CSE1003
Digital Logic and Design

Module 2
BOOLEAN ALGEBRA L7

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Module 2
BOOLEAN ALGEBRA 8 hrs

Boolean algebra

- Properties of Boolean algebra
- Boolean functions
- Canonical and Standard forms
- Logic gates - Universal gates
- Karnaugh map - Don’t care conditions
- Tabulation Method

Tabulation Method

- Also known as the *Quine-McCluskey method.*
- *It is a straightforward, systematic method for producing a minimal function that is less dependent on the designer's ability to recognize patterns than the K-map method.*
- *It is a viable scheme for handling a large number of variables as opposed to the K-map (limited to 5 or 6 variables).*
- *Can also be extended to functions with multiple outputs.*

Tabulation Method

- Step-by-step procedure for simplification by the tabular or Quine-McCluskey method.
  - Identify the ***prime implicants***.
  - **Essential prime implicants** are determined by preparing a **prime implicants chart**.
  - The minterms that are not covered by the essential prime implicants, are taken into consideration by selecting some more prime implications to obtain an optimized Boolean expression.

Prime implicants are expressions with least number of literals that represents all the terms given in a truth table.

Determination of Prime Implicants

1. Each minterm of the function is expressed by its binary representation.

2. The minterms are arranged according to increasing index (index is defined as the number of 1s in a minterm). Each set of minterms possessing the same index are separated by lines.

3. Now each of the minterms is compared with the minterms of a higher index. For each pair of terms that can combine, the new terms are formed. If two minterms are differed by only one variable, that variable is replaced by a '-' (dash) to form the new term with one less number of literals. A line is drawn in when all the minterms of one set is compared with all the minterms of a higher index.

4. The same process is repeated for all the groups of minterms. A new list of terms is obtained after the first stage of elimination is completed.

5. At the next stage of elimination two terms from the new list with the '-' of the same position differing by only one variable are compared and again another new term is formed with a less number of literals.

6. The process is to be continued until no new match is possible.

7. All the terms that remain unchecked *i.e.*, where no match is found during the process, are considered to be the prime implicants.

Prime Implicant Chart

1. After obtaining the prime implicants, a chart or table is prepared where rows are represented by the prime implicants and the columns are represented by the minterms of the function.

2. Crosses are placed in each row to show the composition of the minterms that makes the prime implicants.

3. A completed prime implicant table is to be inspected for the columns containing only a single cross. Prime implicants that cover the minterms with a single cross are called the essential prime implicants.

Quine-McCluskey method

Step 1. List in a column all the minterms of the function to be minimized in their binary representation. Partition them into groups according to the number of 1 bits in their binary representations. This partitioning simplifies identification of logically adjacent minterms since, to be logically adjacent, two minterms must differ in exactly one literal, and therefore the binary representation of one minterm must have either one more or one fewer 1 bit than the other.

Quine-McCluskey method

Step 2. Perform an exhaustive search between neighboring groups for adjacent minterms and combine them into a column of $(n-1)$-variable implicants, checking off each minterm that is combined. The binary representation of each new implicant contains a dash in the position of the eliminated variable. Repeat for each column, combining $(n-1)$-varible implicants into $(n-2)$-variable implicants, and so on, until no further implicants can be combined. Any term not checked off represents a prime implicant of the function, since it is not covered by a larger implicant. The final result is a list of prime implicants of the switching function.

Quine-McCluskey method

Step 3. Construct a prime implicant chart that lists minterms along the horizontal and prime implicants along the vertical, with an × entry placed wherever a certain prime implicant (row) covers a given minterm (column).

Step 4. Select a minimum number of prime implicants that cover all the minterms of the switching function.

Quine-McCluskey method

Minimize the following expression using Q-M method. $f(A,B,C,D) = \sum m(2,4,6,8,9,10,12,13,15)$

Minterms	$ABCD$	
2	0010	
4	0100	Group 1 (a single 1)
8	1000	
6	0110	
9	1001	Group 2 (two 1's)
10	1010	
12	1100	
13	1101	Group 3 (three 1's)
15	1111	Group 4 (four 1's)

List 1			List 2			List 3		
Minterm	$ABCD$		Minterms	$ABCD$		Minterms	$ABCD$	
2	0010	√	2, 6	0-10	PI_2	8, 9, 12, 13	1-0-	PI_1
4	0100	√	2, 10	-010	PI_3			
8	1000	√	4, 6	01-0	PI_4			
6	0110	√	4, 12	-100	PI_5			
9	1001	√	8, 9	100-	√			
10	1010	√	8, 10	10-0	PI_6			
12	1100	√	8, 12	1-00	√			
13	1101	√	9, 13	1-01	√			
15	1111	√	12, 13	110-	√			
			13, 15	11-1	PI_7			

	2	4	6	√ 8	√ 9	10	√ 12	√ 13	√ 15
** PI_1				×	⊗		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
** PI_7								×	⊗

	√ 2	√ 4	√ 6	√ 10
PI_2	×		×	
*PI_3	×			×
*PI_4		×	×	
PI_5		×		
PI_6				×

$$f(A, B, C, D) = PI_1 + PI_3 + PI_4 + PI_7$$
$$= 1\text{-}0\text{-} + \text{-}010 + 01\text{-}0 + 11\text{-}1$$
$$= A\bar{C} + \bar{B}C\bar{D} + \bar{A}B\bar{D} + ABD$$

Rules for PI chart reduction

Rule 1. A row that *is covered* by another row may be eliminated from the chart. When identical rows are present, all but one of the rows may be eliminated.

Rule 2. A column that *covers* another column may be eliminated. All but one column from a set of identical columns may be eliminated.

