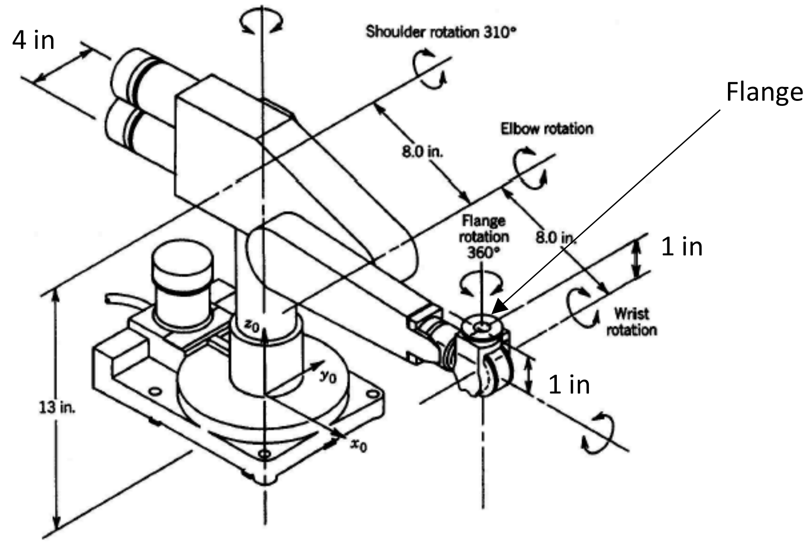
ME604: Practice Problems

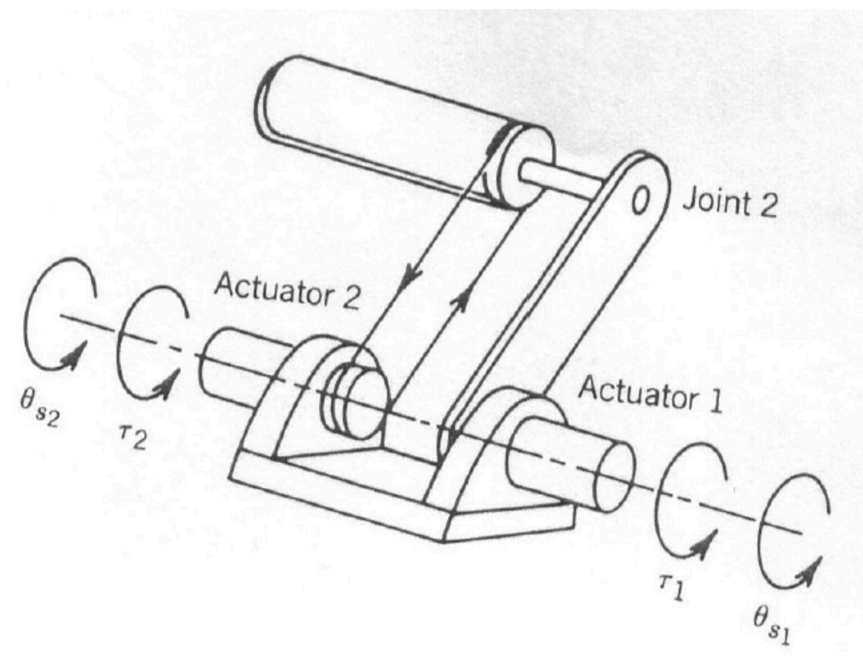
1. Consider the PUMA manipulator shown below:



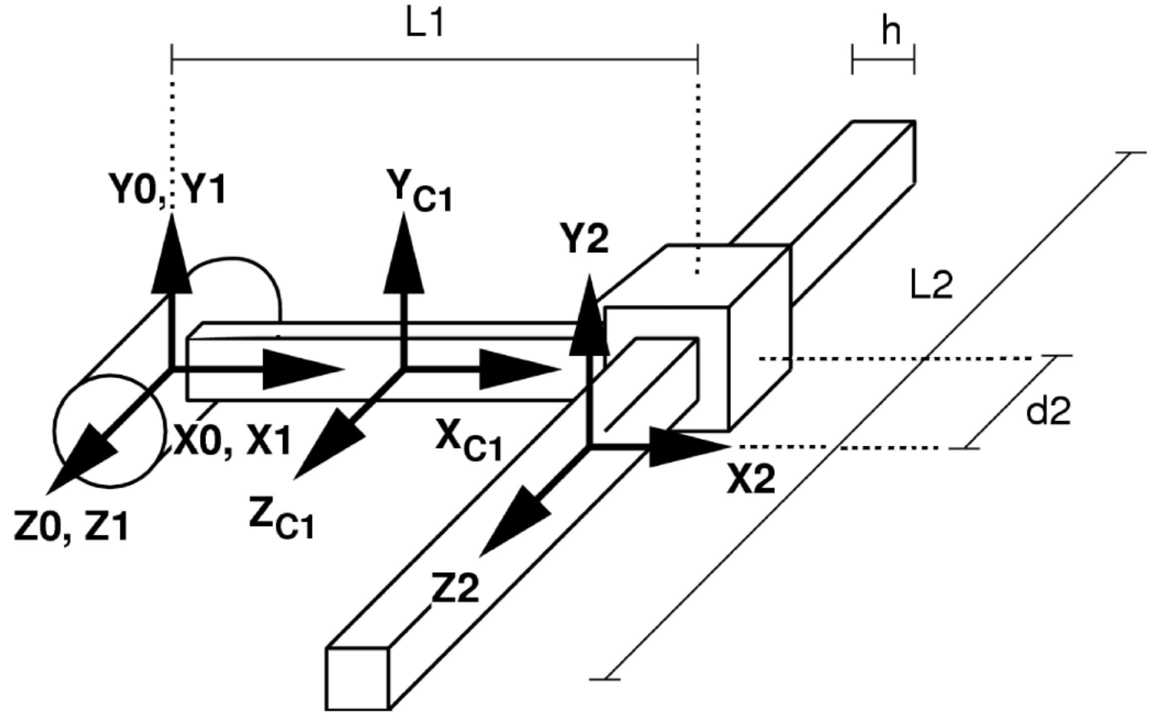
   (a) At the instant shown, a force of 1 kN acts vertically downwards at the end-effector of the robot. Determine the joint torques needed to statically .

