Sampling Techniques

Large and Small Sample Test

Large Sample Test ($n \geq 30$)
Z- test (One Sample)

Test for single Mean - Lower Tail Test of Population Mean with Known Variance

The null hypothesis of the lower tail test of the population mean can be expressed as follows:

$$\boldsymbol{\mu \geq \mu_0}$$

Where $\boldsymbol{\mu_0}$ is a hypothesized lower bound of the true population mean $\boldsymbol{\mu}$

Z- test cont…

Let us define the test statistic z in terms of the <u>sample mean</u>, the sample size and the <u>population standard deviation</u> σ :

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

Then the null hypothesis of the lower tail test is to be rejected if $z < -z_{\alpha}$***, where*** z_{α}***, is the 100(1− α) <u>percentile</u> of the <u>standard normal distribution</u>.***

Problems on Single Mean Test

- ***Suppose the manufacturer claims that the mean lifetime of a light bulb is more than 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assume the population standard deviation is 120 hours. At .05 significance level, can we reject the claim by the manufacturer?***

The null hypothesis is that $\mu \geq 10000$

```
> xbar = 9900             # sample mean
> mu0 = 10000             # hypothesized value
> sigma = 120             # population standard deviation
> n = 30                  # sample size
> z = (xbar-mu0)/(sigma/sqrt(n))
> z                       # test statistic
[1] -4.564355
```

Critical Value

We then compute the critical value at .05 significance level.

```
> alpha = .05
> z.alpha = qnorm(1-alpha)
> -z.alpha                       # critical value
[1] -1.644854
```

Interpretation

The test statistic -4.5644 is less than the critical value of -1.6449. Hence, at .05 significance level, we reject the claim that mean lifetime of a light bulb is above 10,000 hours.

Alterative Comparison P- Value

- The p-value is the level of marginal significance within a statistical hypothesis test representing the probability of the occurrence of a given event. The p-value is used as an alternative to rejection points to provide the smallest level of significance at which the null hypothesis would be rejected. A smaller p-value means that there is stronger evidence in favour of the alternative hypothesis.

P- Value Cont….

- The p-value approach to hypothesis testing uses the calculated probability to determine whether there is evidence to reject the null hypothesis. The null hypothesis, also known as the conjecture, is the initial claim about a population of statistics. The alternative hypothesis states whether the population parameter differs from the value of the population parameter stated in the conjecture. In practice, the p-value, or critical value, is stated in advance to determine how the required value to reject the null hypothesis.

P –Value Comparison

- ***Instead of using the critical value, we apply the pnorm function to compute the lower tail p-value of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that μ ≥ 10000.***

```
> pval = pnorm(z)
> pval                          # lower tail p-value
[1] 2.505166e-06
```

Upper Tail Test of Population Mean with Known Variance:

- *The null hypothesis of the upper tail test of the population mean can be expressed as follows*

$$\mu \leq \mu_0$$

where μ_0 is a hypothesized upper bound of the true population mean μ.

- *Let us define the test statistic z in terms of the <u>sample mean</u>, the sample size and the <u>population standard deviation</u> σ :*

Upper Tailed Test cont…

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

Then the null hypothesis of the upper tail test is to be rejected if $z \geq z_\alpha$ where z_α is the 100(1− α) percentile of the standard normal distribution

Problem

- ***Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the population standard deviation is 0.25 grams. At .05 significance level, can we reject the claim on food label?***

R - Code

- ***The null hypothesis is that μ ≤ 2. We begin with computing the test statistic.***

```
> xbar = 2.1                # sample mean
> mu0 = 2                   # hypothesized value
> sigma = 0.25              # population standard deviation
> n = 35                    # sample size
> z = (xbar-mu0)/(sigma/sqrt(n))
> z                         # test statistic
[1] 2.366432
```

Critical Value Comparison

```
> alpha = .05
> z.alpha = qnorm(1-alpha)
> z.alpha                    # critical value
[1] 1.644854
```

