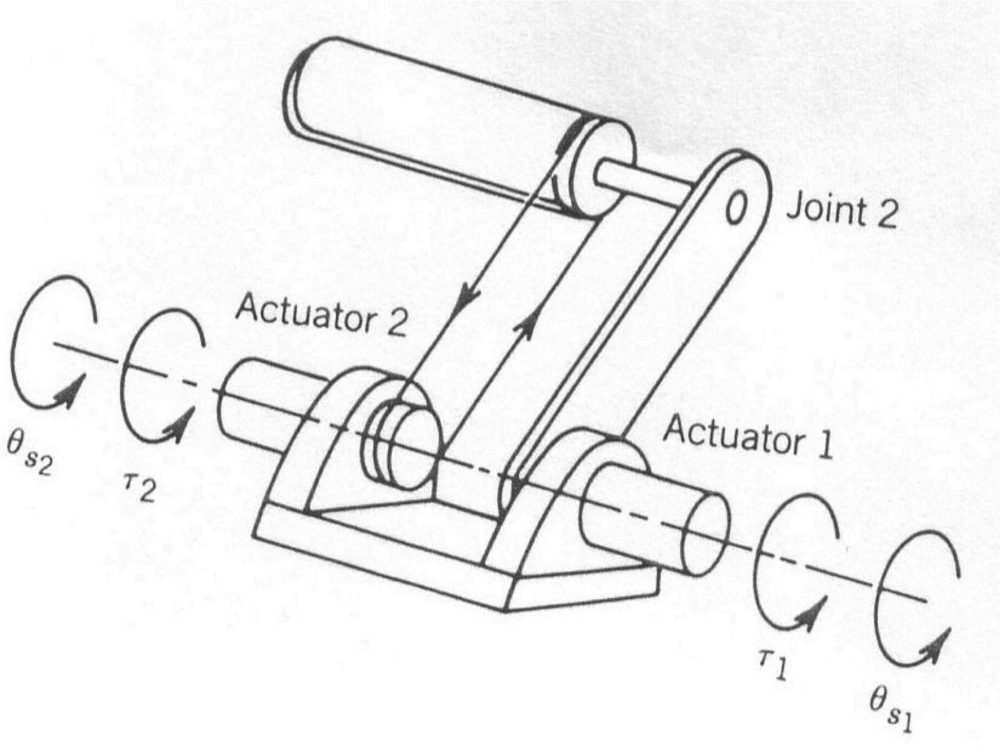
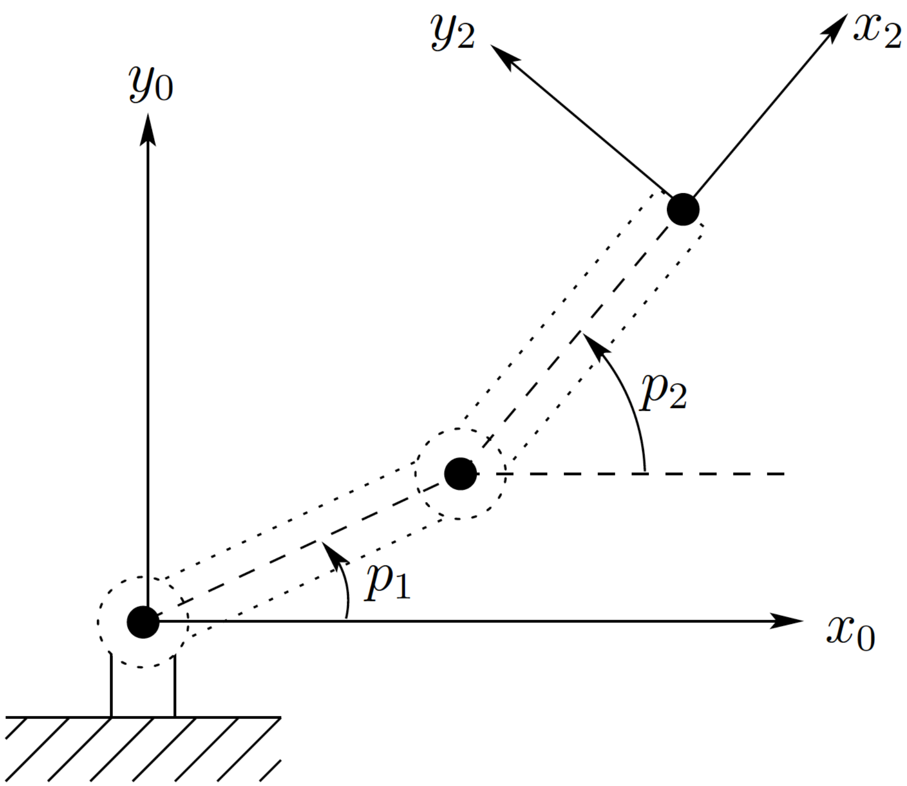
Quiz

Write the expression for kinetic energy, and derive the equations governing the motion of the two link manipulator shown below, with generalized coordinates p_1 and p_2. You may assume links to be of rods with uniformly distributed mass, m_1 and m_2. Neglect gravity.





Independent joint control

Actuator dynamics

- PMDC motor
  - Principle of operation

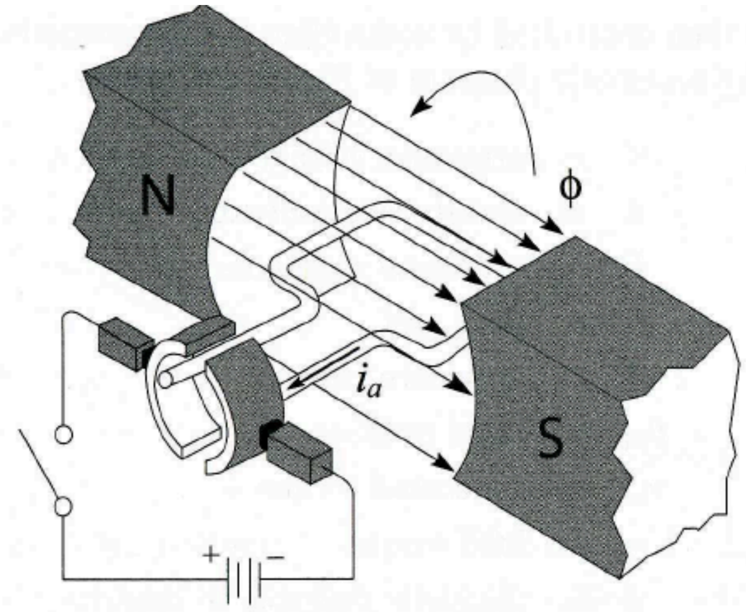
    Current carrying conductor in a magnetic field experiences a force

$$F = i \times \phi$$

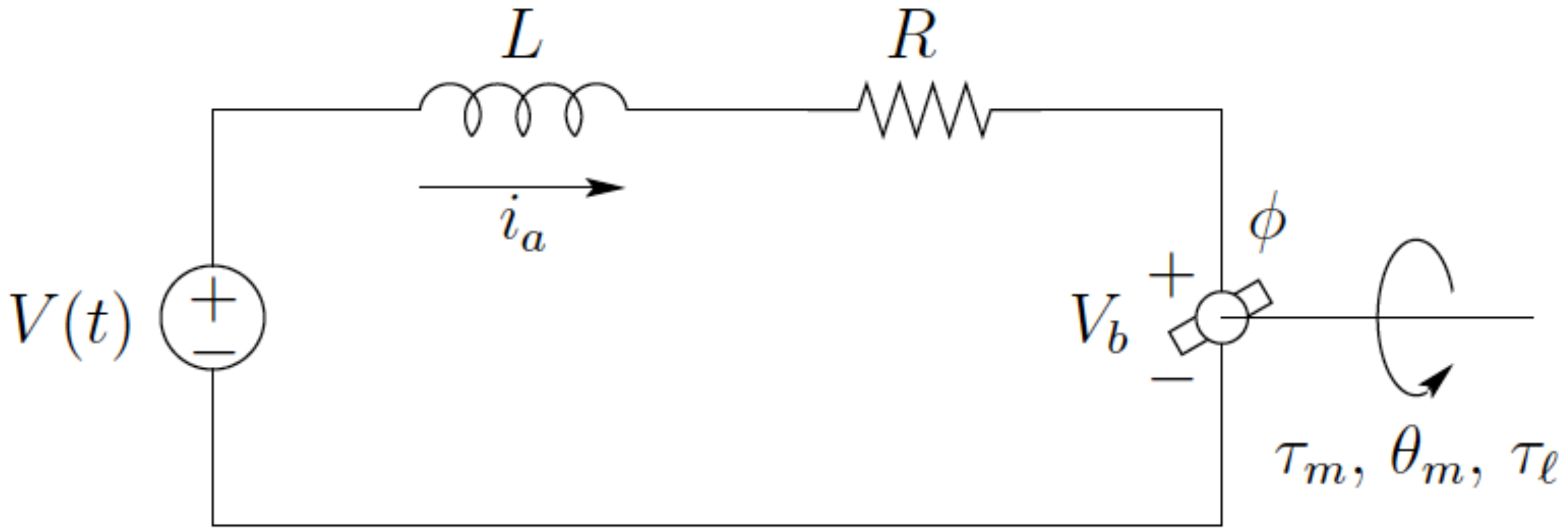
$$\Rightarrow \tau_m = (K.|i|.|\phi|)\, i$$

