DeBroglie Hypothesis

$$\lambda_{DeBroglie} = h/mv = h/p$$

In this case, we are considering the

electron to be a WAVE, and

the **electron wave will "fit" around the orbit**

if the momentum (and energy) is just right

(as in the above relation). But this will happen only for specific cases - and those are the specific allowed orbits (r_n) and energies (E_n) that are allowed in the Bohr Theory!

What we now have is a **wave/particle duality for light** (E&M vs photon), **AND** a **wave/particle duality for electrons**!

DeBroglie Hypothesis

If the electron behaves as a wave, with

λ **= h/mv**, then we should be able to test this wave behavior via interference and diffraction.

In fact, experiments show that electrons **DO EXHIBIT INTERFERENCE** when they go through multiple slits, just as the DeBroglie Hypothesis indicates.

Properties of matter waves

- The lighter the particle, greater the wavelength associated with it
- Smaller the velocity of the particle greater the wavelength associated with it
- When v=0, λ= infinity, this shows that matter waves are generated by the motion of particles. These waves are produced whether the particles are charged or uncharged. Wavelength is independent of charge. This fact reveals that these waves are not electromagnetic and they are new kind of waves
- The velocity of matter waves is not constant while velocity of electromagnetic wave is constant
- Velocity of matter wave is greater than velocity of light

Proof:

A particle in motion with an associated wave has two different velocities

- Due to the mechanical motion of the particle v and

the other related to the propagation of the wave denoted by ω

We know that E=hν

$$E=mc^2, \qquad \nu = \frac{mc^2}{h}$$

The velocity of the wave is given by

$$\omega = \nu\lambda, \qquad \lambda = \left[\frac{h}{mv}\right]$$

$$\omega = \frac{c^2}{v}$$

since the velocity v cannot exceed velocity of light c, Hence ω>c (velocity of light) This is an unexpected result and this can be understood by assuming by the wave velocity equal to phase velocity or group velocity.

- The wave and particle aspects of moving bodies can never appear together in the same experiment

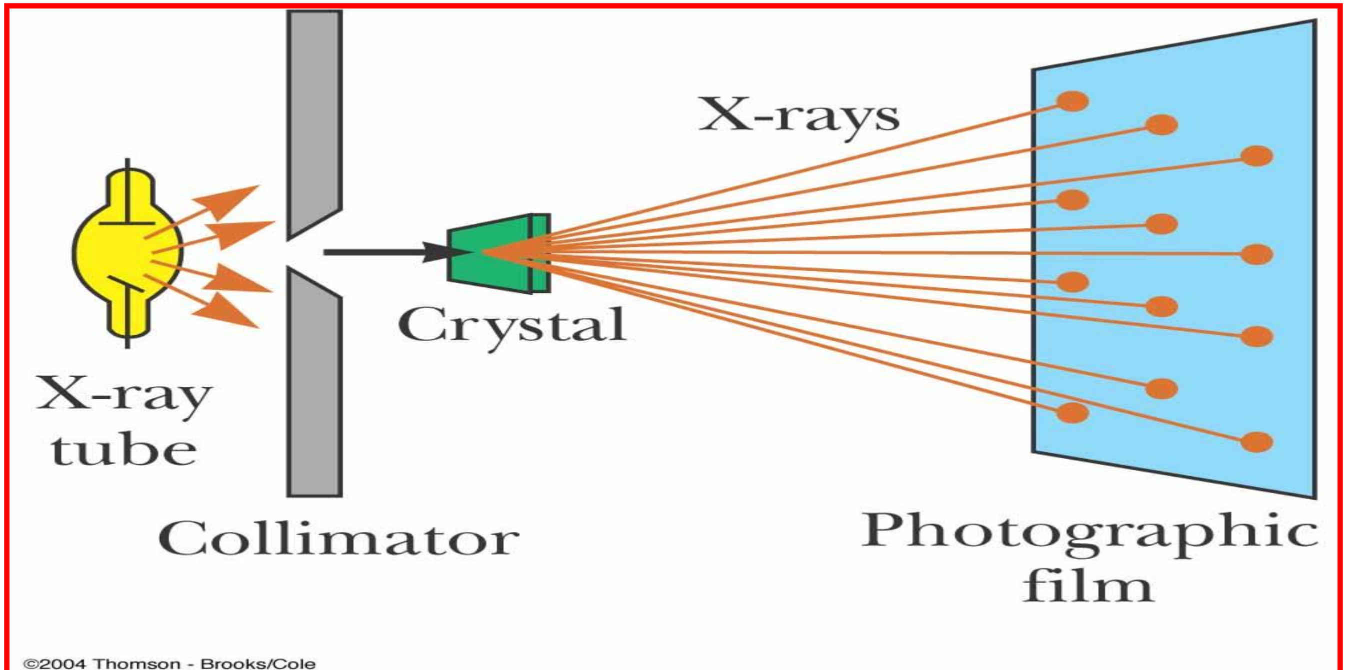
- The wave nature of matter introduces an uncertainty in the location of position of the particle because a wave cannot be exactly taken at a particular point, If the wave is very large, the particle can be identified at a given point whereas if the wave is small, the particle cannot be located easily. Heisenberg's uncertainty principle is based on this concept.

