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Why does preregistration increase the persuasiveness of evidence? A Bayesian

rationalization

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25 Abstract

The replication crisis has led many researchers to preregister their hypotheses and data 26 analysis plans before collecting data. A widely held view is that preregistration is supposed 27 to limit the extent to which data may influence the hypotheses to be tested. Only if data 28 have no influence an analysis is considered confirmatory. Consequently, many researchers 29 believe that preregistration is only applicable in confirmatory paradigms. In practice, researchers may struggle to preregister their hypotheses because of vague theories that necessitate data-dependent decisions (aka exploration). We argue that preregistration 32 benefits any study on the continuum between confirmatory and exploratory research. To 33 that end, we formalize a general objective of preregistration and demonstrate that exploratory studies also benefit from preregistration. Drawing on Bayesian philosophy of science, we argue that preregistration should primarily aim to reduce uncertainty about the inferential procedure used to derive results. This approach provides a principled justification of preregistration, separating the procedure from the goal of ensuring strictly 38 confirmatory research. We acknowledge that knowing the extent to which a study is 39 exploratory is central, but certainty about the inferential procedure is a prerequisite for persuasive evidence. Finally, we discuss the implications of these insights for the practice of 41 preregistration. 42

Keywords: preregistration; confirmation; exploration; hypothesis testing; Bayesian;
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Why does preregistration increase the persuasiveness of evidence? A Bayesian rationalization

The scientific community has long pondered the vital distinction between 48 exploration and confirmation, discovery and justification, hypothesis generation and 49 hypothesis testing, or prediction and postdiction (Hoyningen-Huene, 2006; Nosek et al., 2018; Shmueli, 2010). Despite the different names, it is fundamentally the same dichotomy that is at stake here. There is a broad consensus that both approaches are necessary for science to progress; exploration, to make new discoveries and confirmation, to expose these discoveries to potential falsification, and assess empirical support for the theory. However, mistaking exploratory findings for empirically confirmed results is dangerous. It inflates the likelihood of believing that there is evidence supporting a given hypothesis, even if it is false. A variety of problems, such as researchers' degrees of freedom together with researchers' hindsight bias or naive p-hacking have led to such mistakes becoming commonplace yet unnoticed for a long time. Recognizing them has led to a crisis of 59 confidence in the empirical sciences (Ioannidis, 2005), and psychology in particular (Open Science Collaboration, 2015). As a response to the crisis, evermore researchers preregister their hypotheses and their data collection and analysis plans in advance of their studies (Nosek et al., 2018). They do so to stress the predictive nature of their registered statistical 63 analyses, often with the hopes of obtaining a label that marks the study as "confirmatory". Indeed, rigorous application of preregistration prevents researchers from reporting a set of results produced by an arduous process of trial and error as a simple confirmatory story (Wagenmakers et al., 2012) while keeping low false-positive rates. This promise of a clear distinction between confirmation and exploration has obvious appeal to many who have already accepted the practice. Still, the majority of empirical researchers do not routinely 69 preregister their studies. One reason may be that some do not find that the theoretical advantages outweigh the practical hurdles, such as specifying every aspect of a theory and 71 the corresponding analysis in advance. We believe that we can reach a greater acceptance

of preregistration by explicating a more general objective of preregistration that benefits all kinds of studies, even those that allow data-dependent decisions.

One goal of preregistration that has received widespread attention is to clearly
distinguish confirmatory from exploratory research (Bakker et al., 2020; Mellor & Nosek,
2018; Nosek et al., 2018; Simmons et al., 2021; Wagenmakers et al., 2012). In such a
narrative, preregistration is justified by a confirmatory research agenda. However, two
problems become apparent under closer inspection. First, many researchers do not
subscribe to a purely confirmatory research agenda. Second, there is no strict mapping of
the categories preregistered vs. non-preregistered onto the categories confirmatory
vs. exploratory research.

Obviously, researchers can conduct confirmatory research without preregistration—
though it might be difficult to convince other researchers of the confirmatory nature of
their research, that is, that they were free of cognitive biases, made no data-dependent
decisions, and so forth. The opposite, that is, preregistered but not strictly confirmatory
studies, are also becoming more commonplace (Chan et al., 2004; Dwan et al., 2008; Silagy
et al., 2002).

This is the result of researchers applying one of two strategies to evade the self-imposed restrictions of preregistrations: writing a loose preregistration, to begin with (Stefan & Schönbrodt, 2023) or deviating from the preregistration afterward. Both strategies may be used for sensible scientific reasons or with the self-serving intent of generating desirable results. Thus, insisting on equating preregistration and confirmation has led to the criticism that, all things considered, preregistration is actually harmful and neither sufficient nor necessary for doing good science (Pham & Oh, 2021; Szollosi et al., 2020).

