CSE1003
Digital Logic and Design

Module 2
BOOLEAN ALGEBRA L5

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Module 2
BOOLEAN ALGEBRA 8 hrs

Boolean algebra

- Properties of Boolean algebra
- Boolean functions
- Canonical and Standard forms
- Logic gates - Universal gates
- Karnaugh map - Don’t care conditions
- Tabulation Method

K-map method for simplifying a Boolean expression – step by step procedure

1. Construct the K-map as discussed. Enter 1 in those cells corresponding to the minterms for which function value is 1. Place 0's in other cells.

2. Form the groups of possible 1s as pair, quad and octet. There can be overlapping of groups if they include common cells. While doing this make sure that there are minimum number of groups.

3. Encircle the cells which contain 1s and are not adjacent to any other cell. These are known as isolated minterms and they appear in the expression in same form.

4. Avoid any redundant group.

5. Write the Boolean term for each group and obtain the minimized expression by summing product terms of all the groups.

Grouping Cells (Pair)

C / AB	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	1	0
AB	1	0
$A\bar{B}$	0	0

$X = \bar{A}B\bar{C} + AB\bar{C}$
$= B\bar{C}$

cell adjacency

C / AB	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	1	1
AB	0	0
$A\bar{B}$	0	0

$X = \bar{A}B\bar{C} + \bar{A}BC$
$= \bar{A}B$

wrap around adj

C / AB	$\bar{C}$	C
$\bar{A}\bar{B}$	1	0
$\bar{A}B$	0	0
AB	0	0
$A\bar{B}$	1	0

$X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} = \bar{B}\bar{C}$

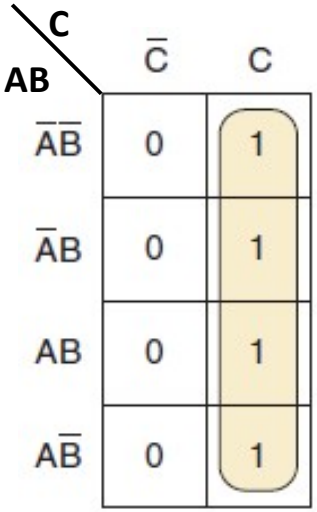
CD / AB	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	0	0	1

$X = \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$
$+ A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$
$= \bar{A}\bar{B}C + A\bar{B}D$

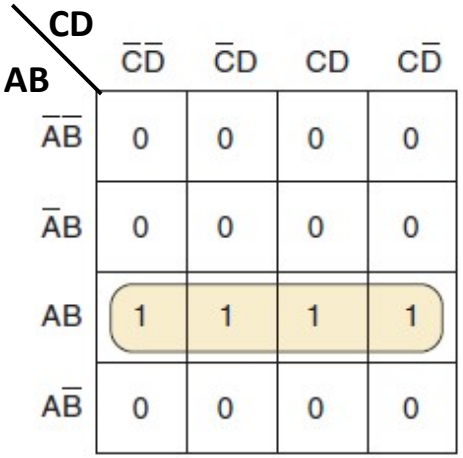
Looping a pair of adjacent 1s in a K map eliminates the variable that appears in complemented and uncomplemented form.

Groups of Four (Quads)

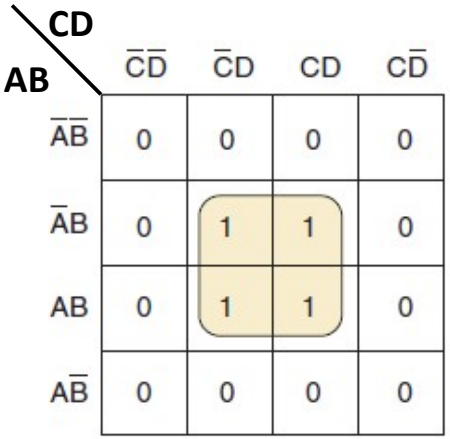
A K map may contain a group of four 1s that are adjacent to each other. This group is called a *quad.*



