(Q4) (a) $x_c(t) = \cos(12\pi t)$

After passing through ideal sampler with $T = \frac{1}{20}$:-

In discrete domain 2π maps to $\frac{1}{T}$ (Sample freq)

$\Rightarrow$ frequency at input = 6 Hz

$\Rightarrow$ In discrete domain 6 Hz maps to $\frac{2\pi}{20} \times 6$

$= \frac{3\pi}{5}$

$\Rightarrow x[n] = \cos\left(\frac{3\pi}{5} n\right) = \frac{e^{j\omega_0} + e^{-j\omega_0}}{2}$

$H(e^{j\omega}) = \frac{j\omega}{T}$

Since $e^{j\omega_0}$ & $e^{-j\omega_0}$ are eigenvalues of system

$\Rightarrow x[n] \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow \frac{j\omega_0}{T}\frac{e^{j\omega_0}}{2} + \frac{j(-\omega_0)}{T}\frac{e^{-j\omega_0}}{2}$

$= \frac{\omega_0}{2T} e^{j(\omega_0 + \frac{\pi}{2})} + \frac{\omega_0}{2T} e^{-j(\omega_0 + \frac{\pi}{2})}$

$= \frac{3\pi \times \cancel{20}^{4}}{\cancel{5} \times \cancel{2}}\left[\cancel{2} \cdot \cos\left(\omega_0 + \frac{\pi}{2}\right)\right]$

$= 12\pi \sin\left(\frac{3\pi n}{5} + \frac{\pi}{2}\right)$

$= -12\pi \sin\left(\frac{3\pi}{5} n\right)$

After perfect reconstruction :-

$-12\pi \cos\left(\frac{3\pi}{5} n\right) \longrightarrow \boxed{-12\pi \sin(12\pi t)}$

$\downarrow$

Derivative of $\cos(12\pi t)$

(b) $x_c(t) = \cos(28\pi t)$

$\Rightarrow$ frequency = 14 Hz

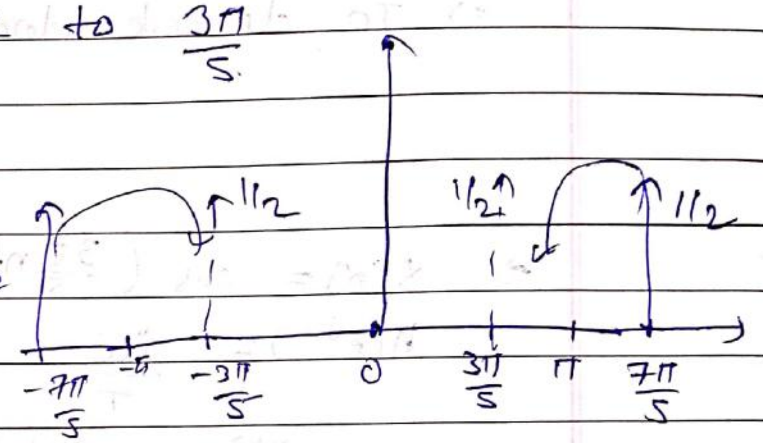
Since sampling frequency is 20 Hz which is

less than $(2 \times B = 28\ Hz)$
$\downarrow$
Nyquist freq.

$\Rightarrow$ There will be aliasing in signal Band.

14 Hz maps to $\frac{7\pi}{5}$

$\frac{7\pi}{5}$ will alias back to $\frac{3\pi}{5}$



$\Rightarrow$ $x[n]$ will be $\cos\left(\frac{3\pi n}{5}\right)$

It is same as in the first part.

$\Rightarrow$ $y_c(t) = -12\pi \sin(12\pi t)$

Yes, Output from both methods are same.