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We argue that such criticism is not directed against preregistration itself but against

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a justification through a confirmatory research agenda (Wagenmakers et al., 2012). When researchers criticize preregistration as being too inflexible to fit their research question, gg they often simply acknowledge that their research goals are not strictly confirmatory. 100 Forcing researchers into adopting a strictly confirmatory research agenda does not only 101 imply changing how they investigate a phenomenon but also what research questions they 102 pose. However reasonable such a move is, changing the core beliefs of a large community is 103 much harder than convincing them that a method is well justified. We, therefore, attempt 104 to disentangle the methodological goals of preregistration from the ideological goals of 105 confirmatory science. It might well be the case that psychology needs more confirmatory 106 studies to progress as a science. However, independently of such a goal, preregistration can 107 be useful for any kind of study on the continuum between strictly confirmatory and fully 108 exploratory.

To form such an objective for preregistration, we first introduce some tools of Bayesian philosophy of science and map the exploration/confirmation distinction onto a dimensional quantity we call "theoretical risk" (a term borrowed from Meehl, 1978, but formalized as the probability of proving a hypothesis wrong if it does not hold), which is inversely related to the type-I error rate in null hypothesis testing.

Further, we outline two interpretations of preregistration. The first one corresponds 115 to the traditional application of preregistration to research paradigms that focus on 116 confirmation by maximizing the theoretical risk or, equivalently, by limiting type-I error 117 (when dichotomous decisions about theories are an inferential goal). We argue that this 118 view on the utility of preregistration can be interpreted as maximizing theoretical risk, 119 which otherwise may be reduced by researchers' degrees of freedom, p-hacking, and suchlike. 120 The second interpretation is our main contribution: We argue that contrary to the classic 121 view, the objective of preregistration is not the maximization of theoretical risk but rather 122 the minimization of uncertainty about the theoretical risk. This interpretation leads to a 123

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broad applicability of preregistration to both exploratory and confirmatory studies.

To arrive at this interpretation, we rely on three arguments. The first is that
theoretical risk is vital for judging evidential support for theories. The second argument is
that the theoretical risk for a given study is generally uncertain. The third and last
argument is that this uncertainty is reduced by applying preregistration. We conclude that
because preregistration decreases uncertainty about the theoretical risk, which in turn
increases the amount of knowledge we gain from a particular study, preregistration is
potentially useful for any kind of study, no matter where it falls on the
exploratory-confirmatory continuum.

Epistemic value and the Bayesian rationale

Let us start by defining what we call expected epistemic value. If researchers plan 134 to conduct a study, they usually hope that it will change their assessment of some theory's 135 verisimilitude (Niiniluoto, 1998). In other words, they hope to learn something from 136 conducting the study. The amount of knowledge researchers gain from a particular study 137 concerning the verisimilitude of a specific theory is what we call epistemic value. 138 Researchers cannot know what exactly they will learn from a study before they run it. 139 However, they can develop an expectation that helps them decide about the specifics of a planned study. This expectation is what we term expected epistemic value. To make our 141 three arguments, we must assume three things about what an ideal estimation process entails and how it relates to what studies (preregistered vs not preregistered) to conduct.

- 1. Researchers judge the evidence for or against a hypothesis rationally.
- 2. They expect other researchers to apply a similar rational process.
 - 3. Researchers try to maximize the expected epistemic value for other researchers.

The assumption of rationality can be connected to Bayesian reasoning and leads to our adoption of the framework. Our rationale is as follows. Researchers who decide to

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conduct a certain study are actually choosing a study to bet on. They have to "place the 149 bet" by conducting the study by investing resources and stand to gain epistemic value with 150 some probability. This conceptualization of choosing a study as a betting problem allows 151 us to apply a "Dutch book" argument (Christensen, 1991). This argument states that any 152 better must follow the axioms of probability to avoid being "irrational," i.e., accepting bets 153 that lead to sure losses. Fully developing a Dutch book argument for this problem requires 154 careful consideration of what kind of studies to include as possible bets, defining a 155 conversion rate from the stakes to the reward, and modeling what liberties researchers have 156 in what studies to conduct. Without deliberating these concepts further, we find it 157 persuasive that researchers should not violate the axioms of probability if they have some 158 expectation about what they stand to gain with some likelihood from conducting a study. 159 The axioms of probability are sufficient to derive the Bayes formula, on which we will heavily rely for our further arguments. The argument is not sufficient, however, to warrant 161 conceptualizing the kind of epistemic value we reason about in terms of posterior 162 probability; that remains a leap of faith. However, the argument applies to any reward 163 function that satisfies the "statistical relevancy condition" (Fetzer, 1974; Salmon, 1970). 164 That is, evidence only increases epistemic value for a theory if the evidence is more likely 165 to be observed under the theory than under the alternative. 166

Please note that our decision to adopt this aspect of the Bayesian philosophy of science does not make assumptions about the statistical methods researchers use. In fact, this conceptualization is intentionally as minimal as possible to be compatible with a wide range of philosophies of science and statistical methods researchers might subscribe to.

Epistemic value and theoretical risk

Our first argument is that theoretical risk is crucial for judging evidential support for theories. Put simply, risky predictions create persuasive evidence if they turn out to be correct. This point is crucial because we attribute much of the appeal of a confirmatory 175 research agenda to this notion.

