CSE1003
Digital Logic and Design

Module 2
BOOLEAN ALGEBRA L2

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Module 2
BOOLEAN ALGEBRA 8 hrs

Boolean algebra

- Properties of Boolean algebra
- Boolean functions
- Canonical and Standard forms
- Logic gates - Universal gates
- Karnaugh map - Don’t care conditions
- Tabulation Method

Boolean Algebra

Logic 0	Logic 1
False	True
Off	On
Low	High
No	Yes
Open switch	Closed switch

1854
George Boole

- Boolean algebra is mathematics of logic.
- The algebra which deals with the logical operations of binary variables is called Boolean Algebra.
- **Boolean algebra** is a means for expressing the relationship between a logic circuit's inputs and outputs.
- The inputs are considered **logic variables** whose logic levels at any time determine the output levels.
- It is one of the most basic tools available to the logic designer and thus can be effectively used for simplification of complex logic expressions.

Boolean algebra

Boolean function

– Is an expression formed with binary variables, Boolean operators and the equality sign.

– Can also be represented in the form of a truth table.

Expression: a set of literals (possibly with repeats) combined with logic operations (and possibly ordered by parentheses)

$AB + C$ $\quad$ $(\bar{A}+B)C$

Equation: expression1 = expression2

$(\bar{A} + B)C = ((\bar{A}) + B)C$

Function of (possibly several) variables: an equation where the left hand side is defined by the right hand side

$f(A, B, C) = ((\bar{A}) + B)C$

Boolean algebra

<5 1
>2
0.8
0 } 0

Variables, Literals and Terms in Boolean Expressions

- ***Variables*** are the letters in a Boolean expression which represents a physical quantity such as a voltage signal. They may take on the value ‘0’ or ‘1’.
- The complement of a variable is not considered as a separate variable. Each occurrence of a variable or its complement is called a ***literal***.
- A **term** is the expression formed by literals and operations at one level.

A B C
$\overline{A}$ $\overline{B}$ $\overline{C}$

$$\overline{A}+A.B+A.\overline{C}+\overline{A}.B.C$$

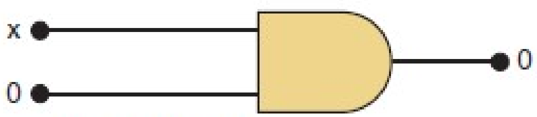
$$(\overline{P}+Q).(R+\overline{S}).(P+\overline{Q}+R)$$

BOOLEAN THEOREMS

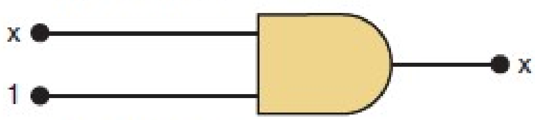
- help to simplify logic expressions and logic circuits

