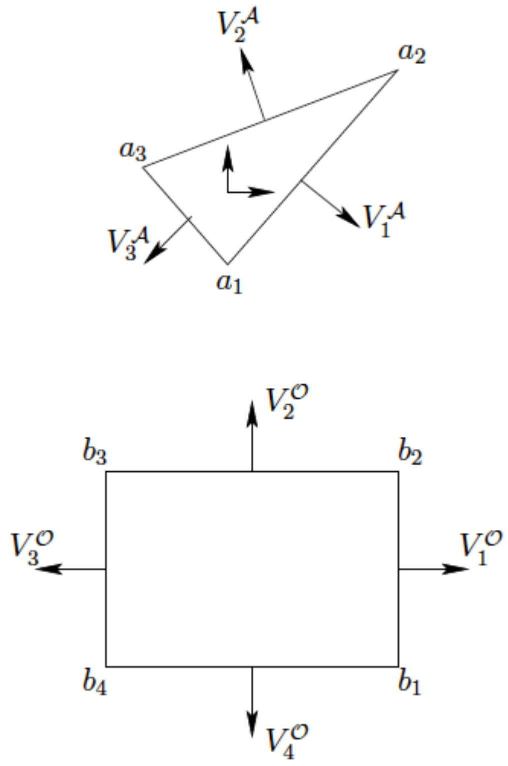
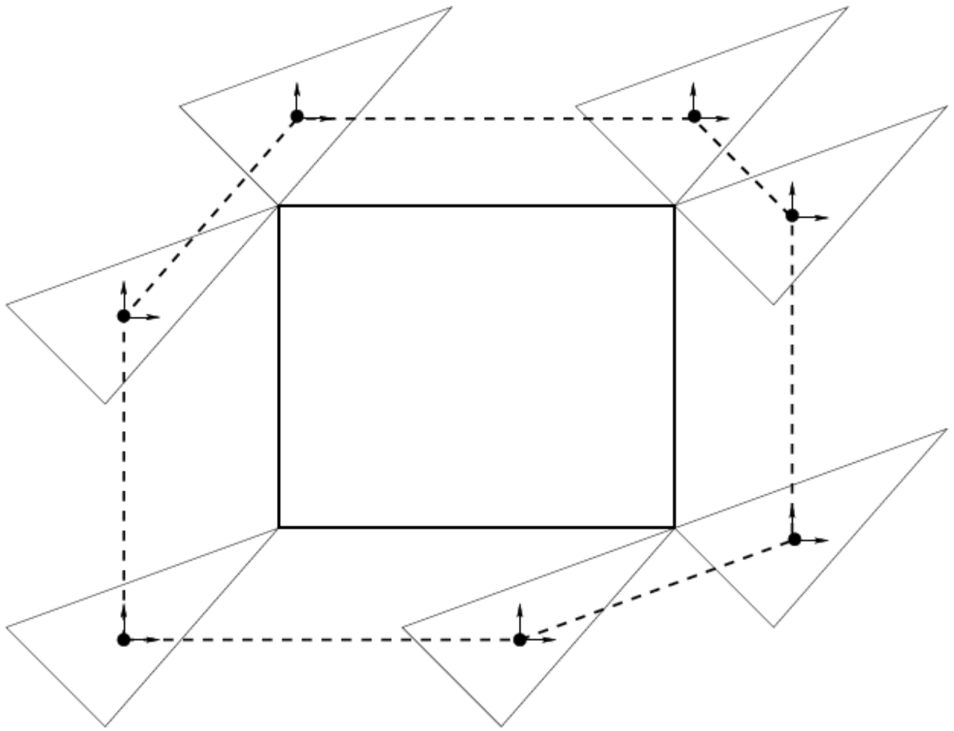
Configuration space

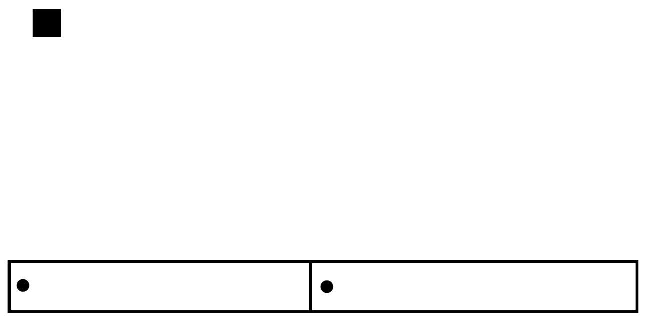
- Configuration space, Q : set of all possible configurations of the robot
  - Single link revolute arm:
  $$Q = S^1\text{, unit circle}$$
  - Two link planar manipulator:
  $$Q = S^1 \mathrm{X}\, \mathrm{S}^1 = \mathrm{T}^2\text{, Torus}$$
  - Cartesian arm:
  $$Q = R^3$$
- Workspace, W: Cartesian space in which robot operates
- $A(q)$: subset of W occupied by the robot at configuration q
- Configuration space obstacle : set of configurations for which robot collides with an obstacle
  $$QO = \{q \in Q: A(q) \cap O \neq 0\}$$
  where, $O = \cup\, O_i$, denote the obstacles in W.
  $$Q_{free} = Q \backslash QO$$

