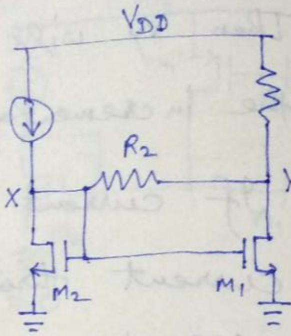
EE 204-2018-2 Analog Circuits
Homework #3 Solution

Question 1:
For each circuit shown below, sketch V_X and V_Y as a function of I_{REF}.

Q1 (a)

Here,

If we apply KCL at node X and node Y, then



At node X,

$$-I_{Ref} + \frac{K}{2}\left(V_{gs2} - V_T\right)^2 + \frac{V_X - V_Y}{R_2} = 0$$

$$\left[\because V_X = V_{gs2} = V_{gs1}\right]$$

$$-I_{Ref} + \frac{K}{2}\left(V_X - V_T\right)^2 + \frac{V_X - V_Y}{R_2} = 0 \longrightarrow (1)$$

At node Y,

$$\frac{K}{2}\left(V_X - V_T\right)^2 + \frac{V_Y - V_X}{R_2} + \frac{V_Y - V_{DD}}{R_1} = 0$$

$$\frac{V_Y}{R_2} + \frac{V_Y}{R_1} = \frac{V_X}{R_2} + \frac{V_{DD}}{R_1} - \frac{K}{2}\left(V_X - V_T\right)^2$$

$$V_Y = \frac{R_1 R_2}{R_1 + R_2}\left[\frac{V_{DD}}{R_1} + \frac{V_X}{R_2} - \frac{K}{2}\left(V_X - V_T\right)^2\right] \longrightarrow (2)$$

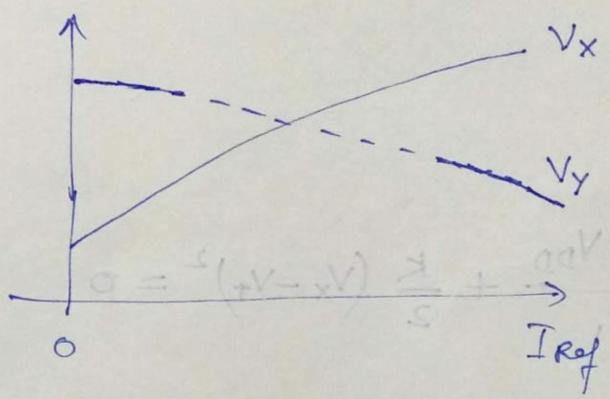
Put equation (2) in (1), then

$$-I_{Ref} + \frac{K}{2}\left(V_X - V_T\right)^2 + \frac{V_X}{R_2} - \frac{R_1}{R_1 + R_2}\left[\frac{V_{DD}}{R_1} + \frac{V_X}{R_2} - \frac{K}{2}\left(V_X - V_T\right)^2\right] = 0$$

$$\frac{K}{2}\left(V_X - V_T\right)^2\left(1 + \frac{R_1}{R_1+R_2}\right) + V_X\left[\frac{1}{R_2} - \frac{R_1}{R_2(R_1+R_2)}\right]$$

$$= \frac{V_{DD}}{R_1+R_2} + I_{Ref} \quad \text{--- (3)}$$

From equation (2) and (3), we can conclude that if I_{ref} increases, then V_X also increases. And if V_X increases, then V_Y decreases effectively.



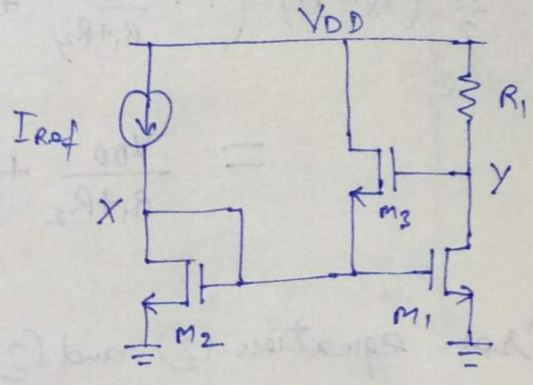
Note :- If we see the circuit intuitively,

~~If I_{Ref} increases~~,

- It is a current mirror circuit.
- If I_{ref} increases, it will increase the voltage at node X.
- $V_X = V_{gs1}$, so current through M_1 will also increase due to V_X.
- To maintain the incremental current through M_1, V_Y should decrease.

Q1 (b)

Here,
We are applying KCL at node X and node Y.



