19BCB0025
Kumar Sparsh

Digital Assignment - 1

MAT2001 - Statistics for Engineers

1. Find the mean, median, mode for the given data:

(i)

Wages	No. of workers (f)
2000-3000	3
3000-4000	5
4000-5000	20
5000-6000	10
6000-7000	5
	$\Sigma f = 43$

(ii)

c.I.	f
20-40	6
40-60	9
60-80	11
80-100	14
100-120	20
120-140	15
140-160	10
160-180	8
180-200	7
	$\Sigma f = 100$

20/01/2020

Sol:- (i)

class interval	f_i	x_i	cf
2000-3000	3	2500	3
3000-4000	5	3500	8
4000-5000	20	4500	28
5000-6000	10	5500	38
6000-7000	5	6500	43
	$\Sigma f_i = 43$		

$$\bar{x}\,(\text{Mean}) = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{7500 + 17500 + 90000 + 55000 + 32500}{43}$$

$$= 202500/43 = 4709.3$$

Median class:

$$\frac{N}{2} = \frac{43}{2} = 21.5$$

$\Rightarrow$ median class: 4000-5000

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - cf\right)$$

$$= 4000 + \frac{1000}{20}\left(\frac{43}{2} - 8\right)$$

$$= 4000 + \frac{50}{2}(43 - 16) = 4000 + 25 \times 27$$

$$= \underline{\underline{4675}}$$

$$\text{Mode} = l + \frac{h(f_m - f_1)}{(2f_m - f_1 - f_2)} = 4000 + \frac{1000(20-5)}{(40-5-10)}$$

Modal class: 4000-5000

$$= 4000 + \frac{\overset{200}{\cancel{1000}} \times \cancel{15}^{\,3}}{\cancel{25}\,\cancel{5}}$$

$$= \underline{\underline{4600}}$$

(ii)

C.I.	x_i	f_i	cf
20-40	30	6	6
40-60	50	9	15
60-80	70	11	26
80-100	90	14	40
100-120	110	20	60
120-140	130	15	75
140-160	150	10	85
160-180	170	8	93
180-200	190	7	100 = N

Median class → 100-120 (Modal class)

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{11000}{100} = \underline{\underline{110}}$$

$\because \frac{N}{2} = 50$

$\Rightarrow$ Median

$$= l + \frac{h}{f}\left(\frac{N}{2} - cf\right)$$

$$= 100 + \frac{20}{20}(50 - 40)$$

$$= \underline{\underline{110}}$$

$$\text{Mode} = l + \frac{h(f_m - f_1)}{(2f_m - f_1 - f_2)} = 100 + \frac{20(20-14)}{(40-14-15)}$$

$$= 100 + \frac{20 \times 6}{11}$$

$$= \underline{\underline{110.91}}$$

2. Calculate mean deviation, standard deviation, Quartile deviations for :-

(i)

	Age (in years)	No. of members (f_i)	x_i	cf	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
	20-30	3	25	3	-29.7	883.1
	30-40	61	35	64	-19.7	388.1
Q_1 class →	40-50	132	45	196	-9.7	94.1
	50-60	153	55	349	0.3	0.1
Q_3 class →	60-70	140	65	489	10.3	106.1
	70-80	51	75	540	20.3	412.1
	80-90	2	85	542 = N	30.3	918.1

Sol:-

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{29660}{542} = 54.72$$

$$\text{Mean deviation} = \frac{\sum f_i |x_i - A|}{\sum f_i} = \frac{5155}{542} = \underline{\underline{9.51}}$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{1}{N} \sum f_i (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{76467.2}{542}} = \sqrt{141.08} = \underline{\underline{11.88}}$$

$$Q_{01} = l + \frac{h}{f}\left(\frac{(1)N}{4} - cf\right)$$

$$\frac{N}{4} = \frac{542}{4} = 135.5$$

$$\frac{3N}{4} = 406.5$$

$$= 40 + \frac{10}{132}(135.5 - 64)$$

$$= \underline{45.42}$$

$$Q_3 = 60 + \frac{10}{140}(406.5 - 349)$$

$$= \underline{64.11}$$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{64.11 - 45.42}{2} = \underline{9.345}$$

