TUTORIAL-3 - Q2

(a) $y[n] = \{x[n] + x[n-1]\}/2$ (b) $y[n] = \{x[n] - x[n-1]\}/2$

(i) $h_a[n] = \{\delta[n] + \delta[n-1]\}/2$

$h_b[n] = \{\delta[n] - \delta[n-1]\}/2$

(ii) $H_a(w) = \{e^{-jw} + 1\}/2$

$H_b(w) = \{1 - e^{-jw}\}/2$

(iii) $x[n] = \cos w_0 n$

$y_a[n] = \{\cos(w_0 n) + \cos(w_0(n-1))\}/2$

$= \cos\left[w_0(n-\tfrac{1}{2})\right]\cos\left[w_0/2\right]$

$= \left(\frac{e^{jw_0 n} + e^{-jw_0 n}}{4}\right) + \left(\frac{e^{jw_0(n-1)} + e^{-jw_0(n-1)}}{4}\right)$

$y_b[n] = \{\cos(w_0 n) - \cos(w_0(n-1))\}/2$

$= -\sin\left[w_0(n-\tfrac{1}{2})\right]\sin\left[w_0/2\right]$

$= \left(\frac{e^{jw_0 n} + e^{-jw_0 n}}{4}\right) - \left(\frac{e^{jw_0(n-1)} + e^{-jw_0(n-1)}}{4}\right)$

$\therefore\ y_a[n] = \frac{e^{jw_0 n}}{4}[1 + e^{-jw_0}] + \frac{e^{-jw_0 n}}{4}[1 + e^{jw_0}]$

$\therefore\ y_b[n] = \frac{e^{jw_0 n}}{4}[1 - e^{-jw_0}] + \frac{e^{-jw_0 n}}{4}[1 - e^{+jw_0}]$

(iv) DTFT of $y_a[n]$: $Y_a(w) = \left[\frac{1+e^{-jw_0}}{2}\right] \times \frac{\delta[w-w_0]}{2}$

$$+ \left[\frac{1+e^{jw_0}}{2}\right] \times \frac{\delta[w+w_0]}{2}.$$

DTFT of $y_b[n]$: $Y_b(w) = \left[\frac{1-e^{-jw_0}}{2}\right] \times \frac{\delta[w-w_0]}{2}$

$$+ \left[\frac{1-e^{jw_0}}{2}\right] \frac{\delta[w+w_0]}{2}$$

DTFT of $x[n]$: $X(w) = \mathcal{F}^{-1}\left[\cos(w_0 n)\right]$

$$= \mathcal{F}^{-1}\left[\frac{e^{jw_0 n} + e^{-jw_0 n}}{2}\right]$$

$$= \frac{\delta[w-w_0]}{2} + \frac{\delta[w+w_0]}{2}$$

$$X(w) \times H_a(w) = \left\{\frac{e^{-jw}+1}{2}\right\} \times \left[\frac{\delta(w-w_0)}{2} + \frac{\delta(w+w_0)}{2}\right]$$

$$= \left(\frac{1+e^{-jw_0}}{2}\right) \times \frac{\delta(w-w_0)}{2} + \left(\frac{1+e^{+jw_0}}{2}\right) \times \frac{\delta(w+w_0)}{2}$$

we can see $Y_a(w) = H_a(w) \times \cancel{X}(w)$

This is true for $Y_b(w)$ too.

∴ Since these are LSI systems, we see that frequency response of the system is equal to ratio of ~~and~~ [DTFT of output sequence] by [DTFT of input sequence].

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(v) $H_a(\omega) = (1 + e^{-j\omega})/2$

$$h_a[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_a(\omega)\, e^{j\omega_a n}\, d\omega_a$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2} \times (1+e^{-j\omega})\, e^{j\omega_a n}\, d\omega$$

$$= \frac{1}{4\pi}\left(\left[\frac{e^{j\omega_a n}}{jn}\right]_{-\pi}^{\pi} + \left[\frac{e^{j\omega(n-1)}}{j(n-1)}\right]_{-\pi}^{\pi}\right)$$

$$= \frac{1}{4\pi}\left[\frac{1}{jn}(2j\sin(n\pi)) + \frac{1}{j(n-1)}(2j\sin((n-1)\pi))\right]$$

