8.9 (a)

CD \ AB	00	01	11	10
00			1	1
01	1		1	1
11	1			
10				

f = AC' + A'B'D

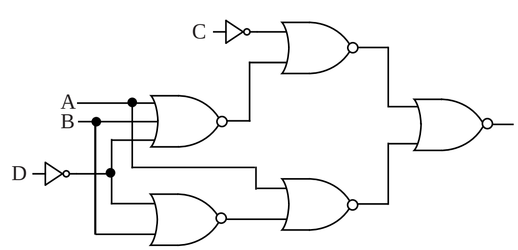
$f = (A'B'+AC')(A+D) = AA'B' + AC' + A'B'D + AC'D) = AA'B' + AC' + A'B'D$
static-1 hazard: 0001↔1001
static-0 hazard: 0010↔1010
potential dynamic hazards:
0000↔1000 and 0011↔1011
dynamic hazard: 0000↔1000
(Note the 0011↔1011 change only propagates over one path in the circuit and is not a dynamic hazard.)

8.9 (b) Since a circuit with NOR gates is desired, start with POS expressions for f that corresponds to a hazard-free OR-AND (NOR-NOR) circuit. From the Karnaugh map, all prime implicants are required,
$f = (A'+C')(A+B')(A+D)(C'+D)(B'+C')$.

CD \ AB	00	01	11	10
00	0	0		
01		0		
11		0	0	0
10	0	0	0	0

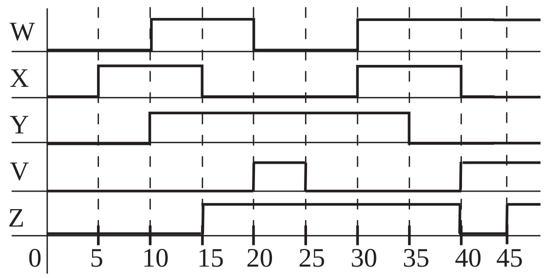
f = (A + D)(A + B')(A'+ C')(C'+ D)(B'+ C')

f can be multiplied out as $f = (A'B'D+C')(A+B'D)$. When this expression is expanded to a POS, it does not contain any sum of the form $(X + X' + ß)$ so the corresponding circuit is free of hazards. The three level NOR circuit is.



It is possible to start with a SOP that is free of hazards, namely, $f = AC' + A'B'D + B'C'D$, and then factor it, e.g., the same result as above is obtained by $f = (A+B'D)C'+A'B'D = (A'B'D+C')(A+B'D)$.

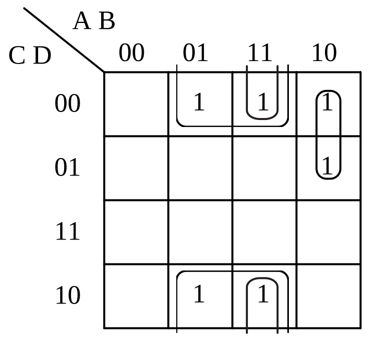
8.10



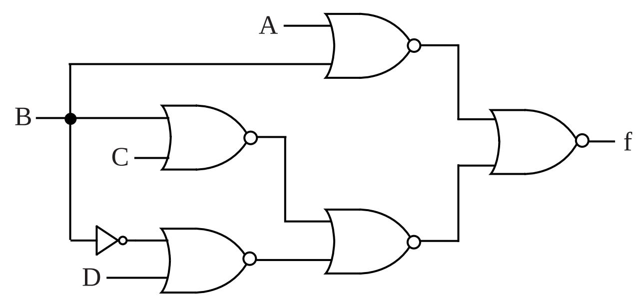
8.11 (a) $f = (A + B)(B'C' + BD')$
$= AB'C' + ABD' + BB'C' + BD'$
$= (A + B)(B' + B)(B' + D')(B + C')(C' + D')$

From the Karnaugh map and the $BB'C'$ term
static-1 hazard: 1100↔1000
static-0 hazard: 0001↔0101
potential dynamic hazards:
0000↔0100 and 1101↔1001
The circuit shows that only 0000↔0100 propagates over three paths.

8.11 (a) contd



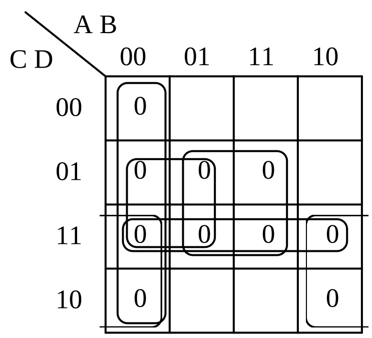
f = A B'C' + A B D' + B D' .



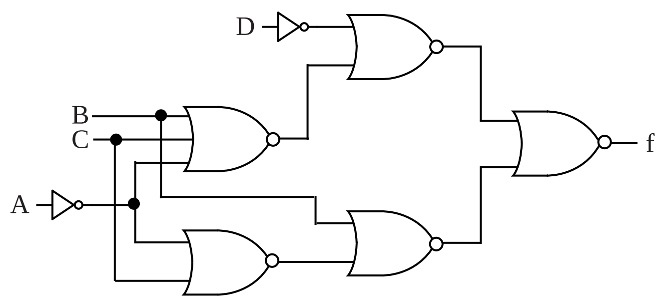
8.11 (a) From the Karnaugh map for f, it is seen that a hazard-free POS expression for f requires all prime implicants.

$f = (A + B)(B' + D')(B + C')(C' + D')(A + D')$

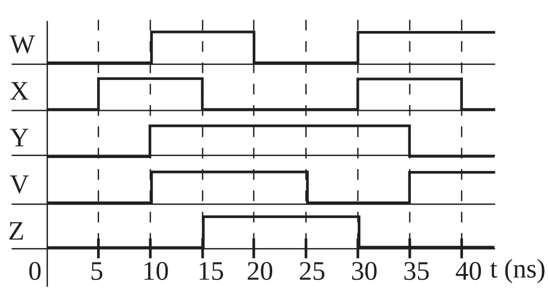
f can be multiplied out as $f = (A + B)(B' + D')(B + C')(C' + D')(A + D') = (AC' + B)(AB'C' + D')$



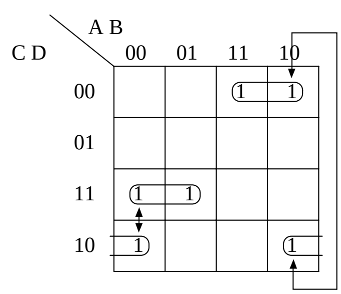
f = (A + B)(B + C')(B'+ D')(C'+ D')(A + D')



8.12



8.13



Z = A'C D + A C'D' + B'C D'

Static 1-hazards lie between 1000↔1010 and 0010↔0011

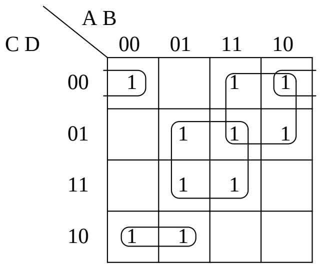
Without hazards: $Z^t = AC'D' + A'CD + B'CD' + A'B'C + AB'D'$

Unit 8 Solutions

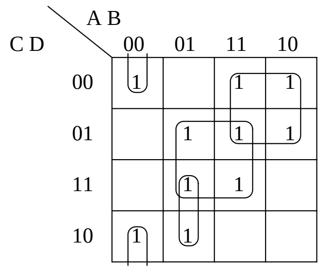
8.14 $A = Z$; $B = 0$; $C = Z' = X$; $D = Z \cdot 0 = 0$;
$E = Z$; $F = 0 + 0 + X = X$; $G = (0 \cdot Z)' = 0' = 1$;
$H = (X + 1)' = 1' = 0$

8.15 $A = B = C = 1$, so $F = (A + B' + C')\,(A' + B + C')\,(A' + B' + C) = 1$
But, in the figure, gate 4 outputs F = 0, indicating something is wrong. For the last NOR gate, F = 1 only when all its inputs are 0. But the output of gate 1 is 1. Therefore, gate 4 is working properly, but gate 1 is connected incorrectly or is malfunctioning.

