PDE

Tuesday, 2 March 2021 9:16 AM

$$f(x,y) = x^2 + 3xy + y - 1$$

$$f_x \text{ (or) } \frac{\partial f}{\partial x} = 2x + 3y \quad — (1)$$

$$f_y \text{ (or) } \frac{\partial f}{\partial y} = 3x + 1 \quad — (2)$$

$$f_x(4,-5) = 2(4) + 3(-5) = -7$$

$$f_y(4,-5) = 13$$

$$f(x,y) = \frac{2y}{y+\cos x}$$

$$f_x = \left[\frac{(y+\cos x)\frac{\partial}{\partial x}(2y) - 2y\frac{\partial}{\partial x}(y+\cos x)}{(y+\cos x)^2}\right] \quad \left[\frac{u}{v} = \frac{vu' - v'u}{v^2}\right]$$

$$= \frac{0 - 2y(-\sin x)}{(y+\cos x)^2}$$

$$\boxed{f_x = \frac{2y\sin x}{(y+\cos x)^2}}$$

$$f_y = \frac{2\cos x}{(y+\cos x)^2}$$

(3)

$$yz - \ln z = x + y$$

$$\boxed{z = f(x,y)}$$

$$\boxed{\frac{\partial z}{\partial x} \to \text{Find}}$$

Take: $yz - \ln z = x + y \quad — (3)$

$$\frac{\partial}{\partial x}(\ \ ) - \frac{\partial}{\partial x}(\ln z) = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial x}(y)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}(\ln z) = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial x}(y)$$

$$y\frac{\partial z}{\partial x} - \frac{1}{2}\frac{\partial z}{\partial x} = 1 + 0 \quad [\because z = f(x,y)]$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{2}\right) = 1$$

$$\frac{\partial z}{\partial x} = \frac{1}{y - 1/2}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{2}{2y-1}}$$

$$\frac{\partial z}{\partial y} =$$ [Try !!!]

Second order derivative:

$\frac{d^{1} x}{dt^{1}}$ = Order is 1

$\frac{d^{2} x}{dt^{2}}$ = order is 2

$\frac{d^{n} x}{dt^{n}}$ = Order is n

$$a_n \frac{d^n x}{dt^n} + a_{n-1}\frac{d^{n-1} x}{dt^n} + \cdots + a_1\frac{dx}{dt} + a_0$$

n^{th} - order.

first order PDF

first order PDF

$$f_x \text{ (or) } \frac{\partial f}{\partial x}.$$

Second order PDF

$$f_{xx} \text{ (or) } \frac{\partial^2 f}{\partial x^2}.$$

$$f_{yy} \text{ (or) } \frac{\partial^2 f}{\partial y^2}.$$

$$f_{xy} = \frac{\partial^2 f}{\partial x \cdot \partial y}$$

$$\boxed{f_{yx} = \frac{\partial^2 f}{\partial y \cdot \partial x}} \Rightarrow \frac{\partial^2 f}{\partial x \cdot \partial y} \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$\therefore \boxed{f_{xy} = f_{yx}.}$$

$$f(x,y) = x\cos y + ye^x$$

To find

$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y^2}, \quad \frac{\partial^2 f}{\partial y \cdot \partial x}.$$

Start with $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

$$\frac{\partial f(x,y)}{\partial x} = \cos y + y \cdot e^x$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\underbrace{\cos y}_{0} + ye^x\right)$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = ye^x}$$

$$\frac{\partial}{\partial x^2}$$

② $\frac{\partial f}{\partial y} = x(-\sin y) + e^x$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left[-x\sin y + \underset{0}{e^x}\right]$$

$$= -x\cdot\cos y$$

$$\boxed{\frac{\partial^2 f}{\partial y^2} = -x\cos y}$$

③ $\frac{\partial^2 f}{\partial x\cdot\partial y}$ (or) $\frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}\right] = f_{xy}$

$$= \frac{\partial}{\partial x}\left[-x\sin y + e^x\right]$$

$$\boxed{\frac{\partial^2 f}{\partial x\cdot\partial y} = -\sin y + e^x} \quad \text{— ④}$$

$f_{xyz} = f_{yzx}$

$\Updownarrow$

$f_{xyz} = f_{zyx}$.

④ $\frac{\partial^2 f}{\partial y\cdot\partial x} = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}\right] = f_{yx}$

$$= \frac{\partial}{\partial y}\left[\cos y + y e^x\right]$$

$$\boxed{\frac{\partial^2 f}{\partial y\cdot\partial x} = -\sin y + e^x} \quad \text{— ⑤}$$

It can be seen that

eqn. ④ = eqn. ⑤

$$\boxed{\frac{\partial^2 f}{\partial x\cdot\partial y} = \frac{\partial^2 f}{\partial y\cdot\partial x}}$$

⑤ $f(x, y, z) = \underline{1 - 2xy^2z + x^2y}$.

(5) $f(x,y,z) = 1 - 2xy^2z + x^2y$

To find f_{yxyz} $\Rightarrow$ $f_{yx} = f_{xy}$.

Start: $f_y = -4xyz + x^2$

$$\frac{\partial}{\partial x}(f_y) = -4yz + 2x$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f_y}{\partial x}\right) = -4z + 0$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 f_y}{\partial y \partial x}\right) = -4$$

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f(x,y,z)\right)\right]\right)$$

Thomas

14.3

f_{xyz} ✓

$$\int\int\int dx\, dy\, dz$$