

Question 3

The exclusive OR (XOR) operation \oplus is defined as follows:

$$(a \oplus b) = \bar{a}.b + a.\bar{b}$$

Show that $a.b = (a + b).(\overline{a \oplus b})$. (2 marks)

Solution: Question 3

$$\begin{aligned}\overline{a \oplus b} &= \overline{\bar{a}.b + a.\bar{b}} \\ &= (\overline{\bar{a}.b}).(\overline{a.\bar{b}}) && \dots(\text{DeMorgan's law}) \\ &= (a + \bar{b}).(\bar{a} + b) && \dots(\text{DeMorgan's law}) \\ &= a.\bar{a} + a.b + \bar{b}.\bar{a} + \bar{b}.b && \dots(\text{Distributivity}) \\ &= a.b + \bar{b}.\bar{a}\end{aligned}$$

$$\begin{aligned}(a + b).(\overline{a \oplus b}) &= (a + b).(a.b + \bar{b}.\bar{a}) \\ &= a.a.b + b.a.b + a.\bar{a}.\bar{b} + b.\bar{a}.\bar{b} && \dots(\text{DeMorgan's law}) \\ &= a.b + a.b + 0 + 0 \\ &= a.b\end{aligned}$$