Single-variable theorems

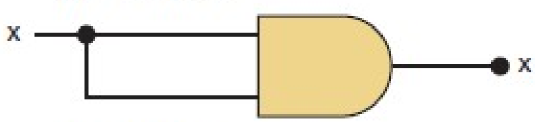
AND



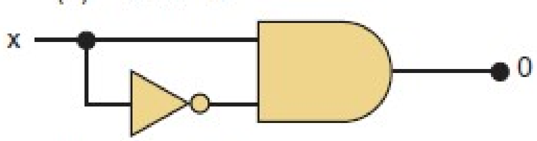
(1) $x \cdot 0 = 0$



(2) $x \cdot 1 = x$

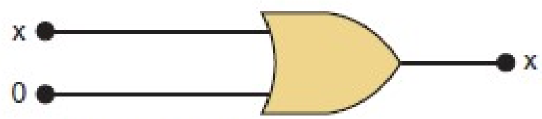


(3) $x \cdot x = x$

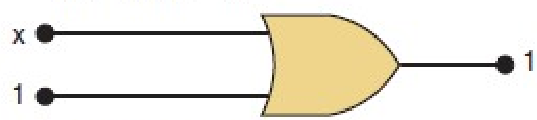


(4) $x \cdot \overline{x} = 0$

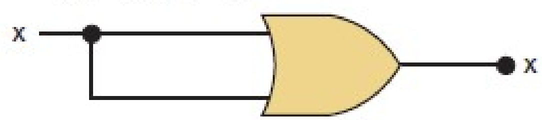
OR



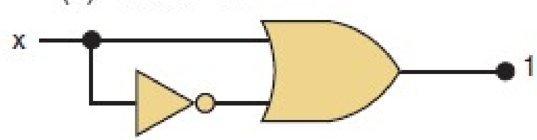
(5) $x + 0 = x$



(6) $x + 1 = 1$



(7) $x + x = x$

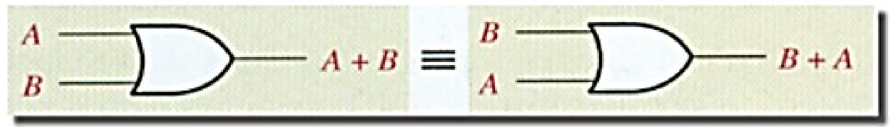


(8) $x + \overline{x} = 1$

Laws of Boolean Algebra

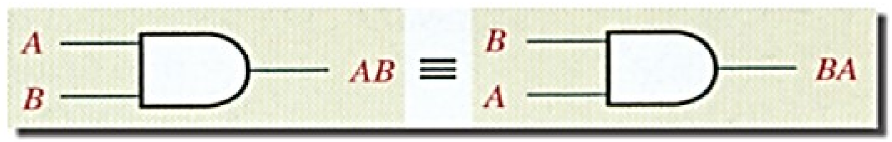
- Commutative Laws

A + B = B + A



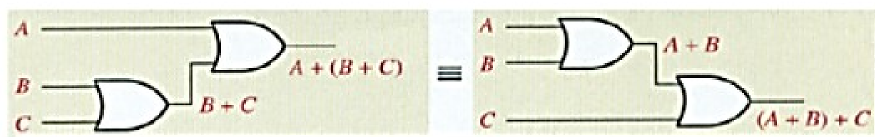
A • B = B • A

AB = BA

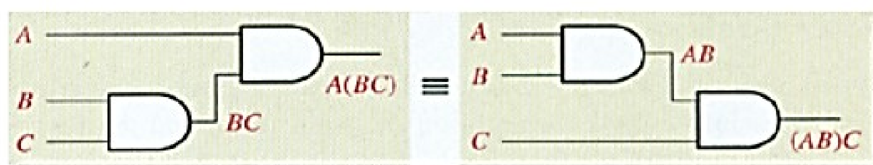


- Associative Laws

A + (B + C) = (A + B) + C

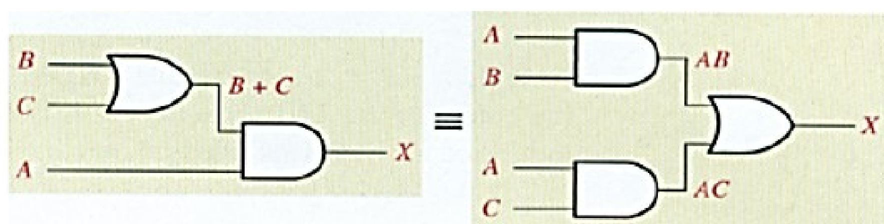


A • (B • C) = (A • B) • C



- Distributive Law

A • (B + C) = A • B + A • C
A (B + C) = A B + A C

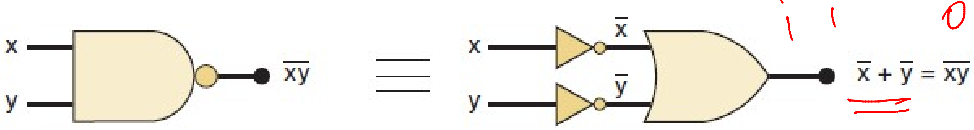


DeMorgan’s Theorem

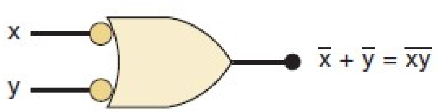
DeMorgan’s theorems are extremely useful in simplifying expressions in which a product or sum of variables is inverted.