   (b) A screw driver is gripped in the end-effector so that its tip is along z_0 at a distance of 9 in from the center of the robot wrist. What is the force and torque the screw driver tip applies when the same joint torques that were determined in part (a) are applied?

2. Consider the two-link manipulator with remotely driven links as shown below. Find an expression for the motor torques needed to balance a force **F** at the end effector. Assume that the gear ratios are r_1 and r_2, respectively.



3. Consider the robot manipulator shown below. The links of this manipulator are modeled as bars of uniform density, having square cross-sections of thickness h, lengths L_1 and L_2 and total masses of m_1 and m_2, with center of mass as shown. Assume that the joints themselves are massless.



(a) For each link i, we have attached a frame $\{C_i\}$ to the center of mass (frame $\{2\}$ is same as frame $\{C_2\}$. Calculate matrices ${}^{0}_{C_1}T$ and ${}^{0}_{C_2}T$.

For this two-link manipulator, the mass matrix has the form

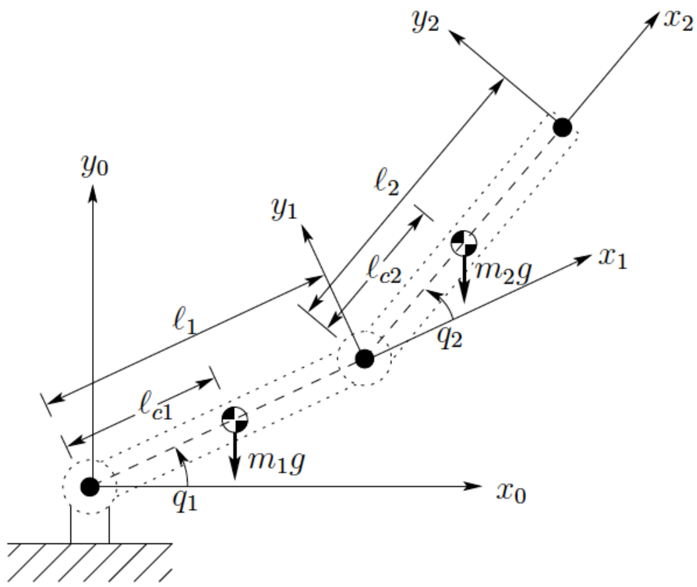
$$M(q) = m_1 J_{v1}^T J_{v1} + m_2 J_{v2}^T J_{v2} + J_{\omega 1}^T I_{c1} J_{\omega 1} + J_{\omega 2}^T I_{c1} J_{\omega 2}$$

where, J_{vi} is the Jacobian of the center of mass of link i, $J_{\omega i}$ is the angular velocity of link i, and I_{ci} is the inertia tensor of link i expressed in frame $\{C_i\}$.

(b) Calculate ${}^{0}J_{v1}$ and ${}^{0}J_{v2}$.
(c) Calculate ${}^{c1}J_{\omega 1}$ and ${}^{c2}J_{\omega 2}$.
(d) Calculate I_{c1} and I_{c2} in terms of the masses and dimensions of the links.
(e) Calculate the mass matrix $M(q)$.
(f) Calculate the other terms (gravity vector, Coriolis and centrifugal terms) and write out the equations of motion as:

$$\tau_1 = f_1(\ddot{q}, \dot{q}, q); \qquad \tau_2 = f_2(\ddot{q}, \dot{q}, q)$$

4. For the two-link manipulator shown below, write the dynamic equation in task space co-ordinates.



5. Describe the natural and artificial constraints associated with the task of opening a box with a hinged lid.

6. Consider a single degree of freedom system described by the equation

$$2\ddot{x} + \dot{x} = F(t)$$

Design a PD trajectory tracking controller to track a reference signal $x_d(t) = \sin t + \cos 2t$. The closed loop system should have a natural frequency less than 10 radians with a damping ratio greater than 0.707.

7. Consider the coupled nonlinear system

$$\ddot{y}_1 + 3y_1y_2 + y_2^2 = u_1 + y_2u_2,$$
$$\ddot{y}_2 + (\cos y_1)\dot{y}_2 + 3(y_1 - y_2) = u_2 - (\cos y_1)^2 y_2 u_1$$

(a) Can these equations be written in the form

$$u_1 = f_1(\ddot{q}, \dot{q}, q); \qquad u_2 = f_2(\ddot{q}, \dot{q}, q)$$

(b) Find an inverse dynamics control so that the closed loop system is linear and decoupled, with each subsystem having natural frequency 10 radians, and damping ratio 0.5.