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★  $x[n] = x[-n]$

$$\Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} x[n] e^{-j\omega n} + \sum_{n=-\infty}^{-1} x[-n] e^{+j\omega n}$$

$$= \sum_{n=1}^{\infty} x[n] (e^{-j\omega n} + e^{+j\omega n}) + x[0]$$

$$= \sum_{n=1}^{\infty} x[n] 2 \cos(\omega n) + x[0]$$

$$\sum 2x[n] \cos(\omega n) + x(0)$$

$$\begin{aligned} x(\omega + \pi) &= \sum_{n=1}^{\infty} 2x[n] \cos(\omega n + n\pi) + x(0) \\ &= \sum_{n \in \text{odd}} -2x[n] \cos(\omega n) + \sum_{n \in \text{even}} 2x[n] \cos(\omega n) \\ &= \sum_{n=1}^{\infty} 2x[n] (-1)^n \cos(\omega n) + x(0) \end{aligned}$$

Only possible if  
 $x[n] = 0 \quad \forall n \in \text{odd}$   
 $\Rightarrow \boxed{x[n] = 0}$

$$\begin{aligned} x[n] &= \sum \\ y_1[n] &= x[2n] \\ y_2[n] &= x[3n] \end{aligned}$$

$$\Rightarrow y_1[0] = x[0] \quad y_1[1] = x[2] \dots$$

Now  $\because x = 0 \quad \forall n \in \text{odd}$

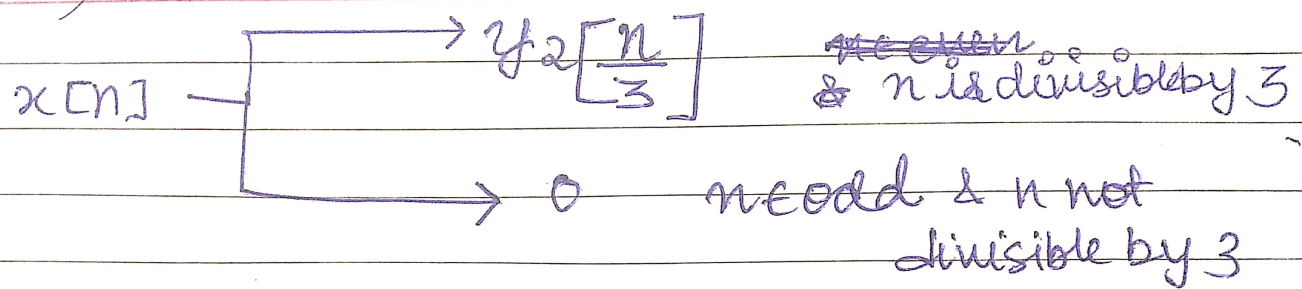
$$\Rightarrow y_1[n] =$$

$$\Rightarrow x[n] \rightarrow y[n] \quad \frac{n}{2} = \text{ev}$$

$$\boxed{x[n] = y_1\left[\frac{n}{2}\right]}$$

$$\begin{aligned} x[n] &\rightarrow y_1\left[\frac{n}{2}\right] \quad n \in \text{even} \\ &\rightarrow 0 \quad \text{otherwise} \end{aligned}$$

Upsampling



For  $n=2, 4, 8, 10 \dots$

$x[n]$  can't be expressed in terms of  $y_2[n]$ .