$\boxed{1.}$

- Not memory less.
- Not linear

Eg:

$$x_1(t) = u(t) \Rightarrow y_1(0) = 1$$

$$x_2(t) = -u(t) \Rightarrow y_2(0) = 0$$

$$\underbrace{x_1(t) + x_2(t)}_{x_3(t)} = 0 \Rightarrow y_3(0) = 0$$

- Shift invariant
  ↳ easy justification

- Not causal.

Eg: $x(t) = u(t)$

$\Downarrow$

$y(-\frac{1}{2}) = 1$.

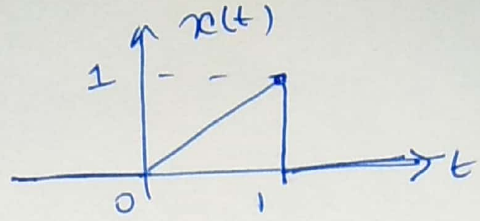
But $y(t) = 0 \; \forall \, t < 0$ if causal.

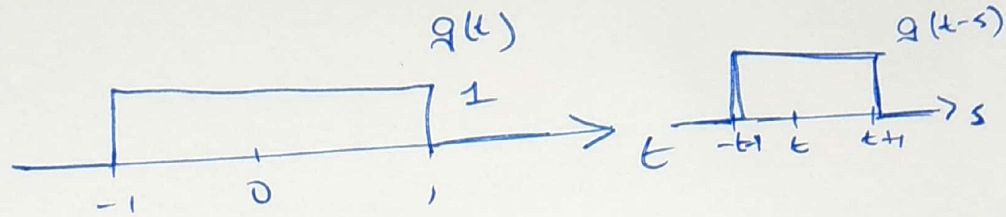
- Stable.

{2 marks each}

{1 if no justification}

(2.)





a) $y = x * g \Rightarrow y(t) = \int x(s)\, g(t-s)\, ds$

$$y(t) = \begin{cases} 0 & t < -1 \quad (1) \\ \int_0^{t+1} s\, ds = \frac{(t+1)^2}{2} & t \in [-1, 0] \quad (1) \\ 1/2 & t \in [0, 1] \quad (1) \\ \int_{t-1}^{1} s\, ds = \frac{1-(t-1)^2}{2} & t \in [1, 2] \quad (1) \\ 0 & t > 2. \quad (1) \end{cases}$$

[5 marks]

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b) $y(t) = \int x(s)\, g(t-s)\, ds$

$y(2t) = \int x(s)\, g(2t-s)\, ds$

$s = 2s'$

$= 2\int x(2s')\, g(2t - 2s')\, ds'$

$$\cancel{\text{??}}\ y(2t) = 2 \cdot \left(x(2t) * g(2t) \right)$$

$$\Rightarrow \quad x(2t) * g(2t) = \frac{1}{2} y(2t)$$

[5 marks]

$$= \begin{cases} 0 & t < -1/2 \\ \frac{(2t+1)^2}{4} & t \in [-1/2, 0] \\ 1/4 & t \in [0, 1/2] \\ \frac{1-(2t-1)^2}{4} & t \in [1/2, 1] \\ 0 & t \geq 1 \end{cases}$$

5. Say k even. $\left(\omega_0 = \frac{2\pi}{T}\right)$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_0 k t}\, dt$$

$$= \frac{1}{T}\int_0^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt + \underbrace{\frac{1}{T}\int_{T/2}^{T} x(t)\, e^{-jk\omega_0 t}\, dt}_{y = t - T/2}$$

$$= \frac{1}{T}\int_0^{T/2} x(t)\, e^{-jk\omega_0 t} + \frac{1}{T}\int_0^{T/2} x(y + T/2) . e^{-jk\omega_0 (y+T/2)}$$

$$= \frac{1}{T}\int_0^{T/2} x(t)\, e^{-jk\omega_0 t} - \frac{1}{T}\int_0^{T/2} x(y) . e^{-jk\omega_0 y} . \underbrace{e^{-j\left(\frac{2\pi}{T}\right)k . \frac{T}{2}}}_{= 1} dt$$

$$= \underline{0.}$$

$\boxed{6.}$ a) $e^{j\omega n} \longrightarrow H(\omega) e^{j\omega n}$

$$\Rightarrow H(\omega) \cdot e^{j\omega n} - \frac{1}{2} H(\omega) e^{j\omega(n-1)} = e^{j\omega n}$$

$$\Rightarrow H(\omega)\left[1 - \frac{e^{-j\omega}}{2}\right] = 1$$

$$\Rightarrow H(\omega) = \frac{1}{\left(1 - \frac{e^{-j\omega}}{2}\right)}$$

[3 marks]

b) Let ~~x[n]=~~ $x[n] = \delta[n]$

$\forall\, n < 0.$

$$\Rightarrow y[n] = 0$$

$$y[0] = 1$$

$$y[1] = \tfrac{1}{2}\, y[0]$$

$$y[2] = \tfrac{1}{2}\, y[1]$$

$\vdots$

$$y[n] = \begin{cases} 0 & n < 0 \\ \frac{1}{2^n} & n \geq 0 \end{cases}$$

$$= 2^{-n} u[n].$$

[4 marks]

c) $\sin(3n) = \dfrac{e^{j3n} - e^{-j3n}}{2j}$

$$\downarrow$$

$$\frac{1}{2j}\left\{ e^{3jn} H(3) - e^{-3jn} H(-3) \right\}$$

$$\parallel$$

$$\frac{1}{2j}\left\{ \frac{e^{3jn}}{1-\frac{1}{2}e^{-3j}} - \frac{e^{-3jn}}{1-\frac{1}{2}e^{+3j}} \right\}$$

[3 marks]