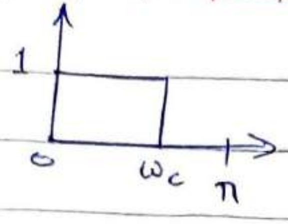
Q3) (i)



Since $h[n]$ is real and $H(\omega) = \overline{H(\omega)}$ (phase response zero), we get $H(-\omega) = H(\omega)$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\omega n}}{jn}\right)_{-\omega_c}^{\omega_c} \qquad (n \neq 0)$$

$$= \frac{2j\sin(\omega_c n)}{2\pi j n} = \frac{\sin(\omega_c n)}{\pi n}$$

for $n=0$, $h[n] = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot d\omega = \frac{\omega_c}{\pi}$

$$\Rightarrow h[n] = \begin{cases} \frac{\sin(\omega_c n)}{\pi n} & n \neq 0 \\ \frac{\omega_c}{\pi} & n = 0 \end{cases}$$

(ii) We can write $H(\omega)$ as $1 - H_{LP}(\omega)$ where $H_{LP}(\omega)$ is DTFT of low pass filter in part (i).

$h[n] =$ Inv. DTFT(1) $-$ Inv. DTFT $(H_{LP}(\omega))$
by linearity / super-position

Page No.
Date / /

$$\text{Inv DTFT}(1) = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1.\, e^{j\omega n}\, d\omega$$

$$(\text{for } n \neq 0) \quad = \frac{1}{2\pi}\left(\frac{e^{j\omega n}}{jn}\right)_{-\pi}^{\pi} = 0 \quad (\text{As } n \in \mathbb{Z})$$

$$(\text{for } n = 0) \quad = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1.\, d\omega = 1$$

$$\Rightarrow \text{Inv DTFT}(1) = \delta[n]$$

$$\Rightarrow h[n] = \begin{cases} -\dfrac{\sin(\omega_c n)}{\pi n} & n \neq 0 \\ 1 - \dfrac{\omega_c}{\pi} & \end{cases}$$

or, in shorthand, $h[n] = \delta[n] - \dfrac{\sin(\omega_c n)}{\pi n}$ (taking limit when $n=0$)

(iii) Observe $H(\omega) = \underbrace{H_{LPF-\omega_{c_2}}(\omega)}_{\text{LPF with cutoff } \omega_{c_2}} - H_{LPF-\omega_{c_1}}(\omega)$

$$\Rightarrow h[n] = \text{Inv. DTFT}\left(H_{LPF-\omega_{c_2}}(\omega)\right) - \text{In DTFT}\left(H_{LPF-\omega_{c_1}}(\omega)\right)$$

$$= \begin{cases} \dfrac{\sin(\omega_{c_2} n)}{\pi n} - \dfrac{\sin(\omega_{c_1} n)}{\pi n} & n \neq 0 \\ \dfrac{\omega_{c_2}}{\pi} - \dfrac{\omega_{c_1}}{\pi} & n = 0 \end{cases}$$

(iv) Observe $H(\omega) = 1 - \underbrace{H_{(iii)}(\omega)}_{\hookrightarrow H(\omega) \text{ of part (iii)}}$

$$\Rightarrow h[n] = \text{Inv DTFT}(1) - h_{(iii)}[n]$$

$$= \begin{cases} \delta[n] - \left(\dfrac{\sin(\omega c_2 n)}{\pi n} - \dfrac{\sin(\omega c_1 n)}{\pi n} \right) & n \neq 0 \\ 1 - \dfrac{\omega c_2}{\pi} + \dfrac{\omega c_1}{\pi} & n = 0 \end{cases}$$

or,

$$h[n] = \begin{cases} \dfrac{\sin(\omega c_1 n)}{\pi n} - \dfrac{\sin(\omega c_2 n)}{\pi n} & n \neq 0 \\ 1 - \dfrac{\omega c_2}{\pi} + \dfrac{\omega c_1}{\pi} & n = 0 \end{cases}$$

<u>Note</u> : We have interchangeably used $H(\omega)$ and $H(e^{j\omega})$ to represent frequency response.