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<u>linearity</u>: if $(x_1[n], y_1[n])$ and $(x_2[n], y_2[n])$ satisfy the given difference eq^n

$$y_1[n] = x_1[n] + \beta y_1[n-1]$$
$$y_2[n] = x_2[n] + \beta y_2[n-1]$$

adding, $(y_1+y_2)[n] = (x_1+x_2)[n] + \beta (y_1+y_2)[n-1]$

Thus $((x_1+x_2)[n], (y_1+y_2)[n])$ satisfies the difference equation

$\therefore$ system is linear

<u>time invariance</u>: if $(x[n], y[n])$ satisfies the difference equation,

$$y[n] = x[n] + \beta y[n-1] \quad \forall n \in \mathbb{Z}$$

$n \to n-k$ $\quad y[n-k] = x[n-k] + \beta y[n-k-1]$

$\therefore$ $(x(n-k), y(n-k))$ satisfies the difference equation

$\therefore$ system is time invariant

<u>causal</u>: at any time instant n, the o/p of the system depends only on the present i/p value $x[n]$ and the past o/p value $y[n]$. $\therefore$ system is causal

<u>memory</u>: clearly, at any point n, $y[n]$ depends on $y[n-1]$ $(\beta \neq 0)$

$\therefore$ system is not memoryless

<u>stability</u>: consider input $x[n] = \delta[n]$

then $y[0] = x[0] + \beta \cdot 0 = 1$

$y[1] = x[1] + \beta y[0] = 0 + \beta \times 1 = \beta$

$y[2] = x[2] + \beta y[1] = \beta^2$

$\therefore$ generalizing, $\underbrace{h[n]}_{\text{impulse response}} = \beta^n$

(note: $h[n] = 0$ for $n < 0$ also implies causality)

consider $S = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |\beta|^n$

The series S converges for $|\beta| < 1$ $\therefore$ the system is stable for $|\beta| < 1$ and unstable for $|\beta| \geq 1$

NOTE: in particular, for $\beta = -1$, $h[n]$ forms a "Grandi series" which is defined to be divergent, although it is interesting to note that it's sum can be defined in atleast 3 possible ways:

(i) $S = (1-1) + (1-1) + (1-1) + \ldots = 0$

(ii) $S = 1 + (-1+1) + (-1+1) + \ldots = 1$

(iii) $S = 1 - 1 + 1 - 1 + 1 - 1 + \ldots$

$S = 0 + 1 - 1 + 1 - 1 + \ldots$

adding $2S = 1 \Rightarrow S = \frac{1}{2}$

(iii) is called a Cesàro sum.