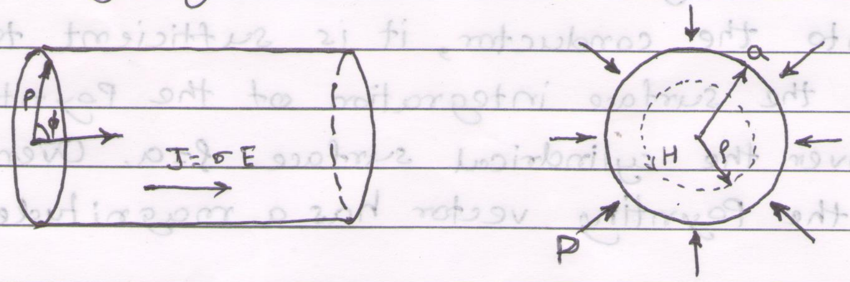
(1) The Poynting vector is $\bar{P} = \bar{E} \times \bar{H}$



We know that, $\bar{J} = \sigma \bar{E}$ $\therefore \bar{E} = \dfrac{\bar{J}}{\sigma}$

$$\therefore \bar{P} = \frac{\bar{J}}{\sigma} \times \bar{H}$$

Assume the current density $\bar{J}$ is in $\hat{z}$ direction then,

$$\bar{J} = J_z \bar{a}_z \quad \text{and} \quad \bar{H} = H_\phi \bar{a}_\phi$$

$$\bar{P} = \frac{J_z}{\sigma} \bar{a}_z \times H_\phi \bar{a}_\phi = -\frac{J_z}{\sigma} H_\phi \bar{a}_\rho \quad \ldots [\because \bar{a}_z \times \bar{a}_\phi = -\bar{a}_\rho$$

Now, the magnetic field intensity H_ϕ at any radius ρ is

$$= \frac{I}{2\pi\rho} = \frac{J_z \times \pi \rho^2}{2\pi\rho} = \frac{J_z \rho}{2}$$

$\ldots [\because I = J_z \times \text{area}$

$$\bar{P} \text{ at any radius} = -\frac{J_z}{\sigma} \times \frac{J_z \rho}{2} \bar{a}_\rho = -\frac{J_z^2}{2} \frac{\rho}{\sigma} \bar{a}_\rho$$

(2) (i) We have $a = 1cm$, $\sigma = 2\times10^{7}$ S/m, $I = 100$ A, $l = 1m$

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$$

$$= \frac{1}{2\times10^{7}\times\pi\times(1\times10^{-2})^2}$$

$$= 1.59\times10^{-4}\ \Omega$$

$$P = I^2 R$$

$$= (100)^2\times1.59\times10^{-4}$$

$$= 1.59\ W$$

(ii) $$J = \frac{I}{A} = \frac{100}{\pi\times(1\times10^{-2})^2}$$

$$= 318.31\times10^{3}\ (A/m^2)$$

$$\therefore \bar{J} = 318.31\times10^{3}\ \bar{a}_z\ (A/m^2)$$

We know that, $\bar{J} = \sigma\bar{E}$

$$\therefore \bar{E} = \frac{\bar{J}}{\sigma} = \frac{318.31\times10^{3}}{2\times10^{7}}\bar{a}_z$$

$$= 15.92\times10^{-3}\ \bar{a}_z\ (V/m)$$

Magnetic field within the conductor is,

$$\bar{H} = I\left(\frac{\rho}{2\pi a^2}\right)\bar{a}_\phi$$

where, a is the radius of the conductor and ρ is the radius of circular path inside, then

$$\bar{H} = 100\times\frac{1}{2\pi\,(10^{-2})^2}\rho\,\bar{a}_\phi$$

$$= 159.2\times10^{3}\rho\,\bar{a}_\phi\ \ (A/m)$$

The Poynting vector is

$$\overline{P} = \overline{E} \times \overline{H}$$

$$= \begin{vmatrix} \overline{a}_\rho & \overline{a}_\phi & \overline{a}_z \\ 0 & 0 & 15.92\times10^{-3} \\ 0 & 159.2\times10^{3}\times\rho & 0 \end{vmatrix}$$

