EE 204-2018-2 Analog Circuits
Homework #1 Solution

Question 1 (a)

The following is the curve for transconductance as a function of gate voltage when $\mathbf{V_{DS} = 5V}$ for NMOS with **W = 10um** and **L = 0.18um**

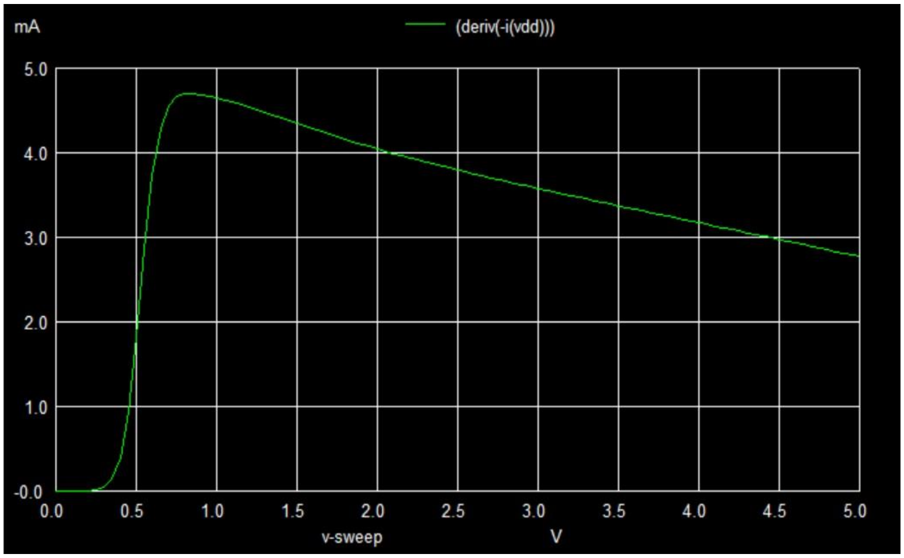


Figure 1: Transconductance as a function of gate voltage applied

The maximum value of transconductance (**gm**) is **4.695mS** at $\mathbf{V_{GS} = 0.83V}$.

SPICE code:

```
∗Transconductance of a n−MOSFET
.include tsmc.txt
M1 2 1 0 0 CMOSN W=10u L=180u
VDD 2 0 DC 5v
VGG 1 0 DC 3.84V
∗ defining the run−time control functions
dc VGG 0 5 0.05
.control
run
plot(deriv(−I (vdd )))
.endc
.end
```

Question 1 (b)

The corresponding $\mathbf{V_{GS}}$ value is **0.83V**.

Question 1 (c)

We get the following curve of output resistance at V_{GS} = 0.83V.

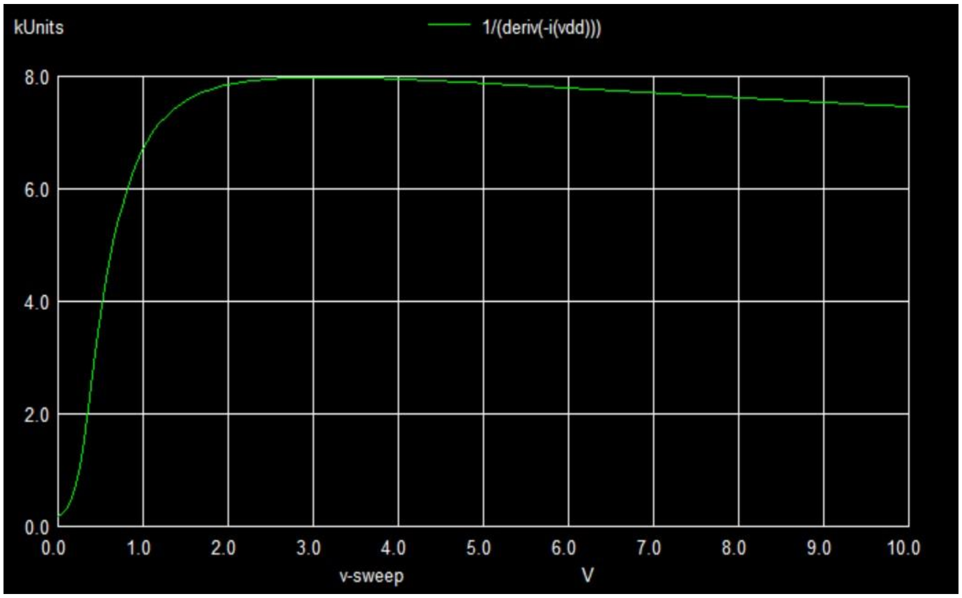


Figure 2: Output resistance as a function of drain voltage applied

The value of **output resistance** at $\mathbf{V_{DS=5V}}$ is **7.878kΩ**.

