EE 204-2018-2: Analog Circuits

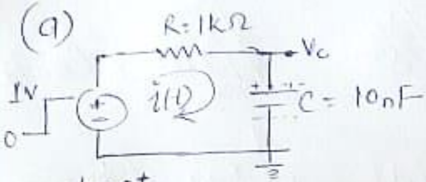
Homework #0 Solution

Question 1:

Solution- Homework #0

Ques 1

(a)



$t \geq 0^+$

$1 - iR - V_c = 0$ — ①

differentiating the eqn ①

$-\frac{di}{dt}R - \frac{dV_c}{dt} = 0$ — ②

$i = C\frac{dV_c}{dt}$ — ③

from ② and ③

$-\frac{di}{dt}\cdot R - \frac{i}{C} = 0$

$\frac{di}{dt} + \frac{i}{RC} = 0$

solving the differential eqn.

$i(t) = Ae^{-\frac{1}{RC}t}$ — ④

at $t=0^+$ Capacitor acts as short ckt, hence $i(0^+) = \frac{V}{R}$

$i(0^+) = \frac{1}{1000} = 1mA$

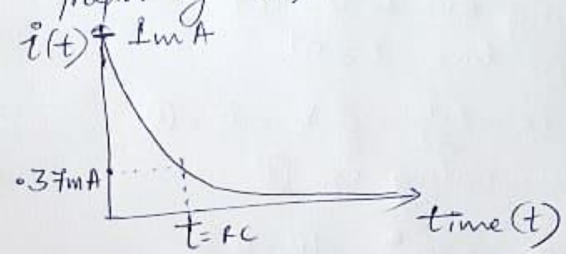
applying initial condition to eqn ④

$i(0^+) = Ae^0 = 1mA$

$\Rightarrow A = 1mA$

$$\boxed{i(t) = 1m\,e^{-\frac{t}{10\times10^{-6}}}}$$

Graph of $i(t)$



Here τ = time constant = RC

$RC = 10\times10^{-9}\times1000$

$= 10\mu s$

$i(10\mu s) = 1m\,e^{-\frac{10\mu}{10\mu}1}$

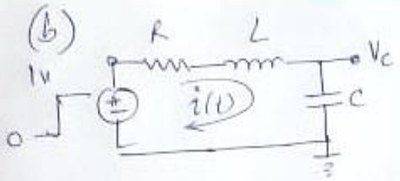
$= 1m\,e^{-1}$

$= .367$

$\approx .37mA$

Hence current is reduced by 67% in one time constant.

(b)



applying KVL to the ckt for time $t \geqslant 0^+$

$$1 - iR - L\frac{di}{dt} - V_c = 0 \quad -①$$

differentiating eqn ①

$$-R\frac{di}{dt} - L\frac{d^2i}{dt^2} - \frac{dV_c}{dt} = 0 \quad -②$$

$$i = C\frac{dV_c}{dt} \quad -③$$

from ③ and ④

$$-R\frac{di}{dt} - L\frac{di^2}{dt^2} - \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \quad -⑤$$

roots of above diff. eqn

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$= -.9\times10^6, \ -.1\times10^6$$

$$\boxed{i(t) = Ae^{-.9\times10^6 t} + Be^{-.1\times10^6 t} + C.}$$

at $t = 0^+$ $\quad i = 0 \Rightarrow A + B + C = 0$

$t = \infty$ $\quad i = 0 \Rightarrow C = 0$

$$i(t) = A\left[e^{-.9\times10^6 t} - e^{-.1\times10^6 t}\right] -⑥$$

finding the value of A.

$$i = C\frac{dV_c}{dt}$$

$$V_c = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \frac{1}{C}\int_{0}^{t} i\,dt \quad \left(\begin{array}{l}\text{because}\\ i = 0 \text{ for}\\ t<0\end{array}\right)$$

$$= \frac{A}{C}\int_0^t \left(e^{-.9\times10^6 t} - e^{-.1\times10^6 t}\right)dt$$

$$V_c(t) = \frac{A}{C}\left[\frac{e^{-.9\times10^6 t}}{-.9\times10^6} + \frac{e^{-.1\times10^6 t}}{.1\times10^6}\right]_0^t$$

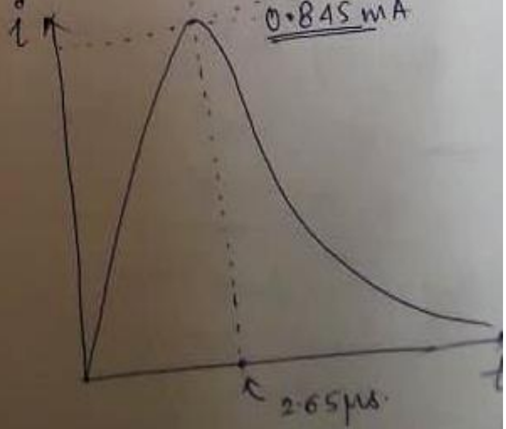
at $t = \infty$ $\quad V_c(\infty) = 1V$

$$V_c(\infty) = \frac{A}{C}\left[0 - \left(\frac{-1}{.9\times10^6} + \frac{1}{.1\times10^6}\right)\right] = 1$$

$$A = -1.25\times10^{-3} \quad -⑦$$

putting the value of A in eqn ⑥

$$i(t) = -1.25m\left(e^{-.9\times10^6 t} - e^{-.1\times10^6 t}\right)$$

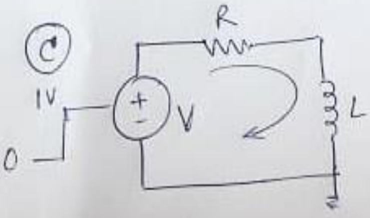


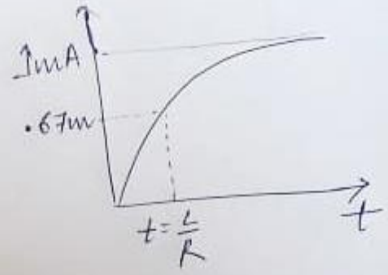
To find the value at which current is max. We need to find '$\frac{di}{dt} = 0$ (to calculate maxima)

at $t \simeq 2.65\,\mu s$ the current is maximum.

