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(1) If $\bar{A} = 10\rho^{1.5}\,\hat{a}_z$ wb/m. in free space. find $\bar{J}$

$\bar{B} = \nabla \times \bar{A}$

$$\nabla \times \bar{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & 10\rho^{1.5} \end{vmatrix}$$

$$= -\frac{\partial}{\partial\rho}(10\rho^{1.5})\,\hat{a}_\phi = -10\times1.5\,\rho^{0.5}\,\hat{a}_\phi$$

$$= -15\rho^{0.5}\,\hat{a}_\phi \text{ Tesla.}$$

$$\bar{H} = \frac{1}{\mu_0}\bar{B} = \frac{-15}{4\pi\times10^{-7}}\rho^{0.5}\,\hat{a}_\phi = -1.193\times10^{7}\rho^{0.5}\,\hat{a}_\phi \text{ A/m.}$$

$\bar{J} = \nabla \times \bar{H}$

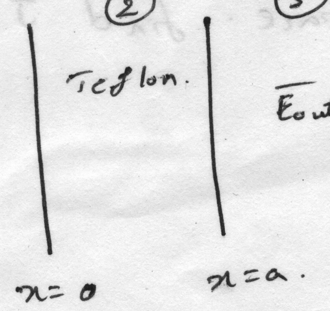
$$\nabla \times \bar{H} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & -1.19\times10^{7}\rho^{0.5} & 0 \end{vmatrix} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(-1.193\times10^{7}\rho^{0.5})\,\hat{a}_z$$

$$= \frac{1}{\rho}(-1.193\times10^{7}\times0.5)\frac{1}{\rho^{0.5}}\,\hat{a}_z$$

$$= -0.5965\times10^{7}\frac{1}{\rho^{3/2}}\,\hat{a}_z \text{ A/m}^2$$

(2) we locate a slab of Teflon in the region $0 \le x \le a$ and assume free space where $x<0$ and $x>a$, outside the teflon there is a uniform field $\overline{E_{out}} = E_0\,\hat{a}_x$ V/m we seek values for $\bar{D}$, $\bar{E}$ everywhere.

① ② ③



$\overline{E_{out}} = E_0 \hat{a_x}$

$D_{1N} = D_{2N} = D_{3N} = \varepsilon_0 E_0$

$\overline{D_{1N}} = \overline{D_{2N}} = \overline{D_{3N}} = \varepsilon_0 E_0 \hat{a_x}$

$D_{2N} = \varepsilon_{Teflon} E_{2N}$

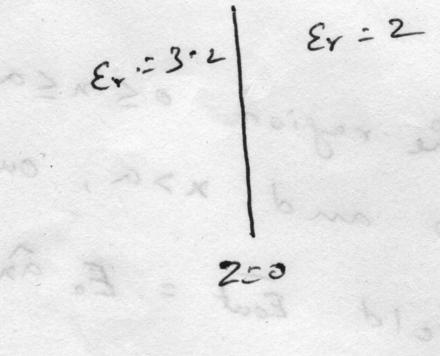
$\overline{E_{2N}} = \dfrac{\varepsilon_0}{2.1} E_0 \hat{a_x}$

$= 0.476\, E_0 \hat{a_x}$

$E_{1N} = \dfrac{D_{1N}}{\varepsilon_0}$

$\overline{E_{1N}} = E_0 \hat{a_x}$

③ let the region $z<0$ be composed of a uniform dielectric material for which $\varepsilon_r = 3.2$. while the region $z>0$ is characterized by $\varepsilon_r = 2$. let $\overline{D_1} = -30\hat{a_x} + 50\hat{a_y} + 70\hat{a_z}\ nC/m^2$ and find ⓐ D_{N1} ⓑ $\overline{D_{t1}}$ ⓒ D_{t1} ⓓ D_1 ⓔ θ_1 ⓕ $\overline{D_{N2}}$ ⓖ $\overline{D_{t2}}$ ⓗ $\overline{D_2}$ ⓘ θ_2



ⓐ $D_{N1} = 70\, nC/m^2$.