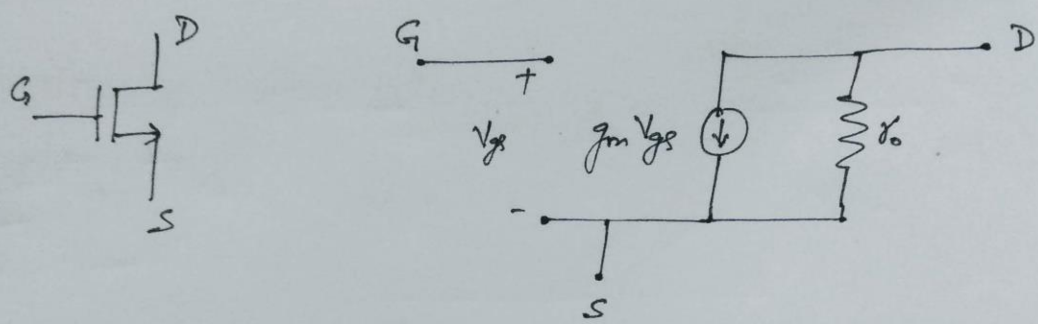
SPICE code:

```
∗Output conductance of a n−MOSFET
.include tsmc.txt
M1 2 1 0 0 CMOSN W=10u L=0.18u
VDD 2 0 DC 5V
VGG 1 0 DC 0.83V
∗ defining the run−time control functions
dc VDD 0 10 0.05
.control
run
∗ plotting input and output voltages
plot 1/(deriv(−I (vdd )))
.endc
.end
```

Question 1 (d)

Ques. 1 (d)

Small Signal equivalent of the MOSFET :→



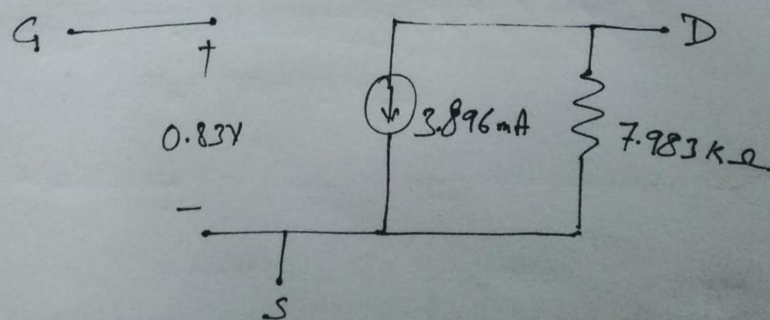
$V_{DS} = 3V$

$V_{gs} = 0.83\ V$ [Q1 (b)]

$g_m = 4.695\ mS$ [Q1 (a)]

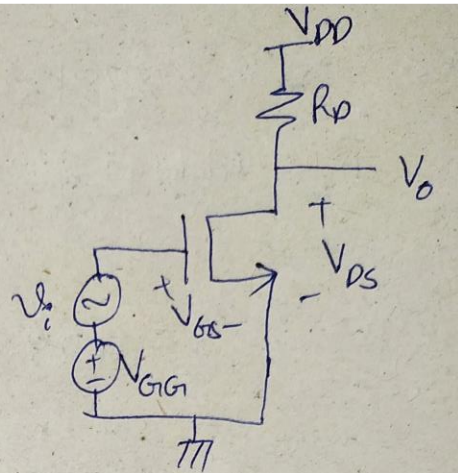
$r_o = 7.983\ k\Omega$ [Q1 (c)]

$V_{TH} = 0.3694\ V$ [TSMC model]



Question 1 (e)

.e.



