4.5 METHODS TO CALCULATE ARITHMETIC MEAN

4.5.1 Arithmetic Mean of an Ungrouped Data

Direct Method. If n observations in the raw data consist of n distinct values denoted by x_1, x_2, x_3, ..., x_n of the observed variable x occurring with frequencies f_1, f_2, f_3, ..., f_n respectively, then the arithmetic mean of the variate x is given by $\bar{x}$, where

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\Sigma f_i} = \frac{\sum_{i=1}^{n} f_ix_i}{N}$$

where $$N = \sum_{i=1}^{n} f_i = f_1 + f_2 + ... + f_n = \textbf{Sum of frequencies}$$

Example 6. *Find the Arithmetic Mean from the following frequency table:*

Marks	52	58	60	65	68	70	75
No. of Students	7	5	4	6	3	3	2

Solution. Let x be the marks and f be the frequency so that we have the following table:

Computation Table

x	f	$f \times x$
52	7	364
58	5	290
60	4	240
65	6	390
68	3	204
70	3	210
75	2	150
Total	**30**	**1848**

Here $N = \Sigma f = 30$ and $\Sigma fx = 1848$.

$$\therefore \textbf{ Mean : } \bar{x} = \frac{\Sigma fx}{N} = \frac{1848}{30} = \frac{616}{10} = \textbf{61.6.}$$

4.5.2 Concealed Frequency

Example 7. *The following table gives the distribution of 100 accidents in New Delhi during seven days of a week of a given month. During that month there were 5 Mondays, 5 Tuesdays and 5 Wednesdays and only four each of the other days. Calculate the number of accidents per day.*

Day:	*Sunday*	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	*Saturday*
No. of accidents:	*26*	*16*	*12*	*10*	*8*	*10*	*18*

Solution. Let us prepare the following table.

Table : Computation of Mean

Day	*No. of Accidents* (x)	*Number* (f)	$f \times x$
Sunday	26	4	104
Monday	16	5	80
Tuesday	12	5	60
Wednesday	10	5	50
Thursday	8	4	32
Friday	10	4	40
Saturday	18	4	72
		$\Sigma f = 31$	$\Sigma fx = 438$

Now **Mean:** $\bar{x} = \dfrac{\Sigma f x}{\Sigma f} = \dfrac{438}{31} = 14.13.$

4.5.3 Arithmetic Mean of a Grouped Data

Direct Method. In this case the raw data is presented in the form of a **Frequency Distribution with Class Intervals.** The arithmetic mean is calculated by the **Direct Method.** The **arithmetic mean** is defined as:

$$\textbf{Mean:} \quad \bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \qquad \text{where } n = \sum_{i=1}^{k} f_i ,$$

and x_i = **class mark** (or **mid-value or mid-point**) of the ***i*th class interval** and is given by:

$$x_i = \frac{\textbf{lower limit of } i \textbf{ th class interval + upper limit of } i \textbf{ th class interval}}{2}$$

Example 8. *The data on number of patients attending a hospital in a month are given below. Find the average number of patients attending the hospital in a day.*

Number of patients	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number days attending the hospital	2	6	9	7	4	2

Solution.

Table: Calculation of Mean

No. of patients (Class Interval)	*Frequency* f_i	*Mid-value* x_i	$f_i \times x_i$
0 – 10	2	5	$2 \times 5 = 10$
10 – 20	6	15	$6 \times 15 = 90$
20 – 30	9	25	$9 \times 25 = 225$
30 – 40	7	35	$7 \times 35 = 245$
40 – 50	4	45	$4 \times 45 = 180$
50 – 60	2	55	$2 \times 54 = 110$
Total	$\Sigma f_i = 30$		$\Sigma f_i \times x_i = 860$

Average $= \bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{860}{30} = 28.67 \approx 29$ **patients per day.**

number of tails (x) are shown in the following table. Calculate the arithmetic mean by the short-cut method.

