① $$u^j(x,t) = \frac{1}{2}\left(\varphi_0^j(x+t) + \varphi_0^j(x-t) + \int_{x-t}^{x+t} \varphi_1^j(\xi)\,d\xi\right)$$

$|x| \leq L,\ 0 \leq t \leq T$:

$$|u^1(x,t) - u^2(x,t)| \leq \frac{1}{2}\Big(|\varphi_0^1(x+t) - \varphi_0^2(x+t)|$$

$$+ |\varphi_0^1(x-t) - \varphi_0^2(x-t)| + \int_{x-t}^{x+t} |\varphi_1^1(\xi) - \varphi_1^2(\xi)|\,d\xi \Big)$$

$$\leq \max_{|\xi| \leq L+T} |\varphi_0^1(\xi) - \varphi_0^2(\xi)|$$

$$+ \frac{1}{2} \max_{|\xi| \leq L+T} |\varphi_1^1(\xi) - \varphi_1^2(\xi)| \cdot |\underbrace{x+t-(x-t)}_{=2t}| \leq 2T$$

$$\leq \max_{|\xi| \leq L+T} |\varphi_0^1(\xi) - \varphi_0^2(\xi)| + T \cdot \max_{|\xi| \leq L+T} |\varphi_1^1(\xi) - \varphi_1^2(\xi)|.$$

(2) $$u(x,t) = f(x+t) + g(x-t)$$

$$u_t(x,t) = f'(x+t) - g'(x-t).$$

$$u_t(x,0) = f'(x) - g'(x) \leq 0 \quad \text{für } 2a \leq x \leq 2b$$

$\Rightarrow$ $f-g$ monoton fallend auf $[2a, 2b]$.

$$u(C) = f(a+b+b-a) + g((a+b)-(b-a))$$

$$= f(2b) + g(2a) = \frac{1}{2}\left(f(2a)+g(2a)\right) + \frac{1}{2}\left(g(2a) - f(2a)\right) + f(2b)$$

$$\underset{g-f \text{ mon. wachsend}}{\leq} \frac{1}{2} u(2a,0) + \frac{1}{2}\left(g(2b) - f(2b)\right) + f(2b)$$

$$= \frac{1}{2}\left(u(A) + u(B)\right). \qquad \square$$

③ $u_{tt} - u_{xx} = f(x,t)$

$\xi = x+t, \ \eta = x-t, \quad v(\xi,\eta) = u(x,t)$

$= u\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right)$

$g(\xi,\eta) = f(x,t) = f\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right).$

$$\frac{\partial^2}{\partial\xi\partial\eta} v(\xi,\eta) = \frac{\partial}{\partial\xi}\left(\frac{1}{2}u_x\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right) - \frac{1}{2}u_t\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right)\right)$$

$$= \frac{1}{4}\left(u_{xx} - u_{tt}\right)\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right)$$

$$= -\frac{1}{4} f\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right) = -\frac{1}{4} g(\xi,\eta)$$

$$u(x,t) = v(\xi,\eta) = a(\xi) + b(\eta) - \frac{1}{4}\int_0^{\xi}\int_0^{\eta} f\left(\frac{\tilde\xi+\tilde\eta}{2}, \frac{\tilde\xi-\tilde\eta}{2}\right) d\tilde\eta\, d\tilde\xi$$

$$= a(x+t) + b(x-t) - \frac{1}{4}\int_0^{x+t}\int_0^{x-t} f\left(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2}\right) d\eta\, d\xi$$