Tuesday, 30 March 2021 9:21 AM

Problem: $\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$$\text{Volume} = \iiint_D f(\rho,\phi,\theta)\,\rho^2\sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_{\theta_1}^{\theta_2}\int_{\phi_{min}}^{\phi_{max}}\int_{\rho_1}^{\rho_2} \quad d\rho\, d\phi\, d\theta$$

Soln:

$$\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{\pi}\int_0^{\pi} \sin\phi \boxed{\int_0^{2\sin\phi} \rho^2 \, d\rho} \, d\phi \, d\theta \quad (1)$$

$$= \int_0^{\pi}\int_0^{\pi} \sin\phi \left[\frac{\rho^3}{3}\right]_0^{2\sin\phi} d\phi \, d\theta$$

$$= \int_0^{\pi}\int_0^{\pi} \sin\phi \left[\frac{8\sin^3\phi}{3} - 0\right] d\phi \, d\theta$$

$$= \frac{8}{3}\int_0^{\pi}\int_0^{\pi} \sin\phi \cdot \sin^3\phi \, d\phi \, d\theta$$

$$= 8/3 \int_0^{\pi}\int_0^{\pi} \sin^2\phi \, \sin^2\phi \, d\phi \, d\theta$$

$$= 8/3 \int_0^{\pi}\int_0^{\pi} (1-\cos^2\phi)\sin^2\phi \, d\phi \, d\theta$$

$$\left[\because \sin^2\phi + \cos^2\phi = 1, \quad \sin^2\phi = 1-\cos^2\phi\right]$$

$$= \underbrace{8/3 \int_0^{\pi}\int_0^{\pi} \sin^2\phi \, d\phi \, d\theta}_{A} - \underbrace{8/3 \int_0^{\pi}\int_0^{\pi} \cos^2\phi \, \sin^2\phi \, d\phi \, d\theta}_{B}$$

Take A & evaluate

$$= \int^{\pi}\int^{\pi} \sin^2\phi \, d\phi \, d\theta \qquad \left[\because \sin^2\phi = \frac{1-\ldots}{}\right.$$

$\frac{\cos 2\phi}{2}$

$$= 8/3 \int_0^{\pi}\int_0^{\pi} \sin^2\phi \, d\phi \, d\theta \qquad \left[\because \sin^2\phi = \frac{1-\cdots}{\cdots}\right.$$

$$= 8/3 \int_0^{\pi}\int_0^{\pi} \frac{(1-\cos 2\phi)}{2} \, d\phi \, d\theta$$

$$= \cancel{8}^{4}/3 \int_0^{\pi}\int_0^{\pi} 1/\cancel{2} \, d\phi \, d\theta - 8/3 \cdot 1/2 \int_0^{\pi}\int_0^{\pi} \cos 2\phi \, d\phi \cdots$$

$$= 4/3 \int_0^{\pi} [\phi]_0^{\pi} \, d\theta - 4/3 \int_0^{\pi} \left[\frac{\sin 2\phi}{2}\right]_0^{\pi} \cdots$$

$$= 4/3 \, \pi \int_0^{\pi} d\theta - 4/3 \left(\frac{\sin 2\pi}{2} - \frac{\sin(\cdots}{2}\right.$$

$$= 4/3 \, \pi \, [\theta]_0^{\pi} - 4/3 \, (0)$$

$$= 4/3 \, \pi^2 \qquad \left[\text{For } A\right]$$

Take B & evaluate

$$= 8/3 \int_0^{\pi}\int_0^{\pi} \sin^2\phi \cos^2\phi \, d\phi \, d\theta$$

$\frac{\cos 2\varphi}{2}$

$d\theta$

$\pi \; d\theta$

$\int_0^{\pi} d\theta$

$$- \frac{8}{3}\int_0^{\pi}\int_0^{\pi} \frac{4}{4}\sin^2\phi\cdot\cos^2\phi\, d\phi\, d\theta$$

$$= \frac{8}{3}\cdot\frac{1}{4}\int_0^{\pi}\int_0^{\pi} 4\sin^2\phi\cdot\cos^2\phi\, d\phi\, d\theta$$

$$= \frac{2}{3}\int_0^{\pi}\int_0^{\pi} (2\sin\phi\cos\phi)^2\, d\phi\, d\theta$$

$$\left[\because 2\sin\theta\cdot\cos\theta = \sin 2\theta\right]$$

$$= \frac{2}{3}\int_0^{\pi}\int_0^{\pi} (\sin 2\phi)^2\, d\phi\cdot d\theta$$

$$= \frac{2}{3}\int_0^{\pi}\int_0^{\pi} \sin^2 2\phi\, d\phi\, d\theta$$

$$\left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right.$$

$$= \frac{2}{3}\int_0^{\pi}\int_0^{\pi} \frac{1-\cos 4\phi}{2}\, d\phi\, d\theta$$

$$= \frac{\cancel{2}}{3}\int_0^{\pi}\int_0^{\pi} \frac{1}{\cancel{2}}\, d\phi\, d\theta - \frac{2}{3}\int_0^{\pi}\int_0^{\pi} -$$

$$\frac{1}{\ }\int^{\pi} [\phi]^{\pi}\, d\theta - \frac{1}{3}\int^{\pi} \left[\frac{\sin}{}\right.$$

$$\frac{\cos 4\phi}{2} \, d\phi \, d\theta$$

$$4\phi \Big]_{0}^{\pi} d\theta$$

$$= \frac{1}{3}\int_0^{\pi} [\phi]_0^{\pi}\, d\theta - \frac{1}{3}\int_0$$

$$= \frac{1}{3}\pi\,[\theta]_0^{\pi} - 0$$

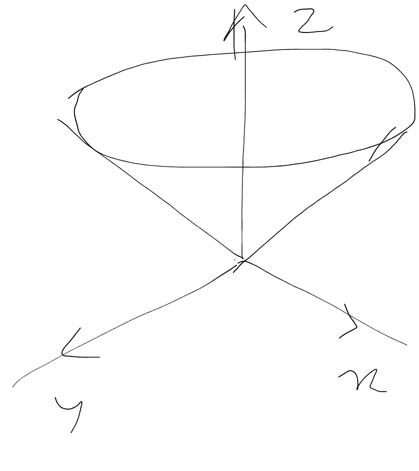
$$= \frac{1}{3}\pi^2 \quad \text{[regarding } \beta\text{]}$$

$$\iiint = \frac{4}{3}\pi^2 - \frac{1}{3}\pi^2$$

$$= \pi^2$$

Problem - 2

Use spherical co-ordinates to find vol. of solid G bounded above by sphere $x^2+y^2+z^2=1$ & below by cone $z=\sqrt{x^2+y^2}$



Find ρ limits: -