$2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$

Put $x+\frac{1}{x}=y$

$\Rightarrow x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=y^2-2$

$\therefore 2\left(y^2-2\right)-9y+14=0$

$\Rightarrow 2y^2-9y+10=0$

$\Rightarrow(2y-5)(y-2)=0$

$\Rightarrow y=\frac{5}{2}$ or 2

When $y=\frac{5}{2},\ \ x+\frac{1}{x}=\frac{5}{2}\Rightarrow x=2$ or $\frac{1}{2}$

When $y=2,\ \ x+\frac{1}{x}=2\ \Rightarrow x=1$

Hence $x=2,\frac{1}{2},1$

$\therefore$ The number of real values = 3

37. Ans: [a]

α,β are the roots of $x^2-x+p=0$

$\Rightarrow \alpha+\beta=1, \alpha\beta=p$

γ,δ are the roots of $x^2-4x+q=0$

$\Rightarrow \gamma+\delta=4, \gamma\delta=q$

Let r be the common ratio of the G.P..

Then $\frac{\gamma+\delta}{\alpha+\beta}=4\Rightarrow r^2=4\Rightarrow r=\pm 2$

When r = 2

We get $\alpha(1+r)=1\Rightarrow\alpha=\frac{1}{1+r}=\frac{1}{3}$

$p=\alpha\beta=\alpha\times\alpha r=\alpha^2 r=\frac{2}{9}$ which is not an integer

When $r=-2$,

$\alpha(1+r)=1\Rightarrow\alpha=-1$

$p=\alpha^2 r=-2$

$q=r\delta=\left(\alpha r^2\right)\left(\alpha r^3\right)=\alpha^2 r^5$

$=(-2)^5=-32$

$\therefore$ The values of p and q are $-2,-32$.

38. Ans: [a]

$ax^2+bx+c=0$

Let the roots be α and 3α.

$\therefore 3\alpha+\alpha=-\frac{b}{a}\Rightarrow 4\alpha=-\frac{b}{a}\Rightarrow\alpha=-\frac{b}{4a}$

$3\alpha^2=\frac{c}{a}$

$\Rightarrow 3\left(\frac{-b}{4a}\right)^2=\frac{c}{a}\Rightarrow\frac{3b^2}{16a^2}=\frac{c}{a}$

$\Rightarrow\frac{3b^2}{16a}=c$

$\Rightarrow 3b^2=16ac$

39. Ans: [b]

$\left(a^2+b^2\right)x^2-2b(a+c)x+\left(b^2+c^2\right)=0$

Since the roots are equal, the discriminant is equal to 0.

$4b^2(a+c)^2-4\left(a^2+b^2\right)\left(b^2+c^2\right)=0$

$\Rightarrow b^2a^2+2b^2ac+b^2c^2-a^2b^2-b^4-a^2c^2-b^2c^2=0$

$\Rightarrow 2b^2ac-b^4-a^2c^2=0$

$\Rightarrow b^4+a^2c^2-2b^2ac=0$

$\Rightarrow\left(b^2-ac\right)^2=0 \qquad \Rightarrow b^2=ac$

$\Rightarrow$ a, b, c are in G.P.

40. Ans: [c]

$x^2-(c+6)x+2(2c-1)=0$

Sum of the roots $=c+6$

Product of the roots $=2(2c-1)$

$c+6=\frac{1}{2}\times 2(2c-1)$

$\Rightarrow c+6=2c-1$

$\Rightarrow c=7$

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FUNCTIONS – I

1. Ans: [d]

$F(x)=max\,(2x+1,3-4x)$ is minimum when $2x+1=3-4x$

i.e.$6x=2$

$x=2/6=1/3$

therefore minimum possible value of f(x) is

$(2x+1)(at\ x=1/3)=2*1/3+1=5/3$

or (3 – 4x) at x = 1/3 = 3 – 4.1/3 = 5/3

2. Ans: [d]

$F(x)=ax2-b|x|$

$ax^2>0$ for $a>0$ and

$(-b|x|)>0$ for $b<0$

$F(x)=ax2-b|x|\ >0\ for\ x\neq 0$

$F(0)=ax2-b|x|=0\ for\ x=0$

If $x=0$ $f(x)$ is minimised whenever, $a>0, b<0$.