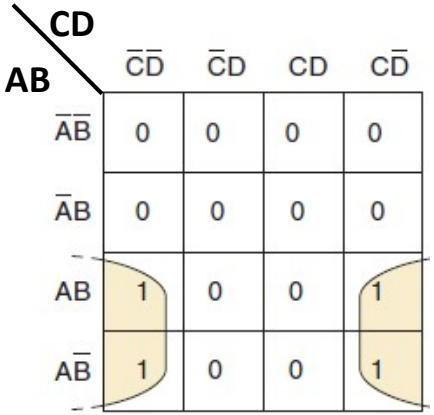
$X = C$



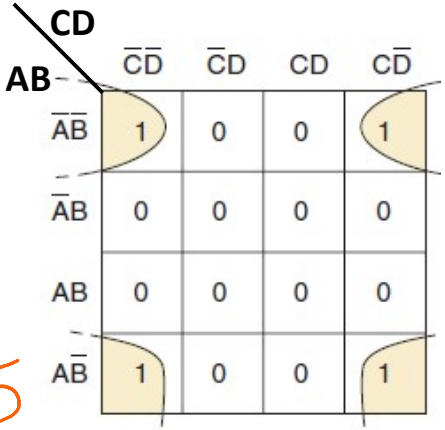
$X = AB$



$X = BD$



$X = A\bar{D}$

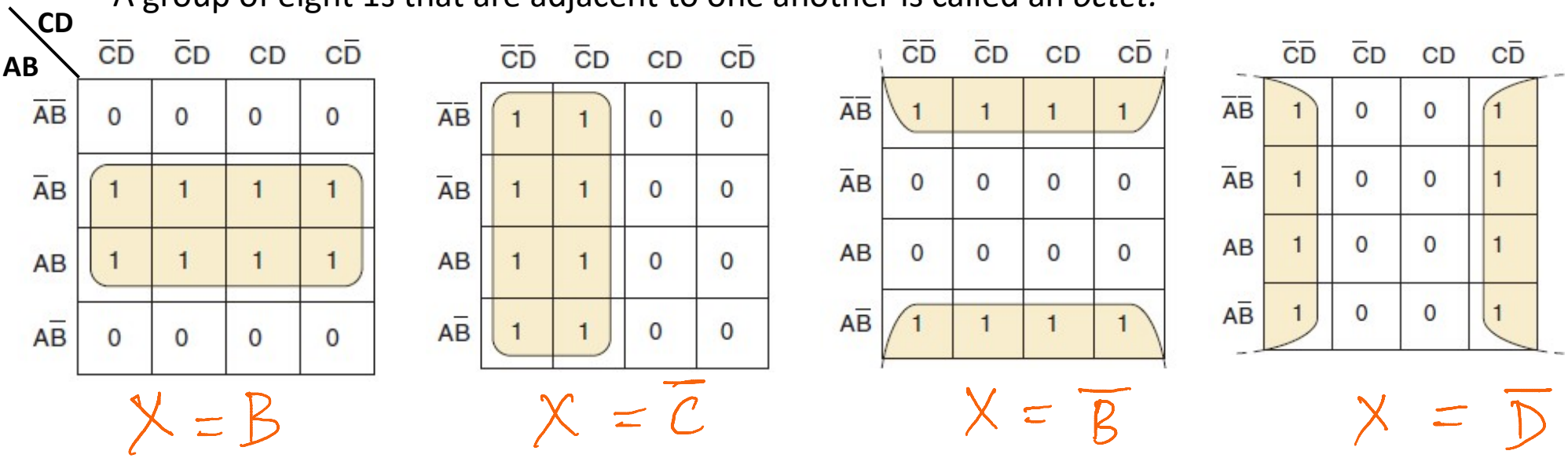


$X = \bar{B}\bar{D}$

Looping a quad of adjacent 1s eliminates the two variables that appear in both complemented and uncomplemented form.

Groups of Eight (Octets)

A group of eight 1s that are adjacent to one another is called an *octet*.