The complement of a product of variables is equal to the sum of the complements of the variables

$$\overline{XY} = \bar{X} + \bar{Y}$$



(a)



(b)

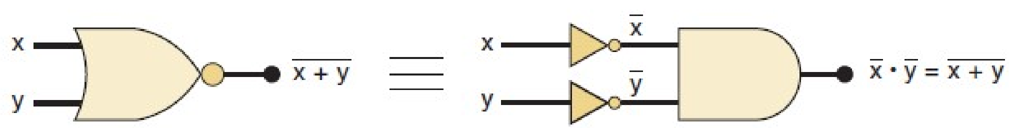
-ve OR

X	Y	$\overline{XY}$
0	0	1
0	1	1
1	0	1
1	1	0

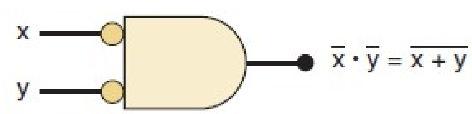
X	Y	$\bar{X} + \bar{Y}$
0	0	1
0	1	1
1	0	1
1	1	0

The complement of a sum of variables is equal to the product of the complements of the variables.

$$\overline{X + Y} = \bar{X}.\bar{Y}$$



(a)



(b)

-ve AND

Boolean Logic

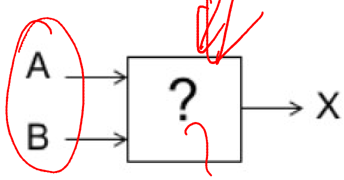
- Truth Table: all combinations of input variables
- k variables ➜ 2^k input combinations

Truth Tables specifies how a logic circuit's output depends on the logic levels present at the inputs.

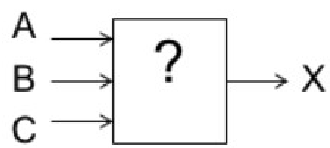
Output

Inputs

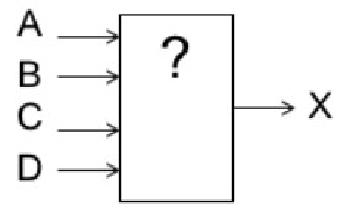
A	B	x
0	0	1
0	1	0
1	0	1
1	1	0



A	B	C	x
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



A	B	C	D	x
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1



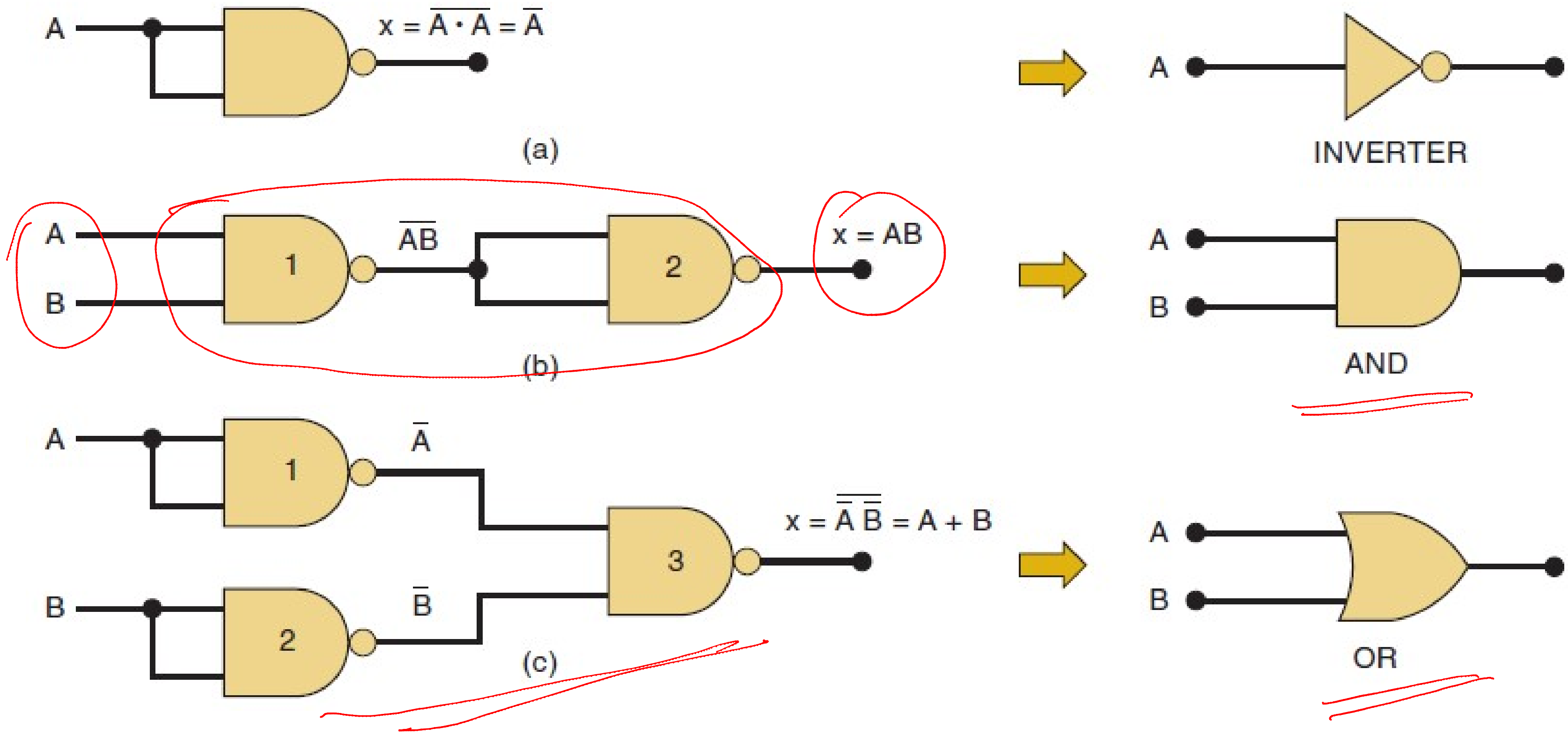
Laws of Boolean Algebra used for simplifying logical expressions