Let us make some simplifying assumptions and define our notation. To keep the notation simple, we restrict ourselves to evidence of a binary nature (either it was observed or not). We denote the probability of a hypothesis before observing evidence as P(H) and its complement as $P(\neg H) = 1 - P(H)$. The probability of observing evidence under some hypothesis is P(E|H). We can calculate the probability of the hypothesis after observing the evidence with the help of the Bayes formula:

$$P(H|E) = \frac{P(H)P(E|H)}{P(E)} \tag{1}$$

The posterior probability P(H|E) is of great relevance since it is often used directly or indirectly as a measure of confirmation of a hypothesis. In the tradition of Carnap, in its direct use, it is called confirmation as firmness; in its relation to the a priori probability P(H), it is called *increase in firmness* Carnap (1950), preface to the 1962 edition]. As noted before, we concentrate on posterior probability as a measure of epistemic value since no measure shows universally better properties than others. However, it is reasonable that any measure of confirmation increases monotonically with an increase in posterior probability P(H|E), and our argument applies to those measures as well.

In short, we want to increase posterior probability P(H|E). Increases in posterior probability P(H|E) are associated with increased epistemic value, of which we want to maximize the expectation. So how can we increase posterior probability? The Bayes formula yields three components that influence confirmation, namely P(H), P(E|H) and P(E). The first option leads us to the unsurprising conclusion that higher a priori probability P(H) leads to higher posterior probability P(H|E). If a hypothesis is more probable to begin with, observing evidence in its favor will result in a hypothesis that is more strongly confirmed, all else being equal. However, the prior probability of a

hypothesis is nothing our study design can change. The second option is equally 198 reasonable; that is, an increase in P(E|H) leads to a higher posterior probability P(H|E). 199 P(E|H) is the probability of obtaining evidence for a hypothesis when it holds. We call 200 this probability of detecting evidence, given that the hypothesis holds "detectability." 201 Consequently, researchers should ensure that their study design allows them to find 202 evidence for their hypothesis, in case it is true. When applied strictly within the bounds of 203 null hypothesis testing, detectability is equivalent to power (or the complement of type-II 204 error rate). However, while detectability is of great importance for study design, it is not 205 directly relevant to the objective of preregistration. Thus, P(E) remains to be considered. 206 Since P(E) is the denominator, decreasing it can increase the posterior probability. In 207 other words, high risk, high reward. 208

If we equate riskiness with a low probability of obtaining evidence (when the 209 hypothesis is false), the Bayesian rationale perfectly aligns with the observation that risky 210 predictions lead to persuasive evidence. This tension between high risk leading to high gain 211 is central to our consideration of preregistration. A high-risk, high-gain strategy is bound 212 to result in many losses that are eventually absorbed by the high gains. Sustaining many 213 "failed" studies is not exactly aligned with the incentive structure under which many, if not 214 most, researchers operate. Consequently, researchers are incentivized to appear to take 215 more risks than they actually do, which misleads their readers to give their claims more 216 credence than they deserve. It is at this juncture that the practice and mispractice of 217 preregistration comes into play. We argue that the main function of preregistration is to 218 enable proper judgment of the riskiness of a study. 219

To better understand how preregistrations can achieve that, let us take a closer look at the factors contributing to P(E). Using the law of total probability, we can split P(E)into two terms:

$$P(E) = P(H)P(E|H) + P(\neg H)P(E|\neg H)$$
(2)

We have already noted that there is not much to be done about prior probability 223 $(P(H), \text{ and hence its counter probability } P(\neg H)), \text{ and that it is common sense to increase}$ 224 detectability P(E|H). The real lever to pull is therefore $P(E|\neg H)$. This probability tells 225 us how likely it is that we find evidence in favor of the theory when in fact, the theory is 226 not true. Its counter probability $P(\neg E|\neg H) = 1 - P(E|\neg H)$ is what we call "theoretical 227 risk", because it is the risk a theory takes on in predicting the occurrence of particular 228 evidence in its favor. We borrow the term from Meehl (1978), though he has not assigned 229 it to the probability $P(\neg E|\neg H)$. Kukla (1990) argued that the core arguments in Meehl 230 (1990) can be reconstructed in a purely Bayesian framework. However, while he did not 231 mention $P(\neg E|\neg H)$ he suggested that Meehl (1978) used the term "very strange 232 coincidence" for a small $P(E|\neg H)$ which would imply, that $P(\neg E|\neg H)$ can be related to or 233 even equated to theoretical risk.

Let us note some interesting properties of theoretical risk $P(\neg E|\neg H)$. First, increasing theoretical risk leads to higher posterior probability P(H|E), our objective. Second, if the theoretical risk is smaller than detectability P(E|H) it follows that the posterior probability must decrease when observing the evidence. If detectability exceeds theoretical risk, the evidence is less likely under the theory than it is when the theory does not hold. Third, if the theoretical risk equals zero, then posterior probability is at best equal to prior probability but only if detectability is perfect (P(H|E) = 1). In other words, observing a sure fact does not lend credence to a hypothesis.

