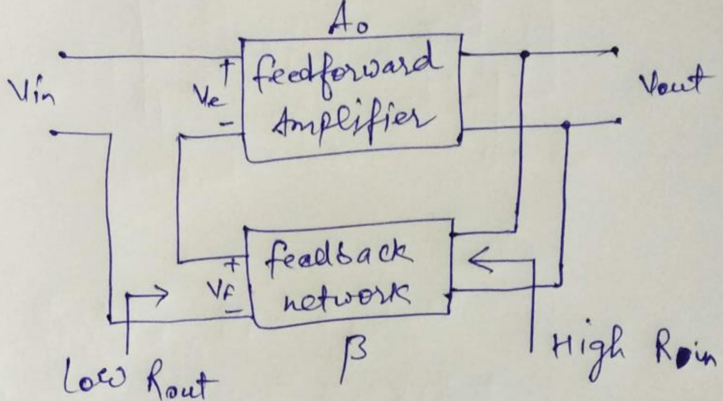
EE 204-2018-2 Analog Circuits
Homework #4 Solution

Question 1:

Ques 1. [Chapter – 8 → Feedback, B. Razavi, Analog CMOS Book]

Four types of feedback network :→

(1) Voltage – Voltage feedback :→



Here, feedback network is connected in parallel with the output and in series with the input port.

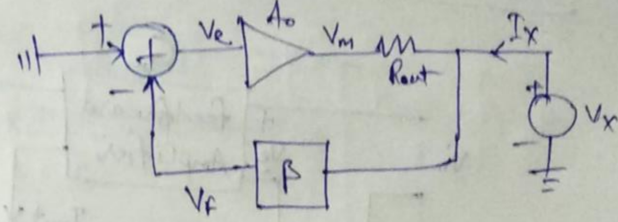
$\therefore \quad V_F = \beta V_{out}$

$V_e = V_{in} - V_F$

and $V_{out} = A_o (V_{in} - \beta V_{out})$

Gain $\boxed{\dfrac{V_{out}}{V_{in}} = \dfrac{A_o}{1 + \beta A_o}}$

Output Resistance



R_{out} = output impedance of feedforward amplifier.

$$V_f = \beta V_f$$

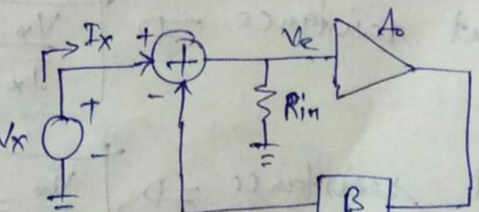
$$V_e = -\beta V_x$$

$$V_m = -\beta A_o V_x$$

Hence $$I_x = \frac{[V_x - (-\beta A_o V_x)]}{R_{out}}$$

$$\boxed{\frac{V_x}{I_x} = \frac{R_{out}}{1+\beta A_o}}$$

Input Resistance :→



$$V_e = I_x R_{in}$$

$$V_f = \beta A_o I_x R_{in},$$

we have

$$V_e = V_x - V_f$$

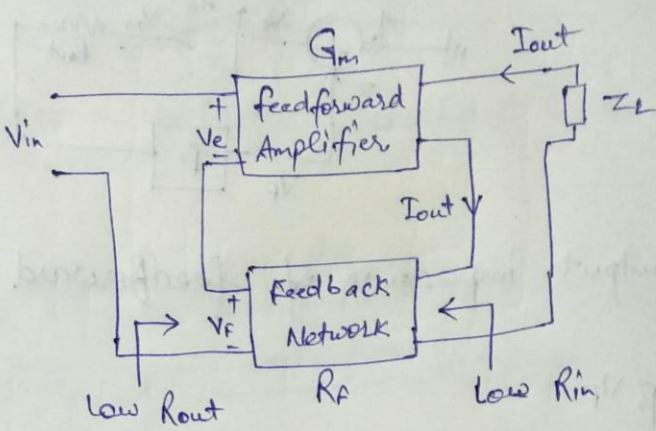
$$= V_x - \beta A_o I_x R_{in}$$

thus

$$I_x R_{in} = V_x - \beta A_o I_x R_{in}$$

$$\boxed{\frac{V_x}{I_x} = R_{in}(1+\beta A_o)}$$

② Current - Voltage Feedback :→



$$V_F = R_F I_{out}$$

$$V_e = V_{in} - R_F I_{out}$$

Hence $\quad I_{out} = G_m (V_{in} - R_F I_{out})$

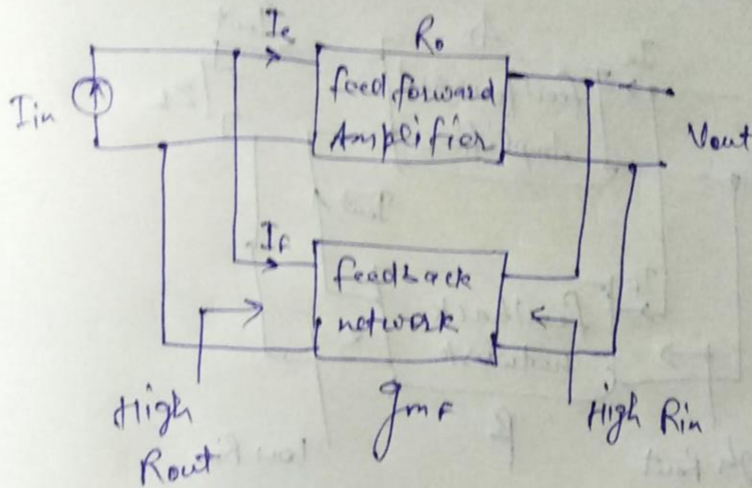
$$\boxed{\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}}$$

Similarly as part ①,

output resistance ⇒ $\boxed{\frac{V_x}{I_x} = R_{out}(1 + G_m R_F)}$

Input resistance ⇒ $\boxed{\frac{V_x}{I_x} = R_{in}(1 + G_m R_F)}$

(3) Voltage - Current feedback :→



$$I_f = g_{mf} V_{out}$$

$$I_e = I_{in} - I_f$$

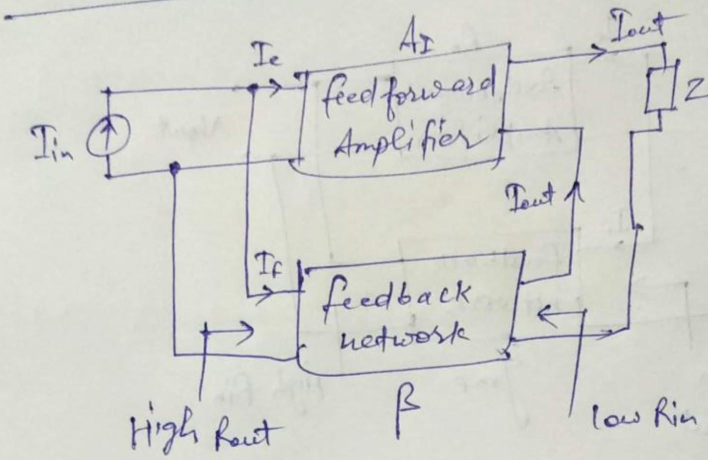
$$\rightarrow V_{out} = R_o I_e = R_o (I_{in} - g_{mf} V_{out})$$

$$\boxed{\frac{V_{out}}{I_{in}} = \frac{R_o}{1 + g_{mf} R_o}}$$

Output ~~inpu~~ resistance → $\boxed{\frac{V_x}{I_x} = \frac{R_{out}}{1 + g_{mf} R_o}}$