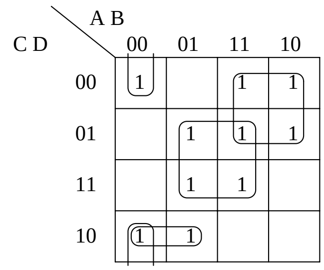
8.16 (a) $F\,(A, B, C, D) = \sum m(0, 2, 5, 6, 7, 8, 9, 12, 13, 15)$
There are 3 different minimum AND-OR solutions to this problem. The problem asks for any two of these.



F = B D + A C' + A'C D' + B'C'D'



F = B D + A C' + A'B'D' + A'B C



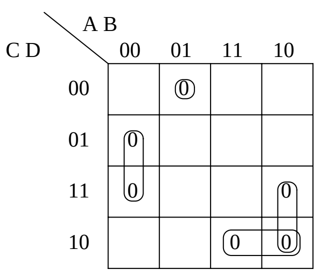
F = B D + A C' + A'B'D' + A'C D'

<u>Solution 1</u>: 1-hazards are between 0000↔0010 and 0111↔0110

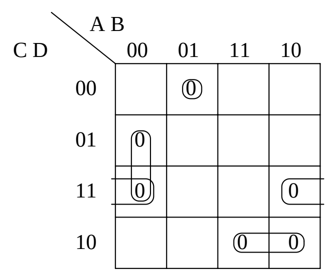
<u>Solution 2</u>: 1-hazards are between 0010↔0110 and 0000↔1000

<u>Solution 3</u>: 1-hazards are between 0111↔0110 and 0000↔1000
Without hazards:
$F^t = BD + AC' + B'C'D' + A'CD' + A'B'D' + A'BC$

8.16 (b)



$F = (A + B + D')\,(A + B' + C + D)\,(A' + C' + D)\,(A' + B + C')$
0-hazard is between 1011↔0011



$F = (A + B + D')\,(A + B' + C + D)\,(A' + C' + D)\,(B + C' + D')$
0-hazard is between 1011↔1010

Either way, without hazard:
$F^t = (A + B + D')\,(A + B' + C + D)\,(A' + C' + D)\,(B + C' + D')\,(A' + B + C')$

Unit 9 Problem Solutions

9.1 See FLD p. 703 for solution.

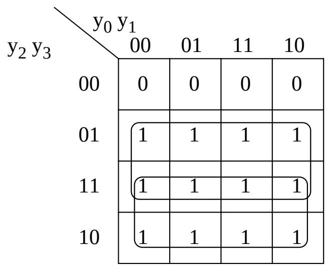
9.2 See FLD p. 703 for solution.

9.3 See FLD p. 704 for solution.

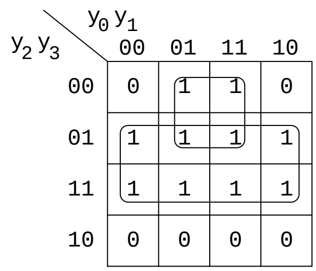
9.4 See FLD p. 704 and Figure 4-4 on FLD p.105.

9.5

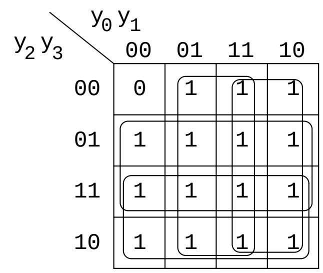
$y_0y_1y_2y_3$	$a\ b\ c$
0 0 0 0	0 0 0
1 0 0 0	0 0 1
X 1 0 0	0 1 1
X X 1 0	1 0 1
X X X 1	1 1 1



$a = y_3 + y_2$



$b = y_3 + y_2' y_1$



$c = y_3 + y_2 + y_1 + y_0$

9.6 See FLD p. 705 for solution.

9.7 See FLD p. 705 for solution.

9.8 See FLD p. 705-706 for solution.

9.9 The equations derived from Table 4-6 on FLD p. 107 are:
$D = x'y'b_{in} + x'yb_{in}' + xy'b_{in}' + xyb_{in}$
$bout = x'b_{in} + x'y + yb_{in}$
See p. p. 706 for PAL diagram.

9.10 Note: $A_6 = A_4'$ and $A_5 = A_4$. Equations for A_4 through A_0 can be found using Karnaugh maps. See FLD p. 707-708 for answers.

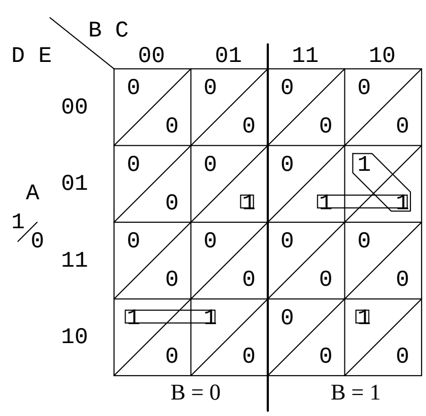
9.11 (a) $F = C'D' + BC' + A'C \rightarrow$ Use 3 AND gates
$F' = [C'D' + BC' + A'C]' = [C'(B + D') + CA']'$
$= [(C + B + D')(A' + C')]'$
$= B'C'D + AC \rightarrow$ Use 2 AND gates

9.11 (b) $F = A'B' + C'D' \rightarrow$ Use 2 AND gates
$F' = (A'B' + C'D')'$
$= (A + B)(C + D)$
$= AC + AD + BC + BD \rightarrow$ Use 4 AND gates

9.12 (a) See FLD p. 708, use the answer for 9.12 (b), but leave off all connections to 1 and 1'.

9.12 (b) See FLD p. 708 for solution.

9.13 Using Shannon's expansion theorem:
$F = ab'cde' + bc'd'e + a'cd'e + ac'de'$
$= b'(acde' + a'cd'e + ac'de') + b(c'd'e + a'cd'e + ac'de')$
$= b'[ade'(c + c') + a'cd'e] + b[(c' + a'c)d'e + ac'de']$
$= b'(ade' + a'cd'e) + b(c'd'e + a'd'e + ac'de')$
The same result can be obtained by splitting a Karnaugh map, as shown to the right.

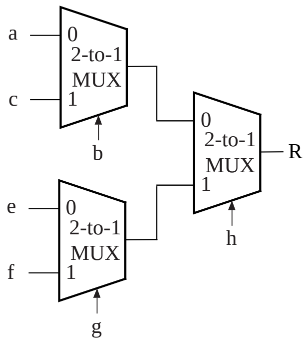


Unit 9 Solutions

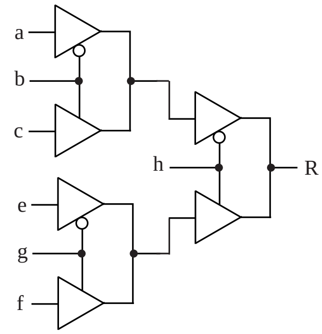
9.14 (a) $R = ab'h' + bch' + eg'h + fgh$

