Compton Shift

The incident monochromatic X-rays with frequency ν_0 are regarded as a stream of particles – the photons, with energy

$$E_0 = h\nu_0$$

The linear momentum p_0 of the photon can be obtained from the relativistic relation between energy and momentum.

$$E = \sqrt{p^2c^2 + m_0^2c^4} \qquad \text{(Relativistic relation between energy and momentum for any particle)}$$

where E = Energy, p = linear momentum, m_0 = rest mass of the particle and c = speed of light in vacuum.

Since the photon has zero rest mass ($m_0 = 0$), $E = pc$ (or) $p = \frac{E}{c}$. The linear momentum of the photon is

$$p_0 = \frac{E_0}{c} = \frac{h\nu_0}{c}$$

The scattering of X-rays by electrons is treated as elastic collision between photons and electrons which are particles. The electrons which are loosely bound can be treated as almost free particles at rest.

So, the initial linear momentum of the electrons is zero and the initial energy is equal to m_ec^2, where m_e is the rest mass of the electron. The Compton scattering of a photon by free electron at rest is shown in Figure 3

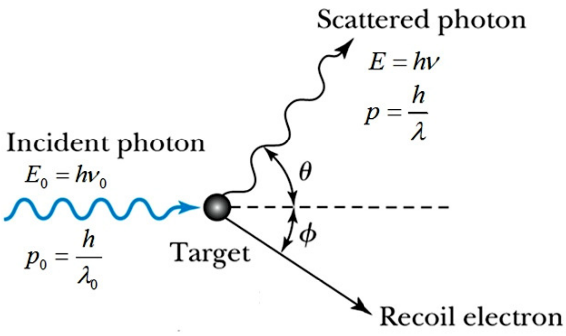

Figure 3. The quantum model for X-ray scattering from an electron. The collision of the photon with the electron displays the particle-like nature of the photon

The photon of energy $E_0 = h\nu_0$ and momentum $p_0 = \frac{h\nu_0}{c}$ hits free electron at rest. The scattered photon moves at an angle θ with direction of incident photon. If ν is the frequency of the scattered X-rays, it has energy $E = h\nu$ and linear momentum $p = \frac{h\nu}{c}$

The electron recoils at an angle ϕ, with the linear momentum p_e, and energy E_e.

The principle of conservation of energy and linear momentum can be applied to the collision process. The sum of the energy of the photon and electron before collision must be the same as the total energy after collision,

$$E_0 + m_ec^2 = E + E_e = E + \sqrt{p_e^2c^2 + m_e^2c^4}$$

The conservation of linear momentum gives

$$\vec{p}_0 + 0 = \vec{p} + \vec{p}_e \qquad \text{or} \qquad \vec{p}_e = \vec{p}_0 - \vec{p}$$

$$\text{or } p_e^2 = (\vec{p}_0 - \vec{p}) \bullet (\vec{p}_0 - \vec{p}) = p_0^2 + p^2 - 2p_0 \bullet p$$

$$= p_0^2 + p^2 - 2p_0 p \cos\theta$$

$$\left[(E_0 - E) + m_e c^2\right] = \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$\left[(E_0 - E) + m_e c^2\right]^2 = p_e^2 c^2 + m_e^2 c^4$$

$$(E_0 - E)^2 + m_e^2 c^4 + 2(E_0 - E) m_e c^2 = p_e^2 c^2 + m_e^2 c^4$$

$$(E_0 - E)^2 + 2(E_0 - E) m_e c^2 = p_e^2 c^2$$

$$= p_0^2 c^2 + p^2 c^2 - 2p_0 p c^2 \cos\theta$$

$$= E_0^2 + E^2 - 2E_0 E \cos\theta$$

$$= E_0^2 + E^2 - 2E_0 E + 2E_0 E - 2E_0 E \cos\theta$$

$$= (E_0 - E)^2 + 2E_0 E(1 - \cos\theta)$$

$$(E_0 - E) m_e c^2 = E_0 E(1 - \cos\theta)$$

$$\frac{1}{E} - \frac{1}{E_0} = \frac{(1 - \cos\theta)}{m_e c^2} \qquad \text{or} \qquad \frac{1}{\nu} - \frac{1}{\nu_0} = \frac{h(1 - \cos\theta)}{m_e c^2}$$

$$\lambda - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_c (1 - \cos\theta)$$

(where Compton wavelength, λ_c = 0.00243 nm)

$$\boxed{\Delta\lambda = \lambda_c (1 - \cos\theta)}$$