EE334
Mutually coupled circuits and transformers

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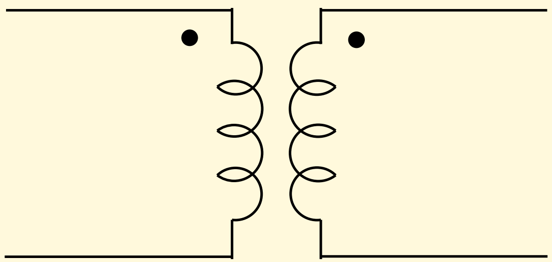
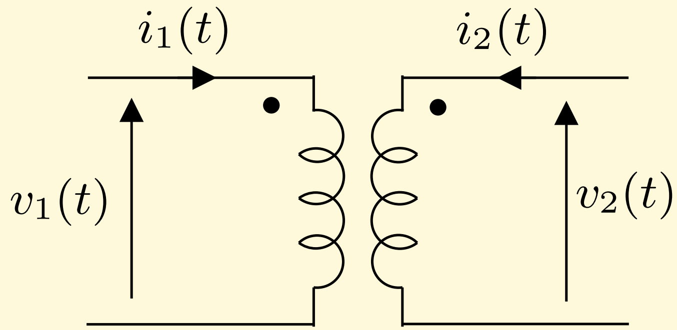
1. Dot convention in coupled circuits



$$\uparrow$$

$$\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}$$

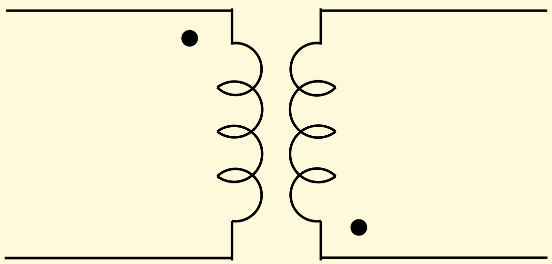


$$\uparrow$$

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

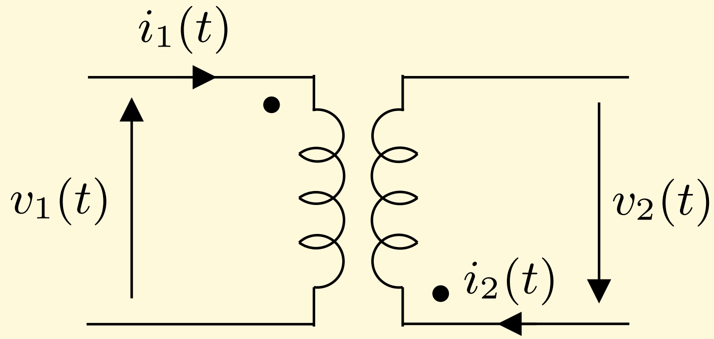
$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix}$$

Dot convention in coupled circuits



$\uparrow$

$$\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}$$



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$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

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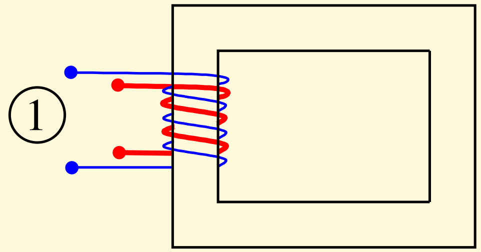
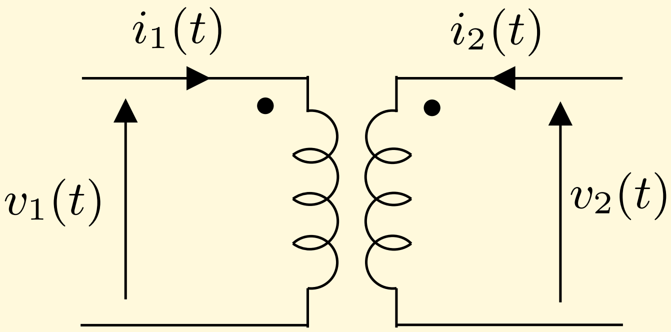
2. Ideal transformer





Ideal transformer

Using Faraday's law:

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

where, ψ_1 and ψ_2 are the flux links in the coils 1 and 2, given as:

Ideal transformer

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where, ψ_1 and ψ_2 are the flux links in the coils 1 and 2, given as:

$$\frac{\psi_1}{N_1} = \phi_1 = \frac{L_{11}}{N_1} i_1 + \frac{M}{N_1} i_2 \tag{1}$$

$$\frac{\psi_2}{N_2} = \phi_2 = \frac{M}{N_2} i_1 + \frac{L_{22}}{N_2} i_2 \tag{2}$$

Ideal transformer

Using Faraday's law:

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

where, ψ_1 and ψ_2 are the flux links in the coils 1 and 2, given as:

$$\frac{\psi_1}{N_1} = \phi_1 = \frac{L_{11}}{N_1} i_1 + \frac{M}{N_1} i_2$$

$$\frac{\psi_2}{N_2} = \phi_2 = \frac{M}{N_2} i_1 + \frac{L_{22}}{N_2} i_2$$

For all $i_1(t)$ and $i_2(t)$ in **Ideal transformers** $\phi_1 = \phi_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$ also $L_{11} = \frac{N_1}{N_2}M \quad L_{22} = \frac{N_2}{N_1}M \quad L_{11}\,L_{22} = M^2$

3. Ideal transformer in phasor domain

In sinusoidal steady-state, the ideal transformer's equations are:

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = j\,\omega \begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

Under **open-circuit secondary**: $\bar{I}_{oc} = \dfrac{\bar{V}_1}{j\omega L_{11}}$

Ideal transformer in phasor domain contd...

In sinusoidal steady-state, the ideal transformer's equations are:

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = j\,\omega \begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

Under **open-circuit secondary**: $\bar{I}_{oc} = \dfrac{\bar{V}_1}{j\omega L_{11}}$

Under loaded condition:

$$\begin{aligned} \bar{V}_1 &= j\,\omega\,L_{11}\,\bar{I}_{1load} + j\,\omega M\,\bar{I}_{2load} \\ \bar{V}_1 &= j\,\omega\,L_{11}\bar{I}_{1load} + j\,\omega\,\frac{N_2}{N_1}L_{11}\,\bar{I}_{2load} \\ \bar{V}_1 &= j\,\omega\,L_{11}\left(\bar{I}_{1load} + \frac{N_2}{N_1}\,\bar{I}_{2load}\right) \qquad (3) \end{aligned}$$

Ideal transformer in phasor domain contd...

$$\bar{V}_1 \;=\; j\,\omega\,L_{11}\left(\bar{I}_{1load} + \frac{N_2}{N_1}\,\bar{I}_{2load}\right)$$

Under **open-circuit secondary**: $\bar{I}_{oc} = \dfrac{\bar{V}_1}{j\omega L_{11}}$

$$\left(\bar{I}_{oc} - \frac{N_2}{N_1}\,\bar{I}_{2load}\right) = \bar{I}_{1load}$$

If $\bar{I}_{oc}$ is **negligible** compared to load currents, then

$$\bar{I}_{1load} = \left(-\frac{N_2}{N_1}\,\bar{I}_{2load}\right)$$

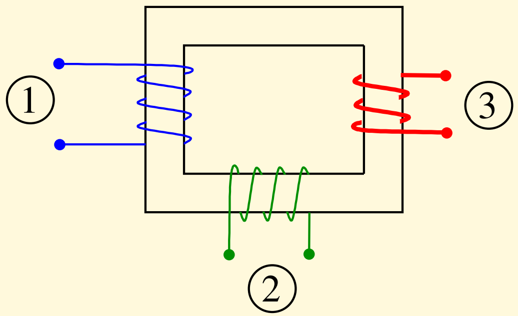
MMF balance

Consider the equation $$\left(\bar{I}_{oc} - \frac{N_2}{N_1}\,\bar{I}_{2load}\right) = \bar{I}_{1load}$$

Multiplying throughout by N_1 and rearranging,

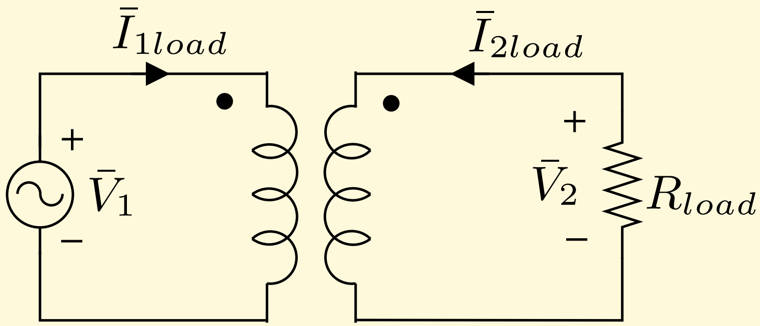
$$N_1\,(\bar{I}_{1load} - \bar{I}_{oc}) = -N_2\,\bar{I}_{2load}$$

For three winding transformer:



$N_1\,(\bar{I}_{1load} - \bar{I}_{oc}) = -N_2\,\bar{I}_{2load} - N_3\,\bar{I}_{3load}$

Ideal transformer in phasor domain contd...



$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = j\,\omega \begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix} \begin{bmatrix} \bar{I}_{1load} \\ \bar{I}_{2load} \end{bmatrix}$$

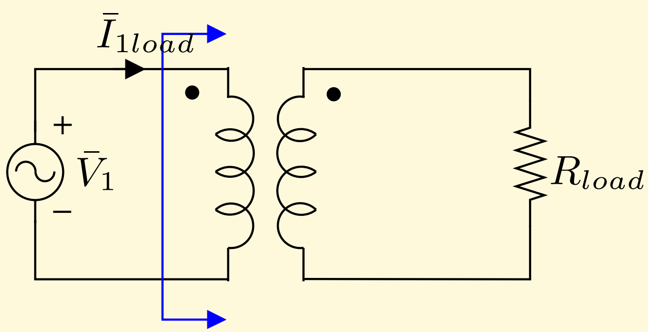
$$\bar{I}_{1load} = \left(\bar{I}_{oc} - \frac{N_2}{N_1}\,\bar{I}_{2load} \right)$$

Note: $\bar{I}_{oc} = \dfrac{\bar{V}_1}{j\omega L_{11}}$ and $\bar{I}_{2load} = \dfrac{-\bar{V}_2}{R_{load}}$

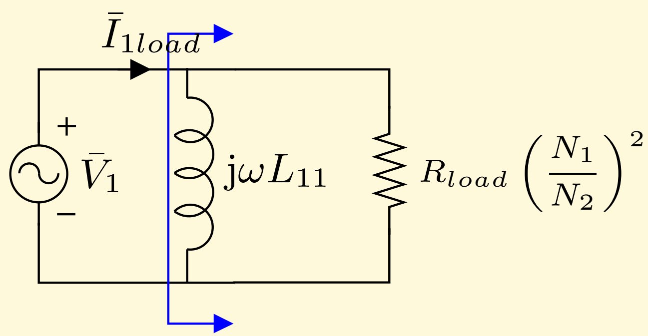
Ideal transformer in phasor domain contd...

$$\bar{I}_{1load} = \left[\frac{1}{j\omega L_{11}} + \frac{1}{R_{load}\left(\frac{N_1}{N_2}\right)^2} \right] \bar{V}_1$$

Original system:

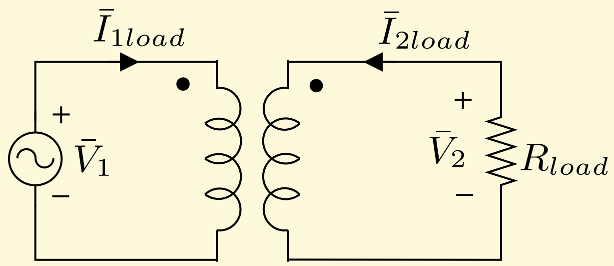


Equivalent system:



Note: above equivalent with ideal transformers **only**.