For DC biasing

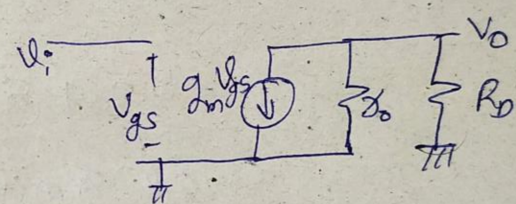
$V_{GG} = V_{GS}$

KVL in output loop

$V_{DD} = V_{DS} + I_D R_D$

Small signal analysis

$v_{gs} = v_i$



$v_o = -g_m v_{gs} [r_o // R_D]$

$A_{v} = \frac{v_o}{v_i} = -g_m [r_o // R_D]$

given $A_v = 2$

$$g_m \frac{r_o R_D}{r_o + R_D} = 2$$

$$R_D [g_m r_o - 2] = 2 r_o$$

$$R_D = \frac{2 r_o}{g_m r_o - 2}$$

we have $g_m = k'(V_{GS} - V_T)$

$$k' = \frac{g_m}{(V_{GS} - V_T)}$$

and $I_D = \frac{k'}{2} (V_{GS} - V_T)^2$

From simulations we have

$g_m = 4.695\,mS$ $\quad V_{DS} = 5V \quad V_T = 0.3694V$

$r_o = 7.878\,k\Omega$ $\quad V_{GS} = 0.83\,V$

so, $R_D = \dfrac{2r_o}{g_m r_o + 2} = \dfrac{2(7.878)k}{(4.695m)(7.878k) - 2}$

$R_D = 450.33\,\Omega$

$k' = \dfrac{g_m}{(V_{GS} - V_T)} = \dfrac{4.695m}{(0.83 - 0.3694)} = 10.19\,m$

$I_D = \dfrac{k'}{2}(V_{GS} - V_T)^2 = \dfrac{10.19m}{2} = 1.08\,mA$

$V_{DD} = V_{DS} + I_D R_D = 5 + (1.08m)(0.45033k)$
$= 5.4863\,V$

$V_{GG} = V_{GS} = 0.83\,V$

From small signal equivalent circuit

output impedence with $R_D = (r_o // R_D) = (7.878k)//0.45k$
$= 425.9\,\Omega$

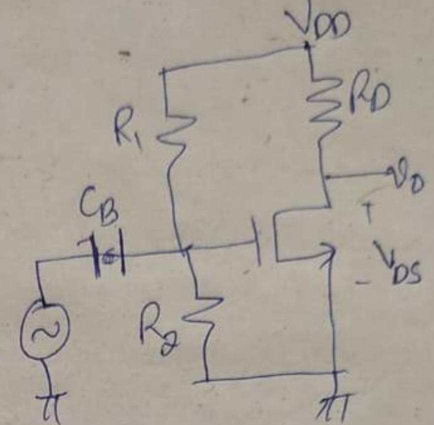
without $R_D = r_o = 7.878\,k\Omega$

Question 1 (f)

$V_{DS} = 3V$

$V_{DD} = 5V$

$V_{GS} = 0.83V$



$$V_{GG} = V_{GS} = \frac{R_2}{R_1+R_2} V_{DD}$$

$$R_2(V_{DD} - V_{GS}) = R_1 V_{GS}$$

$$R_1 = \frac{V_{DD}-V_{GS}}{V_{GS}} R_2$$

$$= \frac{5-0.83}{0.83} R_2$$

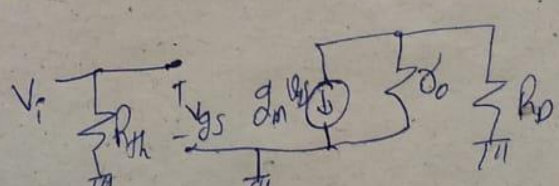
$R_{th} = (R_1 || R_2)$

$$R_1 = 5.024 R_2$$

So, for $R_2 = 10k\Omega$ $R_1 = 50.24 k\Omega$

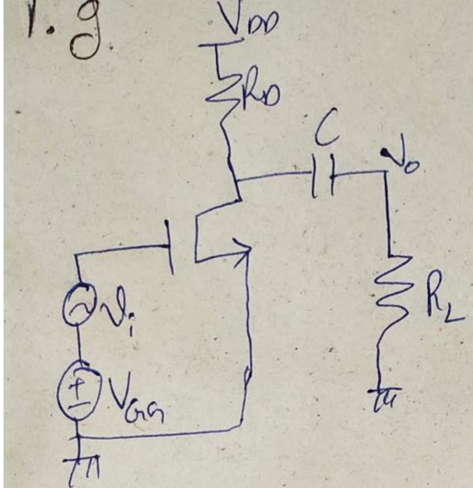
$V_{RD} = V_{DD} - V_{DS} = 5-3$

$V_{RD} = 2V$

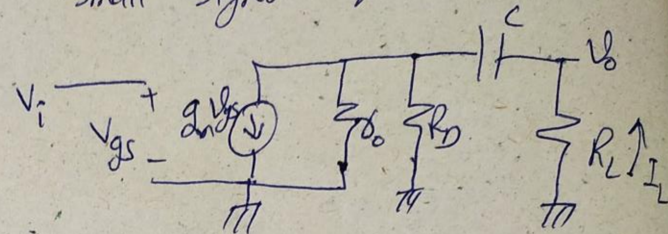


Question 1 (g)

1.g.



Small signal equivalent



Using current division

$$i_L = \frac{(r_o \| R_D)}{(r_o \| R_D) + \left[R_L + \frac{1}{sC}\right]} g_m v_{gs}$$

$$v_o = -R_L i_L$$

$$v_o = \frac{-R_L (r_o \| R_D) sC}{sC\left((r_o \| R_D) + R_L\right) + 1} g_m v_{gs}$$

$$A_v = \frac{v_o}{v_i} = \frac{-R_L (r_o \| R_D) sC}{sC\left((r_o \| R_D) + R_L\right) + 1} g_m \quad (v_{gs} = v_i)$$

We have $r_0 = 7.878\,k\Omega$ $\quad g_m = 4.695\,mS$

$R_D = 404.13\,\Omega$ $\quad R_L = 10\,k\Omega$

$C = 1\,\mu F$

$$r_0 \parallel R_D = \frac{r_0 R_D}{r_0 + R_D} = 425.98\,\Omega$$

$$A_v = \frac{-10k\,(0.42598k)\,1\mu\,S}{S\,1\mu\,[0.42598k + 10k] + 1}\, g_m$$

$$= \frac{-4.259\,S}{S\{10.4259\,m\} + 1}\,[4.695\,m]$$

$$|A_v| = \frac{+19.99\,\omega \times 10^{-3}}{\sqrt{1 + (10.4259\,m\,\omega)^2}}$$

at $f = 1\,MHz$ $\quad \omega = 2\pi f = 2\pi \times 1M$ rad/s

$$(A_v)_{f=1M} = \frac{125.6 \times 10^3}{65507.8617}$$

$$= \underline{1.917}$$

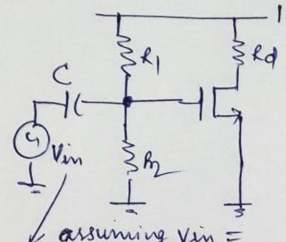
at $f = 1\,mHz$ $\quad \omega = 2\pi f = 2\pi\,m$ rad/s

$$(A_v)_{f=1m} = \frac{125.6 \times 10^{-6}}{1} = \underline{125.6 \times 10^{-6}}$$

Question 2 (a)

Ques 2(a) Design of CS amplifier

let us use potential divider ckt for biasing the ckt.



assuming Vin = 10mV with 1kHz freq

Given $K' = \mu_n C_{ox}\frac{W}{L} = 2m\frac{A}{V^2}$

$\lambda = 0.01\,V$.

lets start by assuming that the drain current is 1mA.