Looping an octet of adjacent 1s eliminates the three variables that appear in both complemented and uncomplemented form.

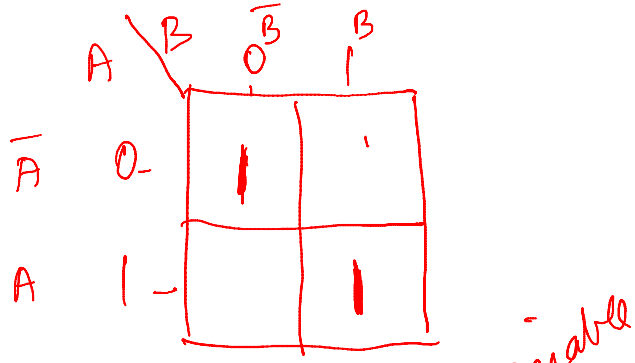
When a variable appears in both complemented and uncomplemented form within a loop, that variable is eliminated from the expression.

Variables that are the same for all squares of the loop must appear in the final expression.

Mapping a standard SOP expression

- Determine the binary value of each product term in the standard SOP expression.
- Place a 1 on the K-map in the cell having the same value as the product term.

$f = \bar{A}\bar{B} + AB$ $\quad 2^2 = 4$



2 variable

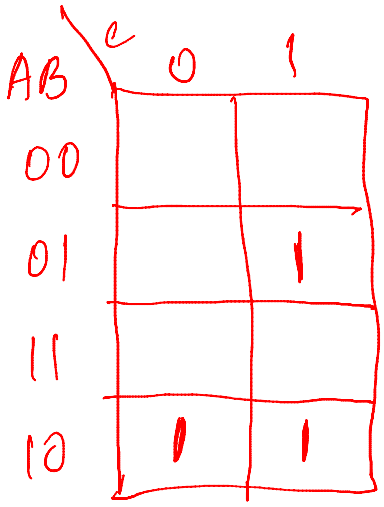
$f = \bar{A}\bar{B} + AB$

0 0 $\quad$ 1 1