Davisson-Germer Experiment

- If particles have a wave nature, then under appropriate conditions, they should exhibit diffraction
- Davisson and Germer measured the wavelength of electrons
- This provided experimental confirmation of the matter waves proposed by de Broglie

Diffraction of X-Rays by Crystals

- X-rays are electromagnetic waves of relatively short wavelength ($\lambda = 10^{-8}$ to 10^{-12} m = 100 – 0.01 Å)
- Max von Laue suggested that the regular array of atoms in a crystal (spacing in order of several Angstroms) could act as a three-dimensional diffraction grating for x-rays

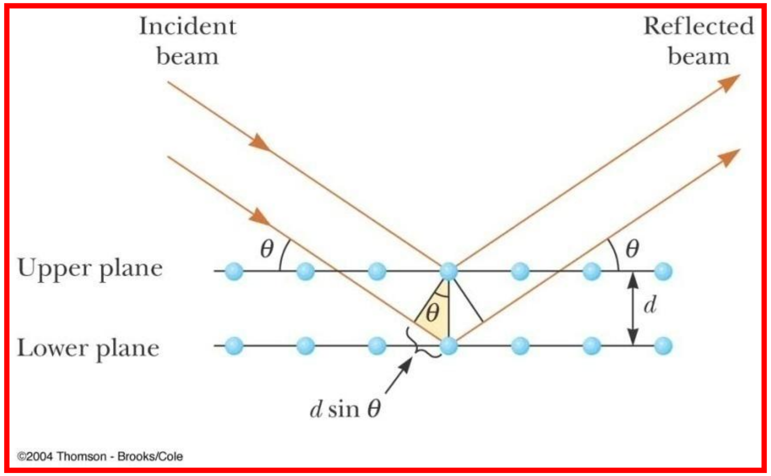


X-Ray Diffraction

- This is a two-dimensional description of the reflection (diffraction) of the x-ray beams
- The condition for *constructive interference* is

