Table 10.3 The ANOVA table for three factors of **classification**

S.V.	*S.S.*	*d.f.*	*M.S.*	*F*
Between rows	Q_1	$n-1$	$Q_1/(n-1)=M_1$	$\left(\dfrac{M_1}{M_4}\right)^{\pm 1}$
Between columns	Q_2	$n-1$	$Q_2/(n-1)=M_2$	$\left(\dfrac{M_2}{M_4}\right)^{\pm 1}$
Between letters	Q_3	$n-1$	$Q_3/(n-1)=M_3$	$\left(\dfrac{M_3}{M_4}\right)^{\pm 1}$
Residual	Q_4	$(n-1)(n-2)$	$Q_4/(n-1)(n-2)=M_4$	–
Total	Q	n^2-1	–	–

Note *The following working formulas may be used to compute the Q's:*

1. $Q=\sum\sum x_{ij}^2-\dfrac{T^2}{n^2}$, *where* $T=\sum\sum x_{ij}$

2. $Q_1=\dfrac{1}{n}\sum T_i^2-\dfrac{T^2}{n^2}$, *where* $T_i=\sum_{j=1}^{n} x_{ij}$

3. $Q_2=\dfrac{1}{n}\sum T_j^2-\dfrac{T^2}{n^2}$, *where* $T_j=\sum_{i=1}^{n} x_{ij}$

4. $Q_3=\dfrac{1}{n}\sum T_k^2-\dfrac{T^2}{n^2}$, *where* T_k *is the sum of all* x_{ij}'s *receiving the* k^{th} *treatment.*

5. $Q_4=Q-Q_1-Q_2-Q_3$.

Also $T=\sum_i T_i=\sum_j T_j=\sum_k T_k$

Analyse the variance in the following Latin square of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields.

We subtract 120 from the given values and work out with the new values of x_{ij}.

$i \backslash j$	1	2	3	4	T_i	T_i^2/n	$\sum_j x_{ij}^2$
1	D2	A1	C3	B2	8	16	18
2	B4	C3	A2	D5	14	49	54
3	A0	B-1	D0	C1	0	0	2
4	C2	D3	B1	A2	8	16	18
T_j	8	6	6	10	$T = 30$	$\sum T_i^2/n = 81$	92
T_j^2/n	16	9	9	25	$\sum T_j^2/n = 59$		
$\sum_i x_{ij}^2$	24	20	14	34	92		

Rearranging the data according to the letters, we have

Letter	x_k				T_k	T_k^2/n
A	1	2	0	2	5	6.25
B	2	4	- 1	1	6	9.00
C	3	3	1	2	9	20.25
D	2	5	0	3	10	25.00
		Total			30	60.50

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 92 - \frac{30^2}{16} = 35.75$$

$$Q_1 = \frac{1}{n}\sum T_i^2 - \frac{T^2}{N} = 81 - 56.25 = 24.75$$

$$Q_2 = \frac{1}{n}\sum T_j^2 - \frac{T^2}{N} = 59 - 56.25 = 2.75$$

$$Q_3 = \frac{1}{n}\sum T_k^2 - \frac{T^2}{N} = 60.50 - 56.25 = 4.25$$

$$Q_4 = Q - Q_1 - Q_2 - Q_3 = 35.75 - (24.75 + 2.75 + 4.25)$$

$$= 4.0$$

ANOVA table

S.V.	*S.S.*	*d.f.*	*M.S.*	F_0
Between rows	$Q_1 = 24.75$	$n - 1 = 3$	8.25	
Between columns	$Q_2 = 2.75$	$n - 1 = 3$	0.92	
Between letters	$Q_3 = 4.25$	$n - 1 = 3$	1.42	$\dfrac{1.42}{0.67} = 2.12$
Residual	$Q_4 = 4.0$	$(n - 1)(n - 2) = 6$	0.67	–
Total	$Q = 35.75$	$n^2 - 1 = 15$	–	–

From the F-tables, $F_{5\%}\ (v_1 = 3, v_2 = 6) = 4.76$.
Since $F_0 (= 2.12) < F_{5\%}\ (= 4.76)$ with respect to the letters, the difference between the methods of cultivation is not significant.

The following data resulted from an experiment to compare three burners B_1, B_2 and B_3. A Latin square design was used as the tests were made on 3 engines and were spread over 3 days.

	Engine 1	Engine 2	Engine 3
Day 1	$B_1 - 16$	$B_2 - 17$	$B_3 - 20$
Day 2	$B_2 - 16$	$B_3 - 21$	$B_1 - 15$
Day 3	$B_3 - 15$	$B_1 - 12$	$B_2 - 13$

Test the hypothesis that there is no difference between the burners.

We subtract 16 from the given values and work out with new values of x_{ij}.

	E_1	E_2	E_3	T_i	$\frac{T_i^2}{n}$	$\sum_j x_{ij}^2$
D_1	$0(B_1)$	$1(B_2)$	$4(B_3)$	5	8.33	17
D_2	$0(B_2)$	$5(B_3)$	$-1(B_1)$	4	5.33	26
D_3	$-1(B_3)$	$-4(B_1)$	$-3(B_2)$	-8	21.33	26
T_j	-1	2	0	$T = 1$	$\sum T_i^2/n = 35$	69
T_j^2/n	0.33	1.33	0	$\sum T_j^2/n = 1.66$		
$\sum_i x_{ij}^2$	1	42	26	69		

Rearranging the data values according to the burners, we have

Burner		x_k		T_k	T_k^2/n
B_1	0	-1	-4	-5	8.33
B_2	1	0	-3	-2	1.33
B_3	4	5	-1	8	21.33
		Total		$T = 1$	$\Sigma \frac{T_k^2}{n} = 31$

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 69 - \frac{1}{9} = 68.89$$

$$Q_1 = \frac{1}{n}\sum T_i^2 - \frac{T^2}{N} = 35 - \frac{1}{9} = 34.89$$

$$Q_2 = \frac{1}{n}\sum T_j^2 - \frac{T^2}{N} = 1.67 - \frac{1}{9} = 1.56$$

$$Q_3 = \frac{1}{n}\sum T_k^2 - \frac{T^2}{N} = 31 - \frac{1}{9} = 30.89$$

$$Q_4 = Q - Q_1 - Q_2 - Q_3 = 1.55$$

ANOVA table

S.V.	S.S.	d.f.	M.S.	F_0
Between rows (days)	$Q_1 = 34.89$	$n - 1 = 2$	17.445	$\frac{17.445}{0.775} = 22.51$
Between Cols. (engines)	$Q_2 = 1.56$	$n - 1 = 2$	0.780	$\frac{0.780}{0.775} = 1.01$
Between letters (burners)	$Q_3 = 30.89$	$n - 1 = 2$	15.445	$\frac{15.445}{0.775} = 19.93$
Residual	$Q_4 = 1.55$	$(n - 1)(n - 2) = 2$	0.775	–
Total	$Q = 68.89$	$n^2 - 1 = 8$	–	–

From the F-tables, $F_{5\%}$ $(v_1 = 2, v_2 = 2) = 19.00$
Since $F_0 (= 19.93) > F_{5\%}$ $(= 19.00)$ for the burners, there is significant difference between the burners.

Incidentally, since $F_0 > F_{5\%}$ for the rows, the difference between the days is significant and since $F_0 < F_{5\%}$ for the columns, the difference between the engine is not significant.