$$= (-15.92\times10^{-3}\times159.2\times10^{3}\times\rho)\cdot\overline{a}_\rho$$

$$\overline{P} = -2534.1\rho\ \overline{a}_\rho$$

(iii) Integrating $\overline{P}$ over the cylindrical surface, enclosing one meter length of conductor, we get,

$$\int_s \overline{P}.\overline{ds} = \int_s \overline{P}.(\rho\, d\phi\, dz\, \overline{a}_\rho)$$

$$= \int_0^1\int_0^{2\pi} -2534.1\,\rho^2 d\phi\, dz$$

$$= -2534.1\times[\phi]_0^{2\pi}\ [z]_0^1\times(1\times10^{-2})^2$$

$$= -1.59\ W$$

and $\therefore |\overline{P}.\overline{ds}| = 1.59\ W$ which is same as in part (i)

(3) Given: $H_0 = 0.2$ and for free space, $\eta = 120\pi\,(\Omega)$

As the expression for field says that the wave is travelling in $+x$ direction,

$$\bar{P}_{avg} = \frac{1}{2\eta} E_0^2\, \bar{a}_x = \frac{1}{2}\eta H_0^2\, \bar{a}_x$$

$$\bar{P}_{avg} = \frac{1}{2}(120\pi)(0.2)^2\, \bar{a}_x$$

Normal to plane with equation $Ax + By + Cz + D = 0$

$$\bar{a}_n = \pm \frac{A\bar{a}_x + B\bar{a}_y + C\bar{a}_z}{\sqrt{A^2 + B^2 + C^2}}$$

For the given plane, $\bar{a}_n = \dfrac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}$

The differential area for the given plane,

$$\bar{ds} = ds\, \bar{a}_n$$

Power crossing the given plane is,

$$P = \int_s \bar{P}_{avg} \cdot \bar{ds} = \int_s \frac{1}{2}(120\pi)(0.2)^2\, \bar{a}_x \cdot ds\left(\frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}\right)$$

$$= \frac{1}{2}(120\pi)(0.2)^2 \frac{1}{\sqrt{2}} \int ds = \frac{1}{2}(120\pi)(0.2)^2 \frac{1}{\sqrt{2}}\, s$$

$$= \frac{1}{2}(120\pi)(0.2)^2 \times \frac{1}{\sqrt{2}} \times (0.1)^2$$

$$= 53.31 \text{ mW}$$

(ii) The given plane is $x=1=$ constant,

$$\overline{a_n} = \overline{a_x}$$

$$\overline{ds} = ds\,\overline{a_n} = ds\,\overline{a_x}$$

Power crossing the circular disc of radius 5 cm

$$P = \int_s \overline{P_{avg}} \cdot \overline{ds}$$

$$= \int_s \frac{1}{2}(120\pi)(0.2)^2\,\overline{a_x} \cdot ds\,\overline{a_x}$$

$$= \frac{1}{2}(120\pi)(0.2)^2\, s$$

$$= \frac{1}{2}(120\pi)(0.2)^2 \times \pi(0.05)^2$$

$$P = 59.23\ mW$$

(4) Given electric field intensity in phasor form

$$\bar{E}_s = E_0 (\bar{a}_y - j\bar{a}_z) e^{-j\beta x}$$

Instantaneous expression of electric field intensity will be,

$$\bar{E} = Re\{E_0 (\bar{a}_y - j\bar{a}_z) e^{j(\omega t - \beta x)}\}$$

$$= Re\{E_0 (\bar{a}_y - j\bar{a}_z)[\cos(\omega t) + j\sin(\omega t)]\} e^{-j\beta x}$$

