CSE1003
Digital Logic and Design

Module 2
BOOLEAN ALGEBRA L6

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Module 2
BOOLEAN ALGEBRA 8 hrs

Boolean algebra

- Properties of Boolean algebra
- Boolean functions
- Canonical and Standard forms
- Logic gates - Universal gates
- Karnaugh map - Don’t care conditions
- Tabulation Method

Plotting a Truth Table on K-map

A truth table and corresponding SOP K-map

Inputs			Output
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

AB \ C		$\bar C$ 0	C 1
$\bar A \bar B$	00	1	1
$\bar A B$	01		1
AB	11		1
$A\bar B$	10		1

$\rightarrow \bar A \bar B$

$\rightarrow C$

$f = \bar A \bar B + C$

Represent the following Boolean function by K-map:

$$F(A,B,C,D) = ABC + \bar B CD + BD \rightarrow \text{non standard form}$$

$$F(A,B,C,D) = ABC(D+\bar D) + \bar B CD(A+\bar A) + BD(A+\bar A)(C+\bar C)$$

$$= ABCD + ABC\bar D + A\bar B CD + \bar A \bar B CD + ABCD$$

$$+ \bar A BCD + AB\bar C D + \bar A B \bar C D$$

standard form

AB \ CD	$\bar C \bar D$	$\bar C D$	CD	$C\bar D$
$\bar A \bar B$			1	
$\bar A B$		1	1	
AB		1	1	1
$A \bar B$			1	

$f = CD + BD + ABC$

K-map For POS Expression

A truth table and corresponding POS K-map

Inputs			Output
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

AB \ C	0	1
00	0	0
01		0
11	0	
10		

$\bar{A}$ 1 $\bar{B}$

A 0 B

Represent the following Boolean expression by K-map:

$$Y(A,B,C,D) = (A+B+\bar{C})(\bar{A}+C+\bar{D})$$ — non standard form

$$Y(A,B,C,D) = (A+B+\bar{C}+D\bar{D})(\bar{A}+C+\bar{D}+B\bar{B})$$
$$= (A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})(\bar{A}+B+C+\bar{D})$$

$(\bar{A}+C+\bar{D}+\bar{B})$

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00			0	0
$A+\bar{B}$ 01				
$\bar{A}+\bar{B}$ 11		0		
$\bar{A}+B$ 10		0		

$$Y(A,B,C,D) = (\bar{A}+C+\bar{D})(A+B+\bar{C})$$

Using K-Map

Minimize the following expression in the POS form

$$F(A,B,C,D) = (\overline{A}+\overline{B}+C+D)(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})(\overline{A}+B+C+D)(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})(A+B+C+D)(\overline{A}+\overline{B}+C+\overline{D})$$

$\downarrow$

1101

- 1100 $\overline{A}+\overline{B}+C+D$
- 1110 $\overline{A}+\overline{B}+\overline{C}+D$
- 1111 $\overline{A}+\overline{B}+\overline{C}+\overline{D}$
- 1000 $\overline{A}+B+C+D$
- 0110 $A+\overline{B}+\overline{C}+D$
- 0111 $A+\overline{B}+\overline{C}+\overline{D}$
- 0000 $A+B+C+D$

AB \ CD	00	01	11	10
00	0			
01			0	0
11	0	0	0	0
10	0			

$$F(A,B,C,D) = (B+C+D)(\overline{B}+\overline{C})(\overline{A}+\overline{B})$$

Don't-Care Conditions

- Some logic circuits can be designed so that there are certain input conditions for which there are no specified output levels, usually because these input conditions will never occur.
- There will be certain combinations of input levels where we "don't care" whether the output is HIGH or LOW.
- A circuit designer is free to make the output for any don't-care condition either a 0 or a 1 to produce the simplest output expression.

A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	"don't
1	0	0	x	care"
1	0	1	1	
1	1	0	1	
1	1	1	1	

AB\C	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	x
AB	1	1
$A\bar{B}$	x 1	1

AB\C	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	0
AB	1	1
$A\bar{B}$	1	1

