Tutorial 1 Question no. 3

Part a : $\boxed{T_p \to 0}$

from Question no. 1, for the given signal p(t) the FT obtained was :

$$P(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} \exp\left(-j\pi k \frac{T_p}{T_s}\right) \text{sinc}\left(\frac{kT_p}{T_s}\right) \delta\left(\omega - \frac{2\pi}{T_s} k\right)$$

for $T_p \to 0$

$$\exp\left(-j\pi k \frac{T_p}{T_s}\right) = 1$$

$$\text{sinc}\left(k\frac{T_p}{T_s}\right) = 1$$

$$\therefore \lim_{T_p \to 0} P(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} \delta\left(\omega - \frac{2\pi}{T_s} k\right)$$

Changes observed : While the Fourier Transform still remains a train of impulses, their amplitudes have changed. While for non-zero T_p the magnitudes of the pulses were proportional to the spectral coeff amplitudes, the magnitudes for T_p infinitesimally small comes out to be of equal amplitudes of $(1/T_s)$.

Intuitively (from our previous experience from EE 210 course), for $T_p \to 0$, p(t) converts into a train of impulses. The Fourier Transform of a periodic train of impulses is another train of impulses with equal magnitudes.

Part b : $\boxed{T_p \to T_s}$

$$\exp\left(-j\pi k \frac{T_p}{T_s}\right) = (-1)^k.$$

$$\text{sinc}\left(k\frac{T_p}{T_s}\right) = \text{sinc}(k) \quad : \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$$

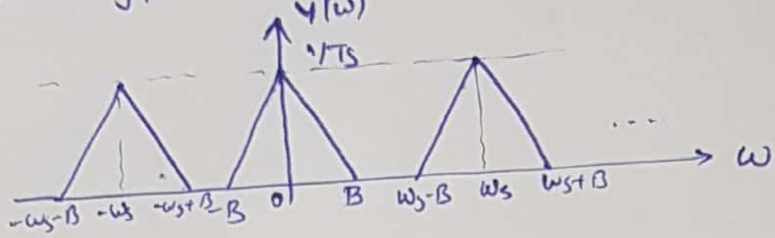
$$\therefore P(\omega) = \frac{1}{T_s}(-1)^0 \,\text{sinc}(0)\, \delta\left(\omega - \frac{2\pi}{T_s} k\right) = \frac{1}{T_s} \delta\left(\omega - \cancelto{0}{\frac{2\pi}{T_s}} k\right)$$

Changes observed : The Fourier transform is a single impulse located at the origin. Intuitively, $T_p \to T_s$ makes p(t) a constant function. The Fourier Transform of a constant function is an impulse located at the origin.

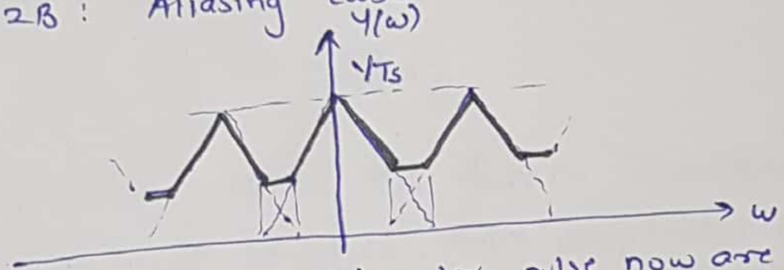
Part a : $\boxed{T_p \to 0}$

As we have already seen that $P(\omega)$ becomes a ~~cha~~ train of impulses of equal magnitude, the resultant spectrums in Question no. 2 are as shown below

(i) $\omega_s > 2B$: Nyquist criteria satisfied



(ii) $B < \omega_s < 2B$: Aliasing case



Changes observed : All the triangulor pulse now are of equal magnitude
Moreover in the case of aliasing, the resultant spectrum is **periodic**

Part b : $\boxed{T_p \to T_s}$

As we have already seen that $P(\omega)$ becomes a ~~train~~ single impulse centered at the origin, the resultant spectrum $Y(\omega)$ in both (i) and (ii) is the same as $X(\omega)$