Interpretation

- ***The test statistic 2.3664 is greater than the critical value of 1.6449. Hence, at .05 significance level, we reject the claim that there is at most 2 grams of saturated fat in a cookie.***

Two-Tailed Test of Population Mean with Known Variance

- *The null hypothesis of the two-tailed test of the population mean can be expressed as follows:*

$$\mu = \mu_0$$

where μ_0 is a hypothesized value of the true population mean . Let us define the test statistic z in terms of the <u>sample mean</u>, the sample size and the <u>population</u> <u>standard deviation</u> σ :

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

- ***Then the null hypothesis of the two-tailed test is to be rejected if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$, where $z_{\alpha/2}$ is the $100(1 - \alpha/2)$ percentile of the standard normal distribution.***

Problem

- ***Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the population standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?***

R – Code

- The null hypothesis is that $\mu = 15.4$.

```
> xbar = 14.6                    # sample mean
> mu0 = 15.4                     # hypothesized value
> sigma = 2.5                    # population standard deviation
> n = 35                         # sample size
> z = (xbar-mu0)/(sigma/sqrt(n))
> z                              # test statistic
[1] -1.893146
```

Comparison using p- value

```
> pval = 2 * pnorm(z)     # lower tail
> pval                    # two-tailed p-value
[1] 0.05833852
```

The p – value turns out to be greater than the .05 significance level, we do not reject the null hypothesis that μ = 15.4.

Lower Tail Test of Population Proportion

- *The null hypothesis of the lower tail test about population proportion can be expressed as follows:*

$$P \geq P_0$$

where p_0 is a hypothesized lower bound of the true population proportion p. Let us define the test statistic z in terms of the sample proportion and the sample size:

$$z = \frac{p - p_0}{\sqrt{p_0 q_0 / n}} \sim N(0,1)$$

- ***Then the null hypothesis of the lower tail test is to be rejected if $z < -z_\alpha$, where z_α is the $100(1-\alpha)$ percentile of the standard normal distribution.***

Problem

Suppose 60% of citizens voted in last election. 85 out of 148 people in a telephone survey said that they voted in current election. At 0.5 significance level, can we reject the null hypothesis that the proportion of voters in the population is above 60% this year?

R – Code

The null hypothesis is that p ≥ 0.6

```
> pbar = 85/148              # sample proportion
> p0 = .6                    # hypothesized value
> n = 148                    # sample size
> z = (pbar-p0)/sqrt(p0*(1-p0)/n)
> z                          # test statistic
[1] -0.6375983
```

P-value comparison

```
> pval = pnorm(z)
> pval
[1] 0.2618676
```

As p-value turns out to be greater than the .05 significance level, we do not reject the null hypothesis that p ≥ 0.6.

Upper Tail Test of Population Proportion

- The null hypothesis of the upper tail test about population proportion can be expressed as follows:

$$P \leq P_0$$

where p_0 is a hypothesized upper bound of the true population proportion p. Let us define the test statistic z in terms of the sample proportion and the sample size:

$$z = \frac{p - p_0}{\sqrt{p_0 q_0 / n}} \sim N(0,1)$$

Problem

- ***Suppose that 12% of apples harvested in an orchard last year was rotten. 30 out of 214 apples in a harvest sample this year turns out to be rotten. At .05 significance level, can we reject the null hypothesis that the proportion of rotten apples in harvest stays below 12% this year?***

R - Code

- The null hypothesis is that $p \leq 0.12$

```
> pbar = 30/214                # sample proportion
> p0 = .12                     # hypothesized value
> n = 214                      # sample size
> z = (pbar-p0)/sqrt(p0*(1-p0)/n)
> z                            # test statistic
[1] 0.908751
```

P- Value comparison

- ***The p- value turns out to be greater than the .05 significance level, we do not reject the null hypothesis that p ≤ 0.12.***

```
> pval = pnorm(z, lower.tail=FALSE)
> pval                    # upper tail p?value
[1] 0.1817408
```

