An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 composition resistors and 2 ceramic capacitors connected in series configuration. The hourly failure rate of each component is given below:

Silicon transistor	:	$\lambda_t = 4 \times 10^{-5}$
Silicon diode	:	$\lambda_d = 3 \times 10^{-5}$
Composition resistor	:	$\lambda_r = 2 \times 10^{-4}$
Ceramic capacitor	:	$\lambda_c = 2 \times 10^{-4}$

Calculate the reliability of the circuit for 10 hours, when the components follow exponential distribution.

Since the components are connected in series, the system (circuit) reliability is given by

$$R_s(t) = R_1(t) \cdot R_2(t) \cdot R_3(t) \cdot R_4(t)$$

$$= e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdot e^{-\lambda_3 t} \cdot e^{-\lambda_4 t}$$

$$= e^{-(5\lambda_t + 3\lambda_d + 10\lambda_r + 2\lambda_c)t}$$

$$\therefore \quad R_s(10) = e^{-(20 \times 10^{-5} + 9 \times 10^{-5} + 20 \times 10^{-4} + 4 \times 10^{-4}) \times 10}$$

$$= e^{-(20 + 9 + 200 + 40) \times 10^{-4}}$$

$$= e^{-0.0269} = 0.9735$$

Thermocouples of a particular design have a failure rate of 0.008 per hour. How many thermocouples must be placed in parallel if the system is to run for 100 hours with a system failure probability of no more than 0.05? Assume that all failures are independent.

If T is the time to failure of the system, it is required that

$$P(T \leq 100) \leq 0.05$$

i.e., $$1 - R_p(100) \leq 0.05 \tag{1}$$

Let the number of thermocouples to be connected in parallel be n.

Then $$R_p(t) = 1 - (1 - R)^n \tag{2}$$

where R is the reliability of each couple.

The failure rate of each couple = 0.008 (constant)

$\therefore$ $$R = e^{-0.008t} \tag{3}$$

Using (3) in (2), we have

$$1 - R_p(t) = (1 - e^{-0.008\,t})^n$$

$\therefore$ $$1 - R_p(100) = (1 - e^{-0.8})^n \tag{4}$$

Using (4) in (1), we have

$$(1 - e^{-0.8})^n \leq 0.05$$

i.e., $$(0.55067)^n \leq 0.05 \tag{5}$$

By trials, we find that (5) is not satisfied when $n = 0, 1, 2, 3, 4$ and 5.

When $n = 6$, $(0.55067)^n = 0.02788 < 0.05$

Hence 6 thermocouples must be used in the parallel configuration.

Six identical components with constant failure rates are connected in (a) high-level redundancy with 3 components in each subsystem (b) low level redundancy with 2 components in each subsystem. Determine the component MTTF in each case, necessary to provide a system reliability of 0.90 after 100 hours of operation.

Let λ be the constant failure rate of each component. Then $R = e^{-\lambda t}$, for each component. For high level redundancy,

$$R_s(t) = 1 - [1 - \{R(t)\}^3]^2$$

$$= 1 - (1 - e^{-3\lambda t})^2$$

$$\therefore \quad R_s(100) = 1 - (1 - e^{-300\lambda})^2 = 0.90$$

i.e., $(1 - e^{-300\lambda})^2 = 0.1$

i.e., $1 - e^{-300\lambda} = 0.31623$

$$\therefore \quad e^{-300\lambda} = 0.68377$$

$$\therefore \quad 300\lambda = 0.38013$$

$$\therefore \text{ MTTF of each component} = \frac{1}{\lambda} = \frac{300}{0.38013} = 789.2 \text{ hours.}$$