$$\boxed{2d\sin\theta = n\lambda}$$

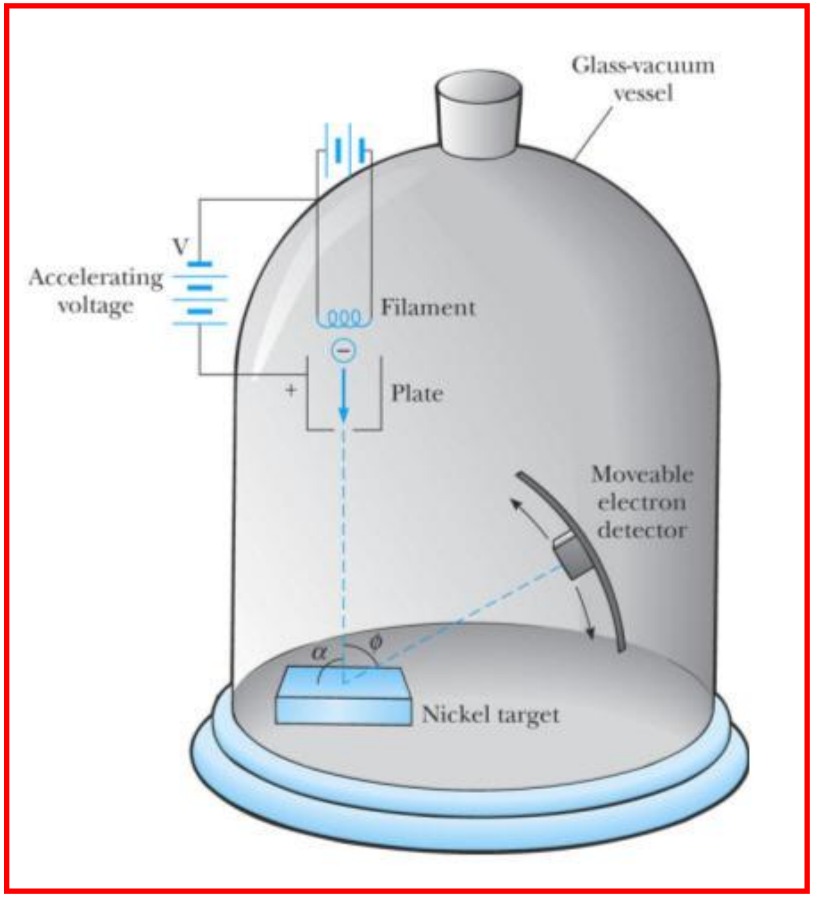
where $n = 1, 2, 3$



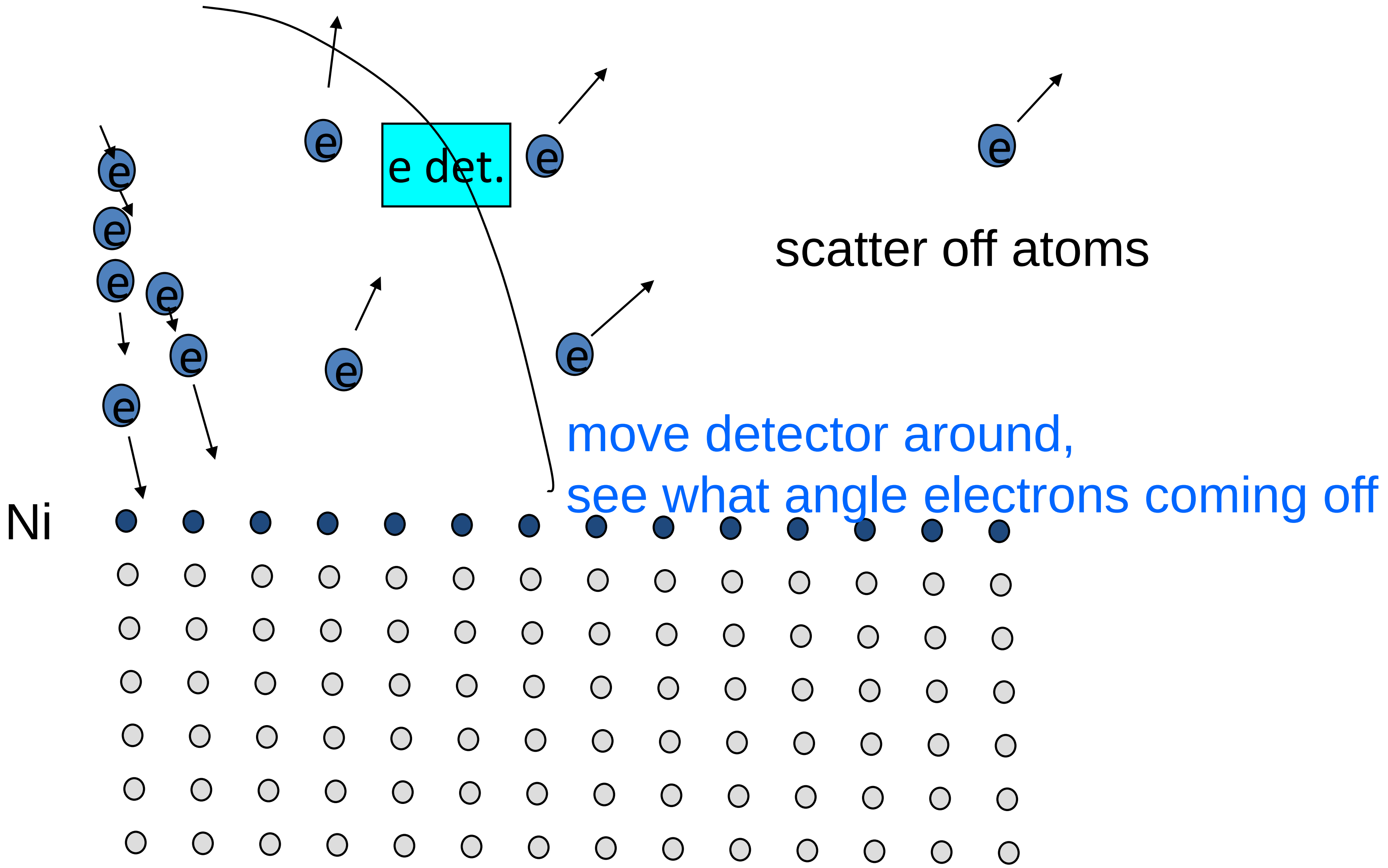
- This condition is known as **Bragg's law**
- This can also be used to calculate the spacing between atomic planes