Input resistance → $\boxed{\frac{V_x}{I_x} = \frac{R_{in}}{1 + g_{mf} R_o}}$

(4) Current – current feedback :→



Gain ⇒ $\boxed{\dfrac{I_{out}}{I_{in}} = \dfrac{A_I}{1+\beta A_I}}$

Output resistance ⇒ $\boxed{R_{out}(1+\beta A_I)}$

Input resistance ⇒ $\boxed{\dfrac{R_{in}}{1+\beta A_I}}$

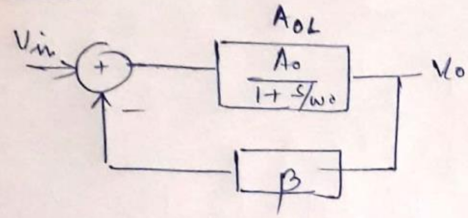
* **Note** :→ You can refer "Design of Analog CMOS Integrated Circuits".

→ Chapter – 8 ⇒ feedback

→ B. Razavi

Question 2:

Ques 2



$$A_{OL} = \frac{A_o}{1+\frac{s}{\omega_o}}$$

$$\frac{V_o}{V_{in}} = \frac{A}{1+A\beta} = \frac{\frac{A_o}{1+s/\omega_o}}{1+\frac{\beta A_o}{1+\frac{s}{\omega_o}}} = \frac{A_o}{1+\frac{s}{\omega_o}+\beta A_o}$$

$$A_{CL} = \frac{\left[A_o/1+A_o\beta\right]}{\left[1+\frac{s}{\omega_o(1+A\beta_o)}\right]}$$

the cut-off freq has now become: $\omega_o(1+A_o\beta)$

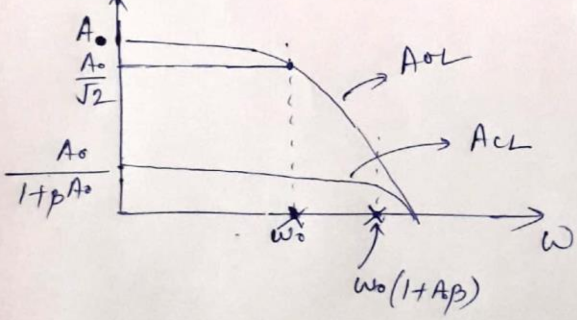
⇒ Gain - Bandwidth product for open loop system at 3 dB frequency.

$$\frac{A_o}{\sqrt{2}} \times \omega_o = \frac{A_o\omega_o}{\sqrt{2}}$$

Gain - Bandwidth product for closed loop system at 3 dB frequency.

$$\frac{A_o}{\cancel{(1+A_o\beta)}\sqrt{2}} \times \omega_o\cancel{(1+A\beta)} = \frac{A_o\omega_o}{\sqrt{2}}$$

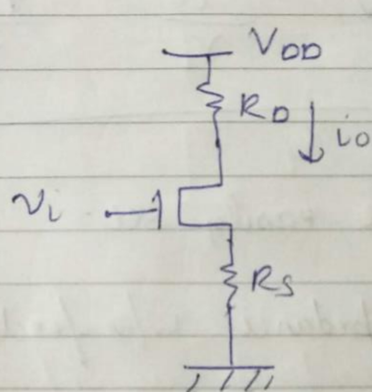
Hence proved. that gain bandwidth product is constant.



Question 4:

Question 4

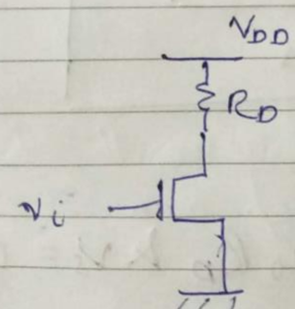
Part a>

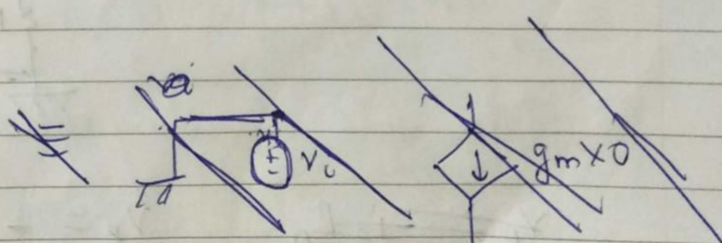


In the above circuit i_o creates a voltage difference across R_S which further changes v_{gs} across mosfet and acts as feedback. R_S is the feedback element.

The type is current - voltage feedback

For open-loop gain





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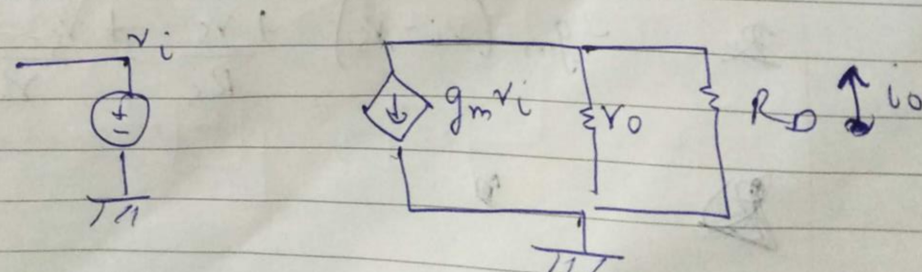