(ii) Find quartile deviation & coeff of skewness:-

Size	f_i	x_i	cf	
4-8	6	6	6	
8-12	10	10	16	
12-16	18	14	34	← Q_1 class
16-20	30	18	64	← Q_2 class
20-24	15	22	79	
24-28	12	26	91	← Q_3 class
28-32	10	30	101	
32-36	6	34	107	
36-40	2	38	109 = N	

Sol^n: $\frac{N}{4} = 27.25$

$\frac{2N}{4} = 54.5$

$\frac{3N}{4} = 81.75$

$$Q_1 = l + \frac{h}{f}\left(\frac{iN}{4} - cf\right) = 12 + \frac{4}{18}(27.25 - 16)$$

$$Q_1 = 14.5$$

$$Q_3 = l + \frac{h}{f}\left(\frac{iN}{4} - cf\right) = 24 + \frac{4}{12}(81.75 - 79)$$

$$Q_3 = 24.92$$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{24.92 - 14.5}{2} = 5.21$$

$$Q_2 = 16 + \frac{4}{30}(54.5 - 34) = 18.73$$

By Bowley's coeff^n of skewness,

$$S_k = \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1} = 0.188$$ = +vely skewed distribution

Ans.

3. Find the mean, standard deviation, coeff of variation of breaking strength of 80 test pieces of certain alloy from the following table:

Breaking strength	No. of pieces (f_i)	x_i	cf	$(x_i-\bar{x})^2$
44-46	3	45	3	16.2
46-48	24	47	27	4.1
48-50	27	49	54	0.0006
50-52	21	51	75	3.9
52-54	5	53	80=N	15.8

Soln:-

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3922}{80} = 49.025$$

$$S.D. = \sqrt{\frac{1}{N}\sum f_i (x_i-\bar{x})^2} = \sqrt{\frac{1}{80}(307.9)} = \sqrt{3.84875}$$

$$\therefore \sigma = S.D. = \underline{1.962}$$

$$\text{coeff of variation} = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.962}{49.025} \times 100$$

$$= \underline{4.002\%}$$

4. Probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width is given below:

x :	0	1	2	3	4
$P(x)$:	0.41	0.37	0.16	0.05	0.01

Find mean,
variance of imperfections per 10 meters of fabric?

Sol:

$$\text{Mean} = \sum_{x=0}^{4} x P(x) = 0(0.41) + 1(0.37) + 2(0.16) + 3(0.05) + 4(0.01)$$

$$= E(x) = \underline{\underline{0.88}}$$

$$\text{Variance} = E(x^2) - ((E(x))^2) = Var(x)$$

$$E(x^2) = \sum_{x=0}^{4} x^2 P(x) = 0^2(0.41) + 1^2(0.37) + 2^2(0.16) + 3^2(0.05) + 4^2(0.01)$$

$$= 0.37 + 0.64 + 0.45 + 0.16$$

$$\underline{\underline{Var(x) = 1.62}}$$

Ans.

5. Impurities in the batch of final product of a chemical process often reflect a serious problem. From considerable plant data gathered, it is known that the proportion Y of impurities in a batch has density func^n given by:

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Verify that above is a valid density func.
(ii) A batch is considered not sellable and then not acceptable if the percentage of impurities exceeds 60%. With the current quality of process, what is the percentage of batches that are not acceptable?

Sol:-

Conditions for a valid density function:-

i) $f(y) \geq 0 \quad \forall x \in R$

ii) $\int_{-\infty}^{\infty} f(y)\,dy = 1$

Cond^n (i) is satisfied as given.

For cond^n (ii),

$$\int_{-\infty}^{\infty} f(y)\,dy = \int_0^1 10(1-y)^9\,dy \qquad \begin{aligned} 1-y &= t \\ -dy &= dt \end{aligned}$$

$$= -\int_1^0 10\,t^9\,dt = \int_0^1 10t^9\,dt = \left[t^{10}\right]_0^1$$

$$= 1$$

$\Rightarrow$ cond^n (ii) is also satisfied.

Unacceptablity requires $\geq 60\%$ impurity.