($A^l = \bar{A}$ denotes the complement/inverse/NOT of A)

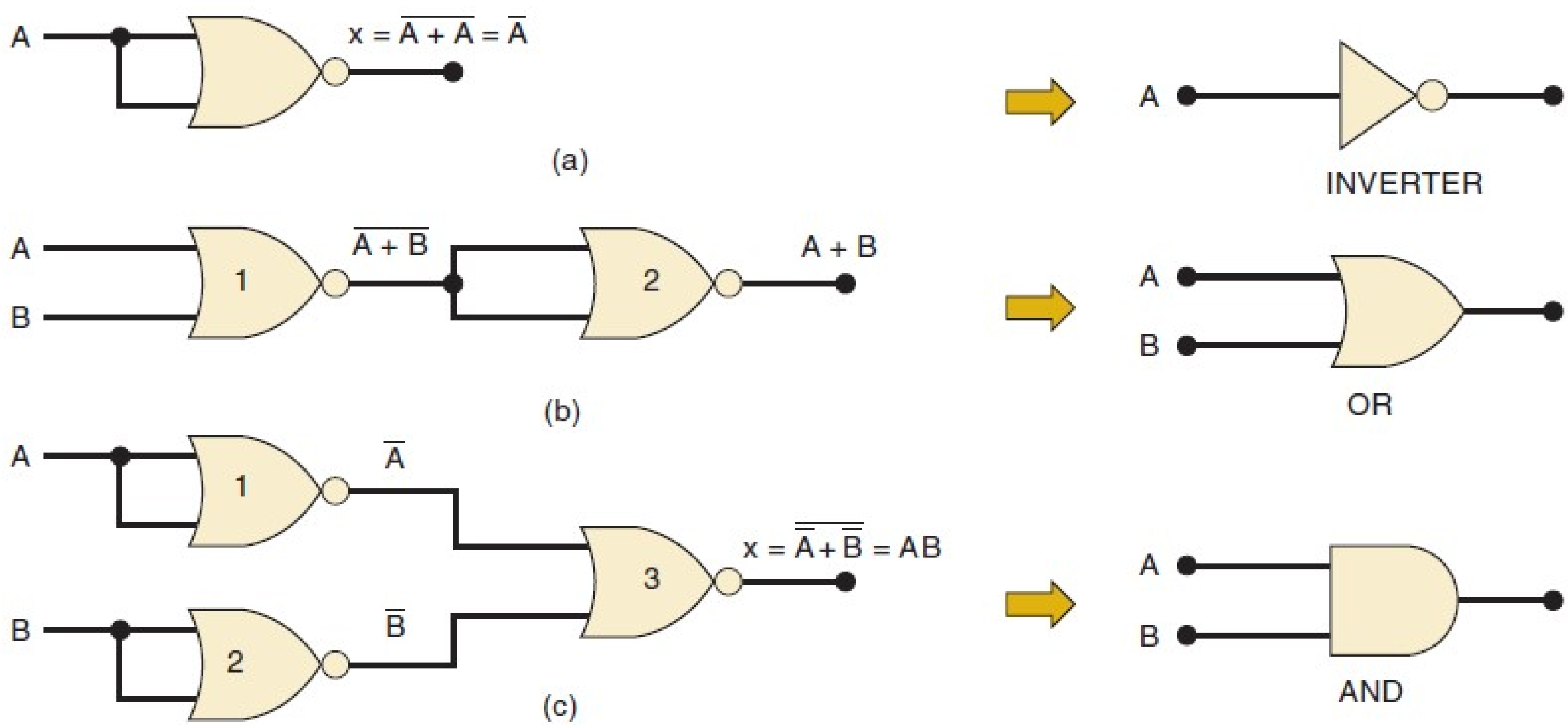
$A+0=A$	$(A^l)^l = A$
$A+1=A$	$A+AB=A$
$A\cdot 0=0$	$A+A^lB=A+B$
$A\cdot 1=A$	$A+B=B+A$
$A+A=A$	$A\cdot B=B\cdot A$
$A+A^l=1$	$\overline{A+B}=\bar{A}\cdot\bar{B}$
$A\cdot A=A$	$\overline{A\cdot B}=\bar{A}+\bar{B}$
$A\cdot A^l=0$	$(A+B)(A+C)=A+BC$