The last statement sounds like a truism but is directly related to Popper's seminal criterion of demarcation. He stated that if it is impossible to prove that a hypothesis is false $(P(\neg E|\neg H) = 0$, theoretical risk is zero), it cannot be considered a scientific

hypothesis (Popper, 2002, p. 18). We note these relations to underline that the Bayesian rationale we apply here is able to reconstruct many commonly held views on riskiness and epistemic value.

Both theoretical risk $P(\neg E|\neg H)$ and detectability P(E|H) aggregate countless influences; otherwise, they could not model the process of evidential support for theories. To illustrate the concepts we have introduced here, consider the following example of a 251 single theory and three experiments that may test it. The experiments were created to 252 illustrate how they may differ in their theoretical risk and detectability. Suppose the 253 primary theory is about the cognitive phenomenon of "insight." For the purpose of 254 illustration, we define it, with quite some hand-waving, as a cognitive abstraction that 255 allows agents to consistently solve a well-defined class of problems. We present the 256 hypothesis that the following problem belongs to such a class of insight problems: 257

Use five matches (IIIII) to form the number eight.

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We propose three experiments that differ in theoretical risk and detectability. All experiments take a sample of ten psychology students. We present the students with the problem for a brief span of time. After that, the three experiments differ as follows:

- 1. The experimenter gives a hint that the problem is easy to solve when using Roman numerals; if all students come up with the solution, she records it as evidence for the hypothesis.
- 265 2. The experimenter shows the solution "VIII" and explains it; if all students come up
 with the solution, she records it as evidence for the hypothesis.
 - 3. The experimenter does nothing; if all students come up with the solution, she records it as evidence for the hypothesis.

We argue that experiment 1 has high theoretical risk $P(\neg E_1|\neg H)$ and high detectability $P(E_1|H)$. If "insight" has nothing to do with solving the problem $(\neg H)$, then

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presenting the insight that Roman numerals can be used should not lead to all students 271 solving the problem $(\neg E_1)$; the experiment, therefore, has high theoretical risk 272 $P(\neg E_1|\neg H)$. Conversely, if insight is required to solve the problem (H), then it is likely to 273 help all students to solve the problem (E_1) , the experiment, therefore, has high 274 detectability $P(E_1|H)$. The second experiment, on the other hand, has low theoretical risk 275 $P(\neg E_2|\neg H)$. Even if "insight" has nothing to do with solving the problem $(\neg H)$, there are 276 other plausible reasons for observing the evidence (E_2) , because the students could simply 277 copy the solution without having any insight. With regard to detectability, experiments 1 278 and 2 differ in no obvious way. Experiment 3, however, also has low detectability. It is 279 unlikely that all students will come up with the correct solution in a short time (E_3) , even 280 if insight is required (H); experiment 3 therefore has low detectability $P(E_3|H)$. The 281 theoretical risk, however, is also low in absolute terms, but high compared to the detectability (statistical relevancy condition is satisfied). In the unlikely event that all 10 283 students place their matches to form the Roman numeral VIII (E_3) , it is probably due to 284 insight (H) and not by chance $P(\neg E_3|\neg H)$). Of course, in practice, we would allow the 285 evidence to be probabilistic, e.g., relax the requirement of "all students" to nine out of ten 286 students, more than eight, and so forth. 287

As mentioned earlier, the we restrict ourselves to binary evidence, to keep the mathematical notation as simple as possible. We discuss the relation between statistical methods and theoretical risk in the Statistical Methods section.

Preregistration as a means to increase theoretical risk?

Having discussed that increasing the theoretical risk will increase the epistemic value, it is intuitive to task preregistration with maximizing theoretical risk, i.e., a confirmatory research agenda. Indeed, limiting the type-I error rate is commonly stated as the central goal of preregistration (Nosek et al., 2018; Oberauer, 2019; Rubin, 2020). We argue that while such a conclusion is plausible, we must first consider at least two

constraints that place an upper bound on the theoretical risk.

First, the theory itself limits theoretical risk: Some theories simply do not make 298 risky predictions, and preregistration will not change that. Consider the case of a researcher contemplating the relation between two sets of variables. Suppose each set is separately well studied, and strong theories tell the researcher how the variables within the set relate. However, our imaginary researcher now considers the relation between these two 302 sets. For lack of a better theory, they assume that some relation between any variables of 303 the two sets exists. This is not a risky prediction to make in psychology (Orben & Lakens, 304 2020). However, we would consider it a success if the researcher would use the evidence 305 from this rather exploratory study to develop a more precise (and therefore risky) theory, 306 e.g., by using the results to specify which variables from one set relate to which variables 307 from the other set, to what extent, in which direction, with which functional shape, etc., to 308 be able to make riskier predictions in the future. We will later show that preregistration 300 increases the degree of belief in the further specified theory, though it remains low till 310 being substantiated by testing the theory again. This is because preregistration increases 311 the expected epistemic value regardless of the theory being tested, as we will show. 312

Second, available resources limit theoretical risk. Increasing theoretical risk $P(\neg E|\neg H)$ will usually decrease detectability P(E|H) unless more resources are invested. In other words, one cannot increase power while maintaining the same type-I error rate without increasing the invested resources. Tasking preregistration with an increase in theoretical risk makes it difficult to balance this trade-off. Mindlessly maximizing theoretical risk would either never produce evidence or require huge amounts of resources.

