Tutorial 4

Q8] i) Causal $\Rightarrow h[n] = 0 \quad \forall n < 0$

ii) $H(e^{j\omega}) = \sum_{0}^{\infty} h[n] e^{-j\omega n}$

$H^*(e^{-j\omega}) = \sum_{0}^{\infty} \overline{h[n]} e^{-j\omega n}$

$\therefore h[n] = \overline{h[n]} \Rightarrow h[n] \in \mathbb{R}$

iii) Let $g[n]$ be a sequence.

$\therefore G(e^{j\omega}) = \sum_{-\infty}^{\infty} g[n] e^{-j\omega n}$

If $G(e^{j\omega}) \in \mathbb{R}$, $G(e^{j\omega}) = G^*(e^{j\omega})$

$\Rightarrow \sum_{-\infty}^{\infty} g[n] e^{-j\omega n} = \sum_{-\infty}^{\infty} \overline{g[n]} e^{j\omega n}$

$= \sum_{-\infty}^{\infty} \overline{g[-n]} e^{-j\omega n}$

$\Rightarrow g[n] = g^*[-n]$.

DTFT of $h[n+1] \in \mathbb{R}$. $\Rightarrow h[1+n] = h^*[1-n]$

$\Rightarrow h^*[0] = h[2] = h[0]$ (as $h[n] \in \mathbb{R}$)

$\Rightarrow h[3] = h[-1] = 0$

$\Rightarrow h[4] = h[-2] = 0$

$\Big\}$ All $h[k]$ $(k > 2) = 0$ as they will be $= h[2-k] = 0$ for $k > 2$

$\therefore$ The only non-zero terms are $h[0], h[1]$ & $h[2]$, & $h[0] = h[2]$.

a) Hence $h[n]$ has finite support.

Support $= 0, 1, 2$. $H(e^{j\omega}) = h[0] + h[1] e^{-j\omega} + h[2] e^{-2j\omega}$

b) $H(e^{j\omega}) = 2$ for $\omega = 0$.

$h[0] + h[1] + h[2] = 2$ —— ①

$H(e^{j\omega}) = 0$ for $\omega = \pi$

$h[0] - h[1] + h[2] = 0.$ —— ②

$\dfrac{dH(e^{j\omega})}{d\omega} = 0$ for $\omega = \pi$

$h[1](-\cancel{j}) e^{-j\pi} + h[2](-2\cancel{j}) e^{-2j\pi} = 0$

$-h[1] + 2h[2] = 0.$ —— ③

Solving ①, ② & ③

$\boxed{h[0] = 1/2}$ $\boxed{h[1] = 1}$ $\boxed{h[2] = 1/2}$

Which is consistent with all the above derived conditions ($h[0] = h[2]$)