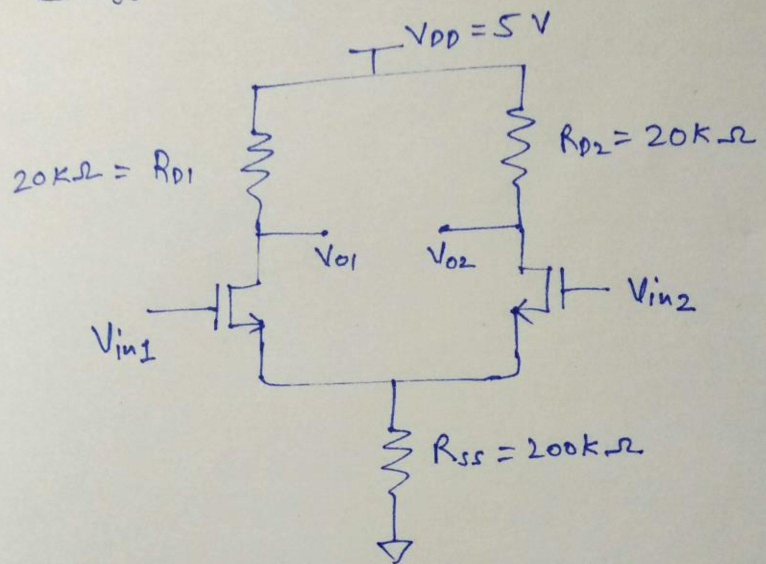
EE 204-2018-2 Analog Circuits
Homework #2 Solution

Question 1A-I

Qus. 1 A

(1) Differential amplifier



Here $g_{m1} = g_{m2} = 2\ mS$, $R_{D1} = R_{D2} = 20k\Omega$

$$(V_{O1} - V_{O2}) = -g_{m1}(V_{in1} - V_{in2})R_{D1}$$

$$\frac{(V_{O1} - V_{O2})}{(V_{in1} - V_{in2})} = -g_{m1}R_{D1}$$

$$= -2 \times 10^{-3} \times 20 \times 10^{3}$$

Differential gain $\boxed{A_V = -40}$

Question 1A-II (a)

homework-2

1A.

II. (a) $g_{m1} = g_{m0} + \frac{\Delta g_m}{2}$

$g_{m2} = g_{m0} - \frac{\Delta g_m}{2}$

$g_{m0} = 2\,mS$ and $\Delta g_m = 0.2\,mS$

$\left.\begin{aligned} R_{d1} &= 20\,k\Omega \\ R_{d2} &= 20\,k\Omega \end{aligned}\right\} = R_d$

$R_{SS} = 200\,k\Omega$

$r_{o1} = r_{o2} = \infty$

$$A_{CM-DM} = -\left[\frac{R_d \Delta g_m}{1 + 2 g_m R_{SS}}\right] = -\left[\frac{20\times10^3 \times 0.2\times10^{-3}}{1 + 2\times2\times10^{-3}\times200\times10^3}\right]$$

$$= -4.993\times10^{-3}$$

$$CMRR = \frac{1}{2}\left|\frac{(g_{m1}R_{d1} + g_{m2}R_{d2}) + 2g_{m1}g_{m2}R_{SS}(R_{d1}+R_{d2})}{g_{m1}R_{d1} - g_{m2}R_{d2}}\right|$$

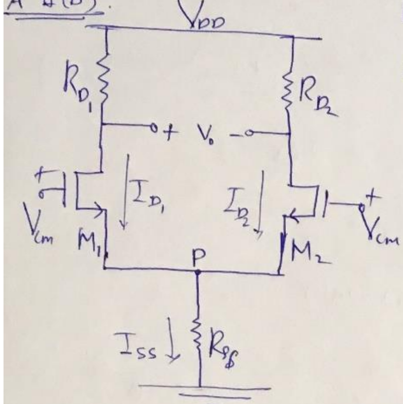
$$= \frac{1}{2}\left|\frac{(2.1\times20 + 1.9\times20) + 2\times2.1\times1.9\times200\times10^{-3}(20+20)\times10^{3}}{2.1\times20 - 1.9\times20}\right|$$

$$= 7990$$

$$= 78.05\,dB$$

Question 1A-II (b)

A II (b).