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

	√ 5	√ 10
* PI_2	×	
*PI_4		×

Simplify the following function using Quine-McCluskey method.

$$Y(A,B,C,D) = \Sigma m(0,2,3,5,7,8,12,13)$$

STEP 1:

- The minterms of the function are represented in binary form as shown in Table 1.
- The binary representation are grouped into a number of sections in terms of the number of 1's index as shown in Table 2.

Minterms	Binary *ABCD*	No. of 1's
m_0	0000	0
m_2	0010	1
m_3	0011	2
m_5	0101	2
m_7	0111	3
m_8	1000	1
m_{12}	1100	2
m_{13}	1101	3

Minterms Group	Index	Binary *ABCD*
0	0	0000 ✓
2	I	0010 ✓
8		1000 ✓
3	II	0011 ✓
5		0101 ✓
12		1100 ✓
7	III	0111 ✓
13		1101 ✓

STEP 2:

- Compare every term of the lowest group with each term in the adjacent group.
- If two minterms differ in only one variable, that variable should be removed and a dash (–) is placed at the position, thus a new term with one less literal is formed.
- If such a situation occurs, a check mark (√) is placed next to both minterms.
- After all pairs of terms have been considered, a horizontal line is drawn under the last terms as depicted in Table.

Step 3:

- Now repeat step-2 with newly formed groups i.e. combine four minterms of adjacent groups if possibilities exist.
- In this case, dashes (–) exist in same position of two groups and only one position will be different.
- No four minterm group is possible here.

Minterms Group	Index	Binary *ABCD*	
0	0	0000	✓
2	I	0010	✓
8		1000	✓
3	II	0011	✓
5		0101	✓
12		1100	✓
7	III	0111	✓
13		1101	✓

The combinations of two minterms

Minterms Group	Binary				
	A	*B*	*C*	*D*	
0, 2	0	0	–	0	PI
0, 8	–	0	0	0	PI
2, 3	0	0	1	–	PI
8, 12	1	–	0	0	PI
3, 7	0	–	1	1	PI
5, 7	0	1	–	1	PI
5, 13	–	1	0	1	PI
12, 13	1	1	0	–	PI

STEP 4:

- All terms which remain unchecked are the PIs.
- Remove repeated prime implicants as shown in Table.
- Now, construct the prime implicant chart as shown in table.

The combinations of two minterms

Minterms Group	Binary				
	A	*B*	*C*	*D*	
0, 2	0	0	–	0	PI
0, 8	–	0	0	0	PI
2, 3	0	0	1	–	PI
8, 12	1	–	0	0	PI
3, 7	0	–	1	1	PI
5, 7	0	1	–	1	PI
5, 13	–	1	0	1	PI
12, 13	1	1	0	–	PI

Prime implicants chart

Prime Implicants	Minterms							
	0	2	3	5	7	8	12	13
0, 2•	×	×						
0, 8	×					×		
2, 3		×	×					
8, 12•						×	×	
3, 7•			×		×			
5, 7				×	×			
5, 13•				×				×
12, 13							×	×

Step 5:

- Here the function does not have any essential prime implication.
- So choose the prime implicants such that all minterms are covered.

Prime Implicants	Binary Representation	Product Term
0, 2	0 0 – 0	$\overline{A}\,\overline{B}\,\overline{D}$
8, 12	1 – 0 1	$A\overline{C}\,\overline{D}$
3, 7	0 – 1 1	$\overline{A}CD$
5, 13	– 1 0 1	$B\overline{C}D$

Minimized SOP expression

$$f(A,B,C,D) = \overline{A}\,\overline{B}\,\overline{D} + A\overline{C}\,\overline{D} + \overline{A}CD + B\overline{C}D$$

Simplify the following function using Quine-McCluskey method.

$$f(A,B,C,D) = \Sigma m(7,9,12,13,14,15) + d(4,11)$$

Minterms	Binary *ABCD*	No. of 1's
m_7	0111	3
m_9	1001	2
m_{12}	1100	2
m_{13}	1101	3
m_{14}	1110	3
m_{15}	1111	4
m_4	0100	1
m_{11}	1011	3

Minterm Group	Index	Binary *ABCD*
m_4	I	0100 ✓
m_9	II	1001 ✓
m_{12}		1100 ✓
m_7	III	0111 ✓
m_{11}		1011 ✓
m_{13}		1101 ✓
m_{14}		1110 ✓
m_{15}	IV	1111 ✓

Minterm Group	Index	Binary $ABCD$	
m_4	I	0100	✓
m_9	II	1001	✓
m_{12}		1100	✓
m_7	III	0111	✓
m_{11}		1011	✓
m_{13}		1101	✓
m_{14}		1110	✓
m_{15}	IV	1111	✓

Combination of two minterms

Minterm Group	Binary A	B	C	D		
4, 12	–	1	0	0		PI
9, 11	1	0	–	1	✓	
9, 13	1	–	0	1	✓	
12, 13	1	1	0	–	✓	
12, 14	1	1	–	0	✓	
7, 15	–	1	1	1		PI
11, 15	1	–	1	1	✓	
13, 15	1	1	–	1	✓	
14, 15	1	1	1	–	✓	

Combination of four minterms

Minterm Group	Binary A	B	C	D	
9, 11, 13, 15	1	–	–	1	PI
9, 13, 11, 15	1	–	–	1	PI Eliminated
12, 13, 14, 15	1	1	–	–	PI
12, 14, 13, 15	1	1	–	–	PI Eliminated

Prime implicant chart

Prime Implicants	Binary Representation	Variable Representation
4, 12	– 1 0 0	$B\overline{C}\,\overline{D}$
7, 15	– 1 1 1	BCD
9, 11, 13, 15	1 – – 1	AD
12, 13, 14, 15	1 1 – –	AB