For low level redundancy

$$R_s(t) = [1 - \{1 - R(t)\}^2]^3$$

$$= [1 - \{1 - e^{-\lambda t}\}^2]^3$$

$$\therefore \quad R_s(100) = [1 - \{1 - e^{-100\lambda}\}^2]^3 = 0.90$$

i.e., $1 - (1 - e^{-100\lambda})^2 = 0.96549$

$$\therefore \quad (1 - e^{-100\lambda})^2 = 0.03451$$

$$\therefore \quad 1 - e^{-100\lambda} = 0.18577$$

$$\therefore \quad e^{-100\lambda} = 0.81423$$

$$\therefore \quad 100\lambda = 0.20551$$

$$\therefore \text{ MTTF of each component} = \frac{1}{\lambda} = \frac{100}{0.20551} = 486.6 \text{ hours.}$$

A computerised airline reservation system has a main computer online and a secondary standby computer. The online computer fails at a constant rate of 0.001 failure per hour and the standby unit fails when on-line at the constant rate of 0.005 failure per hour. There are no failures while the unit is in the standby mode.

(a) Determine the system reliability over a 72 hours period.

(b) The airline desires to have a system MTTF of 2000 hours. Determine the minimum MTTF of the main computer to achieve this goal, assuming that the standby computer MTTF does not change.

(a) $\lambda_1 = 0.001/\text{hour}$; $\lambda_2 = 0.005/\text{hour}$

$$R_s(t) = \frac{1}{\lambda_2 - \lambda_1} [\lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_2 t}\}$$

$$= \frac{1}{0.004} \{0.005\ e^{-0.001t} - 0.001\ e^{-0.005t}\}$$

$$\therefore \quad R_s(72) = \frac{1}{4}\{5\, e^{-0.072} - e^{-0.360}\}$$

$$= 0.9887$$

(b) $$\text{MTTF} = \int_0^{\infty} R(t)\, dt$$

$$= \frac{1}{\lambda_2 - \lambda_1} \int_0^{\infty} (\lambda_2\, e^{-\lambda_1 t} - \lambda_1\, e^{-\lambda_2 t})\, dt$$

$$= \frac{1}{\lambda_2 - \lambda_1}\left(\frac{\lambda_2}{\lambda_1} - \frac{\lambda_1}{\lambda_2}\right) = \frac{\lambda_2 + \lambda_1}{\lambda_1 \lambda_2} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

The requirement is $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = 2000$

$$\therefore \quad \frac{1}{\lambda_1} = 2000 - \frac{1}{0.005}$$ ($\because$ MTTF and hence λ_2 do not change for standby unit)

$$= 1800$$

i.e., MTTF of the main computer should be increased to 1800 hours.

A fuel pump with an MTTF of 3000 hours is to operate continuously on a 500 hour mission.

(a) What is the mission reliability?

(b) Two such pumps are put in standby parallel configuration. If there are no failures of the back up pump while in standby mode, what are the system MTTF and the mission reliability?

(c) If the standby failure rate is75% of that of the main pump (when operational), what are the system MTTF and the mission reliability?

(a) $\text{MTTF} = \dfrac{1}{\lambda} = 3000 \qquad \therefore \lambda = \dfrac{1}{3000}$

$R(t) = e^{-\lambda t} \qquad \therefore R(500) = e^{-(500/3000)} = 0.8465.$

(b) $\lambda_1 = \lambda_2 = \lambda = \dfrac{1}{3000}$

$$R_s(t) = (1 + \lambda t)e^{-\lambda t}$$

$$\therefore R_s(500) = \left(1 + \frac{500}{3000}\right) e^{-(500/300)} = 0.9876$$

$$\text{MTTF} = \frac{2}{\lambda} = 6000 \text{ hours}$$

(c) $\lambda_1 = \dfrac{1}{3000}$; $\lambda_2 = \dfrac{3}{4} \times \dfrac{1}{3000} = \dfrac{1}{4000}$

$$R_s(t) = \frac{1}{\lambda_2 - \lambda_1} (\lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_2 t})$$

$$= 12{,}000 \times \left(\frac{1}{3000} e^{-\frac{1}{4000}t} - \frac{1}{4000} e^{-\frac{1}{3000}t} \right)$$

$$\therefore R_s(500) = (4\, e^{-1/8} - 3\, e^{-1/6})$$

$$= 0.9905$$

$$\text{MTTF} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = 7000 \text{ hours.}$$