$i_{max} \simeq 0.845\ mA$

ⓒ





applying KVL for the time $t \geq 0^+$

$\tau = \frac{L}{R} = 10^{-6} = 1\mu s$

$$1 - iR - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}$$

$$i(t) = Ae^{-\frac{R}{L}t} + B.$$

at $t = 0^+$ inductor is open

$\Rightarrow i = 0,$

$\Rightarrow 0 = A + B$

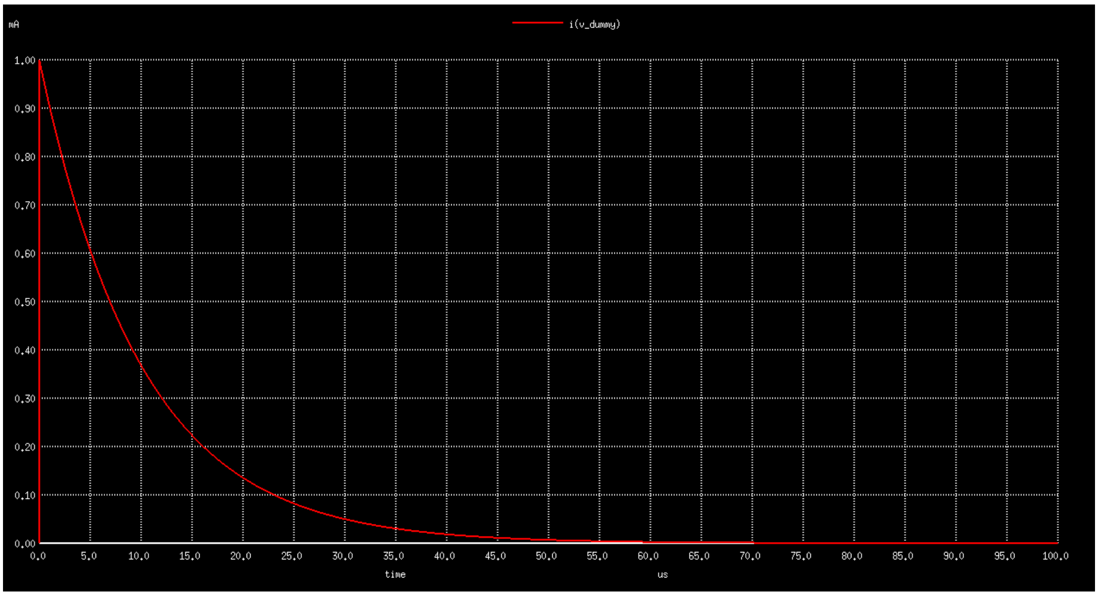
$$i(t) = A\left(e^{-\frac{R}{L}t} - 1\right)$$