$R_{D_1} = R_{D_0} + \frac{\Delta R_D}{2}$

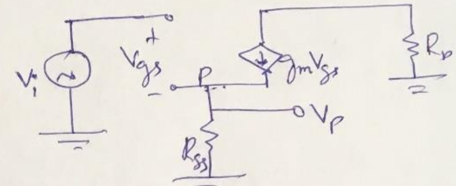
$R_{D_2} = R_{D_0} - \frac{\Delta R_D}{2}$

$\Delta R_D = R_{D_1} - R_{D_2}$

$\therefore R_{D_1} + R_{D_2} = 2R_{D_0}$

Let, $V_{cm} = V_i$

Considering one transistor, with small signal analysis,



Now, $V_g = V_i$ & $V_s = V_P = g_m V_{gs} \cdot R_{SS}$

$\therefore V_{gs} = V_g - V_s = V_i - g_m V_{gs} R_{SS} \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_{SS}}$

& $\therefore \frac{V_P}{V_i} = \frac{g_m R_{SS}}{1 + g_m R_{SS}}$

$\because$ Two transistors are sending current through R_{SS}. In mismatch

$\therefore$ $\Delta V_P = \frac{2R_{SS} g_m}{1 + 2g_m R_{SS}} \cdot \Delta V_{i,cm}$ & $\Delta I_{SS} = \frac{\Delta V_P}{R_{SS}}$

Also, $\Delta I_{D_1} = \Delta I_{D_2} = \frac{\Delta I_{SS}}{2} = \frac{g_m}{1 + 2g_m R_{SS}} \Delta V_{i,cm}$

Hence, $\Delta V_{o,CM-DM} = (-\Delta I_{D_1} R_{D_1}) - (-\Delta I_{D_2} R_{D_2}) = -\left(\frac{\Delta I_{SS}}{2}\right) \Delta R_D$

or $\Delta V_{o,CM-DM} = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}} \Delta V_{i,cm}$ & $A_{CM-DM} = \frac{\Delta V_{o,cm-Dm}}{\Delta V_{i,cm}}$

$$\Rightarrow \boxed{A_{CM-DM} = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}}$$

$\because$ In Differential mode, $V_P \approx 0$ virtual ground.

& $\boxed{A_{DM} = -g_m R_{D_0}}$ {check.}

given $R_{SS} = 200\,k\Omega$
$R_D = 20\,k\Omega$
$\Delta R_D = 2\,k\Omega$
$g_m = 2\,mS$

So, $A_{DM} = -(2\times10^{-3}).(20\times10^{3})$

or $A_{DM} = -40$

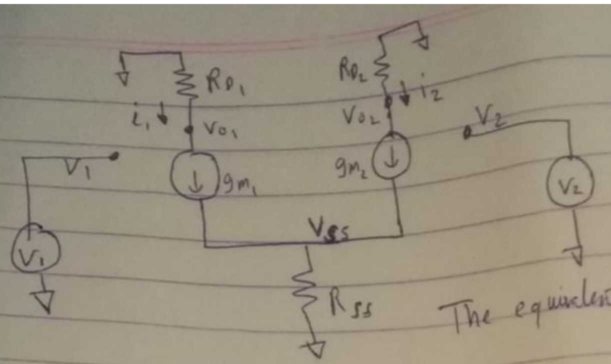
& $A_{cm-DM} = \dfrac{-(2\times10^{-3})(2\times10^{3})}{1+(2\times2\times10^{-3}\times200\times10^{3})}$

$A_{cm-DM} = \dfrac{-4}{1+800} \simeq \dfrac{-4}{800} = -5\times10^{-3}$

$\dfrac{A_{cm-DM}}{A_{DM}} = \dfrac{\cancel{-}\,\overset{1}{\cancel{5}}\times10^{-3}}{\cancel{-}\,\underset{8}{\cancel{40}}} = \dfrac{1}{8000}$

$\therefore$ $CMRR = 20\log\left|\dfrac{A_{DM}}{A_{cm-DM}}\right| = 78.0618\ dB$

Question 1A-II(c)



The equivalent ckt

$V_{SS} = (i_1 + i_2) R_{SS}$ — (1)

$i_1 = (V_1 - V_{SS}) gm_1$, $i_2 = (V_2 - V_{SS}) gm_2$ — (2)

for Common mode,

$V_1 = V_2 = V_C$

Solving for V_{SS} using above equations

$V_{SS} = (gm_1 + gm_2) / (1/R_{SS} + gm_1 + gm_2)$

now $V_{o1} = R_{D_1} gm_1 (V_C - V_{SS})$, $V_{o2} = R_{D_2} gm_2 (V_C - V_{SS})$

$\Rightarrow V_o = V_{o1} - V_{o2} = (R_{D_1} gm_1 - R_{D_2} gm_2)(V_C - V_S)$

$= (R_{D_1} gm_1 - R_{D_2} gm_2) V_C / (1 + (gm_1 + gm_2) R_{SS})$

$\Rightarrow A_{CM} = (R_{D_1} gm_1 - R_{D_2} gm_2) / (1 + (gm_1 + gm_2) R_{SS})$

