Why does preregistration increase the persuasiveness of evidence? A Bayesian rationalization

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17 Abstract

The replication crisis has led many researchers to preregister their hypotheses and data 18 analysis plans before collecting data. A widely held view is that preregistration is supposed 19 to limit the extent to which data may influence the hypotheses to be tested. Only if data 20 have no influence an analysis is considered confirmatory. Consequently, many researchers 21 believe that preregistration is only applicable in confirmatory paradigms. In practice, 22 researchers may struggle to preregister their hypotheses because of vague theories that necessitate data-dependent decisions (aka exploration). We argue that preregistration 24 benefits any study on the continuum between confirmatory and exploratory research. To 25 that end, we formalize a general objective of preregistration and demonstrate that exploratory studies also benefit from preregistration. Drawing on Bayesian philosophy of science, we argue that preregistration should primarily aim to reduce uncertainty about the inferential procedure used to derive results. This approach provides a principled justification of preregistration, separating the procedure from the goal of ensuring strictly 30 confirmatory research. We acknowledge that knowing the extent to which a study is 31 exploratory is central, but certainty about the inferential procedure is a prerequisite for persuasive evidence. Finally, we discuss the implications of these insights for the practice of 33 preregistration. 34

Keywords: preregistration; confirmation; exploration; hypothesis testing; Bayesian;

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# Why does preregistration increase the persuasiveness of evidence? A Bayesian rationalization

The scientific community has long pondered the vital distinction between 40 exploration and confirmation, discovery and justification, hypothesis generation and 41 hypothesis testing, or prediction and postdiction (Hoyningen-Huene, 2006; Nosek et al., 2018; Shmueli, 2010). Despite the different names, it is fundamentally the same dichotomy that is at stake here. There is a broad consensus that both approaches are necessary for science to progress; exploration, to make new discoveries and confirmation, to expose these discoveries to potential falsification, and assess empirical support for the theory. However, mistaking exploratory findings for empirically confirmed results is dangerous. It inflates the likelihood of believing that there is evidence supporting a given hypothesis, even if it is false. A variety of problems, such as researchers' degrees of freedom together with researchers' hindsight bias or naive p-hacking have led to such mistakes becoming commonplace yet unnoticed for a long time. Recognizing them has led to a crisis of 51 confidence in the empirical sciences (Ioannidis, 2005), and psychology in particular (Open Science Collaboration, 2015). As a response to the crisis, evermore researchers preregister their hypotheses and their data collection and analysis plans in advance of their studies (Nosek et al., 2018). They do so to stress the predictive nature of their registered statistical 55 analyses, often with the hopes of obtaining a label that marks the study as "confirmatory". Indeed, rigorous application of preregistration prevents researchers from reporting a set of 57 results produced by an arduous process of trial and error as a simple confirmatory story (Wagenmakers et al., 2012) while keeping low false-positive rates. This promise of a clear distinction between confirmation and exploration has obvious appeal to many who have already accepted the practice. Still, the majority of empirical researchers do not routinely 61 preregister their studies. One reason may be that some do not find that the theoretical advantages outweigh the practical hurdles, such as specifying every aspect of a theory and the corresponding analysis in advance. We believe that we can reach a greater acceptance

of preregistration by explicating a more general objective of preregistration that benefits all kinds of studies, even those that allow data-dependent decisions.

One goal of preregistration that has received widespread attention is to clearly
distinguish confirmatory from exploratory research (Bakker et al., 2020; Mellor & Nosek,
2018; Nosek et al., 2018; Simmons et al., 2021; Wagenmakers et al., 2012). In such a
narrative, preregistration is justified by a confirmatory research agenda. However, two
problems become apparent under closer inspection. First, many researchers do not
subscribe to a purely confirmatory research agenda. Second, there is no strict mapping of
the categories preregistered vs. non-preregistered onto the categories confirmatory
vs. exploratory research.

Obviously, researchers can conduct confirmatory research without preregistration—
though it might be difficult to convince other researchers of the confirmatory nature of
their research, that is, that they were free of cognitive biases, made no data-dependent
decisions, and so forth. The opposite, that is, preregistered but not strictly confirmatory
studies, are also becoming more commonplace (Chan et al., 2004; Dwan et al., 2008; Silagy
et al., 2002).

This is the result of researchers applying one of two strategies to evade the
self-imposed restrictions of preregistrations: writing a loose preregistration, to begin with
(Stefan & Schönbrodt, 2023) or deviating from the preregistration afterward. Both
strategies may be used for sensible scientific reasons or with the self-serving intent of
generating desirable results. Thus, insisting on equating preregistration and confirmation
has led to the criticism that, all things considered, preregistration is actually harmful and
neither sufficient nor necessary for doing good science (Pham & Oh, 2021; Szollosi et al.,
2020).

We argue that such criticism is not directed against preregistration itself but against

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a justification through a confirmatory research agenda (Wagenmakers et al., 2012). When researchers criticize preregistration as being too inflexible to fit their research question, 91 they often simply acknowledge that their research goals are not strictly confirmatory. 92 Forcing researchers into adopting a strictly confirmatory research agenda does not only 93 imply changing how they investigate a phenomenon but also what research questions they pose. However reasonable such a move is, changing the core beliefs of a large community is much harder than convincing them that a method is well justified. We, therefore, attempt to disentangle the methodological goals of preregistration from the ideological goals of confirmatory science. It might well be the case that psychology needs more confirmatory studies to progress as a science. However, independently of such a goal, preregistration can be useful for any kind of study on the continuum between strictly confirmatory and fully 100 exploratory. 101

To form such an objective for preregistration, we first introduce some tools of
Bayesian philosophy of science and map the exploration/confirmation distinction onto a
dimensional quantity we call "theoretical risk" (a term borrowed from Meehl, 1978, but
formalized as the probability of proving a hypothesis wrong if it does not hold), which is
inversely related to the type-I error rate in null hypothesis testing.

Further, we outline two interpretations of preregistration. The first one corresponds 107 to the traditional application of preregistration to research paradigms that focus on 108 confirmation by maximizing the theoretical risk or, equivalently, by limiting type-I error 109 (when dichotomous decisions about theories are an inferential goal). We argue that this 110 view on the utility of preregistration can be interpreted as maximizing theoretical risk, 111 which otherwise may be reduced by researchers' degrees of freedom, p-hacking, and suchlike. 112 The second interpretation is our main contribution: We argue that contrary to the classic 113 view, the objective of preregistration is not the maximization of theoretical risk but rather 114 the minimization of uncertainty about the theoretical risk. This interpretation leads to a 115

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broad applicability of preregistration to both exploratory and confirmatory studies.

