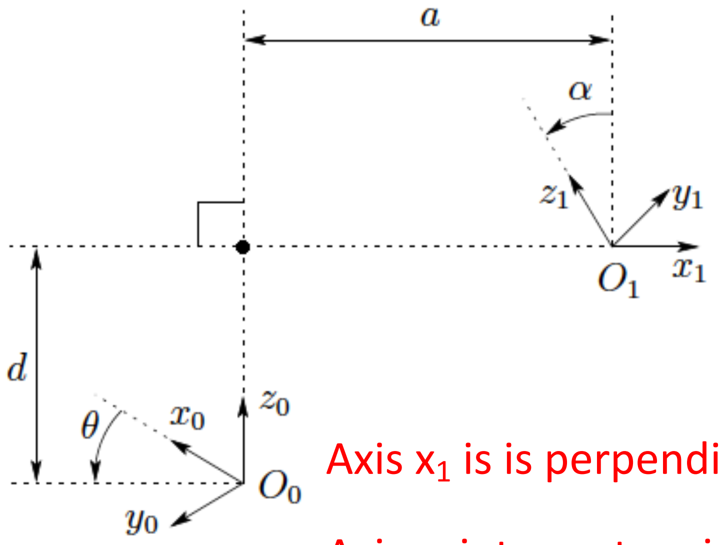
Quiz

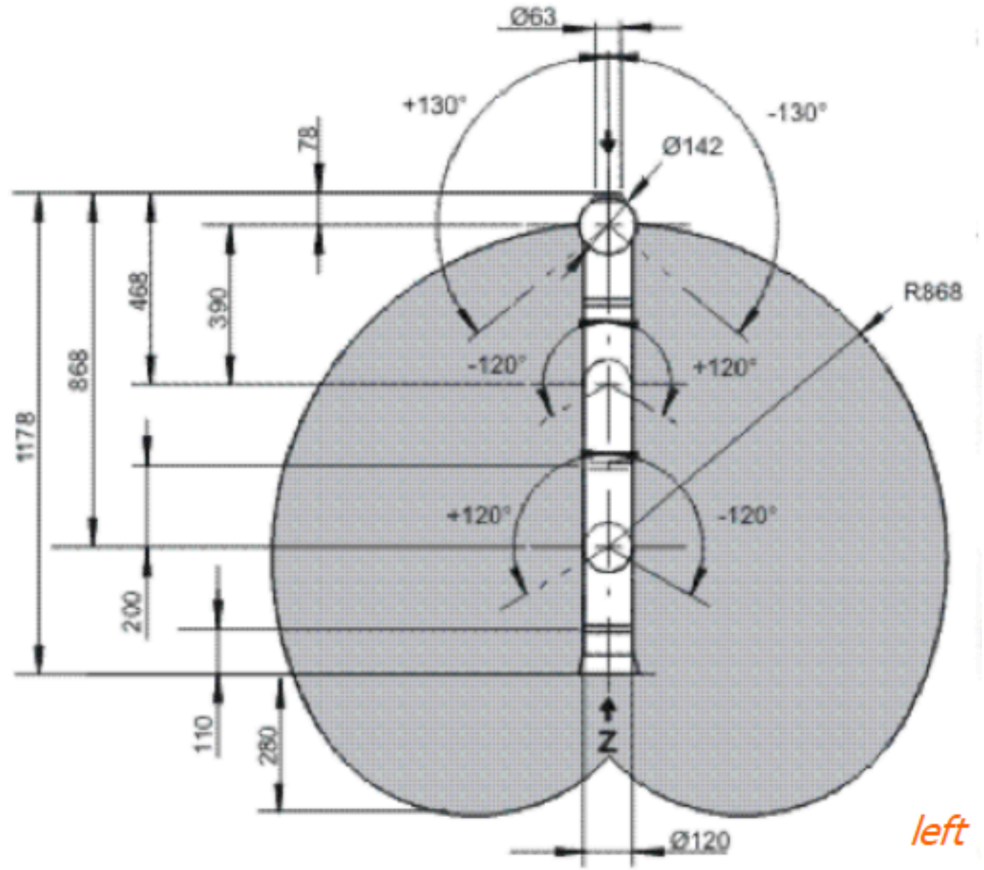
- Determine
  - Frames and table of DH parameters for the 7R manipulator shown.