at $t = \infty$, inductor is short

$$i(\infty) = \frac{1}{1k} = 1mA$$

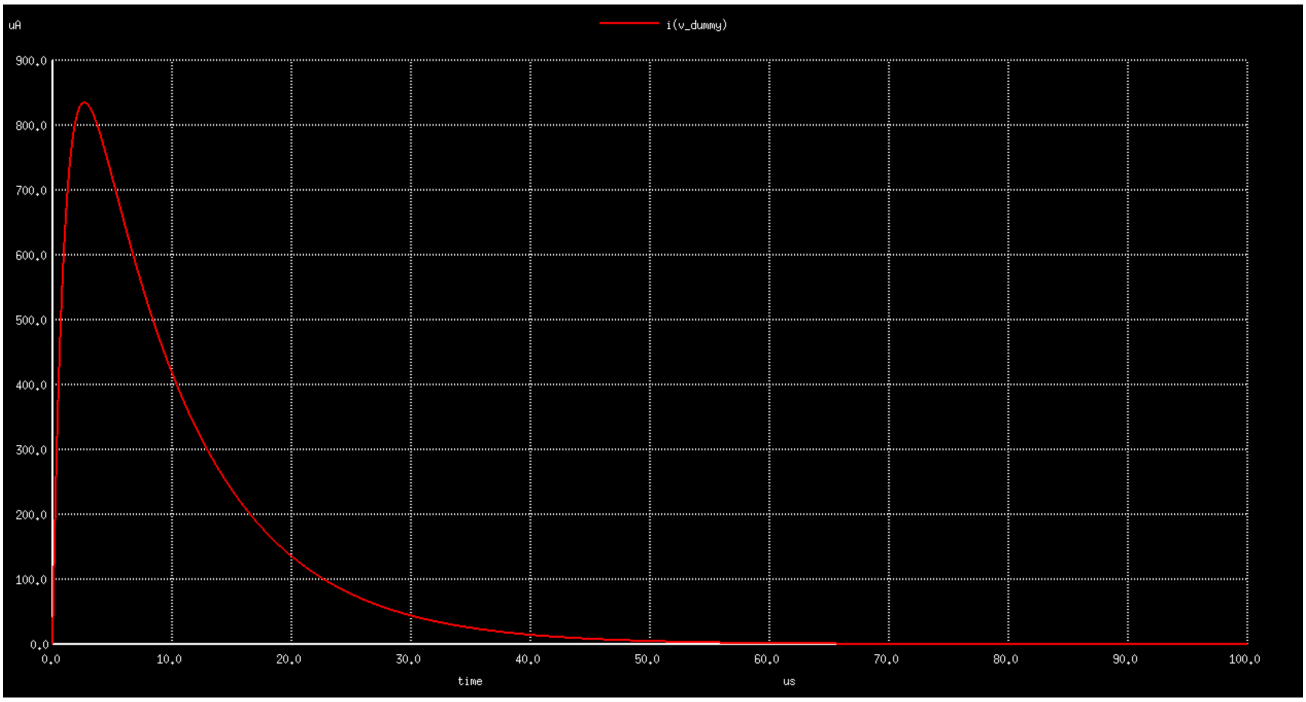
$$1m = -A$$

$$\boxed{i(t) = 1m\left(1 - e^{-\frac{Rt}{L}}\right)}$$



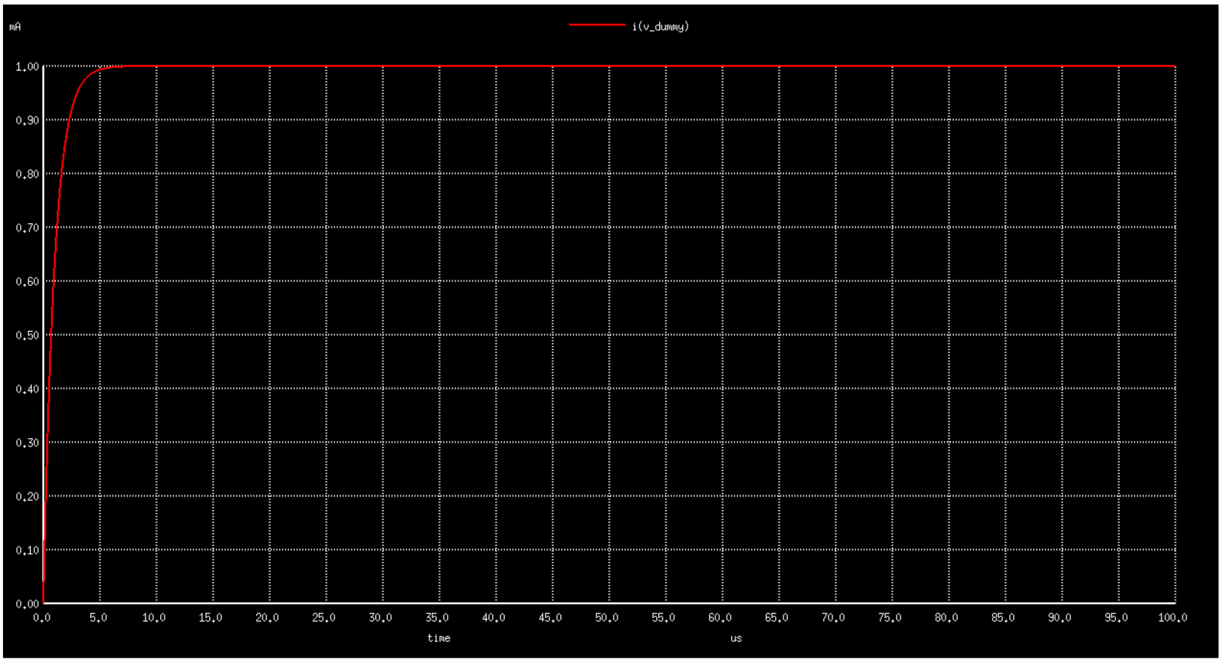
SPICE Code Q1 (a):

```
* Voltage Source
v1 1 gnd dc 1V
* Dummy Voltage Source
v_dummy 1 a 0V
* Resistor
r1 a b 1k
* Capacitor
c1 b gnd 10n ic=0V
.control
run
* Transient Analysis [ tran <Step_Size> <Stop_Time> {<Start_Time>} ]
tran 2n 100u uic
plot i(v_dummy)
.endc
.end
```



SPICE Code Q1 (b)

```
* Voltage Source
v1 1 gnd dc 1V
* Dummy Voltage Source
v_dummy 1 a 0V
* Resistor
r1 a b 1k
* Inductor
l1 b l_end 1m ic=0A
* Capacitor
c1 l_end gnd 10n ic=0V
.control
* Transient Analysis [ tran <Step_Size> <Stop_Time> {<Start_Time>} ]
tran 2n 100u uic
plot i(v_dummy)
.endc
.end
```



SPICE Code Q1 (c)

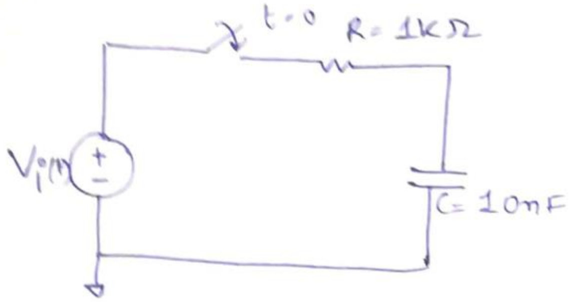
```
* Voltage Source

v1 1 gnd dc 1V

* Dummy Voltage Source

v_dummy 1 a 0V

* Resistor

r1 a b 1k

* Inductor

l1 b gnd 1m ic=0A

.control

run

* Transient Analysis [ tran <Step_Size> <Stop_Time> {<Start_Time>} ]

tran 2n 100u uic

plot i(v_dummy)

.endc

.end
```

Question 2:

Homework 0

(2)

(a)



- $V_i(t) = \sin(2\pi\times10^5 t + \pi/2)$

$\Rightarrow V = 1\angle 90^\circ$

- $R = 1K\Omega$

$\Rightarrow X_R = 10^3\angle 0^\circ$

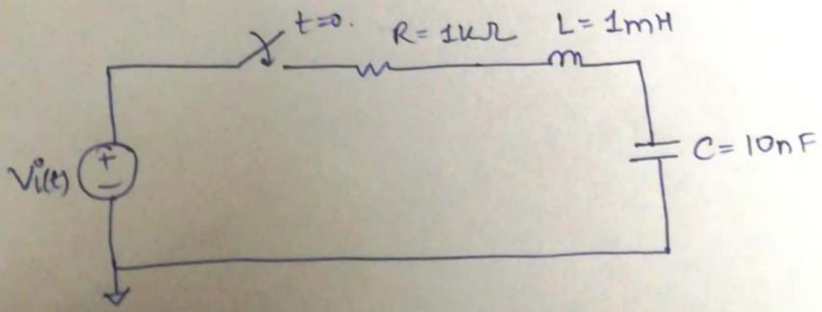
- $C = 10nF$

$\Rightarrow X_c = \dfrac{1\angle -90^\circ}{\omega C} = \dfrac{1\angle -90^\circ}{2\pi\times10^5\times 10\times10^{-9}} = \dfrac{10^3}{2\pi}\angle -90^\circ$

