1 dik :
$$\frac{y}{2x} = 1 - \frac{dy}{2dx}$$

x	y
1	2
1,2	z

dit : $z = ?$

Jawab :

$$\frac{y}{2x} = 1 - \frac{dy}{2dx}$$

$$\frac{y}{2x} - 1 = -\frac{dy}{2dx}$$

$$\frac{dy}{dx} = 2 - \frac{y}{x}$$

(I) $$\int_{x_0}^{x} \frac{dy}{dx} dx = \int_{x_0}^{x} \left(2 - \frac{y}{x}\right) dx$$

$$y - y_0 = \int_{x_0}^{x} \left(2 - \frac{y}{x}\right) dx$$

$$y - 2 = \int_{1}^{x} \left(2 - \frac{y}{x}\right) dx$$

$$y - 2 = \int_{1}^{x} \left(2 - \frac{2}{x}\right) dx$$

$$y = 2 + [2x - 2\ln x]_{1}^{x}$$

$$= 2 + 2x - 2\ln x - 2 + 2\ln 1$$

$$y_1 = 2(x - \ln x) \approx 2,0354$$

(ii) $$\frac{dy}{dx} = 2 - \frac{y_1}{x} = 2 - \frac{2 - (x - \ln x)}{x} = \frac{2\ln x}{x}$$

$$\int_{1}^{x} \frac{dy}{dx} dx = \int_{1}^{x} \frac{2\ln x}{x} dx$$

$$y - 2 = \left[(\ln x)^2\right]_{1}^{x} = (\ln x)^2$$

$$y_2 = 2 + (\ln x)^2$$

$$\approx 2,0332$$

(iii) $$\frac{dy}{dx} = 2 - \frac{y_2}{x} = 2 - \frac{2}{x} - \frac{(\ln x)^2}{x}$$

$$\int_{1}^{x} \frac{dy}{dx} dx = \int_{1}^{x} \left[2 - \frac{2}{x} - \frac{(\ln x)^2}{x}\right] dx$$

$$y - 2 = \left[2x - 2\ln x - \frac{(\ln x)^3}{3}\right]_{1}^{x}$$

$$= 2x - 2\ln x - \frac{(\ln x)^3}{3} - 2$$

$$y_3 = 2x - 2\ln x - \frac{(\ln x)^3}{3}$$

$$\approx 2,0293$$

$$\therefore z = 2,0293 //$$

4.

diketahui : $f(x,y) = dy/dx$

$$= y + x - 0,2$$

x	0,1	0,3	0,5	0,7
y	1,115	1,327	1,468	1,729

Jawab :