To arrive at this interpretation, we rely on three arguments. The first is that
theoretical risk is vital for judging evidential support for theories. The second argument is
that the theoretical risk for a given study is generally uncertain. The third and last
argument is that this uncertainty is reduced by applying preregistration. We conclude that
because preregistration decreases uncertainty about the theoretical risk, which in turn
increases the amount of knowledge we gain from a particular study, preregistration is
potentially useful for any kind of study, no matter where it falls on the
exploratory-confirmatory continuum.

## Epistemic value and the Bayesian rationale

Let us start by defining what we call expected epistemic value. If researchers plan 126 to conduct a study, they usually hope that it will change their assessment of some theory's 127 verisimilitude (Niiniluoto, 1998). In other words, they hope to learn something from 128 conducting the study. The amount of knowledge researchers gain from a particular study 129 concerning the verisimilitude of a specific theory is what we call epistemic value. 130 Researchers cannot know what exactly they will learn from a study before they run it. 131 However, they can develop an expectation that helps them decide about the specifics of a 132 planned study. This expectation is what we term expected epistemic value. To make our 133 three arguments, we must assume three things about what an ideal estimation process 134 entails and how it relates to what studies (preregistered vs not preregistered) to conduct. 135

- 1. Researchers judge the evidence for or against a hypothesis rationally.
- 2. They expect other researchers to apply a similar rational process.
  - 3. Researchers try to maximize the expected epistemic value for other researchers.

The assumption of rationality can be connected to Bayesian reasoning and leads to our adoption of the framework. Our rationale is as follows. Researchers who decide to

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conduct a certain study are actually choosing a study to bet on. They have to "place the 141 bet" by conducting the study by investing resources and stand to gain epistemic value with 142 some probability. This conceptualization of choosing a study as a betting problem allows 143 us to apply a "Dutch book" argument (Christensen, 1991). This argument states that any 144 better must follow the axioms of probability to avoid being "irrational," i.e., accepting bets 145 that lead to sure losses. Fully developing a Dutch book argument for this problem requires 146 careful consideration of what kind of studies to include as possible bets, defining a 147 conversion rate from the stakes to the reward, and modeling what liberties researchers have 148 in what studies to conduct. Without deliberating these concepts further, we find it 149 persuasive that researchers should not violate the axioms of probability if they have some 150 expectation about what they stand to gain with some likelihood from conducting a study. 151 The axioms of probability are sufficient to derive the Bayes formula, on which we will 152 heavily rely for our further arguments. The argument is not sufficient, however, to warrant 153 conceptualizing the kind of epistemic value we reason about in terms of posterior 154 probability; that remains a leap of faith. However, the argument applies to any reward 155 function that satisfies the "statistical relevancy condition" (Fetzer, 1974; Salmon, 1970). 156 That is, evidence only increases epistemic value for a theory if the evidence is more likely 157 to be observed under the theory than under the alternative. 158

Please note that our decision to adopt this aspect of the Bayesian philosophy of science does not make assumptions about the statistical methods researchers use. In fact, this conceptualization is intentionally as minimal as possible to be compatible with a wide range of philosophies of science and statistical methods researchers might subscribe to.

#### Epistemic value and theoretical risk

Our first argument is that theoretical risk is crucial for judging evidential support for theories. Put simply, risky predictions create persuasive evidence if they turn out to be correct. This point is crucial because we attribute much of the appeal of a confirmatory 167 research agenda to this notion.

Let us make some simplifying assumptions and define our notation. To keep the notation simple, we restrict ourselves to evidence of a binary nature (either it was observed or not). We denote the probability of a hypothesis before observing evidence as P(H) and its complement as  $P(\neg H) = 1 - P(H)$ . The probability of observing evidence under some hypothesis is P(E|H). We can calculate the probability of the hypothesis after observing the evidence with the help of the Bayes formula:

$$P(H|E) = \frac{P(H)P(E|H)}{P(E)} \tag{1}$$

The posterior probability P(H|E) is of great relevance since it is often used directly or indirectly as a measure of confirmation of a hypothesis. In the tradition of Carnap, in its direct use, it is called confirmation as firmness; in its relation to the a priori probability P(H), it is called *increase in firmness* Carnap (1950), preface to the 1962 edition]. As noted before, we concentrate on posterior probability as a measure of epistemic value since no measure shows universally better properties than others. However, it is reasonable that any measure of confirmation increases monotonically with an increase in posterior probability P(H|E), and our argument applies to those measures as well.

In short, we want to increase posterior probability P(H|E). Increases in posterior probability P(H|E) are associated with increased epistemic value, of which we want to maximize the expectation. So how can we increase posterior probability? The Bayes formula yields three components that influence confirmation, namely P(H), P(E|H) and P(E). The first option leads us to the unsurprising conclusion that higher a priori probability P(H) leads to higher posterior probability P(H|E). If a hypothesis is more probable to begin with, observing evidence in its favor will result in a hypothesis that is more strongly confirmed, all else being equal. However, the prior probability of a

hypothesis is nothing our study design can change. The second option is equally 190 reasonable; that is, an increase in P(E|H) leads to a higher posterior probability P(H|E). 191 P(E|H) is the probability of obtaining evidence for a hypothesis when it holds. We call 192 this probability of detecting evidence, given that the hypothesis holds "detectability." 193 Consequently, researchers should ensure that their study design allows them to find 194 evidence for their hypothesis, in case it is true. When applied strictly within the bounds of 195 null hypothesis testing, detectability is equivalent to power (or the complement of type-II 196 error rate). However, while detectability is of great importance for study design, it is not 197 directly relevant to the objective of preregistration. Thus, P(E) remains to be considered. 198 Since P(E) is the denominator, decreasing it can increase the posterior probability. In 199 other words, high risk, high reward. 200

If we equate riskiness with a low probability of obtaining evidence (when the 201 hypothesis is false), the Bayesian rationale perfectly aligns with the observation that risky 202 predictions lead to persuasive evidence. This tension between high risk leading to high gain 203 is central to our consideration of preregistration. A high-risk, high-gain strategy is bound 204 to result in many losses that are eventually absorbed by the high gains. Sustaining many 205 "failed" studies is not exactly aligned with the incentive structure under which many, if not 206 most, researchers operate. Consequently, researchers are incentivized to appear to take 207 more risks than they actually do, which misleads their readers to give their claims more 208 credence than they deserve. It is at this juncture that the practice and mispractice of 209 preregistration comes into play. We argue that the main function of preregistration is to 210 enable proper judgment of the riskiness of a study.