The axis of rotation are indicated by (+ -) in the figure



Axis x_1 is is perpendicular to axis z_0

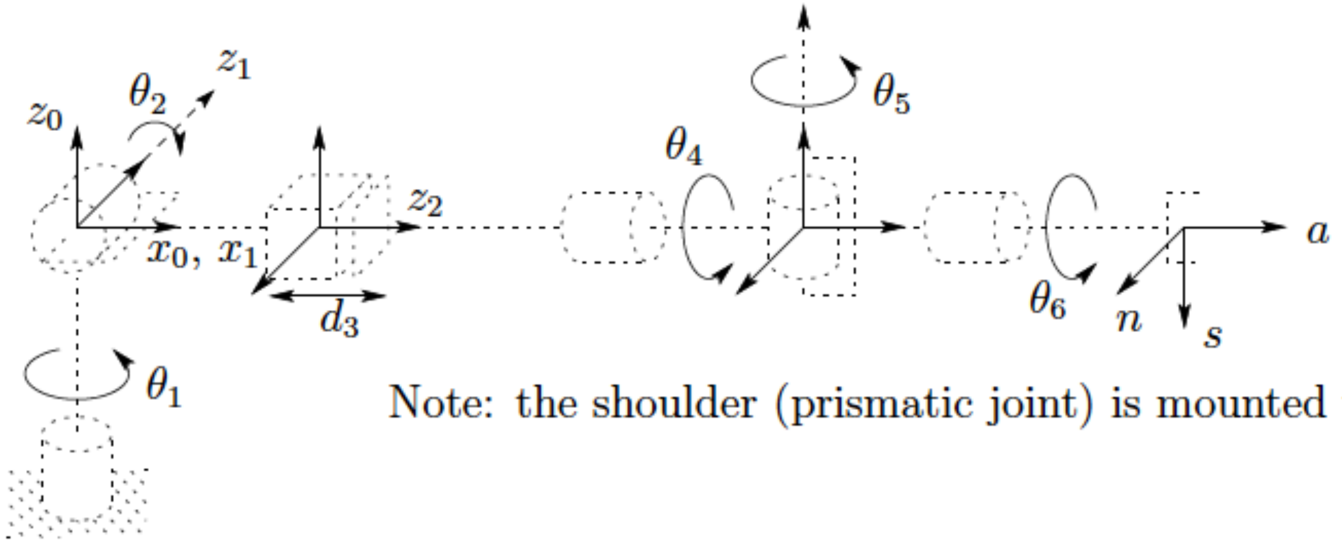
Axis x_1 intersects axis z_0



Side view (from the left)

Front view

DH parameters: zero position



Note: the shoulder (prismatic joint) is mounted wrong.

Link	d_i	a_i	α_i	θ_i
1	0	0	−90	$\theta^\star$
2	d_2	0	−90	$\theta^\star$
3	$d^\star$	0	0	0
4	0	0	−90	$\theta^\star$
5	0	0	+90	$\theta^\star$
6	d_6	0	0	$\theta^\star$

* joint variable

What is the robot configuration when $\theta_2 = 0$?