At node X,

$$-I_{Ref} + \frac{K}{2}(V_x - V_T)^2 - \frac{K}{2}(V_y - V_x)^2 = 0 \quad \text{— (1)}$$

At node Y,

$$\frac{V_y - V_{DD}}{R_1} + \frac{K}{2}(V_x - V_T)^2 = 0$$

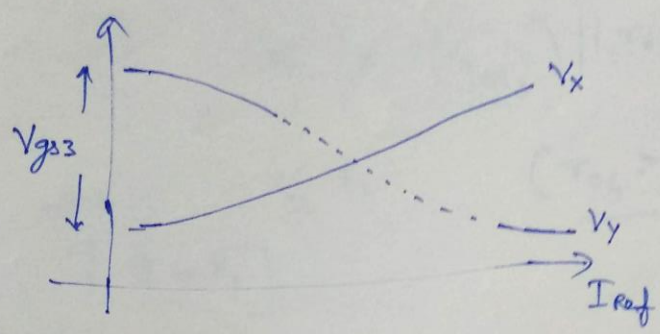
$$V_y = V_{DD} - \frac{K R_1}{2}(V_x - V_T)^2 \quad \text{— (2)}$$

Put equation (2) ~~at nod~~ in (1), then we get

$$I_{Ref} = \frac{K}{2}(V_x - V_T)^2 - \frac{K}{2}\left[V_{DD} - \frac{KR_1}{2}(V_x - V_T)^2 - V_x\right]^2 \quad \text{— (3)}$$

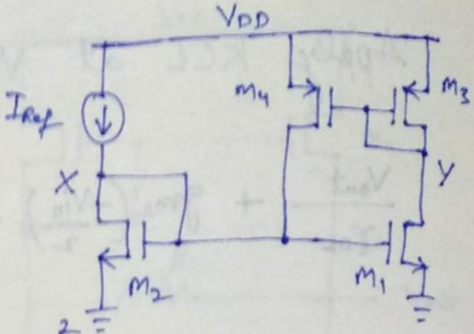
In equation (3), if I_{Ref} is increasing, then to maintain the equality, V_x should increase.

In equation (2), if V_x increases, then V_Y will decrease.



→ Here, you can apply same intuition as Q 1(a).

Q1(c)



Apply KCL at X,

$$-I_{Ref} + \frac{k}{2}(V_X - V_T)^2 - \frac{k}{2}(V_{DD} - V_Y - V_T)^2 = 0$$

$$I_{Ref} = \frac{k}{2}(V_{DD} - V_Y - V_T)^2 - \frac{k}{2}(V_X - V_T)^2 \quad \text{——} (1)$$

Apply KCL at Y,

$$-\frac{k}{2}(V_{DD} - V_Y - V_T)^2 + \frac{k}{2}(V_X - V_T)^2 = 0$$

$$(V_{DD} - V_Y - V_T)^2 = (V_X - V_T)^2 \quad \text{——} (2)$$

Put equation (2) in (1),

$$I_{Ref} = \frac{k}{2}(V_X - V_T)^2 - \frac{k}{2}(V_X - V_T)^2 = 0$$

⇒ Now we will solve this circuit by intuition.

↳ This is the positive feedback circuit with back to back current mirror.

→ If I_{Ref} increases, V_X will increase.

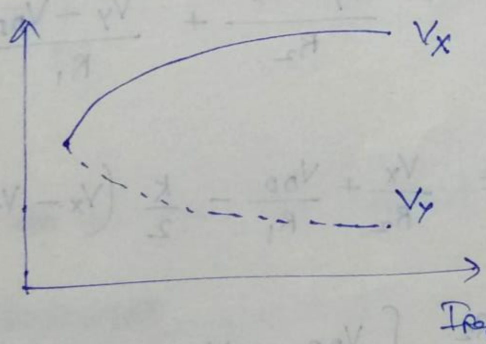
→ V_X will increase the current through M_1 and same ~~...~~ increase in M_3 due to same branch.

→ Then V_Y will decrease slightly to maintain the incremental current.

→ If current through M_3 increases, then current through M_4 increases and it will increase the voltage at node X.

→ Voltage at node Y will start decrease again and so on.