Two-Tailed Test of Population Proportion

- The null hypothesis of the two-tailed test about population proportion can be expressed as follows:

$$P = P_0$$

- where p_0 is a hypothesized value of the true population proportion p. Let us define the test statistic z in terms of the sample proportion and the sample size:

$$z = \frac{p - p_0}{\sqrt{p_0 q_0 / n}} \sim N(0,1)$$

- **Then the null hypothesis of the two-tailed test is to be *rejected* if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$, where $z_{\alpha/2}$ is the 100(1 − α) percentile of the standard normal distribution.**

Problem

- ***Suppose a coin toss turns up 12 heads out of 20 trials. At .05 significance level, can one reject the null hypothesis that the coin toss is fair?***

The null hypothesis is that p = 0.5

```
> pbar = 12/20                  # sample proportion
> p0 = .5                       # hypothesized value
> n = 20                        # sample size
> z = (pbar-p0)/sqrt(p0*(1-p0)/n)
> z                             # test statistic
[1] 0.8944272
```

P- value comparison

```
> pval = 2 * pnorm(z, lower.tail=FALSE)  # upper tail
> pval
[1] 0.3710934
```

P- value turns out to be greater than the .05 significance level, we do not reject the null hypothesis that p = 0.5.

Alternate Calculation for One Sample Proportion Test

- ***To apply the prop.test function to compute the p-value directly***
- ***Syntax:***
- prop.test(x,n,p, alt =c("greater", "lesser", "two.sided"),conf.level=.95, correct = FALSE)

Problem 1

Suppose that 12% of apples harvested in an orchard last year was rotten. 30 out of 214 apples in a harvest sample this year turns out to be rotten. At .05 significance level, can we reject the null hypothesis that the proportion of rotten apples in harvest stays below 12% this year?

```
> prop.test(30, 214, p=.12, alt="greater", correct=FALSE)

        1-sample proportions test without continuity correction

data:  30 out of 214, null probability 0.12
X-squared = 0.82583, df = 1, p-value = 0.1817
alternative hypothesis: true p is greater than 0.12
95 percent confidence interval:
 0.1056274 1.0000000
sample estimates:
        p
0.1401869
```

Problem 2

Suppose a coin toss turns up 12 heads out of 20 trials. At .05 significance level, can one reject the null hypothesis that the coin toss is fair?

```
> prop.test(12, 20, p=0.5, correct=FALSE)

        1-sample proportions test without continuity correction

data:  12 out of 20, null probability 0.5
X-squared = 0.8, df = 1, p-value = 0.3711
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.3865815 0.7811935
sample estimates:
  p
0.6
```

Large Sample Test

Z - Test for Two Samples

Two Proportion Test

R-Code:-

Tests about a proportion using x and n using the prop.test function:

Usage: prop.test(c(x1,x2), c(n1,n2), correct=, alternate =).

1. x1 and x2 are the number of successes in sample 1 and 2 respectively.
2. n1 and n2 are the sample sizes or number of trials.
3. correct = TRUE (use a continuity correction factor) or FALSE (do not).
4. alternate = "two.sided" (default), "less", or "greater".

Problem

A popular cold-remedy was tested for it's efficacy. In a sample of 150 people who took the remedy upon getting a cold, 117 (78%) had no symptoms one week later. In a sample of 125 people who took the placebo upon getting a cold, 90 (75%) had no symptoms one week later. The table summarizes this information. Test the claim that the proportion of all remedy users who are symptom-free after one week is greater than the proportion for placebo users. Test this claim at the 0.05 significance level.

