Group 17

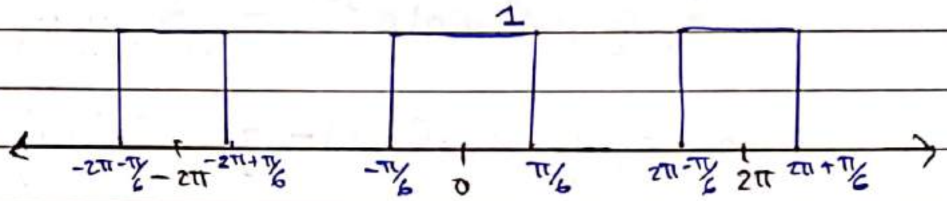
Question 7. Tut 5

Q7] $x[n] = \sin\left(\frac{\pi n}{6}\right) - \cos\left(\frac{\pi n}{3}\right)$

ⓐ $h[n] = \frac{\sin(\pi n/6)}{\pi n}$

$= \frac{1}{6}\frac{\sin(\pi n/6)}{\pi n/6} = \frac{1}{6}\operatorname{sinc}\left(\frac{n}{6}\right)$

$h(e^{j\omega}) \Rightarrow$



this is a low pass filter & if we consider with cutoff freq $\pi/6$.

$\therefore y[n] = \sin\left(\frac{\pi n}{6}\right)$.

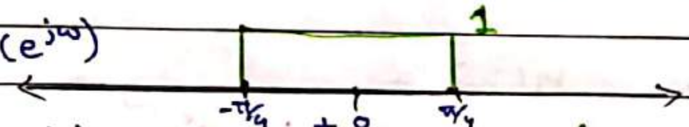
as $\cos\left(\frac{\pi n}{3}\right)$ will get filtered out.

ⓑ $h[n] = \frac{\sin(\pi n/4)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$.

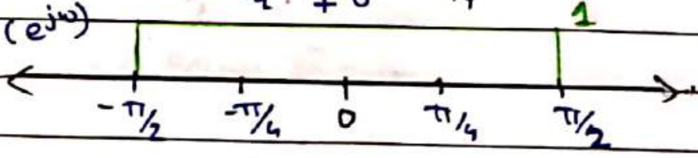
$= \underbrace{\frac{1}{4}\operatorname{sinc}\left(\frac{n}{4}\right)}_{h_1[n]} + \underbrace{\frac{1}{2}\sin\left(\frac{n}{2}\right)}_{h_2[n]}$.

~~$h(e^{j\omega})$~~

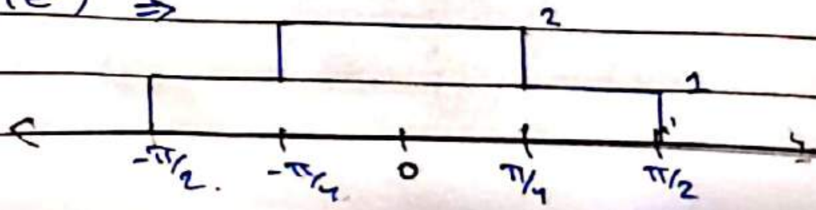
$h_1(e^{j\omega})$



$h_2(e^{j\omega})$



$h(e^{j\omega}) \Rightarrow$



$$\therefore \quad y[n] = \cancel{H(e^{j\omega}} \; |H(e^{j\pi/6})| \sin\left(\frac{\pi n}{6}\right) - |H(e^{j\pi/3})| \cos\left(\frac{\pi n}{3}\right)$$

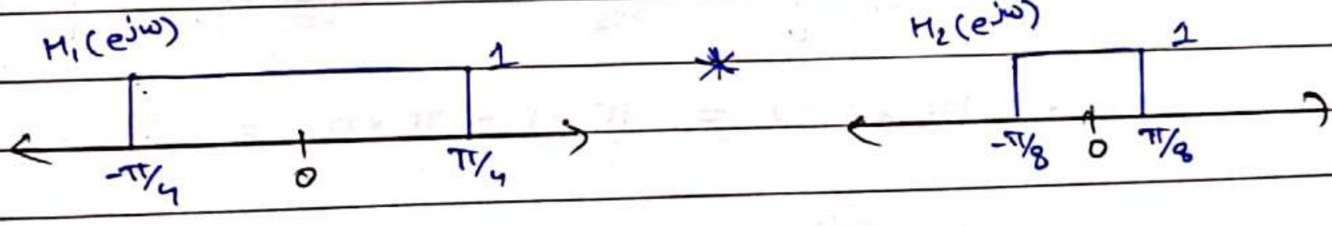
$$\left[\text{ here } \angle H(e^{j\pi/6}) = \angle H(e^{j\pi/3}) = 0 \right].$$

