# Heisenberg Uncertainty Principle

**Dr Rajeshkumar Mohanraman**
Assistant Professor Grade 1
School of Advanced Sciences
VIT Vellore

## Bragg's law

The phases of the beams coincide when the incident angle equals reflecting angle. The rays of the incident beam are in phase and parallel upto point z, which is the point at which top beam strikes the top layer. The second beam passes to next layer and is scattered by B. The second beam travels extra distance AB + BC. This extra distance is an integral multiple of the wavelength.

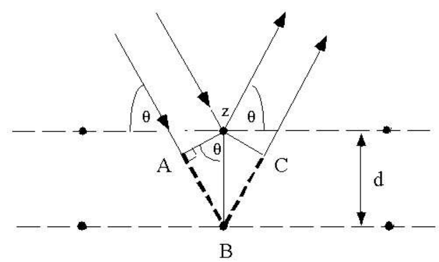
$n\lambda$ = AB+BC.

But AB=BC

$n\lambda$ = 2AB.......................................(1)

d is the hypotenuse of the right triangle Abz. Ab is opposite to angle θ

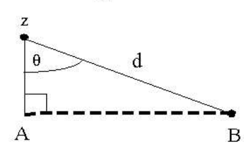
AB = d sinθ....................................(2)




Substitute equation (2) in equation (1)

$n\lambda = 2d \sin\theta$

This is equation for Bragg's law




### Bragg Equation

According to Bragg Equation:

**$n\lambda = 2d \sin\Theta$**

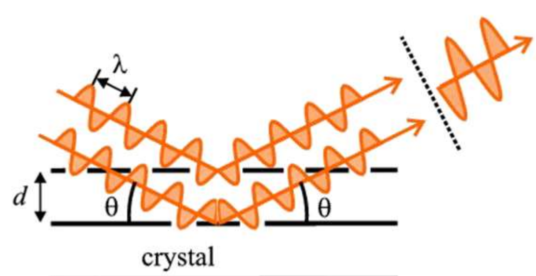
The **law** states that when the x-ray is incident onto a crystal surface, its angle of incidence, θ, will reflect back with a same angle of scattering, θ. And, when the path difference, d is equal to a whole number, n, of wavelength, a constructive interference will occur.

Therefore, according to the derivation of Bragg's Law:

- The equation explains why the faces of crystals reflect X-ray beams at particular angles of incidence (Θ, λ).
- The variable *d* indicates the distance between the atomic layers, and the variable Lambda specifies the wavelength of the incident X-ray beam.
- n as an integer.

*This observation illustrates the X-ray wave interface, which is called X-ray diffraction (XRD) and proof for the atomic structure of crystals.*

If electrons act like waves, we should be able to apply Bragg's Law to the diffraction of electrons.



# Heisenberg Uncertainty Principle

The uncertainty principle says that both the position and momentum of a particle cannot be determined at the same time and accurately. The result of position and momentum is at all times greater than h/4π. The formula for Heisenberg Uncertainty principle is articulated as,

**The Uncertainty Principle Equation**

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

*For example, the location and speed of a moving car can be determined at the same time, with minimum error. But, in microscopic particles, it will not be possible to fix the position and measure the velocity/momentum of the particle simultaneously.*

Where

**h** is the Planck's constant ( $6.62607004 \times 10^{-34}$ $m^2$ kg / s)
**Δp** is the uncertainty in momentum
**Δx** is the uncertainty in position

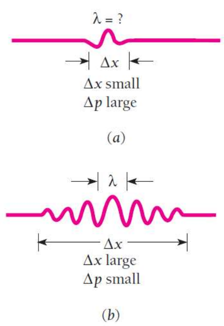



Figure 3.12 (*a*) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (*b*) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

The relationship between the distance $\Delta x$ and the wave-number spread $\Delta k$ depends upon the shape of the wave group

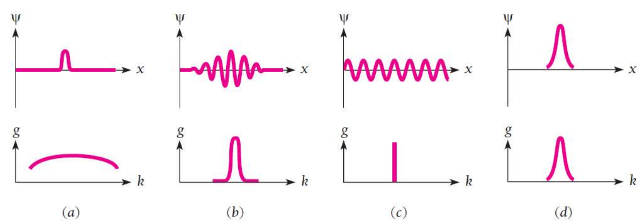



Figure 3.14 The wave functions and Fourier transforms for (*a*) a pulse, (*b*) a wave group, (*c*) a wave train, and (*d*) a Gaussian distribution. A brief disturbance needs a broader range of frequencies to describe it than a disturbance of greater duration. The Fourier transform of a Gaussian function is also a Gaussian function.

wave groups in general do not have Gaussian forms, it is more realistic to express the relationship between $\Delta x$ and $\Delta k$ as

$$\Delta x \, \Delta k \geq \tfrac{1}{2} \quad \text{.......................................(1)}$$

The de Broglie wavelength of a particle of momentum $p$ is $\lambda = h/p$ and the corresponding wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

In terms of wave number the particle's momentum is therefore

$$p = \frac{hk}{2\pi}$$

Hence an uncertainty $\Delta k$ in the wave number of the de Broglie waves associated with the particle results in an uncertainty $\Delta p$ in the particle's momentum according to the formula

$$\Delta p = \frac{h\,\Delta k}{2\pi}$$

Since $\Delta x \, \Delta k \geq \frac{1}{2}$, $\Delta k \geq 1/(2\Delta x)$ and

Uncertainty principle

$$\Delta x \, \Delta p \geq \frac{h}{4\pi}$$

This equation states that the product of the uncertainty $\Delta x$ in the position of an object at some instant and the uncertainty $\Delta p$ in its momentum component in the $x$ direction at the same instant is equal to or greater than $h/4\pi$.

## *Notes*

- Quantum mechanics is the discipline of measurements on the minuscule scale. That measurements are in macro and micro-physics can lead to very diverse consequences. Heisenberg uncertainty principle or uncertainty principle is a vital concept in Quantum mechanics.

- Heisenberg's Uncertainty principle is applicable to energy and time. These uncertainties always exist in conjugate pairs such as momentum/position and energy/time.