ME 604: Introduction to Robotics
Spring 2019

Practice Problems

1. Using the fact that $v_1 . v_2 = v_1^T v_2$, show that the dot product of two free vectors does not depend on the choice of frames in which their coordinates are defined.

2. Suppose A is a 2 X 2 rotation matrix, i.e, $A^T A = I$ and det $A = 1$. Show that there exists a unique θ such that A is of the form

$$A = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix}$$

3. In each of the following cases, write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication)

   a) i. Rotate by ϕ about the world *x*-axis.
      ii. Rotate by θ about the current *z*-axis.
      iii. Rotate by ψ about the world *y*-axis.

   b) i. Rotate by ϕ about the world *x*-axis.
      ii. Rotate by θ about the world *z*-axis.
      iii. Rotate by ψ about the current *x*-axis.

   c) i. Rotate by ϕ about the world *x*-axis.
      ii. Rotate by θ about the current *z*-axis.
      iii. Rotate by ψ about the current *x*-axis.
      iv. Rotate by α about world *z*-axis.

   d) i. Rotate by ϕ about the world *x*-axis.
      ii. Rotate by θ about the world *z*-axis.
      iii. Rotate by ψ about the current *x*-axis.
      iv. Rotate by α about world *z*-axis.

4. Suppose that three coordinate frames $\{A\}$, $\{B\}$ and $\{C\}$ are given, and

$$ {}^{A}_{C}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}; \ {}^{A}_{B}R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

   Find the matrix ${}^{B}_{C}R$.

5. If R be a rotation matrix, show that +1 is an eigenvalue of R. Let k be a unit eigenvector corresponding to the eigenvalue +1. Give a physical interpretation of k.

6. Suppose R represents a rotation of 90° about the fixed y-axis followed by a rotation of 45° about the current z-axis. Find the equivalent axis/angle to represent R. Sketch the initial and final frames, and the equivalent axis vector k. What are the Euler parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ of R.

7. Given the transformation matrix,

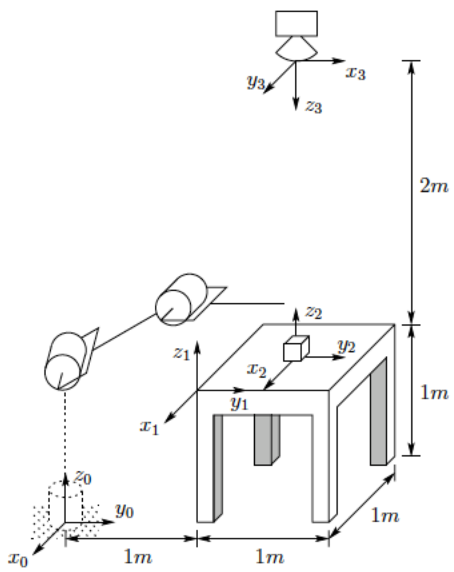
$$ {}^{A}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & c\theta & -s\theta & 1 \\ 0 & s\theta & c\theta & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Find ${}^{B}_{A}T$. Given $\theta = 45^\circ$ and ${}^{B}P = [4 \quad 5 \quad 6]^T$, compute ${}^{A}P$.

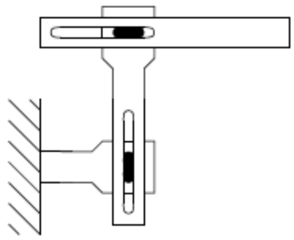
8. Compute the homogenous transformation representing a translation of 3 units along the x-axis, followed by a rotation of $\pi/2$ about the current *z-axis*, followed by a translation of 1 unit along the fixed y-axis. Sketch the frame. What are the coordinates of the origin with respect to the fixed frame in each case?

9. Consider the figure below. The cube measuring 20 cm on a side is placed at the center of the table as shown. The camera is situated directly above the center of the cube, 2 m above the tabletop. Coordinate frames $x_0y_0z_0, x_1y_1z_1, x_2y_2z_2$ and $x_3y_3z_3$ are attached to the robot base, table top, center of the cube and the camera, respectively. Find the homogenous transformation relating each of the frames to the base frame.