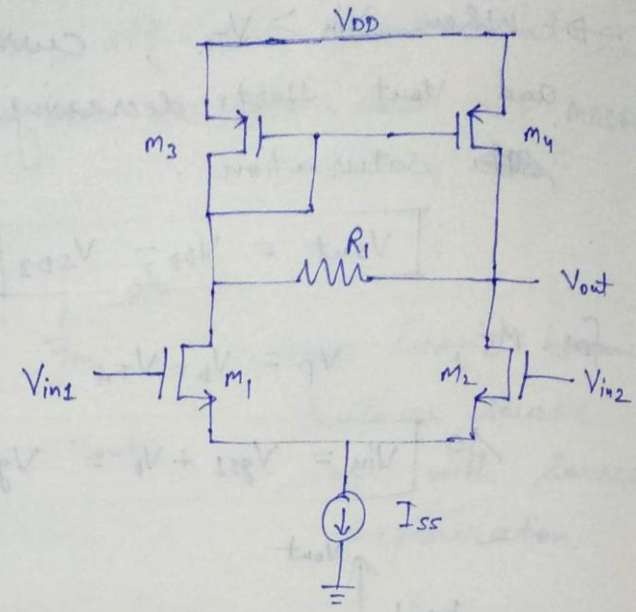
→ If V_X becomes so high, then M_1 & M_4 go in triode region.



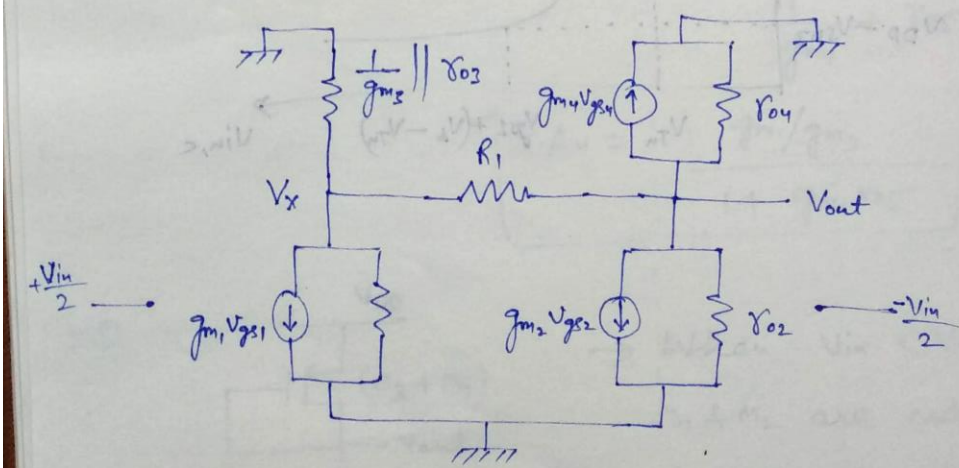
If V_X and V_Y are initially same.

Question 2:
Calculate the gain of the following circuit.

Q.2



Small signal equivalent circuit



Apply KCL at V_x,

$$\frac{V_x}{\frac{1}{g_{m3}}||r_{o3}} + \frac{V_x - V_{out}}{R_1} + \frac{V_x}{r_{o1}} + g_{m1}\frac{V_{in}}{2} = 0$$

$$V_x = \frac{-g_{m1}\frac{V_{in}}{2} + \frac{V_{out}}{R_1}}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}}||r_{o3}\right)} \qquad (1)$$

Apply KCL at Vout,

$$\frac{V_{out}}{r_{o2}} + g_{m2}\left(\frac{-V_{in}}{2}\right) + \frac{V_{out} - V_x}{R_1} + \frac{V_{out}}{r_{o4}} + g_{m4} V_x = 0$$

$$V_{out}\left[\frac{1}{r_{o2}} + \frac{1}{R_1} + \frac{1}{r_{o4}}\right] = g_{m2}\frac{V_{in}}{2} + V_x\left[\frac{1}{R_1} - g_{m4}\right] \quad \text{—— (2)}$$

Put equation (1) in (2), then we get

$$V_{out}\left[\frac{1}{r_{o2}} + \frac{1}{R_1} + \frac{1}{r_{o4}}\right] = g_{m2}\frac{V_{in}}{2} + \left[\frac{1}{R_1} - g_{m4}\right]\left[\frac{-g_{m1}\frac{V_{in}}{2} + \frac{V_{out}}{R_1}}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}} \| r_{o3}\right)}\right]$$

