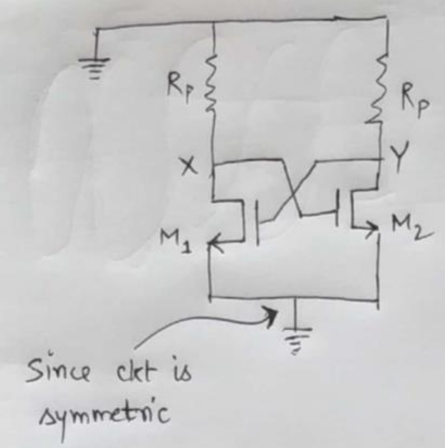
EE 204-2018-2 Analog Circuits
Homework #5 Solution

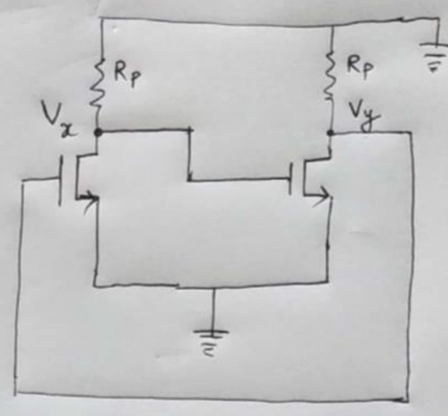
Question 1:

Q1/Solution

(i) At oscillation C_P & L_P cancel each other out. Hence the eqv. ckt. is:



$\equiv$



* Assuming $\lambda = 0$

$$\text{Loop gain} = \text{Gain}\Big|_{\text{First stage}} \times \text{Gain}\Big|_{\text{Second Stage}}$$

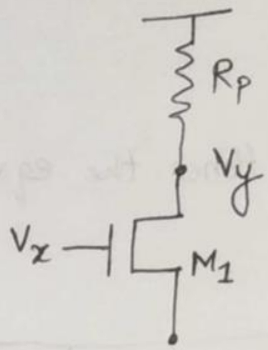
$$= (-g_{m_1} R_P) \times (-g_{m_2} R_P)$$

$$\text{Loop Gain} = g_{m_1} g_{m_2} R_P^2 \equiv g_m^2 R_P^2 \quad (\text{if } g_{m_1} = g_{m_2} = g_m)$$

Applying the Barkhausen criteria,

$$g_m^2 R_P^2 > 1 \Rightarrow 2\mu_n C_{ox} \frac{W}{L} I_D R_P^2 > 1$$

where $I_D = \frac{I_{SS}}{2}$, Hence $\quad I_{SS}\Big|_{min} = \dfrac{1}{\mu_n C_{ox} \frac{W}{L} R_P^2}$

(ii) 