(putting values) $= .01$

for Diff Mode

$V_1 = +\frac{V_d}{2}$, $V_2 = -V_d/2$

$V_o = V_{o1} - V_{o2} = R_{D_1} gm_1 (\frac{V_d}{2} - V_S) - R_{D_2} gm_2 (-\frac{V_d}{2} - V_S)$

$V_S = 0$ for diff mode

$\Rightarrow A_{DM} = \frac{R_{D_1} gm_1 + R_{D_2} gm_2}{2}$

putting values $A_{DM} \simeq 40$

$\Rightarrow CMRR \simeq 4000$

Question 1A-II(d)

Ques 1A II. (d)

$\boxed{\text{Repeating part (a)}}$. Given $r_{o1} = r_{o2}$: $50K\Omega$, $R_{D1} = R_{D2}$: $20K\Omega$

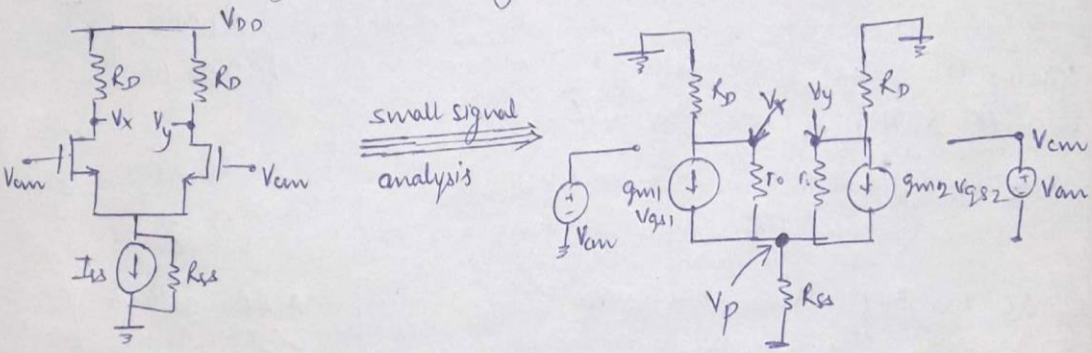
R_{SS}: $200K\Omega$, $g_{m1} = g_{m0} + \frac{\Delta g_m}{2}$

$= 2m + \frac{.2m}{2} = 2.1mS$

$g_{m2} = g_{m0} - \frac{\Delta g_m}{2}$

$= 2m - \frac{.2m}{2} = 1.9mS.$

When r_o comes into effect, we need to apply exact analysis using small signal model of ckt.



Due to mismatch there would be A_{CM-DM}. and we need to find $\frac{V_x - V_y}{V_{cm}}$ i.e. differential o/p due to common mode inputs

Applying KCL at node V_p.

$$\Rightarrow \frac{V_p}{R_{SS}} + \frac{V_p - V_x}{r_o} + \frac{V_p - V_y}{r_o} = g_{m1}(V_{gs1}) + g_{m2}(V_{gs2})$$

$$\Rightarrow \frac{V_p}{200K} + \frac{V_p - V_x}{50K} + \frac{V_p - V_y}{50K} = 2.1m(V_{cm} - V_p) + 1.9m(V_{cm} - V_p)$$

$$\left[V_{gs} = V_{gate} - V_{source} = V_{cm} - V_p \right]$$

$$\Rightarrow \frac{V_p}{200K} + \frac{V_p - V_x}{50K} + \frac{V_p - V_y}{50K} = 4m(V_{cm} - V_p)$$

$$\Rightarrow V_p\left[\frac{1}{200K} + \frac{1}{50K} + \frac{1}{50K} + 4m\right] = 4m\, V_{cm} + \frac{V_x}{50K} + \frac{V_y}{50K}$$

$$\Rightarrow \boxed{809 V_p = 800 V_{cm} + 4V_x + 4V_y} \quad \text{—(1)}$$

Now applying KCL at node V_x.