ⓑ $\overline{D_{t1}} = -30\hat{a_x} + 50\hat{a_y}\ nC/m^2$.

ⓒ $D_{t1} = \sqrt{900 + 2500} = 58.3\, nC/m^2$.

ⓓ $D_1 = \sqrt{900 + 2500 + 4900} = 91.1\, nC/m^2$

ⓔ $D_{N1} = D_1 \cos\theta_1$

$\theta_1 = \cos^{-1}\left(\dfrac{70}{91.1}\right) = 39.8^\circ$

ⓕ $\overline{D_{N2}} = \overline{D_{N1}} = 70\, nC/m^2\ \hat{a_z}$

ⓖ $\overline{D_{t2}} \cdot \dfrac{\varepsilon_2}{\varepsilon_1} = \varepsilon_2 \overline{E_{t2}}$

$$\overline{E_{t2}} = \overline{E_{t1}} = \frac{-30\,\hat{a_x} + 50\,\hat{a_y}\ nC/m^2}{\varepsilon_1}$$

$$\overline{D_{t2}} = \frac{\varepsilon_2}{\varepsilon_1}\left(-30\,\hat{a_x} + 50\,\hat{a_y}\right)\ nC/m^2$$

$$= \frac{2}{3.2}\left(-30\,\hat{a_x} + 50\,\hat{a_y}\right) nC/m^2 = -18.75\,\hat{a_x} + 31.25\,\hat{a_y}\ nC/m^2$$

(h) $\overline{D_2} = \overline{D_{t2}} + \overline{D_{N2}}$

$\overline{D_{N2}} = \overline{D_{N1}} = 70\,\hat{a_z}\ nC/m^2.$

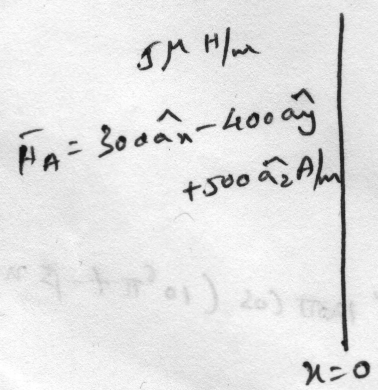
$\overline{D_2} = \left(-18.75\,\hat{a_x} + 31.25\,\hat{a_y} + 70\,\hat{a_z}\right)\ nC/m^2.$

(i) $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}$

$\tan\theta_2 = \dfrac{2}{3.2}\tan(39.8^\circ) \Rightarrow \theta_2 = 27.5^\circ$

(4) let the permittivity be $5\mu H/m$ in region A where $x<0$, and $20\mu H/m$ in region B where $x>0$. If there is a surface current density $\overline{K} = 150\,\hat{a_y} - 200\,\hat{a_z}$ A/m at $x=0$ and if $\overline{H_A} = 300\,\hat{a_x} - 400\,\hat{a_y} + 500\,\hat{a_z}$ A/m find.

(a) $|\overline{H_{tA}}|$ (b) $|\overline{H_{NA}}|$ (c) $|\overline{H_{tB}}|$ (d) $|\overline{H_{NB}}|$



(a) $\overline{H_{tA}} = -400\,\hat{a_y} + 500\,\hat{a_z}$

$|\overline{H_{tA}}| = 640.3$ A/m.

(b) $\overline{H_{NA}} = 300\,\hat{a_x}$

$|\overline{H_{NA}}| = 300$ A/m.

