Test of Significance of Small Samples.

Sample size $n \geq 30$ – Large sample

$n < 30$ – Small sample.

Student's t-distribution.

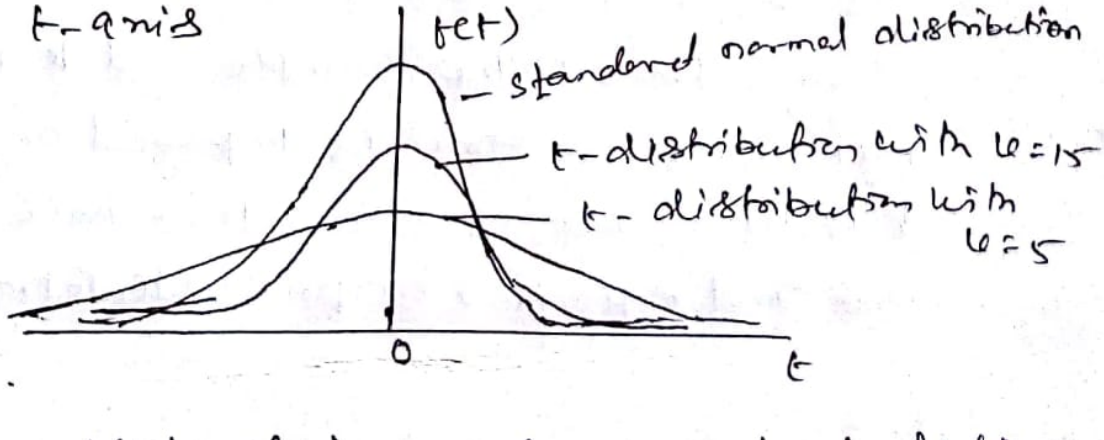
A random variable T is said to be follow Student's t-distribution if its p.d.f is given by

$$f(t) = \frac{1}{\sqrt{\nu}\,\beta\left(\frac{\nu}{2}, \frac{1}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad -\infty < t < \infty$$

where ν is called the number of degrees of freedom.

Properties of t-Distribution

1. The probability curve of the t-distribution is similar to the standard normal curve and is symmetric about $t=0$, bell shaped and asymptotic to the t-axis



2. For sufficiently large value of n, the t-distribution tends to the normal distribution.
3. The mean of the t-distribution is zero
4. The variance of the t-distribution is $\frac{\nu}{\nu-2}$ if $n>2$ and is greater than 1, but it tends to 1 as $\nu \to \infty$.

Uses of t-distribution

The t-distribution is used to test the significance of the difference between

1. The mean of a small sample and the mean of the population
2. The means of two small samples
3. The coefficient correlation in the small sample and that in the population, assumed zero.

Degrees of freedom :-

The number of independent variables used to compute the test statistics is known as the number of degrees of freedom of that statistics.

ie The number of degrees of freedom is given by $\upsilon = n-k$ where n is the number of observations of the sample and k is the number of constraints imposed on them or k is the number of values that have been found out and specified by prior calculations.

Note:-

The critical values of 't' for a single (right or left) tailed test at LOS 'α' corresponding to the 'υ' degrees of freedom is same as that for a two-tailed test at LOS '2α' corresponding to the same degrees of freedom.

Test - I

Test of significance of the difference between sample mean and population mean.

Test statistics $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$ → (1)

where $\bar{x}$ - Sample mean ; μ - Population mean

$s^2 = \frac{1}{n} \sum_i (x_i - \bar{x})^2$; n - sample size

Here d.f $= \nu = n-1$

Result

95% confidence limit for the population μ is

$$\bar{x} - t_{0.05} \times \frac{s}{\sqrt{n-1}} \leq \mu \leq \bar{x} + t_{0.05} \times \frac{s}{\sqrt{n-1}}.$$

where $t_{0.05}$ is the 5% table value of 't' for $\nu = n-1$ degrees of freedom for a two tailed test.

Note: Some times 't' is also taken as $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$

where $S^2 = \frac{n}{n-1} s^2$, s being the s.d of the sample

Problem:- A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter 1.85 cm with a s.d of 0.1 cm. On the basis of this sample, would you say that the work of the machinist is inferior.?

Soln:- Given that

$\bar{x} = 1.85$, $s = 0.1$, $n = 10$, $\mu = 1.75$

$\nu = d.f = n-1 = 10-1 = 9$

1. $H_0: \bar{x} = \mu$

2. $H_1: \bar{x} \neq \mu$ (Two tailed test is used)

3. Let LOS be 5%,

Tabulated value of $t_{0.05}$ for $\nu = 10-1 = 9$ d.f is 2.26

4. $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = 3$

5. Calculated value of $t = 3 >$ Tab. value of t at 5% LOS for $\nu = 9$ is 2.26

Reject H_0

3. Let LOS be 1%.

Tab. value of t at 0.01, $\nu = 9$ d.f is 3.25

Here $t < t_{0.05}$

Accept H_0.