To better understand how preregistrations can achieve that, let us take a closer look at the factors contributing to P(E). Using the law of total probability, we can split P(E) into two terms:

$$P(E) = P(H)P(E|H) + P(\neg H)P(E|\neg H)$$
(2)

We have already noted that there is not much to be done about prior probability 215  $(P(H), \text{ and hence its counter probability } P(\neg H)), \text{ and that it is common sense to increase}$ 216 detectability P(E|H). The real lever to pull is therefore  $P(E|\neg H)$ . This probability tells 217 us how likely it is that we find evidence in favor of the theory when in fact, the theory is 218 not true. Its counter probability  $P(\neg E|\neg H) = 1 - P(E|\neg H)$  is what we call "theoretical 219 risk", because it is the risk a theory takes on in predicting the occurrence of particular 220 evidence in its favor. We borrow the term from Meehl (1978), though he has not assigned 221 it to the probability  $P(\neg E|\neg H)$ . Kukla (1990) argued that the core arguments in Meehl 222 (1990) can be reconstructed in a purely Bayesian framework. However, while he did not 223 mention  $P(\neg E|\neg H)$  he suggested that Meehl (1978) used the term "very strange 224 coincidence" for a small  $P(E|\neg H)$  which would imply, that  $P(\neg E|\neg H)$  can be related to or 225 even equated to theoretical risk.

Let us note some interesting properties of theoretical risk  $P(\neg E|\neg H)$ . First, increasing theoretical risk leads to higher posterior probability P(H|E), our objective. Second, if the theoretical risk is smaller than detectability P(E|H) it follows that the posterior probability must decrease when observing the evidence. If detectability exceeds theoretical risk, the evidence is less likely under the theory than it is when the theory does not hold. Third, if the theoretical risk equals zero, then posterior probability is at best equal to prior probability but only if detectability is perfect (P(H|E) = 1). In other words, observing a sure fact does not lend credence to a hypothesis.

The last statement sounds like a truism but is directly related to Popper's seminal criterion of demarcation. He stated that if it is impossible to prove that a hypothesis is false  $(P(\neg E|\neg H) = 0$ , theoretical risk is zero), it cannot be considered a scientific

hypothesis (Popper, 2002, p. 18). We note these relations to underline that the Bayesian rationale we apply here is able to reconstruct many commonly held views on riskiness and epistemic value.

Both theoretical risk  $P(\neg E|\neg H)$  and detectability P(E|H) aggregate countless influences; otherwise, they could not model the process of evidential support for theories. To illustrate the concepts we have introduced here, consider the following example of a 243 single theory and three experiments that may test it. The experiments were created to 244 illustrate how they may differ in their theoretical risk and detectability. Suppose the 245 primary theory is about the cognitive phenomenon of "insight." For the purpose of 246 illustration, we define it, with quite some hand-waving, as a cognitive abstraction that 247 allows agents to consistently solve a well-defined class of problems. We present the 248 hypothesis that the following problem belongs to such a class of insight problems: 249

Use five matches (IIIII) to form the number eight.

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We propose three experiments that differ in theoretical risk and detectability. All experiments take a sample of ten psychology students. We present the students with the problem for a brief span of time. After that, the three experiments differ as follows:

- 1. The experimenter gives a hint that the problem is easy to solve when using Roman numerals; if all students come up with the solution, she records it as evidence for the hypothesis.
- 2. The experimenter shows the solution "VIII" and explains it; if all students come up with the solution, she records it as evidence for the hypothesis.
- 3. The experimenter does nothing; if all students come up with the solution, she records it as evidence for the hypothesis.

We argue that experiment 1 has high theoretical risk  $P(\neg E_1|\neg H)$  and high detectability  $P(E_1|H)$ . If "insight" has nothing to do with solving the problem  $(\neg H)$ , then

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presenting the insight that Roman numerals can be used should not lead to all students 263 solving the problem  $(\neg E_1)$ ; the experiment, therefore, has high theoretical risk 264  $P(\neg E_1|\neg H)$ . Conversely, if insight is required to solve the problem (H), then it is likely to 265 help all students to solve the problem  $(E_1)$ , the experiment, therefore, has high 266 detectability  $P(E_1|H)$ . The second experiment, on the other hand, has low theoretical risk 267  $P(\neg E_2|\neg H)$ . Even if "insight" has nothing to do with solving the problem  $(\neg H)$ , there are 268 other plausible reasons for observing the evidence  $(E_2)$ , because the students could simply 269 copy the solution without having any insight. With regard to detectability, experiments 1 270 and 2 differ in no obvious way. Experiment 3, however, also has low detectability. It is 271 unlikely that all students will come up with the correct solution in a short time  $(E_3)$ , even 272 if insight is required (H); experiment 3 therefore has low detectability  $P(E_3|H)$ . The 273 theoretical risk, however, is also low in absolute terms, but high compared to the detectability (statistical relevancy condition is satisfied). In the unlikely event that all 10 275 students place their matches to form the Roman numeral VIII  $(E_3)$ , it is probably due to 276 insight (H) and not by chance  $P(\neg E_3|\neg H)$ ). Of course, in practice, we would allow the 277 evidence to be probabilistic, e.g., relax the requirement of "all students" to nine out of ten 278 students, more than eight, and so forth.

As mentioned earlier, the we restrict ourselves to binary evidence, to keep the mathematical notation as simple as possible. We discuss the relation between statistical methods and theoretical risk in the Statistical Methods section.

## Preregistration as a means to increase theoretical risk?

Having discussed that increasing the theoretical risk will increase the epistemic value, it is intuitive to task preregistration with maximizing theoretical risk, i.e., a confirmatory research agenda. Indeed, limiting the type-I error rate is commonly stated as the central goal of preregistration (Nosek et al., 2018; Oberauer, 2019; Rubin, 2020). We argue that while such a conclusion is plausible, we must first consider at least two

constraints that place an upper bound on the theoretical risk.

