# Department of Mathematics
# School of Advanced Sciences
# MAT 1011 – Calculus for Engineers (MATLAB)
# Experiment 4–B
# Triple Integrals

Triple integrals enable us to solve more general problems such as to calculate the volumes of three – dimensional shapes, the masses and moments of solids of varying density, and the average value of a function over a three –dimensional region.

The triple integral of $f(x, y, z)$ over the region $D$ is given by

$$\iiint_D f(x, y, z) dV$$

where the region $D$ is bounded by the surfaces $x = a$, $x = b$, $y = \psi_1(x)$ to $y = \psi_2(x)$, $z = \phi_1(x, y)$ to $z = \phi_2(x, y)$.

Hence

$$\iiint_D f(x, y, z) dV = \int_a^b \int_{\psi_1(x)}^{\psi_2(x)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z) dz dy dx.$$

Similarly when the region $D$ is bounded by the surfaces $y = c$, $y = d$, $x = \psi_1(y)$ to $x = \psi_2(y)$, $z = \phi_1(x, y)$ to $z = \phi_2(x, y)$.

Hence

$$\iiint_D f(x, y, z) dV = \int_a^b \int_{\psi_1(y)}^{\psi_2(y)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z) dz dx dy.$$

**Volume using Triple Integral**

The volume of a closed, bounded region $D$ in space is given by

$$V = \iiint_D dV$$

**Syntax for evaluation of triple integral:**

```
int(int(int(f,z,za,zb),y,ya,yb),x,xa,xb)
```

or

```
I=int(int(int(f,z,za,zb),x,xa,xb),y,ya,yb)
```

**Syntax for visualization of region bounded by the limits of integration:**

```
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)
```

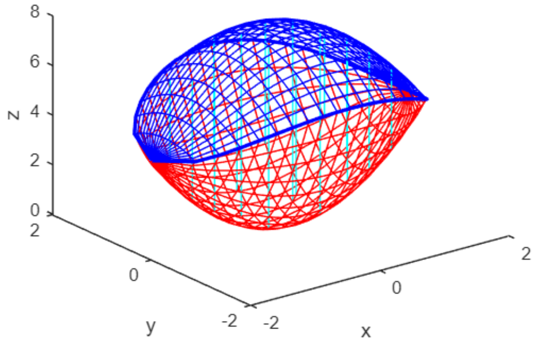
```
viewSolidone(z,za,zb,xa,xb,y,ya,yb)
```

**Example 1.** Find the volume of the region $D$ enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

```
clear
clc
syms x y z
xa=-2;
xb=2;
ya=-sqrt(2-x^2/2);
yb=sqrt(2-x^2/2);
za=x^2+3*y^2;
zb=8-x^2-y^2;
I=int(int(int(1+0*z,z,za,zb),y,ya,yb),x,xa,xb)
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)
```

**Output**

```
I =
8*pi*2^(1/2)
```



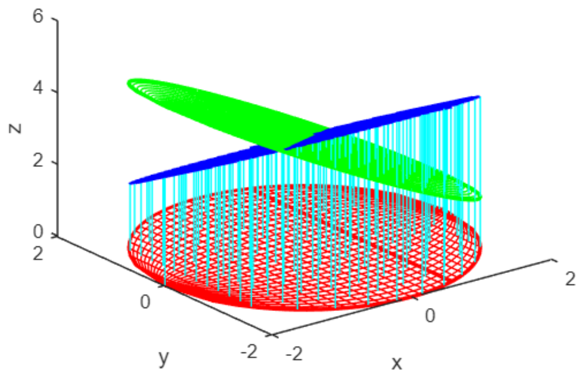

**Example 2.** Find the volume of the region cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$.

The limits of integration are $z = 0$ to $3 - x$, $x = -\sqrt{4-y}$ to $\sqrt{4-y}$, $y = -2$ to $2$.

```
clear
clc
syms x y z
ya=-2;
yb=2;
xa=-sqrt(4-y^2);
xb=sqrt(4-y^2);
za=0+0*x+0*y;
zb=3-x-0*y;
I=int(int(int(1+0*z,z,za,zb),x,xa,xb),y,ya,yb)
viewSolidone(z,za,zb,x,xa,xb,y,ya,yb)
```

**Output**

```
I =
12*pi
```



**Example 3.** Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2)$, $0 \le x \le 1$.

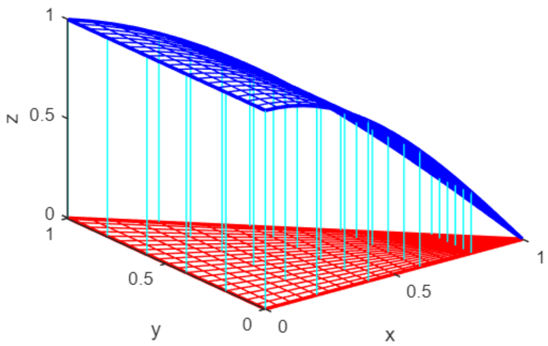
The limits of integration are $z = 0$ to $\cos(\pi x/2)$, $y = 0$ to $1 - x$, $x = 0$ to $1$.

```
clear
clc
syms x y z real
xa=0;
xb=1;
ya=0+0*x;
yb=1-x;
za=0*x+0*y;
zb=cos(pi*x/2)+0*y;
I=int(int(int(1+0*z,z,za,zb),y,ya,yb),x,xa,xb)
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)
```

**Output.**

```
I =

4/pi^2
```




**Exercise.**

1. Find the volume of the region bounded between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.
2. Find the volume of the region cut from the solid elliptical cylinder $x^2 + 4y^2 \le 4$ by the $xy-$ plane and the plane $z = x + 2$.
3. The finite region bounded by the planes $z = x$, $x + z = 8$, $z = y$, $y = 8$ and $z = 0$.