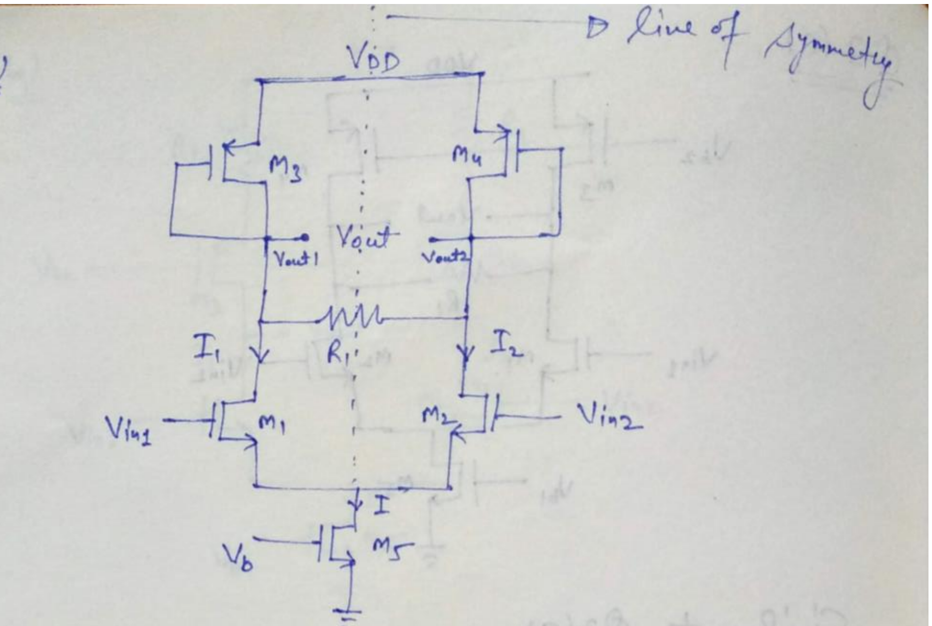
By simplification,

$$V_{out}\left[\frac{1}{r_{o2}} + \frac{1}{R_1} + \frac{1}{r_{o4}} - \frac{\frac{1}{R_1}\left(\frac{1}{R_1} - g_{m4}\right)}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}} \| r_{o3}\right)}\right] = V_{in}\left[\frac{g_{m2}}{2} - \frac{\frac{g_{m1}}{2}\left[\frac{1}{R_1} - g_{m4}\right]}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}} \| r_{o3}\right)}\right]$$

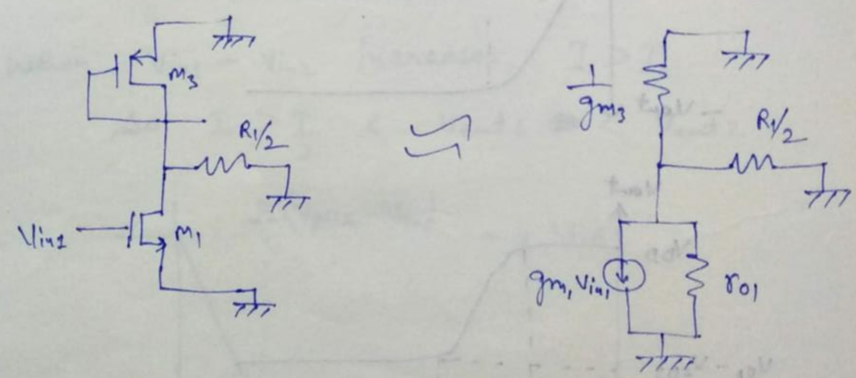
$$A_v = \frac{V_{out}}{V_{in}} = \frac{\frac{g_{m2}}{2} - \frac{\frac{g_{m1}}{2}\left(\frac{1}{R_1} - g_{m4}\right)}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}} \| r_{o3}\right)}}{\frac{1}{r_{o2}} + \frac{1}{R_1} + \frac{1}{r_{o4}} - \frac{\frac{1}{R_1}\left(\frac{1}{R_1} - g_{m4}\right)}{\frac{1}{R_1} + \frac{1}{r_{o1}} + \left(\frac{1}{g_{m3}} \| r_{o3}\right)}}$$

Question 3:
Assuming all of the circuits shown in figure below are symmetric, sketch V_{out} as (a) V_{in1} and V_{in2} vary differentially from zero to VDD, and (b) V_{in1} and V_{in2} are equal and they vary from zero to VDD.

Q.3 (a)



This circuit is symmetrical, so AC potential at line of symmetry would be zero.



Differential gain is

$$\boxed{A_v = g_{m1}\left(\frac{1}{g_{m3}} \,||\, r_{o1} \,||\, R_1/2\right)}$$

To plot the graph, initially we take $V_{in1} = V_{in2}$

$V_{id} = V_{in1} - V_{in2} = 0$

If $V_{in1} = V_{in2}$, then $\boxed{I_1 = I_2 = \frac{I}{2}}$.

$\Rightarrow$ $V_{out} = 0$ because $V_{out1} = V_{out2}$

$\Rightarrow$ As $V_{in1} - V_{in2}$ increases, $I_1 > I_2$.