$$\Rightarrow \frac{V_x}{R_D} + \frac{V_x - V_p}{r_o} + g_{m1} V_{gs1} = 0$$

$$\Rightarrow \frac{V_x}{20K} + \frac{V_x - V_p}{50K} + 2.1m\,(V_{cm} - V_p) = 0$$

$$\Rightarrow \boxed{V_x = 30.29 V_p - 30 V_{cm}} \quad — (2)$$

Applying KCL at node V_y

$$\Rightarrow \frac{V_y}{R_D} + \frac{V_y - V_p}{r_o} + g_{m2} V_{gs2} = 0$$

$$\Rightarrow \frac{V_y}{20K} + \frac{V_y - V_p}{50K} + 1.9m\,(V_{cm} - V_p) = 0$$

$$\Rightarrow \boxed{V_y = 27.43 V_p - 27.14 V_{cm}} \quad — (3)$$

Putting the values of V_x and V_y from eqn (2) and (3) into eqn (1)

$$\Rightarrow 809 V_p = 800 V_{cm} + 4(30.29 V_p - 30 V_{cm}) + 4(27.43 V_p - 27.14 V_{cm})$$

$$\Rightarrow 809 V_p = 571.44 V_{cm} + 230.88 V_p$$

$$\Rightarrow 578.12 V_p = 571.44 V_{cm}$$

$$\boxed{V_p = .988 V_{cm}} \quad — (4)$$

from eqn (2) and (3)

$$V_x - V_y = 2.86 V_p - 2.86 V_{cm}$$

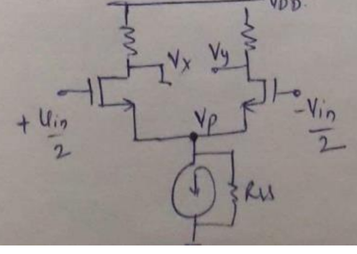
putting V_p from eqn (4)

$$V_x - V_y = 2.86 \times (.988) V_{cm} - 2.86 V_{cm}$$

$$\boxed{\frac{V_x - V_y}{V_{cm}} = -.034 \Rightarrow A_{CM-DM}}$$

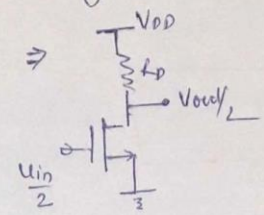
Ans

To calculate CMRR we need to find out A_{DM}.



Applying half ckt. analysis

$\Rightarrow$



$$\Rightarrow A_{DM} = -g_m R_{out}$$

$$\Rightarrow \quad = -g_m (R_D \| r_o)$$

$$= -2m\,[20K \| 50K]$$

$$\boxed{A_{DM} = -28.57}$$

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

$$= \frac{28.57}{.034} = 840.3$$

$$\boxed{CMRR = 840.3}$$

In Previous Case, when $r_{o1} = r_{o2} = \infty$

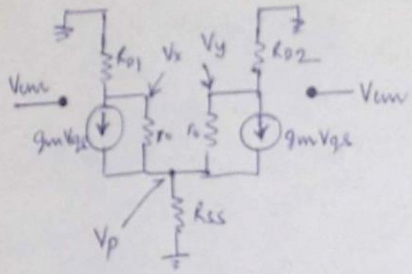
$A_{DM} = 40$, $A_{CM-DM} = .005$

$$CMRR = \frac{40}{.0005} = 8000$$

Repeating part (b) Given $r_{o1}=r_{o2}=50k$, $g_{m1}=g_{m2}=2mS$

$R_{SS}=200k\Omega$ $\quad R_{D1}=R_{D0}+\frac{\Delta R_D}{2}=21k\Omega$

$R_{D2}=R_{D0}-\frac{\Delta R_D}{2}=19k\Omega$



applying KCL at node V_p.

$$\frac{V_p}{R_{SS}}+\frac{V_p-V_x}{r_o}+\frac{V_p-V_y}{r_o}=2g_mV_{gs}$$

$$V_p\left(\frac{1}{R_{SS}}+\frac{2}{r_o}\right)=\frac{V_x}{r_o}+\frac{V_y}{r_o}+2g_m(V_{cm}-V_p)$$

$$V_p\left(\frac{1}{R_{SS}}+\frac{2}{r_o}+2g_m\right)=\frac{V_x}{r_o}+\frac{V_y}{r_o}+2g_mV_{cm}$$

$$V_p\left(\frac{1}{200k}+\frac{2}{50k}+2\cdot 2m\right)=\frac{V_x}{50k}+\frac{V_y}{50k}+4mV_{cm}$$

$$\boxed{809V_p=800V_{cm}+4V_x+4V_y} \quad -(1)$$

applying KCL at node V_x

$$\frac{V_x}{R_{D1}}+\frac{V_x-V_p}{r_o}+g_mV_{gs}=0$$

$$\frac{V_x}{21k}+\frac{V_x-V_p}{50k}+2m(V_{cm}-V_p)=0$$

$$\boxed{V_x=-29.41V_{cm}+29.71V_p} \quad -(2)$$

Similarly, at node V_y

$$\boxed{V_y=-27.40V_{cm}+27.67V_p} \quad -(3)$$

putting the values of (2) and (3) into (1)