3. Ans: [a]

Min $\{f(x2),h(x)\}<3$

$f(x2)<3\ or\ h(x)<3$

$2x^2-1<3\ or\ x^2+x+1<3$

$X2 - 2 < 0$ or $x2 + x - 2 < 0$

$-\sqrt{2} < x < \sqrt{2}$ or $(x + 2)(x - 1) < 0$

$-\sqrt{2} < x < \sqrt{2}$ or $-2 < x < 1$

Both the above ranges for x, satisfy the inequality Min $\{f(x2), h(x)\} < 3$

$-2 < x < \sqrt{2}$ (as $-2 < -\sqrt{2}$ and $\sqrt{2} > 1$)

4. Ans: [b]

$F(x) = |x - 2| + |2.5 - x| + |3.6 - x|$

Sub. from the options, the values of x in the function,

when $x = 2.3$

$F(x) = 0.3 + 0.2 + 1.3 = 1.8$

when $x = 2.5$

$F(x) = 0.5 + 0 + 1.1 = 1.6$

When $x = 2.7$

$F(x) = 0.7 + 0.2 + 0.9 = 1.8$

Thus for any value of x, $f(x)$ will be greater than 1.6

→ f(x) is minimum at x = 2.5.

5. Ans: [b]

$F(x) = |3x - 2| + |2x - 3|$

$= 3|x - 2/3| + 2|x - 3/2|$

For $x \geq 3/2, f(x) = 3x - 2 + 2x - 3$

$= 5(x - 1)$

So minimum value $= 5(3/2 - 1) = 5/2$

For $3/2 \geq x \geq 2/3$,

$F(x) = 3x - 2 + 3 - 2x = x + 1$

So minimum value $= 2/3 + 1 = 5/3$

For $x \leq 2/3, f(x) = 2 - 3x + 3 - 2x$

$= 5(1 - x)$

So, minimum value $= 5(1 - 2/3) = 5/3$

So, minimum value of $f(x)$ is $5/3$.

6. Ans: [b]

Since $|x| \geq -x$ for any value of x,

$k - x \leq |x| + k$ for any value of x

$f(x) = |x| + k$ for any $x \geq k$ as $|x| \geq 0$

and $f(0) = k$

So, minimum value of $f(x)$ is k.

7. Ans: [b]

Given $f(x) = ax2 + bx + c$ $(a \neq 0)$.

3 is a root of $f(x)$

$9a + 3b + c = 0$ ... (1)

Also, $f(5) = -3f(2)$.

$25a + 5b + c = -3(4a + 2b + c)$

$= -12a - 6b - 3c$

$37a + 11b + 4c = 0$ ... (2)

From two equations $a - b = 0$ $a = b$

Thus we get $f(x) = ax2 + ax + c$

Dividing $f(x)$ by $x - 3$, we get $c = -12a$

$F(x) = ax2 + ax - 12a$

$F(x) = 0, -4$ is the second root.

8. Ans: [b]

$F(x) = x3 - 4x + p$

$F(0) = +p$ and

$F(1) = 1 - 4 + p = -3 + p$

$F(0)$ and $f(1)$ are of opposite signs.

If p is positive, $(p - 3)$ has to be negative and p has to take values less than 3 i.e. $0 < p < 3$.

9. Ans: [c]

For D_f, |x| - x > 0,|x| > x i.e., x < |x|

which is true if x < 0 .

D_f = (– ∝, 0).

10. Ans: [d]

Since $f(-x) = \log\frac{1-x}{1+x} = \log\left(\frac{1+x}{1-4}\right)^{-1} = -\log\frac{1+x}{1-x} = -f(x)$

∴ f(x) is odd.

11. Ans: [a]

Let $y = \frac{2x}{x^2+1} \Longrightarrow x^2y - 2x + y = 0$

Since x is real, discriminant $4 - 4y^2 \geq 0$

$1 - y^2 \geq 0$

$y^2 \leq 1$

$|y| \leq 1$

$-1 \leq y \leq 1$.

12. Ans: [d]

As f(x) = max (2x + 1, 3 – 4x)

We know that f(x) would be minimum at the point of intersection of these curves.

i.e., 2x + 1 = 3 – 4x

i.e., $6x = 2 \Longrightarrow x = \frac{1}{3}$

Hence, minimum value of f(x) is $\frac{5}{3}$.