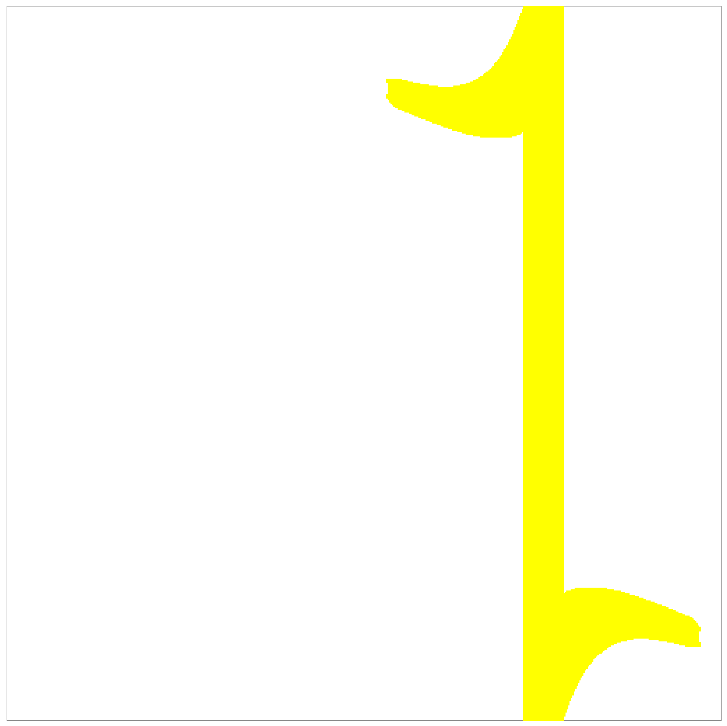
Example: Rigid body translating in space





Example: Two-link manipulator





Path planning problem: Find a path from an initial configuration q_{init} to q_{final}, such that the robot does not collide with any obstacles

Configuration space potential fields

- Robot is a point particle in configuration space
- Basic Idea
  - Robot moves under an artificial potential field U
  - Robot is attracted to the final configuration q_{final}
  - Robot is repelled by boundaries of QO

$$U(q) = U_a(q) + U_{rep}(q)$$

  - Using gradient descent, force acting on the robot (in configuration space)

$$F(q) = -\nabla U(q) = -\nabla U_a(q) - \nabla U_{rep}(q)$$

Attractive potential field

$$\rho_f(\boldsymbol{q}) = ||\boldsymbol{q} - \boldsymbol{q}_{\text{final}}||$$

$$U_{\text{att}}(\boldsymbol{q}) = \begin{cases} \dfrac{1}{2}\zeta\rho_f^2(\boldsymbol{q}) & : \quad \rho_f(\boldsymbol{q}) \leq d \\ \\ d\zeta\rho_f(\boldsymbol{q}) - \dfrac{1}{2}\zeta d^2 & : \quad \rho_f(\boldsymbol{q}) > d \end{cases}$$

$$F_{\text{att}}(q) = -\nabla U_{\text{att}}(\boldsymbol{q}) = \begin{cases} -\zeta(\boldsymbol{q} - \boldsymbol{q}_{\text{final}}) & : \quad \rho_f(\boldsymbol{q}) \leq d \\ \\ -\dfrac{d\zeta(\boldsymbol{q} - \boldsymbol{q}_{\text{final}})}{\rho_f(\boldsymbol{q})} & : \quad \rho_f(\boldsymbol{q}) > d \end{cases}$$

Repulsive potential field

$$\rho_f(\boldsymbol{q}) = ||\boldsymbol{q} - \boldsymbol{q}_{\text{final}}||$$

$$U_{\text{rep}}(\boldsymbol{q}) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho(\boldsymbol{q})} - \frac{1}{\rho_0}\right)^2 & : \quad \rho(\boldsymbol{q}) \leq \rho_0 \\ 0 & : \quad \rho(\boldsymbol{q}) > \rho_0 \end{cases}$$

For a single convex obstacle,

$$F_{\text{rep}}(\boldsymbol{q}) = \begin{cases} \eta\left(\frac{1}{\rho(\boldsymbol{q})} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(\boldsymbol{q})}\nabla\rho(\boldsymbol{q}) & : \quad \rho(\boldsymbol{q}) \leq \rho_0 \\ 0 & : \quad \rho(\boldsymbol{q}) > \rho_0 \end{cases} .$$

