EE224 Midsemester Test

February 22, 1100-1300 hrs

1. Consider a Boolean algebra Ω with operations $+$, $.$, identities 0, 1. For elements a, b of the Boolean algebra, define the operation $a \implies b$ to be $\overline{a} + b$. Recall that the operation $\oplus$ is defined as $a \oplus b = a.\overline{b} + \overline{a}.b$. Starting from the Axioms of Boolean algebra, show that each of the following expressions is equal to 1. (a, b, c, d, e are arbitrary elements of the Boolean algebra):

   (a) $(a \implies a)$. (1 mark)

   (b) $((a \implies b) \implies (\overline{b} \implies \overline{a}))$. (1 mark)

   (c) $(a \implies b).(b \implies c) \implies (a \implies c)$. (1 mark)

   (d) $(((a \implies b).(a \implies c)) \implies (a \implies (b.c)))$. (1 mark)

   (e) $(((a \implies b).(b \implies a)) \implies \overline{a \oplus b})$. (1 mark)

2. Consider the following function f on 3 variables defined by the formula:

$$(x_1.\overline{x_2}) + (x_2.\overline{x_3}) + (x_3.\overline{x1})$$

   Let g be a Boolean function defined by the formula $x_1 + x_2 + x_3$.

   (a) Show that there exists a Boolean function h such that $f = g.h$. (2 marks)

   (b) Find the simplest possible Boolean function h (the one with a sum of products formula which has the fewest literals) such that $f = g.h$. (3 marks)

3. Using 2 to 1 multiplexors, implement the Boolean function with five input bits, defined to be 1 if and only if at least 3 of the input bits are 1. Try to use as few multiplexors as you can. (5 marks)

4. Consider the following Mealy FSM: the input alphabet consists of input symbols $\{RST, U, D\}$ and the output alphabet is $\{TICK, TOCK\}$. Assume that at time instant 0, RST is applied at the input to put the machine into the initial state. Subsequent to the application of RST, suppose that at time instant k, the number of U's seen thus far (including the current input) is A and the number of D's seen thus far B, then the machine outputs a $TOCK$ at instant k if $A = B$ *modulo* 3, else it outputs a $TICK$ at instant k.

   (a) Identify a possible set of states and the next-state and output functions which implement the specified behaviour of the state machine. (2 marks)

   (b) Encode the set of states, input symbols, and output symbols using bits, and implement the next-state and output-functions using Karnaugh maps (that is, identify the simplest possible sum-of-product formulas for these functions). (3 marks)

5. Each of the following statements is either true or false. In each case, decide whether the statement is true or false, and give a justification/proof for your claim.

   (a) There exists a Boolean algebra with seven elements. (1 mark)

   (b) Given just multiplexors, one can implement any Boolean function. (1 mark)

   (c) Given just gates which implement the $\implies$ operator introduced in Question 1, one can implement any Boolean function. (1 mark)

   (d) Let f be a Boolean function on n variables $x_1, x_2, \ldots x_n$. Recall Shannon's expansion, $f = x_1.f_{x_1} + \overline{x_1}.f_{\overline{x_1}}$: then, f is 0 at all points if and only if $f_{x_1} + f_{\overline{x_1}}$ is zero at all points. (1 mark)

   (e) The set of subsets of a finite set is a Boolean algebra. (1 mark)

6. Show that if we have logic gates that implement the $\oplus$ operator and gates that implement the . operator, then using the constant 1 and these gates, we can implement any Boolean function. (2 marks)

   Find an implementation of the formula $x_1 + x_2 + x_3 + x_4$ using only the constant 1, and the $\oplus$, . operations. (3 marks)