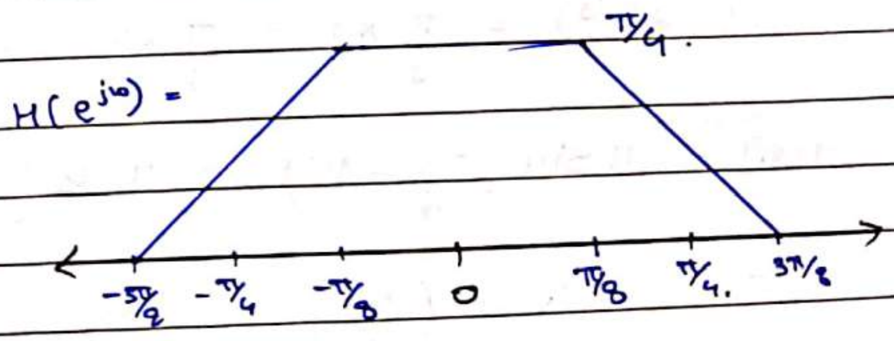
$$\therefore \quad y[n] = 2\sin\left(\frac{\pi n}{6}\right) - \cos\left(\frac{\pi n}{3}\right).$$

(c) $h[n] = \dfrac{\sin\left(\frac{\pi n}{4}\right) \sin\left(\frac{\pi n}{8}\right)}{\pi^2 n^2}$

$$= \underbrace{\frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}}_{h_1[n]} \times \underbrace{\frac{\sin\left(\frac{\pi n}{8}\right)}{\pi n}}_{h_2[n]}.$$

$$H(e^{j\omega}) = H_1(e^{j\omega}) * H_2(e^{j\omega}).$$





$$y[n] \quad |H(e^{j\pi/6})| = \frac{5}{24}\pi.$$

$$|H(e^{j\pi/3})| = \frac{\pi}{24}$$

$$\therefore \quad y[n] = \frac{5}{24}\pi \sin\left(\frac{\pi n}{6}\right) - \frac{\pi}{24}\cos\left(\frac{\pi n}{3}\right)$$

(d) $h[n] = \frac{1}{\pi n} \sin\left(\frac{\pi n}{4}\right) . \sin\left(\frac{\pi n}{8}\right)$

$$= \frac{1}{4} \text{sinc}\left(\frac{\pi n}{4}\right) . \times \sin \frac{\pi n}{8}$$

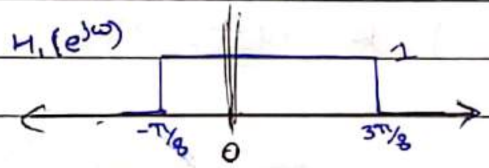
$$\quad H_1(e^{j\omega}) \qquad\qquad H_2(e^{j\omega})$$

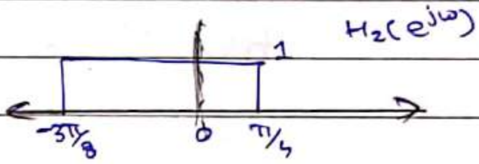
$$H(e^{j\omega}) = H_1(e^{j\omega}) * H_2(e^{j\omega}).$$

$$= H_1(e^{j\omega}) * \left[\frac{\pi}{j}\delta\left(\omega - \frac{\pi}{8}\right) - \frac{\pi}{j}\delta\left(\omega + \frac{\pi}{8}\right)\right]$$

we are considering ωs only in range $-\pi$ to π.

$$H(e^{j\omega}) = \frac{\pi}{j} H_1(e^{j(\omega - \pi/8)}). - \frac{\pi}{j} H_1(e^{j(\omega + \pi/8)})$$





now, $H(e^{j\pi/6}) = \frac{\pi}{j} \times 1 - \frac{\pi}{j} \times 0 = \frac{\pi}{j} = \pi \angle -90^\circ$

$$H(e^{j\pi/3}) = \frac{\pi}{j} \times 1 - \frac{\pi}{j} \times 0 = \frac{\pi}{j} = \pi \angle -90^\circ$$

$$\therefore y[n] = \pi \sin\left(\frac{\pi n}{6} - 90^\circ\right) - \pi \cos\left(\frac{\pi}{3}n - 90^\circ\right)$$

$$y[n] = -\pi \cos\left(\frac{n\pi}{6}\right) - \pi \sin\left(\frac{n\pi}{3}\right).$$