Electromagnetic Wave Equation :-

Let us apply Maxwell's electromagnetic equs. to a homogeneous, isotropic dielectric medium. As the dielectric medium is one which offers infinite resistance to the current, and hence its conductivity $j=0$, In homogeneous isotropic medium there is no volume distribution of charge, thus the charge density is $\rho = 0$. Hence,

$$j=0, \quad \rho=0, \quad D = k\varepsilon_0 E = \varepsilon E \text{ and } B = \mu_0\mu_r H = \mu H$$

Hence Maxwell's equations for a dielectric medium become

$$\nabla \cdot E = 0 \quad \text{——} \; (1)$$

$$\nabla \cdot B = 0 \quad \text{——} \; (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{——} \; (3)$$

$$\nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t} \quad \text{——} \; (4)$$

For obtaining the eqn. of propagation of wave in dielectric medium, 'E' should be eleminated from eqns (3) & (4).

Taking curl of eqn. (4),

$$\nabla \times \nabla \times B = \nabla \times \mu\varepsilon \frac{\partial E}{\partial t}$$

$$= \mu\varepsilon\left(\nabla \times \frac{\partial E}{\partial t}\right)$$

$$= \mu\varepsilon \frac{\partial}{\partial t}(\nabla \times E) \qquad \text{from eqn (3)}$$

$$= \mu\varepsilon \frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla \times \nabla \times B = -\mu\varepsilon \frac{\partial^2 B}{\partial t^2} \quad \text{——} \; (5)$$

$$\nabla(\nabla \cdot B) - \nabla^2 B = -\mu\varepsilon \frac{\partial^2 B}{\partial t^2}$$

$$0 - \nabla^2 B = -\mu\varepsilon \frac{\partial^2 B}{\partial t^2} \qquad \text{from eqn (2)}$$

$$\therefore \nabla^2 B = \mu\varepsilon \frac{\partial^2 B}{\partial t^2} \quad \text{———} \quad (6)$$

||ly from eqn (3), we can show that

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{———} \quad (7)$$

Eqns. (6) and (7) represent the relation betn the space and time variation of magnetic 'B' & electric field 'E'. These are called wave eqn for B and E respectively. These eqns. have the same general form of the differential eqn. of wave motion. The general wave eqn. is represented by

$$\nabla^2 y = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{———} \quad (8)$$

where 'v' is the velocity of wave and y is it amplitude. Comparing eqns (7) & (8), the factor $\mu\varepsilon$ has the same significance as $1/v^2$.

So we find that the variations of E and B are propogated in homogeneous, isotropic medium with a velocity given by

$$\frac{1}{v^2} = \mu\varepsilon \quad \text{or} \quad v^2 = \frac{1}{\mu\varepsilon}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}} \quad \text{———} \quad (9)$$

where μ and ε are permeability and permittivity of the medium.

for free space $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi} \times 9 \times 10^{9}}}$

$$v = 3 \times 10^{8} \text{ m/sec.}$$

Eqns. (6) & (7) involve periodic variations of electric & magnetic fields. So they are called electromagnetic waves. In this wave Maxwell predicted the propogation of electromagnetic waves in three dimensions and prove that they travel with the velocity of light.