Differential kinematics

- Relationship between motion (velocity) in joint space and motion (linear/angular velocity) in task space

$$\theta \xrightarrow{\text{Forward kinematics}} x$$

$$\theta + \delta\theta \xrightarrow{\text{Instantaneous kinematics}} x + \delta x$$

$$\delta\theta \leftrightarrow \delta x$$

$$\dot{\theta} \leftrightarrow \dot{x}$$

Linear velocity

Angular velocity

Jacobian

- Can be obtained through direct differentiation of the forward kinematics
- Linear and angular velocities of a rigid body
- Velocity propagation
- Explicit form
  - based upon the kinematic structure of the robot
- Static forces

Joint coordinates

- Coordinate $- i$: $\begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$

- Joint coordinate $- i$: $q_i = \overline{\varepsilon_i}\theta_i + \varepsilon_i d_i$

$$\text{with } \varepsilon_i = \begin{cases} 0 & \text{revolute} \\ 1 & \text{prismatic} \end{cases}$$

Direct differentiation : Analytical Jacobian

Forward kinematics: $x = f(q)$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} f_1(q) \\ \vdots \\ f_m(q) \end{bmatrix}$$

$$\delta x_1 = \frac{\partial f_1}{\partial q_1}\delta q_1 + \frac{\partial f_1}{\partial q_2}\delta q_2 + \cdots + \frac{\partial f_1}{\partial q_n}\delta q_n$$

$$\vdots$$

$$\delta x_m = \frac{\partial f_m}{\partial q_1}\delta q_1 + \frac{\partial f_m}{\partial q_2}\delta q_2 + \cdots + \frac{\partial f_m}{\partial q_n}\delta q_n$$

$$\begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial q_1} & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \vdots \\ \delta q_n \end{bmatrix}$$

mx1

nx1

$$\delta x = J\delta q$$

Jacobian

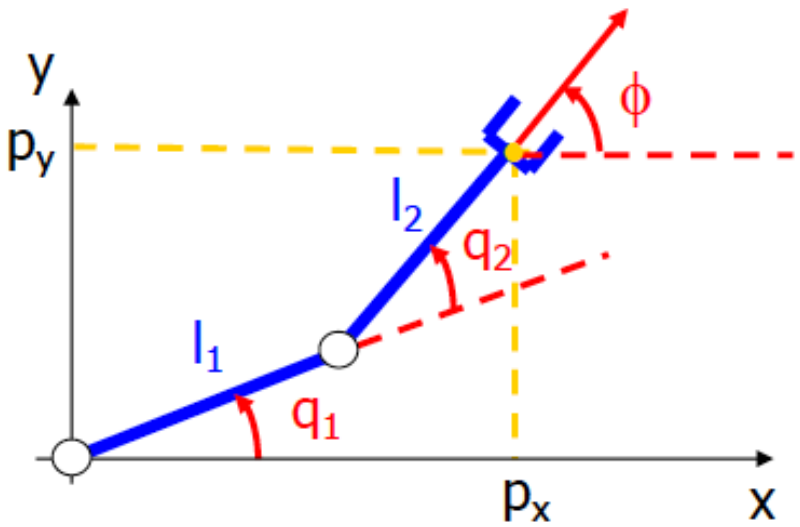
$$\delta x = J\delta q$$

$$\dot{x} = J\dot{q}$$

$$\text{where} \quad J_{ij} = \frac{\partial}{\partial q_j} f_i(q)$$

What does the i-th column of the Jacobian represent?

Example: 2 link manipulator



direct kinematics

$$r \left\{ \begin{array}{l} p_x = l_1 c_1 + l_2 c_{12} \\ p_y = l_1 s_1 + l_2 s_{12} \\ \hline \phi = q_1 + q_2 \end{array} \right.$$