$1m = \frac{1}{2}\times 2m\,(V_{GS}-V_t)^2$

$V_{GST}^2 = 1$

$V_{GS} = 1 + V_{th}$

$V_{GS} = 1 + .4$ ($V_{th} \approx 0.4V$ from Model file)

$= 1.4V.$

$\boxed{V_G = 1.4V}$ (as source is grounded)

to get 1.4V at gate

$\Rightarrow \frac{15R_2}{R_1+R_2} = 1.4V \Rightarrow \frac{R_1}{R_2} = \frac{68}{7}$

$\boxed{\begin{array}{l} R_2 = 7K \\ R_1 = 68K. \end{array}}$

find g_m

$g_m = \frac{2I_D}{V_{GST}}$

$= \frac{2\times 1m}{1}$

$\boxed{g_m = 2m\,A/V}$

gain $\Rightarrow g_m R_D$

$\approx 2m R_D$

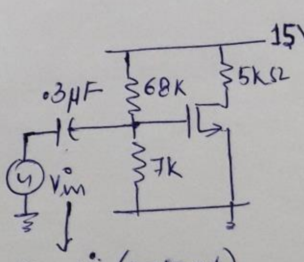
$2m R_D = 10$ ($|A_V| = 10$)

$\boxed{R_D = \frac{10}{2m} = 5k\Omega}$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{.01\times 1m} = 100k\Omega$

Here in our expression of gain we neglected r_o, the reason is $R_D = 5k\Omega$ is much less than $r_o = 100k\Omega$.

So $r_o \| R_D \approx R_D$



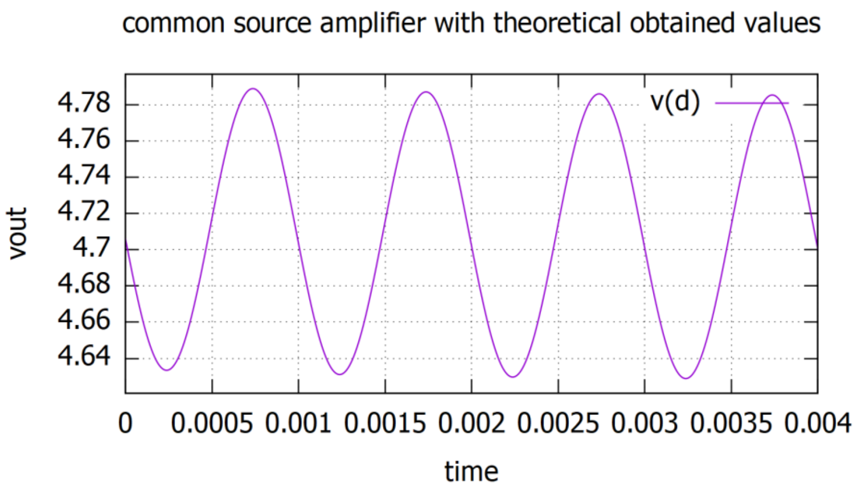
10m sin(2π1000t)

finding C.