$= (ab' + bc)h' + (eg' + fg)h$

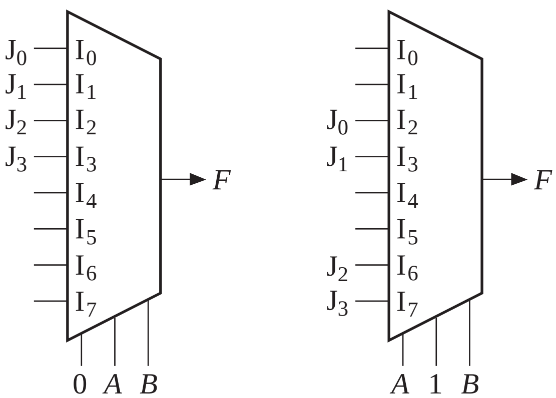
$= [(a)b' + (c)b]h' + [(e)g' + (f)g]h$



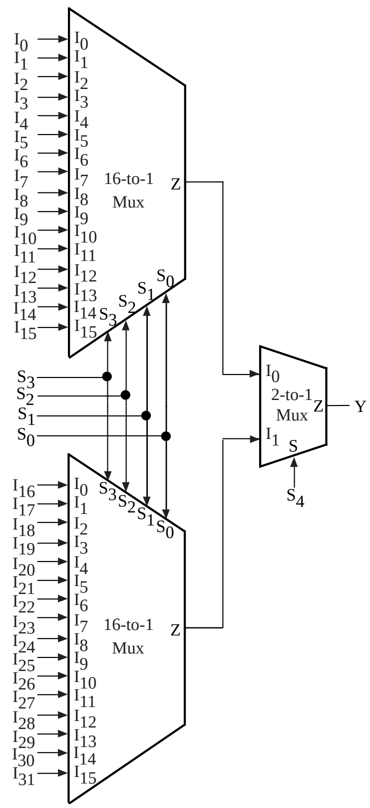
9.14 (b)



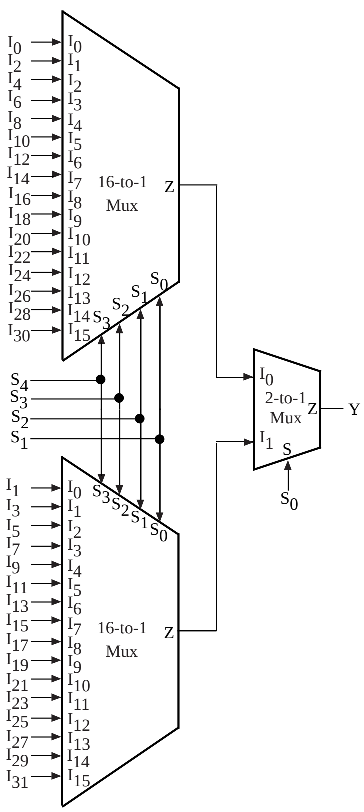
9.15 There are many solutions. For example:



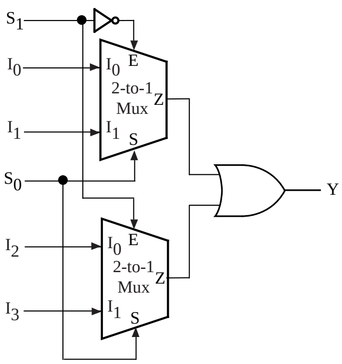
9.16



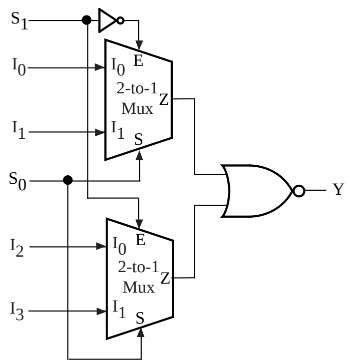
9.16 contd



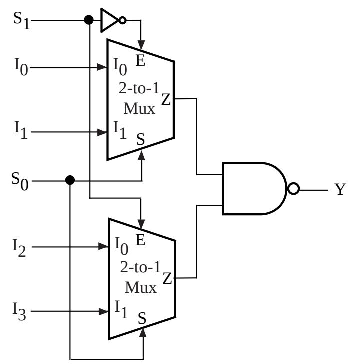
9.17 (a)



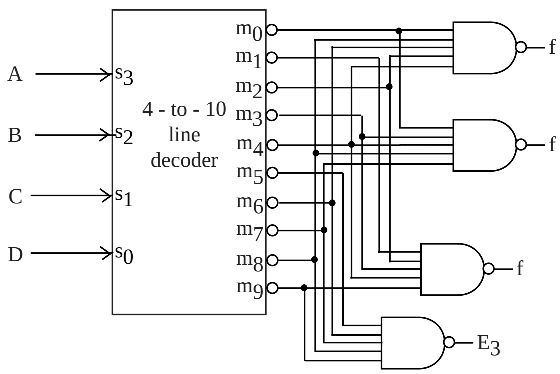
9.17 (b)



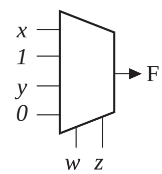
9.17 (c)



9.18 Since the decoder outputs are negative, NAND gates are required. The excess-3 outputs are Σ m(5,6,7,8,9), Σ m(1,2,3,4,9), Σ m(0,3,4,7,8), and Σ m(0,2,4,6,8) so four 5-input NAND gates are needed with inputs corresponding to the minterms of the excess-3 outputs.

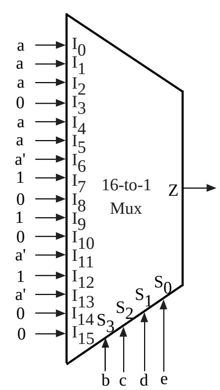


9.19 Using S1 = w and S0 = z, I0 = x, I1 = 1, I2 = y and I3 = 0 which does not require any gates.



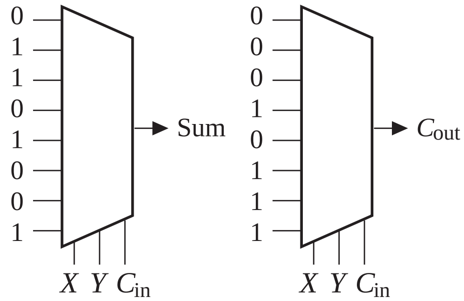
Other answers: Using S1 = w and S0 = y, I0 = x, I1 = z, I2 = 0 and I3 = z' which requires one inverter. Using S1 = w and S0 = x, I0 = z, I1 = 1, I2 = yz' and I3 = y which requires one inverter and one AND gate.

9.20 $f(a, b, c, d, e) = a'b'cde' + a'b'cde + a'bc'd'e + a'bc'de + a'bcd'e' + a'bcd'e + ab'c'd'e' + ab'c'd'e + ab'c'de' + ab'cd'e' + ab'cd'e + ab'cde + abc'd'e + abcd'e'$
$= a(b'c'd'e') + a(b'c'd'e) + a(b'c'de') + a(b'cd'e') + a(b'cd'e) + a'(b'cde') + [a'(b'cde) + a(b'cde)] + [a'(bc'd'e) + a(bc'd'e)] + a'(bc'de) + [a'(bcd'e') + a(bcd'e')] + a'(bcd'e)$
I0 = a, I1 = a, I2 = a, I3 = 0, I4 = a, I5 = a, I6 = a', I7 = 1, I8 = 0, I9 = 1, I10 = 0, I11 = a', I12 = 1, I13 = a', I14 = 0, I15 = 0

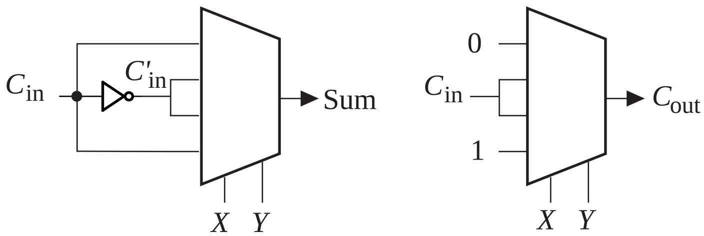


9.21 (a)