13. Ans: [b]

Minimum possible value of any expression inside mod is zero. So we will check for x = 3, – 2 and x = 5. At x = 3 we will get minimum value, which is 7.

14. Ans: [a]

Using property (iii) with x = 1,

f(3) = f(1) + 12(1) + 12 = 1 +12 +12 = 25

since f(1) = 1 by property (i).

Using property (ii) with x = 3,

f(6) = 4f(3) + 6 = 4(25) + 6 = 106

Therefore, the value of f(6) is 106.

15. Ans: [b]

f(x) = |x – 2| + |2.5 – x| + |3.6 – x| attains minimum value when any of the terms = 0.

16. Ans: [b]

If a = 2, the function is constant.

17. Ans: [d]

g(x) = max (5 – x, x + 2)

We have to draw graph and the find the point of intersection.

y = 5 – x

y = x + 2

Hence at the point of intersection of two straight line.

Smallest of g(x) = 3.5

18. Ans: [d]

The denominator x^2 –3x + 2 has real roots. Hence the maximum value of the function f(x) will be infinity.

19. Ans: [d]

g(x + 1) + g(x – 1) = g(x)

g(x + 2) + g(x) = g(x + 1)

Adding these two equation, we get

g(x + 2) + g(x – 1) = 0

g(x + 3) + g(x) = 0 ... (1)

g(x + 4) + g(x + 1) = 0

g(x + 5) + g(x + 2) = 0

g(x + 6) + g(x + 3) = 0

g(x + 6) – g(x) =0 (From (1))

20. Ans: [d]

Given function = f(1) + f(2) + f(3) + f(4)+ .... = n^2 f(n)

Given f(1) = 3600

For n = 2,

f(1) + f(2) = 2^2f(2)

i.e. 2^2f(2) – f(2) = f(1)

f(2) = f(1)/(2^2–1) ... (1)

For n = 3

f(1) + f(2) + f(3) = 3^2f(3)

put the value of f(2) from (1)

→ f(1) + f(1)/(2^2 – 1) = 3^2f(3) – f(3)

→ f(1)+ f(1)/(2^2 – 1) = (3^2 – 1)f(3)

now take f(1) in left side

i.e. f(1) = [1 + 1/(2^2 – 1)] = f(3)(3^2 – 1)

i.e. f(3) = f(1) x 2^2/(2^2 – 1) x 1/(3^2 – 1)

f(3) = 600

Similarly

f(9) = f(1) x (2^2 x 3^2 x 4^2 .... 8^2)/((2^2 – 1)(3^2 – 1)(4^2 – 1) .... (9^2 – 1))

f(9)=80

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FUNCTIONS - II

1. Ans: [c]

$f(1) = a + b + 1 = 4$

$\therefore$ a + b = 3 ... (1)

$f(-2) = 4a - 2b + 1 = 1$

$\therefore$ 4a – 2b = 0 ... (2)

On solving (1) & (2), a = 1 & b = 2

So, the required equation is $x^2 + 2x + 1$.

2. Ans: [b]

Square root of a negative number is not a real number.

So its domain = $(0, \infty)$

Note: $\sqrt{0}$ = 0 and so it is included.

3. Ans: [d]

Let $f(x) = y$

$\Rightarrow \sqrt{16 - x^2} = y$, or $x = \pm\sqrt{16 - y^2}$

x is defined, when $16 - y^2 \ge 0$

$\Rightarrow (4 + y)(4 - y) \ge 0$

or $(y + 4)(y - 4) \le 0$

$\Rightarrow y \in [-4, 4]$

But $f(x)$ gives only non-negative values.

so $y \in [0, 4]$

4. Ans: [c]

For even function, $f(x) = f(-x)$

Option (c) satisfies this condition.

$f(-x) = e^{-3x} + e^{3x} = f(x)$

5. Ans: [b]

$fog(2) = f(\log 2) = e^{2\log 2}$

$= e^{\log 4} = 4$

6. Ans: [b]

Domain of f(x) = R – {0} (As at x = 0, f(x) is not defined)

Domain of g(x) is defined through out R.

Domain of h(x) is defined through out R.

g(x) and h(x) have the same domain.

