Binomial Distribution

Problem

① Two dice are thrown 120 times. Find the average number of times in which the number on the first die exceeds the number on the second die.

Solution:- The number on the first die exceeds that ~~the~~ on the second die, in the following combinations:

(2,1), (3,1), (4,1), (5,1), (6,1), (3,2), (4,2), (5,2)
(6,2) (4,3), (5,3), (6,3) (5,4), (6,4), (6,5)

where the numbers in the parentheses represent the numbers in the first and second die respectively.

P(Success) = P(Number in the first die exceeds the number in the second die)

$$= \frac{15}{36} = \frac{5}{12}$$

If x is the number of successes, then x follows a binomial distribution with $n=120$ and $p=\frac{5}{12}$

$$\text{Mean} = E(x) = np = 120 \times \frac{5}{12} = \underline{50}$$

②. It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing

i) atleast 2 defectives
ii) exactly 2 defectives
iii) at most 2 defectives items in a large consignment of 1000 packets using binomial distribution

Soln:- Let p = probability that an item is defective

$p = 0.05$

$q = 0.95$

n = Number of independent items (trials) considered $= 20$.

Let x denote the number of defectives in the n items considered.

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}.$$

$$P(X=x) = {}^{20}C_{x} (0.05)^{x} (0.95)^{20-x}$$

i) $$P(X=2) = {}^{20}C_{2} (0.05)^{2} (0.95)^{18} = 0.1887$$

If N is the number of sets (packets), each set (packet) containing 20 trials (items) then the number of sets containing exactly 2 successes is given by

$$N(X=2) = N \times P(X=2)$$
$$= 1000 \times 0.1887 = 189 \text{ (nearly)}$$

ii) $$P(\text{atleast 2 defectives}) = P(x \geq 2)$$
$$= 1 - P(X<2)$$
$$= 1 - (P(X=0) + P(X=1))$$
$$= 1 - \left({}^{20}C_{0} (0.05)^{0} (0.95)^{20} + {}^{20}C_{1} (0.05)(0.95)^{19} \right)$$
$$= 1 - (0.3585 + 0.3774)$$
$$= 0.2641$$

$$N(X \geq 2) = N \times P(X \geq 2) = 1000 \times 0.2641 = 264 \text{ (nearly)}$$

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Problem

Poisson Distribution

Fit a Poisson distribution for the following distribution

x:	0	1	2	3	4	5
f:	142	156	69	27	5	1

Soln:- Assume that the given distribution is approximately Poisson and hence find the probability mass function and then find the theoretical frequencies.

Probability mass function $P(X=x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}$, $x = 0,1,2,\ldots\infty$

Here λ is the mean of the Poisson distribution.

x:	0	1	2	3	4	5	Total
f:	142	156	69	27	5	1	400
fx:	0	156	138	81	20	5	400.

$$\text{mean} = \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{400}{400} = 1 = \lambda$$

$$P(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!} = \frac{e^{-1}}{x!}$$

Theoretical frequencies $= N \times P(x) = N \times \frac{e^{-\lambda}\cdot\lambda^x}{x!}$

$= N \times \frac{e^{-1}}{x!}$, $x = 0,1,2\ldots$

x:	0	1	2	3	4	5
f:	147.15	147.15	73.58	24.53	6.13	1.23

Converting the theoretical frequencies into whole numbers consistent with the condition that the total frequency = 400, we get

x:	0	1	2	3	4	5
f:	142	147	74	25	6	1

iii) $P(\text{atmost 2 defectives}) = P(X \le 2)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= 0.3585 + 0.3774 + 0.1887$$

$$= 0.9246$$

$$N(X \le 2) = N \times P(X \le 2)$$

$$= 1000 \times 0.9246$$

$$= 925 \text{ (nearly.)}$$

Problem:-

③. If the chance of running a bus service according to schedule is 0.8, calculate the probability on a day schedule with 10 services

i) exactly one is late

ii) atleast one is late.

Soln:- Probability of a bus running according to schedule $= 0.8 = q$.

probability that a bus is late is $p = 0.2$

$$P(x) = 10_{C_x} \, p^x \, q^{10-x}.$$

i) $P(X=1) = 10_{C_1} (0.2)(0.8)^9 = 2(0.8)^9$

ii) $P(X \ge 1) = 1 - P(X<1) = 1 - P(0)$

$$= 1 - (0.8)^{10}$$