$$\dot{p}_x = - l_1 s_1 \dot{q}_1 - l_2 s_{12} (\dot{q}_1 + \dot{q}_2)$$

$$\dot{p}_y = l_1 c_1 \dot{q}_1 + l_2 c_{12} (\dot{q}_1 + \dot{q}_2)$$

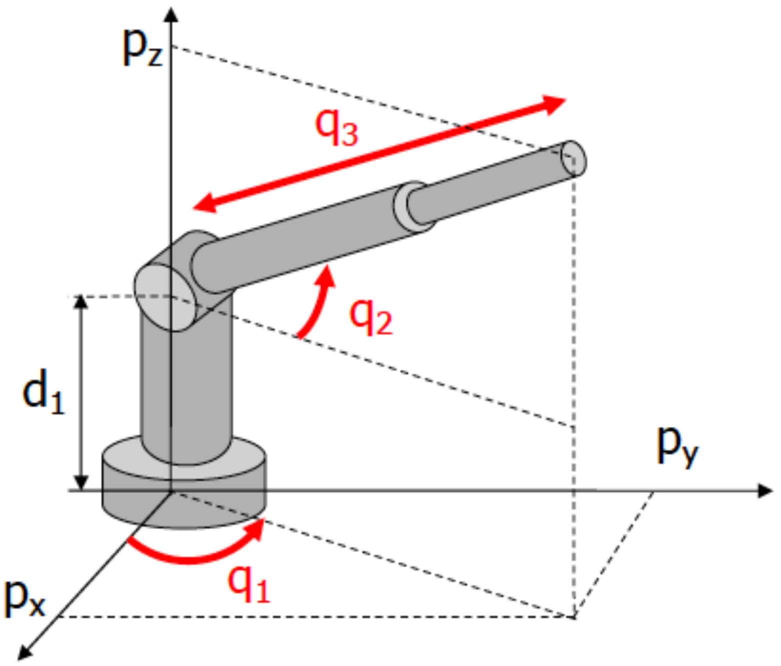
$$\dot{\phi} = \omega_z = \dot{q}_1 + \dot{q}_2$$

here, all rotations occur around the same fixed axis z (normal to the plane of motion)

$$\Rightarrow \quad J_r(q) = \begin{pmatrix} - l_1 s_1 - l_2 s_{12} & - l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ \hline 1 & 1 \end{pmatrix}$$