First, the theory itself limits theoretical risk: Some theories simply do not make 290 risky predictions, and preregistration will not change that. Consider the case of a 291 researcher contemplating the relation between two sets of variables. Suppose each set is separately well studied, and strong theories tell the researcher how the variables within the set relate. However, our imaginary researcher now considers the relation between these two 294 sets. For lack of a better theory, they assume that some relation between any variables of 295 the two sets exists. This is not a risky prediction to make in psychology (Orben & Lakens, 296 2020). However, we would consider it a success if the researcher would use the evidence 297 from this rather exploratory study to develop a more precise (and therefore risky) theory, 298 e.g., by using the results to specify which variables from one set relate to which variables 299 from the other set, to what extent, in which direction, with which functional shape, etc., to 300 be able to make riskier predictions in the future. We will later show that preregistration 301 increases the degree of belief in the further specified theory, though it remains low till 302 being substantiated by testing the theory again. This is because preregistration increases 303 the expected epistemic value regardless of the theory being tested, as we will show. 304

Second, available resources limit theoretical risk. Increasing theoretical risk  $P(\neg E|\neg H)$  will usually decrease detectability P(E|H) unless more resources are invested. In other words, one cannot increase power while maintaining the same type-I error rate without increasing the invested resources. Tasking preregistration with an increase in theoretical risk makes it difficult to balance this trade-off. Mindlessly maximizing theoretical risk would either never produce evidence or require huge amounts of resources.

### Uncertainty about theoretical risk

We have established that higher theoretical risk leads to more persuasive evidence.

In other words, we have reconstructed the interpretation that preregistrations supposedly
work by restricting the researchers, which in turn increases the theoretical risk (or

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equivalently limits the type-I error rate) and thereby creates more compelling evidence.

Nevertheless, there are trade-offs for increasing theoretical risk. Employing a mathematical

framework allows us to navigate the trade-offs more effectively and move towards a second,

more favorable interpretation. To that end, we incorporate uncertainty about theoretical

risk into our framework.

## 320 Statistical methods

One widely known factor is the contribution of statistical methods to theoretical 321 risk. Theoretical risk  $P(\neg E|\neg H)$  is deeply connected with statistical methods, because it is 322 related to the type-I error rate in statistical hypothesis testing  $P(E|\neg H)$  by 323  $P(\neg E|\neg H) = 1 - P(E|\neg H)$ , if you consider the overly simplistic case where the research hypothesis is equal to the statistical alternative-hypothesis because then the nill-hypothesis is  $\neg H$ . Because many researchers are familiar with the type-I error rate, it can be helpful 326 to remember this connection to theoretical risk. Researchers who choose a smaller type-I 327 error rate can be more sure of their results, if significant, because the theoretical risk is 328 higher. However, this connection should not be overinterpreted for two reasons. First, 329 according to most interpretations of null hypothesis testing, the absence of a significant 330 result should not generally be interpreted as evidence against the hypothesis (Mayo, 2018, 331 p. 5.3). Second, the research hypothesis seldomly equals the statistical 332 alternative-hypothesis. We argue that theoretical risk (and hence its complement, 333  $P(E|\neg H)$ ) also encompasses factors outside the statistical realm, most notably the study 334 design and broader analytical strategies. 335

Statistical methods stand out among these factors because we have a large and
well-understood toolbox for assessing and controlling their contribution to theoretical risk.
Examples of our ability to exert this control are the choice of type-I error rate, adjustments
for multiple testing, the use of corrected fit measures (i.e., adjusted R<sup>2</sup>), information
criteria, or cross-validation in machine learning. These tools help us account for biases in

statistical methods that influence theoretical risk (and hence,  $P(E|\neg H)$ ).

The point is that the contribution of statistical methods to theoretical risk can be formally assessed. For many statistical models it can be analytically computed under some assumptions. For those models or assumptions where this is impossible, one can employ Monte Carlo simulation to estimate the contribution to theoretical risk. The precision with which statisticians can discuss contributions to theoretical risk has lured the community concerned with research methods into ignoring other factors that are much more uncertain. We cannot hope to resolve this uncertainty; but we have to be aware of its implications. These are presented in the following.

## 350 Sources of Uncertainty

As we have noted, it is possible to quantify how statistical models affect the 351 theoretical risk based on mathematical considerations and simulation. However, other 352 factors in the broader context of a study are much harder to quantify. If one chooses to 353 focus only on the contribution of statistical methods to theoretical risk, one is bound to 354 overestimate it. Take, for example, a t-test of mean differences in two samples. Under ideal 355 circumstances (assumption of independence, normality of residuals, equal variance), it 356 stays true to its type-I error rate. However, researchers may do many very reasonable things in the broader context of the study that affect theoretical risk: They might exclude 358 outliers, choose to drop an item before computing a sum score, broaden their definition of the population to be sampled, translate their questionnaires into a different language, 360 impute missing values, switch between different estimators of the pooled variance, or any 361 number of other things. All of these decisions carry a small risk that they will increase the 362 likelihood of obtaining evidence despite the underlying research hypothesis being false. 363 Even if the t-test itself perfectly maintains its type I error rate, these factors influence 364  $P(E|\neg H)$ . While, in theory, these factors may leave  $P(E|\neg H)$  unaffected or even decrease 365 it, we argue that this is not the case in practice. Whether researchers want to or not, they

continuously process information about how the study is going, except under strict
blinding. While one can hope that processing this information does not affect their
decision-making either way, this cannot be ascertained. Therefore, we conclude that
statistical properties only guarantee a lower bound for theoretical risk. The only thing we
can conclude with some certainty is that theoretical risk is not higher than what the
statistical model guarantees without knowledge about the other factors at play.