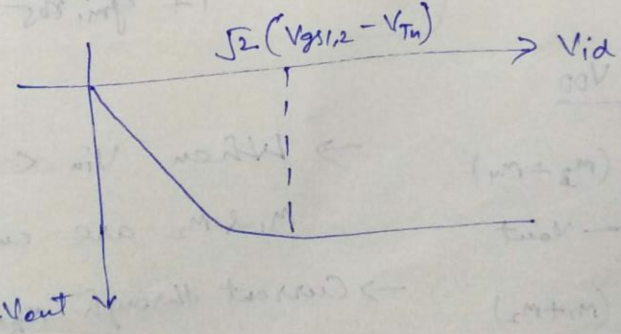
So $I_1 > \frac{I}{2}$

and $V_{out1} < V_{out2}$

then V_{out} will start decreasing.

$\Rightarrow$ After certain point, $I_2 = 0$ & M_2 goes in cut-off region. Now

$\boxed{I_1 = I}$ and V_{out} becomes constant



$\Rightarrow$ When $V_{in1} = V_{in2}$, $I_1 = I_2 = \frac{I}{2}$

$$\frac{I}{2} = \frac{k}{2}(V_{gs1,2} - V_{Tn})^2$$

$$V_{gs1,2} - V_{Tn} = \sqrt{\frac{I}{k}}$$

When $I_1 = I$

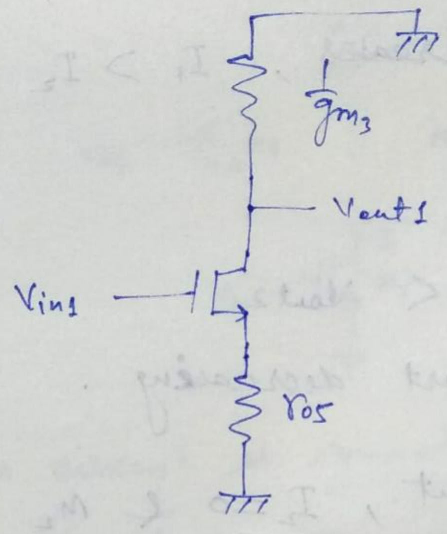
$$I = \frac{k}{2}(V_{gs1} - V_{Tn})^2$$

$$V_{gs1} - V_{Tn} = \sqrt{\frac{2I}{k}}$$

$$\boxed{V_{gs1} - V_{Tn} = \sqrt{2}(V_{gs1,2} - V_{Tn})}$$

$= V_{in2}$

In common mode, all node potentials at line of symmetry is open circuit and all MOSFETs can be added.

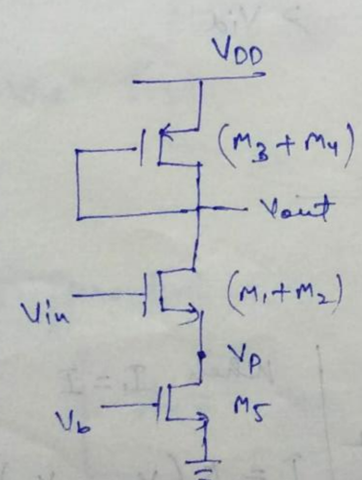


Now circuit becomes common source stage with source degeneration.

$$A_V = \frac{\frac{1}{g_{m_3}}}{\frac{1}{g_{m_1}} + r_{o5}}$$

$$\boxed{A_V = \frac{g_{m_1}/g_{m_3}}{1 + g_{m_1} r_{o5}}}$$

$\Rightarrow$



$\rightarrow$ When $V_{in} < V_{Tn}$

M_1 & M_2 are cutoff.