given r, this is a 3 x 2 matrix

Example: polar manipulator

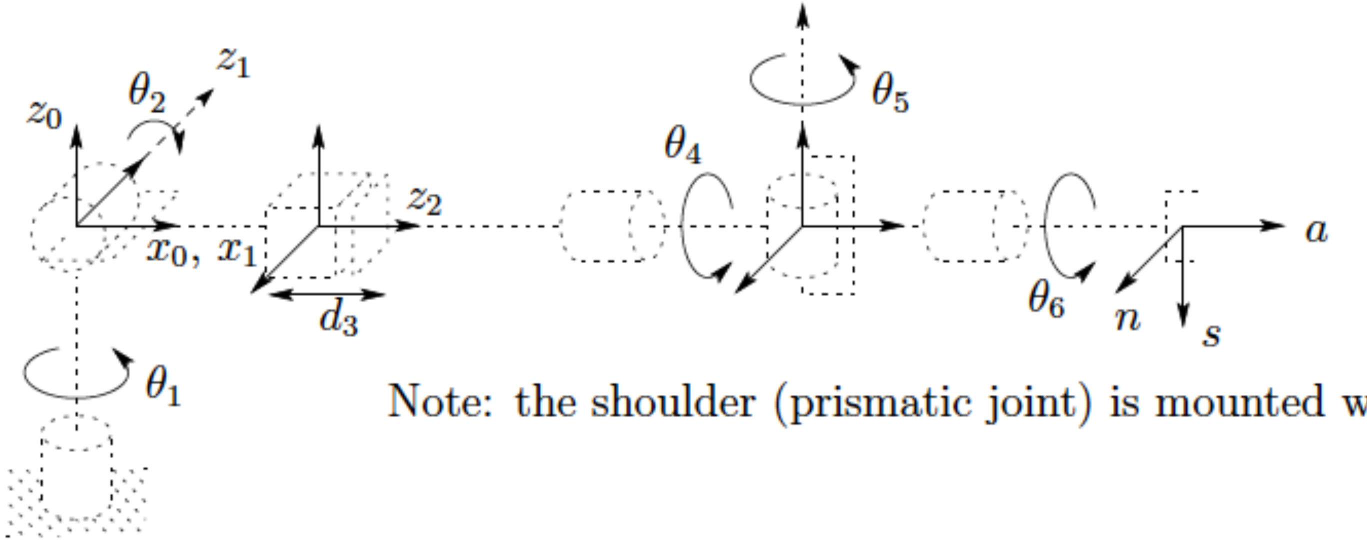


direct kinematics (here, r = p)

$$\left.\begin{aligned} p_x &= q_3\, c_2\, c_1 \\ p_y &= q_3\, c_2\, s_1 \\ p_z &= d_1 + q_3\, s_2 \end{aligned}\right\} f_r(q)$$

$$v = \dot{p} = \underbrace{\begin{pmatrix} -q_3 c_2 s_1 & -q_3 s_2 c_1 & c_2 c_1 \\ q_3 c_2 c_1 & -q_3 s_2 s_1 & c_2 s_1 \\ 0 & q_3 c_2 & s_2 \end{pmatrix}}_{\frac{\partial f_r(q)}{\partial q}} \dot{q} = J_r(q)\, \dot{q}$$

Example: Stanford manipulator

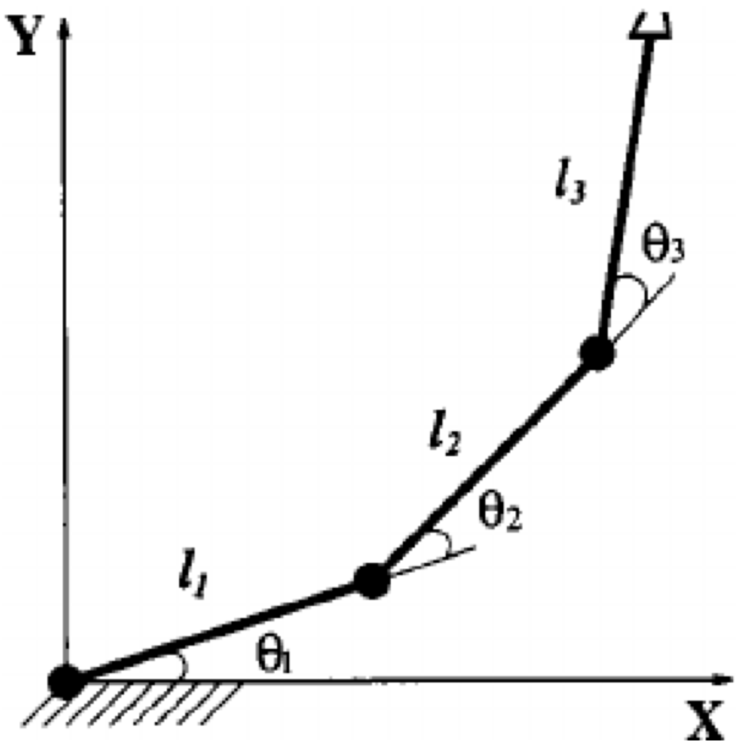


Note: the shoulder (prismatic joint) is mounted wrong.

$$
\begin{aligned}
d_x &= c_1 s_2 d_3 - s_1 d_2 + +d_6(c_1 c_2 c_4 s_5 + c_1 c_5 s_2 - s_1 s_4 s_5) \\
d_y &= s_1 s_2 d_3 + c_1 d_2 + d_6(c_1 s_4 s_5 + c_2 c_4 s_1 s_5 + c_5 s_1 s_2) \\
d_z &= c_2 d_3 + d_6(c_2 c_5 - c_4 s_2 s_5)
\end{aligned}
$$