Uncertainty about theoretical risk

We have established that higher theoretical risk leads to more persuasive evidence.

In other words, we have reconstructed the interpretation that preregistrations supposedly
work by restricting the researchers, which in turn increases the theoretical risk (or

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equivalently limits the type-I error rate) and thereby creates more compelling evidence.

Nevertheless, there are trade-offs for increasing theoretical risk. Employing a mathematical

framework allows us to navigate the trade-offs more effectively and move towards a second,

more favorable interpretation. To that end, we incorporate uncertainty about theoretical

risk into our framework.

328 Statistical methods

One widely known factor is the contribution of statistical methods to theoretical 329 risk. Theoretical risk $P(\neg E|\neg H)$ is deeply connected with statistical methods, because it is 330 related to the type-I error rate in statistical hypothesis testing $P(E|\neg H)$ by 331 $P(\neg E|\neg H) = 1 - P(E|\neg H)$, if you consider the overly simplistic case where the research hypothesis is equal to the statistical alternative-hypothesis because then the nill-hypothesis is $\neg H$. Because many researchers are familiar with the type-I error rate, it can be helpful 334 to remember this connection to theoretical risk. Researchers who choose a smaller type-I 335 error rate can be more sure of their results, if significant, because the theoretical risk is 336 higher. However, this connection should not be overinterpreted for two reasons. First, 337 according to most interpretations of null hypothesis testing, the absence of a significant 338 result should not generally be interpreted as evidence against the hypothesis (Mayo, 2018, 339 p. 5.3). Second, the research hypothesis seldomly equals the statistical 340 alternative-hypothesis. We argue that theoretical risk (and hence its complement, 341 $P(E|\neg H)$) also encompasses factors outside the statistical realm, most notably the study 342 design and broader analytical strategies. 343

Statistical methods stand out among these factors because we have a large and
well-understood toolbox for assessing and controlling their contribution to theoretical risk.
Examples of our ability to exert this control are the choice of type-I error rate, adjustments
for multiple testing, the use of corrected fit measures (i.e., adjusted R²), information
criteria, or cross-validation in machine learning. These tools help us account for biases in

statistical methods that influence theoretical risk (and hence, $P(E|\neg H)$).

The point is that the contribution of statistical methods to theoretical risk can be
formally assessed. For many statistical models it can be analytically computed under some
assumptions. For those models or assumptions where this is impossible, one can employ
Monte Carlo simulation to estimate the contribution to theoretical risk. The precision with
which statisticians can discuss contributions to theoretical risk has lured the community
concerned with research methods into ignoring other factors that are much more uncertain.
We cannot hope to resolve this uncertainty; but we have to be aware of its implications.
These are presented in the following.

358 Sources of Uncertainty

As we have noted, it is possible to quantify how statistical models affect the 359 theoretical risk based on mathematical considerations and simulation. However, other 360 factors in the broader context of a study are much harder to quantify. If one chooses to 361 focus only on the contribution of statistical methods to theoretical risk, one is bound to 362 overestimate it. Take, for example, a t-test of mean differences in two samples. Under ideal 363 circumstances (assumption of independence, normality of residuals, equal variance), it 364 stays true to its type-I error rate. However, researchers may do many very reasonable things in the broader context of the study that affect theoretical risk: They might exclude outliers, choose to drop an item before computing a sum score, broaden their definition of the population to be sampled, translate their questionnaires into a different language, 368 impute missing values, switch between different estimators of the pooled variance, or any 369 number of other things. All of these decisions carry a small risk that they will increase the 370 likelihood of obtaining evidence despite the underlying research hypothesis being false. 371 Even if the t-test itself perfectly maintains its type I error rate, these factors influence 372 $P(E|\neg H)$. While, in theory, these factors may leave $P(E|\neg H)$ unaffected or even decrease 373 it, we argue that this is not the case in practice. Whether researchers want to or not, they

continuously process information about how the study is going, except under strict
blinding. While one can hope that processing this information does not affect their
decision-making either way, this cannot be ascertained. Therefore, we conclude that
statistical properties only guarantee a lower bound for theoretical risk. The only thing we
can conclude with some certainty is that theoretical risk is not higher than what the
statistical model guarantees without knowledge about the other factors at play.