$7k\|68k \geq \frac{10}{2\pi f C}$

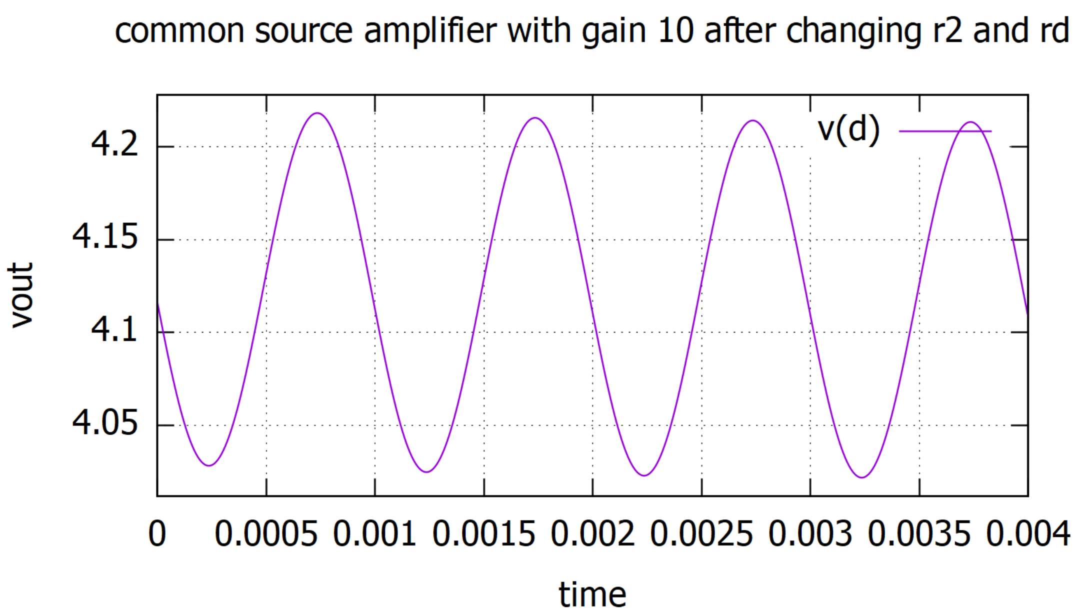
$C \geq .25\mu.$

$\boxed{C = .3\mu F}$



SPICE Code:

```
*Common source amplifier with theoretical obtained values

.include tsmc.txt

m1 d g 0 0  CMOSN w=5u l=.18u

rd   d 1 5k
vdd  1 0 15
r1 g 1 68k
r2 g 0 7k
cb1 g 2 .3u
vin 2 0 sin (0 10m 1k 0 0 )
.tran 1u 4m 0
.control
run
plot v(d) xlabel time ylabel vout
plot v(2) xlabel time ylabel vin
.endc
.end

*In this case the gain = 8, the reason is that
*the modal file is advanced that's why we get little bit deviation
*hence we need to change resistance values.
```

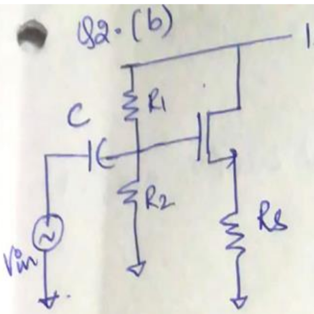


SPICE Code:

```
*Common source amplifier with gain 10 after changing r2 and rd

.include tsmc.txt

m1 d g 0 0  CMOSN w=5u l=.18u

rd   d 1 6.5k
vdd  1 0 15
r1 g 1 68k
r2 g 0 6.2k
cb1 g 2 .3u
vin 2 0 sin (0 10m 1k 0 0 )
.tran 1u 4m 0
.control
run
plot v(d) xlabel time ylabel vout
plot v(2) xlabel time ylabel vin
.endc
.end
```

Question 2 (b)

Q2. (b)



$K' = 2mA/V^2$

$\lambda = 0.01V^{-1}$

Lets assume drain current = 1mA

$I_{DS} = \frac{1}{2} \times 2\times10^{-3} \times (V_{GS}-V_{th})^2 = 1\times10^{-3}$

$V_{GS}-V_{th} = 1$

$V_{GS} = 1+0.4 \quad (V_{th} = 0.4V)$

$V_{GS} = 1.4$

$V_S = I_{DS} \times R_S = 10^{-3} \times R_S$

$V_G = V_S + 1.4 = 10^{-3} \times R_S + 1.4$

$\frac{15R_2}{R_1+R_2} = V_G = 10^{-3}R_S + 1.4$ —— ①

$g_m = \frac{2I_{DS}}{(V_{GS}-V_{th})} = \frac{2\times1\times10^{-3}}{1} = 2mA/V$

$\text{gain} = \frac{g_m r_o R_S}{g_m r_o R_S + r_o + R_S} = \cancel{\frac{g_m R_S}{g_m R_S +}} \frac{g_m}{g_m + \frac{1}{r_o \| R_S}} \simeq \frac{g_m}{g_m + \frac{1}{R_S}}$

$0.9 = \frac{2\times10^{-3}}{2\times10^{-3}+\frac{1}{R_S}}, \quad \frac{1}{R_S} = \frac{2}{9}\times10^{-3}$

$\Rightarrow R_S = \frac{9}{2}\times10^{3} = 4.5K\Omega$

$R_S >> r_o$ hence neglecting r_o is justified.

$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01\times10^{-3}} = 10^{-5}\Omega$ → very small hence neglected

In ①, $\frac{15R_2}{R_1+R_2} = 10^{-3}\times4.5\times10^{3} + 1.4 = 5.9$

$15R_2 = 5.9R_1 + 5.9R_2 \Rightarrow 9.1R_2 = 5.9R_1 \Rightarrow \frac{R_2}{R_1} = \frac{59}{91}$

