Wave Function

- Water waves – the quantity to vary periodically – HEIGHT
- Sound waves – Pressure
- Light waves – electric and magnetic fields
- What is it that varies in the case of matter waves?

Wave Function ψ

- The quantity whose variations make up matter waves is called the wave function

Wave Function

- Wave function ψ cannot be measured directly
- Wave function ψ can be a complex number
- Wave function ψ obeys the principle of superposition

Wave Function cannot be measured directly

- The ψ itself, has no direct physical significance

- Eg. The probability that something be in a certain place at a given time must lie between 0 and 1.
- An intermediate probability, say 0.2, means 20% chance
- But the amplitude of a wave can be –ve and +ve and a –ve probability is meaningless
- This objection does not apply to $|\psi|^2$, Probability density

Probability density

- The probability of experimentally finding the body described by the wave function ψ at the point x,y,z at the time t is proportional to the value of $|\psi|^2$ there at t.

- A large value of $|\psi|^2$ means the strong possibility of the body's presence
- A small value of $|\psi|^2$ means the slight possibility of its presence

Normalization and Probability

- The probability of the particle being between x_1 and x_2 is given by

$$P = \int_{x_1}^{x_2} \Psi^* \Psi \, dx$$

- The wave function must also be normalized so that the probability of the particle being somewhere on the x axis is 1.

$$\int_{-\infty}^{\infty} \Psi^*(x,t)\Psi(x,t)\, dx = 1$$

Wave function can be a complex number

- Wave functions are usually complex with real and imaginary parts
- Probability density $|\psi|^2 = \psi^* \psi$

Wave function $$\Psi = A + iB$$

where A and B are real functions. The complex conjugate Ψ^* of Ψ is

Complex conjugate $$\Psi^* = A - iB$$

and so $$|\Psi|^2 = \Psi^*\Psi = A^2 - i^2B^2 = A^2 + B^2$$

since $i^2 = -1$. Hence $|\Psi|^2 = \Psi^*\Psi$ is always a positive real quantity, as required.

Wave function obeys the principle of superposition

- Wave

$$\Psi = a_1\Psi_1 + a_2\Psi_2$$

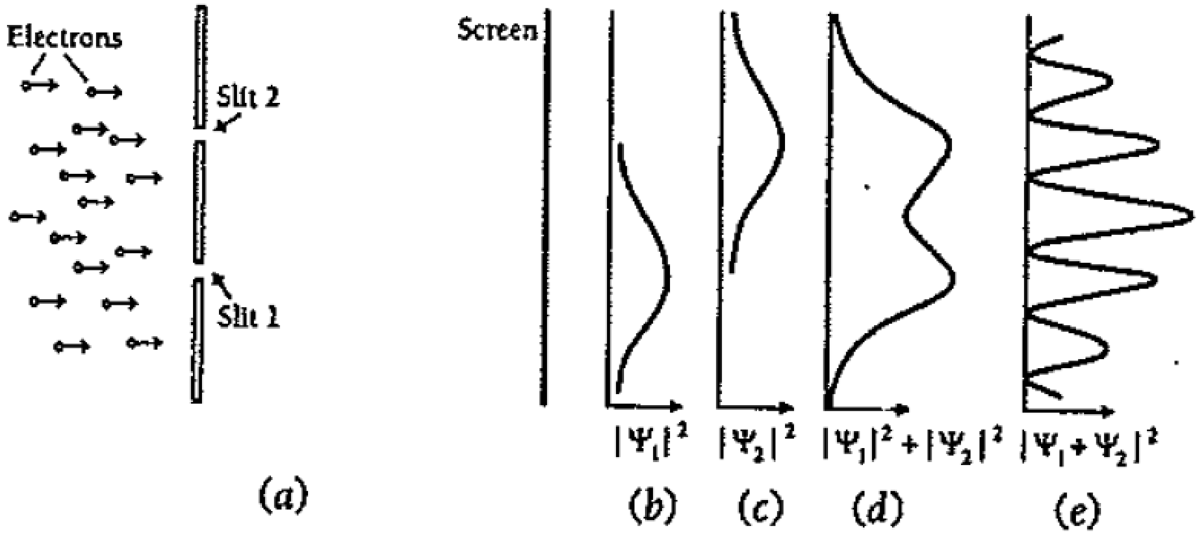

Figure 5.2 (*a*) Arrangement of double-slit experiment. (*b*) The electron intensity at the screen with only slit 1 open. (*c*) The electron intensity at the screen with only slit 2 open. (*d*) The sum of the intensities of (*b*) and (*c*). (*e*) The actual intensity at the screen with slits 1 and 2 both open. The wave functions Ψ_1 and Ψ_2 add to produce the intensity at the screen, not the probability densities $|\Psi_1|^2$ and $|\Psi_2|^2$.

Properties of Valid Wave Functions

Boundary conditions

1) In order to avoid infinite probabilities, the wave function must be finite everywhere.
2) In order to avoid multiple values of the probability, the wave function must be single valued.
3) For finite potentials, the wave function and its derivative must be continuous. This is required because the second-order derivative term in the wave equation must be single valued. (There are exceptions to this rule when V is infinite.)
4) In order to normalize the wave functions, they must approach zero as x approaches infinity.

- Solutions that do not satisfy these properties do not generally correspond to physically realizable circumstances.