(Q7) $x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

(a) $x[n] = \{1, 2, \ldots, 9\}, \qquad h[n] = \{1, 2, 3\}$

$y[n] = x[n] * h[n]$

$$= \sum_{-\infty}^{\infty} x[k] \, h[n-k] = \sum_{-\infty}^{\infty} h[k] \, x[n-k]$$

$\Rightarrow y[n] = (1) x[n-1] + (2) x[n-2] + (3) x[n-3]$

$\Rightarrow y[n] = \{0, 1, 4, 10, 16, 22, 28, 34, 40, 46, 42, 27\}$

(b) $x_1[n] = \{1, 2, 3\}$

$x_2[n] = \{4, 5, 6\}$

$x_3[n] = \{7, 8, 9\}$

$y_1[n] = x_1[n] * h[n] = \sum_{-\infty}^{\infty} h[k] \, x_1[n-k]$

$\Rightarrow y_1[n] = \{0, 1, 4, 10, 12, 9\}$

$y_2[n] = \sum_{-\infty}^{\infty} h[k] \, x_2[n-k]$

$= (1) x_2[n-1] + (2) x_2[n-2] + (3) x_2[n-3]$

$\Rightarrow y_2[n] = \{0, 4, 13, 28, 27, 18\}$

$y_3[n] = \sum_{-\infty}^{\infty} h[k] \, x_3[n-k]$

$= (1) x_3[n-1] + (2) x_3[n-2] + (3) x_3[n-3]$

$\Rightarrow y_3[n] = \{0, 7, 22, 46, 42, 27\}$

$x_1[n] + x_2[n-3] \longrightarrow \boxed{h[n]} \longrightarrow y_1[n] + y_2[n-3] + y_3[n-3]$

$+ x_3[n-3]$

$= x[n]$

$\Rightarrow \quad y_1[n] + y_2[n-3] + y_3[n-3] = y[n]$

$\left\{ \begin{array}{l} \because \text{ System is linear - time invariant} \end{array} \right\}$

$y_1[n] + y_2[n-3] + y_3[n-3] = \{0,1,4,10,12,9,0,0,---\} +$

$\qquad\qquad\qquad \{0,0,0,0,4,13,28,27,18,0,0\}$

$\qquad\qquad + \{0,0,0,0,0,0,0,7,22,46,42,27\}$

$= \{0,1,4,10,16,22,28,34,40,46,42,27\}$

from (a) & (b) we get same $y[n]$.

$\Rightarrow$ System is verified.