Group	#who are symptom Free after one week(x)	Total # in group (n)	Proportion $\hat{p} = x / n$
Remedy	117	150	0.78
Placebo	90	120	0.75

R Code:-

```
> x<-c(117,90)
> n<-c(150,120)
> prop.test(x,n,alternative="greater",correct=FALSE)

        2-sample test for equality of proportions without contin
        correction

data:  x out of n
X-squared = 0.3354, df = 1, p-value = 0.2812
alternative hypothesis: greater
95 percent confidence interval:
 -0.05557192  1.00000000
sample estimates:
prop 1 prop 2
  0.78   0.75
```

We fail reject the null hypothesis because the P-value (.2812) is greater than the significance level. Therefore, we can't support the claim.

Problem 2

- *The Trial Urban District Assessment (TUDA) is a study sponsored by the government of student achievement in large urban school district. In 2009, 1311 of a random sample of 1900 eighth-graders from Houston performed at or above the basic level in mathematics . In 2011, 1440 of a random sample of 2000 eighth-graders from Houston performed at or above the basic level . (The study reports the proportions).*

(A)Is there an increase in the proportion of eighth-graders who performed at or above the basic level in mathematics from 2009 to 2011 at the 5% significance level?

Compute the 95% confidence interval for the difference in proportion of eighth-graders who performed at or above the basic level in mathematics from 2009 to 2011.

Let p1 and p2 be the proportions of eighth-graders that performed at or above he basic level in mathematics in 2011 and 2009,respectively .

H_0: $p1=p2$ *against* $H_1= p1> p2$

```
> prop.test(c(1440,1311),c(2000,1900),alternative="greater",correct=FALSE)

        2-sample test for equality of proportions without continuity
        correction

data:  c(1440, 1311) out of c(2000, 1900)
X-squared = 4.2197, df = 1, p-value = 0.01998
alternative hypothesis: greater
95 percent confidence interval:
 0.005972807 1.000000000
sample estimates:
prop 1 prop 2
  0.72   0.69
```

- The p-value=0.02 < 0.05 so we reject H0. Thus, there is evidence that there is an increase from 2009 to 2011 in the proportion of eighth-graders who performed at or above the basic level at the 5% significance level.

Solution to part (b)

```
> prop.test(c(1440,1311),c(2000,1900),correct=FALSE)

        2-sample test for equality of proportions without continuity
        correction

data:  c(1440, 1311) out of c(2000, 1900)
X-squared = 4.2197, df = 1, p-value = 0.03996
alternative hypothesis: two.sided
95 percent confidence interval:
 0.001369833 0.058630167
sample estimates:
prop 1 prop 2 
  0.72   0.69 
```

Thus, we are 95% confident that the percent of eighth-graders who performed at or above the basic level in mathematics in 2011 is between 0:14% and 5:86% higher than in 2009.

Problem 3

- *The use of helmet among recreational alpine skiers and snowboarders are generally low. A study from Norway wanted to examine if helmet use reduces the risk of head injury. In the study, they compared the helmet use among skiers and snowboarders that was injured with a control group. The control group consisted of skiers and snowboarders that was uninjured. 96 of 578 people with head injuries used a helmet and 656 of 2992 people in the uninjured group used a helmet. Is helmet use lower among skiers and snowboarders who had head injuries?*
-

Let $p1$ be the proportion of helmet use among injured skiers and snowboarders.
Let $p2$ be the proportion of helmet use among uninjured skiers and snowboarders
$H0 : p1 = p2$ against $H1 : p1 < p2$

```
> prop.test(c(96,656),c(578,2992),alternative=c("less"),correct=FALSE)

        2-sample test for equality of proportions without continuity
        correction

data:  c(96, 656) out of c(578, 2992)
X-squared = 8.2336, df = 1, p-value = 0.002056
alternative hypothesis: less
95 percent confidence interval:
 -1.00000000 -0.02482216
sample estimates:
   prop 1    prop 2
0.1660900 0.2192513
```

The p-value= 0.0021 < 0.01 so we have strong evidence that helmet use is lower among skiers and snowboarders who had head injuries compared to uninjured skiers and snowboarders.