2) The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a s.d of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance

Soln:- Given that $n = 25$, $\bar{x} = 1550$, $s = 120$, $\mu = 1600$

$d.f = \nu = 25-1 = 24$

1. $H_0: \bar{x} = \mu$

2. $H_1: \bar{x} < \mu$ (Left tailed test is used)

3. Let the LOS be 5%., Tab. value of $t_{0.05}$ for $\nu = 24$ is 1.71.

4. $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

$= \dfrac{1550 - 1600}{120/\sqrt{24}} = -2.04$

5. Cal. value of $|t| = 2.04 >$ Tab. value of t at 0.05 LOS for 24 d.f is 1.71

Reject Ho

ie The claim of the Company is not accepted at 5% LOS.

3) Tests made on the breaking strength of 10 pieces of a metal wire gave the results:

578, 572, 570, 568, 572, 570, 570, 572, 596, 584 kg.

Test it the mean breaking strength of the wire can be assumed as 577 kg.

Soln:- Let A = 582, $d_i = x_i - 582$

x_i :	578	572	570	568	572	570	570	572	596	584
$d_i = x_i - 582$:	-4	-10	-12	-14	-10	-12	-12	-10	14	2
d_i^2 :	16	100	121	196	100	144	144	100	196	4

$$s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{1}{n}\sum d_i\right)^2$$

$$s^2 = \frac{1144}{10} - \left(\frac{-68}{10}\right)^2 = 68.16$$

$$s = 8.26$$

1. Ho: $\bar{x} = \mu$; 2. H1: $\bar{x} \neq \mu$ (Two tailed test used)

3. Let LOS be 5%.

Tabulated value of $t_{0.05}$ for $\nu = 9$ is 2.26

4. $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = \dfrac{575.2 - 577}{8.26/\sqrt{9}} = -0.65$

5. Cal. value of $|t| = 0.65 < t_{0.05}$ for $\nu = 9$ is 2.26

Accept H_0

Test - II.

Test of significance of the difference between means of two small samples drawn from the same normal population

Test statistics $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$

Here d.f $= \nu = n_1 + n_2 - 2$

Prob. 1 :-

Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	size	mean	s.d
Sample I :	8	1234	36
Sample II :	7	1036	40

Is the difference in the means sufficient to warrant that type I bulbs are superior to type II bulbs?

Soln:- Given that

$n_1 = 8 \quad n_2 = 7$

$\bar{x}_1 = 1234, \quad \bar{x}_2 = 1036, \quad s_1 = 36, \quad s_2 = 40$

1. $H_0 : \bar{x}_1 = \bar{x}_2$;
2. $H_1 : \bar{x}_1 > \bar{x}_2$
3. Let the LOS be 5%, $\quad \nu = n_1 + n_2 - 2 = 15 - 2 = 13$

Tab. value of $t_{0.05}$ for $\nu = 13$ is 1.77

4. $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 9.39$$

5. Cal. value of 9.39 > Tab. value of $t_{0.05}$ for $\nu = 13$ is 1.77

Reject H_0.

ie Type I bulbs may be regarded superior to type II bulbs at 5% LOS.

Prob. 2 Below are given the gains in weights (lbs) of cows fed on two diets X and Y gain in weights (in lbs)

Diet X:	25	32	30	32	24	14	32		
Diet Y:	24	34	22	30	42	31	40	30	32	35

Test at 5% LOS, whether the two diets differ as regards their effect on mean increase in weight (Table value of t for 15 d.f at 5% is 2.131)

Soln:-

1. $H_0: \bar{x}_1 = \bar{x}_2$
2. $H_1: \bar{x}_1 \neq \bar{x}_2$
3. Let LOS be 5%.

Tab. value of $t_{0.05}$ for $\nu = n_1 + n_2 - 2 = 15$ is 2.131

x_1	$d_1 = x_1 - 27$	x_2	$d_2 = x_2 - 32$	d_1^2	d_2^2
25	-2	24	-8	4	64
32	5	34	2	25	4
30	3	22	-10	9	100
32	5	30	-2	25	4
24	-3	42	10	9	100
14	-13	31	-1	169	1
32	5	40	8	25	64
189	0	30	-2	266	4
		32	0		0
		35	3		9
		320	0		350

$\bar{x}_1 = \frac{189}{7} = 27$; $\bar{x}_2 = \frac{320}{10} = 32$

$s_1^2 = \frac{\Sigma d_1^2}{n_1} - \left(\frac{\Sigma d_1}{n_1}\right)^2 = \frac{266}{7} - 0 = 38$

$s_2^2 = \frac{\Sigma d_2^2}{n_2} - \left(\frac{\Sigma d_2}{n_2}\right)^2 = \frac{350}{10} - 0 = 35$

4. $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -1.59$

5. Cal. value of $|t| = 1.59$ < Tab. value of $t_{0.05}$ for $\nu = 15$ is 2.131

Accept H_0