381 The effects of uncertainty

Before we ask how preregistration influences this uncertainty, we must consider the 382 implications of being uncertain about the theoretical risk. Within the Bayesian framework, this is both straightforward and insightful. Let us assume a researcher is reading a study from another lab and tries to decide whether and how much the presented results confirm the hypothesis. As the researcher did not conduct the study (and the study is not 386 preregistered), they can not be certain about the various factors influencing theoretical risk 387 (researcher degrees of freedom). We therefore express this uncertainty about the theoretical 388 risk as a probability distribution Q of $P(E|\neg H)$ (remember that $P(E|\neg H)$ is related to 389 theoretical risk by $P(E|\neg H) = 1 - P(\neg E|\neg H)$, so it does not matter whether we consider 390 the distribution of theoretical risk or $P(E|\neg H)$). To get the expected value of P(H|E)391 that follows from the researchers' uncertainty about the theoretical risk, we can compute 392 the expectation using Bayes theorem: 393

$$\mathbb{E}_{Q}[P(H|E)] = \mathbb{E}_{Q}\left[\frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)}\right] \tag{3}$$

Of course, the assigned probabilities and the distribution Q vary from study to study and researcher to researcher, but we can illustrate the effect of uncertainty with an example. Assuming P(E|H) = 0.8 (relective of the typically strived for power of 80%). Let us further assume that the tested hypothesis is considered unlikely to be true by the research community before the study is conducted (P(H) = 0.1) and assign a uniform distribution for $P(E|\neg H) \sim U([1-\tau,1])$ where τ is set to $1-\alpha$, reflecting our assumption that this term gives an upper bound for theoretical risk $P(\neg E|\neg H)$. We chose this uniform distribution as it is the maximum entropy distribution with support $[1-\tau,1]$ and hence conforms to our Bayesian framework (Giffin & Caticha, 2007).

With this, we derive the expected value of P(H|E) as

$$\mathbb{E}_{Q}[P(H|E)] = \mathbb{E}_{Q}\left[\frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)}\right] \tag{4}$$

$$= \int_{[1-\tau,1]} \tau^{-1} \frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)} dP(E|\neg H)$$
 (5)

$$= \frac{P(H)P(E|H)}{P(\neg H)\tau} \ln \left(\frac{P(H)P(E|H) + P(\neg H)}{P(H)P(E|H) + P(\neg H)(1-\tau)} \right)$$
(6)

Figure 1 shows exemplary the effect of theoretical risk (x-axis) on the posterior
probability (y-axis) being certain (solid line) or uncertain (dashed line) about the
theoretical risk of a study. Our expectation of the gained epistemic value varies
considerably depending on how uncertain we are about the theoretical risk a study took on.
Mathematically, uncertainty about theoretical risk is expressed through the variance (or
rather entropy) of the distribution. The increase in uncertainty (expressed as more entropic
distributions) leads to a decreased expected epistemic value.

The argument for a confirmatory research agenda is that by increasing theoretical 410 risk we increase expected epistemic value, i.e., moving to the right on the x-axis in Figure 1 411 increases posterior probability (on the y-axis). However, if a hypothesis in a certain study 412 has low theoretical risk, there is not much researchers can do about it. However, studies do 413 not only differ by how high the theoretical risk is but also by how certain the recipient is 414 about the theoretical risk. A study that has a very high theoretical risk (e.g., 1.00% chance 415 that if the hypothesis is wrong, evidence in its favor will be observed.) but has also 416 maximum uncertainty will result in a posterior probability of 22%, while the same study 417

with maximum certainty will result in 90% posterior probability. The other factors
(detectability, prior beliefs, measure of epistemic value) and, therefore, the extent of the
benefit varies, of course, with the specifics of the study. Crucially, even studies with some
exploratory aspects benefit from preregistration, e.g., in this scenario with a $\tau = 0.80$ (false
positive rate of 0.20) moving from uncertain to certain increases the posterior from 0.15 to
0.31.

Preregistration as a means to decrease uncertainty about the theoretical risk

We hope to have persuaded the reader to accept two arguments: First, the
theoretical risk is important for judging evidential support for theories. Second, the
theoretical risk is inherently uncertain, and the degree of uncertainty diminishes the
persuasiveness of the gathered evidence. The third and last argument is that
preregistrations reduce this uncertainty. Following the last argument, a preregistered study
is represented by the solid line (certainty about theoretical risk), and a study that was not
preregistered is more similar to the dashed line (maximally uncertain about theoretical
risk) in Figure 1 and Figure 2.

Let us recall our three assumptions:

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- 1. Researchers judge the evidence for or against a hypothesis rationally.
- 2. They expect other researchers to apply a similar rational process.
- 3. Researchers try to maximize the expected epistemic value for other researchers.

The point we make with these assumptions is that researchers aim to persuade
other researchers, for example, the readers of their articles. Not only the original authors
are concerned with the process of weighing evidence for or against a theory but really the
whole scientific community the study authors hope to persuade. Unfortunately, readers of a
scientific article (or, more generally, any consumer of a research product) will likely lack
insight into the various factors that influence theoretical risk. While the authors

themselves may have a clear picture of what they did and how it might have influenced the theoretical risk they took, their readers have much greater uncertainty about these factors. In particular, they never know which relevant factors the authors of a given article failed to disclose, be it intentionally or not. From the perspective of the ultimate skeptic, they may claim maximum uncertainty.