When a conductor moves in a magnetic field, a voltage V_b is induced

$$V_b = (K.|\phi|)\dot{\theta}_m$$



PMDC motor



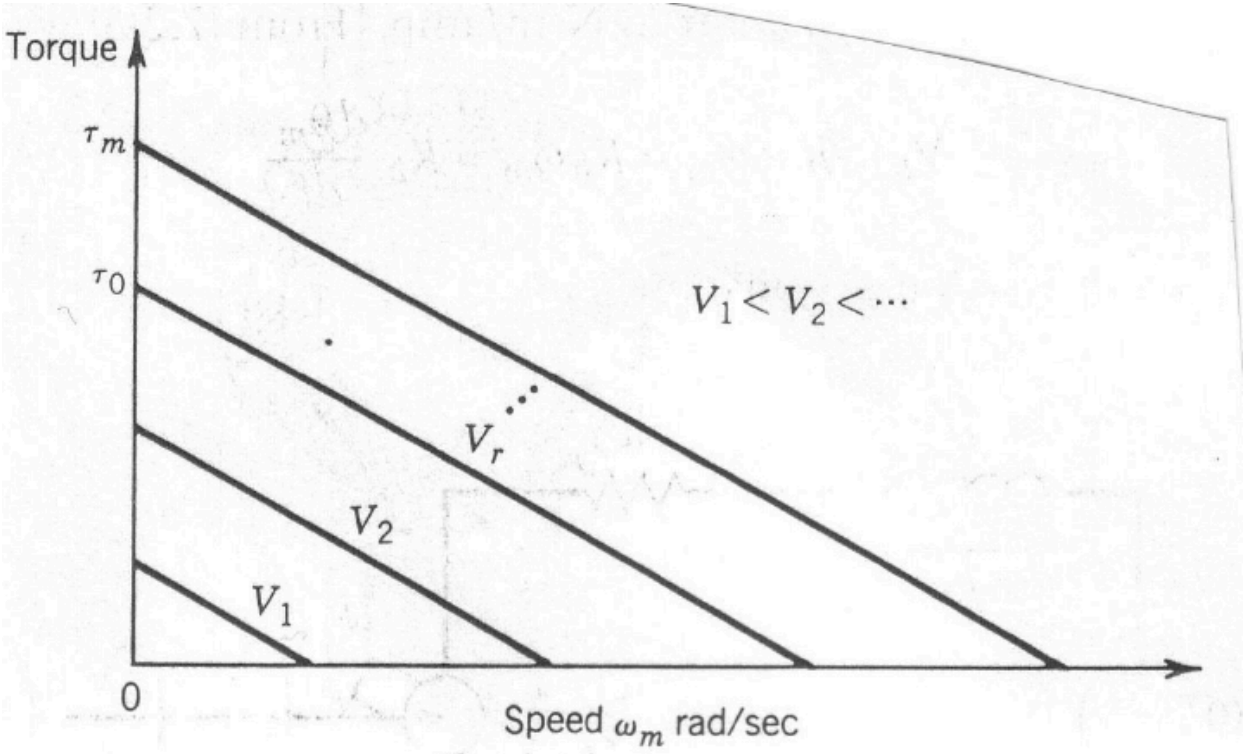
- Relationship between armature current, voltage, rotor velocity and motor torque are

$$L\frac{di}{dt} + Ri = V - V_b$$

$$V_b = (K.|\phi|)\dot{\theta}_m = K_b\dot{\theta}_m$$

$$\tau_m = (K.|\backslash\mathrm{phi}|.|i|)i = K_m i$$

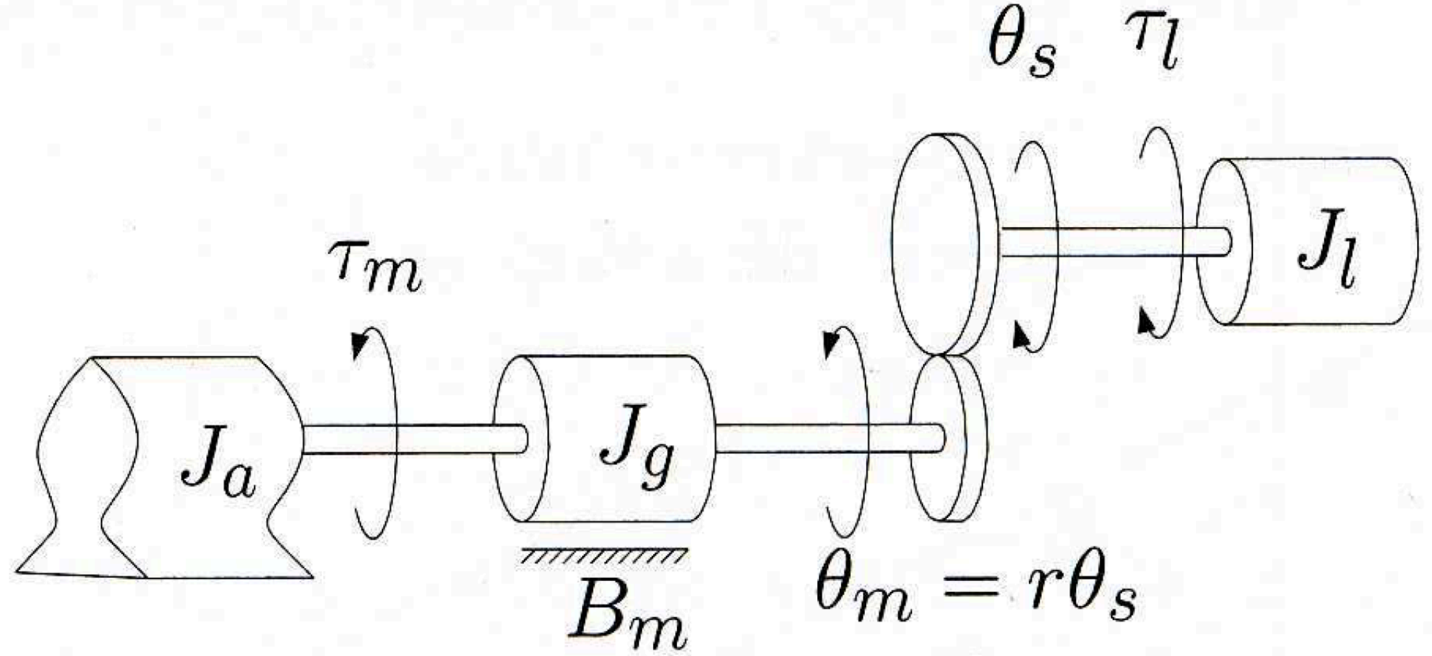
Load-speed curve



- When the motor is stalled ($\omega_m = 0$)