We get $\quad i_o = + g_m v_i \times \dfrac{r_0}{r_0 + R_D}$

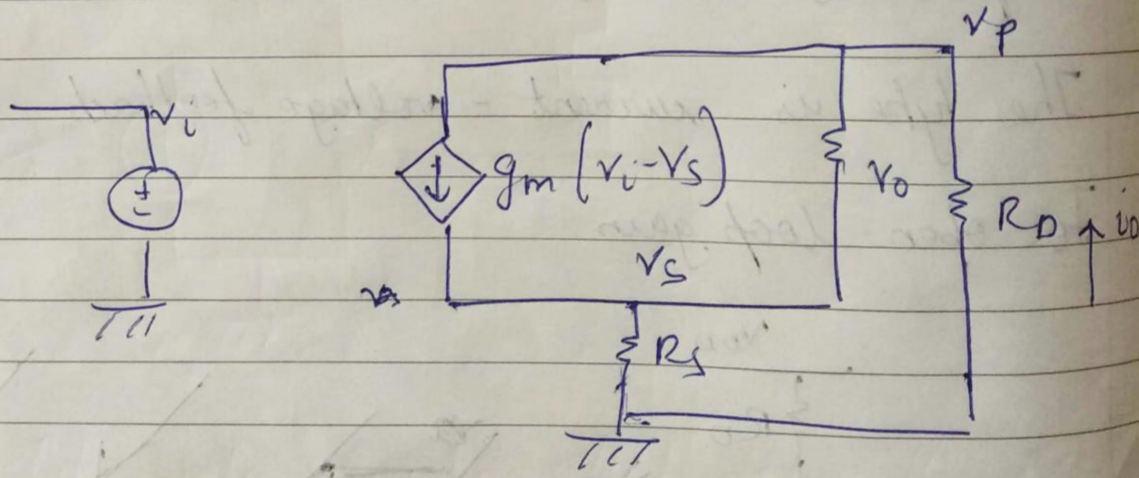
$$\Rightarrow \boxed{A_v = \frac{i_0}{v_i} = \frac{+ g_m r_0}{r_0 + R_D}}$$

From circuit we easily see -

Input impedance w/o feedback $= \infty$

Output impedance w/o feedback $= R_D \| r_0$

For closed loop gain we have



~~$v_p = -i_0 R_D$~~ $\quad v_p = - i_0 R_D \;\&\; v_s = i_0 R_S$

$$-g_m(v_i - v_s) + \frac{v_s}{R_S} + \frac{v_s - v_p}{r_0} = 0$$

$$\Rightarrow -g_m\left(v_i - i_o R_S\right) + \frac{i_o R_S}{R_S} + \frac{i_o R_S + i_o R_D}{r_o} = 0$$

$$\Rightarrow g_m v_i = i_o\left(g_m R_S + 1 + \frac{R_S + R_D}{r_o}\right)$$

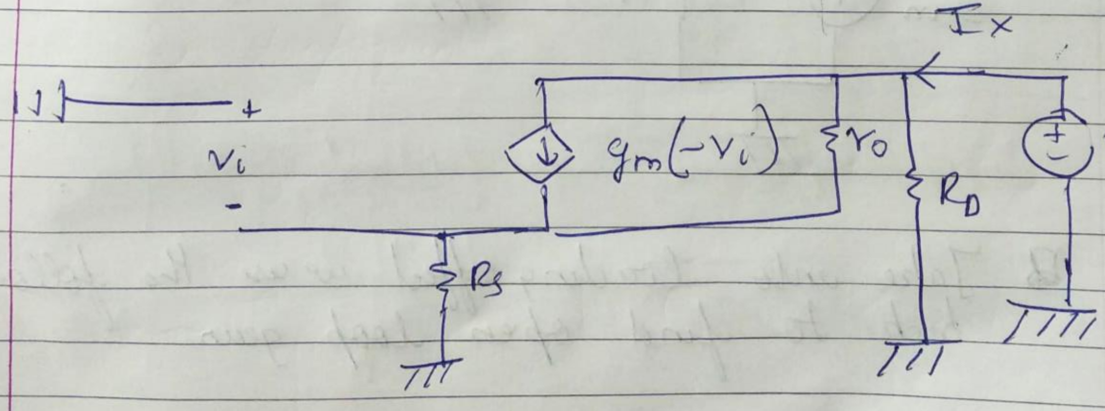
$$\Rightarrow \boxed{\frac{i_o}{v_i} = \frac{g_m r_o}{r_o + g_m R_S r_o + R_S + R_D}}$$

closed-loop gain

From circuit we easily see

Input impedance with feedback = ∞

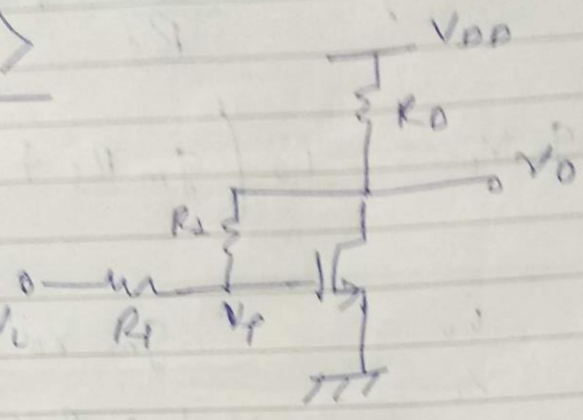
For output impedance:



Solving above circuit we get

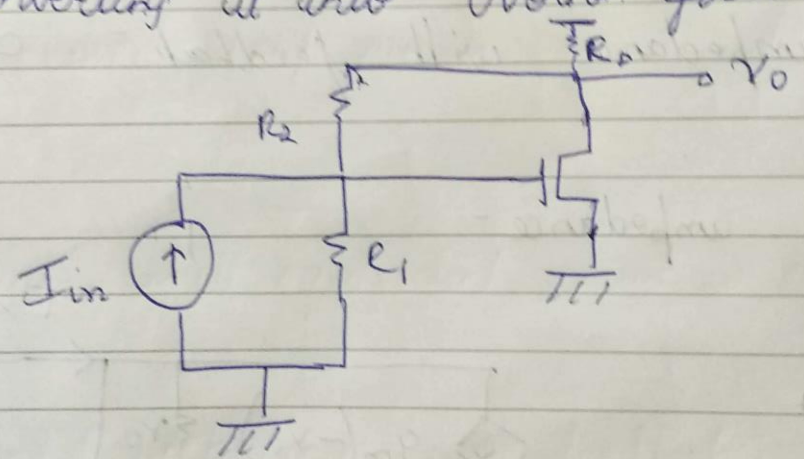
Output impedance with feedback = $(r_o + g_m r_o R_S) \| R_D$

Part b>

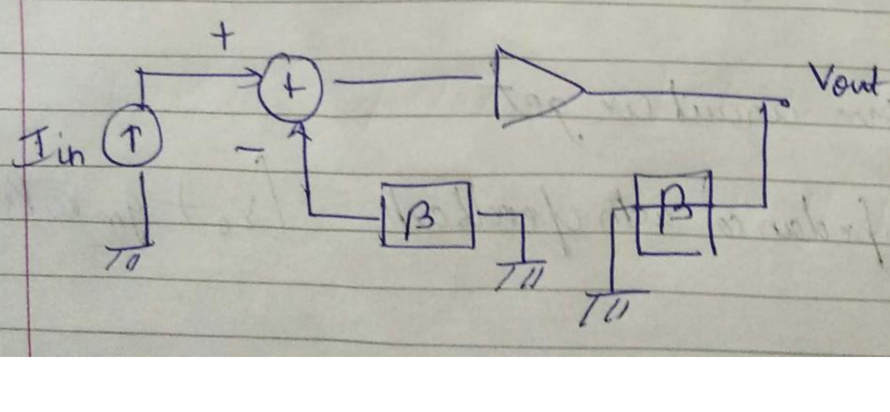


In the above case R_2 senses output voltage & gives appropriate current at node P.