Problem 4

- *A survey is taken two times over the course of two weeks. The pollsters wish to see if there is a difference in the results as there has been a new advertising campaign run. Here is the data*

	Week1	Week2
Favorable	45	56
Unfavorable	35	47

H0: P1 = P2

H1: P1 =P2 (two- sided)

R - Code

```
> prop.test(c(45,56),c(45+35,56+47))

        2-sample test for equality of proportions with continuity correction

data:  c(45, 56) out of c(45 + 35, 56 + 47)
X-squared = 0.010813, df = 1, p-value = 0.9172
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.1374478  0.1750692
sample estimates:
   prop 1    prop 2
0.5625000 0.5436893
```

we observe that the p-value is 0.9172 so we accept the null hypothesis that P1 =P2.

Two mean Test

The following data shows the heights of individuals of two different countries with the population variance of 5 and 8.5 respectively. Is there any significant difference between the average heights of two groups.

A: 175	168	168	190	156	181	182	175	174	179
B: 185	169	173	173	188	186	175	174	179	180

R – Code

```
>
> a = c(175, 168, 168, 190, 156, 181, 182, 175, 174, 179)
> b = c(185, 169, 173, 173, 188, 186, 175, 174, 179, 180)
> n1=length(a)
> n2=length(b)
> zeta = abs((mean(a) - mean(b)) / (sqrt(var(a)/n1 + var(b)/n2)))
> zeta
[1] 0.947373
>
```

P- value comparison

```
> pvalue=2*pnorm(zeta)
> pvalue
[1] 1.656551
```

Since it turns out to be greater than the .05 significance level, we do not reject the null hypothesis

Practice Problems on Large Samples

- In the sample of 1000 people in Maharashtra,540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance

- A particular brand of tires claims that its deluxe tire averages at least 50,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 8000. A survey of owners of that tire design is conducted. From the 28 tires surveyed, the average lifespan was 46,500 miles with a standard deviation of 9800 miles. Do the data support the claim at the 5% level?

Practice problems cont….

- In the large city A,20 per cent of Random sample of 900 School children had defective eye –sight. In the large city B,15 percent of random sample of 1600 school children had the same defective. Is this Difference between the two Proportions Significant? Obtain 95% confidence limits of the difference in the population proportions.
- A cigarette manufacturing firm claims its brand A of the cigarettes outsells its brand B by 8%.if its found that 42 out sample of 200 smoker prefer brand A and 18 out of another random sample of 100 smokers prefers brand B, test whether the 8% difference is a valid cliam.

Practice problems cont….

- The average number of sick days an employee takes per year is believed to be about 10. Members of a personnel department do not believe this figure. They randomly survey 8 employees. The number of sick days they took for the past year are as follows: 12; 4; 15; 3; 11; 8; 6; 8. Let X = the number of sick days they took for the past year. Should the personnel team believe that the average number is about 10?
- In 1955, *Life Magazine* reported that the 25 year-old mother of three worked [on average] an 80 hour week. Recently, many groups have been studying whether or not the women's movement has, in fact, resulted in an increase in the average work week for women (combining employment and at-home work). Suppose a study was done to determine if the average work week has increased. 81 women were surveyed with the following results. The sample average was 83; the sample standard deviation was 10. Does it appear that the average work week has increased for women at the 5% level?

Practice problems cont….

- **A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39, 350 kilo meters with a standard deviation of 3, 260. Could the sample come from a population with mean life of 40, 000 kilometers?**
- **The mean life time of a sample of 400 fluorescent light bulbs produced by a company is found to be 1, 570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean life time of bulbs is 1600 hours against the alternative hypothesis that it is greater than 1, 600 hours at 1% and 5% level of significance**

Small Sample Test

size less than thirty

Small Sample Test

t-test for single mean and t-test for difference of means

- The t.test( ) function produces a variety of t-tests. Unlike most statistical packages, the default assumes unequal variance.