So, proportion of batch that is not acceptable

$$= \int_{0.6}^{1} 10(1-y)^9 \, dy = P(0.6 \leq Y \leq 1)$$

$$= \left[t^{10}\right]_{0}^{0.4} = \left(\frac{4}{10}\right)^{10}$$

Percentage of unacceptable batches $= \left(\frac{4}{10}\right)^{10} \times 10^2 = \frac{4^{10}}{10^8} = \underline{0.0105\%}$

6. (i) If the joint probability distribution of X and Y are given by:

$$f(x,y) = \frac{(x+y)}{30} \quad \text{for } x = 0, 1, 2, 3; \; y = 0, 1, 2.$$

find: (a) $P(X \leq 2, Y = 1)$

(b) $P(X > 2, Y \leq 1)$

(c) $P(X > Y)$

(d) $P(X + Y = 4)$

Sol:

(a) $P(X \leq 2, Y = 1) = \sum_{x=0}^{3} \left(\frac{x+1}{30}\right) = \frac{1+2+3+4}{30}$

$$= \underline{1/3}$$

(b) $P(X > 2, Y \leq 1) = P(X = 3, Y \leq 1)$

$$= \sum_{y=0}^{1} \left(\frac{y+3}{30}\right) = \frac{3+4}{30} = \frac{7}{30}$$

(c) $P(X > Y) = \cancel{P(X=0, Y>0) + P(X=1, Y>1) + P(X=2, Y>2) + P(X=3, Y>3)}$

$$= P(Y < X) = P(Y=0, X>0) + P(Y=1, X>1) + P(Y=2, X>2)$$

$$= \sum_{x=1}^{3} \left(\frac{x}{30}\right) + \sum_{x=2}^{3} \left(\frac{x+1}{30}\right) + \sum_{x=3}^{3} \left(\frac{x+2}{30}\right)$$

$$= \left(\frac{1}{30} + \frac{2}{30} + \frac{3}{30}\right) + \left(\frac{3}{30} + \frac{4}{30}\right) + \left(\frac{5}{30}\right)$$

$$= \frac{18}{30} = \frac{3}{5}$$

(d) Following ordered pairs are possible for $X+Y=4$

$(x,y) = (2,2), (3,1)$

$\therefore P(X+Y=4) = P(X=2, Y=2) + P(X=3, Y=1)$

$$= \frac{2+2}{30} + \frac{3+1}{30} = \frac{8}{30} = \frac{4}{15}$$

Ans.

(ii) joint distribution of X and Y is given by

$$P(x,y) = \frac{2x+3y}{72}, \quad x = 0, 1, 2; \quad y = 1, 2, 3$$

(a) find all marginal distributions of X and Y
(b) find the conditional distribution of X given Y = 1.

Sol:- (a)

X \ Y	Y = 1	Y = 2	Y = 3
X = 0	$\frac{3}{72} = \frac{1}{14}$	$\frac{6}{72} = \frac{1}{12}$	$\frac{9}{72} = \frac{1}{8}$
X = 1	5/72	8/72 = 1/9	11/72
X = 2	7/72	10/72	13/72

$$P(X=0) = \sum_{Y=1}^{3} \left(\frac{2x+3y}{72}\right) = \frac{3+6+9}{72} = \frac{18}{72} = \frac{1}{4}$$

$$P(X=1) = \sum_{Y=1}^{3} P(x,y) = \frac{5+8+11}{72} = \frac{24}{72} = \frac{1}{3}$$

$$P(X=2) = \sum_{Y=1}^{3} P(x,y) = \frac{7+10+13}{72} = \frac{30}{72} = \frac{5}{12}$$

$$P(Y=1) = \sum_{X=0}^{2} P(x,y) = \frac{3+5+7}{72} = \frac{15}{72} = \frac{5}{24}$$

$$P(Y=2) = \frac{6+8+10}{72} = \frac{24}{72} = \frac{1}{3}$$

$$P(Y=3) = \frac{9+11+13}{72} = \frac{33}{72} = \frac{11}{24}$$ Ans.