Ideal transformer in phasor domain contd...



Note: $\bar{I}_{1load} = \left(\bar{I}_{oc} - \frac{N_2}{N_1}\,\bar{I}_{2load}\right)$

For the above system:

$$
\begin{aligned}
\bar{V}_1\,\bar{I}_1^* &= \bar{V}_1\left(\bar{I}_{oc} - \frac{N_2}{N_1}\,\bar{I}_{2load}\right)^* \\
\bar{V}_1\,\bar{I}_1^* &= \bar{V}_1\,\bar{I}_{oc}^* - \frac{N_2}{N_1}\,\bar{V}_1\,\bar{I}_{2load}^* \\
\bar{V}_1\,\bar{I}_1^* &= \bar{V}_1\,\bar{I}_{oc}^* - \bar{V}_2\,\bar{I}_{2load}^*
\end{aligned}
$$

Ideal transformer in phasor domain contd...

Also the real power is given by:

$$\begin{aligned}
\bar{V}_1\, \bar{I}_1^* &= \bar{V}_1\, \bar{I}_{oc}^* - \bar{V}_2\, \bar{I}_{2load}^* \\
Re\{\bar{V}_1\, \bar{I}_1^*\} &= Re\{\bar{V}_1\, \bar{I}_{oc}^* - \bar{V}_2\, \bar{I}_{2load}^*\} \\
Re\{\bar{V}_1\, \bar{I}_1^*\} &= Re\{-\bar{V}_2\, \bar{I}_{2load}^*\} \\
Re\{\bar{V}_1\, \bar{I}_1^*\} &= R_{load}\, |\bar{I}_{2load}|^2
\end{aligned}$$

4. Practical transformer

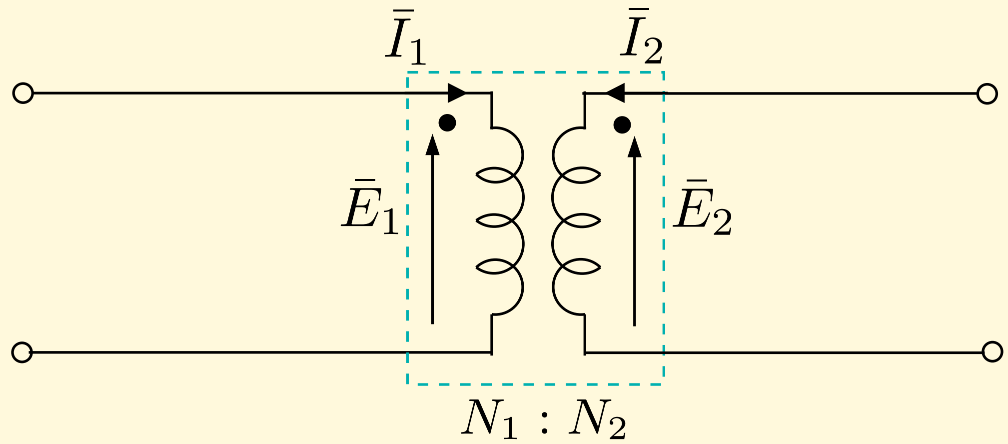


Figure: Ideal transformer ($\bar{I}_{oc} = 0$).

$$\frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2}, \qquad \frac{\bar{I}_1}{\bar{I}_2} = -\frac{N_2}{N_1}$$

Practical transformer contd...

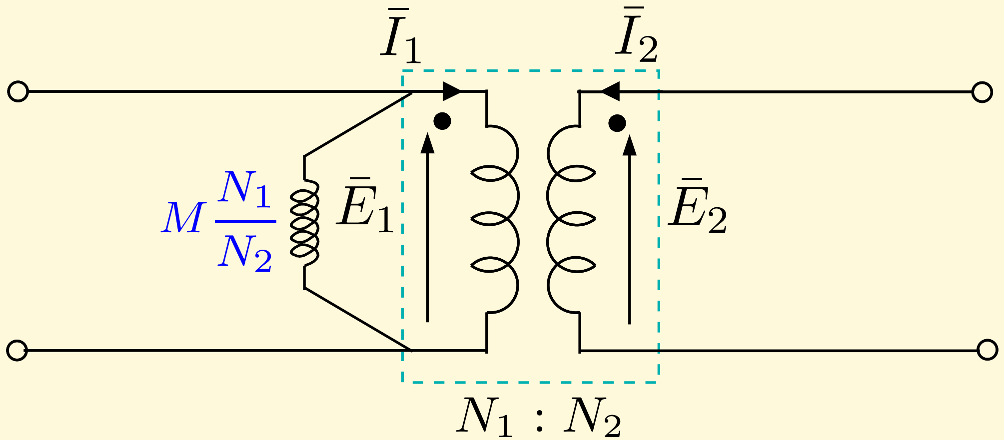


Figure: Ideal transformer ($\bar{I}_{oc} \neq 0$).
[Note: $M\frac{N_1}{N_2}$ is in source side and draws $\bar{I}_{oc}$ from the source.]

$$\frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2}, \quad \frac{\bar{I}_1}{\bar{I}_2} = -\frac{N_2}{N_1}$$

Practical transformer contd...

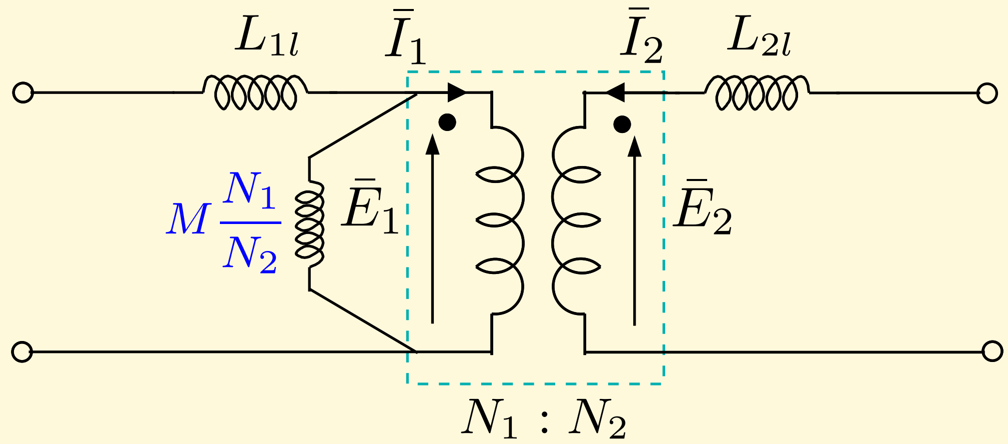


Figure: Practical transformer.
[L_{1l} and L_{2l} are the leakage inductances of the windings.]

$$\frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2}, \quad \frac{\bar{I}_1}{\bar{I}_2} = -\frac{N_2}{N_1}$$

Practical transformer contd...

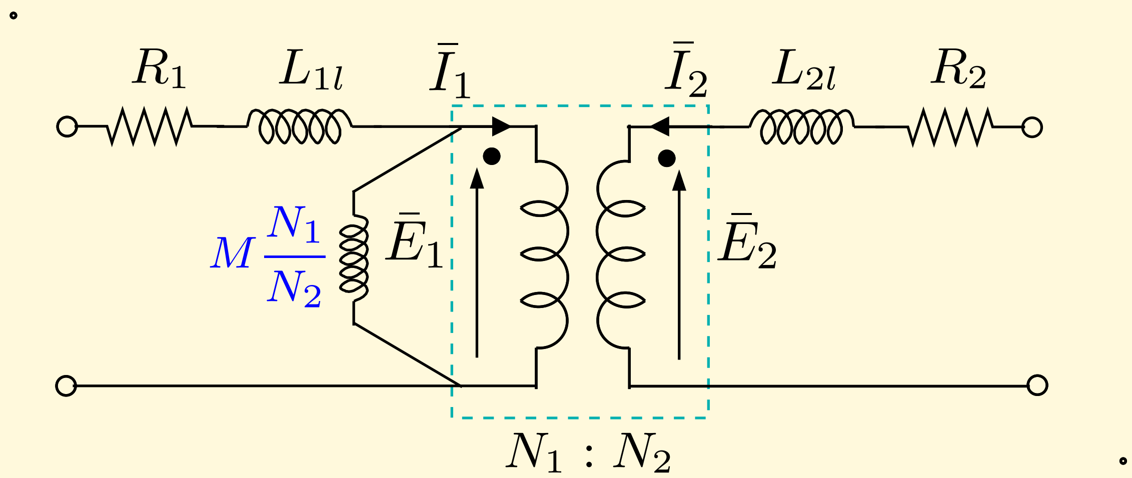


Figure: Practical transformer.
[R_1 and R_2 abstracts the winding resistances.]

$$\frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2}, \quad \frac{\bar{I}_1}{\bar{I}_2} = -\frac{N_2}{N_1}$$

Practical transformer contd...

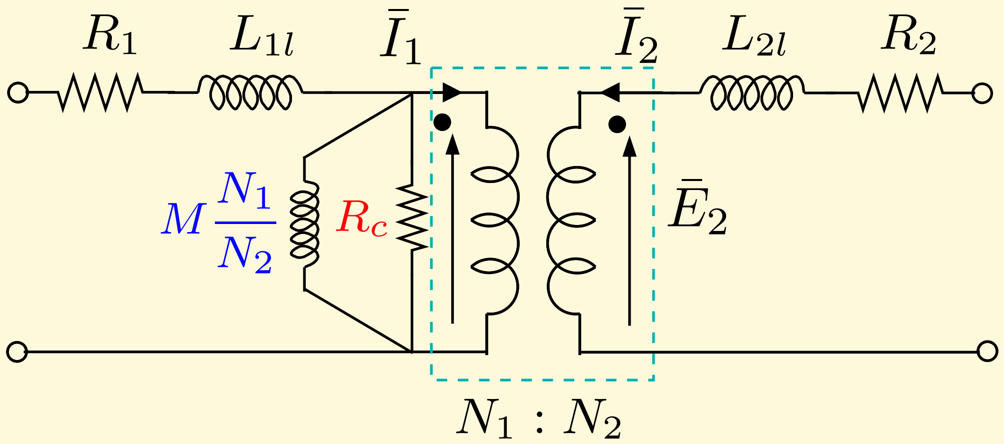


Figure: Practical transformer.