$\rightarrow$ $z = A$

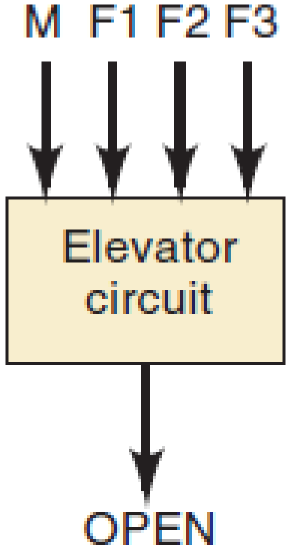
SOP – Replace 'x' by 1

POS – Replace 'x' by 0

Example for Don’t-Care Conditions

Design a logic circuit that controls an elevator door in a three-storey building. The circuit in Figure has four inputs. *M* is a logic signal that indicates when the elevator is moving (*M* = 1) or stopped (*M* = 0). *F1*, *F2*, and *F3* are floor indicator signals that are normally LOW, and they go HIGH only when the elevator is positioned at the level of that particular floor. For example, when the elevator is lined up level with the second floor, *F2*= 1 and *F1*= *F3*= 0. The circuit output is the *OPEN* signal, which is normally LOW and will go HIGH when the elevator door is to be opened.

M=1 elevator moving
M=0 " stopped



M	F1	F2	F3	OPEN
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	0
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

$MF_1 \backslash F_2F_3$	$\overline{F2}\,\overline{F3}$	$\overline{F2}\,F3$	$F2\,F3$	$F2\,\overline{F3}$
$\overline{M}\,\overline{F1}$	0	1	X 1	1
$\overline{M}\,F1$	1	X 1	X 1	X 1
$M\,F1$	0	X	X	X
$M\,\overline{F1}$	0	0	X	0

$MF_1 \backslash F_2F_3$	$\overline{F2}\,\overline{F3}$	$\overline{F2}\,F3$	$F2\,F3$	$F2\,\overline{F3}$
$\overline{M}\,\overline{F1}$	0	1	1	1
$\overline{M}\,F1$	1	1	1	1
$M\,F1$	0	0	0	0
$M\,\overline{F1}$	0	0	0	0

$$OPEN = \overline{M}\,(F1 + F2 + F3)$$

K-Map

Advantages:

- K mapping is a more orderly process with well-defined steps compared with the trial-and-error process sometimes used in algebraic simplification.
- K mapping usually requires fewer steps, especially for expressions containing many terms, and it always produces a minimum expression.

Limitations:

- K-maps are not suitable when the number of variables involved exceed four.
- It may be used with difficulty up to five and six variable systems. But, beyond ‘six variables’ K-maps cannot be physically visualized.
- K-map simplification is a manual technique and simplification process is heavily dependent on the abilities of the designer. It cannot be programmed.

Map the following expression on a K-Map.

$$F = \bar{A} + A\bar{B} + AB\bar{C}$$

AB \ C	0	1
00	1	1
01	1	1
11	1	
10	1	1

I term : $\bar{A}$ → BC are not given
They may take values
as 00, 01, 10, 11

for $\bar{A}$ → possible combinations
for which F can be 1

$$\left.\begin{array}{l} 000 \\ 001 \\ 010 \\ 011 \end{array}\right\}$$ place a '1' in the respective cells

II term $A\bar{B}$ → C can be either '0' or 1
possible combinations : 100 or 101

Determine the product terms for the K-map in fig & write the resulting minimum SOP expression.

AB \ C	0	1
00	1	
01		1
11	1	1
10		

$$f = \bar{A}\bar{B}\bar{C} + BC + AB$$

Determine the product terms for the K-Map in fig & Write the resulting minimum SOP expression.

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

$$f = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}\bar{C} + A\bar{B}C$$

Minimum SOP expression

$$f = \bar{B} + AC + \bar{A}\bar{C}$$

Use a K-Map to minimize the following SOP expression.

$$F = A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

AB \ C	0	1
00	1	1
01		1
11		
10	1	1

$$f = \bar{B} + \bar{A}C$$

Practice problems

① Obtain the simplified Boolean expression from the truth table in the SOP form.

AB \ C	0	1
00	0	0
01	0	0
11	1	1
10	1	1

$$f = A$$

② Obtain the standard SOP expression and simplified SOP expression from the K-Map.

AB \ C	0	1
00	1	1
01	1	1
11		
10		1

(a) $f = \bar{A} + \bar{B}C$

AB \ C	0	1
00	1	1
01		
11	1	
10	1	

(b) $f = \bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C}$

(3) Simplify the logic function F in the following cases :

a) $F(A, B, C) = \sum m(1, 3, 4, 7)$

AB \ C	0	1
00		1
01		1
11		1
10	1	

$$f = A\bar{B}\bar{C} + BC + \bar{A}C$$

$f(A, B, C) = \Sigma m\ (0, 1, 3, 5, 7)$ minimize this by K map.

Ans $f(A, B, C) = C + \overline{A}\,\overline{B}$