$$V_r = Ri_a = R\frac{\tau_0}{K_m} \Rightarrow K_m = \frac{R\tau_0}{V_r}$$

Robot with one joint



- In terms of the motor angle, the equation of motion is

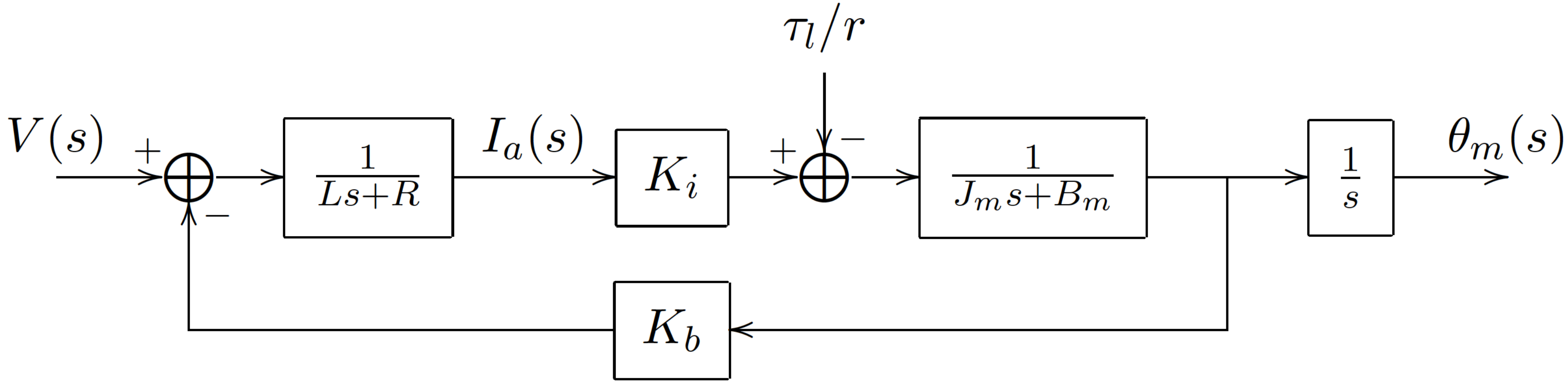
$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - \frac{\tau_l}{r} = K_m i - \frac{\tau_l}{r}, \qquad J_m = J_a + J_g$$

Dynamical model

- Combining the electrical and mechanical models,

$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = K_m i - \frac{\tau_l}{r}$$

$$L\frac{di}{dt} + Ri = V - K_b \frac{d\theta_m}{dt}$$



Transfer function

- The transfer function from $V(s)$ to $\theta_m(s)$ and τ_l to θ_m are

$$\frac{\theta_m(s)}{V(s)} = \frac{K_m}{[(Ls+R)(J_m s + B_m) + K_m K_b]s}, \qquad (\tau_l = 0)$$

$$\frac{\theta_m(s)}{\tau_l(s)} = \frac{1}{r}\frac{-(Ls+R)}{[(Ls+R)(J_m + B_m) + K_m K_b]s}, \qquad (V = 0)$$

- If $L/R \approx 0$, we obtain

$$\frac{\theta_m(s)}{V\,(s)} \approx \frac{K_m}{[R(J_m s + B_m) + K_m K_b]s}$$

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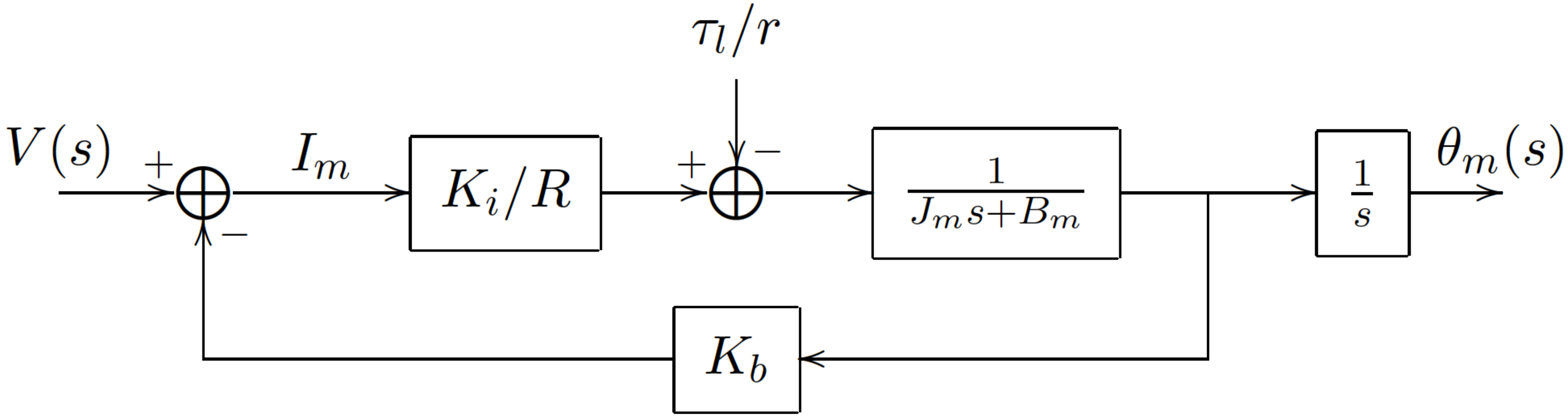
$$\frac{\theta_m(s)}{V\,(s)} \approx \frac{K_m}{[R(J_m s + B_m) + K_m K_b]s}, \qquad \frac{\theta_m(s)}{\tau_l(s)} = \frac{1}{r}\frac{-1}{[(Ls+R)(J_m + B_m) + K_m K_b/R]s}$$

Reduced order system

- Using superposition,

$$J_m\ddot{\theta}_m + (B_m + K_m K_b/R)\dot{\theta}_m = \left(\frac{K_m}{R}\right)V \; - \frac{\tau_l}{r}$$



- In general,

$$q_k = f_k(\theta_{m_1}, \ldots, \theta_{m_n}); \tau_{l_k} = f_k(\tau_1, \ldots, \tau_n)$$

Recall, $D\ddot{q} + C\dot{q} + G = \tau$

Example: two-link manipulator

- Here,

$$q_1 = \theta_{s_1}, \qquad q_2 = \theta_{s_1} + \theta_{s_2}$$

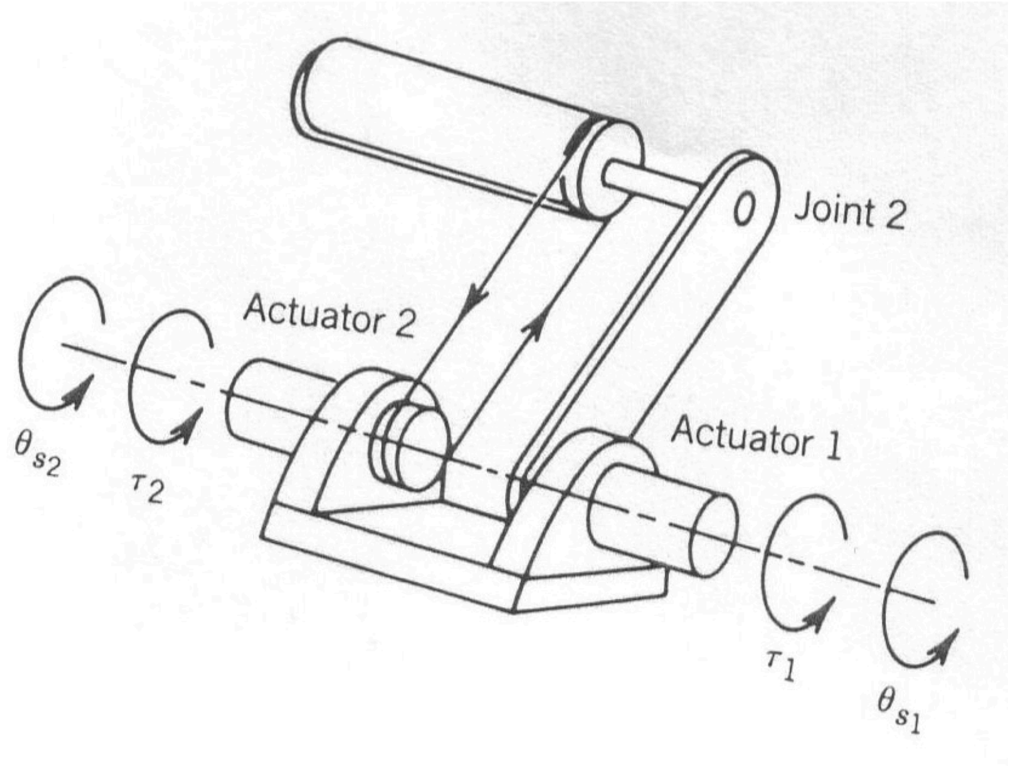
- Relationship between joint torques and loads is

$$\tau_{l_1} = \tau_1, \qquad \tau_{l_2} = \tau_1 + \tau_2$$

- Hence,

$$\theta_{s_1} = q_1, \qquad \theta_{s_2} = q_2$$

$$\tau_1 = \tau_{l_1} + \tau_{l_2}$$



Set point tracking

- Assume

$$q_k = \frac{\theta_{m_k}}{r_k}, \qquad \tau_{l_k} = \tau_k, \qquad k = 1, \dots, n$$

