2 (a) $x(t) \xrightarrow{F.T.} X(j\Omega)$

$$x_s(t) = x(t) \cdot \sum \delta(t - nT)$$

$$= x(t) \cdot \sum \frac{1}{T} e^{jn\left(\frac{2\pi}{T}\right)}$$

$$X_s(j\Omega) = X(j\Omega) * \frac{1}{T} \delta\left(\Omega - n \cdot \frac{2\pi}{T}\right)$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{T} X\left(\Omega - n \cdot \frac{2\pi}{T}\right)$$

$$= \sum a_k e^{jTk\Omega}$$

$$\cancel{a_k = \int_{-T/2}^{T/2} X\left(\Omega - n \cdot \frac{2\pi}{T}\right) \cdot e^{-jTk\Omega}}$$

$$a_k = \frac{1}{\omega_s} \int_{-\infty}^{\infty} X(j\Omega) e^{-jTn\Omega} \, d\Omega$$

$$= \frac{T}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{-j(nT)\Omega} \, d\Omega$$

$$= T \cdot x(nT)$$

$$X_s(j\Omega) = \sum T \cdot x(nT) e^{-jnT\Omega}$$

$$= \sum_{n=-\infty}^{\infty} T \cdot x_d[n] \, e^{-jnT\Omega}$$

$$X_s(j\Omega) = T \cdot X_d(\Omega T)$$

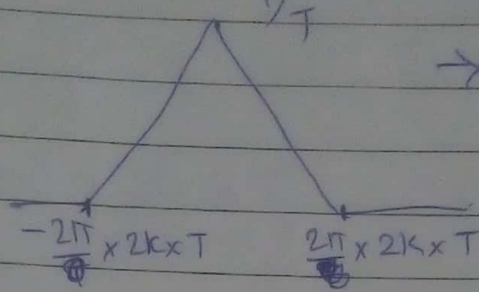
$\frac{2\pi}{T}$ periodic $\qquad$ 2π periodic

$$X_s(j\Omega)\Big|_{\frac{\pi}{T}} = T \cdot X_d(\pi)$$

or $\omega \in (-\pi, \pi)$ $\quad X_d(\omega) = \frac{1}{T} X_c\left(j\frac{\Omega}{T}\right)$

$Y_d(\omega) = X_d(\omega) \cdot H_d(\omega)$

$X_d(\omega) =$

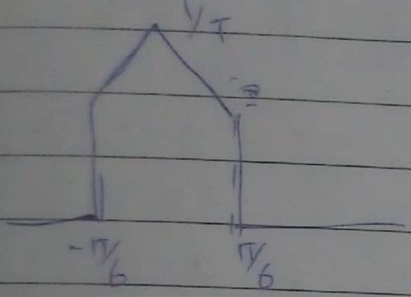


$\rightarrow 2\pi \times 2k \times T < \pi$

or $T < \frac{1}{4kHz}$ ~~0.25ms~~

$\boxed{\therefore T < 0.25ms}$

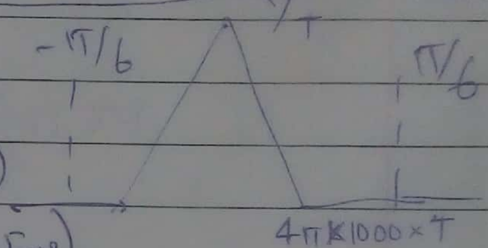
$Y_d(\omega) = $ (i) $\quad 2\pi \times 2k \times T > \frac{\pi}{6} \rightarrow T > \frac{0.25}{6} ms$



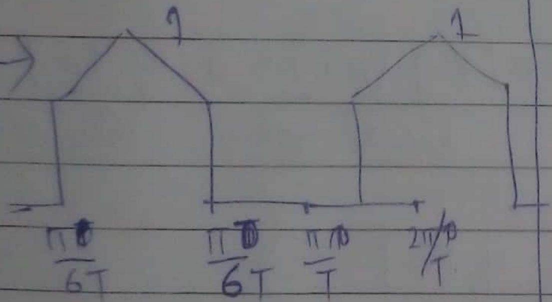
(ii)

Part(b)

$\left(T \leq \frac{0.25ms}{6}\right)$



$Y_s(j\Omega) = T \cdot Y_d(\omega T)$
$= T\, X_d(\omega T)\, H_d(\omega T)$



$\downarrow H_r(j\Omega)$

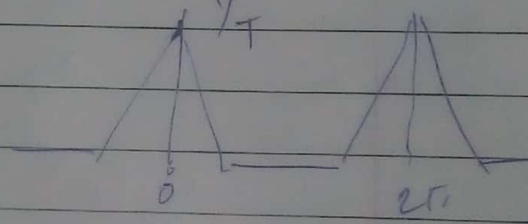
$\Omega_c > \frac{\pi}{6T}$

$T_{max} = 0.25 ms = \frac{1}{4kHz}$

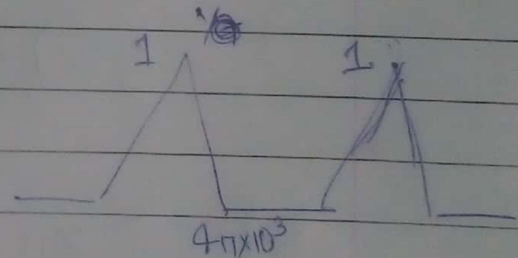
$\therefore \Omega_c > \frac{\pi}{6} \times 4 \times 10^3$

$> \frac{2\pi}{3} \times 10^3$ rad/s

$Y_d(\omega)$



$Y_s(\omega)$



$\downarrow H_r(j\Omega)$

$\Omega_c > 4\pi \times 10^3$

$\downarrow$

No use of LPF.

$\therefore$ ~~$\Omega_c < 4\pi, 2\pi$~~

$\frac{2\pi}{3} \times 10^3 < \Omega_c < 4\pi \times 10^3$