Gradient descent

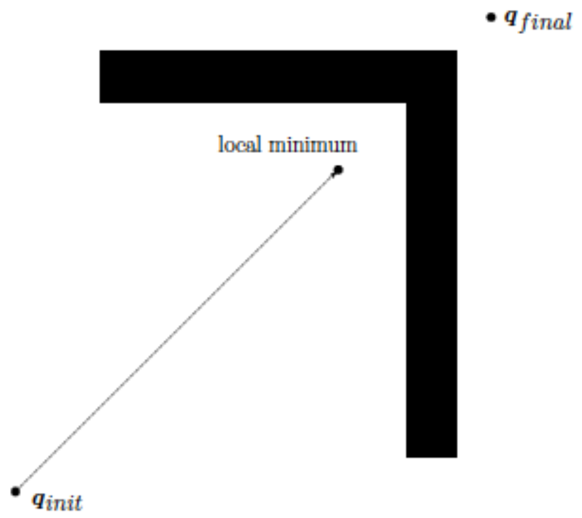
1. $\boldsymbol{q}^0 \leftarrow \boldsymbol{q}_{\text{init}},\ i \leftarrow 0$
2. $\textbf{IF}\ \boldsymbol{q}^i \neq \boldsymbol{q}_{\text{final}}$

$$\boldsymbol{q}^{i+1} \leftarrow \boldsymbol{q}^i + \alpha^i \frac{F(\boldsymbol{q}^i)}{||F(\boldsymbol{q}^i)||}$$

$$i \leftarrow i + 1$$

$\textbf{ELSE}$ return $< \boldsymbol{q}^0, \boldsymbol{q}^1 \cdots \boldsymbol{q}^i >$

3. $\textbf{GO TO}\ 2$



Workspace potential field

$$U_{\mathrm{att}\,i}(\boldsymbol{q}) = \begin{cases} \frac{1}{2}\zeta_i ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})||^2 & : \quad ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})|| \leq d \\ d\zeta_i ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})|| - \dfrac{1}{2}\zeta_i d^2 & : \quad ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})|| > d \end{cases}$$

$$\begin{aligned} \mathcal{F}_{\mathrm{att},i}(q) &= -\nabla U_{\mathrm{att}\,i}(\boldsymbol{q}) \\ &= \begin{cases} -\zeta_i(\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})) & : \quad ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})|| \leq d \\ -\frac{d\zeta_i(\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}}))}{||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})||} & : \quad ||\boldsymbol{a}_i(\boldsymbol{q}) - \boldsymbol{a}_i(\boldsymbol{q}_{\mathrm{final}})|| > d \end{cases} \end{aligned}$$

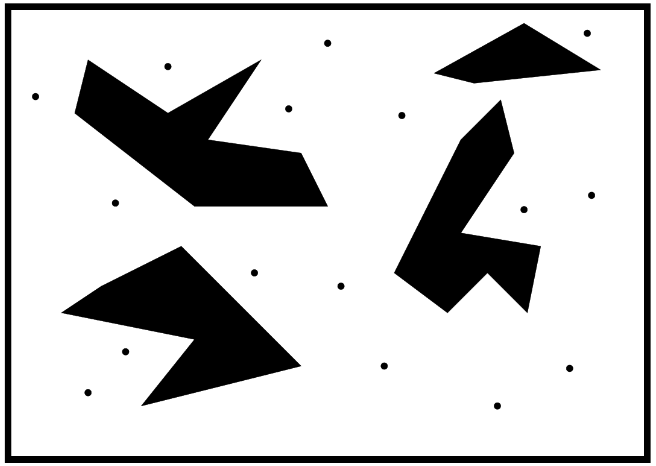
Workspace potential field

$$U_{\text{rep}_j}(\boldsymbol{q}) = \begin{cases} \frac{1}{2}\eta_j \left(\frac{1}{\rho(\boldsymbol{a}_j(\boldsymbol{q}))} - \frac{1}{\rho_0} \right)^2 & : \quad \rho(\boldsymbol{a}_j(\boldsymbol{q})) \leq \rho_0 \\ 0 & : \quad \rho(\boldsymbol{a}_j(\boldsymbol{q})) > \rho_0 \end{cases}$$

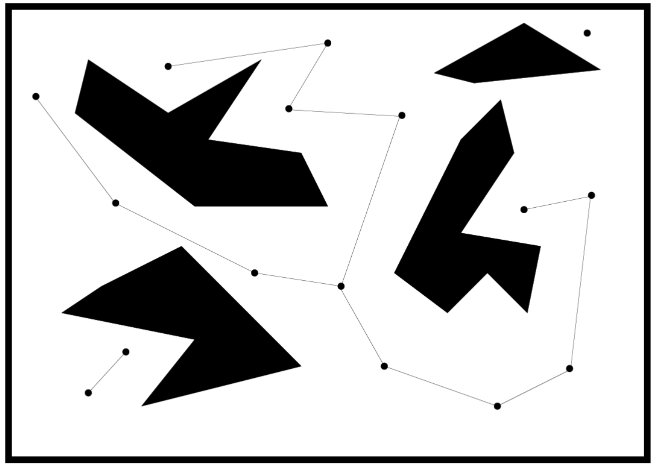
$$\mathcal{F}_{\text{rep},j}(\boldsymbol{q}) = \begin{cases} \eta_j \left(\frac{1}{\rho(\boldsymbol{a}_j(\boldsymbol{q}))} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(\boldsymbol{a}_j(\boldsymbol{q}))} \nabla \rho(\boldsymbol{a}_j(\boldsymbol{q})) & : \quad \rho(\boldsymbol{a}_j(\boldsymbol{q})) \leq \rho_0 \\ 0 & : \quad \rho(\boldsymbol{a}_j(\boldsymbol{q})) > \rho_0 \end{cases}$$

