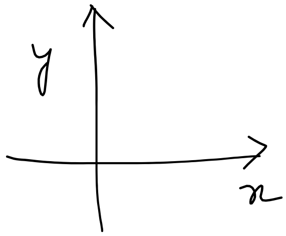
Limits of MVC

Tuesday, 23 February 2021 9:02 AM



R

$f: R^2 \longrightarrow R$

TO find Range:

$f(x, y) = \frac{1}{x^2} + \frac{1}{y^2}$

Domain: $D : \{ x, y \mid x \neq 0 \text{ and } y \neq 0 \}$

$f(5,5) = \frac{1}{25} + \frac{1}{25}$

$= \frac{2}{25}$

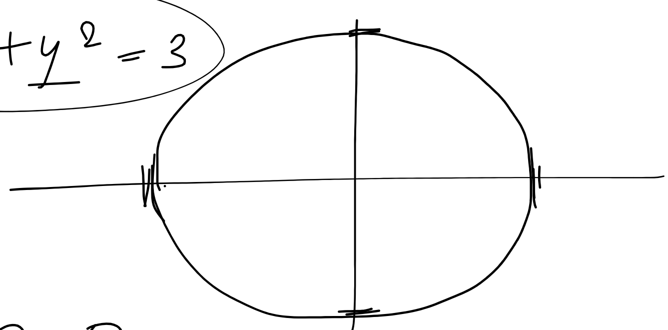
R

$f(-5,+5) = \frac{2}{25}$

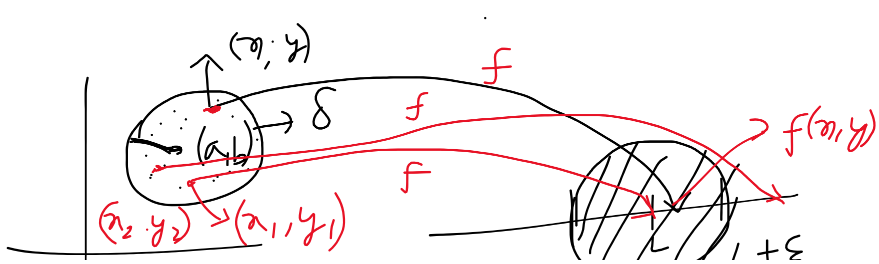
$f(-5,-5) = \frac{2}{25}$

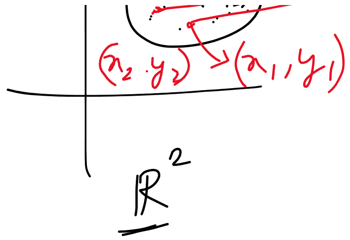
$R : \{ f(x,y) \mid R_+ - \{0\} \}$

$x^2 + y^2 = 3$



$x^2 + y^2 - 3 = 0$

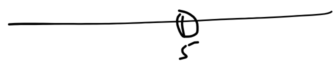




$\varepsilon \to$ You may choose

$\varepsilon = 0.1, \quad L = 5$

$\varepsilon = 0.00001$



$$|x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon$$

$$|f(x,y) - L| < \varepsilon$$

$$|\sin(x)| \leq 1$$

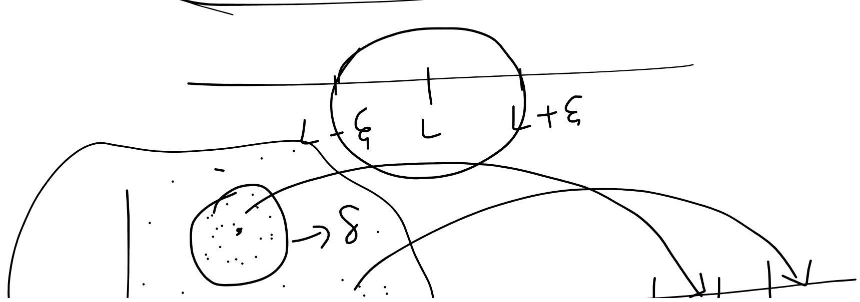
$$-1 \leq \sin(x) \leq 1$$

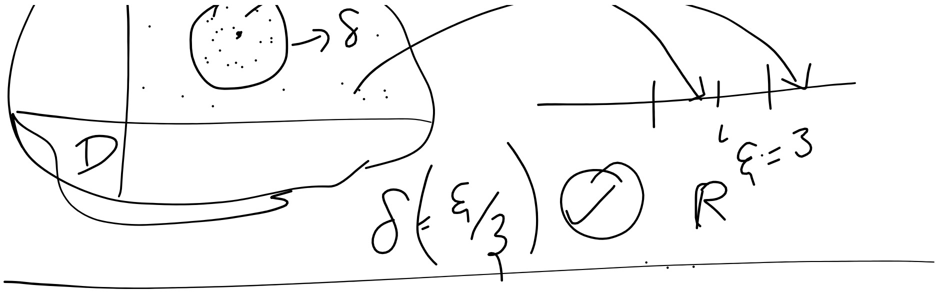
$$|f(x,y) - L| < \varepsilon$$

$$-\varepsilon < f(x,y) - L < \varepsilon$$

$$-\varepsilon + L < f(x,y) - L + L < \varepsilon + L$$

$$L - \varepsilon < f(x,y) < L + \varepsilon$$





$$\lim_{(x,y)\to(0,0)} = \frac{xy}{x^2+y^2}$$

Steps: $y = 0 \Rightarrow \lim_{(x,y)\to(0,0)} = \frac{0}{1} = 0$

$x = 0 \Rightarrow \lim_{(x,y)\to(0,0)} = \frac{0}{1} = 0$

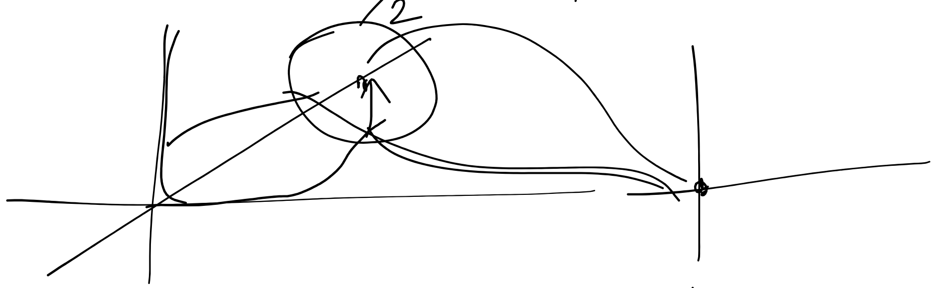
$\boxed{y = mx} \Rightarrow \lim_{(x,y)\to(0,0)} \left[\frac{x(mx)}{x^2+(mx)^2}\right]$

$= \lim_{(x,y)\to(0,0)} \left[\frac{mx^2}{x^2+m^2x^2}\right]$