(C) $\bar{H}_{tA} - \bar{H}_{tB} = \hat{a}_{N12} \times \bar{K}$

$$-400\hat{a_y} + 500\hat{a_z} - \bar{H}_{t2} = \hat{x} \times (150\hat{a_y} - 200\hat{a_z})$$

$$\bar{H}_{t2} = -600\hat{a_y} + 350\hat{a_z}$$

$$|\bar{H}_{t2}| = 694.6\ A/m.$$

(D) $B_{NA} = B_{NB}$

$$5\times10^{-6} \times 300 = 20\times10^{-6} H_{NB} \Rightarrow |\bar{H}_{NB}| = 75\ A/m.$$

(5) What value of A and β are required if the two fields

$$\bar{E} = 120\pi \cos(10^6\pi t - \beta x)\hat{y}\ V/m.$$

$$\bar{H} = A\pi \cos(10^6\pi t - \beta x)\hat{z}\ A/m.$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\mu\frac{\partial}{\partial t}\{H_z\}$$

$$\frac{\partial}{\partial x}\{120\pi\cos(10^6\pi t - \beta x)\} = -\mu\frac{\partial}{\partial t}\{A\pi\cos(10^6\pi t - \beta x)\}$$

$$\Rightarrow 120\pi \sin(10^6\pi t - \beta x)\beta = A\pi\mu \sin(10^6\pi t - \beta x)(10^6\pi)$$

$$\Rightarrow 120\pi\beta = A\pi \times 10^6 \times \pi\mu \Rightarrow \beta = \frac{A\pi\mu\times10^6}{120} \quad -(1)$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = \varepsilon\frac{\partial}{\partial t}\{E_y\}$$

$$-\frac{\partial}{\partial x}\{A\pi\cos(10^6\pi t - \beta x)\} = \varepsilon\frac{\partial}{\partial t}\{120\pi\cos(10^6\pi t - \beta x)\}$$

$$-A\pi\beta \sin(10^6\pi t-\beta x) = -\frac{1}{\ } 120\pi^2 \varepsilon \sin(10^6\pi t-\beta x)\times 10^6$$

$$\Rightarrow A\beta\pi = \varepsilon\, 120\pi^2 \times 10^6 \Rightarrow A\beta = 120\times 10^6 \pi \varepsilon. \quad -(2)$$

Substituting (1) in (2)

$$\frac{A^2\pi\times 10^6\mu}{120} = 120\times 10^6\pi\varepsilon \Rightarrow A = \frac{1}{\pi}$$

$$\beta = \frac{A\pi\times 10^6\mu}{120} = \frac{10^6\mu}{120} = \frac{10^6\times 4\pi\times 10^{-7}}{120} = \frac{\pi}{30}$$

(6) A time dependent electric field intensity is given as $\bar{E} = 10\pi\cos(10^6 t-50z)\,\hat{x}$ V/m. The field exists in a material with properties $\varepsilon_r = 4$ and $\mu_r = 1$. Given that $J=0$ and $\rho=0$, calculate the magnetic field intensity and magnetic flux density in the material.

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

$$-\frac{\partial\bar{B}}{\partial t} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{y}\frac{\partial}{\partial z}E_x.$$

$$\Rightarrow -\frac{\partial\bar{B}}{\partial t} = \frac{\partial}{\partial z}\left\{10\pi\cos(10^6 t-50z)\right\}\hat{y}$$

$$\frac{\partial\bar{H}}{\partial t} = -125\times 10^7 \sin(10^6 t-50z)\,\hat{y}.$$

$$\bar{H} = \cancel{125\times 10^7}\ 1250\cos(10^6 t-50z)\,\hat{y}$$

$$\bar{B} = \mu\bar{H} = 4\pi\times 10^{-7}\times 1250\cos(10^6 t-50z)\,\hat{y}.$$

$$= 5\times 10^{-4}\pi\cos(10^6 t-50z)\,\hat{y}.$$

~~(7) let $\mu = 10^{-5}$ H/m, $\varepsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$. find k (including units) so that each of the following pairs of fields satisfies maxwell's equations.~~

~~(a) $\bar{D} = 6\hat{a}_x - 2y\,\hat{a}_y + 2z\,\hat{a}_z$ nC/m², $\bar{H} = kx\,\hat{a}_x + 10y\,\hat{a}_y - 25z\,\hat{a}_z$ A/m.~~