$I = \dfrac{V}{X_R + X_c} = \dfrac{1\angle 90^\circ}{10^3 + \frac{10^3}{2\pi}\angle -90^\circ} = 9.875\times10^{-4}\angle 99.043$

$I_i(t) = 9.875\times10^{-4}\sin(2\pi\times10^5 t + 0.55\pi)$

(b)



- $V = 1\angle 90^\circ$
- $X_R = 10^3\angle 0^\circ$
- $X_c = \dfrac{10^3}{2\pi}\angle -90^\circ$

$\Big\}$ from part (a)

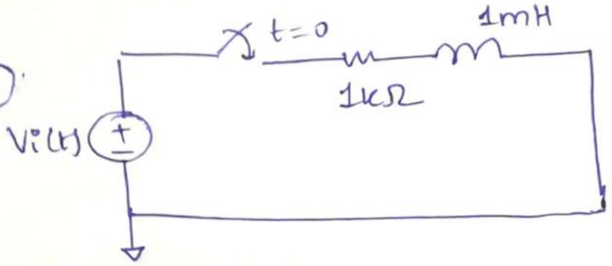
- $L = 1mH \Rightarrow X_L = \omega L\angle 90^\circ = 2\pi\times10^5\times10^{-3}\angle 90^\circ = 2\pi\times10^2\angle 90^\circ$

$$I = \frac{V}{X_R + X_L + X_C}$$

$$I = \frac{1\angle 90^\circ}{10^3 + 2\pi \times 10^2 \angle 90^\circ + \frac{10^3}{2\pi}\angle -90^\circ} = 9.053 \times 10^{-4} \angle 64.865$$

$$I_i(t) = 9.053 \times 10^{-4} \sin(2\pi \times 10^5 t + 0.36\pi)$$

(c)



- $V = 1\angle 90^\circ$
- $X_R = 10^3$
- $X_L = 2\pi \times 10^2 \angle 90^\circ$

} from (a) and (b)

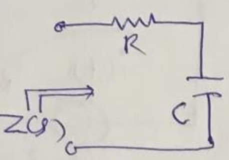
$$I = \frac{V}{X_R + X_L} = \frac{1\angle 90^\circ}{10^3 + 2\pi \times 10^2 \angle 90^\circ} = 8.467 \times 10^{-4} \angle 57.858$$

$$I_i(t) = 8.467 \times 10^{-4} \sin(2\pi \times 10^5 t + 0.321\pi)$$

Question 3:

3.

a)



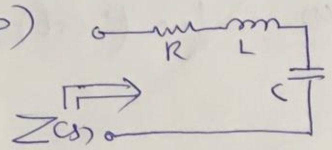
$R = 1k\Omega$

$C = 10nF$

$$Z_a(s) = R + \frac{1}{sC}$$

$$\therefore Z_a(j\omega) = R - j\cdot\left(\frac{1}{\omega C}\right) = 1000 - j\left(\frac{1}{10^{-8}\cdot\omega}\right)$$

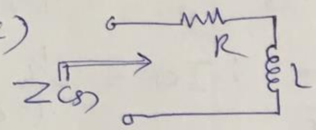
b)



$$Z_b(s) = R + sL + \frac{1}{sC}$$

$$\therefore Z_b(j\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right) = R + j\left(10^{-3}\omega - \frac{1}{10^{-8}\cdot\omega}\right)$$

c)



$$Z_c(s) = R + sL$$

$$\therefore Z_c(j\omega) = R + j\omega L = 1000 + j(10^{-3}\cdot\omega)$$

$\therefore$ Magnitude:

$$|Z_a(j\omega)| = \sqrt{1000^2 + \frac{1}{10^{-16}\cdot\omega^2}}$$

$$|Z_b(j\omega)| = \sqrt{1000^2 + \left(10^{-3}\cdot\omega - \frac{1}{10^{-8}\cdot\omega}\right)^2}$$

$$|Z_c(j\omega)| = \sqrt{1000 + 10^{-6}\cdot\omega^2}$$

with $\omega = 2\pi f$ rad

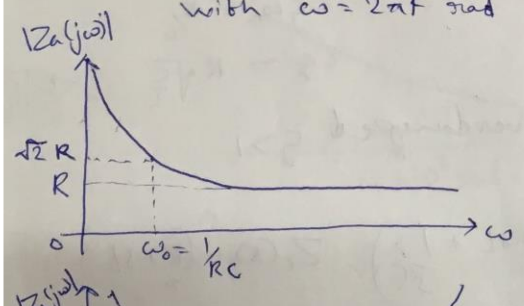
Phase:

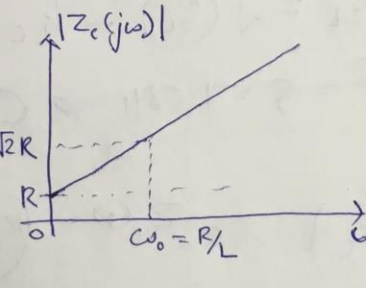
$$\angle Z_a(j\omega) = \tan^{-1}\left(\frac{-1}{\omega RC}\right)$$