$= \lim_{(x,y)\to(0,0)} \left[\frac{m}{1+m^2}\right]$

$m = 1 \quad = \lim_{(x,y)\to(0,0)} \left[\frac{1}{1+1}\right] = \frac{1}{2}$

$= \frac{1}{2}$ ✓ $\quad \boxed{L_1 = L_2 \neq L_3}$



$\lim_{x\to 0} f(x) \to L_1$ on C_1 path

$\lim f(x) \to L_2$ on C_2 path

$\lim_{x \to 0} f(x) \to L_2$ on c_2 path

Suppose $L_1 = L_2$ (Limit exists)

Suppose $L_1 \neq L_2$ (Limit does not exist)

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① $c_1 : y = 0$

② $c_2 : x = 0$

③ $y = mx$

④ $y = x^2$

$y =$

$$\lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2+y^2}$$

$c_1, c_2 \to L_1 = L_2 = 0$

c_3 $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{3x^2 m x}{x^2+m^2x^2}$$

$$= \frac{3m x^3}{x^2+m^2x^2}$$

$$= \frac{3m x^3}{x^2(1+m^2)}$$

$$= \frac{3m x}{1+m^2}$$

$\Rightarrow 0$ //

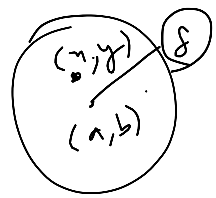
$y = x^2$

$$| \quad\quad | < \delta$$

$$|f(x,y) - L| < \varepsilon$$

$$\lim_{(x,y)\to(0,0)} \Rightarrow \left|\frac{3x^2y}{x^2+y^2} - 0\right| < \varepsilon$$

$$\Rightarrow$$



$$\Rightarrow \sqrt{(x-a)^2+(y-b)^2}$$

$$0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$$

$$(a,b) - (0,0)$$

$$0 < \sqrt{x^2+y^2} < \delta$$

$$\Rightarrow \left|\frac{3x^2y}{x^2+y^2} - 0\right| < \varepsilon$$

$$= \frac{3x^2|y|}{x^2+y^2}$$

$$\Rightarrow x^2 \le x^2+y^2 \quad (y^2 \ge 0)$$

$$\boxed{\frac{x^2}{x^2+y^2} \le 1}$$

$$= \frac{3x^2|y|}{x^2+y^2}$$

$$\le 3 \cdot 1 \cdot |y|$$

$$\leq 3 \cdot \sqrt{y^2}$$

$$\leq 3\sqrt{x^2+y^2}$$

$$< 3\delta$$