x:	0	1	2	3	4	5	6	7	8	9	10
f:	2	8	43	133	207	260	213	120	54	9	1

Solution. Let 5 be the **assumed mean,** *i.e.*, $a = 5$. Let us prepare the following table in order to calculate the arithmetic mean:

x	f	$d = x - 5$	fd
0	2	– 5	– 10
1	8	– 4	– 32
2	43	– 3	– 129
3	133	– 2	– 266
4	207	– 1	– 207
5	260	0	0
6	213	1	+ 213
7	120	2	+ 240
8	54	3	+ 162
9	9	4	+ 36
10	1	5	+ 5
	Σf = 1050		**Σfd = 12**

$\therefore$ **Arithmetic Mean:** $\bar{x} = a + \dfrac{\Sigma fd}{\Sigma f} = 5 + \dfrac{12}{1050} = 5 + 0.0114 =$ **5.0114.**

Example 10. *For the following frequency table, find the mean.*

Class :	*100 – 120*	*120 – 140*	*140 – 160*	*160 – 180*	*180 – 200*	*200 – 220*	*220 – 240*
Frequency :	*10*	*8*	*4*	*4*	*3*	*1*	*2.*

Solution. Let $a = 170$, then we have the following table:

Table: Computation of Mean

Class	*Frequency* (f)	*Mid-value* (x)	$d = x - a = x - 170$	fd
100 – 120	10	110	– 60	– 600
120 – 140	8	130	– 40	– 320
140 – 160	4	150	– 20	– 80
160 – 180	4	170	0	0
180 – 200	3	190	20	60
200 – 220	1	210	40	40
220 – 240	2	230	60	120
	$\Sigma f = 32$			**$\Sigma fd = -780$**

Now **Mean** $= a + \dfrac{\Sigma fd}{\Sigma f} = 170 - \dfrac{780}{32} = 170 - 24.375 =$ **145.625.**

4.5.5 Arithmetic Mean by Step-Deviation Method

Step Deviation Method:

When the class intervals in a grouped data are equal, then the calculation can be simplified further by taking out the common factor from the deviations. This common factor is equal to the width of the class-interval. In such cases, the deviation of variates x from the assumed mean 'a'

Example 13. *In a study on patients, the following data were obtained. Find the arithmetic mean.*

Age (in years) :	*10 – 19*	*20 – 29*	*30 – 39*	*40 – 49*	*50 – 59*	*60 – 69*	*70 – 79*	*80 – 89*
No. of cases :	*1*	*0*	*1*	*10*	*17*	*38*	*9*	*3*

Solution. The data is presented in the form of an inclusive series. We have to **transform, the inclusive series into exclusive series.** It can be transformed as follows:

We find the distance between the lower limit of the second class interval and the upper limit of the first class-interval. This is equal to 20 – 19 = 1. We subtract $\frac{1}{2}$ of this distance (*i.e.*, 0.5) from the lower limit and add it to the upper limit. The new classes will be formed as follows:

The new data is 10 – 0.5 = 9.5; 19 + 0.5 = 19.5 and so on.

Let a = 44.5, and $d = (x - a)/i = (x - 44.5)/10$, where $i = 10$.

Table: Computation of Mean

Age (in years)	*No. of cases* (f)	*Mid-value* (x)	$d = \frac{x - 44.5}{10}$	fd
9.5 – 19.5	1	14.5	– 3	– 3
19.5 – 29.5	0	24.5	– 2	0
29.5 – 39.5	1	34.5	– 1	– 1
39.5 – 49.5	10	44.5	0	0
49.5 – 59.5	17	54.5	1	17
59.5 – 69.5	38	64.5	2	76
69.5 – 79.5	9	74.5	3	27
79.5 – 89.5	3	84.5	4	12
	$N = 79$			$\Sigma fd = 128$

Now $\bar{x} = a + \frac{\Sigma fd}{N} \times i$. Here $a = 44.5$; $i = 10$; $N = 79$.

$\therefore$ $\bar{x} = 44.5 + \frac{128}{79} \times 10 = 44.5 + 16.2 = \mathbf{60.7}$.

... distribution of marks obtained by 60 students in Economics