Davisson and Germer Experiment

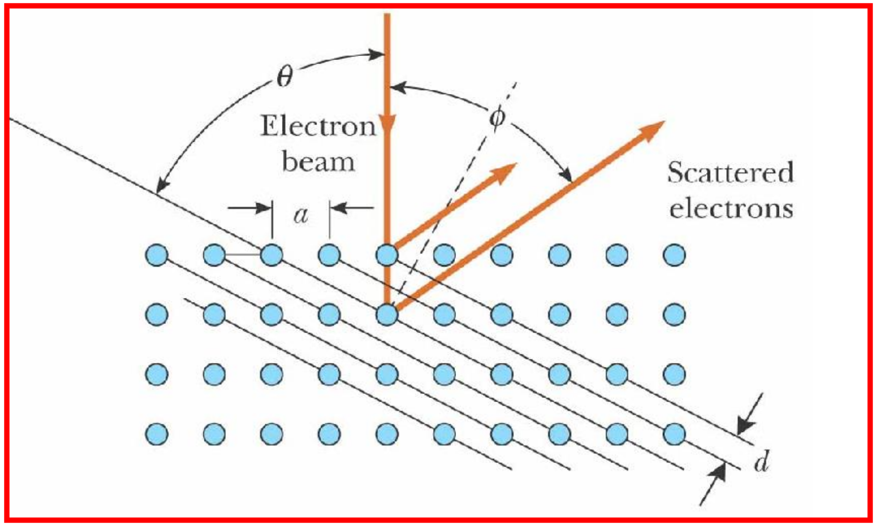
- Electrons were directed onto nickel crystals
- Accelerating voltage is used to control electron energy: $E = |e|V$
- The scattering angle and intensity (electron current) are detected
  - φ is the scattering angle