The type is voltage-current feedback. Converting it into Norton form

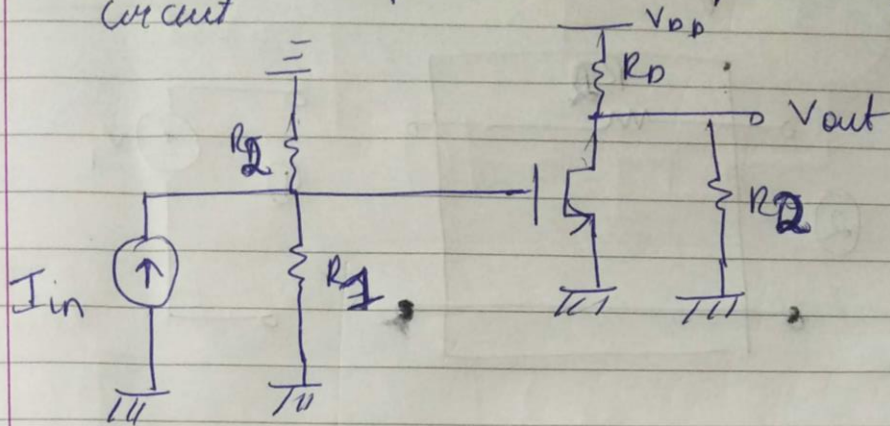


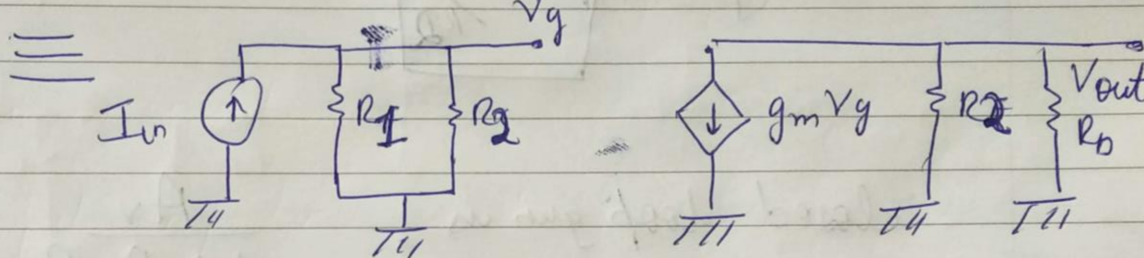
To Take into loading effect we use the following trick to find open loop gain



$$\beta = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

We break the feed back loop and get the following circuit





$$V_{out} = -g_m V_g \left(R_2 \| R_D\right)$$

& $$V_g = I_{in} \times \left(R_1 \| R_2\right)$$

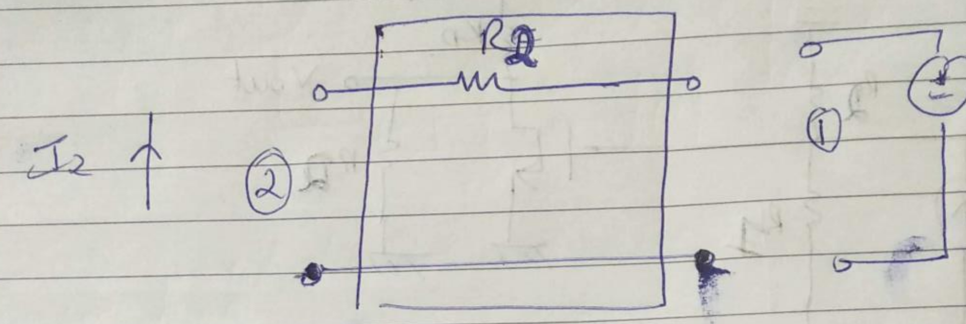
$$\therefore \boxed{\frac{V_{out}}{I_{in}} = -g_m \left(R_1 \| R_2\right)\left(R_2 \| R_D\right)}$$

(A_v) ↓ ~~etos~~ open loop gain

& $$\beta = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

where ① is looking into input of feedback and ② looking from output of feedback

We have the feedback circuit as



We get $\boxed{\beta = -1/R_2}$

∴ closed loop gain is $\dfrac{A_v}{1 + A_v \beta}$

Now from small equivalent circuit we can easily see

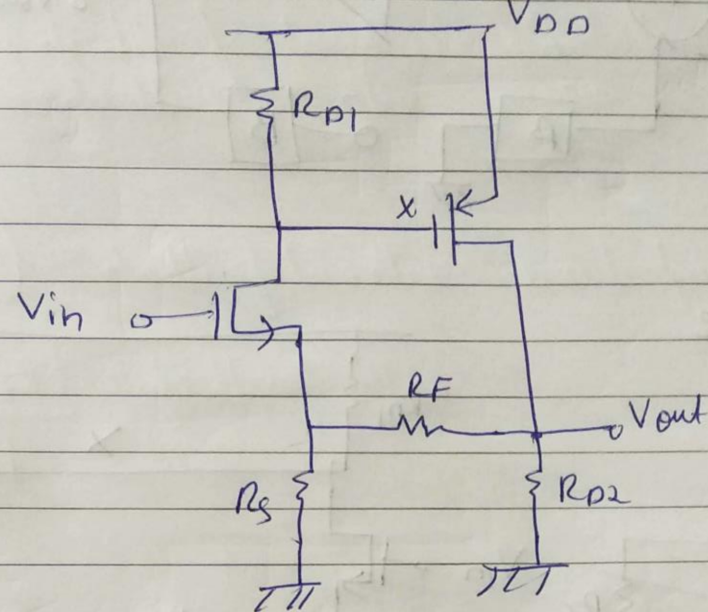
Input impedance w/o feedback $= R_1 \| R_2$

" " w feedback $= \dfrac{R_1 \| R_2}{1 + A_v \beta}$

Output impedance w/o feedback $= R_2 \| R_D$

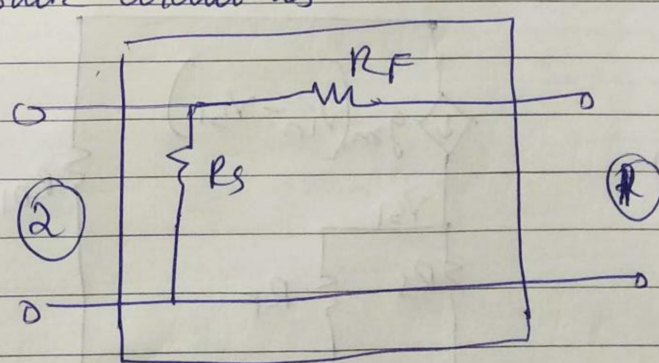
" " w feedback $= \dfrac{R_2 \| R_D}{1 + A_v \beta}$

Part c)