- Then, for joint k,

$$\sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i,j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k$$

J_{eff_k}

$$J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + \frac{K_{b_k}K_{m_k}}{R_k}\right)\dot{\theta}_{m_k} = \frac{K_{m_k}}{R_k}V_k - \frac{\tau_k}{r_k}$$

u_k

Combining,

$$\boxed{\left(J_{m_k} + \frac{1}{r_k^2}\boxed{d_{kk}(q)}\right)}\ddot{\theta}_{m_k} + \boxed{\left(B_{m_k} + \frac{K_{b_k}K_{m_k}}{R_k}\right)}\dot{\theta}_{m_k} = \boxed{\frac{K_{m_k}}{R_k}V_k} - d_k$$

$$d_k = \frac{1}{r_k}\sum_{i\neq j}\ddot{q}_j + \sum_{i} c_{ijk}\dot{q}_i\dot{q}_j + g_k$$

B_{eff_k}

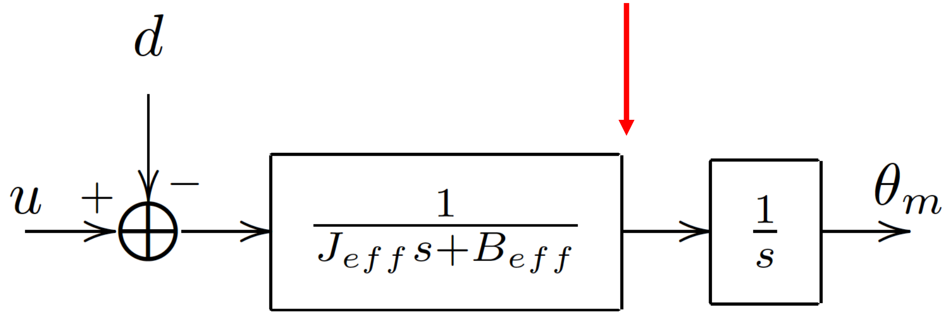
$$J_{eff_k}\ddot{\theta}_{m_k} + B_{eff_k}\dot{\theta}_{m_k} = u_k - d_k$$

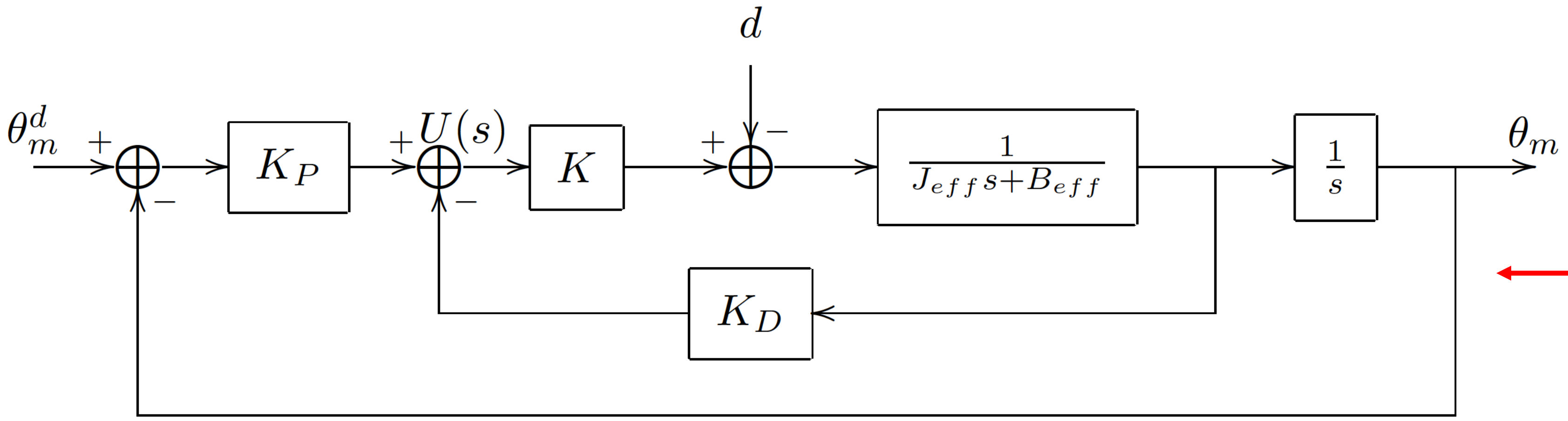
PD compensator

Open loop
system

$$J_{eff_k}\ddot{\theta}_{m_k} + B_{eff_k}\dot{\theta}_{m_k} = u_k - d_k$$

$$u_k(t) = K_p\big(\theta_d - \theta(t)\big) - K_d\dot{\theta}(t)$$





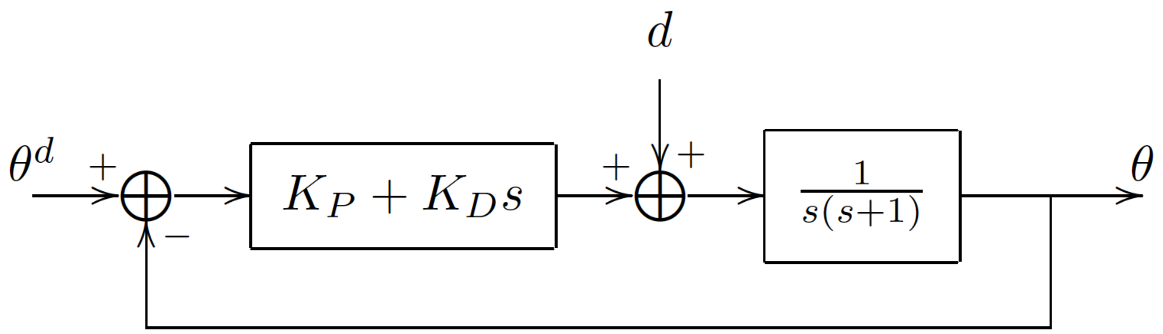
Closed
loop
system

Closed loop system,

$$\ddot{\theta}_{m_k} + \overset{2\zeta\omega_n}{\boxed{\frac{(B_{eff_k} + KK_d)}{J_{eff}}}}\dot{\theta}_{m_k} + \overset{\omega^2}{\boxed{\frac{KK_p}{J_{eff}}}}(\theta - \theta_d) = -d_k$$

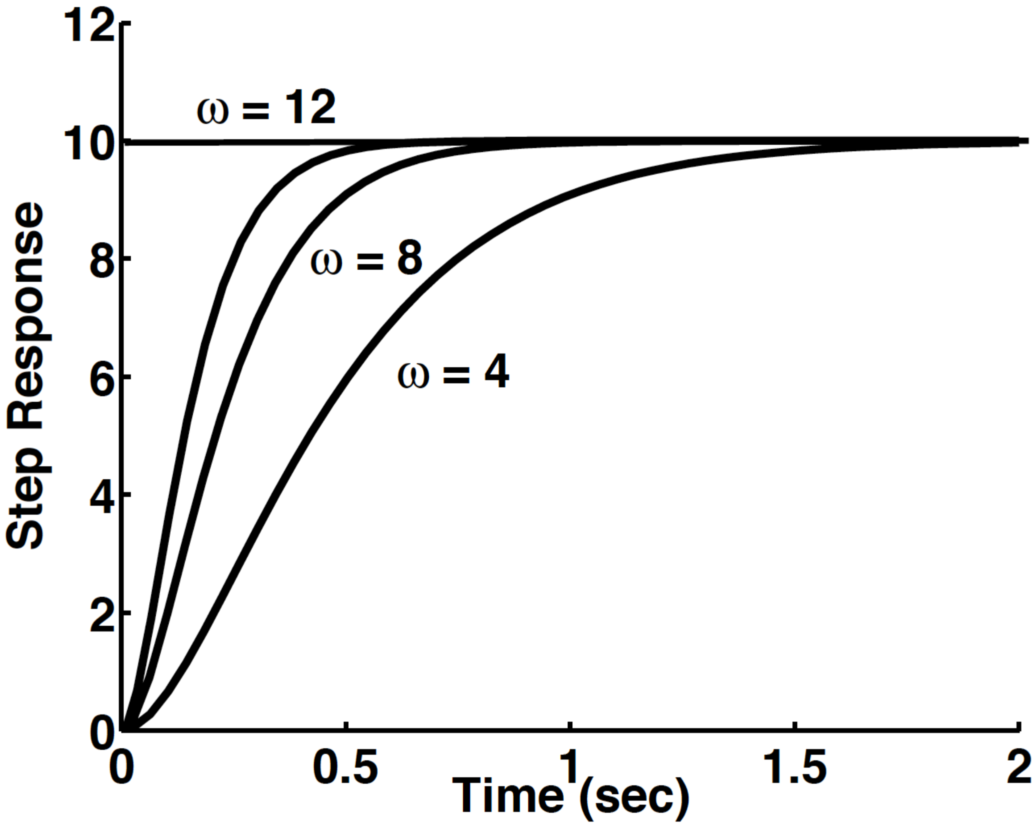
Selection of Kp and Kd Example

Second order system, $G(s) = \frac{1}{s(s+1)}$



$$K_p = \frac{\omega^2 J_{eff}}{K}, \qquad K_D = \frac{2\zeta\omega J_{eff} - B_{eff}}{K}$$

