4.16.4 Computation of Mode in a Continuous Frequency Distribution (Or Method of Interpolation)

(*i*) **Modal class.** It is that **class in a grouped frequency distribution in which the mode lies.** The modal class can be determined either by inspection or with the help of grouping table. After finding the modal class, we calculate the mode by the following formula:

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i,$$

where l = **the lower limit of the modal class**

i = **the width of the modal class**

f_1 = **the frequency of the class preceding the modal class**

f_m = **the frequency of the modal class**

f_2 = **the frequency of the class succeeding the modal class.**

Sometimes, it so happened that the above formula fails to give the mode. In this case, **the modal value lies in a class other than the one containing maximum frequency.** In such cases, we take the help of the following formula:

$$\text{Mode} = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i \text{, where } \Delta_1 = f_m - f_1,\ \Delta_2 = f_m - f_2,$$

where l, f_1, f_2, f_m, and l have usual meanings.

The precedure of finding the mode by the above method is called **Method of Interpolation.**

Example 43. *Find the mode for the following data:*

Marks	*1 – 5*	*6 – 10*	*11 – 15*	*16 – 20*	*21 – 25*
No. of student	*7*	*10*	*16*	*32*	*24*

Solution. From the above table, it is clear that the maximum frequency is **32** and it lies in the **class** 16 – 20. Thus, the **modal class is 16 – 20.**

Here $l = 16,\ f_m = 32,\ f_1 = 16,\ f_2 = 24,\ i = 5.$

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_2 - f_1} \times i = 16 + \frac{32 - 16}{64 - 24 - 16} \times 5$$

$$= 16 + \frac{16}{24} \times 5 = 16 + \frac{10}{3} = 16 + 3.33 = \mathbf{19.33.}$$

Example 44. *Calculate Median and Mode for the following distribution:*

Production per day (in Tons)	*21 – 22*	*23 – 24*	*25 – 26*	*27 – 28*	*29 – 30*
No. of day	*7*	*13*	*22*	*10*	*8*

Solution. We shall first convert the given date into a continuous series.

Table: Calculations of Median and Mode

Class-boundaries	*No. of days (f)*	*Cumulative Frequency (c.f.)*
20.5 – 22.5	7	7
22.5 – 24.5	13	20
24.5 – 26.5	22	42
26.5 – 28.5	10	52
28.5 – 30.5	8	60
Total	$N = \Sigma f = 60$	

Since $\frac{1}{2}N = 30 \Rightarrow$ Median lies in the class **24.5 – 26.5.**

$\therefore$ **Median** $= 24.5 + \frac{30-20}{22} \times 2 = 24.5 + 0.91 =$ **25.41 tonnes.**

Since the maximum frequency is 20 so the modal class is **24.5 – 20.5.**

$$\textbf{Mode} = l + \frac{f_m - f_1}{2f_m - f_2 - f_1} \times i = 24.5 + \frac{22-13}{2\times 22 - 13 - 10} \times 2 = \textbf{25.36 tonnes.}$$

4.17 MERITS, DEMERITS AND USES OF MODE

Merits:

1. *It can be easily understood.*
2. *It can be located in some cases by inspection.*
3. *It is capable of being ascertained graphically.*
4. *It is not affected by extreme values.*
5. *It represents the most frequent value and hence it is very often in practice.*
6. *The arrangement of data is not necessary if the items are a few.*

Demerits:

1. *There are different formulae for its calculations which ordinarily give different answers.*
2. *Mode is determinate. Some series have two or more than two modes.*
3. *It cannot be subjected to algebraic treatments. For example, the combined mode cannot be calculated for the modes of two series.*
4. *It is an unsuitable measure as it is affected more by sampling fluctuations.*
5. *Mode for the series with unequal class-intervals cannot be calculated.*

Uses:

1. *It is used for the study of most popular fashion.*
2. *It is extensively used by businessmen and commercial managements.*

4.18 GRAPHICAL REPRESENTATION OF MODE

Graphically the value of mode can be determined with the help of ***Histogram.*** **A histogram** is a group of rectangles based on ***'class intervals'*** and **frequencies,** where the length of rectangle is proportion to the frequencies and width of the rectangle is proportion to the magnitude of class intervals. The basic condition for this is that: **'the class intervals must be equal'.**

Steps of Construction:

Step 1: Draw a histogram of the given data.

