

Introduction to Robotics

Tutorial

Due: Apr 12th 2019

1. Consider the 1-DOF system described by the equation of motion, $4\ddot{x} + 20\dot{x} + 25x = f$.
 - (a) Find the natural frequency ω_n and the natural damping ratio ζ_n of the natural (passive) system ($f = 0$). What type of system is this (oscillatory, overdamped, etc.) ?
 - (b) Design a PD controller that achieves critical damping with a closed-loop stiffness $k_{CL} = 36$. In other words, let $f = -k_v\dot{x} - k_px$, and determine the gains k_v and k_p . Assume that the desired position is $x_d = 0$.
 - (c) Assume that the friction model changes from linear ($20\dot{x}$) to Coulomb friction, $30\text{sign}(\dot{x})$. Design a control system which uses a non-linear model-based portion with trajectory following to critically damp the system at all times and maintain a closed-loop stiffness of $k_{CL} = 36$. In other words, let $f = \alpha f' + \beta$ and $f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d)$. Then, find $f, \alpha, \beta, f', k'_p$ and k'_v . Note that f is an m -mass control, and f' is a unit-mass control. Use the definition of error, $e = x - x_d$.
 - (d) Given a disturbance force $f_{dist} = 4$, what is the steady-state ($\ddot{e} = \dot{e} = 0$) error of the system in part (c)?
2. For a certain RR manipulator, the equations of motion are given by

$$\begin{bmatrix} 4 + c_2 & 1 + c_2 \\ 1 + c_2 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -s_2(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2) \\ s_2\dot{\theta}_1^2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

- (a) Assume that joint 2 is locked at some value of θ_2 using brakes and joint 1 is controlled with a PD controller, $\tau_1 = -40\dot{\theta}_1 - 400(\theta_1 - \theta_{1d})$. What is the minimum and maximum inertia perceived at joint 1 as we vary θ_2 ? What are the corresponding closed-loop frequencies?
- (b) Still assuming that joint 2 is locked, at what values of θ_2 do the minimum and maximum damping ratios occur? What are the minimum and maximum damping ratios?
- (c) Now assume that both joints are free to move, and that this system is controlled by a partitioned PD controller, $\tau = \alpha\tau' + \beta$. Design a partitioned, trajectory-following controller (one that tracks a desired position, velocity and acceleration) which will provide a closed-loop frequency of 10 rad/sec on joint 1 and 20 rad/sec on joint 2 and be critically damped over the entire workspace. That is, let

$$\tau' = \ddot{\theta}_d - \begin{bmatrix} k'_{v1} & 0 \\ 0 & k'_{v2} \end{bmatrix} (\dot{\theta} - \dot{\theta}_d) - \begin{bmatrix} k'_{p1} & 0 \\ 0 & k'_{p2} \end{bmatrix} (\theta - \theta_d)$$

then find the matrices α and β and the vector τ , along with the necessary gains k'_{vi} and k'_{pi} .

- (d) If $\theta_2 = 180^\circ$, what is the steady-state error vector for a given disturbance torque, $\tau_{dist} = [2 \ 4]^T$?