Syntax: one sample – single mean

- t.test(y, mu=3, alt = "greater"/ "lesser", var.equal = TRUE/FALSE)

Ho: mu=3

Problem

- An outbreak of salmonella-related illness was attributed to ice produced at a certain factory. Scientists measured the level of Salmonella in 9 randomly sampled batches ice crean.The levels(in MPN/g) were:

0.593	0.142	0.329	0.691	0.231	0.793	0.519	0.392	0.418

Is there evidence that the mean level pf Salmonella in ice cream greater than 0.3 MPN/g?

R- Code & Interpretation

```
> x=c(0.593,0.142,0.329,0.691,0.231,0.793,0.519,0.392,0.418)
> t.test(x,alternative="greater",mu=0.3)

        One Sample t-test

data:  x
t = 2.2051, df = 8, p-value = 0.02927
alternative hypothesis: true mean is greater than 0.3
95 percent confidence interval:
 0.3245133       Inf
sample estimates:
mean of x
0.4564444
```

From the output we see that the p-value = 0.029. Hence, there is moderately strong evidence that the mean Salmonella level in the ice cream is above 0.3MPN/g.

Problem

Suppose that 10 volunteers have taken an intelligence test; here are the results obtained. The average score of the entire population is 75 in the same test. Is there any significant difference (with a significance level of 95%) between the sample and population means, assuming that the variance of the population is not known

.

Scores: 65, 78, 88, 55, 48, 95, 66, 57, 79, 81

R- Code & Interpretation

```
> a = c(65, 78, 88, 55, 48, 95, 66, 57, 79, 81)
> t.test (a, mu=75)

        One Sample t-test

data:  a
t = -0.78303, df = 9, p-value = 0.4537
alternative hypothesis: true mean is not equal to 75
95 percent confidence interval:
 60.22187 82.17813
sample estimates:
mean of x
     71.2
```

the p-value with a significance level of 95%. If ***p-value is lesser than 0.05*** *hence w****e reject the null hypothesis***

T- test for two samples (independent)

- Two-Tailed Test: t.test(x, y, mu = ,)
- Right-Tailed Test: t.test(x, y, mu= , alternative="greater")
- Left-Tailed Test: t.test(x, y, mu= ,alternative="less")

Problem

Comparing two independent sample means, taken from two populations with unknown variances. The following data shows the heights of individuals of two different countries with unknown population variances. Is there any significant difference between the average heights of two groups.

A:	175	168	168	190	156	181	182	175	174	179
B:	185	169	173	173	188	186	175	174	179	180

R- Code & Interpretation

```
> a = c(175, 168, 168, 190, 156, 181, 182, 175, 174, 179)
> b = c(120, 180, 125, 188, 130, 190, 110, 185, 112, 188)
> t.test(a,b, var.equal=FALSE, paired=FALSE)

        Welch Two Sample t-test

data:  a and b
t = 1.8827, df = 10.224, p-value = 0.08848
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.95955 47.95955
sample estimates:
mean of x mean of y
    174.8     152.8
```

The p-value > 0.05, we conclude that the means of the two groups are significantly similar

Problem

- Suppose the recovery time for patients taking a new drug is measured (in days). A placebo group is also used to avoid the placebo effect. The data are as follows

with drug	: 15 10 13 7 9 8 21 9 14 8
placebo	: 15 14 12 8 14 7 16 10 15 2

Is there any significant difference between the average effect of these two drugs?