$$\boxed{V_p=0.988V_{cm}} \quad -(4)$$

from eqn (2) and eqn (3)

$$V_x-V_y=-2.01V_{cm}+2.04V_p$$

$$=-2.01V_{cm}+2.04\times .988V_{cm}$$

$$\boxed{\frac{V_x-V_y}{V_{cm}}=.0055=A_{CM-DM}}$$

$$CMRR=\frac{A_{DM}}{A_{CM-DM}}=\frac{28.57}{.0055}$$

$$\boxed{CMRR=5194.5}$$

In previous case, when $r_{o1}=r_{o2}=\infty$, then

$A_{DM}=40$

$A_{CM-DM}=.005$

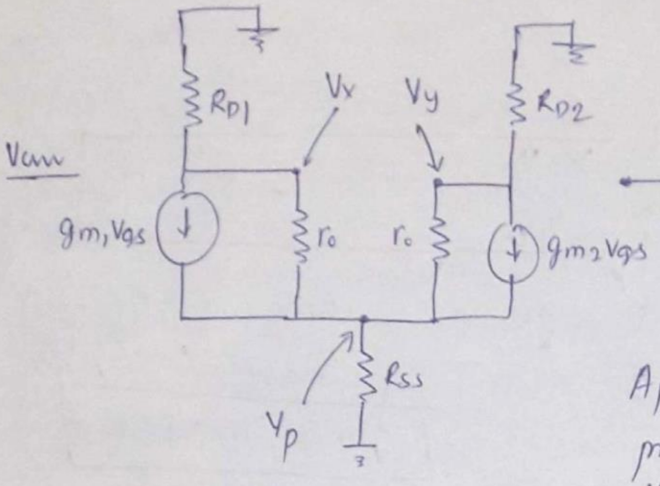
$CMRR=8000$

Repeating part c

Given $r_{o1} = r_{o2} = 50k\Omega$, $R_{ss} = 200k\Omega$

$R_{D1} = 21k\Omega$ $g_{m1} = 2.1mS$

$R_{D2} = 19k\Omega$ $g_{m2} = 1.9mS$



In this case we need to find $\frac{V_x - V_y}{V_{cm}}$ when the mismatch is present in R_D and g_m both.

Applying KCL analysis like previous case. i.e. at node V_x, V_y & V_p.

$$809V_p = 800V_c + 4V_x + 4V_y$$

$$V_x = 31.36V_p - 31.06V_{cm}$$

$$V_y = 26.45V_p - 26.17V_{cm}$$

$$V_p = 0.988V_{cm}$$

$$V_x - V_y = -.04V_{cm}$$

$$\boxed{\frac{V_x - V_y}{V_{cm}} = -.04 = A_{CM-DM}}$$

$$\boxed{CMRR = \left|\frac{-28.57}{-.04}\right| = 714.25}$$

With $r_{o1} = r_{o2} = \infty$

$$\frac{V_x - V_y}{V_{cm}} = A_{CM-DM} = 0.01$$

$$CMRR = \frac{40}{.01} = 4000$$

Question 1A-II(e)

According to the electrical circuit given in d) part we apply KCL at x,y and p nodes to get the following equations

$$V_p\left(\frac{1}{R_{ss}}+\frac{2}{r_o}\right)-\left(\frac{V_x+V_y}{r_o}\right)-(g_{m1}+g_{m2})(V_{cm}-V_p)=0 \tag{1}$$

$$V_x\left(\frac{1}{r_o}+\frac{1}{R_{D1}}\right)+g_{m1}(V_{cm}-V_p)-\frac{V_p}{r_o}=0 \tag{2}$$

$$V_y\left(\frac{1}{r_o}+\frac{1}{R_{D2}}\right)+g_{m2}(V_{cm}-V_p)-\frac{V_p}{r_o}=0 \tag{3}$$