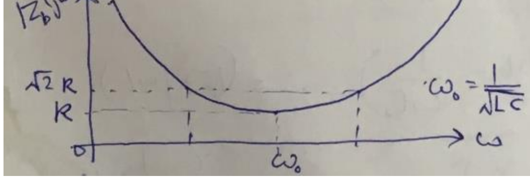
$$\angle Z_b(j\omega) = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$\angle Z_c(j\omega) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

with $\omega = 2\pi f$ rad



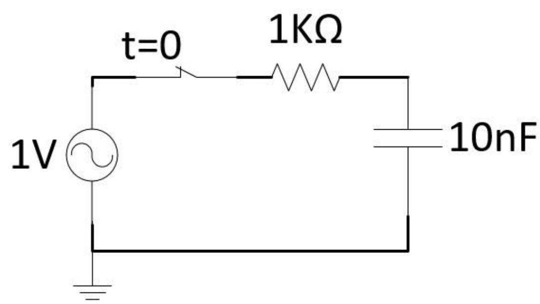




$\omega_{o(a)} = 10^{+5}$ rad/s

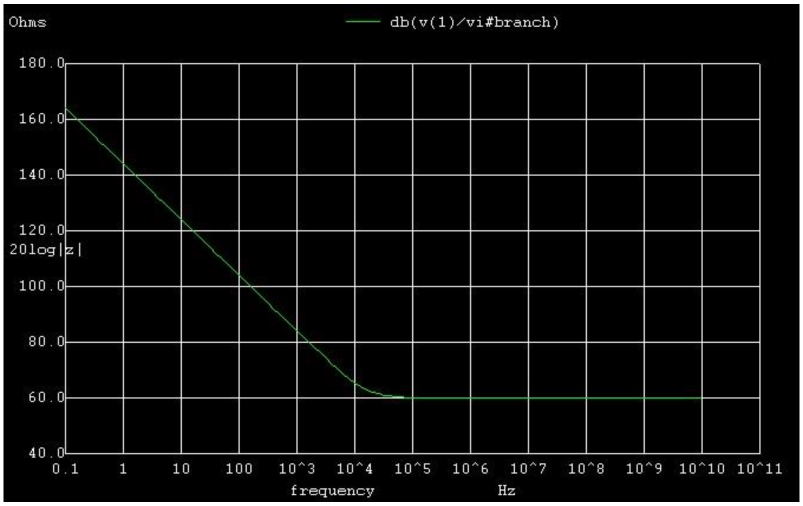
$\omega_{o(c)} = 10^{+6}$ rad/s

$\omega_{o(b)} = 3.1623 \times 10^{5}$ rad/s

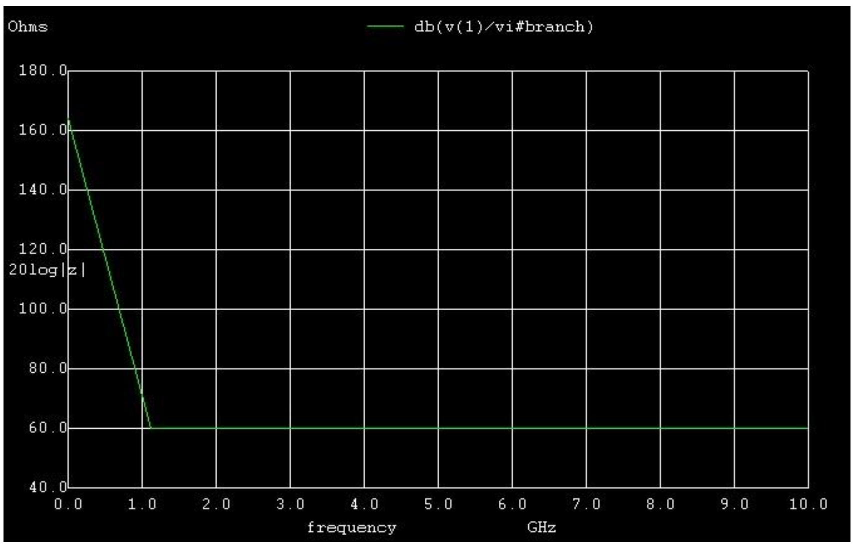


(a)

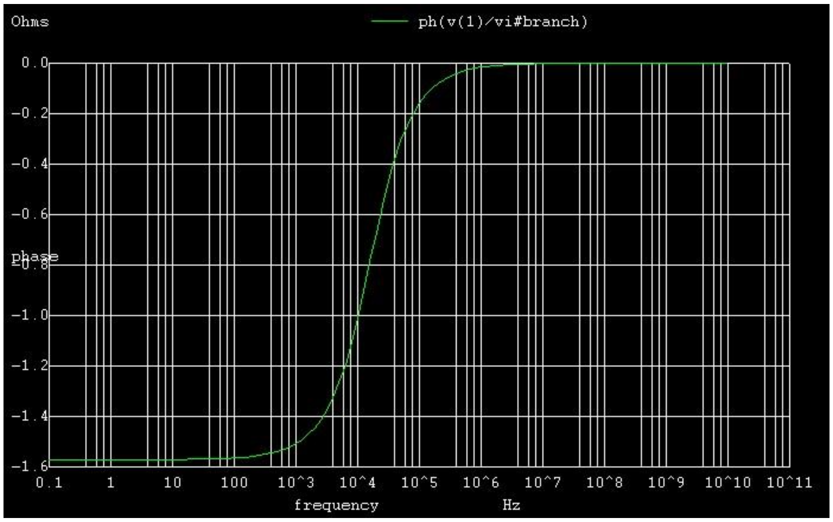
Impedance (Magnitude) as a function of frequency: log scale



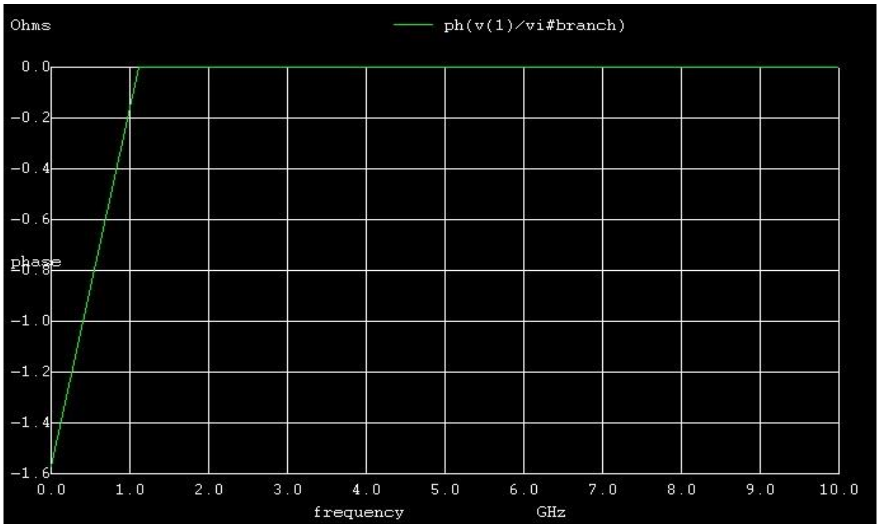
Impedance (Magnitude) as a function of frequency: linear scale