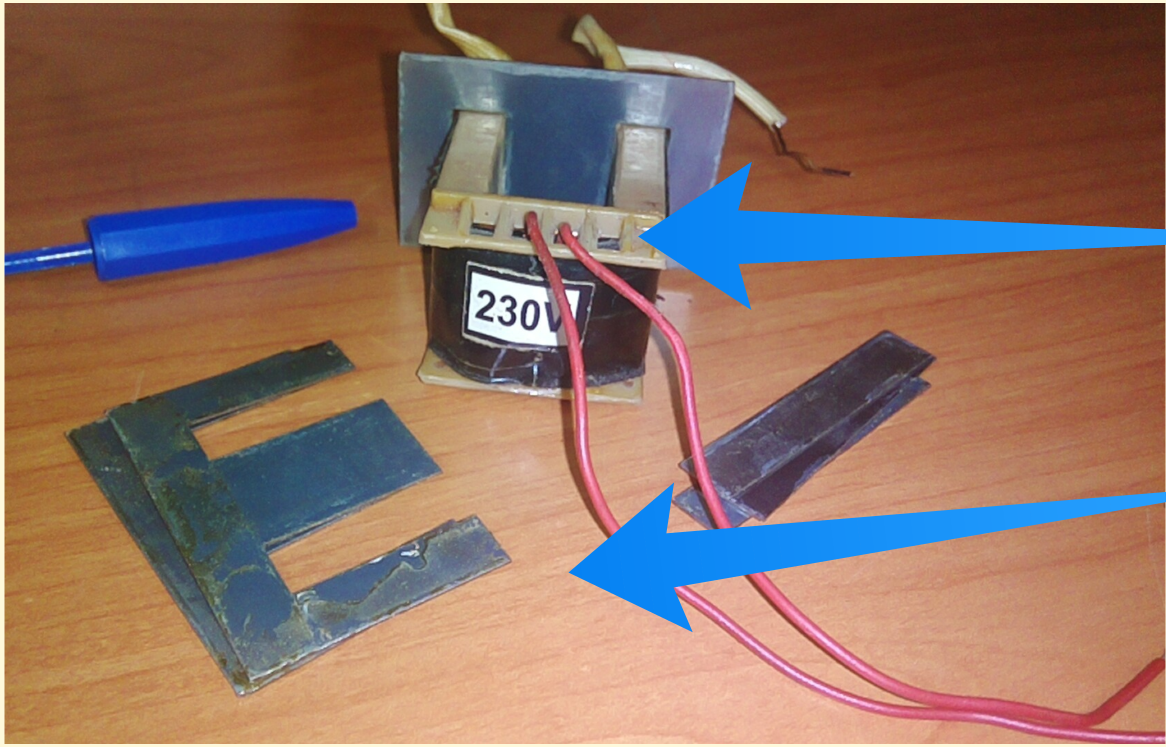
[R_c abstracts core losses (eddy current and hysterisis losses).]

$$\frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2}, \quad \frac{\bar{I}_1}{\bar{I}_2} = -\frac{N_2}{N_1}$$

Single phase 50 Hz transformer (low power ratings)

Name plate rating:
2000 VA, 230 V/ 115 V, 50 Hz

Single phase 50 Hz transformer (low power ratings)



Bobbin holding both the windings

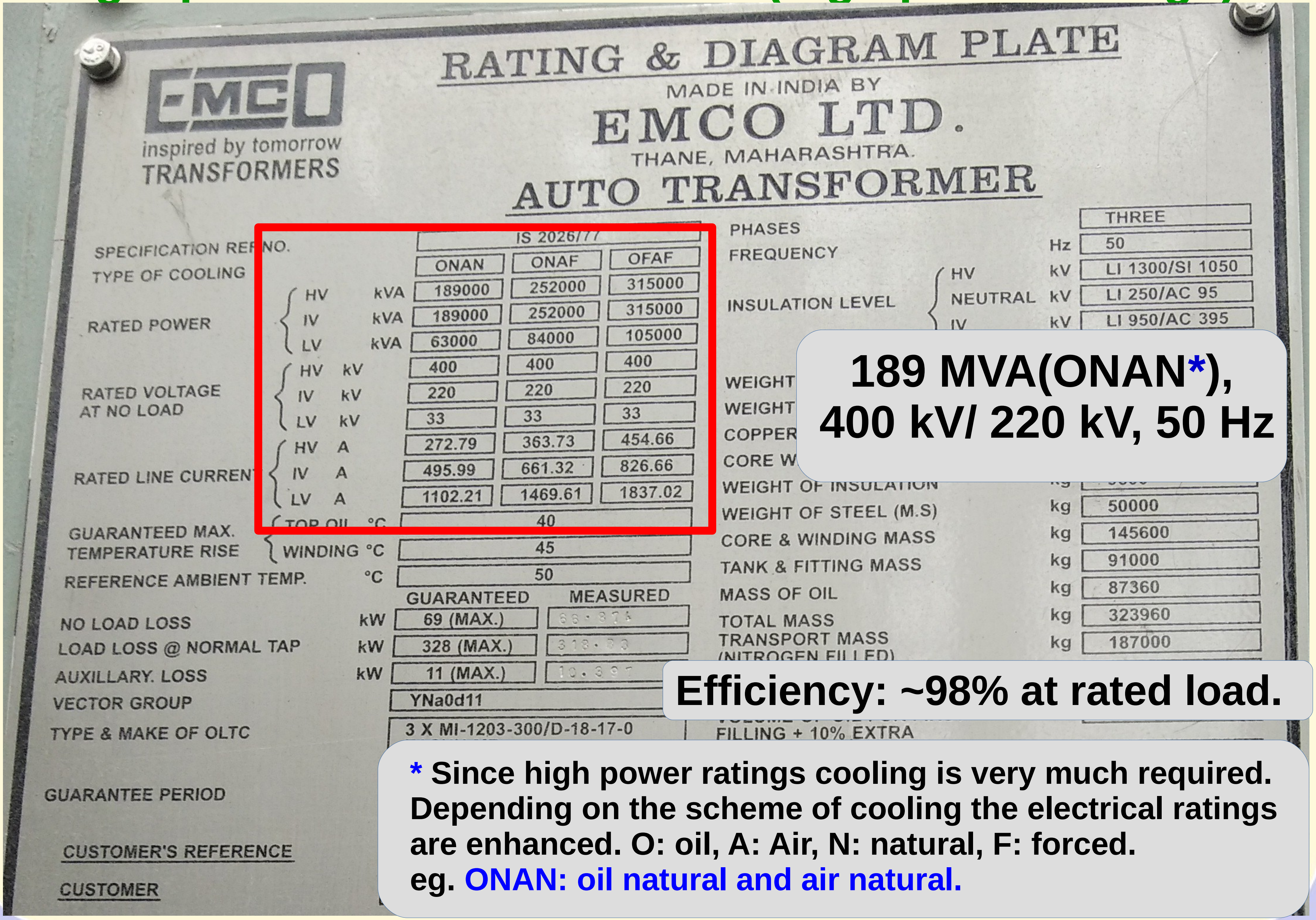
E-I laminated CRGO stacks (cold rolled grain-oriented steel)

Single phase 50 Hz transformer (high power ratings)

12.5 MVA, 33 kV/ 11 kV 50 Hz.

Single phase 50 Hz transformer (high power ratings)

Single phase 50 Hz transformer (high power ratings)

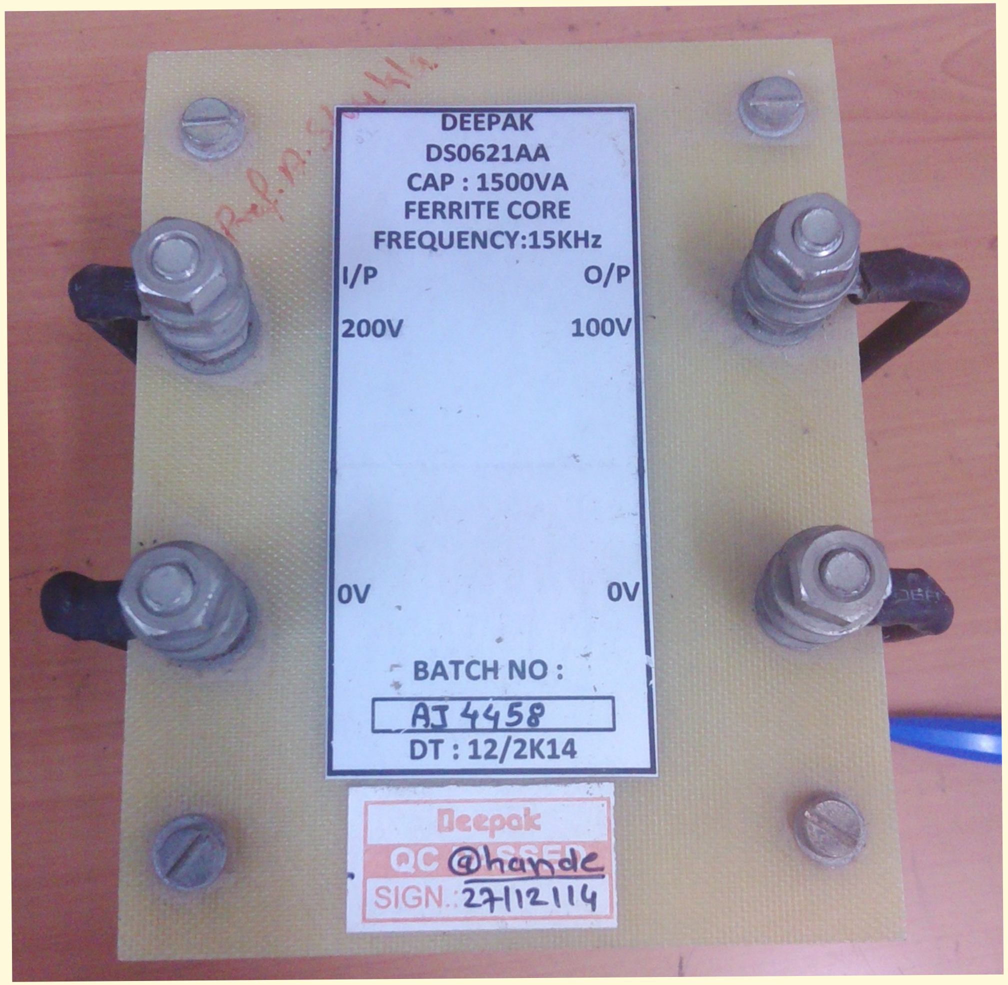


189 MVA(ONAN*), 400 kV/ 220 kV, 50 Hz

Efficiency: ~98% at rated load.

