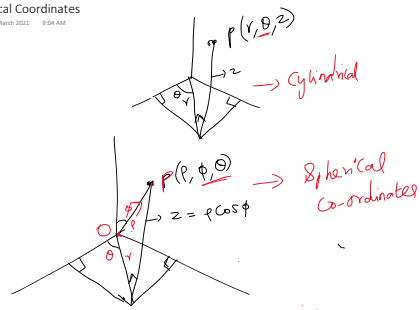

$\rho \rightarrow$ distance from P to origin
$\phi \rightarrow$ angle $\overrightarrow{OP}$ makes with positive z-axis $(0 \le \phi \le \pi)$
$\theta \rightarrow$ angle from cylindrical coordinates $(0 \le \theta \le 2\pi)$

equations relating Spherical Coordinates to cartesian & Cylindrical Coordinates

$r = \rho \sin\phi$

$x = r\cos\theta = \rho\sin\phi\cos\theta$
$y = r\sin\theta = \rho\sin\phi\sin\theta$
$z = z = \rho\cos\phi$

$\phi \in [0, \pi]$
$\theta \in [0, 2\pi]$
ρ - always positive

Cylindrical Co-ordinates

Spherical Co-ordinates

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{r^2+z^2}$$

Problem

Find Spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$ — (1)

Sub $x = \rho\sin\phi\cos\theta$ & $z = \rho\cos\phi$
$y = \rho\sin\phi\sin\theta$

$$\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta + (\rho\cos\phi - 1)^2 = 1$$

$$\rho^2\sin^2\phi\left[\cos^2\theta + \sin^2\theta\right] + \rho^2\cos^2\phi + 1 - 2\rho\cos\phi$$

$$\rho^2\sin^2\phi + \rho^2\cos^2\phi - 2\rho\cos\phi$$

$\rho \cos\phi = 1$

$= 0$

$= 0$

$\rho^2 \sin^2\phi + \rho^2 \cos^2\phi - 2\rho\cos\phi$

$\rho^2 [\sin^2\phi + \cos^2\phi] - 2\rho\cos\phi$

$\rho^2 - 2\rho\cos\phi = 0$

$\rho(\rho - 2\cos\phi) = 0$

$\rho = 0$ (or) $\rho = 2\cos\phi$

$\rho = 0$ × $\rho > 0$

Hence $\boxed{\rho = 2\cos\phi}$