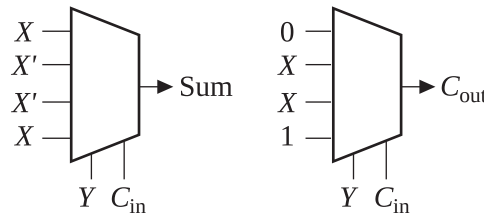
$x\ y\ c_{in}$	$Sum\ C_{out}$
0 0 0	0 0
0 0 1	1 0
0 1 0	1 0
0 1 1	0 1
1 0 0	1 0
1 0 1	0 1
1 1 0	0 1
1 1 1	1 1



9.21 (b)

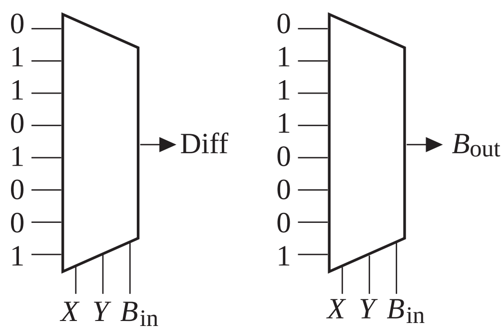


9.21(c)

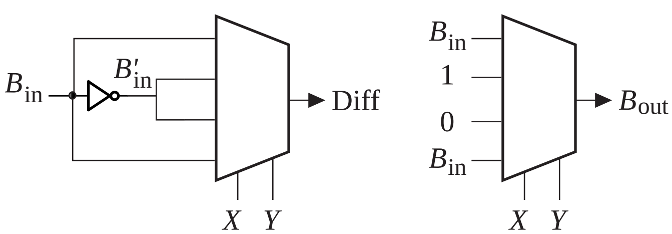


9.22 (a)

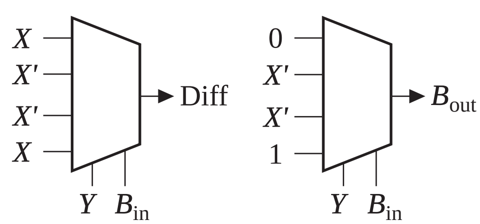
$x\ y\ b_{in}$	$Diff\ B_{out}$
0 0 0	0 0
0 0 1	1 1
0 1 0	1 1
0 1 1	0 1
1 0 0	1 0
1 0 1	0 0
1 1 0	0 0
1 1 1	1 1



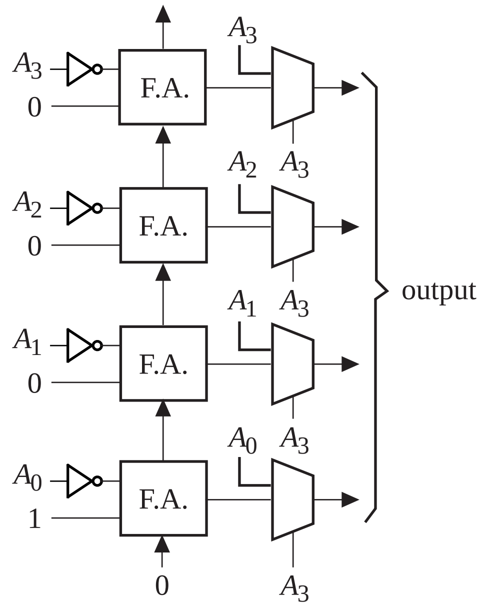
9.22 (b)



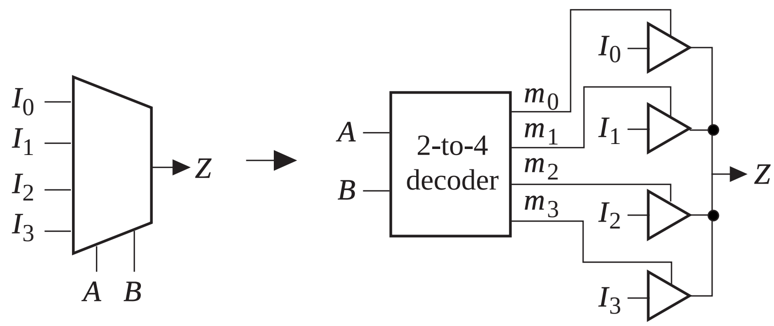
9.22 (c)



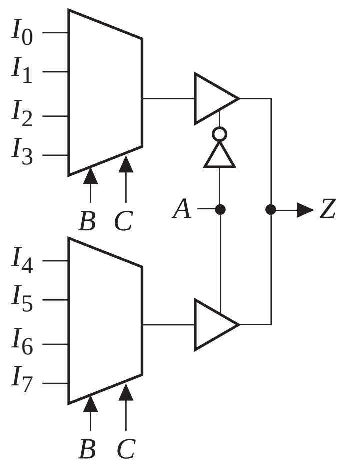
9.23 For a positive number A, |A| = A and for a negative number A, |A| = –A. Therefore, if the number is negative, that is A[3] is 1, then the output should be the 2's complement (that is, invert and add 1) of the input A.



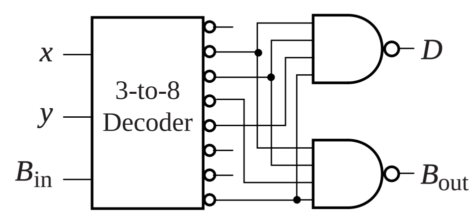
9.24



9.25

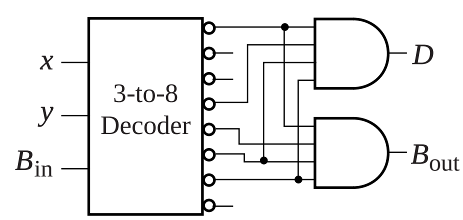


9.26 (a)



$D = \sum m(1, 2, 4, 7)$; $Bout = \sum m(1, 2, 3, 7)$

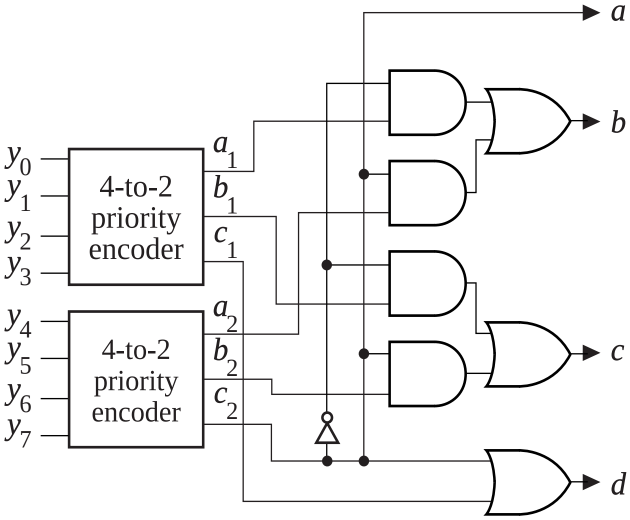
9.26 (b)



$D = \prod M(0, 3, 5, 6)$; $B_{out} = \prod M(0, 4, 5, 6)$

Unit 9 Solutions

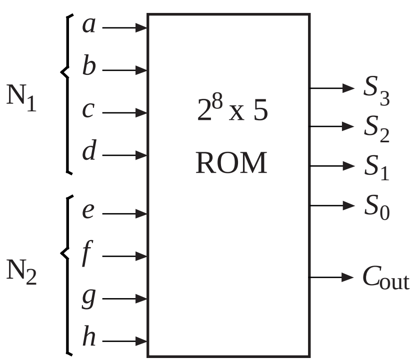
9.27



If any of the inputs y_0 through y_7 is 1, then d of the 8-to-3 decoder should be 1. But in that case, c_1 or c_2 of one of the 4-to-2 decoders will be 1. So $d = c_1 + c_2$.