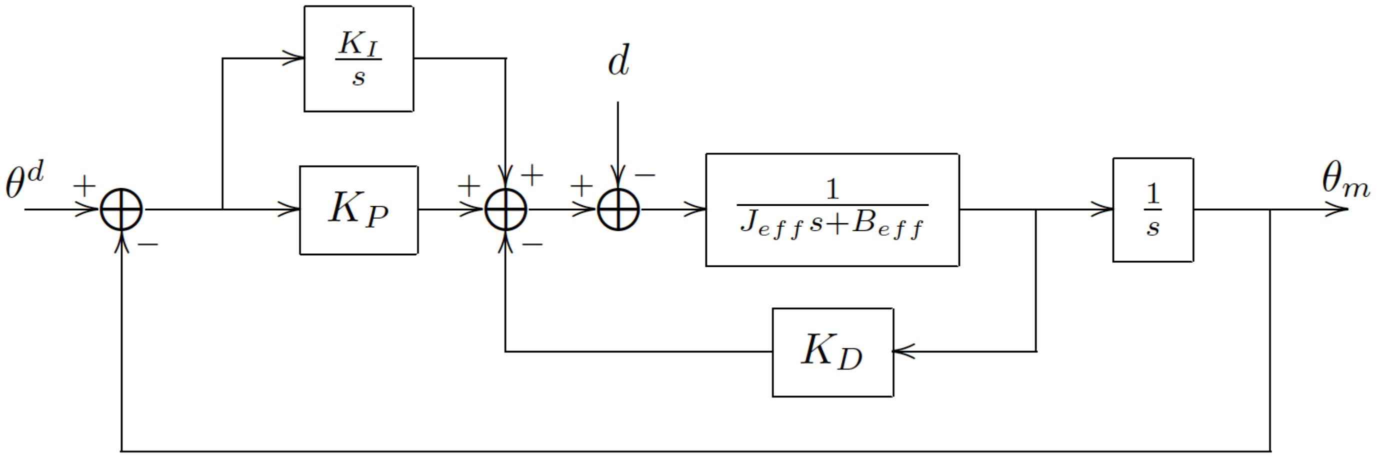
Natural Frequency (ω)	Proportional Gain K_P	Derivative Gain K_D
4	16	7
8	64	15
12	144	23

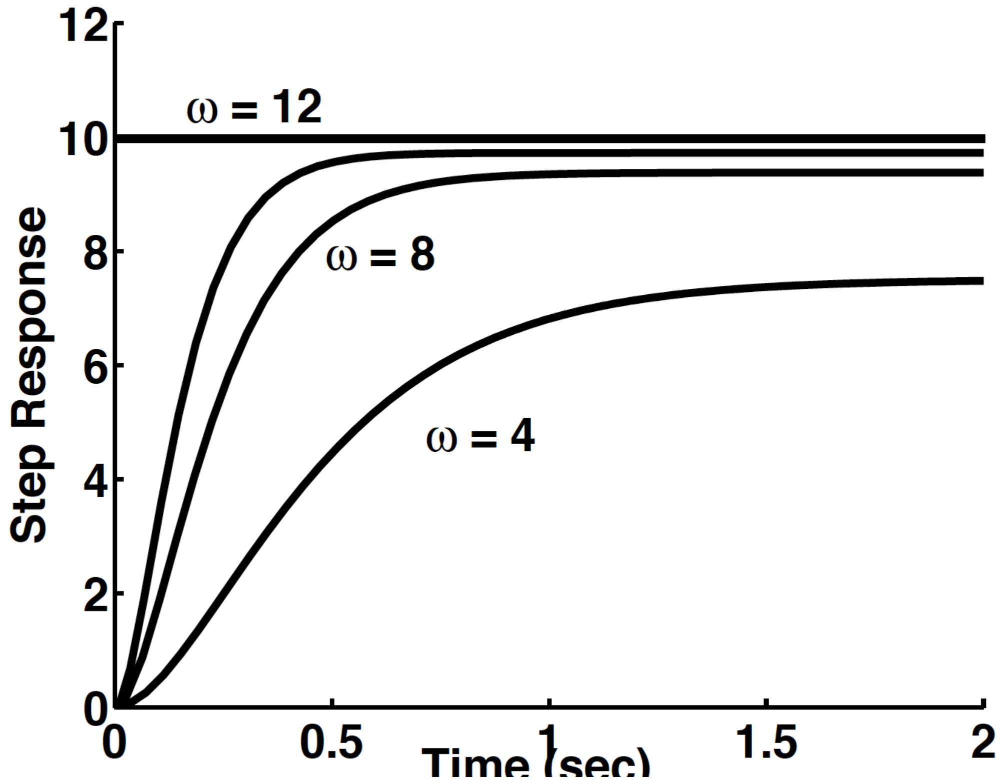


Steady state error and PID compensator

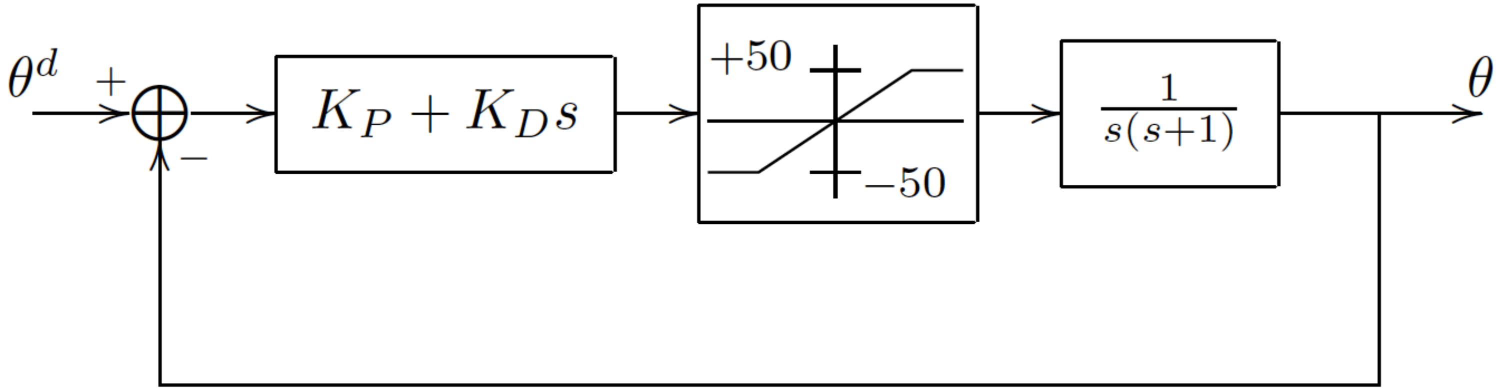
Steady state error, $e_{ss_k} = \frac{d_k}{KK_p}$