### 373 The effects of uncertainty

Before we ask how preregistration influences this uncertainty, we must consider the 374 implications of being uncertain about the theoretical risk. Within the Bayesian framework, this is both straightforward and insightful. Let us assume a researcher is reading a study from another lab and tries to decide whether and how much the presented results confirm the hypothesis. As the researcher did not conduct the study (and the study is not 378 preregistered), they can not be certain about the various factors influencing theoretical risk 379 (researcher degrees of freedom). We therefore express this uncertainty about the theoretical 380 risk as a probability distribution Q of  $P(E|\neg H)$  (remember that  $P(E|\neg H)$  is related to 381 theoretical risk by  $P(E|\neg H) = 1 - P(\neg E|\neg H)$ , so it does not matter whether we consider 382 the distribution of theoretical risk or  $P(E|\neg H)$ ). To get the expected value of P(H|E)383 that follows from the researchers' uncertainty about the theoretical risk, we can compute 384 the expectation using Bayes theorem: 385

$$\mathbb{E}_{Q}[P(H|E)] = \mathbb{E}_{Q}\left[\frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)}\right] \tag{3}$$

Of course, the assigned probabilities and the distribution Q vary from study to study and researcher to researcher, but we can illustrate the effect of uncertainty with an example. Assuming P(E|H) = 0.8 (relective of the typically strived for power of 80%). Let us further assume that the tested hypothesis is considered unlikely to be true by the research community before the study is conducted (P(H) = 0.1) and assign a uniform distribution for  $P(E|\neg H) \sim U([1-\tau,1])$  where  $\tau$  is set to  $1-\alpha$ , reflecting our assumption that this term gives an upper bound for theoretical risk  $P(\neg E|\neg H)$ . We chose this uniform distribution as it is the maximum entropy distribution with support  $[1-\tau,1]$  and hence conforms to our Bayesian framework (Giffin & Caticha, 2007).

With this, we derive the expected value of P(H|E) as

$$\mathbb{E}_{Q}[P(H|E)] = \mathbb{E}_{Q}\left[\frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)}\right] \tag{4}$$

$$= \int_{[1-\tau,1]} \tau^{-1} \frac{P(H)P(E|H)}{P(H)P(E|H) + P(\neg H)P(E|\neg H)} \, \mathrm{d}P(E|\neg H) \tag{5}$$

$$= \frac{P(H)P(E|H)}{P(\neg H)\tau} \ln \left( \frac{P(H)P(E|H) + P(\neg H)}{P(H)P(E|H) + P(\neg H)(1-\tau)} \right)$$
(6)

Figure 1 shows exemplary the effect of theoretical risk (x-axis) on the posterior probability (y-axis) being certain (solid line) or uncertain (dashed line) about the theoretical risk of a study. Our expectation of the gained epistemic value varies considerably depending on how uncertain we are about the theoretical risk a study took on. Mathematically, uncertainty about theoretical risk is expressed through the variance (or rather entropy) of the distribution. The increase in uncertainty (expressed as more entropic distributions) leads to a decreased expected epistemic value.

The argument for a confirmatory research agenda is that by increasing theoretical 402 risk we increase expected epistemic value, i.e., moving to the right on the x-axis in Figure 1 403 increases posterior probability (on the y-axis). However, if a hypothesis in a certain study 404 has low theoretical risk, there is not much researchers can do about it. However, studies do 405 not only differ by how high the theoretical risk is but also by how certain the recipient is 406 about the theoretical risk. A study that has a very high theoretical risk (e.g., 1.00% chance 407 that if the hypothesis is wrong, evidence in its favor will be observed.) but has also 408 maximum uncertainty will result in a posterior probability of 22%, while the same study 409

with maximum certainty will result in 90% posterior probability. The other factors
(detectability, prior beliefs, measure of epistemic value) and, therefore, the extent of the
benefit varies, of course, with the specifics of the study. Crucially, even studies with some
exploratory aspects benefit from preregistration, e.g., in this scenario with a  $\tau = 0.80$  (false
positive rate of 0.20) moving from uncertain to certain increases the posterior from 0.15 to
0.31.

### Preregistration as a means to decrease uncertainty about the theoretical risk

We hope to have persuaded the reader to accept two arguments: First, the
theoretical risk is important for judging evidential support for theories. Second, the
theoretical risk is inherently uncertain, and the degree of uncertainty diminishes the
persuasiveness of the gathered evidence. The third and last argument is that
preregistrations reduce this uncertainty. Following the last argument, a preregistered study
is represented by the solid line (certainty about theoretical risk), and a study that was not
preregistered is more similar to the dashed line (maximally uncertain about theoretical
risk) in Figure 1 and Figure 2.

#### Let us recall our three assumptions:

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- 1. Researchers judge the evidence for or against a hypothesis rationally.
- 2. They expect other researchers to apply a similar rational process.
- 3. Researchers try to maximize the expected epistemic value for other researchers.

The point we make with these assumptions is that researchers aim to persuade
other researchers, for example, the readers of their articles. Not only the original authors
are concerned with the process of weighing evidence for or against a theory but really the
whole scientific community the study authors hope to persuade. Unfortunately, readers of a
scientific article (or, more generally, any consumer of a research product) will likely lack
insight into the various factors that influence theoretical risk. While the authors

themselves may have a clear picture of what they did and how it might have influenced the theoretical risk they took, their readers have much greater uncertainty about these factors. In particular, they never know which relevant factors the authors of a given article failed to disclose, be it intentionally or not. From the perspective of the ultimate skeptic, they may claim maximum uncertainty.

Communicating clearly how authors of a scientific report collected their data and 440 consequently analyzed it to arrive at the evidence they present is crucial for judging the 441 theoretical risk they took. Preregistrations are ideal for communicating just that because 442 any description after the fact is prone to be incomplete. For instance, the authors could 443 have opted for selective reporting, that is, they decided to exclude a number of analytic 444 strategies they tried out. That is not to say that every study that was not-preregistered 445 was subjected to practices of questionable research practices. The point is that we cannot 446 exclude it with certainty. This uncertainty is drastically reduced if the researchers have 447 described what they intended to do beforehand and then report that they did exactly that. 448 In that case, readers can be certain they received a complete account of the situation. 449 They still might be uncertain about the actual theoretical risk the authors took, but to a 450 much smaller extent than if the study would not have been preregistered. The remaining 451 sources of uncertainty might be unfamiliarity with statistical methods or experimental 452 paradigms used, the probability of an implementation error in the statistical analyses, a 453 bug in the software used for analyses, etc. In any case, a well-written preregistration 454 should aim to reduce the uncertainty about the theoretical risk and hence increase the 455 persuasiveness of evidence. Therefore, a study that perfectly adhered to its preregistration will resemble the solid line in Figure 1/2. Crucially, perfect means here that the theoretical 457 risk can be judged with low uncertainty, not that the theoretical risk is necessarily high. 458