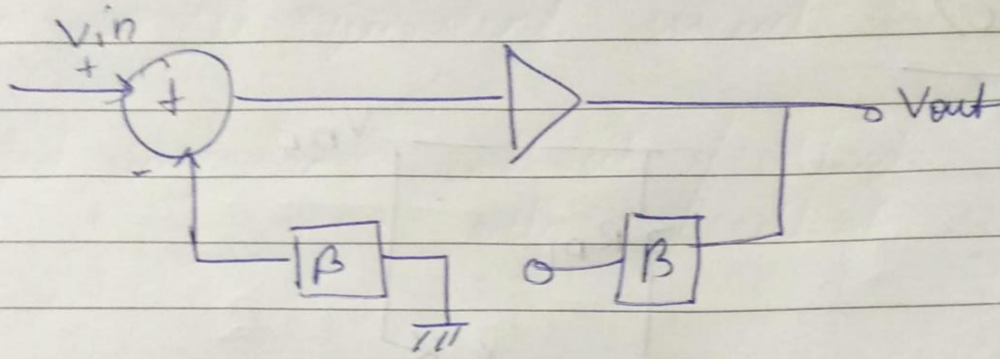
In the above circuit R_F and R_s senses output voltage and returns a fraction to it to the first mosfet as V_{gs}.

The feedback circuit is

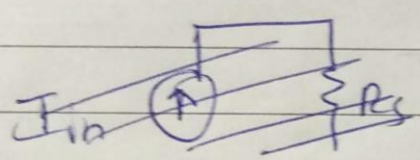


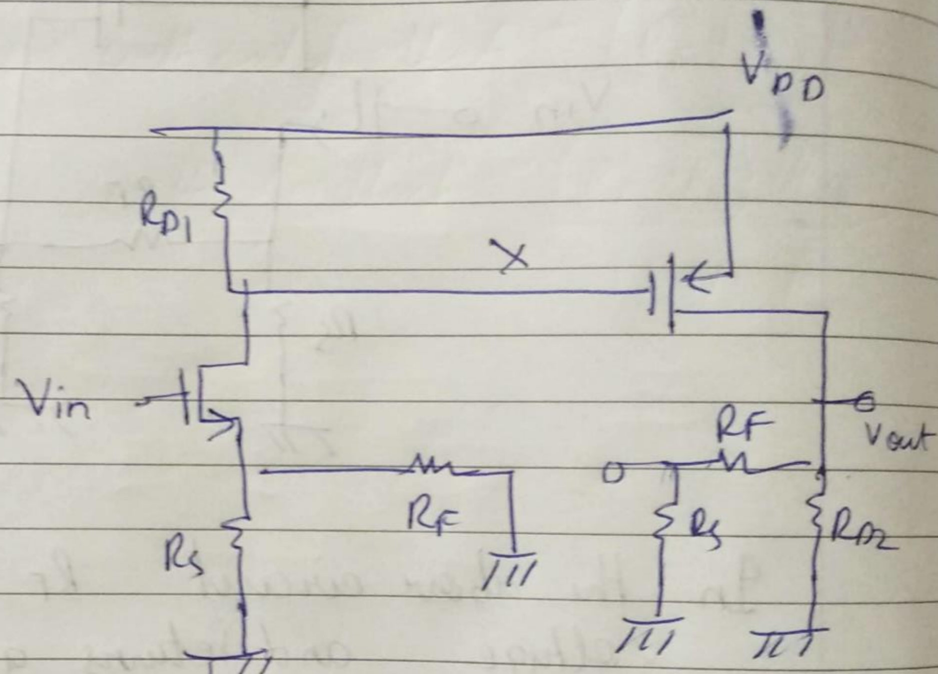
The type is voltage-voltage feedback

We again use another trick for open-loop gain to take care of loading effect

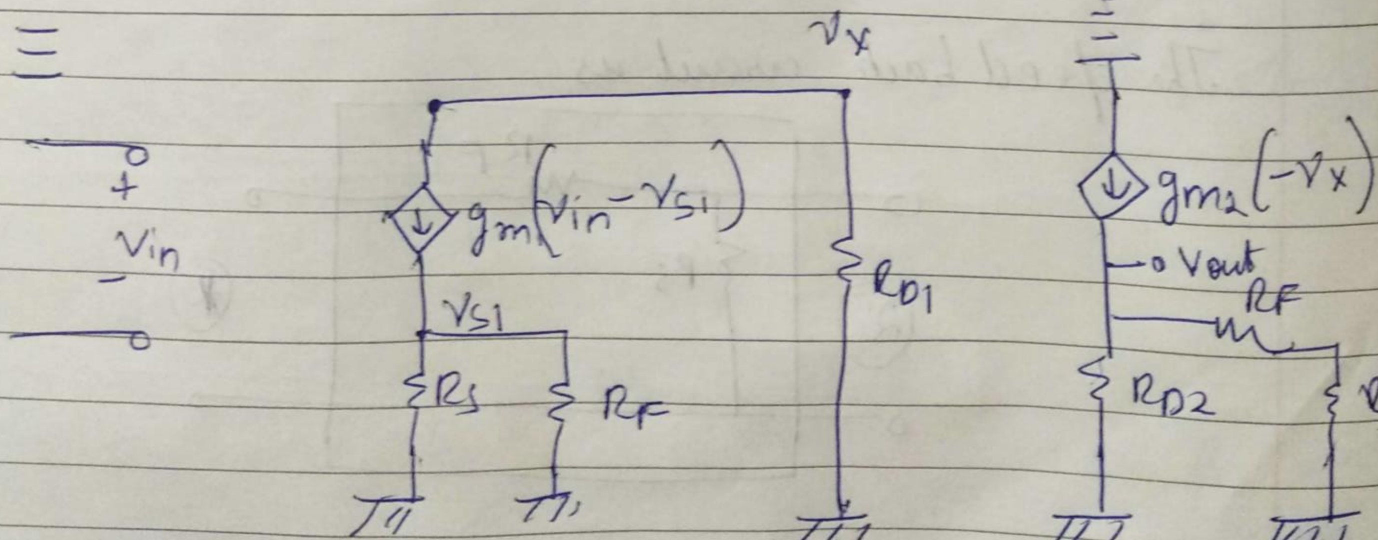

Vin
+
-
β
β
Vout

we have


Iin
Rs


VDD
RD1
X
Vin
RF
Rs
RF
Vout
Rs
RD2

≡


Vin
+
-
vx
gm(vin - vS1)
RD1
vS1
Rs
RF
gm2(-vx)
Vout
RF
RD2
Rs

$$v_{S1} = g_{m1}\left(v_{in} - v_{S1}\right)\left(R_S \| R_F\right)$$

$$\Rightarrow v_{S1}\left(1 + g_{m1}\left(R_S \| R_F\right)\right) = g_{m1} v_{in}\left(R_S \| R_F\right)$$

$$\Rightarrow v_{S1} = \frac{g_{m1} v_{in}\left(R_S \| R_F\right)}{1 + g_{m1}\left(R_S \| R_F\right)}$$

And.

$$v_X = -g_{m1}\left(v_{in} - v_{S1}\right) R_{D1}$$

$$= -\frac{g_{m1} v_{in} R_{D1}}{1 + g_{m1}\left(R_S \| R_F\right)}.$$

And.

$$V_{out} = g_{m2}\left(-v_X\right)\left(R_{D2} \| \left(R_S + R_F\right)\right)$$

$$\boxed{\frac{V_{out}}{v_{in}} = -\frac{g_{m2} g_{m1} R_{D1}\left(R_{D2} \| \left(R_S + R_F\right)\right)}{1 + g_{m1}\left(R_S \| R_F\right)}}.$$

↓ open loop gain (A_v)

Again using $\beta = \left.\frac{V_2}{V_1}\right|_{I_2 = 0}$ $\Rightarrow \beta = \frac{R_S}{R_F + R_S}$.