~~(b) $\bar{E} = (20y - kt)\,\hat{a}_x$ V/m, $\bar{H} = (y + 2\times10^{6}t)\,\hat{a}_z$ A/m.~~

let $\mu = 3\times10^{-5}$ H/m, $\varepsilon = 1.2\times10^{-10}$ F/m and $\sigma = 0$ everywhere. If $\bar{H} = 2\cos(10^{10}t - \beta x)\,\hat{a}_z$ A/m, use maxwell's equations to obtain expressions for $\bar{B}$, $\bar{D}$, $\bar{E}$ and β.

$$\bar{B} = \mu\bar{H} = 3\times10^{-5}\times 2\cos(10^{10}t - \beta x)\,\hat{a}_z \text{ A/m}.$$

$$= 6\times10^{-5}\cos(10^{10}t - \beta x)\,\hat{a}_z \text{ T}. \quad 1$$

$$\nabla\times\bar{H} = \frac{\partial\bar{D}}{\partial t}$$

$$\frac{\partial\bar{D}}{\partial t} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = -\frac{\partial}{\partial x}H_z\,\hat{y}$$

$$= 2\beta\sin(10^{10}t - \beta x)\,\hat{y}.$$

$$\bar{D} = -\frac{2\beta}{10^{10}}\cos(10^{10}t - \beta x)\,\hat{y} \text{ C/m}^2.$$

$$\bar{E} = \frac{\bar{D}}{\varepsilon} = -1.67\beta\cos(10^{10}t - \beta x)\,\hat{y} \text{ V/m}.$$

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

$$\nabla\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \frac{\partial}{\partial x}\left\{-1.67\beta\cos(10^{10}t - \beta x)\right\}\hat{z}$$

$$= 1.67\beta^2\sin(10^{10}t - \beta x)\,\hat{z}.$$

$$\bar{B} = -\int 1.67\,\beta^2 \sin(10^{10}t - \beta x)\,dt\,\hat{z}$$

$$= 1.67\times10^{-10}\beta^2\cos(10^{10}t - \beta x)\,\hat{z}$$

Comparing with eq (1)

$$1.67\times10^{-10}\beta^2 = 6\times10^{-5} \Rightarrow \beta = 599.4 \text{ rad/m.}$$

(8) For a current distribution in free space.

$$\bar{A} = (2x^2y + yz)\,\hat{a}_x + (xy^2 - xz^3)\,\hat{a}_y - (6xyz - 2x^2y^2)\,\hat{a}_z \text{ wb/m.}$$

(a) Calculate $\bar{B}$

(b) Find the magnetic flux through a loop described by

$x = 1,\ 0 < y < 2,\ 0 < z < 2$

(c) Show that $\nabla\cdot\bar{A} = 0$ and $\nabla\cdot\bar{B} = 0$

(a) $$\bar{B} = \nabla\times\bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left(\frac{d}{dy}A_z - \frac{d}{dz}A_y\right)\hat{a}_x - \left(\frac{d}{dx}A_z - \frac{d}{dz}A_x\right)\hat{a}_y + \left(\frac{d}{dx}A_y - \frac{d}{dy}A_x\right)\hat{a}_z$$

$$= (-6xz + 4x^2y + 3xz^2)\,\hat{a}_x + (6yz - 4xy^2 + y)\,\hat{a}_y + (y^2 - z^3 - 2x^2 - z)\,\hat{a}_z \text{ wb/m}^2$$