R- Code & Interpretation

```
> x = c(15, 10, 13, 7, 9, 8, 21, 9, 14, 8)
> y = c(15, 14, 12, 8, 14, 7, 16, 10, 15, 12)
> t.test(x,y,alt="less",var.equal=TRUE)

        Two Sample t-test

data:  x and y
t = -0.53311, df = 18, p-value = 0.3002
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
     -Inf 2.027436
sample estimates:
mean of x mean of y
     11.4      12.3
```

P value (0.3002) > 0.05 then there is no evidence to reject our Null hypothesis

Problem

- **Six subjects were given a drug (trearment group) and an additional 6 subjects a placebo(control group).Their reaction time to stimulus was measured(in ms).We want to perform a two sample t-test for comparing the means of the treatment and control groups**

Control	91	87	99	77	88	91
Treatment	101	110	103	93	99	104

R- Code and Inference

```
> control=c(91,87,99,77,88,91)
> Treat=c(101,110,103,93,99,104)
> t.test(control,Treat,alternative="less",var.equal=TRUE)

        Two Sample t-test

data:  control and Treat
t = -3.4456, df = 10, p-value = 0.003136
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
      -Inf -6.082744
sample estimates:
mean of x mean of y
 88.83333 101.66667
```

```
> control=c(91,87,99,77,88,91)
> Treat=c(101,110,103,93,99,104)
> t.test(control,Treat,alternative="less",var.equal=FALSE)

        Welch Two Sample t-test

data:  control and Treat
t = -3.4456, df = 9.4797, p-value = 0.003391
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
      -Inf -6.044949
sample estimates:
mean of x mean of y
 88.83333 101.66667
```

From both the output we see that the p-value = 0.003136(equal) and 0.003391(Unequal). Therefore, it infers that there is different between treatment and control group.

Paired t-test (Dependent Sample)

```
# paired t-test

> t.test(y1,y2,paired=TRUE)        # where y1 & y2 are numeric
```

Problem

- *A school athletics has taken a new instructor, and want to test the effectiveness of the new type of training proposed by the new instructor comparing the average times of 10 runners in the 100 meters. The results are given below(time in seconds)*

Before training	*12.9*	*13.5*	*12.8*	*15.6*	*17.2*	*19.2*	*12.6*	*15.3*	*14.4*	*11.3*
After training	*12.7*	*13.6*	*12.0*	*15.2*	*16.8*	*20.0*	*12.0*	*15.9*	*16.0*	*11.1*

- *Solu:*
- *In this case we have two sets of paired samples, since the measurements were made on the same athletes before and after the workout. To see if there was an improvement, deterioration, or if the means of times have remained substantially the same (hypothesis H0), we need to make a Student's t-test for paired samples, proceeding in this way*

R- Code & Inference

```
> before = c(12.9, 13.5, 12.8, 15.6, 17.2, 19.2, 12.6, 15.3, 14.4, 11.3)
> after = c(12.7, 13.6, 12.0, 15.2, 16.8, 20.0, 12.0, 15.9, 16.0, 11.1)
> t.test(before,after, paired=TRUE)

        Paired t-test

data:  before and after
t = -0.21331, df = 9, p-value = 0.8358
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5802549  0.4802549
sample estimates:
mean of the differences
                  -0.05
```

Interpretation :-

The p-value is greater than 0.05, then we do not reject the hypothesis H_0 *of equality of the averages and conclude that the new training has not made any significant improvement to the team of athletes.*

Problem

Suppose now that the manager of the team (given the results obtained) fired the coach who has not made any improvement, and take another, more promising. We report the times of athletes after the second training:

Before training:	*12.9*	*13.5*	*12.8*	*15.6*	*17.2*	*19.2*	*12.6*	*15.3*	*14.4*	*11.3*
After the second training:	*12.0*	*12.2*	*11.2*	*13.0*	*15.0*	*15.8*	*12.2*	*13.4*	*12.9*	*11.0*

Solu:

Now we check if there was actually an improvement, ie perform a t-test for paired data, specifying in R to test the alternative hypothesis H1 of improvement in times. To do this simply add the syntax alt = "less" when you call the t-test