459 Discussion

To summarize, we showed that both higher theoretical risk and lower uncertainty 460 about theoretical risk lead to higher expected epistemic value across a variety of measures. 461 The former result that increasing theoretical risk leads to higher expected epistemic value 462 reconstructs the appeal and central goal of preregistration of confirmatory research 463 agendas. However, theoretical risk is something researchers have only limited control over. 464 For example, theories are often vague and ill-defined, resources are limited, and increasing 465 theoretical risk usually decreases detectability of a hypothesized effect (a special instance of 466 this trade-off is the well-known tension between type-I error and statistical power). While 467 we believe that preregistration is always beneficial, it might be counterproductive to pursue 468 high theoretical risk if the research context is inappropriate for strictly confirmatory research. Specifically, appropriateness here entails the development of precise theories and 470 the availability of necessary resources (often, large enough sample size, but also see Brandmaier et al. (2015)) to adequately balance detectability against theoretical risk. 472

In terms of preparing the conditions for confirmatory research, preregistration may 473 at most help to invest some time into developing more specific, hence riskier, implications 474 of a theory. But for a confirmatory science, it will not be enough to preregister all studies. 475 This undertaking requires action from the whole research community (Lishner, 2015). 476 Incentive structures must be created to evaluate not the outcomes of a study but the rigor with which it was conducted (Cagan, 2013; Schönbrodt et al., 2022). Journal editors could 478 encourage theoretical developments that allow for precise predictions that will be tested by other researchers and be willing to accept registered reports (Fried, 2020a, 2020b; van 480 Rooij & Baggio, 2021, 2020). Funding agencies should demand an explicit statement about 481 theoretical risk in relation to detectability and must be willing to provide the necessary 482 resources to reach adequate levels of both (Koole & Lakens, 2012). 483

Our latter result, on the importance of preregistration for minimizing uncertainty,

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has two important implications. The first is, that even if all imaginable actions regarding 485 promoting higher theoretical risk are taken, confirmatory research should be preregistered. 486 Otherwise, the uncertainty about the theoretical risk will diminish the advantage of 487 confirmatory research. Second, even under less-than-ideal circumstances for confirmatory 488 research, preregistration is beneficial. Preregistering exploratory studies increases the 480 expected epistemic value by virtue of reducing uncertainty about theoretical risk. 490 Nevertheless, exploratory studies will have a lower expected epistemic value than a more 491 confirmatory study if both are preregistered and have equal detectability. 492

Focusing on uncertainty reduction also explains two common practices of 493 preregistration that do not align with a confirmatory research agenda. First, researchers seldomly predict precise numerical outcomes, instead they use preregistrations to describe the process that generates the results. Precise predictions would have very high theoretical 496 risk (they are likely incorrect if the theory is wrong). A statistical procedure may have high or low theoretical risk depending on the specifics of the model used. Specifying the process, 498 therefore, is in line with the rationale we propose here, but is less reasonable when the goal of preregistration is supposed to be a strictly confirmatory research agenda. 500

Second, researchers often have to deviate from the preregistration and make 501 data-dependent decisions after the preregistration. If the only goal of preregistration is to 502 ensure confirmatory research, such changes are not justifiable. However, under our rational, 503 some changes may be justified. Any change increases the uncertainty about the theoretical 504 risk and may even decrease the theoretical risk. The changes still may be worthwhile if the 505 negative outcomes may be offset by an increase in detectability due to the change. 506 Consider a preregistration that failed to specify how to handle missing values, and 507 researchers subsequently encountering missing values. In such case, detectability becomes 508 zero because the data cannot be analyzed without a post-hoc decision about how to handle 500 the missing data. Any such decision would constitute a deviation from the preregistration, 510

which is possible under our proposed objective. Note that a reader cannot rule out that the 511 researchers leveraged the decision to decrease theoretical risk, i.e., picking among all 512 options the one that delivers the most beneficial results for the theory (in the previous 513 example, chosing between various options of handling missing values). Whatever decision 514 they make, increased uncertainty about the theoretical risk is inevitable and the expected 515 epistemic value is decreased compared to a world where they anticipated the need to deal 516 with missing data. However, it is still justified to deviate. After all they have not 517 anticipated the case and are left with a detectability of zero. Any decision will increase 518 detectability to a non-zero value offsetting the increase in uncertainty. The researchers also 519 may do their best to argue that the deviation was not motivated by increasing theoretical 520 risk, thereby, decreasing the uncertainty. Ideally, there is a default decision that fits well 521 with the theory or with the study design. Or, if there is no obvious candidate, the researchers could conduct a multiverse analysis of the available options to deal with missings to show the influence of the decision (Steegen et al., 2016). 524

As explained above, reduction in uncertainty as the objective for preregistration 525 does not only explain some existing practice, that does not align with confirmation as a 526 goal, it also allows to form recommendations to improve the practice of preregistration. 527 Importantly, we now have a theoretical measure to gauge the functionality of 528 preregistrations, which can only help increase its utility. In particular, a preregistration should be specific about the procedure that is intended to generate evidence for a theory. Such a procedure may accommodate a wide range of possible data, i.e., it may be exploratory. The theoretical risk, however low, must be communicated clearly. Parts of the process left unspecified imply uncertainty, which preregistration should reduce. However, 533 specifying procedures that can be expected to fail will lead to deviation and, subsequently, 534 to larger uncertainty. 535

We have proposed a workflow for preregistration called *preregistration as code* (PAC)

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elsewhere (Peikert et al., 2021). In a PAC, researchers use computer code for the planned 537 analysis as well as a verbal description of theory and methods for the preregistration. This 538 combination is facilitated by dynamic document generation, where the results of the code, 539 such as numbers, figures, and tables, are inserted automatically into the document. The 540 idea is that the preregistration already contains "mock results" based on simulated or pilot 541 data, which are replaced after the actual study data becomes available. Such an approach 542 dissolves the distinction between the preregistration document and the final scientific 543 report. Instead of separate documents, preregistration, and final report are different versions of the same underlying dynamic document. Deviations from the preregistration 545 can therefore be clearly (and if necessary, automatically) isolated, highlighted, and 546 inspected using version control. Crucially, because the preregistration contains code, it may 547 accommodate many different data patterns, i.e., it may be exploratory. However, while a PAC does not limit the extent of exploration, it is very specific about the probability to generate evidence even when the theory does not hold (theoretical risk). Please note that 550 while PAC is ideally suited to reduce uncertainty about theoretical risk, other more 551 traditional forms of preregistration are also able to advance this goal. 552

Contrary to what is widely assumed about preregistration, a preregistration is not necessarily a seal of confirmatory research. Confirmatory research would almost always be less persuasive without preregistration, but in our view, preregistration primarily communicates the extent of confirmation, i.e., theoretical risk, of a study. Clearly communicating theoretical risk is important because it reduces the uncertainty and hence increases expected epistemic value.

