

3.

$$x[n] \xrightarrow{\text{DTFS}} a_k$$

$$\begin{aligned} (-1)^n x[n] &= e^{j\pi n} x[n] \\ &= e^{j\left(\frac{2\pi}{N}\right) \cdot \left(\frac{N}{2}\right)n} x[n] \end{aligned}$$

$$\downarrow \text{DTFS}$$

$$a_{k-N/2}$$

$$\Rightarrow (-1)^n x[n] \xrightarrow{\text{DTFS}} a_{k-N/2}$$

10 marks for correct ans.  
5 marks for convolution / other approach that also not yield the simple final form of DTFS coefficients

$$a_0 = 0.$$

4.

$$T = 3.$$

$$a_k = \frac{1}{3} \int_0^3 x(t) e^{-j\left(\frac{2\pi}{T}\right)kt} dt$$

$$= \frac{1}{3} \left[ 2 \int_0^1 e^{-j\left(\frac{2\pi}{3}\right)kt} dt - 2 \int_1^2 e^{-j\left(\frac{2\pi}{3}\right)kt} dt \right]$$

$$= \frac{2}{3} \left[ \frac{1}{-j(\frac{2\pi}{3})k} (e^{-j(\frac{2\pi}{3})k} - 1) - \frac{1}{-j(\frac{2\pi}{3})k} (e^{-j(\frac{2\pi}{3}) \cdot 2k} - e^{-j(\frac{2\pi}{3})k}) \right]$$

$$= \frac{2}{3} \left[ \frac{1}{j(\frac{2\pi}{3}k)} \left\{ 1 - 2e^{-j(\frac{2\pi}{3})k} + e^{-j(\frac{2\pi}{3}) \cdot 2k} \right\} \right]$$

$$= \frac{2}{j\pi k} (1 - e^{-j(\frac{2\pi}{3})k})^2$$


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