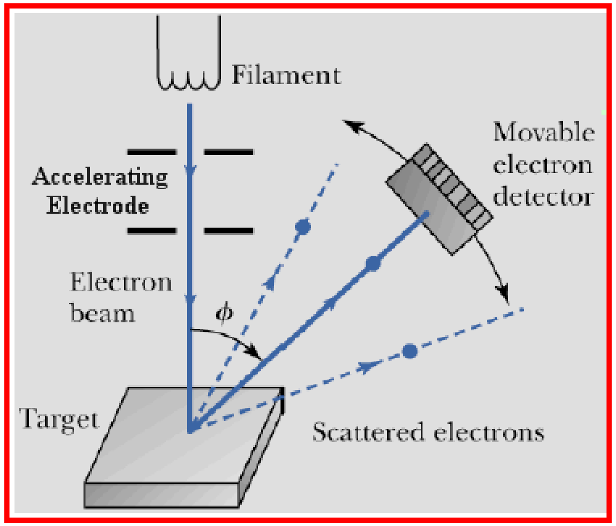


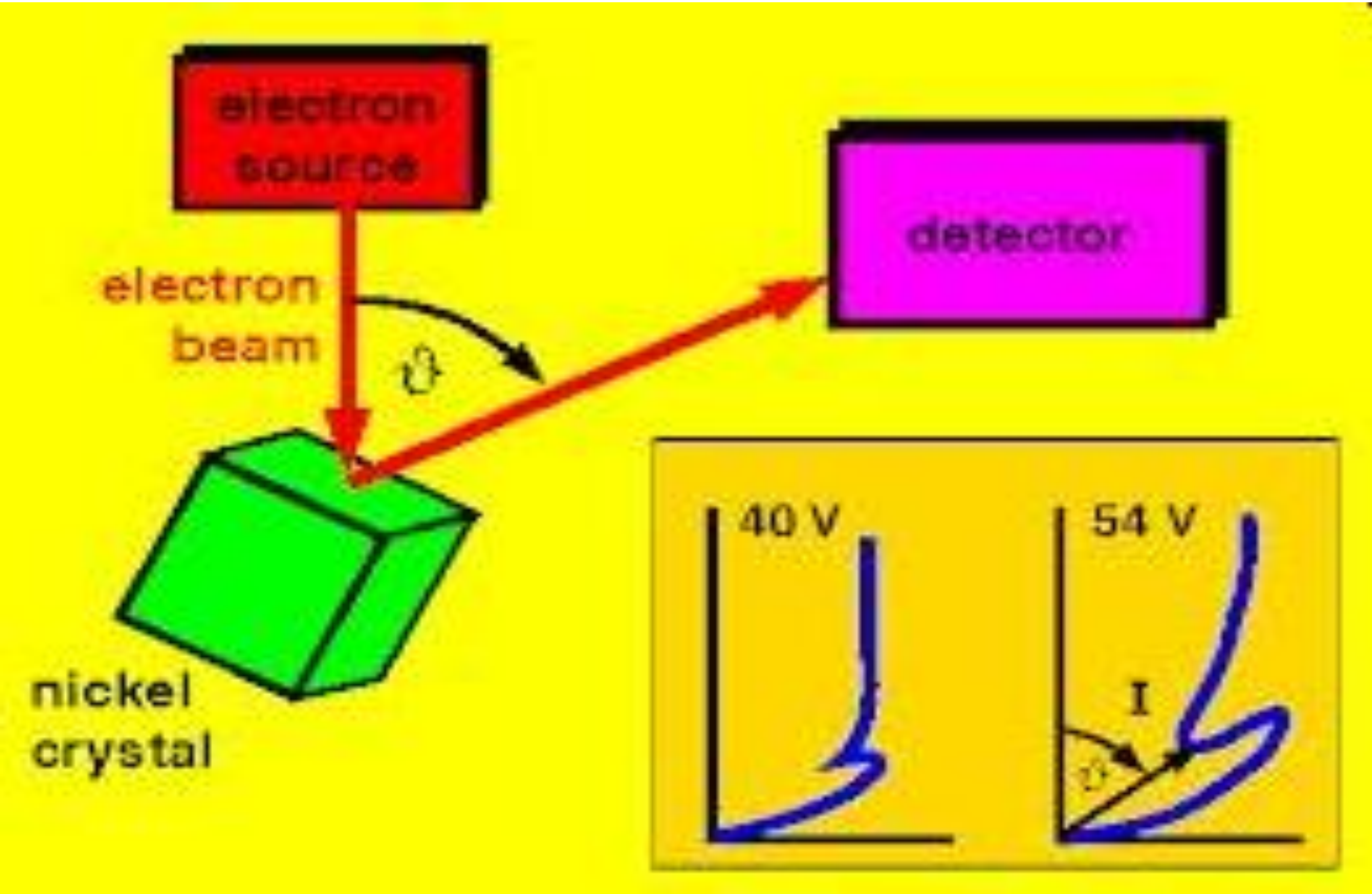
Davisson and Germer -- VERY clean nickel crystal.
Interference is electron scattering off Ni atoms.