$h = x_2 - x_1 = 0,3 - 0,1 = 0,2$

$f_n = f_4 = y_4 + x_4 - 0,2 = 2,229$

$f_{n-1} = f_3 = y_3 + x_3 - 0,2 = 1,768$

$f_{n-2} = f_2 = y_2 + x_2 - 0,2 = 1,427$

$f_{n-3} = f_1 = y_1 + x_1 - 0,2 = 1,015$

a.) Adam

$y_{n+1} = y_n + \dfrac{h}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$

$$= 1,729 + \dfrac{0,2}{24}(55(2,229) - 59(1,768) + 37(1,427) - 9(1,015))$$

$$= 2,2452$$

b.) milne

predaksi

$y_{n+1} = y_{n-3} + \dfrac{4h}{3}(2f_{n-2} - f_{n-1} + 2f_n)$

$$= 1,327 + \dfrac{4(0,2)}{3}(2(1,427) - 1,768 + 2(2,229))$$

$$= 2,5934$$

korekti

$f_{n+1} = y_{n-1} + x_{n-1} - 0,2$

$$= 2,5934 + 0,9 - 0,2 = 3,2934$$

$y_{n+1} = y_{n-1} = \dfrac{h}{3}(f_{n-1} - 4f_n + f_{n+1})$

$$= 1,468 + \dfrac{0,2}{3}(1,768 + 4(2,229) + 1,768)$$

$$= 2,3998$$

c.) Adam - moulton

   preakfi

   y_{n+1} (Adam) $= 2,2452$

   koreki

   $f_{n+1} = y_{n+1} + x_{n+1} - 0,2$

   $= 2,2452 + 0,9 - 0,2$

   $= 2,9452$

   $y_{n+1} = y_n + \dfrac{h}{24} \left(9 f_{n+1} + 19 f_n - 5 f_{n-1} - f_{n-2} \right)$

   $= 1,729 + \dfrac{0,2}{24} \left(9 (2,9452) + 19 (2,229) - 5 (1,768) - 1,427 \right)$

   $= 2,2173$

[2] ODE $= \dfrac{dw}{dt} + aw = b$ =) $\dfrac{dw}{dt} = b - aw$

a) Euler

   $w_{i+1} = w_i + \emptyset_i \Delta t \neq w_i + \dfrac{dw}{dt} \Big|_i \Delta t$

   $w(t=0) = 0$ m/s. Hitungan bertahap selang $\Delta t = 10s$

   $\emptyset_i = \dfrac{dw}{dt} \Big|_i = 9,75 - 0,06 w_i$

b) Heun

   $w_{i+1}^{Pred} = w_i + \emptyset_i \Delta t$

   $w_{i+1} = w_i + \dfrac{1}{2} \left(\emptyset_i + \emptyset_{i+1}^{Pred} \right) \Delta t$

   $\emptyset_i = \dfrac{dw}{dt} \Big|_i = 9,75 - 0,06 w_i$

   $\emptyset_{i+1}^{Pred} \emptyset_i = \dfrac{dw}{dt} \Big|_i = 9,75 - 0,06 w_{i+1}^{Pred}$

c.) Ralston second order

   $w_{i+1} = w_i + \emptyset \Delta t = w_i + (a_1 k_1 + a_2 k_2) \Delta t$

   $k_1 = f(t_i, w_i) = \dfrac{dw}{dt} \Big|_i = 9,75 - 0,06 w_i$

   $k_2 = f(t_i + P_1 \Delta t, w_i + a_{11} k_1 \Delta t)$

   $a_1 = 1 - a_2$

   $P_1 = a_{11} = \dfrac{1}{2 a_2}$

   metode ralston $\left(a_2 = \dfrac{2}{3} \right)$

a.) Ralston third order

$$w_{i+1} = w_i + \emptyset \, \Delta t = w_i \left[\frac{1}{6} (k_1 + 4k_2 + k_3) \right] \Delta t$$

$$k_1 = t(t_1, w_1) = \frac{dw}{dt} \Big|_1 = 9.75 - 0.06 \, w_i$$

$$k_2 = t(t_1 + \frac{1}{2} \Delta t, w_i + \frac{1}{2} k_1 \Delta t)$$

$$k_3 = t(t_1 + \Delta t, w_i - k_1 \Delta t + 2k_2 \Delta t)$$

3. a.) Single corrector Heun

i	x_i	y_i^{EKSAK}	k_1	k_2	$\emptyset$	y_i	E_t
0	0	1	8.5	-1.25	4.875	1	0 %
1	0.5	3.2187	1.25	-1.5	-0.125	3.437	-6.8 %
2	1	3	-1.5	-1.25	-1.375	3.375	-12.5 %
3	1.5	2.2187	-1.25	0.5	-0.375	2.687	-21.1 %
4	2	2	0.5	2.25	1.375	2.5	-25 %
5	2.5	2.7187	2.25	2.5	2.375	3.187	-17.2 %
6	3	4	2.5	-0.25	1.125	4.375	-9.4 %
7	3.5	4.7187	-0.25	-7.5	-3.875	4.937	-4.6 %
8	4	3	...	...	...	...	0 %