A.B.1=A.B
A+B+1=1
A.B.0=0
A+B+0=A+B

UNIVERSALITY OF NAND GATES



UNIVERSALITY OF NOR GATES



Precedence rules in Boolean algebra

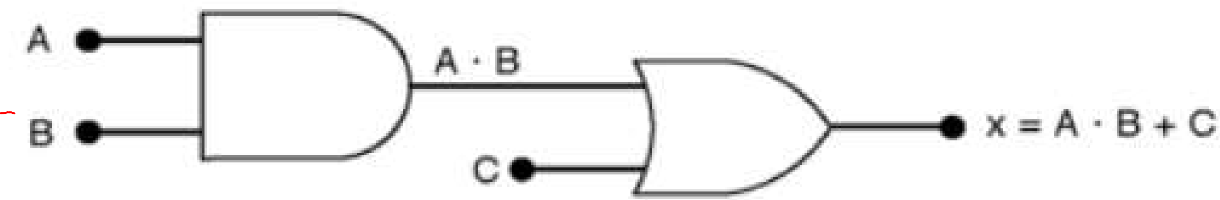
- Scan the expression from left to right.
- First evaluate expression enclosed in parentheses.
- Perform all the complement operations.
- Perform all the AND operations in the order.
- Perform all the OR operations.

DESCRIBING LOGIC CIRCUITS ALGEBRAICALLY FROM BOOLEAN EXPRESSIONS

A Boolean function from an algebraic expression can be realized to a logic diagram composed of logic gates.