If one of the inputs y_4, y_5, y_6, and y_7 is 1, then a should be 1, and b and c should correspond to a_2 and b_2, respectively. Otherwise, a should be 0, and b and c should corresond to a_1 and b_1, respectively. So $a = c_2$, $b = c_2a_2 + c_2'a_1$, and $c = c_2b_2 + c_2'b_1$.

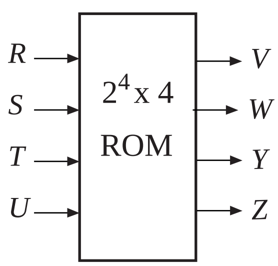
9.28



a	b	c	d	e	f	g	h	S_3	S_2	S_1	S_0	C_{out}	Meaning
0	0	0	0	0	0	0	0	X	X	X	X	X	(0000 is a not valid input)
0	0	1	1	0	0	1	1	0	0	1	1	0	(0 + 0 = 0)
0	1	0	1	0	1	1	0	1	0	0	0	0	(2 + 3 = 5)
1	0	1	0	0	1	1	1	0	1	0	0	1	(7 + 4 = 11)

9.29 (a)

R S T U	*V W Y Z*
0 0 0 0	0 0 0 0
0 0 0 1	0 0 0 1
0 0 1 0	0 0 1 0
0 0 1 1	0 1 0 0
0 1 0 0	0 1 0 1
0 1 0 1	0 1 1 0
0 1 1 0	1 0 0 0
0 1 1 1	1 0 0 1
1 0 0 0	1 0 1 0
1 0 0 1	1 1 0 0
1 0 1 0	X X X X
1 0 1 1	X X X X
1 1 0 0	X X X X
1 1 0 1	X X X X
1 1 1 0	X X X X
1 1 1 1	X X X X



9.29 (b)

TU \ RS	00	01	11	10
00			X	1
01			X	1
11		1	X	X
10		1	X	X

V = S T + R

TU \ RS	00	01	11	10
00	0	1	X	0
01	0	1	X	1
11	1	0	X	X
10	0	0	X	X

W = S'T U + <u>S T'U</u> + R U + <u>S T'U'</u>

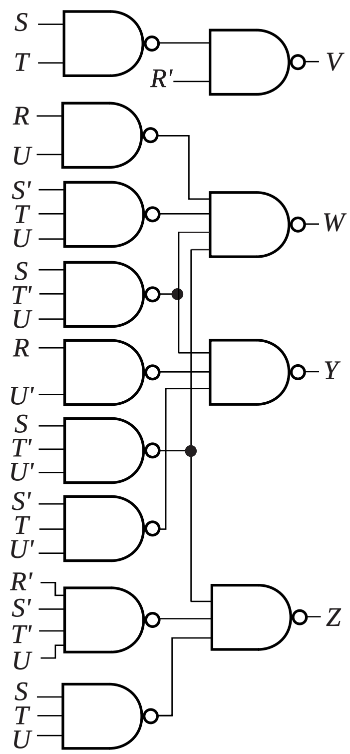
9.29 (b) (contd)

TU \ RS	00	01	11	10
00	0	0	X	1
01	0	1	X	0
11	0	0	X	X
10	1	0	X	X

Y = S'T U' + <u>S T'U</u> + R U'

TU \ RS	00	01	11	10
00	0	1	X	0
01	1	0	X	0
11	0	1	X	X
10	0	0	X	X

Z = R'S'T'U + <u>S T'U'</u> + S T U



9.29 (c)

R S T U	*V W Y Z*
- 1 1 -	1 0 0 0
1 0 0 -	1 0 0 0
1 - - 0	0 0 1 0
1 - - 1	0 1 0 0
- 0 1 1	0 1 0 0
- 1 0 1	0 1 1 0
- 1 0 0	0 1 0 1
- 0 1 0	0 0 1 0
0 0 0 1	0 0 0 1
- 1 1 1	0 0 0 1

9.30 (a)

R S T U	*V W Y Z*
0 0 0 0	0 0 0 0
0 0 0 1	0 0 0 1
0 0 1 0	0 1 0 0
0 0 1 1	0 0 1 0
0 1 0 0	X X X X
0 1 0 1	X X X X
0 1 1 0	0 1 0 1
0 1 1 1	X X X X
1 0 0 0	1 1 0 0
1 0 0 1	1 0 1 0
1 0 1 0	1 0 0 0
1 0 1 1	1 0 0 1
1 1 0 0	X X X X
1 1 0 1	X X X X
1 1 1 0	0 1 1 0
1 1 1 1	X X X X

9.30 (b)

TU \ RS	00	01	11	10
00	0	X	X	1
01	0	X	X	1
11	0	X	X	1
10	0	0	0	1

V = R S'

TU \ RS	00	01	11	10
00	0	X	X	1
01	0	X	X	0
11	0	X	X	0
10	1	1	1	0

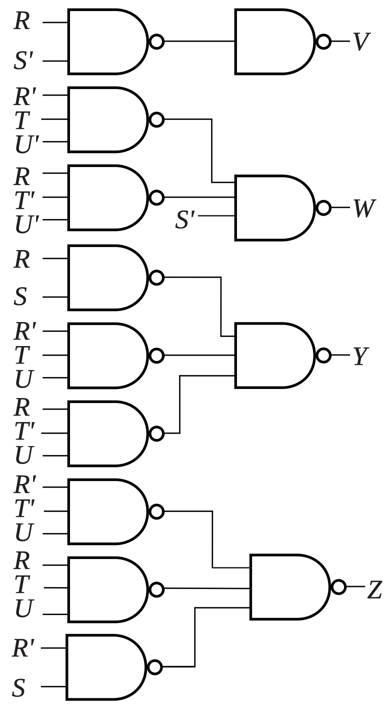
W = R'T U' + R T'U' + S

TU \ RS	00	01	11	10
00	0	X	X	0
01	0	X	X	1
11	1	X	X	0
10	0	0	1	0

Y = R'T U + R T'U + R S

TU \ RS	00	01	11	10
00	0	X	X	0
01	1	X	X	0
11	0	X	X	1
10	0	1	0	0

Z = R'T'U + R'S + R T U



9.30 (c)

R S T U	*V W Y Z*
1 0 - -	1 0 0 0
- 1 - -	0 1 0 0
0 - 1 0	0 1 0 0
1 - 0 0	0 1 0 0
1 1 - -	0 0 1 0
0 - 1 1	0 0 1 0
1 - 0 1	0 0 1 0
0 1 - -	0 0 0 1
0 - 0 1	0 0 0 1
1 - 1 1	0 0 0 1

or

R S T U	*V W Y Z*
1 0 - -	1 0 0 0
0 1 - -	0 1 0 1
1 1 - -	0 1 1 0
0 - 1 0	0 1 0 0
1 - 0 0	0 1 0 0
0 - 1 1	0 0 1 0
1 - 0 1	0 0 1 0
0 - 0 1	0 0 0 1
1 - 1 1	0 0 0 1

9.31 (a)

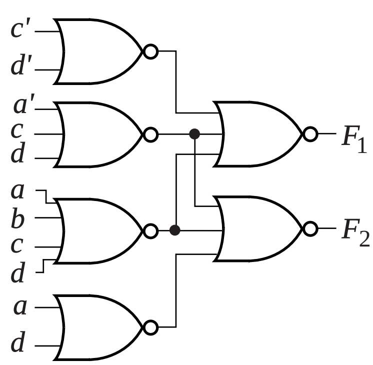
CD \ AB	00	01	11	10
00	0	1	1	1
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

