ALGEBRA - I

Algebra

1. The equation of the form ax + b = 0 is a linear equation in one variable

2. The equations of the form ax + by + c = 0 where a, b ≠ 0 is called the linear equation in two variables x and y

3. Simultaneous equations are a pair of equations of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. The solution of these equations is (x, y) an ordered pair.

4. The set of three equations of the form $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$ is called the system of linear equations in three variables

5. An equation of the form $ax^2 + bx + c = 0, a \neq 0$ is called a quadratic equation. Here x is the variable and a, b, c $\in$ R are the constants.

a, b, c are called the coefficients of the equation.

6. The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The sum of the roots $= -\frac{b}{a}$, the product of the roots $= \frac{c}{a}$.

7. The roots of the quadratic equation depends on $b^2 - 4ac$ which is called the discriminant of the equation.

(i) The equation has real and distinct roots when $b^2 - 4ac > 0$

(ii) The equation has real and equal roots when $b^2 - 4ac = 0$

(iii) The equation has complex roots when $b^2 - 4ac < 0$

(iv) If $b^2 - 4ac$ is a perfect square the equation has two rational roots.

8. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$. The quadratic equation whose roots are α and β is $(x - \alpha)(x - \beta) = 0$.

1. If $7a + 8b = 53$ and $9a + 5b = 47$, find the values of a and b.

(a) (4, 5) (b) (4, 3) (c) (3, 4) (d) (5, 4)

2. If I add 7 times my age seven years from now and 3 times my age three years ago, I get 12 times my current age. How old will I be 3 years from now?

(a) 22 years (b) 23 years

(c) 24 years (d) 25 years

3. A farmer has some chickens and feed for them. The feed lasts for exactly 30 days. He sold 10 of the chickens and purchased some feed such that the entire feed with him will last for 150 days. Now, he has thrice the initial feed. Find the initial number of chickens.

(a) 25 (b) 30

(c) 40 (d) Cannot be determined

4. The cost of 4 chocolates, 6 biscuits and 12 lolly-pops is Rs.36. The cost of 15 biscuits, 9 lolly-pops and 3 chocolates is Rs.48. Find the cost of 1 biscuit.

(a) Rs.4 (b) Rs.3 (c) Rs.2 (d) Rs.1

5. Tree I grows at $3/7^{th}$ of tree II. If both the trees together grow 3 ft for every 3 years, find the time required by tree II to grow 7 ft.

(a) 7 years (b) 10 years (c) 8 years (d) 12 years

6. Find the quadratic equation whose roots are the reciprocals of the roots of the equation $x^2 - 7x + 12 = 0$.

(a) $x^2 - 12x + 7 = 0$ (b) $x^2 + 12x - 7 = 0$

(c) $12x^2 + 7x - 1 = 0$ (d) $12x^2 - 7x + 1 = 0$

7. The equation $\sqrt{4x + 9} - \sqrt{11x + 1} - \sqrt{7x + 4} = 0$ has

(a) no solution (b) 1 solution

(c) 2 solutions (d) more than 2 solutions

8. The equation $(\cos p - 1)\, x^2 + (\cos p)x + \sin p = 0$ has real roots if p lies in the interval

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

9. The set of values of p for which the roots of the equation $5x^2 + 4x + p(p - 2) = 0$ are of opposite sign is

(a) $(-\infty, 0)$ (b) $(0, 2)$

(c) $(0, \infty)$ (d) None of these

10. If α, β are the roots of the equation $(x - a)(x - b) = c$ with $c \neq 0$, find the roots of the equation $(x - \alpha)(x - \beta) + c = 0$.

(a) a, c (b) b, c

(c) a, b (d) a + c, b + c

11. If p and q are the roots of $x^2 - 2x + A = 0$ and r and s are the roots of $x^2 - 18x + B = 0$ and $p < q < r < s$ are in A.P., find (A, B).

(a) (-3, 77) (b) (77, -3)

(c) (-3, -77) (d) None of these

12. The sum of a fraction and its reciprocal equals 85/18. Find the fraction.

(a) 2/6 (b) 2/3 (c) 2/9 (d) 4/9

13. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then

(a) ab^2, ca^2, bc^2 are in A.P.

(b) ab^2, bc^2, ca^2 are in A.P.

(c) ab^2, bc^2, ac^2 are in A.P.

(d) None of these

14. Let a, b, c be $\in R$ and $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(a) $\gamma - \frac{1}{2}(\alpha + \beta)$

(b) $\gamma = \alpha + \frac{\beta}{2}$

(c) $\gamma = \alpha + \beta$

(d) $\alpha < \gamma < \beta$

15. If α, β are the roots of $ax^2 + bx + c = 0$, then the roots of the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ are

(a) $\alpha - 1, \beta - 2$

(b) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$

(c) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$

(d) None of these

16. If for the quadratic equation $x^2 - kx + 1 = 0$, one of the roots is A such that $\tan A = 2\sqrt{5} - 1$, then the other root is

(a) greater than 1 (b) greater than 2

(c) less than 1 (d) None of these

17. If p and q are the roots of the equation $x^2 + px + q = 0$, then

(a) p = 1 (b) p = 1 or 0

(c) p = -2 (d) p = -2 or 0

18. If a, b, c are positive real numbers which are in G.P., then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in

(a) A.P.
(b) G.P.
(c) H.P.
(d) None of these

19. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$, the value of (a – c) (b – c) (a + d) (b + d) is

(a) $p^2 - q^2$

(b) $q^2 - p^2$

(c) $q^2 + p^2$

(d) None of these

20. Find the value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other.

(a) -2/3 (b) 1/3

(c) -1/3 (d) 2/3