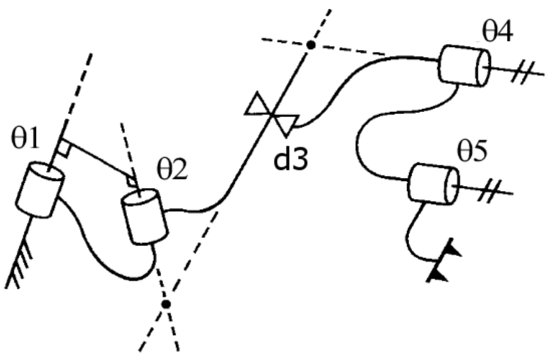
10. Suppose the cube from problem 9 is rotated by 45° about z_2 and moved so that its center has coordinates $(0, 0.8, 0.1)^T$ relative to the frame $x_1y_1z_1$. Compute the homogeneous transformation relating the block frame to the camera frame; the block frame to the base frame.



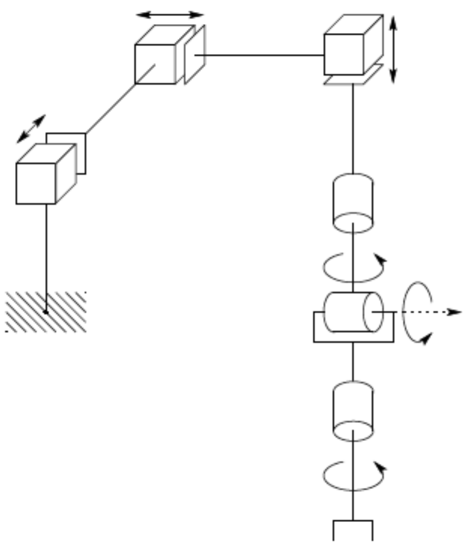
11. Consider the two-link Cartesian manipulator shown below. Derive the forward kinematic equations using the DH-convention.



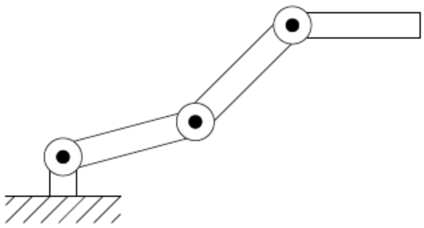
12. Consider the 2RP2R manipulator shown below. Draw a schematic of this manipulator, with the axes for frames {0} through {5} labeled. Assign DH frames, and make a table of link parameters. Include all non-zero DH parameters and joint variables in your schematic. Draw your schematic in the position where, as far as possible, the angles θ_i's are in their zero position.



13. Consider the six-link manipulator shown below. Derive the forward and inverse kinematic relationships for this manipulator. What are the position and orientation of the end-effector when all joint variables are 0?

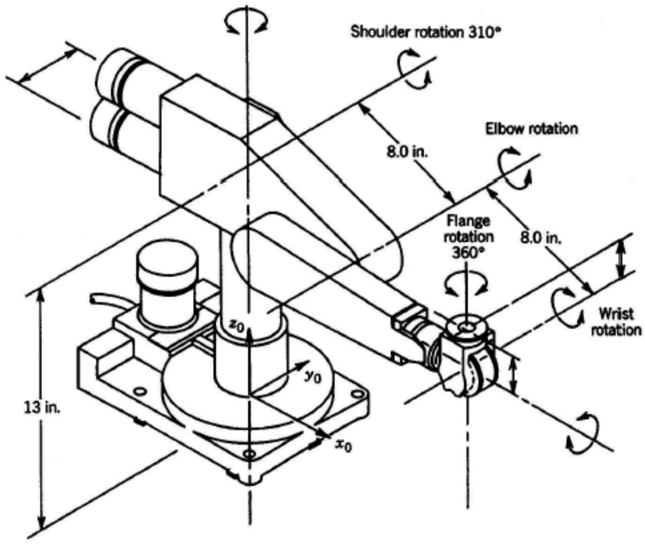


14. Given a desired end-effector position, how many solutions are there to the inverse kinematics of the three-link planar arm shown below. If the orientation of the end-effector is also specified, how many solutions are there? Use a geometric approach to find them.