F_1 = (A + B + C + D)(A'+ C + D')(C' + D')

9.31 (a) (contd)

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	0	0	0
11	0	0	1	1
10	1	1	1	1

F_2 = (A + B + C + D)(A' + C + D')(A + D')



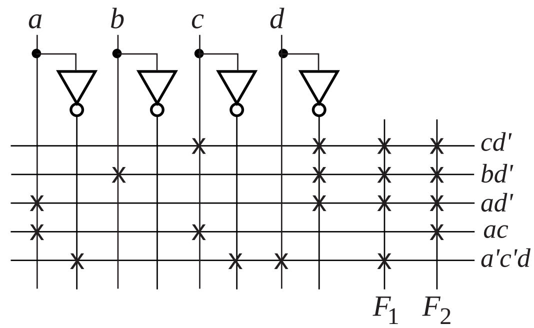
Alternate solution:

$F_1 = (a + b + c + d)\,(a + c' + d')\,(a' + d')$

$F_2 = (a + b + c + d)\,(a + b' + d')\,(c + d')$

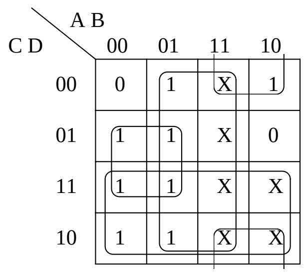
9.31 (b)

	$a\ b\ c\ d$	F_1F_2
(cd')	- - 1 0	1 1
(bd')	- 1 - 0	1 1
(ad')	1 - - 0	1 1
(ac)	1 - 1 -	0 1
($a'c'd$)	0 - 0 1	1 0

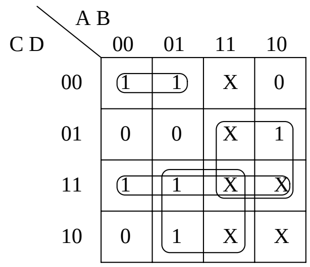


9.32 (a)

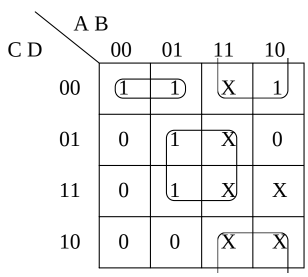
$A\ B\ C\ D$	$W X\ Y\ Z$
0 0 0 0	0 1 1 1
0 0 0 1	1 0 0 0
0 0 1 0	1 0 0 1
0 0 1 1	1 1 0 0
0 1 0 0	1 1 1 0
0 1 0 1	1 0 1 0
0 1 1 0	1 1 0 1
0 1 1 1	1 1 1 1
1 0 0 0	1 0 1 1
1 0 0 1	0 1 0 1



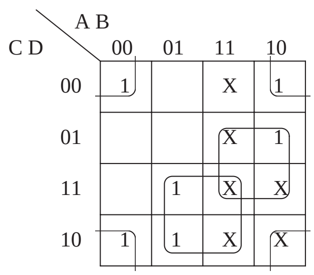
$W = A'D + C + B + \underline{AD'}$



$X = \underline{A'C'D'} + C\,D + \underline{A\,D} + \underline{B\,C}$

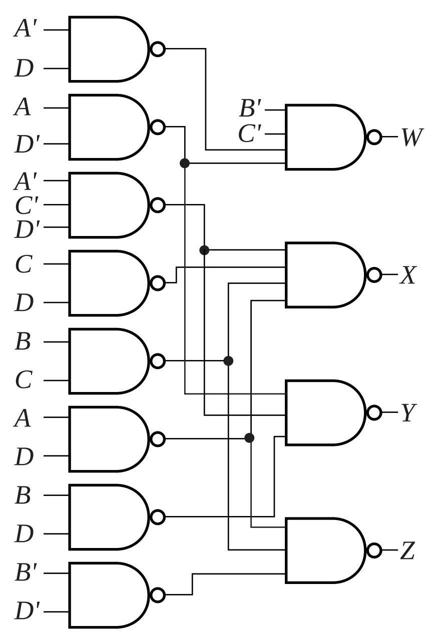


$Y = \underline{A\,D'} + B\,D + \underline{A'C'D'}$



$Z = \underline{A\,D} + \underline{B\,C} + B'D'$

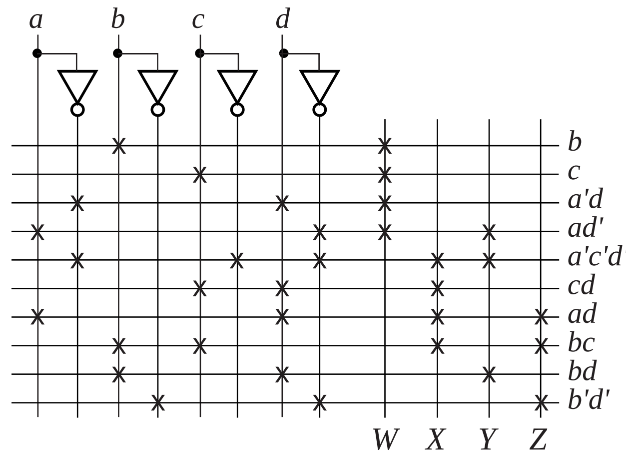
Alt: $Z = A + \underline{B\,C} + B'D'$



9.32 (b)

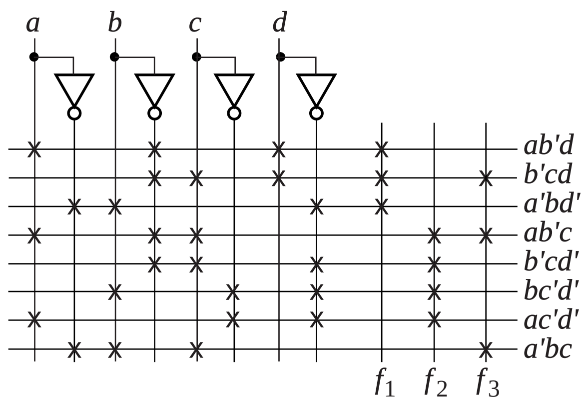
a b c d	*W X Y Z*
- 1 - -	1 0 0 0
- - 1 -	1 0 0 0
0 - - 1	1 0 0 0
1 - - 0	1 0 1 0
0 - 0 0	0 1 1 0
- - 1 1	0 1 0 0
1 - - 1	0 1 0 1
- 1 1 -	0 1 0 1
- 1 - 1	0 0 1 0
- 0 - 0	0 0 0 1

9.32 (c)



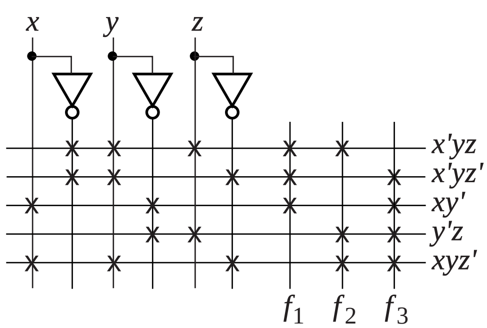
9.33 (a) See solution for 7.10

a b c d	f_1 f_2 f_3
1 0 - 1	1 0 0
- 0 1 1	1 0 1
0 1 - 0	1 0 0
1 0 1 -	0 1 1
- 0 1 0	0 1 0
- 1 0 0	0 1 0
1 - 0 0	0 1 0
0 1 1 -	0 0 1