(b) $$\phi = \int \bar{B}\cdot d\bar{s}$$

$$= \int_0^2\int_0^2 (-6z + 4y + 3z^2)\,dy\,dz$$

$$= \int_0^2 \left[-6\frac{z^2}{2} + 4yz + \frac{3z^3}{3}\right]_0^2 dy = \int_0^2 (8y - 4)\,dy$$

$$= \left[\frac{8y^2}{2} - 4y\right]_0^2 = 16 - 8 = 8\ wb$$

(c) $\nabla \cdot \bar{A} = 0$

$$\cancel{\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}}$$

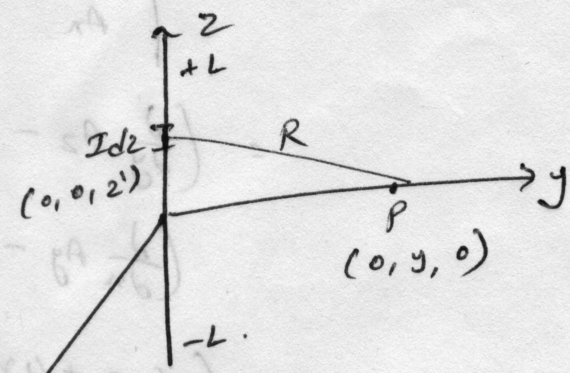
$$\vec{\nabla} \cdot \bar{A} = \frac{\partial}{\partial x}(2x^2y + yz) + \frac{\partial}{\partial y}(x \cdot y^2 - xz^3) - \frac{\partial}{\partial z}(6xyz - 2x^2y^2)$$

$$= 4xy + 2xy - 6xy = 0$$

$\nabla \cdot \bar{B} = 0$

$$\nabla \cdot \bar{B} = -6z + 8xy + 3z^2 + 6z - 8xy + 1 - 3z^2 - 1 = 0$$

(9) Magnetic field about a long straight wire carrying current I using the vector potential.



$$\bar{A} = \int \frac{\mu}{4\pi} \frac{I\, d\bar{l}}{R} \qquad R = \sqrt{z'^2 + y^2}$$

$$= \frac{\mu I}{4\pi} \int_{-L}^{L} \frac{dz}{\sqrt{y^2 + z'^2}} \hat{a}_z$$

$$= \frac{\mu I}{2\pi} \left\{ \ln\left[z' + \sqrt{z'^2 + y^2}\right] \right\}_0^L \hat{a}_z$$

$$= \frac{\mu I}{2\pi} \left\{ \ln\left[L + \sqrt{L^2 + y^2}\right] - \ln(y) \right\} \hat{a}_z$$

for $L \gg y$ vector potential is approximated by.

$$\bar{A} \approx \frac{\mu I}{2\pi}\left[\ln(2L) - \ln y\right] \hat{a}_z$$

$$\bar{B} = \nabla \times \bar{A}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \frac{\partial}{\partial y} A_z \, \hat{a}_x$$

$$\bar{B} = \frac{\mu I}{2\pi}\left(-\frac{1}{y}\right) \hat{a}_x$$

$$\bar{H} = -\frac{I}{2\pi y} \hat{a}_x \quad A/m.$$

(10) If $V = 10 \sin \omega t$, $\mu_r = 1$, $\varepsilon_r = 10$. Find $\nabla \cdot \bar{A}$

(a) $f = 50 Hz$ (b) $f = 100 THz$.

(a) $\nabla \cdot \bar{A} = -\mu \varepsilon \frac{dv}{dt}$.

$$\frac{dv}{dt} = (10 \cos \omega t)\,\omega = 10\omega \cos \omega t.$$

$$\mu \varepsilon \frac{dv}{dt} = \mu \varepsilon (10\, \omega \cos \omega t)$$

if $f = 50 Hz$

$$\omega = 2\pi \times 50$$

$$\nabla \cdot \bar{A} = -4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times 10 \times 10 \times 2\pi \times 50 \cos(100\pi t)$$

$$= -349.54 \times 10^{-15} \cos(100\pi t)$$

if $f = 100 THz$.

$$\omega = 2\pi \times 10^{14}$$

$$\nabla \cdot \bar{A} = -4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times 10 \times 10 \times 2\pi \times 10^{14} \cos(2\pi \times 10^{14} t)$$

$$= -0.699 \cos(2\pi \times 10^{14} t)$$