Impedance (Phase) as a function of frequency: log scale



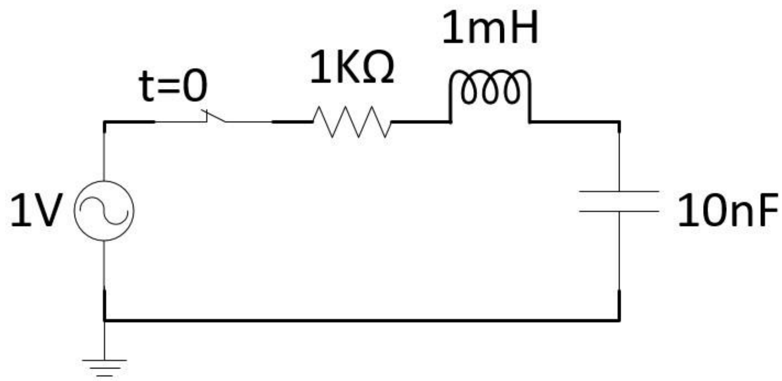
Impedance (Phase) as a function of frequency: linear scale



Spice code for RC circuit: Q3 (a)

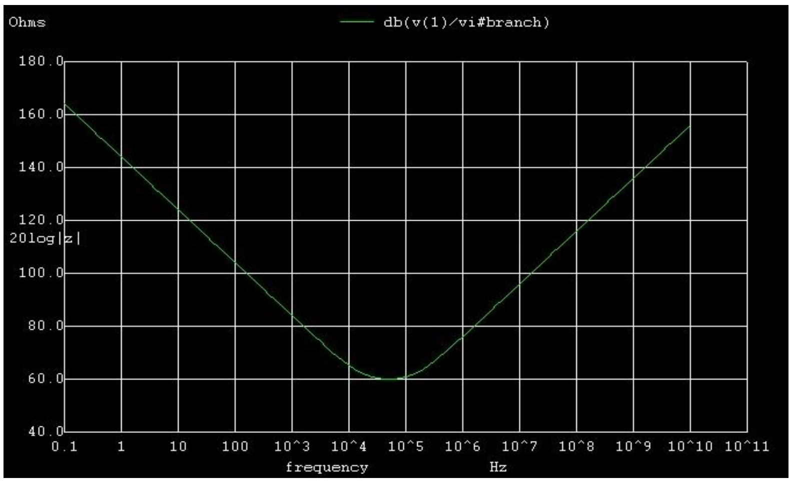
```
*input impedance of RC ckt - log scale

r1 1 2 1k
c1 2 0 10n
vi a 1 0
vin a 0 dc 0 ac 1
.ac dec 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```

```
*input impedance of RC ckt – linear scale

r1 1 2 1k
c1 2 0 10n
vi a 1 0
vin a 0 dc 0 ac 1
.ac lin 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```

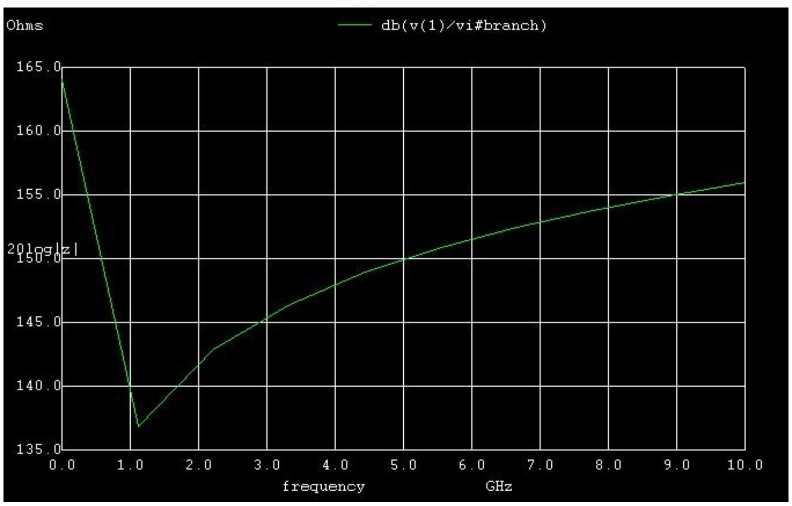


(b)

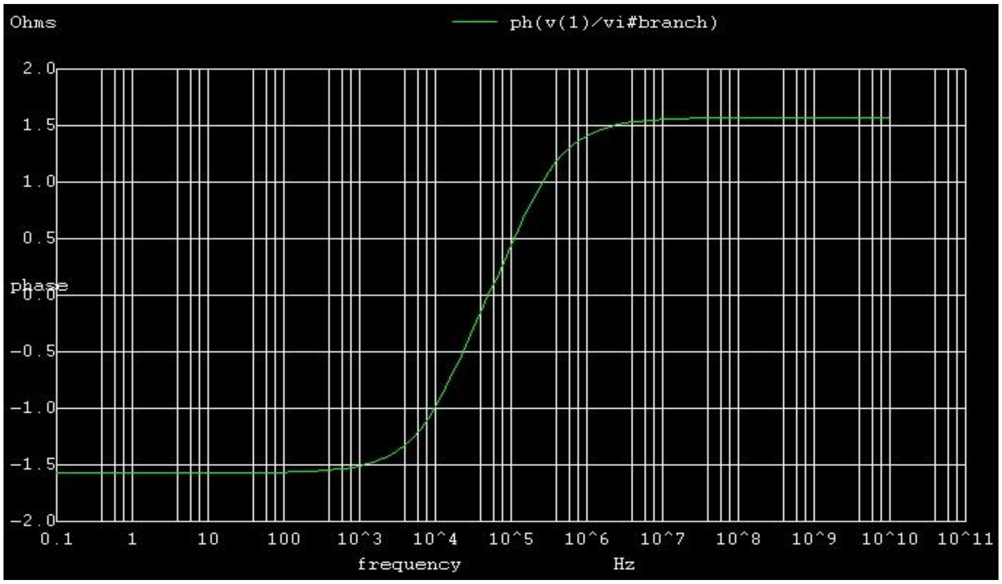
Impedance (Magnitude) as a function of frequency: log scale