(b) $P(X=x_i / Y=1)$:- $\cancel{\frac{P(X=x_i, Y=1)}{P(Y=1)} = \frac{P(X=0, Y=1)+P(X=1, Y=1)+P(X=2, Y=1)}{P(Y=1)}}$

$$\cancel{= \frac{\left(\frac{18}{72}\right)}{\left(\frac{15}{72}\right)}}$$

$$P(X=0/Y=1) = \frac{P(X=0, Y=1)}{P(Y=1)} = \frac{\left(\frac{3}{72}\right)}{\left(\frac{15}{72}\right)} = \frac{1}{5}$$

$$P(X=1/Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{(5/72)}{(15/72)} = \frac{1}{3}$$

$$P(X=2/Y=1) = \frac{P(X=2, Y=1)}{P(Y=1)} = \frac{(7/72)}{(15/72)} = \frac{7}{15}$$ Ans.

7. (i) A fast food restaurant operates both a drive through facility and a walk-in facility. On a randomly selected day, let X and Y, respectively be the proportions of the time that the drive-through and walk-in facilities are in use, and suppose that the joint density function of these random variables is:

$$f(x,y) = \left(\frac{2}{3}\right)(x+2y), \quad \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}$$

(a) Find marginal density of X.
(b) Find marginal density of Y.

Sol:- (a) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 \frac{2}{3}(x+2y)\,dy$

$$= \left[x[y]_0^1 + [y^2]_0^1 \right] \frac{2}{3} = \frac{2}{3}(x+1)$$

(b) $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 \frac{2}{3}(x+2y)\,dx$

$$= \frac{2}{3}\left[\left[\frac{x^2}{2}\right]_0^1 + 2y[x]_0^1 \right] = \frac{2}{3}\left(2y + \frac{1}{2}\right)$$ Ans.

(ii) Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pounds per square inch (psi). Let X denote the actual air pressure for the right tire & Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density func^n:

$$f(x,y) = \begin{cases} k(x^2+y^2), & 30 \le x < 50, \\ & 30 \le y < 50 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) find k.

(b) find $P\begin{pmatrix} 30 \le X \le 40, \\ 40 \le Y \le 50 \end{pmatrix}$

(c) find the probability that both tires are under filled.

Sol^n,-

(a) $\because$ Total probability $= 1$

$$\Rightarrow \int_{30}^{50}\int_{30}^{50} k(x^2+y^2)\,dy\,dx = k\int_{30}^{50}\left[x^2y + \frac{y^3}{3}\right]_{30}^{50} dx$$

$$= k\int_{30}^{50}\left(20x^2 + \frac{98\times10^3}{3}\right)dx$$

$$= k\left[20\frac{x^3}{3} + \frac{98\times10^3}{3}x\right]_{30}^{50} = k\left[20\times\frac{98\times10^3}{3}\times 2\right] = 1$$

$$\Rightarrow k = \frac{3\times10^{-4}}{4\times98} = \frac{3\times10^{-4}}{4\times98}$$

$$\begin{array}{r} 864 \\ -27 \\ \hline 37 \end{array} \quad \begin{array}{r} 125 \\ -27 \\ \hline 98 \end{array} \quad \begin{array}{r} 125 \\ -64 \\ \hline 61 \end{array}$$

15

(b) $$P\begin{pmatrix} 30 \leq X \leq 40 \\ 40 \leq Y \leq 50 \end{pmatrix} = \int_{30}^{40}\int_{40}^{50} k\,(x^2+y^2)\,dy\,dx$$

$$= k\int_{30}^{40} \left[\frac{y^3}{3} + x^2 y\right]_{40}^{50} dx$$

$$= k\int_{30}^{40} \left(\frac{61\times10^3}{3} + 10x^2\right) dx$$

$$= k\left[\frac{61\times10^3}{3}x + \frac{10}{3}x^3\right]_{30}^{40}$$

$$= k\left[\frac{61\times10^4}{3} + \frac{10}{3}\times37\times10^3\right]$$

$$= k\times2\times\frac{98}{3}\times10^4 = \left(\frac{\cancel{3}\times\cancel{10^{-4}}}{4\times\cancel{98}}\right)\left(\frac{2\times\cancel{98}\times\cancel{10^4}}{\cancel{3}}\right)$$

$$P\begin{pmatrix} 30 \leq X \leq 40, \\ 40 \leq Y \leq 50 \end{pmatrix} = \frac{1}{2}$$