$f = \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C}$

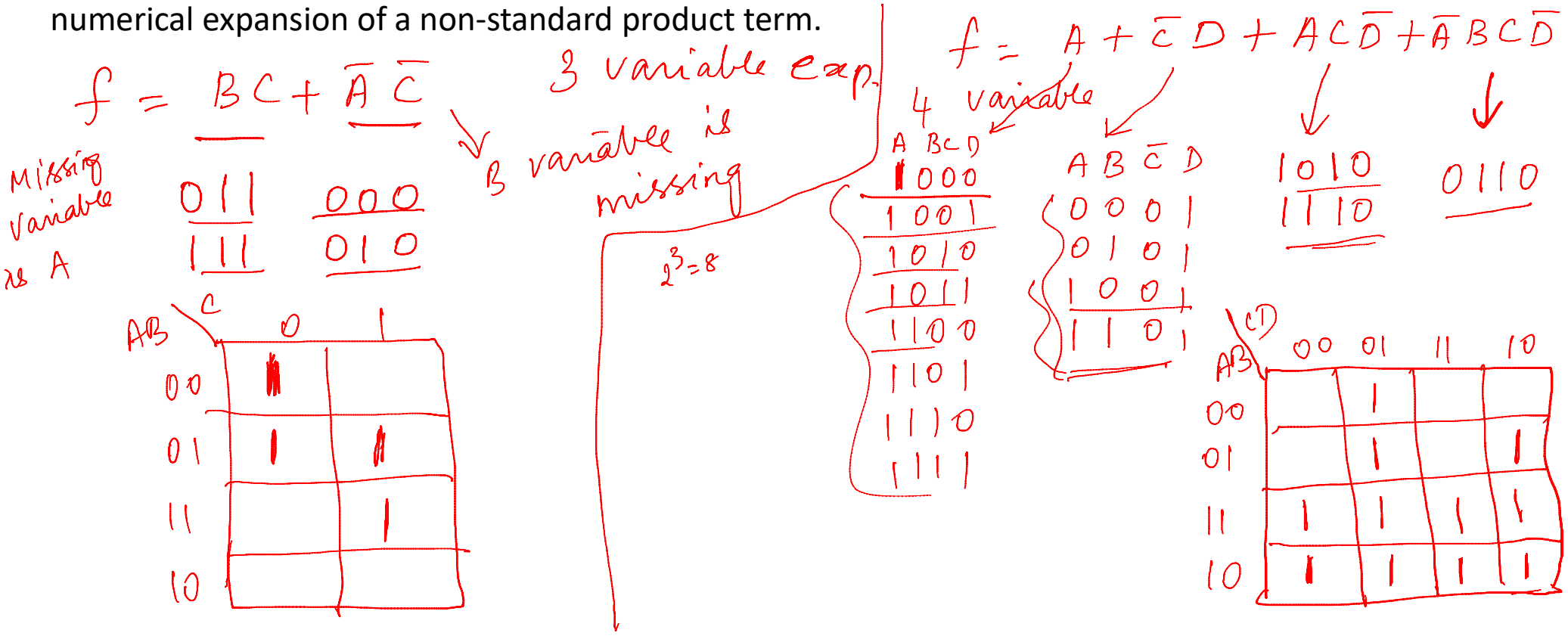
0 1 1 $\quad$ 1 0 1 $\quad$ 1 0 0

3 Variable



Mapping a non-standard SOP expression

If an expression is not in standard form then it must be converted into standard form by numerical expansion of a non-standard product term.



K-map simplification of SOP expressions

- Grouping the cells that have 1s.
- Determine the minimum product term for each group.
- Sum the product term to get the minimum SOP expression.

Eg. $f = \Sigma m(0,2,3,4,5,6)$

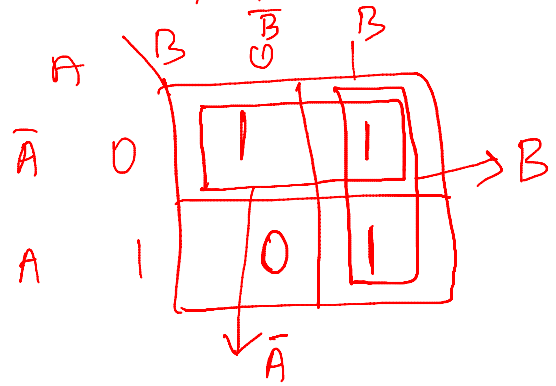
3 Variables

Reduce this expression

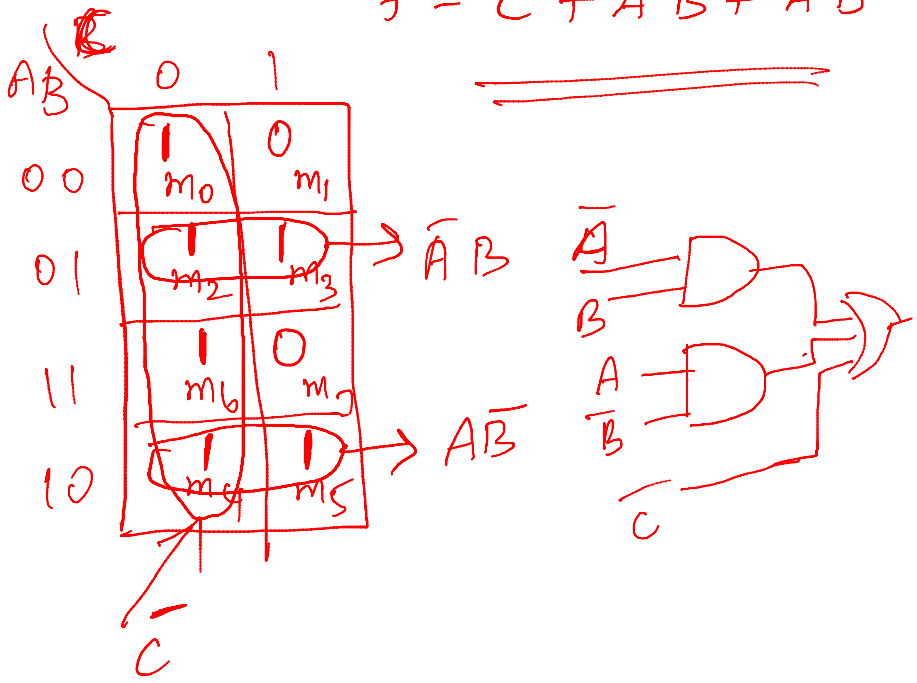
$f = \bar{A}\bar{B} + \bar{A}B + A\bar{B} = \bar{A} + B$

