★ $x[n] = x[-n]$

$$\Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} x[n] e^{-j\omega n} + \sum_{n=1}^{\infty} x[-n] e^{+j\omega n}$$

$$= \sum_{n=1}^{\infty} x[n] \left(e^{-j\omega n} + e^{j\omega n}\right) + x[0]$$

$$= \sum_{n=1}^{\infty} x[n]\, 2\cos(\omega n) + x[0]$$

$\sum 2x[n]\cos(\omega n) + x(0)$

$$X(\omega+\pi) = \sum_{n=1}^{\infty} 2x[n]\cos(\omega n + n\pi) + x(0)$$

$$= \sum_{n\in odd}^{\infty} -2x[n]\cos(\omega n) + \sum_{n\in even}^{\infty} 2x[n]\cos(\omega n) + \ldots$$

$$= \sum_{n=1}^{\infty} 2x[n](-1)^n\cos(\omega n) + x(0)$$

Only possible if

$x[n]=0 \quad \forall\ n\in odd$

$\Rightarrow \boxed{x[1]=0}$

~~$X(\omega) = \sum$~~

$y_1[n] = x[2n]$

$y_2[n] = x[3n]$

$\Rightarrow y_1[0] = x[0] \quad y_1[1] = x[2] \ldots\ldots$

Now $\because x=0 \quad \forall\ n\in odd$

~~$\Rightarrow y_1[n] =$~~

~~$\Rightarrow x[n] \rightarrow y[n] \quad \frac{n}{2} = ev$~~

$$\boxed{x[n] = y_1\left[\frac{n}{2}\right]}$$

$$x[n] = \begin{cases} y_1\left[\frac{n}{2}\right] & n\in even \\ 0 & otherwise \end{cases}$$

Oversampling

$$x[n] \rightarrow \begin{cases} y_2\left[\frac{n}{3}\right] & \text{\sout{n e even}} \text{ \& } n \text{ is divisible by } 3 \\ 0 & n \in \text{odd \& } n \text{ not divisible by } 3 \end{cases}$$

For $n = 2, 4, 8, 10 \ldots$

$x[n]$ can't be expressed in terms of $y_2[n]$.