$\Rightarrow R_1 = 91K\Omega$ and $R_2 = 59K\Omega$

for capacitance,

$$\frac{R_1 R_2}{R_1+R_2} \geq \frac{10}{2\pi f C}$$

Considered input signal as $V_{in} = 10mV \sin(2\pi \times (1kHz) \times t)$

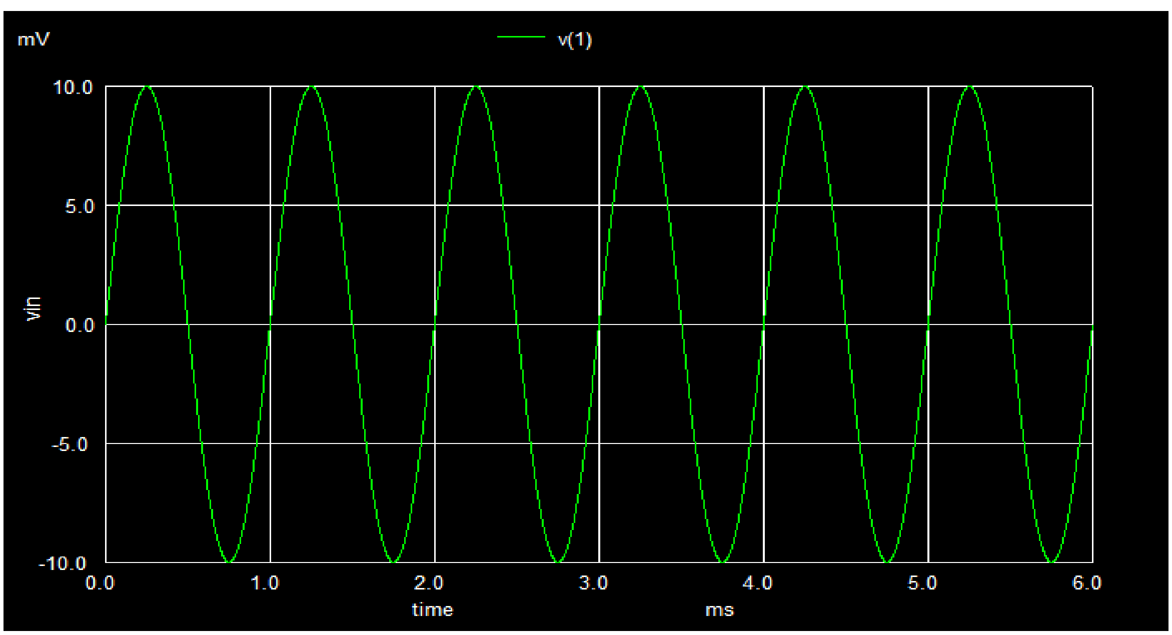
$f = 1KHz$

$$C \geq \frac{10 \times (59+91)}{2\pi \times 10^3 \times 59 \times 91 \times 10^3}$$

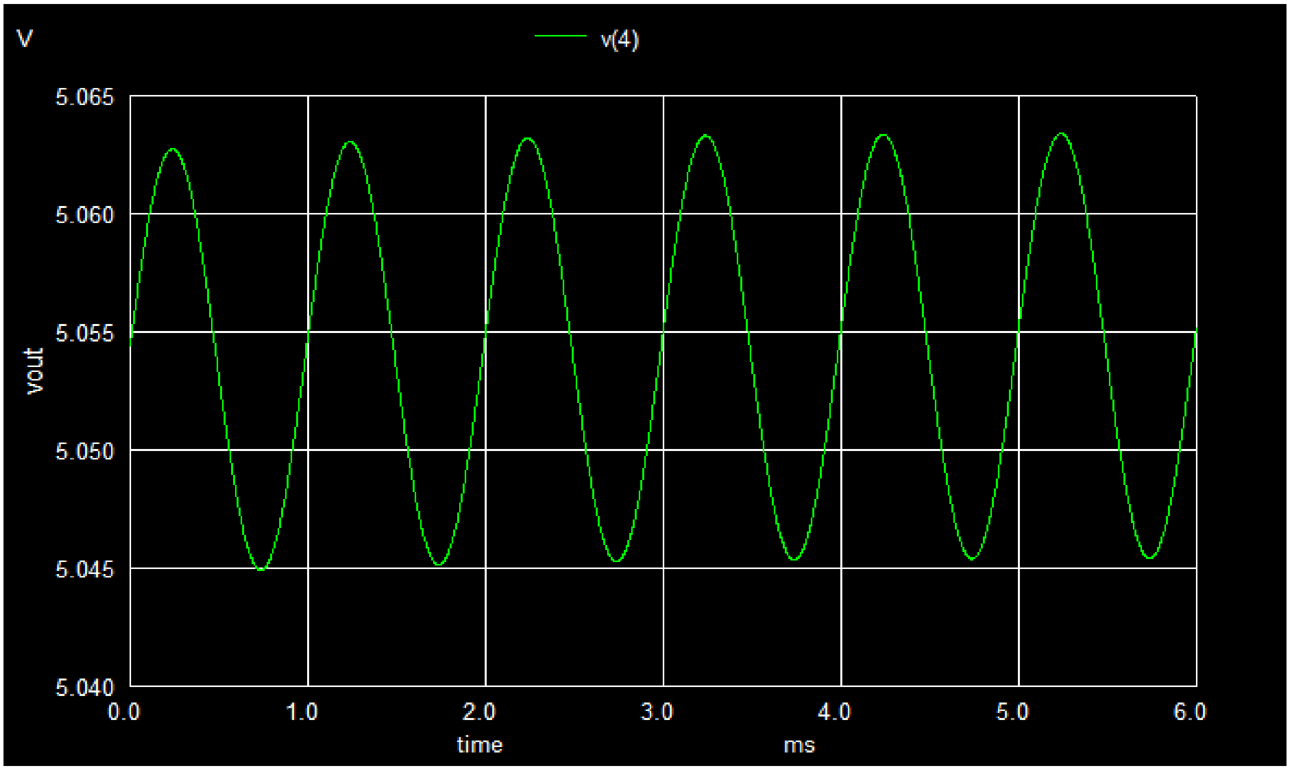
$\Rightarrow C \geq 0.044\,\mu F$

Take $C = 0.05\,\mu F$

Vin:



Vout:



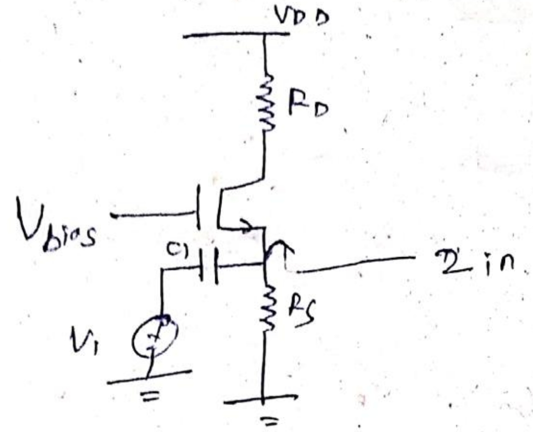
SPICE Code:

```
.include tsmc.txt
vdd 3 0 15
c1 2 1 0.05u
r2 2 0 59k
r1 2 3 91k
m1 3 2 4 4 cmosn w=5u l=0.18u
rs 4 0 4.5k
vin 1 0 sin (0 10m 1k 0 0)
.tran 0.002m 6m
.control
run
plot v(4) xlabel time ylabel vout
plot v(1) xlabel time ylabel vin
.endc
.end
```

Question 2 (c)

2-c

common gate - Amplifier



As this is a Design Problem
Let us consider $I_D = 1mA$

$V_{GS} - V_T = 0.2V$

$$g_m = \frac{2I_D}{V_{GS}-V_T} = \frac{2 \times 1mA}{0.2} = 10\,mS$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 1mA} = 100K\Omega$$

-> Looking in to the source

$$Z = \frac{1}{g_m} \| r_o$$

$$Z \simeq 0.1K\Omega$$

$\frac{1}{g_m} = 0.1K$

we have considered

$V_{GS} - V_T = 0.2$

$V_G - V_S - V_{Th} = 0.2$

~~$V_G = 0.2 + 2.5 + V_{Th}$~~

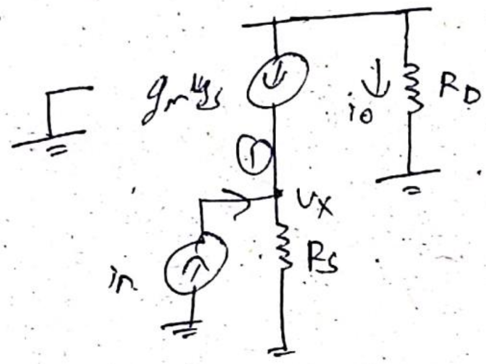
$= 3.2V$

Assumed

$V_{Th} = 0.5$

~~$V_S = 2.5$~~

small-signal Analysis.



$i_0 = -g_m V_{gs}$.