Probability that,

(c) Both tires are underfilled

$$= P\begin{pmatrix} 30 \leq X < 40, \\ 30 \leq Y < 40 \end{pmatrix} = \int_{30}^{40}\int_{30}^{40} k(x^2+y^2)\,dy\,dx$$

$$= \int_{30}^{40} k\left[x^2 y + \frac{y^3}{3}\right]_{30}^{40} dx = k\int_{30}^{40}\left(10x^2 + \frac{37\times 10^3}{3}\right)dx$$

$$= k\left[\frac{10}{3}x^3 + \frac{37\times 10^3}{3}x\right]_{30}^{40} = \frac{3\times 10^{-4}}{98\times 4}\left[\frac{10^4}{3}\times 37\times 2\right] = \left(\frac{37}{98}\right)\frac{1}{2}$$

$$P\left(\text{both tires are under filled}\right) = \frac{37}{196}$$ Ans.

8. The joint probability density function of X and Y is

$$\cancel{f(x,y) = \begin{cases} k(4-x-y) & ; \begin{matrix} 0 \le x \le 2 \\ 0 \le y \le 2 \end{matrix} \end{cases}}$$

$$f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)} & , 0 \le x \le \infty \\ 0 & , 0 \le y \le \infty \end{cases}$$

Verify whether X & Y are independent random variables?

Sol^n:-

$$f_X(x) = \int_0^\infty f(x,y)\,dy = 0$$

$$f_Y(y) = \int_0^\infty f(x,y)\,dx = \int_0^\infty 4xy \cdot e^{-(x^2+y^2)}\,dx$$

$$= 4\int_0^\infty (x \cdot e^{-x^2})(y \cdot e^{-y^2})\,dx = 4ye^{-y^2}\int_0^\infty x \cdot e^{-x^2}\,dx$$

$x^2 = t$

$2x\,dx = dt$

$$= 2ye^{-y^2}\int_0^\infty e^{-t}\,dt$$

$$= -2ye^{-y^2}\left[e^{-t}\right]_0^\infty$$

$$= -2ye^{-y^2}(0-1)$$

$$f_Y(y) = 2ye^{-y^2}$$

$$f(x,y) = \int_0^\infty\int_0^\infty 4xye^{-x^2} \cdot e^{-y^2}\,dy\,dx$$

$$= \int_0^\infty 2ye^{-y^2}\,dy = \int_0^\infty e^{-t}\,dt = -e^{-t}$$

$$= \left[-e^{-t}\right]_0^\infty$$

$$= \left[e^{-t}\right]_\infty^0 = 1$$

$\because f(x,y) \neq f_X(x) \cdot f_Y(y)$

$\Rightarrow$ X and Y are not independent random.

Ans.

9. Two random variables X and Y have the following joint probability density functions:-

$$f(x,y) = \begin{cases} k(4-x-y), & 0 \le x \le 2, \; 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases};$$

Find (i) k;
(ii) marginal density of X and Y;
(iii) conditional density of X and Y;
(iv) Var(X);
(v) Var(Y);
(vi) cov(X, Y).

Sol^n:-

(i) $\because$ Total probability = 1

$$\therefore \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^2\int_0^2 k(4-x-y)\,dy\,dx = \int_0^2 \left(4kx - k\frac{x^2}{2} - kyx\right)_0^2 dy$$

$$= k\int_0^2 \left(4x - \frac{x^2}{2} - yx\right)_0^2 dy = k\int_0^2 (8-2-2y)\,dy$$

$$= 2k\int_0^2 (3-y)\,dy = 2k\left[3[y]_0^2 - \left[\frac{y^2}{2}\right]_0^2\right]$$

$$= 2k\left[6-2\right] = 2k \times 4 = 8k = 1.$$

$$\Rightarrow \underline{\underline{k = 1/8}}$$

(ii) $f_X(x) = \int_0^2 f(x,y)\,dy = k\int_0^2 (4-x-y)\,dy$

$$= \frac{1}{8}\left[6-2x\right] = \underline{\underline{\frac{3-x}{4}}} \; ; \quad 0 \le x \le 2$$

$$f_Y(y) = \int_0^2 f(x,y)\,dx = \frac{1}{8}(6-2y) = \underline{\underline{\frac{3-y}{4}}} \; ; \quad 0 \le y \le 2$$