7. Ans: [c]

$y = 5e^{\sqrt{x^2-1}} \log(x - 1)$

Since, $x^2 - 1 \ge 0$ and $x - 1 > 0$

$\Rightarrow (x + 1)(x - 1) \ge 0$ and $x > 1$

$\Rightarrow x \in R - (-1, 1)$ and $x > 1$

$\therefore$ domain is $(1, \infty)$

8. Ans: [b]

Total number of functions possible $= 3^4 = 81$

A's each element can be mapped to any of the 3 elements in B.

To find number of into functions,

Case i: Let A being mapped to any two elements of B.

So, number of into functions $= 2^4 - 2 = 14$

(Note: Each element of A can be mapped to 2 elements of B and 2 is subtracted to eliminate 2 function which are mapped to single element of B)

So, total number of into functions with element of A mapped to 2 elements of $B = \left(2^4 - 2\right) \times 3 = 14 \times 3 = 42$

Case ii: All elements of A mapped to one element of B.

Number of such into functions = 3

So, total number of onto functions = 81 – 42 – 3 = 36

9. Ans: [c]

Since $A \rightarrow B$ is 1 – 1, not onto, $\Rightarrow$ a < b

Since $B \rightarrow C$ is onto, not 1 – 1, $\Rightarrow$ b > c

Since $C \rightarrow A$ is 1 – 1, not onto, $\Rightarrow$ c < a

So, c < a < b

10. Ans: [b]

$$\frac{x^2+6x+6}{x^2+6x+12} = \frac{x^2+6x+12-6}{x^2+6x+12}$$

$$= 1 - \frac{6}{x^2+6x+12}$$

Range of $x^2+6x+12$

$$= x^2+6x+9+3 = (x+3)^2+3$$

Range of $(x+3)^2+3 = [3, \infty]$

Range of $\frac{6}{(x+3)^2+3} = [0, 2]$

Range of $f(x) = 1 - [0, 2]$

$= [-1, 1]$

11. Ans: [b]

$f(3) = 27 - 18 + 3 + 1 = 13$

$f(13) = 13^3 - 2 \times 13^2 + 13 + 1 = 1873$

12. Ans: [a]

f(x) is not defined for x = 2.

So domain of f(x) = R – {2}

13. Ans: [c]

$f(x)$ is not defined for $x^2 - 25 < 0$

$\Rightarrow (x+5)(x-5) < 0$ (not defined)

$\Rightarrow (x+5)(x-5) \geq 0$ (defined)

$\Rightarrow x \leq -5$ & $x \geq 5$

$\Rightarrow$ Defined in R – (– 5, 5)

14. Ans: [c]

$(fg)x = f(x) \cdot g(x) = e^x \cdot \log x$

$\therefore (fg)(1) = e^1 \cdot \log 1 = 0$

15. Ans: [b]

f(x) is defined only when $x^2 - 3x > 0$ and $x^2 - 3x \neq 0$

or $x^2 - 3x > 0$, $x(x-3) > 0$ and $x \neq 0$ or 3

$\Rightarrow x \notin [0,3]$

$\Rightarrow x \in R - [0, 3]$

16. Ans: [a]

$h(3) = 9$

$goh(3) = g(9) = \frac{1}{9}$

$fogoh(3) = f\left(\frac{1}{9}\right) = \frac{1}{9} + 2 = \frac{19}{9} = 2\frac{1}{9}$

17. Ans: [c]

$f(x) = \frac{|x+3|}{x+3}$

Since, $x + 3 \neq 0$, domain of $f(x) = R - \{-3\}$

18. Ans: [a]

$fog(-x) = f(-g(x)) = fog(x)$

so fog is an even function.

19. Ans: [a]

b – $f(x) = x^5$ is not onto function as $3 \in z$ does not have a preimage in z.

c – $f(x) = 3x + 2$ is not onto function as numbers of the form 3x + 1, 3x do not have pre image in z.

d – $f(x) = x^2 + x + 1$ is not a 1 – 1 function as $f(-4) = f(3) = 13$ but $-4 \neq 3$

$a - f(x) = x + 5$ is a 1 – 1 and on to function.

20. Ans: [d]

$fog(x) = f\left(\sqrt{1-x}\right) = \sqrt{3 - \sqrt{1-x}}$

For fog to be defined, $1 - x \geq 0$ and $3 - \sqrt{1-x} \geq 0$

$1 \geq x$ and $x \geq -8$ $\quad \Rightarrow -8 \leq x \leq 1$