FEM Assignment

Inderjeet Jayakumar Nair,170020013
Ayan Sharma, 170020023
Ameya Anjarlekar,170070013
Aaron John Sabu, 170070050
Vedant Satav, 170070011

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1 Calculation of the Reactance

In this part we attempt to vary various parameters of the transformer and correspondingly compute the leakage inductance with the aid of the provided FEMM software and using the empirical formula.

Template Transformer
We consider the following transformer with the given dimensions and $\mu_r = 50000$. The source current density in LV and HV windings was taken to be $1.708299\frac{MA}{m^2}$ and $1.366639\frac{MA}{m^2}$

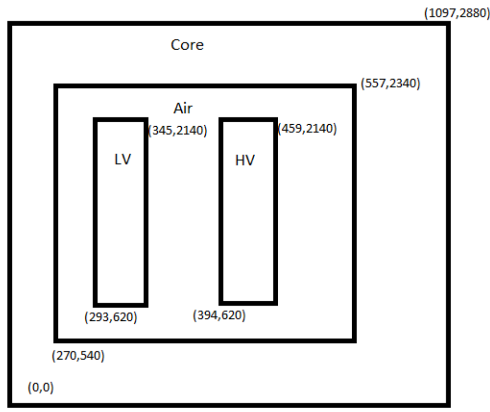

Figure 1: Template Transformer

Using the FEMM software we obtained the contour for vector potential as shown in Figure 2:

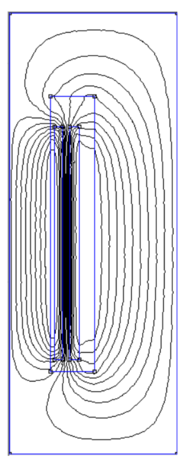

Figure 2: Field lines for vector potential

The following table encapsulates the energy calculation in various regions.

Part	Energy(J/m)	Mean Diameter(m)	Energy(J)
Core	0.02621	0.540	0.0445
Air	359.8	0.739	835.324
LV	114.726	0.638	229.95
HV	139.914	0.853	374.938

$$Total\ Energy = 1440.256J \frac{Li^2}{2} = 1440.256J Thus\ (i = 137.78A) L = 0.1517H$$

Now if we proceed by analytical calculations we have:
Effective area = $0.2066m^2$
Effective height = 1.573m
L(referred to HV side) = $\frac{\mu_o N^2 A}{H_{teff}} = 0.158H$

Effect of increasing LV-HV gap
Now the HV-LV gap is increased to 60mm keeping all the effects constant. The following diagram shows the transformer with necessary changes made.

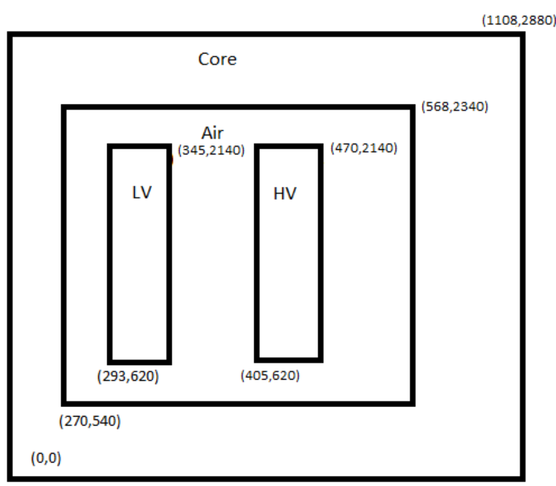

Figure 3: Extending HV-LV gap

The contour for vector potential was obtained using FEMM with all the other parameters kept constant.

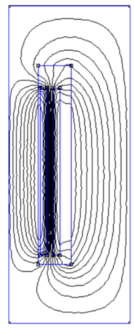

Figure 4: Magnetic Vector potential flux lines

The following table encapsulates the calculation of energy in various regions.

Part	Energy per depth(J/m)	Mean Diameter(m)	Energy(J)
Core	0.00325	0.540	0.00551
Air	439.05	0.750	1034.487
LV	115.405	0.638	231.310
HV	140.642	0.875	386.610

$$\text{Total Energy} = 1652.4125\text{J}$$

$$\frac{1}{2}Li^2 = 1652.4125J$$

$$\text{Thus(i} = 137.78\text{A) L} = 0.1741\text{H}$$

We can see from the above computation that the leakage inductance **increases** if the distance between HV-LV winding **increases**. Proceeding by analytical calculations yields:
Effective area = $0.2356m^2$
Effective Height = 1.576m L(referred to HV side) = $\frac{\mu_o N^2 A}{H_{teff}} = 0.180H$

Effect of increasing yoke clearance In this case we increase the winding to top yoke clearance by 50mm and winding to bottom yoke clearance by 50mm. Thus the transformer model for this case is shown below.

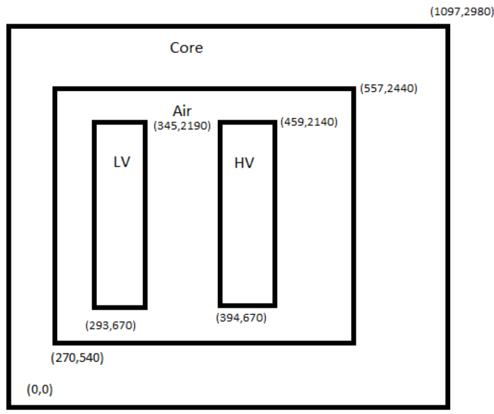

Figure 5: Yoke clearance increment

The contour for vector potential was obtained using FEMM with all the other parameters kept constant.