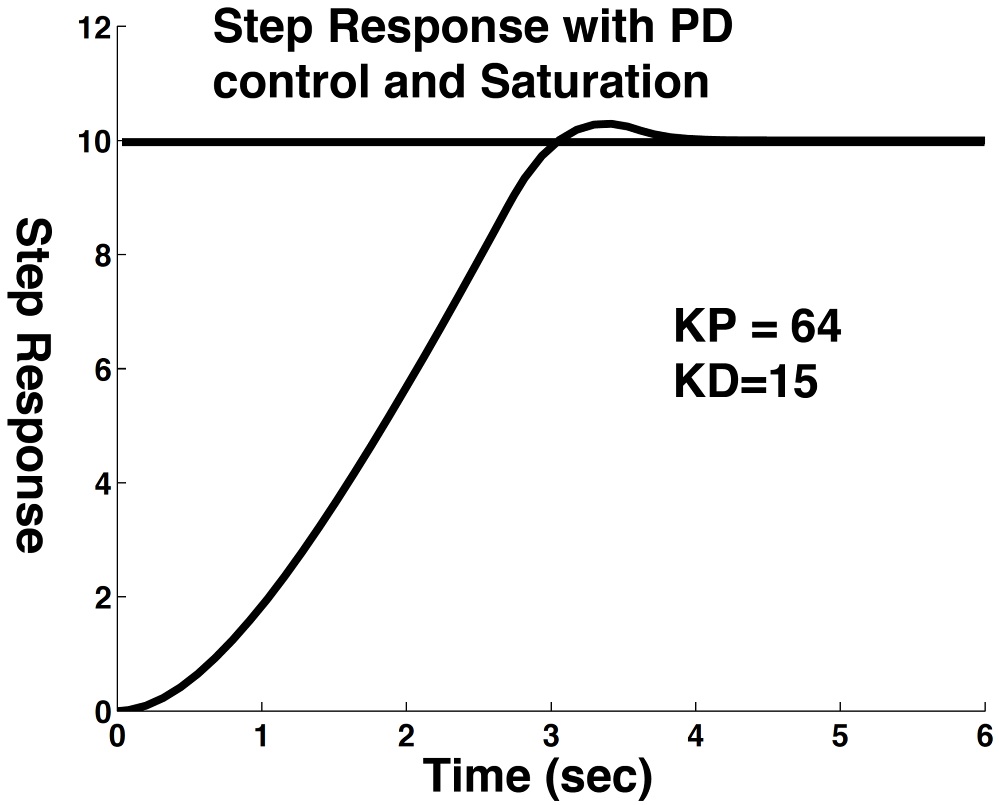
Use an integral term in the controller to eliminate steady-state error



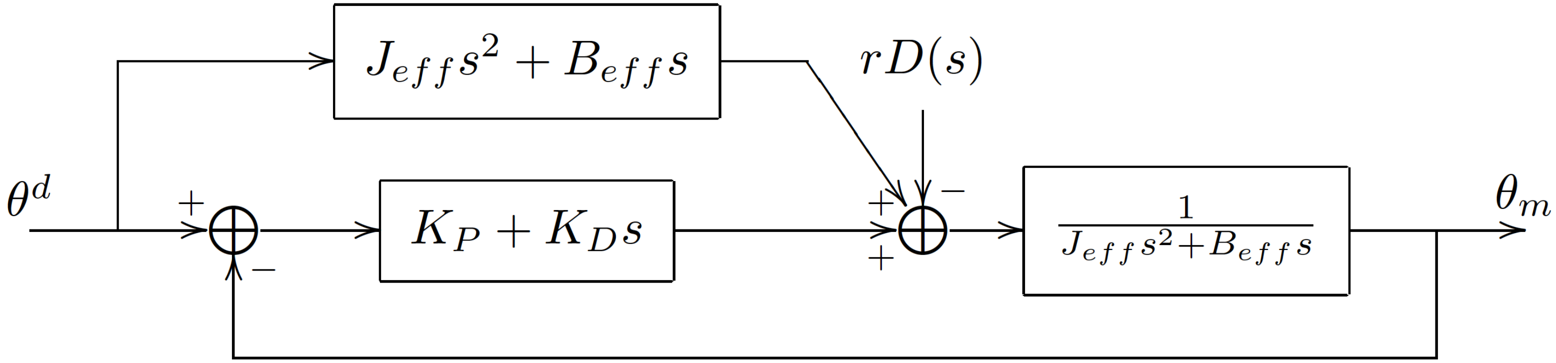


Response with saturation





Trajectory tracking and feedforward control



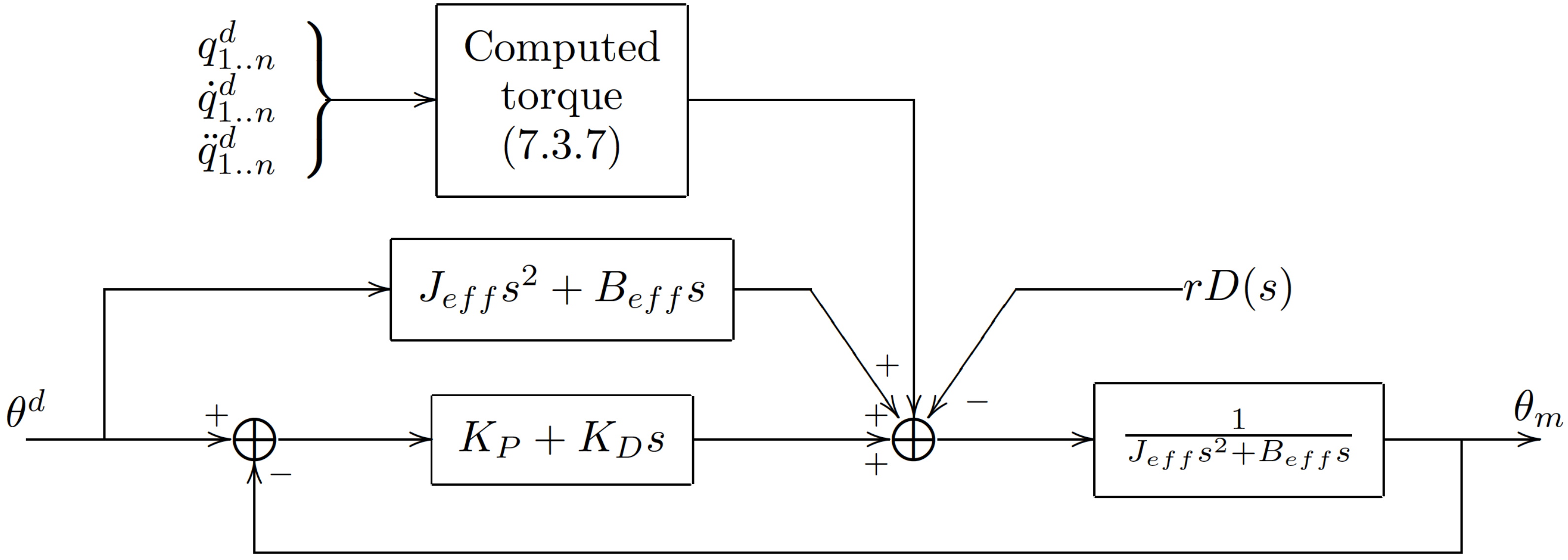
$$\begin{aligned} V(t) &= \frac{J_{eff}}{K}\ddot{\theta}^d + \frac{B_{eff}}{K}\dot{\theta}^d + K_D(\dot{\theta}^d - \dot{\theta}_m) + K_p(\theta^d - \theta_m) \\ &= f(t) + K_D\dot{e}(t) + K_p e(t) \end{aligned}$$

$$f(t) = \frac{J_{eff}}{K}\ddot{\theta}^d + \frac{B_{eff}}{K}\dot{\theta}^d$$

$$J_{eff}\ddot{\theta}_m + B_{eff}\dot{\theta}_m = KV(t) - rd(t)$$

$$J_{eff}\ddot{e} + (B_{eff} + KK_D)\dot{e} + KK_p e(t) = -rd(t)$$

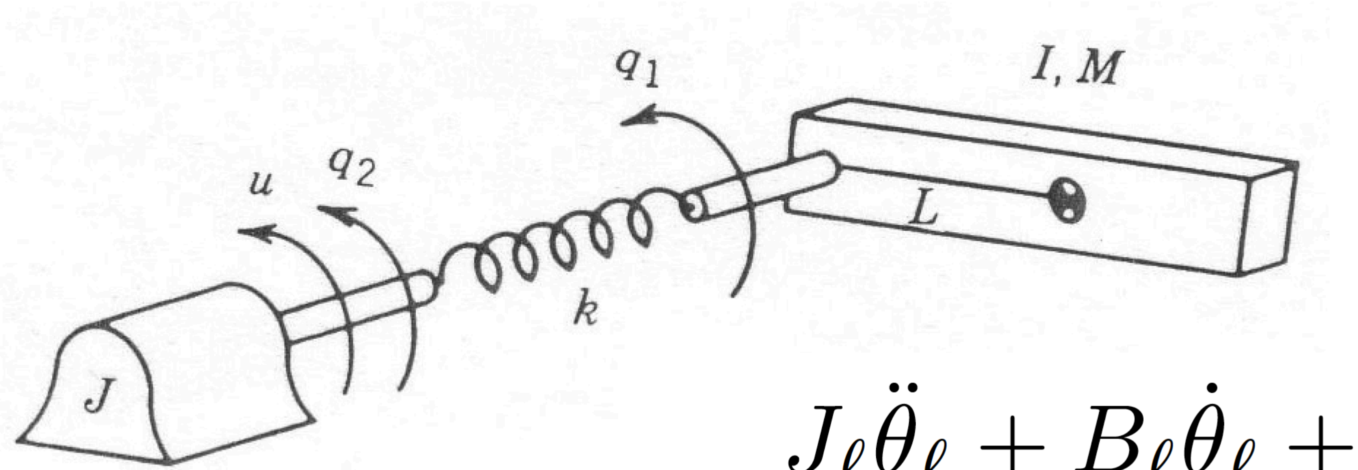
Computed torque disturbance cancellation



$$d^d \quad := \quad \sum d_{jk}(q^d)\ddot{q}^d_j + \sum c_{ijk}(q^d)\dot{q}^d_i\dot{q}^d_j + g_k(q^d)$$