(iii) $f(x/y) = \dfrac{f(x,y)}{f_Y(y)} = \dfrac{\frac{1}{8}(4-x-y)}{\frac{1}{8}(6-2y)} = \underline{\underline{\dfrac{(4-x-y)}{2(3-y)}}}$

$$f(y/x) = \frac{f(x,y)}{f_X(x)} = \frac{\frac{1}{8}(4-x-y)}{\frac{1}{8}(6-2x)} = \underline{\underline{\frac{(4-x-y)}{2(3-x)}}}$$

(iv) $Var(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \Rightarrow E(x) = \int_0^2\int_0^2 \frac{x(4-x-y)}{8}\,dx\,dy = \frac{20}{24} = \frac{5}{6}$$

Ans.

$$\therefore E(x) = \int_0^2 x f_X(x)\,dx =$$

Similary, $\boxed{E(X) = E(Y) = \frac{5}{6}}$

$$E(X^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dx\,dy$$

$$= \int_0^2\int_0^2 x^2\frac{(4-x-y)}{8}\,dy\,dx = \frac{1}{8}\int_0^2\left(\frac{32}{3} - 8y - 4\right)dy = 1$$

$\therefore \boxed{E(X^2) = E(Y^2) = 1}$

Now,

$$Var(X) = E(X^2) - (E(X))^2 = 1 - \left(\frac{5}{6}\right)^2$$

$$= 1 - \frac{25}{36} = \frac{11}{36}$$

Similarly, $\boxed{Var(Y) = Var(X) = \frac{11}{36}}$ Ans.

$\because (E(X^2) = E(Y^2)$ and $E(X) = E(Y))$

(v) $cov(X, Y)$:

covariance between X and Y

$$cov(X, Y) = \int_0^2\int_0^2 (x-\bar{x})(y-\bar{y})\, f(x,y)\,dx\,dy$$

$$= \int_0^2 \int_0^2 (x - E(x))(y - E(y)) f(x,y)\, dx\, dy$$

$$= \int_0^2 \int_0^2 \left(x - \frac{1}{8}\right)\left(y - \frac{1}{8}\right) f(x,y)\, dx\, dy$$

$$= \int_0^2 \int_0^2 \left(x - \frac{1}{8}\right)\left(y - \frac{1}{8}\right)\left(\frac{4-x-y}{8}\right) dx\, dy$$

$$= \iint \frac{(8x-1)(8y-1)(4-x-y)}{8^3}\, dx\, dy = \frac{91}{192}$$?

Ans.

$$\mathrm{cov}(X,Y) = E(XY) - E(X) \cdot E(Y)$$

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy\, f(x,y)\, dy\, dx$$

$$= \frac{1}{8} \int_0^2 \int_0^2 xy\,(4-x-y)\, dy\, dx$$

$$E(X) = \frac{1}{8} \int_0^2 \int_0^2 x(4-x-y)\, dy\, dx$$

$$E(Y) = \frac{1}{8} \int_0^2 \int_0^2 y(4-x-y)\, dy\, dx$$

(v) $\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$

$$E(XY) = \int_0^2\int_0^2 xy\, f(x,y)\, dy\, dx = \int_0^2\int_0^2 \frac{xy}{8}(4-x-y)\, dy\, dx$$

$$= \left(\frac{1}{8}\right)\int_0^2\int_0^2 (4xy - x^2y - xy^2)\, dy\, dx = \frac{1}{8}\int_0^2 \left[\frac{4xy^2}{2} - \frac{x^2y^2}{2} - \frac{xy^3}{3}\right]_0^2 dx$$

$$= \frac{1}{8}\int_0^2 \left(8x - 2x^2 - \frac{8}{3}x\right) dx = \int_0^2 \left(x - \frac{x^2}{4} - \frac{x}{3}\right) dx = \int_0^2 \left(\frac{2x}{3} - \frac{x^2}{4}\right) dx$$

$$= \left[\frac{x^2}{3}\right]_0^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$E(X) = \int_0^2\int_0^2 x\, f(x,y)\, dy\, dx = \frac{5}{6}$$

$$E(Y) = \int_0^2\int_0^2 y\, f(x,y)\, dx\, dy = 5/6$$

So, $\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$

$$= \frac{2}{3} - \frac{5}{6} \times \frac{5}{6} = \frac{2}{3} - \frac{25}{36} = \frac{24-25}{36}$$

$$= \frac{-1}{36}$$

Ans.