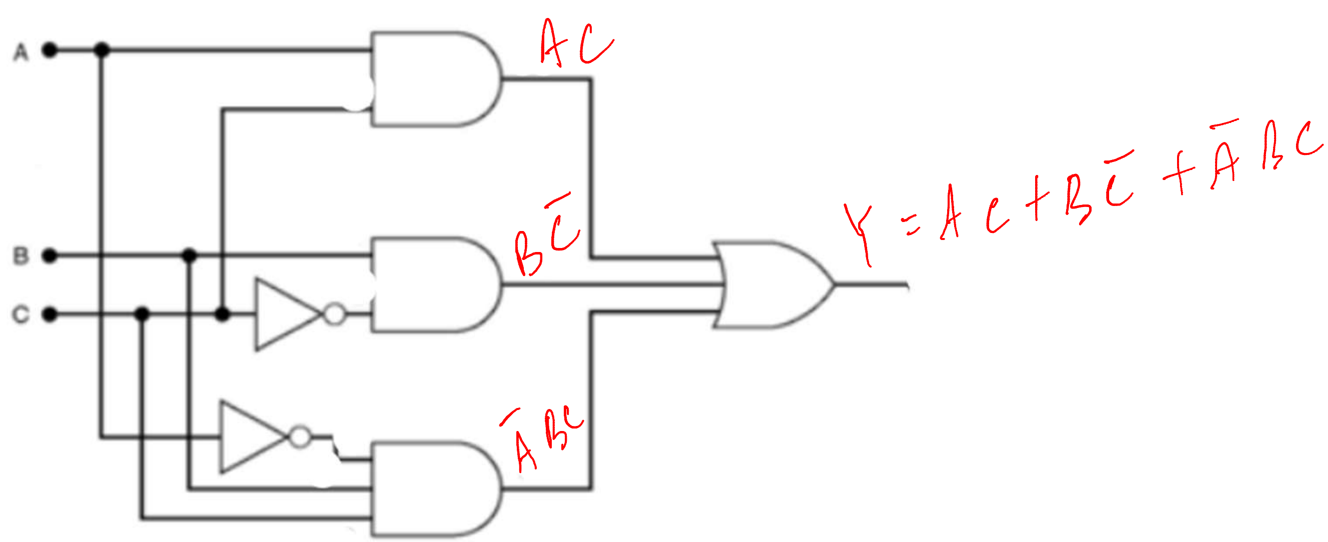
$x = A \cdot B + C$

2 gates
1 AND
1 OR



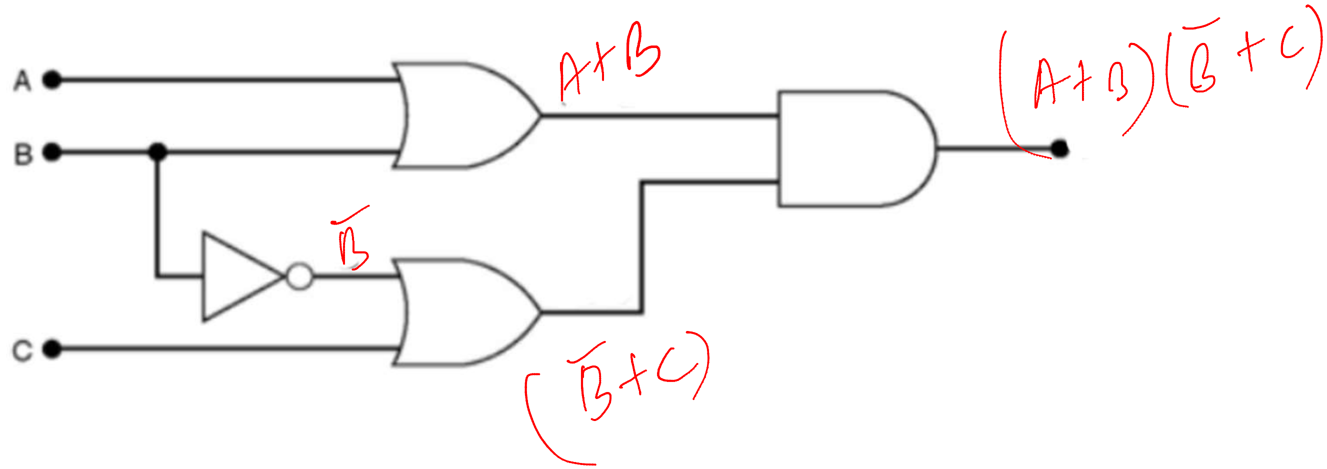
$y = AC + B\bar{C} + \bar{A}BC$

Total ~~8~~
AND : 3
OR : 2
NOT : 2

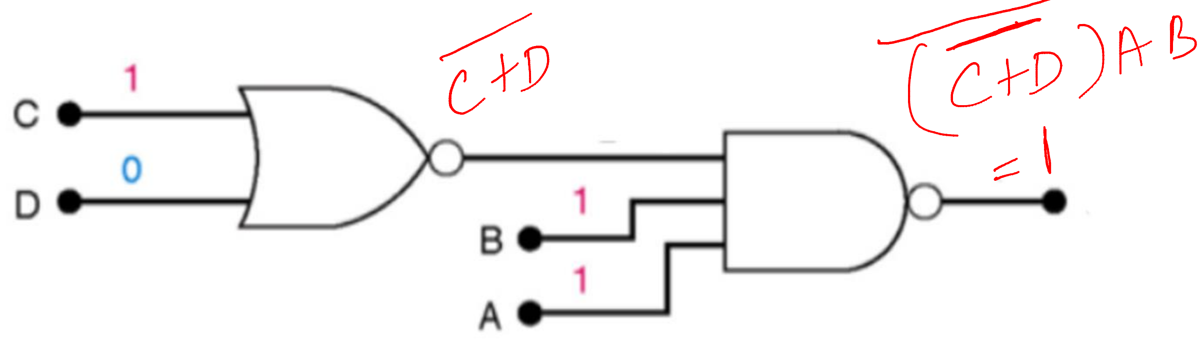


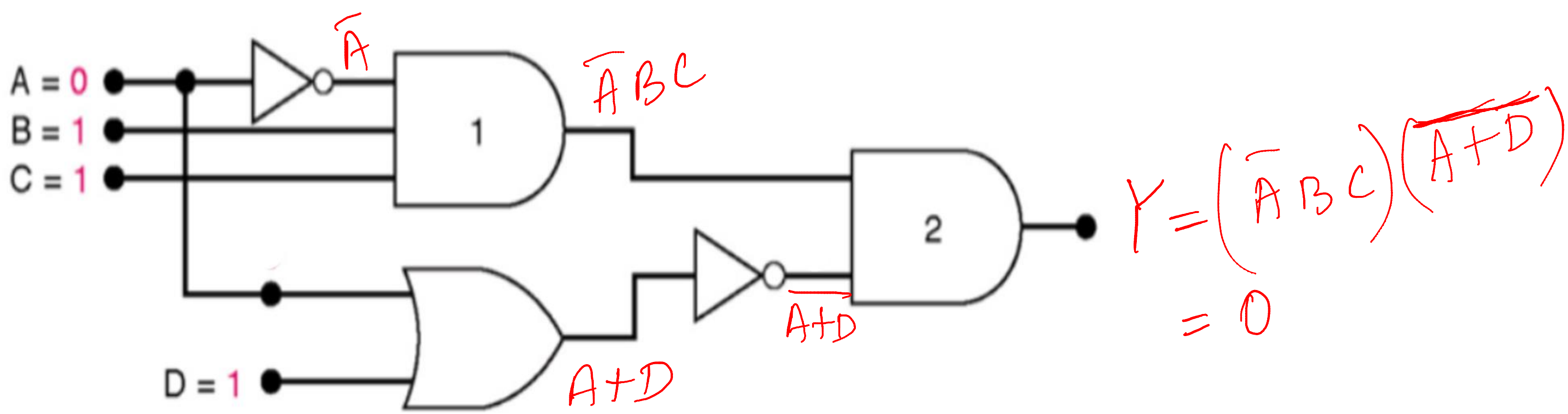
DESCRIBING LOGIC CIRCUITS ALGEBRAICALLY FROM BOOLEAN EXPRESSIONS

$$x = (A + B)(\overline{B} + C)$$



Determining output value from a logic diagram





Simplification Techniques

- The primary objective of all simplification procedures is to obtain an expression that has the minimum number of terms.
- Obtaining an expression with the minimum number of literals is usually the secondary objective.
- If there is more than one possible solution with the same number of terms, the one having the minimum number of literals is the choice.

Algebraic Method

K-Map "

Tabulation

$$Y = ABC + \bar{A}B\bar{C} + \bar{B}CA + A\bar{C}$$

4 term

$A + C$

Simplify using Boolean Algebra

$A\bar{B}(D+\bar{D}) = A\bar{B} \quad D+\bar{D}=1$

$y = A\bar{B}D + A\bar{B}\bar{D}$ → $y = A\bar{B}$

$\bar{A}A + AB + \bar{A}B + BB$
$= 0 + AB + \bar{A}B + B$
$= 0 + (A+\bar{A})B + B$
$= B + B = B$