Stability of PD control

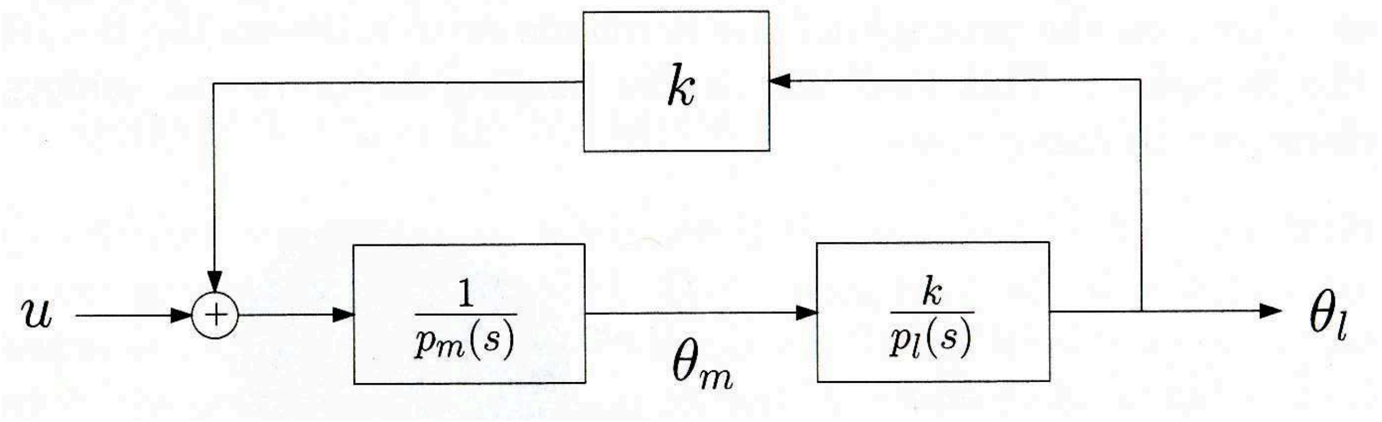
Drive train dynamics



$$\begin{aligned} J_\ell \ddot{\theta}_\ell + B_\ell \dot{\theta}_\ell + k(\theta_\ell - \theta_m) &= 0 \\ J_m \ddot{\theta}_m + B_m \dot{\theta}_m - k(\theta_\ell - \theta_m) &= u \end{aligned}$$

$$\begin{aligned} p_\ell(s)\Theta_\ell(s) &= k\Theta_m(s) \\ p_m(s)\Theta_m(s) &= k\Theta_\ell(s) + U(s) \end{aligned} \qquad \begin{aligned} p_\ell(s) &= J_\ell s^2 + B_\ell s + k \\ p_m(s) &= J_m s^2 + B_m s + k \end{aligned}$$

Open loop system



$$\frac{\Theta_\ell(s)}{U(s)} = \frac{k}{p_\ell(s)p_m(s) - k^2}$$

Characteristic equation

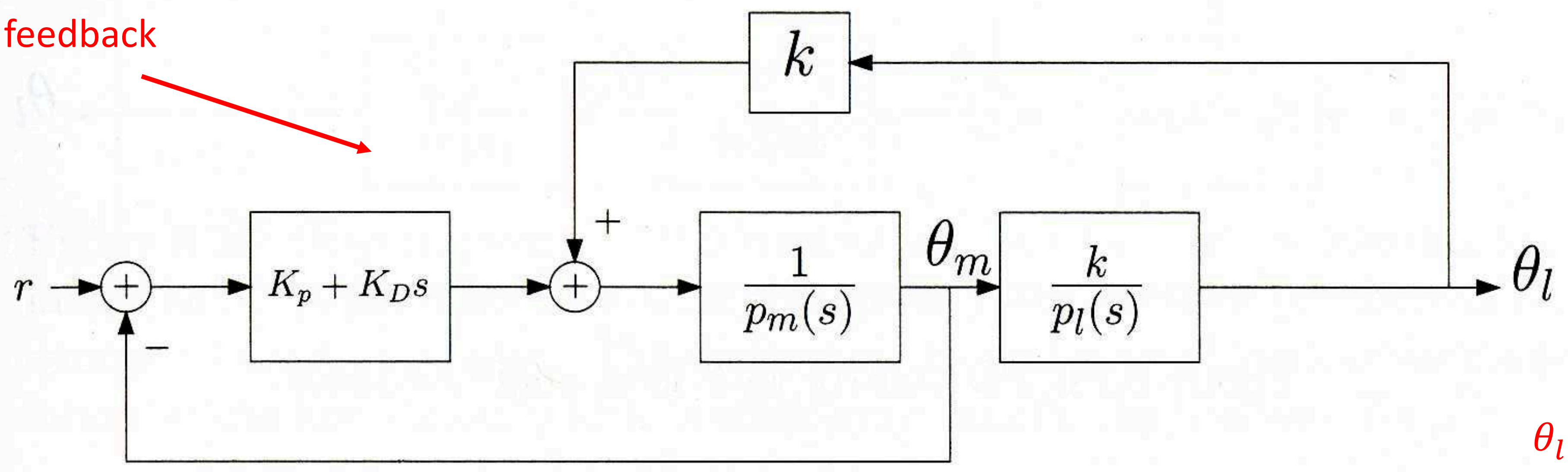
$$J_\ell J_m s^4 + (J_\ell B_m + J_m B_\ell)s^3 + (k(J_\ell + J_m) + B_\ell B_m)s^2 + k(B_\ell + B_m)s.$$