15. Consider the PUMA manipulator shown below.

(a) Draw a schematic of the manipulator with the axes of frames {0} through {6} labeled.
(b) Establish the DH frames, and construct a table of link parameters. Show any non-zero DH parameter on your schematic.
(c) For a desired position and orientation, solve the inverse position and orientation problems for this manipulator.



16. Given the Euler angle transformation $R = R_z(\psi)R_y(\theta)R_z(\phi)$, show that
$\frac{d}{dt}R = S(\omega)R$, where $\omega = \{c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}\}i + \{s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}\}j + \{c_\theta \dot{\phi} + \dot{\psi}\}k$.

17. Two frames $\{A\}$ and $\{B\}$ are related by the homogenous transformation matrix

$$ {}^A_BT = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

A particle has velocity $v(t) = (3,1,0)^T$relative to frame $\{B\}$. What is the velocity of the particle in frame $\{A\}$.

18. Consider three frames $\{0\}$, $\{1\}$ and $\{2\}$ such that their origins coincide. If the angular velocities ${}^0\omega_1$ and ${}^1\Omega_2$ are given as

$$ {}^0\omega_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad {}^1\Omega_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} $$

What is the angular velocity ${}^0\omega_2$ at the instant when

$$ {}^0_1R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} $$

At this instant, a particle located at ${}^2P = (1,1,0)^T$ has a velocity $v(t) = (3,1,0)^T$ relative to frame $\{2\}$. What is the $v_p(t)$ total linear velocity of the particle?

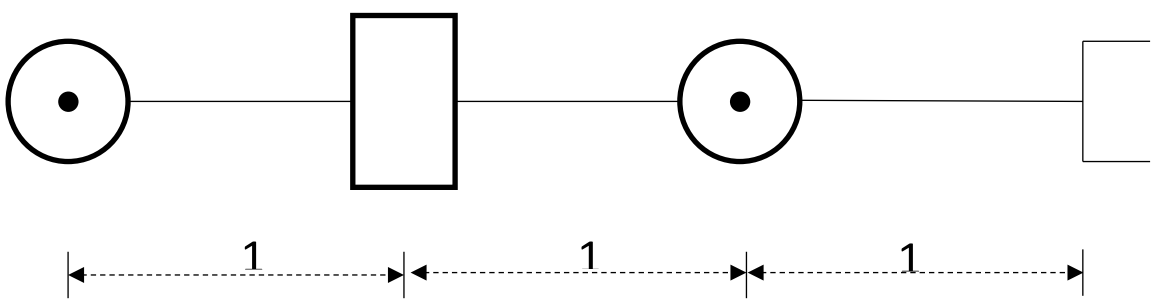
19. A certain RPR manipulator has the following transformation matrices, where {E} is the frame of the end effector.

$$ {}^0_1T = \begin{bmatrix} c_1 & 0 & -s_1 & l_1c_1 \\ s_1 & 0 & c_1 & l_1s_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} c_1 & 0 & -s_1 & l_1c_1 - d_2s_1 \\ s_1 & 0 & c_1 & l_1s_1 + d_2c_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_ET = \begin{bmatrix} -s_1 & c_1s_3 & c_1c_3 & l_1c_1 - d_2s_1 + l_2c_1c_3 \\ c_1 & s_1s_3 & c_3s_1 & l_1s_1 + d_2c_1 + l_2c_3s_1 \\ 0 & c_3 & -s_3 & -l_2s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Derive the basic Jacobian relating joint velocities to the end effector's linear and angular velocities in frame $\{0\}$.

20. Consider the RRR manipulator shown below.



(a) Assign DH frames and find the DH parameters for this manipulator.
(b) Derive the forward kinematics, ${}^0_3T$, of this manipulator (frame $\{4\}$ being the end-effector frame)
(c) Derive the basic Jacobian, *J*, for this manipulator.
(d) Find ${}^1J_v$, the position Jacobian matrix expressed in frame $\{1\}$.

(e) Use the matrix found in part (d) to find the singularities (with respect to linear velocity) of the manipulator.

(f) For each type of singularity found in part (e), explain the physical interpretation of the singularity, by sketching the arm in a singular configuration and describing the resulting limitation on its movement.

21. Find the 6 X 3 Jacobian for the three links of the cylindrical manipulator shown below. Determine the singular configurations of this arm.