$\rightarrow$ Current through $M_3 + M_4$,

$$I_D = \frac{k}{2}(V_{SG} - V_{Tp})^2 = 0$$

$$V_{SG} = V_{Tp}$$

$$V_{DD} - V_{out} = V_{Tp}$$

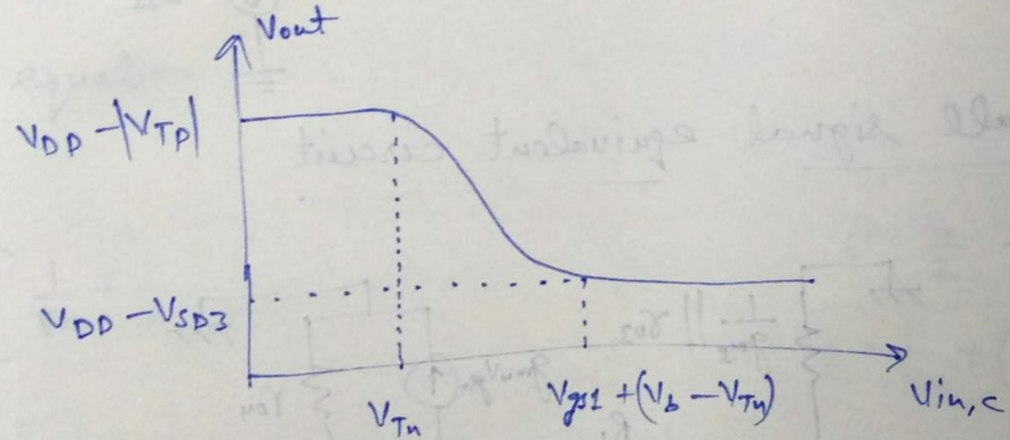
$$\boxed{V_{out} = V_{DD} - V_{Tp}}$$

→ When $V_{in} \geq V_{Tn}$, current starts flowing and V_{out} starts decreasing till M_5 goes into saturation.

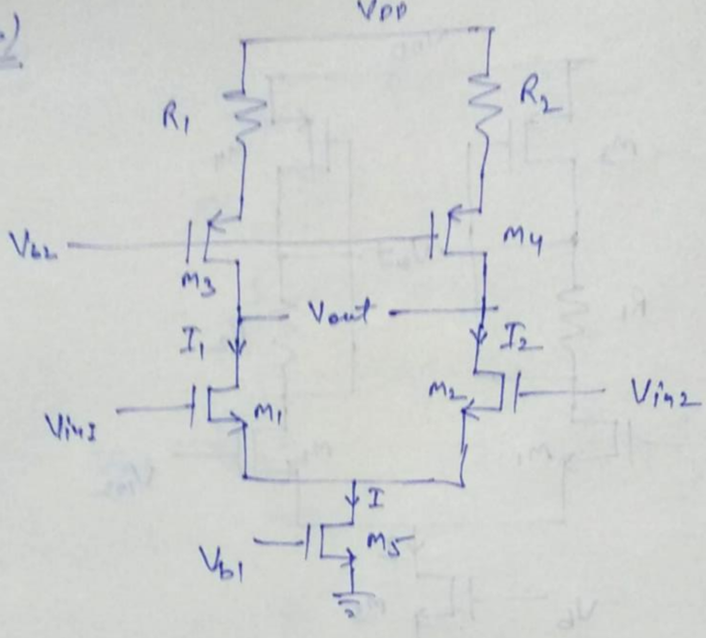
$$\boxed{V_{out} = V_{DD} - V_{SD3}}$$

for M_5, $V_p = V_b - V_{Tn}$

so $\boxed{V_{in} = V_{gs1} + V_p = V_{gs1} + (V_b - V_{Tn})}$



Q3 (b)



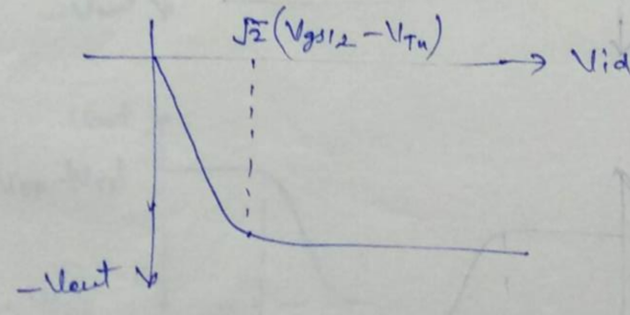
Similar to Q3(a),

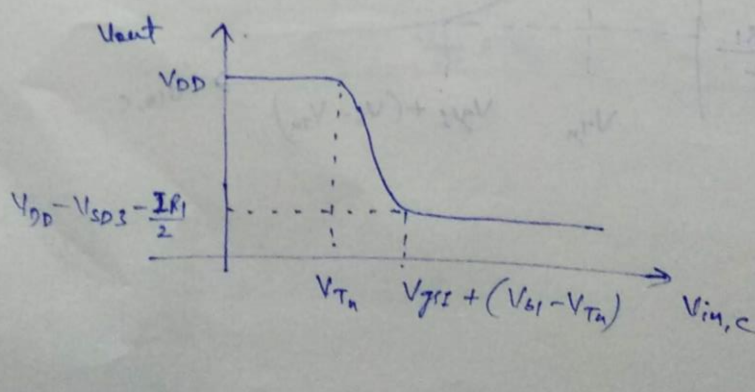
When $V_{in1} = V_{in2}$, then $\boxed{I_1 = I_2 = \frac{I}{2}}$