$$J_\ell J_m s^4 + k(J_\ell + J_m)s^2$$

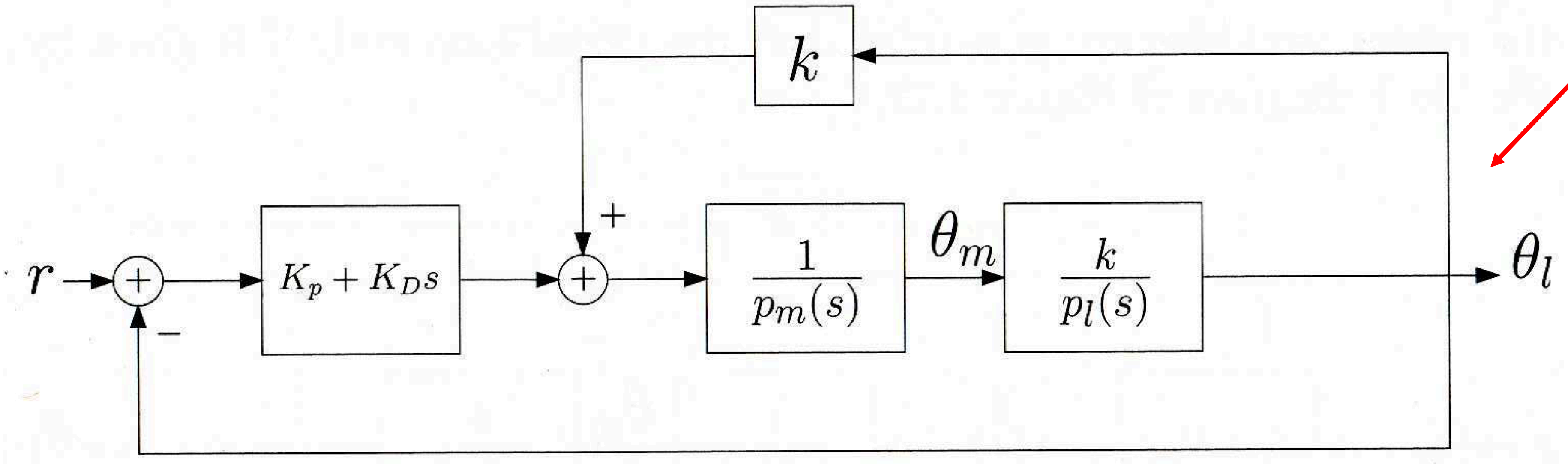
Neglecting damping

PD control

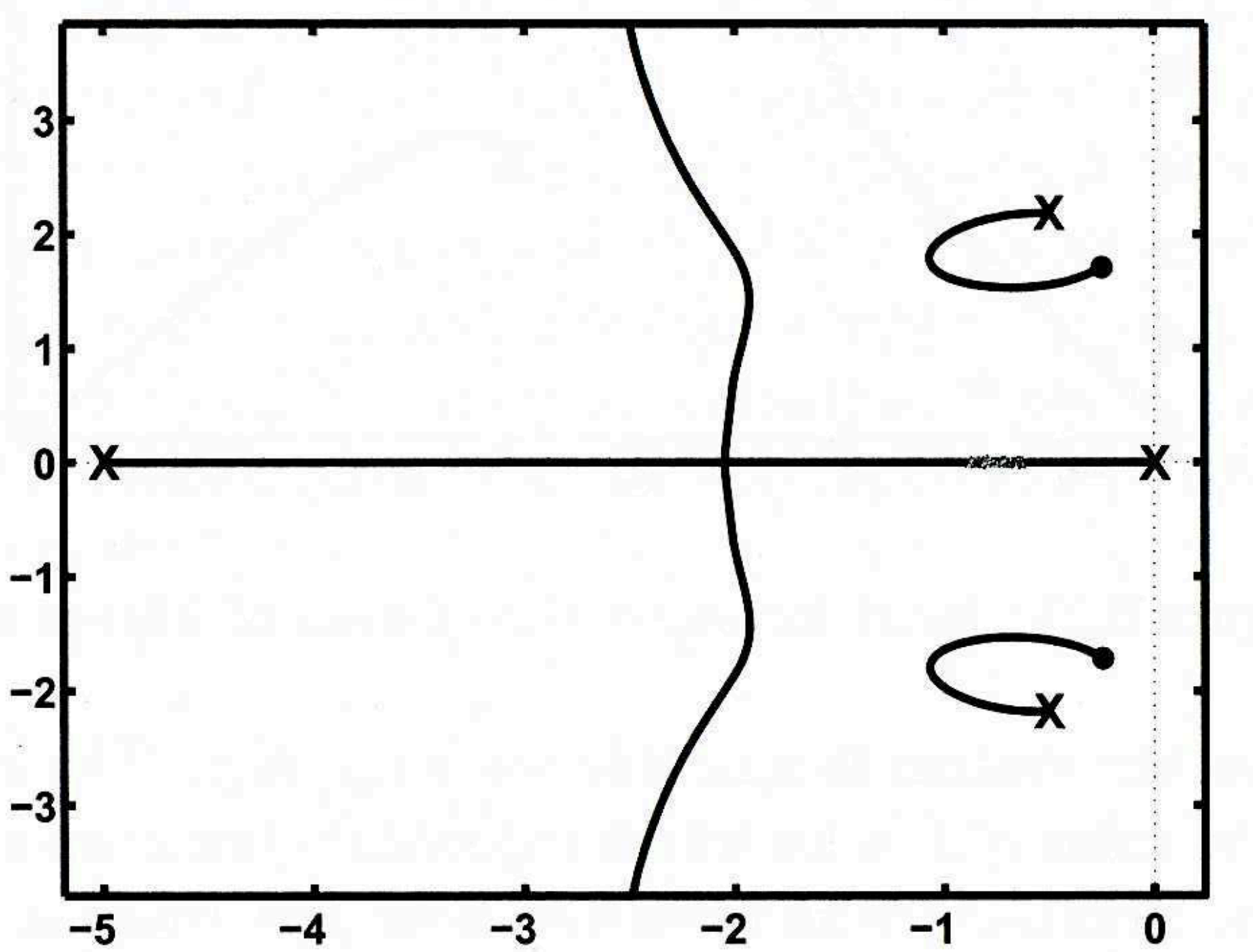
θ_m feedback



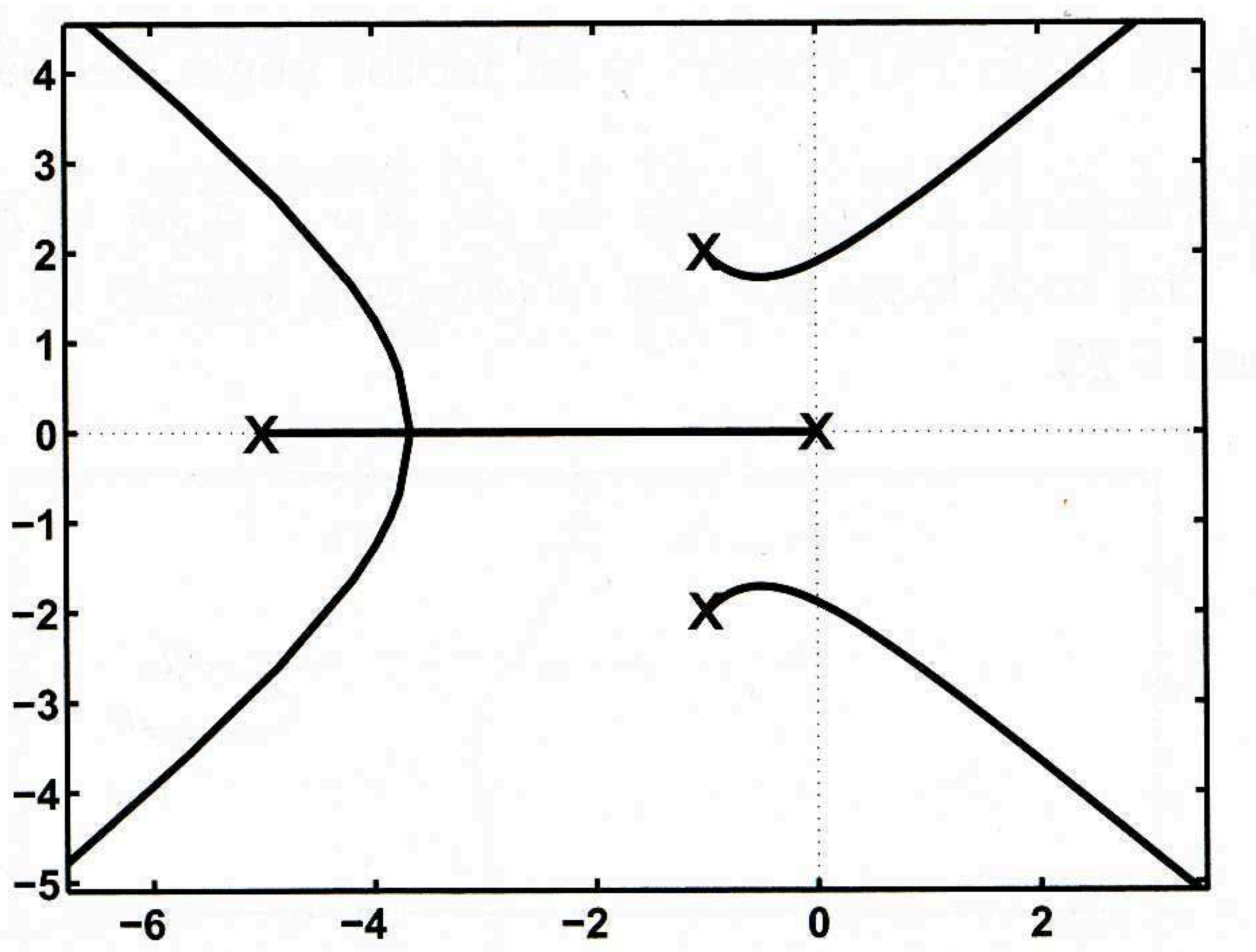
$\theta_l$feedback



Root locus with θ_m feedback



Root locus with θ_l feedback



Step response