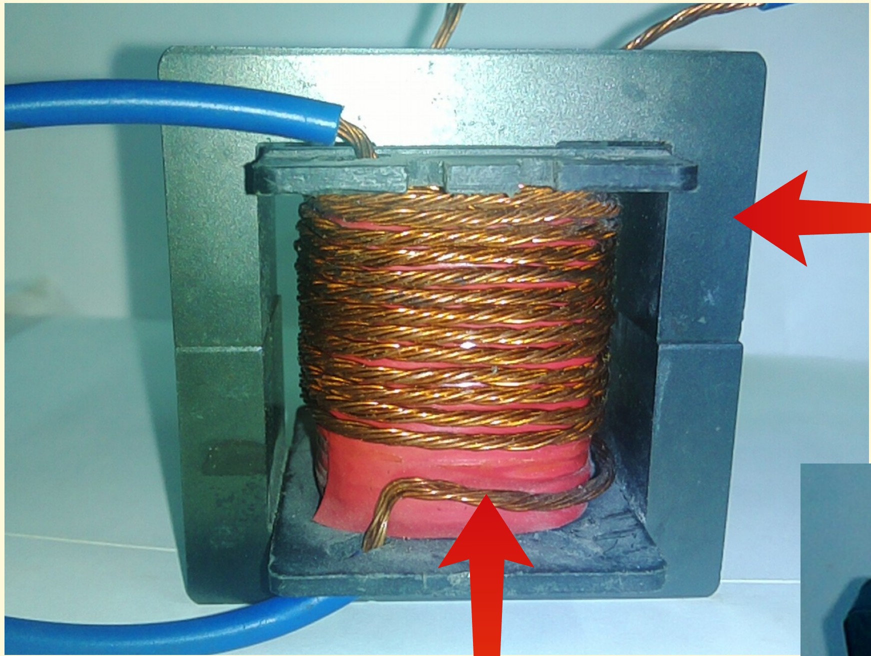
*** Since high power ratings cooling is very much required. Depending on the scheme of cooling the electrical ratings are enhanced. O: oil, A: Air, N: natural, F: forced. eg. ONAN: oil natural and air natural.**

Single phase 15 kHz transformer



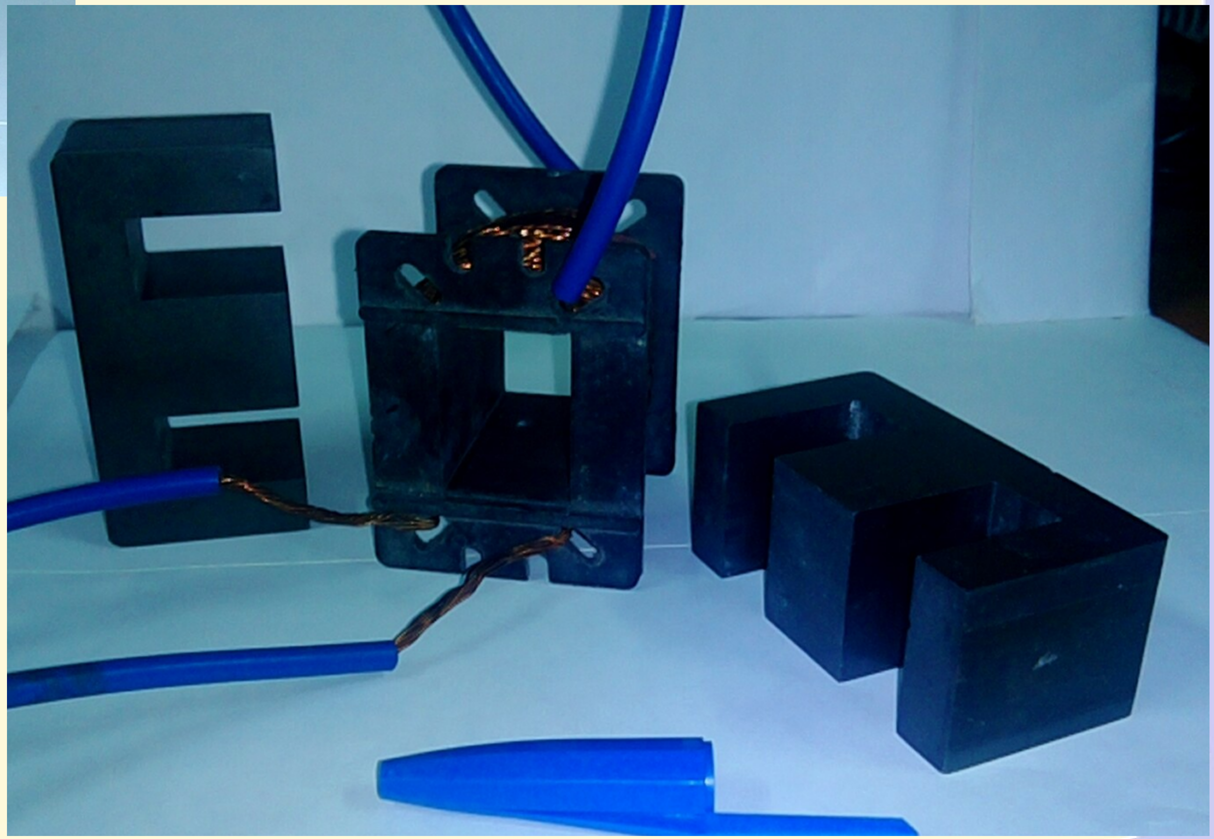
1.5 kVA, 200 V/ 100 V, 15 kHz

Single phase 15 kHz transformer

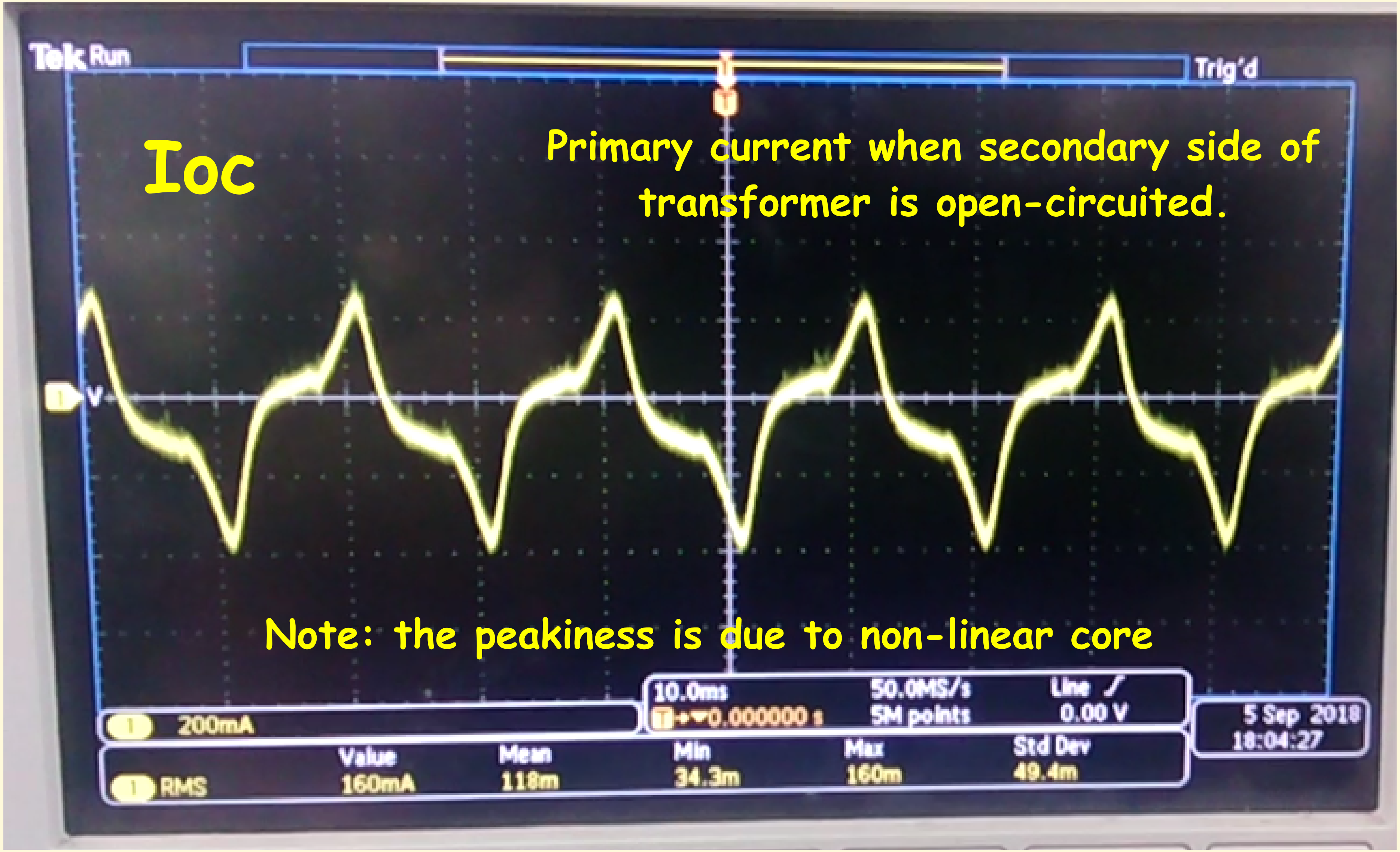


E-E Ferrite core

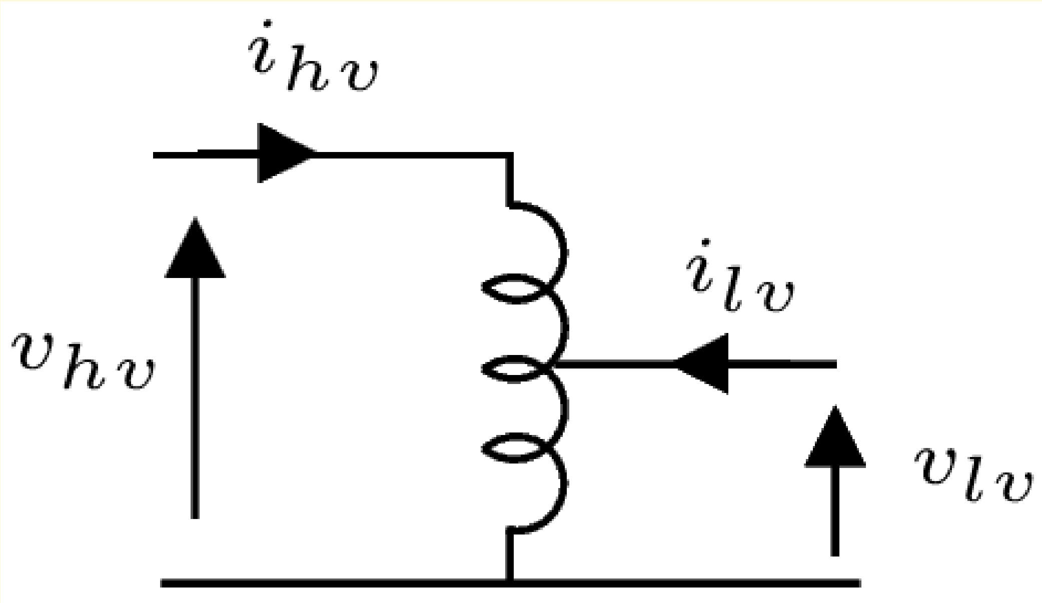
Litz wire, (stranded wire) for utilizing conductor cross section at high frequency



Input current when secondary open-circuited.