Probabilistic Roadmap Method (PRM)

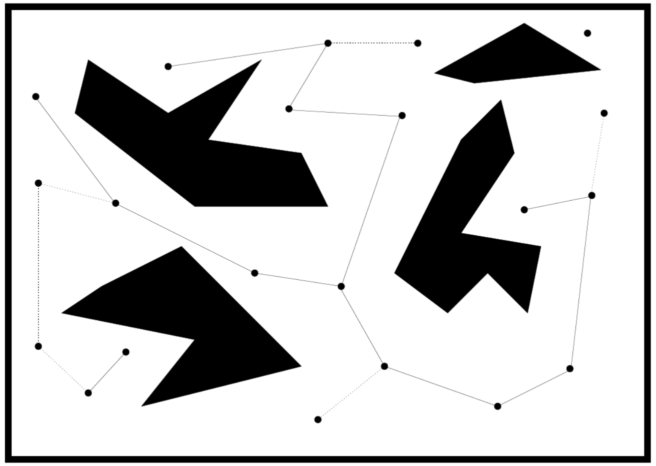
Sampling



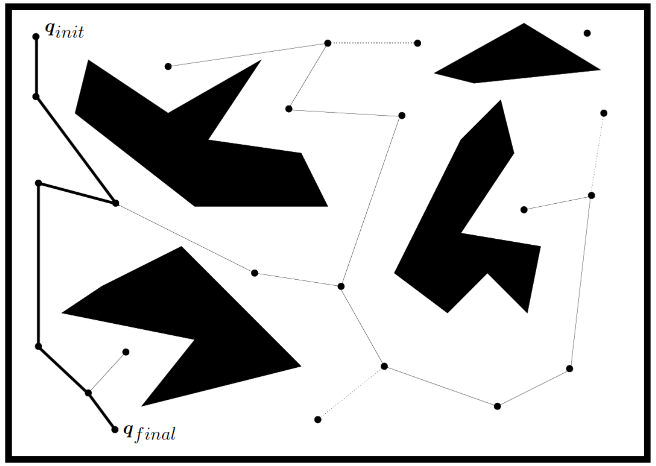
Connecting neighbors



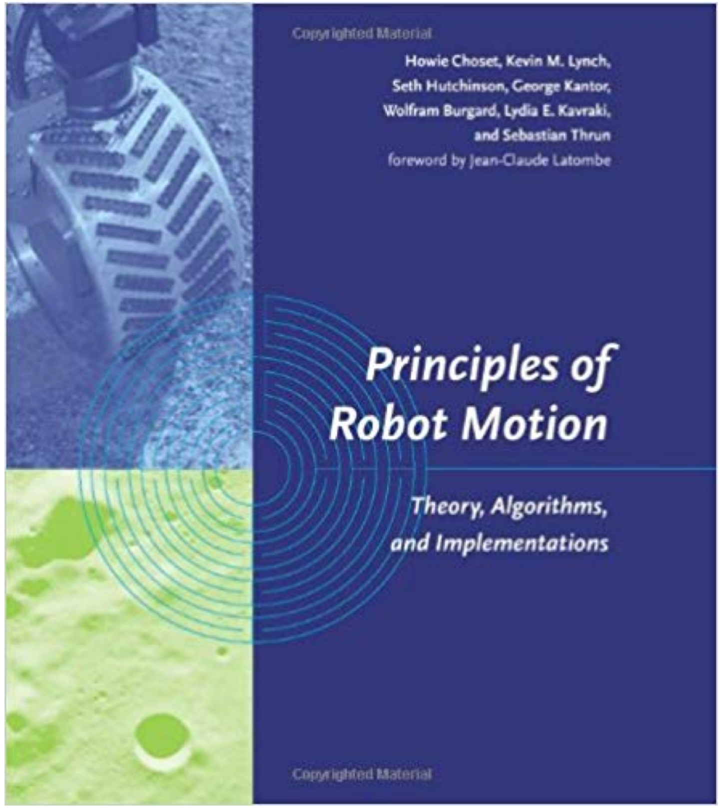
Enhancement



Path search



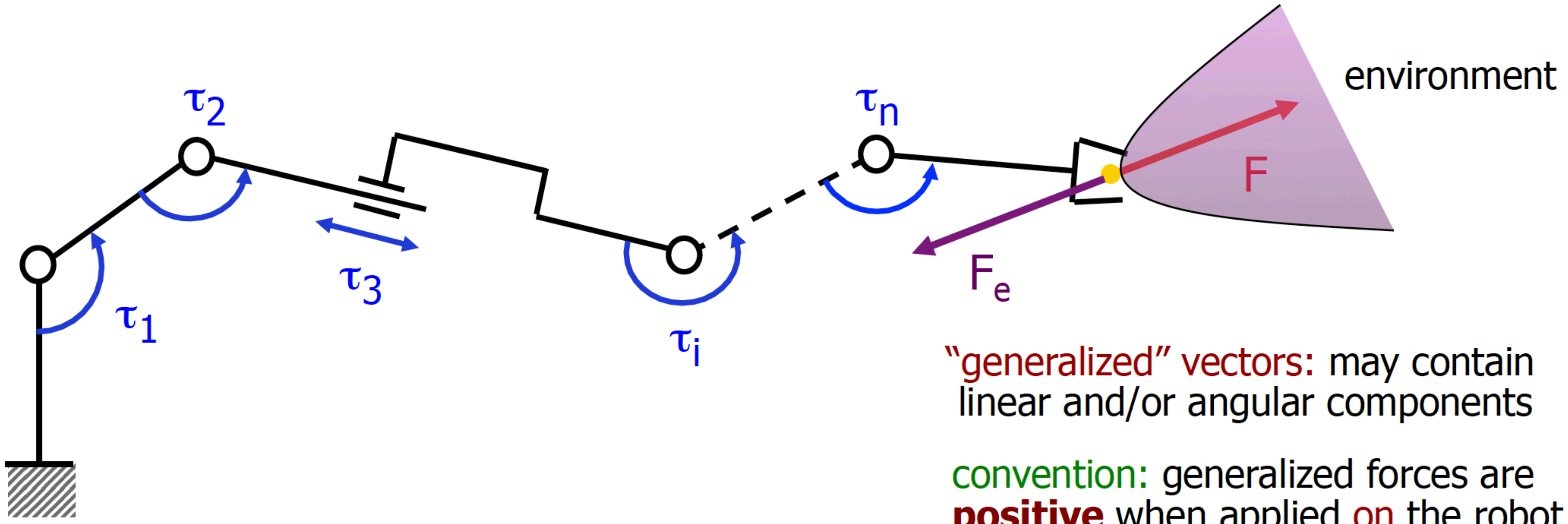
Reference



Principles of robot motion:

Theory, Algorithm, and Implementation

Generalized forces and torques



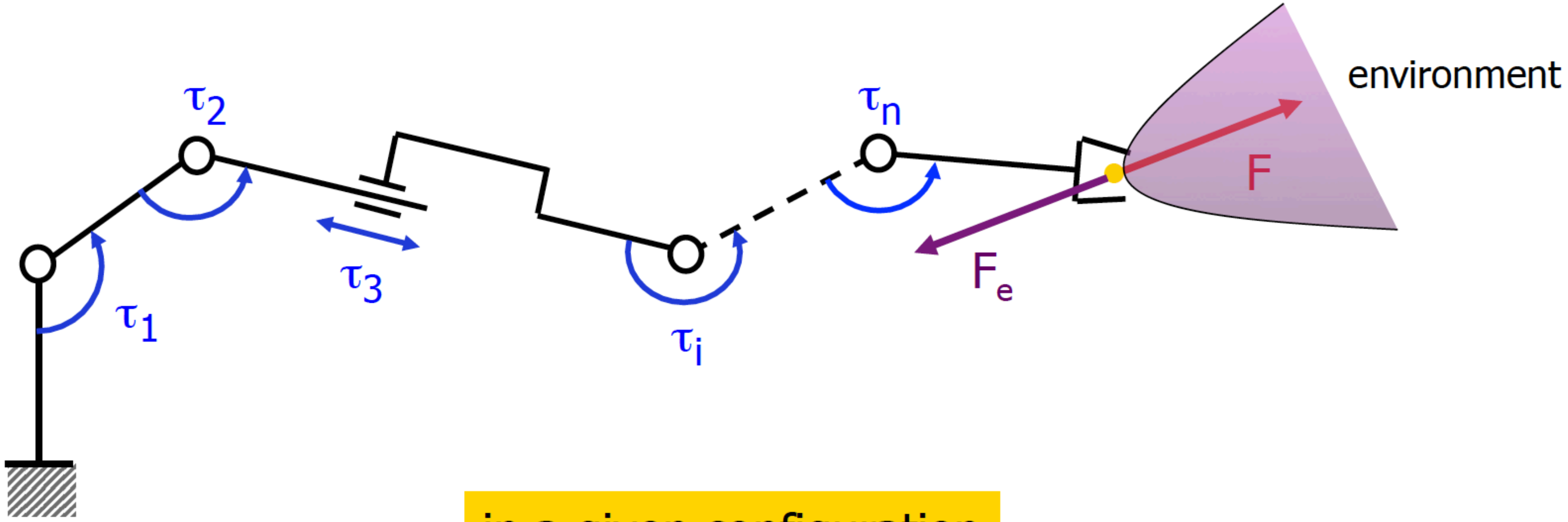
"generalized" vectors: may contain linear and/or angular components

convention: generalized forces are **positive** when applied on the robot

- τ = forces/torques exerted by the motors at the robot joints
- F = equivalent forces/torques exerted at the robot end-effector
- F_e = forces/torques exerted by the environment at the end-effector
- principle of action and reaction: $\boxed{F_e = -F}$