$\therefore$ closed loop gain $= \frac{A_v}{1 + A_v \beta}$

From small equivalent circuit

Input impedance without feedback $= \infty$

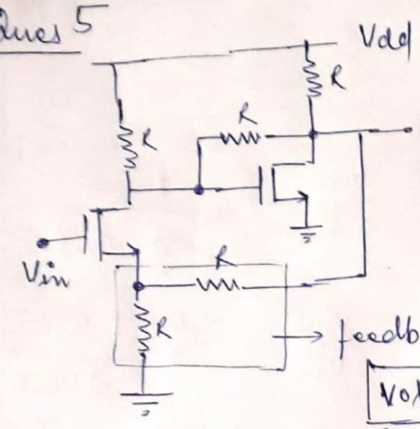
" " with feedback $= \infty$

Output impedance without feedback $= (R_F + R_S) \| R_{D2} \| r_0$

" " with feedback $= \dfrac{(R_F + R_S) \| R_{D2} \| r_0}{1 + A_v \beta}$

Question 5:

Ques 5



→ feedback network

Given: $\lambda = \gamma = 0$

$R = 2K\Omega$

$g_m = \frac{1}{200}$

$g_m R = 10$

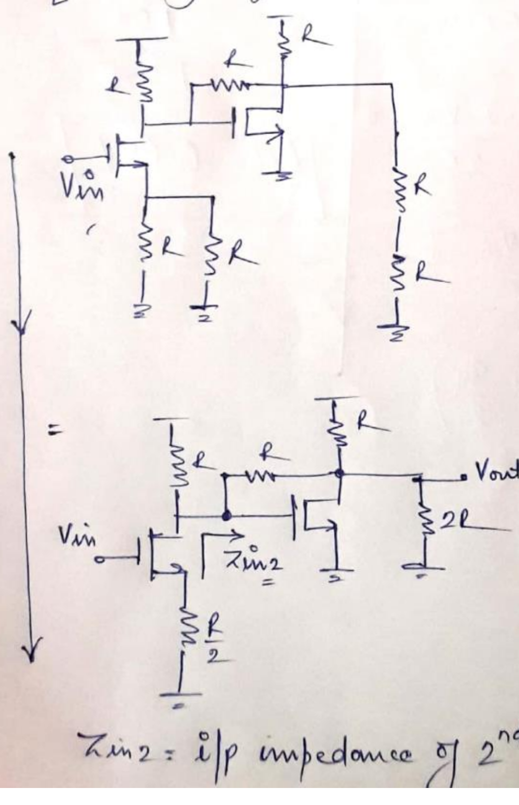
$\boxed{\text{voltage tapping \& voltage mixing feedback}}$

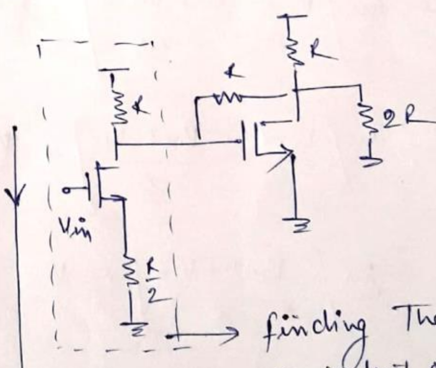
To solve for gain (closed loop), find out the open-loop gain first i.e. A_{OL} and then find out the β value.

$$\boxed{A_{CL} = \frac{A_{OL}}{1+\beta A_{OL}}} \quad , \quad \beta = \frac{R}{R+R} = \frac{1}{2} = .5$$

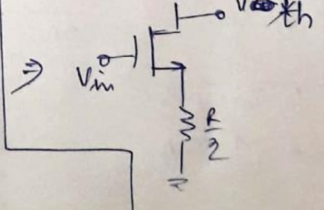
In order to find A_{OL}, we need to break the ckt in such a way so that there should be no loading effect and A_{OL} can be calculated easily.

[kindly Refer to Razavi Book]

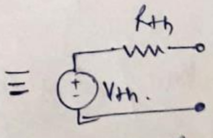




→ finding Thevenin equivalent of this 1st stage.



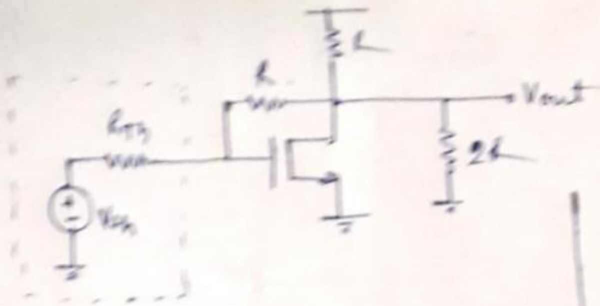
$\equiv$



$$\frac{V_{th}}{V_{in}} = \frac{g_m R}{1+g_m \frac{R}{2}} = \frac{10}{1+5} = \frac{5}{3}$$

Z_{in2} = i/p impedance of 2^{nd} stage

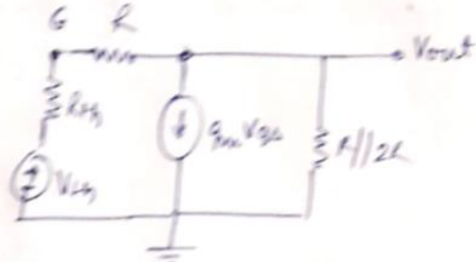
$R_{th} = R.$



$V_{Th} = \frac{5}{3} V_{in}$

$R_{Th} = R = 2k\Omega$

applying small signal analysis



$$\frac{V_{out}}{R\|2R} + g_m V_{gs} + \frac{V_{out} - V_{Th}}{R + R_{Th}} = 0$$

$$\frac{3V_{out}}{2R} + g_m \left[V_{out} - \frac{(V_{out} - V_{Th})R}{R + R_{Th}} \right] + \frac{V_{out} - V_{Th}}{R + R_{Th}} = 0$$