Auto transformer or variac

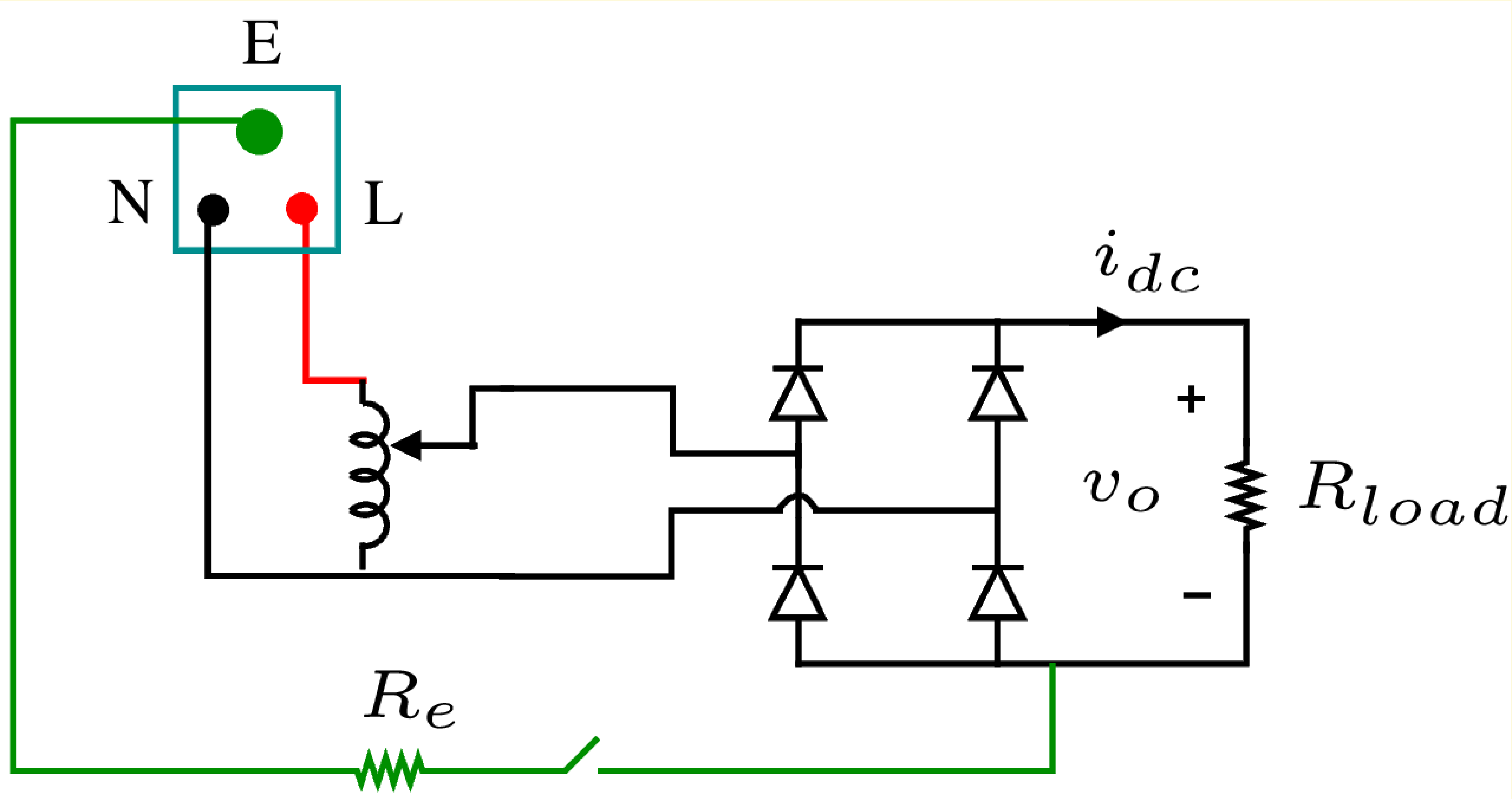


Isolation using transformer

Experiment for illustrating isolation

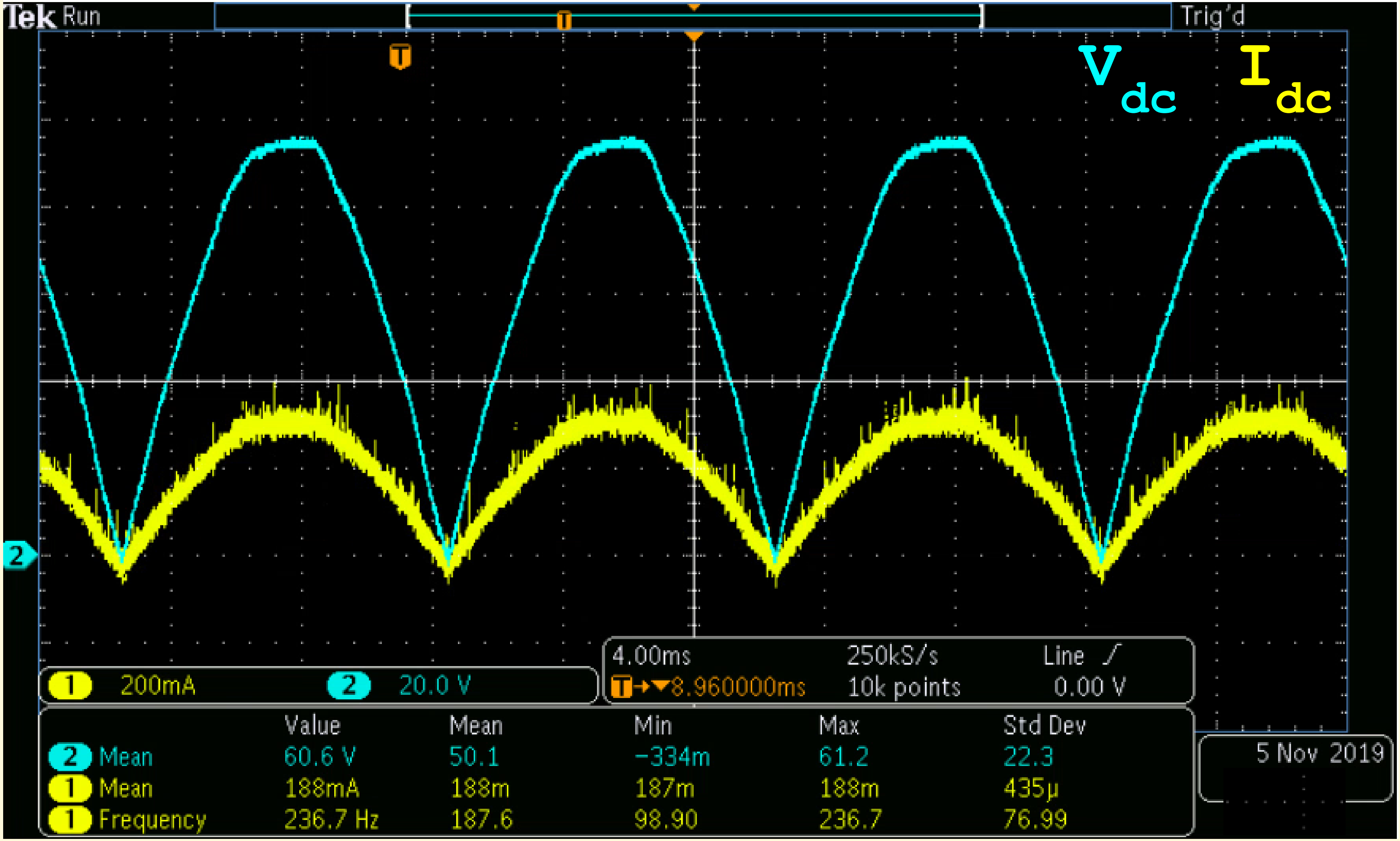
Case-1

Consider the following circuit



Experiment for illustrating isolation

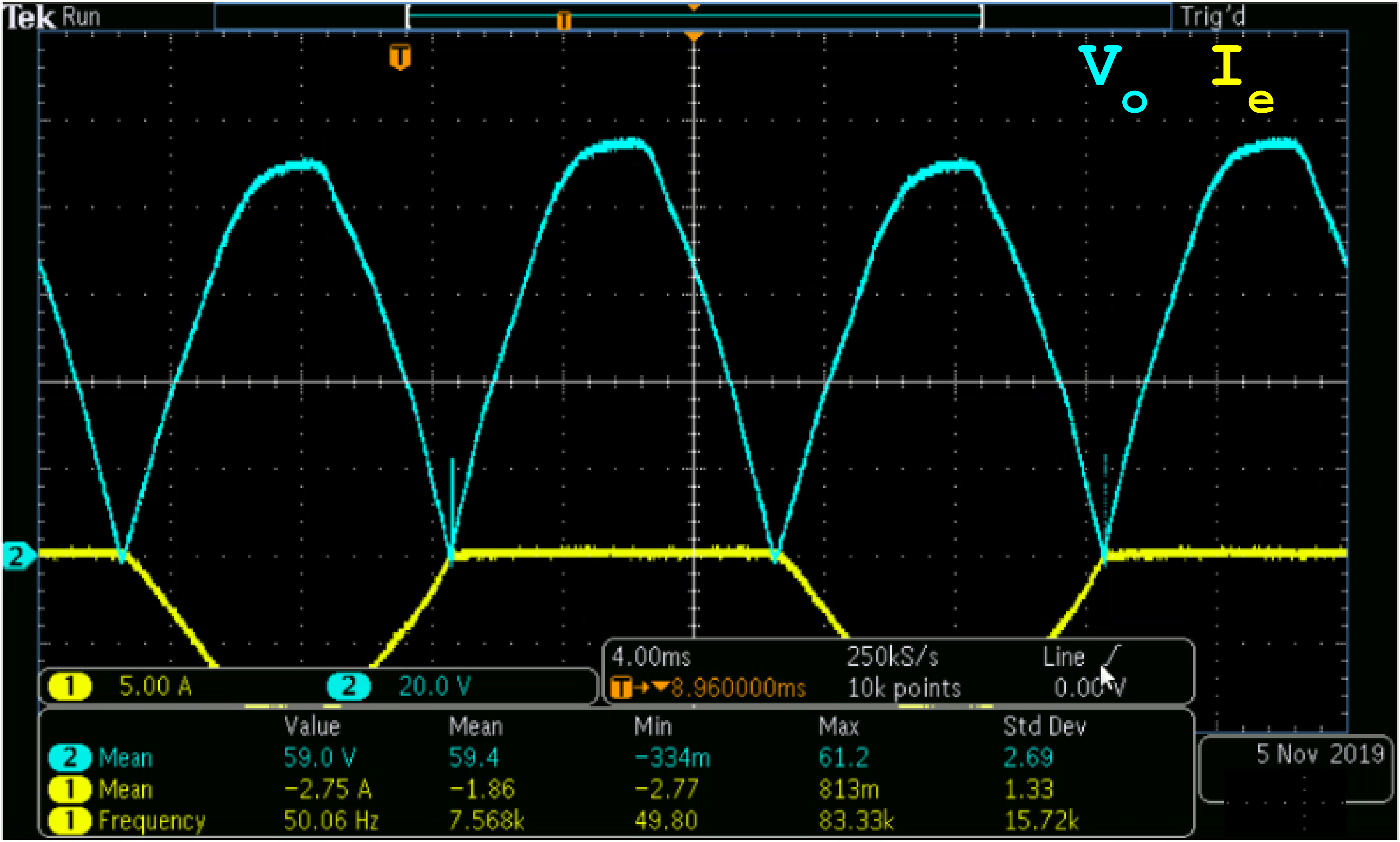
Case-1