$z = (\bar{A}+B)(A+B)$ → $z = B$

$x = ACD + \bar{A}BCD$ → $x = ACD + BCD$

$(A + \bar{A}B)CD = (A+B)CD$
$= ACD + BCD$

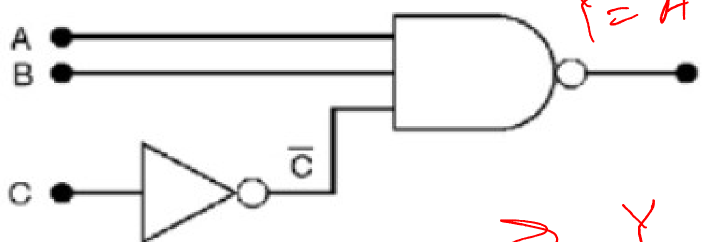
$z = \overline{(\bar{A}+C)\cdot(B+\bar{D})}$ → $z = A\bar{C} + \bar{B}D$

$\overline{(\bar{A}+C)} + \overline{(B+\bar{D})} = \bar{\bar{A}}\cdot\bar{C} + \bar{B}\cdot\bar{\bar{D}}$
$= A\bar{C} + \bar{B}D$

Simplify using Boolean Algebra

Determine the output expression for the circuit below and simplify it using DeMorgan's Theorem

$Y = \overline{A \cdot B \cdot \overline{C}}$



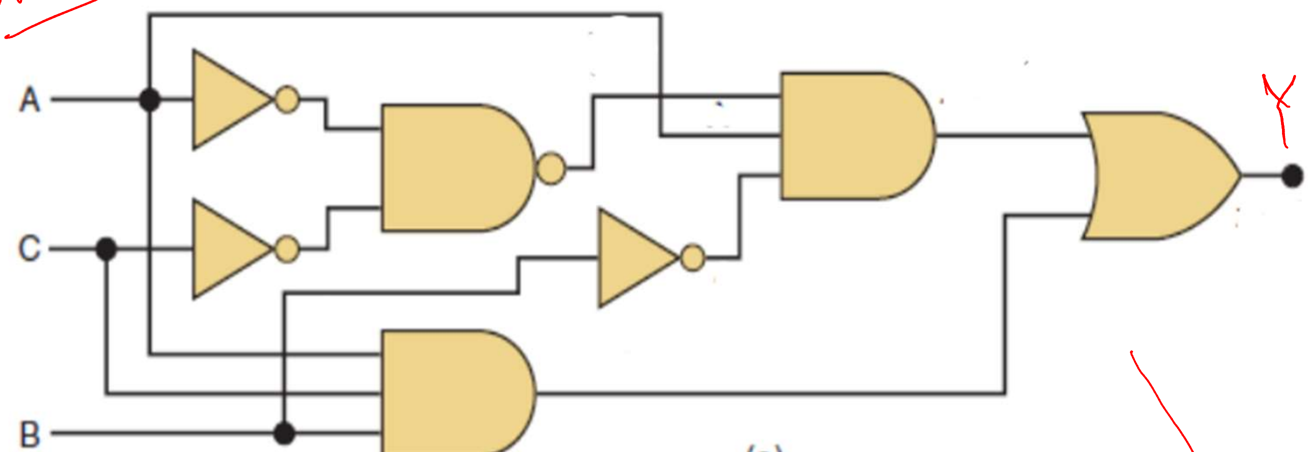
$Y = \overline{A} + \overline{B} + C$

2 NOT
1 OR.

$$z = \overline{A \cdot B \cdot \overline{C}} = \overline{A} + \overline{B} + \overline{\overline{C}} = \overline{A} + \overline{B} + C$$

Simplify the logic circuit shown in Figure

Given circuit

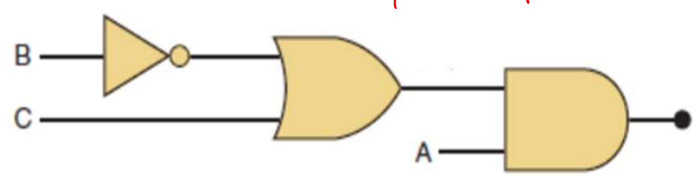


(a)

$Y = ?$

Simplify.

Simplified ckt.



(b)