Solving the above equations gives us the following result

$$V_p=\frac{r_o\left(g_{m2}(R_{D1}+r_o)+g_{m1}(R_{D2}+r_o)\right)R_{SS}V_{cm}}{(R_{D1}+r_o)(R_{D2}+r_o)+\big(R_{D1}+R_{D2}+g_{m2}R_{D1}r_o+g_{m1}R_{D2}r_o+r_o(2+(g_{m1}+g_{m2})r_o)\big)R_{SS}} \tag{4}$$

$$V_x=-\frac{R_{D1}r_o\left(-g_{m2}R_{SS}+g_{m1}(R_{D2}+r_o+R_{SS})\right)V_{cm}}{(R_{D1}+r_o)(R_{D2}+r_o)+\big(R_{D1}+R_{D2}+g_{m2}R_{D1}r_o+g_{m1}R_{D2}r_o+r_o(2+(g_{m1}+g_{m2})r_o)\big)R_{SS}} \tag{5}$$

$$V_y=-\frac{R_{D2}r_o\left(-g_{m1}R_{SS}+g_{m2}(R_{D1}+r_o+R_{SS})\right)V_{cm}}{(R_{D1}+r_o)(R_{D2}+r_o)+\big(R_{D1}+R_{D2}+g_{m2}R_{D1}r_o+g_{m1}R_{D2}r_o+r_o(2+(g_{m1}+g_{m2})r_o)\big)R_{SS}} \tag{6}$$

1.1 Sensitivity with only g_m mismatch

After substituting $R_{D1}=R_{D2}=R_D$, $g_{m1}=g_m+\Delta g_m/2$, $g_{m2}=g_m-\Delta g_m/2$, $r_o=50k$ and $R_{SS}=200k$. We get the following values

$$A_{CM-DM}=-\frac{50\times10^3\Delta g_m R_D\left(450\times10^3+R_D\right)}{(50\times10^3+R_D)\left(50\times10^3(9+400\times10^3 g_m)+R_D\right)} \tag{7}$$

Now the sensitivity is found as

$$Sensitivity_{gm}=\frac{\partial A_{CM-DM}}{\partial g_m}\frac{g_m}{A_{CM-DM}}=-\frac{2\times10^{10}g_m}{50\times10^3(9+4\times10^5 g_m)+R_D} \tag{8}$$

Therefore the value turns out to be **-0.988386**

1.2 Sensitivity with only R_D mismatch

After substituting $g_{m1} = g_{m2} = g_m$, $R_{D1} = R_D + \Delta R_D/2$, $R_{D2} = R_D - \Delta R_D/2$, $r_o = 50k$ and $R_{SS} = 200k$. We get the following values

$$A_{CM-DM} = -\frac{10^{10}\Delta R_D g_m}{\Delta R_D^2 - 4\left(50 \times 10^3 + R_D\right)\left(50 \times 10^3\left(9 + 400 \times 10^3 g_m\right) + R_D\right)} \tag{9}$$

Now the sensitivity is found as

$$Sensitivity_{RD} = \frac{\partial A_{CM-DM}}{\partial R_D}\frac{R_D}{A_{CM-DM}} = -\frac{R_D\left(-4\left(50 \times 10^3 + R_D\right) - 4(50 \times 10^3\left(9 + 400 \times 10^3 g_m\right) + R_D\right)}{\Delta R_D^2 - 4\left(50 \times 10^3 + R_D\right)\left(50 \times 10^3\left(9 + 400 \times 10^3 g_m\right) R_D\right)} \tag{10}$$

Therefore the value turns out to be **-0.286209**

1.3 Sensitivity with R_D and g_m mismatch

After substituting $g_{m1} = g_m + \Delta g_m/2 g_{m2} = g_m - \Delta g_m/2$, $R_{D1} = R_D + \Delta R_D/2$, $R_{D2} = R_D - \Delta R_D/2$, $r_o = 50k$ and $R_{SS} = 200k$. We get the following values

$$A_{CM-DM} = -\frac{50 \times 10^3\left(-200 \times 10^3 \Delta R_D g_M + \Delta g_m\left(\Delta R_D^2 - 4R_D(450 \times 10^3 + R_D)\right)\right)}{20 \times 10^9 \Delta g_m \Delta R_D + \Delta R_D^2 - 4(50 \times 10^3 + R_D)\left(50 \times 10^3\left(9 + 400 \times 10^3\right) + R_D\right)} \tag{11}$$

Now the sensitivity is found as

$$Sensitivity_{RD} = \frac{\partial A_{CM-DM}}{\partial R_D}\frac{R_D}{A_{CM-DM}} = 0.656071 \tag{12}$$