$$\frac{3V_{out}}{2R} + g_m \left[V_{out} - \frac{V_{out} - V_{Th}}{2} \right] + \frac{V_{out} - V_{Th}}{2R} = 0$$

$$\frac{3V_{out}}{2R} + g_m \left[\frac{V_{out} + V_{Th}}{2} \right] + \frac{V_{out} - V_{Th}}{2R} = 0$$

$$V_{out} \left[\frac{3}{2R} + \frac{g_m}{2} + \frac{1}{2R} \right] = V_{Th} \left[\frac{1}{2R} - \frac{g_m}{2} \right]$$

$$V_{out} \left[\frac{2}{R} + \frac{g_m}{2} \right] = V_{Th} \left[\frac{1 - g_m R}{2R} \right]$$

$$\frac{V_{out}}{V_{Th}} = \frac{(1 - g_m R)\cancel{2R}}{\cancel{2R}(4 + g_m R)}$$

$$\Rightarrow \frac{V_{out}}{\frac{5}{3} V_{in}} = \frac{1 - g_m R}{4 + g_m R}$$

$$A_{OL} = \frac{V_{out}}{V_{in}} = \frac{5}{3} \frac{(1 - g_m R)}{4 + g_m R}$$

$$= \frac{5}{3} \times \frac{(1 - 10)}{4 + 10}$$

$$= \frac{5}{3} \times \frac{(-9)}{14} = -1.07$$

$$|A_{CL}| = \frac{A_{OL}}{1 + \beta A_{OL}}$$

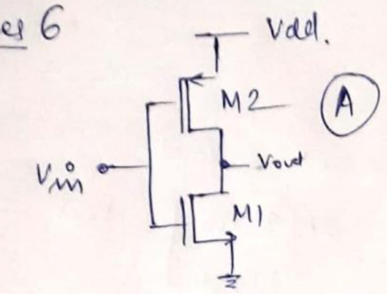
$$= \frac{1.07}{1 + .5(1.07)}$$

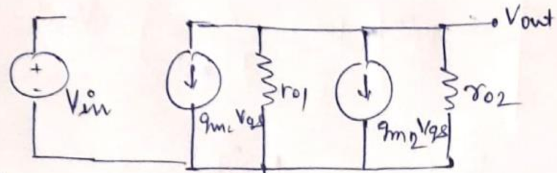
$$= \frac{1.07}{1.535}$$

$$\boxed{A_{CL}\ 0.697}$$

Question 6:

Ques 6





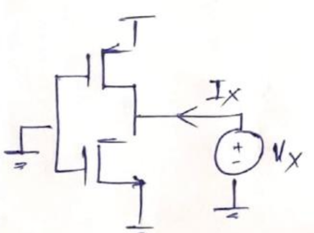
(a) v_{gs} is same for both the transistor M1 & M2. ($V_{gs} = V_{in}$)

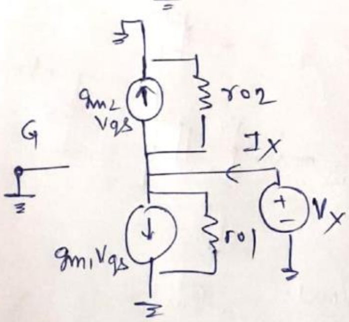
$$V_{out} = -(g_{m1} + g_{m2})\, v_{gs} \cdot (r_{o1} \| r_{o2})$$

$$= -(g_{m1} + g_{m2})\, V_{in}\, (r_{o1} \| r_{o2})$$

$$\boxed{A_v = \frac{V_{out}}{V_{in}} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})}$$

output impedance.



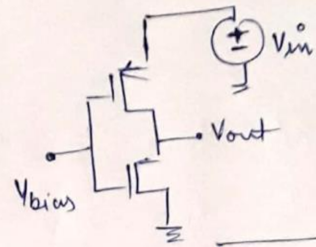


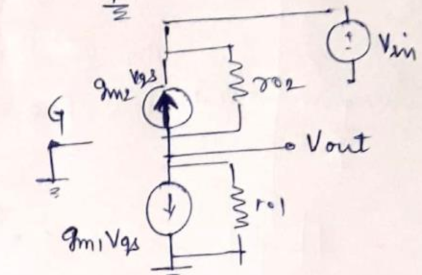
$g_{m1}V_{gs} = g_{m2}V_{gs} = 0$

as $V_{gs} = 0$

$$\boxed{\frac{V_x}{I_x} = r_{o2} \| r_{o1}}$$

(b) sensitivity





$g_{m1}V_{gs} = 0$ as $V_{gs} = 0$

$g_{m2}V_{gs} = g_{m2}[0 - V_{in}]$

$= -g_{m2}V_{in}$

Applying KCL @ node Vout

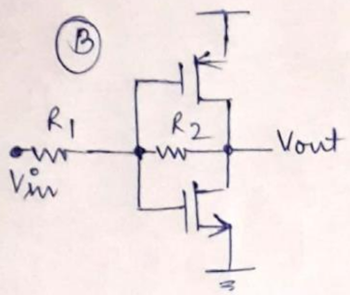
$$\frac{V_{out}}{r_{o1}} + g_{m2}V_{gs} + \frac{V_{out} - V_{in}}{r_{o2}} = 0$$

$$\Rightarrow V_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) = \frac{V_{in}}{r_{o2}} + g_{m2}V_{in}$$

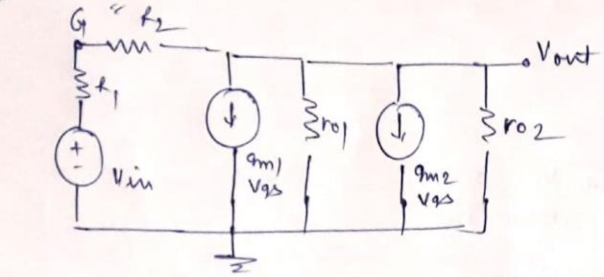
$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{(1 + g_{m2}r_{o2})(r_{o2} \| r_{o1})}{r_{o2}}$$

$$\boxed{\text{sensitivity} \simeq g_{m2}(r_{o2} \| r_{o1})}$$

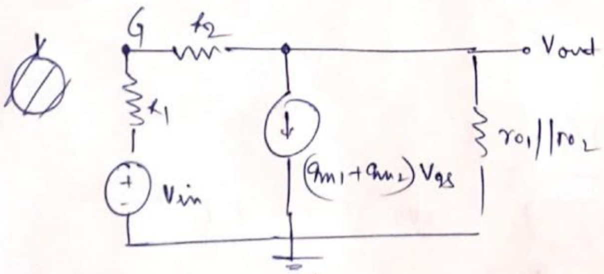
(B)



(a) Gain



since vgs is same for both NMOS & PMOS.



$$\frac{V_{out}}{(r_{o1}||r_{o2})} + (g_{m1}+g_{m2})V_{gs} + \frac{V_{out}-V_{in}}{R_1+R_2} = 0 \quad \text{— (1)}$$