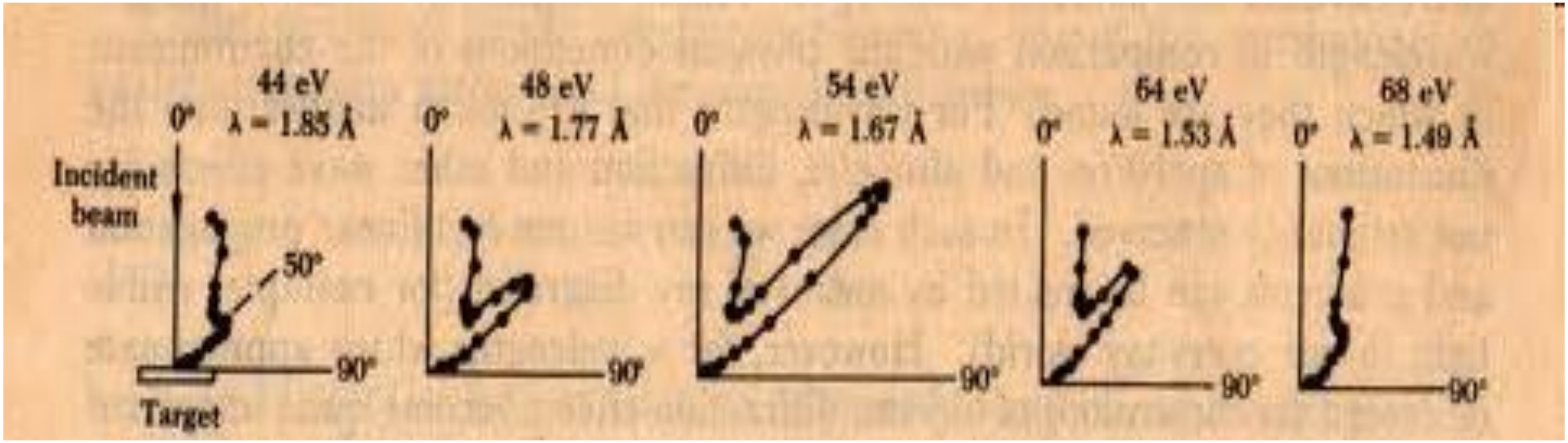


- If electrons are "just" particles, we expect a smooth monotonic dependence of scattered intensity on angle and voltage because only elastic collisions are involved
- Diffraction pattern similar to X-rays would be observed if electrons behave as waves