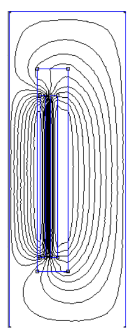

Figure 6: Magnetic Vector Potential Field Lines

The following table encapsulates the calculation of energy in various regions.

Part	Energy(J/m)	Mean Diameter(m)	Energy(J)
Core	0.02558	0.540	0.0434
Air	359.138	0.739	833.788
LV	114.55	0.638	229.60
HV	138.767	0.853	371.86

$$\text{Total Energy} = 1435.2914\text{J}$$

$$\frac{1}{2}Li^2 = 1435.2914J$$

$$\text{Thus(i} = 137.78\text{A) L} = 0.1512\text{H}$$

Thus Leakage inductance **decreases** if winding to yoke clearance is **increased**. Now if we proceed by analytical calculations we have:
Effective area = $0.2066m^2$
Effective height = 1.573m
L(referred to HV side) = $\frac{\mu_o N^2 A}{H_{teff}} = 0.158H$

Effect of increasing relative permeability of core The transformer dimensions are same as that of in first case with $\mu_r = 75,000$. The figure below is the cross section of the transformer considered.

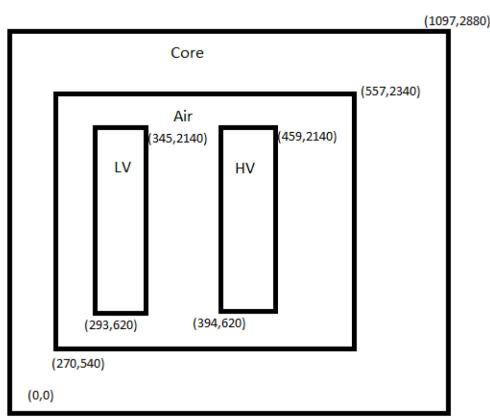

Figure 7: Increment of relative permeability

The contour for vector potential was obtained using FEMM with all the other parameters kept constant.

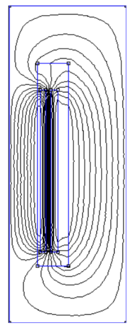

Figure 8: Magnetic Vector Potential Field Lines

The following table encapsulates the calculation of energy in various regions.

Part	Energy(J/m)	Mean Diameter(m)	Energy(J)
Core	0.01705	0.540	0.0289
Air	359.139	0.739	833.790
LV	114.551	0.638	229.60
HV	138.768	0.853	371.87

$$\text{Total Energy} = 1435.2889\text{J}$$

$$\frac{1}{2}Li^2 = 1435.2889J$$

$$\text{Thus(i} = 137.78\text{A) L} = 0.1512\text{H}$$

Now if we proceed by analytical calculations we have:
Effective area = $0.2066m^2$
Effective height = 1.573m
L(referred to HV side) = $\frac{\mu_o N^2 A}{H_{teff}} = 0.158H$

2 Finite Element Method

2.1 Question 1

Consider the potential (or in general, an arbitrary function to be approximated) A at point (x,y) inside an element. Then, if e denotes the element:

$$A^e = \sum_i N_i(x,y)A_i^e \tag{1}$$

where i denotes the i^{th} node of element 'e'.
Now, if we are on the edge of joining the k^{th} and the e^{th} node, then the expansion functions N_i satisfy:

$$N_i(x,y) = 0 \tag{2}$$

for i

2.2 Question 2: Aspects of FEM

Energy Minimization
When we approximate the potential in a region, we use interpolation to find its value given the value at the vertices of the sub-region. Thus, we would get an expression for the potential with a set of constants. These constants would be optimal if the energy of the system is minimum. Thus, Energy minimization is done to determine the value of the potential at the nodes of the elements in terms of the global stiffness and global source matrix constants which in turn give us the value of the constants assumed during interpolation to finally give n linear equations with n unknowns(the assumed constants). This could then be solved to find out the required constants.
Whole-domain and Sub-domain Approximation
To find out the magnetic potential in a region, we discretize the domain into a finite number of elements of dimensions based on the requirements. Within each element, the potential is approximated by interpolating it from the value of the nodes using a suitable function, typically a polynomial with some with no. of unknown constants that agree with the type of the element. This is the Whole-domain and Sub-domain Approximation in FEM.
Numerical Error Reduction
In FEM, we calculate the approximate magnetic potential by dividing the area into various small parts and approximating it with a suitable function.

As it is an approximation, it will have some numerical error.
Ways to reduce numeric error : 1)For simplicity of computation, we take the approximation of the magnetic potential over a small region as a linear function.However, the magnetic potential will be non-linear. Thus, taking higher order polynomial terms in the expression of magnetic potential will help to approximate the non-linearity better and thus reduce the numeric error.
2)If we increase the number of sub-parts in which the area is divided, we have greater known values for the magnetic potential now, and thus we would be able to approximate the field over this small area in a more accurate way. Thus, increasing the no. of elements into which the domain is divided will reduce the numerical error.