  reaction from environment is equal and opposite to the robot action on it

Transformation of forces – statics



in a given configuration

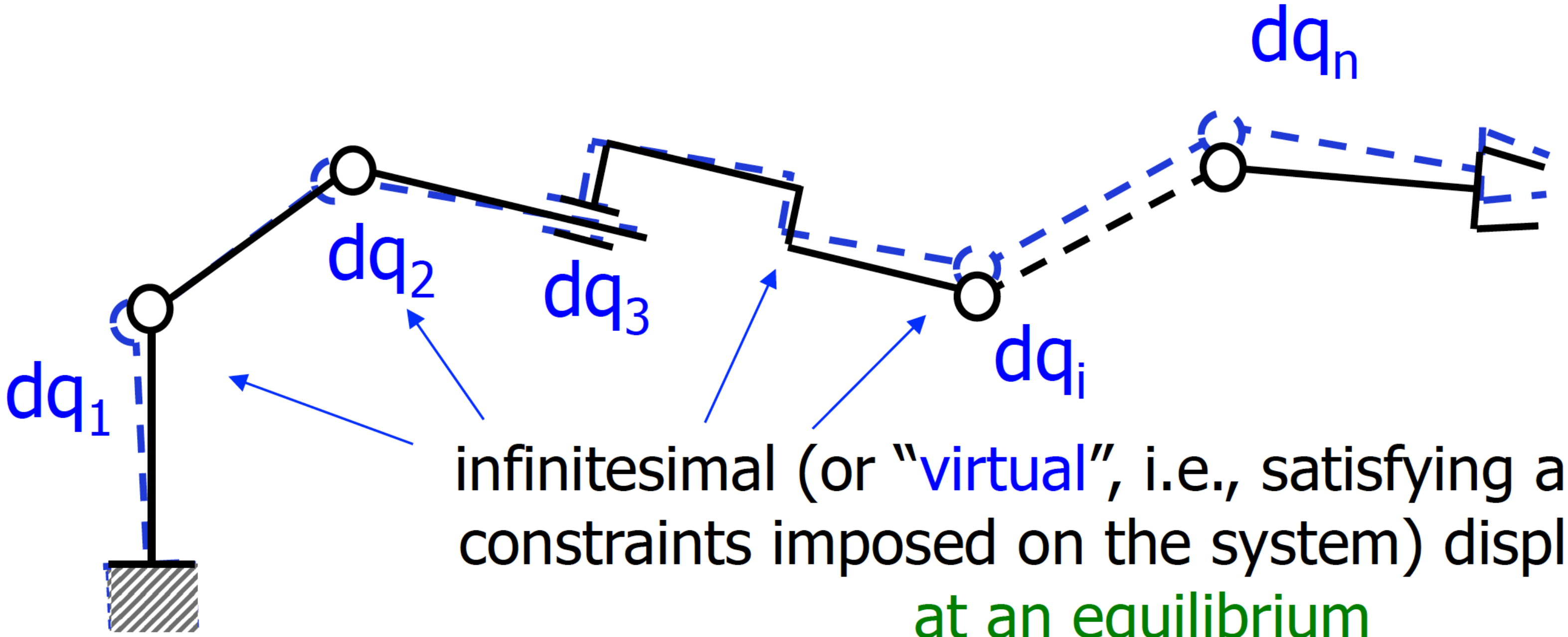
- what is the transformation between F at robot end-effector and τ at joints?

in **static equilibrium** conditions (i.e., **no motion**):

- what F will be exerted on environment by a τ applied at the robot joints?
- what τ at the joints will balance a F_e (= -F) exerted by the environment?

all equivalent formulations

Virtual displacement and work



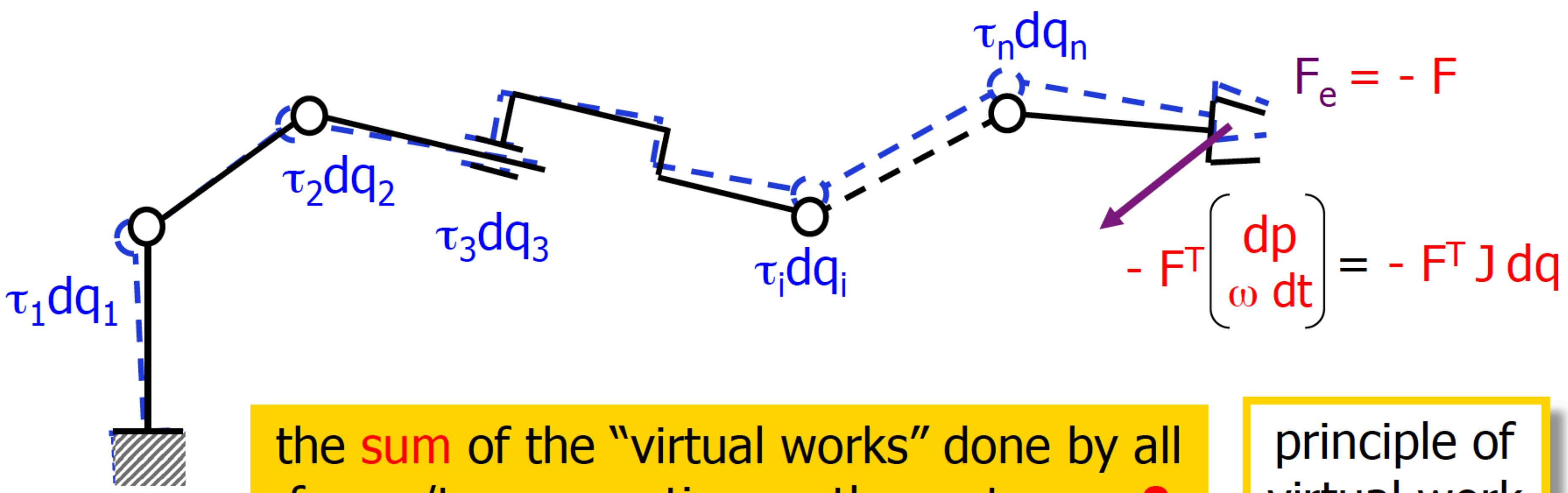
$$\begin{pmatrix} dp \\ \omega\, dt \end{pmatrix} = J\, dq$$