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- References
- Bakker, M., Veldkamp, C. L. S., Assen, M. A. L. M. van, Crompvoets, E. A. V., Ong, H.
- H., Nosek, B. A., Soderberg, C. K., Mellor, D., & Wicherts, J. M. (2020). Ensuring the
- quality and specificity of preregistrations. *PLOS Biology*, 18(12), e3000937.
- https://doi.org/10.1371/journal.pbio.3000937
- Brandmaier, A. M., Oertzen, T. von, Ghisletta, P., Hertzog, C., & Lindenberger, U. (2015).
- LIFESPAN: A tool for the computer-aided design of longitudinal studies. Frontiers in
- 571 Psychology, 6, 272.
- <sup>572</sup> Cagan, R. (2013). San Francisco Declaration on Research Assessment. *Disease Models &*
- 573 Mechanisms, dmm.012955. https://doi.org/10.1242/dmm.012955
- <sup>574</sup> Carnap, R. (1950). Logical Foundations of Probability. Chicago, IL, USA: Chicago
- University of Chicago Press.
- Chan, A.-W., Hróbjartsson, A., Haahr, M. T., Gøtzsche, P. C., & Altman, D. G. (2004).
- Empirical Evidence for Selective Reporting of Outcomes in Randomized
- TrialsComparison of Protocols to Published Articles. JAMA, 291(20), 2457–2465.
- https://doi.org/10.1001/jama.291.20.2457
- <sup>580</sup> Christensen, D. (1991). Clever Bookies and Coherent Beliefs. The Philosophical Review,
- 100(2), 229-247. https://doi.org/ 10.2307/2185301
- Dwan, K., Altman, D. G., Arnaiz, J. A., Bloom, J., Chan, A.-W., Cronin, E., Decullier, E.,
- Easterbrook, P. J., Elm, E. V., Gamble, C., Ghersi, D., Ioannidis, J. P. A., Simes, J., &
- Williamson, P. R. (2008). Systematic Review of the Empirical Evidence of Study
- Publication Bias and Outcome Reporting Bias. *PLOS ONE*, 3(8), e3081.
- https://doi.org/10.1371/journal.pone.0003081
- Fetzer, J. H. (1974). Statistical Explanations. In K. F. Schaffner & R. S. Cohen (Eds.),
- PSA 1972: Proceedings of the 1972 Biennial Meeting of the Philosophy of Science
- Association (pp. 337–347). Springer Netherlands.
- 590 https://doi.org/10.1007/978-94-010-2140-1 23

```
Fried, E. I. (2020a). Lack of Theory Building and Testing Impedes Progress in The Factor
591
       and Network Literature. Psychological Inquiry, 31(4), 271–288.
592
       https://doi.org/10.1080/1047840X.2020.1853461
593
   Fried, E. I. (2020b). Theories and Models: What They Are, What They Are for, and What
594
       They Are About. Psychological Inquiry, 31(4), 336–344.
595
       https://doi.org/10.1080/1047840X.2020.1854011
596
   Giffin, A., & Caticha, A. (2007). Updating Probabilities with Data and Moments. AIP
597
       Conference Proceedings, 954, 74–84. https://doi.org/10.1063/1.2821302
598
   Hoyningen-Huene, P. (2006). Context of Discovery Versus Context of Justification and
599
       Thomas Kuhn. In J. Schickore & F. Steinle (Eds.), Revisiting Discovery and
600
       Justification: Historical and philosophical perspectives on the context distinction (pp.
601
       119–131). Springer Netherlands. https://doi.org/10.1007/1-4020-4251-5_8
602
   Ioannidis, J. P. A. (2005). Why Most Published Research Findings Are False. PLOS
603
       Medicine, 2(8), e124. https://doi.org/10.1371/journal.pmed.0020124
604
   Koole, S. L., & Lakens, D. (2012). Rewarding Replications: A Sure and Simple Way to
605
       Improve Psychological Science. Perspectives on Psychological Science, 7(6), 608–614.
606
       https://doi.org/10.1177/1745691612462586
607
   Kukla, A. (1990). Clinical Versus Statistical Theory Appraisal. Psychological Inquiry, 1(2),
608
       160–161. https://doi.org/10.1207/s15327965pli0102_9
609
   Lishner, D. A. (2015). A Concise Set of Core Recommendations to Improve the
610
       Dependability of Psychological Research. Review of General Psychology, 19(1), 52–68.
611
       https://doi.org/10.1037/gpr0000028
612
   Mayo, D. G. (2018). Statistical Inference as Severe Testing: How to Get Beyond the
613
       Statistics Wars (First). Cambridge University Press.
614
       https://doi.org/10.1017/9781107286184
615
   Meehl, P. E. (1990). Appraising and Amending Theories: The Strategy of Lakatosian
616
```

Defense and Two Principles that Warrant It. Psychological Inquiry, 1(2), 108–141.

617

```
https://doi.org/10.1207/s15327965pli0102 1
618
   Meehl, P. E. (1978). Theoretical risks and tabular asterisks: Sir Karl, Sir Ronald, and the
619
       slow progress of soft psychology. Journal of Consulting and Clinical Psychology, 46(4),
620
```

627

- Mellor, D. T., & Nosek, B. A. (2018). Easy preregistration will benefit any research. 622
- Nature Human Behaviour, 2(2), 98–98. https://doi.org/10.1038/s41562-018-0294-7 623
- Niiniluoto, I. (1998). Verisimilitude: The Third Period. The British Journal for the 624
- Philosophy of Science, 49(1), 1-29. https://doi.org/10.1093/bjps/49.1.1 625

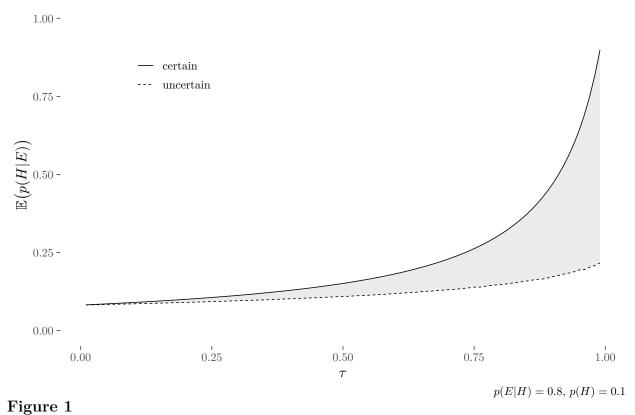
806-834. https://doi.org/10.1037/0022-006X.46.4.806