$$= E_0 (\bar{a}_y \cos(\omega t) + \bar{a}_z \sin(\omega t)) e^{-j\beta z}$$

Therefore, the magnitude of the field is

$$|\bar{E}| = \sqrt{(E_0 \cos\omega t)^2 + (E_0 \sin\omega t)^2}$$

or, $|E_1|^2 + |E_2|^2 = E_0$

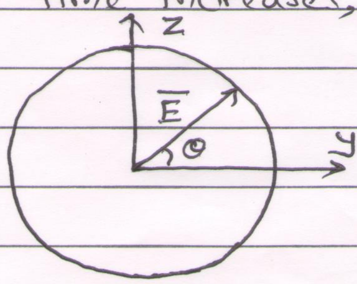
which is a circular equation i.e. the wave is circularly polarized.

Now, the instantaneous angle θ that the field $\bar{E}$ makes with y-axis is given as

$$\tan\theta = \frac{E_0 \sin\omega t}{E_0 \cos\omega t}$$

or, $\theta = \omega t$

$\therefore$ As time increases, $\bar{E}$ rotates from y to z as shown



and since the direction of wave propagation is in $+\bar{a}_z$ direction so, the rotation from y to z obeys the right hand rule.

$\therefore$ The field is right hand circularly polarized.

(5) Given the phasor form of electric field intensity,

$$\bar{E}_s = 4(\bar{a}_z - j\bar{a}_x)e^{-j\beta y}$$

So, the electric field intensity of the reflected wave will be

$$\bar{E}_{rs} = \Gamma[4(\bar{a}_z - j\bar{a}_x)]e^{j\beta y}$$

where Γ is the ~~electric~~ reflection coefficient at the interface.

For perfect conductor $\Gamma = -1$,

$$\therefore \quad \bar{E}_{rs} = 4(-\bar{a}_z + j\bar{a}_x)e^{j\beta y}.$$

$\therefore$ Instantaneous expression of the electric field of reflected wave will be

$$\bar{E} = Re\{4(-\bar{a}_z + j\bar{a}_x)(\cos\omega t + j\sin\omega t)\}e^{j\beta y}$$

$$= 4(-\cos(\omega t)\bar{a}_z - \sin(\omega t)\bar{a}_x)e^{j\beta y}$$

Therefore, the magnitude of the reflected field is

$$|\bar{E}| = \sqrt{(4\cos\omega t)^2 + (4\sin\omega t)^2}$$

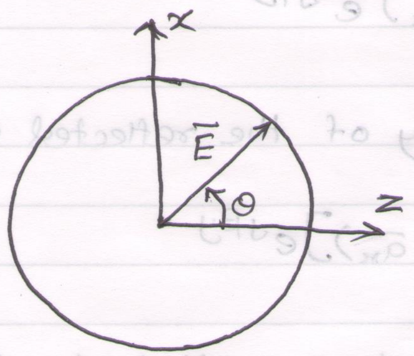
or, $|E_1|^2 + |E_2|^2 = 4$

which is a circular equation i.e. the wave is circularly polarized.

Now, the instantaneous angle θ ~~tan~~ that $\bar{E}$ makes with z-axis is given as,

$$\tan\theta = \frac{-4\sin\omega t}{-4\cos\omega t} \quad \therefore \quad \theta = \omega t$$

So, as time increases, electric field $\bar{E}$ rotates from z to x as shown in the figure below



Since the direction of wave propagation is along $-\bar{a}_y$, so, the rotation from z to x follows left hand rule. Thus, we conclude that the EM wave is LHC (left hand circularly) polarized.

