

Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 1–B
Maxima and Minima of a function of one variable

Absolute/Global Extrema

- Let $f(x)$ be a function defined in $[a, b]$. Then f has an absolute maximum value on $[a, b]$ at a point $c \in [a, b]$ if $f(x) \leq f(c)$ for all $x \in [a, b]$.
- Let $f(x)$ be a function defined in $[a, b]$. Then f has an absolute minimum value on $[a, b]$ at a point $c \in [a, b]$ if $f(x) \geq f(c)$ for all $x \in [a, b]$.

Extreme value theorem

If f is continuous on a closed interval $[a, b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a, b]$.

Relative/Local Extreme values

- A function f has a local maximum value at an interior point c of its domain if $f(x) \leq f(c)$ for all x in some open interval containing c .
- A function f has a local minimum value at an interior point c of its domain if $f(x) \geq f(c)$ for all x in some open interval containing c .
- A function f has a local maximum or minimum value at an interior point c of its domain, and if f is defined at c , then $f'(c) = 0$.

MATLAB syntax used in the code:

Here we used the inbuilt function `findpeaks` to find the peak values (maxima) of the function $f(x)$ at the interior points of the given interval.

For finding minimum value of $f(x)$, we have considered $f_1(x) = -f(x)$ and used the `findpeaks` function.

The syntax `[lmax_f, loc]=findpeaks(f)` finds the local maximum value(s) of the discrete function i.e., `lmax_f` and the location (index of x-value) `loc` of the local maxima.

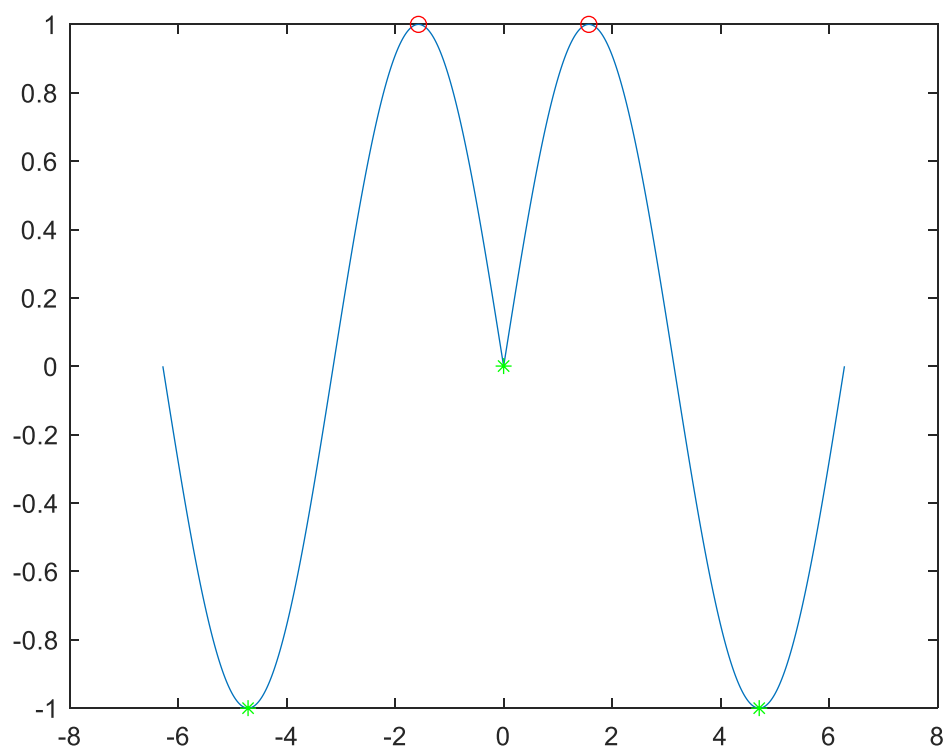
The global maximum and minimum of the function can be clearly visualised from the generated graph.

The following MATLAB code illustrates the evaluation and visualization of local extrema of a function $f(x) = \sin |x|$ in an interval $I = (-2\pi, 2\pi)$.

```
clear
clc
syms x
f(x)=sin(abs(x));
I=[-2*pi,2*pi];
f1(x)=-f(x);
a=I(1);b=I(2);
t=linspace(a,b,10000); %Discretizing the interval I
g=double(f(t)); %Finding the values of f(x) at t values
[lmax_f,loc]=findpeaks(g);
lmax_x=round(t(loc),4);
h=double(f1(t));
[lmin_f,loc]=findpeaks(h);
lmin_x=round(t(loc),4);
disp('Local maximum occur at x=')
disp(lmax_x)
disp('The Local Maximum value(s) of the function are ')
disp(double(f(lmax_x)))
disp('Local minimum occur at x=')
disp(lmin_x)
disp('The Local Minimum value(s) of the function are ')
disp(double(f(lmin_x)))
plot(t,f(t));hold on; %Plotting the function
plot(lmax_x,double(f(lmax_x)),'or');%Pointing the local
maxima on the curve of f(x)
plot(lmin_x,double(f(lmin_x)),'*g');%Pointing the local
minima on the curve of f(x)
hold off
```

Output

```
Local maximum occur at x=
-1.5703    1.5703
The Local Maximum value(s) of the function are
1.0000    1.0000
Local minimum occur at x=
-4.7122   -0.0006    4.7122
The Local Minimum value(s) of the function are
-1.0000    0.0006   -1.0000
```



EXERCISE

1. Find the local and global maxima and minima for the function $x^3 - 12x - 5$ on $x \in (-4, 4)$.
2. Find the local and global maxima and minima for the function $x + \sin 2x$ on $x \in (-5, 5)$.