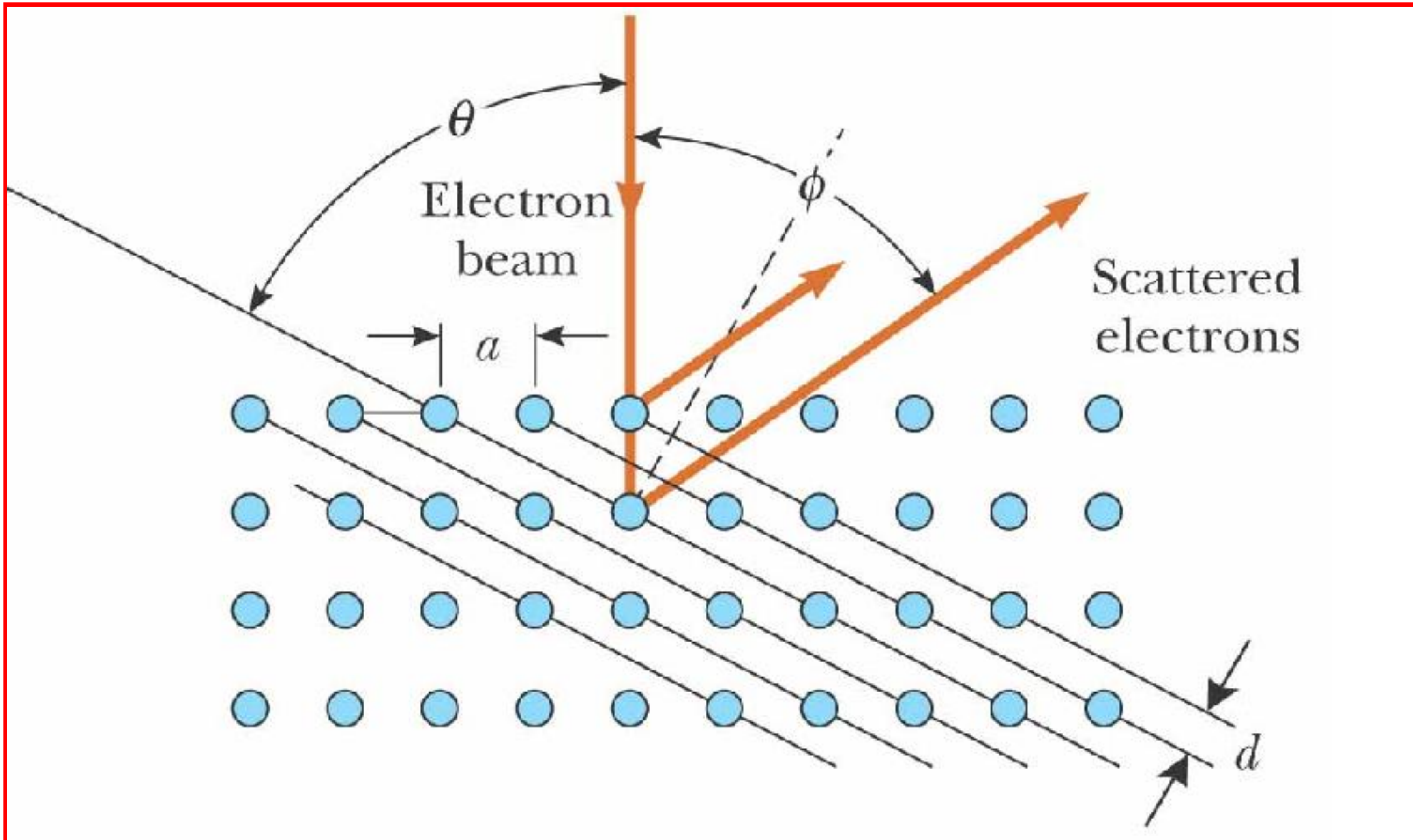





electron
source
detector
electron
beam
ϑ
nickel
crystal
40 V
54 V
I
ϑ


44 eV
0°
λ = 1.85 Å
Incident beam
50°
90°
Target
48 eV
0°
λ = 1.77 Å
90°
54 eV
0°
λ = 1.67 Å
90°
64 eV
0°
λ = 1.53 Å
90°
68 eV
0°
λ = 1.49 Å
90°

Davisson and Germer Experiment



Davisson and Germer Experiment

- **Observations:**
  - Intensity was stronger for certain angles for ***specific*** accelerating voltages (i.e. for specific electron energies)
  - Electrons were reflected in almost the same way that X-rays of comparable wavelength

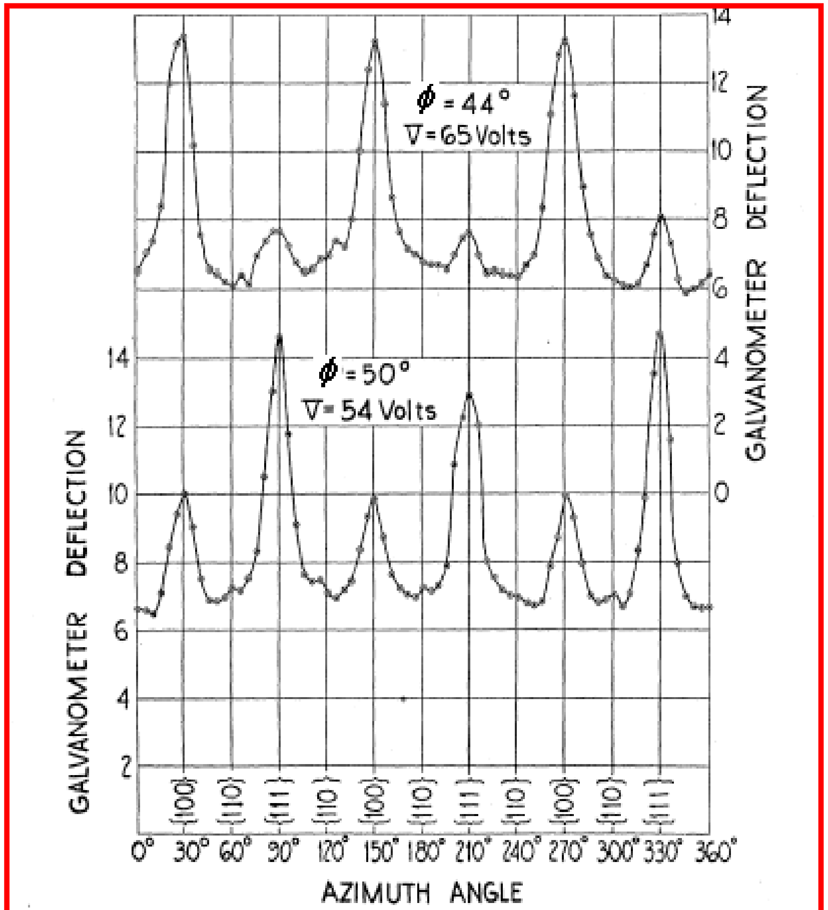
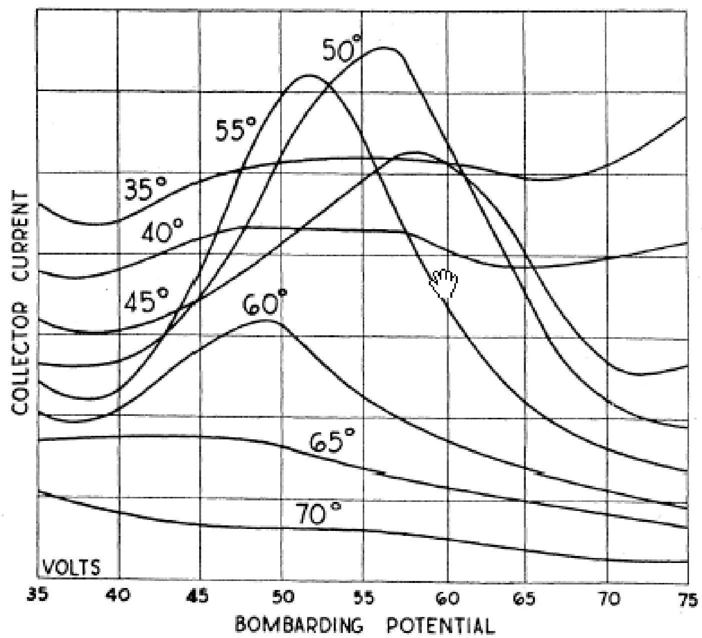