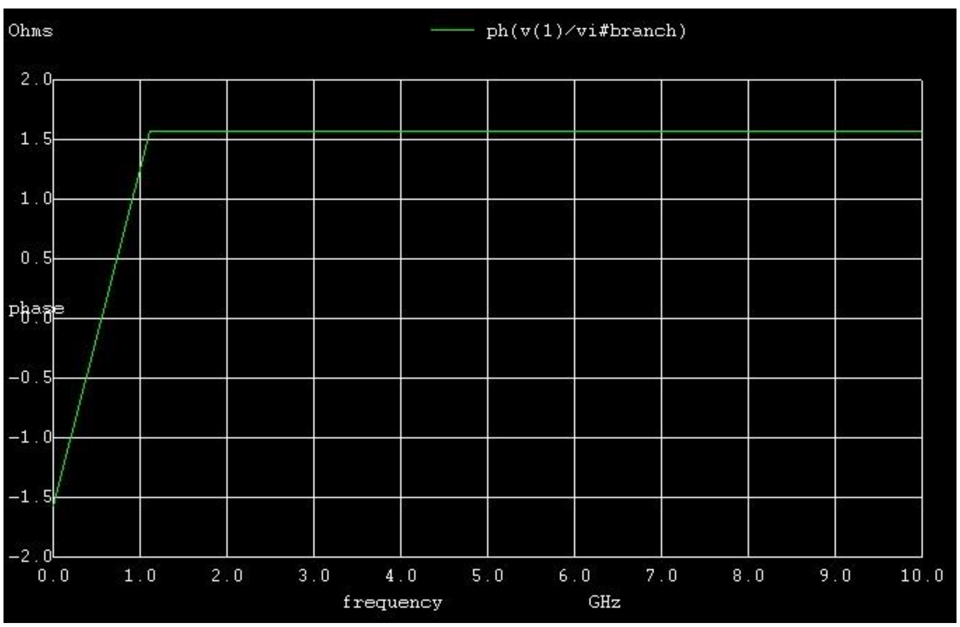
Impedance (Magnitude) as a function of frequency: linear scale



Impedance (Phase) as a function of frequency: log scale



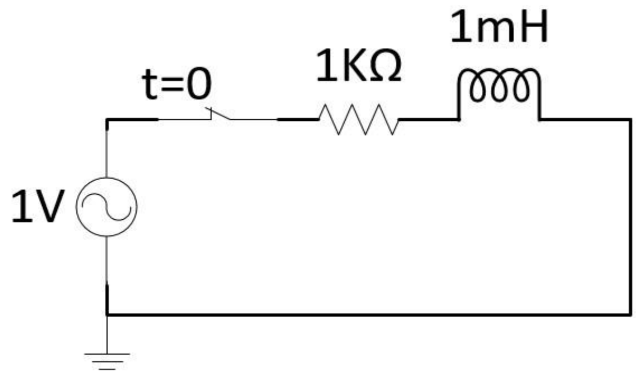
Impedance (Phase) as a function of frequency: linear scale



Spice code for RLC circuit: Q3 (b)

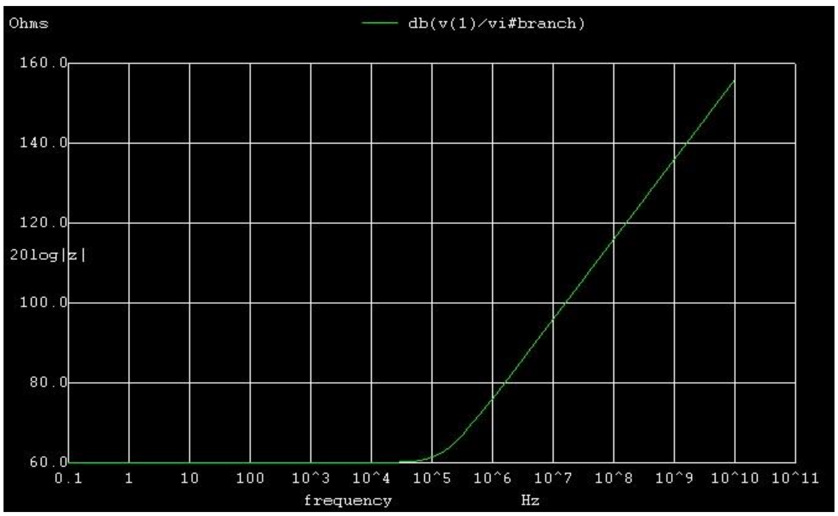
```
*input impedance of RLC ckt – log scale

r1 1 2 1k
l1 2 3 1m
c1 3 0 10n
vi a 1 0
vin a 0 dc 0 ac 1
.ac dec 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```

```
*input impedance of RLC ckt – linear scale

r1 1 2 1k
l1 2 3 1m
c1 3 0 10n
vi a 1 0
vin a 0 dc 0 ac 1
.ac lin 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```

