Tut 2 Ques 5

$x[n] \rightarrow$ non-zero $\rightarrow N_0 \leq n \leq N_1$

$h[n] \rightarrow$ non-zero $\rightarrow N_2 \leq n \leq N_3$

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$\therefore x[n]$ is non-zero only in $N_0 \leq n \leq N_1$

$$y[n] = x[n] * h[n] = \sum_{k=N_0}^{N_1} x[k] h[n-k]$$

For $y[n]$ to be non-zero.

$h[n-k]$ need to be non-zero in this range.

Hence, $N_2 \leq n-k \leq N_3$

$\therefore k$ goes from N_0 to N_1

$\Rightarrow y[n]$ is non-zero in this range $N_0 + N_2 \leq n \leq N_1 + N_3$

Example: consider $x[n] = \begin{cases} 1 & -1 \leq n \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$h[n] = \begin{cases} 1 & -4 \leq n \leq -3 \\ 0 & \text{otherwise} \end{cases}$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= \sum_{k=-1}^{1} x[k] h[n-k]$$

$$= x[1] h[n-1] + x[0] h[n] + x[-1] h[n+1]$$