Communicating clearly how authors of a scientific report collected their data and 448 consequently analyzed it to arrive at the evidence they present is crucial for judging the 449 theoretical risk they took. Preregistrations are ideal for communicating just that because 450 any description after the fact is prone to be incomplete. For instance, the authors could 451 have opted for selective reporting, that is, they decided to exclude a number of analytic 452 strategies they tried out. That is not to say that every study that was not-preregistered 453 was subjected to practices of questionable research practices. The point is that we cannot 454 exclude it with certainty. This uncertainty is drastically reduced if the researchers have 455 described what they intended to do beforehand and then report that they did exactly that. 456 In that case, readers can be certain they received a complete account of the situation. 457 They still might be uncertain about the actual theoretical risk the authors took, but to a 458 much smaller extent than if the study would not have been preregistered. The remaining 459 sources of uncertainty might be unfamiliarity with statistical methods or experimental 460 paradigms used, the probability of an implementation error in the statistical analyses, a 461 bug in the software used for analyses, etc. In any case, a well-written preregistration 462 should aim to reduce the uncertainty about the theoretical risk and hence increase the 463 persuasiveness of evidence. Therefore, a study that perfectly adhered to its preregistration will resemble the solid line in Figure 1/2. Crucially, perfect means here that the theoretical 465 risk can be judged with low uncertainty, not that the theoretical risk is necessarily high.

467 Discussion

To summarize, we showed that both higher theoretical risk and lower uncertainty 468 about theoretical risk lead to higher expected epistemic value across a variety of measures. 469 The former result that increasing theoretical risk leads to higher expected epistemic value 470 reconstructs the appeal and central goal of preregistration of confirmatory research 471 agendas. However, theoretical risk is something researchers have only limited control over. 472 For example, theories are often vague and ill-defined, resources are limited, and increasing 473 theoretical risk usually decreases detectability of a hypothesized effect (a special instance of 474 this trade-off is the well-known tension between type-I error and statistical power). While 475 we believe that preregistration is always beneficial, it might be counterproductive to pursue 476 high theoretical risk if the research context is inappropriate for strictly confirmatory research. Specifically, appropriateness here entails the development of precise theories and 478 the availability of necessary resources (often, large enough sample size, but also see Brandmaier et al. (2015)) to adequately balance detectability against theoretical risk. 480

In terms of preparing the conditions for confirmatory research, preregistration may 481 at most help to invest some time into developing more specific, hence riskier, implications 482 of a theory. But for a confirmatory science, it will not be enough to preregister all studies. 483 This undertaking requires action from the whole research community (Lishner, 2015). 484 Incentive structures must be created to evaluate not the outcomes of a study but the rigor with which it was conducted (Cagan, 2013; Schönbrodt et al., 2022). Journal editors could encourage theoretical developments that allow for precise predictions that will be tested by other researchers and be willing to accept registered reports (Fried, 2020a, 2020b; van Rooij & Baggio, 2021, 2020). Funding agencies should demand an explicit statement about 480 theoretical risk in relation to detectability and must be willing to provide the necessary 490 resources to reach adequate levels of both (Koole & Lakens, 2012). 491

Our latter result, on the importance of preregistration for minimizing uncertainty,

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has two important implications. The first is, that even if all imaginable actions regarding 493 promoting higher theoretical risk are taken, confirmatory research should be preregistered. 494 Otherwise, the uncertainty about the theoretical risk will diminish the advantage of 495 confirmatory research. Second, even under less-than-ideal circumstances for confirmatory 496 research, preregistration is beneficial. Preregistering exploratory studies increases the 497 expected epistemic value by virtue of reducing uncertainty about theoretical risk. 498 Nevertheless, exploratory studies will have a lower expected epistemic value than a more 490 confirmatory study if both are preregistered and have equal detectability. 500

Focusing on uncertainty reduction also explains two common practices of 501 preregistration that do not align with a confirmatory research agenda. First, researchers 502 seldomly predict precise numerical outcomes, instead they use preregistrations to describe 503 the process that generates the results. Precise predictions would have very high theoretical 504 risk (they are likely incorrect if the theory is wrong). A statistical procedure may have high 505 or low theoretical risk depending on the specifics of the model used. Specifying the process, 506 therefore, is in line with the rationale we propose here, but is less reasonable when the goal 507 of preregistration is supposed to be a strictly confirmatory research agenda. 508

Second, researchers often have to deviate from the preregistration and make 509 data-dependent decisions after the preregistration. If the only goal of preregistration is to 510 ensure confirmatory research, such changes are not justifiable. However, under our rational, 511 some changes may be justified. Any change increases the uncertainty about the theoretical 512 risk and may even decrease the theoretical risk. The changes still may be worthwhile if the 513 negative outcomes may be offset by an increase in detectability due to the change. 514 Consider a preregistration that failed to specify how to handle missing values, and 515 researchers subsequently encountering missing values. In such case, detectability becomes 516 zero because the data cannot be analyzed without a post-hoc decision about how to handle 517 the missing data. Any such decision would constitute a deviation from the preregistration, 518

which is possible under our proposed objective. Note that a reader cannot rule out that the 519 researchers leveraged the decision to decrease theoretical risk, i.e., picking among all 520 options the one that delivers the most beneficial results for the theory (in the previous 521 example, chosing between various options of handling missing values). Whatever decision 522 they make, increased uncertainty about the theoretical risk is inevitable and the expected 523 epistemic value is decreased compared to a world where they anticipated the need to deal 524 with missing data. However, it is still justified to deviate. After all they have not 525 anticipated the case and are left with a detectability of zero. Any decision will increase 526 detectability to a non-zero value offsetting the increase in uncertainty. The researchers also 527 may do their best to argue that the deviation was not motivated by increasing theoretical 528 risk, thereby, decreasing the uncertainty. Ideally, there is a default decision that fits well 529 with the theory or with the study design. Or, if there is no obvious candidate, the researchers could conduct a multiverse analysis of the available options to deal with missings to show the influence of the decision (Steegen et al., 2016). 532