$$i_0 = -g_m(0 - V_X)$$

$$i_0 = g_m V_X$$

$$V_X = \frac{i_0}{g_m}$$

KCL At node - I

$$-i_{in} + \frac{i_0}{R_S \cdot g_m} + i_0 = 0$$

$$I_{in} = I_0\left(\frac{1}{g_m R_S} + 1\right)$$

$$\frac{I_0}{I_{in}} = \frac{1}{1 + \frac{1}{g_m R_S}}$$

given

$$0.8 = \frac{g_m R_S}{1 + g_m R_S}$$

$$0.8 = \frac{10m \times R_S}{1 + 8.10m R_S}$$

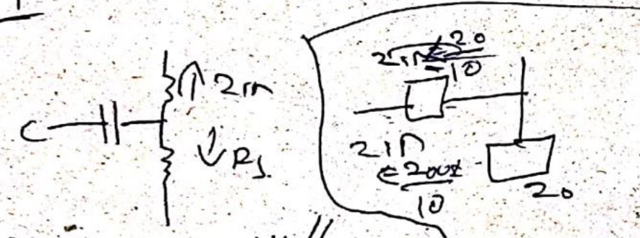
$R_S = \frac{0.8}{2} K$

$= 0.4K$

$V_G = 0.2 + V_S + V_{TN}$ $\qquad V_S = I_S R_S$

$= 0.2 + 0.4 + 0.5$ $\qquad = I_D R_S$

$= 1.1$



$.1K // .4K$

$0.08K$

$\cancel{0.8C} = \frac{1}{2\pi \times fC}$ $\qquad = \cancel{0.8K}\ 0.08K.$

$C = \frac{1}{2 \times 3.14 \times 1000 \times 0.006 \times 10^3}$

$= \cancel{26}\ 26\ nf$

$C = 26 nf$

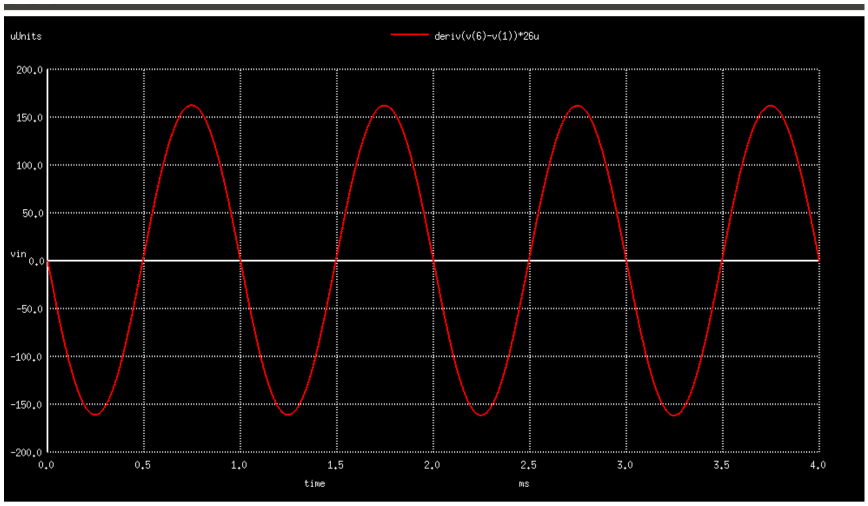
$\rightarrow\ r_o = r_o + R_S(1 + g_m r_o)$

$= 100K + 5K(1 + 10mS \times 100K)$

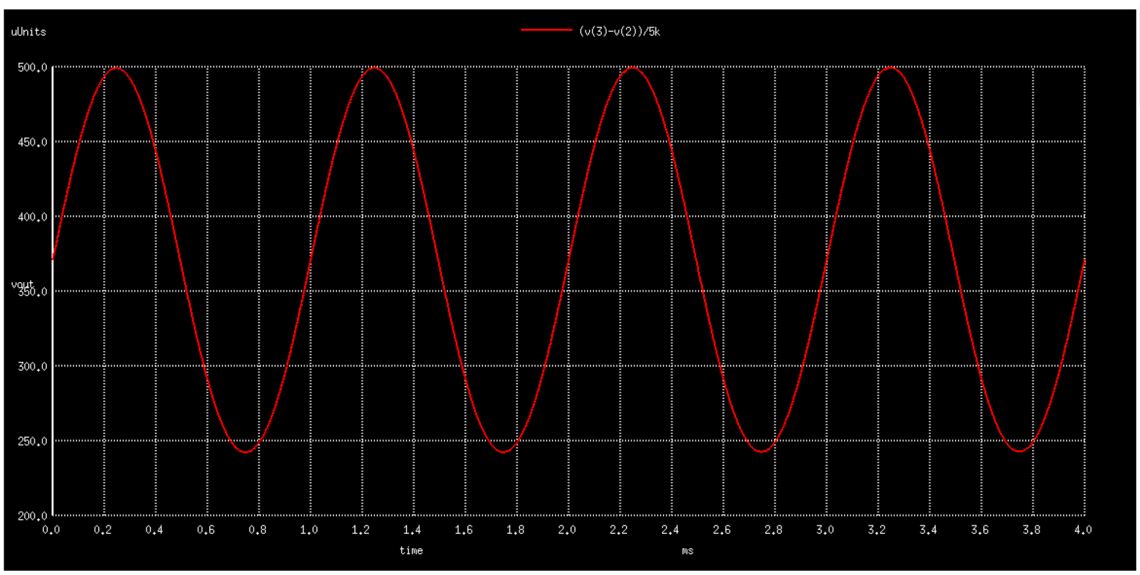
$= 5.105 \times 10^6\ \Omega$

With designed R_S we can't get the required current gain, so we tweek R_S to $1.5K$ to get required A_I.

Iin:



Iout:



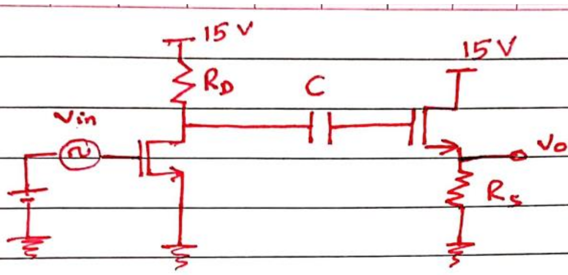
Current Gain = (499-371)/162 = 0.79

SPICE Code:

```
*common gate
.include tsmc.txt
M1 2 4 1 1 CMOSN w=5u l=.18u
VDD 3 0 DC 15v
c1 6 1 26u
VGG 4 0 DC 1.1
rs 1 0 1.5k
rd 3 2 5K
vn 0 6  sin (0 50m 1k 0 0 )
.tran 1u 4m 0
.control
run
plot  (v(3)-v(2))/5k xlabel time ylabel vout
plot  deriv(v(6)-v(1))*26u xlabel time ylabel vin
.endc
.end
```

Question 2 (d)

Q2 d)



Coupling Capacitor needs to be very high.

Overall gain = gain of CS × gain of CD
= −10 × 0.9
= −9.

Output impedence = (Z_o) common drain

$$= \frac{1}{g_m + \frac{1}{r_o \| R_s}} \approx \frac{1}{g_m} = 202\ \Omega$$

$= 183\ \Omega$