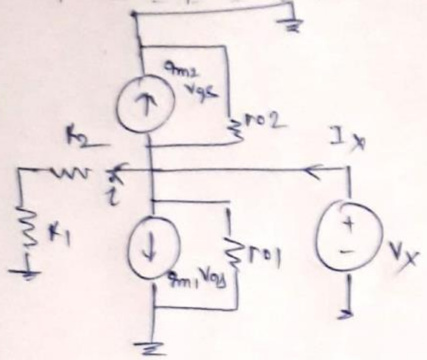
$$V_g = V_{out} - \frac{(V_{out}-V_{in})R_2}{R_1+R_2}$$

$$V_g = \frac{V_{out}R_1 + V_{in}R_2}{R_1+R_2} \quad \text{— (2)}$$

solving (1) & (2)

$$\frac{V_{out}}{V_{in}} = \frac{1 - (g_{m1}+g_{m2})R_2}{(R_1+R_2)\left[\frac{1}{r_{o1}||r_{o2}} + \frac{(g_{m1}+g_{m2})R_1}{R_1+R_2} + \frac{1}{R_1+R_2}\right]}$$

Output impedance



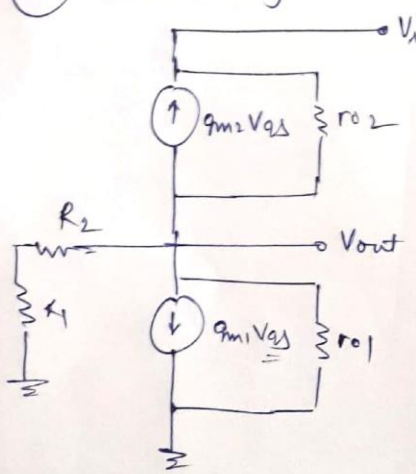
$$I_x = i + g_{m1}V_{gs1} + g_{m2}V_{gs2} + \frac{V_x}{r_{o1}} + \frac{V_x}{r_{o2}}$$

$$= \frac{V_x}{R_1+R_2} + g_{m1}\left[\frac{V_x R_1}{R_1+R_2}\right] + g_{m2}\left(\frac{V_x R_1}{R_1+R_2}\right) + \frac{V_x}{r_{o1}} + \frac{V_x}{r_{o2}}$$

$$\frac{I_x}{V_x} = \frac{1+(g_{m1}+g_{m2})R_1}{R_1+R_2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}$$

$$\frac{V_x}{I_x} = r_{o1} \,||\, r_{o2} \,||\, \frac{(R_1+R_2)}{[1+(g_{m1}+g_{m2})R_1]}$$

(b) Sensitivity



from eq1 (1) (2) & (3)

$$\frac{V_{out}}{V_{in}} = \frac{(g_{m2}r_{o2}+1)}{r_{o2}}\left(r_{o1}||r_{o2} \,||\, \frac{R_2+R_1}{1+(g_{m1}+g_{m2})R_1}\right)$$

★ Ckt 2 is less sensitive

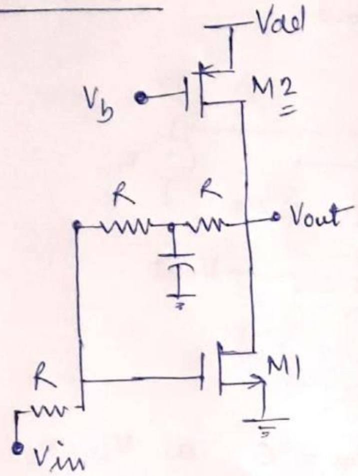
$$\frac{V_{out}-V_{in}}{r_{o2}} + \frac{V_{out}}{r_{o1}} + g_{m1}V_{gs1} + g_{m2}V_{gs2} + \frac{V_{out}}{R_1+R_2} = 0 \quad -(1)$$

$$V_{gs1} = \frac{V_{out}R_1}{R_1+R_2} \quad -(2)$$

$$V_{gs2} = \frac{V_{out}R_1}{R_1+R_2} - V_{in} \quad -(3)$$

Question 7:

Ques 7

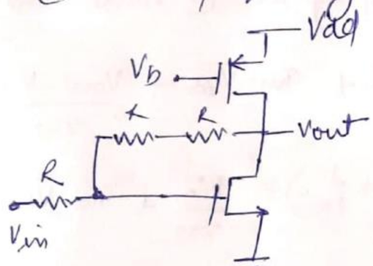


$g_{m1,2} : \frac{1}{200} = g_m$

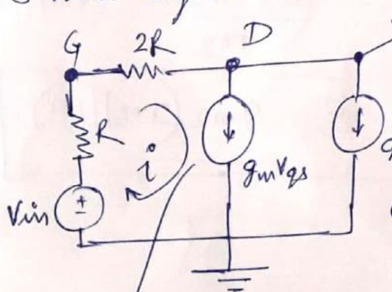
$R_{1-3} : 2k\Omega = R$.

A_{CL} @ high & low frequencies.

(1) Gain @ very low frequency.
The Capacitor will become open @ low frequency.



⇒ Small signal ckt



for PMOS

$g_m v_{gs} = 0$

as $[V_g$ and $V_s = 0]$

for NMOS

$i = g_m v_{gs}$

$i = g_m[V_{in} - iR]$

$$i = \frac{g_m V_{in}}{1 + g_m R}$$

$V_{in} - 3iR = V_{out}$

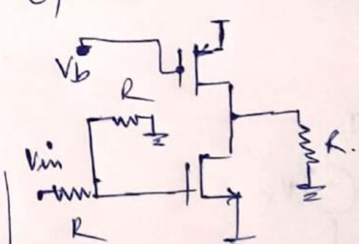
$$V_{in} - 3R\left[\frac{g_m V_{in}}{1+g_m R}\right] = V_{out}$$

$$V_{in}\left[1 - \frac{3Rg_m}{1+g_m R}\right] = V_{out}$$

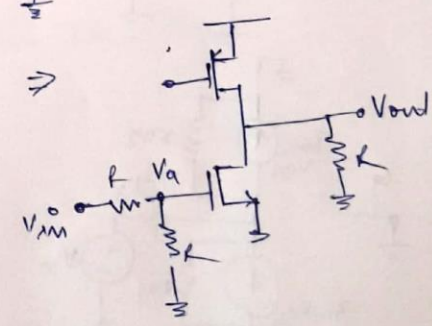
$$\frac{V_{out}}{V_{in}} = \frac{1 - 2g_m R}{1 + g_m R}$$

$$= \frac{1 - 2\times\frac{1}{200}\times 2000}{1+\frac{1}{200}\times 2000} = \frac{-19}{11}$$

(2) Gain @ very high frequency
Capacitor will become short ckt



⇒



$$\frac{V_{out}}{V_a} = -g_m R$$

$$\frac{V_{out}}{\frac{V_{in}}{2}} = -g_m R$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_m R}{2} = -5$$