and $\boxed{V_{out} = 0}$

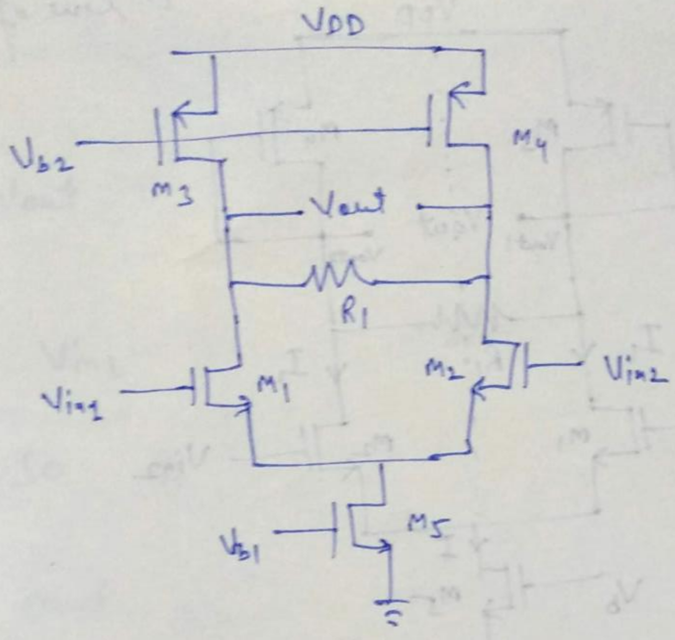
When $V_{in1} - V_{in2}$ increases, $I_1 > I_2$

So $I_1 > \frac{I}{2}$ & $V_{out1} < V_{out2}$

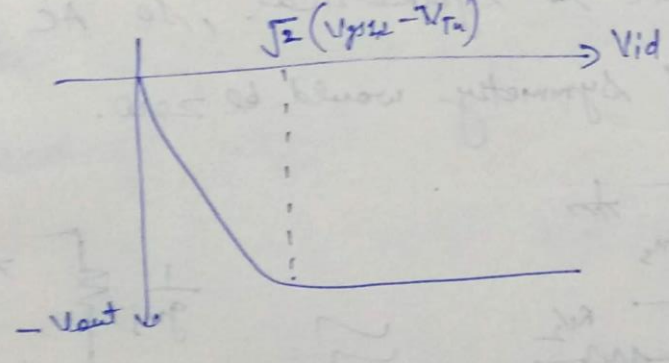


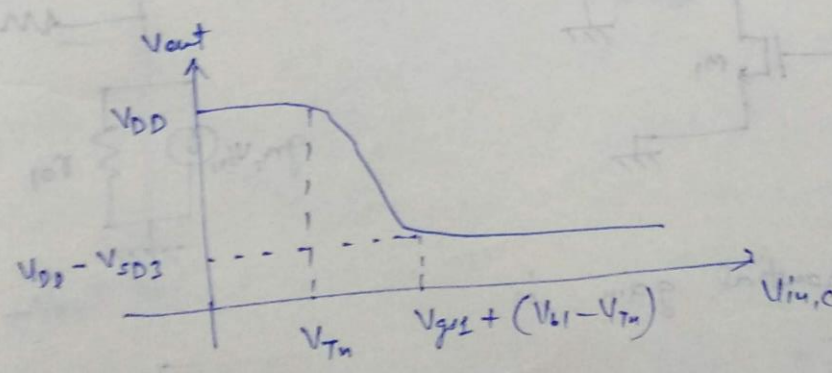


Q3 (C)