Negative bus not earthed

Experiment for illustrating isolation

Case-1

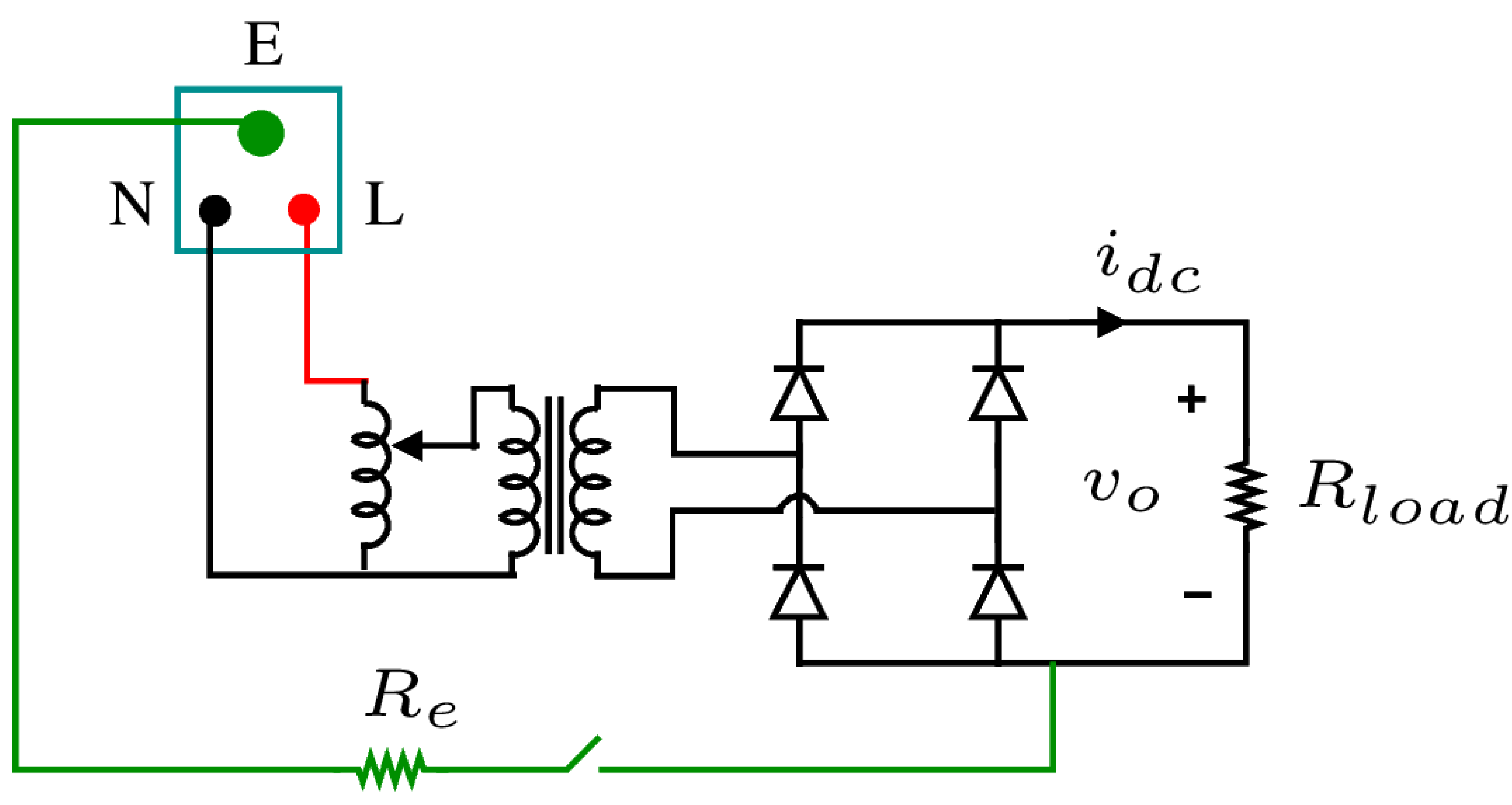


Negative bus earthed through a resistor

Experiment for illustrating isolation

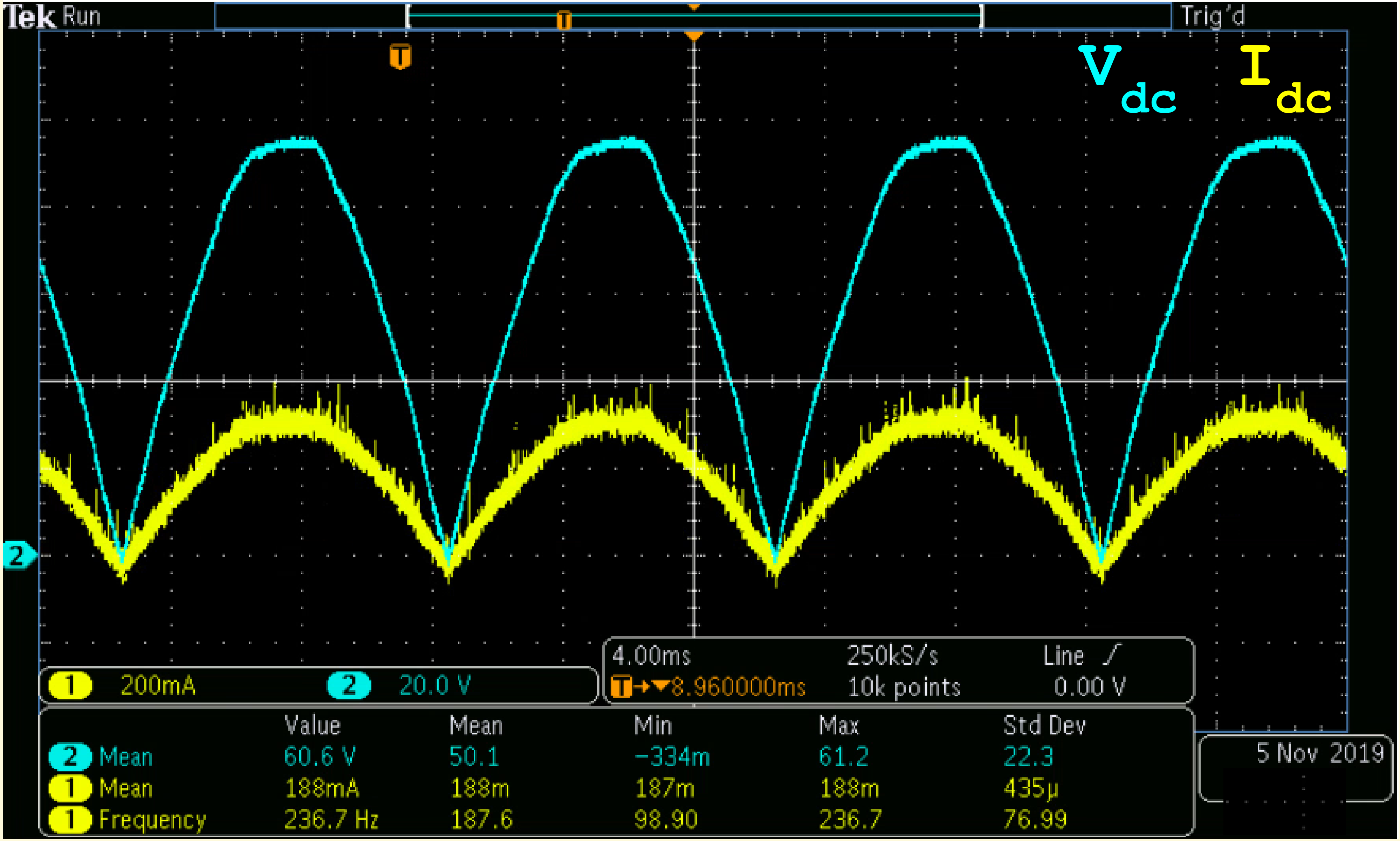
Case-2

Consider the previous circuit but with a isolation transformer:



Experiment for illustrating isolation

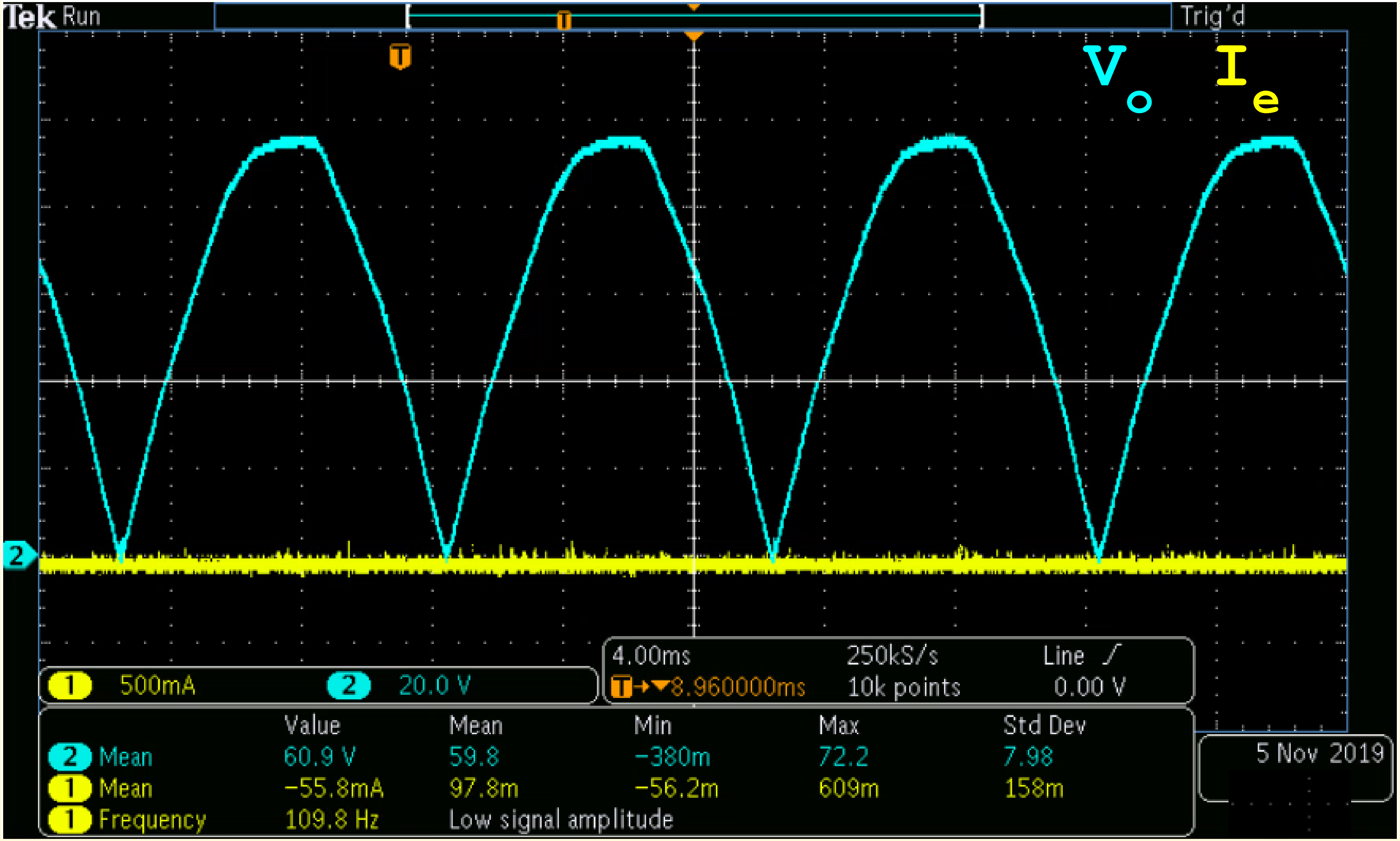
Case-2



Negative bus not earthed

Experiment for illustrating isolation

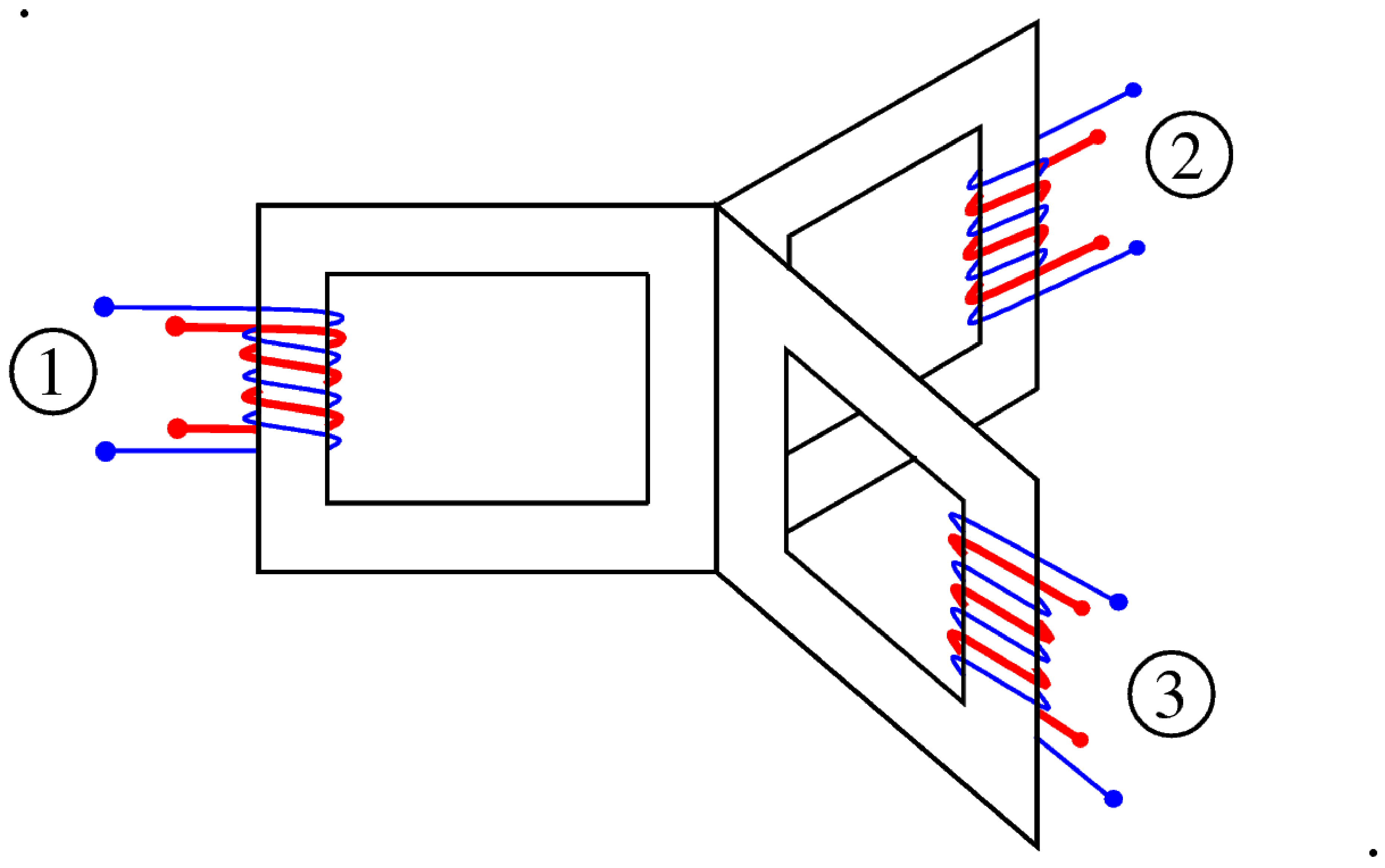
Case-2



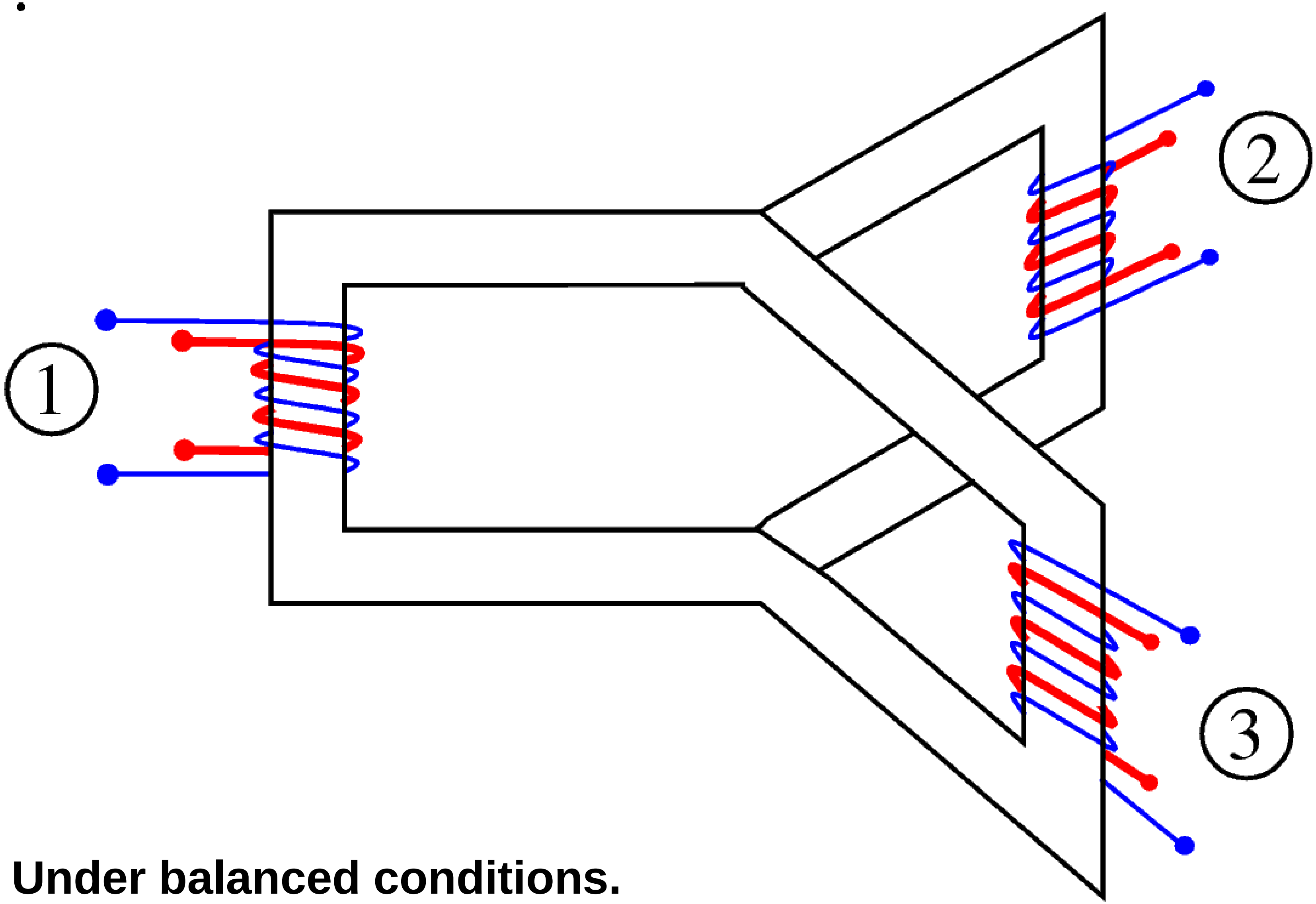
Negative bus earthed through a resistor

Three-phase transformer

Three single-phase two winding transformers:

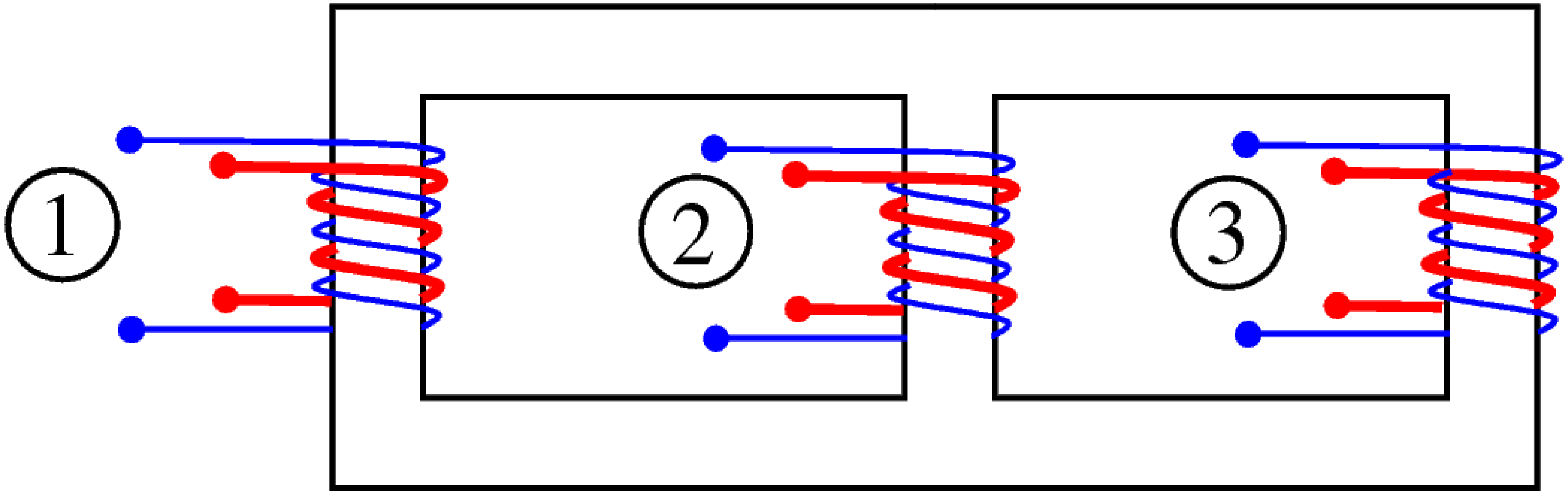


Three single-phase two winding transformers:



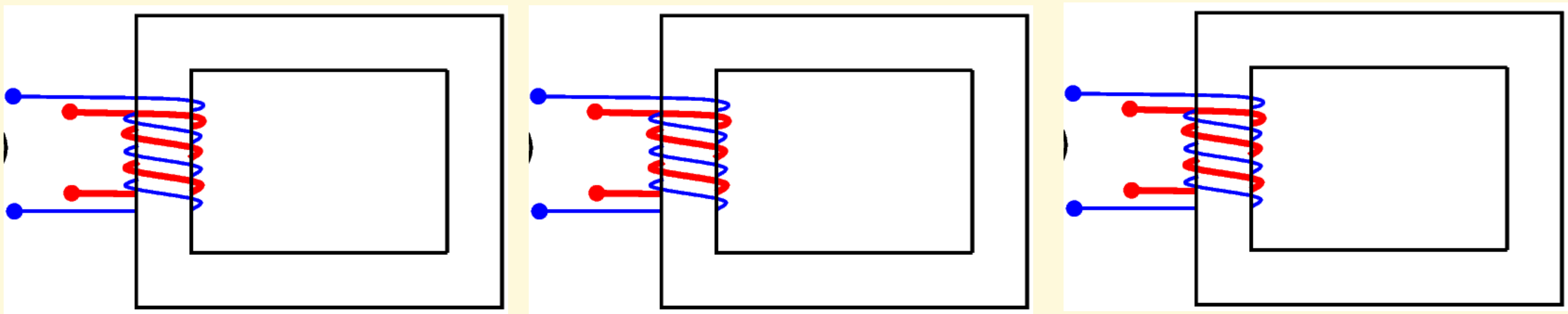
Under balanced conditions.

Three phase two winding transformers:



Three phase (two winding) transformer

Consider the MMF equation:



If $\bar{I}_{oc}$ is **negligible**,

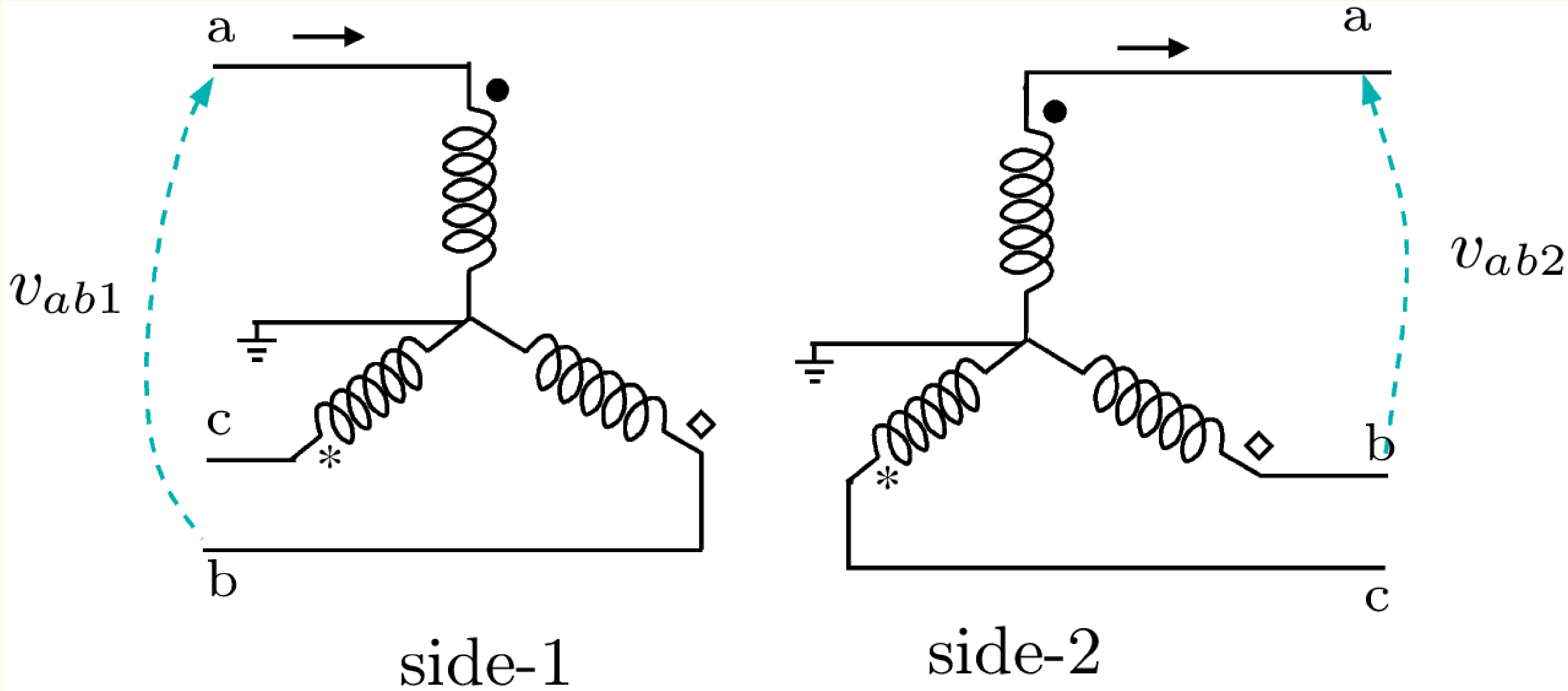
$$N_{lv}.\bar{I}_{lva} + N_{hv}.\bar{I}_{hva} = 0$$

$$N_{lv}.\bar{I}_{lvb} + N_{hv}.\bar{I}_{hvb} = 0$$

$$N_{lv}.\bar{I}_{lvc} + N_{hv}.\bar{I}_{hvc} = 0$$

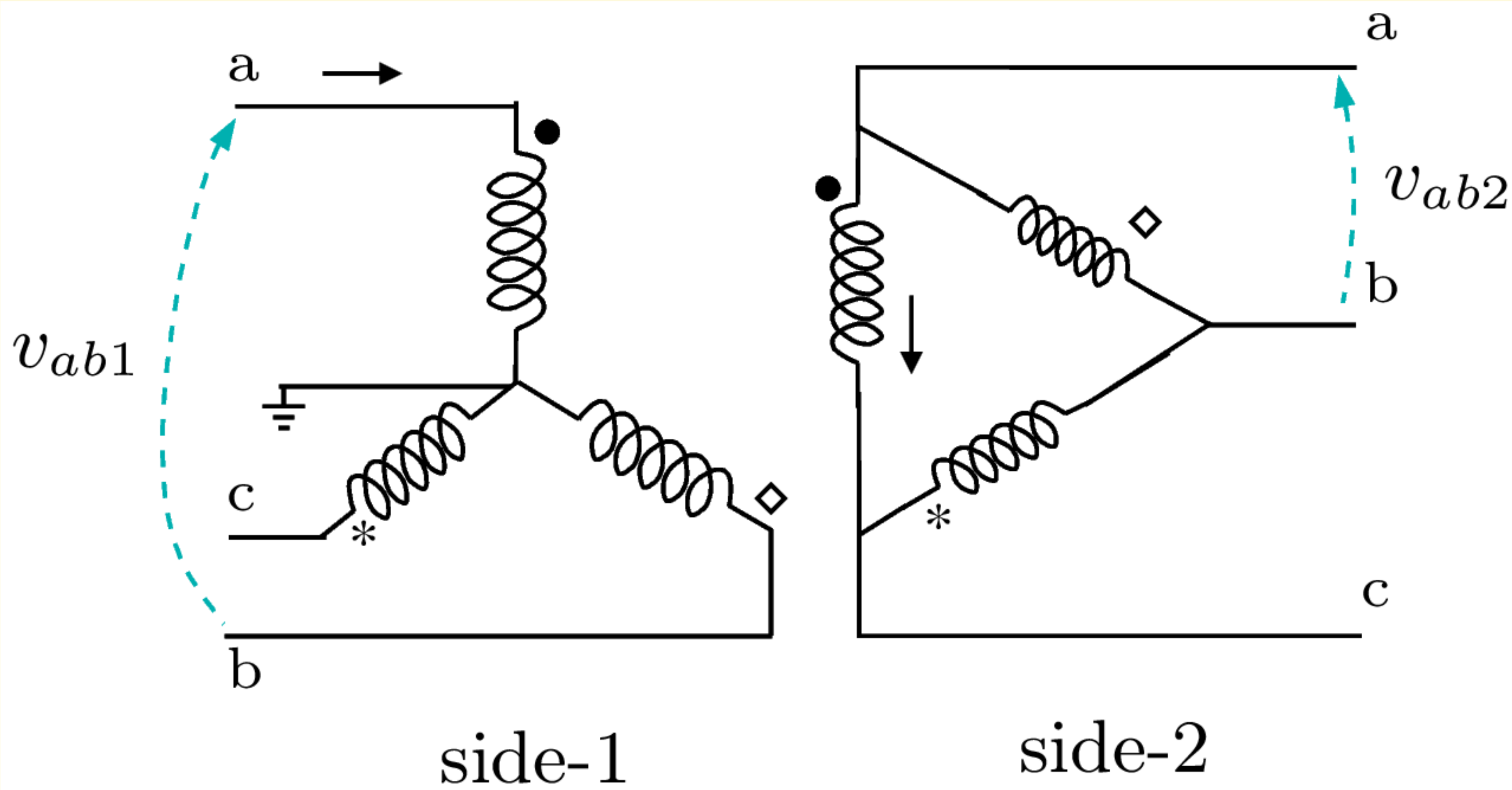
Some aspects of three-phase transformer

Consider the following circuit



Some aspects of three-phase transformer

Consider the following circuit



Some aspects of three-phase transformer

Consider the following circuit