R- Code & Inference

```
> before=c(12.9, 13.5, 12.8, 15.6, 17.2, 19.2, 12.6, 15.3, 14.4, 11.3)
> after = c(12.0, 12.2, 11.2, 13.0, 15.0, 15.8, 12.2, 13.4, 12.9, 11.0)
> t.test(before,after, paired=TRUE, alt="less")

        Paired t-test

data:  before and after
t = 5.2671, df = 9, p-value = 0.9997
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
     -Inf 2.170325
sample estimates:
mean of the differences
                   1.61
```

In response, we obtained a p-value well above 0.05, which leads us to conclude that we can reject the null hypothesis H_0 *in favour of the alternative hypothesis H1: the new training has made substantial improvements to the team*

Problem

- *Consider the paired data below that represents cholesterol levels on 10 men before and after a certain medication Test the claim that, on average, the drug lowers cholesterol in all men. i.e., test the claim that* $\mu_d > 0$. *Test this at the 0.05 significance level*

Before(x)	237	289	257	228	303	275	262	304	244	233
After(y)	194	240	230	186	265	222	242	281	240	212

R- Code and Interpretation

```
> before=c(237,289,257,228,303,275,262,304,244,233)
> after=c(194,240,230,186,265,222,242,281,240,212)
> t.test(before,after,paired=TRUE,alternative="greater",mu=0)

        Paired t-test

data:  before and after
t = 6.5594, df = 9, p-value = 5.202e-05
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 23.05711      Inf
sample estimates:
mean of the differences
                     32
```

We can reject the null hypothesis and support the claim because the P-value ($5.2\ X10^{5}$) is less than the significance level

F- Test (Variance Ration Test)

- Syntax:

```
var.test(x, y)
```

Problem

- *Five Measurements of the output of two units have given the following results (in kilograms of material per one hour of operation) .Assume that both samples have been obtained from normal populations, test at 10% significance level if two populations have the same variance*

Unit A	14.1	10.1	14.7	13.7	14.0
Unit B	14.0	14.5	13.7	12.7	14.1

$H_0: S_1^2 = S_2^2$

$H_1: S_1^2 \neq S_2^2$

R- Code and Inference

```
> Unit_A=c(14.1,10.1,14.7,13.7,14.0)
> Unit_B=c(14.0,14.5,13.7,12.7,14.1)
> var.test(Unit_A,Unit_B)

        F test to compare two variances

data:  Unit_A and Unit_B
F = 7.3304, num df = 4, denom df = 4, p-value = 0.07954
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
  0.7632268 70.4053799
sample estimates:
ratio of variances
          7.330435
```

Here p valuve >0.05 ,then there is no evidence to reject the null hypothesis

Practice Problems

- A certain stimulus administered to each of the 13 patients resulted in the following increase of blood pressure: 5, 2, 8,-1, 3, 0, -2, 1, 5, 0, 4, 6, 8. Can it be concluded that the stimulus, in general, be accompanied by an increase in the blood pressure.
- The manufacturer of a certain make of electric bulbs claims that his bulbs have a mean life of 25 months with a standard deviation of 5 months. Random samples of 6 such bulbs have the following values: Life of bulbs in months: 24, 20, 30, 20, 20, and 18. Can you regard the producer's claim to valid at 1% level of significance

Practice Problems cont…

- The life time of electric bulbs for a random sample of 10 from a large consignment gave the following data: 4.2, 4.6, 3.9, 4.1, 5.2, 3.8, 3.9, 4.3, 4.4, 5.6 (in '000 hours). Can we accept the hypothesis that the average life time of bulbs is 4, 000 hours
- Data on weight (grams) of two treatments of NMU (nistroso- methyl urea) are recorded. Find out whether these two treatments have identical effects by using t test for sample means at 5% level of significance.

Sample	1	2	3	4	5	6	7	8	9	10	11	12
Treatments 0.2 %	2.0	2.7	2.9	1.9	2.1	2.6	2.7	2.9	3.0	2.6	2.6	2.7
0.4%	3.2	3.6	3.7	3.5	2.9	2.6	2.5	2.7				