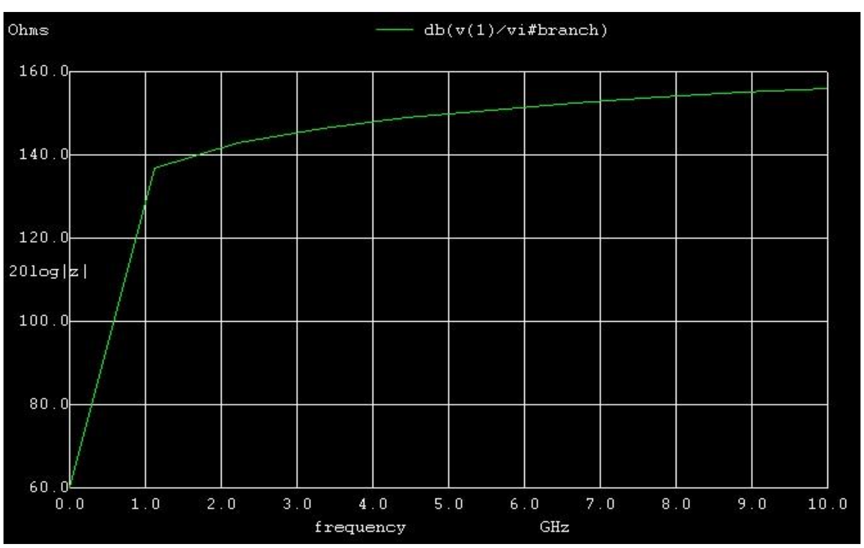


(c)

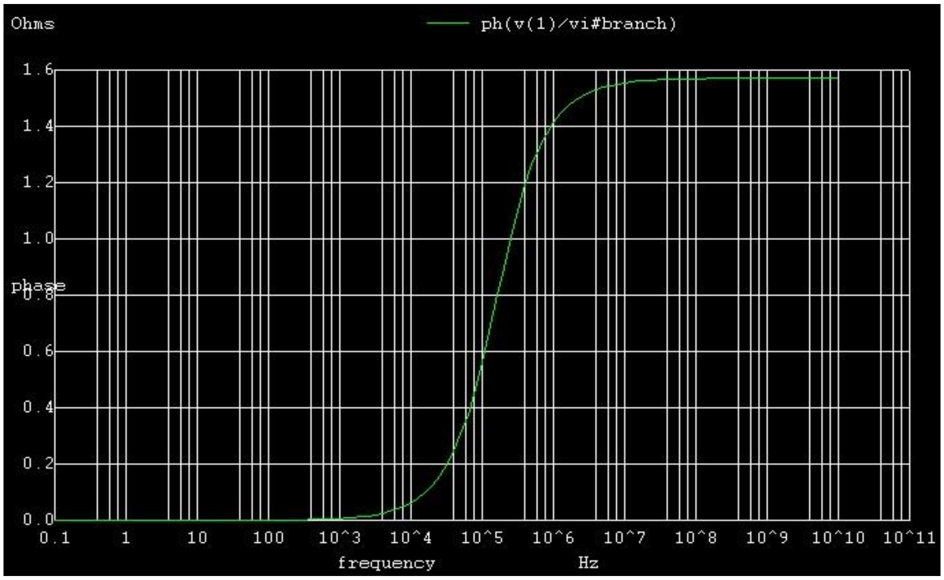
Impedance (Magnitude) as a function of frequency: log scale



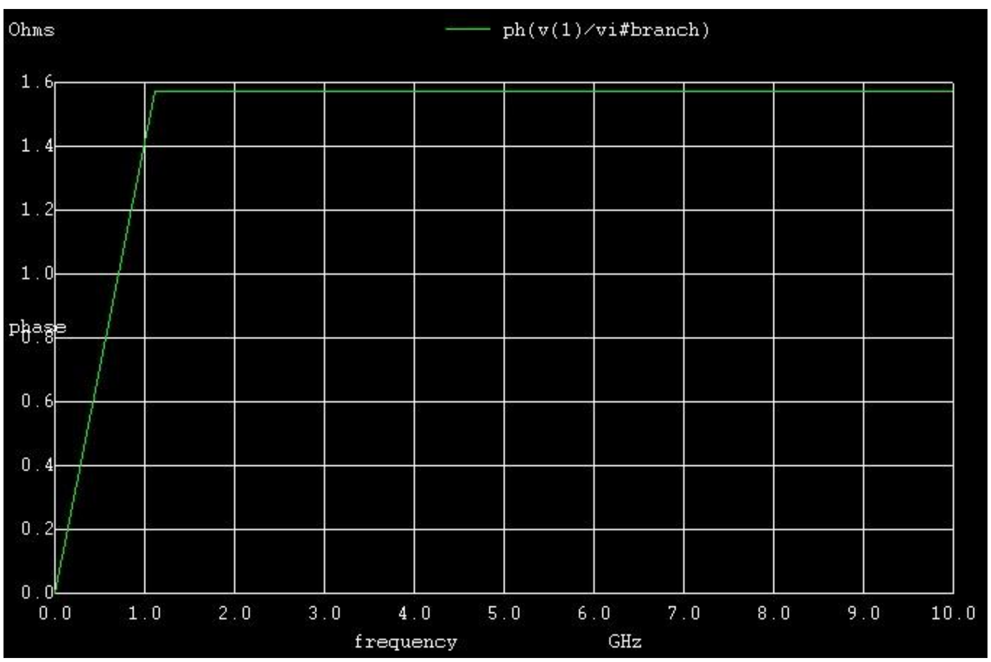
Impedance (Magnitude) as a function of frequency: linear scale



Impedance (Phase) as a function of frequency: log scale



Impedance (Phase) as a function of frequency: linear scale



Spice code for RL circuit: Q3 (c)

```
*input impedance of RL ckt – log scale

r1 1 2 1k
l1 2 0 1m
vi a 1 0
vin a 0 dc 0 ac 1
.ac dec 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```

```
*input impedance of RL ckt – linear scale

r1 1 2 1k
l1 2 0 1m
vi a 1 0
vin a 0 dc 0 ac 1
.ac lin 10 0.1 10g

.control
run

plot db(v(1)/vi#branch) xlabel frequency ylabel 20log|Z|
plot ph(v(1)/vi#branch) xlabel frequency ylabel Phase
.endc
.end
```