$\boxed{\because \lim_{x\to 0}\frac{\sin x}{x} =}$

we know that $\frac{2\pi\sin(n\pi)}{n\pi} = \begin{cases} 2\pi & \text{when } n=0 \\ 0 & \text{else} \end{cases}$

similarly, $\frac{2\pi}{(n-1)\pi}\sin((n-1)\pi) = \begin{cases} 2\pi & \text{when } n=1 \\ 0 & \text{else} \end{cases}$

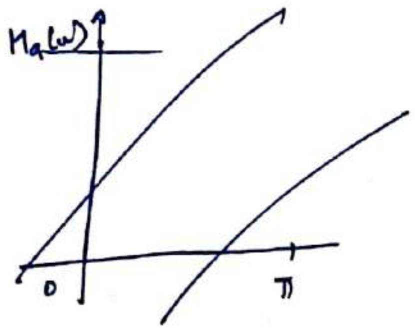
$$\Rightarrow h_a[n] = \frac{1}{4\pi}\left[2\pi \times \delta[n] + 2\pi \times \delta[n-1]\right]$$

$$= \frac{1}{2}[\delta[n] + \delta[n-1]]$$

similarly for $H_b(\omega)$ and $h_b[n]$.

Hence verified.

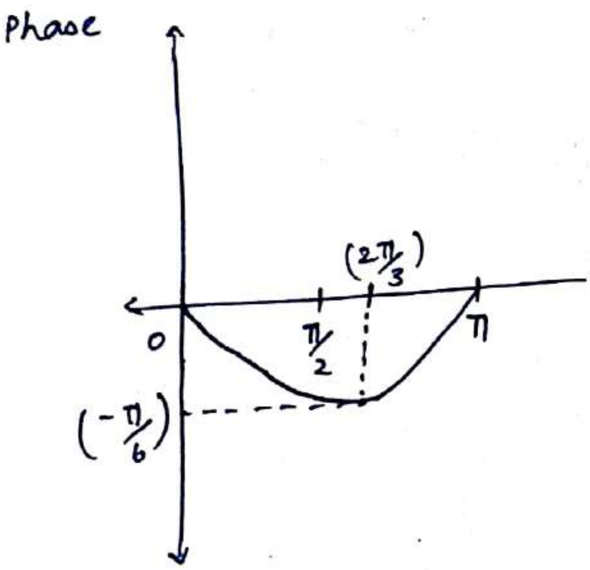
~~(vi) $H_a(w) = \frac{(1+e^{-jw})}{2}$~~ ~~$H_3(w) = \frac{(1-e^{-jw})}{2}$~~

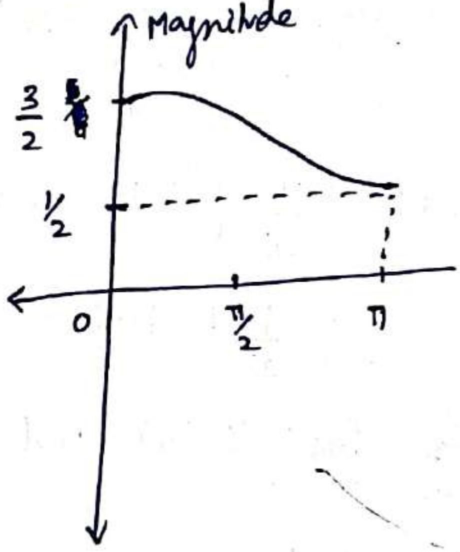


(vi) $H_a(w) = \frac{(1+e^{-jw})}{2} = 1 + \frac{(\cos w - j\sin w)}{2}$

Phase $= -\tan^{-1}\left(\frac{\sin(w)}{2+\cos(w)}\right)$

Magnitude $= \sqrt{1 + \frac{\cos^2 w}{4} + \cos w + \frac{\sin^2 w}{4}} = \sqrt{\frac{5}{4} + \cos w}$





$$H_b(w) = \frac{1 - e^{-jw}}{2} = 1 - \frac{\cos w}{2} + j\frac{\sin w}{2}$$

$$\text{Phase} = -\tan^{-1}\left(\frac{\sin w}{2 - \cos w}\right)$$

$$\text{Magnitude} = \sqrt{1 + \frac{\cos^2 w}{4} - \cos w + \frac{\sin^2 w}{4}} = \sqrt{\frac{5}{4} - \cos w}.$$