$V_{DS} > V_{GS} - V_{Th}$

$V_y > V_x - V_{Th}$

$V_y - V_x > -V_{Th}$

Similarly from M_2 we get $V_x - V_y > -V_{Th}$

i.e $-V_{Th} < V_y - V_x < V_{Th}$

Max diff of $V_y - V_x$ is $I_{SS} R_P$ hence

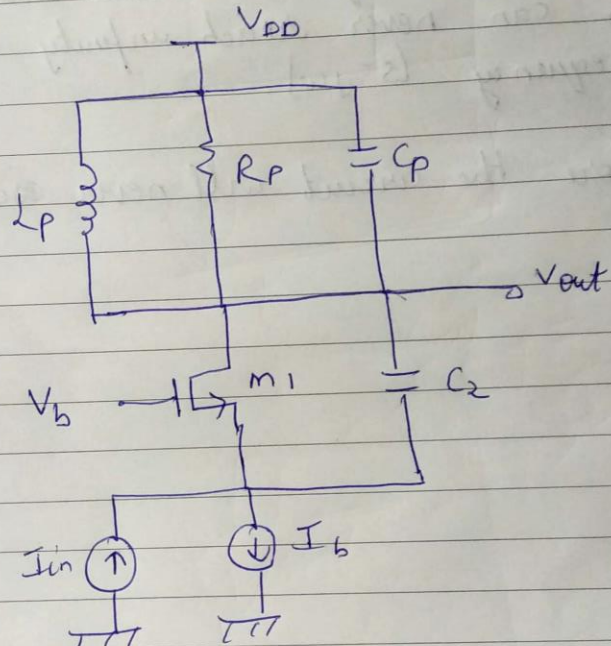
$$I_{SS} R_P < V_{Th}$$

$$I_{SS}\Big|_{max} = \frac{V_{Th}}{R_P}$$

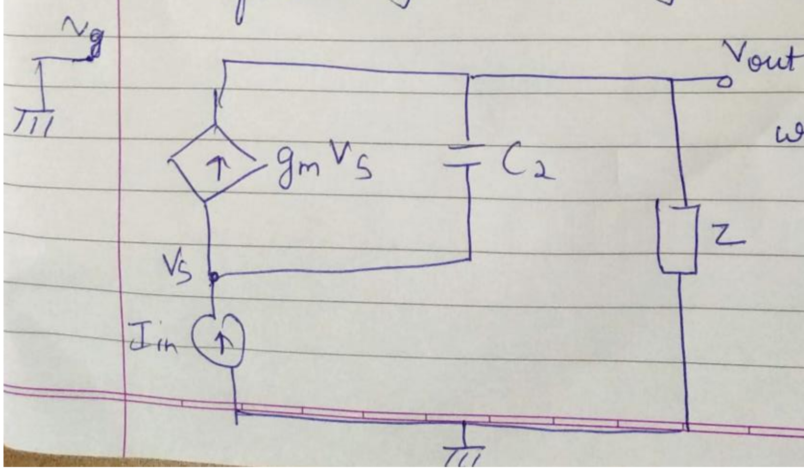
Question 2:

Question 2

To obtain the closed-loop gain, the following circuit is analysed :-



In the above case, I_{in} is the input current. To obtain the gain; V_b and V_{DD} are grounded whereas I_b is opened. We have the following small signal circuit



~~where Z=~~

where $Z = sL_p \parallel \frac{1}{sC_p} \parallel R_p$

From the above figure we find -

$$\boxed{\frac{V_{out}}{I_{in}} = Z = sL_p \parallel \frac{1}{sC_p} \parallel R_p}$$

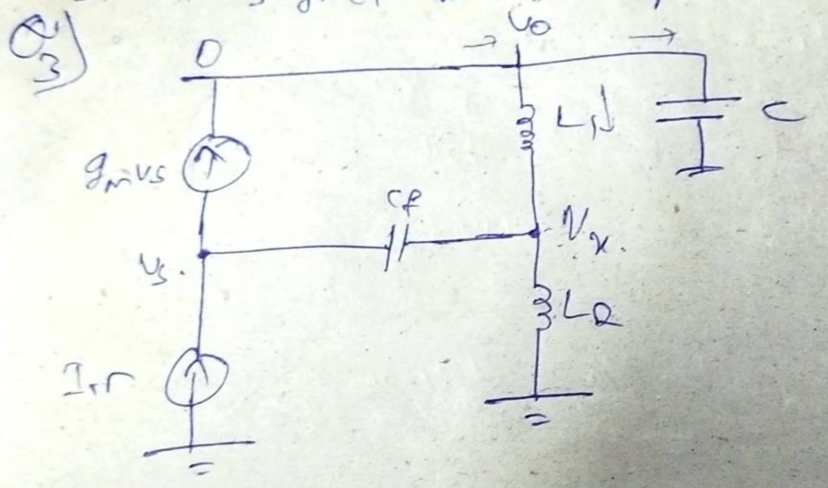
From the above equation we see that the gain can never reach infinity at any frequency $(s = j\omega)$.

Therefore the circuit will never oscillate.