infinitesimal (or "virtual", i.e., satisfying all possible constraints imposed on the system) displacements at an equilibrium

- without kinetic energy variation (zero acceleration)
- without dissipative effects (zero velocity)

the "virtual work" is the work done by all forces/torques acting on the system for a given virtual displacement

Principle of virtual work



the sum of the "virtual works" done by all forces/torques acting on the system **= 0**

principle of virtual work

$$\tau^T dq - F^T \begin{bmatrix} dp \\ \omega dt \end{bmatrix} = \tau^T dq - F^T J dq = 0 \quad \boxed{\forall dq}$$

$$\Rightarrow \quad \boxed{\tau = J^T(q) F}$$

Velocity and force mappings

$J(q)$

velocity $\dot{q}$
(or displacement dq)
in the joint space

generalized velocity v
(or e-e displacement $\begin{bmatrix} dp \\ \omega dt \end{bmatrix}$)
in the Cartesian space

forces/torques τ
at the joints

generalized forces F
at the Cartesian e-e

$J^T(q)$

the singular configurations
for the velocity map are the same
as those for the force map

$\rho(J) = \rho(J^T)$

Force manipulability

- in a given configuration, evaluate how "effective" is the transformation between joint torques and end-effector forces
  - "how easily" can the end-effector apply generalized forces (or balance applied ones) in the various directions of the task space
  - in singular configurations, there are directions in the task space where external forces/torques are balanced by the robot without the need of any joint torque
- we consider all end-effector forces that can be applied (or balanced) by choosing joint torque vectors of unit norm

$$\tau^T \tau = 1 \quad \Rightarrow \quad \boxed{F^T J J^T F = 1}$$

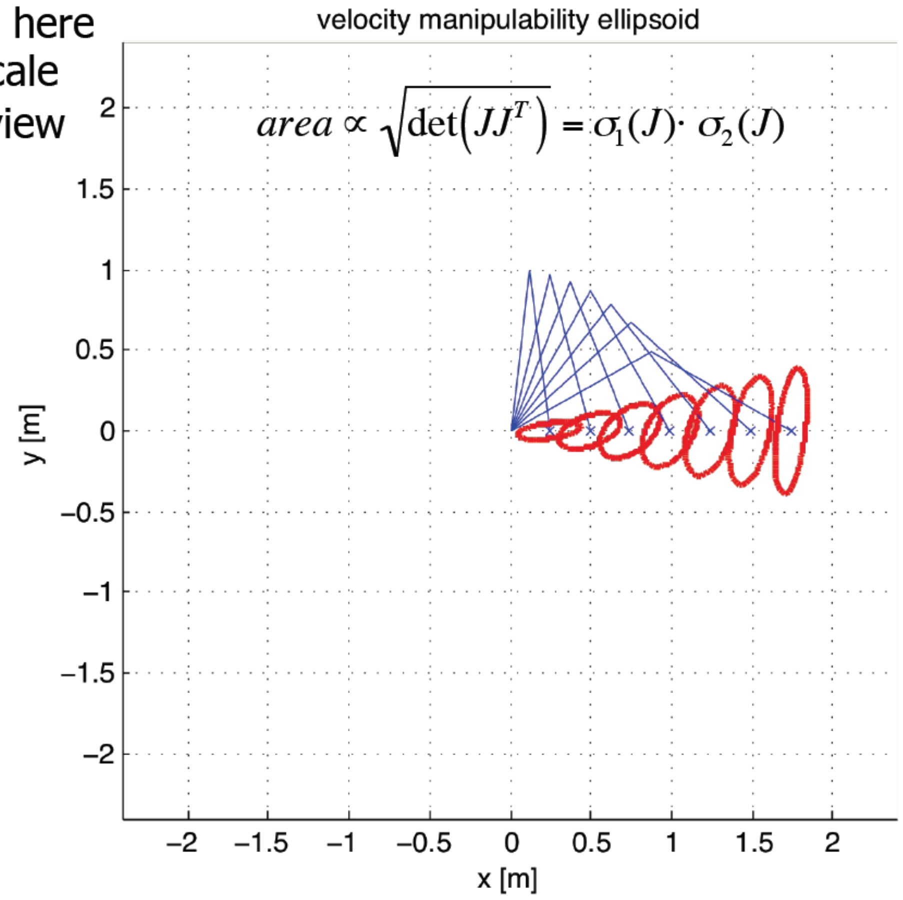
same directions of the principal axes of the velocity ellipsoid, but with semi-axes of inverse lengths