- Nosek, B. A., Ebersole, C. R., DeHaven, A. C., & Mellor, D. T. (2018). The preregistration 626 revolution. Proceedings of the National Academy of Sciences, 115(11), 2600–2606.
- https://doi.org/10.1073/pnas.1708274114 628
- Oberauer, K. (2019). Preregistration of a forking path What does it add to the garden of evidence? In Psychonomic Society Featured Content. 630
- Open Science Collaboration. (2015). Estimating the reproducibility of psychological 631 science. Science, 349(6251), aac4716. https://doi.org/10.1126/science.aac4716 632
- Orben, A., & Lakens, D. (2020). Crud (Re)Defined. Advances in Methods and Practices in 633 Psychological Science, 3(2), 238-247. https://doi.org/10.1177/2515245920917961 634
- Peikert, A., & Brandmaier, A. M. (2023a). Supplemental materials for preprint: Why does 635 preregistration increase the persuasiveness of evidence? A Bayesian rationalization. 636
- Zenodo. https://doi.org/10.5281/zenodo.7648471 637
- Peikert, A., & Brandmaier, A. M. (2023b). Why does preregistration increase the 638 persuasiveness of evidence? A Bayesian rationalization. PsyArXiv; PsyArXiv. 639
- https://doi.org/10.31234/osf.io/cs8wb 640
- Peikert, A., van Lissa, C. J., & Brandmaier, A. M. (2021). Reproducible Research in R: A 641
- Tutorial on How to Do the Same Thing More Than Once. Psych, 3(4), 836–867. 642
- https://doi.org/10.3390/psych3040053 643
- Pham, M. T., & Oh, T. T. (2021). Preregistration Is Neither Sufficient nor Necessary for

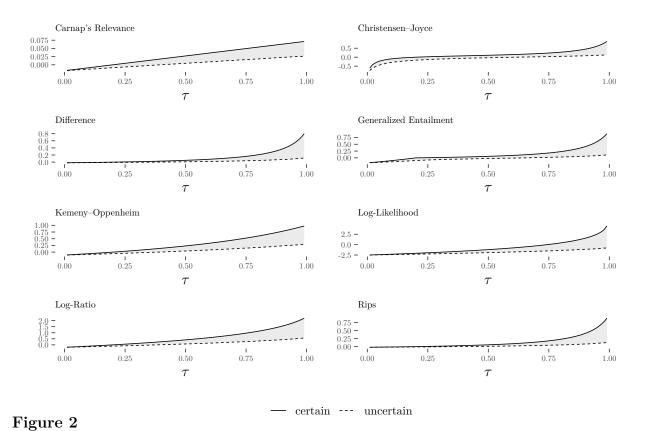
- Good Science. Journal of Consumer Psychology, 31(1), 163–176.
- https://doi.org/10.1002/jcpy.1209
- Popper, K. R. (2002). The logic of scientific discovery. Routledge.
- Rubin, M. (2020). Does preregistration improve the credibility of research findings? The
- Quantitative Methods for Psychology, 16(4), 376–390.
- https://doi.org/10.20982/tqmp.16.4.p376
- Salmon, W. C. (1970). Statistical Explanation. In The Nature & function of scientific
- theories: Essays in contemporary science and philosophy (pp. 173–232). University of
- 653 Pittsburgh Press.
- Schönbrodt, F., Gärtner, A., Frank, M., Gollwitzer, M., Ihle, M., Mischkowski, D., Phan, L.
- V., Schmitt, M., Scheel, A. M., Schubert, A.-L., Steinberg, U., & Leising, D. (2022).
- Responsible Research Assessment I: Implementing DORA for hiring and promotion in
- psychology. PsyArXiv. https://doi.org/10.31234/osf.io/rgh5b
- 658 Shmueli, G. (2010). To Explain or to Predict? Statistical Science, 25(3), 289–310.
- https://doi.org/10.1214/10-STS330
- 660 Silagy, C. A., Middleton, P., & Hopewell, S. (2002). Publishing Protocols of Systematic
- Reviews Comparing What Was Done to What Was Planned. JAMA, 287(21),
- 2831–2834. https://doi.org/10.1001/jama.287.21.2831
- 663 Simmons, J. P., Nelson, L. D., & Simonsohn, U. (2021). Pre-registration: Why and How.
- Journal of Consumer Psychology, 31(1), 151–162. https://doi.org/10.1002/jcpy.1208
- Steegen, S., Tuerlinckx, F., Gelman, A., & Vanpaemel, W. (2016). Increasing Transparency
- Through a Multiverse Analysis. Perspectives on Psychological Science, 11(5), 702–712.
- https://doi.org/10.1177/1745691616658637
- 668 Stefan, A. M., & Schönbrodt, F. D. (2023). Big little lies: A compendium and simulation
- of p-hacking strategies. Royal Society Open Science, 10(2).
- https://doi.org/10.1098/rsos.220346
- Szollosi, A., Kellen, D., Navarro, D. J., Shiffrin, R., Rooij, I. van, Zandt, T. V., & Donkin,

- 672 C. (2020). Is Preregistration Worthwhile? Trends in Cognitive Sciences, 24(2), 94–95.
- https://doi.org/10.1016/j.tics.2019.11.009
- van Rooij, I., & Baggio, G. (2021). Theory Before the Test: How to Build
- High-Verisimilitude Explanatory Theories in Psychological Science. Perspectives on
- 676 Psychological Science, 16(4), 682–697. https://doi.org/10.1177/1745691620970604
- van Rooij, I., & Baggio, G. (2020). Theory Development Requires an Epistemological Sea
- Change. Psychological Inquiry, 31(4), 321–325.
- https://doi.org/10.1080/1047840X.2020.1853477
- Wagenmakers, E.-J., Wetzels, R., Borsboom, D., van der Maas, H. L. J., & Kievit, R. A.
- 681 (2012). An Agenda for Purely Confirmatory Research. Perspectives on Psychological
- Science, 7(6), 632–638. https://doi.org/10.1177/1745691612463078





Posterior probability (confirmation as firmness) as a function of theoretical risk  $\tau$ , where  $\tau$  is either certain (solid line) or maximally uncertain (dotted line).



Several measures for confirmation as an increase in firmness as a function of  $\tau$ , where  $\tau$  is either certain (solid line) or maximally uncertain (dotted line).