Question 3:

Small signal Analysis of

Q3)



Nodal-Analysis.

$$\frac{V_o - V_x}{sL_1} + V_o sC - g_m V_s = 0 \quad -(1)$$

$$I_{in} = g_m V_s + \underline{(V_s - V_x)} sC_f \quad -(2)$$

$$\frac{V_x}{sL_2} + (V_x - V_s) sC_f + \left(\frac{V_x - V_o}{sL_1}\right) = 0 \quad -(3)$$

$\cancel{(1)}$ (2)

$$I_{in} = g_m V_s + sC_f V_s - sV_x C_f$$

$$I_{in} = V_s (g_m + sC_f) - sV_x C_f.$$

$$V_s = \frac{I_{in} + sV_x C_f}{g_m + sC_f} \quad -(4)$$

$$\frac{V_o - V_x}{sL_1} + V_o sC - g_m V_s = 0$$

$$\frac{V_o - V_x}{sL_1} + V_o sC - g_m \left(\frac{I_{in} + sV_x C_f}{g_m + sC_f} \right) = 0$$

$$\frac{V_o}{sL_1} + V_o sC - \frac{V_x}{sL_1} - \frac{g_m I_{in}}{g_m + sC_f} - \frac{g_m sV_x C_f}{g_m + sC_f} = 0.$$

$$\frac{V_o}{sL_1} + V_o sC - \frac{g_m I_{in}}{g_m + sC_f} = V_x \left\{ \frac{1}{sL_1} + \frac{g_m sC_f}{g_m + sC_f} \right\}$$

$$\frac{\left\{ \frac{V_o}{sL_1} + V_o sC - \frac{g_m I_{in}}{g_m + sC_f} \right\}}{\frac{1}{sL_1} + \frac{g_m sC_f}{g_m + sC_f}} = V_x \quad \text{—— (5)}$$

from eq. ②

$$V_S = \frac{I_{in}}{g_m + sC_f} + \frac{sC_f}{g_m + sC_f} \frac{\left\{ \frac{V_o}{sL_1} + V_o sC - \frac{g_m I_{in}}{g_m + sC_f} \right\}}{\frac{1}{sL_1} + \frac{g_m sC_f}{g_m + sC_f}}$$

$$V_S = \frac{I_{in}}{g_m + sC_f} + \frac{sC_f}{g_m + sC_f} \cdot \frac{\left\{ V_o(g_m + sC_f) + V_o s^2 L_1 C - g_m I_{in} sL_1 \right\}}{g_m + sC_f + g_m sC_f}$$

—— (6)

$$\rightarrow V_o\left(\frac{1}{sL_1}+sC\right)-\frac{V_x}{sL_1}-g_mV_s=0$$

$$\rightarrow V_o\left(\frac{1}{sL_1}+sC\right)-\frac{1}{sL_1}\frac{\left\{\frac{V_o}{sL_1}+V_osC-\frac{g_mI_{in}}{g_m+sC_f}\right\}}{\frac{1}{sL_1}+\frac{g_msC_f}{g_m+sC_f}}$$

$$-g_m\left(\frac{I_{in}}{g_m+sC_f}+\frac{sC_f}{g_m+sC_f}\left\{\frac{V_o(g_m+sC_f)+V_os^2L_1C-g_mI_{in}sL_1}{g_m+sC_f+g_m s\cdot C_f}\right\}\right)$$

$$\rightarrow V_o\left(\frac{1}{sL_1}+sC\right)+V_o\frac{\left\{\frac{-1}{s^2L_1^2}+sC\right\}}{\frac{1}{sL_1}+\frac{g_msC_f}{g_m+sC_f}}$$

$$+I_{in}\left\{\frac{-\frac{g_m}{g_m+sC_f}}{\frac{1}{sL_1}+\frac{g_msC_f}{g_m+sC_f}}\right\}$$

$$-g_m\left(\frac{I_{in}}{g_m+sC_f}+\frac{sC_f}{g_m+sC_f}\left\{\frac{V_o(g_m+sC_f)+V_os^2L_1C-g_mI_{in}sL_1}{g_m+sC_f+g_msC_f}\right\}\right)$$

Solve for $\frac{V_o}{I_{in}}$ from above equation and equate I_o and gives the condition for frequency of oscillations.