Fig. 11. Azimuth scattering curves through the "54-volt" electron beam and through the "65-volt" electron beam.

Davisson and Germer Experiment

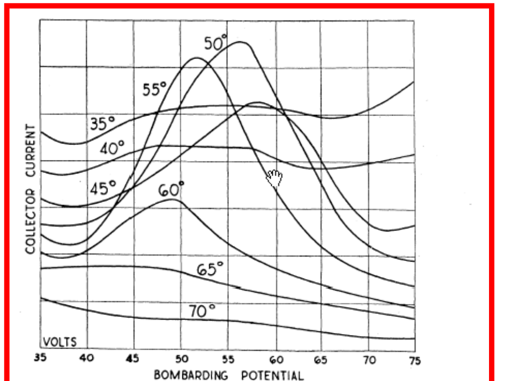
- **Observations:**
  - Current vs accelerating voltage has a maximum, i.e. the highest number of electrons is scattered in a specific direction
  - This can't be explained by particle-like nature of electrons $\Rightarrow$ electrons scattered on crystals behave as waves



Curves of collector current vs. bombarding potential—showing the development of the "54 volt beam." Azimuth {111}.

For ϕ ~ 50° the maximum is at ~54V

- Assuming the wave nature of electrons **we can use de Broglie's approach** to calculate wavelengths of a matter wave corresponding to electrons in this experiment
- $V = 54\text{ V} \Rightarrow E = 54\text{ eV} = 8.64\times10^{-18}\text{J}$



Curves of collector current vs. bombarding potential—showing the development of the "54 volt beam." Azimuth {111}.

$$E = \frac{p^2}{2m}, \quad p = \sqrt{2mE}, \quad \lambda_B = \frac{h}{\sqrt{2mE}}$$

$$\lambda_B = \frac{6.63\times10^{-34}\text{J-sec}}{\sqrt{2\times9.1\times10^{-31}\text{kg}\times8.6\times10^{-18}\text{J}}} = 1.67\,\mathring{\text{A}}$$

This is in excellent agreement with wavelengths of X-rays diffracted from Nickel!

Davisson and Germer Experiment

- **For X-ray Diffraction on Nickel**

$$2d\sin\theta = \lambda$$

$$d_{<111>} = 0.91\,\overset{\circ}{\mathrm{A}};\ \lambda_{\text{X-ray}} = 1.65\,\overset{\circ}{\mathrm{A}}$$

$$\Downarrow$$

$$\theta = 65^\circ \Rightarrow \phi = 50^\circ$$