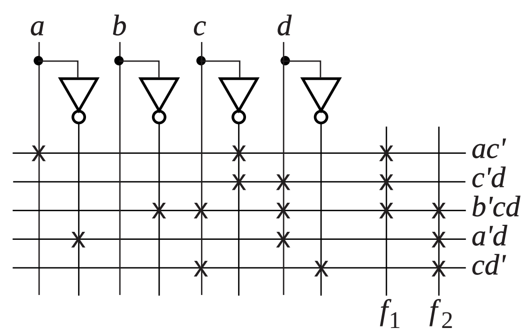
9.33 (b) See solution for 7.41

x y z	f_1 f_2 f_3
0 1 1	1 1 0
0 1 0	1 0 1
1 0 -	1 0 1
- 0 1	0 1 1
1 1 0	0 1 1

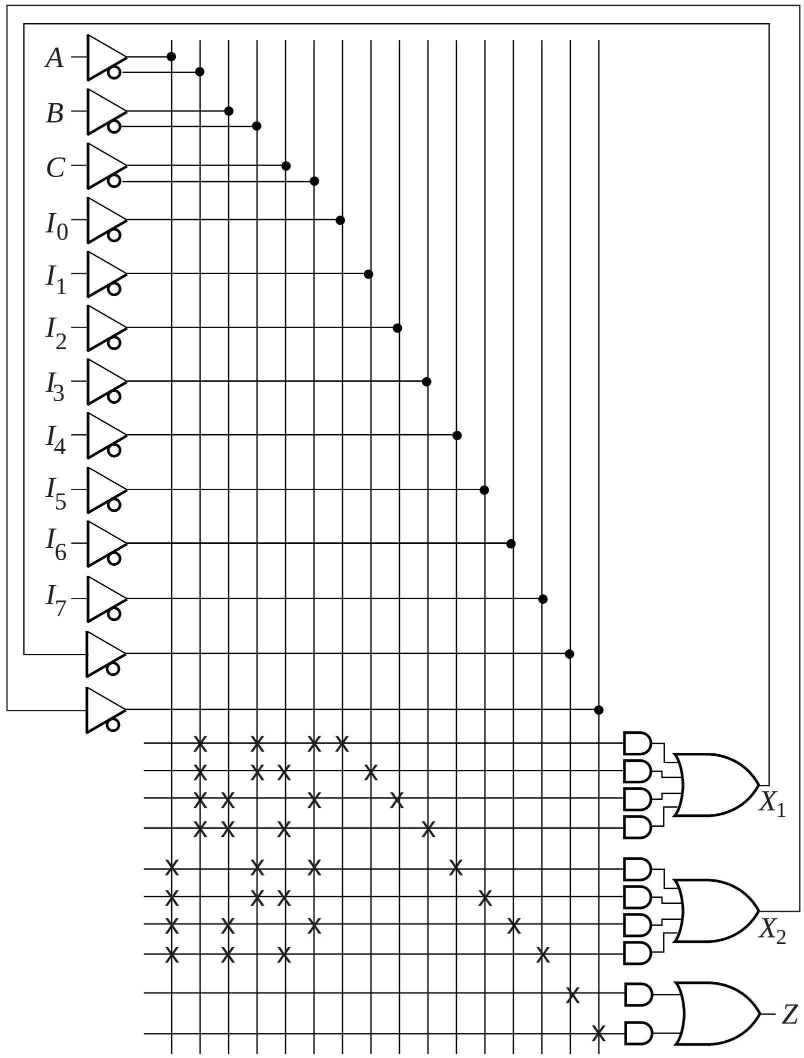


9.33 (c) Because a PLA works with a sum-of-products expression, see solution for 7.43(b), not (a).

a b c d	f_1 f_2
1 - 0 -	1 0
- - 0 1	1 0
- 0 1 1	1 1
0 - - 1	0 1
- - 1 0	0 1



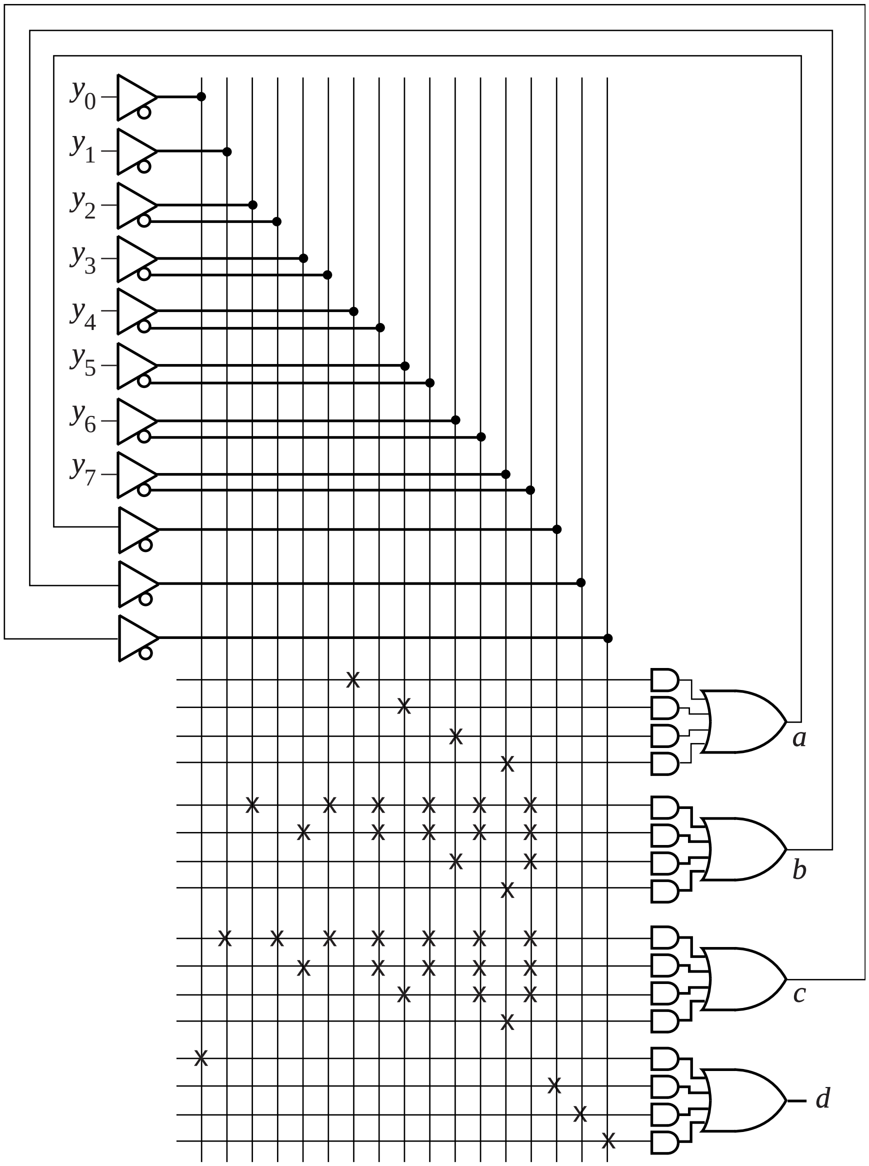
9.34 $Z = I_0A'B'C' + I_1A'B'C + I_2A'BC' + I_3A'BC + I_4AB'C' + I_5AB'C + I_6ABC' + I_7ABC$
$= X_1A' + X_2A$ where $X_1 = I_0B'C' + I_1B'C + I_2BC' + I_3BC$ and $X_2 = I_4B'C' + I_5B'C + I_6BC' + I_7BC$



Note: Unused inputs, outputs, and wires have been omitted from this diagram.