$f = m_0 + m_1 + m_3$

$f = \Sigma m(0,1,3)$



$f = \bar{C} + \bar{A}B + A\bar{B}$

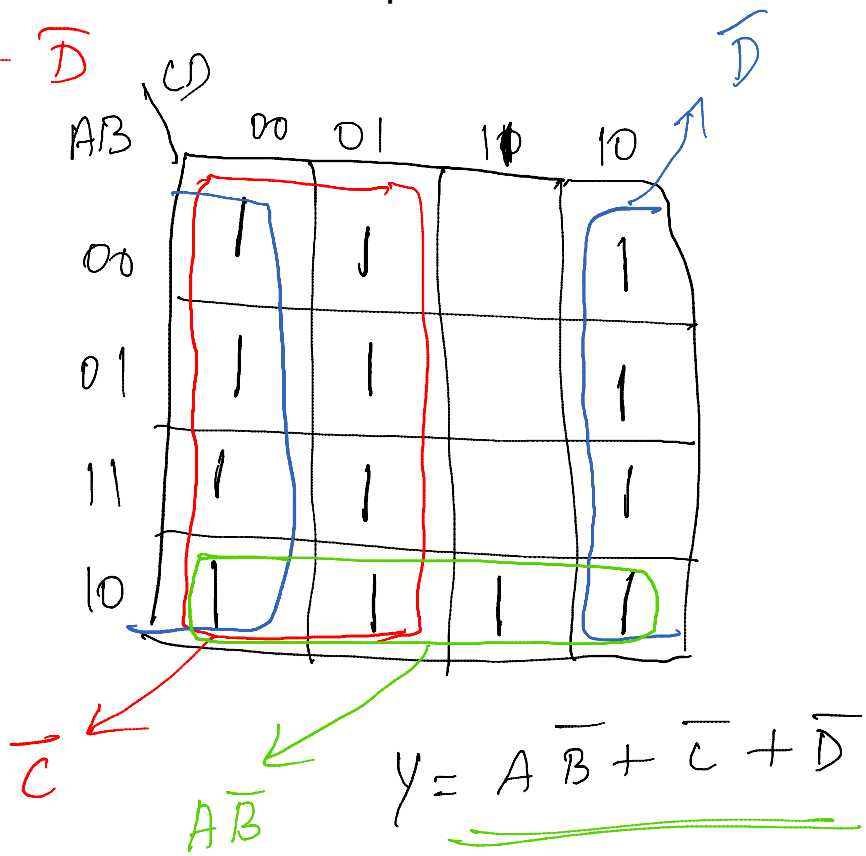


Draw the Karnaugh map for ~~$Y = \bar{A}B + A\bar{B}$~~, and then use it to obtain a minimised expression.

$$Y = \bar{C}(\bar{A}\bar{B}\bar{D} + D) + A\bar{B}C + \bar{D}$$

$$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{C}D + A\bar{B}C + \bar{D}$$

0 0 0 0 0 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0 1 0 1 1 0 0 0 0 0 0 1 0 0 1 0 0 0 1 1 0 1 0 1 0 0 0 1 0 1 0 1 1 0 0 1 1 1 0



$$Y = A\bar{B} + \bar{C} + \bar{D}$$