(6) Given, the electric field intensity of the propagating wave,

$$\bar{E} = \bar{a}_x \sin(\omega t - \beta z) + \bar{a}_y \sin(\omega t - \beta z + \pi/2)$$

So, we conclude that the wave is propagating along $\bar{a}_z$ direction and the field components are along $\bar{a}_x$ and $\bar{a}_y$ and equal.

i.e. $E_x = E_y$

$\therefore$ The wave is circularly polarized. Now we will determine the field is either right circular or left circular. The angle between the electric field $\bar{E}$ and x-axis is given as,

$$\theta = \tan^{-1}\left(\frac{\cos \omega t}{\sin \omega t}\right) = \frac{\pi}{2} - \omega t$$

So, with increase in time the tip of the field intensity moves from y to x-axis and as the wave is propagating in $\bar{a}_z$ direction therefore, the wave is left hand circularly polarized.

~~8~~ 7 Linearly polarized wave A in the $\hat{x}$ direction can be written as,

$$E_A(z) = \bar{a}_x E_0 e^{-j\beta z} \quad [V/m]$$

Wave B has equal components in x and y direction,

$$\therefore \quad E_B(z) = (\bar{a}_x + \bar{a}_y) \frac{E_0}{\sqrt{2}} e^{-j\beta z} e^{-j\theta} \quad [V/m]$$

The factor $\sqrt{2}$ ensures equal amplitudes of wave A and B. "$e^{-j\theta}$" term in the equation for wave B implies the ~~lg~~ lag compared to wave A. Superposition gives,

$$E_A + E_B = \bar{a}_x \left(E_0 e^{-j\beta z} + \frac{E_0}{\sqrt{2}} e^{-j\beta z} e^{-j\theta} \right) + \bar{a}_y \frac{E_0}{\sqrt{2}} e^{-j\beta z} e^{-j\theta}$$

$[V/m]$

In time domain,

$$E_A + E_B = \bar{a}_x \left(E_0 \cos(\omega t - \beta z) + \frac{E_0}{\sqrt{2}} \cos(\omega t - \beta z - \theta) \right)$$

$$+ \bar{a}_y \frac{E_0}{\sqrt{2}} \cos(\omega t - \beta z - \theta) \quad [V/m]$$

when $z=0$ and $\omega t = 0$:

$$\left[E_A + E_B\right]\Big|_{z=0,\omega t=0} = \hat{a}_x \left[E_0 + \frac{E_0}{\sqrt{2}} \cos(\theta) \right] + \hat{a}_y \frac{E_0}{\sqrt{2}} \cos\theta \quad [V/m]$$

— (1)

when $z=0$ and $\omega t = \pi/2$

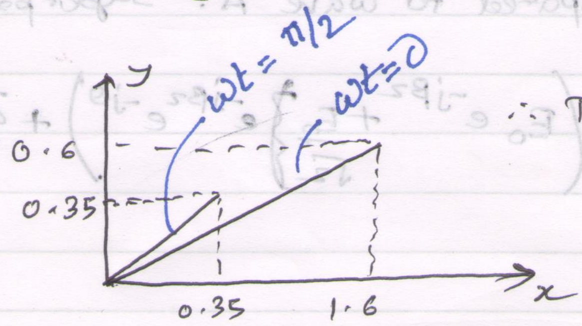
$$\left[E_A+E_B\right]\Big|_{z=0,\,\omega t=\pi/2} = \bar{a}_x\left[\frac{E_0}{\sqrt{2}}\sin\theta\right] + \bar{a}_y\,\frac{E_0}{\sqrt{2}}\sin\theta \quad \left[V/m\right]$$

$-②$

Substituting $\theta = 30°$ in ① and ②, we get,

$$\left[E_A+E_B\right]\Big|_{z=0,\,\omega t=0,\,\theta=30} = \bar{a}_x\, 1.612\,E_0 + \bar{a}_y\, 0.612\,E_0$$

$$\left[E_A+E_B\right]\Big|_{z=0,\,\omega t=\pi/2,\,\theta=30} = \bar{a}_x\, 0.354\,E_0 + \bar{a}_y\, 0.354\,E_0$$



$\therefore$ The wave superposition is right hand elliptically polarized