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Tutorial 5. question 6.

6] $y[n] + \frac{1}{3} y[n-1] = x[n]$. Causal system.

ⓐ $x[n] = \delta[n] + 3\delta[n-1]$.

$x[0] = 1$; $x[1] = 3$; $x[n] = 0 \ \forall \ n \neq 1, 3$.

$y[0] = x[0] - \frac{1}{3} y[-1] = 1 - 0 = 1$.

$\because$ $y[-1] = 0$ because system is causal.

$y[1] = x[1] - \frac{1}{3} y[0]$

$= 3 - \frac{1}{3} = \frac{8}{3}$.

$y[2] = x[2] - \frac{1}{3} y[1] = 0 - \frac{1}{3}\left(\frac{8}{3}\right) = \frac{-1}{3} \cdot \frac{8}{3}$

$y[3] = \frac{-1}{3} \cdot y[2] = \left(\frac{-1}{3}\right)^2 \cdot \frac{8}{3}$

$\vdots$

$y[n] = \left(\frac{-1}{3}\right)^{n-1} \cdot \frac{8}{3}$.

ⓑ $x[n] = \frac{1}{3} u[n]$.

$\cancel{y[0] = \frac{1}{3} x[0]}$ $y[0] = x[0] - \frac{1}{3} y[-1] = \frac{1}{3}$.

$y[1] = x[1] - \frac{1}{3} y[0] = \frac{1}{3} - \frac{1}{3^2}$

$y[2] = x[2] - \frac{1}{3} y[1] = \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3}$

$\vdots$

$y[n] = \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} \cdots + \frac{(-1)^n}{3^{n+1}} = \cancel{\frac{1}{4}\left[1 - \left(\frac{-1}{3}\right)^{n+1}\right]}$.

$$y[n] = \frac{1}{3} \cdot \frac{1-\left(-\frac{1}{3}\right)^{n+1}}{1-\left(-\frac{1}{3}\right)} = \frac{1}{4} \cdot \left[1 - \left(\frac{-1}{3}\right)^{n+1}\right]$$

(c) now, $y[n] + \frac{1}{3} y[n-1] = x[n]$.

$$\downarrow \mathcal{F}$$

$$Y(j\omega) + \frac{1}{3} e^{-j\omega} Y(j\omega) = X(j\omega).$$

$$Y(j\omega) = \frac{X(j\omega)}{1 + \frac{1}{3} e^{-j\omega}}$$

$$\Rightarrow H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{1 + \frac{1}{3} e^{-j\omega}}$$

(c) input $= X(j\omega) = \dfrac{1 - \frac{1}{4} e^{-j\omega}}{1 + \frac{1}{2} e^{-j\omega}}$.

$$Y(j\omega) = X(j\omega) \times H(j\omega)$$

$$= \frac{1 - \frac{1}{4} e^{-j\omega}}{1 + \frac{1}{2} e^{-j\omega}} \cdot \frac{1}{1 + \frac{1}{3} e^{-j\omega}}$$

using partial fract^n,

$$\frac{\left(1 - \frac{1}{4} e^{-j\omega}\right)}{\left(1 + \frac{1}{2} e^{-j\omega}\right)\left(1 + \frac{1}{3} e^{-j\omega}\right)} = \frac{A}{1 + \frac{1}{2} e^{-j\omega}} + \frac{B}{1 + \frac{1}{3} e^{-j\omega}}$$

$$1 - \frac{1}{4} e^{-j\omega} = A\left(1 + \frac{1}{3} e^{-j\omega}\right) + B\left(1 + \frac{1}{2} e^{-j\omega}\right)$$

$$A + B = 1, \quad \frac{A}{3} + \frac{B}{2} = \frac{-1}{4}.$$

on solving, we get -

$$B = -7/2 \qquad A = 9/2.$$

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$$\therefore\ Y(j\omega) = \frac{9}{2} \cdot \frac{1}{1 + \frac{1}{2}e^{-j\omega}} - \frac{7}{2} \cdot \frac{1}{1 + \frac{1}{3}e^{-j\omega}}$$

$$Y(j\omega) = \frac{9}{2} \sum_{n=0}^{\infty} \left[\left(\frac{-1}{2}\right) e^{-j\omega}\right]^n - \frac{7}{2} \sum_{n=0}^{\infty} \left(-\frac{1}{3} e^{-j\omega}\right)^n$$

$$= \frac{9}{2} \sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n e^{-j\omega n} - \frac{7}{2} \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{9}{2} \cdot \left(\frac{-1}{2}\right)^n \cdot u[n]\, e^{-j\omega n} - \sum_{n=-\infty}^{\infty} \frac{7}{2} \left(\frac{-1}{3}\right)^n u[n]\, e^{-j\omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{9}{2}\left(\frac{-1}{2}\right)^n u[n] - \frac{7}{2}\left(\frac{-1}{3}\right)^n u[n] \right) e^{-j\omega n}$$

$$\therefore\ y[n] = \left[\frac{9}{2}\left(\frac{-1}{2}\right)^n - \frac{7}{2}\left(\frac{-1}{3}\right)^n\right] u[n].$$

(d) $X(j\omega) = 1 + 2e^{-3j\omega}.$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$= (1 + 2e^{-3j\omega})(H(j\omega))$$

$$= H(j\omega) + 2e^{-3j\omega} \cdot H(j\omega).$$

$$y[n] = h[n] + 2h[n-3]$$

now, $H(j\omega) = \dfrac{1}{1 + \frac{1}{3}e^{-j\omega}} = \displaystyle\sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n e^{-j\omega n}.$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{-1}{3}\right)^n u[n]\, e^{-j\omega n}.$$

$$\therefore\ h[n] = \left(\frac{-1}{3}\right)^n u[n].$$

$$\therefore\ y[n] = \left(\frac{-1}{3}\right)^n u[n] + 2 \cdot \left(\frac{-1}{3}\right)^{n-3} u[n-3].$$