$$\dot{x}_p = J_{(3\text{x}6)}\dot{q}_{(6\text{x}1)}$$

Planar 3-link manipulator: velocity analysis



Orientation: Direction cosines

$$x_R = J_{x_R}(q)\dot{q}$$

Depends upon representation

$$x_R = \begin{bmatrix} r_1(q) \\ r_2(q) \\ r_3(q) \end{bmatrix}$$

Direction cosines

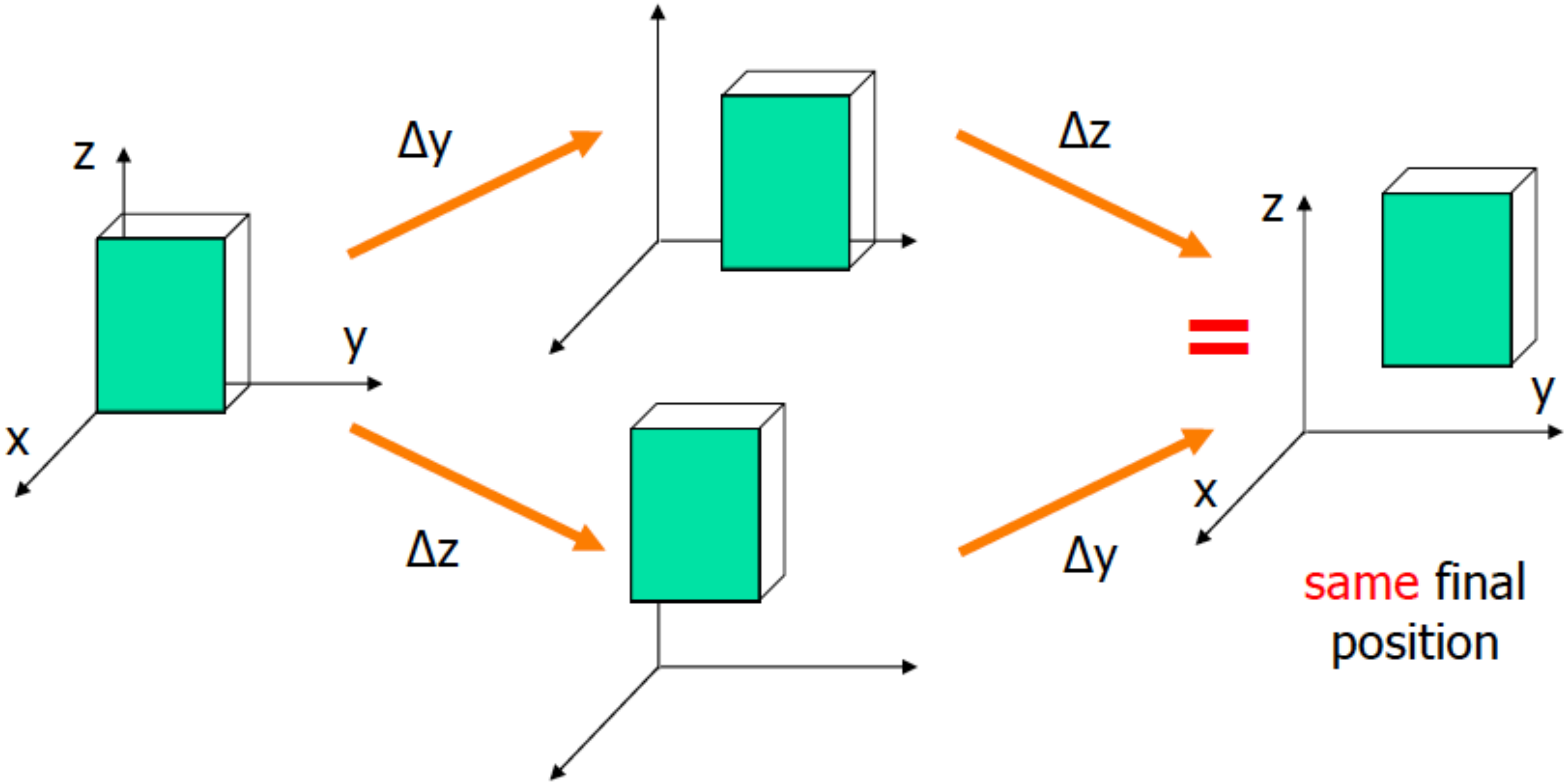
$$\begin{aligned}
r_{11} &= c_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] - d_2(s_4c_5c_6 + c_4s_6) \\
r_{21} &= s_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6) \\
r_{31} &= -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6 \\
r_{12} &= c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5s_6 + c_4c_6) \\
r_{22} &= -s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5s_6 + c_4c_6) \\
r_{32} &= s_2(c_4c_5s_6 + s_4c_6) + c_2s_5s_6 \\
r_{13} &= c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5 \\
r_{23} &= s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5 \\
r_{33} &= -s_2c_4s_5 + c_2c_5 \\
d_x &= c_1s_2d_3 - s_1d_2 + +d_6(c_1c_2c_4s_5 + c_1c_5s_2 - s_1s_4s_5) \\
d_y &= s_1s_2d_3 + c_1d_2 + d_6(c_1s_4s_5 + c_2c_4s_1s_5 + c_5s_1s_2) \\
d_z &= c_2d_3 + d_6(c_2c_5 - c_4s_2s_5)
\end{aligned}$$