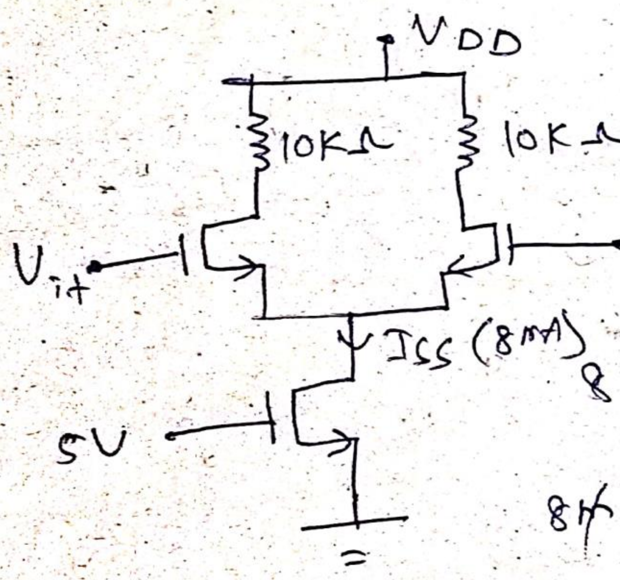
$$Sensitivity_{gm} = \frac{\partial A_{CM-DM}}{\partial g_m}\frac{g_m}{A_{CM-DM}} = -0.892405 \tag{13}$$

Note the change in signs in sensitivity of R_D.

Question 1B-I

1B

1



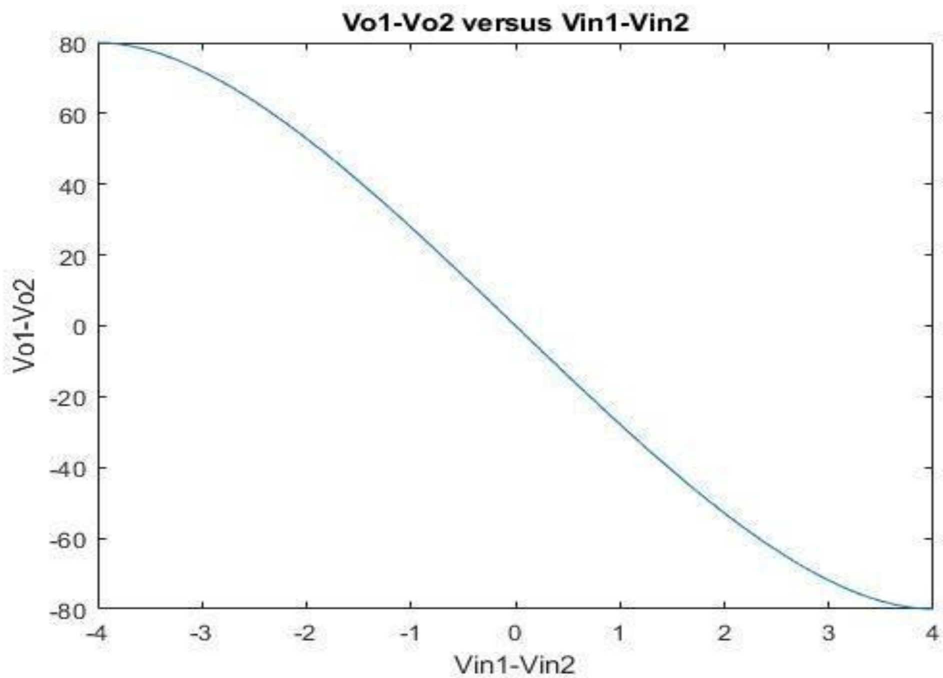
$$8\,mA = \frac{1}{2}\beta(V_{GS} - V_T)^2$$