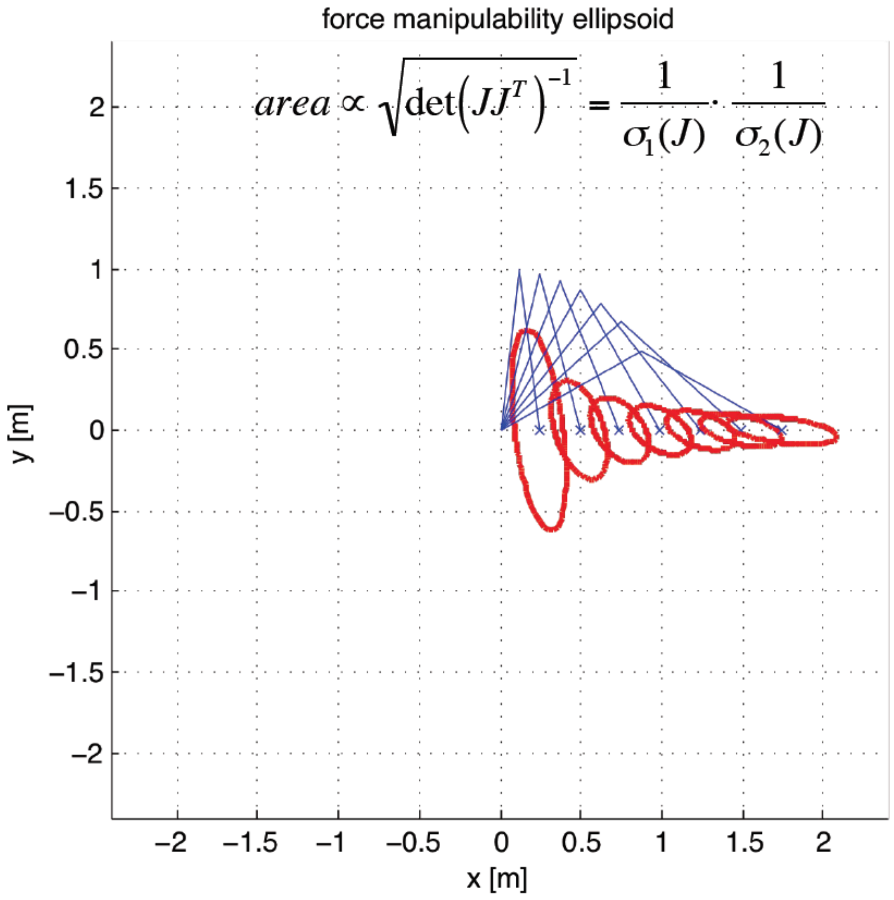
task force manipulability ellipsoid

Example

note:
velocity and force ellipsoids have here a different scale for a better view

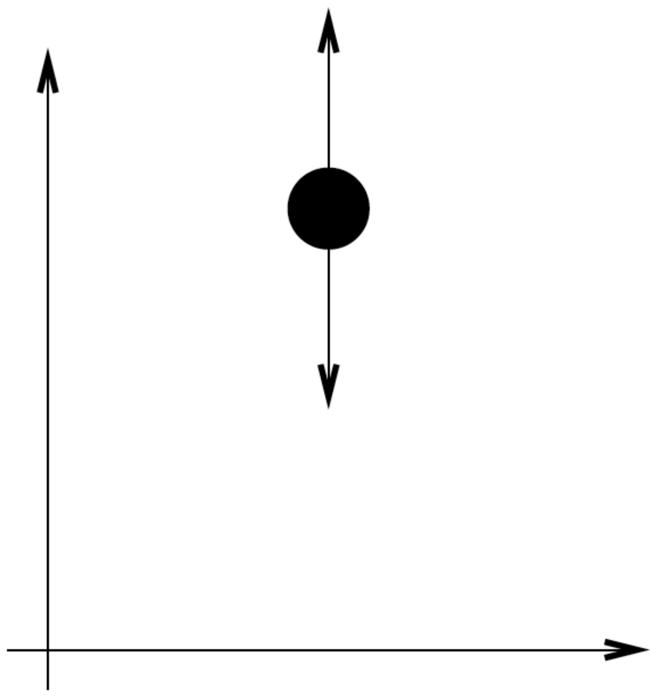
planar 2R arm with unitary links





Dynamics: Euler-Lagrange Equations

One DOF system



Single link robot arm