9.35 For an 8-to-3 encoder, using the truth table given in FLD Figure 9-16, we get
$a = y_4 + y_5 + y_6 + y_7$
$b = y_2y_3'y_4'y_5'y_6'y_7' + y_3y_4'y_5'y_6'y_7' + y_6y_7' + y_7$
$c = y_1y_2'y_3'y_4'y_5'y_6'y_7' + y_3y_4'y_5'y_6'y_7' + y_5y_6'y_7' + y_7$
$d = a + b + c + y_0$
Alternative solution for simplified expressions:
$b = y_2y_4'y_5' + y_3y_4'y_5' + y_6 + y_7$
$c = y_1y_2'y_4'y_6' + y_3y_4'y_6' + y_5y_6' + y_7$

9.35 (contd)



Note: Unused inputs, outputs, and wires have been omitted from this diagram.

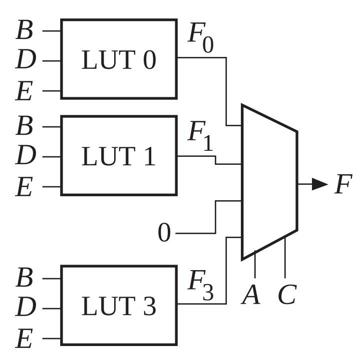
9.36 $F = CD'E + CDE + A'D'E + A'B'DE' + BCD$

9.36 (a) $F = A'B'(CD'E + CDE + D'E + DE') +$
$A'B\,(CD'E + CDE + D'E + CD) +$
$AB'\,(CD'E + CDE) + AB\,(CD'E + CDE + CD)$

9.36 (b) $F = B'C'\,(A'D'E + A'DE') +$
$B'C\,(D'E + DE + A'D'E + A'DE') +$
$BC'\,(A'D'E) + BC\,(D'E + DE + A'D'E + D)$

9.36 (c) $F = A'C'\,(D'E + B'DE') +$
$A'C\,(D'E + DE + D'E + B'DE' + BD) +$
$AC'(0) + AC(D'E + DE + BD)$

9.36 (d) Use the expansion about A and C
$F = A'C'(F_0) + A'C(F_1) + AC(F_3)$
where F_0, F_1, F_3 are implemented in lookup tables:



BDE	$F_0 F_1 F_3$
0 0 0	0 0 0
0 0 1	1 1 1
0 1 0	1 1 0
0 1 1	0 1 1
1 0 0	0 0 0
1 0 1	1 1 1
1 1 0	0 1 1
1 1 1	0 1 1

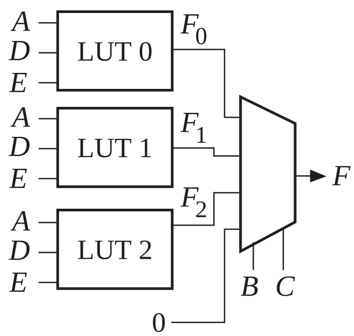
9.37 $F = B'D'E' + AB'C + C'DE' + A'BC'D$

9.37 (a) $F = A'B'(D'E' + C'DE') + A'B(C'DE' + C'D) + AB'(D'E' + C + C'D) + AB(C'DE')$

9.37 (b) $F = B'C'(D'E' + DE') + B'C(D'E' + A) + BC'(DE' + A'D) + BC(0)$

9.37 (c) $F = A'C'(B'D'E' + DE' + BD) + A'C(B'D'E') + AC'(B'D'E' + DE') + AC(B'D'E' + B')$

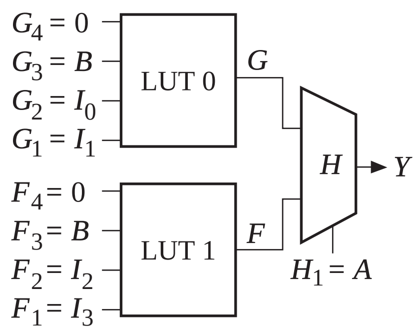
In this case, use the expansion about B and C to implement the function in 3 LUTs:
$F = B'C'(F_0) + B'C(F_1) + BC'(F_2) + BC(0)$
Here we use the LUTs to implement F_0, F_1, F_2 which are functions of A, D, E



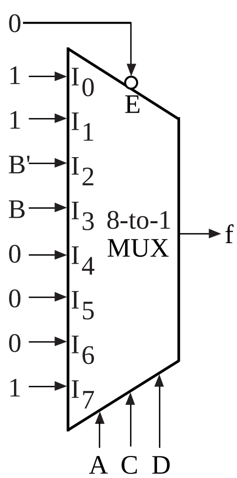
ADE	$F_0 F_1 F_2$
0 0 0	1 1 0
0 0 1	0 0 0
0 1 0	1 0 1
0 1 1	0 0 1
1 0 0	1 1 0
1 0 1	0 1 0
1 1 0	1 1 1
1 1 1	0 1 0

9.38 For a 4-to-1 MUX:
$Y = A'B'I_0 + A'BI_1 + AB'I_2 + ABI_3$
$= A'(B'I_0 + BI_1) + A(B'I_2 + BI_3)$
$= A'G + AF$, where $G = B'I_0 + BI_1$; $F = B'I_2 + BI_3$

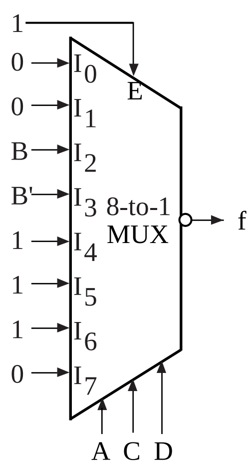
Set programmable MUX so that Y is the output of MUX H.



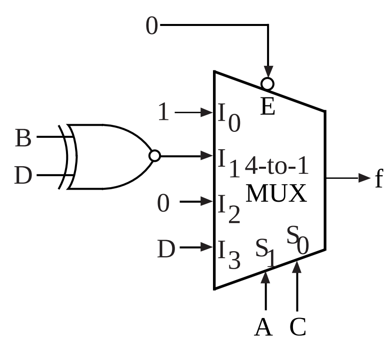
9.39 (a)



9.39 (b)



9.39 (c)

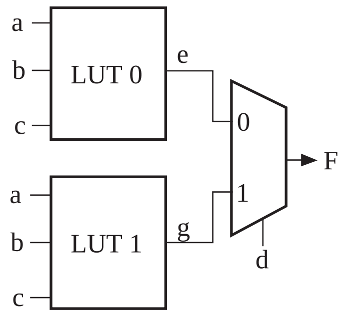


9.40 Same answer as 9.39 except connect E to the enable input in parts (a) and (c) and E' in part (b).

Unit 9 Solutions

9.41 (a) $F = a' + ac'd' + b'cd' + ad$
$= d'(a' + ac' + b'c) + d(a' + a)$
$= d'(a' + ac' + b'c) + d(1)$
$= d'(e) + d(g)$

9.41 (b)



9.41 (c)

a b c	*e g*
0 0 0	1 1
0 0 1	1 1
0 1 0	1 1
0 1 1	1 1
1 0 0	1 1
1 0 1	1 1
1 1 0	1 1
1 1 1	0 1

9.42 (a) $F = cd' + ad' + a'b'cd' + bc'$
$= d'(c + a + bc') + d(a'b'c + bc')$
$= d'(e) + d(g)$

9.42 (b) Same as 9.41 (b).

9.42 (c)

a b c	*e g*
0 0 0	0 0
0 0 1	1 1
0 1 0	1 1
0 1 1	1 0
1 0 0	1 0
1 0 1	1 0
1 1 0	1 1
1 1 1	1 0

9.43 (a) $F = bd + bc' + ac'd + a'd'$
$= d'(bc' + a') + d(b + bc' + ac')$
$= d'(bc' + a') + d(b + ac')$
$= d'(e) + d(g)$

9.43 (b) Same as 9.41 (b).

9.43 (c)

a b c	*e g*
0 0 0	1 0
0 0 1	1 0
0 1 0	1 1
0 1 1	1 1
1 0 0	0 1
1 0 1	0 0
1 1 0	1 1
1 1 1	0 1