Step 2: Inside the rectangle which represents the modal class or maximum frequency, draw two lines diagonally from the upper corners of the bar to the upper corners of two adjacent bars on either side of the rectangle representing maximum frequency.

Step 3: From the point P, where the two diagonals cut each other, draw a perpendicular PM on x-axis meeting it at M. This point M on the horizontal scale determines the value of modes as shown in the figure.

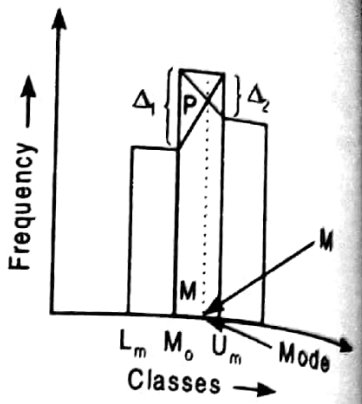

Fig. 4.1.

Example 45. *The monthly profits (in Rs.) of 100 shops are distributed as follows :*

Profit per Shop :	*0 – 100*	*100 – 200*	*200 – 300*	*300 – 400*	*400 – 500*	*500 – 600*
No. of Shops :	*12*	*18*	*27*	*20*	*17*	*6*

Determine the modal value of the distribution graphically and verify the result by calculation.

Solution. The given distribution is regular, so that modal class would be a class having the highest frequency. The modal class, of the given distribution is: 200 – 300.

Graphical Location of Mode

To locate mode we will first draw a histogram of the given frequency distribution. The mode is located at **256** as shown in the figure.

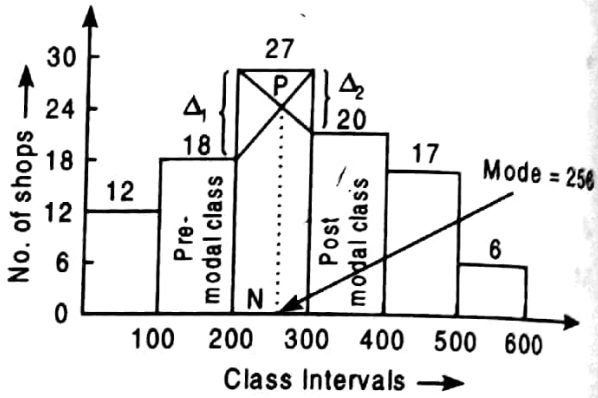

Fig. 4.2.

Draw two lines diagonally from the upper corners of the bar with maximum frequencies, to the upper corners of the two adjacent bars on either side of the rectangle representing maximum frequency meeting each other at P. From P draw PN perpendicular to x–axis meeting it at N. The point $N = 256$, where PN touches the base line is the Mode. **Mode = Rs. 256.**

Determination of Mode by interpolation formula.

Here the modal class is: 200 – 300, $L = 200$, $\Delta_1 = (f_m - f_1) = -27 - 18 = 9$,

$\Delta_2 = (f_m - f_2) = 27 - 20 = 7$ and $h = 100$.

$$\therefore \textbf{ Mode : } \quad M_0 = L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) \times h$$

or

$$M_0 = 200 + \frac{9}{9+7} \times 100 = \textbf{Rs. 256.25}$$

4.19 EMPIRICAL RELATION BETWEEN MEAN, MEDIAN AND MODE

A distribution in which **mean, median and mode coincide is called asymmetrical distribution.** If the distribution is **moderately asymmetrical,** then mean, median and mode are connected by the formula:

$$\textbf{Mode} = 3\,\textbf{Median} - 2\,\textbf{Mean}$$

or

$$\textbf{Median} = \frac{1}{3}\,(\textbf{Mode} + 2\,\textbf{Mean})$$