$$\begin{bmatrix} \dot{r}_1(q) \\ \dot{r}_2(q) \\ \dot{r}_3(q) \end{bmatrix} = \begin{bmatrix} \frac{\partial r_1}{\partial q_1} & \cdots & \frac{\partial r_1}{\partial q_6} \\ \vdots & \vdots & \vdots \\ \frac{\partial r_3}{\partial q_1} & \cdots & \frac{\partial r_3}{\partial q_6} \end{bmatrix} \begin{bmatrix} \dot{q}_6 \\ \vdots \\ \dot{q}_6 \end{bmatrix}$$

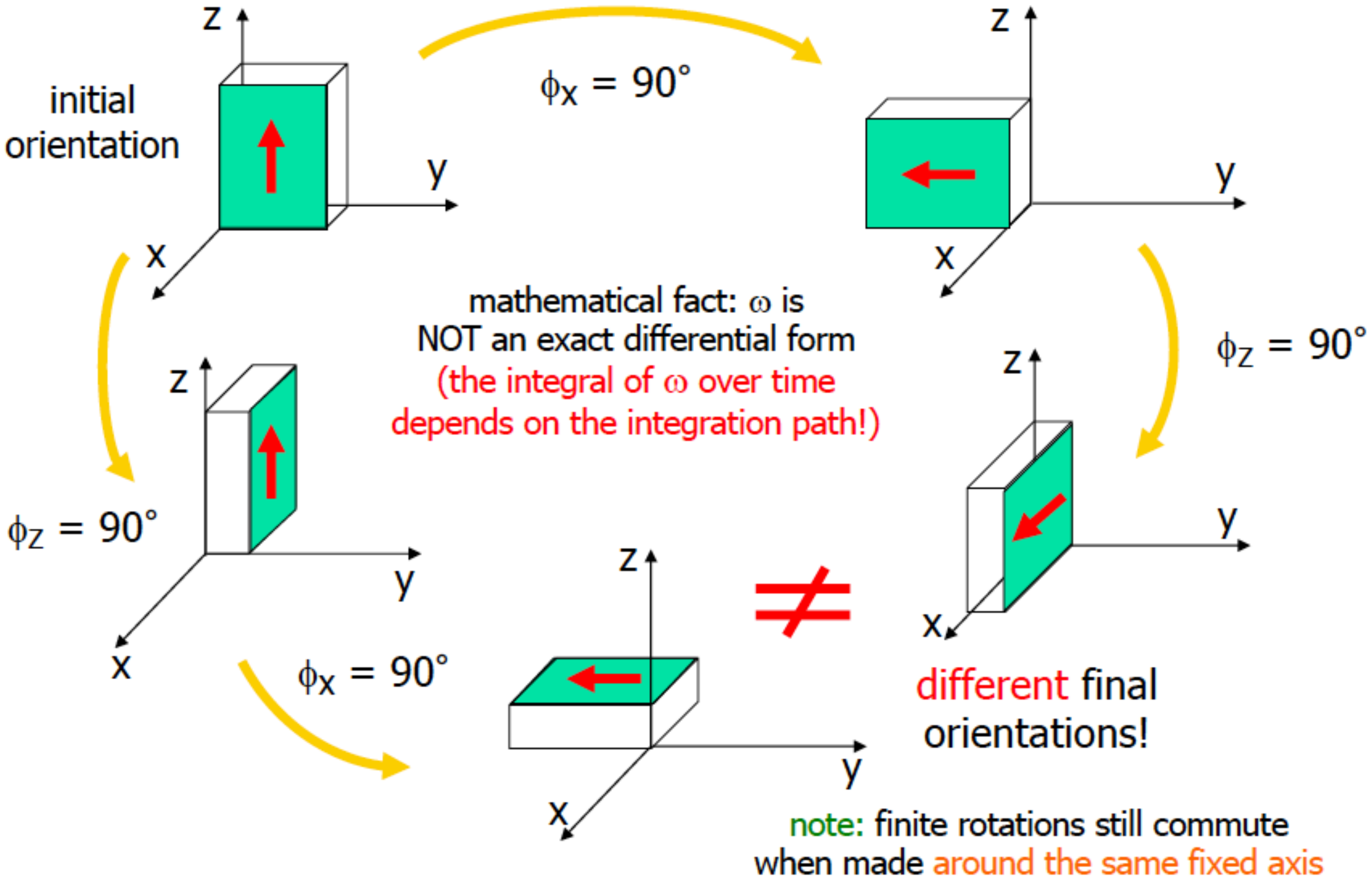
9x6

The Jacobian is dependent upon the representation!

Finite and infinitesimal translations



Finite rotations do not commute



Infinitesimal rotations commute

- infinitesimal rotations $d\phi_X, d\phi_Y, d\phi_Z$ around x, y, z axes

$$R_X(\phi_X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_X & -\sin\phi_X \\ 0 & \sin\phi_X & \cos\phi_X \end{bmatrix} \Rightarrow R_X(d\phi_X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -d\phi_X \\ 0 & d\phi_X & 1 \end{bmatrix}$$

$$R_Y(\phi_Y) = \begin{bmatrix} \cos\phi_Y & 0 & \sin\phi_Y \\ 0 & 1 & 0 \\ -\sin\phi_Y & 0 & \cos\phi_Y \end{bmatrix} \Rightarrow R_Y(d\phi_Y) = \begin{bmatrix} 1 & 0 & d\phi_Y \\ 0 & 1 & 0 \\ -d\phi_Y & 0 & 1 \end{bmatrix}$$

$$R_Z(\phi_Z) = \begin{bmatrix} \cos\phi_Z & -\sin\phi_Z & 0 \\ \sin\phi_Z & \cos\phi_Z & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow R_Z(d\phi_Z) = \begin{bmatrix} 1 & -d\phi_Z & 0 \\ d\phi_Z & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $$R(d\phi) = \underset{\uparrow \atop \text{in any order}}{R(d\phi_X, d\phi_Y, d\phi_Z)} = \begin{bmatrix} 1 & -d\phi_Z & d\phi_Y \\ d\phi_Z & 1 & -d\phi_X \\ -d\phi_Y & d\phi_X & 1 \end{bmatrix} \leftarrow \text{neglecting second- and third-order (infinitesimal) terms}$$

$$= I + S(d\phi)$$