$$8m = \frac{1}{2} \times 1m (V_{GS} - 1)^2$$

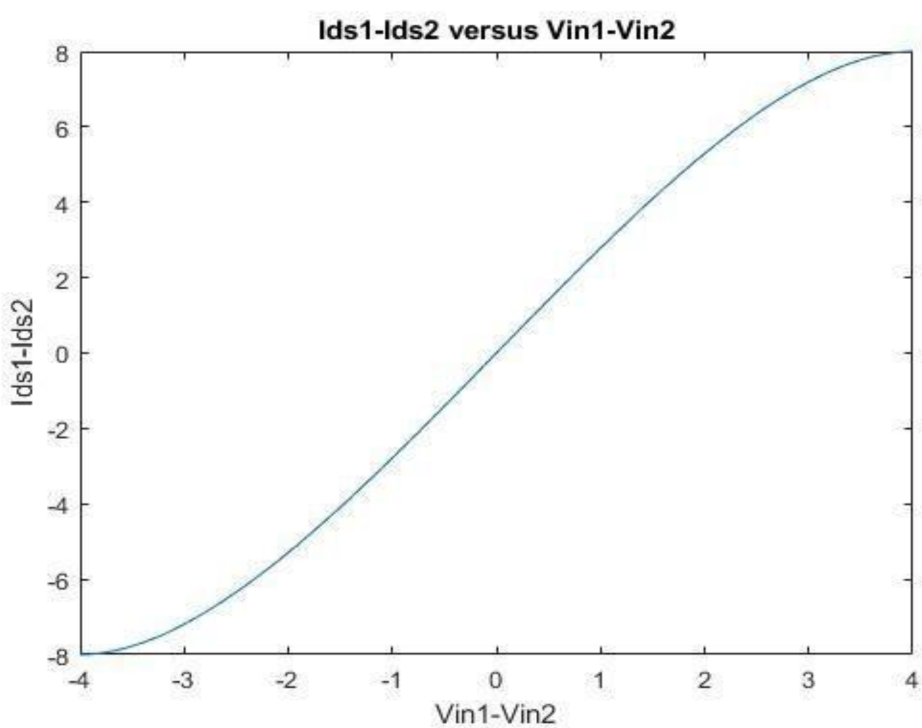
$$16 = (V_{GS} - 1)^2$$

$$\underline{\underline{V_{GS} = 5V}}$$

Question 1B-II



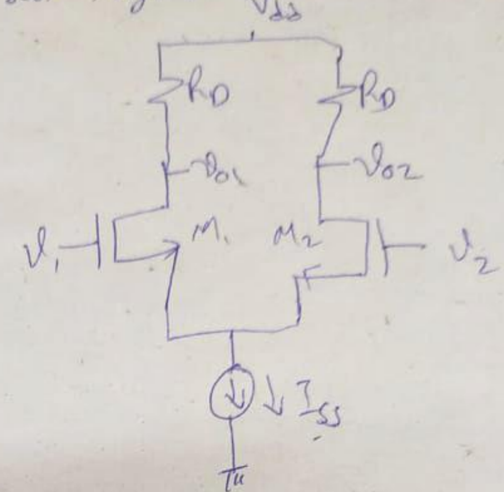
Question 1B-III



Question 1B-IV

1B (iv)

Assuming M_1 & M_2 are in sat region



$$I_D = \frac{k'}{2}(V_{GS}-V_T)^2$$

$$V_{GS} = \sqrt{\frac{2I_D}{k'}} + V_T$$

$$V_{GS1} = \sqrt{\frac{2I_{D1}}{k'}} + V_T$$

$$V_{GS2} = \sqrt{\frac{2I_{D2}}{k'}} + V_T$$

$$V_{GS1} - V_{GS2} = \sqrt{\frac{2I_{D1}}{k'}} - \sqrt{\frac{2I_{D2}}{k'}}$$

$$\cancel{V_{GS2}}\ (v_1 - V_S) - (v_2 - V_S) = \sqrt{\frac{2}{k'}}\left(\sqrt{I_{d1}} - \sqrt{I_{d2}}\right)$$

$$(v_1 - v_2)^2 = \frac{2}{k'}\left[I_{d1} + I_{d2} - 2\sqrt{I_{d1} I_{d2}}\right]$$

$$v_d^2 = \frac{2}{k'}\left[I_{SS} - \sqrt{(I_{d1}+I_{d2})^2 - (I_{d1}-I_{d2})^2}\right]$$

$$\frac{k'}{2}v_d^2 - I_{SS} = -\sqrt{I_{SS}^2 - (I_{d1} - I_{d2})^2}$$

$$(I_{d1} - I_{d2})^2 = I_{SS}^2 - \left(I_{SS} - \frac{k'}{2}v_d^2\right)^2$$

$$\boxed{I_{d1} - I_{d2} = \sqrt{k' I_{ss} v_d^2 - \left(\frac{k'}{2}\right)^2 v_d^4}}$$

$v_{o1} = V_{dd} - I_{d1} R_D \qquad v_{o2} = V_{dd} - I_{d2} R_D$

$$v_{o1} - v_{o2} = R_D (I_{d2} - I_{d1})$$

$$v_o = -R_D \sqrt{k' I_{ss} v_d^2 - \left(\frac{k'}{2}\right)^2 v_d^4}$$

$$v_o = -R_D \sqrt{k' I_{ss} v_d^2} \left[1 - \frac{k' v_d^2}{4 I_{ss}}\right]^{1/2}$$

For $R_D = 10k\Omega$, $I_{ss} = 8mA$, $k' = 1mA/V^2$

$$v_o = v_d \left[-10k \sqrt{1m \times 8m} \left(1 - \frac{1m\, v_d^2}{4 \times 8m}\right)^{1/2}\right]$$

$$\boxed{v_o = v_d \left[-28.28 \left(1 - \frac{v_d^2}{32}\right)^{1/2}\right]}$$

after expanding root with taylor series we g

→ first ~~order~~ harmonic gain $A_{DM} = -28.28$

second harmonic gain $= 0$