As explained above, reduction in uncertainty as the objective for preregistration 533 does not only explain some existing practice, that does not align with confirmation as a 534 goal, it also allows to form recommendations to improve the practice of preregistration. 535 Importantly, we now have a theoretical measure to gauge the functionality of 536 preregistrations, which can only help increase its utility. In particular, a preregistration should be specific about the procedure that is intended to generate evidence for a theory. Such a procedure may accommodate a wide range of possible data, i.e., it may be exploratory. The theoretical risk, however low, must be communicated clearly. Parts of the process left unspecified imply uncertainty, which preregistration should reduce. However, specifying procedures that can be expected to fail will lead to deviation and, subsequently, 542 to larger uncertainty.

We have proposed a workflow for preregistration called *preregistration as code* (PAC)

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elsewhere (Peikert et al., 2021). In a PAC, researchers use computer code for the planned 545 analysis as well as a verbal description of theory and methods for the preregistration. This 546 combination is facilitated by dynamic document generation, where the results of the code, 547 such as numbers, figures, and tables, are inserted automatically into the document. The 548 idea is that the preregistration already contains "mock results" based on simulated or pilot 549 data, which are replaced after the actual study data becomes available. Such an approach 550 dissolves the distinction between the preregistration document and the final scientific 551 report. Instead of separate documents, preregistration, and final report are different 552 versions of the same underlying dynamic document. Deviations from the preregistration 553 can therefore be clearly (and if necessary, automatically) isolated, highlighted, and 554 inspected using version control. Crucially, because the preregistration contains code, it may 555 accommodate many different data patterns, i.e., it may be exploratory. However, while a PAC does not limit the extent of exploration, it is very specific about the probability to generate evidence even when the theory does not hold (theoretical risk). Please note that 558 while PAC is ideally suited to reduce uncertainty about theoretical risk, other more 559 traditional forms of preregistration are also able to advance this goal. 560

Contrary to what is widely assumed about preregistration, a preregistration is not necessarily a seal of confirmatory research. Confirmatory research would almost always be less persuasive without preregistration, but in our view, preregistration primarily communicates the extent of confirmation, i.e., theoretical risk, of a study. Clearly communicating theoretical risk is important because it reduces the uncertainty and hence increases expected epistemic value.

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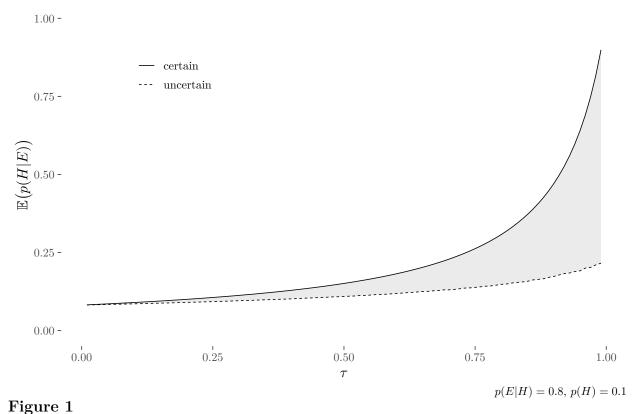
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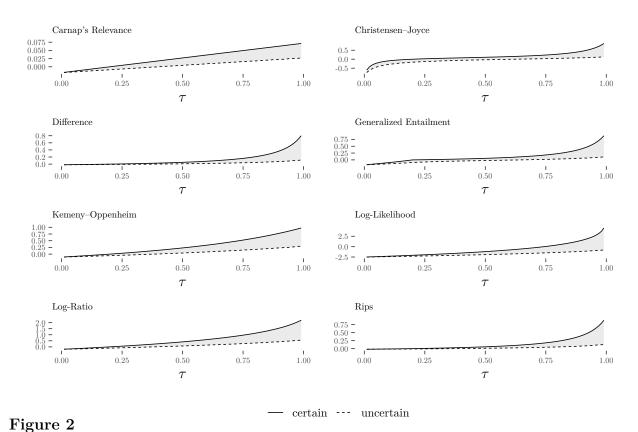
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Posterior probability (confirmation as firmness) as a function of theoretical risk τ , where τ is either certain (solid line) or maximally uncertain (dotted line).



Several measures for confirmation as an increase in firmness as a function of τ , where τ is either certain (solid line) or maximally uncertain (dotted line).