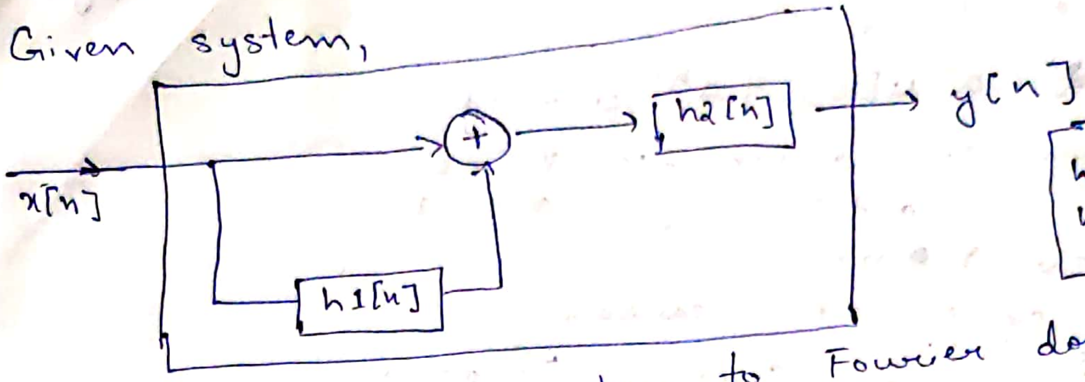
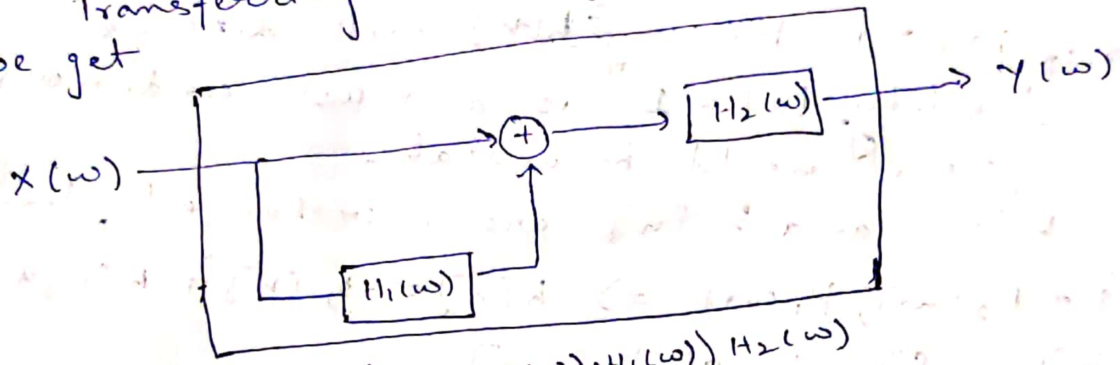
Tut 5 Q-1

Given system,



$h_1[n] = \delta[n-1]$
$h_2[n] = a^n u[n]$

Transferring this system to Fourier domain, we get



Now, $Y(\omega) = (X(\omega) + X(\omega) \cdot H_1(\omega))\, H_2(\omega)$

$\Rightarrow \dfrac{Y(\omega)}{X(\omega)} = (1 + H_1(\omega))\, H_2(\omega) \longrightarrow (1)$

$\Rightarrow H(\omega) = (1 + H_1(\omega)) \cdot H_2(\omega)$

Going back to time domain,

$h[n] = h_2[n] + h_2[n] * h_1[n]$

$= a^n u[n] + (\underbrace{a^n u[n] * \delta[n-1]}_{\text{shifts } n \to n-1})$

$\Rightarrow \boxed{h[n] = a^n u[n] + a^{n-1} u[n-1]}$

For frequency response, let's first calculate $H_1(\omega)$ and $H_2(\omega)$

So $H_1(\omega) = e^{-j\omega}$ and $H_2(\omega) = \sum_{n=-\infty}^{+\infty} e^{-j\omega n} \times a^n u[n]$

$= \sum_{0}^{\infty} a^n e^{-j\omega n}$

Now $\boxed{\text{if } |a| < 1}$ then $H_2(\omega) = \dfrac{1}{1 - ae^{-j\omega}}$

using (1)

$\Rightarrow \boxed{H(\omega) = \dfrac{1 + e^{-j\omega}}{1 - ae^{-j\omega}}} \longrightarrow$ It exist only if $|a| < 1$

PTO

For commenting on stability and causality let's focus on $h[n]$.

$h[n] = a^n u[n] + a^{n-1} u[n-1]$

as we can see $a^n u[n] = 0 \ \forall n < 0$
and $a^{n-1} u[n-1] = 0 \ \forall n < 1$

$\Rightarrow h[n] = 0 \ \forall n < 0$

hence system is causal

Now $\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=0}^{\infty} a^n + \sum_{n=1}^{\infty} a^{n-1}$

Now if $|a| > 1$ then $\sum_{n=-\infty}^{\infty} h[n]$ will diverge

hence for stability, $|a| < 1$ is a must condition.
otherwise system would be unstable.

→ Now if $h_1[n]$ and $h_2[n]$ are interchanged, then $\Rightarrow h[n] = h_1[n] + h_1[n] * h_2[n]$

$H(\omega) = (1 + H_2(\omega)) H_1(\omega)$

So, now $h[n] = \delta[n-1] + a^{n-1} u[n-1]$.

Here again $\delta[n-1] = 0 \ \forall n < 0$

and $a^{n-1} u[n-1] = 0 \ \forall n < 0 \Rightarrow$ system is causal

hence $h[n] = 0 \ \forall n < 0$

Now $\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=-\infty}^{\infty} \delta[n-1] + \sum_{n=-\infty}^{\infty} a^{n-1} u[n-1]$

$= 1 + \sum_{n=1}^{\infty} a^{n-1} \cancel{u[n-1]}$

Now $\sum_{n=1}^{\infty} a^{n-1}$ will converge only if $|a| < 1$, hence system is stable, or

here again we can't say system is stable or not without knowing the value of a

frequency response in this case $(H(\omega)) = \left(1 + \frac{1}{1 - ae^{-j\omega}}\right) e^{-j\omega}$

$= \frac{(2 - ae^{-j\omega}) e^{-j\omega}}{1 - ae^{-j\omega}}$

$\Rightarrow \boxed{H(\omega) = \frac{2e^{-j\omega} - ae^{-2j\omega}}{1 - ae^{-j\omega}}}$ ← here again It exist if $|a| < 1$

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