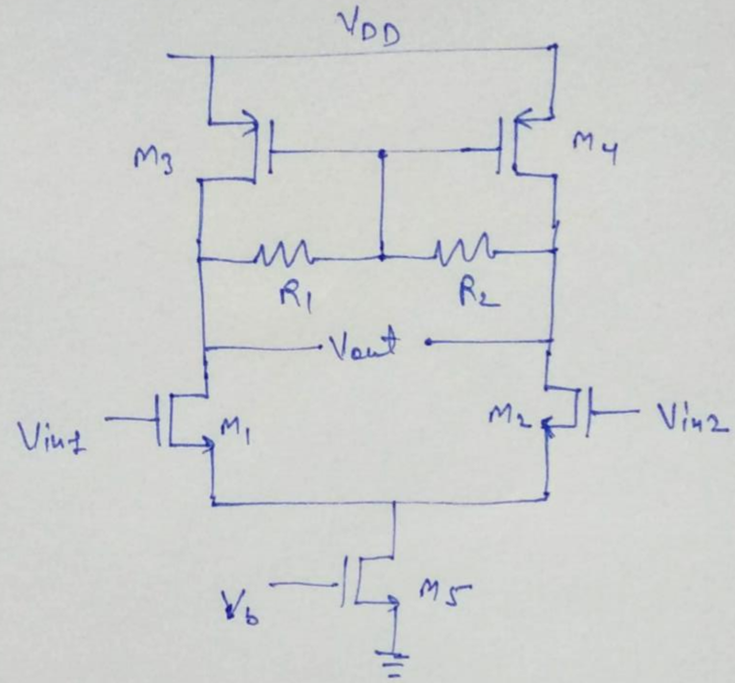


Similar to Q3(a),

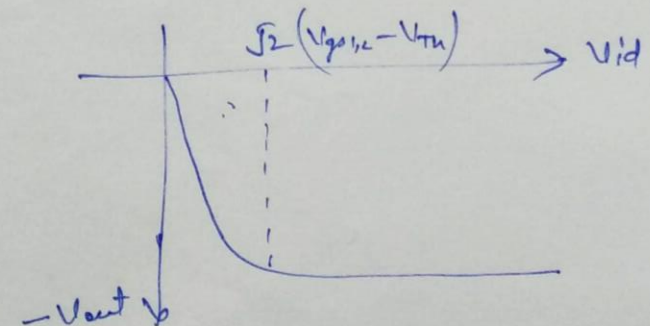


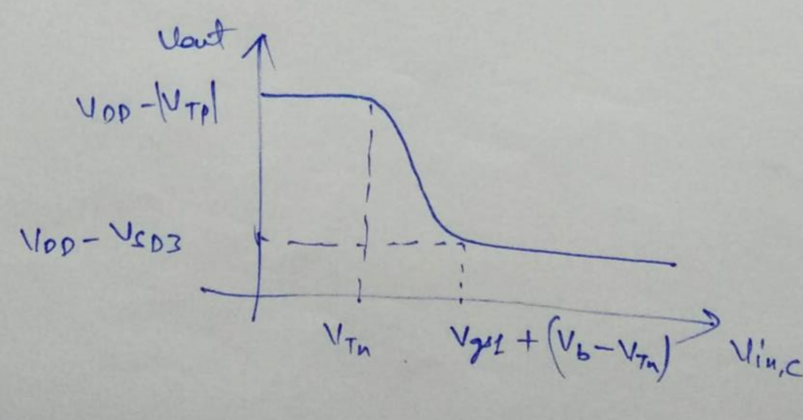


Q3 (d)

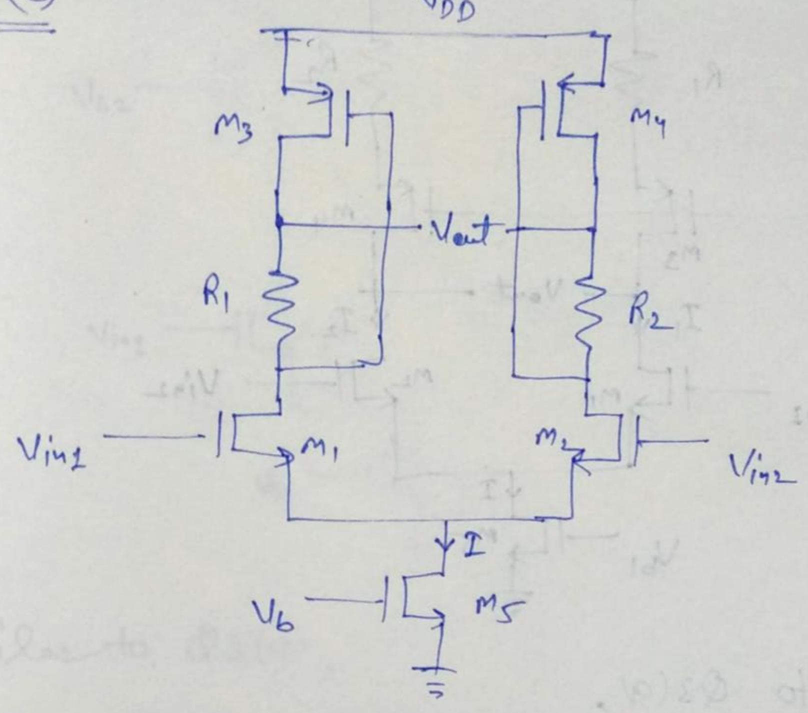


Similar to Q3 (a),





Q3 (e)



Similar to Q3(a)